# ABSTRACT

Title of Dissertation: ESSAYS IN
(I) STRATEGIC ORDERING WITH ENDOGENOUS SEQUENCE OF EVENTS IN SUPPLY CHAIN
(II) STRATEGIC MANAGEMENT OF NEW PRODUCT INNOVATION AND PROCESS IMPROVEMENT

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This dissertation discusses two research problems. First topic is strategic information management in supply chain, and second topic is analytical modeling approach in productivity dilemma. The first two chapters of dissertation discuss the impact of information asymmetry and competition on vertical contractual relationships, and risk neutral firms' strategic ordering decisions with minimal assumptions. Modern business environment caused by competition and information asymmetry plagues most firms across industries, often leading to suboptimal outcomes. Given the lead times in planning capacity, suppliers prefer earlier orders from their downstream partners (retailers). Much attention has been given in the literature to Advance Purchase Discount (APD), where the supplier lowers the wholesale price to entice the retailers to order early. In this dissertation, we suggest another avenue of early purchase model considering more realistic ways - competition between downstream retailers and information flows (from

information acquisition to dissemination) in supply chain. We show that with one retailer having "better" market demand information on uncertain demand than the other, the supplier can induce earlier ordering from the better-informed retailer without any reduction in the wholesale price, or creating rationing risk. In addition, we investigate firm's information investment decisions corresponding to the timing of the orders. We extend the model with different information structures of firms such as imperfect and evolving information. In reality, firms can have more accurate market information near the selling season by acquiring it from more diverse resources. Consistent with practice, we explorer firm's equilibrium outcomes of endogenous sequencing game with this setting.

The third chapter of dissertation is in the trade-off between production efficiency and new product innovation. A firm's ability to compete over time has been rooted not only in improved efficiency, but also in its ability to be simultaneously innovative (Abernathy (1978)). This trade-off between efficiency and innovation has long been discussed in the business context, but limited analytical research has been done using the 'extreme value theory' (Dahan & Mendelson (2001)) to investigate this issue. Our model considers important exogenous innovation factors such as innovation characteristics (Benner & Tushman (2003)) and degree of competition, which has yielded the following theoretical results and practical implications. First, we highlight new product characteristics. If R&D projects are paradigm-shifting innovations, there is a stronger adverse effect between efficiency and innovation than incremental innovation. Second, competition results in underinvestment effort in innovation performance for the firms. For example, in the symmetric firms' competition, the optimal size of R&D projects decreased, as competition increases. On the other hand, firms are more likely to focus on process improvement activities.

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by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2012

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# Acknowledgements

I would like to express my deepest and sincere gratitude to my advisors, Dr. Yi Xu and Dr. Manu Goyal, for their excellent guidance, continuous encouragement, and support during my PhD Program. I would never have been able to complete my dissertation without their support and help. I have been extraordinarily fortunate to have not only excellent academic advisors but also great mentors who provided priceless advices and instructions, resulting in fruitful learning process.

I would also like to thank the advisory committee members, Dr. Kislaya Prasad and Dr. Wedad Elmaghraby for their continuous support and comments. From the first and second year of doctoral program, their classes broadened my intellectual horizon, and absolutely, what they taught me contributed to all of my research.

I am earnestly grateful to Dr. Bongtae Han from Department of Mechanical Engineering, A. James Clark School of Engineering for kindly agreeing to serve as Dean's representative and generously sharing his expertise and thoughts.

Besides my advisors and committee members, many thanks go to my doctoral colleagues, PhD program coordinator, and all other staffs for their care, support, and friendship.

Finally, I would like to thank all of my family members, especially my parents, and wife (Suyoung Han). Their constant love, encouragement, and patience became my energy and inspiration in my doctoral program. Without their endless support, I would not have successfully finished this long journey.

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# Chapter 1

# Strategic Ordering with Endogenous Sequence of Event in Supply Chain

## 1.1 Introduction

Timely orders with valuable market information are one of the critical success factors for both suppliers and retailers especially in modern dynamic business environments with ever-shortened product life cycles, and longer production times caused by global outsourcing. In this context, ways to induce early orders from retailers in order to better manage their production plans, have received considerable attention. The so-called carrot-and-stick approach is used to entice retailers (customers) to order early in operations management literature. Much attention has been given to Advance Purchase Discounts (APD) (cf. Cachon, 2004; Dong & Zhu, 2007), where the supplier lowers the wholesale price to lure the retailers to order early (the carrot). Another remedy for inducing early orders is to create capacity rationing risk - a product may not be available in the future (cf. Liu & Ryzin, 2008; Su, 2008) (the stick).

In the past decade, the main argument in the literature for inducing early order is considering risk between supply chain partners. For instance, in the APD literature, inventory risk to a supplier caused by a supply-demand mismatch is a key drive for early purchase discounts. The harder but more realistic problem, however, exists since in some industries the supplier has no inventory risk. Instead, in many cases, the supplier starts production only after receiving customers' Firm Orders (FO). The following example from the fashion industry helps to motivate this research: VF Corporation is the world's largest apparel company, with more than 30 leading brands including Wrangler, The North Face, Lee, Nautica, etc. VF has outsourced more than half of its production to third party suppliers outside of the United States. The majority of the VF brands' products are available only for one season, with clearly defined and extremely short life cycles. This makes it very difficult for the firm to estimate market demand. As a result, the demand planning is one of the most important processes in their industry. More importantly, production lead time after ordering is up to six months. The main reason for long lead times lies in the start time of production for the supplier. Ellen Martin - the vice president of Supply Chain Systems at VF - argues that the problem is in acquiring the raw material for the supplier because no supplier is going to make fabric without a purchase order (PO) in hand (Supply chain leader (2006)).

In this paper, we suggest another avenue for an early purchase model considering more realistic ways - competition between downstream retailers and information flows (from information acquisition to dissemination) in the supply chain. We mainly analyze the case of competitive markets with demand uncertainty, wherein the choices of supply chain configuration - decisions about the timing of orders and market information acquisition - are all endogenous to the competing firms. More specifically, two competing firms' timing of orders is completely endogenous in a given time horizon, and information acquisition decisions are also choice variables for each firm. We characterize each firms' choices in equilibrium and analyze the effects of these choices on quantities ordered and profits.

From a broader supply chain perspective, optimal timing of orders is important not only for a firm itself but also for its partners because the quality of market information affects the performance of the entire supply chain including vertical partners as well as horizontal competitors (cf. Chen (2003)). Moreover, the current business environment caused by competition and information asymmetry plagues most firms across industries, often leading to suboptimal outcomes. So, one of the strategic decisions any firm (both informed and uninformed) faces centers on when to order their product or key component. A firm can place an order either well in advance of the selling season or relatively close to the selling season. Timing of the order is more critical when production lead time is much longer than the selling season such as in the apparel industry, and it is the situation we consider in this research. To find the optimal timing for ordering, the ordering mechanism is made up of endogenous timing of events: firms are not constrained to placing orders in the feasible time horizon despite the different quality of information. Further, from realistic settings where information endowment is not determined by default, information acquisition decisions necessarily precede endogenous sequencing of events, and they are another choice for the firms before deciding the timing of the order. Hence, a more rooted question to consider is whether the endogenous sequencing model is beneficial for acquiring information by investing in a sizable budget. Surprisingly, we prove that with one retailer having "better" information on uncertain demand than the other, the supplier can induce earlier ordering from the better-informed retailer without any reduction in the wholesale price, or creating rationing risk.

The main model for this research is motivated from a strategic information management model by Anand and Goyal (2009). In their model of horizontal competition between an informed and an uninformed firm with a common upstream supplier, material and information flows intersect through leakage of order information to unintended recipients. Thus, an added complication in our model is that the upstream supplier is strategic such as in the Newbury Comics case (Singer(1999)). In the current study, we model timing of orders and information acquisition decisions as decision variables in the competitive scenarios. Hence, with minimal assumptions, this paper explores generalized strategic information management models by analyzing the impact of market information and competition on procurement and timing of events. We show how firms' ordering behavior for the upstream partner is differentiated by the quality of information they have, and what the maximum investment level is for resolving uncertainty.

With timing flexibility, understanding this concept holds the key to addressing the following questions, which are the focus of this paper: (i) When both firms can acquire demand information, for instance, by investing in information systems or by requesting consulting, which firm is going to invest in market information, and how is it related to the timing of orders and the firm's profits? (ii) What are the optimal (equilibrium) timing and quantity of ordering for two competing firms? (iii) When intensity of competition is changed (for example, a substitute market), how do the optimal timing and quantity of ordering change?

The rest of this paper is organized as follows: Next section reviews the related literature, and section 3 presents our modeling framework with each player's tradeoffs; Sections 4 and 5 present analysis for the Endogenous Sequencing Game (ESG); Section 6 is analysis of the Information Acquisition Game (IAG). Concluding remarks are in Section 7.

## 1.2 Literature Review

We review a variety of research streams about the timing of ordering in both operations management and economics. Plenty of literature in the Operations Management field analyzes optimal sales timing of the seller considering various types of factors such as demand uncertainty, inventory risk, production lead time, capacity constraints, and so on. Thus, many papers deal with sales timing as a key influence on a firm's profit.

One stream of research is referred to as the Advanced Purchase Discount (APD) or Advance Booking Discount (ABD) model. Tang et al (2004) considered a scheme to entice customer's early commitment at a discount price prior to the commencement of the sales season. They analyze how the degree of demand uncertainty, correlation, and the level of market share affect the optimal discount factor. McCardle et al (2004) extended the ABD concept to a duopoly competition model taking into account consumer risk preferences and degree of brand disloyalty. Under this situation, they introduce possible scenarios about whether to introduce the ABD program for two competing retailers. Cachon (2004) also considered the seller's optimal contract timing decision in the presence of inventory risk between a seller and a buyer. He came up with a Pareto set combining push and pull contracts. Push contract refers to a system inventory that is completely pushed to the retailer when demand is uncertain and the supplier merely responds to the retailer's early order. Pull contract means that the supplier takes the full inventory responsibility in the supply chain and that the retailer merely pulls inventory from the supplier as demand occurs, though the retailer is constrained by the supplier's inventory availability. Dong & Zhu (2007) generalized Cachon's model with exogenous wholesale prices in a two-periods game. Depending upon the wholesale price in two periods, they show the Pareto optimal contract mechanism. They also identified conditions that enable Pareto improvement by introducing a new ordering opportunity to firms who were bound by a single ordering opportunity. Milner & Kouvelis (2005) considered the impact of different demand characteristics on contract mechanisms in similar settings with Cachon (2004) and Dong & Zhu (2007).

Another stream of research regarding sales timing issues is pricing mechanisms between retailers and consumers. Several papers investigated how to induce an early purchase using different prices. Liu and Ryzin (2008) claim that rationing risk - a product may not be available in the future - is a possible option for the risk averse customers for inducing early purchase with a high price. They argue whether the restricting supply is a profitable strategy or not, and if it is profitable, the ideal rationing parameter has been obtained. Su (2008) also modeled pricing mechanisms considering strategic customer behavior, which included a level of patience and individual value for the product. When high-value customers are proportionately less patient, markdown pricing policies are effective because the high-value customers would buy early at high prices (as in the fashion industry) while the low-value customers are willing to wait. He also claimed that pure markdown conditions, meaning that prices are never increased during the selling season, are based on the customer's patience level and the cost of waiting.

From the supplier's (seller) perspective, uncertainty about future demand and information asymmetry among supply chain players are important factors for optimal sales timing decisions. Cvsa and Gilbert (2002) modeled a supplier's optimal wholesale price commitment with sales timing using a single supplier and two identical competing buyers. They demonstrated how a supplier can reduce his risk by offering early order opportunities to downstream buyers. Their main finding is that there is a trade-off for buyers between early purchase with uncertain demand (Stackelberg leader effect) and late purchase with no demand uncertainty (production flexibility). Using the Stackelberg model with an uncertainty factor, they claimed that the supplier can benefit (diminishing risk) from providing adequate pricing incentives (discount) to entice downstream buyers to commit to purchase quantity before demand information is revealed. In other words, there is a direct trade-off between the level of demand uncertainty and price discounts. Ferguson (2003) also considered timing of commitment under demand uncertainty. He set up a single supplier - a manufacturer newsvendor model with different lead-times regarding timing of orders. He claimed that the manufacturer might be better off with a contract that requires an early commitment to its order quantity. The choice of commitment time depends upon the probability that uncertainty over demand is resolved prior to the buyer setting its production quantity. Similar with Cvsa and Gilbert (2002), the possibility of a demand information update is also a key role of the model. Taylor (2006) also considered sales timing for a supplier considering the impact of information asymmetry between the supplier and a retailer - the level of effort for accurate forecasting for the retailer. If the retailer exerts sales efforts prior to the selling season or has superior information about market demand, the supplier may prefer to sell early. It means that when the manufacturer sells early, the retailer consequently sets the high effort level to forecast demand accurately.

The economics side of research also considers endogenous timing of ordering using game theory models. A seminal work by Gal-Or (1987) showed that an informed firm might have higher profits by not moving first in a model with a privately informed Stackelberg leader-follower game, which is also considered in our analysis. Maggi (1996) considered an endogenous investment decision game that generates a trade-off between commitment (early investment with uncertainty) and flexibility (late investment with certainty). Surprisingly, the author argued that an investment game yields asymmetric equilibria: one firm chooses a preemptive strategy and the other firm follows a wait-and-see strategy resulting in (approximately) a Stackelberg outcome. Hamilton and Slutsky (1990) modeled endogenous timing of moves in a duopoly game using Stackelberg and Cournot equilibria with complete information by allowing firms to choose the time of play in addition to choosing a specific action. They claimed that in the observable delay game, multiple equilibria might happen depending on preplay communication, flexibility of moves, or observable delays by a rival. Mailath (1993) is a somewhat similar topic with our research. He considered an endogenous move for the informed firm with an asymmetric information game. He modeled a two-period game assuming that movement for the incumbent is fully observable. He assumed that the informed firm has only two choices, either being a leader by moving in the first period or moving simultaneously with the uninformed firm in the second period. His main claim is that even if there is an advantage for the informed firm to move in the second period with complete information, the informed firm still prefers to move first rather than simultaneously in the second period because the informed firm has only a limited advantage by moving in the second period.

Various issues related to information acquisition and demand forecasting in supply chain have been studied in the literature. Taylor & Xiao (2009) explored contract mechanisms with rebate and return contract. They showed the retailer, manufacturer, and total system may benefit from the retailer having even inferior forecasting. Also, they showed that the retailer may overinvest in forecasting. Lariviere (2002) considered the relationship between information acquisition and contract mechanism. He showed a screening mechanism for inducing investing retailers to take a different contract from non-investing retailers requires restricting returns. Shin & Tunca (2009) showed that with common pricing schemes, downstream firm under Cournot competition overinvest in demand forecasting.

In addition, several operations management papers deal with the value of information in a supply chain. Gavirneno et al (1999) studied partial and complete information sharing in a supplier-retailer setting, and compared these to a base case of no information. They concluded that information is always beneficial. Lee et al (2000) also considered the benefits of information sharing in a two-level supply chain. By information sharing, the supply chain can be more efficient with regard to inventory reduction and expected cost reduction. Whang et al (1998) also considered value of postponement. They claimed that postponement can improve both flexibility and demand forecast for a supply chain.

In sum, although all papers mentioned in the Operations Management (OM) field consider timing of contract sales, most papers also consider the upstream seller's (supplier) optimal sales timing. The main difference in this paper from the above-mentioned literature is that we consider information asymmetry between two downstream competitors as well as endogenous timing of orders for two competing buyers. Also, all models except McCardle (2004) and Cvsa and Gilbert (2002) do not consider competition between downstream partners. Moreover, our model claims that without providing a discount for early purchasing, the supplier can induce an early purchase by using information leakage. We will discuss this further in the next section. In addition, most of the OM side of research considers the seller's perspective whereas our model also reflects the buyer's trade-off and equilibrium analysis. Compared with the economic side of research, our paper considers every possible endogenous move of two competing firms - both the informed and uninformed firms. Mailath's (1993) paper is similar to ours, but he also fixed the uninformed firm to place its order in the second period. Also, his analysis does not consider every possible difference in the demand information (intercept). We analyze every possible situation regarding variability of demand ($\theta$, in our model) with a holistic point of view, whereas Mailath modeled a single limited case ($\theta$ is less than 3). Our paper considers every player's (the incumbent, the entrant, and the supplier) trade-off and equilibrium set including the supplier's possible information leakage in a more realistic fashion.

## 1.3 The Model

We formalize the impact of information asymmetry and competition on procurement and the timing of events by analyzing a supply chain consisting of one supplier and two horizontally competing firms. The two firms produce a seasonal product, which are substitutes. We assume that the demand curve is linear[1] and downward-sloping, with an uncertain intercept $\widetilde{A}$. The inverse demand curve for product $y$ is linear of the form: $P_y(q_y, q_{3-y}) = \widetilde{A}_y - q_y - \beta q_{3-y}$, where $q_y$ is the quantity put in the market for product $y$. The demand intercept $\widetilde{A}$ is random and can take one of two values[2]: a high value $A_H$ with probability $p$, and a low value $A_L$ with probability

[1]The linear demand curve has been widely used in the modeling literature. It has an appealing interpretation as the demand arising from the utility-maximizing behavior of consumers with quadratic, additively separable utility functions (Singh and Vives (1984)). Other demand curve such as exponential also works in our model. However, it adds only complication without further insight.

[2]In some of scenarios in our model, market demand information can be considered as a signal between informed and uninformed players. Thus, intercept $\widetilde{A}$ leads to "types" of player in the signaling game. Indeed, model can be extended to multiple demand states (with piecewise differentiable functions similar to continuous function) $\widetilde{A} = \{A_L, A_1, A_2, ..., A_H\}$ with different probability $(p_L, p_1, p_2, ..., p_H)$ (We proved multiple demand states in the appendix.) Its managerial insight is same with two-type model without finding additional implication. (Laffont and Martimort (2002), chapter 2, Appendix 2.1)

$(1-p)$; these values are common knowledge. We refer to the parameter $\beta \in (0,1]$ as the substitutability parameter, where $\beta > 0$ signifies that the two products are strategic substitutes.[3] When $\beta = 1$, two products are perfect substitutes (essentially identical). Product substitutability implies that the demand for a product increases with an increase in the price of the other product (Singh & Vives (1984)). The mean demand intercept is $\mu = pA_H + (1-p)A_L$ and the proxy of demand interval $\theta = A_H/A_L$. We also assume that all players are risk neutral firms.

The two firms source their product (or a key component) from a common supplier. The lead time for delivering orders is $l$. Hence, if $T$ marks the start of the selling season, then the latest time to place orders with the supplier is $l$ time units before $T$. We denote this time epoch by $L$ (Figure 1). We also assume that there are no capacity constraints.

Once the firms start selling, demand uncertainty is resolved by default– i.e., once the selling season begins, the firms observe the realization of the random intercept $\tilde{A}$. However, firms can acquire market information early (at time $E$)[4] by investing in various market research activities[5]. Information acquisition is expensive, and firms incur a cost $K$ to acquire (perfect) information.

The sequence of events is as follows. Firms decide whether to acquire informa-

[3] The canonical example of substitute market in fashion industry is leather jacket and viscose rayon jacket.

[4] Acquiring information betwen times $L$ and $T$ is useless since the information cannot be leveraged to change the order quantities given the lead time. Hence we focus on the more interesting case where $E < L$.

[5] Newbury Comics case (Singer (1999)) details how acquiring information before the selling season is key to the survival of firms.

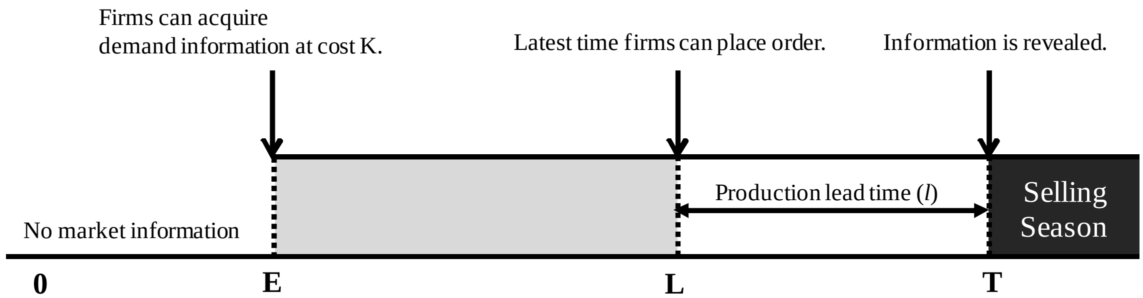



Figure 1.1: Sequence of Events

tion at time E. This marks the start of the Information Acquisition Game (IAG). Thereafter, based on information acquisition decision in IAG, firms decide *when* to order (anytime between 0 and $L$ is feasible). We call this the Endogenous Sequencing Game (ESG). The supplier then delivers the products before the start of the selling season. Demand and profits are realized at time $T$.

A key feature of our model is that we impose very limited structure on the sequence of events – except for the sequence of the two meta games, IAG followed by ESG. Within the ESG, the sequence is completely endogenous. This is in stark contrast with extant literature where most papers assume a well defined and immovable sequence of events (*cf.* Anand & Goyal (2009); Mailath (1993)). Instead, various possible sequences of events can be created, and we will discuss it further in analysis section.

In IAG, there are three possible outcomes: (I, I) when both firms acquire information; (I, NI) or (NI,I) when one firm invests in information acquisition, and the other firm did not; (NI, NI) when no firm invests in information. These information acquisition decisions are observable to all. We index two types of buyers - the informed firm (incumbent), the uninformed firm (entrant) - and the supplier, by $i$, $e$, and $s$ respectively.

The information structure imposed by the IAG results in two broad kinds of games under ESG between the two firms: a game of complete information where either both firms are informed or none are; and a game of incomplete information where only one firm is informed. The appropriate equilibrium concept for the dynamic game of complete information is the subgame perfect Nash equilibrium, whereas for the game of incomplete information, we use the Perfect Bayesian Nash Equilibrium.

An added complication in ESG is that the informed firm and the supplier are *strategic,* i.e., the informed firm's incentives to share information with the supplier and the supplier's incentives to leak this information to the uninformed firm. At the beginning of the ESG with incomplete information, only the informed firm has access to demand information. This setting allows us to analyze the effect of (potential) information leakage on the incentives for information revelation and dissemination. We also assume that the wholesale price $w$ is exogenously fixed throughout time horizon, and without loss of generality, we normalize $w$ to zero[6][7].

The equilibrium in our model has three components that are intertwined with each other: ($i$) Information flows (from information acquisition to information dis-

[6]Wholesale price is exogenously fixed throughout time horizon in our model. Given a proof of the zero whole sale price case, it is trivial to show that the conclusions follow from the full premise.

[7]When the supplier controls and sets the different wholesale price at time E and L, he has a second instrument (other than leakage) by which to influence material and information flows and, hence, to optimize his own profits. Depending on different wholesale price, equilibrium outcome of ESG can be changed and pure insight of model is faded. Thus, different wholesale price can be remained for future research direction.

semination in the supply chain) (*ii*) Material flows (the order quantities of the two suppliers), and (*iii*) Timing of orders.

### 1.3.1 Discussion of Assumptions

The analysis in this paper relies on three key assumptions whose impact on our results we discuss in this section. First, given buyers order timing decisions, the supplier starts production with a purchase (firm) order, and there is no capacity constraints as briefly mentioned in the introduction part (VF Inc. example). Thus, there is no risk of early production for the supplier in our model. It is a contrast from the Advanced Purchase Discount (APD) model (*cf.* Cachon (2004), Dong & Zhu (2007)). In APD model, one of the reasons that the supplier has inventory risk is due to capacity constraints, and it is the motivation that the supplier should have pre-book inventory. It implicitly assumes that the supplier starts production without the buyer's order. Furthermore, the APD model mainly analyzes the Pareto optimal supply chain mechanism by risk sharing and how to coordinate to maximize total supply chain efficiency. In contrast with the APD literature, our model analyzes *profit maximizing firms' (buyers and a supplier) strategic ordering behaviors.*

Second, we assume that in the time interval $[0, L]$, both firms can modify their order quantity multiple times[8]. However, we assume that multiple orders after time L are not available throughout the paper for the following two reasons. First, addi-

[8]For example, when both parties order at time E, the uninformed firm can change his order quantity after the supplier's leakage.

tional orders near the selling season with shorter production lead time are typically much more expensive than when ordering at regular intervals and this might have an extreme effect on a firm's profitability. The best example of this is also the textile and fashion industry. Lead times for design, production, and distribution are normally longer than the selling season, so buyers (or retailers) must commit to their order quantities well in advance of observing sales. Even though sometimes they use rush ordering called 'Quick Response (QR)' (*cf.* Krishnan et al (2010), Cachon & Swinney (2009)) with a short lead time, it is usually much more expensive than the regular order. Multiple orders with a multiple periods model might be a subject for future research. Second, to see the pure effect of endogenous sequencing and information acquisition effect, multiple ordering opportunities are not appropriate in our model.

Third, our model assumes very limited observability for more practical point of view. The only observable is that one firm can notice whether the order has been placed from his competitor after time E, but not the specific order quantity. For example, *the uninformed firm can only observe after time E whether the informed firm placed order at time E*[9]. In the existing literature of the endogenous move game, they assume that the competitor's every movement is fully and perfectly observable (for example, Mailath (1993) two-periods sequencing model). Even in the strategic inventory models such as Saloner (1987) and Moolgard (2000), they have very strong assumptions that production and inventory from a previous period is totally observ-

[9]It is different from imperfect observability in that the informed firm's action is perfectly observable, but the only observable is whether order was placed or not. Detailed order quantity can be acquired by supplier's leakage decision.

able to the competitor. (They claim that the previous period's inventory can restore a Stackelberg leader advantage in the next period.) In addition, in all literature analyzing sequencing model (*cf.* Gal-OR (1987), Dowrick (1986), Huck et al (2001), and Liu (2005)), the following action is always based on fully observed precedence.

## 1.4 Endogenous Sequencing Game (ESG)

### 1.4.1 Analysis of feasible set of time

The expected sales and profits with order timing $[0, L]$ are useful starting point in the Endogenous Sequencing Game (ESG). The following proposition compares the expected order quantity in the different feasible sets of time.

**Proposition 1** *(Comparison between time set* $[0, L]$ *and* $[E, L]$*)*

*I. If feasible timing of order is* $[0, L]$*, there exists the only equilibrium point in Endogenous Sequencing Game (ESG), and is as follows:*

*(i) Both competing firms place orders at time 0 without private information. Equilibrium order quantity for both firms is*

$$\widehat{q} = \frac{\mu}{2+\beta}, \text{ where } \mu = pA_H + (1-p)A_L$$

*(ii) The supplier's expected profit is*

$$E[\pi_s^{[0,L]}] = \frac{2\mu}{2+\beta} \cdot w$$

*II. (Renegotiation-Proofness) At time E, both firms have a chance to acquire real demand information. Since both firms have the opportunities to revise order quantity with the supplier, the supplier's expected profit in equilibrium after time E are related to* $E[\pi_s^{[0,L]}] \leq Min[E[\pi_s^{S[E,L]}, E[\pi_s^{C[E,L]}]]$[10]*. Since renegotiation after time E cannot be prevented,* $\widehat{q}$ *is not subgame-perfect equilibrium and order at time 0 is not credible.*

In our model, because it is not robust against renegotiation after acquiring demand information, the supplier's expected sales quantity and profit are no longer subgame-perfect equilibrium[11]. In other words, profit maximizing supplier has no reason to accept order before time E. Hence, from now on, we assume that the feasible set of timing of the order is from time E (Earliest) to time L (Latest), as seen in Figure 1.

### 1.4.2 Analysis of Information symmetry

Consider the case of information symmetry: either both firms have market demand information, or none has this information. In either case, the only equilibrium with the complete information game is to place orders simultaneously at time E since both players' best responses are always ordering at time E regardless of the competitor's order timing. Thus, the optimal order quantity ends up with simultaneous (Cournot type) equilibrium point, and optimal order timing is at time E. As it is a trivial

[10] $E[\pi_s^{S[E,L]}]$ refers the supplier's expected profit of sequential order, and $E[\pi_s^{C[E,L]}]$ means that of simultaneous order (Cournot).

[11] This result still hold in both constant and increasing wholesale price with respect to time, and even in imperfect information model.

question, the following lemma formalizes equilibrium order quantities and profits for two players in the complete information model.

**Lemma 2** *(i) If both firms have market demand information,*[12] *the only equilibrium point of endogenous sequencing game follows simultaneous move (Cournot) game with ordering at time E. The subgame-perfect Nash equilibrium quantity and profit of two informed firms are:*

$$\begin{aligned} q_{H1} &= q_{H2} = \frac{A_H - w}{2+\beta}; \pi_{H1} = \pi_{H2} = \left(\frac{A_H - w}{2+\beta}\right)^2 \\ q_{L1} &= q_{L2} = \frac{A_L - w}{2+\beta}; \pi_{L1} = \pi_{L2} = \left(\frac{A_L - w}{2+\beta}\right)^2 \end{aligned}$$

*(ii) If both firms do not have market demand information, the only equilibrium point of endogenous sequencing game still follows simultaneous move (Cournot) game with ordering at time E. The subgame-perfect Nash equilibrium quantity and profit of two informed firms are:*

$$q_1 = q_2 = \frac{\mu - w}{2+\beta}; \pi_1 = \pi_2 = \left(\frac{\mu - w}{2+\beta}\right)^2$$

[12] Even if two frms have imperfect information about demand, this still holds under the information symmetry. Only parameters are changed.

### 1.4.3 Analysis of Information asymmetry

Hence, the most challenging part in the Endogenous Sequencing Game (ESG) is the case of having different demand information: one firm invests in information acquisition, and the other firm does not - (I, NI) or (NI, I) in IAG, and it is one of the main thrusts in this paper. In the case of information asymmetry, we analyze the problem in the context of the elements of an organization structure (Anand & Mendelson (1997)). There are three elements, who decides what (the decision rights), who knows what (the information endowment) and what are the incentives of each player. For example, the decision rights of the informed firm are when to order, how much to order, and whether to reveal information. He has the market information in terms of the information endowment. The incentives are to maximize his own profits. Since our model does not have a fixed sequence of events in that each player's timing of ordering is endogenous, we can expect three kinds of scenarios regarding timing of order, corresponding order quantity, and the information flows. The possible outcomes are as follows:

*Scenario 1.* Both the informed and uninformed firms place orders simultaneously at the same point in time (the supplier has no chance to leak order information).

*Scenario 2.* The informed firm places an order before the uninformed firm with the supplier, knowing that the supplier might leak this order information to the uninformed firm, who in turn can use this to tailor his order for the market. The supplier starts production with the informed firm's purchase order. Then, the uninformed firm places an order with the supplier using the information disseminated from the supplier.

*Scenario 3.* The uninformed firm places an order before the informed firm with the supplier, knowing that the supplier might leak this order information to the informed firm. The supplier starts production with the uninformed firm's purchase order. Then the informed firm places an order with the supplier, knowing that the uninformed timing of order is earlier than him.

The first scenario can be explained as a simultaneous move (Cournot) game with incomplete information. The second and third scenarios can be explained as a sequential-move (Stackelberg) game. In the second scenario, the informed firm enters the market first as a Stackelberg leader followed by the uninformed firm. Using three scenarios, we will further analyze the firms' profit maximizing order timing and quantity in next section.

### 1.4.4 The Equilibriums in ESG

Consider payoff functions of the informed firm between scenario 1 and 2. The informed firm faces the following options: If he places an order simultaneously with the uninformed firm, there is no chance of information leakage from the supplier. But, he cannot take first mover's advantage. On the other hand, if the informed firm orders earlier than the uninformed firm, he takes market leader's advantage. However, he faces the possibility of information leakage since he has revealed market information by early order.

The most important point in the informed firm's perspective is that low type informed firm always prefers to move earlier than uninformed firm by revealing his demand information. It means that the low type informed firm strictly prefers to take market leader's advantage despite of the possibility of information leakage. By comparison between first two scenarios, the low type informed firm's payoff function as a Stackelberg leader is always greater than first scenario with all parameter value of $\beta$[13].

The uninformed firm also faces following trade-off: allowing market leader's advantage to the informed firm and taking market information, or none of these. The main consideration is whether market follower's disadvantage can be compensated by information leaked from the supplier, if possible. Based on two scenarios, if he places an order simultaneously with the informed firm at time E, he does not allow the informed party to take market leader's advantage. But, he cannot acquire real market information. On the other hand, if he orders later than the informed firm, he allows the competitor to take market leader's advantage. However, he can get realized market information by information leakage. In some region, we observe that the uninformed firm prefers to be a follower in spite of second mover's disadvantage,

[13] The substitute parameter ($\beta$) also affects the informed firm's preference: the less substitutable the products, the less the two markets overlap, and therefore the larger the effective size of the total market (Roller & Tombak (1993), Singh & Vives (1984)). Thus, the lower substitute parameter, the lower threshold of demand parameter ($p$) for the high type informed firm because of less degree of competition. In addition, low type informed firm's preference is not changed by $\beta$. He always prefers the sequential move regardless of the value $\beta$.

and it formalizes an essential building block for the Information Acquisition Game (IAG).

In the endogenous sequencing game with incomplete information, the choice of timing plays a role as an additional *signal* about the informed player's demand information, although it is different from a quantity signal (Mailath (1993), Saloner (1987).) Thus, the choice of order timing by the informed firm can convey demand information in the endogenous move game. For instance, if a type of informed firm places an order *late*, then the uninformed firm revises his strategy profile, in that no *early* order is placed: No action is considered as an action. By appropriately specifying each player's belief structure captured in the previous section, we can examine if each player's deviation is profitable or not. Finally, we come to the equilibrium timing of order and order quantity from three scenarios.

The following theorem formalizes the uninformed firm's belief structure, proves the existence of equilibrium in endogenous sequencing game (ESG) for all parameter values, and derives the equilibrium.

**Theorem 3** *There are equilibrium points in Endogenous Sequencing Game (ESG) for all $\theta$, $p$, and $\beta$ are as follows.*

Case I. *A separating Perfect Baysian Nash Equilibrium exists:*

*(i) The informed firm places order at time E. Order quantity is:*

$$q_{iH} = \frac{(2-\beta)A_H}{2(2-\beta^2)}, \textit{ if demand is high, and}$$

$$q_{iL} = \begin{Bmatrix} \frac{(2-\beta)A_L}{2(2-\beta^2)}, \textit{ when } \theta \geq \frac{2+\beta}{2-\beta} \\ q_{iL}^{cs*}, \textit{ when } \theta < \frac{2+\beta}{2-\beta} \end{Bmatrix}, \textit{ if demand is low}$$

*(ii) Supplier always leaks.*

*(iii) The uninformed firm orders at time L. Order quantity is:*

$$q_{eH} = \frac{(4-2\beta-\beta^2)A_H}{4(2-\beta^2)}, \textit{ if } Pr_e(\widetilde{A} = A_H) = 1$$

$$q_{eL} = \begin{Bmatrix} \frac{(4-2\beta-\beta^2)A_L}{4(2-\beta^2)}, \textit{ when } \theta \geq \frac{2+\beta}{2-\beta} \\ q_{eL}^{cs*}, \textit{ when } \theta < \frac{2+\beta}{2-\beta} \end{Bmatrix}, \textit{ if } Pr_e(\widetilde{A} = A_H) = 0$$

*consistent with his beliefs that:*

$$Pr_e(\widetilde{A} = A_H) = \begin{bmatrix} 1, \textit{if } q_i = \frac{(2-\beta)A_H}{2(2-\beta^2)} \textit{ and the informed firm orders at time E} \\ \textit{and the supplier leaks} \\ 0, \textit{if } q_i = \begin{Bmatrix} \frac{(2-\beta)A_L}{2(2-\beta^2)}, \textit{ when } \theta \geq \frac{2+\beta}{2-\beta} \\ q_{iL}^{cs*}, \textit{ when } \theta < \frac{2+\beta}{2-\beta} \end{Bmatrix} \\ \textit{and the informed firm orders at time E \& the supplier leaks,} \\ \textit{or, the supplier does not leak} \end{bmatrix}$$

*(iv) The profits of the informed firm and the uninformed firm are as follows:*

$$\begin{aligned}
\pi_{iH} &= \frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)}, \textit{ if demand is high} \\
\pi_{iL} &= \left\{\begin{matrix} \frac{(2-\beta)^2 A_L^2}{8(2-\beta^2)}, \textit{ when } \theta \geq \frac{2+\beta}{2-\beta} \\ \pi_{iL}^{cs}, \textit{ when } \theta < \frac{2+\beta}{2-\beta} \end{matrix}\right\}, \textit{ if demand is low.} \\
\pi_{eH} &= \frac{(4-2\beta-\beta^2)^2 A_H^2}{16(2-\beta^2)^2}, \textit{ if } Pr_e(\widetilde{A} = A_H) = 1 \\
\pi_{eL} &= \left\{\begin{matrix} \frac{(4-2\beta-\beta^2)^2 A_L^2}{16(2-\beta^2)^2}, \textit{ when } \theta \geq \frac{2+\beta}{2-\beta} \\ \pi_{eL}^{cs}, \textit{ when } \theta < \frac{2+\beta}{2-\beta} \end{matrix}\right\}, \textit{ if } Pr_e(\widetilde{A} = A_H) = 0
\end{aligned}$$

Case II. *When* $\theta \leq \frac{4-\beta^2+2\beta^2 \cdot p-\beta^2 p^2}{(2-\beta)^2+4\beta \cdot p-\beta^2 p^2}$*, a pooling equilibrium exists:*

*(i) The informed firm places order at time E. Order quantity is:*

$$q_{ip}^* = \frac{2A_L - \beta\mu}{2(2-\beta^2)}$$

*(ii) The supplier always leaks.*

*(ii) The uninformed firm orders at time L. Order quantity is:*

$$q_{ep}^* = \frac{(4-\beta^2)\mu - 2\beta A_L}{4(2-\beta^2)}$$

*consistent with his beliefs that:*

$$Pr_e(\widetilde{A} = A_H) = \begin{bmatrix} 0, \textit{ if the supplier leaks and } q_i < q_p^{\min}, \textit{ or the supplier does not leak;} \\ p, \textit{ if the supplier leaks and } q_p^{\min} \leq q_i \leq q_{ip}^*; \\ 1, \textit{ otherwise.} \end{bmatrix}$$

*where* $q_p^{\min} = \frac{A_H}{2-\beta^2} - \frac{\beta\mu}{2\left(2-\beta^2\right)} - \frac{\sqrt{\beta(A_H-\mu)((4-\beta)A_H-\beta\mu)}}{2\left(2-\beta^2\right)}$

*(iv) The profits of the informed firm and the uninformed firm are as follows:*

$$\begin{aligned}
\pi_{ipH} &= \left(A_H - \frac{\beta\mu}{4} - \frac{A_L}{2}\right)\left(\frac{2A_L - \beta\mu}{2(2-\beta^2)}\right) \text{ when demand is high, and} \\
\pi_{ipL} &= \frac{(2A_L - \beta\mu)^2}{8(2-\beta^2)} \text{ when demand is low.} \\
\pi_{eP} &= \left(\frac{(4-\beta^2)\mu - 2\beta A_L}{4(2-\beta^2)}\right)^2
\end{aligned}$$

Under the sequential moves, the low type informed firm prefers to order at time E, and its deviation is always not profitable. So, the uninformed firm believes that the informed firm's deviation from time E to time L can happen only in high demand state. Thus, when the informed firm deviates, the uninformed firm, consequently, can infer right after time E that market demand is high state. Thus, market information is disseminated to the uninformed firm and the uninformed firm updates his priors: the game is changed to *Simultaneous move game with complete information* ($\widetilde{A} = A_H$). The informed firm's profit for the deviation is strictly less than the profit of the early order[14]. Also, uninformed firm's deviation to time E is not profitable one.

Now suppose proposed equilibrium is scenario 1. Given informed firm's action (right after time E) - this means that the informed firm finished his order with scenario 1 order quantity (the informed firm's order quantity remain unchanged), the uninformed firm can get market information, and he revises his belief structure. It implies that his *expected* profit by deviation is always greater than ordering at time

[14] $\frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)}$ (profit of order at time E) $- \left(\frac{A_H}{2+\beta}\right)^2$(profit of order at time L) $= \frac{\beta^4}{8(2+\beta)^2(2-\beta)} > 0$

E. Since the uninformed firm's deviation is profitable, simultaneous order (scenario 1) fails to be an equilibrium outcome in ESG.

In the sequential move game, for the quantity choice $q_i$ by the informed firm, the uninformed firm infers that the state of demand is either $A_H$ or $A_L$. Since the quantity choices by the informed firm are best responses, the order quantity has to satisfy *the incentive compatibility* constraints. When $\theta$ is large ($\geq \frac{2+\beta}{2-\beta}$), the high and low demand states are widely separated in that the penalty for the high type informed firm to mimic the low type is quite large. However, when when $\theta < \frac{2+\beta}{2-\beta}$, if the low type informed firm were to order $\frac{(2-\beta)A_L}{2(2-\beta^2)}$, the high type would prefer mimicking the low type by ordering $\frac{(2-\beta)A_L}{2(2-\beta^2)}$. So, if the low type informed firm wants to send a *credible* signal, he needs to order sufficiently smaller than $\frac{(2-\beta)A_L}{2(2-\beta^2)}$.

Even if the costly separating equilibrium is played, the equilibrium is the same with previous analysis: the informed firm's order at time E and the uninformed firm's order at time L. For example, the uninformed firm's deviation from scenario 1 is profitable, and the informed firm's deviation from scenario 2 is not profitable. It implies that *ex ante* profit of the sequential move game is a dominant strategy for both players as long as a credible signal is disseminated.

In addition to separating equilibrium, we might think about pooling equilibrium for the sequential move game. In pooling equilibrium for the Stackelberg game, the informed firm orders the same quantity from the supplier in both demand states. In ESG, pooling equilibrium is also not dominated equilibrium. Given the informed firm's output decision, the uninformed firm cannot gain by deviating from time L to E. His best response to the informed firm's order remains same with order at time L.

*Theorem* 3 has important implications in terms of timing of order. The extant literature in operations management has investigated the ways to induce early order (*cf.* Cachon (2004); Liu & Ryzin (2008)). In all literature, inducing early order is a challenging problem for the upstream firm. The down stream partner could be placed to order either by providing discount of whole sale price (Cachon (2004); Dong & Zhu (2007)), or by creating capacity constraints (Liu & Ryzin (2008), Su (2008)). In contrast, by explicitly considering incentives in terms of timing of order, we have shown that for the complete range of parameter values ($\theta$, $\beta$, and $p$), so long as one firm has better information than the other the informed firm naturally orders earlier than the uninformed party. Interestingly, in our model, the supplier can charge a premium for the early ordering firm rather than providing lower wholesale price.

For a theoretical contribution, we first showed the relationship between market demand information and timing of order *endogenously*, which was often assumed as immovable sequence of events in the extant literature (*cf.*Cvsa & Gilbert (2002)) - indeed the less informed firm is assumed to order late. This also generalizes the results of Anand & Goyal (2009): We formally prove that their model is still hold in the endogenous sequence of events. Further, we formally prove that the model is robust to partial substitute model, while they considered only perfect substitutes. The substitute parameter ($\beta$) affects firm's preference, but our model shows on the equilibrium path that sequential move is still dominant strategy for both firms.

We also found that there is another sequential equilibrium in which the uninformed firm moves first and all types of informed firm moves at time L. Given the uninformed firm's order, the informed firm cannot be profitable by deviating to time E. The informed firm should have to order same quantity at time E, which does not change his payoff. The uninformed firm's deviation is also not profitable. He just lose the first mover's advantage by deviation[15].

### 1.4.5 Value of information

We analyze Cournot model with complete and incomplete information to investigate value of information: (i) when a firm has demand information and (ii) when a firm has no demand information.[16] Value of information is the difference of profits between above two cases. After simplification, value of information for the uninformed firm is a function of $p$, $\beta$, and the gap between two demand intercept. When $\beta$ is decreased from 1, total value of information is increased as a quadratic fashion.

$$\text{Total value of information} = \frac{p(1-p)\left(A_H - A_L\right)^2}{(2+\beta)^2}$$

There are two effects in the partial substitute market: first, when $\beta$ (intensity of competition) is decreased, total value of information is increased. Second, because two

[15] Normann (2002) found smilar results with us although the model is different from us. First, he assumed full observability for both firms. Second, he uses specific case demand interval ($A_H < 2A_L$). Also, he uses three types of demand information ($H, M, L$).

[16] $\left[p\left(\frac{A_H}{2+\beta}\right)^2 + (1-p)\left(\frac{A_L}{2+\beta}\right)^2\right] - \left(\frac{\mu}{2+\beta}\right)^2$

market sizes are increased with lower $\beta$, total value of information is also increased. To see pure (relative) value of information, each expected profit can be divided by total profit of each market. This can be analyzed as follows.[17]

$$\begin{aligned} & \frac{E\,[\text{Profit of an informed firm}]}{\text{Total profit of complete information game}} - \frac{E\,[\text{Profit of an uninformed firm}]}{\text{Total profit of incomplete information game}} \\ = \; & \frac{1}{2} - \frac{4\mu^2}{(4-4\beta-\beta^2)\mu^2 + (2+\beta)^2(p\cdot A_H^2 + (1-p)A_L^2)} \end{aligned}$$

**Corollary 4** *When $\beta$ is decreased from 1, the uninformed firm is more likely to wait for the informed firm information. The uninformed firm can take advantage of the informed firm's information when $\beta$ is low. With the same $\beta$, value of information is maximized when demand uncertainty is highest ($p = 0.5$).*

## 1.5 Information Acquisition Game (IAG)

In this section, we analyze the grand model of the paper - when information acquisition is a choice variable for two competing firms. In the Information Acquisition Game (IAG), two competing firms decide whether to acquire market information with a fixed cost ($K$). We assume that these decisions are common knowledge and observable to each other. So, IAG in table 1 is represented as a $2 \times 2$ matrix form for strategic game. Each firm is endowed with two strategies (Invest ($I$) and No Invest ($NI$)), and based on these decisions, the rest of events follow ESG.[18]

[17] When $p = 0$ or 1 (no uncertainty) and $\beta = 0$ (independent market), there is no pure value of information.

[18] To clarify, we call two competing firms as firm I and firm II in this section.

| | | Firm II | | |
|---|---|---|---|---|
| | | Invest | No Invest | |
| Firm I | Invest | Simultaneous order with Complete Info.(E, E) $(\pi^{Cournot} - K, \pi^{Cournot} - K)$ | Sequential order (E, L) $(\pi^{Leader} - K, \pi^{Follower})$ | $K_1$ |
| | No Invest | Sequential order (L, E) $(\pi^{Follower}, \pi^{Leader} - K)$ | Simultaneous order with No Info. (E, E) $(\pi^{Cournot}, \pi^{Cournot})$ | $K_2$ |
| | | $K_1$ | $K_2$ | |

Table 1.1: Information Acquisition Game (IAG)

The most important point in this section is the trade-off between information acquisition cost and timing of order. If one firm invests in market information, he takes advantage of market leader with incurred cost $K$. For the uninformed firm, even though he has market follower's disadvantage, he can also acquire market information by leakage, and revise his order quantity. In the information symmetry cases - if both firms acquire or none do - the timing of orders are at time E for both firms, and the simultaneous move game is played.

We seek a pure strategy Nash equilibrium of $2 \times 2$ matrix non-cooperative game. As is the typical way of finding equilibrium path, the solution can be obtained by considering the best response function of each firm given investment choice of his competitor. Each firm's best response depends on the value of information acquisition cost $K$. Let each firm's distinction of two payoff functions $K_1$ when competitor does invest in information, and let each firm's distinction of two payoff functions $K_2$ when competitor does not invest. The following proposition derives equilibriums in IAG.

**Proposition 5** *The equilibrium in Information Acquisition Game is divided by 4 cases and as follows:*

*(i) When* $0 \leq K < \min[K_1, K_2]$, *(Invest, Invest) is the only equilibrium for two competitors.*

*(ii) When* $K_1 < K < K_2$, *(Invest, No Invest) and (No Invest, Invest) are equilibriums.*

*(iii) When* $K_2 < K < K_1$, *(Invest, Invest) and (No Invest, No Invest) are equilibriums.*

*(iv) When* $K > \max[K_1, K_2]$, *(No Invest, No Invest) is the only equilibrium.*

We explain the equilibriums in IAG with Figure 2, which illustrates possible equilibriums depending on $K, K_1$,and $K_2$. The value of information can be measured by the difference between two payoff functions: a firm has market information and a firm does not have it. In the both monopoly and competition market, the value of information is maximized when market uncertainty is highest ($p = 0.5$). Also, as is expected, the value of information for monopoly case is higher than competitive market because of the lack of competition and substitute goods (i.e. higher chance to make more profit.) Thus, monopoly is more likely to invest in market information. As the number of competitors are increased (two in figure 2), expected profit for each firms is decreased, and there are less rooms to invest in information.[19]

[19]Figure 3 also explains the investment levels between two firms' competition and that of three

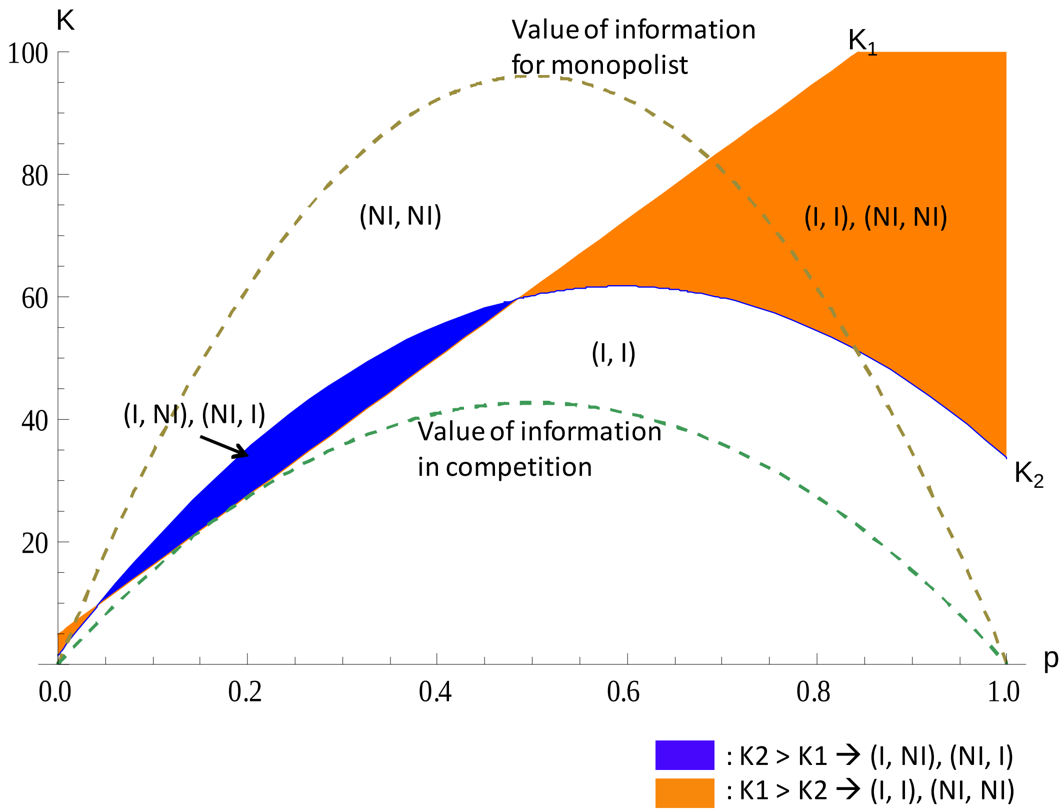



Figure 1.2: Equilibria of Information Acquisition Game

In IAG, investment decision is a little different from pure value of information. Basically, when $p$ is increased, the market size is increased, and market leader's advantage is also increased. Therefore, firms are more likely to invest in information. For instance, in the area of (I, I) (NI, NI), there is a huge advantage as a leader even though information acquisition cost is high. So, both firms do not allow market leader's advantage to the competitor. On the other hand, in the (I, NI), (NI, I) area, advantage as a leader is not significant because information acquisition cost is not ignorable. So, strategically, one firm does invest on it, but the other firm does not. We call this area strategic substitute area.

In addition, we might think about the situation when two firms have different information acquisition cost $(\widehat{K}_I, \widehat{K}_{II})$. Different cost concept is applicable when a firm has cheaper information acquisition cost by experiencing a relatively longer history of sales data, and the other firm is newly entering the market. Depending on the values of $\widehat{K}_I$ and $\widehat{K}_{II}$, the firm's investment strategy could be changed. As seen in the following lemma, while information acquisition cost is increased, firms are more likely not to invest in information.

**Lemma 6** *When two competitors have different information acquisition cost $\widehat{K}_I$, $\widehat{K}_{II}$, equilibriums are in the following tables.*

*(i)* $K_1 \geq K_2$

firms in IAG. Investment level for three firms game is less than two firms's game.

| | | | | |
|---|---|---|---|---|
| | $\widehat{K}_I > K_1$ | *(NI, I)* | *(NI, I)* | *(NI, NI)* |
| *Firm I* | $K_2 < \widehat{K}_I < K_1$ | *(I, I)* | *(I, I), (NI, NI)* | *(I, NI)* |
| | $\widehat{K}_I < K_2$ | *(I, I)* | *(I, I)* | *(I, NI)* |
| | | $\widehat{K}_{II} < K_2$ | $K_2 < \widehat{K}_{II} < K_1$ | $\widehat{K}_{II} > K_1$ |
| | | | *Firm II* | |

*(i)* $K_2 < K_1$

| | | | | |
|---|---|---|---|---|
| | $\widehat{K}_I > K_2$ | *(NI, I)* | *(NI, I)* | *(NI, NI)* |
| *Firm I* | $K_1 < \widehat{K}_I < K_2$ | *(NI, I)* | *(I, NI), (NI, I)* | *(I, NI)* |
| | $\widehat{K}_I < K_1$ | *(I, I)* | *(I, NI)* | *(I, NI)* |
| | | $\widehat{K}_{II} < K_1$ | $K_1 < \widehat{K}_{II} < K_2$ | $\widehat{K}_{II} > K_2$ |
| | | | *Firm II* | |

We also generalized our result with the multiple player game. We generalize our result with the multiple player game. Suppose there are a total of $n+m$ firms, and $n$ firms invest in information and $m$ firms are uninformed.[20] Thus, the informed firms place orders with the supplier at time E, and uninformed firms place orders at time L.

**Lemma 7** *(i) In the n+m player game with n informed firms and m uninformed firms, when all players orders at time E, the game is simultaneous move (Cournot) game. The subgame-perfect Nash equilibrium quantity and profit of n informed firms and m uninformed firms are:*

[20] Without the loss of generality, we assume that $\beta = 1$ for the simplicity of the model

$$\begin{aligned}
q_{iH} &= \frac{A_H}{(n+1)} - \frac{m\cdot\mu}{(n+1)(n+m+1)}; q_{iL} = \frac{A_L}{(n+1)} - \frac{m\cdot\mu}{(n+1)(n+m+1)} \\
q_u &= \frac{\mu}{(n+1)} \\
\pi_{iH} &= \left(\frac{A_H}{(n+1)} - \frac{m\cdot\mu}{(n+1)(n+m+1)}\right)^2; \pi_{iL} = \left(\frac{A_L}{(n+1)} - \frac{m\cdot\mu}{(n+1)(n+m+1)}\right)^2 \\
\pi_u &= \left(\frac{\mu}{n+1}\right)^2 \\
\textit{where } i &= \{1,2,...,n\} \textit{ and } u = \{1,2,...,m\}
\end{aligned}$$

*(ii) In the n+m player game with n informed firms and m uninformed firms, if the informed firms orders before the uninformed firms, and if the supplier commits to leak order information with uninformed firms, the game is sequential move (Stackelberg) game. The subgame-perfect Nash equilibrium quantity and profit of n informed firms and m uninformed firm are:*

$$\begin{aligned}
q_{iH} &= \frac{A_H}{(n+1)}; q_{uH} = \frac{A_H}{(n+1)(m+1)} \\
q_{iL} &= \frac{A_L}{(n+1)}; q_{uL} = \frac{A_L}{(n+1)(m+1)} \\
\pi_{iH} &= \frac{A_H^2}{(n+1)^2(m+1)}; \pi_{uH} = \frac{A_H^2}{(n+1)^2(m+1)^2} \\
\pi_{iL} &= \frac{A_L^2}{(n+1)^2(m+1)}; \pi_{uL} = \frac{A_L^2}{(n+1)^2(m+1)^2} \\
\textit{where } i &= \{1,2,...,n\} \textit{ and } u = \{1,2,...,m\}
\end{aligned}$$

For the illustration purpose, we analyze three player's information acquisition game in the multiple player IAG. Upon information acquisition decision, three competitors can have demand information with cost of $K$, and each firm faces the following

| Outcome | Invest | No Invest | |
|---|---|---|---|
| All competitors invest | $\left(p\left(\frac{A_H^2}{16}\right)+(1-p)\left(\frac{A_L^2}{16}\right)-K\right)$ | $\left(p\left(\frac{A_H^2}{36}\right)+(1-p)\left(\frac{A_L^2}{36}\right)\right)$ | $K_1$ |
| 1 of 2 competitors invests | $\left(p\left(\frac{A_H^2}{18}\right)+(1-p)\left(\frac{A_L^2}{18}\right)-K\right)$ | $\left(p\left(\frac{A_H^2}{36}\right)+(1-p)\left(\frac{A_L^2}{36}\right)\right)$ | $K_2$ |
| No one invest | $\left(p\left(\frac{A_H^2}{12}\right)+(1-p)\left(\frac{A_L^2}{12}\right)-K\right)$ | $\left(\frac{\mu^2}{16}\right)$ | $K_3$ |

Table 1.2: Possible outcomes in Three player Information Acquisition Game (IAG)

six cases of outcomes and each player's *expected* payoffs are as follows.

In the three-player game, upon information acquisition decision, three competitors can have demand information with cost of $K$, and each firm faces the six cases of outcomes. Figure 3 (a) illustrates possible equilibria depending on $K, K_1, K_2$ and $K_3$.[21] The result has a similar pattern with two player game. First, when $0 \leq K < \min[K_1, K_2, K_3]$, (I, I, I) is the only equilibrium for three players. When $K_2 < K < \min[K_1, K_3]$, (I, I, I), (I, NI, NI), (NI, I, NI), and (NI, NI, I) are equilibria. When $K_2 < K_1 < K < K_3$, (I, NI, NI), (NI, I, NI), and (NI, NI, I) are equilibria. When $K_3 < K < K_1$, (I, I, I) and (NI, NI, NI) are equilibria. When $K > \max[K_1, K_2, K_3]$, (NI, NI, NI) is the only equilibrium. The second graph in Figure 3 illustrates the difference between a two-player and three-player game. Interestingly, in the multiple player game, there is an area that only one firm does invest on information. More than one firms' investment, for example, (I,I,NI), is never an equilibrium.

Information Acquisition Game has the following implications. First, when demand

[21] Let each firm's difference of payoff function $K_1$ when all competitors do invest in , and let each firm's difference of payoff function $K_2$ when one of two competitors invests. $K_3$ represents firm's difference of payoff function when none invests.

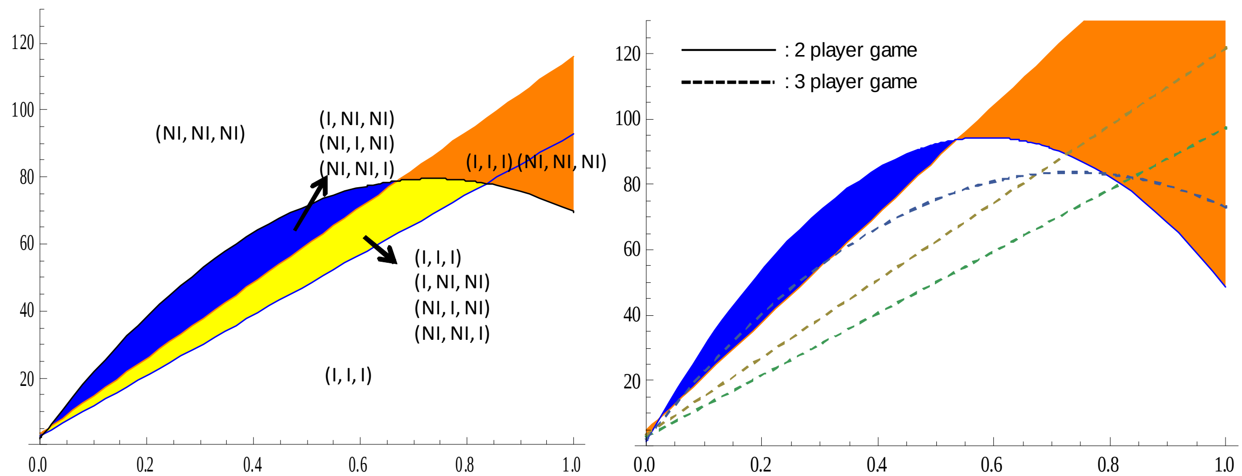



Figure 1.3: Equilibria of Multiple Player Game

interval $\theta$ $(= \frac{A_H}{A_L})$ is increased, value of information is increased. Thus, firms are more likely to invest in market information. Second, due to the effect of competition, firms investment decisions are different from pure value of information. Where the market size is increased - for example, $p$ is close to 1, firms are more likely to invest in the information not to allow huge market leader's advantage. Third, also due to the effect of competition, firms are less likely to invest in information when the number of firms in the market are increased. We understand it as expected market share effect for each firm : When the number of firms are increased, the expected market share and payoff for each firms is decreased. Therefore, firms are less likely to invest in the information.

### 1.5.1 Comparative statics

We study the behavior of each firms investment decision numerically in this section. Figure 4 (a) describes firms' investment decision behaviors in different gaps between $A_H$ and $A_L$. When $\theta$ $(= \frac{A_H}{A_L})$ is small or zero, both firms are not going to invest in information due to the small value of information. When $\theta$ is increased, the value of information is also increased, and therefore investment area ((I, I), (I, NI), (NI, I)) is increased. However, even if $\theta$ is increased, there is an area that not all firms invest in information. It is a strategic substitute area in our model. When $p$ is increased (x-axis), then (I, I) and (NI, NI) area is increased. This is because firms have more probability to make profit by high probability of high demand. Thus, firms are more likely to pay on information acquisition. Figure 4 (b)$\sim$(d) illustrate firms' behavior in different information acquisition cost and demand uncertainty. For $K = 40$ (figure 4(b)), (NI, NI) is an equilibrium when $p = 0.1$. When $p = 0.2$, (I, NI) or (NI, I) is equilibriums. And, $p$ is increased, (I, I) or (NI, NI) is equilibrium. For $K = 60$ (figure 4(c)), (NI, NI) is an equilibrium when $p = 0.2$. When $p = 0.45$, (I, NI) or (NI, I) is equilibriums. As $K$ is increased, firms do not invest in most cases. But, (I, I) or (NI, NI) is still an equilibrium because there is a huge advantage as a market leader.

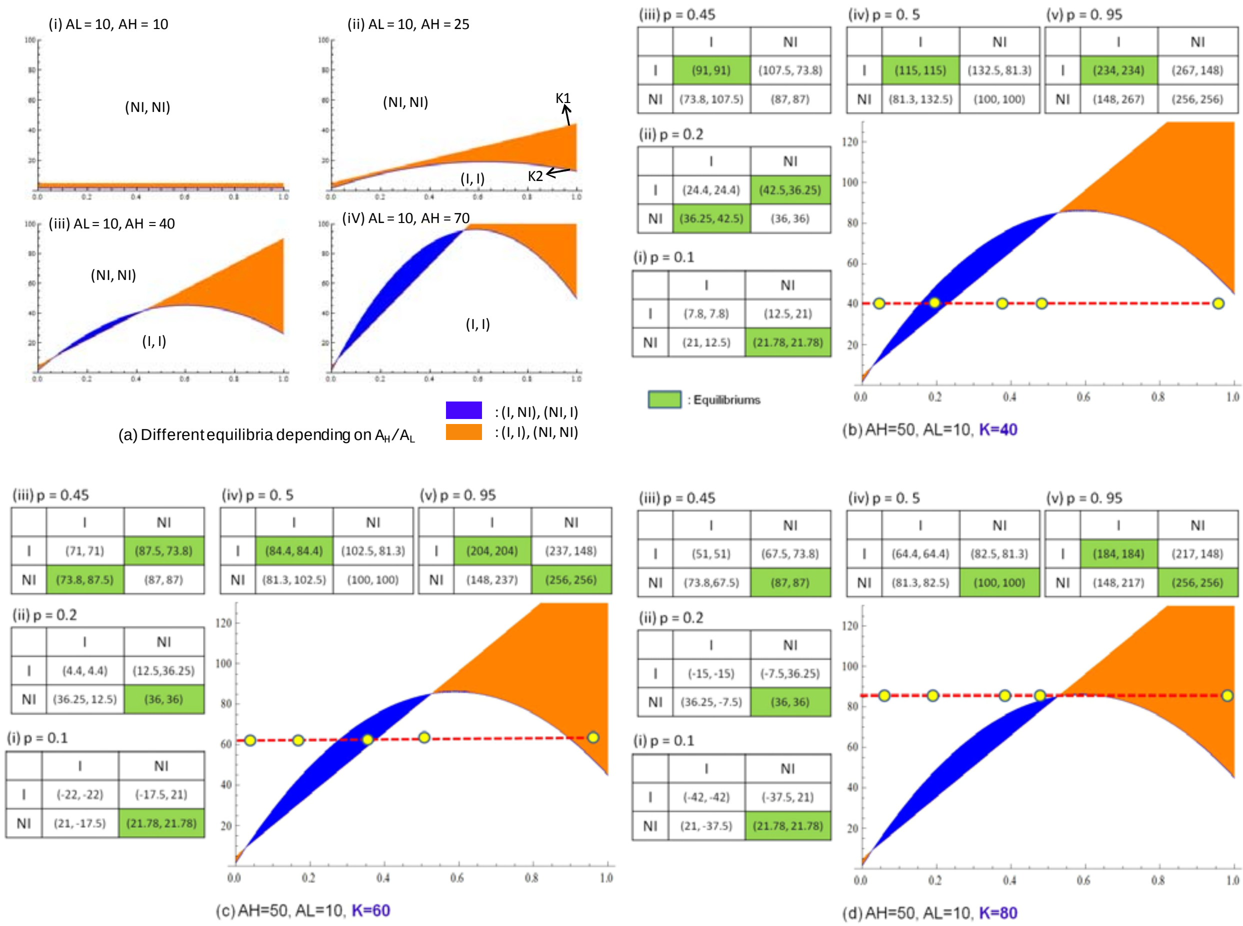



Figure 1.4: Investment decision behavior with different demand variety and cost $K$.

## 1.6 Conclusion

We prove that when one firm acquires market information, the informed firm always places an order *early*. In fact, the uninformed firm strictly prefers to order *late* after the informed firm. This result is interesting for the following reasons:

($i$) Extant literature has highlighted benefits for the downstream firms to order early (*cf.* Cachon, 2004; Dong & Zhu, 2007), which range from better capacity planning by the supplier to getting demand information earlier. However, in all of extant literature, inducing the retailer to order early is a challenge. The retailer can be made to order early either by the lure of a lower wholesale price or by the threat of capacity constraints (*cf.* Liu & Ryzin, 2008; Su, 2008). However, we prove that as long as one firm is better informed than the other, the *better informed* firm automatically orders early. In fact, far from charging a lower wholesale price to induce early ordering, the supplier can in fact charge a premium.

($ii$) From a more theoretical perspective, we generalize the results of Anand and Goyal (2009) in three ways: ($a$) The sequence of moves that was exogenously imposed in Anand and Goyal (2009) emerges endogenously – the better informed firm indeed orders early. ($b$) While they considered perfect substitutes, we extend their results to products that may be partial substitutes. ($c$) While they allowed only one firm to acquire information, both firms in our model can do so. We show that, consistent with practice, not all firms are equally informed even within the same industry.

Our results on endogenous sequencing (where the informed firm orders early) are robust to *(i)* when the firms are imperfectly informed – all that is needed is one firm to be *better* informed than the other. *(ii)* multiple demand states and/or multiple players, and *(iii)* endogenous wholesale price.

Further, we analyzed the imperfect information with information evolution as a extensive research. It is natural that firms have imperfect information in the earlier period, and this imprecision can be resolved in the later period. In this model, we found two important equilibriums which has quite meaningful managerial implication. First case is that the informed firm still places order early in spite of certain degree of noise for the demand information. We confirm that the base model is still hold as long as imprecision factor of the informed firm is greater than certain threshold value. Even if the informed firm can have better information later, market leader advantage is still dominant in the competition model. Second equilibrium is when the informed firm places order late allowing market leader advantage to the uninformed firm. These two equilibria imply that optimal timing of order has a important relationship with information evolution slope.

# Chapter 2

# Endogenous Sequencing Game in Imperfect and Evolving Information

## 2.1 Introduction

More accurate market demand information has a strong relationship with firm's success in the selling season. Enterprises across all industries and market segments invest large percentages of their budgets in information systems and market research to improve accuracy in forecasting demand. However, due to fairly longer production and delivery lead time, it is more difficult to estimate accurate market demand information well in advance of the selling season. In reality, firms can usually have more accurate market demand information near the selling season by acquiring market information from more diverse resources. So, it is natural that firms have inaccurate market information in the earlier period, and that imprecision can be resolved in near the selling season. One of limitations from previous model is that the informed firm has perfect market information by default throughout the feasible time horizon.

In this chapter, we explorer the imperfect information and information evolution model as an extensive research. In imperfect information model, the informed firm has better information than the uninformed firm, but the informed firm has certain degree of noise. This base model is extended with information evolution concept. With information evolution model, two firms have different imprecision factors (for example, $\alpha_1$, $\alpha_2$) with early period, and its information evolution slopes (information exploitation) as time goes to the selling season are also different. We investigate equilibrium outcomes of Endogenous Sequencing Game with this setting.

Research questions are as follows. $(i)$ What are the equilibrium timing and quantity of order for two competing firm when the informed firm's demand information also has a certain degree of noise? How could the equilibrium solution be changed compared to previous chapter's result? $(ii)$ Firms can acquire more accurate picture of demand, if they delays orders. What is the best timing of order and quantity for two competing firm in endogenous sequencing game? How firms' profit functions are changed by different information evolution slope? We compare every possible scenario for three risk-neutral supply chain participants (a single supplier and two competing firms).

We found two important equilibriums which has quite meaningful managerial implication. First case is that the informed firm still places order early in spite of certain degree of noise in market demand information. We confirm that the base model is still hold as long as imprecision factor of the early ordering informed firm is greater than certain threshold level of $\alpha^*$. Even if the informed firm can have better information later, market leader's advantage is still dominant than information acquisition effect by ordering late in the competition model. Second case is when the informed firm places order late allowing market leader's advantage to the uninformed firm. These two equilibria imply that optimal timing of order has a important relationship with information evolution slope. When the slope is quite steep during ordering period, the informed firm can delay orders until this imprecision is cleared. On the other hand, when the quality of information is gradually improved, the informed firm places order as early as possible in spite of some degree of noise. In addition, we analyze the supplier's preference between two equilibrium. Assuming that the supplier's production cost is directly proportional to time, we found that the supplier prefers one equilibrium than the other strictly.

The paper is organized as follows. In the next section, we present models with imperfect information and information evolution. In section 3, we analyze imperfect information game and its equilibrium point. Section 4 investigates information evolution game and its equilibria. In section 5, we compare the supplier's preference between two equilibrium.

## 2.2 The Model

We consider the imperfect and information evolution model and its impact on competition on procurement and the timing of events by analyzing a supply chain consisting of one supplier and two horizontally competing firms. The two firms produce a seasonal product, which are perfect substitutes for the ease of analysis. We assume that the demand curve is linear and downward-sloping, with an uncertain intercept $\widetilde{A}$. The inverse demand curve for product $y$ is linear of the form: $P_y(q_y,q_{3-y}) = \widetilde{A}_y - q_y - q_{3-y}$, where $q_y$ is the quantity put in the market for product $y$. The demand intercept $\widetilde{A}$ is random and can take one of two values: a high value $A_H$ with probability $p$, and low value $A_L$ with probability $(1-p)$; these values are common knowledge. We refer to the parameter $\alpha \in (\frac{1}{2}, 1]$ as an imprecision parameter, where it indicates the accuracy of belief: the informed firm has some belief what demand is, but does not know it exactly. We further assume that $\alpha$ is common knowledge. When $\alpha = 1$, it is the perfect information game - the same model with ESG in previous section. Base model of this chapter analyzes that this imprecision parameter is not changed. We call this Imperfect Information Game (ESG-IIG) However, we further consider the case that $\alpha$ is cleared at time L. We call this Information Evolution Game (ESG-IEG). Thus, In IEG, if the informed firm waits until time L, he observes the realization of the random intercept $\tilde{A}$. The other setting is the same with previous

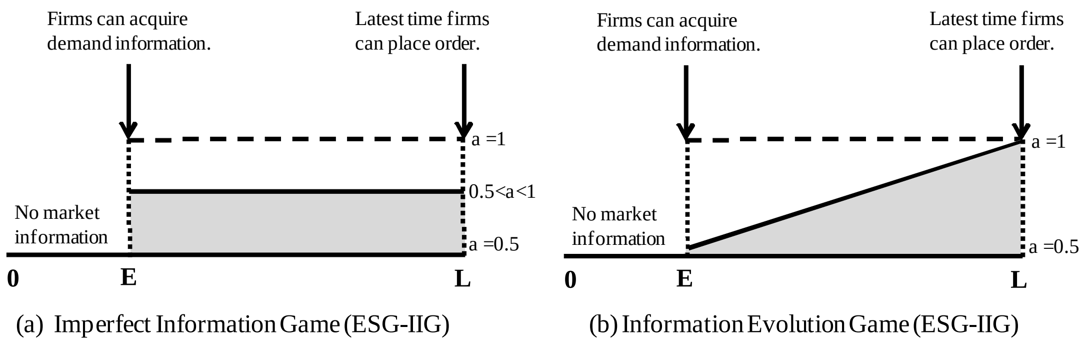



Figure 2.1: Time line of Events and Information Structure of Models.

chapter. The two firms source their product (or a key component) from a common supplier. The lead time for delivering orders is $l$. Hence, if $T$ marks the start of the selling season, then the latest time to place an order with the supplier is $l$ time units before $T$. We denote this time epoch by $L$. Thus, We also assume that there are no capacity constraints.

Once the selling season begins, demand uncertainty is resolved by default. The informed firm can acquire market information early (at time E) by various market research activities. Since it is quite well in advance of the selling season, and there are quite limited resources to get high quality market information. Thus, the informed firm's market information is not perfect market information. In the extension section, we further assume that this imprecision is resolved at near the selling season (at time L). So, we restrict the time horizon of analysis into $[E, L]$ in this paper.

The sequence of event is as follows. Both the informed and uninformed firm have two option of ordering either at time E or time L, knowing that the supplier might leak order information to the competitor, who can revise this order information for his order for the market. This decision is made before time E for both firms. The supplier then delivers the products before the start of the selling season. Demand and profits are realized at time $T$.

Additional feature of the model in this chapter is that we impose realistic information structure - No firm has a perfect market information well in advance of the selling season (time E), and this information can be realized at near the selling season (in the extension part) - without changing endogenous sequence of events. We mainly analyze the information structure of incomplete information where only one firm is informed with noisy signal. We still assume that uninformed firm has the only general market information throughout the time horizon. The appropriate equilibrium concept for the dynamic game of incomplete information is the Perfect Bayesian Nash Equilibrium.

The equilibrium in our model has three components which are intertwined with each other: ($i$) Information flows (from information acquisition to information dissemination in the supply chain) ($ii$) Material flows (the order quantities of the two suppliers), and ($iii$) Timing of orders.

## 2.3 Imperfect Information Game (ESG-IIG)

Suppose the informed firm has some belief as to what demand is - but does not know it exactly. For this setting, we introduce imprecision factor $\alpha$ ($\frac{1}{2} < \alpha < 1$)

where $\Pr(Signal = High\ /\ A = A_H) = \Pr(Signal = Low\ /\ A = A_L) = \alpha$. It means that the informed firm knows true demand with a noise factor $\alpha$. Consequently, $\Pr(Signal = Low\ /\ A = A_H) = \Pr(Signal = High\ /\ A = A_L) = 1 - \alpha$. Using Baye's rule, $\Pr(A = A_H/Signal = High) = \frac{\alpha p}{\alpha p+(1-\alpha)(1-p)}$, $\Pr(A = A_L/Signal = High) = \frac{(1-\alpha)(1-p)}{\alpha p+(1-\alpha)(1-p)}$, $\Pr(A = A_L/Signal = low) = \frac{\alpha(1-p)}{\alpha(1-p)+(1-\alpha)p}$, and $\Pr(A = A_H/Signal = low) = \frac{(1-\alpha)p}{\alpha(1-p)+(1-\alpha)p}$. Thus, the high type informed firm's expected profit, for example, is formulated as follows:

$$E\left[\pi_{iH}\right] = \Pr(A = A_H/S = H)[(A_H - q_{iH} - q_e)q_{iH}] + \Pr(A = A_L/S = H)[(A_L - q_{iH} - q_e)q_{iH}]$$

The following lemma explains equilibrium solution of both simultaneous and sequential move game for imperfect information. Detailed proof are provided in appendix.

**Lemma 8** *(i) The subgame-perfect Nash equilibrium quantity and profit for the simultaneous move game for imperfect information model are:*

$$\begin{aligned}
q_{iH} &= \frac{(\Pr(Signal = High/A = A_H) \cdot A_H + \Pr(Signal = Low/A = A_H) \cdot A_L)}{2} - \frac{\mu}{6} \\
&= \frac{(\alpha p A_H + (1-\alpha)(1-p)A_L))}{2(\alpha p + (1-\alpha)\,(1-p))} - \frac{\mu}{6} \\
q_{iL} &= \frac{(\Pr(Signal = Low/A = A_L) \cdot A_L + \Pr(Signal = High/A = A_L) \cdot A_H)}{2} - \frac{\mu}{6} \\
&= \frac{(\alpha(1-p)A_L + (1-\alpha)pA_H))}{2(a(1-p) + (1-\alpha)p)} - \frac{\mu}{6} \\
q_E &= \frac{\mu}{3}
\end{aligned}$$

$$
\begin{aligned}
E\left[\pi_{iH}\right] &= \left(\frac{(\alpha p A_H + (1-\alpha)(1-p)A_L))}{2(\alpha p + (1-\alpha)\,(1-p))} - \frac{\mu}{6}\right)^2 \\
E\left[\pi_{iL}\right] &= \left(\frac{(\alpha(1-p)A_L + (1-\alpha)pA_H))}{2(a(1-p) + (1-\alpha)p)} - \frac{\mu}{6}\right)^2 \\
E\left[\pi_E\right] &= \left(\frac{\mu}{3}\right)^2
\end{aligned}
$$

*(ii) The subgame-perfect Nash equilibrium quantity and profit for the sequential move game for imperfect information model are:*

$$
\begin{aligned}
q_{iH} &= \frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{2(\alpha p + (1-\alpha)(1-p)}, \; q_{iL} = \frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{2(\alpha(1-p) + (1-\alpha)p)} \\
q_{eH} &= \frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{4(\alpha p + (1-\alpha)(1-p)}, \; q_{eL} = \frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{4(\alpha(1-p) + (1-\alpha)p)}
\end{aligned}
$$

$$
\begin{aligned}
E\left[\pi_{iH}\right] &= \frac{1}{8}\left(\frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{(\alpha p + (1-\alpha)(1-p)}\right)^2, \; E\left[\pi_{iL}\right] = \frac{1}{8}\left(\frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{2(\alpha(1-p) + (1-\alpha)p)}\right)^2 \\
E\left[\pi_{eH}\right] &= \frac{1}{16}\left(\frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{(\alpha p + (1-\alpha)(1-p)}\right)^2, \; E\left[\pi_{eL}\right] = \frac{1}{16}\left(\frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{2(\alpha(1-p) + (1-\alpha)p)}\right)^2
\end{aligned}
$$

### 2.3.1 Analysis of Imperfect Information Game

Our model has endogenous sequence of moves for two players, equilibrium points have 1. Timing of order, 2. Material flows, and 3. Information flows. The difference from previous chapter is whether the informed firm still reveal noisy signal, and the uninformed firm delay his order for more accurate but not perfect market information. The supplier also has decision right whether to leak this noisy signal or not. Thus, IIG is also comprised with three types of scenarios: 1. Simultaneous order, 2. Informed firm's order at time E and uninformed firm's order at time L, and 3. Uninformed firm's order at time E and the informed firm's order at time L. The first scenario can be explained by first part of above lemma, and the second scenario can be explained by second part.

Each player's preference has similar pattern with previous chapter (only parameter is changed), and it can be summarized as follows. As long as imprecision factor ($\alpha$) is greater than half, low type informed firm always prefers sequential order by ordering at time E. Some of high type informed firm prefers to order simultaneously at time L. The uninformed firm prefers sequential order by ordering at time L, and as compared to previous section, the range of $[p_1, p_2]$ is decreased. But, as long as $\alpha > \frac{1}{2}$, there exists the uninformed firm's preference of sequential order.

### 2.3.2 Equilibrium of Imperfect Information Game

We analyze whether three feasible timing of orders are sustained as an equilibrium outcome in the imperfect information environment. By appropriately specified each player's belief structure captured in the previous section, we examine if each player's deviation is profitable or not. Also, we come up with the equilibrium timing of order and the volume of order from three possible scenarios.

#### Supplier's leakage decision

Followed by same procedure from previous chapter's analysis, the supplier's leakage decision can be analyzed in case of imperfect information model. *In the high demand state*, profit maximizing supplier's strategy is to leak informed firm's information to the uninformed firm. Selling quantity for sequential move $(\frac{3\alpha p A_H+(1-\alpha)(1-p)A_L}{4(\alpha p+(1-\alpha)(1-p)})$ is strictly greater than simultaneous move game $(\frac{(\alpha p A_H+(1-\alpha)(1-p)A_L))}{2(\alpha p+(1-\alpha)(1-p))}+\frac{\mu}{6})$. *In the low demand state*, the supplier will not leak the informed firm's ordering directly to the uninformed firm when demand uncertainty is located at $[p_1^i, p_2^i]$ where $p_1^i$ and $p_2^i$ is when two game's ordering quantity is same (when $\frac{(\alpha(1-p)A_L+(1-\alpha)pA_H))}{2(a(1-p)+(1-\alpha)p)}+\frac{\mu}{6}=\frac{3\alpha(1-p)A_H+(1-\alpha)pA_L}{4(\alpha(1-p)+(1-\alpha)p)}$). Since it is the only case when the supplier will not leak directly, the uninformed firm will infer that demand state is low. Therefore, previous chapter's result is still hold in the imperfect information case.

**Corollary 9** *Even if the informed firm's market information has certain degree of noisy signal, the supplier will always leak order information to the uninformed firm.*

The key consideration for the equilibrium analysis is that even if the informed firm has an imperfect signal at the start of a period, market leader's advantage is still dominant as long as the informed firm has better information ($\alpha > \frac{1}{2}$) than the uninformed firm. For example, when real demand is low, the low type informed firm's best action is to reveal his demand information into the uninformed firm to convey information at any time. In other words, the low type informed firm should always places order early even when *demand is low with noise.* This is still hold whenever the uninformed firm places order. On the other hand, the informed firm might not order when *demand is high with high uncertainty.* However, the uninformed firm will infer that it is high demand state based on his belief structure if the informed firm does not order early. So, the game is changed to simultaneous move game with complete information, and high type informed firm's profit $E\left[\pi_{iH}\right]=\left(\frac{(\alpha pA_H+(1-\alpha)(1-p)A_L))}{3(\alpha p+(1-\alpha)(1-p))}\right)^2$, which is strictly less than his initial profit of order at time E $(\left(\frac{(\alpha pA_H+(1-\alpha)(1-p)A_L))}{2(\alpha p+(1-\alpha)(1-p))}-\frac{\mu}{6}\right)^2)$ . So, we conclude that the informed firm always places order at time E.

The uninformed firm also consider following trade-off: simultaneous order at time E without acquiring market information and ordering at time L allowing market leader's advantage to the informed firm. Instead, by ordering at time L, the uninformed firm can acquire market information with certain degree of noise. Other procedures are same with perfect information game - The uninformed firm's deviation from ordering at time E is profitable, but his deviation from time L is not profitable. The intuition behind the argument is as follows: even though the value of information by ordering at time L for the uninformed firm is less than perfect information game, there is still information gap between two firms, and it plays a role conforming equilibrium. We found that as long as imprecision parameter is greater than half, previous chapter's Endogenous Sequencing Game is still holds, and sequential move game is still dominant for both firms.

**Proposition 10** *In the imperfect information game with certain imprecision parameter $\alpha$,there is only equilibrium for all $\alpha$, $\theta$ and $p$, and is as follows.*

*(i) The informed firm places order early. Order quantity is:*

$$\begin{aligned}
q_{iH} &= \frac{\alpha pA_H+(1-\alpha)(1-p)A_L}{2(\alpha p+(1-\alpha)(1-p)}, \text{ \textit{if demand is high, and}} \\
q_{iL} &= \frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{2(\alpha(1-p)+(1-\alpha)p)}, \text{ \textit{if demand is low}}
\end{aligned}$$

*(ii) Supplier leaks information both direct and indirect way.*

*(iii) The uninformed firm orders*

$$\begin{aligned} q_{eH} &= \frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{4(\alpha p + (1-\alpha)(1-p)}, \text{ if } Pr_e(\widetilde{A} = A_H) = 1 \\ q_{eL} &= \frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{4(\alpha(1-p) + (1-\alpha)p)}, \text{ if } Pr_e(\widetilde{A} = A_H) = 0 \end{aligned}$$

*consistent with his beliefs that:*

$$Pr_e(\widetilde{A} = A_H) = \begin{bmatrix} 1, \textit{if } q_i = \frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{2(\alpha p + (1-\alpha)(1-p)} \\ \textit{and the informed firm orders early and the supplier leaks} \\ 0, \textit{if } q_i = \frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{2(\alpha(1-p) + (1-\alpha)p)} \\ \textit{and the informed firm orders early and the supplier leaks,} \\ \textit{or, the supplier does not leak} \end{bmatrix}$$

*(iv) The profits of the informed firm and the uninformed firm are as follows:*

$$\begin{aligned} E[\pi_{iH}] &= \frac{1}{8}\left(\frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{(\alpha p + (1-\alpha)(1-p)}\right)^2, \textit{ if demand is high, and} \\ E[\pi_{iL}] &= \frac{1}{8}\left(\frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{2(\alpha(1-p) + (1-\alpha)p)}\right)^2, \textit{ if demand is low.} \\ E[\pi_{eH}] &= \frac{1}{16}\left(\frac{\alpha p A_H + (1-\alpha)(1-p)A_L}{(\alpha p + (1-\alpha)(1-p)}\right)^2, \text{ if } Pr_e(\widetilde{A} = A_H) = 1 \\ E[\pi_{eL}] &= \frac{1}{16}\left(\frac{\alpha(1-p)A_L + (1-\alpha)pA_H}{2(\alpha(1-p) + (1-\alpha)p)}\right)^2, \text{ if } Pr_e(\widetilde{A} = A_H) = 0 \end{aligned}$$

## 2.4 Information Evolution Game (ESG-IEG)

Suppose early ordering informed firm has very noisy signal about the demand ($\alpha$ is slightly greater than 0.5). In that period, if the informed firm waits, he can get a more accurate picture of demand. If he waits until '*Late*', then he will know demand with certainty. However, the informed firm has to trade-off waiting with the fact that waiting might cause him to act as a follower against the uninformed firm. If the uninformed firm waits, he cannot acquire more accurate signal of demand: recall that the uninformed firm can only acquires demand information by observable delay after the informed firm's order. But, if the uninformed firm places an order at time E, he might take market leader's advantage in case of late order from the informed firm.

The key fact in this setting is whether the low type informed firm still always prefers to order at time E. If he still prefers, the equilibrium path is similar pattern with previous results. If not, the uninformed firm cannot revise his belief structure when the informed firm delays his order. If the informed firm places order at time L with the uninformed firm, the uninformed firm cannot guarantee that demand is high state (therefore, simultaneous move game with incomplete information is played in this case).

For the simplicity, we assume that this signal become accurate when selling season is coming, and finally this imprecision is cleared at time L ($a = 1$). Also, for the simplicity of the model and to see pure effect of information evolution, we further assume that the uninformed firm has a perfectly noisy signal ($\alpha = 0.5$) throughout the time horizon $[E, L]$.

**Lemma 11** *$(i)$ If the informed firm places an order at time $E$ in Information Evolution Game (IEG), the games still follow lemma 1 of Imperfect Information Game (IIG). Cleared imprecision factor $(\alpha)$ does not affect the games of early ordering informed firm.*

*$(ii)$ If both the informed firm and the uninformed firm place orders at time $L$, the game is simultaneous move game. The subgame-perfect Nash equilibrium quantity of the informed firm and uninformed firm are:*

$$q_{iH} = \frac{3A_H - \mu}{6}; q_{iL} = \frac{3A_L - \mu}{6}; q_e = \frac{\mu}{3}$$

*and expected profit of both firms are:*

$$\begin{aligned}
E\left[\pi_{iH}\right] &= \Pr(A = A_H/S = H)\left(\frac{3A_H - \mu}{6}\right)^2 + \Pr(A = A_L/S = H)\left(\frac{3A_L - \mu}{6}\right)^2 \\
&= \frac{\alpha p}{\alpha p + (1-\alpha)(1-p)}\left(\frac{3A_H - \mu}{6}\right)^2 + \frac{(1-\alpha)(1-p)}{\alpha p + (1-\alpha)(1-p)}\left(\frac{3A_L - \mu}{6}\right)^2 \\
E\left[\pi_{iL}\right] &= \Pr(A = A_L/S = L)\left(\frac{3A_L - \mu}{6}\right)^2 + \Pr(A = A_H/S = H)\left(\frac{3A_H - \mu}{6}\right)^2 \\
&= \frac{\alpha(1-p)}{\alpha(1-p) + (1-\alpha)p}\left(\frac{3A_L - \mu}{6}\right)^2 + \frac{(1-\alpha)p}{\alpha(1-p) + (1-\alpha)p}\left(\frac{3A_H - \mu}{6}\right)^2 \\
E\left[\pi_e\right] &= \frac{\mu^2}{9}
\end{aligned}$$

*$(iii)$ If the uninformed firm orders before the informed firm, the game is sequential move (Stackelberg) game. The subgame-perfect Nash equilibrium quantity and profit*

*or the informed firm and uninformed firm are:*

$$q_{eH} = \frac{\mu}{2}; q_{iH} = \frac{2A_H - \mu}{4}; q_{iL} = \frac{2A_L - \mu}{4}$$

*and expected profit of both firms are:*

$$\begin{aligned} E\left[\pi_{iH}\right] &= \frac{\alpha p}{\alpha p + (1-\alpha)(1-p)} \left(\frac{2A_H - \mu}{4}\right)^2 + \frac{(1-\alpha)(1-p)}{\alpha p + (1-\alpha)(1-p)} \left(\frac{2A_L - \mu}{4}\right)^2 \\ E\left[\pi_{iL}\right] &= \frac{\alpha(1-p)}{\alpha(1-p) + (1-\alpha)p} \left(\frac{2A_L - \mu}{4}\right)^2 + \frac{(1-\alpha)p}{\alpha(1-p) + (1-\alpha)p} \left(\frac{2A_H - \mu}{4}\right)^2 \\ E\left[\pi_e\right] &= \frac{\mu^2}{8} \end{aligned}$$

In this section, we analyze if all feasible timing of orders are sustained as an equilibrium in the Information Evolution Game. By appropriately specified belief structure from each firm's trade-offs, we examine whether each player's one shot deviation is profitable or not. Since the only equilibrium of IIG is the informed firm's order at time E and the uninformed firm's order at time L (E, L), we start from (E, L) model as a proposed equilibrium and verify whether each player's deviation is profitable.

### 2.4.1 The informed firm's Early order, and the uninformed firm's Late order : (E, L)

First, we analyze the informed firm's preference in timing of order. In the base model, low type informed firm always prefers to order at time E. However, in the information evolution model, it is not. There is a threshold $\alpha^*$ where the low type informed firm also prefers to order at time L as well as high type informed firm. The threshold $\alpha^*$ can be estimated by comparing between the expected profit of low type informed firm's order at time E and expected profit of order at time L. Thus, when $\alpha \geq a^*$, low type informed firm still always prefers order at time E. This threshold is found where

$$\begin{aligned}\frac{1}{8}\left(\frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{(\alpha(1-p)+(1-\alpha)p)}\right)^2 &= \frac{\alpha(1-p)}{\alpha(1-p)+(1-\alpha)p}\left(\frac{3A_L-\mu}{6}\right)^2\\ &+\frac{(1-\alpha)p}{\alpha(1-p)+(1-\alpha)p}\left(\frac{3A_H-\mu}{6}\right)^2\end{aligned}$$

For example, when $A_H = 30, A_L = 10, p = 0.5$, threshold $\alpha^*$is 0.87, and when $A_H = 50, A_L = 10, p = 0.5$, threshold $\alpha^*$is 0.95. Thus, when $\alpha \geq a^*$ and the informed firm deviated from time E to L, the uninformed firm revises his belief structure, and he infers that it is high demand state. Then, the game is changed to complete information game. However, if $\alpha \leq a^*$, the uninformed firm cannot revises his belief structure *because both high and low type informed firm prefers to order late.* Finally, the informed firm's deviation is profitable with low $\alpha$. Therefore, we conclude that with threshold value of $\alpha^*$ the informed firm still places order early at every time, and the below of threshold value enables the informed firm to deviate to time L. In addition, the uninformed firm's deviation from time L to E is no profitable deviation with same procedure in base model since the uninformed firm couldn't get any valuable information by deviation. We conclude that (E, L) equilibrium is hold when $\alpha^*$ is greater than threshold value. Otherwise, it is not an equilibrium.

**Corollary 12** *With threshold value of information imprecision factor $\alpha$, sequential move game is still hold for two firms. threshold value $\alpha^*$ is the point where low type informed firm always prefers to place order at time E.*

### 2.4.2 The informed firm's Late order, and the uninformed firm's Late order : (L, L)

In the previous section, the informed firm does place order early when $\alpha \geq \alpha^*$. But, when $\alpha < \alpha^*$, the informed firm's deviation is profitable. In this part, we verify that whether both firms' order at time L is another equilibrium or not. If both firms place order at time L, it is simultaneous move game with perfect information. Expected profit for both firms are

$$\begin{aligned}
E\left[\pi_{iH}\right] &= \frac{\alpha p}{\alpha p + (1-\alpha)(1-p)}\left(\frac{3A_H - \mu}{6}\right)^2 + \frac{(1-\alpha)(1-p)}{\alpha p + (1-\alpha)(1-p)}\left(\frac{3A_L - \mu}{6}\right)^2 \\
E\left[\pi_{iL}\right] &= \frac{\alpha(1-p)}{\alpha(1-p) + (1-\alpha)p}\left(\frac{3A_L - \mu}{6}\right)^2 + \frac{(1-\alpha)p}{\alpha(1-p) + (1-\alpha)p}\left(\frac{3A_H - \mu}{6}\right)^2 \\
E\left[\pi_e\right] &= \frac{\mu^2}{9}
\end{aligned}$$

It is easy to verify that the uninformed firm's deviation from 'Late' to 'Early' is always profitable. When the uninformed firm deviates from time L to E, his optimal order quantity is $\frac{\mu}{2}$ (as a Stackelberg leader), and expected profit is $\frac{\mu^2}{8}$ as long as the informed firm places order at time L. It is always greater than late ordering profit ($\frac{\mu^2}{9}$). Therefore, it is not an equilibrium.

### 2.4.3 The informed firm's Late order, and the uninformed firm's Early order : (L, E)

When the uninformed firm place order at time E and the informed firm orders at time L, expected profit for both firms are

$$\begin{aligned}
E\left[\pi_{iH}\right] &= \frac{\alpha p}{\alpha p+(1-\alpha)(1-p)}\left(\frac{2A_H-\mu}{4}\right)^2+\frac{(1-\alpha)(1-p)}{\alpha p+(1-\alpha)(1-p)}\left(\frac{2A_L-\mu}{4}\right)^2\\
E\left[\pi_{iL}\right] &= \frac{\alpha(1-p)}{\alpha(1-p)+(1-\alpha)p}\left(\frac{2A_L-\mu}{4}\right)^2+\frac{(1-\alpha)p}{\alpha(1-p)+(1-\alpha)p}\left(\frac{2A_H-\mu}{4}\right)^2\\
E\left[\pi_e\right] &= \frac{\mu^2}{8}
\end{aligned}$$

With low $\alpha$ which is less than $\alpha^*$, the informed firm's deviation from time L to E is never profitable. If the informed firm deviates, both high and low type informed firm's objective functions are

$$\begin{aligned}
E\left[\pi_{iH}\right] &= \Pr(A=A_H/S=H)[(A_H-q_{iH}-\frac{\mu}{2})q_{iH}]+\Pr(A=A_L/S=H)[(A_L-q_{iH}-\frac{\mu}{2})q_{iH}]\\
E\left[\pi_{iL}\right] &= \Pr(A=A_L/S=L)[(A_L-q_{iL}-\frac{\mu}{2})q_{iL}]+\Pr(A=A_H/S=H)[(A_H-q_{iH}-\frac{\mu}{2})q_{iH}]
\end{aligned}$$

and optimal order quantity and profits are

$$\begin{aligned}
q_{iH} &= \frac{(\alpha pA_H+(1-\alpha)(1-p)A_L))}{2(\alpha p+(1-\alpha)(1-p))}-\frac{\mu}{4}\\
q_{iL} &= \frac{(\alpha(1-p)A_L+(1-\alpha)pA_H))}{2(a(1-p)+(1-\alpha)p)}-\frac{\mu}{4}
\end{aligned}$$

$$\begin{aligned}
E\left[\pi_{iH}\right] &= \left(\frac{(\alpha pA_H+(1-\alpha)(1-p)A_L))}{2(\alpha p+(1-\alpha)(1-p))}-\frac{\mu}{4}\right)^2\\
E\left[\pi_{iL}\right] &= \left(\frac{(\alpha(1-p)A_L+(1-\alpha)pA_H))}{2(a(1-p)+(1-\alpha)p)}-\frac{\mu}{4}\right)^2
\end{aligned}$$

and it is strictly less than initial profits. It means that he does not allow market leader's advantage to the uninformed firm, but he cannot acquire clear market information. Thus, the informed firm's deviation is not profitable. Also, the uninformed firm's deviation is never profitable. Since the uninformed firm cannot take any benefit from delaying ordering, he just lose market leader's advantage. When the uninformed firm deviates from time E to L, the game becomes simultaneous move with incomplete information. Expected profit for the uninformed firm is $\frac{\mu^2}{9}$,which is strictly less than initial profit. Therefore, we conclude that (L, E) is an equilibrium when imprecision factor $\alpha$ is strictly less than threshold value $\alpha^*$.

### 2.4.4 The informed firm's Early order, and the uninformed firm's Early order : (E, E)

It is easy to confirm that the uninformed firm will deviate from time E to L as long as two conditions are hold: $(i)$the informed firm has better information than uninformed firm, and $(ii)$ the informed firm places order at time E. Suppose the informed firm has better information and he places order early. The uninformed firm's deviation from 'Early' to 'Late' is always profitable. Initial profit for the both players are $\pi_{iH} = \left(\frac{(\alpha p A_H+(1-\alpha)(1-p)A_L))}{2(\alpha p+(1-\alpha)(1-p))} - \frac{\mu}{6}\right)^2$, $\pi_{iL} = \left(\frac{(\alpha(1-p)A_L+(1-\alpha)pA_H))}{2(a(1-p)+(1-\alpha)p)} - \frac{\mu}{6}\right)^2$, and $\pi_E = \left(\frac{\mu}{3}\right)^2$ in previous lemma.

If the uninformed firm deviates from 'Early' to 'Late', his objective functions are

$$\begin{aligned}
E\left[\pi_{eH}\right] &= \Pr(A = A_H/S = H)[(A_H - q_{iH}^{C*} - q_{eH})q_{eH}] \\
+\Pr(A &= A_L/S = H)[(A_L - q_{iH}^{C*} - q_{eH})q_{eH}] \\
E\left[\pi_{eL}\right] &= \Pr(A = A_L/S = L)[(A_L - q_{iL}^{C*} - q_e)q_{iL}] \\
+\Pr(A &= A_H/S = H)[(A_H - q_{iL}^{C*} - q_e)q_{iH}]
\end{aligned}$$

(where $q_{iH}^{C*}$ and $q_{iL}^{C*}$ are the informed firm's optimal order quantity for simultaneous move game) and optimal order quantity and profits for the uninformed firm after deviation are

$$\begin{aligned}
q_{eH} &= \frac{3(\Pr(A = A_H/S = H) \cdot A_H + \Pr(A = A_L/S = H) \cdot A_L) + \mu}{12} \\
q_{eL} &= \frac{3(\Pr(A = A_L/S = L) \cdot A_L + \Pr(A = A_H/S = H) \cdot A_H) + \mu}{12} \\
E\left[\pi_{eH}\right] &= \left(\frac{3(\Pr(A = A_H/S = H) \cdot A_H + \Pr(A = A_L/S = H) \cdot A_L) + \mu}{12}\right)^2 \\
E\left[\pi_{eL}\right] &= \left(\frac{3(\Pr(A = A_L/S = L) \cdot A_L + \Pr(A = A_H/S = H) \cdot A_H) + \mu}{12}\right)^2
\end{aligned}$$

Expected profit $p \cdot E\left[\pi_{eH}\right] + (1-p)E\left[\pi_{eL}\right]$ is always greater than initial $\pi_e$. Therefore, it is an profitable deviation, and (E, E) is not an equilibrium.

In sum, we conclude that there are two possible equilibriums depending on imprecision factor $\alpha$, and are as follows.

**Proposition 13** *With threshold value $\alpha^*$ where low type informed firm always prefers* *to place order early with uninformed firm's late order,i.e.*

$$\begin{aligned}\frac{1}{8}\left(\frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{2(\alpha(1-p)+(1-\alpha)p)}\right)^2 &= \frac{\alpha(1-p)}{\alpha(1-p)+(1-\alpha)p}\left(\frac{3A_L-\mu}{6}\right)^2\\ &\quad+\frac{(1-\alpha)p}{\alpha(1-p)+(1-\alpha)p}\left(\frac{3A_H-\mu}{6}\right)^2\end{aligned}$$

*there are two equilibriums for all $\alpha$, $\theta$ and $p$, and are as follows.*

***Case (i)*** *When $\alpha \geq \alpha^*$*

*(i) The informed firm places order early. Order quantity is:*

$$\begin{aligned} q_{iH} &= \frac{\alpha p A_H+(1-\alpha)(1-p)A_L}{2(\alpha p+(1-\alpha)(1-p)}, \text{ if demand is high, and}\\ q_{iL} &= \frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{2(\alpha(1-p)+(1-\alpha)p)}, \text{ if demand is low}\end{aligned}$$

*(ii) Supplier leaks information both direct and indirect way.*

*(iii) The uninformed firm orders*

$$\begin{aligned} q_{eH} &= \frac{\alpha p A_H+(1-\alpha)(1-p)A_L}{4(\alpha p+(1-\alpha)(1-p)}, \text{ if } Pr_e(\widetilde{A}=A_H)=1\\ q_{eL} &= \frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{4(\alpha(1-p)+(1-\alpha)p)}, \text{ if } Pr_e(\widetilde{A}=A_H)=0\end{aligned}$$

*consistent with his beliefs that:*

$$Pr_e(\widetilde{A}=A_H)=\left[\begin{array}{c} 1, \textit{if } q_i = \frac{\alpha p A_H+(1-\alpha)(1-p)A_L}{2(\alpha p+(1-\alpha)(1-p)} \\ \\ \textit{and the informed firm orders early and the supplier leaks} \\ \\ 1, \textit{if the informed firm does not place order at time } E \\ \\ 0, \textit{if } q_i = \frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{2(\alpha(1-p)+(1-\alpha)p)} \\ \\ \textit{and the informed firm orders early and the supplier leaks,} \\ \\ \textit{or, the supplier does not leak} \end{array}\right]$$

*(iv) The profits of the informed firm and the uninformed firm are as follows:*

$$\begin{aligned}
E\left[\pi_{iH}\right] &= \frac{1}{8}\left(\frac{\alpha p A_H+(1-\alpha)(1-p)A_L}{(\alpha p+(1-\alpha)(1-p)}\right)^2, \textit{ if demand is high, and} \\
E\left[\pi_{iL}\right] &= \frac{1}{8}\left(\frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{2(\alpha(1-p)+(1-\alpha)p)}\right)^2, \textit{ if demand is low.} \\
E\left[\pi_{eH}\right] &= \frac{1}{16}\left(\frac{\alpha p A_H+(1-\alpha)(1-p)A_L}{(\alpha p+(1-\alpha)(1-p)}\right)^2, \ \textit{if } Pr_e(\widetilde{A}=A_H)=1 \\
E\left[\pi_{eL}\right] &= \frac{1}{16}\left(\frac{\alpha(1-p)A_L+(1-\alpha)pA_H}{2(\alpha(1-p)+(1-\alpha)p)}\right)^2, \ \textit{if } Pr_e(\widetilde{A}=A_H)=0
\end{aligned}$$

***Case (ii)*** *When* $\alpha<\alpha^*$

*(i) The uninformed firm places order early. Order quantity is:*

$$q_e=\frac{\mu}{2}$$

*(ii) Supplier leaks information.*

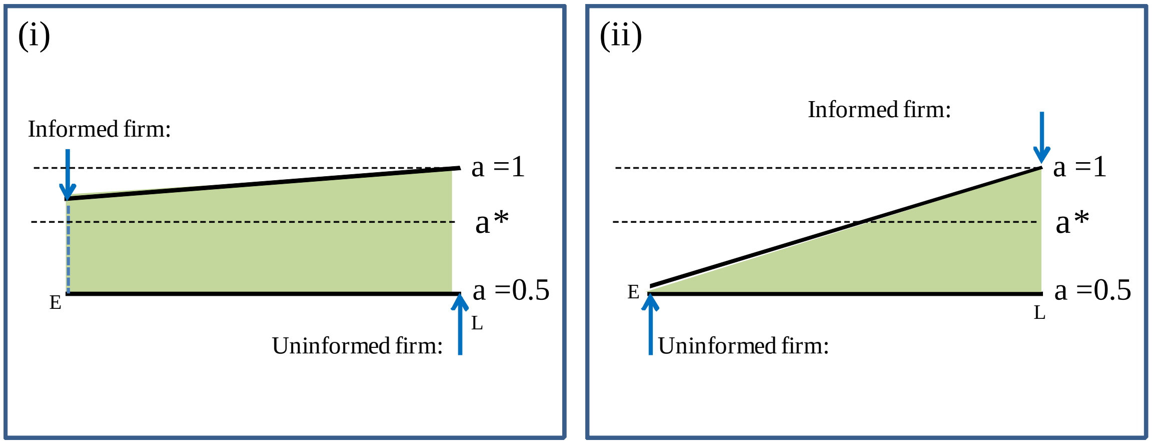



Figure 2.2: Two Equilibria in Information Evolution Game

*(iii) The informed firm orders*

$$\begin{aligned} q_{iH} &= \frac{(\alpha p A_H + (1-\alpha)(1-p)A_L))}{2(\alpha p + (1-\alpha)(1-p))} - \frac{\mu}{4}, \textit{ if demand signal is high} \\ q_{iL} &= \frac{(\alpha(1-p)A_L + (1-\alpha)pA_H))}{2(a(1-p) + (1-\alpha)p)} - \frac{\mu}{4}, \textit{ if demand signal is low} \end{aligned}$$

*(iv) The profits of the informed firm and the uninformed firm are as follows:*

$$\begin{aligned} \pi_e &= \frac{\mu^2}{8} \\ \pi_{iH} &= \left(\frac{(\alpha p A_H + (1-\alpha)(1-p)A_L))}{2(\alpha p + (1-\alpha)(1-p))} - \frac{\mu}{4}\right)^2, \textit{ if demand signal is high} \\ \pi_{iL} &= \left(\frac{(\alpha(1-p)A_L + (1-\alpha)pA_H))}{2(a(1-p) + (1-\alpha)p)} - \frac{\mu}{4}\right)^2, \textit{ if demand signal is low} \end{aligned}$$

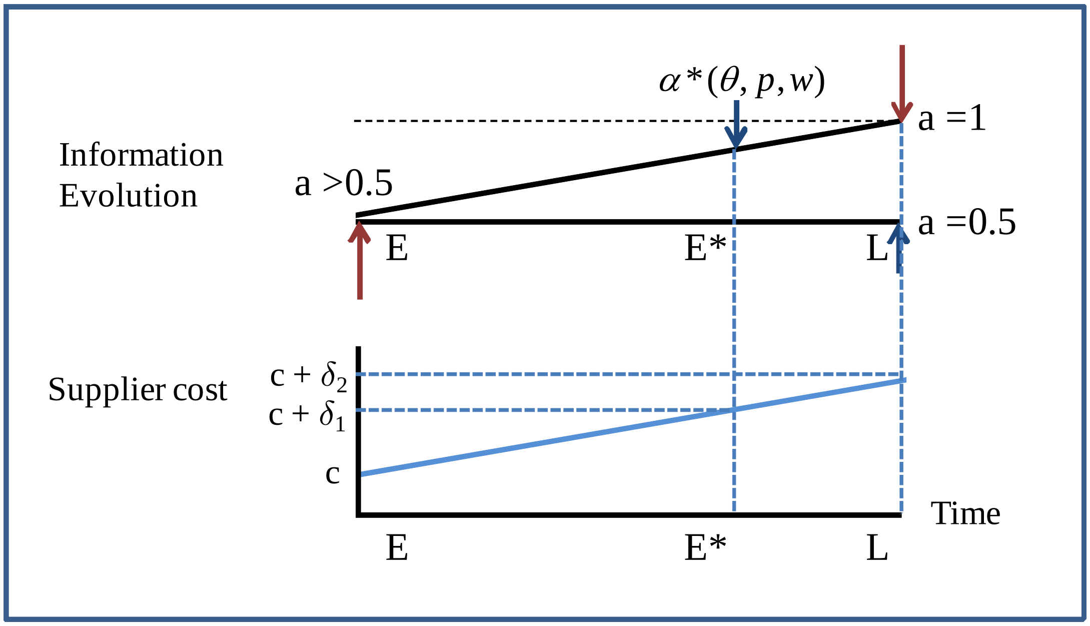



Figure 2.3: The relationship between evolution of market information and the supplier's production cost.

## 2.5 Supplier's preference about two equilibria

Suppose that the supplier can choose opening up and closing point of ordering, and let this time window $[E, L]$. Also, suppose that the supplier's production cost is minimum with the order at time E, and it is increased and maximum with the order at time L. Also suppose that at time E, the informed firm's $\alpha < \alpha^*$, and $\alpha$ is reached to $\alpha^*$ and time $E^*$, and finally it is one at time L. Suppose the supplier's wholesale price is $w$, and the cost of production for the earliest ordering is $c$, and production cost of the timing at $E^*$ is $c + \delta_1$, and that of time L is $c + \delta_2$ $(\delta_1 < \delta_2)$. Figure 2 illustrates the relationship among time horizon, evolution of market information, and the supplier's production cost.

Without any kind of private demand information for the supplier, his expected profit for the first equilibrium (the informed firm's order at time $E^*$, the uninformed firm's order at time L) is

$$\begin{aligned} E[\pi_{s1}] &= [\text{Expected order quantity for the informed firm at time } E^*](w-(c+\delta_1)) \\ &\quad +[\text{Expected order quantity for the uninformed firm at time L}](w-(c+\delta_2)) \end{aligned}$$

The supplier's expected profit for the second equilibrium (the informed firm's order at time L, the uninformed firm's order at time E) is

$$\begin{aligned} E[\pi_{s2}] &= [\text{Expected order quantity for the uninformed firm at time } L](w-c) \\ &\quad +[\text{Expected order quantity for the informed firm at time E}](w-(c+\delta_2)) \end{aligned}$$

After simplification, expected profit of second equilibrium ($E[\pi_{s2}]$) is always greater than first equilibrium ($E[\pi_{s1}]$). Its difference is minimized at $p=0$, and maximized at $p=1$. For example, when $p=0.5$,

$$\begin{aligned} E[\pi_{s1}] &= \frac{1}{4}(A_H+A_L)(w-c-\delta_1)+\frac{1}{8}(A_H+A_L)(w-c-\delta_2) \\ &= \frac{3}{8}(A_H+A_L)(w-c)-\frac{1}{4}(A_H+A_L)\cdot\delta_1-\frac{1}{8}(A_H+A_L)\cdot\delta_2 \\ E[\pi_{s2}] &= \frac{A_H+A_L}{4}\cdot(w-c) \\ &\quad + \left[\begin{array}{c} \frac{1}{2}\left(\frac{(\alpha A_H+(1-\alpha)A_L))}{2}-\frac{A_H+A_L}{8}\right) \\ +\frac{1}{2}\left(\frac{(\alpha A_L+(1-\alpha)A_H)}{2}-\frac{A_H+A_L}{8}\right) \end{array}\right](w-(c+\delta_2)) \\ &= \frac{3}{8}(A_H+A_L)\cdot(w-c)-\frac{1}{8}(A_H+A_L)(w-(c+\delta_2)) \end{aligned}$$

$$\begin{aligned}
E[\pi_{s2}] - E[\pi_{s1}] &= \frac{3}{8}(A_H + A_L)\cdot(w-c) - \frac{1}{8}(A_H + A_L)(w - (c+\delta_2)) \\
&\quad - \left(\frac{3}{8}(A_H + A_L)(w-c) - \frac{1}{4}(A_H + A_L)\cdot\delta_1 - \frac{1}{8}(A_H + A_L)\cdot\delta_2\right) \\
&= \frac{1}{4}(A_H + A_L)\cdot\delta_1
\end{aligned}$$

which is always positive number regardless of any other variables. Therefore, the supplier prefers to accept order at time E.

## 2.6 Discussion / Conclusion

In this chapter, we prove that informed firm's early order is robust to imperfect information, and even certain condition of information evolution setting. Consistent with practice, firms can usually have more accurate market demand information near the selling season by acquiring market information from more diverse resources. So, it is natural that firms have inaccurate market information in the earlier period, and that imprecision can be resolved in near the selling season. One of limitations from previous model is that the informed firm has perfect market information by default throughout the feasible time horizon. Our results on endogenous sequencing where the informed firm orders early are robust to $(i)$ when the firms are imperfectly informed - all that is needed is one firm to be better informed than the other, and $(ii)$ when the early acquired information has better quality of signal than threshold level even if this information is evolving.

Another interesting finding is the case that the uninformed firm orders early and the informed firm orders late with information evolution. When the informed firm has low quality of signal, he allows market leader's advantage to the uninformed firm and waits for more accurate market information. Thus, there is a trade-off between market leader's advantage and more accurate signal. The informed firm has more benefit from cleared information, and the uninformed firm has leader's advantage. This also holds the reverse case. The informed firm has market leader's advantage, and the uninformed firm has better demand information by observable delay. Surprisingly, we found that simultaneous move game is never formalizing equilibrium in endogenous sequencing game in various information structures.

In Operations Management research, all extant literature to induce early order had to have either create incentive or risk (carrot or stick). However, we show that much simple mechanism to induce early order in supply chain as long as one firm is better informed than the other.

# Chapter 3

# Strategic Management in New Product Innovation and Process Improvement

## 3.1 Introduction

The trade-off between firm's efficiency and creativity (also called *productivity dilemma*) has long been discussed in the business context. Abernathy (1978) first suggested that a firm's ability to compete over time was rooted not only in increasing efficiency, but also in its ability to be simultaneously innovative. In both operations and strategic management research, balancing between exploration and exploitation has been a consistent theme for more than 30 years after Abernathy's arguments. Not only in academic area, but also in industry practice, whether to focus on efficiency or creativity is one of the most important and difficult decisions for the top management. Recent industry example - Struggle between creativity and efficiency (Business Week (2007)) - illustrates how process management affects firm's ability in new product innovation. 3M has built a reputation for being an outstanding corporate innovator over its 100-plus-year history. However, after importing company-wide process management (Six Sigma) program by new CEO in early 2000, the company's reputation as an innovator has been sliding[1]. However, to the best of knowledge, there was no clear cut answer with analytical modeling approach explaining this issue. Thus, in this paper, we develop analytical models to explain productivity dilemma issue. We mainly analyze the relationship between process management and different type of new product innovation using extreme value theory with generalized cost function in innovation, focusing on how innovation performance and process management affect firm's optimal investment decision and the market equilibrium outcome. Although there are some other qualitative issues which are hard to be in line with the mathematical modeling framework, we build a stylized modeling approach considering some of important factors such as innovation characteristics of product and competition.

In spite of consistent debates among industry practitioners about productivity dilemma, less attention has been given in analytical approach of Operations Management area. Also, there has been limited formal academic research on whether

[1] In 2004, 3M was ranked No.1 on Boston consulting Group's most innovative companies list. It dropped No. 2 in 2005, No. 3 in 2006, No. 7 in 2007, No. 22 in 2008, No. 41 in 2009, and not ranked (NR) in 2010.

the trade-off between process management and innovation is inevitable. Benner & Tushman (2003) shows seminal work with an empirical approach that process management will promote more incremental innovation, but it will decreases blue-sky innovation. They show that process management activities must be buffered from exploratory activities and that ambidextrous forms provide the complex context for these inconsistent activities to coexist. This phenomenon is further claimed by Ruffa (2009), describing a related approach known as *lean dynamics.* Lean dynamics takes a different approach from lean manufacturing: it does not directly target the desired outcome of waste elimination. i.e. zero defects. Instead, it promotes a different way of structuring the business that creates an inherent "dynamic and optimal stability" of firm. Both Benner & Tushman (2003) and Ruffa (2009) are supported by interview of Arthur Fry, inventor of the Post-it note and a more than 30 years of 3M scientist, with BusinessWeek. We take note of his argument: *"Innovation is a numbers game. You have to go through 5,000 to 6,000 raw ideas to find one successful business. Six Sigma asks to eliminate all that waste at the first time and just come up with the right idea ."*

Process management, first introduced by Deming (1986), became popular as a central element of quality management, and these practices have been continued as ISO 9000, and Six Sigma recently. A large number of organization have adopted process management practices to improve quality and ultimately reduce cost. While Process Management's contribution to improve manufacturing efficiency is widely demonstrated (ex. 3M's short term profit increase), it has led not only to excellence in operations but also affect to adjacent process for selecting and developing technological innovations (Brown & Duguid (2000)). As variation decreasing and efficiency oriented focus of process management spread to core of innovation, or variation creating activities, it's effect also migrates firm's capability of creativity. For instance, Benner & Tushman (2002) empirically shows that after applying process management, firm's number of patent of incremental innovation (by citing prior patent) is increased whereas that of radical innovation (no prior patent) is sharply decreased.

While process management demands precision and consistency, innovation calls for variation and serendipity. Different from process management, firm's innovation structure depends on it's profit extremes. Dahan & Mendelson (2001) first introduces three types innovation projects using extreme value distribution in New Product Development (NPD). They classify innovation along proximity to the current technological trajectory on the technological dimension. A firm undertakes multiple R&D projects, and choose a project with highest performance. A key theorem in extreme value distribution states that when the distribution of the maximum of multiple independent, identically distributed random variables converges to a limit, and the limit belongs to one of three families of distribution (Coles (2001)). Two extreme case is a product category with great upside uncertainty (*Frechet*) and that of predictably finite bounds on the upside profit potential (*Weibull*[2]). For the general case (*Gum-*

[2] We call these type of innovations as New Frontier (or paradigm shifting) and Incremental innovation respectively throughout the paper.

*bel*), there are no specific limits on the profit potential, but the probability of outside of central range is unlikely. To capture innovation performance, we also apply these extreme value distributions.

In this study, main problem centers on process management's effect on technological innovation. More specific research questions are as follows.

• What is the optimal investment decision for firms between process efficiency and new product innovation?

• How could this optimal investment decision be changed by product characteristics (type of innovation) and the degree of competition?

First, we highlight the characteristics of type of innovation using extreme value theorem. If R&D projects are new frontier innovation or independent with previous activities - larger variance and fat-tailed distribution, there exist stronger adverse relationship between efficiency and new frontier innovation than incremental innovation, which is relatively small variance and finite bound. Second, competition effect forces to decrease firm's investment volume. We apply these two effects - innovation and efficiency - into competition model using multinomial logit (MNL) market share model. Basically, we find similar result with extant literature in innovation explaining that competition result in underinvestment effort from the firms (*cf.* Fullerton and McAfee(1999), Taylor (1995)). Thus, firms are more likely to focus on process improvement. For example, in the symmetric firms' competition, their optimal number of R&D project sharply decreased compared to that of monopoly case.

Structure of paper is as follows. In next section, we briefly review related literature. In Section 3 and 4, we introduce key building blocks of innovation and process management characteristics. In section 5, we apply innovation competition model and proceed with equilibrium analysis. Concluding remarks are in Section 6.

## 3.2 Literature Review

The role of innovation and process management has been examined in the business strategy side of research. Benner & Tushman (2003) is the key motivation of our paper. They claimed that process management is fundamentally inconsistent with innovation but incremental innovation and change. They defined type of innovation and its relationship with process management, and process management is only suitable to exploitative innovations involving improvements in existing components and build on the existing technological trajectory, whereas exploratory innovation involves a shift to a different technological trajectory. We do not review the literature on productivity dilemma issue in empirical side of research as it is quite vast, but, refer to the excellent surveys in Gupta et al (2006), Adler et al (2009). The necessity of balancing efficiency and innovation has not lessen after Abernaty's observation. For example, O'Reilly et al (2008) suggested how ambidexterity acts as a dynamic capability. They suggest that efficiency and innovation need not be strategic trade-offs and highlight the substantive role of top management in building dynamic capabilities.

Our model consider statistical point of view on new product development model combining the effect of process management. Dahan and Mendelson (2001), which is starting point of our analysis, first developed a model from parallel draws from extreme value distribution (Galambos (1978)). The expected performance outcome of innovation is the highest realization of the parallel draw. The major accomplishment of Dahan and Mendelson is that optimal solutions in innovation can be varied with type of innovations, and we employ this model to identify the statistical properties that influence the best performance. Many literature used this concept to develop the models. For example, Girotra et al (2010) showed that independent structure of innovation has higher performance than group of brainstorming. Terwiesch and Xu (2008) modeled innovation contest in that one solver's solution is the highest outcome of parallel draws. Kavadias and Sommer (2009) showed similar research with Girotra et al (2010), and they analyzed relative performance of two different group using simulation model. All these literature considered Gumbel distribution for their analysis. A economics literature Fullerton and McAfee (1999) explored the optimal design of R&D tournament focusing on symmetric competitors. They also find that the seeker suffers from underinvestment in effort by the participants.

To best of our knowledge, our paper is the first analytical modeling research in productivity dilemma. A stream of research has investigated process management effect in manufacturing/assembling area. Analytical models on process management is divided by two main groups: cost reduction and quality improvement. Bernstein and Kok (2009) considered investment level in process management for cost reduction. They explores the multi-periods model of supplier's investment in cost reduction activities done by Lean Production and Six Sigma during the life-cycle of the product. Fine and Porteus (1989) considered single firm process improvement setting using dynamic programming. They characterize optimal investment policy resulting in set up cost reduction and process quality improvement. Li and Rajagopalan (1998) demonstrated comprehensive model in process management, quality, and learning effect. Their model shows that after considerable amount of investment in quality improvement, this investment effort is decreasing.

## 3.3 The Model

We formalize investment strategy in new product innovation and process management. First, we assume that innovation performance and process management compete for scarce resources (March (1991), Gupta & Smith (2006)). Thus, by definition, more resources devoted to process management imply fewer resources left over for innovation, and vice versa. - that is, high values of one will necessarily imply low values of the other. Accordingly, the logic dictates that innovation and process management can be viewed as two ends of a continuum. Second, we assume that firm's innovation performance can be estimated by maximum of multiple innovation projects. As briefly mentioned in 3M inventor Arthur Fry, the goal of idea generation is to maximize the performance of the best idea. Firms prefer to have 1 outstanding idea rather than 10 fair ideas. Former research on innovation has modeled the process of creating ideas from extreme value distribution with parallel draws. Extreme value theory explains the maximum value of multiple sample from an underlying distribution can be modeled as a function of three variables: the number of sample size, the average, and the variance. The generalized extreme value distribution is:

$$F(x;\mu,\sigma,\xi) = Exp\left[-\left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{\frac{-1}{\xi}}\right]$$

where $\mu$ is average performance, $\sigma$ is variance, and $\xi$ is shape parameter[3]. Expected performance of $n$ number of project result in

$$E[V] = n\int_0^{\infty} x\cdot[F(x)]^{n-1}f(x)dx = \int_0^{\infty}(1-[F(x)]^n)dx$$

We classify the type of innovation according to the tail shape of their innovation performance. Frechet distribution has fat right hand tailed innovation outcome, and it is characterized as new frontier and paradigm shifting innovation. This type of innovation has breakthrough potential in their product. (ex. iPhone, 3D TV) Weibull distribution is short tailed and bounded in upside potential, and it is classified as incremental innovation in well matured industry. Thus, any gain due to innovation in Weibull type of innovation tends to be bounded in innovation potential. Gumbel distribution is considered as normally distributed R&D performance, and many academic papers in innovation use Gumbel distribution for ease of analysis. This paper

[3] The distribution is Frechet (Frontier Innovation in our model) if $\xi > 0$, Weibull (Incremental Innovation) if $\xi < 0$, and Gumbel as $\xi \to \infty$.

also considers Gumbel distribution for analytical tractability issue.

### Discussion about cost function

In our model, variance $\sigma$ is important decision variable. $\sigma$ parameterizes innovation potential and large $\sigma$ implies higher potential to create cutting edge innovative product. Moreover, $\sigma$ intervenes in the unit innovation costs, and is an indicator of degree of process management for firm. The main argument of process management is to reduce variance. It might result in cost reduction, increasing speed to market, improving R&D processes, etc (Kwak and Anbari (2006)). In this paper, we analyze the effect of process management with cost saving point of view. As firm reduce its variance ($\sigma$) by various practice of process management tools, firms cost for innovation is decreased. Thus, we assume that firm's unit innovation cost is a function of variance ($\sigma$). The coefficient $A$ is a scale parameter, and $\alpha$ indicates generalized cost function in innovation is following form: $(A, \alpha, K > 0,\ 0 < c < 1$, and $\sigma >> c)$

$$C(\sigma) = A \cdot \sigma^{\alpha} + K$$

In many Operations Management literature, cost estimation models typically posit a relationship of the form $C(\sigma) = A \cdot \sigma^{\alpha}$ (*cf.* Jones & Mendelson (2011), Bhaskaran & Krishnan (2009)[4], etc.)[5] Large $\sigma$ implies higher possibility to have better innovation

[4]Many other literature (ex. Cohen & Klepper (1996)) estimated convex cost function for innovation quality. Since convexity is not a driving factor for our analysis, we open to other kind of cost function such as linear or cancave in this analysis.

[5]Some models used quality as a key decision variable in cost function commonly assuming that higher quality pays higher cost. Coincidence with these papers, we also assume that higher performance in innovation pays higher R&D cost by intervening of $\sigma$ in the cost function.

performance whereas higher unit R&D cost in innovation. Large $\sigma$ also implies lower level of process management. Conversely, small $\sigma$ implies reduced unit innovation cost as a result of process management. But, firms can expect limited potential in innovation performance. We further assume that $\sigma$ also includes organizational culture implicitly. For example, many recent innovative firms such as Google and Gore-Tex give its employee one day out of five working days to pursue their own creative ideas. It enables firm to create paradigm shifting innovation, but more expensive unit R&D cost.

Therefore, a generalized firm's expected profit in innovation can be summarized as follows:

$$\begin{aligned} E[\pi] &= E[V] - n \cdot C(\sigma) \\ &= \int_0^{\infty} (1 - [F(x)]^n) dx - n \cdot C(\sigma) \end{aligned}$$

We extend this model in competition. In the competition model, two firms decide how many R&D project will be chosen in earlier stage, knowing that his competitors are doing same kind of innovation projects. We start with two symmetric firm case, and further analyze asymmetric firm's innovation competition. In asymmetric firm's competition, we mainly focus process management effect with different variances ($\sigma_1$, $\sigma_2$) and cost ($C(\sigma_1)$, $C(\sigma_2)$).

The time line of model is as follows. First, the firm collects all possible idea of new product (estimating type of innovation ($\xi$) and variance ($\sigma$).) Then, decides how

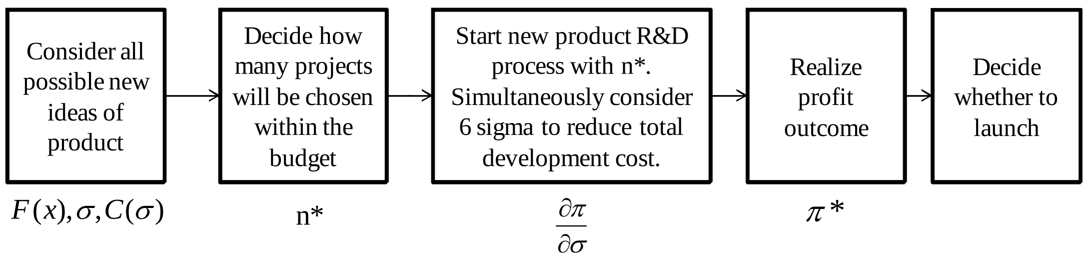



Figure 3.1: Time line of the Model

many projects will be selected within the budget[6]. Selected projects start innovation process, and the firm considers process management activities to reduce total R&D cost. Also, this R&D process is engaged in competition. After realization of profit and new product's performance, firms decide to launch new product or not. This time line is also depicted in Figure 1.

In next section, we analyze characteristics of extreme value theory, and impact of process management to firm's profitability. And we continue with competition effect on both innovation and process management.

### 3.3.1 Effect of $\sigma$ in firm's innovation performance

Firm's innovation performance can be determined by a project with the highest outcome, our analysis depends on the distribution of the maximum of $n$ draws from the outcome distribution. Extreme value theory provides a relationship between the number of projects a firm undertakes and the expected performance of innovation.

[6]Until this stage, it is an usual innovation process with parallel draws (Dahan & Mendelson (2001)).

A central notion in extreme value theory explains that when the limit exists, the distribution of the maximum of i.i.d. random variables converges to one of three types of distributions (Coles (2001)), known as the extreme value distributions according to their tail shapes. Frechet distribution is considered when a innovation category with high upside uncertainty such as paradigm shifting product. In such cases, products might become 'hot-seller', characterized by fat right hand tail distribution. Weibull distribution can be measured by highly predictable innovation. It has finite bound on the upside profit potential of new product like incremental innovation. Gumbel distribution is considered when there are no specific limits on the profit potential from a new product innovation, but it has profit performance with central tendency. Gumbel distribution is the asymptotic distribution for the maximum of multiple draws from exponential-tailed distributions such as *normal*. These three extreme value distribution and properties are summarized in Appendix.

In this section, we analyze innovation performance with different extreme value distribution. To see the effect of variance, let $x_1, x_2, ..., x_n$ i.i.d random variables with probability distribution $F(x)$ where $F(x)$ is generalized three extreme value distributions (von Mises (1936)). The cumulative distribution of the maximum outcome of $n$ independent innovation projects is $[F(x)]^n$ . Using tail distribution, expected performance of $n$ project is

$$E[V] = n\int_0^\infty x \cdot [F(x)]^{n-1} f(x)dx = \int_0^\infty (1 - [F(x)]^n)dx$$

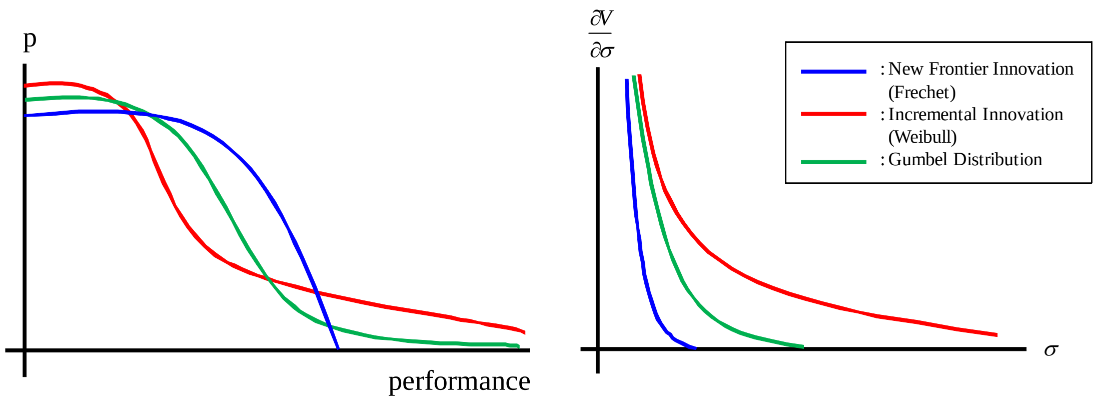
Marginal effect of variance is

$$
\begin{aligned}
\frac{\partial E[V,\sigma]}{\partial \sigma} &= \frac{\partial}{\partial \sigma}\int_0^\infty (1-[F(x)]^n)dx = \int_0^\infty \frac{\partial}{\partial \sigma}(1-[F(x)]^n)dx \\
&= \int_0^\infty \frac{\partial}{\partial \sigma}(1-[e^{-(1+\frac{x}{\alpha\sigma})^{-\alpha}}]^n)dx \\
&= \frac{1}{\sigma^2}\int_0^\infty n\cdot x\cdot[e^{-(1+\frac{x}{\alpha\sigma})^{-\alpha}}]^n\cdot(1+\frac{x}{\alpha\sigma})^{-\alpha-1}dx \\
&= \frac{1}{\sigma}E[V]
\end{aligned}
$$

Let expected performance for new frontier innovation (Frechet) and incremental innovation (Weibull) distribution are $E[V_F]$ and $E[V_W]$, respectively. Since expected performance for new frontier innovation is higher than incremental innovation ($E[V_F] > E[V_W]$) with same $\sigma$, $\frac{\partial E[V_F,\sigma]}{\partial \sigma} > \frac{\partial E[V_W,\sigma]}{\partial \sigma}$. Marginal effect by increasing variance (decreasing process management activity) is, *ceteris paribus*, higher in Frontier innovation than incremental innovation. On the other hand, process management such as six sigma is less adverse effect in incremental innovation. Thus, we come up with following lemma.

**Theorem 14** *By increasing variance ($\sigma$) with reducing process management, its impact for expected performance is higher for frontier innovation than incremental innovation. In other word, process management activities such as six sigma have more adverse effect on performance of frontier innovation than incremental innovation.*

Thus, in the innovation performance side, variation creating activities being widely practiced in the top innovative firms have higher impact to firm's capability to increase innovation performance. On the other hand, if firm's upside potential is limited such




(i) Probability Density Function of Extreme Value Distribution

(ii) Marginal performance of three extreme distributions

Figure 3.2: Characteristics of Extreme Value Distribution

as incremental type of innovation, variance increasing activities have limited potential for firm's performance. Consequently, variance reduction activities such as decreasing creative activities or process management like six sigma has more reverse effect to innovative firms. If a firm's characteristics are a kind of efficiency oriented firm with limited innovation potential, process improvement activities might helpful for firm's overall profitability. In the next section, we proceed firm's process management activities and its relationship with its overall profit.

### 3.3.2 Process management and firm's Profit

Continued from previous section, we analyze process management effect on firm's profit in this section. We add cost term from previous section to estimate firm's profitability (Expected Profit = Expected performance - Total cost in innovation).

As briefly mentioned in the model section, we assume that a firm's cost function is directly related with variance of innovation project ($C(\sigma) = A \cdot \sigma^a + K$). Firms expected profit can be expressed by

$$E[\pi] = \int_0^\infty (1 - [F(x)]^n)dx - n \cdot C(\sigma)$$

Due to mathematical tractability issue, we employ Gumbel distribution in the performance part from now on. Gumbel distribution is widely used to analyze parallel innovation test ($cf.$ Terwiesch and Xu (2009), Kavadias & Sommer (2009), Girotra et al (2010)). Even in Gumbel distribution, we can still distinguish new frontier innovation from incremental innovation in that different values of variable $\sigma$ stand for the type of innovation. In Gumbel distribution, $\sigma$ is the scale of profit uncertainty. Large $\sigma$ indicates frontier type of innovation (blue-sky research) since expected performance with large variance is also higher. Relatively small $\sigma$ is for the incremental innovation.

In Gumbel distribution, expected performance can be remarkably simplified as follows:

$$E[V(n, \sigma)] = \mu + \sigma \log n + \sigma \cdot \gamma$$

where $\gamma$ is Euler's constant ($\backsimeq 0.57722$). Thus, firm's expected profit can be expressed as:

$$\begin{aligned} E[\pi(\sigma, n)] &= E[V] - n \cdot C(\sigma) \\ &= \mu + \sigma \log n + \sigma \cdot \gamma - n \cdot C(\sigma) \end{aligned}$$

At initial stage, a firm decides how many projects will be chosen within the budget. After using first order condition with respect to $n$,

$$n^* = \frac{\sigma}{c(\sigma)}$$

Thus, optimal number of projects is increased when cost of unit project is smaller. By Rewriting profit function with optimal $n^*$, expected profit can be simplified as follows.

$$\begin{aligned} E[\pi(\sigma, n^*)] &= \mu + \sigma \log \frac{\sigma}{c(\sigma)} + \sigma \cdot \gamma - \sigma \\ &= \mu + \sigma \left( \log \frac{\sigma}{c(\sigma)} + \gamma - 1 \right) \end{aligned}$$

To analyze six sigma effect on innovation, we apply generalized cost functions ($C(\sigma) = A \cdot \sigma^a + K$) and measure marginal effect of $\sigma$.

$$\begin{aligned} E\left[\pi(\sigma, n^*)\right] &= \mu + \sigma \left( \log \frac{\sigma}{A \cdot \sigma^a + K} + \gamma - 1 \right) \\ \frac{\partial E\left[\pi(\sigma, n^*)\right]}{\partial \sigma} &= \gamma + a \left( \frac{K}{K + A\sigma^a} - 1 \right) + \log \left( \frac{K}{K + A\sigma^a} \right) \end{aligned}$$

Its marginal effect can be divided by the following three cases.

*Case 1.* Cost saving effect is high by applying process management activities (i.e. $C(\sigma) = A \cdot \sigma^2$). (Recall that $0 < A < 1$, and $\sigma >> A$ : this condition ensure that there are more than 1 optimal project.)

$$\begin{aligned} n^* &= \frac{1}{A \cdot \sigma} \\ \frac{\partial E\left[\pi(\sigma, n^*)\right]}{\partial \sigma} &= \log \frac{1}{A \cdot \sigma} + \gamma - 2 \end{aligned}$$

By analyzing F.O.C of $\pi(\sigma, n^*)$, when $\sigma > \frac{0.236}{A}$, $\frac{\partial E[\pi(\sigma,n^*)]}{\partial\sigma} < 0$. When $\sigma$ decreases, $\pi$ increases. Therefore, six sigma effect is higher than innovation effect when $\sigma$ is greater than above threshold level. However, if $\sigma \leq \frac{0.236}{A}$, $\frac{\partial E[\pi(\sigma,n^*)]}{\partial\sigma} \geq 0$: Process management effect is lower than innovation effect. As a conclusion, process management effect does not have clear cut solution when cost saving effect is high. It depends on the value of variance and coefficient of unit cost.

*Case 2.* Cost saving effect is linear by applying process management activities $(C(\sigma) = A \cdot \sigma)$

$$
\begin{aligned}
n^* &= \frac{1}{A} \\
\frac{\partial E\left[\pi(\sigma, n^*)\right]}{\partial\sigma} &= \log\frac{1}{A} + \gamma - 1
\end{aligned}
$$

First order condition for linear cost function is independent with $\sigma$. So, Process management effect on linear cost function depends on unit cost parameter. If $A \geq 0.642$, $\frac{\partial E[\pi(\sigma,n^*)]}{\partial\sigma} < 0$. Therefore, process management is effective when unit cost is high. On the other hand, process management has adverse effect in innovation.

*Case 3.* Cost saving effect is low by applying process management activities. i.e. $C(\sigma) = A \cdot \sqrt{\sigma}$

$$
\begin{aligned}
n^* &= \frac{\sqrt{\sigma}}{A} \\
\frac{\partial E\left[\pi(\sigma, n^*)\right]}{\partial\sigma} &= \log\frac{\sqrt{\sigma}}{A} + r - \frac{1}{2}
\end{aligned}
$$

$\frac{\partial E[\pi(\sigma,n^*)]}{\partial \sigma}$ is always greater than 0. When $\sigma$ decreases (increases), $\pi$ decreases (increases). So, when cost saving effect is low in process management, six sigma is not appropriate tool to apply. Also, since $\log \frac{\sqrt{\sigma}}{A}$ is increasing function, adverse effect of process management is higher in new frontier innovation. (less harm to the incremental innovation.)

**Proposition 15** *(Process management effect and degree of cost savings)*

$(i)$ *Even if process management effect is high in cost saving, it is not obvious that process management effect such as six sigma is beneficial to new product innovation. It depends on coefficient of cost function and variance of the performance.*

$(ii)$ *When six sigma effect is low in cost saving, it is* not *beneficial to apply process management to increase profit for both incremental and frontier innovation. Their marginal* adverse *effect by applying process management is higher in new frontier innovation than incremental innovation. (process management effect < NPD effect)*

$(iii)$ *When there are large amount of fixed costs in product development, six sigma is less beneficial than small fixed cost.*

Thus, when the variation of innovation potential is quite large, process management effect is less than innovation effect. It implies that new frontier type of innovation (large $\sigma$) has more adverse effect for firm's profit by applying process improvement activities. With all three cases of cost functions, its marginal effect for firm's profit is always negative. However, if cost saving effect is high such as case 1 and there is limited upside potential, process management effect is favorable to firm's profit.

## 3.4 Competition effect in Innovation

In this section, we consider competition effect in innovation. Suppose two firms are competing in a next generation product. In the competition, each firm should consider number of projects, and six sigma effect in their innovation. Thus, each firm decides in investment level of innovation and process management activities. To analyze competition between two firms, we use Bell-Keeney-Little's Multinomial logit (MNL) market share theorem (Bell, Keeney, and Little (1975)). Let innovation performance of firm $i$ be specified as: $V_i = x_0 + \sigma \log n + \sigma \cdot \gamma$ from previous section (using Gumbel distribution). The market share for firm $i$ in a market with a competitor is given by: $M_i = \frac{e^{V_i}}{e^{V_i} + e^{V_{3-i}}}$. Thus, profit function for firm $i$ is

$$E[\pi_i^c] = K \cdot M_i - n_i \cdot c_i(\sigma)$$

where $K$ is parameter of market size.

### 3.4.1 Analysis of Symmetric firms

We start with two symmetric firms' competition. Suppose two firms are identical in innovation potential ($\sigma$), and unit cost of innovation ($c(\sigma)$). Then, Market share of firm 1 is reduced to following form.

$$\begin{aligned}
M_1 &= \frac{e^{V_1}}{e^{V_1}+e^{V_2}} = \frac{e^{\mu+\sigma\log n_1+\sigma\cdot\gamma}}{e^{\mu+\sigma\log n_1+\sigma\cdot\gamma}+e^{\mu+\sigma\log n_2+\sigma\cdot\gamma}} = \frac{n_1^\sigma}{n_1^\sigma+n_2^\sigma} \\
M_2 &= \frac{e^{V_2}}{e^{V_1}+e^{V_2}} = \frac{n_2^\sigma}{n_1^\sigma+n_2^\sigma}
\end{aligned}$$

Profit functions of each firm are reduced to

$$\begin{aligned}
E\left[\pi_1^c\right] &= K\cdot\frac{n_1^\sigma}{n_1^\sigma+n_2^\sigma} - n_1\cdot c(\sigma) \\
E\left[\pi_2^c\right] &= K\cdot\frac{n_2^\sigma}{n_1^\sigma+n_2^\sigma} - n_2\cdot c(\sigma)
\end{aligned}$$

First order conditions for each profit function are

$$\begin{aligned}
\frac{K\cdot\sigma\cdot n_1^\sigma\cdot n_2^\sigma - n_1\cdot c(\sigma)(n_1^\sigma+n_2^\sigma)^2}{n_1(n_1^\sigma+n_2^\sigma)} &= 0 \\
\frac{K\cdot\sigma\cdot n_1^\sigma\cdot n_2^\sigma - n_2\cdot c(\sigma)(n_1^\sigma+n_2^\sigma)^2}{n_2(n_1^\sigma+n_2^\sigma)} &= 0
\end{aligned}$$

Optimal number of projects $n_1^*, n_2^*$ are

$$n_1^* = n_2^* = K\cdot\frac{\sigma}{4c(\sigma)}$$

When $K = 1$, two firms' optimal number of projects is only one–quarter of monopoly firm. With same cost function, the marginal benefit by increasing one project ($\frac{\partial V_M}{\partial n}$) of monopoly firm is $\frac{\sigma}{n}$. It means that firm's performance (revenue) can be increased by $\frac{\sigma}{n}$ as monopoly firm increases one additional innovation project. However, in competition, the marginal benefit by increasing one project ($\frac{\partial V_1}{\partial n}$) of one firm (firm 1) is $\frac{\sigma}{n_1}\cdot\frac{n_1^\sigma n_2^\sigma}{\left(n_1^\sigma+n_2^\sigma\right)^2}$. In the symmetric firms, after applying $n_1 = n_2$, competing firm's marginal benefit $\frac{\partial V_1}{\partial n} = \frac{1}{4}\frac{\sigma}{n_1} = \frac{1}{4}\frac{\partial V_M}{\partial n}$. In addition, when $m$ firms are competing in innovation, optimal number of project for each firm is $\frac{(m-1)\sigma}{m^2 \cdot c(\sigma)}$. For example, in three firms game, $m_i^*$ is $\frac{2\sigma}{9c(\sigma)}$.

**Proposition 16** *Competition in new product innovation causes firms to invest less effort to their R&D investment. When there are m firms competing in a kind of new product, each firm's generalized investment volume in equilibrium is* $\frac{(m-1)\sigma}{m^2 \cdot c(\sigma)}$.

Counter intuitively, competition effect in new product innovation results in firm's underinvestment decision in equilibrium. This result is coincident with prior research in economics ($cf.$ Fullerton & McAfee (1999)): Many solver on an innovation problem will lead lower equilibrium level of investment (effort). Intuition behind this is that negative externality is involved resulting in underinvestment in innovation.

### 3.4.2 Process Management effect in Innovation

Suppose two firms have strategy space $(n_1, \sigma_1)$ and $(n_2, \sigma_2)$. Then, market share of each firm are

$$\begin{aligned} M_1 &= \frac{e^{V_1}}{e^{V_1}+e^{V_2}} = \frac{e^{\mu+\sigma_1 \log n_1+\sigma_1 \cdot \gamma}}{e^{\mu+\sigma_1 \log n_1+\sigma_1 \cdot \gamma}+e^{\mu+\sigma_2 \log n_2+\sigma_2 \cdot \gamma}} = \frac{e^{\sigma_1 \log n_1+\sigma_1 \cdot \gamma}}{e^{\sigma_1 \log n_1+\sigma_1 \cdot \gamma}+e^{\sigma_2 \log n_2+\sigma_2 \cdot \gamma}} \\ M_2 &= \frac{e^{V_2}}{e^{V_1}+e^{V_2}} = \frac{e^{\sigma_2 \log n_2+\sigma_2 \cdot \gamma}}{e^{\sigma_1 \log n_1+\sigma_1 \cdot \gamma}+e^{\sigma_2 \log n_2+\sigma_2 \cdot \gamma}} \end{aligned}$$

$$\begin{aligned} E\left[\pi_1^c\right] &= K \cdot \frac{e^{\sigma_1 \log n_1+\sigma_1 \cdot \gamma}}{e^{\sigma_1 \log n_1+\sigma_1 \cdot \gamma}+e^{\sigma_2 \log n_2+\sigma_2 \cdot \gamma}} - n_1 \cdot C(\sigma_1) \\ E\left[\pi_2^c\right] &= K \cdot \frac{e^{\sigma_2 \log n_2+\sigma_2 \cdot \gamma}}{e^{\sigma_1 \log n_1+\sigma_1 \cdot \gamma}+e^{\sigma_2 \log n_2+\sigma_2 \cdot \gamma}} - n_2 \cdot C(\sigma_2) \end{aligned}$$

The objective function is strictly concave in $n_1$ and $n_2$, and the globally optimal number of projects $n^*$ can be determined by differentiating $E\left[\pi^c\right]$ with respect to $n$. The objective function is also strictly concave in convex cost function ($C(\sigma_1) = A\sigma_1^2$), and optimal $\sigma^*(\frac{\partial^2 \pi_1^c}{\partial \sigma_1^2} < 0)$ can be found. Thus, we apply convex cost function for the analysis. Differentiating $E\left[\pi^c\right]$ with respect to $n$ and $\sigma$ are following forms.

$$
\begin{aligned}
\frac{\partial E\left[\pi_1^c(n_1,\sigma_1)\right]}{\partial n_1} &= K \cdot \left[\frac{e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} \cdot \sigma_1}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right) n_1} - \frac{e^{2(\sigma_1 \log n_1 + \sigma_1 \cdot \gamma)} \cdot \sigma_1}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2 n_1}\right] \\
&\quad - A \cdot \sigma_1^2 \\
\frac{\partial E\left[\pi_2^c(n_2,\sigma_2)\right]}{\partial n_2} &= K \cdot \left[\frac{e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma} \cdot \sigma_2}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right) n_2} - \frac{e^{2(\sigma_2 \log n_2 + \sigma_2 \cdot \gamma)} \cdot \sigma_2}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2 n_2}\right] \\
&\quad - A \cdot \sigma_2^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E\left[\pi_1^c(n_1,\sigma_1)\right]}{\partial \sigma_1} &= K \cdot \left[\frac{e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)} - \frac{e^{2(\sigma_1 \log n_1 + \sigma_1 \cdot \gamma)}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2}\right] \\
&\quad - 2n_1 \cdot A \cdot \sigma_1 \\
\frac{\partial E\left[\pi_2^c(n_2,\sigma_2)\right]}{\partial \sigma_2} &= K \cdot \left[\frac{e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)} - \frac{e^{2(\sigma_2 \log n_2 + \sigma_2 \cdot \gamma)}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2}\right] \\
&\quad - 2n_2 \cdot A \cdot \sigma_2
\end{aligned}
$$

After applying $n_1 = n_2 = n$ and $\sigma_1 = \sigma_2 = \sigma$,

$$
\begin{aligned}
\frac{\partial E\left[\pi_1^c(n_1,\sigma_1)\right]}{\partial n_1} &= \frac{\partial E\left[\pi_2^c(n_2,\sigma_2)\right]}{\partial n_2} = \frac{\sigma}{4}\left(\frac{K}{n} - 4A\sigma\right) \\
\frac{\partial E\left[\pi_1^c(n_1,\sigma_1)\right]}{\partial \sigma_1} &= \frac{\partial E\left[\pi_2^c(n_2,\sigma_2)\right]}{\partial \sigma_2} = \frac{1}{4}\left[K(\log n + \gamma) - 8An\sigma\right]
\end{aligned}
$$

As a result, optimal $n^*$ and $\sigma^*$ are

$$
\begin{aligned}
n_1^* &= n_2^* = \frac{K \cdot \sigma}{4c(\sigma)} = \frac{K}{4A\sigma} = K \cdot e^{2-\gamma} \\
\sigma_1^* &= \sigma_2^* = \frac{Ke^{\gamma-2}}{4A} = \frac{0.059K}{A}
\end{aligned}
$$

**Proposition 17** *Competition effect increases process improvement activities rather than innovation activities. Size of innovation projects and optimal level of variation in competition are decreased. When there are m firms competing in a kind of new product, each firm's generalized $\sigma^*$ in equilibrium is $\frac{(m-1)e^{\gamma-\frac{2}{K}}}{m^2 A}$.*

For a firm has higher variance than $\sigma^*$, process management is beneficial to firm's profit. But, if firms have less variation than optimal $\sigma^*$, process management is adverse effect to the firm. As the number of firms are increased, firms are more likely to encourage process improvement activities rather than increasing innovation activities.

For the reference, marginal effect of process management for other cost functions are as follows.

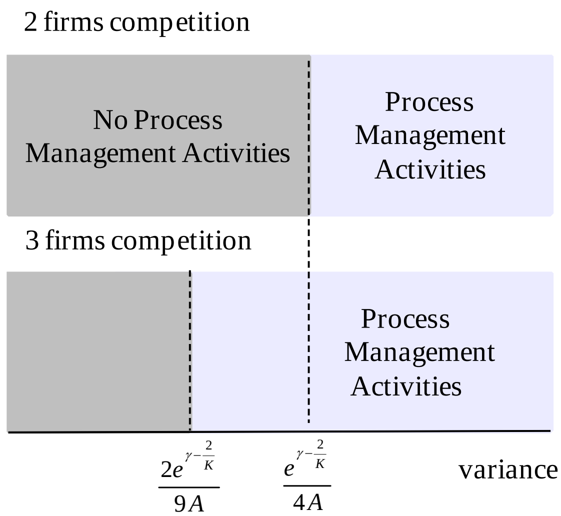



Figure 3.3: Optimal level of process improvement activities in competition model

$$
\begin{aligned}
\frac{\partial E\left[\pi_1^c(n_1,\sigma_1)\right]}{\partial \sigma_1} &= \frac{\partial E\left[\pi_2^c(n_2,\sigma_2)\right]}{\partial \sigma_2} = \frac{1}{4}(\log n + \gamma - 4AnC'(\sigma)) \text{ (generalization)} \\
&= \frac{1}{4}(\log n + \gamma - 8An\sigma) \text{ (Convex cost function)} \\
&= \frac{1}{4}(\log n + \gamma - 4An) \text{ (Linear cost function)} \\
&= \frac{1}{4}(\log n + \gamma - \frac{2An}{\sqrt{\sigma}}) \text{ (Concave cost function)}
\end{aligned}
$$

After applying optimal number of projects $n^* = \frac{\sigma}{4c(\sigma)}$,

$$
\begin{aligned}
\frac{\partial E\left[\pi^c(n^*,\sigma)\right]}{\partial \sigma} &= \frac{1}{4}(\log \frac{1}{4A} + \gamma - 1) \text{ (Linear cost function)} \\
&= \frac{1}{8}(2\log \frac{1}{4A\sigma} + 2\gamma - \frac{1}{\sigma^{\frac{3}{2}}}) \text{ (Concave cost function)}
\end{aligned}
$$

It is impossible to find optimal level of $\sigma$ variable in the linear and concave cost function, but we can see marginal effect by differentiating profit with respect to $\sigma$

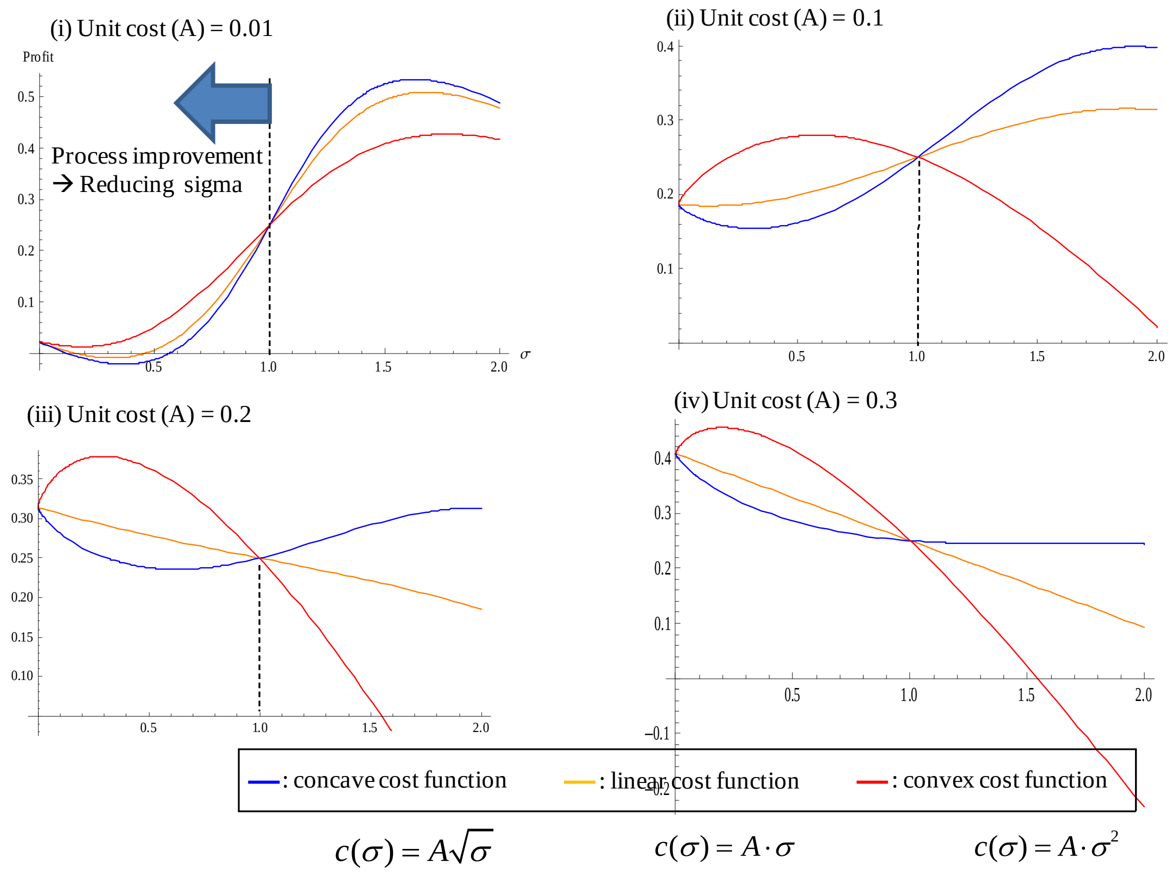



Figure 3.4: Plot of profit functions with different cost coefficients

$(\frac{\partial E[\pi^c(n^*,\sigma)]}{\partial \sigma})$. Marginal effect of process management is independent of variance in linear cost function, and it is increasing function in concave cost function from above. Also, for the illustration purpose, we provide each cost functions profit level with different cost coefficients from unit sigma level ($\sigma = 1$)[7] in figure 5.

When unit cost is small ($A = 0.01$), every function's profit is decreased with

[7]Different initial sigma values besides $\sigma = 1$ have same pattern with figure 5.

process improvement activities. It means that, in the relatively small cost of R&D, process management activities are not beneficial for firm's profitability. When unit cost is increased, for example, $A = 0.1$, convex cost function's profit is switched to increasing function, and certain level of process management activities are maximizing firm's profit. But, for other functions, profit is increasing function in this range. When unit scale parameter become larger, certain level of process improvement for all cost fnctions, profit is increased.

## 3.5 Asymmetric Firms Competition

### 3.5.1 Different cost parameter

Among various asymmetric environments, we mainly analyze different cost coefficients $A_1$ and $A_2$ since cost coefficient is important role in investigating optimal size of innovation projects and process improvement level. $A_1$ and $A_2$ indicate scale parameter of unit innovation cost. Different values in scale parameter imply two firm's initial unit R&D costs are different from each other. We further assume that two firms cost parameters are observable to each other. Two firms have strategy space $(n_1, \sigma_1)$ and $(n_2, \sigma_2)$, and each firm's profit functions are as follows (From now on, we applied convex cost function ($A\sigma^2$) to find out optimal level of $\sigma$).

$$E[\pi_1^c] = K \cdot \frac{e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma}}{e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}} - n_1 \cdot A_1 \sigma_1^2$$

$$E[\pi_2^c] = K \cdot \frac{e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}}{e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}} - n_2 \cdot A_2 \sigma_2^2$$

Its first order condition with respect to $n_1$ and $n_2$ are

$$\frac{\partial E[\pi_1^c(n_1, \sigma_1)]}{\partial n_1} = \frac{e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} \cdot \sigma_1}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right) n_1} - \frac{e^{2(\sigma_1 \log n_1 + \sigma_1 \cdot \gamma)} \cdot \sigma_1}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2 n_1} - A_1 \cdot \sigma_1^2$$

$$\frac{\partial E[\pi_2^c(n_2, \sigma_2)]}{\partial n_2} = \frac{e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma} \cdot \sigma_2}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right) n_2} - \frac{e^{2(\sigma_2 \log n_2 + \sigma_2 \cdot \gamma)} \cdot \sigma_2}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2 n_2} - A_2 \cdot \sigma_2^2$$

First order condition with respect to $\sigma_1$ and $\sigma_2$ are

$$\frac{\partial E[\pi_1^c(n_1, \sigma_1)]}{\partial \sigma_1} = \frac{e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)} - \frac{e^{2(\sigma_1 \log n_1 + \sigma_1 \cdot \gamma)}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2} - 2n_1 \cdot A_1 \cdot \sigma_1$$

$$\frac{\partial E[\pi_2^c(n_2, \sigma_2)]}{\partial \sigma_2} = \frac{e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)} - \frac{e^{2(\sigma_2 \log n_2 + \sigma_2 \cdot \gamma)}(\gamma + \log[n_1])}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2} - 2n_2 \cdot A_2 \cdot \sigma_2$$

Above forms are further reduced as following.

$$\frac{\partial E[\pi_1^c(n_1, \sigma_1)]}{\partial n_1} = \frac{\sigma_1(A_1 e^{2\sigma_1 \gamma} n_1^{2\sigma_1+1} \sigma_1 + A_1 e^{2\sigma_2 \gamma} n_1 n_2^{2\sigma_2} \sigma_1 + e^{(\sigma_1+\sigma_2)\gamma} n_1^{\sigma_1} n_2^{\sigma_2}(2A_1 n_1 \sigma_1 - 1))}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2 n_1}$$

$$- - - - - - - - - - - - - - - - - - - - - - - - - - - - \quad (1)$$

$$\frac{\partial E[\pi_2^c(n_2, \sigma_2)]}{\partial n_2} = \frac{\sigma_2(A_2 e^{2\sigma_1 \gamma} n_1^{2\sigma_1} n_2 \sigma_2 + A_2 e^{2\sigma_2 \gamma} n_2^{2\sigma_2+1} \sigma_2 + e^{(\sigma_1+\sigma_2)\gamma} n_1^{\sigma_1} n_2^{\sigma_2}(2A_2 n_2 \sigma_2 - 1))}{\left(e^{\sigma_1 \log n_1 + \sigma_1 \cdot \gamma} + e^{\sigma_2 \log n_2 + \sigma_2 \cdot \gamma}\right)^2 n_2}$$

$$- - - - - - - - - - - - - - - - - - - - - - - - - - - - \quad (2)$$

$\sigma_1, \sigma_2, n_1, n_2 \neq 0$, and numerator should be zero in equilibrium. Thus, equation (1) and (2) can be further simplified in optimality condition.

$$A_1 e^{2\sigma_1\gamma} n_1^{2\sigma_1+1}\sigma_1 + A_1 e^{2\sigma_2\gamma} n_1 n_2^{2\sigma_2}\sigma_1 + e^{(\sigma_1+\sigma_2)\gamma} n_1^{\sigma_1} n_2^{\sigma_2}(2A_1 n_1\sigma_1 - 1) = 0 \quad (3)$$

$$A_2 e^{2\sigma_1\gamma} n_1^{2\sigma_1} n_2\sigma_2 + A_2 e^{2\sigma_2\gamma} n_2^{2\sigma_2+1}\sigma_2 + e^{(\sigma_1+\sigma_2)\gamma} n_1^{\sigma_1} n_2^{\sigma_2}(2A_2 n_2\sigma_2 - 1) = 0 \quad (4)$$

In equation (3), and (4) we find important relationship between number of project and variance and unit cost parameter. (3)-(4) can be reduced to

$$(A_1 n_1\sigma_1 - A_2 n_2\sigma_2)(e^{\sigma_1\gamma} n_1^{\sigma 1} + e^{\sigma_2\gamma} n_2^{\sigma 2})^2 = 0 \quad (5)$$

Therefore, in equilibrium, $A_1 n_1\sigma_1 = A_2 n_2\sigma_2$ condition should be satisfied between two firms. After simplification, optimal number of projects for each firm can be explained as following forms[8].

$$n_1^* = \frac{A_1^{\sigma_1} A_2^{\sigma_2}}{A_1\sigma_1(A_1^{\sigma_1} + A_2^{\sigma_2})^2}$$

$$n_2^* = \frac{A_1^{\sigma_1} A_2^{\sigma_2}}{A_2\sigma_2(A_1^{\sigma_1} + A_2^{\sigma_2})^2}$$

In equation (5), we find important relationship among number of projects, degree of process management, and cost coefficient.

**Proposition 18** *(The relationship between three variables)*

$(i)$ *Smaller cost firm in innovation has larger number of projects than higher cost firm in equilibrium, while higher cost firm has less number of projects. Also, higher cost*

[8] It is not tractible to find optimal $\sigma_1^*$ and $\sigma_2^*$ in asymmetric case because variable $\sigma_1$ and $\sigma_2$ are also located in power side.

*firm is more likely to reduce variance ($\sigma$) using process management activities.*

*(ii) With same cost coefficient, balanced number of project and process management level for two competing firms in equilibrium is $n_1\sigma_1 = n_2\sigma_2$.*

The most important insight in (5) is that in equilibrium, two firms are balanced with cost coefficient, number of projects, and degree of process management. For example, in the relatively cheaper unit cost of innovation, firms are more likely to increase innovation activities, whereas expensive cost firm is more likely to spur in their process improvement activities. Furthermore, when both firm has same $\sigma = \sigma_1 = \sigma_2$ (such as same industry in innovation environment), optimal number of projects for each firm are

$$\begin{aligned} n_1^* &= \frac{(A_1A_2)^\sigma}{A_1\sigma(A_1^\sigma + A_2^\sigma)^2} \\ n_2^* &= \frac{(A_1A_2)^\sigma}{A_2\sigma(A_1^\sigma + A_2^\sigma)^2} \end{aligned}$$

If $A_1 = A_2 = A$ (in symmetric case), $n_1^* = n_2^* = \frac{1}{4A\sigma}$, which is same result in symmetric firms' analysis.

In addition, we also analyzed different average in innovation performance ($\mu_1 \neq \mu_2$) between two firms. Interestingly, it does not affect optimal number of project and variance level (independent with average).

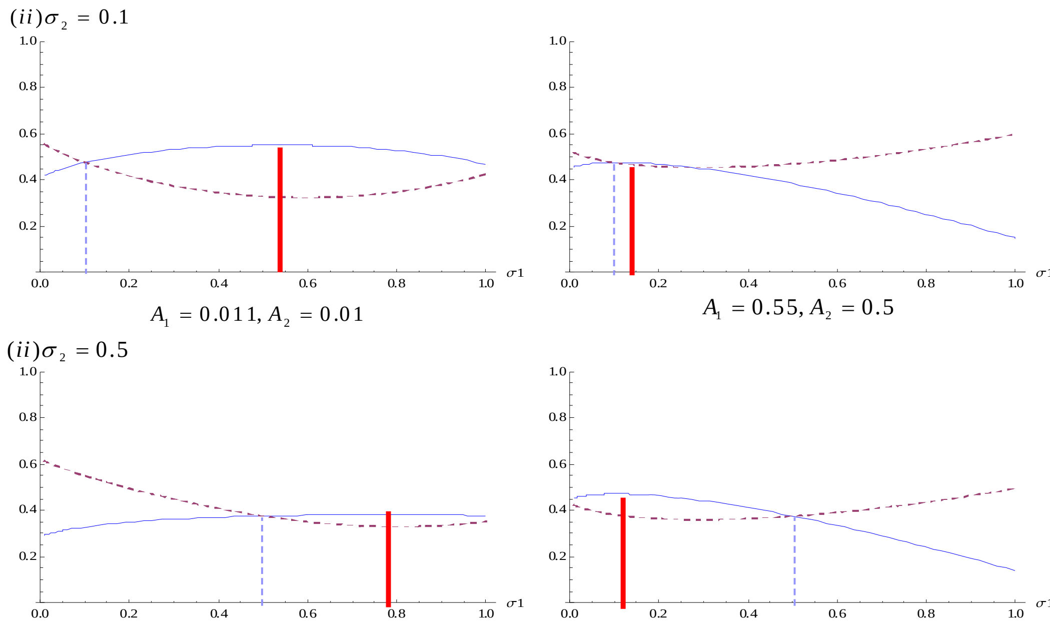



Figure 3.5: Cavature of profit function with different $\sigma$ value

### 3.5.2 Comparative Statics

Firms can make use of different scale parameters in cost functions by increasing(decreasing) the number of projects or by reducing variance ($\sigma$) from process improvement activities. As mentioned before, it is impossible to find optimal $\sigma$ level with closed form solution. Thus, in this section, we illustrate process management effect by varying $\sigma$ for asymmetric firms.

Figure 6 depicts firm's behavior with different scale parameters in cost functions. When scale parameters $(A_1, A_2)$ are small, it is slightly increasing function until a certain value by increasing $\sigma$. For the relatively small cost of R&D, process management activities are not beneficial for firm's profitability. When unit scale parameter become larger ($A_1 = 0.55, A_2 = 0.5$), curve is switched to decreasing function when scale parameter is larger. Certain level of process improvement increases firm's profit. It has similar results with symmetric firm's analysis. In addition, this tendency is still hold with different second firm's $\sigma_2$ value.

## 3.6 Conclusion

In this paper, we develop analytical models to explain productivity dilemma issue. We mainly analyze the relationship between process management and new product innovation performance using extreme value theory with generalized cost function in innovation, focusing on how innovation performance and process management affect firm's optimal investment decision and the market equilibrium outcome. As seen in 3M case, there have been numerous debates about effectiveness of process management activities. Although there are some other qualitative issues, we build a stylized modeling approach considering some of important factors such as innovation characteristics of product and competition.

The most important finding in this paper is that effectiveness of process improvement depends the type of innovation. By increasing variance ($\sigma$) with reducing process management, its impact for expected performance is higher for frontier innovation than incremental innovation. In other word, process management activities such as six sigma have more adverse effect on performance of frontier innovation than incremental innovation. We also find that competition in new product innovation causes firms to invest less effort to their R&D investment. Competition effect increases process improvement activities rather than innovation activities. Size of innovation projects and optimal level of variation ($\sigma$) in competition are sharply decreased.

# Appendix A

# Technical Appendix

## A.1 Proofs of Chapter 1

### A.1.1 Preliminary

**Lemma 19** *(i) In a sequential move (Stackelberg) game with substitute parameter $\beta$, the subgame-perfect Nash equilibrium outcome in terms of the order quantity (q) and the profits ($\pi$) of the leader (informed firm) and the follower (uninformed firm) are:*

$$\begin{aligned} q_{iH} &= \frac{(2-\beta)A_H}{2(2-\beta^2)}, q_{iL} = \frac{(2-\beta)A_L}{2(2-\beta^2)} \\ q_{eH} &= \frac{(4-2\beta-\beta^2)A_H}{4(2-\beta^2)}, q_{eL} = \frac{(4-2\beta-\beta^2)A_L}{4(2-\beta^2)} \end{aligned}$$

$$\begin{aligned} \pi^L_{iH} &= \frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)}, \pi^L_{iL} = \frac{(2-\beta)^2 A_L^2}{8(2-\beta^2)} \\ \pi^L_{eH} &= \frac{(4-2\beta-\beta^2)^2 A_H^2}{16(2-\beta^2)^2}, \pi^L_{eL} = \frac{(4-2\beta-\beta^2)^2 A_L^2}{16(2-\beta^2)^2} \end{aligned}$$

*(ii) In a simultaneous move (Cournot) game with incomplete information with substitute parameter $\beta$, the subgame-perfect Nash equilibrium outcome in terms of the order quantity ($q$) and the profits ($\pi$) of the leader (informed firm) and the follower (uninformed firm) are:*

$$\begin{aligned} q_{iH} &= \frac{A_H}{2} - \frac{\beta\mu}{2(2+\beta)}, q_{iL} = \frac{A_L}{2} - \frac{\beta\mu}{2(2+\beta)} \\ q_e &= \frac{\mu}{2+\beta} \end{aligned}$$

$$\begin{aligned} \pi_{iH} &= \left(\frac{A_H}{2} - \frac{\beta\mu}{2(2+\beta)}\right)^2, \pi_{iL} = \left(\frac{A_L}{2} - \frac{\beta\mu}{2(2+\beta)}\right)^2 \\ \pi_e &= \left(\frac{\mu}{2+\beta}\right)^2 \end{aligned}$$

**Proof.** (i) If the informed firm puts in $q_{iH}$ and $q_{iL}$, the uninformed firm solves

$$\begin{aligned} q^*_{eH} &= \arg\max_{q_{iH}}(A_H - \beta q_{iH} - q_{eH})q_{eH} = \frac{A_H - \beta q_{iH}}{2} \\ q^*_{eL} &= \arg\max_{q_{iL}}(A_L - \beta q_{iL} - q_{eL})q_{eL} = \frac{A_L - \beta q_{iL}}{2} \end{aligned}$$

informed firm in turn solves for

$$\begin{aligned} \pi_{iH} &= \max_{q_{iH}}(A_H - q_{iH} - \beta q^*_{eH}(q_{iH}))q_{iH} \\ \pi_{iL} &= \max_{q_{iL}}(A_L - q_{iL} - \beta q^*_{eL}(q_{iL}))q_{iL} \end{aligned}$$

(ii) The informed firm and the uninformed firm correspondingly solve the following simultaneous move game of incomplete information.

$$\begin{aligned}
\pi_{iH} &= \max_{q_{iH}} (A_H - q_{iH} - \beta q_e) q_{iH} \\
\pi_{iL} &= \max_{q_{iL}} (A_L - q_{iL} - \beta q_e) q_{iL} \\
\pi_e &= \max_{q_e} p\{(A_H - \beta q_{iH} - q_e) q_e\} + (1-p)\{(A_L - \beta q_{iL} - q_e) q_e\}
\end{aligned}$$

The solution to the first order condition gives the order quantities and profits as noted. ■

**Proof of normal separating equilibrium**

For existence of separating equilibrium, $q_{iL}^*$ and $q_{iH}^*$ should satisfy the following optimization problem.

$$\begin{aligned}
\pi_{iL} &= \max_{qiL} (A_L - q_{iL} - \beta q_{eL}^*(q_{iL})) q_{iL} \\
\pi_{iH} &= \max_{qiH} (A_H - q_{iH} - \beta q_{eH}^*(q_{iH})) q_{iH}
\end{aligned}$$

subject to

$$(A_L - q_{iH} - \beta q_{eH}^*(q_{iH})) q_{iH} \leq (A_L - q_{iL}^* - \beta q_{eL}^*(q_{iL})) q_{iL}^* \tag{1}$$

$$(A_H - q_{iL} - \beta q_{eL}^*(q_{iL})) q_{iL} \leq (A_H - q_{iH}^* - \beta q_{eH}^*(q_{iH})) q_{iH}^* \tag{2}$$

where

$$
\begin{aligned}
q_{eL}^* &= \arg\max_{q_{eL}}(A_L - \beta q_{iL} - q_{eL})q_{eL} = \frac{A_L - \beta q_{iL}}{2} \\
q_{eH}^* &= \arg\max_{q_{eH}}(A_H - \beta q_{iH} - q_{eH})q_{eH} = \frac{A_H - \beta q_{iH}}{2}
\end{aligned}
$$

Constraint (1) and (2) are incentive compatibility constraint. There is no incentive to mimic the other. Also, since only constraint (2) is binding constraint, above problem can be simplified as follows.

$$
\pi_{iL} = \max_{qiL}(A_L - q_{iL} - \beta q_{eL}^*(q_{iL}))q_{iL} = \max_{qiL}\left(\frac{(2-\beta)}{2}A_L - \frac{(2-\beta^2)}{2}q_{iL}\right)q_{iL}
$$

$$
\text{s.t.} \quad \left(A_H - \frac{\beta}{2}A_L - \frac{(2-\beta^2)}{2}q_{iL}\right)q_{iL} \leq \frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)}
$$

Lagrangian of above formula is

$$
\begin{aligned}
L(q_{iL}, \lambda) &= \max_{qiL}\left(\frac{(2-\beta)}{2}A_L - \frac{(2-\beta^2)}{2}q_{iL}\right)q_{iL} \\
&\quad -\lambda\left(\frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)} - \left(A_H - \frac{\beta}{2}A_L - \frac{(2-\beta^2)}{2}q_{iL}\right)q_{iL}\right)
\end{aligned}
$$

First order condition is

$$
\begin{aligned}
\frac{\partial L(q_{iL}, \lambda)}{\partial q_{iL}} &\leq 0 \Rightarrow \frac{(2-\beta)}{2}A_L - (2-\beta^2)q_{iL} + \lambda\left(A_H - \frac{\beta}{2}A_L - (2-\beta^2)q_{iL}\right) + v_1 = 0 \\
\frac{\partial L(q_{iL}, \lambda)}{\partial \lambda} &\geq 0 \Rightarrow \frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)} = \left(A_H - \frac{\beta}{2}A_L - \frac{(2-\beta^2)}{2}q_{iL}\right)q_{iL} + v_2 = 0
\end{aligned}
$$

Solving above equations,

$$
\frac{(A_H - A_L)(2-\beta)(A_H(\beta-2) + A_L(\beta+2))}{8(\beta^2-2)} \geq 0
$$

After manipulating above formula, condition for separating equilibrium is

$$\theta = \frac{A_H}{A_L} \geq \frac{2+\beta}{2-\beta}$$

**Analysis of Costly separation for substitute case**

When $\theta < \frac{2+\beta}{2-\beta}$, sequential move's order quantity and profit is as follows.

(i) informed firm orders:

$$\begin{aligned} q_{iH}^* &= \frac{(2-\beta)A_H}{2(2-\beta^2)} \\ q_{iL}^* &= \frac{(2-\beta^2)(2A_H-\beta A_L)-\sqrt{\beta(\beta^2-2)^2(A_H-A_L)((4-\beta)A_H-\beta A_L)}}{2(\beta^2-2)^2} \end{aligned}$$

(ii) uninformed firm orders:

$$\begin{aligned} q_{eH}^* &= \frac{(4-2\beta-\beta^2)A_H}{4(2-\beta^2)} \\ q_{eL}^* &= \frac{8A_L+\beta\left(2(\beta^2-2)A_H+\beta(\beta^2-6)A_L+\sqrt{\beta(\beta^2-2)^2(A_H-A_L)((4-\beta)A_H-\beta A_L)}\right)}{4(\beta^2-2)^2} \end{aligned}$$

(iii) The profits for each party are:

$$\begin{aligned}
\pi_{iH} &= \frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)} \\
\pi_{iL} &= \frac{(4+(4-\beta)\beta(\beta^2-2)A_H^2}{8(\beta^2-2)^2} \\
&+ \frac{4A_L\left(\beta(\beta^2-2)A_L - \sqrt{\beta(\beta^2-2)^2(A_H-A_L)((4-\beta)A_H-\beta A_L)}\right)}{8(\beta^2-2)^2} \\
&+ \frac{4A_H\left((2+\beta)(2-\beta^2)A_L + \sqrt{\beta(\beta^2-2)^2(A_H-A_L)((4-\beta)A_H-\beta A_L)}\right)}{8(\beta^2-2)^2} \\
\pi_{eH} &= \frac{(4-2\beta-\beta^2)^2 A_H^2}{16(2-\beta^2)^2} \\
\pi_{eL} &= (\frac{8A_L}{4(\beta^2-2)^2} \\
&+ \frac{\beta\left(2(\beta^2-2)A_H + \beta(\beta^2-6)A_L + \sqrt{\beta(\beta^2-2)^2(A_H-A_L)((4-\beta)A_H-\beta A_L)}\right)}{4(\beta^2-2)^2})^2
\end{aligned}$$

**Proof of pooling equilibrium**

Let $q_{ip}^H$ and $q_{ip}^L$ the optimal pooling quantities for the informed firm. Suppose $\exists q_{ip}$ such that on observing this quantity, the uninformed firm cannot tease the true demand state and sticks to his priors. Thus, the uninformed firm orders $q_{ep}$ where

$$q_{ep}^*(q_{ip}) = \arg\max_{q_{ep}} p\{(A_H - \beta q_{iH} - q_e)q_e\} + (1-p)\{(A_L - \beta q_{iL} - q_e)q_e\} = \frac{\mu - \beta q_i}{2}$$

The high type informed firm's objective function is

$$\pi_{iH}^p = \max_{qip}(A_H - q_{ip} - \beta q_{ep}^*(q_{ip}))q_{ip}$$

and the low type informed firm maximizes

$$\pi_{iL}^{p} = \max_{qip}(A_L - q_{ip} - \beta q_{ep}^{*}(q_{ip}))q_{ip}$$

The obtained optimal order quantities for the high and low type are as follows:

$$\begin{aligned} q_{ip}^{H*} &= \frac{2A_H - \beta\mu}{2(2-\beta^2)} \\ q_{ip}^{L*} &= \left(\frac{2A_L - \beta\mu}{2(2-\beta^2)}\right)^{+} \end{aligned}$$

Pooling equilibrium must satisfy $q_{ip} \le \min(q_{ip}^{H*}, q_{ip}^{L*}) = q_{ip}^{L*} = \left(\frac{2A_L-\beta\mu}{2(2-\beta^2)}\right)^{+} = q_{ip}^{*}$. The low type informed firm do not agree to pool at a order quantity $q_{ip}^{*}$ for any reasonable belief structure.

Next, the lowest order quantity for both high and low type informed firm can be analyzed as follows. First, the high type informed firm prefers to pool as long as profit of pooling dominate that of separating.

$$(A_H - q_{ip} - \beta q_{ep}^{*}(q_{ip}))q_{ip} \ge \max_{qiH>qip}(A_H - q_{iH} - \beta q_{eH}^{*}(q_{iH}))q_{iH}$$

where

$$q_{eH}^{*} = \arg\max_{q_{eH}}(A_H - \beta q_{iH} - q_{eH})q_{eH} = \frac{A_H - \beta q_{iH}}{2}$$

It is simplified to

$$(A_H - q_{ip} - \beta\frac{\mu - \beta q_{ip}}{2})q_{ip} \ge \frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)}$$

We get $q_{ip} \in \left[\frac{A_H}{2-\beta^2} - \frac{\beta\mu}{2\left(2-\beta^2\right)} - \frac{\sqrt{\beta(A_H-\mu)((4-\beta)A_H-\beta\mu)}}{2\left(2-\beta^2\right)}, \frac{A_H}{2-\beta^2} - \frac{\beta\mu}{2\left(2-\beta^2\right)} + \frac{\sqrt{\beta(A_H-\mu)((4-\beta)A_H-\beta\mu)}}{2\left(2-\beta^2\right)}\right]$.

Thus, the lowest pooling quantity for the high type is $\frac{A_H}{2-\beta^2} - \frac{\beta\mu}{2\left(2-\beta^2\right)} - \frac{\sqrt{\beta(A_H-\mu)((4-\beta)A_H-\beta\mu)}}{2\left(2-\beta^2\right)}$.

For the low type,

$$(A_L - q_{ip} - \beta q^*_{ep}(q_{ip}))q_{ip} \geq \max_{qiL>qip}(A_L - q_{iL} - \beta q^*_{eL}(q_{iL}))q_{iL}$$

where

$$q^*_{eL} = \arg\max_{q_{eL}}(A_L - \beta q_{iL} - q_{eL})q_{eL} = \frac{A_L - \beta q_{iL}}{2}$$

After simplification, We get $q_{ip} \in [\frac{A_L}{2-\beta^2} - \frac{\beta\mu}{2(2-\beta^2)} - \frac{\sqrt{\beta(A_H-\mu)((5-\beta)A_L-A_H-\beta\mu)}}{2(2-\beta^2)}$, $\frac{A_L}{2-\beta^2} - \frac{\beta\mu}{2(2-\beta^2)} + \frac{\sqrt{\beta(A_H-\mu)((5-\beta)A_L-A_H-\beta\mu)}}{2(2-\beta^2)}]$. Hence, the lowest pooling quantity for the low type is $\frac{A_L}{2-\beta^2} - \frac{\beta\mu}{2(2-\beta^2)} - \frac{\sqrt{\beta(A_H-\mu)((5-\beta)A_L-A_H-\beta\mu)}}{2(2-\beta^2)}$

Finally, we find the lowest demand interval $\theta(= A_H/A_L)$ for pooling equilibrium. The pooling equilibrium exists if

$$\frac{A_H}{2-\beta^2} - \frac{\beta\mu}{2\left(2-\beta^2\right)} - \frac{\sqrt{\beta(A_H-\mu)((4-\beta)A_H-\beta\mu)}}{2\left(2-\beta^2\right)} < \left(\frac{2A_L-\beta\mu}{2(2-\beta^2)}\right)^+$$

(the minimum quantity of pooling is less than separating). It is simplified to

$$\beta(\theta-(p\theta+(1-p)))((4-\beta)\theta-\beta(p\theta+(1-p))-4(\theta-1)^2 > 0$$

The above inequality is satisfied for $\theta \in (1, \frac{4-\beta^2+2\beta^2\cdot p-\beta^2p^2}{(2-\beta)^2+4\beta\cdot p-\beta^2p^2})$.Therefore, the pooling equilibrium exists for $\theta \leq \frac{4-\beta^2+2\beta^2\cdot p-\beta^2p^2}{(2-\beta)^2+4\beta\cdot p-\beta^2p^2}$.

### A.1.2 Each play's preference and the supplier's leakage decision

#### The informed firm's trade-offs

First, *in the high demand state*, there is trade-off between demand uncertainty($p$) and substitute coefficient($\beta$). When $\beta$ is decreased from 1, threshold level of uncertainty parameter $p$ for the preference of simultaneous move is increased. Consequently, preference of the sequential moving area is decreased and the informed firm is more likely to prefers to simultaneous move without information leakage. In certain level (for example, $\beta \simeq 0.5$), the informed firm try not to leak their order information (wait until time T) regardless of $p$. Second, *in the low demand state*, When $\beta = 0$, both sequential and simultaneous move has same profit caused by monopoly situation. However, when $\beta$ is increased, the preference of sequential move area is increased. When $\beta$ and $p$ converges to 1, there is some region that simultaneous move is preferred. To check existence of probability of simultaneous move, first we should check whether the informed firm orders positive quantity. To have positive $q_{iL}^*$:

$$A_L + \beta \cdot p \cdot A_L - \beta \cdot p \cdot A_H > 0 \Rightarrow p < \frac{2}{\beta(\theta - 1)}$$

$\frac{2}{\beta(\theta-1)}$ is always less than the probability of intersect of two profit function $\left(\frac{2}{\beta(\theta-1)} + \frac{(4-\beta)}{\sqrt{2(2-\beta)}\beta(\theta-1)}\right)$. Thus, *the low type informed firm always prefers sequential move.* Also, informed firm can take advantage of monopoly position with low $\beta$.

**The uninformed firm's trade-offs**

By comparing profit of simultaneous move$\left(\left(\frac{\mu}{2+\beta}\right)^2\right)$ and expected value of sequential move

$\left(p\left(\frac{(4-2\beta-\beta^2)^2A_H^2}{16(2-\beta^2)^2}\right)+(1-p)\left(\frac{(4-2\beta-\beta^2)^2A_L^2}{16(2-\beta^2)^2}\right)\right)$, there are two intersection in terms of $p$.

$$p_1^{'} = \frac{A_H(\beta^2(4+\beta)-8)^2+A_L(-64+\beta^2(64+\beta(-16+\beta(-16+\beta(8+\beta)))))}{32(A_H-A_L)(\beta^2-2)^2}$$
$$\frac{-(\beta^2(4+\beta)-8)\sqrt{\begin{array}{c}A_H(\beta^2(4+\beta)-8)^2+A_L(\beta^2(4+\beta)-8)^2\\+2A_HA_L(-64+\beta^2(64+\beta(-16+\beta(-16+\beta(8+\beta)))))\end{array}}}{32(A_H-A_L)(\beta^2-2)^2}$$

$$p_2^{'} = \frac{A_H(\beta^2(4+\beta)-8)^2+A_L(-64+\beta^2(64+\beta(-16+\beta(-16+\beta(8+\beta)))))}{32(A_H-A_L)(\beta^2-2)^2}$$
$$\frac{+(\beta^2(4+\beta)-8)\sqrt{\begin{array}{c}A_H(\beta^2(4+\beta)-8)^2+A_L(\beta^2(4+\beta)-8)^2\\+2A_HA_L(-64+\beta^2(64+\beta(-16+\beta(-16+\beta(8+\beta)))))\end{array}}}{32(A_H-A_L)(\beta^2-2)^2}$$

Condition for these two profit function have intersection is

$$\theta^{'} \geq \frac{(\beta^2(4+\beta)-8)^2}{64-\beta^2(64+\beta(\beta(\beta(\beta+8)-16)-16))-8\sqrt{\beta^3(\beta^2-2)^2(16-\beta^2(8+\beta))}} \tag{A.1}$$

When $\beta = 1$, threshold $\theta$ is 4.9733 (same with perfect substitute case), and when $\beta$ goes to 0, $\theta$ is also converges to 0.

An observation is as follows: When $\beta$ is decreased from 1, preference of the sequential moving area is increased as quadratic fashion. It means that Stackelberg follower's profit is increased faster than Cournot model's profit when $\beta$ is decreased. As a result, when $\beta$ is decreased from 1, the uninformed firm's strategy is strictly preferred by sequential move.

**The supplier's decision**

When demand is high state, the supplier always leak order information (same with perfect substitute case). When Low demand state, in perfect substitute case, if demand uncertainty is high, the supplier does not leak order information. If demand uncertainty is low, supplier leak the order information. When $\beta$ is decreased from 1, the supplier's preference of leakage area is decreased. The supplier can take advantage of two monopolists - one is perfectly informed, and the other is uninformed - with low $\beta$.

**Proof of Supplier's leakage.** Suppose informed firm's order quantity is $\widetilde{q}_{iH}$ and $\widetilde{q}_{iL}$. If supplier leaks, uninformed firm orders $\frac{A_H - \beta q_{iH}}{2}$ when demand is high and $\frac{A_L - \beta q_{iL}}{2}$ when demand is low. For supplier not to leak, the following inequalities should hold,

$$
\begin{aligned}
w \times \left( \widetilde{q}_{iH} + \frac{(A_H - w) - \beta \widetilde{q}_{iH}}{2} \right) &< w \times \left( \frac{A_H - w}{2} - \frac{\beta(\mu - w)}{2(2+\beta)} + \frac{\mu - w}{2+\beta} \right) \\
w \times \left( \widetilde{q}_{iL} + \frac{(A_L - w) - \beta \widetilde{q}_{iL}}{2} \right) &< w \times \left( \frac{A_L - w}{2} - \frac{\beta(\mu - w)}{2(2+\beta)} + \frac{\mu - w}{2+\beta} \right)
\end{aligned}
$$

This simplifies to

$$\begin{aligned}\widetilde{q}_{iH} &< \frac{\mu - w}{2+\beta} \\ \widetilde{q}_{iL} &< \frac{\mu - w}{2+\beta}\end{aligned}$$

To prevent leakage, the informed firm should order less than $\frac{\mu-w}{2+\beta}$ for both high/low demand state. But, the informed firm's order at leakage case $\frac{(2-\beta)(A_H-w)}{2(2-\beta^2)}$ has always more profitable than ordering $\frac{\mu-w}{2+\beta}$ for preventing leakage. Therefore, orders per profit maximizing quantity $\frac{(2-\beta)(A_H-w)}{2(2-\beta^2)}$, supplier always leaks in equilibrium. ∎

### A.1.3 Equilibrium outcome of possible scenarios

**The informed firm's order and the uninformed firm's order = (E, E)**

Suppose the proposed equilibrium is Simultaneous move (Cournot model) with incomplete information. Order quantities and profits are

$$\begin{aligned}q_{iH} &= \frac{A_H}{2} - \frac{\beta\mu}{2(2+\beta)}, q_{iL} = \frac{A_L}{2} - \frac{\beta\mu}{2(2+\beta)} \\ q_e &= \frac{\mu}{2+\beta}\end{aligned}$$

$$\begin{aligned}\pi_{iH} &= \left(\frac{A_H}{2} - \frac{\beta\mu}{2(2+\beta)}\right)^2, \pi_{iL} = \left(\frac{A_L}{2} - \frac{\beta\mu}{2(2+\beta)}\right)^2 \\ \pi_e &= \left(\frac{\mu}{2+\beta}\right)^2\end{aligned}$$

First, when the informed firm deviate from early to late, its profits are

$$\pi_{iH} = \left(\frac{A_H}{2} - \frac{\beta\mu}{2(2+\beta)}\right)^2, \pi_{iL} = \left(\frac{A_L}{2} - \frac{\beta\mu}{2(2+\beta)}\right)^2$$

It is the same with ordering early. Therefore, it is not a profitable deviation. Second, when the uninformed firm deviate from early to late, its optimal order quantity and profits are

$$\begin{aligned} q_{eH} &= \frac{(4-\beta^2)A_H + \beta^2\mu}{4\,(2+\beta)}, q_{eL} = \frac{(4-\beta^2)A_L + \beta^2\mu}{4\,(2+\beta)} \\ \pi_{eH} &= \left(\frac{(4-\beta^2)A_H + \beta^2\mu}{4\,(2+\beta)}\right)^2, \pi_{eL} = \left(\frac{(4-\beta^2)A_L + \beta^2\mu}{4\,(2+\beta)}\right)^2 \end{aligned}$$

Expected Profit is

$$\pi_{eH} = p\left(\frac{(4-\beta^2)A_H + \beta^2\mu}{4\,(2+\beta)}\right)^2 + (1-p)\left(\frac{(4-\beta^2)A_L + \beta^2\mu}{4\,(2+\beta)}\right)^2$$

It is always greater than original profit($\left(\frac{\mu}{2+\beta}\right)^2$). As a conclusion, ordering at (Early, Early) is not an equilibrium point.

**The informed firm's order and the uninformed firm's order = (Early, Late)**

When the informed firm places order early, and the uninformed firm places order late, their equilibrium order quantity and profit is as follows.

$$\begin{aligned} q_{iH} &= \frac{(2-\beta)A_H}{2(2-\beta^2)}, q_{iL} = \frac{(2-\beta)A_L}{2(2-\beta^2)} \\ q_{eH} &= \frac{(4-2\beta-\beta^2)A_H}{4(2-\beta^2)}, q_{eL} = \frac{(4-2\beta-\beta^2)A_L}{4(2-\beta^2)} \end{aligned}$$

$$\pi_{iH}^{L} = \frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)}, \pi_{iL}^{L} = \frac{(2-\beta)^2 A_L^2}{8(2-\beta^2)}$$
$$\pi_{eH}^{L} = \frac{(4-2\beta-\beta^2)^2 A_H^2}{16(2-\beta^2)^2}, \pi_{eL}^{L} = \frac{(4-2\beta-\beta^2)^2 A_L^2}{16(2-\beta^2)^2}$$

First, when the informed firm deviates from early to late, game is changed with simultaneous move with complete information (Cournot game). The informed firm's profit is

$$\pi_{iH} = \left(\frac{A_H}{2+\beta}\right)^2, \pi_{eH} = \left(\frac{A_H}{2+\beta}\right)^2$$

Since, original profit function $\left(\frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)}\right)$ is strictly greater than the profit of deviation $\left(\left(\frac{A_H}{2+\beta}\right)^2\right)$ $(\frac{(2-\beta)^2 A_H^2}{8(2-\beta^2)} - \left(\frac{A_H}{2+\beta}\right)^2 = \frac{\beta^4}{8(2+\beta)^2(2-\beta)} > 0)$, it is not a profitable deviation for the informed firm. Second, when the uninformed firm deviates from late to early, its order quantity and profit are

$$q_{eH} = \frac{(2-\beta)\mu}{4}$$
$$\pi_{eH} = \left(\frac{(2-\beta)\mu}{4}\right)^2$$

Expected Profit of ordering late (original profit) is

$$p\left(\frac{(4-2\beta-\beta^2)^2 A_H^2}{16(2-\beta^2)^2}\right) + (1-p)\left(\frac{(4-2\beta-\beta^2)^2 A_L^2}{16(2-\beta^2)^2}\right) = \frac{(4-2\beta-\beta^2)^2 \mu^2}{16(2-\beta^2)^2}$$

Difference between original profit and deviated profit is

$$\frac{(2-\beta)^2\mu^2}{16} - \frac{(4-2\beta-\beta^2)^2\mu^2}{16(2-\beta^2)^2} \leq 0$$

It is also not profitable deviation for the uninformed firm.

**Proof of Theorem 3 using Mixed Strategy Analysis.** Suppose the uninformed firm uses mixed strategy. $r$ is the probability that the uninformed firm places order early, and $(1-r)$ is the probability that the uninformed firm orders late. $q_e^C$ and $q_e^S$ denote optimal simultaneous and sequential move order quantity for the uninformed firm respectively. Then the informed firm's objective function is

$$\begin{aligned}
\pi_{iH} &= \max_{q_{iH}} r \cdot (A_H - q_{iH} - q_e^C)q_{iH} + (1-r)(A_H - q_{iH} - q_{eH}^S)q_{iH} - wq_{iH} \\
\pi_{iL} &= \max_{q_{iL}} r \cdot (A_L - q_{iL} - q_e^C)q_{iL} + (1-r)(A_L - q_{iL} - q_{eL}^S)q_{iL} - wq_{iL}
\end{aligned}$$

Where

$$\begin{aligned}
q_{eH}^S &= \arg\max_{q_{eH}} (A_H - q_{iH} - q_{eH})q_{eH} - wq_{eH} \\
q_{eL}^S &= \arg\max_{q_{eL}} (A_L - q_{iL} - q_{eL})q_{eL} - wq_{eL}
\end{aligned}$$

$$\pi_e^C = \max_{q_e^C} p \cdot \left[(A_H - q_{iH} - q_e^C)q_e^C\right] + (1-p)\left[(A_L - q_{iL} - q_e^C)q_e^C\right] - wq_e^C$$

Optimal solutions are

$$\begin{aligned}
\pi_{iH} &= \frac{(1+r)(2A_H + r(1-p)(A_H - A_L) - 2w)^2}{8(2+r)^2} \\
\pi_{iL} &= \frac{(1+r)(p \cdot r \cdot A_H - (2 + p \cdot r)A_L - 2w)^2}{8(2+r)^2} \\
\pi_{eH}^S &= \frac{((2 + r + p \cdot r)A_H + r(1-p)A_L - 2w)^2}{16(2+r)^2} \\
\pi_{eL}^S &= \frac{(p \cdot r \cdot A_H + (2 + 2r - p \cdot r))A_L - 2w)^2}{16(2+r)^2} \\
\pi_e^C &= \frac{(\mu - w)^2(1+r)^2}{4(2+r)^2}
\end{aligned}$$

Difference between expected profit of sequential move and simultaneous move for the uninformed firm is

$$
\begin{aligned}
& p\cdot\pi_{eH}^{S}+(1-p)\pi_{eL}^{S}-\pi_{e}^{C} \\
= \ & p\cdot\frac{((2+r+p\cdot r)A_H+r(1-p)A_L-2w)^2}{16(2+r)^2} \\
& +(1-p)\frac{(p\cdot r\cdot A_H+(2+2r-p\cdot r))A_L-2w)^2}{16(2+r)^2}-\frac{(\mu-w)^2(1+r)^2}{4(2+r)^2} \\
= \ & \frac{\begin{array}{c} p\cdot((2+r+p\cdot r)A_H+r(1-p)A_L-2w)^2 \\ +(1-p)(p\cdot r\cdot A_H+(2+2r-p\cdot r))A_L-2w)^2-4(\mu-w)^2(1+r)^2 \end{array}}{16(2+r)^2} \\
= \ & \frac{1}{16(2+r)^2}\left(\begin{array}{c} p\cdot(r\cdot\mu+(2+r)A_H-2w)^2+(1-p)(r\cdot\mu+(2+r)A_L-2w)^2 \\ -4(\mu-w)^2(1+r)^2 \end{array}\right) \\
= \ & \frac{1}{16(2+r)^2}\left(p\cdot(2+r)^2A_H^2+(1-p)(2+r)^2A_L^2-(2+r)^2\mu^2\right) \\
= \ & \frac{1}{16}p(1-p)(A_H-A_L)^2
\end{aligned}
$$

and it is always positive regardless of $r$ and $w$. Therefore, The uninformed firm strictly prefer sequential move game to simultaneous game. ■

### A.1.4 Endogenous Sequencing Game of Multiple demand states

In this section, we model multiple demand states. First, we analyze three demand states case, and this can be generalized to more than three states such as $A_{H1}, A_{H2}, ..., A_L$. For the simplicity, we analyze three state case for the analysis. The demand intercept A is random and can take one of three values: a high value $A_H$

with probability $p_H$, a middle value $A_M$ with probability $p_M$, and low value $A_L$ with probability $(1 - p_H - p_M)$. The mean demand intercept $\mu' = p_H A_H + p_M A_M + (1 - p_H - p_M)A_L$.

**Lemma 20** *(i) If the supplier commit not to leak the informed firm's order information with the uninformed firm, the game is simultaneous move (Cournot) game with incomplete information. The subgame-perfect Nash equilibrium quantity and profit of the informed firm and the uninformed firm are:*

$$\begin{aligned} q_{iH} &= \frac{3A_H - \mu'}{6}; q_{iM} = \frac{3A_M - \mu'}{6}; q_{iL} = \frac{3A_L - \mu'}{6}; q_e = \frac{\mu'}{3} \\ \pi_{iH} &= \left(\frac{3A_H - \mu'}{6}\right)^2; \pi_{iM} = \left(\frac{3A_M - \mu'}{6}\right)^2; \pi_{iL} = \left(\frac{3A_L - \mu'}{6}\right)^2; \pi_e = \left(\frac{\mu'}{3}\right)^2 \end{aligned}$$

*(ii) If the informed firm places order before the uninformed firm, and if supplier commit to leak order information with the uninformed firm, the game is sequential move (Stackelberg) game. The subgame-perfect Nash equilibrium quantity and profit of the informed firm and the uninformed firm are:*

$$\begin{aligned} q_{iH} &= \frac{A_H}{2}; q_{iM} = \frac{A_M}{2}; q_{iL} = \frac{A_L}{2}; \\ q_{iH} &= \frac{A_H}{4}; q_{iM} = \frac{A_M}{4}; q_{iL} = \frac{A_L}{4}; \\ \pi_{iH} &= \frac{A_H^2}{8}; \pi_{iM} = \frac{A_M^2}{8}; \pi_{iL} = \frac{A_L^2}{8}; \\ \pi_{eH} &= \frac{A_H^2}{16}; \pi_{eM} = \frac{A_M^2}{16}; \pi_{eL} = \frac{A_L^2}{16}; \end{aligned}$$

**Proof.** (i) The informed firm and the uninformed firm's profit function for simultaneous move game is

$$
\begin{aligned}
\pi_{iH} &= \max_{q_{iH}} (A_H - q_{iH} - q_e) q_{iH} \\
\pi_{iM} &= \max_{q_{iM}} (A_M - q_{iM} - q_e) q_{iM} \\
\pi_{iL} &= \max_{q_{iL}} (A_L - q_{iL} - q_e) q_{iL} \\
\pi_e &= \max_{q_e} p_H (A_H - q_{iH} - q_e) q_e + p_M (A_M - q_{iM} - q_e) q_e + (1 - p_H - p_M)(A_L - q_{iL} - q_e) q_e
\end{aligned}
$$

Using first order condition, its order quantity and profit are as noted.

(ii) The sequential move game's model is as follows.

$$
\begin{aligned}
\pi_{iH} &= \max_{q_{iH}} (A_H - q_{iH} - q^*_{eH}(q_{iH})) q_{iH} \\
\pi_{iM} &= \max_{q_{iM}} (A_M - q_{iM} - q^*_{eM}(q_{iM})) q_{iM} \\
\pi_{iL} &= \max_{q_{iL}} (A_L - q_{iL} - q^*_{eL}(q_{iL})) q_{iL}
\end{aligned}
$$

where

$$
\begin{aligned}
q^*_{eH} &= \arg\max_{q_{eH}} (A_H - q_{iH} - q_{eH}) q_{eH} = \frac{A_H - q_{iH}}{2} \\
q^*_{eM} &= \arg\max_{q_{eM}} (A_H - q_{iH} - q_{eM}) q_{eM} = \frac{A_M - q_{iM}}{2} \\
q^*_{eL} &= \arg\max_{q_{eL}} (A_L - q_{iL} - q_{eL}) q_{eL} = \frac{A_L - q_{iL}}{2}
\end{aligned}
$$

Using first order condition, its order quantity and profit are as noted. ∎

**Supplier's leakage decision**

First, when demand is high, supplier always leak the informed firm's order information with the uninformed firm. ($\frac{3A_H}{4} > \frac{A_H}{2} + \frac{\mu'}{6}$ for all probability). If demand is either middle or low, supplier might not leak in some conditions. If the supplier does not leak, the uninformed firm can infer that it is not high demand state (either middle or low). Thus, the uninformed firm's objective function is changed as follows.

$$\begin{aligned}\pi_e &= \max_{q_e} \frac{p_M}{1-p_H}(A_M - q_{iM} - q_e)q_e + \frac{(1-p_H-p_M)}{1-p_H}(A_L - q_{iL} - q_e)q_e \\ &= \max_{q_e} p'_M(A_M - q_{iM} - q_e)q_e + (1-p'_M)(A_L - q_{iL} - q_e)q_e \quad \text{where } p'_M = \frac{p_M}{1-p_H}\end{aligned}$$

Equilibrium order quantity for each player is

$$q_{iM} = \frac{3A_M - \mu''}{6}; q_{iL} = \frac{3A_L - \mu''}{6}; q_e = \frac{\mu''}{3} \text{ where } \mu'' = p'_M A_M + (1-p'_M)A_L$$

Expected order quantity for middle and low demand state are $\frac{3A_M+\mu''}{6}$and $\frac{3A_L+\mu''}{6}$. Regardless of demand probability $p'_M$ and $p'_L$, the supplier can better off by leaking order information in the middle demand. Also, by same reason with previous section, the uninformed firm can infer low demand state when the supplier does not leak. Therefore, we can conclude that supplier will always leak order information even in the multiple stages.

**Equilibrium analysis**

In the lowest state, the informed firm strictly should place order early to maximize profit. If the informed firm does not order early, then the uninformed firm can infer that demand state is one of high state. So, the uninformed firm places order earlier than the informed firm, and as a result, the informed firm's profit function can be hurt. Therefore, the informed firm always place order early to maximize profit.(Proposition 1 is still hold.)

### A.1.5 Proofs of Information Acquisition Game

#### Two player game

When both two competitors can have demand information with cost of $K$, four possible scenarios and each player's *expected* payoffs are in the Table 1.1.

Each firm's best response depends on the value of $K$. Let each firm's profit difference when competitor does invest $K_1$, and let each firm's profit difference when competitor does not invest $K_2$.

$$\begin{aligned}
K_1 &= p\left(\frac{A_H^2}{9}\right) + (1-p)\left(\frac{A_L^2}{9}\right) - \left(p\left(\frac{A_H^2}{16}\right) + (1-p)\left(\frac{A_L^2}{16}\right)\right) \\
&= \frac{7}{144}\left(p \cdot A_H^2 + (1-p)A_L^2\right) \\
K_2 &= p\left(\frac{A_H^2}{8}\right) + (1-p)\left(\frac{A_L^2}{8}\right) - \frac{\mu^2}{9} \\
&= \frac{1}{72}\left(A_H^2(9-8p)p - 16A_HA_L(1-p)p + A_L^2(1+(7-8p)p)\right)
\end{aligned}$$

**Proof of Proposition 5.** First, when $0 \leq K < \min[K_1, K_2]$, (Invest, Invest) is the only equilibrium for two competitors. When $K_1 < K < K_2$, (Invest, No Invest) and (No invest, Invest) are equilibria. When $K_2 < K < K_2$, (Invest, Invest) and (No

Invest, No Invest) are equilibria. When $K > \max[K_1, K_2]$, (No Invest, No Invest) is the only equilibrium. Sample figure of equilibrium is illustrated in figure 1.2. ■

**Lemma 21** *Even if the supplier can have a chance to acquire demand information, the supplier's expected profit by acquiring information is always less than expected profit without acquiring information as long as positive information acquisition cost.*

***Proof.*** *Suppose the case when the supplier acquires demand information, and let two competitors that information. Also, let supplier's wholesale price $w$, and information acquisition cost $K$. First, when the supplier acquires information with cost $K$, selling quantity in high demand is $\frac{A_H - w}{3}$ for the both buyers, and selling quantity in low demand is $\frac{A_L - w}{3}$ for the both buyers. Ex Ante expected profit for the supplier is*

$$\left(p\left(2 \times \frac{A_H - w}{3}\right) + (1-p)\left(2 \times \frac{A_L - w}{3}\right)\right) w - K = \frac{2}{3}(\mu - w)w - K$$

*Next, when the supplier does not acquire information, expected profit is*

$$\left(p\left(2 \times \frac{\mu - w}{3}\right) + (1-p)\left(2 \times \frac{\mu - w}{3}\right)\right) w = \frac{2}{3}(\mu - w)w$$

*and it is always greater than the profit of information acquisition.* ■

### Three player game

**Lemma 22** *(i) In the 3 player game with one informed firm and two uninformed firms, when all players orders early, the game is simultaneous move (Cournot) game. The subgame-perfect Nash equilibrium quantity and profit of one informed firm and* *two uninformed firms are:*

$$
\begin{aligned}
q_{iH} &= \frac{4(A_H - w) - 2(\mu - w)}{8}; q_{iL} = \frac{4(A_L - w) - 2(\mu - w)}{8} \\
q_{u1} &= q_{u2} = \frac{(\mu - w)}{4} \\
\pi_{iH} &= \left(\frac{4(A_H - w) - 2(\mu - w)}{8}\right)^2; \pi_{iL} = \left(\frac{4(A_L - w) - 2(\mu - w)}{8}\right)^2 \\
\pi_{u1} &= \pi_{u2} = \frac{(\mu - w)^2}{16}
\end{aligned}
$$

*(ii) In the 3 player game with one informed firm and two uninformed firms, if the informed firm orders before the uninformed firms, and if the supplier commits to leak order information with uninformed firms, the game is sequential move (Stackelberg) game. The subgame-perfect Nash equilibrium quantity and profit or one informed firm and two uninformed firms are:*

$$
\begin{aligned}
q_{iH} &= \frac{A_H - w}{2}; q_{u1H} = q_{u2H} = \frac{A_H - w}{6} \\
q_{iL} &= \frac{A_L - w}{2}; q_{u1L} = q_{u2L} = \frac{A_L - w}{6} \\
\pi_{iH} &= \frac{(A_H - w)^2}{12}; \pi_{u1H} = \pi_{u2H} = \frac{(A_H - w)^2}{36} \\
\pi_{iL} &= \frac{(A_L - w)^2}{12}; \pi_{u1L} = \pi_{u2L} = \frac{(A_L - w)^2}{36}
\end{aligned}
$$

**Proof.** (i) High type informed firm, low type informed firm, and two uninformed firms solve the following simultaneous move game of incomplete information.

$$\begin{aligned}
\pi_{iH} &= \max_{q_{iH}}(A_H - q_{iH} - q_{u1} - q_{u2})q_{iH} - wq_{iH} \\
\pi_{iL} &= \max_{q_{iL}}(A_L - q_{iL} - q_{u1} - q_{u2})q_{iL} - wq_{iL} \\
\pi_{u1} &= \max_{q_{u1}} p\left[(A_H - q_{iH} - q_{u1} - q_{u2})q_{u1}\right] + (1-p)\left[(A_L - q_{iL} - q_{u1} - q_{u2})q_{u1}\right] - wq_{u1} \\
\pi_{u2} &= \max_{q_{u2}} p\left[(A_H - q_{iH} - q_{u1} - q_{u2})q_{u2}\right] + (1-p)\left[(A_L - q_{iL} - q_{u1} - q_{u2})q_{u2}\right] - wq_{u2}
\end{aligned}$$

Using first order condition, its order quantity and profit are as noted.

(ii) If supplier leaks order information with uninformed firm, the game is sequential move game with complete information (Stackelberg model). The model is as follows.

$$\begin{aligned}
\pi_{iH} &= \max_{q_{iH}}(A_H - q_{iH} - q^*_{u1H}(q_{iH}) - q^*_{u2H}(q_{iH}))q_{iH} - wq_{iH} \\
\pi_{iL} &= \max_{q_{iL}}(A_L - q_{iL} - q^*_{u1L}(q_{iL}) - q^*_{u2L}(q_{iL}))q_{iL} - wq_{iL}
\end{aligned}$$

Where

$$\begin{aligned}
q^*_{u1H} &= \arg\max_{q_{u1H}}(A_H - q_{iH} - q_{u1H} - q_{u2H})q_{u1H} - wq_{u1H} \\
q^*_{u1L} &= \arg\max_{q_{u1L}}(A_L - q_{iL} - q_{u1L} - q_{u2L})q_{u1L} - wq_{u1L} \\
q^*_{u2H} &= \arg\max_{q_{u2H}}(A_H - q_{iH} - q_{u1H} - q_{u2H})q_{u2H} - wq_{u2H} \\
q^*_{u2L} &= \arg\max_{q_{u2L}}(A_L - q_{iL} - q_{u1L} - q_{u2L})q_{u2L} - wq_{u2L}
\end{aligned}$$

Using first order condition, its order quantity and profit are as noted. ■

**Lemma 23** *(i) In the 3 player game with two informed firms and one uninformed firm, when all players orders early, the game is simultaneous move (Cournot) game.*

*The subgame-perfect Nash equilibrium quantity and profit of two informed firms and one uninformed firm are:*

$$
\begin{aligned}
q_{i1H} &= q_{i2H} = \frac{4(A_H - w) - (\mu - w)}{12}; q_{i1L} = q_{i2L} = \frac{4(A_L - w) - (\mu - w)}{12} \\
q_u &= \frac{(\mu - w)}{4} \\
\pi_{i1H} &= \pi_{i2H} = \left(\frac{4(A_H - w) - (\mu - w)}{12}\right)^2; \pi_{i1L} = \pi_{i2L} = \left(\frac{4(A_L - w) - (\mu - w)}{12}\right)^2 \\
\pi_u &= \frac{(\mu - w)^2}{16}
\end{aligned}
$$

*(ii) In the 3 player game with two informed firms and one uninformed firm, if the informed firms orders before the uninformed firms, and if the supplier commits to leak order information with uninformed firms, the game is sequential move (Stackelberg) game. The subgame-perfect Nash equilibrium quantity and profit or two informed firms and one uninformed firm are:*

$$
\begin{aligned}
q_{i1H} &= q_{i2H} = \frac{A_H - w}{3}; q_{uH} = \frac{A_H - w}{6} \\
q_{i1L} &= q_{i2L} = \frac{A_L - w}{3}; q_{uL} = \frac{A_L - w}{6} \\
\pi_{i1H} &= \pi_{i2H} = \frac{(A_H - w)^2}{18}; \pi_{uH} = \frac{(A_H - w)^2}{36} \\
\pi_{i1L} &= \pi_{i2L} = \frac{(A_L - w)^2}{18}; \pi_{uL} = \frac{(A_L - w)^2}{36}
\end{aligned}
$$

**Lemma 24** *(Generalization) (i) In the n+m player game with n informed firms and m uninformed firms, when all players orders early, the game is simultaneous move*

*(Cournot) game. The subgame-perfect Nash equilibrium quantity and profit of n informed firms and m uninformed firms are:*

$$
\begin{aligned}
q_{iH} &= \frac{A_H}{(n+1)} - \frac{m \cdot \mu}{(n+1)(n+m+1)}; q_{iL} = \frac{A_L}{(n+1)} - \frac{m \cdot \mu}{(n+1)(n+m+1)} \\
q_u &= \frac{\mu}{(n+1)} \\
\pi_{iH} &= \left(\frac{A_H}{(n+1)} - \frac{m \cdot \mu}{(n+1)(n+m+1)}\right)^2; \pi_{iL} = \left(\frac{A_L}{(n+1)} - \frac{m \cdot \mu}{(n+1)(n+m+1)}\right)^2 \\
\pi_u &= \left(\frac{\mu}{n+1}\right)^2 \\
\text{where } i &= \{1, 2, ..., n\} \text{ and } u = \{1, 2, ..., m\}
\end{aligned}
$$

*(ii) In the n+m player game with n informed firms and m uninformed firms, if the informed firms orders before the uninformed firms, and if the supplier commits to leak order information with uninformed firms, the game is sequential move (Stackelberg) game. The subgame-perfect Nash equilibrium quantity and profit of n informed firms and m uninformed firm are:*

$$
\begin{aligned}
q_{iH} &= \frac{A_H}{(n+1)}; q_{uH} = \frac{A_H}{(n+1)(m+1)} \\
q_{iL} &= \frac{A_L}{(n+1)}; q_{uL} = \frac{A_L}{(n+1)(m+1)} \\
\pi_{iH} &= \frac{A_H^2}{(n+1)^2(m+1)}; \pi_{uH} = \frac{A_H^2}{(n+1)^2(m+1)^2} \\
\pi_{iL} &= \frac{A_L^2}{(n+1)^2(m+1)}; \pi_{uL} = \frac{A_L^2}{(n+1)^2(m+1)^2} \\
\text{where } i &= \{1, 2, ..., n\} \text{ and } u = \{1, 2, ..., m\}
\end{aligned}
$$

When three competitors can have demand information with cost of $K$, each firm faces following six case of scenarios and each player's *expected* payoffs are in Table 1.2 with $K_1 = \frac{7}{144}\left(p \cdot A_H^2 + (1-p)A_L^2\right)$, $K_2 = \frac{1}{36}\left(p \cdot A_H^2 + (1-p)A_L^2\right)$, $K_3 = \frac{1}{48}A_L^2 + 4A_H^2 \cdot p + 2A_L(A_L - 3A_H)p - 3(A_H - A_L)^2 p^2$. As a reference, $K_2$ is always less than $K_1$.

**Proof of three player game.** We check the best response of firm for every possible cases. For example, when two competitors are assumed to invest, the best response of firm is (Invest) when $K$ is trivial. On the other hand, when $K$ is higher than Max $[K_1, K_2, K_3]$, the best response is (No Invest). With same procedure, following equilibrium can be found. First, when $0 \leq K < \min[K_1, K_2, K_3]$, (I, I, I) is the only equilibrium for two competitors. When $K_2 < K < \min[K_1, K_3]$, (I, I, I), (I, NI, NI), (NI, I, NI), and (NI, NI, I) are equilibria. When $K_2 < K_1 < K < K_3$, (I, NI, NI), (NI, I, NI), and (NI, NI, I) are equilibria. When $K_3 < K < K_1$, (I, I, I) and (NI, NI, NI) are equilibria. When $K > \max[K_1, K_2, K_3]$, (NI, NI, NI) is the only equilibrium. Sample comparative statics are in figure #. ■

### A.1.6 Relationship between the supplier's different wholesale price and timing of order

**Lemma 25** *When the monopolist has a following trade-off between timing of order and demand uncertainty - i.e. early order with demand uncertainty and late order with certain demand, the monopolist prefer to order late as long as the supplier charges* *same wholesale price* ($w = w_e = w_l$)*.*

**Proof.** Let $q$ be the selling quantity of the monopolist (retailer) and $Q$ be the order quantity from the supplier. $w$ is wholesale price. Monopolist's payoff function is

$$\begin{aligned} Max\ \pi_r &= P(Q)\cdot q - Q\cdot w \\ s.t.\ \ q(Q) &\leq Q \end{aligned}$$

First, suppose the monopolist places *order early* with demand uncertainty, his profit functions are divided into three cases. First case is to order high enough quantity, and third case is to order minimum quantity. Second case is to order between two extreme order quantities.

(i) $Q_1 = \frac{A_H}{2}$

$$\begin{aligned} \pi_{r1} &= p(A_H - Q_1)Q_1 + (1-p)(A_L - Q_1)Q_1 - w\cdot Q_1 \\ &= p\left(\frac{A_H}{2}\right)^2 + (1-p)\left(\frac{A_L}{2}\right)^2 - w\cdot\frac{A_H}{2} \end{aligned}$$

(ii) $\frac{A_L}{2} < Q_2 < \frac{A_H}{2}$

$$\pi_{r2} = p(A_H - Q_2)Q_2 + (1-p)(A_L - Q_2)Q_2 - w\cdot Q_2$$

Since the optimal order quantity for above profit function is $Q_2 = \frac{A_H - w/p}{2}$, profit function is simplified as follows.

$$\pi_{r2} = p\left[\left(\frac{A_H}{2}\right)^2 - \left(\frac{w}{2p}\right)^2\right] + (1-p)\left(\frac{A_L}{2}\right)^2 - w\left[\left(\frac{A_H}{2}\right) - \left(\frac{w}{2p}\right)\right]$$

(iii) $Q_3 = \frac{A_L}{2}$

$$\begin{aligned}\pi_{r3} &= p(A_H - Q_3)Q_3 + (1-p)(A_L - Q_3)Q_3 - w \cdot Q_3 \\ &= p\left(A_H - \frac{A_L}{2}\right)\frac{A_L}{2} + (1-p)\left(\frac{A_L}{2}\right)^2 - w \cdot \frac{A_L}{2}\end{aligned}$$

Since $\pi_{r1}$ is always less than $\pi_{r2}$ , Monopolist's profit functions are summarized as follows.

a. $p > \frac{w}{A_H - A_L}$

$$\pi_{REH} = p\left[\left(\frac{A_H}{2}\right)^2 - \left(\frac{w}{2p}\right)^2\right] + (1-p)\left(\frac{A_L}{2}\right)^2 - w\left[\left(\frac{A_H}{2}\right) - \left(\frac{w}{2p}\right)\right]$$

b. $p < \frac{w}{A_H - A_L}$

$$\pi_{REL} = p\left[A_H - \frac{A_L}{2}\right]\frac{A_L}{2} + (1-p)\left(\frac{A_L}{2}\right)^2 - w \cdot \frac{A_L}{2}$$

Consequently, the supplier's profits for early ordering monopolist are

$$\begin{aligned}\pi_{SEH} &= \left[\left(\frac{A_H}{2}\right) - \left(\frac{w}{2p}\right)\right](w - c) \\ \pi_{SEL} &= \left(\frac{A_L}{2}\right)(w - c)\end{aligned}$$

Second, when the monopolist *orders late* with updated demand information, his order quantity depends on the supplier's production quantity. (Since we will show the supplier produces either two points ($\frac{A_H - w}{2}$ or $\frac{A_L - w}{2}$) of quantity for the case of late order in the next lemma, the monopolist's profit function depends on unit availability.)

1. When the supplier produces high demand case or quantity $(\frac{A_H-w}{2})$ (When $p \geq \frac{c}{w}$), then the late order monopolist's expected profits are

$$\pi_{RLH} = p\left(\frac{A_H - w}{2}\right)^2 + (1-p)\left(\frac{A_L - w}{2}\right)^2$$

2. When the supplier produces low demand case or quantity $(\frac{A_L-w}{2})$ (When $p < \frac{c}{w}$), then the late order monopolist's expected profits are

$$\pi_{RLL} = p\left(\frac{2A_H - A_L - w}{2}\right)\left(\frac{A_L - w}{2}\right) + (1-p)\left(\frac{A_L - w}{2}\right)^2$$

After comparing each profit functions, *the monopolist can expect highest profit with late order when the supplier produced high demand state quantities.* One more thing we need to consider is when the supplier produces $\frac{A_L-w}{2}$ units when realized demand is high (push challenge). When realized demand is low, this 'push challenge' never happen because the monopolist can order just $\frac{A_L-w}{2}$ even if the supplier produces $\frac{A_H-w}{2}$. The supplier will bear whole excessive production cost. So, push challenge mentioned in the Cachon (2004) can happen if the supplier produces low demand quantity. ■

**Lemma 26** *When the monopolist has demand uncertainty at early period, and that demand is realized at late period, the supplier prefer to sell early.*

**Proof.** Let $q$ be the order quantity of the monopolist and $Q$ be the production quantity for the supplier. Supplier's objective function is

$$\begin{aligned} Max\ \pi_S &= q(Q) \cdot w - Q \cdot c \\ s.t.\ \ q(Q) &\leq Q \end{aligned}$$

When monopolist places order early, supplier can just produce monopolist's order quantity. The supplier's profits for early ordering monopolist are

$$\begin{aligned}\pi_{SEH} &= \left[\left(\frac{A_H}{2}\right) - \left(\frac{w}{2p}\right)\right](w-c) \\ \pi_{SEL} &= \left(\frac{A_L}{2}\right)(w-c)\end{aligned}$$

When monopolist places order late, supplier's optimal production quantity is divided into 3 cases.

(i) $Q_1 \geq \frac{A_H - w}{2}$

When the supplier produces greater than or equal to maximum level of ordering quantity, the supplier's profit function is

$$\pi_{s1} = \left[p\left(\frac{A_H - w}{2}\right) + (1-p)\left(\frac{A_L - w}{2}\right)\right] w - c \cdot Q_1$$

(ii) $\frac{A_L - w}{2} \leq Q_2 \leq \frac{A_H - w}{2}$

when the supplier produces between two extreme points, the supplier's profit function is

$$\pi_{s2} = \left[p \cdot Q_2 + (1-p)\left(\frac{A_L - w}{2}\right)\right] w - c \cdot Q_2$$

(iii) $Q_3 \leq \frac{A_L - w}{2}$

When the supplier produces less than or equal to minimum level of ordering quantity, the supplier's profit function is

$$\pi_{s3} = [p \cdot Q_3 + (1-p)Q_3]\, w - c \cdot Q_3$$

Let $\Delta_{1-2}$ is the difference between $\pi_{s1}$ and $\pi_{s2}$, and $\Delta_{2-3}$ is the difference between $\pi_{s2}$ and $\pi_{s3}$.

$$\begin{aligned}\Delta_{1-2} &= p \cdot w\left(\frac{A_H - w}{2} - Q_2\right) - c(Q_1 - Q_2) \\ \Delta_{2-3} &= (p \cdot w - c)(Q_2 - Q_3) + (1-p)w\left(\frac{A_L - w}{2} - Q_3\right)\end{aligned}$$

When $p \geq \frac{2c(Q_1 - Q_2)}{w(A_H - 2Q_2 + w)}$, $\Delta_{1-2}$ is positive. So, the supplier would produce $Q \geq \frac{A_H - w}{2}$ (high quantity), and $\pi_{s1}$ is decreasing function of $Q_1$. Therefore, the supplier will produce $Q = \frac{A_H - w}{2}$. when $p \leq \frac{(w-c)(A_L - 2Q_3 - w)}{w(w + 2Q_2 - A_L)}$, $\Delta_{2-3}$ is negative. So, the supplier would produce $Q \leq \frac{A_L - w}{2}$ (low quantity), and $\pi_{s3}$ is increasing function of $Q_3$. Therefore, the supplier will produce $Q = \frac{A_L - w}{2}$. When $p$ is located in between two value, the supplier will produce $\frac{A_L - w}{2} \leq Q \leq \frac{A_H - w}{2}$. Rewriting $\pi_{s2}$ is $(p \cdot w - c)Q_2 + (1-p)w\left(\frac{A_L - w}{2}\right)$. When $p \geq \frac{c}{w}$, it is increasing function. Optimal production quantity is $\frac{A_H - w}{2}$. When $p \leq \frac{c}{w}$, optimal production quantity is $\frac{A_L - w}{2}$. Therefore, the supplier's profit functions end up with two cases.

a. When $p \geq \frac{c}{w}$,

$$\pi_{SLH} = w\left(\frac{\mu - w}{2}\right) - c\left(\frac{A_H - w}{2}\right)$$

b. When $p \leq \frac{c}{w}$,

$$\pi_{SLL} = \left(\frac{A_L - w}{2}\right)(w - c)$$

After comparing each profit functions($\pi_{SEH}/\pi_{SEL}$ and $\pi_{SLH}/\pi_{SLL}$), *the supplier can expect highest profit with early order with high demand state order quantities.* ■

**Lemma 27** *The supplier can provide discount to induce the retailer (monopolist) to place early order. The maximum discount amount is the minimum of the following: until when the retailer is indifferent with timing of order, and the supplier's early order profit is same with late order profit.*

**Proof.** First, suppose the supplier's decision is producing high demand state quantity ($\frac{A_H - w}{2}$ when $p > \frac{c}{w}$. i.e. relatively low production cost and high wholesale price) if the retailer places order late. Expected profit of the retailer and the supplier are

$$\begin{aligned} \pi_{RLH} &= p\left(\frac{A_H - w}{2}\right)^2 + (1-p)\left(\frac{A_L - w}{2}\right)^2 \\ \pi_{SLH} &= w\left(\frac{\mu - w}{2}\right) - c\left(\frac{A_H - w}{2}\right) \end{aligned}$$

If the retailer places order early with demand uncertainty, both players' expected profits are

a. $p > \frac{w}{A_H - A_L}$

$$\begin{aligned} \pi_{REH} &= p\left[\left(\frac{A_H}{2}\right)^2 - \left(\frac{w}{2p}\right)^2\right] + (1-p)\left(\frac{A_L}{2}\right)^2 - w\left(\left(\frac{A_H}{2}\right) - \left(\frac{w}{2p}\right)\right) \\ \pi_{SEH} &= \left[\left(\frac{A_H}{2}\right) - \left(\frac{w}{2p}\right)\right](w - c) \end{aligned}$$

b. $p < \frac{w}{A_H - A_L}$

$$\begin{aligned} \pi_{REL} &= p\left(A_H - \frac{A_L}{2}\right)\frac{A_L}{2} + (1-p)\left(\frac{A_L}{2}\right)^2 - w \cdot \frac{A_L}{2} \\ \pi_{SEL} &= \left(\frac{A_L}{2}\right)(w - c) \end{aligned}$$

If $p > \frac{c}{w}$ and $p > \frac{w}{A_H - A_L}$, the retailer can provide discount to induce early order. (since $\pi_{RLH} > \pi_{REH}$). After replacing $w$ with $w - \delta$ for $\pi_{REH}$, there is a $\delta$ ($\delta_1$) which makes two profit function in equal. $\delta_1 = w - p \cdot A_H + \sqrt{p^2 A_H (A_H - 2w) + w^2 - 2wA_L(1-p)}$. The supplier's profit is also affected by discount. Comparing both supplier's profit function ($\pi_{SLH}$ and $\pi_{SEH}$), the supplier's maximum discount is $\delta_2 = \frac{1}{2}(2w - c - p \cdot A_H + \sqrt{(c + p \cdot A_H)^2 - 4p(c+\mu)w + 4p \cdot w^2})$. So, maximum discount the supplier can provide is $\min[\delta_1, \delta_2]$. If $p > \frac{c}{w}$ and $p < \frac{w}{A_H - A_L}$, Retailer can also provide discount (since $\pi_{RLH} > \pi_{REL}$). The maximum discount $\delta_3 = \frac{p(A_H - A_L)^2 - 2p(A_H - A_L)w + w^2}{2A_L}$. With same logic the supplier's maximum discount (for $\pi_{SLH}$ and $\pi_{SEL}$) $\delta_4 = \frac{c(A_H - A_L) - w(c + p(A_H - A_L)) + w^2}{A_L}$. So, maximum discount the supplier can provide is $\min[\delta_3, \delta_4]$

Second, when $p < \frac{c}{w}$, the supplier produces low demand state quantity if the retailer places order late. Expected profit of the retailer and the supplier are

$$\begin{aligned}
\pi_{RLL} &= p\left(\frac{2A_H - A_L - w}{2}\right)\left(\frac{A_L - w}{2}\right) + (1-p)\left(\frac{A_L - w}{2}\right)^2 \\
\pi_{SLL} &= \left(\frac{A_L - w}{2}\right)(w - c)
\end{aligned}$$

If the retailer places order early with demand uncertainty, both players' expected profits are same with previous case. If $p < \frac{c}{w}$ and $p > \frac{w}{A_H - A_L}$, $\pi_{REH}$ is always greater than $\pi_{RLL}$. So, the retailer always prefers to order early. (no need to provide discount). If $p < \frac{c}{w}$ and $p < \frac{w}{A_H - A_L}$, it is divided into two cases. When $\frac{w}{2(A_H - A_L)} < p < \frac{w}{A_H - A_L}$, $\pi_{REL}$ is always greater than $\pi_{RLL}$. (no need to provide discount). If $0 < p < \frac{w}{2(A_H - A_L)}$, $\pi_{REL}$ is smaller than $\pi_{RLL}$. Maximum discount $\delta_5 = \frac{w(w - 2p(A_H - A_L))}{2A_L}$. For the supplier's perspective, when $0 < p < \frac{w}{2(A_H - A_L)}$, the maximum discount $\delta_6 =$

$\frac{w(w-c)}{A_L}$.So, maximum discount the supplier can provide is $\min\left[\delta_5, \delta_6\right]$.

| | | |
|---|---|---|
| $p > \frac{c}{w}$ | $p > \frac{w}{A_H - A_L}$ | $\min\left[\delta_1, \delta_2\right]$ |
| $p > \frac{c}{w}$ | $p < \frac{w}{A_H - A_L}$ | $\min\left[\delta_3, \delta_4\right]$ |
| $p < \frac{c}{w}$ | $p > \frac{w}{A_H - A_L}$ | No discount |
| $p < \frac{c}{w}$ | $p < \frac{w}{A_H - A_L}$ | $\min\left[\delta_5, \delta_6\right]$ |

■

**Discount for the uninformed firm in competition**

**Lemma 28** *Profit by providing any discount for the uninformed firm to induce early purchase is always less than profit of sequential move for the supplier.*

**Proof.** Suppose the supplier charges discounted wholesale price $w-\delta$ for the early purchasing customers and charges $w$ for the late purchasing customers. When the informed firm orders early and the uninformed firm orders late, each firm's optimal order quantity is

$$\begin{aligned} q_{iH} &= \frac{A_H - w + 2\delta}{2}; q_{iL} = \frac{A_L - w + 2\delta}{2} \\ q_{eH} &= \frac{A_H - w - 2\delta}{4}; q_{eL} = \frac{A_L - w - 2\delta}{4} \end{aligned}$$

The supplier's expected profit is

$$\begin{aligned} \pi_{Sseq} &= \left(p \cdot \frac{A_H - w + 2\delta}{2} + (1-p) \cdot \frac{A_L - w + 2\delta}{2}\right)(w - \delta) \\ &\quad + \left(p \cdot \frac{A_H - w - 2\delta}{4} + (1-p) \cdot \frac{A_L - w - 2\delta}{4}\right) w \\ &= \left(\frac{\mu - w + 2\delta}{2}\right)(w - \delta) + \left(\frac{\mu - w - 2\delta}{4}\right) w \end{aligned}$$

When both informed and uninformed firm places order early, order quantity is

$$q_{iH} = \frac{3A_H - \mu - 2(w - \delta)}{6}; q_{iL} = \frac{3A_L - \mu - 2(w - \delta)}{6}; q_e = \frac{\mu - (w - \delta)}{3}$$

The supplier's expected profit is

$$
\begin{aligned}
\pi_{Ssim} &= \left(p \cdot \frac{3A_H - \mu - 2(w-\delta)}{6} + (1-p) \cdot \frac{3A_L - \mu - 2(w-\delta)}{6} + \frac{\mu - (w-\delta)}{3}\right)(w-\delta) \\
&= \frac{2}{3}\left(\mu - (w-\delta)\right)(w-\delta)
\end{aligned}
$$

The difference between first and second profit function ($\pi_{Sseq} - \pi_{Ssim}$) is $\frac{1}{12}(w+2\delta)(\mu - w - 2\delta)$ and it is always greater than zero as long as low type uninformed firm places order positive quantity ($A_L - w - 2\delta > 0$). *Therefore, it is never optimal for the supplier to announce a discount in the early period to induce the uninformed firm to order early.* ■

## A.2 Proof of Chapter 2

### A.2.1 Proof of Imperfect Information Game

**Proof of lemma 8.** (i) The informed firm and the uninformed firm's profit function for simultaneous move game is

$$
\begin{aligned}
\pi_{iH} &= \Pr(A = A_H/S = H)[(A_H - q_{iH} - q_e)q_{iH}] + \Pr(A = A_L/S = H)[(A_L - q_{iH} - q_e)q_{iH}] \\
\pi_{iL} &= \Pr(A = A_L/S = L)[(A_L - q_{iL} - q_e)q_{iL}] + \Pr(A = A_H/S = H)[(A_H - q_{iH} - q_e)q_{iH}] \\
\pi_e &= pE[\pi_e/A = A_H] + (1-p)E[\pi_e/A = A_L]
\end{aligned}
$$

where

$$
\begin{aligned}
E[\pi_e/A &= A_H] \\
&= \Pr(S = H/A = A_H)(A_H - q_{iH} - q_e)q_e + \Pr(S = L/A = A_H)(A_H - q_{iL} - q_e)q_e \\
E[\pi_e/A &= A_L] \\
&= \Pr(S = L/A = A_L)(A_L - q_{iL} - q_e)q_e + \Pr(S = H/A = A_L)(A_L - q_{iH} - q_e)q_e
\end{aligned}
$$

Equilibrium order quantity and profit for each player are as noted.

(ii) For the sequential move game,

$$
\begin{aligned}
\pi_{iH} &= \Pr(A = A_H/S = H)[(A_H - q_{iH} - q^*_{eH}(q_{iH}))q_{iH}] \\
+\Pr(A &= A_L/S = H)[(A_L - q_{iH} - q^*_{eH}(q_{iH}))q_{iH}] \qquad (A.2) \\
\pi_{iL} &= \Pr(A = A_L/S = L)[(A_L - q_{iL} - q^*_{eL}(q_{iL}))q_{iL}] \\
+\Pr(A &= A_H/S = H)[(A_H - q_{iL} - q^*_{eL}(q_{iL}))q_{iL}] \qquad (A.3)
\end{aligned}
$$

where

$$
\begin{aligned}
q^*_{eH} &= \arg\max_{q_{iH}} \Pr(A = A_H/S = H)[(A_H - q_{iH} - q_{eH})q_{eH}] \\
+\Pr(A &= A_L/S = H)[(A_L - q_{iH} - q_{eH})q_{eH}] \\
q^*_{eL} &= \arg\max_{q_{iL}} \Pr(A = A_L/S = L)[(A_L - q_{iL} - q_{eL})q_{eL}] \\
+\Pr(A &= A_H/S = H)[(A_H - q_{iL} - q_{eL})q_{eL}]
\end{aligned}
$$

Equilibrium order quantity and profit for each player are as noted. ■

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