

ABSTRACT

Title of Dissertation: ACTIVE NON-RECIPROCAL ACOUSTIC
METAMATERIALS

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This dissertation presents different configurations of active Acoustic MetaMaterials (*AMM*) which are proposed in order to control the flow of vibration and acoustic wave propagations in various applications. Distinct among these configurations is a 1-dimensional (1D) periodic array which consists of an assembly of active acoustic unit cells which are provided with programmable piezoelectric elements. By tuning the structural properties of these cells, the 1D array can impede the wave propagation over specific frequency ranges. In order to achieve non-reciprocal acoustic wave transmission of the *AMMs*, three different methodologies are introduced including active control of the piezoelectric elements using virtual gyroscopic control actions, eigenstructure shaping controller, and finally spatial-temporal modulation algorithm.

Theoretical models are developed to investigate the fundamentals and the underlying physical phenomena associated with all the considered three *AMM*

configurations. Experimental prototypes of all these *AMM* configurations are built and tested to demonstrate their effectiveness in controlling the propagation of vibration and noise through these materials. Furthermore, the experimental results are used to validate the developed theoretical models.

The developed theoretical and experimental approaches are envisioned to be valuable tools in the design of arrays of *AMM* for various applications which are only limited by our imagination.

ACTIVE NON-RECIPROCAL ACOUSTIC METAMATERIALS

by

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Dedication

To my beloved parents.

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Chapter 1: Introduction

1.1 Overview

Acoustic metamaterials (*AMMs*) are artificial materials which consists of subwavelength unit cells and have some acoustic properties that are unusual as compared to natural materials, such as negative effective bulk modulus, negative effective mass density and anisotropic mass density. Hence, *AMMs* can be used to realize some novel acoustic phenomena like acoustic cloaking, acoustic lenses and negative refraction, etc. Recently, extensive research in the field of acoustic metamaterials has been focused on developing non-reciprocal acoustic systems which can break the time-reversal symmetry of the acoustic wave propagation. Various strategies have been developed to break the acoustic reciprocity, including the use of system nonlinearities, spatiotemporal modulation, active control systems, and circulator or gyroscopic elements.

In this dissertation, we proposed three configurations of active non-reciprocal acoustic metamaterial with different control strategies. Firstly, an active non-reciprocal gyroscopic metamaterial (*NGMM*) cell was presented as the non-reciprocal behavior was realized by using a physical gyroscopic dynamic controller. Secondly, we extend our work to *NGMM* cells with a simplified version of the virtual gyroscopic dynamic controller. For the last two cases, non-reciprocal acoustic wave transmission in a waveguide was demonstrated via two different control strategies, including eigenstructure assignment and spatiotemporal modulation.

1.2 Literature Review

In recent years, the development and application of *AMMs* have drawn significant attention of researchers around the world since it has some unique properties that cannot be found from natural materials. In order to control the follow of vibration and acoustic wave propagation, researches on *AMMs* are diversified including theory, structural design and manufacture, and various corresponding applications. Comprehensive literature review on *AMMs* has been conducted by Haberman and Guild [1], Liu *et al.* [2] and Liao *et al.* [3]. One challenging area in *AMMs* is non-reciprocal *AMMs*, as wave propagation is reciprocal in the conventional linear time-invariant acoustic medium. With the aim of breaking acoustic reciprocity, many different approaches have been introduced in recent years, which are summarized in the literature reviews by Fleury *et al.* [4], Nassar *et al.* [5], and Rasmussen *et al.* [6].

In this section, we present a brief literature review on historical background and developments of *AMMs*, theorems of acoustic reciprocity and different strategies for realization of acoustical non-reciprocity.

1.2.1 Acoustic Metamaterials

In 1968, Veselago [7] illustrated the possibility of electromagnetic wave to have negative refractive index for a medium with negative permittivity and negative permeability. However, it was until late 1990s Pendry *et al.* [8, 9] explained how to manufacture an artificial material with negative magnetic permeability. And then the experimental verification of metamaterial with negative refraction index was

demonstrated by Smith *et al.* [10] and Shelby [11]. Since then, extensive efforts have been devoted to the study of metamaterial in different fields.

As a subset of metamaterial, *AMM* is an artificial material that is used to control acoustic wave propagation in medium. The first *AMM* was developed by Liu *et al.* [12] called locally resonant sonic materials, which has a negative effective elastic constant. In acoustics, wave propagation can be characterized as following equation:

$$\frac{\partial^2 P}{\partial t^2} = \frac{K}{\rho} \nabla^2 P \quad (1.1)$$

where P is the acoustic pressure, K is the bulk modulus and ρ is the fluid mass density. The propagation speed of acoustic waves through media is given by $c = \sqrt{K/\rho}$ and the acoustic wave impedance is $Z = \sqrt{K\rho}$. Since natural materials do not have negative bulk modulus and negative density, numerous researchers are trying to develop *AMMs* with negative parameters in order to regulate the acoustic behavior.

For spring-mass systems, Milton and Willis [13] explained the concept of effective mass and later negative effective mass was experimentally captured by Yao *et al.* [14] when the excitation frequency is near the resonant frequency of the system. Analogous to spring-mass system, acoustic negative mass density can be obtained with membrane-like system [15-18] and result in a high transmission loss at certain frequencies. Negative bulk modulus can be realized with open and closed cavity resonators [19-22]. Based on the above two single negative parameter system, Lee *et al.* [23] introduced a system with double negative parameters simultaneously, which consists of a periodic array of interspaced membranes and side holes. Another type of

double negative system is called space-coiling *AMMs* by coiling up space with labyrinthine structures [24-28], which has a negative refractive index.

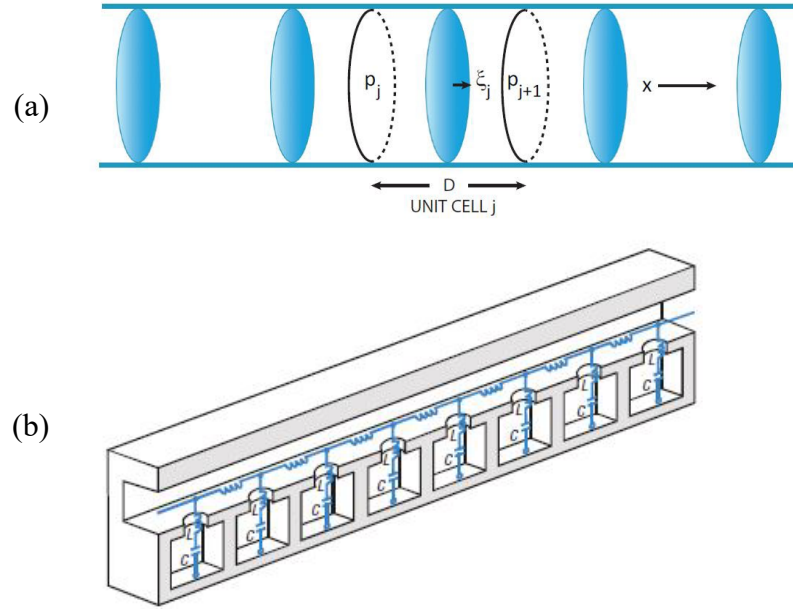


Figure 1.1 Acoustic metamaterials with negative density and negative modulus [18, 19] (a) schematic drawing of a *1D* membrane-based *AMM* with periodically spaced membranes in a tube [18]; (b) schematic drawing of a *1D AMM* consisting of periodically spaced Helmholtz resonators [19].

Furthermore, tunable *AMMs* has been an emerging field which involves active control. Baz [29, 30] introduced a new class of active acoustic metamaterial (*AAMM*) configured with flexible piezo diaphragm. According to the piezoelectric effect, different configurations of *AAMM* were proposed by Akl and Baz [31-34] which have effective density and bulk modulus and can be utilized to realize acoustic cloaking. For Helmholtz resonator based *AMMs*, Lee *et al.* [35] demonstrated a tunable bulk modulus system by modifying resonator cavities with terminal plungers. For membrane-like system, acoustic properties can be tuned through various approaches, including

pressurized air control [36, 37], electric control [38], magnetic and electromagnetic control [39-41].

1.2.2 Acoustic Reciprocity

Reciprocity is a fundamental property of wave propagation in an unmodulated acoustic medium, which states that the sound received at point B that transmitted from point A is the same as the sound received at point A that transmitted from point B. The concept of acoustic reciprocity was first introduced by Helmholtz [43] in 1860 on sound transmission in ducts. However, it was until 1873 Rayleigh [44] provided the theorem of reciprocity for elastic vibration systems and in 1877 he further extended the reciprocity theorem to acoustic system provided in volume II of *The Theory of Sound* [45]. Other than acoustics, wave propagation is reciprocal in many different physical systems, such as electromagnetics, electronics and optics [46, 47].

One example of reciprocity statements in acoustics was illustrated by Rasmussen *et al.* [6]. In an acoustic field $(p_i, \mathbf{v}_i)$, where $i = 1$ or 2 that represents acoustic source and receiver, respectively. The sound equation can be formulated as:

$$-i\omega \frac{p_i}{\rho_0 c_0^2} + \nabla \cdot \mathbf{v}_i = -i\omega \frac{m_i}{\rho_0} \quad (1.2)$$

$$-i\omega \rho_0 \mathbf{v}_i + \nabla p_i = -\omega^2 \mathbf{d}_i \quad (1.3)$$

where m_i is the monopole source distribution and $\mathbf{d}_i$ is the dipole distribution.

According to Eq. (1.2) and (1.3), reciprocity can be derived as:

$$\iiint_V (p_1 m_2 - p_2 m_1 + i\omega \rho_0 \mathbf{v}_1 \cdot \mathbf{d}_2 - i\omega \rho_0 \mathbf{v}_2 \cdot \mathbf{d}_1) = 0 \quad (1.4)$$

By interchanging the monopole sound source, the sound pressure at the receiver is the same, $p_1 = p_2$, which is shown in Figure 1.2(a). For an acoustic dipole source by assuming $\mathbf{D}_1 = \mathbf{D}_2$, the acoustic reciprocity is described as $\mathbf{D}_2 \cdot \mathbf{v}_1 = \mathbf{D}_1 \cdot \mathbf{v}_2$.

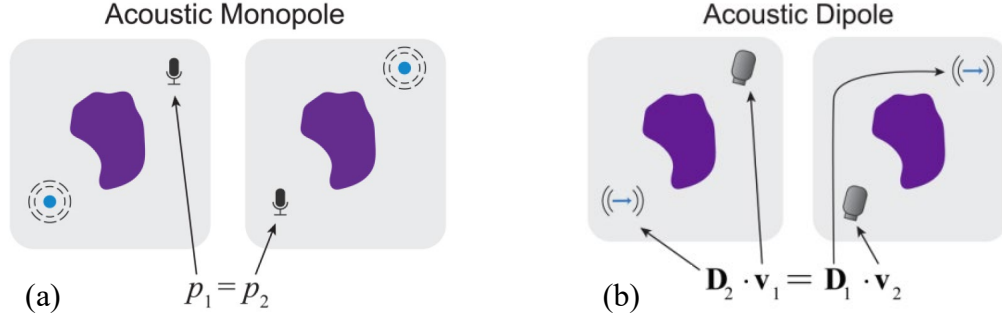


Figure 1.2 Description of the acoustic reciprocity for monopole sources [6]

1.2.3 Acoustic Non-Reciprocity with Circulators and Gyroscopic Elements

Extensive research is currently focusing on breaking the reciprocity theorem of the wave propagation in different physical systems. For example, in electromagnetics, unidirectional wave propagation can be achieved by magnetic biasing [48-53]. In optics, non-reciprocal devices can be developed by using the polarization rotation of magneto-optically active material [54-58].

For an acoustic system, the utilization of circulators and gyroscopic elements (CGE) is one approach that can realize the non-reciprocal acoustic transmission [59-70]. In 2014, Fleury *et al.* [59] developed a fluid-filled acoustic ring cavity coupled with three acoustic waveguides, shown in Figure 1.3. Compared with stationary system, acoustic isolation was observed by rotating the ring cavity as the ring's azimuthal resonant modes were split. Based on the work of Fleury *et al.* [59], Beli *et al.* [63] proposed a flexible rotating ring which can produce non-reciprocal response as

a result of Coriolis acceleration and material damping. In 2018, Zangeneh-Nejad and Fleury [62] proposed a non-reciprocal acoustic phase shifter where forward and backward wave transmissions have different phase shifts when the fluid in the waveguide has a constant moving velocity.

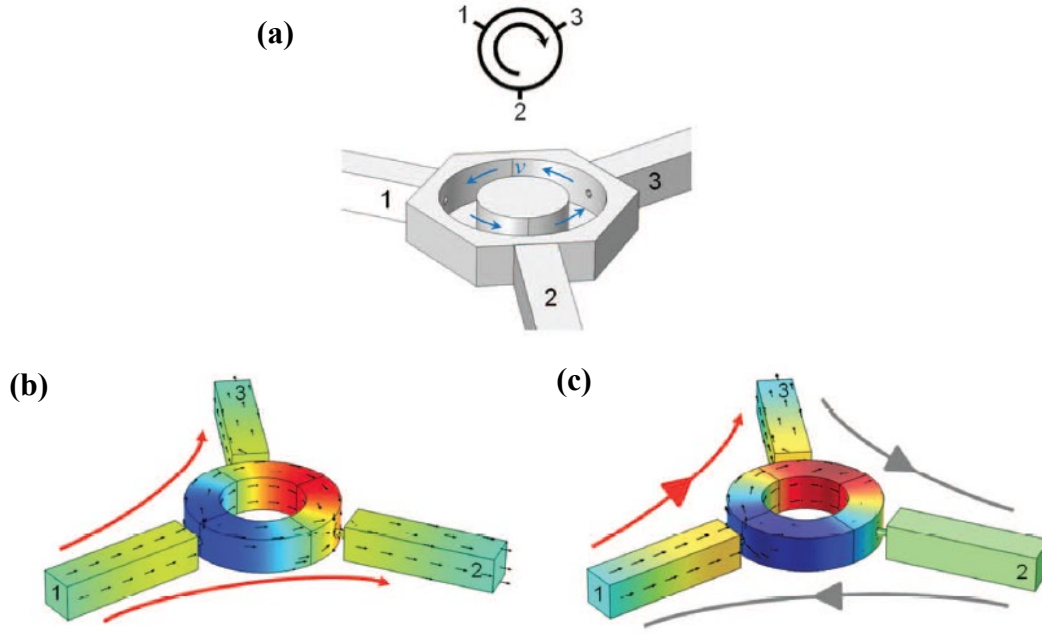


Figure 1.3 Acoustic non-reciprocity in a compact acoustic circulator [59], (a) schematic drawing of the circulator; (b) wave simulation for stationary fluid; (c) wave simulation for circulating fluid

In 2015, Nash *et al.* [60] introduced a new concept of metamaterial as gyroscopic metamaterial which composed of rapid spinning objects that are coupled to each other, and the time-reversal symmetry of wave propagation was broken because of the gyroscopic effect. Recently, unsymmetrical dispersion relation and directional bandgaps have been detailed discussed in different configurations of lattices involved with gyroscopic effect [61, 64, 65].

Furthermore, non-reciprocal wave transmission can be achieved in periodic rotating structures [66, 67]. Attarzadeh *et al.* [66] developed a class of space-time-periodic gyric metamaterial (*GMM*) lattices with embedded spinning rotors and wave transmission symmetry was broken by tuning the rotational speed or direction of periodic placed rotors. Brandenbourger *et al.* [67] created a robotic mechanical metamaterials whereby non-reciprocal interactions was observed with an active control of the rotation of a robotic unit cell, which is coupled to its neighbors via pre-stretched elastic beams. In addition, Zhao *et al.* [68] demonstrated the non-reciprocal wave propagation in elastic gyroscopic medium based on the Stroh formalism. However, all the above-proposed non-reciprocal systems [59-69] with *CGE* require physical gyrators, circulators or gyroscopic components inside the system or medium. In 2020, Baz [70] proposed a non-reciprocal gyroscopic metamaterial (*NGMM*) cell comprised of an acoustic cavity with dynamically programmable flexible boundaries. Without any embedded moving components, non-reciprocal behavior was realized by virtually synthesizing a gyroscopic control action from *NGMM* boundaries.

1.2.4 Acoustic Non-Reciprocity by Spatiotemporal Modulation

Apart from using *CGEs*, recently the idea of spatiotemporal modulation has been introduced to break time-reversal symmetry for the wave propagation in different fields. In photonics, Fang *et al.* [71] in 2012 created a photonic Aharonov-Bohm effect by inducing a dielectric phase modulation in the photonic transition, which results in an on-chip nonmagnetic photonic isolation. As an extension of their work, Fang *et al.*

[72] obtained an effective magnetic field for photons that breaks time-reversal symmetry by dynamically modulating photonic resonator lattices. For circulatory systems, Estep *et al.* [73] and Fleury *et al.* [74] demonstrated that non-reciprocity can be realized by parametric modulation which aims at introducing an effective angular momentum into the system. In 2015, Hadad *et al.* [75] proposed space-time gradient metasurfaces with non-reciprocal electromagnetically induced transparency effect by temporally modulating the electronic properties of surface impedance.

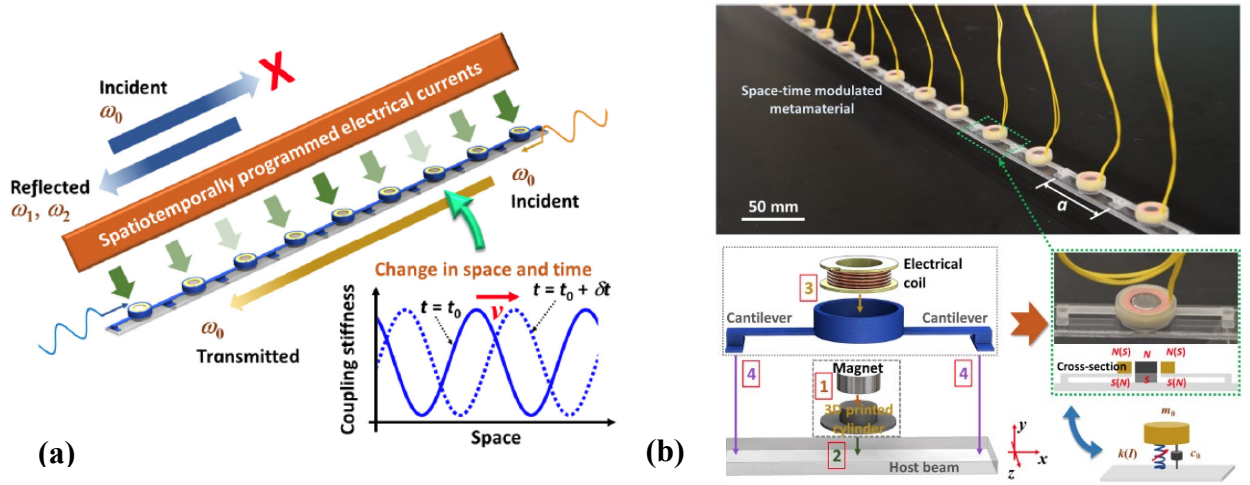


Figure 1.4 Non-reciprocal wave propagation in a continuum-based metamaterial [82],
 (a) schematic drawing of an elastic metamaterial with inner stiffness electrically modulated in space and time; (b) design of the modulated metamaterial

For elastic wave propagation, the non-reciprocal behavior was explored by changing system's properties in both space and time [76-82]. Dispersion relations and directional band gaps of the corresponding system [76-82] by using the Bloch-Floquet wave solution, where spatial modulation results in universal band gaps of the wave propagation and temporal modulation further breaks the symmetry of the dispersion relationship. In 2019, Chen *et al.* [82] designed a tunable elastic metamaterial beam

with an array of permanent magnets and oscillating electrical coils shown in Figure 1.4. Non-reciprocal wave transmission was captured by manipulating electric currents with different phases between electrical coils, which aimed at spatiotemporally modulating the coupling stiffness of the beam.

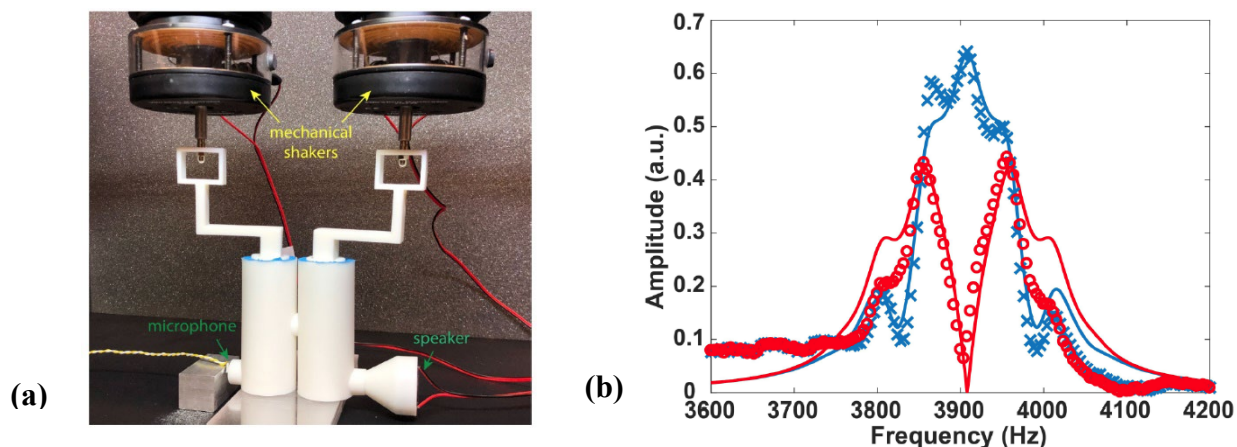


Figure 1.5 Non-reciprocal acoustic transmission in space-time modulated coupled resonators [84], (a) experimental setup of the coupled resonators under modulation; (b) experimental result of nonreciprocal transmission in forward and backward directions

In addition, non-reciprocal acoustic transmission can be obtained by modulating the acoustic medium [83-87]. In 2019, Shen *et al.* [83, 84] proposed and experimentally demonstrated a one-dimensional non-reciprocal waveguide with two cascaded Helmholtz resonators shown in Figure 1.4. Non-reciprocal acoustic transmission was demonstrated based on spatiotemporally modulating the size of the resonator cavities, as the energy of the incident wave converted from the fundamental mode to higher order modes. For space-time-modulated acoustic systems, Li *et al.* [86] generalized a transfer matrix method for solving wave propagation problems in time-varying acoustics. For the above-mentioned approaches [83-86], non-reciprocal sound

propagation was developed by modulating the effective density and bulk modulus of the acoustic medium. In 2020, Zhu *et al.* [87] proposed a tunable unidirectional acoustic amplifier based on a space-time modulated membrane system. Acoustic one-way amplification was demonstrated by temporally modulating the surface tensions of two membranes that are clamped in a waveguide.

1.2.5 Acoustic Non-Reciprocity with Nonlinearities and Active Controls

Other than the use of *CGEs* and spatiotemporal modulation, different configurations non-reciprocal devices have developed based on nonlinearities. In thermodynamics, Li *et al.* [88, 89] provided a thermal diode model consisted of one-dimensional nonlinear lattices and demonstrated its rectifier effect as heat flux is allowed in one direction. The first non-reciprocal acoustic device based on nonlinear effect was introduced by Liang *et al.* [90] in 2009. They presented a so-called acoustic diode formed by pairing a nonlinear acoustic medium with a superlattice. The superlattice is formed by laminating two linear media periodically and is capable of reflecting the fundamental frequency of incident wave. In such configuration, acoustic transmission from superlattice to nonlinear medium can be fully blocked at certain frequency range, while in an opposite direction some the acoustic energy is transmitted as it is converted into higher order harmonics. The proposed acoustic diode [90] was experimentally validated by Liang *et al.* [91] with stacking water and glass layers as superlattice and ultrasound contrast agent microbubbles as the nonlinear medium shown in Figure 1.6.

In 2011, Boechler *et al.* [92] developed and experimentally demonstrated an acoustic rectifier by using bifurcations and chaos to generate nonlinear vibration modes in a granular crystal. In 2014, Popa and Cummer [93] realized a broadband sound isolation by a non-reciprocal active metamaterial unit cell, which is consisted of a piezoelectric membrane coupled with a nonlinear electronic circuit and sandwiched between two asymmetric tunable acoustic cavities.

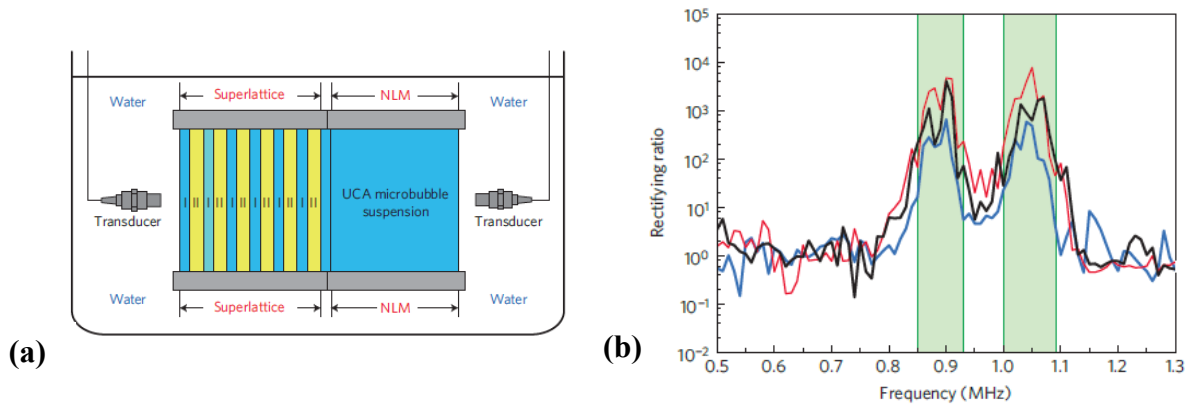


Figure 1.6 Acoustic diode with nonlinear medium [91], (a) experimental configuration of the acoustic diode structure; (b) corresponding rectifying ratios of the acoustic diode with different nonlinear medium samples

However, acoustic isolation realized by the above-proposed nonlinear acoustic devices [90-93] requires converting sound to non-fundamental frequencies. In 2015, Liu *et al.* [94] proposed an acoustic diode model composed by a weakly nonlinear phononic crystal with asymmetric linear structures at two ends. With a harmonic excitation at one end of the nonlinear medium, wave rectification was observed and the frequency of acoustic waves was preserved in forward direction with a relatively high power-transmission rate. In 2016, Gu *et al.* [95] developed a non-reciprocal acoustic

device constructed by an acoustic nonlinear material and a pair of acoustic gain and lossy materials while one-way transmission of acoustic waves with invariant frequency is allowed.

Recently, Baz [96-99] introduced three different classes of active non-reciprocal acoustic metamaterial (*ANAM*) which consist of acoustic cavities provided with active piezoelectric boundaries connected to switching resistor networks, programmable nonlinear virtual resistors, or anti-parallel diodes. Asymmetric acoustic transmission and energy flow distribution of these systems were demonstrated and the control of these nonlinear resistor configurations is carried out by using appropriate Lyapunov-based controllers.

1.3 Scope of this Dissertation

In this dissertation, four different configurations of *ANAM* are presented in order to control the flow of vibration and acoustic wave propagations in various applications. Based on the work of Baz [70], an experimental realization of an active *NGMM* single cell is presented. The proposed active *NGMM* system consists of a one-dimensional acoustic cavity provided with piezoelectric boundaries that act as sensors and actuators. Desirable non-reciprocal characteristics of tunable magnitude and phase shift were generated by gyroscopic control action synthesized by using a dummy *NGMM* cell to act as a physical dynamic controller. Furthermore, a simplified version of *NGMM* cell unit is presented using a virtual gyroscopic dynamic controller and

acoustic non-reciprocity is experimentally demonstrated through a single unit cell system.

In addition, two different active control methodologies are presented for achievement of non-reciprocal transmission in an acoustic waveguide. Relied on the theoretical work of Baz [100], non-reciprocal transmission in a one-dimensional duct is experimentally validated using an eigen-structure assignment control strategy, with a focus on adjusting the eigenvectors in a way that enables the spatial control and redistribution of the wave propagation along the duct. With the idea of spatiotemporal modulation, we proposed an *ANAM* consisted of a one-dimensional duct provided with an array of controllers integrated with space-time modulated feedback gains. Unsymmetrical wave propagation is numerically and experimentally demonstrated by shifting the phase of the harmonic time-varying gain between adjacent controllers in the acoustic duct.

This dissertation is organized in six chapters. Chapter 1 briefly summarizes the literature review on acoustic metamaterials, concept of acoustic reciprocity and recent research on acoustic non-reciprocity. Chapter 2 presents an extensive theoretical and experimental realization of a new class of active *NGMM* cell with active piezoelectric boundaries. These active boundaries are capable of virtually synthesizing the gyroscopic control via the connected physical dynamic controller. In Chapter 3, a simplified *NGMM* cell is developed and experimentally demonstrated using virtual gyroscopic dynamic controller. System gyroscopic effect is induced by the direct interaction of the active boundaries at the two ends of the cell via analog control circuits. In Chapter 4, we present a comprehensive experimental realization of a class

of active non-reciprocal metamaterial (*ANMM*) acoustic duct that is provided with a controller designed by simultaneous shaping of both the eigenvalues and eigenvectors of the closed-loop system. In Chapter 5, we present a class of *ANMM* comprised of the acoustic duct with periodically placed active control diaphragms. Asymmetric wave propagation of the system is theoretically and experimentally demonstrated by introducing a space-time modulation active feedback gain of the controllers. Chapter 6 summarizes the conclusions of theoretical and experimental results of the proposed *ANAM* configurations. In addition, a brief summary of the future extension of this dissertation is outlined and the major contributions of this dissertation are presented.

1.4 Summary

This chapter has presented a brief review of relevant literature of acoustic metamaterials and their various applications. The concept of acoustic reciprocity has been introduced and recent research on developing acoustic non-reciprocity has been discussed. Different active non-reciprocal acoustic metamaterials have been briefly introduced as the scope of this dissertation.

Chapter 2: Active Synthesis of a Gyroscopic-Nonreciprocal Acoustic Metamaterial Cell with a Dummy Cell

2.1 Introduction

In this chapter, experimental realization of a new class of active *Nonreciprocal Gyroscopic Meta-Material (NGMM)* is presented. In Section 1.2.3, different configurations of the non-reciprocal acoustic diodes have been introduced to break the reciprocity by relying in their operation on the use of circulators or gyroscopic elements (*CGEs*) [59-69]. However, it is essential to mention here that all the presently proposed non-reciprocal acoustic configurations with *CGEs* [59-69] are implemented only by the physical placement of circulators, gyroscopes, or gyrators in each cell or at the vertices of the different lattices making up the acoustic metamaterial structure. Such a physical implementation of these configurations renders their practical realization and operation rather difficult and unreliable particularly with the small size constraints of these metamaterials.

In view of these challenges, we provide a completely different approach whereby the metamaterial cells are implemented using acoustic cavities with actively programmable flexible boundaries. These boundaries are controlled, in such a manner, to introduce virtually synthesized gyroscopic coupling with the acoustic cavities that tends to break reciprocity of energy flow through *NGMM* cell without the inclusion of any moving parts as those needed by most of the currently available *CGE*.

This chapter presents accordingly a comprehensive investigation of the theory of operation, the manufacturing of an experimental prototype, and detailed monitoring

of the performance characteristics of the proposed *NGMM* under different operating conditions.

This chapter attempts mainly at experimentally demonstrating the effectiveness of the *NGMM* in controlling and tuning the directivity and distribution of acoustic energy flowing through the metamaterial both in the time and frequency domains. The paper complements and validates experimentally the work of Baz [70] which introduced theoretically the concept of *NGMM*, analyzed its stability, and established the bandgap characteristics of periodic *NGMM* for different periodicity configurations and gyroscopic gain.

2.2 Concept of the Nonreciprocal Gyroscopic Meta-Material (*NGMM*)

The concept of the *NGMM* can best be understood by considering the heart of the proposed metamaterial that is the “*dynamic controller*” shown in Figure 2.1, This controller is coupled with a simple meta-material cell model and relies in its operation on a constant control gain, k_g .

Consider a dynamic model of the dynamics of a meta-cell which is described by a state variable vector X and subjected to an excitation U with a control action F_x such that

Meta-cell:

$$[M_x]\ddot{X} + [D_x]\dot{X} + [K_x]X = U + F_x \quad (2.1)$$

where $[M_x]$, $[D_x]$, and $[K_x]$ are the equivalent mass, damping, and stiffness matrices of the cell, respectively. Also, the control action F_x is formulated such that it is given by

Control law:

$$F_x = k_g C \dot{Y} \quad (2.2)$$

where k_g is the controller gain, Y is the state variable vector of the dynamic controller, and C is the output measurement matrix. Eq. (2.2) indicates that the control action F_x is generated by a negative feedback of the derivative of the output of the dynamic controller ($C \dot{Y}$). Hence, the dynamics of the meta-cell are coupled with those of the dynamic controller which are described as follows:

Dynamic controller:

$$[M_y]\ddot{Y} + [D_y]\dot{Y} + [K_y]Y = F_y \quad (2.3)$$

where $[M_y]$, $[D_y]$, and $[K_y]$ are the equivalent mass, damping, and stiffness matrices of the dynamic controller, respectively.

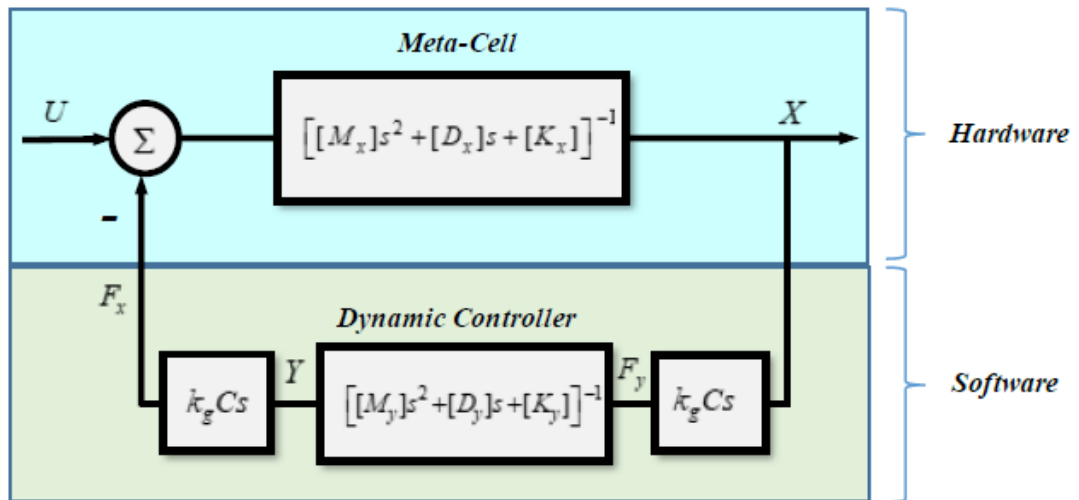


Figure 2.1 Structure of the "dynamic controller"

Also, the control action F_y of the dynamic controller is formulated such that is given by

$$F_y = -k_g C \dot{X} \quad (2.4)$$

Combing Eqs. (2.1) through (2.4) yields:

$$\begin{aligned} \begin{bmatrix} [M_x] & 0 \\ 0 & [M_y] \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{Y} \end{Bmatrix} + \begin{bmatrix} [D_x] & 0 \\ 0 & [D_y] \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix} \\ + \begin{bmatrix} 0 & -k_g C \\ k_g C & 0 \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \end{Bmatrix} + \begin{bmatrix} [K_x] & 0 \\ 0 & [K_y] \end{bmatrix} \begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{Bmatrix} U \\ 0 \end{Bmatrix} \end{aligned}$$

i.e.

$$[M]\{\ddot{Z}\} + [D]\{\dot{Z}\} + [G]\{\dot{Z}\} + [K]\{Z\} = F \quad (2.5)$$

where $[M]$, $[D]$, $[G]$, and $[K]$ are the overall mass, damping, gyroscopic, and stiffness matrices of the closed-loop meta-cell system, respectively. Also, $\{Z\} = \{X \ Y\}^T$ and $\{F\} = \{U \ 0\}^T$.

Eq. (2.5) indicates that positively feeding the output of the meta-cell back into the dynamic controller in order to generate a control action F_y which is used to generate the output of the dynamic controller CY . This, in turn, is utilized to generate the control action F_x which is then fed negatively into the meta-cell. With such positive and negative feedback of the control actions F_y and F_x , a gyroscopic coupling between the meta-cell and the dynamic controller is developed as can be seen in Eq. (2.5) and denoted by control matrix $[G]$. Such a combined coupling is summarized in Figure 2.1 with s denoting the Laplace complex number.

In summary, Eq. (2.5) represents the combined dynamic equations of the meta-cell and controller system. This combined system is structured such that the controller generates a closed-loop system with a built-in gyroscopic matrix to induce controlled non-reciprocal characteristics.

2.3 The Acoustic Metamaterial

Figure 2.2 displays a simple configuration of a unit cell of the non-reciprocal gyroscopic metamaterial (*NGMM*), which includes an acoustic cavity provided with piezoelectric discs mounted on brass substrates. The piezo-discs are connected to the controller.

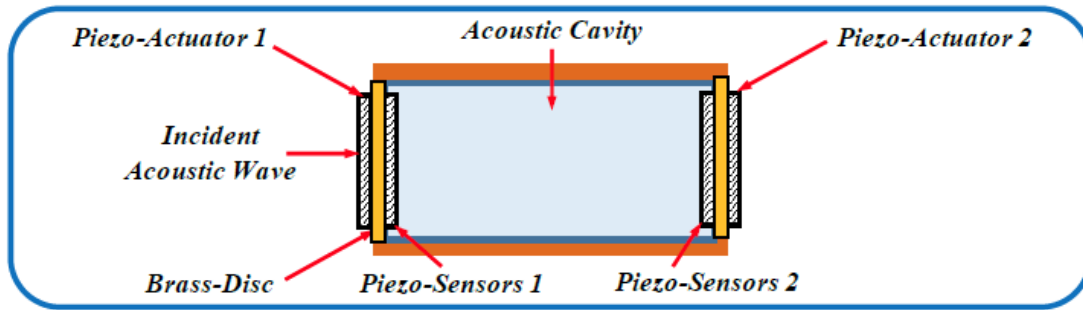


Figure 2.2 The acoustic metamaterial (*NGMM*) cell

A photograph of the active acoustic metamaterial cell, investigated in this work, is shown in Figure 2.3. Water, in a cylindrical pipe [Figure 2.3(a)], is confined from both ends by two piezoelectric (*PZT4* – lead, zirconium, titanate) bimorphs. The bimorph itself is composed on an intermediate brass disc with *PZT* layers deposited on both surfaces [Figure 2.3(b)].

2.3.1 The Acoustic Cavity

The acoustic cavity is manufactured from acrylic with an outer diameter of 0.041 m, inner diameter of 0.035 m, and overall length of 0.045 m.

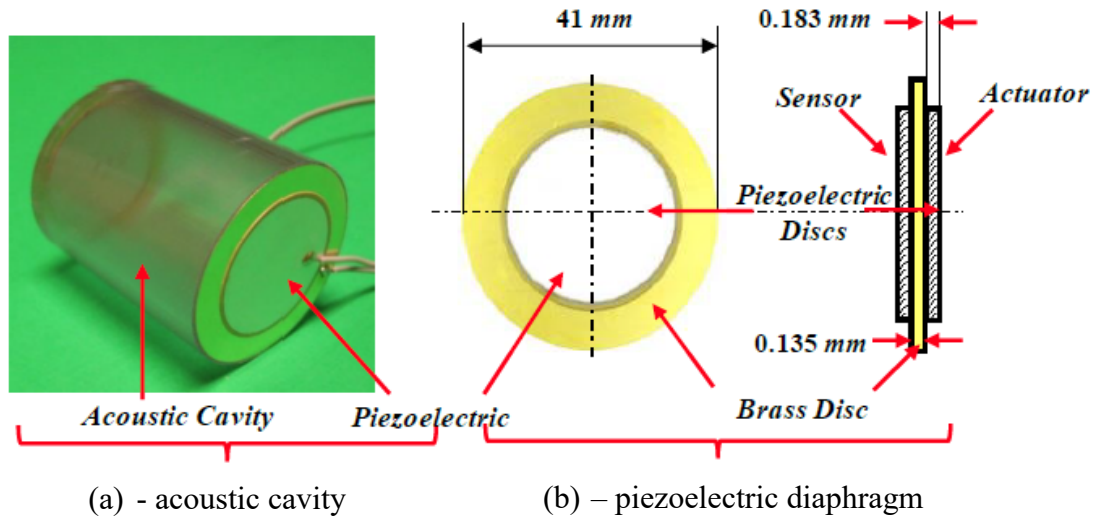


Figure 2.3 Photograph of the acoustic metamaterial cell

2.3.2 The Piezoelectric Diaphragm

The piezo-diaphragm (model AB4113B, Digi-Key Corporation) has an outer diameter of 41 mm, overall thickness of 0.5 mm of which 0.135 mm is brass intermediate disc thickness as shown in Figure 2.3(b).

The material properties of the piezoelectric bimorph components are listed in Table 2.1.

Table 2.1 Physical properties of the piezoelectric bimorph

Piezo Material	<i>PZT – 4</i> (lead, zirconium, titanate)
Frequency	<i>1.3 kHz</i>
Capacitance @ Frequency	<i>150 000 pF @ 1 kHz</i>
Voltage – Input, Max	<i>30 V p-p</i>
Impedance	<i>200 Ohm</i>

2.3.3 Modal Characteristics of the Meta-Cell

A finite element model is developed for the metamaterial cell using *ANSYS®*. Axisymmetric elements (*FLUID29*) are used to model the water inside the metamaterial cell. The piezoelectric bimorphs were modeled using (*PLANE223*) for the piezoelectric components and *PLANE183* for the brass intermediate disc. The finite element mesh of the developed meta-cell is illustrated in Figure 2.4.

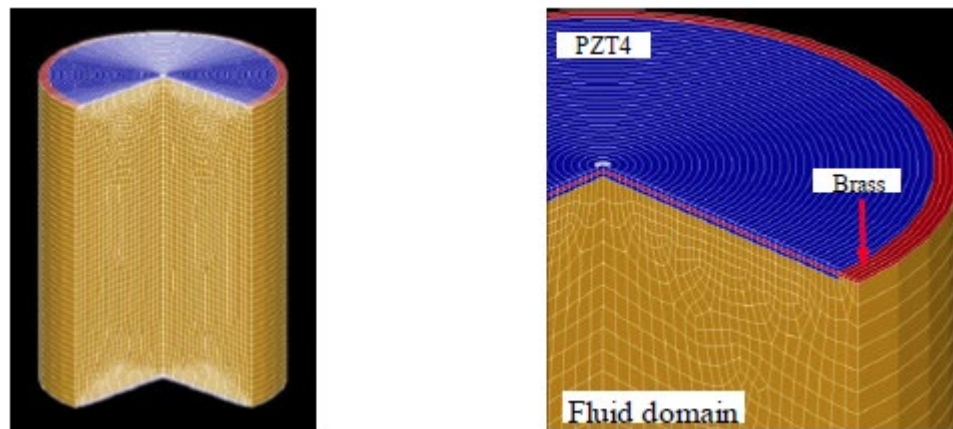


Figure 2.4 The finite element mesh of the engineered metamaterial *NGMM* cell

The first eight modes of vibration of the meta-cell are listed in Table 2.2. The corresponding mode shapes of the first two modes are displayed in Figure 2.5. These modes are primarily dominated by the piezoelectric disc oscillations.

Table 2.2 Natural frequencies of the metamaterial *NGMM* cell

Mode	1	2	3	4	5	6	7	8
Frequency (Hz)	556	1078	1082	1582	1817	2011	2291	2294

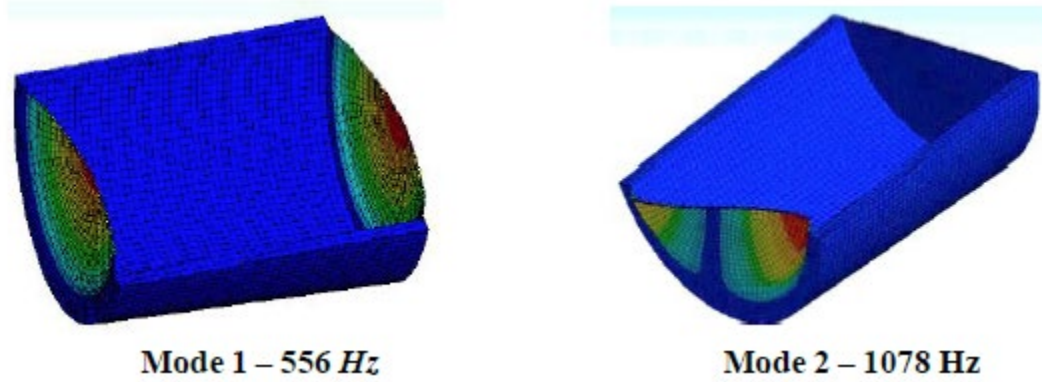


Figure 2.5 The shapes of the first two modes of the metamaterial *NGMM* cell

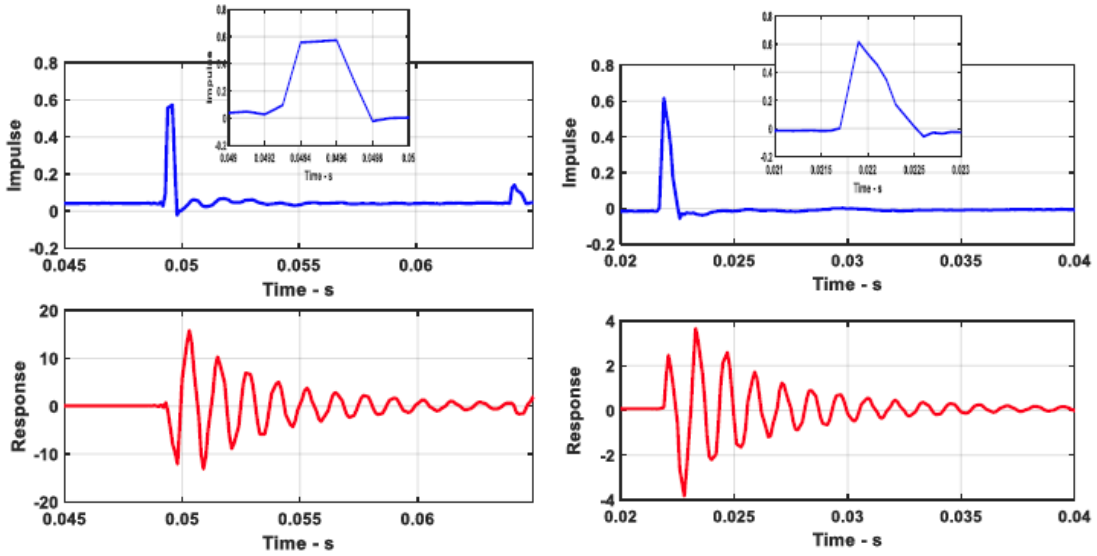
2.3.4 Identification of the Model of the Metamaterial Cell

An impulse hammer is employed to apply impulse input to the piezo-actuator 1 of the meta-cell. The impulse signal is captured using an analyzer. The outputs of the piezo-sensors 1 and 2 of the meta-cell, as shown in Figure 2.2, are monitored. The combinations of the impulse forces and the corresponding sensors outputs for the two

cases (input at actuator 1/output at sensor 1 and input at actuator 1/output at sensor 2) are shown in Figures 2.6(a) and 2.6(b), respectively.

These two sets of input/output sets are used to identify the transfer functions T_{11} and T_{12} between the actuator 1 and sensors 1 and 2, respectively. Such an identification process aims at developing a 4-port model of the meta-cell in the form:

$$\begin{Bmatrix} X_{input} \\ X_{output} \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} F_{input} \\ F_{output} \end{Bmatrix} \quad (2.6)$$



(a) – Input at 1 and output at 1

(b) – Input at 1 and output at 2

Figure 2.6 The input/output signals for identifying of the meta-cell dynamic models

In Eq. (2.6), note that $T_{22} = T_{11}$ and $T_{21} = T_{12}$ because of the symmetrical structure of the meta-cell. Such a 4-port model enables the direct implementation of the proposed active synthesis of a gyroscopic-nonreciprocal controller. Furthermore, the 4-port form allows easily the predictions of the response of the meta-cell during forward (F_{input} , X_{output}) and backward (F_{output} , X_{input}) wave propagation.

Using this input/output information displayed in Figures 2.6(a) and 2.6(b), the meta-cell 4-port model is identified using *MATLAB* Identification Toolbox (*ident*) assuming the transfer functions T_{ij} has one zero and two poles. In this manner, a low order model is identified to simplify the implementation of the controller. The locations of the poles and zeros of an assumed model are extracted by minimizing the sum of the square of the errors between the experimental measurements and the output of the identified system as determined by using the measured input as outlined by Ljung [101,102]. The work of Ljung [101,102] provides the theoretical bases for the identification process which is adopted in the *MATLAB* toolbox “*ident*”.

Figures 2.7(a) and 2.7(b) display comparisons between the experimental responses of the meta-cell and the identified response. It is evident that the assumed simple model captures adequately the response of the cell in the time domain.

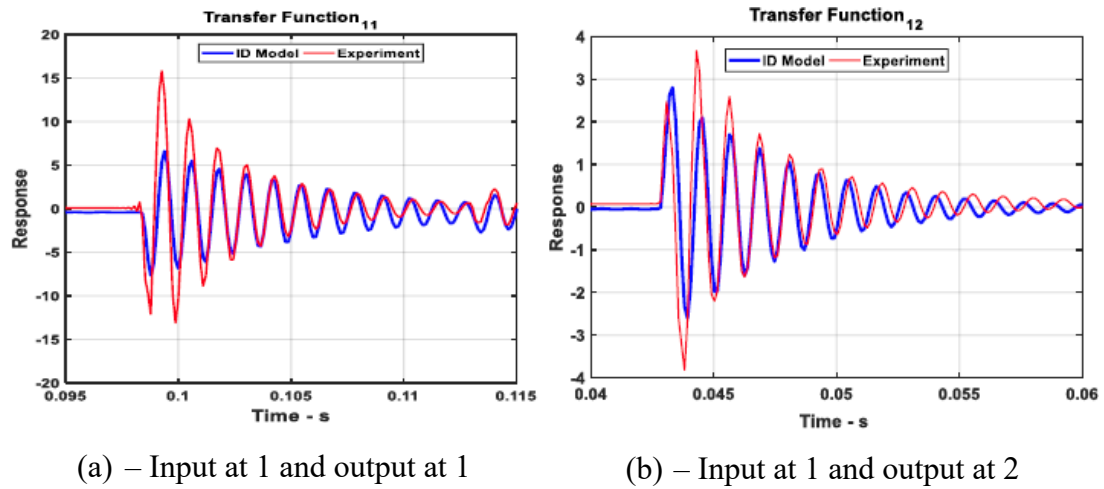


Figure 2.7 Comparison between the experimental and identified response of the meta-cell in the time domain

Figure 2.8 display another comparison between the experimental and identified response of the meta-cell in the frequency domain. Not that the experimental frequency content is obtained by applying the fast Fourier transform (*FFT*) on the time domain measurements displayed in Figure 2.6. Furthermore, the frequency response of the identified model is computed by using the *bode* plot.

The results displayed in Figure 2.8 suggest a close agreement between the experimental measurements and the identified predictions. The figure indicates also that the identified models predict accurately only the first mode of the meta-cell because these models are low-order models.

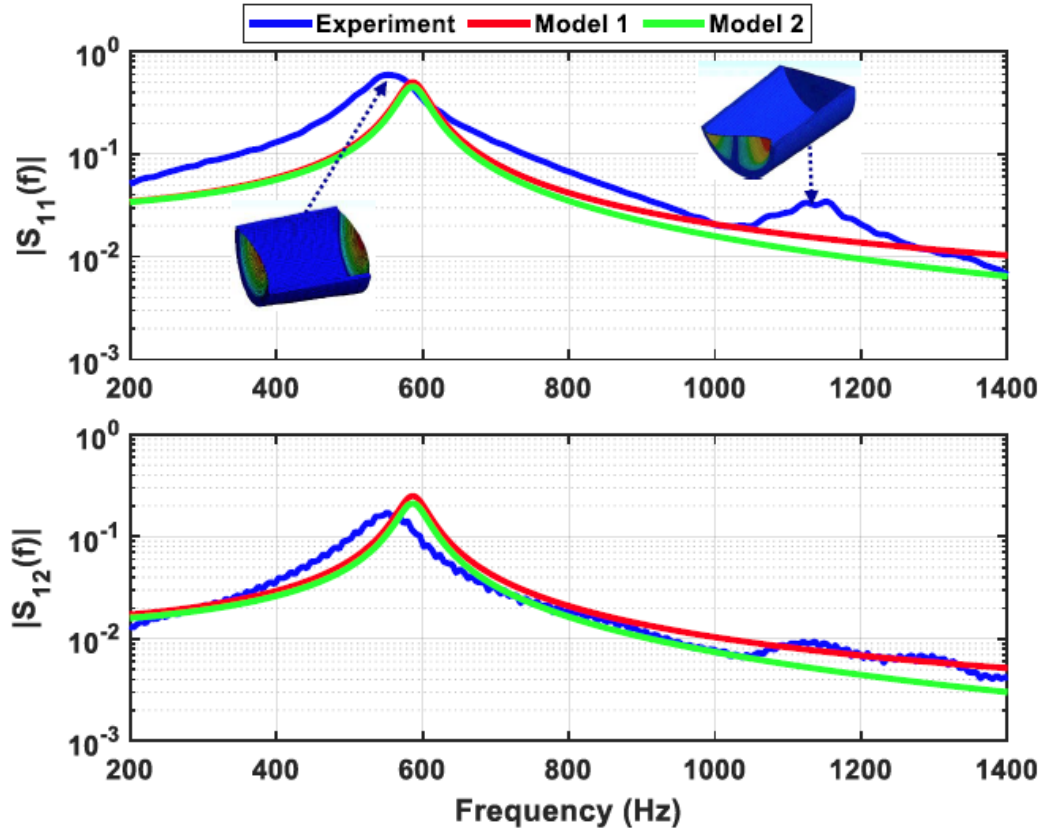


Figure 2.8 Comparison between the experimental and identified response of the meta-cell in the frequency domain

Table 3 lists a comparison between the experimental and identified modes of the meta-cell.

Table 2.3 Comparison between the experimental and identified response of the meta-cell in the frequency domain

<i>Meta-cell</i>	<i>Mode 1 (Hz)</i>	<i>Damping ratio (ζ) of mode 1</i>	<i>Mode 2 (Hz)</i>	<i>Damping ratio (ζ) of mode 2</i>
Experimental	558	0.0362	1121	0.0182
Identified	584	0.0336	-----	-----
FEM	556	-----	1078	-----

In Figure 2.8, two identified models are displayed. In the first model (Model 1):

$$T_{11} = \frac{a_1 s^2 + a_2 s + 1.37 \times 10^8}{s^2 + 246.8s + 1.36 \times 10^7} \quad (2.7)$$

and

$$T_{12} = 0.537 \frac{b_1 s^2 + b_2 s + 1.278 \times 10^8}{s^2 + 334.8s + 1.42 \times 10^7}$$

where a_1 , a_2 , b_1 , and b_2 are 4 to 7 orders of magnitudes smaller than the numerators constants (1.37×10^8 and 1.278×10^8).

In the second model (Model 2), the constants a_1 , a_2 , b_1 , and b_2 are neglected resulting in the following transfer functions:

$$T_{11} = \frac{a}{s^2 + bs + c} \quad \text{and} \quad T_{12} = d \frac{a}{s^2 + bs + c} \quad (2.8)$$

where $a = 1.37 \times 10^8$, $b = 334.8$, $c = 1.42 \times 10^7$, and $d = 0.537$.

In this model, the zeros of T_{11} and T_{12} are eliminated and the poles pf both are set to be equal. Such approximations are intended to simplify the implementation of the controller without appreciable loss of accuracy. This intention is clear in the comparisons established in Figure 2.8.

The details of integration of these equations [Eqs. (2.7) and (2.8)] with the equations of the corresponding dynamic controller to form the closed-loop model of the active meta-cell are presented in the Appendix A for both forward and backward propagations.

2.4 Experimental Characteristics of the Acoustic Metamaterial Coupled with an Active Gyroscopic Nonreciprocal Controller

2.4.1 Preferred Configuration of the Controller

The meta-cell described and characterized in Section 2.3 is integrated with an active gyroscopic-nonreciprocal controller. Figure 2.9 shows a preferred configuration for the implementation of the block diagram of the controller displayed in Figure 2.1. In the preferred configuration, the dynamic controller is replaced by a “*dummy meta-cell*” which is intended to physically replicate the dynamics of the main meta-cell. Such a replacement aims at avoiding the use of a digital equivalent in order to enhance the implementation speed of the controller which is also implemented in an analog manner. The inputs and outputs to the *dummy cell* (or the dynamic controller) are manipulated by using two analog circuits *A* and *B*. In circuit *A*, an operation amplifier-based analog differentiator is used to obtain $\dot{X}$. Then, the output is amplified by an analog amplifier which is placed in series with the differentiator to obtain $k_g \dot{X}$. Similar operations are carried out by the analog circuit *B* which operates on the output Y of the “*dummy cell*” to generate an output $k_g \dot{Y}$. The sign of this output is inverted using an operation

amplifier-based inverter to produce $-k_g \dot{Y}$, which is necessary to generate the gyroscopic effect.

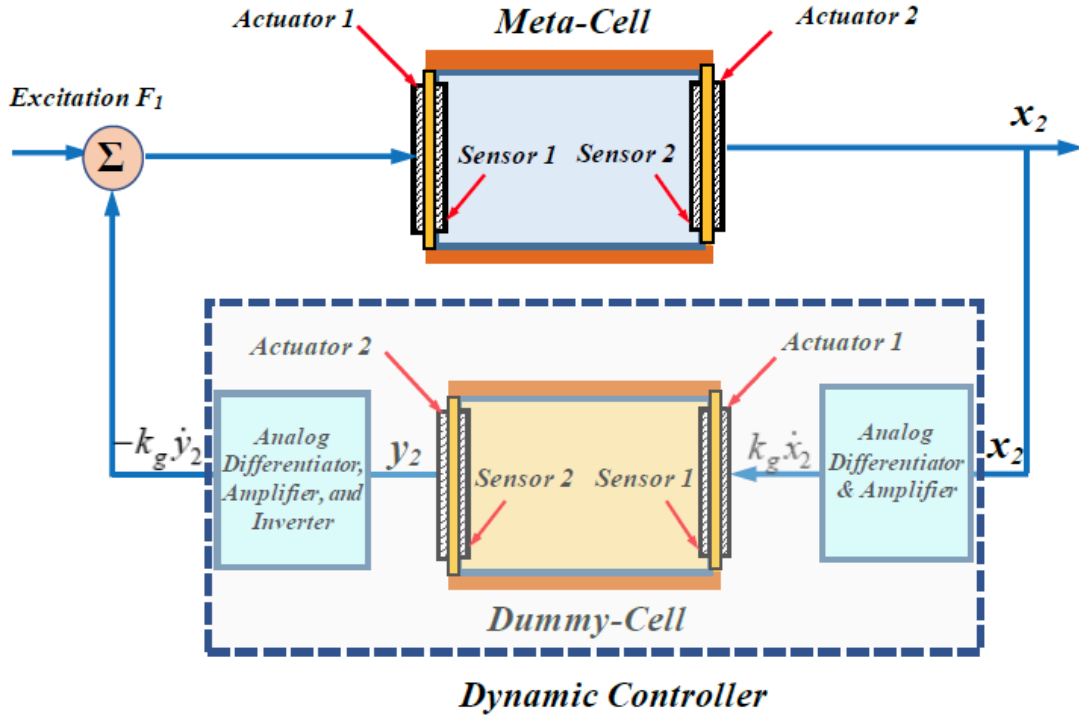
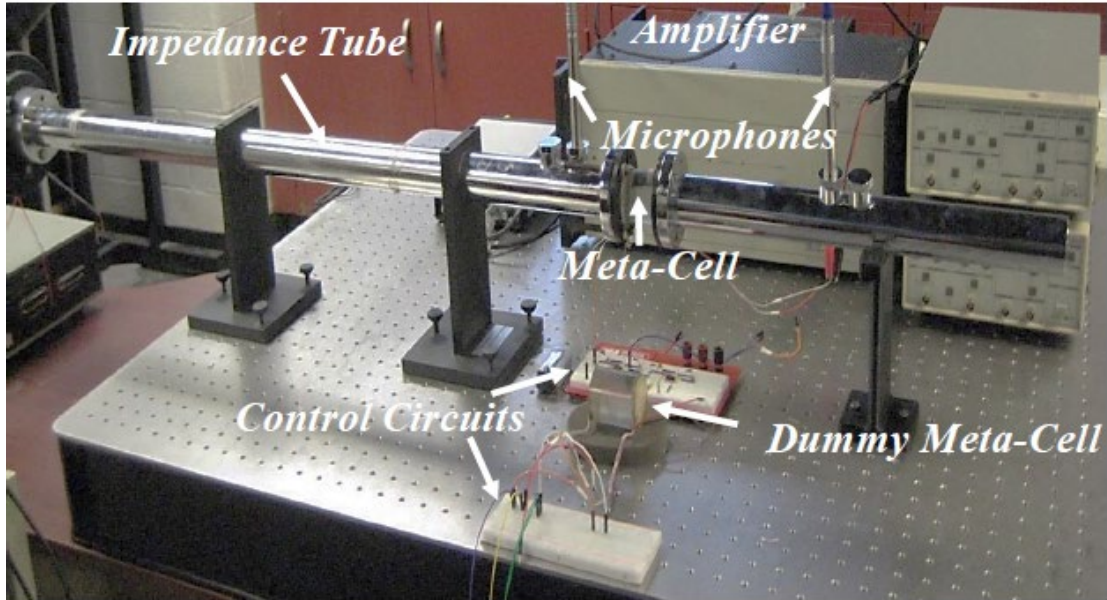
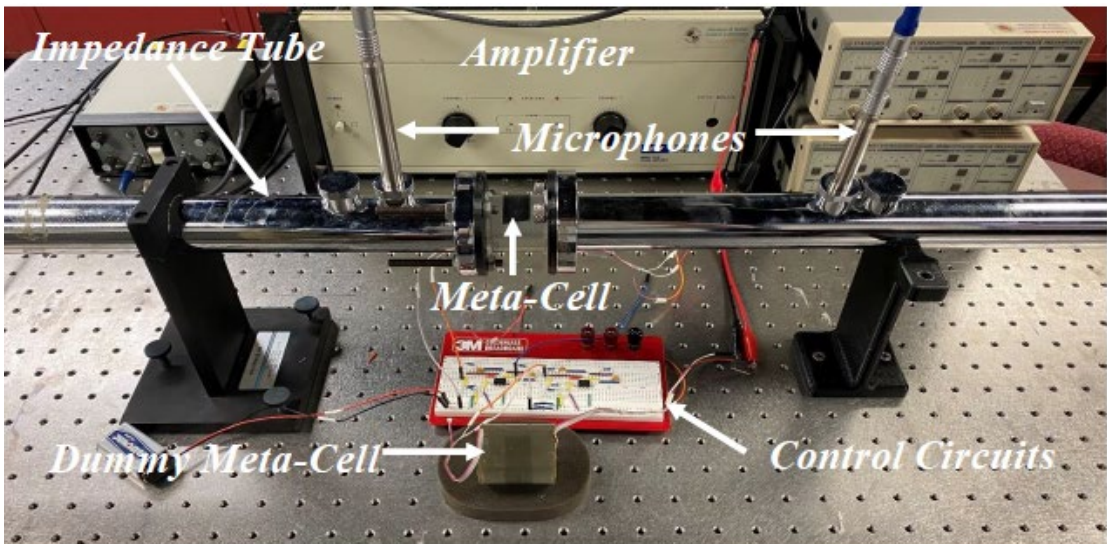


Figure 2.9 Analog control of the meta-cell using an active gyroscopic nonreciprocal controller

Figures. 2.10(a) and 2.10(b) display photographs of the experimental setup including the meta-cell, dummy cell, and the analog circuits with the meta-cell placed in the ACUPRO impedance and transmission tube.



(a) – Overall view



(b) – zoom in view

Figure 2.10 Photographs of the analog gyroscopic nonreciprocal controller

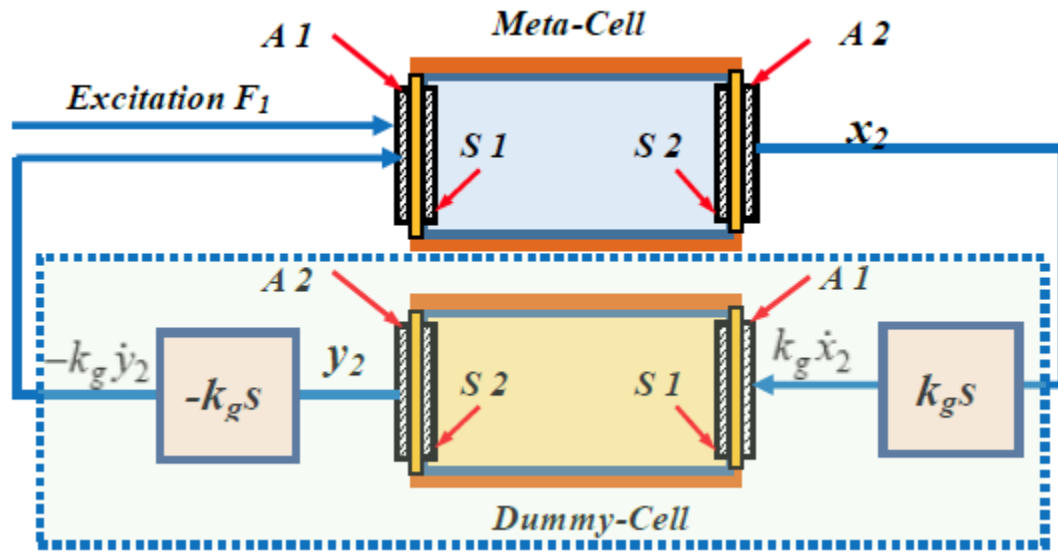
2.4.2 Theoretical Performance of the Controller

Figure 2.11(a) displays the controller configuration during forward wave propagation. Also, Figure 2.11(b) shows the theoretical performance characteristics in the frequency domain for both magnitude and phase shift. These predictions are based on the mathematical structure outlined in the Appendix A.

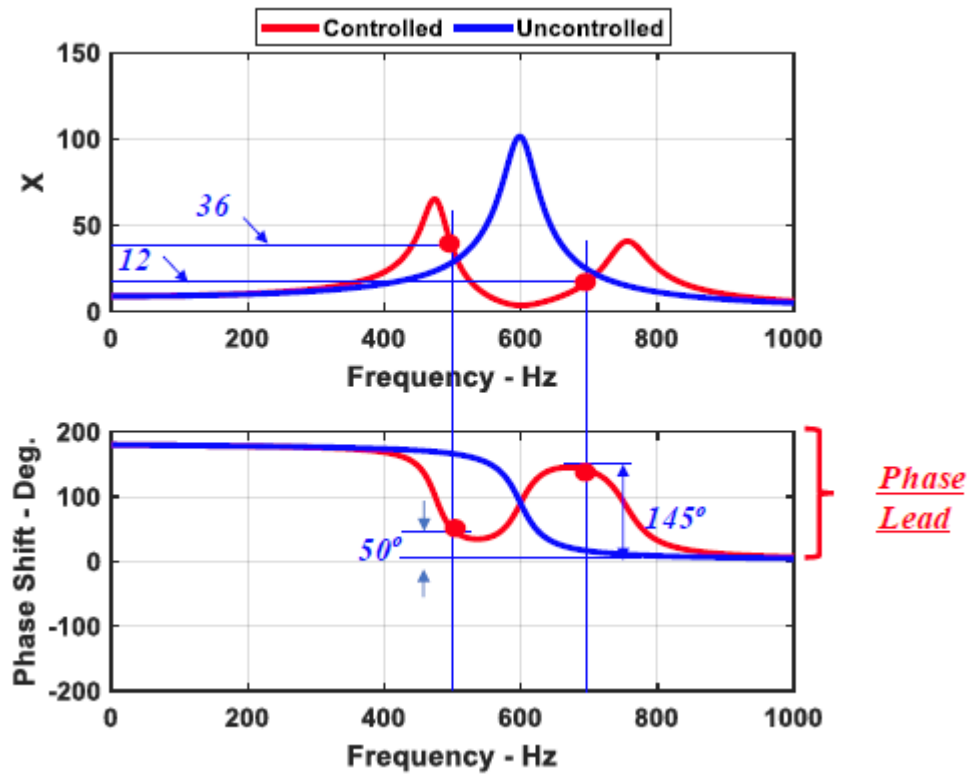
Figure 2.11(b) presents the most important features that contribute to the unique behavior of the virtual gyroscopic controller. In the figure, the uncontrolled meta-cell is distinguished by a single dominant frequency at 600 *Hz* as implied by the structure of the simplified 4-port model.

Once the meta-cell is augmented by the dynamic controller, the frequency response of the closed-loop system exhibits two-dominant frequencies characteristics that in effect constitute side band frequencies of the open-loop frequency. For a gyroscopic gain $k_g = 1,750$, the side band frequencies occur at 465 and 752 *Hz*.

More important than the frequency split phenomenon is the associated phase shift characteristics. The displayed phase shift indicates that a phase lag is encountered relative to uncontrolled case. The phase lead between the input excitation and uncontrolled response are 50° and 145° for excitations of 500 and 675 *Hz*, respectively. This phase lead is attributed to the gyroscopic control action.



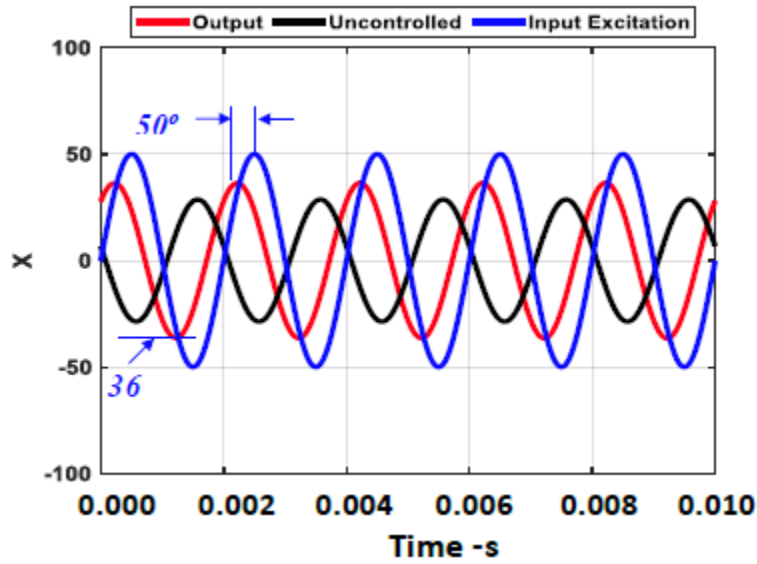
(a) – Forward configuration



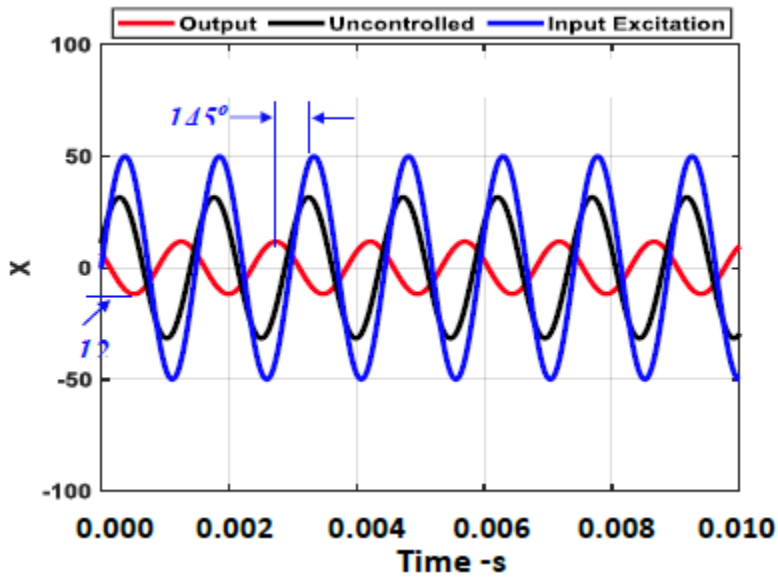
(b) – Frequency response

Figure 2.11 Forward configuration and frequency response of the gyroscopic controller

It is important to note that the frequency domain results displayed in Figure 2.11(b) are confirmed by and conform with the time domain results displayed in Figures 2.12(a) and 2.12(b).



(a) – 500 Hz



(b) – 675 Hz

Figure 2.12 Time responses of the cell with the gyroscopic controller during forward propagation

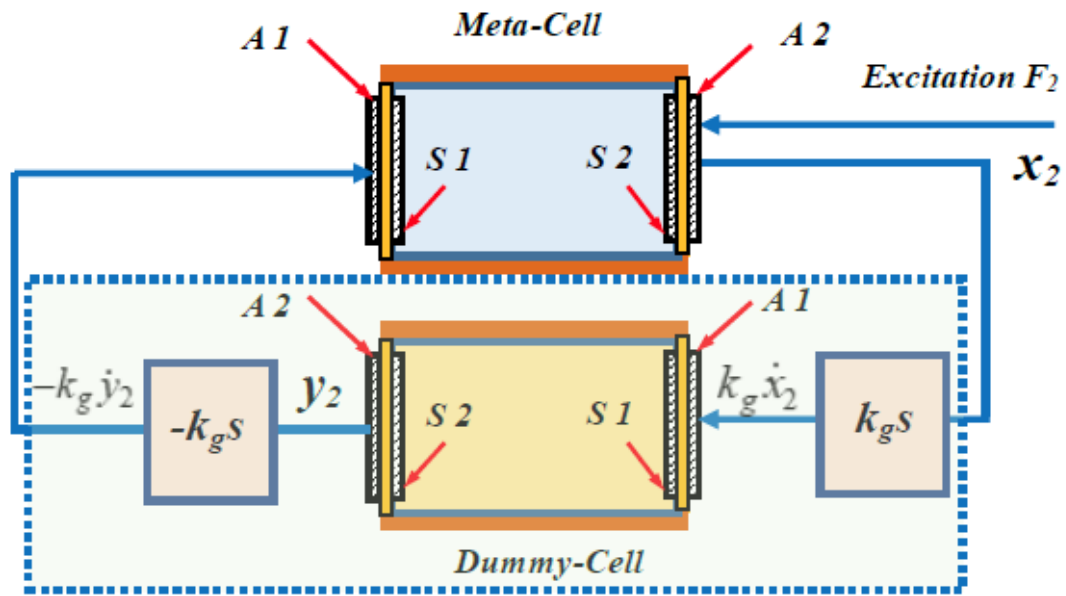
Figure 2.13(a) and 2.13(b) display the controller configuration and the frequency response of the meta-cell during the backward propagation, respectively.

The observed features of frequency split and phase shift are similar to those experienced during the forward propagation. The predicted frequency split, for a gyroscopic gain $k_g = 1,750$, results in sideband frequencies occurring at 452 and 752 *Hz*.

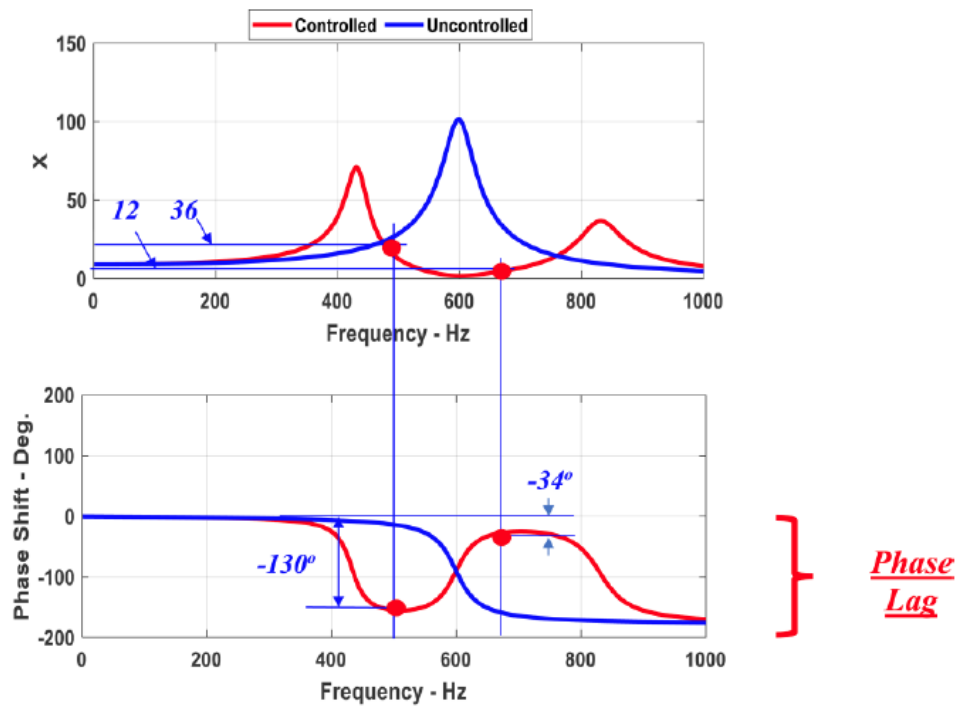
However, in this case, the phase shift experienced is phase lag instead of the phase lead experienced with forward propagation. The predicted phase shifts are -130° and -34° for excitations of 500 and 675 *Hz*, respectively.

It is important to note that the frequency domain results displayed in Figure 2.13(b) are confirmed by and conform with the time domain results displayed in Figures 2.14(a) and 2.14(b).

Figures 2.14(a) and 2.14(b) display the phase shifts resulting during backward wave propagation for frequencies of 500 and 675 *Hz*, respectively. The obtained results are in the agreement with results shown in Figure 2.13.

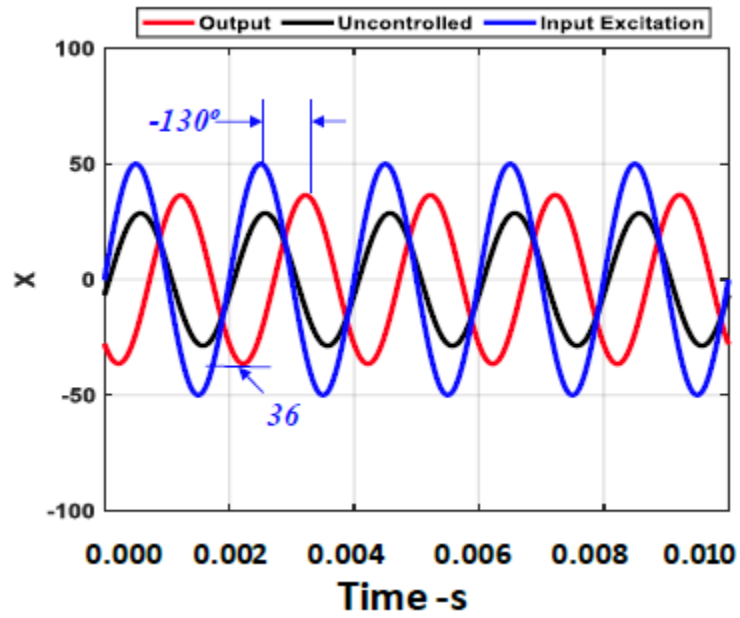


(a) – Backward configuration

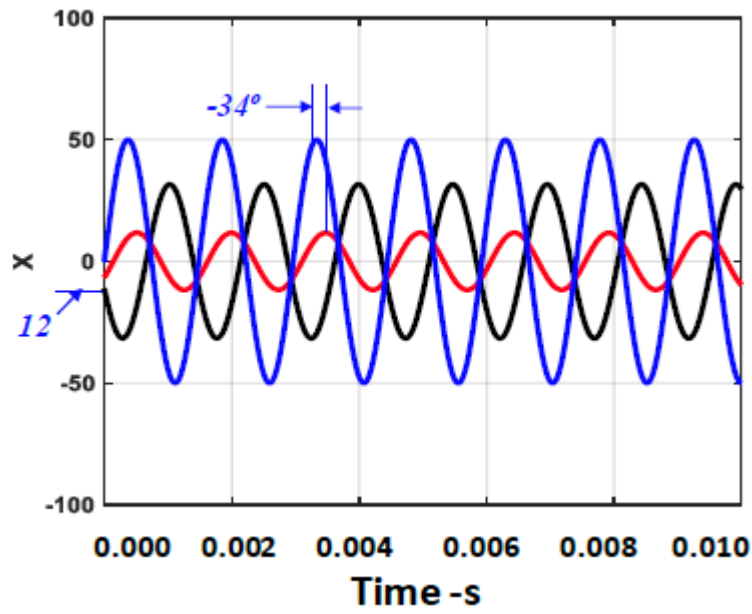


(b) – Frequency response

Figure 2.13 Backward configuration and frequency response of the gyroscopic controller



(a) – 500 Hz



(b) – 675 Hz

Figure 2.14 Time responses of the cell with the gyroscopic controller during backward propagation

Figure 2.15 shows a summary of the effect of changing the gyroscopic gain on the frequency-split phenomenon of the gyroscopic controller. The figure displays the common and well-known characteristics of gyroscopic systems which is the “Campbell diagram”.

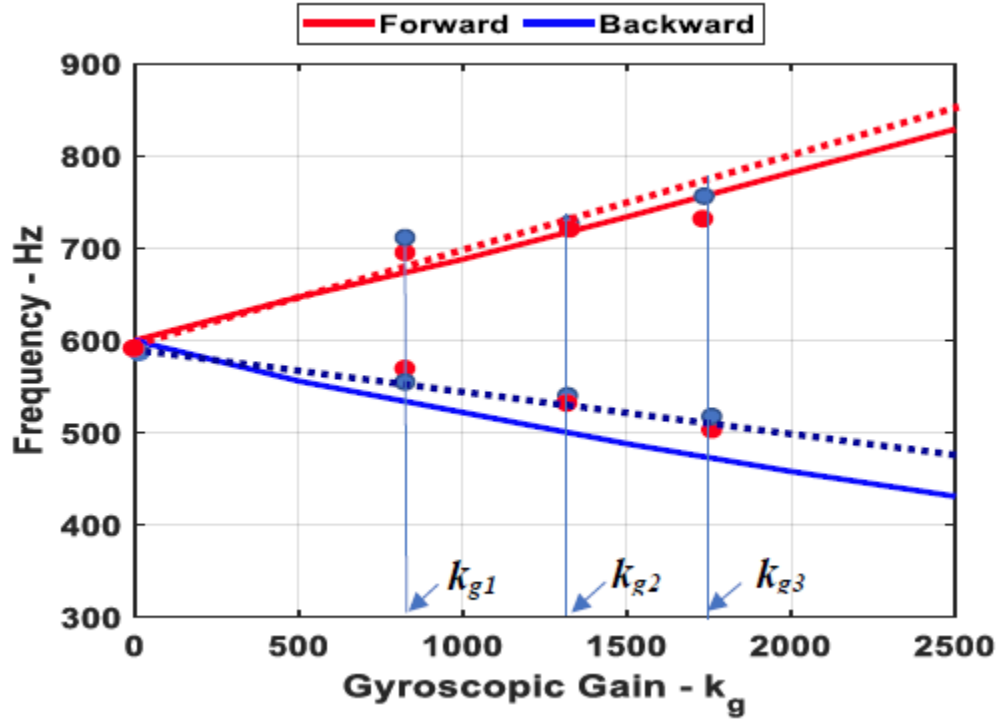


Figure 2.15 Effect of the gyroscopic gain on the frequency split phenomenon (red circles: forward experiment; blue circles: backward experiment)

The effect of the gyroscopic gain on the closed-loop damping ratio is negligible, and it remains nearly between 0.044 and 0.042 for all considered gains. Such a negligible effect is attributed to the additional damping matrix D_g that is explained in the Appendix A.

Figure 2.15 displays along with the model prediction of the frequency splits, drawn in solid lines, comparisons with corresponding experimental characteristics. The

experimental results are displayed in dashed lines. These results are presented in details in Section 2.4.3.

2.4.3 Experimental Performance of the Controller

Figure 2.16 displays the experimental frequency response of the controller for forward and backward propagations along with comparisons with the theoretical predictions as displayed in Figure 2.11 for forward propagation and in Figure 2.13 for backward propagation. It is evident that there is a close agreement between theoretical predictions and experimental results for both the magnitude and phase shift.

Figures 2.17(a) and 2.17(b) display the experimental time response of the controller for forward and backward propagations at excitation frequencies of 500 and 675 *Hz*, respectively. The displayed results are a close agreement with the theoretical predictions, for both the magnitude and phase shift, which are displayed in Figures 2.12 and 2.14 respectively.

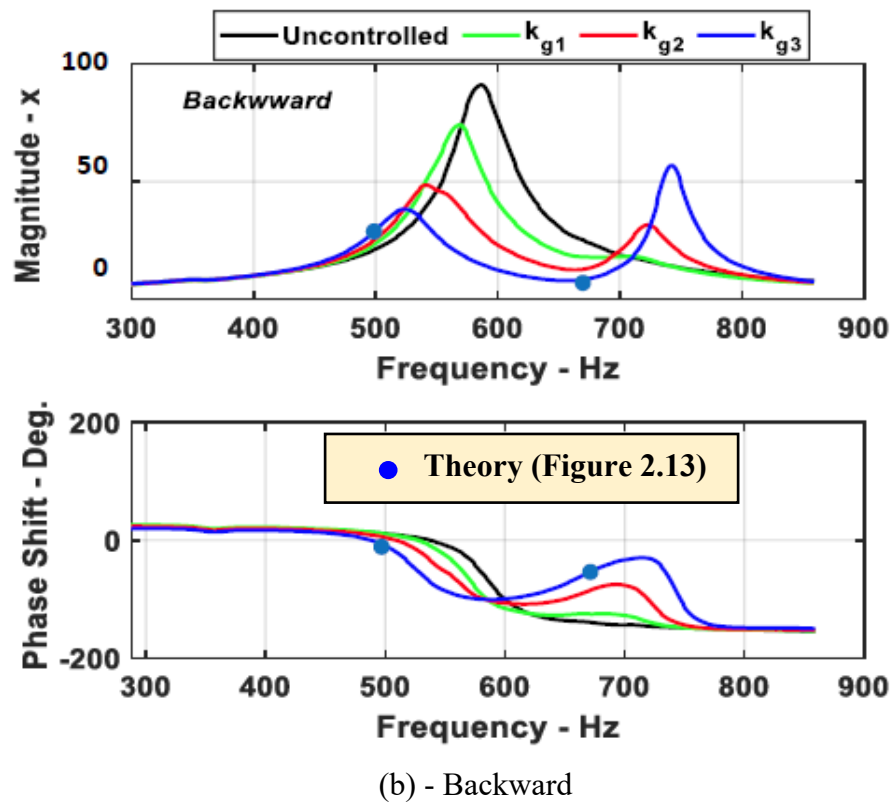
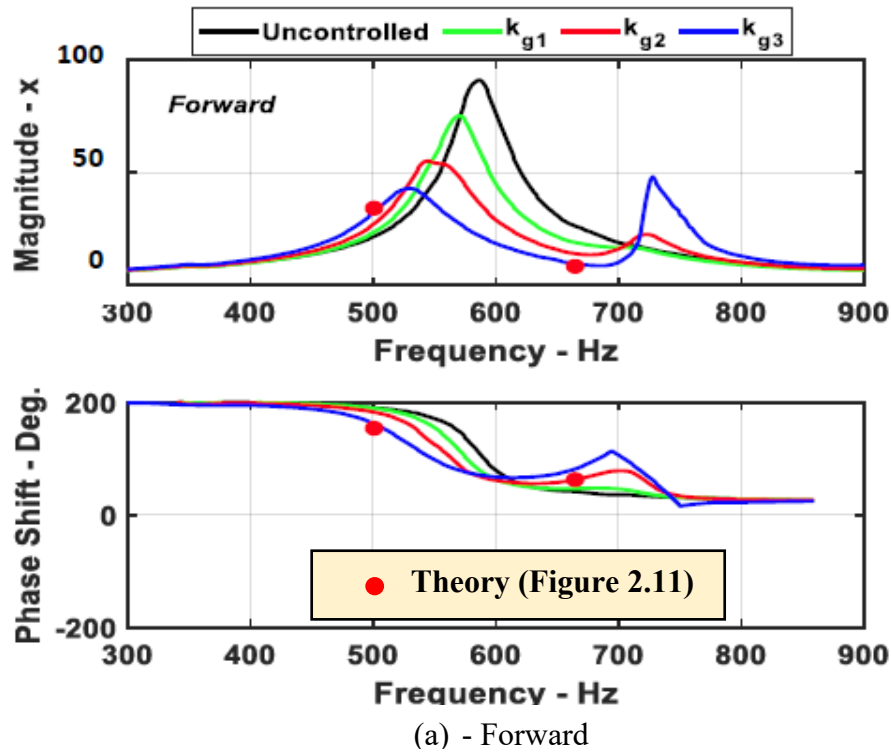
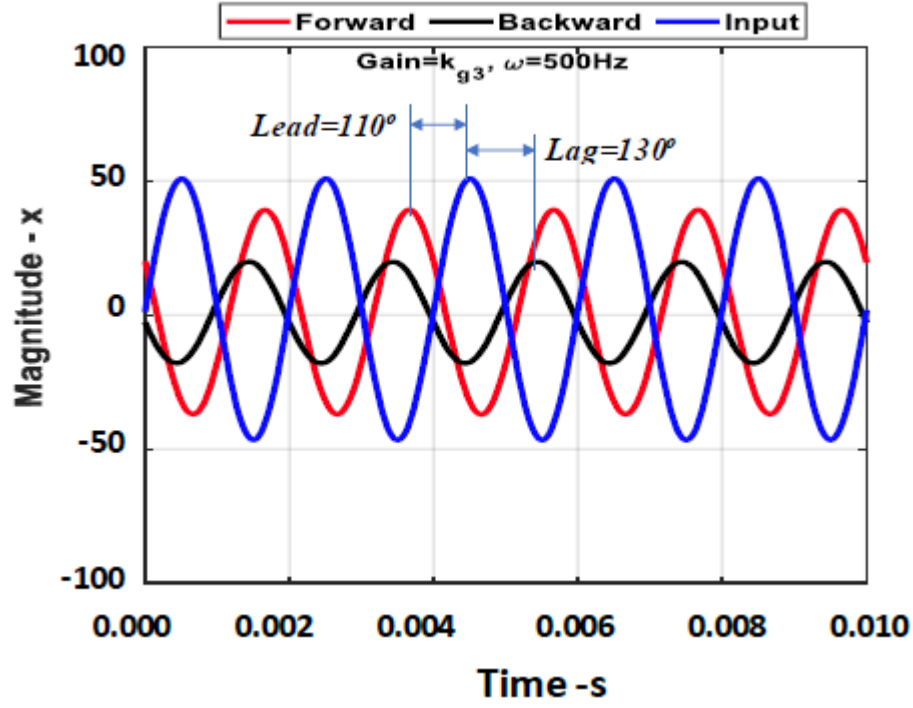
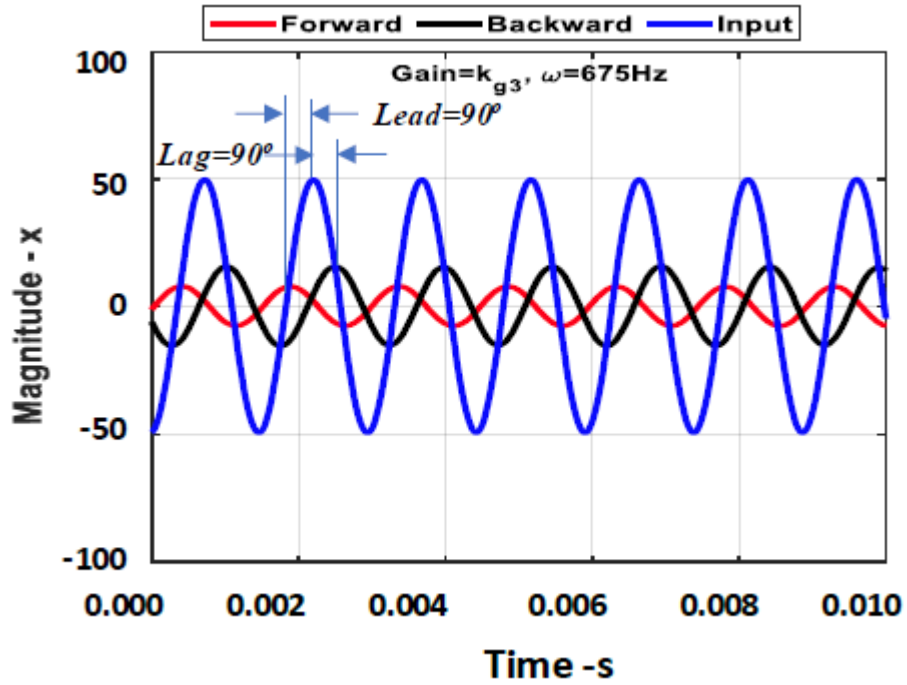


Figure 2.16 Frequency responses of the controller meta-cell during forward and backward propagation



(a) – 500 Hz



(b) – 675 Hz

Figure 2.17 Time responses of the controlled meta-cell during forward and backward propagation for excitations of (a) 500 Hz and (b) 675 Hz

2.5 Summary

This chapter has presented an extensive theoretical and experimental realization of a new class of active *Nonreciprocal Gyroscopic Meta-Material (NGMM)*. The proposed active *NGMM* system consists of an acoustic cavity provided with piezoelectric boundaries that serve as sensors and actuators. These active boundaries are integrated with active control capabilities that virtually synthesize a gyroscopic control action in order to generate desirable non-reciprocal characteristics with tunable magnitude and phase shift.

The dynamics of a prototype of the *NGMM* cell are identified experimentally using impulse excitations and the *MATLAB “ident”* toolbox to extract the cell dynamic equations. The theoretical characteristics of the identified cell when integrated with the virtual gyroscopic controller are predicted for various control gains for both forward and backward propagations.

The theoretical predictions are validated experimentally using a dummy *NGMM* cell to act as a physical dynamic controller. Such a preferred arrangement is coupled with analog controllers in order to enable fast execution of the controller and, in turn, enhance its frequency bandwidth capabilities. The time and frequency response characteristics of the *NGMM* cell are measured for different control gain and the behavior is evaluated for both forward and backward propagations.

It is demonstrated that the gyroscopic controller results in splitting the modes of oscillations of the open-loop *NGMM* into sideband frequencies with the extent of separation is influenced by the gyroscopic gain. Collectively, the sidebands of the closed-loop *NGMM* form the classical Campbell diagram characteristics. Furthermore,

it has been demonstrated the forward wave propagation of the *NGMM* cell exhibits phase lead characteristics whereas backward propagation results in phase lag behavior. It is important to note that extents of the phase lead and lag are influenced by the gyroscopic gain, the inherent damping of the cell, and the excitation frequency of the propagation waves. For effective achievement of non-reciprocal behavior, it is essential to consider the collective effects of all these factors.

This chapter has further demonstrated that the obtained experimental results are found to be in close agreement with the theoretical predictions.

Furthermore, the presented concept, controller design, and implementation of the *NGMM* laydown the foundation for designing effective gyroscopic/non-reciprocal metamaterial cells.

Also, it is envisioned that the presented work can be extended to various critical structures to achieve realistic configurations in a simple and programmable manner. Note that the frequency and time spectra studied in this paper are comparable to those covered in the literature of tunable acoustic metamaterials [103, 104].

In next chapter, the present work is extended to *NGMM* cells with a simplified version of the virtual gyroscopic dynamic controller.

Chapter 3: Active Synthesis of a Gyroscopic-Nonreciprocal Acoustic Metamaterial Cell without a Dummy Cell

3.1 Introduction

Acoustic metamaterials with nonreciprocal characteristics have been at the focus of various investigations for the past decade as reported, for example, by Fleury *et al.* [4] and Nassar *et al.* [5]. It has been demonstrated, in these studies, that breaking the reciprocity can be achieved by relying on the use of passive nonlinearities [90, 93-95], spatio-temporal modulation [76-77,80,105], active control systems [96-98,100,106-109], and circulators or gyroscopic elements (*CGEs*) [59- 60,63-66,70].

In this chapter, the emphasis is placed on the novel use of virtual gyroscopic forces as a viable means for introducing the non-reciprocity in a manner that distinguishes itself from other previous studies utilizing circulators or gyroscopic elements. In the previous studies, the implementation is achieved only through physical placement of circulators, gyrators or gyroscopes at different locations inside the cells of the acoustic metamaterial structure. Such a physical implementation approaches makes their realization and operation difficult, unreliable, and impractical.

In view of these shortcomings, this paper has been intended to provide a radically different approach whereby the metamaterial cells are controlled by introducing virtually synthesized gyroscopic control forces in order to break the reciprocity without the need for any moving parts in contrast to most of the currently available *CGE*.

Therefore, the paper introduces the details of the design, implementation, and testing of the performance characteristics of the proposed *NGMM* under different operating conditions.

The present study extends the work of Baz [70] which introduced theoretically the concept of the *NGMM*, established the bandgap characteristics and analyzed its stability. It further simplifies and extends the work of Raval *et al.* [110] which experimentally tested the *NGMM* concept using a dummy *NGMM* cell that acted as a physical dynamic controller.

3.2 Concept of the Nonreciprocal Gyroscopic Meta-Material (*NGMM*)

The concept of the new *NGMM* configuration can best be understood by considering the different configurations of the proposed metamaterial shown in Fig. 3.1.

The proposed active acoustic metamaterial (*AAMM*) cell consists of an acoustic cavity controlled by two bimorph piezoelectric diaphragms bonded to the opposite ends of the cavity. Each bimorph has a piezo-layer that acts as a sensor and another layer acts as an actuator as shown in Fig. 3.1(a). In this manner, the meta-cell can be represented by a 4-port model that relates the inputs (F_x and F_y) to the outputs (x and y) in terms of the transfer function *TF* of the cell as shown in Figs. 3.1(b) and 3.1(c).

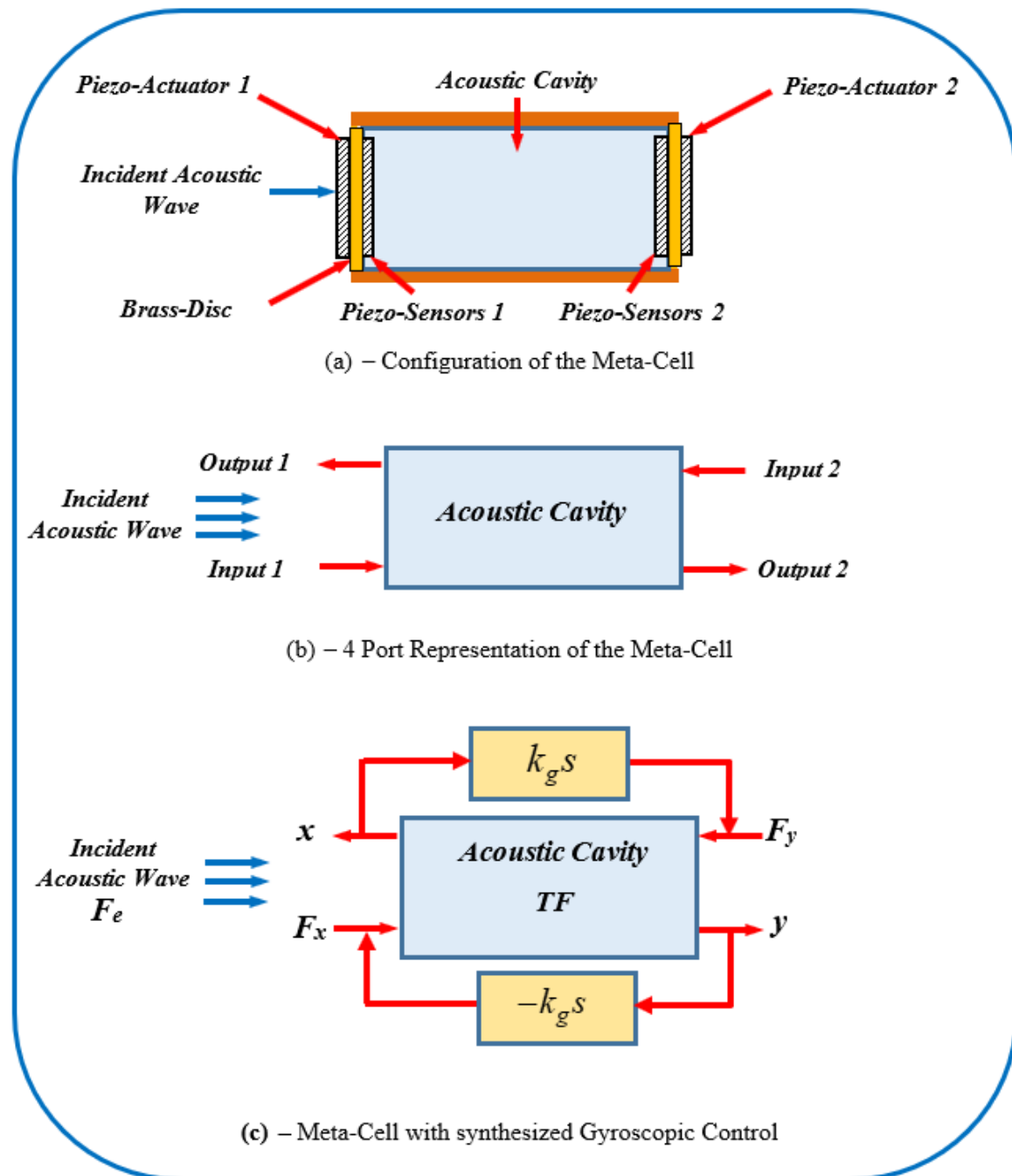


Figure 3.1 The meta-cell with synthesized gyroscopic control

Mathematically, a single-mode model can be written as follows:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = a \begin{bmatrix} 1 & -d \\ -d & 1 \end{bmatrix} \begin{Bmatrix} F_x + eF_e \\ F_y + fF_e \end{Bmatrix} \quad (3.1)$$

or

$$[M]\ddot{Z} + [D]\dot{Z} + [K]Z = BF$$

where

$$[M] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}, [K] = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, [B] = a \begin{bmatrix} 1 & -d \\ -d & 1 \end{bmatrix}$$

$$F = \{F_x + eF_e \quad F_y + fF_e\}^T, Z = \{X \quad Y\}^T$$

where the parameters a , b , c , and d are identified experimentally. Also, $\{e \ f\} = \{1 \ 0\}$ for forward propagation or $\{0 \ 1\}$ for backward propagation with F_e is the applied external acoustic loading.

Such a configuration enables the introduction of any coupling between the two sides of the cell via appropriate control strategy. In this paper, the coupling is established to generate a “*virtual gyroscopic control*” action in order to break the non-reciprocity. This is achieved by feeding the first output x , of the left piezo-sensor, to generate the second right input F_y such that $F_y = k_g \dot{x}$ with k_g denoting the gyroscopic gain. This action is fed into the right piezo-actuator. At the same time, the second output y , of the right piezo-sensor, to generate the left input F_x such that $F_x = -k_g \dot{y}$. With such dual coupling between the input and output ports of the meta-cell, a gyroscopic control action is generated. This action is quantified by a skew symmetric gyroscopic matrix G such that Eq. (3.1) becomes:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \left(\begin{bmatrix} b - adk_g & 0 \\ 0 & b + adk_g \end{bmatrix} + \begin{bmatrix} 0 & -ak_g \\ ak_g & 0 \end{bmatrix} \right) \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = B \begin{Bmatrix} e \\ f \end{Bmatrix} F_e$$

or

$$[M]\{\ddot{Z}\} + ([D] + [D_g] + [G])\{\dot{Z}\} + [K]\{Z\} = [B]B_e F_e \quad (3.2)$$

with $B_e = \{e \ f\}^T$ and the matrices $[D_g]$ and $[G]$ are given by:

$$D = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}, [D_g] = \begin{bmatrix} -adk_g & 0 \\ 0 & adk_g \end{bmatrix}, \text{ and } [G] = \begin{bmatrix} 0 & -ak_g \\ ak_g & 0 \end{bmatrix} \quad (3.3)$$

Eq. (3.2) indicates that the system with the augmented gyroscopic control action has an augmented damping matrix $[D_T] = ([D] + [D_g])$.

In summary, Eq. (3.2) represents the combined dynamic equations of the meta-cell and controller system. This combined system is structured such that the controller generates a closed-loop system with built-in gyroscopic matrix to induce controlled non-reciprocal characteristics.

3.3 The Acoustic Metamaterial

Fig. 3.2 displays a photograph of a prototype of the active acoustic metamaterial cell considered in this work. The cylindrical cavity (Fig. 3.2(a)) is water-filled which is contained between the space formed between the piezoelectric (*PZT4* – lead, zirconium, titanate) bimorphs. The bimorph forms a pair of collocated sensor and actuator separated by a brass substrata as shown in Fig. 3.2(b).

3.3.1 The Acoustic Cavity

The acoustic cavity is manufactured from an acrylic cylinder that has an outer diameter of $0.041m$, inner diameter of $0.035m$ and overall length of $0.045m$.

3.3.2 The Piezoelectric Diaphragm

The piezoelectric bimorph has an outer diameter of 41mm, overall thickness of 0.5mm. The bimorph is model AB4113B (Digi-Key Corporation) which forms a pair of collocated sensor/actuator pair deposited on a brass substrata disk of thickness 0.135mm as shown in Fig. 2(b).

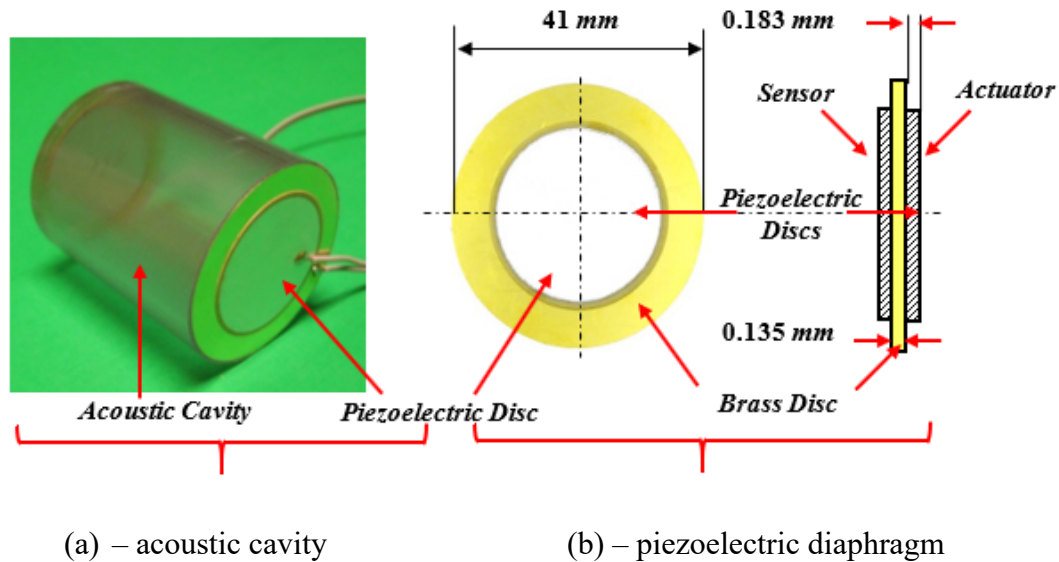


Figure 3.2 Photograph of the acoustic metamaterial cell

The material properties and main characteristics of the piezoelectric bimorph are listed in Table 3.1.

Table 3.1 Physical properties of the piezoelectric bimorph

Piezo Material	Frequency	Capacitance @ Frequency	Voltage - Input, Max	Impedance
PZT-4 (lead, zirconium, titanate)	1.3kHz	150000pf @ 1kHz	30V p-p	200 Ohm

Table 3.2 lists a comparison between the experimental and identified modes and modal damping ratios of the meta-cell.

Table 3.2 Comparison between the experimental and identified modes of the meta-cell

<i>META-CELL</i>	<i>Mode 1 (Hz)</i>	<i>Damping Ratio (ζ) of Mode 1</i>	<i>Mode 2 (Hz)</i>	<i>Damping Ratio (ζ) of Mode 2</i>
Experimental	558	0.0362	1121	0.0182
Identified	584	0.0336	-----	-----

Fig.3 displays a comparison the experimental and identified response of the meta-cell in the frequency domain. The displayed results suggest a close agreement between the experimental measurements and the identified predictions. In Fig. 3, the identified model indicates that the parameters are: $a = 1.36\text{E}8$, $b = 246.8$, $c = 1.46\text{E}7$, and $d = 0.5$.

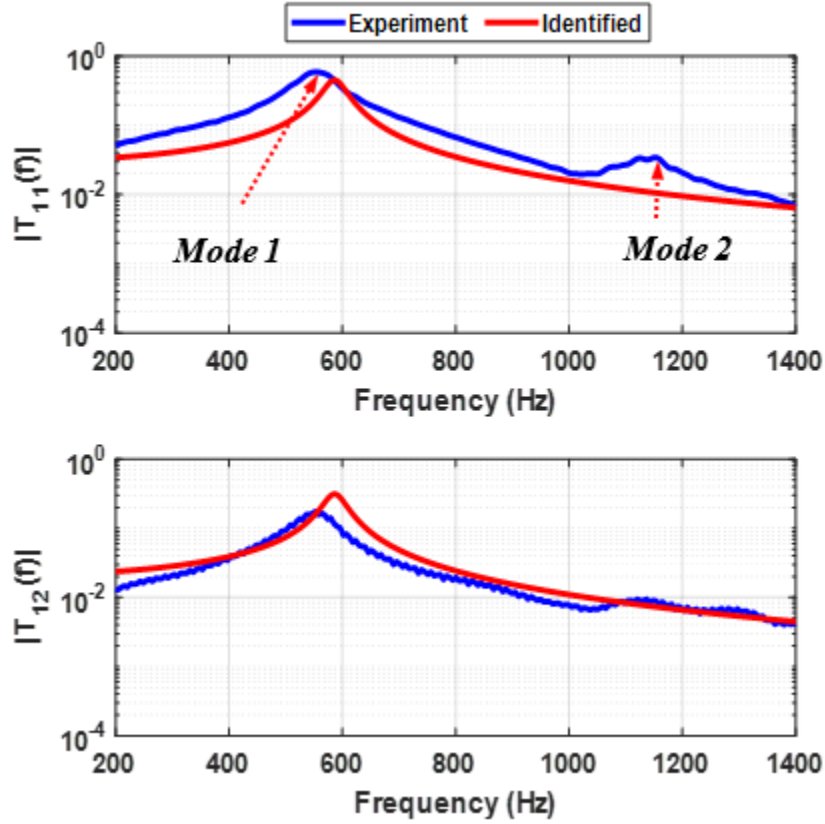


Figure 3.3 Comparison between the experimental and identified response of the meta-cell in the frequency domain

3.4 Stability of the Gyroscopic Nonreciprocal Controller/Cavity System

In this section, the stability analysis of the meta-cell/controller assembly is considered by using the Lyapunov stability criterion. Consider the following Lyapunov function V_L , which is a combination of the energies of the unit cell as given by:

$$V_L = \frac{1}{2} \dot{Z}^T [M] \dot{Z} + \frac{1}{2} Z^T [K] Z \quad (3.4)$$

where M and K are the mass and stiffness matrices as defined by Eq. (3.2).

Then, then the time derivative of the Lyapunov function V_L is given by:

$$\dot{V}_L = \dot{Z}^T([M]\dot{Z} + [K]Z) = -\dot{Z}^T([D] + [D_g] + [G])\dot{Z} \quad (3.5)$$

This yields:

$$\begin{aligned} \dot{V}_L &= -\dot{Z}^T([D] + [D_g] + [G])\dot{Z} \\ &= -\begin{Bmatrix} \dot{x} & \dot{y} \end{Bmatrix} \begin{bmatrix} (b - adk_g) & -ak_g \\ ak_g & (b + adk_g) \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} \end{aligned} \quad (3.6)$$

Or

$$\dot{V}_L = -\left[(b - adk_g)\dot{x}^2 + (b + adk_g)\dot{y}^2 \right]$$

Hence, Eq. (3.6) implies that the time rate of decay of the Lyapunov function V_L can be strictly negative and accordingly the system is stable only if $b > adk_g$, i.e., if the gyroscopic gain k_g is bounded from above by b/ad .

3.5 Breaking the Reciprocity with the Virtual Gyroscopic Controller

In this section, the capability of the virtual gyroscopic controller in breaking the reciprocity is established by analyzing the transfer function of meta-cell during forward and backward propagation.

Eq. (3.2) is cast in the following state-space representation:

$$\textbf{\underline{Meta-Cell:}} \quad \dot{X} = \bar{A}X + \bar{B}F_e, \quad (3.7)$$

$$\textbf{\underline{Output Measurements:}} \quad Y = \bar{C}X.$$

where

$$\bar{A} = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ -[M]^{-1}[K] & -[M]^{-1}[D_o] \end{bmatrix}, [D_o] = ([D] + [D_g] + [G]), [\bar{B}] = \begin{bmatrix} 0_{2 \times 1} \\ [M]^{-1}[B]B_e \end{bmatrix}, X = \{x \ y \ \dot{x} \ \dot{y}\}^T$$

$$\begin{aligned} \bar{C} &= [0 \ 1 \ 0 \ 0] && \text{For Forward Propagation,} \\ \text{and} \quad \bar{C} &= [1 \ 0 \ 0 \ 0] && \text{For Backward propagation.} \end{aligned}$$

Hence, the transfer functions relating the inputs F_e to the outputs Y for forward and backward propagations are:

Forward Propagation:

$$\left. \frac{Y}{F_e} \right|_{\text{Forward}} = -ad \frac{s^2 + (b - a k_g [d - 1/d])s + c}{(s^4 + 2bs^3 + [2c + (b^2 - a^2 d^2 k_g^2) + a^2 k_g^2]s^2 + 2bcs + c^2)} \quad (3.8)$$

and

Backward Propagation:

$$\left. \frac{X}{F_e} \right|_{\text{Backward}} = ad \frac{s^2 + (b + a k_g [d - 1/d])s + c}{(s^4 + 2bs^3 + [2c + (b^2 - a^2 d^2 k_g^2) + a^2 k_g^2]s^2 + 2bcs + c^2)} \quad (3.9)$$

Equations (3.8) and (3.9) clearly indicate the break of the reciprocity as the numerators of the forward and backward responses are distinctly different while the denominators are the exactly the same. Note that the numerator polynomial during forward propagation: is $s^2 + (b - a k_g [d - 1/d])s + c = 0$, whereas that during backward propagation is: $s^2 + (b + a k_g [d - 1/d])s + c = 0$. The distinct sign change of the gain k_g as it appears in both polynomials affects the locations of the zeros and hence the phase shift of the outputs relative to the inputs.

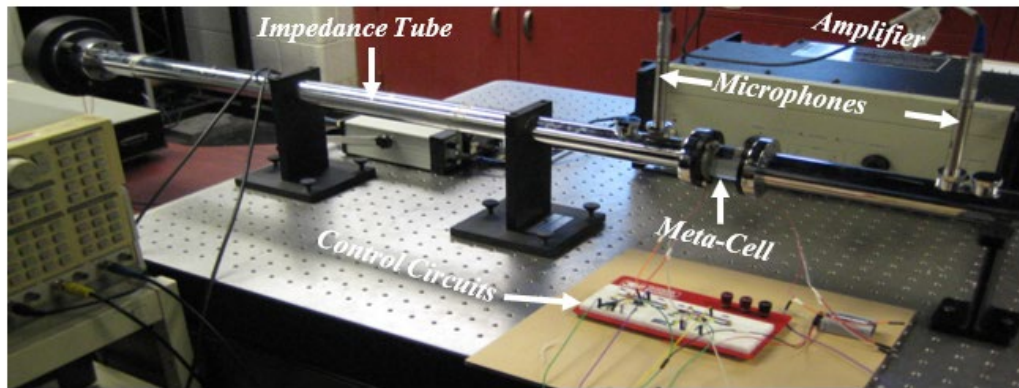
It is important also to note that the numerator defines the interaction between the locations of the sensors and actuators whereas the denominator is related to the physical properties and the nature of the assembly of the components of the meta-cell. A study of the pole-zero maps, as outlined in Appendix B, highlights and reveals the

basic differences between the forward and backward characteristics. In particular, these maps present an additional insight about relationships between the zeros and the resulting lead and lag phase shift characteristics generated by the virtual gyroscopic controller which are the main reason behind breaking the reciprocity

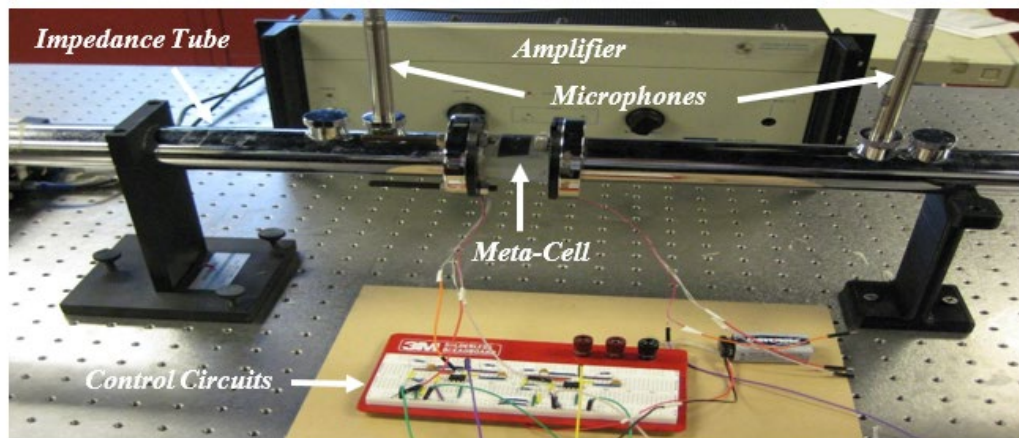
3.6 Experimental Characteristics of the Acoustic Metamaterial Coupled with an Active Gyroscopic Nonreciprocal Controller

3.6.1 Simple Configuration of the Controller

The meta-cell described and characterized in Section 3.3 is integrated with an analog active gyroscopic-nonreciprocal controller. Figs. 3.4(a) and 3.4(b) displays photographs of the experimental set-up including the meta-cell and the associated analog circuits when the meta-cell is placed in the *ACUPRO* Impedance and Transmission tube.



(a) – overall view



(b) – zoom in view

Figure 3.4 Photographs of the analog gyroscopic nonreciprocal controller

3.6.2 Theoretical Performance of the Controller

Fig. 3.5(a) displays the controller configuration for forward wave propagation. Also, Fig. 3.5(b) shows the predictions of the theoretical frequency response characteristics both in magnitude and phase shift. These predictions are based on using a value of k_g that ensures system stability such that: $k_g < 3.6 \times 10^{-6}$ according to the parameters of the identified meta-cell.

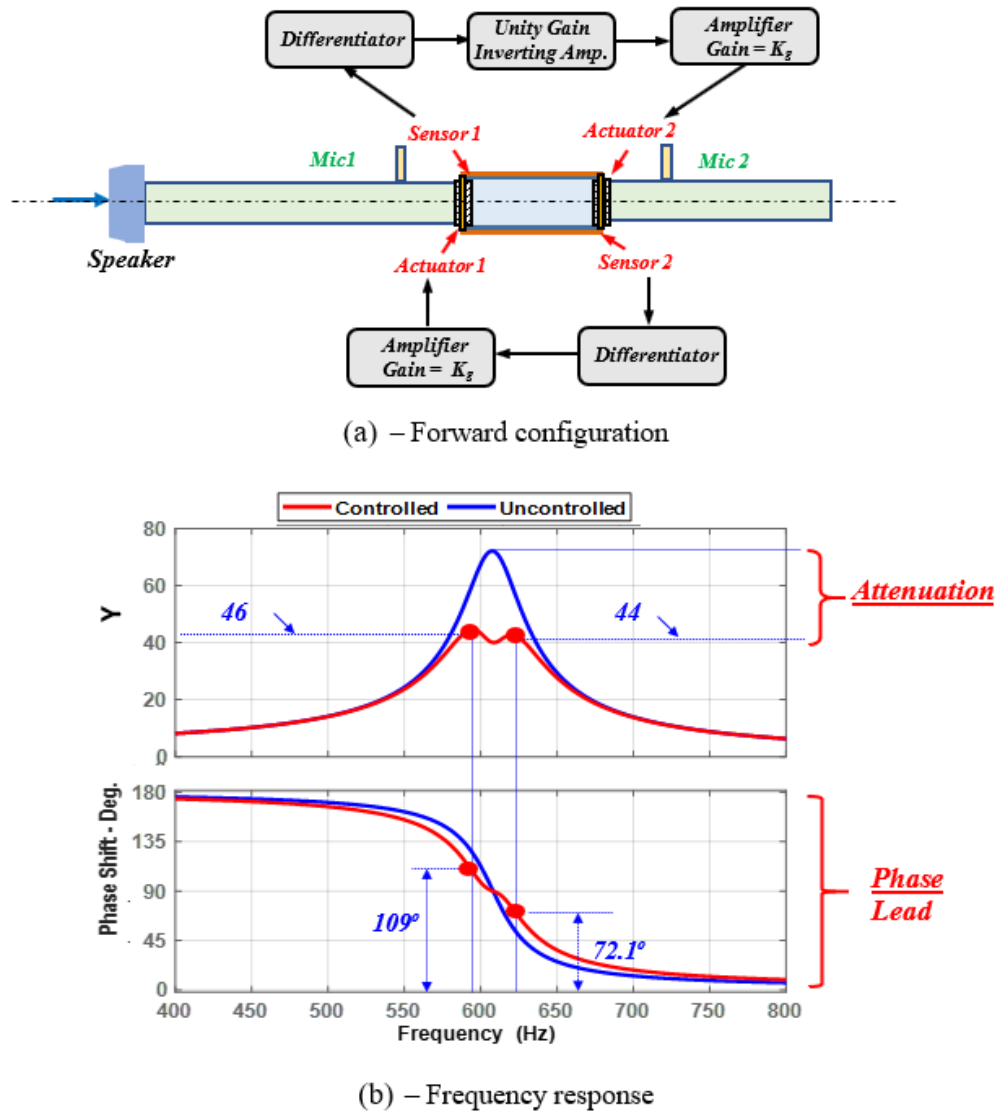


Figure 3.5 Forward configuration and frequency response of gyrosopic controller

Fig. 3.5(b) presents the distinct features that are behind the unique behavior of the virtual gyroscopic controller. In the figure, the uncontrolled meta-cell exhibits a single dominant frequency at 608Hz as predicted by the simplified 4-port model.

Once the meta-cell is integrated with the controller, the frequency response of the closed-loop system reveals two-dominant frequencies that appear as side band frequencies of the open-loop frequency. For a gyroscopic gain $k_g=0.5 \times 10^{-6}$, the side band frequencies occur at 594 and 622 Hz .

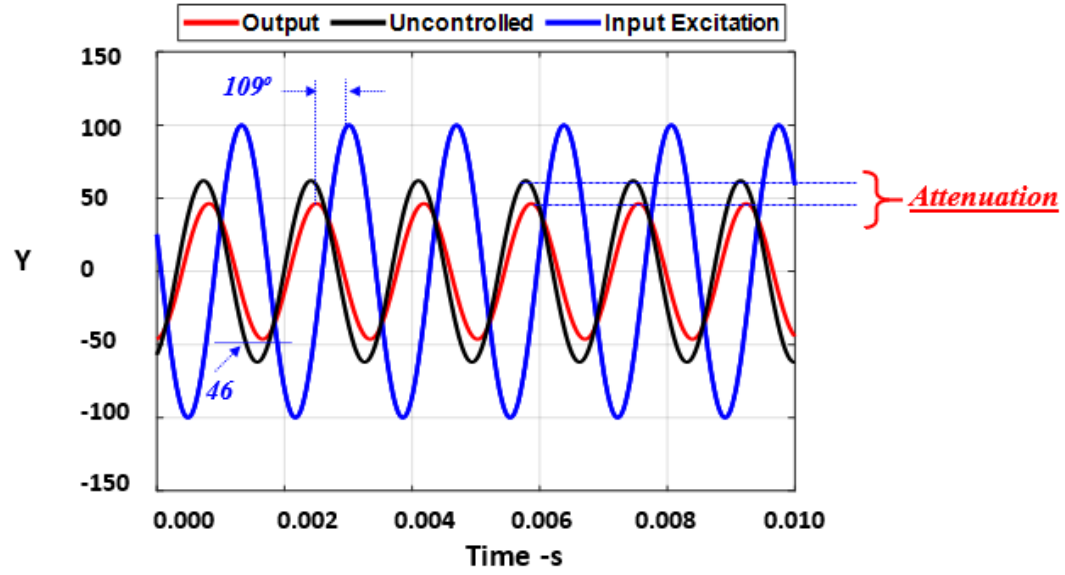
Equally important to the frequency split phenomenon is the developed phase shift characteristics. The displayed phase shift indicate that a phase lead is encountered relative to the uncontrolled case. The phase lead between the input and uncontrolled output response are 109 and 72 degrees at excitations of 594 and 622 Hz , respectively. This phase lead is developed by virtue of the gyroscopic control action. Furthermore, there is a distinct amplitude attenuation associated with the forward propagation as shown in Figure 3.5(b). This attenuation is related to the induced phase shift by the gyroscopic controller as explained in detail in Appendix *B*.

Note that the frequency domain results displayed in Fig. 3.5(b) match the time domain results displayed in Figs. 3.6(a) and 3.6(b) both in amplitudes and phase shifts.

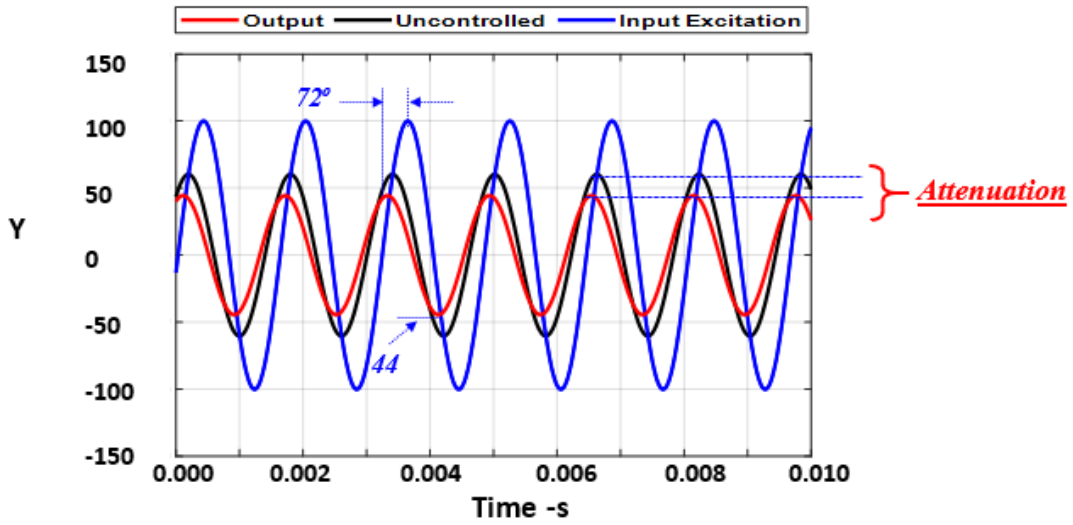
Figs. 3.7(a) and 3.7(b) display the controller configuration and the frequency response of the meta-cell during the backward propagation, respectively.

In this case, there is no observed frequency split. However, the phase shift experienced is phase lag unlike the lead phase shift experienced during forward propagation. The predicted phase shifts are -47 and -132 degrees for excitations of 594 and 622 Hz , respectively for comparison purposes with the forward propagation.

Furthermore, there is a distinct amplitude amplification associated with the backward propagation as shown in Figure 3.7(b). This attenuation is related to the induced phase shift by the gyroscopic controller as explained in detail in Appendix B.

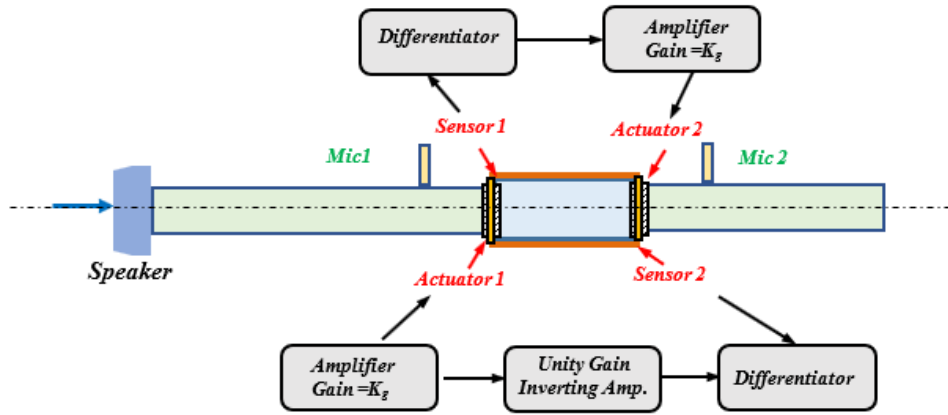


(a) -594 Hz

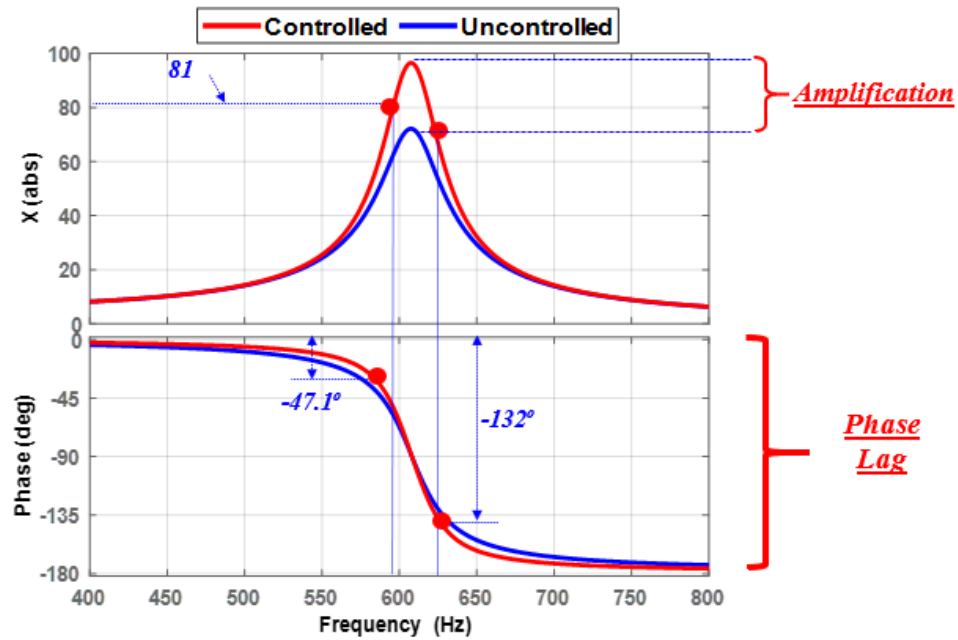


(b) -622 Hz

Figure 3.6 Time responses of the cell with gyroscopic controller during forward propagation



(a) – Backward Configuration



(b) – Frequency response

Figure 3.7 Performance of gyroscope controller during backward propagation

It is important to note that the frequency domain results displayed in Fig. 3.7(b) are confirmed by and conform with the time domain results displayed in Figs. 3.8(a) and 3.8(b).

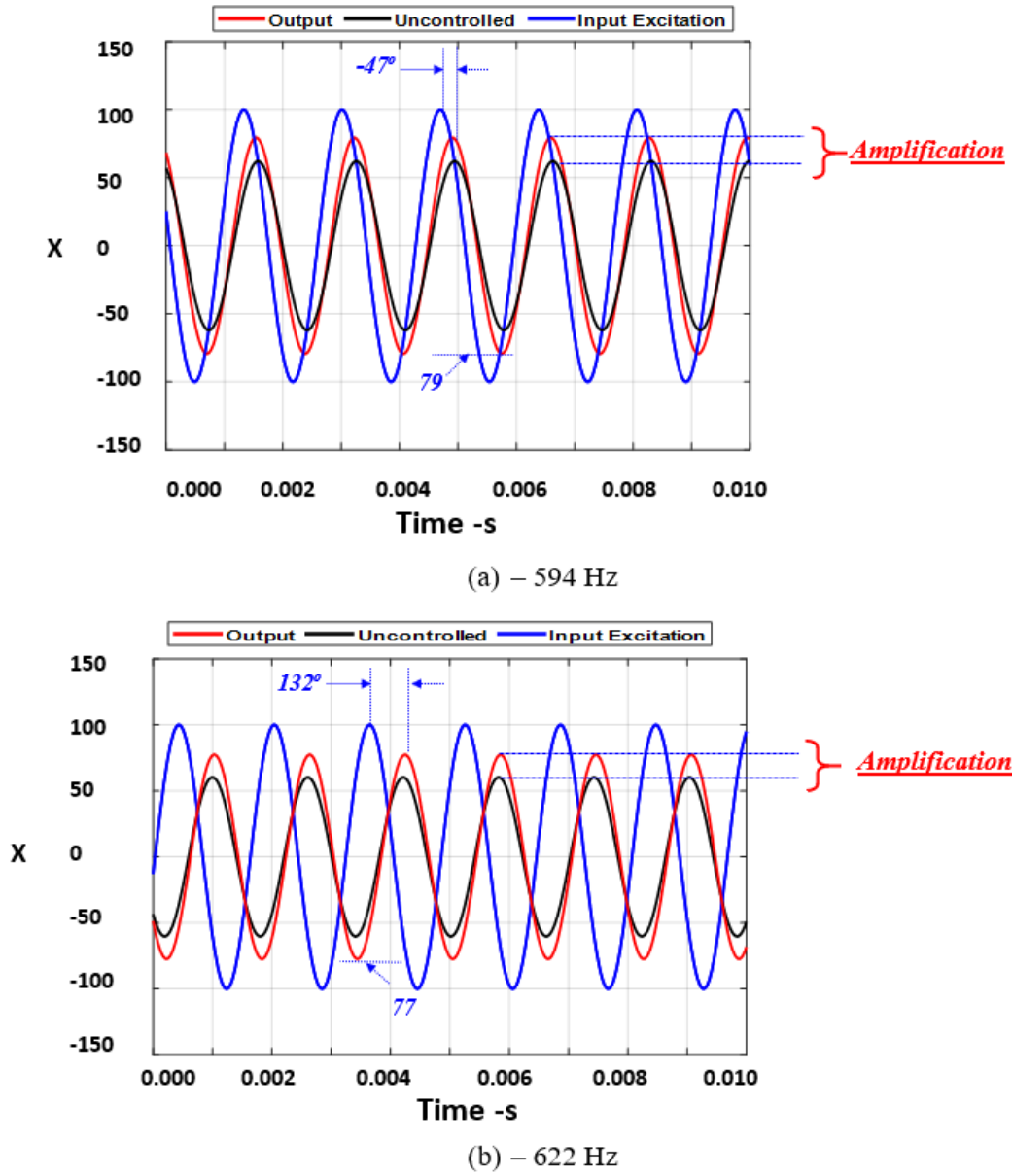


Figure 3.8 Time responses of the cell with gyroscopic controller during backward propagation

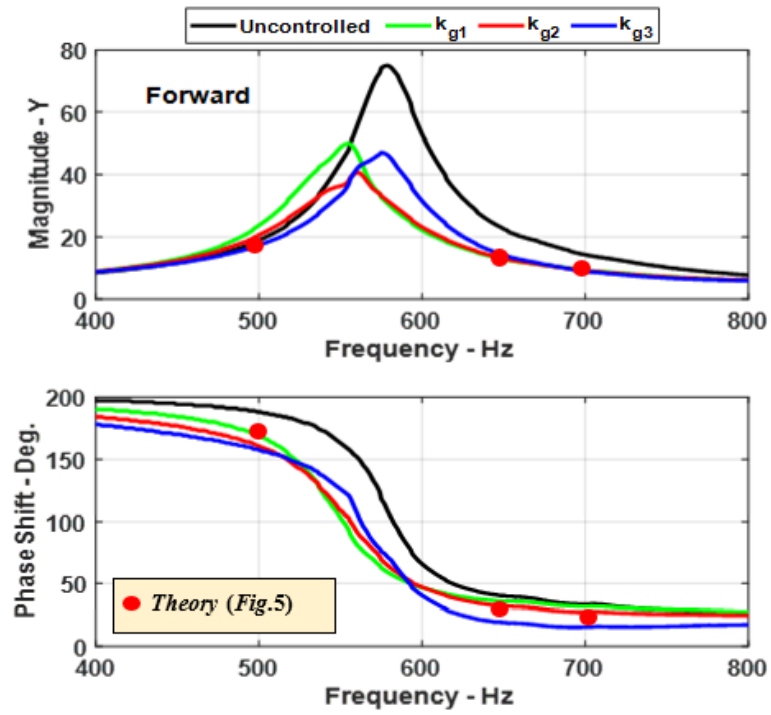
Figs. 3.8(a) and 3.8(b) display the phase shifts resulting during backward wave propagation for frequencies of 594 and 622Hz, respectively. The obtained results are in agreement with the results shown in Fig. 3.7.

Also, note that a comparison between the amplitudes of both the frequency and time response characteristics appearing in Figures 3.5 and 3.7 indicate that the forward propagation results in attenuation whereas the backward propagation is accompanied with amplitude amplification. These features are generated by the nature of the controller as can be easily explained by considering Figures 3.5(a) and 3.7(a). In Figure 3.5(a), the negative feedback component of the gyroscopic controller which fed to actuator 1 attenuates the response during forward propagation. This attenuation is reinforced by the positive feedback component sent to actuator 2 which resists further forward propagation. The reverse effects occur during the backward propagation as shown in Figure 3.7(a).

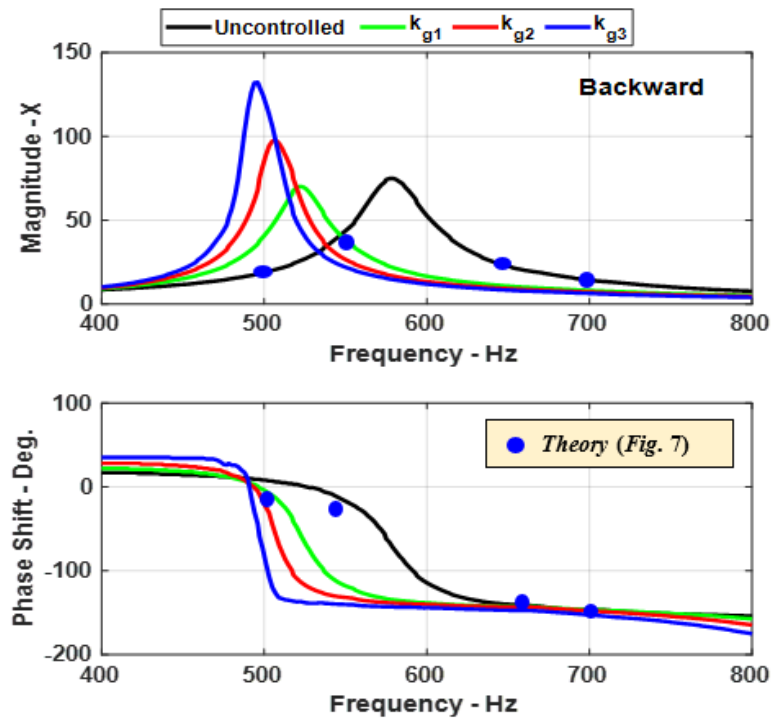
Note that there are reversals of the signs of the signals every time the sensor signals pass through a differentiator or an inverting amplifier as dictated by the operating nature of the used operational amplifiers.

3.6.3 Experimental Performance of the Controller

Fig. 3.9 displays the experimental frequency response characteristics of the controller for both the forward and backward propagations. The figure shows also comparisons with the theoretical predictions as illustrated in Figs. 3.5 and 3.7 for forward and backward propagations, respectively. The displayed results suggest a close agreement between theoretical predictions and experimental results for both the magnitude and phase shift.



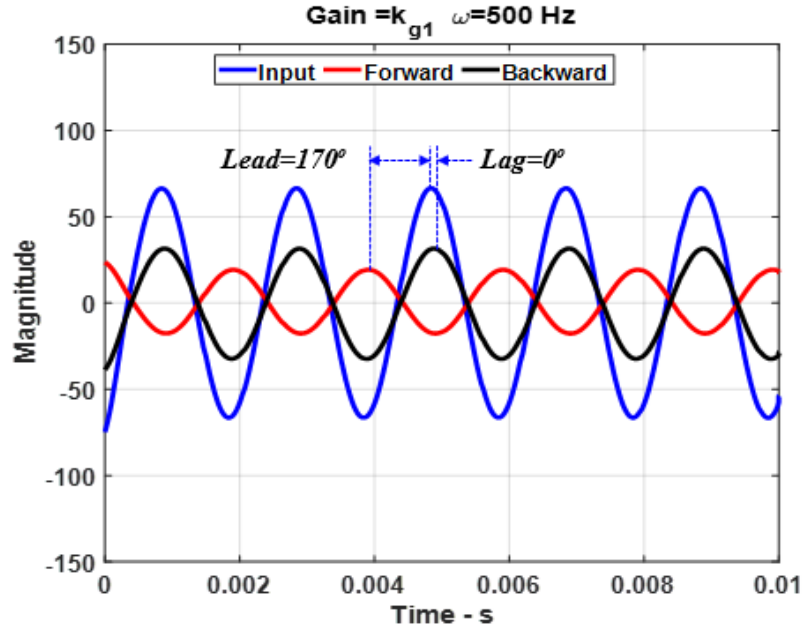
(a)



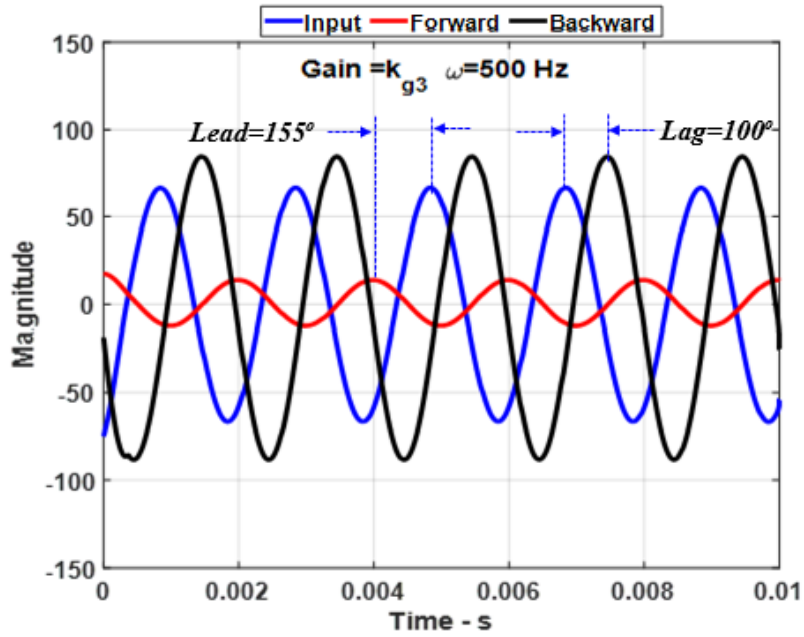
(b)

Figure 3.9 Frequency responses of the controlled meta-cell during forward and backward propagations

Figs. 3.10(a) and 3.10(b) show the experimental time response of the meta-cell during forward and backward propagations at excitation frequency 500 Hz when the controller gains are $k_{g1} = -0.5 \times 10^{-6}$ and $k_{g3} = -2.0 \times 10^{-6}$, respectively.



(a) – Gain k_{g1}



(a) – Gain k_{g3}

Figure 3.10 Time response of the controlled meta-cell during forward and background propagation for excitations of k_{g1} (a) and k_{g3} (b)

3.7 Summary

This chapter has presented a comprehensive theoretical and experimental investigation of an active *Nonreciprocal Gyroscopic Meta-Material (NGMM)*. The proposed metamaterial is manufactured from an acoustic cavity which is integrated with piezoelectric diaphragms that act as collocated pairs of sensors and actuators. These piezo-diaphragms are controlled by a virtually synthesized gyroscopic control action to produce the desirable non-reciprocal characteristics. In this manner, the non-reciprocity is generated without the need for any moving gyroscopic elements as the currently available approaches.

The dynamic behavior of a prototype of the meta-cell is experimentally identified using MATLAB “*ident*” toolbox in order to generate the cell dynamic equations. The developed mathematical model of the cell is integrated with the proposed virtual gyroscopic controller in order to theoretically and experimentally demonstrate its effectiveness in breaking the non-reciprocity.

The theoretical model of the meta-cell is utilized to establish the basis for breaking the non-reciprocity by analyzing the interaction of the poles and zeros of the transfer function. These interactions are shown to develop a unique “*zero migration*” phenomenon. This phenomenon is shown to lead to phase lead and attenuation during forward propagation and phase lag and amplification during backward propagation. Such phase shift and amplitude modulation are the reasons behind breaking the reciprocity.

The theoretical predictions are validated experimentally using a simple configuration of the *NGMM* cell which eliminates the need for a dummy cell as carried

out by Raval *et al* [110]. Such a preferred arrangement when coupled with an analog controller is shown to enable fast execution of the controller and demonstrate the break of the non-reciprocity during forward and backward propagations. The characteristics of the non-reciprocity, as quantified by the extent of the phase shift and amplitude modulation, are shown to be primarily influenced by the gyroscopic gain.

Furthermore, the present work can be extended to various critical applications where realistic non-reciprocal metamaterial configurations can be achieved in a simple and programmable manner. Presently, work is in progress to develop a multi-cell acoustic metamaterial configuration with non-reciprocal characteristics capitalizing on the simplified version of the virtual gyroscopic dynamic controller demonstrated in this paper.

Chapter 4: Active Non-Reciprocal Metamaterial Using an Eigen-Structure Assignment Control Strategy

4.1 Introduction

In this chapter, we present a class of active non-reciprocal metamaterials (*ANMMs*) in an attempt to control the flow of acoustic waves along a one-dimensional acoustic duct. This work is initiated to validate experimentally the theoretical work of Baz [100], which has proposed an active control strategy based on the simultaneous assignment of both the eigenvalues and eigenvectors, i.e., the entire eigen-structure, of the closed-loop system. The study is guided by the wealth of information related to eigen-structure controllers [111-118]. In these controllers, the emphasis is placed on allocating the eigenvalues to improve the closed-loop damping and system's response as well as eigenvectors to provide means for controlling the spatial distribution of the vibrational energy along the structure. In this study, the focus is placed on adjusting the eigenvectors in a way that enables the spatial control and redistribution of the wave propagation along the acoustic duct whereby non-reciprocity with controlled magnitude and direction can be achieved. During this entire process, the system continues to behave in a linear fashion. The theory governing the operation of this proposed approach is introduced, and a comprehensive experimental validation effort is presented to demonstrate the basic features, non-reciprocal behavior, and control characteristics.

4.2 Concept of the Active Non-Reciprocal Metamaterial

In this section, the emphasis is placed on an *ANMM* configuration consisting of an array of periodic systems, which is provided with a set of collocated microphones and mini-speakers as shown in Figures 4.1-4.3. The mini-speakers generate control actions ($u_i s$) that are generated by appropriate assignment of both the eigenvalues and eigenvectors of the closed-loop system [111-118]. Such an approach is more general and powerful than the restrictive approach of just the allocation of the eigenvalues, which has a limit ability to improve only the damping and system's response [119-120]. However, with the simultaneous assignment of the entire eigen-structure, i.e., both the eigenvalues and eigenvectors, the *ANMM* becomes capable of both temporal and spatial control of the vibrational energy flow along the structure to achieve any desirable non-reciprocal characteristics.

Figure 4.1(b) displays the upper bounds of the open-loop eigenvectors, which are maintained the same as for the closed-loop system. Under such conditions, the system exhibits classical reciprocal behavior. However, if the bounds of the closed-loop eigenvectors are altered and shaped to be either gradually decreasing and decreasing in steps as shown in Figure 4.1(c), the wave propagation along the forward direction can be amplified in a manner that matches the assigned profiles of the eigenvectors. More importantly, if the direction of wave propagation is reversed as shown in Figure 4.1(d), the wave propagation along the backward direction will experience attenuation as implied by the shape profile. Hence, non-reciprocal wave propagation behavior will be generated actively without any physical modification of the waveguide parameters. As the shaping of the eigenvector profiles is implemented

completely in a virtual manner, the extent and direction of the introduced non-reciprocity can be programmable and altered on demand by the user end.

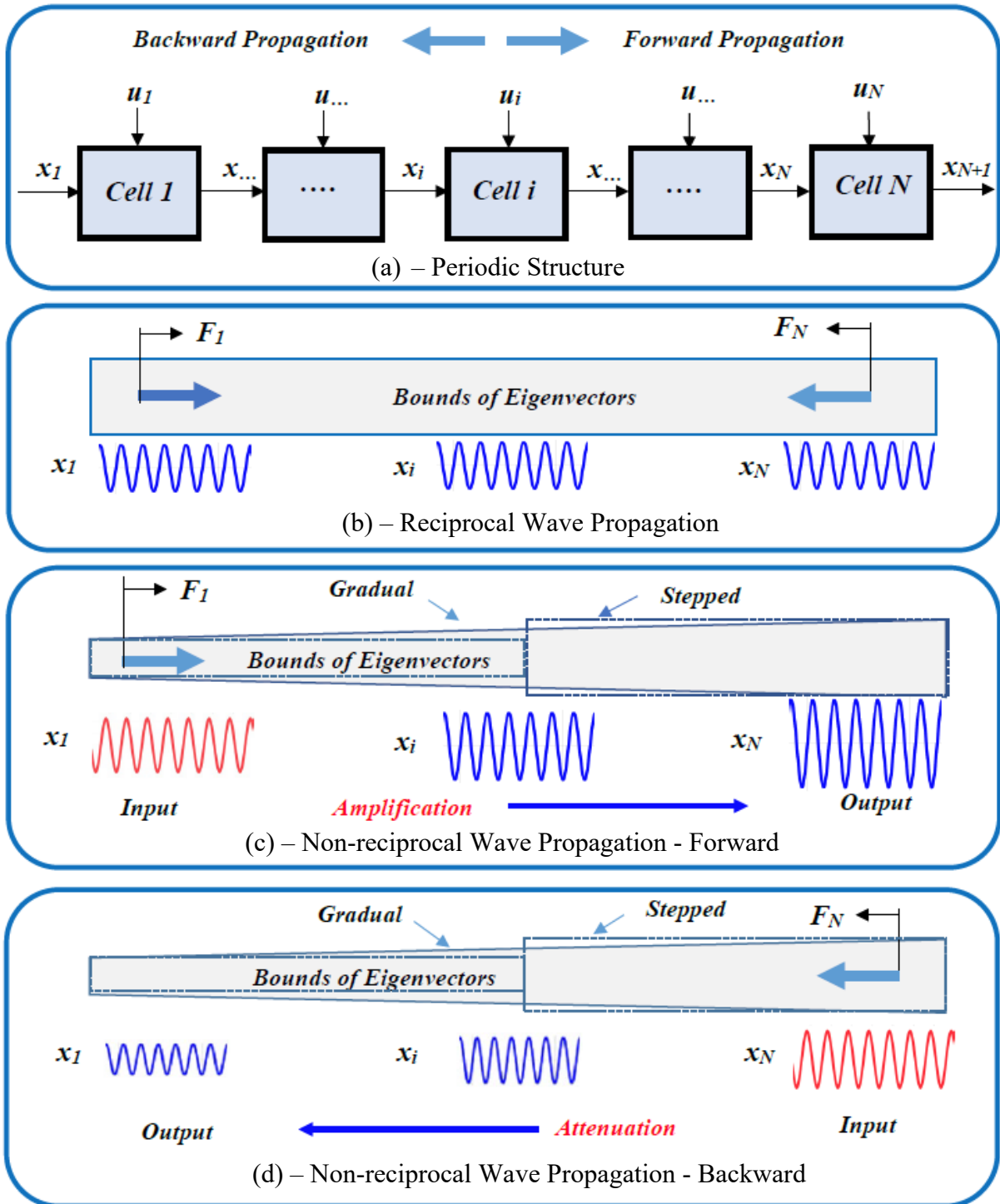


Figure 4.1 A schematic drawing of a one-dimensional (1D) periodic dynamic system

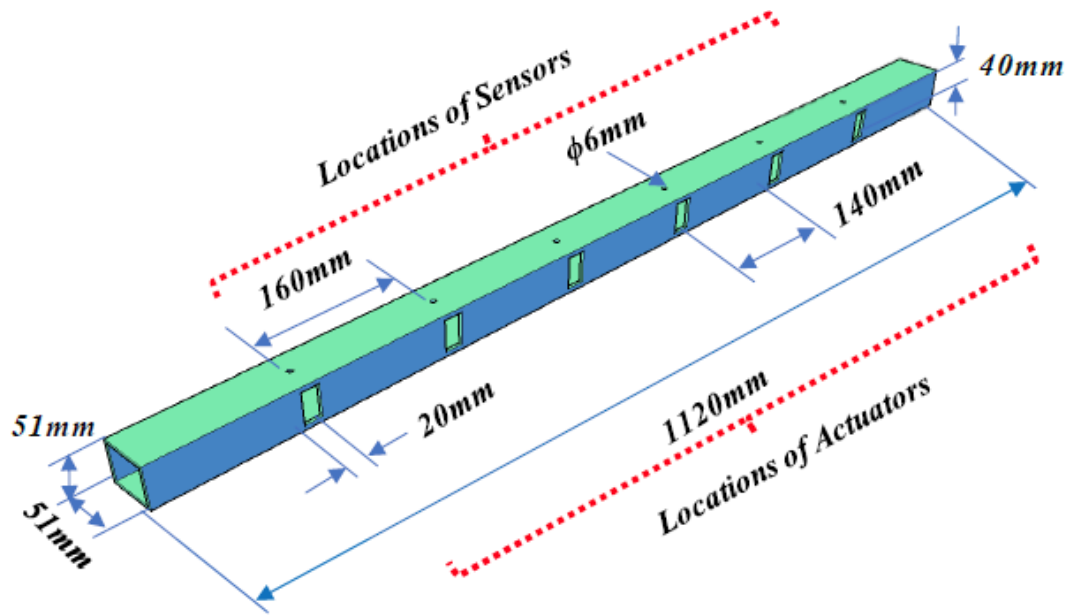
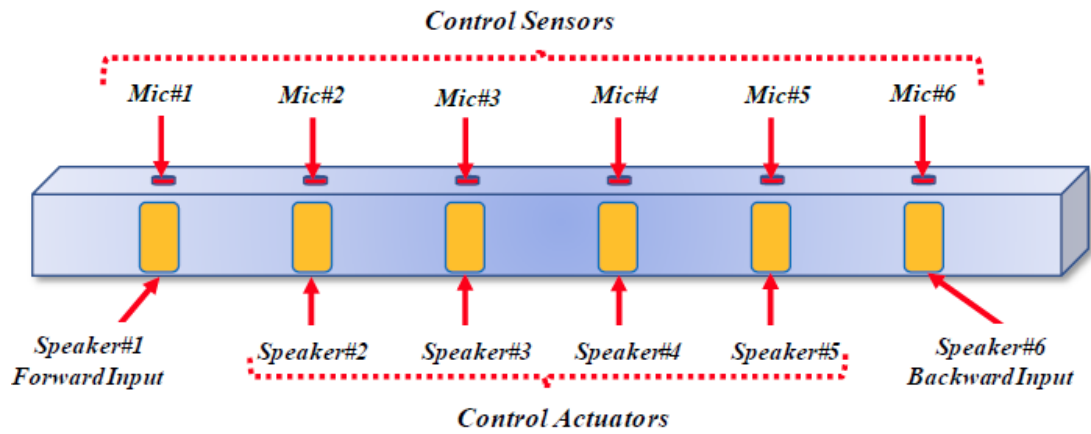
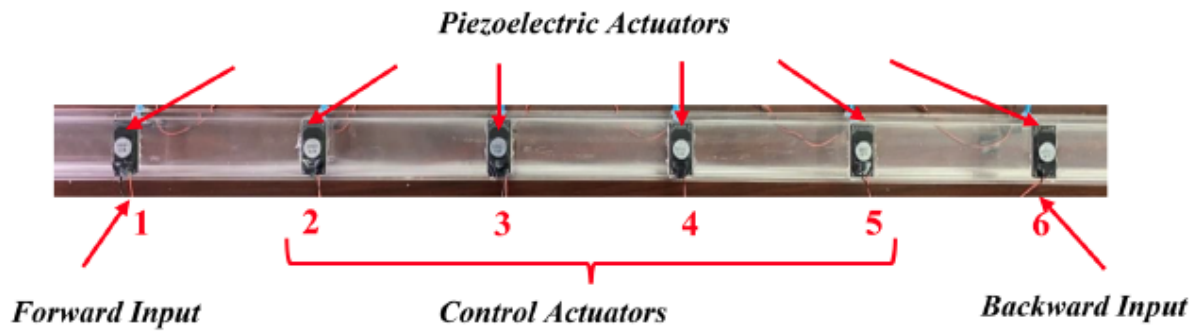


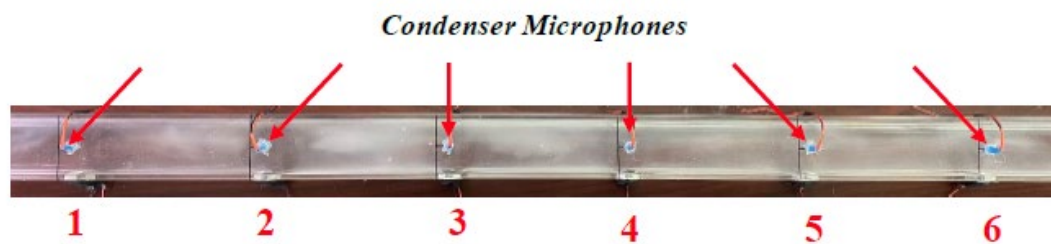
Figure 4.2 Geometrical dimension of the test acoustic duct



(a) – Schematic drawing



(b) – Placement of mini-speakers



(c) – Placement of pressure sensors

Figure 4.3 Acoustic duct with periodically placed control diaphragm

4.3 Theoretical Basis of the Eigen-Structure Assignment with the Output Feedback

This section summarizes briefly the theoretical basis of the eigen-structure assignment with output feedback, which is presented in detail by Baz [100].

The system dynamics are identified experimentally in the form of the following state-space model such that:

$$\begin{array}{ll} \text{System} & [\dot{X}] = [A][X] + [B][u] \\ \text{Measurements} & [y] = [C][X] \\ \text{Control Law} & [u] = [G][y] \end{array} \quad (4.1)$$

where $[X]$, $[y]$, $[A]$, $[B]$, $[C]$, and $[G]$ denote the state vector, output measurement vector, system matrix, input matrix, output matrix, and feedback gain matrix, respectively.

Then the closed-loop system is described by:

$$[\dot{X}] = ([A] + [B][G][C])[X] \quad (4.2)$$

For the closed-loop system defined by Eq. (4.2), the associated eigenvalue problem is given by:

$$([A] + [B][G][C])\{\phi_i^d\} = \lambda_i^d\{\phi_i^d\} \quad (4.3)$$

where λ_i^d and $\{\phi_i^d\}$ denote i th desired eigenvalue and corresponding eigenvector, respectively. Note that $\{\phi_i^d\} = [S]\{\phi_i\}$ is obtained by scaling the actual eigenvector $\{\phi_i\}$ by a scaling matrix $[S]$.

Eq. (4.3) can be rearranged as follows:

$$\{\phi_i^d\} = (\lambda_i^d[I] - [A])^{-1}[B][G][C]\{\phi_i^d\} = [N_i][u_i] \quad (4.4)$$

where $N_i = (\lambda_i^d[I] - [A])^{-1}[B]$ and $[u_i] = [G][C]\{\phi_i^d\}$.

For the desired eigenvector $\{\phi_i^d\}$, the achievable eigenvector $\{\phi_i^a\}$ is determined by [114]:

$$\{\phi_i^a\} = [N_i]([N_i]^T[N_i]^{-1})[N_i]^T\{\phi_i^d\} \quad (4.5)$$

Then the system achievable closed-loop matrix $[A_c]$ is given by:

$$[A_c] = ([A] + [B][G][C])[\Phi^a]^T[\Phi^a] \quad (4.6)$$

where $[\Phi^a] = [\{\phi_1^a\} \quad \{\phi_2^a\} \quad \dots \quad \{\phi_n^a\}]$.

Hence, the control gain matrix $[G]$ is extracted from Eq. (4.6) as follows:

$$[G] = [B^\dagger]([A_c] - [A])[C^\dagger] \quad (4.7)$$

where $[B^\dagger]$ and $[C^\dagger]$ are the pseudo-inverse of $[B]$ and $[C]$, respectively.

4.4 Experimental Acoustic Duct

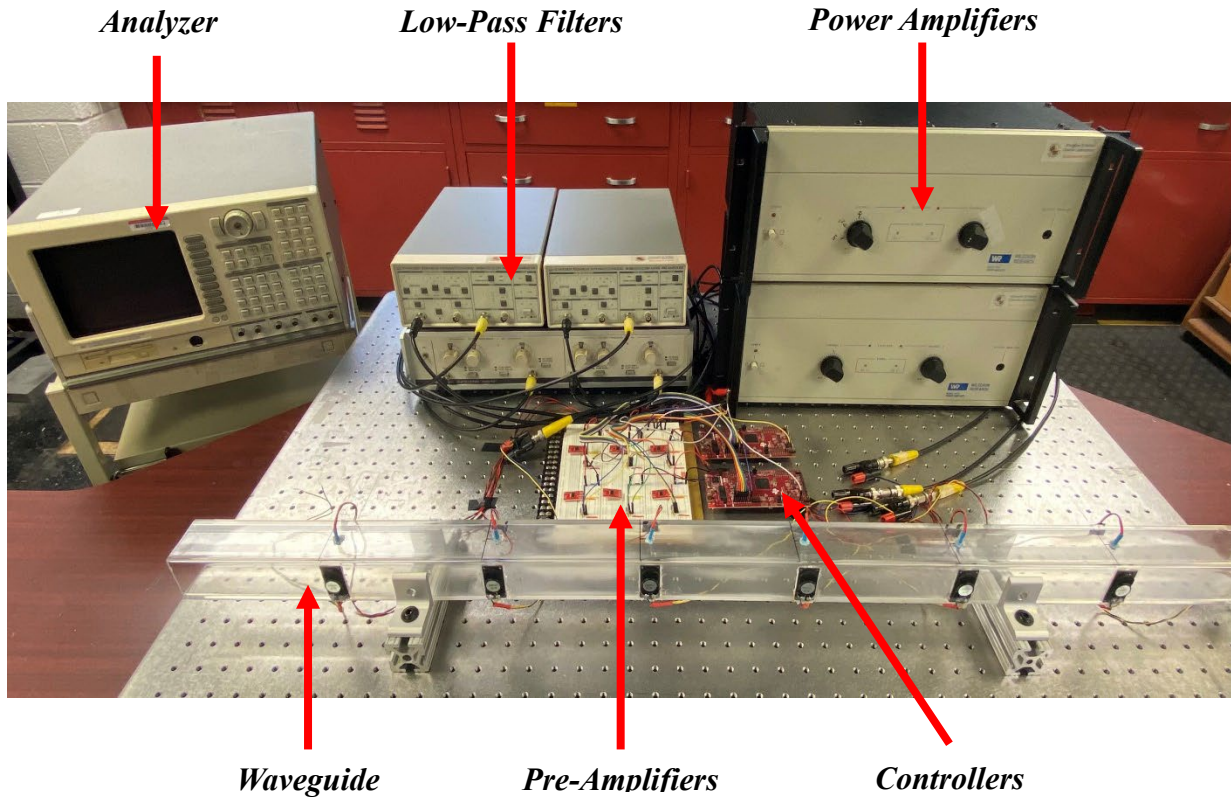
4.4.1 The Prototype

In this section, a prototype of an acoustic duct system is manufactured. The duct is provided with control diaphragms that are periodically placed, as shown in Figures 4.2-4.4, to control the wave and energy propagation in an attempt to induce controllable and reversible non-reciprocal behavior.

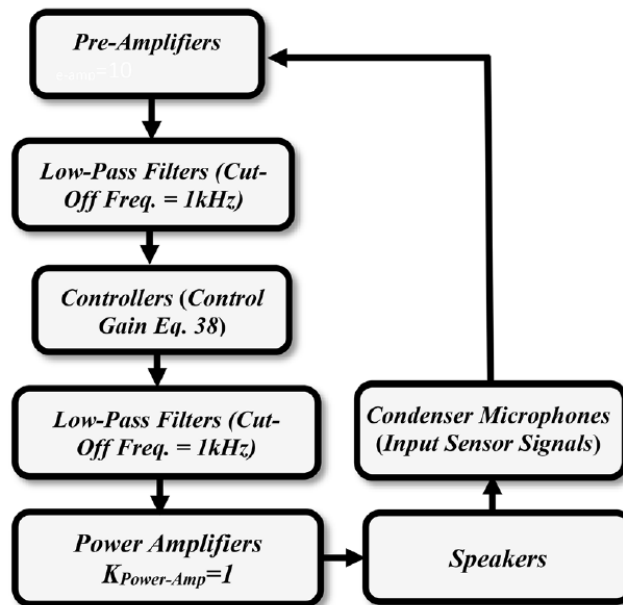
Figure 4.3(a) shows the basic configuration of the acoustic duct with six mini-speakers and six collocated condenser microphone sensors. The forward and backward wave propagations are generated by the excitation of mini-speaker 6 and mini-speaker 1, respectively, as shown in Figure 4.5(b). Figure 4.3(c) shows six collocated microphones that are used for measuring the sound propagation along with the waveguide. Also, the mini-speaker actuators 2-5 are designed for controlling the wave propagation for both forward and backward directions.

4.4.2 Mini-Speakers and Microphones

The mini-speakers used in this chapter are of the electromagnetic type, are configured as shown in Figure 4.5, and have the physical characteristics listed in Table 4.1. These actuators are manufactured by DB Unlimited (model SC400608-1). The actuators have length, width and thickness of 40, 20, and 0.6 *mm*, respectively. The main characteristics of the mini-speaker are given in Appendix C.1.



(a)



(b)

Figure 4.4 Photograph and block diagram of the experimental acoustic duct with periodically placed control actuators and sensors

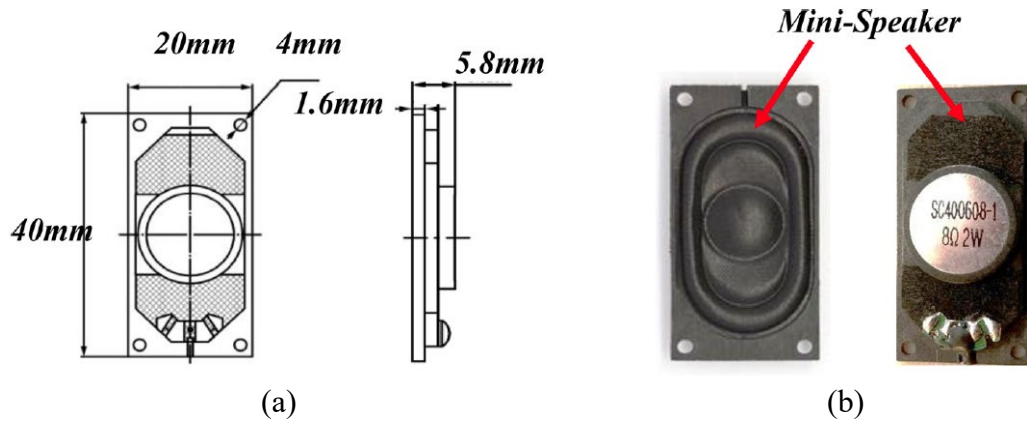


Figure 4.5 The mini-speaker (DB Unlimited)

Table 4.1 The physical and geometrical characteristics of the mini-speakers (DB Unlimited, Dayton, OH)

Resonant frequency (Hz)	Frequency bandwidth (Hz)	SPL at 10 cm (dBA)	Total impedance (Ω)	Nominal power (W)	Weight (g)
650	20 – 20,000	90	10.8	2	7.6

4.4.3 The Measurements and Control Circuits

Based on the calculated control gain matrix, the experiments were carried out by using the conditioning circuit of the condenser microphones, given in Appendix C, at the controller of a Texas Instruments (Dallas, TX) F28379D C2000 Delfino LaunchPad. The control algorithm was implemented by the SIMULINK block diagrams shown in Figure 4.6 with a sampling rate of 10,000 Hz.

Input Sensor Signals

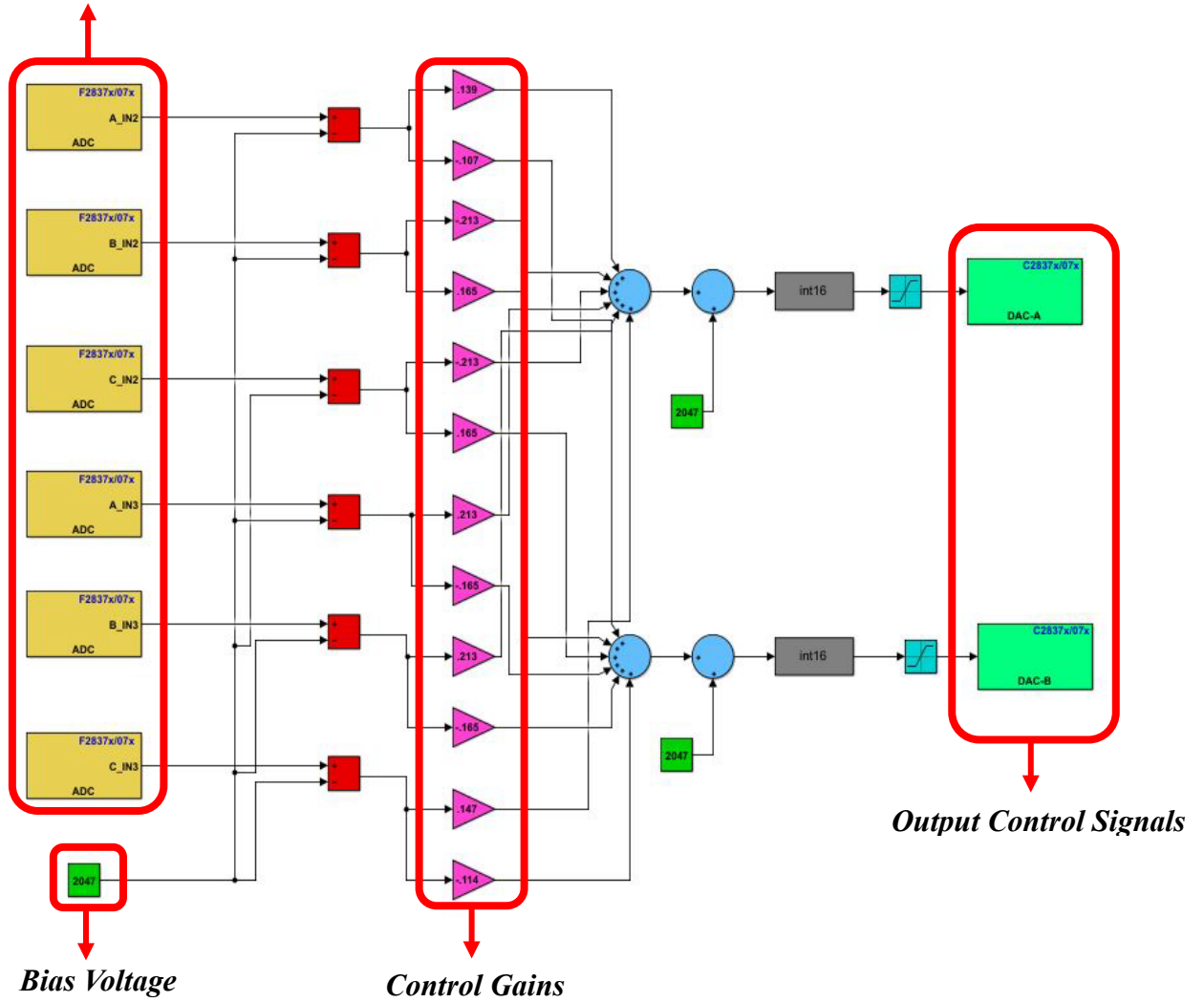


Figure 4.6 Controller configurations programmed by SIMULINK

4.5 System Identification

The state-space equations describing the dynamics of the ANMM system shown in Figures 4.2-4.4 are identified using the identification toolbox of MATLAB[®] “*ident*”. The model of the *ANMM* system is assumed to be described by the following state-space representation:

$$[\dot{X}] = [A][X] + [B][u] + [B_f]u_f + [B_b]u_b$$

and

$$[\dot{y}] = [C][X] \quad (4.8)$$

where $[A]$ is the state matrix (10×10). The assumed 10^{th} order model is considered as the state in which vector $[X]$ consists of the output of the six microphones and their time derivatives such that:

$$[X] = \{x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10}\}^T \quad (4.9)$$

where x_1 through x_6 indicate the pressure p_i at the locations of the microphones ($i = 1, \dots, 6$). Also, $[B]$ denotes the input matrix (10×4), and $[u]$ denotes the control effort, which is defined as $[u] = \{u_1 \ u_2 \ u_3 \ u_4\}^T$ with u_1 through u_4 defining the control efforts of the mini-speaker actuators 2-5.

Furthermore, $[B_f](10 \times 1)$ and $[B_b](10 \times 1)$ are the forward and backward input matrices, with u_f and u_b denoting the inputs from the mini-speaker actuators 1 and 6 to define the forward and backward excitations, respectively. Also, the measurement matrix $[C]$ is defined as

$$C = [I_6 \mid 0_{6 \times 4}] \quad (4.10)$$

The system identification is based on the experimental frequency response of the *ANMM* system between 100 and 800 *Hz*. The identified matrices $[A]$, $[B]$, $[B_f]$, $[B_d]$, and $[C]$ are listed in Appendix D.

Table 4.2 lists the natural frequencies and damping ratios of the uncontrolled as compared to those of the controlled system.

The pole map of the uncontrolled and controlled systems is shown in Figure

4.7.

Table 4.2 The natural frequencies and damping ratios of the uncontrolled and controlled systems

Mode	Uncontrolled			Controlled		
	Pole Location (rad/sec)	Frequency (Hz)	Damping Ratio	Pole Location (rad/sec)	Frequency (Hz)	Damping Ratio
1	$-150.9 \pm 948.7i$	152.9	0.1571	$-408.2 \pm 852.6i$	150.4	0.4318
2	$-122.9 \pm 1794.6i$	286.3	0.0683	$-389.8 \pm 1560.8i$	256.0	0.2423
3	$-157.6 \pm 2693.0i$	429.3	0.0584	$-184.6 \pm 2671.5i$	426.2	0.0689
4	$-183.7 \pm 3581.5i$	570.8	0.0512	$-945.8 \pm 3489.5i$	575.4	0.2616
5	$-198.9 \pm 4510.8i$	718.6	0.0441	$-44.7 \pm 4546.2i$	723.6	0.0098

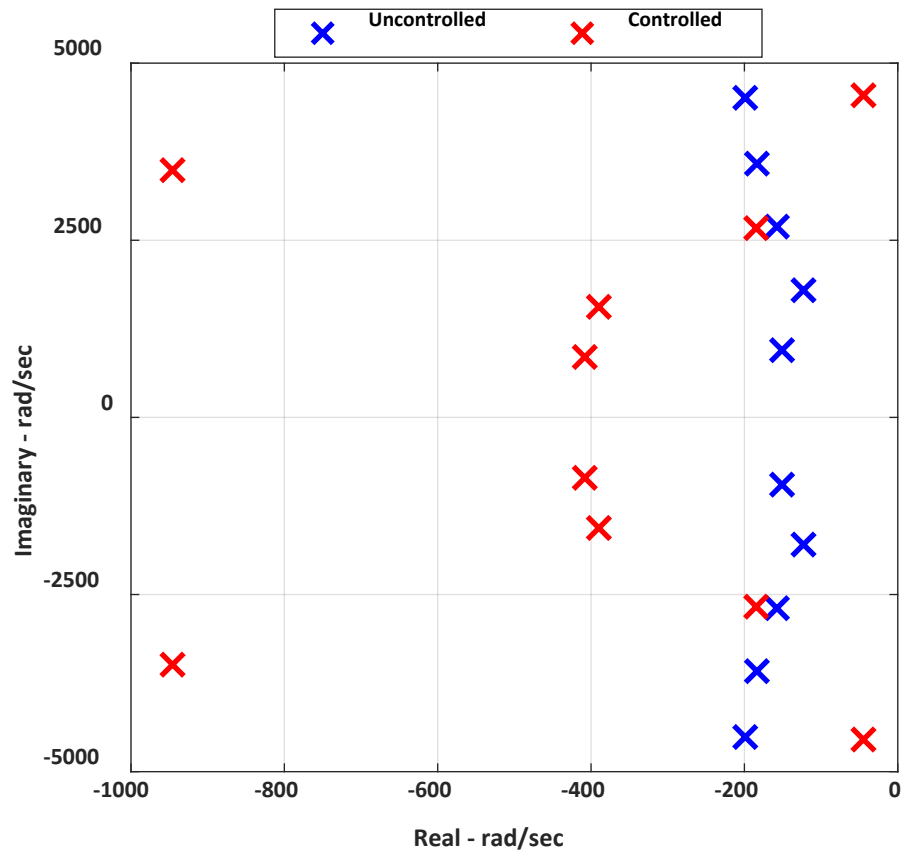


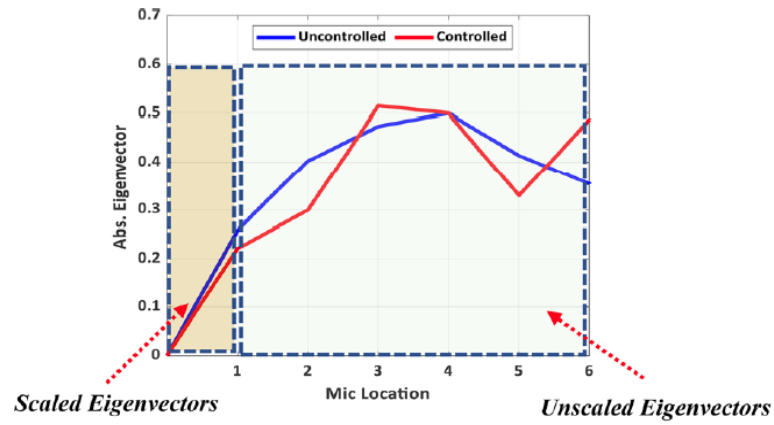
Figure 4.7 System pole location for uncontrolled and controlled systems

Also, Figure 4.8 displays a comparison between the corresponding experimentally obtained eigenvectors of the uncontrolled as compared to those of the controlled systems. Note that the controlled eigenvectors are scaled and reduced at the location of microphone number 1. The scaling is carried out to replicate the eigenstructure profile shown in Figure 4.1(c) to induce asymmetry and non-reciprocity in the wave propagation. The scaling is carried out such that the closed-loop eigenvector is scaled to be 0.01 of that of the open-loop at microphone 1. Hence, the forward propagation would result in amplification and the backward propagation would result in attenuation as shown in Figure 4.1(d).

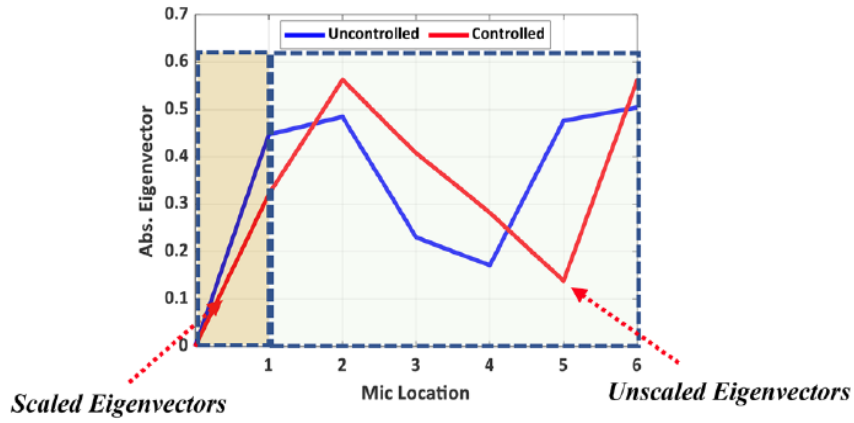
Note that the selected scaling is limited at microphone 1 as scaling at several other location is found to result in higher gain matrix $[G]$ and, in turn, higher control voltages that would be excessively high to be used to drive the control speakers.

Accordingly, the obtained control gain matrix $[G]$, according to Eq. (4.7), is given by:

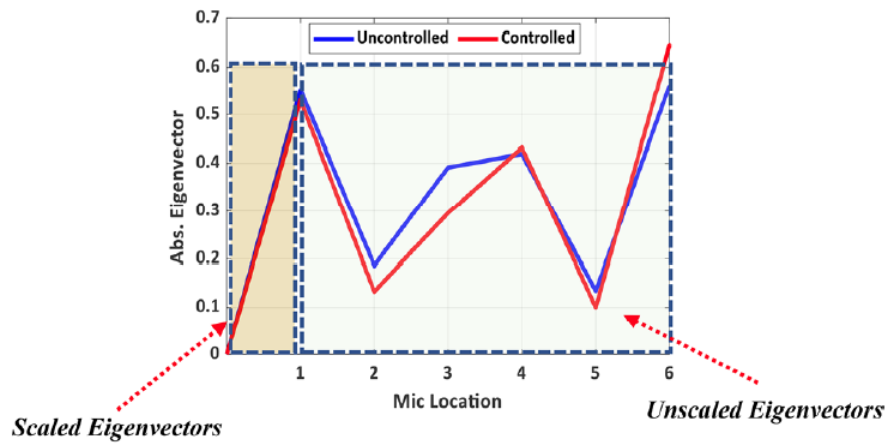
$$G = \begin{bmatrix} 0.1388 & -0.2129 & -0.2129 & 0.2129 & 0.2129 & 0.1468 \\ -0.1074 & 0.1647 & 0.1647 & -0.1647 & -0.1647 & -0.1135 \\ 0.1954 & -0.2998 & -0.2998 & 0.2998 & 0.2998 & 0.2066 \\ 0.3108 & -0.4766 & -0.4766 & 0.4766 & 0.4766 & 0.3286 \end{bmatrix} \quad (4.11)$$



(a) – First mode (152 Hz)



(b) – Second mode (289 Hz)



(c) – Third mode (429 Hz)

Figure 4.8 A comparison between the eigenvectors of the uncontrolled and controlled systems

Figure 4.9 displays comparisons between the frequency responses of original system and identified systems both along the forward and backward directions. The response along the forward direction is measured at the location of microphone 6 with an excitation of mini-speaker 1, while the response along the backward direction is measured at the location of microphone 1 due to an excitation of mini-speaker 6. Close agreements are evident between the characteristics of the original and the identified systems.

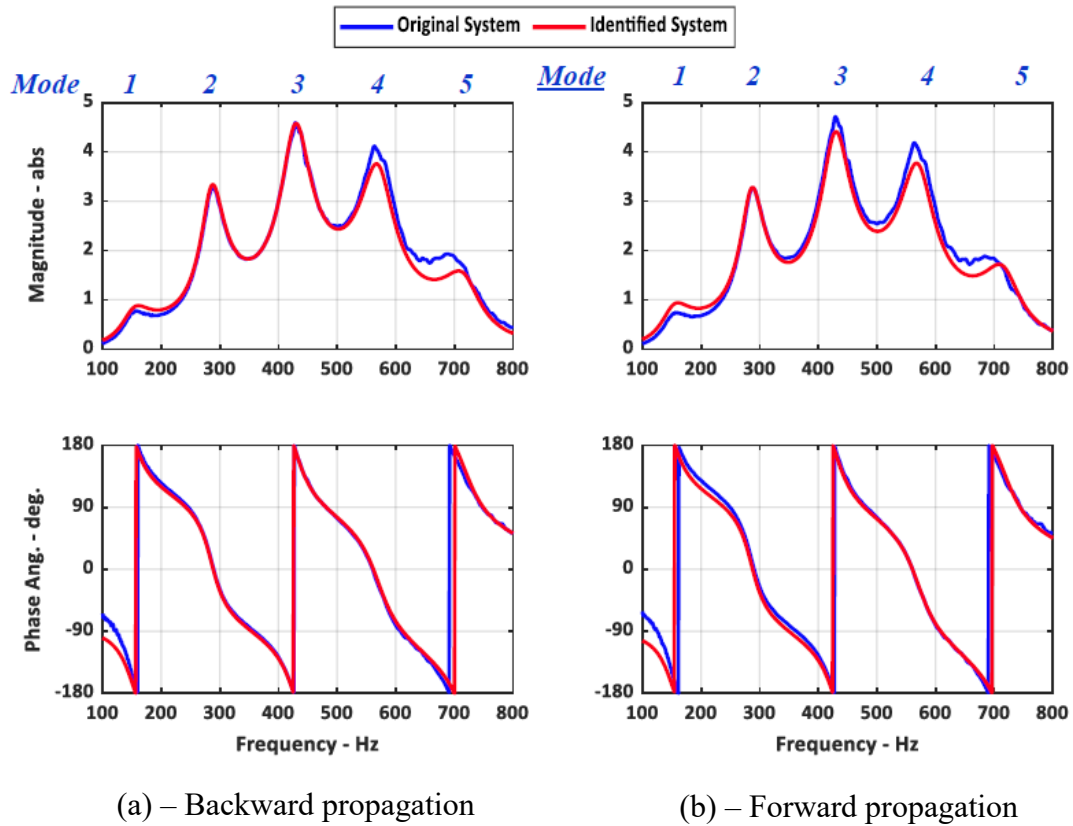


Figure 4.9 Comparison of backward and forward frequency response between original system and identified system

Figure 4.10 displays a sample of the comparisons between the frequency response of the original system and identified system for six different pairs of the inputs

(mini-speakers 1-6) and output of the system at microphone 1. The figure indicates a close agreement between the characteristics of the identified model and the original system, shown in Figure 4.10, can be further reduced by increasing the order of the model to be higher than 10. This would result in an adverse effect of the speed of implementation and bandwidth of the controller.

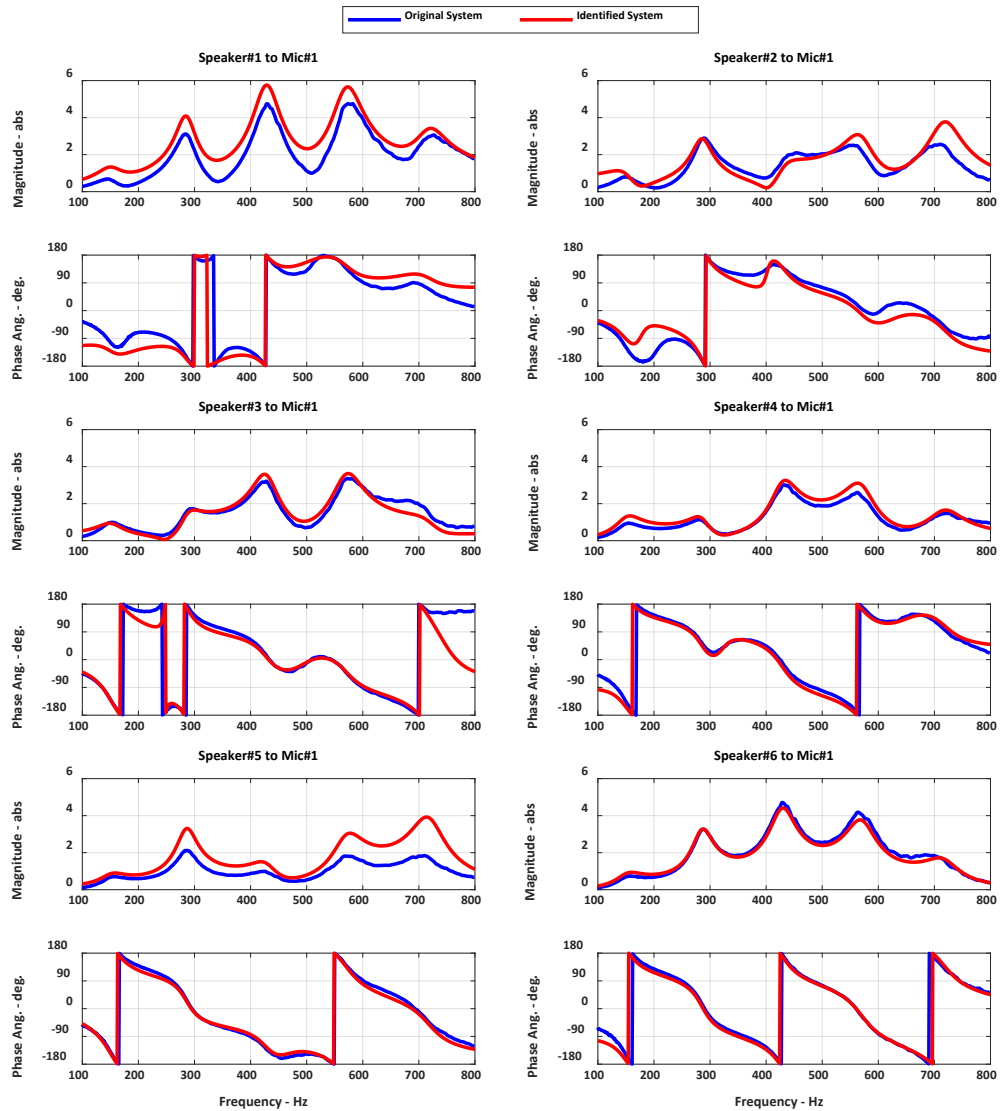


Figure 4.10 Comparison of frequency response between original system and identified system (speakers 1-6 to microphone 1)

4.6 Performance of the Controlled System

Figure 4.11 shows the frequency response of forward and backward wave propagation for the system with and without the controller. The forward response is defined by measuring the response of microphone 6 with an excitation of mini-speaker 1. Also, the backward response is defined by measuring the response of microphone 1 with an excitation of mini-speaker 6.

The figure emphasizes the amplification feature of the forward propagation and the attenuation characteristics of the backward propagation as outlined in Figure 5.1 and conforming with the desired eigenvector profiles displayed in Figure 4.8

Also, the figure identifies and displays the dominant modes of both the uncontrolled and controlled systems, which conform with the results listed in Table 4.2.

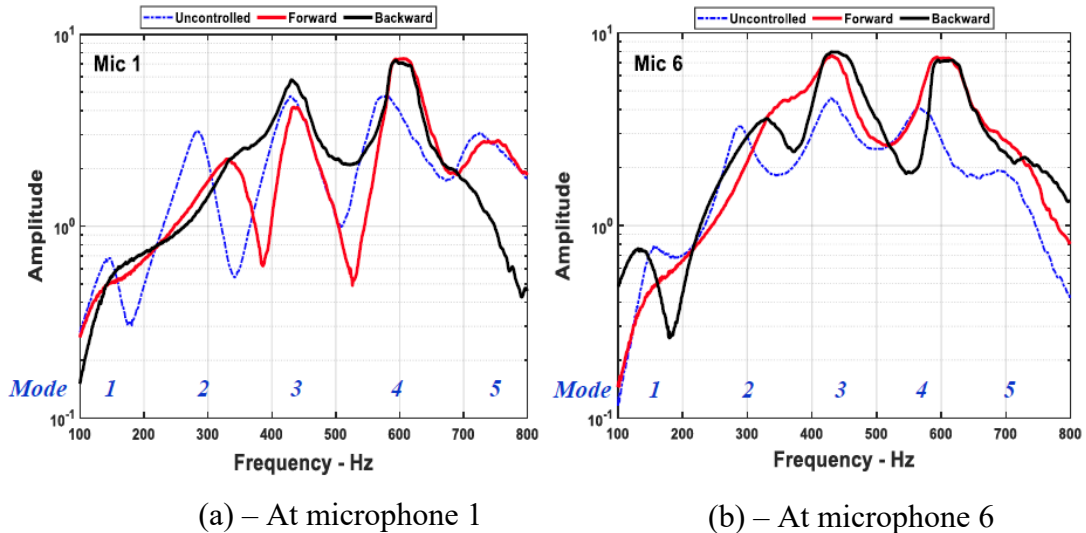


Figure 4.11 Frequency response of forward and backward wave propagation for uncontrolled and controlled systems

Note also that the uncontrolled system is *reciprocal*, whereas the controlled system is *non-reciprocal*, as the forward and backward wave propagations are distinctly different.

Figure 4.12 shows the distribution of the pressure along the duct during the forward and backward wave propagation for the system with the eigen-structure controller when the excitation is at 288 Hz. Again, the figure emphasizes the amplification generated during the forward propagation and the attenuation obtained during the backward propagation.

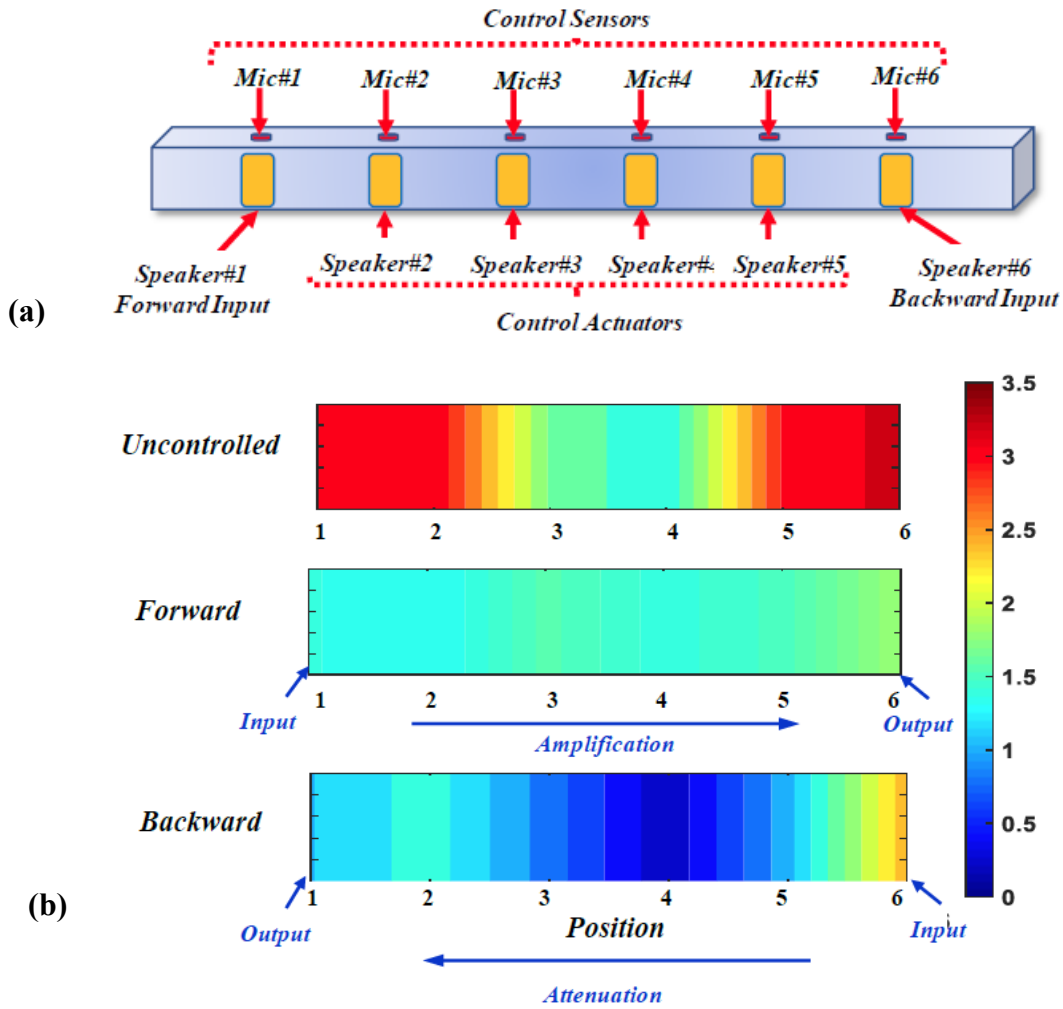


Figure 4.12 Experimental distribution of the pressure along the duct during forward and backward wave propagation at excitation frequency of 288 Hz

Figure 4.13 shows the distribution of the pressure along the duct during the forward and backward wave propagation for the system with the eigen-structure controller when the excitation is at 429 Hz. Again, the figure emphasizes the amplification generated during the forward propagation and the attenuation obtained during the backward propagation.

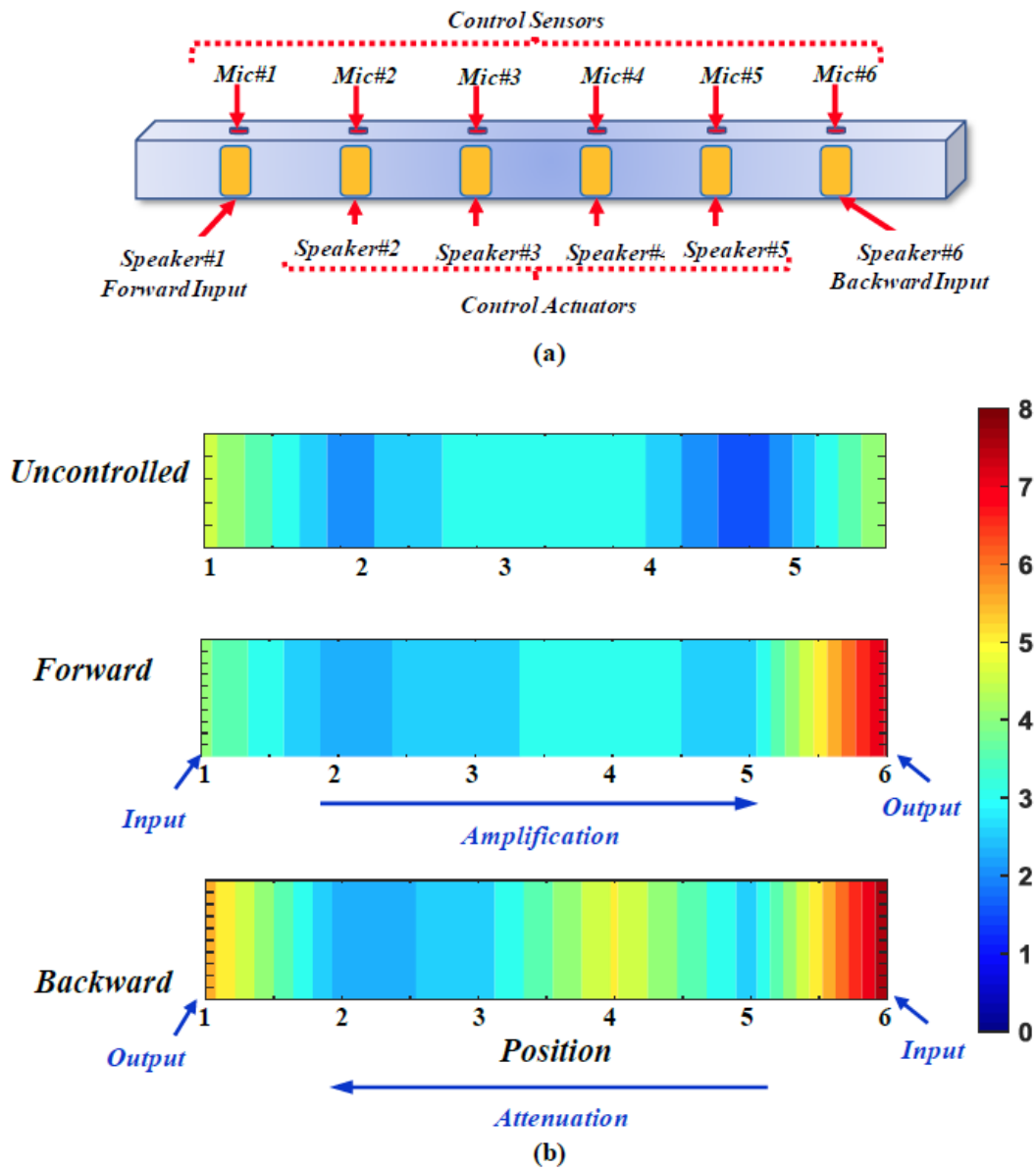


Figure 4.13 Experimental distribution of the pressure along the duct during forward and backward wave propagation at excitation frequency of 429 Hz

Figure 4.14 shows the distribution of the pressure along the duct during the forward and backward wave propagation for the system with the eigen-structure controller when the excitation is at 562 Hz. Again, the figure emphasizes the amplification generated during the forward propagation and the attenuation obtained during the backward propagation.

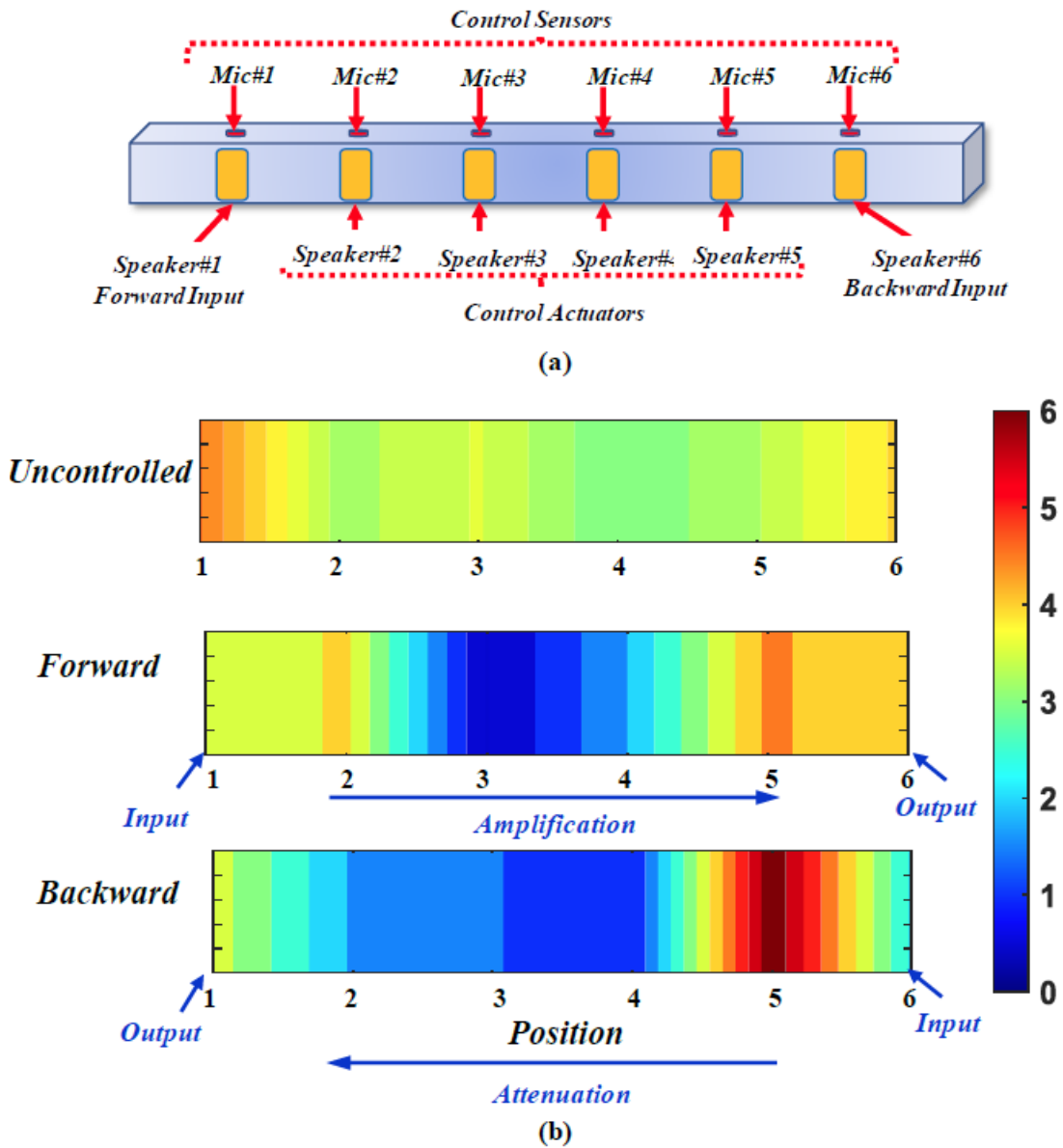


Figure 4.14 Experimental distribution of the pressure along the duct during forward and backward wave propagation at excitation frequency of 562 Hz

Figure 4.15 displays the detailed frequency response during the forward direction for both controlled and uncontrolled systems at the different locations along the duct.

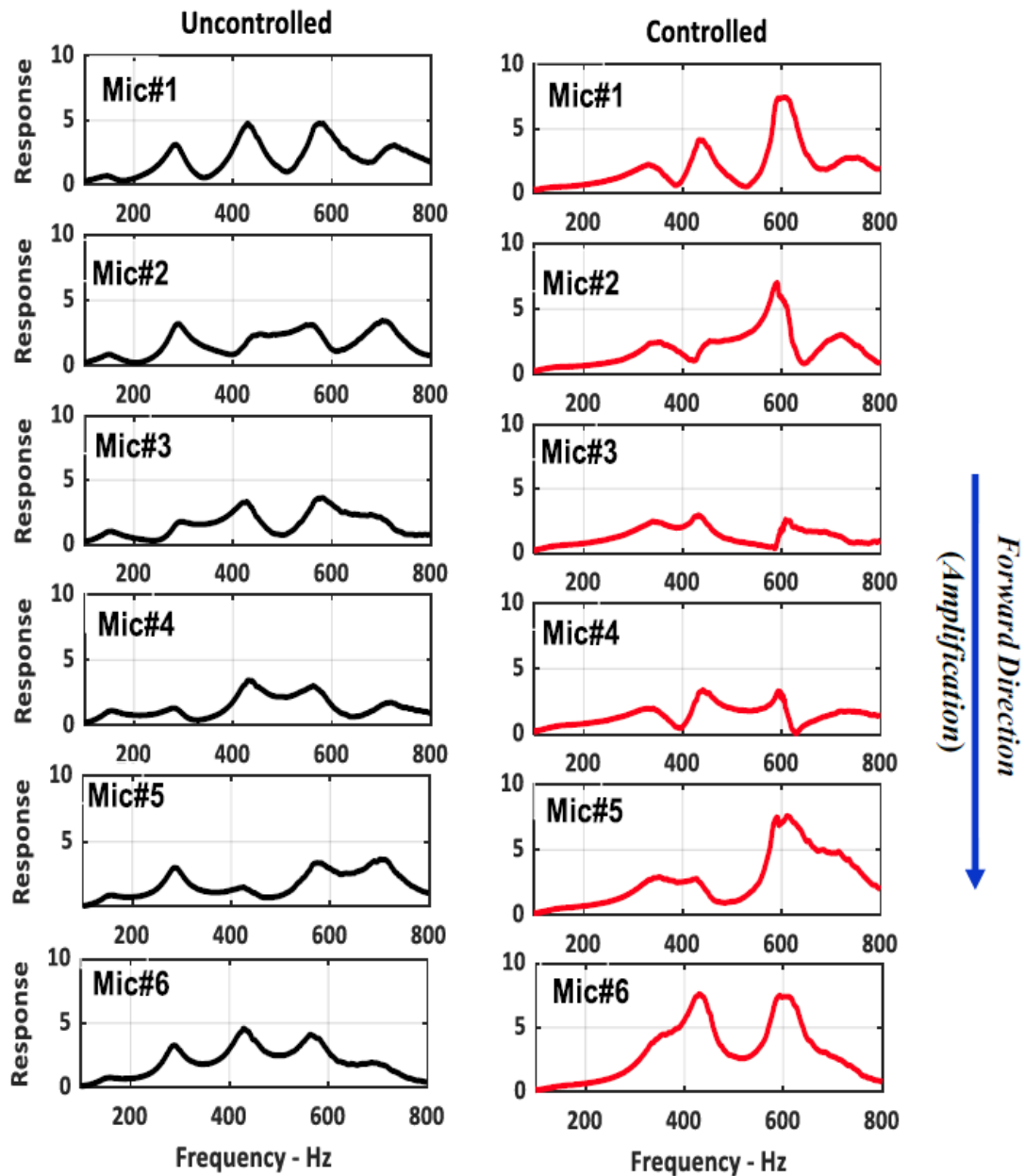


Figure 4.15 The frequency response characteristics along the duct during forward direction for both the uncontrolled and controlled systems

Figure 4.16 displays the corresponding control voltages provided to each mini-speaker during the forward propagation direction.

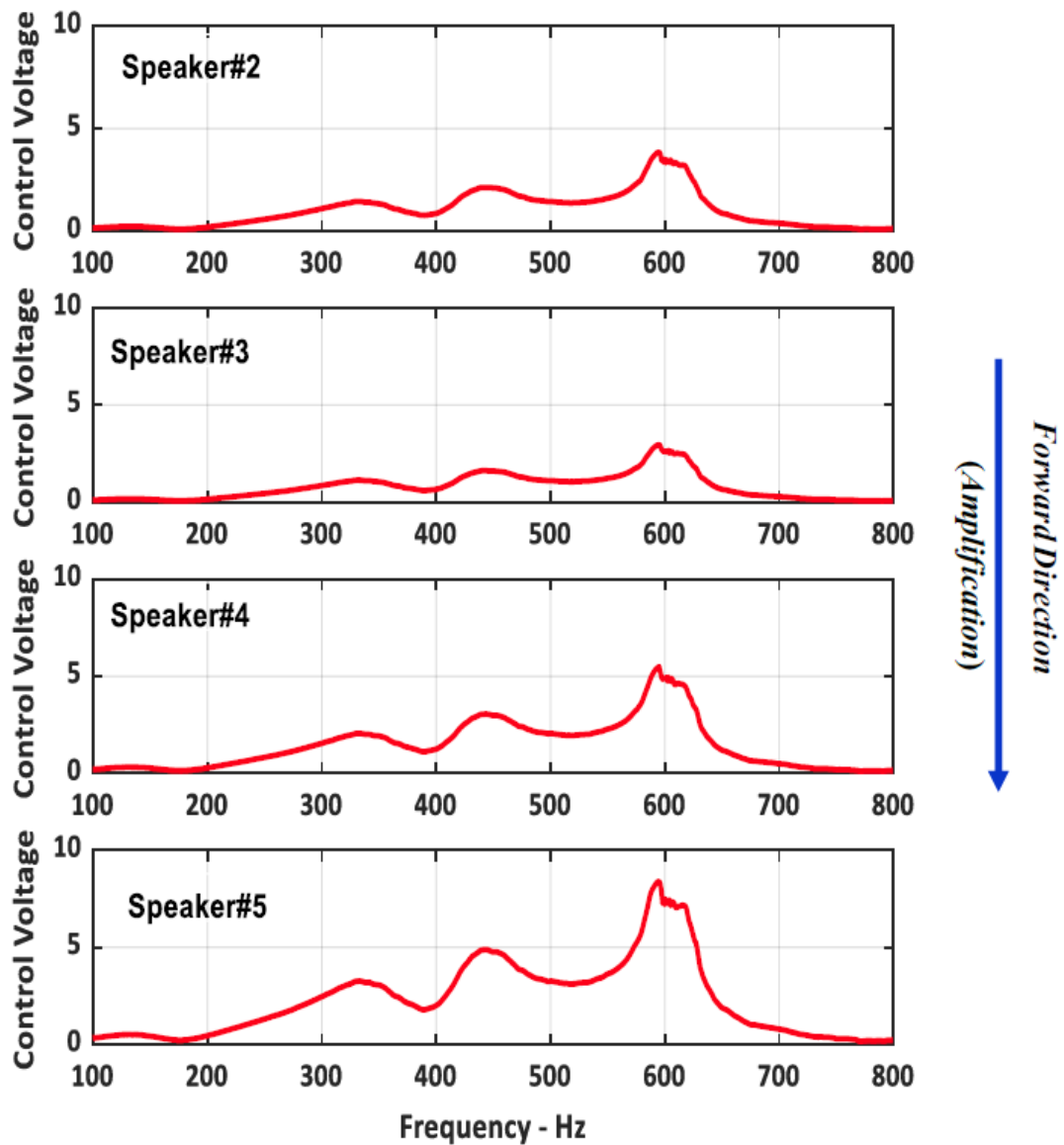


Figure 4.16 Control voltage provided to the mini-speakers during forward propagation direction

Figure 4.17 displays the detailed frequency response during the backward direction for both controlled and uncontrolled systems at the different locations along the duct.

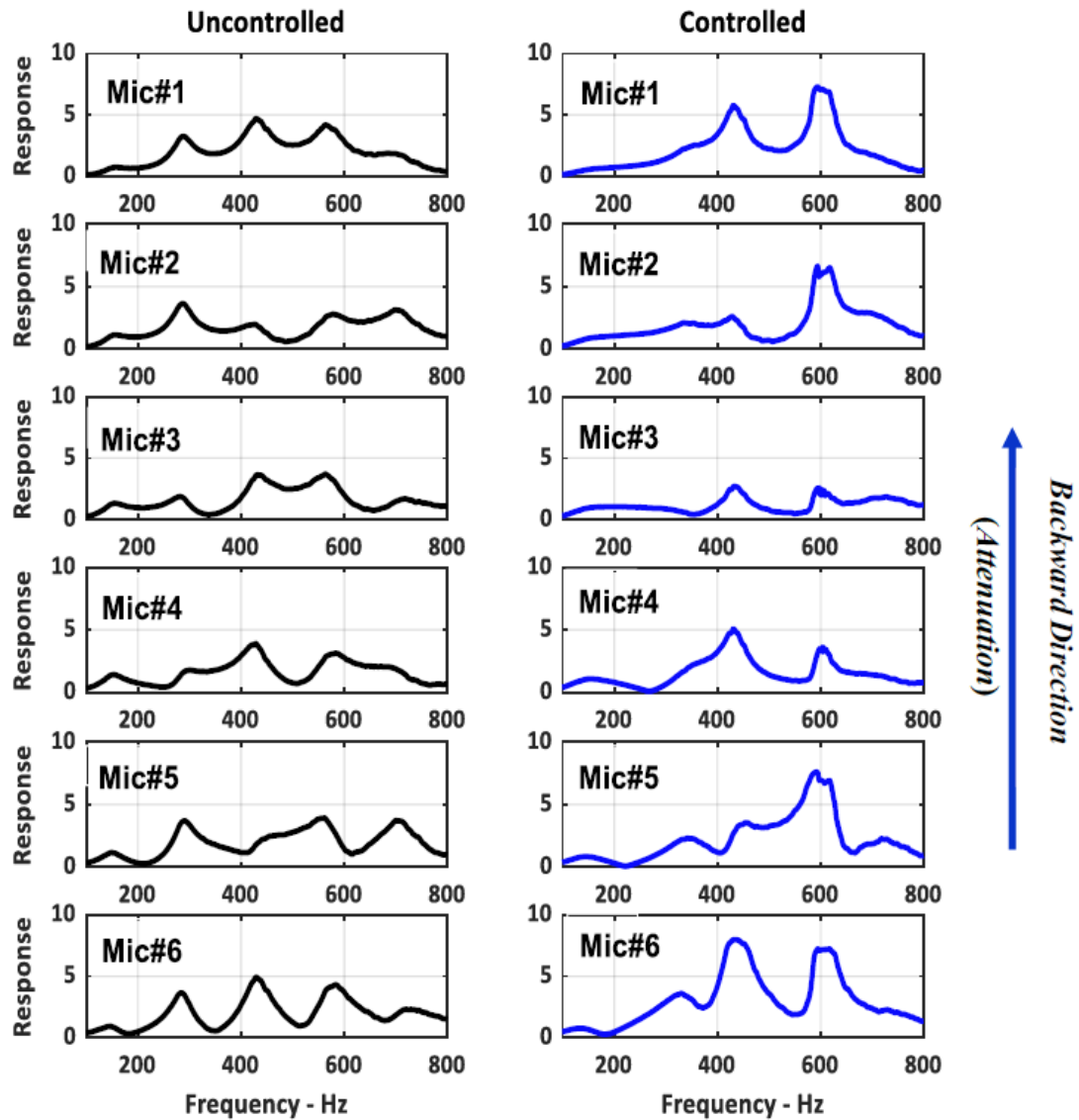


Figure 4.17 The frequency response characteristics along the duct during backward direction for both the uncontrolled and controlled systems

Figure 4.18 displays the corresponding control voltages provided to each mini-speaker during the backward propagation direction.

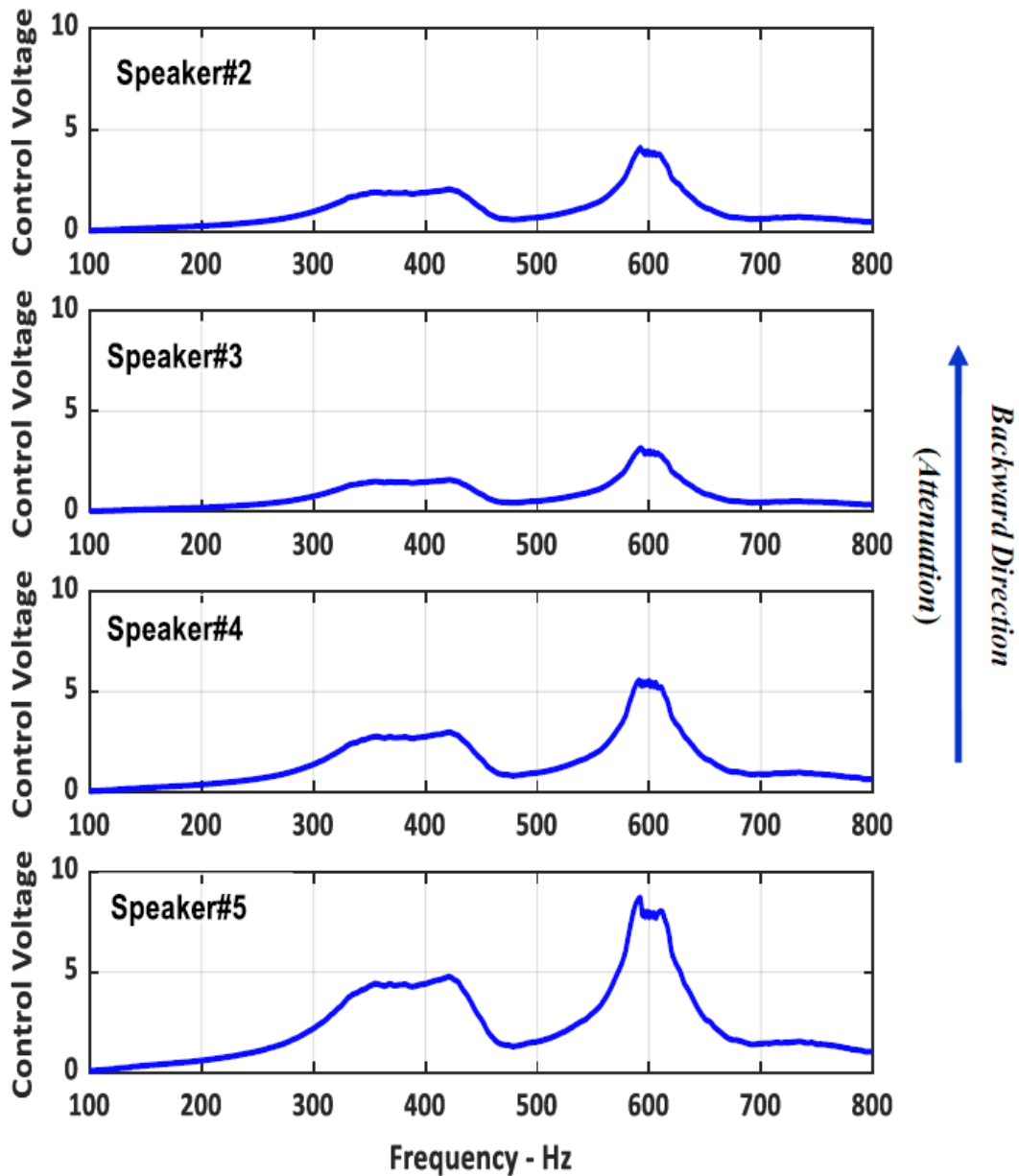


Figure 4.18 Control voltage provided to the mini-speakers during backward propagation direction

In this section, the detailed performance characteristics of the *ANMM* system are monitored experimentally and presented in both the time and frequency domains for various locations along the system. The reported results demonstrated clearly the consistent and effective control of the energy of the wave propagation along the forward and backward directions. More importantly, the displayed results clearly demonstrate the development of non-reciprocal wave propagation patterns along the *ANMM* duct. It is also shown that, with appropriate shaping of the eigenvectors, the forward propagation results in amplification of the propagating wave energy along the propagation direction. At the same time, the propagation along the backward direction yields extensive attenuation of the transmitted wave energy. These experimental patterns conform with the initially selected and shaped profiles of the eigenvectors as outlined in Figures 4.1(c) and 4.1(d).

These unique characteristics of shaping the energy transmission along the *ANMM* acoustic duct are demonstrated for various excitation frequencies (288, 429, and 562 *Hz*) as shown in Figures 4.12, 4.13, and 4.14, respectively. These characteristics are monitored for both forward and backward propagations in comparison with the uncontrolled *ANMM* duct. The non-reciprocal propagation is clearly seen in all these figures. Furthermore, the intended amplification along the forward direction and the attenuation along the backward direction are evident for all the considered frequencies.

It is important to note that the control effort associated with the implementation of the proposed eigen-structure-based controller is monitored for forward and

backward propagations, indicating that effective non-reciprocal behavior is achieved by activating the control speakers with voltages less than 10 V .

4.7 Transmission Loss Characteristics of the Controlled System

The transmission loss (TL) developed by the uncontrolled duct system is displayed in Figure 4.19(a) as measured by using the transmission tube shown in Figure 4.4. The corresponding TL of the controlled system is shown in Figure 4.19(b) for forward and backward propagations.

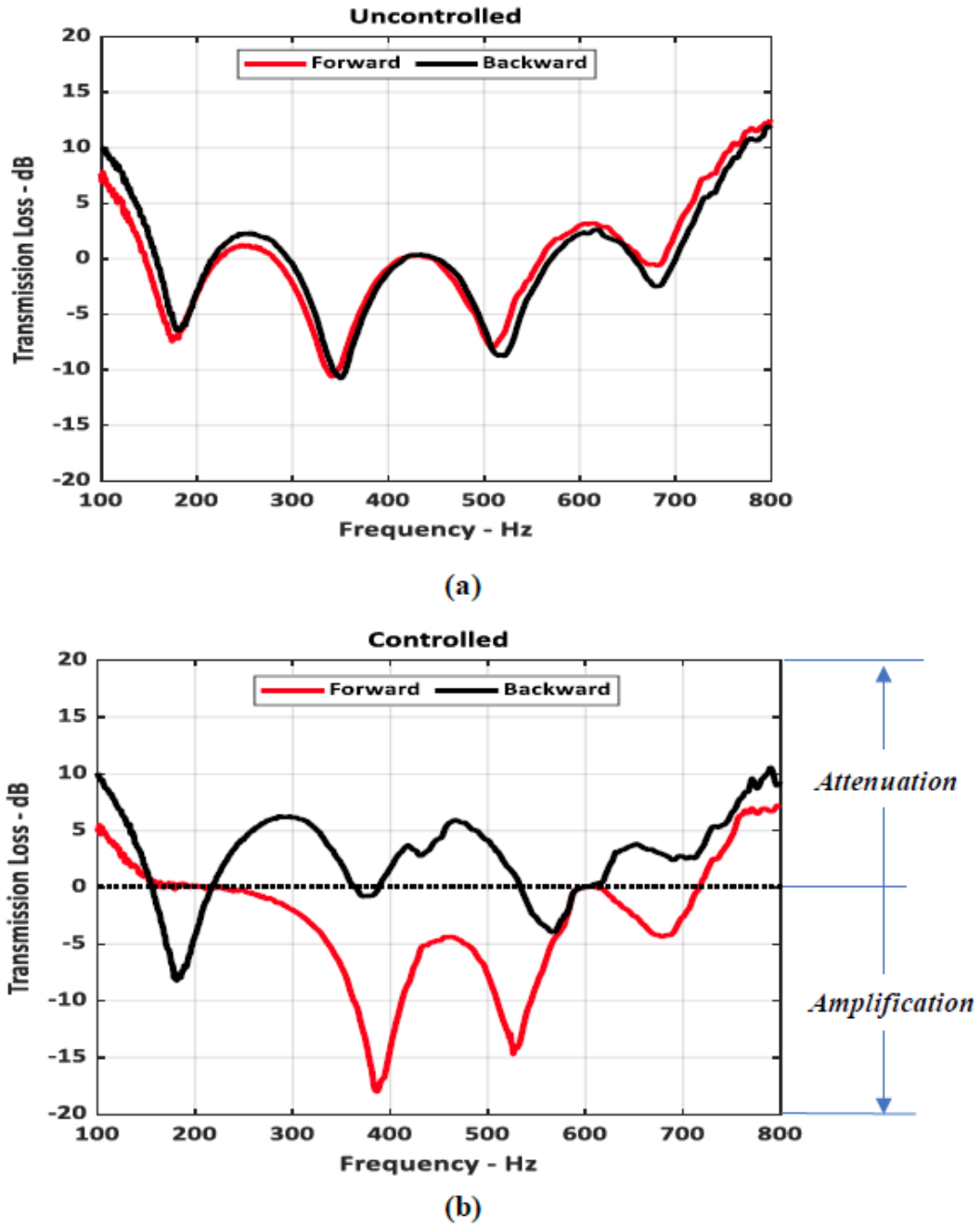


Figure 4.19 Transmission loss during uncontrolled and controlled propagations for forward and backward directions

4.8 Summary

In this chapter, we present a comprehensive experimental realization of a class of *ANMM* acoustic duct that is provided with a controller designed by simultaneous shaping of both the eigenvalues and eigenvectors of the closed-loop system. In this manner, a complete control of the wave propagation can be achieved both temporally and spatially to achieve any desirable non-reciprocal behavior and wave propagation patterns. For example, appropriate attenuation can be generated in one propagation direction, whereas amplification can be achieved in the opposite direction completely based on the user's demand and in a programmable fashion.

Detailed experimental identification, design, and implementation of an actively controlled acoustic duct have been presented to demonstrate the basic features, non-reciprocal behavior, and energy flow characteristics.

The experimental effort includes a multi-input and multi-output (*MIMO*) controller, which demonstrates the viability of the developed concepts in controlling the flow of acoustic energy in an acoustic duct. The appropriate identified model is found to have an order $n = 10$ with six sensors (*i.e.*, microphones) and four actuators (*i.e.*, mini-acoustic speakers). The selected order of the identified model ensures a balance between the accuracy of the model in capturing the experimental system's dynamics and the computational efficiency of implementation of the controller in real time.

The unique characteristics of the proposed eigen-structure-based controller in shaping the energy propagation along the experimental *ANMM* acoustic duct are demonstrated for various excitation frequencies coinciding with the system modes of

oscillations. These characteristics are monitored both forward and backward propagations in comparison with the uncontrolled *ANMM* duct. The non-reciprocal propagation is clearly seen in Figure 4.19, which summarizes in a comprehensive manner the main results of this chapter. The figure indicates that the intended amplification along the forward direction and the attenuation along the backward direction are evident for the considered frequency range that covers the first five modes of oscillations. In the figure, it is shown that the forward wave propagation results in amplifications reaching 15-20 *dB* for the third and fourth modes 5 *dB* for the fifth mode. Furthermore, the figure shows also that the backward wave propagation results in attenuations reaching 5-10 *dB* over the considered frequency range. The obtained results can be easily seen in relation to the behavior of the uncontrolled *ANMM*, which is shown in Figure 4.19(a). The figure emphasizes the reciprocity of the *ANMM* in the absence of the eigen-structure-based controller.

It is essential here to note that the effectiveness of the proposed eigen-structure-based controller is further demonstrated by the low control effort necessary for the implementation of the controller. This control effort is seen to be less than 10 *V* for activating the control speakers for both forward and backward propagations.

Chapter 5: Active Non-Reciprocal Metamaterial Using a Spatiotemporal Modulation Control Strategy

5.1 Introduction

In this chapter, we present a radically different class of active non-reciprocal metamaterial (*ANMM*) comprised of the acoustic duct with periodically placed active control diaphragms using spatiotemporal modulation control strategy. As mentioned in Section 1.2.4, acoustic non-reciprocities can be realized by modulating system's properties spatiotemporally [83-87]. However, most of the proposed non-reciprocal acoustic wave propagations with the method of spatiotemporal modulation are only theoretical, or can be experimentally investigated with a very low modulation frequency. Our proposed *ANMM* distinguishes itself from the abovementioned spatiotemporal modulation methods by introducing the actively tunable space-time modulated feedback gain of the controllers. By discretizing a *1D* acoustic duct into multiple acoustic unit cavities, we introduced a time-varying gain with a phase difference between adjacent acoustic cavities. Directional band gaps of the modulated system are numerically analyzed as the asymmetric acoustic wave propagation can be realized by converting the acoustical energy from the fundamental mode to higher order modes. In addition, non-reciprocal behavior of the proposed *ANMM* was experimentally demonstrated using a waveguide with periodically placed condenser microphones (sensors) and speakers (actuators).

5.2 Concept of the Active Non-Reciprocal Metamaterial

The proposed active nonreciprocal metamaterial (*ANMM*) consists of a one-dimensional (*1D*) array of periodic acoustic cavities with periodically placed active control diaphragms, as shown in Figure 5.1, for controlling the wave and energy propagation in an attempt to induce controllable and reversible non-reciprocal behavior where a unit acoustic cavity is defined as spatial modulation period which consisted of N sub-unit cavities.

Figure 5.2 displays the equivalent electrical analog that simulates the dynamics of the acoustic duct with periodically placed control diaphragms. In this figure, L_c and C_c denote the acoustic inductance (mass) and acoustic capacitance (stiffness) of each acoustic cell. Also, q_n , q_{cn} , and V_i denote the electrical charge (i.e., integral of mass flow rate) flowing through the inductance, the charge flowing through the capacitance, and the voltage (i.e., pressure) at the n th cell, respectively.

In Figure 5.2, the dynamics of the control diaphragms are neglected because of their high frequency response characteristics compared to the dominant and slow characteristics of the acoustic duct.

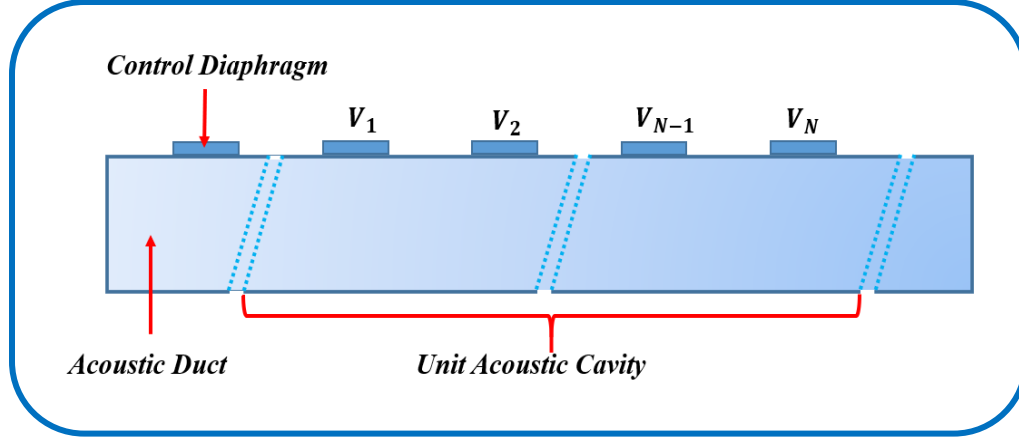


Figure 5.1 Acoustic duct with periodically placed control diaphragms

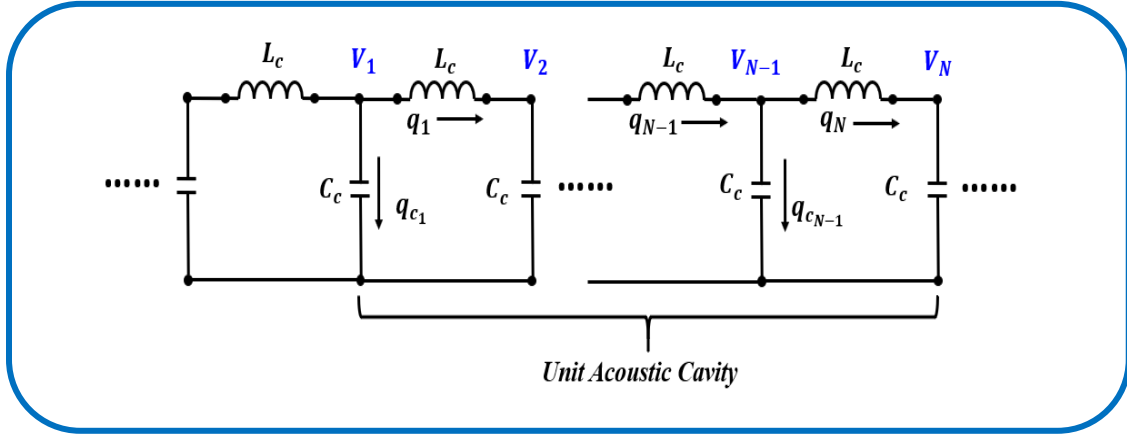


Figure 5.2 A lumped-parameter electrical analog of the dynamics of the acoustic duct with periodically placed control diaphragms

5.2.1 Dispersion Relations

As shown in Figure 5.1, the *ID* acoustic duct is discretized into finite acoustic cavities. For the n th cavity, it can be modeled by the following equation:

$$L_c \ddot{q}_n - \frac{1}{C_c} q_{n-1} + \frac{2}{C_c} q_n - \frac{1}{C_c} q_{n+1} = V_{cn} \quad (5.12)$$

where V_{cn} is the dynamic feedback control action of the n th sub-cell generated by the flush mounted diaphragms.

Due to the periodicity of the acoustic cavities, dynamic relations between adjacent cavities can be formulated by Bloch-Floquet theorem shown as following:

$$q_{n-1}(t) = e^{-i\mu} q_n(t), \quad q_{n+1}(t) = e^{i\mu} q_n(t) \quad (5.13)$$

where μ is the dimensionless wavenumber. For a unit cell, the equations governing the behavior of the controllable acoustic duct can be described as:

$$\ddot{Q} + \omega_0^2 A(\mu) Q = \frac{1}{L_c} V_c \quad (5.14)$$

where $\omega_0^2 = 1/L_c C_c$,

$$Q = \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \\ \vdots \\ q_N \end{Bmatrix}, \quad A = \begin{bmatrix} 2 & -1 & 0 & \dots & e^{-i\mu} \\ -1 & 2 & -1 & & \\ 0 & -1 & 2 & & \\ \vdots & & & \ddots & -1 \\ -e^{i\mu} & & & -1 & 2 \end{bmatrix}, \text{ and } V_c = \begin{Bmatrix} V_{c1} \\ V_{c2} \\ \vdots \\ V_{cn} \\ \vdots \\ V_{cN} \end{Bmatrix}$$

For illustrative purposes, the considered acoustic cavity for each cavity has a cross sectional area of $S_a=0.0001\text{m}^2$ and the length $L=0.01\text{m}$. When the cavity is filled with air, the capacitance $C_c = LS_a/(\rho c^2) = 7 \times 10^{-12} \text{ m}^4\text{s}^2/\text{kg}$ and the inductance $L_c = \rho L/S_a = 125 \text{ kg/m}^4$, with c denoting the sound speed along the cavity. If the considered cavity is excited with input pressure waves at a maximum frequency f_e for example, of 5 kHz, this results in a wavelength of 0.07 m, which is much larger than the cavity length, to justify the use of the lumped parameter modeling approach. According to Eqs. (5.1)-(5.2), dispersion relation for an uncontrolled duct was calculated within the one period of the wavenumber as $\mu \in [-\pi, +\pi]$ shown in Figure 5.3. The only one acoustic branch is non-dispersive between 0 - 7kHz and implies the reciprocal behavior of the unmodulated duct because of the symmetrical dispersion diagram.

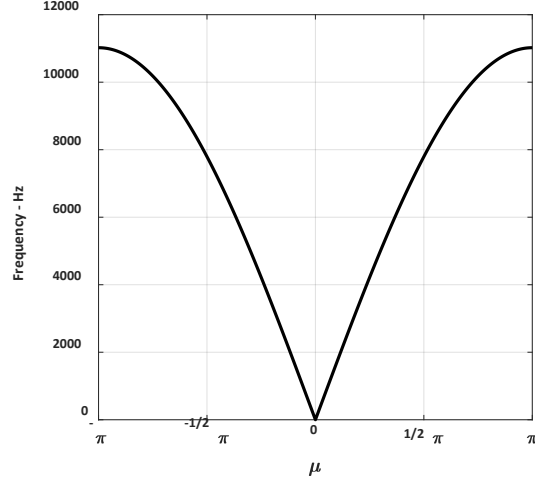


Figure 5.3 Dispersion characteristics of the uncontrolled acoustic duct

5.2.2 Space-Time Modulation

In order to break the symmetry of dispersion characteristics, the system is designed to be spatiotemporally and periodically modulated by the control diaphragms with time-varying feedback control gains. For n th sub-unit, the control effort can be denoted as following configuration:

$$V_{cn} = -\bar{k}_g \left(1 + \cos \left(\omega_m t + \frac{2\pi n}{N} \right) \right) q_n \quad (5.15)$$

where $\bar{k}_g$ is modulated amplitude of the feedback control gains, respectively. $\omega_m = 2\pi f_m$ and f_m is the modulation frequency.

By assembling the control effort from Eq. (5.4) for a single unit and substituting into Eq. (5.3):

$$\ddot{Q} + \omega_0^2 A(\mu) Q + \frac{1}{L_c} G(\mu, t) Q = 0 \quad (5.16)$$

where $G(\mu, t)$ is the modulated gain matrix which can be expressed as the following Fourier series:

$$G(\mu, t) = \sum_{k=-\infty}^{+\infty} G_k e^{ik\omega_m t} \quad (5.17)$$

where G_k is the corresponding Fourier coefficient of the gain matrix. It is noted that Eq. (5.5) is a system of Mathieu-Hill equations, where its stability region in regard of modulation parameters has been studied extensively [111-112]. In addition, unstable interactions of a time-space periodically-modulated medium was detailed explained by Cassedy [113-114]. as system can be potentially unstable with high frequency modulators. In this paper, stability of the system is ensured by adopting a relatively low modulation frequency, ω_0 , and an all-time negative feedback control as shown in Eq. (5.4).

According to the Hill-determinant method, a solution for the purposed modulated system is given by:

$$Q(t) = e^{i\omega t} \sum_{k=-\infty}^{+\infty} Q_k e^{ik\omega_m t} \quad (5.18)$$

Substituting Eqs. (5.6) and (5.7) into Eq. (5.5), and limiting the order of the solution to $[-M, M]$, dispersion relation can be determined by the following quadratic eigenvalue problem (*QEP*):

$$(-(\omega + k\omega_m)^2 + \omega_m^2 A(\mu))Q_k + \sum_{m=-M+k}^{M+k} G_m Q_{k-m} = 0 \quad (5.19)$$

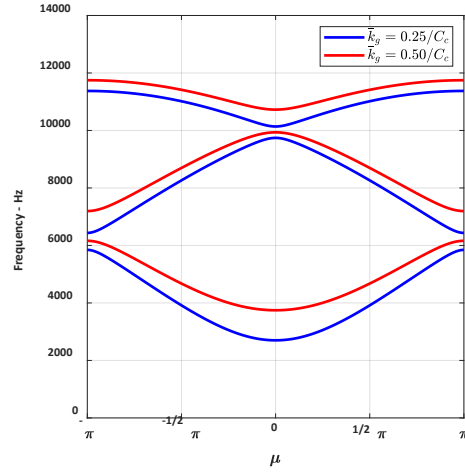
For this above *QEP*, the associated eigenvector is:

$$\bar{Q} = \begin{Bmatrix} \bar{Q}_{-M} \\ \vdots \\ \bar{Q}_0 \\ \vdots \\ \bar{Q}_M \end{Bmatrix}$$

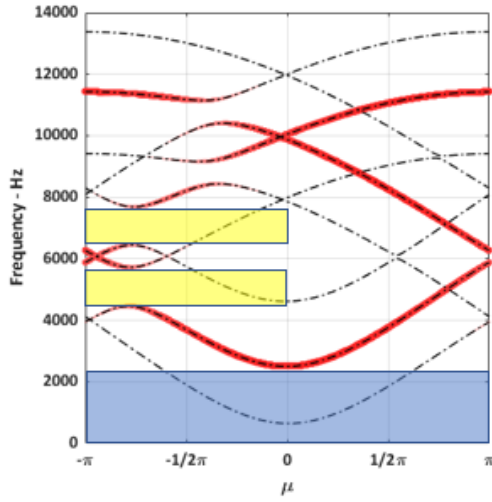
Consider the same system shown in Figure 5.3, firstly we consider a spatially-only modulated system with $N=3$, and Figure 5.4(a) displays system's dispersion relations with different $\bar{k}_g$. The system generates three bounded and complete stop bands which are adjustable by tuning the feedback gain for the reason that control efforts break the uniformity of the medium. Furthermore, the system is modulated with following parameters: $N=3$, $\bar{k}_g = 0.25/C_c$, and $f_m = 2000$ Hz. In Figures 5.4(b) and 5.4(c), the black dashed lines indicate the non-reciprocal dispersion relations for the order of $[-1, +1]$ of a 3-sub-cell unit. It can be seen that the dispersion relation becomes unsymmetrical with respect to the wavenumber $\mu=0$. At $\mu=0$, each spatially modulated branch (blue line in Figure 5.4(a)) was divided into 3 separate branches with a distance of the modulation frequency f_m which are corresponding limited harmonic orders. Clearly, by introducing a time modulated factor, we break the symmetry of the dispersion relation. Furthermore, the band structure of the fundamental mode (0^{th} order) for this controlled system can be filtered by weighting the corresponding eigenvectors from Eq. (5.8) as:

$$W = \frac{|\bar{Q}_0|}{\sum_{k=-M}^M |\bar{Q}_k|} \quad (5.20)$$

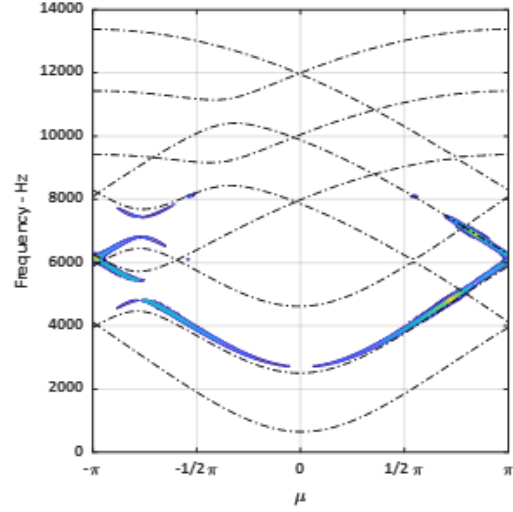
By applying a threshold value of the weighting function, such as $W>0.8$, red line in Figure 5.4(b) indicates the fundamental mode dispersion relation. By removing the redundant branches, a complete stop band (blue colored) was found to be in the range of $[0, 2500\text{Hz}]$ and multiple directional stop bands (yellow colored), for example, $[4461\text{Hz}, 5724\text{Hz}]$ and $[6443\text{Hz}, 7692\text{Hz}]$ as one-way wave propagation can be realized with an incident wave falls in certain frequency ranges.



(a) - Dispersion relation of a spatially modulated system for different $\bar{k}_g$



(b) - Comparison of the dispersion relations of the space-time modulated acoustic duct with fundamental mode by evaluating the eigenvectors fundamental modes



(c) - Comparison of the dispersion relations of the space-time modulated acoustic duct with 2D-FT contours of structural space-time responses fundamental modes

Figure 5.4 Dispersion characteristics of the spatially modulated and space-time modulated acoustic duct

For further validation of the dispersion relation, transient analysis was performed for the same modulated acoustic duct with 401 sub-cells, as the duct was assumed to be long enough to prevent end reflections. And broadband excitation with a centered frequency at 5000 Hz was introduced at the mid-space of duct. Dispersion characteristics was calculated by two-dimensional Fourier transform (*2D-FT*) from space-time responses of acoustic pressure of each sub-cell, $V(x,t)$:

$$V(\mu, \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} V(x, t) e^{-i(\mu x + \omega t)} dx dt \quad (5.21)$$

Figure 5.4(c) displays the *2D-FT* dispersion contour, which shows a good agreement with solution determined by Eqs. (5.8-5.9) shown in Figure 5.4(b).

5.2.3 Time Response Characteristics

In order to verify the band gap characteristics discovered in Figure 5.4, the space-time modulated duct was excited at the mid-span by a narrow band signal with a centered frequency of 5125 Hz, which locates inside the directional band gaps. Figure 5.5 displays the space-time responses of the acoustic potential energy flow for the spatiotemporally modulated system along the length of the duct. Non-reciprocal transmission is apparent as in forward direction, the wave propagates without any attenuation but in backward direction most energy is constrained near the location of excitation.

To further illustrating the non-reciprocal behaviors, Figure 5.6 shows the corresponding wavelet analysis of the cavity located at ± 0.25 m. away from the center of the excitation. Excitation frequency dominates the response in forward direction

while in the backward direction, fundamental mode response almost vanished as it converted to higher order harmonics.

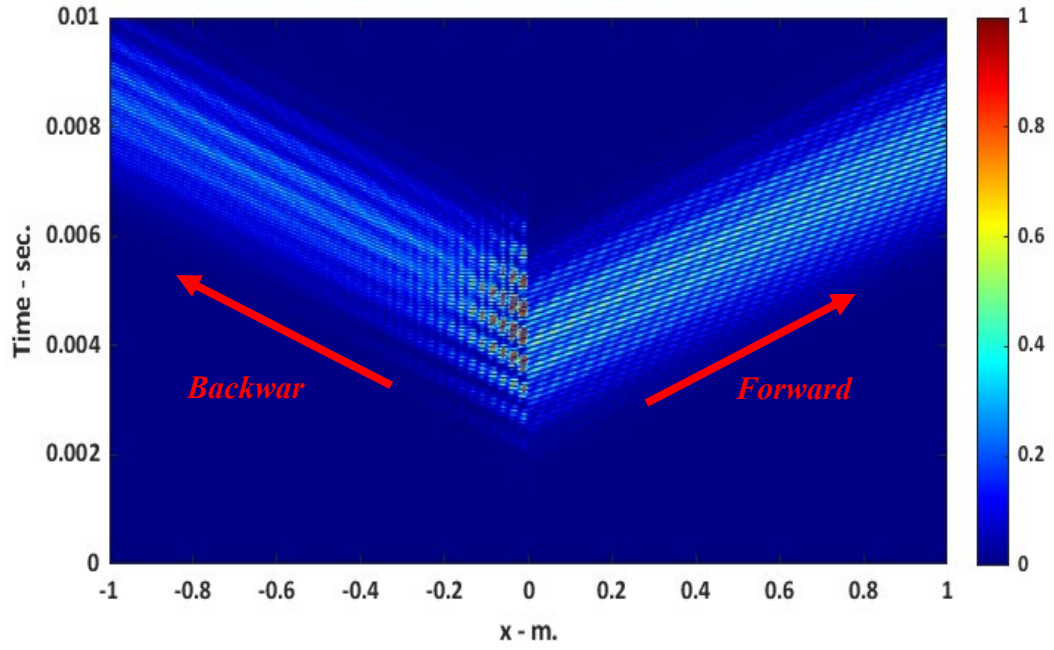
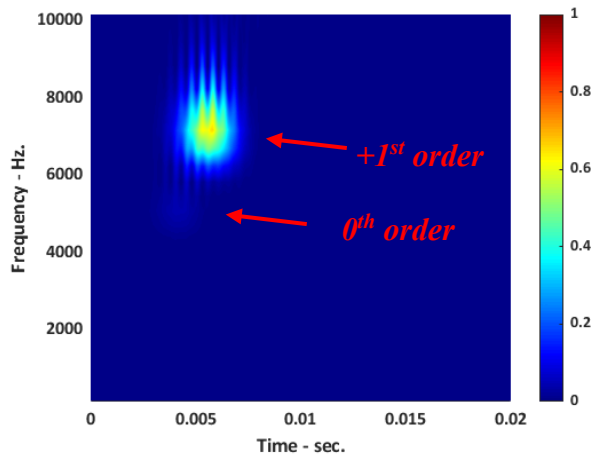
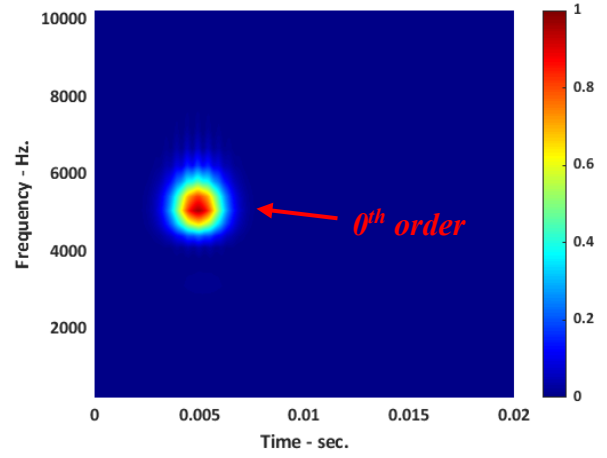


Figure 5.5 Space-time response of acoustic potential energy flow for the modulated duct ($\bar{k}_g = 0.25/C_c$, $f_m = 2000\text{Hz}$) with a narrow excitation frequency of 5.125 kHz



(a) Backward



(b) Forward

Figure 5.6 Wavelet analysis of space-time modulated system at 0.25 m. away from the center of the excitation ($\bar{k}_g = 0.25/C_c$, $f_m = 2000\text{Hz}$) with a narrow band excitation frequency centered at 5.125 kHz

5.3 Experimental Acoustic Duct

The proposed *ANMM* described in Section 5.2 is experimental demonstrated by a waveguide with periodically placed condenser microphones and speakers, which are served as sensors and actuators respectively. Non-reciprocal transmission characteristics were monitored by introducing a harmonic modulated gain between each pair of collocated sensor and actuator.

5.3.1 The Prototype

A prototype of an acoustic duct system is manufactured, as shown in Figures 5.7 and 5.8, with one speaker fixed at one end of the duct for generating incident wave, and collocated microphones and speakers that are equally spaced on the side of duct. Figure 5.7 shows the dimension of the duct, which is made of acrylic, and the location of sensors and actuators.

Fig. 8 shows the basic configurations of the acoustic duct with 6 mini-speaker controllers and 6 collocated condenser microphone sensors. In Fig. 8(a), the mini-speaker#0 is designed for exciting the waveguide and the mini-speaker#1-6 are designed for controlling the non-reciprocal characteristics of the wave propagation. Fig. 8(b) shows 6 collocated microphones which are used for measuring the sound propagation along with the waveguide.

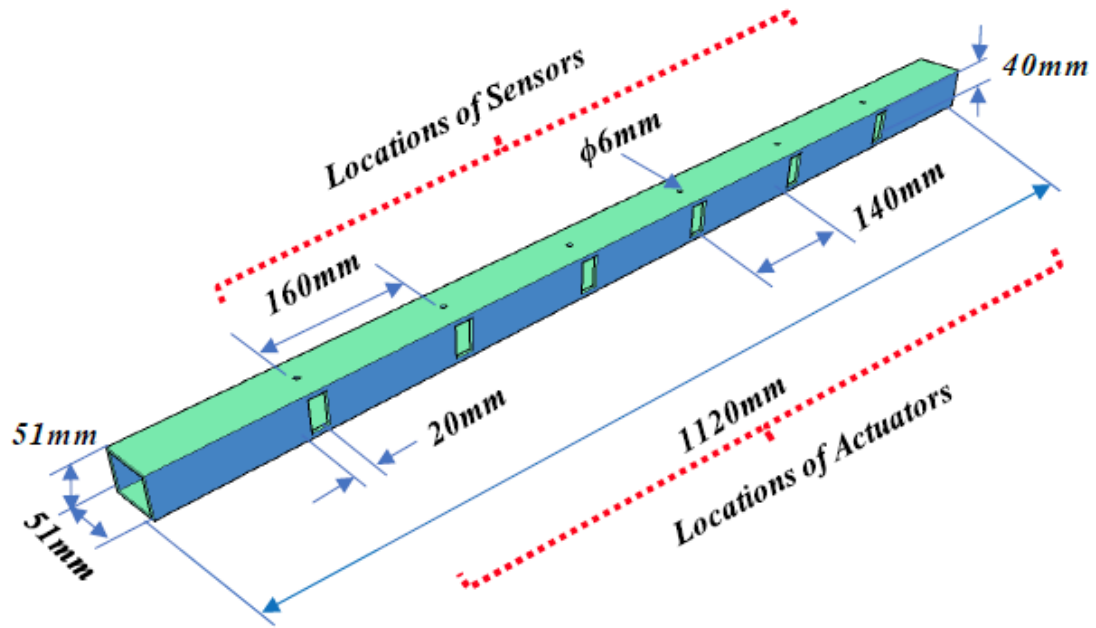
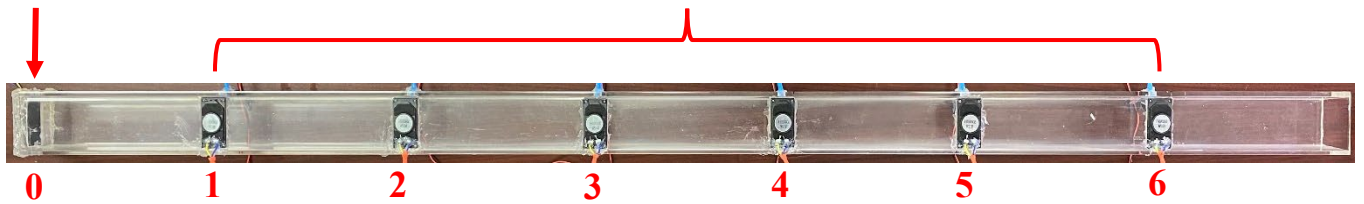


Figure 5.7 Geometrical dimension of the test acoustic duct

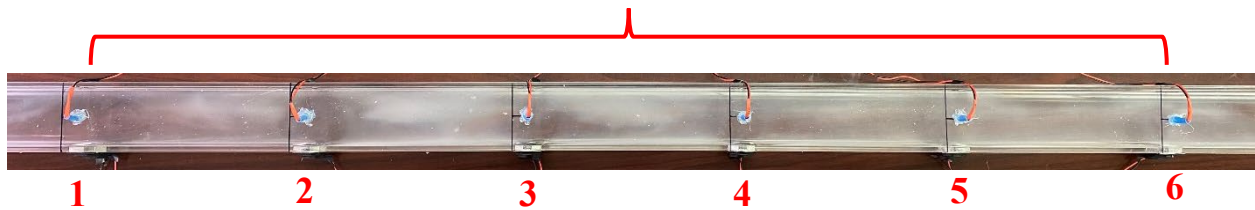
Input Excitation

Speaker Controllers



(a) – placement of speaker actuators

Condenser Microphones



(b) – placement of pressure sensors

Figure 5.8 Acoustic duct with periodically placed control diaphragms

5.3.2 Mini-Speakers and Microphones

The mini-speakers (*DB Unlimited*, Model SC400808-1) used in this chapter are the same model used in Chapter 3, which have the physical characteristics listed in Table 3.1. The acoustic pressure sensors are electret condenser microphones (*CUI devices*, model CMC-6015-47P). The main characteristics of the mini-speakers and properties of the microphones are provided in Appendix C.

5.3.3 Measurements and Control Circuits

Figure 5.9 displays the photographs of the experimental set-up of the controlled acoustic duct, where a pair of one mini-speaker and one condenser microphone constructs a sub-unit for the modulated system. Sound pressure along the waveguide was measured by the attached microphone which is connected to the pre-amplifiers with a gain of 10. Figure 5.10 displays the detail of the control circuit of the n th sub-unit, where the harmonic space-time modulated gains are provided by the function generators (*Texas Instruments*, F28379D C200 Delfina LaunchPad) as:

$$k_{g_n} = \bar{k}_g + \bar{k}_g \sin(\omega_0 t + \phi_n) \quad (5.22)$$

and the control output signals V_{cn} , are generated by the multiplication between gain signals, k_{g_n} , and microphones sensor signals, q_n , through the analog multipliers (*Analog Devices*, AD633ANZ) as following:

$$V_{cn}(t) = k_{g_n}(t)q_n(t) \quad (5.23)$$

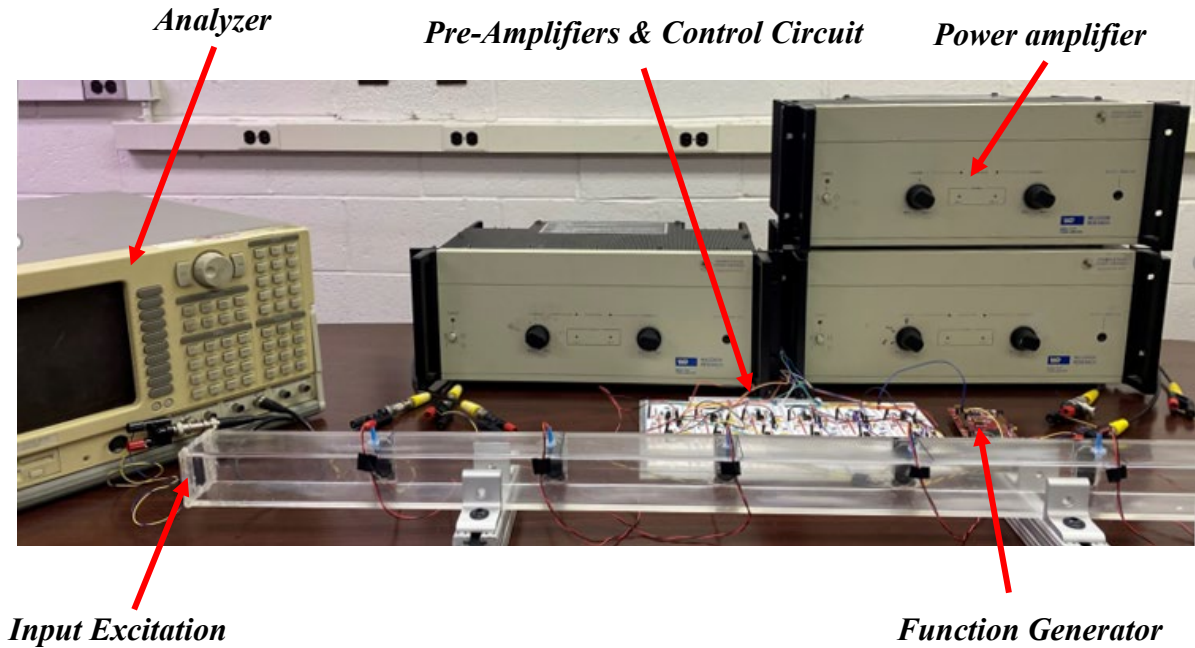


Figure 5.9 Photograph of the experimental acoustic duct with periodically placed control actuators and sensors

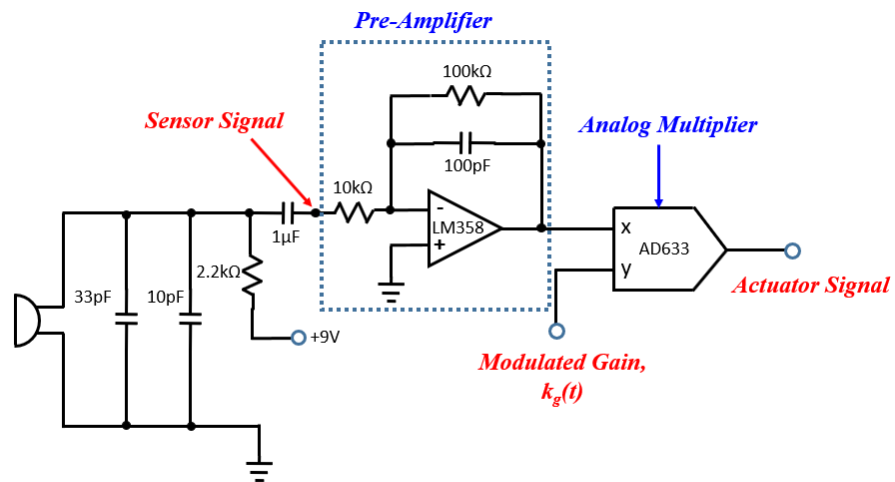


Figure 5.10 Detail of the pre-amplifier and control circuit

5.3.4 Experimental Performance of the Modulated System

As shown in Figure 5.9, experiments of the modulated waveguide were carried out by involving the space-time modulated feedback gain for the entire system. For each sub-unit, the time-varying gain value was set to be $k_{g_n} = 1 + \sin(\omega_0 t + \phi_n)$ with a modulation frequency of 200 Hz. The phase difference between adjacent sub-units is $\Delta\phi = \phi_n - \phi_{n-1} = \pm 2\pi/3$ such that 3 sub-unit cells form a single modulation unit. As the incident wave was generated from mini-speak#0 from one end of the duct, phase differences of forward and backward directions are defined by $\Delta\phi = +2\pi/3$ and $\Delta\phi = -2\pi/3$ respectively.

Figure 5.11 displays the fundamental mode frequency response in a range of [100Hz 1500Hz] of each microphone for forward and backward directions of a single unit modulation along with uncontrolled system, which is done by analyzer swept sine measurement. Between 1100Hz and 1250Hz, it can be seen that forward responses were barely changed but backward responses significantly dropped. Figure 5.12 displays the detailed frequency components of fundamental mode and related sub-order modes at the end of the unit cell with excitations of 800Hz and 1000Hz.

Similarly, Figures 5.13 and 5.14 display the fundamental mode and FFT responses of a double-cell modulation by making the full utilization of 6 sensors and actuators, and same directional band gap can be found in a range of [1100Hz 1250Hz].

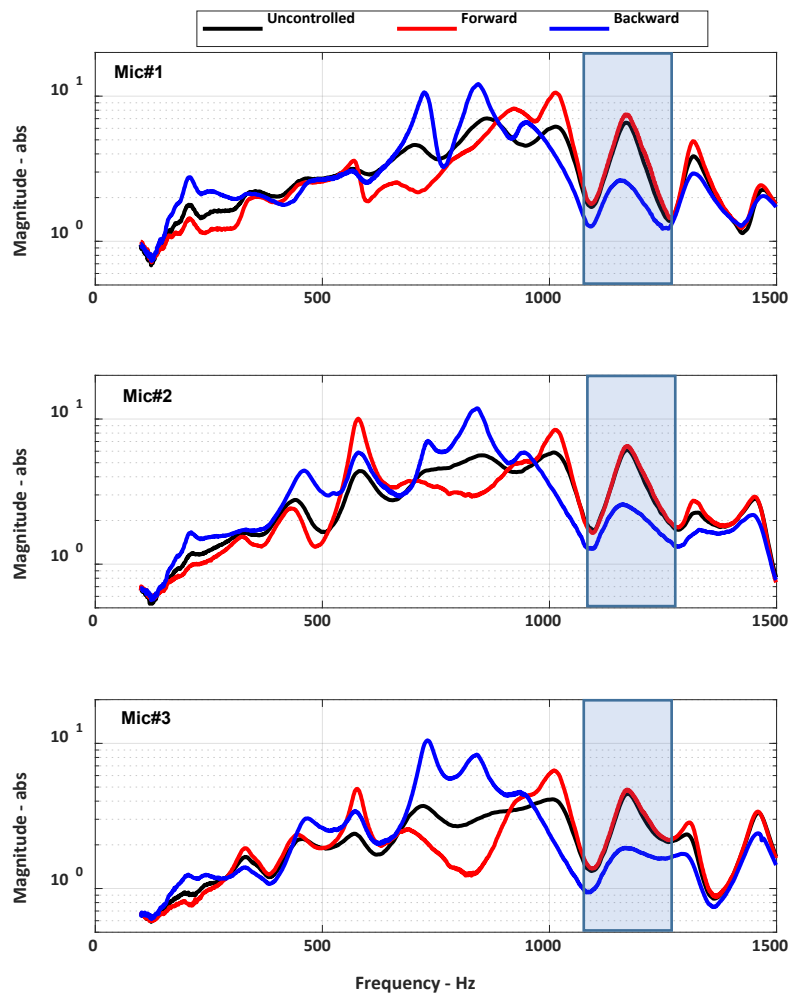
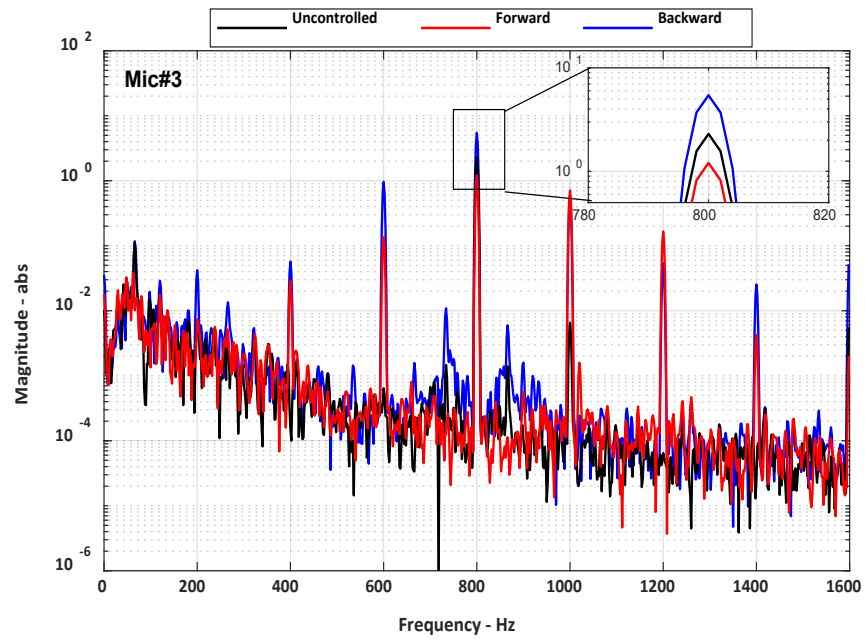
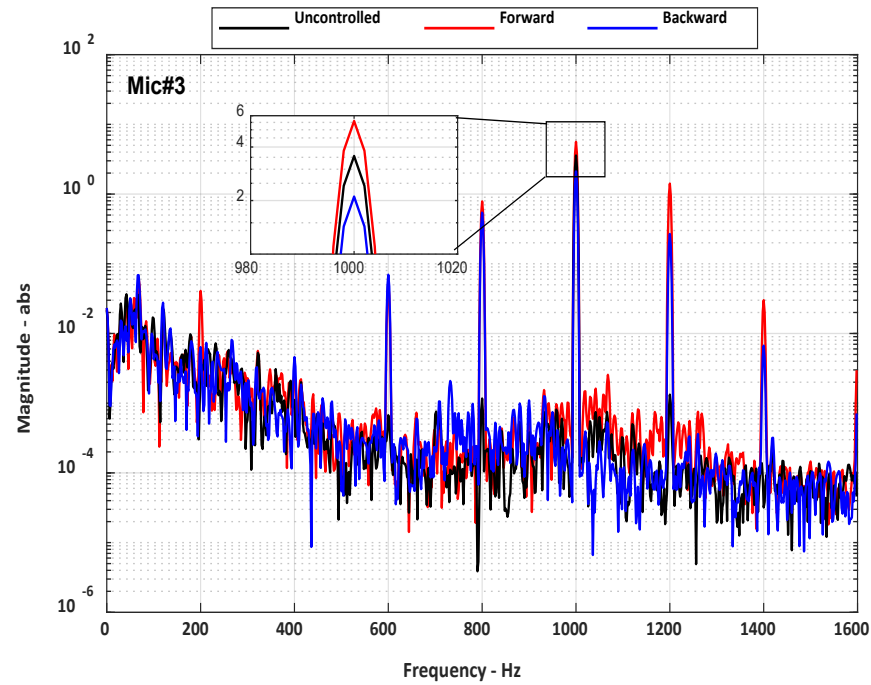


Figure 5.11 Fundamental mode frequency response for a single-unit-cell modulation



(a)



(b)

Figure 5.12 FFT response of Mic#3 for a single-unit-cell modulation with excitations of (a) 800Hz, and (b) 1000Hz

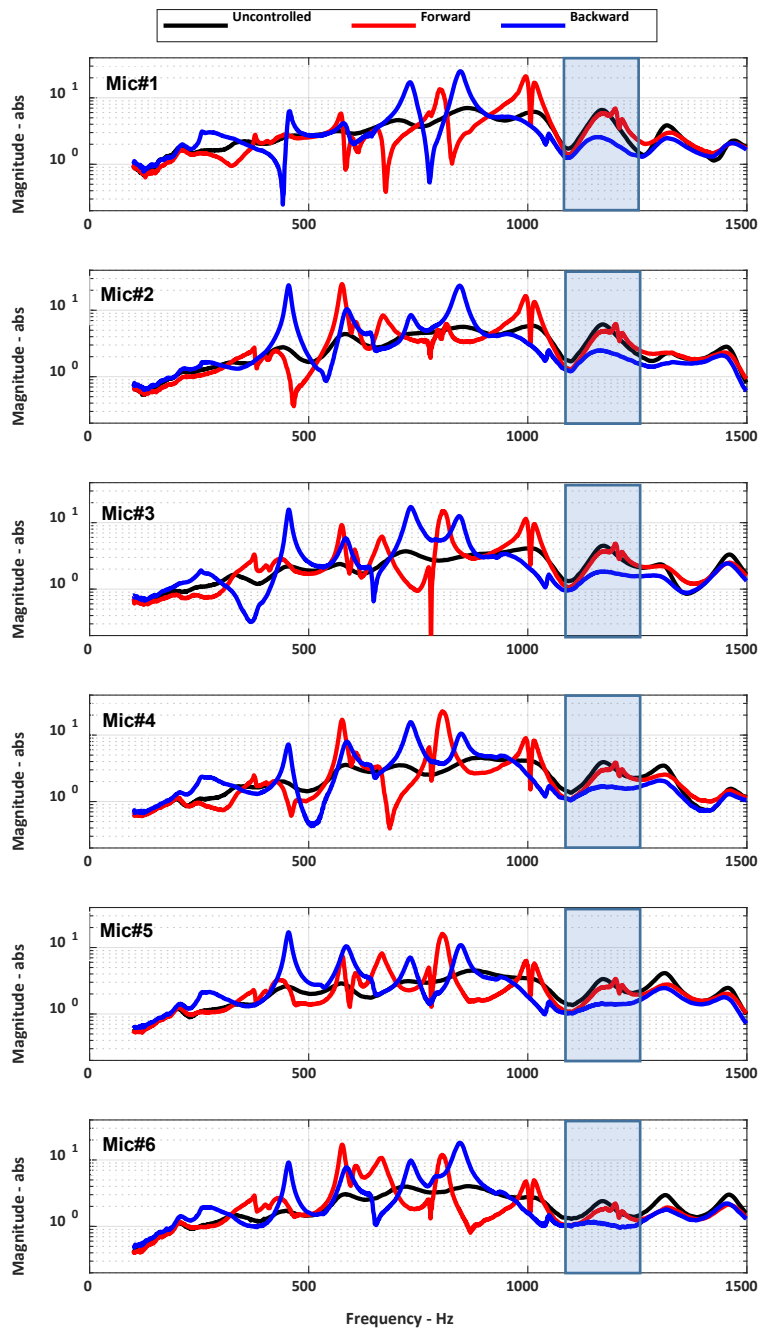
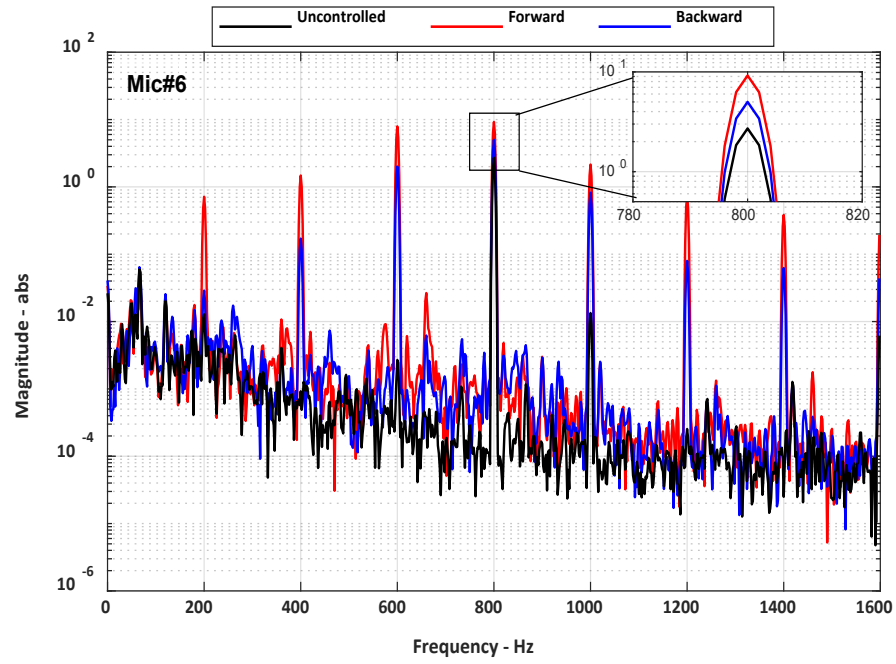
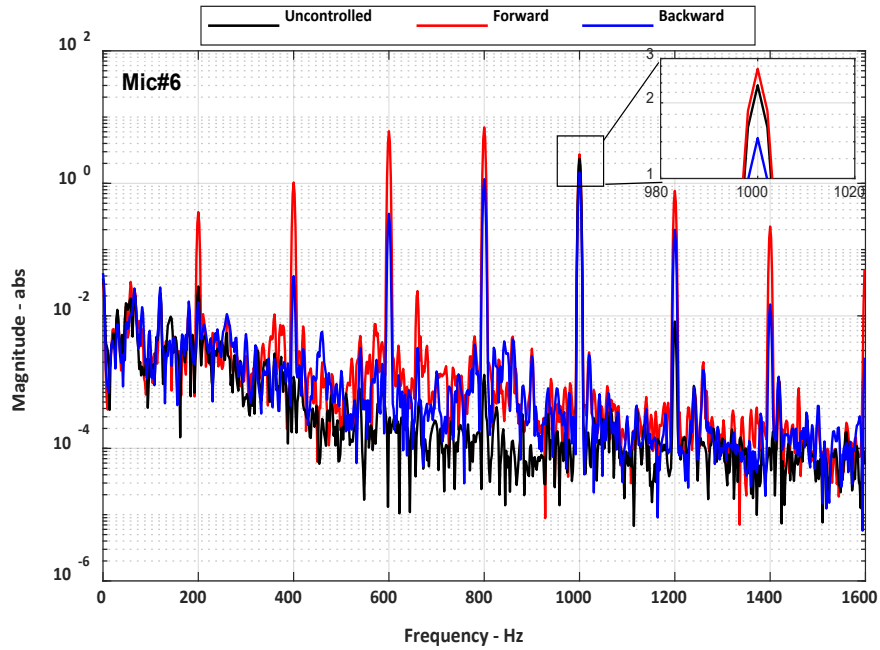


Figure 5.13 Fundamental mode frequency response for a double-unit-cell modulation



(a)



(b)

Figure 5.14 FFT response of Mic#6 for a double-unit-cell modulation with excitations of (a) 800Hz, and (b) 1000Hz

5.4 Summary

This chapter has presented a new class of active non-reciprocal metamaterial (*ANMM*) with theoretical and experimental realization. The proposed *ANMM* consists of a one-dimensional duct provided with an array of controllers integrated with space-time modulated feedback gains, which is designed for break the symmetry of the wave propagation.

Dispersion characteristics of the proposed *ANMM* have been derived based on a linear model of acoustic transmission lines in terms of the wavenumber, μ . For a spatial modulated waveguide, reciprocal stop band zones are found and tunable by adjusting the gain, k_g . By considering a harmonic time-varying gain, unsymmetrical dispersion relation of *ANMM* is identified and non-reciprocal stop band zones are developed, which are numerically validated by a transient analysis example.

Furthermore, experiments are conducted and demonstrated the unsymmetrical wave propagation behavior by shifting the phase of the time-varying gain in an acoustic duct. In certain frequency ranges, one-way propagation is allowed by converting the fundamental mode excitations to sub-order harmonics.

Also, the presented concept and controller design of the *ANMM* can be further extended to various critical structures to achieve realistic acoustic diodes configurations in a simple and programmable manner.

Chapter 6: Conclusions and Recommendations

6.1 Introduction

This chapter summarizes the main conclusions that have been arrived at following the comprehensive investigations of four active non-reciprocal acoustic metamaterial configurations. These configurations included actively synthesized gyroscopic metamaterial with and without dynamic controllers, active metamaterial with tailored eigen-structure assignments, and metamaterials with modulated space and time control capabilities.

The presentation of these new classes of active non-reciprocal metamaterials in such a comprehensive manner constitutes the main and original contributions of this dissertation.

6.2 Conclusions

In Chapter 2, the new class of active Nonreciprocal Gyroscopic Meta-Material (*NGMM*) has been introduced and investigated in a comprehensive manner both theoretically and experimentally when employing a dynamic controller. The material is shown to be capable of virtually synthesizing a gyroscopic control action in order to generate desirable non-reciprocal characteristics of tunable magnitude and direction. The proposed controller is demonstrated and shown to be capable of spatially control and redistribute the energy flow of the acoustic wave propagation along an active acoustic duct. During this entire process, the system remains behaving in a linear fashion.

The theory governing the operation of the proposed *NGMM* is developed along with establishing the necessary structure and proving the stability of the associated dynamic controller. The dispersion characteristics of the *NGMM* are derived for various gyroscopic parameters Ω_g and gyroscopic distribution matrices G . These characteristics quantify the pass and stop band zones as influenced by both Ω_g and G . The relationship between the dispersion characteristics and the classical Campbell diagram for conventional gyroscopic systems is established.

Experimental validation of the behavior of the *NGMM* is implemented demonstrated the effectiveness of the *NGMM* in spatial controlling the distribution of the wave propagation energy flow along an experimental active acoustic duct.

Detailed comparisons between the experimental characteristics and the theoretical predictions are presented revealing close agreements.

The configuration of the *NGMM* is simplified by eliminating the dynamic controller and the associated dummy cell to achieve a more practical configuration of this class of metamaterial. With such simplifications, it was possible to reveal the inherent capabilities and features of the *NGMM* that contribute to and explain, in a rigorous manner, the ability of the synthesized gyroscopic controller in breaking the reciprocity. The extensive theoretical and experimental study of this class of simplified *NGMM* is presented in Chapter 3.

The developed theoretical model of the meta-cell is utilized to establish the basis for breaking the non-reciprocity by analyzing the interaction of the poles and zeros of the transfer function. These interactions are shown to develop a unique “*zero migration*” phenomenon. This phenomenon is shown to lead to phase lead and

attenuation during forward propagation and phase lag and amplification during backward propagation. Such phase shift and amplitude modulation are the reasons behind breaking the reciprocity.

The theoretical predictions are validated experimentally using a simple configuration of the *NGMM* cell which eliminates the need for a dummy cell as carried out by Raval *et al.* [110]. Such a preferred arrangement when coupled with an analog controller is shown to enable fast execution of the controller and demonstrate the break of the non-reciprocity during forward and backward propagations. The characteristics of the non-reciprocity, as quantified by the extent of the phase shift and amplitude modulation, are shown to be primarily influenced by the gyroscopic gain.

Chapter 4 has presented a comprehensive theoretical and experimental investigation of a class of active nonreciprocal metamaterial (*ANMM*) acoustic duct which is provided with a controller designed by simultaneous shaping of both the eigenvalues and eigenvectors of the closed-loop system.

In this manner, a complete control of the wave propagation can be achieved both temporally and spatially to achieve any desirable non-reciprocal behavior and wave propagation patterns. For example, appropriate attenuation can be generated in one propagation direction whereas amplification can be achieved in the opposite direction completely based on the user's demand and in a programmable fashion.

Detailed experimental identification, design, and implementation of an actively controlled acoustic duct have been presented to demonstrate the basic features, non-reciprocal behavior, as well as the energy flow characteristics.

The experimental effort includes multi-input and multi-output (*MIMO*) controller which demonstrates the viability of the developed concepts in controlling the flow of acoustic energy in an acoustic duct. The appropriately identified model is found to have an order $n=10$ with six sensors (*i.e.*, microphones) and 4 actuators (*i.e.*, mini-acoustic speakers). The selected order of the identified model ensures a balance between the accuracy of the model in capturing the experimental system's dynamics and the computational efficiency of implementation of the controller in real time.

The unique characteristics of the proposed eigenstructure-based controller in shaping the energy propagation along the experimental *ANMM* acoustic duct are demonstrated for various excitation frequencies coinciding with the system modes of oscillations. These characteristics are monitored both for forward and backward propagations in comparison with the uncontrolled *ANMM* duct. The non-reciprocal propagation has been clearly demonstrated in a comprehensive manner which constitutes the main contribution of this approach. It is shown that the intended amplification along the forward direction and the attenuation along the backward direction are evident for the considered frequency range that covers the first five modes of oscillations. Further, it is shown that the forward wave propagation results in amplifications reaching 15-20dB for the third and fourth modes and 5dB for the fifth mode. Furthermore, the figure shows also that the backward wave propagation results in attenuations reaching 5-10dB over the considered frequency range. The obtained results can be easily seen in relation to the behavior of the uncontrolled *ANMM*. Such a comparison emphasizes the reciprocity of the *ANMM* in the absence of the eigenstructure-based controller.

It is essential here to note that the effectiveness of the proposed eigen-structure-based controller is further demonstrated by the low control effort necessary for the implementation of the controller. This control effort is seen to be less than 10 volts for activating the control speakers for both the forward and backward propagations.

Chapter 5 presented an active non-reciprocal class of *ANMM* which consists of a one-dimensional duct provided with an array of controllers integrated with space-time modulated feedback gains. This configuration of *ANMM* is designed to demonstrate theoretically and experimentally the break the symmetry of the wave propagation. This investigation is, to our knowledge, the first real *MIMO* implementation of the space-time modulation controller on a real acoustic duct.

The dispersion characteristics of the proposed *ANMM* have been developed using a linear model of acoustic transmission lines in terms of the wavenumber. For a spatially-modulated waveguide, the reciprocal stop band zones are extracted and tuned by adjusting the control gain, k_g .

By considering a harmonic time-varying gains, unsymmetrical dispersion relation of *ANMM* is identified and non-reciprocal stop band zones are developed, which are numerically validated by a transient analysis example.

Furthermore, experiments are conducted and demonstrated the unsymmetrical wave propagation behavior by shifting the phase of the time-varying gain in an acoustic duct. In certain frequency ranges, one-way propagation is allowed by converting the fundamental mode excitations to sub-order harmonics.

6.3 Recommendations for Future Studies

This dissertation has established new and practical directions in the development of active non-reciprocal acoustic metamaterials that are currently not known or yet been considered or pursued in the open-literature. The technology developed and presented in this dissertation definitely lay the foundation for a new era in the development of active non-reciprocal acoustic metamaterials (*ANAM*) and in particular active acoustic diodes.

In spite of the fact that such major development has been introduced, the present work has opened the door for numerous extensions for future studies that are needed to transfer such developed technology to the implementation in many critical applications.

Distinct among these extensions is the miniaturization of the *ANAM* and its associated control circuitry in order to achieve higher attenuations and/or transmission loss as well as extending the bandwidth of its applicability.

A more natural and pressing needed extension is to generate configurations of the *ANAM* that are applicable to two and three-dimensional applications. These configurations will be invaluable to numerous critical situations for both civilian and military applications.

The simulation efforts need also to improved considerably, as all the presented simulations are based on simplified and lumped-parameters models of the metamaterial systems. Simulations should be carried out on both *ANSYS* and *COMSOL* platforms to include fluid/structure/control interactions. With such capabilities, better designs of these new class of metamaterials can be achieved leading to better understanding of the associated physical interactions. More importantly, the performance characteristics of

these classes of metamaterials can be optimized to match more sophisticated and critical applications.

Furthermore, efforts should be exerted to include computer-controlled shunted networks as means for proving the gyroscopic, eigen-structure, and spatial modulation control actions. This can considerably simplify and speed-up the implementation of the controllers and further provide additional capabilities with synthesized components such as programable resistors and negative capacitances.

Additional efforts are needed to improve the measurements strategies by including sound intensity measurements, both along and over the metamaterial, in order to rigorously quantify the power flow along and over these meta-structures for both forward and backward propagation. Further measurements may be carried out using the new technology of “*acoustic camera*” which has been recently acquired in our laboratory.

With all the recommended extensions, it is envisioned that the important and cutting-edge technology developed in this dissertation can be transferred to direct implementation in many critical applications where it can have a direct impact on their silencing and improving their acoustic signatures.

Appendices

Appendix A: The Structure of the Virtual Gyroscopic Controller

The identified model presented in Section 2.3 implies that the simplified model of the meta-cell alone is given in the Laplace domain by:

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = a \frac{1}{s^2 + bs + c} \begin{bmatrix} -1 & d \\ d & -1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

In the time domain, the model becomes:

$$\begin{Bmatrix} \ddot{x}_1 + b\dot{x}_1 + cx_1 \\ \ddot{x}_2 + b\dot{x}_2 + cx_2 \end{Bmatrix} = a \begin{bmatrix} -1 & d \\ d & -1 \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (\text{A.1})$$

where $a = 1.37 \times 10^8$, $b = 334.8$, $c = 1.42 \times 10^7$, and $d = 0.537$.

A.1 Forward Propagation Model

Coupling the model with the virtual gyroscopic controller during forward propagation, with $F_2 = 0$, yields:

$$\begin{Bmatrix} \ddot{x}_1 + b\dot{x}_1 + cx_1 \\ \ddot{x}_2 + b\dot{x}_2 + cx_2 \\ \ddot{y}_1 + b\dot{y}_1 + cy_1 \\ \ddot{y}_2 + b\dot{y}_2 + cy_2 \end{Bmatrix} = a \begin{bmatrix} -(F_e - k_g \dot{y}_2) \\ d(F_e - k_g \dot{y}_2) \\ -k_g \dot{x}_2 \\ dk_g \dot{x}_2 \end{bmatrix} \quad (\text{A.2})$$

Not that x_1 and x_2 are the input and output of the meta-cell. Also, y_1 and y_2 are the input and output of the dummy meta-cell as illustrated in Figure 2.10(a). Also, F_e denotes the external input loading and k_g is the gyroscopic gain.

In an expanded matrix form, Eq. (A.2) takes the following form:

$$M\ddot{z} + D_d\dot{z} + D_g\dot{z} + Kz = F_f \quad (\text{A.3})$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, D_d = \begin{bmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \end{bmatrix}, D_g = ak_g \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & d \\ 0 & 1 & 0 & 0 \\ 0 & -d & 0 & 0 \end{bmatrix},$$

and $K = \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & c \end{bmatrix}$, with M , D_d , D_g , and K denoting the mass, damping,

gyroscopic feedback, and stiffness matrices, respectively.

Also, F_f denotes the external loading matrix for forward propagation such that:

$$F_f = a[-F_e \quad dF_e \quad 0 \quad 0]^T \quad (\text{A.4})$$

Note that D_g matrix, in its present form, is not skew symmetric matrix. But, this matrix can be expanded into a sum of symmetric and skew symmetric matrices as follows:

$$\begin{aligned} D_g &= \frac{1}{2}(D_g + D_g^T) + \frac{1}{2}(D_g - D_g^T) \\ &= \frac{1}{2}ak_g \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} + \frac{1}{2}ak_g \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2d \\ 0 & 1 & 0 & 0 \\ 1 & -2d & 0 & 0 \end{bmatrix} \\ D_g &= D_{dg} + G \end{aligned} \quad (\text{A.5})$$

with D_{dg} is a symmetric damping-like matrix and G is a skew symmetric gyroscopic matrix. Hence, the closed-loop model of the meta-cell with the virtual gyroscopic controller becomes:

$$M\ddot{z} + (D_d + D_{dg})\dot{z} + G\dot{z} + Kz = F_f \quad (\text{A.6})$$

A.2 Backward Propagation Model

Coupling the model with the virtual gyroscopic controller during backward propagation, with $F_1 = 0$, yields:

$$\begin{Bmatrix} \ddot{x}_1 + b\dot{x}_1 + cx_1 \\ \ddot{x}_2 + b\dot{x}_2 + cx_2 \\ \dot{y}_1 + b\dot{y}_1 + cy_1 \\ \dot{y}_2 + b\dot{y}_2 + cy_2 \end{Bmatrix} = a \begin{bmatrix} d(F_e - k_g \dot{y}_1) \\ -(F_e - k_g \dot{y}_1) \\ dk_g \dot{x}_1 \\ -k_g \dot{x}_1 \end{bmatrix} \quad (\text{A.7})$$

In a similar manner as in the case of forward propagation, Eq. (A.7) yields the following equation:

$$M\ddot{z} + (D_d + D_{dg})\dot{z} + G\dot{z} + Kz = F_b \quad (\text{A.8})$$

where

$$D_{dg} = \frac{1}{2}ak_g \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, G = \frac{1}{2}ak_g \begin{bmatrix} 0 & 0 & 2d & -1 \\ 0 & 0 & -1 & 0 \\ -2d & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

and

$$F_f = a[dF_e \quad -F_e \quad 0 \quad 0]^T \quad (\text{A.9})$$

It is worth noting that feedback laws are generally very simple and require only the measurements of $(\dot{x}_2, \dot{y}_2)$, in the case of forward propagation, instead of full measurement of the input and output velocities $(\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2)$. But, in spite of this simplicity, the resulting gyroscopic matrix in this case G is fully populated and enables the full interactions of all the system states. Similar not can be easily stated for the case of backward propagation as the measurement involve only $(\dot{x}_1, \dot{y}_1)$ as implied in Figure 2.11(a).

The closed-loop system equations for forward and backward propagation are the same but the only difference in both cases is the forcing vectors F_f and F_b as described by Eqs. (A.4) and (A.9), respectively.

Appendix B: The Pole/Zero Maps of the Transfer Functions of Forward & Backward Propagations

B.1 The Zeros

Equations (3.8) and (3.9) indicate that the numerator polynomials for:

Forward Propagation: $s^2 + (b - a k_g [d - 1/d])s + c = 0$ (B.1)

has zeros at:

$$s_{1,2}|_{forward} = -\frac{1}{2}(b - a k_g [d - 1/d]) \pm \sqrt{\left[\frac{1}{2}(b - a k_g [d - 1/d])\right]^2 - c} \quad (B.2)$$

Backward Propagation: $s^2 + (b + a k_g [d - 1/d])s + c = 0$ (B.3)

has zeros at:

$$s_{1,2}|_{backward} = -\frac{1}{2}(b + a k_g [d - 1/d]) \pm \sqrt{\left[\frac{1}{2}(b + a k_g [d - 1/d])\right]^2 - c} \quad (B.4)$$

For the meta-cell on hand with $a = 1.36\text{E}8$, $b = 246.8$, $c = 1.46\text{E}7$, and $d = 0.5$, leads to a radical that is dominated by the parameter c . For example, using the gyroscopic gain on the verge of stability such that $k_g = 3.6 \times 10^{-6}$, the radical $= -1.435\text{E}7$ and $-1.454\text{E}7$ for forward and backward propagations, respectively indicating that the radical is nearly equal to $-c$.

Hence, the difference between the zeros for forward and backward propagation is controlled primarily by the real part. It is very clear from Equations (A.1) and (A.3), for $d = 0.5$, that the real parts of the zeros are given by:

$$\textbf{Forward: } z_{1,2f}|_{real} = -(b - 1.5 a k_g) > z_{1,2Uncontrolled}|_{real} = -b \quad (\text{B.5})$$

where $z_{1,2f}|_{real}$ and $z_{1,2Uncontrolled}|_{real}$ denote the real parts of the zeros of the forward propagation and the uncontrolled cell, respectively.

$$\textbf{Backward: } z_{1,2b}|_{real} = -(b + 1.5 a k_g) < z_{1,2Uncontrolled}|_{real} = -b \quad (\text{B.6})$$

where $z_{1,2b}$ denote the zero of the backward propagation.

Equation (B.5) implies the migration of the zeros during the forward propagation to the right of the zeros of the uncontrolled cell, whereas Equation (B.6) indicates that the zeros during the backward propagation migrate to the left of the zeros of the uncontrolled cell.

B.2 The Pole/Zero Maps

The pole/zero maps of the meta-cell during forward and backward propagations are drawn for a gain $k_g = 0.5 \times 10^{-6}$ as shown in Figure B.1 in comparison with that of the uncontrolled cell.

Figure B.1(a) indicates that the uncontrolled cell has collocated pairs of poles and zeros. But, integrating the virtual gyroscopic controller with the cell results in migration of the zeros to the right during forward propagation and to the left during the backward propagation as shown in Figures B.1(b) and B.1(c), respectively. This migration conforms with the results presented in Section B.1. However, the effect of gyroscopic control gain on the locations of the poles is insignificant.

This migration results in lead and lag phase shifts between the inputs and outputs which is the prime reason for breaking the reciprocity. Such an important

feature of the virtual gyroscopic controller can be explained by casting Eqns. (3.8) and (3.9) in the following general:

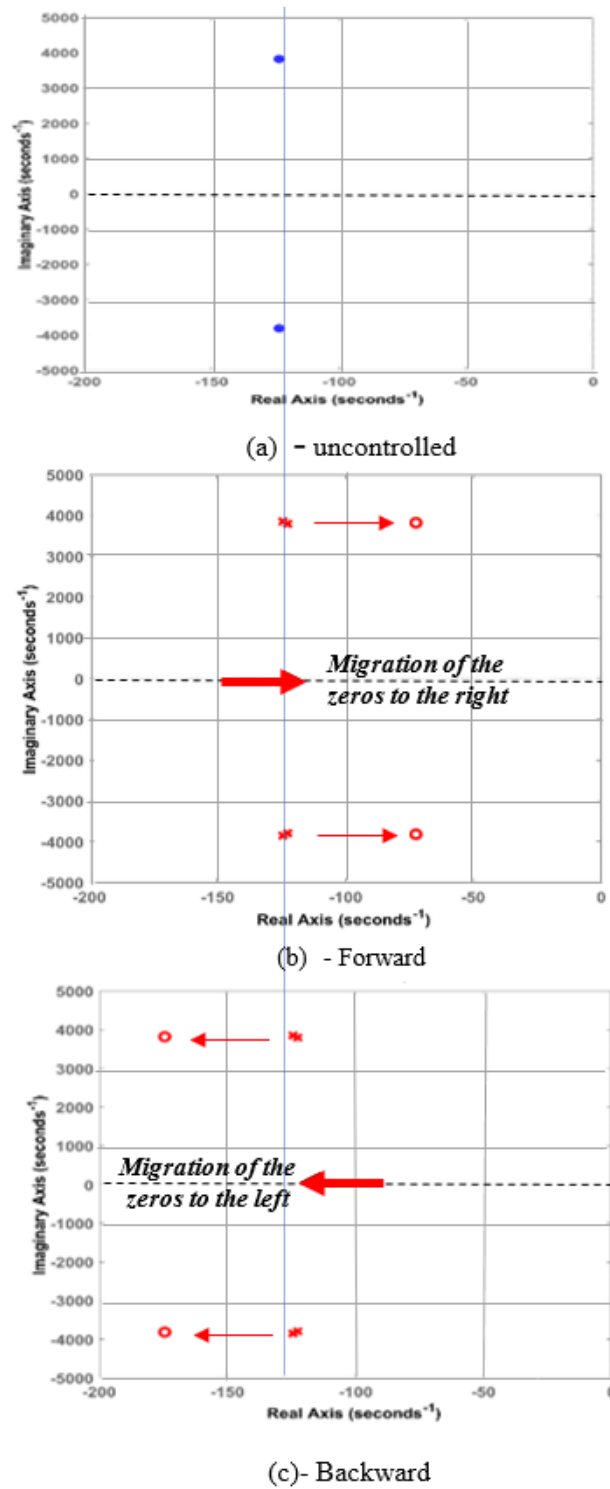


Figure B.1 Pole-zero maps

B.3 Effect of Location of the Zeros on Phase Shift and Amplitude Characteristics

The effect of migration of the zeros of the cell transfer function, during forward and backward propagation, can best be understood by considering amplitude and phase shift characteristics as given by:

Amplitude:
$$|TF| = K \frac{\prod_{i=1}^2 |(s - z_i)|}{\prod_{j=1}^4 |(s - p_j)|} = K \frac{r_1 r_2}{q_1 \dots q_4} \quad (\text{B.7})$$

and

Phase Angle:
$$\angle TF = \sum_{i=1}^2 \angle(s - z_i) - \sum_{j=1}^4 \angle(s - p_j) = \sum_{i=1}^2 \phi_i - \sum_{j=1}^4 \theta_j \quad (\text{B.8})$$

where the z_i and p_j denote the i^{th} zero (o) and the j^{th} pole (x). Also, $|TF|$ and $\angle TF$ denote the magnitude and phase shift of the transfer function TF . Furthermore, r_i and q_j define the magnitudes of the vectors from the i^{th} zero to any point s (i.e., $i\omega$, corresponding to a frequency ω) and the j^{th} pole to point s , respectively. Similarly, ϕ_i and θ_j define the associated phase angles of these vectors as shown in Figure B.2.

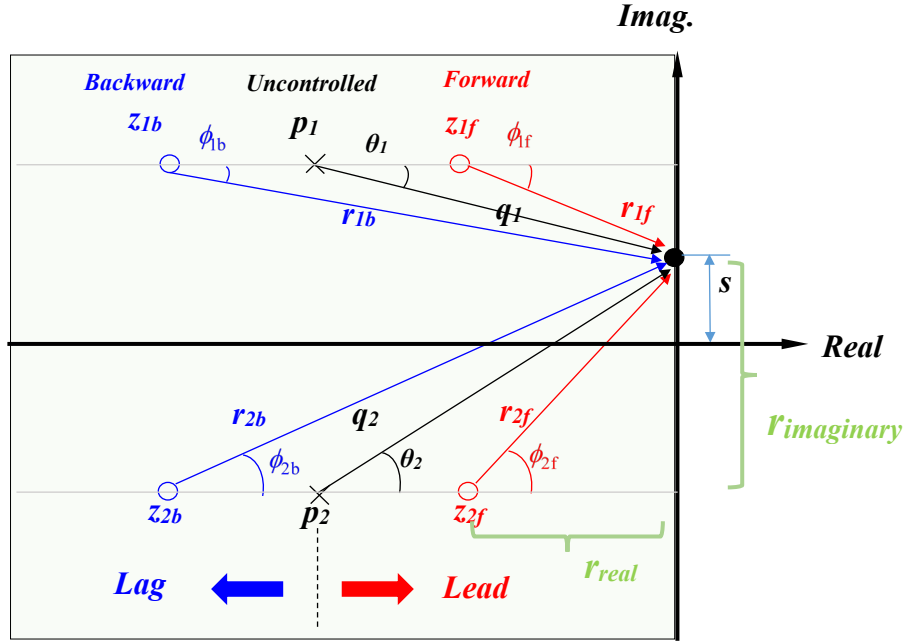


Figure B.2 Definition of parameters of the pole-zero map

B.3.1 Phase Shift Characteristics

Equation (B.8) and Figure B.2 indicate that:

a. For forward Propagation:

$\phi_{1f} > \theta_1$ and $\phi_{2f} > \theta_2$ due to migration of the zeros to the right of the uncontrolled zeros.

From Equation (B.8), this results in:

$$\angle TF|_{Forward} = \left(\sum_{i=1}^2 \phi_{if} - \sum_{j=1}^4 \theta_j \right) > \angle TF|_{Uncontrolled} = \left(\sum_{i=1}^2 \theta_i - \sum_{j=1}^4 \theta_j \right) \quad (B.9)$$

Note that in the uncontrolled case, $\phi_i = \theta_j$ as the poles and zeros are collocated. In effect Equation (B.9) implies that forward propagation results in “Phase Lead”.

b. Backward Propagation

$\phi_{1f} < \theta_1$ and $\phi_{2f} < \theta_2$ due to migration of the zeros to the left of the uncontrolled zeros.

From Equation (B.8), this results in:

$$\angle TF|_{Backward} = \left(\sum_{i=1}^2 \phi_{ib} - \sum_{j=1}^4 \theta_j \right) < \angle TF|_{Uncontrolled} = \left(\sum_{i=1}^2 \theta_i - \sum_{j=1}^4 \theta_j \right) \quad (B.10)$$

Note that in the uncontrolled case, $\phi_i = \theta_j$ as the poles and zeros are collocated. In effect Equation (B.10) implies that backward propagation results in “Phase Lag”.

B.3.2 Amplitude Attenuation and Amplification Characteristics

Figure (A.2) indicates that the magnitude of any vector r_i is given by:

$$r_i = \sqrt{r_{imaginary}^2 + r_{real}^2} \quad (B.11)$$

where $r_{imaginary}$ and r_{real} denote the real and imaginary components of the vector as shown in Figure B.2. Hence, the following two phenomena which associated with the gyroscopic controller can be easily explained in view of the outlined and observed “zero migration” feature:

a. Amplitude Attenuation During Forward Propagation:

As the real parts of the zeros are reduced by virtue of the “zero migration” phenomenon to the right as explained in Section B.1, then $r_{1,2f_{real}}$ become smaller than those of the uncontrolled cell $r_{1,2Uncontrolled_{real}}$. This leads to magnitude attenuation, as implied by Equation (B.11), and by the fact that the imaginary components of the all

the zeros as well as all real and imaginary components of the poles q_i 's remain unaffected by the gyroscopic controller.

b. Amplitude Amplification During Backward Propagation:

As the real parts of the zeros are increased by virtue of the “*zero migration*” phenomenon to the left as explained in Section B.1, then $r_{1,2b_{real}}$ become larger than those of the uncontrolled cell $r_{1,2Uncontrolled_{real}}$. This leads to magnitude amplification, as implied by Equation (B.11), and by the fact that the imaginary components of the all the zeros as well as all real and imaginary components of the poles q_i 's remain unaffected by the gyroscopic controller.

Appendix C: Main Characteristics of the Mini-Speakers and Microphones

C.1 Electrical Impedance of Speakers

Figure B.1 displays the magnitude and phase shift of the electrical impedance of the actuators, which indicates a resonate frequency at 650 *Hz*, as measured by an impedance analyzer (model ZX70AX-200k) (Changzhou Zhixin Precision Electronics Co., Ltd., Changzhou, Jiangsu, China).

C.2 Conditioning Circuits of the Microphones

The acoustic pressure sensors are electret condenser microphones [CUI devices (Lake Oswega, OR), model CMC-6015-47P]. The microphones are of the unidirectional type, with a diameter and height of 6.0 and 2.2 *mm*, respectively. These microphones have sensitivity *S* of -47 ± 3 *dB* at a frequency of 1 *kHz* and frequency bandwidth of 20 *kHz*.

The conditioning circuits of the condenser microphones are shown in Figure B.2.

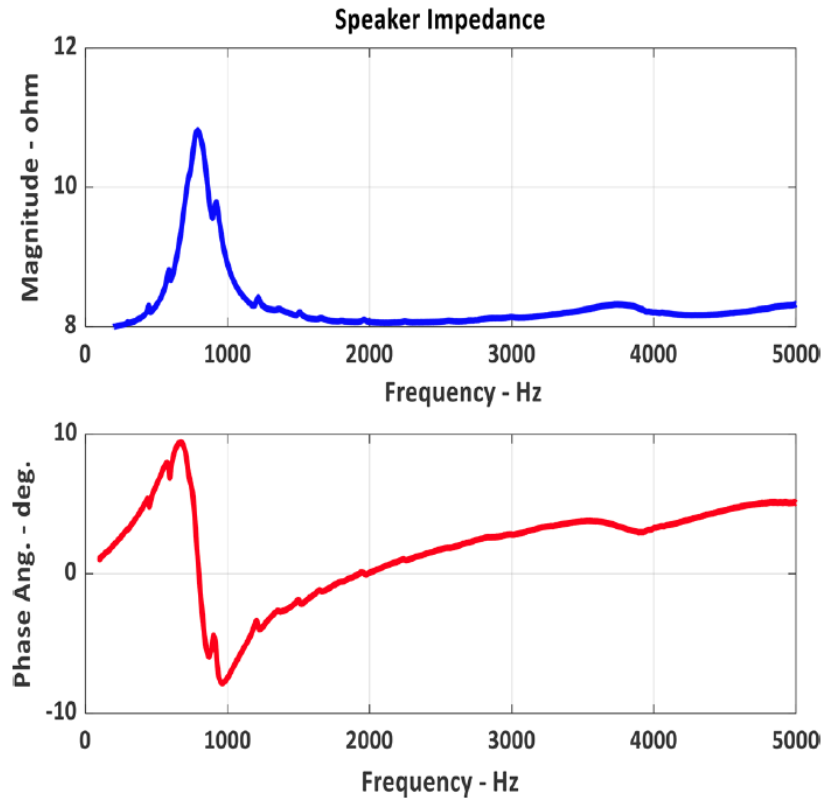


Figure C.3 The magnitude of the electrical impedance of the mini-speakers

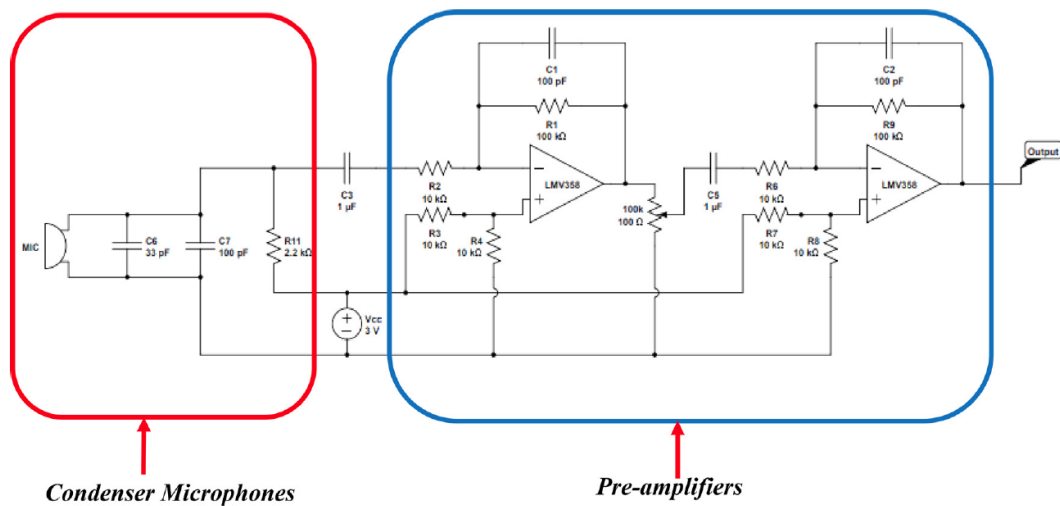


Figure C.4 Conditioning circuits of the condenser microphones

Appendix D: Identified System's Matrices

$$A = 1E6 \begin{bmatrix} 0.0476 & -0.0261 & 0.0107 & -0.0566 & 0.0366 & 0.0156 & -1.3618 & -0.2459 & -0.0840 & -0.3391 \\ -0.0815 & 0.0357 & -0.0121 & 0.0976 & -0.0553 & -0.0344 & 2.4951 & 0.3596 & 0.2682 & 0.3723 \\ 0.0456 & -0.0265 & 0.0127 & -0.0546 & 0.0357 & 0.0131 & -1.2383 & -0.3172 & -0.1101 & -0.2724 \\ 0.0477 & -0.0273 & 0.0152 & -0.0608 & 0.0376 & 0.0155 & -1.3404 & -0.2332 & -0.1507 & -0.2713 \\ -0.0829 & 0.0374 & -0.0084 & 0.0931 & -0.0567 & -0.0324 & 2.4764 & 0.5024 & 0.2214 & 0.4568 \\ 0.0566 & -0.0303 & 0.0160 & -0.0707 & 0.0429 & 0.0188 & -1.6236 & -0.3105 & -0.0994 & -0.2480 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -0.0006 & 0.0006 & 0.0026 & -0.0003 \\ 0.0002 & -0.0001 & 0.0001 & -0.0004 & 0.0002 & 0.0002 & -0.0087 & 0.0009 & -0.0014 & -0.0018 \\ 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0000 & 0.0000 & -0.0006 & -0.0004 & 0.0004 & 0.0004 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0006 & 0.0032 & -0.0009 & 0.0002 \end{bmatrix}$$

$$B = 1E3 \begin{bmatrix} -0.2045 & 1.3900 & -0.1494 & -0.0063 \\ -2.5112 & -0.6656 & 1.2539 & -0.4194 \\ -0.6109 & -2.2738 & -0.4500 & 0.8885 \\ 0.9863 & -0.5426 & -2.0483 & -0.3691 \\ -0.3482 & 1.4639 & -0.3852 & -2.3062 \\ -0.1033 & -0.1943 & 0.9911 & -0.0513 \\ -0.0314 & 0.0791 & 0.0699 & -0.0207 \\ 0.0466 & 0.0894 & -0.0565 & -0.0429 \\ -0.0185 & -0.0053 & 0.0376 & 0.0118 \\ 0.0179 & 0.0323 & -0.0257 & -0.0227 \end{bmatrix},$$

$$B_f = 1E3 \begin{bmatrix} -4.2869 \\ 0.0657 \\ 1.3511 \\ -0.1164 \\ -0.0518 \\ 0.1161 \\ -0.1067 \\ -0.1139 \\ 0.0053 \\ -0.0227 \end{bmatrix}, B_b = 1E3 \begin{bmatrix} 0.0742 \\ 0.0670 \\ -0.1942 \\ 1.0532 \\ 0.5379 \\ -4.7299 \\ -0.1034 \\ 0.1006 \\ -0.0707 \\ 0.0254 \end{bmatrix} \text{ and } C = \begin{bmatrix} I_{6 \times 6} & 0_{6 \times 4} \end{bmatrix}$$

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