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**Placement of Integrated Circuits  
For Reliability on Conductively  
Cooled Printed Wiring Boards**

**by**

**M.D. Osterman and M. Pecht**

# PLACEMENT OF INTEGRATED CIRCUITS FOR RELIABILITY ON CONDUCTIVELY COOLED PRINTED WIRING BOARDS

Michael D. Osterman  
Systems Research Center

Michael Pecht  
Mechanical Engineering Department  
University of Maryland, College Park, MD 20742

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## ABSTRACT

This article presents a mathematical theory for component placement for reliability based on the thermal response of conductively cooled printed wiring board. Placement procedures based on the theory are then developed and a general placement methodology is discussed.

## INTRODUCTION

In order to establish uniform methods for predicting the reliability of electronic equipment, there are a number of handbooks, specifications, and guidelines which can be utilized. In these reliability prediction methods, the failure rate  $\lambda_i$  of an electronic component is generally [1] dependent on the temperature  $T_{jc_i}$  in the form

$$\lambda_i(T_{jc_i}) = D_i + B_i e^{-A_i \left\{ \frac{1}{T_{jc_i}} \right\}} \quad (1)$$

where  $A_i$ ,  $B_i$ , and  $D_i$  are constants determined by package type, environmental considerations, and electrical characteristics. When the components are connected in series, the total failure rate for the printed wiring board (PWB) unit is the summation of the component failure rates.

In the placement of electronic components on a PWB, reliability prediction and analysis have typically been treated as post-processes. However, reliability is a critical part of the PWB design process and has been the topic of several recent articles [15-18]. In particular, Pecht, Palmer, and Naft [17] have examined placement for reliability on convection cooled boards and developed placement routines for determining near optimum placement configurations. Dancer, Pecht, and Palmer [15] have examined and compared several optimization schemes for convectively cooled PWBs for computational accuracy and speed, and Mayer [18] has examined optimizing reliability and life-cycle cost based on the thermal design in avionics.

The reliability of the PWB unit is dependent on the failure rates of the individual components, which are monotonically increasing function of the component junction temperature. The thermal response is, in turn, dependent on the heat dissipation and locations of the individual components. In the general case, where the heat dissipation rates and the temperature sensitivities of all the components on the board are not equal, determining the optimal placement requires  $n$  factorial permutations between the  $n$  locations.

In this paper, a scheme is developed for component placement for reliability in terms of the thermal response of conductively cooled PWBs operating at steady state conditions. The idealized situation in which  $N$  components are to be placed in  $N$  locations along a single row is considered. A general methodology for component placement for the board is then discussed. We assume that components are placed on the PWB in relatively thermally independent rows, and that heat is transferred from the components to constant temperature ( $T_s$ ) heat sinks located at each end of the row. This assumption is good if heat rails are employed on the individual rows or if the rows are thermally matched.

## FORMULATION

To optimize for reliability the goal is to determine the placement of  $N$  components in a row on a PWB which yields the minimum total failure rate  $\lambda_T$ . If the components are optimally placed, then switching any two components will result in an increased total failure rate  $\lambda_T'$ . Assuming components  $i$  and  $i+1$  are switched, the difference in the total failure rate is

$$(\lambda_T' - \lambda_T) = \sum_{j=1}^N \{ \lambda_j(T_{jc'_j}) - \lambda_j(T_{jc_j}) \} \quad (2)$$

where  $T_{jc'_j}$  is the junction temperature of component  $j$  which occurs as a result of interchanging  $i$  and  $i+1$ , and  $T_{jc_j}$  is the junction temperature of component  $j$  in

the original placement configuration. Since  $\lambda_T$  is assumed to be the minimum total failure rate, the right hand side of equation (2) must not be negative. Therefore, the objective of placement for reliability is to ensure that the result of interchanging any two components on the PWB results in a non-negative value for equation (2).

To determine the failure rate for each component on the PWB, it is necessary to calculate the component junction temperature. This requires calculation of each component case temperature and the board temperature. If a component  $j$  at position  $x_j$ , is assumed to be a point source dissipating heat at a rate of  $q_j$ , then

$$q_{jL} = \frac{T_j}{\frac{x_j}{kA}} = \frac{T_j}{R_{jL}} \quad (3)$$

and

$$q_{jR} = \frac{T_j}{\frac{(L-x_j)}{kA}} = \frac{T_j}{R_{jR}} \quad (4)$$

where  $q_{jL}$  is the rate at which heat is transferred to the left hand sink,  $q_{jR}$  is the rate at which heat transferred to the right hand sink,  $T_j$  is the board temperature generated at position  $x_j$ ,  $k$  is the thermal conductivity of the board,  $L$  is the length of the board,  $A$  is the cross-sectional area of the board, and  $R_{jL}$  and  $R_{jR}$  are the thermal resistances between the source and the left and right sink, respectively. From conservation of energy principles, the rate of heat  $q_j$  dissipated by a component  $j$  must be equal to the rate at which heat is transferred to the sinks. Therefore,

$$q_j = q_{jL} + q_{jR} \quad (5)$$

Substituting equations (3) and (4) into equation (5) and solving for the board temperature yields

$$T_j = \frac{q_j R_{jL} R_{jR}}{(R_{jL} + R_{jR})} = \frac{q_j (x_j L - x_j^2)}{kAL} \quad (6)$$

Without loss of generality, the two heat sinks are assumed to be at  $0^\circ\text{C}$ . The temperature contribution of component  $j$  at position  $x_k$  is then be written as

$$T_j(x_k) = T_j \cdot \text{MIN} \left\{ \frac{x_k}{x_j}, \frac{(L-x_k)}{(L-x_j)} \right\} \quad (7)$$

The board temperature,  $T(x)$ , at any position  $x$  on the row resulting from  $N$  components is determined by

$$T(x) = T_s + \sum_{j=1}^N T_j(x) \quad (8)$$

The junction temperature  $T_{jk}$  of component  $k$  is equal to the board temperature under the component plus the

temperature increase between the board and the junction,

$$T_{jk}(x) = T(x) + q_k(R_{xxk} + R_{jc_k}) \quad (9)$$

where  $R_{xxk}$  is a constant which specifies the thermal resistance between the case and the board for component  $k$ , and  $R_{jc_k}$  is a constant which specifies the thermal resistance between the case and the junction for component  $k$ . Thus, the junction temperature of any component  $k$  is a function of its position on the board. The goal is thus to develop the board temperature equations are developed for the  $N$  components on the row for the original and the new placement configurations, and then introduce these equations into a Taylor series approximation of equation (2).

For the original placement configuration, the board temperature at any position  $x_k$  excluding positions  $x_1$  and  $x_{i+1}$  is given by

$$\begin{aligned} T(x_k) = & \sum_{j=1}^{i-1} \frac{q_j}{kAL} \{x_j L - x_j^2\} \text{MIN} \left\{ \frac{x_k}{x_j}, \frac{L-x_k}{L-x_j} \right\} \\ & + \frac{q_i}{kAL} \{x_i L - x_i^2\} \text{MIN} \left\{ \frac{x_k}{x_i}, \frac{L-x_k}{L-x_i} \right\} \\ & + \frac{q_{i+1}}{kAL} \{x_{i+1} L - x_{i+1}^2\} \text{MIN} \left\{ \frac{x_k}{x_{i+1}}, \frac{L-x_k}{L-x_{i+1}} \right\} \\ & + \sum_{j=i+2}^N \frac{q_j}{kAL} \{x_j L - x_j^2\} \text{MIN} \left\{ \frac{x_k}{x_j}, \frac{L-x_k}{L-x_j} \right\} \quad (10) \end{aligned}$$

For the original placement configuration, the board temperatures at  $x_1$  and  $x_{i+1}$  are given by

$$\begin{aligned} T(x_1) = & \sum_{j=1}^{i-1} \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{L-x_1}{L-x_j} \right\} \\ & + \frac{q_i}{kAL} \{x_i L - x_i^2\} + \frac{q_{i+1}}{kAL} \{x_{i+1} L - x_{i+1}^2\} \left\{ \frac{x_1}{x_{i+1}} \right\} \\ & + \sum_{j=i+2}^N \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{x_1}{x_j} \right\} \quad (11) \end{aligned}$$

and

$$\begin{aligned} T(x_{i+1}) = & \sum_{j=1}^{i-1} \frac{q_j}{kAL} \{x_j L - x_j^2\} \text{MIN} \left\{ \frac{x_k}{x_j}, \frac{L-x_{i+1}}{L-x_j} \right\} \\ & + \frac{q_i}{kAL} \{x_i L - x_i^2\} \left\{ \frac{L-x_{i+1}}{L-2x_i} \right\} + \frac{q_{i+1}}{kAL} \{x_{i+1} L - x_{i+1}^2\} \\ & + \sum_{j=i+2}^N \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{x_{i+1}}{x_j} \right\} \quad (12) \end{aligned}$$

When components  $i$  and  $i+1$  are interchanged, the failure rate of the system is given by

$$T(x_k) = \sum_{j=1}^{i-1} \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{x_k}{x_j} \cdot \frac{L - x_k}{L - x_j} \right\} + \frac{q_{i+1}}{kAL} \{x_i L - x_i^2\} \text{MIN} \left\{ \frac{x_k}{x_i}, \frac{L - x_k}{L - x_i} \right\} + \frac{q_i}{kAL} \{x_{i+1} L - x_{i+1}^2\} \text{MIN} \left\{ \frac{x_k}{x_{i+1}}, \frac{L - x_k}{L - x_{i+1}} \right\} + \sum_{j=i+2}^N \frac{q_j}{kAL} \{x_j L - x_j^2\} \text{MIN} \left\{ \frac{x_k}{x_j}, \frac{L - x_k}{L - x_j} \right\} \quad (13)$$

For the new placement configuration, the board temperatures at  $x_i$  and  $x_{i+1}$  are given by

$$T(x_i) = \sum_{j=1}^{i-1} \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{L - x_k}{L - x_j} \right\} + \frac{q_{i+1}}{kAL} \{x_i L - x_i^2\} + \frac{q_i}{kAL} \{x_{i+1} L - x_{i+1}^2\} \left\{ \frac{x_i}{x_{i+1}} \right\} + \sum_{j=i+2}^N \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{x_i}{x_j} \right\} \quad (14)$$

and

$$T(x_{i+1}) = \sum_{j=1}^{i-1} \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{L - x_{i+1}}{L - x_j} \right\} + \frac{q_{i+1}}{kAL} \{x_i L - x_i^2\} \left\{ \frac{L - x_{i+1}}{L - x_i} \right\} + \frac{q_i}{kAL} \{x_{i+1} L - x_{i+1}^2\} + \sum_{j=i+2}^N \frac{q_j}{kAL} \{x_j L - x_j^2\} \left\{ \frac{x_{i+1}}{x_j} \right\} \quad (15)$$

respectively.

The failure rate prediction equation can be approximated by a Taylor series expansion around a junction temperature  $T_o$ , such that

$$\lambda_i(T) = \lambda_i(T_o) + \sum_{m=1}^{\infty} \frac{d\lambda_i(T_o)}{dT^m} \frac{(T - T_o)^m}{m!} \quad (16)$$

Using the Taylor series expansion to express the failure rates of all the components resulting from switching components  $i$  and  $i+1$  to the positions  $i+1$  and

$i$ , the difference between the failure rates is given by

$$\Delta \lambda_i = \lambda_i(T) - \lambda_i(T_o) = \sum_{m=1}^{\infty} \frac{d\lambda_i(T_o)}{dT^m} \frac{(T - T_o)^m}{m!} + \frac{d\lambda_i(T_{jc_i})}{dT^m} \frac{\{T_{jc_i}' - T_{jc_i}\}^m}{m!} - \frac{d\lambda_{i+1}(T_{jc_{i+1}})}{dT^m} \frac{\{T_{jc_{i+1}}' - T_{jc_{i+1}}\}^m}{m!} + \sum_{k=i+2}^N \frac{d\lambda_k(T_{jc_k})}{dT^m} \frac{\{T_{jc_k}' - T_{jc_k}\}^m}{m!} \quad (17)$$

Recalling that the junction temperatures are solely a function of the board temperatures, the difference terms,  $\{T_{jc_k}' - T_{jc_k}\}$ , of equation (17) are the difference between the board temperature resulting from switching components and the board temperature with components in their initial position. For any component  $k$  at  $x_k$ , the difference between the switched and initial junction temperature is given by

$$\{T_{jc_k}' - T_{jc_k}\} = \frac{q_{i+1}}{kAL} (L - x_k)(x_i - x_{i+1}) + \frac{q_i}{kAL} (L - x_k)(x_{i+1} - x_i) \quad (18)$$

where  $x_k > x_{i+1}$ , and

$$\{T_{jc_k}' - T_{jc_k}\} = \frac{q_{i+1}}{kAL} (x_k)(x_{i+1} - x_i) + \frac{q_i}{kAL} (x_k)(x_i - x_{i+1}) \quad (19)$$

where  $x_k < x_i < x_{i+1}$ . The difference between the switched and the initial junction temperatures for component  $i$  is given by

$$\{T_{jc_i}' - T_{jc_i}\} = \frac{q_i [L(x_{i+1} - x_i) - x_{i+1}^2 - x_i^2]}{kAL} + \sum_{j=1}^{i-1} q_j (L - x_j) \frac{(x_{i+1} - x_i)}{kAL} - \sum_{j=i+2}^N q_j x_j \frac{(x_{i+1} - x_i)}{kAL} \quad (20)$$

and the difference between the switched and the initial junction temperature for component  $i+1$  is given by

$$(T_{jc_{i+1}} - T_{jc'_{i+1}}) = \frac{q_{i+1} [L(x_{i+1} - x_i) - x_{i+1}^2 + x_i^2]}{kAL} + \sum_{j=1}^{i-1} q_j (L - x_j) \frac{(x_{i+1} - x_i)}{kAL} - \sum_{j=i+2}^N q_j x_j \frac{(x_{i+1} - x_i)}{kAL} \quad (21)$$

Substituting the difference equations (18)-(21) into equation (17) yields

$$\lambda'_T - \lambda_T = \sum_{m=1}^{\infty} \frac{d^m \lambda_1 (T_{jc_1})}{dT^m} \frac{\delta x^m}{m! (kAL)^m} [q_1 (L - 2x_i - \delta x) + \sum_{j=i+2}^N q_j (L - x_j) - \sum_{j=1}^{i-1} q_j x_j]^m - \frac{d^m \lambda_{i+1} (T_{jc'_{i+1}})}{dT^m} \frac{\delta x^m}{m! (kAL)^m} - [q_{i+1} (L - 2x_i - \delta x) + \sum_{j=i+2}^N q_j (L - x_j) - \sum_{j=1}^{i-1} q_j x_j]^m + \sum_{k=1}^{i-1} \left[ \frac{d^m \lambda_k (T_{jc_k})}{dT^m} \frac{(-1)^m \delta x^m (x_k)^m}{m! (kAL)^m} (q_i - q_{i+1})^m \right] + \sum_{k=i+2}^N \left[ \frac{d^m \lambda_k (T_{jc_k})}{dT^m} \frac{\delta x^m (L - x_k)^m}{m! (kAL)^m} (q_i - q_{i+1})^m \right] \quad (22)$$

where  $\delta x$  is the distance between  $x_{i+1}$  and  $x_i$ .

Since the original placement configuration is assumed to generate the minimum total failure rate, the right hand side of equation (22) must be greater than or equal to zero. If this is not true, then the original placement of components cannot be optimal with respect to the total failure rate. Equation (22) can thus be used to prove that a particular placement arrangement is optimal with respect to the total failure rate.

In the range of operating temperatures utilized in MIL-HDBK-217E[1], the derivatives of the failure rates in equation (22) with respect to the junction temperature are always positive. The sign of the terms being multiplied by the derivatives are dependent on the positions of  $x_i$  and  $x_{i+1}$ , the magnitudes of  $q_i$  and  $q_{i+1}$ , and the heat dissipation rates and positions of all other components. For example, the quantity

$$(L - 2x_i - \delta x) \quad (23)$$

has a sign change when  $x_i$  passes through the point  $x_p$ , when  $x_p$  is defined by

$$x_p = (L - \delta x)/2 \quad (24)$$

For

$$x_p < (L - \delta x)/2, \quad (25)$$

the value of the quantity is positive. Either end of the row can be assumed to be the starting point. If the placement is symmetric on both sides of the board in terms of position and heat dissipation rates, the value of the summations defined by

$$\sum_{j=i+2}^N q_j (L - x_j) - \sum_{j=1}^{i-1} q_j x_j \quad (26)$$

goes to zero as  $x_i$  goes to  $L/2$ . If the placement is nearly symmetric, then these summations generally result in a positive quantity for  $x_i$  between zero and  $L/2$  and a negative quantity for  $x_i$  between  $L/2$  and  $L$ . However, if the position reference is reversed by setting zero equal to  $L$  and  $L$  equal to zero and the positions of the components are redefined then both sign changing quantities defined by (23) and (26) can be maintained as positive values for any  $x$  between zero and  $(L - \delta x)/2$ . Finally, a sign change in equation (22) can also be produced by the difference in the heat dissipation rates of components  $i$  and  $i+1$  in the failure rate derivative terms of all components excluding  $i$  and  $i+1$  in the right hand side of equation (22).

Equation (22) can be simplified if second and higher order terms are assumed negligible with respect to the first order term. Thus

$$\frac{(\lambda'_T - \lambda_T)kAL}{\delta x} \approx \frac{d\lambda_1 (T_{jc_1})}{dT} [q_1 (L - 2x_i - \delta x)$$

$$+ \sum_{j=i+2}^N q_j (L - x_j) - \sum_{j=1}^{i-1} q_j x_j]$$

$$- \frac{d\lambda_{i+1} (T_{jc'_{i+1}})}{dT} [q_{i+1} (L - 2x_i - \delta x)$$

$$+ \sum_{j=i+2}^N q_j (L - x_j) - \sum_{j=1}^{i-1} q_j x_j]$$

$$- \sum_{k=1}^{i-1} \frac{d\lambda_k (T_{jc_k})}{dT} x_k (q_i - q_{i+1})$$

$$+ \sum_{k=i+2}^N \frac{d\lambda_k(Tj c_k)}{dT} (L - x_k)(q_i - q_{i+1}) \quad (27)$$

In equation (27), the quantities multiplied by the failure rate derivatives of both components  $i$  and  $i+1$  have the same form with opposite signs. Thus for the placement to be optimal in terms of reliability with respect to the thermal response of the PWB, the following inequality based on equation (27) must be valid.

$$\begin{aligned} & \frac{d\lambda_i(Tj c_i)}{dT} [q_i(L - 2x_i - \delta x) + \sum_{j=i+2}^N q_j(L - x_j) - \sum_{j=1}^N q_j x_j] \\ & + \sum_{k=i+2}^N \frac{d\lambda_k(Tj c_k)}{dT} (L - x_k)q_i - \sum_{k=1}^{i-1} \frac{d\lambda_k(Tj c_k)}{dT} x_k q_i \\ & > \frac{d\lambda_{i+1}(Tj c_{i+1})}{dT} [q_{i+1}(L - 2x_{i+1} - \delta x) \\ & + \sum_{j=i+2}^N q_j(L - x_j) - \sum_{j=1}^N q_j x_j] \\ & + \sum_{k=i+2}^N \frac{d\lambda_k(Tj c_k)}{dT} (L - x_k)q_{i+1} \\ & - \sum_{k=1}^{i-1} \frac{d\lambda_k(Tj c_k)}{dT} x_k q_{i+1} \quad (28) \end{aligned}$$

Equation (28) can be used to verify that a given placement configuration is optimal. It does not explicitly predict the optimum solution. In regards to any constructive placement scheme, the positions of components are unknown until they are assigned to a position on the board. Thus, all quantities which are determined by the positions of the components on the board are unknown. However, a placement scheme can be developed using approximations for the effect of the unknown placement configuration on the junction temperatures of the components and the unknown positions of the components.

The placement scheme is based on a priority metric defined by

$$\begin{aligned} PR_j &= \frac{d\lambda_j(Tj c_j)}{dT} [q_j(L - 2x_j - \delta x) + \sum_{k=i+2}^N q_k(L - x_k) \\ & - \sum_{k=1}^N q_k x_k] + \sum_{k=i+2}^N \frac{d\lambda_k(Tj c_k)}{dT} (L - x_k)q_j \end{aligned}$$

$$- \sum_{k=1}^{i-1} \frac{d\lambda_k(Tj c_k)}{dT} x_k q_j \quad (29)$$

To simplify the priority metric, we note that derivatives of the components on each side of component  $j$  are multiplied by the heat dissipation rate of component  $j$ . In addition, the quantity resulting from the other component derivatives reflects the effect of the heat dissipation rate of component  $j$ . The effect of the heat dissipation rate of component  $j$  is also reflected in the first quantity in the priority number. Therefore, the effect of the derivatives of the other components can be neglected. Thus the priority metric is simplified to

$$\begin{aligned} PN_j &= \frac{d\lambda_j(Tj c_j)}{dT} [q_j(L - 2x_j - \delta x) \\ & + \sum_{k=i+2}^N q_k(L - x_k) - \sum_{k=1}^N q_k x_k] \quad (30) \end{aligned}$$

However, the unknown placement configuration must still be approximated for a placement scheme to be employed. It is possible to further simplify equation (30) by neglecting the unknown placement terms for all components not under consideration. If we neglect the effect of the heat dissipation rates, the positions, and the derivatives of the components not under consideration, equation (27) simplifies to

$$\begin{aligned} \frac{(\lambda_T - \lambda_T)kAL}{\delta x[L - 2x_i - \delta x]} &\approx \frac{d\lambda_i(Tj c_i)}{dT} q_i \\ &- \frac{d\lambda_{i+1}(Tj c_{i+1})}{dT} q_{i+1} \quad (31) \end{aligned}$$

Equation (31) allows us to develop a placement procedure that is dependent only on the component under consideration. If  $\lambda_T$  is the minimum total failure rate, the left hand side of equation (31) must be positive for  $x_i$  less than  $(L - \delta x)/2$  referenced from either heat sink. Thus, a placement scheme can be developed using the priority metric,  $PRN_j$  defined by

$$PRN_j = \frac{d\lambda_j(Tj c_j)}{dT} q_j \quad (32)$$

Here, only the approximate junction temperature of component  $j$  is needed to calculate the priority number. The ordering of components is only dependent on the individual component derivative of the failure rate with respect to temperature, and the heat dissipation rate of the component.

Limitations in the use of the priority numbers defined by equations (29), (30), and (32) may arise due to the first order approximation. Therefore, any method based on the priority metrics will not guarantee an optimum placement solution. Furthermore, the ability to accurately predict the junction temperatures of the components without knowing the actual placement will also reduce the accuracy of any placement scheme.

However, once an initial placement is generated, it can be checked for optimality and improved.

#### PLACEMENT PROCEDURES

In this section, three constructive placement procedures for minimizing the total failure rate on a row are introduced. In each procedure, a target location is determined and a component is selected from the set of the unplaced components, placed in the target location, and a new target location is determined. Once a component is placed, it is removed from any further considerations. Initially, all components are assumed to be in the unplaced component set. Selection is based on the priority metrics developed earlier.

##### PROCEDURE 1

In this placement procedure, the priority metric  $PRN_j$  given by equation (32), defines the selection criterion. The first target location is the open position closest to either of the two heat sinks. The board temperature at the target location is approximated by assuming that the heat dissipated by all of the components to be placed, excluding the component under consideration, are added together, and the result is treated as a single source located at the center of the board. Thus, the board temperature at the target location is

$$T_{b_1}(x_1) = \frac{(q_{sum_T} - q_1)x_1}{2kA} + \frac{q_1(x_1L - x_1^2)}{kAL} + T_s \quad (33)$$

where  $q_{sum_T}$  is the sum of the heat dissipation rates of all components and  $q_1$  is the heat dissipation rate of the component under consideration. The junction temperature for component  $i$  is determined by

$$T_{jc_i}(x_1) = T_{b_1}(x_1) + q_1(R_{xx_i} + R_{jc_i}) \quad (34)$$

The priority number,  $PRN_j$ , of equation (32) is evaluated for all components in the unplaced set, and the component with the maximum priority number is selected and placed in the target location.

The second target location is the open position closest to the other heat sink. The board temperature is determined by an equation similar to equation (33) that includes the thermal contribution resulting from the placed component. The junction temperature for any component  $i$  placed in the target location  $x_n$  is approximated by

$$T_{jc_i}(x_n) = \frac{(q_{sum_T} - q_1)(L - x_n)}{2kA} + \frac{q_1(x_nL - x_n^2)}{kAL} + \frac{q_1x_1(L - x_n)}{kAL} + T_s + q_1(R_{xx_i} + R_{jc_i}) \quad (35)$$

where  $q_{sum_T}$  is the sum of the heat dissipation rates of all unplaced components,  $q_1$  is the heat dissipation rate of the component under consideration,  $x_n$  is the position closest to the heat sink, and  $q_1$  and  $x_1$  correspond to the heat dissipation and position of the first component selected.

Again, the priority number for each unplaced component is evaluated and the component with the maximum priority number is selected. The sum of the heat dissipations of placed components multiplied by their positions relative to the closest sink on both sides of the board should be tabulated by

$$QXsum_1 = \sum_A q_i x_i \quad (36)$$

where  $A$  is the set of all placed components on the range  $0 < x < L/2$ ,

and

$$QXsum_2 = \sum_B q_i (L - x_i) \quad (37)$$

where  $B$  is the set of all placed components on the range  $L/2 < x < L$ . By comparing values of  $QXsum_1$  and  $QXsum_2$ , the next target location is selected on the side with the smaller  $QXsum$ . Target locations on the left side are order  $[x_2, x_3, x_4, \dots]$  and target locations on the right side of the board are order  $[x_{n-1}, x_{n-2}, x_{n-3}, \dots]$ . For components in the unplaced set, the components' junction temperature is approximated by

$$T_{jc_i}(x_1) = T_s + \frac{(q_{sum_T} - q_1)x_1}{2kA} + \frac{q_1(x_1L - x_1^2)}{kAL} + QXsum_1 \frac{(L - x_t)}{kAL} + QXsum_2 \frac{x_t}{kAL} + q_1(R_{xx_i} + R_{jc_i}) \quad (38)$$

where  $x_t$  is the new target location,  $q_{sum_T}$  is again the sum of the heat dissipation rates of all unplaced components, and  $i$  is the component under consideration.

The placement procedure selects the target location on the side with the lower  $QXsum$ . Once the target location is determined, the board temperature is approximated at the target location for each unplaced component. Then, the priority number is evaluated for each unplaced component. The unplaced component with the highest priority number at the target location is selected and placed in the target location. The procedure continues until all components are placed.

When  $x_t$  is at  $(L-\delta x)/2$  as measured from either heat sink or inside the region around  $L/2$  (i.e. the blackened region in figure 1), ordering takes place by selecting components with the lowest priority numbers. At the zero point, only two components are now in question and a thermal check of the entire row will reveal the actual positioning.

##### PROCEDURE 2

A similar placement procedure can be carried out using the priority metric defined by equation (30). In this case, the position of the target location along with the heat dissipation rates and positions of the placed components must be considered in evaluating the priority metric. For the selection of the first two

components which are placed at  $x_1$  and  $x_n$ , the procedure follows the same method as in Procedure 1. The selection of target locations is also the same. The difference lies in the evaluation of the priority metric after the first two components have been selected.

For all placed components, the priority number is thus evaluated by

$$PN_j = \frac{d\lambda_j(Tjc_j)}{dT} \{ q_j (L - 2x_s - \delta x) + (qsum_T - q_j)L/2 + QXtot \} \quad (39)$$

where

$$QXtot = \begin{cases} QXsum_2 - QXsum_1, & \text{for } x_t < L/2 \\ QXsum_1 - QXsum_2, & \text{for } x_t > L/2 \end{cases}$$

$x_s$  is defined by

$$x_s = \begin{cases} x_t, & \text{for } x_t < L/2 \\ (L - x_t), & \text{for } x_t > L/2 \end{cases}$$

$x_t$  is the target location, and  $QXsum_1$  and  $QXsum_2$  were defined in Procedure 1. Again the selection is made based on the maximum value of the priority number. At the zero points, it is necessary to keep the  $QXsum$ 's of both sides approximately equal.

#### PROCEDURE 3

This placement procedure employs the priority metric defined by equation (29). The difference from the other procedures is in the approximation of the priority metric; namely the approximation of the derivatives of the components not under consideration. From equation (29), we approximate the priority number by the following equation.

$$PN_j = \frac{d\lambda_j(Tjc_j)}{dT} [q_j (L - 2x_s - \delta x) + (qsum_T - q_j)L/2 + QXtot] + DFsum \quad (40)$$

where  $j$  is the component under consideration,  $DFsum$  is defined by

$$DFsum = \sum_{k \in B} \frac{d\lambda_k(Tjc_k)}{dT} (L - x_k) q_j - \sum_{k \in A} \frac{d\lambda_k(Tjc_k)}{dT} x_k q_j + \sum_{k \in A \in B} \frac{d\lambda_k(Tjc_k)}{dT} (L/2) q_j \quad (41)$$

for  $x_t < L/2$ , and

$$DFsum = \sum_{k \in A} \frac{d\lambda_k(Tjc_k)}{dT} x_k q_j - \sum_{k \in B} \frac{d\lambda_k(Tjc_k)}{dT} (L - x_k) q_j$$

$$+ \sum_{k \in A \in B} \frac{d\lambda_k(Tjc_k)}{dT} (L/2) q_j \quad (42)$$

for  $x_t > L/2$ . For the first two summations on the placed component sets A and B, the junction temperatures of these components are approximated by

$$Tjc_k = T_s + \frac{(qsum_T - q_j)x_k}{2kA} + q_k (Rxx_k + Rjc_k) + \frac{q_j (x_t L - x_t^2)}{KAL} \min \left[ \frac{x_k}{x_t}, \frac{(L - x_k)}{(L - x_t)} \right] + \sum_{1 \in A \in B} \frac{q_1 (x_1 L - x_1^2)}{KAL} \min \left[ \frac{x_k}{x_1}, \frac{(L - x_k)}{(L - x_1)} \right] \quad (43)$$

where  $x_t$  is the target position under consideration, and  $q$  is the heat dissipation rate of the component under consideration. For the summation of unplaced set of components, the junction temperature is approximated by

$$Tjc_k = T_s + \frac{(qsum_T - q_1)L}{4kA} + \frac{q_1 (x_t L - x_t^2)}{kAL} \frac{L}{2x_s} + \sum_{1 \in A \in B} \frac{q_1 (x_1 L - x_1^2)}{KAL} \min \left[ \frac{L}{2x_1}, \frac{L}{2(L - x_1)} \right] + q_k (Rxx_k + Rjc_k) \quad (44)$$

Initially, all components are assumed to be in the unplaced set. The first two target locations are adjacent to the heat sinks. The first two components should be selected using Procedure 1. Selection of components is based on the maximum priority number defined by equation (40). A record of the priority numbers of the selected components is obtained in the following manner

$$PNsum_1 = \sum_{i \in A} \frac{d\lambda_i(Tjc_i)}{dT} q_i x_i \quad (45)$$

and

$$PNsum_2 = \sum_{i \in B} \frac{d\lambda_i(Tjc_i)}{dT} q_i x_i \quad (46)$$

where A represents the set of components placed between zero and  $L/2$ , and B represents the set of components placed between  $L/2$  and  $L$ . After the first two components are selected to be placed adjacent to each sink, the remaining target locations are selected by comparing  $PNsum_1$  and  $PNsum_2$ . The open location on the side of the board with the smallest  $PNsum$  is selected. Target locations on the left side are ordered  $\{x_2, x_3, x_4, \dots\}$  and target locations on the right side of the board are ordered  $\{x_{n-1}, x_{n-2}, x_{n-3}, \dots\}$ . Selections are always made based on the component with the largest priority number. The procedure continues until all components are placed.



## PLACEMENT FOR AN ENTIRE PWB

In placement for reliability for a PWB, the board is divided into slots that form rows and columns on the board to accommodate individual components. The method for placing the components on the board is performed by using placement procedures similar to those previously defined.

Initially, all components are assigned to the unplaced set. The board temperatures at the open slots adjacent to one of the heat sinks is calculated following equation (33). The heat dissipated by all the components is considered as a single source at the center of the board. The priority metric defined by equation (32) is employed to evaluate a priority number for all of the components. Components are then alternatively assigned to the open slots adjacent to the heat sinks based on the highest priority number in the unplaced set. Target locations are selected from the columns of open locations adjacent to the heat sinks. The procedure is to fill the edge locations and equally distribute the components with respect to the sum of the heat dissipation rates along each individual row. For example, the first two components selected are placed in opposite corners on the board. The procedure continues until the two columns adjacent to the heat sinks are filled. During this process, the  $QXsum_1$ 's and  $QXsum_2$ 's are calculated as in Procedure 1 for each individual row.

The selection of the subsequent target locations is based on  $QXsum_t$ , the sum of the  $QXsum_1$  and  $QXsum_2$  for each individual row. The row with the lowest  $QXsum_t$  is selected. Column selection is based on  $QXsum_1$  and  $QXsum_2$ . The open location nearest to the placed component on the side with the lowest  $QXsum$  is selected. The board temperatures at this open slot is calculated following equation (38). For components on the selected row, equation (38) is not modified. For placed components not on the selected row, the temperature contributions are scaled by the distance of the placed components to the selected row. Those placed components closest to the row have a greater effect than those placed further away from the selected row.

Once the board temperature at the selected position is approximated, the priority metric defined by equation (30) is employed to evaluate the priority numbers of the unplaced components in the target location. The component with the highest priority number is selected and assigned to the target location. The selected component is removed from the unplaced set and the  $QXsum$ 's are recalculated and a new target location is selected. The selection of a new target location and the evaluation of board temperature at the new target location are repeated as previously described. Selections are always made based on the component in the unplaced set with the highest priority number. The process continues until all components are placed on the board.

## DISCUSSION

To examine the assumptions and the approximations made in developing the procedures, test cases were examined by computer simulation. These cases consisted of sets of seven components taken randomly from the MIL-HDBK-217E [1]. The board was assumed to be constructed from multilayer epoxy fiberboard with a

thermal conductivity in the plane of the heat sinks of  $34.3 \text{ W/m}^{\circ}\text{C}$ . The heat sink temperature were set at  $20^{\circ}\text{C}$ . The available positions were evenly spaced along the row. The thermal resistance between the case and the board,  $R_{xx}$ , was assumed to be a function of the number of pins of the component. For each set of seven components, the minimum total failure rate based on a serially connected system, along with optimum placement configuration for the minimum total failure rate, was determined by examining all  $7! = 5040$  possible arrangements. The placement configurations and the corresponding total failure rates were also determined using Procedure 1, 2, and 3 for each set of components.

In the majority of examples tested, each of the three procedures accurately predicted the optimum placement for reliability based on the thermal response of the board. In some cases, the procedures predicted the mirror image of the optimal placement configurations. In the cases where the procedures failed, the inequality given by equation (28) was found to be violated. This inequality can be used to check the results quickly to determine if the placement is indeed optimal. When the placement procedures failed the resulting total failure rate was normally only a few percentage points off the optimum. The error is attributed to the approximation of the junction temperatures and the assumptions made in deriving the priority number equations.

Each prediction method required less than 5 seconds on an IBM AT computer, compared to approximately forty-five minutes required to examine  $7! = 5040$  possible arrangements. The fact that predicted failure rates are either optimal or near optimal gives weight to the use of such ordering schemes when high reliability is required.

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## REFERENCES

1. "Reliability prediction of electronic equipment", Feb. 1987, MILITARY HANDBOOK MIL-HDBK-217E, Rome Air Development Center.
2. M. Hanan and J. Kurtzberg, 1972, "Placement techniques" in Design Automation of Digital Systems: Theory and Techniques, Ed. M. A. Breuer, New York, Prentice-Hall, chp. 5.
3. G. Odaware, K. Iijima, and K. Wakabayashi, 1985, "Knowledge based placement technique for printed wiring board", 22th Design Automation Conference, pp. 616-622.
4. G. Wippler, M. Wiesel, and D. Mlynski, 1982, "A combined force and cut algorithm for hierarchical VLSI layout", Pro. 19th Automation Conf., pp. 671-677.
5. M. Hanan, P. Wolff, and B. Agule, June 1976, "A study of placement techniques for computer logic graphs", Proc. 13th Design Automation Conf., pp. 214-224.

6. N. Quinn and M. Breuer, 1979, "A force directed component placement procedure for printed circuit boards", IEEE Trans. CAS, vol. CAD-26, no. 6, pp. 377-388.
7. N. R. Quinn, 1975, "The placement problem as viewed from physics of classical mechanics", Proc. 12th Design Auto. Conf., June 23-25, pp. 173-178.
8. L. Steinberg, Jan. 1961, "A backboard wiring problem: A placement algorithm", SIA Review, vol. 3, no. 1, pp. 37-50.
9. L. Corrigan, 1979, "A placement capability based on partitioning", Proc. 16th Design Automation Workshop, pp. 406-413.
10. D. Schweikert, 1976, "A 2-dimensional placement algorithm for the layout of electrical circuits", Proc. 14th Design Automation Workshop, pp. 408-416.
11. D. Schmidt and L. Druffel, 1975, "An iterative algorithm for placement of integrated circuits", Proc. 12th Design Automation Workshop, pp. 361-368.
12. Design Automation of Digital Systems: Theory and Techniques, 1972 Ed. M. A. Breuer, New York, Prentice-Hall.
13. M. A. Breuer, Dec. 1966, "General survey of design automation of digital computers", Proc. of the IEEE, vol. 54, no. 12, pp. 1706-1721.
14. J. Sloan, 1985, Design and Packaging of Electronic Equipment, Van Nostrand Rienhold Company Inc., New York, New York.
15. D. Dancer, M. Pecht, M. Palmer, March 1987, "Reliability optimization scheme for convection cooled printed circuit boards", submitted: IEEE Trans on Reliability.
16. M. Pecht, May 2-5, 1987, "Computer aided design for PCB reliability", Proc. 33rd Annual Institute of Environmental Sciences Conference.
17. Pecht, Palmer, and Naft, Jan. 27-29, 1987, "Thermal reliability management in PCB design", Proc.: 1987 Annual Reliability and Maintainability Symposium.
18. Mayer, A., Mar. 1978, "Computer-aided thermal design of avionics for optimal reliability and minimum life-cycle cost", AFFDL-TR-7848.
19. M. Rothkopf, Jan, 1966, "Scheduling independent tasks on parallel processors", Mgmt. Sci., Vol. 12, No. 8, pp.437-447.
20. M. Rothkopf and S. Smith, March-April 1984, "There are no undiscovered priority index sequencing rules for minimizing total delay costs", Operation Research, Vol. 32, No. 2, pp. 451-456.
21. R. McNaughton, Oct. 1959, "Scheduling with deadlines and loss functions", Mgmt. Sci., Vol.6, No.1, pp. 1-12.