

ABSTRACT

Title of Dissertation: FREQUENCY AND VOLTAGE STABILITY
OF POWER SYSTEMS WITH HIGH
PENETRATION OF RENEWABLE
SOURCES

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The growing integration of converter-interfaced renewable energy sources has fundamentally transformed modern power systems, leading to a significant decrease in system inertia and presenting urgent challenges for frequency control. Distributed energy resources, which represent a substantial portion of the emerging power generation landscape, are anticipated to play a vital role in facilitating frequency regulation in future low-inertia systems. Nevertheless, their effectiveness is still limited due to constraints in active and reactive power generation from droop control, varying response capabilities, and communication difficulties. This dissertation explores the application of reinforcement learning control as a pivotal technology to overcome these challenges by providing real-time active and reactive power generation from distributed energy resources and predictive capabilities for distributed energy generation. This approach enables the development of new frequency service dispatch and control schemes for future power systems.

Three control strategies for power systems and microgrids are proposed, namely modified finite control set model predictive control, Heaviside Pontryagin's optimal control, and reinforcement learning, each offering unique benefits for the dynamic control of distributed energy resource behavior.

The suggested methodologies consider the fidelity and efficiency of control for distributed energy resources applications, and the control strategies developed are the first of their kind to facilitate real-time management of frequency and active power responses in distributed energy resources. Furthermore, a new strategy for managing rapid frequency control is introduced, which is based on the optimization of the controller or goal function, leading to a significant enhancement in the robustness of frequency and voltage control within microgrids and power systems. Simulation experiments were performed using MATLAB and Simulink. The results of the tests validate that the proposed control strategies can effectively manage distributed energy resources in both microgrids and power systems.

The microgrid control methods based on neural networks within leader-follower configurations allow the system to evaluate the leader-follower responses of distributed energy resources, while also pinpointing the discrepancies between anticipated and actual time differences error in real-time. Simulation outcomes indicate that the suggested reinforcement learning control framework minimizes control errors and improves system resilience during frequency disturbances.

A coordinated control scheme based on an energy internet is proposed to further improve the frequency and voltage control capabilities of distributed energy resources. Both centralized (cloud-hosted) and distributed (edge-hosted) architectures are examined to overcome local area voltage and frequency control. The energy internet-based coordinated control scheme facilitates traditional coordinated control of distributed energy resources by eliminating communication delays in the optimal power flow transmission of distributed energy resources. Simulation results confirm the effectiveness of this approach, demonstrating enhancements over conventional control regarding speed, overshoot, and accuracy of active power responses. The power system and microgrid frequency are supported more effectively by coordinated distributed energy resources, leading to increased frequency stability during potential events.

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HIGH PENETRATION OF RENEWABLE SOURCES

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List of Abbreviations

ADP	Adaptive Dynamic Programming
AI	Artificial Intelligence
BESS	Battery Energy Storage System
CEM	Cloud Energy Manager
DERs	Distributed Energy Resources
DG	Distributed Generation
DP	Dynamic Programming
DR	Demand Response
EMS	Energy Management System
ES	Energy Storage
ES-PV	Energy Storage and Photovoltaic
FA-PI	Fixed Admissible Policy Iteration
FCS-MPC	Finite Control Set Model Predictive Control
FFC	Fast Frequency Control
FFR	Fast Frequency Response
FMS	Frequency Management System
FSM	Finite State Machine
HESS	Hybrid Energy Storage System
HJB	Hamilton Jacobi Bellman
HJI	Hamilton-Jacobi-Isaacs
HVDC	High-Voltage-DC
HVS-PMP	Heaviside Pontryagin's Minimum Principle
IRC	Integral Reinforcement Control
IRL	Integral Reinforcement Learning
ISS	Input-to-State Stable
LA	Load Aggregator
LFC	Load Frequency Control
LPV	Linear Parameter Varying
MAE	Mean Absolute Error
MDP	Markov Decision Process
MPC	Model Predictive Control
NN	Neural Network
NPC	Neutral Point Clamped
OITC	Optimal Inverter Transmit Control
PI	Policy Iteration
PnP	Plug-And-Play
PV	Photovoltaic
RES	Renewable Energy Sources
ROCOF	Rate of Change of Frequency
RL	Reinforcement Learning
SC	Supercapacitor
SG	Synchronous Generator
SMG	Smart Micro-Grid
SOC	State of Charge

SW	Switch
TD	Temporal Difference
VSC	Voltage Source Converter
VSG	Virtual Synchronous Generator
VSI	Voltage Source Inverter
VSM	Virtual Synchronous Machine

Chapter 1. Introduction

1.1 Research Context

The global energy landscape is experiencing unprecedented changes, primarily driven by decarbonization targets aimed at reducing greenhouse gas emissions [1]. In April 2020, the US Clean Energy States Alliance (CESA) introduced the 100% clean energy collaborative program, designed to assist U.S. states and other entities in achieving their ambitious objectives of transitioning to 100% clean electricity. This is accomplished by facilitating information exchange, sharing best practices, and addressing technical challenges through webinars, publications, and resource libraries [2]. The program acts as a platform for states such as California, New York, and Washington to collaborate on policy development, program design, and implementation, thereby accelerating progress toward decarbonizing their power sectors. The primary focus of the program is to reduce emissions from electricity generation, followed by the displacement of fossil fuels in other sectors. This transition is reshaping the energy mix, resulting in a significant increase in the integration of renewable energy sources (RES), such as wind and solar, into electrical power systems. The growing share of converter-based renewables not only transforms the energy mix but also impacts the fundamental dynamics of the grid, with considerable implications for the control and stabilization of system frequency.

The Conventional frequency control primarily relies on large synchronous generators (SGs), which are mainly thermal power plants. These generators are usually fitted with turbine governors that adjust mechanical power input in response to frequency fluctuations, adhering to the droop characteristics [3].

The rotating machines also possess substantial kinetic energy within their rotors, which is essential in limiting the rate of change in frequency (ROCOF) immediately following a disturbance. In contrast to traditional thermal power plants, RES tend to be less controllable and are frequently intermittent and variable due to their reliance on weather conditions and seasonal changes. Furthermore, certain types of RES, such as solar and wind, are integrated into the power grid through power electronics converters, which do not naturally provide rotational inertia. Consequently, while the ongoing energy transition promotes the decarbonization agenda, it has also brought about significant operational challenges due to a dramatic decrease in system inertia throughout the power grid.

Low inertia can lead to a more rapid deviation in system frequency during power imbalance events, such as generation loss or demand fluctuations. This situation heightens the risk of exceeding the statutory frequency limits, which range from 59.5 to 60.5 Hz in the US power system, consequently increasing operational risks and costs [4]. Furthermore, numerous Distributed Energy Resources (DERs) integrated into the distribution systems are equipped with protection mechanisms that monitor the ROCOF. These systems are designed to automatically disconnect from the grid when the measured ROCOF surpasses established thresholds [5]. A reduction in system inertia will lead to a higher ROCOF during the same power imbalance event, potentially causing unexpected disconnections of DERs. This disconnection can result in cascading failures, further exacerbating the conditions of the system [6].

In conjunction with the extensive transformation of the transmission network, future power systems will be characterized by a swift rise in DERs. These encompass both small-scale synchronous generators, such as hydroelectric plants and gas turbines,

as well as converter-interfaced sources like solar energy, energy storage (ES), and wind generation. In the United States, approximately 33% of the total distribution-connected capacity is derived from renewable generation, largely attributable to the rapid installation of photovoltaic (PV) panels and wind turbines [7]. The capacity of DERs in the United States has increased by over 28% annually from 2009 to 2020 [8]. Furthermore, the DER market in the US is projected to nearly double in capacity between 2022 and 2027 [9]. This trend is anticipated to persist in the foreseeable future. The significant transformations in power systems, as outlined above, lead to the following key observations regarding frequency control: 1) traditional frequency control mechanisms, which depend on large thermal plants designed for centralized, inertia-rich networks, may no longer be effective or sufficient for future low-inertia systems. 2) DERs, owing to their rapidly growing numbers, have the potential to play an increasingly vital role in supporting the overall system, including frequency control. 3) A new framework and control solutions are necessary to enable DERs to participate more effectively in the frequency control of future grids.

However, there are several key challenges resulting from the increasing use of DERs in current and future power systems:

- Limited Controllability

DERs are generally spread throughout the distribution networks and are owned by various entities. Due to financial limitations, most systems do not possess adequate measurement devices or communication infrastructures to accurately

capture their real-time dynamic behavior [10]. Consequently, system operators frequently experience restricted real-time visibility regarding the status, location, and overall capability of DERs in delivering frequency control services. In contrast to large synchronous generators (SGs), the frequency control ancillary services provided by DERs are often facilitated through service aggregators. As a result, it becomes challenging for system operators to gain direct insight and control over the behaviors of these DERs. When implemented on a large scale, the aggregated behavior may deviate from the expectations of the system operators, leading to ineffective responses. This deficiency in observability further undermines the system operator's capacity to efficiently dispatch the DERs to fulfill the associated technical requirements.

➤ Dynamic Responses of DERs

DERs can vary significantly in their types and capabilities when it comes to responding to frequency events. For instance, both hydroelectric plants and gas turbines are classified as synchronous machines; however, they exhibit markedly different speeds of response and dynamic characteristics when it comes to increasing active power. Converter-interfaced resources also demonstrate a wide range of frequency response behaviors, both in comparison to SGs and among themselves, which is influenced by their source of supply and the internal control algorithms they employ. These control algorithms can differ greatly among manufacturers and may even vary within the same category of devices. Certain DERs might be set up to provide synthetic inertia [11], while

others could adhere to predefined control strategies that involve step changes, such as firm frequency response pre-agreed provider ability to adjust generation or load in response to frequency deviation [12], or ramp-based responses like dynamic containment [13]. Furthermore, a significant number of DERs currently do not respond to frequency fluctuations due to their limited control capabilities. Even among those that do offer support, allowing each to operate independently can result in fragmented and ineffective responses across the system. Coordinating a large array of DERs to achieve a cohesive and dependable frequency response continues to pose a significant challenge, primarily due to the complexities involved and the need for infrastructure upgrades.

➤ Communication Requirements

Coordinating DERs to ensure a rapid response for effective frequency regulation necessitates timely and precise measurements of the active power of the DERs, in addition to low-latency communication between the DERs and the control center. This presents significant challenges in practice, as numerous distribution-level assets are presently lacking high-resolution measurement devices or dependable communication links.

The emerging challenges require the necessity for innovative solutions that can enhance the controllability and visibility of DERs, as well as coordinate their varied

responses in real time. Such strategies are crucial for facilitating effective frequency response in future low inertia systems.

1.2 Research Contributions

While reinforcement learning and advance control technology have shown considerable promise in facilitating DERs to deliver efficient frequency control and has been successfully implemented across various industrial sectors, its use in power systems is still in a relatively nascent phase with only a few limited applications [14].

As will be elaborated in Chapter 2, the provision of frequency control services is increasingly reliant on energy storage (ES); however, current control methodologies do not adequately account for the dynamic behavior of hybrid ES concerning voltage and power constraints during rapid frequency response [15]. Chapter 2 proposes the Heaviside-Pontryagin Minimum Principle as a solution to address the voltage and power constraint limitations of hybrid energy storage systems within the context of fast frequency response control in microgrids. Chapter 2 discusses an optimal frequency control strategy for managing a hybrid energy storage system (HESS). The hybrid energy storage system comprises both batteries and supercapacitors. In this chapter, a novel approach is introduced that utilizes a Heaviside step function in conjunction with Pontryagin's minimum principle (HVS-PMP) to improve the microgrid's fast frequency response (FFR). This is achieved by transforming the supercapacitor voltage constrained region of the HESS into an unconstrained region control using HVS-PMP.

Chapter 3 indicates that the documented use of reinforcement learning in Diesel-engine generators is predominantly restricted due to the unpredictable nature of disturbances, fluctuations in load, and variations in mechanical parameters [16]. Furthermore, Chapter 3 suggests a model-free reinforcement Q-learning feedback control approach to overcome the existing challenges associated with the rapid frequency control of Diesel-engine generators in microgrids. Chapter 3 discusses the control of reinforcement frequency for the Diesel-engine generator within the microgrid. In the context of the smart grid, the Diesel-engine generator serves as a backup for frequency support and is commonly employed for rapid frequency response in the microgrid. Difficulties related with this important task are as follows. 1) Actual uncertainties of the mechanical parameters, 2) Occasion of uncertain disturbances which are coming from other control loops and electrical variables and uncertainty of electrical loads. The author introduces a novel approach to frequency control in the Diesel-engine generator by utilizing reinforcement learning control via continuous Q-learning.

Chapter 4 introduces microgrid inverter voltage fault ride-through problem. It is hard to control microgrid voltage fault ride-through due to significant rapid disturbance and PID controller performance limitations [17]. Chapter 4 suggests a model free exploration with coefficient update concurrent controller via using reinforcement learning with neural network approach to solve the voltage fault ride-through problem via actor-critic approach in the microgrid. Chapter 4 addresses the concept of policy iteration in relation to microgrids aimed at alleviating grid frequency issues. Within the microgrid,

the frequent fluctuations in disturbances complicate the management of voltage fault ride-through. The author suggests the implementation of advanced integral reinforcement learning utilizing Q-learning, which enables the microgrid inverter to effectively manage the voltage fault ride-through challenges present in the microgrid. Chapter 5 discusses the unpredictability associated with DERs and the challenges it poses for regulating system frequency in generator and load areas.

Chapter 5 recommends real-time management of both the governor and ES systems to aid in frequency recovery amidst the uncertainties of a microgrid. An integral reinforcement zero-sum game approach is proposed for controlling the governor and ES. A leader-follower distributed dynamic event-triggered control mechanism is proposed to address power systems with high penetration of DERs and hence reduced inertia achieve rapid convergence and synchronization of ES systems. Chapter 5 discusses the frequency control of microgrids through an event-triggered control mechanism that involves distributed loads and energy storage systems. This chapter introduces a method for controlling synchronous generators using governors alongside the control of energy storage systems within the microgrid. It employs an integral zero-sum game reinforcement learning approach to manage both the governor and the control of energy storage systems. Furthermore, this chapter proposes a dynamic event-triggered mechanism for the distributed control of energy storage systems via leader-follower approach, aimed at reducing the frequency of control command updates for these systems.

Chapter 6 suggests converting the model predictive control cost function into a linear quadratic cost function to computation overhead. Additionally, chapter 6 implements a geographical switch factor scheme that eliminates the need for a cost function, thus enabling swift frequency control via optimal switching methods during low inertia frequency disturbances in power systems. Chapter 6 discusses the agile frequency control of power systems utilizing voltage source inverters within a low inertia environment. Chapter 6 introduces an innovative method in which the model predictive control does not modify the parameters of the cost functions at each sampling interval. This approach reformulates the model predictive cost function into a linearized form to enhance computational efficiency. Furthermore, in the context of inverter control within the power system, the author utilizes a finite control set model predictive control strategy, specifically the SW (switch) factor scheme, to manage the voltage source inverter according to a geographical framework that obviates the necessity for a cost function. This advancement facilitates rapid frequency control through optimal switching techniques during low inertia frequency disturbances in power systems.

Chapter 7 discusses the limitation of the alternating direction method of multipliers (ADMM), specifically its lack of guaranteed convergence when addressing optimal power flow in power systems. Chapter 7 also introduces a steady state voltage security area to account for the interactions of optimal power flow that arise from the voltage off limit voltage between energy storage systems and photovoltaic inverters. To ensure guaranteed convergence and optimized power flow within a decentralized distributed energy management framework in local power systems, an energy internet

utilizing consensus control through all-to-all communication is established between central energy managers and inverters. Chapter 7 addresses the control of energy storage-photovoltaic (ES-PV) through energy internet based distributed optimal power flow within the local power system. The current optimal power flow method utilizing the ADMM presents a limitation due to its relatively low convergence rate. In this chapter, an energy internet based distributed optimal power flow is introduced, employing a central energy manager alongside distributed ES-PV residential inverter. Furthermore, Chapter 7 proposes a steady state voltage security area (SSVSA) for the calculation of voltage limits prior to the decentralized optimal power flow phase among the distributed ES-PV residential inverters. The proposed method demonstrates its ability to address the convergence rate issues associated with the ADMM.

In light of the challenges mentioned previously, this dissertation offers an extensive examination of the application of reinforcement learning and sophisticated control methods as the foundational approaches to address these problems. By developing reinforcement learning and advanced control strategies that can deliver real-time frequency and voltage regulation along with predictive functionalities for DERs, this research seeks to create innovative frequency service dispatch and control frameworks that empower DERs to deliver efficient frequency control services for low inertia systems.

1.3 Principal Contributions

The primary contributions of this research are as follows:

- The development and verification of two modeling strategies for reinforcement learning and advanced control of DERs, including physics-based and system identification methods, aimed at real-time frequency control objectives. These strategies enhance comprehension of the balance between modeling accuracy and the overall performance of reinforcement learning and advanced control systems in frequency control scenarios.
- The creation of a synchronized control strategy for DERs by leveraging the features of reinforcement learning alongside advanced control techniques. This proposed control strategy harmonizes DERs with varying attributes to deliver efficient frequency and voltage regulation.
- A supplementary approach has also been created to alleviate communication delays in reinforcement learning in conjunction with advanced control techniques. Implementing an event-triggered control strategy improves the adaptability of reinforcement learning while utilizing advanced control, all under minimal communication requirements.

1.4 Dissertation Overviews

The layout of this dissertation is illustrated in Fig. 1.1.

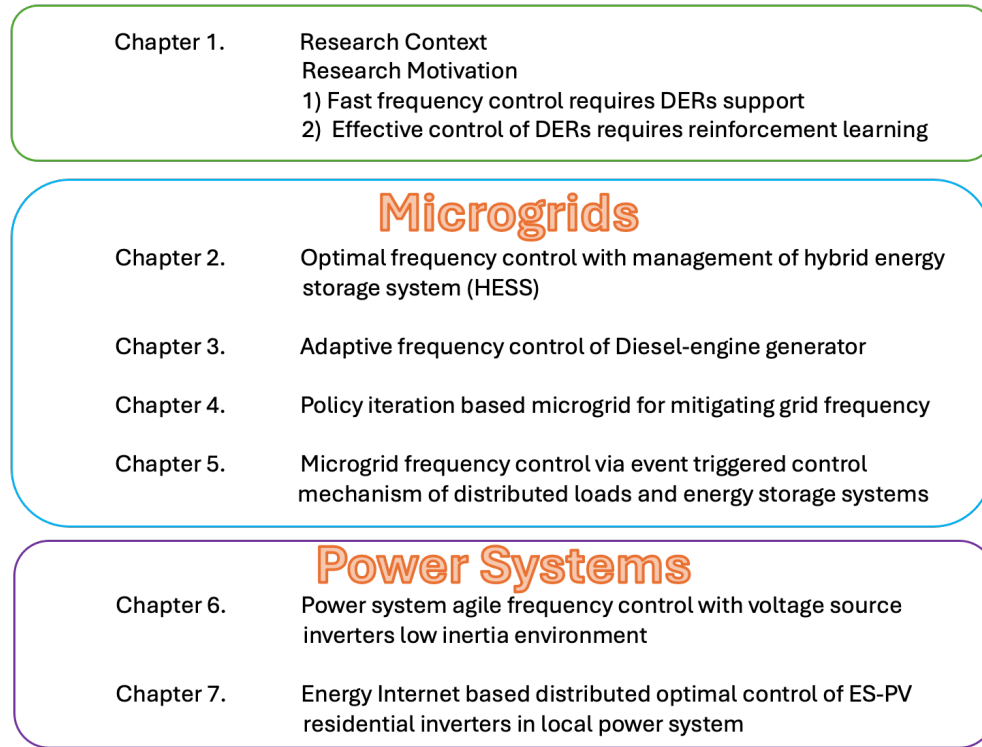


Figure 1.1 Structure of the dissertation

1.5 Publications

The publications listed below have emerged from the research presented in this dissertation:

List of Journal Papers

- Byungchul Kim, “Microgrid Frequency Control via Event Triggered Leader-follow Game of Distributed Energy Storage Systems and Generators,” in IEEE Journal of Emerging and Selected Topics in

Industrial Electronics, vol. 7, pp. 484-497, 12 January 2026. (related to Chapter 5)

- Byungchul Kim, “Cloud Energy Internet Based High Optimum Control of Residential ES-PV Inverters in Local Power Systems,” in IEEE Open Journal of Industry Applications, vol. 6, pp. 821-835, 17 November 2025. (related to Chapter 7)

List of Conference Papers

- Byungchul Kim, “Frequency Control in the Islanded Smart Microgrid Via Diesel-engine Generator,” IEEE 15th International Conference on Power, Energy, and Electrical Engineering (CPEEE), Fukuoka, Japan, Feb. 2025, pp. 280-285. (related to Chapter 3)
- Byungchul Kim, “Decentralized Distributed Oline Multiagent Systems Frequency Control,” IEEE 15th International Conference on Power, Energy, and Electrical Engineering (CPEEE), Fukuoka, Japan, Feb. 2025, pp. 307-311. (related to future work)
- Byungchul Kim, Eyad H. Abed, “Power System Fast Frequency Control in Low Inertia Environment using MPC,” IEEE 16th GreenTech, Springdale, USA, Apr. 2024, pp. 69-73. (related to Chapter 6)
- Byungchul Kim, “Frequency Management Strategy with Management of a Hybrid Energy Storage System in Island Microgrid,” IEEE PES General

Meeting, Seattle, USA, Jul. 2024, pp. 1-5. (related to Chapter 2)

- Byungchul Kim, Eyad H. Abed, “Policy Iteration Based Microgrid Frequency Control,” IEEE 3rd International Conference on Power Systems and Electrical Technology (PEST), Tokyo, Japan, Aug. 2024, pp. 582-587. (related to Chapter 4)
- Byungchul Kim, Eyad H. Abed, “Microgrid Frequency Management via Dynamic Event Control,” IEEE 3rd International Conference on Power Systems and Electrical Technology (PSET), Tokyo, Japan, Aug. 2024, pp. 613-618. (related to Chapter 5)
- Byungchul Kim, Eyad H. Abed, “Cloud Distributed Optimal Control of ES-PV Residential Inverters,” IEEE 3rd International Conference on Power Systems and Electrical Technology (PEST), Tokyo, Japan, Aug. 2024, pp. 58-63. (related to Chapter 7)

Chapter 2. Optimal Frequency Control with Management of a Hybrid Energy Storage System in an Islanded Microgrid

Abstract – The extensive employment of renewable energy source in microgrids presents challenges due to the lack of rotational inertia that serves to stabilize system frequency in traditional power systems. This results in added fluctuations in frequency and voltage. In a hybrid energy storage system, the super capacitor control in voltage and power output constrained mode involves significant computation requirements. To mitigate fluctuations and enhance stability, in this chapter a novel approach is proposed the Heaviside step function in conjunction with Pontryagin's minimum principle (HVS-PMP) to optimize the frequency management system for HESSs comprising batteries and supercapacitors. This innovative method reformulates the problem thereby establishing a state-constrained optimal control reformulated as an unconstrained problem. By applying HVS-PMP to a penalty function, the state constraints are re-expressed, leading to an unconstrained problem that significantly accelerates computation compared to traditional dynamic programming.

2.1. Background

Frequency fluctuations are more likely to happen with high penetration of solar and wind energy sources in microgrids Fig. 2.1. Increased presence of renewable sources like solar and wind along with the associated power converters implies significantly reduced system inertia. This results in a tendency toward increased fluctuations in power and frequency. Many control approaches are available for the microgrids. An overall summary of existing control approaches is provided in [18] where diverse power-sharing methods are compared and categorized. Due to its efficiency in power-sharing between multi-parallel

connected power converters, droop control has been used in an extensive variety of areas [19]. An organized architecture of islanded AC microgrids with soft switching droop control has been suggested to offer a flexible power control [20].

To remedy for the lack of inertia in droop control, the use of so-called virtual synchronous generator has been suggested (e.g., [21], [22]). Extensive research has been conducted on the use of virtual synchronous generators that produce active and reactive power to sustain system power and frequency stability by imitating the dynamics of a synchronous generator [23]-[25].

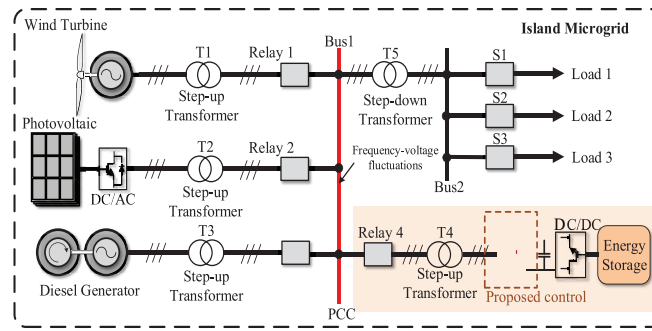


Fig. 2.1. Islanded microgrid.

By using an energy storage (ES) system along with renewable energy sources, microgrid power system droop control contributes to reducing frequency oscillations. For ES system to be of practical use, it is important to increase the battery lifetime. Battery degradation is a major problem in ES systems because the battery contributes a large share of system price. Thus, extending the battery lifetime and mitigating frequency fluctuations are an important topic bringing efforts from both industrial and academic sectors in the hybrid energy storage systems [26]-[28].

While anticipating the commercialized next generation of durable battery,

hybridization is a promising midterm solution. Battery, as a high certain energy source, can be mixed together with a high specific power source such as supercapacitors (SCs) resulting in a HESS [29]. The combination could be in active, semi-active, or passive topologies, where the first two normally provide more degrees of freedom which can bring about better global performances than the third configuration [30]. The semiactive and active HESSs must be organized by management strategies which distribute power flows between storage devices.

Real-time frequency management strategies can be improved using optimization approaches, for instance approaches based on optimal control, model predictive control (MPC), and other methods. Optimal solutions can be determined off-line and then executed using lookup tables in real-time implementation. Linear MPC has been employed for the energy management system (EMS) of a SC/fuel-cell HESS in [31]. It is improved with a model in which only SCs state of charge (SOC) are taken as state variables. The local control level is separated from the energy management supervisory level. Multi-objective approach is examined with two cases of weighting factor for the SCs SOC, known as “low SOC penalty” and “high SOC penalty.” Metaheuristic real-time EMSs or frequency control for SC/battery HESS have been researched using a multilevel hierarchical structure [32].

There is a need for benchmarks for performance evaluation. Another approach is using dynamic programming (DP). However, DP suffers from several drawbacks. First, the solution accuracy relies on the resolutions of the quantization and discretization of the problem. There is always a tradeoff between accuracy and required computational resources. In addition, DP offers its solutions in the form of data lookup tables. Thus, the

insights it may give for the development of real-time EMSs or frequency regulation are often limited and strongly heuristic.

However, PMP can yield analytical solutions that are much less time-consuming and optimal for constrained problems [43]. By using PMP, the numerical computation is only required at the boundaries and constraints of the optimal control problems instead of the whole evolutions as in DP [33]. The optimal control problem with two independent control laws is solved simultaneously. Pareto fronts in the cases without and with battery are generated. Moreover, thanks to use of an analytical approach, PMP solutions can be adapted to develop real-time strategies. Different from DP, the main challenge in PMP is the difficulty in dealing with complex nonlinear systems such as battery/SC HESS, especially with SCs' voltage energy limitations that present an obstacle to state-constrained optimal control. In previous research, HESSs are often modeled in terms of voltage and current which led to nonlinear models [34], [35]. In this chapter, we control frequency via the electrical power active power generation. Table 2.1. summarizes characteristics of the PMP approach in comparison with other methods.

This chapter suggests a Heaviside function method of PMP (HVS-PMP) for developing optimal frequency management system (FMS) for SC/battery HESS. To get the apparent analytical form of the solution, the optimal frequency management problem is first formed in terms of power and energy to assume a germane state-constrained optimal frequency control problem. The SCs energy limitations are thus fully incorporated. Then, the penalty function method is used to present an additional state variable and as a result an extra costate variable. Whereby, the state-constrained problem is converted into a new unconstrained one, which is then solved by using PMP. The

obtained solution has a form of piecewise constant. The evolution of the optimal solution is then determined through iterative numerical simulations.

DP is a local optimization, but it consumes a lot of optimization time. PMP performs a local optimization, but an optimal trajectory obtained from PMP is a unique trajectory that satisfies the necessary and boundary conditions. The method proposed here applies the penalty function to the convex Hamiltonian function using a Heaviside step function and considers how to replace a constrained optimization with an unconstrained problem. We propose to break away from local optimization of system control by using the HVS-PMP method and a Hamiltonian function including dummy variables. The PMP algorithm can be used for real-time optimal control because it is based on the instantaneous minimization of the Hamiltonian. We show that costate of the PMP can be considered as a constant parameter, simplifying the optimal control problem. Optimal control based on PMP with a constant costate guarantees optimality and, in particular, local optimality.

The paper [45] uses the Hamiltonian which is a convex function and the constraint on the state is presented by a linear summation of SOC. In this case, the mathematically optimized problem possibly possesses an appropriate cost that satisfies the boundary condition, which implies that we can find the appropriate cost to satisfy (2.1). This is a situation of strong duality in optimization. In [45] the control concept is based on instantaneous minimization, but the optimal control can be determined only when an appropriate, or optimal, cost is given.

The proposed approach formalizes the amount of frequency change in relation to the amount of energy produced for the Hamiltonian convex function. The constraint

of optimization considers the voltage variation range of a realistic supercapacitor as a constraint due to SC state-constrained Heaviside PMP. The solution is controlled to avoid the realistic maximum and minimum value areas of the supercapacitor in constrained conditions by controlling the on and off of the HVS step function in the boundary condition of energy, resulting in a better optimization than PMP local optimization. The convex Hamiltonian function uses Heaviside step functions, so if the energy of the supercapacitor is greater than the MAX value or the energy of the supercapacitor is less than the MIN value, the entire convex Hamiltonian function uses a penalty function. Therefore, the maximum and minimum value of the supercapacitor were controlled so as not to violate the boundary condition to obtain the optimal frequency change.

Table 2.1. Comparison of approaches for development of FMS.

Methods	Pros	Cons	Reference
Real-Time	Applicable for real-world → For practical implementation	Sub-optimal strategies → need to be evaluated by optimal benchmarks	N/A
Rule-based	Often easy to implement	Not systematic, depend on expertise	[21]
Optimization-based	Systematic, less dependent on human expertise	Often complicated, often high computational cost	[41]
Offline optimal	Local optimal solution → used for bench-marking purpose	Not implementable in real-time platforms → validated by simulation	N/A
Dynamic programming (DP)	Universally applicable to a wide range of nonlinear systems	Time consuming, accuracy depends on fitness of discretization and quantization, give little understanding on how the system should work	[20]
Pontryagin's minimum principle (PMP)	Analytical solution → More insightful understanding (than in look-up table form like DP), faster computation (than DP)	Complicated when apply to complex nonlinear systems, each system configuration needs specific strategy development → Aim of This chapter to improve	[45]

The contributions of this chapter are as follows.

1) The proposed approach solves the state-constrained optimal control problem, which is necessary for HESS applications owing to the energy limit of auxiliary storages such as SCs. The literature on the optimal control problem has not clearly dealt with these constraints.

2) Moreover, owing to the analytical approach, the new optimal FMS is better than DP in terms of both computational efficiency and accuracy since the iterative numerical computations are required only for boundary conditions and for the constraints. The approach can offer an effective tool for FMS developers to use as an optimal benchmark to judge their real-time suboptimal strategies.

3) Finally, the suggested method is germane to both active and semiactive format of HESSs due to its general problem frame in terms of power and frequency. The approach facilitates simultaneous solution for the battery control variable and the SC energy and state variable trajectories. Subsequently, the particular current and voltage evaluations can be inferred for each topology.

2.2. Frequency control based on inertia

In this chapter, frequency control is divided into three different stages: the inertia control, primary control, and secondary control stages. When a frequency fluctuation occurs, the inertia response seeks equilibrium by balancing power through the kinetic energy from a generating unit. Then, the primary response settles frequency to a

new steady-state condition for a time duration between 10 s to 25 s after the appearance of a fluctuation. Finally, the secondary response, also called load frequency control, returns the frequency to its nominal equilibrium value within from 30s to 30 min after the appearance of a fluctuation [37].

1) Primary Control Strategy: The primary control strategy focuses on maintaining voltage and current control via control of the converters, in which link with the distributed resources and energy storage devices. The primary control also contains maximum power extraction, sharing of power among other devices and architectures. The goals of the primary control level are controlling the value and frequency of the voltage, passing on the inputs for inner voltage and current loops, and circulating current limitation.

Droop control is a popular primary control scheme that is based on the decoupling of real and reactive power on the transmission circuit due to line reactance. Droop control enhances the distributed energy resource based microgrid's reliability by guaranteeing that each source delivers an output power according to the amount decided by the linked power electronic converter.

In the case of unbalanced load, the control scheme guarantees system operation in stable condition. In voltage droop control, for reference output power:

$$P_{ref} = G(s) \left[V_{ref} - \left(\frac{\omega_{LP}}{s + \omega_{LP}} \right) V_{dc} \right] \cdot V_{dc}, I_{ref} = \frac{P_{ref}}{V_s} \quad (2.1)$$

where V_{dc} is the converter voltage output, ω_{LP} is the low-pass filter cut off-frequency, V_{ref} is the reference voltage, I_{ref} is the converter's reference current, and V_s is the DC source voltage.

$$\Delta\omega = \frac{1}{Js+D}\Delta P \quad (2.2)$$

Equation (2.2) describes the dynamic nature of system frequency according to the equivalent inertia model for various system configurations, such as diesel engine generator and renewable energy systems such as photovoltaic, wind turbine, and energy storage system. ΔP is the deviation between total power supply among all of diesel-generator, renewable systems and load. J is the equivalent moment of inertia, D denotes the frequency damping coefficient. $\Delta\omega$ is the frequency deviation caused by the power imbalance.

The frequency characteristics of diesel-engine generator and energy storage system differ from one another. Low-pass and high-pass filters are needed to disperse the original oscillation into different time cycles. Diesel-engine generator operates in the lowest frequency region, however energy storage system mostly operates in the middle and high frequency regions. Fig. 2.2 illustrates the microgrid frequency control model.

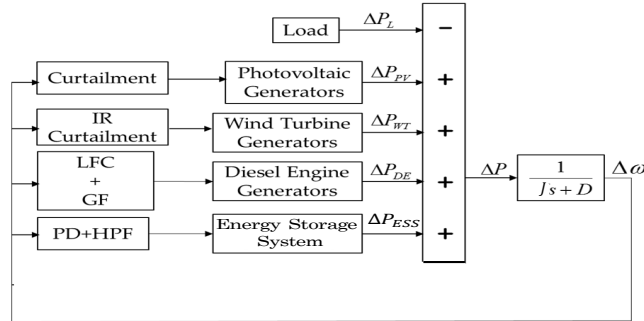


Fig. 2.2. Microgrid frequency control models.

Distributed energy resource-based droop control works according to the droop characteristics of synchronous generators in a power system when the frequency and voltage of the grid drops, the reactive power within the system also drops. The system

output frequency and voltage are determined as follows:

$$f = f' - m_p(P - P'), V = V' - n_q(Q - Q') \quad (2.3)$$

where f' is the generation's frequency output and V' is the generator's voltage output at zero loading. P and Q are the active and reactive power generation. P' is the reference active power and Q' is the reference reactive power. Due to the primary control, the frequency and voltage shift from the rated values bringing about the need for secondary control actions.

Secondary Control Strategy: The secondary control has the purpose of monitoring the microgrid and stabilizing frequency and local voltage [178]. The secondary control with communication scheme builds upon the primary control approach [179]. Accurate frequency management, reactive power-sharing, active and reactive power compensation are some of the goals of secondary control [180]. Fig. 2.3 illustrates microgrid primary and secondary control through a schematic diagram.

The secondary control compares rated voltage and frequency to the particular measured values and transmit the calculated error as an input to the PI controller. The PI droop controller for distributed energy resources results in:

$$\delta_i^\omega = K_p(\omega^* - \omega_{bus}) + K_i \int (\omega^* - \omega_{bus}) dt, \delta_i^v = \alpha_p(V^* - V_{bus}) + \alpha_i \int (V^* - V_{bus}) dt \quad (2.4)$$

where K_p and K_i are PI control parameters for angular frequency and α_p, α_i are PI control gains for voltage control.

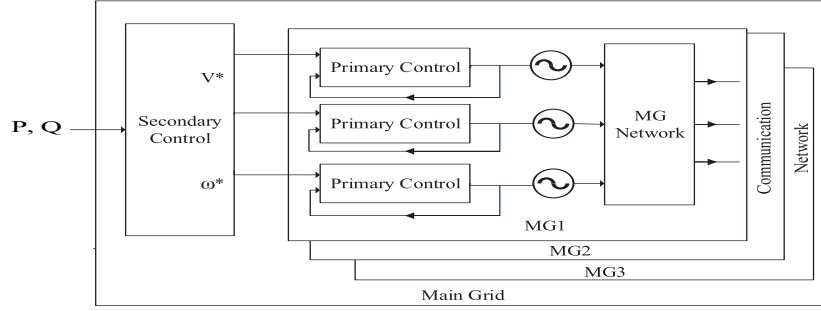


Fig. 2.3. Microgrid primary and secondary control schematic.

By considering the dynamic effects of generation and loads including inertia, droop, and converter control of PVs, wind, diesel generator, the system frequency deviation explained in [42], [43] is given as,

$$\Delta F(s) = \left(\frac{1}{2JD}\right) \sum_{i=1}^{N_G} \Delta P_{m,i} - \sum_{l=1}^{N_L} \Delta P_{L,l} = \left(\frac{1}{2JD}\right) (\sum \Delta P_G + \sum \Delta P_{PV} + \sum \Delta P_{WIND} + \sum P_{ESS} - \sum_{l=1}^{N_L} \Delta P_{L,l}). \quad (2.5)$$

where $\{\Delta P_G, \Delta P_{PV}, \Delta P_{WIND}, \Delta P_{ESS}\} \in \Delta P_{m,i}$, J is inertia, D is damping coefficient, P_m is mechanical power, P_L is load power, respectively. ΔP_G is diesel generator, ΔP_{PV} is photovoltaic power, ΔP_{WIND} is wind turbine power, ΔP_{ESS} is power output of the energy storage system, respectively.

2.3 Battery and supercapacitor-based HESS

2.3.1 System topology

The HESS is commonly taken as a combination of a battery and a super capacitor to prolong use of HESS. The energy storage system (ESS) performs as a voltage source, offering DC power to the inverter and inverter subsystem. As this section concentrates on frequency management of the VSG side, the inverter and inverter subsystem are regarded as an equivalent current source that interacts with

the HESS via dc-link voltage u_{dc} and the traction current i_{sub} . All the system dynamics are considered via the inverter/inverter subsystem current profile placed on the HESS.

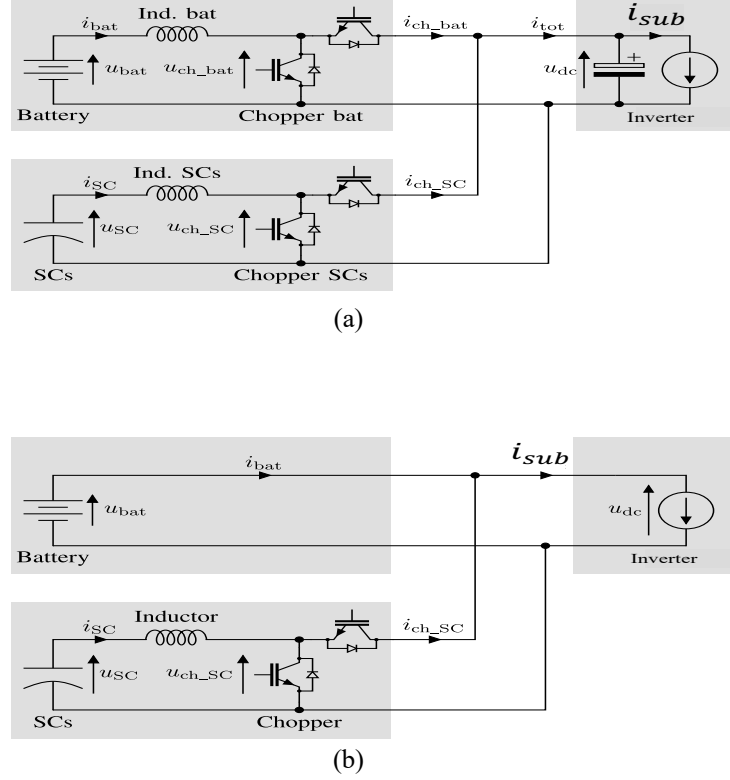


Fig. 2.4. Battery/SC HESS design (a) active topology (b) semiactive topology.

In the HESS, this section models hybridization of Li-ion battery and SCs connected in active and semi-active topologies, as shown in Fig. 2.4. In the active configuration, bidirectional DC/DC converters are in use for both energy storage devices, although in the semiactive one, only SCs are connected to the DC bus through the power electronics converter. The converter is consistent with an inductor connected to the low-voltage side of a power electronic chopper, although the high-voltage side is linked to the DC bus. In a semi-active topology, the DC bus voltage is directly maintained by the battery. In the meantime, the DC bus of the active topology is isolated

from the sources, and therefore, the voltage u_{dc} requires to be controlled by a capacitor.

2.3.2 HESS modeling and control scheme

The battery is reflected on an equivalent circuit model as shown in Fig. 2.5, which is borrowed from [39]. To concentrate on the energy of the battery, the transient impedances are neglected, i.e., only the equivalent series resistance r_{bat} is contemplated. The terminal voltage u_{bat} is comprised of the open circuit voltage $u_{bat_{OC}}$ and the dropped voltage on the resistance, in which $u_{bat} = u_{bat_{OC}}(u_{SOC}) - r_{bat}i_{bat}$ is a nonlinear function of the SOC indication voltage u_{SOC} .

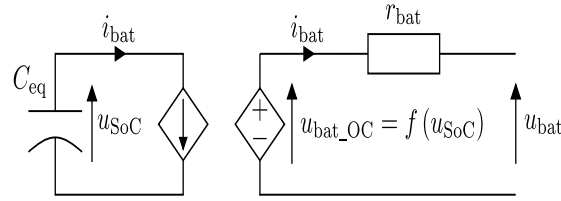


Fig. 2.5. Battery equivalent circuit.

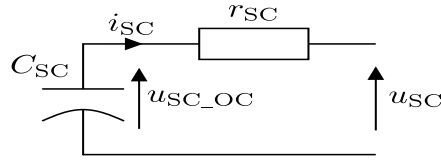


Fig. 2.6. Super capacitor equivalent circuit.

$$u_{bat} = u_{bat,OC}(u_{SOC}) - r_{bat}i_{bat} \quad (2.6)$$

where SOC is an integral of the battery current i_{bat} , viz.

$$u_{SOC} = u_{SOC,init} - \frac{1}{C_{eq}} \int_0^t i_{bat} dt \quad (2.7)$$

Here the equivalent capacitance C_{eq} is 3600 times the battery capacity in Ah.

The SC's model is simpler due to the fact that the SC's voltage is proportional to their SOC. As a result, the SC's equivalent circuit model can be regarded as a big capacitor C_{SC} connected in series with the internal resistor r_{SC} , as shown in Fig. 2.6. The SC's mathematical model is thus given by

$$u_{SC} = u_{SC,init} - \frac{1}{C_{SC}} \int_0^t i_{SC} dt - r_{SC} i_{SC} \quad (2.8)$$

The inductor linear model includes a constant inductance $L_{ind,x}$ and a constant parasitic resistance $r_{ind,x}$:

$$u_x = L_{ind,x} \frac{d}{dt} i_x + r_{ind,x} i_x + u_{ch,x}; x \in \{bat, SC\} \quad (2.9)$$

where x indicates “bat” or “SC” regarding the associated source of DC/DC. converters. In this research, the power electronic choppers are approximated using the averaged model

$$\begin{cases} u_{ch,x} = m_{ch,x} u_{dc} \\ i_{ch,x} = m_{ch,x} i_x \eta_{ch,x}^{k_{ch,x}} \end{cases} \text{ with } k_{ch,x} = \begin{cases} 1 & \text{if } i_x \geq 0 \\ -1 & \text{if } i_x < 0 \end{cases} \quad (2.10)$$

in which $x \in \{bat, SC\}$, $m_{ch,x}$ is the modulation function of the chopper associated with the x source, and η measures efficiency.

In the semi-active topology, the DC-bus voltage is the battery voltage

$$u_{dc} = u_{bat} \quad (2.11)$$

In the active topology, u_{dc} is linked to the DC-bus capacitor model that is given by

$$u_{dc} = \frac{1}{C_{dc}} \int_0^t (i_{tot} - i_{sub}) dt \quad (2.12)$$

where i_{sub} is the inverter/subsystem current assessed by the inverter/subsystem and i_{tot} is the total current consisted with the battery and SCs subsystems as

$$i_{tot} = i_{ch,bat} + i_{ch,SC} \quad (2.13)$$

However, in the semi-active topology, the DC bus is given by

$$i_{sub} = i_{bat} + i_{ch,SC} \quad (2.14)$$

The proposed HESS control strategy is shown in Fig. 2.5. As the DC-bus capacitor model is an integrator, a proportional (P) controller with the reward of the measured subsystem current $i_{sub,mea}$ is enough for the voltage control loop. The chopper current calculation block is the inversion of the current relationship of the chopper model as follows:

$$i_{x,ref} = \frac{i_{ch,x,ref}}{m_{ch,x} \eta_{ch,x}}; x \in \{bat, SC\} \quad (2.15)$$

In the same way, the modulation is the reversal of the voltage relationship of the chopper model

$$m_{ch,x} = \frac{u_{ch,x,ref}}{u_{dc,mea}}; x \in \{bat, SC\} \quad (2.16)$$

For the semi-active topology, there is no requirement of a voltage controller and the regular battery voltage $u_{bat,mea}$ plays the function of the DC-bus one. The current distribution strategy block is to calculate and form the optimal electrical power for the frequency management strategy (FMS) to be developed in Fig. 2.7.

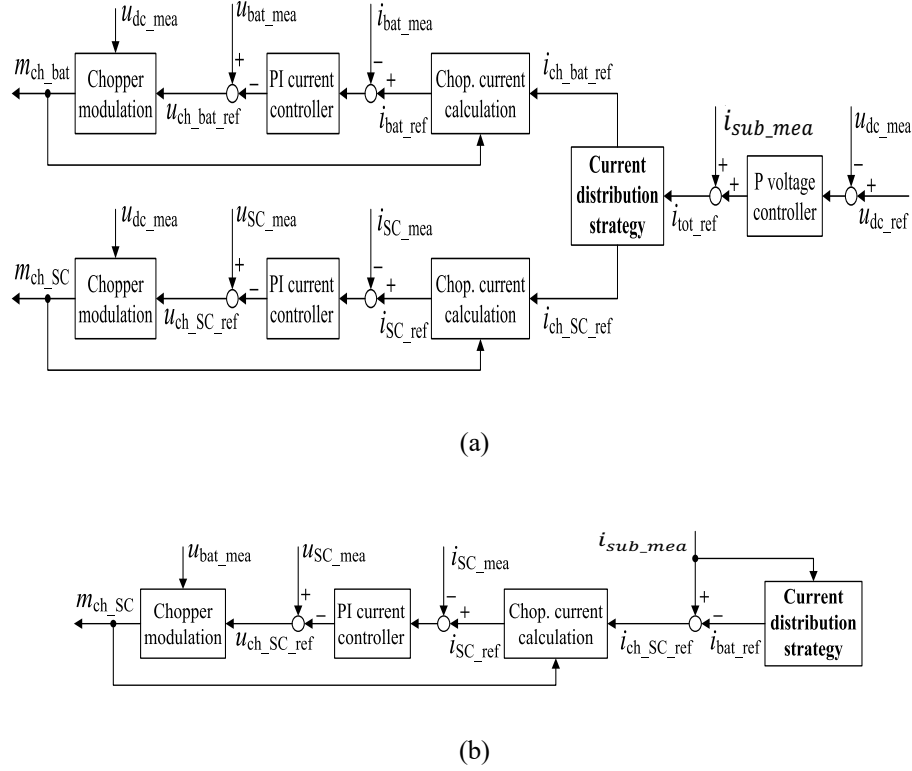


Fig. 2.7. Control plan of the proposed HESS. (a) active topology (b) semi-active topology.



Fig. 2.8. Overall electrical power flow of the battery/SC HESS.

This section talks about system topology such as battery and supercapacitor HESS ($= P_{bat}$) design with active and semi-active topology. Next, HESS modeling of battery and supercapacitor are performed using equivalent circuits. Each equivalent circuit can be represented with voltage and current relationship. Fig. 2.7 shows the active and semi-active topologies of the control plan of HESS. In the active topology control both battery and supercapacitor voltages and current are controlled. In Fig. 2.7, at the initial stage, P voltage controller controls the voltage for the active topology. And then, it

controls the current distribution with battery and super capacitor. In the active topology, controlling voltage and current distribution, it is possible to control both battery and SC. In the semi-active topology only the supercapacitor's voltage and current are controlled. SC power is only controlled in the semi-active topology. With active and semiactive topology control one can generate the HESS power. By using generated power from all the HESS, diesel generator, photovoltaic, and wind turbine generators subtract the home load power and subtract delta of frequency times damping factor and divide all power generations minus load powers with inertia we obtain the frequency deviation and rate of change of frequency. Thus, this chapter discusses how HESS's components such as battery and supercapacitor can be controlled via the cost function of (2.18) by using a HESS ($= P_{bat}$) in section 2.4 with the current and voltage modulation from section 2.3 in Fig. 2.7.

2.4 Optimal approach using PMP

2.4.1 Formulation of the PMP problem

In order to get a clear analytical solution for the FMS, the problem should be stated in terms of power and energy. The desired FMS will be relevant to both active and semi-active configurations. Fig. 2.4 presents the overall power flow of the battery/SC HESSs for both topologies. Every subsystem is as depicted in the corresponding shaded area in Fig. 2.4.

The inverter/subsystem power P_{sub} is requested by the inverter/subsystem, although the battery and the SCs provide P_{bat} and P_{SC} , respectively. The SCs are charged and discharged by their power with an average efficiency η as follows:

$$\frac{d}{dt}E_{SC} = -P_{SC}\eta^{-k_{SC}}; K_{SC} = \begin{cases} 1 & \text{if } P_{SC} \geq 0 \\ -1 & \text{if } P_{SC} < 0 \end{cases} \quad (2.17)$$

in which k_{SC} depends on the direction of power flow, the global model of the battery/SC HESSs is given by

$$\frac{d}{dt}E_{SC} = (P_{bat} - P_{sub})\eta^{-K_{SC}} \quad (2.18)$$

The energy of the SCs E_{SC} is the state variable to be controlled, the battery power P_{bat} is the control variable and the inverter/subsystem power P_{sub} is the output. It should be noted that, by reformulating the problem in terms of power and energy, the nonlinearity of current and voltage relationships of the system has been discarded. The optimal control problem is thus independent of the HESS topologies. As a result, the proposed method is applicable for both active and semi-active formats. In this research, the average efficiency η is regarded as a constant. The coefficient k_{SC} presents a switching behavior to the system model. Alternatively, this behavior is then fully regarded in the iterative numerical simulation to get the optimal evolutions of the control and state variables. The dynamic model of the island microgrid, DG, RES station and massive DRs are presented. The dynamic frequency response of an islanded microgrid can be formulated as follows. The microgrid rate of frequency change is taken as the cost function. This cost function is given in equation (2.19) and depends on frequency deviation, battery power, synchronous generator power, rest of renewable system generations and loads variance.

$$\Delta \dot{f} = \frac{1}{j} \int_0^{t_f} (-D\Delta f + \Delta P_{bat}^2 + \sum_{i=1}^{N_G} \Delta P_G^2 + \sum_{i=1}^{N_{PV}} \Delta P_{PV}^2 + \sum_{i=1}^{N_{WIND}} \Delta P_{WIND}^2 - \Delta P_{LOAD}^2) dt. \quad (2.19)$$

This formulated problem is suitable for applying optimal control theory since it is a model with a quadratic cost function of the control variable. Also, it includes the frequency fluctuation terms with the diesel generator, all of the renewable energy sources, energy storage systems power generations, and the loads. By minimizing this cost function, the frequency fluctuation and battery SOC reduction can also be minimized in contrast to other strategies applied to the same HESS topology. However, minimization of P^2 does not lead to any limit of the VSG dynamic performance. It is due to the inverter/subsystem power demand P_{sub} is always assured by the allocation of the battery and the SCs subsystems, as shown in Fig. 2.8. The reduction of P_{bat} is due to the support of P_{SC} , but not an influence on P_{sub} .

By using generated power from all the HESS, diesel generator, photovoltaic, and wind turbine generator subtract the home load power and subtract deviation of frequency times damping factor and divide all power generations minus load powers with inertia can get the value of derivative of frequency deviation. This can be a part of the cost function for deviation of derivative for the frequency.

2.4.2. Unconstrained solution

In this section, the above problem is solved without any constraint on the SCs energy. This is certainly an unrealistic solution; but the unconstrained solution is the base to solve the state-constrained problem of section 2.3.3. The Hamiltonian of the problem under study is associated with the SC state (2.18) and is given by

$$H = P_{bat}^2 + \lambda(P_{bat} - P_{sub})\eta^{-k_{sc}} \quad (2.20)$$

where λ is the costate variable, P_{bat} the battery output power, is the control variable,

and P_{sub} is an output variable representing the power input to the inverter which converts the HESS output voltage from DC to AC as depicted in Fig. 2.4 (a).

Equation (2.20) is the Hamiltonian function for PMP and consists of two terms. The first term is battery power, and the second is supercapacitor power. Equation (2.20) uses square of the battery power because when battery discharges the power then P_{bat} will have a positive value. However, when the battery is charging, mode P_{bat} will have a negative value. Thus, by taking the square of P_{bat} , we remove the effect of its sign. The second term is supercapacitor power, which is the same as $P_{bat} - P_{sub}$. P_{sub} is the output variable representing the power input to the inverter, which converts the HESS output voltage from DC to AC, as in Fig. 2.4 (a). With costate variable λ , it is possible to weight the both first term P_{bat} and second term supercapacitor. η is the efficiency of the supercapacitor and k_{SC} is using for power direction. It is possible to change the current direction if k_{SC} is 1 then $P_{SC} \geq 0$, and if k_{SC} is -1 then $P_{SC} < 0$. By making the Hamiltonian function with major term P_{bat} , it is possible to control the HESS easily.

It is seen that the Hamiltonian does not contain the state variable E_{SC} , which is adaptable to get the constant costate variable λ as presented in the following. The three necessary conditions of the optimal control are given as follows:

$$\frac{d}{dt}E_{SC} = \frac{\partial H}{\partial \lambda}, \frac{d}{dt}\lambda = -\frac{\partial H}{\partial E_{SC}}, \frac{\partial H}{\partial P_{bat}} = 0 \quad (2.21)$$

As λ is the arbitrarily introduced costate variable, it is independent of the control variable P_{bat} and the output P_{sub} . Thus, the partial derivative of the Hamiltonian with respect to λ can be calculated as

$$\frac{\partial H}{\partial \lambda} = \frac{\partial (P_{bat}^2 + \lambda(P_{bat} - P_{sub})\eta^{-k_{SC}})}{\partial \lambda} = (P_{bat} - P_{sub})\eta^{-k_{SC}} = \frac{d}{dt} E_{SC} \quad (2.22)$$

Thus, the first condition of (2.21) is satisfied. For the second condition of (2.21), battery power P_{bat} and the inverter/subsystem power P_{sub} are independent of the SCs energy E_{SC} . Therefore, the Hamiltonian does not depend on the state variable, i.e.,

$$\frac{d}{dt} \lambda = -\frac{\partial H}{\partial E_{SC}} = 0 \quad (2.23)$$

As a result, the costate variable λ of the unconstrained problem is a constant $\lambda = \lambda_0$.

Considering that the inverter/subsystem power is independent of the battery power, the last condition of (2.21) which is the Hamiltonian minimization can be expanded as follows:

$$\frac{\partial H}{\partial P_{bat}} = \frac{\partial (P_{bat}^2 + \lambda(P_{bat} - P_{sub})\eta^{-k_{SC}})}{\partial P_{bat}} = 2P_{bat} + \lambda\eta^{-k_{SC}} = 0 \quad (2.24)$$

This guide to the form of optimal control law of the unconstrained problem

$$P_{bat} = -\frac{\lambda\eta^{-k_{SC}}}{2} = -\frac{\lambda_0\eta^{-k_{SC}}}{2} \quad (2.25)$$

Some remarks on the solution (2.25) are in order. If the efficiency of the SCs subsystem is negligible, the optimal battery power is a constant. It is the average of the inverter/subsystem power plus the power to linearly move the SCs energy from the initial state $E_{SC,init}$ to the final state $E_{SC,final}$. In general, the SCs should be charge sustaining, i.e., $E_{SC,final} = E_{SC,init}$.

In the more practical case with high efficiency, the optimal battery power is a piecewise constant outline. There are two constant values that switch to each other regarding the changes of the SCs power direction. This chapter regards this practical case.

2.4.3. State-constrained Solution

In practice, the SCs energy is limited, which results in the following limitations on the state variable:

$$E_{SC} - E_{SCmin} \geq 0, E_{SCmax} - E_{SC} \geq 0 \quad (2.26)$$

Here, the upper limit is usually at the nominal voltage of the SCs, although the lower one is at the half of u_{SC-nom} as follows:

$$E_{SCmax} = \frac{1}{2} C_{SC} u_{SCnom}^2, E_{SCmin} = \frac{1}{2} C_{SC} (0.5 u_{SCnom})^2 \quad (2.27)$$

In order to deal with the state-constrained problem, the penalty function method [57] is useful. This entails introducing an additional dummy state variable E_{dum} based on (2.26) as

$$\frac{d}{dt} E_{dum} = (E_{SCmax} - E_{SC})^2 \mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2 \mathbb{H}(g_2) \quad (2.28)$$

where $\mathbb{H}(g_1)$ and $\mathbb{H}(g_2)$ are Heaviside step functions:

$$\mathbb{H}(g_1) = \begin{cases} 0 & \text{if } E_{SC} \leq E_{SCmax} \\ 1 & \text{if } E_{SC} > E_{SCmax} \end{cases}, \mathbb{H}(g_2) = \begin{cases} 0 & \text{if } E_{SC} \geq E_{SCmin} \\ 1 & \text{if } E_{SC} < E_{SCmin} \end{cases} \quad (2.29)$$

which means that the Heaviside functions are activated when the constraints are violated.

A new dynamical system is formed from (2.18) and (2.28) with E_{SC} and

E_{dum} the two state variables. The new Hamiltonian of this system is constructed as

$$H = P_{bat}^2 + \lambda_1(P_{bat} - P_{sub})\eta^{-k_{SC}} + \lambda_2((E_{SCmax} - E_{SC})^2\mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2\mathbb{H}(g_2)) \quad (2.30)$$

in which λ_1 and λ_2 are the two costate variables are related with E_{SC} and E_{dum} , respectively.

Regarding the arbitrary independence of the costate variables, the partial derivatives of the Hamiltonian to λ_1 and λ_2 are given by (2.30) and (2.31). Therefore, the first necessary condition of optimal control is satisfied.

$$\begin{aligned} \frac{\partial H}{\partial \lambda_1} &= \frac{\partial(P_{bat}^2 + \lambda_1(P_{bat} - P_{sub})\eta^{-k_{SC}} + \lambda_2((E_{SCmax} - E_{SC})^2\mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2\mathbb{H}(g_2)))}{\partial \lambda_1} \\ &= (P_{bat} - P_{sub})\eta^{-k_{SC}} = \frac{d}{dt} E_{SC} \end{aligned} \quad (2.31)$$

$$\begin{aligned} \frac{\partial H}{\partial \lambda_2} &= \frac{\partial(P_{bat}^2 + \lambda_1(P_{bat} - P_{sub})\eta^{-k_{SC}} + \lambda_2((E_{SCmax} - E_{SC})^2\mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2\mathbb{H}(g_2)))}{\partial \lambda_2} \\ &= (E_{SCmax} - E_{SC})^2\mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2\mathbb{H}(g_2) = \frac{d}{dt} E_{dum} \end{aligned} \quad (2.32)$$

Regarding the second necessary condition, the partial derivatives of the Hamiltonian to the two state variables are managed as (2.33) and (2.34). It is due to the dummy variable E_{dum} is independent of all the other variables of the system.

$$\begin{aligned} \frac{d}{dt} \lambda_1 &= -\frac{\partial H}{\partial E_{SC}} = \\ &= \frac{\partial(P_{bat}^2 + \lambda_1(P_{bat} - P_{sub})\eta^{-k_{SC}} + \lambda_2((E_{SCmax} - E_{SC})^2\mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2\mathbb{H}(g_2)))}{\partial E_{SC}} = \end{aligned}$$

$$\lambda_2((E_{SCmax} - E_{SC})^2 \mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2 \mathbb{H}(g_2)) \quad (2.33)$$

$$\frac{d}{dt} \lambda_2 = -\frac{\partial H}{\partial E_{dum}} =$$

$$\frac{\partial(P_{bat}^2 + \lambda_1(P_{bat} - P_{sub})\eta^{-k_{SC}} + \lambda_2((E_{SCmax} - E_{SC})^2 \mathbb{H}(g_1) + (E_{SC} - E_{SCmin})^2 \mathbb{H}(g_2)))}{\partial E_{SC}} = 0. \quad (2.34)$$

Apply the third condition, similar to the above unconstrained problem, and the optimal control law has the form of

$$P_{bat} = -\frac{\lambda_1 \eta^{-k_{SC}}}{2} \quad (2.35)$$

With λ_1 calculated by (2.33) where λ_2 is a constant regarding (2.34). Thus, the solution of the state-constrained problem can be executed in three cases with regard to the constraints as follows.

Case 1: The SCs energy is within the two boundaries, i.e.,

$$(E_{SCmax} - E_{SC} \geq 0) \text{ and } (E_{SC} - E_{SCmin} \geq 0) \rightarrow (\mathbb{H}(g_1) = 0) \text{ and } (\mathbb{H}(g_2) = 0) \quad (2.36)$$

$$\left\{ \begin{array}{l} \frac{d}{dt} \lambda_1 = 0 \\ \frac{d}{dt} \lambda_2 = 0 \\ P_{bat} = -\frac{\lambda_1 \eta^{-k_{SC}}}{2} \end{array} \right. \quad (2.37)$$

Case 2: The SCs energy violates the upper boundaries, i.e.,

$$(E_{SCmax} - E_{SC} < 0) \text{ and } (E_{SC} - E_{SCmin} \geq 0) \rightarrow (\mathbb{H}(g_1) = 1) \text{ and } (\mathbb{H}(g_2) = 0) \quad (2.38)$$

$$\begin{cases} \frac{d}{dt}\lambda_1 = 2\lambda_2(E_{SC-max} - E_{sc}) \\ \frac{d}{dt}\lambda_2 = 0 \\ P_{bat} = -\frac{\lambda_1\eta^{-k_{sc}}}{2} \end{cases} \quad (2.39)$$

Case 3: The SCs energy disrupts the lower boundaries, i.e.,

$$(E_{SCmax} - E_{sc} \geq 0) \text{ and } (E_{sc} - E_{SCmin} < 0) \rightarrow (\mathbb{H}(g_1) = 1) \text{ and } (\mathbb{H}(g_2) = 1) \quad (2.40)$$

$$\begin{cases} \frac{d}{dt}\lambda_1 = -2\lambda_2(E_{SC-max} - E_{sc}) \\ \frac{d}{dt}\lambda_2 = 0 \\ P_{bat} = -\frac{\lambda_1\eta^{-k_{sc}}}{2} \end{cases} \quad (2.41)$$

This indicates that between the two boundaries, the solution of the constrained problem has the same form as the unconstrained one with a certain constant λ_1 . When the SCs energy reaches either lower or upper boundary, λ_1 exchanges to the other constant value. These exchanging processes are executed by constant values of λ_2 , which should be chosen very high, the derivative of λ_1 should be very high. Idealistically, when E_{SC} outrages the boundaries, λ_2 should be infinite, which means the step changes of λ_1 . The optimal P_{bat} of the constrained problem has a piecewise constant profile, which is composed of the solutions of the unconstrained sub-problems. The sub-problems are solved each time the SCs energy E_{SC} gets to its limitations. This is the technique of separate computation of unconstrained arcs [57]. Below is the control step to find out when the state variable E_{SC} should reach the

constraints. Fig. 2.9, demonstrates the control chart of state constraints process.

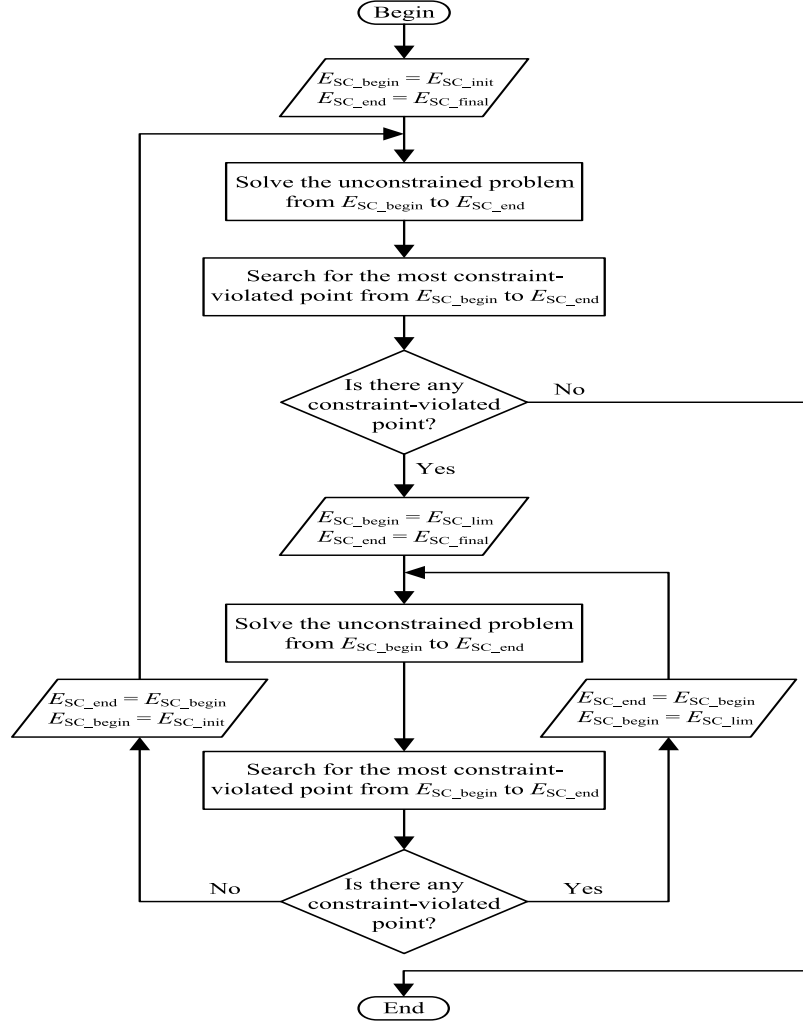


Fig. 2.9. Control chart of proposed algorithm.

2.4.4. Control constraints

Unlike the SCs, the battery can offer a large amount of energy, but its power, SOC, and degradation are bounded as follows:

$$P_{bat,min} \leq P_{bat} \leq P_{bat,max}, SOC_{min} \leq SOC \leq SOC_{max}, Degrad. = Con. \quad (2.42)$$

In the optimal control theory, the control variable constraints are a crucial problem as they can be dealt with by using a saturation function [41] given by

$$P_{bat} = \begin{cases} P_{bat}^* & \text{if } P_{bat-min} \leq P_{bat}^* \leq P_{bat-max} \\ P_{bat-max} & \text{if } P_{bat}^* > P_{bat-max} \\ P_{bat-min} & \text{if } P_{bat}^* < P_{bat-min} \end{cases} \quad (2.43)$$

where P_{bat}^* is the optimal solution of the state-constrained problem addressed above. In this research, the control constraints conditions are used to prove the soundness of the solution satisfying the battery physical limitations. It is due to the optimal solution P_{bat}^* is the minimum trajectory of the battery power. If the trajectory remains within the constraints, then the problem is feasible. If P_{bat}^* contravene the limitations, the optimal control problem is regarded as infeasible, which means, the HESS dimensions are inappropriate for under test. The focus of this section is to suggest an alternative approach of using optimal control theory to develop an offline FMS benchmark. Thus, the HESS is assumed to be properly sized.

2.5 Discussion and numerical results

2.5.1 Simulation results

Table 2.2. VSG parameter specification.

PARAMETER	SYMBOL	VALUE
NOMINAL POWER	P_{VSG}	0 kVA
NOMINAL VOLTAGE (LINE TO LINE)	u_{AB}, u_{BC}, u_{CA}	$380 V_{rms}$
DC-LINK VOLTAGE	U_{dc}	800 V
NOMINAL FREQUENCY OF VSG	f_{VSG}	60 Hz
VIRTUAL INERTIA	J_{VSG}	$1.5 \text{ KG} \cdot \text{m}^2$
VIRTUAL DAMPING	D_{VSG}	3.2
VIRTUAL IMPEDANCE	Z_{VSG}	$0.5 + j0.004 \Omega$
P DROOP COEFFICIENT	M	1.5×10^{-4}
Q DROOP COEFFICIENT	N	2.2×10^{-4}
SWITCHING FREQUENCY	f_{sw}	5 kHz

Table 2.3. Diesel synchronous generator parameter specification.

PARAMETER	SYMBOL	VALUE
NOMINAL POWER	P_{SG}	20 kVA
NOMINAL VOLTAGE (LINE TO LINE)	V_n	380 V_{rms}
NOMINAL FREQUENCY	f_{SG}	60 Hz
INERTIA COEFFICIENT	J_{SG}	0.7 $\text{KG} \cdot \text{m}^2$
DAMPING COEFFICIENT	D_{SG}	0
NUMBER OF POLE PAIRS	N	1

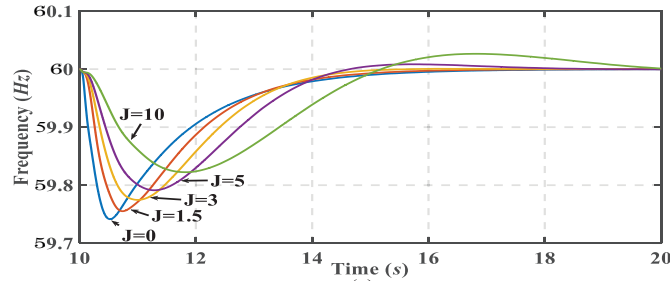


Fig. 2. 10. Diverse virtual inertia $J(D=2)$ effects on main frequency.

To confirm the efficiency of the suggested approach, a MATLAB/Simulink model is developed and tested. To show the power compensate of the proposed method more obviously, the output power waveform is processed by a low-pass filter with a cut-off frequency of 120 Hz.

The topology of the microgrid is shown in Fig. 2.1, It is an AC microgrid. Table 2.2 shows the distributed energy resources characteristics. The load includes the residential homes connected to the distribution networks. It spent 5KW/hr. The BESS is also classified as a load during the charging operation. The power converters are used as interconnection interface for RES and BESS with the grid. Also, no assumption is made for the frequency and voltage of the microgrid, as these are controlled by the system.

Table 2.4. AC microgrid renewable energy sources data.

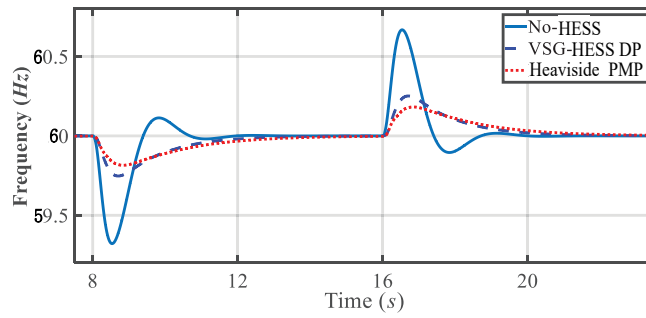
Photovoltaic	
Module-model	Solynda – SL001-157
Model-unit	$157W_p$ at STC
Power-rating	$1.88kW_p$
Wind Turbine	
Model	TN-1.5 Nozzi Nord (1.5kW)
Cut-in / Cut-out speed	4 m/s & 20 m/s
Generator type and ratings	1.5kW PMSG @ 60 Hz
Diesel Generator	
Model	YANIMARL 100N
Rated freq. & Voltage	60 Hz / 400 V
Max Power	3.5 kW
Super Capacitor	
Capacitance	99.5 F
Rated Voltage	48 V
Energy Storage	
Battery (lead acid)	4.6kWh at 96 V

Before proving the performance of the suggested approach under different conditions, the suitable inertia and damping coefficients need to be found. The effects of distinct inertia coefficients on a frequency transient are shown in Fig. 2.10. After studying the effects of multiple coefficient values on frequency fluctuation suppression, the selected coefficients are shown in tables 2.2 and 2.3.

In Fig. 2.11., to contrast the voltage and frequency fluctuations with loading transitions, a 5.5 kW load is connected at $t = 8$ s and detached at $t = 16$ s. Nonidentical working states are tested for comparing purposes. The “No-HESS” line shows, the load is powered by the diesel synchronous generator. The “VSG-HESS DP” line indicates the case in which a HESS controlled by DP with VSG approach is parallel-connected with SG to power the load. The “Heaviside PMP” line indicates the case in which a HESS controlled by the Heaviside step function of PMP approach is

parallel connected with SG to power the load.

Fig. 2.11. exhibits the frequency and ROCOF under loading shift. From Fig. 2.11. (a), the frequency variations of point of common coupling are about 0.69, 0.2 and 0.24 Hz with NO-HESS, VSG-HESS DP and Heaviside PMP approach, respectively. The frequency variation is about 23%, which clearly enhances the power quality and better the system stability. In contrast with the capacity of MGs, the loading modification is low. Thus, the frequency variation is also very low, which should not transcend ISO 8528 standard for generator limit (± 1.5 Hz). Along with the frequency changes, the ROCOF is disserving respect. As shown in Fig. 2.11. (b), before adding the HESS, the maximum ROCOF is about 2.26 Hz/s. The maximum ROCOF of the traditional VSG method is about 0.74 Hz/s, which does not meet the standard, $\text{ROCOF} \leq 0.6$ Hz/s. It should also be renowned, maximum ROCOF with the proposed method is about 0.57 Hz/s, which shows a decrease of about 21% and meets the requirements of the ISO standard.



(a)

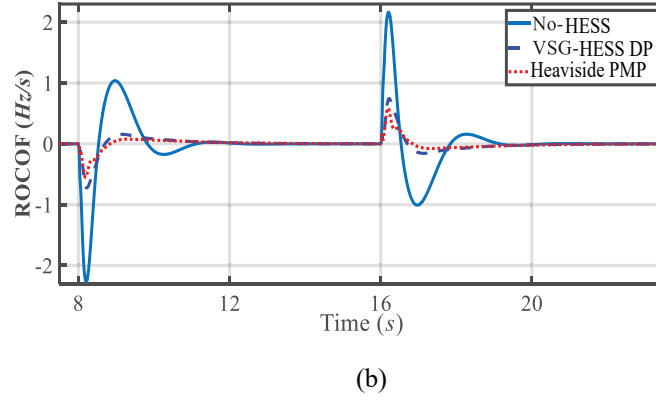


Fig. 2.11. Frequency and ROCOF under loading transitions.

Fig. 2.12, gives the output power of the Heaviside step function of PMP. When the frequency deviates, the Heaviside PMP controller calculates the power changes caused by the frequency changes. For instance, when a step-up load is given at $t=8$ s, the frequency decreases, by discovering the frequency changes, the Heaviside PMP calculates the required power change within a limited range. Then, Heaviside PMP outputs the optimized active power variations $\Delta P_e^*(k)$. By turning off $\Delta P_e^*(k)$, the rated power of VSG can be regulated adaptively.

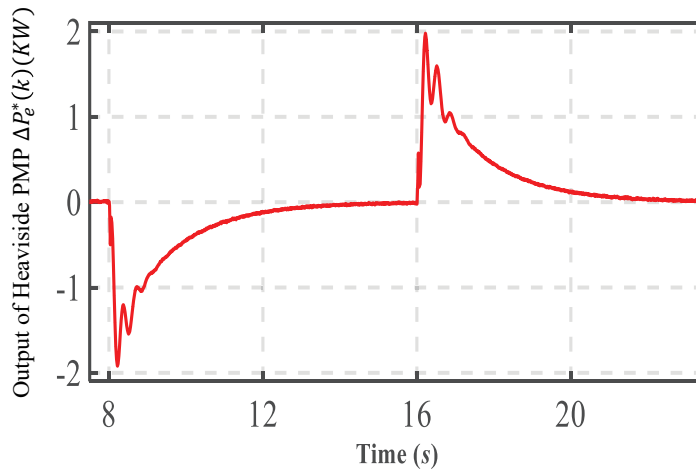


Fig. 2.12. Output of Heaviside step function PMP controller.

Fig. 2.13 shows the total power, battery power and SC power. As is clear from the figure, total power is the summation of battery and SC. As you may check that when SC power is low then battery power is high, and battery power is high then SC power is low. It is operated with complementarity due to the Heaviside PMP controller within a state-constraint solution.

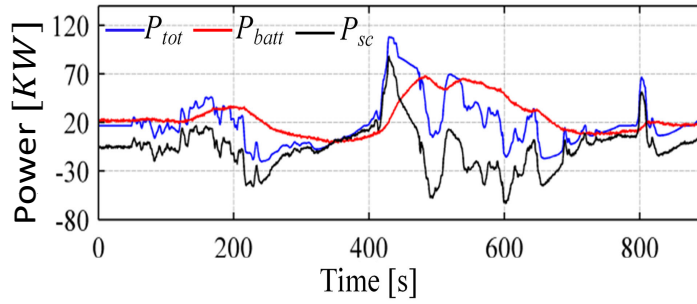


Fig. 2.13. Output of total power, battery power, and supercapacitor power.

Fig. 2.14. illustrates the super capacitor voltage operating range. It shows that it is operated within the maximum and minimum values owing to the SCs energy limitation due to SC state-constraint HVS-PMP approach. It avoids boundary areas which will increase computational requirement for the system.

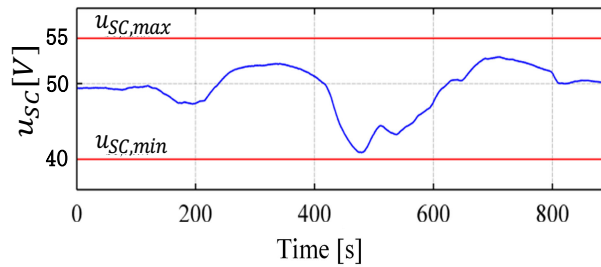


Fig. 2.14. Supercapacitor voltage operation range within the SCs energy. (Limited amount due to SC state-constraint Heaviside PMP solution)

Also, owing to its analytical approach, the proposed Heaviside step function PMP strategy is superior to DP regarding computational effort. It is tested on the same computer environment such as 2 GHZ four-core i5 CPU and 16 GB RAM. It is apprehended; the novel approach is faster than DP 1.2 times regarding the analysis testing cycles.

According to equation (2.4) in [42], [43] the change in frequency is calculated by subtracting the change in load power from the change in mechanical power production of synchronous generators, solar power, wind power, and ESS and dividing the inertia and damping coefficients. The formula applied in my paper reference [42], [43] is cited. The indirect method of measuring computation speed is the ROCOF [181] or frequency control performance and the greater the change in frequency in a limited time, the higher the computation speed is. When proposed controlling the system with the same amount of power production and consumption compared to control using DP, control is possible more than 1.2 times frequency control performance faster than DP, and it can be confirmed that overshoot is small, and convergence occurs with a fast-settling time. Indirect method of measuring the computation speed was measured by the ROCOF or frequency control performance. It is possible to say that proportional derivative control has the least overshoot, then HVS PMP control method, and then DP control approach.

Reference [46] by Ma, Zhang and Wang aims at ensuring stable and secure microgrid operation. Therein, a set of transient stabilization under control constraints are researched. This chapter convexified the problem and used training of explicit neural network-based controllers. The method we suggest does not require offline

training time and state observer for stability. The paper [46] needs to control transitory states and a nonlinear state observer is necessary to estimate the microgrid states such as voltages. The main problem caused by the violation of separation characteristics. The paper [46] requires coupling between the microgrid dynamics and nonlinear state observer. The approach we suggest can turn on and off the convex Hamiltonian penalty function within the range of the energy maximum and minimum values of the supercapacitor. It is possible to controls a cost function that is sensitive to changes in the frequency of the microgrid.

By using MATLAB and Simulink, it is possible to determine the frequency deviation for each of the renewable resources and diesel generator. Fig. 2.15 simulation graph shows, in which frequency control using diesel-engine generator, and other renewable resources such as photovoltaic, and wind frequency deviation. It also shows the frequency control via diesel generator and all the three renewable resources.

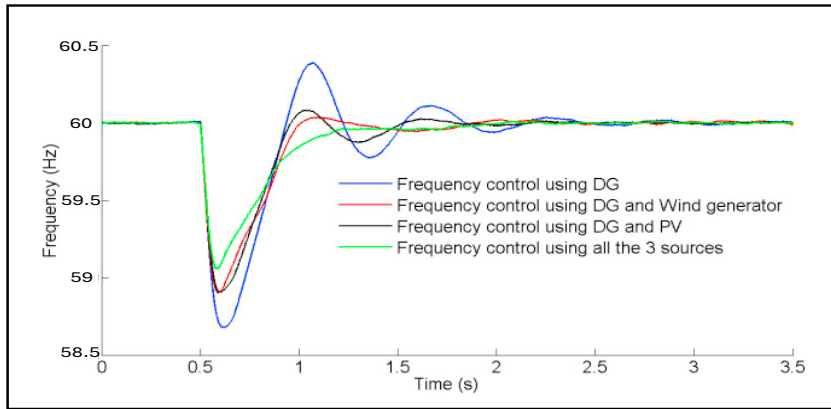


Fig. 2. 15. Frequency control using various of renewable energy resources and diesel-engine generator.

When the general evolutions of optimal power and energy profiles are

obtained, they can be used to deduce the detailed results for each hybridization setting, in which the battery SOC and current performance indexes are the most important criteria because they reflect the energy consumption and the battery aging stress elements of each topology and FMS. Fig. 2.16 indicates the battery SOC evolutions regarding the test cases of active and semi-active configurations with DP and the suggested Heaviside PMP approach over the tested cycle.

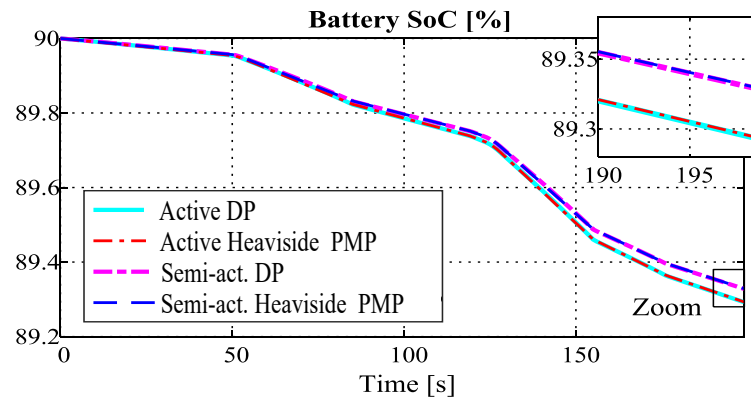


Fig. 2.16. HESS SOC variations of active and semiactive topologies with DP and proposed Heaviside PMP approach over tested cycle.

The results indicate DP and Heaviside PMP approaches are mostly the same in terms of energy consumption because their curves' volume line up well with each other. This confirms the good agreement between DP and the suggested method. It can also be seen, the active topology put away more energy than the semiactive counterpart. It is due to the additional DC/DC converter required to isolate the battery for the DC bus, which presents more losses to the system.

2.6 Summary

This chapter presents a Heaviside step function PMP control method for a

HESS in islanded MGs. Based on the real-time detection of the system frequency, the PMP is established to predict the optimal output power of ESS and VSG, thereby further enhancing the frequency dynamics of the system. Also, considering that changes in active power may lead to frequency variation, we proposed a HVS-PMP power control method which can reduce the power fluctuation by changing the rate's active power of VSG via Heaviside step function of PMP. The method can be generally applied for both active and semi-active topologies by reformulating the problem in terms of power and energy. The suggested method manages the state-constrained optimal control problem, considering the effect of the SCs subsystem efficiency. The derived optimal control law has a piecewise constant profile, which is smoother than what is obtained by DP, HVS-PMP converts state-constrained optimal control problem to state-unconstrained control problem. Also, frequency control with various renewable energy resources with diesel-engine generator has the minimum overshoot and settling time.

Chapter 3. Adaptive Frequency Control in Islanded Smart Microgrids with Diesel-engine Generator through Linear Parameter Varying Q-learning

Abstract –Diesel-engine generators play a crucial role in power systems by providing electrical energy and frequency support. Within an islanded microgrid, the primary ancillary function of these engine generators is to maintain frequency stability. To ensure a reliable and stable smart micro-grid (SMG), it is essential to implement an adaptive primary frequency controller specifically designed for islanded operations. This task presents significant challenges due to the unpredictable nature of disturbances, variations in load, and changes in mechanical torque parameters. This chapter proposes an innovative approach to frequency control for diesel-engine generators. The proposed method leverages reinforcement learning control through continuous Q-learning, aiming to overcome the linear parameter varying (LPV) nature of the system without prior knowledge of the system dynamics.

3.1 Background

One of the crucial elements in SMGs is having more high-technologies, modern controls, along with advanced communications. Among many technologies, inner control loops, as well as the primary controls, aim to regulate the frequency and voltage to their reference values. Diesel-engine generators are an innate part of SMGs which offer primary frequency responses to restore system frequency in response to variations in generation and load.

One of the important SMG operation modes is the islanded mode, which requires the SMG to control the frequency within the microgrid [47]. On this matter,

diverse frequency control methods of different hierarchical levels are being considered and deployed elaborated in the expansion, implementation, and intelligent of current operating microgrids. Therefore, it is necessary to make the primary frequency control of SMG more robust and reliable. This influences the whole dynamic system as it is the most inner loop from the viewpoint of the total closed-loop dynamic system, and it tries to reach overall stability. Among different distributed energy resources having the duty of primary frequency control, Diesel-engine-based distributed energy resources are frequently used in many microgrid projects, such as hospitals, and university campuses for controlling the frequency of their islanded microgrids [48].

Their dynamics need dedicated attention from the viewpoint of both robust and optimal controls due to the following points.

First, the transient frequency response of the islanded power system needs to be optimized in regard to performance while considering the error. Second, many mechanical parameters are not exactly measurable since the parameters of their associated dynamic systems are uncertain. Thus they can be taken to lie within a predefined range. Third, voltage control loops have uncertain disturbances which come from the fast dynamics of electrical variables. Fourth, direct uncertain disturbance of the load significantly affects output performance. Fifth, owing to load variations, the frequency of SMG will fluctuate in a wide range due to the limited value of rotational inertia of the generators.

Therefore, the issues mentioned above can abruptly affect the frequency regulation of SMGs during islanding, and hence protections and other control are

needed. In [71], authors propose a control method based on PID controllers for a wind-diesel system. However, they did not include system parameter uncertainties in their control design. Thus, their control performance may not be robust against all parameter variations in the system. So, one of the main motivations of this research is to propose a systematic method to account for parameter uncertainties in designing SMG controllers. To solve these issues, considering practical aspects of both power and control disciplines, this chapter suggests a Q-learning frequency control for a Diesel-engine generator with ensured disturbance rejection and tracking capability via online LPV continuous Q-learning without having any knowledge of the system dynamics. Deep Q-learning based torque control has been shown to be effective for permanent magnet synchronous machine with known system dynamics [72]. Mechanical and electrical torque are used for calculating frequency of Diesel-engine generator as shown below equation (3.1).

In control engineering, disturbance rejection with asymptotic tracking such as output regulation problem has been researched (e.g., [49]). However, such publications tend not to seek optimal solutions.

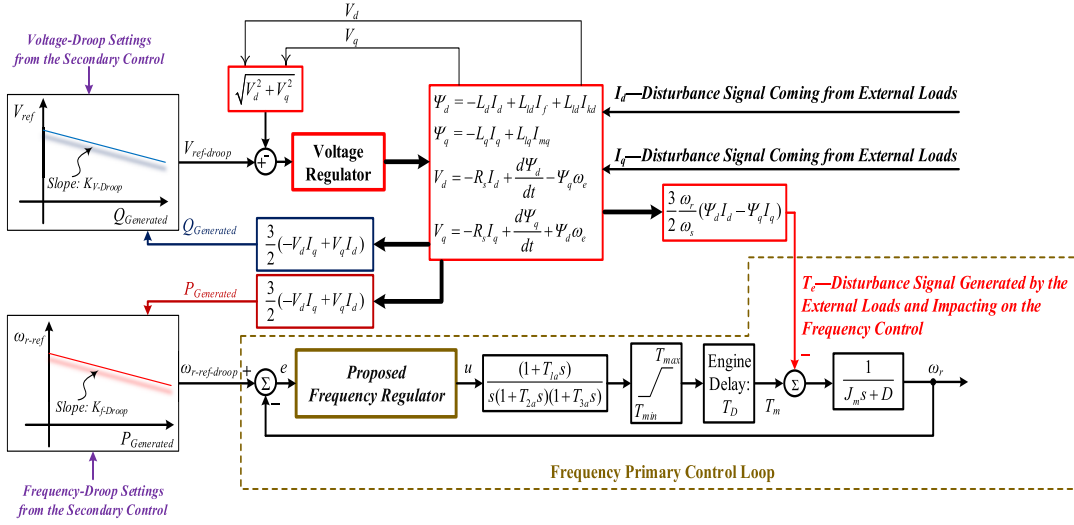


Fig. 3.1. Diagram of dynamics of the primary frequency control of an engine generator within an islanded SMG.

Model-based linear optimal output regulation has been researched for enhancing the transient response of systems [50]. However, the optimal control policies proposed in these papers usually presume the precise knowledge of the control system. Discrete time Q-learning is a practically sound, data-driven approach which offers a way to solve the optimal control problem in a discrete iterative manner. Basically, it is an approximate optimal control method that can estimate the optimal controller via online/real time data [51], [52]. For continuous systems, Chun [53] developed a Q-learning algorithm by discretizing the state as a semi-continuous form of the discrete Q-function. However, the precision of this algorithm depends on the number of discretized samples and discretizing the state space for high dimensional systems requires a large memory space and computationally expensive learning

process. Mehta and Meyn [54] approximated the optimal control policy by using Q-learning. However, they did not present an explicit and naturally basis set for approximating the Q-function.

In this chapter, a continuous Q-learning algorithm is developed for solving the discounted cost LPV system problem. The major contributions of this chapter are summarized as follows:

- 1) Proposes algorithm is an adaptive learning algorithm that converges online to the optimal control solution for completely unknown continuous-time linear systems. This algorithm does not require an initial policy for controlling the system. Frequency regulation problem is formulated in LPV state-space representation applicable to the continuous Q-learning design problem considering all dynamics and disturbance. It is analytical design of robust controllers for a SMG Diesel-engine generator system in the continuous Q-learning framework.
- 2) In order to implement the proposed Q-learning algorithm, the Q-function is parameterized as a quadratic form in terms of the state, the input, and the input derivatives. It is shown that such a parameterization is fundamental for Q-functions and can be precisely justified mathematically. This chapter apprehends the control problem regarding completely unknown LPV system which is good matched with the existing problems under study for SMGs. Also, closed-loop systems with reinforcement control strategies are used.
- 3) Moreover, it states that the new approximate parameterization yields approximation errors that go to zero as more terms are added. This renders the SMG controller resilient

to changes in all the system parameters of the Diesel-engine generator, as well as variations of the load powers. The proposed continuous Q-learning approach is based on a value iteration (VI) which differs from the policy iteration approach, the proposed learning strategy does not depend on knowledge of an initial possible control policy.

4) The proposed value iteration Q-learning algorithm does not require an initial stabilizing policy. The convergence of this algorithm can be explained by using the Banach contraction principle. Input and output electrical torque control performance are found to be better than with the PID approach via online continuous Q-learning without having knowledge of the system dynamics.

3.2 Frequency dynamics of islanded SMG modeling

We model an islanded SMG in the context of benefiting from two parallel engine generators. An islanded SMG's engine generator supplying an electric load has been shown above. This work benefits from previous works in this area [55]-[57]. Equation (3.1) below specifies the dynamic model used in primary frequency controls, as shown in Fig. 3.1. Fig. 3.1 is based on the synchronous machine model used in this work and is adopted from the IEEE 1110-2019 standard [58].

Moreover, it should be noted that if other new technologies such as energy storage systems and renewable energy systems are used, based on the necessity of implementing virtual inertia ([59], [60]) a dynamic model similar to (3.1) will be derived for the frequency dynamics in islanded SMGs.

$$\frac{d}{dt}\omega_r = \frac{1}{J}(T_m - T_e - D\omega_r), T_e = \frac{3}{2}\frac{\omega_r}{\omega_s}(\psi_d I_d - \psi_q I_q) \quad (3.1)$$

where ω_r is mechanical frequency (rotor speed), J is inertia, T_m mechanical torque, T_e electrical torque which is disturbance signal generated by the external loads and impacting on the frequency control, D is friction coefficient, ω_s is the system base frequency for electrical variables. In Fig. 3.1, ω_e is the electrical frequency for electrical variables, $\psi_{\frac{d}{q}}$ is d/q -axis of the flux linkage matrix, $I_{\frac{d}{q}}$ d/q -axis of the current matrix, I_f is the field current in the rotor winding, J is the inertia of machine, $L_{\epsilon d}$ is the leakage inductance of d-winding, $L_{\epsilon q}$ is the leakage inductance of q-winding, $L_{\frac{d}{q}}$ is the d/q -axis of the inductance matrix, L_{ld} is $L_d - L_{\epsilon d}$, L_{lq} is $L_q - L_{\epsilon q}$, r_s is the both d -axis and q -axis of the resistance matrix, $V_{\frac{d}{q}}$ d/q -axis of the voltage matrix.

In this chapter, the time delay is approximated by a nonminimum phase system whose transfer function $H(s)$ is

$$H(s) = \frac{1-0.5T_D s}{1+0.5T_D s} \quad (3.2)$$

From (3.1) and Fig. 3.1, we can represent the Deisel-engine generator frequency control for LPV diesel-engine generator in a LPV state-space representation as below:

$$\frac{d}{dt}x(t) = A_c(\rho(t))x(t) + B_c(\rho(t))u(t), y = C(\rho(t))x(t) \quad (3.3)$$

where the state $x \in R^5$, $\rho \in R^5$ the input $u \in R^5$ regarding $V_d, V_q, I_d, I_q, \omega_r$ is shown in Fig. 3.1. Eq. (3.3) is the state representation of linear parameter varying of $V_d, V_q, I_d, I_q, \omega_r$. Eq. (3.1) governs the rotational angular frequency and electrical torque. The dynamics of renewable energy systems are much slower than proposed

modeling sampling interval. Thus, this premise is reasonable for the scope of our modeling. The pair (A_c, B_c) is controllable, and (C, A_c) is observable.

3.3 Solving LPV problems with Q-function based iterative algorithm

The horizontal reduction cost LPV problem's Q-function is present here. To find the solution of the algebraic riccati equation (ARE), an iterative algorithm-based Q-function is presented here. This algorithm is offline approach. By using Banach contraction principle, the convergence of this algorithm can be proven [182]. This algorithm is a basis to present an online Q-learning algorithm for continuous systems which is presented in the next section.

We consider the problem of designing an LPV state-feedback control law $\mu(t) = K_s(\rho(t), p(t))x(t)$ where $\rho(t)$ is a vector scheduling $A(\rho(t))$, $B(\rho(t))$, and $C(\rho(t))$ parameter defined in the following and $p(t) \in \Pi \subset R^n$ is used to achieve the shifting linear quadratic control's (SLQC) $S(p(t))$ and $R(p(t))$. It is described as admissible concerning performance LPV value function, if $K_s(\rho(t), p(t))x(t)$ is continuous, $\mu(0) = 0$, $\mu(t) = K_s(\rho(t), p(t))x(t)$ stabilizes the system dynamics (3.3), and discounted performance index LPV value function index is finite. Given the continuous LPV system (3.3), the SLQC problem aims at designing the controller $K_s(\rho(t), p(t))$ in $K_s(\rho(t), p(t))x(t)$ such that for $p(t) = p, \rho(t) = \rho$ it minimizes for a given admissible control policy $\mu(t) = K_s(\rho(t), p(t))x(t)$, the continuously differentiable value function is

$$V^\mu(x, p) = \int_t^\infty e^{-\gamma(\tau-t)} \left(x^T(\tau) S(p) x(\tau) + \mu^T(x(\tau)) R(p) \mu(x(\tau)) \right) d\tau \quad (3.4)$$

Notice that since $S(p)$ and $R(p)$ are scheduled with respect to p , and the controller as well, the characteristics of optimality in the sense of the considered criterion can be varied online. Thus, the performance of the closed-loop system can be changed online, this being the main feature of the SLQC. Then, by guaranteeing that $V^\mu(x, p) < \gamma$, and using the Lyapunov function $V(x, p) = x^T (K_0 + \sum_{i=1}^{n_p} K_i P_i) x = x^T P x$ by considering $S(p) = H(p)^T H(p), \forall p \in \Pi$, and $V(x_0, p_0) = x_0^T K_0 x_0 < \gamma$, and $M + M^T$ will be indicated as $He\{M\}$. $V^\mu(x, p) < V(x_0, p_0) < \gamma$, minimization of γ would minimize $V^\mu(x, p)$ as well. Combining the inequality $\dot{V}(x, p) + x^T(t)S(p)x(t) + \mu^T(t)R(p)\mu(t) < 0$ and $\mu(t) = K_s(\rho(t), p(t))x(t)$,

$$\begin{aligned} & (A(\rho) + B(\rho)K(\rho, p))^T P x(t) + x(t)^T P (A(\rho) + B(\rho)K(\rho, p)) + x(t)^T \dot{P} x(t) \\ & + x(t)^T S(p)x(t) + u(t)^T R(p)u(t) < 0. \end{aligned} \quad (3.5)$$

This leads to the parameter dependent LMI:

$$\begin{aligned} & (A(\rho) + B(\rho)K(\rho))^T P + P(A(\rho) + B(\rho)K(\rho)) + \dot{P} + S(p) + K(\rho)^T R(\rho)K(\rho) \\ & < 0, \forall \rho \in \Theta. \end{aligned} \quad (3.6)$$

Then, considering the results about dual matrix inequalities obtained by [86] it can be shown that (3.6) is equivalent to:

$$\begin{aligned} & P(A(\rho) + B(\rho)K(\rho))^T + (A(\rho) + B(\rho)K(\rho))P + P\dot{P}P + PS(p)P \\ & + PK(\rho)^T R(\rho)K(\rho)P < 0, \forall \rho \in \Theta. \end{aligned} \quad (3.7)$$

Equation (3.7), after a change of variable $\Gamma(\rho) \triangleq K(\rho)P$ becomes:

$$\begin{aligned}
& PA(\rho)^T + \Gamma(\rho)^T B(\rho)^T + A(\rho)P + B(\rho)\Gamma(\rho) + (PH^T \ \Gamma(\rho)^T) \begin{pmatrix} I & O \\ O & R \end{pmatrix} \begin{pmatrix} HP \\ \Gamma(\rho) \end{pmatrix} \\
& < O, \forall \rho \in \Theta.
\end{aligned} \tag{3.8}$$

By applying Schur complements matrix [69], (3.5) and (3.8) can be reshaped as:

$$\begin{pmatrix} \gamma & x_0^T \\ x_0 & P \end{pmatrix} \succ 0 \tag{3.9}$$

$$\begin{pmatrix} He\{A(\rho)P + B(\rho)\Gamma(\rho, p)\} & PH(p)^T & \Gamma(\rho, p)^T \\ H(p)P & -I & O \\ \Gamma(\rho, p) & O & -R(p)^{-1} \end{pmatrix} < O, \forall \rho \in \Theta, \forall p \in \pi \tag{3.10}$$

Since (3.10) represents infinite constraints, the assumptions that B is constant, and that:

$$\begin{pmatrix} H(p) \\ R(p)^{-1} \end{pmatrix} = \sum_{j=1}^P \pi_j(p) \begin{pmatrix} H_j \\ \tilde{R}_j \end{pmatrix}, \forall p \in \Pi \tag{3.11}$$

with $\sum_{j=1}^P \pi_j(p) = 1, \pi_j(p) \geq 0$ is done. Then by choosing $K_s(\rho(t), p(t))$ as:

$$K_s(\rho(t), p(t)) = \sum_{i=1}^N \alpha_i(\rho(t)) \sum_{j=1}^p \pi_j(p) K_{ij} \tag{3.12}$$

(3.10) can be reduced to:

$$\begin{pmatrix} He\{A_i P + B \Gamma_{ij}\} & PH_j^T & \Gamma_{ij}^T \\ H_j P & -I & O \\ \Gamma_{ij} & O & -\tilde{R}_j \end{pmatrix} < O, \forall i = 1, \dots, N, \forall j = 1, \dots, P \tag{3.13}$$

the vertex controller gains can be calculated as $K_{ij} = \Gamma_{ij} P^{-1}$. Then $\min_{P, \Gamma_i, i=1, \dots, N} \gamma$ can be

minimized subject to (3.9) and (3.13).

Equation (3.4) can be represented with the integral reinforcement control (IRC) from:

$$V^\mu(x, p) = \int_t^{t+T} e^{-\gamma(\tau-t)} (x^T S(p)x + \mu^T R(p)\mu) d\tau + e^{-\gamma T} V^\mu(x(t+T), p) \quad (3.14)$$

for any $T > 0$. This is referred to as the IRC Bellman equation. Using this IRC Bellman equation, we can develop a partially model-free algorithm for solving the LPV problem as [78].

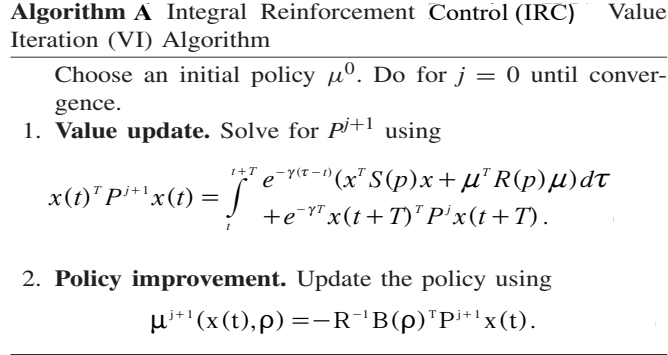


Fig. 3.2. Integral reinforcement control (IRC) value iteration algorithm.

The initial admissible control policy is not necessary for IRC value iteration Algorithm A. The updated value kernel P^{j+1} in (15) is calculated to minimize the temporal difference (TD) error.

$$e(t) = e^{-\gamma T} x(t+T)^T P^j x(t+T) - x(t)^T P^{j+1} x(t) + \int_t^{t+T} e^{-\gamma(\tau-t)} (x^T S(p)x + \mu^{jT} R(p)\mu^j) d\tau \quad (3.15)$$

To apply Algorithm A, the data is estimated at each iteration. The estimated data at each time increment is $(x(t), x(t+T), \varphi[t:t+T])$, where the integral reinforcement measured on each time interval is:

$$\varphi[t:t+T] = \int_t^{t+T} e^{-\gamma(\tau-t)} r(x(\tau), \mu(\tau), p) d\tau \quad (3.16)$$

where $r(x(\tau), \mu(\tau), p) = x^T(\tau)S(p)x(\tau) + \mu^T(\tau)R(p)\mu(\tau)$. After processing much data, the least squares approach can be used to solve the value update [62].

Integral reinforcement control (IRC)-based policy iteration (PI) algorithms applied to LPV problem for completely or partially unknown system dynamics [63]. This algorithm must start with an admissible control policy. Convergence of IRC-based PI algorithms was proven in [61]. Jiang showed how to carry out IRC-based PI with no knowledge of the system dynamics [64]. The approach of [60], [63], and [64] determines the optimal control solution online without knowing complete system dynamics. However, these methods are based on the value function. Our approach is to apply the idea into continuous systems based on continuous Q-learning.

Integral reinforcement learning VI Algorithm A needs the knowledge of the input matrix dynamics $B(p)$. The goal is to look for an algorithm for reinforcement learning (RL) which necessary no system dynamics information. To reach this goal, a Q-learning algorithm will be improved for solving the LPV problem. Based on the IRC definition (3.14), the Q-function for an admissible policy $u(x) = \mu(x)$ is defined as

$$Q^\mu(x(t), \bar{u}[t:t+T], p) := \int_t^{t+T} e^{-\gamma(\tau-t)} (x^T S(p)x + \mu^T R(p)\mu) d\tau + e^{-\gamma T} V^\mu(x(t+T), P) \quad (3.17)$$

where $\bar{u}[t:t+T] := \{u(\tau): t \leq \tau < t+T\}$. This definition was also used by Chun [53]. Drive by this Q-function, the following iterative algorithm is presented to solve

the infinite horizon discounted LPV problem.

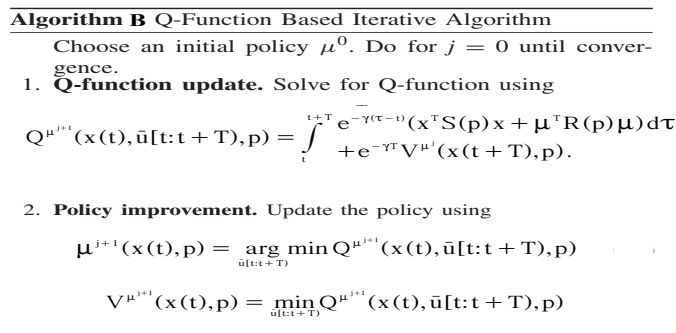


Fig. 3.3. Q-function based iterative algorithm.

By incorporating the value update step 1 and policy enhancement step 2, we can obtain the equivalent value function iteration as

$$V^{\mu^{j+1}}(x(t), p) = \min_{\bar{u}[t:t+T]} \left\{ \int_t^{t+T} e^{-\gamma(\tau-t)} (x^T S(p)x + \mu^T R(p)\mu) d\tau + e^{-\gamma T} V^{\mu^j}(x(t+T), p) \right\}. \quad (3.18)$$

If Algorithm B converges, it converges to the optimal solution given by the Bellman equation. Algorithm B converges to the solution of the Bellman equation which is equivalent to ARE.

Let (X, d) be a metric space. Then a map $T: X \rightarrow X$ is called a contraction mapping on X if there exists $\Gamma \in [0, 1)$ such that $d(T(x), T(y)) \leq \Gamma d(x, y)$ [64]. Consider the set of bounded functions $B(G, \mathbb{R}) \equiv \{V^\mu: G \rightarrow \mathbb{R}\}$. For every $x \in G$, $V^\mu(x)$ is a finite real valued function. Define the operator $\Gamma: B(G, \mathbb{R}) \rightarrow B(G, \mathbb{R})$ such that:

$$(\Gamma V^\mu)(x(t), p) = \min_{\bar{u}[t:t+T]} \left\{ \int_t^{t+T} e^{-\gamma(\tau-t)} r(x(\tau), \mu(\tau), p) d\tau + e^{-\gamma T} V^\mu(x(t+T), p) \right\} \quad (3.19)$$

Then, 1) there exists a unique $V^\mu(x(t), p) = (\Gamma V^\mu)(x(t), p)$, 2) for $j=0, 1, \dots$, $V^{\mu^{j+1}}(x(t), p) = (\Gamma V^{\mu^j})(x(t), p)$ converges to $V^\mu(x(t), p)$ for every $V^{\mu_0}(s(t), p)$.

Proof: Part 1) is proven by using the Banach contraction principle. It is shown that Γ is a contraction map with norm $\|V^\mu\| = |V^\mu(x, p)|$ for every $x \in G$. The space $B(G, \mathbb{R})$ is a Banach space. For all $x \in G$ there exists a control $u \in U$ such that $V^\mu, W^\mu \in B(G, \mathbb{R})$. By using (3.16) and (3.19) one has

$$(\Gamma V^\mu)(x(t), p) - (\Gamma W^\mu)(x(t), p) \leq \varphi[t: t+T] + e^{-\gamma T} V^\mu(x(t+T), p) - \varphi[t: t+T] - e^{-\gamma T} W^\mu(x(t+T), p) \leq \sup e^{-\gamma T} (V^\mu(x, p) - W^\mu(x, p)) \quad (3.20)$$

$\|\Gamma V^\mu - \Gamma W^\mu\| \leq e^{-\gamma T} \|V^\mu - W^\mu\|$. It is seen that Γ is a contraction map whenever the value of discount factor $\gamma > 0$. Now, by applying the Banach contraction principle, Γ has a unique fixed point. Thus, there exists a unique $V^\mu(x(t), p) \in B(G, \mathbb{R})$ which holds $V^\mu(x(t), p) = (\Gamma V^\mu)(x(t), p)$. Part 2) can be proven by the direct application from the proof of Banach fixed point theorem.

3.4 Continuous system Q-learning algorithm

For discrete time Q-learning, a natural basis set for Q-function approximation is given by quadratic terms of the action and state. Natural basis set has not yet been talked about continuous Q-functions. In this section, it is addressed that for continuous Q-function approximation one should apply quadratic terms of the states, the input derivatives, and the input. It is shown which a parameterization is

natural for continuous Q-functions and can be precisely give grounds for mathematically. Based on this result, this section delivers a Q-learning algorithm to solve the horizontal LPV problem. This possibly gives an adaptive control algorithm which converges online to the optimal control solution for completely unknown systems. The online version of this algorithm is given next.

3.4.1 Continuous systems Q-learning algorithm

First, a Q-function Bellman equation is obtained using the Q-function

(3.17). Then, based on this Bellman equation and Algorithm B, a Q-learning algorithm is expanded. To find the solution to the Q-function based Bellman equation, the Q-function is properly parameterized in section IV-B by using the Taylor series expansion as a quadratic form in terms of control input, state, and its derivatives. For any fixed control policy $\mu(x)$, one has $V^\mu(x(t+T), p) = Q^\mu(x(t+T), \bar{u}[t+T:\infty), p)$. Thus, using $V^\mu(x(t+T), p) = Q^\mu(x(t+T), \bar{u}[t+T:\infty), p)$ in the IRC Bellman equation (3.14), Define the Q-function based Bellman equation as:

$$Q^\mu(x(t), \bar{u}[t:t+T], p) = \int_t^{t+T} e^{-\gamma(\tau-t)} (x^T S(p)x + \mu^T R(p)\mu) d\tau + e^{-\gamma T} Q^\mu(x(t+T), \bar{u}[t+T:\infty), p) \quad (3.21)$$

Using the Bellman equation (3.21) in Algorithm B, the following Q-learning algorithm is given. To find the solution to the Bellman equation, a parameterization of the Q-function is presented in the next subsection.

Algorithm C Q-Learning Algorithm for Continual Systems
Choose an initial policy μ^0 . Do for $j = 0$ until convergence.
1. Q-function update. Solve for Q-function using
$Q^{\mu^{j+1}}(x(t), \bar{u}[t:t+T], p) = \int_t^{t+T} e^{-\gamma(\tau-t)} (x^T S(p)x + \mu^T R(p)\mu) d\tau + e^{-\gamma T} Q^{\mu^j}(x(t+T), \bar{u}[t+T:\infty), p).$
2. Policy improvement. Update the policy using
$\mu^{j+1}(x(t), p) = \arg \min_{u(t:t+T)} Q^{\mu^{j+1}}(x(t), \bar{u}[t:t+T], p)$
$V^{\mu^{j+1}}(x(t), p) = \min_{u(t:t+T)} Q^{\mu^{j+1}}(x(t), \bar{u}[t:t+T], p)$

Fig. 3.4. Q-learning algorithm for continuous systems.

3.4.2 Continuous Q-function parametrization

Parameterization for the Q-function is given which admits practical implementation of Q-learning Algorithm C. The parameterization is in terms of the $x(t)$, $u(t)$, and the derivatives of $u(t)$. Given the system dynamics of a continuous-time system over the time interval $[t, t+T]$ in terms of $x(t)$, $u(t)$, and the derivatives of $u(t)$ as in $Z_{k-1}(t)$. This approach is based on Taylor series expansion of $x(t+\tau)$ around $x(t)$ for every τ , $0 \leq \tau \leq T$ and $t \leq t + \tau < t + T$. Given the system dynamics and its state vector $x(t)$. Then for every τ , $t \leq t + \tau < t + T$,

$$x(t + \tau) = \bar{w}_k^T(\tau) M_k Z_{k-1} + E_k^x(t + \tau), \quad (3.22)$$

$$\text{where} \quad x(t + T) \cong x(t) + \frac{\tau}{1!} \dot{x}(t) + \frac{\tau^2}{2!} \ddot{x}(t)$$

$$= \begin{bmatrix} \frac{\tau}{1!} \\ \vdots \\ \frac{\tau^k}{k!} \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ A & B & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ A^k & A^{k-1}B & A^{k-2}B & \dots & B \end{bmatrix} \begin{bmatrix} x \\ u \\ \vdots \\ u^{(k-1)} \end{bmatrix}$$

$E_k^x(t + \tau)$ is the Taylor series approximation error which converges to zero as $k \rightarrow \infty$.

For any admissible policy $\mu(t) = K_s(\rho(t), p(t))x(t)$ the value of the LPV quadratic performance index (cost function) value function equation (3.4) is quadratic so $V^\mu(x(t + T)) = x^T(t + T)Px(t + T)$. Then, the parameterized Q-function is

$$Q^\mu(x(t), \bar{u}[t: t + T], p) = Z_{k-1}^T(t) \left(\left(\bar{M}_{k-1}^T S_k(p) \bar{M}_{k-1} + \bar{R}_k(p) \right) + \right.$$

$$e^{-\gamma T} M_k^T \bar{W}_k(T) P \bar{W}_k^T(T) M_k \Big) Z_{k-1}(t) + E_k^Q(t+T) \quad (3.23)$$

where the error $E_k^Q(t+T) \rightarrow 0$ as $k \rightarrow \infty$.

3.5 Online Q-learning

Parameterization of the continuous Q-function comes from equation (3.23)

is used to implement the Q-learning Algorithm C. This permits the online solution of the LPV problem in real time for completely unknown system dynamics.

3.5.1 Online continuous Q-learning

Write the parameterized Q-function in equation (3.23) as

$$Q^\mu(x(t), \bar{u}[t:t+T], p) \equiv Z_{k-1}^T(t) H Z_{k-1}(t) + E_k^Q(t+T) \quad (3.24)$$

where $H = \left(\left(\bar{M}_{k-1}^T S_k(p) \bar{M}_{k-1} + \bar{R}_k(p) \right) + e^{-\gamma T} M_k^T \bar{W}_k(T) P \bar{W}_k^T(T) M_k \right)$

Define H in (3.24) as $H \equiv \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$ where $H_{11} \in \mathbb{R}^{n \times n}, H_{22} \in \mathbb{R}^{m \times m}, H_{12} \in$

$\mathbb{R}^{n \times m}, H_{21} \in \mathbb{R}^{m \times n}$.

Then according to (3.24), the approximated Q-function can be written as

$$\hat{Q}^\mu(x(t), \bar{u}[t:t+T], p) = [X^T(t) \quad \bar{u}_{k-1}^T(t)] \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} X(t) \\ \bar{u}_{k-1}(t) \end{bmatrix} \quad (3.25)$$

For a fixed control policy, using this parameterized form of the Q-function,

the Bellman equation (3.21) can be approximated

$$\begin{aligned} \hat{Q}^{\mu^{j+1}}(X(t), \bar{u}[t:t+T], p) &= \int_t^{t+T} e^{-\gamma(\tau-t)} (X^T S(p) X + u^T R(p) u) d\tau \\ &+ e^{-\gamma T} \hat{Q}^{\mu^j}(X(t+T), \bar{u}[t+T:\infty), p) \end{aligned} \quad (3.26)$$

where $\hat{Q}^{\mu^{j+1}}(x(t), \bar{u}[t: t+T], p) = Z_{k-1}^T(t) H^j Z_{k-1}(t)$

Given the system dynamics of LPV system, and its control input vector $u(t)$. Then for every $\tau, t \leq t + \tau < t + T$,

$$u(t + \tau) = \bar{W}_{k-1}^T(\tau) \bar{u}_{k-1}(t) + E_k^u(t + \tau), \forall \tau, 0 \leq \tau < T. \quad (3.27)$$

where $E_k^u(t + \tau)$ is the Taylor series approximation error which converges to zero as $k \rightarrow \infty$.

Proof: By using the Taylor's series expansion of $u(t + \tau)$ about $u(t)$, it yields

$$u(t + \tau) = u(t) + \frac{\tau}{1!} \dot{u}(t) + \frac{\tau^2}{2!} \ddot{u}(t) + \dots + \frac{\tau^{k-1}}{(k-1)!} u^{(k-1)}(t) + E_k^u(t + \tau) \quad (3.28)$$

or equivalently

$$u(t + \tau) = \left[1 \quad \frac{\tau}{1!} \quad \frac{\tau^2}{2!} \dots \frac{\tau^{k-1}}{(k-1)!} \right] \times \left[u(t)^T \quad \dot{u}(t)^T \dots u^{(k)}(t)^T \right]^T + E_k^u(t + \tau) \quad (3.29)$$

$$\cong \bar{W}_{k-1}^T(\tau) \bar{u}_{k-1}(t) \quad \forall \tau, 0 \leq \tau \leq T. \quad (3.30)$$

Lagrange Error Bound: The error value in (3.28) can be got by using Lagrange method [64]. Let $E_k^u(t + \tau), 0 \leq \tau \leq T$ is error of the k -th order Taylor polynomial of $u(t + \tau)$. Then the absolute value is

$$|E_{k-1}^u(t + \tau)| = |u(t + \tau) - \bar{W}_{k-1}^T(\tau) \bar{u}_{k-1}(t)| \quad (3.31)$$

Let x be a function such that all of its derivatives are continuous, then the error is bounded by

$$|E_{k-1}^u(t + \tau)| \leq \frac{M}{k!} |T|^k \quad (3.32)$$

where M is some value satisfying $\left| u^{(k)}(t + \tau)|_{\tau=0} \right| \leq M$. On the interval between t and $t+T$.

To apply the policy improvement step of Algorithm C, it is required to. Find the minimum of $\hat{Q}(x(t), \bar{u}[t:t+T], p)$ over $\bar{u}[t:t+T]$. Thus, according to equation (3.27), differentiate $\hat{Q}(x(t), \bar{u}[t:t+T], p)$ with respect to $\bar{u}_{k-1}(t)$ to yield

$$\frac{\partial \hat{Q}(x, \bar{u}[t:t+T], p)}{\partial \bar{u}_{k-1}(t)} = 2H_{21}X(t) + 2H_{22}\bar{u}_{k-1}(t) = 0 \quad (3.33)$$

This yields

$$\bar{u}_{k-1}(t) = -H_{22}^{-1}H_{21}x(t) \quad (3.34)$$

Substituting (3.34) in (3.30)

$$\bar{u}[t:t+T] \cong -\bar{W}_{k-1}^T(\tau)H_{22}^{-1}H_{21}X(t), \forall \tau, 0 \leq \tau \leq T \quad (3.35)$$

Algorithm D below employs Eqs. (3.26), (3.27) and (3.35), to apply the policy.

Evaluation and policy improvement steps of Algorithm D is shown below.

Algorithm D Online Continual Q-Learning

Pick an initial policy μ^0 . Do for $j = 0$ until convergence

1. **Q-function update.** Solve for H^{j+1} using

$$\begin{aligned} & \begin{bmatrix} x^T(t) \\ \bar{u}_{k-1}^{Tk}(t) \end{bmatrix}^T H^{j+1} \begin{bmatrix} x(t) \\ \bar{u}_{k-1}(t) \end{bmatrix} = \\ & \int_t^{t+T} e^{-\gamma(\tau-t)} (x^T(\tau)S(p)x(\tau) + (\mu(x(\tau)))^T R(p)\mu(x(\tau))) d\tau \\ & + e^{-\gamma T} \begin{bmatrix} x^T(t+T) \\ \bar{u}_{k-1}^{Tk}(t+T) \end{bmatrix}^T H^j \begin{bmatrix} x(t+T) \\ \bar{u}_{k-1}(t+T) \end{bmatrix}. \end{aligned}$$

2. **Policy improvement.** Update the policy using

$$\begin{aligned} \bar{u}_{k-1}(t) &= -H_{22}^{-1}H_{21}x(t) \\ \mu^{j+1}(x(\tau)) &= -\bar{w}_k^T(\tau)H_{22}^{-1}H_{21}x(t). \end{aligned}$$

Fig. 3.5. Online continuous Q-learning.

The updated value kernel H^{j+1} in (3.29) is computed to minimize the temporal difference (TD) error.

$$e(t) = e^{-\gamma T} [\bar{u}_{k-1}^T(t+T)]^T H^j \begin{bmatrix} X(t+T) \\ \bar{u}_{k-1}(t+T) \end{bmatrix} - [\bar{u}_{k-1}^T(t)]^T H^{j+1} \begin{bmatrix} X(t) \\ \bar{u}_{k-1}(t) \end{bmatrix} + \int_t^{t+T} e^{-\gamma(\tau-t)} (X^T S(p)X + \mu^{jT} R(p) \mu^j) d\tau \quad (3.36)$$

To solve (3.35) for H^{j+1} the quadratic basis set $[x(t)^T, \bar{u}_{k-1}(t)^T]^T \otimes [x(t)^T, \bar{u}_{k-1}(t)^T]^T$ must be persistently exciting (PE).

3.5.2 Input derivatives computation

The Q-function update (3.29) in Algorithm D requires knowledge of the derivatives of the control input $u(t)$ and $u(t+T)$. Next, it is shown that these input derivatives can be got from $x(t)$. Using $V^{\mu^{j+1}}(x(t), p) =$

$\min_{\bar{u}[t:t+T]} Q^{\mu^{j+1}}(x(t), \bar{u}[t:t+T], p)$ and taking the derivatives of control input, the

following matrix from is got:

$$\begin{bmatrix} u(t+\tau) \\ \dot{u}(t+\tau) \\ \vdots \\ u^{(k-1)}(t+\tau) \end{bmatrix} \cong \begin{bmatrix} 1 & \tau & \frac{\tau^2}{2!} & \cdots & \frac{\tau^{k-1}}{(k-1)!} \\ 0 & 1 & \tau & \cdots & \frac{(k-1)\tau^{k-2}}{(k-1)!} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \tau \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} u(t) \\ \dot{u}(t) \\ \vdots \\ u^{(k-1)}(t) \end{bmatrix} \quad (3.37)$$

Indicating

$$\bar{U}_k(\tau) = \begin{bmatrix} 1 & \tau & \frac{\tau^2}{2!} & \dots & \frac{\tau^{k-1}}{(k-1)!} & \frac{\tau^k}{k!} \\ 0 & 1 & \tau & \dots & \frac{(k-1)\tau^{k-2}}{(k-1)!} & \frac{k\tau^{k-1}}{k!} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \tau & \frac{k(k-1)\dots 3\tau^2}{k!} \\ & & & & 1 & \frac{k!}{\tau} \end{bmatrix} \quad (3.38)$$

and using (3.34) in (3.37) becomes

$$\bar{u}_{k-1}(t + \tau) \cong -U_{k-1}(\tau)H_{22}^{-1}H_{21}x(t)$$

This is used to compute the input derivative vector necessary by the Q-function.

update in

$$\begin{aligned} & [\bar{u}_{k-1}^T(t)]^T H^{j+1} \begin{bmatrix} X(t) \\ \bar{u}_{k-1}(t) \end{bmatrix} \\ &= \int_t^{t+T} e^{-\gamma(\tau-t)} \left(X^T(\tau)S(p)X(\tau) + \mu(x(\tau))^T R(p)\mu(x(\tau)) \right) d\tau \\ &+ e^{-\gamma T} [\bar{u}_{k-1}^T(t+T)]^T H^j \begin{bmatrix} X(t+T) \\ \bar{u}_{k-1}(t+T) \end{bmatrix}. \end{aligned} \quad (3.39)$$

3.6 Simulation results

To test the suggested primary controller, we consider an SMG as in Fig. 3.1 with a diesel engine generator with parameters from Table 3.1, and simulate the system with the primary controller including the suggested controller.

To this end, the aforementioned smart grid utilizing the proposed primary frequency controller has been examined for analysis under the islanded mode of operation. The implemented frequency regulator for Fig. 3.1 is shown in Fig. 3.6.

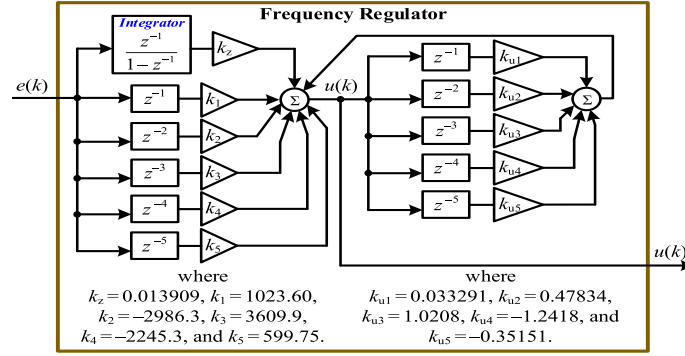


Fig. 3.6. Diagram of implemented frequency regulator with coefficients.

Table 3.1. Parameters of each set of diesel engine generators in for Fig. 3.1

Parameter	Value	Parameter	Value
Nominal power	1.0 MVA	R_s	$9.4 \times 10^{-4} \Omega$
Line-to-line nominal voltage	480 V	L_{ts}	$1.3 \times 10^{-5} H$
Nominal electrical frequency	60 Hz	L_{md}	$6.5 \times 10^{-4} H$
Machine number of pole pairs	2	J_m	$92.88 \text{ kg} \cdot \text{m}^2$
Actuator gain (K)	10	D	$1.0 \text{ N} \frac{\text{ms}}{\text{rad}}$
Actuator time constant 1 (T_{a1})	$2.5 \times 10^{-2} s$	L_{mq}	$4.7 \times 10^{-4} H$
Actuator time constant 2 (T_{a2})	$9.0 \times 10^{-4} s$	R_f	$2.1 \times 10^{-4} \Omega$
Actuator time constant 3	$5.7 \times 10^{-3} s$	L_{lfd}	$1.4 \times 10^{-4} H$

(T_{a3})			
Engine delay (T_D)	$2.4 \times 10^{-2} s$	R_{kd}	$4.4 \times 10^{-2} \Omega$
Exciter PI proportional gain	2.5×10^{-2}	R_{kq}	$4.0 \times 10^{-3} \Omega$
Exciter PI integral gain	7.0×10^{-3}	L_{lkd}	$1.5 \times 10^{-3} H$
Exciter time constant (T_e)	$1.0 \times 10^{-8} s$	L_{lkq}	$9.9 \times 10^{-3} H$
Exciter gain (K_e)	1.0	$\frac{N_s}{N_{fd}}$	2.5×10^{-2}
$\frac{N_s}{N_{kd}}$	1.0	$\frac{N_s}{N_{kq}}$	1.0

We apply the suggested continuous Q-learning control algorithm with the structure shown in Fig. 3.6. including controller settings for regulating the frequency of the SMG. Its parameters have been given in Table 3.1. The optimal control is computed as follows based on precise knowledge of Diesel engine generator system model of Woodward DEGOV [85], [86] and processed:

$$\begin{aligned} \mathcal{K}^* = & [0.136 - 0.398 \ 0.481 - 0.299 \ 0.079 \\ & 4.43 \times 10^{-6} \ 6.37 \times 10^{-5} \ 1.37 \times 10^{-4} \\ & -1.65 \times 10^{-4} - 4.68 \times 10^{-5} \ 1.85 \times 10^{-6}]. \end{aligned}$$

The VI algorithm is tested without the precise knowledge of the system model. To be specific, we pick the data along the system trajectory to promote learning of the optimal control gain. The stopping guideline is satisfied after 180

iterations with the matching Q-learning control gain:

$$\begin{aligned} \mathcal{K}_{180} = & [0.129 - 0.376 \ 0.455 - 0.283 \ 0.075 \\ & 4.11 \times 10^{-6} \ 6.02 \times 10^{-5} \ 1.29 \times 10^{-4} \\ & -1.57 \times 10^{-4} - 4.49 \times 10^{-5} \ 1.67 \times 10^{-6}]. \end{aligned}$$

The error between $\mathcal{K}_j$ at the j th iteration and its optimal value is shown in Fig. 3.7. It is apparent that the deviation between the learned control gain $\mathcal{K}_j$ and the optimal control gain $\mathcal{K}^*$ is decreasing as the iteration j rises. The convergence speed of $\frac{|\mathcal{K}_j - \mathcal{K}^*|}{|\mathcal{K}^*|}$ rises in the first 40 iterations and decreases afterward. When the convergence standard is satisfied, the error is 5.56%.

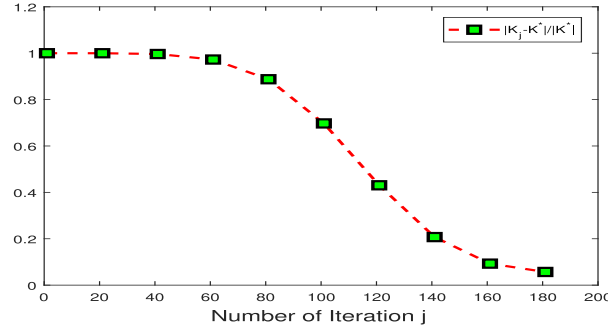
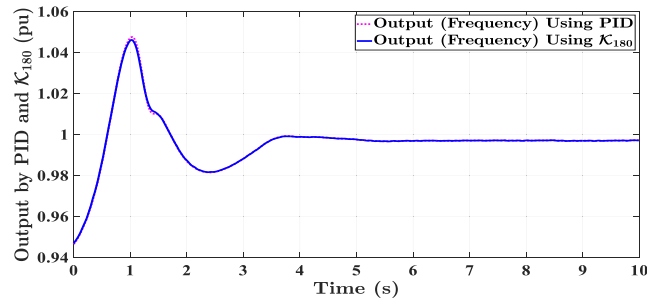


Fig. 3.7. Comparing control gain $\mathcal{K}_j$ with optimal gain $\mathcal{K}^*$.

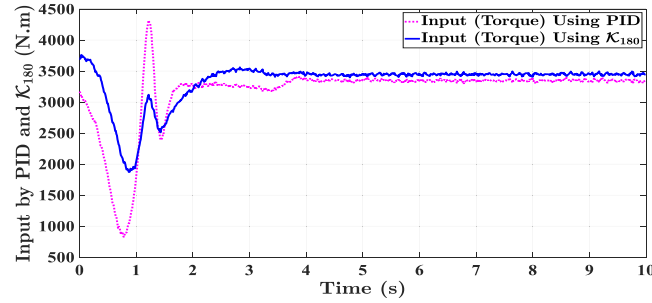
In addition, the response of the traditional PID approach of the frequency controls in islanded microgrids has been studied. A tuned PID controller has been used to conduct the same test scheme under which proposed controller has been tested.

Fig. 3.8. (a) and (b) illustrates the simulation test results of the SMG islanding grid mode simulation. They emulate the case that microgrids needs to be self-

operating without being linked to the main utility grid. Fig. 3.8. (a) demonstrates the output of the closed-loop system, and Fig. 3.8. first graph shows the input to the plant dynamics with the x-axis of time in second. Also, Fig. 3.8. second graph shows an inadequate overshoot and undershoot torque produced by the PID controller. This overshoot and undershoot torque are not being made by the proposed controller under grid connected and islanded modes.



(a)



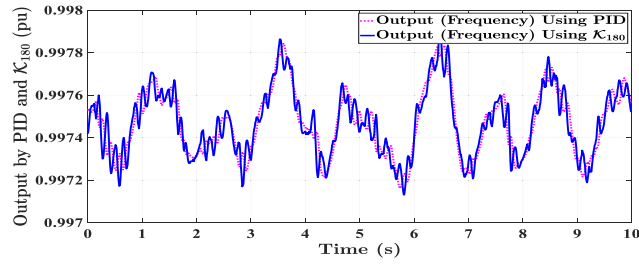
(b)

Fig. 3.8 Simulation test results of the islanding test of the SMG under control a PID and $\mathcal{K}_{180}$

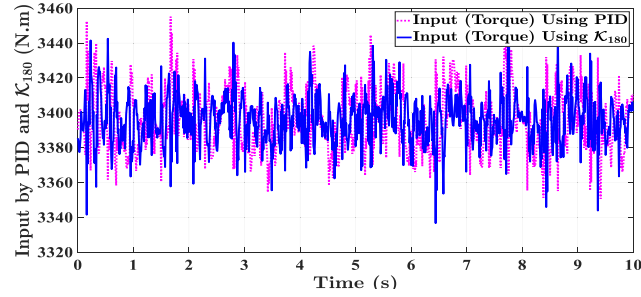
(a) frequency output in pu. (b) Torque input in N.m.

In addition, it is necessary to check the microgrid's reaction when it is in the steady-state condition after being islanded. In this respect, the steady-state test case has been implemented, and its results have been shown in Fig. 3.9, (a) and (b). Fig. 3.9 (a), graph shows the output of the closed-loop dynamic system frequency in pu.,

and (b) graph shows the input to the plant dynamics of the torque in N.m. with the x-axis of time in s. It is remarkable that the droop mechanism's impact on the islanding test response has been reduced since we have been interested in rather purely measuring the effect of the primary frequency control loop on frequency dynamics. The droop algorithm makes each engine generator's active power accurately balanced as shown in Fig. 3.9.



(a)



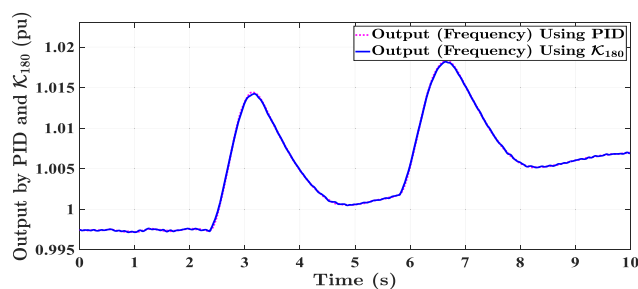
(b)

Fig. 3.9 Simulation test results of the steady state response (a) frequency output in pu.,

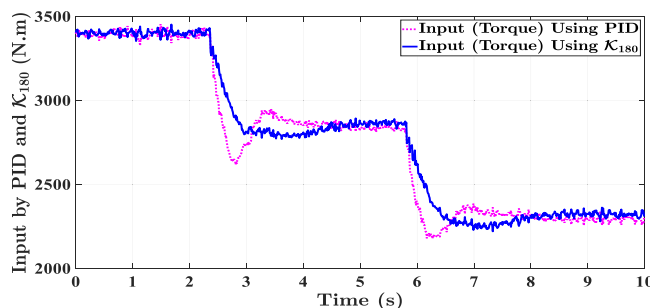
(b) torque input in N.m.

Output frequency and input torque should be studied in response to load changes, in detail when the microgrids are islanded. In this respect, different loads are connected to the grid. Load 6.5kW increase needs to be researched as they stimulate the system. Thus, simulation shown in Fig. 3.10. has been added. Fig. 3.10. shows the simulation test results related with the increasing load in the islanded SMG

mode, which are controlled by either the tuned PID or $\mathcal{K}_{180}$. Fig. 3.10. (a) graph shows the output frequency in pu., and (b) graph shows the input to the plant dynamics torque in N.m. with the x-axis of time is s. We can review that the performance of the proposed Q-learning controller is better than that of the tuned PID controller. Particularly, from the viewpoints of the input torque signal applied to the system, it is apparent that the tuned PID makes overshoot/undershoot torque adjust to the system. Be aware that drastic torques variations are not beneficial in mechanical systems. Another important attention from Fig. 3.10. is that the PID controller cannot perform sturdy against load variations. Namely, no matter the system needs to provide increasing load, the suggested controller has maintained the time response of the closed-loop system.



(a)

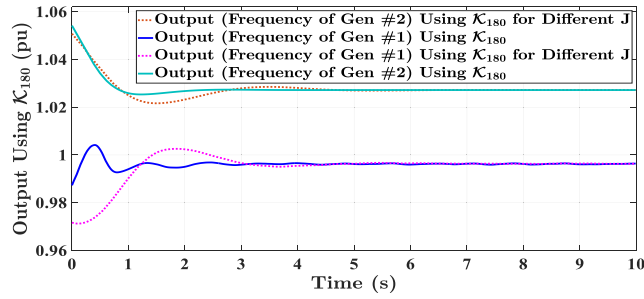


(b)

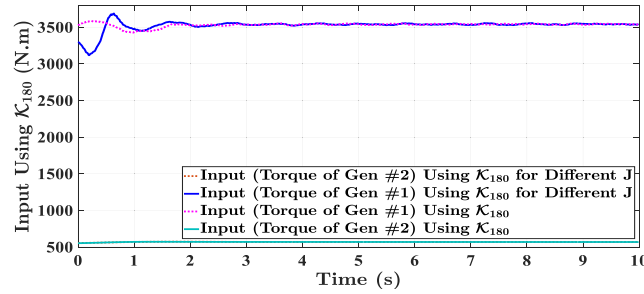
Fig. 3.10 Simulation test results of sequent step changes in increasing load of the SMG under islanded mode controlled by PID controller and $\mathcal{K}_{180}$.

(a) frequency output in pu. (b) torque input in N.m.

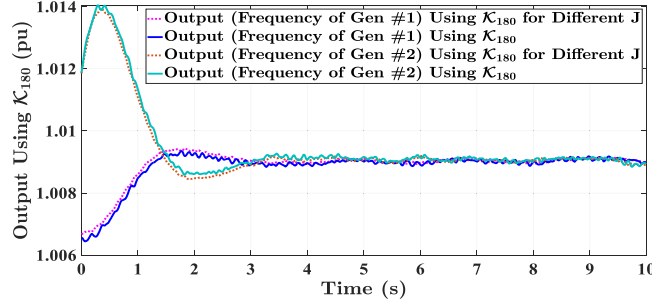
In addition, another important test case in the islanded microgrid is when an outage occurs. This test is very often as SMGs may undergo generation unit outage during fault cases. Fig. 3.11. demonstrates the response of our proposed control algorithm under the blackout of the synchronous generator #2, in addition to its restore. Fig. 3.11. shows that the suggested controller can stabilize the frequency and power after the failure of one of the generations and resume the load optimally and appropriately. Also, the moment of inertia of machines has been reduced by 55%. Though the system needs to refurbish the gains for finding the best possible adjusted gains for the newly updated controller, the system time reaction is acceptable. Updating the gains can aid the time response approach to the previous one.



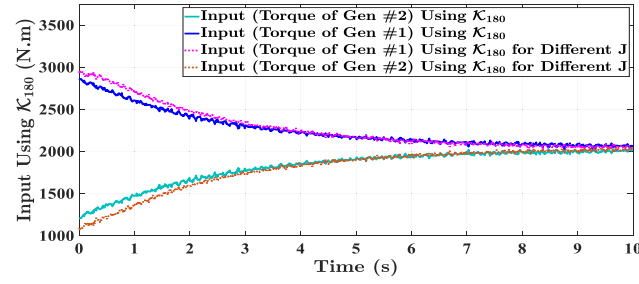
(a)



(b)



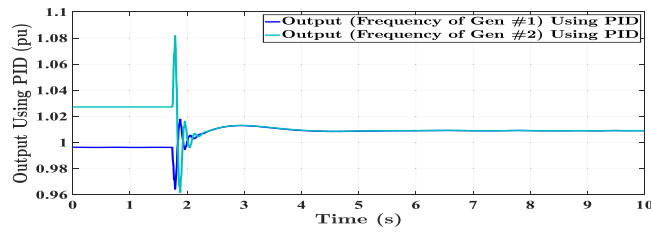
(c)



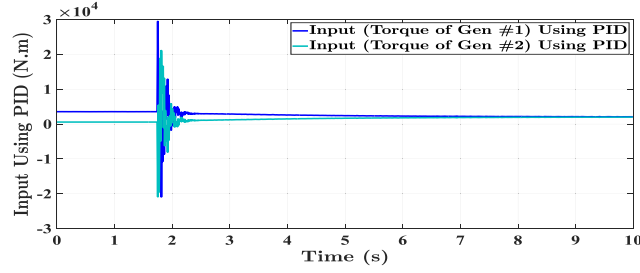
(d)

Fig. 3.11 Simulation test results of the SMG controlled by $\mathcal{K}_{180}$ for generators with the same J and with 55% less J. (a) frequency output in pu. during the failure of generator #2 (b) torque input in N.m. during the failure (c) output in pu. during restoration of generator #2 (d) torque input in N.m. during the restoration.

In the end, after disconnecting generator #2, a restoration test case was run while being controlled by a PID controller for comparison. As anticipated, the PID controller produces an input with violent fluctuations that are unacceptable for mechanical systems. Fig. 3.12, illustrates this test scenario, which shows the unacceptability of the PID controller.



(a)



(b)

Fig. 3.12 Simulation test results after failure and restoration of the generator # 2 in the SMG which is controlled by PID. (a) frequency output in pu. (b) torque input in N.m.

Fig. 3.8. - Fig. 3.11. give a comparison of results with a PID controller versus those achieved with the suggested controller. The performance of the Q-learning control is seen to be superior from the viewpoint of the time response relative to a tuned PID controller.

3.7 Summary

Primary frequency control of future SMG is an important objective in the smart grid, when the SMG is in the islanded mode. Many smart grid research efforts are addressing primary frequency control, the engine generators are being used in many pioneer microgrid projects in industrial research and development sectors. Difficulties related with this important task are as follows: 1) actual uncertainties of the mechanical parameters, 2) uncertain disturbances which are coming from other control loops of electrical variables and uncertainty of electrical loads, 3) manipulating point variations due to load changes. This chapter offers a novel primary frequency control of Diesel-engine generators of SMGs of the future via Q-learning feedback control. Simulation tests have been executed on a SMG using two sets of engine generators to demonstrate effectiveness of the suggested methods.

Chapter 4. Policy Iteration Based Microgrid Control for Mitigating Grid Frequency Fluctuations

Abstract – Due to unknown disturbance, it is hard to control optimally with grid connected inverter under-voltage ride through. This chapter introduces a model-free integral reinforcement learning (IRL) method called exploration policy iteration, along with two entirely model-free integral reinforcement learning techniques referred to as integral Q-learning A and B. The primary goal is to identify the online solution for the specified continuous time nonlinear optimal control problem involving input-affine systems. All proposed methods are designed to explore the state space concurrently and stably. It has demonstrated that the proposed integral reinforcement learning techniques are equivalent to integral policy iteration, as dictated by excitation condition on exploration, which highlights the significance of the integral temporal difference solutions. Additionally, the neural network-based application methods of the proposed strategies were presented, and their effectiveness was validated through voltage source inverter fault ride-through simulations, compared against other methods.

4.1 Background

RL is motivated by biological animal learning process and is devised to learn the best policy by having relations with a given unknown environment to magnify their long-term performance [73]. From the very first moment of the research, RL approaches have been widely researched in the fields of computational intelligence especially concentrated on the finite Markov decision process (MDP) [73]. Thus, a diversity of RL algorithms in MDP surroundings have been suggested, including Q-learning, and actor critic approaches with effective applications [74]. These RL

approaches were expanded to be rooted in two main ideas: 1) temporal difference (TD) and 2) exploration/exploitation, all of which now evolve into the basic components in RL [73].

TD error is one-step prediction error, implying how far the approximate value function is from the true one within the current policy [73]. The value function and its action dependent form, called Q-function, practically reveal the long-term performance index for the current policy and act a part in modifying agent's current policy in RL. All RL approaches furnish at least one update rule, the objective of which is to approximate the value function by decreasing the related TD error [74]. Corresponded with the exploration/exploitation in RL, there is an exploration versus exploitation predicament [73]: to accomplish an enhance response, the RL agents should exploit the information they get, but simultaneously explore the entire environments to enhance future actions. In RL methods for MDPs, a plenty number of explorations of each state-action pair is necessary for the learning of best reply, and therefore exploration and exploitation should be appropriately matched during the learning period [73]. In This chapter, we concentrate on the exploration/exploitation issues and TD learning methods in RL useable to continuous time dynamical systems for adaptive integral Q-learning control.

4.1.1 Policy iteration in optimal control

The optimal control policy and value function can be attained by solving the HJB equation. However, this is a challenging duty even in any case of totally known dynamics due to the inflexibility of the HJB partial differential equation and a problem recognized as the curse of dimensionality. To reduce these problems,

scientist in the research area of both machine learning and controls have researched the computational approaches recognized as policy iteration (PI) [75] and value iteration (VI) [76]. Both approaches comprise of two processes — policy evaluation and policy improvement. The difference between VI and PI rests in policy evaluation [73]. In contrast to VI, PI reduces the related TD error at each step of policy evaluation to calculate the exact value function.

Blending the forward-in-time computational approaches with neural networks in discrete time domain, Werbos [77] suggested adaptive dynamic programming methods for optimal neuro control. In these approaches, one neural network (NN) called actor discovers the enhanced policy as a role of policy enhancement, and the other NN called critic acts a part of policy evaluation for estimating the value function. Separately of these works, the inverse optimal neuro control approaches [78] have been developed, in which the cost function is a posteriori fixed for regulating feedback. In the inverse optimal method, it is not necessary to solve the HJB equation but cannot freely select the needed cost function. The suggested RL approaches in this chapter are a kind of online PI merged with neural networks in which effectively find, by forward-in-time computation, the continuous time adaptive optimal control solution fulfilling the basic HJB equation, where the cost function is given a priori.

4.1.2 Adaptive nonlinear optimal control

Integral reinforcement learning approaches are subject to value iteration and policy iteration in the time domain, which reduce or lessen the related integral TD error at every step. For the integral reinforcement learning approaches advanced from

policy iteration [75] especially, which we call integral policy iteration in our paper, the stability and convergence to the optimal solution are attested under the beginning admissible policy. The execution approaches due to least squares (LS) and the Galerkin neural network estimation. However, all the basic integral reinforcement learning approaches need exact knowledge of the input-coupling dynamics [79], which restrain the pragmatic use of the algorithms. Second, different from reinforcement learning approaches for discrete time dynamical systems [80], there is by no means to concurrently explore the state space for the satisfaction of the permanent excitation conditions during online learning — re-establishing the state variables to some non-zero points is the sole method in these integral reinforcement learning approaches [79] to give extra excitations to the state variables. Finally, the specific initial admissible region in integral policy iteration is not unchanged for the intermediate closed loop systems, so the state trajectory may take off the well-defined stable region during the learning stage, particularly at the time instant the policy is updated.

For the same continuous time optimal control problems, these ideas of integral reinforcement learning are blended with adaptive control approaches, and the resulting online actor-critic reinforcement learning approaches, called synchronous policy iteration, were suggested in [80]. These approaches concurrently update the actor and critic neural networks in an online fashion, and the stability of the reinforcement learning control system is attested under the presumption of persistent excitation. Though presenting new ideas, they need partial/complete knowledge of the system dynamics [81] and an additional neural network to recognize the system

dynamics [82]. In previous literature [83], it blends integral policy iteration with both continuous time Q-function and exploration to improve two online high-level integral reinforcement learning algorithms: integral Q-learning and explored policy iteration with model free algorithm. Both algorithms can explore the entire state space, and the former can be adjusted to completely unknown dynamics.

This research suggests three completely/partially model free integral reinforcement learning algorithms which is continuous time optimal control and use neural networks to estimate the control policy and value function. Integral policy iteration and fixed admissible policy iteration (FA-PI) provide the fundamental ideas of integral-TD and fixed admissibility respectively. All the suggested approaches can concurrently and stably explore the state space during online learning and can refurbish the given admissible region and the exploration signal to ensure input to state stability and fixed admissibility. Also, the two approaches out of the three, named integral Q-learning A and B, respectively. In the suggested approaches, the cost function is a priori in which can be freely selected without such restrictions as in inverse optimal methods [83].

To improve the integral reinforcement control methods, this research stretches out the concepts of exploration, and to the nonlinear system that is commanded by a control policy and probing signal called an exploration. Then we show the excitation condition on the exploration to uniquely solve the high-level integral-TD equation. This remarkable solution is equal to the solution to the integral-TD equation used in integral policy iteration and the related Lyapunov equation used in policy iteration and FA-PI. We suggest one partially model free integral

reinforcement learning approach named explorized integral policy iteration, and two model-free integral policy iteration approaches named integral Q-learning A and B, respectively. Convergence properties of the suggested integral reinforcement learning approaches, related with the design of explorations, are also given under condition. Finally, neural network-based actor-critic least squares execute methods are shown, and we simulate the proposed integral reinforcement learning approaches within the microgrid inverter fault ride-through on transient stability of autonomous environment to verify their performance and to compare them with other approaches.

4.2 Element of reinforcement learning

4.2.1 Nonlinear systems exploration

We reflect on the nonlinear system explored by an unknown time-different probing signal e_τ :

$$\dot{X}_\tau = f(X_\tau) + g(X_\tau)[u(X_\tau) + e_\tau], X(t) = z \in \Omega \quad (4.1)$$

where $e: [t, \infty] \rightarrow \mathbb{R}^m$ is called an exploration, and presume piecewise continuous and uniformly bounded, $X_\tau(z; \mu, e)$ indicates the state trajectory $X(\tau)$ at time $\tau \geq t$ created by the nonlinear system (4.1) under the given policy $u = \mu(x)$ and exploration e_τ . Different from a finite MDP [73] or linear systems [73], we require the following concept of fixed admissibility of an exploration e .

For a given policy $\mu \in A_I(\Omega)$, an exploration e is fixed admissible on Ω , indicated by $e \in A_I(\Omega; \mu)$ or $(\mu, e) \in A_I(\Omega)$,

$$z \in \Omega \Rightarrow X_\tau(z; \mu, e) \in \Omega, \forall \tau \geq t. \quad (4.2)$$

If Ω is compact and $(\mu, e) \in A_I(\Omega)$, then the existence of $X_\tau(z; \mu, e) \forall z \in \Omega$

and $\forall \tau \geq t$ is warranted by (4.2), and (4.2) also hint $X_\tau(\cdot, \mu, e)$ is possible on Ω . Also, input-to-state stable (ISS) for the explorization system (4.1), the stability is accurately defined in this research as follows. For a given policy μ and an exploration e , the nonlinear system (4.1) is input-to-state stable on Ω if $X_\tau \equiv X_\tau(z; \mu, e)$ exists $\forall z \in \Omega$ and $\forall \tau \geq t$, X_τ is feasible on Ω , and there exist $\alpha(\cdot) \in \mathcal{K}$ and $\beta(\cdot, \cdot) \in \mathcal{KL}$ such that for any $z \in \Omega$ and all $\tau \geq t$,

$$\|X_\tau\| \leq \beta(\|z\|, \tau - t) + \alpha(\sup\|e(s)\|) \quad (4.3)$$

It is mentioned in the following theorem in which ISS and fixed admissibility of e are maintained under the small exploration if the policy is created by either FA-PI or integral policy iteration.

Regard $\{\mu_i\}$ and $\{V^{\mu_i}\}$ created by FA-PI or integral policy iteration under assumptions that it is locally Lipschitz on D and there is a policy μ which stabilizes the system. If the exploration e meets

$$\sup\|e(\tau)\| < \sqrt{\frac{\alpha_s \circ \bar{\alpha}_{\mu_i}^{-1}(d_i)}{\bar{\gamma}(R)}} \quad (4.4)$$

Then $(\mu_{i+1}, e) \in A_I(\Omega_{i+1})$ and the nonlinear system (4.1) under $u = \mu_{i+1}(x)$ is input-to-state stable on Ω_{i+1} . In addition, if $\mathcal{D} = \Omega_i = \mathbb{R}^n$ and $\underline{\alpha}_{\mu_i}, \underline{\alpha}_s \in \mathcal{K}_\infty$, then ISS holds globally for any $z \in \mathbb{R}^n$ and any bounded exploration e_τ .

4.2.2 Proceeded integral temporal difference and exploration principles

If X_τ is created by (4.1) with non-zero exploration, then policy evaluation of integral policy iteration does function correctly. Meantime, if $g(X)$ is not known a priori, the next policy μ_{i+1} can be updated by policy enhancement. To figure out these

two problems, the next e -dependent integral temporal difference is contrived as:

$$V(X_t) = \int_t^{t+T} [r(X, \mu(X)) + 2v^T(X)Re(\tau)]d\tau + V(X_{t+T}) \quad (4.5)$$

where $(\mu, e) \in A_I(\Omega)$ is a given policy-exploration pair in which is fixed admissible on a set $\Omega \subseteq \mathcal{D}$; X indicates the trajectory $X_\tau(z; \mu, e)$ for $z=X_t \in \Omega$; $V(x) \in \mathbb{R}$ is positive definite and C^1 on Ω ; $v(x) \in \mathbb{R}^m$ is a policy to be determined. All the integral reinforcement learning approaches suggested in this dissertation will be created based on this integral temporal difference (4.5). The exploration term $v^T(X)Re(\tau)$ is included to the effects of e on integral temporal difference, and to obtain the new policy $v(X) = \mu^+(X)$ without knowing $g(X)$ a priori. $\mu^+(X)$ is the aspired next policy defined in terms of $g(X)$ and $\nabla V(X)$ as

$$\mu^+(X) = -\frac{1}{2}R^{-1}g^T(X)\nabla V(X) \quad (4.6)$$

We presume that e is T-periodic such as $e_\tau = e_{\tau+T}$ for all $\tau \geq t$. Finding $V \in C^1(\Omega)$ and a policy v comforting (4.5) for all $z = X_t \in \Omega$ is identical to solving:

$$H(X, \mu(X), \nabla V(X)) = 2\varphi^T(X)Re_\tau \quad (4.7)$$

for all $X \in \Omega$ and $\tau \in [t, t+T]$, $\varphi(X) := \mu^+(X) - v(X)$ is the policy error.

Using (4.7), we can efficiently prove in which $V^\mu \in C^1(\Omega)$,

$$V(X) = V^\mu(X), v(X) = \mu^+(X)|_{V=V^\mu} \quad (4.8)$$

Are a solution to the integral temporal difference equation (4.5) and meet $H(X, \mu(X), \nabla V(X)) = 0, \phi(X) = 0, \forall X \in \Omega$. For instance, if $m = 1$ and e_τ is constant, the is $e_\tau \equiv c \forall \tau \in [t, t+T]$, then equation (4.7) means v can be obtained

from $V(X)$ and c as $v(X) = \mu^+(X) + \frac{H(X, \mu(X), \nabla V(X))}{Rc}$. This implies that for a given $V(X)$, there are unique solutions depending on the constant c . For the case when $g(x)$ is known, $v = \mu^+$ can be replaced with (4.5) to get the following straightforward integral temporal difference:

$$V(X_t) - V(X_{t+T}) = \int_t^{t+T} [r(X, \mu(X)) - \nabla^T v(X) \cdot g(X) e_\tau] d\tau \quad (4.9)$$

Here, the solution $V = V^\mu$ to (4.9).

Assumption 1a. There exist $k_1, k_2 > 0$ such that $k_1 I \leq \sum_{j=1}^{L-1} (c_j - c_{j+1}) (c_j - c_{j+1})^T \leq k_2 I$. Assume e_τ is given by $e_\tau = c_j, \forall \tau \in [t_j, t_{j+1}]$ and $V^\mu \in C^1(\Omega)$, then the solution to the integral temporal difference (4.5) is determined by (4.8) under there exist $k_1, k_2 > 0$ such that $k_1 I \leq \sum_{j=1}^{L-1} (c_j - c_{j+1}) (c_j - c_{j+1})^T \leq k_2 I$. Under the replacement $V = V^\mu$, (4.5) can be revised as

$$V^\mu(X_t) = \int_t^{t+T} [r(X, \mu(X)) + 2v^T R e_\tau] d\tau + V^\mu(X_{t+T}) \quad (4.10)$$

$$\forall X \in \Omega, \forall j \in \{1, 2, \dots, L\} \quad (4.11)$$

In this context, the equation of (4.8) is true under the following simple excitation condition.

Assumption 1b. There exist $k_3, k_4 > 0$ such that $k_3 I \leq \sum_{j=1}^L c_j c_j^T \leq k_4 I$.

Assumption 1b is identical to the following excitation condition for some constants $\pi_3, \pi_4 > 0$.

$$\pi_3 I \leq \int_t^{t+T} e_\tau e_\tau^T d\tau \leq \pi_4 I \quad (4.12)$$

Identically, if $t_{j+1} - t_j = \frac{T}{L}$ for all $j \in \{0, 1, 2, \dots, L-1\}$ then Assumption 1a is identical to the existence of $\pi_1, \pi_2 > 0$ such that

$$\pi_1 I \leq \int_t^{t+\frac{L-1}{L}T} \left(e_\tau - e_{\tau+\frac{T}{L}} \right) \left(e_\tau - e_{\tau+\frac{T}{L}} \right)^T d\tau \leq \pi_2 I \quad (4.13)$$

For assumption 1a, there should exists a subsequence $\{c_{jk}\}_{k=1}^{m+1}$ whose difference $\{c_{jk} - c_{jk+1}\}_{k=1}^m$ is linearly independent. For this, $L \geq m+1$ is necessary. However, the existence of linearly independent subsequence $\{c_{jk}\}_{k=1}^m$ be adequate for e_τ to meet presumption 1b. In this context, we need $L \geq m$. Thus, when V is given by V^μ a priori, the composition of the exploration e_τ turns into relatively easy and simple. For example, if $m = 1$, then two constants $c_1 \neq c_2$ are required to construct e_τ with obeying Assumption 1a. However, for the case of $V = V^\mu$ and $m = 1$, e_τ satisfying Assumption 1b can be created using only one constant $c_1 \neq 0$. Assumption 1a or Assumption 1b with $V = V^\mu$ is necessary to ensure the uniqueness of the solution (4.8).

4.3 Integral policy iteration and fixed admissible policy iteration without explorations

The goal of the integral reinforcement learning algorithms showed in this dissertation is to search the best admissible policy μ^* minimizing the performance index $J(X_t, u(\cdot)) = \int_t^\infty r(X_\tau, u_\tau) d\tau$ and matching optimal value function $V^*(X) := V^{\mu^*}(X)$. Minimizing the Hamiltonian $H(X, \mu, \nabla V^*)$ among all applicable policies, we can obtain the optimal policy $\mu^*(X)$ as follows:

$$\mu^*(x) = -\frac{1}{2}R^{-1}g^T(x)\nabla V^*(x) \quad (4.14)$$

Moreover, exchange (4.14) into $H(x, \mu(x), \nabla V^*(x)) = 0, \forall x \in \Omega$ and rearranging the equation produce well known HJB equation:

$$0 = S(x) + \nabla V^{*T} f(x) - \frac{1}{4} \nabla V^{*T} g(x) R^{-1} g^T(x) \nabla V^*, V^*(0) = 0. \quad (4.15)$$

The existence of $V^* \in C^1$ pleasing (4.14) is the required and adequate condition for optimality.

The goal of policy iteration [75] and integral reinforcement learning approaches is to find the solution V^* to the HJB equation (4.15) and the matching optimal policy (4.14) by iterations. All the integral reinforcement learning approaches suggested in the dissertation have the same purpose, finding the optimal solution V^* and μ^* . We briefly discuss existing policy iteration integral reinforcement methods [84] without considering explorations.

Fig. 4.1, represents the entire process of a policy iteration method called fixed admissible policy iteration (FA-PI). Usually, policy iteration methods involve of policy evaluation for solving the nonlinear Lyapunov equation and policy improvement for updating the policy by the rule [84].

$$\mu_{i+1}(x) = -\frac{1}{2}R^{-1}g^T(x) \cdot \nabla V^{\mu_i}(x) \quad (4.16)$$

Also, the FA-PI in Fig. 4.1, upgrades the next region Ω_{i+1} in the inverse admissible region upgrade step for the fixed admissibility of μ_i and μ_{i+1} on Ω_{i+1} . This step can be excluded if one can find a universal admissible set Ω on which $\mu_i \in A(\Omega), \forall i \in$

$\mathbb{Z}_+$ and let Ω_i be equivalent to $\Omega, \forall i \in \mathbb{Z}_+$. In this context, FA-PI turn into the policy iteration [84].

$V^\mu(x)$ is a Lyapunov function for the system then we derive:

$$\dot{V}^\mu(x_\tau) \equiv (\nabla V^\mu(x_\tau))^T (f(x_\tau) + g(x_\tau)\mu(x_\tau)) = -r(x_\tau, \mu(x_\tau)) < 0 \quad (4.17)$$

Afterward, integrate (4.17) from t to $t+T$ to explain integral policy iteration. Then, we obtain the subsequent integral temporal difference equation for a given $\mu \in A(\Omega)$:

$$V^\mu(x_t) = \int_t^{t+T} r(x_\tau, \mu(x_\tau)) d\tau + V^\mu(x_{t+T}) \quad (4.18)$$

This integral temporal difference equation is well-defined for all $x_t \in \Omega$ and for any $T > 0$ if μ is fixed admissible on Ω since $x_t \in \Omega$ and $\mu \in A_l(\Omega)$ insures $x_\tau(x_t, \mu, 0)$ persist in the admissible set Ω for all $\tau \in [t, t+T]$. Similarly, FA-PI can be changed by integrating $\dot{V}^\mu(x_\tau) = -r(x_\tau, \mu(x_\tau))$ over the time interval $[t, t+T]$. This modification results in integral policy iteration represent in Algorithm 1, where only the policy evaluation step is shown; the other process is precisely the same to FA-PI represent in Fig. 4.1.

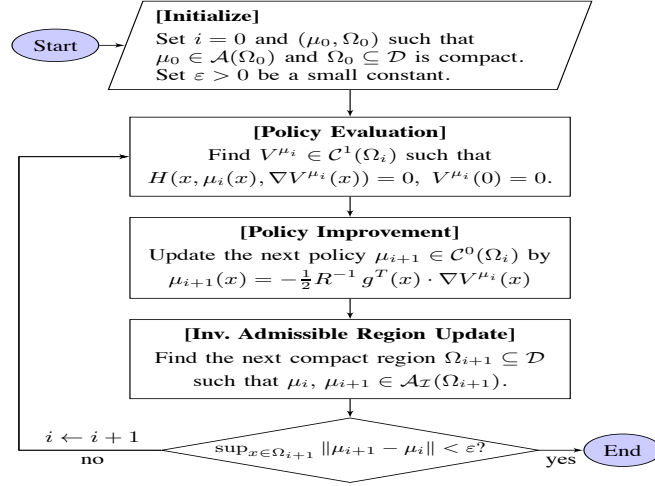


Fig. 4.1 Representation of FA-PI algorithm.

Alg. 4.1. Policy Evaluation of I-PI without Explorations

[Policy Evaluation]

Given $\mu_i \in \mathcal{A}(\Omega_i)$, find $V^{\mu_i} \in \mathcal{C}^1(\Omega_i)$ by solving

$$V^{\mu_i}(x_t) = \int_t^{t+T} r(x_\tau, \mu_i(x_\tau)) d\tau + V^{\mu_i}(x_{t+T}) \quad (19)$$

on the domain Ω_i , where $x_\tau \equiv x_\tau(x_t; \mu_i, 0)$.

Fig. 4.2 Representation of Algorithm 4.1.

When a general admissible set Ω is given a priori and $\Omega_i = \Omega, \forall i \in \mathbb{Z}_+$, then Algorithm 1 turn into the integral policy iteration given in [85]. If both integral policy iteration and FA-PI use the same initial policy μ_0 which is admissible on the same region Ω_0 , and generate the same $\{\Omega_{i+1}\}_{i \in \mathbb{Z}_+}$, then they create the same process $\{V^{\mu_i}\}$ and $\{\mu_i\}$. Thus, integral policy iteration takes over the properties of FA-PI regarding convergence and fixed admissibility considered above [85].

4.4 Integral reinforcement learning with explorations

Inspired by the high-level integral temporal difference (4.5), (4.9), (4.10). we

suggest three completely/partially model free integral reinforcement learning algorithms in which exploit the exploration e_τ to concurrently excite the states and learn the next policy without knowing nonlinear systems. Meanwhile, integral policy iteration is an online approach, all the proposed integral reinforcement learning algorithms can operate in online fashion even when the nonlinear system (f, g) is partially/completely unknown and experience exploration e_τ . Fig. 4.2, presents the entire process of the suggested approaches in a combined way, which are similar to but different from integral policy iteration and FA-PI represented in Fig. 4.1 and Algorithm 1, as explained below.

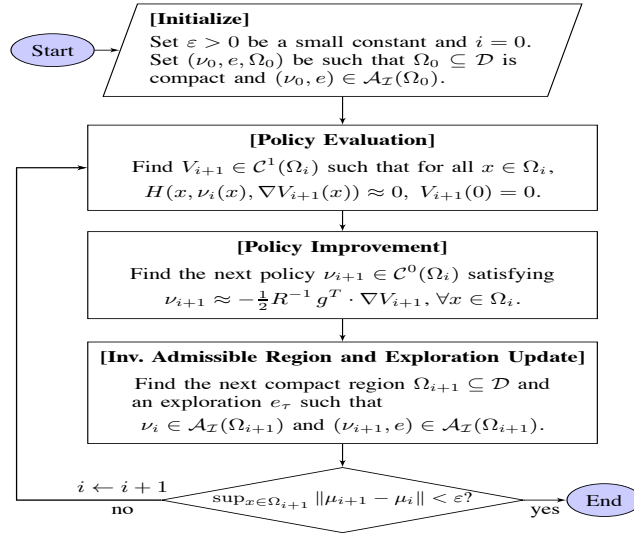


Fig. 4.3 Suggested online integral reinforcement learning approach.

As long as FA-PI and integral policy iteration cannot explore the state space in exploitation fashion, the suggested approaches use admissible explorations to concurrently excite the state variables. To conserve fixed admissibility of the exploration, the suggested approaches create e_τ both at the beginning step and after each policy enhancement, as represented in Fig. 4.3. The three integral reinforcement

learning approaches are fabricated based on the specific high-level integral temporal differences (4.5), (4.9), and (4.18) which produces differences in policy evaluation and improvement steps. The other segments are precisely the same to those shown in Fig. 4.3. Thus, they are excluded in the representation of the suggested integral reinforcement learning approaches (Algorithms 4.2-4.4).

4.4.1 Exploration policy iteration

The first one is called as exploration integral policy iteration, whose policy evaluation and enhancement are presented in Algorithm 4.2. Exploration integral policy iteration stems from the integral temporal difference (4.9) and it can concurrently stimulate the states during policy evaluation by using the exploration e_τ . Different from Algorithm 4.1, the high-level integral temporal difference (4.20) holds the explorized term ' $\nabla^T V_{i+1}(x) \cdot g(x)e_\tau$ ' to offset the outcomes of the exploration e . When $e \equiv 0$, exploration policy iteration (Algorithm 4.2) change into the integral policy iteration presented in Algorithm 4.1. It supplied that (4.20) keeps for all $z = x_t \in \Omega_i$. In explorized integral policy iteration, e_τ needless to meet the excitation conditions such as in Assumptions 1a and 1b, neither does the integral temporal difference (4.9).

Alg. 4.2. Explorized Integral Policy Iteration

Policy Evaluation: Given $(\nu_i, e) \in \mathcal{A}_T(\Omega_i)$ and $z \in \Omega_i$, find $V_{i+1} \in C^1(\Omega_i)$ such that

$$\begin{aligned}
V_{i+1}(x_t) - V_{i+1}(x_{t+T}) \\
= \int_t^{t+T} [r(x, \nu_i(x)) - \nabla^T V_{i+1}(x) \cdot g(x)e_\tau] d\tau \quad (20)
\end{aligned}$$

where $x \equiv x_\tau(z; \nu_i, e)$.

Policy Improvement: Update the next policy ν_{i+1} by

$$\nu_{i+1}(x) = -\frac{1}{2}R^{-1}g^T(x) \cdot \nabla V_i(x).$$

Fig. 4.4 Representation of Algorithm 4.2.

4.4.2 Model free integral Q-learning A, B

The other two integral reinforcement learning algorithms suggested in this dissertation are named integral Q-learning A and B. It is stem from the integral temporal difference (4.5), (4.10). It can be realized without knowing the system dynamics (f, g) . All algorithms, the exploration e_τ act a part in relaxing the prerequisite of the knowledge of $g(x)$.

Algorithm 3 represents policy evaluation and improvement of the suggested integral Q-learning A, the other steps of the algorithms are precisely same to those in Fig. 4.3. In this approach, the integral reinforcement learning agent look for the solution $V_{i+1} \in C^1(\Omega_i)$ and $\nu_{i+1} \in C^0(\Omega_i)$ of the high-level integral temporal difference (4.21) simultaneously. However, integral Q-learning B represented in Algorithm 4 accomplishes policy evaluation and improvement separately. Here, policy evaluation employs the exploration e , policy enhancement of Algorithm 4.4 is improved from the integral temporal difference (4.10) to concurrently explore the state space at once to look for the next policy ν_{i+1} fulfilling (4.21) without using the knowledge of system dynamics (f, g) .

Alg. 4.3. Integral Q-learning A

Policy Evaluation & Improvement: Given $(\nu_i, e) \in \mathcal{A}_{\mathcal{T}}(\Omega_i)$ and $z \in \Omega_i$, find $V_{i+1} \in \mathcal{C}^1(\Omega_i)$ and $\nu_{i+1} \in \mathcal{C}^0(\Omega_i)$ such that

$$\begin{aligned} V_{i+1}(x_t) - V_{i+1}(x_{t+T}) \\ = \int_t^{t+T} \left[r(x, \nu_i(x)) + 2\nu_{i+1}^T(x) Re_\tau \right] d\tau \end{aligned} \quad (21)$$

where $x \equiv x_\tau(z; \nu_i, e)$.

Fig. 4.5 Representation of Algorithm 4.3.

Alg. 4.4. Integral Q-learning B

Policy Evaluation: Given $\nu_i \in \mathcal{A}_{\mathcal{T}}(\Omega_i)$ and $z \in \Omega_i$, find $V_{i+1} \in \mathcal{C}^1(\Omega_i)$ such that

$$V_{i+1}(x_t) = \int_t^{t+T} r(x_\tau, \nu_i(x_\tau)) d\tau + V_{i+1}(x_{t+T})$$

where $x_\tau \equiv x_\tau(z; \nu_i, 0)$.

Policy Improvement: Given $(\nu_i, e) \in \mathcal{A}_{\mathcal{T}}(\Omega_i)$, $z \in \Omega_i$, and $V_{i+1} \in \mathcal{C}^1(\Omega_i)$, find the next policy $\nu_{i+1} \in \mathcal{C}^0(\Omega_i)$ such that (21) holds, where $x \equiv x_\tau(z; \nu_i, e)$.

Fig. 4.6 Representation of Algorithm 4.4.

4.4.3 Characteristics of exploration, input-to-state stability, and fixed admissibility

Concerning the explorations relate to the suggested approaches, we have concentrated on excitation circumstance to remarkably get $V_{i+1} \approx V^{V_i}$ and $\nu_{i+1} \approx \nu_{i+1}^+$ at each iteration. The essential excitation environment for each suggested approach can be succinct as follows: 1) explorized policy iteration: none, 2) integral Q-learning A and 3) integral Q-learning B. In this paragraph, we assumed that at i-th iteration, V_{i+1} and ν_{i+1} has no error, $V_{i+1} = V^{V_i}$ and $\nu_{i+1} \approx \nu_{i+1}^+$ on Ω_i , and deem the next policy ν_{i+1} and next fixed admissible region Ω_{i+1} . Let $\underline{\alpha}_{i+1}$ and $\bar{\alpha}_{i+1}$ be of class $\mathcal{K}$ fulfilling $\underline{\alpha}_{i+1}(|x|) \leq V_{i+1}(x) \leq \bar{\alpha}_{i+1}(|x|)$.

In this ideal case, the suggested integral reinforcement learning approaches

are equivalent to FA-PI, so μ_i switched to v_i , the policies v_i and v_{i+1} are fixed admissible on Ω_{i+1} . Also, equation (4.4) indicates that (v_{i+1}, e) is fixed admissible on Ω_{i+1} , and in which the system $\dot{X} = f + g(v_{i+1} + e)$ is ISS on Ω_{i+1} if the exploration e is limited by

$$\sup ||e(\tau)|| < \sqrt{\underline{\alpha}_s \circ \frac{\bar{\alpha}_{i+1}^{-1}(d_i)}{\bar{\gamma}(R)}} \quad (4.22)$$

If $\mathcal{D} = R^n$ and $\underline{\alpha}_{i+1}, \underline{\alpha}_s \in \mathcal{K}_\infty$, then this ISS keeps globally for any bounded exploration e_τ by (4.4). In preparation for e is created from some constant vectors $\{c_j\}_{j=1}^N$ and fulfills $e_\tau = c_j, \forall \tau \in [t_j, t_{j+1}]$, the boundedness condition (4.22) is changed by

$$||c_j|| < \sqrt{\underline{\alpha}_s \circ \frac{\bar{\alpha}_{i+1}^{-1}(d_i)}{\bar{\gamma}(R)}}, \forall j \in \{1, 2, \dots, N\} \quad (4.23)$$

The question is what to do when the state x is outside the fixed admissible region Ω_{i+1} during online learning at $(i+1)$ th step. Ω_{i+1} can be decided as $\Omega_{i+1} = \Omega_{d_i}^{v_i}$ under $V_{i+1} = V^{v_i}$ and $v_i \in A(\Omega_i)$ by selecting d_i in the interval $(0, \min V_{i+1}(x))$. Here, we have $\Omega_{i+1} \subseteq \Omega_i$; $(V_{i+1}, e) \in A_I(\Omega_{i+1})$ for e fulfilling (4.22). Particularly, the best way to maintain fixed admissibility and ISS is to put in the current policy $u = v_i$ and zero exploration $e \equiv 0$ until which x_τ goes into Ω_{i+1} like $x_\tau \in \Omega_{i+1}$. The next policy $u = v_{i+1}$ and non-zero exploration e fulfilling (4.22) or (4.23) can be put in subsequently without violating fixed admissibility and ISS on Ω_{i+1} . However, if we can find Ω_{i+1} in which holds Ω_i and fulfills $v_i \in A_I(\Omega_{i+1})$, then the next policy v_{i+1} and non-zero exploration e fulfilling (4.22) can be put in any time since x_τ is

already in Ω_{i+1} .

4.4.4 Application of neural network based least square

The suggested integral reinforcement learning approaches (Algorithms 4.2-4.4) can be fulfilled in the least square by using neural networks to estimate V_{i+1} and v_{i+1} . Let $\{\phi_j^a \in C^0(\mathcal{D})\}_{j=1}^\infty$ and $\{\phi_j^c \in C^1(\mathcal{D})\}_{j=1}^\infty$ be the series of real valued neural network activation functions in which are linearly independent and accomplish on their respective function spaces $C^0(\mathcal{D})$ and $C^1(\mathcal{D})$. Here, the superscripts ‘a’ and ‘c’ indicate actor and critic, respectively. Utilization of these activation functions, $v_{i+1} \in C^0$ and $V_{i+1} \in C_1$ can be shown as

$$\begin{cases} v_{i+1}(x) = \sum_{j=1}^\infty v_{ij} \phi_j^a(x) \\ V_{i+1}(x) = \sum_{j=1}^\infty w_{ij} \phi_j^c(x) \end{cases} \quad (4.24)$$

where $V_i := [v_{i1}, v_{i2}, \dots, v_{iN_a}]^T \in \mathbb{R}^{N_a \times m}$, $w_i := [w_{i1}, w_{i2}, \dots, w_{iN_c}]^T \in \mathbb{R}^{N_c}$, $\phi_a(x) := [\phi_1^a(x), \phi_2^a(x), \dots, \phi_{N_a}^a(x)]^T \in \mathbb{R}^{N_a}$, $\phi_c(x) := [\phi_1^c(x), \phi_2^c(x), \dots, \phi_{N_c}^c(x)]^T \in \mathbb{R}^{N_c}$.

Using these equations, (4.24) can be rewritten as

$$\begin{cases} v_{i+1}(x) = v_i^T \phi_a(x) + \varepsilon_i^a(x) \\ V_{i+1}(x) = w_i^T \phi_c(x) + \varepsilon_i^c(x) \end{cases} \quad (4.25)$$

where $\varepsilon_i^a(x)$ and $\varepsilon_i^c(x)$ are neural network resetting errors. $\mathcal{D}$ is compact. There exist $N_a, N_c \in \mathbb{N}$ such that the neural network errors ε_i^a and ε_i^c in (4.25), and $\nabla \varepsilon_i^c$ are all bounded on the compact set $\mathcal{D}$ if $v_{i+1} \in C^0$ and $V_{i+1} \in C^1$ are finite on $\mathcal{D}$. This boundedness characteristic maintains if the domain is restricted to a compact subset of $\mathcal{D}$ such as $\Omega_{a_i}^{v_i}$ in the suggested integral reinforcement learning algorithms. Since

$v_{i+1}(0) = 0$ and $V_{i+1}(0) = 0$, we have $\phi_a(0) = 0$ and $\phi_c(0) = 0$ in generally.

Reflect on integral Q-learning A (Algorithm 4.3) as an application instance.

Here replacing (4.25) into the integral temporal difference (4.21), we get

$$\delta_i(x_t, e) = [\phi_c(x_{t+T}) - \phi_c(x_t)]^T w_i + \int_t^{t+T} [r(x, v_i(x)) + 2\phi_a^T(x) V_i R e_\tau] d\tau \quad (4.26)$$

where $\delta_i(x_t, e) \in R$ is the integral temporal difference error given by

$$\delta_i(x_t, e) = \varepsilon_i^c(x_t) - \varepsilon_i^c(x_{t+T}) - 2 \int_t^{t+T} (\varepsilon_i^a(x))^\tau R e_\tau d\tau \quad (4.27)$$

Define $v_i \in R^{N_{am}}$ as $v_i := \text{col}\{v_{i1}, v_{i2}, \dots, v_{im}\}$. Then, applying $\phi_a^T(x) v_i R e = (R e \otimes \phi_a(x))^T v_i$ to (4.26) and then rearranging the equation, one can get the following equation regarding (4.21):

$$\delta_i(x_t; e) = \psi^T(x_t; e) \cdot \theta_i + Z(x_t; v_i) \quad (4.28)$$

where $\theta_i = \text{col}\{w_i, v_i\}$ is the vector of unknown weights; $\psi(x_t; e)$ and $Z(x_t; v_i)$ are given in Table 4.1. The other high-level integral temporal difference in Algorithms 4.2 and 4 can be also represented with (4.28) with θ_i , $\psi(x_{t;2})$, and $Z(x_t; v_i)$ given in Table 4.1, for each integral temporal difference. In Table 4.1, the integral temporal difference errors $\delta_i(x_t; e)$ for Algorithms 4.2 and 4.4 were left out, however it can be easily got by the similar process. For policy evaluation of Algorithm 4, check [101]; in policy enhancement of Algorithm 2, the next neuro policy $\hat{v}_{i+1}$ can be renewed by

$$\hat{v}_{i+1}(x) = -\frac{1}{2} R^{-1} g^T(x) \nabla \phi_c(x) w_i \quad (4.29)$$

Using $\hat{v}_{i+1}$, instead of v_{i+1} , as was done in [101].

N_θ be the number of elements of θ_i such as $N_\theta = N_a + N_c$ for (4.21). One may have N_θ unknowns in the 1-dimensional equation (4.28). In the executions, θ_i will be remarkably decided in least square sense. Define $\psi[k]$, $\delta_i[k]$, and $Z[k]$ as

$$\begin{cases} \psi[k] := \psi(x_{t+kT}, e), \\ \delta_i[k] := \delta_i(x_{t+kT}, e), \\ Z[k] := Z(x_{t+kT}, v_i) \end{cases} \quad (4.30)$$

Then, relate to $x_{t+(k-1)T}$ as an initial point of the integral temporal differences, the following generalized integral temporal difference error equation can be coming from (4.28):

$$\delta_i[k] = \psi^T[k] \cdot \theta_i + Z[k] \quad (4.31)$$

which holds for any $k \in \mathbb{N}$ since x_τ stays in the admissible region Ω_i for all $\tau \geq t$ by $(v_i, e) \in A_I(\Omega_i)$. Assume the data $(\psi[k], Z[k])$ for $k = 1, 2, \dots, N$ are all usable and describe the least square error E to be lessened as $E^2 := \frac{1}{2} \sum_{k=1}^N \delta_i^2[k]$. Here, differentiating E^2 with regard to θ_i with the replacement of (4.31) produces

$$\frac{\partial E^2}{\partial \theta_i} = \sum_{k=1}^N \frac{\partial \delta_i[k]}{\partial \theta_i} \delta_i[k] = \sum_{k=1}^N [\psi[k] \psi^T[k] \cdot \theta_i + \psi[k] Z[k]] \quad (4.32)$$

Making this equation to zero and rearranging the equation, one can obtain the least square solution of the form

$$\theta_{i,LS} = -(\sum_{k=1}^N \psi[k] \psi^T[k])^{-1} (\sum_{k=1}^N \psi[k] Z[k]) \quad (4.33)$$

For the existence of the remarkable LS solution $\theta_{i,LS}$, we need the following excitation condition:

Assumption 2. There exist $k_5, k_6 > 0$ such that $k_5 I \leq \sum_{k=1}^N \psi[k] \psi^T[k] \leq k_6 I$. The existence of the inverse in (4.33) is ensured by Assumption 2. Similar to Assumption 1a and 1b, $N \geq L_\theta$ is required to satisfy Assumption 2. At least L_θ number of data should be gathered to accomplish the least square (4.33) at each iteration.

The entire control plan with integral Q-learning A and its least square accomplishment is shown in Fig. 4.7. At each iteration, the least square solver gathers the data required to calculate $\psi[k]$, $\delta_i[k]$ and $Z[k]$ for $k = 1, 2, \dots, N$ and then finds the weight vectors w_i and v_i fulfilling (4.33), both of which are shifted to the corresponding critic and actor neural networks to update their weights. The actor neural network creates the control input, the output of the critic neural network $\hat{V}_{i+1}(x)$ is utilized in the exploration generator part to calculate the bound (4.22) on the exploration e_τ . In exploration generator, the exploration e_τ is assembled, and changed if required, that act an important part in exciting the signal $\psi(x_t; e)$ in (4.33), and contents 1) Assumption 1a or (4.13) and 2) the boundedness condition (4.22). The entire control plan with exploration integral policy iteration or integral Q-learning B can be described in a comparable manner by adjusting least square solver and actor neural network blocks.

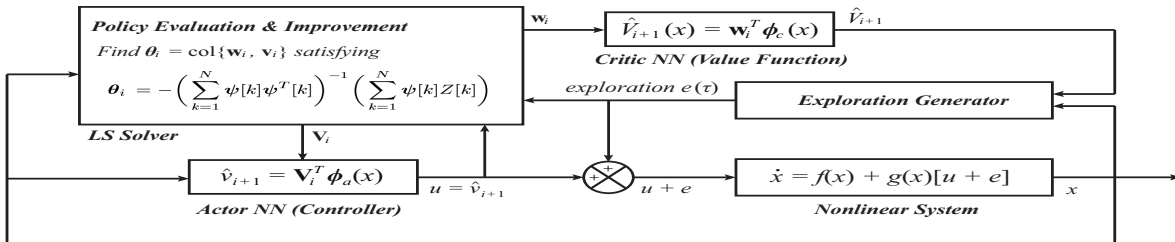


Fig. 4.7 Integral Q-learning A (Algorithm 3) and its least square control scheme and entire control scheme.

Table 4.1 Each high-level integral temporal difference error equation (4.28) for neural network of algorithm 2-4.

Algorithm No.	Process Type	θ_i	$\psi(x_t; e)$	$Z(x_t; v_i)$
2	Policy Evaluation	w_i	$\phi_c(x_{t+T}) - \phi_c(x_t) - \int_t^{t+T} \nabla^T \phi_c(x) \cdot g(x) e_\tau d\tau$	$\int_t^{t+T} r(x, v_i(x)) d\tau$
3	Policy Evaluation & Improvement	$col\{w_i, v_i\}$	$col\{\phi_c(x_{t+T}) - \phi_c(x_t), \int_t^{t+T} 2\phi_a(x) \otimes (Re_\tau) d\tau\}$	$\int_t^{t+T} r(x, v_i(x)) d\tau$
4	Policy Improvement	v_i	$\int_t^{t+T} 2\phi_a(x) \otimes (Re_\tau) d\tau$	$\hat{V}_i(x_{t+T}) - \hat{V}_i(x_t) + \int_t^{t+T} r(x, v_i(x)) d\tau$

4.5 Microgrid inverter frequency fault ride through simulations

Nonlinear controlled inverter based microgrids contemplates the power-sharing and voltage controlling loops. This matter is crucial in autonomous microgrids since the grid forming voltage source inverters are governed through $f - P$ controllers for power-sharing goals which dynamically control the phase angles at the voltage source inverters output voltages.

4.5.1 Fault ride through voltage source inverter

The frequency renewal loop is inserted to the $f - P$ droop loop in the grid-forming voltage source inverters to recover frequency variance originated owing to operation of the droop rules [85] as given as follows:

$$\omega = \omega_0 - k_p \left(\frac{\omega_c}{s + \omega_c} \right) p - \frac{k_f}{s} (\omega - \omega_0) \quad (4.34)$$

Thus, the differential equations expressing the $f - P$ loop dynamics are improved for inductive output impedance as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\omega_c x_2 - k_p \omega_c K \sin(x_1) \end{bmatrix} - k_f \begin{bmatrix} 0 & 0 \\ \omega_c & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (4.35)$$

where $x_1 = \Delta\delta$, $x_2 = \Delta\dot{\delta} = \Delta\omega$, k_f is the feedback gain, ω_c is the cutoff frequency of the low pass filter, k_p is the droop gain of P , $K = k_c \omega_c \frac{V_i V_j}{Z_{ij}}$. To protect the stability of the system during/after current limiting, the system routes might be slide to the stable $\gamma = \alpha x_1 + x_2 = 0$ for a given period of T . Choosing $\alpha > 0$ makes $x(t) \rightarrow 0$ as $t \rightarrow \infty$ and the rate of convergence is managed by selection of α . In this fashion, $x_2 = -\alpha x_1$ and thus $\dot{x}_1 = -\alpha x_1$ manage the motion of the system. This can be recognized by adaptively tuning the feedback gain k_f as addressed next. The sliding surface γ meets the following:

$$\dot{\gamma} = \alpha \dot{x}_1 + \dot{x}_2 = 0 \quad (4.36)$$

Plugging (4.35) into (4.36) results in:

$$\alpha x_2 - (\omega_c + k_f) x_2 - \omega_c k_f x_1 - k_p \omega_c K \sin(x_1) = 0 \quad (4.37)$$

Using sliding surface γ , $x_2 = -\alpha x_1$, (4.37) is updated as:

$$\begin{aligned} -\alpha^2 x_1 + \alpha(\omega_c + k_f) x_1 - \omega_c k_f x_1 - k_p \omega_c K \sin(x_1) &= 0 \\ \rightarrow -\alpha(\alpha - \omega_c) x_1 + k_f(\alpha - \omega_c) x_1 - k_p \omega_c K \sin(x_1) &= 0 \end{aligned} \quad (4.38)$$

and k_f is accomplished from (4.38):

$$k_f = \frac{k_p \omega_c K \sin(x_1) + \alpha(\alpha - \omega_c) x_1}{\alpha - \omega_c x_1} \quad (4.39)$$

To examine the stability of (4.35) with the control law given in (4.39), one may take the following Lyapunov function:

$$V(x) = \frac{1}{2} \gamma^T(x) \gamma(x) = \frac{1}{2} \|\gamma(x)\|_2^2 \quad (4.40)$$

Then $\dot{V}(x) = \gamma^T \dot{\gamma}$ which is expanded as follows:

$$\begin{aligned} \dot{V}(x) &= (\alpha x_1 \dot{x}_1 + x_2 \dot{x}_2) \rightarrow \dot{V}(x) = \alpha x_1 x_2 + x_2 \left(-(\omega_c + k_f) x_2 - \omega_c k_f x_1 - \right. \\ &\quad \left. k_p \omega_c K \sin(x_1) \right) \end{aligned} \quad (4.41)$$

From the sliding manifold $x_2 = -\alpha x_1$:

$$\begin{aligned} \rightarrow \dot{V}(x) &= -\alpha^2 x_1^2 - \alpha^2 (\omega_c + k_f) x_1^2 + \alpha \omega_c k_f x_1^2 + \alpha k_p \omega_c K \sin(x_1) x_1 \\ \rightarrow \dot{V}(x) &= -\alpha^2 x_1^2 - \alpha^2 \omega_c x_1^2 - \alpha (\alpha - \omega_c) k_f x_1^2 + \alpha k_p \omega_c K \sin(x_1) x_1 \end{aligned} \quad (4.42)$$

Plugging k_f from (4.39) into (4.42) results in:

$$\begin{aligned} \rightarrow \dot{V}(x) &= -\alpha^2 x_1^2 - \alpha^2 \omega_c x_1^2 - \alpha^2 (\alpha - \omega_c) x_1^2 - \alpha k_p \omega_c K \sin(x_1) x_1 + \\ &\quad \alpha k_p \omega_c K \sin(x_1) x_1 \\ \rightarrow \dot{V}(x) &= -\alpha^2 x_1^2 - \alpha^2 \omega_c x_1^2 - \alpha^3 x_1^2 + \alpha^2 \omega_c x_1^2 \rightarrow \dot{V}(x) = -\alpha^2 x_1^2 - \alpha^3 x_1^2 \\ \rightarrow \dot{V}(x) &= -\alpha^2 x_1^2 - \alpha x_2^2 < 0 \text{ for } \alpha > 0 \end{aligned} \quad (4.43)$$

which proofs the asymptotic stability of the system. ■

A nonlinear C^1 function $h(x_1)$, and the cost $r(x, u)$ with $S(x) = x_1^2 + x_2^2$ and $R = 1$. By the converse HJB approach [103], the optimal solution is given by $V^*(x) = \frac{1}{2} \|\alpha x_1 + x_2\|_2^2$ and $\mu^*(x) = -x_2 h(x_1)$. In this context, $S(x)$ is quadratic,

and nonlinear system (4.35) can be estimated near the origin by linear system. Thus, by the linear quadratic regulator theory, V^{μ_i} can be estimated by a quadratic function near the origin [101]; we select w_i and $\phi_c(x)$ in the critic neural network $\hat{V}_{i+1}(x) = w_i^T \phi_c(x)$ as

$$w_i = [w_{i1}, w_{i2}, w_{i3}]^T, \phi_c(x) = [x_1^2, x_1 x_2, x_2^2]^T \quad (4.44)$$

To decide the proper actor neural network structure, we replace (4.44) $R = 1$, and $g(x) = [0, h(x_1)]^T$ to (4.29), which achieve:

$$\hat{v}_{i+1}(x) = -\frac{1}{2} w_{i2} \cdot x_1 h(x_1) - w_{i3} \cdot x_2 h(x_1) \quad (4.45)$$

Here, presume that $h(x_1)$ can be stand for

$$h(x_1) = \theta^T \psi(x_1) + \varepsilon_h(x_1) \quad (4.46)$$

with the weight vector $\theta \in \mathbb{R}^M$, the nonlinear regression function $\psi(x_1) \in \mathbb{R}^M$, and the bounded estimation error $\varepsilon_h(x_1)$; M is the number of weights. Then, (4.45) change into $\hat{v}_{i+1}(x) = (v_i^+)^T \rho(x) + \bar{\varepsilon}_h(x)$, where $(v_i^+), \rho(x) \in \mathbb{R}^{2M}$ are described

$$V_i^+ := -\left[\frac{w_{i2}}{2}, w_{i3}\right]^T \otimes \theta \text{ and } \rho(x) := x \otimes \psi(x_1) \quad (4.47)$$

$\bar{\varepsilon}_h(x)$ is given by $\bar{\varepsilon}_h(x) = -\left(\frac{w_{i2}x_1}{2} + w_{i3}x_2\right) \varepsilon_h(x_1)$, which is clearly bounded in a compact set. Consequently, we select V_i and $\phi_a(x)$ in the actor neural network $\hat{v}_{i+1} = v_i^T \phi_a(x)$ as $V_i = [V_{i1}, V_{i2}, \dots, V_{i2M}]^T$ and $\phi_a(x) = \rho(x)$.

The actor neural network is employed in integral Q-learning A and B to find the next policy $\hat{v}_{i+1}$ with $V_i \approx V_i^+$ when $h(x_1)$ is not known. It $h(x_1)$ is correctly known, then

(4.29) can be employed to straightly compute the next policy $\hat{v}_{i+1}$ as was done in policy enhancement of explorized integral policy iteration. At each i th step, w_i and v_i will be retrofitted by the least square solution (4.33) after N data samples $(\psi[k], Z[k])(k = 1, 2, \dots, N)$ are gathered. In the MATLAB, the goal of the suggested integral reinforcement learning approaches is to find the optimal weight vectors w^* and V^* given by $w^* = \left[\frac{1}{2, 0, 1} \right]^T$ and $v^* = v_i^+|_{w_i=w^*} = -[0^T, \theta^T]^T$.

4.5.2 Instance for inverter fault ride through integral reinforcement learning approach simulation

For the contrast with integral policy iteration (Algorithm 1) in [102], we reflect on the nonlinear system (4.35) with $h(x_1) = \sin(x_1)$ which can be demonstrated by (4.46) with $\theta = 1$, $\psi(x_1) = \sin(x_1)$ and $\varepsilon_h(x_1) \equiv 0$. In this context, V_i and $\phi_a(x)$ and given by $V_i = [V_{i1}, V_{i2}]^T$ and $\phi_a(x) = [x_1 \sin x_1, x_2 \sin x_1]^T$.

The beginning admissible policy is given by $\mu_0 = -\frac{3}{2} \sin(x_1)(x_1 + x_2)$; the beginning state x_0 at $t = 0$ and the sampling period T are set to $x_0 = [0.5, -0.5]^T$ and $T = 1$ [s], respectively. The number of data, N gathered per iteration is decided by ' $N = 10 \times N_a$ ' for each process, as shown in Table. 4.2, where $N = 30$ was used under $N_0 = 3$ as well. For the Algorithms 2 and 3, we employed e_τ given by

$$e_\tau = \begin{cases} c & \text{for all } \tau \in \left[t, t + \frac{NT}{2} \right] \\ -c & \text{for all } \tau \in \left[t + \frac{NT}{2}, t + \frac{NT}{2} \right] \end{cases} \quad (4.48)$$

with $c = 2.5$. In policy enhancement of Algorithm 4, we employed:

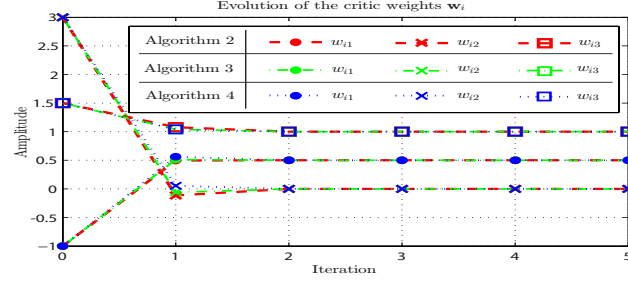
$$e_\tau = 3.5 \text{ for all } \tau \in [t, t + NT) \quad (4.49)$$

Here, these explorations (4.48) and (4.49) content Assumption 1a and 1b, respectively. The update step of FA-PI (Ω_i, e) is excluded in these testing for simplicity. Thus, the same e will be adjusted for all iteration steps.

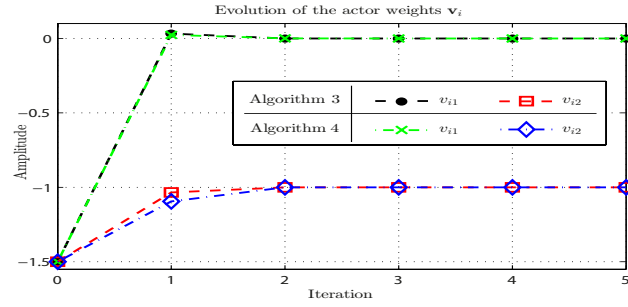
Fig. 4.8 shows the simulation results in the iteration domain, first graph shows the variations of w_i created by the suggested integral reinforcement learning approaches (Algorithm 2-4), and second graph shows the evolution of V_i created by model-free integral Q-learning A and B (Algorithms 3 and 4). As shown in Fig. 4.8 first graph, all the critic weights w_i at each iteration are very close, displaying the equivalent of all the suggested integral reinforcement learning methods and integral policy iteration in the iteration domain under the remarkable condition. The actor neural network weights v_i in Fig. 4.8 second graph shows the coefficients. Also, as displayed in Fig. 4.8, both weights v_i and w_i merge to their optimal values within a few iterations, which is owing to second order convergence of integral policy iteration and policy iteration in the iteration domain [95].

Table 4.2. The number of gathered data per iteration.

Process Type & Algorithm No.	N	N_θ
Policy Evaluation of Algorithm 2	30	3
Policy Evaluation & Improvement of Algorithm 3	50	5
Policy Evaluation of Algorithm 4	30	3
Policy Improvement of Algorithm 4	20	2



(a) Critic neural network weights w_i



(b) Actor neural network weights v_i

Fig. 4.8. Algorithm 2-4 actor/critic weights evolution.

4.5.3 Instance for inverter fault ride through exploration integral Q-learning A approach simulation

In this simulation, integral Q-learning A is adjusted to the nonlinear inverter system (4.35) with $h(x_1) = \cos(2x_1) + 2$. This system was also employed in [80] to test their synchronous actor critic learning approaches. The weight vector w_i of the critic neural network is activated to $w_0 = [1, 1, 1]^T$; the identical beginning actor neural network weight vector is given by $v_0 = v_0^+ = -\left[\frac{1}{2, 1}\right] \otimes \theta$. The states are established to zero, $x_0 = [0, 0]^T$; we established $N = 40$ and $T = 25 [ms]$, thus the least square solution (4.33) is computing every 1 [s]. The exploration plan (4.48) with

$c = 1$ is employed to hold Assumption 1a.

Case 1: We presume $h(x_1)$ is linearly setup as $h(x_1) = \theta^T \psi(x_1)$ with zero estimation error $\varepsilon_h(x) \equiv 0$, where θ and $\psi(x_t)$ are given by $\theta = [1, 2]^T$ and $\psi(x_1) = [\cos(2x_1), 1]^T$. From this, v_i and ϕ_a in the actor neural network are selected as $v_i \in \mathbb{R}^4$ and $\phi_a(x) = [x_1 \cos(2x_1), x_1, x_2 \cos(2x_1), x_2]^T$, and the optimal actor neural network weights V^* are given by $v^* = -[0^T, \theta^T]^T = [0, 0, 1, 2]^T$.

The simulation results are displayed in Fig. 4.9 and Fig. 4.10. As shown in Fig. 4.9 and Fig. 4.10 first graph, the weights in the actor and critic neural networks merge to their optimal ones within 4 iterations; Fig. 4.10 second graph shows the optimal policy estimation error at $i = 6$. It is very small less than 10^{-10} which shows the productiveness of the suggested integral Q-learning A.

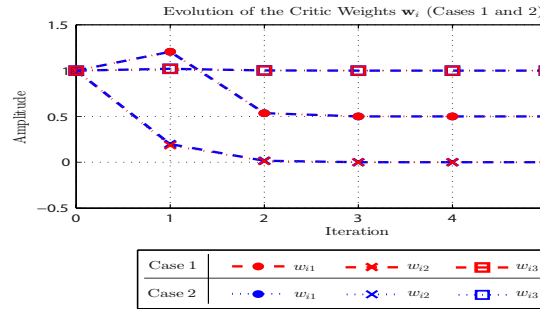


Fig. 4.9. Integral Q-learning A critic weights w_i evolution via case 1 and 2.

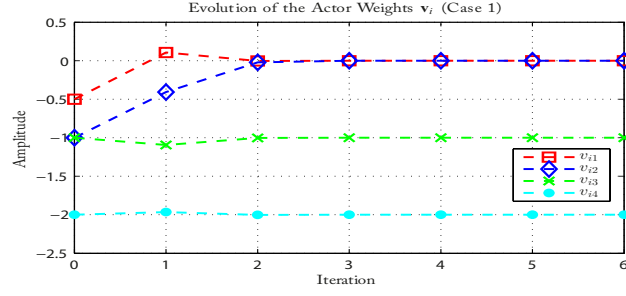
Case 2: Let's presume $h(x_1)$ in $g(x)$ is fully unknown and suppose its expression (4.46) with $M = 7$ and $\psi(x)$ given by $\psi(x_1) = [1 \ x_1 x_1^2 \ x_1^3 \ x_1^4 \ x_1^5 \ x_1^6]^T$. From this and (4.47), the activation function $\phi_a(x) \in \mathbb{R}^{14}$ of the actor neural network can be decided as $\phi_j^a(x) = x_1^j$ for $1 \leq j \leq 7$ and $\phi_j^a(x) = x_1^{j-8} x_2$ for $8 \leq j \leq 14$. The unknown vector θ in (4.46) and in the optimal actor neural network weight v^*

can be also get as $\theta = \left[3, 0, -\frac{2,0,2}{3,0}, -\frac{4}{45}\right]^T$ from the Taylor expansion of $h(x_1)$ at $x_1 = 0$. $h(x_1) = \cos(2x_1) + 2 = 3 - 2x_1^2 + \frac{2x_1^4}{3} - \frac{4x_1^6}{45} + O(x_1^8)$.

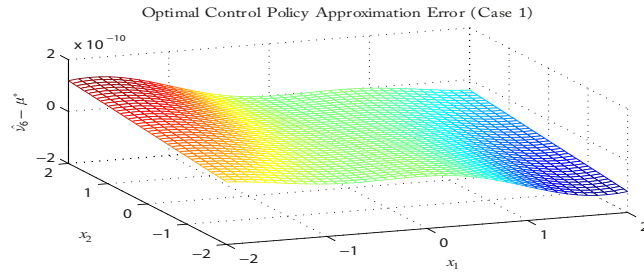
The test results in Case 2 are shown in Fig. 4.9 and Fig. 4.11. From Fig. 4.9, critic neural network weights at each iteration are almost the same as those in Case 1. Final critic neural network weights w_i at $i = 6$ in Case 2 is $w_6 = [0.5, 0.0, 1.0]^T$ and Fig. 4.11 first graph demonstrates the corresponding estimation error of the optimal value function. Here, the maximum error is smaller than 10^{-8} which presents the good performance of integral Q-learning A. The optimal and final weights v^* and v_i at $i = 6$ of the actor neural network are displayed in Table 4.3. Error $\|v_{6j} - v_6^*\|$ for each j is very small. The estimation errors of the optimal policy are shown in Fig. 4.11 second graph is fairly high contrast with those in Fig. 4.10 second graph. These errors are owing to the estimation of the unknown $h(x_1)$, however, can be reduced by integrating higher order terms like x_1^9 into the actor neural network.

Table 4.3. The actor neural network weights v_6 and v^* case 2.

j	v_{6j}	v_j^*	j	v_{6j}	v_j^*	j	v_{6j}	v_j^*
1	0.0000	0	6	0.0000	0	11	0.0000	0
2	0.0000	0	7	0.0001	0	12	- 0.6667	-2/3
3	0.0000	0	8	-3.0000	-3	13	0.0000	0
4	0.0000	0	9	0.0000	0	14	0.0889	4/45
5	0.0000	0	10	2.0000	2			



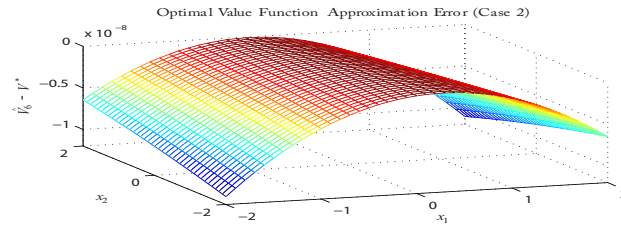
(a) Actor weights evolution



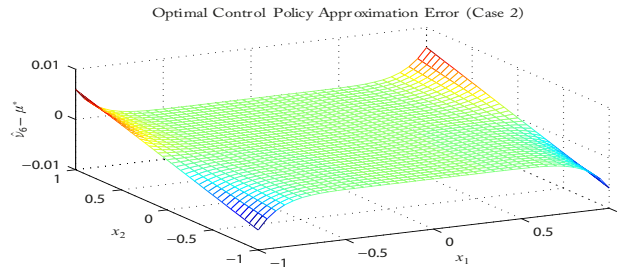
(b) Estimation error of optimal policy

Fig. 4.10. Case 1: Accurately parameterized policy simulation results (a) actor weights evolution

(b) estimation error of optimal policy.



(a) Estimation error of optimal value function

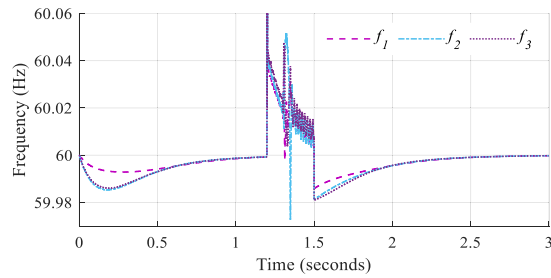


(b) Estimation error of optimal policy

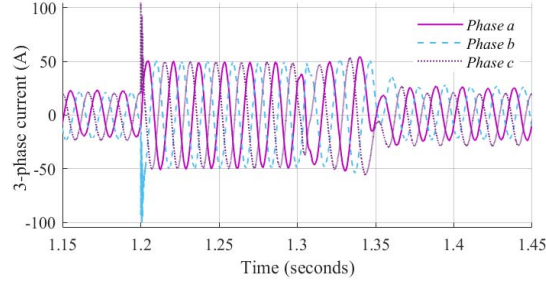
Fig. 4.11. Case 2: Online optimal solution estimation errors when approximation of x_1 is not identically zero

(a) estimation error of optimal value function (b) estimation error of optimal policy.

The performance of the suggested explorized Q-learning A approach can be stabilizing the distributed inverter's fault ride through microgrid in transients is represented in Fig. 4.12. The system is stable during and after the fault ride through for a fault scenario at $t = 1.2$ [s]. Fault period is 120 [ms]. Fault current is limited to 2 [pu]. After fault event and during the explorized Q-learning A mode, the system trajectory is controlled and managed. Even though the voltage source inverter shows transient impedance due to current limiting, instability condition $|\Delta\delta| > |\Delta\omega|$ is not founded. The frequency restoration sequence [86], [87] reconstitutes the frequency back to the nominal value as shown in Fig. 4.12 first graph microgrid's distributed three different inverter frequency recover to nominal frequency within 2 [s]. Also, from the Fig. 4.12 second graph, three-phase current is increased about 0.15 [s] during the transient state when the fault occurs at 1.2 [s]. However, after the transient state, all three-phase current amplitude and phase are turn into normal mode of operation. We may assume in which voltage source inverter's fault ride through is working well.



(a) Voltage source inverter's grid forming frequencies



(b) Three phase current during/after the fault

Fig. 4.12. Frequency and current performance of the suggested explorized Q-learning A approach voltage source inverter's fault ride through.

4.6 Summary

This chapter proposed a model free integral reinforcement learning approach names explorized policy iteration and two entirely model-free integral reinforcement learning approaches named integral Q-learning A and B. The goal of all of this chapter is to find the online solution to the given continuous time nonlinear optimal control problem with input-affine systems. All the suggested approaches are capable to explore the state space concurrently and stably during the learning phase. We have represented that the suggested integral reinforcement learning approaches are all identical to integral policy iteration and fixed admissible policy iteration in the iteration domain, covered by the given excitation condition on the exploration for the remarkableness of the integral temporal difference solutions. The neural network-based application approaches of the suggested plans were also showed, and their performance was confirmed by voltage source inverter fault ride through simulations and contrast with the other approaches. Finally, a proper solution to the transient stability problem of the voltage source inverter microgrids was suggested using explorized Q-learning A approach and confirmed through simulation results.

Chapter 5. Microgrid Frequency Control via Event Triggered Control of Distributed Energy Storages and Generation

Abstract –The unpredictability of generator-load areas presents challenges for regulating system frequency. To address this issue, we propose a real-time flexible frequency control strategy that manages both the governor and energy storage (ES) systems to facilitate frequency recovery in uncertainty of microgrid. Author proposes an advanced integral reinforcement learning method, wherein the reinforcement signal employs a Lipschitz function to navigate the constraints associated with ES control. Additionally, we enhance a novel dynamic event-triggered mechanism to reduce the frequency of control updates for the ES. Furthermore, paper introduce power partition control approach that promote the efficient use of distributed ES. A leader-follower distributed dynamic event-triggered control mechanism is established to ensure rapid convergence and precise synchronization, enabling power dispatch according to demand to reduce limited network resources. The stability of the distributed ES is confirmed through the Lyapunov theory.

5.1 Background

The increasing integration of renewable energy systems, such as photovoltaic (PV) technology, has led to a greater uncertainty in power supply [88]. This heightened uncertainty results in a mismatch between energy demand and supply, which adversely affects frequency stability [89]. The rapid development of demand response strategies, which optimize the use of flexible resources on the demand side through advanced information technologies, has emerged as an effective approach to provide frequency support [90].

The load as an energy consumer may encompass more flexible and

convenient smart devices within smart homes. However, the disparity between variable load demand and unpredictable energy supply leads to frequency instability in microgrids. A significant amount of research has focused on frequency recovery. Variations in frequency can adversely affect the stable operation of the microgrid and may lead to system failures in severe situations. Typically, frequency deviations are addressed by managing microturbines to compensate for power mismatches. However, this approach is not ideal, as the long-term extensive frequency control responsibilities can lead to wear and tear on microturbines, resulting in irregular fluctuations of internal variables during measurement and transmission. Furthermore, the system inertia of the microgrid is reduced due to the substantial fluctuations in renewable energy sources, indicating a need for enhanced robustness in frequency control measures [91].

From the perspective of control equipment, ES demonstrates remarkable capabilities in subsidiary frequency control due to its adaptable management, rapid response times, and high precision [92]. The microturbine benefits from ES, which not only reduces the support energy required by the frequency modulation unit, thereby lowering operational costs, but also functions as either a power producer or consumer. ES contributes to improved power quality while simultaneously reducing electricity bills. With the rapid advancement of demand response on the load side, electric vehicles and temperature-regulated loads are now able to participate in load frequency control [93]. Additionally, the integrated management of smart home devices, the evolution of energy routers, and appropriately interfaced converters have further bolstered the effectiveness of subsidiary frequency control systems [94].

From a control methodology standpoint, adaptive dynamic programming (ADP) facilitates model-free control and has garnered significant attention in industrial applications [95], [96]. Research has been conducted on a data-driven ADP approach aimed at detecting small leaks in pipelines [97], while another study proposed a goal depiction ADP method to derive subsidiary control signals. Nonetheless, these methodologies necessitate the accumulation of extensive historical data and demand substantial storage capacity. Additionally, [98] the identification of unknown systems has been explored through the use of recurrent neural networks, with considerations given to system state limitations, and the approach has been extended to stochastic nonlinear systems [99]. However, inaccuracies in system state observation and identification can adversely affect control precision. To address this issue, a control strategy based on integral reinforcement learning has been validated, which alleviates the stringent requirements of the system model and enables error-free control despite the presence of undiscovered system drift dynamics. Furthermore, [100] the challenges associated with tracking control in partially unknown nonlinear systems subject to disturbances within a finite time frame have been investigated, leading to solutions for the time-varying Hamilton-Jacobi-Isaacs (HJI) equation. However, the controllers developed in this research are regularly enhanced based on the state, which may introduce transmission weight discrepancies between the control device and the control center in practical applications.

In the context of integrating load-side resources into energy storage frequency control, a centralized control system reliant on a unified transmission

center necessitates a robust two-way communication network, which significantly increases both communication demands and computational requirements [101]. As the system's complexity and data communication capabilities grow, it becomes essential to enhance the speed and computational power of the central controller to effectively manage information. In scenarios where communication networks have limited bandwidth, transmitting data at a high sampling rate can lead to network congestion, negatively impacting real-time control operations [102]. To improve transmission efficiency, various studies have implemented event-triggered control strategies aimed at alleviating communication burdens [103], [104] study proposes an event-triggered distributed control framework designed to collaboratively manage energy storage systems in response to power fluctuations while ensuring equitable power distribution among different energy storage units. Nonetheless, these event-triggered methods often involve extensive calculations and monitoring, resulting in a significant operational burden. Consequently, it is imperative to establish an efficient event-triggered process for distributed resources to optimize computational efficiency.

This chapter introduces the challenge of robust optimal control in microgrids as a zero-sum game that comprehensively addresses the uncertainties associated with renewable energy sources, system variables, and load demands. It proposes a dynamic event-triggered integral reinforcement learning method aimed at resolving the online adaptive optimal frequency control issue while adhering to input constraints. The primary objective is to enhance load-side frequency regulation, emphasizing the distributed cooperative control of energy storage systems (ESs)

within a load aggregator framework, enabling ESs to respond effectively to higher-level directives while minimizing communication overhead. The key contributions of this study include the reformation of the microgrid's robust frequency control challenge, which incorporates stochastic uncertainties, into a reinforcement integral-zero sum game format. This is achieved through the application of adaptive primary frequency control by solving the Hamilton-Jacobi-Isaacs equation under uncertain conditions. Furthermore, an advanced integral reinforcement learning strategy is developed to address the challenges posed by ES control limitations and the uncertainties of system variables, utilizing a non-quadratic reinforcement signal that alleviates the precision requirements of the microgrid while considering input constraints in advance, with proven convergence. Additionally, a hierarchical frequency control framework is established, promoting cooperative interactions between load aggregators and ESs for load-side management, alongside the introduction of uniform allocation principles to ensure efficient resource utilization in response to directives. Lastly, a leader-follower control strategy is proposed, leveraging a dynamic event-triggered mechanism where a virtual leader disseminates instructions to all agents via a designated communication network, thereby reducing communication demands while ensuring rapid convergence and synchronization accuracy, ultimately enhancing the stability of the distributed control system.

5.2 Representation of microgrid system

5.2.1 Uncertainty model of microgrid system

The microgrid model is shown in Fig. 5.1. Microgrid consists of ES, photovoltaic, governor, microturbine, and load. The transfer function of each unit can

be explained as:

$S_E = 1/(T_e s + 1)$, $S_G = \frac{1}{T_g s + 1}$, $S_T = \frac{1}{(T_t + \delta(t))s + 1}$, $S_p = \frac{1}{T_p s + 1}$. Among them T_e, T_g, T_t and T_p stands for the time constants of ES, governor, turbine, and power system respectively [92]. R stand for speed regulation coefficient, K_p stand for the power system gain coefficient. It is notable in which the variable of turbine and speed regulation coefficient may change in operation. Therefore, $\varepsilon(t)$ and $\delta(t)$ stand for uncertainty of the variable. The frequency dynamic change of microgrid is explained by a set of differential equations, as follows:

$$\begin{aligned} \Delta \dot{f}(t) = & -\frac{1}{T_p} \Delta f(t) + \frac{K_p}{T_p} \Delta P_e(t) + \frac{K_p}{T_p} \Delta P_t(t) + \frac{K_p}{T_p} \Delta P_{pv}(t) - \frac{K_p}{T_p} \Delta P_{lo}(t), \quad \Delta \dot{P}_e(t) = \\ & -\frac{1}{T_e} \Delta P_e - \frac{1}{(\varepsilon(t) + R)T_e} \Delta f(t) + \frac{1}{T_e} u_e(t), \quad \Delta \dot{P}_t(t) = -\frac{1}{\delta(t) + T_t} \Delta P_t + \\ & \frac{1}{\delta(t) + T_t} \Delta X(t), \Delta \dot{X}(t) = -\frac{1}{T_g} \Delta X(t) - \frac{1}{(\varepsilon(t) + R)T_g} \Delta f(t) + \frac{1}{T_g} u_p(t) \end{aligned} \quad (5.1)$$

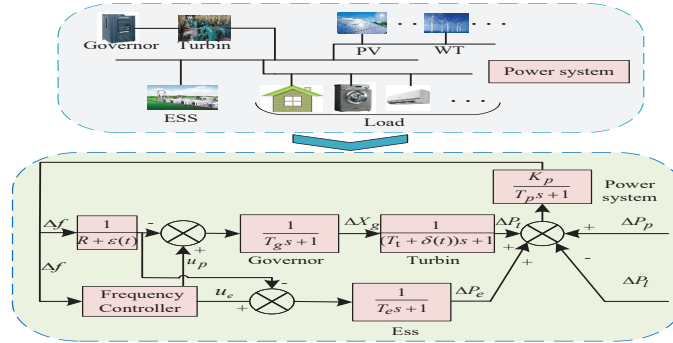


Fig. 5.1. Microgrid smart frequency control diagram.

To describe system status $x(t) = [\Delta f(t), \Delta P_e(t), \Delta P_t(t), \Delta X(t)]$, they stand for frequency deviation, ES power deviation, microturbine output, and governor position value, respectively. System control input $u(t) = [u_g(t), u_e(t)]$

stand for control input of governor and ES, respectively. $\Delta P_{pv}(t)$ and $\Delta P_{lo}(t)$ stand for the power fluctuation of photovoltaic and load, respectively. The uncertainty of photovoltaic and load $\Delta P_{pv}(t) - \Delta P_{lo}(t)$ is described as external disturbance $v(t)$, and the frequency model of microgrid can be represented as:

$$\dot{x}(t) = Ax(t) + B_1 u_g(t) + B_2 u_e(t) + F v(t) \quad (5.2)$$

$$\text{where } A = \begin{bmatrix} \frac{-1}{T_p} & \frac{K_p}{T_p} & \frac{K_p}{T_p} & 0 \\ \frac{-1}{(\varepsilon(t)+R)T_e} & \frac{-1}{T_e} & 0 & 0 \\ 0 & 0 & \frac{-1}{\delta(t)+T_t} & \frac{1}{\delta(t)+T_t} \\ \frac{-1}{(\varepsilon(t)+R)T_g} & 0 & 0 & \frac{-1}{T_g} \end{bmatrix} \quad (5.3)$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_g} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_e} \\ 0 \end{bmatrix}, F = \begin{bmatrix} \frac{K_p}{T_p} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (5.4)$$

5.2.2 Affirmation of integral zero-sum game

For microgrid with ES, the frequency controller is planned to manage the ES and the governor to reconstitute the frequency to the rated frequency. We reflect on is the real time frequency management problem of microgrid with uncertainty, in which contains system variable and disturbance uncertainty. Reflecting on the control input u_e is constrained, a non-quadratic function which is suggested into the performance index function represented with:

$$J(x(0)) = \int_0^\infty (x^T M x + u_g^T N_1 u_g + S_e - \gamma^2 v^T v) d\tau. \quad (5.5)$$

$$S_e = 2u_{emax} \int_t^{u_e} \tanh^{-T} \left(\frac{w}{u_{emax}} \right) N_2 dw \quad (5.6)$$

where $M \in \mathbb{R}^{4 \times 4}$ is positive definite and symmetric matrix. $N_1 \geq 0, N_2 \geq 0$, and γ is the attenuation coefficient and u_{emax} is the maximum control magnitude of ES. $L_2 - Gain$ definition [105] can be applied here, there are some $u_g(t)$ and $u_e(t)$ to insure the closed loop system (5.2) content $L_2 - Gain \leq \gamma, \gamma > 0$ if

$$\int_t^\infty (x^T M x + u_g^T N_1 u_g + S_e) d\tau \leq \gamma^2 \int_t^\infty (v^T v) d\tau \quad (5.7)$$

for all $0 < t < \infty$ and all $v \in L_2[0, \infty]$.

The optimal control requires to be acquired to stabilize the system, goal to the minimum performance indicator (5.5). The robust optimal frequency control of microgrid is represented as an integral zero-sum game, where the control input u_g and u_e are positive players, which reduces the performance indicator. Disturbance v is the adversarial player, which reduces the performance indicator. The Nash equilibrium solution of multiplayer integral zero-sum game requires satisfying the following:

$$J^*(x) = \min_{u_g, u_e} \max_v J(x, u_g, u_e, v) = \max_v \min_{u_g, u_e} J(x, u_g, u_e, v) \quad (5.8)$$

5.3 Integral reinforcement learning with dynamic event triggered approach

5.3.1 Integral reinforcement learning approach

The adaptive dynamic programming approach can work out the optimal control dilemma, for any given control input u_g and u_e , the value function can be represented with:

$$V(x(t)) = \int_t^\infty (x^T M x + u_g^T N_1 u_g + S_e - \gamma^2 v^T v) d\tau \quad (5.9)$$

Differential equation of (5.9) in reference to t can be acquired as the following Bellman equation:

$$0 = (\nabla V)^T \left(Ax(t) + B_1 u_g(t) + B_2 u_e(t) + F v(t) \right) + \int_t^\infty (x^T M x + u_g^T N_1 u_g + S_e - \gamma^2 v^T v) d\tau \quad (5.10)$$

where $\nabla V = \frac{\partial V}{\partial x}$, and the Hamiltonian function can be represented as:

$$H(x, u_g, u_e, v, \nabla V) = (\nabla V)^T \left(Ax(t) + B_1 u_g(t) + B_2 u_e(t) + F v(t) \right) + x^T M x + u_g^T N_1 u_g + S_e - \gamma^2 v^T v \quad (5.11)$$

The optimal value function can be represented with:

$$V^*(x(t)) = \min_{u_g, u_e} \max_v \left\{ \int_t^\infty (x^T M x + u_g^T N_1 u_g + S_e - \gamma^2 v^T v) d\tau \right\} \quad (5.12)$$

Optimal control input/disturbance can be represented with:

$$u_g^*(t) = -\frac{1}{2} N_1^{-1} B_1^T \nabla V, u_e^*(t) = -u_{emax} \tanh \left(\frac{1}{2u_{emax}} N_2^{-1} B_2^T \nabla V \right), v^*(t) = \frac{1}{2\gamma^2} F^T \nabla V \quad (5.13)$$

The optimal control (5.13) is accepted into the Hamilton function to acquire the Hamilton-Jacobi-Isaacs equation:

$$0 = x^T M x + (\nabla V^*)^T A x(t) - \frac{1}{4} (\nabla V^*)^T B_1 N_1 B_1 (\nabla V^*) - (\nabla V^*)^T B_2 u_{emax} \tanh \left(\frac{1}{2u_{emax}} N_2^{-1} B_2^T \nabla V^* \right) + 2 \sum_{j=1}^{n_2} \int_0^{u_j} u_{emax} \operatorname{artanh} \left(\frac{u_j}{u_{emax}} \right) + \frac{1}{4\gamma^2} (\nabla V^*)^T F F^T (\nabla V^*). \quad (5.14)$$

The Hamilton-Jacobi-Isaacs equation (5.14) is a partial differential equation,

and it is hard to acquire an analytical solution. Adaptive dynamic programming-based policy iteration approach [101] can accomplish policy evaluation by (5.10) and policy enhancement by (5.13), which depend on accurate system model variables (A, B_1, B_2, F) . Nevertheless, the system equation in the previous part has uncertain dynamic transition matrix, which influences on the exactness of the control. Thus, motivated by the integral reinforcement learning algorithm, we modify the Bellman equation into an integral form. In time period T , the value function contains the following integral Bellman function:

$$V(t - T) = V(t) + \int_{t-T}^t (x^T M x + u_g^T N_1 u_g + S_e - \gamma^2 v^T v) d\tau \quad (5.15)$$

It is clear that equation (5.15) does not need exact variables of the microgrid. The policy iterative algorithm depended on integral reinforcement learning represented in Algorithm 1. Algorithm 1 is a supplement of integral reinforcement learning underneath policy iterative approach to the integral zero-sum game. The reinforcement signal in policy evaluation stage involves a non-quadratic function a priori to mention the control restrictions of the ES. The accurate control of the governor and ES can be acquired without using information of the system internal dynamics. Also, the following theorem ensures that the algorithm can converge to the optimal solution of the Hamilton-Jacobi-Isaacs equation.

Define a new value function Λ^{i+1} , which begin with $\Lambda^0 \equiv 0$ under the control input μ_g and μ_e .

$$\Lambda^{i+1}(t - T) = \Lambda^i(t) + \int_{t-T}^t (x^T M x + \mu_g^T N_1 \mu_g + S_e - \gamma^2 v^T v) \quad (5.16)$$

$$S_e = 2\mu_{emax} \int_t^{u_e} \tanh^{-T} \left(\frac{w}{\mu_{emax}} \right) N_2 dw \quad (5.17)$$

for $\forall i, V^i(T) \leq \Lambda^i(T)$, i indicates the number of iterations.

Theorem 1: Conforming to the value function described by (5.9), the control described by (5.13) and the new value function described by (5.16). Then, $V^i(t)$ will merge to $V^*(t)$.

In this research, the difficulty of internal variable disturbance and confined ES control is resolved by an integral reinforcement learning algorithm with input restrictions. (5.15) slacken the prerequisite for model variables by building the reward with non-quadratic function and integral Bellman equation. Also, the input restriction difficulty is solved via the continuous bounded characteristic of the hyperbolic tangent function. It does not impact on the conclusions of this research if it is substituted by other functions.

Algorithm 1: Policy iteration algorithm based on IRL

1: Data initialization: Let the iteration index $i = 0$, select a appropriate time period T , and give an admissible control strategy u_g, u_e .

2: Policy evaluation: $V^{(i)}(t - T) = V^{(i)}(t) + \int_{t-T}^t (x^\top Mx + u_g^\top N_1 u_g + S_e - \gamma^2 v^\top v) d\tau$, $V^{(0)}(0) = 0$.

3: Policy update: The update of the two control strategies is based on the following two formulas: $u_g^{(i+1)}(t) = -\frac{1}{2} N_1^{-1} B_1^\top \nabla V^{(i)}$. $u_e^{(i+1)}(t) = -u_{emax} \tanh(\frac{1}{2u_{emax}} N_2^{-1} B_2^\top \nabla V^{(i)})$. $v^{i+1}(t) = \frac{1}{2\gamma^2} F^\top \nabla V^i$.

4: End condition: If $\|V^{(i+1)} - V^{(i)}\| \leq \varrho$ or $i \geq i_{max}$, stop the iteration. Otherwise, $i = i + 1$, and repeat 2 and 3 until 4 is satisfied.

Fig. 5.2. Policy iteration algorithm based on integral reinforcement learning.

5.3.2 Static event trigger control mechanism

The time trigger control approach ends in a squander of supplies. Thus, this part mentions event trigger control mechanism for u_e to lessen the control refurbish times of ES, to decreasing the communication load and calculation.

Here, $\{t_r\}_{r=0}^{\infty}$ stands for the triggering instants, and $t_0 = 0, r \in \mathbb{N}^+$. The system accepts T as the sampling period; however, the system state is modernized only at the trigger moment. The trigger state can be represented with:

$$\hat{x}_r(t) = x(t_r), t_r \leq t < t_{r+1} \quad (5.18)$$

By utilizing the zero-order holder, the piecewise continuous optimal control strategy can be represented with:

$$\begin{aligned} u_e^*(t) &= u_e^*(\hat{x}_r), t_r \leq t < t_{r+1}, u_e^*(t) = \\ &-u_{emax} \tanh\left(\frac{1}{2u_{emax}} N_2^{-1} B_2^T \nabla V^*(\hat{x}_{r+1})\right), t = t_{r+1} \end{aligned} \quad (5.19)$$

When deciding if an event trigger happens, the error condition is very crucial and can be described as:

$$\Pi_{ec}(t) = x(t) - \hat{x}_r(t) \quad (5.20)$$

To be rooted in the above analysis, it is required to allow the following assumptions.

Assumption 1: The optimal control method is locally Lipschitz continuous, which contents: $\|u_e^* - \hat{u}_e^*\|^2 = \|u_e^*(x) - \hat{u}_e^*(\hat{x}_r)\|^2 \leq k_u \|x - \hat{x}_r\|^2 = k_u \|\Pi_{ec}(t)\|^2$.

For system (5.2), the static event trigger mechanism can be represented as follows:

$$\begin{aligned} t_0 = 0, t_{e+1} &= \{t \in \mathbb{R}^+ | t > t_e \cap -2u_{emax} L_{artanh} k_u^2 \|\Pi_{ec}(t)\|^2 + x^T M x + \\ &u_g^* N_1 u_g^* + S_e^*(x_r) - \gamma^2 \|v^*\|^2 \} \end{aligned} \quad (5.21)$$

where L_{artanh} is the Lipschitz constant of function $artanh(\cdot)$, k_u is a positive constant and the closed-loop system (5.2) is asymptotically stable.

Proof: The system is asymptotically stable by choosing $Y(t) = \int_{t-T}^t V^*(x(\tau))d\tau$ as the Lyapunov function.

$$\dot{Y}(t) = \int_{t-T}^t (\nabla V^*(x))^T (Ax + B_1 u_g + B_2 u_e(\hat{x}_r) + Fv) d\tau \quad (5.22)$$

Based on (5.10), we have

$$\begin{aligned} \dot{Y}(t) = \int_{t-T}^t & \left(-x^T Mx - u_g^* N_1 u_g^* - S_e^* + \gamma^2 \|v^*\|^2 + (\nabla V^*(x))^T B_2 (u_e(\hat{x}_r) - \right. \\ & \left. u_e(x)) \right) d\tau \end{aligned} \quad (5.23)$$

Based on (5.13)

$$(\nabla V^*(x))^T B_2 = -2u_{emax} \tanh^{-1} \left(\frac{u_2^*(x)}{u_{emax}} \right)^T \quad (5.24)$$

Put (5.24) into (5.23),

$$\begin{aligned} \dot{Y}(t) = \int_{t-T}^t & (-x^T Mx - u_g^* N_1 u_g^* - S_e^* + \gamma^2 \|v^*\|^2 - \\ & 2u_{emax} \tanh^{-1} \left(\frac{u_2^*(x)}{u_{emax}} \right)^T (u_e^*(\hat{x}_r) - u_e^*(x))) d\tau = \int_{t-T}^t (-x^T Mx - u_g^* N_1 u_g^* - S_e^* + \\ & \gamma^2 \|v^*\|^2 + 2u_{emax} \int_{u_e^*(\hat{x}_r)}^{u_e^*(x)} (\text{artanh} \left(\frac{u_2^*}{u_{emax}} \right)^T - \text{artanh} \left(\frac{u}{u_{emax}} \right)^T) dM) d\tau \end{aligned} \quad (5.25)$$

Based on Assumption 1, we may get

$$\begin{aligned} \dot{Y}(t) \leq \int_{t-T}^t & ((2u_{emax} \|u_e^*(x) - u_e^*(\hat{x}_r)\|^T \cdot \|\text{artanh} \left(\frac{u_e^*(x)}{2u_{emax}} \right) - \\ & \text{artanh} \left(\frac{u_e^*(\hat{x}_r)}{2u_{emax}} \right)\| + \gamma^2 \|v^2\|^2 - x^T Mx - u_g^* N_1 u_g^* - S_e^*) d\tau \leq \end{aligned}$$

$$- \int_{t-T}^t (-2u_{emax}L_{artanh}k_u ||\Pi_{ec}(t)||^2 - \gamma^2 ||v^*||^2 + x^T Mx + u_g^* N_1 u_g^* + S_e^*) d\tau (5.26)$$

5.3.3 Dynamic event trigger control mechanism

To more expand the trigger interval, this part plans a dynamic event trigger control via presenting a dynamic parameter. Dynamic parameters require to content the following equation:

$$\begin{aligned} \dot{\eta} = & -\lambda\eta - 2u_{emax}L_{artanh}k_u^2 ||\Pi_{ec}||^2 + x^T Mx + u_g^* N_1 u_g^* + S_e^*(x_r) - \\ & \gamma^2 ||v^*||^2, \eta(0) = \eta_0 \geq 0. \end{aligned} \quad (5.27)$$

The dynamic parameter involves the past trigger data as well as system state and error at the current time. The dynamic event trigger rule can be represented with:

$$\begin{aligned} t_0 = 0, t_{e+1} = & \{t \in \mathbb{R}^+ | t > t_e \cap \eta(t) + \theta \left(-2u_{emax}L_{artanh}k_u^2 ||\Pi_{ec}||^2 + x^T Mx + \right. \\ & \left. u_g^* N_1 u_g^* + S_e^*(x_r) - \gamma^2 ||v^*||^2 \right) \} \end{aligned} \quad (5.28)$$

where $\theta \geq 0$, and $\lambda > 0$, when $\theta \rightarrow \infty$, dynamic event triggering (5.28) changes into static event triggering (5.21). For ES manage with restrictions, regarding the continuous outside agitation, a dynamic event triggering control is created, which guarantee the $L_2 - gain$ accomplishment under agitation as well as enhance the triggering threshold. If the dynamic parameter η described in (5.27) is adjusted to the outline of dynamic event trigger environment (5.28), $\eta \geq 0$ can be acquired.

According to (5.21), after adding dynamic parameter, the state and dynamic parameter of the system are asymptotically stable.

Proof: Lyapunov equation is chosen as bellows:

$$Y_\eta(t) = Y(t) + \int_{t-T}^t \eta(\tau) d\tau \quad (5.29)$$

More derivation of the above equation can be acquired:

$$\begin{aligned} \dot{Y}_\eta(t) \leq & - \int_{t-T}^t \left(-2u_{emax}L_{artanh}k_u \|\Pi_{ec}\|^2 - \gamma^2 \|v^*\|^2 + x^T M x + u_g^* N_1 u_g^* + \right. \\ & \left. S_e^*(x_r) \right) d\tau + \int_{t-T}^t \left(-\lambda\eta - 2u_{emax}L_{artanh}k_u^2 \|\Pi_{ec}\|^2 + x^T M x + u_g^* N_1 u_g^* + \right. \\ & \left. S_e^*(x_r) - \gamma^2 \|v^*\|^2 \right) d\tau \leq - \int_{t-T}^t \lambda\eta(\tau) d\tau \end{aligned} \quad (5.30)$$

According to Theorem 1 and $\lambda > 0$, we can obtain $\dot{Y}_\eta(t) < 0$, thus, it is supposed in which the dynamic parameter and states of the outline can converge to 0. ■

Via the design of trigger mechanism in (5.21) and (5.28), we can obtain the trigger threshold of static event trigger control and dynamic event trigger control respectively.

$$T_e^s = \frac{x^T M x + u_g^* N_1 u_g^* + S_e^*(x_r) - \gamma^2 \|v^*\|^2}{2u_{emax}L_{artanh}k_u^2} \quad (5.31)$$

$$T_e^d = \frac{x^T M x + u_g^* N_1 u_g^* + S_e^*(x_r) - \gamma^2 \|v^*\|^2}{2u_{emax}L_{artanh}k_u^2} + \frac{\eta(t)}{2u_{emax}L_{artanh}k_u^2} \quad (5.32)$$

where T_e^s stands for the threshold triggered by static events, and T_e^d stands for threshold triggered by dynamic events.

An innovative dynamic event triggered based integral reinforcement learning is created to lessen the control renew times of ES control. Different from event triggered control schemes in [106], which requires continuously monitoring, the proposed dynamic trigger control have a dynamic parameter associated with the outside disturbance and past trigger data, not just the current control and state, which

furthermore broaden the triggering threshold. Different from current dynamic event triggered control in [107], dynamic event triggering changes the integral reinforcement learning algorithm from cycle-based sampling to N-cycle sampling, which can evade external actions.

5.4 Application of event-based neural network

Fig. 5.3. demonstrates the scheme of the event triggered integral reinforcement learning frequency controller; performance indicator function is estimated by actor critic neural network frame [108]. The ES control u_e is renewed according to aperiodically event trigger control.

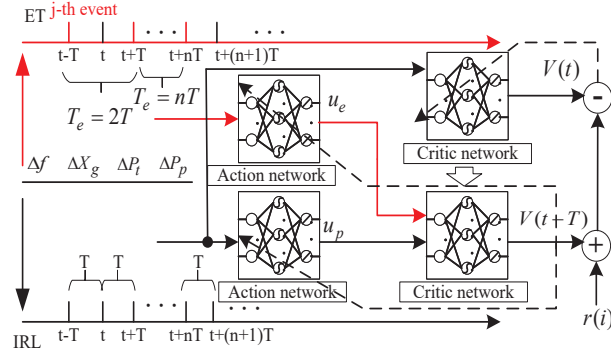


Fig. 5.3. Event triggered integral reinforcement learning control scheme.

5.4.1 Critic neural network

The critic neural network includes three layers: input, hidden, and output layers. The input layer data is the state of the system. Output layer data is a value function. For easy analysis, the weight from the input to the hidden layer is set to identity matrix. The configuration of the critic network can be represented as follows:

$$V(x) = W_c^T \varphi_c(x) + \xi(x) \quad (5.33)$$

$$\nabla V = \nabla \varphi_c^T(x) W_c + \nabla \xi \quad (5.34)$$

where W_c stands for the weight from the hidden layer to output layer, $\varphi_c(x)$ stands

for the critic function, $\xi(x)$ stands for neural network error, and $\nabla \varphi = \frac{\partial \varphi}{\partial x}$, $\nabla \xi = \frac{\partial \xi}{\partial x}$.

Take the value function (5.33) into the integral Bellman equation (5.15), its error can be acquired as follows:

$$W_c^T \varphi_c(x(t)) - W_c^T \varphi_c(x(t-T)) + \int_{t-T}^t (x^T M x + u_g N_1 u_g + S_e - \gamma^2 v^T v) d\tau = \xi_c \quad (5.35)$$

Among them, $\int_{t-T}^t (x^T M x + u_g N_1 u_g + S_e - \gamma^2 v^T v) d\tau$ is described as an increment

signal $r(t)$, $\varphi_c(x(t)) - \varphi_c(x(t-T)) = \Delta \varphi_c(x(t))$ and (5.35) can be redescribed as:

$$r(t) + W_c^T \Delta \varphi_c(x(t)) = \xi_c \quad (5.36)$$

Because the optimal weight is unknown, the current neural network output weight $\widehat{W}_c$

is employed to approximate W_c , and the actual output of the critic network is as follows:

$$\widehat{V}(x) = \widehat{W}_c^T \varphi_c(x) \quad (5.37)$$

$$\nabla \widehat{V} = \nabla \varphi_c^T(x) \widehat{W}_c \quad (5.38)$$

Take (5.37) into (5.15) to get the Bellman error at the current time:

$$\widehat{W}_c^T \Delta \varphi_c(x(t)) + r(t) \triangleq e_c \quad (5.39)$$

The goal of the critic network is to reduce $E_c = 1/2 e_c^T e_c$. Via

gradient descent approach and chain rule, the setting rate of weight can be acquired as follows:

$$\dot{\hat{W}}_c = -l_c \frac{\partial E_c}{\partial e_c} \frac{\partial e_c}{\partial \hat{W}_c} = -l_c \frac{\Delta \varphi}{(\Delta \varphi^T \Delta \varphi + 1)^2} \times (r(t) + \Delta \varphi_c^T(x) W_c) \quad (5.40)$$

where l_c stands for the learning rate, $(\Delta \varphi^T \Delta \varphi + 1)^2$ is employed for normalization.

5.4.2 Actor neural network

The organization of the actor network is three layers. The real actor network approximation output can be represented with:

$$\hat{u}_g(x) = -\frac{1}{2} N_1^{-1} B_1^T \nabla \varphi_a^T \hat{W}_{A1} \quad (5.41)$$

$$\hat{u}_e(x_r) = -u_{emax} \tanh \left(\frac{1}{2u_{emax}} N_2^{-1} B_2^T \nabla \varphi_a^T(x_r) \hat{W}_{A2} \right) \quad (5.42)$$

The statement of estimate disturbance can be acquired based on the critic network:

$$\hat{v}(x) = \frac{1}{2\gamma^2} F^T \nabla \varphi_c^T(x) \hat{W}_c \quad (5.43)$$

The errors of the two actor networks can be represented as:

$$e_{a1} = -\frac{1}{2} N_1^{-1} B_1^T \nabla \varphi_a^T (\hat{W}_{A1} - \hat{W}_c) \quad (5.44)$$

$$e_{a2} = -u_{emax} \tanh \left(\frac{1}{2u_{emax}} N_2^{-1} B_2^T \nabla \varphi_a^T (\hat{W}_{A2} - \hat{W}_c) \right) \quad (5.45)$$

The goal of two actor networks is to reduce $E_{a1} = 1/2 e_{a1}^T e_{a1}$ and $E_{a2} = 1/2 e_{a2}^T e_{a2}$.

Via the gradient descent approach and chain rule, two setting rates are:

$$\dot{\hat{W}}_{A1} = -l_{a1} \frac{\partial E_{a1}}{\partial e_{a1}} \frac{\partial e_{a1}}{\partial \hat{W}_{A1}} = -l_{a1} e_{a1}^T \times \left(-\frac{1}{2} N_1^{-1} B_1^T \nabla \varphi_a^T \right) \quad (5.46)$$

$$\dot{\hat{W}}_{A2} = -l_{a2}(\nabla\varphi_a B_2 e_{a2} + \nabla\varphi_a B_2 \tanh^2(\hat{B}) e_{a2}), t = t_{r+1} \quad (5.47)$$

where $\hat{B} = 1/2u_{emax}N_2^{-1}B_2^T\nabla\varphi_a^T\hat{W}_{A2}$.

The estimate procedure of the neural network is succinctly explained as: evaluating if the dynamic event triggering status are fulfilled. Practicing the actor network and critic network one by one based on the setting rate until the convergence conditions are satisfied.

5.5 Energy storage side power partition principles

Load side accepts the total power reference commands from the upper layer, the first job to be managed is how to allot the commands among the agents to create the actual entire power of all agents uniform to the accepted reference commands. Thus, the power output among the flexible agents needs following convinced principles. Two power partition principles are suggested, respectively.

5.5.1 Power sharing golden rule

Define $\alpha_{j0} = \frac{P_{j0}}{P_{j,max}}$ and $\alpha_j = \frac{P_j}{P_{j,max}}$ which stands for the initial power ratio

and current power ratio value of j th agent, respectively. P_{j0}, P_j and $P_{j,max}$ demonstrate the initial output power, current output power, and maximum power output of j th agent, respectively.

Then, all adaptable agents content the equilibrium status [109]:

$$\frac{P_1}{P_{1,max}} = \frac{P_2}{P_{2,max}} = \frac{P_3}{P_{3,max}} \dots = \frac{P_M}{P_{M,max}} = \alpha^* \quad (5.48)$$

where the value α^* represents for the reference ratio in which gets to the equilibrium status. The equation (5.48) exhibits, in which all the taking part agents cause in equal

proportion.

A virtual captain is defined as a controller can accept the area control signal and supplying a virtual iterative signal to handle all agents toward the aimed goals. Different from [109], the virtual captain $\alpha_0(t)$ refurbishes the citation ratio in response to diverse initial states in the following way:

$$\dot{\alpha}_0(t) = k((1 - a_i)\Delta P_{ci} - \sum_{j=1}^N (\alpha_j(t) - \alpha_{j0})P_{j,max}) \quad (5.49)$$

where k is the gain, $1 - a_i$ is the participation ratio of flexible ESs, ΔP_{ci} is governor. and ES actor error of power in which secondary frequency controller by integral reinforcement learning represented as: $\Delta P_{ci} = -\hat{W}_{A1}e_{a1} - \hat{W}_{A2}e_{a2}$.

When all the agents are uniform, the α^* contented:

$$\alpha^* = ((1 - a_i)\Delta P_{ci} - \sum_{j=1}^N \alpha_{j0}P_{j,max}) / \sum_{j=1}^N P_{j,max} \quad (5.50)$$

The initial values of diverse agents may differ greatly, end in large power variation in fulfilling consensus, which is not favoring to frequency modify. To evade this problem, This chapter suggests the second golden rule.

5.5.2 Adaptable capacity sharing golden rule

Define $\beta_j = \frac{\Delta P_j}{\Delta P_{j,max}} = \frac{P_{j0} - P_j}{P_{j,max} - P_{j0}} = \frac{\alpha_{j0} - \alpha_j}{1 - \alpha_{j0}}$, which stands for the current ratio value of adaptable capacity for j th agent. The stabilize point satisfies the following prerequisites:

$$\frac{\Delta P_1}{\Delta P_{1,max}} = \frac{\Delta P_2}{\Delta P_{2,max}} = \frac{\Delta P_3}{\Delta P_{3,max}} \dots = \frac{\Delta P_M}{\Delta P_{M,max}} = \beta^* \quad (5.51)$$

where β^* describes the reference ration of the stabilize point changed based on adaptable capacity ratio.

Likewise, the virtual captain refurbishes the reference ration, like below:

$$\dot{\beta}_0(t) = k_b((1 - a_i)\Delta P_{ci} - \sum_{j=1}^N(\beta_j(t)(1 - \alpha_{j0})P_{j,max}) \quad (5.52)$$

where k_b is the gain.

Then, when all the agents are uniform, the stability point β^* contents:

$$\beta^* = \sum_{j=1}^N \frac{(1 - \alpha_{j0})P_{j,max}}{(1 - a_i)\Delta P_e} \quad (5.53)$$

Also, similar power output can be acquired:

$$\alpha_j = \alpha_{j0} - (1 - \alpha_{j0})\beta^* \quad (5.54)$$

The two-power partition golden rule declared above can fulfill the following goals: 1. The equations (5.48) and (5.51) can be employed to distribute power to each agent. 2. Equation (5.49) and (5.52) ensure in which the total output of the ES side emulates the received reference commands ΔP_{c2i} .

Contrasted to the golden rule showed in [109], which presumed that the initial values are 0, the suggested golden rule related with the diverse initial states of agents owing to the divergence in power consumption of ESs in reality. Also, equation (5.49) and (5.52) follow the commands allocated by the control error signal, which considers the real time execution of tracking commands better than the disturbance fluctuation. Thus, this suggested golden rule is an enlargement and augment of [109].

It can be checked from equation (5.54) in which when the initial states

$\alpha_{j0} (j = 1, \dots, N)$ are the same, the power output can be uniform if the system is uniform based on the adaptable capacity. Thus, the adaptable capacity agreement golden rule involves the power-sharing golden rule.

According to the adaptable capacity consensus golden rule, the following control goal are recapitulated as:

$$\begin{cases} \lim_{t \rightarrow \infty} (\beta_0(t) - \beta^*) = 0 \\ \lim_{t \rightarrow \infty} (\beta_j(t) - \beta_0(t)) = 0 \end{cases} \quad (5.55)$$

Via the above control goals, the adaptable capacity of all agents can be divided up and modified to the reference value.

5.6 Dynamic event triggered distributed control approach

In this part, a data sampled based dynamic event trigger control is created using leader-follower consensus principle to accomplish the suggested consensus control goals and decrease the communication load.

5.6.1 Communication graph topology

The communication network topology is represented as a directed graph $DG = \{G, W\}$ to express the communication topology involving a set of nodes $G = \{g_1, g_2, \dots, g_M\}$ and a set of edge $W = G \times G$ which express a communication link for data exchange [110]. The proximity matrix $A = [a_{ij}]$ express the relationship of the edges. In A , the a_{ij} express the weight of data exchange between agents i, j . a_{ij} describes in which agent i accepts the information of agent j . If $(g_i, g_j) \in W$, then $a_{ij} = 1$, otherwise, $a_{ij} = 0$. The degree matrix of a graph can be described as $D = \text{diag}\{d_i\}$, $d_i = \sum_{j=1}^M a_{ij}$, Laplacian matrix $L = [\tilde{l}_{ij}] \in \mathbb{R}^{M \times M}$ and $L = D - A$ [111].

5.6.2 Control approach of dynamic event triggered mechanism

According to reference [109], the j th agent state and controller are set as below:

$$\begin{cases} \dot{\beta}_j(t) = u_j(t) \\ u_j(t) = k_u(\sum_{k=1}^N a_{ij}(\beta_j(t) - \beta_i(t)) - S_j(\beta_j(t) - \beta_0(t))) \end{cases} \quad (5.56)$$

where $S_j = 1$ implies in which agent accepts the upper control command, or $S_j = 0$. Assume that at any rate one agent can accept the reference commands. via the communication network, only a handful of agents accept the ratio values of virtual captain as given in equation (5.52). Most of the unused distributed agents get command about their nearby community through the local communication network.

Also, define $n_j(t) = \beta_j(t) - \beta_0(t)$, $\varsigma(t) = \beta_0(t) - \beta^*$.

The subsequent equation is given:

$$\begin{cases} \dot{\eta}_j(t) = u_j(t) + k_b(\sum_{j=1}^N (\eta_j(t) + \varsigma(t))(1 - \alpha_{j0})P_{j,max}) \\ \dot{\varsigma}(t) = -k_b(\sum_{j=1}^N (\eta_j(t) + \varsigma(t))(1 - \alpha_{j0})P_{j,max}) \end{cases} \quad (5.57)$$

Define the sampling period h , and the sample time sequence is kh . Then, the transmission sequence $\{t_k^j h\}$ is decided by the following event triggered process:

$$t_{k+1}^j h = \inf\{kh > t_k^j h | e_j^2(t) \geq \delta \chi_j(t_k^j h) + \nu \varphi_j(t)\} \quad (5.58)$$

where $e_j(t) = \beta_j(kh) - \beta_j(t_k^j h)$, $t \in [kh, kh + h)$, $t_k^j = \max\{t_k^j | t_k^j h \leq t\}$, $\chi_j(t_k^j h) =$

$$\sum_{j=1}^N a_{j1} (\beta_1(t_k^j h) - \beta_j(t_k^j h))^2 - S_j (\beta_j(kh) - \beta_0(kh))^2.$$

The scalar $\delta > 0, \nu > 0$, and the positive definite scalar function $\varphi_j(t)$ can be represented with:

$$\begin{cases} \dot{\varphi}_j(t) = -\mu\varphi_j(t) + \varepsilon(\delta\chi_j(t_k^j h) - e_j^2(t)) \\ \mu > 0, \varepsilon > 0, \varphi(0) > 0 \end{cases} \quad (5.59)$$

The precise proof can be indicated to [112].

To confirm the control goals of agents, the event triggered controller can be represented with:

$$u_j(t) = k_u \{ \sum_{j=1}^N a_{jl} (\beta_l(t_k^j h) - \beta_j(t_k^j h)) - S_j (\beta_l(kh) - \beta_0(kh)) \} \quad (5.60)$$

where k_u is the control gain.

When $\{t_k^j h\}$ contents the predetermined event triggered condition (5.58), it right away triggers the sending of current sampling date to the community over the communication topology. In this manner, only the sampled data at the event triggered prompt can refurbish the controller and system state.

5.6.3 Distributed event trigger system stability

Combining (5.58) – (5.60), the subsequent equation can be represented with.

define $x(t) = \text{col}[\eta(t), \varsigma(t)]$ and we take:

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}x(t - \tau(t)) + \bar{F}e(t) \quad (5.61)$$

where $\bar{A} = k_b \begin{bmatrix} 1_M D & \tilde{D} \\ -D & -D 1_M \end{bmatrix}$, $\tilde{B} = k_u \begin{bmatrix} -(L+S) & 0_{M \times 1} \\ 0_{1 \times M} & 0 \end{bmatrix}$, $\bar{F} = k_u \begin{bmatrix} L \\ 0_{1 \times M} \end{bmatrix}$, $D = [(1 - \alpha_{10})P_{1,max}, \dots, (1 - \alpha_{M0})P_{m,max}]$, $\tilde{D} = 1_M D 1_M$, $1_M = [1, 1, \dots, 1]$, $\tau(t) = t - kh$, $\tau(t) \in [0, h)$.

This section investigates the stability of the suggested control approach for the system (5.61), according to Theorem 2, the control objective (5.55) can be ensured, which is verified as below.

Theorem 2: The system (5.61) is asymptotically stable, if there subsist symmetric positive definite matrices P, R_1, R_2, Q_1, Q_2 , and proper dimension matrices U such that the following linear matrix inequality supports:

$$\psi_l < 0, (l = 1, 2), \begin{bmatrix} \tilde{R}_2 & U \\ * & \tilde{R}_2 \end{bmatrix} \geq 0 \quad (5.62)$$

Proof of Theorem 2: Select a Lyapunov-Krasovskii function:

$$\begin{aligned} V(t) = & x^T(t)Px(t) + \int_{t-\tau(t)}^t x^T(s)Q_1x(s)ds + \sum_{j=1}^N \varphi_j(t) + \\ & \int_{t-h}^{t-\tau(t)} x^T(s)Q_2x(s)ds + h \int_{t-\tau(t)}^t \int_{t+\theta}^t \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \\ & h \int_{-h}^{-\tau(t)} \int_{t+\theta}^t \dot{x}^T(s)R_2\dot{x}(s)dsd\theta + h^2 \int_{t-\tau(t)}^t \dot{x}^T(s)T\dot{x}(s)ds - \frac{\pi^2}{4} \int_{t-\tau(t)}^t [x(s) - \\ & x(t - \tau(t))]^T T [x(s) - x(t - \tau(t))] ds \end{aligned} \quad (5.63)$$

Define the augmented vector:

$$\begin{aligned} \zeta(t) = & col\{x(t), x(t - \tau(t)), x(t - \\ & h), \frac{1}{\tau(t)} \int_{t-\tau(t)}^t x(s)ds, \frac{1}{h-\tau(t)} \int_{t-\tau(t)}^{t-d(t)} x(s)ds, e(t)\} \end{aligned} \quad (5.64)$$

The derivative of $V(t)$ along the trajectories of the system (5.61) can be calculated as below:

$$\begin{aligned}\dot{V}(t) = & \zeta^T(t) \left\{ r_1^T P \Phi_2 + r_1^T Q_1 r_1 - r_3^T Q_2 r_3 + \Phi_2^T (\tau(t) h R_1 + (h - \tau(t)) h R_2) \Phi_2 + \right. \\ & \left. h^2 \Phi_2^T T \Phi_2 - \Pi^T \bar{R} \Pi - \frac{\pi^2}{4} r_1^T T r_1 + 2 \frac{\pi^2}{4} r_1^T T r_2 - \frac{\pi^2}{4} r_2^T T r_2 \right\} \zeta(t) - \mu \sum_{j=1}^N \varphi_j(t) + \\ & \sum_{j=1}^N \varepsilon \left(\delta \chi_j(t_k^j h) - e_j^z(t) \right)\end{aligned}\quad (5.65)$$

where

$$\begin{aligned}\psi_l = & \begin{bmatrix} \Phi_1 & * & * \\ h R_l \Phi_2 & -R_l & * \\ h T \Phi_2 & 0 & -T \end{bmatrix}, \Pi_1 = \begin{bmatrix} r_1 - r_2 \\ r_1 + r_2 - 2r_4 \end{bmatrix}, \Pi_2 = \begin{bmatrix} r_2 - r_3 \\ r_2 + r_3 - 2r_5 \end{bmatrix}, \Pi = \\ & \text{col}\{\Pi_1, \Pi_2\}, \Phi_2 = \bar{A} r_1 + \bar{B} r_2 + \bar{F} r_6, \Sigma_1 = E^T (L + S)^T (L + S) E, \Sigma_2 = E^T (L + \\ & S)^T L, \Psi_l = r_1^T \left(\text{sym}(P \bar{A}) + Q_1 - \frac{\pi^2}{4} T \right) r_1 + r_2^T \left(\theta_1 \Sigma_1 - \frac{\pi^2}{4} T \right) r_2 + 2 r_1^T \left(P \bar{B} + \right. \\ & \left. \frac{\pi^2}{4} T \right) r_2 - r_3^T Q_2 r_3 - \Pi^T \bar{R} \Pi + 2 r_1^T (P \bar{F} - \theta_2 \Sigma_2) r_6 + \theta_3 r_6^T L^T L r_6, \bar{R} = \\ & \begin{bmatrix} \tilde{R}_2 & U \\ * & \tilde{R}_2 \end{bmatrix}, \tilde{R}_2 = \begin{bmatrix} R_2 & 0 \\ 0 & 3R_2 \end{bmatrix}, E = [I_{(n-1) \times (n-1)}, 0_{(n-1) \times 1}], \tilde{R}_2 = \\ & \text{diag}\{R_2, 3R_2\}, \theta_1 = \frac{\delta \mu}{\nu} + \varepsilon \delta, \theta_2 = -\left(\frac{\delta \mu}{\nu} + \varepsilon \delta \right), \theta_3 = (\delta - 1) \left(\frac{\mu}{\nu} + \varepsilon \right), r_g = \\ & [0_{n \times (g-1)}, I_{n \times n}, 0_{n \times (6-g)n}, 0_{n \times (n-1)}], g = 1, 2, \dots, 6.\end{aligned}$$

Applying Schur complement and Lemma 3 in [112], it is possible to get:

$$\dot{V}(t) \leq \zeta^T \psi_l \zeta \quad (5.66)$$

Therefore, if (5.62) holds, the system (5.61) is asymptotically stable.

5.7 Simulation results

5.7.1 Basic data setting

According to the microgrid system scheme in Fig. 5.1., The microgrid system has the rated capacity of 210 kW and the renewable energy system rated capacity is 160 kW. The power constraints of ES are set as $u_{emax} = 0.15$ and the variables of microgrid system [92] are set as $T_e = 0.5, T_g = 0.4, T_p = 11, R = 0.09$. Suggested in which the simulation step $t_s = 0.002s$, and the sample period $h = 0.02s, \alpha_1 = 0.6, \alpha_2 = 0.4$.

The variables according to the control algorithm are given as: $M = 2.6I_{4 \times 4}, N_1 = 6.6, N_2 = 5.6$. Neural network variables contain learning rate $l_c = l_{a1} = l_{a2} = 1$, activation function are $[x_1^2, x_1x_2, x_1x_3, x_1x_4, x_2^2, x_2x_3, x_2x_4, x_3^2, x_3x_4, x_4^2]$. After training, the weights of two actor neural networks and one critic neural network can converge well under the uncertainty perseverance of excitation environment as shown in Fig. 5.4 (a), (b), (c). All three graphs can converge after 260 s. Throughout the training process, the state representation graph of the system can ultimately converge to 0 as represented in Fig. 5.4 (d), fourth graph, which confirmed the stability of the system. The frequency variance graph with and without ES contrast is represented in Fig. 5.4, (e). From (e), under the same load condition, the most frequency deviation range with ES is 0.16 p.u., meanwhile the most frequency deviation range without ES is 0.26 p.u. The maximum frequency deviation magnitude is lessened by 30%. Concurrently, the frequency recuperate speed with ES is 10 s rapider than that of without ES, which confirms the property of fast reply. According to convergent weights in Fig. 5.4 (a), (b), (c) the optimal control approach of governor and ES can be acquired based on equation (5.41) and (5.42). Control graphs are shown in Fig. 5.4 (f). The control

magnitude of ES is less than 0.1 in which satisfies the control restraints.

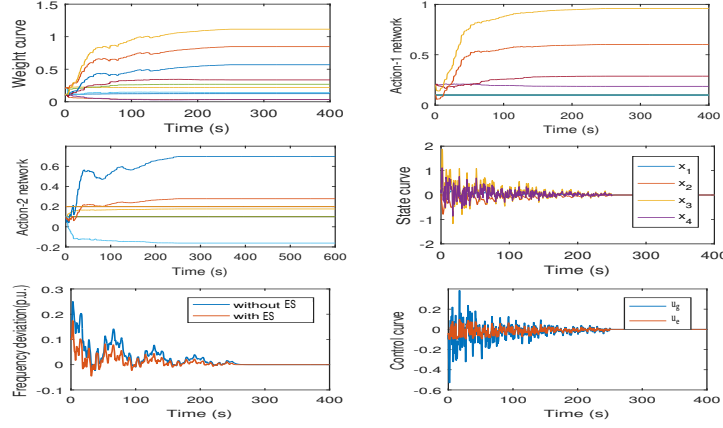


Fig. 5.4. Simulation results (a) Critic network weight curve (b) actor 1 network curve (c) actor 2 network curve (d) state curve (e) frequency deviation graph (f) governor and ES control curve.

5.7.2 Event triggered mechanism

Static event triggered control mechanism is static event triggered mechanism of sampling period, trigger threshold, and frequency fluctuation sampling according to static event triggering are represented in Fig. 5.5. According to Fig. 5.5. (a), integral reinforcement learning approach relative to time triggered control accepts 0.015s as the sampling interval, meanwhile static event triggered control is even extend sampling interval than 0.015s. Thus, the effectiveness of static event triggered control mechanism is greater than time triggered control mechanism. According to Fig. 5.5. (b), trigger threshold and state deviation qualify the circumstance. While the system deviation is larger than the trigger threshold, the system condition is refurbished. According to Fig. 5.5. (c) represents that the frequency fluctuation is refurbished at the trigger time.

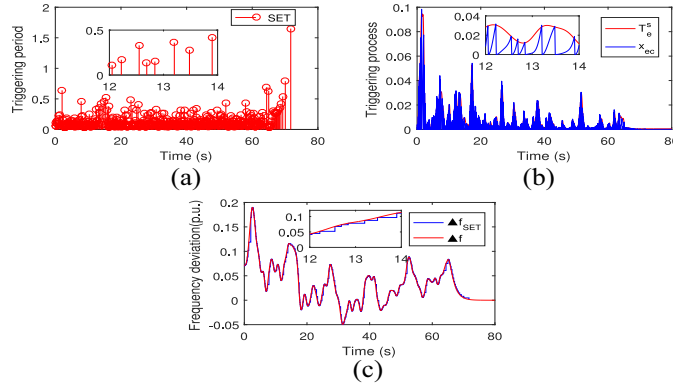


Fig. 5.5. Simulation results (a) Sampling of static event triggering control (b) Trigger threshold and sampling error of static event triggering control (c) Frequency deviation of static event triggering control.

Dynamic event triggered control mechanism is according to Fig. 5.6, dynamic event triggered mechanism of sampling period, trigger threshold, dynamic parameter and frequency fluctuation sampling are represented in figure. Owing to statistics and computation, the many samples triggered by static events is 1730, and several samples triggered by dynamic events mechanism is 1530. To consider dynamic event triggered control with static event trigger control, the sampling period in Fig. 5.6 (a) is greater than in Fig. 5.5. (a). This understanding can be well elucidated by the greater trigger threshold value in Fig. 5.6 (b). According to Fig. 5.6 (c), dynamic parameter little by little converges to zero as time passed, in which confirms properness of the theory. The frequency fluctuation samplings are decreased by comparison Fig. 5.6 (d) with Fig. 5.5. (c).

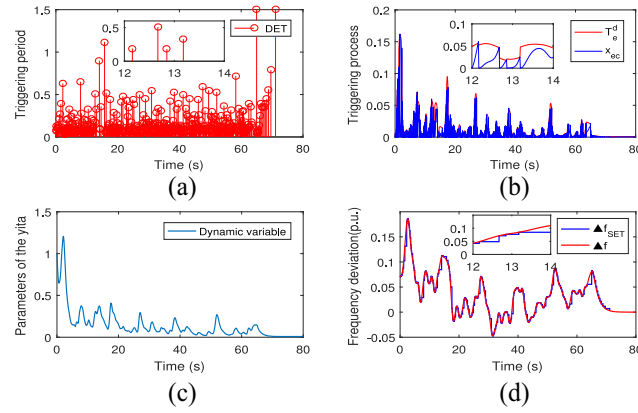


Fig. 5.6. Simulation results (a) Sampling of dynamic event triggering control (b) Trigger threshold and sampling error of dynamic event triggering control (c) dynamic parameter (d) Frequency deviation of dynamic event triggering control.

For contrast, the accurate distinction of time triggered control, static event triggered control, dynamic event triggered control is summed up in Table 1. It is transparent in which dynamic event trigger control has smaller trigger times, lesser trigger rate, and lengthen sampling interval, which devoted in storing resources of the controller. After computation, dynamic event triggering can diminish 31% of sampling points.

Table 5.1. Comparison of triggering approaches.

Trigger control	Time triggered control	Static event triggered control	Dynamic event triggered control

Trigger frequency	7000	1710	1530
Smallest period	0.01	0.025	0.03
Normal period	0.01	0.045	0.053
Trigger rate	100	21	18

5.7.3 Comparison of frequency deviation

To show the productivity of the frequency deviation, two groups of PV and energy storage system load fluctuations are given at 2.2s and 12.2s, respectively. The three-control plot are relative to proportional-integral-derivative control, integral reinforcement learning approach, and integral zero-sum game based integral reinforcement learning algorithm. The variable coefficient choosing of proportional integral-derivative controller is $k_p = 2.2, k_i = 1.02, k_d = 0.03$. After then, weight acquired by the integral reinforcement learning algorithm and integral zero-sum game integral reinforcement learning algorithm are attained respectively via convergence. Control input of ES and governor is gained owing to system state and reserved weight. Fig. 5.7, represents the frequency deviation performance. Three plots can accomplish frequency recovery about 5s at the end of fluctuation. The PID approach has big fluctuation scope compared to other algorithms. Integral zero-sum game integral reinforcement learning algorithm has a smaller deviation scope. The Integral zero-sum game integral reinforcement learning algorithm is faster among three approaches.

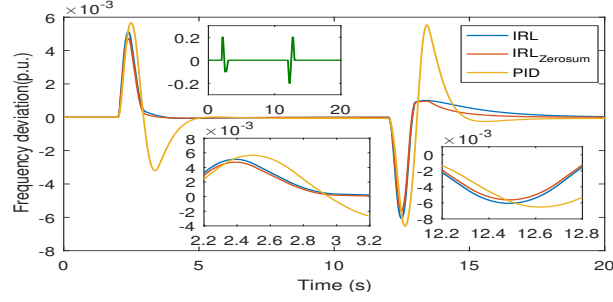


Fig. 5.7. Comparison of frequency deviation.

5.7.4 Power partition golden rules

To test the productiveness of the suggested partition golden rules, author contrasts the frequency control results of diverse partition principles in a single microgrid power system without of triggerd control. System testing variables are getting from [109]. Adapting five load aggregators (LAs) and an ES as an instance, the communication formation is represented in Fig. 5.8, (a). The initial value $\alpha = [0.76, 0.7, 0.83, 0.61, 0.43, 0]$, and the load aggregators maximum power $P_{max}(MW)$ is $P_{max}^L = [17, 13, 23, 11, 21]$, $P_{min}^{ES} = 0.92 MW$, $P_{max}^{ES} = 9.2 MW$, $k_u = 21$, $k_{b1} = 0.15$. The real load frequency control system manages with sampling data and refurbishing the control signal every 3 s [114]. Thus, to contemplate the discrete sampling in the power system area, the ΔP_{ci} area control error signal is refurbished every 3 s.

All agents reply to the power area control error (ACE) signal with adaptable capacity employment ratio β and power output employment ratio α as consensus goal, as represented in Fig. 5.9, and Fig. 5.10. According to Fig. 5.9 (b), the load aggregator units (graph α_1 to α_5) decrease the power output, meanwhile the energy storage (graph α_6) take part in frequency control by modifying its output power based on the reference

value. The power output under diverse principles is represented in Fig. 5.11, meanwhile the frequency reaction is represented in Fig. 5.12. The below graphs represent the performance of the suggested distributed control scheme in the following approaches – First, all the resources α_1 to α_6 and β_1 to β_6 are tracking the predictable reference commands α^* and β^* owing to a reference command is always predictable. Second, the tracking of the entire reference commands is ultimately completed. Third, the fluctuations in the system frequency inclined to be zero.

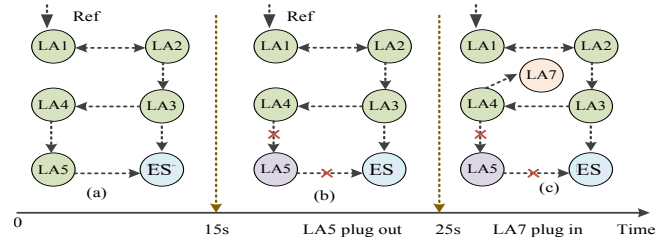
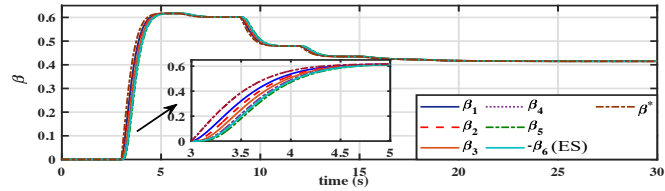
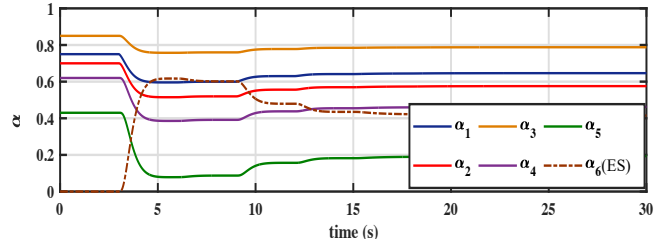


Fig. 5.8. The agent communication formation.



(a) Adaptive capacity employment ratio.



(b) Power employment ratio.

Fig. 5.9. Progression of agents' utilization ratios (a) adaptable capacity employment ratio β

(b) power employment ratio α .

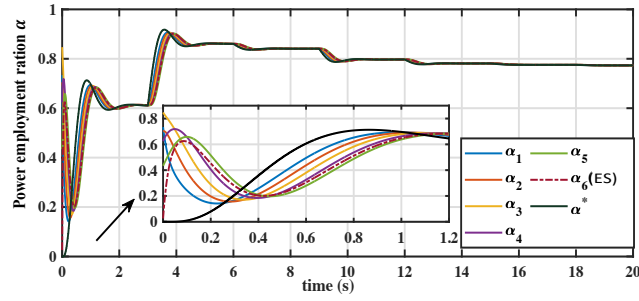
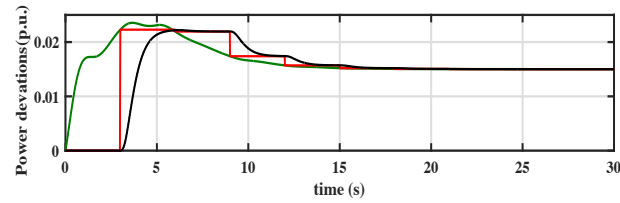
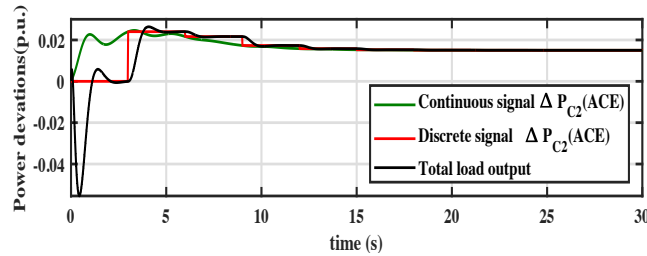


Fig. 5.10. Power-sharing golden rule power employment ratio α .

As illustrated in Fig. 5.11 (a), the suggested scheme can recognize the fast tracking of the area control error signal, meanwhile a big power deviation will happen with the golden rule of power-sharing according to Fig. 5.11 (b). This is due to the total load fluctuation in the microgrid. Also, Fig. 5.12 represent the lowest frequency deviation point -0.25 Hz via the suggested approach that is 23 % higher than power sharing golden rule approach in [108], which is -0.32 Hz.



(a) Chasing area control error (ACE) commands.



(b) Chasing area control error (ACE) with total load fluctuation.

Fig. 5.11. Power output of energy storage load side with continuous and discrete signals.

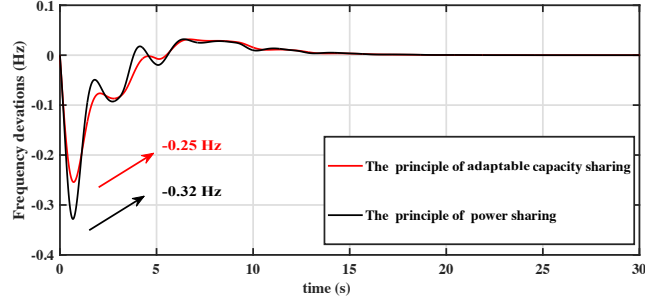
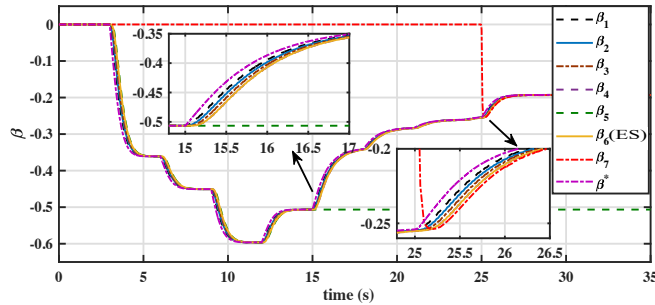


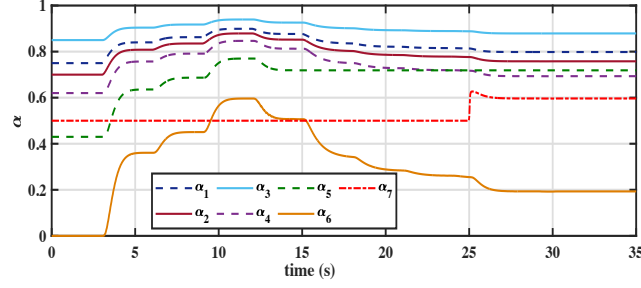
Fig. 5.12. Power output of load side with different power sharing golden rules.

5.7.5 Plug-And-Play (PnP) ability

This section introduces the strength and flexibility of the suggested distributed scheme for Plug-And-Play (PnP) ability. It is presumed in which load aggregator 5 is extracted at $t = 16s$ and then load aggregator 7 is sum up at $t = 26s$, as represented in Fig. 5.8 (b), (c). The variables are $\delta = 0.25, \nu = 0.02, \mu = 14, \varepsilon = 0.25$, and presume that loads, and energy storages have step fluctuations of -0.04 p.u. and -0.03 p.u. at $0s, 5s, 10s$, individually. Adaptable capacity employment ratio β is represented in Fig. 5.13, in which all agents rapidly accomplish the consensus beneath alterations in communication topology, as represented in Fig. 5.13 (a). Owing to the power decrease of the load side, ES (α_6, β_6) is engaged in frequency management of power output.



(a) Adaptable capacity employment ratio β .



(b) Power employment ratio α .

Fig. 5.13. Progression of agents' utilization ratios (a) adaptable capacity employment ratio β
(b) power employment ratio α .

5.8 Summary

This chapter introduces an adaptive frequency control strategy designed for the governor and energy storage (ES) systems to achieve frequency recovery in microgrids characterized by uncertainty. An advanced integral reinforcement learning algorithm is proposed to meet the control constraints of the ES while effectively managing the variable uncertainties inherent in microgrid operations, thereby minimizing diagnostic errors. Additionally, a new dynamic event-triggered control integral reinforcement learning algorithm is developed to address the challenges associated with the integral zero-sum game, facilitating aperiodic updates of control in the presence of fluctuations and mitigating external influences. Furthermore, a distributed cooperative control framework for energy storage load-side management is proposed to enhance frequency control. Through a hierarchical frequency control structure, a uniform partitioning method for load aggregators and ES is recommended to ensure proportional distribution of the adaptable capacities of each resource. This approach leads to a dynamic event-triggered distributed cooperative control mechanism for the system, enabling swift tracking of upper-level reference commands while ensuring effective frequency control. Notably, the dynamic event-triggered control significantly reduces communication burdens, while the stability of the proposed system

is maintained. Simulation results validate the effectiveness of the proposed algorithm.

Chapter 6. Power System Agile Frequency Control with Voltage Source Inverters in Low Inertia Environment

Abstract – Swift advancement of renewable energy systems leads to a notable decrease in inertia within power system networks. This chapter proposes control strategy designed to address the low inertia challenges, employing a no-coefficients in the cost function for model predictive control. Also, by transforming the objective function into a linear quadratic cost function, the approach aims to streamline computational demands. For inverter control for low inertia, author proposes the SW (switch) factor scheme, to manage voltage source inverters based on the no-coefficients in the inverter weighting factor for finite control set model predictive control. This method eschews the use of a cost function, thereby facilitating rapid frequency control through optimal switching techniques. The controller efficiently regulates the power outputs of the inverter to maintain operations within frequency constraints and to optimize power distribution during disturbances. The implementation of optimized model predictive controls without cost functions is validated.

6.1 Background

Large area electric energy generation employing inverters and voltage source converters brings about new problems in real-time power system operation and management. Reduced inertia and damping induce large-scale frequency deviations [115], [116]. To enhance the flexibility of low-inertia systems and alleviate possible stability problems, a new fast frequency control (FFC) ancillary service is needed [116]. This can be achieved through use of DC energy storage systems via voltage source inverters (VSIs) by effectively modulating the output power in response to frequency fluctuations [117].

Grid-forming voltage source inverter (VSI) and voltage source converter (VSC)

control methods include virtual synchronous machines (VSM) modeled after the dynamics of a synchronous machine [118], and a droop-based control which uses the inverters active and reactive power output and the droop control property [119]. However, most of the control methods focus only on the converter's AC side, neglecting the DC link dynamics such as energy storage systems. These approaches make the simplifying assumption that limited energy, and power is available at the DC side capacitor [120]. Also, under a small frequency deviation, a constant droop gain causes unsatisfactory VSI/VSC performance [121]. The inverter and converter are impeded from utilizing their full power capacity when responding to disturbances and faults.

Finite control set model predictive control (FCS-MPC) is an online finite set of optimization-based control technique which combines all system constraints and the objective goal function into a unified framework [122]. Using FCS-MPC, one can calculate optimal inputs based on predicted future state evolution using a state space representation of the system and incorporating constraints within the objective function. Due to these features, FCS-MPC is viewed as an appealing technique for controlling frequency with the VSI in the power electronics and power systems [123].

Recently, several papers have applied MPC in centralized [124], distributed [125], hierarchical [126] power system control, and it was found to yield better performances in the robust control of frequency and frequency uncertainty modulation in comparison with PI control. However, few publications have discussed using MPC in fast frequency control [127]- [129]. In [127], real-time optimal frequency and inertia response control was used with explicit MPC. However, that work did not demonstrate converter-based power generation, and used a simplified power system model for fast frequency control.

Also, there has been a journal paper [130] which utilizes the voltage source inverter with model predictive control in power systems frequency control in low inertia. However, it does not use multi-level inverter control for inner loop which is not suitable for large scale-based power system [130]. Inverters like the neutral point clamped inverter are appropriate for controlling large-scale power generation through a multi-level approach with low harmonics. They can be utilized in the proposed method of minimizing the cost function of neutral point clamped voltage (NPCV) based on finite control set Model Predictive Control (MPC).

Disadvantages of MPC based frequency control in a High-voltage-DC (HVDC) grid were discussed in [128], [129]. Decentralized MPC-based frequency control has also been proposed as a measure to be applied in emergency situations to avoid triggering under frequency load shedding relays [128]. However, it uses only locally available measurements. Each converter is capable of recognizing emergency situations that may result in unacceptable frequency values. Then appropriate control actions are taken to restrain the frequency decline and prevent it from reaching the thresholds of load shedding relays. Based on measurement of ROCOF, frequency predictions are created and the VSC output is updated when the frequency violations are anticipated or captured. Although decentralized, this method requires grid topology and HVDC converter locations to compute sensitivity factors connected to DC voltage drop. Increased computational burden in online solution of the MPC problem and tuning parameters are limitations of this method. A VSC based HVDC link furnished with a centralized MPC controller is promising for stabilization of large power systems [129]. VSC inputs are modulated to damp out oscillations in the power system using global measurements. However,

communication delays and failures may potentially lead to stability problems. Reliable and rapid communication links are necessary for the system to benefit from MPC.

[130] studies frequency control with voltage source inverters in low inertia environment in a power system. The author used synchronous generator and grid following inverters, the system was controlled using P-f and V-Q of droop control. The control method sticks only to the classic droop control method. However, here author proposes a method that includes advanced control methodologies such as the use of a linearized MPC cost function for fast frequency control in power systems and the geometrically designed SW (switching)-based finite control set-MPC proposed for fast control of multi-level inverters. Controlling by dividing small and large signals in [130] paper has a technique to minimize external disturbances, but it is insufficient to operate the entire power system in an agile manner. The second research paper is [131], and the author expresses frequency and voltage as equations through modeling of a synchronous converter and controls it through frequency and voltage equations. However, the results should be studied further for practical and economical consideration. The linearized MPC and finite control set MPC methodologies of the inverter in the paper we proposed can supplement the economical consideration of the proposed research due to MPC characteristics. The third research paper [132] is published June 2024, and the author is trying to realize inverter synchronization in a low-inertia microgrid by using data driven, scalable digital twins and observer-based system control. For control state transitions, the finite state machines (FSM) method is used. Typically, FSM stores each unique state through code links for the current state or consecutive condition states as a global variable. In order to use FMS, it is necessary to define state transition for state change. If the

inverter continuously detects the same frequency and phase or is signifying a synchronization state, the FSM may need to continuously change unsynchronization to synchronization and may use memory resources during communication between inverters or program operation. System computation efficiency may deteriorate due to excessive use of global variables and computation memories. The method I proposed uses multiparameter regional free MPC to divide the control area problem of the system into multiple sub problems and perform like a distributed processing, which can use less system memory and communication memory resources and increase system computation efficiency.

The first method we propose is to change the cost function for frequency deviation in the virtual synchronous generator. The cost function for frequency deviation and power consumption was changed to a secondary cost function based on a no cost function parameters estimation, which is called non-weighting factor scheme, significantly reducing the amount of MPC calculation. The second method is not to calculate using a cost function when performing 3-stage switching using a neutral point clamped inverter, but to use redundant switching states and non-redundant switching states using the geometric switching factor scheme we suggested. Calculate the switching state for each phase and change the resulting value into the optimal switching state divided into 19 for optimal switching state conversion through table programming, which is better than inverter control using MPC cost function. We proposed a stable and fast MPC operation based on no cost function parameters estimation, which is called non-weighting factor scheme. As a result, the method we proposed made it possible to quickly control the system through optimization of the frequency control part and the inverter control part.

In the MPC cost function of the frequency control part, linear optimization of the cost function was performed through no cost function parameters estimation, which is called non-weighting factor scheme, functionization through linearization. Optimization of the inverter part is done by tabulating the switch states for the three phases of the inverter using the geometric switching factor scheme author proposed, for the 19 operating point states, and using no cost function parameters estimation, which is called non-weighting factor scheme, and the no-cost function instead of using the MPC cost function. This allows the inverter to operate quickly. Multiparameter explicit model predictive control still limits the problem complexity mainly with regard to the prediction horizon length and number of decision variables. The centralized model predictive control plan solves a plant-wise optimization problem in a centralized computer and has been the primary method in the control system sector. In this situation, it is beneficial to decompose a large-centralized optimization problem into smaller multiple subproblems. Centralized requires communication infrastructure for a large power system, however, it is better oversight with holistic views, increased efficiency via streamlined processes, easier compliance/auditing, and cost savings through unified management, leading to clearer direction and smoother communication across an organization. Author proposes a centralized MPC based FFC methodology which can be integrated as an additional layer for simplified quadratic cost goal function calculation to the outer loop virtual synchronous generator frequency control with no cost function parameters estimation, which is called non-weighting factor scheme calculation, and inner loop geometric SW (switch) factor scheme for the FCS-MPC. With outer loop, we can convert MPC goal function into linear quadratic cost function to control the optimized MPC, which makes

the problem less burdensome. In the event of large disturbances MPC keeps the frequency deviation and ROCOF within limits prescribed by the user. Moreover, with inner loop, author proposes neutral point clamped geometric SW (switch) factor scheme for the FCS-MPC to reduce computational burden and minimize the cost function via no cost function parameters estimation, which is called non-weighting factor scheme for cost function and optimal switching. Geometric SW (switch) factor scheme is used to select the switching state and minimize the cost function, which is close to the predicted reference inverter voltage vector. This chapter uses a novel SW (switch) factor scheme in which the optimal voltage vector is obtained to controlling the grid current and the neutral point voltage without the necessity of no cost function parameters estimation, which is called non-weighting factor scheme for cost function. This scheme reduces the overall execution time. First, it has fewer calculations to perform as compared to other conventional FCS-MPC available in the literature.

Second, this scheme completely avoids any tuning parameter to control neutral point voltage. Moreover, reduce the computational burden, we propose a systematic centralized MPC method. Finally, the proposed control design is verified through time domain simulations for a low-inertia system within a large-scale power system.

6.2 Virtual synchronous generator frequency control

6.2.1 MPC based VSG for outer loop

For the traditional virtual synchronous generator (VSG), frequency regulation is difficult to regain once it deviates from normal [133]. Even if the frequency is steered back to normal under small inertia, the frequency may be forced to stabilize quickly which can possibly cause oscillations. Frequency regulation capability is difficult to enhance.

Moreover, when the frequency deviates from normal, the dynamic response of the frequency is expected to decelerate. Conversely, the system responds quickly when the post-disturbance frequency returns to normal [134]. A dual MPC scheme is proposed here to improve the precision of frequency control, as illustrated in Fig. 6.1. This scheme does not incorporate a cost function, representing a linearization of the MPC-based VSG, and it also features a no-weighting FCS-MPC SW (switch) factor scheme. P_N is nominal active power, Q_N is nominal reactive power, ω is the angular frequency, V_{REF} is the reference voltage, SW is the switching factor scheme, v is the voltage source inverter's output voltage, i_o is the voltage source inverter's output current.

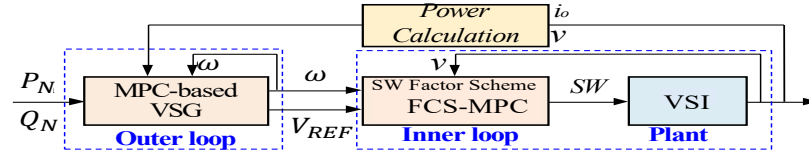


Fig. 6.1. Diagram of proposed control scheme.

Simple modeling of a VSG is illustrated in Fig. 6.2. This provides inertia for stability support and voltage reference with reactive droop control and active power calculation with active droop control.

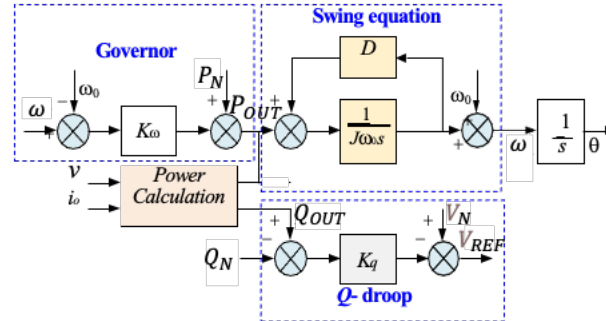


Fig. 6.2. Illustrating simple model of voltage-controlled VSG.

Given a frequency variation, the major function of the governor is to respond

by appropriately managing the active power output. Governor is a component of the turbine control system that regulates rotational speed in response to changing load conditions. The Q-droop is used in output voltage control, since the reactive power is associated with AC bus voltage. The governor and Q-droop formulation result in a droop control technique written as

$$\begin{cases} P_{OUT} = P_N - K_\omega(\omega - \omega_0) \\ V_{REF} = V_N - K_q(Q_{OUT} - Q_N) \end{cases} \quad (6.1)$$

where P and ω are virtual active power of the governor and generated angular frequency of the VSG, respectively. Also P_N and V_N are nominal active power and nominal voltage, $P_N := P(k)$; K_ω and K_q are frequency deviation gain and Q-droop coefficient; V_{REF} and Q_{OUT} are reference voltage and instantaneous output reactive power; ω_0 and Q_N are nominal angular frequency and nominal reactive power, respectively.

The swing equation as the main part of the VSG, is included to emulate rotor inertia that would be provided by a synchronous generator (SG). The swing equation can be represented as

$$J\omega_0 \frac{d}{dt}(\omega - \omega_0) = P - P_{OUT} - D(\omega - \omega_0) \quad (6.2)$$

where J , P_{OUT} and D are moments of virtual inertia, output power, and damping coefficient. ω is the angular frequency, ω_0 is the initial angular frequency, P is the virtual active power, respectively.

The discrete MPC model for the VSG is represented here to achieve efficient control during a sampling period. A discrete model can be obtained from (6.2) using Tustin approximation:

$$J\omega_0[\omega(k+1) - \omega(k)] = T_s[P(k) - P_{OUT}(k)] - T_s D[\omega(k) - \omega_0] \quad (6.3)$$

where T_s is the sampling interval.

To reduce the static error which is the difference between the best measured value and the true value of the quantity, an incremental discrete model can be obtained through algebraic manipulations of (6.3), yielding

$$\Delta\omega(k+1) = A_e\Delta\omega(k) + B_e\Delta P_{OUT}(k) + B_u\Delta P(k) \quad (6.4)$$

$$\text{where } A_e = 1 - \frac{T_s D}{j\omega_0}, B_e = -\frac{T_s}{j\omega_0}, B_u = \frac{T_s}{j\omega_0}.$$

The output follows the dynamics

$$y(k+1) = y(k) + \Delta\omega(k+1) \quad (6.5)$$

Setting the prediction horizon at n steps, and assuming it takes l steps to reach steady state, the output $Y_n(k+1|k)$ can be represented as

$$Y_n(k+1|k) = Iy(k) + S_x\Delta\omega(k) + S_u\Delta P(k) + S_e\Delta P_{OUT}(k) \quad (6.6)$$

where $I = [1 \ 1 \ \dots \ 1]_{1 \times n}^T$, $P_N := P(k) = [\Delta P(k) \ \Delta P(k+1) \ \dots \ \Delta P(k+l-1)]_{1 \times n}^T$,

$$S_x = [A_e \sum_{i=1}^2 A_e^i \ \dots \ \sum_{i=1}^n A_e^i]_{1 \times n}^T, S_e = [B_e \sum_{i=1}^2 A_e^{i-1} B_e \ \dots \ \sum_{i=1}^n A_e^{i-1} B_e]_{1 \times n}^T,$$

$$S_u = \begin{bmatrix} B_u & \dots & 0 \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^n A_e^{i-1} B_u & \dots & \sum_{i=1}^{n-l+1} A_e^{i-1} B_u \end{bmatrix}_{n \times l}$$

Finally, $Y_n(k+1|k)$ can be optimized by modulating $\Delta P(k)$. Equation (6.6) is the future

output representation. Future output is determined by current output ($y(k)$) plus state of x matrix times deviation of frequency omega ($\Delta\omega(k)$) plus state of input matrix times deviation of virtual active power ($\Delta P(k)$) plus state of error matrix times deviation of output power ($\Delta P_{OUT}(k)$). Summation of all of these values will determine the future output value.

6.2.2 Frequency departure from nominal value

When the frequency departs from its nominal value, the control objective response to the frequency change, ω changed little ($\omega \approx 0$). Set the frequency deviation ω_{depart} as

$$\omega_{depart}(i) = \omega(i) - \omega(i-1), i = k, k+1, \dots, k+n-1 \quad (6.7)$$

Thus, the discrete model for ω_{depart} can be represented as

$$\begin{cases} \Delta\omega_{depart}(i) = \Delta\omega(i) \\ \omega_{depart}(i+1) = \Delta\omega(i+1) + \omega_{depart}(i) \end{cases} \quad (6.8)$$

Also, output process $Y_{depart}(i+1|i)$ associated with ω_{depart} can be represented as

$$Y_{depart}(i+1|i) = I\omega_{depart}(i) + S_x\Delta\omega(i) + S_u\Delta P(i) + S_e\Delta P_{OUT}(i) \quad (6.9)$$

$\omega_{depart}(k) = 0$ when $i = k$ is clear. Then, optimized cost function with ω_{depart} can be represented as

$$G_{depart} = \min \{ || \sum_{depart} [Y_{dep.}(k+1|k) - Y_{desire}(k+1)] ||^2 + || \sum_1 \Delta P(k) ||^2 \} \quad (6.10)$$

where $\Sigma_{depart} = \text{diag}(\lambda_D, \lambda_D, \dots, \lambda_D), \Sigma_1 \text{diag}(\lambda_1, \lambda_1, \dots, \lambda_1), \lambda_D$ and λ_1 are weighting factors for frequency and power increment when frequency departs from the nominal.

Desired steady state ω_{desire} is zero vectors. So, reference output is $Y_{desire}(k+1) = [0 \ 0 \ \dots \ 0]^T$. Also cost function (6.10) results in a quadratic programming problem.

Introduce the intermediate variable τ :

$$\begin{aligned} \tau &= \begin{cases} \left[\sum_{depart} (Y_{depart}(k+1)) - Y_{desire}(k+1) \right] \\ \sum_1 \Delta P(k) \end{cases} \\ &= \begin{bmatrix} \Sigma_{depart} S_u \\ \Sigma_1 \end{bmatrix} \Delta P(k) - \begin{bmatrix} \Sigma_{depart} E_{depart}(k+1|k) \\ 0 \end{bmatrix} = Ax - b \end{aligned} \quad (6.11)$$

where $E_{depart}(k+1|k) = Y_{desire}(k+1) - S_x \Delta \omega(k) - S_e \Delta P_{OUT}$

By using a quadratic cost function, the MPC based cost function can be written simply as a no-weighting factor cost function G_{depart} :

$$G_{depart} = \min (\tau^T \tau) = \min [(Ax - b)^T (Ax - b)]. \quad (6.12)$$

The aim is then to minimize departure from nominal frequency over a brief look-ahead period. It is straightforward to see that G_{depart} is optimized when

$$\begin{cases} 2A^T(Ax - b) = 0 \\ 2A^T A > 0 \end{cases} \quad (6.13)$$

Substituting (6.11) into (6.13), algebraic simplification yields the optimized $\Delta P^*(k)|_{depart}$ as

$$\Delta P^*(k)|_{depart} = (S_u^T \Sigma_d^T \Sigma_d S_u + \Sigma_1^T \Sigma_1)^{-1} S_u^T \Sigma_d^T \Sigma_d E_{depart}(k+1|k) \quad (6.14)$$

where S_u is the state of input matrix, $\Sigma_{depart} = \text{diag}(\lambda_D, \lambda_D, \dots, \lambda_D), \Sigma_1 \text{diag}(\lambda_1, \lambda_1, \dots, \lambda_1)$, $E_{depart}(k+1|k) = Y_{desire}(k+1) - S_x \Delta \omega(k) - S_e \Delta P_{OUT}$.

6.2.3 Frequency recovery to nominal value

For the frequency recovery to the nominal value, the control objective is to accelerate the frequency dynamics to return the frequency to normal, which entails. that ω changes quickly. Define the recovery difference as

$$\omega_{recover}(i) = \omega(i) - \omega_{REF}, i = k, k+1, \dots, n \quad (6.15)$$

where ω_{REF} is calculated via droop control which is the steady state post-disturbance value. A discrete model for $\omega_{recover}$ can be obtained as

$$\begin{cases} \Delta \omega_{recover}(i) = \Delta \omega(i) \\ \omega_{recover}(i+1) = \Delta \omega(i+1) + \omega_{recover}(i) \end{cases} \quad (6.16)$$

Also, the output process $Y_{recover}(i+1|i)$ is seen to be

$$Y_{recover}(i+1|i) = I\omega_R(i) + S_x \Delta \omega(i) + S_u \Delta P(i) + S_e \Delta P_{OUT}(i) \quad (6.17)$$

The optimized cost function for $\omega_{recover}$ is

$$\begin{aligned} G_{recov.} = \min \{ & || \sum_{rec.} [Y_{rec.}(k+1|k) - Y_{desire}(k+1)] ||^2 \\ & + || \sum_2 \Delta P(k) ||^2 \end{aligned} \quad (6.18)$$

where $\Sigma_{recover} = \text{diag}(\lambda_R, \lambda_R, \dots, \lambda_R), \Sigma_2 \text{diag}(\lambda_2, \lambda_2, \dots, \lambda_2)$, λ_R and λ_2 are weighting

factors for frequency and power increment when frequency recovers to the nominal. The desired steady state $\omega_{recover}$ is 0, and the reference output is $Y_{desire}(k+1) = [0 \ 0, \dots 0]^T$.

The quantity $G_{recover}$ can be optimized by simple optimization, and optimization of $\Delta P^*(k)|_{recover}$ for $G_{recover}$ can be represented as

$$\Delta P^*(k)|_{recov.} = (S_u^T \sum_r^T \sum_r S_u + \sum_2^T \sum_2)^{-1} S_u^T \sum_r^T \sum_r E_{recover}(k+1|k) \quad (6.19)$$

6.3 SW factor scheme without weighting factor FCS-MPC with neutral point clamped inverter

The dynamic model of a three-level inverter shown in Fig. 6.3 takes a part in designing the FCS-MPC, and it is given by the following:

$$\frac{d\vec{i}_g}{dt} = \frac{1}{L} (-R\vec{i}_g + \vec{V}_{inv} - \vec{v}_g) \quad (6.20)$$

where $\vec{i}_g = [i_{gx}, i_{gy}]^T$, $\vec{v}_{inv} = [v_{invx}, v_{invy}]^T$, $\vec{v}_g = [v_{gx}, v_{gy}]^T$ are the grid currents, the inverter voltages, and grid voltages. L is the inductor. The voltages and currents having subscripts x or y indicate that the quantities have been got from their original values using Clark's transformation.

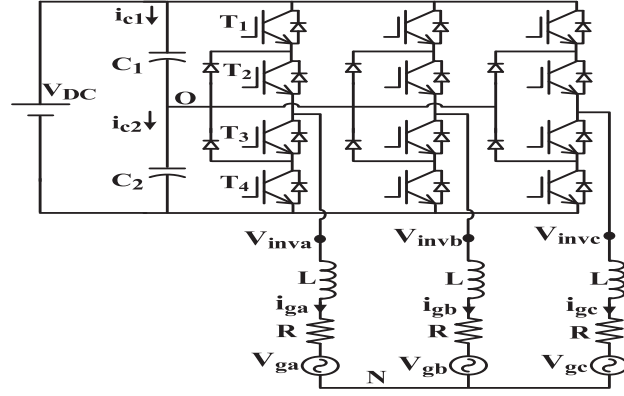


Fig. 6.3. Three-level neutral point clamped inverter.

The predicted grid current at the next sampling instant, which is calculated by discretizing (6.20) is given by the following [135]:

$$\vec{i}_g(n+1) = \frac{1}{L+RT_s} \{L\vec{i}_g(n) + T_s[\vec{V}_{inv}(n+1) - \vec{v}_g(n+1)]\} \quad (6.21)$$

where the sampling time is T_s and the sampling instant by n . R is the resistor. Rearranging (6.21), we can get the following:

$$\vec{v}_{inv}(n+1) = \frac{L+RT_s}{T_s} \vec{i}_g(n+1) - \frac{L}{T_s} \vec{i}_g(n) + \vec{v}_g(n+1). \quad (6.22)$$

$\vec{i}_g(n+1)$ can be substituted by $\vec{i}_g^*(n+1)$ to get the predicted reference inverter voltage vector [136], [137], [141]- [143] as follows:

$$\vec{v}_{inv}^*(n+1) = \frac{L+RT_s}{T_s} \vec{i}_g^*(n+1) - \frac{L}{T_s} \vec{i}_g(n) + \vec{v}_g(n+1) \quad (6.23)$$

From (6.23), it is sure that the predicted reference inverter voltage vector can be come from the predicted reference current $\vec{i}_g^*(n+1)$, the contemporary value of the actual grid current $\vec{i}_g(n)$, and the predicted grid voltage $\vec{v}_g(n+1)$. The predicted grid voltage and the predicted reference current can be derived as follows [138]:

$$\vec{v}_g(n+1) = 3\vec{v}_g(n) - 3\vec{v}_g(n-1) + \vec{v}_g(n-2) \quad (1.24)$$

$$\vec{i}_g^*(n+1) = 3\vec{i}_g^*(n) - 3\vec{i}_g^*(n-1) + \vec{i}_g^*(n-1) \quad (1.25)$$

Similarly, the predicted capacitor voltage can be acquired as follows [139]:

$$v_{c1}(n+1) = v_{c1}(n) + \frac{T_s}{C_1} i_{c1}(n) \quad (1.26)$$

$$v_{c2}(n+1) = v_{c2}(n) + \frac{T_s}{C_2} i_{c2}(n) \quad (1.27)$$

where v_{c1} is the voltage at the C_1 capacitor, v_{c2} is the voltage at the C_2 capacitor.

After predicting all the desired variables using (23)-(27), the following procedure can be followed to achieve neutral point voltage control and grid current control of the neutral point clamped inverter.

The space vector representation of the three-level neutral point clamped inverter is shown in Fig. 6.4 and the matching switching states are listed in Table 6.1. The set of switching states of the neutral point clamped can be divided into two categories, such as redundant switching states (v_0 to v_{14}) and nonredundant switching states (v_{15} to v_{26}). The redundant switching states can be further separated into two groups, such as zero redundant switching states (v_0 to v_2) and nonzero redundant switching states (v_3 to v_{14}). Zero switching states do not influence on the neutral point voltage and produce zero line to line voltage [140]. Thus, any one of the zero switching states can be chosen.

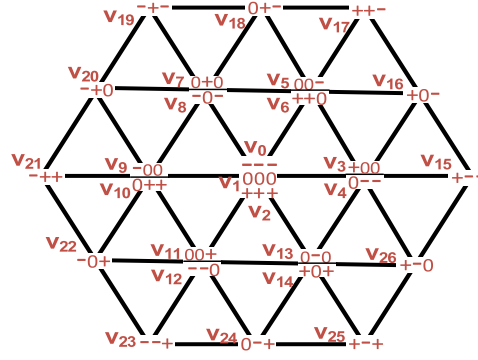


Fig. 6.4. Three-level neutral point clamped inverter space vector representation.

Table 6.1. Representation of switching state.

Representation	T_1	T_2	T_3	T_4
+	ON	ON	OFF	OFF
0	OFF	ON	ON	OFF
-	OFF	OFF	ON	ON

Similarly, each pair of nonzero redundant switching states makes an equal line to line voltage. However, the current direction flowing into the neutral point is the contrary. Out of all the nonzero redundant switching states, the positive nonzero redundant switching states ($v_3, v_6, v_7, v_{10}, v_{11}$, and v_{14}) increase the neutral point voltage (v_{C2}) and the negative nonzero redundant switching states ($v_4, v_5, v_8, v_9, v_{12}$ and v_{13}) lessen the neutral point voltage. Thus, the neutral point voltage balance can be accomplished with a suitable selection of nonzero redundant switch states depending upon the predicted capacitor voltages [140]. As the nonredundant switching states are not used in accomplishing neutral point voltage balance, the total harmonic distortion of the grid current remains unaffected.

After choosing suitable 19 switching states out of 27 switching states for achieving the voltage balancing task, the next step is to decide on the optimal switching state which gives the minimum cost function for managing the grid current.

The cost function for managing the grid current is given as below [136], [137], [141]-[143]:

$$G(n+1) = ||\vec{v}_{inv}^*(n+1) - \vec{v}_{inv}(n+1)||^2 \quad (6.28)$$

The above cost function is the square of the distance between two vectors. Minimization of the given cost function can be alternatively finished by finding the optimal voltage vector which is close to the predictive reference inverter voltage vector ($\vec{v}_{inv}^*(n+1)$). In This chapter, this objective is accomplished by using a novel scheme, which makes use of the geometry and the space vector diagram of multilevel inverters to get the optimal voltage vector.

In this approach, three imaginary lines SW_1 (switch factor scheme), SW_2 , and SW_3 slide along the direction as shown in Fig. 1.5 and only take the marked integral position [144]. For instance, if the values of sliding lines are taken as $SW_1=0.3$, $SW_2=0.4$, and $SW_3=-1.3$, they will slide to take the position of $SW_1=0$, $SW_2=0$, and $SW_3=-2$, which are the nearest integers less than their real values. The direction of sliding motion of SW_1 , SW_2 , SW_3 is inclined at 180° , -60° , and 60° with the respect to the x-axis, respectively. These three imaginary sliding lines have two interesting characteristic which are mentioned next, and by advantage of these, the calculation load in FCS-MPC is reduced.

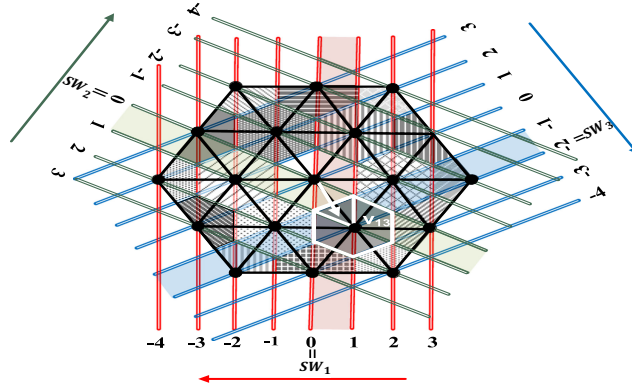


Fig. 6.5. Concept of SW factor scheme.

The first characteristic is seen regarding all the integral positions of the three imaginary sliding lines, it can be from Fig. 6.5 which every possible switching states of the three-level inverter marked by black dots are divided from one another by its optimal area highlighted by different patterns. It can be check that every optimal area has only one switching state. The optimal area of a switching state is so regarded as if the predicted reference inverter voltage vector falls in that area, it will be closest to that switching state. Thus, that switching state can be regards as the optimal switching state. Consequently, the optimal switching state can be easily found out as below.

$$\begin{aligned}
 SW_1 &= \left(\frac{1}{V_{DC}} \right) (4V_{inv}^*(n+1)) \\
 SW_2 &= \left(\frac{1}{V_{DC}} \right) (-2V_{invx}^*(n+1) - 2\sqrt{3}v_{invy}^*(n+1)) \\
 SW_3 &= \left(\frac{1}{V_{DC}} \right) (-2V_{invx}^*(n+1) + 2\sqrt{3}v_{invy}^*(n+1))
 \end{aligned} \tag{6.29}$$

The second fascinating characteristic is that if the values of the three imaginary lines for the inverter are obtained by using (6.29), they will always confine the peak of the predicted reference inverter voltage vector within a triangle, which is a part of

an optimal area. The positions of sliding lines can be obtained from the values of sliding lines by using a floor function. For instance, if the predicted reference inverter voltage vector is the vector shown by an arrow in Fig. 6.5 , the position of SW_1, SW_2 , and SW_3 can be calculated as 0, 0, and -2 from (6.29). This suggests which the predicted reference inverter voltage vector is enclosed in a triangle, which is a part of the optimal area of v_{13}/v_{14} . Thus, once the position of SW_1, SW_2 , and SW_3 are got, we can easily find out the optimal switching state. Because three imaginary lines slide along three different directions to locate the optimal switching state, this scheme has been called the “SW” factor scheme. Via optimal switching, one can obtain reduced cost function with no-weighting factor correction which reduces harmonics, and fast optimal control the inverter. Quick inverter control makes the fast response to the frequency changes, which can adapt to fast frequency control with minimum costs. The whole algorithm to implement the proposed scheme is shown in Fig. 6.6 as a flowchart.

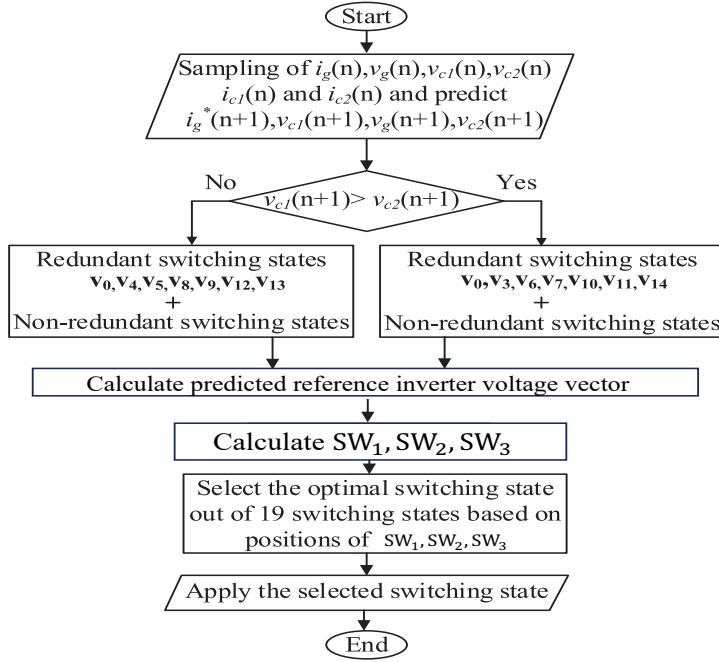


Fig. 6.6. SW factor scheme-based FCS-MPC algorithm.

6.4 Centralized control of frequency

We consider the rapid dynamics of inverters in distributed generation as the cost associated with changes in the voltage vector set point of the Voltage Source Inverter (VSI). Additionally, we view the dynamics of the stored energy level in Battery Energy Storage Systems (BESSs) as the cost related to the state of charge (SOC) of the energy storage. Furthermore, we assess the cost of voltage deviation from its reference set point and the cost of deviation in net tie-line bus power within the framework of the proposed centralized MPC. This chapter mainly accomplishes the primary and secondary frequency control and minimization of the operational cost of the proposed power systems during the power flow optimization is reflected on tertiary frequency control. This is the reason why we use MPC for the FFC which can take full advantage of different inverter-based devices such as inverters and VSCs.

The power flow optimization only considers a steady-state optimization.

Centralized controller determines the power output of each converter to take part in fast frequency control. The goal function's purpose is to minimize the total control effort over the full horizon $k \in K$ and over all device's units i (6.30). ROCOF is computed for all generators in (6.31), equality constraint each individual VSI's charging power in (6.32), frequency limit in (6.38), and SOC in equation (6.33). Active and reactive power generated by the bus in the distributed generation deduct, and ESS charge/discharge, and load power are represented with the (6.34) and (6.35). Active power of net power tie-line is in (6.36). VSI's charging and discharging active power limit, and battery energy storage system SOC limit are in (6.37). Distributed generation (DG) active and reactive power limit, and bus v_{ik} deviation from its reference set points are in (6.39). All these equality and inequality constraints are required for the fast frequency control within the IEEE 39-bus.

$$\min \sum_{k \in K} \left(\sum_{i \in N^G} C_{pi k}^{VSI} \|\Delta P_{ci}^*(k)\| + \sum_{i \in N^{ESS}} C_{ik}^{ESS} + \sum_{i \in N} C_{ik}^V + \sum_{s \in S} C_{sk}^{TIE} \right) \quad (6.30)$$

$$\frac{d}{dt} f_j(k) = \frac{f_j(k+1) - f_j(k)}{T_s} \quad (6.31)$$

$$P_{ci}(k) = P_{ci}^* + \sum_{l=1}^k \Delta P_{ci}^*(l) + R_{ci}^P (\omega_{ci}^* - \omega_{ci}(k)) \quad (6.32)$$

$$SOC_{ik} = SOC_{i(k-1)} + (\eta_{ci} P_{cik} - \eta_{di}^{-1} P_{dik}) T \quad (6.33)$$

$$P_{Gik} - P_{cik} + P_{dik} - P_{Lik} = P_i^C + \sum_{j \in \{i\} \cup N_i} (A_{ij} \Delta V_{jk} + B_{ij} \Delta \theta_{jk}) \quad (6.34)$$

$$Q_{Gik} - Q_{Lik} = Q_i^C + \sum_{j \in \{i\} \cup N_i} (C_{ij} \Delta V_{jk} + D_{ij} \Delta \theta_{jk}) \quad (6.35)$$

$$P_{sk}^{TIE} = \sum_{(i,j) \in L_s} (P_{ij}^C + E_{ij}^j \Delta V_{jk} + F_{ij}^j \Delta \theta_{jk} + Y_{ij}^i \Delta V_{ik} + Z_{ij}^i \Delta \theta_{ik}) \quad (6.36)$$

$$0 \leq P_{cik}(k) \leq \bar{P}_{ci}, 0 \leq P_{dik}(k) \leq \bar{P}_{di}, \underline{SOC}_i \leq SOC_{ik} \leq \overline{SOC}_i \quad (6.37)$$

$$\underline{f} \leq f_i(k) \leq \bar{f} \quad (6.38)$$

$$0 \leq P_{Gik} \leq \bar{P}_{Gi}, 0 \leq Q_{Gik} \leq \bar{Q}_{Gi}, \underline{V}_i \leq V_{ik} \leq \bar{V}_i \quad (6.39)$$

where k is the horizon step, $\sum_{i \in NG} C_{Pik}^{VSI} \parallel \Delta P_{ci}^*(k) \parallel$ is the number of generations for voltage source inverter (VSI) optimal charging cost (For simplicity we only considered charging cost), $\sum_{i \in N^{ESS}} C_{ik}^{ESS}$ is the number of energy storage system ESS's SOC managing cost, $\sum_{i \in N} C_{ik}^V$ is the number of bus for managing cost for reference V bus, $\sum_{s \in S} C_{sk}^{TIE}$ is the number of tie-line managing cost. $P_{ci}(k)$ is the charging power for the VSI, P_{ci}^* is the initial-charged power, $\sum_{l=1} \Delta P_{ci}^*(l)$ is the VSI's charging power deviation, $R_{ci}^P(\omega_{ci}^* - \omega_{ci}(k))$ is the VSI's resistor's part power which is resistor times deviation of angular frequency. SOC_{ik} is the ESS's SOC, $SOC_{i(k-1)}$ is the previous ESS's SOC, $\eta_{ci} P_{cik}$ is the charging efficiency times charging power, $\eta_{di} P_{dik}$ is the discharging efficiency times discharging power. P_{Gik} is the generated power, P_{cik} is the charging power, P_{dik} is the discharged power, P_{Lik} is the load power. P_i^C is the initial bus power, $A_{ij} \Delta V_{jk}$ is the A coefficient times deviation of voltage, $B_{ij} \Delta \theta_{jk}$ is the B coefficient times deviation of phase.

For (6.36), P_{sk}^{TIE} is the tie line power which is consisted with P_{ij}^C is the bus i to j power

plus admittance voltage and phase plus impedance voltage and phase. For (6.38), frequency upper and lower limit which is consisted with current frequency plus VSI's resistor times deviation of charging power.

6.5 Model testing and control simulation

The suggested FFC methods have been carried out and tested on the IEEE 39-bus system illustrated in Fig. 6.7. This is a popular 10-machine representation of the New England power system with generator at node 10-machine depiction of the New England power system with generator at node 10 meaning the collection of a large number of generators. Inverter based generation is placed at nodes 1, 2, and 3. The red symbol represents disturbance location. The pertinent load and generation parameters can be found in [145], [146]. The author conducted the simulation using MATLAB, which encompasses a comprehensive depiction of the dynamics of the generator and transmission line as referenced in [147]. The examined system consists of seven conventional generators, and three synchronous generators from the initial system exactly at nodes 1, 2, and 3, have been displaced by inverter interfaced units of 1000 MW installed power and 10 MWh battery energy storage system for the purpose of this study. The individual power ratings and output limits of the stay behind synchronous generators have been conserved. All VSIs perform in grid forming mode and are provided with the fast frequency control layer.

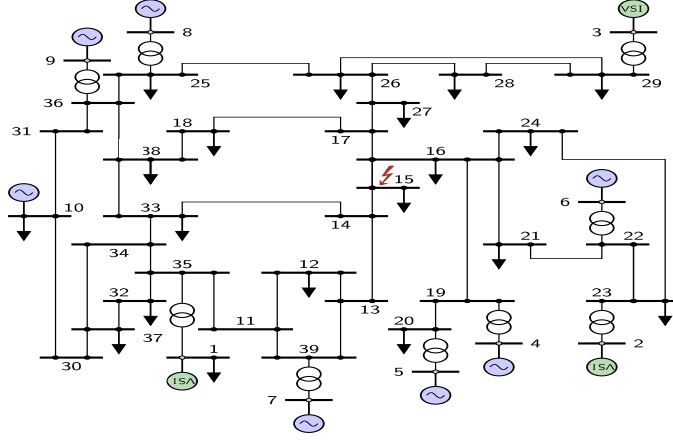


Fig. 6.7. IEEE 39-bus test system.

The disturbances are produced by step changes in active power at network buses of 15, therefore imitating either a loss of load or a loss of generator. First stage of automatic load shedding is established in the case of frequency variation beyond ± 0.5 Hz, although the ROCOF protection is activated at ± 1 Hz/s for the ROCOF estimations mean value over a 240 ms cycle. Thus, the frequency related threshold values are established as follows (1.38): $\bar{f}_{lim} = 60.5$ Hz, $\underline{f}_{lim} = 59.5$ Hz, $\dot{\bar{f}}_{lim} = \frac{1\text{Hz}}{s}$, $\dot{\underline{f}}_{lim} = -\frac{1\text{Hz}}{s}$. The battery energy storage system SOC and VSI power output are defined in per unit, and the maximum and minimum limits are established to 1 and 0, respectively.

The prediction horizon of the MPC based controller is set to three-time steps with a sampling period of 240 ms. On the other hand, the prediction horizon length of 720 ms considers a trade-off between computational effort and controller performance. The MPC sampling period is decided on it surpasses all delays related with the supervisory layer and converter, also the time required to compute the optimal control decisions. The real-time ROCOF measures $\dot{\omega}_{max}$ necessary for the

computation of the disturbance magnitude ΔP is attained from averaging the internal ROCOF state signal $\dot{\omega}$ over a time interval of 12 ms in the instant consequence of a disturbance.

6.5.1 Simulation of frequency control

Centralized grid MPC controller acquires an accurate disturbance estimate across the independent of disturbance location, wide area measuring, and consequently with low error [134]. The values of estimated and practical disturbance magnitudes for each reflected on bus and every VSI are showed in Table 6.2. Let us consider a power disturbance of 1500 MW with 720 ms at node 15. Fig. 6.8 demonstrate frequencies, derivative of the frequencies, active and reactive power outputs of separate generators for FFC methods. The first graph is frequencies in the VSI bus one through three, and synchronous generator units. As shown in the graph, it did not violate the frequency limit, such as 60.5. The second graph shows the derivative of the frequency. It demonstrates, in which it does not violate the limit of derivative of frequency $\dot{f}_{lim} = -\frac{1Hz}{s}$. The third graph shows the active and reactive power change when there was a fault on the 15 bus.

The VSI at node 3 remain in a state of inactivity because of the large electrical distance to the disturbance location and resulting underestimation of the disturbance. Nonetheless, the assistance from the other two converters is enough to pay off the disturbance and inhibit load shedding. However, the global MPC based grid controller transmits all three units equally. Typically, a reduced level of control effort is adopted in the centralized approach. ROCOF is within the critical set points, active and reactive power is increased after the frequency disturbance based on the VSI's location points.

Due to SW (switch) factor scheme based no-weighting FCS-MPC methods, VSI generations are well operated with the connection of VSI bus 1 and VSI bus 2 within the disturbances.

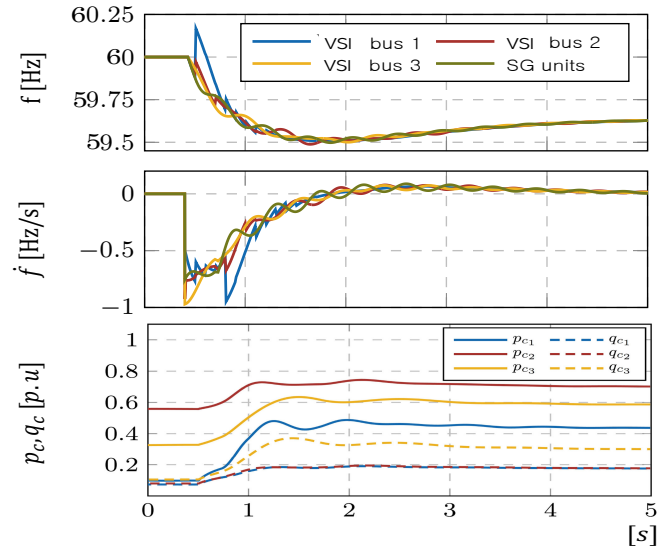


Fig. 6.8. Centralized control individual frequency, frequency change at unit time, active and reactive power output for FFC proposed method following a disturbance at bus 15.

Fig. 6.9 gives results of the centralized PID (proportional integral derivative) / proposed-MPC control approach for IEEE 39-bus system. The first graph shows that through VSI linearized MPC cost function and neutral point clamped (NPC) inverter FCS-MPC method, centralized MPC control performance is nearly identical to centralized PID control on the left side of the Fig. 6.9. The second and the third graph show the NPC inverter active charged power set point in p.u. and active deviation of charged power of optimal ESS set point changes. Compared with the only droop control, proposed centralized MPC control and neutral point clamped (NPC) inverter FCS-MPC have the better performance than only droop control, and its performance is comparable to PID approach due to the less computational burden due to linearized cost function of

MPC and NPC inverter SW (switch) factor scheme FCS-MPC approach.

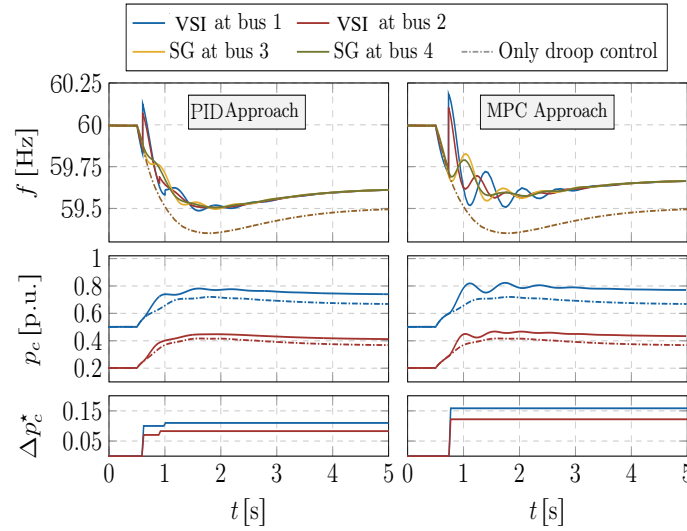


Fig. 6.9. Centralized control performance of PID and proposed MPC within individual frequency, NPC inverter active charging power set point, and optimal active power set point changes following a disturbance at bus 15.

Table 6.2. Fault case at bus with denoted applied disturbance size and estimated asymmetry for each VSI.

BUS	Disturbance [MW]	Estimated disturbance [MW]		
		VSI 1	VSI 2	VSI 3
15	1550	1505	1900	825

6.5.2 DC side dynamics analysis

The most pertinent variables for the dynamics of the DC side circuit, especially the neutral point voltage (NPV) (U_{NPV}), and the battery SOC (U_{dc}) are depicted in Fig. 6.11 for individual VSI units and disturbance at bus 15. The set-up energy capacity of the batteries is presumed to be 10 MWh, with the initial BESS's SOC set to 0.5 p.u. The disturbance and the inverter setting adjust due to power imbalances at the capacitor node,

bring about DC voltage plunge which are rapidly restored by the DC side controls using available energy of the battery. In the end, the input DC voltage indicates faster dynamics compared to the output power, which could potentially bring about high current injections. From Fig. 6.10, the first graph shows, neutral point voltage with no cost function parameters estimation that is formed by the FCS-MPC SW (switch) scheme which is faster than nominal FCS-MPC, and it has less overshoot/undershoot and fast settling time compared to nominal FCS-MPC when there is a one-time impulse disturbance. The second graph shows, BESS's DC side voltage is set to 0.5 SOC; thus, it is controlled within 0.5 SOC even if there is the disturbance. During the control period, neutral point voltage of DC side is effectively controlled with the disturbance. The last graph neutral point voltage of the inverter's DC side is controlled within the 0.06 p.u. voltage difference.

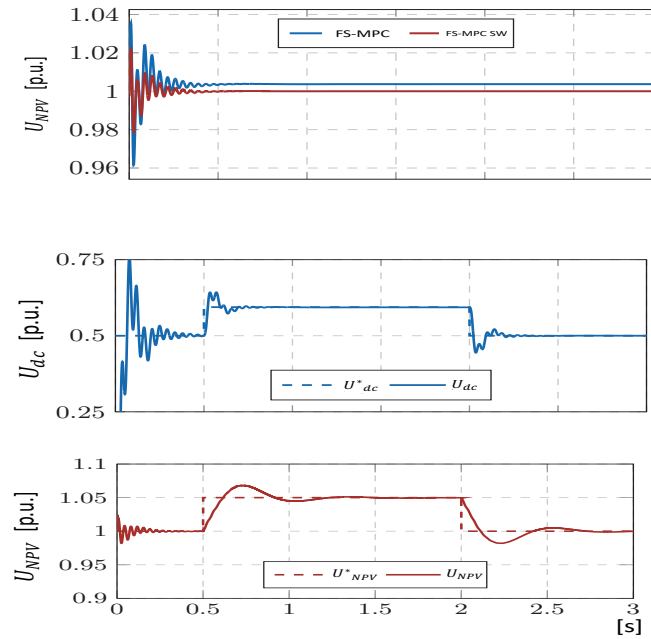


Fig. 6.10. Centralized control one-time step disturbance of FCS-MPC SW factor scheme U_{NPV} voltage compared with FCS-MPC voltage, overall maintenance of BESS U_{dc} , and overall maintenance of BESS U_{NPV} voltage reaction for proposed method following a disturbance at bus 15.

6.5.3 Computation efficiency analysis

The solution of the linear MPC optimization problems (6.34) was accomplished via LP solver and CPLEX solver, and MPC interior point solver were used. For numerical computation and optimization, YALMIP is used for high-level modeling [148]. The centralized MPC problem is solved within 330 ms on average. The MPC solution was created via YALMIP for modeling and MPT3 toolbox for low-level numerical solution of the multi-parametric optimization [149]. The time necessary for obtaining the optimal control inputs decreases to 15 ms.

We proposed to use multiparameter model predictive control for reducing computational requirement. Explicit model predictive control was stimulated by the encouragement to win the primary disadvantage of implicit model predictive control. Implicit model predictive control requires to calculate the optimal control law online at every sampling instant by solving the matching operating point. Explicit model predictive control uses parametric programming to pre-calculate the optimal control law for all permitted values of parameters. Thus, the explicit representation of the optimizer is created offline as a function of the model predictive control parameters. The online parameterization of the optimal control action constricts function evaluations for particular measurements. This significantly lessen computational requirement of the implementation. From a mathematical viewpoint, the problems to be solved in the case of linear model predictive control is multi parameter program. One possible treatment to get over this large memory footprint disadvantage is to use the recently introduced approach explicit model predictive control. The solution complexity of this approach no longer depends on the number of parameters, but rather on the number of optimization variables, for example up to 20.

This still limits the problem complexity mainly with regarding to the prediction horizon length and number of decision variables. The centralized model predictive control plan solves a system-wise optimization problem in a centralized computer and has been the primary method in the power system sector. It is important not to make increased computational complexity and trustworthiness issues. In this situation, it is beneficial to decompose a large-centralized optimization problem into smaller multiple subproblems.

The computational speed for settling time was fastest with a PD (proportional derivative) control, followed by the proposed method, whereas the droop control approach was the slowest. This inefficiency is why droop control is not an effective strategy for addressing the low inertia issues in power systems. The proposed method demonstrates superior performance compared to the PID approach in terms of generating more active power than PID control, as illustrated in Fig. 6.9. Regarding settling time, PD control provides a swift control response, as it does not incorporate errors within the system. From Fig. 6.11, frequency depart from regular frequency and inertia variation in percentage graph shows, 100 % low inertia variation in percentage takes a more constant time than the 40% low inertia variation in percentage. In Fig. 6.11, red point in the 100% low inertia variation is the mid-point of x value 1.8 which has τ value 1.3. It is a stable operation point for the 100 % low inertia variation. To make the system fast frequency control to the low inertia, cost function can be altered to a quadratic programming problem method which uses the $\min(\tau^T \tau)$ function for the cost function which can be transfer into no-weighting factor goal function such as G_{depart} , $G_{recover}$ which are depart and recover from regular frequencies in the equation (6.12), (6.18). In Fig. 6.12, x unit is percent change, τ is intermediate variable. In Fig. 6.12,

graph is centralized control ROCOF analysis. As shown in the graph, PD control has the fastest settling time, and low undershoot and overshoot. Proposed method has the better performance in ROCOF compared with droop control method. Proposed method has 0.23 pick point ROCOF which is much smaller than droop control, which is 0.4, and it is also comparable with the PD control. This shows that the proposed method is adaptable into large scale power system low inertia problem with in/out loop control with no-weighting factor. Proposed method has lower ROCOF and comparable overshoot/undershoot settling time with PID as illustrated in Fig. 6.9. However, proposed approach has lower ROCOF and less overshoot / undershoot than droop control.

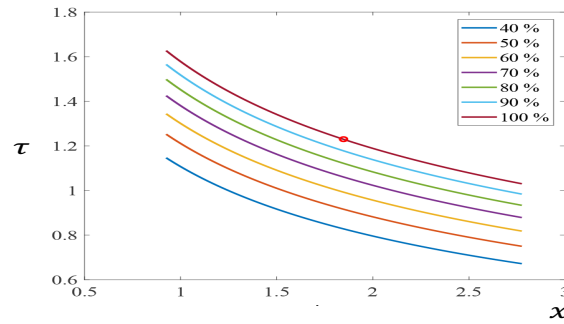


Fig. 6.11. Minimization intermediate variable (τ) which is the component value for goal function $\min(\tau^T \tau)$ and x with the variation of inertia in percentage.

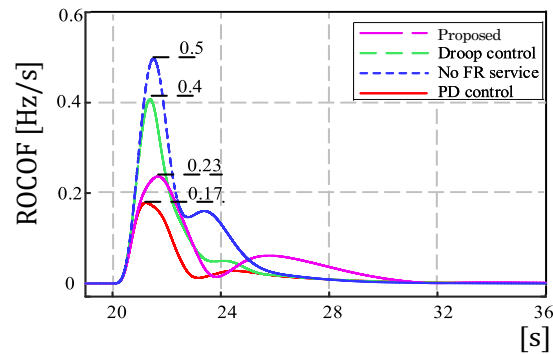


Fig. 6.12. Centralized control ROCOF analysis based on different control approaches.

6.6 Summary

This chapter proposes a novel fast frequency control approach for low inertia distributed generation inverters in the power system. It takes advantage of their agile response to prevent frequency shedding cases. An outer loop of MPC based linearized cost function control layer is added to the management scheme to make no weighting factor. For the inner loop, no-weighting geographical switching factor scheme based finite control set MPC is proposed to avoid the computational burden for weighting factor correction in the traditional neutral point clamped MPC based voltage source inverter. In response to a large disturbance, the system controls the inverter setting value to keep the frequency within the decided set limits. A multi-parameter MPC centralized control method was considered and found to be effective. A novel no-weighting prediction finite control set model predictive control was developed and compared with the other method, such as proportional derivative control, proportional integral derivative control, and droop control. It showed an improvement in fast frequency prediction accuracy and computational efficiency with SW (switch) factor scheme neutral point clamped inverters with the proposed approaches.

Chapter 7. Cloud Energy Internet Based Optimal Control of Residential ES-PV Inverters in Local Power Systems

Abstract – Alternating direction method of multipliers (ADMM) is effective for addressing optimal power flow convex optimization. However, ADMM significant drawback is absence of guaranteed convergence. Considering optimal power flow interactions resulting from the unstable voltage relationship among energy storage -photovoltaic (ES-PV) inverters, a steady state voltage security area (SSVSA) is established for these inverters. To assess the voltage security margin between the operational state and the security boundary, a voltage excursion indicator is proposed for processing optimal power flow within the local area. The proposed approach enables a fully guaranteed convergence and optimized power flow within a decentralized distributed energy management framework, systematically via multi-block ADMM handling of large-scale and distributed data via energy internet. This approach guaranteed convergence of optimal power flow via local consensus control by distributed ES-PV systems with energy internet via all-to-all communication between centralized energy managers and inverters.

7.1 Background

The growth of numerous residential energy storage system integrated with photovoltaic (ES-PV) technology has significant challenges in managing and controlling low voltage distribution systems. Implementing secondary control measures for residential inverters associated with ES-PV can alleviate issues such as overvoltage, particularly when the generation from these systems exceeds the transient voltage levels of the building and the demand during rapidly changing weather conditions [150]. Efforts are currently underway to improve inverter control

mechanisms and broaden the market for additional services, aiming to facilitate the large-scale integration of renewable energy sources while ensuring reliable operation of existing distribution networks [151].

In 2020, nearly 40 countries had a total photovoltaic (PV) capacity exceeding 1 gigawatt, with PV energy contributing over 8% to global electricity production [152]. The increasing adoption of PV technology has resulted in a significantly lower environmental impact compared to diesel engine generators; however, this impact is not negligible, and the utilization of PV generation remains relatively low [153]. The integration of supercapacitors to accommodate large electrical load peaks substantially improves energy self-consumption and self-sufficiency. Nevertheless, achieving optimal power flow and energy consumption at a community scale necessitates advanced control of residential inverters, as without such measures, harmonious management is unattainable [154].

The amount of reactive power absorbed or delivered by inverters can be determined based on droop control laws [155] or optimal power flow methodologies [156]. In both scenarios, the implementation of voltage control through this framework results in low power factors and elevated grid currents, which in turn leads to significant power losses within the grid. Alternative strategies require inverters to operate at a unity power factor, which slightly diminishes the available active power [157]. For instance, there has been an increase in literature regarding droop control methods aimed at quantifying the reduction in active power contributed by individual inverters within a community system. Active power curtailment techniques are particularly effective in low-voltage distribution feeders, where the high resistance to inductance ratio of low-

voltage lines makes voltage measurements more sensitive to changes in the active power being supplied [158]. Alternating direction method of multipliers (ADMM) is effective for addressing optimal power flow convex optimization [168]. However, ADMM significant drawback is absence of guaranteed convergence. Considering the optimal power flow interactions that arise from the unstable voltage relationship between ES-PV inverters.

The optimal control of energy internet systems involving energy storage systems (ESS) and photovoltaic (PV) residential inverters presents significant challenges, primarily due to the necessity of managing these inverters in a decentralized fashion. A robust control algorithm is essential, which can either centralize or distribute the management of the inverters. The implementation of multi-agent systems for control, coupled with effective communication protocols, is crucial. Additionally, it is imperative to assess the steady-state voltage security region of the inverters to ensure effective management of the distributed systems. By evaluating voltage levels and the active/reactive power losses on a point-to-point basis, one can achieve efficient control of both centralized and distributed inverters.

This chapter proposes an optimal inverter transmit control (OITC) strategy that identifies the subdivision of key ES and PV residential inverters that significantly impact grid performance objectives, along with the calculation of their real and reactive power settings. The optimal inverter transmit control (OITC) method is utilized to manage the optimal power distribution among the ES-PV residential inverters, focusing on the calculation of both real and reactive power settings within the local power system. The proposed OITC strategy offers improved resilience compared to volt/volt ampere

reactive methods [155] and advanced process control techniques [157]. Firstly, it enables real-time control of inverters that are required to participate in supplementary operations. Secondly, it facilitates the collective maximization of both real and reactive power output from the participating inverters. For a visual representation of the inverter control regions under the OITC framework, please refer to Fig. 7.3.

In this study, the objective function encompasses the generation costs of inverters along with the active and reactive power losses associated with both centralized and decentralized control approaches for inverters. The Optimal Inverter Transmission Control (OITC) is strategically divided into a subproblem that can be addressed in a decentralized fashion by the utility-owned energy supervisor and customer-owned ES-PV inverter systems, with fully command exchanges. This methodology is referred to as decentralized optimal inverter transmission control (DOITC). The paper introduces two innovative decentralized strategies. Firstly, all customer-owned ES-PV residential inverters are designed to interface with the utility, allowing the utility to enhance network performance, such as voltage regulation and power loss minimization, while individual customers focus on optimizing their financial objectives, particularly the amount of active power they wish to curtail. To ensure both financial viability and robust control for community power systems, a contract-based distributed maximization strategy has been proposed to facilitate power allocation, which is adaptable to the time-varying nature of communication [159]. In terms of data communication security, a distributed robust financial dispatch framework has been recommended for the integrated energy system [160]. Given the bidirectional power flow management among autonomous ES-PV residential inverters, voltage boundary

issues frequently arise within energy internet technology. Therefore, further investigation into energy control is essential to ensure the operational security of ES-PV residential inverters. Traditionally, point-wise approaches have been employed to maintain voltage security; however, these methods necessitate offline computations involving one or more fault scenarios [155]. Consequently, the application of this method in intricate systems is deemed unsuitable for online use [162]. Additionally, accurately determining the voltage security limit presents significant challenges [163]. In light of its precision and efficiency, the security region approach has been explored for practical engineering applications. This method translates voltage constraints into linear inequalities and characterizes the voltage security limit through hyperplanes within the power embedding volume [164]. A distribution system security region based on optimal load capability has been developed for the delivery grid, addressing the issue of real-time voltage measurement [165]. Therefore, the security region approach is well-suited for centralized energy management in power systems. Furthermore, an offline strategy is essential for establishing the security region. However, the proposed utility of the ES-PV residential inverter-based energy internet operates as a distributed system that does not rely on an offline method, instead utilizing an online approach. Thus, it is imperative to focus on the development of the steady state voltage security area (SSVSA).

In the evolution of metering frame standards, both utility and customer-owned EMSs exchange relevant information to determine the optimal settings for energy storage (ES) and photovoltaic (PV) inverters. Once the distribution strategies align, the inverter governor executes the active and reactive setting points. The DOITC

methodology segments the distribution network into clusters, each comprising a centralized cloud energy manager (CEM) and a collection of customer-owned ES-PV residential inverters. A distribution strategy is then formulated to enhance coordination within each cluster, allowing for fully voltage-related communication exchanges, which enables the interconnected clusters to agree on the system voltage profile. The DOITC frameworks are refined through the application of the alternating direction method of multipliers [166]. Related studies address nonconvex optimal power flow solutions in a decentralized context [167]. In distribution systems, a decentralized algorithm based on the multiple alternating direction method of multipliers has been proposed, which addresses a reactive power compensation challenge in relation to a projected power flow scenario [168]. The proposed approach enables a fully guaranteed convergence and optimized power flow within a decentralized distributed energy management framework, systematically via multi-block ADMM handling of large-scale and distributed data via energy internet. This approach guaranteed convergence of optimal power flow via local consensus control by distributed ES-PV systems with energy internet via all-to-all communication between centralized energy managers and inverters.

This chapter proposes a fully distributed maximum energy management strategy that incorporates voltage security constraints for energy internet systems based on ES and PV technologies in residential settings. The key findings of this research can be summarized as follows. Firstly, in contrast to the distributed reactive power compensation method referenced in [168], the proposed approach utilizes a precise AC power flow model, enabling a simultaneous calculation of both active and reactive

power set points. Secondly, to ensure system integrity, the alternating direction method of multipliers is applied for decentralized multi-region state approximation within the transmission framework, utilizing two decentralized optimal integrated transmission control (DOITC) mechanisms. Thirdly, a voltage security verification and stability assessment (SSVSA) for ES-PV inverters is developed to ensure voltage process security, while also addressing the uncertainties associated with voltage limits and fluctuations. Additionally, a voltage excursion index for ES-PV residential inverters is established to evaluate the voltage security thresholds within operational and safety parameters. Consequently, the nonlinear power flow equations are reformulated into linear equivalent expressions to alleviate computational burdens.

The rest of this section is organized as follows. The centralized OITC is proposed in section 7.2. SSVSA for ESS-PV residential inverter with the voltage excursion indicator is suggested in section 7.3. DOITC approach with utility and customer communication passing is presented in section 7.4. DOITC with network cluster allocation is suggested in section 7.4. Effectiveness of the simulation is represented in section 7.5. Section 7.6 summarizes this section.

7.2 Centered optimal inverter transmit control

7.2.1 ESS-PV inverter framework

Regarding a distributed system consists of $N + 1$ nodes gathered in the set $\mathcal{N} := \{0, 1, \dots, N\}$ in which node 0 indicates the secondary of step-down transformer, and lines presented by the group of edges $\varepsilon := \{(m, n)\} \subset \mathcal{N} \times \mathcal{N}$. For simplicity of description, a stabilized system is deliberated, both the centered and decentralized schemes suggested. Subdivision $\mathcal{U}, \mathcal{H} \subset \mathcal{N}$ amass nodes accord with utility poles, and

those with set up residential ES-PV residential inverters via the Fig. 7.1.

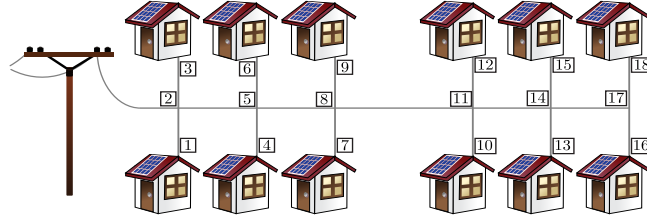


Fig. 7.1. Residential grid network with high penetration of ESS-PV inverters, node 0 represent secondary step-down transformer for the community power systems, $\mathcal{U} = \{2, 5, 8, 11, 14, 17\}$ gathers nodes related to distributed poles, houses are linked to nodes in the set of $\mathcal{H}$. $\mathcal{H} = \{1, 3, 4, 6, 7, 9, 10, 12, 13, 15, 16, 18\}$.

Phasers for the line to ground current and voltage imbued at node $n \in \mathcal{N}$ are I_n and V_n , respectively. It can be described $i := [I_1, \dots, I_N]^T \in \mathbb{C}^N$ and $v := [V_1, \dots, V_N]^T \in \mathbb{C}^N$. By using Kirchhoff's current law and Ohm's law, the linear $i = Yv$ relationship can be formed, where the system admittance matrix $Y \in \mathbb{C}^{N+1 \times N+1}$ is founded according to the system, and π -equivalent circuit of lines $(m, n) \in \varepsilon$ can be formed as shown in Fig. 7.2. In particular, y_{mn}^{sh} and y_{mn} represent shunt and series admittance of line (m, n) . Admittance of Y are represented with:

$$[Y]_{m,n} := \begin{cases} \sum_{j \in \mathcal{N}_m} y_{mj}^{sh} + y_{mj}, & \text{if } m = n \\ -y_{mn}, & \text{if } (m, n) \in \varepsilon \\ 0, & \text{otherwise} \end{cases} \quad (6.1)$$

where $\mathcal{N}_m := \{j \in \mathcal{N} : (m, j) \in \varepsilon\}$ represents the set of nodes connected to m th one through distributed line.

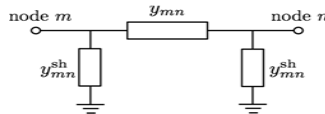


Fig. 7.2. Low voltage distribution line π -equivalent circuits.

A constant active and reactive power model [158] is used for the load, $P_{l,h}$ and $Q_{l,h}$ represent the active and reactive load demands at the node $h \in \mathcal{H}$. P_h^{av} represents the optimal available active power from the ES-PV at node $h \in \mathcal{H}$. The suggested scheme requests the mutual control of both real and reactive power produced by the ES-PV residential inverters. Especially, the permitted operating scheme on the complex power surface for the ES-PV inverters is shown in Fig. 7.3. fourth graph and represented with:

$$F_h^{OITC} := \begin{cases} 0 \leq P_{c,h} \leq P_h^{av} \\ P_{c,h}, Q_{c,h}: Q_{c,h}^2 \leq S_h^2 - (P_h^{av} - P_{c,h})^2 \\ |Q_{c,h}| \leq \tan\theta (P_h^{av} - P_{c,h}) \end{cases} \quad (7.2)$$

where $P_{c,h}$ is curtailed active power, and $Q_{c,h}$ is the imbued/absorbed reactive power by the node h inverter. If there is no limit to the power factor, such as $\theta = \frac{\pi}{2}$, then the operating region can be as Fig. 7.3. third graph.

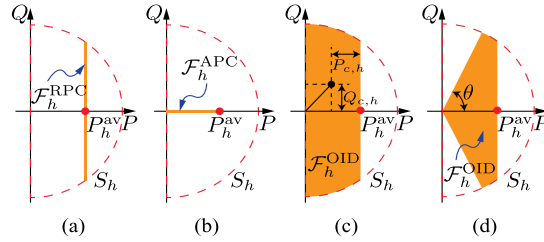


Fig. 7.3. Achievable operating area for the ESS-PV residential inverter (a) reactive power management (b) active power management (c) optimal inverter transmit with mutual management of real and reactive power (d) optimal inverter transmit with a low confined power factor.

6.2.2 ESS-PV residential inverter steady state voltage security Area

This part creates the steady state voltage security area for ESS-PV residential inverter with the voltage excursion indicator to calculate the operation security limit. Presume that energy internet makes up k ESS-PV residential inverters

and a secondary step-down transformer. Also, the z th residential inverter has n_z nodes. Total numbers of the branches and nodes in the energy internet are B and $N = \sum_{z=1}^k n_z$ where z is the series number of residential inverter. The node series group of energy internet is $C_p = \{1, 2, \dots, N\}$ and secondary step-down transformer is set as the 0^{th} node. Delimit the positive directions of power imbuements at nodes as the orientation from the distributed generators and loads to the nodes. Steady state voltage security area in power imbuement boundary is represented with:

$$\Omega_w = \left\{ x_w \middle| V \in R_w, \theta \in R^N, f_{(V_0, \theta_0)}(V, \theta) = x_w \right\} \quad (7.3)$$

where $x_w = (P, Q)^T \in R^{2N}$ is the power injection vector. $P = [P_1, \dots, P_k]$ and $Q = [Q_1, \dots, Q_k]$ are the active and reactive power imbuement groups of energy internet. $P_z = [P_{z1}, \dots, P_{n_{nz}}]$ and $Q_z = [Q_{z1}, \dots, Q_{n_{nz}}]$ are the active and reactive power imbuement groups of the z th residential inverter. V and θ stand for the node voltage size and node voltage angle vectors. $f_{(V_0, \theta_0)}(V, \theta) = x_w$ represents the power flow function with the V_0 and θ_0 of secondary step-down transformer. R_w is the voltage size restriction at nodes of restriction at nodes of residential inverters, in which represented with:

$$R_w = \left\{ V \in R^N \middle| V_{zj}^m \leq V_{zj} \leq V_{zj}^M, z_j \in C_p \right\} \quad (7.4)$$

where superscripts M and m are the upper and lower limits of parameters. z_j is the series number of the j th node in the z th residential inverter, and contents $z_j = (n_1 + n_2 + \dots + n_{z-1}) + j$. For the z th residential inverter, the node series group of the energy internet can be separate into the internal node series set $\Omega_{ln}^z = \{1, \dots, n_z\}$, and

the external node series set Ω_{Ex}^z , in which $\Omega_{ln}^z \cup \Omega_{Ex}^z = C_p$ and $\Omega_{ln}^z \cap \Omega_{Ex}^z = \emptyset$. Regarding the nodes in residential inverter-based energy internet separate into the internal and external nodes. z_i th node and the z_j node both be held by the same z th residential inverter. z' is used to represent the general series of residential inverter, which may be equal to z or not. For the branch $b_{zj}^{z_i} \in \{b_{0,1}, b_{1,2} \dots, b_{N-1,N}\}$, the z_i th node and z_j th node are the first end node and endpoint node. The relationship between V_{z_i} and V_{z_j} can be represented as

$$V_{z_j} = V_{z_i'} - \frac{R_{z_i'z_j}P_{z_i'z_j} + X_{z_i'z_j}Q_{z_i'z_j}}{V_{z_j}} \quad (7.5)$$

where $R_{z_i'z_j} + jX_{z_i'z_j}$ is the impedance value of the branch $b_{z_i'z_j}$. $R_{z_i'z_j}$ and $X_{z_i'z_j}$ stand for the resistance and reactance, $P_{z_i'z_j}$ and $Q_{z_i'z_j}$ stand for active and reactive power flows from the z_i' th node to the z_j th node

For creating the steady state voltage security area for residential inverter, it has the subsequent presumptions.

Presumption 1: In contrast to the effect of the power imbuements of residential inverters, the impact of the power losses is very small, which can be negligible.

For the z_j th node, where $z_j \in C_p$, the voltage size can be rewritten as:

$$\begin{cases} V_{z_j} = V_{z_i'} + (R_{z_i'z_j}P_{z_j,eq} + X_{z_i'z_j}Q_{z_j,eq})/V_0 \\ P_{z_j,eq} = P_{z_j} + \sum_{K \in D_{z_j}} P_K \\ Q_{z_j,eq} = Q_{z_j} + \sum_{K \in D_{z_j}} Q_K \end{cases} \quad (7.6)$$

where $P_{z_j,eq}$ and $Q_{z_j,eq}$ are the equivalent active and reactive power imbuelement of the z_j th node. P_{z_j} and Q_{z_j} are the active and reactive power imbuements of the z_j th node.

Apply (7.6) to all branches of residential inverter, then it is:

$$V_{zj} = V_0 + \sum_{z_i \in \Omega_{ln}^z} \frac{r_{z_i z_j} P_{z_i} + X_{z_i z_j} Q_{z_i}}{V_0} + \sum_{z_{i'} \in \Omega_{Ex}^z} \frac{r_{z_{i'} z_j} P_{z_{i'}} + X_{z_{i'} z_j} Q_{z_{i'}}}{V_0} \quad (7.7)$$

where $r_{z_i z_j}$ and $X_{z_i z_j}$ represents the resistance and the reactance between the z_j th node and z_i th node. $r_{z_{i'} z_j}$ and $X_{z_{i'} z_j}$ represents the resistance and reactance between the z_j th node and $z_{i'}$ th node. Impedance of lines can be computed by given methodology of the energy internet [169], [170] in which can be represented with:

$$r_{z_{i'} z_j} + jX_{z_{i'} z_j} = \begin{cases} R_{0,s} + jX_{0,s} & z_{i'} \notin D_{z_j} \text{ and } z_j \notin D_{z_{i'}} \\ R_{0,z_j} + jX_{0,z_j} & z_{i'} \notin D_{z_j} \text{ or } z_{i'} = z_j \\ R_{0,z_{i'}} + jX_{0,z_{i'}} & z_j \notin D_{z_{i'}} \end{cases} \quad (7.8)$$

where $R_{0,s} + jX_{0,s} = \sum_{b_{z_j}^{z_i} \in B_s} (R_{z_{i'} z_j} + jX_{z_{i'} z_j})$ is equal to the impedance of the branch set B_s , B_s is the branches on the path from the s th node to the 0^{th} node in the energy internet, where the s th node is the first intersection node of the upstream node set of the $z_{i'}$ th node and z_j th node. $R_{0,z_j} + jX_{0,z_j} = \sum_{b_{z_j}^{z_i} \in B_s} (R_{z_{i'} z_j} + jX_{z_{i'} z_j})$ is equal to the impedance of the branch set B_{z_j} is the branches on the path from the z_j th node. $D_{z_{i'}}$ and D_{z_j} represent the downstream node set of the $z_{i'}$ th node and z_j the node.

Regarding the lower and upper limits of voltage restrictions at the z_j th node, the voltage security limit according to (7.7) can be standardized as below:

$$\begin{aligned} \sum_{z_i \in \Omega_{ln}^z} (\alpha_{z_i z_j}^M P_{z_i} + \beta_{z_i z_j}^M Q_{z_i}) + \sum_{z_{i'} \in \Omega_{Ex}^z} (\alpha_{z_{i'} z_j}^M P_{z_{i'}} + \beta_{z_{i'} z_j}^M Q_{z_{i'}}) &\leq 1 \\ \sum_{z_i \in \Omega_{ln}^z} (\alpha_{z_i z_j}^m P_{z_i} + \beta_{z_i z_j}^m Q_{z_i}) + \sum_{z_{i'} \in \Omega_{Ex}^z} (\alpha_{z_{i'} z_j}^m P_{z_{i'}} + \beta_{z_{i'} z_j}^m Q_{z_{i'}}) &\leq 1 \end{aligned} \quad (7.9)$$

where $\alpha_{zi'zj}^M = \frac{r_{zi'zj}}{[(V_{zj}^M - V_0)V_0]}$, $\alpha_{zi'zj}^m = \frac{r_{zi'zj}}{[(V_{zj}^m - V_0)V_0]}$ represent the upper and lower susceptibility coefficients of active power imbuements, $\beta_{zi'zj}^M = \frac{x_{zi'zj}}{[(V_{zj}^M - V_0)V_0]}$ and $\beta_{zi'zj}^m = \frac{x_{zi'zj}}{[(V_{zj}^m - V_0)V_0]}$ represent the upper and lower susceptibility coefficients of reactive power imbuements. The values of $\alpha_{zi'zj}^m$, $\alpha_{zi'zj}^M$, and $\beta_{zi'zj}^m$, $\beta_{zi'zj}^M$ represent the voltage effect of the active and reactive power imbuement at the z_i 'th node to the z_j th node. They are decided by the given topology of the energy internet with online calculation.

To determine the operation security border between the current voltage state and security limit at node $z_j \in \Omega_{in}^z$, a voltage excursion indicator ΔV_{zj} is created which can be represented as (7.10). Especially, $\Delta V_{zj} < 0$ stands for the ES-PV residential inverter-based energy internet is in the operation security state region, and $\Delta V_{zj} \geq 0$ stands for the voltage off limit condition will happen. Also, the suggested steady state voltage security area for ES-PV residential inverter can be represented as linear paired formulas, in which is dissimilar from the traditional nonlinear power flow equations. The dimension of parameters is lessened.

$$\Omega_{zj}^w = \begin{cases} \sum_{z_i \in \Omega_{in}^z} (\alpha_{zi'zj}^M P_{zi} + \beta_{zi'zj}^M Q_{zi}) + \sum_{z_i' \in \Omega_{Ex}^z} (\alpha_{zi'zj}^M P_{zi'} + \beta_{zi'zj}^M Q_{zi'}) + \Delta V_{zj}^M = 1 \\ \sum_{z_i \in \Omega_{in}^z} (\alpha_{zi'zj}^m P_{zi} + \beta_{zi'zj}^m Q_{zi}) + \sum_{z_i' \in \Omega_{Ex}^z} (\alpha_{zi'zj}^m P_{zi'} + \beta_{zi'zj}^m Q_{zi'}) + \Delta V_{zj}^m = 1 \\ P_{zj}^m \leq P_{zj} \leq P_{zj}^M, Q_{zj}^m \leq Q_{zj} \leq Q_{zj}^M, \forall z_j \in C(V, p_c) \end{cases} \quad (7.10)$$

7.3 Decentralized optimal inverter transmit control

In view of separating the cost, $C(V, p_c)$ is consisted with utility voltage fluctuations with network power losses, and generation costs. $C(V, p_c) =$

$C_{utility}(V, P_c) + \sum_h R_h(P_{c,h})$, where $C_{utility}(V, P_c)$ represents utility minimization goals, in which voltage fluctuation and power losses in the grid [156], and $R_h(P_{c,h})$ is a cost incurred by customer h when residential ES-PV inverter is essential to curtail power. In general, a quadratic generation function $R_h(P_{c,h}) := a_h P_{c,h}^2 + b_h P_{c,h}$ is accommodated here, in which the selection of the coefficients is according to utility and customer preplanning [151].

Assume in which customer h dispatches to the utility company net active power $\bar{P}_h := P_h^{av} - P_{l,h}$ and the reactive load $Q_{l,h}$, then customer and utility will consent to the ES-PV inverter setting point according to the optimization goals related by $C_{utility}$ and $\{R_h\}$. By using the extra optimization parameters $\bar{P}_c, \bar{q}_c$, the OITC can be developed as:

$$\min_{V, \bar{p}_c, \bar{q}_c} \bar{C}(V, \bar{p}_c, \bar{q}_c) + \sum_{h \in \mathcal{H}} R_h(P_{c,h}), V \geq 0 \quad (7.11)$$

$$T_r(A_h V) = \bar{P}_h - \bar{p}_{c,h}, T_r(B_h V) = \bar{Q}_{c,h} - Q_{l,h}, \forall h \in \mathcal{H} \quad (7.12)$$

$$T_r(A_n V) = 0, T_r(B_n V) = 0, \forall n \in \mathcal{U} \quad (7.13)$$

$$V_{min}^2 \leq T_r(M_h V) \leq V_{max}^2, (P_{c,h}, Q_{c,h}) \in \mathcal{F}_h^{OITC}, \forall h \in \mathcal{H} \quad (7.14)$$

$$\bar{P}_{c,h} = P_{c,h}, \bar{Q}_{c,h} = Q_{c,h}, \forall h \in \mathcal{H} \quad (7.15)$$

where restrictions (7.15) make sure, in which utility and customer agree upon the inverters' setting points, and $\bar{C}(V, \bar{p}_c, \bar{q}_c) := C_{utility}(V, \bar{p}_c) + G(\bar{p}_c, \bar{q}_c)$.

Modern optimization technique such as alternating direction method of multipliers [170] can't directly solve (7.11) in a decentralized manner. To solve the problem disintegrated, it needs to introduce accessory paraments x_h, y_h per inverter h .

By using these accessory parameters, (7.11) can be redeveloped as

$$\min_{\substack{V, \bar{p}_c, \bar{q}_c \\ \{x_h, y_h\}}} \bar{C}(V, \bar{p}_c, \bar{q}_c) + \sum_{h \in \mathcal{H}} R_h(P_{c,h}), V \geq 0, (7.12) - (7.14) \quad (7.16)$$

$$(\bar{P}_{c,h} = x_h, x_h = P_{c,h}), (\bar{Q}_{c,h} = y_h, y_h = Q_{c,h}), \forall h \in \mathcal{H} \quad (7.17)$$

Optimization problem (7.16) is equivalent to (7.11), it is manageable to an alternating direction method of multipliers decentralized solution [170]. Alternating direction method of multipliers is better than distributed gradient methods due to its fast convergence and flexibility to communication errors [171]. To reflect on the partial quadratically augmented Lagrangian of (7.11), it can be described as below:

$$\begin{aligned} \mathcal{L}(\bar{P}, \{P_h\}, P_{xy}, D) := & \bar{C}(V, \bar{p}_c, \bar{q}_c) + \sum_{h \in \mathcal{H}} [R_h(P_{c,h}) + \bar{\gamma}_h(\bar{P}_{c,h} - x_h) + \gamma_h(x_h - \\ & P_{c,h}) + \bar{\mu}_h(\bar{Q}_{c,h} - y_h) + \mu_h(y_h - Q_{c,h}) + (\frac{k}{2(\bar{P}_{c,h} - x_h)^2}) + (\frac{k}{2(x_h - P_{c,h})^2}) + (\frac{k}{2})(\bar{Q}_{c,h} - \\ & y_h)^2 + (\frac{k}{2})(y_h - Q_{c,h})^2] \end{aligned} \quad (7.18)$$

where $\bar{P} := \{V, \bar{p}_c, \bar{q}_c\}$ gathers the optimization parameters of the utility,

$P_h = \{P_{c,h}, Q_{c,h}\}$ are the selection parameters for the customer h ,

$P_{xy} := \{x_h, y_h, \forall h \in \mathcal{H}\}$ is the set of supplementary parameters,

$\mathcal{D} := \{\bar{r}_h, r_h, \bar{\mu}_h, \mu_h, \forall h \in \mathcal{H}\}$ gathers the dual parameters, $k > 0$ is a given constant.

According to (7.18), alternating direction method of multipliers constitute iterating performance steps [Step 1]-[Step 3] explained below, in which i represents the iteration indicator. [Step 1] Upgrade parameters $\bar{P}$ as follows:

$$\bar{P}[i+1] := \arg \min_{V, \{P_{c,h}, Q_{c,h}\}} \mathcal{L}(\bar{P}, \{P_h[i], P_{xy}[i], \mathcal{D}[i]\}), V \geq 0, (7.12)-(7.15) \quad (7.19)$$

Also, inverter h , upgrade $P_{c,h}, Q_{c,h}$ as below:

$$P_h[i+1] := \arg \min_{P_{c,h}, Q_{c,h}} \mathcal{L}(\bar{P}[i], P_{c,h}, Q_{c,h}, P_{xy}[i], \mathcal{D}[i]) (P_{c,h}, Q_{c,h}) \in \mathcal{F}_h^{OITC} \quad (7.20)$$

[Step 2] Upgrade supplementary parameters P_{xy} :

$$P_{xy}[i+1] := \arg \min_{(x_h, y_h)} \mathcal{L}(\bar{P}[i+1], \{P_h[i+1]\}, \{x_h, y_h\}, \mathcal{D}[i]) \quad (7.21)$$

[Step 3] Dual update:

$$\bar{\gamma}_h[i+1] = \bar{\gamma}_h[i] + k(\bar{P}_{c,h}[i+1] - x_h[i+1]) \quad (7.22)$$

$$\gamma_h[i+1] = \gamma_h[i] + k(x_h[i+1] - P_{c,h}[i+1]) \quad (7.23)$$

$$\bar{\mu}_h[i+1] = \bar{\mu}_h[i] + k(\bar{Q}_{c,h}[i+1] - y_h[i+1]) \quad (7.24)$$

$$\mu_h[i+1] = \mu_h[i] + k(y_h[i+1] - Q_{c,h}[i+1]) \quad (7.25)$$

In [Step 1], the prime parameter $\bar{P}, \{P_h\}$ are acquired by minimizing (7.18), where the supplementary parameters P_{xy} and multipliers $\mathcal{D}$ are remaining their current iteration values. In addition, the supplementary parameters are upgraded in [Step 2] by amend $\bar{P}, \{P_h\}$ to their recent values. Finally, the dual parameters are upgraded in [Step 3] via dual gradient ascent.

It can be acknowledged, in which [Step 2] agreeably decouples into $2|\mathcal{H}|$ scalar and unconstrained quadratic with $x_h[i+1], y_h[i+1]$ solvable closed form.

Using this trait, the following lemma can be proved.

Lemma 1: Assume, in which multipliers are established as $\bar{\gamma}_h[0] = \gamma_h[0] = \bar{\mu}_h[0] = \mu_h[0] = 0$. Then, all the iterations can hold in which:

$$(1) \bar{\gamma}_h[i] = \gamma_h[i], \bar{\mu}_h[i] = \mu_h[i], (2) x_h[i] = \frac{1}{2}\bar{P}_{c,h}[i] + \frac{1}{2}P_{c,h}[i],$$

$$(3)y_h[i] = \frac{1}{2}\bar{Q}_{c,h}[i] + \frac{1}{2}Q_{c,h}[i].$$

Via the lemma 1, the traditional alternating direction method of multipliers [Step 1]-

[Step 3] can be cleared as below.

[Step 1] At the utility side, parameters $\bar{P}$ are upgraded by solving the following convex optimization:

$$\bar{P}[i+1] := \arg \min_{V, \{\bar{P}_{c,h}, \bar{Q}_{c,h}\}} \bar{C}(V, \bar{p}_c, \bar{q}_c) + F(\bar{p}_c, \bar{q}_c, \{P_h[i]\}), V \geq 0, \quad (7.26)$$

where function $F(\bar{p}_c, \bar{q}_c, \{P_h[i]\})$ can be defined as

$$\begin{aligned} F(\bar{p}_c, \bar{q}_c, \{P_h[i]\}) := & \sum_{h \in \mathcal{H}} \left[\frac{k}{2} (\bar{P}_{c,h}^2 + \bar{Q}_{c,h}^2) + \bar{P}_{c,h} \left(\gamma_h[i] - \frac{k}{2} \bar{P}_{c,h}[i] - \frac{k}{2} P_{c,h}[i] \right) + \right. \\ & \left. \bar{Q}_{c,h} \left(\mu_h[i] - \frac{k}{2} \bar{Q}_{c,h}[i] - \frac{k}{2} Q_{c,h}[i] \right) \right] \end{aligned} \quad (7.27)$$

At the customer side, the ES-PV inverter setting points are upgraded by solving the below quadratic restriction:

$$\begin{aligned} P_h[i+1] := & \arg \min_{P_{c,h}, Q_{c,h}} \left[R_h(P_{c,h}) + \frac{k}{2} (P_{c,h}^2 + Q_{c,h}^2) \right. \\ & - P_{c,h} \left(\gamma_h[i] + \frac{k}{2} \bar{P}_{c,h}[i] + \frac{k}{2} P_{c,h}[i] \right) \\ & \left. - Q_{c,h} \left(\mu_h[i] + \frac{k}{2} \bar{Q}_{c,h}[i] + \frac{k}{2} Q_{c,h}[i] \right) \right] (P_{c,h}, Q_{c,h}) \in \mathcal{F}_h^{OITC} \end{aligned} \quad (7.28)$$

[Step 2] At the utility and customer sides, the dual parameters are upgraded as

$$\gamma_h[i+1] = \gamma_h[i] + \frac{k}{2} (\bar{P}_{c,h}[i+1] - P_{c,h}[i+1]) \quad (7.29)$$

$$\mu_h[i+1] = \mu_h[i] + \frac{k}{2} (\bar{Q}_{c,h}[i+1] - Q_{c,h}[i+1]) \quad (7.30)$$

The consequent decentralized approach includes a two-way communication

between the utility and customers of the ongoing repeats $\bar{p}_c[i], \bar{q}_c[i], p_c[i], q_c[i]$. Especially, at each iteration, the utility owned instrument figure out the OITC execution (7.26) to upgrade the intended ES-PV inverter setting points according to the performance goals related to $\bar{C}(V, \bar{p}_c, \bar{q}_c)$ in which repeated with the term $F(\bar{p}_c, \bar{q}_c, \{P_h[i]\})$ compelling agreement with the setting points calculated at the customer side. Instantly (7.26) is solved, the utility conveys to each customer a duplicate of the iterative values $(\bar{P}_{c,h}[i + 1], \bar{Q}_{c,h}[i + 1])$. For the moment, the ES-PV inverter setting points are concurrently upgraded by using (7.28), and afterward sent to the utility. Instantaneously, the upgraded local repeats are interchangeable, utility and customers upgrade the local dual parameters (7.29), (7.30). The consequent decentralized approach is written in Algorithm 1 shown in Fig. 7.4. and topology is shown in Fig. 7.5.

The problem outlined in (7.26) can be reformulated into a standard semidefinite programming framework, which involves the minimization of a linear function subject to a set of linear equalities and inequalities. This is achieved by introducing additional optimization parameters and employing the Schur complement technique [172]. With a specified tolerance for agreement errors, where $0 < \varepsilon \ll 1$, the process concludes when the condition $\|\bar{p}_c[i] - p_c[i]\|_2^2 + \|\bar{q}_c[i] - q_c[i]\|_2^2 \leq \varepsilon$ is satisfied. The utility company has established a consensus that outlines a uniform strategy for addressing the OITC challenge, which includes the management of supply across different voltage levels and power flows while ensuring compliance with the constraints of the electrical grid. Once the decentralized method achieves stability, the real and reactive set points are effectively applied to the ES-PV inverters. Algorithm 1 facilitates online execution, allowing for the immediate transmission and availability of

the adjusted ES-PV inverter setting points $\bar{p}_c[i], \bar{q}_c[i]$, rather than waiting for the convergence of the approach.

Algorithm 1 OITC: Utility-customer communication topology

Set $\gamma_h[0] = \mu_h[0] = 0$ for all $h \in \mathcal{H}$.
for $i = 1, 2, \dots$ (repeat until convergence) **do**
 1. **[Utility]**: update $\mathbf{V}[i+1]$ and $\{\bar{P}_{c,h}[i+1], \bar{Q}_{c,h}[i+1]\}$ via(7.18).
 [Customer- h]: update $\bar{P}_{c,h}[i+1], \bar{Q}_{c,h}[i+1]$ via(7.19).
 2. **[Utility]**: send $\bar{P}_{c,h}[i+1], \bar{Q}_{c,h}[i+1]$ to h ;
 repeat for all $h \in \mathcal{H}$.
 [Customer- h]: receive $\bar{P}_{c,h}[i+1], \bar{Q}_{c,h}[i+1]$ from utility;
 send $P_{c,h}[i+1], Q_{c,h}[i+1]$ to utility;
 repeat for all $h \in \mathcal{H}$.
 [Utility]: receive $P_{c,h}[i+1], Q_{c,h}[i+1]$ from h ;
 repeat for all $h \in \mathcal{H}$.
 3. **[Utility]**: update $\{\gamma_h[i+1], \mu_h[i+1]\}_{h \in \mathcal{H}}$ via(7.20).
 [Customer- h]: update dual variables $\gamma_h[i+1], \mu_h[i+1]$ via(7.20);
 repeat for all $h \in \mathcal{H}$.
end for if stopping criterion ϵ is meet
Perform setting points in the ES-PV inverters.

Fig. 7.4. DOITC: Utility-customer communication topology algorithm.

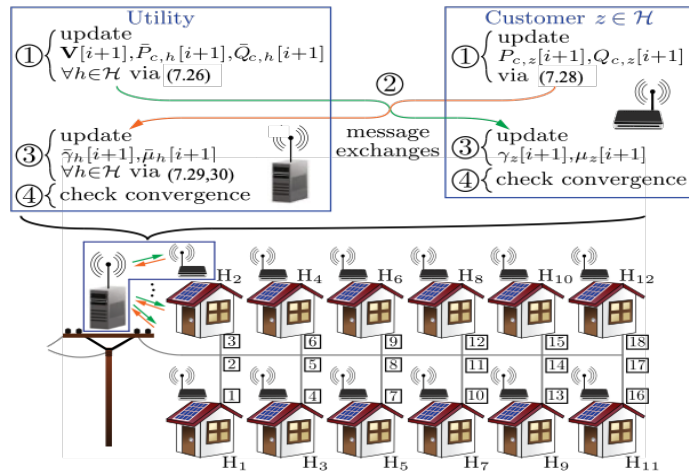


Fig. 6.5. DOITC: Utility-customer communication topology.

7.4 DOITC: Network cluster allocation

Assuming distributed network is allocated into clusters, $\mathcal{C}^a \subset \mathcal{N}$ indicates

the set of nodes within cluster a . Moreover, describe $\tilde{C}^a := C^a \cup \{n | (m, n) \in \varepsilon, m \in C^a, n \in C^j, a \neq j\}$. $\tilde{C}^a$ contains the nodes be owned by not identical clusters, in which are linked to the a th one by an allocation line, please check the Fig. 7.6. for demonstration. Hence, superscript $(\cdot)^a$ will be employed to indicate amount relate to cluster a . $\mathcal{H}^a$ is the set of homes positioned with cluster C^a . Vector $\bar{p}_c^a, \bar{q}_c^a$ gathers duplicates of setting points of ES-PV inverters $h \in \mathcal{H}^a$ accessible with the a th cloud energy manager. An anomaly is V^a , in which represents the submatrix of V equivalent to nodes in the stretched-out cluster $\tilde{C}_a$.

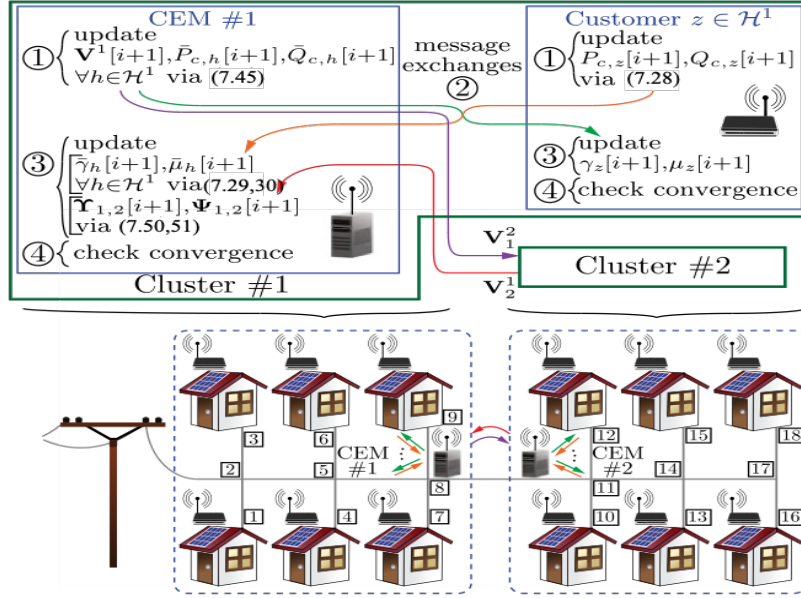


Fig. 6.6. Grid network divided into two clusters to perform algorithm 2. In this allocation, $C^1 = \{1, \dots, 9\}$, $\tilde{C}^1 = \{1, \dots, 9, 11\}$, $C^2 = \{10, \dots, 18\}$, and $\tilde{C}^2 = \{8, 10, \dots, 18\}$.

According to this network allocation, decoupling the grid network related cost $\bar{C}(V, \bar{p}_c, \bar{q}_c)$ in (7.11) is:

$$\bar{C}(V, \bar{p}_c, \bar{q}_c) = \sum_{a=1}^{N_a} [C^a(V^a, \bar{p}_c^a) + \lambda^a \sum_{h \in \mathcal{H}^a} \|\bar{P}_{c,h}, \bar{Q}_{c,h}\|_2] := \bar{C}^a(V^a, \bar{p}_c^a, \bar{q}_c^a) \quad (7.31)$$

where N_a is the number of clusters, $C^a(V^a, \bar{p}_c^a)$ seizes maximization goals of the

ath cluster such as power losses within the cluster [173]. Insufficiency encouraging regulation function is employed to decide which ES-PV inverters in $\mathcal{H}^a$ supply ancillary services. Also, per cluster $a = 1, \dots, N$ describe the region of possible power flows as:

$$R^a := \begin{cases} Tr(A_h^a V^a) = \bar{P}_h - \bar{P}_{c,h}, \forall h \in \mathcal{H}^a \\ Tr(B_h^a V^a) = \bar{Q}_{c,h} - Q_{l,h}, \forall h \in \mathcal{H}^a \\ Tr(A_n^a V^a) = 0, \forall n \in \mathcal{U}^a \\ Tr(B_n^a V^a) = 0, \forall n \in \mathcal{U}^a \\ V_{min}^2 \leq Tr(M_n^a V^a) \leq V_{max}^2, \forall n \in \mathcal{C}^a \end{cases} \quad (7.32)$$

where A_h^a, B_h^a, M_h^a are submatrices of A_h, B_h, M_h , shaped by taking out rows and columns matching to nodes in $\tilde{\mathcal{C}}^a$. With these descriptions, problem (7.11) can be redescrbed as:

$$\min_{\substack{V, \bar{p}_c, \bar{q}_c \\ \bar{p}_c, \bar{q}_c}} \sum_a [\bar{\mathcal{C}}^a(V^a, \bar{p}_c^a, \bar{q}_c^a) + \sum_{h \in \mathcal{H}^a} R_h(P_{c,h})], V \succcurlyeq 0 \quad (7.33)$$

$$\{V^a, \bar{p}_c^a, \bar{q}_c^a\} \in R^a, \forall a \quad (7.34)$$

$$(P_{c,h}, Q_{c,h}) \in \mathcal{F}_h^{OITC}, \forall h \in \mathcal{H}^a, \forall a \quad (7.35)$$

$$\bar{P}_{c,h} = P_{c,h}, \bar{Q}_{c,h} = Q_{c,h}, \forall h \in \mathcal{H}^a, \forall a \quad (7.36)$$

Restrictions (7.36) guarantee that the cloud energy manager and customer owned ES-PV inverter setting points. (7.33) - (7.36) productively separates power flow restrictions, cost, ES-PV correlated agreement restrictions (7.36) on a per-cluster basis. The major difficulty toward solving (7.33) - (7.36) in a distributed manner rests in positive semidefinite restraint $V \succcurlyeq 0$, in which surely matches the matrices $\{V^a\}$. To figure out this difficulty, outcome on finishing partial Hermitian matrices from [173] will support to recognize partitions of the distributed network in clusters, in which the positive semidefinite restraint on V would separate to $V^a \succcurlyeq 0, \forall a$. This separation

certainly promotes the decomposability of (7.33) - (7.36) in cluster subproblems.

First describe the set of adjacent clusters for the a th one as $\bar{B}^a := \{j | \bar{C}^a \cap \bar{C}^j \neq \emptyset\}$. Also, let $\mathcal{G}_c$ be a graph taking the control architecture of the distributed network, in which nodes indicate the clusters and edges link adjacent clusters. The graph $\mathcal{G}_c$ related to the grid network in Fig. 7.6. has two nodes, linked through an edge because two area are linked. Largely, it is obvious, in which if clusters a and j are adjacent, then cloud energy manager a , and cloud energy manager j must consent to the voltages at the two end points of the distributed line linking the two clusters. From Fig. 7.6. line (8,11) links clusters 1 and 2. Thus, cloud energy managers 1 and 2 must consent to voltages V_8 and V_{11} . Let V_j^a indicate the submatrix of V^a comparable to the two voltages on the line linking clusters a and j . Consent to V_8 and V_{11} is equivalent to setting $V_2^1 = V_1^2$, in which V_2^1 is a 2×2 matrix embodying the outer product $[V_8, V_{11}]^T [V_8, V_{11}]^*$.

Suggestion 1: Assume, (1) the cluster graph $\mathcal{G}_c$ is a tree, and (2) clusters are not nested such as $|\tilde{C}^a \setminus (\tilde{C}^a \cap \tilde{C}^j)| > 0, \forall a \neq j$. Then, (7.33) is the same to the following:

$$\min_{\substack{V^a, \bar{p}_c^a, \bar{q}_c^a \\ \bar{p}_c, \bar{q}_c}} \sum_a [\bar{C}^a (V^a, \bar{p}_c^a, \bar{q}_c^a) + \sum_{h \in \mathcal{H}^a} R_h(P_{c,h})] \quad (7.34) - (7.36) \quad (7.37)$$

$$V^a \succcurlyeq 0, \forall a \quad (7.38)$$

$$V_j^a = V_a^j, \forall j \in \bar{B}^a, \forall a \quad (7.39)$$

Under (7.1) and (7.2), there exists a rank 1 matrix V^{opt} solving (7.33) optimally presuming that $\text{rank}\{V^a\} = 1, \forall a = 1, \dots, N_a$. The suggestion 1 insists, in which for the positive semidefinite matrix V to have rank 1, all matrices V^a must have rank 1. If $\text{rank}\{V^a\} = 1$ for all clusters, then $\{V^a\}$ embodies a global optimal solution for given

ES-PV inverter setting points.

Supplementary parameters are brought in to facilitate decomposability of (7.37) in per cluster subproblems. With parameters x_h, y_h affiliated with inverter h , and $W^{a,j}, Q^{a,j}$ with adjacent clusters a and j , (7.37) is redeveloped to:

$$\min_{\substack{\{V^a, p_c^a, q_c^a\} \\ \{\bar{p}_c^a, \bar{q}_c^a\} \\ \{W^{a,j}, Q^{a,j}, x_h, y_h\}}} \sum_a [\bar{C}^a(V^a, \bar{p}_c^a, \bar{q}_c^a) + \sum_{h \in \mathcal{H}^a} R_h(P_{c,h})], (7.34) - (7.35), V^a \geq 0 (7.40)$$

$$\Re\{V_j^a\} = W^{a,j}, W^{a,j} = W^{j,a}, \forall j \in \tilde{\mathcal{B}}^a, \forall a \quad (7.41)$$

$$\Im\{V_j^a\} = Q^{a,j}, Q^{a,j} = Q^{j,a}, \forall j \in \tilde{\mathcal{B}}^a, \forall a \quad (7.42)$$

$$\bar{P}_{c,h} = x_h, x_h = P_{c,h}, \forall h \in \mathcal{H}^a, \forall a \quad (7.43)$$

$$\bar{Q}_{c,h} = y_h, y_h = Q_{c,h}, \forall h \in \mathcal{H}^a, \forall a \quad (7.44)$$

This optimization can be solved crosswise clusters by employing alternating direction method of multipliers. To achieve this goal, a partial quadratically augmented Lagrange acquired by dualizing restraints $\Re\{V_j^a\} = W^{a,j}, \Im\{V_j^a\} = Q^{a,j}, \bar{P}_{c,h} = x_h, \bar{Q}_{c,h} = y_h$ is defined. Then, the traditional alternating direction method of multipliers steps relate to a cyclic minimization of the resulting Lagrange in regard to $\{V^a, p_c^a, q_c^a, \bar{p}_c^a, \bar{q}_c^a\}$ the supplementary parameters $\{W^{a,j}, Q^{a,j}, x_h, y_h\}$ at last, a dual ascent step. It concludes, in which Lemma 1 still valid in the present case. It can show, in which alternating direction method of multipliers steps can be easier and cleared as explained next.

[Step 1] Each ES-PV system upgrades the local duplicate $P_h[i+1]$ by using (7.27), during each cloud energy manager upgrades the voltage profile of its cluster, and the local duplicates of the setting points of inverters $\mathcal{H}^a$ by solving the

following optimization:

$$\begin{aligned} \bar{P}^a[i+1] := \arg \min_{\substack{V^a, \bar{p}_c^a, \bar{q}_c^a \\ \{\alpha_j \geq 0, \beta_j \geq 0\}}} & [\bar{C}^a(V^a, \bar{p}_c^a, \bar{q}_c^a) + F^a(\bar{p}_c^a, \bar{q}_c^a, \{P_h[i]\}) \\ & + F_V^a(V^a, \{V^a, \{V^j[i]\})] \end{aligned} \quad (7.45)$$

$$\{V^a, \bar{p}_c^a, \bar{q}_c^a\} \in R^a, V^a \succcurlyeq 0 \quad (7.46)$$

$$\begin{bmatrix} -\alpha_j & a_j^T \\ a_j & -I \end{bmatrix} \preccurlyeq 0, \forall j \in \tilde{B}^a \quad (7.47)$$

$$\begin{bmatrix} -\beta_j & b_j^T \\ b_j & -I \end{bmatrix} \preccurlyeq 0, \forall j \in \tilde{B}^a \quad (7.48)$$

where vector a_j and b_j gather the real and imaginary parts of the entrances of the

matrix $V_j^a = \frac{1}{2}(V_j^a[i] + V_a^j[i]), V_j^b = \frac{1}{2}(V_j^b[i] + V_b^j[i])$. Normalization function

$F^a(\bar{p}_c^a, \bar{q}_c^a, \{P_h[i]\})$ executing consensus on the ES-PV inverter setting points

described in (7.27), and $F_V^a(V^a, \{V^a, \{V^j[i]\})$ is as follows:

$$\begin{aligned} F_V^a(V^a, \{V^j[i]\}) := & \sum_{j \in \tilde{B}^a} \left[\frac{k}{2}(\alpha_j + \beta_j) + Tr(\gamma_{a,i}^T[i] \{\Re V_j^a\}) + \right. \\ & \left. Tr(\Psi_{a,i}^T[i] \Im \{V_j^a\}) \right] \end{aligned} \quad (7.49)$$

[Step 2] Upgrade dual parameters $\{\gamma_h, \mu_h\}$ by (7.29), (7.30) at both of the cloud

energy manager and customer. Parameters $\{\gamma_{a,j}, \Psi_{a,j}\}$ are upgraded locally per-cluster

$a = 1, \dots, N_a$ as:

$$\gamma_{a,j}[i+1] = \gamma_{a,j}[i] + \frac{k}{2}(\Re\{V_j^a[i+1]\} - \Re\{V_a^j[i+1]\}) \quad (7.50)$$

$$\Psi_{a,j}[i+1] = \Psi_{a,j}[i] + \frac{k}{2}(\Im\{V_j^a[i+1]\} - \Im\{V_a^j[i+1]\}) \quad (7.51)$$

The consequent decentralized approach is summarized as Algorithm 2, shown in Fig. 7.6. It includes a swap of the local submatrices $\{V_j^a[i+1]\}$ among adjacent cloud energy managers to decide the voltages on lines linking clusters. Local

duplicates of the ES-PV inverter setting points between the cloud energy managers and customer-owned ES-PV. By premises similar to suggestion 2, convergence of the approach can be guaranteed.

Suggestion 2: With a $k > 0$ condition, the repeats $\{P^a[i]\}, \{P_h[i]\}, \mathcal{D}[i]$ created by [Step 1]-[Step 2] are convergent, and it converges to a solution of the OITC (7.11) and (7.33). One time the decentralized approach has converged; the real and reactive setting points are executed by the ESS-PV inverter controllers.

Eventually, worst case complexity of a semidefinite programming is on the order $O(\max\{N_c, N_v\}^4 \sqrt{N_v} \log(\frac{1}{\varepsilon}))$ for overall purpose solvers. N_c represents the total number of restraints, N_v represents the total number of parameters, and $\varepsilon > 0$ is given solution accuracy [172]. It abides by the worst-case complexity of (7.45) - (7.49) is greatly lower than the centralized problem. Also, the insufficiency of $\{A_n, B_n, M_n\}$ can be employed to get considerable computation savings [175].

Algorithm 2 DOITC: Multicluster distributed optimization.

Set $\gamma_h[0] = \mu_h[0] = 0$ for all $h \in \mathcal{H}^a$ and for all clusters.
Set $\Upsilon_{a,j}[0] = \Psi_{a,j}[0] = \mathbf{0}$ for all pair of neighboring clusters.

for $i = 1, 2, \dots$ (repeat until convergence) **do**

1. **[CEM- a]:** update $\mathbf{V}^a[i+1]$ and $\bar{\mathbf{p}}_c^a, \bar{\mathbf{q}}_c^a$ via (7.24).
[Customer- h]: update $\bar{P}_{c,h}[i+1], \bar{Q}_{c,h}[i+1]$ via (7.19).
2. **[CEM- a]:** send $\mathbf{V}_j^a[i+1]$ to CEM j ;
[CEM- a]: receive $\mathbf{V}_a^j[i+1]$ from CEM j ;
repeat $\forall j \in \bar{B}^a$
3. **[CEM- a]:** send $\bar{P}_{c,h}[i+1], \bar{Q}_{c,h}[i+1]$ to EMS- h ;
repeat for all $h \in \mathcal{H}^a$.
[Customer- h]: receive $\bar{P}_{c,h}[i+1], \bar{Q}_{c,h}[i+1]$ from CEM a ;
send $P_{c,h}[i+1], Q_{c,h}[i+1]$ to CEM a ;
repeat for all $h \in \mathcal{H}^a$.
[CEM- a]: receive $P_{c,h}[i+1], Q_{c,h}[i+1]$ from h ;
repeat for all $h \in \mathcal{H}^a$.
4. **[CEM- a]:** update $\{\gamma_h[i+1], \mu_h[i+1]\}_{h \in \mathcal{H}}$ via (7.20).
[CEM- a]: update $\{\Upsilon_{a,j}[i+1], \Psi_{a,j}[i+1]\}$ via (7.25);
[Customer- h]: update dual variables $\gamma_h[i+1], \mu_h[i+1]$ via (7.20);

end for
Perform setting points in the ES-PV inverters.

Fig. 7.7. DOITC: Multicluster distributed optimization.

7.5 Simulation case results

7.5.1 DOITC with utility customer communication algorithm

In this section, the effectiveness of the proposed energy internet-based

distributed ESS-PV inverter energy management system is evaluated using the IEEE 33 bus grid and the data presented in Fig. 7.1. [157]. The evaluated ESS-PV system comprises photovoltaic (PV) components and an energy storage system, facilitating bidirectional communication of both information and power. The simulation data is sourced from the National Renewable Energy Laboratory (NREL) to ensure consistency between the outcomes of centralized and decentralized algorithms. Each residence is equipped with fixed rooftop PV systems, which have a DC/AC derating coefficient of 0.77 [175]. The DC ratings for the residences are as follows: 5.52 kW for homes H_1, H_9, H_{10} , 5.7 kW for H_2, H_6, H_8, H_{11} , and 8kW for the rest of the homes. As assumed in [177], it is presumed, in which the PV inverters are oversized 10% of the consequent AC rating. The active powers $\{P_h^{av}\}$ created by the ESS-PV inverters with DC rating are graphed in Fig. 7.8. first graph. Also, the minimum of power factor for the inverter is set to 0.9 to calculate the reactive loads $\{Q_{l,h}\}$. The usable active powers during the day are calculated using the System Advisor Model (SAM) of the NREL according to the typical weather year data for Washington D.C., USA, during the month of July. The residential load data is got from Open Energy Information, during the month of July is employed for test case. To create diverse load profiles, the active power profile is disturbed using a Gaussian random variable with zero mean. Standard deviation of 200 W and power factor of 0.9 is assumed. The consequent active loads are graphed in Fig. 7.8. second graph. Finally, the voltage at the secondary of the transformer is set at 1.03 p.u., to guarantee a minimum voltage size of 0.92 p.u. at all homes when the ESS-PV inverters don't generate power. The pole-to-pole distance is set to 52 m, distances of the drop lines are set to 22m, and voltage limits V^{max}, V^{min} are set to 1.05 and 0.92 p.u.

[136]. In all the managed numerical tests, the rank of matrices V and $\{V^a\}$ was 1, in which global optimal solutions were acquired for given ES-PV inverter setting points.

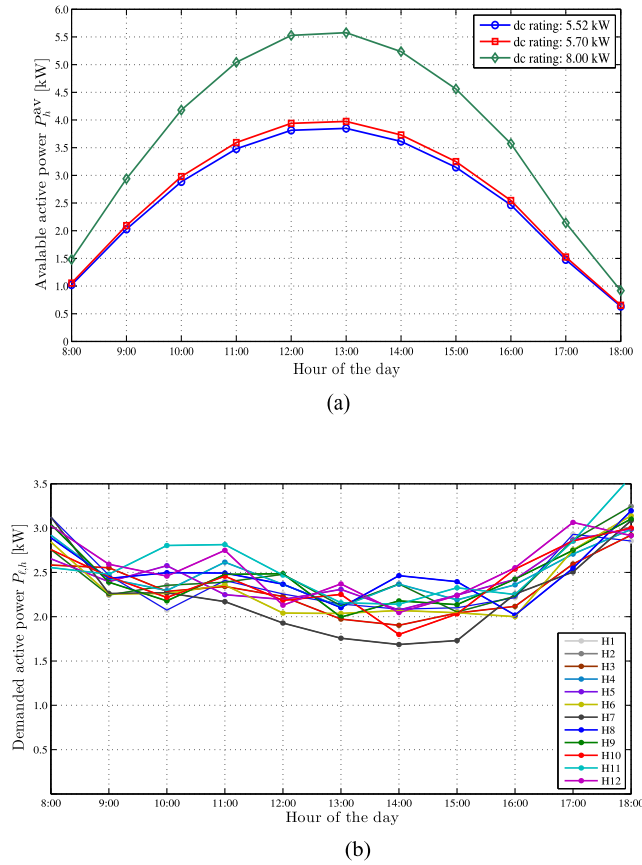


Fig. 7.8. Proposed approach optimization inputs (a) usable active powers $\{P_h^{av}\}$ from different inverters DC ratings (b) Demand of active loads at the home. Reactive demand is calculated by a power factor of 0.9. H_1 through H_{12} indicates the home inverter locations shown in Fig. 7.1

The goal of the utility company is to minimize the power losses in the grid network, in which describing the commensurable matrix $L_{mn} := \Re\{y_{mn}\}(e_m - e_n)(e_m - e_n)^T$ per distributed line $(m,n) \in \varepsilon$, function $C_{utility}(V, \bar{p}_c)$ is set to $C_{utility}(V, \bar{p}_c) = Tr(LV)$, with $L := \sum_{(m,n) \in \varepsilon} L_{mn}$. At the customer's part, function $R_h(P_{c,h})$ is set to $R_h(P_{c,h}) = 0.1P_{c,h}$. The influence of diverse the variable λ is examined in and shown in Fig. 7.8, in which the solution of the centralized OITC optimization (7.11) is proclaimed for nonidentical values of the variable λ . Fig. 7.9.

shows the active power reduced from each inverter during the whole day when $\lambda = 0.8$, although the consequence in Fig. 7.9. was acquired by set the parameter $\lambda = 0$. It is obvious, in which in the (b) case, all of the inverters are controlled, they all reduced active power from 8 AM to 6 PM. In case of $\lambda = 0.8$, the OITC seeks to establish a balance between the achievable objective function and the quantity of regulated ES-PV inverters. It is evident that as solar irradiation intensifies, the number of active ES-PV inverters also rises, reaching an optimal configuration of seven ES-PV inverters operating at 1 PM.

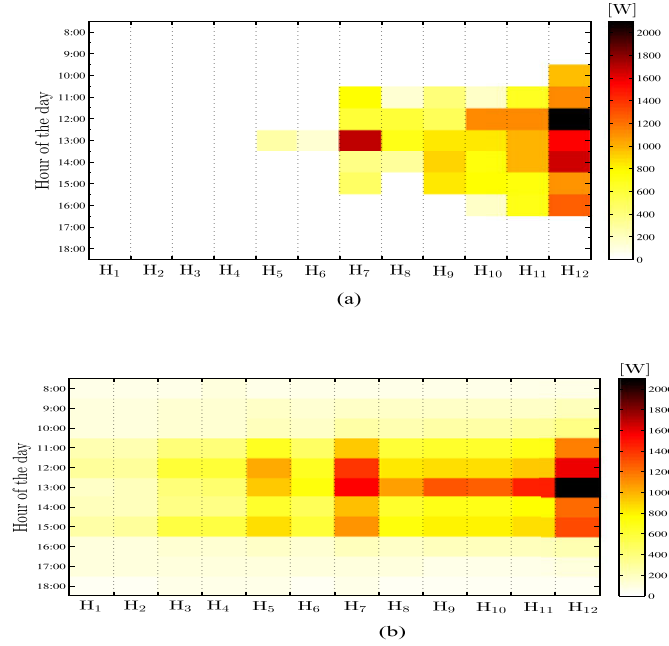


Fig. 7.9. Proposed centralized OITC solution with reduced active power per each home (a) $\lambda=0.8$ and (b) $\lambda=0$. H_1 through H_{12} indicates the home inverter locations shown in Fig. 7.1.

7.5.2 Steady State Voltage Security Area (SSVSA) Analysis

In this section, it is essential to demonstrate the effectiveness of the analysis on steady state voltage security area (SSVSA) for the energy storage system-photovoltaic (ESS-PV) inverter, utilizing the voltage excursion indicator. To validate the implementation of the SSVSA, two scenarios of the energy management system

(EMS) are considered. The first scenario involves an EMS that operates without the SSVSA for the ESS-PV inverter, while the second scenario incorporates the SSVSA for the same inverter. Generally, the total load demands of the energy internet power are set at [2.5, 3.0] MW, and the optimal solutions for both scenarios can be derived using Algorithm 1. Additionally, the voltage security states across 33 nodes are illustrated in Fig. 7.10.

In Fig. 7.10., the security state of under the first and second cases have been demonstrated under the commensurate optimal EMS solutions. Based on Fig. 7.10., it may propose, in which the voltage digression indicator of the whole nodes in the cloud energy manager accomplish varying degrees of voltage off limit with the elongation of loads demands. Also, with the contrast of the voltage digression indicator between the first and the second cases, it can be found that the voltage off limit degree is increased with the increase of load demands without the consideration of the SSVSA for ES-PV inverter. Finally, it can be certified, in which the optimal EMS, obtained by the centralized strategy suggested in Algorithm 1, executes excellent capacity to guarantee computation accuracy and voltage security, in which the voltage size of nodes is always in the secure boundary.

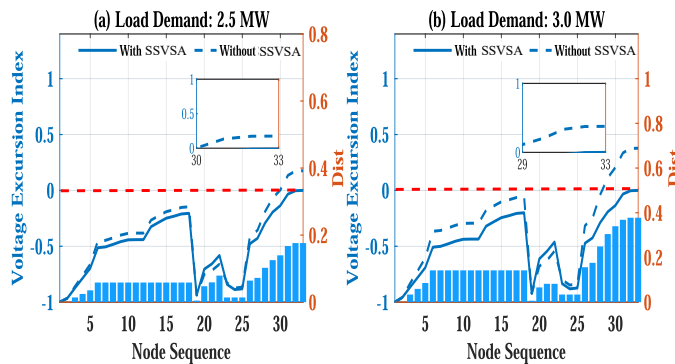


Fig. 7.10. Voltage security state analysis of ESS-PV inverters in SSVSA.

The convergence of Algorithm 1 is exemplar for $\lambda = 0.8$, $C_{utility}(V, \bar{p}_c) = Tr(LV)$, and $R_h(P_{c,h}) = 0.1P_{c,h}$, and by employing the solar irradiation conditions at 12 PM. Fig. 7.11 (a) describe the trajectories of the repeats $\{P_{c,h}[i]\}_{h \in \mathcal{H}}$ with dashed lines, and $\{\bar{P}_{c,h}[i]\}_{h \in \mathcal{H}}$ with solid lines for all the homes $H_1 - H_{12}$. The simulation results match the ones in Fig. 7.11., for the iterations with $i \geq 20$, it meets the convergence, simply inverters at home $H_7 - H_{12}$ are controlled. The active power curtailment setting points are in unanimity. This simulation result is predictable, since (7.11) is in a form, in which is appropriate for the application of the alternating direction method of multipliers to stem from distributed optimization solution approaches [162]. At the last, the lines of the setting point consensus error $|P_{c,h}[i] - \bar{P}_{c,h}[i]|$ is shown in Fig. 7.9. (b) in response to the alternating direction method of multipliers iteration indicator.

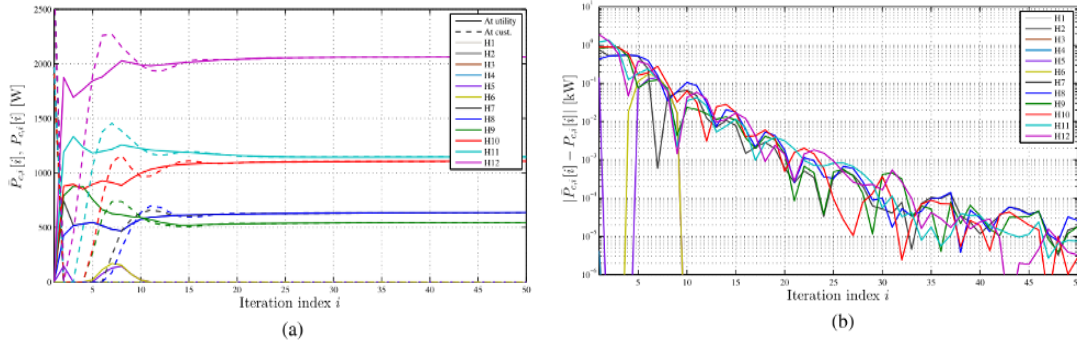


Fig. 7.11. Convergence of Algorithm 1 (a) $\{P_{c,h}[i]\}_{h \in \mathcal{H}}$ with dashed lines, and $\{\bar{P}_{c,h}[i]\}_{h \in \mathcal{H}}$ with solid line, accordant with the alternating direction method of multipliers iterative index (b) consensus error for all homes $h \in \mathcal{H}$ accordant with i .

7.5.3 DOITC with multi-cluster distributed algorithm

Fig. 7.11. indicates the disparity between local voltage on the line (8, 11).

The lines of the voltage errors $|V_8^1[i] - V_8^2[i]|$ and $|V_{11}^1[i] - V_{11}^2[i]|$ are proclaimed in response to the alternating direction method of multipliers iteration indicator i . The

findings indicate that both cloud energy managers concur on the voltage of the connection that links the two clusters. The irregular pattern observed is typical of the alternating direction method of multipliers. Additionally, relevant graphs were obtained for the inverter setting points as well.

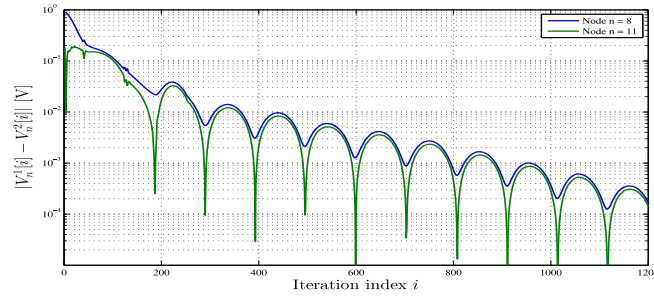


Fig. 7.12. Convergence of Algorithm 2, and consensus error in response to the multi-alternating direction method of multipliers iteration indicator i .

7.6 Summary

This chapter proposes a fully decentralized methodology for the management of energy within a cloud energy internet framework, utilizing the alternating direction method of multipliers alongside the steady state voltage security area (SSVSA) specifically for energy storage (ES) and photovoltaic (PV) inverters. A voltage excursion indicator has been developed within the SSVSA to mitigate the risk of voltage violations and to delineate the voltage security boundary. The model for the cloud energy manager, designed for minimal utility operation, has been effectively illustrated. Additionally, a decentralized approach has been established for determining the optimal real and reactive power set points for residential ES-PV inverters. The proposed decentralized optimal inverter transmit control (OITC) scheme offers a comprehensive strategy for distributing computational tasks and optimization objectives across the entire network. The algorithm emphasizes a future oriented energy internet-based management system for ESS-PV, enabling consumer

participation in energy distribution market activities. Simulation results validate the accuracy of the optimal solutions obtained through the proposed distributed scheme, ensuring the secure operational state of the ES-PV inverters without any instances of voltage violations. Overall, the analysis indicates that the developed fully multi distributed cloud-based energy management strategy is adaptable to the energy internet, guaranteeing both operational security and the reliability of ES-PV inverters.

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