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Throughput of Hybrid (DS-SFH) Spread-Spectrum Random-Access Communications

by

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THROUGHPUT OF HYBRID (DS-SFH) SPREAD-SPECTRUM RANDOM-ACCESS COMMUNICATIONS

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ABSTRACT

The multiple-access capability of hybrid direct-sequence slow frequency hopped (DS-SFH) spread-spectrum systems with forward-error control coding is investigated. Frequency-slot synchronous and asynchronous schemes, packet slotted schemes with several chips of the signature sequence employed in each bit (or symbol, if not binary) and varying hopping rates are considered and different data modulation schemes (BPSK, M-ary, FSK) with coherennt or noncoherent demodulation are examined. The performance of forward-error-control coding schemes, such as Reed-Solomon codes with errors-only and errors and erasures decoding, with or without side information about the presence or absence of multiple-access interference, is evaluated.

Subsequently the throughput versus packet error probability tradeoff for hybrid spread-spectrum random-access schemes with finite or infinite populations, modeled as binomial and Poisson traffic respectively, is examined and an attempt of global comparison between different spread-spectrum systems in the context of our analysis is made. Finally, a specific decentralized retransmission controlled protocol is adopted, whose coupling with the system presented earlier implies a stable throughput.

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1. Introduction

In this work we investigate the performance of Coded Hybrid Direct-Sequence Slow Frequency Hopped (DS-SFH) Spread Spectrum communications, when employed in a network environment in which many sources are contending for the use of a common communication channel. Typical examples constitute satellite communications and terrestrial radio networks, corrupted by Additive White Gaussian Noise (AWGN) and disturbed by non-hostile multi-access interference.

A most commonly used performance measure, is the probability of a symbol error, computed at the receiving end of the channel. In the context of a multiple access channel, in which we focus on the performance of a particular receiver trying to decode correctly one out of several incoming symbol streams, that has been corrupted by AWGN and disturbed by the transmissions that co-exist in the vicinity of the receiver, an approximation to the probability of a symbol error for our system has already been derived in [4]. In addition to the features mentioned above, forward-error-control coding is considered in order to further improve the behavior of our system. The most important performance measure to be examined, is the number of simultaneous transmissions that can be supported under the constraint that the resulting probability of correct reception should not fall below a prespecified level. Both binary and M-ary modulation schemes are considered, respectively coupled with either coherent or non-coherent demodulation. When coherent demodulation is considered, we need to restrict our attention to hopping rates much slower than the data rate, so that adequate time is provided to the receiver to acquire phase synchronization. Nevertheless, in many cases, hopping rates of the same order of magnitude as the data rate, are of interest and so we need to employ modulation schemes that allow for non-coherent reception. In this case the hopping rates are only limited by the current technology of frequency synthesizers.

Moreover, time sychronization may or may not be feasible in a particular application, so we need to consider both synchronous and asynchronous models for our system.

Additional care needs to be taken when comparisons are about to be made with other systems. Since it is quite obvious that Spread-Spectrum Systems generally consume considerably greater bandwidth and that the employment of modulation and/or coding may require some further bandwidth expansion, normalization should be performed, whenever necessary, so that conceptually valid conclusions are made. This is true even in the context of comparison of two different Spread-Spectrum systems and the specifications must be selected with precision in order to meet this requirement. Whenever normalization is not meaningful, care must be taken to compare systems that are a priori equivalent bandwidth-wise.

Based on the performance of coded systems, we consider the employment of our scheme in a packet slotted random-access network environment, in which the actively transmitting population may be either finite or infinite. In this case, the performance measure will naturally be the normalized throughput versus packet error probability tradeoff, where the length of a packet has been defined to be equal to the length of a codeword, a simplifying assumption that can easily be relaxed. Again various modulation, demodulation and frequency hopping schemes are considered along with the different population models.

Finally a recursive retransmission controlled protocol described in [1], is considered as an application of our signalling scheme in a network that establishes for each user some form of feedback information concerning its status. The protocol is modified to fully exploit our system capabilities and key parameters, such as the probability of packet collision and the probability of frequency slot occupation are shown to be upperbounded by the purely frequency hopped scheme for which stability has already been established, thus conditionally implying stability for our case, too.

The paper is organized as follows. In Section 2 the system models are briefly described. In Section 3 expressions yielding the average error probabilities that determine the performance of our system, are cited and in Section 4 throughput calculations involved in our random-access schemes are performed. Then in Section

5 a comparison between different Spread-Spectrum signalling schemes is attempted, while in Section 6 the employment of our system in a recursive retransmission controlled protocol environment is considered. Finally, in Section 7, several numerical results are presented and conclusions about the performance of our system are drawn.

2. System Models

Our models for the Hybrid (DS-SFH) SSMA systems incorporate features from both the slow frequency-hopped and the direct-sequence SSMA models described in the literature, as for instance in [4] or [5].

We shall assume that the reader is already familiar with the functional blocks of our system and we shall proceed to introduce a modified version that will take into account coding of the information sequence. Specifically, as shown in Figure 1, prior to entering the DS/SS modulator module of the transmitter, the information sequence is appropriately coded, being processed by an encoder module. At the receiving end of our system and in cascade with the Despreader-Demodulator module as shown in the same figure, a decoder is introduced, that will consequently decode the received symbol stream. Throughout most of this work we shall assume that Reed-Solomon forward error control coding is employed and that our encoder-decoder module pair is functioning accordingly.

We shall also assume that the reader is familiar with the expressions developed for the description of signals along their course from the transmitting end, through the channel to the receiving end, as for instance in [4] and [5]. Consequently, our starting point will be the evaluation of the average error probability of our system so that it will reflect the effect of coding that we have already discussed.

3. Average Error Probability of Coded Hybrid (DS-SFH) SSMA

We shall proceed to the evaluation of the average error probability at the output of the receiver of the hybrid SSMA systems. The average should be computed with respect to all the random variables involved. We use a simplified version of the techniques introduced in [4] and [5]. In particular, we decouple the effect of hits from other users due to frequency-hopping from the multiple-access interference due to the direct-sequence spread-spectrum signals. This is done by first evaluating the conditional probability of error given the number of full hits (only) and then averaging with respect to the distribution of hits assuming that only full hits occur. In this way an upper bound on the system performance is obtained. Given that a number of full hits from other users has occurred the hybrid SSMA systems under consideration are equivalent to the DS/SSMA systems with noncoherent reception which we analyzed in [7].

As in [4] and [5] we analyze hybrid DS-SFH/SSMA systems which employ *random signature sequences and random frequency-hopping patterns* and assume that the powers of the transmitted signals are nearly equal.

We can write an upper bound on the error probability of the hybrid SSMA system $P_e(K)$ (employing independent random signature sequences and frequency hopping patterns) as

$$P_e(K) = \sum_{k=0}^{K-1} \binom{K-1}{k} P_h^k (1-P_h)^{K-1-k} \; P_a(k) \tag{1}$$

In (11) $P_a(k)$ denotes the conditional probability of error given that k full hits occured and P_h is the probability of a hit. This probability has been calculated in [8] and for asynchronous systems, first-order Markov random hopping patterns, and AWGN channels is given by $P_h = (1 + N_b^{-1})q^{-1}$. As discussed in [8] for moderately large values of q P_h is also a close approximation and bound to the probabilities of hits of the memoryless random hopping patterns and the Reed-Solomon periodic hopping patterns, respectively. Therefore we can safely use it for most applications. For systems employing MFSK N_b should be replaced by $N_s = N_b / \log_2 M$.

The conditional error probability $P_a(k)$ when k full hits occured, was obtained in [6] for BPSK modulation with coherent demodulations as

$$P_a(k) = Q\left\{ \left[\left(\frac{2E_b}{N_0} \right)^{-1} + k\frac{m_\psi}{N} \right]^{-\frac{1}{2}} \right\} \tag{2}$$

and in [7] for BFSK and MFSK modulation with noncoherent demodulation as

$$P_a(k) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1} \exp\left\{ -\frac{m}{2(m+1)} \left[\left(\frac{2E_b \log_2 M}{N_0} \right)^{-1} + k\frac{m_\psi}{MN'} \right]^{-1} \right\} \tag{3}$$

Equation (3) gives an expression for the symbol (bit) error probability of the MFSK (BFSK) hybrid system. It can also serve as a conservative estimate of the bit error probability.

In the synchronous case any user k ($k \neq i$) can cause only full hits with probability $P_h = q^{-1}$ for first-order Markov and memoryless random hopping patters [for periodic Reed-Solomon hopping patterns it is a tight approximation for moderately large q]. Then, the above results are modified to

$$P_a(k) = Q\left\{ \left[\left(\frac{2E_b}{N_0} \right)^{-1} + \frac{k}{2N} \right]^{-\frac{1}{2}} \right\} \tag{4}$$

for BPSK modulation and

$$P_a(k) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1} \exp\left\{ -\frac{m}{2(m+1)} \left[\left(\frac{2E_b \log_2 M}{N_0} \right)^{-1} + \frac{k}{2MN'} \right]^{-1} \right\} \tag{5}$$

for BFSK and MFSK modulation.

In all cases the result for $P_a(k)$ has to be coupled with (1) to provide the overall symbol (or bit) error probability.

Next we consider the performance of extended (n,k) Reed-Solomon (RS) code with $n=M^m$ (n is the codeword length, k is the number of information bits per codeword). When BPSK modulation with coherent demodulation is employed, each code symbol contains m bits [the code alphabet is $GF(2^m)$]. In this case the hopping is slow (i.e., $N_b >> 1$) and interleaving at depth N_b/m is required within each dwell time to guarantee the independence of the errors on the code symbols. When MFSK modulation with noncoherent demodulation is employed, each code symbol contains m M-ary symbols [the code alphabet is $GF(M^m)$] and interleaving at depth N_s/m is required. When bounded distance decoding is employed, the probability of correct decoding for each symbol is given by [12] as:

$$\overline{P}_c(K) = \sum_{l=0}^{t} \frac{l}{n} \binom{n}{l} [p_s(K)]^l [1-p_s(K)]^{n-l}, \tag{6}$$

where $t = [(n-k)/2]$ denotes the error-correcting capability of the RS(n,k) code and $p_s(K)$ is the probability of a symbol error for the uncoded system. Recall that K is the total number of active users in the system. We can approximate $p_s(K)$ by the expression:

$$p_s(K) = 1-\left[1-P_e(K)\right]^m \tag{7}$$

where for asynchronous systems $P_e(K)$ is given by (1) and (2) for BPSK modulation and by (1) and (3) for MFSK modulation; for synchronous systems $P_e(K)$ is given by (1) and (4) for BPSK modulation, and by (1) and (5) for MFSK modulation. This approximation is based on an "independance assumption" about the errors of different M-ary (or binary) symbols within the RS symbol.

When there is available information about the presence or absence of other-user interference, it can be used by the decoder to improve the performance of the system. Assume that the number of interfering users during the transmission of a packet remains constant (packet-slotted system) during the packet duration (a packet = a codeword of the RS code). Also assume that during each frequency-slot (dwell-time) when one RS code symbol is transmitted the number of interfering users in that slot is known (or it can be estimated accurately). Then, the following decoding algorithm is proposed: compare the number of interferers in the particular frequency slot with a fixed threshold K_{TH}; erase these RS symbols (or slots) for which this number is larger than K_{TH}; otherwise attempt to correct the symbols of the RS code.

The threshold K_{TH} is determined, by the condition that $P_a(K_{TH}) \leq P_{TH}$ where $P_a(\cdot)$ is taken from (2) for BPSK modulation and from (3) for MFSK modulation and P_{TH} is the maximum tolerable error probability of the DS/SSMA system to which the hybrid system becomes equivalent when hits occur.

In this case the erasure probability becomes

$$\epsilon_s(K) = \sum_{l=K_{TH}+1}^{K-1} \binom{K-1}{l} P_h^l (1-P_h)^{K-1-l} \tag{8}$$

and the error probability becomes

$$p_s(K) = \sum_{l=0}^{K_{TH}} \binom{K-1}{l} P_h^l (1-P_h)^{K-1-l} \left\{ 1 - \left[1-P_a(l) \right]^m \right\} \tag{9}$$

where $P_a(\cdot)$ is obtained from (2) or (3) for asynchronous systems with BPSK modulation or MFSK modulation, respectively. Finally, the probability of decoding correctly a symbol now becomes [12]

$$\overline{P}_c(K) = \sum_{\substack{2l+j \le e \\ l+j \le n}} \binom{n}{l} \binom{n-l}{j} \frac{l+j}{n} [p_s(K)]^l \, [\epsilon_s(K)]^j \, [1 - p_s(K) - \epsilon_s(K)]^{n-l-j} \tag{10}$$

where $e = n-k$ is the erasure-correction capability of the RS code.

An important performance measure is the multiple-access capability of the hybrid spread-spectrum systems. This is defined as the maximum number of simultaneously transmitted signals from users in the vicinity of a particular receiver so that the resulting symbol error probability $\overline{P}_e(K) = 1 - \overline{P}_c(K)$ does not exceed a prespecified level.

4. Throughput of Coded Hybrid (DS-SFH) Spread Spectrum Random Access Schemes

We shall consider packet slotted, frequency-slot asynchronous or synchronous hybrid (DS-SFH) systems, employing Reed-Solomon error control coding. The network environment, in which our hybrid schemes will be employed, is operating in a packet broadcasting mode, in the sense that each transmitted packet is heard by each receiver, a situation that arises for instance in a packet radio network. The above mentioned analysis, reflects a particular receiver performance, when the latter is affected by the traffic monitored in the vicinity of this receiver. Then, the throughput observed, is referred to as the local throughput and gives a good measure of the signaling scheme when employed in the context of a communication network. This measure is opposed to the end-to-end throughput which characterizes the entire network.

Since a slotted random-access mode is considered, the timing uncertainties are required to be small compared to the packet duration.

Nevertheless, there is no particular synchronicity requirement at the chip, the data and/or the frequency hopping level and hence the timing errors need not be small compared to the chip period, the data period and/or the dwell time. Side information may be used, but as we have already demonstrated in Section 3, our Hybrid (DS-SFH) system does not benefit from such an option. Specifically, it has been verified that an erasure correcting policy performs worse than a policy based in error correction only, this effect attributed to the considerable error-correcting capability of the Hybrid scheme.

Both finite and infinite population models are considered. Channel traffic from the finite population is modeled with the binomial distribution, while channel traffic originating from the infinite population is modeled with the Poisson distribution.

The motivation behind considering the employment of the Hybrid (DS-SFH) Spread-Spectrum system in a network environment, stems from the fact that this system demonstrates some very good performance in terms of the probability of both coded and uncoded symbol error, over a variety of operating conditions, the most prominent of which is the level of the multi-access interference.

The most important performance measure in this section will be the throughput versus packet-error probability tradeoff, which will be shown to be uniformly superior to the conventional narrowband slotted and unslotted ALOHA random-access schemes and locally superior to such schemes with DS/SS or FH/SS signalling.

In the context of the network environment that has just been described, a model packet consists of a single (n, k) Reed-Solomon codeword. Each codeword consists of n code symbols each of which contains $log_m M$ M-ary symbols.

Then, expressions identical to (16) coupled with (17), (11) and either of (12), (13), (14) or (15) of Section 3 hold, thus yielding the packet error probability for the system under consideration. Still, errors of different binary or M-ary symbols within a single RS symbol are considered independent, this approximation reflected in (17). For $n = M$ the expression is exact, since there is only one M-ary symbol per code symbol.

Having determined the packet error probability, given the number of active users in the system, one may proceed to determine the unconditional packet error probability P_E by using the total probability rule which may be written in the form:

$$P_E = \sum_{\text{all}K} Pr(K \text{ active users in the system, Packet error}) \tag{11}$$

or equivalently

$$P_E = \sum_{\text{all}K} Pr(K \text{ active users in the system}) \times$$

$$\times \; Pr(\text{Packet error} \mid K \text{ active users in the system}) \tag{12}$$

where

$$Pr(\text{Packet error} \mid K \text{ active users in the system}) =$$

$$= 1 - \overline{P_c}(K) =$$

$$= \sum_{\ell=t+1}^{n} \binom{n}{\ell} p_s(K)^{\ell}[1 - p_s(K)]^{n-\ell}$$

and $p_s(K)$ is the probability of a symbol error for the uncoded system given by (17)

$t = [(n-k)/2]$ denotes the error-correcting capability of the $RS(n,k)$ code.

Finally, the remaining term of the summation (2): $Pr(K$ active users in the system) depends upon the population models considered.

As we have already mentioned, we shall consider both infinite and finite population models.

4.1. Finite Population

Traffic generated by this population is modeled by the binomial distribution with parameters (K_s, δ) where K_s is the total population of users and δ is the duty cycle i.e. the packet transmission probability of each user. Then

$$Pr(K \text{ active users in the system}) = \binom{K_s}{K} \delta^K (1-\delta)^{K_s - K} \tag{13}$$

and hence the unconditional packet error probability may be rewritten as

$$P_E = \sum_{i=0}^{K_s} \binom{K_s}{i} \delta^i (1-\delta)^{K_s - i}[1 - \overline{P_c}(i)] = \tag{14}$$

$$= 1 - \sum_{i=0}^{K_s} \binom{K_s}{i} \delta^i (1-\delta)^{K_s - i}\overline{P_c}(i) = \tag{15}$$

$$= 1 - Pr(\text{unconditional correct packet reception})$$

Based on the above, the resulting throughput is derived by multiplying the unconditional packet correct reception probability by the average population

$$E\{K\} = K_s\delta \tag{16}$$

and furthermore by a normalization factor which will account for the fact that our scheme has made use of increased bandwidth due to modulation, time and frequency hopping and coding.

Hence, the normalized throughput takes the form

$$S = K_s \times \delta \times \frac{k}{n} \times \frac{1}{N \cdot q} \times \frac{log_2 M}{M} \times (1 - P_E) \tag{17}$$

for the case of M-ary FSK modulation coupled with non-coherent demodulation and the form

$$S = K_s \times \delta \times \frac{k}{n} \times \frac{1}{N \cdot q} \times (1 - P_E) \tag{18}$$

for the case of $BPSK$ modulation coupled with coherent demodulation.

4.2. Infinite Population

This population is modeled by the Poisson distribution with parameter G where G is the average population size. Then

$$Pr(K \text{ active users in the system}) = \frac{e^{-G} G^K}{K!} \tag{19}$$

and hence the unconditional packet error probability may be rewritten as

$$P_p = \sum_{i=0}^{\infty} \frac{e^{-G} G^i}{i!} [1 - \overline{P_c}(i)] = 1 - \sum_{i=0}^{\infty} \frac{e^{-G} G^i}{i!} \overline{P_c}(i) = \qquad (20), (21)$$

$$= 1 - Pr(\text{unconditional correct packet reception})$$

Based on the above, the resulting throughput is derived by multiplying the unconditional packet correct reception probability by the average population

$$E\{K\} = G \tag{22}$$

and furthermore by a normalization factor which will account as in the previous case for the fact that our signalling scheme has made use of increased bandwidth due to modulation, time and frequency hopping and coding.

Hence, the unnormalized throughput takes the form

$$S = G \times \frac{k}{n} \times \frac{1}{N \cdot q} \times \frac{log_2 M}{M} \times (1 - P_E) \tag{23}$$

for the case of M-ary FSK modulation coupled with non-coherent demodulation and the form

$$S = G \times \frac{k}{n} \times \frac{1}{N \cdot q} \times (1 - P_E) \tag{24}$$

for the case of $BPSK$ modulation coupled with coherent modulation.

5. Hybrid (DS-SFH) vs. Purely Frequency Hopped SSMA Systems

There is no established procedure that enables a standard comparison between the two systems. A starting point should evidently be, to impose the constraint that both sytsems consume the same amount of bandwidth. That is, in our case, to require that the number of hopping frequencies of the Purely Frequency-Hopped System under consideration, would equal the product of the number of hopping frequencies times the number of chips per bit in the signature sequence of the Hybrid system. While an upper bound on the performance of Purely FH systems has been used in the past, as for instance in [2], a more realistic approximation can be obtained, by considering this class of systems as a subcase of the class of the Hybrid systems that we have been analyzing so far. Evidently, for this subclass, suffices to set the number of chips per bit in the signature sequence of the Hybrid systems to one, that is degenerate the DS part to yield a Purely FH system. Then it is easy to witness that the upper bound [2] can be derived from our expression (1), by setting the conditional error probability

$$P_a(k) = \begin{cases} 1 & k \neq 0 \\ 0 & k = 0 \end{cases} \tag{25}$$

That is only when no interferers at the frequency hopped level are present, we are to receive correctly the information sequence for the opposite case, we declare the received sequence totally mistaken. Clearly, our approximations are more realistic since they readily acknowledge the fact that even if potential interference does exist at the FH level, there is still some distinct possibility to receive correctly any portion of the information. In our case

$$0 < P_a(k) < 1 \qquad \text{all} \qquad k$$

since even in the case where multiple-access interference is totally absent, the conditional probability of error is greater than zero due to the AWGN channel assumption.

Now, since the codeword error probability is always an increasing function of the symbol error probability, when bounded distance decoding is employed, suffices to compare the symbol performance between the two systems. Then, the conclusions drawn, can only be amplified under the effect of coding. In the context of the comparison that we are seeking to perform, it would be of great interest to include the Purely DSM SSMA system as analyzed and approximated in [8] or [7].

Thus, we are essentially attempting to compare expressions of the form (1) coupled occasionally with (2),(3),(4) or (5) and (2),(3),(4) and (5) alone, under the equal bandwidth consumption constraint. That is, systems with N_{ds}, N_n, q_n, q_{fh} such that

$$N_{ds} = N_n q_n = q_{fh} \tag{26}$$

where

q is the number of hopping frequencies

N is the number of chips per bit in the signature sequence

and

ds stands for Purely DS

h stands for Hybrid (DS-SFH)

fh stands for Purely FH systems

Although analytically it has been impossible to draw some valid conclusions, due to the high degree of complexity that those expressions demonstrate, many interesting observations where made when a numerical evaluation has been attempted.

At this point, it is important to underline the fact that the observed behavior has been completely uniform, irrespectable of the choice of the system parameters. The only effect that a different choice would have, would be to amplify or weaken the overall behavior but never to reverse its nature.

Namely, the Purely DS system has been observed to perform better than the Hybrid, which in its merit has been observed to outperform the Purely FH system. Both these observations are valid as long as the number of potential interferers is relatively small. As this number is reversed, thus making the Purely FH system more advantageous and the Hybrid in between the critical number of interferers K_c, where the situation is reversed, is strongly a function of the system parameters, that is not only the spread-spectrum related parameters but also the modulation-demodulation parameters as well.

Alternatively, this result yields that for a prespecified sufficiently low, symbol error probability, the maximum number of simultaneously transmitted signals in the vicinity of a particular receiver (user tolerance) is substantially in favor of systems whose DS part is accordingly stengthened, in direct agreement with results published in [4] and [5].

As a consequence of the fact that the above observed advantage occurs to substantially low symbol (codeword) error probabilities, this advantage cannot be correctly depicted in terms of throughput versus packet (codeword) error probability performance, since this measure is directly proportional to the probability of *correct* reception which is very close to unity in this case.

Finally, a comparison with the upper bound [1], is surely enough in favor of the Hybrid system, but for reasons we already stated at the beginning of this section, we do not consider worthy mentioning in this work.

6. Application to a Recursive Retransmission Controlled Protocol

In this section we are considering the employment of our Hybrid (DS-SFH) Spread Spectrum signalling scheme in a packet slotted network environment.

In this environment, many remote users compete for the use of a common resource, that is accessing the broadcast channel. If two or more users, each transmit an information packet during the same slot, the packets 'collide' and the transmission is unsuccessful. Such packets join the backlog of packets which must be rebroadcast at a later time. During each time slot each backlogged user must decide in a stochastic fashion to retransmit the backlogged packet or not. The decision mechanism has to be decentralized.

The motivation behind employing our signalling scheme in the context of the above network environment, lies on the fact that this scheme has a considerably greater multi-access capability and yields a better normalized throughput when compared to the conventional signalling schemes that are employed in network environments. Moreover, it is more powerful even than conventional spread-spectrum schemes as we have pointed out in the previous section.

In the discussion that follows we shall focus only on infinite population models generating traffic that can be described by the poisson distribution. We shall assume that the reader is familiar with the description of the protocol in [1] and thus in the outset of our discussion we need only to emphasize the differences between the Purely FH and the Hybrid systems, when both are employed in the environment under consideration. In the purely frequency-hopped scheme, as discussed in [1], a frequency-slot occupation event is defined as the event that takes place when *one or more* users choose a certain frequency simultaneously, to hop their packets in the channel.

Nevertheless, as we have already demonstrated, our system is particularly strong, in terms of packet error probability, even when a certain number of packet transmissions takes place, because the direct-sequence part of it, can overcome successfully the multi-access interference involved.

Therefore, in order to fully exploit the capability of our system a redefinition of the frequency-slot occupation event is necessary. Since our system can sustain a number of simultaneous packet transmissions and still yield a considerably low probability of packet error, we are as flexible as to declare a frequency slot occupied, only when the number of simultaneous transmissions equals or exceeds a prespecified threshold K_{TH}.

In other words, when less than K_{TH} simultaneous transmissions are attempted, our performance measures show, that still we may count upon the reliability of our system as far as correct packet reception is concerned.

In order to determine the threshold K_{TH}, we need to compute the largest K such that

$$Pr(\text{symbol error probability} \mid K \text{ simultaneous transmission})\Big|_{K=K_{TH}} \leq P \quad (27)$$

where P is a prespecified symbol error probability.

In other words, the threshold is computed by fulfilling the constraint that the conditional symbol error probability is small enough to make our system reliable.

This conditional symbol error probability is given by either of (2), (3), (4) or (5) of Section 3.

Also, the prespecified symbol error probability corresponds, naturally to a packet error probability via (1), (7) and (6) of Section 3, when all the parameters of a certain system are fully specified. Hence, although our constraint originates from a symbol performance measure, definitely satisfies an implicit perpformance requirement at the packet level, since the conditional packet error probability is strictly increasing with the conditional symbol error probability. At this particular point, a redefinition of the 'packet collision' event seems apropriate. Following the considerations exposed in the previous paragraph, it seems natural to declare a packet transmission unsuccessful if and only if it coincides with other K_{TH} packet transmissions in the same time slot the reason still being that our system, as opposed to others, can sustain K_{TH} simultaneous transmission yielding a considerably lower packet error probability. The prespecified error probabilities that will yield both thresholds, need not be the same and are certainly subject to more general design considerations.

Under those considerations, the purely frequency hopped case can be viewed as a special case of the above model with $K_{TH} = 1$.

We shall now determine the probability P that a frequency slot is occupied given that the overall incoming traffic is poisson with mean G.

This probability may be computed both for dwell-time synchronous and also asynchronous systems, the latter being one of our initial assumptions.

For both cases we shall make use of the total probability rule, which may be written in the form

$$P = \sum_{\text{all } K} Pr(\text{frequency slot occupation, } K \text{ total transmission}) = \tag{28}$$

$$= \sum_{\text{all } K} Pr(K \text{ total transmissions}) \times$$

$$\times \, Pr(\text{frequency slot occupation} \mid K \text{ total transmissions}) \tag{29}$$

where the hopping frequency selection for each user, during each frequency slot, is assumed to be random, uniformly distributed among the alphabet of q available hopping frequencies, this assumption being valid almost throughout this work.

Now observe that following our definition of the occupation event

$$Pr(\text{frequency slot occupation} \mid K \text{ total transmissions})$$

is zero if $K < K_{TH}$ that is if the total number of transmission is less than the threshold computed such as to fulfill the requirement for low packet error probability.

Hence, our summing of (28) or equivalently (29) actually involves terms for $K \geq K_{TH}$.

Since the overall incoming traffic is poisson with mean G

$$Pr(K \text{ total transmissions}) = \frac{e^{-G} G^K}{K!} \tag{30}$$

Moreover, since during each frequency slot, each user randomly selects one out of q hopping frequencies to hop part of his packet around the channel with probability $\frac{1}{q}$

$$Pr(\text{frequency slot occupation} \mid K \text{ total transmissions}) =$$

$$= \sum_{i=0}^{K} \binom{K}{i} \left(\frac{1}{q}\right)^{i} \left(1 - \frac{1}{q}\right)^{K-i} \tag{31}$$

and hence substituting (31) and (30) into (29) and wiping out the first $K_{TH} - 1$ terms, we get that:

$$P = \sum_{\text{all } K: K \geq K_{TH}} \frac{e^{-G} G^K}{K!} \sum_{i=0}^{K} \binom{K}{i} \left(\frac{1}{q}\right)^{i} \left(1 - \frac{1}{q}\right)^{K-i} = \tag{32}$$

$$= \sum_{K=K_{TH}}^{\infty} \frac{e^{-G} G^K}{K!} \sum_{i=K_{TH}}^{K} \binom{K}{i} \left(\frac{1}{q}\right)^{i} \left(1 - \frac{1}{q}\right)^{K-i} = \tag{33}$$

$$= \sum_{K=K_{TH}}^{\infty} \frac{e^{-G} G^K}{K!} \left[1 - \sum_{i=0}^{K_{TH-1}} \binom{K}{i} \left(\frac{1}{q}\right)^{i} \left(1 - \frac{1}{q}\right)^{K-i}\right] \tag{34a}$$

$$= \sum_{K=K_{TH-1}}^{\infty} \frac{e^{-G} G^K}{K!} \left[1 - \sum_{i=0}^{K_{TH-1}} \binom{K}{i} \left(\frac{1}{q}\right)^{i} \left(1 - \frac{1}{q}\right)^{K-i}\right] \tag{34b}$$

As we have already readily abserved, the purely frequency hopping case is a subcase of the above, for which $K_{TH} = 1$, that is a *single* transmission suffices to occupy one of q frequencies during each dwell time.

In this case (34) yields a frequency occupation probability P_{fo}:

$$P_{fo} = 1 - \sum_{K=0}^{\infty} \frac{e^{-G} G^K}{K!} \left(1 - \frac{1}{q}\right)^{K} =$$

$$= 1 - e^{-G} \sum_{K=0}^{\infty} \frac{\left(G - \frac{G}{q}\right)^{K}}{k!} =$$

$$= 1 - e^{-\frac{G}{q}} \tag{35}$$

or equivalently

$$P_{fo} = 1 - e^{-\lambda} \tag{36}$$

Now, focusing on the dwell time asynchronous model, we observe that random selection of one out of q hopping frequencies, performed at each dwell time, will actually occupy two consecutive frequency slots, (31) thus being modified to:

$$Pr(\text{frequency slot occupation} \mid K \text{ total transmissions}) =$$

$$= \sum_{i=0}^{K} \binom{K}{i} \left(\frac{2}{q} - \frac{1}{q^2}\right)^i \left[\left(1 - \frac{1}{q}\right)^2\right]^{K-i} \tag{37}$$

and hence employing the total probability law for this case yields the result for the probability of a frequency slot occupation:

$$P = \sum_{K=K_{TH}}^{\infty} \frac{e^{-G}G^K}{K!} \sum_{i=K_{TH}}^{K} \binom{K}{i} \left(\frac{2}{q} - \frac{1}{q^2}\right)^i \left[\left(1 - \frac{1}{q}\right)^2\right]^{K-i} = \tag{38}$$

$$= \sum_{K=K_{TH}}^{\infty} \frac{e^{-G}G^K}{K!} \left\{1 - \sum_{i=0}^{K_{TH-1}} \binom{K}{i} \left(\frac{2}{q} - \frac{1}{q^2}\right)^i \left[\left(1 - \frac{1}{q}\right)^2\right]^{K-i}\right\} \tag{39a}$$

$$= \sum_{K=K_{TH-1}}^{\infty} \frac{e^{-G}G^K}{K!} \left\{1 - \sum_{i=0}^{K_{TH-1}} \binom{K}{i} \left(\frac{2}{q} - \frac{1}{q^2}\right)^i \left[\left(1 - \frac{1}{q}\right)^2\right]^{K-i}\right\} \tag{39b}$$

Still, the frequency hopping case is a subcase of the above, for which $K_{TH} = 1$.

In this case (39) yields a frequency occupation probability P_{fo}:

$$P_{fo} = 1 - \sum_{K=0}^{\infty} \frac{e^{-G}G^K}{K!} \left[\left(1 - \frac{1}{q}\right)^2\right]^K =$$

$$= 1 - e^{-G} \sum_{K=0}^{\infty} \left(\frac{G - \frac{2G}{q} + \frac{G}{q^2}}{K!}\right)^K =$$

$$= 1 - e^{\left(-\frac{2G}{q} + \frac{G}{q^2}\right)} \tag{40}$$

or equivalently using (18)

$$P_{fo} = 1 - e^{(-2\lambda + \frac{\lambda}{q})} \qquad (41)$$

When q is sufficiently large, the term $\frac{\lambda}{q}$ may be neglected and hence an approximation to the probability of frequently slot occupation, in this case, may well be:

$$P_{fo} = 1 - e^{-2\lambda} \qquad (42)$$

in agreement with the expression computed in [1] for this quantity.

Let us focus finally, on cases where the total packet incoming traffic is less or equal to the maximum attainable throughput for a given system, employing Hybrid signalling, or equivalently less than the maximum attainable throughput for an 'equivalent' Purely FH system, as has been observed in all cases throughout this work. Let us also set both thresholds involved in the 'collision' and the 'frequency slot occupation' events, to zero. In this 'worst case' operating environment for our system, a stability proof has already been demonstrated in [1] following the procedures set in [10]. That is our system, when employed in the context of the network environment already discussed, following the rules of the recursive retransmission controlled protocol, is proved to be stable, conditioned on the most unfavorable operating situation.

7. Numerical results and conclusions

The maximum number of simultaneous transmissions, in the vicinity of a particular receiver, under the constraint that the coded symbol error probability does not rise beyond a preset value, is tabulated in Tables 1,2 and 3, for varying parameters of the systems, such as the number of chips per symbol, number of hopping frequencies, number of modulating frequencies and so on.

In Table 1, we present the maximum numbers of asynchronous users that can be supported by the hybrid DS-SFH/SSMA spread-spectrum system, for different values of the error probability, both for the errors -only and the combined errors

-and -erasures decoding. A variety of combinations of values of chips per bit versus hopping frequencies are shown, however maintaining the overall bandwith expansion constant. Also shown, are selected values of the threshold probabilities and the corresponding threshold values of the interferers, which in turn determine the decision level for the erasures operation at the decoder. The modulation scheme is BPSK, coherent demodulation is employed and a RS(32,16) code has been used.

Clearly, the errors -only decoding outperforms the errors -and -erasures decoding in all but the purely frequency -hopping schemes. On the other extreme, the purely direct-sequence scheme yields the maximum multi-user capability in all cases. Also, we may observe the decreasing behaviour of the errors and erasures scheme along with the decrease of interferers threshold when the rest of the parameters remain constant.

In Table 2, the same structure has been followed for comparison purposes. 32-ary FSK modulation coupled with noncoherent demodulation has been chosen. The results are consistent with the ones discussed above. Nevertheless, some significant improvement is present, when compared to the previous case, due to the use of larger bandwith.

In this table, users are assumed to be asynchronous, while in the following Table 3, they are assumed to be synchronous. Nevertheless, the same modulation and demodulation schemes, as the ones in the previous tables, are maintained.

In Figures 2-5 selected numerical results are presented demonstrating the normalized throughput versus the packet error probability tradeoff of our systems.

In Figure 2, systems with 50 hopping frequencies and 14 chips per M-ary symbol are considered, while MFSK modulation coupled with non-coherent demodulation is incorporated. For a finite population of 1000 users and for various coding rates, our results show that high coding rates perform better than lower ones. However, the most important observation to be made at this end, is that maximum throughput is *not* attained with the parameters employed although throughput figures have been computed up to a packet transmitting duty cycle of one. The reason for this, is that those specific parameters make our system far more powerful than the environment requires and being such, it is not as much effective as it should. To demonstrate this more clearly, let us compare directly with Figure 5, where the AWGN and multi-user environment has been assumed to remain as in the previous case. The same holds for the modulation scheme and the parameters involved therein. However, observe that our scheme is four times more effective in terms of bandwith, by using half as many hopping frequencies and chips per symbol. Throughput figures now clearly span the whole packet error probability domain, including the region yielding the maximum normalized throughput values. Hence, although for a given duty cycle it is possible that the former and more powerful system might yield a higher throughput, it is the latter system that can attain the maximum throughput by suitable adjustment of the duty cycle. Obviously, in the second case, the selection of the spreading parameters are better tailored to fit the needs of the environment. As before, the general rule that higher rate codes are the most effective, still holds.

Let us pick now another pair of figures, in order to focus on a different point. Figures 4 and 5 show the performance of two identical in every parameter systems, when employed in different traffic environments. For the Binomial traffic, a finite population of 1000 users is asssumed, while MFSK modulation coupled with non-coherent demodulation is considered in both cases. The comparison demonstrates that throughput curves are almost identical in direct agreement with the fact that a large Binomial traffic environment asymptotically approaches that of a Poisson traffic. In both figures, higher rate codes still outperform the lower rate ones.

While in all the numerical results presented so far, one M-ary symbol per RS symbol has been employed, in Figure 3, results involving BPSK modulation coupled with coherent demodulation, based on the assumption of the independence of interference hitting each of the m bits within the same RS symbol, are presented. Also, an infinite population is considered. In this case, although it is true that higher rate codes yield higher *maximum* throughput, this is not uniformly true over the whole packet error probabilities domain. Therefore, the code rate is more of a design parameter here, than in the previous cases and depends strongly on the average size of the population considered.

A more general remark, valid for all cases examined so far, is that larger bandwith consuming schemes perform *uniformly* better than lower ones, even taking into account that our performance measures are already being penalized appropriately.

Finally, we proceed to discuss results presented in Figure 6, where packet error probabilities are plotted versus average population size of a Poisson traffic, for different Spread-Spectrum schemes. *Equivalent* schemes, in the sense outlined in Section 5, are considered, ranging from Purely Frequency Hopped to Purely Direct Sequence. As already mentioned, as long as the number of potential interferers is below some critical point, systems with their DS part more strengthened, perform better than those with a dominant FH part. A moderately Hybrid (DS-SFH) SS scheme, is in between the two extremes. After the critical point, which is strongly a function of the system parameters, the situation is reversed, the Hybrid scheme remaining always bounded, this time with the reverse order.

Overall, our schemes yield high throughput figures, over a wide range of packet error probabilities, and fit particularly in applications at which high reliability with uncompromised performance are required. They outperform globally all narrowband packet slotted schemes and locally the other spreading alternatives. Also, especially when coding is considered, they are mostly attractive, since they can be more efficiently interleaved than DS schemes and have a larger multiple-access capability than FH schemes.

Table 1

Maximum number of asynchronous users that can be supported by a hybrid DS-FH/SSMA employing BPSK with coherent demodulation and RS(32,16) coding

(N_b =100, E_b/N_0 =12dB)

K_{TH}	q	N	P_{TH}	$P_e = 10^{-3}$		$P_e = 10^{-5}$	
				Errors	Erasures	Errors	Erasures
0	700	1	<0.1	76	191	146	125
∞	700	1	1	276	276	146	146
1	350	2	0.1	304	230	174	63
3	100	7	0.1	343	208	227	48
1	100	7	0.01	343	90	227	30
0	100	7	0.001	343	28	227	18
0	50	14	0.001	351	14	241	9
1	25	28	0.001	355	23	248	7
0	25	28	10^{-4}	355	7	248	0
0	10	70	10^{-4}	357	3	260	0
0	1	700	10^{-5}	376	0	271	0
0	1	700	$<10^{-4}$	376	0	271	0
35	1	700	10^{-2}	376	34	271	21

Table 2

Maximum number of asynchronous users that can be supported by
a hybrid DS-FH/SSMA employing 32-ary MFSK
with noncoherent demodulation and RS(32,16) coding
($N_s = 1$, $E_b/N_0 = 12dB$, $P_{e,s} \leq 10^{-3}$)

K_{TH}	q	N	P_{TH}	Errors	Erasures
0	700	1	0.8	1931	201
1	100	7	0.8	2128	96
9	100	7	0.9	2128	69
0	50	14	0.5	2145	15
1	50	14	0.7	2145	49
3	50	14	0.8	2145	127
1	25	28	0.6	2154	25
7	25	28	0.8	2154	150
0	10	70	0.1	2180	3

Table 3

Maximum number of synchronous users that can be supported by

a hybrid DS-FH/SSMA employing 32-ary MFSK

with noncoherent demodulation and RS(32,16) coding

($N_s = 1$, $E_b/N_0 = 12dB$, $P_{e,s} \leq 10^{-3}$)

K_{TH}	q	N	P_{TH}	Errors	Erasures
0	700	1	0.5	1503	101
1	700	1	0.9	1503	336
0	100	7	0.1	1605	15
0	50	14	0.1	1621	8
0	25	28	0.1	1630	4
0	10	70	0.1	1675	0

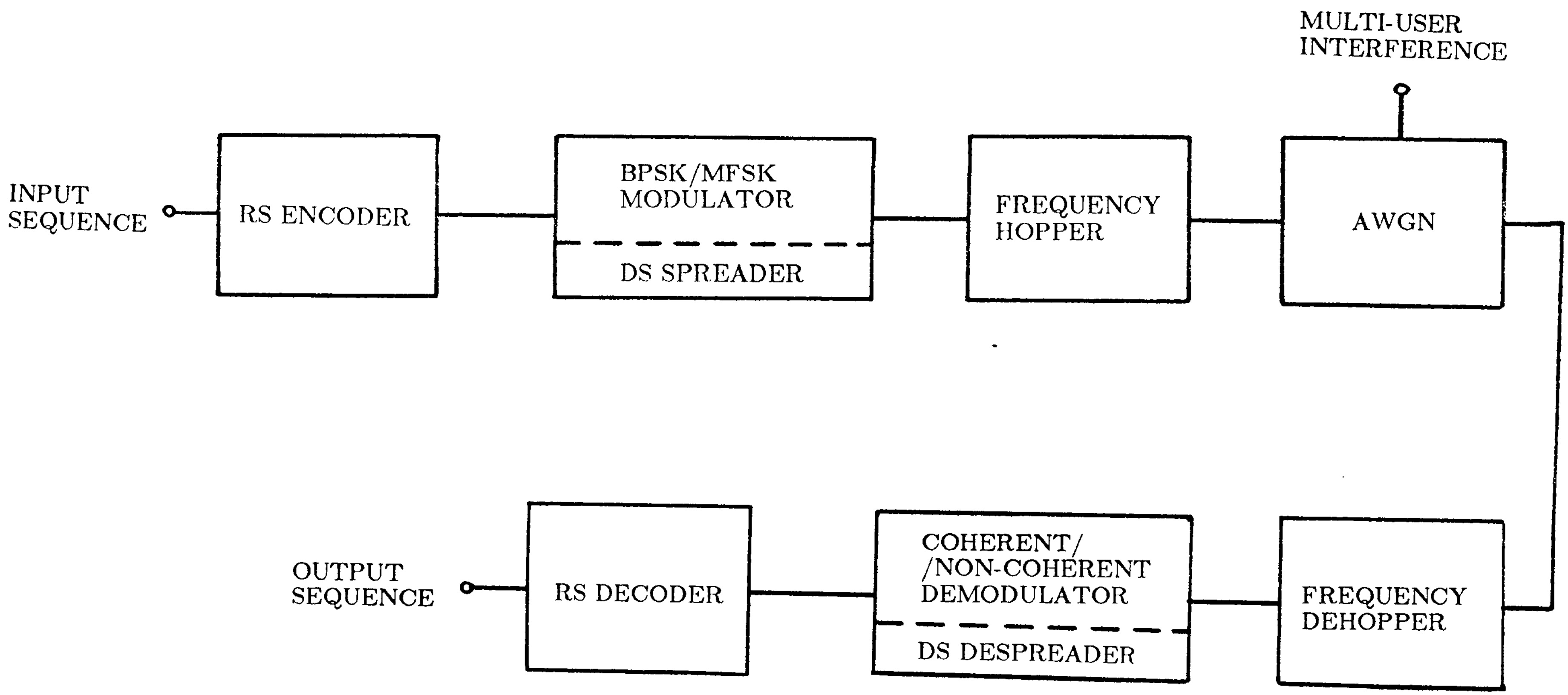

Figure 1. Simplified block diagram of a communication link employing hybrid DS-FH/SS signalling coupled with Reed-Solomon forward error-control coding.

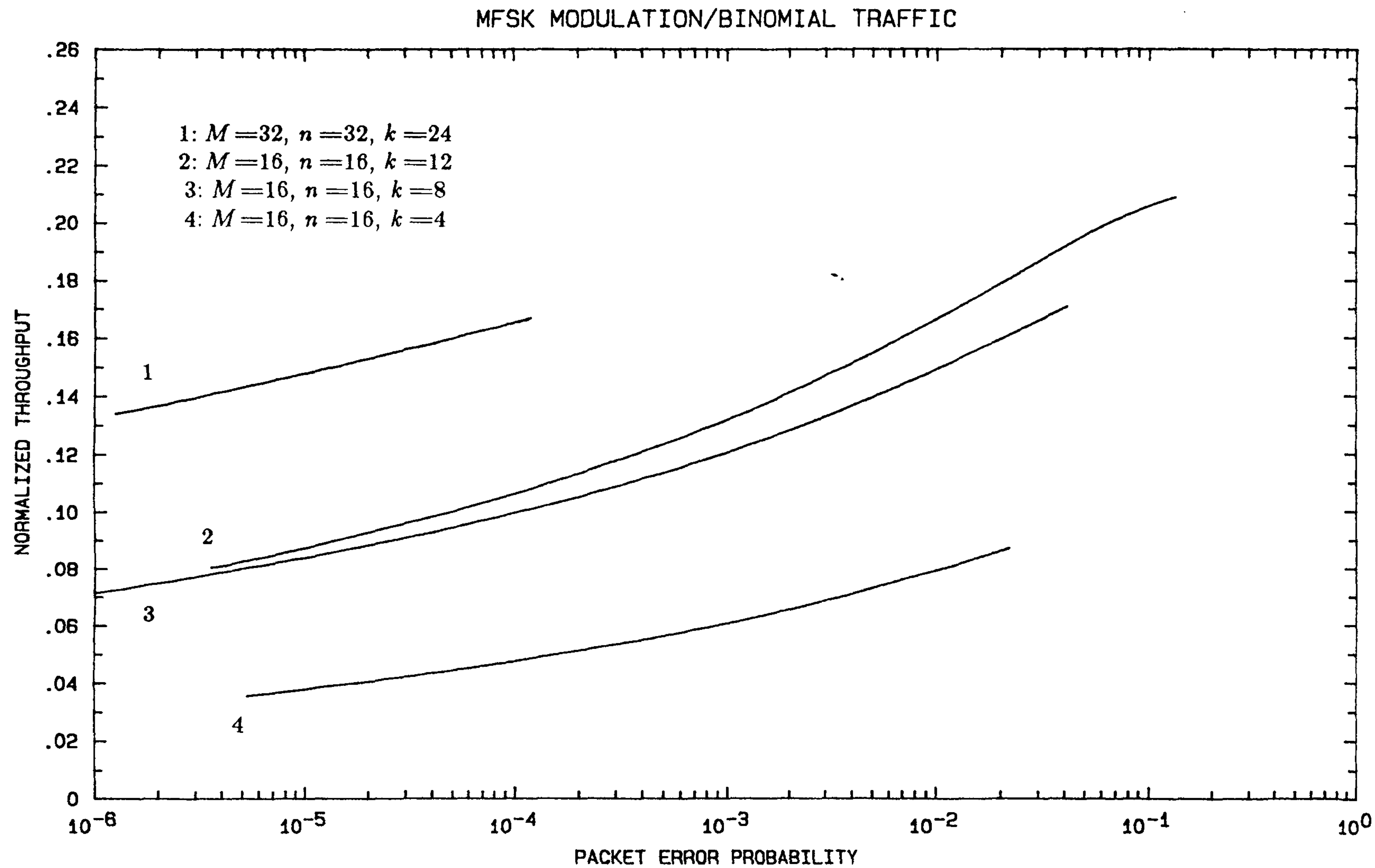

Figure 2. Normalized throughput versus packet error probability for finite population employing hybrid DS-FH/SS random access asynchronous systems with RS coding, errors-only decoding and *MFSK* modulation with non-coherent demodulation.
(q =50, N =14, m =1, $AWGN$, $E_b/N_0 = 12\ dB$, $K_S = 1000$ *users*)

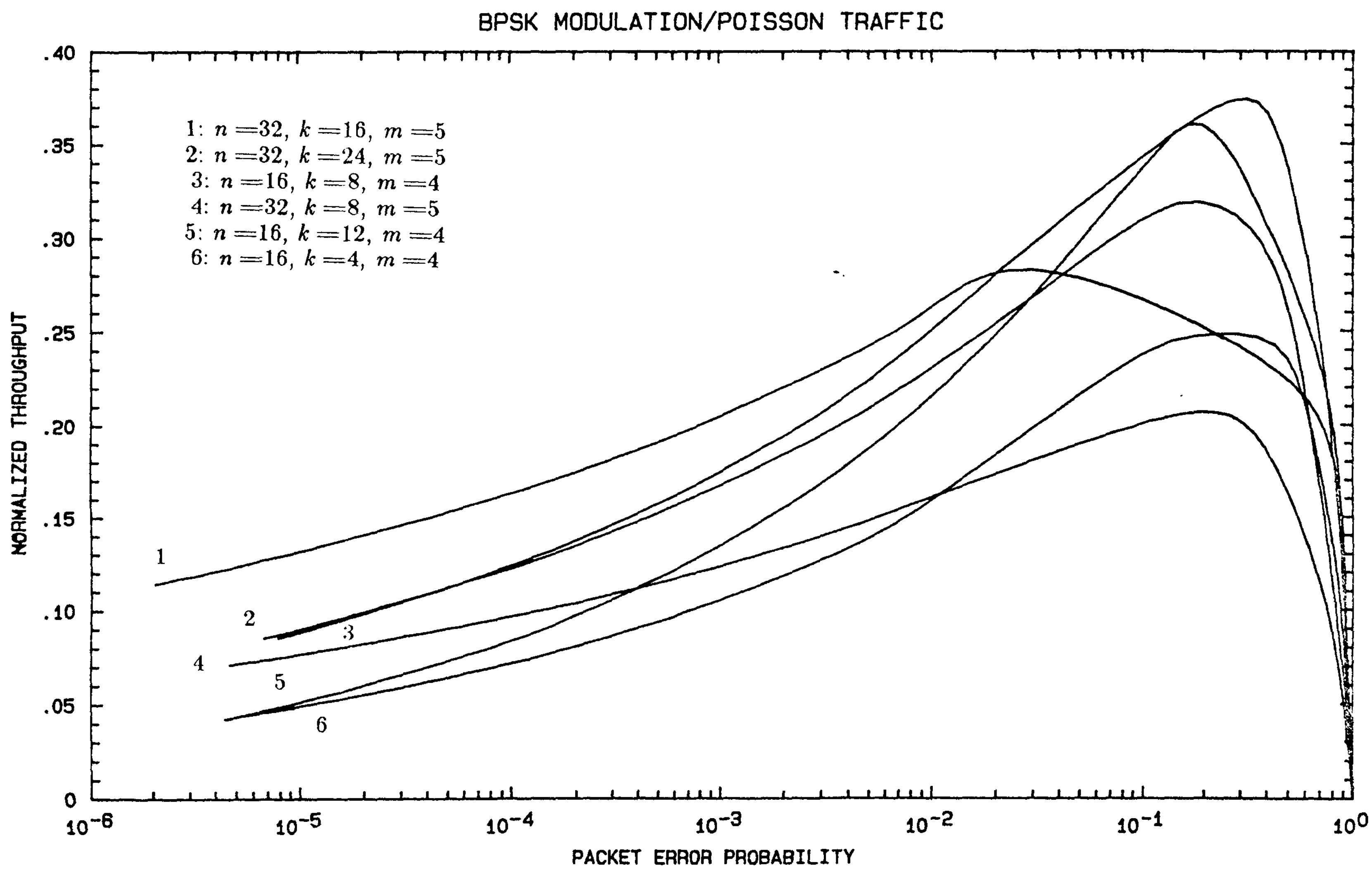

Figure 3. Normalized throughput versus packet error probability for infinite population employing hybrid DS-FH/SS random access asynchronous systems with RS coding, errors-only decoding and *BPSK* modulation with coherent demodulation.
($q = 25$, $N = 7$, $N_b = 100$, $AWGN$, $E_b/N_0 = 12\ dB$)

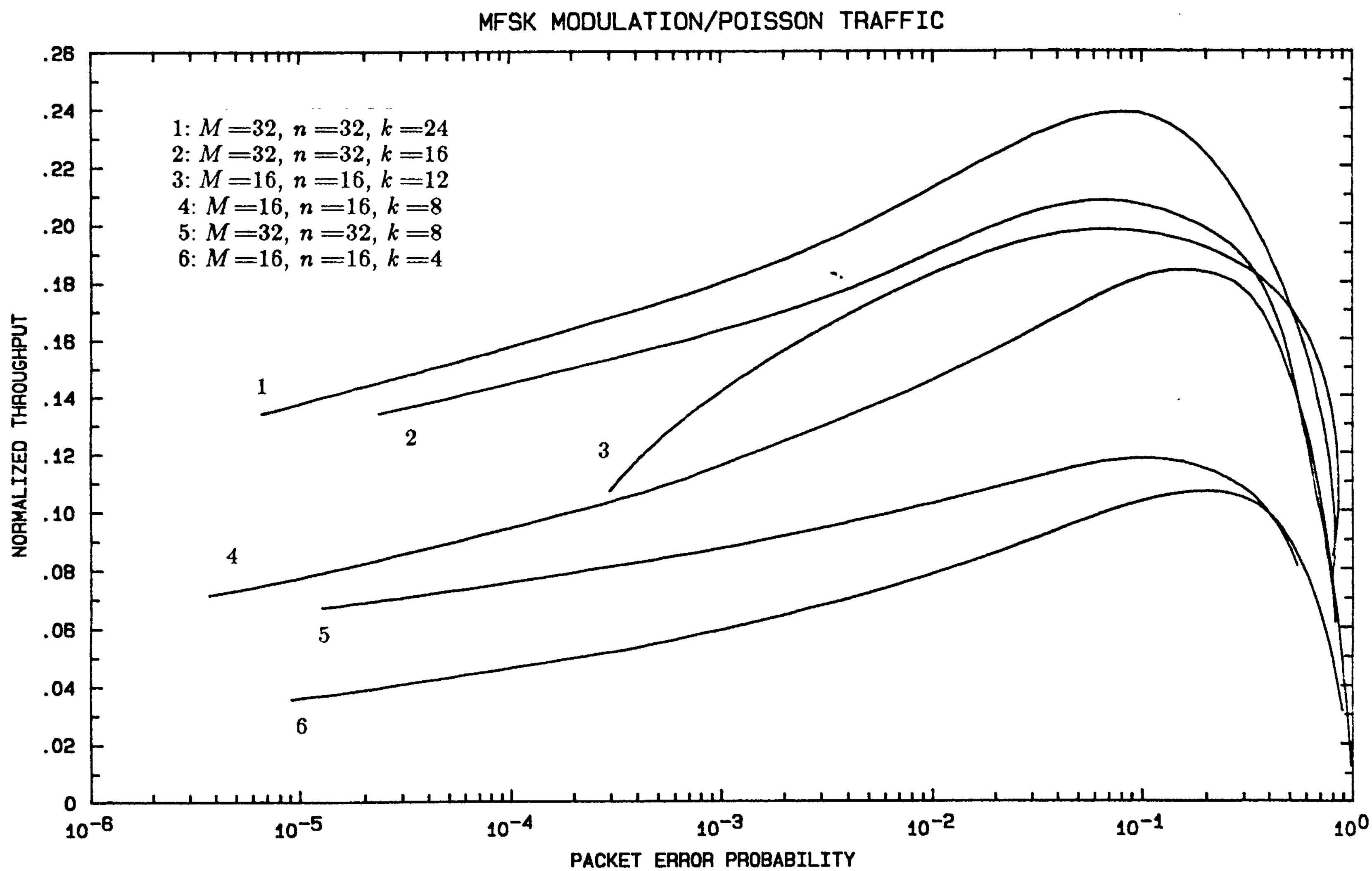

Figure 4. Normalized throughput versus packet error probability for infinite population employing hybrid DS-FH/SS random access asynchronous systems with RS coding, errors-only decoding and *MFSK* modulation with non-coherent demodulation. ($q=25$, $N=7$, $m=1$, $AWGN$, $E_b/N_0 = 12\ dB$)

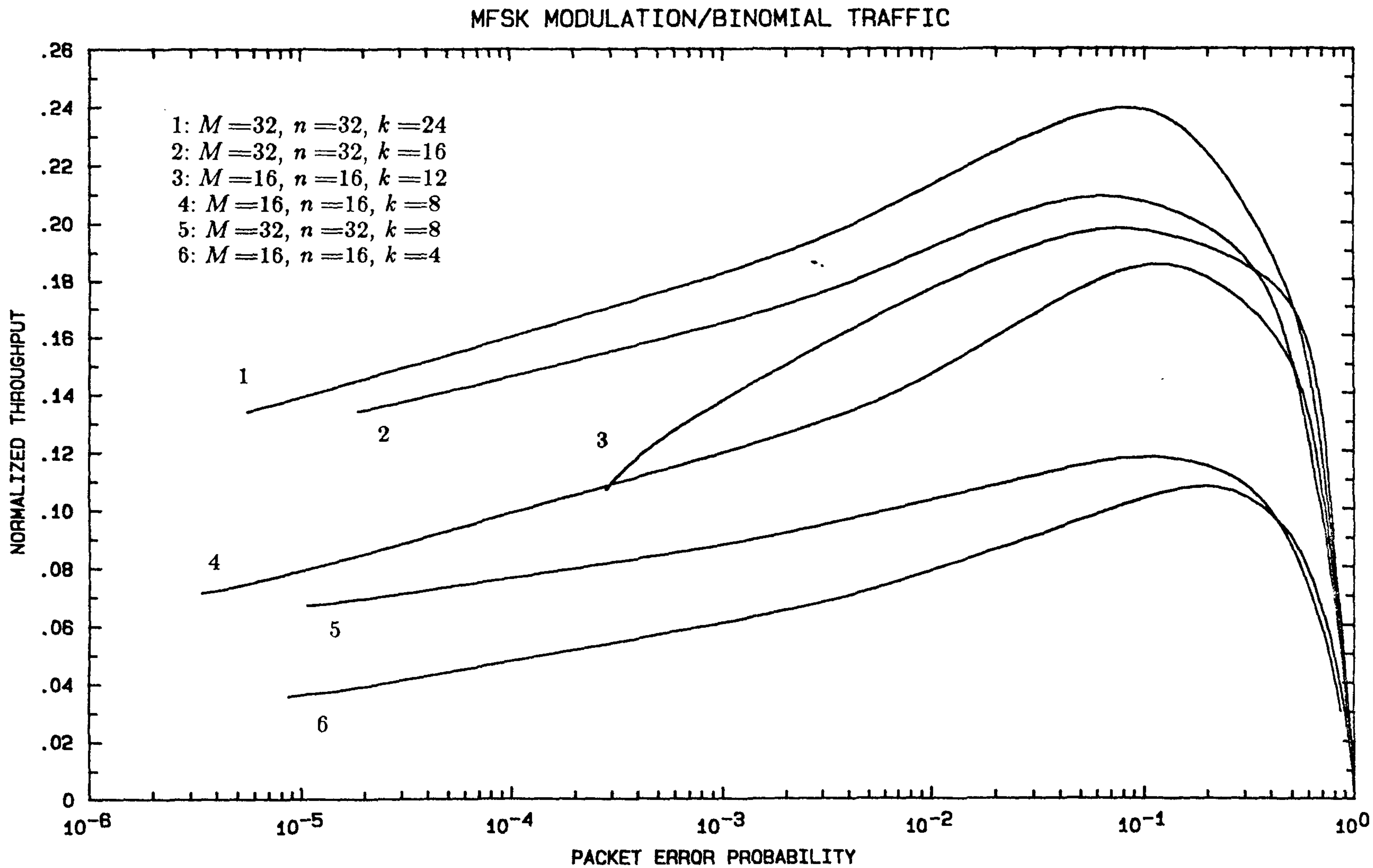

Figure 5 Normalized throughput versus packet error probability for finite population employing hybrid DS-FH/SS random access asynchronous systems with RS coding, errors-only decoding and *MFSK* modulation with non-coherent demodulation.
(q =25, N =7, m =1, $AWGN$, $E_b/N_0 = 12\ dB$, $K_S = 1000$ *users*)

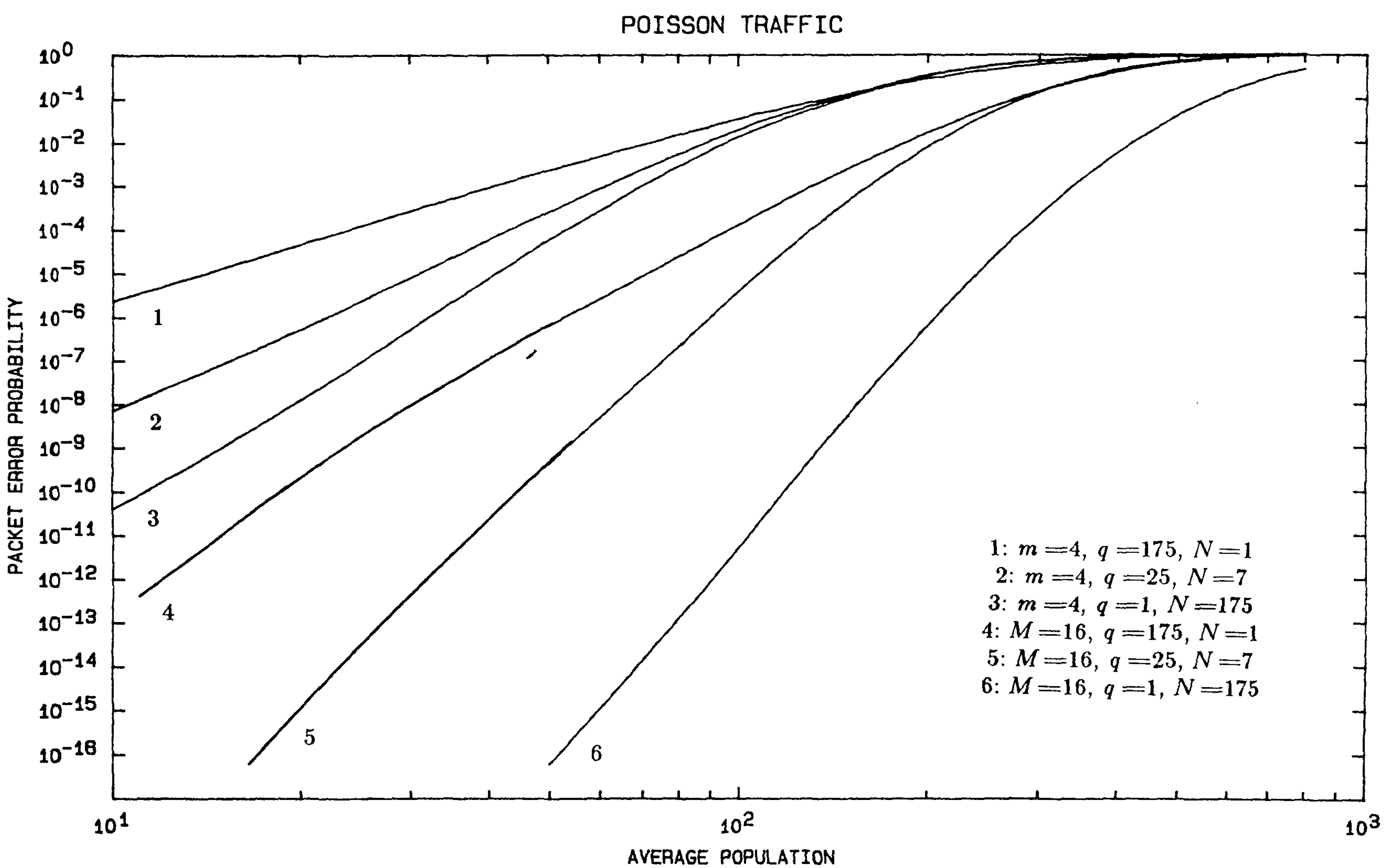

Figure 6. Packet error probabilities versus average population for various spread-spectrum random-access schemes employing RS coding and errors-only decoding.
($n=16$, $k=8$, $AWGN$, $E_b/N_0 = 12\ dB$)

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