

ABSTRACT

Title of dissertation: DEVELOPMENT OF TWO-POINT
FOCUSED LASER
DIFFERENTIAL INTERFEROMETRY
FOR APPLICATIONS IN HIGH-SPEED
WIND TUNNELS

Andrew Paul Ceruzzi
Doctor of Philosophy, 2022

Dissertation directed by: Professor Christopher P Cadou
Department of Aerospace Engineering

Focused laser differential interferometry (FLDI) and its relative two-point focused laser differential interferometry (2pFLDI) are completely non-intrusive (i.e. seedless) optical techniques for measuring density fluctuations and velocity respectively that offer high frequency response ($> 10MHz$). Developed in the 1970s, FLDI is receiving renewed attention today for its potential usefulness in measuring turbulent fluctuations and velocity in hypersonic flows. In the technique, two focused, closely-spaced ($\sim 100\mu m$), orthogonally-polarized beams pass through a region of interest and are subsequently combined and focused onto a photodetector. Differences in refractive index between the two focal volumes cause a phase shift, thus interference, between the beams which is measured by the detector. In this way the instrument is sensitive to the gradient in refractive index along a line between the two focal volumes perpendicular to the beams ($\partial n / \partial x$). Since gradients in index of refraction arise from gradients in density (in homogeneous flows), fluctuations in

the FLDI signal are proportional to local fluctuations in density. If the fluctuations are due to localized eddies convecting through the FLDI measurement volume, then the cross-correlation of the FLDI signal with a that from a second FLDI instrument located a known distance downstream of the first provides a measure of convection velocity (2pFLDI). The ability to measure density fluctuations and velocity simultaneously and at the same point in the flow is critical because it enables one to relate the temporal scales measured by the instrument to the spatial scales present in the flow.

In spite of the technique's age, a unified theory for the FLDI operation and sensitivity limits which is simple and easy to use does not exist so the first objective of this thesis is to develop such a theory. It does so using transfer functions that enable one to isolate the effects of focusing, beam separation, and disturbance frequency on the performance (i.e. sensitivity and spatial resolution) of the instrument. While the transfer functions have been previously proposed by others, an application of these functions which accounts for velocity variation in space ($u_c(z)$) and frequency ($u_c(f)$) is unique to this work.

The theory is validated via comparison to experimental measurements in a canonical turbulent jet where the distributions of velocity and density fluctuations are well known. Measurements made using different FLDI instruments collapse when the differences between them are accounted for, indicating that the unified theory is correctly capturing the effects of instrument parameters like beam separation and beam diameter. FLDI response to the jet is also modeled by substituting the velocity distribution for a dispersion relation, $u_c(f)$, measured by 2pFLDI. The advantage

of the latter procedure is it allows for signal interpretation in flows where historical measurements are unavailable. This is demonstrated by comparing modeled FLDI response to experimental measurements in the flow downstream of a ramp in a small (6.4cm square) Mach 3 wind tunnel.

The second objective of this thesis is to demonstrate 2pFLDI in other industrially-relevant flows. To this end, density fluctuations and convection velocities are measured in the near-wall flow in a 61cm square Mach 4 wind tunnel (Ludwig tube) and in the free-stream flow of a 1.5m diameter Mach 18 tunnel. In each case, a method for estimating the spatiotemporal resolution using transfer functions is demonstrated. The spatiotemporal resolution of the instrument was not well understood prior to this work so quantifying it is an important contribution. Achieving acceptable signal/noise at Mach 18 was difficult because densities were so low. However, convection velocities of $\sim 75 - 80\%$ of the freestream velocity are measured above $200kHz$ in two runs. Spatiotemporal analysis suggests these measurements are the result of freestream disturbances; the first measurement of its kind in a Mach 18 flow.

DEVELOPMENT OF TWO-POINT FOCUSED LASER
DIFFERENTIAL INTERFEROMETRY FOR APPLICATIONS IN
HIGH-SPEED WIND TUNNELS

by

Andrew Paul Ceruzzi

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2022

Advisory Committee:

Professor Christopher Cadou, Chair/Advisor

Dr. Stuart Laurence

Dr. Kenneth Yu

Dr. Christoph Brehm

Dr. Christopher Davis

© Copyright by
Andrew Paul Ceruzzi
2022

Acknowledgments

First, a huge thank you to my advisor, Prof. Christopher Cadou, for many years of guidance, mentorship, and advice. I greatly appreciate his commitment not just to his students, but to teaching and to his family; he is a role model I look up to. Next, thank you to Prof. Stuart Laurence who has been my un-official co-advisor. He has included me in his research group, provided endless advice, and is an important mentor in my life. Thank you to Prof. Kenneth Yu for many years of encouragement and support as well. Thanks as well to the rest of my committee, Prof. Christoph Brehm and Prof. Chris Davis, for advice and perspective.

Thank you to the many other faculty who I have interacted with at the University of Maryland, including Prof. Pino Martin, Prof. James Baeder, and Prof. Christopher Lobb. Thanks to the Aerospace Engineering department staff, they are a team of all-stars.

A massive thanks goes to my numerous lab-mates and class-mates over the years: Between Cadou-lab, HAPL, and everyone else there are too many of you to name. Thank you for advice, friendship, and camaraderie.

Next, I want to thank the team at AEDC Hypervelocity Wind Tunnel 9. Their partnership with the University of Maryland gave me access to resources, mentorship, and a world-class facility. Thank you Mike Smith, Brianne McManamen, John Lafferty, Dan Marren, Eric Marineau, AJ Spicer, tunnel operators, engineers, and staff. Thank you also to Tunnel 9 interns (at the time) Christoph Nieves, Dan Weber, and Braeden Callis who helped a tremendous amount with experiments.

A big thank you to John Schmisser and Mark Gragston for inviting me to the University of Tennessee Space institute. I greatly appreciate the opportunity to collaborate with and learn from other institutions, and I had a great time in Tennessee. Thanks as well to Lauren Lester and Jack Coburn.

Thanks to the Air Force Office of Scientific Research support through the University of Maryland Hypersonic Center of Testing Excellence program (Grant Number: FA9550-17-1-0085), and the University of Maryland Clark Doctoral Fellowship Program.

Finally, thank you to my Mom and Dad, the rest of my family, and my friends. Your support means the world and I love you all!

List of Publications

1. **Ceruzzi, A. P.**, McManamen, B., & Cadou, C. P. “Demonstration of Two-Point Focused Laser Differential Interferometry (2pFLDI) in a Mach 18 flow,” *AIAA Scitech 2021 Forum*. January, 2021. <https://doi.org/10.2514/6.2021-0983>
2. **Ceruzzi, A. P.**, Neisess, C., McManamen, B., & Cadou, C. P. “Investigation of Focused Laser Differential Interferometry (FLDI) Sensitivity Function,” *AIAA Scitech 2021 Forum*. January, 2021. <https://doi.org/10.2514/6.2021-1299>
3. **Ceruzzi, A. P.**, Callis, B. O., Weber, D. C., Cadou, C. P. “Application of Focused Laser Differential Interferometry (FLDI) in a Supersonic Boundary Layer,” *AIAA Scitech 2020 Forum*. January, 2020. <https://doi.org/10.2514/6.2020-1973>
4. **Ceruzzi, A.**, & Cadou, C. P. “Simultaneous Velocity and Density Gradient Measurements using Two-Point Focused Laser Differential Interferometry,” *AIAA Scitech 2019 Forum*. January, 2019. <https://doi.org/10.2514/6.2019-2295>
5. **Ceruzzi, A.**, & Cadou, C. P. “Turbulent Air Jet Investigation using Focused Laser Differential Interferometry,” *53rdAIAA/SAE/ASEE Joint Propulsion Conference*. July, 2017. <https://doi.org/10.2514/6.2017-4834>
6. Gillespie, G., **Ceruzzi, A.**, & Laurence, S. “Multi-point Focused Laser Differential Interferometry for Noise Measurements in High-Speed Tunnels,” *AIAA Aviation 2021 Forum*. August, 2021.

List of Contributions

1. Demonstrated the efficacy of the transfer function approach for interpreting FLDI signals.
2. Demonstrated the efficacy of 2pFLDI for making non-intrusive measurements of velocity.
3. Developed a methodology for identifying the limits of FLDI's spatiotemporal response based on Signal to Noise Ratio.
4. Demonstrated the ability to measure the velocity dispersion relation within these limits.
5. Developed and demonstrated methods for converting between frequency and wavenumber in FLDI measurements.
6. Improved our understanding of the capabilities and limitations of FLDI.
7. Reported first ever non-intrusive measurements of turbulence intensity and convection velocity in a Mach 18 flow.

Table of Contents

Acknowledgements	ii
Publications	iv
Contributions	v
List of Tables	ix
List of Figures	x
List of Abbreviations	xiv
List of Symbols	xv
1 Introduction	1
1.1 Boundary Layer Transition	1
1.2 Free Stream Turbulence in Ground Test Facilities	3
1.3 Measurement Techniques for Tunnel Noise	4
1.4 Focused Laser Differential Interferometry	5
1.5 Objectives and Approach	9
2 Laser Differential Interferometry Theory	12
2.1 Introduction	12
2.2 LDI Diagnostic	12
2.2.1 Relation between voltage and phase difference	14
2.2.2 Relation between phase difference and density	16
2.2.3 Modeling the density field	17
2.2.4 Reduced analytical relation between phase and density	19
2.3 Focused Laser Differential Interferometry (FLDI)	22
2.3.1 Spatiotemporal resolution	27
2.3.2 Wavenumber spectra	29
2.3.3 A general sensitivity function	30
2.4 Two-Point FLDI (2pFLDI)	31

3	FLDI model validation experiments	35
3.1	Overview	35
3.2	Turbulent Jet	37
3.2.1	FLDI & 2pFLDI equipment and data reduction	38
3.2.2	Adjustment and measurement of beam separation Δx_1	41
3.2.3	Measurement of focal spot radius, w_0	42
3.2.4	Measuring beam-pair separation, Δx_2	45
3.2.5	Experiment and Modeling	45
3.2.6	Uncertainty and sensitivity analysis	48
3.2.7	Results using modeled velocity distribution, $u_c(z)$	49
3.2.7.1	Sensitivity to beam separation (Δx_1)	49
3.2.7.2	Sensitivity to position along the optical axis (z)	54
3.2.7.3	Comparison to uniform velocity assumption	57
3.2.8	Measuring the dispersion relation	59
3.2.9	Results using measured dispersion relation, $u_c(f)$	60
3.2.10	Conclusions	66
3.3	Mach 3 Tunnel	66
3.3.1	FLDI and 2pFLDI equipment and data reduction	67
3.3.2	Modeling	69
3.3.3	Dispersion relation	70
3.3.4	Results	71
3.3.5	Comparison to uniform velocity assumption	75
3.4	Chapter Conclusion and Takeaways	77
4	2pFLDI Applications	79
4.1	Overview	79
4.2	UTSI Mach 4 Ludwig Tube	81
4.2.1	2pFLDI equipment and data reduction	81
4.2.2	Experiment, Modeling and Spatiotemporal resolution	82
4.2.3	Results	86
4.2.3.1	Convection Velocity	87
4.2.3.2	Wavenumber Spectra	89
4.2.3.3	Boundary Layer profiles	93
4.2.4	Conclusion and Takeaways	96
4.3	AEDC T9 Mach 18 Freestream	97
4.3.1	2pFLDI equipment and data reduction	98
4.3.2	Experiment and Modeling	99
4.3.3	Results	102
4.3.3.1	Convection Velocity	104
4.3.3.2	Wavenumber Spectra	107
4.4	Chapter Conclusions and Takeaways	112

5	Summary and Future Work	114
5.1	Interpreting FLDI	114
5.2	Validating the FLDI model	115
5.3	Demonstrations of 2pFLDI	116
5.4	Future Research	117
A	Derivations	119
A.1	Derivation of FLDI transfer functions, Eq.2.24	119
A.1.1	Evaluation of integral over beam radius, r	121
	Bibliography	122

List of Tables

2.1	Constants used to convert from index of refraction to density.	17
3.1	Selected parameters of FLDI experiments in this chapter. See Fig. 2.3 for reference	36
3.2	Selected parameters of 2pFLDI experiments in this chapter. See Fig. 2.6 for reference	37
3.3	Parameter typical value and uncertainty	48
3.4	Free stream conditions at measurement location in wind tunnel.	67
4.1	Parameters of 2pFLDI experiments in this chapter. See Fig. 2.6 for reference	80
4.2	Facility calculated free stream (subscript ∞) conditions.	98
4.3	2pFLDI High Pass (HP) filter settings, and convection velocity averaged across all frequencies	107

List of Figures

1.1	Paths to transition in boundary layer. From Reshotko [1].	2
1.2	Schematic of LDI.	6
1.3	Schematic of FLDI in a wind tunnel.	6
1.4	Schematic of 2pFLDI in a wind tunnel.	6
2.1	Schematic illustration of a Laser Differential Interferometer. $(P1_u/P1_d)$ upbeam/downbeam birefringent prism with splitting angle (ϵ_1) , (FL_u/FL_d) upbeam/downbeam field lens with focal length f_{FL} , (Δx_1) beam separation, (L_{SR}) length of most sensitive region.	13
2.2	Relationship between measured voltage (V) and phase difference between beams $(\Delta\phi)$	15
2.3	Schematic illustration of a Focused Laser Differential Interferometer. Labels for the various components are as follows: (OL) objective lens, $(P1_u/P1_d)$ upbeam/downbeam birefringent prism with splitting angle (ϵ_1) , (FL_u/FL_d) upbeam/downbeam field lens with focal length f_{FL} , (Pol_1) polarizer rotated 45° from x-axis, (L_o) distance from objective lens focus to field lens, (L_i) distance from field lens to focus, (Δx_1) beam separation.	23
2.4	Variation of the product of FLDI transfer functions $H_{\Delta x}H_w$ with wavenumber and z for $\lambda_0 = 633nm$, $w_0 = 7.3\mu m$	25
2.5	Illustration of regions and sensitivity functions used to compute spatiotemporal resolution	29
2.6	Schematic illustration of a Two-Point Focused Laser Differential Interferometer. Additional components and parameters beyond single-point FLDI are: $(P2)$ birefringent prism with splitting angle (ϵ_2) , (Pol_2) polarizer, (CL) collimating lens with focal length f_{CL} , (Δx_2) separation between beam pairs	32
3.1	Diagram of air nozzle. $d = 0.125''$ (3.2mm), $L_n = 2.5''$, $D_p = 1.9''$, P_0 is controlled with the regulator upstream of the plenum	39
3.2	Comparison of beam profiles at the focus for three setups. $\Delta x_1 = 290\mu m$ is produced using the Wollaston prism. The other two beam spacings are produced using the Sanderson prism.	42
3.3	Beam intensity profiles along the z axis	43

3.4	Measurements of the beam radius parameter at various points along the z axis with the fit used to determine w_0 (the beam radius at the focus).	44
3.5	Results from knife-edge test used to determine beam pair separation (Δx_2)	44
3.6	Schematic of turbulent jet in relation to FLDI beams	46
3.7	Raw FLDI phase difference spectra for three beam separations acquired with the instrument focus located at the jet centerline ($y/d = 0$ and $z_0/d = 0$)	50
3.8	FLDI sensitivity function to the jet (Eq. 3.3) for three beam separations at jet centerline ($y/d = 0$) and instrument focus ($z_0/d = 0$). $w_0 = 7.3\mu m$, and $\lambda_o = 632.8nm$.	51
3.9	Average amplitude spectra interrogated by FLDI, found from Eq. 3.2. Measurements are acquired at the jet centerline ($y/d = 0$) and the instrument focus ($z_0/d = 0$)	51
3.10	Difference between amplitude spectra for three beam separations, normalized by amplitude from $\Delta x_1 = 35\mu m$. Measurements are acquired at the jet centerline ($y/d = 0$) and the instrument focus ($z_0/d = 0$)	52
3.11	Percent uncertainty in H_{jet}	53
3.12	Raw FLDI phase difference spectra acquired at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)	54
3.13	FLDI sensitivity function at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)	56
3.14	Average amplitude spectra interrogated by FLDI for jet centerline ($y/d = 0$) and for six jet positions along the optical axis (z_0)	56
3.15	Difference between amplitude spectra recovered for six jet positions along the optical axis (z_0), normalized by amplitude at $z_0 = 0$	57
3.16	Percent uncertainty in H_{jet}	57
3.17	Comparison of amplitude spectra computed using uniform velocity profile (a & b) and Gaussian velocity profile (c) for $x/d = 30$, $y/d = 0$, and six z_0 positions.	58
3.18	Comparison of amplitude spectra difference for uniform velocity profile (a & b) and Gaussian velocity profile (c) for $x/d = 30$, $y/d = 0$, and six z_0 positions.	59
3.19	Frequency-dependent convection velocities along the centerline ($y = 0$, $z_0 = 0$) of a round jet. Computed from the cross-spectra of 2pFLDI measurements (Eq. 2.48-2.50).	61
3.20	2pFLDI measured velocity vs. frequency acquired at jet centerline ($y/d = 0$) and instrument focus ($z_0 = 0$)	62
3.21	Phase difference spectra vs. wavenumber acquired at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)	63
3.22	Sensitivity function vs. wavenumber at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)	63

3.23	Amplitude spectra vs. wavenumber at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)	64
3.24	Difference between amplitude spectra for six jet positions along the optical axis (z_0) normalized by amplitude at $z_0 = 0$	65
3.25	Percent uncertainty in H_{jet} vs. wavenumber	65
3.26	Diagram of the M3CT flow path showing the location of the FLDI measurement volume, ramp, and the coordinate directions.	67
3.27	Illustration of the location of M3CT (disturbance generator) within the FLDI optical path.	68
3.28	Dispersion relation in M3CT flow field downstream of the ramp ‘disturbance generator’ measured with 2pFLDI	71
3.29	Dispersion relation in M3CT plotted against log scale	71
3.30	Phase difference spectra vs. frequency in M3CT for nine tunnel positions along optical axis (z_0). Flow off measurement taken prior to $z_0 = 48cm$ run	72
3.31	Phase difference spectra vs.wavenumber in M3CT for nine tunnel positions along optical axis (z_0)	73
3.32	Sensitivity function vs.wavenumber in M3CT for nine tunnel positions along optical axis (z_0)	74
3.33	Amplitude spectra vs. wavenumber in M3CT for nine tunnel positions along optical axis (z_0)	74
3.34	Difference between amplitude spectra for nine tunnel positions along optical axis (z_0) normalized by amplitude at $z_0 = 0$	75
3.35	Comparison of amplitude spectra computed using uniform velocity profile (a) and using measured frequency-dependent velocity (b) for nine z_0 positions	76
3.36	Comparison of amplitude spectra difference for uniform velocity profile (a) and frequency-dependent velocity (b)	77
4.1	The UTSI Mach 4 Ludwig tube.	81
4.2	Schematic of experiment in UTM4 floor turbulent boundary layer	83
4.3	Estimate of $g(z)$ in UTM4 floor turbulent boundary layer	85
4.4	Estimate of FLDI signal to noise ratio for a $2L_c = 44cm$ depth of focus in UTM4 boundary layer	86
4.5	Phase difference vs. time from FLDI (Ch.A) for six wall normal positions (y)	87
4.6	Phase difference vs. frequency from FLDI (Ch.A) for six wall normal positions (y)	88
4.7	Phase difference vs. time from 2pFLDI (Ch.A & Ch.B) for $y/\delta = 0.4$	89
4.8	2pFLDI convection velocity vs. frequency for six wall normal positions (y)	90
4.9	2pFLDI convection velocity vs. frequency normalized by mean convection velocity over all frequencies for six wall normal positions (y)	90
4.10	Phase difference vs. wavenumber from FLDI (Ch.A) for six wall normal positions (y)	91

4.11	Sensitivity and transfer functions for uniform disturbance in UTM4	92
4.12	Amplitude spectra in UTM4 for six wall normal positions (y)	93
4.13	Turbulence intensity profile in UTM4 floor BL for three wavenumber bands	95
4.14	Velocity profile in UTM4 floor BL computed using three wavenumber bands	96
4.15	Schematic illustration of experiment in AEDC T9 Mach 18 test section	100
4.16	Estimate of $g(z)$ in AEDC T9 Mach 18 test section	101
4.17	Estimate of FLDI signal to noise ratio for a $2L_c = 73cm$ depth of focus in T9M18	102
4.18	Phase difference vs. time from FLDI (Ch.A) for five tunnel runs	103
4.19	Phase difference vs. frequency from FLDI (Ch.A) for five runs. flow off taken prior to run.	104
4.20	Phase difference vs. time from 2pFLDI (Ch.A & Ch.B) for Run 4671	105
4.21	2pFLDI convection velocity normalized by freestream velocity, $U_\infty =$ $200m/s$, vs. frequency	106
4.22	2pFLDI convection velocity vs. f , normalized by average value over all f	108
4.23	Phase difference vs. wavenumber from FLDI (Ch.A)	109
4.24	Sensitivity function for uniform disturbance in T9M18	109
4.25	Amplitude spectra in T9M18 for four runs	110
4.26	Amplitude spectra in T9M18 normalized by freestream density for four runs	111

List of Abbreviations

LDI	Laser Differential Interferometry
FLDI	Focus Laser Differential Interferometry
2pFLDI	Two-point Focus Laser Differential Interferometry
AEDC	Arnold Engineering Development Center
M3CT	Mach 3 Calibration Tunnel
UTSI	University of Tennessee Space Institute
UTM4	UTSI Mach 4 Ludwig Tube
T9	AEDC Hypervelocity Wind Tunnel 9
T9M18	T9 Mach 18 test section
FL	Field lens
OL	Objective lens
Pol	Polarizer
P1	FLDI Prisms
P2	2pFLDI Prism
CL	Collimating lens

List of Symbols

z	optical axis
f	frequency
n	Index of refraction
ρ	Density of fluid
β	Dimensionless Gladstone Dale constant
K	Gladstone Dale constant with units of volume/mass
λ_0	Wavelength of laser
$\Delta\phi$	Phase difference between FLDI interferometer legs
V	Voltage output of photodiode
I	Intensity of light
ϵ_1	Beam splitting angle of FLDI prisms
ϵ_2	Beam splitting angle of 2pFLDI prism
Δx_1	Separation distance between FLDI interferometer legs
Δx_2	Separation distance between 2pFLDI beam pairs
w	Beam radius parameter
w_0	Beam radius at focus
u_c	Disturbance convection velocity
U	mean flow velocity
$g(z)$	non-dimensional amplitude of density fluctuations
σ	Standard deviation
$A(f)$	Amplitude of density fluctuations
H	FLDI transfer functions
k_x	x-axis wavenumber component
ψ	Disturbance phase
$\mathcal{F}$	Fourier transform
R_{AB}	Cross-correlation
Γ_{AB}	Cross-spectrum

Chapter 1: Introduction

1.1 Boundary Layer Transition

Next generation hypersonic vehicles such as long range cruise missiles, space planes and reentry vehicles are critical technologies for our military and offer exciting opportunities for space exploration and civil transport. One of the largest challenges facing designers of such vehicles is predicting the location of laminar to turbulent boundary layer transition on the surface of the vehicle. In hypersonic boundary layers, transition to turbulence is associated with significant increases in skin friction and heat transfer [2], which in turn affects predictions of drag, engine inlet conditions, and thermal protection requirements. Uncertainty in the location of transition leads to over-designed thermal protection systems and unpredictable performance.

An active area of study for several decades, hypersonic boundary layer transition research involves the study of laminar boundary layer receptivity to outside disturbances, the growth of disturbances inside the boundary layer, and the breakdown of a transitioning boundary layer to fully turbulent flow. Fig. 1.1, A popular flowchart, first produced by Morkovin et al. [3] and reproduced here from Reshotko [1] outlines the ways in which laminar boundary layers transition. Read-

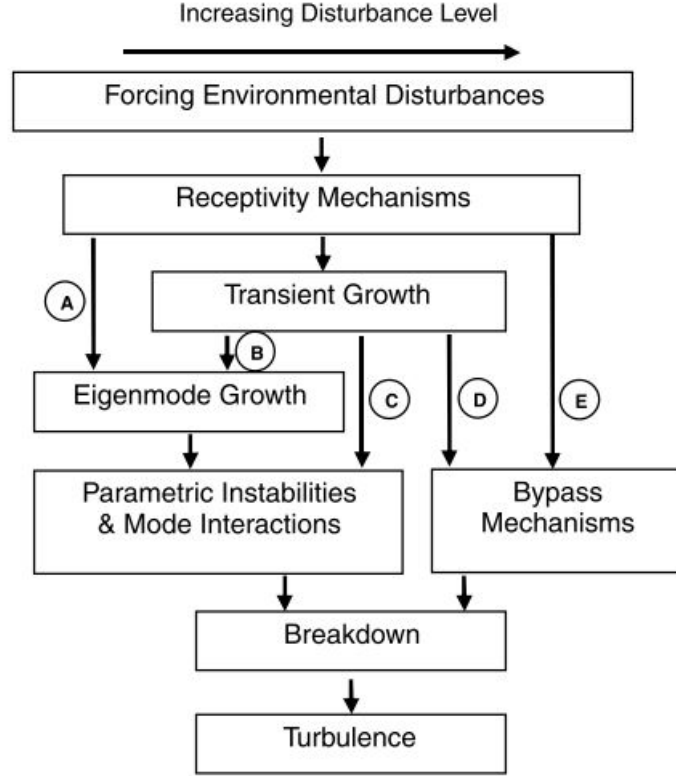


Figure 1.1: Paths to transition in boundary layer. From Reshotko [1].

ing from left to right, as outside disturbance amplitude increases, the paths to transition go from slow modal growth (A), to transient growth (B-D), to bypass mechanisms (E). For a detailed explanation of these paths, the reader is directed to Reshotko [1,4] and the references within. For the purpose of this thesis, we note that understanding the disturbance environment and receptivity mechanisms is critical to understanding which regime of transition a boundary layer will enter. Thus it is critical for predicting transition.

1.2 Free Stream Turbulence in Ground Test Facilities

Ideal boundary layer transition experiments would involve flows at design altitudes with turbulence levels matching those in the atmosphere. The best way to approximate such conditions is via free flight experiments but these are very expensive. As a result, most transition experiments are conducted in ground test facilities like wind tunnels. While tunnels can simulate free-flight Mach numbers, their free-stream turbulence environments are often much harsher than experienced at high altitudes. As a result, it has become widely accepted that free stream turbulence, also called “tunnel noise”, is a principal initiator of transition in wind tunnels [5]. It has also been shown that transition Reynolds numbers measured at the same conditions on the same geometry often vary widely between facilities [6]. Therefore, no study of transition is complete without a firm understanding of the tunnel noise environment.

The origins of tunnel noise include vorticity and entropy fluctuations from the reservoir as well as acoustic waves radiating from the turbulent boundary layers along the tunnel walls [7, 8]. Laufer [9–11] investigated the effect of tunnel noise on transition in supersonic ground facilities. One important takeaway is that the effects of reservoir vorticity are negligible in comparison to the noise field radiated from the walls at $M \geq 2.5$. This observation led to the development of “quiet” supersonic and hypersonic tunnels. An overview of quiet tunnels and the role they play in transition prediction is given by Schneider [12]. Building quiet tunnels is expensive, and while weaker free stream turbulence better simulates free flight, a

proper characterization of the free stream is still essential for meaningful studies of transition in ground test facilities.

1.3 Measurement Techniques for Tunnel Noise

The most common instruments for characterizing tunnel noise are hot wire anemometers (HWA) and pitot pressure probes. Early examples of HWA studies include those referenced in the previous section by Morkovin [8] and by Laufer [10]. Stainback and Wagner [13] used both HWA and pitot probes, providing a useful comparison of the techniques and a method for interpreting the pitot data. Modern measurements of tunnel noise using these intrusive probes include work by Lafferty and Norris [14], Bounitch et al. [15], Masutti et al. [16], Gromyko et al. [17], Schilden et al. [18], and Chaudhry and Candler [19], and many others [20]. Some takeaways from these measurements include:

- Hot-wires are fragile and may not withstand large forces in many Hypersonic facilities.
- Pitot measurements are dependent on the probe geometry, complicating analysis and increasing uncertainty [15].
- Both techniques are intrusive and can not be placed directly upstream of a transition experiment.

These limitations motivate the use of non-intrusive techniques for characterizing tunnels; many are reviewed by Danehy et al. [21]. Most techniques reviewed in

this reference can measure mean flow quantities, but few can measure fluctuations, i.e. tunnel noise. Fluctuations relevant to hypersonic boundary layer instabilities such as the 2nd mode or Mack mode are high frequency (> 400 kHz in some cases [22]) and weak ($< 0.5\%$ of free stream in some cases [23]) requiring an instrument with a unique combination of high bandwidth, sensitivity, and spatial resolution. These requirements motivate the application of a non-intrusive diagnostic tool called Focused Laser Differential Interferometry (FLDI).

1.4 Focused Laser Differential Interferometry

Focused Laser Differential Interferometry (FLDI) is a diagnostic tool that has great potential to advance understanding of tunnel noise and transition. Its relative, Laser Differential Interferometry (LDI), is a method of measuring density differences in fluid flows first developed and documented in the 1970s by Smeets and George [24–27]. The LDI signal is proportional to the difference in average density along the optical paths of two closely-spaced laser beams as illustrated in Fig. 1.2. While very sensitive and capable of operating at bandwidths limited only by the detector, it is an integrated line of sight measurement and thus offers no spatial resolution along the optical (z) axis. Smeets and George sought to correct this deficiency by introducing lenses that brought the two beams to focus (Fig. 1.3) in such a way that the beams overlap everywhere except in a small region near the beams’ foci. This modification is called FLDI and has the effect of reducing sensitivity to density differences away from the focal region. Smeets and George also

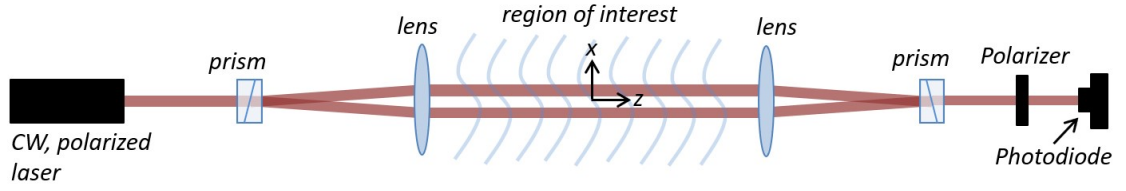


Figure 1.2: Schematic of LDI.

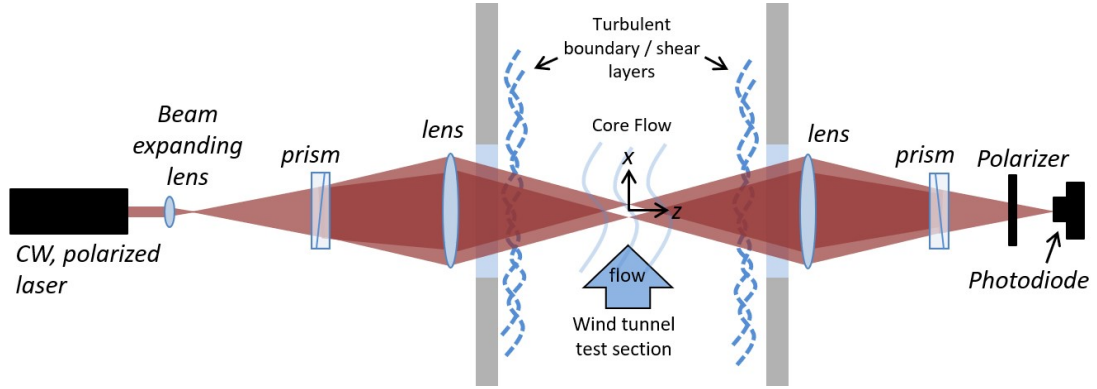


Figure 1.3: Schematic of FLDI in a wind tunnel.

imagined that it would be possible to measure the convection velocity of disturbances by computing the correlation between signals from two FLDI measurement volumes a known distance apart. We call this technique two-point FLDI (2pFLDI) and provide a diagram of the instrument in Fig. 1.4.

The modern advancement and accessibility of lasers with long coherence lengths

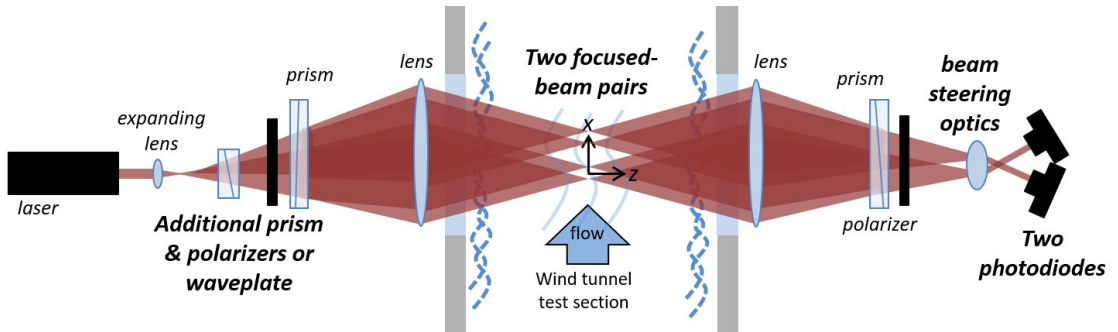


Figure 1.4: Schematic of 2pFLDI in a wind tunnel.

and low noise and data acquisition technology with $> 10\text{MHz}$ bandwidth has led to renewed interest in FLDI as diagnostic for high speed flows. This FLDI “revival” was spurred on by Parziale et al. [23, 28–30] in the early 2010s, who demonstrated measurements of free stream disturbances and 2nd-Mode instabilities in a shock tunnel. In the ten years following Parziale’s first paper, FLDI has been employed by many others. Notable early applications were by Fulghum [31], Settles and Fulghum [32], and Schmidt and Shepherd [33] who sought to characterize the instrument’s transfer functions, recognizing that uncertainty in these functions would hamper usefulness of the technique.

Other recent applications of FLDI include work by Benitez et al. [34], who modified optics to measure 2nd-mode instability amplitudes inside a quiet tunnel with contoured windows, Houpt et al. [35], who employed a cylindrical FLDI in order to probe disturbances near the surface of flat walls, and Birch et al. [36] who used shrouds to minimize the effect of sidewall turbulence on the FLDI signal. Recent applications of 2pFLDI include those by Jewell et al. [37], who used a Koester prism to multiplex the FLDI and probe a circular jet, Bathel et al. [38], who used a Nomarski prism to double the FLDI and measure the disturbance speed of a 2nd mode wave in a Mach 6 flow, and by the author [39–41] (results to be discussed in this work). Also conceived by Smeets and George [26], multiplexing the FLDI offers an attractive method for simultaneously probing a larger spatial domains and making two-dimensional disturbance velocity measurements. Gragston et al. [42, 43] demonstrated a method to achieve this using a diffracting optic. Weisberger et al. [44] demonstrated a multi-point “line FLDI” which uses ovoid beam profiles that

are discretely sampled by a photodiode array.

However, and despite all of these applications, there remains no simple, widely accepted method for quantitatively interpreting the FLDI signal and the disturbance velocity measured by 2pFLDI in turbulent flows. In most of the applications described above, FLDI spectra are presented in arbitrary units or there are large uncertainties in the density fluctuations reported. Parziale’s measurements of free stream fluctuations and 2nd mode waves are an exception but they rely on a separate experiment on a small jet to characterize the FLDI’s response function [30]. While this approach is reasonable for this particular application, it is not clear that it would work in other flow geometries and it is not clear how the parameters of the jet and of the FLDI affect the calibration. Nevertheless, there have been many excellent efforts to characterize the response of the instrument over the past eight years [31–33, 45–48]. Fulghum’s dissertation [31] is filled with insights on the FLDI’s response including the use of transfer functions to describe how the instrument filters an arbitrary density field. Schmidt and Shepherd also explored similar analytical methods for interpreting the signal [33] but ultimately concluded that full simulation via ray tracing is the best way to quantitatively interpret the signal.

The most comprehensive, modern review of FLDI is Lawson’s dissertation [49], published less than one year ago. Lawson thoroughly validates the ray tracing model introduced by Schmidt and Shepherd. He then introduces an analytical method for interpreting the FLDI response to acoustic waves of arbitrary orientation, in an effort to model the noise environment in hypersonic facilities. Using this method good agreement between FLDI and pitot measurements is observed, representing a major

step forward in the technique’s practicality. This work serves as a complimentary piece to Lawson’s and builds off of the framework, terminology and notations used there. His work gives confidence that the analytical transfer function methods can be successful, leading to reliable methods for interpreting the FLDI signal which are computationally cheap and easy to use.

In summary, previous work illustrates the need for a simple, easy to use procedure for interpreting the FLDI instrument. Analytical transfer functions derived from ray tracing Gaussian beams appear to be a good candidate for this procedure but validation of these functions in complex turbulent flows is minimal [32,33,48,49]. 2pFLDI provides a direct measure of convection velocity, a critical parameter of the transfer functions. However, an investigation of how these measured velocities can reduce the uncertainty in the single-point FLDI measurement has yet to be conducted because applications of 2pFLDI are relatively novel compared with single-point FLDI. The need for these procedures, validations, and investigations motivates the goals of this work, which are outlined next.

1.5 Objectives and Approach

The objectives of this work are to:

1. develop a simple quantitative theory for interpreting the single point FLDI signal
2. validate the theory via comparison to experiments in complex, flows such as jets and wind tunnels

3. quantify the FLDI's spatiotemporal resolution
4. expand our understanding of the 2pFLDI velocity measurement
5. apply 2pFLDI in industrial high-speed facilities.

To accomplish these objectives, we begin in Chapter 2 by reviewing LDI theory and proposing a density fluctuation model for turbulent flows. The model assumes density fluctuations are sinusoidal, convect parallel to the flow, and amplitude variations along the optical (z) axis and with frequency (f) are independent of one another. This model is chosen for simplicity: It allows the governing equation proposed by Schmidt and Shepherd [33] to reduce analytically into a tractable expression. The key difference between this model and previous work is we allow the disturbance convection velocity, u_c , to vary with z and f , i.e we accommodate velocity gradients and dispersion. We propose a method for determining the FLDI spatiotemporal resolution to an arbitrary flow using the model.

Next, in Chapter 3 we quantify our ability to remove instrument effects from the signal measured in a turbulent air jet and the flow downstream of a ramp in a Mach 3 wind tunnel. In the jet, two approaches for determining the convection velocity are explored: (1) model velocity profiles $u_c(z)$ based on historical jet measurements, and (2) measure dispersion relation $u_c(f)$ using 2pFLDI. The latter approach can be conducted in flows where historical measurements are unavailable or uncertain, such as in the Mach 3 tunnel. In many cases, our model matches measured amplitudes within $\pm 50\%$, which we take as sufficient validation.

In Chapter 4 we demonstrate applications of 2pFLDI in a Mach 4 Ludwig

tube and Mach 18 blowdown facility. Using our method for determining spatial-temporal resolution, results from each of these facilities are put in proper context. The latter facility represent first-of-their-kind application of 2pFLDI.

Finally, in Chapter 5 we conclude, review implications of our results, and discuss future directions for research and development.

Chapter 2: Laser Differential Interferometry Theory

2.1 Introduction

This chapter is organized as follows: first we discuss the Laser Differential Interferometer (LDI) including how it works, what a set-up looks like, and how to relate the output signal measured to the index of refraction field probed by the instrument. Next we move to the Focused Laser Differential Interferometer (FLDI) and explain the instrument's set-up and methods for interpreting the signal. Much of the FLDI set-up and interpretation builds off of ideas introduced by the LDI, so proper understanding of both instruments should begin with a firm understanding of the LDI. Finally, we explain the two-point FLDI and how it improves upon the capabilities of a single-point FLDI.

2.2 LDI Diagnostic

The Laser Differential Interferometry (LDI) signal is proportional to the finite-difference approximation of the index of refraction gradient ($\partial n / \partial x$) integrated along the line of sight. Fig. [2.1](#) shows the basic LDI set-up which provides the foundation for FLDI and 2pFLDI. The light source is a monochromatic continuous wave (CW)

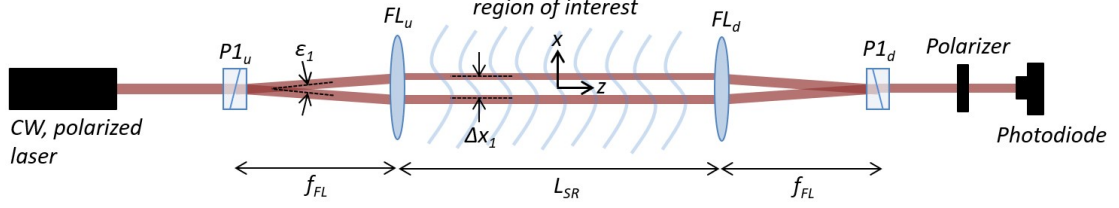


Figure 2.1: Schematic illustration of a Laser Differential Interferometer. ($P1_u/P1_d$) upbeam/downbeam birefringent prism with splitting angle (ϵ_1), (FL_u/FL_d) upbeam/downbeam field lens with focal length f_{FL} , (Δx_1) beam separation, (L_{SR}) length of most sensitive region.

laser linearly polarized at 45° with respect to the axis that defines the gradient of interest. A circularly polarized beam is also acceptable. If the laser does not output a linearly polarized beam at 45° or a circularly polarized beam, a polarizer [28] or wave-plate [38] is required up-beam of the first birefringent prism ($P1_u$). This prism splits the beam into two equally intense orthogonally polarized beams that diverge at a small angle (ϵ_1). A field lens (FL_u) located one focal length (f_{FL}) down-beam of the prism arrests the divergence and fixes the distance between the two beams (Δx_1). This distance is given by:

$$\Delta x_1 = 2f_{FL} \tan\left(\frac{\epsilon_1}{2}\right). \quad (2.1)$$

The beams propagate through a region of interest of length (L_{SR}) where index of refraction gradients create an optical path difference - and thus a phase difference ($\Delta\phi$) - between the beams. Beyond the region of interest, the orthogonally polarized beams are recombined by a field lens (FL_d) and birefringent prism ($P1_d$) identical to FL_u and $P1_u$ but operating in reverse. The recombined beams pass through a

polarizer aligned with the laser polarization (45° from the splitting axis) allowing the beams to interfere.

2.2.1 Relation between voltage and phase difference

The intensity of light (I_{PD}) incident on the photodiode is a function of the phase difference as follows:

$$I_{PD} = 2I_b(1 + \cos(\Delta\phi)), \quad (2.2)$$

where (I_b) is the intensity of a single beam (p. 259 of [50]). In practice, perfect interference contrast is not achievable and the intensity will not reach zero when $\Delta\phi = \pi$. Also, high-frequency signals will be attenuated by the photodiode circuit's response. Thus, the voltage output from the photodiode (V) is expressed as:

$$V = \frac{1}{2} [V_{max} + V_{min} + (V_{max} - V_{min}) \cos(\Delta\phi) H_{PD}], \quad (2.3)$$

where H_{PD} is a transfer function that accounts for the photodiode circuit's response.

Voltage is plotted as a function of phase difference in Fig. 2.2. Using the same terminology as Parziale [28], we define

$$V_0 = \frac{V_{max} + V_{min}}{2}. \quad (2.4)$$

Adjusting the phase by translating $P1_u$ or $P1_d$ along the x-axis so that $V = V_0$ ensures that the instrument's response is approximately linear over its sensitive region i.e. $V - V(t = 0) \approx \Delta\phi - \Delta\phi(t = 0)$. This adjustment is critical because it maximizes sensitivity, simplifies interpretation of the signal, and minimizes the likelihood of phase ambiguity which occurs when the signal reaches V_{max} or V_{min}

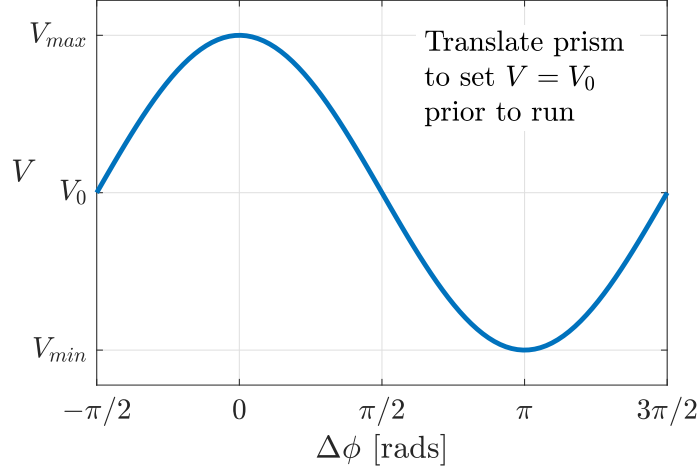


Figure 2.2: Relationship between measured voltage (V) and phase difference between beams ($\Delta\phi$)

during an experiment. The relationship between voltage measured over time and phase difference can now be written as

$$V(t) = V_0 + V_0 \sin(\Delta\phi(t))H_{PD}. \quad (2.5)$$

Fulghum measured the response of the same detector system used in this work (a Thorlabs DET36 photodiode with a $1k\Omega$ resistor) to a LED pulsed square wave for frequencies to $500kHz$ [31] and found it to be flat. Thus, we assume

$$H_{PD} = 1 \text{ for } f < 500kHz \quad (2.6)$$

which means that we can ignore the response of the photodiode circuit for frequencies below $500kHz$. This allows us to write

$$\Delta\phi(t) = \arcsin\left(\frac{V(t)}{V_0} - 1\right), \quad (2.7)$$

which is a common assumption in other FLDI work [23, 33, 34, 36].

2.2.2 Relation between phase difference and density

For two beams traversing paths s_1 and s_2 respectively, the phase difference at the detector face ($s = D$) associated with the index of refraction field $n(\mathbf{x})$ is given by

$$\Delta\phi(t) = \frac{2\pi}{\lambda_0} \int \int_D I_0 \left[\int_S^D n(\mathbf{x}_1, t) ds_1 - \int_S^D n(\mathbf{x}_2, t) ds_2 \right] d\xi d\eta, \quad (2.8)$$

where λ_0 is the laser wavelength, S is the location of the light source, $\mathbf{x}_i(s_i) = (x_i, y_i, z_i)$, and $I_0(\xi, \eta)$ is the intensity on the face when there is no phase difference between the two beams. ξ and η are coordinates which map the detector surface. Eq. 2.8 describes a general interferometer and is taken directly from Schmidt and Shepherd [33] who derived it from Born and Wolf [50]. The relationship between index of refraction and gas density is given by:

$$n = 1 + \beta \frac{\rho}{\rho_s}, \quad (2.9)$$

where β , a dimensionless constant, is a function of gas composition and light wavelength (p. 154 of [51]) and ρ_s is density at $0^\circ C$ and $760 mmHg$. β is often shorted to the Gladstone-Dale constant, $K = \beta/\rho_s$ [52, 53]. Table 2.1 gives the constants used in this work. Thus, we can express Eq. 2.8 in terms of the density field, $\rho(\mathbf{x}, t)$:

$$\Delta\phi(t) = \frac{2\pi K}{\lambda_0} \int \int_D I_0 \left[\int_S^D \rho(\mathbf{x}_1, t) ds_1 - \int_S^D \rho(\mathbf{x}_2, t) ds_2 \right] d\xi d\eta. \quad (2.10)$$

One can solve for the exact phase change provided a complete, well-resolved solution for $\rho(\mathbf{x}, t)$ is available. However, there is no way to solve Eq. 2.10 in the other direction ie. for $\rho(\mathbf{x}, t)$ from $\Delta\phi(t)$. This is because many density fields can result

Parameter	Air	Nitrogen	Units
β	2.91	2.96	10^{-4} (-)
ρ_s	1.29	1.25	kg/m^3
K	2.25	2.37	$10^{-4} m^3/kg$

Table 2.1: Constants used to convert from index of refraction to density.

in the same measured phase difference. Thus, in order to invert Eq. 2.10 and solve for some aspect of the density field, we must make assumptions about the field itself.

2.2.3 Modeling the density field

Previous work [33] has modeled disturbances as sinusoidal waves with frequency-dependent amplitudes that vary in time and in space but only along the flow direction (x). Lawson [49] extends this analysis to account for waves propagating at any angle. While this may be appropriate for homogeneous free-stream turbulence and acoustic noise, it may be less so for flows with substantial variations along the beam direction (z). Settles and Fulghum [32] suggested modeling an amplitude variation with z , such as a Gaussian model for a jet. Building on these previous works, we propose the following model for the density field

$$\rho(x, y, z, t) = g(z) \int_{-\infty}^{\infty} A(f) \exp \{i[k_x x - 2\pi f t + \psi]\} df \quad (2.11)$$

where k_x is the disturbance wavenumber oriented parallel to the flow direction (x), $\psi = \psi(f)$ is the disturbance phase, $A(f)$ is the disturbance amplitude (which depends on the frequency f), and $g(z)$ is a unit-less function which models the distur-

bance amplitude variation with z , averaged across all frequencies. Wavenumber is defined as

$$k_x = \frac{2\pi f}{u_c}. \quad (2.12)$$

where u_c is the streamwise convection velocity of a disturbance. Thus, Eq. 2.11 can also be expressed as

$$\rho(x, y, z, t) = g(z) \int_{-\infty}^{\infty} A(f) \exp \{i [k_x (x - u_c t) + \psi]\} df. \quad (2.13)$$

While it is not explicitly clear in Eq. 2.13, allowing the disturbance convective velocity to vary with z and f , i.e. $u_c = u_c(z, f)$ differentiates this model from those used in previous work. In other words, we are allowing for velocity gradients and dispersion.

For comparison, we also include the model used by Lawson [49] (Eq. 5.45 in his dissertation), given by

$$\rho(x, y, z, t) = \int_{-\infty}^{\infty} A(f) \exp \{i [\mathbf{k} \cdot \mathbf{x} - 2\pi f t + \psi]\} df. \quad (2.14)$$

$\mathbf{k} = (k_x, k_y, k_z)$ is the full wavevector with magnitude k and direction defined by angles α and β such that

$$k_x = k \cos \alpha \cos \beta \quad (2.15)$$

$$k_y = k \sin \alpha \quad (2.16)$$

$$k_z = k \cos \alpha \sin \beta. \quad (2.17)$$

A similar model is also explored by Hameed and Parziale [48]. Note that this model does not include $g(z)$, while our model does not include k_y and k_z . In other words,

Lawson assumes disturbance magnitudes are uniform along the optical axis and we assume disturbances are oriented in the x-direction. While accounting for the full wavevector is more physically accurate, it also leads to a more complex equation. Our analysis will use the density field given by Eq. 2.11 because our goal is to find a concise reduction of Eq. 2.10 that leads to a simple model of the instrument.

2.2.4 Reduced analytical relation between phase and density

In order to reduce Eq. 2.10, we also need to model the intensity distribution in the beam. We use a Gaussian profile given by

$$I_0(r, \theta) = \frac{2}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right), \quad (2.18)$$

where w is the Gaussian beam radius parameter [54]. The polar coordinates r and θ are related to the Cartesian coordinates

$$x = r \cos \theta \quad (2.19)$$

$$y = r \sin \theta. \quad (2.20)$$

If we restrict ourselves to the region between field lenses FL_u and FL_d , the paths of complimentary rays in the interferometer can be parameterized by the z-axis and written as

$$\mathbf{x}_1(z) = \left(x + \frac{\Delta x_1}{2}, y, z\right) \quad (2.21)$$

$$\mathbf{x}_2(z) = \left(x - \frac{\Delta x_1}{2}, y, z\right). \quad (2.22)$$

Returning to Eq. 2.10, re-writing the 2-D integral in polar coordinates, and exchanging the order of integration yields the following expression for the phase difference

measured by the instrument in terms of the local density field:

$$\Delta\phi(t) = \frac{2\pi K}{\lambda_0} \int_{-L_{SR}/2}^{L_{SR}/2} \left[\int_0^{2\pi} \int_0^\infty I_0\rho(\mathbf{x}_1, t) r dr d\theta - \int_0^{2\pi} \int_0^\infty I_0\rho(\mathbf{x}_2, t) r dr d\theta \right] dz. \quad (2.23)$$

Introducing the model of the density field (Eq. 2.13), Eq. 2.18-Eq. 2.22, and simplifying the integrand analytically leads to:

$$\Delta\phi(t) = \frac{2\pi K}{\lambda_0} \int_{-L_{SR}/2}^{L_{SR}/2} \left[g(z) \int_{-\infty}^\infty 2A(f) \times \exp\left(-\frac{k_x^2 w^2}{8}\right) \sin\left(\frac{k_x \Delta x_1}{2}\right) e^{-2\pi i f t} e^{i\psi} df \right] dz. \quad (2.24)$$

The details of this derivation are presented in Appendix A.1. This form is useful because it isolates the effects of the beam radius (w) and the beam spacing (Δx_1) on the signal through the exponential and sinusoidal functions in the integrand. It is convenient to represent these dependencies as transfer functions $H_{\Delta x}$ and H_w respectively where

$$H_{\Delta x} = \sin\left(\frac{k_x \Delta x_1}{2}\right) = \sin\left(\frac{\pi f \Delta x_1}{u_c}\right), \quad (2.25)$$

and

$$H_w = \exp\left(-\frac{k_x^2 w^2}{8}\right) = \exp\left(-\frac{\pi^2 w^2 f^2}{2u_c^2}\right) \quad (2.26)$$

These functions have been derived in previous works [23, 31, 33, 45] using slightly different definitions and terminologies. For example, $H_{\Delta x}$ is equivalent to the correction factor c in [28] and $(H_s \frac{i\omega \Delta x}{2c_r})$ in [33]. Schmidt and Shepherd refer to this as the differentiation performed by the instrument. H_w describes the spatial filtering caused by integrating over the local beam intensity profile $I_0(r, \theta)$. Fulghum

and Schmidt and Shepherd [31, 33] derive the version of Eq. 2.26 based on the disturbance wavenumber (k_x) and beam radius parameter (w). The version of Eq. 2.26 that uses the disturbance frequency (f) and its convective velocity (u_c) is not presented in these works, though it is implied. All approaches assume that the disturbances are sinusoidal. A concise validation of Eq. 2.26 was recently performed by Lawson et al. [45] using acoustic waves.

Returning to Eq. 2.24, we exchange the order of integration again and use the transfer functions as short-hand to write:

$$\Delta\phi(t) = \frac{4\pi K}{\lambda_0} \int_{-\infty}^{\infty} A(f) \int_{-L_{SR}/2}^{L_{SR}/2} g(z) H_{\Delta x} H_w dz e^{-2\pi i f t} i e^{i\psi} df \quad (2.27)$$

Note the similarity of Eq. 2.27 to the Fourier transform of a time dependent signal:

$$\mathcal{F}[\] = \int_{-\infty}^{\infty} [\] e^{2\pi i f t} dt. \quad (2.28)$$

Therefore, taking the Fourier transform of Eq. 2.27 eliminates the $e^{-2\pi i f t}$ term. If we restrict our interest to the magnitude of disturbances (i.e the magnitude of the Fourier transform) and make the additional assumption that phases of the disturbances $\psi(f)$ are random and uncorrelated - which is a reasonable assumption in many turbulent flows (see Chapter 5 of [49]) - then $|ie^{i\psi}| = 1$ and 2.27 becomes:

$$\mathcal{F}[\Delta\phi] = \frac{4\pi K}{\lambda_0} A(f) \int_{-L_{SR}/2}^{L_{SR}/2} |g(z) H_{\Delta x} H_w| dz. \quad (2.29)$$

Eq. 2.29 is a concise description of the response of the LDI instrument to a flow whose density field can be modeled by Eq. 2.11. The left-hand-side is the Fourier transform of the measured signal. The right hand side is the response of the instrument which is a function of three instrument parameters:

1. λ_0 - the laser wavelength
2. w - the local beam radius
3. Δx_1 - the beam separation

and three flow field parameters:

1. $g(z)$ - frequency-averaged density disturbance amplitude variation with z .
2. $u_c(z, f)$ - convective (x -axis) disturbance velocity variation with z or f .
3. $A(f)$ - density disturbance amplitude spectra.

All instrument parameters are constant across the interval $-L_{SR}/2 \leq z \leq L_{SR}/2$ in the LDI system illustrated in Fig. 2.1. Thus, any variation in sensitivity along the optical axis is caused by flow parameters only. This poses a problem in wind tunnel applications where strong fluctuations associated with turbulence along the tunnel walls can overwhelm the signals from the weaker fluctuations in the core flow that are of interest. However, this is a problem that affects all line of sight optical techniques.

2.3 Focused Laser Differential Interferometry (FLDI)

FLDI is a variant of LDI designed to overcome the line of sight problem. It works by expanding the interferometer beams to large diameters and then focusing them such that they overlap along most of their optical paths save for small regions near the focal points. This has the effect of limiting the sensitive region (L_{SR}) to a

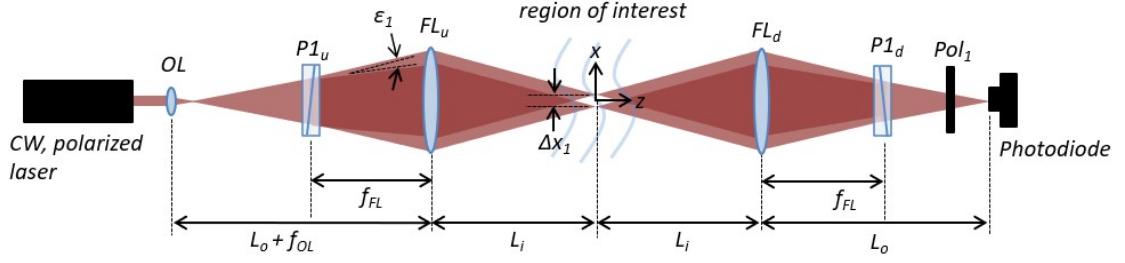


Figure 2.3: Schematic illustration of a Focused Laser Differential Interferometer. Labels for the various components are as follows: (OL) objective lens, ($P1_u/P1_d$) upbeam/downbeam birefringent prism with splitting angle (ϵ_1), (FL_u/FL_d) up-beam/downbeam field lens with focal length f_{FL} , (Pol_1) polarizer rotated 45° from x-axis, (L_o) distance from objective lens focus to field lens, (L_i) distance from field lens to focus, (Δx_1) beam separation.

small area around the focal points. A schematic illustration of the FLDI instrument is presented in Fig. 2.3. An objective lens (OL) with short focal length (f_{OL}) located at a distance $f_{OL} + L_o$ up-beam of the field lens expands the beams so that they overlap substantially reaching a max diameter (D_{FL}) at the field lens. The field lens focuses the beams at a distance L_i down-beam of the field lens. The relationship between L_o , L_i , and f_{FL} is given by the thin lens equation:

$$\frac{1}{L_o} + \frac{1}{L_i} = \frac{1}{f_{FL}}. \quad (2.30)$$

The only difference between LDI and FLDI is that the beam diameter is now a function of distance along the optical axis, i.e. $w = w(z)$. This function is given by

$$w(z)^2 = w_0^2 \left[1 + \left(\frac{\lambda_0 z}{\pi w_0^2} \right)^2 \right], \quad (2.31)$$

where w_0 is the beam radius at the focus ($z=0$) [54]. w_0 can be determined by measuring the beam diameter at any known position z and solving Eq. 2.31 for w_0 . Several measurements are better than one for determining w_0 . Substituting Eq. 2.31 into Eq. 2.26 gives:

$$H_w = \exp\left(-\frac{k_x^2 w_0^2}{8} - \frac{k_x^2 \lambda_0^2 z^2}{8\pi^2 w_0^2}\right) = \exp\left(-\frac{\pi^2 f^2 w_0^2}{2u_c^2} - \frac{f^2 \lambda_0^2 z^2}{2w_0^2 u_c^2}\right) \quad (2.32)$$

With this context, let us re-examine Eq. 2.29 applied to the region between the field lenses of an FLDI:

$$\mathcal{F}[\Delta\phi] = \frac{4\pi K}{\lambda_0} A(f) \int_{-L_i}^{L_i} |g(z) H_{\Delta x} H_w| dz. \quad (2.33)$$

The right hand side describes the attenuation of the true density fluctuation spectrum by the instrument. $H_{\Delta x}$ captures the effect of beam separation which attenuates response as the disturbance wavenumber moves away from $\pi/\Delta x_1$. H_w captures the effect of focusing which causes high wavenumbers to be attenuated faster than smaller ones as one moves away from the focus. The combined effects of $H_{\Delta x}$ and H_w on the response of one FLDI set-up used in this work are illustrated in Fig. 2.4 which is a contour plot of the response of the instrument as a function of distance along the optical axis (z -direction) and the wavenumber of the disturbance. The spatial resolution in the z -direction (taken as the full-width half max of $H_{\Delta x} H_w$) is $\sim \frac{\pi w_0}{2\lambda_0} mm$ for $k_x = \pi/\Delta x_1$ and improves slightly as wavenumber increases (wavelength decreases). As wavenumber decreases below $\pi/\Delta x_1$, resolution degrades while the overall response decreases. This has the effect of reducing the range of wavenumbers to which the instrument is sensitive far from the focus.

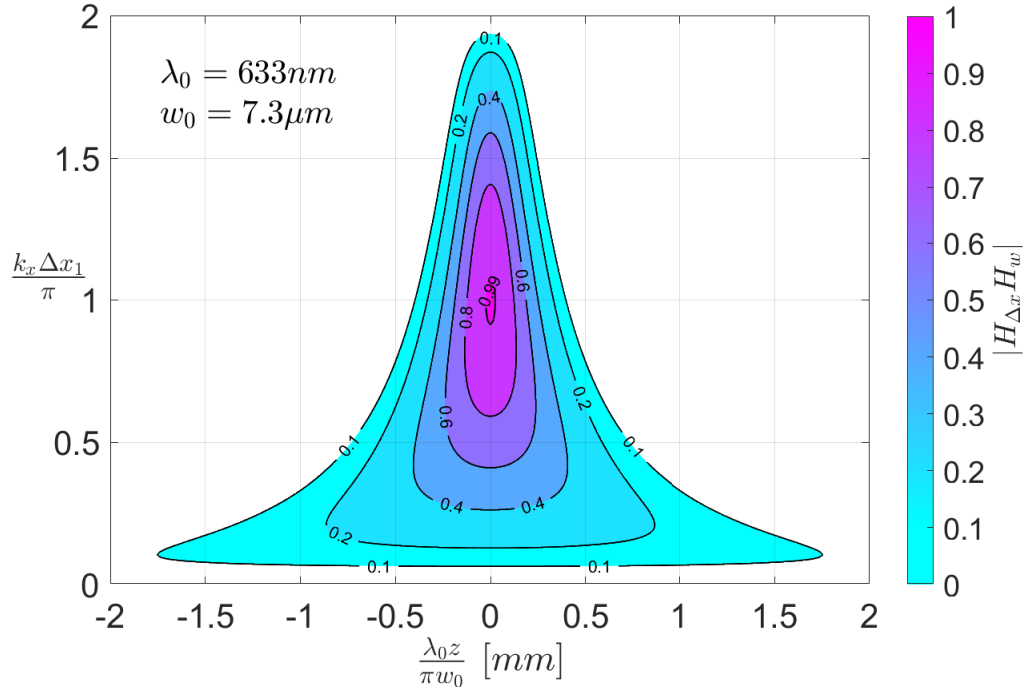


Figure 2.4: Variation of the product of FLDI transfer functions $H_{\Delta x}H_w$ with wavenumber and z for $\lambda_0 = 633nm$, $w_0 = 7.3\mu m$

One objective of this work is to understand if the combined transfer function, $H_{\Delta x}H_w$ accurately models the FLDI's response to an arbitrary flow field. To do this, we define the “FLDI-flow-field sensitivity function”:

$$H_{flow} = \int_{-L_i}^{L_i} |g_{flow}H_{\Delta x}H_w| dz. \quad (2.34)$$

H_{flow} has units of length, and is proportional to FLDI signal strength. In this way it is similar to an “effective” integration length of the instrument. However, it *does not* represent a physical integration length or spatial resolution, because it models signal sensitivity to beam separation in addition to sensitivity along the optical axis. Thus, two FLDIs with different beam separations could have the same H_{flow} , but would not necessarily have the same spatial resolution. Determining the FLDI's spatiotemporal resolution is discussed in section 2.3.1.

To evaluate this integral, we need estimates of $g_{flow}(z)$ and $u_{c,flow}(z, f)$ for the flow field. While these may be readily available for well-studied flows such as turbulent jets, they are far less so for complex environments such as high-speed wind tunnels. We will investigate the accuracy of this model for both examples. H_{flow} is evaluated by plugging in the instrument parameters, λ_0 , w_0 , Δx_1 , as well as the flow models, $g_{flow}(z)$ and $u_{c,flow}(z, f)$ into the right side of Eq. 2.34. If the flow models are simple, the integral may be evaluated analytically. More generally, the integral can be evaluated numerically. The result will be a function of frequency, i.e. $H_{flow}(f)$, and once found Eq. 2.33 can be re-arranged to give an expression for the density fluctuation spectrum in the flow:

$$A_{flow}(f) = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi](f)}{H_{flow}(f)} \quad (2.35)$$

$A_{flow}(f)$ is the amplitude of the density fluctuation at frequency f . Eq. 2.35 deconvolves the effects of beam size and beam separation from the signal so A_{flow} may also be thought of as the spectrum measured by an ‘ideal’ LDI with a beam radius and beam separation of zero. Note that by deconvolving the spatial filtering associated with beam size, we are reversing the focusing effects of the instrument. Therefore, Eq. 2.35 alone is not enough to interpret the FLDI signal when spatial resolution along the optical axis is desired.

2.3.1 Spatiotemporal resolution

While the transfer functions $H_{\Delta x}$ and H_w , plotted in Fig. 2.4, comprehensively describe FLDI’s response to disturbance wavenumbers modeled by Eq. 2.11, they do not tell us which frequencies in the signal can be resolved at what depth of focus, i.e. the spatiotemporal resolution. In order to determine this, we need to know how wavenumbers in the flow map to frequencies in the signal (from the convection velocity $u_c(z, f)$) and whether signals from disturbances far from the focus can overwhelm signals from disturbances near the focus (from $g(z)$ and the transfer functions). The following is a method for determining spatiotemporal resolution when u_c and g are known:

1. Choose a desired depth of focus with length $2L_c$.
2. Break the flow field sensitivity function integral (Eq. 2.34) into three parts giving:

$$H_{flow} = \int_{-L_i}^{-L_c} |gH_{\Delta x}H_w|dz + \int_{-L_c}^{L_c} |gH_{\Delta x}H_w|dz + \int_{L_c}^{L_i} |gH_{\Delta x}H_w|dz. \quad (2.36)$$

Define the following sensitivity functions as illustrated in Fig. 2.5:

$$H_m = \int_{-L_c}^{L_c} |gH_{\Delta x}H_w|dz \quad (2.37)$$

$$H_{rl} = \int_{-L_i}^{-L_c} |gH_{\Delta x}H_w|dz \quad (2.38)$$

$$H_{rr} = \int_{L_c}^{L_i} |gH_{\Delta x}H_w|dz \quad (2.39)$$

such that Eq. 2.36 is expressed as

$$H_{flow} = H_{rl} + H_m + H_{rr}, \quad (2.40)$$

H_m represents sensitivity to the region of the flow field you want to measure while H_{rl} and H_{rr} represent the sensitivity to regions of flow you want to reject (to the left and right of the focus, respectively). Abbreviate further by defining $H_r = H_{rl} + H_{rr}$ as the sensitivity to both regions of flow you want to reject. The overall FLDI-flow-field sensitivity function is now:

$$H_{flow} = H_m + H_r. \quad (2.41)$$

3. Compute the ratio H_m/H_r as a function of frequency. This is a signal to noise ratio where the numerator represents the contribution to the signal from disturbances within the desired depth of focus ($-L_c < z < L_c$) and the denominator represents the contribution to the signal from disturbances outside of the depth of focus. We achieve our desired spatial resolution to disturbance frequencies when $H_m/H_r \gg 1$.

This process give us a band (or bands) of frequencies over which we are able to achieve the desired resolution, $2L_c$. In other words, it provides the spatiotemporal

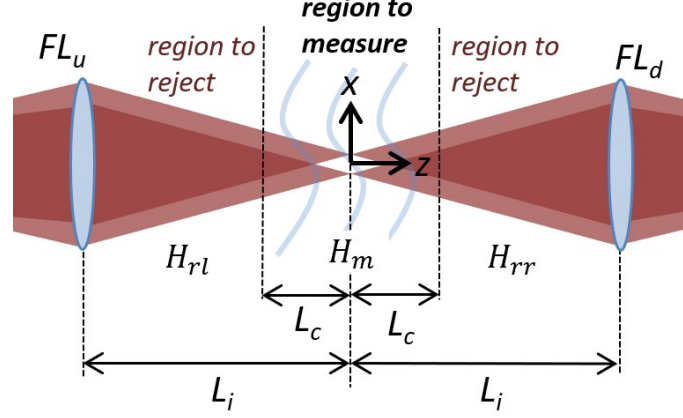


Figure 2.5: Illustration of regions and sensitivity functions used to compute spatiotemporal resolution

resolution of the instrument. In Chapter 4 we will demonstrate how this method is applied in real flows. Finally, because FLDI rejects high wavenumbers (and thus high frequencies) more rapidly than low wavenumbers, the band of frequencies resolved at a desired depth of focus is likely to be centered at higher frequencies than the bands which are not resolved. Put another way, low frequency signals are more likely than high frequency signals to be corrupted by disturbances away from the focus.

2.3.2 Wavenumber spectra

A consequence of allowing for velocity gradients in our model (i.e. $u_c(z)$) is that there is not a one-to-one relationship between wavenumber and frequency. This is because two different disturbance wavenumbers can convect at different speeds thereby mapping to the same frequency and vice versa. In this case, the expression for $H_{\Delta x}$ and H_w in terms of frequency must be used and the sensitivity function,

H_{flow} , and amplitude spectra, A_{flow} , cannot be expressed in terms of wavenumber.

Wavenumber spectra may be more desirable because they are easier to compare to turbulence models and spatial features in the flow. These can only be obtained from FLDI measurements by neglecting velocity gradients (assuming $\partial u_c / \partial z = 0$). In this case, the convection velocity can be constant, or dispersive, i.e. $u_c(f)$ and the frequencies in the signal can be converted directly to wavenumbers per Eq. 2.12.

The governing equation can then be expressed as

$$A_{flow}(k_x) = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi](k_x)}{H_{flow}(k_x)}, \quad (2.42)$$

where $A_{flow}(k_x)$ is the wavenumber spectra.

2.3.3 A general sensitivity function

Up to this point, our analysis has assumed that the flow field can be modeled by Eq. 2.11. As mentioned in section 2.2.3, an alternative model of acoustic waves convecting at an arbitrary angle (Eq. 2.14) is explored by Lawson [49]. In this case the governing FLDI equation (Eq. 2.8) can be reduced to the same form as Eq. 2.35 such that we can write

$$A_{aco} = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi]}{H_{aco}} \quad (2.43)$$

where A_{aco} is the amplitude spectra of acoustic waves spanning a length, L , and H_{aco} is the FLDI sensitivity function to those acoustic waves, given by

$$H_{aco} = \frac{2\sqrt{2}\pi^{3/2}w_0}{\sqrt{k_x^2 + k_y^2}\lambda_0} H_{\Delta x} \exp \left\{ -\frac{w_0^2}{8} \left[k_x^2 + k_y^2 + \frac{16\pi^2 k_z^2}{(k_x^2 + k_y^2)\lambda_0} \right] \right\} \times \mathcal{R} \left\{ \left[\frac{(k_x^2 + k_y^2)L\lambda_0^2 + i \cdot 4\pi^2 k_z w_0^2}{2\sqrt{2}\pi \sqrt{k_x^2 + k_y^2}\lambda_0 w_0} \right] \right\}. \quad (2.44)$$

Eq. 2.44 is adapted from Eq. 5.21 in Lawson’s dissertation and readers are directed there for more details on the derivation [49]. The bigger takeaway is that a general form exists. The FLDI signal can be modeled as

$$\mathcal{F}[\Delta\phi] = A * H. \quad (2.45)$$

where H represents a general sensitivity function that is not necessarily described by H_{flow} (Eq. 2.34) or H_{aco} (Eq. 2.44). Alternative sensitivity functions are found by altering the flow field model (see section 2.2.3) and then analytically reducing the governing equation (Eq. 2.8) in a process similar to the one described in this chapter. We find the flow-parallel disturbance model (Eq. 2.11) and resulting sensitivity function, H_{flow} , strikes an appropriate balance between simplicity and accuracy (accuracy is evaluated in Chapter 3). While further investigation of a more general form is intriguing, it is considered outside of the scope of this work.

2.4 Two-Point FLDI (2pFLDI)

Fig. 2.6 is a schematic illustration of a typical 2pFLDI used in this work. Many components are shared with, and in the same location as in the single point FLDI, shown in Fig. 2.1. Down-beam of the objective lens (OL), an additional birefringent prism ($P2$) splits the beam to create a second FLDI ‘channel’. A polarizer (Pol_2) (or a quarter-wave-plate rotated at 45° or a half-wave plate rotated at 22.5°) is needed to reset the polarization before the up-beam prism ($P1_u$). Many variants of 2pFLDI set-ups exist: $P2$ can be placed up-beam of the objective lens (OL) [55], or replaced with a Koester prism [37], Nomarski prism [38], or diffracting optic [43]. For the

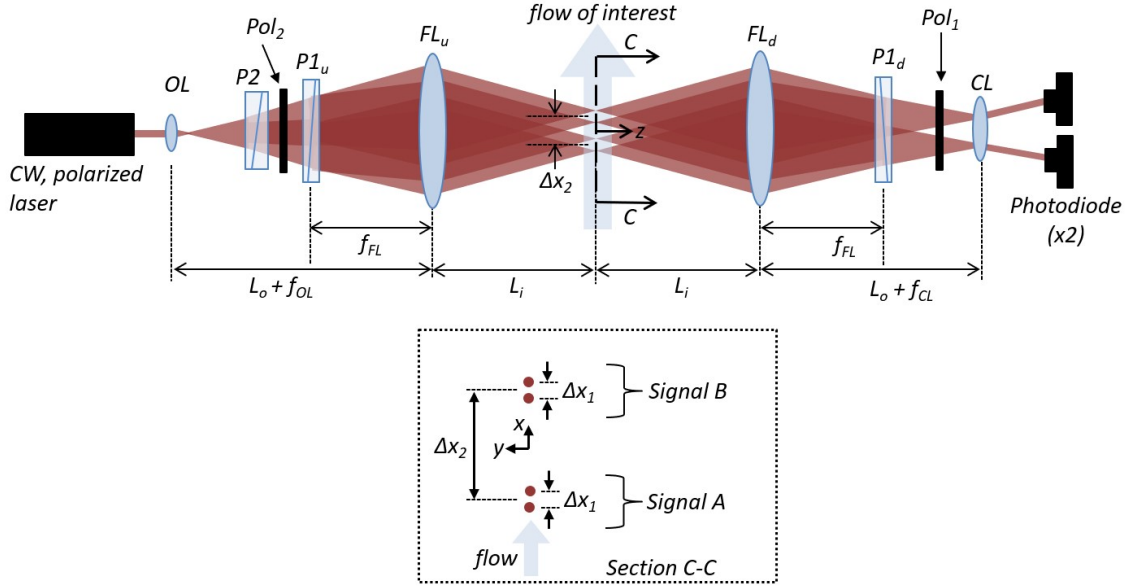


Figure 2.6: Schematic illustration of a Two-Point Focused Laser Differential Interferometer. Additional components and parameters beyond single-point FLDI are: ($P2$) birefringent prism with splitting angle (ϵ_2), (Pol_2) polarizer, (CL) collimating lens with focal length f_{CL} , (Δx_2) separation between beam pairs

set-up diagrammed here, P2 should be as close to $P1_u$ as possible (with room for the polarizer/wave plate in between).

The desired result is a set of beam pairs that come to focus along one line on the x-axis as shown in Section C-C of Fig.2.6. Down-beam of the focal volumes, another optical element like a collimating lens (CL) is required to divert the two FLDI beams to separate photodetectors. We now have two independent FLDI signals A & B with phase differences $\Delta\phi_A(t)$ and $\Delta\phi_B(t)$ respectively. The beam pair foci are separated by a distance (Δx_2).

The normalized cross-correlation between signals A & B is given by:

$$R_{AB}(\Delta t) = \int_0^T \frac{\Delta\phi_A(t)\Delta\phi_B(t + \Delta t)}{\sqrt{|\Delta\phi_A|^2}\sqrt{|\Delta\phi_B|^2}} dt, \quad (2.46)$$

where Δt is the time shift between signals. In general, convection velocities are found from cross (also called space-time) correlations between signals in turbulent flow [56]. The convective velocity (u_c , in the direction of the x-axis) is found by dividing the beam separation by the time shift which maximizes the cross-correlation:

$$\langle u_c \rangle = \frac{\Delta x_2}{\Delta t[\max(R_{AB})]}. \quad (2.47)$$

The use of angled brackets, $\langle \rangle$, emphasizes that this measurement is a time-averaged velocity over the sample period T . Taylor famously hypothesized and later showed [57] that turbulent structures in flows with low turbulence intensity ($u'/U \ll 1$) and negligible mean shear ($k_x U \gg \frac{dU}{dy}$) [58] such as the those produced behind a grid in a wind tunnel convect with the mean flow velocity such that $u_c = U$ regardless of probe separation, Δx_2 .

In shear-generated turbulent flows like those found in boundary layers and jets, the convection velocity is more complex and varies with probe separation as well as turbulent length and time scales [59, 60]. Wills [61] discusses the ambiguity of defining a convection velocity in shear flows and suggests that either a wavenumber-dependent velocity, $u_c(k_x)$, or a frequency-dependent velocity, $u_c(f)$, is more physically meaningful. The 2pFLDI can measure frequency-dependent convection velocity either by bandpass filtering signals A and B [40] or by computing the cross-spectrum of A and B [62]. The cross-spectrum is given by the Fourier transform of the cross-correlation:

$$\Gamma_{AB}(f) = \int_{-\infty}^{\infty} R_{AB}(\Delta t) e^{-2\pi i f \Delta t} d\Delta t. \quad (2.48)$$

The phase of the the cross-spectrum is given by

$$\phi_{\Gamma}(f) = \arctan \left[\frac{\mathcal{I}(\Gamma_{AB})}{\mathcal{R}(\Gamma_{AB})} \right], \quad (2.49)$$

where $\mathcal{I}$ and $\mathcal{R}$ denote the imaginary and real parts, respectively. The frequency-dependent convection velocity is given by:

$$u_c(f) = \frac{2\pi \Delta x_2 f}{\phi_{\Gamma}}. \quad (2.50)$$

Both methods (ie. $\langle u_c \rangle$ and $u_c(f)$) are used to compute the convective velocity from the 2pFLDI signals.

Chapter 3: FLDI model validation experiments

3.1 Overview

In this chapter, we seek to quantify the accuracy of our governing equation (Eq. 2.33 which is derived from our FLDI model, Eq. 2.8) and our flow field model, Eq. 2.11. The FLDI model has been validated by others [33,49,63] but our flow field model has not. Thus we expect the results obtained in this chapter to be a reflection of the accuracy of the latter. To accomplish this, we apply FLDI in two flow fields: (1) a round transonic turbulent jet and (2) the wake behind an 24 degree ramp inside a small Mach 3 calibration wind tunnel (M3CT). In both the jet and the tunnel, we vary the location of the flow along the instrument's optical axis (in our coordinate system this is the z-axis) from the focus ($z = 0$) to some maximum value, z_{max} . In the jet, we also vary the single-point FLDI beam separation, Δx_1 . The right-hand side of Eq. 2.33 models how instrument parameters affect the measurement while the left-hand side, A_{flow} , should be independent of instrument parameters like Δx_1 and w (the local beam radius) which varies along the z-axis according to Eq. 2.31. In the following experiments, we will check to see if this is the case. The instrument parameter we do not vary in this work is the laser beam wavelength, λ_0 . All lasers in this work are Helium Neon (HeNe) with $\lambda_0 = 632.8nm$. Checking to see that the

Parameter	Jet	M3CT	Units
max velocity	300	600	m/s
flow width	3.2 - 25	64	mm
laser power	0.8	21	mW
f_{OL}	15	9	mm
f_{FL}	260	400	mm
Δx_1	35, 100, 290	470	μm
L_i	51	140	cm
z_{max}	22	48	cm
w_0	7.3	14	μm
$w(z_{max})$	6	7	mm

Table 3.1: Selected parameters of FLDI experiments in this chapter. See Fig. 2.3 for reference

model can also account for variations in λ_0 is a good subject for future work.

Table 3.1 gives an overview of selected flow and FLDI parameters for the experiments in this chapter. For the max velocity of the jet and tunnel we use the velocity at the nozzle exit and the freestream velocity, respectively. For the jet, we take measurements at several locations downstream of the nozzle, which has a diameter $d = 3.2\text{mm}$. Since the jet broadens as one moves downstream, the width of the flow increases from 3.2mm to approximately $2\sigma_{jet,\rho} = 25\text{mm}$ at the furthest downstream position measured (50 jet diameters downstream).

As discussed in the previous chapter, the convection velocity, u_c , is a critical

Parameter	Jet	M3CT	Units
f_{OL}	15	9	mm
f_{FL}	125	400	mm
Δx_1	145	470	μm
L_i	17	140	cm
w_0	3	14	μm
Δx_2	1.2	1.3	mm

Table 3.2: Selected parameters of 2pFLDI experiments in this chapter. See Fig. 2.6 for reference

parameter in our governing equation. For the jet, we investigate two approaches for determining u_c . The first is to model a velocity distribution, $u_c(z)$, based on historical measurements by others [64–67]. The second is to measure the dispersion relation, $u_c(f)$, using 2pFLDI. For the M3CT, only the dispersion relation is used. Table 3.2 gives an overview of the 2pFLDI parameters used in both experiments. Note that the 2pFLDI in the jet is different than the single point FLDI while the M3CT uses the exact same set-up (detailed in table 3.1).

3.2 Turbulent Jet

The flow discussed in this section is produced by the round jet illustrated in Fig. 3.1. Compressed air enters a 48mm (1.9”) ID plenum and accelerates through a smoothly converging nozzle with an exit diameter of 3.175mm (0.125”). A diaphragm-type pressure regulator equipped with a strainer to trap debris and a

Bourdon tube gauge (accuracy $\pm 2\%$) maintains a constant pressure of 12 ± 1 psig (26.7 psia, 1.8 atm, 184 kPa) in the plenum. This pressure is chosen to maximize Reynolds number while avoiding supersonic flow at the jet exit. The resulting nozzle exit velocity, Mach number, and Reynolds number based on jet diameter are 306 ± 7 m/s, 0.96 ± 0.01 , and $8.55 \pm 0.34 \times 10^4$ respectively assuming isentropic flow through the nozzle.

The air jet is oriented vertically and mounted on a motorized 3-axis Velmex Bi-Slide stage. This enables the jet to be moved in three dimensions through the FLDI measurement volume which remains fixed. The x-axis is the centerline of the jet and parallel to the line connecting the centers of the four focal volumes. Positive x points up with respect to the laboratory and downstream with respect to the jet flow. The y and z axes are perpendicular to the x-axis with z parallel to the FLDI optical axis and y perpendicular.

3.2.1 FLDI & 2pFLDI equipment and data reduction

The FLDI discussed in this section uses a $0.8mW$ HeNe laser, N-BK7 lenses (OL , FL) from ThorLabs, Polarizers from Thorlabs (LPVISE100-A), and a $13mm^2$, $14ns$ rise-time photodetector from ThorLabs (DET36A). For beam splitting/recombining ($P1_u$ & $P1_d$) we alternate between Wollaston prisms from United Crystals with a splitting angle of 4arcmin and custom made stress-birefringent ‘Sanderson’ prisms [31, 32, 68] which allow splitting angles below 4arcmins.

The 2pFLDI uses much of the same equipment as the single point FLDI. All

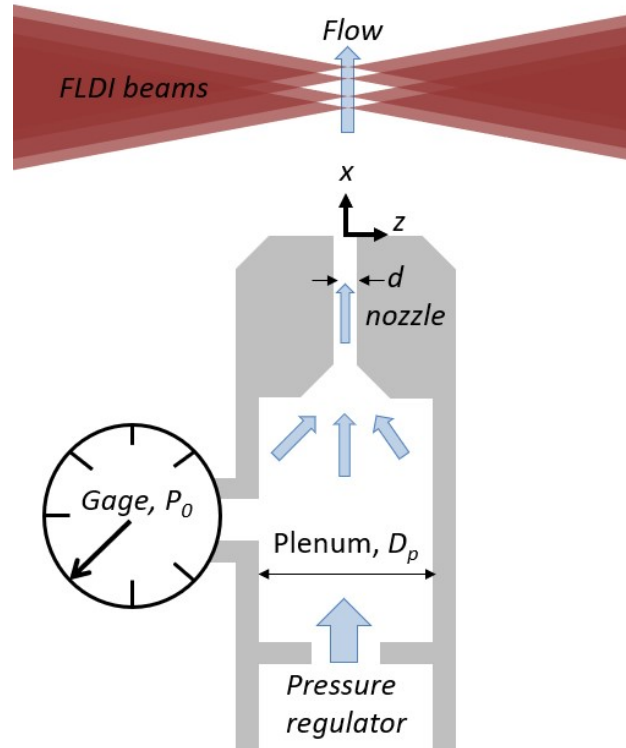


Figure 3.1: Diagram of air nozzle. $d = 0.125''$ (3.2mm), $L_n = 2.5''$, $D_p = 1.9''$, P_0 is controlled with the regulator upstream of the plenum

beam splitting is accomplished with custom made quartz Wollaston prisms ($P1_u$, $P1_d$ with 4arcmin divergence angle & $P2$ with 10° divergence angle) from United Crystals and the optical axis length of the set-up is shortened to increase the convergence angle of the beam thereby improving rejection of disturbances away from the focus. $P2$ is located 15mm up-beam of OL . Tables 3.1 & 3.2 give all relevant parameters for both set-ups.

The photodetector signals are terminated at 1 k Ω and digitized by a 16-bit Picoscope 5444A. The sampling frequency varies but is always between 1 and 4 MHz. Raw voltages are downloaded from the scope and processed using MATLAB. Voltage is converted to phase difference using Eq. 2.7.

Fourier transforming the phase difference fluctuations is required for the signal analysis described in chapter 2. This is accomplished by taking the square-root of the power spectral density, which we estimate using Welch’s method [69] via the MATLAB function `pwelch` with 3.3ms Hamming windows and 50% overlap. This window length is chosen in order to have a sufficiently large number windows to average spectra over, while still maintaining resolution to $\sim 1kHz$ time scales.

For 2pFLDI, the MATLAB function `xcorr` computes the normalized cross-correlation (Eq. 2.46) between two signals. If the correlation peak is less than 0.1, the signals are considered noise-dominated and velocity is not computed. This process reduces the appearance of non-physical velocities (say, much larger than the local mean velocity or less than zero) which could arise from weak correlation between unknown noise sources. Otherwise, the index of maximum correlation divided by the sampling frequency gives the time lag (Δt) between the two signals enabling

the disturbance velocity, $\langle u_c \rangle$, to be calculated using Eq. 2.47. For the dispersion relation (Eq. 2.50), the MATLAB function `cpsd` computes the cross-spectrum (Eq. 2.48). Like the power spectrum, this method also uses Welch's method with $3.3ms$ windows and 50% overlap. The cross spectra phase (Eq. 2.49) can only take values from $-\pi$ to π . This will give accurate velocity measurements for $k_x < \pi/\Delta x_2$. Phase ambiguity will occur for wavenumbers above this range. This is corrected by assuming that the velocity variation with wavenumber is smooth and continuous. In practice, as frequency increases (hence wavenumber increases), 2π is added to the phase every time there is a discontinuous jump from π to $-\pi$. This process is described in more detail by Buxton et al. [70].

3.2.2 Adjustment and measurement of beam separation Δx_1

Beam separation is varied by replacing the Wollaston prisms (whose splitting angles are fixed) with variable-angle stress-birefringent Sanderson Prisms [68]. FLDI measurements are made using three values of Δx_1 . Beam separation is measured by imaging the beams at the focus using a beam profiler camera (Newport LBP2) with a $3.69\mu m \times 3.69\mu m$ pixel size. Sample images are presented in Fig. 3.2. Two Gaussian distributions (one for each beam) are fit to the intensity distributions in the x-y plane. The difference between the centers of each Gaussian distribution is the beam separation. The uncertainty is approximately equal to the pixel size ($3.7\mu m$) resulting in an uncertainty of $\sim 10\%$ for the smallest beam separation. Note that the Wollaston prism ($\Delta x_1 = 290\mu m$) produces beams that are much more

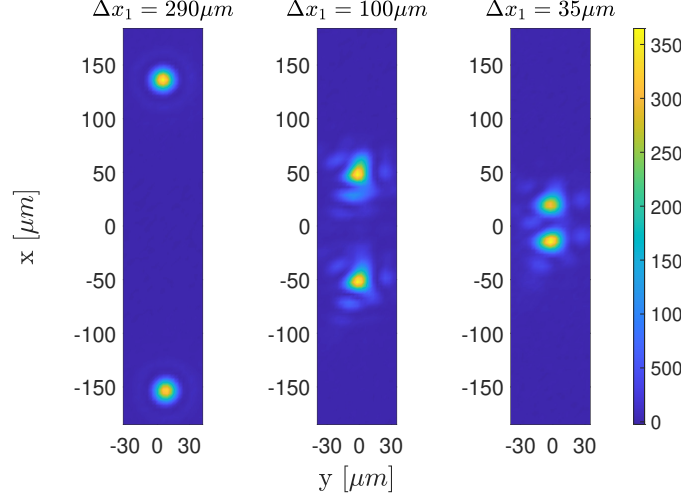


Figure 3.2: Comparison of beam profiles at the focus for three setups. $\Delta x_1 = 290\mu m$ is produced using the Wollaston prism. The other two beam spacings are produced using the Sanderson prism.

uniform/Gaussian-like than those produced by the Sanderson prism ($\Delta x_1 = 100\mu m$ and $\Delta x_1 = 35\mu m$).

3.2.3 Measurement of focal spot radius, w_0

An accurate measure of the beam spot size at the focus is needed to compute the FLDI sensitivity function as well as the spot size at any z -location via. Eq. 2.31. Unfortunately, the pixel size of the beam profiler camera is too large to directly measure the spot size at the focus. However, since the beam diameter increases linearly with z far from $z = 0$, we can use the beam profiler camera to measure $w(z)$ at seven non-uniformly spaced distances ranging from $z = 3.175mm$ to $z = 25.4mm$

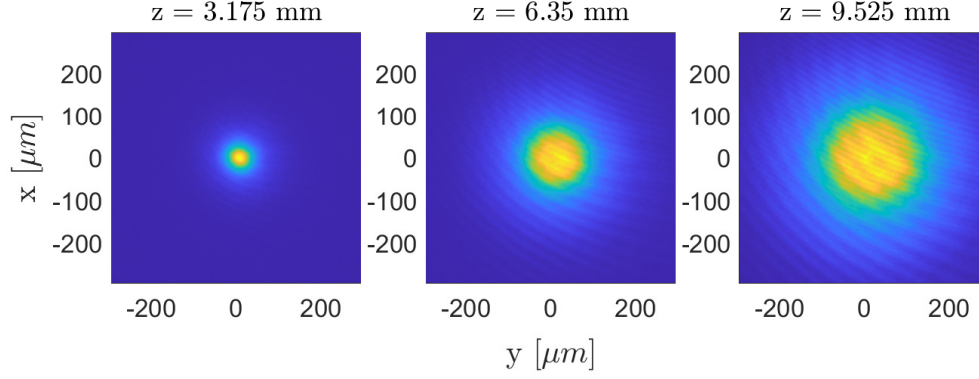


Figure 3.3: Beam intensity profiles along the z axis

and infer the beam radius at the focus (w_0) using Eq. 3.1 with $\lambda_0 = 633nm$.

$$w(z) \approx \frac{\lambda_0}{\pi w_0} z, \quad (3.1)$$

Examples of beam intensity profiles measured at three locations are presented in Fig. 3.3. Gaussian intensity distributions fit to each image are used to find the Gaussian beam radius parameter w where $w = 2\sigma$ and σ is the standard deviation of the Gaussian distribution [54]. Note that the up beam prism is temporarily removed so a single beam is being measured, not two beams split by $290\mu m$ as in the actual setup. This makes no difference since the divergence angles of the two beams are the same as that of the original.

The results of the beam radius parameter measurements are plotted in Fig. 3.4. A linear fit correlates the measurements extremely well ($R^2 = 0.999$) and enables us to determine that the beam radius at the focus is $7.3 \pm 0.3\mu m$ ($\sim 5\%$).

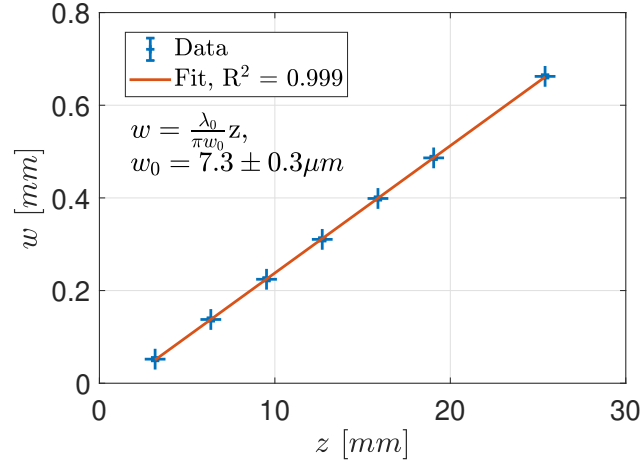
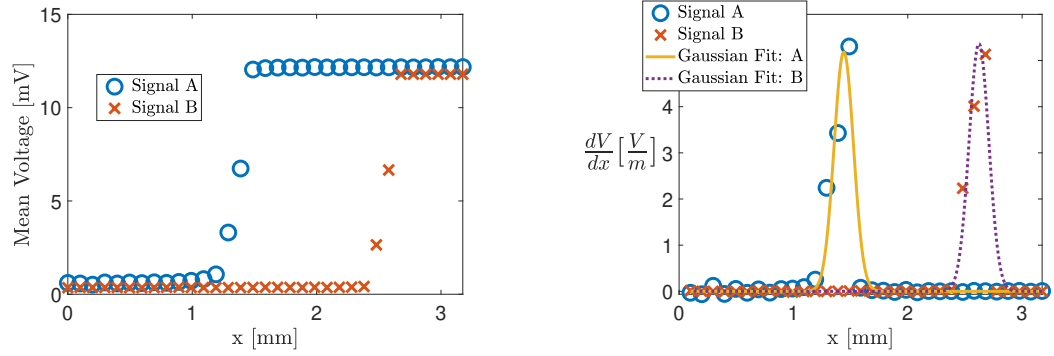


Figure 3.4: Measurements of the beam radius parameter at various points along the z axis with the fit used to determine w_0 (the beam radius at the focus).



(a) Mean voltage vs. knife edge position. (b) Voltage difference and Gaussian fits.

Figure 3.5: Results from knife-edge test used to determine beam pair separation (Δx_2)

3.2.4 Measuring beam-pair separation, Δx_2

The 2pFLDI beam pair separation (Δx_2) can be estimated using paraxial ray-tracing [71] or ray-tracing software such as Code-V or ZEMAX but it is more accurate to measure it. This is best accomplished using a beam profiler camera as done in section 3.2.2. Unfortunately, this camera was not available when the 2pFLDI experiments were performed so the beam separation was determined by translating a knife-edge in the x-direction through the focal region while measuring both detectors' output voltage at 33 equally spaced points over 3.175mm. Fig. 3.5.a shows the results. The knife edge initially blocks both signals so the output voltages are both zero. As the knife edge is moved away, first detector A and then detector B are uncovered. Fig. 3.5.b, shows the derivative of the signals dV/dx with respect to x. Gaussian functions are fit to the derivatives of each signal in order to reduce uncertainty in locating the peak. The distance between the Gaussian peaks is taken to be the beam spacing. The results show that $\Delta x_2 = 1.18 \pm 0.1mm$.

3.2.5 Experiment and Modeling

The density fluctuation in the turbulent jet is given (via Eq. 2.35) by:

$$A_{jet} = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi]}{H_{jet}}, \quad (3.2)$$

where

$$H_{jet} = \int_{-L_i}^{L_i} |g_{jet} H_{\Delta x} H_w| dz. \quad (3.3)$$

density distribution in the jet is given by:

$$g_{jet}(x, z, z_0) = \exp \left(-\frac{(z - z_0)^2}{2\sigma_{jet,\rho}^2(x)} \right). \quad (3.4)$$

where $\sigma_{jet,\rho}$ is the jet spreading angle based on density. Similarly, measurements by Hinze, Wygnanski & Fiedler and others [64, 65, 67] show that axial velocities also follow Gaussian distributions about the jet axis. Therefore, the velocity distribution in the jet is given by:

$$u_c(x, z, z_0) = U_{CL} \exp \left(-\frac{(z - z_0)^2}{2\sigma_{jet,u}^2(x)} \right), \quad (3.5)$$

where $\sigma_{jet,u}(x)$ is the jet spreading angle based on velocity and $U_{CL}(x)$ is the velocity distribution along the jet center line. Hinze's measurements showed that $\sigma_{jet,\rho}(x) = 0.080x$ and $\sigma_{jet,u}(x) = 0.075x$ in the $Re_d = 6.7 \times 10^4$ turbulent jet. The jet centerline velocity distribution is found using Witze's velocity decay model [66], which is based on historical pitot and hot-wire measurements. This model is expressed as

$$U_{CL}(x) = 1 - \exp \left(-\frac{1}{\alpha(x/d) - X_c} \right), \quad (3.6)$$

where X_c , a universal parameter describing core length, takes a value of 0.7 and α , a parameter describing decay rate based on jet exit Mach number and ratio of jet exit density to ambient density, takes a value of 0.13 for a Mach number of 0.96 and a density ratio of 0.84 [66].

H_{jet} is evaluated by substituting Eq. 3.5 into Eq. 2.25 and Eq. 2.32, and then plugging those and Eq. 3.4 into Eq. 3.3 and evaluating the integral numerically using trapezoidal integration (MATLAB function `trapz`) with 1001 linear-spaced z values between $z_0 - 4\sigma_{jet,\rho}$ and $z_0 + 4\sigma_{jet,\rho}$.

Parameter	Index, i	Typical Value, P_i	Uncertainty, b_{P_i}	Units
Δx_1	1	290	3.7	μm
w_0	2	7.3	0.3	μm
$\sigma_{jet,\rho}$	3	6.4	0.1	mm
$\sigma_{jet,u}$	4	6	0.1	mm
U_{CL}	5	100	2	m/s

Table 3.3: Parameter typical value and uncertainty

3.2.6 Uncertainty and sensitivity analysis

The uncertainty analysis uses standard methods outlined by the American Society of Mechanical Engineers [72]. The measurable parameters, P_i , of H_{jet} are the beam separation Δx_1 and the beam radius at the focus w_0 . The free parameters are the jet standard deviation based on density, $\sigma_{jet,\rho}$, the jet standard deviation based on velocity $\sigma_{jet,u}$, and the jet centerline velocity U_{CL} (which is based on two free parameters per Eq. 3.6). Table 3.3 gives indices, typical values (taken from $x/d=25$), and the systematic (or bias) uncertainty b_{P_i} in these parameters. The sensitivity of H_{jet} to parameter P_i is estimated by computing a first order finite difference as follows:

$$\frac{\partial H_{jet}}{\partial P_i} \approx \frac{H_{jet}(P_i + \Delta P_i) - H_{jet}(P_i)}{\Delta P_i}, \quad (3.7)$$

where $\Delta P_i/P_i = 0.03$, or 3% is chosen for the approximation. The total systematic uncertainty $b_{H_{jet}}$ to N parameters is found by summing the uncertainties associated

with each parameter in a root mean square manner:

$$b_{H_{jet}} = \sqrt{\sum_1^N \left(\frac{\partial H_{jet}}{\partial P_i} b_{P_i} \right)^2}. \quad (3.8)$$

3.2.7 Results using modeled velocity distribution, $u_c(z)$

3.2.7.1 Sensitivity to beam separation (Δx_1)

As its name suggests, the differential interferometer performs a finite difference on the index of refraction field. Dividing the signal by the beam separation (Δx_1) yields a signal proportional to the first order approximation of the density gradient. However, the instrument is also sensitive to wavenumber as explained in the previous section and by Parziale in much of his FLDI work (see for example Fig. 3 in [23]). This is accounted for here using a transfer function developed for sinusoidal variations (Eq. 2.25). Lawson et al. [45] uses FLDI simulations to illustrate the effect of varying the beam separation when the disturbance field is static. They note that the signal magnitude increases and x-axis spatial resolution decreases as Δx_1 increases. These effects are qualitatively consistent with those predicted by $H_{\Delta x}$ (Eq. 2.25). The question we seek to answer in this section is to what extent Eq. 2.25 models the FLDI's response to the dynamic, turbulent flow field produced by the jet.

In each experiment, the turbulent jet is aligned with the beam's focus ($z_0 = 0$) and is translated in the streamwise direction along the x-axis. Measurements are made at five uniformly spaced locations (ranging from $x/d=5$ to $x/d=25$) where the FLDI signal is sampled for $20ms$ at $2MHz$. Measurements of signal strength (which

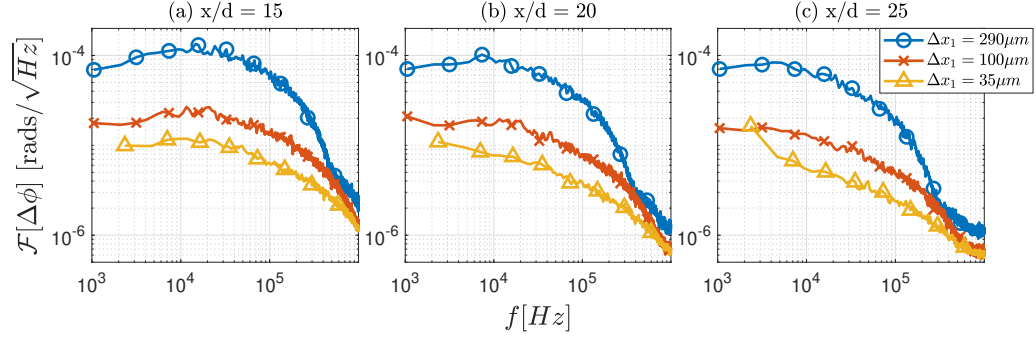


Figure 3.7: Raw FLDI phase difference spectra for three beam separations acquired with the instrument focus located at the jet centerline ($y/d = 0$ and $z_0/d = 0$)

is proportional to phase difference) at $x/d=15, 20$ and 25 are presented in Fig. 3.7 for three different values of Δx_1 . The results show that the phase difference increases with beam separation at low frequencies. This can be explained by thinking of the signal magnitude divided by the beam separation as proportional to the density gradient, i.e. $\Delta\phi/\Delta x_1 \propto \partial\rho/\partial x$. Thus for a fixed density gradient magnitude, phase difference increases linearly with beam separation. We observe this trend for frequencies below $\sim 10^5 Hz$. Above $\sim 10^4 Hz$ phase difference decreases with frequency. Above $\sim 10^5 Hz$, This decline becomes more rapid for the largest beam separation, $\Delta x_1 = 290\mu m$. This behavior represents spatial filtering of wavenumbers which exceed $\pi/\Delta x_1$. This disturbance scale corresponds to a range of frequencies because the convection velocity varies through the jet. Note that spectral content below $\Delta\phi \sim 2 \times 10^{-6}$ is assumed to be noise and is removed prior to subsequent data processing. This “noise floor” is evident at the highest frequencies in Fig. 3.7 where the spectra flatten and coalesce with spectra measured when the flow is off (not shown here).

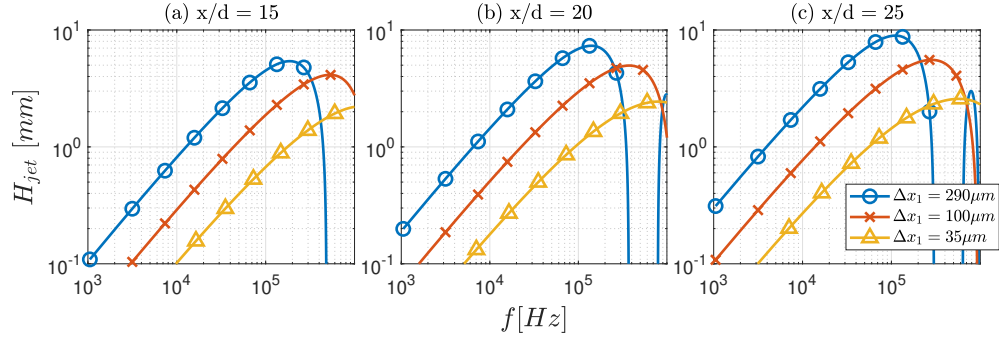


Figure 3.8: FLDI sensitivity function to the jet (Eq. 3.3) for three beam separations at jet centerline ($y/d = 0$) and instrument focus ($z_0/d = 0$). $w_0 = 7.3\mu m$, and $\lambda_o = 632.8nm$.

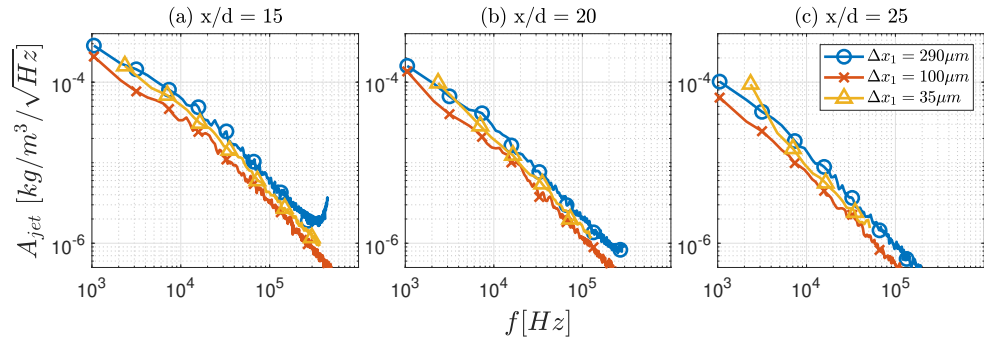


Figure 3.9: Average amplitude spectra interrogated by FLDI, found from Eq. 3.2. Measurements are acquired at the jet centerline ($y/d = 0$) and the instrument focus ($z_0/d = 0$)

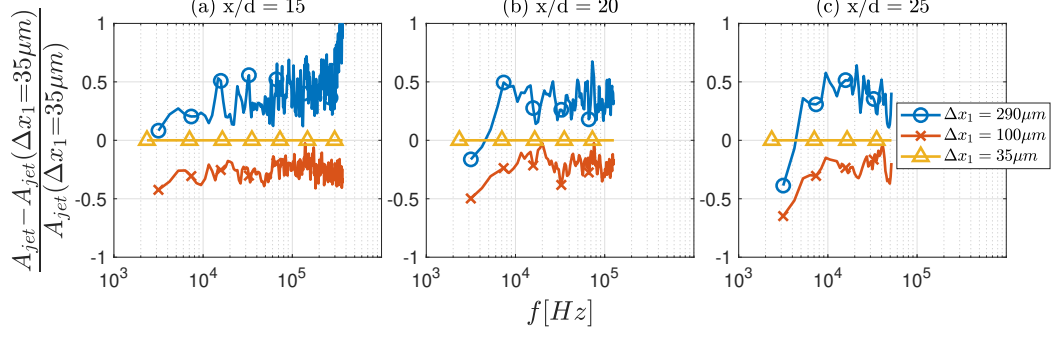


Figure 3.10: Difference between amplitude spectra for three beam separations, normalized by amplitude from $\Delta x_1 = 35\mu m$. Measurements are acquired at the jet centerline ($y/d = 0$) and the instrument focus ($z_0/d = 0$)

Fig. 3.8 shows the overall transfer function H_{jet} as a function of frequency and beam spacing for the measurements presented in Fig. 3.7. The filtering behavior is apparent as H_{jet} increases linearly with frequency until the frequency of the disturbance approaches the point of maximum sensitivity. The true average density fluctuation spectrum is recovered by dividing the signal by H_{jet} as per Eq. 3.2. The results are presented in Fig. 3.9. The collapse of all spectra indicates H_{jet} (Eq. 2.34) and thus $H_{\Delta x}$ (Eq. 2.25) is a good model for the effects of beam separation on this flow field. While results from measurements at only three locations are presented here, similar collapses are observed at all jet locations.

Fig. 3.10 shows the difference between amplitude spectra (A_{jet}) measured at the three beam separations normalized by the value from $\Delta x_1 = 35\mu m$ as a function of frequency. The amplitudes recovered from $\Delta x_1 = 100\mu m$ & $35\mu m$ differ by $\sim 25\%$ over the band $5kHz < f < 200kHz$ while the amplitudes recovered from $\Delta x_1 = 290\mu m$ & $35\mu m$ differ by $\sim 50\%$ over the same band. The effect of beam

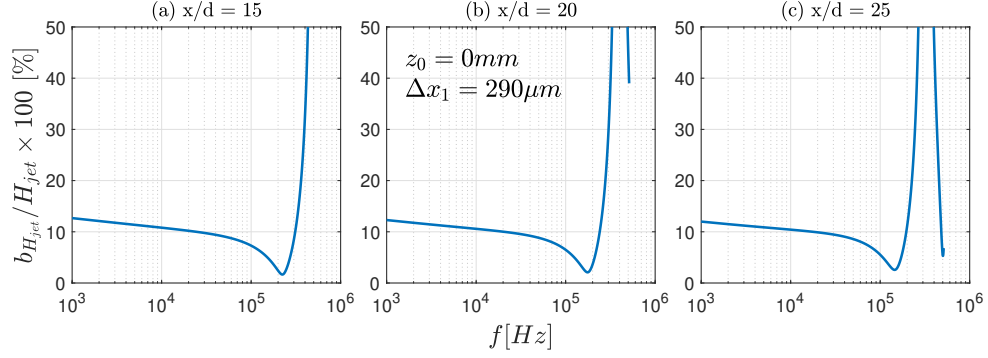


Figure 3.11: Percent uncertainty in H_{jet}

separation appears to be independent of x/d . Notice also that there appears to be no consistent trend in the difference between amplitude spectra as Δx_1 is increased.

The increase in error at the highest frequencies can be explained by the uncertainty and the relatively small values of H_{jet} at these frequencies. Percent uncertainty in H_{jet} is plotted in Fig. 3.11, found from Eq. 3.8. For example, at $x/d = 15$ and $\sim 400kHz$, H_{jet} drops rapidly to small values. To find A_{jet} , we divide by these small values, thus any uncertainty will be magnified. This is illustrated by the rapid increase of uncertainty in Fig. 3.11.a at $400kHz$.

In the next section we show that errors associated with varying the local beam diameter are significantly smaller ($\sim 10-20\%$). This, as well as the fact that there is no consistent trend in the difference between amplitude spectra as Δx_1 is increased, suggests that the differences observed here are mainly due to changes in the optical setup required to change Δx_1 and not Δx_1 itself or discrepancies between our model (Eq. 2.11) and the true density gradient fluctuations in the flow.

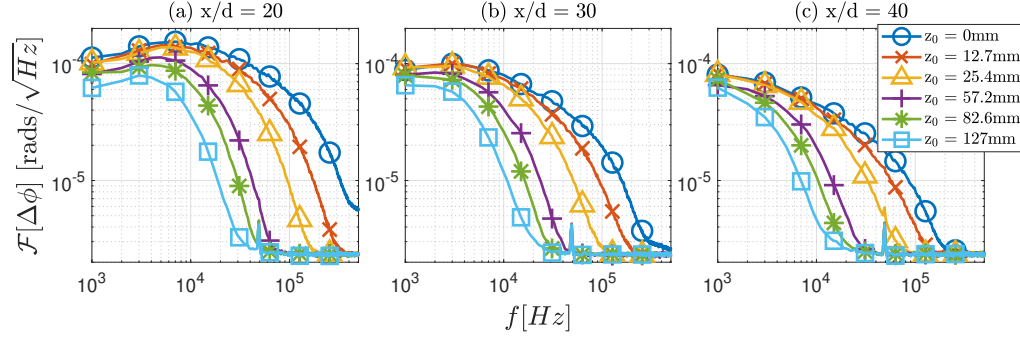


Figure 3.12: Raw FLDI phase difference spectra acquired at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)

3.2.7.2 Sensitivity to position along the optical axis (z)

In this section, we investigate how FLDI sensitivity changes along the optical axis (z -direction) which is important for evaluating the improvement in spatial resolution brought about by focusing. A single beam separation of $\Delta x_1 = 290\mu m$ (which provides the largest overall signal strength) is considered in order to isolate the effect of z . The turbulent jet is translated in the streamwise (x -axis) and spanwise (z -axis) directions. Data are collected at ten uniformly spaced x - locations ranging from $x/d=5$ to $x/d=50$ and 21 non-uniformly spaced locations in the z -direction ranging from $z_0/d = -5$ ($z_0 = -15.9mm$) to $z_0/d = 70$ ($z_0 = 222.2mm$). At each location, the FLDI signal is sampled for $400ms$ at $1MHz$. A sketch of the experiment is shown in Fig. 3.6.

Phase difference spectra are presented in Fig. 3.12 for eighteen measurement locations ($x/d=20, 30, 40$ and six z_0 locations). The results show that signal strength decreases as the jet moves away from the instrument's focus (increasing z_0). Higher

frequencies are more strongly affected by the movement of the jet away from the focus because these frequencies correspond to small wavelength disturbances which are spatially averaged to a greater extent than relatively larger wavelengths (lower frequencies). Fig. 3.13 shows the values of H_{jet} corresponding to the eighteen measurement locations plotted in Fig. 3.12. Prior to calculating H_{jet} , frequencies which contain spectral content below $\Delta\phi \sim 2 \times 10^{-6}$ are assumed to be noise and removed. H_{jet} is not evaluated or plotted at these frequencies in order to de-clutter the plots. These curves predict the instrument's sensitivity to the jet at different values of z_0 . As in the previous section, we use Eq. 3.2 to recover the true average density fluctuation spectra. The results, presented in Fig. 3.14, show that measurements at all z-locations collapse indicating that H_{jet} has correctly captured the effect of position along the optical axis. While data for only three streamwise and six z-axis positions are presented here - for clarity - collapse is observed for all frequencies measured ($f > 500Hz$) and for all streamwise positions above $x/d=15$. Below $x/d=15$, the assumption that disturbances are normally distributed breaks down and the model is not expected to work.

Once again, we assess the model's effectiveness - i.e. its ability to predict and compensate for the effects of disturbances away from the focal volume - by computing the difference between amplitudes recovered for the various z-axis positions and the amplitude at the instrument's focus ($z_0 = 0$) as illustrated in Fig. 3.15. The results show that the error is less than $\pm 20\%$ for almost all frequencies and cases presented here. The error only exceeds 20% for the higher frequencies at $x/d = 20$. In fact, for $x/d=30$ & 40 and $f < 10kHz$, the error is below $\pm 10\%$. This is a significant

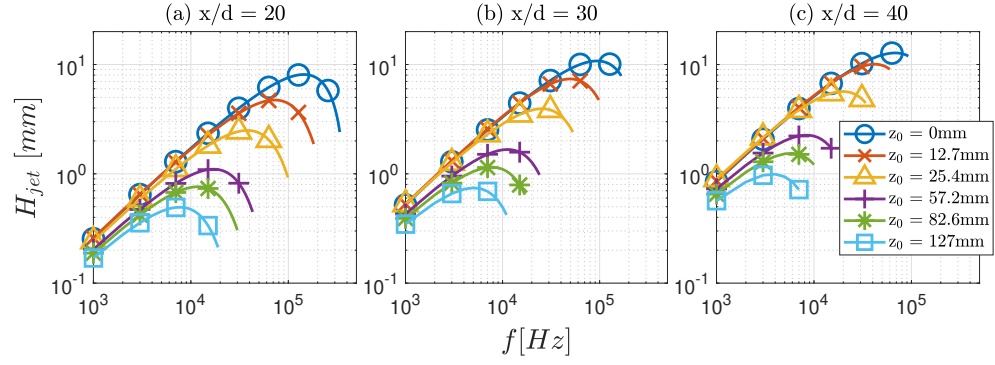


Figure 3.13: FLDI sensitivity function at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)

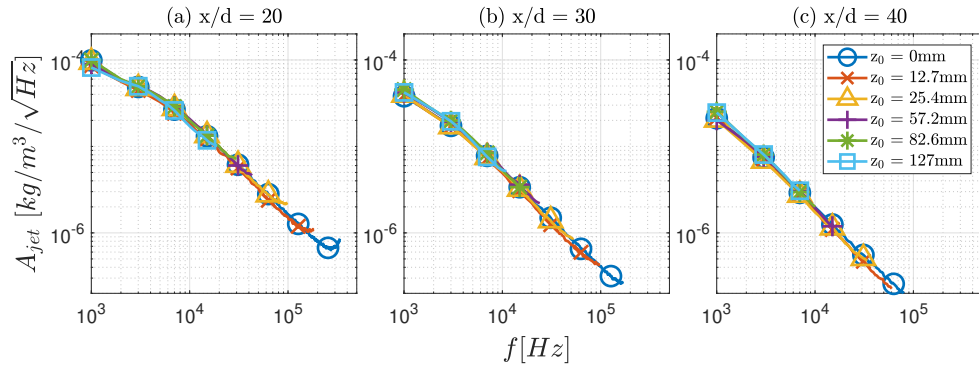


Figure 3.14: Average amplitude spectra interrogated by FLDI for jet centerline ($y/d = 0$) and for six jet positions along the optical axis (z_0)

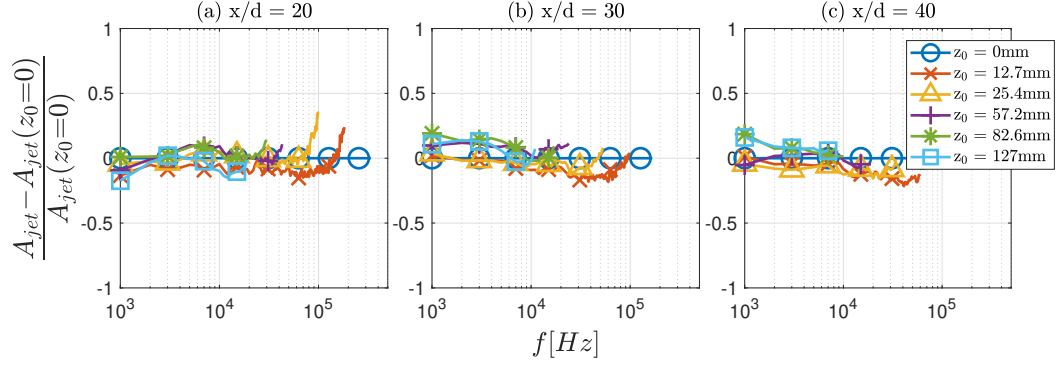


Figure 3.15: Difference between amplitude spectra recovered for six jet positions along the optical axis (z_0), normalized by amplitude at $z_0 = 0$

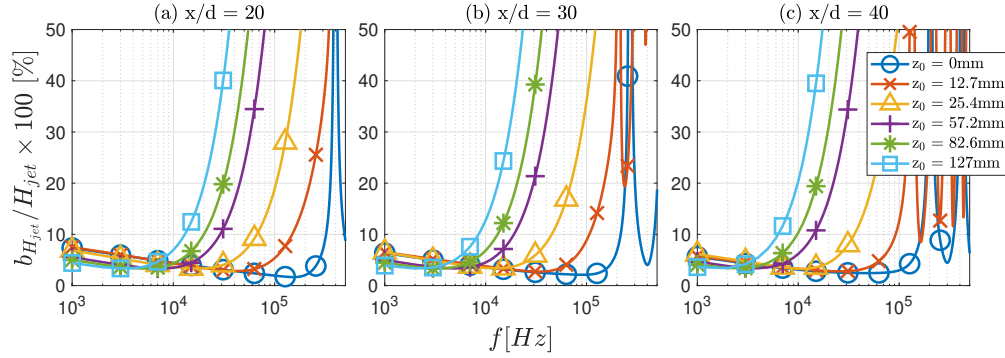


Figure 3.16: Percent uncertainty in H_{jet}

improvement over the error associated with varying the beam separation (25-50%).

The larger errors for higher frequencies at $x/d=20$ can be explained by the relatively small values of H_{jet} at these high frequencies, which we divide by to find A_{jet} , causing large increases in uncertainty. This behavior is illustrated in Fig. 3.16

3.2.7.3 Comparison to uniform velocity assumption

A significant difference between this work and previous is the inclusion of a modeled convection velocity distribution (given by Eq. 3.5). To illustrate how

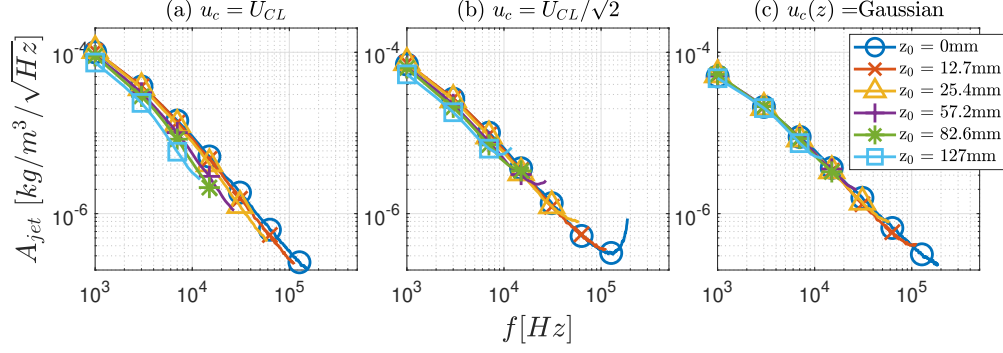


Figure 3.17: Comparison of amplitude spectra computed using uniform velocity profile (a & b) and Gaussian velocity profile (c) for $x/d = 30$, $y/d = 0$, and six z_0 positions.

this approach differs from one where velocity is assumed constant Fig. 3.17 & 3.18 compare the amplitude spectra and amplitude spectra difference, respectively, between a uniform velocity assumption and the Gaussian distribution model at $x/d=30$. For each figure, (a) shows the results when convection velocity is equal to the local jet centerline velocity everywhere, (b) shows the results when convection velocity is equal to the local jet centerline velocity divided by the square root of two, and (c) shows the results using the Gaussian distribution, which are the same as those presented in the previous section, repeated here to aid comparison.

The difference between collapsed amplitude spectra in Fig. 3.17.a, Fig. 3.17.b, & Fig. 3.17.c is hard to discern. Amplitude appears to decrease slightly with z_0 in Fig. 3.17.a where the convection velocity is equal to the centerline mean velocity, indicating the result is not independent of z_0 . Better agreement is observed in Fig. 3.17.b where the convection velocity is equal to the centerline mean velocity divided by $\sqrt{2}$. This collapse looks comparable to Fig. 3.17.c. Difference between

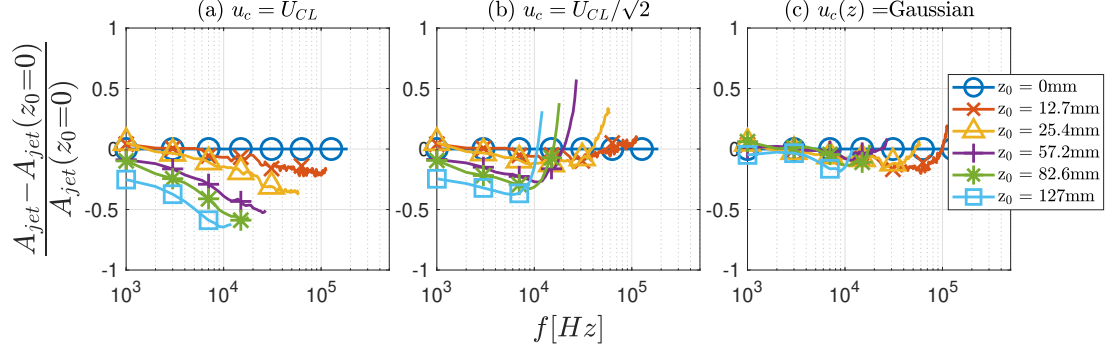


Figure 3.18: Comparison of amplitude spectra difference for uniform velocity profile (a & b) and Gaussian velocity profile (c) for $x/d = 30$, $y/d = 0$, and six z_0 positions.

amplitudes are easier to see when not plotted on a log scale as in Fig. 3.18. Fig. 3.18.a shows a clear trend of decreasing amplitude with z_0 where differences exceed 50% near 10kHz for the larger z_0 plotted. Less variation in amplitude with z_0 is observed in Fig. 3.18.b, compared to Fig. 3.18.a, indicating $u_c = U_{CL}/\sqrt{2}$ is a better choice than $u_c = U_{CL}$ for a uniform convection velocity in the flow. The best collapse is illustrated in Fig. 3.18.c where the convection velocity is modeled as Gaussian with z . Differences between amplitudes are near 10%, compared with 20-40% in Fig. 3.18.b. Amplitudes are clearly independent of z_0 in Fig. 3.18.c. This affirms the importance of accurately modeling convection velocities when interpreting the FLDI signal.

3.2.8 Measuring the dispersion relation

Fig. 3.19 shows frequency-dependent convection velocities computed from our 2pFLDI measurements using Eq. 2.48-2.50. Measurements are made at the jet

centerline using $13ms$ of data (collected at $3MHz$) at each position downstream (x/d). The figure shows that velocity increases with frequency, decreases with x for $f > 5kHz$, and becomes independent of x as frequency goes to zero. This behavior is the result of two phenomena:

1. The scale dependence of FLDI's sensitivity discussed in the previous section causes disturbances with low wavenumbers (corresponding to low frequency signals) to be more averaged across the span of the jet than disturbances with high wavenumber (high frequencies) causing the latter to be closer to the local centerline value.
2. low wavenumber disturbances convect slower than high wavenumbers along the jet centerline [56, 61, 65] because the larger scales carry more momentum from the outer regions of the jet than the smaller scales.

The purpose of these measurements is to evaluate the accuracy of our model for FLDI's response when the dispersion relation measured by 2pFLDI is used in place of the modeled velocity distribution, i.e when substituting these measurements in place of Eq. 3.5. Further discussion of these results is considered outside the scope of this work.

3.2.9 Results using measured dispersion relation, $u_c(f)$

Taken from Fig. 3.19, convection velocities vs. frequency for $x/d=20, 30$ and 40 , corresponding to the locations discussed previously in section 3.2.7.2, are re-plotted in Fig. 3.20. These velocities are used to convert frequency to wavenumber,

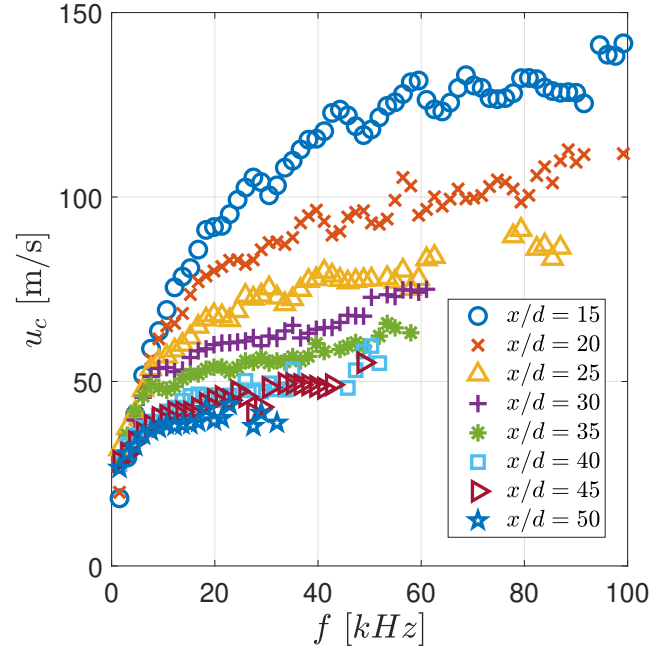


Figure 3.19: Frequency-dependent convection velocities along the centerline ($y = 0$, $z_0 = 0$) of a round jet. Computed from the cross-spectra of 2pFLDI measurements (Eq. 2.48-2.50).

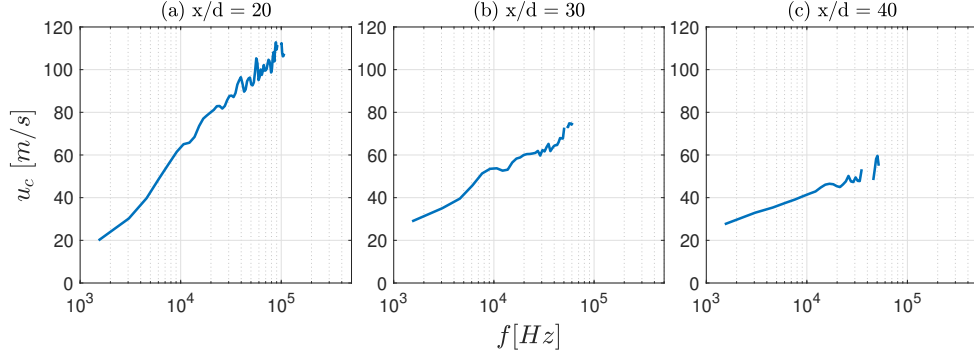


Figure 3.20: 2pFLDI measured velocity vs. frequency acquired at jet centerline ($y/d = 0$) and instrument focus ($z_0 = 0$)

k_x , (Eq. 2.12) for the data presented previously in Fig. 3.12. This is presented in Fig. 3.21. This conversion allows us to use the versions of the transfer functions, $H_{\Delta x}$ and H_w which use wavenumber rather than frequency (Eq. 2.25 and 2.32). Thus, H_{jet} is computed using Eq. 3.3 in terms of wavenumber and plotted in Fig. 3.22 for $x/d=20, 30$ and 40 . As discussed in section 2.3.2, we can now find the amplitude of disturbances vs. wavenumber which is more useful than frequency for comparing to turbulence models and spatial features in the flow. We are also investigating to what extent wavenumber spectra, $A_{jet}(k_x)$, are independent of jet position along the optical axis, z_0 . A_{jet} is plotted in Fig. 3.23, which shows the recovered amplitudes are independent of z_0 over the wavenumber band $[0.5 < k_x < 5]mm^{-1}$ for $x/d=20, 30$, and 40 . At $x/d=40$ and $z_0 = 0$, a linear fit to the log of A_{jet} vs the log of k_x yields a slope of -1.45 , which is plotted as a black line in Fig. 3.23.c.

To quantify the degree to which recovered amplitude wavenumber-spectra are independent of z_0 , we compute the percent difference between $A_{jet}(z_0)$ and its value at $z_0 = 0$, shown in Fig. 3.24. This is a measure of the accuracy of our model

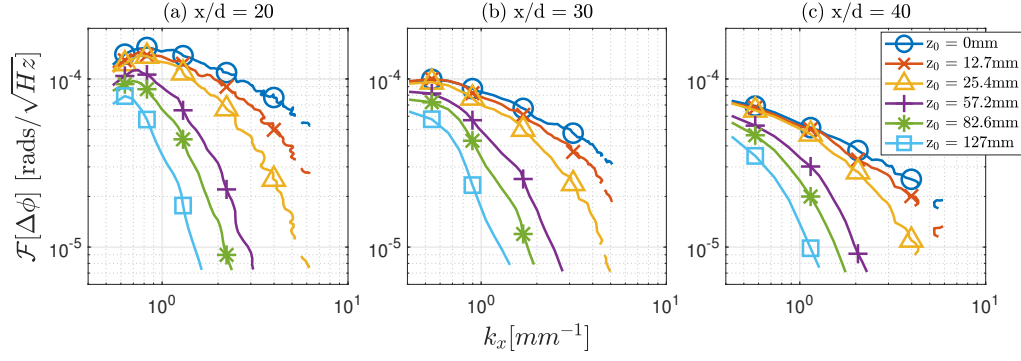


Figure 3.21: Phase difference spectra vs. wavenumber acquired at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)

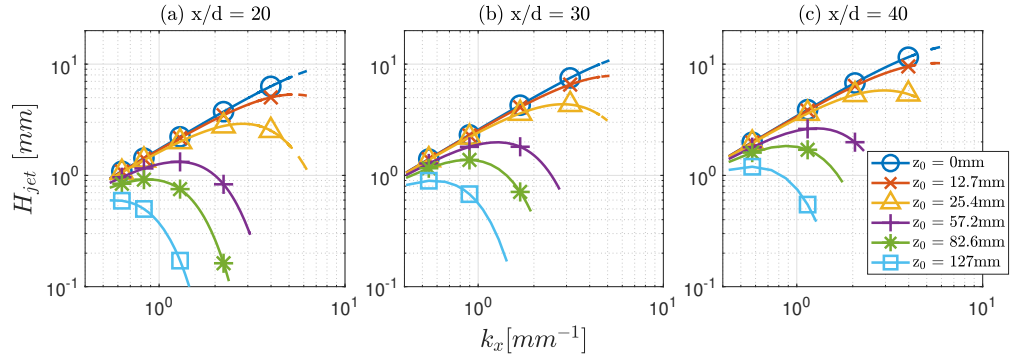


Figure 3.22: Sensitivity function vs. wavenumber at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)

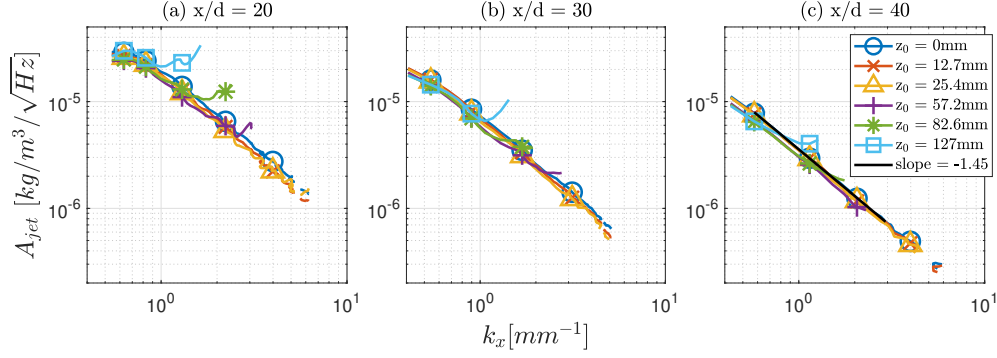


Figure 3.23: Amplitude spectra vs. wavenumber at jet centerline ($y/d = 0$) for six jet positions along the optical axis (z_0)

for FLDI's response to the jet when the 2pFLDI-measured dispersion relation is used to convert frequencies in the signal to wavenumbers. Less than 20% error is observed over the wavenumber band $[0.5 < k_x < 5]mm^{-1}$ when the jet is not too far from the focus $z_0 = 12.7mm$ and $z_0 = 25.4mm$ for all x/d . At $x/d=40$, error is below 20% for $z_0 = 57mm$ and $z_0 = 82mm$ also. For smaller x/d , larger z_0 , and larger wavenumbers, error increases rapidly, as observed in Fig. 3.24.a for $z_0 \geq 57mm$. This behavior might be explained by the uncertainty in H_{jet} when the magnitude of H_{jet} becomes small. Percent uncertainty in H_{jet} for the same 18 jet positions is plotted in Fig. 3.25. It shows that model uncertainty increases correspond to the same conditions in which the largest errors are observed: smaller x/d , larger z_0 , and larger k_x . However, the fact that larger errors are consistently positive (over-predicting A_{jet}) and consistently occur for larger distances away from the focus suggest our model is under-predicting the value of H_{jet} for large z . This could indicate a breakdown of our flow-parallel disturbance model (Eq. 2.11).

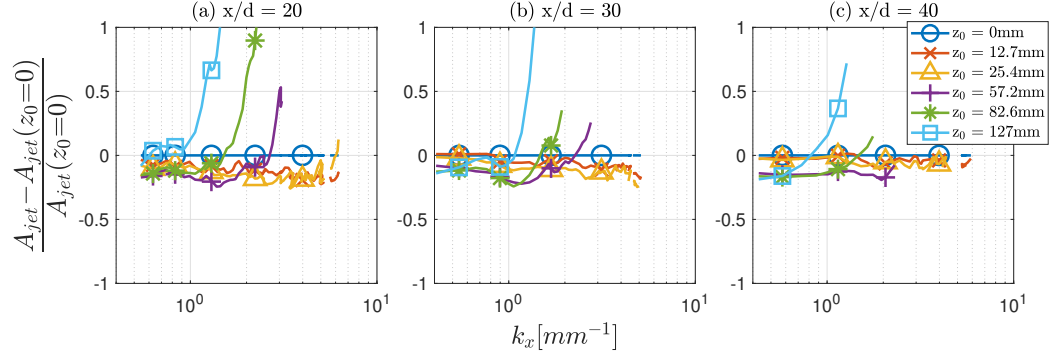


Figure 3.24: Difference between amplitude spectra for six jet positions along the optical axis (z_0) normalized by amplitude at $z_0 = 0$

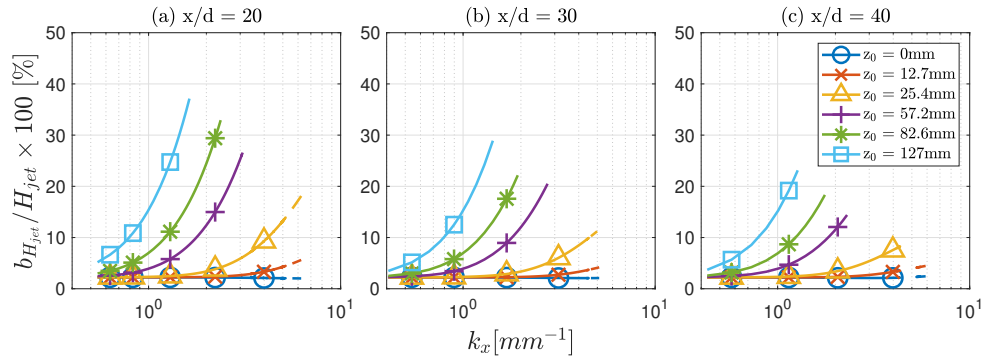


Figure 3.25: Percent uncertainty in H_{jet} vs. wavenumber

3.2.10 Conclusions

The response of a single-point FLDI to a round turbulent jet is modeled by ray tracing (Eq. 2.8) through flow parallel disturbances whose magnitude and velocity vary with z and f (Eq. 2.11). These equations, when combined, reduce analytically to Eq. 2.35 (re-written as Eq. 3.2 for the jet) which allows us to solve for the disturbance amplitude spectra in the jet. The fluctuation magnitude spatial variation is modeled as a Gaussian based off of historical jet studies (Eq. 3.4). Two approaches for extracting the disturbance velocity are explored: (1) modeling velocity variation in space using historical studies (Eq. 3.5), and (2) measuring velocity variation with frequency using 2pFLDI (section 3.2.8). The advantage of the second method is that it eliminates several free parameters in the governing equation and enables FLDI signal interpretation in flows where velocity data are not readily available.

Both approaches illustrate how the signal attenuates as the jet is translated away from the focus. Errors are on the order of 20% when the sensitivity function, H_{jet} , is sufficiently large (roughly $> 1mm$ in this example). Next, we investigate to what extent Eq. 2.35 can describe the FLDI signal in a complex, ‘wind-tunnel-like’ disturbance environment produced in a Mach 3 facility.

3.3 Mach 3 Tunnel

AEDC’s Mach 3 atmospheric temperature/pressure in-draft calibration tunnel (M3CT) is used evaluate the performance of the FLDI instrument in a ‘wind tunnel-like’ disturbance environment. Some of this work has been presented at con-

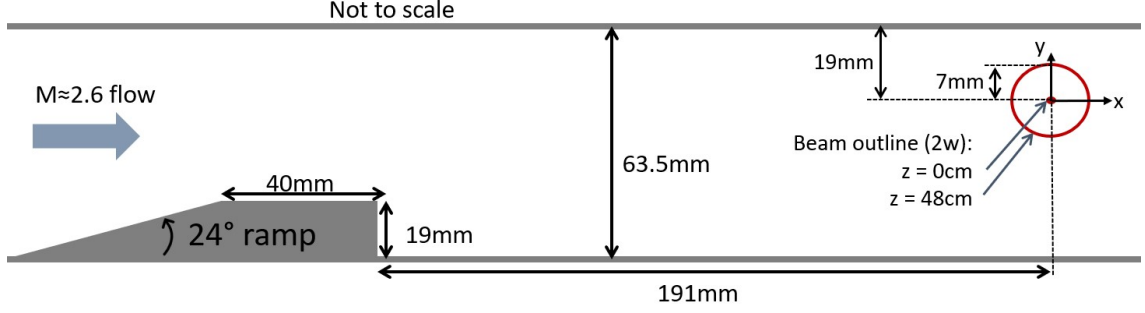


Figure 3.26: Diagram of the M3CT flow path showing the location of the FLDI measurement volume, ramp, and the coordinate directions.

M_∞ (-)	p_∞ (Pa)	T_∞ (K)	ρ_∞ (kg/m ³)	Re_∞^{unit} (1/m)	U_∞ (m/s)
2.62	4165	123	0.118	8×10^6	580

Table 3.4: Free stream conditions at measurement location in wind tunnel.

ferences [41]. The tunnel produces approximately 10 seconds of steady flow. More information about the tunnel itself is available elsewhere [73]. Conditions for all tunnel runs are provided in table 3.4. The probe volume location is approximately 500mm downstream of the nozzle and 19mm below the tunnel ceiling. Runs are conducted with the 24°, 19mm tall ramp ‘disturbance generator’ illustrated in Fig. 3.26 installed 191mm upstream of the probe location. The ramp is chosen simply as a mechanism for generating a disturbance environment with large amplitudes. The details of the flow field downstream of the ramp are not considered in this work.

3.3.1 FLDI and 2pFLDI equipment and data reduction

The experimental set-up is shown in a side view in Fig. 3.26 and in a top-down view in Fig. 3.27. Parameters of the FLDI and 2pFLDI are given in tables 3.1

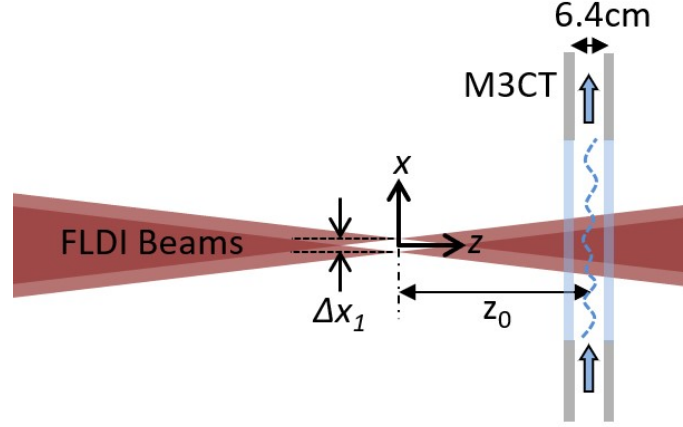


Figure 3.27: Illustration of the location of M3CT (disturbance generator) within the FLDI optical path.

& 3.2 respectively. The beams are focused over 140cm from a maximum diameter of $2w = 40\text{mm}$ to a minimum diameter of $2w_0 = 28\mu\text{m}$ at the focus [54]. 48cm from the focus, the beam diameter ($2w$) is approximately 14mm . The FLDI uses a 21mW HeNe laser (THORLABS HNL210LB), a 20X microscope objective lens, (OL), large plano-convex condenser lenses (FL) with 100mm diameter and a 400mm focal length (Edmund Optics 27-503), Polarizers from Thorlabs (LPVISE100-A), and a 13mm^2 , 14ns rise-time photodetector from ThorLabs (DET36A). Wollaston prisms from United Crystals with splitting angle of 4 arcmin are used for beam splitting/recombining ($P1_u$ & $P1_d$). The photodetector signal is terminated at $1\text{k}\Omega$ and digitized by a 16-bit Picoscope 5442D. The sampling frequency is 4MHz .

Raw voltages are downloaded from the scope and processed using MATLAB. Voltage is converted to phase difference using Eq. 2.7. All analyses are performed on 4×10^6 data points acquired starting at $t = 4\text{s}$ where $t = 0$ is the time the

value opens to start the run. The flow in the tunnel is steady during this interval. Once again, Fourier transforms are computed by taking the square-root of the power spectral density which we estimate using Welch's method [69] and the MATLAB function `pwelch` with $125\mu s$ Hamming windows and 50% overlap.

Two trials are conducted. In each trial, 17 runs are conducted at 17 equally spaced z-axis positions ranging from $z_0 = 0$ (tunnel centerline at focus) to $z_0 = 48cm$. The tunnel is fixed and the FLDI equipment is translated to vary z_0 . Both trials yielded similar results so only one trial is discussed here.

3.3.2 Modeling

Re-expressing Eq. 2.35 for the M3CT gives

$$A_{M3CT} = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi]}{H_{M3CT}}, \quad (3.9)$$

where

$$H_{M3CT} = \int_{-L_i}^{L_i} |g_{M3CT} H_{\Delta x} H_w| dz. \quad (3.10)$$

The schematic illustration of the FLDI probe beam relative to the tunnel presented in Fig. 3.27 is useful for understanding the relationship between the FLDI and the width of the tunnel, $L = 6.4cm$. We assume disturbances are uniform across the tunnel width such that

$$g_{M3CT}(z) = \begin{cases} 1 & \text{for } -L < z - z_0 < +L \\ 0 & \text{elsewhere.} \end{cases}$$

This allows us to solve the integral in Eq. 3.10 analytically giving:

$$H_{M3CT} = \frac{\sqrt{2}\pi^{3/2}w_0}{\lambda_0 k_x} H_{\Delta x} \exp\left(-\frac{w_0^2 k_x^2}{8}\right) \times \left\{ \operatorname{erf}\left[\frac{\lambda_0 k_x}{2\sqrt{2}\pi w_0}(z_0 + L)\right] - \operatorname{erf}\left[\frac{\lambda_0 k_x}{2\sqrt{2}\pi w_0}(z_0 - L)\right] \right\}. \quad (3.11)$$

Similar expressions for modeling FLDI response to uniform disturbances which are symmetrically distributed with respect to the focus were explored by Fulghum [31] and Schmidt and Shepherd [33]. Our expression allows for disturbances offset from the focus by distance z_0 . We measure $u_{c,M3CT}(f)$ using 2pFLDI to convert wavenumbers to frequencies and vice versa. This approach was motivated by its success in the turbulent jet.

3.3.3 Dispersion relation

The 2pFLI experiment used to measure $u_{c,M3CT}$ is conducted with the instrument's focus located at the tunnel centerline ($z_0 = 0$). The two measurement volumes are separated by approximately $1.3mm$ along the x-axis (streamwise) and centered approximately around the location of the single FLDI probe illustrated in Fig. 3.26. 4×10^6 points of data are acquired (with the ramp present in the tunnel) from $t = [4 - 5]s$ just like all other single point FLDI experiments. The dispersion relation (a plot of disturbance velocity vs. frequency) is computed from the cross-spectra of the FLDI signals, i.e. Eq. 2.48-2.50. Fig. 3.28 shows that the disturbance velocity is near $U_\infty = 580m/s$ over most of the frequency range ($> 100kHz$). Plotting this on a logarithmic frequency scale in Fig. 3.29 shows that velocity is near $350m/s$ at $1kHz$, decreases to $\sim 180m/s$ at $4kHz$, and then

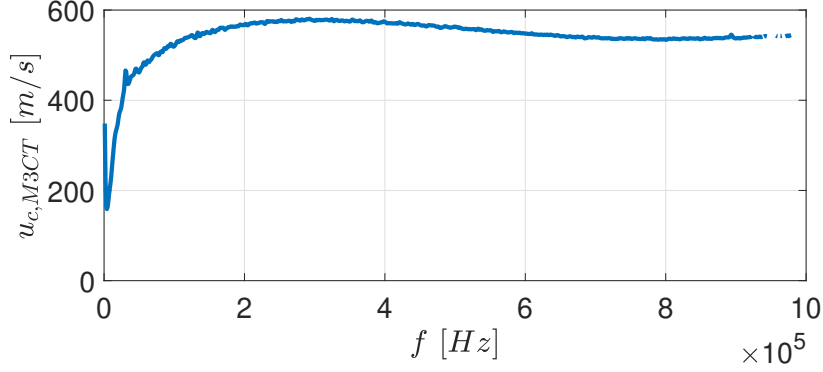


Figure 3.28: Dispersion relation in M3CT flow field downstream of the ramp ‘disturbance generator’ measured with 2pFLDI

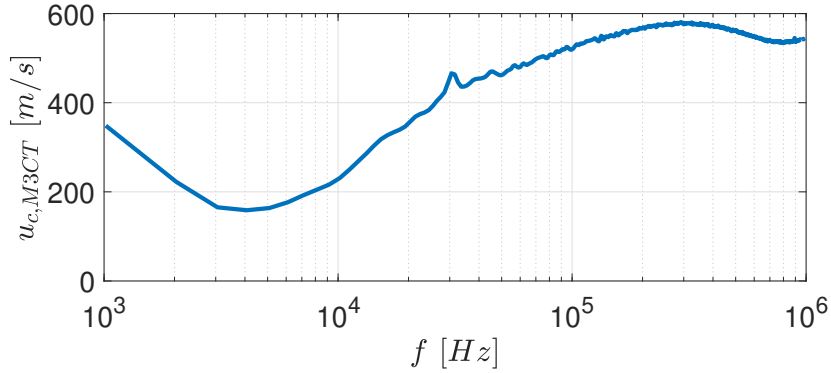


Figure 3.29: Dispersion relation in M3CT plotted against log scale

increases with frequency until $\sim 300kHz$.

3.3.4 Results

Phase difference spectra are presented in Fig. 3.30 for nine optical axis positions (z_0) of tunnel centerline with respect to FLDI focus. $z_0 = 0$ represents tunnel centerline coincident with the focus. For the entire frequency band measured here ($4 - 1000kHz$), signal strength decreases as the $6.4cm$ -wide tunnel moves away from

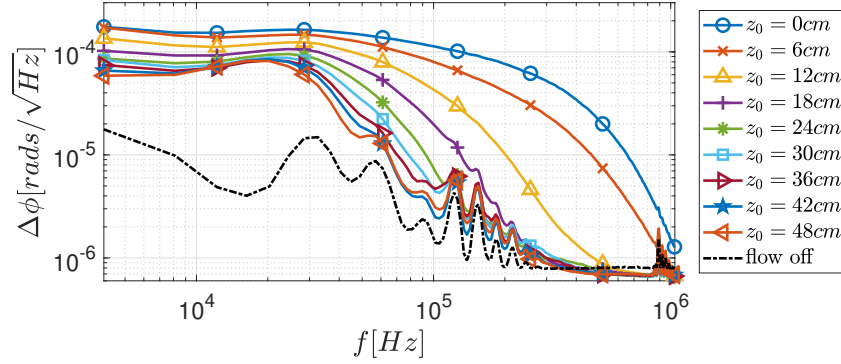


Figure 3.30: Phase difference spectra vs. frequency in M3CT for nine tunnel positions along optical axis (z_0). Flow off measurement taken prior to $z_0 = 48cm$ run the focus until $z_0 = 30cm$. This trend continues beyond $z_0 = 30cm$ for all frequencies except $\sim 10 - 20kHz$, where signal strength becomes independent of z_0 . Fluctuations observed for $z_0 \geq 18cm$ and $100kHz < f < 250kHz$ are also present in the signal taken when the tunnel is not running (labeled “flow off” and plotted with a dashed line). Thus, these fluctuations are not considered relevant to the flow features under investigation.

Phase difference spectra are plotted in Fig. 3.31. Frequencies are converted to wavenumbers using the convection velocities plotted in Fig. 3.29. Wavenumbers which contain spectral content below $5 \times 10^{-6} rads/\sqrt{Hz}$ are assumed to be noise and removed. Fig. 3.31 shows the same trends as Fig. 3.30, but with a spatial, rather than temporal, perspective. It illustrates that signal strength becomes independent of z_0 for $0.3mm^{-1} < k_x < 0.35mm^{-1}$ which corresponds to a wavelength of $\sim 19mm$. $19mm$ is the height of the ramp as well as the distance between the tunnel ceiling and FLDI probe.

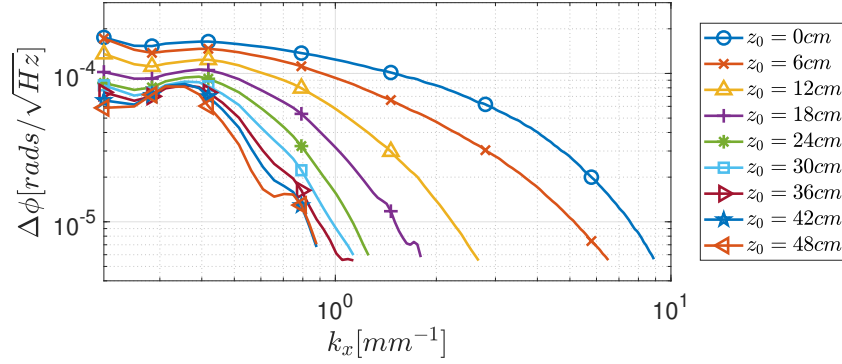


Figure 3.31: Phase difference spectra vs.wavenumber in M3CT for nine tunnel positions along optical axis (z_0)

Our model for the instrument's response (H_{M3CT} , Eq. 3.11) is plotted for nine z_0 values in Fig. 3.32. Recall that these curves enable us to deconvolve instrument effects from the true disturbance amplitudes present in the flow (A_{M3CT}) via Eq. 3.9. The measured disturbance amplitudes are plotted in Fig. 3.33. The spectra generally collapse over the $0.3mm^{-1} < k_x < 0.6mm^{-1}$. The collapse extends to $k_x \sim 3mm^{-1}$ when the tunnel is not far ($z_0 < 18cm$) from the focus. This collapse demonstrates the general efficacy of the model in compensating for the effect of z_0 although it is not as good as in the jet (see figure 3.2.1).

The degree to which A_{M3CT} is independent of z_0 is quantified by computing the difference between $A_{M3CT}(z_0)$ and its value at the focus, $A_{M3CT}(z_0 = 0)$. This is illustrated in Fig. 3.34 which shows that all amplitudes are within 50% of one-another for $0.2mm^{-1} < k_x < 0.65mm^{-1}$ and for $0.2mm^{-1} < k_x < 3mm^{-1}$ when $z_0 < 18cm$. Rapid increases in error for larger z_0 are reminiscent of trends observed in the jet (see Fig. 3.24 for example). Once again, this could be explained by

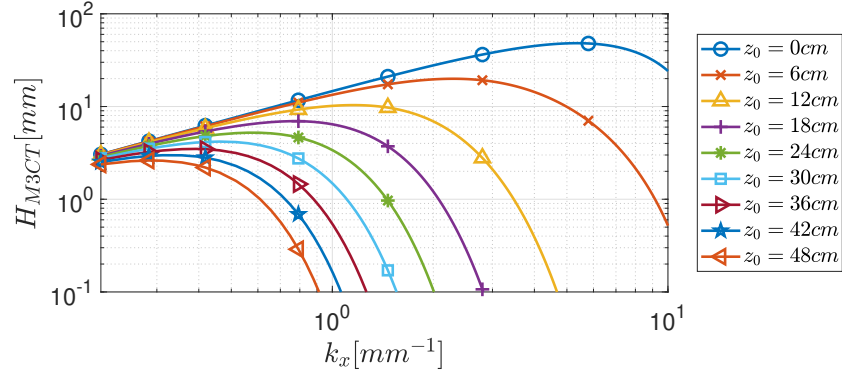


Figure 3.32: Sensitivity function vs.wavenumber in M3CT for nine tunnel positions along optical axis (z_0)

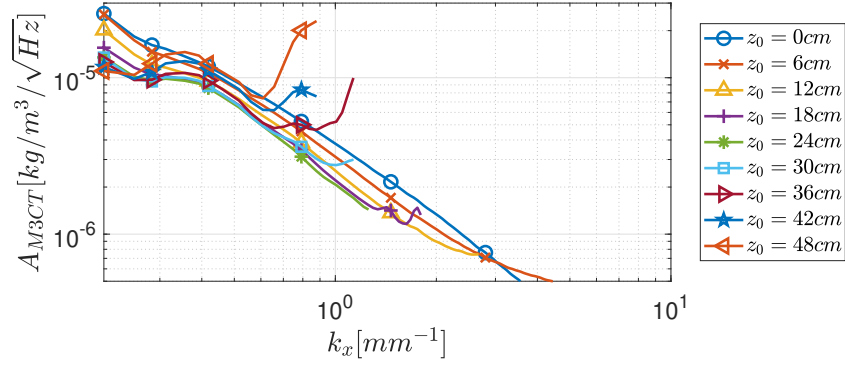


Figure 3.33: Amplitude spectra vs. wavenumber in M3CT for nine tunnel positions along optical axis (z_0)

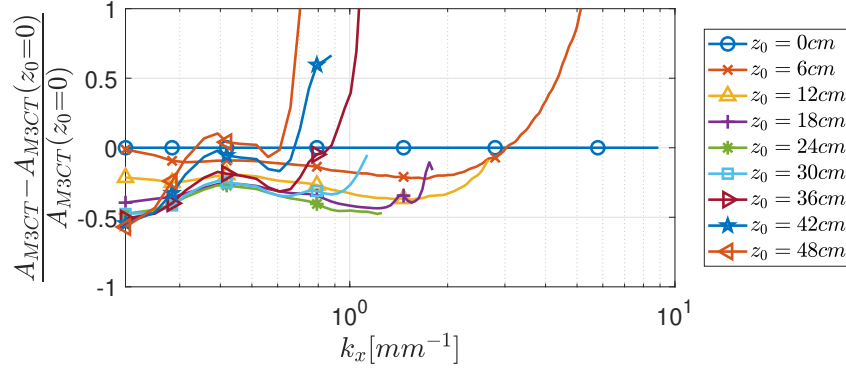


Figure 3.34: Difference between amplitude spectra for nine tunnel positions along optical axis (z_0) normalized by amplitude at $z_0 = 0$

uncertainty in the sensitivity function, H_{M3CT} , which is magnified when its value becomes small ($\sim < 1mm$ for $z_0 > 30cm$). Like the jet, our model consistently over-predicts amplitudes thereby under-predicting the sensitivity function in the M3CT when it is far from the focus and suggesting that this trend is due to bias and not random error.

3.3.5 Comparison to uniform velocity assumption

Similar to section 3.2.7.3 for the jet, in this section we compare our results to those found when assuming the convection velocity is uniform. Amplitude spectra and amplitude spectra difference are plotted in Fig. 3.35 and Fig. 3.36, respectively. For each, sub-figure (a) shows the results when convection velocity is assumed a constant value of 550m/s while (b) shows the results when convection velocity varies with frequency according to the relation measured by 2pFLDI. Thus, Fig. 3.35.b and Fig. 3.36.b are the same as Fig. 3.33 and Fig. 3.34. 550m/s is chosen because

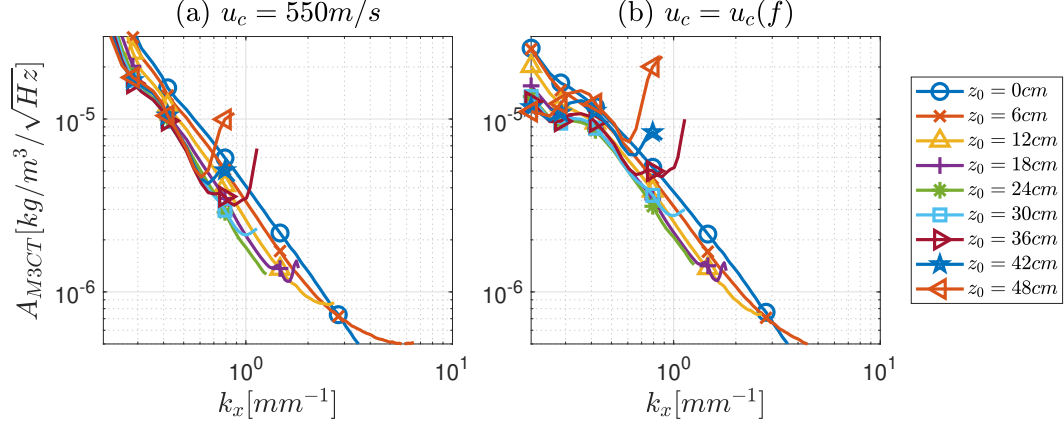


Figure 3.35: Comparison of amplitude spectra computed using uniform velocity profile (a) and using measured frequency-dependent velocity (b) for nine z_0 positions this is approximately the average velocity measured over $100 - 900kHz$, i.e. $\sim 90\%$ of the bandwidth. Note that 550m/s is $\sim 94\%$ of the freestream velocity in the tunnel.

In general, Fig. 3.35 and Fig. 3.36 show that the difference between the two approaches to modeling velocity are minimal. This is not surprising, because the velocity is measured as nearly constant over a wide range: $100 - 900kHz$, i.e. $\sim 90\%$ of the bandwidth. Thus, for frequencies above $100kHz$ corresponding to wavenumbers above $\sim 1\text{mm}^{-1}$ for $u_c = 550\text{m/s}$, the uniform velocity assumption is a good one. Below $100kHz/1\text{mm}^{-1}$, we expect to see significant differences in the plots, because the measured velocity is much lower than 550m/s here (see Fig. 3.28). Indeed, we observe better collapse and smaller amplitude difference in Fig. 3.35.b and Fig. 3.36.b vs Fig. 3.35.a and Fig. 3.36.a for $0.2\text{mm}^{-1} < k_x < 0.5\text{mm}^{-1}$.

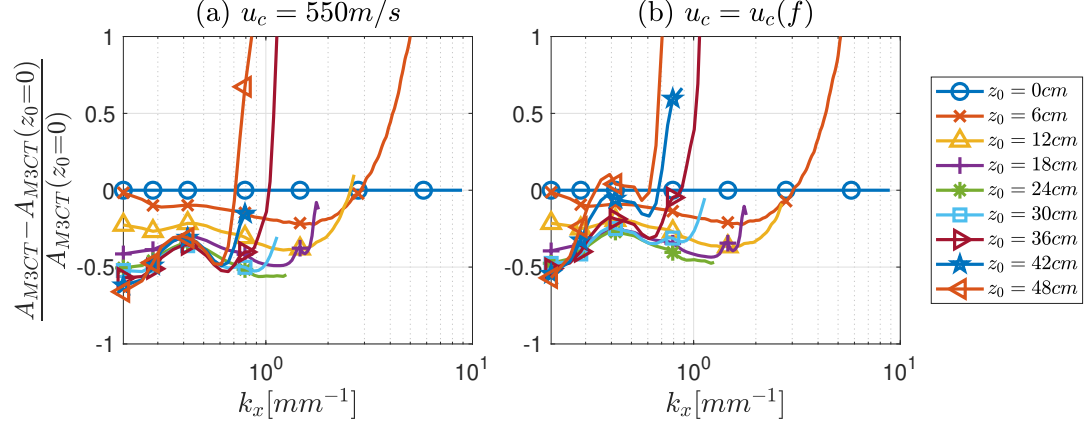


Figure 3.36: Comparison of amplitude spectra difference for uniform velocity profile (a) and frequency-dependent velocity (b)

3.4 Chapter Conclusion and Takeaways

In this chapter, the ability to model FLDI's response to a turbulent jet and to the flow downstream of a ramp in a Mach 3 tunnel using transfer functions derived from a flow parallel disturbance assumption is investigated and quantified. Particular emphasis is placed on the response as the flow is translated along the optical axis (z_0). Response to FLDI beam separation, Δx_1 , is also investigated in the jet.

Methods for measuring FLDI and 2pFLDI instrument parameters Δx_1 , Δx_2 , and w_0 are demonstrated. A procedure for estimating uncertainty in the model of the response in the jet is detailed. A critical parameter needed to relate frequency to wavenumber is the convection velocity (u_c) which is allowed to vary either spatially with z , or temporally with f . Both of these approaches to relating frequency to disturbance wavenumber in jets are novel and able to capture instrument re-

sponse with $\sim 20\%$ error across a significant band of frequencies/wavenumbers in the jet: $100kHz$ when historical measurements are used to predict $u_c(z)$ and $4.5mm^{-1}/50kHz$ when the dispersion relation, $u_c(f)$ measured by 2pFLDI is used instead. Instrument response is captured over a smaller bandwidth with larger error $\sim 40\%$ when the velocity is assumed constant.

Instrument response to disturbances in the Mach 3 tunnel is captured with $< 50\%$ error over $3mm^{-1}/200kHz$ when the $6.4cm$ tunnel is translated within $18cm$ of the focus. Further away from the focus, a large band of the FLDI signal is significantly attenuated. The remaining low wavenumber/low frequency band is captured by the transfer functions with $< 50\%$ error.

In these experiments, ‘error’ refers to the ability of transfer functions to account for the location of the disturbance (jet or M3CT) with respect to the FLDI focus. Errors of 20% to 50% are not surprising, because we are using a simple disturbance model (Eq. 2.11) for complex flows. These ‘error’ values quantify the effectiveness of our approach. No previous work known to the authors has quantified this approach before in flows similar to a turbulent jet or Mach 3 tunnel, thus the results are a significant contribution. Improved modeling, model evaluation over a wider parameter space, and evaluation against other jets and complex flows are good candidates for future work.

Chapter 4: 2pFLDI Applications

4.1 Overview

In this chapter, we apply 2pFLDI in two high speed wind tunnels in order to demonstrate how to interpret the measurements using the transfer function framework. The first tunnel is the University of Tennessee Space Institute’s Mach 4 Ludwig tube (UTM4). The second is Arnold Engineering Development Center’s Hypervelocity Wind Tunnel 9 Mach 18 test section (T9M18). In UTM4 we probe 12 wall-normal locations in the floor boundary layer and one location above the boundary layer. In T9M18 we probe the freestream of the test section at the centerline at two unit Reynolds numbers. Table 4.1 gives an overview of the conditions and dimensions in each tunnel and the parameters and specs of each 2pFLDI set-up. Note that the 2pFLDI used in T9M18 has the same parameter values and specs as the 2pFLDI used in the M3CT discussed previously in section 3.3. Just as in the previous chapter all lasers are Helium Neon (HeNe) with $\lambda_0 = 633nm$.

Parameter	UTM4	T9M18	Units
Re_∞ (unit)	20	3.8 - 4.7	$\times 10^6 \text{ m}^{-1}$
U_∞ (nominal)	670	2000	m/s
ρ_∞	140	4 - 5	g/m^3
test section width	61	150	cm
laser power	10	21	mW
f_{OL}	-9	9	mm
f_{FL}	200	400	mm
Δx_1	300	464	μm
L_i	41	140	cm
w_0	7	14	μm
Δx_2	2.6	1.2	mm

Table 4.1: Parameters of 2pFLDI experiments in this chapter. See Fig. 2.6 for reference

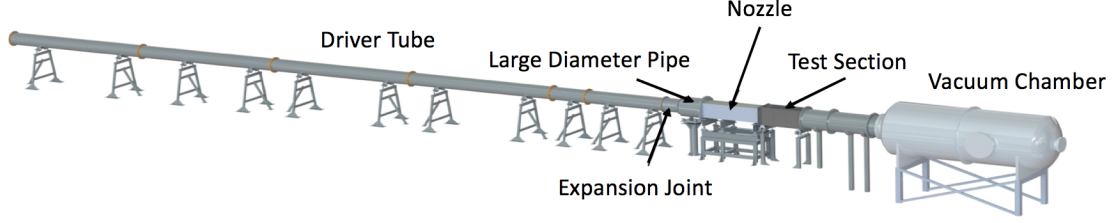


Figure 4.1: The UTSI Mach 4 Ludwieg tube.

4.2 UTSI Mach 4 Ludwieg Tube

The UTSI Mach 4 Ludwieg tube is shown in Fig. 4.1. The planar nozzle expands into a square test section with a 61 cm x 61 cm (24" x 24") cross-section and a length of 1.82 meters (6 ft.). The facility has a run time of roughly 200 ms for the first wave passage. Of that 200 ms, approximately 130 ms is at steady-state conditions suitable for testing. Per length Reynolds numbers range from approximately $5.6e6$ to $4.7e7$ m^{-1} . No heating is applied to the driver tube and the stagnation temperature is room temperature. Optical access in the test-section is provided by BK7 windows.

4.2.1 2pFLDI equipment and data reduction

The parameters of the optical setup for the 2pFLDI system are given in table 4.1. A 10 mW HeNe laser (Newport) beam expanded using an -9mm focal-length ($f_{OL} = -9mm$) plano-concave lens before passing through a polarizer (Thorlabs LPVISE100-A). Next, the beam continues through a 30 mm x 30 mm 1.5° double Wollaston prism (United Crystal) and is split based on polarization. A second

polarizer (Thorlabs LPVISE200-A) is then used to change the beam polarization to 45° so that a 50 mm x 50 mm 5 arcminute Wollaston prism (United Crystals) again splits the beams before the 75 mm diameter $f = 200$ mm plano convex focusing lens. The collection optics beyond the focus consist of a 75 mm diameter $f = 200$ mm plano convex lens, a 50 mm x 50 mm 5 arcminute Wollaston prism (United Crystals), a polarizer (Thorlabs LPVISE100-A), a -50 mm focal length ($f_{CL} = -50mm$) plano concave lens, and a neutral density filter. Two photodetectors (Thorlabs PDA36A2) are used for FLDI monitoring beam intensities. To reduce the effect of vibrations during tunnel operation, the system was built on a single 95 mm extruded rail. This also allowed easier translation to the region of interest without much realignment.

Voltages from the photodetectors are sampled with a 16-bit Picoscope 5442D at 20MHz. The max and min voltages are found by translating the up-beam Wollaston prism ($P1_u$) along the x-axis and then the voltage is set to the mean, V_0 , prior to each run. A baseline is established by collecting 0.5sec of data are prior to each run. Voltages are converted to phase differences via. Eq. 2.7. Phase difference amplitude spectra are found from the square-root of the power spectral density which is estimated using Welch's method with $129\mu s$ hamming windows and 50% overlap.

4.2.2 Experiment, Modeling and Spatiotemporal resolution

Fig. 4.2 is a schematic illustration of the UTSI Mach 4 Ludwig tube test section cross section showing the location of the 2pFLDI probe. 13 non-uniformly

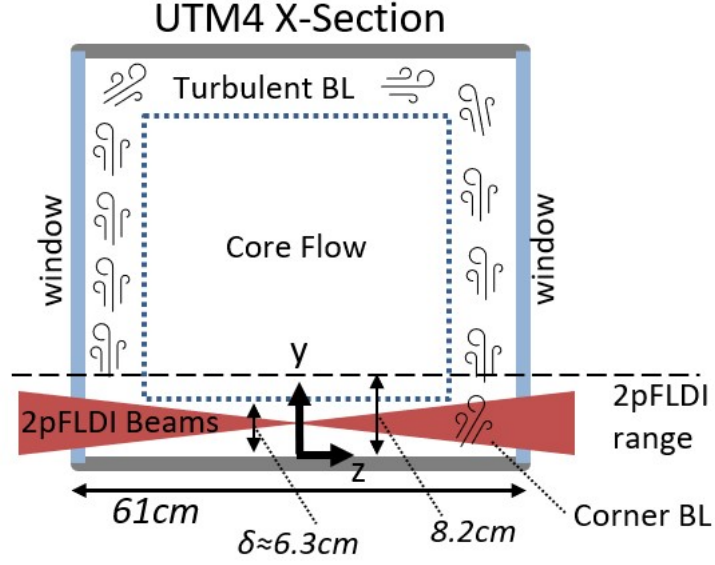


Figure 4.2: Schematic of experiment in UTM4 floor turbulent boundary layer

spaced wall-normal (y) locations are probed ranging from $y = 0.4cm$ to $y = 8.2cm$. The boundary layer thickness is estimated to be $6.3cm$ using a RANS simulation. The single point beam separation, beam radius parameter at the focus, and two-point beam pair separation are found using a beam profiler camera (Newport LBP2) and the same methods used in the jet (see sections 3.2.2 - 3.2.4). They are $\Delta x_1 = 300\mu m$, $w_0 = 7.1\mu m$, and $\Delta x_2 = 2.6mm$.

The governing equation for the FLDI signal (Eq.2.35) expressed using symbols for the UTM4 tunnel is given by

$$A_{UTM4} = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi]}{H_{UTM4}}, \quad (4.1)$$

where

$$H_{UTM4} = \int_{-L_i}^{L_i} |g_{UTM4} H_{\Delta x} H_w| dz. \quad (4.2)$$

For g_{UTM4} , we assume disturbances are uniform magnitude ($g = 1$) across a $44cm$

span centered about the y-axis. Outside of that span, but inside the tunnel, (i.e. $-30.5cm < z < -22cm$ & $22cm < z < 30.5cm$) we assume $g = 2$, representing disturbances in the corner flow boundary layers (Corner BL in Fig. 4.2). Everywhere else (outside of the tunnel) $g = 0$. g_{UTM4} is symmetric about the y axis and plotted for half the tunnel width in Fig. 4.3.

These assumptions are intended to be simple and conservative with the goal of demonstrating a method and procedure for analyzing FLDI. If more accuracy is desired, a more complex form of $g(z)$ should be chosen based on historical studies, as is demonstrated previously in the turbulent jet (section 3.2). Historical studies were not available for this flow, thus uniform assumptions are made. The assumption of $g = 2$ in the corner BL is intended to be conservative, representing a worst case scenario where large, strong disturbances in the tunnels corner could corrupt the measurement. Next, we chose a desired depth of focus and compute the wavenumber above which we can confidently attribute signal to disturbances within that depth of focus. Thus, the details of regions outside of that depth of focus are not important, only that they represent a worst case scenario.

Following the procedure for determining FLDI spatiotemporal resolution, outlined in section 2.3.1, we choose a spatial resolution of $2L_c = 44cm$. Now that we have $g(z)$ and L_c we can compute H_m , the sensitivity to disturbances we want to *measure*, and H_r , the sensitivity to disturbances we want to *reject*, using Eq. 2.36 - 2.41. The ratio H_m/H_r is plotted in Fig. 4.4 vs. wavenumber, which shows that the ratio is near unity until $k_x = 0.1mm^{-1}$, after which it increases rapidly. The ratio exceeds five at $k_x = 0.5mm^{-1}$, thus we take this to be the cut-off wavenumber,

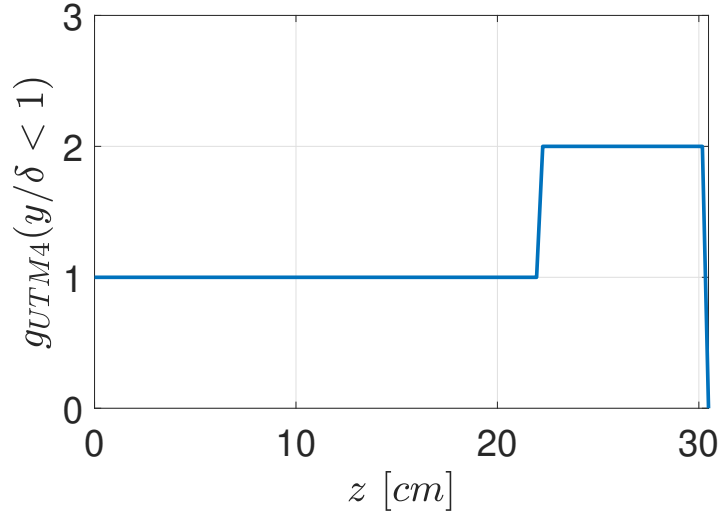


Figure 4.3: Estimate of $g(z)$ in UTM4 floor turbulent boundary layer

above which we achieve our desired depth of focus. The temporal resolution varies based on the convection velocity of the disturbance, u_c , which we will measure with 2pFLDI. As an example, for a disturbance convecting at the freestream velocity (670m/s, $k_x = 0.5mm^{-1}$) corresponds to a cutoff frequency of 53kHz.

For wavenumbers above this cut-off and for uniform disturbances of length $2L_c$ centered about the focus:

$$H_{UTM4} = \frac{\sqrt{2}\pi^{3/2}w_0}{\lambda_0 k_x} H_{\Delta x} \exp\left(-\frac{w_0^2 k_x^2}{8}\right) \operatorname{erf}\left[\frac{\lambda_0 k_x L_c}{2\sqrt{2}\pi w_0}\right]. \quad (4.3)$$

Note that the sensitivity to corner boundary layers is ignored as long as we restrict ourselves above the wavenumber cut-off. Below this cut-off, signal should not be analyzed.

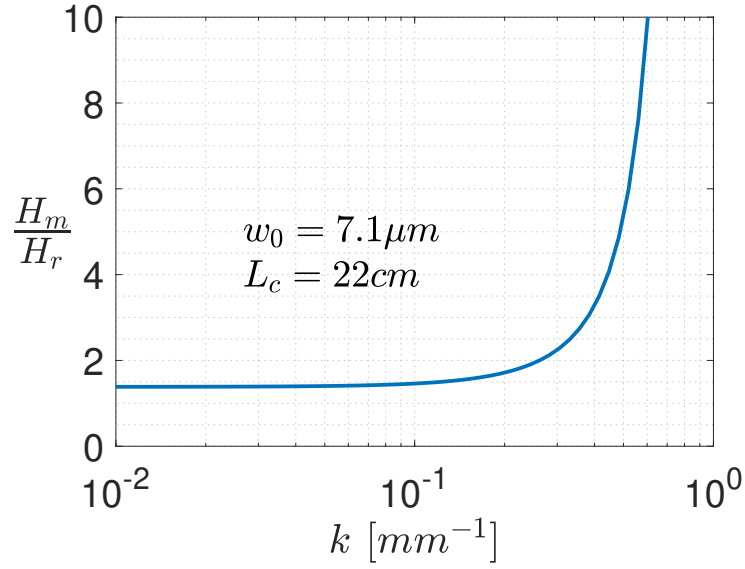


Figure 4.4: Estimate of FLDI signal to noise ratio for a $2L_c = 44cm$ depth of focus in UTM4 boundary layer

4.2.3 Results

Time traces of single point FLDI phase difference (Ch.A) are displayed in Fig. 4.5 for six tunnel runs corresponding to six different wall normal (y/δ) positions of the FLDI. Each trace is offset by 2.5 radians so that all traces can be seen on one plot. For each run, the largest fluctuations are observed from $0 < t < 15ms$ and for $t > 240ms$. These correspond to the periods of tunnel start up and shut-down, respectively. The period of steady state test time used for subsequent analysis is $130ms < t < 220ms$ - indicated in the figure as “good flow used”. During this time, fluctuations from the run when FLDI is positioned just outside of the boundary layer at $y/\delta = 1.3$ are clearly smaller than those measured inside of the boundary layer (all other traces). These six runs will be analyzed in detail next.

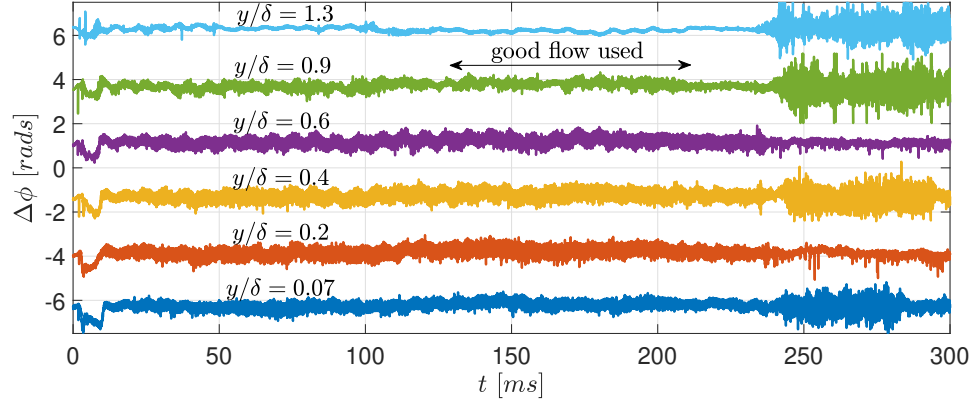


Figure 4.5: Phase difference vs. time from FLDI (Ch.A) for six wall normal positions (y)

The amplitude spectral density of phase difference is shown in Fig. 4.6 for the “good flow” period of each of the six runs. The dashed line shows the amplitude spectral density of phase difference prior to tunnel start-up in the $y/\delta = 1.3$ run. Below $\sim 80kHz$, humps of increased amplitude are observed for all measurements. As frequency increases, the slope decreases slightly. This behavior is similar to predictions of how low frequency signal might appear if it is corrupted by side-wall boundary layer disturbances by Settles and Fulghum [32]. Near $500kHz$, all spectra decrease rapidly, which resembles behavior seen when wavenumbers exceed $\pi/\Delta x_1$. To interpret these signals further, we need to convert frequencies to wavenumbers with the convection velocity.

4.2.3.1 Convection Velocity

Fig. 4.7 shows a zoom in of the phase difference time trace for both signals (Ch.A and B) for the run when the 2pFLDI is located at $y/\delta = 0.4$. Ch.B looks like a

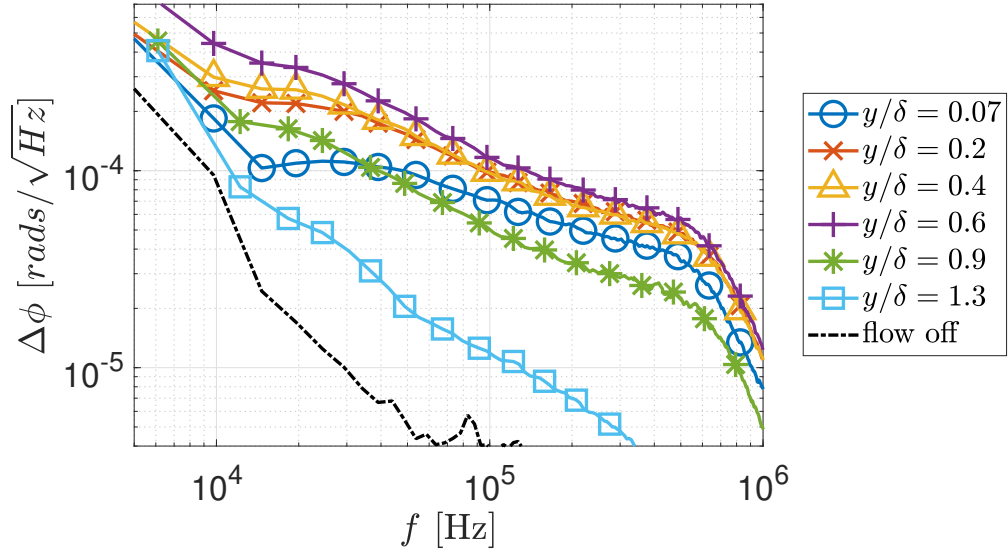


Figure 4.6: Phase difference vs. frequency from FLDI (Ch.A) for six wall normal positions (y)

a copy of Ch.A, shifted in time by Δt , indicating the passage of coherent disturbances through both measurement volumes. As described in section 2.4, mean convection velocity, $\langle u_c \rangle$, is found from the cross-correlation between Ch.A and Ch.B. (Eq. 2.46). Frequency dependent convection velocity, $u_c(f)$, is found from the phase of the cross-spectra between Ch.A and Ch.B, which is the Fourier transform of the cross-correlation (Eq. 2.48-2.50).

Convection velocity vs. frequency is plotted for six runs in Fig. 4.8. It shows velocity that increases rapidly with frequency until 100kHz for all runs. Above 100kHz, velocity increases gradually or is constant and ranges from 500m/s to 630m/s. In this range, velocity increases with y , as is expected in a boundary layer. The rapid changes observed below 100kHz are likely the result of signal originating from outside the desired depth of focus per our previous analysis (section 4.2.2). At

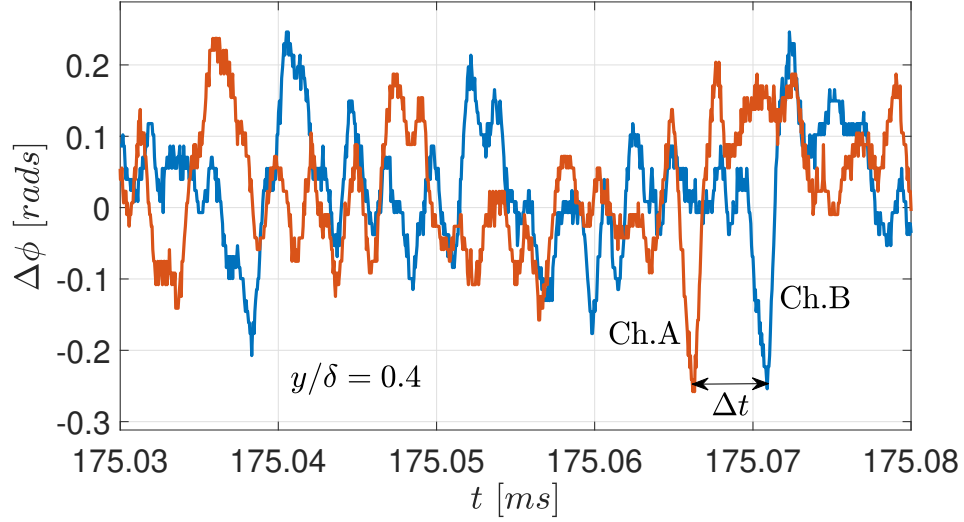


Figure 4.7: Phase difference vs. time from 2pFLDI (Ch.A & Ch.B) for $y/\delta = 0.4$

frequencies below $\sim 53kHz$, slower moving structures in the corner boundary layers cause measured velocities to be slower.

Fig. 4.9 shows the same velocities normalized by each run's respective velocity computed from the cross-correlation. This appears to collapse the data between $0.2 \leq y/\delta \leq 0.9$ although slight deviations from the trend are apparent in the measurements closest to the wall and outside the boundary layer. In general, Fig. 4.9 shows that the velocities for $f > 50kHz$ only deviate from the cross-correlation velocity by a few percent showing that dispersion is minimal above 50kHz.

4.2.3.2 Wavenumber Spectra

Fig. 4.10 shows the single-point FLDI (Ch.A) amplitude spectral density of phase difference plotted against wavenumber computed using the measured convection velocities in Fig. 4.8. The trends observed here are nearly identical to those

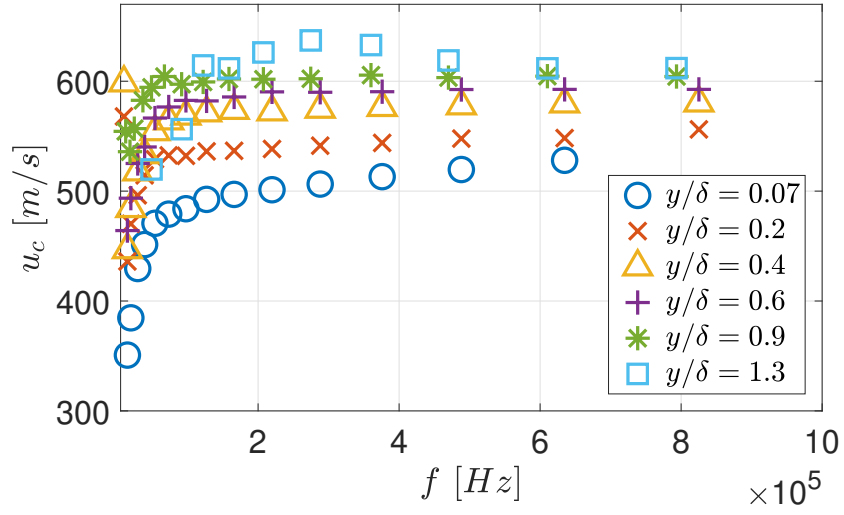


Figure 4.8: 2pFLDI convection velocity vs. frequency for six wall normal positions (y)

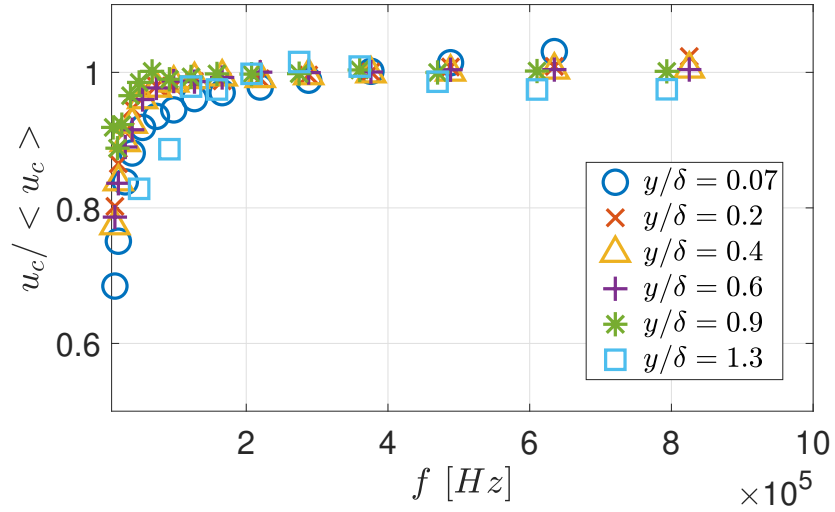


Figure 4.9: 2pFLDI convection velocity vs. frequency normalized by mean convection velocity over all frequencies for six wall normal positions (y)

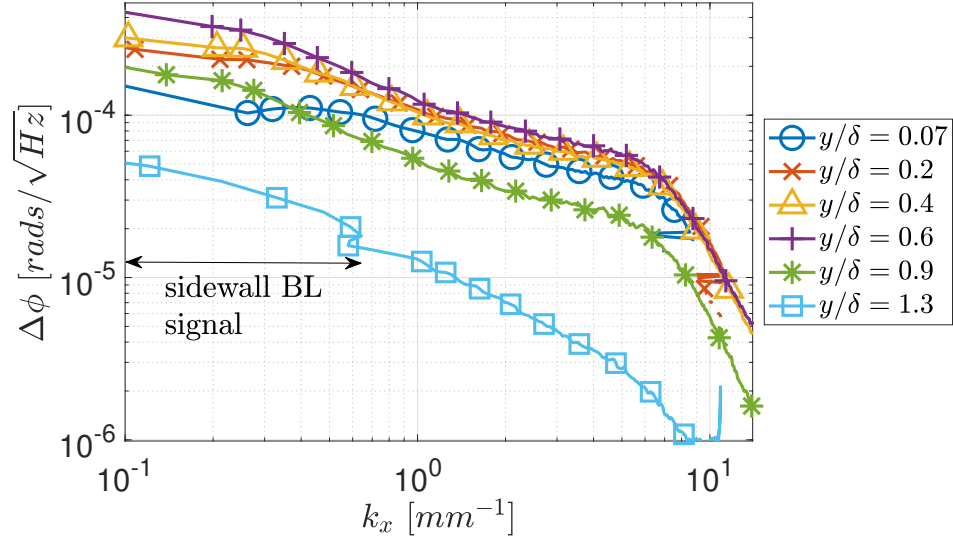


Figure 4.10: Phase difference vs. wavenumber from FLDI (Ch.A) for six wall normal positions (y)

observed in Fig. 4.6 because the convection velocities are nearly constant across f . Fig. 4.10 puts the signal in the context of the spatial features of the flow. This allows us to determine what parts of the signal may be corrupted by disturbances outside of the desired depth of focus. Per the analysis in section 4.2.2, we expect signals with wavenumbers below 0.5mm^{-1} to be corrupted by the corner flow. For the one measurement above the boundary layer ($y/\delta = 1.3$), this cut-off may be larger because disturbances within the depth of focus will be relatively weaker than those within the boundary layer. The 'bump' in the $y/\delta = 1.3$ curve at $k_x = 0.6\text{mm}^{-1}$ for the curve corresponding to $y/\delta = 1.3$ suggests that our analysis is valid because significant changes in the spectrum are observed near the wavenumber predicted.

Moving forward, we restrict our analysis to wavenumbers above 0.5mm^{-1} which represent disturbances within the 44cm depth of focus. The sensitivity func-

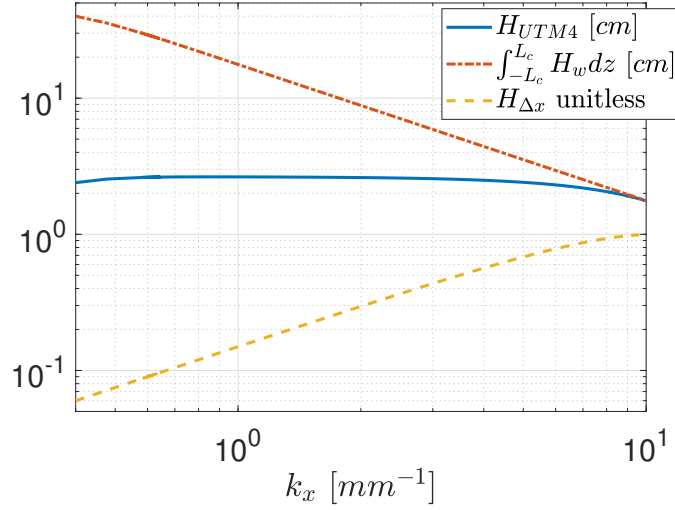


Figure 4.11: Sensitivity and transfer functions for uniform disturbance in UTM4. The sensitivity function H_{UTM4} is plotted vs. wavenumber in Fig. 4.11, which shows H_{UTM4} is between 20cm and 30cm and nearly constant from $0.5mm^{-1} < k_x < 10mm^{-1}$. This is initially surprising, because in general the instrument's response varies strongly with wavenumber. Plotting the two transfer functions explains the behavior, shown in Fig. 4.11 as dashed lines. For this particular FLDI setup, the depth of focus and the wavenumber band the rate at which the transfer function due to beam separation $H_{\Delta x}$ increases with wavenumber. This is nearly equal to the rate at which the transfer function due to beam diameter H_w decreases with wavenumber. Thus the product, the overall transfer function, remains approximately constant.

Fig. 4.12 shows density-based amplitude spectra (found by evaluating Eq. 4.1) normalized by the freestream density in the tunnel ($\rho_\infty = 0.14kg/m^3$). The trends are similar to those present in the phase difference spectra because the sensitivity function is nearly constant across this wavenumber band. Above $k_x = 5mm^{-1}$, a

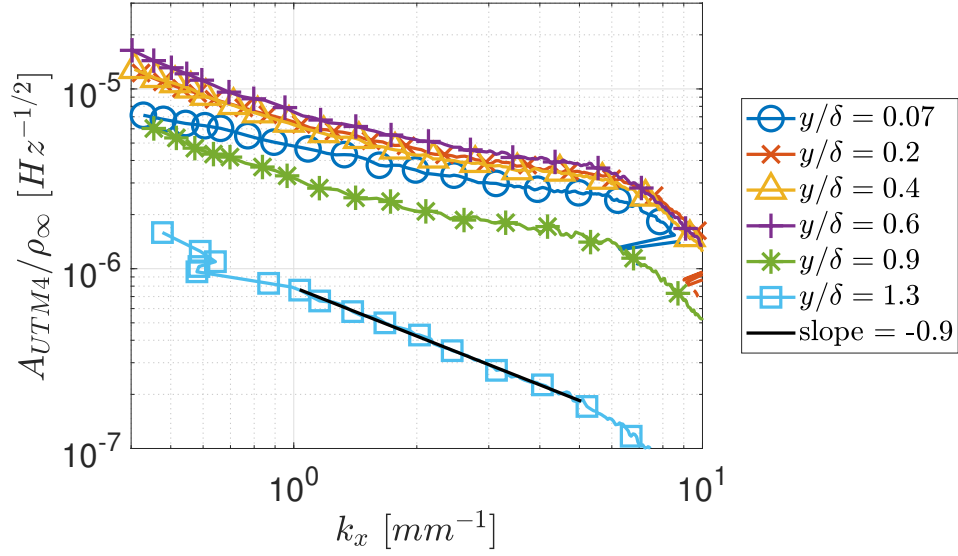


Figure 4.12: Amplitude spectra in UTM4 for six wall normal positions (y)

steep roll-off is present in all spectra as well as some scatter in the $y/\delta = 0.07$ data. This region occurs before the predicted roll off of $\pi/\Delta x_1 \approx 10 \text{ mm}^{-1}$. It is not clear whether this behavior is a flow feature or an instrument effect. For $1 \text{ mm}^{-1} < k_x < 5 \text{ mm}^{-1}$ and $y/\delta = 1.3$, the decrease of amplitude with wavenumber is linear (in a logarithmic scale) according to $k_x^{-0.9}$. For all other measurements in the boundary layer, the slope is more gradual

4.2.3.3 Boundary Layer profiles

The FLDI-measured turbulence intensity can be found for specific wavenumber bands by taking the square-root of the power spectral density integrated over the wavenumber band of interest. This is the same as the root-mean squared value over a specific band. For a band from k_1 to k_2 , wavenumbers are converted back to frequencies corresponding to f_1 and f_2 using the measured convection velocities.

The turbulence intensity is given by:

$$RMS(A/\rho_\infty) = \sqrt{\int_{f_1}^{f_2} (A/\rho_\infty)^2 df}. \quad (4.4)$$

This quantity is computed using the trapezoidal rule (MATLAB `trapz`) to evaluate the integral. This is done for three different wavenumber bands and 13 runs of the tunnel corresponding to all 13 y positions interrogated with 2pFLDI in this campaign. The results are plotted in Fig. 4.13 and show that turbulence intensity increases rapidly from a minimum value at $y/\delta = 1.3$ to a maximum at $y/\delta = 0.5$. All bands have similar shapes. With the exception of $y/\delta = 0.5$ & 0.6 , turbulence intensity in the boundary layer below $y/\delta = 0.9$ is $\sim 10\times$ larger than the value measured outside the boundary layer at $y/\delta = 1.3$.

For perspective, Zhang and Duan [74] find that the magnitudes of density based turbulence intensity across all wavenumbers in a Mach 2 and Mach 6 boundary layer simulated by DNS are on the orders of 10 and 20%, respectively. The magnitudes measured by our FLDI over the $0.5mm^{-1} < k_x < 5mm^{-1}$ band are $\sim 0.3\%$ for this Mach 4 boundary layer. We believe the reason for this discrepancy is that the wavenumbers containing the majority of the energy ($< 0.5mm^{-1}$) are not resolved with sufficient depth of focus by the FLDI. A comparison between DNS and FLDI in the comparable wavenumber bands is intriguing future work.

The mean velocity over a wavenumber band k_1 to k_2 is found by converting wavenumbers back to frequencies f_1 and f_2 and then using the data in Fig. 4.8 to computing the mean value of $u_c(f)$ in the interval f_1 to f_2 . The results are plotted in Fig. 4.8 for all 13 y positions in Fig. 4.14. The bands used are the same as those

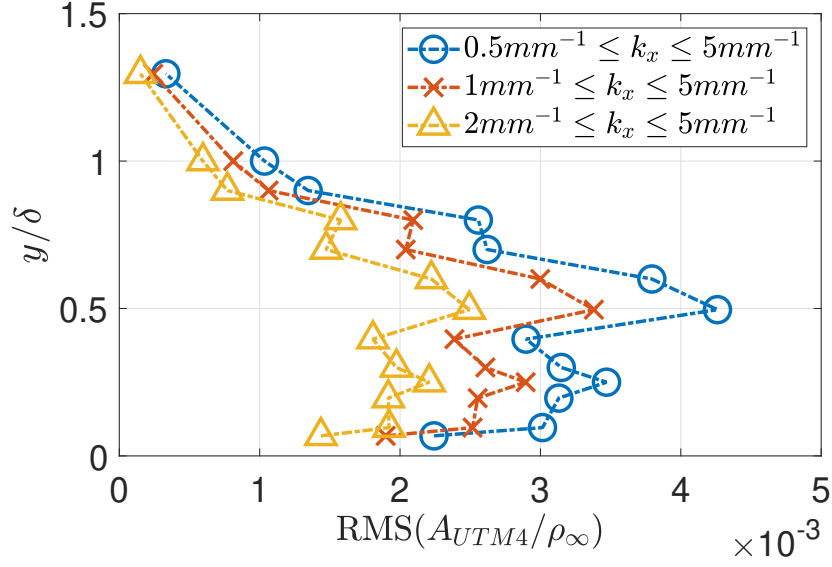


Figure 4.13: Turbulence intensity profile in UTM4 floor BL for three wavenumber bands

used to determine turbulence intensity. Also plotted is the cross-correlation velocity which is the mean value over all wavenumbers (denoted “all” in the figure) weighted by signal strength. Error bars represent uncertainty in velocity introduced by small variations ($< 5\%$) in the beam pair separation Δx_2 (measured with the beam profiler camera) over the tunnel width. The mean velocity boundary layer profile predicted by a RANS simulation is also included in Fig. 4.14. Convection velocities measured by 2pFLDI agree with the computations in the region $0.2 < y/\delta < 0.5$. Velocities measured using 2pFLDI exceed the RANS predictions for $y/\delta < 0.2$ and are smaller than the RANS predictions for $y/\delta > 0.5$. All velocities for $y/\delta > 0.2$ are between 80 and 95% of the RANS values. 2pFLDI velocity measurements using the high wavenumber bands (corresponding to structures lying within the depth of focus) are nearly identical. This suggests that the cross-correlation velocity is not significantly

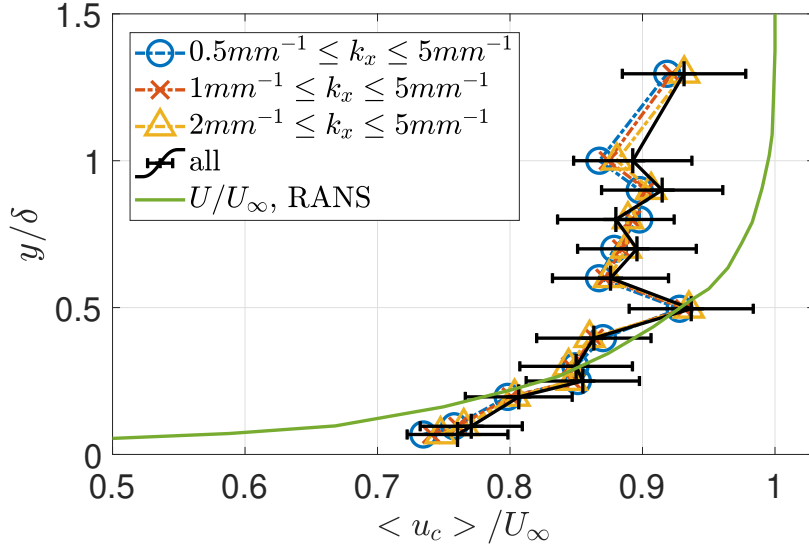


Figure 4.14: Velocity profile in UTM4 floor BL computed using three wavenumber bands

corrupted by disturbances away from the instrument's focus. Measurements in the freestream of AEDC T9's Mach 18 wind tunnel presented later in this chapter will show this is not always the case.

4.2.4 Conclusion and Takeaways

A method for modeling the response and determining the spatiotemporal resolution of 2pFLDI in the floor boundary layer of a Mach 4 Ludwig Tube is presented. The method predicts that the FLDI can resolve disturbances with wavenumbers greater than 0.5mm^{-1} at 44cm depth of focus. For disturbances convecting near the freestream velocity, this corresponds to frequencies greater than 50kHz. Evidence of this wavenumber/frequency cut-off is present in the measured spectra in the form of increased amplitude “humps” below the cut-off and in the measured velocities in

the form of a rapid roll-off with decreasing frequency below 50kHz. Above the cut-off, density-based turbulence intensity is recovered using transfer functions which assume uniform flow parallel waves across the depth of focus. Turbulence intensity peaks at half the boundary layer thickness and is on the order of $10\times$ stronger in the boundary layer than outside. Convection velocity profiles measured using only signal from inside the depth of focus do not significantly deviate from those measured using the entire signal. Almost all velocities measured are between 80 and 95% of RANS predictions of the freestream velocity except for measurements made closest to the floor ($y/\delta < 0.2$).

4.3 AEDC T9 Mach 18 Freestream

AEDC’s Hypervelocity Tunnel 9 (T9) in White Oak, Maryland is a Nitrogen gas blowdown facility capable of producing Mach numbers from 7 to 18 in its 1.5m wide circular test section. For more detail on the wide range of capabilities at T9, readers are directed to Marren and Lafferty [75]. Mach 18 flows at two nominal unit Reynolds numbers ($1, 1.5 \times 10^6/ft$) are investigated. This high Mach number flow results in very low free stream density ($< 5g/m^3$) and very large ($\sim 40cm$) sidewall boundary layers thus presenting a unique challenge for FLDI measurements: density changes are especially small and disturbances away from the focal volume are especially strong and widespread. Nominal free stream and reservoir conditions are provided in table 4.2. “Core size” refers to the approximate width of “clean” flow free from sidewall boundary layers. Details on how these facility conditions are

Nom. Re_∞	Re_∞	M_∞	p_∞	ρ_∞	U_∞	Core Size
$1/ft \times 10^6$	$1/m \times 10^6$	(-)	(Pa)	(g/m^3)	(m/s)	(m)
1.0	3.79	18.2	35.4	3.97	2032	0.737
1.5	4.74	18.3	45.0	4.96	2064	0.737

Table 4.2: Facility calculated free stream (subscript ∞) conditions.

found is given here [76, 77].

4.3.1 2pFLDI equipment and data reduction

The parameters of the optical setup for the 2pFLDI system are given in table 4.1. The equipment is the same as that used previously in the M3CT (see section 3.3.1). The photodetector signals are terminated at $1k\Omega$, amplified and filtered by Stanford SR560 low-noise pre-amplifiers, and digitized by a 16-bit Picoscope 5442D. A $1k\Omega$ as opposed to a 50Ω resistor is used because the signal strength is low. Various gain and high-pass filter settings for the SR560 are used with adjustments made run-to-run in an effort to reduce noise. The recommended settings for future FLDI experiments are $20\times$ gain, $100Hz$ high pass filter, and 50Ω resistor to reduce high frequency roll off. The sampling frequency is 7MHz. Raw voltages are downloaded from the scope and processed using MATLAB. Voltage is converted to phase difference using Eq. 2.7. V_0 is determined experimentally prior to each run by measuring the maximum and minimum voltage associated with translating $P1_d$ along the x-axis using a remote-operated picomotor.

All spectral analyses are based on 2 seconds of “good-flow” (defined where

the freestream Reynolds number is essentially constant [78]) starting at $t = 2s$ where $t = 0$ is the time the start-up trigger signal is received. Fourier transforming the phase difference fluctuations is computed by taking the square-root of the power spectral density. The latter is estimated using Welch’s method [69] via the MATLAB function `pwelch` with $1ms$ Hamming windows and 50% overlap.

4.3.2 Experiment and Modeling

Fig. 4.15 is a cross-section of the AEDC T9 Mach 18 test section at the location where the 2pFLDI probes the freestream flow. The beams are focused through $10cm$ diameter windows which set an upper limit on the convergence angle of the beams toward the focus - and thus the depth of field - achievable. In future FLDI experiments in T9, we recommend using larger windows and filling them as much as possible to maximize the convergence angle of the beams. The windows are in recessed wells where sidewall boundary layers on the tunnel walls become shear layers denoted “Turbulent SL” in the schematic illustration. Just as in previous experiments, the single point beam separation $\Delta x_1 = 464\mu m$, the beam radius parameter at the focus $w_0 = 14\mu m$, and the two-point beam pair separation $\Delta x_2 = 1.22mm$ are found with a beam profiler camera (Newport LBP2) using the methods described in sections 3.2.2 - 3.2.4.

Re-expressing Eq. 2.35 for T9 gives

$$A_{T9M18} = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi]}{H_{T9M18}}, \quad (4.5)$$

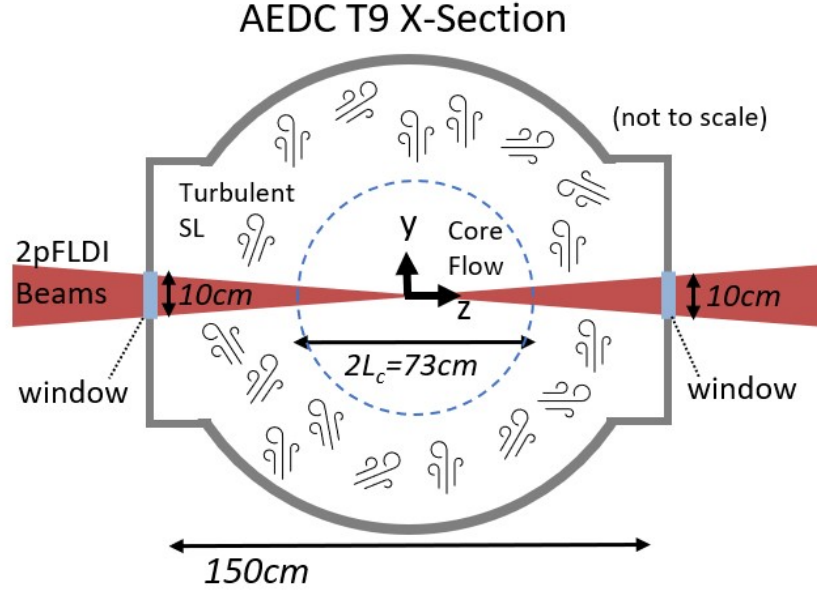


Figure 4.15: Schematic illustration of experiment in AEDC T9 Mach 18 test section

where

$$H_{T9M18} = \int_{-L_i}^{L_i} |g_{T9M18} H_{\Delta x} H_w| dz. \quad (4.6)$$

For g_{T9M18} , we assume disturbances are uniform magnitude ($g = 1$) across the core flow (73cm) span centered about the y -axis. The size of the core flow is determined here [76]. Outside of that span, but inside the tunnel, (i.e inside the turbulent shear layers $-75\text{cm} < z < -37\text{cm}$ & $37\text{cm} < z < 75\text{cm}$) we assume $g = 15$ representing disturbances in the shear layers which are 15 times stronger than disturbances in the freestream. Everywhere else (outside of the tunnel) $g = 0$. g_{T9M18} is symmetric about the y axis and plotted for half the tunnel width in Fig. 4.16.

Once again, $g(z)$, is intended to be conservative. Free stream turbulence measured with pitot probes in the same facility but at mach numbers of 14 or lower is on the order of 3-5% [14]. If this is also the case in the Mach 18 test section, our

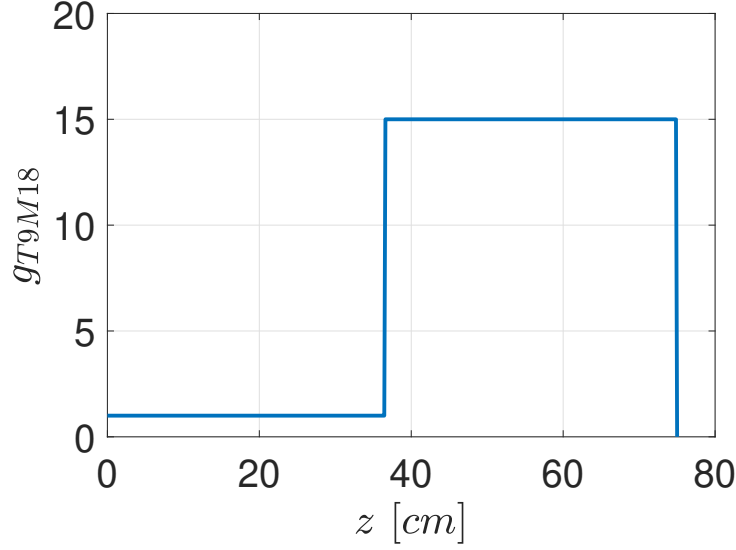


Figure 4.16: Estimate of $g(z)$ in AEDC T9 Mach 18 test section

estimate of $g(z)$ would imply 45-75% turbulence intensity in the shear layers. This is high compared to magnitudes near 45% measured by Zhang and Duan [74] in a Mach 14 turbulent boundary layer with DNS. It is hard to say how accurate our assumption is, due to a lack of measurements in the test section shear layers and in Mach 18 flows generally.

Next, we choose the core flow width as our desired spatial resolution such that $2L_c = 73\text{cm}$. The methods outlined in section 2.3.1 are employed to compute the sensitivity to disturbances we want to measure (H_m) and those we want to reject (H_r). Changing notation slightly we write $H_{core} = H_m$ and $H_{SL} = H_r$ for the core flow and shear layer respectively. The ratio H_{core}/H_{SL} as a function of wavenumber (graphed in Fig. 4.17) is near zero until $k_x \sim 0.5\text{mm}^{-1}$, exceeds unity at $\sim 0.7\text{mm}^{-1}$, and exceeds five near 0.9mm^{-1} . Thus we take $k_x = 0.9\text{mm}^{-1}$ as the cut-off wavenumber above which we achieve our desired spatial resolution. To

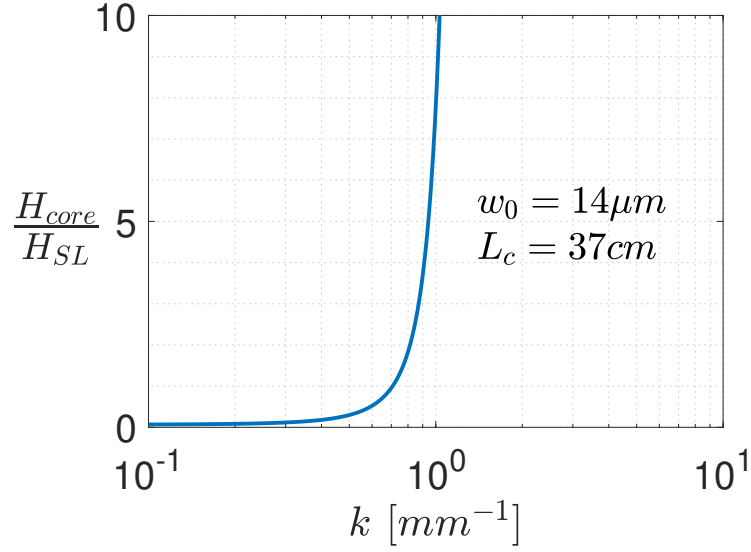


Figure 4.17: Estimate of FLDI signal to noise ratio for a $2L_c = 73cm$ depth of focus in T9M18

give a sense of the corresponding temporal resolution, a $k_x = 0.9mm^{-1}$ disturbance convecting at 1600m/s corresponds to a 230kHz signal. Signal above this cut-off is processed using Eq. 4.5 where $H_{T9M18} = H_{core}$, and H_{core} is equivalent to the analytic sensitivity function for a uniform, flow parallel disturbance expressed in Eq. 4.3.

4.3.3 Results

Time traces of phase difference measured by single point FLDI (Ch.A) are plotted for five tunnel runs in Fig. 4.18. Runs 4659 and 4667 correspond to a unit Reynolds number of $1 \times 10^6/ft$ and runs 4671, 4658, and 4662 correspond to a unit Reynolds number of $1.5 \times 10^6/ft$. The absence of low frequency oscillations in run 4671 may be the result of a 3Hz high pass filter applied by the pre-amplifiers. A

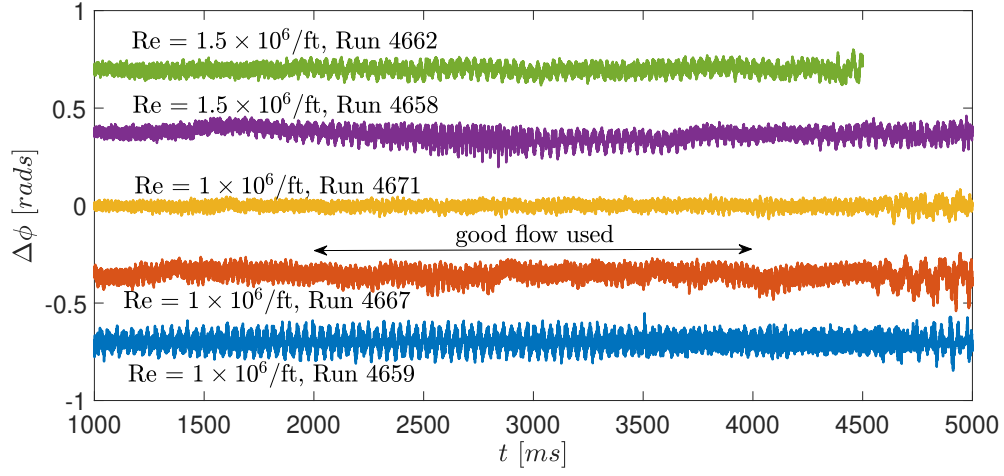


Figure 4.18: Phase difference vs. time from FLDI (Ch.A) for five tunnel runs

100Hz high pass filter is applied for run 4659. The other runs do not employ a high pass filter which may explain the larger oscillations observed in run 4658. Vary large amplitude fluctuations are evident in several runs for $t > 4.5s$ which indicates tunnel shut-down. The steady state test time used in subsequent analysis is indicated as “good flow used” in the plot.

Fig. 4.19 shows the amplitude spectral density of phase difference from $2 < t < 4s$ plotted in various colors and symbols for the five runs. Measurements of phase spectra taken ~ 15 minutes prior to each run are plotted in black with symbols matching those used in the respective run data. For all runs, signal strength is significantly larger than the flow off noise for $20kHz < f < 200kHz$ and decreases with frequency. The amplitudes of the flow off spectra vary significantly between runs for $f > 100kHz$. It is largest for run 4667 where signal to noise reaches unity near 300kHz. Noise is smallest for the two runs which employed a high pass filter. Wave packets are observed above 200kHz for runs 4658 and 4659. These features

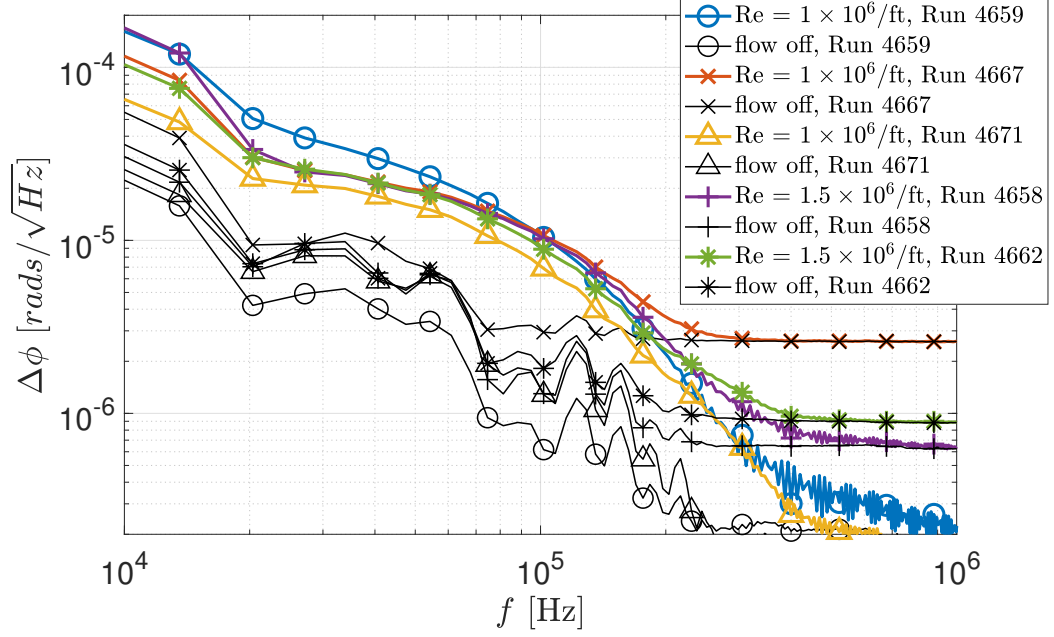


Figure 4.19: Phase difference vs. frequency from FLDI (Ch.A) for five runs. flow off taken prior to run.

can not be explained by features in the flow and are assumed to be signal processing artifacts or electronic noise.

4.3.3.1 Convection Velocity

Time traces of phase difference computed from Ch.A (upstream probe) and Ch.B (downstream probe) for the 2pFLDI employed in run 4671 are plotted in Fig. 4.20. Ch.B (blue) appears very similar to Ch.A (red) except that it is shifted in time by a tiny $\Delta t \sim 1\mu s$, indicated on the graph. The time shift here is smaller than that observed in UTM4 because the flow is faster and the beam pair separation Δx_2 is smaller. Next, cross-correlation and cross-spectra are used to compute convection

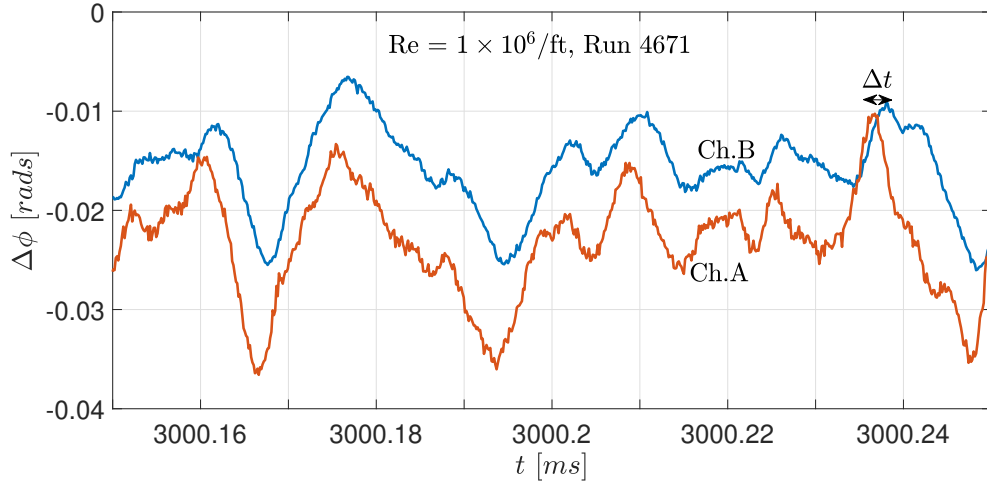


Figure 4.20: Phase difference vs. time from 2pFLDI (Ch.A & Ch.B) for Run 4671 velocity.

Fig. 4.21 shows convection velocity normalized by freestream velocity vs. frequency for the five runs. Free stream velocity is estimated by a methodology involving pitot pressure probes as well as other measurement techniques detailed here [76, 77]. Velocities range from 50 to 100% of the freestream value for $25kHz < f < 150kHz$ and increase with frequency for $100kHz < f < 200kHz$ in all runs. For $200kHz < f < 350kHz$, velocities measured in run 4671 and 4662 remain near a constant value of 80% of the free stream value whereas those measured in runs 4659 and 4658 exceed the free stream value and show a large degree of variance. SNR in run 4667 was too small to measure velocities associated with frequencies $> 230kHz$. Velocities above the freestream value are non physical in a supersonic flow and the behavior seen in runs 4659 and 4658 may be the result of the wavepacket noise observed in the spectra at the same frequencies (Fig. 4.19). The modeling and spatiotemporal resolution analysis presented in section 4.3.2 suggests

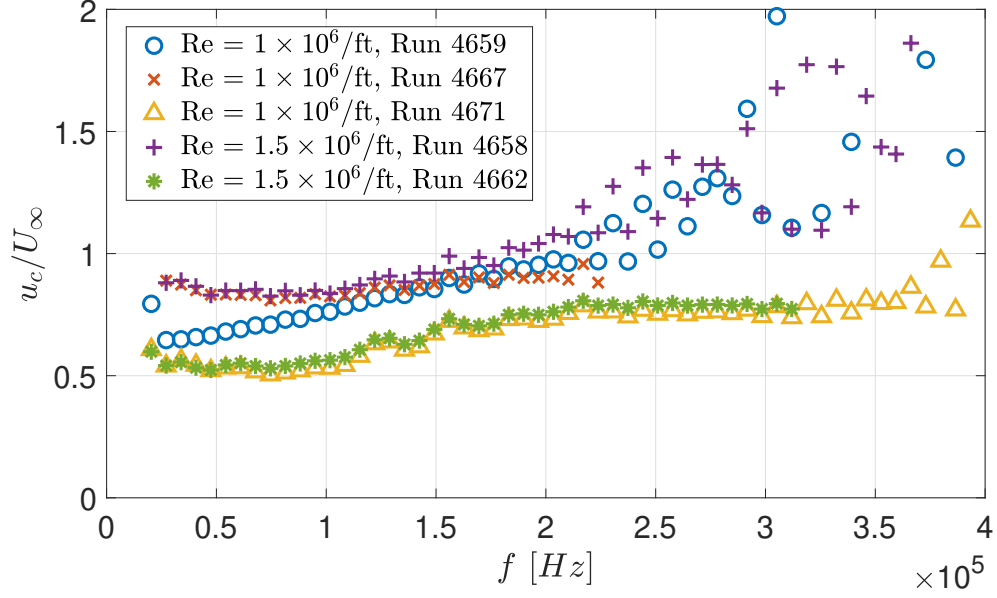


Figure 4.21: 2pFLDI convection velocity normalized by freestream velocity, $U_\infty = 200m/s$, vs. frequency

that signals below 200kHz will be significantly influenced by disturbances in the shear layers while signals above 200kHz will be largely the result of disturbances in the freestream.

Table 4.3 gives the convection velocity computed from the cross-correlation (Eq. 2.46) for each run as well as the high pass filter settings used. The velocities range from 52 to 84% of the freestream value. No trend with filter setting is apparent. Fig. 4.22 shows the frequency dependent velocities normalized by the cross-correlation values which represent a signal-strength-weighted average across all frequencies. $u_c/\langle u_c \rangle$ is near unity at 50kHz and increases with frequency for all runs. This indicates that the velocities found from the cross-correlation, $\langle u_c \rangle$ are associated with disturbances having frequencies around 50kHz. These are likely to

Run #	Re [$1/m \times 10^6$]	HP filter	$\langle u_c \rangle$ [m/s]	$\langle u_c \rangle / U_\infty$
4659	3.8	100Hz	1320	0.65
4667	3.8	none	1680	0.83
4671	3.8	3Hz	1060	0.52
4658	4.7	none	1730	0.84
4662	4.7	none	1100	0.53

Table 4.3: 2pFLDI High Pass (HP) filter settings, and convection velocity averaged across all frequencies

be located in the sidewall shear layers and not within the depth of focus. Therefore, the key takeaway from these experiments is that the velocities found by taking the cross-correlation of the 2pFLDI signals may not represent velocities of disturbances near the focus of instrument.

It appears that only two runs (4671 and 4662) produced physically meaningful convection velocities for freestream disturbances. Both measured $u_c/U_\infty \sim 0.75 - 0.82$.

4.3.3.2 Wavenumber Spectra

The phase difference amplitude spectral density vs. wavenumber is plotted in Fig. 4.23 for all five runs. For runs 4671 and 4662 frequencies are converted to wavenumbers using the measured convection velocities plotted in Fig. 4.21. For the other three runs, $u_c = 0.8U_\infty$ is used for all frequencies. Our modeling and spatiotemporal resolution analyses predict that signals below a wavenumber

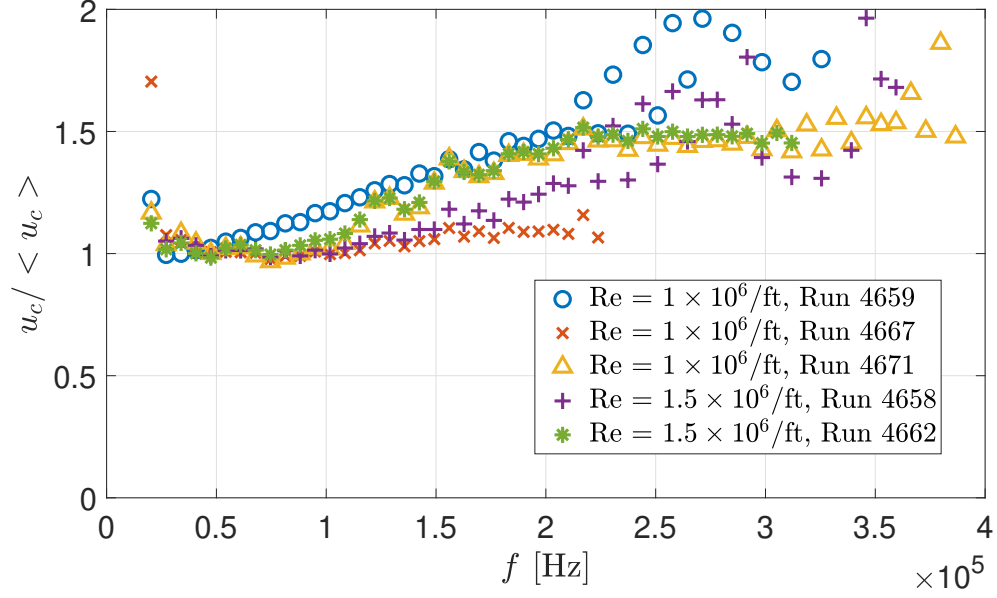


Figure 4.22: 2pFLDI convection velocity vs. f , normalized by average value over all f

of approximately $k_x = 0.9$ will be corrupted by disturbances outside of the 73cm depth of focus. For run 4662, there is a change in slope near the predicted cut-off wavenumber $k_x = 0.9$, indicating the presence of a low wavenumber hump similar to those seen in the UTM4 measurements and predicted by Settles and Fulghum [32]. This wavenumber region is indicated in Fig. 4.23 as “sidewall SL signal”. Above this wavenumber we can be confident that the signal originates from disturbances within the depth of focus until the point at which the signal and the flow off noise floor become coincident, i.e. where SNR reaches unity. The region of the spectra between these limits is circled and labeled “core flow signal” on the plot. Note that this corresponds to a small wavenumber band of $0.9\text{mm}^{-1} < k_x < 1.3\text{mm}^{-1}$. We will restrict our analysis to this region from now on.

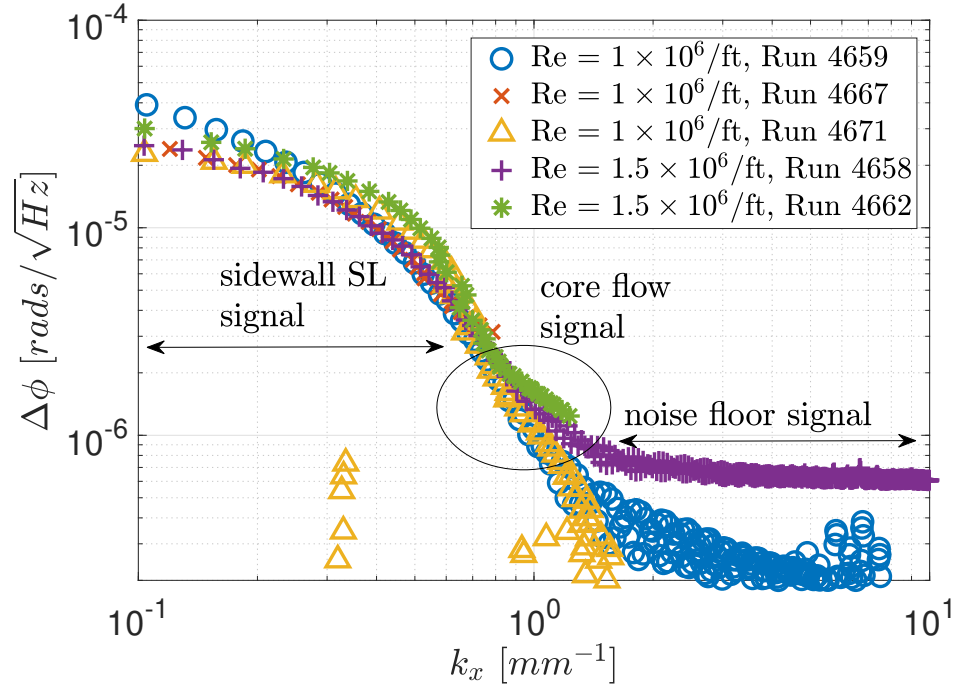


Figure 4.23: Phase difference vs. wavenumber from FLDI (Ch.A)

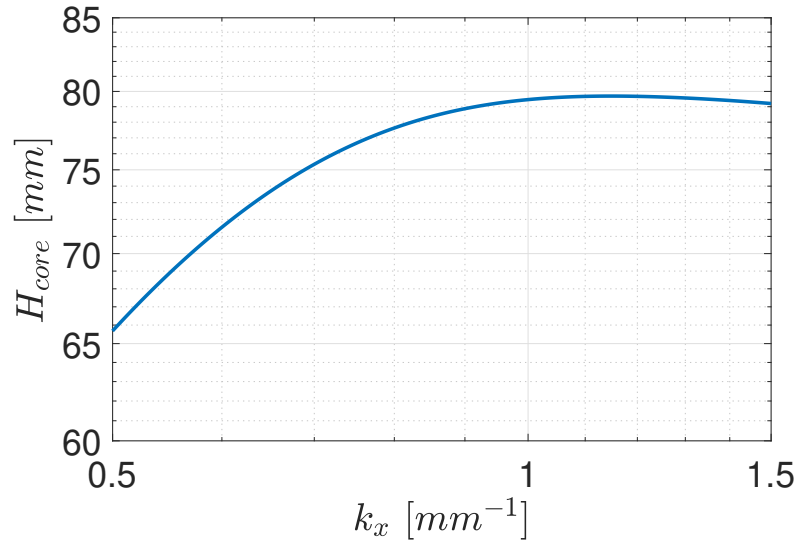


Figure 4.24: Sensitivity function for uniform disturbance in T9M18

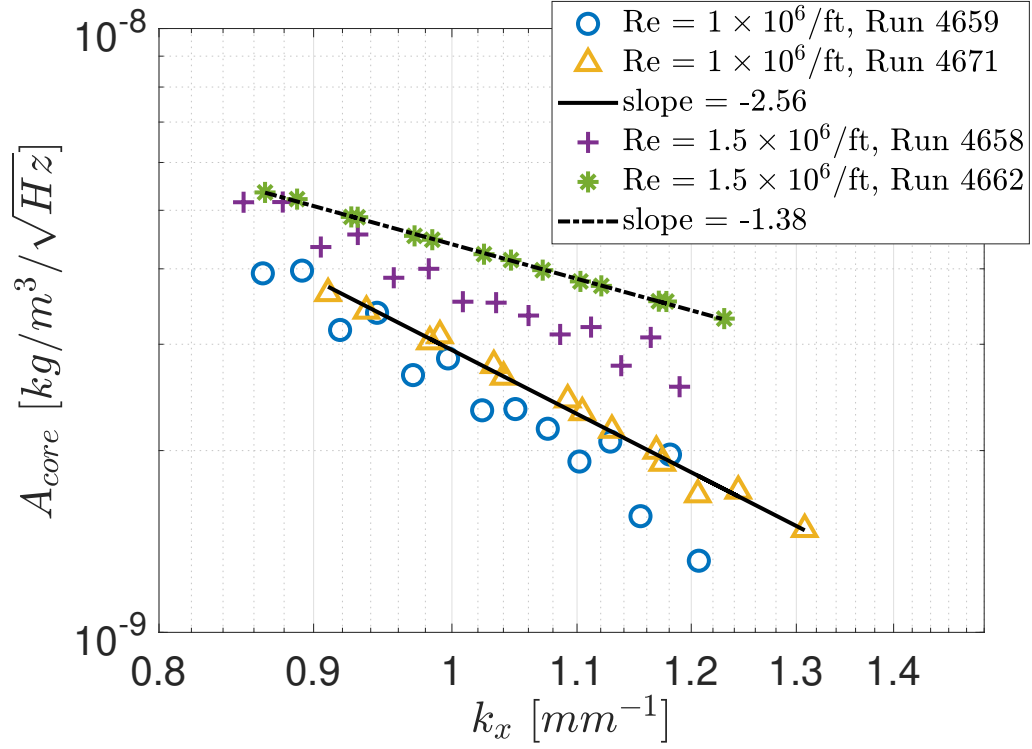


Figure 4.25: Amplitude spectra in T9M18 for four runs

The FLDI sensitivity function to uniform, flow parallel disturbances for this experiment is graphed in Fig. 4.24. Over the wavenumber band of interest ($0.9 \text{ mm}^{-1} < k_x < 1.3 \text{ mm}^{-1}$), the value of H_{core} is constant and near 80mm. This function is used along with the measured spectra in Eq. 4.5 to recover the true amplitudes of density fluctuations in the core flow of the Mach 18 test section. Amplitude spectral density, labeled A_{core} , is shown in Fig. 4.25 for four of the five runs. The signal measured during run 4667 does not have sufficient SNR at these high wavenumbers. Note that the amplitudes are on the order of 10^{-9} kg/m^3 which gives an idea of the lower limit of what FLDI can measure.

A linear (logarithmic scale) trend is observed in the data, and linear fits to

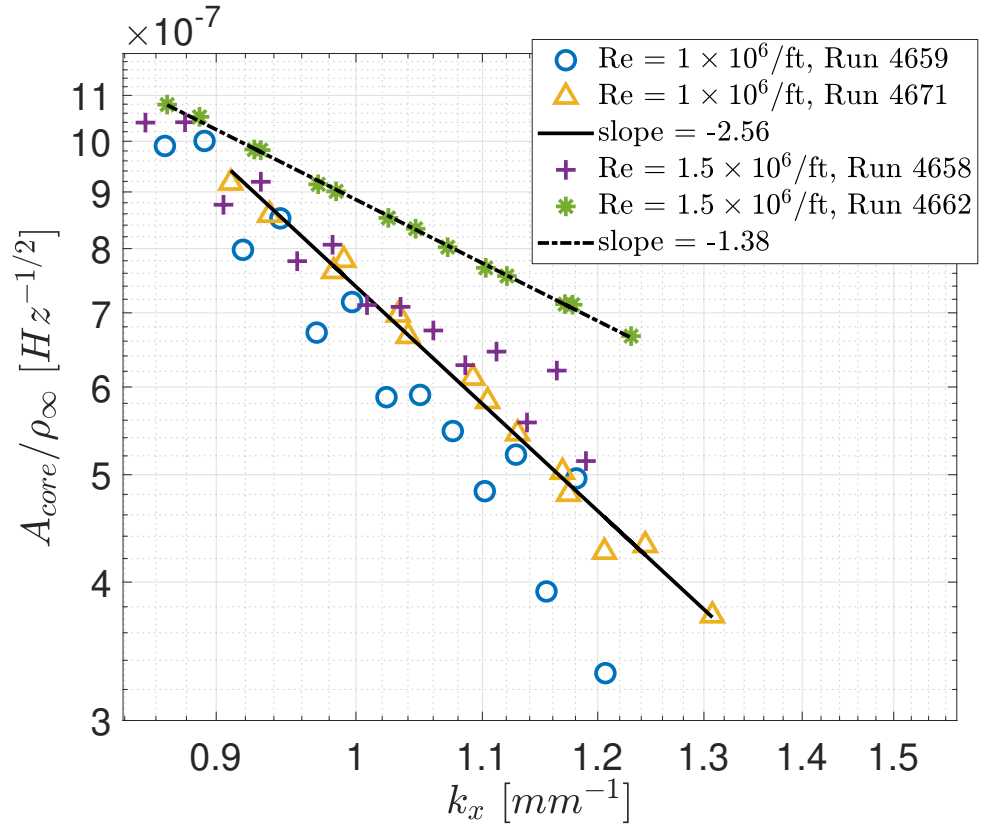


Figure 4.26: Amplitude spectra in T9M18 normalized by freestream density for four runs

the amplitudes measured in runs 4671 and 4662 yield $k_x^{-2.56}$ and $k_x^{-1.38}$, respectively. These linear fits are illustrated in Fig. 4.25 as solid and dashed lines. Fig. 4.26 gives the amplitude spectra normalized by respective freestream density, which is equivalent to a density-based turbulence intensity. Turbulence intensity is near agreement for all runs at $k_x = 0.9mm^{-1}$. As wavenumber increases, the measurements deviate. The data set is too small to draw any conclusions about how turbulence intensity varies with Reynolds number in this Mach 18 facility. Analysis of these amplitudes using transfer functions which account for acoustic waves at inclination angles with respect to freestream [49] is a good candidate for future work.

4.4 Chapter Conclusions and Takeaways

In this chapter, methods for interpreting and reducing the signal from 2pFLDI in high speed facilities are demonstrated. In UTSI's 61cm Mach 4 Ludwig tube, the 2pFLDI ($w_0 = 7\mu m$) can resolve wavenumbers larger than $0.5mm^{-1}$, corresponding to frequencies larger than $\sim 50kHz$, with 44cm depth of focus. In AEDC's Mach 18 tunnel, the 2pFLDI ($w_0 = 14\mu m$) can resolve wavenumbers larger than $0.9mm^{-1}$, corresponding to frequencies larger than $\sim 200kHz$, with 73cm depth of focus. A change in spectra slope is observed near these wavenumber and frequency cut-offs in both flows. Measurements of convection velocity as a function of frequency, i.e. the dispersion relation, are novel. Minimal dispersion (a few percent) is measured above 50kHz in the Mach 4 turbulent boundary layer and above 200kHz in T9 for runs 4671 and 4662. A freestream disturbance velocity near 75-82% of the mean velocity

is measured in the Mach 18 flow. Amplitude spectra of freestream disturbances are resolved over a $0.9mm^{-1} < k_x < 1.3mm^{-1}$ band using transfer functions based on the assumption that disturbances are uniform and flow parallel. The procedures and data reduction methods outlined here serve as a framework for future 2pFLDI and multi-point FLDI experiments.

Chapter 5: Summary and Future Work

In this chapter we review and summarize the contributions of this thesis which can be grouped into three main categories:

- Development of analytical transfer function methods for interpreting FLDI signals including a method for determining spatiotemporal resolution.
- Validation and quantification of the accuracy of transfer function methods in two ‘canonical’ flows: a turbulent jet and a Mach 3 wind tunnel.
- Demonstration of FLDI and 2pFLDI in high speed facilities including signal interpretation methods and the first ever nonintrusive measurements of freestream disturbances in a Mach 18 flow.

After these conclusions, future avenues for FLDI development and application are suggested.

5.1 Interpreting FLDI

Chapter 2 gives an overview of focused laser differential interferometry theory. The instrument is modeled by ray tracing the beams. Assuming that the beams are Gaussian and that disturbances are sinusoidal along a known propagation direction

enables significant analytical reduction of the model. Assuming that disturbances propagate parallel to the flow direction yields two transfer functions, $H_{\Delta x}$ and H_w which account for the ways in which beam separation and beam size affect the measured signal, respectively. While the foundational work on FLDI transfer functions was performed by others [32, 33, 49], the unique contribution of this work is the development of a method for accounting for spatiotemporal variations in disturbance amplitude and velocity via the parameters $g(z)$ and $u_c(z, f)$. Also unique in this work is a procedure for determining FLDI spatiotemporal resolution from the transfer functions. These methods provide simple and easy to use standards for future FLDI experiments.

5.2 Validating the FLDI model

Chapter 3 describes experiments in two canonical flows - a turbulent jet and Mach 3 flow in a small wind tunnel - that quantify the accuracy of the transfer function model. Specifically, we show how instrument parameters, Δx_1 and $w(z)$, affect the measurement. We also present a method for determining the beam radius parameter at the focus (w_0) using a beam profiler camera and show that accurately measuring w_0 is critical for application of the transfer functions.

For our model of the jet, convection velocity u_c is varied spatially ($u_c(z)$) based on historical jet studies or temporally ($u_c(f)$) based on 2pFLDI measurements. For the z-dependent case, the z-axis integral of the transfer functions is resolved numerically. Both cases yield errors between model and measurement on the order

of 10-50% when the jet is translated along the optical axis. This suggests we have a good understanding of the way in which FLDI rejects signal away from its focal region. The advantage of $u_c(f)$ as opposed to $u_c(z)$ is that it can be measured directly by 2pFLDI, enabling signal analysis in flows where historical measurements are not available. This is demonstrated in the Mach 3 tunnel which also yields errors between model and measurement on the order of 10-50% when the FLDI is translated along the optical axis through the tunnel. These successful applications of the the simple FLDI model in the complex jet and tunnel flows represent a significant validation of the model.

5.3 Demonstrations of 2pFLDI

Chapter 4 details the application and signal analysis of 2pFLDI measurements made in two separate campaigns. The first is in University of Tennessee Space Institute’s Mach 4 Ludwig tube (UTM4) and the second is in AEDC T9’s Mach 18 test section (T9M18). In each campaign, a procedure for determining the FLDI spatiotemporal resolution and interpreting the signal is described. This procedure sets an example for future FLDI experiments to follow, thus it is an important contribution. Transfer functions illustrate how FLDI achieves a spatial resolution of $\mathcal{O}(50cm)$ to disturbance wavenumbers greater than $\mathcal{O}(1mm^{-1})$, corresponding to frequencies greater than $\mathcal{O}(100kHz)$ in these facilities. With this context, measurements of turbulence intensity and convection velocity in the floor boundary layer of the Mach 4 tunnel are presented. Measurements of freestream disturbances in

T9’s Mach 18 test section are reported as well. Successfully applying 2pFLDI in this large-scale industrial setting is a significant achievement, and the results represent a contribution to the understanding of disturbance convection velocity in this hypersonic facility.

5.4 Future Research

One avenue of future research is to explore alternatives to the flow parallel (Eq. 2.11) and acoustic wave (Eq. 2.14) disturbance models employed in this work and by Lawson [49], respectively. Recent work by Hameed and Parziale [48] offers an example of a model which accounts for non-uniform disturbances with non flow parallel inclinations angles. Investigating alternative models will lead to different forms of the transfer functions which may be more accurate, depending on the specific flow of interest. Alternative models can be validated using the same experiments and procedures employed in chapter 3.

The FLDI depth of focus is improved by increasing the convergence angle of the beams which in turn reduces w_0 . The smallest w_0 achieved in this work is on the order of 3 microns. In theory, this could be reduced to as small as twice the laser wavelength [54]. A study which seeks to maximize FLDI’s spatial resolution by employing very large, short focal length field lenses would illuminate the limitations of the technique.

Finally, the 2pFLDI velocity measurement does not need to share the signal-point FLDI measurement spatial resolution. An optical arrangement where the two

FLDI optical axes are not parallel, but the focus still coincide along the x-axis could yield velocity measurements with significantly higher spatial resolution. Similar to a cross-beam Schlieren set-up [31], a cross-beam FLDI could yield measurements of dispersion with spatial resolution on the order of the beam pair separation, Δx_2 .

Appendix A: Derivations

A.1 Derivation of FLDI transfer functions, Eq.2.24

Beginning with the argument of the z-axis integral in Eq.2.23, which we will call Z , we have:

$$Z = \int_0^{2\pi} \int_0^\infty I_0(r, \theta) \rho(\mathbf{x}_1, t) r dr d\theta - \int_0^{2\pi} \int_0^\infty I_0(r, \theta) \rho(\mathbf{x}_2, t) r dr d\theta \quad (\text{A.1})$$

Plug in the definition of beam intensity profile I_0 (Eq.2.18), the density field model $\rho(x, y, z, t)$ (Eq.2.13) and the definitions of the beam paths $\mathbf{x}_1$ and $\mathbf{x}_2$ (Eq.2.21 and Eq.2.22):

$$\begin{aligned} Z = & \int_0^{2\pi} \int_0^\infty \frac{2}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) g(z) \int_{-\infty}^\infty A(f) \exp\left[i\left(k_x\left(x + \frac{\Delta x_1}{2}\right) - 2\pi f t + \psi\right)\right] df r dr d\theta - \\ & \int_0^{2\pi} \int_0^\infty \frac{2}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) g(z) \int_{-\infty}^\infty A(f) \exp\left[i\left(k_x\left(x - \frac{\Delta x_1}{2}\right) - 2\pi f t + \psi\right)\right] df r dr d\theta \end{aligned} \quad (\text{A.2})$$

Next, exchange the order of integration and re-arrange some terms:

$$\begin{aligned}
Z = & \\
& g \int_{-\infty}^{\infty} A \int_0^{\infty} \frac{2r}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) \int_0^{2\pi} \exp\left[i\left(k_x\left(x + \frac{\Delta x_1}{2}\right) - 2\pi ft + \psi\right)\right] d\theta dr df - \\
& g \int_{-\infty}^{\infty} A \int_0^{\infty} \frac{2r}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) \int_0^{2\pi} \exp\left[i\left(k_x\left(x - \frac{\Delta x_1}{2}\right) - 2\pi ft + \psi\right)\right] d\theta dr df
\end{aligned} \tag{A.3}$$

Re-arrange more terms and convert from Cartesian to polar coordinates using $x = r \cos(\theta)$:

$$\begin{aligned}
Z = & g \int_{-\infty}^{\infty} A \times \\
& \left\{ \exp\left[i\left(k_x\left(+\frac{\Delta x_1}{2}\right) - 2\pi ft + \psi\right)\right] \int_0^{\infty} \frac{2r}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) \int_0^{2\pi} \exp(ik_x r \cos \theta) d\theta dr - \right. \\
& \left. \exp\left[i\left(k_x\left(-\frac{\Delta x_1}{2}\right) - 2\pi ft + \psi\right)\right] \int_0^{\infty} \frac{2r}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) \int_0^{2\pi} \exp(ik_x r \cos \theta) d\theta dr \right\} df
\end{aligned} \tag{A.4}$$

Re-arrange terms and evaluate the integral over θ :

$$\begin{aligned}
Z = & g \int_{-\infty}^{\infty} A \exp[i(-2\pi ft + \psi)] \left(\exp\left[\frac{ik_x \Delta x_1}{2}\right] - \exp\left[\frac{-ik_x \Delta x_1}{2}\right] \right) \times \\
& \int_0^{\infty} \frac{2r}{\pi w^2} \exp\left(-\frac{2r^2}{w^2}\right) 2\pi J_0(k_x r) dr df \tag{A.5}
\end{aligned}$$

where J_0 is the Bessel function of the first kind. Next, we evaluate the subtraction of the two exponential terms and evaluated the integral over beam radius, r . For more detail on this integral evaluation, see the next subsection, appendix [A.1.1](#).

$$Z = g \int_{-\infty}^{\infty} A \exp[i(-2\pi ft + \psi)] 2i \sin\left(\frac{k_x \Delta x_1}{2}\right) \exp\left(-\frac{k_x^2 w^2}{8}\right) df. \tag{A.6}$$

Finally, re-arrange to get the argument of the z-axis integral in Eq. [2.24](#).

A.1.1 Evaluation of integral over beam radius, r

Start with the integral over r in Eq.A.5, which we will call $\mathcal{R}$:

$$\mathcal{R} = \int_0^\infty \frac{4r}{w^2} \exp\left(-\frac{2r^2}{w^2}\right) J_0(k_x r) dr \quad (\text{A.7})$$

Using the definition of the Bessel function of the first kind, J_0 :

$$\mathcal{R} = \frac{4}{w^2} \int_0^\infty r \exp\left(-\frac{2r^2}{w^2}\right) \left[\sum_{n=0}^\infty \frac{(-k_x^2 r^2/4)^n}{(n!)^2} \right] dr \quad (\text{A.8})$$

Expand out the sum:

$$\mathcal{R} = \frac{4}{w^2} \int_0^\infty r \exp\left(-\frac{2r^2}{w^2}\right) \left[1 - \frac{k_x^2 r^2}{4} + \frac{k_x^4 r^4}{64} - \dots \right] dr \quad (\text{A.9})$$

Re-arrange:

$$\mathcal{R} = \frac{4}{w^2} \left[\int_0^\infty r \exp\left(-\frac{2r^2}{w^2}\right) dr - \frac{k_x^2}{4} \int_0^\infty r^3 \exp\left(-\frac{2r^2}{w^2}\right) dr + \frac{k_x^4}{64} \int_0^\infty r^5 \exp\left(-\frac{2r^2}{w^2}\right) dr - \dots \right] \quad (\text{A.10})$$

Evaluate the integrals over r :

$$\mathcal{R} = \frac{4}{w^2} \left[\frac{w^2}{4} - \frac{k_x^2}{4} \left(\frac{w^4}{8} \right) + \frac{k_x^4}{64} \left(\frac{w^6}{8} \right) - \dots \right] \quad (\text{A.11})$$

Finally, we have:

$$\mathcal{R} = 1 - \frac{k_x^2 w^2}{8} + \frac{k_x^4 w^4}{128} - \dots \quad (\text{A.12})$$

Recognizing the Taylor's series expansion, we have

$$\mathcal{R} = \exp\left(-\frac{k_x^2 w^2}{8}\right). \quad (\text{A.13})$$

Bibliography

- [1] Eli Reshotko. Transition Issues for Atmospheric Entry. *Journal of Spacecraft and Rockets*, 2008.
- [2] John D. Anderson. *Hypersonic and High-Temperature Gas Dynamics*. American Institute of Aeronautics and Astronautics, Reston ,VA, 2nd edition, 2006.
- [3] M. V. Morkovin, E. Reshotko, and T. Herbert. Transition in Open Flow Systems: A Reassessment. *47th Annual Meeting of the Division of Fluid Dynamics*, 39, 1994.
- [4] Eli Reshotko. Boundary layer instability, transition and control. In *32nd Aerospace Sciences Meeting and Exhibit*, 1994.
- [5] S. R. Pate and C. J. Schueler. Radiated aerodynamic noise effects on boundary-layer transition in supersonic and hypersonic wind tunnels. *AIAA Journal*, 7(3):450–457, 1969.
- [6] Steven P. Schneider. Effects of High-Speed Tunnel Noise on Laminar-Turbulent Transition. *Journal of Spacecraft and Rockets*, 38(3):323–333, 2001.
- [7] L. S. G. Kovasznay. Turbulence in Supersonic Flow. *Journal of Aeronautical Sciences*, 20(10):657–674, 1953.
- [8] Mark V. Morkovin. On Transition Experiments at Moderate Supersonic Speeds. *Journal of the Aeronautical Sciences*, 24(7):480–486, jul 1957.
- [9] J. Laufer. Factors Affecting Transition Reynolds Numbers on Models in Supersonic Wind Tunnels. *Journal of the Aeronautical Sciences*, 21(7), 1954.
- [10] J. Laufer. Aerodynamic noise in supersonic wind tunnels. *Journal of the Aerospace Sciences*, 28(9):685–692, 1961.
- [11] John Laufer. Some Statistical Properties of the Pressure Field Radiated by a Turbulent Boundary Layer. 1191, 1964.

- [12] Steven P. Schneider. Developing mechanism-based methods for estimating hypersonic boundary-layer transition in flight: The role of quiet tunnels. *Progress in Aerospace Sciences*, 2015.
- [13] P. Stainback and R. Wagner. A comparison of disturbance levels measured in hypersonic tunnels using a hot-wire anemometer and a pitot pressure probe. In *7th Aerodynamic Testing Conference*, Reston, Virginia, sep 1972. American Institute of Aeronautics and Astronautics.
- [14] John Lafferty and Joseph Norris. Measurements of Fluctuating Pitot Pressure, "Tunnel Noise," in the AEDC Hypervelocity Wind Tunnel No. 9. In *2007 U.S. Air Force T&E Days*, Reston, Virginia, feb 2007. American Institute of Aeronautics and Astronautics.
- [15] Alex Bounitch, Daniel Lewis, and John Lafferty. Improved Measurements of "Tunnel Noise" Pressure Fluctuations in the AEDC Hypervelocity Wind Tunnel No. 9. *49th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, (9):1–12, 2011.
- [16] Davide Masutti, Emanuele Spinosa, Olivier Chazot, and Mario Carbonaro. Disturbance level characterization of a hypersonic blowdown facility. *AIAA Journal*, 50(12):2720–2730, 2012.
- [17] Yu V Gromyko, P A Polivanov, A A Sidorenko, and D A Buntin. OF THERMOPHYSICS AND PHYSICAL HYDRO- An experimental study of the natural noise in the Transit-M hypersonic wind tunnel. 20(4):13–16, 2013.
- [18] Thomas Schilden, Wolfgang Schröder, Syed Raza Christopher Ali, Anne Marie Schreyer, Jie Wu, and Rolf Radespiel. Analysis of acoustic and entropy disturbances in a hypersonic wind tunnel. *Physics of Fluids*, 2016.
- [19] Ross S. Chaudhry and Graham V. Candler. Computing Measured Spectra from Hypersonic Pitot Probes with Flow-Parallel Freestream Disturbances. *AIAA Journal*, 2017.
- [20] Lian Duan, Meelan M. Choudhari, Amanda Chou, Federico Munoz, Rolf Radespiel, Thomas Schilden, Wolfgang Schröder, Eric C. Marineau, Katya M Casper, Ross S. Chaudhry, Graham V. Candler, Kathryn A Gray, and Steven P Schneider. Characterization of Freestream Disturbances in Conventional Hypersonic Wind Tunnels. *Journal of Spacecraft and Rockets*, 56(2):357–368, mar 2019.
- [21] P M Danehy, J Weisberger, C Johansen, D. Reese, T. Fahringer, N. J. Paraziale, C. Dedic, J. Estevadeordal, and B. A. Cruden. Non-Intrusive Measurement Techniques for Flow Characterization of Hypersonic Wind Tunnels. pages 1–140, 2019.
- [22] Eric C. Marineau. Prediction Methodology for Second-Mode-Dominated Boundary-Layer Transition in Wind Tunnels. *AIAA Journal*, 2017.

- [23] N. J. Parziale, J. E. Shepherd, and H. G. Hornung. Free-stream density perturbations in a reflected-Shock tunnel. *Experiments in Fluids*, 55(2), 2014.
- [24] G. Smeets and A. George. Gas-Dynamic Investigations in a Shock Tube using a Highly Sensitive Interferometer. 298(0704), 1971.
- [25] G. Smeets. Laser Interferometer for High Sensitivity Measurements on Transient Phase Objects. *IEEE Transactions on Aerospace and Electronic Systems*, AES-8(2):186–190, 1972.
- [26] G. Smeets and A. George. Laser-Differential Interferometer Applications in Gas Dynamics. 298(0704), 1973.
- [27] G. Smeets. Flow Diagnostics by Laser Interferometry. *IEEE Transactions on Aerospace and Electronic Systems*, 13(2), 1977.
- [28] Nicholas J Parziale, J E Shepherd, and Hans G. Hornung. Reflected Shock Tunnel Noise Measurement by Focused Differential Interferometry. *42nd AIAA Aerospace Sciences Meeting*, (June):1–10, 2012.
- [29] N. J. Parziale, J. E. Shepherd, and H. G. Hornung. Differential Interferometric Measurement of Instability in a Hypervelocity Boundary Layer. *AIAA Journal*, 2013.
- [30] Nick Parziale. *Slender-Body Hypervelocity Boundary-Layer Instability*. PhD thesis, California Institute of Technology, 2013.
- [31] Matthew R Fulghum. *Turbulence Measurements in High-Speed Wind Tunnels using Focusing Laser Differential Interferometry*. PhD thesis, The Pennsylvania State University, 2014.
- [32] Gary S. Settles and Matthew R. Fulghum. The Focusing Laser Differential Interferometer, an Instrument for Localized Turbulence Measurements in Refractive Flows. *Journal of Fluids Engineering*, 2016.
- [33] B. E. Schmidt and J. E. Shepherd. Analysis of focused laser differential interferometry. *Applied Optics*, 2015.
- [34] Elizabeth K. Benitez, Joseph S. Jewell, and Steven P. Schneider. Focused laser differential interferometry for hypersonic flow instability measurements with contoured tunnel windows. *AIAA Journal*, 1 PartF:1–22, 2020.
- [35] Alec Houpt and Sergey Leonov. Cylindrical Focused Laser Differential Interferometer. *AIAA Journal*, pages 1–9, 2021.
- [36] Byrenn Birch, David Buttsworth, and Fabian Zander. Measurements of freestream density fluctuations in a hypersonic wind tunnel. *Experiments in Fluids*, 61(7):1–13, 2020.

- [37] Joseph S. Jewell, Ahsan Hameed, Nick J. Parziale, and Sivaram P. Gogineni. Disturbance Speed Measurements in a Circular Jet via Double Focused Laser Differential Interferometry. *AIAA Scitech 2019 Forum*, (January), 2019.
- [38] Brett F. Bathel, Joshua M. Weisberger, Gregory C. Herring, Rudolph A. King, Stephen B. Jones, Richard E. Kennedy, and Stuart J. Laurence. Two-point, parallel-beam focused laser differential interferometry with a Nomarski prism. *Applied Optics*, 59(2):244, 2020.
- [39] Andrew Ceruzzi and Christopher P. Cadou. Simultaneous Velocity and Density Gradient Measurements using Two-Point Focused Laser Differential Interferometry. *AIAA Scitech 2019 Forum*, (January), 2019.
- [40] Andrew P Ceruzzi, Braeden O Callis, and Daniel C Weber. Application of Focused Laser Differential Interferometry (FLDI) in a Supersonic Boundary Layer. In *AIAA Scitech Forum*, pages 1–6, 2020.
- [41] Andrew P Ceruzzi, Brianne McManamen, and Christopher P. Cadou. Demonstration of Two-Point Focused Laser Differential Interferometry (2pFLDI) in a Mach 18 flow. In *AIAA Scitech 2021 Forum*, number January, pages 1–14, 2021.
- [42] Mark Gragston, Theron Price, Kirk Davenport, Zhili Zhang, and John D. Schmisser. Linear array focused-laser differential interferometry for single-shot multi-point flow disturbance measurements. *OPTICS LETTERS*, 46(1):154–157, 2021.
- [43] Mark Gragston, Theron J Price, Kirk Davenport, and D John. An $m \times n$ FLDI Array for Single-Shot , Multipoint Disturbance Measurements in High-Speed Flows. *AIAA Scitech 2021 Forum*, (January):1–16, 2021.
- [44] Joshua M Weisberger, Brett F Bathel, Gregory C Herring, and Gregory M Buck. Two-Line Focused Laser Differential Interferometry of a Flat Plate Boundary Layer at Mach 6. In *AIAA Scitech 2021 Forum*, number January, pages 1–29, 2021.
- [45] Joel M. Lawson, Mallory C. Neet, Ilan J. Grossman, and Joanna M. Austin. Static and dynamic characterization of a focused laser differential interferometer. *Experiments in Fluids*, 2020.
- [46] Brett F Bathel, Joshua M Weisberger, Gregory C Herring, Rangesh Jagannathan, Craig T. Johansen, Stephen B. Jones, and Angelo A. Cavone. Analysis of the Amplitude Response of a Two-Point and a Multi-Point Focused Laser Differential Interferometer. In *AIAA Scitech 2021 Forum*, number January, pages 1–17, 2021.
- [47] A. Hameed and N. J. Parziale. Focused Laser Differential Interferometer Response to a Controlled Phase Object. In *AIAA Scitech 2021 Forum*, number January, pages 1–10, 2021.

- [48] Ahsan Hameed and Nick J. Parziale. Focused Laser Differential Interferometry Transfer Functions for Complex Density Disturbance Fields. In *AIAA Aviation*, number 3, pages 1–14, 2021.
- [49] Joel Michael Lawson. *Focused Laser Differential Interferometry*. PhD thesis, California Institute of Technology, 2021.
- [50] Max Born and Emil Wolf. *Principles of Optics*. Pergamon Press, New York, 6th edition, 1980.
- [51] H. W. Liepmann and A. Roshko. *Elements of Gasdynamics*. John Wiley & Sons, Inc., New York, 1 edition, 1957.
- [52] J. H. Gladstone and T. P. Dale. Researches on the Refraction, Dispersion and Sensitiveness of Liquids. 1863.
- [53] W Merzkirch. *Techniques of flow visualization*. 1987.
- [54] Anthony E. Siegman. *Lasers*. University Science Books, 1986.
- [55] Mallory C. Neet, Joel M. Lawson, and Joanna M. Austin. Design, alignment, and calibration of a focused laser differential interferometer. *Applied Optics*, 60(26):7903, 2021.
- [56] James M. Wallace. Space-time correlations in turbulent flow: A review. *Theoretical and Applied Mechanics Letters*, 4(2):022003, 2014.
- [57] G. I. Taylor. The spectrum of turbulence. *Proceedings of the Royal Society*, 164(919), 1938.
- [58] C. C. Lin. On Taylor’s Hypothesis and the Acceleration Terms in the Navier-Stokes Equations. *Quarterly of Applied Mathematics*, 10(4):295–306, 1953.
- [59] A. Favre, J. Gaviglio, and R. Dumas. Structure of velocity space-time correlations in a boundary layer. *Physics of Fluids*, 10(S138), 1967.
- [60] Marc A. Kolpin. The flow in the mixing region of a jet. *Journal of Fluid Mechanics*, 18(4):529–548, 1964.
- [61] J. A.B. Wills. On convection velocities in turbulent shear flows. *Journal of Fluid Mechanics*, 20(3):417–432, 1964.
- [62] Graeme Gillespie, Andrew Ceruzzi, and Stuart Laurence. Multi-point Focused Laser Differential Interferometry for Noise Measurements in High-Speed Tunnels. 2021.
- [63] Joel M. Lawson and Joanna M. Austin. Expansion Tube Freestream Disturbance Measurements using a Focused Laser Differential Interferometer. (January):1–16, 2020.

- [64] J. O. Hinze and B. G. Van Der Hegge Zijnen. Transfer of heat and matter in the turbulent mixing zone of an axially symmetrical jet. *Applied Scientific Research*, 1(1):435–461, 1949.
- [65] I. Wygnanski and H. Fiedler. Some measurements in the self-preserving jet. *Journal of Fluid Mechanics*, 38(03):577, 1969.
- [66] Peter O Witze. Centerline Velocity Decay of Compressible Free Jets. *AIAA Journal*, 12(4):417–418, 1974.
- [67] Ching Jen Chen and Wolfgang Rodi. *Vertical turbulent buoyant jets: a review of experimental data*, volume 80. Pergamon Press, 1980.
- [68] S. R. Sanderson. Simple, adjustable beam splitting element for differential interferometers based on photoelastic birefringence of a prismatic bar. *Review of Scientific Instruments*, 76(11):113703, 2005.
- [69] Peter D Welch. The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms $I_k(f_n) = I_A h(\%) [a.k]$. Technical Report 2, 1967.
- [70] O. R.H. Buxton, R. de Kat, and B. Ganapathisubramani. The convection of large and intermediate scale fluctuations in a turbulent mixing layer. *Physics of Fluids*, 25(12), 2013.
- [71] Ting-Chung Poon and Taegeun Kim. *Engineering Optics with MATLAB*. World Scientific Publishing Co., Singapore, 2006.
- [72] ASME. Test Uncertainty. Technical report, 2006.
- [73] Jonathan M. Brooks, Ashwani K. Gupta, Michael Smith, and Eric C. Marineau. Development of Particle Image Velocimetry in a Mach 2.7 Wind Tunnel at AEDC White Oak. In *53rd AIAA Aerospace Sciences Meeting*, 2015.
- [74] Chao Zhang and Lian Duan. Direct Numerical Simulation Database for Supersonic and Hypersonic Turbulent Boundary Layers. *AIAA Journal*, 56(11), 2018.
- [75] Dan Marren and John Lafferty. The AEDC Hypervelocity Wind Tunnel 9. In *Advanced Hypersonic Test Facilities*, pages 467–478. American Institute of Aeronautics and Astronautics, Reston ,VA, jan 2002.
- [76] Inna Kurits, Nicholas J. Fredrick, and John F. Lafferty. Initial calibration of the mach 18 capability at the aedc hypervelocity wind tunnel no. 9. *AIAA Scitech 2021 Forum*, (January):1–23, 2021.
- [77] John J. Korte, John F. Lafferty, Michael S. Smith, Daniel R. Lewis, Cameron W. Butcher, Nicholas J. Fredrick, Mariusz Zarzecki, and Kenneth E. Tatum. Determination of hypervelocity freestream conditions for a vibrationally frozen nitrogen flow. *AIAA Scitech 2021 Forum*, (January):1–20, 2021.

- [78] John F Lafferty and Joseph D Norris. Measurements of Fluctuating Pitot Pressure, "Tunnel Noise," in the AEDC Hypervelocity Wind Tunnel No. 9 *.