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On the Performance and Complexity of Channel-Optimized Vector Quantizers

by

N. Farvardin

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Nariman Farvardin and Vinay Vaishampayan
Electrical Engineering Department
Institute for Advanced Computer Studies
and
Systems Research Center
University of Maryland
College park, MD 20742

Abstract

In this correspondence, the performance and complexity of channel-optimized vector quantizers are studied for the Gauss-Markov source. Some interesting observations on the geometric structure of these quantizers are made which have an important implication on the encoding complexity. For the squared-error distortion measure, it is shown that while the optimum partition is not described by the nearest-neighbor rule, an operation equivalent to a Euclidean distance measurement with respect to an appropriately defined set of points (used to identify the encoding regions) can be used to perform the encoding. This implies that the encoding complexity is proportional to the number of encoding regions. It is then demonstrated that for very noisy channels and a heavily correlated source, when the codebook size is large the number of encoding regions is considerably smaller than the codebook size - implying a reduction in encoding complexity.

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I. Introduction

Vector quantization, as a means of data compression, has received a tremendous amount of attention in the past decade. This is, primarily, due to dramatic performance improvements obtained in many image and speech coding situations when scalar quantizers are replaced by vector quantizers [1]-[3].

A data compression system removes the redundancy in the source and retains the useful information for subsequent transmission and/or storage. This removal of redundancy, in turn, introduces a great deal of sensitivity to the transmission noise or storage device errors. Since vector quantization is now finding applications in practical situations and since some type of channel noise is present in any practical communication system, the analysis and design of vector quantizers for noisy channels is receiving increasing attention. In [4]-[8], various techniques for assigning binary codewords to the vector quantizer codevectors are proposed. On the other hand, a modification of Lloyd's algorithm for the design of vector quantizers for noisy channels is discussed in [9]-[13].

In this correspondence, besides providing explicit numerical results on the performance of channel-optimized vector quantizers for Gauss-Markov sources, we provide some insight on the geometric structure of the channel-optimized vector quantizer and discuss its implication on the encoding complexity. We will show that while the optimum partition is not described by the nearest-neighbor rule, an operation equivalent to a Euclidean distance measurement with respect to an appropriately defined set of points can be used to perform the encoding. We will show that the complexity of this operation is proportional to the number of encoding regions (instead of the codebook size). Further, we will demonstrate that, in general, when the channel gets noisier the number of encoding regions associated with the optimum system gets smaller - hence resulting in lower complexity.

The rest of this paper is organized as follows: In Section II notation is introduced followed by a brief description of the problem. In Section III the necessary conditions for optimality are presented. Section IV includes a discussion of the geometric structure of the optimum partition and the implications of this on the encoding complexity. In Section V specific numerical results on the performance and complexity of the channel-optimized vector quantizer for Gauss-Markov sources are presented. Finally, a summary is presented in Section VI.

II. Notation and Problem Statement

Let us suppose that the source to be encoded is a real-valued, stationary and ergodic process $\{X_t;\ t = 0, 1, \ldots\}$ with zero mean and variance σ_X^2.

The source is to be encoded by means of a vector quantizer (VQ) whose output is transmitted over a channel. We consider a k-dimensional, M-level VQ and a discrete memoryless channel (DMC) with input and output alphabets $\mathcal{J} = \{1, 2, \ldots, M\}$. The overall encoder-decoder, as illustrated in Fig. 1, can be described in terms of an *encoder* mapping γ, a *channel index* mapping b and a *decoder* mapping g. The encoder $\gamma : \mathbb{R}^k \mapsto \mathcal{J}$ is described in terms of a partition $\mathcal{P} = \{S_1, S_2, \ldots, S_M\}$ of $\mathbb{R}^k$

according to [1]

$$\gamma(\mathbf{x}) = i, \text{ if } \mathbf{x} \in S_i, \; i \in \mathcal{J}, \tag{1.a}$$

where $\mathbf{x} = (x_{nk}, x_{nk+1}, \ldots, x_{nk+k-1})$ is a typical source output vector. The channel index assignment is a one-to-one mapping $b : \mathcal{J} \mapsto \mathcal{J}$ which assigns to the encoder output i, an index $i' = b(i) \in \mathcal{J}$ which is then delivered to the channel. The channel is a DMC with $P(j|i')$ denoting the probability that the index j is received given that i' is transmitted. Finally, the decoder mapping $g : \mathcal{J} \mapsto \mathbb{R}^k$, is described in terms of a finite reproduction alphabet (codebook) $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_M\}$, according to

$$g(j) = \mathbf{c}_j, \; j \in \mathcal{J}. \tag{1.b}$$

Let us assume that the distortion caused by representing the source vector $\mathbf{x}$ by a reproduction vector (also called a codevector) $\mathbf{y}$, is given by a non-negative distortion measure $d(\mathbf{x}, \mathbf{y})$. The performance of this coding system is generally measured by the *average distortion* per sample $D(\mathcal{P}, \mathcal{C}; b)$ and the encoding rate R. The average distortion is described by

$$D(\mathcal{P}, \mathcal{C}; b) = \frac{1}{k} \sum_{i=1}^{M} \sum_{j=1}^{M} P(j|b(i)) \int_{S_i} p(\mathbf{x}) d(\mathbf{x}, \mathbf{c}_j) d\mathbf{x} \;, \tag{2.a}$$

where $p(\mathbf{x})$ is the k-fold probability density function (p.d.f.) of the source and the encoding rate is given by

$$R = \frac{1}{k} \log_2 M, \text{ bits/sample.} \tag{2.b}$$

For a given source, a given noisy channel, a fixed dimension k and a fixed codebook size M, we wish to choose $\mathcal{C}$, $\mathcal{P}$ and b in such a way as to minimize $D(\mathcal{P}, \mathcal{C}; b)$.

III. Necessary Conditions and Algorithm

For now, let us consider the simpler problem of minimizing $D(\mathcal{P}, \mathcal{C}; b)$ when b is fixed. This problem is a straightforward extension of the channel-optimized scalar quantization problem [10] and a special case of the trellis vector quantization of [11], [12]. In fact, upon rewriting (2.a) as

$$D(\mathcal{P}, \mathcal{C}; b) = \frac{1}{k} \sum_{i=1}^{M} \int_{S_i} p(\mathbf{x}) \{ \sum_{j=1}^{M} P(j|b(i)) d(\mathbf{x}, \mathbf{c}_j) \} d\mathbf{x} \;, \tag{3}$$

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[1] Notice that the notation used for $\mathcal{P}$ does not exclude the possibility that some of the encoding regions be empty. This corresponds to the situation where the encoder simply does not transmit some of the indices in $\mathcal{J}$; due to the channel noise, however, any index in $\mathcal{J}$ may be received and hence the size of the reproduction alphabet must be exactly M.

it becomes clear that for a fixed b, the problem of minimizing the average distortion is identical to the VQ design problem with a modified distortion measure [11] (the term in the braces in (3)). Specifically, for a fixed b and a fixed $\mathcal{C}$, the optimum partition $\mathcal{P}^* = \{S_1^*, S_2^*, \ldots, S_M^*\}$ is such that

$$S_i^* = \{\mathbf{x} : \sum_{j=1}^{M} P(j|b(i))d(\mathbf{x}, \mathbf{c}_j) \ \leq \ \sum_{j=1}^{M} P(j|b(l))d(\mathbf{x}, \mathbf{c}_j), \ \forall l\}, \ i \in \mathcal{J}. \tag{4.a}$$

We remark here that any change in $b(i)$ in (4.a) will only result in a relabeling of the elements of $\mathcal{P}^*$. In fact, the MSE obtained after the application of (4.a) will be independent of the index assignment b. For this reason, we believe the choice of b is only of limited importance in the VQ design. The numerical results in Section V support this claim.

Similarly, it is easy to show that for a fixed b and a fixed $\mathcal{P}$, the optimum codebook $\mathcal{C}^* = \{\mathbf{c}_1^*, \mathbf{c}_2^*, \ldots, \mathbf{c}_M^*\}$ must satisfy

$$\mathbf{c}_j^* = \arg \min_{\mathbf{y} \in \mathbb{R}^k} E\{d(\mathbf{X}, \mathbf{y})|V = j\}, \ j \in \mathcal{J}, \tag{4.b}$$

where V is used to denote the random variable at the channel output.

A successive application of equations (4.a) and (4.b) results in a sequence of encoder-decoder pairs for which the corresponding average distortions form a non-increasing sequence of non-negative numbers which has to converge. Therefore, a straightforward extension of the algorithm in [1] can be used for optimizing the partition $\mathcal{P}$ and the codebook $\mathcal{C}$. From now on, we will refer to the encoder-decoder pair obtained from this modified algorithm as the channel-optimized VQ (COVQ).

Let us now focus attention on the squared-error distortion criterion where $d(\mathbf{x}, \mathbf{y}) = \| \mathbf{x} - \mathbf{y} \|^2$. In this case, the optimum partion in (4.a) is given by

$$S_i^* = \{\mathbf{x} : \sum_{j=1}^{M} P(j|b(i))||\mathbf{x} - \mathbf{c}_j||^2 \ \leq \ \sum_{j=1}^{M} P(j|b(l))||\mathbf{x} - \mathbf{c}_j||^2, \ \forall l\}, \ i \in \mathcal{J}. \tag{5.a}$$

Similarly, the optimum codebook is easily shown to simplify to

$$\mathbf{c}_j^* = \frac{\sum_{i=1}^{M} P(j|b(i)) \int_{S_i} \mathbf{x} p(\mathbf{x}) d\mathbf{x}}{\sum_{i=1}^{M} P(j|b(i)) \int_{S_i} p(\mathbf{x}) d\mathbf{x}}, \ j \in \mathcal{J}. \tag{5.b}$$

For the Gauss-Markov source, the binary symmetric channel and the squared-error distortion measure, performance results at different bit rates and for various values of k are presented in Section V. In what follows, for the squared-error distortion measure we will concentrate on the geometric structure of the channel-optimized vector quantization scheme and its implications on the encoding complexity.

IV. Geometric Structure and Complexity Implications

For fixed b and $\mathcal{C}$, the ith optimum encoding region described by (5.a), can be written as

$$S_i^* = \bigcap_{l \neq i} S_{il}^*, \tag{6}$$

where S_{il}^* is described by

$$S_{il}^* = \{\mathbf{x} : 2\sum_{j=1}^{M}[P(j|b(l)) - P(j|b(i))]\langle \mathbf{x}, \mathbf{c}_j\rangle \;\leq\; \sum_{j=1}^{M}[P(j|b(l)) - P(j|b(i))]\; ||\mathbf{c}_j||^2\}, \tag{7}$$

and $\langle \mathbf{x}, \mathbf{y}\rangle$ denotes the inner product of $\mathbf{x}$ and $\mathbf{y}$. In view of the expression in (7), the region S_{il}^* is characterized by a *hyperplane* H_{il}, separating the regions S_i^* and S_l^*, described as follows:

$$H_{il} = \{\mathbf{x} : 2\sum_{j=1}^{M}[P(j|b(l)) - P(j|b(i))]\langle \mathbf{x}, \mathbf{c}_j\rangle \;=\; \sum_{j=1}^{M}[P(j|b(l)) - P(j|b(i))]\; ||\mathbf{c}_j||^2\}. \tag{8}$$

Note that the hyperplane H_{il} is not necessarily the *perpendicular bisector* of the cord connecting $\mathbf{c}_i$ and $\mathbf{c}_l$. This observation has an important implication: unlike the conventional VQ (noiseless channel case), to determine whether $\mathbf{x}$ belongs to S_{il}^* or ${S_{il}^*}^c$ (one side of H_{il} or the other), it is not enough to perform a distance measurement from $\mathbf{x}$ to $\mathbf{c}_i$ and $\mathbf{c}_l$. In fact, to make this determination, as it can be seen from (7), distance measurements from $\mathbf{x}$ to all $\mathbf{c}_i$'s is necessary. This corresponds to a major increase in encoding complexity which is merely due to the presence of the noisy channel. In the following we will show that this encoding complexity can be reduced in two ways.

First, let us define for all nonempty S_i's

$$\mathbf{y}_i \triangleq E(\mathbf{Y}|\mathbf{X} \in S_i) = \sum_{j=1}^{M} P(j|b(i))\mathbf{c}_j, \; i \in \mathcal{J}, \tag{9}$$

where $\mathbf{Y}$ denotes the random variable at the output of the decoder. Also, for brevity of notation we will define

$$\alpha_i \triangleq E(||\mathbf{Y}||^2|\mathbf{X} \in S_i) = \sum_{j=1}^{M} P(j|b(i))||\mathbf{c}_j||^2, \; i \in \mathcal{J}. \tag{10}$$

Then, it is easy to show that the hyperplane H_{il} is described by

$$H_{il} = \{\mathbf{x} : 2\langle \mathbf{x}, \mathbf{y}_l - \mathbf{y}_i\rangle \;=\alpha_l - \alpha_i\}. \tag{11}$$

It is interesting to note that while H_{il} is not necessarily perpendicular to the cord connecting $\mathbf{c}_i$ to $\mathbf{c}_l$, according to (11), it is *perpendicular* to (but not necessarily the bisector of) the cord connecting $\mathbf{y}_i$ to $\mathbf{y}_l$. This means that the distances of $\mathbf{x}$ to $\mathbf{y}_i$ and $\mathbf{y}_l$ can be used to determine the side of H_{il} to which the point $\mathbf{x}$ belongs. More precisely, to determine which region a given point $\mathbf{x}$ belongs to, it suffices to compute $d_i'(\mathbf{x}) \triangleq \alpha_i - 2\langle \mathbf{x}, \mathbf{y}_i \rangle$, for all $i \in \mathcal{J}$ which correspond to nonempty regions, and select that i which minimizes $d_i'(\mathbf{x})$. Notice that this involves only M inner product computations and M scalar subtractions. Hence, the complexity of encoding is essentially proportional to the number of *nonempty* encoding regions in the partition $\mathcal{P}^*$.

The second important point is that, in fact, when the channel is noisy, the number of nonempty encoding regions associated with the locally optimal encoder-decoder pair may turn out to be smaller than the cardinality of the codebook, M. Roughly speaking, this means that the optimum system trades quantization accuracy for less sensitivity to channel noise. The same observation was made in the case of channel-optimized scalar quantization [10]. To be more specific, let us go back to (5.a) and consider the final optimum codebook $\mathcal{C}^*$. Then, for some i, it may be the case that for any $\mathbf{x} \in \mathbb{R}^k$ there exists an $l = l(\mathbf{x})$ such that $\sum_j P(j|b(l))||\mathbf{x} - \mathbf{c}_j||^2 \leq \sum_j P(j|b(i))||\mathbf{x} - \mathbf{c}_j||^2$; this corresponds to $S_i^* = \emptyset$. Generally, the more noisy the channel is, the smaller the number of nonempty encoding regions will be. To illustrate this point, for a two-dimensional ($k = 2$), three-level ($M = 3$) VQ with a fixed codebook $\mathcal{C} = \{\mathbf{c}_1 = (1,0),\ \mathbf{c}_2 = (-1,0),\ \mathbf{c}_3 = (0,2)\}$ and a DMC described by the diagram in Fig. 2, we have depicted the shape of the optimum encoding regions and the corresponding $\mathbf{y}_i$'s for $\epsilon = 0.00,\ 0.10,\ 0.15$ and 0.20, in Figs. 3.a - 3.d. Here, we have assumed that $b(i) = i,\ i = 1,2,3$. It can be seen from these figures, that for the fixed codebook $\mathcal{C}$, the encoding region S_2^* becomes smaller as ϵ gets larger; this is because the channel is less reliable in transmitting $i = 2$ than $i = 1$ or 3. It is seen (Fig. 3.d) that when $\epsilon = 0.20$, S_2^* vanishes, implying that no source output vector should be mapped to $i = 2$.

Suppose for a given source, a given channel and a given codebook size M, the locally optimum COVQ is found. Let us assume that the partition associated with this COVQ consists of N ($N \leq M$) encoding regions. Then, despite the fact that there are M codevectors in the codebook $\mathcal{C}$, we have only N $\mathbf{y}_i$'s as defined in (9) and hence the encoding complexity is proportional to N. As our numerical results in Section V indicate, for very noisy channels and for a correlated source, the value of N is noticeably smaller than M, resulting in considerable complexity reductions.

V. Numerical Results

In this section, we present numerical results on the performance and complexity of the COVQ scheme and make comparison with the Linde, Buzo and Gray VQ (LBGVQ) whose design is based on a noiseless channel assumption [1].

We consider a Gauss-Markov source with two different correlation coefficients: $\rho = 0.0$ and $\rho = 0.9$. The channel is assumed to be a Binary Symmetric Channel (BSC) with crossover probability ϵ. The binary codeword delivered to the channel is the binary representation of the index at the output of the channel index assignment mapping.

For $R = 1$ bit/sample, Signal-to-Noise Ratio (SNR) performance results are presented in Tables 1 and 2 for $\rho = 0.0$ and 0.9, respectively. These results include $k = 1, 2, 4$ and 8 and $\epsilon = 0.00, 0.005, 0.01, 0.05$ and 0.10. Also, the number of encoding regions (as a measure of encoding complexity) for these different cases is included in Tables 3 and 4.

Before we discuss these results, some details about how they are obtained are necessary. The LBGVQ results are obtained by using the algorithm in [1] with a stopping threshold of 10^{-3}. The assignment of binary codewords to the codevectors of the designed LBGVQ is done via a simulated annealing algorithm described in [7]. The COVQ results are obtained by means of the same algorithm with the modified distortion measure [9]-[11]. The LBGVQ and its corresponding binary codeword assignment are used as the starting point for the design of the COVQ for $\epsilon = 0.005$. Then, the COVQ obtained for $\epsilon = 0.005$ is used as the starting point for the COVQ for $\epsilon = 0.01$, and so on. Our experiments indicate that the final results for the COVQ are not very sensitive to the initial choice of the binary codeword assignment for the LBGVQ. This implies that in minimizing the average distortion in (3) the initial choice of b does not play a significant role.

The results in Tables 1 and 2 indicate that the COVQ performs better than the LBGVQ in all cases; the performance improvements are more noticeable for larger dimensions, higher source correlation values and very noisy channels. It is interesting to note that, in fact, it is for these cases (e.g., $k = 8$, $\rho = 0.9$ and $\epsilon = 0.1$) that the largest reduction in the number of encoding region (hence, encoding complexity) is observed.

VI. Summary and Conclusions

We have described a VQ-based system in a noisy channel situation. It is shown that in some cases (especially, when the channel is very noisy) useful performance gains can be obtained by using a COVQ instead of an LBGVQ. Furthermore, we have shown that the encoding complexity can be made proportional to the number of nonempty encoding regions; this is done by defining an appropriate set of points with respect to which an operation equivalent to a Euclidean distance measurement is performed. Finally, it is shown that when the channel is very noisy and for correlated sources, the number of encoding regions is much smaller than the codebook size and hence the encoding complexity of COVQ is smaller than that of LBGVQ.

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		$\epsilon = 0.00$	$\epsilon = 0.005$	$\epsilon = 0.01$	$\epsilon = 0.05$	$\epsilon = 0.10$
$k = 1$	LBGVQ	4.40	4.25	4.10	3.09	2.09
	COVQ	4.40	4.25	4.11	3.15	2.27
$k = 2$	LBGVQ	4.38	4.23	4.08	3.06	2.05
	COVQ	4.38	4.23	4.11	3.15	2.26
$k = 4$	LBGVQ	4.58	4.36	4.15	2.82	1.64
	COVQ	4.58	4.43	4.24	3.17	2.28
$k = 8$	LBGVQ	5.08	4.64	4.25	2.15	0.70
	COVQ	5.08	4.64	4.34	3.19	2.29

Table 1: SNR (in dB) Performance Results;
Memoryless Gaussian Source; $R = 1$ bit/sample.

		$\epsilon = 0.00$	$\epsilon = 0.005$	$\epsilon = 0.01$	$\epsilon = 0.05$	$\epsilon = 0.10$
$k = 1$	LBGVQ	4.40	4.25	4.10	3.09	2.09
	COVQ	4.40	4.25	4.11	3.15	2.27
$k = 2$	LBGVQ	7.87	7.31	6.81	4.13	2.19
	COVQ	7.87	7.31	6.83	4.37	2.76
$k = 4$	LBGVQ	10.18	9.10	8.24	4.37	2.00
	COVQ	10.18	9.15	8.37	6.23	4.65
$k = 8$	LBGVQ	11.49	9.99	8.87	4.46	2.00
	COVQ	11.49	10.31	9.70	7.44	5.73

Table 2: SNR (in dB) Performance Results;
Gauss-Markov Source; $\rho = 0.9$; $R = 1$ bit/sample.

		$\epsilon = 0.00$	$\epsilon = 0.005$	$\epsilon = 0.01$	$\epsilon = 0.05$	$\epsilon = 0.10$
$k = 1$	LBGVQ	2	2	2	2	2
	COVQ	2	2	2	2	2
$k = 2$	LBGVQ	4	4	4	4	4
	COVQ	4	4	4	4	4
$k = 4$	LBGVQ	16	16	16	16	16
	COVQ	16	16	16	16	16
$k = 8$	LBGVQ	256	256	256	256	256
	COVQ	256	256	256	256	256

Table 3: Number of Encoding Regions;
Memoryless Gaussian Source; $R = 1$ bit/sample.

		$\epsilon = 0.00$	$\epsilon = 0.005$	$\epsilon = 0.01$	$\epsilon = 0.05$	$\epsilon = 0.10$
$k = 1$	LBGVQ	2	2	2	2	2
	COVQ	2	2	2	2	2
$k = 2$	LBGVQ	4	4	4	4	4
	COVQ	4	4	4	4	4
$k = 4$	LBGVQ	16	16	16	16	16
	COVQ	16	16	16	11	9
$k = 8$	LBGVQ	256	256	256	256	256
	COVQ	256	249	230	98	61

Table 4: Number of Encoding Regions;
Gauss-Markov Source; $\rho = 0.9$; $R = 1$ bit/sample.

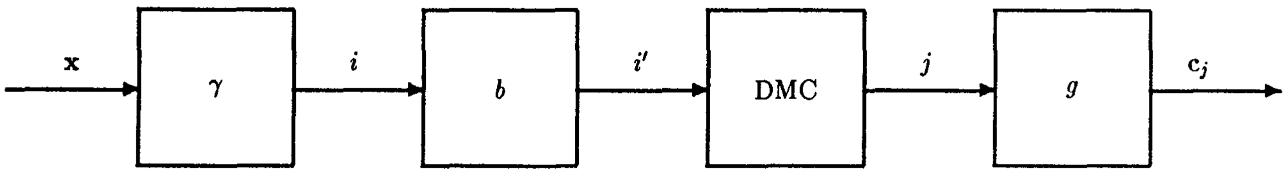

Figure 1: Block Diagram of the Coding System.

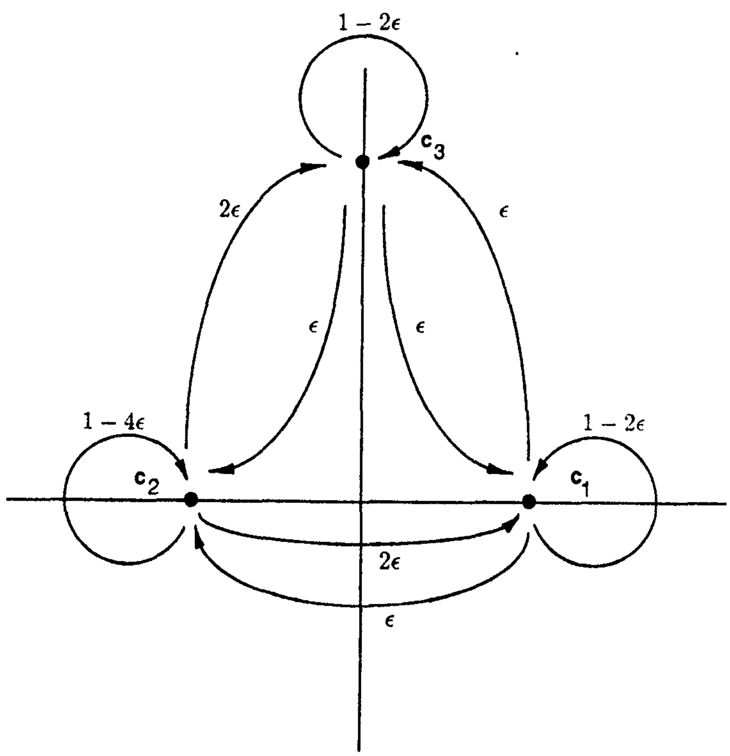

Figure 2: Codevectors and Channel Transition Probabilities.

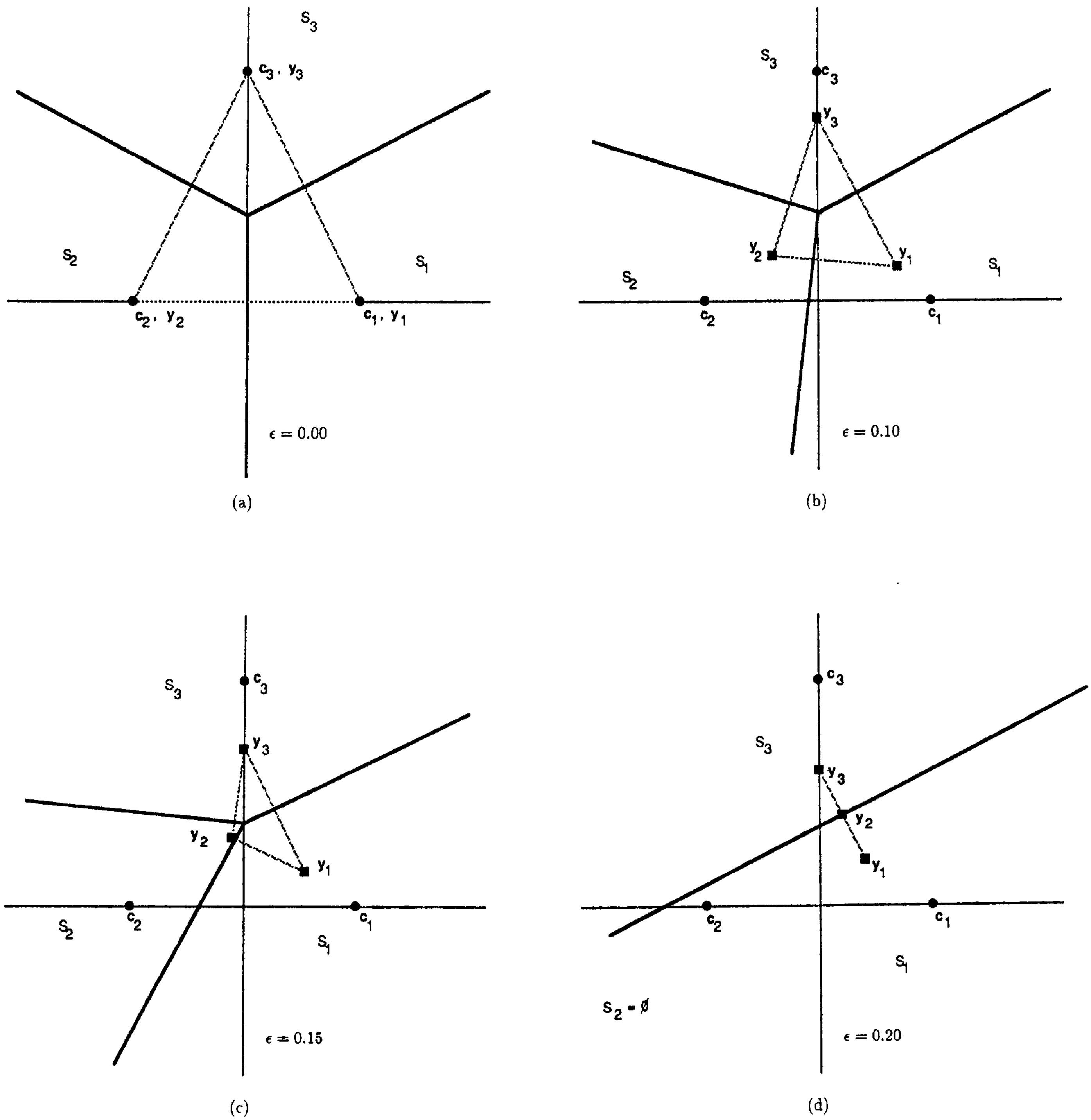

Figure 3: Shape of Encoding Regions for Different Values of ϵ.