

ABSTRACT

Title of Dissertation: **PARTICLE INDUCED TRANSITION IN
HIGH-SPEED BOUNDARY-LAYER FLOWS**
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Boundary-layer transition to turbulence presents a critical challenge in aerospace engineering due to its impact on thermal load, especially for hypersonic vehicles. This transition, influenced by various disturbances such as acoustic waves, entropy waves, and particle impingement, follows complex and non-unique pathways to turbulence. It significantly affects the surface heat flux and thus will impact the design of thermal protection systems. This dissertation focuses on the transition process initiated by particle impingement, which introduces small-scale disturbances through a complex receptivity process that typically initiates a natural transition path.

Using direct numerical simulations, this study explores the particle-induced transition process. The disturbance spectrum, consisting of both stable and unstable modes along with continuous acoustic contributions, is meticulously reconstructed near the particle impingement site using biorthogonal decomposition to assess the contributions of different eigenmodes to the initial disturbance spectrum. A large number of discrete and continuous eigenmodes are seeded, but the dominant eigenmodes capture only a small fraction of the disturbance energy, with the majority

reflected into the freestream through the continuous modes associated with the continuous acoustic branches.

The modeling fidelity is also investigated, particularly the particle-source-in-cell (PSIC) approach, commonly used due to its efficiency in capturing particle-flow interactions. Comparisons with the Immersed-Boundary-Method (IBM), however, reveal that PSIC inadequately captures particle-wall interactions and needs correction for accurate disturbance modeling.

Finally, a reduced-order model is developed for the prediction of particle-induced transition. This model integrates data from high-fidelity simulations, linear stability theory, and a saturation amplitude model while also considering particle characteristics like size, density and concentration. The model's capability is demonstrated for a wide range of transition scenarios, including data from the HIFiRE-1 flight test, offering a robust tool for rapid transition prediction in hypersonic vehicle design.

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HIGH-SPEED BOUNDARY-LAYER FLOWS

by

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Dedication

To my family

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List of Papers

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2. Saikia B., A. Al Hasnine S. M., Brehm C. On the role of discrete and continuous modes in a cooled high-speed boundary layer flow. Journal of Fluid Mechanics. 2022;942:R7. doi:10.1017/jfm.2022.380
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4. Russo V., A. Al Hasnine S. M. and Brehm C. Towards Understanding Particle Cloud Interactions with High-Speed Boundary Layer Flows, AIAA 2021-2829. AIAA AVIATION 2021 FORUM. August 2021.
5. A. Al Hasnine S. M., Russo V., Tumin A. and Brehm C. Three-Dimensional Spatio-Temporal Disturbance Flow Field Analysis of Particulate-Induced High-Speed Boundary-Layer Transition, AIAA 2021-1657. AIAA Scitech 2021 Forum. January 2021.
6. Saikia B., A. Al Hasnine S. M., Dueñas L. and Brehm C. On the energy transfer mechanisms for the supersonic mode., AIAA 2021-1517. AIAA Scitech 2021 Forum. January 2021.
7. Russo V., A. Al Hasnine S. M. and Brehm C. Particle-Impingement Simulations for a Blunt Cone in Hypersonic Flow, AIAA 2021-0967. AIAA Scitech 2021 Forum. January 2021.

8. A. Al Hasnine S. M., Russo V., Browne O. M., Tumin A. and Brehm C. Disturbance Flow Field Analysis of Particulate Interaction with High Speed Boundary Layers, AIAA 2020-3046. AIAA AVIATION 2020 FORUM. June 2020.
9. Browne O. M., A. Al Hasnine S. M., Russo V. and Brehm C. Fully-Resolved Particulate-Induced Transition Simulations for High-Speed Boundary-Layers with an Immersed Boundary Method, AIAA 2020-1795. AIAA Scitech 2020 Forum. January 2020.
10. Browne O. M., A. Al Hasnine S. M., and Brehm C. Towards modeling and simulation of particulate interactions with high-speed transitional boundary-layer flows. AIAA Aviation 2019 Forum. 2019.

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List of Abbreviations

2D	Two Dimensional
3D	Three dimensional
AMR-WPT	Adaptive Mesh Refinement Wavepacket Tracking
AMR	Adaptive Mesh Refinement
BDF	Backwards Differentiation Formula
BES	Biorthogonal Eigenfunction System
BOD	Biorthogonal Decomposition
CHAMPS	Cartesian Higher-order Adaptive Multi-physics Solver
CFD	Computational Fluid Dynamics
CNSE	Compressible Navier-Stokes Equations
COP	Coefficient of Projection
DNS	Direct Numerical Simulation
EVP	Eigenvalue Problem
F mode	Fast Mode
FA	Fast Acoustic
FFT	Fast Fourier Transform
HIFiRE	Hypersonic International Flight Research Experimentation
IBM	Immersed Boundary Method
KEC	Kinetic Energy Consistent
L	Linear
LDE	Linear Disturbance Equations
LOAC	Light Optical Aerosol Counter
LST	Linear Stability Theory
NLDE	Non-linear Disturbance Equations
NL	Non-linear
N-S	Navier-Stokes
PSIC	Particle-Source-in-Cell
S mode	Slow Mode
SA	Slow Acoustic
TPS	Thermal Protection System
T-S	Tollmien–Schlichting
USA	United States of America
WENO	Weighted Essentially Non-oscillatory

Chapter 1: Introduction

1.1 Motivation

Traveling at hypersonic speeds would reduce the flight times for commercial air traffic such that it would only take 110 minutes to travel from Tokyo to Los Angeles [14]. In some scenarios there are no alternatives but to travel at high speed, *e.g.* atmospheric reentry. The history of hypersonic research and development is several decades old with both military and commercial interests shaping the path forward. German V-2 rockets are often credited as the earliest use of hypersonic projectile during the Word War II [15]. Today, any ballistic missile with a range greater than 400km, travels at hypersonic speed [15]. In 1961, the Vostok-I was the first ever spaceship to carry human beings to the outer space and return to the Earth at a speed of about 25 times the speed of sound [16]. Vostok-I carried Major Yuri Gagarin to orbit and back to earth, making him the first person to fly at hypersonic speed [16]. Later in the same year, the X-15 hypersonic vehicle was flown at Mach 5.3 carrying Major Robert White. The X-15 flew 199 flights between 1959 to 1968 with a maximum Mach number of 6.7 [15, 17]. The X-15 had a nearly spotless flight record with the exception of one fatality during its lifetime [17]. More recently, the X-43A was designed by NASA with an air-breathing scramjet engine for the first time to demonstrate sustained, powered flight at hypersonic conditions. In 2004, the X-43A achieved a maximum Mach number of 9.68 for a 10 second duration [18].

The flow field around hypersonic vehicles is composed of several complex and coupled phenomena, such as transition, turbulence, chemical reactions, shocks, ionization, dissociation, etc. At high speed flight, the heat transfer to the surface can be a major concern. The conversion of kinetic energy into thermal energy via viscous effects is referred to as viscous dissipation [16]. Additionally, the turbulent boundary layers increase the surface heating and skin friction by as much as five times [19]. To counteract this heating issue, both passive and active thermal controls may be used in hypersonic vehicles, such as the use of thermal protection shields (TPS) which may remove some of the thermal load via thermo-chemical ablation process. The transition to turbulence is a complex non-unique process as it involves wide range of parameters and can proceed through different paths based on the initial disturbance amplitude.

The TPS can be heavy and undermine the effective design criteria if not accurately designed. In particular, predicting the transition onset location is critical for TPS design efforts. This onset location may be studied via wind tunnel campaigns, in free flight, or with computational fluid dynamics (CFD). There are only four quiet wind tunnels in the world [20] which has a maximum operating Mach number as high as 6 for a short time while conventional (noisy) wind tunnels can produce conflicting results [19, 21, 22]. Flight tests provide an ideal environment to study the transition process under design conditions, however their costs is substantial and the risk is often prohibitive [19]. With the advent of ever-increasing computational power, CFD has become a tool to perform numerical experiments which can study high-speed flows at any design conditions which could hardly be done in wind tunnels. One such design environment is particle-laden flights which has the potential to impact the breakdown to turbulence.

1.2 Particles in Atmosphere

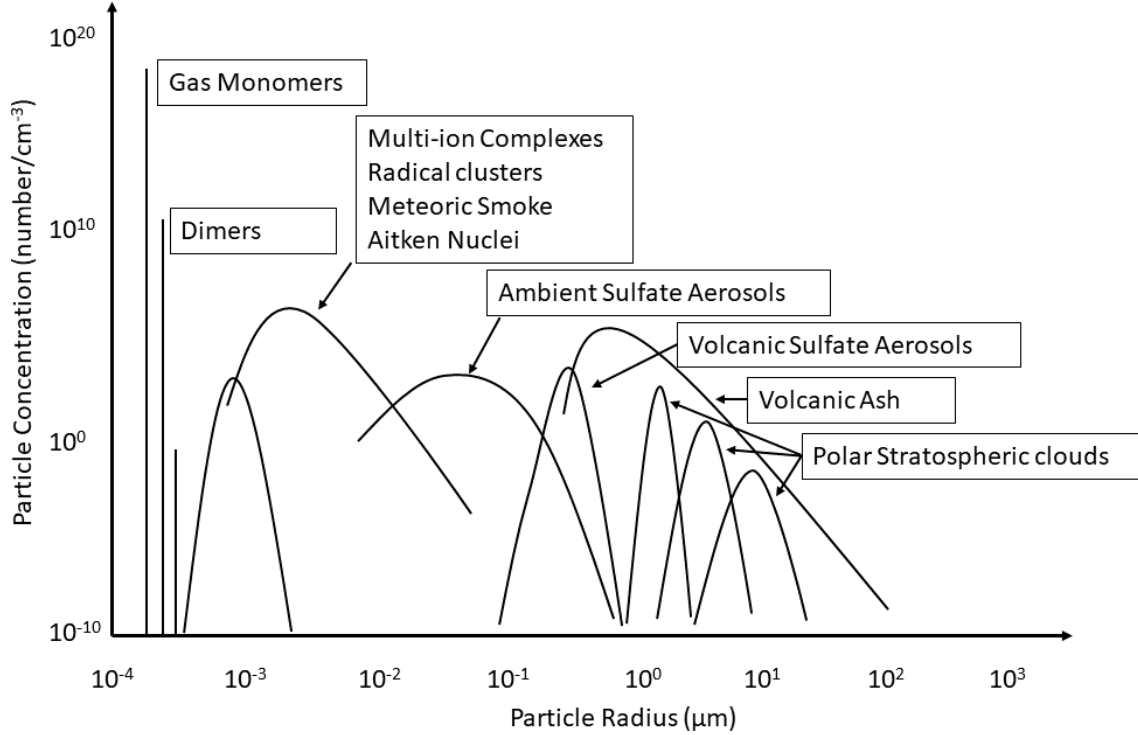


Figure 1.1: Particle size and concentration distribution in the Earth's middle atmosphere (reproduced and approximated from Turco [5]).

Fig. 1.1 illustrates the concentrations of different types and sizes of particles in the stratosphere. There are a significant number of particles with a radius of $10^{-1}\mu\text{m}$ to $10\mu\text{m}$ [23]. Refs. [24] and [25] reported early balloon experiments to measure the particle concentration in the atmosphere at different altitudes in different cities of Europe. All the studies reported smaller concentrations of larger particles (greater than $10\mu\text{m}$). Recently, Habeck et al. [26] conducted balloon experiments with Light Optical Aerosol Counter (LOAC) to measure the particle concentration in the atmosphere up to an altitude of 34km in southern Minnesota, USA. They reported that the majority of the particles are submicron in size and particles greater than $10\mu\text{m}$ were not detected during the

experiment [26].

These atmospheric particles can impinge on the surface of a high-speed vehicles and trigger transition to turbulence. The particles in the stratosphere are small, so they will not create turbulent spots but they can initiate the transition process by seeding disturbances inside the boundary layer [23]. Particles can affect low-and high speed transition by different mechanisms as noted by Bushnell [23, 27]: (1) particles can produce surface roughness effects by sticking to or damaging the surface, (2) vortex or vorticity shedding of the wake of the particle when the particle is inside the boundary layer, (3) particle rotation can be caused by boundary-layer mean shear, (4) reverse shocklet effects as the particle crosses the shock front, and (5) the elastic particles can interact with the bow shock to form jets and shear layers.

1.3 Transition to Turbulence

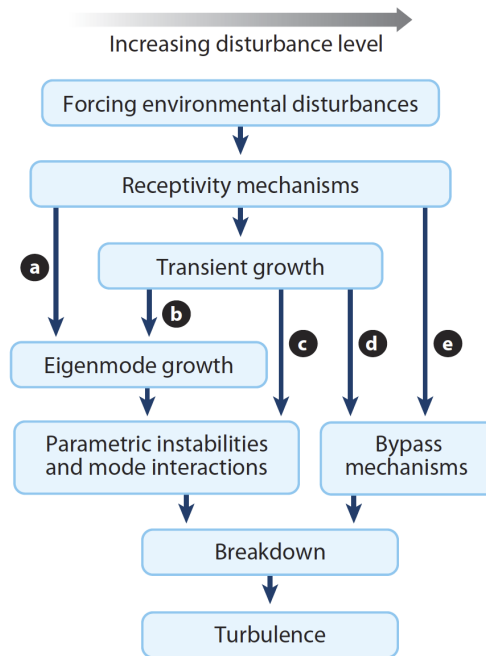


Figure 1.2: Paths to turbulence in boundary-layer flows (from Morkovin *et al.*[6, 7]).

The transition to turbulence can follow different paths based on the flow field parameters and types of disturbances [7]. Fig. 1.2 shows the different paths of transition to turbulence with the first stage governed by receptivity mechanisms. The external disturbances are converted to instability waves inside the boundary layer through the receptivity process [20]. Most common disturbance sources are acoustic, entropy and vorticity waves, surface roughness, bow-shock oscillations, thermal noise, kinetic fluctuations and particles [23]. Path “a” is associated with a low disturbance amplitude environment which can be observed in free flight [19]. The disturbances grow linearly with exponential growth rates. When the amplitude of the disturbances reaches the saturation limit, the non-linear interactions take over. This excites secondary mechanisms which lead to final breakdown to turbulence [19]. The disturbance growth may also follow path “b”, “c” or “d” of algebraic growth which are all subsets of transient growth. Transient growth excites other modes, secondary instability, or can lead directly to bypass transition.

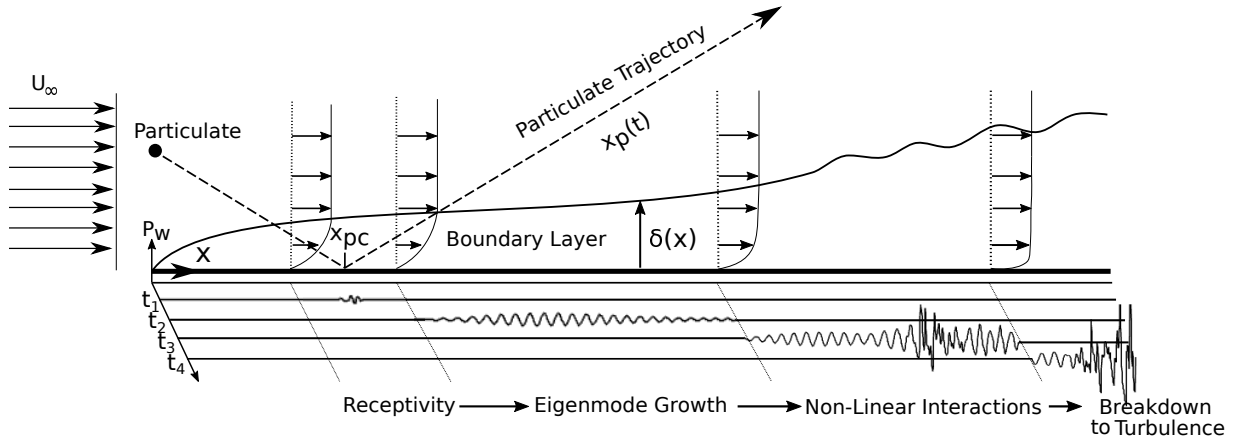


Figure 1.3: Particle-induced laminar-turbulent transition process following path “a” (adjusted from ref. [2]).

Particle induced transition usually follows path “a” of very small scale disturbance amplitudes. Fig. 1.3 displays the particle induced transition scenario for a flat plate. When the particle impinges on the surface of the flat plate, it seeds instabilities inside the boundary layer. The

instability waves grow exponentially and convect in the downstream direction as a wavepacket. After the linear regime, the flow field undergoes non-linear interactions before finally breaking down to turbulence.

High-speed transition is very different from low-speed transition ($Mach < 1$). In parallel low speed attached boundary layer, only one instability mode, the Tollmien–Schlichting (T–S) wave, dominates the flow field. In high speed inviscid flows, second and higher modes exist in addition to the viscous first mode [7, 20]. These modes are also known as Mack modes. For boundary layer the second mode is a two dimensional wave which shows maximum growth for 2-D wave modes. On the other hand, the first mode wave may display maximum growth at wavenumbers greater than zero.

1.4 The Scope of the Current Research and Structure of the Dissertation

The primary objective of the current research is to study the receptivity process of the particle induced disturbance flow field. First, the receptivity process is studied by comparing a particle-induced disturbance to a pulse disturbance in a Mach 5.35 flat plate boundary-layer flow. The aim is to identify the differences in the flow fields by comparing the frequency spectra of the particle-induced disturbance flow field with the frequency spectra of the disturbance flow field generated by a pulse disturbance.

Next, the disturbance flow field near the particle impingement location is decomposed with the biorthogonal eigenfunction system (BES). BES can be employed when the disturbance flow field can be represented by a superposition of eigenmodes and an orthogonality relation among these modes can be established. BES can be applied near to the particle impingement location

to study the receptivity process. Previously, BES has been employed to gain insight into the receptivity process in refs. [28, 29, 30]. Ref. [31] discusses the use of a 3D perturbation analysis in a spatial framework for a 2D incompressible boundary layer flow. In a later work in ref. [32], the 3D perturbation analysis in a temporal framework for a 2D compressible flow was studied for a Mach 5.6 sharp cone. The differences between the 2D and 3D perturbations have been studied by looking at the synchronization between the discrete modes, which may be broken down into fast mode (F) and slow mode (S). The study noted that the synchronization is absent at higher wave angles ($\Psi = \tan^{-1}\beta/\alpha$). The synchronization between the mode F and S can give rise to an unstable mode. It was also demonstrated that the initial value problem can be expanded in BES as a superposition of discrete and continuous spectra modes. The flow field of a Mach 8 5.3° sharp wedge is decomposed by employing the BES in ref. [29]. In a further work, it is shown in ref. [33] that the 3D solution of the linearized Navier-Stokes equations can be expanded in BES, and biorthogonal decomposition (BOD) of the flow field can be carried out in three dimensions. In addition, the disturbance flow field of a Mach 5.92 and Mach 8 flow over a sharp wedge is decomposed by using the BES in ref. [30]. Recently, the author also employed BOD to decompose the disturbance flow field in Mach 6 and Mach 5.35 flat plates in Refs. [34] and [2], respectively. It was demonstrated that the disturbance flow field can be fully reconstructed from the contributions from the discrete and continuous modes [2].

In the current study, the disturbance flow field near the particle impingement location is projected on the most dominant discrete mode S to identify the contributions to the disturbance flow field. The amplitude plots are compared with linear stability theory for solution accuracy. The frequency spectra is also plotted above the wall. The disturbance flow field is projected on the acoustic branches to identify the contributions in the freestream disturbance flow field. An energy

norm is used to combine all the flow field variables into a single scalar. A full reconstruction of the disturbance flow field is carried out to identify the contributions of different discrete and continuous modes to the total disturbance flow field. The error between the projection and disturbance flow fields are calculated to establish the differences. The disturbance flow fields in a Mach 5.35 flat plate, Mach 4 14° wedge and Mach 9.5 14° wedge are considered for the above analysis.

Next, the particle source in cell (PSIC) based simulation is compared with the fully resolved particle simulation by using the immersed boundary method (IBM). The PSIC method was previously proposed and validated by Chuvakhov et al. [35]. They simulated the particle induced disturbance flow field in a Mach 4 14° wedge by considering the compressible Navier-Stokes equations and compared it with the theoretical results of Fedorov [36]. The PSIC approach is adopted here by using an efficient adaptive mesh refinement wavepacket tracking (AMR-WPT) framework for particle induced disturbance flow field simulations [8]. The PSIC and IBM simulation approaches are compared at three different particle impingement locations with two different particle sizes. The wall-pressure envelope is calculated by employing the Hilbert transformation. The disturbance spectra of the disturbance wall pressure from the PSIC approach are compared with the disturbance spectra from the IBM simulations.

The final objective is to develop a low-fidelity particle model to simulate particle induced transition to turbulence. The model considers particle properties, *i.e.*, particle radius, concentration and density considering particle concentrations from three different measurement locations. The model is informed by direct numerical simulation (DNS) and linear stability theory (LST) and employs the e^N method to predict transition to turbulence.

This dissertation is organized in the following way, chapter 2 outlines the governing equations

and numerical methods. It discusses the compressible Navier-Stokes equations (CNSE), non-linear disturbance equations (NLDE), PSIC method, BES, AMR-WPT approach and immersed boundary method for fully resolved particle simulations. Chapter 3 discusses the validation and verification study for the PSIC, IBM and biorthogonal decomposition (BOD). The particle induced transition simulation results for a Mach 5.35 flat plate, Mach 4 14° wedge and Mach 9.5 14° wedge are presented in chapter 4. The projections on discrete and continuous branches are also presented in this chapter. The chapter discusses the full reconstruction of the disturbance flow field from the contributions of the discrete and continuous modes. Chapter 5 presents the comparison between PSIC and IBM simulation approaches with the particle model for simulating particle-induced transition to turbulence being presented in chapter 6. Final conclusions are drawn in chapter 7.

Chapter 2: Governing Equations and Numerical Methods

In the current study, an existing in house solver, namely the Cartesian Higher-order Adaptive Multiphysics Solver (CHAMPS), has been extended to simulate particle-laden flows for high-speed boundary layers. The solver employs a higher order conservative finite difference scheme which is based on a block structured Cartesian grid for domain decomposition. Adaptive mesh refinement (AMR) has been adopted to maintain the finest grid level only in the desired regions of the computational domain. These regions are usually the flow field around the particle and the evolving wavepacket of the disturbance flow field. An immersed boundary method (IBM) has also been employed to fully resolve the particle. The IBM framework has been verified and validated in the previous works of Brehm et al. [37, 38, 39]. The present work solves the compressible Navier-Stokes equations (CNSE) and non-linear disturbance equations (NLDE) coupled with a particle model. The NLDE along with AMR implementation has been validated and verified in prior works of Browne et al. [40, 41].

2.1 Compressible Navier-Stokes Equations

The first step of simulating particle-induced transition is to obtain the base state of the flow field which is obtained by solving the CNSE. The CNSE are solved in conservative form for an

ideal gas and can be written in vector form as

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = 0, \quad (2.1)$$

where $\mathbf{W}$ represents the conservative state vector and $\mathbf{E}$, $\mathbf{F}$, and $\mathbf{G}$ are the convective and viscous flux vectors. The full forms of state and flux vectors $\mathbf{W}$, $\mathbf{E}$, $\mathbf{F}$, and $\mathbf{G}$ are given by

$$\mathbf{W} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E_t \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho uv - \tau_{xy} \\ \rho uw - \tau_{xz} \\ (\rho E_t + p)u - u\tau_{xx} - v\tau_{xy} - w\tau_{xz} + q_x \end{bmatrix},$$

$$\mathbf{F} = \begin{bmatrix} \rho v \\ \rho uv - \tau_{xy} \\ \rho v^2 + p - \tau_{yy} \\ \rho vw - \tau_{yz} \\ (\rho E_t + p)v - u\tau_{xy} - v\tau_{yy} - w\tau_{yz} + q_y \end{bmatrix}, \text{ and} \quad (2.2)$$

$$\mathbf{G} = \begin{bmatrix} \rho w \\ \rho u w - \tau_{xz} \\ \rho v w - \tau_{yz} \\ \rho w^2 + p - \tau_{zz} \\ (\rho E_t + p)w - u\tau_{xz} - v\tau_{yz} - w\tau_{zz} + q_z \end{bmatrix}. \quad (2.3)$$

where p , E_t , ρ , u , v and w are the pressure, total energy, density, streamwise, transverse and spanwise velocity components, respectively. The total energy, E_t , can be written as

$$E_t = e + \frac{u^2 + v^2 + w^2}{2}, \quad (2.4)$$

where $e = R_{gas}T/(\gamma - 1)$ is the internal energy. The viscous shear-stress tensor components, τ_{ij} , are given as

$$\tau_{xx} = \frac{2\mu}{3} \left(2\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \frac{\partial w}{\partial z} \right), \quad \tau_{yy} = \frac{2\mu}{3} \left(2\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} - \frac{\partial w}{\partial z} \right), \quad (2.5)$$

$$\tau_{zz} = \frac{2\mu}{3} \left(2\frac{\partial w}{\partial z} - \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right), \quad \tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad (2.6)$$

$$\tau_{xz} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \quad \text{and} \quad \tau_{yz} = \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right), \quad (2.7)$$

where μ is the dynamic viscosity. The viscosity, μ , is evaluated with Sutherland's law including

a low-temperature correction

$$\mu = \begin{cases} S_1 T_1, & \text{for } T < T_1, \\ S_1 T & \text{for } T_1 \leq T \leq T_2, \text{ and} \\ \frac{S_2 T^{3/2}}{T+T_2} & \text{for } T > T_2, \end{cases} \quad (2.8)$$

where $S_1 = 6.93873 \times 10^{-8} \text{Ns/m}^2 \text{K}$, $S_2 = 1.458 \times 10^{-6} \text{Ns/m}^2 \text{K}^{1/2}$, $T_1 = 40 \text{K}$ and $T_2 = 110.4 \text{K}$.

The heat-flux vector, q_i , components are given by

$$q_x = -k \frac{\partial T}{\partial x}, \quad q_y = -k \frac{\partial T}{\partial y}, \quad \text{and} \quad q_z = -k \frac{\partial T}{\partial z}, \quad (2.9)$$

where k and T are the thermal conductivity and temperature, respectively. The Prandtl, Pr , number is set to 0.71. The equation of state for an ideal gas, $p = \rho R_{gas} T$, is used to close the CNSE.

2.2 Non-Linear Disturbance Equations

Next, the disturbance flow field is simulated by solving the NLDE. NLDE is derived from the CNSE by decomposing the state vector and flux vectors into steady base flow components and unsteady components. The state vector, $\mathbf{W}$, is decomposed as follows

$$\mathbf{W} = \bar{\mathbf{W}} + \tilde{\mathbf{W}} = \underbrace{\begin{bmatrix} \bar{\rho} \\ \bar{\rho}\bar{u} \\ \bar{\rho}\bar{v} \\ \bar{\rho}\bar{w} \\ \bar{\rho}\bar{E}_t \end{bmatrix}}_{\text{steady baseflow}} + \underbrace{\begin{bmatrix} \tilde{\rho} \\ \tilde{\rho}\bar{u} + \bar{\rho}\tilde{u} \\ \tilde{\rho}\bar{v} + \bar{\rho}\tilde{v} \\ \tilde{\rho}\bar{w} + \bar{\rho}\tilde{w} \\ \tilde{\rho}\tilde{E}_t + \bar{\rho}\bar{E}_t \end{bmatrix}}_{\text{linear disturbance}} + \underbrace{\begin{bmatrix} 0 \\ \tilde{\rho}\tilde{u} \\ \tilde{\rho}\tilde{v} \\ \tilde{\rho}\tilde{w} \\ \tilde{\rho}\tilde{E}_t \end{bmatrix}}_{\text{nonlinear disturbance}}, \quad (2.10)$$

where $\bar{\mathbf{W}}$ is the steady baseflow and $\tilde{\mathbf{W}}$ is the unsteady disturbance vector, which contains the linear and nonlinear terms. In addition, the total flux vectors, $\mathbf{E}$, $\mathbf{F}$ and $\mathbf{G}$, in the CNSE (eq. (2.1)) are decomposed into steady ($\bar{\cdot}$) and unsteady ($\tilde{\cdot}$) components in the following manner

$$\mathbf{E} = \bar{\mathbf{E}} + \tilde{\mathbf{E}}, \quad \mathbf{F} = \bar{\mathbf{F}} + \tilde{\mathbf{F}}, \quad \text{and} \quad \mathbf{G} = \bar{\mathbf{G}} + \tilde{\mathbf{G}}. \quad (2.11)$$

The fluxes can also be decomposed into convective ($\cdot_c$) and viscous ($\cdot_v$) components. In the x-direction, this decomposition leads to $\mathbf{E} = \mathbf{E}_c + \mathbf{E}_v$ and can be written as

$$\mathbf{E}_c = \underbrace{\begin{bmatrix} \bar{\rho}\bar{u} \\ \bar{\rho}\bar{u}^2 + \bar{p} \\ \bar{\rho}\bar{u}\bar{v} \\ \bar{\rho}\bar{u}\bar{w} \\ (\bar{\rho}\bar{E}_t + \bar{p})\bar{u} \end{bmatrix}}_{\text{steady baseflow}} + \underbrace{\begin{bmatrix} \tilde{\rho}\bar{u} + \bar{\rho}\tilde{u} \\ \tilde{\rho}\bar{u}^2 + 2\bar{\rho}\bar{u}\tilde{u} + \tilde{p} \\ \tilde{\rho}\bar{u}\bar{v} + \bar{\rho}\bar{u}\tilde{v} + \bar{\rho}\tilde{u}\bar{v} \\ \tilde{\rho}\bar{u}\bar{w} + \bar{\rho}\bar{u}\tilde{w} + \bar{\rho}\tilde{u}\bar{w} \\ (\tilde{\rho}\bar{E}_t + \bar{\rho}\bar{E}_t + \tilde{p})\bar{u} + (\bar{\rho}\bar{E}_t + \bar{p})\tilde{u} \end{bmatrix}}_{\text{linear disturbance}} + \underbrace{\begin{bmatrix} \tilde{\rho}\tilde{u} \\ \tilde{\rho}\bar{u}^2 + \tilde{\rho}\tilde{u}^2 + 2\bar{\rho}\bar{u}\tilde{u} \\ \tilde{\rho}\bar{u}\tilde{v} + \tilde{\rho}\tilde{u}\bar{v} + \bar{\rho}\bar{u}\tilde{v} + \bar{\rho}\tilde{u}\bar{v} \\ \tilde{\rho}\bar{u}\tilde{w} + \tilde{\rho}\tilde{u}\bar{w} + \bar{\rho}\bar{u}\tilde{w} + \bar{\rho}\tilde{u}\bar{w} \\ (\tilde{\rho}\bar{E}_t + \bar{\rho}\bar{E}_t + \tilde{\rho}\tilde{E}_t + \tilde{p})\tilde{u} + \tilde{\rho}\tilde{E}_t\bar{u} \end{bmatrix}}_{\text{nonlinear disturbance}}, \quad \text{and}$$

$$\mathbf{E}_v = - \underbrace{\begin{bmatrix} 0 \\ \bar{\mu}\bar{\tau}_{xx} \\ \bar{\mu}\bar{\tau}_{xy} \\ \bar{\mu}\bar{\tau}_{xz} \\ \bar{\mathcal{E}} \end{bmatrix}}_{\text{steady baseflow}} - \underbrace{\begin{bmatrix} 0 \\ \bar{\mu}\tilde{\tau}_{xx} + \tilde{\mu}\bar{\tau}_{xx} \\ \bar{\mu}\tilde{\tau}_{xy} + \tilde{\mu}\bar{\tau}_{xy} \\ \bar{\mu}\tilde{\tau}_{xz} + \tilde{\mu}\bar{\tau}_{xz} \\ \tilde{\mathcal{E}}_l \end{bmatrix}}_{\text{linear disturbance}} - \underbrace{\begin{bmatrix} 0 \\ \tilde{\mu}\tilde{\tau}_{xx} \\ \tilde{\mu}\tilde{\tau}_{xy} \\ \tilde{\mu}\tilde{\tau}_{xz} \\ \tilde{\mathcal{E}}_{nl} \end{bmatrix}}_{\text{nonlinear disturbance}},$$

where

$$\bar{\mathcal{E}} = -\bar{\mu}(\bar{u}\bar{\tau}_{xx} + \bar{v}\bar{\tau}_{xy} + \bar{w}\bar{\tau}_{xz}) - \bar{k}\frac{\partial\bar{T}}{\partial x}, \quad (2.12)$$

$$\begin{aligned} \tilde{\mathcal{E}}_l = & -\bar{\mu}(\bar{u}\tilde{\tau}_{xx} + \tilde{u}\bar{\tau}_{xx} + \bar{v}\tilde{\tau}_{xy} + \tilde{v}\bar{\tau}_{xy} + \bar{w}\tilde{\tau}_{xz} + \tilde{w}\bar{\tau}_{xz}) - \\ & \tilde{\mu}(\bar{u}\bar{\tau}_{xx} + \bar{v}\bar{\tau}_{xy} + \bar{w}\bar{\tau}_{xz}) - \bar{k}\frac{\partial\tilde{T}}{\partial x} - \tilde{k}\frac{\partial\tilde{T}}{\partial x}, \quad \text{and} \end{aligned} \quad (2.13)$$

$$\begin{aligned} \tilde{\mathcal{E}}_{nl} = & -\bar{\mu}(\tilde{u}\tilde{\tau}_{xx} + \tilde{v}\tilde{\tau}_{xy} + \tilde{w}\tilde{\tau}_{xz}) - \\ & \tilde{\mu}(\bar{u}\tilde{\tau}_{xx} + \tilde{u}\bar{\tau}_{xx} + \tilde{u}\tilde{\tau}_{xx} + \bar{v}\tilde{\tau}_{xy} + \tilde{v}\bar{\tau}_{xy} + \tilde{v}\tilde{\tau}_{xy} + \bar{w}\tilde{\tau}_{xz} + \tilde{w}\bar{\tau}_{xz} + \tilde{w}\tilde{\tau}_{xz}) - \tilde{k}\frac{\partial\tilde{T}}{\partial x}. \end{aligned} \quad (2.14)$$

From eqs. (2.1), (2.10) and (2.11) the following form of the CNSE is achieved

$$\frac{\partial\tilde{\mathbf{W}}}{\partial t} + \frac{\partial\tilde{\mathbf{W}}}{\partial t} + \frac{\partial(\tilde{\mathbf{E}} + \tilde{\mathbf{E}})}{\partial x} + \frac{\partial(\tilde{\mathbf{F}} + \tilde{\mathbf{F}})}{\partial y} + \frac{\partial(\tilde{\mathbf{G}} + \tilde{\mathbf{G}})}{\partial z} = 0. \quad (2.15)$$

Rearranging eq. (2.15) with steady baseflow terms on the right hand side, the following form of the CNSE is achieved

$$\frac{\partial\tilde{\mathbf{W}}}{\partial t} + \frac{\partial\tilde{\mathbf{E}}}{\partial x} + \frac{\partial\tilde{\mathbf{F}}}{\partial y} + \frac{\partial\tilde{\mathbf{G}}}{\partial z} = -\left(\frac{\partial\bar{\mathbf{W}}}{\partial t} + \frac{\partial\bar{\mathbf{E}}}{\partial x} + \frac{\partial\bar{\mathbf{F}}}{\partial y} + \frac{\partial\bar{\mathbf{G}}}{\partial z}\right). \quad (2.16)$$

The right hand side of eq. (2.16) itself is a solution of the CNSE. So, the right hand side can be regarded as equivalent to zero. With this assumption, the final form of the NLDE is achieved as

$$\frac{\partial \tilde{\mathbf{W}}}{\partial t} + \frac{\partial \tilde{\mathbf{E}}}{\partial x} + \frac{\partial \tilde{\mathbf{F}}}{\partial y} + \frac{\partial \tilde{\mathbf{G}}}{\partial z} = 0. \quad (2.17)$$

The solution of the NLDE depends on the baseflow quantities and gradients, so a steady baseflow solution must be obtained first before solving the NLDE. Information from the baseflow quantities are then used by the NLDE solver to evaluate the convective and viscous fluxes. The convective fluxes are evaluated by employing a 5th-order accurate WENO scheme [42] and the viscous fluxes are evaluated by employing a 2nd-order centered conservative finite difference scheme. The WENO scheme helps to deal with the shock for high-speed flow simulations. The convective and viscous fluxes are further divided into linear and nonlinear disturbance terms. The nonlinear fluxes can be neglected when performing a linear study. However, in the present work, the full form of the NLDE is considered.

2.3 Particle-Source-In-Cell method

Next, a particle model is coupled to the NLDE solver. The particle is tracked in the Lagrangian frame in the continuous flow field. Fig. 2.1 illustrates the particle-flow interactions as the particle crosses the shock and impinges on the surface of a wedge and bounces away after an elastic collision. The particle velocity in the flow field is updated based on the force exerted on it by the flow field through Newton's second law. Then the particle position is determined from the particle velocity. This leads to a system of equations as follows

$$\frac{d\mathbf{x}_p}{dt} = \mathbf{v}_p, \quad \text{and} \quad (2.18)$$

$$m_p \frac{d\mathbf{v}_p}{dt} = \mathbf{F}_p, \quad (2.19)$$

where $\mathbf{x}_p = (x, y, z)^T$, $\mathbf{v}_p = (u, v, w)^T$ and $\mathbf{F}_p$ are the particle position, velocity and force vectors, respectively. $m_p (= 4/3\pi R_p^3 \rho_p)$ is the particle mass. The particle density, ρ_p , and radius, R_p , are considered as the input variables from which the particle mass is determined. Throughout, the subscript p represents particle properties.

The force on the particle may include added mass effects, Basset force, pressure gradient [43] etc. However, these forces are may be less dominant [8] for the current work. Hence, only the drag force acting on the particle is considered in the current study which can be written as

$$\mathbf{F}_p = -\frac{1}{2} C_D \rho |\mathbf{v}_p - \mathbf{v}| (\mathbf{v}_p - \mathbf{v}) \pi R_p^2. \quad (2.20)$$

where C_D is the drag coefficient and ρ is the fluid density. The Crowe drag correlation [44] is

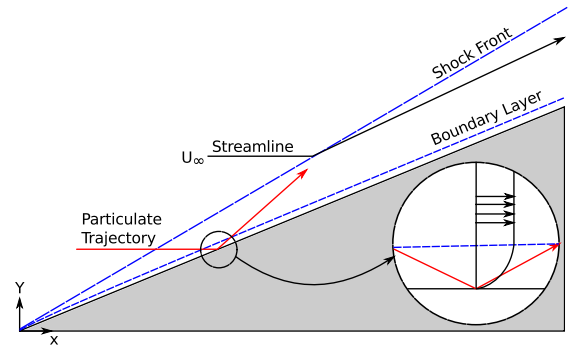


Figure 2.1: Schematic of the coordinate system with streamline, shock front, boundary-layer, and particle trajectory of flow past a wedge (adjusted from ref. [8]).

used for determining the drag coefficient which can be written as

$$C_D = (C_{D,inc} - 2) e^{(-3.632((Ma_p G)/Re_p))} + \frac{H}{1.183 Ma_p} e^{(-Re_p/(2Ma_p))} + 2, \quad (2.21)$$

$$\log_{10}(G) = 1.25 \left(1 + \tanh \left(0.77 \log_{10} (Re_p) - 1.92 \right) \right), \quad \text{and} \quad (2.22)$$

$$H = 2.3 + 1.7 \left(\frac{T_p}{T} \right)^{0.5} - 2.3 \tanh \left(1.17 \log_{10} (Ma_p) \right). \quad (2.23)$$

where $Ma_p = |\mathbf{v}_p - \mathbf{v}|/c$ is the particle Mach number, $Re_p = |\mathbf{v}_p - \mathbf{v}| 2R_p/\nu$ is the particle Reynolds number, c is the speed of sound, ν is the kinematic viscosity ($\nu = \mu/\rho$), $C_{D,inc}$ is the drag coefficient in incompressible flow, T_p is the temperature of the particle and T is the temperature of the fluid. $C_{D,inc}$ is evaluated based on the Morrison model [45]

$$C_{D,inc} = \frac{24}{Re_p} + \frac{2.6(Re_p/5)}{1 + (Re_p/5)^{1.52}} + \frac{0.411 \left(Re_p/(2.63 \times 10^5) \right)^{-7.94}}{1 + \left(Re_p/(2.63 \times 10^5) \right)^{-8}} + \frac{0.25 \left(Re_p/10^6 \right)}{1 + \left(Re_p/10^6 \right)}, \quad (2.24)$$

$C_{D,inc}$ can also be evaluated following the correlation from Schiller and Naumann [46] as

$$C_{D,inc} = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}). \quad (2.25)$$

Next, the profiles of relative Mach number (Ma_p), relative Reynolds number (Re_p) and the drag coefficient C_D along the boundary layer edge of a Mach 4 14° wedge are compared for the above two drag correlations. The particle size and density of $R_p = 50\mu m$ and $1000 kg/m^3$ are considered, respectively. The aftershock conditions are considered as the baseflow for this analysis. The results are compared with the study of Fedorov [47]. Figs. 2.2a-c show the relative Mach number, relative Reynolds number and the drag coefficient at the boundary layer edge in

a Mach 4 14° wedge. A good agreement with Fedorov [47] is obtained when the Crowe model is used with the Schiller and Naumann correlation [46]. Small differences are observed with the Crowe and Morrison model which increases in the downstream direction.

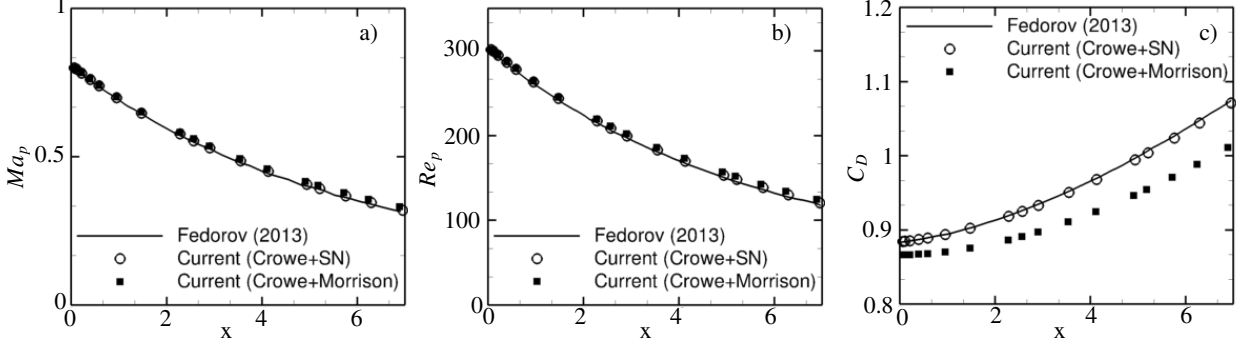


Figure 2.2: Particle a) relative Mach number b) relative Reynolds number c) drag coefficient at the upper boundary layer edge in Mach 4 14° wedge. SN stands for Schiller and Naumann (adjusted from ref. [8]).

The particle can have elastic or inelastic collisions with the wall, which can be modelled as

$$\mathbf{v}_{p,t+\Delta t} = (\mathbf{I} - (1 + \alpha)\mathbf{N})\mathbf{v}_{p,t}, \quad (2.26)$$

where $\mathbf{v}_{p,t}$ is the particle velocity before collision, $\mathbf{v}_{p,t+\Delta t}$ is the particle velocity after collision, $\mathbf{I}$ is the identity matrix, $\mathbf{N}$ is the directional matrix and α is the coefficient of restitution. The value of α ranges from 0 to 1. The particle is having an elastic collision with the wall when $\alpha = 1$. The effect of the particle on the flow field is modelled by adding source terms on the right hand side of the NLDE. The momentum source terms, which is based on the drag force, can be written as

$$\mathbf{S}_m = C_D \frac{\rho}{2} |\mathbf{v}_p - \mathbf{v}| (\mathbf{v}_p - \mathbf{v}) \pi R_p^2 \delta(\mathbf{x} - \mathbf{x}_p), \quad (2.27)$$

where the delta function $\delta(\mathbf{x} - \mathbf{x}_p)$ ensures a Gaussian distribution of the force in the flow field.

The delta function can be written in three dimensions on the left and two dimensions on the right as

$$\delta_{3D}(\mathbf{x} - \mathbf{x}_p) = \begin{cases} \frac{1}{(\sigma\sqrt{2\pi})^3} e^{[-\frac{|\mathbf{x}-\mathbf{x}_p|^2}{2\sigma^2}]}, \\ 0, \end{cases} \quad \text{and} \quad \delta_{2D}(\mathbf{x} - \mathbf{x}_p) = \begin{cases} \frac{1}{(2\pi\sigma^2)} e^{[-\frac{|\mathbf{x}-\mathbf{x}_p|^2}{2\sigma^2}]}, & |\mathbf{x} - \mathbf{x}_p| < 4\sigma, \\ 0, & |\mathbf{x} - \mathbf{x}_p| \geq 4\sigma, \end{cases} \quad (2.28)$$

where σ is the half width of the Gaussian distribution. The energy source term is also based on the drag force and particle velocity which can be written as

$$S_e = C_D \frac{\rho}{2} |\mathbf{v}_p - \mathbf{v}| \left((\mathbf{v}_p - \mathbf{v}) \cdot \mathbf{v}_p \right) \pi R_p^2 \delta(\mathbf{x} - \mathbf{x}_p), \quad \text{or} \quad S_e = \mathbf{S}_m \cdot \mathbf{v}_p, \quad (2.29)$$

The energy source term can also be derived from the dot product of the momentum source term and the particle velocity. By adding the momentum and energy source terms to the right hand side of eq. (2.17) the following final form of the NLDE is achieved

$$\frac{\partial \tilde{\mathbf{W}}}{\partial t} + \frac{\partial \tilde{\mathbf{E}}}{\partial x} + \frac{\partial \tilde{\mathbf{F}}}{\partial y} + \frac{\partial \tilde{\mathbf{G}}}{\partial z} = -\mathbf{S}(\tilde{\mathbf{W}}, \tilde{\mathbf{W}}), \quad (2.30)$$

where vector $\mathbf{S}(\tilde{\mathbf{W}}, \tilde{\mathbf{W}})$ represents both the momentum and energy source terms.

2.4 Biorthogonal Eigenfunction System

The governing equation for the biorthogonal eigenfunction system is the linearized version of the CNSE. The linearized CNSE is Fourier transformed in time to get the following

$$\frac{\partial}{\partial y} \left(\mathbf{L}_0 \frac{\partial \mathbf{A}}{\partial y} \right) + \mathbf{L}_1 \frac{\partial \mathbf{A}}{\partial y} = \mathbf{H}_1 \mathbf{A} + \mathbf{H}_2 \frac{\partial \mathbf{A}}{\partial x} + \mathbf{H}_3 \frac{\partial \mathbf{A}}{\partial z}, \quad (2.31)$$

where the vector $\mathbf{A}$ has sixteen disturbance quantities with their spatial derivatives in three dimensions

$$\begin{aligned}\mathbf{A}(x, y, z) = & (\tilde{u}, \partial\tilde{u}/\partial y, \tilde{v}, \tilde{p}, \tilde{T}, \partial\tilde{T}/\partial y, \tilde{w}, \partial\tilde{w}/\partial y, \partial\tilde{u}/\partial x, \partial\tilde{v}/\partial x, \\ & \partial\tilde{T}/\partial x, \partial\tilde{w}/\partial x, \partial\tilde{u}/\partial z, \partial\tilde{v}/\partial z, \partial\tilde{T}/\partial z, \partial\tilde{w}/\partial z)^T,\end{aligned}\quad (2.32)$$

and solution of eq. (2.31) depends on the boundary conditions at the wall and in the freestream as

$$y = 0 : \tilde{u} = \tilde{v} = \tilde{w} = \tilde{T} = 0, \quad (2.33)$$

$$y \rightarrow \infty : |A_m| \rightarrow 0, \quad m \in [1, 16], \quad (2.34)$$

Eq. (2.31) contains five matrices, $\mathbf{L}_0, \mathbf{L}_1, \mathbf{H}_1, \mathbf{H}_2$ and $\mathbf{H}_3$, with the dimensions of sixteen by sixteen. The details of these matrices have been outlined in ref. [33].

To satisfy the orthogonality relation, the eigenvalue problem (EVP) needs to account for both the direct and adjoint problem. The direct and adjoint EVP for the BES can be written as

$$\frac{\partial}{\partial y} \left(\mathbf{L}_0 \frac{\partial \mathbf{A}_{\alpha\beta}}{\partial y} \right) + \mathbf{L}_1 \frac{\partial \mathbf{A}_{\alpha\beta}}{\partial y} - \mathbf{H}_1 \mathbf{A}_{\alpha\beta} - i\alpha \mathbf{H}_2 \mathbf{A}_{\alpha\beta} - i\beta \mathbf{H}_3 \mathbf{A}_{\alpha\beta} = 0, \quad (2.35)$$

$$y = 0 : A_{\alpha\beta 1} = A_{\alpha\beta 3} = A_{\alpha\beta 5} = A_{\alpha\beta 7} = 0,$$

$$y \rightarrow \infty : |A_{\alpha\beta i}| < \infty, \text{ and}$$

$$\frac{\partial}{\partial y} \left(\mathbf{L}_0^T \frac{\partial \mathbf{B}_{\alpha\beta}}{\partial y} \right) - \mathbf{L}_1^T \frac{\partial \mathbf{B}_{\alpha\beta}}{\partial y} - \mathbf{H}_1^T \mathbf{B}_{\alpha\beta} - i\alpha \mathbf{H}_2^T \mathbf{B}_{\alpha\beta} - i\beta \mathbf{H}_3^T \mathbf{B}_{\alpha\beta} = 0, \quad (2.36)$$

$$y = 0 : B_{\alpha\beta 2} = B_{\alpha\beta 4} = B_{\alpha\beta 6} = B_{\alpha\beta 8} = 0,$$

$$y \rightarrow \infty : |B_{\alpha\beta i}| < \infty,$$

where $\mathbf{A}_{\alpha\beta}$ and $\mathbf{B}_{\alpha\beta}$ are the eigensolutions to the direct and adjoint EVPs. The orthogonality relation for the EVPs is given by

$$\langle \mathbf{H}_2 \mathbf{A}_{\alpha\beta}, \mathbf{B}_{\alpha^*\beta} \rangle \equiv \int_0^\infty (\mathbf{H}_2 \mathbf{A}_{\alpha\beta}, \mathbf{B}_{\alpha^*\beta}) dy = \Gamma \Delta_{\alpha\alpha^*}, \quad (2.37)$$

where Γ is a constant which depends on the normalization of $\mathbf{A}_{\alpha\beta}$ and $\mathbf{B}_{\alpha\beta}$. $\Delta_{\alpha\alpha^*}$ is the Kronecker delta function if any of the eigenvalues is from the discrete spectra. If both of the eigenvalues are from the continuous spectra, $\Delta_{\alpha\alpha^*}$ is a Dirac delta function. Using this orthogonality relation, the projection on mode α is determined from

$$\hat{\mathbf{A}} = \frac{\langle \mathbf{H}_2 \mathbf{A}, \mathbf{B}_{\alpha\beta} \rangle}{\langle \mathbf{H}_2 \mathbf{A}_{\alpha\beta}, \mathbf{B}_{\alpha\beta} \rangle} \mathbf{A}_{\alpha\beta}, \quad (2.38)$$

where $\langle \mathbf{H}_2 \mathbf{A}, \mathbf{B}_{\alpha\beta} \rangle / \langle \mathbf{H}_2 \mathbf{A}_{\alpha\beta}, \mathbf{B}_{\alpha\beta} \rangle$ is the coefficient of projection (*COP*) [48]. The projection on the continuous spectra is done by performing an integration over a continuous spectra parameter, k . The combined solution of the discrete and continuous spectra represent the final solution of the LNSE

$$\mathbf{A}_\beta(x, y) = \underbrace{\sum_d \chi_d \mathbf{A}_{\alpha_d}(y) e^{i\alpha_d x}}_{\text{contribution from discrete spectrum}} + \underbrace{\sum_c \int_0^\infty \chi_c(k) \mathbf{A}_{\alpha_c}(y) e^{i\alpha_c(k)x} dk}_{\text{contribution from the continuous branches}}, \quad (2.39)$$

where d refers to the discrete modes, c refers to the continuous branches, χ_d is found from the

projection on discrete modes and χ_c can be obtained from the projection on continuous branches.

The non-dimensional flow properties are defined as follows, frequency parameter as $F = \omega \nu_\infty / U_\infty^2$, where $\omega = 2\pi f$, f is the dimensional frequency, Reynolds number as $Re = U_\infty H / \nu_\infty$, angular frequency $\Omega = FRe$ or $\Omega = \omega \tau$. τ is the time scale, $\tau = \sqrt{(\nu_\infty x)} / U_\infty^{1.5}$ and H is the length scale, $H = \sqrt{(\mu_\infty x / (\rho_\infty U_\infty))}$ and kinematic viscosity $\nu_\infty = \mu_\infty / \rho_\infty$ where μ_∞ is the freestream dynamic viscosity and ρ_∞ is the freestream density of the fluid.

2.5 Adaptive Mesh Refinement Wavepacket Tracking Simulation Approach

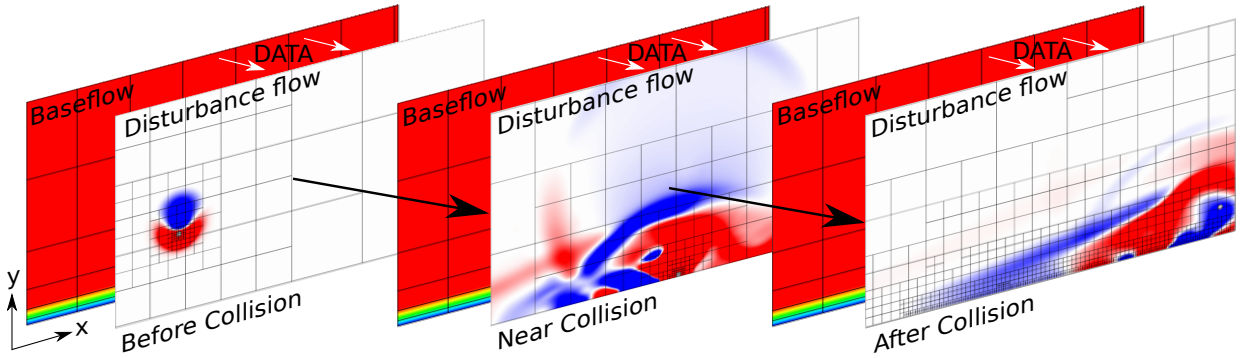


Figure 2.3: Example of AMR mesh tracking of particle before, near, and after collision in a $M=5.35$ flat plate boundary-layer flow (adjusted from ref. [9]).

The particle-flow interaction is simulated by following an established wavepacket tracking strategy, adaptive mesh refinement wavepacket tracking (AMR-WPT). More details of AMR-WPT approach can be found in refs. [40, 41, 49, 50]. The AMR-WPT employs an overset grid approach in which two different grids are used to solve for the disturbance flow field. The baseflow, $\bar{\mathbf{W}}$, is obtained by solving the CNSE on a curvilinear grid. A stretching function is used to ensure enough grid points are present in the boundary layer. The NLDE is solved on a block structured Cartesian mesh with appropriate boundary conditions. When the baseflow is converged, $d\bar{\mathbf{W}}/dt \approx 0$, the information from the curvilinear mesh is transferred to the structured

Cartesian mesh by means of a higher order interpolation. The interpolation from a curvilinear mesh to Cartesian structured mesh occurs every time the Cartesian mesh gets updated. The Cartesian mesh is adapted at a user-specified frequency to track the wavepacket developing in the boundary layer [41]. The overset dual mesh approach is shown in fig. 2.3. It shows three refinement steps, *i.e.* before particle collision, near collision and after collision.

The AMR approach is implemented within the PARAMESH library [51]. The Cartesian mesh has several grid levels extending from the coarser mesh level to the finest mesh level. The refinement of a coarser block, also known as the parent block, will create new child blocks [40]. Each parent block will create four child blocks in two dimensions and eight child blocks in three dimensions assuming isotropic refinement. The information from the parent block is transferred to the child block. An analogous process may be followed for a derefinement step [40]. The refinement-derefinement of the Cartesian mesh is controlled by the flow field variables. The refinement-derefinement function, ϕ , is based on the flow field pressure, pressure gradient, vorticity and temperature [41]. These variables show peak values in different regions of the boundary layer [41]. The refinement function can be defined as [41]

$$\phi = \max_{\tilde{\Omega}} \left(\frac{|\tilde{\phi}_i(x, t)|}{\max_{\tilde{\Omega}} |\tilde{\phi}_i(x, t)|} \right), \quad i = 1, 2, 3, \dots, \quad (2.40)$$

where $\tilde{\phi}_i(x, t)$ is any solution quantity. The parameter $|\tilde{\phi}_i(x, t)|$ is normalized with the maximum norm. The value of ϕ lies within the range $0 < \phi < 1$. When ϕ at any point within any block is less than a predefined value, ϕ_l , the block is derefined. On the other hand, if ϕ is greater than a predefined value, ϕ_u , the block is refined.

2.6 Immersed Boundary Method for Fully-Resolved Particle Simulations

The immersed boundary method (IBM) in modeling particle-fluid interaction simulations is advantageous as it does not require any mesh motion to deform the computational mesh according to the surface of the spherical particles. The flow field around the particle is fully resolved by using the IBM. This way, additional approximations and complications are avoided. As the particle moves in the flow field, the IBM ensures that the boundary layer at the surface of the particle is resolved with appropriate grid resolution.

The IBM for NLDE is based on the method outlined in refs. [4, 37, 39]. The IBM-NLDE approach uses locally stabilized finite difference stencils for ensuring the stability of the numerical method [3]. An upwind biased finite difference scheme is employed with higher accuracy for the IBM-NLDE approach [3]. A sharp interface is maintained at the particle-fluid interface. The irregular points are treated differently than the regular points where irregular points are the points in the vicinity of the particle-fluid interface whose computational stencil would access solution data behind the immersed boundary. A different strategy for the computational stencil is followed for the irregular points. More details are provided in refs. [3, 4, 37].

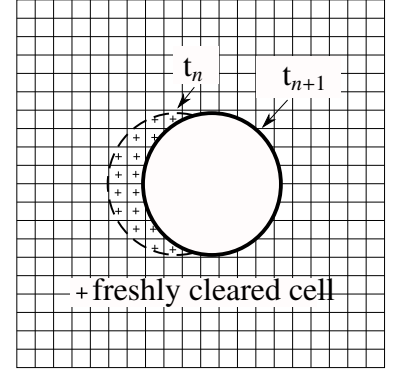


Figure 2.4: Freshly-cleared cells emerge as particle moves from time step t_n to t_{n+1} (adjusted from ref. [10]).

When the particle moves in the flow field, a new set of grid points emerges at the wake of the particle. These grid points do not have any valid flow field data. These grid points are referred to as freshly cleared cells [3]. Fig. 2.4 displays that the freshly cleared cells emerge as the particle moves from time t_n to t_{n+1} . A higher order interpolation method is employed to fill

up these freshly cleared cells with appropriate values. A cloud of points is considered around the freshly cleared cells for the interpolation purpose. More details of this procedure can be found in refs. [3, 39, 52, 53]. Appropriate boundary conditions are implemented at the wall of the particle. The flow velocity at the surface is set equal to the velocity of the particle and the pressure as well as the temperature is determined from a higher order interpolation method [3]. The fully-resolved particle moving at a Reynolds number of $O(100)$ in the flow field is displayed in fig. 2.5 at three time instances. As the particle moves in the flow field, vortex shedding is observed in the wake of the particle.

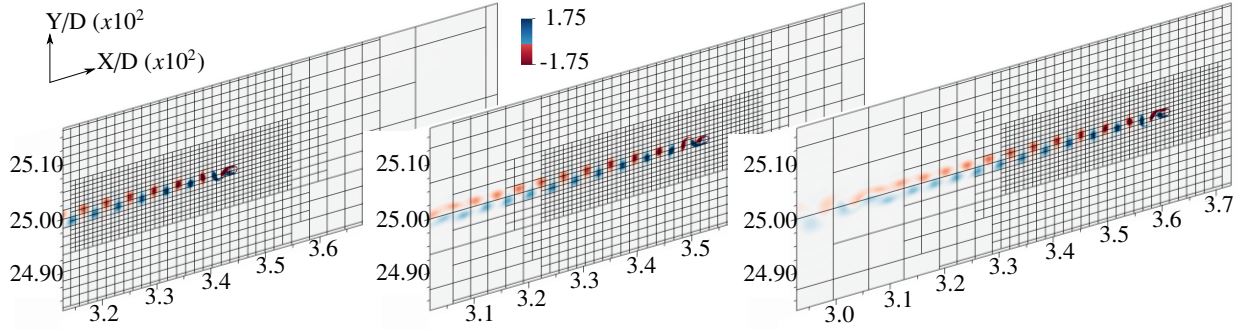


Figure 2.5: Instantaneous pseudocolor plots of spanwise vorticity component with AMR mesh following the moving cylinder (adjusted from ref. [3]).

Chapter 3: Validation and Verification

The particle induced simulation approach is validated and verified in this section. Two different approaches have been followed to simulate the particle-flow interactions, PSIC and by fully resolving the particle using the IBM. PSIC is validated by comparing with another established CFD solver US3D and also by simulating the flow in three different ways inside CHAMPS, solving CNSE on a curvilinear grid, solving the NLDE on a curvilinear grid as well as solving the NLDE on an AMR grid. In addition, BOD is applied to study the receptivity process. The implementation of BOD is validated by showing that the prescribed coefficient of projection can be recovered from the projection on mode S or F.

3.1 Validation of Particle-Source-in-Cell approach

The implementation of the PSIC approach in CHAMPS is validated by comparing with an established CFD code, US3D. US3D employs a finite volume method to solve the CNSE [54]. It uses a low-dissipation kinetic energy consistent (KEC) flux scheme to maintain stability and resolve small fluctuations [54, 55]. It also employs implicit time integration using the backwards differentiation formula (BDF2) [54].

A stationary particle with radius $10\mu m$ is considered in a Mach 2 flow field at an altitude of 20km. The freestream properties at 20km altitude are provided in table 3.1 [35]. Figs. 3.1a

R_p	T_∞	P_∞	ρ_∞	M_∞	σ
$10\mu m$	216.7K	5530Pa	0.0889kg/m^3	2	$5R_p$

Table 3.1: Particle and flow properties at 20km altitude

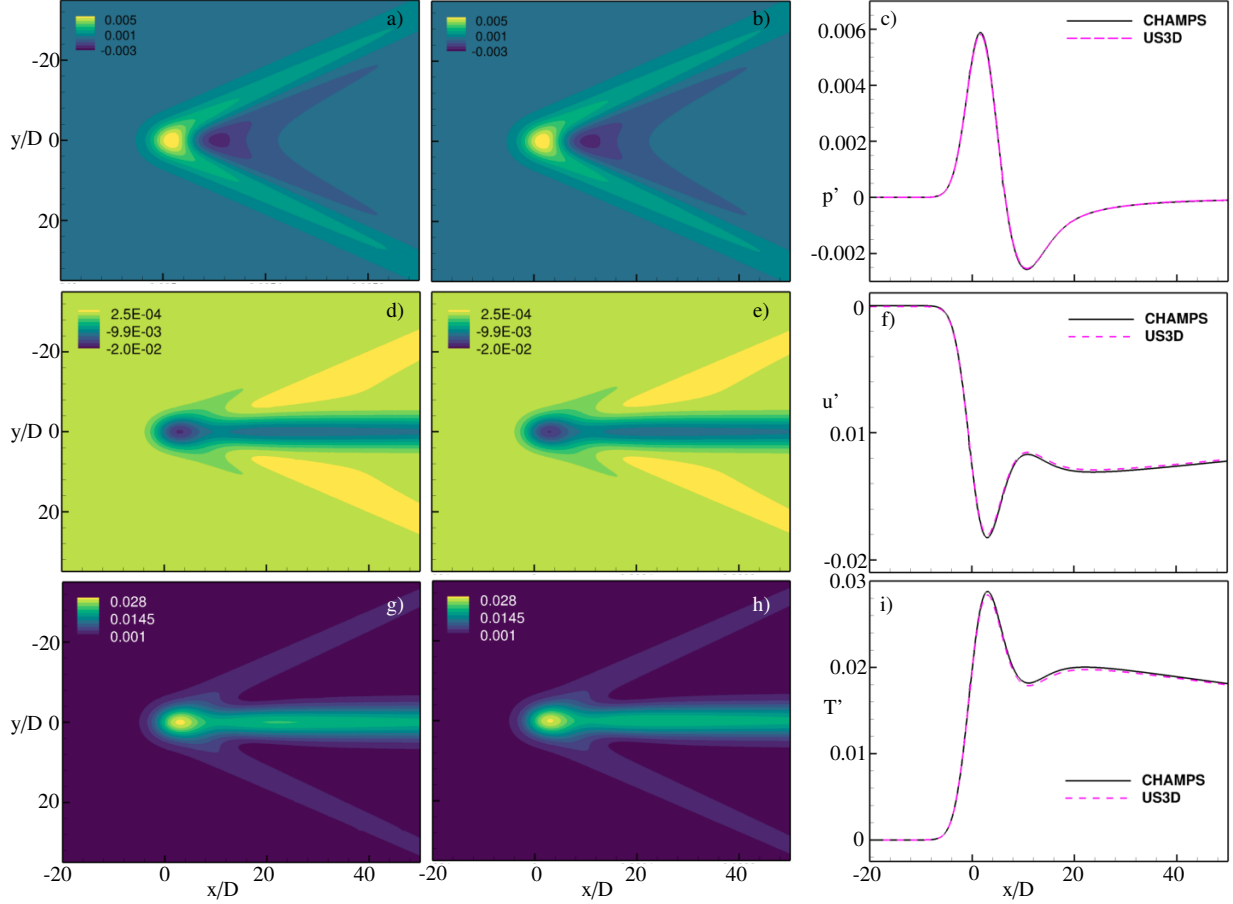


Figure 3.1: Comparison with US3D (a,d,g) CHAMPS, (b,e,h) US3D, (a,b) pressure contour, (d,e) streamwise velocity contour, (g,h) temperature contour. (c,f,i) line extracted at $y/D=0$.

and 3.1b show the disturbance pressure contour plot from the CHAMPS and US3D solvers, respectively. Curves are extracted at the symmetry plane and at $y/D = 0$ for a direct comparison in fig. 3.1c and show close agreement between the two solvers. The streamwise velocity and temperature are compared in figs. 3.1d-f and figs. 3.1g-i, respectively, with a similarly good agreement.

Next, the particle-induced transition simulation is performed by following three approaches,

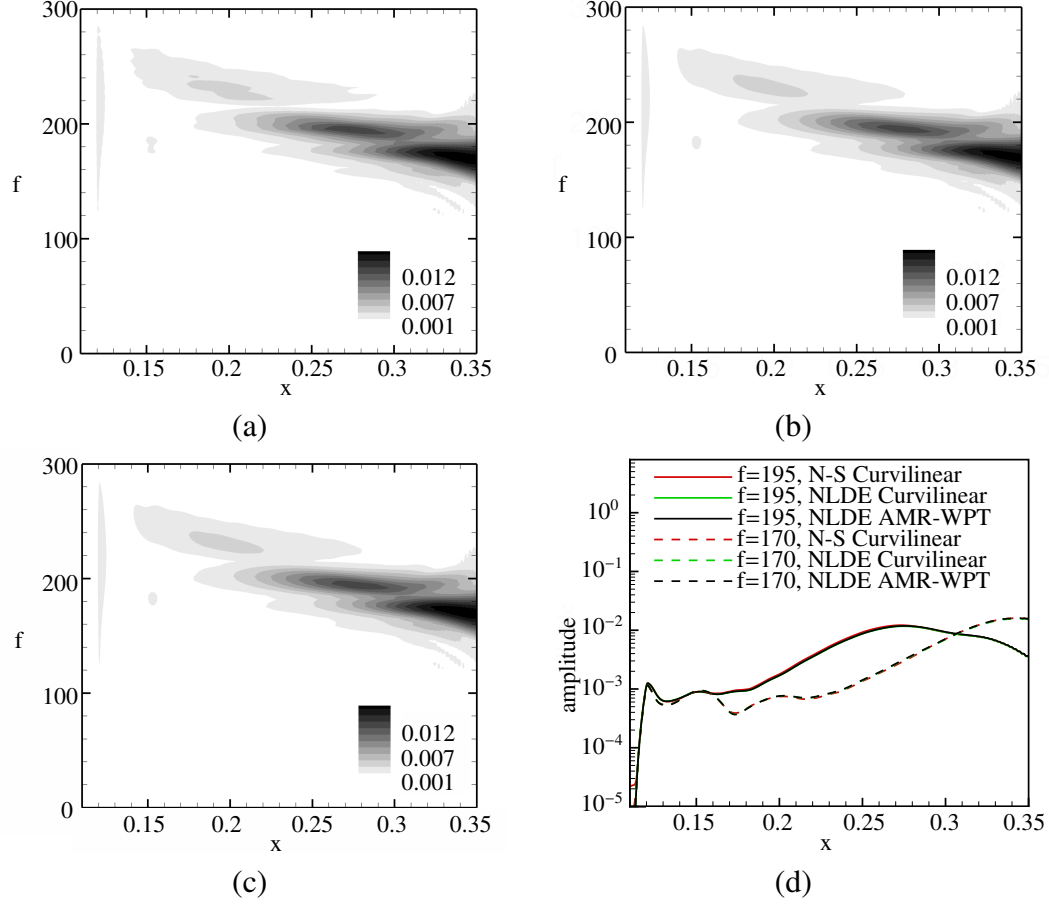


Figure 3.2: Amplitude plot of the wall disturbance pressure field by solving (a) Navier-Stokes equations on a curvilinear grid, (b) NLDE on curvilinear grid, (c) NLDE with the AMR-WPT approach, and (d) amplitude curves extracted at different frequencies for the different approaches.

(1) solving the CNSE on a curvilinear grid, (2) solving the NLDE on a curvilinear grid and (3) solving the NLDE with the Adaptive Mesh Refinement-Wavepacket Tracking (AMR-WPT) approach. The particle impinges at $x_{pc} = 0.12m$ in a Mach 5.35 flow field. The freestream properties of the flow field are provided in table 4.1. Figs. 3.2a-c show the wall disturbance pressure amplitude plots for the simulation performed by solving the CNSE on a curvilinear grid, the NLDE on a curvilinear grid and the NLDE on the AMR grid, respectively. The amplitude peak is found at $x=0.35m$ and $f=168.75$ for all the cases. Fig. 3.2d shows the development of the wall pressure amplitude in the streamwise direction. A close agreement is observed among the

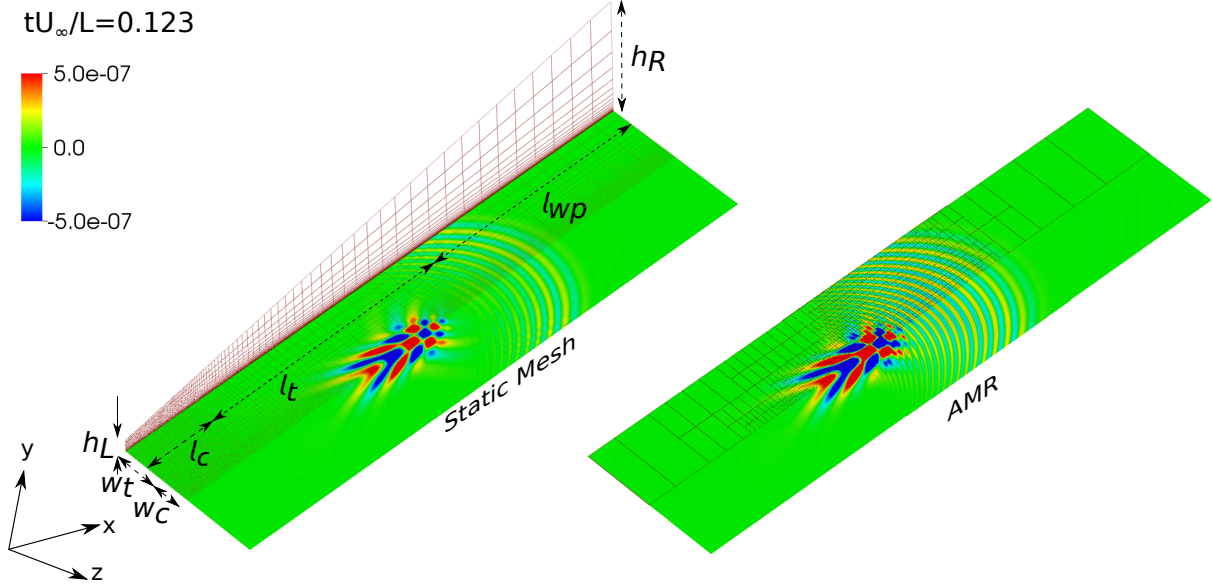


Figure 3.3: Wall pressure normalized by $(\rho_\infty U_\infty^2)$ at $tU_\infty/L = 0.123$ in (left) static mesh and (right) AMR approach in a Mach 4 14° wedge (adjusted from ref. [8]).

different approaches.

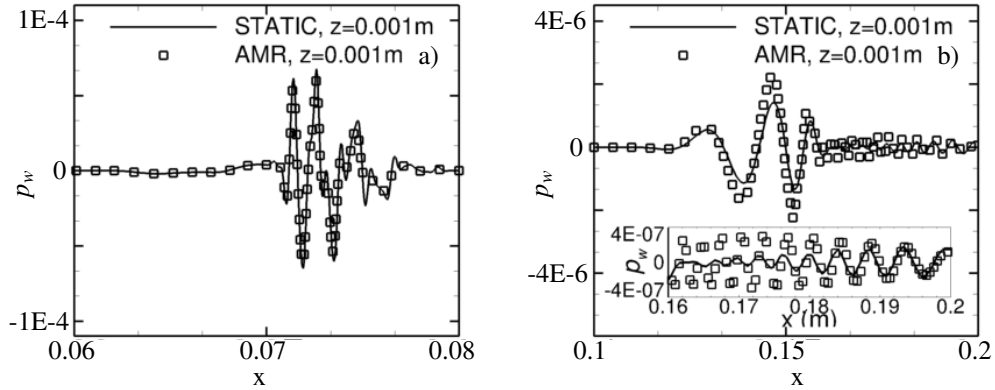


Figure 3.4: Wall pressure normalized by $(\rho_\infty U_\infty^2)$: (a) shortly after particulate impingement at $tU_\infty/L = 0.0161$ and (b) some later time at $tU_\infty/L = 0.123$ for static mesh and AMR approaches. Axis is rotated 14° clockwise (adjusted from ref. [8]).

Next, the AMR approach is validated by comparing with the static mesh approach in 3D for a Mach 4 14° wedge. The computational domain resembles the subdomain study of Chuvakhov et al. [35]. Different grid resolutions are used in different regions of the subdomain. The subdomain is divided into the particle collision ($\cdot_c$), transitional ($\cdot_t$) and wavepacket ($\cdot_{wp}$)

regions. The collision region has a length of $l_c = 0.0206m$ and a width of $w_c = 0.004m$. Beyond the collision region the width is extended to $w_t = 0.026m$. The lengths of the transitional and wavepacket regions are $l_t = 0.1158m$ and $0.0821m$, respectively. Near the wall, the smallest grid spacing in the wall-normal direction is $\Delta y = 5 \times 10^{-6}m$. The grid spacings in the streamwise and spanwise directions are $\Delta x_c = \Delta z_c = 5 \times 10^{-5}m$. The streamwise grid spacing is increased to $16\Delta x_c$ in the region where wavepacket is fully developed. The subdomain heights at the inflow and outflow are $h_L \approx 8\delta_{99,pc}$ and $h_R \approx 81\delta_{99,pc}$, respectively. $\delta_{99,pc}$ is the boundary-layer height at the particle impingement location. Finally, the domain is discretized with 1021, 211 and 251 points in the streamwise, wall-normal and spanwise directions.

A $R_p = 10\mu m$ particle is introduced at position (0.058m,0.016m,0.0m) which impinges at the wall approximately at $x_{pc} = 0.065m$. The particle is assumed to stick to the surface after impingement. So, the momentum and energy source term is zero after the particle impingement. To ensure similar forcing, the energy and source terms from the AMR simulation are used in the static mesh simulation along the trajectory of the particle. A time step size of $2.5 \times 10^{-9}s$ is used for both the AMR and static mesh approach. Fig. 3.3 displays the disturbance wall pressure field at $tU_\infty/L = 0.123$. A length scale of $L = 1m$ is considered. Figs. 3.4a and 3.4b show the wall pressure signature at two time instances near the symmetry plane. The AMR simulation shows larger acoustic wave amplitude ahead of the wavepacket than the static mesh approach. Overall, a good agreement exists between the AMR and static mesh simulation approach.

3.2 Validation of Immersed Boundary Method for Particle

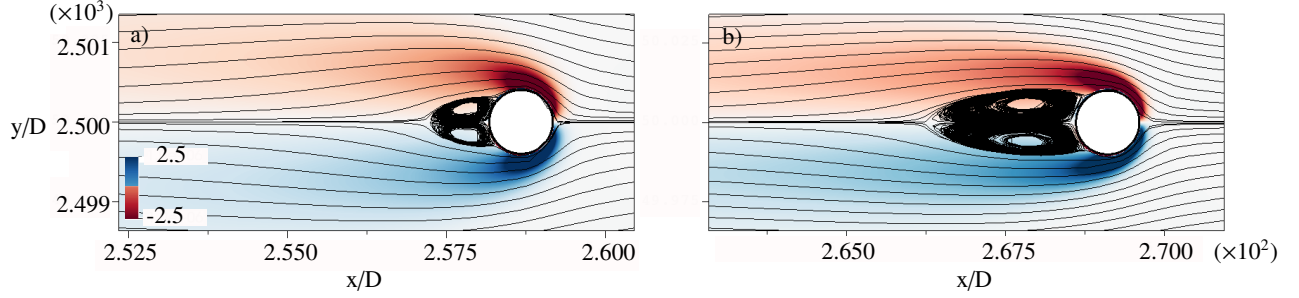


Figure 3.5: Instantaneous pseudocolor contours of spanwise vorticity and streamlines for the cylinder flow with a) $Re_D=20$ and b) $Re_D=40$ (adjusted from ref. [3]).

In this section, the IBM-NLDE solver for particle-flow interaction simulation is validated by comparing the flow field parameters with available literature data. 2D and 3D

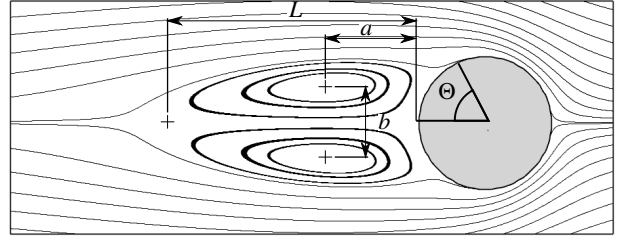


Figure 3.6: Dimensions of cylinder wake particles are considered traveling in the flow field with finite velocities. Reynolds numbers, $Re_D = 20, 40, 100$ and 200 are considered for the cylinder-flow interactions and $Re_D = 100$ is considered for the sphere-flow interaction simulations. The relative Reynolds number is defined as $Re_D = (\rho_\infty |v_p - U_\infty| D) / \mu_\infty$, where U_∞ is the freestream velocity, v_p is the velocity of the particle, ρ_∞ is the freestream density, μ_∞ is the freestream dynamic viscosity and D is the diameter of the particle. The flow around the particle is steady for the cases with $Re_D = 20$ and 40 in 2-D and $Re_D = 100$ in 3-D [3]. On the other hand, for the larger $Re_D = 100$ and 200 the wake of the particle becomes unsteady and vortex shedding can be observed[3]. Different dimensions of the cylinder wake that are used for comparison purposes are illustrated in fig. 3.6. The different parameters of the wake are x- and y-locations of the vortex centers, a, b , length of separation, L and angle of separation, Θ . The simulations are performed with grid resolution of $\Delta x = \Delta y = \Delta z \approx D/60$. Similar grid resolution was also used in the reference simulation by Brehm et al. [4]. The computational domain has dimension of $[0m, 4m]$ in the streamwise

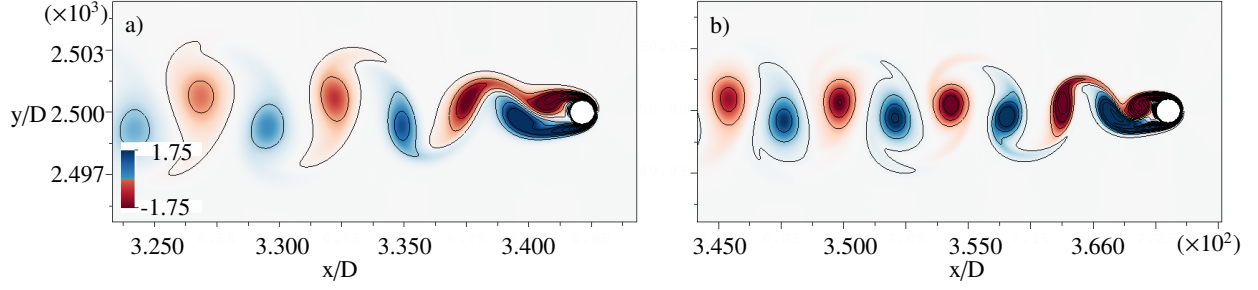


Figure 3.7: Instantaneous pseudocolor contours of spanwise vorticity for the cylinder flow with a) $Re_D=100$ and b) $Re_D=200$ (adjusted from ref. [3]).

direction and $[0m, 1m]$ in the transverse direction for the cylinder simulations. For the 3-D simulation, the computational domain extends over $[0m, 0.05m]$ in the all three directions. The computational domain and freestream parameters are provided in table A.1.

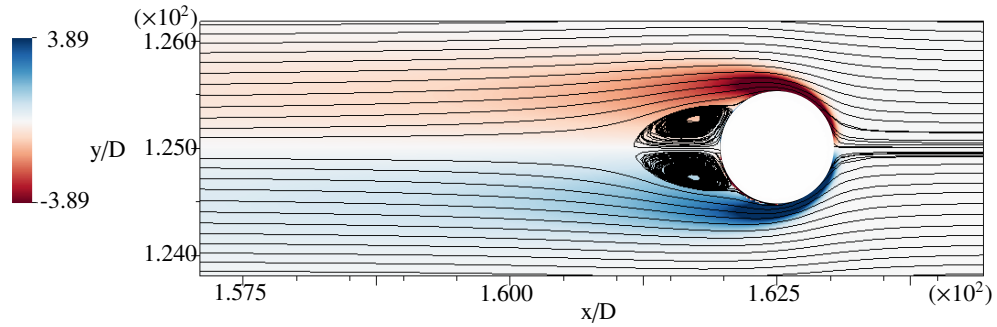


Figure 3.8: The instantaneous pseudocolor contours of spanwise vorticity and streamlines in $z=0$ plane for the sphere flow with $Re_D=100$ (adjusted from ref. [3]).

Figs. 3.5a and 3.5b show the spanwise vorticity contours for the cylinders with $Re_D = 20$ and $Re_D = 40$, respectively. The wake parameters are provided in table A.2 with comparison to the data in the literature. The wake parameters from the simulations are in close agreement with the literature. Figs. 3.7a and 3.7b show the spanwise vorticity contour for the cylinder traveling with $Re_D = 100$ and 200 , respectively. The unsteady wake can be characterized by a non-dimensional number, the Strouhal number. The Strouhal number can be defined as $St = fD/|U_\infty - v_p|$, where f is the dimensional frequency of the vortex shedding [3]. The time-averaged drag coefficient, C_D , lift coefficient C_L and Strouhal number, St , of the flow field are provided

in table A.3 including the data from the literature. The coefficients and Strouhal numbers are in good agreement.

Next, the flow field is studied in three dimensions. Fig. 3.8 shows the spanwise vorticity contour at the symmetry plane for a sphere traveling with $Re_D = 100$. The wake contains two counter-rotating steady vortices at the symmetry plane. The length of vortices, L_b and the center of the vortices x_c and y_c are provided in table A.4. Good agreement can be observed between the literature and simulated data.

3.3 Validation of Biorthogonal Decomposition

Case	Re	f (kHz)	C_F	C_S	COP of F	COP of S	α (mode F)	α (mode S)
1	1432.38	204.849	-1	1	-1.000-6.616e-5i	1.000-1.697e-5i	0.1727+0.01219i	0.1715-0.004542i
2	1432.38	190.524	-1	1	-1.000-8.383e-5i	1.000-3.207e-5i	0.1557+0.009738i	0.1575-0.00354i
3	1754.29	167.604	1	-1	1.000+5.976e-5i	-1.000+2.568e-5i	0.1727+0.01159i	0.1718-0.004767i
4	1754.29	157.576	1	-1	1.000+7.517e-5i	-1.000+3.907e-5i	0.1591+0.009931i	0.1596-0.003963i

Table 3.2: Cases for S and F mode superposition and corresponding COP for $\beta = 10^{-7}$ (adjusted from ref. [1]).

In this section, the biorthogonal decomposition method is validated. The solution to the linearized Navier-Stokes equation is formulated by the combination of eigenfunctions of modes F and S. The solution $\mathbf{A}$ in eq. (2.38) can be obtained as following

$$\mathbf{A} = C_S \mathbf{A}_{S\alpha\beta} + C_F \mathbf{A}_{F\alpha\beta}, \quad (3.1)$$

where C_S and C_F are the prescribed COP of the eigenfunctions of mode S and F, respectively. $\mathbf{A}_{S\alpha\beta}$ and $\mathbf{A}_{F\alpha\beta}$ are the eigenfunctions of the mode S and F, respectively. Four different cases have been considered to validate the projection. The Reynolds numbers and frequencies are provided in table 3.2 for different cases. The phase speeds of the cases are provided in fig. 3.9. Cases 2 and

4 are taken before synchronization and cases 1 and 3 are taken after synchronization. Figs. 3.10a-d show the streamwise eigenfunctions and superposition of S and F mode. Table 3.2 lists the COP after projecting the solution $\mathbf{A}$ on F and S modes. The coefficient of projections are in close agreement to the prescribed values, C_F and C_S , for the F and S modes.

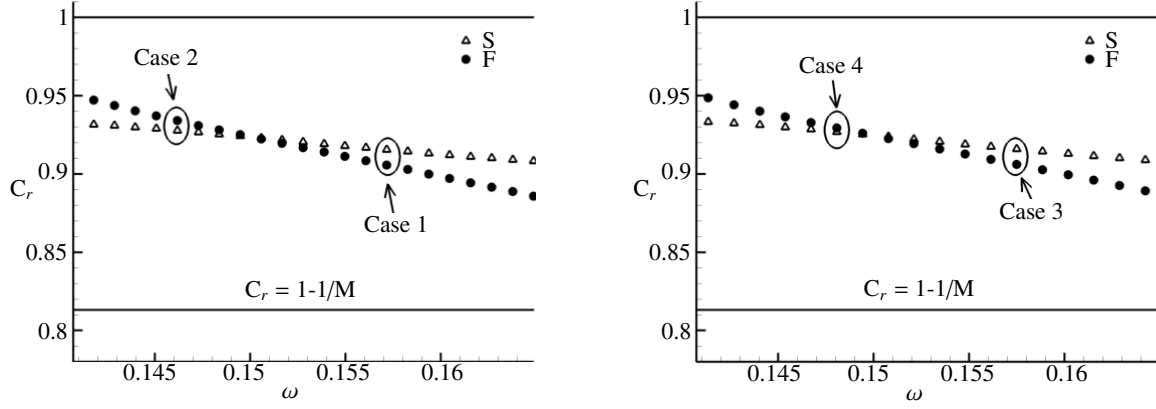


Figure 3.9: Phase speed of different cases (adjusted from ref. [1]).

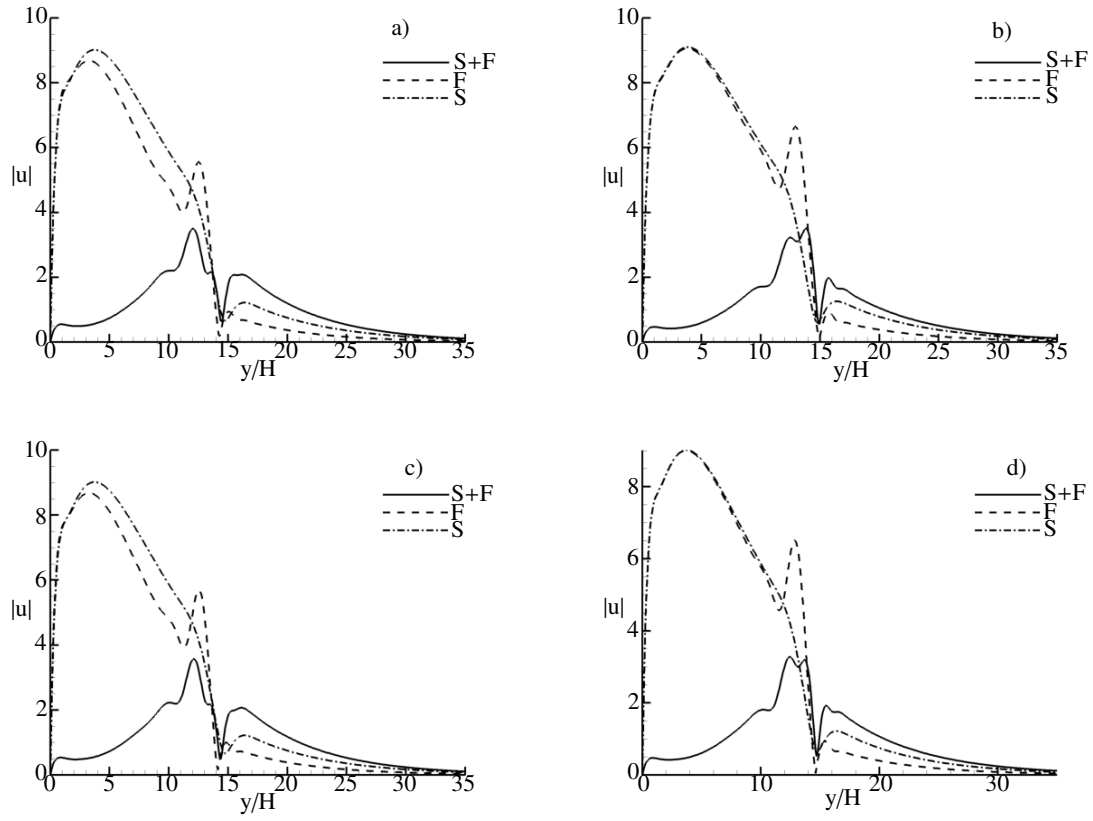


Figure 3.10: Streamwise eigenfunctions and superposition of S and F mode for different cases a) case 1, b) case 2, c) case 3 and d) case 4 (adjusted from ref. [1]).

Chapter 4: Disturbance Flow Field Generated by Particle Impingement

In this section, the particle induced disturbance flow fields simulated by following the PSIC approach are discussed for the Mach 5.35 flat plate, Mach 4 14° wedge and Mach 9.5 14° wedge. The disturbance flow field is studied in the frequency domain to understand the spectral characteristics of the flow field. BOD is applied to study the most dominant and other stable modes inside the boundary layer. The disturbance field in the freestream is also studied by identifying the contributions from the stable modes. Finally, the disturbance flow field is reconstructed from the contributions of the discrete and continuous eigensolutions of the boundary layer.

4.1 Properties of the Disturbance Spectrum

The disturbance spectra of the flow field are associated with the eigenvalue spectra of the flow field. The eigenvalue spectrum contains modes from the discrete and continuous branches [33]. Fig. 4.1a shows the modes S and F from the discrete branch as well as vorticity, entropy, fast acoustic (FA) and slow acoustic (SA) modes from the continuous branch. In three dimensions the vorticity branch again has two sub categories, branches A and B. The vorticity and entropy branches are distinct even though they seem to situate together in the eigenvalue spectrum [33]. The naming convention of the discrete modes emerges from the behavior of these modes. For

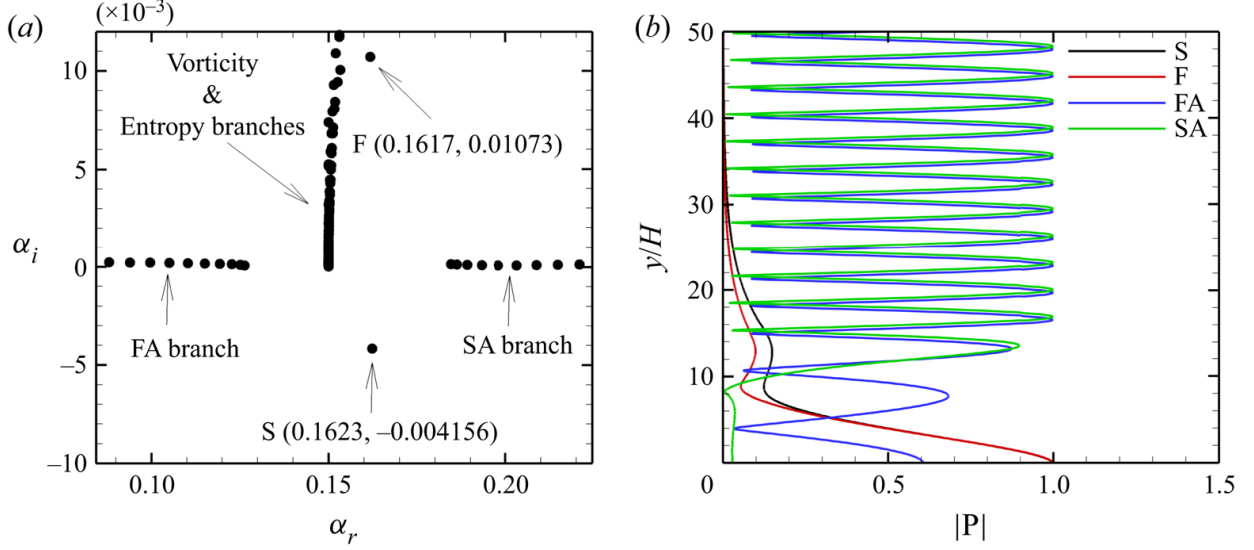


Figure 4.1: (a) Eigenvalue spectrum with modes from the discrete and continuous branches. (b) eigenfunctions with continuous spectra parameter, $k = 1$, for the discrete and continuous modes in $M=5.35$ flat plate for $Re = 1500$, $F = 10^{-4}$ and $\beta = 10^{-7}$ (adjusted from ref. [2]).

example, discrete mode F synchronizes with the fast acoustic (FA) mode at low frequency and low Re , so it is called the F mode. A similar property of the discrete mode S is observed as it synchronizes with the slow acoustic (SA) mode at low frequency and low Re . In 2-D, The phase speed of SA and FA modes are $1 - 1/M$ and $1 + 1/M$, respectively [56]. The phase speed of SA and FA modes are $1 - \sqrt{(1 + \beta^2/\alpha_r^2)}/M$ and $1 + \sqrt{(1 + \beta^2/\alpha_r^2)}/M$ in three dimensions, respectively. Fig. 4.1b shows the eigenfunctions for the discrete and continuous modes. The peak of the discrete modes resides inside the boundary layer and it rapidly decays outside. In general, the peak of the continuous mode is found oscillating outside the boundary layer.

The synchronization of the discrete modes can give rise to an unstable or stable mode. This phenomenon is particularly important in high-speed flows. At high-speed, the synchronization of mode F and S at high Re and frequency can give rise to the unstable mode S which is also known as second mode or second Mack mode. More details of the synchronization method of the discrete modes are outlined in section 4.2.3.3. Mode synchronization happens due to the

non-parallel nature of the mean flow [2, 34, 57].

4.2 Disturbance Flow Field in a Mach 5.35 Flat Plate

4.2.1 Particle Induced Disturbance Flow Field

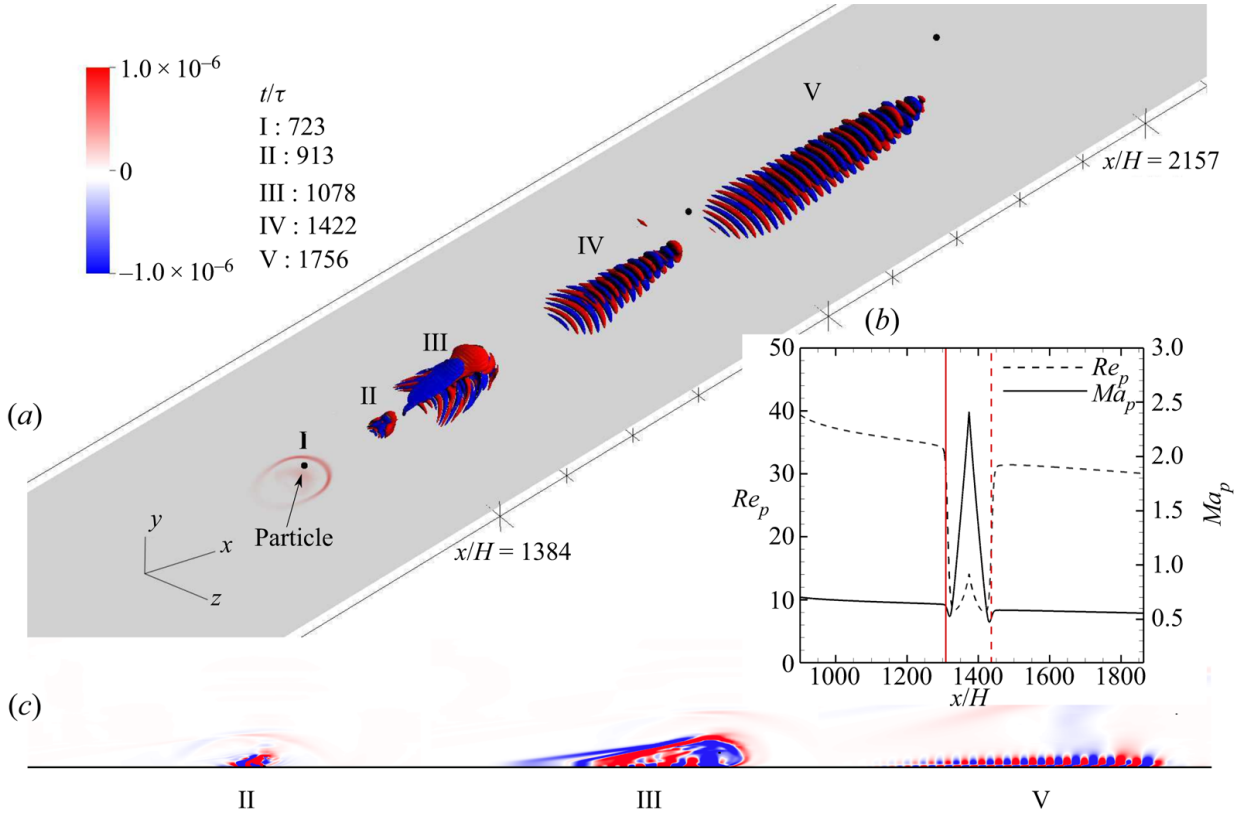


Figure 4.2: (a) Pseudocolor plot of disturbance pressure in a Mach 5.35 flat plate at five different times: (I) before impingement (II) near impingement and (III,IV,V) after impingement (b) particle Reynolds number and Mach number, red solid vertical line marks the entry of the particle to the boundary layer, red dashed vertical line marks the exit of the particle from the boundary layer. (c) disturbance pressure at the symmetry cut plane at three time instances (adjusted from ref. [2]).

The flow past a Mach 5.35 flat plate has been considered to study the transition process initiated by the particle impingement. This disturbance flow field has been studied previously for a different kind of disturbance source, *i.e.* pulse disturbance, in refs. [40, 41]. The freestream and particle properties are provided in table 4.1.

M_∞	$Re_\infty(1/m)$	$p_\infty(N/m^2)$	$T_\infty(K)$	$T_w(K)$	$U_\infty(m/s)$	$R_p(\mu m)$	σ
5.35	13.678×10^6	1.298×10^3	63.923	300	857.633	10	$5R_p$

Table 4.1: Flow field and particle properties in a flat plate boundary layer.

A particle of radius $R_p/H_{pc} = 0.1$ and density $1000kg/m^3$ is considered. The subscript pc denotes the properties at the particle impingement location. The particle is introduced at $\mathbf{x}/H \approx (810.3, 180.6, 0)^T$ with a velocity of $\mathbf{v}_p = (U_\infty \cos \theta_p, U_\infty \sin \theta_p)^T$ and at an angle of $\theta_p = -7^\circ$. The particle collides with the wall at $x_{pc}/H = 1374$. The particle bounces off the wall after an elastic collision with the wall ($\alpha=1$). The computational domain has dimensions $[0, 2865H]$ in the streamwise direction, $[0, 477H]$ in the transverse direction and $[0, 2865H]$ in the spanwise direction. The simulation is performed for half of the flow field ($z > 0$) assuming spanwise symmetry at $z = 0$. A total of nine AMR grid levels are considered with grid spacing $\Delta y = 0.0234H$ in the transverse direction and $\Delta x = \Delta z = 10\Delta y$ in the remaining two directions.

Fig. 4.2a shows the disturbance pressure field at five time instances before, near and after particle impingement. The particle impingement introduces disturbance which triggers a convective wavepacket. The wavepacket grows in size and travels in the downstream direction. This wavepacket is dominated by second mode waves. Fig. 4.2b shows the particle Mach number, Ma_p , and Reynolds number, Re_p . Re_p lies within the range of 10 to 40. The Re_p reduces when the particle enters the boundary layer. The particle Mach number lies within the range of 0.5 to 2.5. The flow around the particle remains subsonic before the particle enters the boundary layer. The particle reaches supersonic speed inside the boundary layer and a shock forms ahead of the particle. Fig. 4.2c shows the disturbance pressure in a cut plane, $z = 0$ at three time instances.

First, the wall disturbance pressure is Fourier transformed in the spanwise direction to plot the space-time diagram. Figs. 4.3a and b illustrate the space-time diagram at $\beta = 0$ and

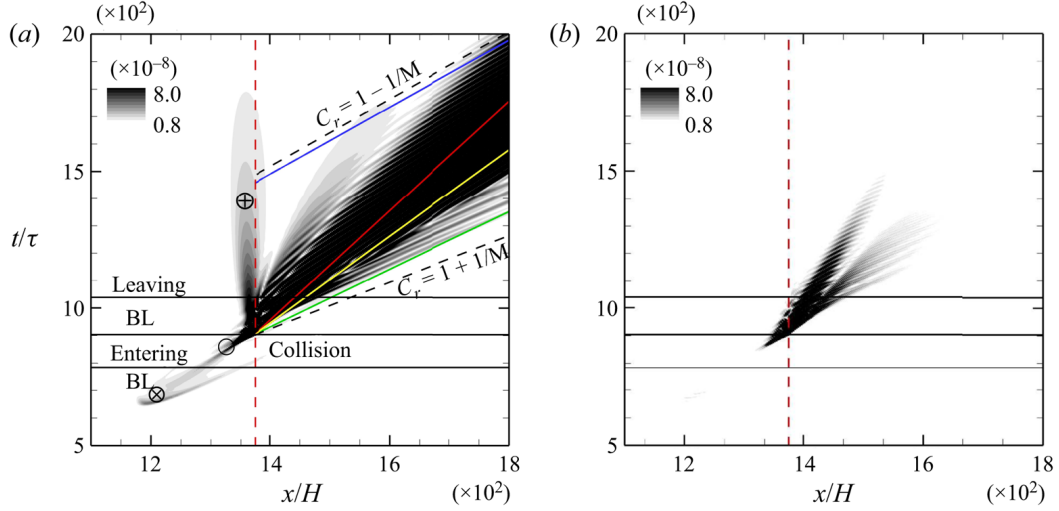


Figure 4.3: Space-time plot of disturbance wall pressure at different planes (a) $\beta = 0$ and (b) $\beta_{x/H=1374} = 0.28$. Red vertical dashed line marks the particle impingement location. The time instances of particle entering, colliding and leaving the boundary layer are marked as solid black horizontal lines. $\otimes$ marks acoustic waves entering the boundary layer before particle, $\circ$ marks the shock wave at the wall, $\oplus$ marks the reflected acoustic waves. The solid lines in (a) correspond to, blue - minimum phase speed with $C_{r_{min}} = 0.82$, green - maximum phase speed with $C_{r_{max}} = 0.95$, red - minimum group velocity with $C_{g_{min}} = 0.5$ and yellow - maximum group velocity with $C_{g_{max}} = 0.63$ (adjusted from ref. [2]).

$\beta = 0.28$. The time instances of the particle entering the boundary layer, colliding with the wall, and leaving the boundary layer are marked by black horizontal lines. Even before the particle enters the boundary layer there is wall pressure disturbance caused by the acoustic wave formed in front of the particle, which is marked as $\otimes$ in the space-time diagram. The strong wall pressure fluctuation before the particle impingement is marked by $\circ$ in the space-time diagram in fig. 4.3a. This strong pressure fluctuation is caused by the shock front formed in front of the particle when it enters the boundary layer and becomes supersonic. The space-time diagram at $\beta = 0.28$ shows higher disturbance pressure in the vicinity of the particle impingement. However, the disturbance pressure decays at $\beta > 0$ spanwise wavenumbers. The disturbance pressure can be bounded by the fastest traveling disturbance wave $C_r = 1 + 1/M$. The maximum and minimum group velocities are $C_{g_{max}} = 0.63$ and $C_{g_{min}} = 0.5$ and marked as yellow and red lines in fig. 4.3a, respectively. The

maximum, $C_{r_{max}}$, and minimum, $C_{r_{min}}$, phase speeds are marked as green and blue solid lines with values $C_{r_{max}} = 0.95$ and $C_{r_{min}} = 0.82$.

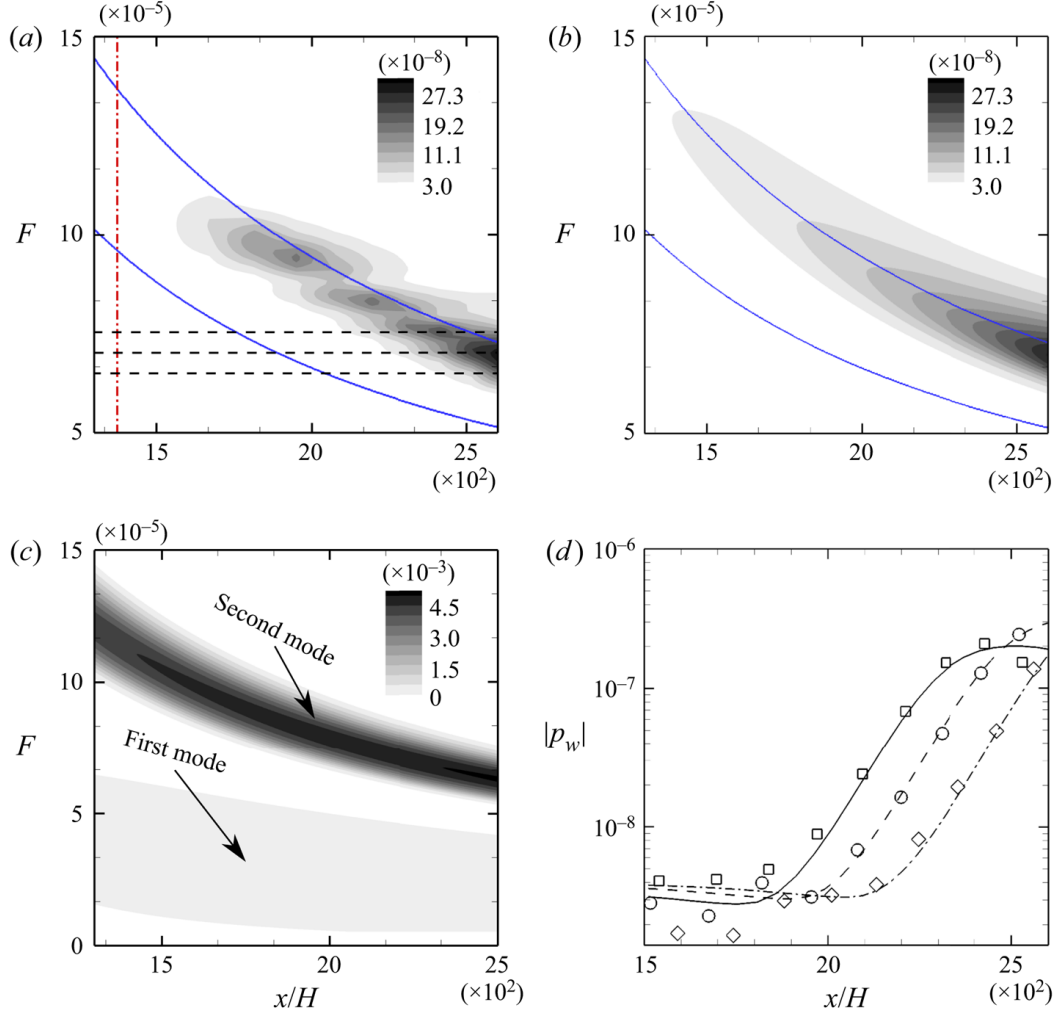


Figure 4.4: (a) FFT amplitude plot at $\beta = 0$, horizontal dashed lines mark the frequencies for comparison, vertical red dashed line marks the position of particle impingement, (b) amplitude plot from LST analysis, (c) growth rate ($-\alpha_i$) plot, (d) Comparison of FFT with LST amplitude for different frequencies, $\square$, —: $F = 7.543 \times 10^{-5}$, $\circ$, ---: $F = 7.023 \times 10^{-5}$ and $\diamond$, -.-.-: $F = 6.502 \times 10^{-5}$. Symbols and lines represent FFT and LST data, respectively. Blue lines mark the neutral curves for second mode (adjusted from ref. [2]).

Next, the disturbance flow field is studied in the frequency domain by performing FFT in the spanwise direction and in time. Fig. 4.4a shows the amplitude plot of wall disturbance pressure at $\beta = 0$. The second mode neutral curve is included as solid blue lines. The growth

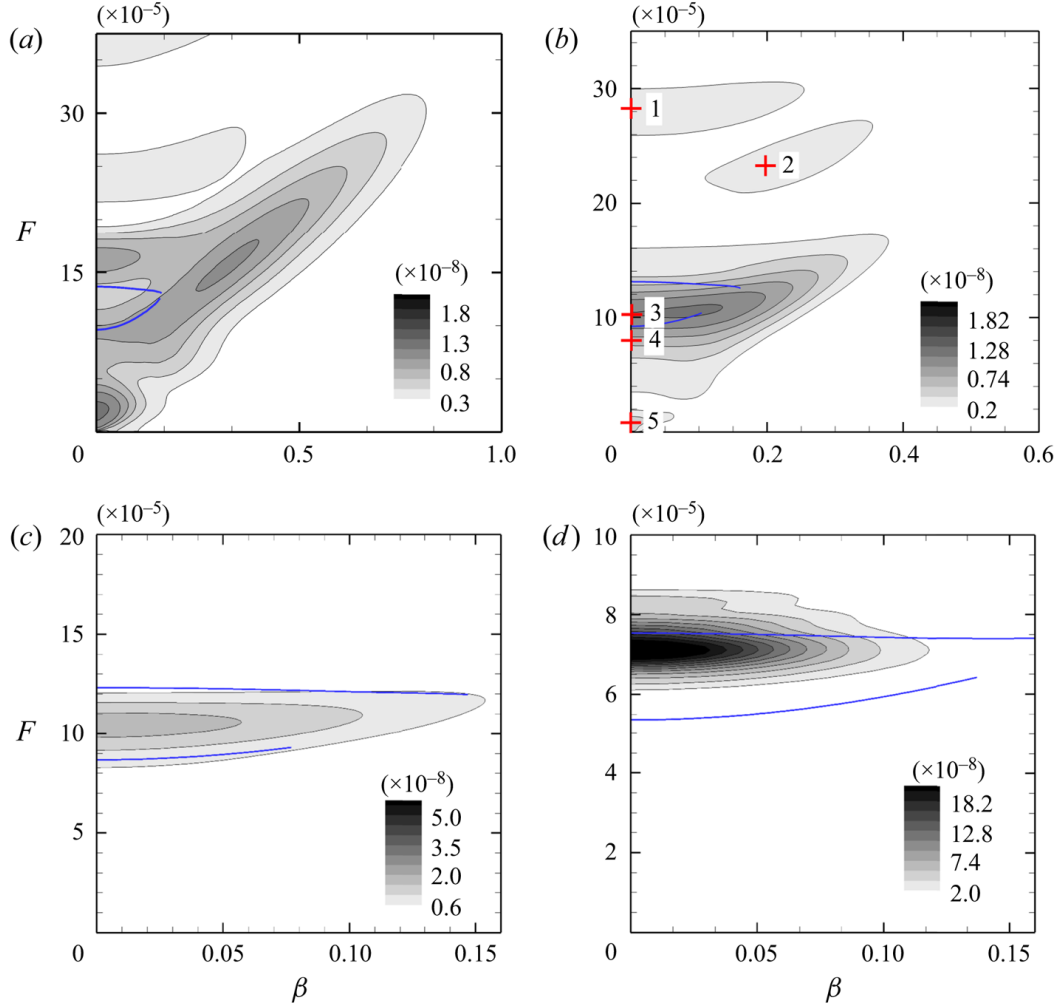


Figure 4.5: Wall pressure FFT in $F - \beta$ plane for (a) $x_{pc}/H = 1374$, (b) $x/H = 1432$, (c) $x/H = 1525$ and (d) $x/H = 2517$. Red "+" marks the cases considered for reconstruction in section 4.2.3.3, blue curves mark the neutral curve for second mode (adjusted from ref. [2]).

rate plot is shown in fig. 4.4c. The plot suggests that there are first mode waves in addition to the second mode. However, the growth rate of second mode is higher than the first mode (where the first mode is more amplified for 3-D disturbance). Fig. 4.4b shows the scaled version of the amplitude plot from the LST. The peak occurs within the second mode region as can be observed from both figs. 4.4a and 4.4b. Amplitude curves are extracted at three different frequencies, $F = 7.543 \times 10^{-5}$, $F = 7.023 \times 10^{-5}$ and $F = 6.502 \times 10^{-5}$ and plotted in fig. 4.4d. The curves from the FFT and LST prediction are in close agreement inside the unstable second mode region.

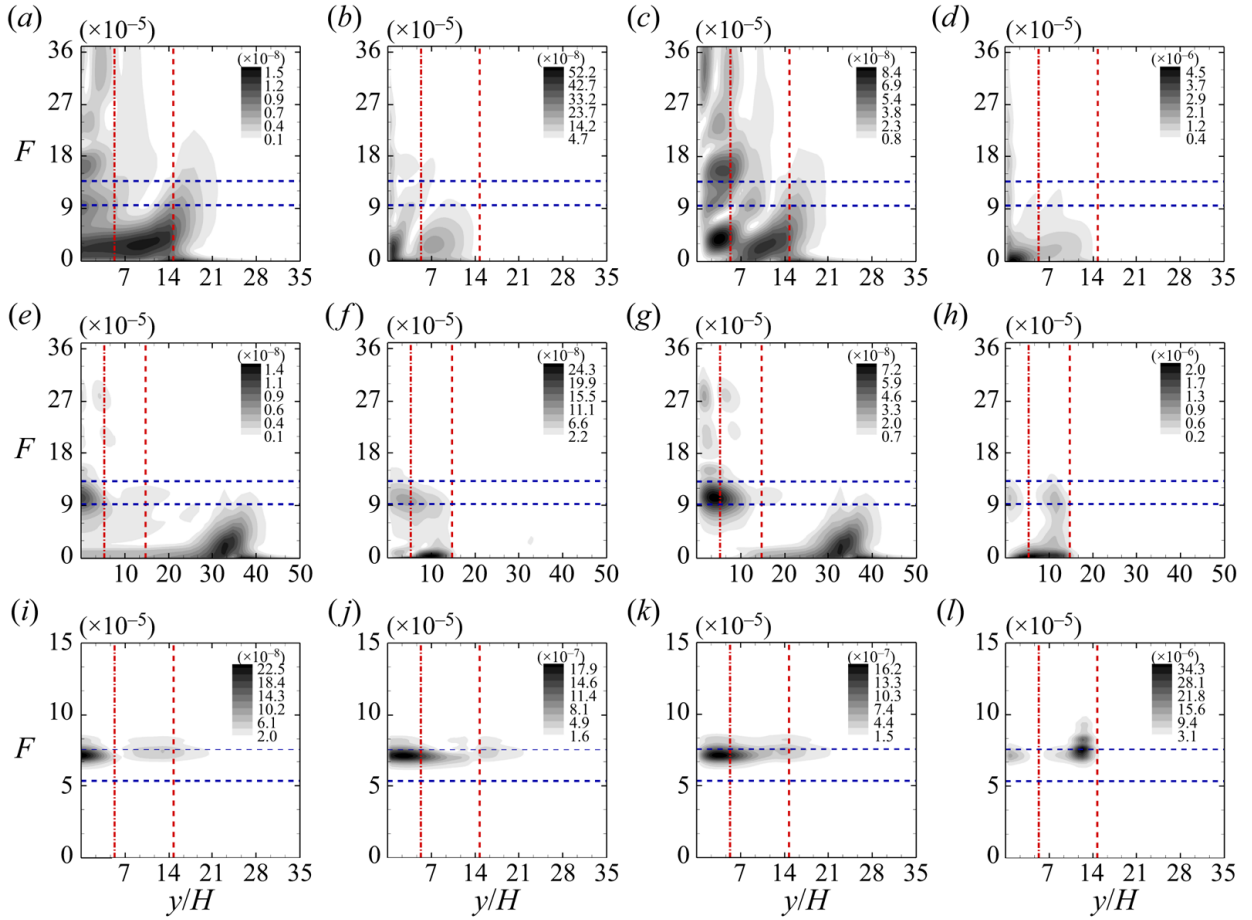


Figure 4.6: Disturbance flow frequency spectra in wall-normal direction at $\beta = 0$ for (a) $|p|, x_{pc}/H = 1374$; (b) $|u|, x_{pc}/H = 1374$, (c) $|v|, x_{pc}/H = 1374$, (d) $|T|, x_{pc}/H = 1374$, (e) $|p|, x/H = 1432$; (f) $|u|, x/H = 1432$, (g) $|v|, x/H = 1432$, (h) $|T|, x/H = 1432$, (i) $|p|, x/H = 2517$, (j) $|u|, x/H = 2517$, (k) $|v|, x/H = 2517$, (l) $|T|, x/H = 2517$. The horizontal blue dashed lines mark the neutral curve for second mode, the vertical red dashed-dotted line marks the position of the sonic line, the vertical red dashed line marks the boundary-layer height (adjusted from ref. [2]).

However, near the particle impingement location there are differences between the FFT and LST prediction. This suggests that near particle impingement there are other modes from the discrete and continuous spectra in addition to the second mode that contribute to the disturbance flow field.

Next, the disturbance flow field is studied in the $F - \beta$ plane for four different streamwise directions. Figs. 4.5a-c show the frequency spectra in the $F - \beta$ plane for $x_{pc}/H = 1374$, $x/H =$

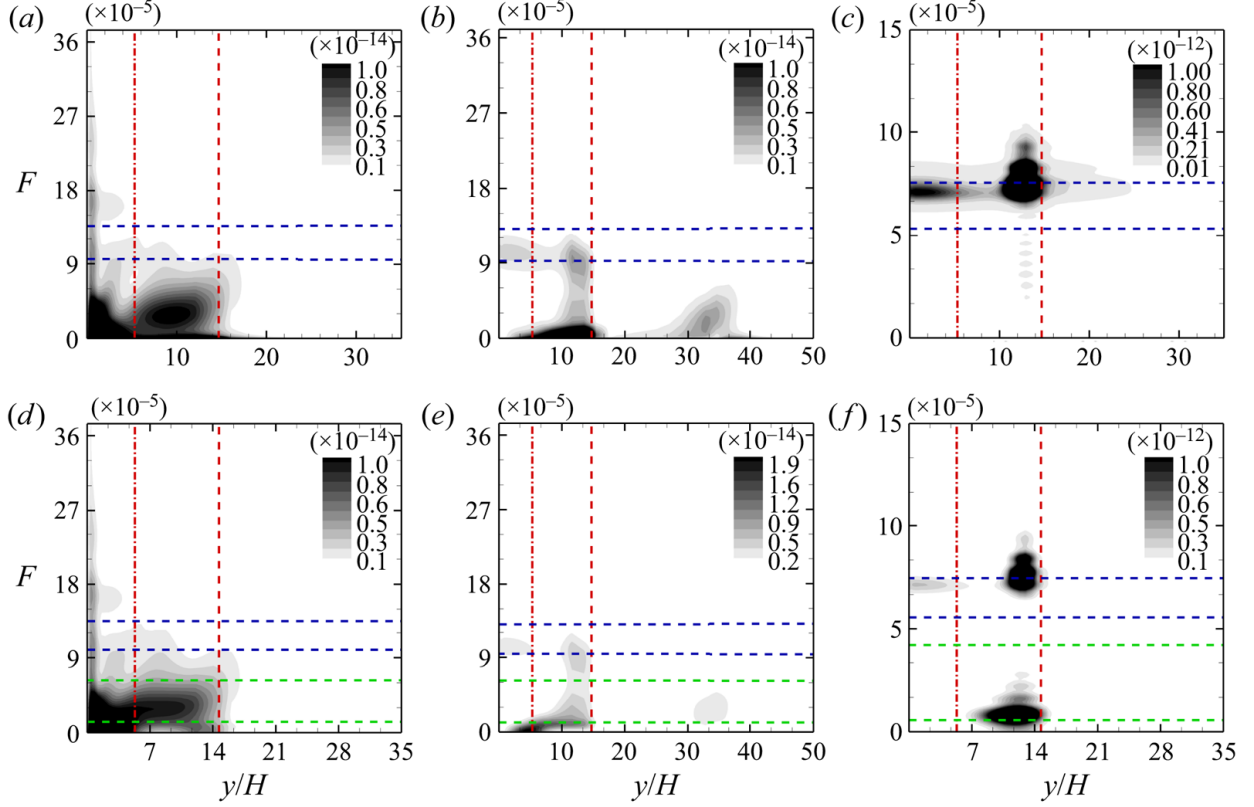


Figure 4.7: Disturbance energy frequency spectra at (a) $x_{pc}/H = 1374, \beta = 0$, (b) $x/H = 1432, \beta = 0$, (c) $x/H = 2517, \beta = 0$, (d) $x_{pc}/H = 1374, \beta = 0.055$, (e) $x/H = 1432, \beta = 0.05$, (f) $x/H = 2517, \beta = 0.058$. Blue dashed lines mark the second mode frequency range, red dashed-dotted line marks the sonic line, red dashed line marks the boundary layer height and the green dashed lines mark the first mode frequency range (adjusted from ref. [2]).

1432, $x/H = 1525$ and $x/H = 2517$. The particle introduces disturbances with a wide range of frequencies and wavenumbers at the impingement location as shown in fig. 4.5a. The second mode neutral curve is included as blue solid lines. As the disturbances travel downstream the peak is found within the second mode region near branch II. This agrees with the LST that the second mode dominates the flow field far from the impingement location. Five cases (marked in fig. 4.5b) will be considered later to fully reconstruct the disturbance flow field from the different eigenmodes from the discrete and continuous branches.

The disturbance flow field FFT is plotted in the $F - y$ plane to study the flow field in the

wall-normal direction. Figs. 4.6a-l show disturbance pressure, streamwise velocity, wall-normal velocity and temperature at three locations, $x_{pc}/H = 1374$, $x/H = 1432$ and $x/H = 2517$. At the particle impingement location, the peak of the frequency spectrum occurs at lower frequencies, $F < 9 \times 10^{-5}$ and the peak is not found at the dominant second mode frequency range at this location. A transient growth mechanism may be observed with similar characteristics as can be observed in the streamwise velocity and temperature disturbances. A more detailed study of particle induced non-modal growth mechanism can be found in ref. [58] which is also consistent with the results of ref. [59]. Near to the impingement location at $x/H = 1432$, a peak starts to appear within the second mode frequency range. Also, an amplitude peak is found outside the boundary layer at this location. This peak is associated with the acoustic waves traveling outside the boundary layer and is studied in detail later in section 4.2.3.3. At a later downstream location at $x/H = 2517$, the peak of the frequency spectrum is contained within the second mode frequency range. The peak for the pressure, streamwise velocity and wall-normal velocity is found below the sonic line and peak for the temperature is found above the sonic line but below the boundary layer height. At this location the second mode has become the dominant mode of the disturbance flow field.

An energy norm is considered which is based on the five flow parameters, *i.e.* pressure, streamwise velocity, wall-normal velocity, spanwise velocity and temperature. This way a scalar quantity is obtained which depends on individual contributions of the different flow parameters.

The energy norm from Chu [60] is adopted here

$$E_{dist} = \frac{1}{2} \bar{\rho} (\langle \check{u}, \check{u} \rangle + \langle \check{v}, \check{v} \rangle + \langle \check{w}, \check{w} \rangle) + \frac{1}{2} \frac{\bar{a}^2 \langle \check{\rho}, \check{\rho} \rangle}{\gamma \bar{\rho}} + \frac{1}{2} \frac{\bar{\rho} c_v \langle \check{T}, \check{T} \rangle}{\bar{T}}, \quad (4.1)$$

where $\bar{\rho}$ and $\bar{T}$ are the baseflow density and temperature, respectively. $\check{\rho}, \check{T}, \check{u}, \check{v}$ and $\check{w}$ are the FFT of the density, temperature, streamwise velocity, wall-normal velocity and spanwise velocity, respectively. c_v and $\bar{a}$ are evaluated as $c_v = [\gamma(\gamma - 1)M^2]^{-1}$ and $\bar{a}^2 = \bar{T}/M^2$, respectively. $\langle \cdot, \cdot \rangle$ denotes the inner product such as $\langle \check{u}, \check{u} \rangle \equiv (\check{u}^* \check{u} + \check{u} \check{u}^*)/2$, and $(\cdot)^*$ represents the complex conjugate. This energy norm is also known as ‘‘Mack norm’’ in the literature [2].

Figs. 4.7a-f show the energy spectra at three streamwise locations, $x_{pc}/H = 1374$, $x/H = 1432$ and $x/H = 2517$. Two different spanwise wavenumbers are considered for each location for the energy spectra. At the impingement location and $\beta = 0$, the peak is found at low frequency within the boundary layer. A similar observation can also be made for $\beta > 0$. At a position $x/H = 1432$, slightly downstream, a peak appears in the second mode frequency range and another peak is found outside the boundary layer which is associated with the acoustic wave as already discussed before. At a further downstream location at $x/H = 1432$, the peaks are contained within the second mode frequency range at $\beta = 0$. At $\beta = 0.058$, a peak of the energy spectra is also observed at a lower frequency range of the first mode. The first mode frequency range is marked by green dashed line.

4.2.2 Comparison with Pulse Disturbance

In this section, the particle induced disturbance flow field is compared with the disturbance flow field generated by the pulse disturbance. The pulse is introduced into the flow field via a blowing-suction slot as wall forcing [41]:

$$\tilde{W}_F(x, t) = H(\mathbf{x})\Lambda(t), \quad (4.2)$$

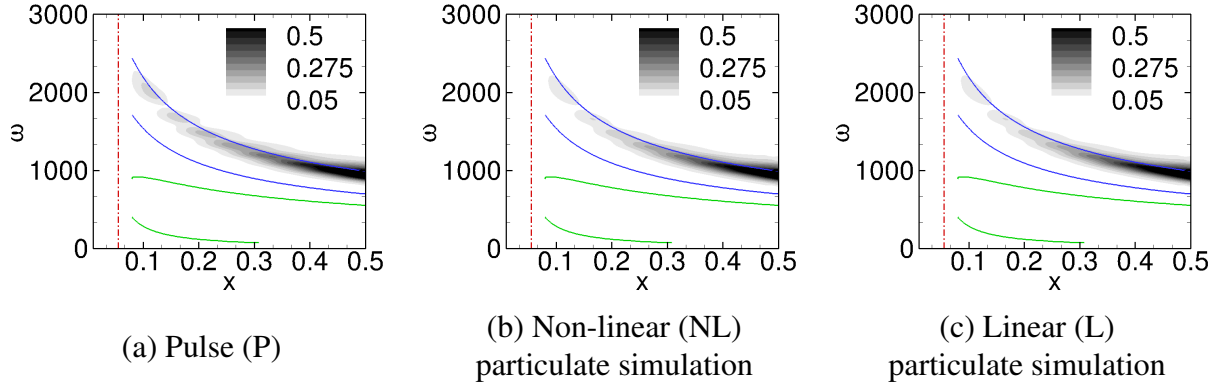


Figure 4.8: Amplitude plots normalized by maximum at $x = 0.5m$ for the $M=5.35$ flat plate (x in meter). Vertical dashed line marks the particulate impingement location, $x_{pc} = 0.055m$ (adjusted from ref. [8]).

where the $H(\mathbf{x})$ is the disturbance shape and $\Lambda(t)$ is the discrete delta function. The disturbance shape $H(\mathbf{x})$ is as following [41]

$$H(\mathbf{x}) = \cos(\pi x_l)^3 \cos(\pi z_l)^3, \quad (4.3)$$

where x_l and z_l are the disturbance shapes in streamwise and spanwise directions, respectively. x_l and z_l can be written as in [41]

$$x_l = \frac{x - 0.5(x_s + x_e)}{x_e - x_s}; \quad z_l = \frac{z}{z_e - z_s}. \quad (4.4)$$

The subscripts “s” and “e” denote the starting and end position of the pulse. The discrete delta function is given by

$$\Lambda(t) = \begin{cases} \hat{\Lambda} \sin(-\omega t) & t < T_f, \\ 0 & t > T_f, \end{cases} \quad (4.5)$$

where T_f is the forcing period which is taken as $3.9 \times 10^{-6} s$ and $\hat{\Lambda}$ is the pulse amplitude which is $1 \times 10^{-2} U_\infty$ for the current simulations.

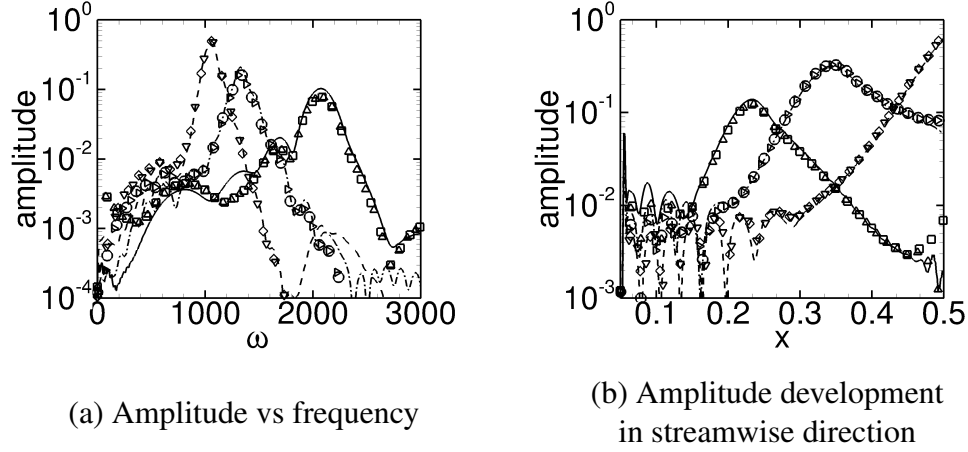


Figure 4.9: Amplitude profiles for fig. 4.8 pulse (P), non-linear (NL), and linear (L) flat plate-particulate interaction simulations (x in meter). $\parallel x=0.1/\omega = 1400$: P — ; NL $\square$; L $\triangle$ $\parallel x=0.25/\omega = 1150$: P - - - -; NL $\circ$; L $\triangleright$ $\parallel x=0.4/\omega = 900$: P - - -; NL $\diamond$; L ∇ $\parallel$ (adjusted from ref. [8]).

Figs. 4.8a-c show the amplitude plots of the pulse disturbance simulation, non-linear simulation and linear simulation of particle impingement, respectively. The particle impinges at $x=0.055m$ which is marked as a red dashed line in the amplitude plot. The impingement location is chosen such that it impinges upstream of the second mode neutral curve. The first and second mode neutral curves are included as green and blue solid lines respectively. It should be noted here that the amplitude plots are normalized based on the peak value at $x=0.5m$. This way the pulse disturbance is directly compared with the particle impingement simulations. The frequency is calculated as $\omega = 2\pi f$, where f is in Hz. The most amplified region is within the second mode neutral curve. The peak of the amplitude plot is found near to the second branch of the neutral curve. Near to the impingement location some differences can be attributed to the different receptivity process. After the initial transient period when the receptivity is complete, the amplitude plots show similar behavior. Fig. 4.9a shows the amplitude development in frequency

domain at three streamwise locations, $x=0.1m$, $x=0.25m$ and $x=0.4m$. A close agreement is found at the most amplified regions between the pulse simulation and particle impingement simulation. Fig. 4.9b shows the amplitude curves for different frequencies, $\omega = 1400, 1150$ and 900 . The amplitude curves are in close agreement in the amplified regions. In addition, the good agreement between the linear and non-linear simulations suggests that the particle introduces small scale disturbances, which can be simulated by considering the linear disturbance equations (LDE).

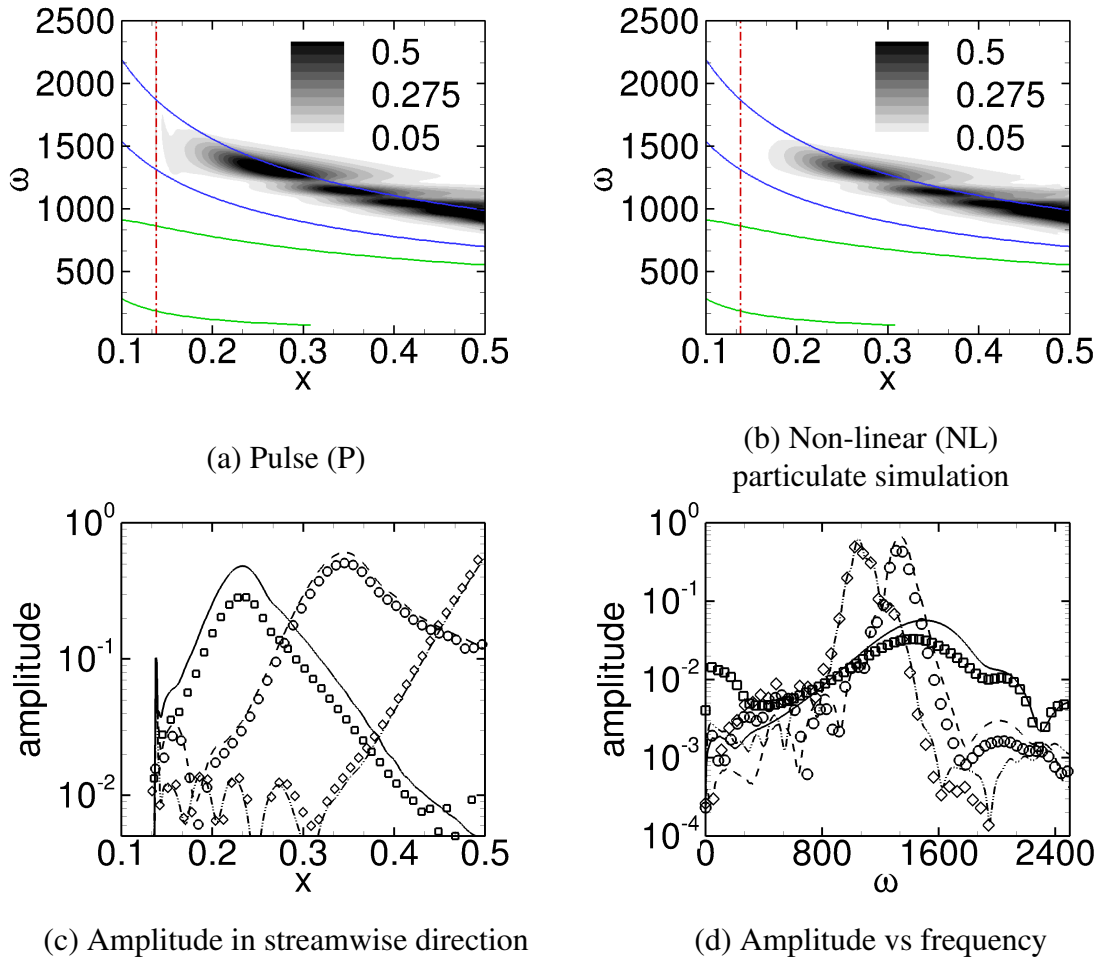


Figure 4.10: Amplitude plots (a), (b) and profiles (c), (d) (normalized by peak at $x=0.5m$, x in meter). Vertical dashed line marks impingement location, $x_{pc} = 0.138m$; $\parallel x=0.15/\omega = 1400$: P —; NL $\square$ $\parallel x=0.25/\omega = 1150$: P - - -; NL $\circ$ $\parallel x=0.4/\omega = 900$: P - · - · - · -; NL $\diamond$ $\parallel$ (adjusted from ref. [8]).

Next, the simulations are performed for the particle impinging further downstream inside the neutral curve at $x=0.138\text{m}$. This position is chosen to introduce particle induced disturbances well inside the unstable flow region. Figs. 4.10a and 4.10b show the amplitude plots for the pulse disturbance and particle impingement, respectively. The first and second mode neutral curves are added as green and blue solid lines, respectively. The amplitude is normalized by the peak value at $x=0.5\text{m}$. The most amplified region is within the second mode neutral curve for both the simulations. Fig. 4.10c shows the amplitude development in the streamwise direction for three frequencies, $\omega = 1400, 1150$ and 900 . Fig. 4.10d shows the amplitude versus frequency plots at three streamwise locations, $x=0.15\text{m}$, 0.25m and 0.4m . The results suggest that the particle induced disturbance flow field is slightly different than the disturbance flow field generated by the pulse near the disturbance source. Overall, the particle impingement simulations can be imitated by a carefully designed pulse disturbance for the particle impinging upstream of the neutral curve. There are differences between the two simulation approaches for the particle impinging at a further downstream location within the second mode neutral curve.

Finally, the disturbance flow field induced by the particle is compared with the disturbance flow field generated by the pulse disturbance in three dimensions. The particle impinges at $x=0.138\text{m}$ and the pulse is introduced at the same location. The computational domain extends over $[0\text{m}, 0.6\text{m}]$ in the streamwise direction, $[0\text{m}, 0.1\text{m}]$ in the wall-normal direction and $[0\text{m}, 0.6\text{m}]$ in the spanwise direction. The AMR simulations have nine levels. The finest level is discretized with $\Delta x = \Delta z = 2.5 \times 10^{-5}\text{m}$ and $\Delta y = 5 \times 10^{-6}\text{m}$. The particle is assumed to have an elastic collision with the surface of the flat plate. The pulse disturbance is considered with amplitude of $\hat{\Lambda} = 1 \times 10^{-5}U_\infty$ and a forcing period of $T_f = 3.19 \times 10^{-6}\text{s}$. More details about the pulse disturbance can be found in ref. [50].

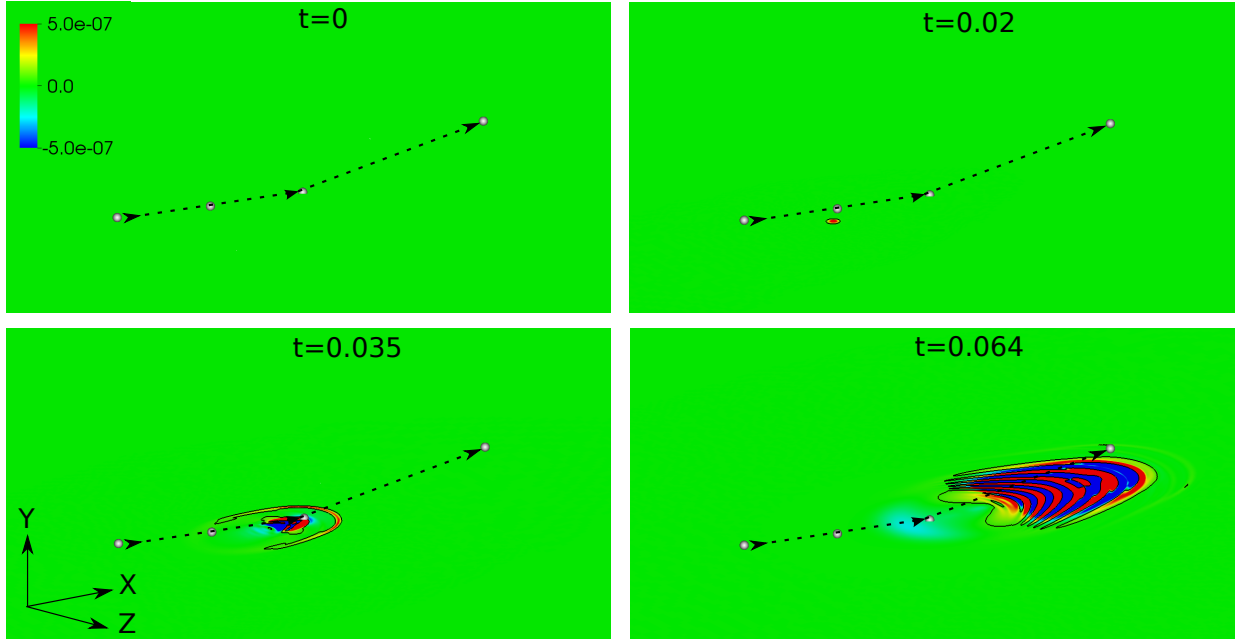


Figure 4.11: Instantaneous visualization of particulate trajectory and upstream collision (wall pressure disturbance $\tilde{p}/(\rho_{\infty}U_{\infty}^2)$) with 3-D $M=5.35$ flat plate at four time instances. Isoline at 5×10^{-8} . Time (t) normalized by L/U_{∞} (adjusted from ref. [8]).

Fig. 4.11 shows the initial stages of the disturbance pressure flow field and the particle trajectory at four time instances. The particle introduces disturbances inside the boundary layer which give birth to a wavepacket. The wavepacket grows in size and propagates in the downstream direction. Figs. 4.12a-f show the disturbance spectra in the frequency-wavenumber plane for the particle induced disturbance pressure field and the disturbance pressure field generated by the pulse disturbance at wall. Near to the particle impingement at $x=0.14\text{m}$ in fig. 4.12a a wide range of frequencies and wavenumbers get excited unlike the pulse disturbance in fig. 4.12b. Amplitude curves are extracted at $\beta = 0$ and 1000 in fig. 4.12g. There are differences between the amplitude curves extracted from the disturbance spectra of particle impingement and pulse disturbance. Near the particle impingement the disturbance spectra clearly shows differences. As the wavepacket travels downstream the second mode dominates the flow field as expected from LST. The peak in the disturbance spectrum shifts to a lower frequency with an inverse relationship

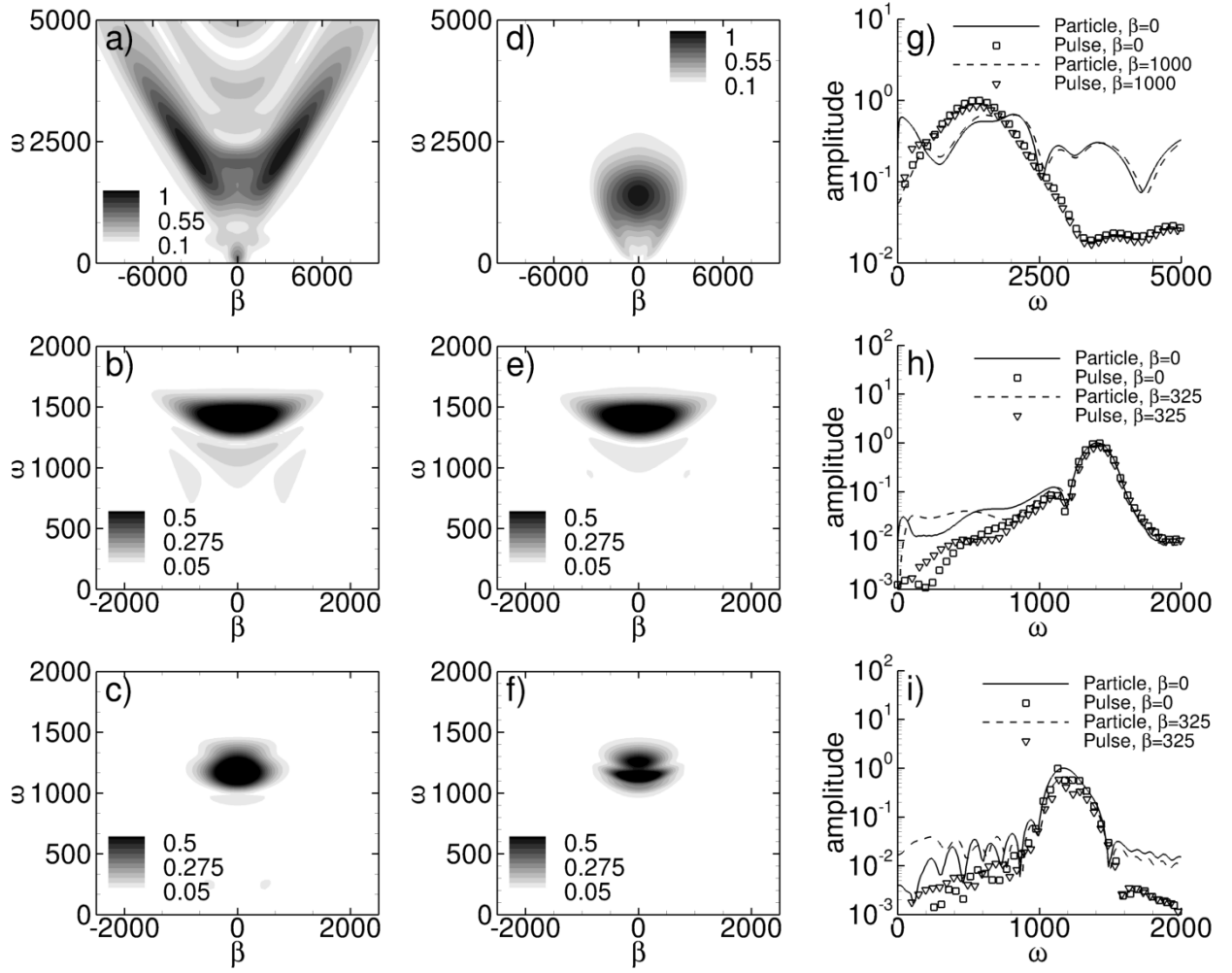


Figure 4.12: Amplitude plots: a),b),c) - particulate impingement; d),e),f) - pulse. Amplitude vs frequency plots: g),h),i) a),d),g) - $x=0.14\text{m}$; b),e),h) - $x=0.2\text{m}$; c),f),i) - $x=0.33\text{m}$. Note that amplitude is normalized by peak value (adjusted from ref. [8]).

with the increasing boundary-layer height. Figs. 4.12b and 4.12e display the disturbance spectra of the particle induced disturbance pressure field and pulse generated disturbance pressure field at $x=0.2\text{m}$, respectively. The peak is found within the same frequency range for both the particle and pulse simulations. Fig. 4.12h shows the amplitude curves extracted at $\beta = 0$ and 325. The amplitude curves show similar behavior in the second mode frequency range. However, there are differences between the particle and pulse simulation at lower frequencies. Similarly, at a position further downstream, $x=0.33\text{m}$, the disturbance spectra are in good agreement within the second

mode frequency range in figs. 4.12c, 4.12f and 4.12h. Nevertheless, there are differences at higher and lower frequencies as shown in fig. 4.12i. Overall, the disturbance flow field induced by particle impingement is different from the disturbance flow field generated by pulse disturbance near to the disturbance source and these simulations show some similarity in the second mode exponential growth regime.

4.2.3 Biorthogonal Decomposition of the Disturbance Flow Field

Biorthogonal decomposition (BOD) can elucidate the receptivity process during particle impingement. The contribution of the discrete and continuous modes in disturbance flow generation is explained in detail in this section. The BOD is performed by employing the orthogonality relation in eq. (2.38) to retrieve the amplitude of the different modes from the discrete and continuous branches. The disturbance flow field is first projected onto the most unstable mode S to study the disturbance flow field inside the boundary layer. Then the projections on continuous branches are performed to explain the disturbance flow field outside the boundary layer. Finally, the contributions from the discrete and continuous branches are combined together to fully reconstruct the disturbance flow field.

4.2.3.1 Projection on Discrete Modes S

The growth rate of the unstable mode S dominates the disturbance flow field leading to high N -factor. The disturbance flow field contribution to the mode S can help in understanding the receptivity process. The disturbance flow field is projected on mode S in the vicinity of the particle impingement location and further downstream.

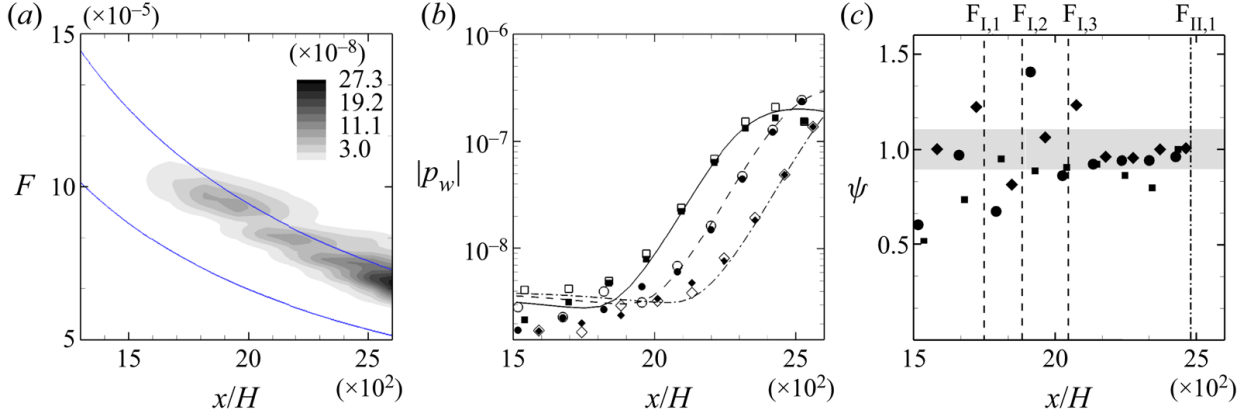


Figure 4.13: (a) Disturbance wall pressure projected on mode S, blue lines mark the neutral curve of second mode, (b) Amplitude curves extracted at different frequencies (F), $\blacksquare$; $\square$; —: $F_1 = 7.543 \times 10^{-5}$, $\bullet$; $\circ$; ---: $F_2 = 7.023 \times 10^{-5}$ and $\blacklozenge$; $\diamond$; -.-.: $F_3 = 6.502 \times 10^{-5}$, lines represent LST, filled symbols represent projection on mode S, hollow symbols represent disturbance pressure FFT. (c) projection ratio (ψ), vertical dashed lines and dashed dotted line mark the branch I and branch II of the neutral curve, respectively (adjusted from ref. [2]).

Fig. 4.13a shows the wall disturbance pressure projected on mode S at $\beta = 0$. The blue curves mark the neutral curves for the second mode. The peak of the amplitude plot is found within the second mode region. This figure can be directly compared with fig. 4.4a, which shows similar characteristics. Amplitude curves are extracted in fig. 4.13b at three distinct frequencies, $F = 7.543 \times 10^{-5}$, 7.023×10^{-5} and 6.502×10^{-5} , from the FFT, projection on mode S and LST prediction. The amplitude developments are in close agreement within the second mode growth region. Near particle impingement there are discrepancies among the disturbance flow FFT, projection on mode S and LST prediction. Fig. 4.13c illustrates the projection ratio. The projection ratio can be defined as the ratio of amplitude of projected value to the disturbance flow FFT value, $\psi = P_{proj}/P_{fft}$. The branch I (lower neutral branch), for the three frequencies considered here, are marked as black dashed lines and branch II (upper neutral branch) is marked as black dashed dotted line. When the disturbance flow passes the branch I the projection ratio becomes unity. This happens because mode S becomes dominant here and the projection on

mode S represents the disturbance flow field. Before the branch I the projection ratio deviates from unity which suggests that there are other modes that contribute to the disturbance flow field. These modes belong to the stable discrete branch and continuous branch of the eigenvalue spectrum.

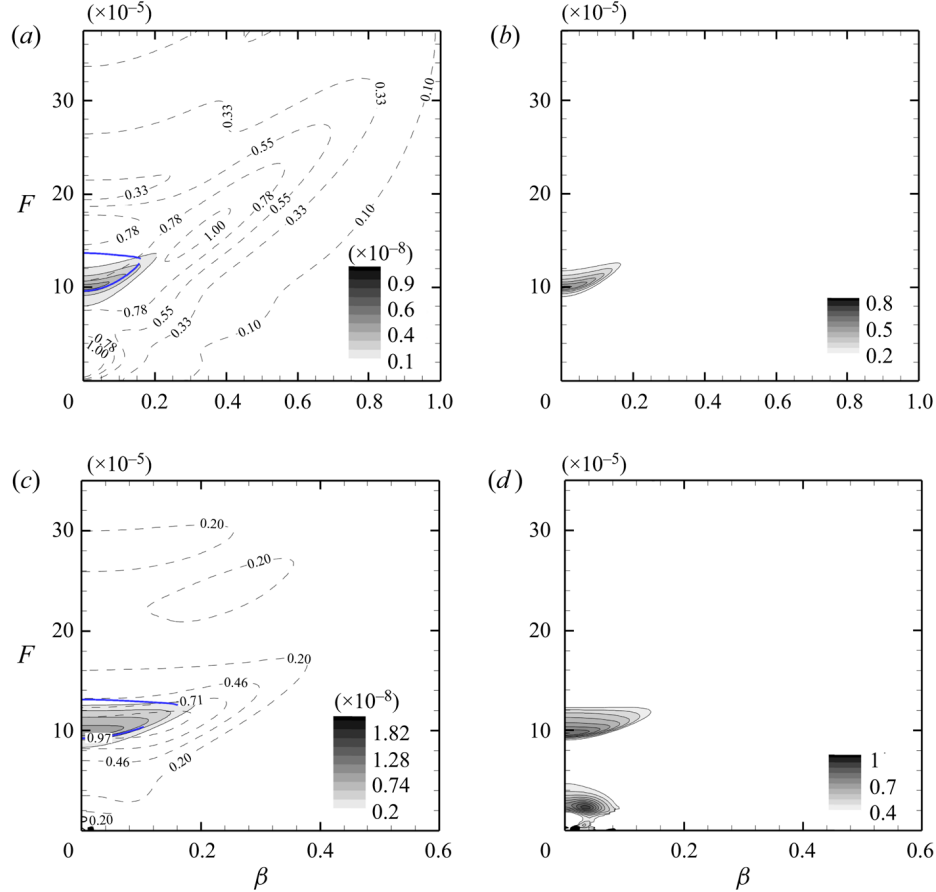


Figure 4.14: $F - \beta$ spectra of the FFT of disturbance wall pressure projected on mode S at (a) $x_{pc}/H = 1374$ (c) $x/H = 1432$. Projection ratio at (b) $x_{pc}/H = 1374$ and (d) $x/H = 1432$ (adjusted from ref. [2]).

Next, the $F - \beta$ spectra of the disturbance flow pressure projected on mode S are shown in the vicinity of the particle impingement location in figs. 4.14a and 4.14c at $x_{pc}/H = 1374$ and $x/H = 1432$, respectively. The disturbance flow FFT is included as dashed contour lines in the figures. The second mode neutral curve is included as a blue line. A small amount of

the disturbance flow ends up as mode S near to the impingement location. It reiterates that near the particle impingement location there are significant contributions from the other discrete and continuous modes. The projection ratio, ψ , is shown in figs 4.14b and 4.14d. The projection ratio is close to unity within the second mode region. For the location at $x/H = 1432$, a peak is also found in the first mode frequency range. This suggests that the first mode is also activated at this location.

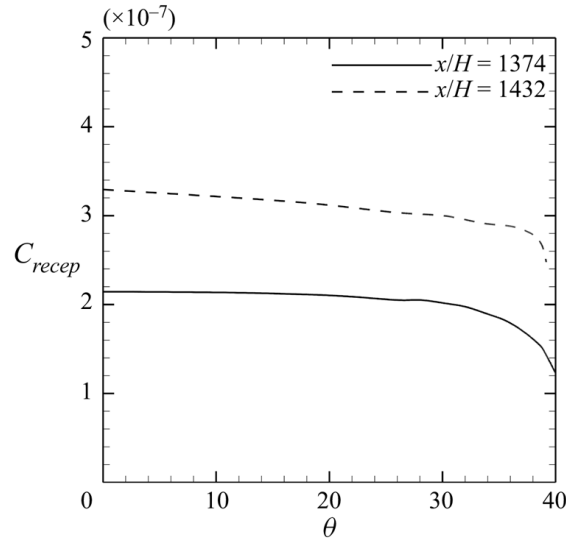


Figure 4.15: Receptivity coefficient (C_{recep}) at the lower neutral branch for different wave angles at two streamwise locations, $x_{pc}/H = 1374$ and $x/H = 1432$ (adjusted from ref. [2]).

The receptivity coefficient is calculated along the lower neutral branch as following

$$C_{recep} = \frac{|P_{ws}|}{\rho U_{rel}^2}, \quad (4.6)$$

where ρ is the fluid density, U_{rel} is the relative velocity of particle and $|P_{ws}|$ is the projected wall pressure on mode S. Fig. 4.15 shows the receptivity coefficient at different wave angles, $\theta = \tan^{-1}(\beta/\alpha_r)$, for two streamwise locations, $x_{pc}/H = 1374$ and $x/H = 1432$. The receptivity coefficient at $x/H = 1432$ shows higher amplitude than the impingement location.

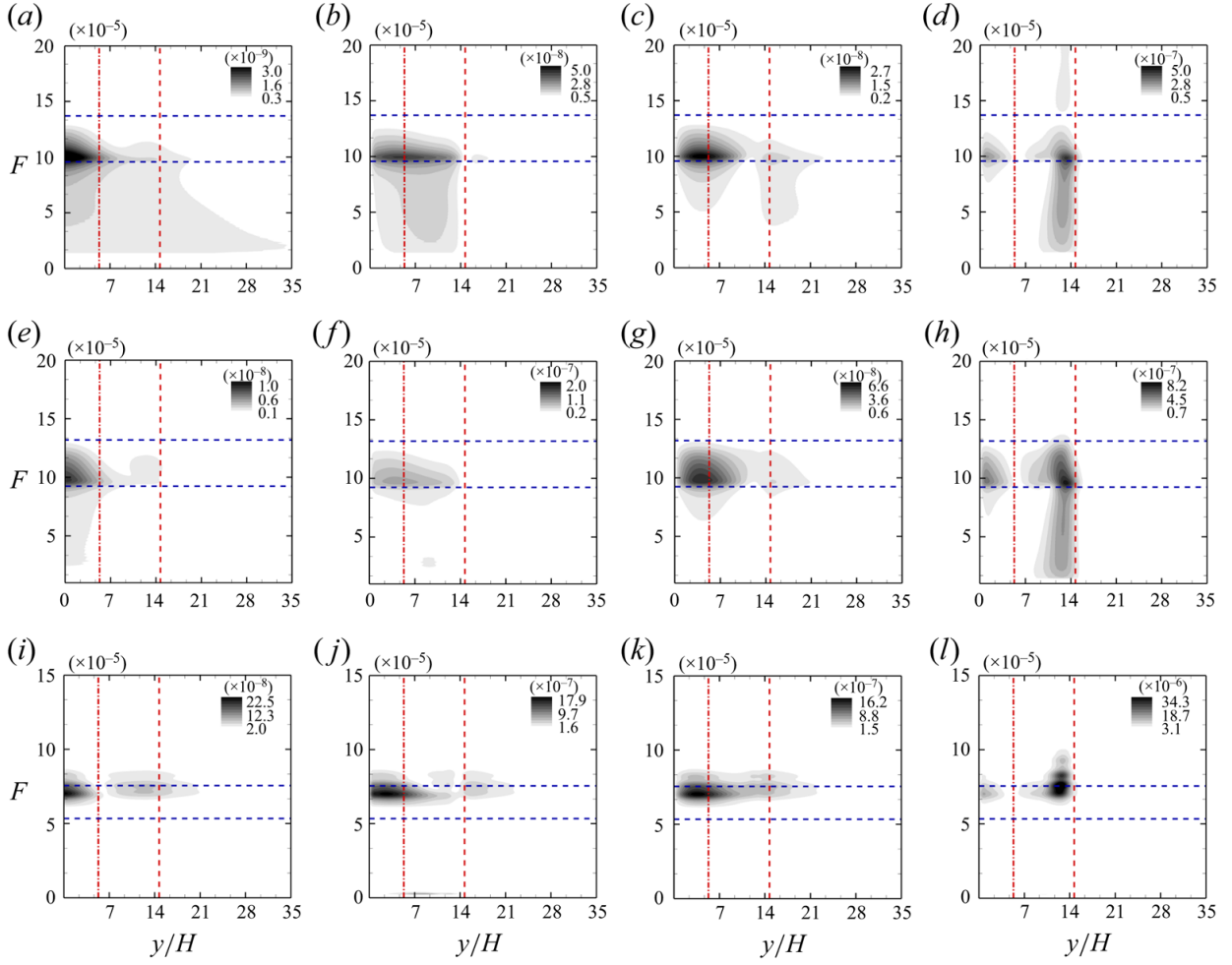


Figure 4.16: Disturbance flow FFT projected on mode S in the wall-normal direction at (a) $|p|$, $x_{pc}/H = 1374$; (b) $|u|$, $x_{pc}/H = 1374$; (c) $|v|$, $x_{pc}/H = 1374$; (d) $|T|$, $x/H = 1374$; (e) $|p|$, $x/H = 1432$; (f) $|u|$, $x/H = 1432$; (g) $|v|$, $x/H = 1432$; (h) $|T|$, $x/H = 1432$; (i) $|p|$, $x/H = 2517$; (j) $|u|$, $x/H = 2517$; (k) $|v|$, $x/H = 2517$; (l) $|T|$, $x/H = 2517$. Horizontal blue dashed line mark the frequency range of second mode, vertical red dot-dashed line marks the sonic line, vertical red dashed line marks the boundary layer height (adjusted from ref. [2]).

Next, the frequency spectra of the disturbance flow projected on mode S are plotted in wall-normal direction in figs. 4.16a-l for the disturbance pressure, streamwise velocity, wall-normal velocity and temperature at $\beta = 0$. The horizontal blue lines mark the frequency range of the second mode. The red dashed dotted line marks the sonic line and the red dashed line marks the boundary-layer height. At the impingement location, $x_{pc}/H = 1374$, the peak is found below the sonic line and near lower neutral branch. Similarly, at a slightly downstream location

at $x/H = 1432$ the peak is found near the lower neutral branch for all the flow quantities. At a further downstream location, $x/H = 2517$, the peak shifts towards the upper neutral branch. Figs. 4.6i-l can be directly compared with figs. 4.16i-l. Projection on mode S almost represents the disturbance flow FFT in figs. 4.6i-l. Mode S contributes significantly to the disturbance flow field at this location.

The projection on mode S is compared with the disturbance flow FFT profiles of pressure, streamwise velocity and temperature in figs. 4.17a-i for different locations. The disturbance flow FFT profiles are significantly different than the projection on mode S at the impingement location. As mode S becomes dominant in the downstream locations, a closer match is observed at $x/H = 2517$ in figs. 4.17e, 4.17c and 4.17i. The disturbance flow field in the vicinity of the impingement location has contributions from other stable discrete and continuous modes, which will be demonstrated later in section 4.2.3.3 while reconstructing the disturbance field from the contributions from discrete and continuous branches.

Next, the disturbance energy is calculated based on the projection on mode S. Figs. 4.18a-f show the disturbance energy of the projection on mode s for three locations and two different spanwise wavenumbers. At the impingement location, $x_{pc}/H = 1374$, the peak of disturbance energy spectra is found within the second mode frequency range. A small amount of the disturbance energy ends up as mode S at $\beta = 0$, comparing with fig. 4.7a. At $\beta = 0.55$ another peak is observed at the first mode region. At a further downstream location, ($x/H = 2517$, fig. 4.18c), the peak shifts towards the upper neutral branch. The disturbance energy of the flow field in fig. 4.7c is in close agreement with the projection on mode S in fig. 4.18c. The disturbance energy of projection on mode S at this location almost matches the disturbance energy of the flow field. The mode S is dominant at this location which is in agreement with the LST prediction.

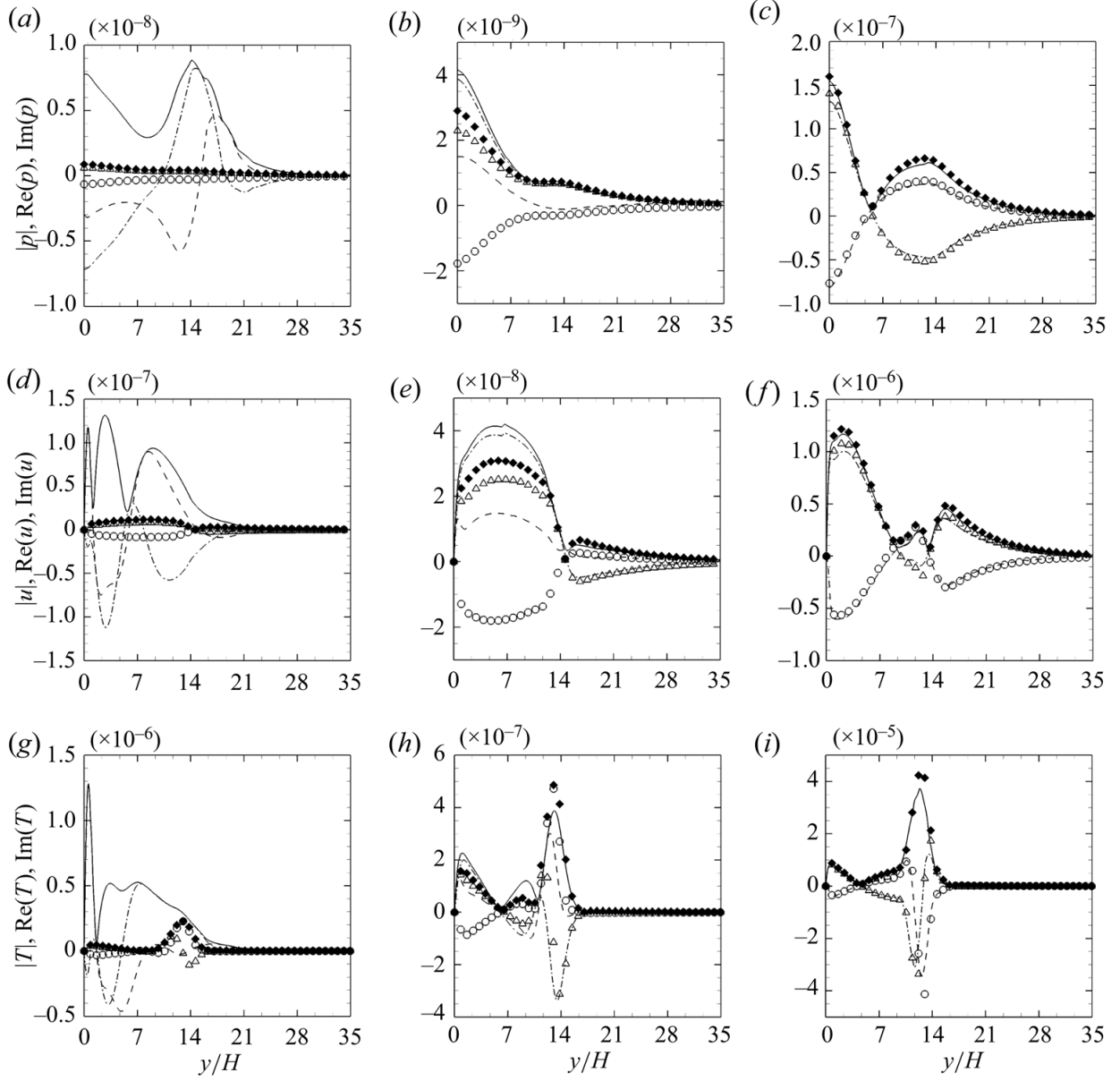


Figure 4.17: Disturbance flow FFT profile of pressure, streamwise velocity and temperature at (a,d,g) $x_{pc}/H = 1374$, (b,e,h) $x/H = 1670$, (c,f,i) $x/H = 2517$. —, $\blacklozenge$ for $|\cdot|$; ---, $\circ$ for $\text{Re}(\cdot)$; ····, Δ for $\text{Im}(\cdot)$. Lines represent the FFT of the disturbance flow field and symbols represent the projection on mode S (adjusted from ref. [2]).

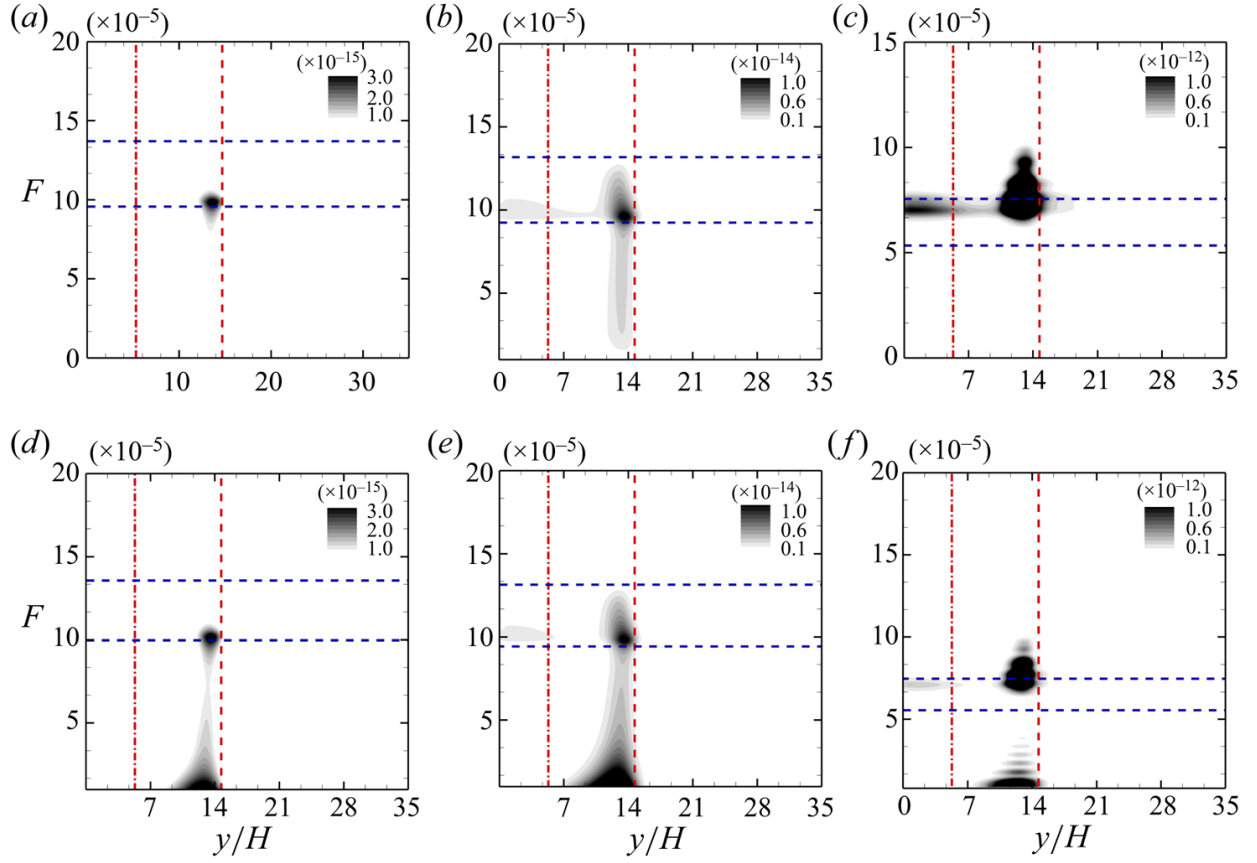


Figure 4.18: Disturbance energy projected on mode S at (a) $x_{pc}/H = 1374, \beta = 0$; (b) $x/H = 1432, \beta = 0$; (c) $x/H = 2517, \beta = 0$; (d) $x_{pc}/H = 1374, \beta = 0.055$; (e) $x/H = 1432, \beta = 0.05$; (f) $x/H = 2517, \beta = 0.058$. The horizontal blue dashed line marks the frequency range of the second mode. The red dot-dashed line marks the sonic line. The red dashed line represent the boundary-layer height (adjusted from ref. [2]).

4.2.3.2 Projection on Continuous Spectra

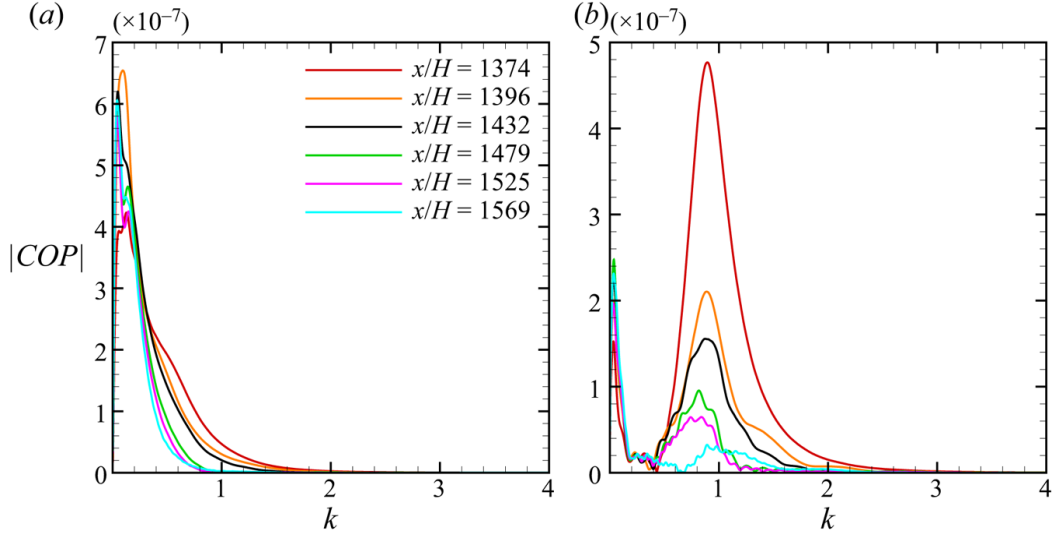


Figure 4.19: Coefficient of projection of (a) SA and (b) FA branch at different locations at $\beta = 0$ and $F = 1.764 \times 10^{-5}$ (adjusted from ref. [2]).

In this section, the projection of the disturbance flow field on the continuous slow and fast acoustic modes is discussed. The contribution of the vorticity and entropy branch to the disturbance flow field has been found to be negligible. The continuous modes are different from the discrete modes as they contribute differently to the flow field. Continuous acoustic modes contribute significantly to the disturbance flow field outside the boundary layer. The disturbance flow field is projected on the continuous branch for different continuous spectra parameter ' k '. The continuous spectra parameter, k , is also known as the wall-normal component of the wavenumber vector. k is considered to have values between 0.001 to 4.0 with a step size of $\Delta k = 0.001$. Figs. 4.19a and 4.19b show the coefficient of projection at six streamwise locations for the slow acoustic (SA) and fast acoustic (FA) modes, respectively. The SA modes show a peak COP for the continuous spectra parameter $k < 1$ and the FA modes show peak COP for $0.5 < k < 2$. Figs. 4.20a-d show the FFT of the disturbance pressure field, projection on SA and

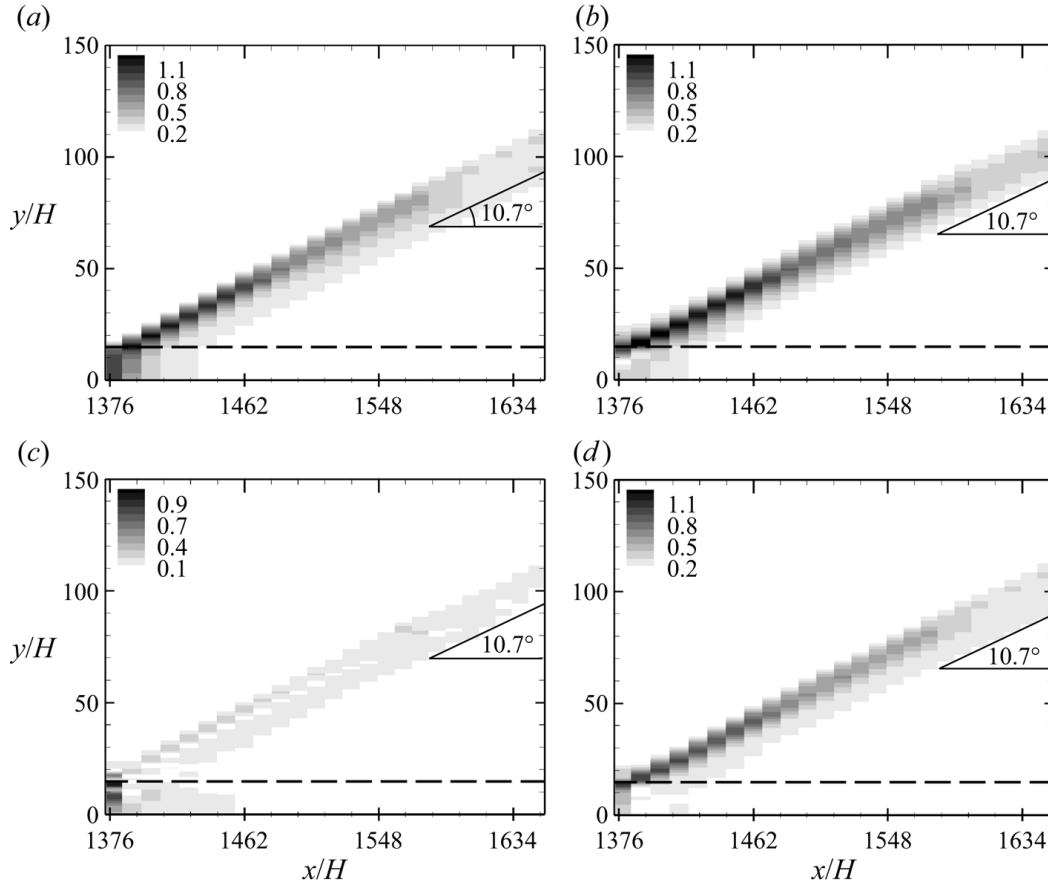


Figure 4.20: (a) FFT of pressure disturbance, (b) disturbance flow field projected on SA modes, (c) disturbance flow field projected on FA modes, (d) combination of SA and FA modes at $\beta = 0$ and $F = 1.764 \times 10^{-5}$ with $k = 0.001 - 4.0$. The boundary layer height is marked as a black dashed line and Mach angle is included as 10.7° (adjusted from ref. [2]).

FA modes and superposition of SA and FA modes at $\beta = 0$ and $F = 1.764 \times 10^{-5}$. The projections on SA and FA modes show dominance in different regions of the flow field. The combination of SA and FA modes fully reconstructs the flow field outside the boundary layer as shown in fig. 4.20d.

4.2.3.3 Full Reconstruction of the Disturbance Flow Field

In this section, the disturbance flow field is reconstructed from the contributions of the discrete and continuous branches. Five cases with different frequencies, F , and spanwise wavenumbers,

Case	$F(\times 10^{-4})$	β	S	F1	F2	F3
1	2.822	0.0	(0.425, 0.0076)	(0.722, 0.063)	(0.422, 0.014)	(0.34, 0.00107)
2	2.315	0.206	(0.35, 0.0072)	(0.529, 0.042)	(0.293, 0.0066)	-
3	1.014	0.0	(0.156, -0.0034)	(0.155, 0.0095)	-	-
4	0.7938	0.0	(0.125, 0.00027)	(0.107, 0.0014)	-	-
5	0.1764	0.0	(0.0301, -5.22×10^{-5})	(0.0213, 4.96×10^{-6})	-	-

Table 4.2: Eigenvalues of discrete modes for different cases (adjusted from ref. [2])

β , have been considered for reconstruction near to the particle impingement location at $x/H = 1432$. The frequencies and spanwise wavenumbers are provided in table 4.2 for different cases. Figs. 4.21a and 4.21c show the development of the discrete modes S and F with respect to the frequency, F , at $\beta = 0$ and $\beta = 0.206$, respectively. Multiple F modes exist in the eigenvalue spectrum. These modes are numbered according to their emergence in the eigenvalue spectrum. Figs. 4.21b and 4.21d show the phase speed, c_r , versus frequency, F , development. The cases are marked as black vertical lines in the figs. 4.21a-d. The phase speeds of the FA and SA branches are marked by black horizontal lines at $1 + 1/M$ and $1 - 1/M$ in fig. 4.21b for $\beta = 0$, respectively. Mode S synchronizes with SA branch and mode F synchronizes with FA branch as frequency approaches to zero. Mode S synchronizes with mode F1 at $F \approx 1.07 \times 10^{-4}$, which gives rise to the second mode S. A second synchronization of mode S with mode F2 happens at $F \approx 2.88 \times 10^{-4}$, which remains stable. For $\beta = 0.206$ in figs. 4.21c and 4.21d, mode S synchronizes with mode F1 at $F = 1.32 \times 10^{-4}$ and with mode F2 at $F = 3.04 \times 10^{-4}$. Both of this synchronization remains stable. Synchronization can transfer energy from one mode to another through phase speed matching, giving rise to the unstable mode [2].

Eq. (2.39) is employed to reconstruct the disturbance flow field from the contribution of the discrete and continuous branches. The continuous spectrum parameter, k , is considered within the range of 0.001-0.6 with step size of 0.001 for projecting the disturbance flow field on the continuous branches. Then the integration is performed by following the trapezoidal method for

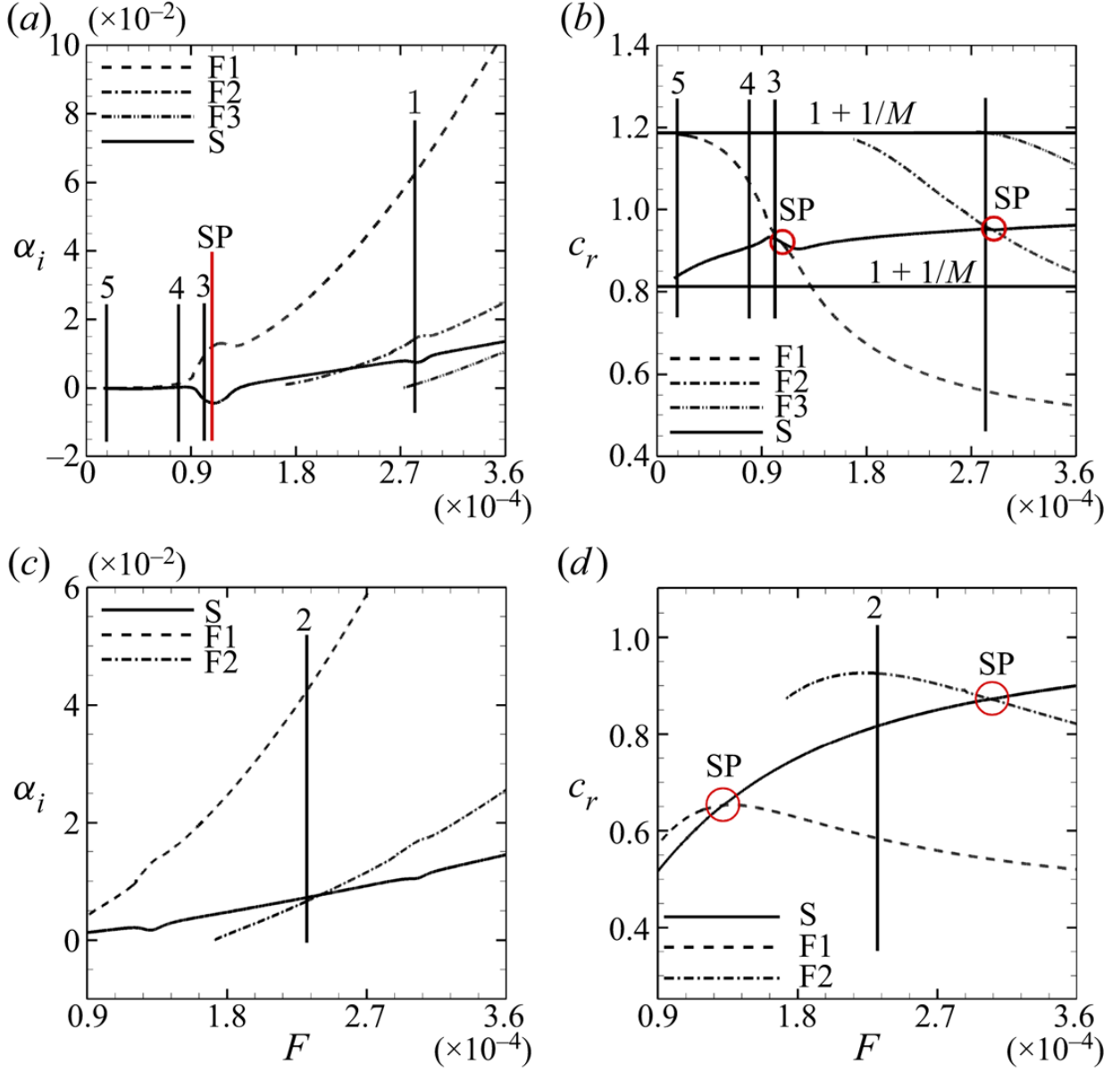


Figure 4.21: (a) α_i vs. F for different discrete modes at $\beta = 0$, (b) c_r vs. F at $\beta = 0$, (c) α_i vs. F for different discrete modes at $\beta = 0.206$ and (d) c_r vs. F at $\beta = 0.206$ (adjusted from ref. [2]).

the k range to obtain the total contribution of the continuous branch. Figs. 4.22a-e show the COP of the SA and FA branches for the five cases. The COP amplitude of FA branch is higher for higher frequency. For low frequency, SA branch shows higher amplitude of COP.

Fig. 4.23a shows the FFT of the disturbance pressure and its projection on modes S, F and SA, FA branches for case 1. Combination of modes S, F, and SA, FA branches reconstruct the

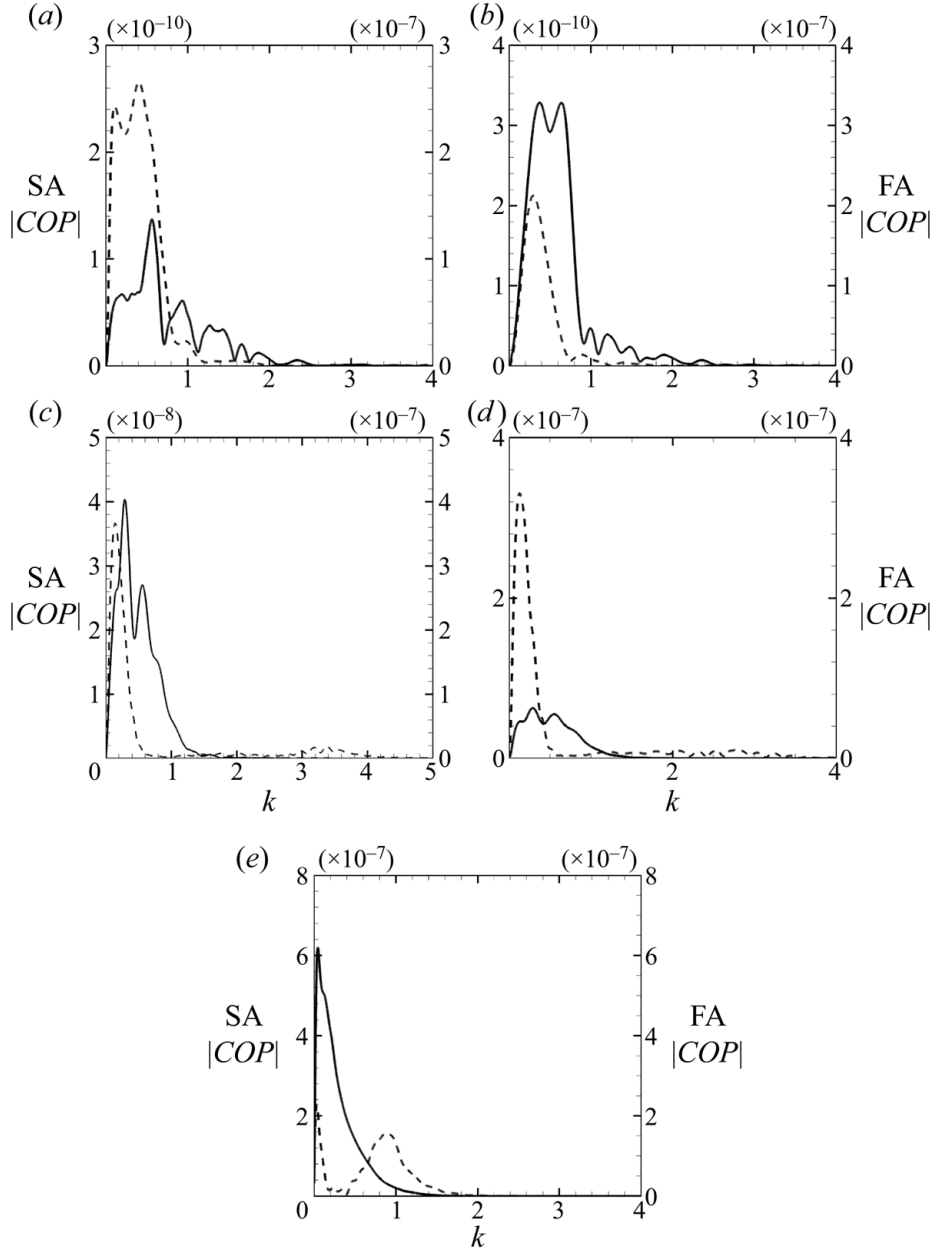


Figure 4.22: COP of SA and FA branches for (a) $F = 2.82 \times 10^{-4}, \beta = 0$; (b) $F = 2.315 \times 10^{-4}, \beta = 0.206$; (c) $F = 1.014 \times 10^{-4}, \beta = 0$; (d) $F = 7.94 \times 10^{-5}, \beta = 0$; (e) $F = 1.76 \times 10^{-5}, \beta = 0$. Solid line represents the SA branch and dashed line represents the FA branch (adjusted from ref. [2]).

disturbance flow FFT. Inside the boundary layer mode F2 has the highest contribution. Outside the boundary layer FA branch as well as mode F3 show higher contribution. The modes are marked by red squares in the eigenvalue spectrum in fig. 4.23b. Table 4.3 lists the projection coefficient, relative error and maximum error. The relative error and maximum error are determined

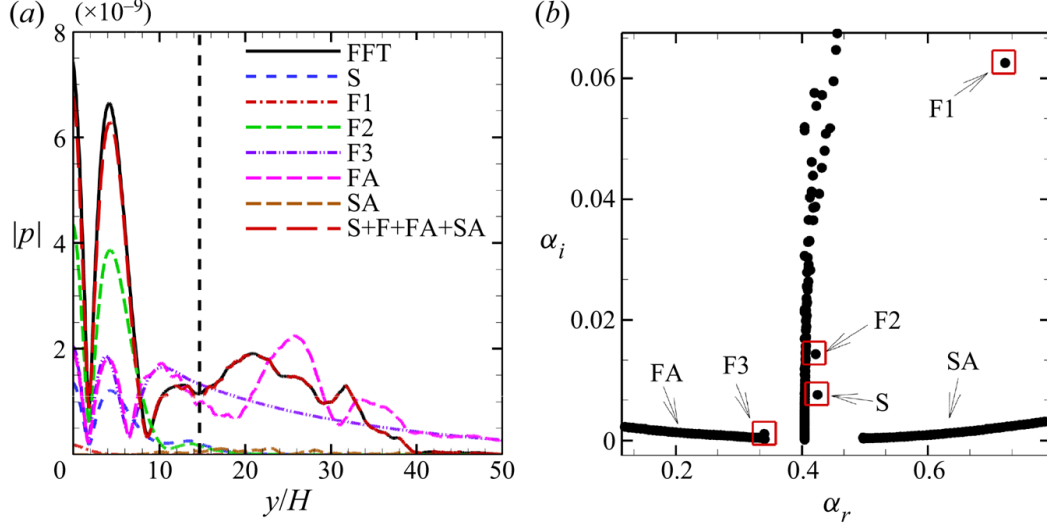


Figure 4.23: (a) FFT of the disturbance pressure field and its projection on mode S, F, SA and FA, and (b) eigenvalue spectrum for case 1 at $x/H = 1432$ for $F = 2.822 \times 10^{-4}$ and $\beta = 0$. Black vertical dashed line marks the boundary-layer height (adjusted from ref. [2]).

by following

$$\text{Rel. Error} = \sqrt{\frac{\sum (P_{FFT} - P_{proj})^2}{N \times \max(P_{FFT})}}, \text{ and} \quad (4.7)$$

$$\text{Max Error} = \frac{|\max(P_{FFT} - P_{proj})|}{\max(P_{FFT})}, \quad (4.8)$$

where P_{proj} the projection on discrete and continuous modes, P_{FFT} is the disturbance flow FFT and N is the number of wall-normal points. For case 1, the maximum projection ratio at wall, relative error and maximum error are 0.588 from mode F2, 2.3% and 8.8%, respectively.

Case	F	β	S	F1	F2	F3	FA	SA	Rel. Error	Max Error
1	2.822×10^{-4}	0.0	0.183	0.025	0.588	0.276	0.262	1.125×10^{-4}	0.023	0.088
2	2.315×10^{-4}	0.206	0.009	0.026	1.03	-	0.159	6.255×10^{-4}	0.011	0.043
3	1.014×10^{-4}	0.0	0.75	0.36	-	-	0.078	0.0042	0.0071	0.032
4	7.938×10^{-5}	0.0	0.292	0.925	-	-	0.132	0.011	0.011	0.051
5	1.764×10^{-5}	0.0	0.352	0.047	-	-	0.401	0.568	0.016	0.026

Table 4.3: Projection ratios of wall pressure for discrete and continuous modes for cases 1-5 (adjusted from ref. [2])

Fig. 4.24a shows the reconstruction of the disturbance pressure FFT for case 2. Inside the

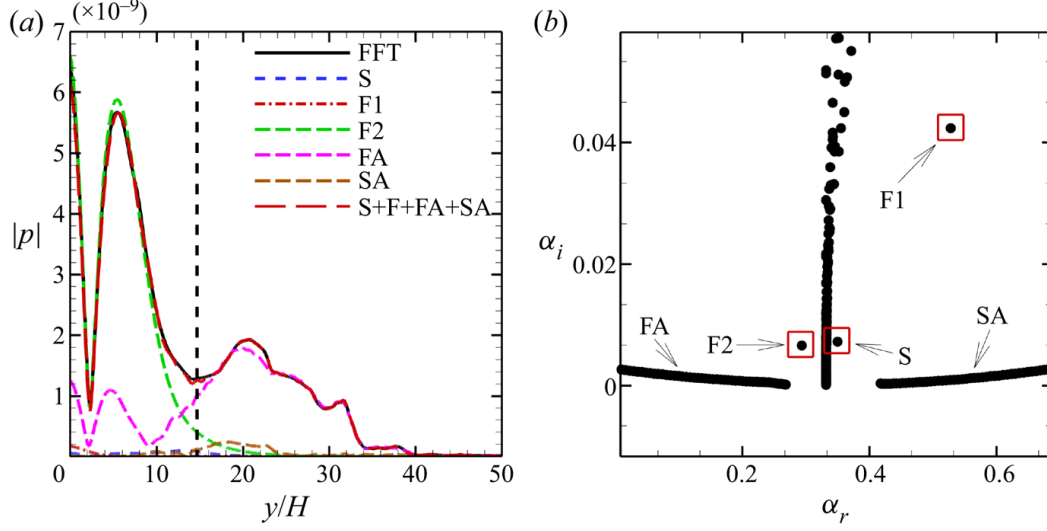


Figure 4.24: (a) FFT of disturbance pressure field and it's projection on mode S, F, SA and FA, and (b) eigenvalue spectrum for case 2 at $x/H = 1432$ for $F = 2.315 \times 10^{-4}$ and $\beta = 0.206$. Black vertical dashed line marks the boundary-layer height (adjusted from ref. [2]).

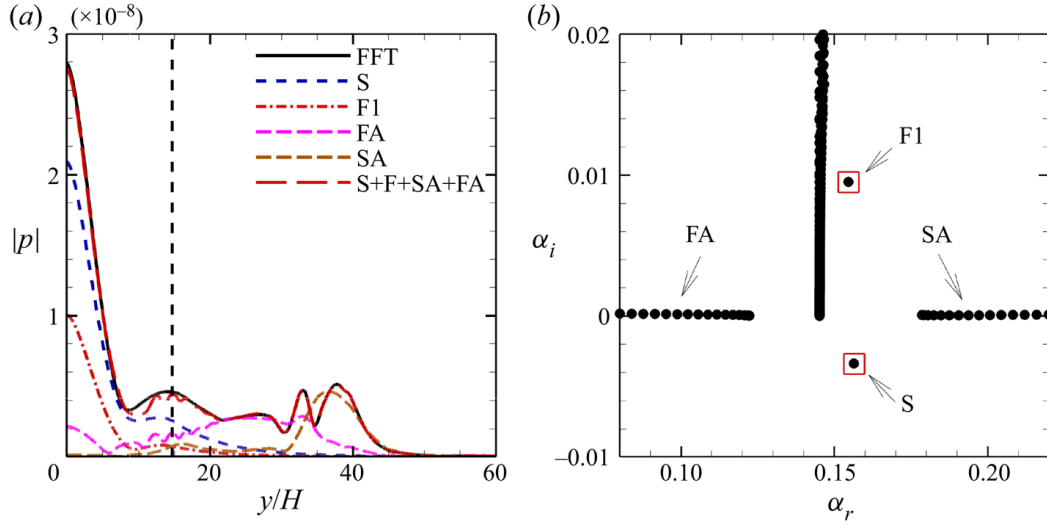


Figure 4.25: (a) FFT of disturbance pressure field and it's projection on mode S, F, SA and FA, and (b) eigenvalue spectrum for case 3 at $x/H = 1432$ for $F = 1.014 \times 10^{-4}$ and $\beta = 0$. Black vertical dashed line marks the boundary-layer height (adjusted from ref. [2]).

boundary layer mode F2 almost represent the disturbance pressure FFT. Outside the boundary layer the FA branch dominates and is in close agreement with the disturbance flow FFT. Fig. 4.24b shows the eigenvalue spectrum for case 2. All the modes are stable for case 2. For case 2, the maximum projection ratio at the wall, relative error and maximum error are 1.03 from mode F2,

1.1% and 4.3%, respectively.

Fig. 4.25a shows the reconstruction of the disturbance pressure field for case 3. Mode S dominates the flow field inside the boundary layer. Outside the boundary layer both FA and SA branches show dominance. Fig. 4.25b shows the eigenvalue spectrum for case 3. Mode S is unstable and all other modes are stable for case 3. Mode S is the second Mack mode for this case at $x/H = 1432$, $F = 1.014 \times 10^{-4}$ and $\beta = 0$. For case 3, the maximum projection ratio at the wall, relative error and maximum error are 0.75 from mode S, 0.7% and 3.2%, respectively.

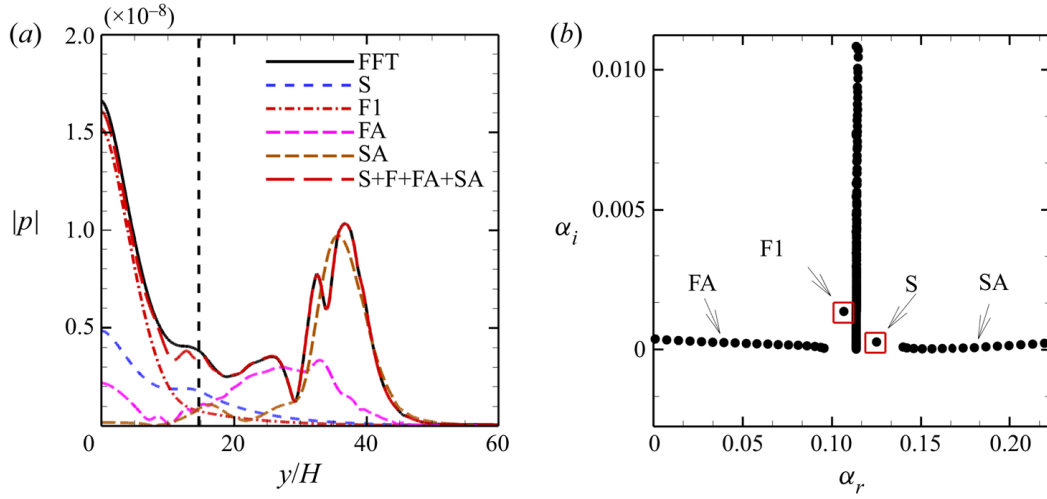


Figure 4.26: (a) FFT of disturbance pressure field and its projection on mode S, F, SA and FA, and (b) eigenvalue spectrum for case 4 at $x/H = 1432$ for $F = 7.938 \times 10^{-5}$ and $\beta = 0$. Black vertical dashed line marks the boundary-layer height (adjusted from ref. [2]).

The reconstruction of the disturbance pressure FFT for case 4 is illustrated in fig. 4.26a. Mode F1 dominates the flow field inside the boundary layer. The SA branch dominates the flow field outside the boundary layer. Fig. 4.26b shows the eigenvalue spectrum for the case 4. All the modes are stable for case 4 with $x/H = 1432$, $F = 7.938 \times 10^{-5}$ and $\beta = 0$. For case 4, the maximum projection ratio at the wall, relative error and maximum error are 0.925 from mode F1, 1.1% and 5.1%, respectively.

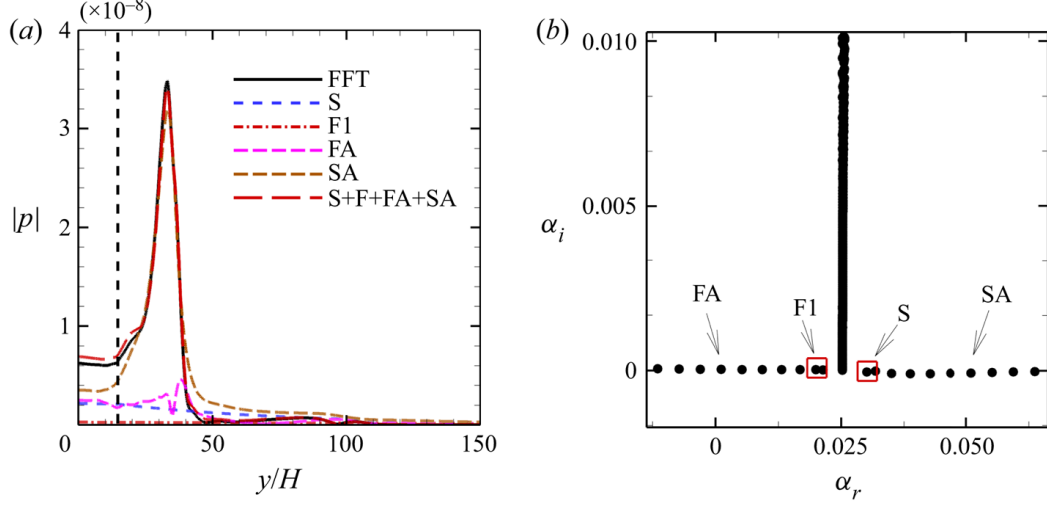


Figure 4.27: (a) FFT of disturbance pressure field and its projection on mode S, F, SA and FA, and (b) eigenvalue spectrum for case 5 at $x/H = 1432$ for $F = 1.764 \times 10^{-5}$ and $\beta = 0$. Black vertical dashed line marks the boundary-layer height (adjusted from ref. [2]).

The disturbance pressure FFT and its reconstruction from different discrete and continuous modes are illustrated in fig. 4.27a for case 5. The peak of the FFT profile is found outside the boundary layer. The SA branch dominates the flow field outside the boundary layer and almost represents the disturbance pressure FFT. The eigenvalue spectrum is provided in fig. 4.27b. All the modes from discrete and continuous branches are stable for the case 5 at $x/H = 1432$, $F = 1.764 \times 10^{-5}$ and $\beta = 0$. For case 5, the maximum projection ratio at the wall, relative error and maximum error are 0.568 from SA branch, 1.6% and 2.6%, respectively.

4.3 Disturbance Flow Field in a Mach 4 14° wedge

4.3.1 Particle Induced Disturbance Flow Field

M_1	M_2	p_2 [Pa]	T_2 [K]	T_{wall} [K]	T_{wall}/T_2
4.0	3.0	19044.75	325.5	823.4	2.53

Table 4.4: Freestream after-shock conditions and wall temperature ratios for Mach 4 14° wedge.

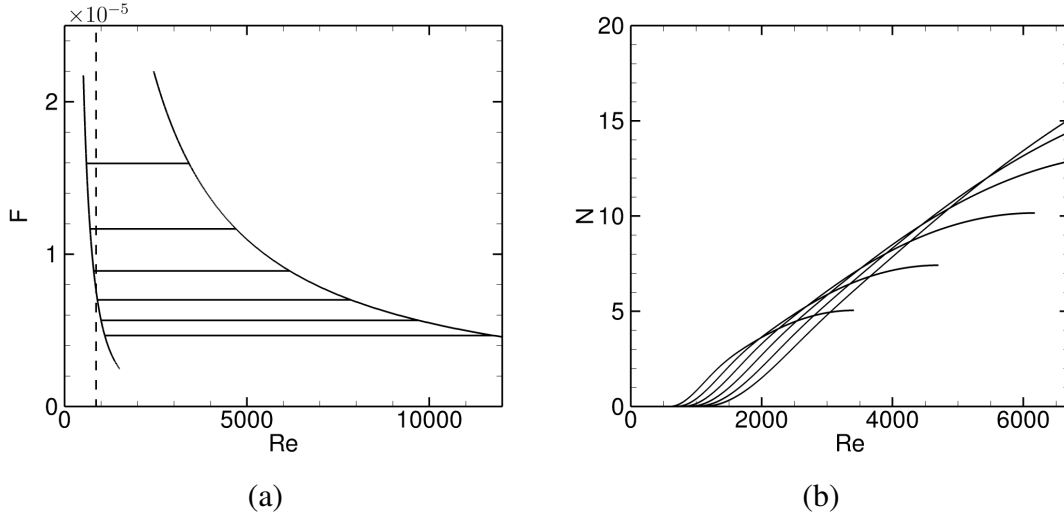


Figure 4.28: (a) Neutral curves at $\beta = 0.075$ with frequencies used for calculating N-factors, vertical dashed line marks the particle impingement location, and (b) N-factor distributions at $\beta = 0.075$ for the Mach 4 14° wedge (adjusted from ref. [11]).

In this section, the disturbance flow field induced by a particle in a Mach 4 14° wedge is discussed. The following analysis is reported in ref. [11]. Particle impingement on a Mach 4 14° wedge was previously studied by Chuvakhov et al. [35]. The freestream properties are provided in table 4.4. The wall of the wedge is assumed isothermal with adiabatic temperature. The wall temperature to aftershock freestream temperature ratio is 2.53. This a first mode dominated flow field based on the LST prediction [11, 35]. Figs. 4.28a and 4.28b display the neutral curves and the N-factor distributions at $\beta = 0.075$ for the freestream conditions in table 4.4. N-factor is calculated as

$$N(x) = - \int_{x_l}^{x_u} \alpha_i(x, \beta, F) dx, \quad (4.9)$$

where $-\alpha_i$ is the growth rate, x_l and x_u are the locations of the lower and upper neutral branch, respectively. The integration is performed for each frequency, F , and spanwise wavenumber, β , pair. The spanwise wavenumber $\beta = 0.075$ is chosen because this is the wavenumber at which maximum amplification is observed at $x=0.072m$, which is also near to the particle impingement

location. N-factor curve at approximately $x/H = 800$ reaches to N-factor of approximately 10 at $x/H \approx 6000$. So, the transition to turbulence will take place within the domain of interest.

The simulation is performed in a computational domain which extends over $[0, 64195.9H]$ in the streamwise direction, $[0, 10354.2H]$ in the wall-normal direction and $[0, 64195.9H]$ in the spanwise direction. Scale H is evaluated at the particle impingement location. The AMR grid is employed with nine levels. The finest level is discretized with wall-normal grid spacing of $\Delta y = 0.0632H$. Streamwise and spanwise grid spacings of $\Delta x = \Delta z = 10\Delta y$ are considered. Particle radius of $R_p = 0.12943H$ and Gaussian half width of $\sigma = 5R_p$ are considered. It was previously demonstrated in refs. [8, 35] that a Gaussian half width of $5R_p$ was able to provide converged results. The particle is introduced at $(x, y) = 791.44H, 19.285H$ upstream of shock with the freestream velocity. The particle collides at $x_{pc}/H = 867.16$ and is assumed to stick to the surface upon impingement. After particle impingement, the energy and momentum source terms are assumed zero in eq. 2.30. A similar assumption was also made in ref. [35]. Point probes are placed in the domain to record the flow quantities and their derivatives. After the simulation the data is prepared for the biorthogonal decomposition. Fast Fourier Transform (FTT) is applied in the spanwise direction as well as in time for the BOD analysis.

Fig. 4.29 displays the pressure disturbance field at four time instances. When the particle impinges on the wall of the wedge, it introduces disturbances inside the boundary layer. This disturbance grows in size and propagates in the downstream direction. Initially, an acoustic wave forms in front of the wavepacket, propagating in the downstream direction and eventually decaying. When the particle enters the boundary layer, the relative Mach number reaches supersonic speed [8, 11] and a shock forms ahead of the particle.

Next, disturbance spectra are plotted in $F - \beta$ plane at three streamwise locations, $x/H =$

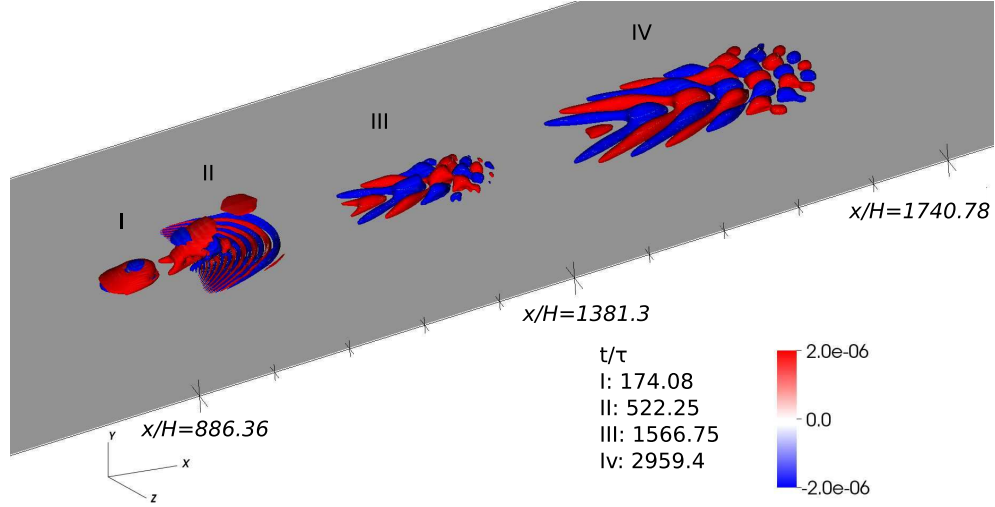


Figure 4.29: Pressure ($p/\rho U_\infty^2$) disturbance flow field in a $M=4$ 14° wedge with $T_{wall} = 823.4\text{K}$. τ is based on the particle impingement location (adjusted from ref. [11]).

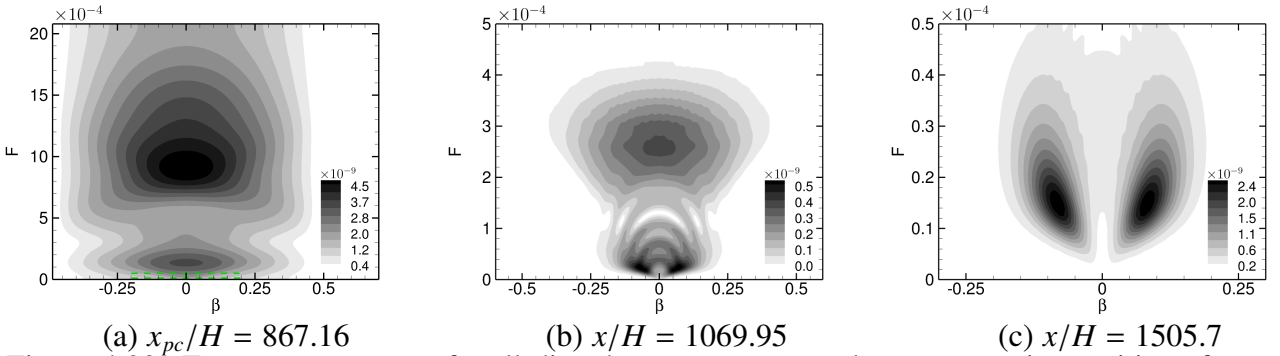


Figure 4.30: Frequency spectra of wall disturbance pressure at three streamwise positions for $M=4$ 14° boundary layer flow with $T_{wall} = 823.4\text{K}$. Green horizontal dashed lines mark the unstable region (adjusted from ref. [11]).

867.16, 1069.95 and 1505.7 in figs. 4.30a-c. Fig. 4.30a displays the disturbance spectrum at the particle impingement location. A wide range of frequencies and wavenumbers is introduced by the particle impingement. Peaks in the disturbance spectrum occur at $F \approx 1 \times 10^{-3}$ and $F \approx 1.5 \times 10^{-4}$ at $\beta \approx 0$. However, as we go in the downstream direction, these peaks vanish in fig. 4.30b as first mode starts to dominate. In fig. 4.30c the peaks in disturbance spectrum occurs at $F \approx 1.5 \times 10^{-5}$ and $\beta \approx \pm 0.075$. These are within the first mode frequency and wavenumber range.

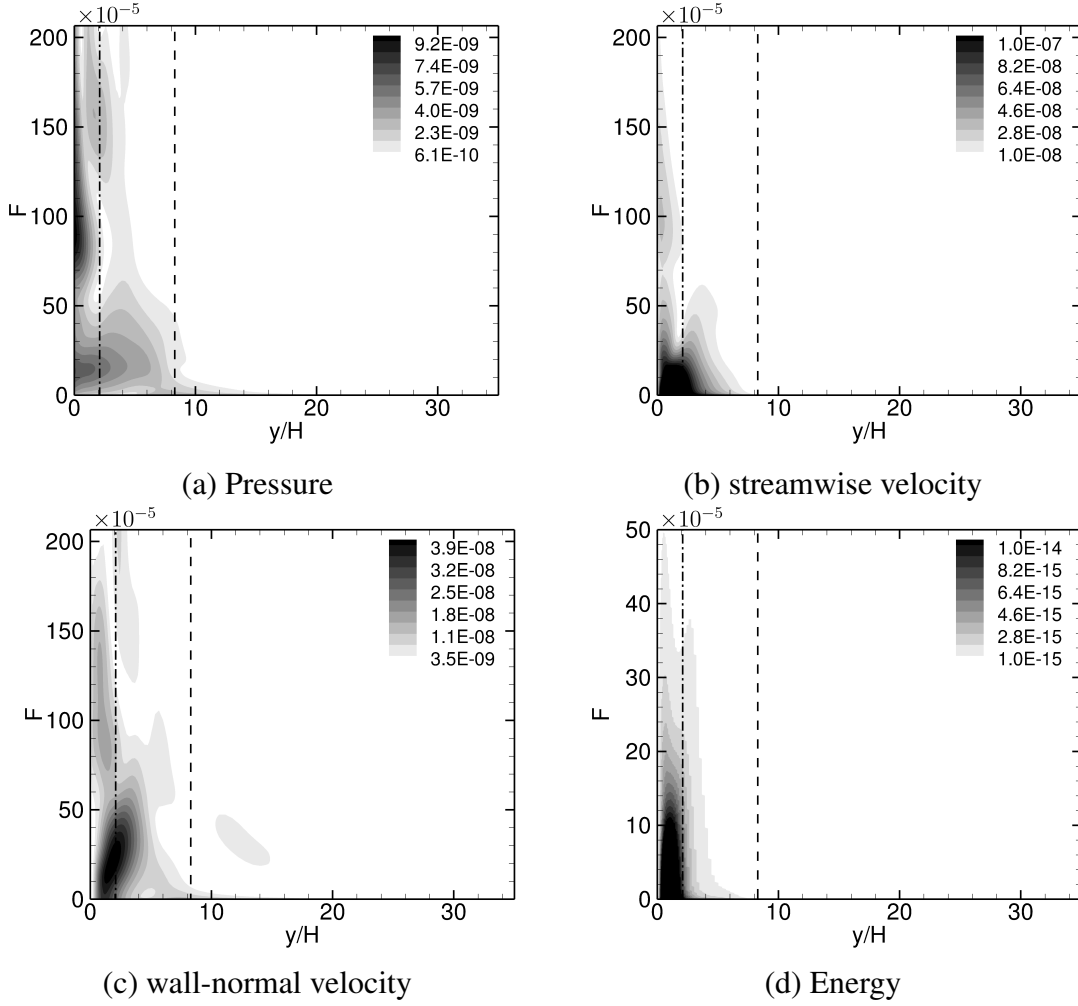


Figure 4.31: Frequency spectra of different disturbance flow quantities (a)-(c) and energy (d) at $x_{pc}/H = 867.16$ and $\beta = 0$ in $M=4$ 14° wedge with $T_{wall} = 823.4\text{K}$. Dashed line and dash dot line mark the boundary layer height and sonic line, respectively (adjusted from ref. [11]).

Next, the frequency spectra are plotted in $F - y$ plane. Figs. 4.31a-d show the frequency spectra of pressure, streamwise velocity, wall-normal velocity and disturbance energy, respectively, at $x_{pc}/H = 867.16$ and $\beta = 0$. The sonic line and boundary layer edge are marked by the black dashed line and dash dotted line, respectively. The peaks in the frequency spectra occur below the boundary-layer edge and at low frequencies for all the flow quantities, while the peak in the energy spectrum (fig. 4.31) occurs below the sonic line and at low frequency.

The frequency spectra are plotted in $F - y$ plane in figs. 4.32a-d for a further downstream

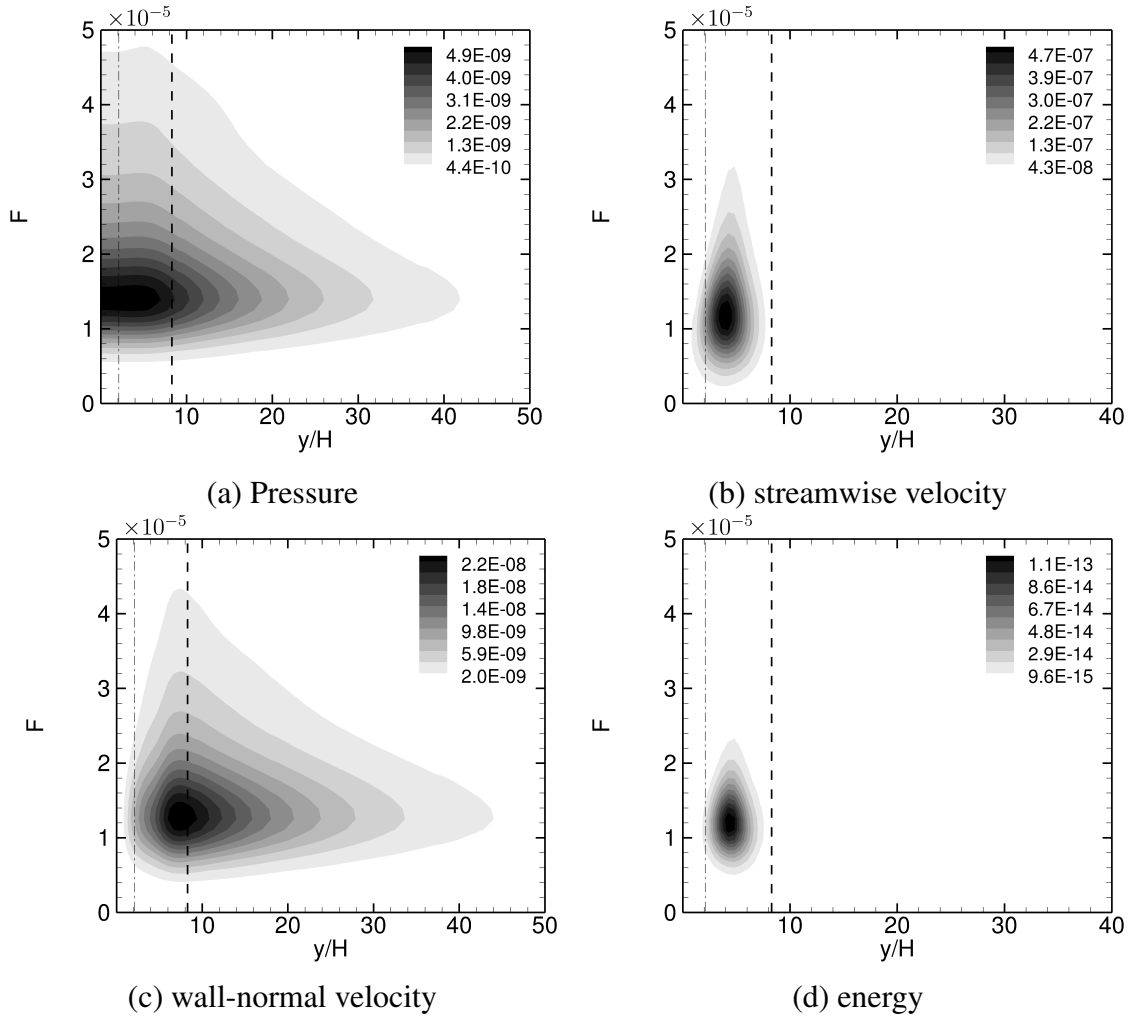


Figure 4.32: Frequency spectra of different disturbance flow quantities (a)-(c) and energy (d) at $x/H = 1505.7$ and $\beta = 0.075$ in $M=4$ 14° wedge with $T_{wall} = 823.4\text{K}$. Dashed line and dash dot line mark the boundary layer height and sonic line, respectively (adjusted from ref. [11]).

location at $x/H = 1505.7$ and $\beta = 0.075$. At this location and wavenumber the first mode is already dominating the flow field. The peaks in the frequency spectra are found within the first mode frequency range and below the boundary-layer edge.

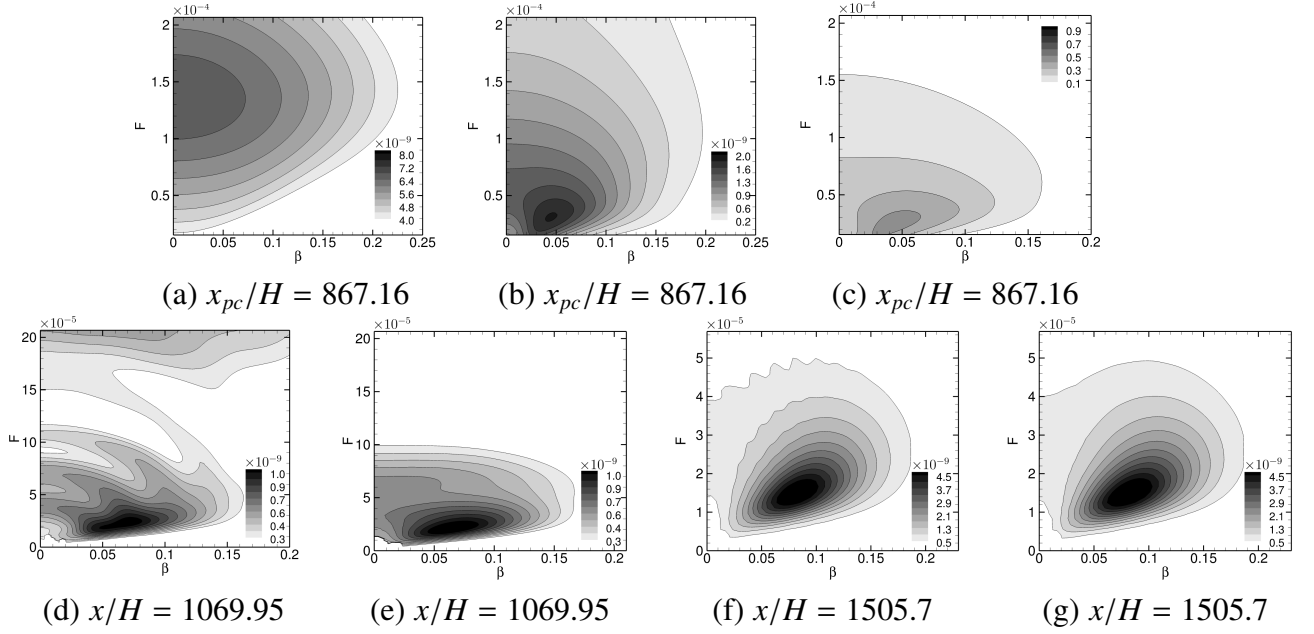


Figure 4.33: Frequency spectra of disturbance pressure at the wall projected on mode S at three different streamwise locations: (a,d,f) FFT of disturbance pressure, (b,e,g) projection of pressure on mode S and (c) Projection ratio (projection/FFT) at the impingement location (adjusted from ref. [11]).

4.3.2 Biorthogonal Decomposition of the Disturbance Flow Field

4.3.2.1 Projection on Discrete Mode S

In this section, the disturbance flow field is projected on the mode S, the dominant mode for the flow conditions considered here. Figs. 4.33b, 4.33e and 4.33g display the projection on mode S in $F - \beta$ plane for $x_{pc} = 867.16$, $x/H = 1069.95$ and $x/H = 1505.7$. At the impingement location, $x_{pc} = 867.16$, the peak in frequency spectrum of the disturbance flow field occurs at a different frequency and wavenumber pair than the projection on mode S. Fig. 4.33c shows the projection ratio (projection over FFT) at the impingement location. The peaks in the frequency spectra of projection on mode S and projection ratio occur at the similar frequency and wavenumber range. Further downstream of the particle impingement location, the mode S starts

to dominate the flow field. This is evident at location $x/H = 1069.95$, where the disturbance pressure and projection on mode S show peak at similar frequency and wavenumber range. Further downstream, where mode S dominates the flow field, a close agreement between the FFT and projection on mode S is achieved at $x/H = 1505.7$.

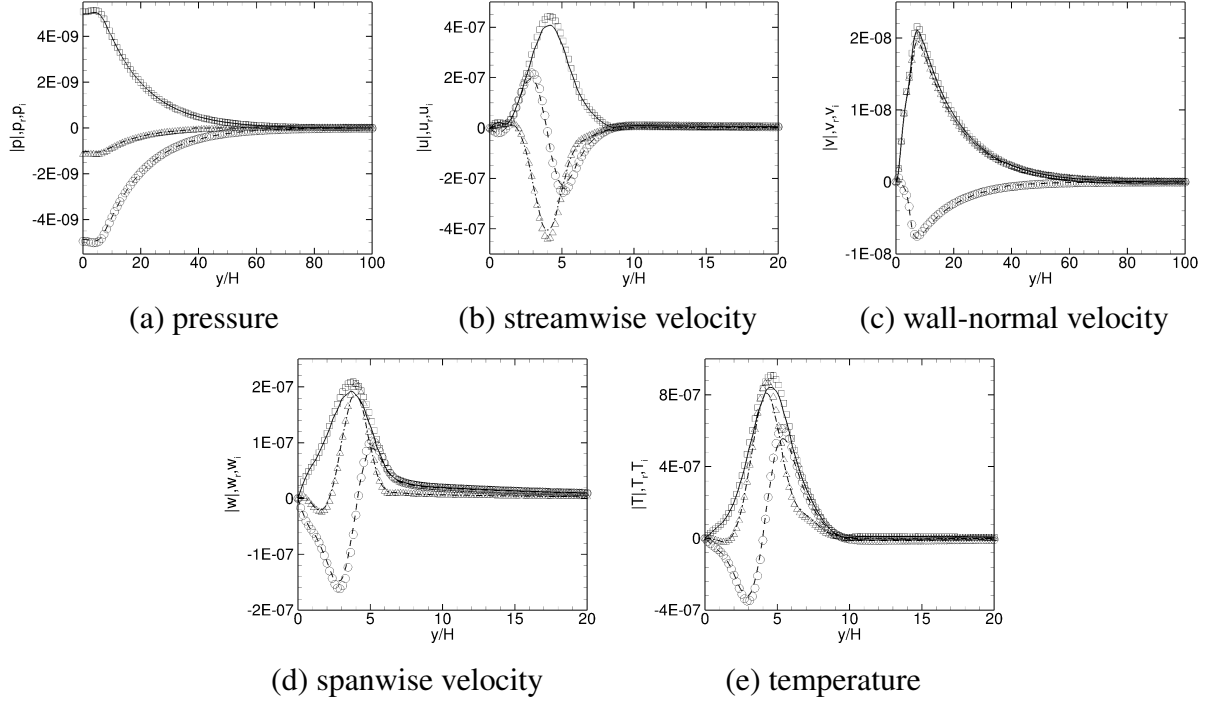


Figure 4.34: Profiles of disturbance flow FFT and its projection on mode S: (a) pressure, (b) streamwise velocity, (c) wall-normal velocity, (d) spanwise velocity and (e) temperature at $x/H = 1505.7$, $F = 1.526 \times 10^{-5}$ and $\beta = 0.075$. Lines represent FFT and symbols represent projection: —& $\square$: $|\cdot|$, ---& $\circ$: $\text{Re}(\cdot)$ and ----& Δ : $\text{Im}(\cdot)$ (adjusted from ref. [11]).

Next, the disturbance flow field and its projection on mode S are evaluated above the wall. The disturbance flow field is projected on mode S for the location $x/H = 1505.7$. Figs. 4.34a-e show the FFT of disturbance pressure, streamwise velocity, wall-normal velocity, spanwise velocity and temperature at $x/H = 1505.7$, $F = 1.526 \times 10^{-5}$ and $\beta = 0.075$. An excellent match between the FFT of disturbance flow field and projection on mode S is achieved at this location, frequency and wavenumber. At this location mode S is already dominating the flow field, so

contributions from mode S almost represent the disturbance flow field at this location.

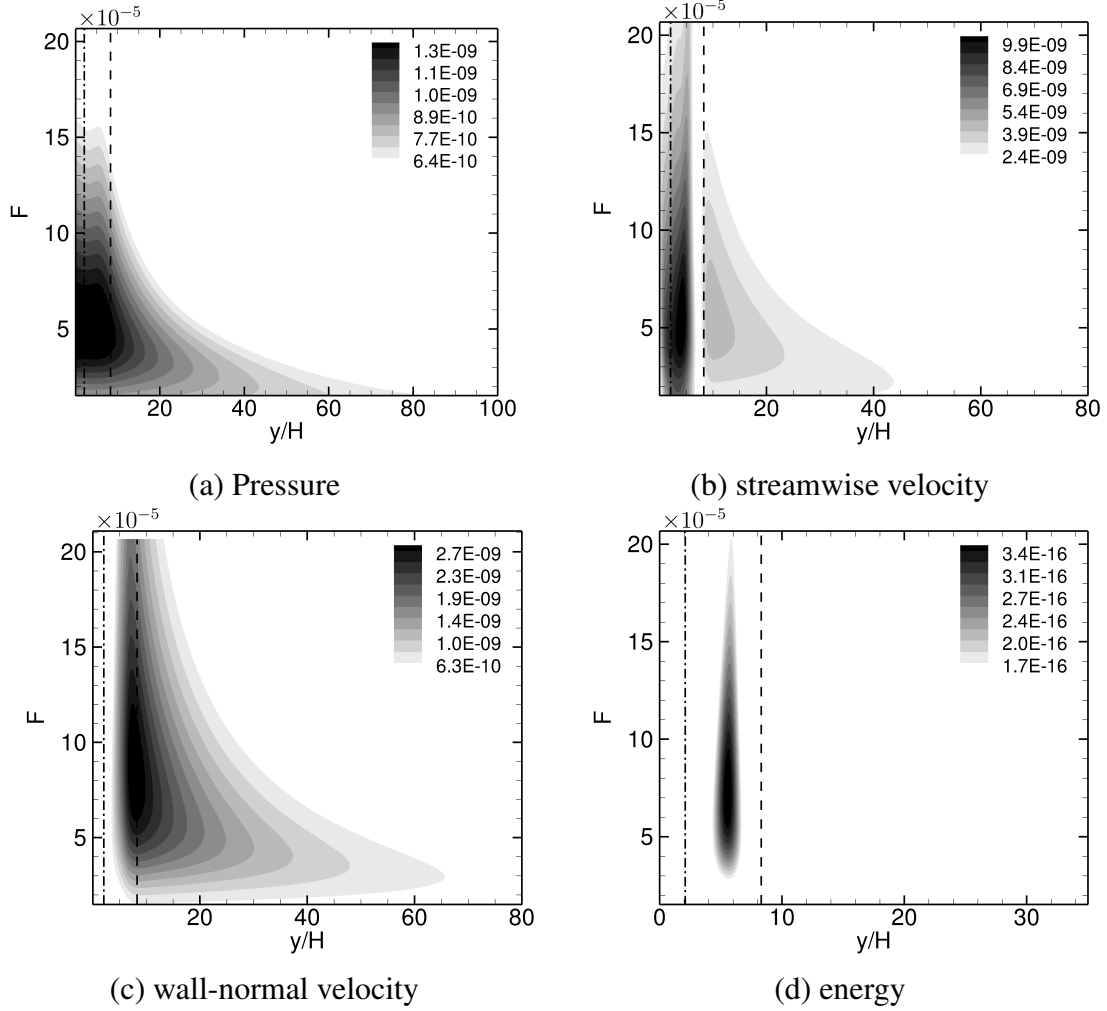


Figure 4.35: Frequency spectra of disturbance flow field projected on mode S (a)-(c) and energy (d) at $x_{pc}/H = 867.16$ and $\beta = 0$. Dashed line and dash dot line mark the boundary layer height and sonic line, respectively (adjusted from ref. [11]).

Figs. 4.35a-d show the disturbance pressure, streamwise velocity, wall-normal velocity and energy projected on mode S, respectively, at $x_{pc} = 867.16$ and $\beta = 0$ in $F - y$ plane. The pressure, streamwise velocity and energy peak below the boundary layer edge. Peak in the frequency spectrum of wall-normal velocity is found near the boundary layer edge. Figs. 4.36a-d display the disturbance pressure, streamwise velocity, wall-normal velocity and energy projected on mode S at $x/H = 1505.7$ in the $F - y$ plane. Mode S dominates the flow field downstream of the particle

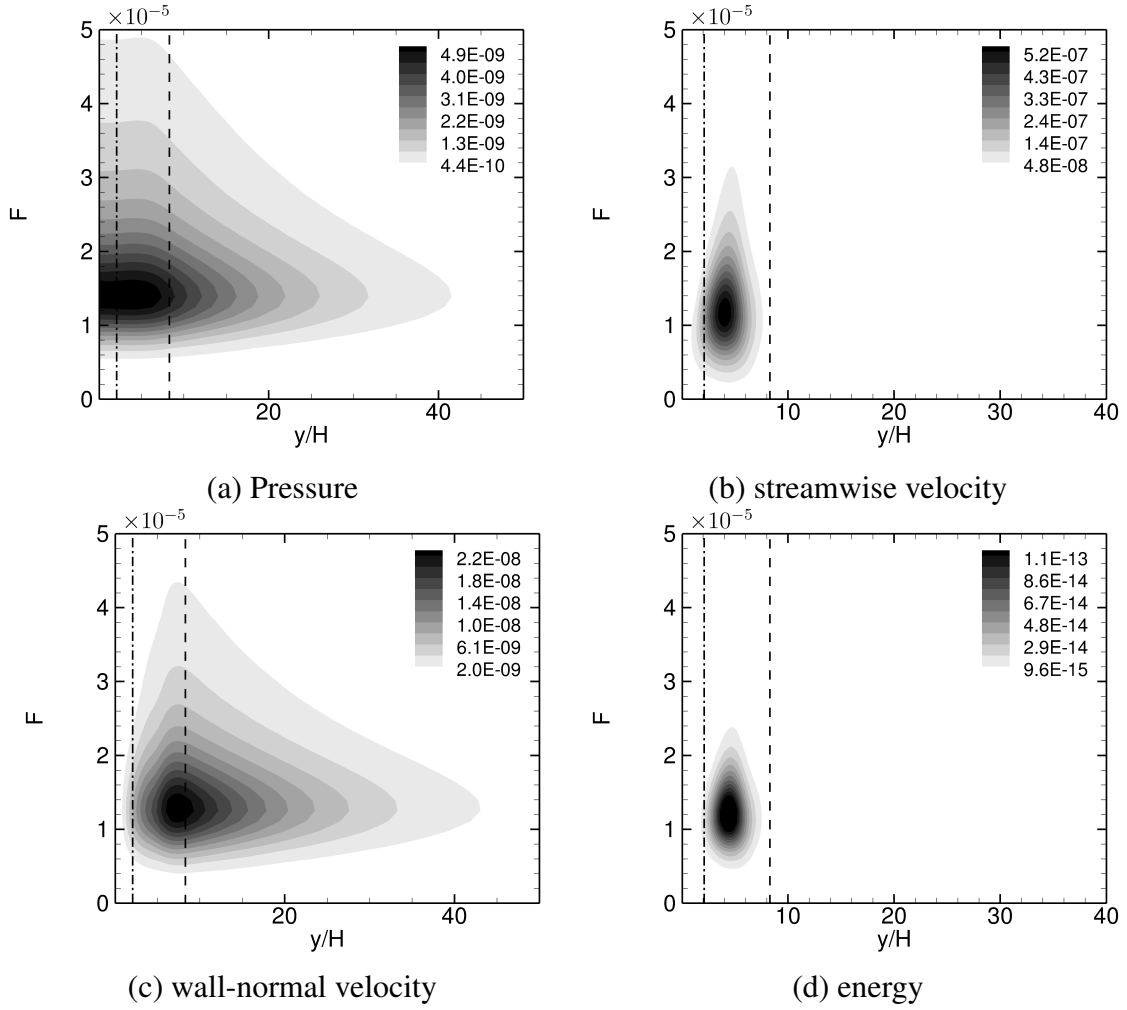


Figure 4.36: Frequency spectra of disturbance flow field projected on mode S (a)-(c) and energy (d) at $x/H = 1505.7$ and $\beta = 0.075$. Dashed line and dash dot line mark the boundary layer height and sonic line, respectively (adjusted from ref. [11]).

impingement location at $x/H = 1505.7$, showing that there is close agreement between figs. 4.32 and 4.36.

4.3.2.2 Full Reconstruction of the Disturbance Flow Field

In this section, the disturbance flow field is reconstructed from the contributions of discrete and continuous branches. Figs. 4.37a-d show the eigenvalue spectra, disturbance flow FFT and its projection on the discrete and continuous branches, coefficient of projection of SA and FA

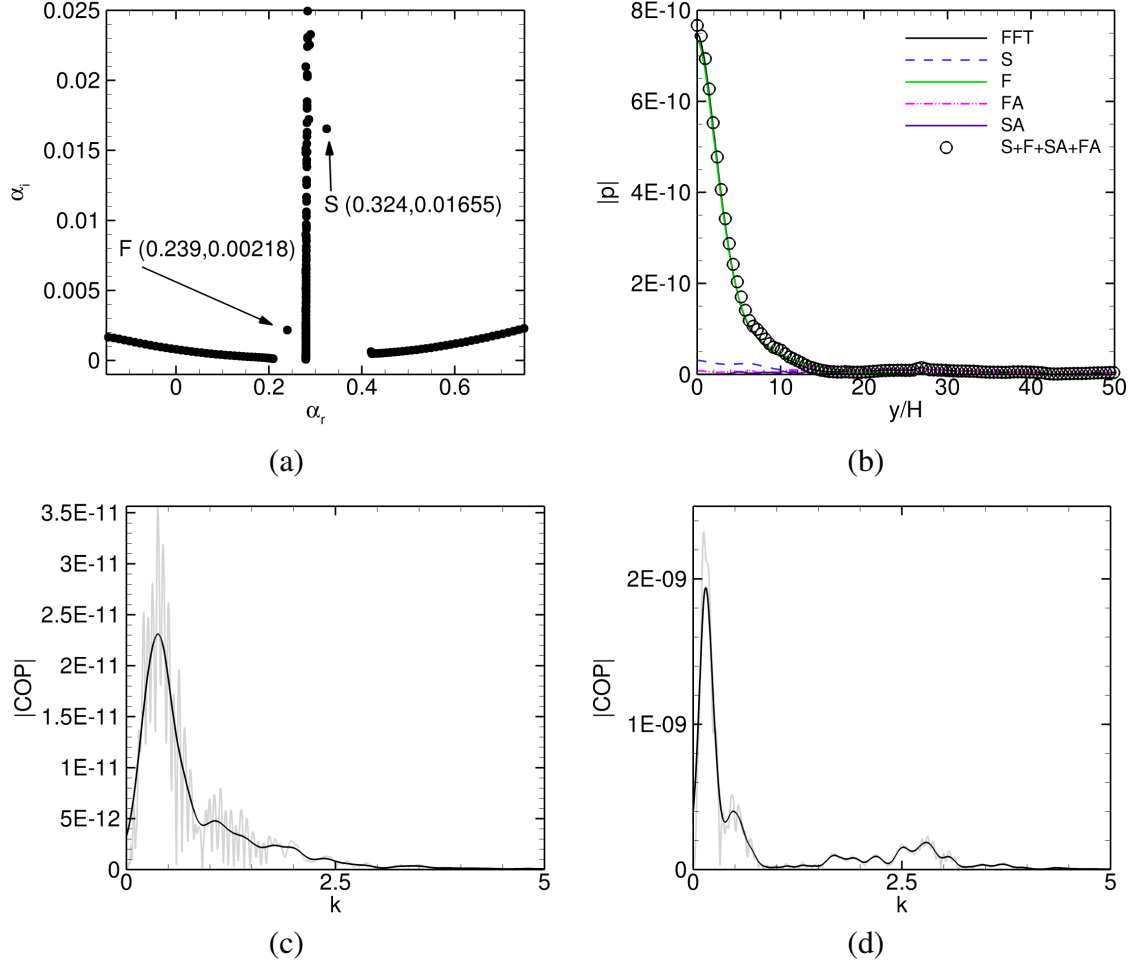


Figure 4.37: $x/H = 1069.95$, $F = 2.61 \times 10^{-4}$ and $\beta = 0$: (a) eigenvalue spectrum (b) FFT amplitude of disturbance pressure and its projection on mode F, SA and FA, (c) amplitude of coefficient of projection of SA, (d) amplitude of coefficient of projection of FA (adjusted from ref. [11]).

branches at $x/H = 1069.95$, $F = 2.61 \times 10^{-4}$ and $\beta = 0$. All the modes are stable for this condition. Most of the contribution to the disturbance flow field comes from mode F, which almost fully represents the disturbance flow FFT. Mode S contributes less to the disturbance flow field, and contributions from the SA and FA branches are almost negligible. The continuous branches consider $k = 0.001 - 5.0$ with $\Delta k = 0.001$. The integration within the k range is performed by employing the trapezoidal method.

A different frequency, F , and wavenumber, β pair is considered for the same location.

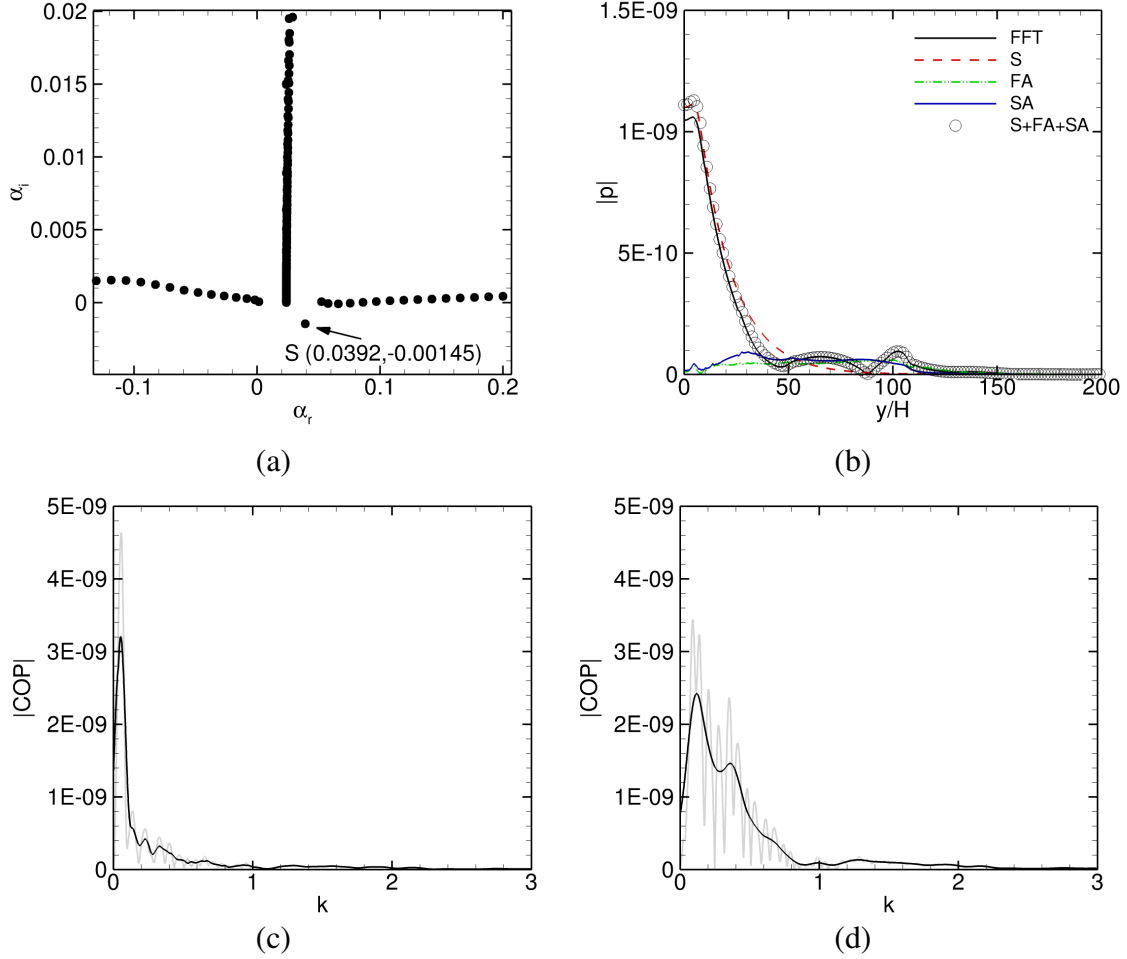


Figure 4.38: $x/H = 1069.95$, $F = 2.22 \times 10^{-5}$ and $\beta = 0.0656$: (a) eigenvalue spectrum (b) FFT amplitude of disturbance pressure and its projection on mode F, S, SA and FA, (c) amplitude of coefficient of projection of SA, (d) amplitude of coefficient of projection of FA branch (adjusted from ref. [11]).

Figs. 4.38a-d display the eigenvalue spectra, disturbance flow FFT and its projection on discrete and continuous branches, coefficient of projection of SA and FA branches at $x/H = 1069.95$, $F = 2.22 \times 10^{-5}$ and $\beta = 0.0656$. Mode S is unstable at this frequency and wavenumber. Inside the boundary layer mode S has major contribution to the disturbance flow field. Outside the boundary layer both SA and FA branches contribute to the development of the disturbance flow field. The continuous branches are integrated within the range of $k = 0.001 - 3.0$ with step size of $\Delta k = 0.001$.

4.4 Disturbance Flow Field in a Mach 9.5 14° wedge

4.4.1 Particle Induced Disturbance Flow Field

M_1	M_2	p_2 [Pa]	T_2 [K]	T_{wall} [K]	T_{wall}/T_2
9.5	5.4	22951.65	621.14	300	0.48

Table 4.5: Freestream after-shock conditions and wall temperature ratios for Mach 9.5 14° wedge.

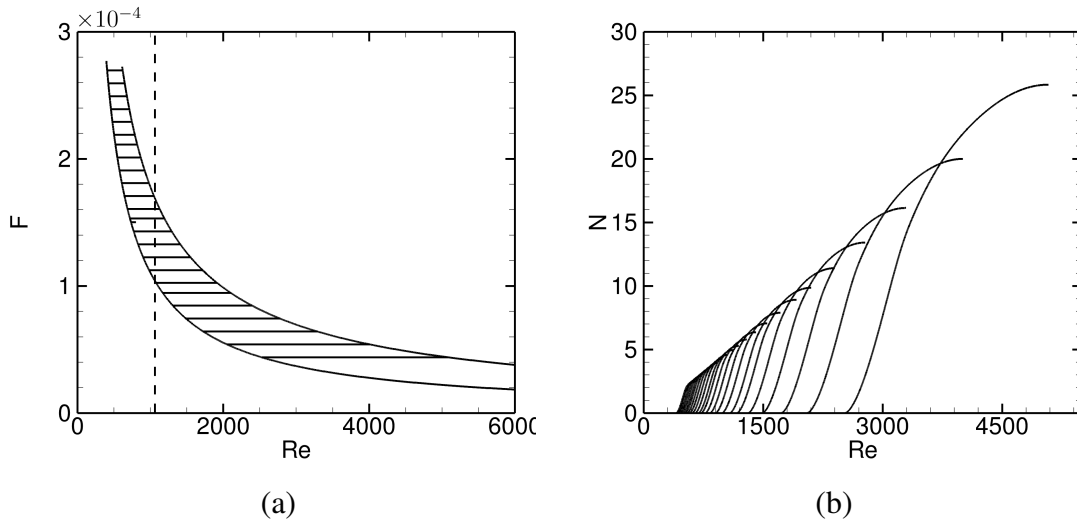


Figure 4.39: (a) Neutral curves at $\beta = 0$ with frequencies used for calculating N-factors, vertical dashed line marks the particle impingement location, and (b) N-factor distributions at $\beta = 0$ for the Mach 9.5 14° wedge (adjusted from ref. [11]).

In this section the particle is assumed to impinge on a Mach 9.5 14° wedge boundary layer and follows the analysis as reported in ref. [11]. The freestream properties are provided in table 4.5. An isothermal wall is considered with the temperature of 300K. The wall to freestream aftershock temperature ratio is 0.48. As predicted by the LST analysis in Ref [11], this is a second mode-dominated flow field, which means it is dominated by 2D waves at $\beta = 0$. Figs. 4.39a and 4.39b show the neutral curves and the N-factor distributions at $\beta = 0$. The frequencies considered for N-factor curves are marked as horizontal lines inside the neutral curves, and the

particle impingement location is marked by vertical dashed line. The particle impinges on the wall at $x/H = 1060.89$. The particle undergoes an elastic collision with the wall. The N-factor curve at $x/H \approx 1200$ reaches to the N-factor of 10 at $x/H \approx 1800$, which shows that transition to turbulence will take place within the computational domain.

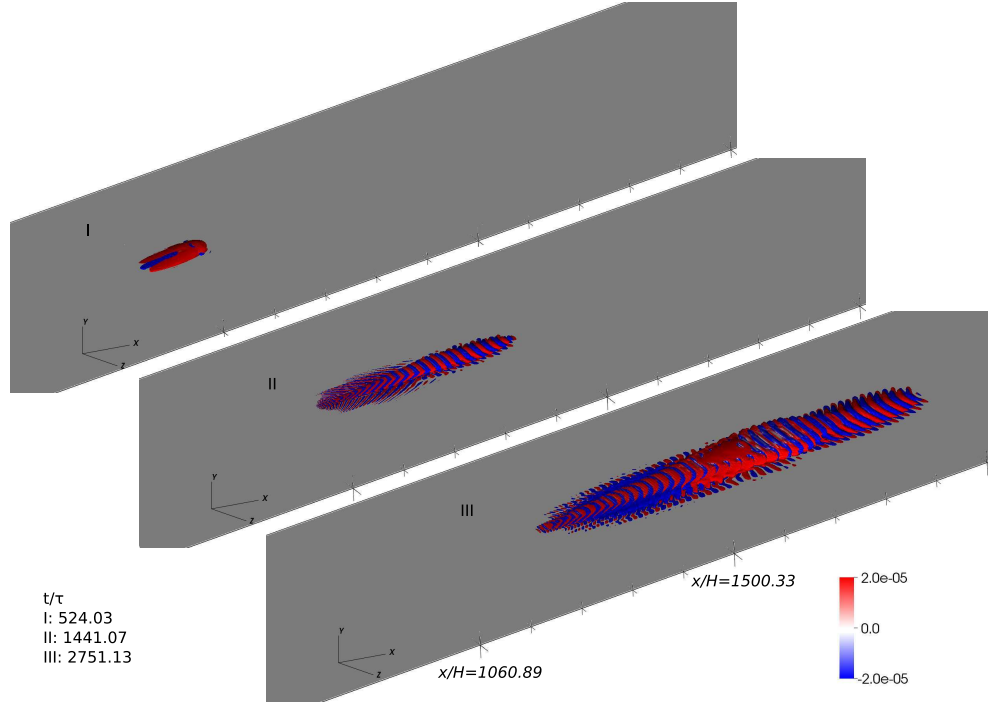


Figure 4.40: Pressure ($p/\rho U_\infty^2$) disturbance flow field in a $M=9.5$ 14° wedge with $T_{wall} = 300K$. τ is based on the particle impingement location (adjusted from ref. [11]).

Fig. 4.40 displays the disturbance pressure field at three time instances. As can be seen, the particle impingement introduces small scale disturbances inside the boundary layer, which grow in size from instance I to III and form a wavepacket. This second mode dominated wavepacket propagates in the downstream direction.

Figs. 4.41a-c show the disturbance spectra in $F - \beta$ plane for $x/H = 1055.58$, 1073.02 and 1166.5 . Near the particle impingement a wide range of frequencies and wavenumbers get excited. The unstable frequency range is marked as green dashed line in fig. 4.41a. Further downstream

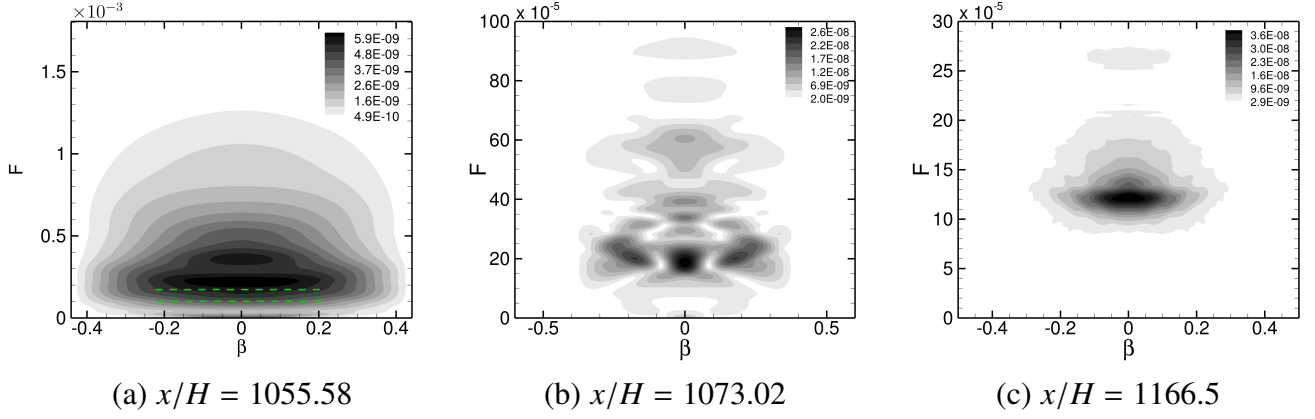


Figure 4.41: Frequency spectra of wall disturbance pressure at three streamwise positions for $M=9.5$ 14° boundary layer flow. Green horizontal dashed lines mark the unstable region (adjusted from ref. [11]).

the non-linear mechanisms are already in seen as can be observed from the disturbance spectra broadening [11].

Next, the frequency spectra are studied above the wall. Figs. 4.42a-d display the frequency spectra of disturbance pressure, streamwise velocity, wall-normal velocity, and energy at $x/H = 1112.67$, respectively. The frequency spectra of pressure and wall-normal velocity show peaks below the boundary layer edge as well as in the freestream. The peak in the freestream is related to the acoustic waves reflected at the wall and traveling in the freestream. The streamwise velocity shows a peak below the boundary layer edge at low frequency. Similarly, the energy peaks at a lower frequency.

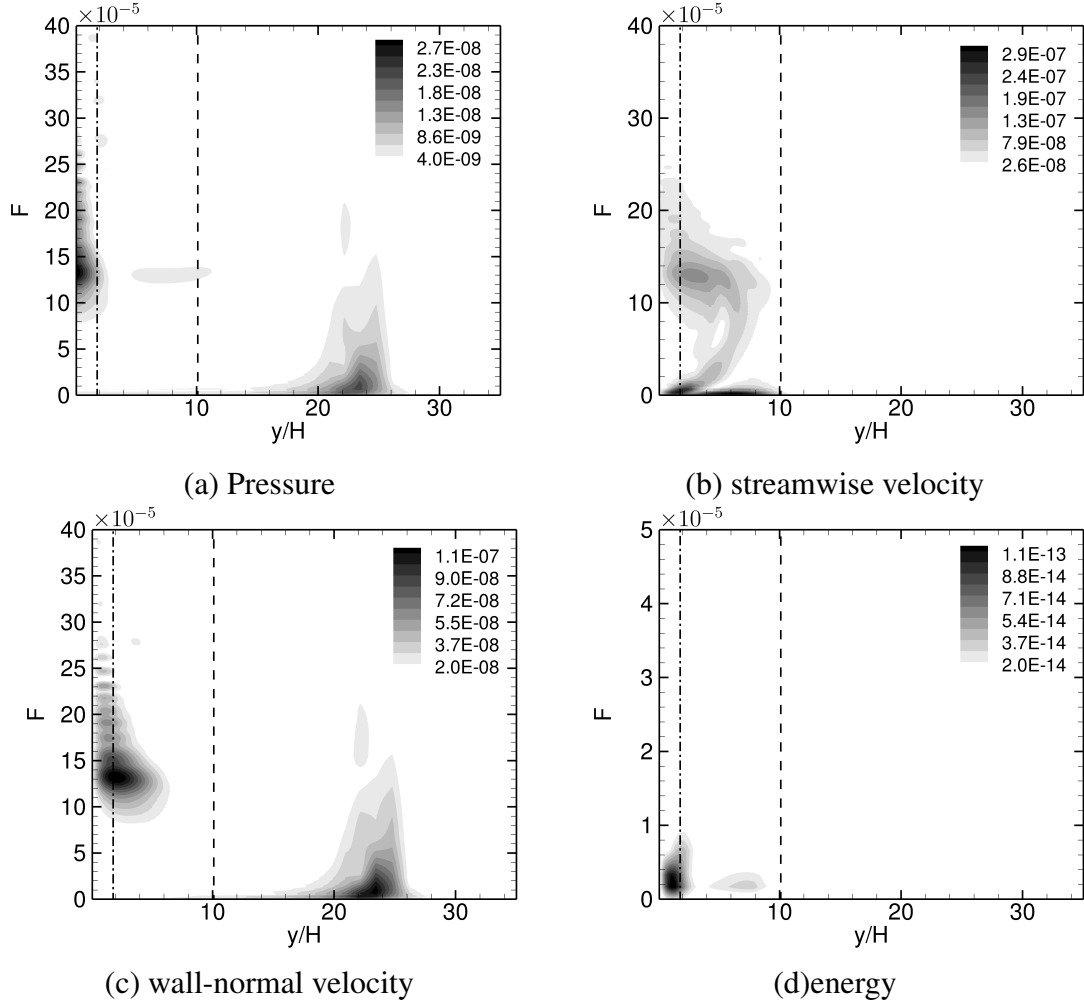


Figure 4.42: Frequency spectra of different disturbance flow quantities (a)-(c) and energy (d) at $x/H = 1112.67$ and $\beta = 0$ in $M=9.5$ 14° wedge with $T_{wall} = 300K$. Dashed line and dash dot line mark the boundary layer height and sonic line, respectively (adjusted from ref. [11]).

4.4.2 Biorthogonal Decomposition of the Disturbance Flow Field

The disturbance flow field is projected on mode S in $F - \beta$ plane. Figs. 4.43b and 4.43e show the projection on mode S at $x/H = 1112.67$ and $x/H = 1299.3$, respectively. The projection on mode S at $x/H = 1112.67$ shows peaks at $F \approx 1.3 \times 10^{-4}$ and $F \approx 1.4 \times 10^{-4}$. Fig. 4.43c shows the projection ratio. The projection ratio is high at this location, which implies that mode S is already dominating at this location [11]. Further downstream at $x/H = 1299.3$, the projection on

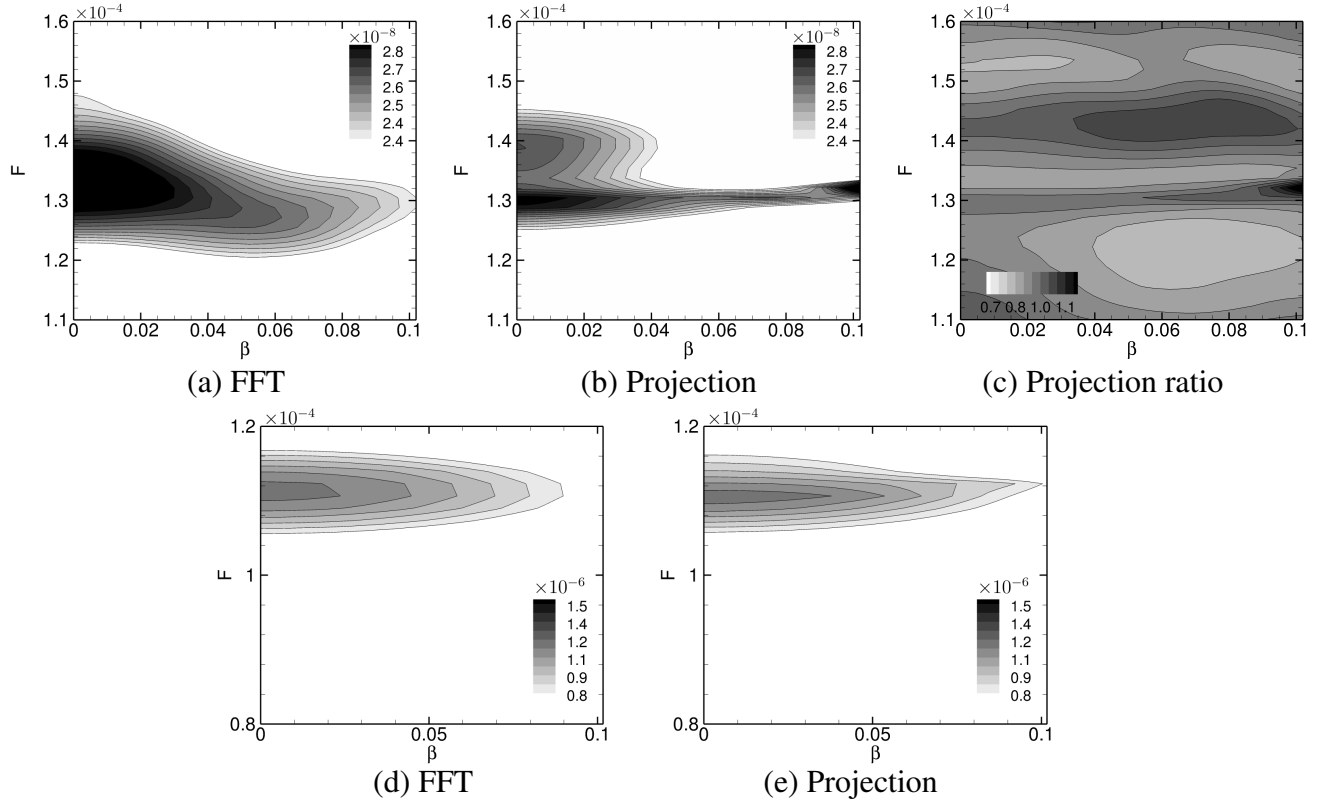


Figure 4.43: (Case 3) Frequency spectra of wall pressure disturbance at (a) $x/H = 1112.67$, (d) $x/H = 1299.3$, projection on mode S at (b) $x/H = 1112.67$, (e) $x/H = 1299.3$, and (c) projection ratio (projection/FFT) at $x/H = 1112.67$ (adjusted from ref. [11]).

mode S and FFT of the disturbance wall pressure are in close agreement as mode S dominates the flow field.

Next, the disturbance flow field above the wall is projected on mode S. Figs. 4.44a-d show the pressure, streamwise velocity, wall-normal velocity, and energy projected on mode S, respectively, at $\beta = 0$ and $x/H = 1112.67$. The peak occurs below the boundary-layer edge and near to the sonic line for all the quantities.

Finally, the contributions from the discrete and continuous branches are considered to fully reconstruct the disturbance flow field. Fig. 4.45a-d show the eigenvalue spectra, FFT of disturbance field as well as projection on discrete and continuous branches, coefficient of projection of SA branch and FA branch, respectively, at $x/H = 1112.67$, $F = 1, 34 \times 10^{-4}$ and

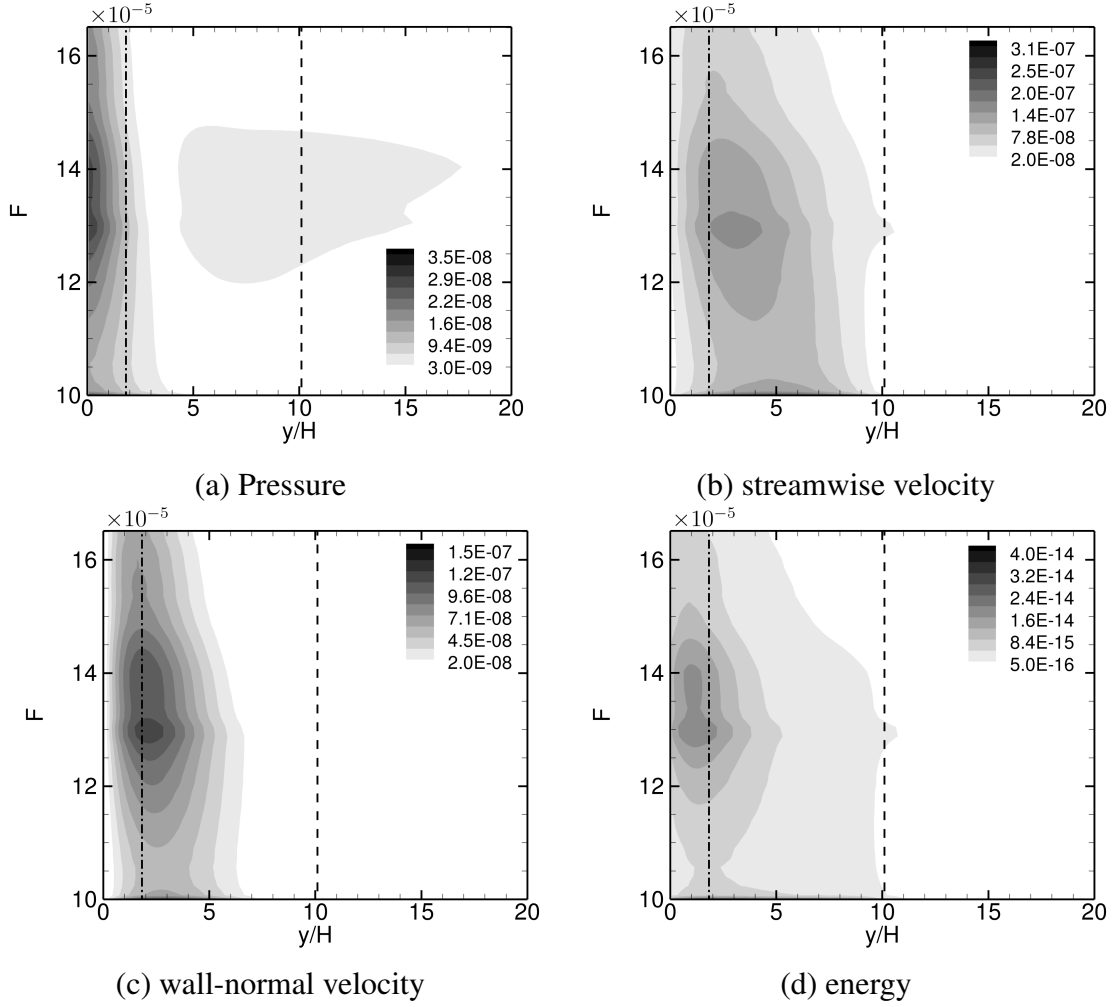


Figure 4.44: Frequency spectra of disturbance flow field projected on mode S (a)-(c) and energy (d) at $x/H = 1112.67$ and $\beta = 0$. Dashed line and dash dot line mark the boundary layer height and sonic line, respectively (adjusted from ref. [11]).

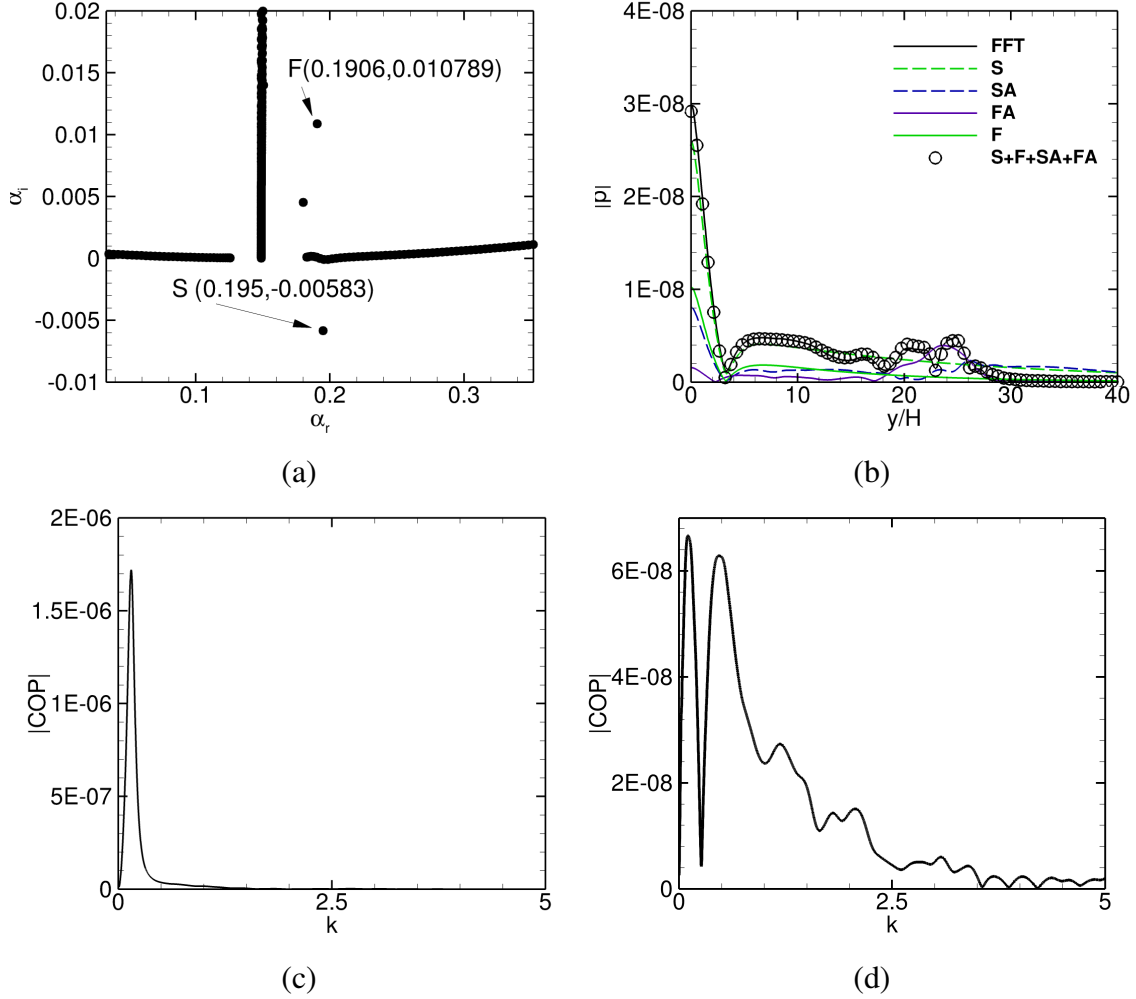


Figure 4.45: (a) Eigenvalue spectrum at $x/H = 1112.67$, $F = 1.34 \times 10^{-4}$ and $\beta = 0$, (b) FFT pressure amplitude and its projection on S, F, SA and FA and amplitude of COP of (c) SA branch and (d) FA branch (adjusted from ref. [11]).

$\beta = 0$. In fig. 4.45b, the mode S contributes significantly to the disturbance flow field inside the boundary layer. The boundary-layer edge is at $\delta_{99} = 10.1H$ [11]. Mode F, SA branch and FA branch also contributes to the disturbance flow field inside the boundary layer. In the freestream, mode S and FA contributes significantly to the development of the disturbance pressure field. The pressure profile is reconstructed fully by considering the contributions from the discrete and continuous branches.

Chapter 5: Fully Resolved Particle Induced Disturbance Flow Field

In this section, the PSIC simulation approach is directly compared with the fully resolved particle simulation by using a higher-order IBM for a Mach 5.35 flat plate boundary-layer flow. A detailed grid sensitivity study is done for solution accuracy. The particle is assumed to impinge at different locations inside the neutral curve with different particle sizes.

5.1 Grid Sensitivity Study

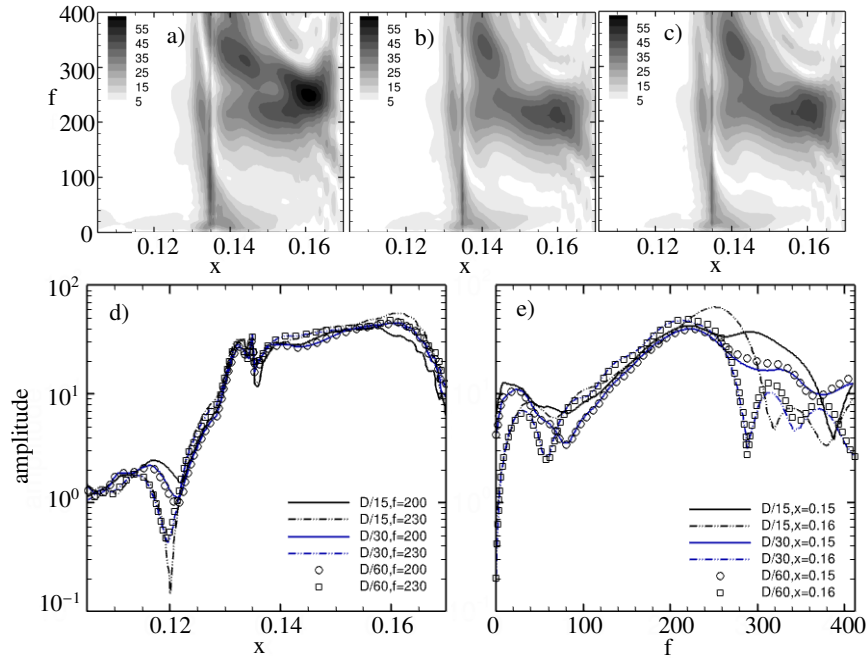


Figure 5.1: Amplitude plots for the IBM simulations with different grid resolutions: (a) D/15, (b) D/30 and (c) D/60 as well as amplitude vs x -location in (d) and amplitude vs frequency in (e). x -location is in meters (adjusted from ref. [3]).

Three different grid resolutions have been considered to study the fully resolved particle induced disturbance flow field simulation by the IBM in two dimensions. The particle of radius $50\mu\text{m}$ is considered for the grid sensitivity study. The initial position of the particle is at $\mathbf{x}_p = (0.048, 0.01073)^T$ in the computational domain. The particle moves with the freestream velocity with an approach angle of $\theta_p = -7^\circ$ and is assumed to have an elastic collision with the surface of the flat plate. Figs. 5.1a-c show the amplitude plot of the disturbance flow simulation performed by considering D/15, D/30 and D/60 grid resolutions. Figs. 5.1d and 5.1e display the amplitude curves extracted at different frequencies and streamwise locations. D/30 and D/60 grid resolutions are in close agreement in these figures. For further study, the D/30 grid is employed to simulate a $R_p = 100\mu\text{m}$ particle traveling in a stationary fluid with $\text{Re}_D = 40$. The steady wake dimension is measured as length of separation, $L/D = 2.28$, distance from cylinder wall to the vortex center, $a/D = 0.75$ and $b/D = 0.6$, separation angle, $\Theta = 55.1^\circ$, and drag coefficient of $C_D = 1.38$. These wake parameters are in close agreement with the simulation performed with D/60 grid resolution (see table 3.5). Fig. 5.2 displays the spanwise vorticity contour with streamlines from the simulation of $100\mu\text{m}$ particle traveling with $\text{Re}_D = 40$.

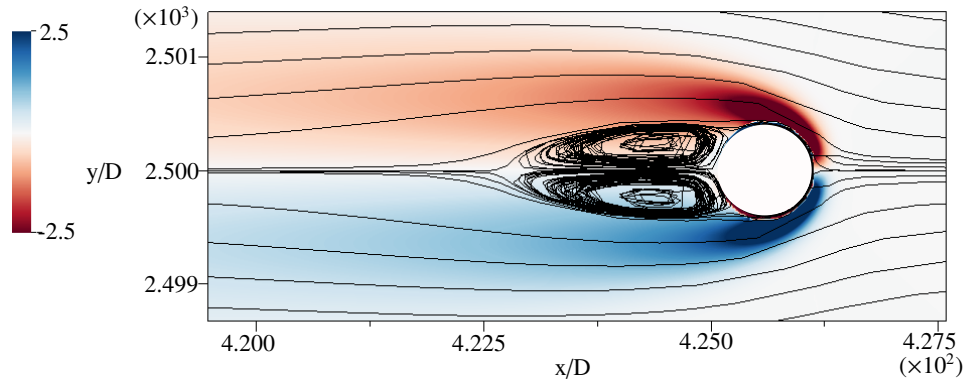


Figure 5.2: Instantaneous pseudocolor plot of spanwise vorticity with streamlines for the cylinder flow with $\text{Re}_D=40$, simulated with D/30 grid resolution (adjusted from ref. [3]).

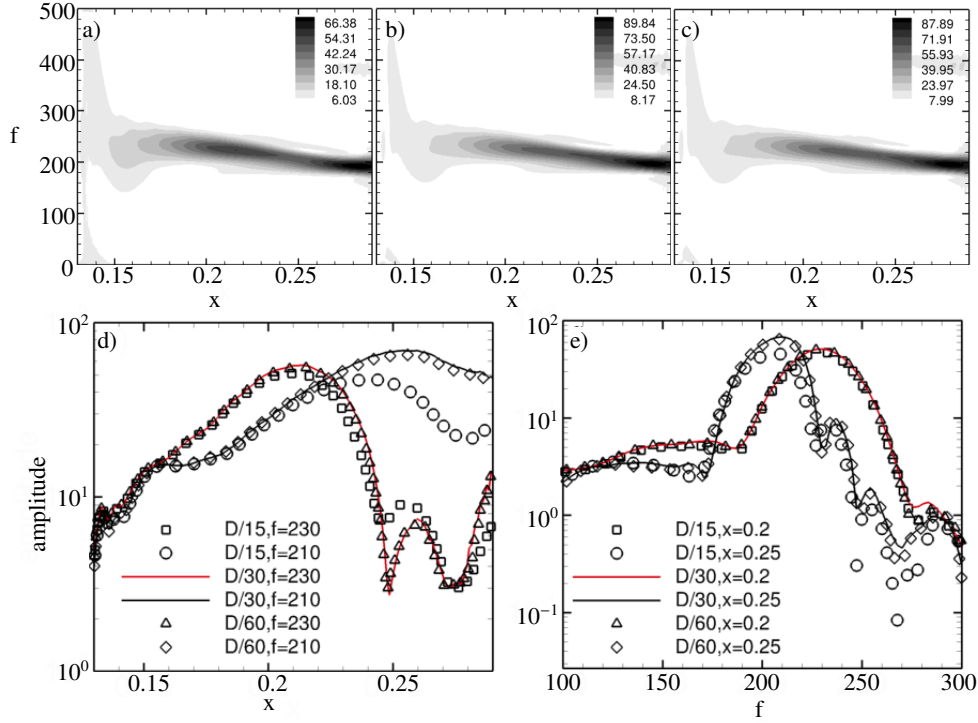


Figure 5.3: Amplitude plots for the PSIC simulations with $\sigma = 2.5R_p$ and different grid resolutions: (a) D/15, (b) D/30 and (c) D/60 as well as amplitude vs x-location in (d) and amplitude vs frequency in (f). x-location is in meters (adjusted from ref. [3]).

Next, the grid sensitivity study is performed for the PSIC approach. Similar to the IBM simulations the PSIC simulations consider three different grid resolutions, e.g., D/15, D/30 and D/60. The particle is introduced at the same location as the IBM simulation above. A particle of radius $R_p = 100\mu\text{m}$ is considered for the grid sensitivity study. A Gaussian half width of $\sigma = 2.5R_p$ is considered for the simulations. The particle undergoes an elastic collision with the wall. Figs. 5.3a-c display the amplitude plot of the simulation performed by D/15, D/30 and D/60 grid resolution, respectively. The amplitude plots show similar characteristics. Figs. 5.3d and 5.3e show the amplitude development at two frequencies and two streamwise locations, respectively. Amplitude curves for D/15 resolution show slightly lower amplitude than the other two grid resolution. Because D/30 and D/60 grid resolutions show close agreement, D/30 grid resolution is considered for further study in three dimensions for the PSIC approach.

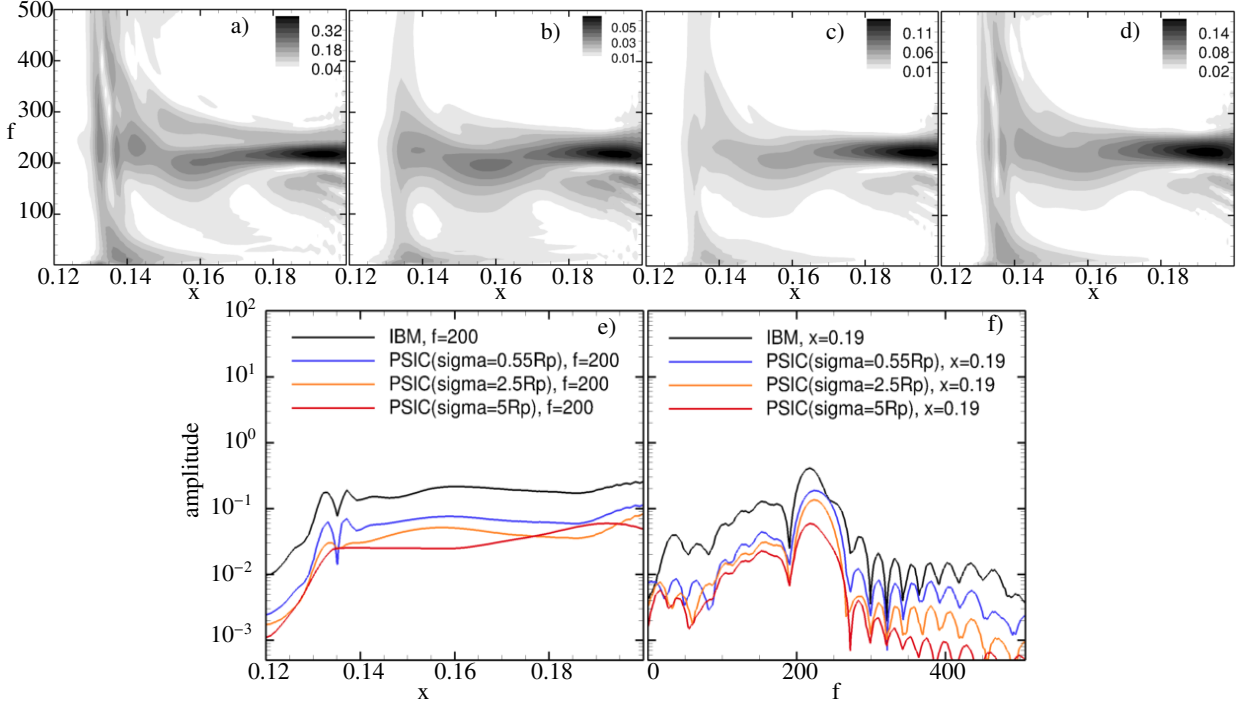


Figure 5.4: Amplitude plots for the PSIC simulations with different σ : (a) IBM , (b) $5R_p$, (c) $2.5R_p$ (d) $0.55R_p$ as well as amplitude vs x-location in (e) and amplitude vs frequency in (f). x-location is in meters (adjusted from ref. [3]).

Finally, different half widths of the Gaussian distribution are considered. Half widths of $0.55R_p$, $2.5R_p$ and $5R_p$ are considered to perform a sensitivity study. The particle and simulation setups are similar to previous PSIC and IBM simulations except the simulations are performed in three dimensions now. As previously mentioned, D/30 grid resolution is considered for this study. The amplitude plot is shown at $k=0$ for IBM and PSIC simulations in figs. 5.4a-d. The peak occurs at the same frequency range for all four simulations. However, the IBM shows higher amplitude than the other three simulations. Fig. 5.4e shows the amplitude development in the streamwise direction. All the PSIC simulations show lower amplitude than the IBM simulations. Fig. 5.4f shows the amplitude curves at $x=0.19$ m. The amplitude curves show lower amplitude than the IBM simulations, similar as before. Gaussian half width of $2.5R_p$ is considered for the comparison of IBM and PSIC approach.

5.2 Comparison of PSIC and IBM Results

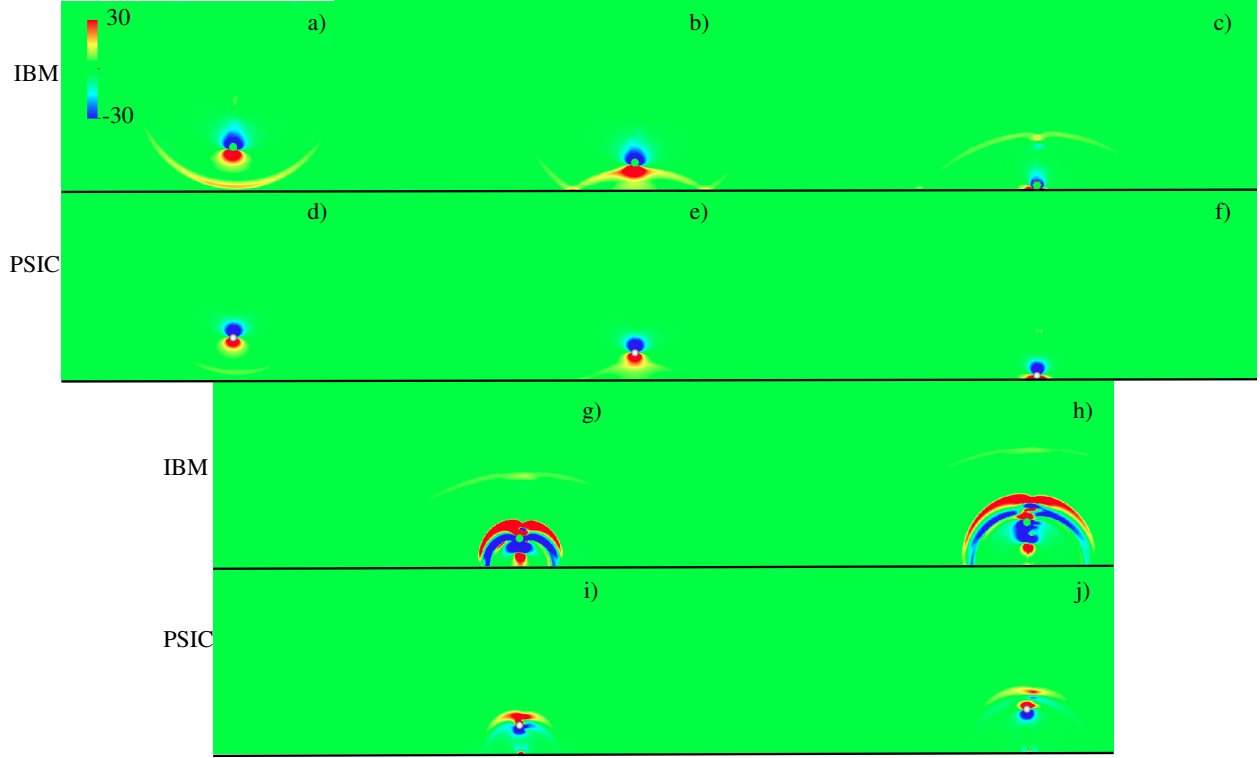


Figure 5.5: Disturbance pressure field of (a,b,c,g,h) IBM and (d,e,f,i,j) PSIC simulations, (a,b,d,e) before particle impingement, (e,f) near particle impingement and (g,h,i,j) after particle impingement for a particle traveling with $Ma_p = 0.65$.

The simulations are performed in a computational domain with dimensions $[0.01m, 0.61m]$ in the streamwise direction, $[0.0m, 0.1m]$ in the wall-normal direction and $[0.0m, 0.05m]$ in the spanwise direction. The particle is introduced at $y_p = 0.0173m$ with an approach angle of $\theta_p = -7^\circ$. Eight different simulation cases are considered. The details of the simulation setup are provided in table 5.1.

First, as the receptivity process is very complicated, particle impingement simulations have been performed removing the presence of the boundary layer. A free slip boundary condition is considered at the wall. Figs. 5.5a-j show the disturbance pressure field at five time instances

for the IBM and PSIC approaches with $Ma_p = 0.65$. Two time instances are considered prior to particle impingement, one near particle impingement, and the last two additional snapshots after the particle impingement. Initially, the fully resolved simulation shows a strong acoustic wave ahead of the particle in fig. 5.5a. After particle impingement, the IBM simulations show higher amplitudes as well. The wall pressure signature is shown in fig. 5.7b. There are differences in the wall pressure signature between the IBM and PSIC approach. The IBM simulation shows a higher amplitude than the PSIC simulation and the difference is maximal near to the particle impingement location (t6).

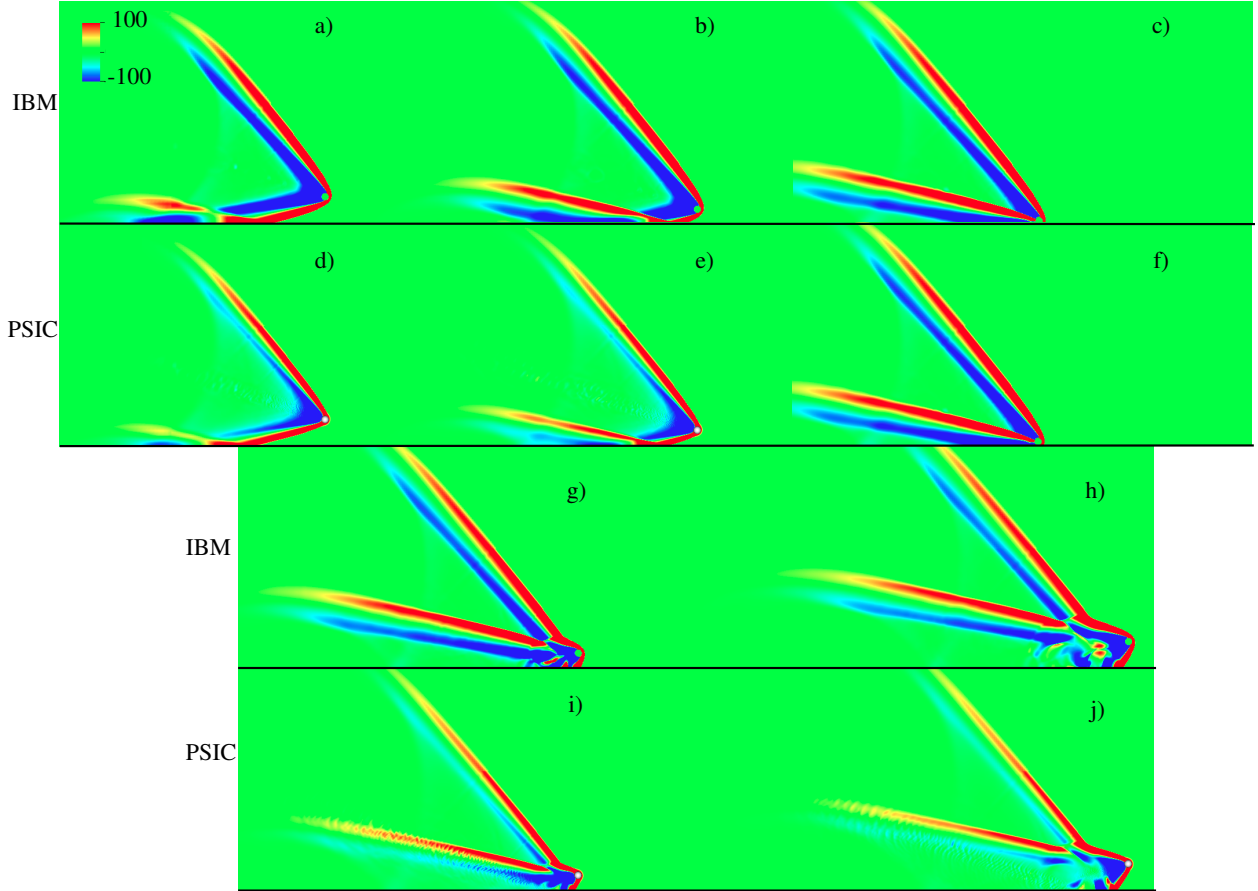


Figure 5.6: Disturbance pressure field of (a,b,c,g,h) IBM and (d,e,f,i,j) PSIC simulations, (a,b,d,e) before particle impingement, (e,f) near particle impingement and (g,h,i,j) after particle impingement for a particle traveling with $Ma_p = 2$.

Next, the relative Mach number is increased to study the differences between the IBM and PSIC approaches. Figs. 5.6a-j display the disturbance pressure fields at five time instances for the particle traveling with $Ma_p = 2$. A shock forms ahead of the particle. The general flow features look very similar while some differences can be seen very close to the wall. Fig. 5.7b shows the wall pressure signature at thirteen time instances, including before particle impingement, near particle impingement, and after particle impingement. The IBM approach shows larger amplitudes than the PSIC simulation at all steps. The difference is large near to the particle impingement location (t_7). These results when neglecting the effect of boundary layer clearly demonstrate the difference between the two simulation approaches.

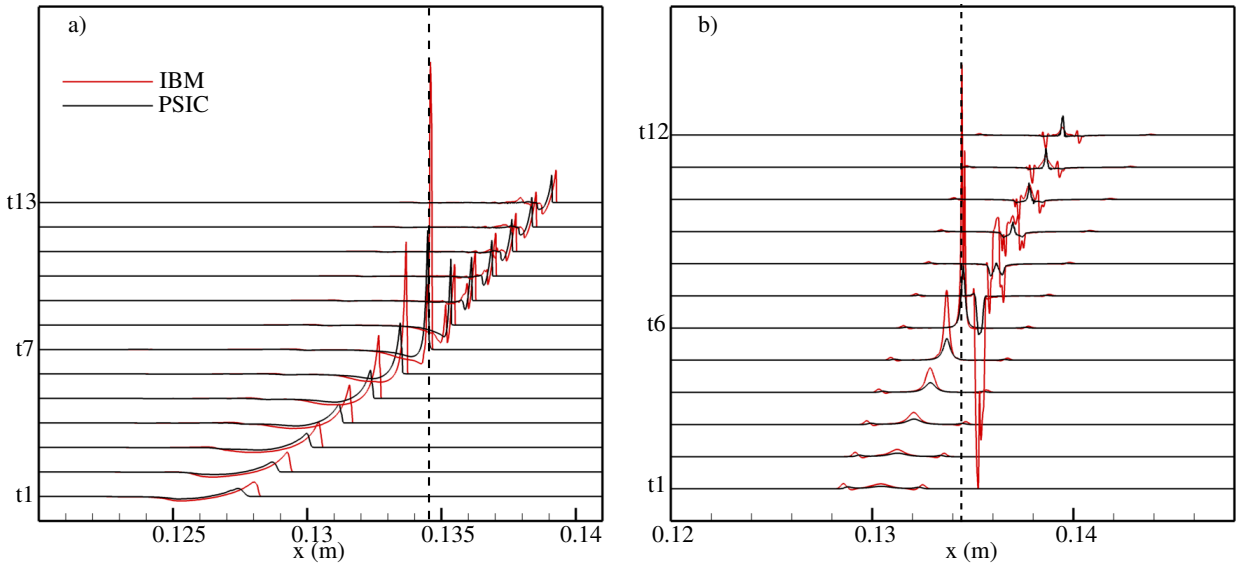


Figure 5.7: Wall pressure signature for particle traveling with (a) $Ma_p = 2$ and (b) $Ma_p = 0.65$. Vertical dashed lines mark the impingement location.

Next, the particle drag coefficient is compared for the cases I-X048R100 and P-X048R100. Fig. 5.8a shows the drag coefficients, C_D , along the trajectory of the particle. The drag coefficient varies as the particle travels in the freestream, impinges on the boundary layer and bounces off the wall. The drag coefficients are approximately 0.79 and 0.71 for the PSIC and IBM approaches

Case	Type	$R_p(\mu m)$	$x_p(t=0)$	x_{pc}
I-X048R100	IBM	100	0.048	0.135
P-X048R100	PSIC	100	0.048	0.135
I-X183R100	IBM	100	0.183	0.27
P-X183R100	PSIC	100	0.183	0.27
I-X318R100	IBM	100	0.3185	0.405
P-X318R100	PSIC	100	0.3185	0.405
I-X048R50	IBM	50	0.048	0.135
P-X048R50	PSIC	50	0.048	0.135

Table 5.1: Details of the particle induced disturbance flow simulations in 3D.

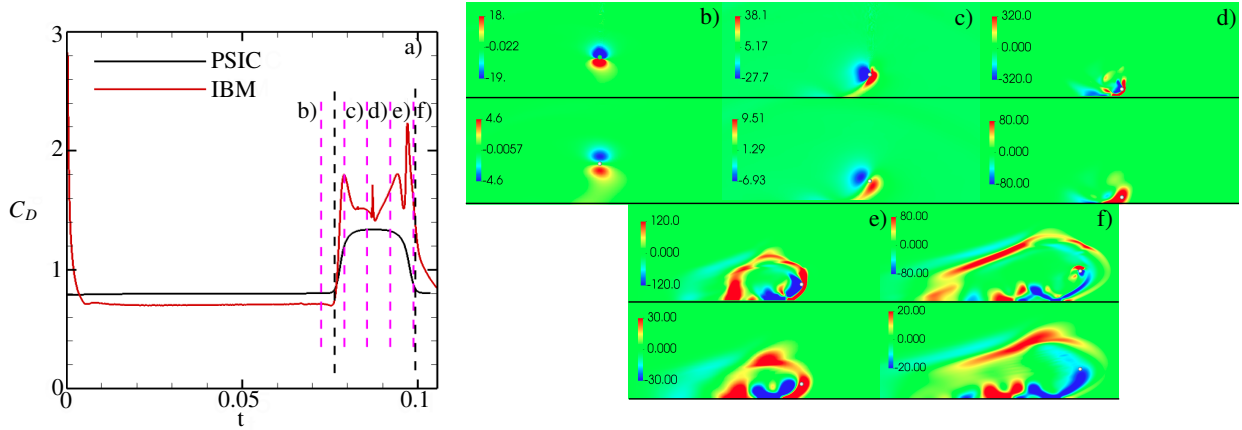


Figure 5.8: a) Drag coefficient, C_D , in time following different approaches for the cases I-X048R100 and P-X048R100. Vertical black dashed lines mark near the time when particle enters and leaves the boundary layer. Vertical purple dashed lines mark the position of the pressure flow field to the right. Time is normalized by L/U_∞ . b)-f) Disturbance pressure field corresponding to the vertical purple dashed lines. Top pseudocolor plots are for the IBM and bottom is for the PSIC approach.

in the freestream, respectively. The C_D reaches a peak value of 1.3 inside the boundary layer for the PSIC approach. Inside the boundary layer the IBM simulation shows a peak value of 2.23. Figs. 5.8b-f show the disturbance pressure field at five time instances. Figs. 5.8b and 5.8c show the disturbance flow field before particle impingement, fig. 5.8d shows the disturbance flow field near the particle impingement, and figs. 5.8e and 5.8f show the disturbance flow fields after particle impingement. While the general flow features again look very similar the fully-resolved simulation shows higher amplitudes than the PSIC simulation.

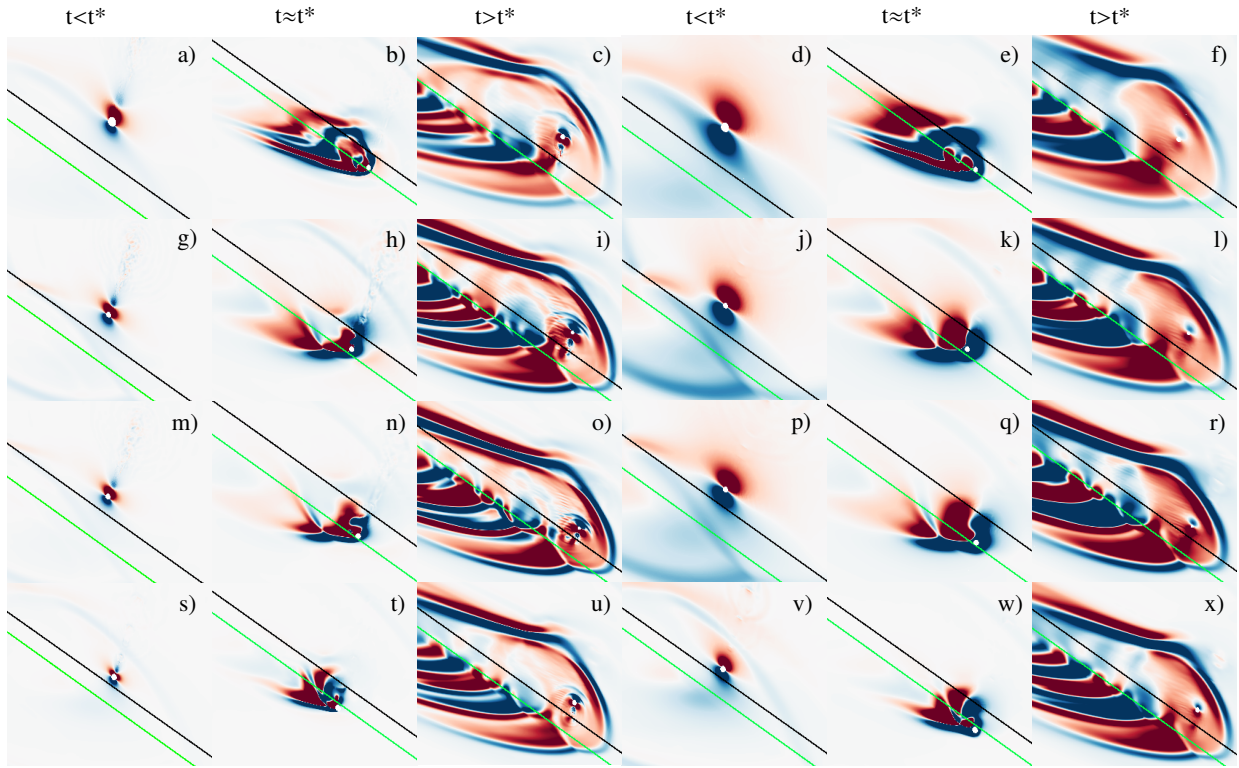


Figure 5.9: Instantaneous pseudocolor plot of pressure disturbance flow field a,g,m,s,d,j,p,v: before particle impingement, b,h,n,t,e,k,q,w: near particle impingement and c,i,o,u,f,l,r,x: after particle impingement for IBM (a,b,c,g,h,i,m,n,o,s,t,u) and PSIC (d,e,f,j,k,l,p,q,r,v,w,x) for the cases I-X048R100 (a,b,c), P-X048R100 (d,e,f), I-X183R100 (g,h,i), P-X183R100 (j,k,l), I-X318R100 (m,n,o), P-X318R100 (p,q,r), I-X048R50 (s,t,u) and P-X048R50 (v,w,x) (adjusted from ref. [3]).

Figs. 5.9a-x display the disturbance pressure field for all eight cases. The disturbance flow fields are shown for before particle impingement, near particle impingement, and after particle impingement. The green line marks the sonic line and the black line marks the boundary layer height δ_{99} . The pseudocolor plots are in close agreement before and after particle impingement. There are, however, some differences between the IBM and PSIC approaches near to the particle impingement location which could be due to the fine details resolved by the IBM simulations [3].

Next, to obtain a better understanding about the pressure signature perceived on the surface, the envelopes of the pressure disturbances at the wall are compared by employing the Hilbert transformation. First, the Hilbert transformation is applied to the wall pressure in the spanwise

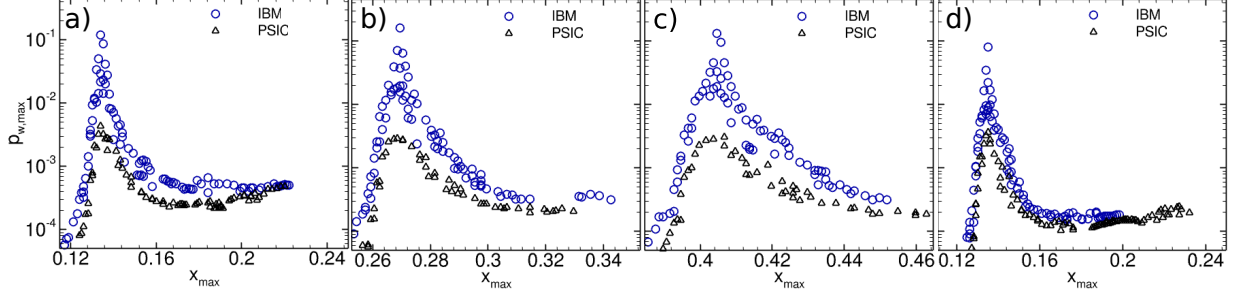


Figure 5.10: Maximum of absolute Hilbert transformation of wall disturbance pressure (normalized by $\rho_\infty U_\infty^2$) for cases: a) I-X048R100 and P-X048R100, b) I-X183R100 and P-X183R100, c) I-X318R100 and P-X318R100, d) I-X048R50 and P-X048R50 (adjusted from ref. [3]).

direction, z , for each streamwise location, x , [8]

$$\tilde{p}_w(x, z_{max}) = \max_{z, x=const.} |\mathbf{H}[\tilde{p}_w(x, z, t_n)]|. \quad (5.1)$$

then the maximum in x is determined as

$$\tilde{p}_{w,max} \equiv \tilde{p}_{w,max}(x_{max}, z_{max}) = \max_x \tilde{p}_w(x, z_{max}). \quad (5.2)$$

Figs. 5.10a-d show the maximum of the absolute Hilbert transformation of wall pressure for all eight cases considered in this study. The IBM simulations show higher amplitudes than the PSIC simulations. The difference is maximum at the particle impingement location and less further downstream.

Next, the disturbance flow field is studied in the frequency-wavenumber domain. Figs. 5.11a-c show the disturbance spectra for the IBM simulations at $x=0.135\text{m}$, 0.155m and 0.185m , respectively. The disturbance spectra for the PSIC simulations at the same locations are displayed in figs. 5.11e-g. The frequency is calculated as, $f=f^*L/U_\infty$. f^* is the dimensional frequency in

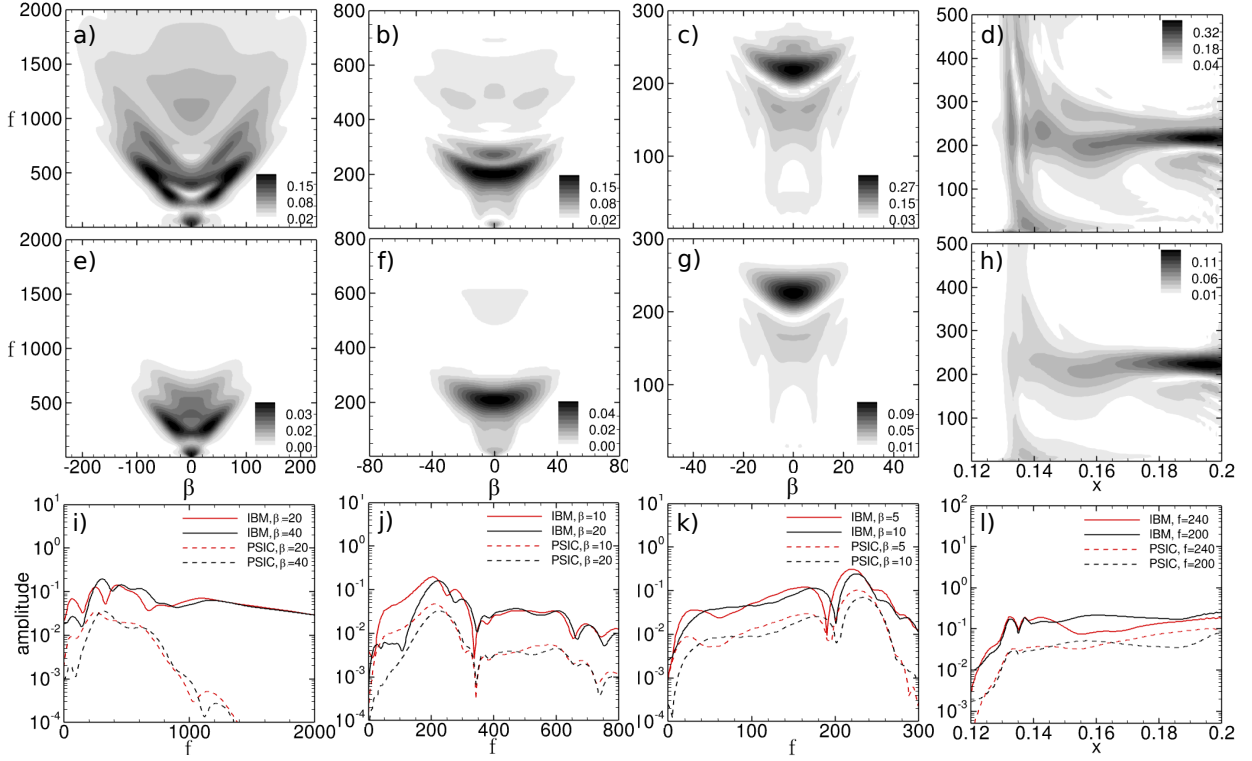


Figure 5.11: Disturbance spectra of (a,b,c,d) IBM and (e,f,g,h) PSIC simulations at a,e) $x=0.135$, b,f) $x=0.155$ and c,g) $x=0.185$ in the spanwise modenumber-nondimensional frequency plane, d,h) at $\beta = 0$, amplitude vs frequency at i) $x=0.135$, j) $x=0.155$, k) $x=0.185$ and l) amplitude vs x -location plot at $\beta = 0$ for cases I-X048R100 and P-X048R100 (adjusted from ref. [3]).

Hz, U_∞ is the freestream velocity, and length scale of $L = 1\text{m}$ is used. The particle introduces a wide range of frequencies and wavenumbers during impingement. At $x=0.135\text{m}$, the peak on the disturbance spectra is found within $-100 < \beta < 100$. However, the peak is found at higher frequencies and wavenumbers in the disturbance spectra for the IBM simulation, and the amplitude is also higher for the IBM case. A further downstream locations at $x=0.155\text{m}$ and $x=0.185\text{m}$, the second mode starts to dominate the flow field. Therefore, the peak occurs within the second mode frequency range for both the simulations at this location. However, the IBM simulation shows a higher amplitude than the PSIC simulation. Amplitude curves are extracted at different wavenumbers, β , and plotted in figs. 5.11i-k for a direct comparison, in which a similar observation can be made. The amplitude plots at $\beta = 0$ are shown in figs. 5.11d and 5.11h for

the IBM and PSIC approaches, respectively. Fig. 5.11l shows the amplitude curves extracted at different frequencies. The amplitude differences between the IBM and PSIC simulations are observed in the amplitude plots and in amplitude curves of different frequencies.

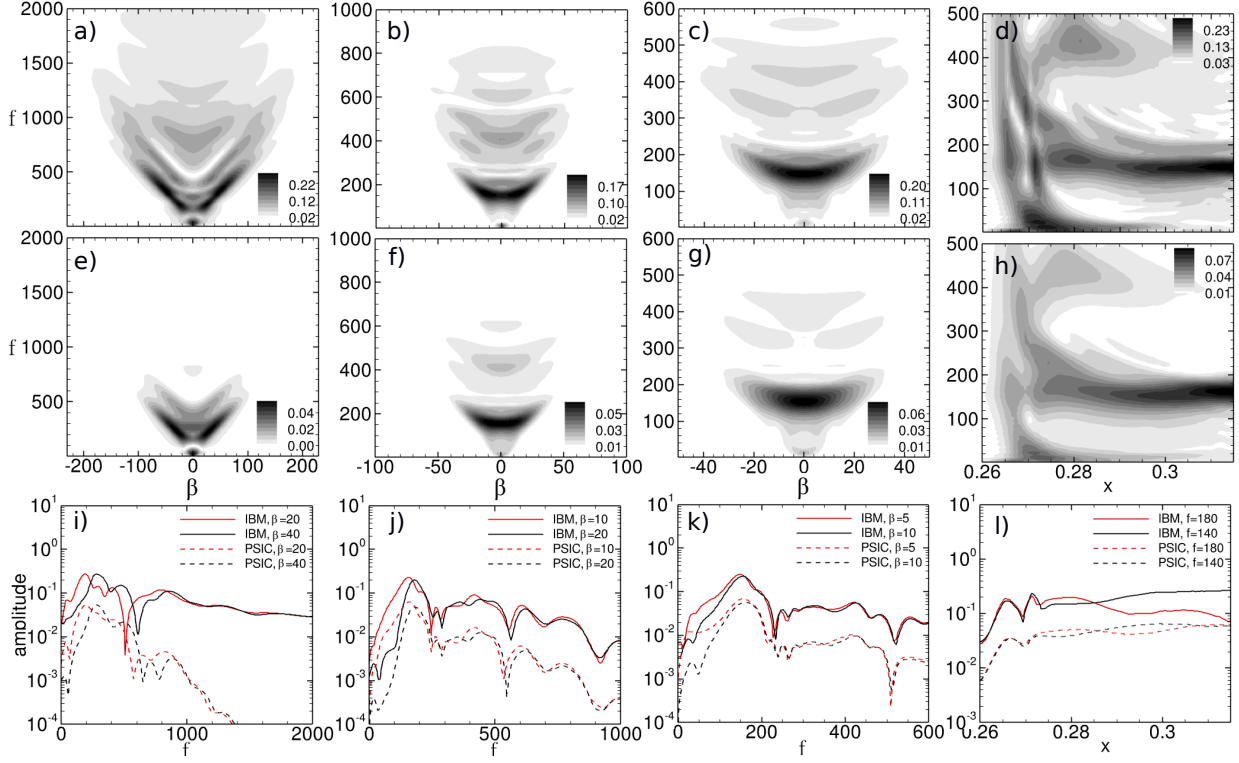


Figure 5.12: Disturbance spectra of (a,b,c,d) IBM and (e,f,g,h) PSIC simulations at a,e) $x=0.27$, b,f) $x=0.29$ and c,g) $x=0.3$ in the spanwise modenumbr-nondimensional frequency plane, d,h) at $\beta = 0$, amplitude vs frequency at i) $x=0.27$, j) $x=0.29$, k) $x=0.3$ and l) amplitude vs x -location plot at $\beta = 0$ for cases I-X183R100 and P-X183R100 (adjusted from ref. [3]).

Particle impingement further downstream is considered and the disturbance spectra are studied. Figs. 5.12a-c show the disturbance spectra for the case I-X183R100 at $x=0.27\text{m}$, 0.29m and 0.3m , respectively. Figs 5.12e-g show the disturbance spectra for the case P-X183R100 at $x=0.27\text{m}$, 0.29m and 0.3m , respectively. The amplitude curves are extracted at different wavenumbers, β , and plotted in figs. 5.12i-k. The peaks in the disturbance spectra occur within the frequency range $0 < f < 500$ and wavenumber range $-100 < \beta < 100$ for both the PSIC and IBM approaches at the impingement location. However, the IBM simulation also shows peaks at

higher frequencies. At $x=0.29\text{m}$, the peak is found within $100 < f < 200$ and $-50 < \beta < 50$. As the second mode starts to dominate disturbance flow field the peak is expected to occur within the same frequency and wavenumber range. Once again, the IBM approach shows higher amplitudes than the PSIC approach. Figs. 5.12d and 5.12h display the amplitude plot at $\beta = 0$ for the IBM and PSIC approaches, respectively. Fig. 5.12i shows the amplitude development at two frequencies.

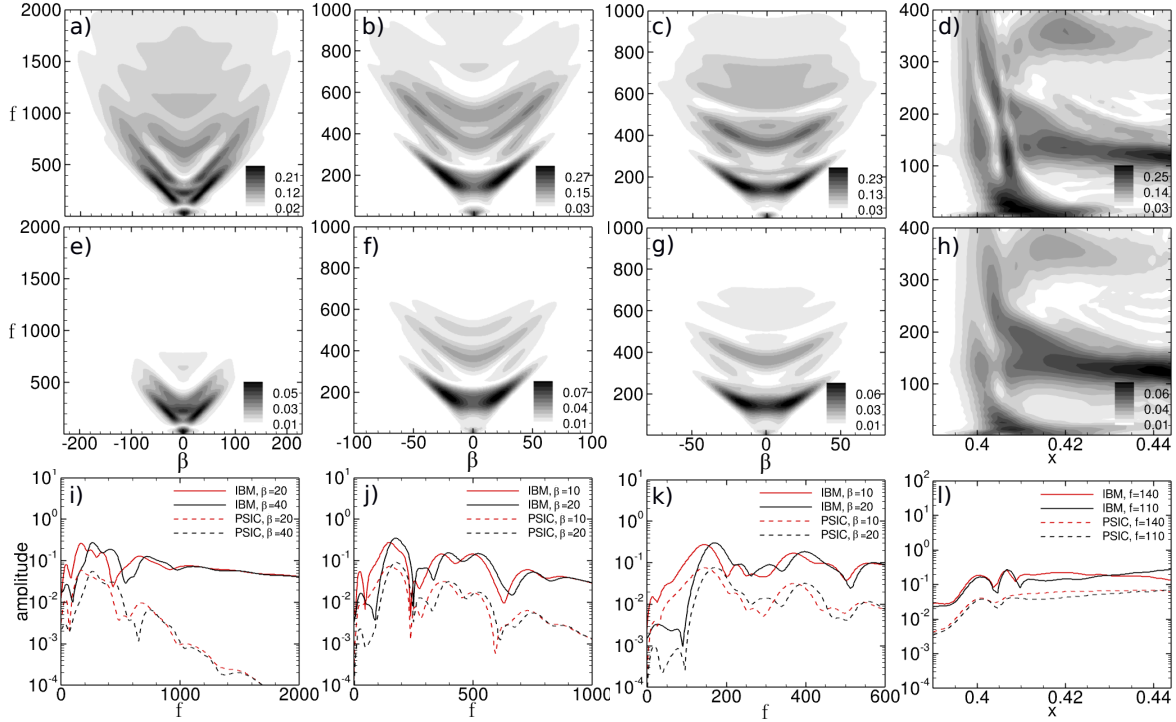


Figure 5.13: Disturbance spectra of (a,b,c,d) IBM and (e,f,g,h) PSIC simulations at a,e) $x=0.405$, b,f) $x=0.415$ and c,g) $x=0.42$ in the spanwise modenumbr-nondimensional frequency plane, d,h) at $\beta = 0$, amplitude vs frequency at i) $x=0.405$, j) $x=0.415$, k) $x=0.42$ and l) amplitude vs x -location plot at $\beta = 0$ for cases I-X318R100 and P-X318R100 (adjusted from ref. [3]).

Next, a location further inside the unstable flow region is considered for particle impingement.

Figs. 5.13a-c display the disturbance spectra of the case I-X318R100 at $x=0.405\text{m}$, 0.415m and 0.42m , respectively. The disturbance spectra for the case P-X318R100 are shown in figs. 5.13e-g. The peaks in the disturbance spectra occur within $-100 < \beta < 100$ and $0 < f < 500$ at the particle impingement location for both the simulations. However, the IBM simulation also shows

significant amplitudes at higher frequencies. At the downstream location $x=0.415\text{m}$ and 0.42m the peaks shift to lower frequencies with an inverse relationship to the increasing boundary layer height. Figs. 5.13i-k show the difference between the amplitude curves of the IBM and PSIC simulations at different wavenumbers, β . The amplitude plots at $\beta = 0$ are shown in figs. 5.13d and 5.13h for the IBM and PSIC approaches, respectively. Fig. 5.13l displays the amplitude development at two frequencies for the IBM and PSIC simulations. In all of these figures, the IBM simulation shows higher amplitude than the PSIC simulation.

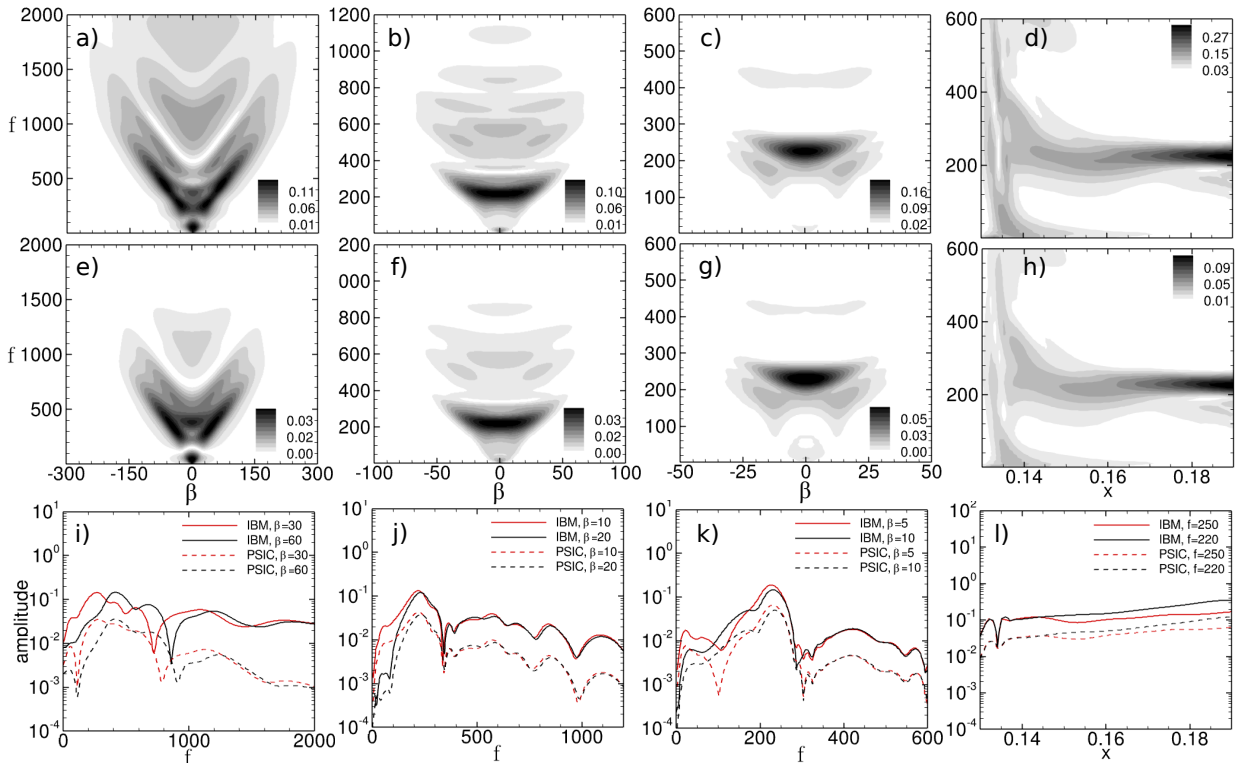


Figure 5.14: Disturbance spectra of (a,b,c,d) IBM and (e,f,g,h) PSIC simulations at a,e) $x=0.135$, b,f) $x=0.15$ and c,g) $x=0.17$ in the spanwise modenumbr-nondimensional frequency plane, d,h) at $\beta = 0$, amplitude vs frequency at i) $x=0.135$, j) $x=0.15$, k) $x=0.17$ and l) amplitude vs x -location plot at $\beta = 0$ for cases I-X048R50 and P-X048R50 (adjusted from ref. [3]).

A different particle size is considered for the cases I-X048R50 and P-X048R50. Figs 5.14a-c show the disturbance spectra for the case I-X048R50 at $x=0.135\text{m}$, 0.15m and 0.17m , respectively. The peak in the disturbance spectra is found within $-150 < \beta < 150$ and $0 < f < 100$ for both the

simulations. However, the IBM simulation also shows large amplitude at higher frequencies. The peaks shift to lower frequencies at the downstream locations $x=0.15\text{m}$ and 0.17m . At $x=0.15\text{m}$, the peak is achieved within the frequency range $f=200$ to $f=300$. The differences between amplitude curves from the IBM and PSIC simulations are shown in figs. 5.14i-k. Figs. 5.14d and 5.14h display the amplitude plots of the IBM and PSIC approaches at $\beta = 0$, respectively. Fig. 5.14l shows the amplitude development at two frequencies at $\beta = 0$. The IBM simulation shows higher amplitude than the PSIC simulation for a particle size of $R_p = 50\mu\text{m}$ also.

For all the simulations, the disturbance spectra of the IBM simulations are different than the PSIC simulations at the particle impingement location. As the generated wavepacket propagates downstream and the second mode starts to grow faster than the other modes the disturbance spectra show similarity in the peak regions. However, the IBM simulations show larger amplitudes than the PSIC simulations at all locations. The maximum amplitude ratio ($\gamma_p = P_{\max,IBM}/P_{\max,PSIC}$) is calculated at $\beta = 0$ at different locations and provided in table 5.2. The average amplitude ratio of $\gamma_p \approx 4$ is observed.

Cases	x_{pc}	Second position	Third position
I,P-X048R100	4.47	3.88	3.04
I,P-X183R100	4.57	3.22	3.55
I,P-X318R100	4.16	5.08	4.2
I,P-X048R50	3.82	3.05	2.88

Table 5.2: Amplitude ratio, $\gamma_p = p_{\max,IBM}/p_{\max,PSIC}$, of IBM to PSIC at $\beta = 0$ (adjusted from ref. [3]).

Chapter 6: A Model for Particle Induced Transition to Turbulence

In this section, a model for particle induced transition prediction is presented which considers different aspects of the transition process. The model takes into account the flow field stability properties via linear stability theory and the amplitude of the disturbance flow field is determined from direct numerical simulations (DNS) of different flow fields. The e^N method [61] is employed to estimate the transitional flow regime with the transitional N-factors being determined from the saturation limit of the disturbance amplitude. Different particle properties have been considered, such as particle size, concentration, density, trajectory, etc. Model demonstration for two different flow conditions have been considered: a Mach 9.5 14° sharp wedge at a 26km altitude and a Mach 5.07 7° HIFiRE-1 cone at a 16.75km altitude.

6.1 Flow Field and Particle Properties

Case	Altitude (km)	M_∞	$Re_\infty(1/m)$	$p_\infty(N/m^2)$	$T_\infty(K)$	$U_\infty(m/s)$
14° Wedge	26	9.5	1.126×10^7	2090.7	222.84	2842.13
HIFiRE-1	16.75	5.07	1.846×10^7	9851.9	201.0	1441.2

Table 6.1: The freestream conditions for the 14° wedge and HIFiRE-1 geometry.

The freestream conditions for the Mach 9.5 14° wedge and HIFiRE-1 flow fields are provided in table 6.1. The freestream condition for Mach 9.5 14° wedge is based on the US standard atmosphere model (1976) at a 26km altitude. The dominant mode for both flow fields is

the second mode.

Particles in the atmosphere originate from different sources: volcanic ashes, clouds, etc. A number of measurement campaigns have been carried out by various research groups to measure the particle concentration of different particle sizes in the atmosphere [26, 62, 63]. The concentrations of the particles are reported for particle diameters within the range of $0.1\mu m$ to $50\mu m$. The concentration of the sub-micron particles are almost an order of magnitude larger than the micron size particles, however, small particles tend to deviate from the straight line trajectory and avoid hitting the surface of the geometry as a result of the flow field and particle properties. Some particles will graze the surface without hitting it. It should be noted that these grazing particles can still introduce disturbance amplitudes that are comparable to amplitudes generated during particle impingement.

The particle concentrations from three experimental studies performed in Aire-sur-l'Adour (France), southern Minnesota (USA) and Ile du Levant (France) are considered for the current simulations [26, 62]. The particle sizes are considered for eleven particle diameter bins that are reported in refs. [26, 62] which are relevant for the altitudes considered in the current study, $0.2-0.3\mu m$, $0.3-0.4\mu m$, $0.4-0.5\mu m$, $0.5-0.6\mu m$, $0.6-0.7\mu m$, $0.7-0.9\mu m$, $0.9-1.1\mu m$, $1.1-3.0\mu m$, $3.0-5.0\mu m$, $5.0-7.5\mu m$ and $7.5-10.0\mu m$. The relation between the particle diameter bin and concentration is expressed as

$$\frac{dN_p}{d\text{Log}(D)} = \frac{\text{particle concentration}}{\log_{10}(D_{up}) - \log_{10}(D_{low})}, \quad (6.1)$$

where, D_{up} and D_{low} are the upper and lower limit of the size bin, respectively and N_p is the number density. Fig. 6.1a shows the number density versus particle radius at three different locations at an altitude of 26km and 16.75km. Overall, the measurements in Aire-sur-l'Adour

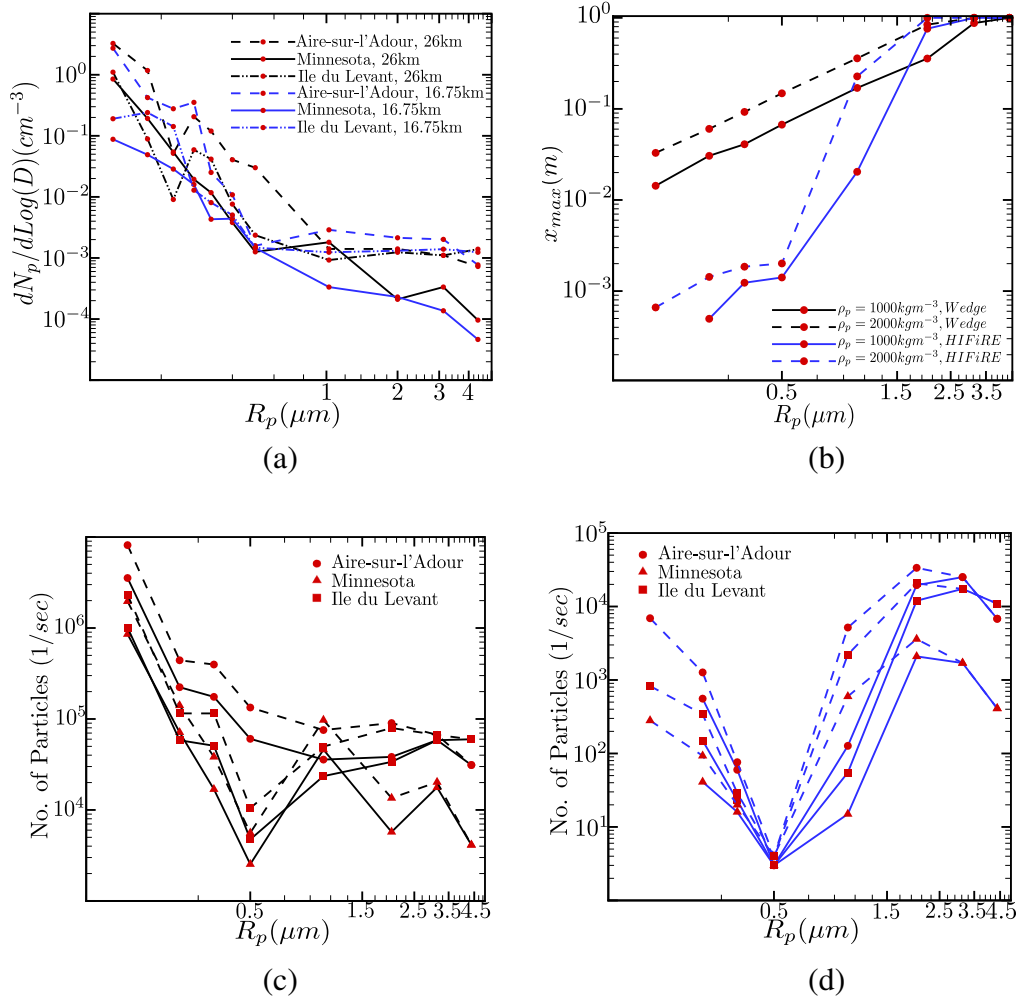


Figure 6.1: (a) Number densities of particles for three different regions at 26km and 16.75km, (b) maximum particle impact streamwise distance measured from the leading edge/nose of 14° wedge (black lines) and HIFiRE-1 cone (blue lines), (c) number of particles hitting the surface of 14° wedge per second (solid and dashed lines mark $\rho_p = 1000 \text{ kg/m}^3$ and 2000 kg/m^3 , respectively), (d) number of particles hitting the surface of the HIFiRE-1 cone per second (solid and dashed lines mark $\rho_p = 1000 \text{ kg/m}^3$ and 2000 kg/m^3 , respectively). Adjusted from ref. [12].

show higher particle concentrations than the other two locations. The number of particles hitting the surface of the geometry is determined by finding the maximum distance along the geometry a particle from the bin size would impact and the corresponding cross-sectional area. Fig. 6.1b shows the maximum streamwise distance a particle would hit versus particle radius. Only particles hitting the surface up to a streamwise distance of $L=1\text{m}$ have been considered here. Smaller

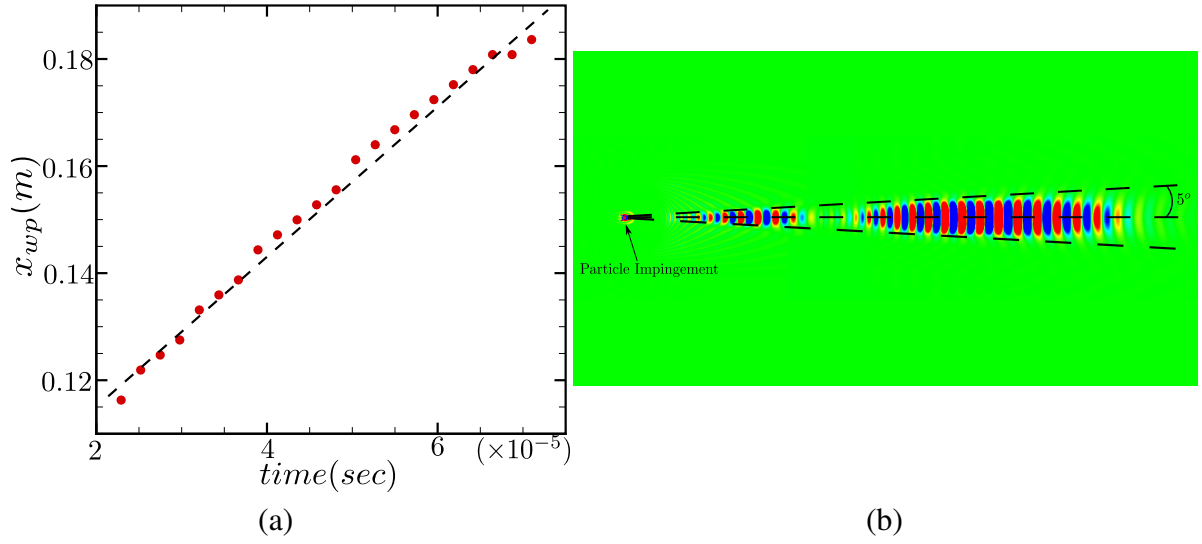


Figure 6.2: (a) Wavepacket hump location versus time and (b) pressure disturbance wavepacket (adjusted from ref. [12]).

particles impact near the nose of the geometry with the maximum impact location shifting downstream as the radius of the particle increases. This trend is expected because as the size of the particle increases, the particle gets heavier and impinges at a further downstream location. The particles with a radius greater than $2\mu m$ can hit all along the surface of the wedge or cone. Figs. 6.1c and 6.1d show the number of particles hitting the surface of the 14° wedge and 7° HIFiRE-1 cone, respectively. The cross-sectional area of the 14° wedge is larger than the HIFiRE-1 cone, therefore, the total number of particles impacting the surface is higher for the wedge geometry. Fig. 6.1d shows a minimum in the number of particles hitting the surface at the $0.5\mu m$ particle radius. The minimum is established through opposing trends with a large concentration of smaller particles which hit only a small cross-section of the HIFiRE-1 cone and a lower concentration of larger particles that hit the entire cone.

The wedge has a width of 0.5m and length of 1.5m. The number concentration is $149923m^{-3}$ for the bin size of $0.2-0.3\mu m$ for the Minnesota measurements. On the other hand, the cross-

sectional area of maximum impact is only $0.0017m^2$ for the $0.2-0.3\mu m$ bin size and particle density of $1000kg/m^3$. This leads to approximately 738077 particles hitting the surface of the wedge per second. One second is large considering the velocity of typical hypersonic vehicles and therefore, the simulation is performed over several convective time units. One convective flow time is the time required by the wavepacket to travel from the front of the geometry to the end. Hereby, it is assumed that the wavepacket travels with a constant group velocity. Although, it is clear that this is a rough estimate, it provides a simple approximation of the velocity of the wavepacket. An example wavepacket hump versus time plot is shown in fig. 6.2a. The group velocity can be approximated as the slope of the plot. The group velocity of the wavepacket for the 14° wedge is $1398m/s$ which leads to one flow through time being equivalent to $0.001073s$, approximately. Based on one flow time, approximately 792 particles will impinge on the surface of the 14° wedge for the $0.2-0.3\mu m$ bin size.

6.2 Receptivity and Stability Characteristics

Next, the receptivity mechanism for particle-induced transition is discussed. The initial amplitude of the disturbance inside the boundary layer seeding the instability waves strongly affects the location of the breakdown to turbulence. The disturbance amplitude in particle induced transition is related to the particle size; with increasing particle size, the initial disturbance amplitude also increases. In the current study, the initial amplitude is determined by prior DNS studies considering different particle sizes impinging on the boundary layer. Then the critical N-factor is calculated as $N^* = \ln(A_{sat}/A_o)$, where A_o is the initial amplitude and $A_{sat} = \phi_e, 0.5\phi_e$ and $0.1\phi_e$ for different saturation levels. ϕ_e denotes the boundary layer edge pressure.

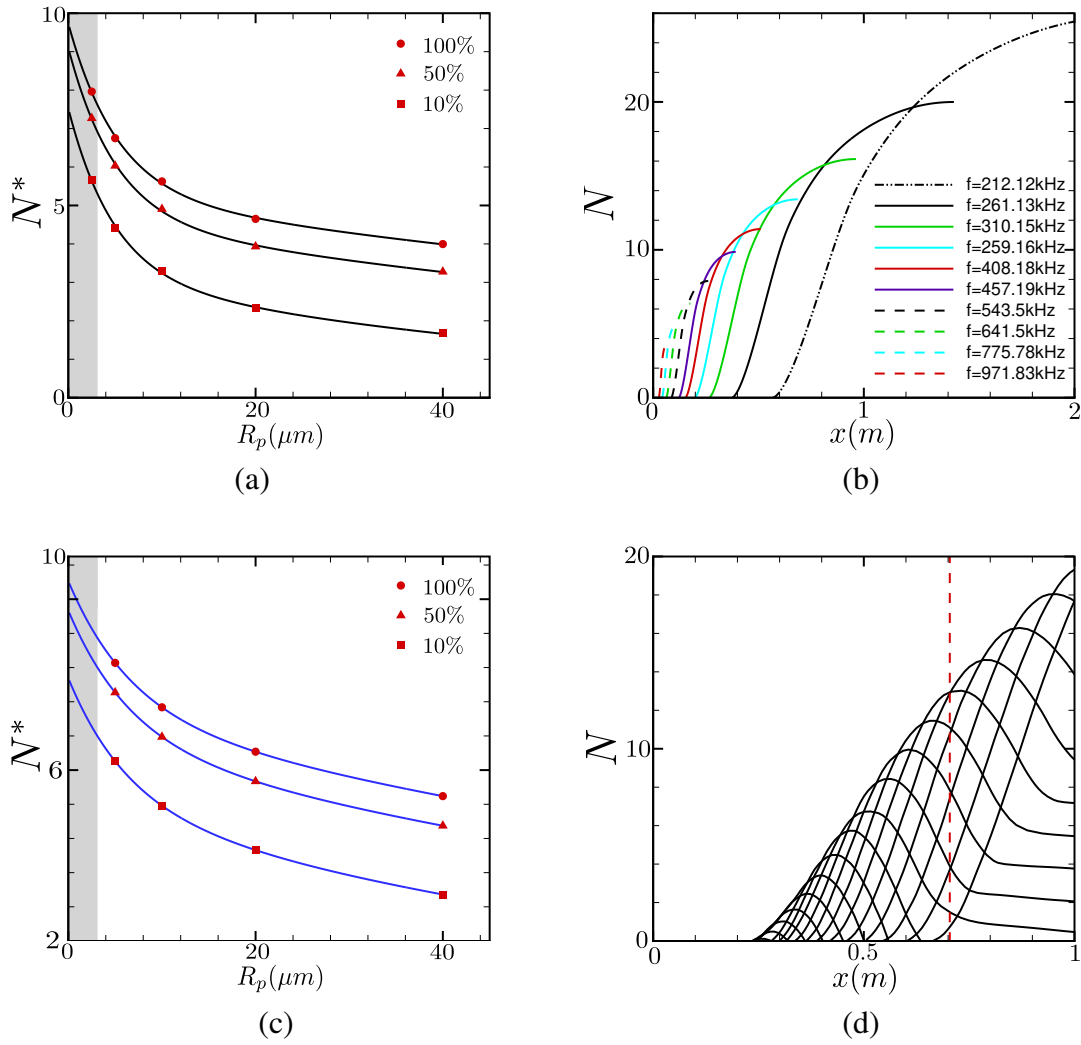


Figure 6.3: (a) Critical N-factor curves versus particle size for different saturation levels for a Mach 9.5 14° wedge flow field. The shaded area is the region of interest in the current study. (b) N-factor curves for the Mach 9.5 14° wedge flow field. (c) Critical N-factor versus particle size for different saturation levels on the HIFiRE-1 cone. The shaded area is the region of interest in the current study. (d) N-factor curves for the HIFiRE-1 flow field adopted from Li *et. al.* [13]. Vertical red line is the location of transition onset observed in flight (adjusted from ref. [12]).

Assuming lower saturation amplitudes, the transition to turbulence will occur earlier. Figs. 6.3a and 6.3c show the critical N-factors for the 14° wedge and HIFiRE-1, respectively for different saturation levels. Figs. 6.3b and 6.3d show the N-factor curves of the 14° wedge and HIFiRE-1 cone. The N-factors are large enough that transition to turbulence can be assumed to occur within

the domain of interest for these geometries. Higher N-factors lead to earlier breakdown of the transitional flow.

When the particle impinges on the surface, it initiates the transition process by creating a wavepacket which grows in size and propagates in the downstream direction. Fig. 6.2b illustrates the wavepacket for the 14° wedge case. The spreading rate of the wavepacket is considered by measuring the spreading angle from the DNS simulations. The spreading angle is measured as 5° which is used for the simulations of current study.

6.3 Transition to Turbulence

In this section, the final stage of the model is discussed. As is common for other transition prediction methods, the e^N -method [61] is employed to finally determine the location of turbulence onset based on when a critical N-factor is achieved. In addition, a difference in N-factor is considered as $\Delta N = N_x - N_{pc}$, where, N_{pc} is the N-factor at the particle impingement location and N_x is the N-factor at any streamwise location, x , of wavepacket position in the downstream direction. A turbulent spot is introduced at the center of the wavepacket when the difference in N-factor, ΔN , reaches the critical N-factor which is determined by the saturation amplitude discussed earlier. The spreading rate of the turbulent spot is approximated from the data available in literature. Estimates for the spreading angle in spanwise and wall-normal directions for turbulent spots can be found in Fischer [64]. The lateral spreading is different in subsonic flows than from hypersonic flows by almost a factor of three. Lateral spreading angles of 3° and 4° are considered for the 14° wedge and HIFiRE-1 cone, respectively.

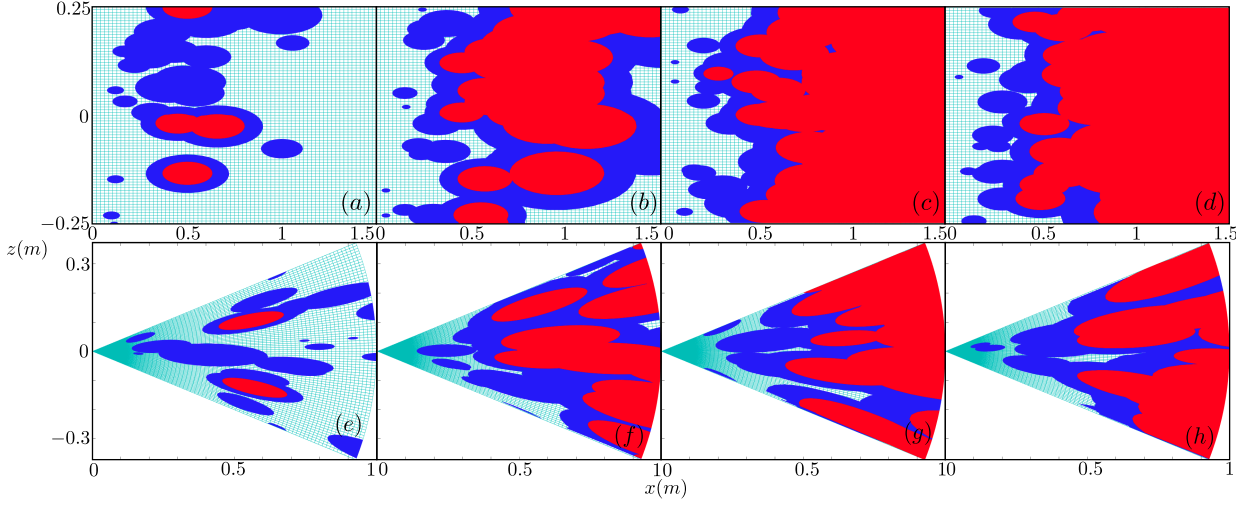


Figure 6.4: (a)-(d) Simulated disturbance flow field for Mach 9.5 14° wedge flow field considering Minnesota particle concentrations with $\rho_p = 1000 \text{ kg/m}^3$ and 100% saturation condition at (a) $3.95 \times 10^{-4} \text{ s}$, (b) $7.18 \times 10^{-4} \text{ s}$, (c) $3.55 \times 10^{-3} \text{ s}$, and (d) $1.073 \times 10^{-2} \text{ s}$. (e)-(h) Simulated disturbance flow field for HIFiRE-1 flow field considering Aire-sur-l'Adour particle concentrations with $\rho_p = 2000 \text{ kg/m}^3$ and 100% saturation condition at (e) $3.53 \times 10^{-4} \text{ s}$, (f) $7.07 \times 10^{-4} \text{ s}$, (g) $2.77 \times 10^{-3} \text{ s}$, and (h) $8.27 \times 10^{-3} \text{ s}$. Blue and red colored area mark transitional and turbulent flow regions, respectively (adjusted from ref. [12]).

6.4 Application of Transition Model to Mach 9.5 14° Wedge Flow

First, the particle model is employed to predict the transition process in a Mach 9.5 14° wedge flow. Two different particle densities have been considered, $\rho_p = 1000 \text{ kg/m}^3$ and 2000 kg/m^3 . These particle densities are relevant for the atmospheric conditions considered here [35, 54]. The simulations consider particle concentrations in three different regions: Aire-sur-l'Adour (France), Minnesota (USA) and Ile du Levant (France).

The simulation domain extends from $[0 \text{ m}, 1.5 \text{ m}]$ in the streamwise direction and $[0 \text{ m}, 0.5 \text{ m}]$ in the spanwise direction. The domain is discretized with 71 points in the streamwise direction and 80 points in the spanwise direction. The simulation does not depend on the discretization of the domain as the only purpose of the discretization is to calculate the intermittency factors.

The particles are introduced randomly up to the maximum impact location at each time

step. Figs. 6.4a-d show the particle induced disturbance flow fields based on the $\rho_p = 1000 \text{ kg/m}^3$, 100% saturation level with the particle concentration from the Minnesota measurements. Blue ellipses represent the transitional wavepacket and red ellipses represent the turbulent flow region. As time progresses, more particles are introduced into the domain. So, the transitional and turbulent regions increase in size for later times. The simulations are run for ten flow through time units.

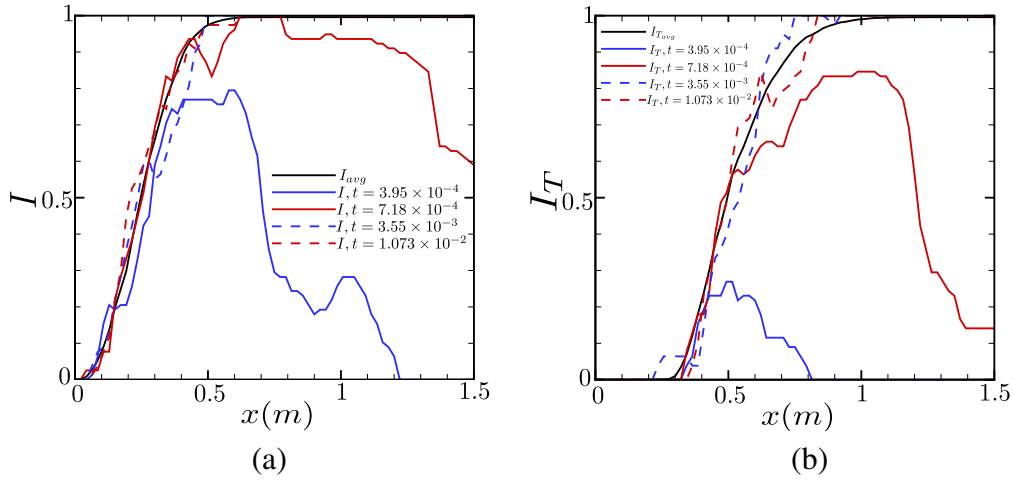


Figure 6.5: (a) Average and instantaneous transitional intermittency factors at different times. (b) Average and instantaneous turbulent intermittency factors at different times for Mach 9.5 14° wedge (adjusted from ref. [12]).

The intermittency factor is measured based on the surface covered by the transitional and turbulent regions with respect to the width of the simulation domain. It is calculated as following

$$I = \frac{\int_0^{L_z} I(z) dz}{L_z}, \quad \text{or} \quad I = \frac{\int_{-\Delta\Theta/2}^{\Delta\Theta/2} I(\phi) d\phi}{\Delta\Theta}, \quad (6.2)$$

where L_z is the width of the wedge, $\Delta\Theta$ is the lateral extent of the unrolled cone surface and $I(z) = 1$ and $I(\phi) = 1$ when the flow is transitional or turbulent. The flow becomes fully turbulent when the intermittency factor, I , reaches a value close to one, here it is assumed as

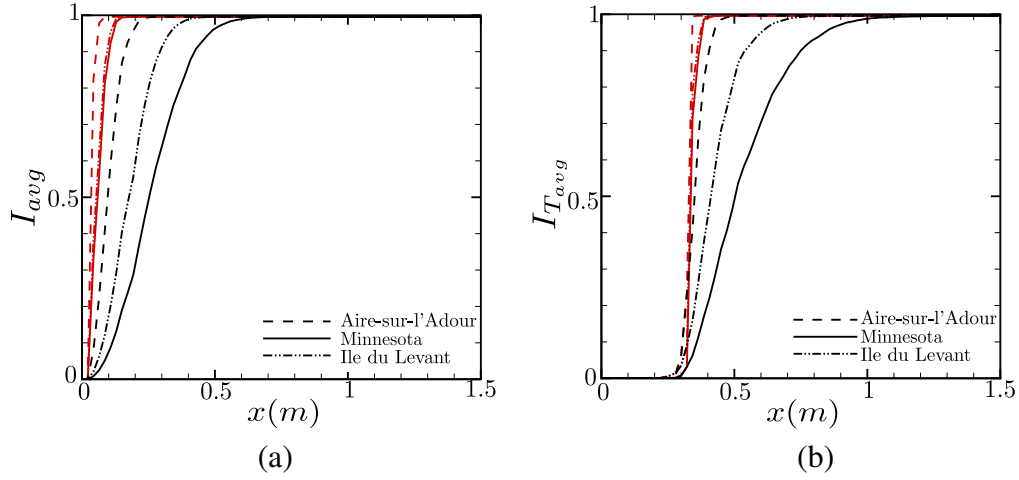


Figure 6.6: (a) Transitional and (b) turbulent intermittency factors for different particle concentrations according to different regions for Mach 9.5 14° wedge flow. Red curves are for the simulations performed with all the 11 particle bin sizes and black curves are for the simulations performed with bins 0.5-0.6 μm and above (adjusted from ref. [12]).

$I \geq 0.9$.

Figs. 6.5a and 6.5b show the transitional and turbulent intermittency factors for instantaneous and average quantities, respectively. As the time progresses more particles impinge on the surface of the wedge, generating more wavepackets. So, the instantaneous intermittency factor tends to match the average intermittency factor at later times. The time-averaged intermittency factor converges after five flow through times. In fig. 6.5b it can be observed that the flow reaches a fully turbulent state at approximately $x=0.7\text{m}$ based on the criteria $I \geq 0.9$.

Next, the disturbance flow field is simulated based on the particle concentration in three different regions: Aire-sur-l'Adour, Minnesota and Ile du Levant. Figs. 6.6a and 6.6b show the transitional and turbulent intermittency factors, respectively. The fully turbulent region is achieved at approximately $x=0.3\text{m}$ considering all eleven bins. Higher concentration of particles from the smallest three bin sizes impinge near the leading edge of the wedge. The smallest three bin sizes are removed from the rest of the analysis due to the small oblique shock angle, and since

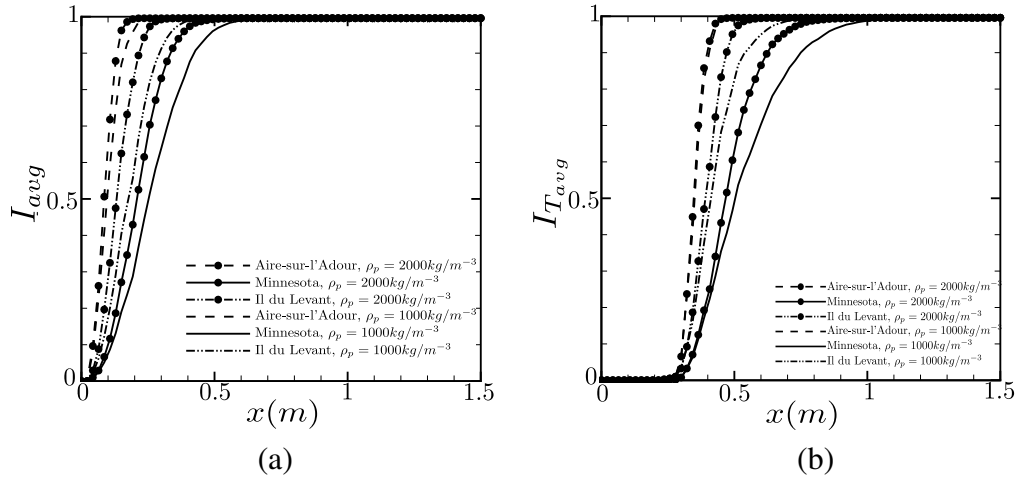


Figure 6.7: (a) Transitional and (b) turbulent intermittency factors for particle densities of $\rho_p = 1000 \text{ kg/m}^3$ and 2000 kg/m^3 for Mach 9.5 14° wedge flow (adjusted from ref. [12]).

there are no sharp corners in real world applications. A higher particle concentration will create more wavepackets, and hence lead to an earlier onset of turbulence for the higher concentration of particle. Aire-sur-l'Adour reported higher particle concentration than the other two regions; so, transition to turbulence occurs earlier for the Aire-sur-l'Adour measurements. Fully turbulent regions begin at $x=0.4\text{m}$, 0.5m and 0.7m for the Aire-sur-l'Adour, Ile du Levant and Minnesota measurements, respectively.

Figs. 6.7a and 6.7b show the average intermittency factors for transitional and turbulent flow regions for two different particle densities, respectively. As the particle density increases, the maximum impact location shifts downstream and more particles impinge on the surface of the wedge generating more wavepackets. So, as the particle density increases the intermittency curves shift upstream indicating an earlier onset of turbulence. The shift is larger for smaller particle concentrations.

Next, the saturation level is varied to study the effect of different critical N-factors. Figs. 6.8a and 6.8b show the average transitional and turbulent intermittency factors, respectively. The

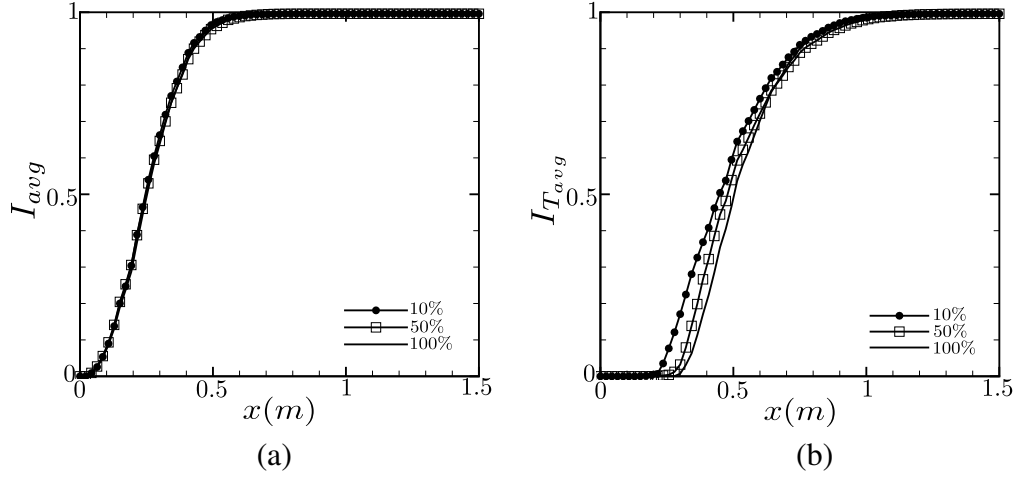


Figure 6.8: (a) Transitional and (b) turbulent intermittency factors for different saturation levels for Mach 9.5 14° wedge flow (adjusted from ref. [12]).

transitional intermittency factor does not vary as it has no dependency on the saturation level. Turbulent spots are generated earlier for smaller critical N-factors. So, as the saturation level decreases the average turbulent intermittency curves shift upstream but reach unity or $I \geq 0.9$ almost at the same location.

6.5 Application of Transition Model to HIFiRE-1

Next, the particle model is applied to simulate the transition process for the HIFiRE-1 flight at one trajectory point. Some input parameters are assumed in order to perform the simulations. The initial disturbance amplitude, A_o , is assumed to scale with the particle radius as $R_p^{3/2}$ [65]. This amplitude-radius scaling is determined by multiple DNS simulations of particle impingement on flat plates, wedges and cones. The wavepacket and turbulent spot spread at an angle 6° and 4° , respectively [64]. The group velocity is approximately 80% of the streamwise velocity at the boundary-layer edge. The width due to curvature is neglected during the calculation of the intermittency factor.

Figs. 6.4e-h show the transitional and turbulent regions at four time instances for the Aire-sur-l'Adour condition with $\rho_p = 2000 \text{ kg/m}^3$ and 100% saturation level. The blue ellipses represent the transitional region and red ellipses represent the turbulent region. Similar to the wedge simulation, the transitional and turbulent regions increase in size for later times.

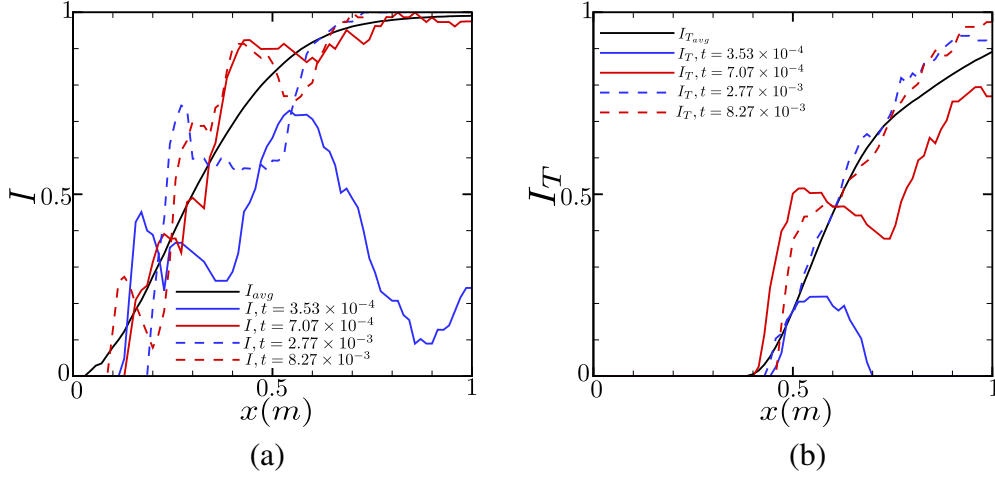


Figure 6.9: (a) Average and instantaneous transitional intermittency factors at different times. (b) Average and instantaneous turbulent intermittency factors at different times for the HIFiRE-1 test case (adjusted from ref. [12]).

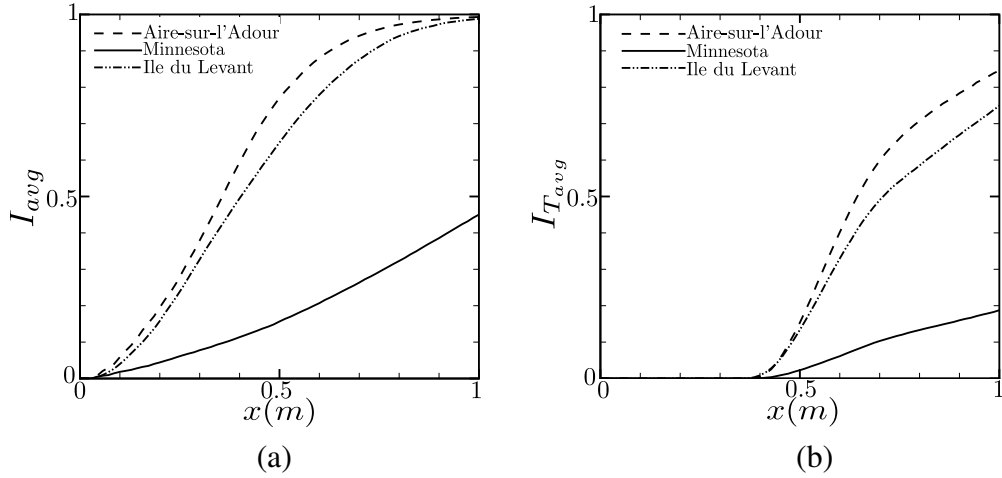


Figure 6.10: (a) Transitional and (b) turbulent intermittency factors for different particle concentrations according to different regions for HIFiRE-1 test case (adjusted from ref. [12]).

Figs. 6.9a and 6.9b show the instantaneous and time-average intermittency factors for the

transitional and turbulent state for the Aire-sur-l'Adour condition, $\rho_p = 2000\text{kg/m}^3$ and 100% saturation level, respectively. The instantaneous intermittency curves converge to the average intermittency curve at approximately five flow through times similar to the wedge case. It should be noted here that as the time progresses, more particles will impinge on the surface of HIFiRE-1 generating more wavepackets. So, instantaneous intermittency factors at later times tend to match the time-averaged intermittency factor.

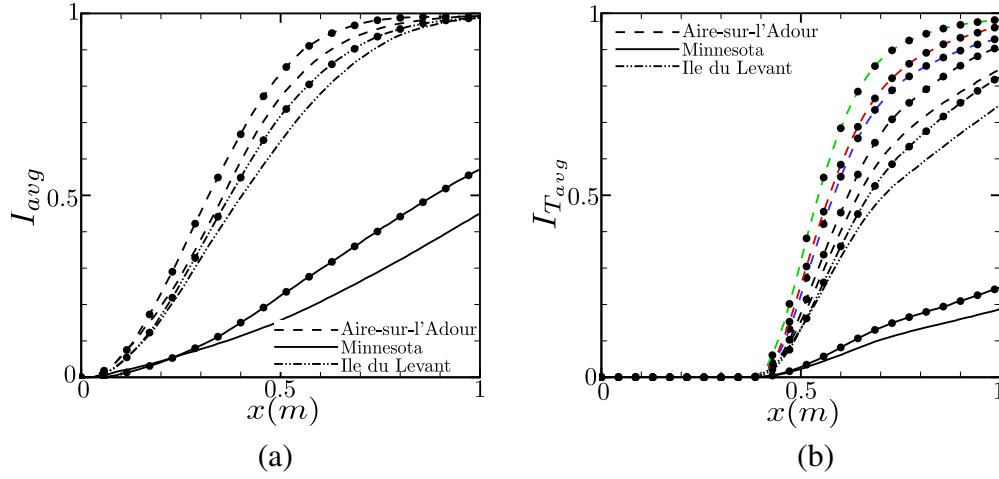


Figure 6.11: (a) Transitional and (b) turbulent intermittency factors for particle densities of $\rho_p = 1000\text{kg/m}^3$ (without symbols) and $\rho_p = 2000\text{kg/m}^3$ (with circles) for HIFiRE-1 test case. Blue, red and green curves represent simulations assuming particle concentrations increased by 20%, 50% and 100% (adjusted from ref. [12]).

Next, the model is used to simulate the flow field at three different locations, similar to what was done for the wedge condition. Figs. 6.10a and 6.10b show the average intermittency factors for the transitional and turbulent regions for three locations, respectively. A saturation level of 100% and particle density of 1000kg/m^3 have been considered for the purpose of the simulation. As before the higher particle concentrations of the Aire-sur-l'Adour measurements yielded higher average intermittency factors than the other two locations. However, none of the cases achieved a fully turbulent state defined by $I \geq 0.9$.

Finally, the particle density is changed to $2000\text{kg}/\text{m}^3$ to compare with the above simulations. Figs. 6.11a and 6.11b display the average intermittency factors for the transitional and turbulent states at three locations, respectively. As the density of the particle increases, the maximum impact location shifts downstream, allowing more particles to impinge on the surface of the HIFiRE-1 cone. So, as the density of particle increases, the average intermittency factor also increases for all three locations. The fully turbulent state is not observed for the Minnesota and Ile du Levant conditions and the turbulent state is only observed at the end of the domain for the Aire-sur-l'Adour condition. As there is some uncertainty in concentration levels, the particle concentration is increased by 20%, 50% and 100%. This increase in particle concentration causes the turbulence onset location to shift upstream. With a 100% increase in the particle concentration, the new turbulence onset location becomes 0.7m for the Aire-sur-l'Adour condition where transition in the actual flight test was observed.

Chapter 7: Conclusion

7.1 Summary

Particles are present in the atmosphere in large numbers as suggested by the experimental measurements of particle concentration at different altitudes in refs. [26, 62]. These particles can reach surface of high-speed vehicles and introduce small-scale disturbances inside the boundary layer. The receptivity process gives rise to wavepackets and as they convect downstream they are amplified due to inherent instability mechanism. When these disturbances reach nonlinear saturation, secondary instability and nonlinear mechanisms become relevant and finally breakdown to turbulence can be observed. Although, researchers have considered the role of particles in the boundary-layer transition process of atmospheric free flight a long time ago [5], the interest in this field has grown recently with the seminal article from Chuvakhov et al. [35], which is followed by a series of research studies from the current author [1, 2, 3, 8, 9, 10, 11, 12, 23, 65, 66].

In this dissertation several aspects relevant to particle-induced transition have been explored. First, the Particle-Source-in-Cell (PSIC) approach in CHAMPS is validated by comparing with the simulation results from the finite volume solver US3D. The particle-induced disturbance flow fields from both the solvers are in close agreement. The disturbance flow fields are also simulated on static and AMR meshes for a direct comparison of the amplitude plots which were found to be in good agreement. However, the AMR approach shows larger amplitudes of acoustic waves

than the static mesh approach because AMR mesh is able to track these disturbances.

Next, the particle-induced disturbance flow field is compared with the disturbance flow field generated by a pulse inside the boundary layer. The particle introduces a wide range of frequencies and wavenumbers in the $F - \beta$ plane at impingement location, which is not observed in pulse disturbance simulation. It is generally not straightforward to design a pulse disturbance which can mimic the disturbance field generated by the particle impingement on boundary layer [8]. Though the pulse disturbance simulation shows good agreement with the particle-induced disturbance flow field in the most amplified second mode region, there are differences in the high and low frequency ranges of the frequency spectra. These differences may affect the final stages of the transition process.

The discrete and continuous branches of the eigenvalue spectra contribute to the development of the disturbance flow field induced by particle impingement on the boundary layer. BOD is employed to determine the contributions from the discrete and continuous branches. The discrete modes contribute primarily to the development of the disturbance flow field inside the boundary layer. Outside the boundary layer, significant contributions stem from the continuous branch. In the current analysis, the main contributions have been found from the SA and FA acoustic modes from the continuous branch. Contributions from the vorticity and entropy branches have been found to be negligible. An energy norm [60] has been adopted to analyse the disturbance flow field by combining all the flow variables into one flow quantity. The projection on mode S shows that a small portion of the disturbance energy ends up in mode S. Most of the disturbance energy is carried away by the acoustic modes into the freestream. Additionally, the disturbance energy excite stable higher Mack modes F2, F3 etc., which decay rapidly in the downstream direction.

Next, the PSIC simulation approach is compared with fully resolved particle simulation by

employing a higher order IBM. The fully resolved simulations are validated by comparing the simulations of particles moving in a stationary fluid with the data available in the literature. Grid sensitivity studies have been performed to achieve grid converged results for both the PSIC and IBM approaches. At the particle impingement location, the IBM and PSIC simulations show some differences in the disturbance spectra. The high frequency contents in the IBM simulations are not observed in the PSIC simulations. Far from the impingement location where the second mode starts to dominate, the development of amplitude curves agree more closely. However, the amplitudes in IBM simulations are higher than those for the PSIC simulations at all locations. The amplitude ratio of maximum IBM amplitude to PSIC amplitude is calculated at the wavenumber of zero for different streamwise locations. The amplitude ratio is found to be approximately 4, which suggests a N-factor difference of $\Delta N \approx 1.4$ [3]. So, the simulations by the PSIC approach may predict a more delayed transition than the IBM approach [3].

Finally, a reduced order model is developed to simulate the particle induced transition process. The model considers the particle and flow field properties, *e.g.* particle size, concentration, density, receptivity and stability characteristics of the flow field. Particle-induced transition is simulated for two different flow fields, *i.e.* a Mach 9.5 14° wedge and a Mach 5.07 7° HIFiRE-1 cone. Particle properties in three different regions have been considered, *i.e.* Aire-sur-l'Adour (France), Minnesota (USA) and Ile du Levant (France). Transition to turbulence in a flow at Mach 9.5 over a 14° wedge is observed for all the locations. As the particle concentration in Aire-sur-l'Adour is higher than the other two regions, the model predicts the earliest transition for this region. Additionally, when the particle density is increased from $\rho_p = 1000 \text{ kg/m}^3$ to $\rho_p = 2000 \text{ kg/m}^3$, more particles impinge on the surface of the geometry and the location of transition onset shifts upstream. The transition in the HIFiRE-1 geometry is only achieved for

the Aire-sur-l'Adour condition with a $\rho_p = 2000\text{kg}/\text{m}^3$ barely at the end of the geometry. The transition is not observed within the domain of interest for the particle properties of Minnesota and Ile du Levant regions.

7.2 Contributions to the Research

The author made some significant contributions to the research in particle induced transition. Though many researchers employed BOD for disturbance flow field analysis in 2-D [29, 30], this is the first time BOD is applied for such a complex flow field induced by the particle impingement on the boundary layer in 3-D. The disturbance flow field is decomposed by employing the BOD to determine the contributions from the first mode, second mode, and higher Mack modes. It was shown that the disturbance flow field near the particle impingement location can be reconstructed from the contributions of the discrete and continuous branches with a maximum error of 8.8%.

The PSIC simulations are compared with the IBM simulations for determining the differences during the receptivity process. This is the first time it has been shown that the amplitudes in fully resolved simulations are greater than the amplitudes of the PSIC simulations. This comparison will help to adjust the amplitudes of the PSIC simulations for the prediction of particle-induced transition process.

A simple particle model has been developed to simulate the particle induced transition process. This is an efficient model which can be run on a workstation in a few minutes. The model considers the particle and flow field properties to simulate the disturbance flow field. This model will help to estimate the transition onset locations for realistic atmospheric conditions and hypersonic flow scenarios.

7.3 Future Scope of Research

While the current research has investigated various aspects of particle-induced transition, there is a lot more research that can be done in this area. For example, BOD can be employed for three dimensional boundary layers and more complex geometries. Currently, the BOD research focuses primarily on simple geometries and flows, such as flat plates and wedges. Future efforts should aim to adapt the coordinate system and extend the local approach to a global analysis to accommodate more complex geometries and flow fields. Additionally, the method of multiple scales could be applied to address significant non-parallel flow effects in these flows [30].

Not all particles make contact with the surface of high-speed vehicles. The particle trajectory depends on the flow field and particle properties. Some of the particles will simply graze the boundary layer above the surface, introducing disturbances within the boundary layer. However, grazing particles have been overlooked in the existing studies. Future work should take these particles into account in the particle model to simulate the transition process.

To date, studies have assumed that the particles are spherical. Particles with different shapes, *e.g.* ellipsoidal shapes, should be considered in future studies. The amplitude of the disturbance flow field induced by the ellipsoidal particles can be compared with the disturbance amplitude induced by the spherical particles. BOD can also be employed to study the receptivity process of the impingement of ellipsoidal particles on the boundary layer.

While current LST studies predict that the transition to turbulence could take place within the computational domain, the nonlinear breakdown to turbulence of particle-induced disturbances has not been explored. Future work could extend beyond the receptivity and linear stages to explore non-linear dynamics and the turbulence onset due to both single and multiple particle

impingements on the boundary layer.

Appendix A: IBM validation data

	Cylinder	Sphere
$L \times H(\times W)$	$4m \times 1m$	$0.05m \times 0.05m \times 0.05m$
initial position	(0.01m,0.5m)	(0.025m,0.025m,0.025m)
D	$200\mu m$	$200\mu m$
V_p	$80.1545m/s$	$10.2834m/s$
U_∞	$0.0m/s$	$0.0m/s$
ρ_∞	$P_\infty/(R_{gas}T_\infty)$	$P_\infty/(R_{gas}T_\infty)$
T_∞	$350K$	$350K$
$P_\infty(Re_D = 20)$	$2.6kPa$	-
$P_\infty(Re_D = 40)$	$5.2kPa$	-
$P_\infty(Re_D = 100)$	$13kPa$	$101.327kPa$
$P_\infty(Re_D = 200)$	$26kPa$	-
γ	1.4	1.4
c_p	$1005.025J/(Kg - K)$	$1005J/(Kg - K)$
μ_∞	$2.0736 \times 10^{-5}Ns/m^2$	$2.0736 \times 10^{-5}Ns/m^2$

Table A.1: Flow parameters and domain setup for the cylinder and sphere simulations.

$Re_D = 20$	L/D	a/D	b/D	Θ°	C_D	$Re_D = 40$	L/D	a/D	b/D	Θ°	C_D
Fornberg [67]	0.91	-	-	45.7	2.0		2.24	-	-	55.6	1.5
Dennis [68]	0.94	-	-	43.7	2.05		2.35	-	-	53.8	1.52
Coutanceau [69]	0.93	0.33	0.46	45	-		2.13	0.76	0.59	55.8	-
Linnick [70]	0.93	0.36	0.43	43.5	2.06		2.28	0.72	0.6	53.6	1.54
Brehm [71]	0.94	0.36	0.43	43.5	2.08		2.29	0.72	0.6	52.4	1.57
Brehm [4]	0.96	0.36	0.42	44	2.02		2.26	0.72	0.58	52.9	1.51
Current study	0.93	0.36	0.44	43.35	1.92		2.32	0.72	0.61	54.84	1.45

Table A.2: Wake dimensions for the cylinders travelling with $Re_D=20$ and 40 (adapted from Ref. 4).

$Re_D = 100$				$Re_D = 200$		
	St	C_D	C_L	St	C_D	C_L
Berger [72]	0.16-0.17	-	-	0.18-0.19	-	-
Liu [73]	0.165	1.35 ± 0.012	± 0.339	0.192	1.31 ± 0.049	± 0.69
Linnick [70] ($\lambda=0.056$)	0.169	1.38 ± 0.01	± 0.337	0.199	1.37 ± 0.046	± 0.7
Linnick [70] ($\lambda=0.023$)	0.166	1.34 ± 0.009	± 0.333	0.197	1.34 ± 0.044	± 0.69
Mittal [74]	-	1.35	-	-	-	-
Henderson [75]	-	1.35	-	-	-	-
Marella [76]	-	1.36	-	-	-	-
Belov [77]	-	-	-	0.193	1.19 ± 0.042	± 0.64
Brehm [71]	0.17	1.38 ± 0.011	± 0.342	0.198	1.38 ± 0.044	± 0.7
Brehm [4]	0.165	1.32 ± 0.01	± 0.32	0.192	1.3 ± 0.04	± 0.66
Current study	0.162	1.4 ± 0.01	± 0.3	0.194	1.33 ± 0.038	± 0.59

Table A.3: Strouhal number, drag coefficient and lift coefficient of the cylinders travelling with $Re_D=100$ and 200 (adapted from Ref. 4).

Author	x_c/D	y_c/D	L_b/D
Marella [76]	-	-	0.88
Taneda [78]	0.745	0.28	0.8
Mittal [74]	0.742	0.278	0.84
Johnson and Patel [79]	0.75	0.29	0.88
Brehm [4]	0.78	0.3	0.9
Current study	0.74	0.27	0.82

Table A.4: Wake dimensions of the sphere travelling with $Re_D = 100$ (adapted from Ref. 4).

Bibliography

- [1] S. M. A. A. Hasnine, V. Russo, O. M. Browne, A. Tumin, and C. Brehm. Disturbance flow field analysis of particulate interaction with high speed boundary layers. *AIAA Aviation 2020 Forum*, 2020.
- [2] S.M. A. Al Hasnine, V. Russo, A. Tumin, and C. Brehm. Biorthogonal decomposition of the disturbance flow field generated by particle impingement on a hypersonic boundary layer. *Journal of Fluid Mechanics*, 969:A1, 2023.
- [3] Sayed Mohammad Abdullah Al Hasnine, Vincenzo Russo, and Christoph Brehm. An assessment of the particle modeling fidelity for particle-induced transition simulations. *AIAA AVIATION 2022 Forum*.
- [4] C. Brehm, C. Hader, and H.F. Fasel. A Locally Stabilized Immersed Boundary Method for the Compressible Navier-Stokes Equations. *J. Comput. Phys.*, 295:475–504, 2015.
- [5] R. P. Turco. Upper-atmosphere aerosols: properties and natural cycles. chapter 3b. *The Atmospheric Effects of Stratospheric Aircraft: A First Program Report*, pages 63–91, 1992.
- [6] M. V. Morkovin, E. Reshotko, and T. Herbert. Transition in open flow systems - a reassessment. *Bull. Am. Phys. Soc.*, 39:1882, 1994.
- [7] A. Fedorov. Transition and stability of high-speed boundary layers. *Annual Review of Fluid Mechanics*, 43:79–95, 2011.
- [8] O. M. F. Browne, S. M. A. A. Hasnine, and C. Brehm. Numerical method for particulate-induced high-speed boundary-layer transition simulations. *AIAA Journal*, 59(4):1196–1213, 2021.
- [9] S. M. A. A. Hasnine, V. Russo, A. Tumin, and C. Brehm. Three-dimensional spatio-temporal disturbance flow field analysis of particulate-induced high-speed boundary-layer transition. *AIAA Scitech 2021 Forum*, 2021.
- [10] O. M. F. Browne, S. M. A. A. Hasnine, V. Russo, and C. Brehm. Fully-resolved particulate-induced transition simulations for high-speed boundary-layers with an immersed boundary method. In *AIAA Scitech 2020 Forum*, page 1795, 2020.

- [11] Sayed Mohammad Abdullah Al Hasnine, Vincenzo Russo, and Christoph Brehm. Particle impingement simulations for first and second mode dominated high-speed boundary layers. *AIAA AVIATION 2021 FORUM*.
- [12] Sayed Mohammad Abdullah Al Hasnine, Vincenzo Russo, Bijaylakshmi Saikia, and Christoph Brehm. A model for particle-induced transition for hypersonic boundary layer flows. In *AIAA AVIATION 2023 Forum*.
- [13] F. Li, M. Choudhari, C.-L. Chang, R. Kimmel, D. Adamczak, and M. Smith. Transition analysis for the ascent phase of hifire-1 flight experiment. *Journal of Spacecraft and Rockets*, 52(5):1283–1293, 2015.
- [14] H. Taguchi, A. Murakami, T. Sato, and T. Tsuchiya. Conceptual study on hypersonic airplanes using pre-cooled turbojet. In *15th AIAA International Space Planes and Hypersonic Systems and Technologies Conference*.
- [15] David M Van Wie. Hypersonics: Past, present, and potential future. *Johns Hopkins APL Technical Digest*, 35(4):335–341, 2021.
- [16] John David Anderson. *Hypersonic and high temperature gas dynamics*. AIAA, 2006.
- [17] Dennis R Jenkins. *Hypersonics before the shuttle: A concise history of the X-15 research airplane*. National Aeronautics and Space Admin., NASA Office of Policy and Plans, NASA, 2000.
- [18] Randall T. Volland, Lawrence D. Huebner, and Charles R. McClinton. X-43a hypersonic vehicle technology development. *Acta Astronautica*, 59(1):181–191, 2006. Space for Inspiration of Humankind, Selected Proceedings of the 56th International Astronautical Federation Congress, Fukuoka, Japan, 17-21 October 2005.
- [19] Helen L. Reed, Eduardo Perez, Joseph Kuehl, Travis Kocian, and Nicholas Oliviero. Verification and validation issues in hypersonic stability and transition prediction. *Journal of Spacecraft and Rockets*, 52(1):29–37, 2015.
- [20] Cunbiao Lee and Shiyi Chen. Recent progress in the study of transition in the hypersonic boundary layer. *National Science Review*, 6(1):155–170, 2019.
- [21] Dennis M Bushnell. Hypersonic flight experimentation-status and shortfalls. *Future aerospace technology in the service of the alliance*, 3, 1997.
- [22] John J Bertin and Russell M Cummings. Critical hypersonic aerothermodynamic phenomena. *Annu. Rev. Fluid Mech.*, 38:129–157, 2006.
- [23] Oliver M Browne, Sayed M Hasnine, and Christoph Brehm. Towards modeling and simulation of particulate interactions with high-speed transitional boundary-layer flows. In *AIAA Aviation 2019 Forum*, page 2971, 2019.

- [24] E. Brattich, E. Serrano Castillo, F. Giulietti, J.-B. Renard, S. N. Tripathi, K. Ghosh, G. Berthet, D. Vignelles, and L. Tositti. Measurements of aerosols and charged particles on the bexus18 stratospheric balloon. *Annales Geophysicae*, 37(3):389–403, 2019.
- [25] J.-B. Renard, F. Dulac, G. Berthet, T. Lurton, D. Vignelles, F. Jégou, T. Tonnelier, M. Jeannot, B. Couté, R. Akiki, N. Verdier, M. Mallet, F. Gensdarmes, P. Charpentier, S. Mesmin, V. Duverger, J.-C. Dupont, T. Elias, V. Crenn, J. Sciare, P. Zieger, M. Salter, T. Roberts, J. Giacomoni, M. Gobbi, E. Hamonou, H. Olafsson, P. Dagsson-Waldhauserova, C. Camy-Peyret, C. Mazel, T. Décamps, M. Piringer, J. Surcin, and D. Daugeron. Loac: a small aerosol optical counter/sizer for ground-based and balloon measurements of the size distribution and nature of atmospheric particles – part 2: First results from balloon and unmanned aerial vehicle flights. *Atmospheric Measurement Techniques*, 9(8):3673–3686, 2016.
- [26] Joseph B. Habeck, James Flaten, and Graham V. Candler. High-altitude balloon measurements of atmospheric particulates. *AIAA Scitech 2020 Forum*, 2020.
- [27] D. Bushnell. Notes on initial disturbance fields for transition problem. *Instability and Transition*, pages 217–232, 1990.
- [28] Tumin M. A normal multimode decomposition method for stability experiments. *Physics of Fluids*, 8:2777–2779, 1996.
- [29] A. Tumin, X. Wang, and X. Zhong. Direct numerical simulation and the theory of receptivity in a hypersonic boundary layer. *Physics of Fluids*, 19:paper 014101, 2007.
- [30] A. Tumin, X. Wang, and X. Zhong. Numerical simulation and theoretical analysis of perturbations in hypersonic boundary layers. *AIAA J.*, 49:463–471, 2011.
- [31] A. Tumin. Multimode decomposition of spatially growing perturbations in a two-dimensional boundary layer. *Physics of Fluids*, 15(69):2525–2540, 2003.
- [32] E. Forgoston and A. Tumin. Initial-value problem for three-dimensional disturbances in a compressible boundary layer. *Physics of Fluids*, 17(8):084106, 2005.
- [33] A. Tumin. Three-dimensional spatial normal modes in compressible boundary layer. *Journal of Fluid Mechanics*, 586:295–322, 2007.
- [34] B. Saikia, S. M. A. A. Hasnine, and C. Brehm. On the role of discrete and continuous modes in a cooled high-speed boundary layer flow. *Journal of Fluid Mechanics*, 942:R7, 2022.
- [35] P. V. Chuvakhov, A. V. Fedorov, and A. O. Obraz. Numerical modelling of supersonic boundary-layer receptivity to solid particulates. *Journal of Fluid Mechanics*, 859:949–971, 2019.
- [36] A. V. Fedorov. Receptivity of a supersonic boundary layer to solid particulates. *Journal of Fluid Mechanics*, 737:105–131, 2013.

- [37] C. Brehm and H.F. Fasel. A novel concept for the design of immersed interface methods. *J. Comput. Phys.*, 242:234–267, 2013.
- [38] C. Brehm, C. Hader, and H.F. Fasel. A Locally Stabilized Immersed Boundary Method for the Compressible Navier–Stokes Equations. *Journal of Computational Physics*, 295:475–504, 2015.
- [39] C. Brehm, M.F. Barad, and C.C. Kiris. Development of Immersed Boundary Computational Aeroacoustic Prediction Capabilities for Open-Rotor Noise. *Journal of Computational Physics*, (38):690–716, 2019.
- [40] O. M. F. Browne, A. P. Haas, H. F. Fasel, and C. Brehm. An efficient linear wavepacket tracking method for hypersonic boundary-layer stability prediction. *J. Comput. Phys.*, 380:243–268, 2019.
- [41] Oliver M.F. Browne, Anthony P. Haas, Hermann F. Fasel, and Christoph Brehm. A nonlinear compressible flow disturbance formulation for adaptive mesh refinement wavepacket tracking in hypersonic boundary-layer flows. *Computers & Fluids*, 240:105395, 2022.
- [42] C. Brehm, M. F. Barad, J. A. Housman, and C. C. Kiris. A Comparison of Higher-order Finite-Difference Shock Capturing Schemes. *Computers & Fluids*, 122:184–208, 2015.
- [43] J.D. Regele, J. Rabinovitch, T. Colonius, and G. Blanquart. Unsteady effects in dense, high speed, particle laden flows. *International Journal of Multiphase Flow*, 61:1–13, 2014.
- [44] C. T. Crowe. Drag coefficient of particles in rocket nozzles. *AIAA J*, 5:1021–1022, 1967.
- [45] F. A. Morrison. An introduction to fluid mechanics. pages 624–625, 2013.
- [46] L. Schiller and A. Naumann. Über die grundlegenden berechnungen bei der schwerkraftaufbereitung. *Zeitschrift des Vereins Deutsches Ingenieure*, 77:318–321, 1933.
- [47] Alexander Fedorov. Theoretical modeling of TS-dominated transition induced by solid particulates. In *51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, 2013.
- [48] P. Gaydos and A. Tumin. Multimode decomposition in compressible boundary layers. *AIAA Journal*, 42(6):1115–1121, 2004.
- [49] O. M. F. Browne, A. P. Haas, H. F. Fasel, and C. Brehm. An efficient strategy for computing wave-packets in high-speed boundary layers. In *47th AIAA Fluid Dynamics Conference, Denver, CO*, 2017.
- [50] O. M. F. Browne, A. P. Haas, H. F. Fasel, and C. Brehm. A nonlinear wavepacket tracking method for hypersonic boundary-layer flows on irregular domains. *AIAA Aviation 2020 Forum*, page 2985, 2020.

- [51] P. MacNeice, K. M. Olson, C. Mobarri, R. de Fainchtein, and C. Packer. PARAMESH: A Parallel Adaptive Mesh Refinement Community Toolkit. *Computer Physics Communications*, 126(3):330–354, 2000.
- [52] J. Boustani, M. F. Barad, C. C. Kiris, and C. Brehm. Fully-coupled fluid-structure interaction simulations of a supersonic parachute. *AIAA Aviation 2019 Forum*, 2019.
- [53] Jonathan Boustani, Michael F. Barad, Cetin C. Kiris, and Christoph Brehm. An immersed boundary fluid–structure interaction method for thin, highly compliant shell structures. *Journal of Computational Physics*, 438:110369, 2021.
- [54] Joseph B. Habeck, Michael Kroells, Anthony Knutson, and Graham V. Candler. Numerical study on the role of aerosol particles in hypersonic boundary layer transition. *AIAA SCITECH 2024 Forum*.
- [55] Pramod K. Subbareddy and Graham V. Candler. A fully discrete, kinetic energy consistent finite-volume scheme for compressible flows. *Journal of Computational Physics*, 228(5):1347–1364, 2009.
- [56] A. Fedorov and A. Tumin. High-speed boundary-layer instability: Old terminology and a new framework. *AIAA Journal*, 49(8):1647–1657, 2011.
- [57] A. Tumin. LST and the Eigenfunction Expansion Method for Linearized Navier-Stokes Equations (a Summary). *AIAA Paper*, (2020 – 0105), 2020.
- [58] V. Russo, S. M. A. A. Hasnine, and C. Brehm. Particle-impingement simulations for a hypersonic flow over a blunt cone. *AIAA SciTech Forum and Exposition*, 2021.
- [59] P. Paredes, M. M. Choudhari, F. Li, J. S. Jewell, and R. L. Kimmel. Nonmodal growth of traveling waves on blunt cones at hypersonic speeds. *AIAA Journal*, 57(11):4738–4749, 2019.
- [60] B. T. Chu. On the energy transfer to small disturbances in fluid flow (part i). *Acta Mechanica*, 1(3):215–234, 1965.
- [61] JL Van Ingen. A suggested semi-empirical method for the calculation of the boundary layer transition region. *Technische Hogeschool Delft, Vliegtuigbouwkunde, Rapport VTH-74*, 1956.
- [62] J. B. Renard, F. Dulac, G. Berthet, T. Lurton, D. Vignelles, F. Jégou, T. Tonnelier, C. Thauray, M. Jeannot, B. Couté, R. Akiki, N. Verdier, M. Mallet, F. Gensdarmes, P. Charpentier, S. Mesmin, V. Duverger, J. C. Dupont, T. Elias, V. Crenn, J. Sciare, J. Giacomoni, M. Gobbi, E. Hamonou, H. Olafsson, P. Dagsson-Waldhauserova, C. Camy-Peyret, C. Mazel, T. Décamps, M. Piringer, J. Surcin, and D. Daugeron. LOAC: a small aerosol optical counter/sizer for ground-based and balloon measurements of the size distribution and nature of atmospheric particles - Part 2: First results from balloon and unmanned aerial vehicle flights. *Atmospheric Measurement Techniques Discussions*, 8(9):10057–10096, September 2015.

- [63] Erika Brattich, Encarnación Serrano Castillo, Fabrizio Giulietti, Jean-Baptiste Renard, Sachi N. Tripathi, Kunal Ghosh, Gwenael Berthet, Damien Vignelles, and Laura Tositti. Measurements of aerosols and charged particles on the BEXUS18 stratospheric balloon. *Annales Geophysicae*, 37(3):389–403, June 2019.
- [64] Michael C. Fischer. Spreading of a turbulent disturbance. *AIAA Journal*, 10(7):957–959, 1972.
- [65] Vincenzo Russo, Sayed Mohammad Abdullah Al Hasnine, and Christoph Brehm. Towards understanding particle cloud interactions with high-speed boundary layer flows. *AIAA AVIATION 2021 FORUM*, 2021.
- [66] Vincenzo Russo, Sayed Mohammad Abdullah Al Hasnine, and Christoph Brehm. Particle-impingement simulations for a blunt cone in hypersonic flow. In *AIAA Scitech 2021 Forum*.
- [67] Bengt Fornberg. A numerical study of steady viscous flow past a circular cylinder. *Journal of Fluid Mechanics*, 98(4):819–855, 1980.
- [68] S.C.R. Dennis and G.-Z. Chang. Numerical solutions for steady flow past a circular cylinder at reynolds numbers up to 100. *Journal of Fluid Mechanics*, 42(3):471–489, 1970.
- [69] Madeleine Coutanceau and Roger Bouard. Experimental determination of the main features of the viscous flow in the wake of a circular cylinder in uniform translation. part 1. steady flow. *Journal of Fluid Mechanics*, 79(2):231–256, 1977.
- [70] Mark N. Linnick and Hermann F. Fasel. A high-order immersed interface method for simulating unsteady incompressible flows on irregular domains. *Journal of Computational Physics*, 204(1):157–192, 2005.
- [71] Christoph Brehm and Hermann Fasel. A non-staggered immersed interface method for solving the incompressible navier-stokes equations. *40th Fluid Dynamics Conference and Exhibit*, 2010.
- [72] E Berger and R Wille. Periodic flow phenomena. *Annual Review of Fluid Mechanics*, 4(1):313–340, 1972.
- [73] C. Liu, X. Zheng, and C.H. Sung. Preconditioned multigrid methods for unsteady incompressible flows. *Journal of Computational Physics*, 139(1):35–57, 1998.
- [74] R. Mittal, H. Dong, M. Bozkurtas, F.M. Najjar, A. Vargas, and A. von Loebbecke. A versatile sharp interface immersed boundary method for incompressible flows with complex boundaries. *Journal of Computational Physics*, 227(10):4825–4852, 2008.
- [75] R.D. Henderson. Details of the drag curve near the onset of vortex shedding. *Physics of Fluids*, 7(9):2102–2104, 1995.
- [76] S. Marella, S. Krishnan, H. Liu, and H.S. Udaykumar. Sharp interface cartesian grid method i: An easily implemented technique for 3d moving boundary computations. *Journal of Computational Physics*, 210(1):1–31, 2005.

- [77] A Belov, L Martinelli, and A Jameson. A new implicit algorithm with multigrid for unsteady incompressible flow calculations. *33rd Aerospace Sciences Meeting and Exhibit*, 1995.
- [78] S. Taneda. Experimental investigation of the wake behind a sphere at low reynolds numbers. *Journal of the Physical Society of Japan*, 11(10):1104–1108, 1956. cited By 360.
- [79] T.A. Johnson and V.C. Patel. Flow past a sphere up to a reynolds number of 300. *Journal of Fluid Mechanics*, 378:19–70, 1999.