ABSTRACT

Title of Dissertation: A GENERAL MODEL OF BARRIER ISLAND EROSION MANAGEMENT—WITH APPLICATION TO OPTIMAL RESPONSE UNDER SEA LEVEL RISE.

Craig Elliott Landry, Doctorate in Agricultural and Resource Economics, 2004

Dissertation Directed By: Professor Kenneth E. McConnell, Department of Agricultural and Resource Economics

This dissertation lays out a conceptual model for managing beach erosion on barrier islands. Households affected by erosion management are identified as beach visitors and coastal homeowners. The returns from beach quality accruing to beach visitors are assessed via travel cost theory, combining revealed preference and contingent behavior data, while the returns from beach quality accruing to coastal homeowners are assessed using hedonic price theory and data from multiple housing markets. An optimal control model is formulated, which takes into account (i) distinct beach user groups, (ii) joint services of beaches (both recreational and loss-mitigating), (iii) active and passive beach management options, (iv) costs of beach maintenance, and (v) the dynamic motion of beach quality. Optimality conditions define efficient beach nourishment operations, as well as the optimal terminal time for active management (i.e. beach nourishment) on barrier island beaches. Empirical results illustrate the optimal beach width for a particular site and the schedule of nourishment operations detailing the amount of sand to be placed on the beach in each time period.

The analysis presents estimates of the terminal time of active management for a particular site, and how the terminal time varies with (i) the rate of sea level rise, (ii) the value of threatened coastal property, and (iii) the magnitude of fixed beach nourishment costs.

A GENERAL MODEL OF BARRIER ISLAND EROSION MANAGEMENT—WITH APPLICATION TO OPTIMAL RESPONSE UNDER SEA LEVEL RISE

By

Craig Elliott Landry

Thesis or Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctorate of Agricultural
and Resource Economics
2004

Advisory Committee:
Professor Kenneth E. McConnell, Chair
Professor Nancy Bockstael
Professor Maureen Cropper
Professor John Horowitz
Professor Lars Olson

Dedication

This work is dedicated to my parents—Jo Ann and Rick—and my beloved wife, Kristen.

Acknowledgements

I express sincere gratitude to members of the Department of Agricultural and Resource Economics at the University of Maryland, College Park for tutelage and inspiration in my tenure as a nascent resource economist at the University of Maryland. I am especially grateful to Dr. Ted McConnell for his guidance in the preparation of my not only my dissertation, but also myself as a research scientist. His insights have been of vital importance in my progress. The numerous criticisms and comments of Dr. Nancy Bockstael have been invaluable to me. She is truly an asset to the Department and the discipline of environmental and resource economics. Dr. Lars Olson is responsible for teaching me the intricacies of dynamic modeling, and was instrumental in helping me learn the application of these models by using them in my dissertation work. I must also thank Dr. Marc Nerlove for encouraging me to explore different kinds of software for optimization and econometrics—a background that was immensely helpful during my dissertation research. I also thank Dr. John Horowitz and Dr. Maureen Cropper for serving on my dissertation committee and providing many useful suggestions. Lastly, I express deep gratitude to Dr. John List. Dr. List taught me how to think like a researcher and has shown me how rewarding this engagement can be. Thank you, John.

Table of Contents

List of Tables

List of Figures

Chapter 1: Coastal Erosion Management as an Economic Problem

There is a consensus among scientists that we are currently in a period of eustatic (world-wide) sea level rise[1], with estimates ranging from 30 to 80 centimeters over the next century (Wigley and Raper 1993). Predictions translate a 30-centimeter rise in sea level into an average 30 meters of coastline recession (Klarin and Hershman 1990). That is, 30 centimeters of sea level rise could result in the shoreline retreating 30 meters landward in some areas if nothing is done to stop it. Managing coastal erosion, however, impacts the quality of coastal resources. The economies of many modest-sized coastal towns are dependent upon the appeal of beach resources, and the development of these towns has been tied to the demand for beach recreation and leisure. To date, there has been no comprehensive model for the management of barrier island beaches in response to erosion and sea level rise.

The purpose of this research is to develop a framework for optimal management of barrier island beach resources when the hinterland is developed. The model provides a framework for planning beach erosion control projects under constant sea level and a constant erosion rate (i.e. the *short run*). The framework is also used to examine beach erosion control under sea level rise, with increasing erosion levels (i.e. the *long run*). It is shown that this conceptual framework can identify the time horizon of management responses under sea level rise. One of the main objectives of this research is to explore whether active management

---

[1] See, e.g. Nummedal, Pilkey, and Howard (1987) or Bird (2000).

(specifically, beach nourishment)[2] might be economically justified in the foreseeable future, or if passive management (shoreline retreat—i.e. letting erosion proceed unabated) is more likely to become optimal in the long run. In the event of the latter, this research aims to estimate the timing of a shift in management regimes. The long-term feasibility of beach nourishment as a management strategy depends upon: (i) the rate of sea level rise, (ii) the costs of nourishment, and (iii) the benefits of preserving the current shoreline.

The long-run application of this model differs from conventional approaches to coastal protection by focusing on the stream of services derived from barrier beaches, rather than the value of threatened property at some future time (i.e. the opportunity cost of protection). The received literature that considers coastal protection largely ignores external costs of such schemes. Structural fortification of the shoreline protects threatened property, but imposes external costs on beach resources. The expected value of threatened property is appropriate as a primary decision criterion if the value of beach resources is a small portion of the total economic value associated with a site (e.g. large coastal cities). The approach is less appropriate for most U.S. barrier islands, for which the value of beach resources can be a significant portion of total economic value, not only locally but nationally as well.[3] The methods utilized in this dissertation place beach resources at the focal point of the analysis, and are designed to address the problem of barrier island

---

[2] Active management may also include shoreline armoring—the construction of large-scale protective devices on the shoreline. This research examines only beach nourishment, as this is the more common form of active management. Shoreline armoring is illegal in a number of coastal states (e.g. North Carolina).

[3] Beaches are a leading U.S. tourist destination. Approximately 180 million "person-visits" are taken annually, and tourism in coastal areas accounts for 85 percent of U.S. tourism revenue (Houston 1996).

management in response to short-term erosion problems and the long-term problem of sea level rise.

An analytical model is developed that characterizes the optimal management response to erosion on barrier islands. The model focuses only on the average beach profile, and thus does not consider the distribution of beach quality along the shore. This simplification is made for analytical tractability. The resource problem is essentially loss of beach width, which is considered a deterministic process. Beach erosion is a dynamic process, and thus the model is necessarily dynamic in nature. Optimality conditions are derived and discussed. The model is used to characterize optimal management under constant sea level and a constant erosion rate, as well as under sea level rise and the attending increases in erosion activity. The management tool is beach nourishment—adding sand to the beach.

An empirical application of the model estimates the optimal schedule of beach nourishment operations for a specific coastline in the southeastern U.S., allowing for a corner solution at any point in time (i.e. no beach nourishment). The empirics also examine whether beach nourishment is a tenable management practice in the long run, given assumptions about sea level rise and costs and benefits. A termination of beach nourishment in the long run implies a policy of shoreline retreat, which would entail gradual migration of barrier islands and associated losses in property and infrastructure. A primary goal of this research is estimation of the optimal timing of such a transition. Information on the optimal timeline of shoreline retreat could be instrumental in allowing the market value of threatened properties to properly adjust to the risk of sea level rise[4] and invaluable for coastal planning and investment purposes. This research does not consider the distribution of cost and benefits engendered by beach erosion control (though this is certainly a topic of concern for many parties).

The Coastal Erosion Problem

The coastal environment is one of the most dynamic places on earth. The position and form of the coastline are influenced by the interaction of the ocean, atmosphere, and coastal landforms; the shore attains a dynamic equilibrium, determined by waves, wind, ocean currents, sediment supply, storms, and sea level. As such, the coast has never been a particularly stable environment. This instability is not obvious to the casual observer, however, because the changes are very gradual in some cases, and sporadic in others.

Barrier islands dominate the eastern and gulf coasts of the U.S. Most of these islands exhibit sandy beaches—a common characteristic of the dynamic coastal equilibrium. Wave energy dissipates as waves strike land, and fine sediments can be deposited on the shore. This process gives rise to the sandy beach, an environmental resource often of considerable economic value. Despite the inherent instability of the coastline, a natural beach usually persists, albeit in possibly different forms and locations.

Coastal erosion is the loss of coastal land, resulting in a recession of the shoreline (the demarcation between land and sea). More specifically, beach erosion is the loss of beach area. From hereon, when I refer to beach quality, I mean beach area

[4] As Yohe, Neumann, and Ameden (1995) recognize, the trajectory of threatened property values is largely dependent upon the perception of the likelihood of abandonment.

or beach width. (Beach width is the dimension of beach quality that can be controlled by the coastal planner, which in turn determines beach area.) There are two main causes of erosion on the coast[5]—coastal storms and sea level rise. Beaches can be decimated by energetic waves associated with coastal storms. These storms can move a great deal of beach sand offshore, but most of the sand subsequently returns to the beach in the majority of cases. This oscillating process, in fact, protects land behind the beach from the direct attack of storm waves, but can create erosion problems in the *short run* as the position of the shoreline fluctuates. Oscillation of the shoreline is difficult to predict, but is only a concern on developed coastlines. When the coast is developed, fluctuations in the shoreline can cause extensive property damage. Thus, many developed barrier islands are currently protected against storms by periodic beach renourishment, sometimes in conjunction with shoreline armoring (e.g. seawalls). It should be noted that there is also substantial movement of sand along the shore due to the longshore (littoral) current. This dimension of the problem is essentially distributive in nature, and is not considered herein.

The stochastic oscillation of the shoreline in response to coastal weather patterns is tied to a baseline sea level, which is currently rising at an average of 1 – 2 millimeters per year (Edgerton 1991).[6] With a rise in sea level, undeveloped barrier islands move landward by rolling over, as sand is transported from the ocean to the land side; the beach, being an equilibrium characteristic, will tend to migrate landward over time as the island recedes (Dean and Maurmeyer 1983; Leatherman 1988; Pilkey and Dixon 1996). Developed barrier islands differ from their

[5] These are aside from the potential problem of reduced sediment supply (e.g. damming of rivers).
[6] The relative rate of sea level rise also depends upon the local rate of land subsidence.

undeveloped counterparts in that they are anchored to a specific location by the existence of infrastructure, housing, businesses, etc. Since these islands are not allowed to migrate, they can become increasingly threatened by inundation and increased storm wave heights. The quality of the sandy beach usually suffers under the increased erosive pressure. If the island is not protected by sea walls and such, the beach may migrate landward though the island is not allowed to rollover due to the presence of development. In this respect, it is the hinterland behind the beach that can be lost in the *long run*; houses that end up on the beach are typically condemned. This can create social pressure to stabilize the shoreline as erosive forces escalate.

These facts characterize an especially difficult public policy problem. Beaches are a source of recreational value for households. Development on the coastal fringe facilitates access to beaches and provides for enjoyment of scenic amenities, but limits the shoreline's ability to evolve in response to coastal hazards. Attempts to fortify property with protective devices can further degrade beaches. As such, the sandy beach—an important recreational amenity—may diminish or disappear. Beach nourishment can ameliorate the effects of coastal armor and provides some protection in its own right, but the improvement is most often only temporary. Once the decision is made to armor the coast, beach quality usually suffers.

Optimal coastal management depends upon balancing benefits and costs of remedial actions, incorporating the dynamic effects of management decisions on all relevant services and user groups. In the *short run* (assuming constant sea level and erosive pressure), one can derive an optimal time path of the management variable (beach nourishment). In the *long run*, sea level rise increases erosive pressure on the shoreline and may render some settlements indefensible. The optimal management strategy in the long run depends on the degree of erosive pressure (i.e. sea level rise) and how this affects management costs. Existing literature on coastal protection has focused on the value of coastal property that is threatened by sea level rise. As Yohe, Neumann, and Ameden (1995) point out, with full information on the risks of sea level rise, market depreciation over 30 years time could drive the value of this property to zero. While this outcome is complicated by uncertainty regarding sea level rise and the inherent lack of reliability in a commitment to abandon property, it can clearly be problematic to rely on such a subjective decision criterion for coastal policy making. Also, this line of research has considered shoreline armoring as the primary policy tool, but has largely ignored the effect of this type of coastal protection on beach resources. The methods utilized herein are different in that they focus on the stream of services derived from barrier island beaches.

Erosion Management Options

Coastal erosion management options can be classified as active or passive. Active shoreline management consists of beach nourishment and/or shoreline armoring. Beach nourishment has come into favor as an active management tool in recent years. This activity consists of placing sand onto the beach face in order to maintain beach area and location. The quality of a nourished beach is generally much better than a structurally protected beach. The beach protects coastal property from hurricanes and nor'easters by absorbing the energy of incoming storm waves. The beach also buffers oceanfront property from storm-induced erosion. Thus, nourishment increases or maintains recreational and leisure opportunities, in addition to offering some protection for coastal properties. The costs of beach nourishment include search costs (for adequate sand resources), extraction and transportation costs, and placement costs. These costs can be significant and are incurred any time sand has to be placed on the beach. Nourished beaches typically last 2-10 years (i.e. after 2-10 years the beach typically returned to its original state), so periodic replenishments are usually required. Thus, the benefits of a single beach nourishment operation are transitory. The dynamic nature of the management model captures the movement of these benefits. For example, nourishment in the current period will bolster beach quality, but that quality will decay in subsequent periods according to the rate of erosion.

Shoreline armoring can involve construction of shore-parallel structural devices, such as seawalls and rockpiles.[7] While indemnifying coastal properties, these structures can engender a number of negative side effects on beach quality. They interrupt sand movement across different parts of the beach face (Everts 1979; Leatherman 1988; Clayton et al. 1992), prevent beach movement in response to storms and sea level rise, and reflect wave energy, scouring the beach and augmenting erosion (Pilkey 1999; Bird 2000). Shoreline armoring is currently present on many shorelines, but has come under heightened scrutiny for its negative side effects, both in terms of safety and aesthetics. Some states have outlawed the use of shoreline armoring (e.g. North Carolina and Maine), and it has generally become an unpopular management tool. For purposes of this research, it is not considered a relevant management option for barrier island beaches.

[7] Other armoring devises include groins, built perpendicular to the shoreline, and breakwaters, built parallel, but offshore. These devises affect the distribution of sand along the shore.

Some coastal geologists and environmentalists have advocated a policy of retreat from the shoreline—a type of passive management (Chasis et al. 1985; Pilkey and Dixon 1996). Moving buildings and infrastructure away from the shoreline would allow natural coastal evolution to continue, which would maintain a natural beach. Terminating active management would save public monies allocated for shoreline management; however costs associated with retreat would include losses of private property and public infrastructure. The size of these losses primarily depends upon the degree of development existing at the time of abandonment and the value of the housing stock. Both of these factors, quantity and price, are obviously determined within the market for coastal housing. Thus, expectations of buyers and sellers and the time horizon in which the market can adjust to policy changes are crucial to an assessment of shoreline retreat.

Beach nourishment can be considered a coastal erosion control—a management variable that is under the discretion of the coastal planner. The sustained absence of control under positive levels of erosion implies a *de facto* policy of shoreline retreat, and under certain regularity conditions it can be shown that this is indeed the case. Following Landry, Keeler, and Kriesel (2003), agents affected by beach management are classified into two groups: beach visitors and coastal property owners. The former includes households that participate in recreational and leisure activities at the beach and are thus concerned about beach quality, but do not own a stake in island property. The latter are similarly concerned with beach quality, but are also concerned about maintaining their property, which can be threatened by coastal storms and sea level rise.

Beaches provide for recreational and leisure services for beach visitors and coastal homeowners alike. Beach area is an important attribute of beach quality because it provides space for recreational activities and contributes to the overall aesthetics of the beach environment. Parsons, Massey, and Tomasi (2000) find an increasing and concave relationship between recreational willingness to pay (WTP) and beach width. The beach also provides protection from coastal storms. If the hinterlands are developed, the beach provides a buffer that protects property improvements from storm waves and storm-induced erosion. Under this characterization, recreation and protection are *joint* services of the beach on developed barrier islands. Both should be considered in beach management decisions.

Economist's Thinking on Coastal Erosion

There are two branches of literature that have examined the economics of coastal management—the first primarily considering the short run problem of efficient management in response to episodic erosion and the second addressing the long run problem of coastal protection under sea level rise. My conceptual model aims to integrate these two branches in a consistent framework, and thus to draw clear link between decisions in the short and long run. This linkage is important because management decisions in the short run affect the state of beach resources, which in turn affects the coastal housing market. A number of papers have shown that the value of coastal property partially capitalizes the value of coastal resource quality. It is precisely the value of housing, however, that has played a central role in evaluating coastal protection in the long run. Clearly short-run decisions impact the long-run problem. If we nourish beaches in the short run, housing values will be bolstered due to both the recreational and loss-mitigating values of beaches. If we let beaches degrade in the short run, the value of the housing stock will be somewhat diminished. The framework proposed in the next chapter allows for an integration of these short and long term aspects of beach management.

Efficient Coastal Management in the Short Run

The existing literature on beach erosion management in the short run is fairly limited in scope. Most studies consider only one beach service (recreation vs. storm protection) or one user group (beach visitors vs. coastal homeowners) in isolation. This branch of the literature has made use of primarily static models. As such, no study has adequately incorporated the dynamics of the beach management problem.

Bell (1986) and Silberman and Klock (1988) estimate the recreational benefits of beach nourishment, but fail to consider its protective benefits thus producing underestimates. Bell (1986) estimates the optimal square-footage of beach space per beach user, and then compares the estimated benefits of maintaining optimal beach area with a rough cost estimate. He concludes that nourishment is efficient at twenty-four Florida beaches. In an attempt to incorporate time, Bell assumes that the beach, once it is nourished, provides for increased benefits at some constant level. This assumption will generally not hold, since erosive forces immediately begin to work on a newly nourished beach. Typically when it is time for a replenishment operation (2-10 years after the initial nourishment), the beach has returned to its original diminished (pre-nourishment) state. Benefit estimates derived under the assumption that the nourished beach maintains its area over the life of the project will typically be upward biased. Silberman and Klock (1988) use a split-sample stated preference survey to estimate the differential WTP for beach recreation before and after a beach nourishment project. They find a stronger effect on visitation than on benefits per trip, but positive net benefits overall. This result suggests that recreational benefit estimates need to consider how visitation changes with beach quality.

Edwards and Gable (1991) and Pompe and Rhinehart (1995) focus their analyses on coastal homeowners, using hedonic property price models to estimate welfare measures. Edwards and Gable explore the recreational benefits of coastal beaches accruing to local residents. They argue that proximity to the beach reveals implicit savings in travel cost that reflect household preferences for beach recreation, and they include distance to the beach as a regressor in their hedonic model. They then use the marginal implicit price of distance (the derivative of hedonic price function with respect to distance variable) to estimate a demand equation for distance. Edwards and Gable use data from a single housing market.[8] As will be discussed in Chapter 4, their procedure does not identify the demand equation because there is no exogenous variable in the demand equation that does not appear in the hedonic price function. Also, their coefficient estimates are biased because distance from the beach is endogenous. Moreover, proximity to the shoreline also affects the magnitude of expected loss in the event of a storm. The selection of optimal distance depends upon the household's valuation of coastal amenities, <u>and</u> its assessment of coastal risks and willingness to tolerate such risks. The point estimate of marginal recreational benefits derived in this manner will almost certainly be downward biased if

[8] Their data does cover multiple years, but they do not estimate separate hedonic models for different time periods.

households are risk averse. That is, the implicit savings of locating further from the beach will be diminished because moving further from the beach also decreases expected storm loss, *ceteris paribus*.

Pompe and Rhinehart examine the dual nature of beach services, attempting to disentangle protective from recreational benefits. Following Edwards (1989), they estimate a hedonic property model including an interaction variable (beach width × distance from the beach) that they claim reflects the recreational aspects of the beach, and include beach width at the nearest shore to account for storm protection benefits. This approach is only valid if the storm protection benefits accruing to homeowners are independent of distance, which they are arguably not. Pompe and Rhinehart fail to control for differences in storm risk across properties, and in doing so cannot disentangle recreational from protective benefits. Moreover, marginal benefit measures are of limited use in beach management policy analysis if large changes in beach quality are of interest. One would prefer to have a preference function that encompasses the relevant levels of beach quality.

Other authors have addressed shoreline retreat. Parsons and Powell (2001) provide an estimate of the costs of shoreline retreat on property owners on the Delaware coast. They use a parameterized hedonic property model to simulate fifty years of unabated shoreline retreat on the coastal housing market. For those houses that are lost to erosion, the value of proximity is purged from housing value before it is removed from the stock of housing (since proximity value is transferred to remaining properties). Their findings suggest that the costs of nourishment are less than the adjusted value of houses that would be lost over the next 50 years. Using a similar simulation approach, Landry, Keeler, and Kriesel (2003) compare beach nourishment, shoreline armoring, and shoreline retreat over 25 years, taking account of property effects and recreational benefits accruing to both beach visitors and coastal homeowners. They find that the efficiency of active management depends upon the erosion rate and how management costs evolve over time (given sea level rise and changes in resource stocks and technology). While both these models incorporate dynamics in the retreat scenario, they are static in other regards. Neither considers the dynamic adjustment of the beach under an active management regime, nor do they allow for adjustment in the value of the housing stock (which would lower the property losses associated with shoreline retreat). Nor do these studies consider storm protection benefits engendered through physical changes in the beach. Moreover, both papers take the time horizon as given, rather than making it an instrument of optimization.

Part of a retreat management scheme can include enforcing mandatory setback provisions (requiring houses to be a certain distance from the shoreline) on new construction or significantly damaged buildings. Frech and Lafferty (1984) examine the effect of setback provisions on the housing market in California. They find that housing prices increased as a result of the institution of setback provisions. Their results suggest that the increase in property values was due to a restriction on the supply of land suitable for building rather than the increase in coastal amenities associated with the setback provision.

The existing literature on erosion management is primarily empirical. Many of the studies are *ad hoc* and lacking of a well-defined conceptual framework for coastal management decisions. Most of these papers are fairly narrow in scope, examining only one beach service and/or one user group. No study has adequately incorporated the dynamic adjustment process of beaches that characterizes the beach erosion management problem.

Switching our attention to the long run, the beach erosion management literature has yet to give serious consideration to drastic changes in sea level. The existing studies are primarily static in nature, making them inappropriate for long-term analysis. For example, Bell (1986) concludes that beach nourishment is a cost effective management tool, but does not consider how rising seas (which Bell cites as a cause of erosion) might increase management costs over time, potentially changing his conclusion. The introduction of changes in sea level provides further justification for a dynamic model, as the problem becomes non-autonomous.

Coastal Protection in the Long Run

Titus, et al. (1991) estimate the nationwide costs of (i) protecting developed property and (ii) losses associated with undeveloped lowlands and wetlands, for a range of sea level rise scenarios in the U.S. Their analysis assumes that raising barrier islands by pumping sand is the preferred response, while levees and pumping systems will protect developed portions of the mainland. They make use of engineering cost estimates, appraised undeveloped land values, a range of estimated values for wetlands, and assume no further development on the coastal lowlands in compiling a current value cost figure. Their results suggest that the cost of protecting developed property from a one-half (one) meter rise in sea level would cost between \$55 and \$123 billion (\$143 and \$305 billion) and that the United States could lose between 20 and 45 percent (29 and 69 percent) of its coastal wetlands. The estimated market value of the lost wetlands is between \$11 and \$82 billion (\$17 and \$128 billion), but the authors state that the environmental effects associated with lost wetlands could be catastrophic. As such, they recommend a gradual abandonment of undeveloped coastal lowlands in order to allow wetland migration. They suggest that confining coastal development to presently developed areas and protecting those areas should be cost-effective, with a price tag of roughly \$2,000 per quarter-acre lot.

The analysis of Titus, et al. presumes that all developed coastal land will be protected and presents a *positive* economic assessment of the costs associated with that scenario. Their estimated price tag of \$2,000 per quarter-acre lot is an average figure, which does not allow for a determination of which areas should be protected. Recognizing this, Yohe (1991b) offers a framework for a *normative* economic assessment of coastal protection schemes. His framework utilizes a stochastic sea level rise trajectory and a corresponding trajectory of marginal property damages. The benefits of coastal protection are the foregone property losses, measured as the net present value of the *expected* change in damages attributable to protection. He uses pre-existing cost estimates for Long Beach Island, NJ to evaluate either raising the island by adding sand or constructing a dike under a range of sea level rise scenarios. He finds raising the island is the best course of action under gradual sea level rise; building a dike is preferred under accelerated sea level rise. This line of research does not consider the external effects of shoreline armoring on beach resources. If barrier island beach resources are indeed the locus of economic value on barrier islands (as I maintain), construction of a dike cannot be optimal. The dike will destroy the beach. Yohe's analysis takes property values as exogenous. His benefit measures are determined by the trajectory of those property values, but these in turn depend crucially upon how the market reacts to the potential for abandonment. Thus, they are endogenous. If policy analysts conclude that property should be protected at all costs, then property values can be fortified by this information, and the prophecy fulfills itself.

This latter problem is considered in Yohe and Neumann (1995) and Yohe, Neumann, and Ameden (1995). These papers explore coastal protection under two property value scenarios: one in which property values increase "business as usual", and one in which property values depreciate in anticipation of sea level rise.[9] Their optimization problem is represented by a simple dynamic model that posits the choice of beginning and terminating active management times as a function of the present value of benefits (property value trajectories) minus the present value of costs (management expenditures and property losses at terminal time). Their model abstracts from the actual micro-level decisions regarding the amount of beach nourishment (or shoreline armoring).

Yohe, Neumann, and Ameden (1995) consider protection of five areas around Charleston, S.C., utilizing three sea level rise trajectories: 33cm, 76cm, and 100cm through the year 2100. They find that protection is warranted in some cases, but not in others. The scenario of adaptation and foresight (i.e. depreciation) in the property market decreases the amount of protection, but the likelihood of this scenario is sensitive to uncertainty regarding sea level rise and the credibility of a retreat policy.

[9] They also consider a scenario under which benefits are ignored and protection is guaranteed regardless of cost.

They assume protection of mainland locations involves the construction of levees, but they note that this option is not feasible on the one barrier island they examine. They find that Sullivan Island should be protected immediately, through beach nourishment. In the case of this barrier island, the only real timing decision is deciding *when to stop protection*. This is precisely the question I intend to address.

The development of the U.S.'s Eastern and Gulf Coast barrier islands has been primarily due to an increased demand for beach and coastal recreation, and the economies of many of these small towns is centered around the appeal of coastal amenities. For barrier islands, coastal beaches are the primary attraction. To consider management of these islands with a disregard for effects on the beach seems a misplaced exercise. The management problem is focused upon land that is not merely located near the beach, but rather land that confers upon owners and occupiers the array of benefits associated with the barrier island environment. The value of the land is implicitly tied to the quality of this environment. Decisions regarding its use and protection should not be made irrespective of the effect that these decisions have on the coastal environment. The theoretical model explored in the next chapter posits management decisions as relying on the stream of services derived from the beach.

The dissertation is organized as follows. Chapter 2 presents a conceptual model of optimal erosion management on barrier islands. Chapters 3 and 4 offer details on the empirical models that are used to assess economic returns of beach quality accruing to beach visitors and coastal homeowners, respectively. Chapter 5 puts together the pieces and derives solutions to the optimal control problem under various scenarios.

Chapter 2: Theory of Optimal Beach Erosion Management

This chapter presents an economic model of optimal beach erosion management on barrier island beaches. The model accounts for distinct household demands for beach services (both recreational and loss-mitigating aspects), and incorporates a number of dynamic phenomena. Most importantly, the dynamic nature of the beach is addressed within a transition equation. For simplicity, I assume that the beach erodes in a uniform and deterministic manner, reflecting the permanent loss of beach width due to storms and the background sea level rise (1-2 mm/year). Additional sea level rise is introduced as an evolving erosion rate. The dynamic nature of the model is critical in accounting for the evolution of beach quality as a dynamic resource.

Economic Returns from Beach Quality

Assume there is a time-dependent variable that represents the quality of the beach in terms of average beach width, q_t. Initial beach width is taken as given, but subject to erosive force that reduces beach width in a deterministic manner. The coastal planner can augment beach width by adding sand to the beach system. I assume that any additional sand is of a similar quality to the existing sand, so there are no other qualitative or aesthetic effects associated with beach nourishment. I take the length of the beach as given and assume that nourishment is applied to the entire length. With length given, beach width determines beach area, which provides space for recreational and leisure activities for both visitors and local residents and contributes to the aesthetics of the coastal landscape. Beach width also provides protection from high velocity waves and erosion associated with coastal storms.

Maintaining the aforementioned distinction, agents affected by beach management are classified into two groups: beach visitors, represented by *b*, and coastal homeowners, represented by *r*. I assume that beach visitors are only concerned with beach quality as it relates to recreation and aesthetics associated with beach use. Household utility for beach visitors is $u^b(v,y;q)$, where v represents visits to the beach, y is a composite good, and q is beach quality. Visitors making trips to a single site cannot freely choose beach quality, but it partially determines the value of the trip. Utilizing the standard expenditure-minimization framework, one can derive a measure of economic welfare arising from changes in beach quality. A beach visitor's willingness-to-pay for a change in beach quality will depend upon the level of prices (p_y, where y represents a composite commodity), initial utility (U^0), and the extent of the change. For the representative beach visitor, the measure is:

$$WTP^b(q^0,q^1) = E^b(p_y,p_v,U^0,q^0) - E^b(p_y,p_v,U^0,q^1), \qquad [2.1]$$

where $E(.)$ is the minimum expenditure function, and p_v is the price of a beach visit. Expression [2.1] is non-negative if the subsequent quality (q^1) is greater than the initial quality (q^0), and non-positive if the subsequent quality is less than the initial quality. I assume that [2.1] is increasing and concave in q. The empirical approach to estimating [2.1] is detailed in Chapter 3.

Deriving a welfare measure for coastal homeowners is slightly more complicated, as beach quality affects the level of risk faced by the household. Coastal homeowners benefit from beach area in terms of both recreational and loss-mitigating aspects. Household utility for coastal homeowners is $u(a,y,q)$, where a is a vector of housing characteristics, y is a composite good, and q is beach quality. Coastal homeowners can choose the quality of their local beach when purchasing a property. I hypothesize that this value will be reflected in housing prices.

For simplicity, assume that coastal households face only two states of nature—(1,0) with associated annual probabilities of ρ and $(1-\rho)$, where 1 represents landfall of a major coastal storm and 0 is the converse. The expected loss in the event of a storm is $L(q_t)$. Assume L is a continuous function of beach quality, which represents the loss-mitigating service of the beach. Thus, $\partial L/\partial q < 0$. In choosing a home associated with higher beach width, households are decreasing their expected loss, thus insuring themselves against coastal storms. Self-insurance of this sort decreases the expected loss conditional on a hurricane making landfall. Ehrlich and Becker (1972) have shown that self-insurance is a substitute for formal insurance.

Both the recreational and loss-mitigating services of beaches should increase the value of coastal homes in the vicinity of good quality beaches, but the relationship is complicated by the existence of other forms of indemnification, including formal flood insurance and other forms of self-insurance. Chapter 4 offers a detailed theoretical model relating property values to beach quality. The model utilizes an expected utility framework to incorporate the probability of storm landfall, and also incorporates other forms of indemnification, including formal flood insurance. For the time being, a preference function for coastal residents is taken as granted. Utilizing the standard expected utility maximization framework, the maximum WTP of the representative coastal resident for changes in beach quality can be represented implicitly as:

$$\rho \times V^{r}(m - L(q^{0}), p, q^{0}) + (1-\rho) \times V^{r}(m, p, q^{0})$$
$$= \rho \times V^{r}(m - L(q^{1}) - WTP^{r}, p, q^{1}) + (1-\rho) \times V^{r}(m - WTP^{r}, p, q^{1}) \qquad [2.2]$$

where $V^{r}(m,p,q)$ is the indirect utility function for coastal homeowners and $WTP^{r}(q^{0},q^{1})$ represents willingness to pay for a change in beach quality. Willingness to pay will also depend on income (*m*), prices, the probability of loss, and the expected loss.

Other factors affect the risk profile of coastal homeowners. The probability of storm landfall (ρ) is assumed constant within a region, while the determinants of an individual household's expected loss will vary across properties. Elevation above base flood elevation (BFE) is a primary structural determinant of flood losses. Houses with more elevation should expect lower storm damage losses. The expected loss can also be affected by building standards and proximity to the shore. Including these characteristics in addition to beach quality, and solving for WTP in [2.2] gives rise to the welfare measure $WTP^{r}\ (q^{0},q^{1},\rho,L(q^{0},q^{1},loss),m)$, where *loss* is a vector of expected loss determinants other than beach quality. Let $q^{1} = q^{0} + \hat{q}$, with q^{0} representing initial beach width and $\hat{q}$ representing the increment to beach width. The slope of the coastal homeowner's preference function in beach quality space is:

$$\partial WTP / \partial\hat{q} = [\partial V / \partial\hat{q} - \rho\mu^{L} \partial L / \partial\hat{q}\ ] / [\rho\mu^{L} + (1-\rho)\mu^{0}], \qquad [2.3]$$

where μ^L and μ^0 are the marginal utility of income in the loss and no-loss states, respectively. The first term in the numerator on the RHS of [2.3] is the direct effect of beach quality on utility. This represents the recreational and aesthetic value of beaches for coastal homeowners. The second term represents beach quality's role as self-insurance. The denominator in [2.3] is the expected value of marginal utility of income. By assumption, $\partial L / \partial \hat{q} < 0$, representing self-insurance. Thus, willingness to pay is an increasing function of $\hat{q}$. Before considering the management problem in detail, we must define the physical processes that give rise to the process of coastal erosion on barrier islands.

Beach Geomorphology

The dominant coastal land feature in the Eastern and Gulf regions of the United States is the barrier island (Nummedal 1983). The balance between sea level and sediment supply generally determines whether these barriers will be transgressive (long-term retreating landward, i.e. shoreline recession) or regressive (long-term expanding seaward) (Nummedal 1983). This distinction is crucial to the geological evolution of the coastline. Transgressive barrier islands currently dominate the coastal landscape, due to sea level rise (and also possibly due to damming of rivers which decreases sediment supply). Transgressive barriers move landward as erosion diminishes their seaward extent and overwash from storms deposits sediment in the back-barrier region (Clayton et al. 1992).

Waves are the primary agent in coastal erosion. They carry energy into the coastal environment, transporting sediment, and generate nearshore currents that disperse the sediment (Komar 1983). Waves can be beach-building or beach-reducing (Hardistay 1990). Swell waves, which are less energetic and shallower, deliver sediments to the beach, building it up. Storm waves are steeper and break more forcefully on the beach, transferring sand to offshore bars. The oscillation between swell and storm waves leads to fluctuations in the width of the beach, but the process protects the hinterland from direct wave attack. In some cases—hurricanes and nor'easters, in particular—property may become threatened by this oscillation of the beach. Most sand that is carried offshore during storms returns to the beach afterwards, but this is not always the case. Sand can be lost from the littoral system for various reasons.

There is a considerable strand of engineering literature related to predicting the two-dimensional, orthogonal profile[10] of the beach (see Figure 2.1) as a function of wave and sediment characteristics. Empirical evidence posits the relationship as a power law:

$$d = a\,x^b, \qquad [2.4]$$

where d is underwater depth of the beach surface, x is horizontal offshore distance, and a and b are coefficients (Hardistay 1990). The b exponent is approximately 2/3. The magnitude of a depends upon the grain size and specific gravity of sediment as well as wave height, but for typical values of beach sand a usually varies between

[10] The orthogonal profile is the side-view transect which is orthogonal to the shoreline. Other geomorphological literature has focused on the influence of tidal and longshore currents, which influence the distribution of sand along the shoreline. This aspect of the erosion problem is not emphasized in this dissertation.

0.04 and 0.11. Finer sand is associated with lower values of a, and thus more gently sloping beaches. Equation [2.4] governs the shape of the beach below the high-water line; this line fluctuates with tides, storm surge, and sea level. Relationship [2.4] describes equilibrium. A natural beach approaches this equilibrium state exponentially with a half-life of approximately "a few hours" (Hardistay 1990). The beach form above the high-water line is affected by wind and wave run-up, but is typically taken as constant.

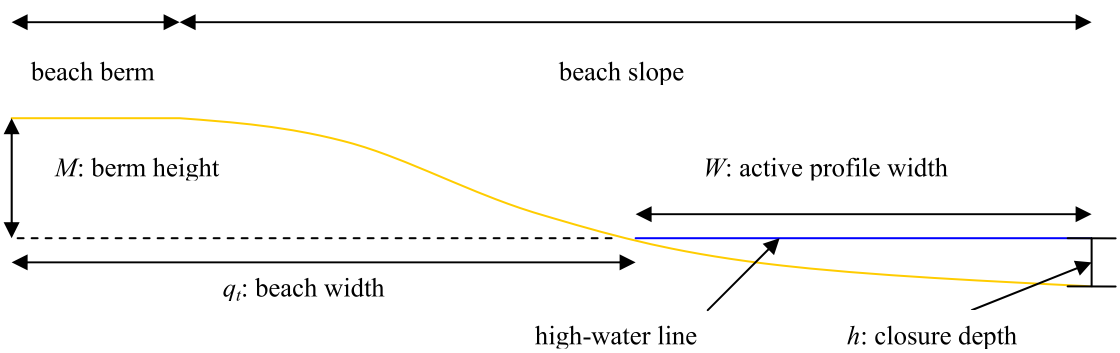


Figure 2.1: The Orthogonal Beach Profile

A typical beach orthogonal profile is presented in Figure 2.1. The beach berm, on the far left, is the highest portion of the beach. The back section of the berm often contains dunes and scrub vegetation. Vegetated dunes provide additional protection from storms. The sloping portion of the beach, in the middle of the figure, is typically the widest. This portion is also the most geomorphically active. While the beach berm is often not compromised except during major coastal storms, the sloping portion can vary over the course of a year. It is the beach slope that responds to storm and swell waves in the short run. Fluctuations in this portion of the beach can affect recreational use. If the fluctuations are great (possibly affecting portions of the berm), coastal property may be threatened. Also depicted in 3.1 is the depth of closure—approximately equal to the depth of breaking waves. At this depth, the beach profile is relatively inactive.

Beach width is defined as the distance from the average high-water line to the back of the beach. The back of the beach is delineated either by where the sand dunes begin, where maritime forest or scrub vegetation begins, or where property development starts. As such, beach width determines the amount of space available for recreation and leisure, per unit shoreline length.

Kana (1993) advocates use of the Ideal Present Profile (IPP) in planning beach nourishment operations. For relatively straight shorelines with predominantly shore-parallel contours, the IPP represents the most likely equilibrium profile on the majority of the shoreline (i.e. away from inlets or other perturbing forces). In the field, development of the IPP requires a detailed geomorphic model of the site and delineation of the littoral cells.[11] The IPP is then constructed by taking depth and angle measurements of multiple transects of the beach in areas where the profile is deemed natural (in terms of approximating equilibrium). These profiles are matched at a common backshore contour, and the IPP is calculated as the average of these representative profiles (Kana 1993). In the theoretical model to follow, the existence of an IPP is presumed. Moreover, I assume that the beach is reasonably well represented by the IPP, and thus only focus on this representation of beach quality.

---

[11] The geomorphic model utilizes an aerial view of the site to identify the orientation and character of incoming waves, location of shoals, dominant tidal currents, and direction of longshore (littoral) transport, and how these elements interact in determining the geomorphology of the site. Littoral cells are compartments along the shore in which the longshore transport is unidirectional.

Dean (1991) shows how to estimate the increment in beach width produced by a certain volume of beach fill (sand) using only the average beach profile (or IPP). The formula depends upon relative grain size of the fill and natural sediment. For simplicity, I assume that the fill sediment is of the same grain as the natural beach sediment. Let M represent the height of the beach berm (in meters above sea level), and h represents the depth of closure (in meters below sea level). Let the increase in beach width per unit length be given by $\hat{q}$. The increase in average beach width is then approximated by:

$$\hat{q}\,(n) = n\,/\,(M + h), \qquad [2.5]$$

where n is the volume of beach fill *per unit of shoreline length* (l) (i.e. if N is total sand volume for a project, $n{=}N/l$). Equation [2.5] is increasing and linear in n. The slope of the equation is determined by the parameters M and h, which describe the vertical dimension of the beach profile.

Equation [2.5] corresponds with a unit of shoreline length, which meshes well with the IPP approach suggested by Kana (1993). I focus on the average beach profile as the representation of resource quality, and use relationship [2.5] to convert beach fill (n, which will serve as the management control) into incremental beach width (which affects the state variable, q_t). The change in beach width per unit length is given by $\hat{q}\,(n)$, and the corresponding increase in total beach area is $\hat{q}\,(n)\times l$. I use a reduced-form equation to represent the costs of beach nourishment as a function of total sand (or "beach fill") added to the beach: $C(N) = C(n\times l)$. Chapter 5 provides a detailed exposition on estimation of a beach nourishment cost function for the southeastern U.S.

Shoreline Recession

Equation [2.5] only applies in the case of constant sea level. Rising seas push the shoreline landward. In this case, the amount of sand required to extend beach width must also take into account the countervailing effects of shoreline recession. Bruun (1962) offered the first model to predict shoreline recession due to sea-level rise. His model assumes a constant equilibrium beach profile and is based on the concepts of conservation of sediment and dynamic adjustment of the beach profile:

$$R = - W\,\Delta S /(M + h) = - \Delta S / \tan \alpha, \qquad [2.6]$$

where R is horizontal retreat distance (conceptually equivalent to $-\hat{q}$), W is the width of the active beach profile (the horizontal distance from the mean high-water line to the depth of closure—see Figure 2.1), ΔS is the increase in sea level, α is the average beach slope (in degrees), M is height of the beach berm, and h is the depth of closure (as before) (Dean and Maurmeyer 1983). Assuming the average beach slope is strictly less than 45 degrees ($\tan \alpha < 1$), equation [2.6] characterizes shoreline recession as an amplification of the change in sea level, with shallow sloped beaches retreating more rapidly for a given sea level change.[12]

[12] The depth of closure is the critical assumption of [2.6]. By limiting the depth of motion for beach sediment, shoreline recession is the only possible response to sea level rise. Dean (1991) also considers a case in which onshore movement of sand is possible. In this case, the horizontal retreat distance need not be as large as that implied by [2.6]. This case is, however, controversial.

The Bruun model has been altered to account for various other considerations and complexities. Dean and Maurmeyer (1983) generalize the Bruun rule to barrier island recession as:

$$R = \Delta S\ (W + w + W_l)\ /[(M + h) - (M_l + h_l)], \qquad [2.7]$$

where W and W_l are active profile width on the ocean and lagoon sides of the island, respectively, w is the width of the island, M and M_l are the ocean and lagoon berm heights, and h and h_l ocean and lagoon breaking wave depths (closure depths), respectively. The model of barrier island recession due to sea-level rise is depicted in Figure 2.2.

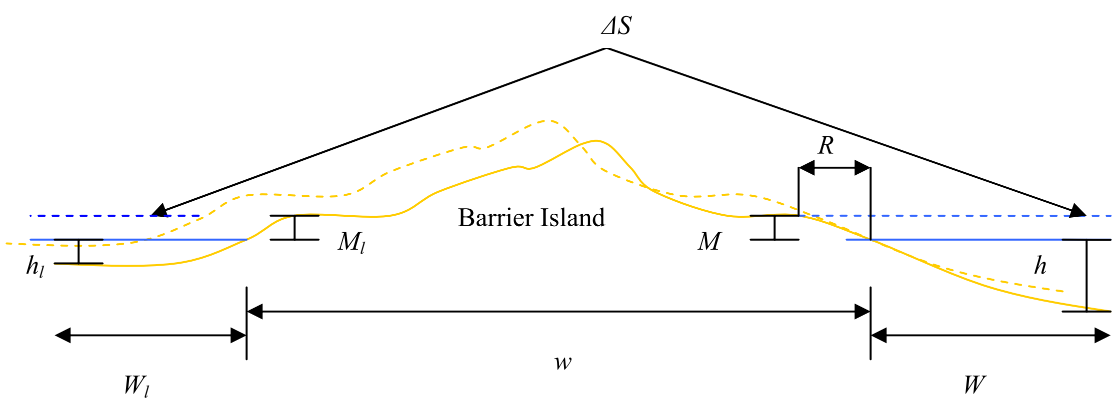


Figure 2.2: Barrier Island Recession due to Sea Level Rise – Initial forms of island (gold) and sea level (blue) are indicated by solid lines; subsequent forms are indicated by dashed lines; ΔS is the change in sea level; R is horizontal retreat distant; M_l and M are berm heights on the lagoon and ocean side; h_l and h are closure depths on the lagoon and ocean side; W_l and W are active profile width on the lagoon and ocean side; and w is island width. [Adapted from Dean and Maurmeyer 1983]

By [2.7], barrier island recession is greater than shoreline retreat on the mainland. Adding a positive quantity to the numerator and subtracting a positive quantity from the denominator cannot decrease the value of the ratio on the RHS of [2.7]. This ratio is multiplied by the change in sea level, ΔS. Equation [2.7] reduces to [2.6] if there is no deposition on the lagoon side (in the back-barrier, i.e., W_l, M_l, $h_l = 0$ and removing w from the equation). Further, if the denominator is zero, i.e., $(M + h) = (M_l + h_l)$, the island will "drown" in-place, as there is no yield of sediment to allow the island to migrate.

Dean (1991) explores the change in beach fill quantities required to maintain average beach width under sea level rise. The relationship is derived by combining [2.5] and [2.6]. For mainland shores (or stabilized barrier island shorelines), the change in beach width per unit length is simply:

$$\hat{q}\,(n) = (n - W\,\Delta S) / (M + h), \qquad [2.8]$$

Equation [2.8] clearly shows that the balance between nourishment and shoreline recession determines net change in beach width. Consider the discrete time analog of [2.8], with n representing nourishment sand per period of time and ΔS representing the change in sea level per unit time. Since $W > 1$ meter, nourishment fill needs to increase at a rate greater than or equal to that of sea level rise in order to prevent beach erosion. For example, for an active profile width of 500 meters and sea level rise of 0.3cm/year, nourishment volume per unit length will need to increase by 1.5 m^3/year to maintain the current position of the shoreline. While equation [2.7] suggests that the active profile of barrier islands is larger than that of mainland shores (essentially encompassing the entire island), equation [2.8] applies to stabilized barrier islands because they are not in a natural state that would allow them to migrate.

Equation [2.8] will hold as long as the position of the island can be maintained by replenishing the beach—that is, as long as mean sea level is well below the height of the beach berm. Once the berm is compromised, overwash will begin to deposit sand in the back-bay, and the island will begin the process of migration. At this critical threshold of sea level rise, nourishment quantities would need to increase dramatically in order to maintain the position of the island. No longer would the aim of shoreline management be simply maintenance of the beach; equation [2.8] would no longer apply. Rather, sand must be added to the entire island, including the surf zone and back-barrier lagoon (Yohe, et al. 1991).[13]

To maintain the elevation of the island relative to sea level, the requisite volume of sand per unit shoreline length is approximately $(W_o + w + W_l)\Delta S$. This would entail quite an engineering feat, and this type of operation is not reflected in any historical beach management data. Nonetheless, I will use the historical cost data to estimate what these costs might be. At significantly high sea level (i.e. above the critical threshold), the change in beach width per unit length is given by:

$$\hat{q}(n) = (n - (W + w + W_l)\,\Delta S) / [(M + h) - (M_l + h_l)], \qquad [2.9]$$

where W and W_l are active profile width on the ocean and lagoon sides, respectively, and w is the width of the island.

[13] Buildings would also have to be elevated, and infrastructure would have to be rebuilt. I don't consider these costs.

The following stylized facts will govern beach dynamics in the analysis to follow. With a relatively constant sea level (i.e. the *short run*), the relationship between average beach quality and nourishment quantities is defined by [2.5], and the erosion parameter, θ, is assumed constant. With a rising sea level (i.e. the *long run*), the sand volume required to produce a unit of beach width is still defined by [2.5], but the erosion parameter evolves following equation [2.6]. The net effect on average beach quality is given by equation [2.8].

Equation [2.8] only applies as long as sea level does not threaten to inundate the island. Once sea level rises to the point of inundation, barrier island migration becomes the natural response and [2.7] becomes the relevant model of shoreline change. The change in the erosion rate under this scenario would be quite large, reflecting not only the width of the active offshore zone, but also the width of the barrier island and the width of the active lagoon zone. Assuming the beach profile parameters remain the same, the sand volume required to produce a unit of beach width is still defined by [2.5]. But, the erosion parameter evolves following equation [2.7], with the net effect on average beach quality is given by equation [2.9]. Clearly, the required nourishment volume per unit length will witness a rather large discrete jump at this critical point. Combining [2.8] and [2.9], one can map a non-stationary transition equation for any barrier island beach, conditional on the beach profile and the rate of sea-level rise.

In summary, the relationships given by [2.5], [2.8], and [2.9] will be used to define the transition equation for beach quality. Equation [2.5] defines the transition equation for beach management with constant sea level. Equations [2.8] and [2.9] define the transition equation for beach management under sea level rise.

Optimal Beach Management

The coastal planner's problem is to maximize the difference between the benefits (as indicated by [2.1] and [2.2]) and management costs subject to the relationship between beach quality, erosion, and beach nourishment. The coastal planner chooses the amount of beach nourishment to be conducted in each period. This problem is a non-renewable resource management problem, but differs from the conventional non-renewable problem because society benefits from preservation rather than extraction. The non-renewable resource exhibits a decaying tendency, and the management control represents a contribution to the level of resource quality that counters the tendency for decay.

A sustained corner solution implies a *de facto* policy of shoreline retreat in the long run. By "sustained corner solution" I mean a lack of control for a period of time sufficient to lead to significant diminution of beach resources and associated losses of property. Returns from beach quality are represented by [2.1] and [2.2], and these preferences are taken as static. Since nourishment costs are expected to increase with sea level, one can define the point at which shoreline retreat becomes the optimal policy response by the balance of benefits and costs. If costs rise to a certain point, they will eclipse the benefits of preservation, thus triggering a policy shift. Interestingly, barrier island beaches may be considered a renewable resource under a retreat scenario, assuming that island migration can keep pace with sea level rise.

The problem as non-renewable is related to maintaining quality *and the present location* of the beach and thus preserving coastal developments as well.

Using control theory, the management problem can thus be represented as:

$$\max_{n_t} \sum_{t=0}^{T-1} \eta^t \{B \times WTP^b(q_t) + R \times WTP^r(q_t, L(q_t)) - C(N_t)\} \qquad [2.10]$$

subject to $\quad q_{t+1} - q_t = -\theta + \tau\, n_t \qquad [2.11]$

$$N_t = n_t \times l \qquad n_t \geq 0 \qquad [2.12]$$

$$q_{t=0} = q_0,\ q_T = free \qquad q_0 \geq 0 \qquad [2.13]$$

where *WTP* functions reflect mean willingness-to-pay[14] of the appropriate user groups; $\eta^t = 1/(1 + \delta)^t$ is a discount factor; *B* is the number of beach visitor households, and *R* is the number of coastal homeowners; $C(N_t)$ represents the costs of beach nourishment,[15] with N_t representing the total volume of sand (or "beach fill") added to the beach in period *t*; τ is a parameter that converts sand volume to incremental beach width; $q_{t+1} - q_t$ describes the dynamic motion for beach width (bolstered by beach nourishment and naturally decaying at some rate θ); and q_0 is the initial beach quality condition. The terminal level of beach quality (q_T) is free, but could be specified as a specific value.

---

[14] The WTP functions in the objective functional represent total WTP for the given quality level, *q*. For example, $WTP^b(q) = E^b(p,L,U^0,0) - E^b(p,L,U^0,q)$, where the '*0*' represents a beach quality level of zero.

[15] Assume $\partial C/\partial N_t > 0$ and $\partial^2 C/\partial N_t^2 \geq 0$.

Equations [2.10] through [2.13] describe an optimal control problem with one control variable (n_t) and one state variable (q_t). As indicated in the discussion on beach geomorphology, the τ parameter is given by:

$$\tau = \hat{q}(n)/n = 1 / (M + h) \qquad [2.5']$$

The erosion parameter, θ, reflects average annual beach erosion caused by coastal storms and the background rate of sea level rise. Assuming relatively constant sea level (i.e. the short run) the erosion control problem may be considered autonomous, and the beach quality transition equation is constant over time. This setup can be used to evaluate beach erosion management programs in the near term.

Alternatively, one may set up the model to examine the problem of sea level rise (i.e. the long run). With rising seas, erosive pressure will be increasing, as the barrier island becomes prone to migrating. This characterization of the problem suggests that the erosion parameter, θ, will be increasing over time, and we must estimate the path of θ_t. Equation [2.6] indicates how θ will change with sea level rise, and equation [2.8] gives us the net effect on beach quality. Let ΔS be the change in sea level per unit time (a positive constant). Equation [2.8] can be separated into two parts:

$$\hat{q}(n_t) = n_t / (M + h) - W \Delta S \times t / (M + h),$$

$$\hat{q}(n_t) = n_t\tau - \Delta\theta_t \qquad [2.8]$$

The first gives us the change in average beach width due to nourishment ($n\tau$), and the second gives us the change in the erosion rate ($\Delta\theta_t$) as a function of sea level change (ΔS), physical characteristics of the beach, and time. Thus we have:

$$\Delta\theta_t = (W\Delta S\times t) / (M+h),$$

which applies only as long as sea level rise is below the critical threshold that would engulf the barrier island. Then, the trajectory of the erosion rate parameter under these conditions is given by:

$$\theta_t = \underline{\theta} + \Delta\theta(t) = \underline{\theta} + (W\Delta S\times t) / (M+h), \qquad [2.14]$$

where $\underline{\theta}$ is the background erosion rate.

Once sea level rise reaches the critical value, the barrier island will be prone to landward migration and [2.7] becomes the model of shoreline response. The net effect on average beach width is given by equation [2.9], which can be separated into two parts:

$$\hat{q}\,(n_t) = n_t / [(M + h)\text{-}(M_l + h_l)] - (W + w + W_l)\ \Delta S\times t / [(M + h)\text{-}(M_l + h_l)]$$

$$\hat{q}\,(n_t) = n_t\tilde{\tau} - \Delta\tilde{\theta}_t \qquad [2.9]$$

The first term on the RHS is the change in average beach width due to nourishment when sea level rises above the critical value, and the second term is the change in the erosion rate due to sea level rise above the critical value. Thus, when aggregate sea level rise reaches a critical value, our τ parameter changes and our erosion trajectory undergoes structural change. The new rate of change for erosion is:

$$\Delta\tilde{\theta}_t = (W + w + W_l)\ \Delta S \times t\ /\ [(M + h)-(M_l + h_l)],$$

which applies after the critical threshold that will engulf the barrier island. The complete trajectory of the erosion rate parameter under these conditions is given by:

$$\theta_t|\ critical\ sea\ level = \underline{\theta} + \Delta\tilde{\theta}_t = \underline{\theta} + (W + w + W_l)\ \Delta S \times t\ /\ [(M + h)-(M_l + h_l)], \quad [2.15]$$

where $\underline{\theta}$ is the background erosion rate.

Under conditions [2.8] and [2.9] the coastal erosion problem is non-autonomous. The erosion trajectories specified in [2.14] and [2.15] are increasing monotonic functions of time. They represent the escalating erosive pressures associated with sea level rise. In the long run, we consider the selection of the terminal time as a management parameter. The time horizon of beach nourishment will depend upon the number of households (*B* and *R*), recreational and protective services of the beach (represented by *WTP*), the rate of sea level rise, the sensitivity of management costs to sea level rise, and initial beach quality.

The present value Hamiltonian for the constant sea level (short run) problem takes the following form:

$$H = \eta^t \{B \times WTP^b(q_t) + R \times WTP^r(q_t, L(q_t)) - C(N_t)\}$$
$$+ \lambda_{t+1}(-\theta + \tau n_t) \qquad [2.16]$$

where λ_{t+1} is the present shadow value of an additional unit of beach width delivered at time $t+1$. Applying the Maximum Principle to this control problem gives the first-order conditions for optimality,[16] which include:

$$\partial H / \partial n_t = -\eta^t \partial C / \partial N_t \times l + \tau\lambda_{t+1} \leq 0 \text{ , } n_t \geq 0 \qquad \forall\ t \qquad [2.17]$$

$$(-\eta^t \partial C / \partial N_t \times l + \tau\lambda_{t+1}) \times n_t = 0$$

$$\lambda_{t+1} - \lambda_t = -\partial H/\partial q_t \qquad \forall\ t \qquad [2.18]$$

$$= -\eta^t \{R \times (\partial WTP^r / \partial q_t + \partial WTP^r / \partial L \times \partial L / \partial q_t) + B \times \partial WTP^b / \partial q_t\}$$

$$q_{t+1} - q_t = \partial H/\partial\lambda_{t+1} = -\theta + \tau n_t \qquad \forall\ t \qquad [2.19]$$

Condition [2.17] allows for a corner solution to the optimal amount of beach nourishment, thus the attendant complementary slackness condition. For beach nourishment to be undertaken in any period t ($n_t > 0$), optimality condition [2.17] requires that the present value of marginal cost of an additional unit of beach width be

[16] The Mangasarian Sufficiency Theorem requires that both $B \times WTP^b(q_t) + R \times WTP^r(q_t, L(q_t)) - C(N_t)$ and $-\theta + \tau n_t$ be differentiable and concave in n_t, jointly.

equal to the present shadow value of beach quality in the next period. That is, $(\eta^t \partial C / \partial N_t \times l) / \tau = \lambda_{t+1}$.

Equation [2.18] is a first-order difference equation that defines how the shadow value of beach quality evolves over time. Assuming aggregate marginal benefits of beach quality are strictly positive, equation [2.18] implies that the present shadow value of beach quality decreases over time, at a rate equal to the discounted value of the marginal benefits of beach quality. If marginal WTP is linear in q_t, then equation [2.18] is a first-order linear difference equation; otherwise, it is non-linear. Equation [2.19] imposes the transition equation on our control solution. It is a first-order, liner difference equation. Combining [2.17] and [2.18], we have:

$$\begin{aligned} &\eta^t (\partial C / \partial N_t \times l) / \tau = \\ &\lambda_t - \eta^t \{R[\partial WTP^r / \partial q_t + \partial WTP^r / \partial L \times \partial L / \partial q_t] + B \times \partial WTP^b / \partial q_t\} \end{aligned} \qquad [2.20]$$

which equates the present value of the marginal cost (RHS) of beach nourishment with the present value of its marginal benefits (LHS).

In addition to [2.17] - [2.19], we require a transversality condition to completely identify the time-paths of the state, control, and costate variables. There are two management strategies in this regard. We can require that the shadow value of beach quality be zero in the last period. That is:

$$\lambda_T \times q_T = 0 \qquad [2.21]$$

This condition requires that either (i) the present value of beach quality be driven to zero in the final period, or (ii) the resource stock be driven to zero in the final period. This condition embodies the notion that efficiency requires all surpluses from beach quality be extracted before the termination of control. Thus, the stock of the resource is determined such that it has no residual value in the final period. Alternatively, we could specify some terminal amount of beach quality. This may be a more reasonable approach for the short run problem, as the beach is a public resource that is not likely to be extirpated in the short run. In this case, the transversality condition is:

$$q_T = \overline{q} \qquad [2.21']$$

The present value Hamiltonian for the increasing sea level (long run) problem takes the following form:

$$\begin{aligned} H &= \eta^t \{B \times WTP^b(q_t) + R \times WTP^r(q_t, L(q_t)) - C(N_t)\} \\ &+ \lambda_{t+1}(-\theta_t + \tau n_t) \end{aligned} \qquad [2.16']$$

where θ_t follows either equation [2.14] or [2.15] and the τ parameter will vary dichotomously, both conditioned upon whether the critical level of sea level rise has been broached. The first-order conditions are equivalent to [2.17]-[2.29], with the exception being that the transition equation for beach quality [2.19] is now:

$$q_{t+1} - q_t = \partial H/\partial \lambda_{t+1} = n_t\tau - \underline{\theta} - \Delta\theta_t \quad \text{for } t < S_{crit}/\Delta S$$

$$q_{t+1} - q_t = \partial H/\partial\lambda_{t+1} = n_t\tilde{\tau} - \underline{\theta} - \Delta\tilde{\theta}_t \text{ for } t \geq S_{crit}/\Delta S, \qquad [2.19']$$

where S_{crit} is the critical level of aggregate sea level rise that threatens to flood the entire barrier island. The state path for beach quality only need be piecewise differentiable, so condition [2.19'] poses no difficulties for the beach erosion control problem.

In the long run, we are also interested in determining the terminal management time. If T is free, it should be chosen such that the Hamiltonian evaluated at the terminal time is zero (Chiang 1991). Since erosion is increasing monotonically with sea level rise, the nourishment costs of producing a given beach width, conditional on some arbitrary starting point, should be increasing monotonically as well. Economic returns from beach quality are represented by concave functions, and the parameters that define the extensive margins of beach use (B and R) are assumed constant. Thus, the benefits of beach quality are bounded, and the balance of benefits and costs implicitly defines the terminal time. In the long run, transversality condition [2.21] applies, but the introduction of a free terminal time, T, necessitates an additional transversality condition. If $\lambda_T=0$ then the condition is:

$$B \times WTP^b(q_{T-1}) + R \times WTP^r(q_{T-1}, L(q_{T-1})) = C(n_{T-1} \times l). \qquad [2.22]$$

Condition [2.22] indicates that the benefits of beach management must equal the costs in the penultimate period. Given that costs will be increasing monotonically and economic returns from beach quality are bounded, this condition implicitly defines the time at which beach nourishment should be abandoned. In the absence of beach nourishment, a policy of shoreline retreat is implicit. The other possible condition associated with [2.21], $q_T=0$, does not give rise to a reasonable transversality condition.

The following chapter focuses on estimating the preference function for beach visitors in [2.1] in the context of recreational beach demand. Chapter 4 presents a hedonic price analysis of coastal properties that is used to estimate the preference function for coastal homeowners defined implicitly in [2.2]. Chapter 5 first focuses on estimation of a beach nourishment cost function. Then, the components are assembled in an empirical analysis of optimal beach erosion management.

Chapter 3: Recreational Demand and Beach Quality

This chapter focuses on welfare effects stemming from changes in beach quality accruing to beach visitors. The goal of the analysis is to estimate an approximation of the preference function in [2.1]. Since this expression is meant to embody the preferences of the representative beach visitor, coastal residents (i.e. those residing on the barrier island) are excluded from the analysis. The approach utilized is that of a single-site recreational demand model using pre-existing survey data from two barrier islands in Georgia.

The data were gathered on-site over three seasons. The data include *ex-post* revealed trips to the beach under current conditions, as well as *ex-ante* stated number of trips under hypothetical improvement scenarios. A separate set of models is estimated for each island. First, the survey and data are described. Then, a theoretical model that incorporates particular aspects of the data is derived. The specific assumptions of the model are then discussed and the empirical results are presented. The last section outlines the procedures used to approximate a marginal value function for changes in beach quality, which is then integrated, producing an estimate of [2.1].

Beach Visit Demand Data

The recreational demand model uses travel cost and contingent behavior data gathered on-site of two barrier islands—Tybee and Jekyll—in Georgia in 1998. Data were gathered in the winter, spring, and summer. Surveyors intercepted individuals recreating at the beach and inquired about participation in the study. Participants' names and addresses were recorded, along with observable characteristics (i.e. gender, race, party size, etc.), age, the number of years they had been visiting the island, whether the beach was their primary destination, and one-way transit time to the island. Participants were given a survey and a postage-paid, return mailing envelope. If the survey was not returned within a month, a reminder postcard was mailed to the participant's home address. This was followed by a replacement survey two weeks later if the original survey was not returned (Dillman 1979).

The survey instrument was designed around a contingent choice question, allowing respondents to choose between current and improved beach conditions. The survey provided (i) background information on beach erosion, including the causes of beach erosion, potential results of erosion and how these results might affect recreation, and (ii) a review of management responses to the erosion problem. The hypothetical improvement scenarios offered amelioration of portions of the beach with the poorest quality (i.e. no beach at high tide or narrow beach at high tide). Most of the surveys were ambiguous as to how the improved beach would be attained, but a sub-sample of the surveys specified either beach nourishment or shoreline retreat as the prospective management tool.

Beach width on each island was measured at various transects during high and low tide; measurements were taken with a laser range finder during the summer of 1997. The distributions of current beach quality conditions, in terms of beach width, were described using maps and pictures, and also summarized in a table. The status quo distributions of beach quality conditions for each island are presented in column A of Tables 3.1a and 3.1b. (Note all measurements are at high-tide.) The improvement scenarios offered a decrease in the frequency of poor beach conditions along the shore of each island, as indicated in columns B and C of Tables 3.1a and 3.1b. As shown in Table 3.1, under Case *I* the worst portions of the beach are improved (i.e., those with no beach at high-tide), while Case *II* offers a more sizeable improvement of not only the worst, but also portions of the beach with moderately poor quality (those with an average beach width of 9 meters at high-tide on Tybee Island, and 11 meters at high-tide on Jekyll Island). Within the survey instruments, these improvement scenarios were described on the same page as the status quo, with the distribution of improved beach quality indicated on a map, described with pictures, and summarized in a table. The status quo case is denoted *'0'*; the moderate improvement scenario is denoted *'I'*, and the greater improvement scenario is denoted *'II'*.

Table 3.1a: Frequency Distributions of Beach Quality on Jekyll Island

Average Beach Width	*A Current (0)*	*B Case I*	*C Case II*
55 meters	0.46	0.46	0.46
22 meters	0.19	0.19	0.54
11 meters	0.11	0.35	0
0 meters	0.24	0	0
sum	1.00	1.00	1.00

Table 3.1b: Frequency Distributions of Beach Quality on Tybee Island

Average Beach Width	*A Current (0)*	*B Case I*	*C Case II*
28 meters	0.76	0.76	0.76
23 meters	0.07	0.07	0.24
9 meters	0.07	0.17	0
0 meters	0.1	0	0
sum	1.00	1.00	1.00

The beach quality indicators focus on average beach width, and its distribution along the shore. This, in turn, implies the amount of beach area available in a given scenario. The state of the beach resource under each scenario can be represented as a weighted average of the distribution of different beach types. For example, on Jekyll Island the average beach width implied by the current distribution of beach types is 31 meters. On Tybee Island, the average beach width under current conditions is 23.5 meters. I calculate these weighted averages by summing the average widths in each cell weighted by the proportions from the beach width distribution under the current scenario. I focus on the change in average beach width as a signal of the change in resource quality between the scenarios. Thus, I assume that average beach width adequately captures the state in the resource from the perspective of beach visitors. The increases in average beach width for Jekyll Island under the alternative scenarios are 2 meters for Case *I* and 6 meters for Case *II*. For Tybee Island, the improvement scenarios specify increases in average beach width of 1 meter for Case *I* and 3.5 meters for Case *II*. Again, these improvement scenarios were based on ameliorating areas of the shore with the poorest beach quality. Thus, the effect on average beach width was determined by the distribution of beaches with poor quality and the degree of improvement. The improvement scenarios were randomly distributed among respondents. Thus, each observation is associated with *only one* of the two improvement scenarios.

Associated with the improvement scenarios were increased parking fees, an element of on-site cost. Increased parking fees serve as the payment vehicle for the contingent choice portion of the survey. Pre-tests of the survey questionnaire indicated that increasing fees associated with the use of beach resources was a more realistic scenario for financing beach improvements than increasing overall taxes or price levels. The survey instrument presents increases in both daily and annual parking fees (ranging from 10 to 1200 percent) in conjunction with the improvements in beach quality. The prospective fee increases were randomly distributed among the subject pool. Increasing the cost of a visit under improved conditions, however, can induce a change in demand not attributable to the quality change. Visitation data, for both before and after the change in beach quality, were collected.

Visitation under the current conditions was elicited on the first page of the survey:

> 1. A. Over the last year, how many days did you visit this island?: _____ days
>   B. How many of these days involved an overnight stay?: ______ days
> 2. Do you currently own an annual parking sticker? ___ Yes ___No.

With this information, one can calculate the number of trips and the average days on site. I refer to this data as *ex-post* trip demand. Question number two tells us whether the household pays for parking on a daily or annual basis. To provide information on how visitation would change, a contingent behavior question inquiring about visitation under the alternative scenario of improved beaches was included after the valuation question. What follows is an excerpt from the survey instrument:

> The **alternative management** scenario would require an increase in the parking meter fee to *X* cents/hour, the city parking lot to \$*Y*/day or the annual pass to \$*Z*/year. This money would be used for beach nourishment or for financing a retreat policy.
>
> 5. Considering the beach conditions and the price of using the beach, which would you prefer to see at this island? (circle one)
>
> a. **Current Conditions** (at \$0.50/hour or \$5/day or \$50/year)
> b. **Alternative Management** (at \$*X*/hour or \$*Y*/day or \$*Z*/year)
>
> 6. In either scenario, residents and frequent users would most likely buy an annual pass while occasional users would not. What option would you choose? (circle one)
> a. Parking meter fee
> b. City parking lot
> c. Annual pass

7. Suppose that the **alternative management plan** happens and beach conditions in the right-hand map result. At the new fees of $*X*/hour or $*Y*/day or $*Z*/year, how would you change the number of days you visit this island in a one-year period? (circle one)
a. Visit the same number of days.
b. Reduce the days you visit. How many fewer days? ____ (fill in blank)
c. Increase the days you visit. How many more days? ____ (fill in blank)

Question five is the contingent choice valuation question, the responses to which are not utilized in this analysis. (See Landry, Keeler, and Kriesel (2003) or Kriesel, Keeler, and Landry (2003) for analysis of this data.) Question six allows us to estimate how payment arrangements change under the alternative scenario, and with responses to question seven, I construct what I refer to as *ex-ante* trip demand. Following the valuation and behavioral questions were a series of questions on recreational experience and the standard socio-economic characteristics. This analysis utilizes only the *ex-post* behavioral travel data and the *ex-ante* stated travel data.

Surveyors made initial contact with 6,101 households during the three seasons on the two barrier islands. Of the initial contacts, 3,228 surveys were returned, for a response rate of 52.9%. The dataset is comprised of 2,341 usable observations (i.e. those with complete data) on respondent's current visitation and stated number of visits under hypothetical, improved beach conditions. The distinct, but functionally equivalent, survey instruments for each island fully describe current beach conditions and offer one of two improvements in beach quality. Overall, there are six quality levels to be found in the pooled data. In addition, there are approximately 300 observations, a sub-sample of the main dataset, on travel and on-site costs incurred with current visitation. These data are used to estimate average trip cost per mile and average on-site cost per person, per day. These procedures are described in the subsequent empirical analysis. First, however, we must review the theory that supports the use of these data.

Model of Recreational Demand with On-site Time

McConnell (1992) explores the problem of on-site time in travel cost models. In contrast to McConnell, I attempt here to apply this model in a setting where multiple-day trips are common. The model is formulated on a single beach recreation site. (The models for each island are estimated separately.) Households choose v, the number of visits to the site, d_{os}, the number of days on-site per visit, and y, a Hicksian composite commodity, to maximize a quasi-concave utility function: $u(v,d_{os},y;q)$. The variable q represents beach quality, and is beyond the control of the household. Assuming no utility is produced by travel, beach visits have no effect on utility unless time is spent on-site. Likewise, on-site time cannot produce utility unless the household invests in at least one visit. We impose joint weak complementarity to account for this: $\partial u(v,0,y;q)/\partial v = \partial u(0,d_{os},y;q)/\partial d_{os} = 0$. Assuming households do not exhibit any non-use value for beach quality, we can make a further restriction involving joint weak complementarity between visits, on-site time, and beach quality: $\partial u(0,d_{os},y;q)/\partial q = \partial u(v,0,y;q)/\partial q = 0$.

Budget and time constraints are given by:

$$m = \overline{m} + wd_w = c_{tr}v + c_{os}vd_{os} + c_y y$$

$$D^* = d_{tr}v + d_{os}v + d_y y,$$

respectively, where c_i is the cost of travel for a visit (for $i=tr$), cost of a day on-site (for $i=os$), and cost of the Hicksian commodity (for $i=y$); m is total income, composed of unearned income, $\overline{m}$, and returns from labor, wd_w; w is the wage rate; d_i is the number of days spent working (for $i=w$), the number of days spent in transit per trip (for $i=tr$), the number of days spent at the recreation site per trip (for $i=os$), and the number of days spent consuming the composite commodity (for $i=y$); and D^* is the number of days available for consumption activities. That is, $D^* = D - d_w$, with D representing the total days in the planning period.

Total days spent at the beach site are calculated as $bdays = v \times d_{os}$. One could posit utility as a function of *bdays*, but this implies no preference over how days are dispersed within the planning period. It is highly plausible that household utility exhibits diminishing returns to both visits and days on-site, and thus the household's willingness to substitute visits for day on-site will depend upon their relative quantities. The form $u(v,d_{os},y)$ is more general.

Assuming the household chooses the number of days to devote to work, and substituting for $d_w = D - D^* = D - (d_{tr}v + d_{os}v + d_y y)$ in the budget constraint, we obtain an expression for full income:

$$m = \overline{m} + wD = (p_{tr} + p_{os}d_{os})v + p_y y,$$

where $p_{tr} = c_{tr} + wd_{tr}$, $p_{os} = c_{os} + w$, and $p_y = c_y + wd_y$. This constraint is nonlinear in visits/on-site time space. McConnell (1992) recognizes that only the interior portion of that constraint is relevant, due to joint weak complementarity. That is, it is not reasonable for a household to consume $d_{os} > 0$ if $v = 0$, because on-site days do not augment utility if no visits are taken; the household can save money by moving to a corner solution in visits/on-site time space. The same argument holds for $v > 0$ and $d_{os} = 0$, and it is thus reasonable to assume interior solutions for the population of current users.

The household's problem is:

$$\max_{v,d_{os},y} u(v, d_{os}, y; q) + \mu(m - (p_{tr} + p_{os} d_{os})v - p_y y) . \qquad [3.1]$$

The result of this optimization process is the indirect utility function $V(p_{tr},p_{os},p_y,m;q)$, or $V(p_{tr}^*,p_{os}^*,m^*;q)$, where the asterisks represent normalization by the price of the Hicksian commodity. I will work with the latter form from here, but drop the asterisks for ease of exposition. The price of a beach visit is $p_v = p_{tr}+p_{os}\times d_{os}$. The nonlinear budget constraint in [3.1] causes a breakdown of the standard system of Marshallian demand functions because one cannot ensure that the first-order conditions for maximization hold (Bockstael and McConnell 1983). One may easily verify, however, that Roy's Identity still applies for trip demand:

$$v(p_{tr},p_{os},m;q) = - [\partial V(p_{tr},p_{os},m;q)/\partial p_{tr}] \,/\, [\partial V(p_{tr},p_{os},m;q)/\partial m] \qquad [3.2]$$

(and we might note also for Hicksian commodity demand). Roy's identity does not apply for on-site time demand, and the on-site time demand function has no welfare significance because the expression for on-site time demand cannot be traced back to a unique on-site-time compensated demand function. McConnell (1992) further shows that measuring the area under the visit demand curve above price will still provide an estimate of consumer surplus attributable to the recreation site. Note that in doing so, the number of days on-site must be adjusted optimally as p_{tr} changes.

If on-site time is indeed endogenous, McConnell suggests treating on-site price as a parameter and directly estimating the visit demand equation on the left hand side of [3.2]. However, if on-site time varies across households and is endogenous, this regression will produce a biased estimate of the coefficient on p_{tr}.[17] This is problematic because β_{tr} figures prominently into estimation of consumer surplus. The standard linear regression model implied by [3.2] is

$$v(p_{tr},p_{os},m;q) = \beta_0 + \beta_{tr}\times p_{tr} + \hat{\beta}_{os} \times p_{os} + \beta_m \times m + \varepsilon,$$

where $\hat{\beta}_{os} = \beta_{os} \times \bar{d}_{os}$, with β_{os} representing the true coefficient and $\bar{d}_{os}$ representing the average days on-site, and $\varepsilon = u + \beta_{os}(p_{os} \times d_{os} - p_{os} \times \bar{d}_{os})$. Because d_{os} is by definition a function of p_{tr}, the coefficient β_{tr} will be biased, as will the coefficients of other variables which are correlated with d_{os}. Rather than the specification implied by [3.2], I specify trip demand as a function of the visit price $p_v = p_{tr} + p_{os} \times d_{os}$.

Using the area under the appropriately specified demand function above price, one can produce an estimate of the value of access to the study sites under various beach quality conditions. The specification of the relevant portion of the demand system is:

---

17 I thank Nancy Bockstael for her help on this important point.

$$v = v(p_{tr}+p_{os}\times d_{os},p_{subs},m,X;q) \quad [3.3]$$

$$d_{os} = d_{os}(p_{tr},p_{os},p_{subs},m,X;q), \quad [3.4]$$

where d_{os} is included in [3.3] to make clear that it is simultaneously determined. Thus, equation [3.3] is a quasi-Marshallian demand curve. An instrumental variables approach can be used to account for the endogeneity of on-site time. The variable p_{subs} represents the price of travel to a substitute site, and X is a vector of socio-economic covariates associated with the household.

As recognized by McConnell (1992), the assumption that on-site price (p_{os}) is constant is made for convenience, and in many cases may be wrong. In general, direct on-site costs (c_{os}—a component of p_{os}) are likely to depend upon endogenous attributes of the trip, such as the quality of accommodations and party size. Endogeneity of the on-site price variable is another argument for dispensing with the demand equation in [3.2], but will also lead to biased and inconsistent estimates for at least some of the parameters in [3.3] and [3.4]. Moreover, lacking detailed information on on-site costs for each observation in the sample necessitates using an estimated value (as will be done in the empirical analysis to follow). This will introduce measurement error into the on-site price variable. Luckily, all that is required for consistent estimation of [3.3] is the total on-site expenditures—$p_{os}\times d_{os}$ – which suggests the following form for [3.4]

$$p_{os}\times d_{os} = p_{os}\times d_{os}(p_{tr},p_{subs},m,X;q) \quad [3.4']$$

Assuming a suitable instrument can be found, the linear prediction from equation [3.4'] can be used to define an exogenous visit price—$p_v = p_{tr} + \Theta$, where Θ is the predicted on-site expenditures–which can be used to consistently estimate the trip demand function in [3.3].

This demand system of [3.3] and [3.4'] is conditional on q, and presumably will change with changes in q and p_{os}. Since beach quality is a viewed as a good, we expect that consumer surplus will increase with q, controlling for on-site price. Since we have measures of *ex-post* visitation and projections of *ex-ante* visitation under different levels of q, we can estimate this demand system for each of the beach quality levels. A marginal value function for beach visitors can be estimated by calculating the change in consumer surplus per unit increase in beach width conditional on the final width.

In mathematics, the approximation of the value of access is:

$$CS = \int_{p_{tr}^{0}}^{p_{tr}^{ch}(q^j)} v(p_{tr}^{j} + \Theta^j, p_{subs}, m^j, X; q^j)\, dp_{tr}, \qquad [3.5]$$

where CS represents consumer surplus, $p_{tr}{}^{0}$ is the current trip price, $p_{tr}{}^{ch}(q^j)$ is the choke price at which trip demand is zero, for $j=0$ (current beach quality) and $j=I$ or $j=II$ (subsequent beach quality). Again, Θ^j is predicted on-site expenditures from the first-stage instrumental variables regression [3.4']. In order to estimate [3.5], we require only the *ex-post* demand and *ex-ante* stated demand curves.

The accuracy and uniqueness of [3.5] as a welfare estimate depends upon: (i) the price flexibility of income (i.e. how does marginal willingness to pay change as income changes?), and (ii) the size of consumer surplus attributable to access to the site relative to consumption of the numeraire (Randall and Stoll 1980), as well as (iii) substitution possibilities among the non-market resource (i.e. recreation site) and private goods (Hanemann 1991). In particular, if marginal willingness to pay for visits to the site does not change with income and the site is not unique (implying lots of substitution possibilities between the non-market resource and private goods) then [3.5] will be equivalent to willingness-to-pay for access. In general, consumer surplus will be a reasonable approximation to the Hicksian compensated welfare measure if marginal willingness to pay for visits to the site are relatively unresponsive to changes in income, if the total value of access is relatively small (less than or equal to about 5 percent) compared to consumption of the composite, and if the site is not particularly unique.

Recreational Demand, Beach Visitors, and Beach Quality

There are a number of practical issues that must be addressed in estimating the single-site demand model of [3.3] and [3.4']. First, residents of Tybee and Jekyll Island are removed from the dataset, since the purpose of the analysis is to focus on beach visitors. Next, the data must be cleaned of observations that do not appear to be primarily related to the demand for beach recreation. This requires a subjective assessment of each household's number of trips, time spent on-site, and distance traveled. Excluded from the analysis are those respondents who spent over six weeks on-site, those that made over 50 trips to the island, and those that traveled from the far western United States or other countries. Households spending over six weeks on-site or making over 50 trips are less likely to be doing so for beach recreation and more likely to be working on the island. Households traveling from great distances are less likely to be doing so for the sole purpose of beach recreation, but rather are likely to be engaging in many types of activities, of which beach recreation is only a portion of the value attributable to travel cost. While these assumptions can be criticized for their subjective nature, they are arguably more innocuous than doing nothing to purge the data of dubious observations.

The sample must also be tailored to address a potential problem for accounting welfare effects from changes in beach quality across the two user groups—visitors and coastal homeowners. Specifically, there exists a potential for double counting when the recreational demand data are combined with the other benefits component of the optimal control model. As indicated in Chapter 2, a hedonic model will be used to estimate coastal homeowners' demand for beach quality. The hedonic approach presumes that beach quality is capitalized in the market price of coastal housing. Asset theory posits the market price of housing as the present discounted value of the stream of property rents in perpetuity. Thus, there is a close theoretical relationship between market price and rental rate; however, the sales and rental markets are, in reality, distinct. One would imagine that the rental rates of coastal properties would similarly adjust with beach quality, but it is difficult to know *a priori* how this adjustment would play out. Nevertheless, the increase in consumer surplus accruing to those visitors that rent beach houses could very well be partially (or wholly) captured by landlords. In this case, the two welfare measures—

that associated with recreational users and that associated with coastal homeowners—would overlap, thus the potential for double counting.

To circumvent this problem, visitors that rent beach houses could be discarded from the analysis, but such visitors must be identified. A stylized fact is helpful in this regard: beach houses are typically only rented on a weekly basis, and weekly rates are usually competitive with (if not cheaper than) other accommodations. It seems reasonable to assume visitors staying over five days will tend to rent beach homes, while those staying five days or less will utilize other accommodations (hotels or campgrounds). To remove the potential for double counting, visitors staying over five days, on average, are removed from the dataset. One can be reasonably sure that the remaining data points represent visitors that are not participants in the beach house rental market.

As aforementioned, a subset of the data included travel expenditures. As an additional survey item to a portion of the intercepted beach visitors, 492 expenditure surveys were randomly distributed. The response rate for the expenditure survey was higher (72%) than the overall contingent choice survey response rate. The expenditure survey apportioned expenditures into transit and on-site costs, so that these costs could be separated accordingly. Average vehicle transit costs per mile and average on-site costs per person, per day were calculated for each island separately. The average automobile transit cost for Jekyll Island visitors is $0.3283 per mile. The average automobile transit cost for Tybee Island visitors is $0.3795 per mile. Average on-site costs per person, per day were $31.18 on Jekyll Island and $24.57 on Tybee Island.

Monetary transit costs were calculated assuming an average rate of speed of 45 miles per hour. Transit and on-site time costs were approximated using a portion of the wage rate and assuming an average workday of 8 hours. Annual income was indicated by categorical response to a survey question, and the midpoint was used as an estimate of net household income. Daily wage per household was calculated as *(annual income)/250*, where 250 represents annual work days minus two-weeks vacation. Full income was estimated as *260×(daily household wage)*. The survey questionnaire inquired about foregone income associated with the beach trip. For respondents that indicated they gave up a chance to earn income, travel and on-site time was valued at the full daily wage. For others, transit and on-site time was valued at $^1/_3$ the daily household wage. The transit and daily on-site prices where calculated as $p_{tr} = c_{tr} + \alpha w d_{tr}$ and $p_{os} = c_{os} + \alpha w$, where $\alpha = 0.33$ or 1 depending upon the respondent. In all, 1946 observations were available for analysis—822 for Jekyll Island and 1124 for Tybee Island.

Parking fees at each island were payable on either a daily or annual basis (see survey excerpt, above). Thus, in some cases parking is part of the on-site cost of visiting the beach, while in others it is a lump-sum investment made before any visits are undertaken (a subtraction from income). The current parking cost arrangement (daily or annual) is elicited in the survey, and the present expenditures on daily parking should be embedded in the on-site cost data. For purposes of assessing the value of beach improvements, information on the desired method of payment (daily or annual) under subsequent conditions was elicited. In order to completely identify prices and income in the status quo and alternative scenario, we must consider four cases: (i) those who pay on a daily basis under the current conditions and will continue to do so under the improvements, (ii) those who currently pay on an annual basis and will continue to, and those who switch one way (iii) or the other (iv). This complexity can be addressed by considering two monetary sources of welfare effects, price and income changes. See Table 3.2 for a breakdown of the cost changes.

Table 3.2: Price and Income Changes Associated with Current and Projected Quality Levels and Parking Fees

Current fee *(q^0)*	Projected fee *(q^j)*	Price change	Income change
Daily	Daily	$P_{daily}{}^{j} - P_{daily}{}^{0}$	0
Annual	Annual	0	$P_{annual}{}^{0} - P_{annual}{}^{j}$
Daily	Annual	$-P_{daily}{}^{0}$	$-P_{annual}{}^{j}$
Annual	Daily	$P_{daily}{}^{j}$	$P_{annual}{}^{0}$

The *'0'* superscript represents current conditions and the *'j'* superscript represents subsequent conditions.

For households that currently pay on a daily basis and will continue to do so under the subsequent conditions, the price change is simply the additional daily cost imposed. There is no income change stemming from the policy scenario for this group. For households that currently pay on an annual basis and will continue to do so, the income change reflects the incremental cost of an annual pass, and there is no relevant on-site price change stemming from the policy scenario. For households that indicate a change in their preferred method of payment, there are both price and income changes associated with the improvement scenario. Those currently paying the daily fee, but preferring the annual fee under subsequent conditions, witness a decrease in an on-site cost parameter equal to the current daily fee and a decrease in income equal to the subsequent pass price. Those currently paying the annual fee, but preferring the daily fee under subsequent conditions, witness an increase an on-site cost parameter equal to the subsequent daily fee and an increase in income equal to the current pass price. Income and price changes combined with the change in beach quality encompass the parametric changes in the model. Variations in visits, on-site time, and utility encompass the endogenous changes.

Other covariates included in the model are the travel price to a substitute site, a dummy variable indicating whether the trip was taken for multiple purposes (i.e. not solely for the purpose of beach recreation), and seasonal dummy variables. The multi-purpose trip dummy variable is meant to represent the unobserved increment in the price of travel due to joint consumption activities (Parsons and Wilson 1999). Substitute price (p_{subs}) measures only the transit cost to an alternative beach site. Data on on-site time and expenditures at the alternative site were not available. Roughly 40 percent of the survey respondents did not indicate a substitute site. These respondents were assigned a substitute site based on their state and city cohort (and the *assign* dummy variable was set to 1). Thus, there is some error in the p_{subs} variable. Other covariates that could be included in the model are the number of years the household has been visiting the island and socio-economic characteristics (age, gender, race and education). Education was measured by a dummy variable set equal to one if the head of household had at least some college, zero otherwise. All of these variables, except the price of substitutes, are included in the vector X. Descriptive statistics for the raw data are displayed in Column A of Tables 3.3a and 3.3b.

On-site expenditures ($p_{os} \times d_{os}$) in equation [3.3] is an endogenous variable. While average days spent on-site is elicited in the survey, on-site price at each island must be estimated. On-site price is calculated as the product of average price per

Table 3.3a: Descriptive Statistics for Jekyll Island

Variable	Definition	Mean A (Raw)	Mean B(Weighted)*
v0	*ex-post* revealed trip demand (visits)	4.26 (7.14)	3.63 (6.43)
v1	*ex-ante* stated trip demand (visits)	2.78 (5.31)	2.48 (4.80)
$d_{os}0$	*ex-post* revealed on-site days demand	2.42 (1.44)	2.53 (1.46)
$d_{os}1$	*ex-ante* stated on-site days demand	2.10 (1.57)	2.23 (1.59)
p_{tr}	transit price (1998$)	412.83 (461.39)	520.46 (561.96)
$p_{os}0$	*ex-post* on-site-day price (1998$)	229.89 (113.72)	240.40 (122.96)
$p_{os}1$	*ex-ante* on-site-day price (1998$)	237.32 (114.55)	247.84 (123.89)
p_v0	*ex-post* price per visit (1998$)	894.00 (672.91)	1050.32 (796.75)
p_v1	*ex-ante* price per visit (1998$)	964.27 (687.60)	1122.80 (794.06)
finc0	*ex-ante* full income (1000s 1998$)	57.190 (29.194)	59.564 (29.954)
finc1	*ex-post* full income (1000s 1998$)	57.179 (29.199)	59.554 (29.959)
p_{subs}	travel price to substitute site (1998$)	366.26 (430.61)	440.92 (529.23)
assign	dummy variable=1 if substitute assigned, 0 otherwise	0.3795 (0.4856)	0.4071 (0.4927)
mpurpose	dummy variable=1 is multiple purpose trip, 0 otherwise	0.0912 (0.2881)	0.1150 (0.3199)
spring	dummy variable=1 if spring season, 0 otherwise	0.2384 (0.4264)	0.1975 (0.3994)
summer	dummy variable=1 if summer season, 0 otherwise	0.6496 (0.4774)	0.6770 (0.4690)
years	Number of years the household has visited Jekyll Island	9.70 (12.23)	8.91 (11.85)
age	age of head of household (in years)	42.76 (12.79)	43.22 (12.86)
male	dummy variable=1 if male respondent, 0 otherwise	0.3929 (0.4887)	0.4023 (0.4918)
nonwhite	dummy variable=1 if nonwhite respondent, 0 otherwise	0.0340 (0.1815)	0.0350 (0.1842)
heduc	dummy variable=1 if college educated, 0 otherwise	0.6594 (0.4742)	0.6693 (0.4718)

* The inverse of the expected value of trips is used as a weight to correct for endogenous stratification.

Table 3.3b: Descriptive Statistics for Tybee Island

Variable	Definition	Mean A (Raw)	Mean B(Weighted)*
v0	*ex-post* revealed trip demand (visits)	9.45 (11.84)	8.14 (11.19)
v1	*ex-ante* stated trip demand (visits)	7.15 (10.51)	6.21 (9.84)
$d_{os}0$	*ex-post* revealed on-site days demand	1.71 (1.16)	1.81 (1.23)
$d_{os}1$	*ex-ante* stated on-site days demand	1.52 (1.25)	2.61 (1.33)
p_{tr}	transit price (1998$)	295.25 (440.99)	426.01 (623.53)
$p_{os}0$	*ex-post* on-site-day price (1998$)	196.59 (100.55)	207.99 (114.23)
$p_{os}1$	*ex-ante* on-site-day price (1998$)	206.79 (101.51)	218.43 (115.22)
p_v0	*ex-post* price per visit (1998$)	587.20 (557.34)	748.66 (752.23)
p_v1	*ex-ante* price per visit (1998$)	657.34 (568.06)	821.12 (753.77)
finc0	*ex-ante* full income (1000s 1998$)	59.053 (29.292)	60.47 (30.334)
finc1	*ex-post* full income (1000s 1998$)	59.021 (29.292)	60.442 (30.335)
p_{subs}	travel price to substitute site (1998$)	332.11 (479.10)	433.24 (674.54)
assign	dummy variable=1 if substitute assigned, 0 otherwise	0.4306 (0.4954)	0.4261 (0.4965)
mpurpose	dummy variable=1 is multiple purpose trip, 0 otherwise	0.1779 (0.3826)	0.2271 (0.4206)
spring	dummy variable=1 if spring season, 0 otherwise	0.3220 (0.4675)	0.3143 (0.4662)
summer	dummy variable=1 if summer season, 0 otherwise	0.5222 (0.4997)	0.5269 (0.5013)
years	Number of years the household has visited Tybee Island	13.46 (15.31)	12.23 (14.99)
age	age of head of household (in years)	40.56 (13.40)	40.81 (13.29)
male	dummy variable=1 if male respondent, 0 otherwise	0.3790 (0.4854)	0.3909 (0.4899)
nonwhite	dummy variable=1 if nonwhite respondent, 0 otherwise	0.0489 (0.2158)	0.0488 (0.2163)
heduc	dummy variable=1 if college educated, 0 otherwise	0.6823 (0.4658)	0.6840 (0.4668)

* The inverse of the expected value of trips is used as a weight to correct for endogenous stratification.

person, per day and party size (recorded in the survey data). The number of children in the household is used as an instrument for on-site expenditures. Number of children should be correlated with on-site expenditures, but not with the number of visits to the island, so assuming it has some explanatory power in equation [3.4'] it should serve as a suitable instrument. An OLS regression of the exogenous covariates of [3.3] and the number of children in the household on on-site expenditures is used to predict on-site expenditures for Case *0*, while the tobit model is used for Cases *I* and *II*. These regression results are included in Tables A.1a and A.1b of the Appendix. The predicted value of on-site expenditures (Θ) is utilized in place of $p_{os} \times d_{os}$ in [3.3]. With the predicted values of on-site expenditures on hand, equation [3.3] can be estimated for the three levels of beach quality (q) on each island. The estimation procedure must take account of correlation among trip decisions, censoring of *ex-ante* trip demand, the truncation of non-users, and endogenous stratification due to on-site sampling. Each observation contains data on status quo visitation, while roughly 30% of the observations contain additional stated preference data associated with Case *I*, with the remaining 70% associated with Case *II*.

After estimating the demand equations for each island, I combine the results of the two models in estimation of a marginal value function for beach quality. Marginal WTP is approximated by the change in consumer surplus per unit change in average beach width. Change in consumer surplus per unit of beach width is estimated for each household in each sample, and is conditioned on the vector of covariates. The marginal value of beach quality is assumed to be decreasing in total quality (beach width). I thus regress subsequent beach quality—33 meters for Jekyll Island Case *I*, 37 meters for Jekyll Island Case *II*, 24.5 meters for Tybee Island Case *I*, and 27 meters for Tybee Island Case *II*—on the estimated marginal valuation of beach quality and expect a negative coefficient on the subsequent beach quality variable. To make use of information from both series of demand equations in estimating the slope of the marginal value function, I use a fixed effects model.

Econometric Specification for Beach Visit Demand Model

We begin by specifying a family of visit demand equations as in [3.3], each associated with a different level of beach quality. We have:

$$v^j = v^j(p_{tr} + \Theta^j, p_{subs}, m^j, X; q^j), \qquad [3.6]$$

where $j=0$ for the status quo conditions, $j=I$ for the first alternative (modest improvement level), and $j=II$ for the second alternative (greater improvement level). The symbol Θ^j represents the predicted value of on-site expenditures from the first-stage regressions. Equation [3.6] could be estimated by two-stage least squares, but this procedure fails to account for certain aspects of the data—correlation of error term across the *ex-post* and *ex-ante* trip demand, censoring of trip demand, truncation of non-users, and endogenous stratification. Recall that we will have two such series of demand curves, one for Jekyll and one for Tybee Island.

The demand equations in [3.6] are assumed additively separable in deterministic and random components, and are given by:

$$v_i^j = z_i'\beta^j + \varepsilon_i^j \qquad [3.7]$$

for $j=0$, I, II, where $i=1,..,N$ demand observations for case 0, $i=1,..,N_I$ demand observations for case I, $i=1,..,N_{II}$ demand observations for case II, with $N_I + N_{II} = N$; z is a $(k+1) \times 1$ vector of covariates, β^j is a $(k+1) \times 1$ vector of constant but unknown coefficients, and ε^j is an independent and identically distributed error term for each j. The errors will, however, be correlated across cases 0 and I, and 0 and II, because each household appears twice in the dataset. Stacking the ε in [3.7] we have E, and assume $E \sim$ bivariate normal $(0,0,\sigma^0,\sigma^j,\hat{\rho})$ for $j = I$ and II. As such,

$$\begin{pmatrix} v_i^0 \\ v_i^j \end{pmatrix} \sim N\left(\begin{pmatrix} z_i'\beta^0 \\ z_i'\beta^j \end{pmatrix}, \begin{pmatrix} (\sigma^0)^2 & \hat{\rho}^j\sigma^0\sigma^j \\ \hat{\rho}^j\sigma^0\sigma^j & (\sigma^j)^2 \end{pmatrix}\right), \qquad [3.8]$$

for $j = I$ and II, where $\hat{\rho}^j$ is the correlation coefficient between Case 0 and Case j. The joint probability density function for beach visits under both the status quo and improvement scenario can be written as the product of the conditional and marginal density functions: $f(v^j,v^0) = f(v^j|v^0)f(v^0)$. The random variable

$$v_i^j \mid v_i^0 \sim N(z_i'\beta^j + \hat{\rho}^j\sigma^j / \sigma^0(v_i^0 - z_i'\beta^0), (\sigma^j)^2(1-(\hat{\rho}^j)^2)). \qquad [3.9]$$

The likelihood function for this problem can be written as:

$$L=\prod_{i=1}^{N}[1/(\sigma^{I}(1-(\hat{\rho}^{I})^{2})^{1/2})\phi((v_i^{I}-z_i{}'\beta^{I}-\hat{\rho}\sigma^{I}/\sigma^{0}(v^{0}-z_i{}'\beta^{0}))/(\sigma^{I}(1-(\hat{\rho}^{I})^{2})^{1/2})\times$$
$$1/\sigma^{0}\phi((v^{0}-z_i{}'\beta^{0})/\sigma^{0})]^{(1-CaseII)}$$
$$\times[1/(\sigma^{II}(1-(\hat{\rho}^{II})^{2})^{1/2})\phi(v_i^{II}-z_i{}'\beta^{II}-\hat{\rho}\sigma^{II}/\sigma^{0}(V^{0}-z_i{}'\beta^{0}))/(\sigma^{II})(1-(\hat{\rho}^{II})^{2})^{1/2})\times$$
$$1/\sigma^{0}\phi((v^{0}-z_i{}'\beta^{0})/\sigma^{0})]^{CaseII}$$

[3.10]

where N is the number of observations, ϕ represents the standard normal probability density, and *CaseII* is a dummy variable = 0 for alternative *I* and =1 for alternative *II*.

The likelihood function in [3.10] accounts for correlation across the *ex-post* and *ex-ante* trip demands, but does not account for censoring or endogenous stratification. I will first address censoring of *ex-ante* trip demand. Corner solutions in *ex-ante* trip demand are found in the following frequencies—13 percent for Jekyll Island Case *I*, 15 percent for Jekyll Island Case *II*, 14 percent for Tybee Island Case *I*, and 12 percent for Tybee Island Case *II*. Applying the tobit model to the *ex-ante* demand data, we have:

$$E(v_i\,|\,censoring)=\Phi(-z_i{}'\beta/\sigma)\times 0+$$
$$(1-\Phi(-z_i{}'\beta/\sigma))\times[z_i{}'\beta+\sigma(\phi(-z_i{}'\beta/\sigma)/(1-\Phi(-z_i{}'\beta/\sigma)))]$$
$$=\Phi(z_i{}'\beta/\sigma)\times z_i{}'\beta+\sigma\times\phi(-z_i{}'\beta/\sigma)$$

$$Var(v_i\,|\,censoring)=\sigma^{2}(1-\Phi(-z_i{}'\beta/\sigma))\times$$
$$[(1-\kappa)+(-z_i{}'\beta/\sigma-\phi(-z_i{}'\beta/\sigma)/(1-\Phi(-z_i{}'\beta/\sigma)))^{2}\Phi(-z_i{}'\beta/\sigma)]$$

[3.11]

where $\kappa = \{\phi(-z_i'\beta/\sigma)/(1-\Phi(-z_i'\beta/\sigma))\}^2 - \phi(-z_i'\beta/\sigma)/(1-\Phi(-z_i'\beta/\sigma))\times(-z_i'\beta/\sigma)$, and Φ represents the standard normal cumulative distribution function. The j superscripts have been suppressed, but equations [3.11] apply for cases I and II. Following Amemiya (1973), the likelihood function can be written as:

$$L = [\prod_{i=1}^{\hat{N}} 1/(\sigma^I (1-(\hat{\rho}^I)^2)^{1/2})\phi((v_i^I - z'_i\beta^I - \hat{\rho}^I\sigma^I/\sigma^0(v_i^0 - z_i'\beta^0))/(\sigma^I(1-(\hat{\rho}^I)^2)^{1/2}))\times$$
$$1/\sigma^0\phi((v_i^0 - z_i'\beta^0)/\sigma^0)$$
$$\times\prod_{i=1}^{N^*}(1-\Phi((z'_i\beta^I + \hat{\rho}^I\sigma^I/\sigma^0(v_i^0 - z_i'\beta^0))/(\sigma^I)(1-(\hat{\rho}^I)^2)^{1/2})\times$$
$$1/\sigma^0\phi((v_i^0 - z_i'\beta^0)/\sigma^0)]^{(1-CaseII)}$$
$$\times[\prod_{i=1}^{\hat{N}} 1/(\sigma^{II}(1-(\hat{\rho}^{II})^2)^{1/2})\phi((v_i^{II} - z_i'\beta^{II} - \hat{\rho}^{II}\sigma^{II}/\sigma^0(v_i^0 - z_i'\beta^0))/(\sigma^{II})(1-(\hat{\rho}^{II})^2)^{1/2}))\times$$
$$1/\sigma^0\phi((v_i^0 - z_i'\beta^0)/\sigma^0)$$
$$\times\prod_{i=1}^{N^*}(1-\Phi((z_i'\beta^{II} + \hat{\rho}^{II}\sigma^{II}/\sigma^0(v_i^0 - z_i'\beta^0))/(\sigma^{II})(1-(\hat{\rho}^{II})^2)^{1/2})\times$$
$$1/\sigma^0\phi((v_i^0 - z_i'\beta^0)/\sigma^0)]^{CaseII}$$

[3.10’]

where $v^j > 0$ for all $v^j \in \hat{N}$, $v^j = 0$ for all $v^j \in N^*$, and $\hat{N} + N^* = N$. The first four lines of [3.10’] correspond with Case I, while the last four correspond with Case II. Within each case, the first product sign relates to the non-limiting observations, while the second relates to the censored observations.

Since the beach visitor data were gathered on-site, we have two inherent problems: (i) frequent users of the resource will be over-represented; and (ii) the data do not include non-users of the resource. The first problem causes biased regression coefficients; the second excludes potential beneficiaries of improvement projects because current non-users may become active users after the improvement. This could be especially important for beaches, as more beach area provides for less congestion and thus more visitors. Shaw (1988) explains how to correct for endogenous stratification due to on-site sampling. He shows that the on-site density of trips is related to the population trip density by:

$$h(v_k) = [v_k \times f(v_k)] / \int_0^\infty \upsilon f(\upsilon) d\upsilon, \qquad [3.12]$$

where v_k represents trips taken, $f(v_k)$ is the population density, and $h(v_k)$ is the observed on-site density. Equation [3.12] indicates that individual observations need to be weighted by the inverse of the expected value of trips. In general, it makes no difference whether we use the expected value of a censored or truncated variable, because the truncation correction factor ($1/Prob(v_k>0)$) appears in both the numerator and denominator of [3.12]. For the likelihood function in [3.10'], however, we are conditioning on the truncated data. In this case, the truncated mean should be used in equation [3.12]. This weight should also be used in calculation of summary statistics; the weighted means are presented in column B of Tables 3.3a and 3.3b.

Incorporating [3.12] into our likelihood function, we arrive at the following:

$$L = [\prod_{i=1}^{\hat{N}} 1/(\sigma^{I}(1-(\hat{\rho}^{I})^{2})^{1/2})\phi((v_i^{I} - z'_i \beta^{I} - \hat{\rho}^{I}\sigma^{I}/\sigma^{0}(v_i^{0} - z_i'\beta^{0}))/(\sigma^{I}(1-(\hat{\rho}^{I})^{2})^{1/2})) \times$$
$$v_i^0 \times 1/\sigma^0 \phi((v_i^0 - z_i'\beta^0)/\sigma^0)/\widetilde{v}_i^0$$

$$\times \prod_{i=1}^{N^*} (1 - \Phi((z'_i \beta^{I} + \hat{\rho}^{I}\sigma^{I}/\sigma^{0}(v_i^{0} - z_i'\beta^{0}))/(\sigma^{I})(1-(\hat{\rho}^{I})^{2})^{1/2}) \times$$
$$v_i^0 \times 1/\sigma^0 \phi((v_i^0 - z_i'\beta^0)/\sigma^0)/\widetilde{v}_i^0]^{(1-CaseII)}$$

$$\times [\prod_{i=1}^{\hat{N}} 1/(\sigma^{II}(1-(\hat{\rho}^{II})^{2})^{1/2})\phi((v_i^{II} - z_i'\beta^{II} - \hat{\rho}^{II}\sigma^{II}/\sigma^{0}(v_i^{0} - z_i'\beta^{0}))/(\sigma^{II})(1-(\hat{\rho}^{II})^{2})^{1/2})) \times$$
$$v_i^0 \times 1/\sigma^0 \phi((v_i^0 - z_i'\beta^0)/\sigma^0)/\widetilde{v}_i^0$$

$$\times \prod_{i=1}^{N^*} (1 - \Phi((z_i'\beta^{II} + \hat{\rho}^{II}\sigma^{II}/\sigma^{0}(v_i^{0} - z_i'\beta^{0}))/(\sigma^{II})(1-(\hat{\rho}^{II})^{2})^{1/2}) \times$$
$$v_i^0 \times 1/\sigma^0 \phi((v_i^0 - z_i'\beta^0)/\sigma^0)/\widetilde{v}_i^0]^{CaseII}$$

[3.10’’]

where $\widetilde{v}_i^0$ represents the expected value of the truncated *ex-post* trips variable. This likelihood function is identical to [3.10’], except that the density for *ex-post* trips has been modified following [3.12]. This likelihood function accounts for correlation across *ex-post* revealed demand and *ex-ante* stated demand, and is thus efficient. The likelihood function incorporates the censored and truncated nature or the recreational demand data. Moreover, applying Shaw’s correction for on-site sampling, our likelihood function should produce unbiased parameter estimates that can be utilized in welfare analysis for current resource users. With additional assumptions, the results can be extrapolated to the population of potential resource users for analysis of coastal erosion management policies.

The improvement scenarios introduce multiple changes that must be considered in computing welfare effects. While the resource is improved, which we might expect could increase visitation on both the intensive and extensive margin, parking fees have increased, which could diminish visitation. In valuation of the resource change, the former effect is what we want to capture, while the latter is something we wish to control. In the raw data, 59% (63%) or respondents indicated that they would continue to visit Jekyll (Tybee) Island in the same frequency after the change (improved beach and higher parking fees). Thirty-four percent (thirty-one percent) indicated they would visit Jekyll (Tybee) less, while 7% (6%) indicated they would increase their visits. It seems reasonable to assume households that exhibit a decrease in *ex-ante* demand are reacting to the price change and those that exhibit increasing demand are reacting to the resource improvement. In the analysis of welfare effects, predicted *ex-ante* demand will be adjusted to reflect *ex-post* visit price and income levels.

Valuation of Changes in Beach Quality

The likelihood function in [3.10''] was maximized using the Newton-Raphson algorithm. The estimation results for each island are presented in Tables 3.4a and 3.4b. A number of household characteristics—years of visitation, age, gender, race and education—were dropped from the model due to statistical insignificance. The likelihood ratio test statistics for joint significance are 233.89 for the Jekyll Island model and 328.63 for the Tybee Island model, each with 23 degrees of freedom, indicating that the estimated coefficients are jointly different from zero at conventional significance levels ($p<0.00001$). For the Jekyll Island demand model,

Table 3.4a: Jekyll Island Visit Demand Equations

	A. *Ex-post* (*0*)	B. *Ex-ante* (*I*)	C. *Ex-ante* (*II*)
p_v	-0.004328***	-0.002234***	-0.001770***
	(0.000509)	(0.000872)	(0.000543)
p_{subs}	0.001577**	0.000414	0.000674
	(0.000732)	(0.001201)	(0.000815)
finc	0.016644***	0.006168	0.014627**
	(0.007145)	(0.011725)	(0.008308)
mpurpose	-2.084514***	-2.477554***	-1.356066**
	(0.665073)	(1.029033)	(0.822407)
assign	-0.722527*	1.150049*	1.695234***
	(0.045765)	(0.644758)	(0.435605)
spring	1.378865**	0.241793	0.775758
	(0.678088)	(1.016182)	(0.889006)
summer	-0.475521	-0.725623	0.097641
	(0.614343)	(0.924572)	(0.824842)
nourish	-------------	-------------	0.194224
			(0.489105)
retreat	-------------	-------------	-0.446991
			(0.505043)
intercept	4.611654***	3.12404***	1.232113***
	(0.713332)	(1.134599)	(0.879124)
sigma	5.196044***	5.167735***	5.085512***
	(0.095290)	(0.219162)	(0.149857)
$(\text{covariance})^{½}$	-------------	15.77768***	10.98662***
		(1.254723)	(0.904818)

N = 822; lnL = -5737.5629; LRT(df=23) = 233.89; *=statistically significant at α=10%; **=statistically significant at α=5%; ***=statistically significant at α=1%

all visit price coefficients are negative and statistically significant under a one-tailed hypothesis test. A $1000 increase in the price of a visit leads to an estimated decrease of 4.3 beach trips before the improvement in beaches (*ex-post* demand), while a $1000 increase in the price of a visit decreases trips by 2.2 and 1.8 trips in the Case *I* and Case *II* visit demand models (*ex-ante* demand), respectively. Each of these coefficients is associated with demand after the beach has been improved, and the shallower slope coefficient is associated with the greatest level of improvement in

Table 3.4b: Tybee Island Visit Demand Equations

	A. *Ex-post* (*0*)	B. *Ex-ante* (*I*)	C. *Ex-ante* (*II*)
p_v	-0.007971***	-0.005427***	-0.005703***
	(0.000677)	(0.000970)	(0.000908)
p_{subs}	-0.000217	0.000022	-0.000329
	(0.000758)	(0.000914)	(0.001233)
finc	0.034040***	0.016507	0.041427***
	(0.009563)	(0.014795)	(0.011655)
mpurpose	-2.785504***	-0.700112	-2.79633***
	(0.711576)	(1.227867)	(0.874234)
assign	0.642668	1.910437**	1.716736***
	(0.530703)	(0.869918)	(0.650779)
spring	-0.231885	1.494251	0.543622
	(0.814111)	(1.148898)	(1.157978)
summer	-1.291379*	-0.213842	-1.345634
	(0.767210)	(1.103982)	(1.101336)
nourish	-------------	-------------	1.680381**
			(0.673106)
retreat	-------------	-------------	0.411526
			(0.683324)
intercept	8.942191***	4.720708***	5.050656***
	(0.928230)	(1.328756)	(1.235609)
sigma	8.852646***	9.009819***	9.551774***
	(0.141481)	(0.316156)	(0.215188)
$(\text{covariance})^{½}$	-------------	52.39367***	54.9174***
		(3.307904)	(2.580985)

N = 1124; lnL = -8827.9851; LRT(df=23) = 328.63; *=statistically significant at α=10%; **=statistically significant at α=5%; ***=statistically significant at α=1%

beach quality. All coefficients are statistically significant and have the expected sign in the *ex-post* demand equation except the summer season dummy variable, which is statistically insignificant.

In general, there are less significant variables in the *ex-ante* demand equations, which is expected since the datasets are smaller, but the signs of the coefficients are in accord with economic theory.[18] Under a one-tailed hypothesis test,

[18] The additional covariates in the *ex-ante* Case *II* model are introduced to control for survey treatment effects. A subset of the surveys included explicit statements about how beaches would be maintained,

income is positive and significant in two of the three equations. Those households making multipurpose trips take less beach trips, reflecting the incremental costs associated with incidental or joint consumption (Parsons and Wilson 1997). Each of the covariance terms is significantly different from zero and positive, indicating a positive correlation between *ex-post* and *ex-ante* demand for beach visits.

For the Tybee Island model, the coefficient on the price of a beach visit is negative and statistically significant in each of the demand equations under a one-tailed hypothesis test. A $1000 increase in the price of a beach visit is estimated to decrease the demand by 7.5 trips before the improvement in beach quality (*ex-post* demand). After the improvement in beach quality, demand for beach visits is less responsive to changes in price. A $1000 increase in the price of a beach visit is estimated to decrease demand by 5.4 trips for the Case *I* model, and by 5.7 trips for the Case *II* model. The coefficient on substitute price is negative in two of the three demand equations—a counter-intuitive result—but none of the coefficients is statistically significant. The remaining coefficients exhibit the expected sign.

Income and the multipurpose trip dummy variable are significant in two of the three demand equations. The *nourish* control variable is statistically significant in the Case *II* equation. The nourish control is a dummy variable identifying those observations for which beach nourishment was indicated in the survey questionnaire as the beach-improving policy. Results indicate that *ex-ante* visitation is higher in this survey treatment, suggesting that foreknowledge on how beaches will be maintained induces higher stated visitation if the management method is beach

---

some identifying beach nourishment as the strategy and others indicating that shoreline retreat would be the management tool. The estimated effects of these treatments do not have a statistically significant effect on *ex-ante* demand for beach visits.

nourishment. The *retreat* control was not statistically significant. Each of the covariance terms is significantly different from zero and positive, indicating a positive correlation between *ex-post* and *ex-ante* demand for beach visits.

The parameters of the demand equations were used to estimate consumer surplus for each household under both current and improved conditions. Consumer surplus is given in [3.5] as the area under the demand curve above the current price. Incorporating our parametric model of [3.6], we have:

$$CS_i^j = \int_{p_{tr}^0}^{p_{tr}^{ch}(q^j)} (\hat{\gamma}_i^j + \beta_{tr}^j p_{tr}) dp_{tr} = -(v_i^j)^2 / 2\beta_{tr}^j \qquad [3.13]$$

where $\hat{\gamma}_i^j$ is the linear combination of covariates for household *i* and parameters, other than own-site travel price (β_{tr}^j), from the demand equation for $j = 0, I, II$. The final expression in [3.13] is attained by integration and simplification of the definite integral.

Since the demand data are censored, v_i^j in [3.13] should be expressed as the expected value of a censored random variable following the definition in [3.9]. To control for the changes in price and income that occur between the revealed and stated scenarios, the expected value of *ex-ante* trips is adjusted to reflect *ex-post* price and income levels. Calculation of CS_i^j in [3.13] produces an estimate of the value of access for household *i* under beach quality q^j.

Average annual consumer surplus for each beach quality level is calculated using the inverse of expected *ex-post* demand, expressed as the mean of a truncated random variable, as a weight. Average CS for access to the beach under status quo conditions is $1,296 for Jekyll Island, and $3,147 for Tybee Island (all values in 1998$). These are sizable welfare measures, but should be put in perspective. The weighted expected numbers of trips under ex-post conditions are 3.00 for Jekyll Island and 6.46 for Tybee Island. This implies consumer surplus measures per trip of $432 and $487, respectively. Also, bear in mind that many of these trips include multiple days on-site. The weighted mean of days on-site is 2.5 for Jekyll Island and 1.8 for Tybee Island, giving rise to daily household consumer surplus measures of $173 and $271 for Jekyll Island and Tybee Island, respectively. Lastly, the average household size in each sample is 3.8 persons,[19] which produces daily individual consumer surplus measures of $46 for Jekyll Island and $71 for Tybee Island. Viewed in light of these facts, the welfare measures seem reasonable.

The weighted averages of consumer surplus for Case *I* are $1,781 for Jekyll Island and $3,463 for Tybee Island. These are sizable increases over the status quo, a $485 increase for Jekyll Island (about 37 percent increase) and a $316 increase for Tybee Island (about 10 percent increase). These numbers may be interpreted as estimates of the welfare change attributable to increasing average beach width—a 2 meter increase for Jekyll Island (about 6 percent increase over the status quo) and a 1 meter increase for Tybee Island (about 4 percent increase over the status quo). For comparison with the status quo, weighted average consumer surplus per household, per trip under the Case *I* scenarios are $667 and $600 for Jekyll and Tybee, respectively.

---

[19] Household size is defined as the number of persons in the beach party for whom the survey respondent was paying expenses, and thus does not reflect the conventional definition of a household.

Weighted average CS measures for Case *II* are \$2,243 for Jekyll Island and \$4,286 for Tybee Island. These numbers represent larger increases in the economic value of access to the beach over the status quo relative to the Case *I* scenarios. The increases over status quo CS are \$947 for Jekyll Island (an increase of 73 percent) and \$1139 for Tybee Island (an increase of 36 percent). The corresponding changes in beach quality are 6 meters for Jekyll Island (about 19 percent over the status quo) and 3.5 meters for Tybee Island (about 15 percent over the status quo). For comparison with the status quo and Case *I*, weighted average consumer surplus per household, per trip under the Case *II* scenarios are \$828 and \$648 for Jekyll and Tybee, respectively.

Overall, the results of the model suggest that the value of access to beaches increases with improvements in beach quality. The magnitudes of the benefit measures do not seem unreasonable when viewed in light of visitation, the number of days spent on-site, and the average household size. Within each model, the relative change in consumer surplus is consistent with the relative change in beach width—larger improvements in beach quality are associated with larger increases in the estimate of willingness to pay for access to the beach.

The change in consumer surplus between j=0 and j=*I* or j=0 and j=*II* will serve as our approximation of willingness to pay for changes in beach quality. In order to standardize this measure, we divide by the change in beach quality producing an approximation of marginal willingness to pay. Thus, our welfare measure is:

$$MWTP_i(\Delta q \mid q^j) = (CS_i^j - CS_i^0)/\Delta q\,, \qquad [3.14]$$

where Δq represents the change in average beach width associated with the improvement scenario. Note that the welfare measure is conditioned on the subsequent level of beach quality, a treatment variable that varied randomly within each sample.

The marginal WTP in [3.14] is estimated for each household in the dataset. I then regress subsequent beach width on marginal willingness to pay, and use the consumer surplus measures associated with initial beach width as constraints on the parameter estimates. That is, I require that the area under the marginal value curve evaluated at the initial beach quality condition be equal to the estimated average consumer surplus from the travel cost model. (Each island has its own intercept, so each constraint relates to the common slope parameter and a unique intercept.) In this way, initial consumer surplus serves as a bench mark to calibrate the marginal value function. I thus estimate a constrained linear regression model with fixed island effects to estimate the slope and intercept of the marginal value function, with the constraints relating the parameters to the estimated welfare measures under initial conditions. The results are given in Table 3.5. The parameter estimates are derived by minimizing the sum of squared errors of the linear regression subject to the following two constraints:

$$31\times(jekyll + q\times 31) + 0.5\times q\times 31 = 1296$$

$$23.5\times(tybee + q\times 23.5) + 0.5\times q\times 23.5 = 3147$$

where the definitions of the variables are defined in Table 3.5. All of the coefficient estimates are statistically significant at the 1% level. This regression is meant to provide a prediction of willingness to pay that varies with average beach quality.

Table 3.5: Beach Quality Marginal WTP Function

Variable	Definition	Coefficient	Standard error
q	subsequent beach width (q^I or q^{II})	-16.81942***	1.351028
tybee	intercept for Tybee Island	726.7994***	47.62375
jekyll	intercept for Jekyll Island	823.9094***	62.82282
	N = 1828; Root MSE = 583.8		

Utilizing the *tybee* intercept from the results in Table 3.5, we have a marginal value function for beach width on Tybee Island:

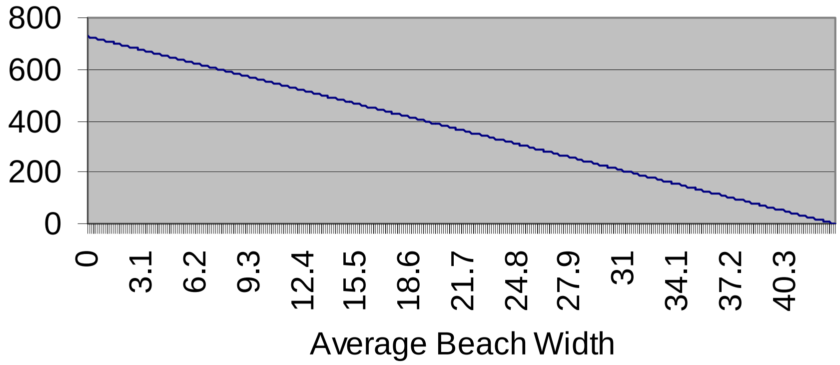
$$MWTP^b\ (q_t) = 726.80 - 16.82\ q_t, \qquad [3.15]$$

where q_t represents average beach width on Tybee Island in meters and WTP is in 1998 dollars. Integrating [3.15], we obtain an expression for total benefits of beach width per household, per year:

$$WTP^b\ (q_t) = 726.80q_t - 8.41\ q_t^{\ 2} + K^b, \qquad [3.16]$$

where K^b is a constant of integration. This marginal value function is depicted in Figure 3.1. This preference function will be used to represent WTP for beach area by beach visitors in the empirical simulation to follow.

Figure 3.1: Marginal Value Function for Beach Visitors



Chapter 4: Hedonic Prices and Beach Quality

This Chapter presents an empirical analysis of the benefits of beaches accruing to coastal homeowners. I use hedonic property models from multiple housing markets to estimate the demand for beach quality. This approach is a valid method for identifying demand only if preferences for beach quality are identical across markets segments; that is, households cannot self-select into the different markets based on the housing attribute we intend to value. I first discuss the difficulties that arise in using housing market data for welfare analysis when risk is a factor, then develop a theoretical model that addresses some of these issues. An econometric model is developed. I discuss the data utilized, and then present the results of the empirical analysis.

House Values, Risk, and Beach Quality

Deriving a welfare measure for coastal homeowners is more complicated than for beach visitors because beach quality also serves to mitigate risk of damage from coastal storms. Houses in the coastal zone are generally susceptible to hurricanes and nor'easters. These coastal storms bring with them flooding (due to storm surge), high velocity waves, high winds, and beach erosion. Beaches offer protection in at least two ways—they inhibit high velocity waves, and they serve as a buffer from storm-induced beach erosion. Aside from beach quality, there exist other forms of indemnification from storm risk—formal flood insurance, self-insurance, and self-protection (Ehrlich and Becker 1972).

In many cases, market insurance for protection against flood hazard is required under the terms of the property mortgage agreement. There is ample evidence, however, that such provisions have not been strictly enforced (FEMA 1997),[20] so even in these cases insurance coverage may be a under household discretion. The price at which formal flood insurance is offered reflects (i) the flood zone in which the house is located, (ii) attributes of the insured structure (location of contents, number of stories, presence of basement, elevation, etc.), and (iii) whether the structure predates the "regular" phase of the National Flood Insurance Program (NFIP) (existing structures are "grandfathered" at subsidized rates) (Kunreuther and Sheaffer 1970; Shilling, Sirmans and Benjamin 1989).

In the parlance of Ehrlich and Becker (1972), self-insurance involves reducing the size of the potential loss should a disaster occur. Common examples are expenditures on protective items, such as a sprinkler system or motorcycle helmet. For coastal properties, there are no such items that can be subsequently purchased by a homeowner to reduce the potential size of storm losses. But homeowners do choose an array of attributes that mitigate the conditional expected loss *when they purchase a property*. Assuming that participants in the coastal housing market are well informed about storm risk and how attributes of the property mitigate expected loss, a hedonic analysis of property prices seems a reasonable way to infer the value of loss-mitigating attributes. The implicit prices of these attributes are determined within the market.

[20] The lack of compliance prompted the U.S. Congress to pass the National Flood Insurance Reform Act of 1994, which provided for stauncher penalties (taking effect in October of 1996).

Beach quality can be viewed as a form of self-insurance because it inhibits wave action and buffers coastal properties from storm-induced erosion.[21] Also, the beach provides for recreational and aesthetic enjoyment during times of fair weather. Beach quality, however, is not under the control of individual homeowners. Rather, it is a natural attribute, which can be augmented by beach nourishment under the discretion of a coastal planner.[22] The only point in time when a household can freely choose the quality of the beach is when purchasing a home. Specifically, they can purchase a home in a jurisdiction with good beach quality or poor beach quality. Since beaches provide for joint benefits, I expect that homes in jurisdictions with better beaches will trade at a premium.

Other factors that affect the degree of potential loss faced by the household are determined at the time of purchase. These include: (i) whether the home was built after the publication of a Flood Insurance Rate Map (FIRM), (ii) whether the home is located in the V-zone, and (iii) the elevation of the home above base flood elevation (BFE). Each of these requires some elaboration. Flood-prone locations that are built upon after the publication of a FIRM are required to meet higher building standards that mitigate the amount of damage the building sustains during a hurricane or storm event. However, the publication of a FIRM coincides with a community entering the "regular" phase of the NFIP, at which time explicit subsidies in flood insurance rates are removed. Shilling, Sirmans, and Benjamin (1989) suggest that these subsidies can be capitalized in the price of housing, which would provide a countervailing effect to self-insurance. Houses located in V-flood zones, or "velocity" zones are

[21] Note the term "storm-induced erosion" is synonymous with the concept of short-run erosion problems in previous chapters.
[22] Beach nourishment is not feasible on a small scale.

expected to suffer greater damage from storm waves than those located in A-zones. The extent of the V-zone is determined by computer simulation of the landward penetration of three-foot waves during the 100-year storm, and varies with local topography. Lastly, elevation of the property in reference to the BFE (height of the "100-year flood") is a form or self-insurance, lowering the expected loss due to flooding.

While each of these attributes fits the definition of self-insurance, they could also be considered self-protection, defined by Ehrlich and Becker as an investment which skews the distribution of income across states, increasing the probability of a favorable outcome. Since these three housing attributes are also considered in the setting of flood insurance premiums (as noted above), it is clear that they must exhibit some protection. The rationale is simple—the possibility of moral hazard in the insurance market leads to a situation in which insurance rates should reflect household self-protection; otherwise the household would not have appropriate incentives for protecting themselves from the loss. If insurance rates reflect the variation in housing attributes, then these attributes should have an effect on the probability of hazard.

It is natural to ask how a housing attribute might serve as both self-insurance and self-protection. An example will help to clarify: Elevation above BFE provides for a lower expected loss in the event of a storm; this is self-insurance (as discussed above). However, the probability of floodwaters reaching the lowest floor of the house *conditional on a storm occurring* would also be affected by elevation; in this case elevation would be a form of self-protection. The distinction turns on how we define the loss event.

In order to focus on the protective aspects of beaches, I define the loss event as the annual probability of a storm occurring, with the understanding that the storm must be of sufficient strength to cause (i) flooding, (ii) high velocity waves, (iii) high winds, and/or (iv) beach erosion, in order to warrant consideration as a hazard. Since beaches offer protection from high velocity waves and beach erosion, considering the probability of only the first hazard (flooding) would not lend itself to consideration of the protective aspects of beaches. Flood insurance covers hazards (i), (ii) and (iv), but not (iii); wind damage is covered under separate insurance. In practice, insurance price varies with the probability of hazards (i) and (ii), but not with (iv). The conditional loss is the expected loss from the coastal storm. This loss is decreasing in "post-FIRM construction", elevation, and beach quality, but increasing in "presence in a V-zone".

Suppose, for the moment, that flood insurance is fair (in the sense that premium equals expected loss) and complete (in the sense that it pays for all damages). Assume that coastal homeowners are risk averse. Since housing attributes that afford self-protection decrease the price of insurance, coastal homeowners still have an incentive to protect themselves from coastal hazards, and their bids on coastal housing with different levels of protection will reflect their assessment of the protection, and their degree of risk aversion. There is no moral hazard problem here because the property attributes are observable by both parties—the insurer and the insured. Similarly, housing bids will also reflect the value of attributes that serve as self-insurance. Since self-insurance lowers the expected loss, we should not see properties with more self-insurance purchasing higher insurance coverage, *ceteris paribus*. In either event, housing prices should vary with both risk-mitigating and loss-mitigating attributes.

In reality, unsubsidized flood premiums will exceed expected loss due to administrative costs, but if coastal homeowners are risk averse they will be willing to pay more than the expected value of the loss (with excess willingness-to-pay depending upon their risk premium). Flood insurance is not complete because coverage is capped at a specific amount[23], and flood insurance does not cover loss of land due to erosion. In this case, housing attributes that afford self-protection and self-insurance will be valued more highly than if insurance offered full coverage. In particular, note that since flood insurance does not cover loss of land, beach quality is the sole protection mechanism against storm-induced erosion. As long as households recognize this aspect of beach quality its value as self-insurance should be reflected in housing prices.

Expected Utility Theory and Hedonic Prices

The household's purchasing decision for homes in the coastal zone can be modeled using expected utility theory. I assume that coastal households face only two states of nature, one in which a storm occurs and the other in which it does not. The associated annual probabilities are ρ and $(1-\rho)$. The probability of storm landfall within a region is exogenous; it cannot be altered through mitigation measures. Loss-mitigating housing attributes include location in a V-zone, whether the house is built

[23] As of 1997, flood insurance for residential property could not exceed $250,000 building coverage and $100,000 contents coverage. The value of many coastal properties exceeds this cap.

to meet post-FIRM building codes, elevation above BFE, and beach quality. An important spatial distinction is made in reference to Coastal Barrier Resource Act (CBRA) zones. Houses located in CBRA zones are not eligible for flood insurance.

Utility of the coastal homeowner is represented by $u^r(y,a,res;\xi)$, where y is a composite good that serves as the numeraire, a is the vector of structural and neighborhood attributes associated with the property, and *res* is a vector of coastal resource quality attributes, including beach quality at the shore nearest the property (q), ocean-frontage (*ocfrnt*), and distance from the beach (*dist*). The ξ vector represents socio-economic variables that vary across households. The function $u^r(.)$ has the usual properties.[24] I drop the r superscript in the subsequent discussion, but the reader should bear in mind that the theory applies to coastal homeowners.

A brief review of the theory of hedonic prices is prudent at this point. The household is a price-schedule taker, and thus the hedonic price function $\hat{H}(a)$ is taken as given. The household's budget constraint is $m = y + H(a)$, where m is income, the price of the numeraire is set to one, and $H(a)$ is the hedonic housing price expressed as an annual rent.[25] Assuming interior solutions for all continuous choice variables, maximization of $u(y,a,res;\xi)$ subject to the budget constraint requires the following:

$$\partial u/\partial a_i = \mu \partial H/\partial a_i \qquad \forall\ a_i \qquad [4.1]$$

$$\partial u/\partial y = \mu \qquad [4.2]$$

[24] I assume $U^r(y,a,res)$ is quasi-concave, twice-differentiable, and increasing in all arguments except *dist* (assuming a is defined in terms of desirable characteristics).

[25] That is, $H(a) = \hat{H}(a) \times \delta$, where $\hat{H}(a)$ is the hedonic sales price (present value of the stream of housing services) and δ is the annual discount rate.

$$m - y - H(a) = 0. \quad [4.3]$$

The symbol μ is the Lagrange multiplier on the budget constraint, and thus represents the marginal utility of income. In equilibrium, the marginal rate of substitution is equal to the marginal implicit price from the hedonic equation: $\partial u/\partial a_i$ / $\partial u/\partial y$ = $\partial H/\partial a_i$.

In order to introduce coastal resource quality and loss-mitigation into the standard hedonic model, two vectors are included as arguments of *H(.)*, a *loss* vector and the *res* vector (already specified in the discussion of coastal homeowner utility). The *loss* vector is composed of property attributes that affect the expected loss—post-FIRM construction (*pFIRM*), location in the V-zone (*vzone*), and elevation above BFE (*elev*). Beach quality is also related to expected loss, but is included in the *res* vector. The household faces storm risk that occurs with probability ρ, and produces a loss *L(q, loss)* (which is a decreasing function of *q*, *pFIRM*, and *elev*, and an increasing function of *vzone*). The coastal resident's optimization problem can be set up as follows:

$$\begin{aligned} \underset{a,res,loss,s}{Max}\ \Omega &= \rho[u(m - H(a,res,loss) - s\pi - L(q,loss) + s, a, res; \xi)] \\ &+ (1-\rho)[u(m - H(a,res,loss) - s\pi, a, res; \xi)] \end{aligned} \quad [4.4]$$

where *s* is the amount of insurance held (which may be constrained), and π is the price of insurance. Note that in practice π varies with the *loss* vector. Elevation above BFE affects the probability of flood loss conditional on a hurricane occurring.

The price of flood insurance will vary systematically with the dichotomous *pFIRM* variable due to the flood insurance subsidy for pre-existing structures. As such, we should expect correlation among $H(.)$ and π. Since I am not considering self-protection, however, I assume that π is exogenous. Note also that the expected loss (L) and coverage amount (s) in [4.4] are expressed in annual dollars.

As indicated by [4.3] the budget constraint is assumed to hold with equality, but as Palmquist (1984) has noted, it is likely non-linear due to the presence of *H(a,res,loss)*. He shows that linearizing the budget constraint at the optimal bundle of housing attributes is necessary for conducting welfare analysis. The hedonic price function *H(a,res,loss)* is dependent upon q as a recreational *and* loss-mitigating attribute of the property. Beach quality provides joint services to households. One might argue that the attributes *ocfrnt* and *dist* should affect not only resource quality, but also expected loss. Ocean-frontage provides for access and view amenities, but could increase the expected loss in the event of a storm. Likewise, distance from the beach can provide protection from storms, but diminishes beach access and view amenities. Note, however, that the specific flood zones (V and A) should account for the spatial distribution of storm losses. As such, *ocfrnt* and *dist* should only reflect the amenity effects of these attributes, assuming that households view flood zones as accurate spatial measures of expected loss.

Assuming interior solutions for all continuous choice variables, maximization of expected utility in [4.4] requires the following first-order conditions:

$$\partial\Omega/\partial a_i = \partial u/\partial a_i - \partial H/\partial a_i\,[\rho\mu^L + (1-\rho)\mu^0] = 0 \quad \forall\ a_i \qquad [4.5]$$

$$\partial\Omega/\partial q = \partial u/\partial q - \partial H/\partial q\,[\rho\mu^L + (1-\rho)\mu^0] - \rho\mu^L \partial L/\partial q = 0 \qquad [4.6]$$

$$\partial\Omega/\partial dist = \partial u/\partial dist - \partial H/\partial dist\,[\rho\mu^L + (1-\rho)\mu^0] = 0 \qquad [4.7]$$

$$\partial\Omega/\partial elev = -\,\partial H/\partial elev[\rho\mu^L + (1-\rho)\mu^0] - \rho\mu^L \partial L/\partial elev = 0 \qquad [4.8]$$

$$\partial\Omega/\partial s = \rho\mu^L(1-\pi) + (1-\rho)\mu^0(-\pi) = 0 \qquad [4.9]$$

where μ^L and μ^0 represent the marginal utilities of income in the loss and no-loss states, respectively, $m^L = m\text{-}H(a,res,loss)+s(1\text{-}\pi)\text{-}L$, and $m^0 = m\text{-}H(a,res,loss)\text{-}s\pi$.[26] Equation [4.9] applies only if the home is located outside of a CBRA zone or if the household is not required to hold some level of flood insurance coverage by its mortgage lender. The three dichotomous variables, *ocfrnt*, *pFIRM* and *V*-zone, are chosen by the following rule: If $\Omega(f=1) > \Omega(f=0)$ then $f=1$, else $f=0$, for f = *ocfrnt, pFIRM* , *vzone*. Location in a V-zone is positively correlated with distance to the shore and ocean-frontage. Households that place a high value on proximity to the ocean may not be able to freely choose whether to locate in or out of this zone. Since this correlation is not perfect, one would expect houses outside of the V-zone to be valued more highly, *ceteris paribus*. But identifying the separate influence of these two variables empirically may be difficult.

Manipulating [4.5] – [4.8],[27] we derive the following equilibrium conditions for the hedonic price function:

[26] If the price of insurance (π) were posited as an endogenous function of the *loss* vector, condition [4.8] would include an additional partial derivative for the effect of elevation on insurance price. This additional term would include the optimal insurance coverage (s), and thus would introduce simultaneity into the first-order contions.

[27] Manipulation of [4.9] implicitly defines the demand for insurance by $\mu^L/\mu^0 = (1-\rho)/\rho \times \pi/(1-\pi)$. This result is not emphasized here because the focus is on the benefits of beach quality via the hedonic price function. We are not interested in the demand for flood insurance, per se, but rather its effect on the demand for beach quality.

$$\partial H/\partial a_i = \partial u/\partial a_i \,/\, [\rho\mu^L + (1-\rho)\mu^0] \qquad \forall\, a_i \qquad [4.5']$$

$$\partial H/\partial q = [\partial u/\partial q - \rho\mu^L \partial L/\partial q] \,/\, [\rho\mu^L + (1-\rho)\mu^0] \qquad [4.6']$$

$$\partial H/\partial dist = \partial u/\partial dist \,/\, [\rho\mu^L + (1-\rho)\mu^0] \qquad [4.7']$$

$$\partial H/\partial elev = -\rho\mu^L \partial L/\partial elev \,/\, [\rho\mu^L + (1-\rho)\mu^0] \qquad [4.8']$$

It is common in applications of hedonic price theory, to assume that the supply of housing attributes is exogenous,[28] and thus only treat the demand side of the housing market. One can interpret the expressions in [4.5'] – [4.8'] as measures of household marginal willingness-to-pay (WTP). Households are differentiated by income and other socioeconomic characteristics, represented by the vector ξ. They take the hedonic price function as given, and the allocation of housing occurs through households equating their marginal rate of substitution between the housing characteristic and the numeraire good to the implicit price of the housing characteristic from the hedonic price function (Rosen 1974).

Equation [4.5'] shows that in equilibrium, the slope of the hedonic price function with respect to a structural or neighborhood attribute is equal to the marginal WTP for that attribute weighted by the expected value of the marginal utility of income across the two states. Condition [4.6'] determines the optimal amount of beach quality associated with the property. The marginal implicit price derived from the hedonic price function is equal to the sum of marginal utility of beach quality

[28] Palmquist (1999) states, "…since the quantities of the characteristics in existing houses are predetermined and costly to alter, the equilibrium price schedule is completely demand-determined…". This may not hold for newly constructed homes, however.

stemming from recreational enjoyment ($\partial u/\partial q > 0$) and the marginal loss-mitigation value stemming from the role of beach quality as self-insurance ($-\rho\mu^L \partial L/\partial q > 0$), weighted by the expected value of the marginal utility of income. The expression on the RHS of [4.6'] is the marginal willingness to pay for beach quality, equivalent to the slope of the preference function derived in [2.3]. Similar to [4.5'], equation [4.7'] shows the marginal willingness to pay for distance from the beach as an attribute of the property. Lastly, condition [4.8'] determines the optimal elevation above BFE. The marginal implicit price of elevation is equal to the product of the probability of loss, the marginal utility of income in the loss state and, the marginal loss-mitigation effect, all weighted by the expected marginal utility of income.

Estimation Issues

Estimation of the hedonic price function is often conducted by ordinary least squares (OLS). A properly specified hedonic price equation can be used to assess welfare effects of marginal and localized, non-marginal changes in public goods (Palmquist 1992) and provides bounds on the welfare effects of widespread, non-marginal changes (Bartik 1988; Palmquist 1988). There are two closely related problems associated with deriving precise uncompensated measures of welfare change (i.e. consumer surplus), in general. The first has to do with identification of the demand equations when only a single point for each agent is observed—that point of tangency with the hedonic price function. The second problem stems from the non-linearity of the hedonic price function; the marginal implicit prices and quantities of attributes are jointly determined.

Rosen's (1974) original exposition on the theory of the hedonic price equation included a discussion of the derivation of demand functions for the attributes of the differentiated good. Since the slope of the hedonic price function reveals the implicit price of an attribute, Rosen envisioned estimating inverse demand equations with the predicted implicit price as the dependent variable and the quantity of the attribute as an explanatory variable. He further recommended household and producer characteristics be used as shift variables to overcome what he believed to be a garden-variety, simultaneous equations identification problem.

Brown and Rosen (1982) subsequently pointed out that these demand equations are not identified because the implicit prices are estimated, not observed. Identification requires restrictions. Mendelsohn (1985) notes that even if the marginal prices were known with certainty, identification would still require restrictions since there are an infinite number of demand functions that could underlie the observed marginal price. Brown and Rosen explore functional form restrictions and excluded variables in the demand equation. With regard to the former, with an m^{th} degree polynomial hedonic price function, demand must be of degree m-2 in the attribute for identification. With regard to the latter, the hedonic price equation must contain exogenous variables that are excluded from the demand equations for identification. (The demand equation must also contain exogenous variables excluded from the hedonic price function, but this is typically not a problem, as demand equations should include socio-economic characteristics as shift variables, which will be excluded from the hedonic price function.) In either case, the restrictions must be defensible, and since the form of the hedonic price function is not known *a priori*, the first option can be difficult to maintain.

Regarding the second option, Brown and Rosen suggest that data from multiple markets could be used to identify the structural demand equations. Let demand be conditioned on observable household characteristics, indexed by ξ. Assume the distribution of household characteristics varies across markets, but that the structure of preferences (conditioned on characteristics) is the same across markets. For the case of exogenous housing supply, the distribution of housing characteristics will also vary exogenously across markets. Within a single market, the hedonic price equation will reveal only one point on the individual households' bid curves. However, since preferences differ only by ξ, observations from multiple markets can be used to identify demand because differences in the distribution of preferences and housing characteristics will give rise to variation in price (Tinbergen 1959; Epple 1987). Multiple markets introduce differences in the matching process between consumers and housing, which give rise to different marginal implicit prices. Since these differences in matching are not reflected in the structure of the demand equation, information from multiple markets can be used to identify the parameters of demand (Kahn and Lang 1988).

The second difficulty in estimating demand for attributes stems from a separate identification problem arising from joint endogeneity of marginal implicit prices and attribute quantities, first recognized by Palmquist (1984) and Mendelsohn (1984). Given a non-linear hedonic price function, consumers choose both the marginal price and the quantities of housing attributes.[29] As such, estimation of inverse housing-characteristic demand equations by OLS will produce biased coefficient estimates. Consistent estimation can be achieved with instrumental variables, though the suitability of instruments depends upon the source and structure of modeling errors.

Epple (1987), Bartik (1987), and Kahn and Lang (1988) show that correlation among producer and consumer attributes is a characteristic of the equilibrium hedonic price equation when demand and supply are endogenous. The correlation arises through the matching process. That is, consumers with a strong preference for attribute *i* will transact with firms that are more efficient in producing attribute *i* (i.e. a separating equilibrium). As such, the seemingly natural approach of using producer and consumer characteristics as instruments in demand and supply equations will not work. At least some of these characteristics will be correlated with the endogenous housing attribute variables. Epple shows that even when supply is exogenous, estimation of the hedonic price function by OLS is inconsistent unless the error term is uncorrelated with the error terms from the demand equations.

Epple (1987) considers the implications of specific orthogonality conditions under a number of cases regarding endogeneity of supply, mutual correlation among measurement errors, and the presence of unmeasured characteristics. He assumes that: (i) unmeasured product characteristics are uncorrelated with all measured agent characteristics, and (ii) unmeasured agent characteristics are uncorrelated with measured agent characteristics but correlated with both measured and unmeasured

[29] Note this problem does not arise if the hedonic price function is linear in attributes. However, if it is linear there will be no variation in marginal implicit price to support second-stage estimation of the demand equation. In any event, the linear functional form is usually rejected in empirical applications.

product characteristics. Under these assumptions he derives conditions for identification of the hedonic price function and demand and supply equations. His results specify a set of relationships between proxy, unmeasured, and observed variables that are necessary for identification.

Kahn and Lang (1988) argue that Epple's results are particular to his specific assumptions regarding the structure of errors, and in their view assumption (ii) will likely not hold. They contend that unobserved agent characteristics are likely to be correlated with observed agent characteristics due to the matching process that underlies the hedonic price function. For example, households with a taste for detailed carpentry work would be matched with suppliers that are carpenters. They show that agent characteristics interacted with dummy variables from *k*-1 markets can be used as instrumental variables in the first stage of two-stage least squares, but recommend a more efficient non-linear three-stage least squares approach. Bartik (1987) takes the same point of view with regard to Epple's assumption (ii). Bartik argues that if the estimation problem stems from unobserved consumer taste, any exogenous variable that shifts the budget constraint is an appropriate instrument. As do Kahn and Lang, he suggests using market-dummy variables interacted with demand shifters in a first-stage regression.

Multiple Hedonic Markets and Beach Quality Demand

To apply the multi-market hedonic model, one must have data on housing purchases across a number of markets. Many researchers take an urban area to be a single market, though some have suggested that multiple sub-markets may exist in a single urban area. Barrier islands, due primarily to their diminutive size, do not typically support large urban centers. Rather, most barrier islands on the east and gulf coast are home to rather small, somewhat bucolic beach towns, composed of vacation and rental homes and beach-related businesses. As such, the local labor market has a much smaller impact on the extent of the housing market. The boundaries of the housing market are determined more by travel distance from metropolitan areas and, perhaps, regional population and income. Markets in this analysis of beach quality are delineated based on a judgment of the extent of the area that regional households might regard as reasonable substitutes in purchasing coastal housing for vacationing, and possibly as a source of rental income.

The dataset used for this analysis is composed of sales of primarily pre-existing housing. Less than 15 percent represent newly constructed homes. Thus, I assume that the hedonic price schedule is primarily demand driven and focus only on the demand side of the market for coastal housing. The theory of Tinbergen (1959) still applies in this case. If the *distributions* of consumer and housing characteristics differ across space, but the structures of consumer preferences are the same, then information from multiple markets can be used to identify the structural parameters of the demand equations. The different distributions of preferences and attributes will give rise to distinct hedonic price functions in the different markets. Thus, the marginal implicit prices will differ, but these differences will be in no way related to the structure of demand. As such, the different prices allow for identification of a demand curve in the second stage regression.

In order for the multiple markets strategy to work, households cannot be self-selecting into different markets based upon the characteristics that the researcher is trying to value. Self-selection into different markets would create a situation in which preferences vary systematically between markets. Suppose, for example, that one market exhibits a much lower degree of risk and households that are more risk-averse are drawn to that market. In this case, risk preferences (and thus the value of loss-mitigating household attributes) will differ across markets, rendering any model that requires identical preferences invalid.

The hedonic price function is $H_j(a,res,loss)$, with the j subscript representing market j. Since the functional form of the hedonic price function is not known *a priori* a flexible form is desirable. The Box-Cox transformation has been used in many empirical analyses and nests the more typical functional forms, namely linear, semi-log, and log-linear. This flexible approach may only be utilized with strictly positive, continuous variables. While the rental rate of the house certainly meets this requirement, the environmental and risk variables in most cases, do not. Beach width can be zero; elevation above BFE and distance from the ERF can both be negative; *pFIRM*, *V*-zone, and *ocfrnt* are dichotomous variables. Thus, only the dependent variable is transformed. The j hedonic price equations are estimated as:

$$\begin{aligned} H_{ij}(a,res,loss) &= \beta_j^0 + a_i'\beta_j^a + res_i'\beta_j^{res} + loss_i'\beta_j^{loss} + \varepsilon_{ij} \\ &= (h_{ij}^{\gamma j}-1)/\gamma_j \quad \text{for } \gamma_j \neq 0 \\ &= ln(h_{ij}) \quad \text{for } \gamma_j = 0, \qquad [4.10] \end{aligned}$$

where h_{ij} is the rental price of housing unit i in market j, β_j^0 and the β_j^l are constant coefficients (for vectors l=*a, res, loss*) that differ across markets, γ_j are Box-Cox parameters to be estimated by maximum likelihood, and ε_{ij} is an identically and independently distributed (i.i.d.) random error term with zero mean. The Box-Cox transformation of the dependent variable is utilized in order to pick the functional form that best represents the data.

As indicated in [4.5'] – [4.8'] the first derivatives of the hedonic price function with respect to housing attributes reveal the marginal implicit prices. This result is conditional on the expected value of the marginal utility of income across the two states being a reasonable representation of the household's marginal utility of income when forming their bid for housing. Under this assumption, the marginal implicit price can be utilized as the dependent variable in a second stage regression equation that embodies the demand function for the housing attribute. To simplify estimation, I assume that housing and the numeraire are weakly separable. I also assume that the expected loss and resource variables are weakly separable from the other housing attributes. This significantly reduces the number of endogenous variables in the second stage, and does not seem an unreasonable assumption. Inverse demand for loss-mitigating attributes is given by:

$$\partial H/\partial loss_{ik} = W_{ik} = \varphi(loss_i,\, res_i, X_i) = \psi_k^{0} + loss_i'\psi_k^{loss} + res_i'\psi_k^{res} + X_i'\psi_k^{x} + \varepsilon_{ik}, \tag{4.11}$$

where W_{ik} is the marginal implicit price of risk attribute k for household i. One can envision a system of demand equations for loss-mitigating (and other) attributes as given in [4.11], though for estimation purposes, one may focus on any portion thereof. The demand equations are functions of the entire vector of loss-mitigating attributes (*loss*) and socio-economic characteristics of the household (X). The residuals in [4.11] are assumed i.i.d. with zero mean, and can be divided into two components, one representing unobservable risk preference of household i and the other a purely random component: $\varepsilon_{ik} = \omega_i + v_{ik}$. Given this decomposition, estimation of [4.11] by OLS will produce biased coefficients because W_{ik} and *loss* are correlated with ω_i. Even if instruments are available to control this endogeneity, the equations will not be identified unless the restrictions identified by Brown and Rosen are utilized.

Identification of the parameters in [4.11] can be obtained by using data from multiple markets. Following Kahn and Lang (1988) and Bartik (1987) demand shifters are interacted with k-1 market dummy variables to be used as instruments. This approach addresses the first identification problem—that of separately identifying the demand equations when only one point is observed on the hedonic price function. Endogeneity of housing attributes can be addressed by utilizing other instruments that are correlated with the optimal bundle of characteristics, but uncorrelated with unobserved tastes. In addition, the household's residual income—$y = m - H(a,res,loss)$ (i.e. that amount of money leftover for the consumption of the numeraire, y)—must be included as a regressor if the marginal utility of income is not constant.

Palmquist (1984) recognizes that since the hedonic price function will most likely be non-linear, the budget constraint in [4.3] will be non-linear as well. Since housing expenditures are usually a large share of the budget, the non-linearity in the residual budget constraint will likely be significant. The presence of a non-linear budget constrain complicates welfare analysis because the standard Marshallian demand functions are ill-defined (Bockstael and McConnell 1983). Palmquist suggests linearizing the budget constraint by adding the predicted housing price. The linearized constraint will be tangent to the indifference curve at the same point as the non-linear one and is consistent with a Marshallian demand function underlying the optimizing behavior of the household. The residual budget constrain is also endogenous because it includes *H(a,res,loss)*. As such, if it is included in [4.11], estimation will require further instrumentation. In the expected utility model set up in [4.4], the expected residual income is given by:

$$\begin{aligned} E(y) = \hat{y} = \rho[m - H(a, res, loss) - s\pi - L(q, loss) + s] \\ + (1 - \rho)[m - H(a, res, loss) - s\pi]. \end{aligned} \quad [4.12]$$

Equation [4.12] is likely nonlinear due to the presence of *H(.)*, but could also be nonlinear in *L(.)*. While *H(.)* is a large portion of the budget, *L(.)* is conditional on ρ, a relatively small number, and likely to be significantly less than *H(.)*. As such, I assume that the non-linearity due to *L(.)* can be ignored. The expected loss in the event of a storm, *L(.)*, must be predicted in order to estimate the expected residual income. Utilizing fairly limited data (only about 120 observations) on flood insurance claims, a simple logit model was used for this purpose. This model is described in the results section.

Multiple Market Data

The hedonic data used in the analysis were compiled for a study of the National Flood Insurance Program and coastal erosion sponsored by the Federal Emergency Management Agency. The complete dataset includes information from 18 coastal counties across the U.S., with structural characteristics, market price, flood insurance premium and coverage, flood and erosion risk proxies, and subjective judgments of beach quality at the time of purchase. The data were compiled from three sources—a field survey of residential properties, a mail survey of homeowners, and insurance records from the Federal Insurance Administration. I focus on six coastal counties in the southeastern U.S., in what I construe as two distinct housing markets. Coverage in the data is somewhat poor; there are many holes. Of the 3,961 observations from the six counties, only about 1,400 have complete data on structural attributes, sales price, and other requisite items. Further confining the analysis to thirteen years of sales (spanning 1986-1998) reduces sample size to around 700 observations.

The first market is in the Carolinas, and includes Dare and Brunswick Counties, in North Carolina, and Georgetown County, South Carolina. The second market includes Glynn County, Georgia, and Brevard and Lee Counties in Florida. A series of Chow tests were run with garden-variety hedonic models in order to test the specification of multiple markets. The restriction of the market segments suggested above could not be rejected at conventional confidence levels (99 percent), but combining the data across these segments was rejected. Descriptive statistics for each market can be found in Tables 4.1.

Table 4.1a: Descriptive Statistics: Dare & Brunswick, NC and Georgetown, SC

Variable	Definition	Mean	Std Dev
rent98	Annual rental rate of housing	22654.13	18461.85
housage	Age of house at time of purchase (years)	11.6459	14.9676
sqft	Square footage	2005.68	1184.72
lotsize	Size of lot (square feet)	14425.21	12490.67
mtstory	Dummy variable equal to 1 if house has multiple stories; 0 otherwise	0.6171	0.5778
airc	Dummy variable equal to 1 if house has central air; 0 otherwise	0.9150	0.3315
firep	Dummy variable equal to 1 if house has fireplace; 0 otherwise	0.3642	0.5720
dmsa	Distance to Charlotte, NC (kilometers)	509.66	168.94
dcbd	Distance to central business district (meters)	3464.61	3715.75
ocfrnt	Dummy variable equal to 1 if house is oceanfront; 0 otherwise	0.5228	0.5937
dist	Distance to beach (meters)	106.85	91.39
medbw	Median beach width (meters)	15.65	12.72
pfirm	Dummy variable equal to 1 if house built post-FIRM; 0 otherwise	0.6982	0.5456
vzone	Dummy variable equal to 1 if house located in V-zone; 0 otherwise	0.4559	0.5920
elev	Elevation above BFE (meters)	1.686	2.230
hugo	Dummy variable equal to 1 if sale occurred after Hurricane Hugo (1989); 0 otherwise	0.7398	0.5215
brun	Dummy variable equal to 1 if house located in Brunswick County; 0 otherwise	0.3230	0.5559
geor	Dummy variable equal to 1 if house located in Georgetown County; 0 otherwise	0.0994	0.3556
cobra	Dummy variable equal to 1 if house located in CBRA zone; 0 otherwise	0.0559	0.2732
pstorm	Probability of storm landfall	0.0012	0.0009
loss	Expected annualized loss due to storm	427.81	569.67
s	Annualized insurance coverage	9564.32	8664.27
π	Annual flood insurance premium	574.84	813.82
expinc	Expected annual income	118023.83	90566.01
insreq	Dummy variable equal to 1 if household indicated insurance was required	0.6198	0.5770
mktins	Dummy variable equal to 1 if household holds flood insurance	0.9076	0.3440

Table 4.1b: Descriptive Statistics: Glynn, GA and Brevard & Lee, FL

Variable	Definition	Mean	Std Dev
rent98	Annual rental rate of housing	40969.32	74442.83
housage	Age of house at time of purchase (years)	23.79	21.00
sqft	Square footage	2490.94	1766.06
lotsize	Size of lot (square feet)	17352.46	23449.06
mtstory	Dummy variable equal to 1 if house has multiple stories; 0 otherwise	0.4429	0.6300
airc	Dummy variable equal to 1 if house has central air; 0 otherwise	0.3417	0.6015
firep	Dummy variable equal to 1 if house has fireplace; 0 otherwise	0.1700	0.4764
dmsa	Distance to Orlando, FL (kilometers)	247.00	152.28
dcbd	Distance to central business district (meters)	5579.14	7036.98
ocfrnt	Dummy variable equal to 1 if house is oceanfront; 0 otherwise	0.4555	0.6316
dist	Distance to beach (meters)	110.05	90.09
medbw	Median beach width (meters)	15.57	26.47
pfirm	Dummy variable equal to 1 if house built post-FIRM; 0 otherwise	0.3326	0.5975
vzone	Dummy variable equal to 1 if house located in V-zone; 0 otherwise	0.1111	0.3986
elev	Elevation above BFE (meters)	1.559	3.385
andr	Dummy variable equal to 1 if sale occurred after Hurricane Andrew (1992); 0 otherwise	0.6187	0.6160
brev	Dummy variable equal to 1 if house located in Brevard County; 0 otherwise	0.4176	0.6254
lee	Dummy variable equal to 1 if house located in Lee County; 0 otherwise	0.4938	0.6341
cobra	Dummy variable equal to 1 if house located in CBRA zone; 0 otherwise	0.0514	0.2802
pstorm	Probability of storm landfall	0.0020	0.0036
loss	Expected annualized loss due to storm	2613.59	8126.27
s	Annualized insurance coverage	8623.70	13322.94
π	Annual flood insurance premium	332.58	919.58
expinc	Expected residual annual income	57637.89	106326.26
insreq	Dummy variable equal to 1 if household indicated insurance was required	0.2169	0.5227
mktins	Dummy variable equal to 1 if household holds flood insurance	0.6220	0.6149

A series of T-shaped sampling frames were used in gathering the coastal housing data. The top portion of the T was oriented parallel to the shoreline at the oceanfront in order to ensure adequate coverage of those properties that face an immediate erosion hazard. In order to correct for this over-sampling of the oceanfront, weighting factors were devised. These weights are used in all calculations (including the descriptive statistics presented in 4.1).

The dependent variable for the hedonic price regressions is rental rate of the property expressed in 1998 dollars (*rent98*). The rental rate was calculated by multiplying the house sales price by the prevailing 30-year, fixed mortgage rate in the month of the sale. Structural covariates include the age of the house (*housage*), square-footage of heated space (*sqft*), the lot size (*lotsize*), presence of air conditioning (*airc*), presence of a fireplace (*firep*), and whether the structure is multiple stories (*mtstory*). The vector of structural characteristics was limited due to the poor coverage in the data. Distance from the nearest metropolitan statistical area (MSA) (*dmsa*) and distance from the cities' central business district (*dcbd*) were included to control for amenities associated with proximity to urbanized areas. The MSA for the Carolina market was Charlotte, North Carolina, and in the Georgia-Florida market the MSA was Orlando, Florida.

Presence on the oceanfront (*ocfrnt*) and distance from the beach (*dist*) were included as environmental amenity variables. Defining beach quality required some exploration of the data. The data include the homeowner's subjective assessment of beach width at the nearest beach at the time the property was purchased. The subjective beach quality measures appeared to be too noisy to be of use in the hedonic model. In order to standardize this attribute measure, the median of the subjective beach width measures (*medbw*) in the beach community over a three-year period during which the house was sold was utilized as the measure of beach quality. The two market segments had an appreciable number of communities (21 in the Carolinas and 12 in the Georgia-Florida market). The numerous communities combined with the time dimension provided sufficient variation in the beach quality measure. The average number of observations used in calculating median beach width was approximately 22. In addition to beach quality, other loss mitigation variables include whether the property was built after the publication of the local jurisdiction FIRM (*pFIRM*), whether the property was located in the V-zone (*vzone*), and elevation of the properties lowest floor above BFE (*elev*). Dummy variables accounted for whether the sale occurred in the period after a major hurricane (Hugo (*hugo*) in the Carolinas and Andrew (*andr*) in Georgia-Florida) and whether the property was located in a CBRA zone (*cobra*) (and thus prohibited from obtaining federally-backed flood insurance). It is expected both of these qualities will reduce the rental price of the house, *ceteris paribus*.

The data include information on flood insurance holdings. A large proportion of households held flood insurance (*mktins*), some voluntarily while others were required to do so by their mortgage lender (*insreq*). The high participation rate in the Carolinas could reflect bias in the sample. Information on premiums (π) and coverage (s) was also available. Insurance coverage was converted to an annualized measure using a 7% discount rate (the prevailing mortgage rate in 1998).

The probability of a major coastal storm was estimated based on historical storm landfall data. A procedure outlined in Green, Walk, and Altay (2003) shows how to calculate a threat index that roughly approximates the probability of a major coastal storm. The threat index for a local jurisdiction is estimated as the ratio of coastline in the jurisdiction to the total coastline in the state multiplied by the average number of storms in the state within some time interval. I calculated the threat index for each local jurisdiction using historical storm data extending back at least 150 years. The result is *pstorm*, displayed in Tables 4.1.

Each market exhibits adequate variation in risk and loss-mitigating attributes, so that self-selection should not be a problem. The average storm "probability" in the Carolina market was 0.118 percent per year, and in the Georgia-Florida market it was 0.206 percent per year. The rough probability measure ranged from 0.018 to 0.262 percent in the Carolinas, and from 0.022 to 1.432 percent in Georgia-Florida. These probabilities are all of a fairly low magnitude. The highest annual percentage in the Georgia-Florida market was associated with St. Simon's Island in Georgia, which has a relatively long coastline.

Median beach width ranged from 1.5 to 60 meters in the Carolinas market and from 0 to 53 meters in the Georgia-Florida market. Likewise, elevation above BFE ranged from –2.8 to 9.2 meters (8.5 meters) in the Carolinas (Georgia-Florida) market. Fifty-two percent of the houses were oceanfront in the Carolinas, while 45% were oceanfront in Georgia-Florida. Seventy percent of the houses were constructed after publication of the FIRM in the Carolinas—and thus were built to be flood- and wind-resistant—and 46% were located in the V-zone. In the Georgia-Florida market, a much smaller proportion—33%—were constructed post-FIRM, and only 11% were located in the V-zone. The smaller proportion of V-zone homes probably reflects a difference in topography in this market. In each market, approximately 5% of the homes were located in CBRA zones; recall these homes are prohibited from obtaining federally-backed flood insurance.

Hedonic Price Equation Results

The hedonic price function in [4.10] was estimated for the two coastal housing markets for a span of 13 years (1986-1998). The dependent variable is the rental rate of the house (in 1998 dollars). The hedonic model is predicated on normalization by the numeraire, depicted in the budget constraint [4.3]. The hedonic rental rates were normalized for variation in the cost of living (net of housing costs) by using price level at the city nearest to the coastal county.[30] The cost of living data are from 2002, and thus could be a source of error, as they do not include historical fluctuations in relative prices across the coastal counties. The Box-Cox transformation parameter is estimated via maximum likelihood. The parameter estimate, confidence intervals, and tests of nested forms are presented with the results. The transformed, normalized rental rate is utilized as the dependent variable in ordinary least squares regression for the two markets.

Regression results are presented in Table 4.2. The Box-Cox parameter estimate for the Carolina market is 0.2555, and for the Georgia-Florida market it is 0.0616. A series of likelihood ratio tests were conducted to test the linear and semi-

[30] These cities were: Wilmington, NC for Brunswick County; Greenville, NC for Dare County, Myrtle Beach, SC for Georgetown County; Albany, GA for Glynn County; Melbourne, FL for Brevard County; and Cape Coral, FL for Lee County.

Table 4.2: Hedonic Price Regression Results

	Carolinas		Georgia-Florida	
Variable	Coefficient	Std Err	Coefficient	Std Err
housage	-0.0351	0.0269	0.0019	0.0021
sqft	0.0026***	0.00031	0.00012***	0.000019
lotsize	0.00012***	0.00003	0.000023***	0.0000016
mtstory	5.0447***	0.5801	0.1518***	0.0522
airc	1.7463***	0.9594	0.1870***	0.0641
firep	1.5356***	0.5735	0.1084	0.0775
dmsa	-0.0029	0.0146	-0.0030***	0.0009
dcbd	-0.00041***	0.00008	-0.000016***	0.000005
ocfrnt	4.6412***	0.8004	0.5265***	0.0588
dist	-0.0058	0.0052	-0.00088**	0.00036
medbw	0.2344**	0.0920	0.0271***	0.0040
medbw2	-0.0045***	0.0016	-0.000201***	0.000034
pfirm	-0.8596	0.7308	0.0663	0.0625
vzone	-0.3463	0.7018	0.0073	0.0826
elev	0.3303*	0.1853	0.0144	0.0147
hugo/andr	-1.9166*	0.9914	-0.3297***	0.0972
time	-0.2482**	0.1207	0.0147	0.0129
brun/brev	-1.1840	3.9161	-1.3866***	0.2871
geor/lee	4.8079	4.7169	-0.0204	0.1367
cobra	-2.9940**	1.1790	-0.1233	0.1332
intercept	34.8970***	9.0752	10.2116***	0.4066
R^2	0.7034		0.8546	
F_{stat}	41.26		82.86	
gamma	0.25546	0.03977	0 .06156	0.02843
LRT	625.46		900.33	

Both regressions utilize data from 1986-1998; dependent variable is transformed rental rate in 1998$; *=statistically significant at α=10%; **=statistically significant at α=5%; ***=statistically significant at α=1%

log forms for each market. The semi-log form could not be rejected for the Georgia-Florida model; both forms were rejected for the Carolina model. Each model has relatively high explanatory power with R-squared measures of 70.34% (Carolinas) and 85.46% (Georgia-Florida). Thirteen (twelve) of the 20 covariates are statistically significant at the 10% level in Carolinas (Georgia-Florida) model, and most coefficients have the expected signs, with the exception of *vzone* in the Georgia-Florida model. I focus, from this point, on the resource and loss mitigation variables.

Ocean-frontage has a positive and significant effect on property rental rate in each of the models. Distance from the shore has a negative effect on property rental rate, all else equal, but this effect is only significant in the Georgia-Florida model. Median beach width is statistically significant in each model, and the hedonic price functions are increasing and concave in this beach quality measure. The dummy variables for post-FIRM construction and presence in the v-zone are not significant in either model. With regard to post-FIRM construction, this result is not surprising due to countervailing effects of a home being built after the publication of the local FIRM. On one hand, the property is constructed to meet higher standards of flood- and wind-resistance, which should increase their value relative to properties that were constructed earlier. On the other hand, flood insurance rates increase after the publication of the FIRM, and pre-existing properties enjoy reduced rates that new construction does not. If some homebuyers recognize this, it could be partially capitalized in the sales price. The V-zone dummy variable is correlated with ocean-frontage, but the degree of correlation is not exceedingly high. Elevation above BFE has a positive effect on rental rate, but this effect is not significant in the Georgia-Florida model. Lastly, properties located in a CBRA zone are discounted, though this discount is not different from zero in the Georgia-Florida model.

Table 4.3 displays descriptive statistics on rental rates and marginal implicit rental prices associated with the continuous resource and loss mitigation variables from the hedonic regressions. These implicit rental rates were calculated as:

$$\partial h_{ij}/\partial res_{ijk} = W_{ijk} = (\gamma_j\, H_{ij}(.)+1)^{(1/\gamma j\text{-}1)} \times \gamma_j\, \partial H_{ij}/\partial res_{ijk}$$

where h_{ij} is the calculated housing rental rate (observed sales price times mortgage rate), $H_{ij}(.) = \beta_j^0 + a_i'\beta_j^a + res_i'\beta_j^{res} + loss_i'\beta_j^{loss}$ is the predicted housing rental rate, γ_j is the Box-Cox parameter, and *i* indexes households, *j* indexes markets, and *k* indexes the resource (or loss-mitigating) variables. In general, cost of living (net of housing) is lower in the Georgia-Florida market, but the predicted rental rates are higher for this market. The average rental rate in the Carolinas is $19,496 per annum, and in the Georgia-Florida market it is $36,724. The difference in the real rental rate is approximately $16,000 per annum. This evidence supports the contention that there are substantial differences in the distribution of consumer characteristics and/or the distribution of housing characteristics across these two markets.

Table 4.3: Predicted Rental Values and Marginal Implicit Prices

	Carolinas		Georgia-Florida	
Variable	Mean	Std Dev	Mean	Std Dev
housing rental rate	19495.55	12916.73	36724.73	67712.00
marginal rental rate for *q*	144.53	189.47	758.92	1452.27
marginal rental rate of *elev*	503.04	242.64	528.73*	974.85
marginal rental rate of dist	-8.94*	4.31	-32.46	59.86
N	369		303	

* Marginal effect not significantly different from zero at the 10% level

Despite this difference, the resulting marginal rental prices seem reasonable: the average marginal rental rate for an additional meter of beach width in the Carolinas is $145 per year, while in the Georgia-Florida market it is $759 per year. An additional meter of elevation above the BFE is worth $503 per year, on average, in the Carolinas. The estimated value of an additional meter of elevation in the Georgia-Florida model was $529 per year, though this implicit price is not statistically significant. The average rental rate for decreasing distance from the shoreline by one meter is $9 in the Carolinas model and $32 in the Georgia-Florida model. The price of distance from the shore, however, was not statistically significant in the Carolinas model. The influence of dichotomous variables can be calculated as the change in predicted rental rate as the indicator variable is changed from ‘0’ to ‘1’. Presence on the oceanfront is estimated to increase the average rental rate by about $5,400 in the Carolinas and approximately $13,200 in Georgia-Florida. These effects are rather large, but not implausible.

Coastal Homeowner Beach Quality Demand

The individual household marginal implicit rental rates for beach quality are utilized as the dependent variable in a second-stage inverse-demand equation for beach quality. I focus only on the demand for beach quality, rather than the demand system implied by [4.11]. I assume that resource and loss-mitigating variables are weakly separable from other housing attributes in the demand equation. The regression equations include the resource quality variables, loss-mitigating variables, linearized residual expected income (*adjexpinc*) (from [4.12]), and household characteristics as explanatory variables. The household characteristics were obtained from the mail survey questionnaire of coastal homeowners. The included household characteristics are: age of the head of household, whether the head of household has college or graduate school as their highest educational attainment, the number of persons under age 19 in the household, and whether the household is required to hold flood insurance by their mortgage lender.

Beach quality is defined as median beach width in the community over a three-year period during which the property was purchased. Due to the nonlinearity of the hedonic price function, both the marginal implicit rental rate and beach quality are jointly determined. The balance of resource (*ocfrnt* and *dist*) and loss-mitigating (*pFIRM*, *vzone*, and *elev*) variables is endogenous as well, as is the residual budget constraint. Due, however, to the survey nature of the data, a wealth of instruments is available to identify parameters of the demand equation.

Following Bartik (1987) and Kahn and Lang (1988) demand shifters are interacted with a dummy variable representing the *Carolina* market to create instruments that are used in the first-stage of two-stage least squares. Three-stage least squares was not utilized due to the potential mis-specification of either [4.10] and/or [4.11]. The demand shifters are: whether the head of household's highest level is college or graduate school; whether the head of household works part-time, is unemployed, or retired; the age of the head of household and the square of age; the size of the household and the square of size; number of children under 19 in the household and the square of children; whether the coastal home is the primary residence of the household; whether the coastal home is primarily an investment property rented out year-round; and whether the homeowner upgraded the property after purchase.

A series of dummy variables were used to specifically address endogeneity of the budget constraint. These included: whether the state of primary residence had no state income tax, whether the state of primary residence had low state income tax (less than 4% for the highest bracket), and whether the state of primary resident had high state income tax (greater than 7% for the highest bracket). The excluded category is medium state income tax rate (greater than 4%, but less than 7% for the highest bracket). Note the state of primary residence can differ from the state in which the property is located. Primary residence was elicited in the homeowner mail survey. To increase efficiency, the cross products of all of these variables are also used as instruments.

Expected residual income was approximated as follows. Gross household income (m) at the time the property was purchased is approximated by the household's response to a categorical question in the homeowner survey. This income figure is inflated to 1988 dollars using the CPI. The probability of storm landfall is approximated by the threat index, previously described. The insurance premium (π) and amount of coverage (s) is obtained from the survey questionnaire. For households without flood insurance, the premium and coverage were set to zero.

The expected storm loss is estimated from a logistic regression model for a sample (118) of properties located on the mid-Atlantic and the Gulf that experienced storm damage. All of these properties held flood insurance and the loss was calculated as the sum of the household's insurance settlement and deductible, divided by the current value of the house. The results of this model (found in Table 4.4) suggest that post-FIRM construction and elevation decrease the proportion of loss to housing value, while median beach width and location in a V-zone are insignificant. Dummy variables for the Carolinas and the Gulf were included. The excluded category is Sussex County, Delaware. These results were used to calculate an (admittedly rough) approximation of expected loss $L(.)$.

Table 4.4: Logistic Regression Equation for Expected Loss

Variable	Coefficient	Std Err	t-stat	p-value
medbw	0.00835	0.00963	0.87	0.3878
pfirm	-0.71381	0.31517	-2.26	0.0255
vzone	-0.29633	0.37798	-0.78	0.4347
elev	-0.10744	0.03806	-2.82	0.0056
gulf	1.18931	0.48505	2.45	0.0158
carolina	-0.41657	0.42131	-0.99	0.3249
intercept	-2.39836	0.55405	-4.33	<.0001

Dependent variable is odds ratio=*ln(p/(1-p))*, where *p*=(settlement+deductible)/(housing value); R^2=0.3042; F_{stat}=8.09

Hedonic price theory assumes that the household bid function is non-linear. As such, we expect the inverse demand function to be non-linear. The results of the two-stage-least-squares estimation procedure are presented in Table 4.5. Descriptive statistics for this equation can be found in Table 4.6. The model has fairly high explanatory power for an empirical demand equation, with an R-squared of 18.64%. Six of the 13 covariates are statistically significant at the 10% level, including median beach width, whether the property was built post-FIRM, whether the property is located in the V-zone, the income measure, age of the head of household and whether the head of household has graduate school as the highest level of educational attainment. The quadratic beach quality term is not statistically significant, thus results suggest that marginal WTP is decreasing and linear in beach quality. This result likely reflects errors in estimation. Post-FIRM housing construction decreases marginal WTP, as does presence in the V-zone. The latter result is unexpected, as one would expect households located in this zone to value the loss-mitigating properties of beach quality more highly. Expected income increases marginal WTP, implying beach quality is a normal good. Age and educational attainment increase marginal WTP.

Table 4.5: Beach Quality Inverse Demand Equation

Variable	Definition	Coefficient	Std Err
medbw	median beach width over 3 year period	-39.2662***	13.37191
medbw2	square of median beach width	0.424343	0.263193
ocfrnt	dummy variable equal to 1 if home located on oceanfront; 0 otherwise	67.00692	111.9406
dist	distance from the shore (meters)	-0.36238	0.791719
elev	elevation above BFE (meters)	-37.6412	24.79946
pfirm	dummy variable equal to 1 if home built post-FIRM; 0 otherwise	-202.289**	80.85934
vzone	dummy variable equal to 1 if home located in V-zone; 0 otherwise	-406.704***	87.67625
adjexpinc	linearized residual expected income	0.001284***	0.000478
age	age of head of household	5.512876**	2.639724
college	dummy variable equal to 1 if head of household had college as highest educational attainment; 0 otherwise	123.1723	76.48735
gradsch	dummy variable equal to 1 if head of household had graduate school as highest educational attainment; 0 otherwise	131.7250*	77.40344
hholdu19	number of household members under age of 19	-9.17153	14.69722
insreq	dummy variable equal to 1 if required to hold flood insurance	7.014555	52.28920
intercept	---	538.5387**	241.2555

Dependent variable is marginal implicit rental rate of q in 1998$; N=467; R^2=0.1864; $F_{stat=}$7.98; *=statistically significant at α=10%; **=statistically significant at α=5%; ***=statistically significant at α=1%

Table 4.6: Descriptive Statistics for Inverse-demand Equation

Variable	Definition	Mean	Std Dev
medbw	median beach width over 3 year period	14.8711	12.6880
ocfrnt	dummy variable equal to 1 if home located on oceanfront; 0 otherwise	0.5342	0.5931
dist	distance from the shore (meters)	109.191	89.107
elev	elevation above BFE (meters)	1.608	2.360
pfirm	dummy variable equal to 1 if home built post-FIRM; 0 otherwise	0.6153	0.5785
vzone	dummy variable equal to 1 if home located in V-zone; 0 otherwise	0.3688	0.5737
adjexpinc	linearized residual expected income	147324.82	100174.52
age	age of head of household	50.42	11.43
college	dummy variable equal to 1 if head of household had college as highest educational attainment; 0 otherwise	0.4160	0.5861
gradsch	dummy variable equal to 1 if head of household had graduate school as highest educational attainment; 0 otherwise	0.4367	0.5898
hholdu19	number of household members under age of 19	2.69	2.07
insreq	dummy variable equal to 1 if required to hold flood insurance	0.6405	0.5706

Integration of the Marginal WTP function gives the total value of beach quality accruing to the coastal household. To make use of the representative agent, all covariates are set to their weighted sample means. The choke price is \$917. The marginal WTP function for the representative agent is:

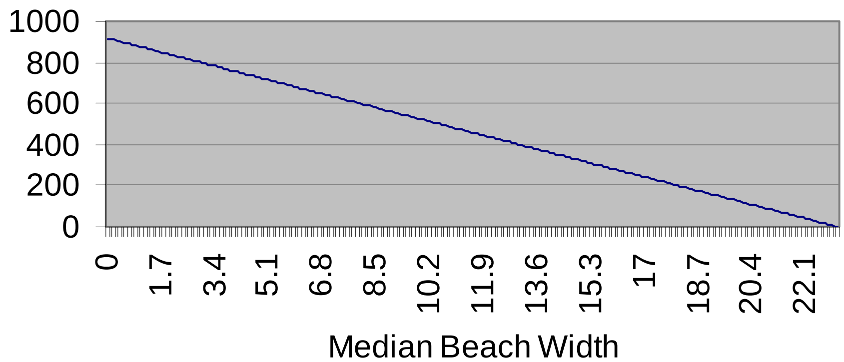
$$MWTP^{r}\,(q_t) = 916.70 - 39.67q_t \qquad [4.13]$$

This equation is depicted in Figure 4.1. The total value of beach quality for the representative coastal household is given by:

$$WTP^{r}(q_t) = 916.70q_t - 19.84q_t^2 + K^r, \qquad [4.14]$$

which is simply the integral of the marginal WTP function in Figure 4.1. K^r is a constant of integration.

Figure 4.1: Marginal Value Function for Coastal Homeowners



Identification of the uncompensated demand function, allows for the calculation of WTP in [4.14]. This function embodies what Bockstael and McConnell have called "pure willingness to pay", which is, in some sense, a generic welfare measure associated with the change in some exogenous public good. It is generic because it is not tied to a specific policy application. This measure of welfare change is revealed in the market data, but it implicitly assumes that the household is constrained to consume the same vector of housing characteristics as before the change (other than the attribute that has changed). As such, it is a lower bound to benefits (Bartik 1988). This measure can be inappropriate if one wishes to value a specific policy or proposal, as an exact welfare measure must take account of household adjustments in response to the exogenous change. This measure is appropriate however if the analyst desires a general representation of preferences, as is the case in the analysis to follow.

Chapter 5: Optimal Beach Erosion Control

This Chapter brings together the empirical estimates of benefits (presented in Chapters 3 and 4) in an application of the theoretical model of optimal beach nourishment (presented in Chapter 2). First, the study site is described, including the specifics on the beach profile. The parameters of the beach profile are then used to specify the transition equations for the study site. Next, historical beach nourishment cost data are used to estimate a beach nourishment cost function. Finally, the pieces of the optimal control problem are assembled. The solutions to this problem are described and discussed for both the short-term and long-term applications. Various specifications of the problem and sensitivity of the results are explored.

Details of the Study Site

The specific application of the optimal control model is to Tybee Island, the northernmost barrier island in Georgia. Tybee Island is located about 19 miles east of the city of Savannah, and has a relatively small year-round population of less than 3,000 people (1998 estimate). The population grows to approximately 10,000 between May and September (1994 estimate), and can exceed 30,000 on peak days in the summer (USACE 1994). It is a primary recreational destination for Savannah residents, as well as visitors from Atlanta and other population centers. Tybee is situated in a meso-tidal (tide-dominated) region, with tides typically ranging from 2 to 3.5 meters. The beach at Tybee Island is 4666 meters long, and the island is 1207 meters wide, on average. Tybee is a fairly typical southeastern barrier island, consisting of geological formations and subjected to climatological conditions that are somewhat similar to other barriers in the region (Clayton et al. 1992). The geographic setting of Georgia is unique in some respects, however, as the bathymetry and orientation of the coastline partially shelter the island from the full brunt of hurricane and tropical storm forces.

Tybee Island's beach has been intensively managed over the past 30 years, primarily by the USACE. There have been six major beach nourishment operations since 1976. As such, detailed information on the average beach profile is available. The average berm height (M) is 3.35 meters (USACE 1994). The depth of closure (h) is estimated at approximately 7 meters, the background erosion rate (θ) is 0.67056 meters per year, and the median sediment grain ranges in size from 0.16mm to 0.22mm in the nearshore region (Applied Technology and Management, Inc. 2002). Based on extreme wave height and the grain size and specific gravity of sediment, the scale parameter for equation [2.4] is a=0.039 (Hardistay 1990; Applied Technology and Management, Inc. 2002). This corresponds with an active profile width of W=2,377 meters (from equation [2.4]). This is a relatively shallow beach profile, which suggests the potential for relatively large shoreline retreat (by equation [2.6]) (Applied Technology and Management, Inc. 2002).

To fully identify the stream of management costs, one must determine the critical sea level—the mean sea level that threatens to engulf the island (Yohe et al. 1991). Given the tidal range on Tybee, 2 – 3.5 meters, and the berm height of 3.35 meters, a reasonable guess at the critical sea level rise is approximately 1.5 meters. Sea level rise of 1.6 meters would put the highest high tide at the height of the beach berm. With wave run-up, this would threaten to flood the island. I assume that sea level rise over 1.5 meters will require raising the island (by adding sand to its entire profile) to maintain Tybee's present location. Thus, I set S_{crit}=1.5.

The lagoon sides of barrier islands are not typically nourished, and thus are not as intensively studied by engineers. As a result, I do not have data on the berm or wave heights on the lagoon side of Tybee Island. This information is required to predict the erosion rate when sea level exceeds S_{crit} (by equation [2.16]). Shore profile on the lagoon side is generally less pronounced than the ocean side (Dean and Maurmeyer 1982). Without guidance on the lagoon-side profile parameters, I assume they are $^1/_3$ that of the ocean-side profile parameters. This assumption gives a berm height (M_l) of 1.117 meters and a closure depth (h_l) of 2.33 meters on the lagoon side. The corresponding lagoon profile width is W_l=462 meters.[31]

These physical dimensions of Tybee Island's shore are used to determine the various parameters of the transition equation for the management problem. For sea level rise below the critical level (S_{crit}) the increment in average beach width due to beach fill (τ) is given by equation [2.5']—$\tau = 1/(M + h)$; plugging in the parameters for Tybee Island, we have $\tau = 0.0966$. For the case of a constant sea level, the only other piece of information we require is the background erosion rate, $\underline{\theta} = 0.67056$ meters per year. Thus, the short-run (constant sea level) transition equation is:

$$q_{t+1} - q_t = - 0.67056 + 0.0966\, n_t. \qquad [5.1]$$

[31] Alternatively, assuming the lagoon side parameters were ½ the ocean side, we have M_l=1.675 meters, h_l=3.5 meters, and W_l=850 meters.

For the case of sea level rise, we have two transition equations, which will closely reflect the dynamic equations in [2.8] and [2.9], the only difference being that the background erosion rate ($-\underline{\theta}$) must be appended. Equation [2.8] applies for sea level rise below the critical level (S_{crit}), and equation [2.9] applies for sea level rise above the critical level. For equation [2.8], the τ parameter is the same as the short-run (constant sea level) case, and the erosion rate trajectory is given by equation [2.16]. The rate of change in the erosion rate is $\Delta\theta_t = W/(M+h) \times \Delta S \times t$, where ΔS is the change in sea level per unit time (measured in meters). Substituting the specific values for Tybee Island's ocean shore, the long-run transition equation is:

$$q_{t+1} - q_t = 0.0966\, n_t - 0.67056 - 229.662 \Delta S \times t \qquad [5.2a]$$

for $\Delta S \times t < S_{crit}$,

where ΔS is the change in sea level per unit time.

Once sea level rise reaches the critical level (S_{crit}), equation [2.9], with the background erosion term ($-\underline{\theta}$) appended, serves as the transition equation. Note, the τ parameter changes in this equation. The new parameter is $\tilde{\tau} = 1/[(M + h)-(M_l + h_l)]$. To parameterize [2.9], we require additional information on island width and the dimensions of the lagoon shore. Recall, the average width of Tybee Island is 1207 meters, and we had to make some assumptions about the lagoon shore profile. For sea level rise above S_{crit}, the rate of change in the erosion rate is $\Delta\tilde{\theta}_t =$

$(W+w+W_l)/[(M+h)-(M_l+h_l)] \times \Delta S \times t$. Inserting the specific measurements for Tybee Island in [2.9] and appending the background erosion parameter, the transition equation for sea level rise past the critical value is:

$$q_{t+1} - q_t = 0.1449\, n_t - 0.67056 - 586.122 \Delta S \times t \qquad [5.2b]$$

for $\Delta S \times t \geq S_{crit}$.[32]

Note that, relative to [5.2a], the τ parameter increases in [5.2b], as does the parameter of the erosion rate trajectory, $\Delta\theta_t$. This is to be expected. The τ parameter converts sand volume into beach width (or island width, as the case may be). If we are applying sand to the entire island, as in [5.2b] (as opposed to only the beach, as in [5.2a]), we would expect the conversion parameter to be larger because the island and lagoon have less slope than the offshore zone. The erosion trajectory parameter must also be greater, as was shown in Chapter 2. For Tybee Island, I assume equation [5.2a] applies for sea level rise below 1.5 meters. Equation [5.2b] applies for sea level rise greater than or equal to 1.5 meters.

For application to the optimal control model, the rate of sea level rise needs to be adjusted because I assume that the background rate (1-2 mm/year) is reflected in the historical erosion rate, $\underline{\theta}$. Assume background sea level rise in the vicinity of

[32] Using the alternate assumption on the relationship between lagoon and ocean shore profile (1/2), produces the following transition equation: $q_{t+1} - q_t = 0.1932\, n_t - 0.67056 - 856.812 \Delta S \times t$ for $\Delta S \times t \geq S_{crit}$.

Tybee Island is 2 mm/year. Thus, 20 cm must be subtracted from any sea level rise trajectory over the course of the next century.

Beach Nourishment Cost Function

Historical data on beach fill volumes and pecuniary costs are used to estimate a reduced-form beach management cost function. While my primary focus is on the Ideal Present Profile (IPP) of the beach as a measure of the state of the resource, I model nourishment costs in aggregate. That is, I focus on nourishment sand per unit of length (n_t) as a choice variable, but estimate the cost function as $C(N_t)$, where $N_t = n_t \times l$. I do this in order to avoid making assumptions about returns to scale in beach production along the shoreline.

The historical cost data includes 365 observations from the Gulf (Texas, Alabama, Mississippi, and Florida), Southeast (Florida, Georgia, South Carolina, and North Carolina), and Mid-Atlantic States (Virginia, Maryland, and Delaware). The data were obtained from Duke University's Program for the Study of Developed Shorelines,[33] and extend back to the early 1960s. Monetary costs were converted to 1998 dollars using the all-industry, producer-price index. The dependent variable is total project cost. Total sand volume (N) and the square of sand volume (N^2) are the chief independent variables of the reduced-form equation.

The cost function was estimated by least squares with state fixed effects and a time trend.[34] The reduced-form equation is:

---

[33] PSDS website: http://www.env.duke.edu/psds/index.html

[34] Other forms were estimated, but results were similar. LSDV estimation provided the best fit to the data.

$$C(N_{ik}) = cost98_{ik} = \gamma_k + \gamma_1 N_{ik} + \gamma_2 N_{ik}^2 + \gamma_3\, time_{ik} + \varepsilon_{ik}, \qquad [5.3]$$

where $N_{ik} = n_{ik} \times l_{ik}$, i represents observations on beach nourishment operations, and k represents jurisdictions (states). The parameter estimates are presented in Table 5.1. The LSDV estimates have fairly high explanatory power (41.37%), and all variables are statistically significant except for the Texas, Mississippi, and Virginia intercept terms. Results suggest that the cost function is increasing and convex. If these data encompass a sufficiently long time period (in which all inputs could be varied), then the results indicate decreasing returns to scale. The time trend is positive, indicating that costs have been increasing with time. This result could reflect dwindling reserves of beach fill sand in close proximity to the shore.

Table 5.1: LSDV Estimates of Beach Nourishment Cost Function

Variable	Definition	Coefficient	Standard error
N	sand volume (cubic meters)	2.70152*	1.60590
N^2	square of sand volume	0.00000134*	0.0000007
texa	intercept for Texas	-3999311	3333382
loui	intercept for Louisiana	-4691012**	2262796
miss	intercept for Mississippi	-5288455	5754290
bama	intercept for Alabama	-5531081*	2982792
flor	intercept for Florida	-3542236***	797561
geor	intercept for Georgia	-5603137**	2674120
scar	intercept for South Carolina	-4803018***	1486330
ncar	intercept for North Carolina	-5141801***	1116034
virg	intercept for Virginia	1917744	1369406
mary	intercept for Maryland	14055147***	2642486
dela	intercept for Delaware	-3103231***	1056543
time	time trend (1961=1; 2002=42)	167171***	27904

Dependent Variable=cost in 1998\$; # obs.=365; R^2=0.4137; F (14)=17.64;
*=statistically significant at α=10%; **=statistically significant at α=5%;
***=statistically significant at α=1%

Solutions to the Optimal Control Model

We now have all elements of the optimal control system (introduced in [2.10]-[2.13]) at hand. We use consumer surplus from beach visits as an approximation of beach visitors' willingness to pay for beach quality. For beach visitors, we assume that beach width is an essential input into the production of "beach days"—the recreational good embodied by a day at the beach. As such, the reservation utility for the study site with zero beach quality is assumed to be zero. I set the constant of integration, K^b from equation [3.15], to zero to reflect this. Our economic return function for beach visitors is:

$$WTP^b\,(q_t) = 726.80q_t - 8.41\,q_t^{\,2}, \qquad [5.4]$$

where *b* represents beach visitors.

The recreational demand model of Chapter 3 takes into account not only changes in the intensive margin of beach use arising from changes in beach quality, but implicitly considers changes in the extensive margin (or how alteration of beach quality affects the propensity to visit for non-users) by controlling for the truncation of non-users. Thus, changes in visitation for both current users and non-users are embodied in [5.4]. This economic return implies large benefits measures for each beach visitor household, but a seemingly reasonable optimal beach width for this group. With no cost, the optimal beach width implied by [5.4] (at which marginal value becomes zero) is 43.2 meter. Willingness-to-pay for this optimal width is $15,720 per household per year. This benefit measure is much larger than expected *a* *priori*. In fact, it represents about a quarter of annual income. Apparently, the constrained linear regression produces unreasonable results for economic returns from beach quality accruing to beach visitors. This caveat notwithstanding, equation [5.4] will be used to represent the preferences of beach visitors in the optimization model to follow. One should bear in mind that the economic returns from beach quality for beach visitors are very likely to be overestimated, perhaps by a significant amount. As such, solution to the optimization problem should be considered illustrative.

For coastal homeowners, we make use of the inverse-demand equation for beach quality presented in Chapter 4. Recall, the economic return function is:

$$WTP^r\,(q_t) = 916.70q_t - 19.84q_t^2 + K^r, \qquad [4.14]$$

where K^r is the constant of integration, and r represents coastal homeowners. Equation [4.14] is a "pure willingness to pay" measure—a general representation of preferences associated with the change in an exogenous public good. The constant of integration will not affect results of the short-run (constant sea level) model, because it drops out of the first-order conditions. The constant will, however, affect model results in the long run, as total benefits partially determine the terminal control time by equation [2.22]. Two cases will be considered.

The constant of integration could be set to zero if we wanted to examine the time horizon of active management with no regard to the residual value of property net of beach quality. This case would imply that the value of coastal property is zero at the terminal time, corresponding with full depreciation of threatened properties. In this case, we have:

$$WTP^{r} (q_t) = 916.70q_t - 19.84q_t^2. \qquad [5.5]$$

On the other hand, the constant of integration could be set to some positive value that we predict to be the average residual value of property, net of beach quality, at the terminal time. This case would imply less than full depreciation, or perhaps appreciation, or the threatened properties. In this case, we have

$$WTP^{r} (q_t) = 916.70q_t - 19.84q_t^2 + \overline{P}_{-q}, \qquad [5.5']$$

where $\overline{P}_{-q}$ is average property price, net of beach quality, at the terminal time. Previous research (Landry, Keeler, and Kriesel 2003) provides a hedonic regression equation that can be used to estimate the average value of property on Tybee Island. I use this approach to predict the value of a house on Tybee Island with all housing characteristics set to their sample means and beach quality set to zero. This measure represents the residual value of coastal housing in 1998 dollars. The estimated value is \$124,926, which is \$23,172 less than the estimated housing value with beach quality set to its sample mean (31.39 meters). I set the intercept in [5.5'] $\overline{P}_{-q}$ = 124,926. This is akin to the notion of "economic vulnerability" studied by Yohe (1991a). Other cases of residual property value could be considered, in particular, letting the average property appreciate or depreciate at different rates. The sensitivity of the terminal time to these cases would provide evidence about how optimal beach management under sea level rise responds to the trajectory of the market value of threatened coastal properties. With no cost, the optimal beach width implied by [5.5] and [5.5'] (at which marginal value becomes zero) is 23.1 meters—a plausible result. Total willingness-to-pay for this level of beach quality (from [5.5]) is $10,598 per coastal-homeowner household per year.

The uncompensated inverse-demand equation on which [5.5] and [5.5'] are based holds expected income and the quantity of other risk and resource variables constant. It does not hold utility constant, as would the Hicksian measure of compensation. As the threat of sea level rise becomes more imminent the threat of erosion loss will become more pronounced, and we might expect that willingness-to-pay for protection would increase. My model does not address this aspect of the beach erosion problem. I do not consider erosion risk, aside from that associated with coastal storms. The benefit functions of [5.5] and [5.5'] are a snapshot of preferences under the current expectations of sea level rise. My intentions are to forecast management decisions based on current preferences in order to provide a conceptual framework for management and to make some predictions about the time horizon of management given what is currently revealed about demand for beach quality in the housing and recreation markets. I feel strongly that the latter will provide useful information for achieving efficiency in the coastal property market and for planning responses to sea level rise.

Beach nourishment cost per meter of shoreline for Tybee Island is calculated from Table 5.1. The intercept for the state of Georgia is utilized, and the time trend is initially set to 38 (corresponding with technology and resources in the year 1998). Under these assumptions, the cost function is:

$$
\begin{aligned}
C(N_t) &= 749{,}361 + 2.7N_t + 0.00000134N_t^2 \\
C(n_t) &= 749{,}361 + 2.7(n_t \times l) + 0.00000134(n_t \times l)^2 \qquad [5.6] \\
C(n_t) &= 749{,}361 + 12598.2n_t + 29.1739n_t^2,
\end{aligned}
$$

where the intercept term reflects the combined influence of the time trend ("locked-in" at the year 1998) and the Georgia state fixed effect. The third line follows by inserting the length of Tybee Island (l=4666m) and simplifying. If we were to allow the time trend to evolve, costs increase over time, as suggested by the results in Table 5.1. This produces the following cost function:

$$
C(n_t) = -\,5{,}603{,}137 + 12598.2n_t + 29.1739n_t^2 + 167{,}171 \times time, \qquad [5.6']
$$

where *time* is count variable, starting at 1 for the year 1961.

Let the discount rate be δ=0.1. Previous research on Tybee Island identifies the number of coastal households, R=2,795, and the total number of annual visitor trips at 899,284 (Landry, Keeler, and Kriesel 2003). I use the weighted truncated mean of *ex-post* trips (9.09 trips per year) to estimate the number of current beach visitor households at 98,931. I then multiply the number of current users by the ratio of the truncated to the censored mean ([9.09 trips per year]/[6.46 trips per year] = 1.407) to produce an estimate of the potential user population, 139,196 households.

Recall, visitors staying over five days on-site were removed from the sample of recreational users in order to avoid double-counting of benefits (see Chapter 3 for more details). The number of beach visitor households must also be adjusted for this correction. The raw data contain 2,467 observations, of which 1,946, or 79 percent, pertained to households staying less than six days on-site. Multiplying this percentage by our potential user population estimate, puts the number of beach visitor households staying less than six days on-site at B=109,965.

Constant Sea Level (Short-term) Control Problem

The short-term control problem, assuming a finite time horizon (25 years) and constant erosion rate, is:

$$\max_{n_t} \sum_{t=0}^{24} \eta^t \{B \times WTP^b(q_t) + R \times WTP^r(q_t, L(q_t)) - C(n_t)\}$$

$$\text{subject to} \qquad q_t - q_{t-1} = -\theta + \tau n_{t-1}, \qquad n_t \geq 0$$

$$q_{t=0} = q_0 \geq 0$$

$$q_T = free.$$

Initial average beach quality in the Tybee Island survey is 23.5 meters. This estimate is based on beach measurements in the spring of 1998. Substituting [5.4], [5.5], [5.6], and the other parameter values, the current-value Hamiltonian is:

$$\widetilde{H} = 109{,}965\times(726.80q_t - 8.41\, q_t^{\,2}) + 2{,}795\times(916.7q_t - 19.84q_t^2)$$

$$-\,(749{,}361 + \; 12598.2n_t + \; 29.1739n_t^2) + \eta\phi_{t+1}(-0.67056 + 0.09662\times n_t),$$

where $\eta = 1/(1 + \delta)$ is the discount factor for one period and $\phi_{t+1} = \lambda_{t+1}\times(1+\delta)^{t+1}$ is the current shadow value in the subsequent period. This Hamiltonian is differentiable and concave in q_t and n_t, jointly. Thus, the Mangasarian Sufficiency Theorem applies, and the necessary conditions for the maximum principle are sufficient for a global maximum. The transition equation is linear in q_t and n_t, requiring no sign restriction on the costate variable (Chiang 1992). The necessary conditions for maximizing $\widetilde{H}$ are:

$$\partial\widetilde{H}/\partial n_t = -12598.2 - 58.348n_t + 0.09662\eta\phi_{t+1} \le 0, \quad n_t \ge 0 \qquad \forall\, t \quad [5.7]$$

$$(-12598.2 - 58.348n_t + 0.09662\eta\phi_{t+1})\times n_t = 0$$

$$\eta\phi_{t+1} - \phi_t = -\partial\widetilde{H}/\partial q_t$$

$$= -109{,}965\times(726.80 - 16.82\, q_t) - 2{,}795\times(916.70 - 39.67q_t) \quad \forall\, t \quad [5.8]$$

$$q_{t+1} - q_t = \partial\widetilde{H}/\partial\eta\phi_{t+1} = -0.67056 + 0.09662\times n_t, \qquad \forall\, t \quad [5.9]$$

as well as a transversality condition. If we intend to extract all surpluses from the control problem in the short run, condition [2.21] is the appropriate transversality condition, and we have:

$$\phi_T\eta^T\times q_T = \phi_{25}\eta^{25}\times q_{25} = \phi_{25}/10.835\times q_{25} = 0, \qquad [5.10]$$

where $\eta^T = 1/(1+\delta)^T$ is the discount factor evaluated in the final period. Under condition [5.10], either the shadow value of beach quality or the stock of beach quality must be zero at the terminal time. On the other hand, the coastal planner may want to leave a certain level of beach quality at the terminal time. In this case, condition [2.21'] is the appropriate transversality condition, and we have:

$$q_T = \overline{q}\,. \qquad [5.10']$$

For illustration purposes, we may assume $\overline{q} = q_0$ = 23.5 meters; the coastal planner intends to leave the resource in a terminal state that is equivalent to the initial state.

With a discount rate of δ=0.1, we have η=0.9091. From condition [5.7], we see that for $n_t > 0$, we must have:

$$n_t = 0.0015\phi_{t+1} - 215.915 \qquad [5.11]$$

Let us assume $n_t > 0$ for t=0, ... T-1, and let us first consider the case of transversality condition [5.10]. If erosion were allowed to proceed unabated over the 25-year time horizon, average beach width at the end of the last period would measure 6.74 meters. (See Figure 5.1.) Thus, $q_{25} \neq 0$, and $\phi_{25} = 0$ must be true by condition [5.10]. With $\phi_{25} = 0$, [5.11] cannot hold for the final period, so $n_{24}=0$. This makes intuitive sense, as the coastal planner would not want to waste resources in the final period since the effects occurring after this period are of no import. In general, we can deduce that

$\phi_{t+1} > 143{,}943.33$ in any period for beach nourishment to be undertaken in the previous period. Assuming an interior solution and substituting [5.11] in [5.9] gives:

$$q_{t+1} - q_t - 0.000145\phi_{t+1} = -21.532, \qquad [5.9']$$

which along with

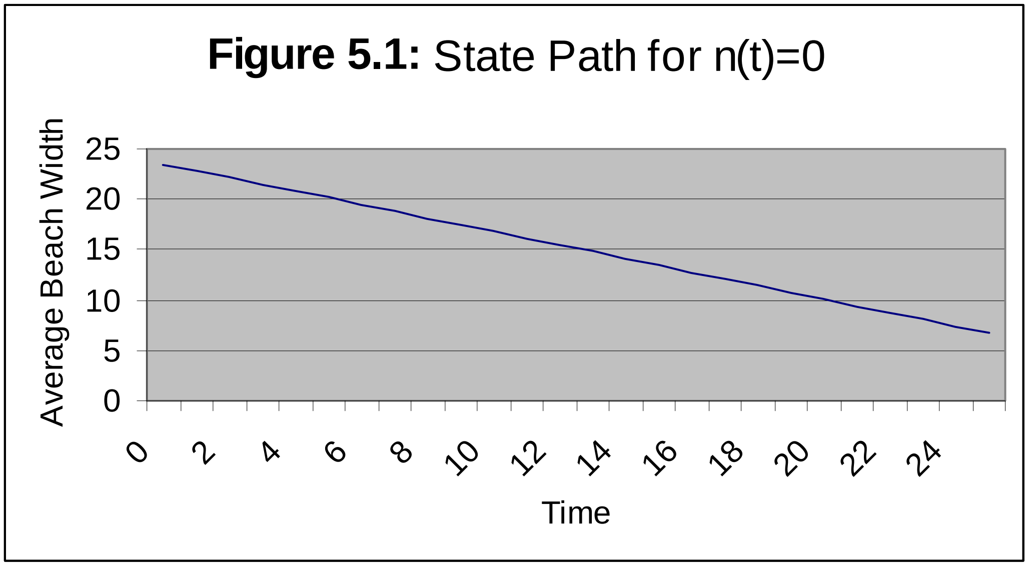
$$\phi_{t+1} - 1.1\phi_t - 2{,}156{,}537.85q_t = -82{,}484{,}738.5 \qquad [5.8']$$

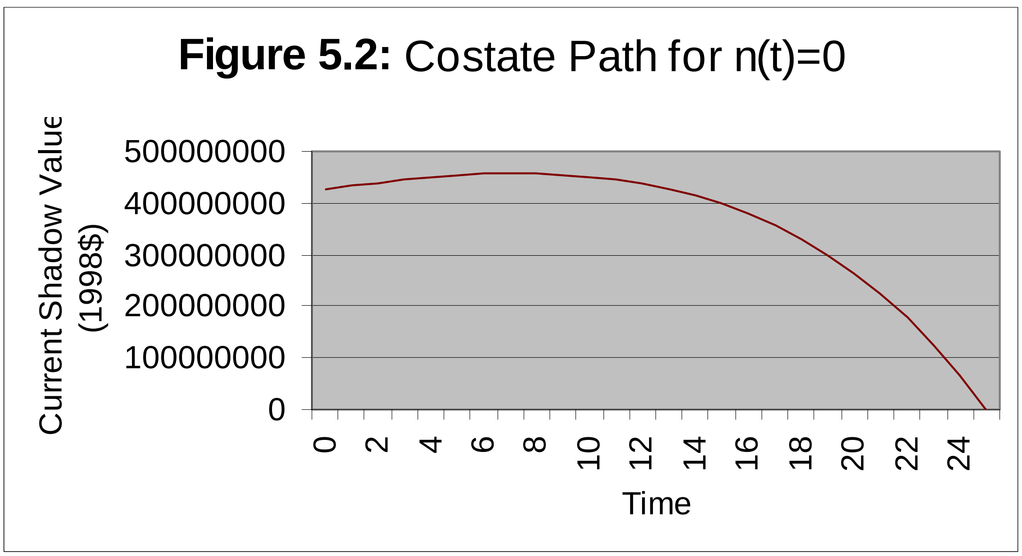
constitutes a system of first-order, linear difference equations with constant coefficients. Equations [5.8'] and [5.9'] comprise the canonical form of the solution to the dynamic optimization problem, but this system only applies for $n_t > 0$ for all t. We have seen that these equations cannot apply for all t for transversality condition [5.10]. The equations can apply for transversality condition [5.10'], but upon further inspection, the system proves to be unstable and does not have a solution.[35]

Next, let us assume that $n_t = 0$ for t=0, ... T-1. Simulating erosion over 25 years produces a final average beach width of 6.74 meters. The uncontrolled dynamic state path is deterministic, and is depicted in Figure 5.1. Transversality condition [5.10] is the relevant boundary condition for the case of "no control", and thus $\phi_{25} = 0$. The dynamic path of the costate variable can be solved recursively, using equation [5.8] and working backwards in time. The costate path is shown in

[35] The roots of the characteristic equation for this system are 0.0035 and 314.796—the first is stable, but the second causes the state path to blow up over time.

Figure 5.2. The costate variable is always greater than \$143,943.33, and thus $n_t = 0$ for t=0, ... T-1 cannot be a solution.





Dynamic Programming

This evidence suggests that the control path is intermittent. In some periods n_t>0, while in others, n_t=0. In order to solve the short-term problem, I use dynamic programming. Numerical dynamic programming is most readily accomplished by discretizing the state and control spaces and applying Bellman's backward recursion algorithm. The approach of backward recursion is based on Bellman's Principle of Optimality, which states that an optimal policy must constitute an optimum with regard to the remaining periods regardless of preceding decisions. As such, one can solve the problem by working backwards. Bellman's equation for the beach erosion management problem is:

$$V_J(q_t) = \max_{n_t \geq 0}\{WTP^*(q_t) - C(n_t) + \eta V_{J-1}(q_{t+1})\}, \qquad [5.12]$$

where WTP^* is the weighted sum across user groups (aggregate measure) of willingness-to-pay, following the above notation $n_t = N_t / l$, and J represents the number of periods remaining. Recall $q_{t+1} = q_t - \theta + \tau n_t$. $V_J(q_t)$ is the value function, which gives the sum of current and discounted future returns to beach quality following the optimal policy of beach nourishment.

Aggregate willingness-to-pay becomes negative at $q \geq 85$ meters, providing a natural upper bound on beach quality for policy purposes. The state space was defined as $0 \leq q \leq 85$, with 0.1 meter increments giving rise to a 1×851 vector of states. Since q may only take on multiples of 0.1, the erosion rate can only be approximated. That is, we cannot use the historical erosion rate of 0.67056 meters per year in the transition equation, as this would send next period's state to an undefined location because q only takes on multiples of 0.1. I use an erosion rate of 0.7 meters per year, which will lead to an approximate solution. Likewise, the τ

parameter must be approximated at 0.1. The control space was defined over $0 \leq n \leq 850$, with 1 square-meter increments giving rise to a 1×851 vector of controls.

Assuming V is differentiable in q, returns from beach management are maximized when

$$-\partial C/\partial n_t + \eta\, \partial V_{J-1}/\partial q_t \times 0.1 = 0 \qquad [5.13]$$

for an interior solution. The τ parameter, which converts sand volume to beach width per meter of shoreline length, is approximated at 0.1 and appears as a multiplicative term in [5.13]. By the envelope theorem,

$$\partial V_J/\partial q_t = \partial WTP^*/\partial q_t + \eta\, \partial V_{J-1}/\partial q_{t+1} \qquad [5.14]$$

Combining [5.13] and [5.14], we have

$$\partial V_J/\partial q_t = \partial WTP^*/\partial q_t + [\partial C/\partial n_t]/0.1 \qquad [5.15]$$

for $t=0,\ldots,T-1$ and $J=T-1,\ldots,0$. Substituting [5.15] for $\partial V_J/\partial q_t$ and $\partial V_{J-1}/\partial q_{t+1}$ in [5.14] we arrive at

$$[\partial C/\partial n_t]/0.1 = \eta\, \partial WTP^*/\partial q_{t+1} + \eta\, [\partial C/\partial n_{t+1}]/0.1 \qquad [5.16]$$

Expression [5.16] is interpreted as follows: at the optimum, the marginal cost of an additional increment to beach width in period t should be equal to the present value of the sum of marginal willingness to pay and the marginal cost of an additional increment to beach width in the subsequent period ($t+1$). The first term on the RHS of [5.16] reflects the benefits of beach nourishment, which begin to accrue in the next period. The second term on the RHS of [5.16] reflects the foregone cost of beach nourishment in the subsequent period due to action in the current period. Rearranging [5.16], we have

$$[\partial C/\partial n_t]/0.1 = \eta/(1-\eta)\ \partial WTP^*/\partial q_{t+1} \qquad [5.16']$$

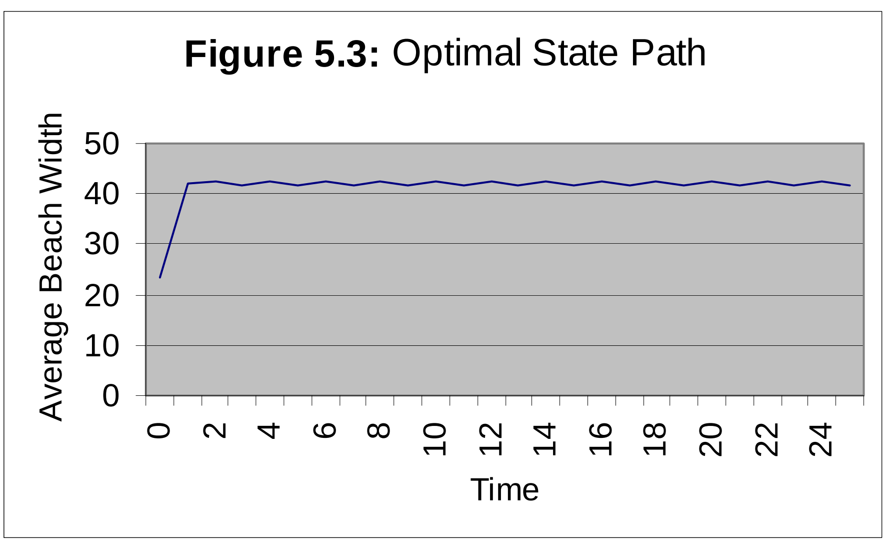
which defines optimal beach nourishment by balancing present marginal cost with the present value of the flow of benefits due to nourishment in perpetuity.

A numerical routine for estimating the value function and optimal beach nourishment policy was adapted following Miranda and Fackler (2002). The program implements value function iteration; backward recursion is used to solve the beach erosion problem by starting in the last period and determining the optimal decision rule and resulting maximum value for each possible state. Working backwards, this procedure is repeated for each previous period. The solution at the first period provides the maximum attainable value, and the entire procedure provides a roadmap of optimal policies for each period conditional on the results of the previous period.

Figures 5.3 and 5.4 depict the optimal state path and control paths, respectively, for initial beach quality of 23.5 meters and a terminal value given by

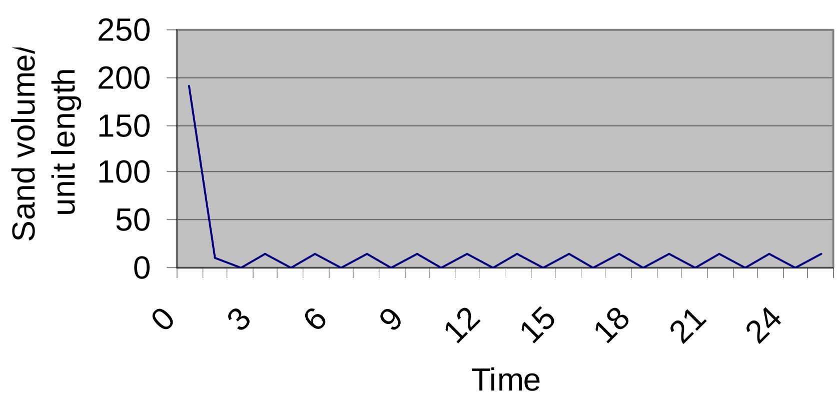
$WTP^*(q_T)$. The beach quality that maximizes aggregate willingness-to-pay is 42.07 meters. The magnitude of the benefits of maintaining this beach quality dominates that of the short-run costs. The optimal beach nourishment policy overshoots this optimal level of quality in the second period and borders it in all subsequent periods, buffeting between 42.4 meters and 41.7 meters. As such, the optimal solution approximates a steady state by maintaining the state variable in close proximity to the optimum.

With a terminal value function, we maintain this pattern until period *T*-1. If we were to neglect the terminal value, this pattern would be maintained only until *T*-2. In the former case, the optimal control is 192m^3 of sand per meter of beach length in the initial period, 11m^3/m in the second period, and buffets between 0m^3/m and 14m^3/m in all subsequent periods until *T*-1, after which it becomes zero. The present value of the stream of returns associated with the optimal policy is an immense $189 billion. This number is upward biased due to the overstatement of benefits associated with beach visitors. (The control path is slightly different for the case of no terminal value, but follows the same basic pattern.)

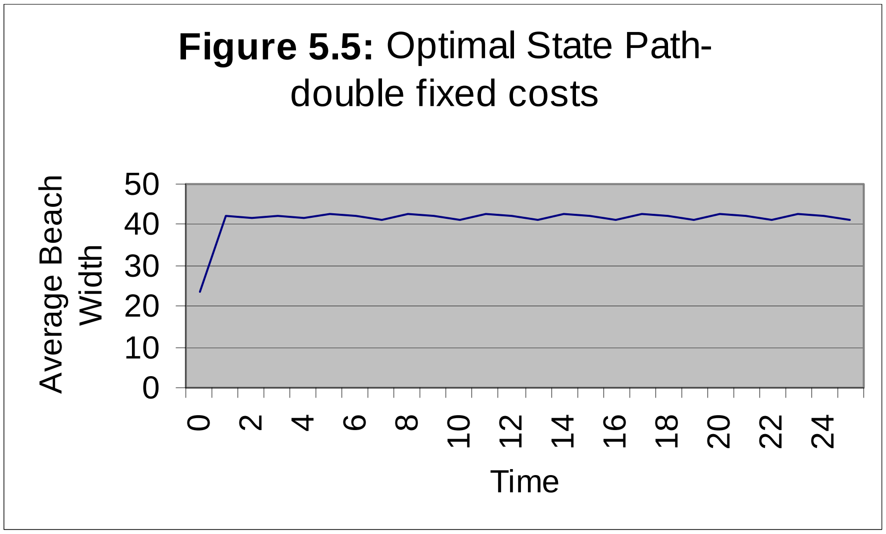


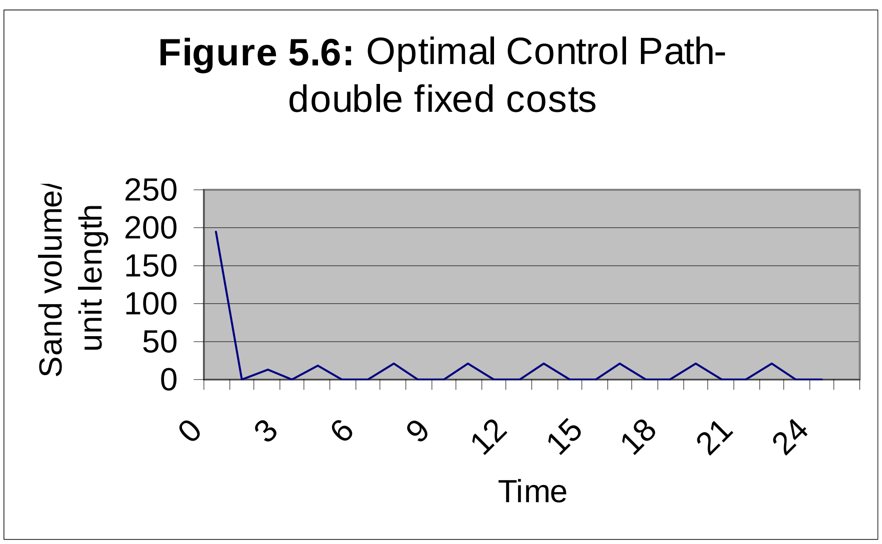
If initial beach quality were 58.7 meters or greater, the optimal policy would be no beach nourishment over the entire time interval. At an initial level of 58.7 meters, annual erosion reduces average beach width to 42 meters only after 24 years, and thus beach nourishment is not warranted. Conversely, if initial beach quality were only 10 meters, the initial nourishment would consist of $327m^3$ of sand per meter of beach length (giving rise to an average of 42 meters of beach width) followed by $11m^3/m$ in the second period, and the following the intermittent pattern of $0m^3/m$ and $14m^3/m$ in all subsequent periods until T-1. This type of solution path is similar to the most rapid approach, and holds for all initial beach widths below 42 meters. The dominance of the magnitude of the benefits over that of the costs leads to this type of solution. This aspect of the problem also makes the solution insensitive to the discount rate. The existence of fixed costs contributes to the intermittent nature of the solution near the optimum state.

Figure 5.4: Optimal Control Path



A doubling of fixed costs alters the solution. Changing the fixed costs to $1,498,722 leads to larger gaps of inactivity in the optimal solution. The optimal state and control paths for this scenario are depicted in Tables 5.5 and 5.6, respectively. The optimal policy still embodies a most-rapid approach, but once the state is in the neighborhood of the optimum the optimal policy is to place $21m^3$ of sand per meter of beach length and forego sand placement in the two subsequent periods. The larger periods of inactivity are due to the increased fixed costs, and this pattern persists for greater increases in fixed costs.





The intermittent nature of the solution mirrors the structure of a typical beach nourishment project. In practice, beach renourishment is usually conducted every 5-10 years for chronically eroded beaches. While my solution suggests more frequent operations are optimal, it could be that the present model misses some of the fixed costs. For example, use of the beach is limited during nourishment operations, and the operations themselves could negatively impact the aesthetics of the coastal zone. These external costs may lead policymakers to increase the period of time between beach nourishment operations.

Rising Sea Level (Long-term) Control Problem

In the long run, sea level rise will cause increasing shoreline erosion. In general, the long-term transition equation is given in [2.8] and [2.9], with the specific long-term transition equation for Tybee Island in equations [5.2a] and [5.2b]. These expressions depend upon the rate of sea level rise—ΔS. I consider three scenarios that span the current estimates of sea level rise over the next century—30cm, 55cm, and 80cm. These sea level rise estimates must be adjusted for the background level of sea level rise, assumed 20cm per century for Tybee Island. Thus, I subtract 20cm from the total rise, and divide by 100 to produce an annual rate, giving 0.001 meters/year, 0.0035 meters/year, and 0.006 meters/year, respectively. In addition, I make the terminal time a choice variable in the long run. Our problem is:

$$\max_{n_t, T} \sum_{t=0}^{T-1} \eta^t \{B \times WTP^b(q_t) + R \times WTP^r(q_t, L(q_t)) - C(n_t, t)\}$$

subject to $\quad q_t - q_{t-1} = -\theta - \Delta\theta(t) + \tau_{t-1} n_{t-1}, \qquad n_t \geq 0$

$$q_{t=0} = q_0 \geq 0$$

$$q_T = free,$$

where the cost function is now an explicit function of time, the τ parameter varies with time, and $\Delta\theta(t) = W/(M+h) \times \Delta S \times t$ for $\Delta S \times t < S_{crit}$, otherwise $\Delta\theta(t) = \Delta\tilde{\theta}_t = (W+w+W_l)/[(M+h)-(M_l+h_l)] \times \Delta S \times t$.

As we have found that the Maximum Principle is of limited use in solving the beach nourishment problem, I dispense with the Hamiltonian expression from here on. The dynamic programming solution algorithm for the long-term problem, however, is complicated by the non-stationary form of the transition equation. I save the numerical solution to the long-term problem as a topic for future research, and from here on focus on illustrating the type of results that can be produced with such a model. To accomplish this, I assume that the solution to the long-term problem is of the same form as the short-term problem. Specifically, I assume that the state variable should bracket the optimal value (42 meters) through alternating periods of beach replenishment and inactivity until T-1, at which time control is terminated. I call this the "short run steady state" (SRSS) solution. Maintaining this level of beach quality is accomplished at escalating cost over time due to the effects of sea level rise on the erosion rate. Also, the empirical cost function exhibits an upward trend, indicating increasing real costs over time. I take this approach so that I can estimate the terminal control time under various scenarios relating to the rate of sea level rise and the residual value of coastal real estate without explicitly solving the non-autonomous long run management problem. The assumption of a SRSS solution may understate the terminal time, because it seems likely that the optimal width will diminish in later periods, reducing costs. However, this will also reduce benefits, so it is difficult to predict exactly how the optimal terminal time will be affected. I use the SRSS assumption only to illustrate the type of output I intend to produce once I solve the long-term dynamic programming problem.

Condition [2.22] implicitly defines the terminal time by the equality of beach nourishment benefits and costs in the penultimate period. To review, the terminal time is implicitly defined as:

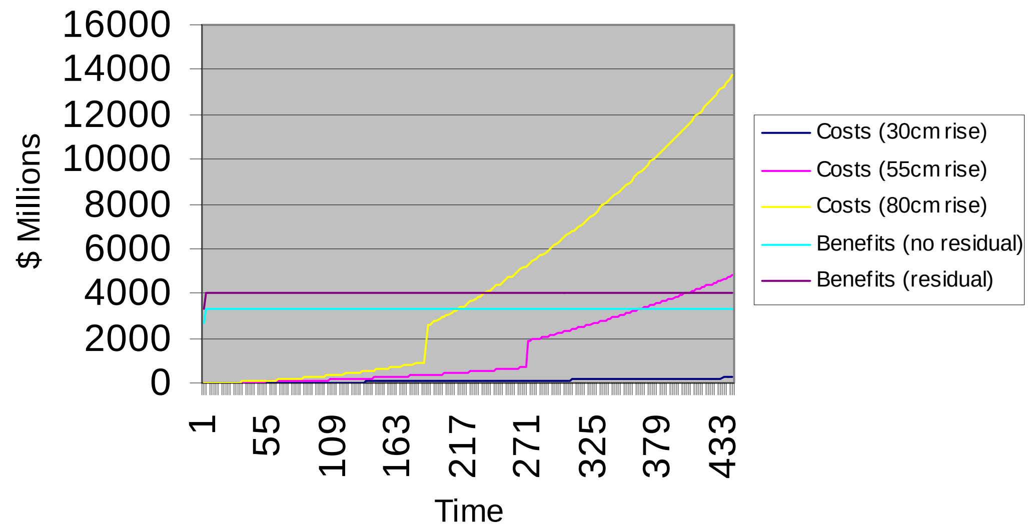
$$B \times WTP^{b}(q_{T-1}) + R \times WTP^{r}(q_{T-1}, L(q_{T-1})) = C(n_{T-1} \times l)\,. \qquad [2.22]$$

Under sea level rise, the erosion rate increases monotonically. Thus, the cost of producing an increment to beach width will rise monotonically as well because beach nourishment must compensate for increasing erosion. Economic returns from beach quality, on the other hand, are bounded. This characterization of the erosion management problem provides the structure by which the terminal time can be determined. In the long run, the sustained absence of beach nourishment implies a policy of shoreline retreat.

Recall, that we have two expressions for the economic returns from beach quality accruing to coastal homeowners, [5.5] and [5.5’]. The former implicitly assumes that residual property values—property values net of beach quality at the terminal time—are zero, while the latter allows for residual property values to maintain their current (1998) levels. Other depreciation or appreciation schemes could be used to map alternate property value trajectories as well. The distinction of residual property value only matters in the long run, since the constant term does not affect marginal values, which drive the results of the model in the short-run.

Figure 5.7 shows the dynamic cost functions for maintaining SRSS beach width under each of the sea level rise trajectories and the aggregate benefit measures for each residual property value scenario. The cost function for Tybee Island is given in [5.6']—costs of sand placement are trending upwards over time. The dynamic cost paths in 5.7 differ only by the rate of sea level rise (ΔS) which determines the long term erosion rate and thus affects the amount of beach nourishment required to maintain the SRSS beach width. Higher rates of sea level rise are associated with more drastic increases in the costs of maintaining the beach. The dynamic motion of beach quality is determined by the transition equations in [5.2a] and [5.2b], with the switching point determined as the year in which aggregate sea level rise reaches 1.5 meters. For the 30cm sea level rise trajectory, this occurs in period 500. For the 55cm sea level rise trajectory this occurs in period 273, and for the 80cm trajectory it occurs in period 188. These time periods witness a discrete jump in beach management costs, as shown in Figure 5.7.

Figure 5.7: Costs and Benefits of Beach Maintenance under Sea Level Rise



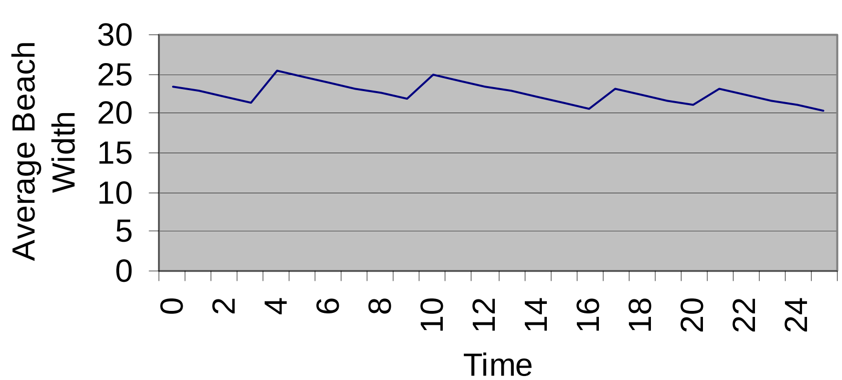
Homeowner returns from beach quality with no residual property value at the terminal time are given by [5.5], while returns from beach quality with a positive constant residual property value (in 1998$) are given by [5.5’]. Returns for beach visitors are given by [5.4]. Each return function is scaled by the relevant number of households for Tybee Island—2,795 coastal homeowners and 109,965 beach visitors. With no residual property value, the current value of aggregate benefits of maintaining the SRSS optimal beach width (42 meters) is just over $1.734 billion per year. Aggregate benefits under the assumption of a positive constant residual property value are $2.083 billion per year. Since the SRSS solution entails beach nourishment every other year, the benefits in Table 5.7 represent the attendant benefits of such operations over a two-year interval. Again, these numbers are upward biased due to the apparent bias in the recreational benefits estimates of [5.4]. Thus, the resulting time horizons should be considered *upper bounds* conditional on the residual property value assumptions.

For the 30cm sea level rise trajectory, projected management costs are always well below the benefits of preserving the present coastline over 500 years, regardless of the assumption on residual property values. Thus, the upper bound time horizon for relatively low sea level rise is indefinite. Assuming no residual value for threatened coastal properties, benefits and costs equate in period 365 (213), for the 55cm (80cm) sea level rise trajectory. These time horizons represent an upper bound on the time frame, in years, under which beach quality can be expected to be maintained under the SRSS management solution and assuming full depreciation of threatened barrier island properties. On the other hand, assume barrier island real estate maintains its average value at the terminal time (measured in constant 1998 dollars). Under these conditions the upper bound on the time horizon on the SRSS management solution extends to 481 years for the 55cm rise trajectory and 235 years for the 80cm sea level rise trajectory. Thus, results of the dynamic simulations suggest that the upper bound on active beach management is on the order of hundreds of years.

To provide a *lower bound* on the time horizon estimates the same cost calculations are compared to the benefits measures derived *for coastal property owners only*, maintaining the aforementioned distinction between full and zero depreciation. Thus, economic returns accruing to beach visitors are ignored in calculating the lower bound. For clarity we will call this the restricted model. Focusing on only returns to coastal households, the optimal beach width with zero cost would be 23.1 meters, and aggregate benefits of maintaining this beach width are $29.596 million per year. This aggregate benefit measure seems much more reasonable than the previous measure associated with both coastal property owners and beach visitors. Moreover, looking at the short run solution to this version of the problem, we see the benefit measure does not dominate the costs as seen in the previous model.

The short run solution to the restricted model is depicted in Figures 5.8 and 5.9. Figure 5.8 shows the optimal state path and Figure 5.9 shows the optimal control path for the beach management problem when economic returns are restricted to coastal property owners. In these results, we see the same pattern of over-nourishment followed by inactivity with at least three notable differences. First, beach nourishment activity effectively brackets a lower level of beach quality, reflecting only the preferences of coastal homeowners. Over the 25-year time horizon, beach quality is 23.35 meters, on average. Secondly, the gaps of inactivity in beach management have increased substantially reflecting the greater relative magnitude of fixed costs compared to the benefits of management. Beach replenishment in this case should occur only every 4-6 years. Lastly, the solution to the restricted model is sensitive to the discount rate. The optimal state is diminishing over time, reflecting the discount of future benefits. The restricted solution does not approximate a steady state as in the previous case. Likewise, the optimal control is diminishing as well.

Figure 5.8: Optimal State Path for Restricted Model



In order to simulate the restricted solution over a longer time horizon, I assume that the state variable should bracket the optimal beach quality (approximately 23 meters), and beach replenishment should be conducted in 5-year intervals. We will call this the "short run restricted steady state" (SRRSS) solution. Thus, the benefits of beach nourishment must be measured as the present value of returns accruing over 5 periods. This approach will provide only an approximate solution, as evidence from the SR problem indicates that the optimal state should be decreasing over time.

Figure 5.9: Optimal Control Path for Restricted Model

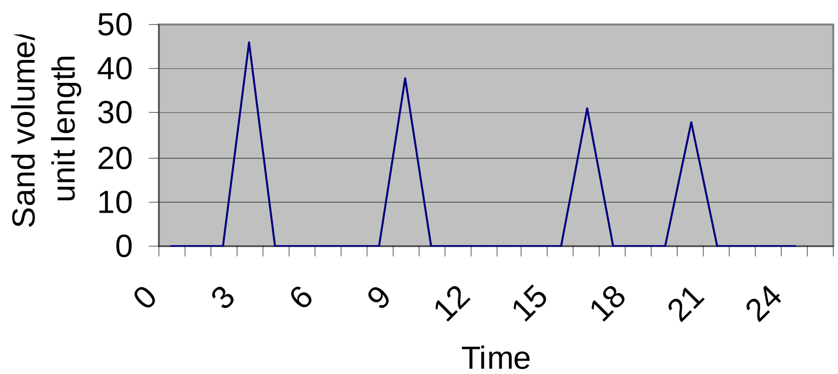
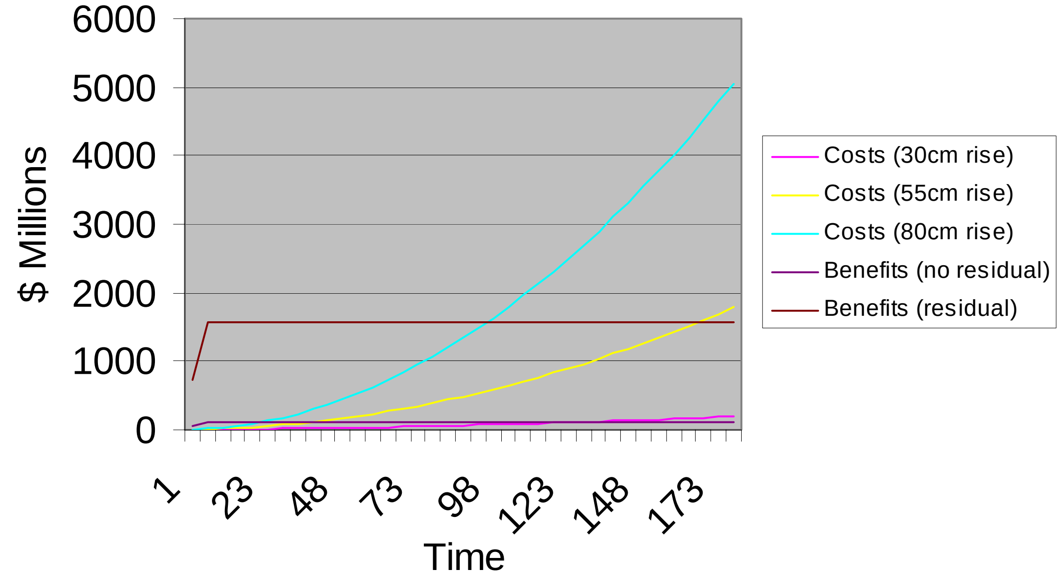


Figure 5.10 indicates costs and benefits of beach nourishment for the restricted model. Assuming no residual property value, the current value of aggregate benefits of maintaining the SRRSS beach width are approximately $26 million per year. Over five years this amounts to a little over $115 million, assuming a 10 percent discount rate. Aggregate benefits under the assumption of a constant residual property value are $375 million per year. Over five years this amounts to about $1.57 billion. Again, these numbers represent lower bounds because the recreational benefits are excluded. The costs of beach management are the same as the previous model.

Figure 5.10: Costs and Benefits of Beach Maintenance under Sea Level Rise - Restricted Model



Consider first the case of no residual property value. The lower horizontal line in Figure 5.10 represents the benefits associated with this case. Under the low sea level rise trajectory of 30cm over the next century, benefits and costs roughly equate in period 128. Under the 55cm sea level rise trajectory, costs exceed benefits in period 38, while under the highest trajectory—80cm over the next century—the costs of beach management exceed the benefits in period 23. Next, consider the case of a constant residual value for threatened barrier island properties. The upper horizontal line in Figure 5.10 represents the benefits associated with this case. The costs of beach management under the low sea level rise trajectory of 30cm are well below the benefits through 500 years into the future. For the 55cm (80cm) sea level rise trajectory, costs exceed benefits in period 168 (98). Thus, the lower bound time horizon for active management on Tybee Island is on the order of decades to hundreds of years, with the most drastic case—full depreciation of threatened properties and 80cm sea level rise over the next century—extending only about 25 years into the future.

Discussion

Using non-market valuation to quantify the returns from beach quality to different user groups and optimal control theory to model the management problem, I have laid out a conceptual model of beach erosion management. By applying dynamic programming I have shown how this model can be used to determine efficient management of barrier island beaches in the short run. Building upon the short run solution, I have also illustrated an assessment of coastal protection on barrier island beaches under sea level rise. My model differs from current approaches to coastal erosion management by incorporating both active and passive management regimes, examining the dual nature of beach services, identifying and incorporating the benefits accruing to different user groups, and explicitly accounting for the dynamic adjustment process of beaches.

I find that the Maximum Principle is of limited use in solving the beach management problem. Explicitly allowing for corner solutions to the optimal nourishment control is a realistic part of the model in the short run and introduces the possibility of shoreline retreat as an optional policy in the long run. Corner solutions, however, complicate the solution of the simultaneous difference equations that result from the Maximum Principle. Dynamic programming was required to solve the dynamic optimization problem.

I use Tybee Island, Georgia as a study site and show the optimal short run management strategy for Tybee's beach. In the short run, the magnitude of the benefits of maintaining the beach far outweighs the costs. In fact, the beach should be maintained close to the level—42 meters wide on average—which maximizes aggregate (<u>not</u> net) benefits across the two user groups. Since the benefits associated with this level of beach quality far outweigh the costs, the solution is analogous to a most-rapid approach. If initial beach quality is below the optimum, beach nourishment in the first period should build up the beach close to the optimal. If initial beach quality is above the optimum, no nourishment should be undertaken until erosion reduces beach width close to the optimum. In either event, once in the vicinity of the optimum beach nourishment should be conducted in two-year intervals. This latter result reflects the fixed costs of beach nourishment. The short term solution is sensitive to changes in fixed costs, but not the discount rate. One important caveat—recreational benefits are biased upwards, so the domination of benefits over costs is very possibly an artifact of this bias.

Exploratory results of the management problem under sea level rise indicate how the model can be used to determine the time horizon of active management under various assumptions about sea level rise and the trajectory of property values. Previous research on coastal protection (Yohe, Neumann, and Ameden (1995)) suggests that the issue for managing barrier islands via nourishment under sea level rise is determining when to stop protection. That is, at what point should we give up trying to preserve barrier beaches? My model offers a framework for making a determination of the terminal management time.

Assuming that costs rise monotonically with sea level but that the number of current users bound benefits, the terminal time can be identified by the equality of total benefits and costs in the penultimate period. Upward bias in the recreational benefits estimates leads to an upper bound on the time horizon if such benefits are included in the model. Due to the non-autonomous nature of the long-term problem, however, the dynamic programming solution is more complicated than the short-term case. I have yet to implement a dynamic programming algorithm to solve the long-term beach erosion management problem. This will be the subject of future research.

To illustrate the type of solution that may be produced, I assume that the form of the long-term solution is equivalent to the short run steady state solution. Under these circumstances, the upper bound on the time horizon of active beach management on Tybee Island ranges from 215 years into the future to indefinite. In the latter case, management costs are well below aggregate benefits into the foreseeable future. The time horizon is clearly sensitive to the rate of sea level rise and the trajectory of property values on Tybee Island. If threatened properties fully depreciate in anticipation of sea level rise, the time horizon of active management is shorter because the benefits of coastal protection are lower.

In order to provide a lower bound on the time horizon estimates, I solve a restricted model in which only benefits to coastal homeowners are considered. When only homeowners are considered beneficiaries, the optimal beach width is considerably lower (about 23 meters). The short run solution to this restricted model is marked by longer intervals of inactivity. About 4-6 years should pass between beach nourishment operations. Also, in the restricted model, the benefits do not dominate the costs as was seen in the previous model. The optimal beach width diminishes over time, reflecting the discounting of future benefits of sand placement.

To project benefits and costs of the restricted model, I assume that the solution should maintain an approximate steady state, bracketing the optimal beach width of 23 meters through 5-year intervals of beach nourishment, though the assumption of an approximate steady state is not supported by the short run solution. Again, solving the dynamic programming problem for the long run is an important topic for future research. Nonetheless, under this assumption the time horizon of active management ranges from approximately 20 to 130 years if properties fully depreciate in anticipation of sea level rise. The 20-year time horizon is associated with the 80cm sea level rise scenario. This is a very short time horizon, and it is quite possible that 20 years is not sufficient time to fully depreciate barrier island real estate assets. Under the medium sea level rise scenario, the time horizon of active management is about 40 years, while for the 30cm trajectory the time horizon is much greater, 128 years.

Under the assumption of positive constant residual property values on Tybee Island, the time horizons of active beach management range from 100 years to indefinite. If sea level were to rise 80cm over the next century and all existing properties maintained their current 1998 values, the costs of beach maintenance would exceed the benefits accruing to property owners in period 98. Reducing the sea level rise trajectory to 55cm over the next century extends the time horizon to about 165 years. The 30cm sea level rise trajectory gives rise to an indeterminate time horizon—the benefits are well above the costs over 500 years into the future.

Clearly, the time horizon is sensitive to the rate of sea level rise and the value of threatened property in the future.

Conclusions

Beach erosion is a significant problem along America's coastline, and the prospects of sea level rise offer more complications and higher stakes. There is a large amount of property exposed to the risks associated with living on the shore, and management of these risks can have dramatic effects on the beach, a valuable public resource. Existing analyses of short run erosion management are fairly limited in terms of fully specifying the dimensions, both temporal and social, of the management problem. Existing studies of coastal protection under sea level rise are rather abstract in terms of the micro-level decisions regarding management and do not consider the non-market effects of management (i.e. on beach resources).

The conceptual framework constructed herein focuses on the stream of services produced by barrier beaches. This distinction is key, as beach resources are of primary importance on the many modest-sized barrier island communities on the East and Gulf Coasts of the U.S. Most of the economies of these communities are centered on beach resources, and ignoring the effect of management decisions on these amenities is foolhardy. Existing research on coastal protection under sea level rise (e.g. Yohe, Neumann, and Ameden 1995) can be appropriately applied to sheltered mainland shores, which may lack beaches, or large metropolitan areas, for which beaches may be a minor consideration, but seems inappropriate for modest-sized barrier island communities.

One might be inclined to view the management of small to medium size beach towns on barrier islands as of secondary importance relative to larger metropolitan areas, which represent much more property wealth. The management of these beach towns, however, could have large impacts on economic welfare through the influence of beach resources. The relative value of beach resources will depend upon how other areas of the coast have responded to sea level rise. As some portions of the mainland are armored, the existing beaches on barrier islands may become more valuable because substitution possibilities become more limited. This fact combined with the prediction that the demand for beach recreation may increase with temperature suggests that the management of these small island communities could be very important.

The model I have laid out focuses on only one barrier island beach, and is thus in the spirit of a partial equilibrium model. Sea level rise is a global phenomenon, and thus will affect all portions of the shoreline simultaneously, but the impacts will vary across locations. The social value of beach preservation in any particular location will depend upon how the total amount of beach resources changes in other locations. Some locations may be more susceptible to erosion loss, and thus more costly to protect, while other locations may respond more favorably to sea level rise. The determination of which areas to maintain and which areas to abandon will depend upon (i) the relative densities and average value of properties in the various locations and (ii) demand for recreation and substitution possibilities at the locations. This type of management problem is not considered herein, but my conceptual model could be useful in making such an assessment.

Appendices

Table A.1a: First Stage On-site Expenditures Regressions: Jekyll Island

	Ex-post demand (*0*)	*Ex-ante* demand (*I*)	*Ex-ante* demand (*II*)
lnptr	0.298965***	190.832***	216.7789***
	(0.0326469)	(45.89128)	(30.09011)
lnpsubs	0.051651*	2.094264	27.42192
	(0.027881)	(40.15171)	(25.36261)
lnfinc	0.4169675***	192.629***	258.3584***
	(0.045896)	(62.65738)	(42.85662)
mpurpose	-0.651262***	-453.6888***	-339.8628***
	(0.085371)	(107.2522)	(82.61514)
assign	-0.057272	15.0147	104.9566**
	(0.046496)	(66.1295)	(42.05494)
summer	0.239782***	144.826	220.2095***
	(0.077011)	(96.00777)	(82.52887)
spring	-0.018428	116.0297	72.28233
	(0.084393)	(102.9214)	(89.31281)
male	0.020959	-17.29273	29.82432
	(0.0457654)	(63.80098)	(42.37136)
nonwhite	-0.063326	110.2624	21.04635
	(0.121906)	(180.3518)	(108.8398)
heduc	-0.081207	80.45553	-71.92113
	(0.049460)	(72.07137)	(45.23557)
nourish	-------------	-------------	-30.16359
			(51.16889)
retreat	-------------	-------------	20.55067
			(52.47384)
children	0.064551***	107.2045*	48.59842***
	(0.019736)	(61.42723)	(18.19849)
children sq	-0.001135**	-32.90295**	-0.7117313
	(0.000513)	(15.03995)	(0.4462935)
Intercept	2.414798***	-1472.409***	-2027.717***
	(0.184594)	(257.0154)	(177.932)
sigma	-------------	449.7548	478.6269
	-------------	(23.37776)	(15.26393)
lnL	-------------	-1478.398	-3943.463
LRT	-------------	86.82	298.63
R^2	0.4635	--------------	--------------
F	58.24	--------------	--------------
N	822	222	600

Dependent variable is $\ln(p_{os} \times d_{os})$ for *Ex-post* model, $p_{os} \times d_{os}$ for *Ex-ante* models; *=statistically significant at α=10%; **=statistically significant at α=5%; ***=statistically significant at α=1%

Table A.1b: First Stage On-site Expenditures Regressions: Tybee Island

	Ex-post demand (*0*)	*Ex-ante* demand (*I*)	*Ex-ante* demand (*II*)
lnptr	0.232007***	96.33689***	107.1138***
	(0.019718)	(21.77892)	(14.52926)
lnpsubs	0.086896***	38.13924	51.00967***
	(0.024091)	(27.1886)	(17.75367)
lnfinc	0.405588***	87.29568**	107.6878***
	(0.033049)	(38.97942)	(23.61688)
mpurpose	-0.441450***	-107.8154*	-217.2262***
	(0.050239)	(59.95545)	(36.1914)
assign	-0.074312**	13.10923	17.82008
	(0.035566)	(41.54195)	(25.64359)
summer	0 .247147***	95.74139*	63.29332
	(0.051748)	(51.45533)	(46.58541)
spring	0.259694***	88.42782	70.21834
	(0.055522)	(55.10793)	(48.97703)
male	-0.012342	-20.45944	-28.05887
	(0.037192)	(43.48117)	(26.53271)
nonwhite	-0.217268***	-29.71418	-81.70359
	(0.081639)	(89.59495)	(59.69414)
heduc	-0.023143	-42.10163	-7.410312
	(0.038304)	(46.59462)	(27.14314)
nourish	-------------	-------------	48.47546
			(32.30185)
retreat	-------------	-------------	32.6004
			(52.47384)
children	0.115515***	51.52318***	40.5708***
	(0.015902)	(18.72135)	(11.35051)
intercept	2.20756***	-789.3818***	-937.968***
	(0.148356)	(165.0714)	(109.5517)
sigma	-------------	354.6931	348.2594
	-------------	(15.47979)	(9.437747)
lnL	-------------	-2054.800	-5247.523
LRT	-------------	81.72	282.03
R^2	0.4562	--------------	--------------
F	84.79	--------------	--------------
N	1124	320	804

Dependent variable is $\ln(p_{os}\times d_{os})$ for *Ex-post* model, $p_{os}\times d_{os}$ for *Ex-ante* models; *=statistically significant at α=10%; **=statistically significant at α=5%; ***=statistically significant at α=1%

Bibliography

Amemiya, T. 1973. "Regression Analysis When the Dependant Variable is Truncated Normal" *Econometrica* 41: 997-1016.

Applied Technology and Management, Inc. 2002. *Savannah Harbor Beach Erosion Study* prepared for Georgia Ports Authority: Charleston, SC.

Attanasi, E.D. and M.R. Karlinger. 1979. "Risk Preferences and Flood Insurance" *American Journal of Agricultural Economics* 61(3): 490-495.

Bartik, Timothy J. 1987. "The Estimation of Demand Parameters in Hedonic Price Models" *Journal of Political Economy* 95(1): 81-88.

Bartik, Timothy J. 1988. "Measuring the Benefits of Amenity Improvements in Hedonic Price Models" *Land Economics* 64(2): 172-183.

Bell, F.W. 1986. "Economic Policy Issues Associated with Beach Nourishment" *Policy Studies Review* 6: 374-381.

Bird, E. 2000. *Coastal Geomorphology: An Introduction* John Wiley and Sons: New York, NY.

Bockstael, Nancy E. and Kenneth E. McConnell. 1983. "Welfare Measurement in the Household Production Framework" *The American Economic Review* 73(4): 806-814.

Bockstael, Nancy E. and Kenneth E. McConnell. 1993. "Public Goods as Characteristics of Non-Market Commodities" *The Economic Journal* 103: 1244-1257.

Brookshire, D.S., M.A. Thayer, J. Tschirhart, W.D. Schulze. 1985. "A Test of the Expected Utility Model: Evidence from Earthquake Risks" *Journal of Political Economy* 93(2): 369-389.

Brown, James N. and Harvey S. Rosen. 1982. "On the Estimation of Structural Hedonic Prices" *Econometrica* 50(3): 765-768.

Bruun, P. 1962. "Sea Level Rise as a Cause of Shore Erosion" *Journal of Waterway, Port, Coastal, and Ocean Engineering* 88: 117-130.

Cameron, A. Colin and Pravin K. Trivedi. 1986. "Econometric Models Based on Count Data: Comparisons and Applications of Some Estimators and Tests" *Journal of Applied Econometrics* 1(1): 29-53.

Cameron, A. Colin and Pravin K. Trivedi. 1990. "Regression Based Tests for Overdispersion in the Poisson Model" *Journal of Econometrics* 46: 347-364.

Chasis, S., R.G. Dean, P.S. Denison, D.R. Godschalk, P. H. F. Graber, H.C. Miller, R.A. Morton, S. Newsome, J. Nicholas and W.D. Pilkey. 1985. "National Strategy for Beach Preservation", *Second Skidaway Institute of Oceanography Conference on America's Eroding Shoreline* Savannah, GA. (www.geo.duke.edu/psds/psds_skidaway.htm)

Chiang, Alpha. 1992. *Elements of Dynamic Optimization* McGraw-Hill Publishing: New York, NY.

Clayton, T.D., L.A. Taylor, Jr., W.J. Cleary, P.E. Hosier, P.H. F. Graber, W.J. Neal and O.H. Pilkey, Sr. 1992. *Living with the Georgia Shore* Duke University Press: Durham, NC.

Daniel, H. 2001. "Replenishment Versus Retreat: The Cost of Maintaining Delaware's Beaches" *Ocean and Coastal Management* 44:87-104.

Dean, R.G. and E.M. Maurmeyer. 1983. "Models for Beach Profile Response" in *Handbook of Coastal Processes and Erosion*, P.D. Komar and J.R. Moore (Eds.) CRC Press: Boca Raton, FL.

Dean, Robert G. 1991. "Equilibrium Beach Profiles: Characteristics and Applications" *Journal of Coastal Research* 7(1): 53-83.

Dorfman, J.H., A.G. Keeler, and W. Kriesel. 1996. "Valuing Risk-Reducing Interventions with Hedonic Models: The Case of Erosion Protection" *Journal of Agricultural and Resource Economics* 21(1): 109-119.

Dolan, R. and B. Hayden. 1983. "Patterns and Prediction of Shoreline Change" in *Handbook of Coastal Processes and Erosion*, P.D. Komar and J.R. Moore (Eds.), CRC Press: Boca Raton, FL.

Edgerton, Lynne T. 1991. *The Rising Tide: Global Warming and World Sea Levels* Island Press: Washington, D.C.

Edwards, S.F. 1989. "On Estimating Household Demand for Outdoor Recreation from Property Values: An Exploration" *Northeastern Journal of Agricultural and Resource Economics* 78: 229-40.

Edwards, S.F. and F.J. Gable. 1991. "Estimating the Value of Beach Recreation from Property Values: An Exploration with Comparisons to Nourishment Costs" *Ocean and Shoreline Management* 15: 37-55.

Ehrlich, I. and G.S. Becker. 1972. "Market Insurance, Self-Insurance, and Self-Protection" *Journal of Political Economy* 80(4): 623-648.

Epple, Dennis. 1987. "Hedonic prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products" *Journal of Political Economy* 95(1): 59-80.

Everts, C.H. 1979. "Beach Behavior in the Vicinity of Groins" *Proceedings of the Specialty Conference on Coastal Structures* U.S. Army Engineering Research Center - Waterway Experiment Station: Vicksburg, MS; 853-876.

Federal Emergency Management Agency (FEMA). 1997. *Mandatory Purchase of Flood Insurance Guidelines* www.fema.gov

Fisher, Anthony C. and W. Michael Hanneman. 1993. "Assessing Climate Change Risks: Valuation of Effects" in *Assessing Surprises and Nonlinearities in Greenhouse Warming*, Joel Darmstadter and Michael A. Toman (Eds.) Resources for the Future: Washington, D.C.

Green, Walter G. III, Ellen M. Walk and Nezih Altay. 2003. "A Simplified Hurricane Threat Index Model for Coastal Jurisdictions" *Notes on the Science of Extreme Situations, Paper No. 9* University of Richmond: Richmond, VA. (www.richmond.edu/~wgreen/papers9(4).pdf)

Haab, Timothy C. and Kenneth E. McConnell. 2002. *Valuing Environmental and Natural Resources: The Econometrics of Non-Market Valuation* Edward Elgar: Northampton, MA.

Hanemann, W. Michael. 1991. "Willingness to Pay and Willingness to Accept: How Much Can They Differ?" *The American Economic Review* 81(3): 635-647.

Hardistay, J. 1990. *Beaches: Form and Process – Numerical Experiments with Monochromatic Waves on the Orthogonal Profile* Unwin-Hyman: London, UK.

H. J. Heinz III Center for Science, Economics, and the Environment. 2000. *Evaluation of Erosion Hazards* Prepared for Federal Emergency Management Agency: Contract EMW-97-CO-0375.

Houston, James R. 1995. "Beach Nourishment" *Shore and Beach* 63(1): 21-24.

Kahn, Shulamit and Kevin Lang. 1988. "Efficient Estimation of Structural Hedonic Systems" *International Economic Review* 29(1): 157-166.

Kana, Timothy W. 1993. "The Profile Volume Approach to Beach Nourishment" in *Beach Nourishment Engineering and Management Considerations* Donald K. Stauble and Nicholas C. Kraus (Eds.) American Society of Civil Engineers: New York, NY.

Klarin, P. and M. Hershman. 1990. “Response of Coastal Zone Management Programs to Sea Level Rise in the United States” *Coastal Management* 18(2): 143-165.

Komar, P.D. 1983. “Beach Processes and Erosion – An introduction” in *Handbook of Coastal Processes and Erosion*, P.D. Komar and J.R. Moore (Eds.) CRC Press: Boca Raton, FL.

Kriesel, Warren, Andrew Keeler and Craig Landry. 2003. “Financing Beach Improvements: Comparing Two Approaches on the Georgia Coast” *Working Paper* University of Georgia: Athens, GA.

Kunreuther, Howard and John R. Sheaffer. 1970. “An Economically Meaningful and Workable System for Calculating Flood Insurance Rates” *Water Resources Research* 6(2): 659-667.

Kyper, T. N. and R. M. Sorenson. 1985. “The Impact of Selected Sea Level Rise Scenarios on the Beach and Coastal Structures at Sea Bright, N. J.” *Coastal Zone ’85* 2: 2645-2661.

Landry, Craig E., Andrew G. Keeler and Warren Kriesel. 2003. “An Economic Evaluation of Beach Erosion Management Alternatives” *Marine Resource Economics* 18(2): 105-127.

Leatherman, S.P. 1988. *Barrier Island Handbook* Coastal Publication Series, Laboratory for Coastal Research, The University of Maryland: College Park, MD.

Long, L. 1990. “Population by the Sea” *Population Today*. 10: 6-8.

Loomis, John B. 1993. “An Investigation into the Reliability of Intended Visitation Data” *Environmental and Resource Economics* 3: 183-191.

Loomis, John and John Crespi. 1999. “Estimated Effects of Climate Change on Selected Outdoor Recreational Activities in the United States” in *The Impacts of Climate Change on the US Economy*, Robert Mendelsohn and James E. Neumann (Eds.) Cambridge University Press: Cambridge, UK.

McConnell, K.E. 1990. “Models for Referendum Data: The Structure of Discrete Choice Models for Contingent Valuation” *Journal of Environmental Economics and Management* 18: 19-34.

McConnell, K.E. 1992. “On-site Time in the Demand for Recreation” *American Journal of Agricultural Economics* : 918-925.

Mendelsohn, Robert. 1984. "Estimating the Structural Equations of Implicit Markets and Household Production Functions" *The Review of Economics and Statistics* 66(4): 673-677.

Mendelsohn, Robert. 1985. "Identifying Structural Equations with Single Market Data" *The Review of Economics and Statistics* 67(3): 525-529.

Miranda, Mario J. and Paul L. Fackler. 2002. *Applied Computational Economics and Finance* MIT Press: Boston, MA.

National Research Council. 1999. "Human Dimensions of Global Climate Change" in *Global Environmental Change: Research Pathways for the Next Decade* National Academic Press: Washington, D.C.

Nordhaus, William D. 1991. "Economic Approaches to Greenhouse Warming" in *Global Warming: Economic Policy Responses*, R. Dornbusch and J.M. Potera (Eds.) MIT Press: Cambridge, MA.

Nummedal, D. 1983. "Barrier Islands" in *Handbook of Coastal Processes and Erosion*, P.D. Komar and J.R. Moore (Eds.) CRC Press: Boca Raton, FL.

Nummedal, D., O.H. Pilkey, Jr., and J.D. Howard. 1987. *Sea-Level Fluctuation and Coastal Evolution* The Society of Economic Paleontologists and Mineralogists: Tulsa, OK.

Oshfeldt, Robert L. and Barton A. Smith. 1985. "Estimating the Demand for Heterogeneous Goods" *The Review of Economics and Statistics* 67(1): 165-171.

Palmquist, Raymond B. 1984. "Estimating the Demand for the Characteristics of Housing" *The Review of Economics and Statistics* 66(3): 394-404.

Palmquist, Raymond B. 1988. "Welfare Measurement for Environmental Improvments Using the Hedonic Model: The Case of Hazardous Waste Sites" *Journal of Environmental Economics and Management* 15: 297-312.

Palmquist, Raymond B. 1992. "Valuing Localized Externalities" *Journal of Urban Economics* 31: 59-68.

Palmquist, Raymond B. 1999. "Hedonic Price Models" in *Handbook of Environmental and Resource Economics* Jeroen C.J.M. van den Bergh (Ed.), Edward Elgar Publishing: Cheltenham, UK.

Parsons, George R. and Aaron J. Wilson. 1997. "Incidental and Joint Consumption in Recreational Demand" *Agricultural and Resource Economics Review* 26(1): 1-6.

Parsons, G.R., D.M. Massey, and T. Tomasi. 2000. "Familiar and Favorite Sites in a Random Utility Model of Beach Recreation" *Marine Resource Economics* 14: 299-315.

Parsons, G.R. and M. Powell. 2001. "Measuring the Cost of Beach Retreat" *Coastal Management* 29: 91-103.

Pilkey, Jr., O.H. and K.L. Dixon. 1996. *The Corps and the Shore*. Island Press: Washington, D.C.

Pilkey, Jr., O.H. 1999. A Presentation on Coastal Erosion Problems. Room 200A - GGS Building, Univeristy of Georgia; Athens, GA: January 19.

Pompe, J.J. and J.R. Rinehart. 1995. "Beach Quality and the Enhancement of Recreational Property Values" *Journal of Leisure Research* 27: 143-154.

Randall, Alan and John R. Stoll. 1980. "Consumer's Surplus in Commodity Space" *The American Economic Review* 70(3): 449-455.

Rosen, Sherwin. 1974. "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition" *Journal of Political Economy* 82(1): 34-55.

Shilling, James D., C.F. Sirmans and John D. Benjamin. 1989. "Flood Insurance, Wealth Redistribution, and Urban Property Values" *Land Economics* 26: 43-53.

Silberman, J., and M. Klock. 1988. "The Recreational Benefits of Beach Renourishment" *Ocean and Shoreline Management* 11: 73-90.

Tinbergen, Jan. 1959. "On the Theory of Income Distribution" in *Selected paper of Jan Tinbergen* L.H. Klaasen, L.M. Koyck, and H.J. Witteveen (Eds.) North Holland: Amsterdam.

Titus, James G., Richard A. Park, Stephen P. Leatherman, J. Richard Weggel, Michael S. Greene, Paul W. Mausel, Scott Brown, Cary Gaunt, Manjit Trehan and Gary Yohe. 1991. "Greenhouse Effect and Sea Level Rise: The Cost of Holding Back the Sea" *Coastal Management* 19: 171-204.

U.S. Army Corps of Engineers. 1994. *Tybee Island Beach Erosion Control Project – Section 934 Reevaluation Report* South Atlantic Division: Savannah, GA

Wigley, T.M.L. and S.C.B. Raper. 1993. "Thermal Expansion of Sea Water Associated with Global Warming" *Nature* 330: 127-131.

Yohe, Gary W. 1991a. "The Cost of Not Holding Back the Sea – Economic Vulnerability" *Ocean and Shoreline Management* 15: 233-255.

Yohe, Gary W. 1991b. "Uncertainty, Climate Change and the Economic Value of Inormation: An Economic Methodology for Evaluating the Timing and Relative Efficacy of Alternative Responses to Climate Change with Application to Protecting Developed Property from Greenhouse Gas Induced Sea Level Rise" *Policy Studies* 24: 245-269.

Yohe, Gary W. 1993. "Economic Response to Physical Effects of Climate Change" in *Assessing Surprises and Nonlinearities in Greenhouse Warming*, Joel Darmstadter and Michael A. Toman (Eds.) Resources for the Future: Washington, D.C.

Yohe, Gary and James Neumann. 1995. "Planning for Sea Level Rise and Shore Protection under Climate Uncertainty" in *Climate Change and Water Resources Planning Criteria*, Kenneth D. Frederick, David C. Major, and Eugene Stakhiv (Eds.) Kluwer Academic Publishers: Boston, MA.

Yohe, Gary W., James Neumann, and Holly Ameden. 1995. "Assessing the Economic Cost of Greenhouse-Induces Sea Level Rise: Methods and Application in Support of a National Survey" *Journal of Environmental Economics and Management* 29: S-78 – S-97.

Yohe, Gary and James Neumann. 1997. "Planning for Sea Level Rise and Shore Protection Under Climate Uncertainty" in *Climate Change and Water Resources Planning Criteria*, Kenneth D. Frederick, David C. Major, and Eugene Z. Stakhiv (eds), Klewer Academic Publishers: Boston, MA.

Yohe, Gary, James Neumann, and Patrick Marshall. 1999. "The Economic Damage Induced by Sea Level Rise in the United States" in *The Impacts of Climate Change on the US Economy*, Robert Mendelsohn and James E. Neumann (Eds.) Cambridge University Press: Cambridge, UK.