ABSTRACT

Title of Dissertation:	EXPLORING EMBODIED MATHEMATICAL COGNITION THROUGH *FROM HERE TO THERE!*
	Nihal Katirci Doctor of Philosophy 2023
Dissertation directed by:	Dr. Caro Williams-Pierce, Assistant Professor, College of Information Studies

This dissertation seeks to investigate how digital gestures connect to students' mathematical understanding when playing *From Here to There!* (FH2T). This investigation explores the intersection of three fields, game-based learning, embodied cognition, and mathematics education. I used three studies which break down the different aspects of the overall research: Study 1 (The Game Interaction Study) covers the interaction between the game and the researcher; Study 2 (The Quantitative Gesture Study) is based on an analysis of the quantitative data gathered by the developers; and Study 3 (The Student Observations Study) focuses on collecting qualitative data and analyzing it through embodied mathematical cognition and failure and feedback lenses. These three studies illuminate the understanding of digital gestures and mathematical learning.

Keywords: Algebra, video game, mathematics learning, *From Here to There!* (FH2T)

EXPLORING EMBODIED MATHEMATICAL COGNITION THROUGH
FROM HERE TO THERE!

by

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This dissertation is dedicated to my grandmother, whom I lost this year and who was illiterate. I would like to thank my dear family members – my mom, my dad, my sisters, and my nephews – for their support, even though there is a distance and time difference between us.

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Chapter 1: Introduction

Overview

Mathematical skills are gatekeepers that either enable students to go to college and make successful careers or keep students away from educational and occupational opportunities relevant to mathematics (Cobb, 2017). One factor influencing the lack of mathematical abilities is tstudents' thoughts about mathematics, such as thinking that mathematics is boring and they don't like it (Yılmaz et al., 2010). Some authors state that making the process of learning mathematics fun and enjoyable can be the solution to engaging students during math lessons (Gresham, 2008; Rossnan, 2006; Strauss, 2016). Additionally, embodied cognitive science researchers claim that students' simple actions, like gestures (simple hand movements such as pointing), can influence students' mathematical learning (Alibali & Nathan, 2012; Cook et al., 2008). Moreover, game-based learning researchers assert that games are practical instructional tools to improve mathematical learning (Gresalfi & Barnes, 2015; Kebritchi et al., 2010; Williams-Pierce, 2016). Therefore, game-based learning and embodied cognition have a similar impact on students' mathematical learning through digital gestures (such as dragging and tapping, e.g., Dubé & McEwen, 2015; Sinclair & Heyd-Metzuyanim, 2014). Through this dissertation, I aim to analyze the combined impacts of game-based learning and embodied cognition within the context of mathematics education through digital gestures.

Statement of Problem

Video games can be a ground-breaking instructional apparatus for students to connect with and improve their mathematical understanding (Gresalfi & Barnes, 2015; Katirci et al., 2020; Kebritchi et al., 2010; Lee et al., 2012; Sinclair & Heyd-Metzuyanim, 2014; McLaren et al., 2017; Ottmar et al., 2015; Williams-Pierce, 2016). For this reason, some scholars use Game-Based Learning (GBL) (de Freitas, 2006; Garris et al., 2002, Squire et al., 2005) or Digital Game-Based Learning (DGBL) (Kiili, 2005; Prensky, 2003; Van Eck, 2006) frameworks to explore the adequacy of computer games for using them in learning. Researchers claim that mathematical reasoning is embodied, and the body's association with the environment can advance mathematical understanding (Alibali & Nathan, 2012; Cook et al., 2008; Goldin-Meadow et al., 2001; Nathan & Walkington, 2017; Nathan et al., 2016; Núñez, 2004; Walkington et al., 2014; Williams-Pierce et al., 2017). The focus of my dissertation is to examine how these two learning techniques — game-based learning and embodied cognition in mathematics education — can be combined to advance the comprehension of their expected advantages for mathematics education, especially regarding algebraic understanding.

There are some algebra-based digital learning tools that are used by teachers and students and analyzed by researchers. For instance, *GeoGebra* is an interactive software that has been heavily researched (e.g., Arbain & Shukor, 2015; Diković, 2009; Edwards & Jones, 2006; Hohenwarter & Jones, 2007; Williams-Pierce, 2019). Hohenwarter and Fuchs (2004) claim that GeoGebra supports students in exploring algebra in experimental and geometrical systems. Another popular educational game is *DragonBox12 +* (Cates, 2018; Katirci et al., 2020; Long, Y., & Aleven, 2017; Siew et al., 2016). Katirci (2017) states that *DragonBox* helped students gain more confidence in their algebraic problem-solving ability. In addition to GeoGebra and DragonBox12, there is another educational game, *From Here to There!* (FH2T), which has only been examined by the design team. According to the design team, FH2T helps students interact with algebraic expression elements (Ottmar et al., 2015). I posit that playing this educational video game through touchscreen technology also supports embodied mathematical cognition, and FH2T can effectively be used as a learning tool as has been pointed by design team.

My overall research question is — *How do digital gestures connect to students' mathematical understanding when playing From Here to There! (FH2T)?* This question explores the intersection of the two fields, game-based learning and embodied cognition in mathematics education, using the math game, FH2T, as a case study. I plan to answer this question using three studies that break down the different aspects of the overall research question: Study 1 (The Game Interaction Study); Study 2 (The Quantitative Gesture Study): Quantitative Data; and Study 3 (The Student Observations Study): Case Study.

Background and Studies' Frame

Games can be examined through different theoretical frameworks or lenses (e.g., MDA, Hunicke et al., 2004; Fernández-Vara, 2009). Hunicke et al. (2004) introduced the Mechanics, Dynamics and Aesthetics (MDA) framework as a way for designers, researchers, and scholars to understand games. In other words, they aim to bridge the two sides – design/develop and research/critique. However, in their schema

(Figure 1.1), they only cover the designer's and the player's role, and the role of the researchers is missing. The authors state that "games are created by designer/teams of developers and consumed by players" (p. 1). I extend their schema by adding the researcher's role as a form of critique of the game (Figure 1.2).

Figure 1.1: Designer-Game-Player schema by Hunicke et al. (2004, p.1)

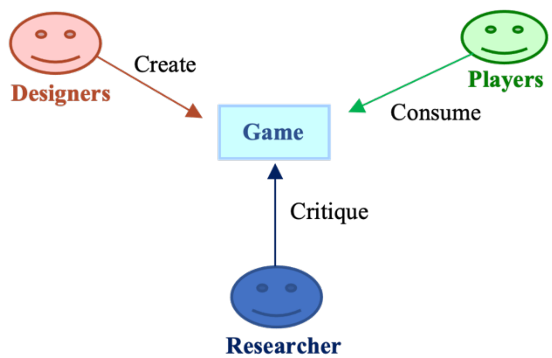


Figure 1.2: Researcher's role in Designer-Game-Player schema

As a researcher, I investigate this extended schema's elements using FH2T and perform three interrelated research studies to unpack the different relationships between the person (designer, consumer, researcher) and the game. In the literature review, I offer an overview of the publications of the developers in two parts: designers create the game (e.g., Ottmar et al., 2015) and designers collect and analyze quantitative data on players consumption of the game (e.g., Ottmar et al., 2015; Hulse et al., 2019; Chan et al., 2021). Figure 1.3 illustrates these two parts.

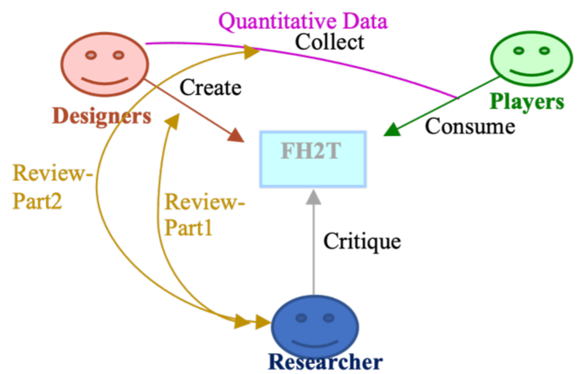


Figure 1.3: Literature Review

After reviewing articles related to FH2T, I develop three studies (Figure 1.4) to investigate three separate constituents of the schema: The First Study (The Game Interaction Study) covers the researcher's emergent mathematical game play experience — the researcher's critique of the game; The Second Study (The Quantitative Gesture Study) will focus on that the ways embodied cognition can give us insight into mathematical understanding through quantitative representation — the researcher critique quantitative data on players consumption of the game; and The Third Study (Student Observations Study) will analyze the players' emergent experiences while engaging with the game — the researcher collects and critiques qualitative data on players consumption of the game. To make clear how these three studies relate to each other, I developed a representation (Figure 1.4) that is built upon the Hunicke et al.'s (2004) illustration.

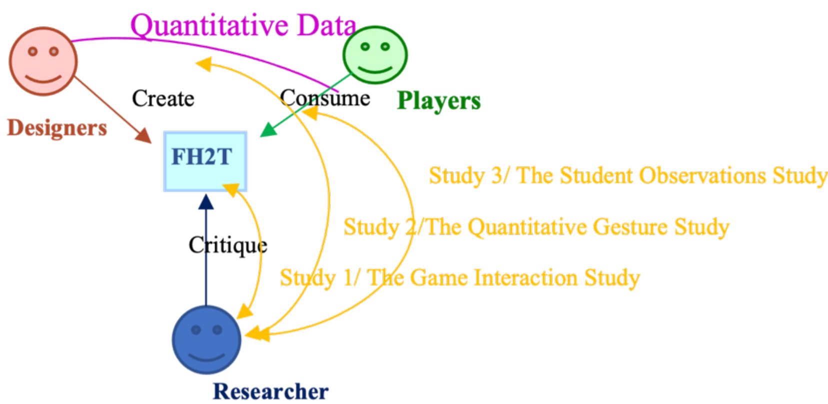


Figure 1.4: Studies' Organization

I expound on each study separately. The Game Interaction study aims to explore the game through the researcher's playing and then analyzes the interaction between the game and the researcher. In the Quantitative Gesture Study, through the embodied mathematical cognition perspective, I investigate the quantitative data gathered and represented by the developers. In the Student Observations Study, through the embodied mathematical cognition perspective and lenses of failure and feedback, I observe students when they are playing FH2T.

Research Questions

Considering the purpose of the studies, I developed the following answerable research questions that put together answer overall research question:

Overarching:

- How do digital gestures connect to students' mathematical understanding when playing FH2T?

Study 1 (The Game Interaction Study):

- How do the implication and affordances offered through playing FH2T match with Game-Based Learning's structures?

- How do failure and feedback manifest in mathematical play within playing the game from the researcher's perspective?
- Do the learning outcomes of the game endorse the Common Core State Standards of mathematics required by the state of Maryland?

Study 2 (The Quantitative Gesture Study):

- Do students' digital gesture clusters within playing FH2T affect their keep playing or not?
- What conditions affect their decision to keep playing or not?

Study 3 (The Student Observations Study):

- What is the learner's mathematical gameplay experience in playing FH2T?
- How do failure and feedback manifest in mathematical play within playing the game through the learner's perspective?
- What sort of physical gestures do they use to explain what they are doing with their digital gesture?

The Game: *From Here to There!* (FH2T)

In this study, I use *From Here to There! (FH2T),* a game designed by the Graspable Math Team (Founders: David Landy – Professor, Erik Weitnauer – Software Developer, Erin Ottmar – Professor, and Developers: David Brokaw, Thad Martin, Christian Achgill, Dan Manzo). The designers defined FH2T as "a self-paced interactive application that introduces students to mathematical content through discovery-based puzzles" (Ottmar et al., 2015, p.1793). FH2T is a dynamic mathematics game designed for students to learn algebra through perceptual intervention. FH2T has 14 worlds placed on a symbolic tree (see Figure 1.5) and each world contains 18 puzzles. The different worlds focus on specific mathematics/algebraic concepts ranging from addition to linear equations. Learners start with simple mathematical content and construct knowledge and skills throughout the game (See Table 1.1 - Worlds and Contents). In the First Study's findings section, I analyze the game in more detail through the Game-Based Learning framework.

Figure 1.5: Worlds on Tree (© Graspable Math)

Table 1.1: Worlds and Contents

World Number	Contents
World 1	Addition
World 2	Multiplication
World 3	Order of operations + and ×
World 4	Subtraction and negative numbers
World 5	Mixed Practice of + and -
World 6	Division
World 7	Order of operations
World 8	Equation + and -
World 9	Inverse operations + and -
World 10	Distributions
World 11	Factoring
World 12	Equation +, -, ×, and ÷
World 13	Inverse operation
World 14	Final Review

Definition of Terms

Game: According to Salen and Zimmerman (2003), 'A *game* is a system in which players engage in an artificial conflict, defined by rules, that results in a quantifiable outcome.' (p. 80)

Algebra: From the Dictionary (n.d.), 'The branch of mathematics that deals with general statements of relations, utilizing letters and other symbols to represent specific sets of numbers, values, vectors, etc., in the description of such relations.'

Mathematical Play: According to Williams-Pierce (2019), 'mathematical play is voluntary engagement in cycles of mathematical hypothesis with occurrences of failure.' (p. 591)

Educational Game: Commercial or non-commercial game which is designed for educational purpose instead of just entertainment.

Failure: From the Dictionary (n.d.), 'nonperformance of something expected' and 'an act of failing.'

Failure and Feedback in the context of educational game: Failing and getting feedback is a cycle in the context of video game that players learn the game and the topic through their actions (Williams-Pierce, 2019). Simply, it is an action (Players' behavior)- reaction (Game's systematic feedback) cycle.

Digital-Touchscreen Gesture: The simple hand motions to control or interact with digital-touchscreen devices. For example, *tapping:* "briefly touching the surface with fingertip"; *dragging:* "moving fingertip over surface without losing contact" (Villamor et al., 2010, p. 1).

Relevance and Contributions

My work here is incorporated from the following published materials:

1. Katirci, N., Shokeen, E., & Williams-Pierce, C. (2022, April). *From Here to There!: Game-Based Learning*. Submitted to the 2022 American Educational Research Association Annual Meeting and Exhibition. https://aera22-aera.ipostersessions.com/Default.aspx?s=B7-0F-51-C2-6B-39-0F-A7-52-42-22-74-C2-9C-B7-32
2. Katirci, N. (2021, Nov). *Game-Based Learning: From Here to There!* Submitted to the 2021 Learning Sciences Graduate Student Conference.

Organization of the Dissertation

This chapter (Introduction) includes general information about the dissertation, the statement of the problem, the research questions, and the definition of terms. Chapter 2 provides the background literature to the dissertation. The literature review is presented in three subsections; using technology in mathematics education, embodied mathematical cognition, and the intersection of embodied mathematical cognition and technology. Chapter 3, Chapter 4, and Chapter 5 cover the three studies: The Game Interaction Study, The Quantitative Gesture Study, and The Student Observation Study, respectively. Chapter 6 addresses discussion, opportunities for future research, and a conclusion.

Chapter 2: Literature Review

Overview

This literature review examines how two learning methods — game-based learning and embodied cognition in mathematics education — can be integrated to advance our understanding of their potential benefits for mathematics learning. Mathematics education is the primary area of my proposal (illustrated in Figure 2.1) and it infuses into the other two areas: educational technology and embodied cognition. The intersection of these three areas — mathematics education, educational technology, and embodied cognition — is a specific area of specialization in my research. This dissertation's information comes from two fields: Human-Computer Interaction (HCI) and Learning Sciences (LS). My overall research question — *How do digital gestures when playing FH2T connect to students' mathematical understanding?* — belongs to the intersection of these two fields. It belongs in HCI because digital/touchscreen gestures refer to the embodied interaction between students and touchscreen technologies such as tablets and smartphones. It belongs in LS because the focus is on the process of students' mathematical learning through playing a video game.

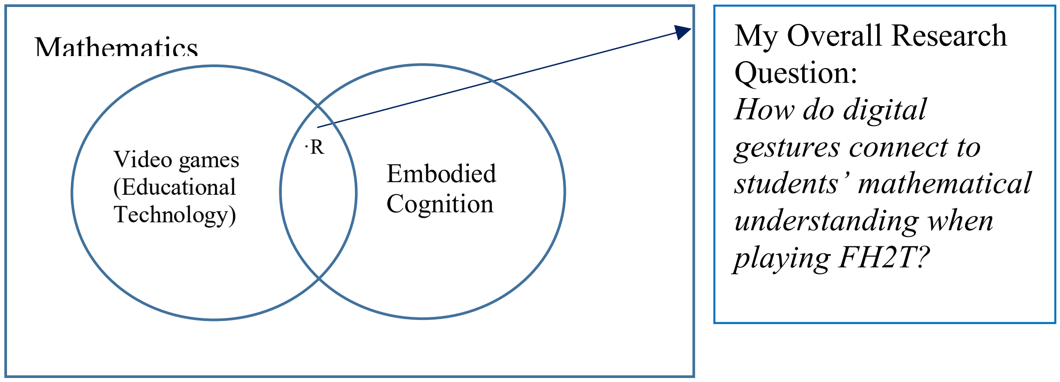


Figure 2.1*: The intersection of three interest fields*

Creswell (2014) lists three reasons for doing a literature review: "to present results of similar studies, to relate the present study to an ongoing dialogue in the literature, and to provide a framework for comparing results of a study with other studies" (p. 51). Through this section, I will present related studies of my research interest, identify a gap in the literature, and introduce the reasoning behind my research question. I use different analyses like thematic (Maguire & Delahunt, 2017) and chronological (Hart, 1998) analysis through the narrative synthesis in different sections and the referral (snowball) technique (Biernacki & Waldorf, 1981) to further explore the literature.

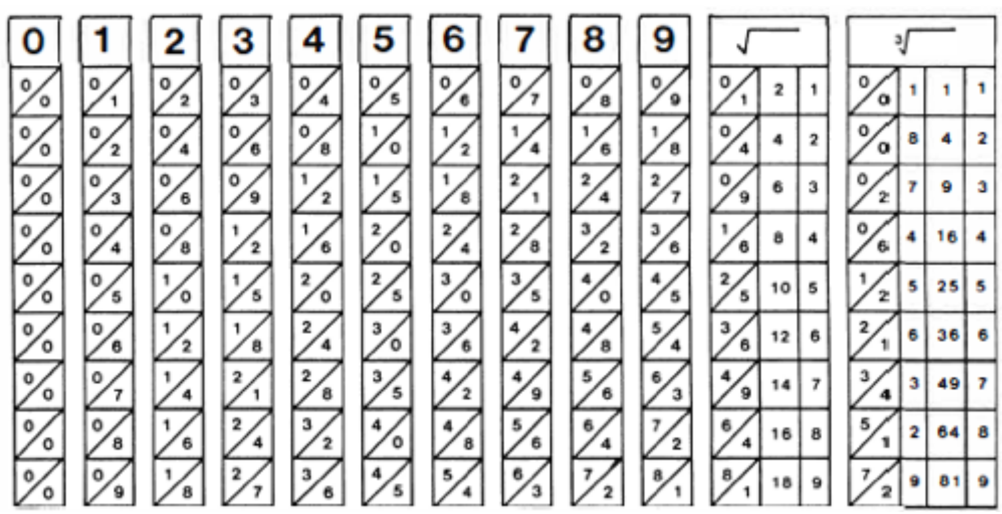
My inclusive theme is mathematics learning: when I search the literature, I always include "mathematics learning/education" as a keyword. Then, in the short history of using computers in the mathematics education section of the paper, I use both chronological and thematic analysis. The chronological analysis helps me to categorize literature by its dates. Thematic analysis helps me to identify different instructional methods related to mathematics learning (e.g., Programming, drill-practice; Computer Based Training (CBT); Internet-based training (IBT); e-learning;

Social software, free and open content). Additionally, in the subsequent sections, thematic analysis through the snowball technique is applied. For example, I use Garris et al.'s (2002) paper for the literature about game-based learning in mathematics education, then use Hulse et al.'s (2019) article for the publications about FH2T, Wilson's (2002) piece for embodied cognition, Goldin-Meadow et al.'s (2001) study for embodied mathematical cognition, and Sinclair and Heyd-Metzuyanim's (2014) work for embodied mathematical cognition and touchscreen technology. I use them as a starting point in the snowball technique because these articles are directly related to my research.

This literature review includes three main sections (Using Technology in Mathematics Education, Embodied Mathematical Cognition, and Embodied Mathematical Cognition and Technology) and their subsections. In the first section and subsection, I review how technology and video games have been used as instructional tools in mathematics education (Figure 2.1, left set in the Venn Diagram). In the second section, I investigate how researchers describe embodied mathematical cognition (Figure 2.1, right set in the Venn Diagram). The third section examines the literature that combines video games and embodied mathematical cognition (Figure 2.1, the intersection of two sets). As a subsection, I review the intersection of embodied cognition and touchscreen games. Lastly, I introduce my research question (Figure 2.1, intersection-·RQ).

Using Technology in Mathematics Education

When we hear the word technology, we probably think of computers, smartphones, and machines. However, technology is not limited to electrical and computerized systems. For instance, Napier's bones (Figure 2.2) are used as a calculating instrument (Aspray, 2000; Yuriana & Suwardi, 2018). According to Kumar et al. (1999), one element of technology is a "physical component which comprises items such as products, tools, equipment, blueprints, techniques, and process" (p. 82). Given this definition, Napier's bones would count as a technology. Napier's bones are also a good example of how tools were used in mathematics and mathematics education throughout history (e.g., Peeples, 2007). However, while I am aware that technology can refer to analog devices, in this paper, I focus on using computers/computing tools/software as technological tools in mathematics education.



Source: Aspray (2000, p. 18).

Figure 2.2: A Modern set of Napier's Bones

Short History of Using Computers in Mathematics Education

There are five phases in the history of using computers in education (Leinonen, 2005):

1. Programming/drill and practice - late 1970s and early 1980s, which I have modified to be *Before and during the early 1980s* because there are some examples before 1970s.

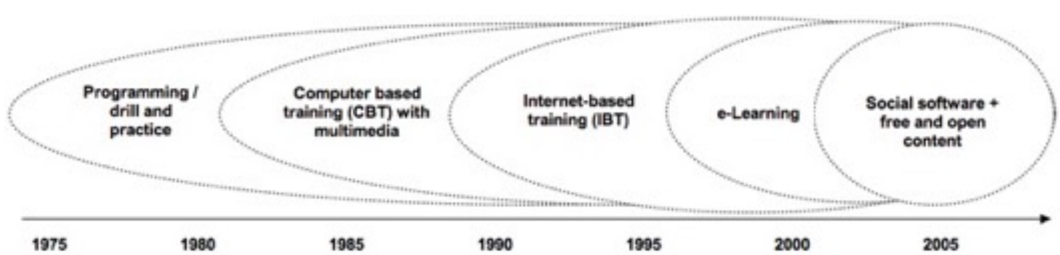
2. Computer-based training (CBT) with multimedia - late 1980s and early 1990s.

3. Internet-based training (IBT)- early 1990s.

4. e-learning - late 1990s and early 2000s

5. Social software + free and open content – late 2000s (Figure 2.3).

In this section, I focus on these five phases in mathematics education. To build upon Leinonen's phases, I have added a new phase: Video Games. Using video games for educational purposes became popular after Leinonen's (2005) categorization (e.g., Quest Atlantis - Barab et al., 2005), and it has had an influential effect on mathematics education, so it should have a place in nowadays' classification of computer use in education. In the next section, I review video games in mathematics education.



Source: Leinonen (2005).

Figure 2.3: A history of computers in education

Phase 1: Before and during the early 1980s

Programming and drill-practice materials were the first computerized instructional tools used in education. Hartley (1974) described programming learning as individualized instruction that builds on logically sequenced small steps; students follow this learning actively and at their own pace. Although Leinonen's (2005) phases started in the late 1970s, there are various programming learning examples before the 1970s. For instance, Pressey's testing machine (Pressey, 1927, as cited in Skinner, 1958) (Figure 2.4) and Skinner's (1958) programmed instruction meet Hartley's (1974) description.

In mathematics education, programming was used before the 1980s. Skinner (1958) mentioned that instructional materials were designed for teaching mathematics, such as arithmetic, the operations of addition, subtraction, multiplication, and division. The success of the machine depended on these materials and also when and how students used them. Furthermore, Stolurow (1963) reported that, in a study with underachieving students using programmed machines to learn arithmetic, they preferred to use programmed machines instead of traditional instruction. Programming materials were one of the alternatives for traditional materials in mathematics learning.

Source: Skinner (1986).

Figure 2.4: Pressey's machine

The second part of Leinonen's (2005) first phase is drill-and-practice. According to Lim et al. (2012), drill-and-practice "is a method of instruction characterized by the *systematic repetition* of concepts, examples, and practice problems" (p. 1040). There are both paper-based and computer-based drill-and-practice programs (Landeen & Adams, 1988); a paper-based form of drill-and-practice exercises is called a worksheet. Hence, in the computer-based part, drill-and-practice programs are designed as courseware programs for learning (Streibel, 1985). Moreover, Hasselbring et al. (1988) claimed that drill-and-practice activities could be used for developing mathematical automaticity. Programmed and drill-practice instructions were used to help students learn and continue to be used today. Video games also cover the characteristics of programming and drill-practice because they involve logically sequenced small steps (Hartley, 1974) and systematic repetition (Lim et al, 2012).

Phase 2: Late 1980s- Early 1990s: Computer-Based Training (CBT) with Multimedia

The second phase of the Leinonen's (2005) classification was computer-based training with multimedia. Kulik and Kulik (1991) also categorized computer-based instruction into three types of applications: computer-assisted instructions in the form of microworlds (e.g., Edwards, 1991; Papert, 1980; Thompson, 1992), tutorials (e.g., Tilidetzke, 1992), and simulations (e.g., Van Eck & Dempsey, 2002); computer-managed instructions, in which computers evaluate students' test performance (e.g., Winters et al., 1993); and computer-enriched instructions (e.g., Mitra & Steffensmeier, 2000), such as computers serving as problem-solving tools.

The first type of computer-based training is computer-assisted instruction which is in the form of microworlds. The term microworld was used by Papert (1980), who designed the Turtle, a computer-controlled cybernetic animal in the LOGO (programming language) environment. Papert (1980) defined microworld as a place "where certain kinds of mathematical thinking could hatch and grow with particular ease" (p. 125). Other researchers also used LOGO programming and microworlds in teaching different mathematics topics such as geometry (Lehrer et al., 1988; Olive, 1991), fractions (Harel, 1990), and problem-solving (Nastasi et al., 1990). These studies show that microworlds and computer-assisted instruction can be used to teach different mathematics topics and improve learning.

The second type of computer-based training is computer-managed instruction. According to Baker (1978), the three themes of computer-managed instruction (CMI) are individualization, behavioral objectives, and educational technology, which are similar to Skinner's (1958) programmed instructional components (behavioral objectives, small frames of instruction, self-pacing, active learner response to an inserted question, and immediate feedback). For example, individualization meets self-pacing and active learner components, behavioral objectives are in both, and educational technology covers machines and giving immediate feedback as a characteristic of teaching machines. Moreover, CMI programs are designed to evaluate and monitor learners' improvement, locate and manage resources, and record and generate reports about learners' performance (Wee et al., 2012). Nowadays, Learning Management Systems (LMSs) are used as computer-managed instructional tools to follow students' progress.

The third type of computer-based training is computer-enriched instruction. Kulik and Kulik (1991) described computer-enriched instruction (CEI) as when a "computer (a) serves as a problem-solving tool, (b) generates data at the student's request to illustrate relationships in models of social or physical reality, or (c) executes programs developed by the student" (p. 79). See et al. (2010) created and used a CEI program to teach the central limit theorem (CLT) in a biostatistics course through a tutorial, story, simulated learning, summary, Q&A (question and answer), exercise, and exit pages. According to Kulik and Kulik's (1991) description, See et al.'s (2010) program would serve as both a problem-solving tool and data generator. See et al. (2010) also designed a 2 x 2 cross-over method (participants in two groups receive materials in a different sequence) to evaluate the timing of using CEI in the course. They found that CEI was good for previewing content before teaching the topic.

All three instructions, computer-assisted, computer-managed, and computer-enriched, are part of computer-based training to help students learn. These methods are also used in video games, but game designers denominate them differently. For instance, each video game has its own microworld (computer-assisted environment). Each video game also takes advantage of the computer-managed system to show the leaderboard or give individual rewards to each player differently.

Phase 3: Early 1990s: Internet-Based Training (IBT)

After the internet became popular in the 1990s, instructional designers talked about using it to deliver instruction. According to Tsai and Machado (2002), e-learning, online learning, web-based learning, and distance learning had been used interchangeably. Tsai and Machado (2002) differentiated these terms by how learning materials were delivered and stated that "web-based learning is associated with learning materials delivered in a Web browser, including when the materials are packaged on CD-ROM or other media" (p. 2). Moreover, Driscoll (1997) defined internet-based and/or web-based training as "any skill or knowledge transfer that takes place using the World Wide Web as the distribution channel" (p. 5). These two definitions show that there is no sharp distinction between the terminology of web-based or internet-based training, and they are used interchangeably.

There are some examples of web-based trainings in mathematics education. Wang et al. (2004) designed a Web-based Mathematics Education (WME) framework and they claimed that "*The Mathematics Education Markup Language* (MeML) was the centerpiece of WME. MeML aims to provide an effective and expressive means for authoring and to deliver mathematics education content on the Web" (p. 1).

Furthermore, Baki and Güveli (2008) developed web-based mathematics teaching (WBMT) materials on functions and evaluated these materials' effectiveness for high school students. According to their results, WBMT improved students' learning in the experimental group (Baki & Güveli, 2008). Consequently, researchers (Baki & Güveli, 2008; Tsai & Machado, 2002; Wang et al., 2004) have upheld that internet-based instructions effectively teach mathematics. Internet-based training has been used to help students learning and will continue to be used. Educational video games also cover internet-based training characteristics because they involve learning materials delivered through a Web browser or CD-ROM as Tsai and Machado's (2002) description.

Phase 4: Late 1990s-Early 2000s: e-learning

E-learning is the fourth phase of Leinonen's (2005) classification of using computers in education. Tsai and Machado (2002) claimed that "E-learning is mostly associated with activities involving computers and interactive networks simultaneously" (p. 2). Hence, e-learning's definition is not so different from that of internet-based training: web-based, asynchronous, distance education (Smith & Ferguson, 2005). E-learning students are self-motivated, enjoy interacting online with peers, and are knowledgeable of the e-learning format (Roblyer, 1999).

In mathematics education, according to Smith and Ferguson (2005), dropout rates in mathematics e-learning courses are higher than in non-mathematics courses. To create an ideal, math-friendly, e-learning environment, they suggested these environments should have a digital whiteboard for the instructor to draw diagrams, formulas, and explanations. Students can easily reach the board and use it. Hence, e-learning environments should also include real-time chat opportunities (Smith & Ferguson, 2005). This study shows that students benefit from social interaction between themselves and their teachers when learning mathematics in an e-learning environment.

Phase 5: Late 2000s: Social Software, Free and Open Content

The final phase in Leinoen's (2005) categorization includes both social software as blogs, and free and open content as *OpenCourseWare.* Blogs are informal in this category, but open content is used as an educational resource. For instance, UNESCO defines Open Educational Resources as "technology-enabled, open provision of educational resources for consultation, use, and adaptation by a community of users for non-commercial purposes" (UNESCO, 2002). It became popular when the Massachusetts Institute of Technology (MIT) announced that it would make all course materials available free for the community through their program *MIT OpenCourseWare* (Pirkkalainen & Pawlowski, 2010). Then in 2012, MIT and Harvard universities launched edX (edX.org) as a massive open online course (MOOC) provider, which generally covers college-level courses (Breslow et al., 2013).

For the high school level in mathematics education, Canessa and Pisani (2013) preferred to use HOOC as an acronym for High School Open Online Courses. Their study aimed to reinforce Italian high school students' knowledge of science and mathematics and facilitate students' entrance to university. The high school students did not watch new courses; they could re-watch the same lessons they received in their own classroom online, at their own place and pace. Researchers called this an open online course because the webpage (www.opendante.com) and videos are available for the public in Italian. The study results showed that students who watched online videos and attended the class scored higher than other students who only participated in the class. Moreover, students stated that HOOC lectures helped them with their homework (Canessa & Pisani, 2013).

In the above sections, I summarize Leinonen's (2005) five phases of using technology in education with examples in mathematics education. It is crucial to understand how computers in education have evolved and why educators prefer tablet-based video games in education. I plan to investigate using video games in mathematics education through this proposed study. The next section summarizes using video games (and the game, FH2T) in mathematics education, which I consider to be the sixth phase in using technology in education.

Using Video Games in Mathematics Education

Esposito (2005) defined a video game as "a *game* which we *play* thanks to an *audiovisual apparatus,* and that can be based on a *story*" (p. 2). Similarly, according to Salen and Zimmerman (2003), "a *game* is a system in which players engage in an artificial conflict, defined by rules, that results in a quantifiable outcome" (p. 80). The definition shows that players are fascinated by video games. Video games can be powerful instructional tools to engage students and improve their learnings in mathematics (Gresalfi & Barnes, 2015; Kebritchi et al., 2010; Williams-Pierce, 2016). For this reason, some scholars use Game-Based Learning (GBL) (de Feritas, 2006; Garris et al., 2002, Squire et al., 2005) or Digital Game-Based Learning (DGBL) (Kiili, 2005; Prensky, 2003; Van Eck, 2006) frameworks to investigate the effectiveness of video games. I explain GBL and DGBL in more detail in the third chapter and investigate FH2T through the GBL framework in the third chapter (findings of the First Study).

Studies show that video games affect students' mathematical understanding (Kebritchi et al., 2010, McLaren et al., 2017). *DimensionM^{TM}* is one example of how computer games can be used in high school students' mathematics learning (Kebritchi et al., 2010). Kebritchi and colleagues'(2010) impact research took place in an urban high school in the United States with 193 students using quantitative (motivation surveys, school district-wide benchmark exams) and qualitative (interview) instruments. Their quantitative test results for mathematics achievement were significantly higher for the treatment group who played the game and attended the mathematics classes than the control group who just participated in the classes. Their interview results showed that the game positively affect students' motivation and achievement, the game positively changed their mood; the game was exciting, challenging, and attractive.

Another computer game used for mathematical achievement is *Decimal Point,* a single-player educational game designed for middle school students (McLaren et al., 2017). McLaren et al.'s (2017) impact research took place in two middle schools in the U.S. among 153 6th-grade students. It used experimental (game and non-game conditions) and quantitative (pre-test, post-test, and delayed post-test) instruments. Their results showed that the game increased student learning about decimals in game-playing students more than in the non-game group students (McLaren et al., 2017).

Both *DimensionM*^{TM} and *Decimal Point* can be categorized as computer games, and the studies about them showed that computer games are useful tools in mathematics learning. However, there are also many web-based games that students can play on their computers or other devices. For instance, coolmathgames.com and coolmath4kids.com are two sibling sites with mathematics games, which are grouped into several categories like strategy, skills, numbers, logic, and trivia. Zhang (2015) investigated the relationship between interest in online mathematics games based on these two sibling websites and fourth-grade students' academic performance and found a significant negative correlation between them through impact research. This finding shows that the design and the quality of games are important. Therefore, desired learning is not guaranteed in mathematics video games.

Furthermore, there are other video game devices, consoles such as Sony PlayStation, Nintendo Switch, Nintendo Wii, and Microsoft Xbox, which are generally used for entertainment but can be leveraged for educational purposes. Researchers have investigated the learning in some games designed for entertainment and played through a computer or platforms, such as Lineage (Steinkuehler, 2006) and World of Warcraft (Nardi & Harris, 2006). In addition to games designed for entertainment but used for learning, some game designers have designed their educational games for specific topics by using these game platforms (e.g., Lee et al., 2012; Williams-Pierce, 2016). For instance, *Xdigit* (Lee et al., 2012) is a kinetic arithmetic game to improve mathematical learning through the motion-sensing controller, Microsoft Kinect for the Xbox, and *Rolly's Adventure* (Williams-Pierce, 2016) is designed to support fractions learning through the game on PlayStation.

Games for tablets and other mobile devices have also been researched. These devices are cheaper and easier to implement in a classroom. For instance, *TouchCounts* (Sinclair & Heyd-Metzuyanim, 2014) and *DragonBox 12+* (Katirci et al., 2020) are examples of tablet-based games selected and investigated by researchers. I explain these studies in the last section of this chapter in more detail because researchers did not just focus on games; they also examined the gestures while playing a game and mathematical communication between students and teachers. In this dissertation, I am also analyzing another computer game, *From Here to There!* (FH2T) to further contribute to the literature on gaming in mathematics education.

Using FH2T in Mathematics Education

As mentioned in the introduction, *From Here to There! (FH2T)* was developed by the Graspable Math research team. Each problem in FH2T starts with a given form of an expression and asks for the goal form of the expression. The players must convert the expression from the starting form (here) to the goal form (there). The players solve these expression or algebra problems to collect clovers, unlock new problems, and pass levels. In Ottmar et al.'s (2015) paper, they present the initial findings of the research on the game. In their pilot study, 110 middle school students (6^{th}, 7^{th}, and 8^{th} grade) participated and solved FH2T's problem at their own pace. The pilot study contains both retrieval practice and fluid visualization versions of the game. Students need to remember and write the correct form of the algebraic and arithmetic expressions in the retrieval practice mode. In the fluid visualization mode, the players just tap to the sign to get the correct form of the expression. According to the pilot study result, the mathematical understanding of the students improved by the end of the study (Ottmar et al., 2015).

In another study, Ottmar and colleagues (2015) divided players into three conditions: a control group, a retrieval practice group, and a fluid visualization group. In this study, 85 middle school students (6^{th} and 7^{th} grade) from 5 classes participated. The classes were randomly assigned as control and intervention classes. Then, within the intervention classes, students were randomly assigned retrieval practice and fluid visualization conditions. According to this study's results, there is a significant difference between the fluid visualization group and both the retrieval practice and control group. Moreover, Ottmar et al. (2015) stated that "these results … suggest promise for tablet-based technologies for teaching abstract algebraic content" (p.1797). In my study, I combine the embodied cognition development feature of tablet-based technologies and the mathematical understanding enhancement by playing FH2T. I explain embodied mathematical cognition in the next section.

Embodied Mathematical Cognition

Before defining embodied mathematical cognition, I will first explain embodied cognition through the views of two scholars, Shapiro (2011) and Wilson (2002), and illustrate the concept with examples. Then, in the following section, I review studies of embodied mathematical cognition (see Figure 2.1 - the right set in the Venn Diagram).

Embodied Cognition

Embodied cognition is a theory in cognitive science claiming that "states of the body modify states of the mind" (Wilson & Golonka, 2013, p.1). In other words, cognition is rooted in the body's potential interaction with the physical world (Wilson, 2002). Cognitive scientists think that there are many co-existing views of embodiment (Shapiro, 2011; Wilson, 2002).

Shapiro (2011) presents his three themes of embodiment: Conceptualization, Replacement, and Constitution. In brief, conceptualization refers to the concepts in which an organism depends on understanding the surrounding world based on the type of body it has. So, if organisms differ in their bodies, they also differ in how they understand the world. Shapiro (2011) cites Varela et al.'s (1991) example of color vision to explain their ideas; we experience color regarding how our color detection systems interact with light (Varela et al., 1991, as cited in Shapiro, 2011). This example shows that not just the color of the object but also the limitation of organisms' vision affects how they see. This theme is important because it explains that the concepts a human can gain are finite and limited by the properties of a human organism's body.

Through the second theme, Replacement*,* Shapiro (2011) suggests that the interaction between an organism's body and its environment replaces the need for the world's computational processes. Therefore, cognition can be explained without the appeal to computational processes. Shapiro (2011) uses the robotic works of Brooks (1991) as an example because robots "have bodies and experience the world directly – their actions are part of a dynamic with the world, and the actions have immediate feedback on the robot's own sensations" (Brooks, 1991; p. 1227, as cited in Shapiro, 2011; p. 139).

In the third theme, Constitution*,* the elements of cognition are beyond the mind; the body and world play not just a causal role but also a constitutive role in cognitive processing. Shapiro (2011) explains this theme with the help of O'Regan and Noë's (2001) sensorimotor theories of perceptual experiences like visual experiences. For instance, the shape of the object (e.g., circle, sphere); the color of the object (e.g., red, green), and the kind of the object (e.g., ball, apple) look to have through changes in perspective. This theme emphasizes that we experience the world in the manner in which we do on the grounds that we get familiar with the different possibilities between how we move and how we perceptual data changes thus.

Wilson (2002) took a different approach and stated six claims of embodied cognition:

1. Cognition is situated.
2. Cognition is time pressured.
3. We off-load cognitive work onto the environment.
4. The environment is part of the cognitive system.
5. Cognition is for action.
6. Offline cognition is body-based.

When cognition is situated, activities take place in the context of the environment*.* Wilson (2002) promoted this claim through real-life examples such as driving. Moreover, Lave (1988) provides a strong example of situated cognition through mathematics education, examining how arithmetic was used in situ in the Adult Math Project. For example, while shopping, cognitive activities like deciding to buy a product take place in the context of a real-world environment, in a shopping store. It might be challenging to create this kind of situation for every student in a real-world environment and build the connection between mathematics education.

However, game characteristics (e.g., fantasy) may make it easier to create different situations, and games might help students take advantage of situated cognition. Then, they can improve their mathematical cognition..

As humans, we need to solve our problems in real-life situations, and time-pressured environments influence our performance and decisions. Wilson (2002) explained this circumstance through her second claim: cognition is time pressured. Cognition is time pressured and can extend beyond real-world applications and be explored in game-based learning as well. Kirsh and Maglio (1994) studied the actions of players while playing the game Tetris. In Tetris, players need to select the best place to put a shape (called Zoid) before touching any other Zoids that have already landed. Speed and the right choice of actions help players to continue the game. This study illustrates the time pressure component of cognition.

Wilson’s (2002) third claim is that we off-load cognitive work onto the environment by using equipment in the environment. According to Risko and Gilbert (2016), cognitive offloading means the “use of physical action to alter information processing requirements of a task so as to reduce cognitive demand” (p. 677). For instance, we use different kinds of calendars (paper-based or digital) to avoid or offload cognitive load work from our minds (Wilson, 2002). Offloading cognitive work onto the environment also applies to game-based learning. For example, in the game Tetris, players do not need to think about which shape of the Zoid fills the blank because they can change the direction of the Zoid while playing and offload some cognitive work onto the game. Therefore, I posit that we can learn and practice how we offload cognitive work onto the environment through playing games.

The fourth claim, the environment is part of the cognitive system, assumes that the environment also has a function in cognitive activity. Wilson (2002) stated that “The forces that drive cognitive activity do not reside solely inside the head of the individual, but instead are distributed across the individual and the situations as they interact” (p. 630). For example, Thomas and Lleras’ (2009) explain the environmental effects on the participant’s cognitive activity. Thomas and Lleras (2009) have studied how participants solve Maier’s two-string problem (to tie two strings hanging from opposite ends of the room) by using only their arms (swing group and stretch group) and the objects that are given (a wrench, a paperback book, two dumbbells, and a plate). The results showed that the participants who were directed to swing their arms were more likely to solve Maier’s problem. This study ties into Wilson’s (2002) fourth claim because, to solve this problem, participants interact with the objects as well as those who told them to swing their arms. I consider this claim a crucial component of the learning environment; teachers should consider creating a learning environment or choosing instructional tools like educational games for our students because the social and physical environment influences how the students learn.

The fifth claim is that cognition is for action. According to Wilson (2002), “the function of the mind is to guide action, and cognitive mechanisms such as perception and memory must be understood in terms of their ultimate contribution to situation-appropriate behavior” (p. 626). I am particularly interested in this claim because researchers found that gestures are representational or simulated actions

(Hostetter & Alibali, 2008; Novack & Goldin-Meadow, 2017). In the next section, I will discuss in more detail how gestures are related to mathematical understanding.

According to Wilson (2002), cognitive processing is tied in some way to bodily processes of immediate sensation and motor control, a process she refers to as offline cognition is body-based. Moreover, cognitive processing involves mental imagery and working, episodic, and implicit memories (Wilson, 2002). Thomas and Lleras' (2009) study is a great example of this claim because participants use their bodies to solve the problem after being directed to do specific body movements (stretching and swinging). Based on arguments that body movements affect thinking, I argue that playing video games helps to improve motor skills and mental imagery ability.

In summary, Shapiro (2011) and Wilson (2002) both broke down embodied cognition differently. Still, both centered on the environment and pointed out that body-environment interaction and perceptual-sensual experiences have roles in cognitive decisions. In the next section, I review the influence of body-environment interaction and perceptual experiences on mathematical understanding.

Embodied Mathematical Cognition

Lakoff and Núñez (2000) argue that mathematical ideas (e.g., numbers, arithmetic operations, set theory, algebra, and infinity) could be understood as image schemas, aspectual schemas, conceptual metaphors, and conceptual blends grounded in normal language usage and the sensory-motor system. Researchers refer to embodied mathematical cognition in different ways; Lakoff and Nunez (2000) use the terminology the theory of embodied mathematics in their analysis, while other scholars use terminology like Embodied Mathematical Imagination and Cognition (EMIC) (Nathan et al., 2016) or Grounded and Embodied Mathematical Cognition (GEMC) (Nathan & Walkington, 2017). Despite naming differences, each claim is similar to Wilson's (2002) and Shapiro's (2011) because they both focus on body-environment interaction. In essence, mathematical thinking is embodied, and the body's interaction with the environment can promote mathematical understanding (Cook et al., 2008; Nathan & Walkington, 2017; Nathan et al., 2016; Núñez, 2004; Williams-Pierce et al., 2017).

According to McNeill (1992), a gesture is one form of embodiment in language and thought. McNeill (1992) categorized gestures into four groups: iconic, metaphoric, deictic (pointing), and beat gestures. Iconic gestures represent the semantic content of speech; for example, a rising hand with index and middle fingers wiggling to depict climbing. Metaphoric gestures represent semantic content by metaphor, for example, cupping hands to explain holding an idea. Deictic (pointing) gestures show objects, places, or events, for example, pointing the place in front of the person to show "here." Lastly, beat gestures do not depict semantic content and instead show a rhythm (McNeill, 1992).

When Alibali and Nathan (2012) analyzed teachers' and learners' gestures through exploratory study, they identified three types of gestures: pointing (deictic), representational (iconic), and metaphoric. Alibali and Nathan (2012) used McNeill's (1992) typology of gesture to influence their categorization. Alibali and Nathan (2012) argued that pointing gestures indicate the base of cognition, representational gestures demonstrate action and perception, and metaphoric gestures display conceptual body metaphors. They noted that mathematics teachers and students use all these gesture types when they are teaching and learning. For example, both teachers and students were using pointing gestures to show the starting point of a line, then representational gestures to show the slope of the line, and metaphoric gestures to show the abstract concept of time when they are working (Alibali & Nathan, 2012). Through pointing gestures, students are able to reduce cognitive load, and through representational gestures, are able to visualize the model of the issue. Lastly, metaphoric gestures helped students understand the psychological reality shaped by subjective facts, experiences, and observations of the abstract concept's facts (Alibali & Nathan, 2012).

In addition to Alibali and Nathan's (2012) categorization, Walkington et al. (2014) classified gestures into two types: dynamic depictive gestures ("display a mathematical object being transformed using the affordance of the body," p. 479) and static depictive gestures ("display an unmoving, unchanging mathematical object in bodily form," p. 479). Walkington et al. (2014) conducted two empirical impact studies to explore how spontaneously performing these gestures and being directed to perform these gestures are related to justifying proofs in geometry. They found that spontaneously performing dynamic gestures substantiated mathematical understanding in the first study, and the second study consolidated that there is a relationship between dynamic gestures and valid proofs (Walkington et al., 2014).

Students' gestures and speech when they produced proofs for the triangle inequality conjecture (the sum of the length of any two sides of the triangle must be higher than the measure of the third side) was also researched (Williams-Pierce et al., 2017). Williams-Pierce et al. (2017) made two claims in their report: 1) “gestures are a powerful tool for mathematics education research because researchers can investigate students’ mathematical thinking by attending to gestures, and 2) students’ gestures can elicit new forms of mathematical reasoning” (p. 249). Consequently, Alibali and Nathan’s (2012), Walkington et al.’s (2014), and Williams-Pierce et al.’s (2017) analyses show that performing gestures have roles in developing mathematical reasoning. Additionally, each study (Alibali and Nathan, 2012; Walkington et al., 2014; Williams-Pierce et al., 2017) shows that teachers' use of gestures influences their students’ mathematical understanding. Some scholars (Katirci et al., 2020; Nathan & Walkington, 2017; Sinclair & Heyd-Metzuyanim, 2014) include technology (video games or applications) in their interactions with students. Hence, in the next section, I review the intersection of embodied mathematical cognition and educational technology (see Figure 2.1 - the intersection of two sets).

Embodied Mathematical Cognition and Technology

The development of technology influenced educational practices. The relationship between technology and education encourages me to look at human-computer interaction or human-technology interaction. Some researchers have investigated embodied interaction between humans and computers. Dourish (2004) defines embodied interaction as “the creation, manipulation, and sharing of meaning through engaged interaction with artifacts” (p. 126). In his definition, artifacts contain technological and computer-based products.

Furthermore, in the educational field, Abrahamson and Trninic (2011) designed *The Mathematical Imagery Trainer* to explore embodied interactions for learning the concept of proportionality. In their design, participants hold the device (infrared beams) with their hands to control the crosshairs on the computer screen. Their aim was to make the screen green, which means that the two crosshairs ratio must be one over two. When participants were raising one hand, they needed to keep the ratio constantly to stay in the green. Based on the study results, the authors stated that after engaging with the trainer, students "develop and rehearse multimodal image schema supporting and understanding mathematical content" (Abrahamson and Trninic, 2011, p. 7).

Another embodied mathematical interaction tool, *The Hidden Village* video game, was designed by Nathan and Walkington (2017) using grounded theory and embodied mathematical cognition. The purpose of this game was to engage students while supporting their mathematical proof skills in geometry. *The Hidden Village* is a kinetic-based computer game where players use their arms to make depictive gestures (dynamic or static) to prove and disprove geometric conjectures. For example, students were bending their arms in and out near their ears to show different angles of a triangle. According to their study results, middle and high school students performing dynamic depictive gestures help students formulate mathematical insights and informal proof; however, non-dynamic (static) gestures only help in the mathematical insight. Consequently, these two design studies (Abrahamson & Trninic, 2011; Nathan & Walkington, 2017) show that the embodied interaction between technology and humans supports students in developing mathematical understanding.

Studies discussed above examined the types of gestures that were performed between teachers and students (Alibali and Nathan, 2012; Walkington et al., 2014; Williams-Pierce et al., 2017) and between students and computers via gesture recognition devices (infrared beams in Abrahamson and Trninic (2011), or camera in Nathan and Walkington (2017)). In addition to studies on student-teacher gestures and embodied cognition in gameplay, scholars also define touchscreen gestures (Villamor et al., 2010) and investigate touchscreen gestures in mathematical reasoning (Dubé & McEwen, 2015; Katirci et al., 2020; Sedaghatjou & Campbell, 2017; Sinclair & Heyd-Metzuyanim, 2014). The next section reviews the intersection of embodied mathematical cognition and touchscreen technology (see Figure 2.1 - intersection-·RQ).

Embodied Mathematical Cognition and Touchscreen Technology

Tablets and mobile phones have touchscreen technology that promote interaction between touchscreen devices and humans through touchscreen gestures. These touchscreen gestures are tapping (touching the screen with a fingertip), dragging (moving fingertip over the surface without losing contact), and pinching (touching the surface with two fingers and bringing them closer together) (Villamor et al., 2010). Since touchscreen devices are easier to use and carry, they receive considerable attention from education stakeholders. Additionally, teachers are using touchscreen devices and apps or games in their classrooms. Many scholars have studied how touchscreen devices influence students' mathematics learning (Dubé & McEwen, 2015; Katirci et al., 2020; Sedaghatjou & Campbell, 2017; Sinclair & Heyd-Metzuyanim, 2014).

Sinclair and Heyd-Metzuyanim (2014) claimed that the body, fingers and hands, and emotions play an essential role in learning mathematics. They supported their claim by designing and investigating an educational application, *TouchCounts*, which helps kindergarten students develop number sense through counting and adding. For one of the participants, the educator realized that their goal and the participant's goal were different from each other. The player was just playfully tapping on the iPad while the educator's goal was to support learning by having the player complete specific exercises. This case shows that the player's purpose of playing the game might be different from the teacher's objectives and also from the designer's intention. Hence, this differentiation might cause different learning outcomes.

The communication between educator, student and a touchscreen application was another element in Sinclair and Heyd-Metzuyanim's (2014) research. According to the authors, "communication as something that involves not just spoken or written words, but also gesture, facial expressions, and exclamations" (Sinclair & Heyd-Metzuyanim, 2014, p. 82). Consequently, their study shows that teacher-student interactions through speech, gestures, and expressed emotions can influence student-technology interaction and mathematical thinking. One of the gaps in this field is that most research takes place in an elementary (K-6) context with the teacher's involvement. Therefore, the processes of learning that occur solely between the player and technology are under-examined.

In higher education, Dubé and McEwen (2015) compared the two gestures (tapping and dragging) while participants (adults; mean age of 26 years) were using the *EstimationLine* application for mathematics tasks to determine whether participants' interaction on the touchscreen device has consequences for their mathematical understanding. According to the authors' results from a practice trial, both participants (tapping groups and dragging groups) performed similarly. However, participants who were assigned to drag performed better than other participants. Consequently, Dubé and McEwen's (2015) impact study shows that touchscreen gestures are effective in mathematical understanding without any human interaction. Even though they used an app instead of a game; I think the distinction between app and game is nuanced because their study is still applicable to touch screen gestures.

Furthermore, Katirci et al. (2020) conducted an impact study to determine the students' embodied interaction and touchscreen technology. According to their findings, participants used touchscreen gestures when playing the game (*DragonBox 12+*). Participants also performed similar gestures (e.g., dragging) during the discussion phase after gameplay to support and build upon each other's mathematical reasoning. Consequently, Katirci et al.'s (2020) study indicates that the embodied interaction between student-peers along with the student-touchscreen device influenced students' mathematical understanding.

Most studies on mathematical video games do not include embodied cognition through touchscreen technology. When researchers do think about embodied cognition in mathematics learning via video games, there is generally a teacher involved in supporting students' learning in some way when students are playing the game. Therefore, there is considerable research still to be done regarding embodied mathematical cognition and touchscreen games. In particular, more research is needed to understand how students learn mathematics through gesture interactions with touchscreen games without teacher intervention or guidance. My contribution to the field will be fitting in this identified gap from both sides, embodied mathematical cognition and touchscreen video games, through different participants (middle school and college students) and in a different learning environment (without teacher supervision).

Chapter 3: The Game Interaction Study

Overview

With the increasing use of video games for educational purposes, it is important to understand what kind of affordance a particular game provides to learners. This study examined different design elements within an educational video game, *From Here to There!* (FH2T). To understand the implication and affordances of playing FH2T, data was analyzed using analysis. The study results show that FH2T's failure and feedback system helps players to learn from their failure by providing formative, summative, and informative feedback when solving algebraic equations. To support researchers and teachers, I identified a list of possible learning outcomes of playing FH2T by making a comparison of the game content with the Common Core State Standards for mathematics.

Introduction

Recent research reports that poor mathematical academic performance is seen in a significant percentage of youth (Kastberg et al., 2016). One influencing factor of a lack of mathematical abilities is that students don't enjoy mathematics because of how it is being taught to them. Most of the students complain that mathematics is boring, difficult, and they do not like it (Yılmaz et al., 2010). Many educators argue that making the process of learning mathematics fun and enjoyable can be the potential solution to engaging students in mathematics (Gresham, 2008; Rossnan, 2006; Strauss, 2016). Game-Based Learning (GBL) researchers assert that games are practical instructional tools to make mathematical learning enjoyable for engaging students (Gresalfi & Barnes, 2015; Kebritchi et al., 2010; Williams-Pierce, 2016).

Within mathematics, algebra is complex for students to understand. Game-based learning can be used to help learners build algebraic understanding. However, in the literature, there is a lack of research on what affordances within games help learners to develop algebraic understanding. To fill this gap, I did an investigation of the mathematics learning game, *From Here to There!* (FH2T), by identifying its game-based learning structures including failure and feedback cycles. I described how Garris et al. 's (2002) GBL Model maps to FH2T, identified potentially productive moments of failure and feedback in the game, and linked the game elements directly to Common Core State Standards of mathematics education. This study is the first step in a longer trajectory of research that involves further investigation of how players learn in interaction with FH2T.

Research questions:

- How do the implications and affordances offered through playing FH2T match with Game-Based Learning's structures?
- How do failure and feedback manifest in mathematical play while playing the game from the researcher's perspective?
- Do the learning outcomes of the game endorse the Common Core State Standards of mathematics required by the state of Maryland?

Theoretical Framework

In this section, I share the theoretical foundations for the study. The components of the learning theory — Game-Based Learning - GBL (de Feritas, 2006; Garris et al., 2002; Squire et al., 2005) — and the lenses of failure and feedback (Hattie & Timperley, 2007; Juul, 2009; Williams-Pierce, 2019) frame the research analysis of this study.

Game-Based Learning (GBL)

Game-Based Learning (GBL) refers to students' learning through playing games. In GBL, researchers typically study serious games (designed for training) or educational games (designed for learning) which also cover digital games (e.g., de Freitas, 2018; Egenfeldt-Nielsen et al., 2011; Kiili, 2005). In this study, I refer to FH2T as an educational game and follow the GBL model (Figure 3.1) by Garris et al. (2002) to analyze elements of FH2T.

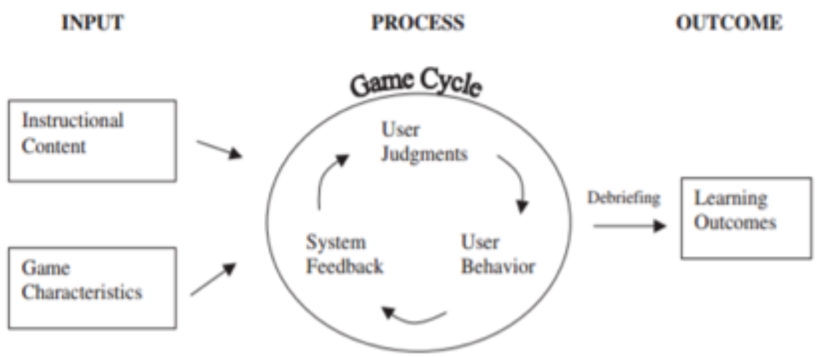


Figure 3.1: Game-Based Learning model by Garris et al. (2002)

Garris et al. (2002) use the phrase "instructional games" when explaining GBL through an input-process-output system in their foundational study (Figure 3.1). In their system, instructional content (e.g., numbers, geometrical shapes, measurement, etc.) and game characteristics (fantasy, rules, goals, feedback, challenges, mystery, and control) are the two essential categories of the input stage. In the process stage, the game cycle is the circular part consisting of user behavior, system feedback, and user judgment (Garris et al., 2002). User behavior refers to their action and manner while playing the game, persistence for solving a level, or appropriate use of the game's rules (Garris et al., 2002). System feedback is how the game gives feedback to players (Garris et al., 2002), such as sound or pop-up messages (Yang et al., 2012). User judgment refers to players' reactions to a game, enjoyment, or interest in being a gamer (Garris et al., 2002). The game cycle stage is the core of GBL, representing the repetition of students' behavior and the system's immediate feedback to help them reach learning outcomes (Garris et al., 2002).

Failure and Feedback

When playing a game, there are generally two possible results to an action: success (acceptance of the action) and failure (denial of the action). Whenever a player makes an action in the game, the game reacts to the players' actions with feedback (Game Cycle, Garris et al., 2002). Every action, notwithstanding success or failure, is paired with feedback (Williams-Pierce, 2019). The contradiction between being successful or failing might lead players to enjoy the game and keep playing or to quit playing (Juul, 2009). While failure might mean that there is still something to understand or learn, which is presented in the form of a new challenge in the game (Ramirez, 2015). Players seek help or feedback to overcome a challenging situation or repeated failure to change as a success in their next attempts (Williams-Pierce, 2019).

While the game developers designed in-game failure and feedback, they called the moments of failure as *errors*. There are three different types of errors in FH2T: shaking, keypad, and snapping errors. Shaking errors are made by attempting to incorrectly use existing operators (i.e., not following the order of operations). Keypad errors are made by attempting to enter a non-equivalent expression using the keypad (i.e., writing '31' by mistake instead of '32'). Lastly, snapping errors are made by attempting to incorrectly reorder terms (i.e., exceeding the minimum number of steps to solve an expression). I exemplify these errors and their feedback on selected puzzles in the Finding Section.

In the next section, I explain my methodology for the study.

Methods

This section describes the method of the study. It provides details ofthe purpose of the study, research questions, and study design.

Purpose of the Study and Research Questions

This study, which is the first study in a larger research project that investigates how students learn in interaction with an educational game (FH2T) and analyzes FH2T through an analysis methodological lens (Figure 3.2- The Game Interaction). My overall research question for the dissertation, "How do digital gestures connect to students' mathematical understanding when playing FH2T?" focuses on digital gestures and students' mathematical understanding through the game, FH2T. Before collecting data or analyzing pre-existing data about students' mathematical understanding, I needed to learn more about the game's characteristics. For example, I needed to understand FH2T's technical and mathematical constructions as well as the gestures, and their connections to potential mathematical learning. In my opinion, the best way for investigating the game is to play it instead of reading articles about it or watching when someone else is playing. Therefore, the first study of this dissertation project applies an analysis to my own gameplay. The findings from this study informed the design of the subsequent second (The Quantitative Gesture) and third (The Student Observations) studies. This the Game Interaction study seeks to answer the following research questions:

- How do the implications and affordances offered through playing FH2T match with Game-Based Learning's structures?
- How do failure and feedback manifest in mathematical play while playing the game from the researcher's perspective?
- Do the learning outcomes of the game endorse the Common Core State Standards of mathematics required by the state of Maryland?

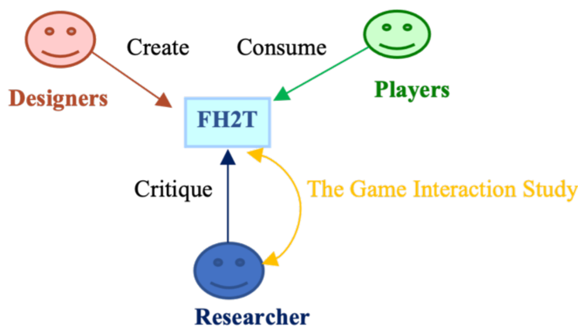


Figure 3.2: The Game Interaction Study

Study Design

I played FH2T as a researcher. While playing, I recorded the screen by pressing the "Windows + G" option to display and record the game play (Murray, 2018). Then, I created an Excel spreadsheet to list the worlds by columns:

1. World Number and Topic (String variables[1]),
2. Puzzle Number (Numeric variables[2]),
3. New Rule (Binary variables[3]),
4. The New Rule (See Appendices 3 for all rules of the game) (String variables),
5. Numbers of Clovers (Numeric variables),
6. Start (String variables),
7. Goal (String variables),

[1] String Variables: "Variables involving words (i.e., letter strings)." (Field, 2013, p. 884)
[2] Numeric Variables: "Variables involving numbers." (Field, 2013, p. 880)
[3] Binary Variable: "A categorical variable that has only two mutually exclusive categories (e.g., being dead or alive)." (Field, 2013, p. 871)

8. Steps (Numeric variables),
9. Hint (String variables),
10. Gesture Number (Numeric variables),
11. Gesture Explanation (See Appendices 4 for all gestures and descriptions of the game) (String variables),
12. How I solved the Puzzle (String variables),
13. Note (String variables).

Columns 1 to 11 represent the foundational information about FH2T. The first (World Number and Topic) and second (Puzzle Number) columns were recorded to specify each puzzle and its algebraic topics. The third (New Rule) and fourth (The New Rule) columns were recorded to keep track of where the new rule was introduced (in which puzzle) and what the new rule said. In the new rule puzzle, there is also an animated tutorial (e.g., Puzzle 1.1, Figure 3.3-F). Players can easily solve the new rule puzzle by repeating the gesture in the animated tutorial. However, in later puzzles, players might not be able to solve similar expressions by using the same rule because there is no animated explanation. These (3^{rd} and 4^{th}) columns are crucial for understanding the game and the mathematical knowledge needed to play FH2T. For this reason, I recorded in which puzzle the new rules were introduced (e.g., Puzzle 1.1) and what the new rules were (e.g., "Drag terms to commute. Blue lines show you where you can drop the term. Make the expression look like the goal.") in two columns.

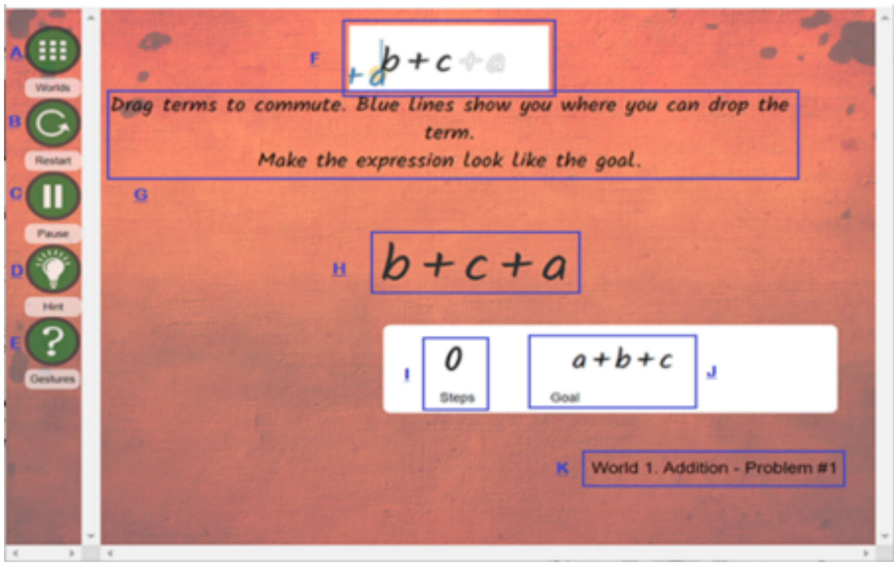


Figure 3.3: The Puzzle1.1 Labeled in Blue (© Graspable Math)

The fifth column (Number of Clovers) contains a numeric variable to record the number of clovers that could be earned from the puzzle. The sixth column (Start) includes a string variable to record the starting point of the puzzle. The seventh column (Goal) covers a string variable to record the goal of the presented puzzle. The eighth column (Steps) contains a numeric variable to record the least number of steps for solving the puzzle. The ninth column (Hint) includes a string variable to record the hint given if the player clicks the Hint icon on the left bar (see Figure 3.3-D). The tenth (Gesture Number, Figure 3.4-Left, there is one gesture) and the eleventh (Gesture Explanation, see Figure 3.4-Right; “drag terms to commute”) columns provide information about how the player solves the puzzle. If players make the gestures shown, they will be successful in completing the puzzle. If they make additional gestures than what is required, they will fail. From this information, success is clear (making the same gesture on the gesture column) while other actions cause failure. Feedback depends on the different gestures that the players make.

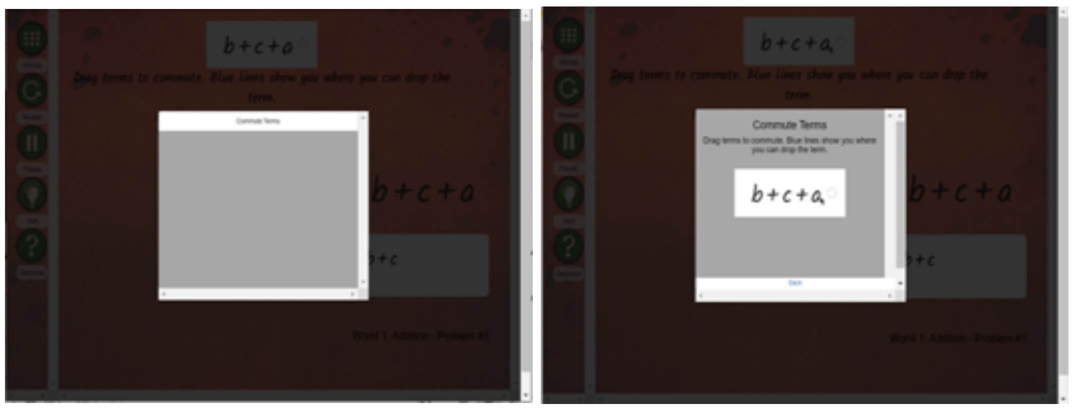


Figure 3.4: Gesture- Commute Terms (Left); Commute Terms in Details (Right)

The twelfth column covers string variables and lists the gestures I used to solve the puzzle. For example, for Puzzle 1.1 (See Figure 3.3), I wrote: “drag a in front of b.” Moreover, when I was playing the game, I was surpassing the least number of steps to solve some puzzles and I thought that some dynamics of some puzzles might be tricky for the students. For these challenging puzzles, I highlighted their respective rows in the excel sheet and marked (13th column) these puzzles for further analysis in the Quantitative Gesture and the Student Observations studies. I considered how students will solve these puzzles. Tracking my playing (screen records and excel spreadsheet) led me to decide critical factors to examine in future studies.

Findings

I did an analysis to investigate the game and to build upon my overarching research question: How do digital gestures connect to students’ mathematical understanding when playing *From Here to There!* (FH2T)? I interacted with FH2T and interpreted the game under the three frameworks – Game-Based Learning (GBL), Failure and Feedback, and Mathematics Learning Standards – to answer the research questions of this study:

1. How do the implications and affordances offered through playing FH2T match with Game-Based Learning's structures?

2. How do failure and feedback manifest in mathematical play while playing the game from the researcher's perspective?

3. Do the learning outcomes of the game endorse the Common Core State Standards of mathematics required by the state of Maryland?

This Game Interaction Study includes three finding sections (GBL Analysis, Failure and Feedback, Mathematics Learning Standards) and their subsections. In the first section, I present FH2T according to Garris et al.'s (2002) Game-Based Learning approach. In the second section, I use Williams-Pierce's (2019) lenses of failure and feedback to investigate FH2T's failure and feedback system. The third section examines FH2T over the Maryland College and Career-Ready Standards for mathematics.

GBL Analysis Findings

Garris et al. (2002) propose three stages to approach game-based learning (GBL): Input, Process, and Outcome (shown in Figure 3.5). In this section, I follow their categorization to investigate FH2T.

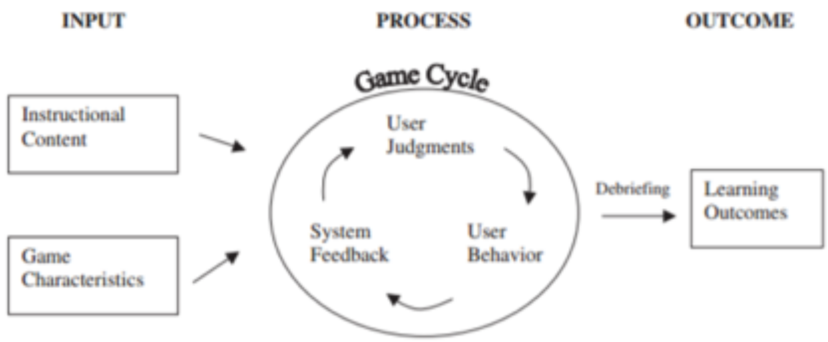


Figure 3.5: Game-Based Learning model by Garris et al. (2002).

The Input Stage

The first stage is the *Input* stage. Instructional content and game characteristics are the two essential categories in the Input stage (Garris et al., 2002).

The Instructional Content of the Game

FH2T is a dynamic mathematics game designed for students to learn algebra through perceptual-based intervention (Ottmar et al., 2015). FH2T has 14 worlds (Figure 3.6), and each world contains 18 puzzles (Figure 3.7 Left-Started and Right-Finished). The worlds focus on specific mathematical/algebraic concepts ranging from addition to linear equations (Table 3.1). Learners start with simple mathematical content and build up knowledge and skills throughout the game.

Figure 3.6: Worlds on Tree

Figure 3.7: Outline of the World 1- Started and Finished

Table 3.1: Worlds and Contents

World Number	Contents
World 1	Addition
World 2	Multiplication
World 3	Order of operations + and ×
World 4	Subtraction and negative numbers
World 5	Mixed Practice of + and -
World 6	Division
World 7	Order of operations
World 8	Equation + and -
World 9	Inverse operations + and -
World 10	Distributions
World 11	Factoring
World 12	Equation +, -, ×, and ÷
World 13	Inverse operation
World 14	Final Review

Game Characteristics

The game characteristics are another vital element of the *input* stage (Garris et al., 2002). The features of FH2T are explained in two subsections: Beginning the Game and Getting to Know the Game's Icons Better. In these subsections, I use the following terminology to explain each feature: "player" is the person who is playing the game; "puzzle" is the specific expression that players are solving; and "world" is the level or unit in which players are solving puzzles about it. While explaining the learning or teaching aspects of FH2T in the Outcome Stage, I will refer to "learner," "problem," and "topic/unit," respectively.

Beginning the Game

All players need to first register for the game. For registration, FH2T only requires a username and password. Then players can log into FH2T (see Figure 3.8).

Figure 3.8: Intro of the Game

The game welcomes players with a pop-up message (see Figure 3.9) by saying, "In this game, you will discover *algebra* by playing with numbers and symbols. There are several worlds arranged *on a tree*, with *puzzles* in each. Each world explores a particular *mathematical idea*. Start *by tapping on the colored world* in the lower right. *Solve enough puzzles,* and the next world will become colored. Tap any colored world to solve its puzzles. Have *fun*!" [emphasis added].

Figure 3.9: Pop-up Message/ Welcome Note

Through this note, players can find the answer to possible questions about the game. For instance, they can learn the instructional content of the game: *algebra* and *mathematical ideas*. Then, the game outlines where the worlds are located: *on a tree* (see Figure 3.10 - Main Page). Players start the levels *by tapping on the colored worlds* (see Figure 3.11) and can move on through the next puzzle or onto the next levels *by solving enough puzzles (14) in the world* (Figure 3.12). When the players solve 14 puzzles in the world, the pop-up message, YOU DID IT, appears on the screen (Figure 3.12). By tapping the "Explore Next World" icon on the screen, the players go to the main page. The next world's icon on the main page will become colored so the players can tap the next world icon and solve puzzles in the next world. The following section (Getting to Know the Game's Icons Better) covers more information about the game's icons. There is also missing information in this welcome note: how do players solve the puzzles? This missing piece of information is intentional; players learn how to solve each puzzle by playing. Learning by playing is the *fun* part of the game. The process and output stages will cover the fun and learning aspects of the game.

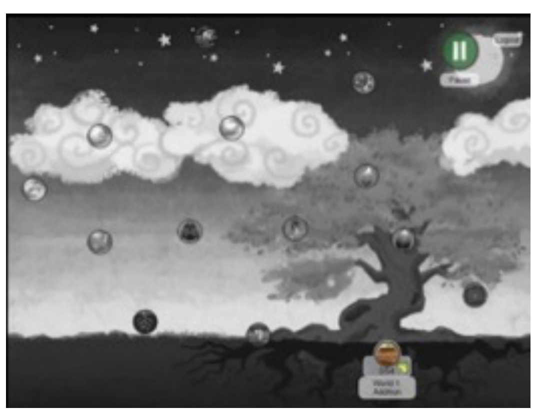


Figure 3.10: Worlds on Tree- Main Page

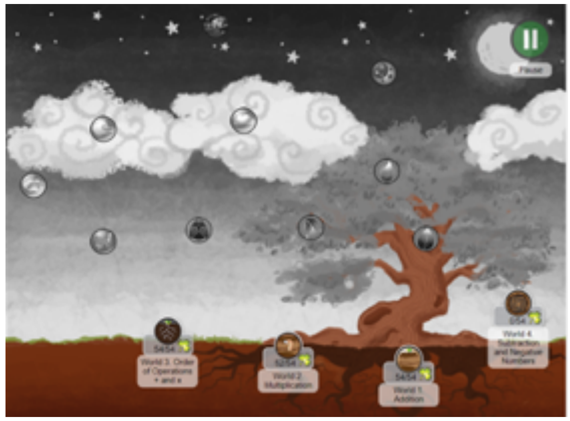


Figure 3.11: Colored Worlds

Figure 3.12: Next World is unlocked - Pop-up Message

Getting to Know the Game's Icons Better

After the Welcome Note (see Figure 3.9), the players can start the game by tapping the World 1 colored icon on the tree (see Figure 3.10). Then, players can see the World's outline and various icons on the screen (see Figure 3.13, blue boxes and underlined texts are added). Players can see all puzzles (e.g., The World consists of 18 puzzles, and to see all puzzles, the players need to scroll the bar down) and icons in the World's outline (see Figure 3.13). The icons and their function are explained in more detail in Appendix 3.1- Icons on the World 1 Screen.

Figure 3.13: Outline of World 1

After reviewing the World's outline (Figure 3.13), the players can start the puzzle by tapping the first puzzle, which is unlocked. Then players can see the puzzle and various icons on the screen (see Figure 3.14, blue underlined texts and blue boxes are added). The icons and their functions are explained in more detail in Appendix 3.2- Icons on the Puzzle Screen.

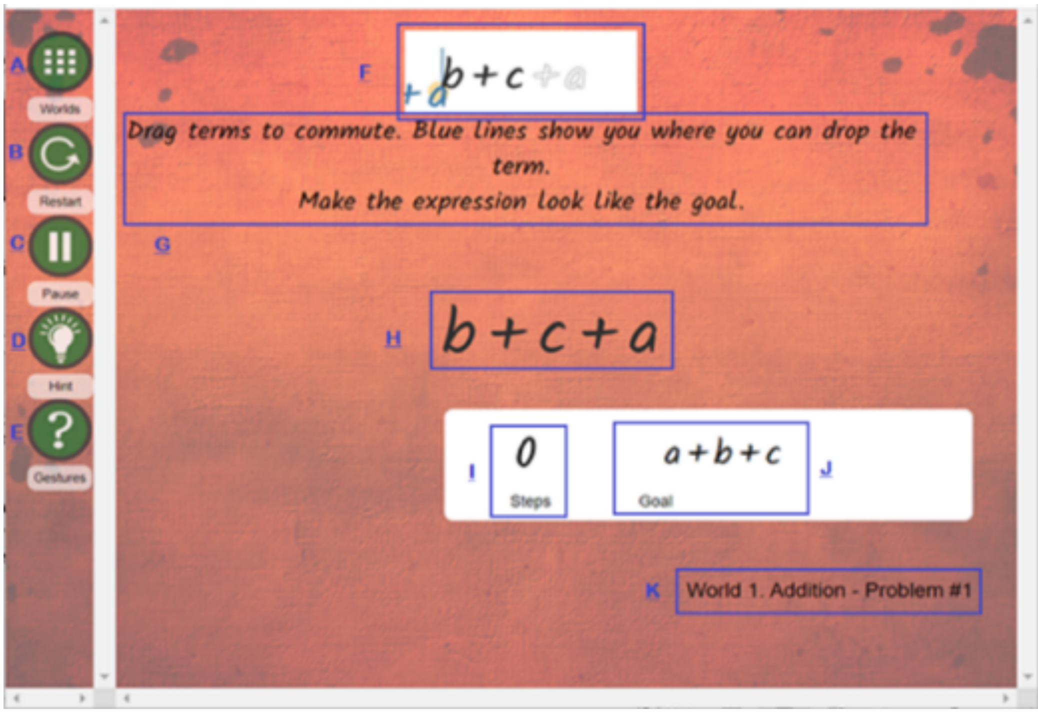


Figure 3.14: World 1-1

The Process Stage

The *process* stage is the second stage of GBL (Garris et al., 2002). In the process stage, the *game cycle* is a circular part consisting of *user behavior*, *system feedback,* and *user judgment* (Garris et al., 2002). *User behavior* refers to the player's action while playing the game. *System feedback* refers to how the game gives players feedback while *user judgment* refers to players' reactions to the game (Garris et al., 2002).

Playing the Game - Player Behavior

To play FH2T and to solve the puzzles, players need to apply procedures to re-write different expressions. Ottmar et al. (2015) describe this process as "the user's goal is to transform the equation *from* the starting form (*here*) *to* the ending form (*there*)" (p. 1794) [emphasis added]. This process of transforming equations *from* given expression *to* goal form gives also the name of the game, *From Here to There!*. In Figure 3.15, starting form and ending form (goal) are emphasized by blue underlined texts. To re-write the expression, players need to act by following the rules of the game (All rules are listed in Appendix 3.3- Rules of the Game). In other words, players need to apply digital gestures to solve the puzzle. For example, to solve Puzzle 1.1, players drag the term -a- to rewrite the expression, see Figure 3.14-F (All gestures and descriptions are listed in Appendix 3.4- Gestures and Descriptions). This study, the Game Interaction, includes my (as a researcher) behaviors, see the Failure and Feedback Analysis section, and in future studies, The Quantitative Gesture and The Student Observations, I will observe the learners' behaviors.

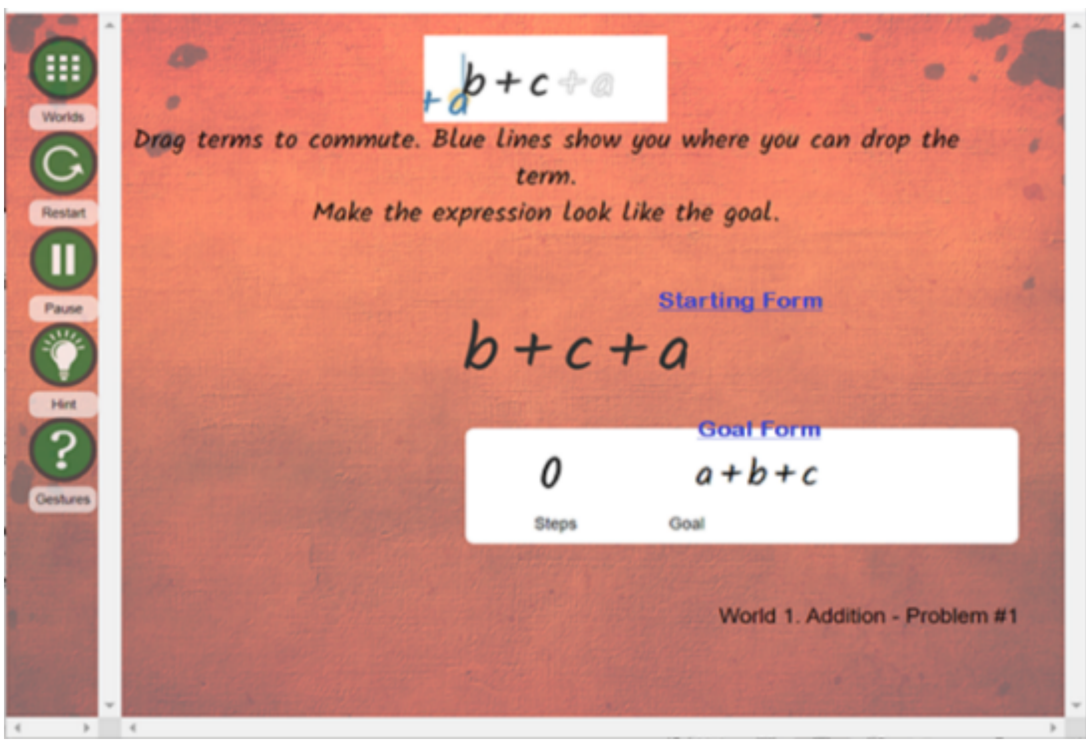


Figure 3.15: <u>From</u> Starting Form (<u>Here</u>) <u>to</u> Goal Form (<u>There</u>)

System Feedback

FH2T provides feedback in three ways: formative (Heron, 2010), summative (Reynolds & Kao, 2019), and informative (Kim et al., 2018) feedback. All kinds of feedback are given visually, text-based, and appear on the screen through pop-up messages. Formative feedback is given during the solving of the puzzle, and it helps players avoid or learn from failure. For example, blue lines show players where they are going to drop the term (see Figure 3.15). Players get informative feedback during the puzzle-solving process. To obtain informative feedback, players need to click icons. For instance, when players need extra information about the puzzle, they can click the Hint (see Figure 3.14-D) and the Gestures icons (see Figure 3.14-E) on the screen's left banner. Feedback from these Hint and Gestures icons are an example of informative feedback (for all Hints and Gestures, see Appendix 2 and 4, respectively). The Failure and Feedback Analysis section gives more details about informative feedback from my perspective. The Quantitative Gesture and The Student Observations studies will be done from the learners' view to analyze the learners' iterative action-reflection cycles of getting this informative feedback. Summative feedback is given after the player solved the puzzle. For instance, symbolic rewards (Deci et al., 2001) like collecting clovers (see Figure 3.17) and symbolically growing a tree from land to sky (see Figure 3.10 and 3.11) are two summative feedback examples from FH2T. The next section explains how players collect symbolic rewards, or clovers, and lose them.

How are clovers collected?

Clovers are collected by solving the puzzles. In World 1- Puzzle 5 (see Figure 3.16), there is a warning that says, "The step counter turns red when you use more steps than are required. Click restart on the left to start over." As the warning states, if a player solves the puzzle with more steps than are required, the pop-up message – YOU DID IT – appears on the screen (Figure 3.17-Left). As seen in Figure 3.17-Left, there are just two clovers in the pop-up message, and written motivations as "Good Job!" is checked, but "Best Solution" was struck through. The player loses one of three clovers and gets just two clovers because s/he could not get the best solution (Figure 3.17-Left). If the player wants to get three clovers, s/he can click the "Retry" icon and start the puzzle over. If the player solves the puzzle with the required number of steps in more trials, she gets three clovers (Figure 3.17-Right). In both situations, the player solved and passed the puzzle and can continue to the next puzzle by clicking the "Next Problem" icon in Figure 3.17-Left or directly in Figure 3.17-Right.

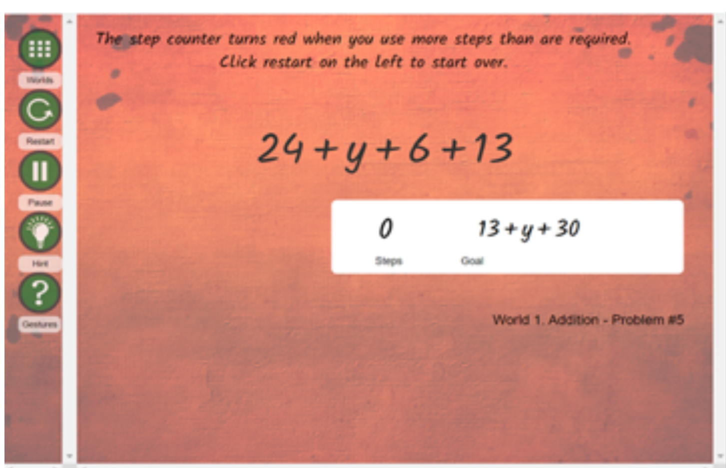


Figure 3.16: World 1-Puzzle 5

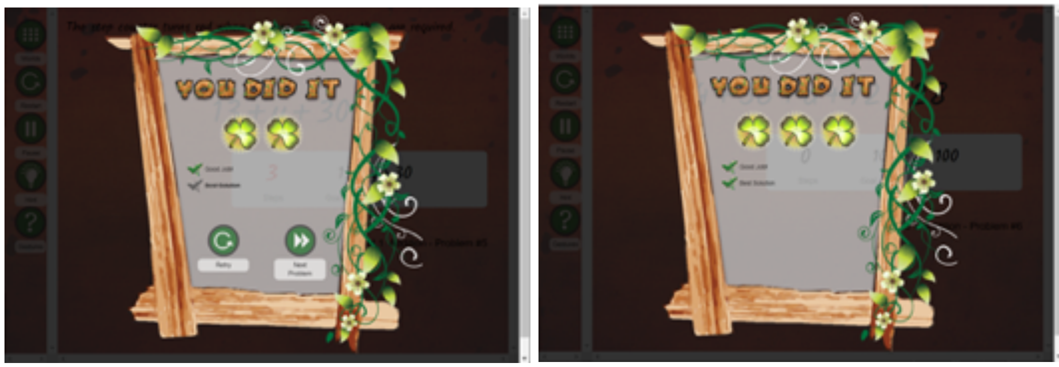


Figure 3.17: Rewards: 2 Clovers (Left) and 3 Clovers (Right)

User Judgments

User judgment is important because players decide to keep playing or quit the game according to their enjoyment and interest level during the play. The Game Interaction Study has been done to analyze the game from, as researcher, my perspective. I finished all worlds in FH2T to be able to analyze it. Next, The Quantitative Gesture and The Student Observations studies will be done to observe the learners' iterative action-reflection or behavior-judgment cycles. Hence, the next studies will give more meaningful information on user judgments like their satisfaction and attraction level to the game.

The Outcome Stage

The *outcome* stage is the last stage of GBL (Garris et al., 2002). The *game cycle* and *process* are completed when the player solves all of the levels or quits the game. The learning outcomes emerge by debriefing the process. Debriefing depends on the learner or the teacher/researcher. For example, finishing the level might be enough information for the learners to decide whether the game is effective to learn that topic. However, the teacher/researcher might want to recheck learning using a post-test after playing the game. In the Mathematics Learning Standards Analysis section, I compare the Maryland College and Career Ready Standards and FH2T to list possible learning outcomes. The Quantitative Gesture Study will be done to generalize learners' mathematical understanding. In The Student Observations Study, I will interrogate learning outcomes through post-tests.

Failure and Feedback Analysis Findings

The first two features of provocative objects are "consistent and useful feedback; and high enough levels of difficulty and ambiguity that players experience frequent failure that is closely paired with the feedback" (Williams-Pierce, 2019, p. 590). In this analysis, failure is used as a "nonperformance of something expected" and "an act of failing" (Dictionary, n.d.). This section analyzes the game's feedback system and players' possible failures under the two subtitles: *Failure and Feedback on The Game* and *Failure and Feedback on Mathematics*. In this section, I am the player. For this reason, I used she/her pronouns to indicate the player.

Failure and Feedback on The Game

This section analyzes the failure and feedback system of the game mechanics under five subtitles: 1. Tap to solve the level or the puzzle; 2. Blue lines and blue boxes; 3. Tap the number; 4. Tap the keypad first, then tap the number; 5. Use the keypad twice.

1. Tap to solve the level or the puzzle.

To open the level, the player needs to tap the colored World, see Figure 3.18. In Figure 3.18 she has already solved World 1 to 7, and she is starting to solve World 8 (Ladybug icon). She must tap World 8's icon to enter the level.

In this situation, one failure (nonperformance of something expected) might be tapping on other Worlds' icons instead of World 8's. Tapping on either Worlds 9's icon, which is still black and white, is considered a failure because the player should follow the number queue. She could not skip any world without solving it. In response to pressing the black and white Worlds icons (e.g., Worlds 9 in Figure 3.18), she would not go anywhere. In other words, the game screen would not change. The game is giving feedback by not accepting the action or doing nothing. (Note: sometimes when playing games and interacting with an object where the object does not do anything might show a design flaw or indicate that the game froze. However, in this game, it is an indicator that the player is not interacting with interactable objects- black and white icons in the Worlds' outline.)

Figure 3.18: Colored Worlds

If the player taps on Worlds 1 to 7, which are colored and solved, a potential failure may depend on how many clovers were collected. There are two possibilities: all clovers have been collected, or some clovers have not been collected in the World.

For the first possibility, if she has already collected all clovers, tapping on these worlds is redundant. The second possibility is that all clovers in the worlds have not been collected (e.g., in Figure 3.18, she did not take one of the clovers in World 5): she made a mistake (surpassed the best solution steps) and lost a clover in the specific puzzle (e.g., Puzzle 18 in World 5). If the aim of players is to re-tap the World to get all clovers, she taps the World' icon and as feedback the game will react by allowing the player to re-enter the selected world where too few clovers were collected.

Similarly, to open the puzzle, the player needs to tap the unlocked puzzle (see Figure 3.19; Puzzle 11 is unlocked). In Figure 3.19, the player has already solved Puzzles 1 to 10; she is starting to solve Puzzle 11, so she needs to tap Puzzle 11's box.

Figure 3.19: Locked-Unlocked Puzzles

In this section, I describe the possible failure of tapping on the other puzzle's boxes instead of Puzzle 11. When Puzzle 12 to 18's boxes are still locked, tapping on them is a failure because players should not skip any puzzle without solving puzzles before Puzzle 15. (Note: after solving Puzzle 14, the next world is unlocked, and players can continue solving puzzles in the next world- see Figure 3.8). As feedback, the player will not go anywhere. The game is giving feedback by not accepting the action or by doing nothing.

Moreover, when the player is tapping on Puzzles 1 to 10, puzzles that she has already solved, the failure may depend on how many clovers were collected from these puzzles; if she took all clovers in the previous puzzles, tapping on these puzzles is a redundant action. If she did not collect all clovers in the previous puzzles, she made a mistake in the specific puzzle. As feedback for tapping on Puzzle 1 to 10, the game will allow the player to re-enter the puzzle.

In summary, failure is tapping on other worlds/puzzles instead of the next one on the queue. One type of feedback response if the worlds/puzzles are still locked is that nothing happens. Another feedback is re-entering the solved worlds/puzzles if the worlds/puzzles are unlocked.

2. **Blue lines and blue boxes**

Players are learning the game rules while playing FH2T. In other words, there is no tutorial at the beginning of the game. The first rule of the game is: “Drag terms to commute. Blue lines show you where you can drop the term. Make the expression look like the goal.” (See Figure 3.20-Text). The rule is presented by words and animated graphics together. This rule contains three crucial pieces of information: the first one – “Drag terms to commute.” – tells players to solve this puzzle using the “drag” gesture; the second one – “Blue lines show you where you can drop the term.” – explains to players how the game gives feedback about their actions; and the last one – “Make the expression look like the goal.” – implies the game's aim and how players should solve the puzzle by rewriting the expression to look like the goal.

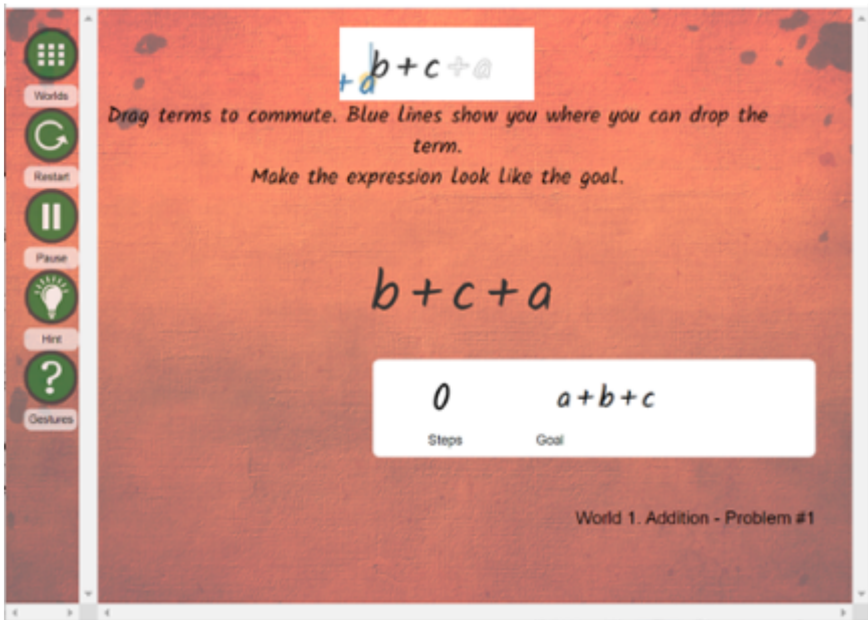


Figure 3.20: Blue Line

When the player is trying to solve the puzzle, she commutes variables as shown in the animated graphic in Puzzle 1 (Figure 3.20). She needs to be sure that she puts the variable in the correct position required by the game. Mathematically, the order of the variables in addition does not matter (commutative property) so these two expressions (starting: $b + c + a$, and goal $a + b + c$) are equal. In other words, there is no mathematical difference among $b + c + a$, $a + b + c$, $b + a + c$, $c + b + a$, $c + a + b$, and $a + c + b$. However, in the game, the goal is to rewrite the expression $b + c + a$ as $a + b + c$ using the least number of steps. If the player could not pay attention to follow the blue line (Figure 3.21-1) and put a to the right side of b like Figure 3.21-2, she did not reach the goal. To solve the puzzle, she needs an extra step: drag a to the left side of b. (Figure 3.21-3). She solved the puzzle with two steps and got feedback with red color as two steps were not the best solution for the puzzle (Figure 3.21-4).

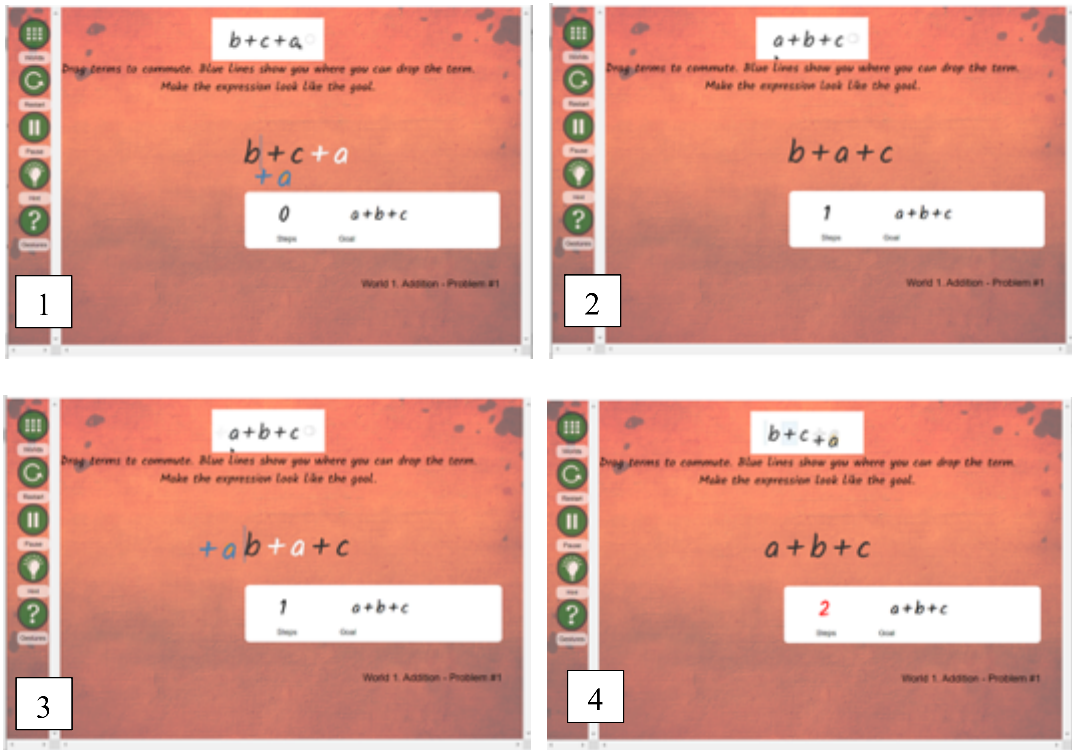


Figure 3.21: Solve Problem 1.1 – 1-4

In this situation, putting the variables in an incorrect position is an in-game failure instead of a misconception of mathematics because the choice is still mathematically correct. However, the goal is different from the player's action. Feedback is provided in the next step (Figure 3.21-4) when the "Steps" number turns red; this means the puzzle can be solved in fewer steps – this puzzle is solvable in one step. Moreover, she got summative feedback when she made the goal expression – "YOU DID IT" pop-up message (Figure 3.22) – with two clovers and a "Good Job!" checkpoint. However, "Best Solution" was struck through because she solved the puzzle with extra steps.

Figure 3.22: YOU DID IT

In addition to the “blue line,” another feedback related to the player's action is the “blue box.” In later puzzles, the player needs to perform operations by tapping on an operation sign or dragging one number on top of the other number; this is also notified as a rule in the game (Figure 3.23). When dragging the number on top of different numbers, the dragged number and operation sign is colored from black to blue. There is a ghost number colored white in its old position, and the other addend is indicated with a blue box (Figure 3.23). In this puzzle, making mistakes may seem strange because addends are small numbers and close to each other, but making mistakes is still possible. One possible failure is dragging 3 onto 15 to make 18 (Figure 3.24-1 and Figure 3.24-2) and dragging 2 onto 6 to make 8 (Figure 3.24-3 and Figure 3.24-4). There is immediate visual feedback; the player should realize that the goal expression is different from her final expression. When she realizes it, she should click the “Restart” icon (a curved arrow) to start over the puzzle.

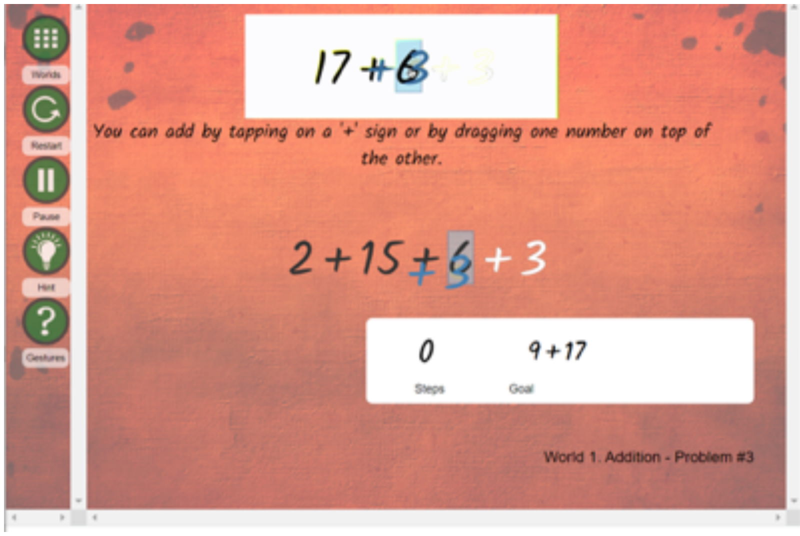


Figure 3.23: Addend- Blue Box

Figure 3.24: Addend – Blue Box – 1-4

In summary, an example of failure is putting the integer in the incorrect position and dragging a number onto the incorrect number. The feedback is given through the red “Steps” number, and the reaction of the player should be to click the “Restart” icon and solve the puzzle again.

3. Tap the number.

The previous example (See Figure 3.23) of the rule says, "You can add by tapping on a '+' sign…" As a rule, players can tap on the operation sign or addends to sum two numbers. For instance, in Figure 3.23, the starting form of the puzzle is $2 + 15 + 6 + 3$, and the goal form is $9 + 17$. There are many ways that the player can solve the puzzle. One way is tapping the + sign between 2 and 15 to make 17, then tapping 3, which interacts with 6 to become 9, and lastly dragging 9 to the left side of 17. However, if the player taps on 6, it interacts with 17 and makes 23 (Figure 3.25). In other words, tapping on the number 6 also leads to the sum of 17 and 6 which equals 23. However, this action was not announced as a rule like "You can perform operations by tapping on a number." before this puzzle, so the player needs to be careful when tapping on numbers. When she taps on a number, the number interacts with another number on its left.

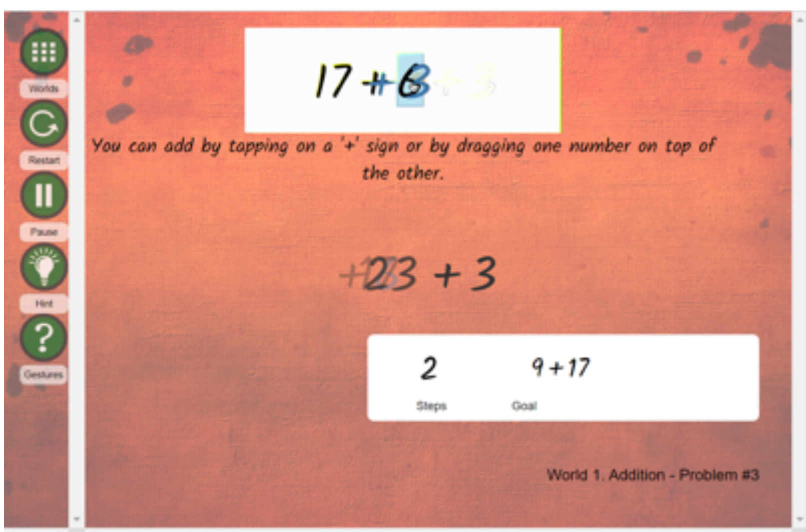


Figure 3.25: 17+6 = 23

In this situation, the failure is tapping on 6 instead of 3 to make 9. Since the feedback has not been given to indicate this additional error, the player should realize that she made a mistake and "Restart" the puzzle.

4. Tap the keypad first, then tap the number.

In later puzzles, the player needs to substitute numbers by using the keypad. This is also notified as a rule in the game with text and animated graphics (Figure 3.26). When the rule or keypad is introduced to the players, it is explained through a simple example (see Figure 3.26). In Puzzle 1.8 (Figure 3.26), there is just one integer, 9, and the player needs to substitute 9. It is not clarified in the rule/text, but the animated graphic shows that the player needs to tap the keypad icon first on the right banner and then tap the number to substitute. When the player taps on the keypad icon, the figure in the icon turns yellow and the name of the icon turns to "Keypad On" (Figure 3.27-a). Once this happens, she then taps on the number, 9, which activates a keypad option (Figure 3.27-b). The blue box helps the player to figure out which number is going to be substituted (Figure 3.27-b).

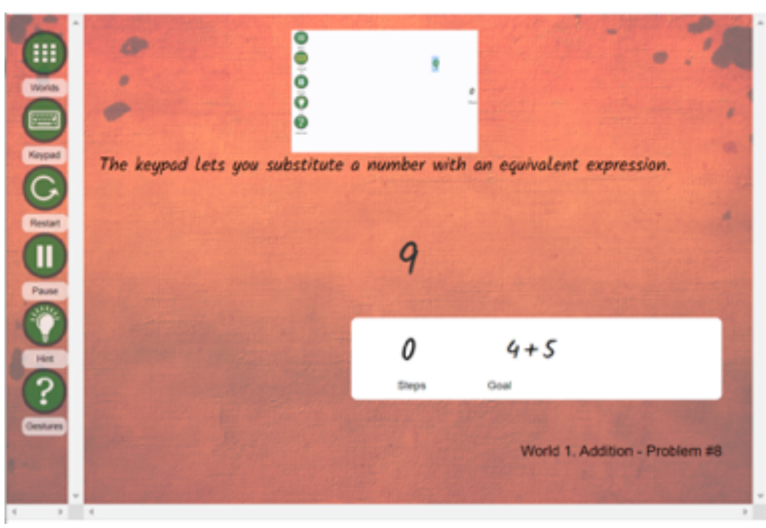


Figure 3.26: Keypad

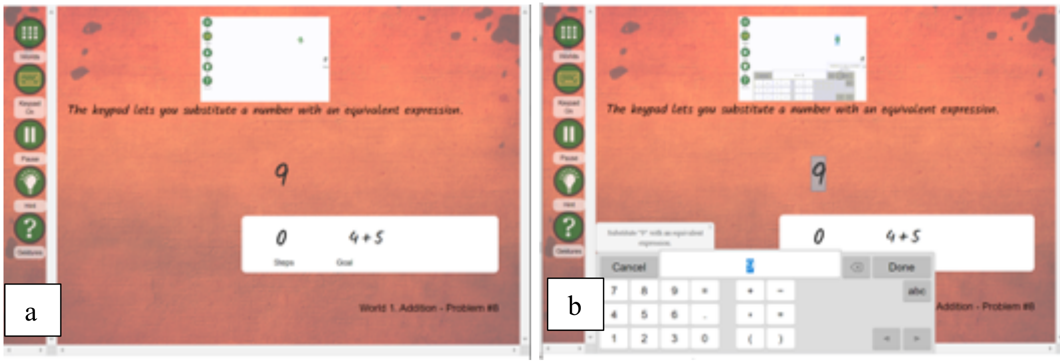


Figure 3.27: Keypad On

In another puzzle (Puzzle 1.9), the player needs to select the number she will substitute (Figure 3.28). In this puzzle, she can substitute any numbers to make the expression look like the goal. For instance, she can substitute 7 into 1 and 6; or 6 into 1 and 5; or 10 into 5 and 5. In this situation, a possible failure is tapping on 6 first to substitute instead of tapping on the keypad first (Note: tapping on 7 or 10 does not cause failure because there are no numbers on their left). This action leads to the sum of 7 and 6, which equals 13 (Figure 3.29-a); after this point, the player can still solve the puzzle by substituting 13 into 12 and 1 (Figure 3.29-b-c-d).

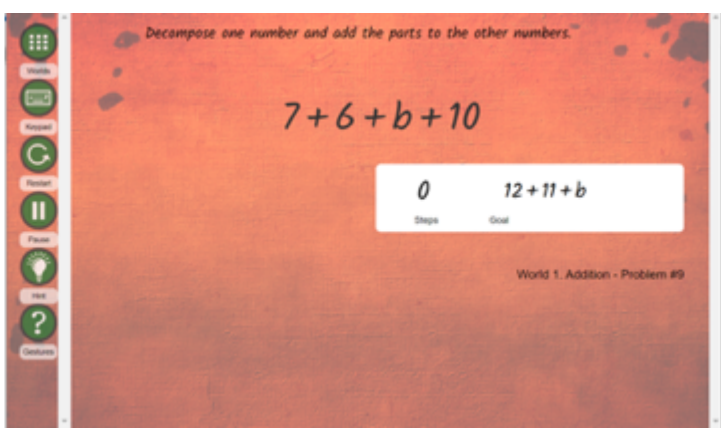


Figure 3.28: Select Number to Substitute

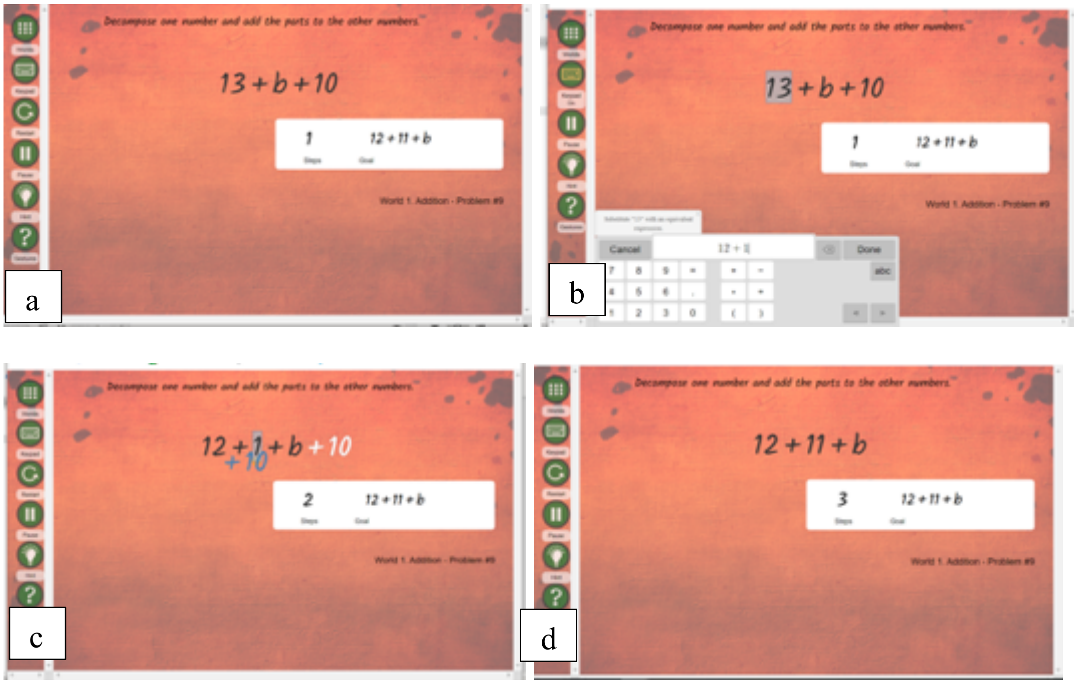


Figure 3.29: 7+6=13

5. **Use the keypad twice.**

In later puzzles, the player needs to substitute a number into more than two variables (see Figure 3.30). She needs to factor 64 into 2.4.8. Mathematically, we can write it in just one step by using the keypad. However, in the game, she needs to follow the order; when she tries to split the number into the factor in one step, she gets feedback on the box of the keypad: “You have split too much. Slow down.” Due to the feedback, she needs to factor 64 in two steps. As shown in the animated graphics, she should first tap on the keypad to activate it and then tap on 64 to re-write it as 2. 32 (Figure 3.31-a). Then, she should tap on the keypad again and tap 32 to factor it as 4.8 (Figure 3.31-b).

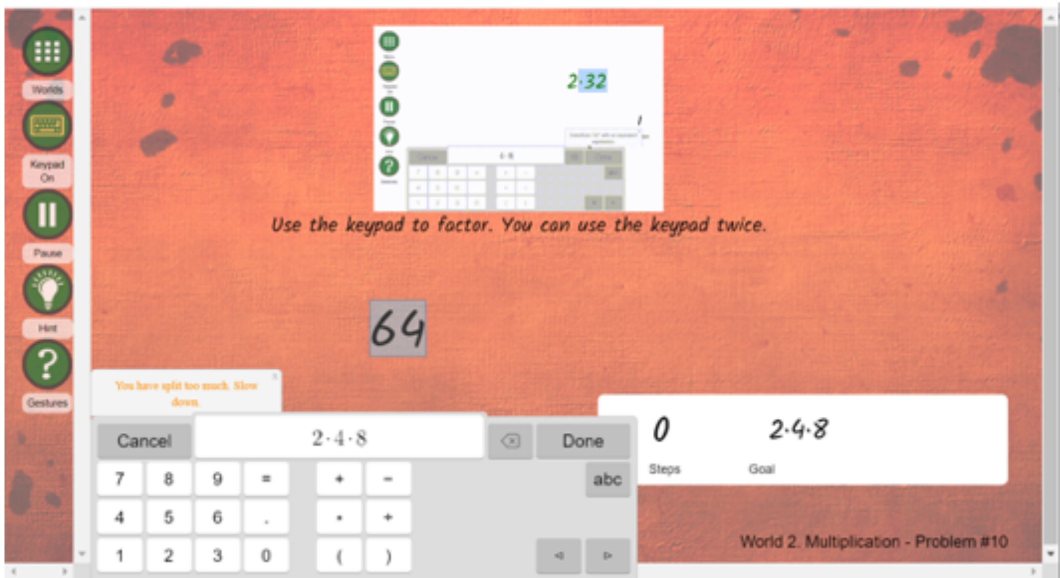


Figure 3.30: Use Keypad Twice

Figure 3.31- a: 64=2.32

Figure 3.31-b: 32=4.8

In summary, the failure of this puzzle is trying to substitute the number just in one step, and the feedback is given on the top of the keypad as "You have split too much. Slow down."

Failure and Feedback on Mathematics

This section analyzes the failure and feedback system of the game on mathematical issues under the two subtitles: 1. Learn and then apply the rule, 2. Mathematically correct expression.

1. Learn and then apply the rule.

In general, before the rule is published, the rule did not work on previous puzzles. For example, before Puzzle 1.8, players do not need to substitute the item, so they do not have the keypad icon on the screen. Figure 3.32-a shows Puzzle 1.7, and Figure 3.32-b shows the screenshot of Puzzle 1.8 to highlight the difference in their rules.

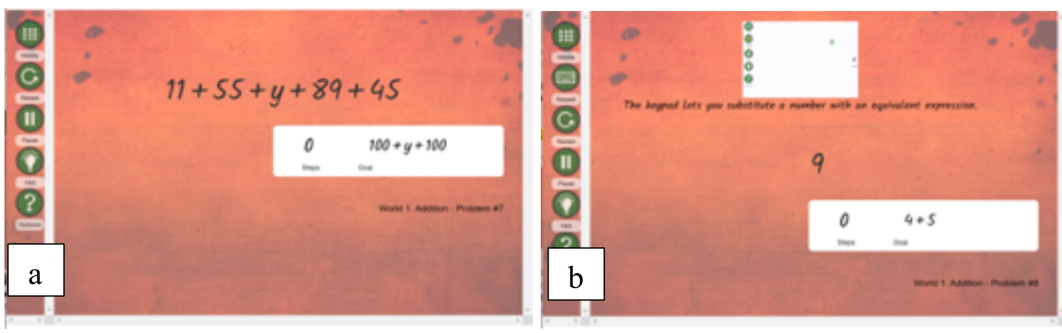


Figure 3.32: Introduce Keypad

2. Mathematically correct expression

The new expression must be mathematically correct especially when players substitute or factor the number. For example, when the player substitutes 64, she made a mistake by writing 31 instead of 32 (see Figure 3.33). However, because the new expression is not correct, when she taps the done button to submit her work, the done button did not transfer the new expression on the screen to substitute 64 like 2. 31. The player then gets feedback: “The total of the new expression should be the same as the original." Another feedback related to the keypad is that when the player deleted 32 mistakenly and tapped on the done button (see Figure 3.34); the done button did not work again and did not permit her to substitute 64 as 2 . $(empty)$ resulting in the feedback, "Could not parse expression."

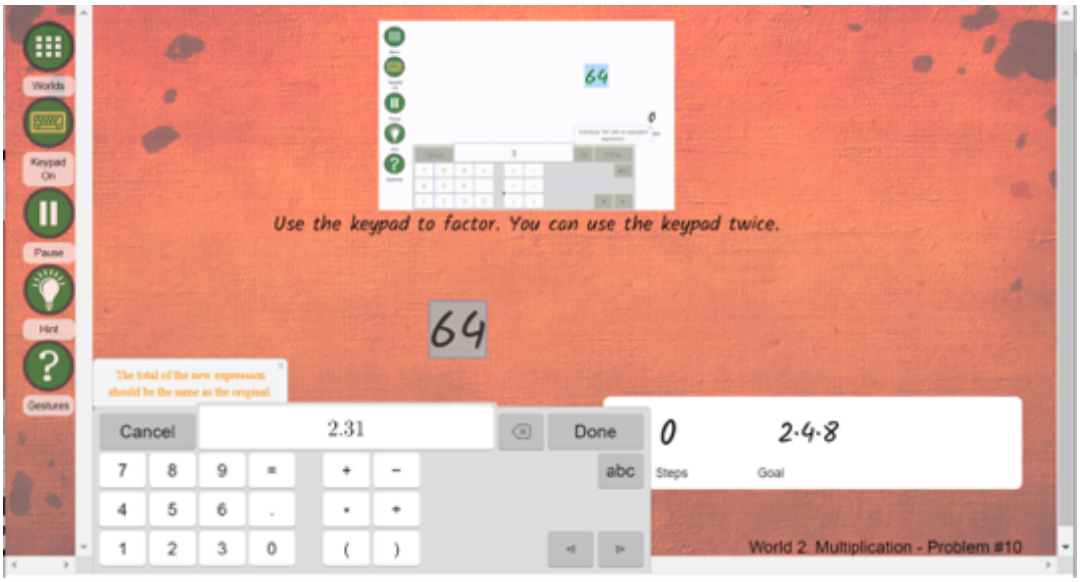


Figure 3.33: New Expression

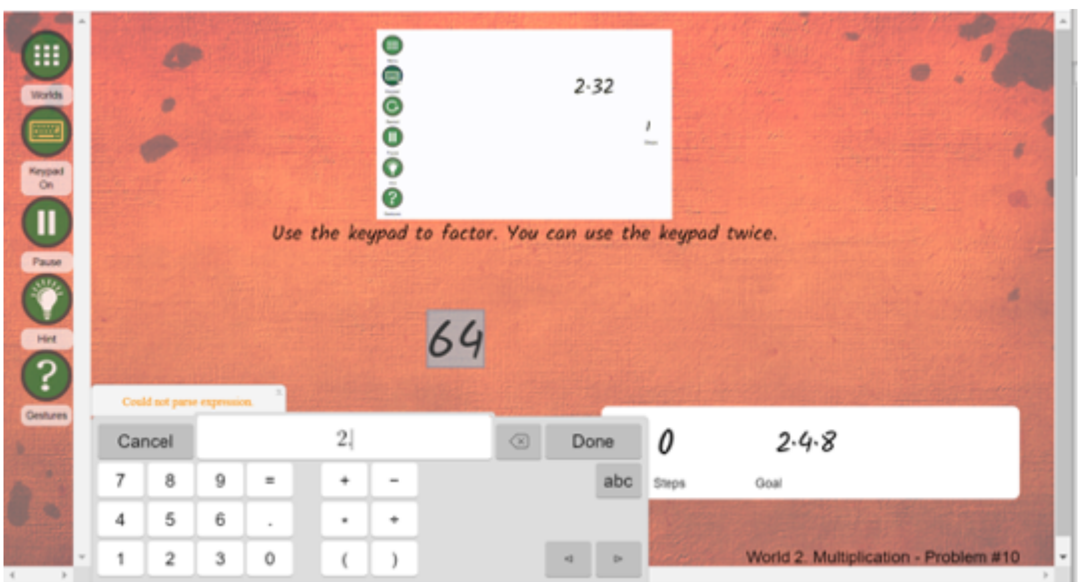


Figure 3.34: Not Parse

Mathematics Learning Standards Analysis Findings

In this study, I analyze the game *From Here to There!.* Before playing educational games in public schools, teachers/managers are asking to confirm that the game covers the mathematics learning standards of their respective states. In the future study, I am planning to observe students in a public school environment. For this reason, I check the Maryland College and Career-Ready Standards framework for 6th and 7th grades' (Maryland State Department of Education, 2017a, 2017b) and prepare Table 3.2 to show which problems in FH2T met the 6th and 7th grades' standards.

Table 3.2: Maryland State P-12 Common Core Math Learning Standards and FH2T

CCLS	State Standard	FH2T ex.
6.EE.A	Apply and extend previous understandings of arithmetic to algebraic expressions	1.1 – 14.18
6.EE.2.b.1 / 6-Y.8	Identify terms and coefficients	2.4
6.EE.2.b.1 / 6-Y.9	Sort factors of variable expressions	3.11
6.EE.2.c / 6-O.3	Evaluate numerical expressions involving whole numbers	1.8
6.EE.3.a / 6-Y.11	Properties of addition	1.1 – 1.18
6.EE.3.a / 6-Y.12	Properties of multiplication	2.1 – 2.18
6.EE.3.a / 6-Y.13	Multiply using the distributive property	10.2
6.EE.4 / 6-Y.17	Add and subtract like terms	3.10
6.EE.4 / 6-Y.19	Identify equivalent expressions II	1.1 – 14.18
6.EE.5 / 6-Z.9	Solve one-step equations with whole numbers	8.4
7.EE.1.a / 7-R.8	Identify terms and coefficients	2.4
7.EE.1.c / 7-R.10	Properties of addition and multiplication	1.1 – 2.18
7.EE.1.c / 7-R.13	Multiply using the distributive property	10.2
7.EE.1.c / 7-R.14	Add and subtract linear expressions	1.1 – 1.18 & 4.1 – 4.18
7.EE.1.c / 7-R.19	Identify equivalent linear expressions II	1.1 – 14.18
7.EE. 2.a	Ability to utilize Properties of Operations in order to rewrite expressions in different forms	1.1 – 14.18
7.EE. 2.b	Ability to develop understanding of equivalent forms of numbers, their various uses and relationships, and how they apply to a problem	1.1 – 14.18

Discussion

This study explored whether FH2T covers all elements of the Game-Based Learning model: instructional contents, feedback system, and learning outcomes - Common Core Standards for our state. The goal of this study was also to determine interesting and challenging puzzles for analyzing students' reactions in the forthcoming Quantitative Gestures and Student Observations studies. Through the analysis exploring FH2T under the GBL model and failure and feedback lens, I have determined that FH2T's failure and feedback system is well designed and helps players avoid or learn from failure to solve algebraic equations.

Limitations

I designed this study as a first part of the dissertation research. The overarching aim of the research is to explore how digital gestures connect to students' mathematical understanding when playing FH2T. However, there are some limitations of the research and this study. First, the participants for each study will be different – in the analysis the participant is myself, in the Quantitative Gesture study there are 358 sixth and seventh grade students, and in the Student Observations study there are only seven undergraduate students. The variability in research participants throughout the research project might affect the research findings because each participant will bring his/her own previous gameplay experiences and mathematical understanding to the study. Moreover, the playing time will be limited in the subsequent studies; I was able to finish all level for this study but in the Quantitative Gesture and Student Observations studies, students may not complete all worlds in FH2T.

Future Studies

The second aim of investigating FH2T is to determine puzzles to look at in the second (Quantitative Gesture) and third (Student Observations) studies. For the second study, examining all puzzles (FH2T contains 252 puzzles) in detail will take too much time. When I was playing the game, I was surpassing the best solution steps of some puzzles, and I thought that some dynamics of some puzzles might be tricky for the students. Therefore, I highlighted these puzzles in the excel sheet (see Table 3.3). Students can skip to play Puzzles 15, 16, 17, and 18 in each world and go to the next world in the game. I will not focus on these puzzles in detail because, after solving Puzzle 14, students could go to the next world. For the third (Student Observations) study, participants will have a time restriction on the data collection process, students could not play World 11 and the following worlds, because, in my playing experience, it took more than 2 hours to finish all worlds. Therefore, I will not focus on Worlds 11, 12,13, and 14. I will only analyze the bolded puzzles in Table 3.3.

Table 3.3: Worlds and Highlighted Puzzles

World Number	Puzzle Number
1	**1.9, 1.12, 1.14**
2	**2.7, 2.10, 2.13, 2.14**
3	**3.11, 3.13, 3.14,** 3.16
4	**4.6, 4.7, 4.10,** 4.15
5	**5.6, 5.7, 5.9, 5.10, 5.11, 5.12,** 5.17
6	**6.6, 6.8, 6.9, 6.10,** 6.15, 6.16, 6.18
7	**7.7, 7.8, 7.10, 7.13, 7.14,** 7.17
8	**8.1, 8.8, 8.11,** 8.15
9	**9.3, 9.4, 9.5, 9.6, 9.9, 9.13**
10	**10.3, 10.5, 10.12,** 10.18
11	11.5, 11.6, 11.12
12	12.3, 12.5, 12.8, 12.9,
13	13.6, 13.13, 13.15
14	14.4, 14.10, 14.16, 14.17, 14.18

My two goals for the second study – Quantitative Gesture – will be to explore whether students' digital gesture clusters in playing FH2T determine if they keep playing or not. I also look at the conditions that affect their game play. With information about students' gestures, I can highlight potential interview questions for the Student Observations study. My two goals for the third – Student Observations – study will be to investigate learners' mathematical emergent experience in playing FH2T and build upon my overall question of how digital gestures connect to students' mathematical understanding when playing FH2T.

Conclusion

I explored FH2T through Game-Based Learning, failure and feedback, and Mathematics Learning Standards frameworks as a first study in a larger research project to answer 1. How do the implications and affordances offered through playing FH2T match with Game-Based Learning's structures? 2. How do failure and feedback manifest in mathematical play while playing the game from the researcher's perspective? 3. Do the learning outcomes of the game endorse the Common Core State Standards mathematics required by the state of Maryland? I used an analysis method to analyze the data.

Results indicate that FH2T serves as an inclusive educational game that effectively engages the student in algebraic content. The structures and affordances implicit in the game are promising for developing an algebraic understanding of learners who play this game. Its failure and feedback system works clearly for the game and mathematical understanding, and FH2T covers Maryland College and Career Ready Standards for mathematics.

In future studies, I will investigate students' gameplay and learning experiences within FH2T through embodied mathematical cognition perspective. In the Quantitative Gesture Study, I will use the quantitative data to investigate if there is any connection among digital gesture clusters' patterns, the length of gameplay, and mathematical understanding. Lastly, in the Student Observations study, I will use the qualitative analysis method to explore students' mathematical gameplay experiences under the embodied mathematical cognition framework.

Chapter 4: The Quantitative Gesture Study

Overview

This study explores how players' digital gestures affect their decision to keep playing *From Here to There!* (FH2T). To understand the implication and affordance of digital gestures within playing FH2T, quantitative data from FH2T development team is analyzed under the embodied mathematical cognition perspective. The study results show that the increase in the number of digital gestures, or the number of steps, causes a greater number of students to quit playing FH2T.

Introduction

Games are increasingly used in education, especially on touchscreen devices such as tablets or other mobile devices. Many researchers argue that touchscreen devices influence students' mathematics learning (Dubé & McEwen, 2015; Katirci et al., 2020; Sedaghatjou & Campbell, 2017; Sinclair & Heyd-Metzuyanim, 2014). In particular, games and applications that are designed for learning mathematics through touchscreen devices are increasingly being used for mathematics education in formal and informal environments. For instance, *TouchCounts* (Sinclair & Heyd-Metzuyanim, 2014), *EstimationLine* (Dube & McEwen, 2015), and *DragonBox 12+* (Katirci et al., 2020) are examples of touchscreen-based mathematics learning games investigated by researchers. Touchscreen technology promotes the interaction between touchscreen devices and humans through digital gestures. These digital gestures include tapping (touching the screen with a fingertip), dragging (moving fingertip over the surface without losing contact), and pinching (touching the surface with two fingers and bringing them closer together) (Villamor et al., 2010).

In this study, I analyze a research-based technology game, *From Here to There!* (FH2T) to further contribute to the literature on mathematics education games and embodied cognition in mathematics education. I investigate the quantitative data gathered by the team of developers in their 2019 study (Chan et al., 2021). This study is one part of a longer trajectory of research (covers three studies to overarching research question: *How do digital gestures connect to students' mathematical understanding when playing FH2T?)* that involves further investigation of how players use their gestures to learn while playing *From Here to There!* (FH2T).

Research questions:

- Do students' digital gestures within playing FH2T affect their decision to keep playing or not?
- What conditions affect their decision to keep playing or not?

Theoretical Frameworks

This section describes the theoretical frameworks of the study. In this study, I will focus on the ways action, or embodied cognition, provides insight into students' mathematical understanding and present my findings through quantitative visual representations. The learning theory of embodied mathematical cognition (Alibali & Nathan, 2012; Cook et al., 2008; Goldin-Meadow et al., 2001; Lakoff & Núñez, 2000; Nathan & Walkington, 2017; Nathan et al., 2016; Núñez, 2004; Walkington et al., 2014; Williams-Pierce et al., 2017) and the lens of visual learning analytics (Lee et al., 2021; Vieira et al., 2018) frame the study.

Embodied Mathematical Cognition (EMC)

Embodied cognition is defined as the ways in which “states of the body modify states of the mind” (Wilson & Golonka, 2013, p.1). Kirsh (2013) claims that cognition is connected between the body and the brain. Additionally, the interaction between mind and environment plays a critical role in the cognitive system (Hollan et al., 2000; Wilson, 2002). Hence, gestures (hand movements) and bodily action should be viewed as cognitive components because they form some of the thinking of the person (Kirsh, 2013).

Gestures play an essential role in mathematics education (Nathan et al., 2013; Reynolds & Reeve, 2001). Researchers agree that mathematical thinking is embodied by the body’s interaction with educational tools or the environment (Cook et al., 2008; Nathan & Walkington, 2017; Nathan et al., 2016; Núñez, 2004; Williams-Pierce et al., 2017). Mathematics teachers also use gestures in their instruction and students express their knowledge through gestures in their mathematics classrooms (Alibali & Nathan, 2012).

Digital gestures (e.g., dragging, tapping; Villamor et al., 2010) are required to use touchscreen tools. Scholars have studied how touchscreen devices influence students’ mathematics learning (Dubé & McEwen, 2015; Katirci et al., 2020; Sinclair & Heyd-Metzuyanim, 2014). Sinclair and Heyd-Metzuyanim (2014) claimed that bodies, fingers, hands, and emotions play an important role in learning mathematics. Dubé and McEwen's (2015) impact study shows that touchscreen gestures are effective in mathematical understanding without any human interaction. Katirci et al.’s (2020) study indicates that the embodied interactions between student-peers and between the student-touchscreen device influenced students' mathematical understanding. Hence, touchscreen gestures play an essential role in learning mathematics through touchscreen equipment (e.g., mobile phones and tablets).

In the game – From Here to There (FH2T) – developers called the game actions *gestures*. Participants use their hands (e.g., fingers on an iPad) to play FH2T and to learn or practice different algebraic concepts. As portrayed on the left bar of Figure 4.1, there are icons that help the players solve puzzles. The fifth icon (Figure 4.1-E) is the 'Gestures' icon figured with a question mark symbol. The Gesture icon gives the players clues as to which gesture (action or rule) is needed to solve the puzzle. Each gesture appears in a listed format with a pop-up message. For example, Figure 4.2-Left shows the name of the rule/gesture (Commute Terms) in Puzzle 1.1 (World 1 Puzzle 1) (See Appendix #4 for more information about all the gestures in FH2T). Figure 4.2-Right shows the detail of the gesture. The gesture explanation is given by text and animated graphics together, just like when the rule was introduced. According to the text-based explanation, the players need to "*drag* terms to commute" to solve the puzzle.

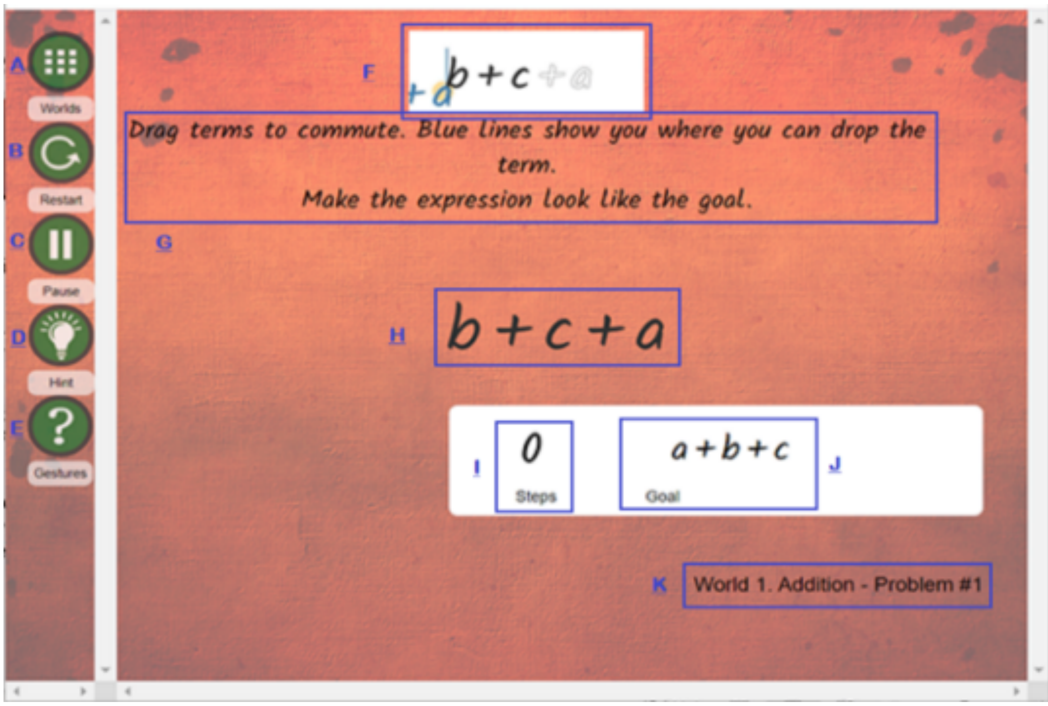


Figure 4.1: E - Gestures

Figure 4.2: Gesture - Commute Terms (Left); Commute Terms in Details (Right)

Visual Learning Analytics

According to The Society for Learning Analytics Research, learning analytics is defined as “the measurement, collection, analysis and reporting of data about learners and their contexts, for purposes of understanding and optimizing learning and the environments in which it occurs” (Siemens & Gasevic, 2012, p.1). Moreover, Thomas and Cook (2005) defined visual analytics as "the science of analytical reasoning facilitated by interactive visual interfaces" (p.4). The combination of these two areas, *visual learning analytics,* is defined as "the use of computational tools and methods for understanding educational phenomena through interactive visualization techniques" (Vieira et al., 2018, p. 120). In this study, I used visual learning analytics to understand students' behavior.

Students who played the game generated large datasets comprised of clickstream data. These data give information about students' behavior during gameplay. Processing and interpreting data on students' behavior also provides information about their learning in both the game and mathematics. The game developers provided visual learning analytics from the large dataset and shared them with me.

In this study, there are four different types of visual representations of the data: Measure Charts, Treemaps, Sankey Diagrams, and Indivisualizer (I explain each of them in detail in the next section). Through these visual representations, I investigate the impact of gestures on participants' puzzle solving strategies. The visual representations provide clues into participants' gestures, and this means that they were engaging in these gestures. However, I could not understand participants' gestures in a detailed way because I did not have enough information about participants' specific gestures. For example, there are two ways to add terms in the game. One is dragging the item over to the other item and one is tapping to the operation sign (+ symbol) between two items. In the data I have, I can see that the player added, but not which of those two approaches they took. In this study, I only consider gestures that I know occurred. In future studies, I will observe participants' actions or strategies to be able to consider any gestures that they made.

Methods

In this study, I am using secondary data collected by the designer team of the game (Graspable Math Team). This section describes the methods of the Quantitative Gesture study and provides a detailed explanation of the purpose of the study as well as the research questions, participants, data (corpus in use, the visual representation of the quantitative analysis available with the corpus), and study design.

Purpose of the Study and Research Questions

I propose that students' gestures are correlated to the length of FH2T gameplay. While playing FH2T in the previous (the Game Interaction) study, I realized that making gestures in incorrect orders caused errors such as keypad[4], shaking[5], and snapping[6] errors and that these errors might lead players to quit the game. Conversely, making gestures in correct order may lead students to keep playing the game and to understand the mathematical concepts better. To test this hypothesis, I (as a researcher) use an embodied cognition perspective to analyze the quantitative data (gathered by the team of developers in their fall 2019 study) (Figure 4.3) to answer the following research questions:

- Do students' digital gestures within playing FH2T affect their decision to keep playing or not?

[4] Keypad Errors: Errors that the student made by attempting to enter a non-equivalent expression using the keypad.
[5] Shaking Errors: Errors that the student made by attempting to incorrectly use existing operators.
[6] Snapping Errors: Errors that the student made by attempting to incorrectly reorder terms.

- Under what conditions affect their decision to keep playing or not?

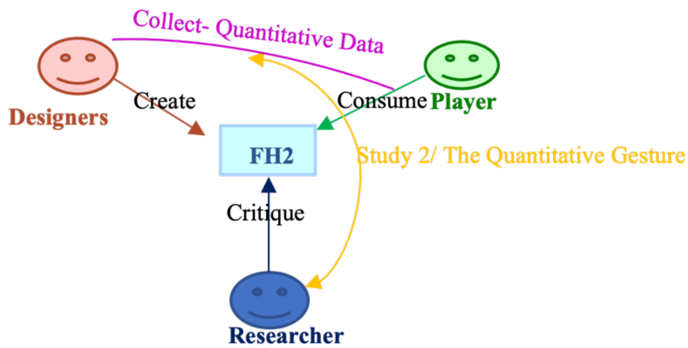


Figure 4.3: The Quantitative Gesture Study

Participants

In fall 2019, quantitative data (e.g., students ID, completed problems, time interactions, number of steps, errors, etc.) was collected from eleven teachers and 358 students (6^{th} and 7^{th} grades) across six middle schools in the Southeastern United States. Dr. Ottmar and colleagues, who analyze FH2T data, released this student clickstream data from the game for researchers (Chan et al., 2021). For this study, I use their data visuals — Measure Charts, Treemaps and Sankey Diagrams, and Indivisualizer— to investigate my research questions: Do students' digital gestures within playing FH2T affect their decision to keep playing or not?, Under what conditions affect their decision to keep playing or not? These four visuals have different features. Measure Charts, Treemaps, and Sankey Diagrams cover all students' data in one graph. I investigated these representations for specific puzzles and analyzed them in detail. The Indivisualizer is conducted for each student individually for each puzzle. I randomly selected 15 students by using Excel's random number generator (Quirk et al., 2020) and analyzed their data in detail.

The graphics (Figure 4.4-Left) from the research dashboard illustrate that 358 students solved Puzzle 1.1 (World 1 Puzzle 1). However, in Puzzle 11.6 (World 11 Puzzle 6) (Figure 4.4-Right), only 16 participants (less than 5%) completed the 6th Puzzle in World 11. The number of students solving the puzzles gradually decreases. Since only 5% of participants solved Puzzle 11.6, I decided to investigate if there is any connection between patterns of digital gestures used and the length of gameplay – from Puzzle 1.1 to Puzzle 11.6. In other words, are students discouraged by making unsuccessful gesture patterns that cause them to make an error, and do they stop playing FH2T because of these gestures?

Number of Students Completed	358
Measures	Sample mean
Completion (1 = Yes / 0 = No)	1.000
Number of steps	1.435
Number of go-backs	0.086
Number of resets	0.059
Step-efficiency_first	0.879
Step-efficiency_last	0.902
Time taken (sec)	21.8349
Pause time - first (%)	0.726
Pause time - last (%)	0.738
Use of hints (1 = Yes / 0 = No)	0.009
Number of total errors	0.537
Number of keypad errors	0.000
Number of shaking errors	0.114
Number of snapping errors	0.463

The proportion of students who completed the problem by successfully reaching the goal state at least once

Number of Students Completed	16
Measures	Sample mean
Completion (1 = Yes / 0 = No)	0.049
Number of steps	6.941
Number of go-backs	0.000
Number of resets	2.000
Step-efficiency_first	0.867
Step-efficiency_last	0.823
Time taken (sec)	67.0929
Pause time - first (%)	0.596
Pause time - last (%)	0.276
Use of hints (1 = Yes / 0 = No)	0.012
Number of total errors	1.529
Number of keypad errors	0.529
Number of shaking errors	0.118
Number of snapping errors	1.118

The proportion of students who completed the problem by successfully reaching the goal state at least once

Figure 4.4: Measure Chart for Puzzle 1.1 - Left and Puzzle 11.6 - Right.

Data and Study Design

Quantitative data (e.g., Figure 4.4) was collected by the developers of FH2T in 2019 (Chan et al., 2021). The developers also established a researcher dashboard to provide information about students while they played FH2T. I accessed the data for this study through the dashboard. The game dashboard has its own data logging system which records each student’s moment-by-moment problem-solving steps such as time spent, mouse clicks, and errors (See Figure 4.4). In the dashboard, students’ in-game behaviors are displayed through different kinds of data visualizations: Measure Chart (See Figure 4.5); Treemap (See Figure 4.6); Sankey Diagram (See Figure 4.7) and Indivisualizer (See Figure 4.8). In the Quantitative Gesture study, I used an embodied mathematical cognition perspective to code these four visualizations of different puzzles of the game and looked for emergent patterns (See Appendix 4.3 Flow Chart).

Measure Chart

The Measure Chart displays information about the puzzle and the players’ actions. I used the Measure Chart to understand the likelihood and the conditions of students quitting the game. Figure 4.5 shows the chart of Puzzle 1.1 for the 358 players of the study. In my analysis, I am interested in the labeled rows: ‘Number of students completed’, ‘Number of steps’, ‘Number of total errors’, ‘Number of keypad errors’, ‘Number of shaking errors’, and ‘Number of snapping errors’ (marked by blue stars in Figure 4.5). The rows labeled ‘number of students completed’ provides information about how many students solved the puzzle. The differences in the ‘Number of students completed’ between two puzzles explains how many students quit playing the game, or how many students decided to stop solving the second puzzle. Another row, ‘the number of steps’, will give me the average number of steps it took students to solve the puzzle. In the game, each step also means an action or gesture. Therefore, the ‘the number of steps’ row displays how many gestures had been done by players to solve the puzzle. Other rows in Figure 4.5, ‘number of total errors,’ ‘number of keypad errors’, ‘number of shaking errors’, and ‘number of snapping errors’ give information about what kind of errors students made. In the game, errors also indicate the possibility of the player doing incorrect gestures while solving the puzzles. I looked at the Measure Charts of puzzles[7] from the Game Interaction Study which has a high number of total errors, keypad errors, shaking errors, and snapping errors.

Number of Students Completed	358
Measures	Sample mean
Completion (1 = Yes / 0 = No)	1.000
Number of steps	1.435
Number of go-backs	0.086
Number of resets	0.059
Step-efficiency_first	0.879
Step-efficiency_last	0.902
Time taken (sec)	21.8349
Pause time - first (%)	0.726
Pause time - last (%)	0.738
Use of hints (1 = Yes / 0 = No)	0.009
Number of total errors	0.537
Number of keypad errors	0.000
Number of shaking errors	0.114
Number of snapping errors	0.463

Note: Stars mark relevant information.

Figure 4.5: Measure Chart for Puzzle 1.1.

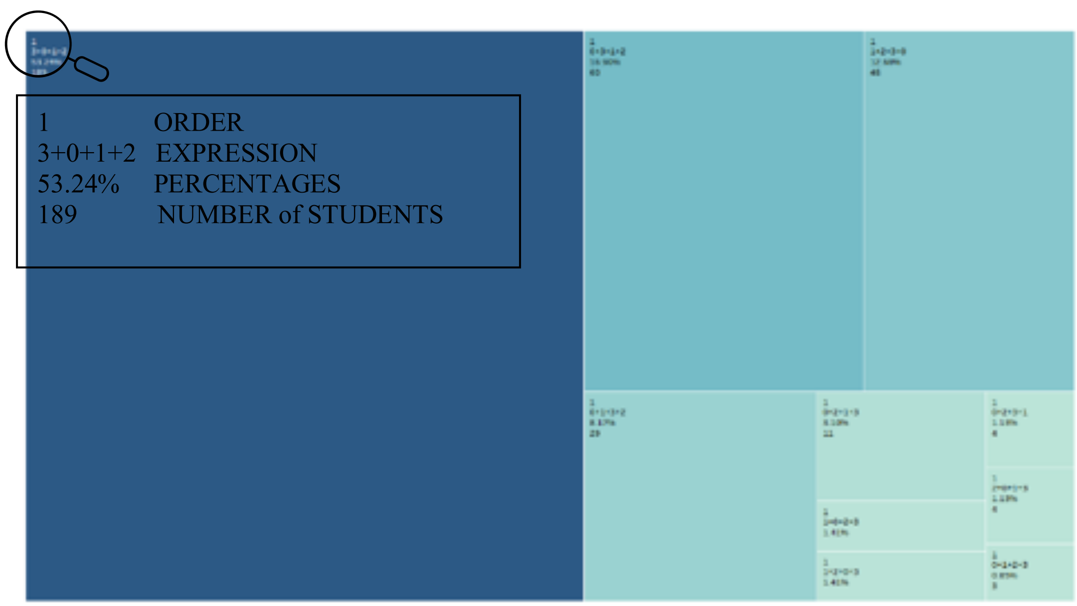
Treemap

Treemap is another model of representation that displays the first actions made by all participants for a single puzzle. Figure 4.6 shows the Treemap of Puzzle 1.2 which illustrates frequencies for each action by area, size, and color across all the students. The size and color of the rectangles are determined by the number of

[7]Puzzles: 1.9, 1.12, 1.14, 2.7, 2.10, 2.13, 2.14, 3.11, 3.13, 3.14, 3.16[*], 4.6, 4.7, 4.10, 4.15[*], 5.6, 5.7, 5.9, 5.10, 5.11, 5.12, 5.17[*], 6.6, 6.8, 6.9, 6.10, 6.15[*], 6.16[*], 6.18[*], 7.7, 7.8, 7.10, 7.13, 7.14, 7.17[*], 8.1, 8.8, 8.11, 8.15[*], 9.3, 9.4, 9.5, 9.6, 9.9, 9.13, 10.3, 10.5, 10.12, 10.18[*].

[*]Player can skip the puzzles 15, 16, 17 and 18 in each World.

students and by the percentages of students. Darker and larger rectangles indicate a greater number of students who took that specific step. In Figure 4.6, each rectangle introduces four pieces of information: the first line (e.g., 1, see the black text box on the figure) shows in which order the students used the expression; the second line (e.g., 3+0+1+2, see the black text box on the figure) represents the first mathematical expression made by students; the third line (e.g., 53.24%, see the black text box on the figure) indicates the percentages of students; and the fourth line (e.g., 189, see the black text box on the figure) represents the number of students who made that expression. In other words, 53.24% of all students prefer to first drag ‘3’ or ‘the last item on the right’ to the left. In the analysis, I explain in detail how I code the Treemaps of different puzzles.

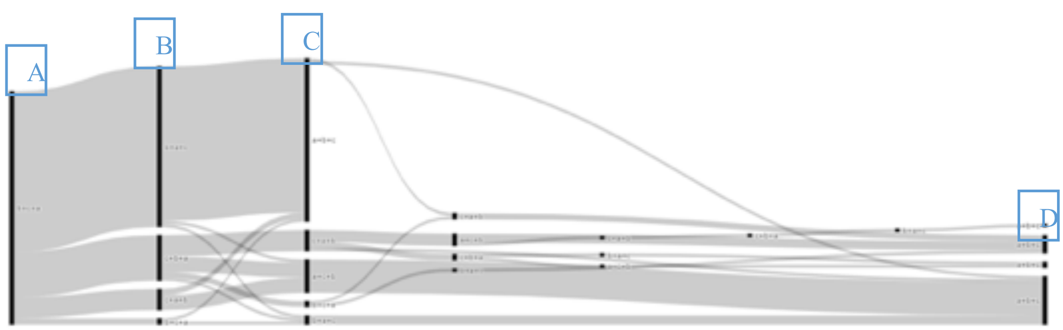


Note: Magnified glass and black box are added to help readers.

Figure 4.6: Treemap for Puzzle 1.2.

Sankey Diagram

The Sankey Diagram (Flow Diagram) displays the flow of the strategies for solving the puzzles. The designers use the Sankey Diagram to visualize different strategies of students into one graph (Lee et al. 2021). Figure 4.7 shows the diagram of Puzzle 1.1 which represents the flow of solutions (the steps of solutions) and their frequencies (student density) from one set of values to another by using line widths. In the diagram, the vertical bars (e.g., b+c+a – labeled as A; b+a+c – labeled as B; a+b+c – labeled as C – for Puzzle 1.1) show the students' mathematical problem-solving steps or their choice of mathematical expressions. The thickness of the path illustrates the number of students who took that path. The Sankey Diagram gives me a representation of the quantitative data, but it does not clearly explain the problem-solving strategies or patterns of students. To understand the strategies of students, I recorded Sankey Diagrams of selected puzzles[8] (from the Game Interaction Study) in folders categorized by puzzles and coded students' gestures through diagrams. In the analysis section, I explain in detail how I code the Sankey Diagrams of puzzles.



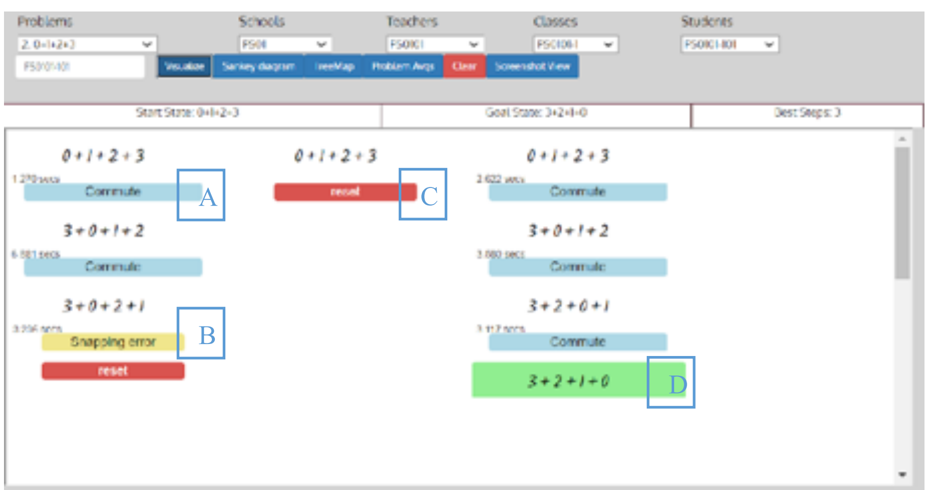
Note: A, B, C, D labels are added to help readers.

Figure 4.7: Sankey Diagram for Puzzle 1.1

[8]Puzzles: 1.12, 2.13, 3.11, 3.16*,
*Player can skip the puzzles 15, 16, 17 and 18 in each World.

Indivisualizer

The Indivisualizer is another representation that illustrates information about the participants individually. Figure 4.8 is an Indivisualizer showing one participant's actions between the start state and the goal state in Puzzle 1.2. Figure 4.8 represents the start states, goal states, and best steps of Puzzle 1.2. The light blue boxes (labeled as A in Figure 4.8) represent the player's actions leading to the transformation and the number above the light blue box indicates the time (in seconds) taken between transformations. The light-yellow boxes (labeled as B in Figure 4.8) represent the player's actions as an error (keypad error, snapping error, shaking error). The red boxes (labeled as C in Figure 4.8) indicate that the player hit the reset button to restart the puzzle. The green box (labeled as D in Figure 4.8) indicates that the participant reached the goal state of the puzzle. In the analysis section, I explain in detail how I code the participants' indivisualizers.



Note: A, B, C, D labels are added to help readers

Figure 4.8: A Student's Actions in Puzzle 1.2 - Indivisualizer.

In summary, the first purpose of this study is to investigate the students' strategies for solving the highlighted puzzles in the game interaction study then to understand the patterns of players' gestures in those puzzles, for to answer the research questions: *Do students' digital gestures within playing FH2T affect their decision to keep playing or not? What conditions affect their decision to keep playing or not?* This study will also help me identify potential interview questions for the student observation study conducted in Chapter 5, building upon my overarching research question: *How do digital gestures connect to students' mathematical understanding when playing FH2T?*

Analysis

I discuss my analysis in four analytical parts: (1) Analyzing Measure Charts, (2) Analyzing Indivisualizers, (3) Analyzing Treemaps, and (4) Analyzing Sankey Diagrams. This section provides a detailed explanation of these analyses.

Analyzing Measure Charts

The first part involved using Measure Charts to examine patterns of progress for all the players. I analyzed if there is any correlation among labeled rows in the Measure Charts for each puzzle (See Figure 4.9). To analyze, I created an Excel Sheet (See Table 4.1) to record the labeled rows I am interested in (See blue stars in Figure 4.9 – 'Number of students completed', 'Number of steps', 'Number of total errors', 'Number of keypad errors', 'Number of shaking errors', 'Number of snapping errors'). Then to answer the research question – "Are there digital gestures that relate to whether or not students keep playing?" – I added the columns named 'Number of students left,' 'Best Step,' and 'Mean - Best Steps'. 'Number of students left' is the difference in the ‘Number of students completed’ between two puzzles. ‘Best Step’ is the minimum number of steps to solve the puzzle. ‘Mean - Best Steps’ is the difference between the ‘Number of steps’ and ‘Best Step’ for the puzzle. I had selected 49 puzzles[9] from over 256 puzzles of the game based on my play experience (see the Game Interaction Study – Chapter 3 – for my selection parameters). I checked the rows of these puzzles (except Puzzles 3.16, 4.15, 5.17, 6.15, 6.16, 6.18, 7.17, 8.15, and 10.18 because students can pass these puzzles and start the new world).

Number of Students Completed	358
Measures	Sample mean
Completion (1 = Yes / 0 = No)	1.000
Number of steps	1.435
Number of go-backs	0.086
Number of resets	0.059
Step-efficiency_first	0.879
Step-efficiency_last	0.902
Time taken (sec)	21.8349
Pause time - first (%)	0.726
Pause time - last (%)	0.738
Use of hints (1 = Yes / 0 = No)	0.009
Number of total errors	0.537
Number of keypad errors	0.000
Number of shaking errors	0.114
Number of snapping errors	0.463

Note: Stars mark relevant information.

Figure 4.9: Measure Chart for Puzzle 1.1.

[9] Puzzles: 1.9, 1.12, 1.14, 2.7, 2.10, 2.13, 2.14, 3.11, 3.13, 3.14, 3.16[*], 4.6, 4.7, 4.10, 4.15[*], 5.6, 5.7, 5.9, 5.10, 5.11, 5.12, 5.17[*], 6.6, 6.8, 6.9, 6.10, 6.15[*], 6.16[*], 6.18[*], 7.7, 7.8, 7.10, 7.13, 7.14, 7.17[*], 8.1, 8.8, 8.11, 8.15[*], 9.3, 9.4, 9.5, 9.6, 9.9, 9.13, 10.3, 10.5, 10.12, 10.18[*]

[*] Player can skip the puzzles 15, 16, 17 and 18 in each World.

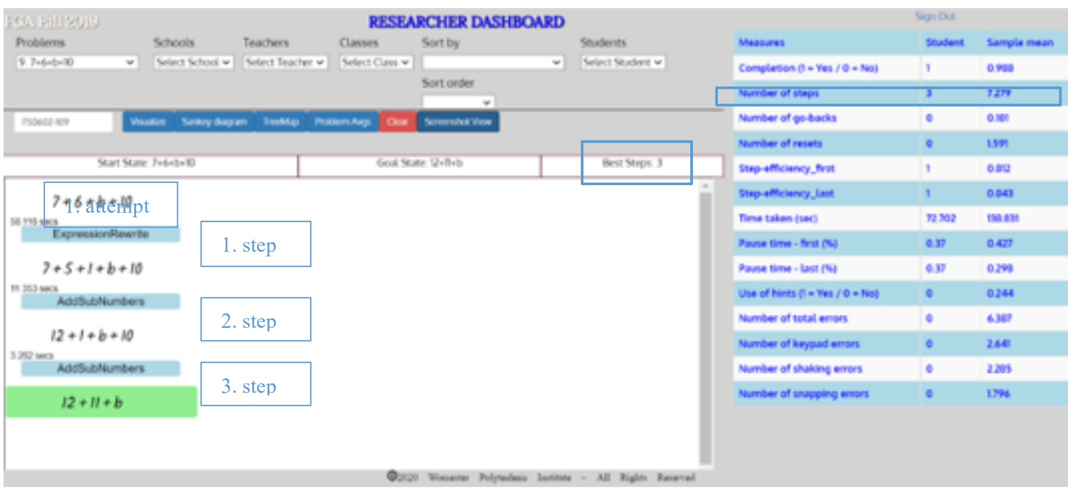
Then, I used the Excel *CORREL* function (Salkind, 2017) to understand the relationship between:

a) The number of students left and the number of steps
b) The number of students left and the number of differences between mean and best steps
c) The number of students left and the total number of errors
d) The mean number of steps and the total number of errors
e) The number of differences between mean and best steps and the total number of errors

Analyzing Indivisualizers

In the second analytical part, Indivisualizers were used to review students' actions. The Indivisualizer is conducted for each student individually for each puzzle. I randomly selected 15 students (~5% of participants) by using Excel's random number generator (Quirk et al., 2020) and analyzed each of their data in detail. For the current analysis, I checked the performance of the selected students in these puzzles by using their measure charts. Depending on their status, I labeled the students into three categories: I marked students as *Solved* (S) when they solved the puzzle, as *Not Attempted* (NA) when they did not try to solve the puzzle, or as *Not Solved* (NS) when they started to solve but did not finish it. Student labels were added to an excel spreadsheet that displayed student progress up to World 11 (see Appendix 4.1 Players' Status).

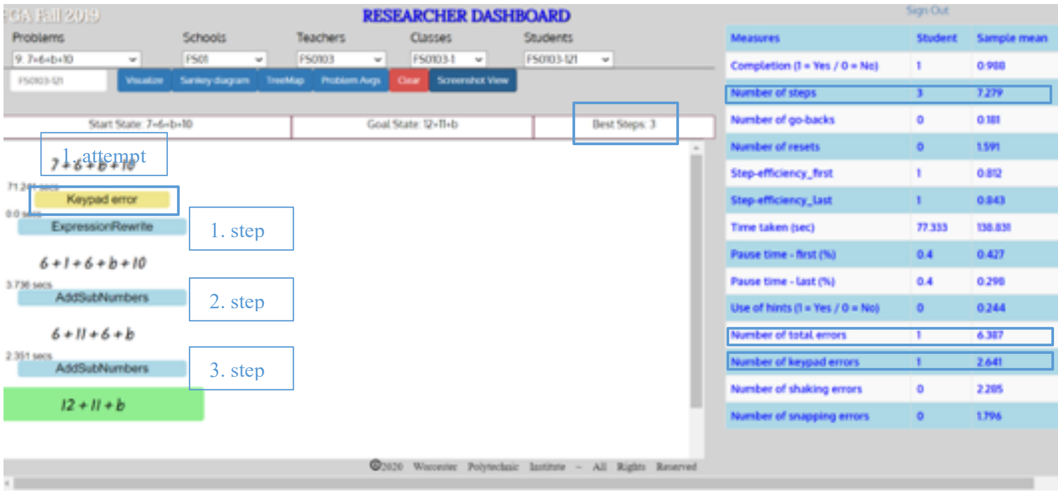
I then coded each student's Indivisualizers of the selected puzzles to determine how many attempts and steps the student took to solve each puzzle. Depending on the number of steps, I categorized students as *Good – G* (who solved in the best number of steps, Figure 4.10), *Moderate – M* (who solved between the best and mean number of steps, Figure 4.11), or *Developing – D* (who solved in more than the mean number of steps, Figure 4.12). I realized that some students in the *Good* category solved without any errors while other students in the same category made errors when solving the puzzle. In order to distinguish between the two, I did a second round of analysis that included the number of errors students made in the coding and separated these errors by type (keypad, shaking, snapping errors). I then split the category of *Good* students in two as *Good – G* and *Good+Error – G+E* (Figure 4.13). Hence the order for the codes from successful to effortful is *G, G+E, M*, and *D* (see Figures 4.10 – 4.13; see Appendix 4.2 Performance of Players).



Note: Blue box labels are added to help readers.

Figure 4.10: Good - G Code

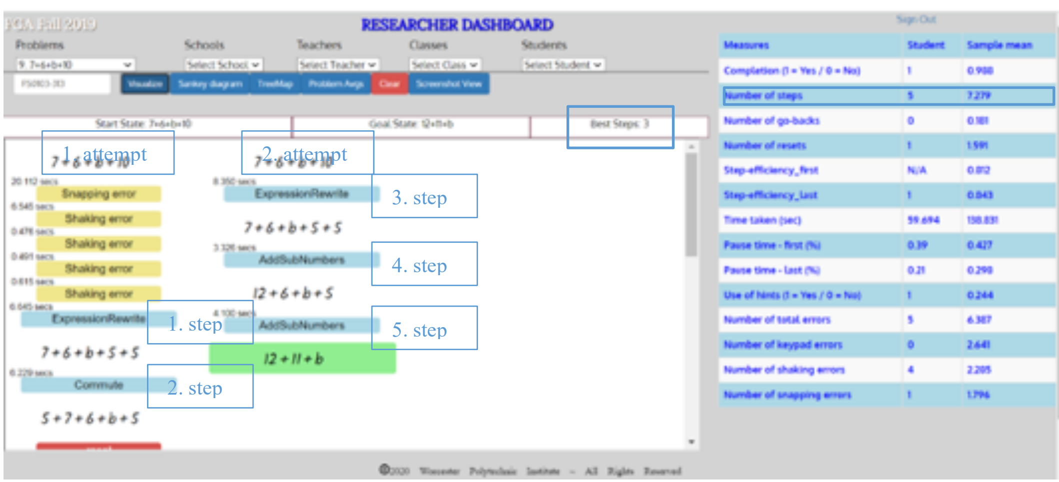
Figure 4.10 shows that the student (FS0602-109) solved Puzzle 1.9 in one attempt and three steps and the best number of steps for this puzzle is also three. As a result, this player was coded as a *Good - G* student for Puzzle 1.9.



Note: Blue box labels are added to help readers.

Figure 4.11: Good + Error - G+E Code

Figure 4.11 shows that one student (FS0103-121) solved Puzzle 1.9 in one attempt and three steps. The best number of steps for this puzzle is also three, however, the student made a keypad error. As a result, this player was coded as a *Good + Error - G+E* student for Puzzle 1.9.

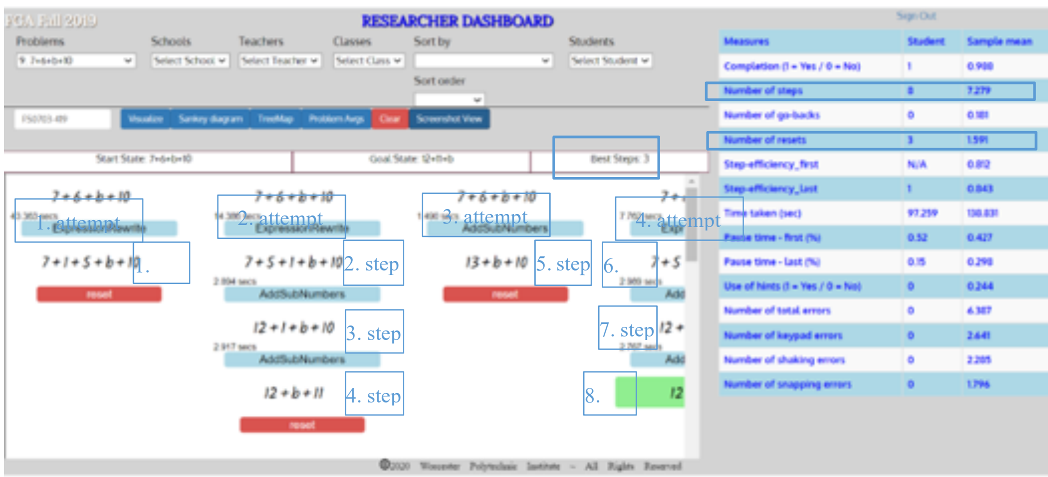


Note: Blue box labels are added to help readers.

Figure 4.12: Moderate - M Code

Figure 4.12 shows that one of the students (FS0103-313) solved Puzzle 1.9 in two attempts and five steps. The best number of steps for this puzzle is three and the sample mean step number is 7.279. As a result, this student was coded as a *Moderate - M* student for Puzzle 9.

Figure 4.13 shows that another student (FS0703-419) solved Puzzle 9 (1.9) in four attempts and eight steps. The best number of steps for this puzzle is three and the sample mean step number is 7.279. Therefore, this student was coded as a *Developing - D* player for Puzzle 9.



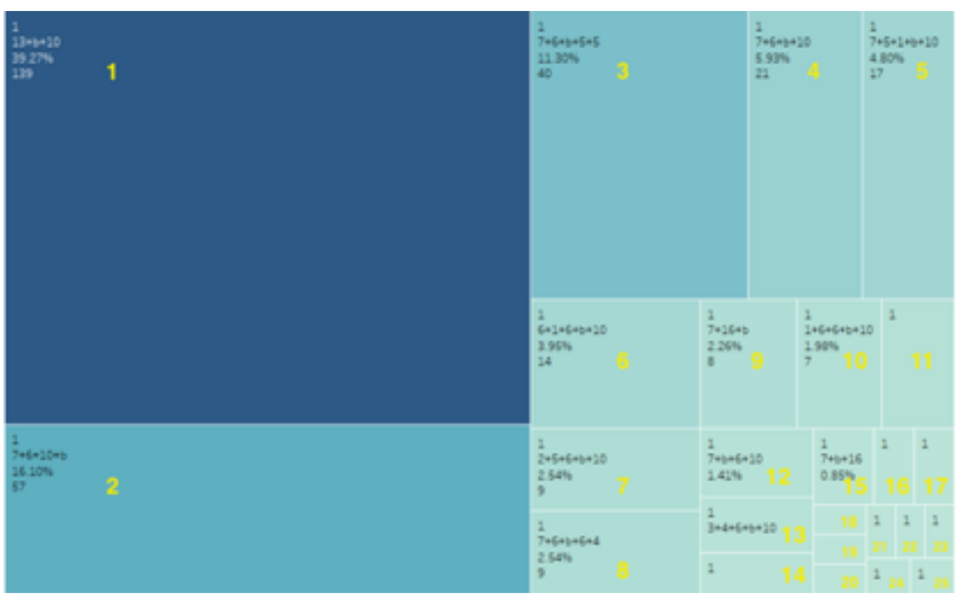
Note: Blue box labels are added to help readers.

Figure 4.13: Developing - D Code

I marked the students' actions in this second analytical part based on what were their situation on the puzzle (e.g., *Solved (S), Not Solved (NS),* and *Not Attempt (NA)*), then coded their performance in the puzzle – *Good (G), Good +Error (G+E), Moderate (M),* and *Developing (D),* see Appendix 4.2 Performance of Players. I needed more data to understand the gestures of the players. Hence, I screen-captured Treemaps and Sankey Diagram of each puzzle in the next analysis because these graphs provide more insight into students' gestures.

Analyzing Treemaps

The third analytical part compares each player's Indivisualizers and Treemaps of the puzzles. For the comparison, I first numbered the Treemaps for each puzzle[10] from largest to smallest percentage (Figure 4.14) and added the total numbers into the coding sheet (for example, for the Puzzle 1.9 there are 25 different first actions, see Table 4.2). Then, I used this number system to code each participant's first action in the puzzles (For example, student FS0103-121 – Player 1–, Figure 4.11, did 6+1+6+b+10 which belongs the 6th box, Figure 4.14, see Table 4.2 for all coding). In the second round of this analytical part, I calculated the Pearson correlation coefficients to understand if there is any relation between: 1) the variety in the types of first actions taken by students and the mean number of steps of the puzzle and 2) the first action of the player and the performance of the player.



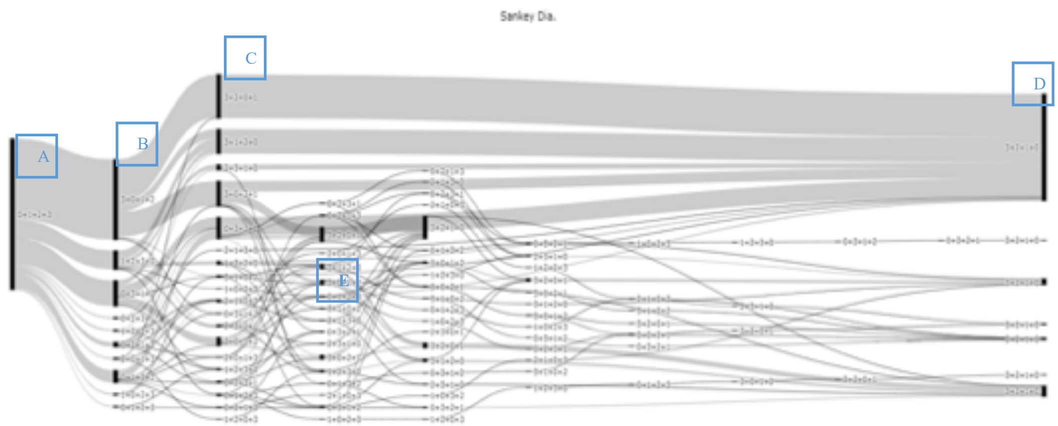
Note: Numbered from largest to smallest percentages.

Figure 4.14: Treemap for Puzzle 1.9.

[10] The dashboard provided Treemaps of the following selected Puzzles: 1.9, 1.12, 1.14, 2.7, 2.10, 2.13, 2.14, 3.11, 3.14, 3.16, 4.7, 4.10, 4.15, 5.17.

Analyzing Sankey Diagrams

I then investigated the Sankey Diagrams to understand the different problem-solving approaches students use to complete the puzzles; in other words, their variability in puzzle solving. For instance, Figure 4.15 illustrates the students' different steps to solve Puzzle 1.2. The puzzle consists of two mathematically equivalent expressions, a start state (labeled as A in Figure 4.15) and a goal state (labeled as D in Figure 4.15). For Puzzle 1.2, the minimum – best – number of steps to solve it is 3. Some students were able to solve in 3 steps, however others exceeded 3 steps because they did what I am calling *nonproductive gestures*. I define productive gestures as gestures that help students solve the puzzle in the best number of steps. Conversely, nonproductive gestures are extra steps that are not necessary for solving the puzzle. Hence, I coded the Sankey Diagrams of the Puzzle for productive (ex. B and C) and nonproductive (ex. E) gesture steps. For example, to solve Puzzle 1.2 in 3 steps, most students preferred to drag the last item on the right to the left, in order of 3, then 2, then 1; Starting → 0+1+2+3 (labeled as A in Figure 4.15), First Step → 3+0+1+2 (labeled as B in Figure 4.15), Second Step → 3+2+0+1 (labeled as C in Figure 4.15), Goal → 3+2+1+0 (labeled as D in Figure 4.15).



Note: A, B, C, D, E labels are added to help readers.

Figure 4.15: Sankey Diagram for Puzzle 1.2.

I recorded the Sankey Diagrams of each puzzle, then selected one puzzle from each categorization of players performance; *developing* (e.g., Puzzle 1.12), *moderate* (e.g., Puzzle 2.13), *good* (e.g., Puzzle 3.11) and *not attempt* (e.g., Puzzle 3.16), and analyzed these four in detail to understand the student gesture in different categorization.

Findings

This section covers the findings of each analytic section separately. (See Appendix 4.4 Concept Map for the Summary)

Analyzing Measure Charts

In the first analytical session, Analyzing Measure Charts, I created an Excel Sheet (Table 4.1) to record the labeled rows that I am interested in.

Table 4.1: Measure Charts' Data

Puzzle #	# Student completed	# Student left	# Best Steps	# Steps	Mean - Best Steps #	# Total errors	# Keypad errors	# Shaking errors	# Snapping errors
9	352	6	3	7.279	4.279	6.387	2.641	2.285	1.796
12	351	7	5	9.863	4.863	3.900	1.903	1.447	0.788
14	350	8	2	4.194	2.194	0.721	0.041	0.295	0.505
25	348	10	2	3.057	1.057	1.658	0.000	1.348	0.468
28	345	13	2	3.386	1.386	3.339	2.136	1.066	0.282
31	344	14	3	5.051	2.051	1.206	0.400	0.511	0.387
32	343	15	2	4.701	2.701	1.003	0.366	0.255	0.503
47	309	49	1	1.210	0.210	2.068	1.146	0.549	0.756
49	304	54	2	2.864	0.864	8.514	4.687	1.425	2.687
50	300	58	3	6.354	3.354	3.784	2.076	1.038	0.780
60•	278	80	4•	15.068•	11.068•	4.367	0.604	2.309	2.000
61	274	84	1	3.121	2.121	0.721	0.121	0.191	0.423
64	269	89	1	1.075	0.075	0.616	0.511	0.082	0.060
78	246	112	3	8.306	5.306	1.253	0.437	0.188	0.849
79	243	115	2	2.905	0.905	0.148	0.004	0.037	0.165
81	239	119	2	3.188	1.188	2.017	0.613	1.042	0.650
82	233	125	3	4.954	1.954	2.903	1.765	0.727	0.550
83	233	125	2	2.164	0.164	0.599	0.517	0.069	0.043
84	232	126	3	4.953	1.953	1.129	0.228	0.250	0.780
96	208	150	2	2.212	0.212	0.962	0.590	0.278	0.255
98•	195	163	2•	10.500•	8.500•	7.293	0.976	4.938	1.971
99•	188	170	4•	14.431•	10.431•	6.292	0.426	4.097	2.815
100•	182	176	5•	31.293•	26.293•	9.654	0.495	6.149	5.128
115•	149	209	8•	19.307•	11.307•	4.493	0.040	3.460	2.500
116•	144	214	7•	21.698•	14.698•	11.678	0.376	9.591	11.577
118•	133	225	6•	20.745•	14.745•	26.596	3.525	19.745	4.950
121•	127	231	6•	19.176•	13.176•	19.183	0.725	13.969	6.267
122	126	232	6	11.969	5.969	2.969	0.071	1.898	1.717
127	117	241	1	1.516	0.516	3.885	2.180	0.713	1.869
134	91	267	6	8.781	2.781	1.063	0.375	0.146	0.844
137	80	278	10	18.875	8.875	11.216	1.318	7.420	3.125
147	57	301	6	8.034	2.034	1.431	0.000	0.897	0.638
148•	53	305	7•	40.193•	33.193•	5.246	0.333	2.842	2.789
149	51	307	8	22.528	14.528	3.415	0.283	1.717	3.057
150	48	310	7	12.922	5.922	1.392	0.000	0.941	4.804
153•	40	318	6•	18.500•	12.500•	2.452	0.095	1.071	1.738
157	37	321	8	12.256	4.256	0.872	0.026	0.487	0.846
165	35	323	1	1.657	0.657	1.314	0.000	0.743	0.657
167	35	323	5	7.143	2.143	4.343	0.000	3.314	1.257
174	31	327	5	9.406	4.406	8.156	0.000	3.594	6.719

Note: • mark the results which is twice as much of best step numbers.

I used the Excel *CORREL* function (Salkind, 2017) to calculate various Pearson correlation coefficients. Separate Pearson correlation coefficients were computed to assess various relationships (see Table 4.2).

Table 4.2: Pearson Correlation Coefficients among variables

Variables	Coefficient Constant r	T-value t	Significance p
The number of students left and the number of steps	0.4785	3.3595	0.0018
The number of students left and the number of differences between mean and best steps	0.3696	2.4516	0.0189
The number of students left and the total number of errors	0.2036	1.2821	0.2076
The mean number of steps and the total number of errors	0.5029	3.5864	0.0009
The number of differences between mean and best steps and the total number of errors	0.4912	3.4765	0.0013

According to Table 4.2, there was a significant positive but weak correlation between the number of students left and the number of steps variables, $r = .4785$, $N = 40$, $p = .0018$. Moreover, the correlation between the number of students left and the number of differences between mean and best steps was significantly positive but weak, $r = .3696$, $N = 40$, $p = .0189$. These two relationships show that an increase in the number of steps to solve the puzzle was correlated to an increase in the number of students who left the game. As the number of steps increases in each new puzzle, more steps also cause more time to play, which might mean that students stopped playing because time was over. Hence, there is a correlation between steps and leaving the game however, there is another variable that I did not include in my models which is the time.

The relationship between the number of students left and the total number of errors was weak and positive, $r = .2036$, $n = 40$; however, the relationship was not significant ($p = .2076$). This shows that the number of students left did not appear to be associated with the number of errors. On the other hand, there was a significant moderate, positive correlation between the mean number of steps and the total number of errors, $r = .5029$, $n = 40$, $p = .0009$. The relation between the number of differences between mean and best steps and the total number of errors was a significant positive and moderate relationship, r = .4912, n = 40, p = .0013. This shows that an increase in the total number of errors was correlated to an increase in the number of steps for solving the puzzles. Similarly, as the number of errors increases in each new puzzle, more errors also cause students to think the problem is difficult for them, then cause more steps to solve it.

In addition, players were particularly struggling in Puzzles 60 (4.6), 98 (6.8), 99 (6.9), 100 (6.10),115 (7.7), 116 (7.8), 118 (7.10), 121 (7.13), 148 (8.4), and 153 (9.9) because the mean number of steps is more than twice the best step amount. I will observe students' reactions and strategies to solve these puzzles in the next study.

Analyzing Indivisualizers

In analyzing Indivisualizers, I first created an Excel sheet (See Appendix 4.1 Player' Status) to record the players' actions for each puzzle based on if they were able to solve the puzzle or if they did not solve it. Figure 4.16 is a bar chart of the data that shows that all students solved the puzzles selected from World 1 and World 2. The number of players who solved puzzles in subsequent worlds decreases as the number of worlds increases. Only one (Player 2- See Appendix 4.1 Player's Status) player solved some puzzles in World 10.

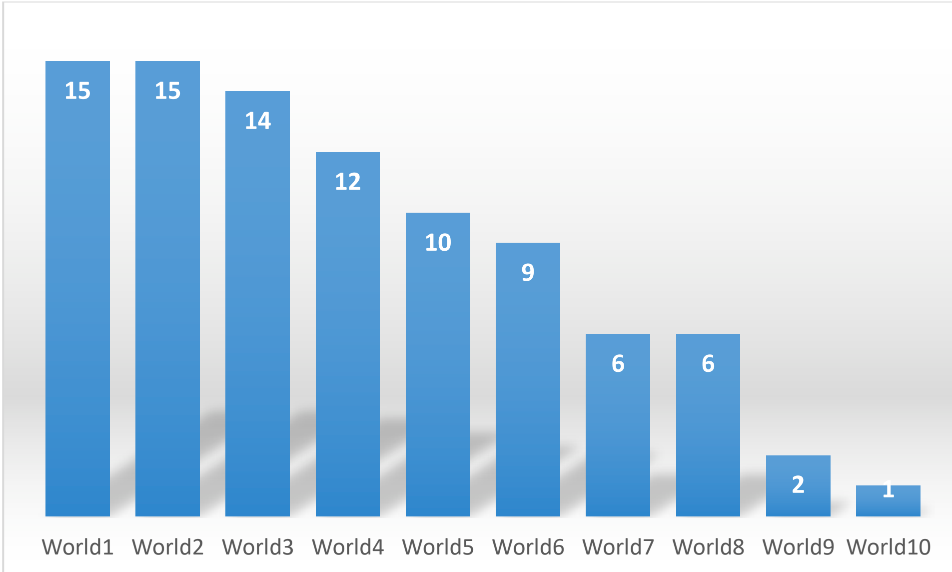
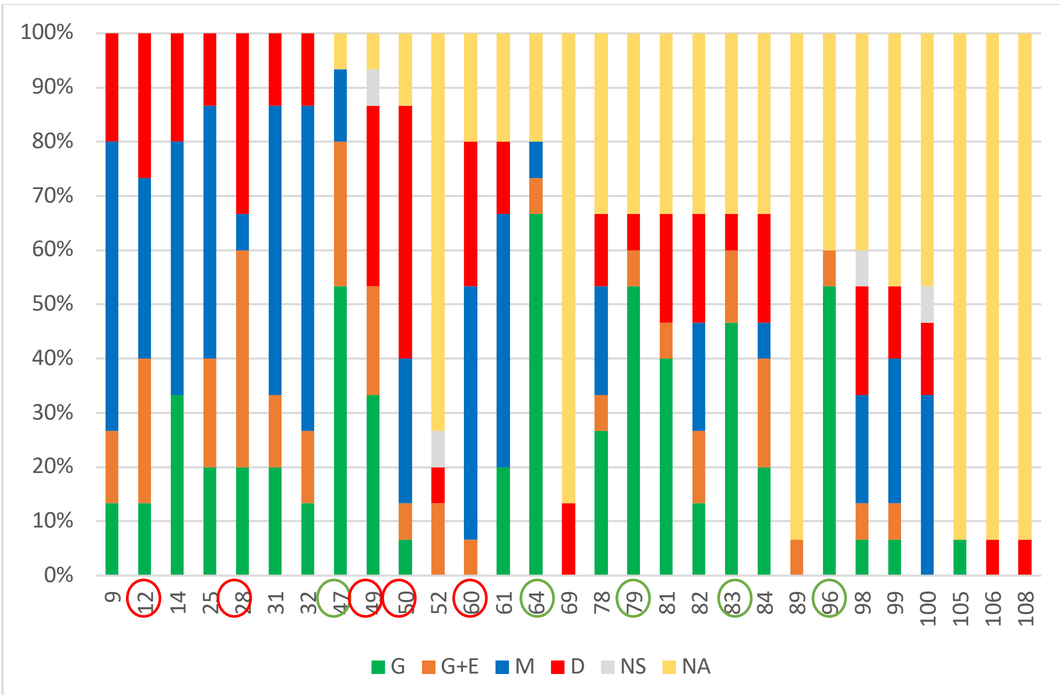


Figure 4.16: Number of Players Solved Each World.

I coded each player's Indivisualizers and created another excel spreadsheet (See Appendix 4.2 Performance of Players) to record the player's categorization of their successfulness for each puzzle. Figure 4.17 illustrates a bar chart of the Performance of Players (Good – G (GREEN), Good+ Error – G+E (ORANGE), Moderate – M (BLUE), Developing – D (RED), Not Solved – NS (GRAY), Not Attempt – NA (YELLOW)). Figure 4.17 shows that players were mostly categorized as Moderate– M (BLUE) because they solved between the best and mean number of steps. Most of the players who solved Puzzle 47 (3.11), Puzzle 64 (4.10), and Puzzle 96 (6.6) were labeled as G (GREEN) and no players struggled (D - RED) with these puzzles. In Puzzle 28 (2.10), some players were labeled as G+E (ORANGE) because they solved the puzzle in the best number of steps but also made some errors. In Puzzle 52 (3.16[11]), Puzzle 69 (4.15[8]), Puzzle 89 (5.17[8]), Puzzle 105 (6.15[8]), Puzzle

[11]Player can pass the puzzles 15, 16, 17and 18 in each World.

106 (6.16[8]), and Puzzle 108 (6.18[8]), I labeled most players as NA (YELLOW) because they preferred to skip these puzzles without attempting to solve them. In Puzzle 49 (3.13), Puzzle 52 (3.16[8]), Puzzle 98 (6.8), and Puzzle 100 (6.10), at least one player was labeled as NS (GRAY) because they attempted to solve the puzzle but not finished it.



Note: Green circles and red circles are added to help readers

Figure 4.17: Performance of Players

According to Figure 4.17, players had the highest performance in these puzzles: Puzzle 47 (3.11) – 8 players, Puzzle 64 (4.10) - 10 players, Puzzle 79 (5.7) - 8 players, Puzzle 83 (5.11) - 7 players, Puzzle 96 (6.6) - 8 players (Green circles in Figure 4.17); and in which players were struggling: Puzzle 12 (1.12) - 4 players, Puzzle 28 (2.10) - 5 players, Puzzle 49 (3.13) - 5 players, Puzzle 50 (3.14) - 7 players, Puzzle 60 (4.6) - 4 players (Red circles in Figure 4.17). In the next chapter, I will observe students' reactions and strategies to solve these puzzles.

In analyzing Sankey Diagrams part, I looked at specific puzzles in detail; Puzzle 12 (1.12) where players were struggling, Puzzle 31 (2.13) where players were mostly coded as moderate, Puzzle 47 (3.11) where players were coded as good, and Puzzle 52 (3.16) is another selected puzzle, but players preferred to skip it.

Analyzing Treemaps

I created an Excel sheet (Table 4.3) to record the players' first action code in the puzzles. The first row in Table 4.3 shows how many different first actions occurred in each puzzle. For example, there are 25 different first actions in Puzzle 9 (1.9), 10 different first actions in Puzzle 12 (1.12), etc. Other rows represent each player and their first action in each puzzle. For example, in Puzzle 9 (1.9), Player 1 did the sixth rectangle' action in the Treemap (For Puzzle 9 (1.9), the start state is 7+6+b+10 and Player 1 used the keypad and rewrote 7 into 6+1, so their action was 6+1+6+b+10), but Player 2 did the third rectangles' action in the Treemap (For Puzzle 9 (1.9), the start state is 7+6+b+10 and Player 2 used the keypad and rewrote 10 into 5+5, so their action was 7+6+b+5+5) (See Figure 4.4). In Puzzles 25 (2.7), 47 (3.11), and 64 (4.10), 12 players, 12 players, and 11 players, respectively, used the most preferred action (1s) to solve the puzzle.

Table 4.3: Treemaps' Data

Puzzle #	9	12	14	25	28	31	32	47	50	52	61	64
Total	25	10	22	13	10	9	18	6	11	9	10	9
P.1	6	3	1	1	2	1	8	1	4	NA	1	1
P.2	3	1	1	6	2	2	1	1	1	1	2	1
P.3	1	3	4	1	1	1	7	1	6	NA	2	1
P.4	2	1	2	2	1	1	1	NA	NA	NA	NA	NA
P.5	3	1	2	1	1	4	X	1	1	NA	2	1
P.6	1	3	X	1	5	3	2	2	2	NA	6	1
P.7	1	4	2	1	1	2	4	1	4	1-1	4	8
P.8	5	2	6	1	2	1	3	2	5	NA	3	1
P.9	6	1	X	4	1	2	2	1	2	1	1	1
P.10	7	8	3	1	2	1	1	1	1	NA	4	1
P.11	2	1	2	1	2	2	4	1	1	NA	3	1
P.12	1	3	4	1	1	4	8	1	NA	NA	NA	NA
P.13	X	1	4	1	2	2	3	1	2	NA	2	1
P.14	3	1	1	1	1	1	1	1	1	NS	NA	NA
P.15	1	1	3	1	3	1	1	1	5	NA	1	1

Code Note: X: Unable to read the section of the Treemap. NA: Player did not attempt to solve. NS: Player started to solve the puzzle but did not finish.

I used the Excel *CORREL* function (Salkind, 2017) to calculate the Pearson correlation coefficient. A Pearson correlation coefficient was computed to assess two relationships (see Table 4.4).

According to Table 4.4, there was a weak, positive correlation between the amount of the first actions in the Treemap and the mean number of steps of the puzzle, r = .2968, n = 12. However, the relationship was not significant (p = .3488). This shows that the amount of the first actions reflected in the Treemap did not appear to be associated with the mean number of steps students took to complete the puzzle.

Table 4.4: Pearson Correlation Coefficients Among Variables

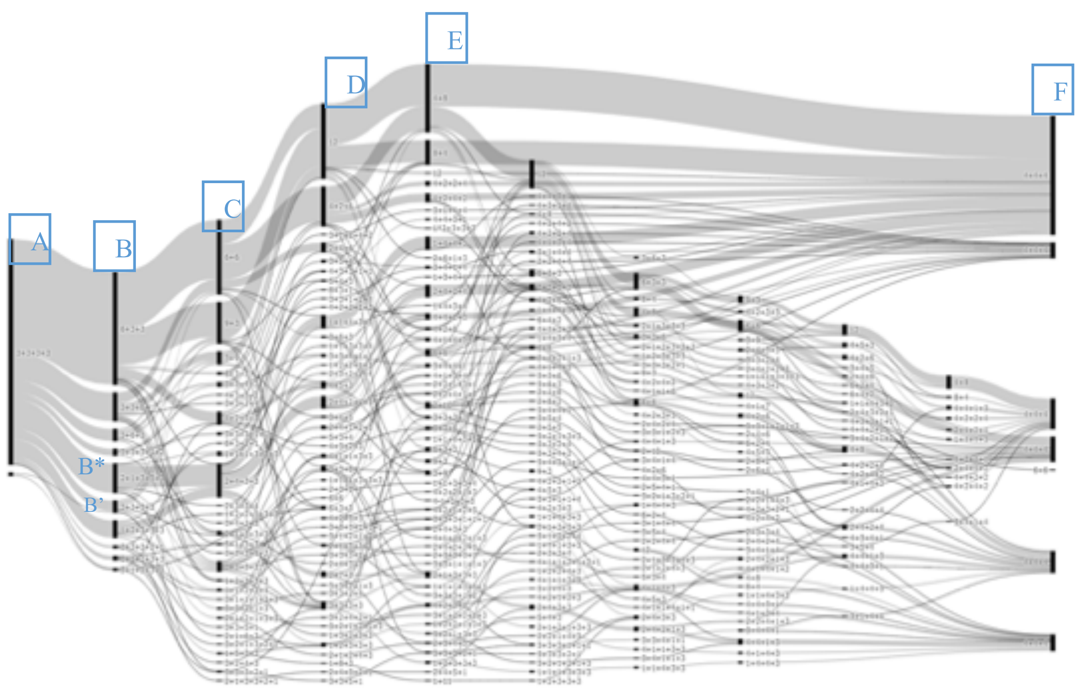
Variables		Coefficient Constant r	T-value t	Significance p
The amount of the first action coded in the Treemap and the mean number of steps		0.2968	0.9830	0.3488
The first action of each player and the performance of the player.	P.1	0.3705	1.2614	0.2358
	P.2	0.5486	2.0747	0.0647
	P.3	0.4603	1.6398	0.1321
	P.4	-0.3746	-1.2778	0.2302
	P.5	0.1401	0.4476	0.6640
	P.6	0.5640	2.1602	0.0560
	P.7	0.1511	0.4835	0.6391
	P.8	-0.1870	-0.6023	0.5604
	P.9	0.4599	1.6380	0.1325
	P.10	0.2129	0.6890	0.5064
	P.11	0.4121	1.4301	0.1831
	P.12	0.5727	2.2092	0.0516
	P.13	0.4195	1.4616	0.1745
	P.14	-0.0762	-0.2023	0.8454
	P.15	0.32	1.0681	0.3105

For the correlation between the first action of the player and the performance of the player, I calculated the Pearson coefficient for each player separately. According to Table 4.4, with the exception of 3 players (Player 4, Player 8 and Player 14), there was a weak, positive correlation between the variables (for example, Player 1's result is $r = .3705$, $n = 12$). However, the relationships were not significant ($p > 0.05$, for example, for Player 1 $p= .2358$). This shows that the players' first actions for solving the puzzles did not appear to be associated with the performance of the players.

The next section, Analyzing Sankey Diagrams, gives more information on the flow of the actions for solving each puzzle.

Analyzing Sankey Diagrams

I recorded the Sankey Diagrams of the puzzles from the dashboard provided. I then used my Indivisualizers analysis to select one puzzle from each category of the performance of players (e.g., from the developing, moderate, good, and not attempted) and analyzed these puzzles in detail. Puzzle 12 (1.12) (Figure 4.18) is a puzzle where players were struggling, Puzzle 31 (2.13) (Figure 4.19) is a puzzle where players were mostly coded as moderate, Puzzle 47 (3.11) (Figure 4.20) is a puzzle where players performed in the good category, and Puzzle 52 (3.16) (Figure 4.21) is a puzzle that players preferred to not attempt to solve or skip. I coded the Sankey diagram of these puzzles for productive and nonproductive gestures and the impression of the paths that the majority of players followed.



Note: A, B, C, D, E, F labels are added to help readers.

Figure 4.18: Sankey Diagram for Puzzle 1.12

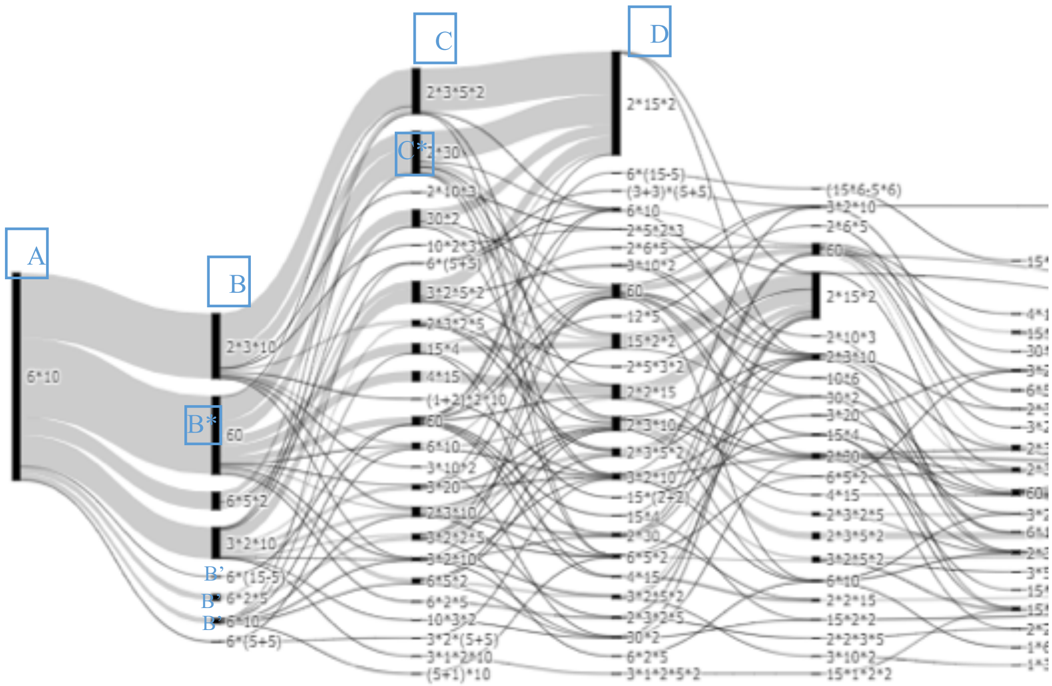
Before starting to analyze Puzzle 1.12, I wanted to clarify the mathematical rules students learned in previous puzzles. World 1 covers Addition, so the rules are related to addition. Students had four rules until Puzzle 1.12: 1. “Drag terms to commute. Blue lines show you where you can drop the term” (i.e., Commute Terms). 2. “You can add by tapping on a '+' sign or by dragging one number on top of the other” (i.e., Perform Operations by Tapping or Dragging). 3. “The keypad lets you substitute a number with an equivalent “(i.e., Substitute Numbers Using the Keypad). 4. “This keypad decomposes a number into two addends. Use the keypad twice to decompose a number into 3 addends.” Students can change the place of numbers by dragging, they can add numbers by tapping '+' sign or dragging them onto another number and they can rewrite a number by using the keypad button.

For Puzzle 1.12, the starting state is 3+3+3+3 (labeled as A in Figure 4.18) and the goal state is 4+4+4 (labeled as F in Figure 4.18). To solve the puzzle in 5 steps, most students prefer to add the first two 3’s together, then add the last two 3’s together, then add these two 6’s, and then substitute the number twice: Starting → 3+3+3+3 (labeled as A in Figure 4.18), First Step → 6+3+3 (labeled as B in Figure 4.18), Second Step → 6+6 (labeled as C in Figure 4.18), Third Step → 12 (labeled as D in Figure 4.18), Fourth Step → 4+8 (labeled as E in Figure 4.18), Goal → 4+4+4 (labeled as F in Figure 4.18).

The nodes in Figure 4.18 illustrate a different mathematical expression made by students to solve Puzzle 1.12. The column of the B label shows that there are 10 different first steps. For this puzzle, 9 steps are productive first steps and students are still able to solve the puzzle in 5 steps. There is just one nonproductive step, 3+3+3+3 (labeled as B’ in Figure 4.18) which is the same as the starting state and it needs an extra step to solve the puzzle. The top three preferred productive first steps are 6+3+3 (48.72%), 3+3+6 (13.96%), 2+1+3+3 (12.25% labeled as B* in Figure 4.18) while all other steps are chosen at less than 10%. This data indicates that students prefer to start operating from left to right and to combine numbers first and then substitute them. To understand more about students’ gesture preferences, I will conduct a future qualitative study to observe student interactions in the next chapter.

Similarly, before starting to analyze Puzzle 2.13, I analyzed the rules that students learn in this World. World 2 covers Multiplication and the rules are related to multiplication. Students receive 3 new rules in this World: 1. “Drag a number to commute! Does the order of multiplication matter?” 2. “You can multiply by tapping the dot (i.e., multiplication sign) or dragging one number on top of the other.” 3. “Use the keypad to factor. You can use the keypad twice.” After seeing the rules, students can multiply numbers by tapping the '×' sign or dragging the number onto others and rewriting a number by using the keypad button.

For Puzzle 2.13, the starting state is 6×10 (labeled as A in Figure 4.19) and the goal state is $2 \times 15 \times 2$ (labeled as D in Figure 4.19). To solve the puzzle in three steps, most students prefer to multiply the numbers, then substitute the number twice, i.e. Starting $\rightarrow 6 \times 10$ (labeled as A in Figure 4.19), First Step $\rightarrow 60$ (labeled as B* in Figure 4.19), Second Step $\rightarrow 2 \times 30$ (labeled as C* in Figure 4.19), Goal $\rightarrow$ $2 \times 15 \times 2$ (labeled as D in Figure 4.19).



Note: A, B, C, D labels are added to help readers, and the right side of the diagram had cut to make it seeable.

Figure 4.19: Sankey Diagram for Puzzle 2.13

The nodes in Figure 4.19 illustrate a different mathematical expression made by students to solve Puzzle 31 (2.13). The column of the B label shows that there are eight different first steps. For this puzzle, five steps are productive first steps and students are still able to solve the puzzle in three steps. There are three nonproductive first steps: $6 \times (15 - 5)$, 6×10, and $6 \times (5 + 5)$ (labeled as B' in Figure 4.19). 6×10 is the same as the starting state and is an extra step to solving the puzzle. $6 \times (15 - 5)$ and $6 \times (5 + 5)$ also cause extra steps because students need to put the number which is inside the parenthesis out of the parenthesis. The top three preferred productive first steps are 60 (37.93%), $2 \times 3 \times 10$ (32.47%), $3 \times 2 \times 10$ (15.52%), and the others are less than 10%. These steps show us students prefer to combine numbers first then substitute them and start operating from left to right (I will observe these gesture preferences in the next study).

Before starting to solve Puzzle 47 (3.11), students learned a new rule in this Puzzle: “Use the keypad and highlight the whole term '2y', then substitute with y + y” with animation (Figure 4.20a). Hence this is the new rule that players deal with rewriting variables via keypad. For Puzzle 47 (3.11), the starting state is $2y$ (labeled as A in Figure 4.20b) and the goal state is $y + y$ (labeled as C in Figure 4.20). To solve the puzzle in one step, students need to use the keypad function and rewrite $2y$ as $y + y$, i.e. Starting → $2y$ (labeled as A in Figure 4.20b), First Step → $y + y$ (labeled as C* in Figure 4.20b).

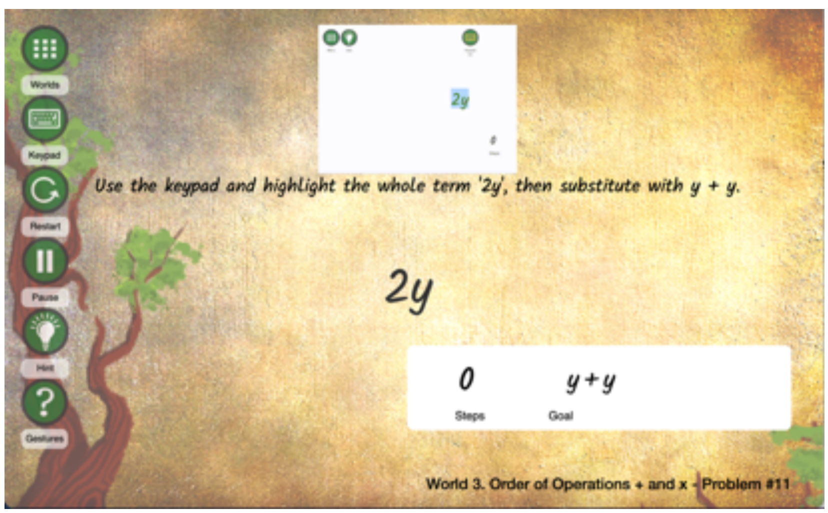
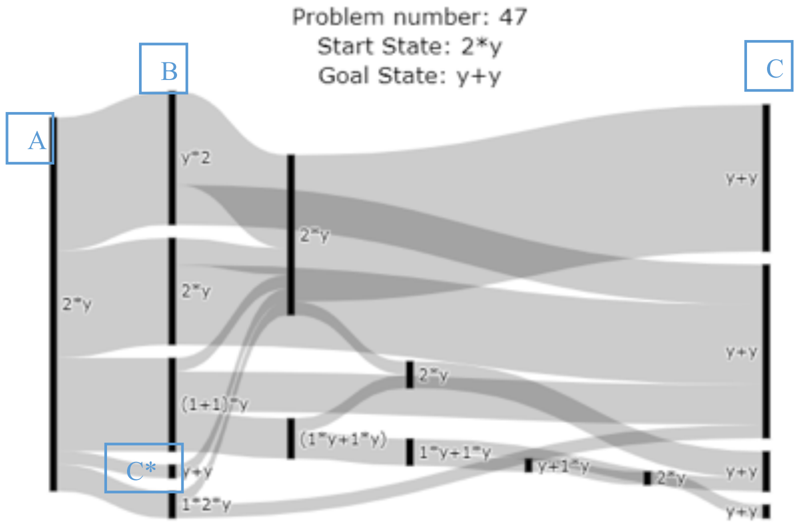


Figure 4.20a: Puzzle 47 (3.11)

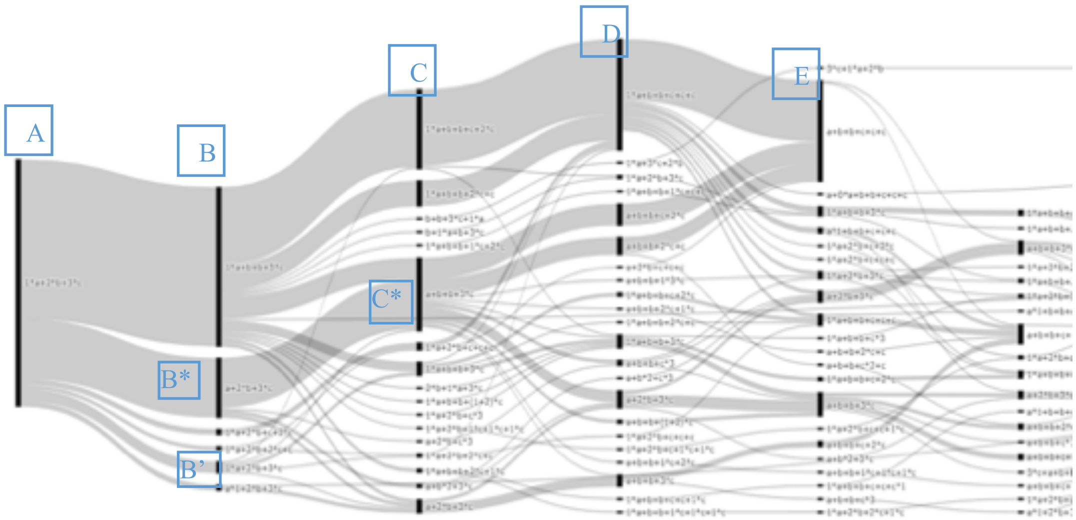


Note: A, B, C labels are added to help readers, and the right side of the diagram had cut to make it seeable.

Figure 4.20a: Sankey Diagram for Puzzle 47 (3.11).

However, students made different mathematical expressions to solve Puzzle 47 (3.11). The nodes in Figure 4.20b illustrate these different steps. The column of the B label shows that there are five different first steps. One step is a productive first step because the Puzzle is solvable in one step. There are four nonproductive first steps: $y2$, $(1 + 1) \times y$, $1 \times 2 \times y$, and $2y$. $2y$ is the same as the starting state and it needs an extra step to solve the puzzle, and $(1 + 1) \times y$ also causes an extra step because students need to put y to the inside of the parenthesis. These steps show that although the selected students in my data were coded as good, the majority of students struggled (I will observe these gesture preferences in the next study.).

Before starting to analyze Puzzle 3.16, students must follow specific rules related to the World. World 3 covers Order of Operations $+$ and $\times$ and each rule relates to this concept. Students receive 3 new rules on World 3: 1. “If the terms shake, you are trying to do something that is not mathematically possible”. 2. “To move 2y, select the 2, then drag it down until it is joined by the y, then move 2y as one object". 3. "Use the keypad and highlight the whole term '2y', then substitute with y + y". Following an introduction to the rules, students can rewrite a number by using the keypad button.



Note: A, B, C, D, E labels are added to help readers, and the right side of the diagram had cut to make it seeable.

Figure 4.21: Sankey Diagram for Puzzle 3.16.

For Puzzle 3.16, the starting state is $1a + 2b + 3c$ (labeled as A in Figure 4.21) and the goal state is $a + b + b + c + c + c$ (labeled as E in Figure 4.21). To solve the puzzle in four steps, most students prefer to decompose the variable $2b$ first, then substitute $3c$ in two steps, then multiply $1a$ to eliminate 1: Starting $\rightarrow 1a + 2b + 3c$ (labeled as A in Figure 4.21), First Step $\rightarrow 1a + b + b + 3c$ (labeled as B in Figure 4.20), Second Step $\rightarrow 1a + b + b + c + 2c$ (labeled as C in Figure 4.21), Third Step $\rightarrow 1a + b + b + c + c + c$, Goal $\rightarrow a + b + b + c + c + c$ (labeled as E in Figure 4.21).

The column of the B label nodes in Figure 4.21 illustrates a different mathematical expression made by students to solve Puzzle 52 (3.16). There are six different first steps. For this puzzle, four steps are productive first steps and students are able to solve the puzzle in four steps. There are just two nonproductive steps: $a1 + 2b + 3c$ (labeled as B' in Figure 4.21) and $1a + 2b + 3c$. $1a + 2b + 3c$ is the same as starting state and causes an extra step to solve the puzzle. The top two preferred productive first steps are: 1. $1a + b + b + 3c$ (64.74%), 2. $a + 2b + 3c$ (21.79%), and the others are less than 10%. These steps show that students prefer to start operating from left to right and substitute left items first. I will observe these gesture preferences in next chapter.

Discussion

This study explored whether digital gestures within playing FH2T affect players' decisions to keep playing or not. The goal of this study was also to determine interesting and challenging puzzles for analyzing students' reactions as well as interview questions for the forthcoming Student Observation studies. Through the analysis exploring students' gesture while playing FH2T under the embodied mathematical cognition and visual learning analytics perspective, I have determined that digital gestures cause errors and affect players' decisions to keep playing the game.

Limitations

I designed this study as a second study of my dissertation research. The overarching aim of the research is to explore how digital gestures connect to students' mathematical understanding when playing FH2T. However, there are some limitations of the research and this study. First, the participants for each study will be different – in the Analysis, the participant is myself, in the Quantitative Gesture study there are 358 sixth and seventh-grade students, and in the Student Observation study there will be at least six undergraduate students. The variability in research participants throughout the research project might affect the research findings because each participant will bring his/her own previous gameplay experiences and mathematical understanding to the study. Moreover, the playing time will be limited in the subsequent studies; I was able to finish all levels for the first, the Game Interaction study, but in this study, each player spent different amounts of time on the game while in the Student Observation studies, students will have restricted time and may not complete all of the worlds in FH2T.

Future Studies

My two goals for this Quantitative Gesture study are to explore whether students' digital gesture clusters in playing FH2T determine if they keep playing or not and to look at the conditions that affect their game play. With information about students' gestures, I highlight potential interview questions for the Student Observation study. My two goals for the next study – Student Observation – will be to investigate learners' mathematical emergent experience in playing FH2T and build upon my overall question of how digital gestures connect to students' mathematical understanding when playing FH2T.

Conclusion

I explored players' digital gestures while playing FH2T through embodied mathematical cognition and visual learning analytics frameworks to determine how digital gestures affect players' decisions to keep playing or not. I used quantitative data and four different visual learning analytics; Measure Charts, Indivisualizer, Treemap, and Sankey Diagrams. The study's research questions were: Do students' digital gestures within playing FH2T affect their decision to keep playing the game or not? What conditions affect their decision to keep playing or not? Results indicate that the increase in the number of digital gestures, in other words, the number of steps, increases the number of students who quit playing FH2T.

In the next Students Observation study, I will use the qualitative analysis method to observe students' gameplay and learning experiences within FH2T under the embodied mathematical cognition framework.

Chapter 5: The Student Observation Study

Overview

This exploratory study investigates how student gestures while playing a mathematical education game, *From Here to There!* (FH2T), affect their gameplay experience. Data for this research was collected from undergraduate students. The interviews and the gameplay observations were recorded on videotape. The research results show that *pointing gestures* demonstrate the particular objects in the game participants were mentioning, *representational gestures* (dynamic and static) indicate participants' mental reflection, *metaphoric gestures* show body-based metaphors of the topic, and *feedback gestures* convey participants' emotional response.

Introduction

Computers are increasingly used in education (Leinonen, 2005). Students learn through educational technologies (Hefzallah, 2004). Videogames are one way to engage students and improve their learning, especially in mathematics (Gresalfi & Barnes, 2015; Kebritchi et al., 2010; Williams-Pierce, 2016). While playing mathematical video games, students use both their minds and bodies (Gee, 2008), a process called embodied mathematical cognition. Embodied mathematical cognition is a theory claiming that the body's interaction with the environment can promote mathematical understanding (Cook et al., 2008; Nathan & Walkington, 2017; Nathan et al., 2016; Núñez, 2004; Williams-Pierce et al., 2017). Games and applications that are designed for learning mathematics through touchscreen devices increase the effect of embodied mathematical cognition.

This study is the final stage of a longer trajectory of research on the overall research question: *How do digital gestures connect to students' mathematical understanding when playing From Here to There! (FH2T)?* The research previously involves two studies; The Game Interaction (Katirci et al., 2022), and The Quantitative Gesture (Katirci & Williams-Pierce, in submission). In the Game Interaction study (Katirci et al., 2022) where I played the game *From Here to There!* (FH2T), I realized that some puzzles might be challenging for students. In the Quantitative Gesture study (Katirci & Williams-Pierce, in submission) I analyzed *what* digital gesture clusters students made and *how* students solved the puzzles in the game. However, in the previous studies, I did not observe students. The questions of *why* students make some gesture clusters while solving puzzles and *how* these digital gestures affect students' mathematical understanding could not be answered with the previous data.

Ideas around embodied mathematical cognition guide me to design the Student Observation Study to explore three additional sub-research questions: 1) What is the student's mathematical gameplay experience in playing FH2T? 2) What sort of physical gesture clusters do they use to explain what they are doing with their digital gesture? And 3) How do failure and feedback manifest in mathematical gameplay within playing the game from the student's perspective? Hence, in this study, I observed and video-recorded students while they played the game and used an embodied cognition perspective to analyze their actions (Figure 5.1). I investigated how students interacted with the game to further contribute to the literature on mathematics education games and embodied cognition in mathematics education by analyzing qualitative student data (pre-interview, observation, play-aloud, and post-interview).

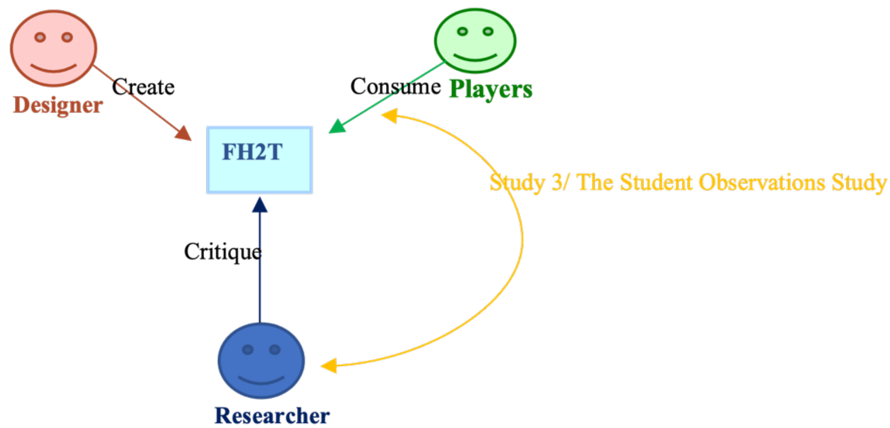


Figure 5.1: The Student Observations Study Organization

Theoretical Frameworks

This section describes the theoretical frameworks of the study. Embodied mathematical cognition (Alibali & Nathan, 2012; Cook et al., 2008; Goldin-Meadow et al., 2001; Lakoff & Núñez, 2000; Nathan & Walkington, 2017; Nathan et al., 2016; Núñez, 2004; Walkington et al., 2014; Williams-Pierce et al., 2017) frames this study because FH2T covers mathematical content and involves embodied cognition (Shapiro, 2011; Wilson, 2002; Wilson & Golonka, 2013) through interactions with touchscreen devices. In this study, I focus on students' gestures while they play FH2T.

Embodied Cognition

Embodied cognition is a theory in cognitive science claiming that “states of the body modify states of the mind” (Wilson & Golonka, 2013, p.1). In other words, cognition is rooted in the body’s potential interaction with the physical world

(Wilson, 2002). Cognitive scientists think that there are many co-existing views of embodiment (Shapiro, 2011; Wilson, 2002).

Shapiro (2011) presents his three themes of embodiment: Conceptualization, Replacement, and Constitution. *Conceptualization* refers to the concepts that an organism depends on to understand the surrounding world based on its body type. So, if organisms differ in their bodies, they also differ in how they understand the world. *Replacement* refers to the system that represents the interaction between an organism's body and its environment. Replacing the environment helps to understand the world's computational processes. Therefore, cognition can be explained without appealing to computational processes. *Constitution* means that the elements of cognition are beyond the mind; the body and world play not just a causal role but also a constitutive role in cognitive processing.

Wilson (2002) took a different approach and stated six claims of embodied cognition:

1. Cognition is situated.
2. Cognition is time pressured.
3. We off-load cognitive work onto the environment.
4. The environment is part of the cognitive system.
5. Cognition is for action.
6. Offline cognition is body-based.

She claims that when cognition is situated, activities occur in the environment's context. As humans, we need to solve our problems in real-life situations, and time-pressured environments influence our performance and decisions. We off-load cognitive work onto the environment by using equipment from the environment. Therefore, the environment is part of the cognitive system and is assumed to have a function in cognitive activity. I am particularly interested in the fifth claim, cognition is for action, because researchers found that gestures are representational or simulated actions (Hostetter & Alibali, 2008; Novack & Goldin-Meadow, 2017). Cognitive processing is tied in some way to bodily processes of immediate sensation and motor control, a process she refers to as offline cognition is body-based (Wilson 2002).

Embodied Mathematical Cognition (EMC)

Lakoff and Núñez (2000) argue that mathematical ideas (e.g., numbers, arithmetic operations, set theory, algebra, and infinity) can be understood as image schemas (e.g., images of subsets), aspectual systems (the structures of events, e.g., tapping), conceptual metaphors (e.g., conceptualize numbers as points on a line), and conceptual blends (combination of two structures, e.g., the center of the circle, and the radius of the circle) grounded in normal language usage and the sensory-motor system (e.g., addition as object collection). Mathematical thinking is embodied, and the body's interaction with the environment can promote mathematical understanding (Cook et al., 2008; Nathan & Walkington, 2017; Nathan et al., 2016; Núñez, 2004; Williams-Pierce et al., 2017).

According to McNeill (1992), a gesture is one form of embodiment in language and thought. McNeill (1992) categorized gestures into four groups: iconic (i.e., the semantic content of speech), metaphoric (i.e., the semantic content by metaphor), deictic (i.e., pointing), and beat (i.e., showing a rhythm) gestures. Alibali and Nathan (2012) used McNeill's (1992) typology of gesture to influence their categorization. When Alibali and Nathan (2012) analyzed mathematics teachers' and learners' gestures through an exploratory study, they identified three types of gestures: pointing (deictic), representational (iconic), and metaphoric. Alibali and Nathan (2012) argued that pointing gestures indicate the base of cognition, representational gestures demonstrate action and perception, and metaphoric gestures display conceptual body metaphors. Through pointing gestures, students reduce cognitive load, through representational gestures, students visualize the model of the issue, and through metaphoric gestures, students are able to understand the psychological reality shaped by subjective facts, experiences, and observations of the abstract concept's facts (Alibali & Nathan, 2012). Building on Alibali and Nathan's (2012) categorization, Walkington et al. (2014) classified gestures into two types: dynamic depictive gestures ("display a mathematical object being transformed using the affordance of the body," p. 479) and static depictive gestures ("display an unmoving, unchanging mathematical object in bodily form," p. 479). These classifications seem different but, after McNeill's general categorizations of revealing gestures about thoughts, they are just subclassifications to understand people deeply when they are talking about mathematical conceptions.

Students' gestures and speech have also been researched (e.g., Williams-Pierce et al., 2017). In the mathematics classroom, when students produced proofs for the triangle inequality conjecture (the sum of the length of any two sides of the triangle must be higher than the measure of the third side), they used hand gestures to justify themselves (Williams-Pierce et al., 2017). Some scholars (Katirci et al., 2020; Nathan & Walkington, 2017; Sinclair & Heyd-Metzuyanim, 2014) also include technology (video games or applications; e.g. *DragonBox*, *The Hidden Village, and TouchCounts,* respectively) in their interactions with students. Sinclair and Heyd-

Metzuyanim (2014) claimed that the body, fingers, hands, and emotions play an essential role in learning mathematics. They supported their claim by designing and investigating an educational application, *TouchCounts*. They found that *TouchCounts* helps kindergarten students develop number sense through counting and adding by using their fingers in other words by using their embodied cognition. Moreover, Katirci et al. (2020) conducted an impact study to determine students' embodied interaction and touchscreen technology when playing another game (*DragonBox 12+*). According to their findings, students used touchscreen gestures and performed similar gestures (e.g., dragging) during the discussion phase after gameplay to support and build upon each other's mathematical reasoning. Evidence of the relationship between embodied mathematical cognition and gameplay also led Nathan and Walkington (2017) to design an embodied mathematical interaction tool, *The Hidden Village* video game. *The Hidden Village* is a kinetic-based computer game where players use their arms to make depictive gestures (dynamic or static) to prove and disprove geometric conjectures. According to Nathan and Walkington’s (2017) finding, gesturing while playing the game helps students to formulate mathematical insights and to make informal proof of the geometrical conjectures.

Studies focusing on mathematical video games do not investigate embodied cognition through touchscreen technology. More research is needed to understand how students learn or practice mathematics through gesture interactions with touchscreen games without teacher intervention or guidance. I posit that FH2T might be used as a digital learning tool for promoting students’ embodied mathematical cognition when teachers are not present. More specifically, digital gestures in FH2T might enhance student learning of mathematical concepts. In the next section, I explain the methods of the study.

Methods

This section describes the methods of the Student Observations study and is separated into four sections: 1) description of case study methodologies, 2) the study design, 3) data collection and 4) data analysis.

Case Study

This study draws on qualitative case study analysis; Yin (1994) claimed that case study research is used to answer "how" and "why" questions within a real-life contemporary setting. Case studies focus on a detailed investigation of a single person or several people, an event, or a problem to describe-explore-explain (Yin, 1994). Creswell (2013) stated that "case study research begins with the identification of a specific *case*" (p. 98). In this study, the students played the game, FH2T, and verbally expressed their mathematical play experience to the researcher. Since playing FH2T is a kind of activity that students engage within, each student will be a case study and investigated in detail.

Study Design

The data was collected from seven undergraduate students. Students were representative of different departments (e.g., Journalism, Mechanical Engineering, Letters and Science, Public Health Science, Information Science (2), and Computer Science). Before playing FH2T, students were asked about their gameplay experiences and habits. Students ranged from having extensive gameplay experience to none. At the end of the study, participants received payment (a $20 Gift Card) for being a part of the research study. Sample demographics of the participants are provided in Table 5.1. Pseudonyms are used to protect their identities.

Table 5.1: Participants

Name (Pseudonyms)	Gender	Department	Year
Ava	She	Letter and Science	Freshman
Bella	She	Information Science	Senior
Carl	He	Public Health Science	Senior
Demi	She	Computer Science	Freshman
Evan	He	Information Science	Junior
Fin	He	Mechanical Engineering	Freshman
Gina	She	Journalism	Senior

Data collection consisted of three 40 minutes sessions (120 minutes) and was broken down into four different phases: pre-test (~20 min, on the first session), gameplay (~60 min, 20 min on the first session, and 40 min on the second session), playing aloud (Pellicone et al., 2022) (~20 min in the third session) and post-test (~20 min, in the third session). At the beginning of the study, participants answered interview questions about their gameplay experiences and habits and solved four algebraic equations as a pre-test. At the end of the study, participants answered game-related interview questions and solved four similar algebraic equations as a post-test. After completing the pre-test, participants started playing FH2T for the first and second sessions. In the third session, called the Play Aloud session, participants kept playing the game as the researcher observed and interviewed them; they played aloud, and as a researcher, I asked each participant to explain how they solved the puzzles and had them describe the reasons for the digital gestures or actions they were using.

Data Collection

This study used qualitative data collection methods – observations, documents, and interviews (Creswell, 2013). Data were collected in three different ways to enhance the credibility and quality of the research, triangulate information, and provide validity to the findings (Creswell, 2013).

The first type of data collection consisted of pre-interview and pre-test scores of the participants (approximately 20 minutes). Participants completed a paper and pencil pre-test that contained four algebraic equations related to the game. In the second data collection process, I observed participants while they played FH2T and took field notes on participants' digital and physical gestures as well as their reactions to the gameplay elements. Participants played *FH2T* on an iPad (for the touchscreen experience of game playing) for approximately 60 minutes. In the third data collection process, the participants kept playing the game but were asked to play aloud (Pellicone et al., 2022). The final part of the data collection was a post-interview focusing on student opinions regarding FH2T and how the embodied components of the game playing influenced their ability to solve mathematical problems. All parts were video, and audio recorded. The video and audio recording occurred using two standalone cameras which were placed on both sides of the students. These cameras captured the touchscreen devices and the students' gestures and algebraic patterns in the game while they were playing.

Data Analysis

The qualitative data includes personal documents (Pre-test), observations (Game Play), play-aloud conversations, and post-interview transcripts. Each part of the data collection process was video-recorded and transcribed, and personal data were added to the transcript to triangulate the information. The data was coded solely by the researcher using open coding, to code the data with open description (e.g, what participants said in pre- and post-interview and how they played the game in gameplay and play-aloud sessions) (Creswell, 2013). The analysis of each data source consists of four sections: 1) Pre-interview, 2) Game Play, 3) Play Aloud, and 4) Post-interview.

Pre-Interview Protocol

The first section involved a pre-interview consisting of two parts: 1) general opinions about playing games and habits and 2) ability with mathematics. For each participant, it took approximately 20 minutes. There were ten questions in total:

Q1. Do you like to play games in your free time?
Q2. What's your average time of playing video games within a week?
Q3. What sort of game do you typically play?
Q4. How well do you think you do/did in math class?
Q5. How much do you like doing algebra?
Q6. Have you ever played educational mathematics games?
  a) What games?
  b) Did you play at school or somewhere else?
  c) How much did you like playing that game/s?
Q7. $3 + 4 = 7$ What does this symbol ($=$) mean?
Q8. Solve
  $4 + 3 + a + 9 = 8 + \_\_ + a$
Q9. Solve
  $10 \cdot 10a \cdot 20 \cdot 5 = a \cdot 100 \cdot \_\_$
Q10. Solve
  $120 = 5 \cdot 12 + 12 \cdot \_\_$

The answer to the first six questions were short answers. Students' answers to these questions were transferred to a Microsoft Word document to analyze. I used these questions specifically to get a sense of each participant's background in gameplay by looking patterns among them.

The last four are mathematics problems. I picked these questions which cover fundamental rules of algebra (e.g., the meaning of equality, addition property, multiplication property, and distributive property). Students' answers were photographed first and then were classified into three categories: (1) larger error, which results from the wrong operation because of a misunderstanding of the principles of solving equations; (2) small error which results from a technical mistake; and (3) perfect, which means a completely correct answer. I divided it into three categories to use a more inclusive spectrum instead of binary labeling them wrong and right.

Game Play

Participants played the game for 20 minutes after the pre-interview, took a 5-minute break, and continued to play for another 40 minutes non-stop without any researcher questioning. The analysis procedure of this section consists of two steps. First, as a researcher, I took field notes while participants played the game. These field notes were transferred to a Microsoft Word document. Second, I created a Microsoft Excel Spreadsheet to transcribe the students' digital gestures puzzle by puzzle. On the Microsoft Excel Spreadsheet, I sorted the information by columns. The columns created were as follows:

1. Notes about the play,
2. World Number and Topic,
3. Puzzle Number,
4. Start,
5. The goal,
6. Min Step,
7. In which attempt was solved,
8. Attempt 1,
9. How solved,
10. #Step,
11. #Clover[12],
12. Notes about the attempt,

[12] The game gives Clovers as symbolic rewards after solved the puzzle. (See Chapter 3- How are clovers collected? part for more information.)

13-17 same as 8-12 (if could not solve in the first attempt), and continues until they are solved.

This Excel Spreadsheet helped me to organize the students' gameplay experience in detail (e.g., to see in how many attempts students solved the problem – Column #7, how many clovers they won – Column #11, and if they lost any clover, what was their reaction – Column #12, etc.) I started analyzing first with open coding, how the student solved the puzzle, then comparing different puzzles within the same student, looking at patterns and categorizing to understand students' action in more detail and answer the first and third research questions which are both related to students' mathematical gameplay experience.

Play-Aloud Protocol

After playing the game for 60 minutes in silence, participants started the Play-Aloud session (Pellicone et al., 2022). In this session, students kept playing the game while simultaneously explaining their thoughts and describing out loud their gameplay decision-making. As a researcher, I started the session by asking "If you had to describe this game to a friend, how would you describe it? Could you play like you are doing a Livestream to your friends?" I needed to ask additional questions when the students did not mention the key components of FH2T (e.g., gesture and hint buttons) in their talk aloud. Flexible follow-up questions varied depending on each student's gameplay actions. Students' answers to these questions transcribed, and the result from these questions will be analyzed in later papers. The following are some example questions:

- How would you solve that problem?
- Why did you drag that item first?
- Why did you tap that sign first?

- Why did you tap the **Hint** button?
  - How would you describe tapping the Hint button?
  - What would you recommend to your friend about using the Hint button?
- Why did you tap the **Gestures** button?
  - How would you describe tapping the Gesture button?
  - What would you recommend to your friend about using the Gesture button?
- Why did you start over?
  - How would you describe starting over?
  - What would you recommend to your friend about starting over?
- Why did you turn back?

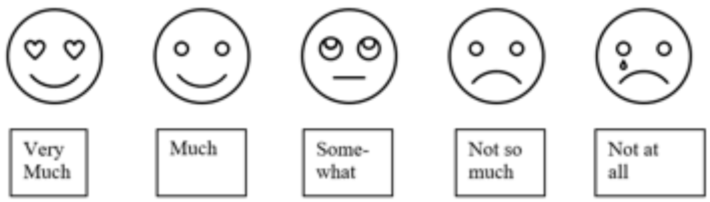
The analysis procedure of this section consisted of two steps. First, I took field notes when participants were playing the game. These field notes were transferred to a Microsoft Word document. Second, I transcribed the students' spoken language and physical and digital gestures within the game into Excel spreadsheets individually.

Alibali and Nathan (2012) used McNeill's (1992) typology of gesture to influence their categorization. When Alibali and Nathan (2012) analyzed mathematics teachers' and learners' gestures through an exploratory study, they identified three types of gestures: *pointing*, *representational* (dynamic and static), and *metaphoric*. While I analyze the data, I used their typology. For example, I observed participants show or point to an object on the game screen with their index finger, coded as a *pointing gesture*. For the code *representational gesture*, for instance, one of the participants drew a zero in the air to represent a number and showed its location. Moreover, when participants use conceptual metaphors and show gestures coded as *metaphoric.* I also added another category as *feedback* when participants express their emotions (positive or negative) through gestures.

Post-Interview Protocol

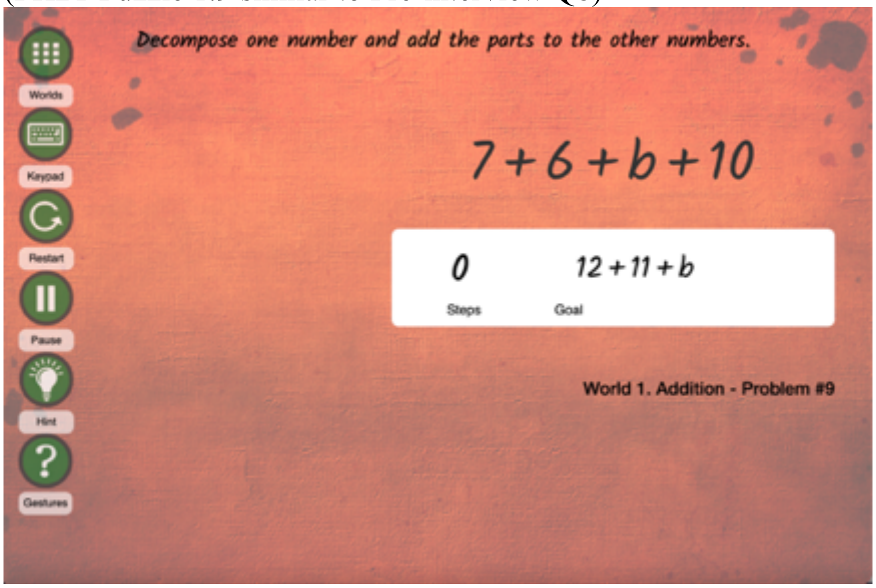
The final data collection process, the post-interview, took approximately 20 minutes for each participant. The questions in the post-interview focused on students' 1) general opinions of gameplay sessions and 2) ability with mathematics and mathematics gaming. There were 16 questions in total.

Q1) What was your general impression of the game?
Q2) How will you describe this game to your friends/or family? OR if you were describing this game to a friend, what would you say?
Q3) How much did you like playing this game?

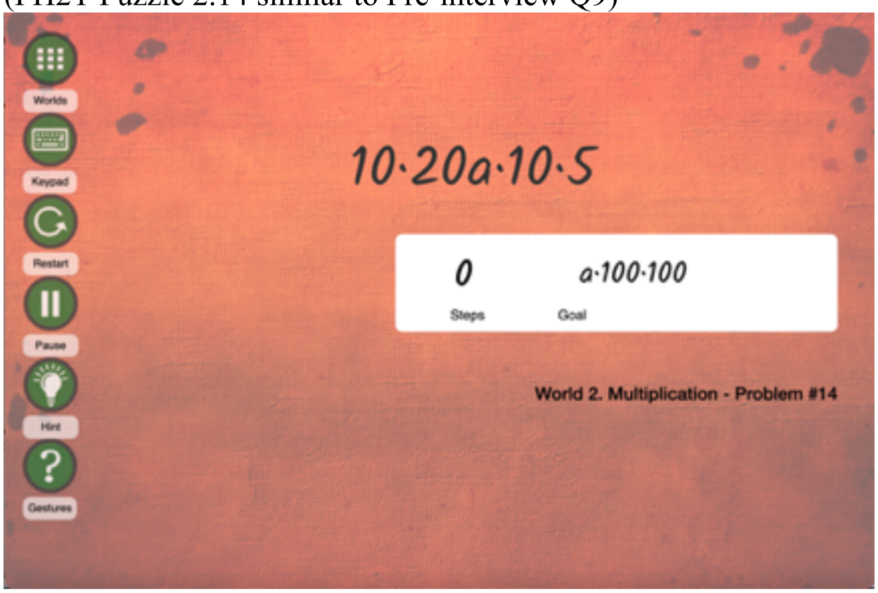


Q4) Was it fun? Would you keep playing it if you could?
Q5) Does this game remind you of any game that you have played before?
Q6) This game was designed for you to learn something. What do you think this game is designed for you to learn?
Q7) Did the game remind you of anything else in your life?
Q8) Was it like other games you've played before?
Q9) Was it liking any math you've done before?
Q10) How similar or different were these puzzles with respect to something you have done before?
Q11) Is there something that you might not have thought about before that occurred during this gameplay session?
Q12) Do you think you learned anything from playing this game? What?

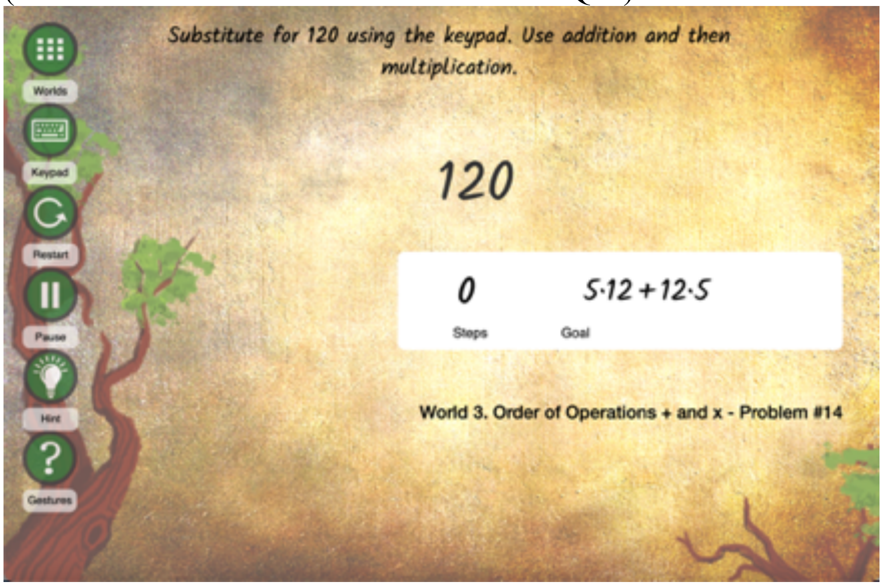
Q13) How did you solve this puzzle? Could you describe and solve it again from the tablet?
(FH2T Puzzle 1.9 similar to Pre-interview Q8)



Q14) How did you solve this puzzle? Could you describe and solve it again from the tablet?
(FH2T Puzzle 2.14 similar to Pre-interview Q9)



Q15) How did you solve this puzzle? Could you describe and solve it again from the tablet?
(FH2T Puzzle 3.14 similar to Pre-interview Q10)



Q16) How did you solve your latest puzzle? Could you describe and solve it again from the tablet?

The first 12 questions (Q1-Q12) were short answers. Students' answers to these questions will be transcribed, and the result from these questions will be analyzed in later papers. The last four questions (Q13-Q16) are mathematics problems from the game. These questions are similar or the same as the pre-interview math questions. Students solved these problems through the iPad. The students' spoken language, relevant digital gestures within the game, and physical gestures were transcribed to a Microsoft Word document with their respective video recordings.

Findings

This qualitative study aimed to explore three sub-research questions: 1) What is the learner's mathematical gameplay experience in playing FH2T? 2) What sort of physical gesture clusters do they use to explain what they are doing with their digital gesture? And 3) How do failure and feedback manifest in mathematical gameplay within playing the game from the learner's perspective? This section presents the findings of the study in four sub-sections: pre-interview, gameplay, play-aloud, and post-interview.

Pre-Interview

According to the pre-interview (Q1- Q3), all participants liked playing games in their free time. Their weekly amounts of play time ranged between 1 (Evan[13]) to 15 (Fin) hours per week. Participants' choice of game genres also varied. For example, Ava mentioned that she likes playing fighting and adventure/open-world games. Bella likes free-play games such as the Sims and Animal Crossing. Carl, Demi, and Gina said they like RPG [Role Playing Games] and multiplayer games. Evan mentioned sports and action games. Fin also included tabletop and board games that he likes playing. Almost all of them have played educational mathematics games (Q6) in the past except Bella. Ava and Carl mentioned that they played "CoolMath" games which is a network started in 1997 (CoolMath, n.d.) that offers interactive math games (Agosto, 2004). Evan gave an example of "IXL" which is a personalized learning website founded in 1998, offering online games and hands-on activities in math, language arts, science, and social studies for K-12 students (IXL, n.d.).

For the participants' mathematical self-evaluation (Q4 and Q5), they thought they were above average in math class (e.g.; Ava: "I'm pretty decent with math.", Bella: "Moderate", Evan: "Above Average") and moderately liked algebra (e.g.; Ava: "I dislike algebra the least.", Bella: "not much but I'm okay with it," Evan: 'Not but favorite but I don't hate it"). To understand more about their ability to solve

[13] All participants' names pseudonyms

algebraic problems, I asked them to solve four mathematical equations in the pre-interview:

Q7. $3 + 4 = 7$ What does this symbol (=) mean?

Q8. Solve

$4 + 3 + a + 9 = 8 + \_\_ + a$

Q9. Solve

$10 \cdot 10a \cdot 20 \cdot 5 = a \cdot 100 \cdot \_\_$

Q10. Solve

$120 = 5 \cdot 12 + 12 \cdot \_\_$

All participants correctly answered the first question (Table 5.2). Five of the participants gave a short answer that simply means "equals to" whereas Carl and Fin gave a long answer: "To equate, it ~~gives the what~~[14] shows how both sides of an equation are numerically equal to each other" and "The combinations of symbols on each side effectively mean the same thing. That is to say we know they are equivalent." Hence, they all know the meaning of the = symbol, which will help them to solve other questions in the pre-interview and also in the game. Furthermore, when students say "equals", mathematics educators know that when there is a symbol (=), it is not necessarily equivalence both sides, it needs perform and action. While giving detailed answer, Carl and Fin provide stronger evidence of their understanding of the symbol (=).

[14] The player, Carl, struck through the words when he was solving the pre-test.

Table 5.2: Pre-interview Q7- Mathematical Equation Solutions

Name	Answer
Ava	"Is equal to".
Bella	Equals to
Carl	To equate, it ~~gives the what~~ shows how both sides of an equation are numerically equal to each other
Demi	Equals, is equal to
Evan	It means "equal to"
Fin	The combinations of symbols on each side effectively mean the same thing. That is to say we know they are equivalent.
Gina	Equals

All participants solved Q8 correctly except Gina who made a small error (Table 5.3). There are small differences in the solutions of the participants who solved correctly. For example, Ava (Figure 5.2) named the blank "x", rewrote the equation by adding two numbers (7 and 9) together, and solved it by subtracting the number (8) first, then the letter (a). On the other hand, Fin (Figure 5.3) solved it correctly but without any explanation on the paper. In Gina's case (Figure 5.4), she made a small error when she preferred to transfer the numbers to one side and the letters to the other (she wrote $8 = a$ instead of $blank = 8$ or $8 = blank$).

Table 5.3: Pre-interview Q8-Q9-Q10 Codes

Name	Q8	Q9	Q10
Ava	Perfect/3	Perfect/3	Perfect/3
Bella	Perfect/3	Large Error/1	Perfect/3
Carl	Perfect/3	Perfect/3	Perfect/3
Demi	Perfect/3	Large Error/1	Large Error/1
Evan	Perfect/3	Large Error/1	Perfect/3
Fin	Perfect/3	Perfect/3	Perfect/3
Gina	Small Error/2	Large Error/1	Large Error/1

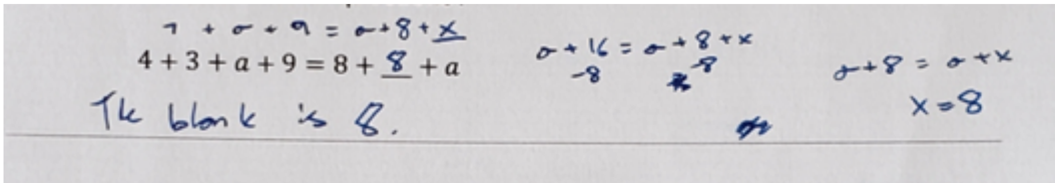


Figure 5.2: Ava’s Pre-interview Q8-Perfect/3 Solution

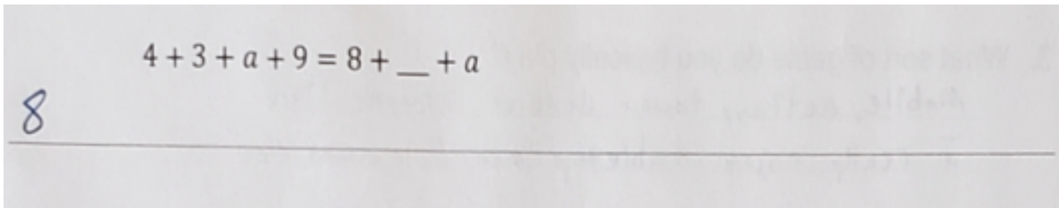


Figure 5.3: Fin’s Pre-interview Q8-Perfect/3 Solution

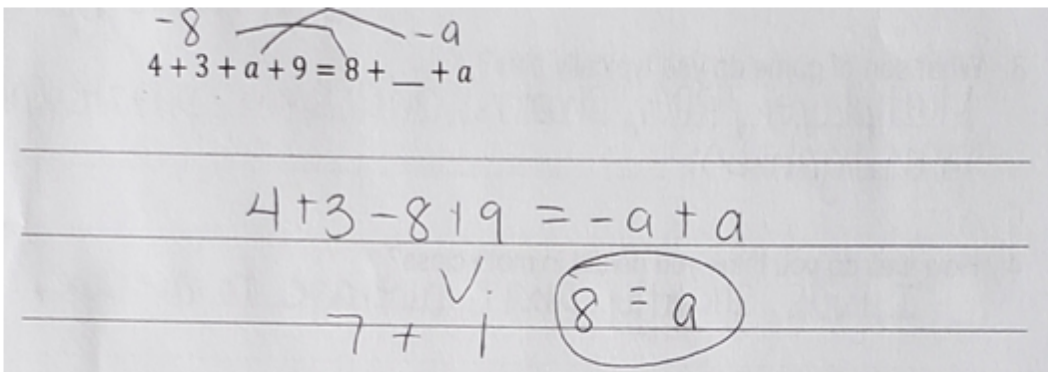


Figure 5.4: Gina’s Pre-interview Q8-Small Error/2 Solution

For Q9 (Table 5.3), three participants (Ava, Carl, and Fin) solved it correctly. Similar to Q8, Ava solved Q9 by naming the blank “x”, rewriting the equation by multiplying four numbers (10, 10, 20, and 5), solving it by dividing from the mind 10000 to 100, and finally, checking her work by multiplying 100 and 100 (Figure 5.5). Fin solved it correctly without any solution and just wrote the correct answer (Figure 5.6). Carl just wrote 10000 which is supposed to be the multiplication of 10, 10, 20, and 5 then the correct answer (Figure 5.7).

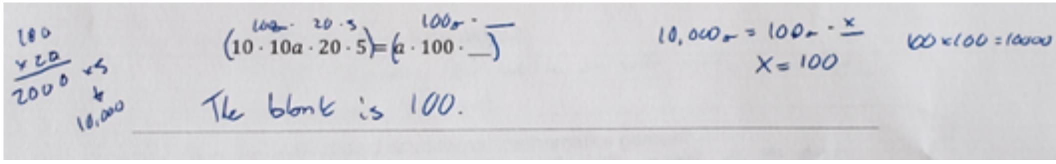


Figure 5.5: Ava's Pre-interview Q9-Perfect/3 Solution

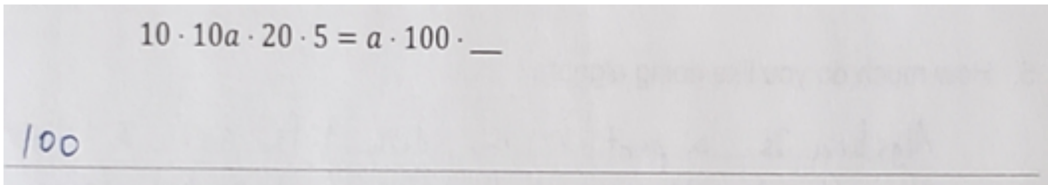


Figure 5.6: Fin's Pre-interview Q9-Perfect/3 Solution

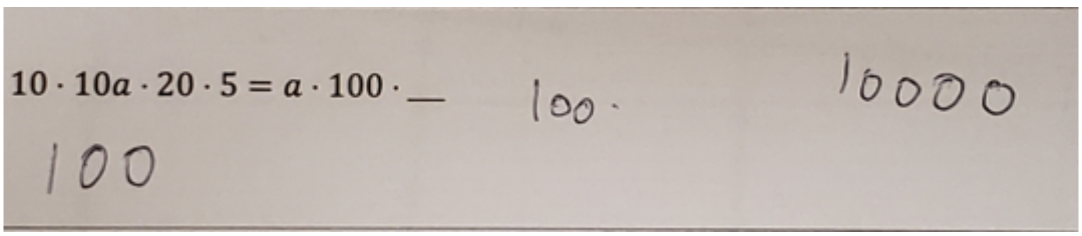


Figure 5.7: Carl's Pre-interview Q9-Perfect/3 Solution

For Q9, four participants (Bella, Demi, Evan, and Gina) made large errors. Their errors were different from each other. For example, Bella (Figure 5.8) did not realize there was a blank in the question of that she needed to solve the equation for the blank. As a result, she solved the equation for a. Similarly, Gina (Figure 5.9) also did not solve for the blank. She made conceptual errors like moving 20 and 5 from the left side of the equation to the right side, but their operations were multiplication. She moved them like addition, changing the sign of 20 and 5 from positive to negative but kept them as multipliers. Evan (Figure 5.10) just wrote a 10 in the blank, possibly because he misinterpreted the first 10 as being the number of the question rather than a number in the actual equation. Demi (Figure 5.11) rewrote the equation for calculating the left side numbers that were equal to 400, then from this calculation arrived at an answer of 4.

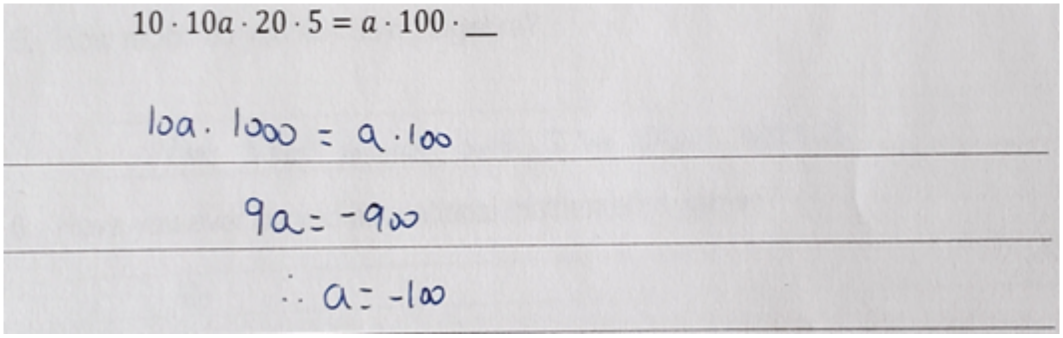


Figure 5.8: Bella's Pre-interview Q9-Large Error/1 Solution

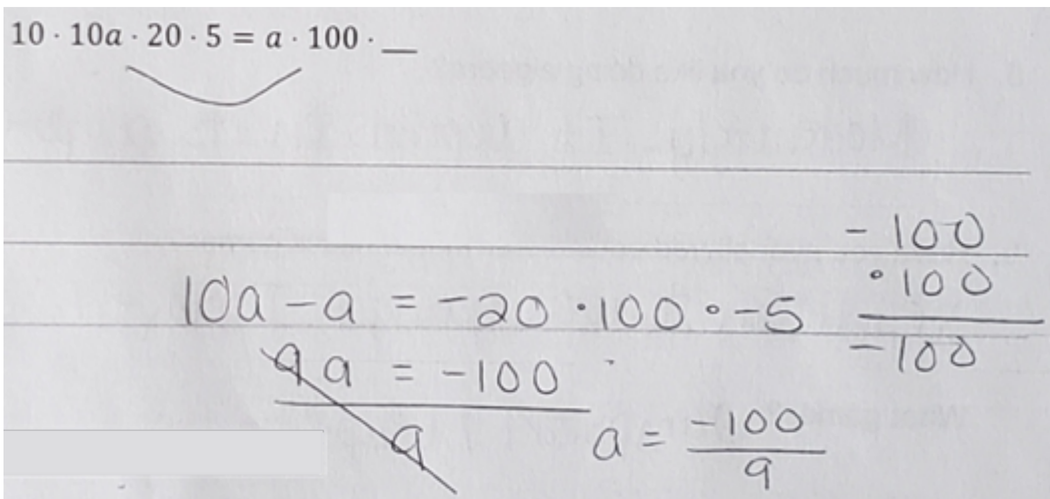


Figure 5.9: Gina's Pre-interview Q9-Large Error/1 Solution

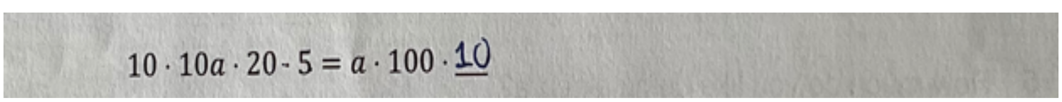


Figure 5.10: Evan's Pre-interview Q9-Large Error/1 Solution

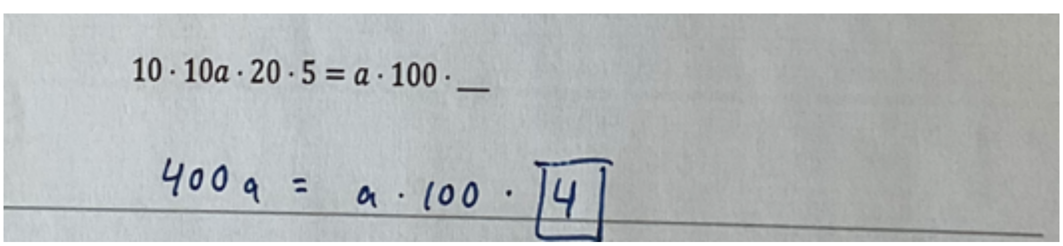


Figure 5.11: Demi's Pre-interview Q9-Large Error/1 Solution

Five participants (Ava, Bella, Carl, Evan, and Fin) solved Q10 correctly while Demi and Gina did not (Table 5.3). Carl (Figure 5.12) and Fin (Figure 5.13) solved the question without any calculations written on the paper and just wrote the correct answer, 5. Ava (Figure 5.14) first wrote parentheses onto the equation to simplify the orders of operation, multiplied 5 and 12 (equal to 60) in her head to write the equation as $60 + 12.\_$, then wrote 5 as the answer for the blank. She also checked her work by confirming that $60 + 60 = 120$. Bella (Figure 5.15) first multiplied 5 and 12 (equal to 60) in her head, then rewrote the equation as $120 = 60 + 12.\_$, then wrote $\_ = 5$ as a correct answer. Evan (Figure 5.16) also multiplied 5 and 12, equal to 60 and wrote 60 above the equation. He made some other calculations as $60 + 12 = 72$, $120 - 72 = 48$, and wrote 48 on the blank, but then he realized he made a mistake he scrubbed out 48 and wrote 5 on the blank.

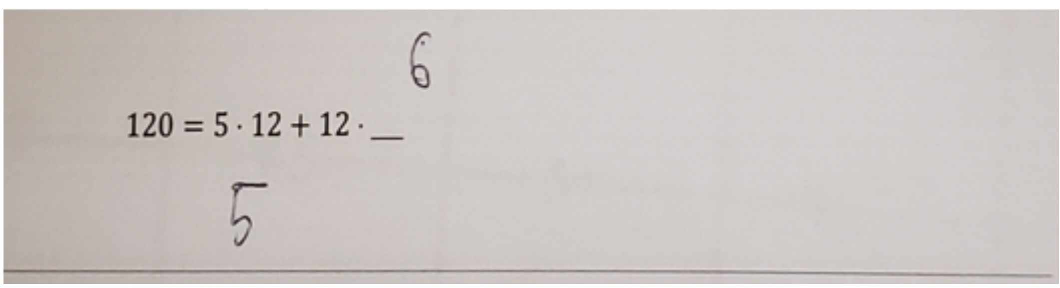


Figure 5.12: Carl's Pre-interview Q10-Perfect/3 Solution

Figure 5.13: Fin's Pre-interview Q10-Perfect/3 Solution

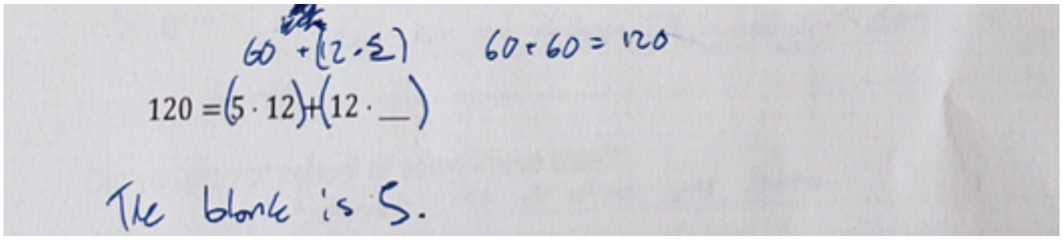


Figure 5.14: Ava's Pre-interview Q10-Perfect/3 Solution

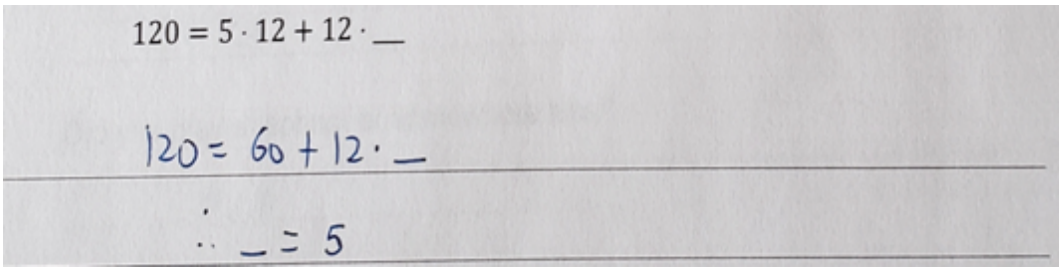


Figure 5.15: Bella's Pre-interview Q10-Perfect/3 Solution

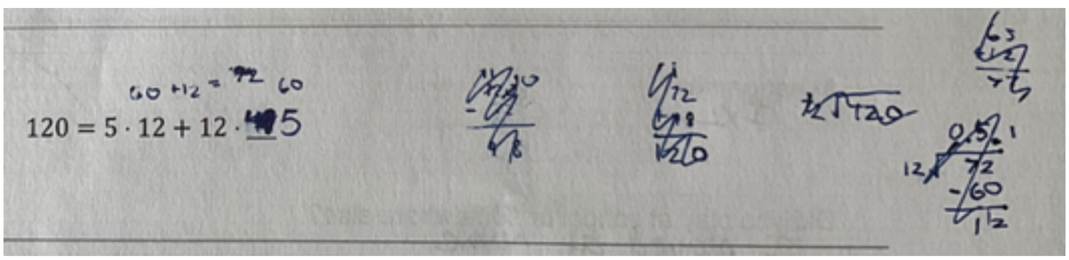


Figure 5.16: Evan's Pre-interview Q10-Perfect/3 Solution

Demi and Gina made large errors for Q10. Both made a conceptual error and a calculation error. Demi (Figure 5.17) first multiplied 5 by 12, equal to 60 in her head, then rewrote the equation as $120 = 60 + 12.\_$ writing a 2 instead of a 5. Similarly, Gina (Figure 5.18) multiplied 12 by 5 equaling to 60, but then added 12 to 60 and found 72 (which is a conceptual error as an order of operation). She then subtracted 72 from 120 to get 38 (which is another calculation error). She rewrote the equation as $120 = 5.12 + 12.38$, and underlined 38 as her answer.

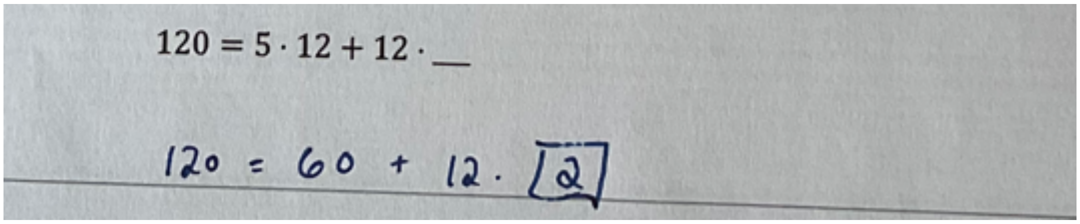


Figure 5.17: Demi's Pre-interview Q10-Large Error/1 Solution

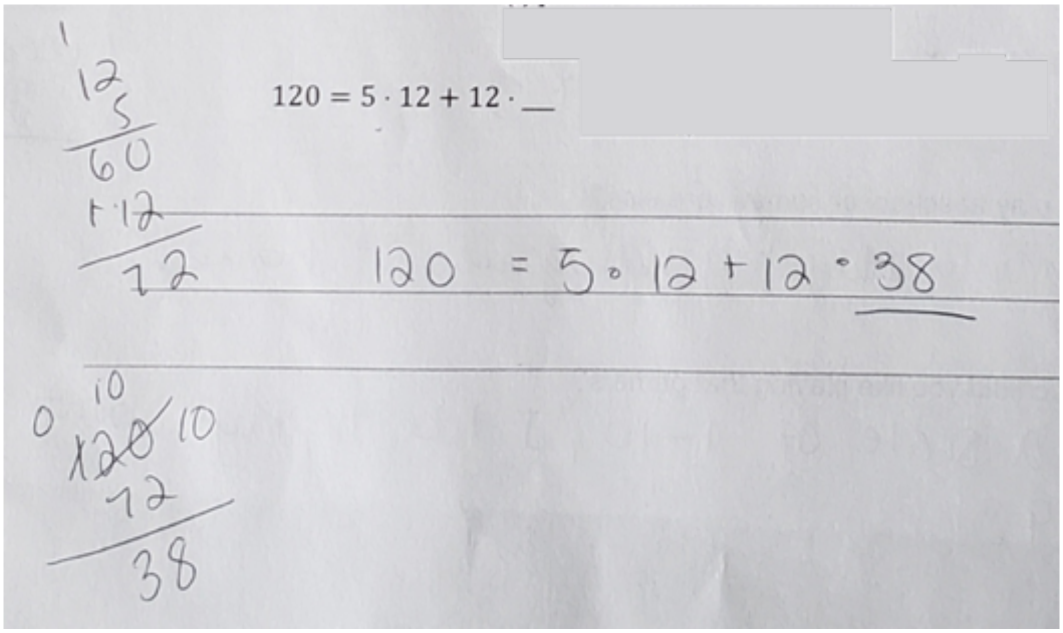


Figure 5.18: Gina's Pre-interview Q10-Large Error/1 Solution

In summary, Ava perfectly solved all three mathematical equations in the pre-test. She showed her work by rewriting the equations and also checked her work by redoing the calculation. Ava's problem-solving approach showed me that she understands mathematics and prefers to write out her thinking while solving equations. Carl and Fin also successfully solved all of the pre-test equations, but they did not write out their thinking processes on the paper. Instead, they just wrote the correct answers. Their problem-solving approaches suggest that they are familiar with basic algebra and prefer solving problems in their heads rather than writing down their ideas. Bella and Evan solved the first and third equations perfectly, but they made large errors on Q9. Bella's error was conceptual, while Evan's error was probably notation-based because he misinterpreted the first 10 in the question. Meanwhile, Demi and Gina could not solve Q9 and Q10 on the pre-test. Their conceptual errors suggest that they may have not solved algebraic equations in a while or that other factors are at play. More information about the participants and further analysis are needed to understand their mathematical knowledge. In the next section, Game Play, I present participants' game-playing experiences.

Game Play

While participants played *From Here to There!*, their gameplay actions were videotaped. I analyzed the participants' physical and digital gestures with an iPad to understand their mathematical gameplay experiences; I looked at how they used different icons on the game screen and how they reacted to their failure (for example, keypad[15], shaking[16], and snapping errors[17]) and feedback (i.e., formative, summative, informative feedback[18]) from the game. There were seven participants, and each of them had a different playing style. None of them finished all of the levels within the time limit, and their final levels ranged from World 3 Puzzle 18 to World 10 Puzzle 10. I analyzed each player's data individually.

Ava's Actions. As mentioned in the pre-interview, Ava likes playing games in general and played educational games when she was in middle school. This game was not difficult for her because she had previously played games via iPad and

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[15] Keypad Errors: Errors that the student made by attempting to enter a non-equivalent expression using the keypad.
[16] Shaking Errors: Errors that the student made by attempting to incorrectly use existing operators.
[17] Snapping Errors: Errors that the student made by attempting to incorrectly reorder terms.
[18] See Chapter 3 - System Feedback part for more information.

educational games. She solved Q8 to Q10 with a "perfect" score in the pre-interview, suggesting a good mathematical background. She reached the 8th World's 18th puzzle by the end of the gameplay session.

In the first world of the game, she tried to earn as many clovers as possible for each puzzle; if she was able to solve the puzzle but only earned 1 or 2 clovers, she would restart the puzzle and try to decrease her problem-solving steps to earn 3 clovers. For example, to solve Puzzle 1.4 (Figure 5.19), she first dragged y to the front (i.e.; $y + 5 + x + 2$), then dragged x next to y (i.e.;$y + x + 5 + 2$), and dragged 2 next to x (i.e.; $y + x + 2 + 5$).

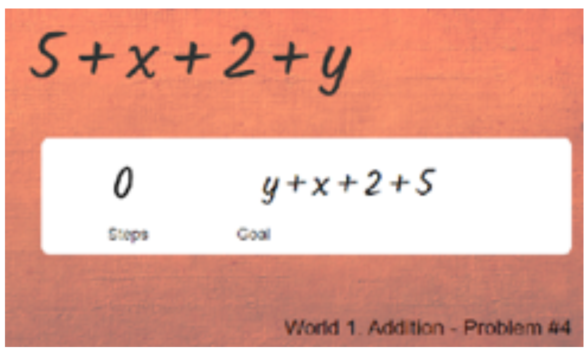


Figure 5.19: Puzzle 1.4

In this solution, she exceeded the minimum number of steps for the puzzle (Figure 5.20, highlighted by a blue circle). However, since she solved the puzzle, she still got 2 clovers. She made a *snapping error* (dragging 5 to the end was the correct action for this case) in her second step (i.e., dragging x next to y) which led her to exceed the minimum number of steps. Her reaction to getting two clovers was to retry the puzzle. In her second attempt, she first dragged 5 next to 2, then dragged y to the front and received three clovers as a reward.

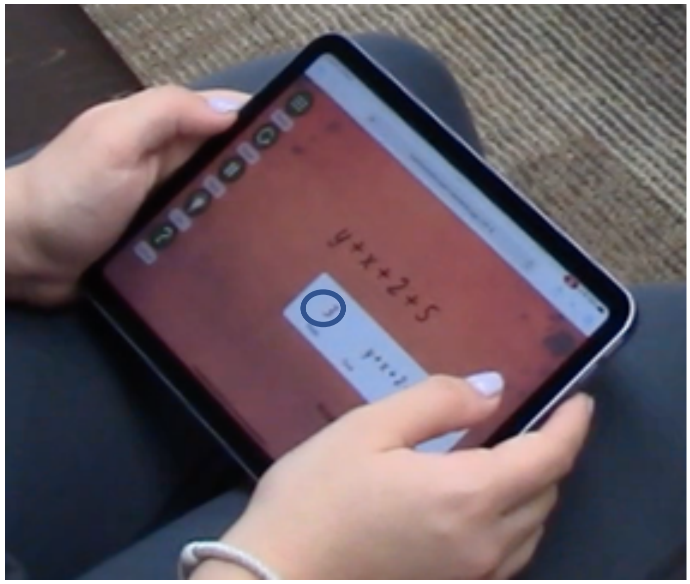


Figure 5.20: Ava exceeded step numbers in Puzzle 1.4

As a feature of the game, players are able to skip the last four puzzles (i.e., Puzzle 15,16,17, and 18) in each world; Ava took advantage of this feature in Worlds 1, 2, and 3 and did not solve those puzzles. In World 1, Ava tried to perfectly solve each puzzle. However, in World 2, she stopped retrying puzzles even if she received one or two clovers. For example, she attempted to solve Puzzle 2.5 three times, but she got two clovers each time. Similarly, she attempted Puzzle 2.7 two times and got two clovers.

For Puzzle 2.8 (Figure 5.21), she was able to solve the puzzle on her fifth attempt, got one clover, and then moved on to the next puzzle. Puzzle 2.8 was the first puzzle that she really struggled with. In her first three attempts, she got different answers. For instance, in the first attempt, she multiplied 24 and 24 making 576, in the second, she multiplied 24 and 16 making 384, and in the third, she multiplied 4 and 16 making 64. None of these numbers were correct, so she restarted the puzzle. In her fourth attempt, she made a *shaking error* by tapping the multiplication sign between variable (c) and number (24). On the fourth attempt, she followed these steps:

Start $4 \cdot 6c \cdot 24 \cdot 16$ Goal $96 \cdot 96c$ Minimum Step 2

1. Drag 6 between 24 and 16, made
   $4c \cdot 24 \cdot 6 \cdot 16$
2. Tap on the multiplication sign between 6 and 16, made
   $4c \cdot 24 \cdot 96$
   *Tap on the multiplication sign between $4c$ and 24 [**Shaking Error**]
   *Drag 4 between c and 24 then drop it, and it turns to its previous place.
3. Drag c next to 24, made
   $4 \cdot 24c \cdot 96$
4. Tap on the multiplication sign between 4 and $24c$, made
   $96c \cdot 96$
5. Drag 96 onto $96c$, made $9216c$
   Tap on the restart icon (NOTE: * *Participant did an action but not count as a step)*

Figure 5.21: Puzzle 2.8

On the fifth attempt, she did not make any shaking errors and was satisfied with earning only one clover. Her steps were:

Start $4 \cdot 6c \cdot 24 \cdot 16$ Goal $96 \cdot 96c$ Minimum Step 2

1. Drag 6 between 24 and 16, made
   $4c \cdot 24 \cdot 6 \cdot 16$
2. Tap on the multiplication sign between 6 and 16, made
   $4c \cdot 24 \cdot 96$
3. Drag c next to 24, made
   $4 \cdot 24c \cdot 96$
4. Tap on the multiplication sign between 4 and $24c$, made
   $96c \cdot 96$

5. Drag 96 to the front, made
   $96 \cdot 96c$
   Get one clover as a reward

Her gameplay suggests that the reason behind her extra steps was that she did not pay attention to the rule stating, “You can multiply by tapping the dot (i.e., multiplication sign) or *dragging one number on top of the other*.” (Italic added). Instead of dragging a number (e.g., 16) on top of another number ($6c$), she preferred to first move one number next to another number (e.g., first steps in her fifth attempt) and then tap the multiplication sign (e.g., second steps in her fifth attempt), which caused extra steps. She remembered this rule in the next puzzles and reduced her number of steps to receive three clovers. When making extra steps in the earlier Worlds, she did not use the Hint or Gesture icons to remind herself about the rule or to learn the clue for solving the problem in fewer steps. It was only in the later worlds (ex., Puzzles 5.16, 7.3, 7.13) when she began to tap the Gesture and/or Hint icons for extra information (Figure 5.22).

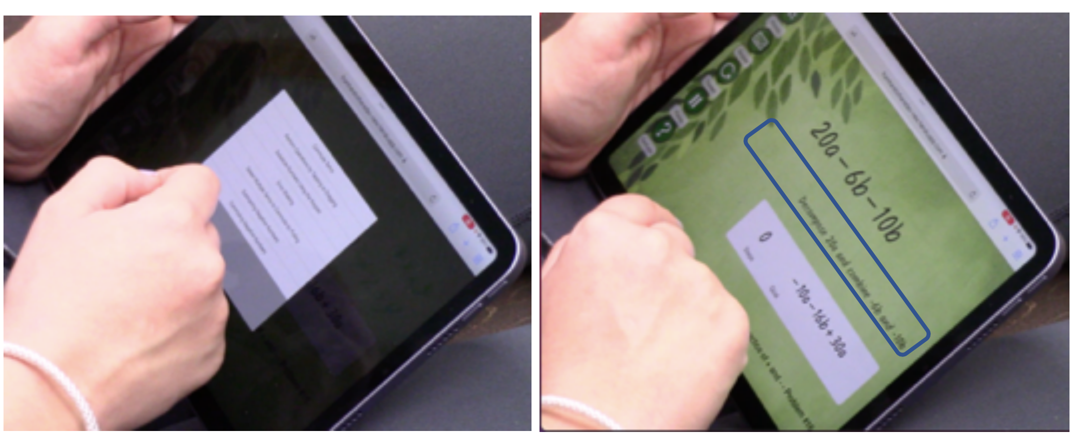


Figure 5.22: Puzzle 5.16 - After the Gesture Icon (Left) and Hint Icon (Right)

In summary, the observation and Ava's gameplay records show that she was learning the game's rules by playing the game. In the beginning, she re-tried the puzzle to eliminate extra steps and get the maximum rewards. She also used Gesture and Hint icons to obtain informative feedback when she could not solve the puzzle and needed clue to solve it, especially in later puzzles.

Bella's Actions. In the pre-interview, Bella mentioned that she likes to play games but did not have any previous educational gameplay experience. She solved Q8 and Q10 with a "perfect" score but made a large error in Q9 in the pre-test. She reached Puzzle 10.8 at the end of the gameplay session.

Unlike Ava, Bella did not re-try any puzzles after solving and getting one or two clovers as a reward. This gameplay approach influenced her to solve more puzzles and go far worlds in the game. Bella solved the first World's last four puzzles, but not the other Worlds'. Bella also seemed to not pay attention to the information or tutorial on the screen. For example, in Puzzle 6.4 (Figure 5.23), the game already gives information on how the puzzle is solved, such as "divide by tapping the division bar." However, Bella preferred to drag 7 (denominator) onto 56 (numerator) which decomposed 56 as 7 and 8 in the numerator, then she dragged 7 (denominator) again onto 7 (numerator) to cancel the 7's and got 8. This procedure was also repeated for other numbers ($\frac{56}{4}$ and $\frac{56}{28}$), and these repetitions caused three extra steps, resulting in one clover reward.

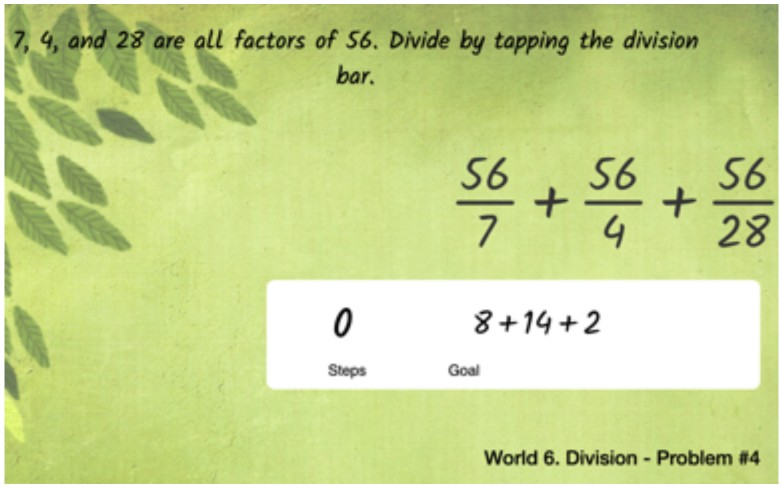


Figure 5.23: Puzzle 6.4

Bella also solved puzzles in one or two attempts until World 7 Puzzle 10. She was able to solve this puzzle on her 11th attempt with one clover and did not use the Hint or Gesture icons. On the first attempt, she followed these six steps:

Start $\frac{xy\cdot\frac{1}{x}-y}{z}\cdot 0z$ Goal $0\cdot 0$ Minimum Step 2

1. Tap on the multiplication sign between xy and $\frac{1}{x}$, made $\frac{x\cdot\frac{y\cdot 1}{x}-y}{z}\cdot 0z$
2. Tap on the multiplication sign between x and $\frac{y\cdot 1}{x}$, made $\frac{\frac{xy\cdot 1}{x}-y}{z}\cdot 0z$
3. Tap on the x (numerator) x's are gone, made $\frac{y\cdot 1-y}{z}\cdot 0z$
4. Tap on the multiplication sign between y and 1, made $\frac{y-y}{z}\cdot 0z$
5. Tap on the minus sign between y and y, made $\frac{0}{z}\cdot 0z$
6. Tap on the multiplication sign between $\frac{0}{z}$ and $0z$, made 0

   Tap on the restart icon.

Her reaction to getting 0 after her sixth step was to restart the puzzle. In her second attempt, she tapped on the z (which is next to 0) and made 0. After reaching 0, she restarted the puzzle again. Instead, she should have used the keypad to rewrite 0 as 0.0. On her third attempt, she started making *shaking errors* by tapping the division sign before simplifying the numerator. On her sixth attempt, out of 11 attempts, she made *shaking errors* by not paying attention to the order of operations. She may have made these errors because she did not realize what the failure was, or she was more focused on just passing the puzzle and did not care about making errors. Unlike Ava, Bella did not use any Gesture or Hint icons to get informative feedback from the game.

Carl's Actions. Carl was one of two participants who gave a long answer to Q7 and solved Q8 to Q10 with a "perfect" score in the pre-interview, suggesting a strong math ability. He reached the 8th World's 3rd puzzle by the end of the gameplay session, and he was able to solve the last four puzzles (i.e., Puzzle 15,16,17, and 18) of all the Worlds except World 3.

Unlike Bella and Ava, Carl made silent gestures while playing the game. For example, while completing Puzzle 1.9, he made hand and head gestures in step 2:

<u>Start</u> $7+6+b+10$ <u>Goal</u> $12+11+b$ <u>Minimum Step</u> 3

*Tap on 7 (nothing happened)

1. Tap on the keypad, tap 7, write $1+6$, tap done, made
   $1+6+6+b+10$
2. Tap the plus sign between 6 and 6, made
   $1+12+b+10$ [**Hand and Head Gesture**]
3. Drag 10 onto 1, made
   $11+12+b$
4. Drag 12 to the front, made
   $12+11+b$
   Get two clovers as a reward

**Participant did an action but not count as a step*

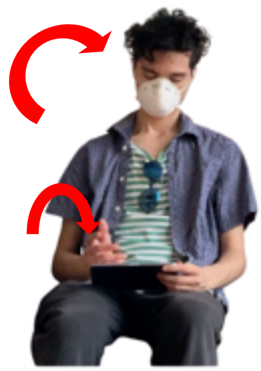


Figure 5.24: Carl's Reaction-Gesture

After adding 6 and 6 (in the second step), he realized that he made a mistake (because the order of the numbers need to change) and showed this with his hand and head gestures (Figure 5.24).

In Puzzle 7.10, unlike Bella, Carl preferred to use the Hint icon in his second attempt (Figure 5.25 – Hint: Multiply x with its inverse add $-y$ with its opposite). In his first attempt, he tapped the multiplication sign before 0 and made 0 the whole equation. In his second attempt, he followed these ten steps:

Start $\frac{xy \cdot \frac{1}{x} - y}{z} \cdot 0z$ Goal $0 \cdot 0$ Minimum Step 2

1. Drag z in front of 0, made
   $$\frac{xy \cdot \frac{1}{x} - y}{z} \cdot z \cdot 0$$
2. Tap on the multiplication sign between $\frac{xy \cdot \frac{1}{x} - y}{z}$ and z, made
   $$\frac{\left(\frac{xy \cdot 1}{x} - y\right) \cdot z}{z} \cdot 0$$
3. Drag z (numerator) onto z (denominator) z's are gone, made
   $$(\mathrm{xy} \cdot \frac{1}{x} - \mathrm{y}) \cdot 0$$
   *Drag the screen down, and tap on the Hint icon (Hint: "Multiply x with its inverse add -y with its opposite" come to the screen – Figure 5.25)
   *Drag y (next to x) to next to the other y, drop y back next to x
4. Drag $-y$ to the front, made

$(-y + xy \cdot \frac{1}{x}) \cdot 0$

5. Drag x next to $\cdot \frac{1}{x}$, made

   $(-y + y \cdot \frac{1}{x} \cdot x) \cdot 0$

6. Tap on the multiplication sign between $\frac{1}{x}$ and$x$, made

$(-y + y \cdot \frac{1x}{x}) \cdot 0$

   *Tap on the plus sign between $-y$ and $y \cdot \frac{1x}{x}$ [**Shaking Error**]

7. Tap on the multiplication sign between y and $\frac{1x}{x}$, made

   $(-y + \frac{y1x}{x}) \cdot 0$

   *Tap on the division sign between $y1x$ and x [**Shaking Error**]

8. Drag x (numerator) onto x (denominator) x's are gone, made
   $(-y + y \cdot 1) \cdot 0$
9. Tap on the multiplication sign between y and 1, made
   $(-y + y) \cdot 0$
10. Tap on the plus sign between $-y$ and y, made
    $0 \cdot 0$

   Get one clover as a reward.

**Participant did an action but not count as a step*

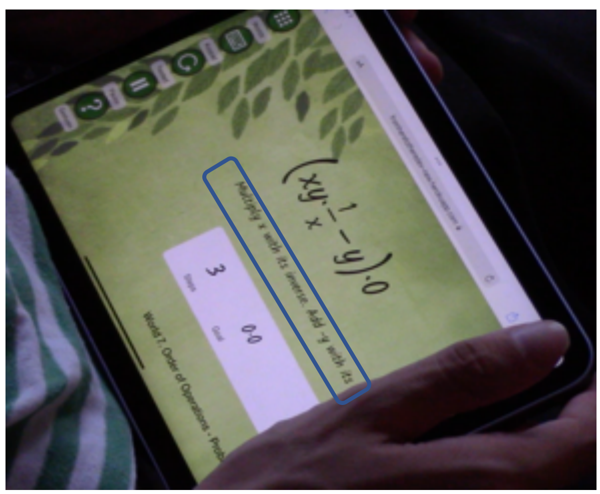


Figure 5.25: Puzzle 7.10 Hint

He followed the Hint which says “multiply x with its inverse” he did this in the 8th step and “add -y with its opposite” he did this in the 10th step and finally he reached the goal. He made some shaking errors because he did not pay attention to the order of operations; he tried to do addition before multiplication (6th step). There were also extra steps (the minimum step for a solution in this puzzle is 2), so he got only one clover.

Carl’s gameplay shows that he did not pay attention to collecting rewards and instead focused on solving the puzzles. When he needed informative feedback, he did not hesitate to use the Hint icon. He also showed his reactions to the game through his gestures when he was thinking, when he thought he made a mistake, or when he was able to solve the puzzle.

Demi’s Actions. Demi was one of the participants who struggled with the pre-test questions. She also only reached the 6^{th} World’s 14^{th} puzzle by the end of the gameplay session because she turned back and replayed puzzles if she previously skipped them or only received one or two clovers. For example, when she was solving Puzzle 3.13, she could not solve it after her second attempt and decided to tap the World icon to go back to Puzzle 2.15, which she skipped when she was solving World 2’s Puzzles.

Demi also used her finger to point at numbers on the screen before tapping them or to do an imaginary calculation (*pointing gestures*). For example, in Puzzle 2.18 she used her finger when following these three steps:

Start 120·77 Goal 15·56·11 Minimum Step 3

*Tap on the keypad, tap 120, tap cancel
*Tap on the keypad, tap 77, tap cancel

1. Tap on the multiplication sign between 120 and 77, made
   9240
2. Tap on the keypad, tap 9240, write 15 · 571, and tap done, [**Keypad Error** “The total of the new expression should be the same as the original.” Figure 5.26 – highlighted by blue circle], Delete 571 [Do math on her leg Figure 5.27], Write 616, tap done, made
   15 · 616
3. Tap on the keypad, tap 616, write 56 · 11, tap done, made

$15 \cdot 56 \cdot 11$

Get three clovers as a reward.

**Participant did an action but not count as a step*

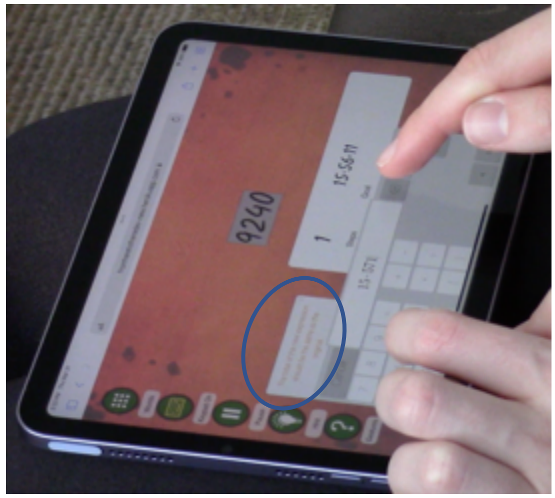


Figure 5.26: Puzzle 2.18- Demi's Keypad Error

In the second step, she made a keypad error because she could not decompose 9240 correctly. After the keypad error, she needed to re-do the calculations because 571 was not correct, so she used her right leg as a support (Figure 5.27) and did the calculation with the help of her right index finger. It appeared that she was using her finger to do a calculation of 56 multiplied by 11 on her leg.

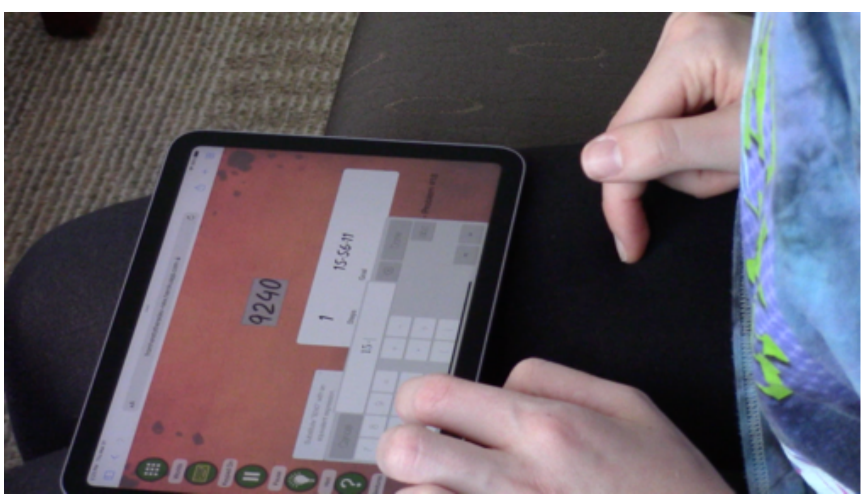


Figure 5.27: Demi used her finger and leg to do imaginary calculation.

Demi used hand gestures and the Gesture icon to solve the puzzles. When she was solving Puzzle 3.13, she kept getting keypad errors (Figure 5.28). She first used the Hint icon to get informative feedback (Figure 5.29) but it was not enough for her to solve the puzzle. She then decided to return to previous puzzles like 3.10, 2.17, and 2.18. After some time, she went back to Puzzle 3.13 and immediately tapped on the Gesture icon to get informative feedback. When she was solving Puzzle 3.13, she saw five rules or gestures for the game (Figure 5.30) and she tapped the link and reread four of them: Perform Operations by Tapping or Dragging (Figure 5.31-Left), Substitute Numbers Using the Keypad (Figure 5.31-Right), Commute Terms (Figure5.32-Left), Select Multiple Terms to Commute by Pulling (Figure 5.32-Right). Remembering the usefulness of these rules, she solved Puzzle 3.13 by following these two steps:

Start $bc + c + b$ Goal $c + cb + b$ Minimum Step 2

1. Drag b in bc after c, made
   $cb + c + b$
2. Drag c to the front, made
   $c + cb + b$

Get the three clovers as a reward.

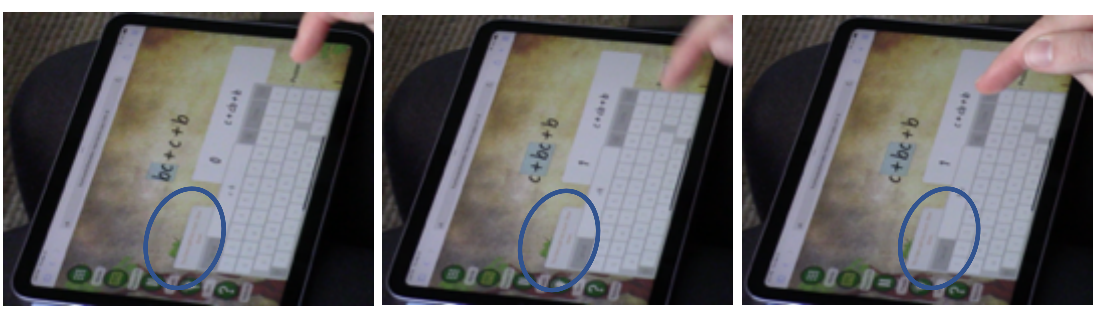


Figure 5.28: Puzzle 3.13- Keypad errors

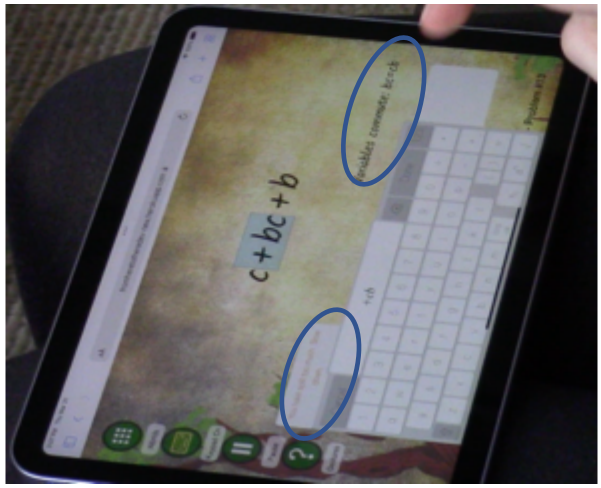


Figure 5.29: Puzzle 3.13- Hint

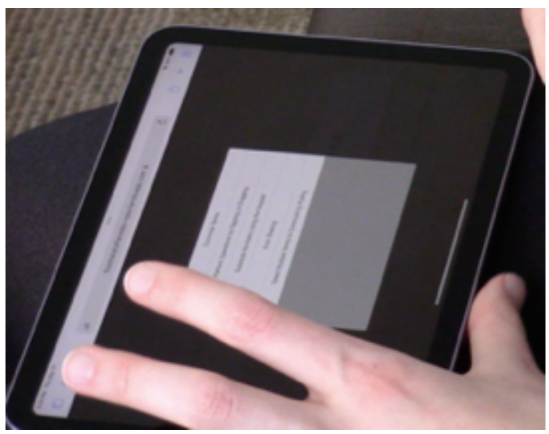


Figure 5.30: Puzzle 3.13- Gesture icon

Figure 5.31: Puzzle 3.13- Gestures: Perform Operations by Tapping or Dragging and Substitute Numbers Using the Keypad

Figure 5.32: Puzzle 3.13- Gestures: Commute Terms and Select Multiple Terms to Commute by Pulling

In summary, Demi's gameplay shows that she cared more about the rewards than solving the puzzles and did her best to collect all of the clovers. She also used her hand gestures to help her solve each puzzle as well as the Hint and Gesture icons when she needed extra informative feedback.

Evan's Actions. According to Evan's pre-interview, he likes games but does not spend a lot of time playing them. He had experience playing educational and personalized learning games. He solved Q8 and Q10 with a "perfect" score but made a large error in Q9 in the pre-test. He reached Puzzle 6.16 at the end of the gameplay session. Evan did not retry any puzzle if he got one or two clovers and he solved the last four puzzles of all five Worlds.

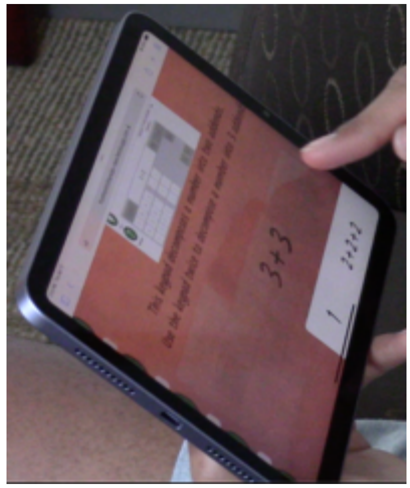


Figure 5.33: Puzzle 1.11, 6=3+3

Evan sometimes seemed to not pay attention to the tutorials on the game screen. For example, in Puzzle 1.11, the animated tutorial showed how players can decompose the number 6 into $2 + 2 + 2$ in two steps. However, Evan preferred to decompose 6 into $3 + 3$ (Figure 5.33), which is mathematically correct but not what the game asked for. Evan appeared to solve the puzzle by playing without caring about the rules of the game, the tutorial, or other information sources on the screen.

Like Carl, Evan made hand gestures to express his thinking or reactions while playing the game. This behavior was most evident in Puzzle 2.8 where he made three attempts; in his first attempt, he reached a number by dragging 6 onto 24 to create 144, which was not in the goal. As a result, he restarted the puzzle. In his second attempt, he followed these steps while using hand gestures:

tart $4 \cdot 6c \cdot 24 \cdot 16$ Goal $96 \cdot 96c$ Minimum Step 2

1. Drag 16 onto 6, made
   $4 \cdot 96c \cdot 24$
2. Tap on the keypad, tap 24, write $12 \cdot 12$, tap done
   [**Keypad Error**] Delete 12, write 2, tap done, made $4 \cdot 96c \cdot 12 \cdot 2$
3. Drag 4 onto 12, made
   $96c \cdot 48 \cdot 2$

4. Drag 48 onto 2, made
   $96c \cdot 96$ [**Hand Gesture-Figure 5.34**]
   Tap on the restart icon

Figure 5.34: Evan's Gesture

After dragging 48 onto 2 (in the fourth step), he realized he made a mistake (because the order of the numbers needed to change) and showed this with his hand gesture (Figure 5.34). In response to his mistake, he tapped on the restart icon. He then solved the puzzle in his third attempt with two steps (Step 1: Drag 16 onto 6, made $96c$, Step 2: Drag 24 onto 4, made 96) and got three clovers. He also used *pointing* gestures before tapping any numbers or variables on the game screen (Figure 5.35).

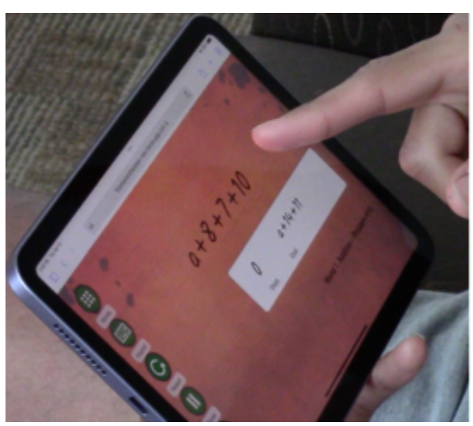


Figure 5.35: Puzzle 1.10 Pointing Gestures

Evan solved the game puzzle by playing and sometimes did not pay attention to the information/tutorials that the game provided. He used pointing gestures and showed his reactions to the game through his gestures when he thought he made a mistake. He did not pay much attention to rewards, search for any extra informative feedback, or use the Hint or Gesture icons.

Fin's Actions. Fin was the second participant who gave a long answer to Q7 and solved with a "perfect" score to Q8, Q9, and Q10 in the pre-interview. He reached the 10^{th} World's 10^{th} puzzle by the end of the gameplay session because he skipped all of the last four puzzles (i.e., Puzzle 15,16,17, and 18) of the Worlds and did not retry any puzzles when he got one or two clovers.

Unlike other participants, he did not use any Hint or Gesture icons in the gameplay session. He sometimes used body gestures as a type of *thinking* gesture, like putting his hand to his chin (Figure 5.36 Puzzle 1.9-Right and Puzzle 9.7-Left).

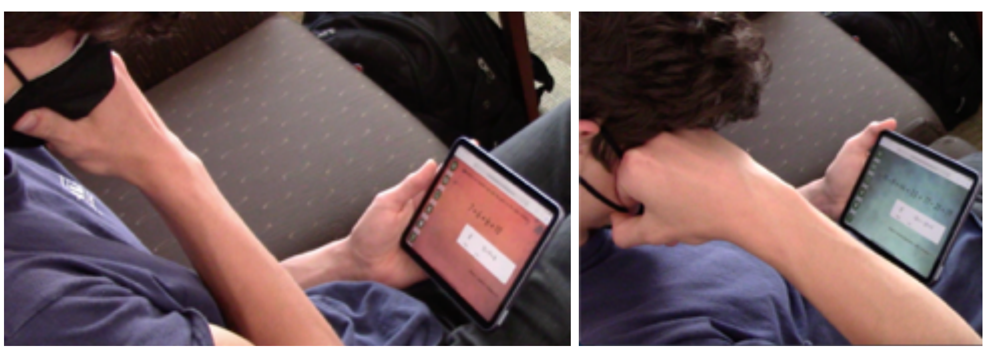


Figure 5.36: Fin's Thinking Gestures Puzzle 1.9 and Puzzle 9.7

Fin struggled the most with Puzzle 9.7. He was able to solve it in 16 steps which was the highest step number for him. He followed these steps:

Start $-91-9+44+33=77-21-79$ Goal $-12=-21+9$ Minimum Step 7

1. Drag 77 from the right to the left side of the equation, made
   $-91-9+44+33-77=77-77-21-79$
2. Drag -91 from the left to the right side of the equation, made
   $-91+91-9+44+33-77=77-77-21-79+91$
3. Tap on the plus sign between -91 and 91, made 0
   $0-9+44+33-77=77-77-21-79+91$
4. Tap on the plus sign between 44 and 33, made 77
   $0-9+77-77=77-77-21-79+91$
5. Tap on the minus sign between 77 and 77, made 0
   $0-9+0=77-77-21-79+91$
6. Tap on the plus sign between -79 and 91, made $+12$
   $0-9+0=77-77-21+12$
7. Tap on the minus sign between 77 and 77, made 0
   $0-9+0=0-21+12$
8. Tap on the minus sign between 0 and 21, 0 gone, made
   $0-9+0=-21+12$
9. Tap on the minus sign between 0 and -9, 0 gone, made
   $-9+0=-21+12$
10. Tap on the plus sign between -9 and 0, 0 gone, made
   $-9=-21+12$
11. Drag 12 from the right to the left side of the equation, made
   $-9-12=-21+12-12$
12. Drag -9 from the left to the right side of the equation, made
   $-9+9-12=-21+12-12+9$
13. Tap on the minus sign between 12 and 12, made 0
   $-9+9-12=-21+0+9$
14. Tap on the plus sign between -9 and 9, made 0
   $0-12=-21+0+9$
15. Tap on the plus sign between 0 and -12, 0 gone, made
   $-12=-21+0+9$
16. Tap on the plus sign between -21 and 0, 0 gone, made
   $-12=-21+9$
   Got one clover as a reward.

In this world (World 9), Fin got a new rule in Puzzle 9.1: "*A new way to work with equations - dragging. Drag the 2 across the equals sign to add its opposite to both sides. Notice the sign change*!" Fin was able to drag a number from one side to another side of the equation as he did in steps 1, 2, 11, and 12. He kept using this rule, but he needed to use it wisely. Instead of dragging 77 from the right to the left side in the first step, he should have added 77 and -79 on the right side and then added 44 and 33, and -91 on the left side. Hence, he could reach the equation $-9-14=$

$-21-2$ with three steps. He also needed to use previous rules like one from Puzzle 8.1 “Tap and hold the equals sign to do the same thing to both sides. The changes you make to E in the keypad will happen on both sides.” With this rule, he should have added 11 to both sides. Hence, this old rule needed to be remembered to solve Puzzle 9.7 in seven steps. If he used the Hint icon to get extra information, he had got the hint as "You need -21 and 9 on the right. Isolate -21 and drag -9 across the equals sign to add 9 to both sides.” hence he would not need to do steps 1, 2, 11, and reduce the step numbers at least 3 steps.

Fin’s gameplay shows that he paid attention to the new rules in each new world and applied the rules to the puzzles. He did not pay attention to collecting rewards and instead just focused on solving the puzzles to move forward. He did not use any Hint or Gesture icons to get informative feedback from the game.

Gina’s Actions. Gina was another participant who struggled with the pre-test questions and solved the least number of puzzles in the group. She reached Puzzle 3.18 by the end of the gameplay session, and never skipped the last four puzzles of the worlds.

Gina preferred to use her finger while she was reading the instructions. (Figure 5.37). She used her index finger to read in almost in every puzzle. Gina was able to solve only 54 puzzles (reached Puzzle 3.18) and was the participant to solve the least number of puzzles. It is possible that reading the tutorials with her index finger may have slowed her down.

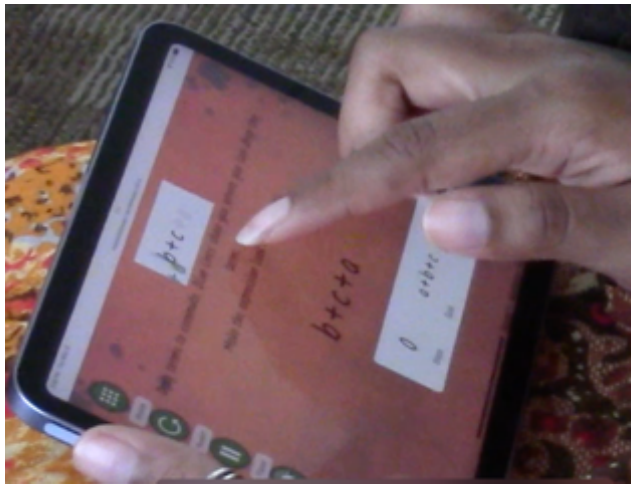


Figure 5.37: Gina's Following Finger Gesture

In the second puzzle, Puzzle 1.2, after reading the instruction "Click the lightbulb on the left if you need a hint." she followed the information and tapped the Hint icon (Figure 5.38) and got informative feedback: "Click on numbers and drag them to rearrange". Puzzle 1.2 was similar to the previous puzzle, as in the Hint, the numbers needed to be rearranged. Therefore, her tapping showed that she read the instruction and followed it, so she was learning the game.

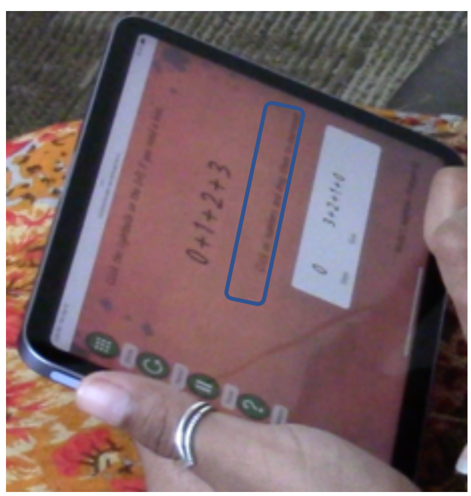


Figure 5.38: Gina Following Information

However, in some puzzles, she seemed to not pay attention to the goal or animated instructions of the puzzle. For example, in Puzzle 1.11 (Figure 5. 39) the tutorial animation showed that the starting point is 6 and the goal is $2 + 2 + 2$ (Figure 5.39 –Blue Box). However, Gina approached the puzzle differently: $3 + 3$ (in the first attempt), $2 \cdot 3$ (in the second attempt), $2 \cdot (1 + 2)$ and $1 \cdot 2 \cdot (1 + 2)$ (in the third attempt), $2 \cdot 2 + 2$ (in the fourth attempt) (Figure 5.40). These steps are mathematically correct but not what the game asked for. On the fourth attempt, after the incorrect decomposing of 4 (i.e.: rewriting 4 as $2 \cdot 2$ instead of $2 + 2$), she realized that she made a mistake and tapped the multiplication sign between 2 and 2 to make 4, again, and rewrote 4 as $2 + 2$ by using a keypad icon. If there were no tutorials in the puzzle in two modalities (e.g., written form and animation), I would say that Gina explored the game and tried out different options. However, there was a tutorial and Gina solved the puzzle in her fourth attempt. Gina appeared to solve the puzzle without caring about the tutorial on the screen and without caring about the number of steps and/or attempts and errors.

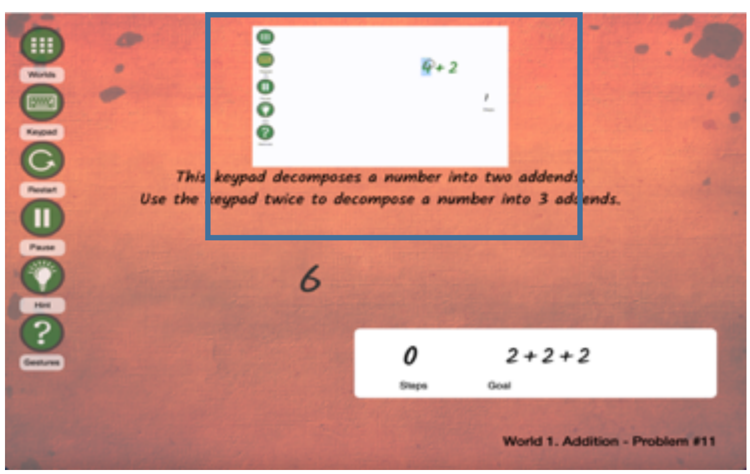


Figure 5.39: Puzzle 1.11

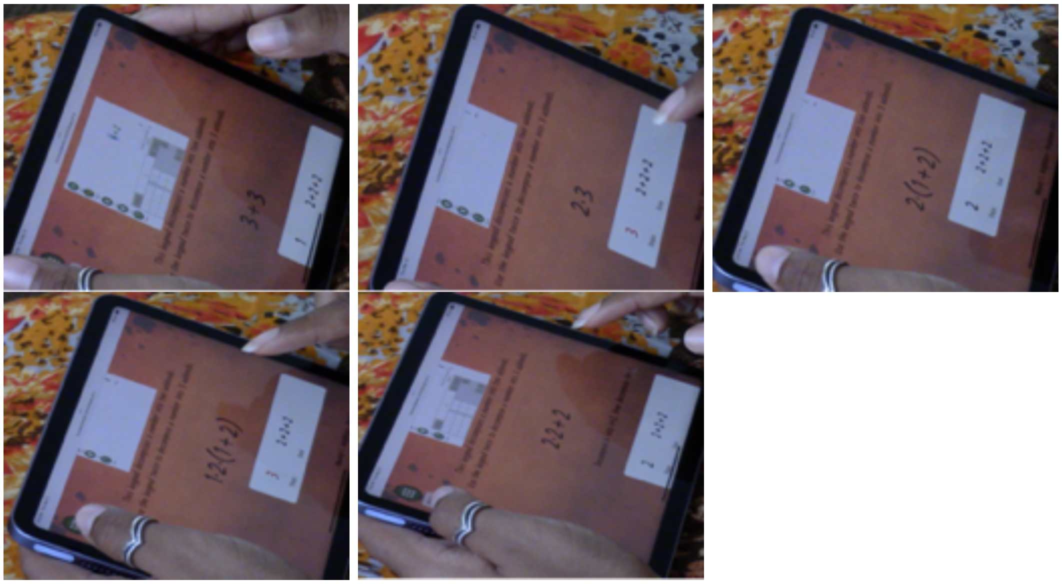


Figure 5.40: Puzzle 1.11 Gina's Different Decomposition of 6

Gina worked the slowest through the puzzle and was able to reach just Puzzle 3.18 at the end of the play sessions. She solved the puzzle by reading the instruction and playing but she did not pay attention to the animated tutorials or rewards. She used the Hint icon more than once after already using the offered Hint gesture. She used pointing gestures while solving the puzzles.

In summary, participants had different gameplay strategies and paces, and showed different kinds of gestures while playing the game. Ava, Carl, Demi, and Gina preferred to use the Hint icon to get informative feedback, while Bella, Evan and Fin did not use any Hints. Ava and Demi also used the Gesture icon. Ava and Demi re-solved puzzles if they got one or two clovers, but Bella, Carl, Evan, Fin, and Gina did not re-solve puzzles after completing them and earning clovers. Ava skipped solving the last four puzzles in Worlds 1,2, and 3 but solved puzzles in the 4th to 8th Worlds. Bella just solved the last four puzzles in the 1st World, then skipped to other worlds. Carl skipped the last four puzzles in 3rd World, and Demi also just skipped the 5th World's last four puzzles. Meanwhile, Fin preferred to skip those puzzles. Evan and Gina solved all of the last four puzzles in the Worlds they reached. Evan, Fin, and Gina used pointing gestures, and Carl and Fin showed their thinking through gestures. Fin (10.10), Bella (10.8), Ava (8.18), and Carl (8.3) went the farthest in the game while Evan (6.16), Demi (6.14), and Gina (3.18) solved puzzles in the earlier worlds. In the next section, Play Aloud, I present how players express themselves while playing the game.

Play-Aloud

After gameplay, I observed the students in a play-aloud session (Pellicone et al., 2022) to understand what sort of physical gesture clusters the participants used to explain their digital gestures. The participants started playing the puzzle that was their last puzzle at the end of the gameplay session, but this time they explained out loud what they were doing. At the beginning of the session, I also asked each participant "If you had to describe this game to your friends, how would you describe it, could you play like you are doing live streams to your friends?" I asked this question to clarify the meaning of the play-aloud session by giving the known example of live stream videos.

My open-coding process included some priori codes which were defined by Alibali and Nathan (2012): Pointing gestures, Representational gestures (dynamic and static), and Metaphoric gestures. Moreover, I added the gesture code and labeled each gesture as: Feedback gestures; thinking gestures, and negative and positive emotion gestures.

Pointing Gestures. Participants played the game on an iPad tablet, a touchscreen device. Hence, to play the game, they generally used their index finger to *tap* ("briefly touching the surface with a fingertip", Villamor et al., 2010, p.1) on the number and to *drag* ("moving fingertip over the surface without losing contact", Villamor et al., 2010, p.1) the number to the new place. Tapping and dragging are examples of touchscreen gestures. On the other hand, similar gestures with an index finger were defined as *pointing* gestures, "gestures that serve to indicate objects or locations, often with an extended index finger" (Alibali & Nathan, 2012, p.251) because players used their index finger or any other finger to show the variable before tapping or dragging it.

For example, Demi's first time through Puzzle 7.3. The topic of World 7 is Order of Operations, and in Puzzle 7.3, the given equation is $\frac{39}{2+1} - 2\cdot(4+1)$ and the goal is $-7+1+9$. Demi was seeing addition as another operation in the denominator for the first time. In Figure 5.41, Demi expressed her feelings about the situation. To solve this Puzzle 7.3, the player should start with parentheses like $2+1$ or $(4+1)$ to follow the order of operations. $2+1$ did not have visual parentheses, however, to follow the order of operations – Parentheses, exponents, multiplication-division (from left to right), and addition-subtraction (from left to right) – or to do the division, she needed to tap the plus sign in $2+1$ first. After pointing at $2+1$, Demi tapped the plus sign between 2 and 1, and made 3, then kept solving the puzzle.

I haven't seen, I don't think I've seen this yet,
pointing to the denominator, 2+1
which is cool
tapping keypad icon, re-tap keypad icon
Let's set this up then
tapping plus sign between 2 and 1 made 3

Note: For this and the following figures, gestures are shown indented and in bold, and actions are shown indented and italics.

Figure 5.41: Demi – Pointing New Concept.

Another example of a *pointing gesture* is pointing to two variables at the same time. In World 10, the topic is Distribution. In Puzzle 10.13 (Figure 5.42), the goal is to end with $24xy + 28x - 30y - 35$ when the starting equation is $(4x - 5)\cdot(6y + 7)$. To solve the puzzle, players need to multiply $4x$ and $6y$ by distributing $(6y + 7)$ across $(4x - 5)$. In Figure 5#2, Fin pointed together $4x$ and $6y$, then he pointed separately $4x$ and $6y$, and after thinking a little, he dragged $6y + 7$ into $4x - 5$ to make $24xy$ as the first variable of the goal. He then proceeded with solving the puzzle.

We want to combine
pointing 4x and 6y together
$4x$
pointing 4x
and $6y$
pointing 6y
into $24xy$…
So, let's see, it's gonna be like this, then combine that
dragging $(6y + 7)$ *in the* $(4x - 5)$

Figure 5.42: **Fin – Pointing Two Numbers Together**

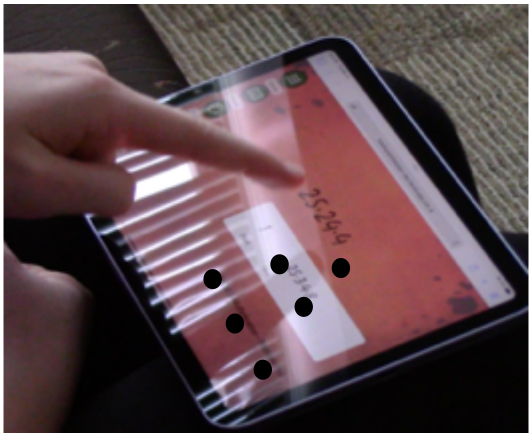
Representational Gestures. Representational Gestures are defined as "gestures that depict semantic content directly via the shape or motion trajectory of the hand" (Alibali & Nathan, 2012, p. 251). There are two subcategories of representational gestures; dynamic and static (non-dynamic) representational gestures. Dynamic representational gestures are used to investigate a variety of an object's attributes (e.g., adjusting the angle of the triangle, Nathan et al., 2021). Static representational gestures are used to describe the characteristics of objects (e.g., location or shape, like showing two sides of a triangle with two hands, Nathan et al., 2021). In this study, participants did a live stream of their game-playing to demonstrate their actions and perceptions as they expressed themselves through both dynamic and static representational gestures.

Figure 5.43 highlights both dynamic and static features of representational gestures. It is dynamic because the participant showed the object's properties like drawing 0 in the air, and it is static because the participant showed the location of 0. In Puzzle 7.13 (Figure 5.43)**,** the goal is $0 - a \cdot -a + 0$ and the starting equation is $-z + y \cdot \frac{4 - a \cdot -a - 4}{y} + z$. Demi got 0 at the end of the equation by dragging $-z$ onto z (Figure 5.#3). After getting 0, to solve the puzzle she needed 0 in the beginning of the equation. She represented 0 with a gesture by drawing 0 in the air (i.e., dynamic representation) at the beginning of the equation (i.e., static representation). Then she kept solving the puzzle, first canceling $y's$ in the equation and then canceling the 4's to equal zero. So, Demi solved the puzzle in three steps. Hence, this example shows that some participants, like Demi, used representational gestures to cover mathematical knowledge and express themselves.

We get our plus zero on the end
pointing 0 at the goal
pointing 0 at the equation
and yes, technically, there should be a zero
drawing 0 in the air over y
in the beginning
dragging y (numerator) onto y (denominator) – canceling y's
in order to get our zero
drawing 0 in the air
in the least amount of step, I'm going to bring the negative four to the positive four]
dragging −4 *onto* 4 *– canceling* 4*'s*

Figure 5.43: **Demi – Drawing 0, Tracing a Circle in the Air.**

Representational gestures can also convey imaginative perceptions. In this game, *From Here to There!*, player skill is measured by step numbers. The purpose of these step numbers is to mimic the traditional step-by-step process of solving mathematics problems. Demi depicted these steps as gestures in Puzzle 2.16 (Figure 5.44). The goal of the puzzle is $25\cdot3\cdot4\cdot8$ and the start equation is $5\cdot24\cdot4\cdot5$. After dragging 5 (at the end) onto 5 (at the beginning) to make 25, Demi needed to decompose 24. In this example, Demi represented the steps by showing not an object but the imaginary place of the factors of 24; first 12 and 2 then 3, 4, and 2. This example includes representational gestures, "*split up … into…*" which cover mathematical concepts like the factors of 24.



Normally I would just split up the 24

pointing 24 (first-row, one point)

into other 12 and 2

pointing two places for 12 and 2 below 24 (second-row, two points)

and then go into three four two

pointing three places for 3 4 2 below 12 and 2 (third-row, three points)

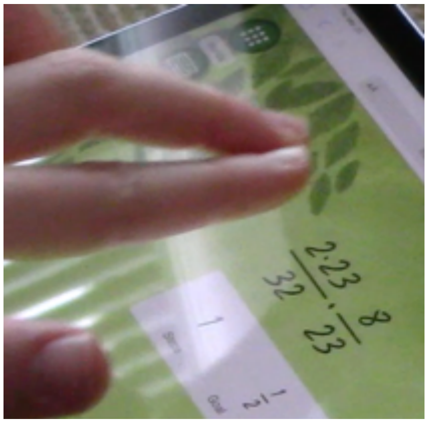
and then multiply the 2 by 4

pointing a place for 2 and pointing 4 in the equation

Figure 5.44: Demi – Trajectory the Steps

Metaphoric Gestures. There is an argument that “metaphoric gestures reveal speakers’ body-based conceptual metaphors” (Alibali & Nathan, 2012, p. 267). As humans, we are understanding abstract concepts through concrete terms, in other words, conceptual metaphors (Lakoff and Núñez, 2001). Lakoff and Núñez (2001) presented some conceptual metaphors for mathematical concepts: Numbers Are Things in the World, Arithmetic is Object Collection, Arithmetic is Motion Along a Path, Arithmetic is Object Construction, etc.

In this study, I observed the “Numbers are Things in the World” and “Arithmetic Is Object Collection (putting a collection together or taking smaller parts from the larger collection)” metaphors. Figure 5.45 shows how Demi solved Puzzle 6.10, where the main topic of World 6 is division. In Puzzle 6.10, the given equation is $\frac{46}{32}\cdot\frac{8}{23}$ and the goal is $\frac{1}{2}$. Demi used metaphors like numbers are things that can move or be manipulated (line 3), multiplication brings these objects together, and division splits up these objects (line 8). Moreover, Demi expressed herself through gestures (line 4 for moving the numbers and lines 10 and 13 for bringing and separating the numbers).



But these fractions
pointing fractions
are cool, because I liked, I could move them
moving her hand from up and down
and like like if I moved (23) up here
dragging 23 (denominator) onto 46 (numerator) made 2.23
then it would split this one (the 23 in the denominator) or this one (46 – 2.23 – NUMERATOR)
bringing together her index and middle finger and then
separate
like into what is it called like common factors
bringing together her left and right hand and then
separate
and then I could bring them together (23's) and
dragging 23 (numerator) onto 23 (denominator) 23's gone

Figure 5.45: Demi – Putting Together

Another metaphoric gesture example is not related to mathematical understanding. This metaphoric gesture refers to the idea that brains can be colloquially considered machines, where Person 1 may chuckle at Person 2, who is thinking really hard, and say: "I can see your gears are turning!" After solving Puzzle 11.13, Fin made a twirling motion reminiscent of gears turning with his index finger in the air and said, "talk it through, it really helps me get the [silence a moment – twirling motion in the air], I guess the best solution" (Figure 5.46). I coded this gesture as a metaphoric gesture because of what I observed and the alignment of his gestures with the definitions of metaphoric gestures. First of all, he did the gesture after solving the puzzle, then was silent for a second, and then made the gesture like he wanted to remember something or wanted to work the "gears in his brain." His gesture looked like he was turning a gear.

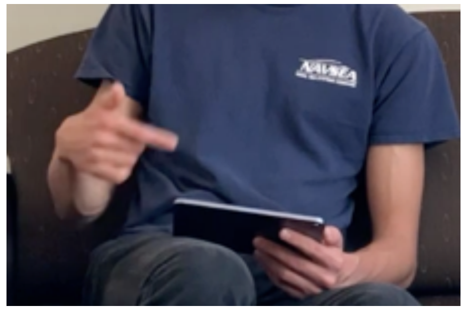


Figure 5.46: "Brain is a Machine"

Feedback Gestures. One of the research questions is "How do failure and feedback manifest in mathematical gameplay within playing the game from the student's perspective?" Hence, after some successful or unsuccessful puzzle-solving action, players would react to their achievement (e.g., by celebrating) or their failure (e.g., sunken shoulders). Some gestures meant that the player was thinking, and expressing either negative emotions (feeling confusion, getting bored, getting defensive, feeling nervousness, etc.) or positive emotions (feeling pride, enjoyment, etc).

Thinking Gestures. In this study, the players' thinking gestures were actually silent gestures; players did not say that they were thinking or what they were thinking. Instead, they just played silently while enacting gestures as a way to show that they were thinking. For example, they scratched their head (Figure 5.47 – Left) and put their hand to their chin (Figure 5.47 – Right).

Figure 5.47: Thinking Gesture – Scratching Head (Left) and Putting Hand to Chin (Right)

Negative Emotions Gestures. The players' negative emotions were also silent gestures; they did not say they were getting bored, getting defensive, or feeling nervous, but they did show these emotions through various kinds of gestures as a clue. In other words, their gestures helped to explain the negative emotions they were possibly feeling while playing. For example, cracking their fingers; meaning dealing with nervous energy (Shmerling, 2020) (Figure 5.48 – Left), crossing their arms; meaning feeling uncomfortable (Glass, 2002) (Figure 5.48 – Right), drumming their fingers; meaning nervousness (Glass, 2002) (Figure 5.49 – Left), self-touching; meaning uncomfortable (Glass, 2002) (Figure 5.49 – Right).

Figure 5.48: Negative Emotions Gestures – Cracking Fingers (Left) and Crossing Arms (Right)

Figure 5.49: Negative Emotions Gestures – Drumming Fingers (Left) and Self-touching (Right)

Positive Emotions Gestures. Besides negative emotions, players showed some positive emotions. However, these positive emotions were not expressed through gestures. For example, players sometimes said “Yes!”, and “Perfect!” after getting three clovers from the puzzle. They also showed some enjoyment while solving puzzles and this joyfulness affected their voices. Even though they were wearing masks, I could still see them smile and laugh sometimes.

In summary, these pointing, representational, metaphoric, and feedback gesture examples show that participants used these physical gesture clusters to explain themselves and to express their emotions. Moreover, their gestures helped us to understand what they meant by their comments about the game, and vice versa, their comments helped us understand their gestures.

Post-Interview

According to the players’ post-interview responses, they generally liked playing the game during the gameplay session. Their responses will be analyzed in detail in later papers. In this study, the last four questions were analyzed.

Q13) How did you solve this puzzle? Could you describe and solve it again from the tablet?

(FH2T Puzzle 1.9, Figure 5.50)

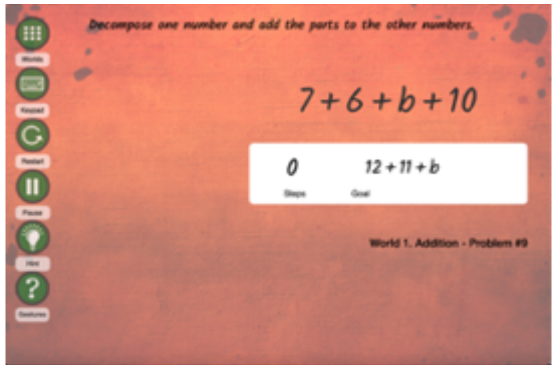


Figure 5.50: Post-Interview Q13 – Puzzle 1.9

Q14) How did you solve this puzzle? Could you describe and solve it again from the tablet?

(FH2T Puzzle 2.14, Figure 5.51)

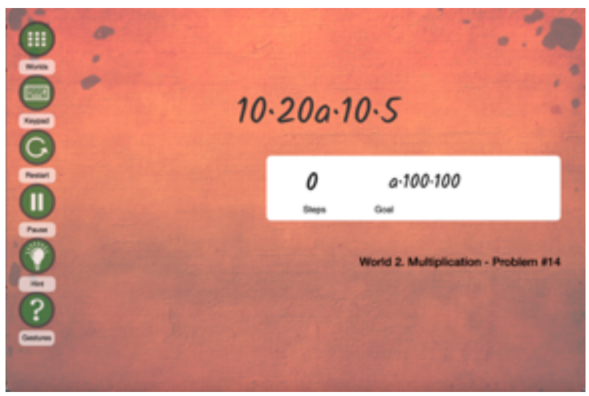


Figure 5.51: Post-Interview Q14 – Puzzle 2.14

Q15) How did you solve this puzzle? Could you describe and solve it again from the tablet?

(FH2T Puzzle 3.14, Figure 5.52)

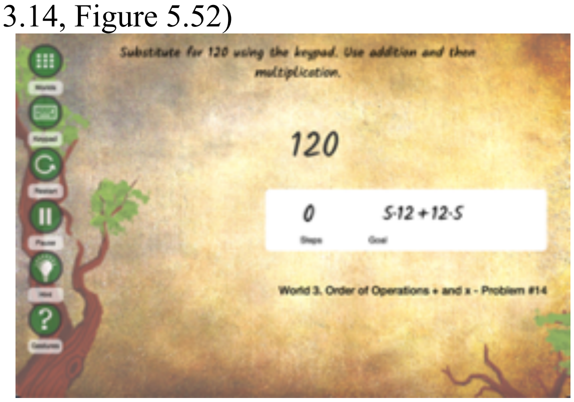


Figure 5.52: Post-Interview Q15 – Puzzle 3.14

Q16) How did you solve your latest puzzle? Could you describe and solve it again from the tablet?

Q13 Responses. The post-interview Q13 (Puzzle 1.9 Figure 5.50) is similar to pre-interview Q8 (Figure 5.53). The purpose of these questions is to understand participant knowledge of properties of addition like closure, commutative, associative, etc. In Puzzle 1.9, players need to first decompose 10, then add the factor of 10 to 7 and 6.

$$4 + 3 + a + 9 = 8 + \_\_ + a$$

Figure 5.53: Pre-interview Q8

Ava solved Puzzle 1.9 in the gameplay session and got three clovers. In the post-interview, she chose to decompose 10 into $5 + 5$ (Figure 5.54). She paid attention to the order of her digital gestures by tapping the "keypad" button to access the keyboard and then tapping the number (10). She then substituted the number correctly ($5 + 5$) to avoid a keypad error and tapped done to activate the substitution from the keypad to the game screen. To solve the puzzle and earn three clovers, she dragged the first 5 onto 6 to make 11 and then dragged the second 5 onto 7 to make 12. Carl followed the same strategy as Ava (decomposing 10 into $5 + 5$, then adding 5's to 6 and 7).

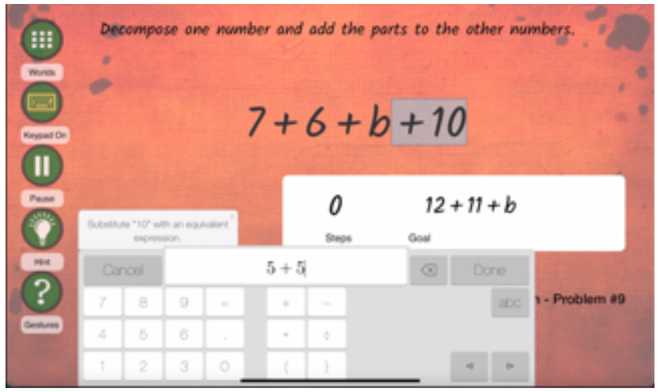


Figure 5.54: Ava and Carl's Strategy on Puzzle 1.9

Bella solved the puzzle in the gameplay session and got two clovers. In the gameplay session, she preferred to decompose 7 into 6 + 1, then she dragged 6 next to 6, and 10 next to 1, then tapped the plus sign between these number groups. This strategy involved extra two steps. However, in the post-interview, Bella decomposed 6 as 5 + 1 (Figure 5.55), then dragged 10 onto 1 to make 11, and completed the problem by dragging 5 onto 7 to make 12. Since she solved the puzzle in three steps, she received three clovers.

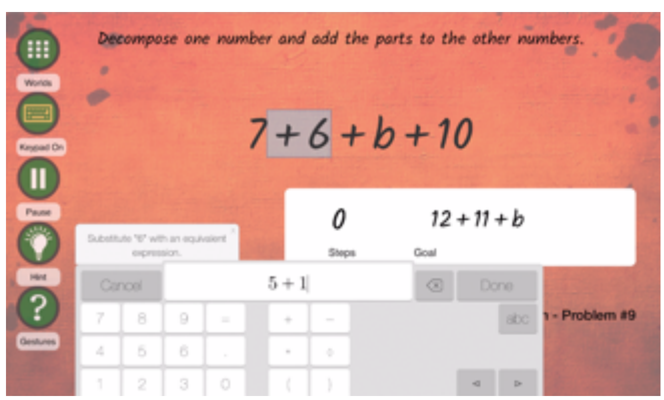


Figure 5.55: Bella's Strategy on Puzzle 1.9

Demi solved the puzzle in the gameplay session and got three clovers. In the post-interview, on her first attempt, she got two clovers. She first dragged b to the end, which caused an extra step. To complete the problem, she then tapped the plus sign between 7 and 6 to make 13, decomposed 13 into 12 + 1 (Figure 5.56 – Left), and tapped the plus sign between 1 and 10 to make 11. Therefore, she solved the puzzle in four steps and got two clovers. However, in her second attempt, she dragged 10 onto 7 made 17, then decomposed 17 into 12 + 5 (Figure 5.56 –Right), and ended with tapping the plus sign between 5 and 6 to make 11. In this attempt, she solved the puzzle in three steps and got three clovers.

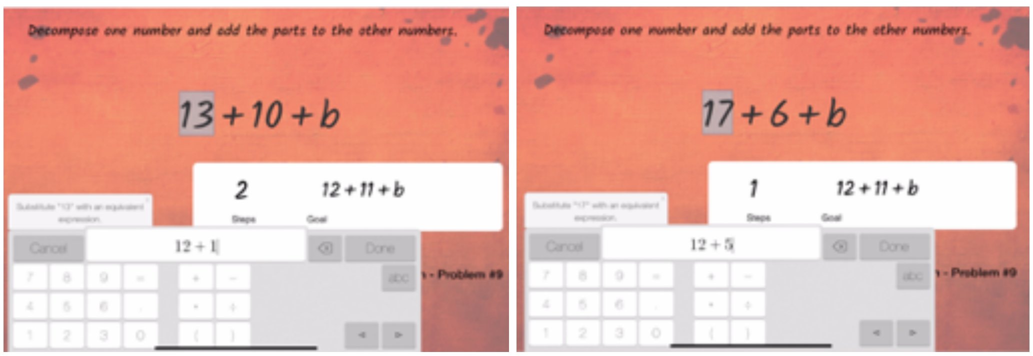


Figure 5.56: Demi's Strategies on Puzzle 1.9

Evan solved the puzzle in the gameplay session and got two clovers. In the post-interview, on his first attempt, he tapped plus sign between 7 and 6 made 13, then restarted the puzzle. In his second attempt, he tried to decompose 6 into 3 + 2 but received keypad error feedback: "The total of the new expression should be the same as the original" (Figure 5.57). After this feedback, he deleted 2 and tapped 3 on the keyboard. Then he tapped the plus sign between 7 and 3 and got 10. Then he decomposed 10 into 5 + 5, but then tapped the restart icon. In his third attempt, he decomposed 10 into 5 + 5, and then dragged one 5 onto 6 making 11, and the second 5 onto 7 to make 12, solving the puzzle in three steps and getting three clovers.

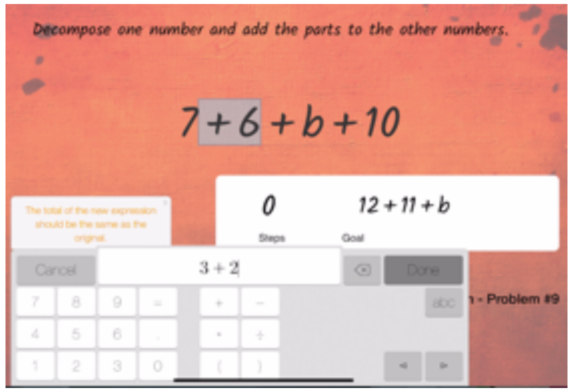


Figure 5.57: Evan's Strategy on Puzzle 1.9

Fin solved the puzzle in the gameplay session and got two clovers. In the post-interview, he preferred to decompose 7 into 1 + 6 (Figure 5.58– Left), then drag 6 next to 6 making 12. After performing this step, he restarted the puzzle. In his second attempt, he decomposed 7 into 6 + 1 (Figure 5.58 – Right), dragged 10 onto 1 to make 11, and then 6 onto 6 to make 12, solving the puzzle in three steps and getting three clovers.

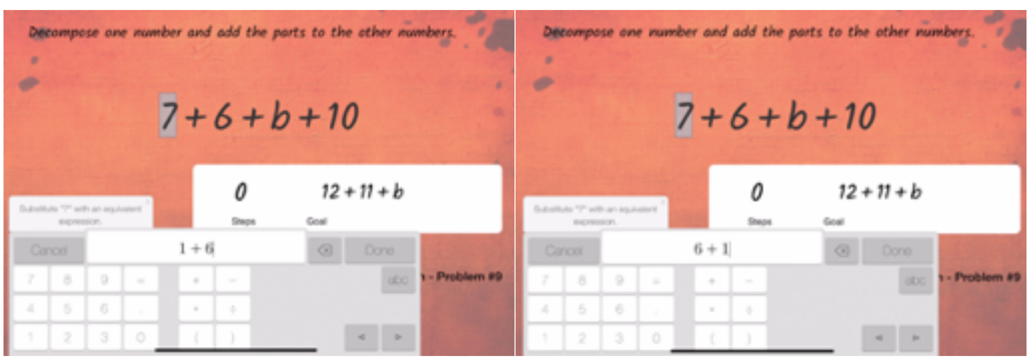


Figure 5.58: Fin's Strategy on Puzzle 1.9

Gina solved the puzzle in the gameplay session and got two clovers. In the post-interview, she got 2 clovers again. She, like Fin, preferred to decompose 7 into 6 + 1 (Figure 5.59), then drag 6 onto 6 to make 12. After performing these steps, she dragged b to the end of the equation, which caused an extra step. To complete the problem, she dragged 1 onto 10 and made 11. She was able to solve the puzzle in four steps and got two clovers.

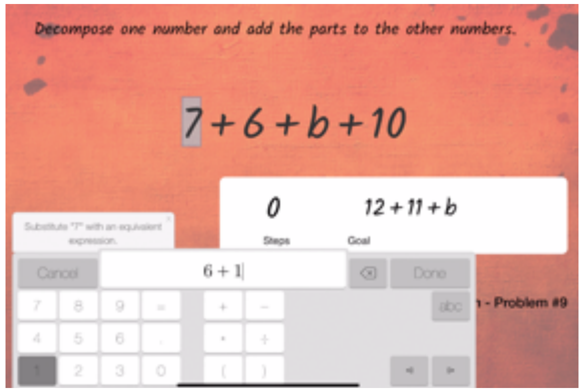


Figure 5.59: Gina's Strategy on Puzzle 1.9

Q14 Responses. The post-interview Q14 (Puzzle 2.14, Figure 5.51) is similar to pre-interview Q9 (Figure 5.60). These questions help to understand participant knowledge of properties of multiplication like the commutative and associative properties. In Puzzle 2.14, players need to multiply 10 by 10 to make 100 and 20 by 5 to make 100. Bella, Carl, Demi, Evan, and Fin each solved the puzzle as required by following these steps.

$$10 \cdot 10a \cdot 20 \cdot 5 = a \cdot 100 \cdot \_\_$$

Figure 5.60: Pre-interview Q9

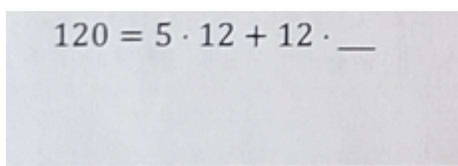
Ava solved Puzzle 2.14 in the gameplay session and got one clover. In the interview, she mentioned she treated the blank as an x during the pre-interview and solved for x. She did one side was equal to ten thousand a and the other side one hundred a times x. She solved the question by merging numbers together a, 100, and 100 to multiply with each other. In the post-interview, while solving Puzzle 2.14, she dragged a to the beginning of the equation first, then tried to factor 20 as 10 times 5, but this caused a keypad error (Figure 5.61). The feedback (Keypad yellow popup message: “The total of the new expression should be the same as the original) helped Ava realize that she made an error. After receiving this feedback, she changed 5 to 2, then dragged 2 onto 5 made 10, then multiplied 10’s each other to make $100 \cdot 100$. Dragging a to the front, decomposing 20 into 10 and 2, and multiplying 2 with 5 caused her to make extra three steps, so she solved the Puzzle in five steps while the minimum step number is two. She got just one clover in the end. In her second attempt, she did not decompose 20, directly multiplied 5 with 20, and made 100, avoiding the extra two steps. However, instead of dragging 20 onto 5, she dragged 5 onto 20a, creating an extra step to get a alone. In this attempt, she solved the puzzle in three steps and got two clovers.

Figure 5.61: Ava’s Strategy on Puzzle 2.14

Gina solved the puzzle in the gameplay session and got two clovers. In the post-interview, she again got two clovers because she preferred to drag a to the front of the equation which caused an extra step (Figure 5.62). She then solved the puzzle in three steps by dragging 5 onto 20 making 100 and dragging 10 onto 10 making 100.

Figure 5.62: Gina's Strategy on Puzzle 2.14

Q15 Responses. The post-interview Q15 (Puzzle 3.14, Figure 5.52) is the same as the pre-interview Q10 (Figure 5.63). The question aims to understand participant knowledge of properties of multiplication like the associative and distributive properties. In Puzzle 3.14, players need to decompose 120 to 60 and 60, factor the first 60 into $5 \cdot 12$, and then the second 60 into $12 \cdot 5$. Except for Gina, other participants – Ava, Bella, Carl, Demi, Evan, and Fin –solved the puzzle as required by following these steps.



$120 = 5 \cdot 12 + 12 \cdot \_\_$

Figure 5.63: Pre-interview Q10

Gina solved Puzzle 3.14 in the gameplay session and got two clovers. In the post-interview, she again got two clovers but solved them in two attempts. In the first attempt, she decomposed 60 into 6.10 (Figure 5.64-Left), which is mathematically correct but not the goal of the puzzle. In her second attempt, she also followed the needed steps: 1. Decomposing 120 into $60 + 60$, and 2. Decomposing the first 60 as $5 \cdot 12$. However, in the third step, she again decomposed the second 60 as $5 \cdot 12$

instead of as $12 \cdot 5$, thus needing to change the order of 5 and 12 (Figure 5.64-Right) and causing an extra step.

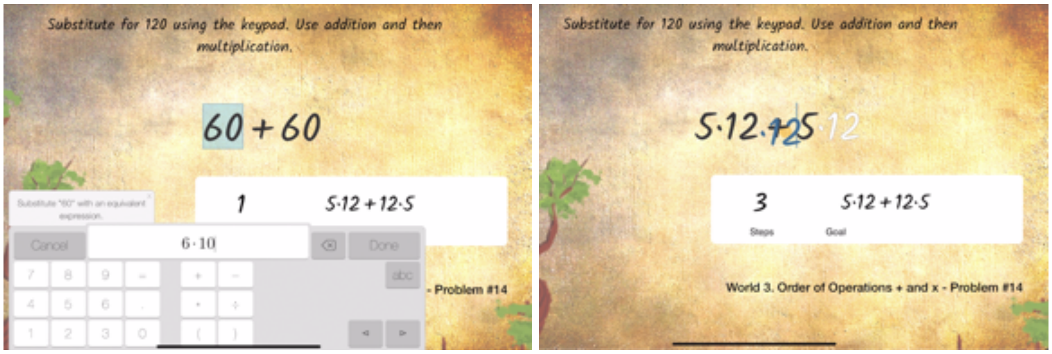


Figure 5.64: Gina's Strategy on Puzzle 3.14

Q16 Responses. The post-interview Q16 is the participants' last puzzle in the play-aloud session. Ava solved Puzzle 9.10 as her final puzzle. In Puzzle 9.10, the given equation is $z - 19 + 38 = z + 80 - 61$ and the goal is $-99 = -99$. Nine steps are enough to solve the puzzle, but Ava said she did not care about the step numbers or the number of clovers she gets. She first got rid of z's from the equation by dragging z from one side to another, which required five steps already. Then she realized 80 and 19 make 99 on the left side, so she dragged 80 from the right to the left side. She also realized 61 and 38 make 99 too, so she dragged 38 from the left to the right side (Figure 5.65). She then got rid of like terms (e.g., 38 and -38 on the left side and 80 and -80 on the right side). Finally, she solved the puzzle with 13 steps and got one clover.

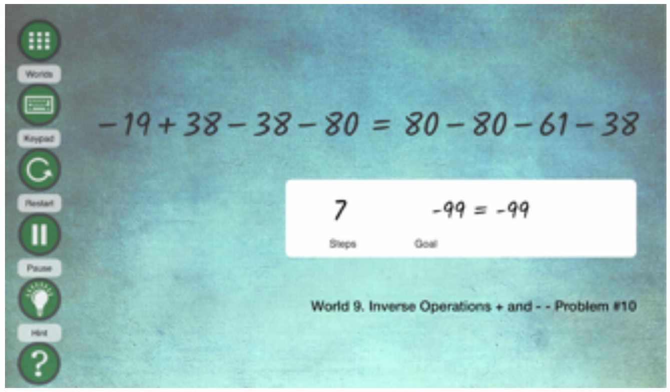


Figure 5.65: Ava's Strategy on Puzzle 9.10

Bella solved Puzzle 12.2 as her final puzzle (Figure 5.66). In Puzzle 12.2, the given equation is $18 = 18$ and the goal is $\frac{18}{6} = \frac{18}{6}$. Puzzle 12.2 is the first puzzle introducing the feature of allows players to divide the variable to the same number in both sides of the equation by tapping and holding an equal sign, which opens the keyboard. For this reason, the puzzle has an animated solution that Bella followed by first tapping and holding the equal sign to open the keyboard, then tapping the division sign, then 6, and then done on the keyboard. She solved the puzzle with one step and got three clovers.

Figure 5.66: Bella's Strategy on Puzzle 12.2

Carl solved Puzzle 8.1 as his final puzzle (Figure 5.67). Puzzle 8.1 is the first puzzle introducing the feature of tapping and holding an equal sign, which opens the keyboard and allows players to add a variable to both sides of the equation (the property of equality). Since the puzzle is the first puzzle that introduces the property of equality, it has an animated solution. Carl followed the animated solution by first tapping and holding the equal sign to open the keyboard, then tap the plus sign, then tap 1, and then done on the keyboard. So, he solved the puzzle with one step and got three clovers.

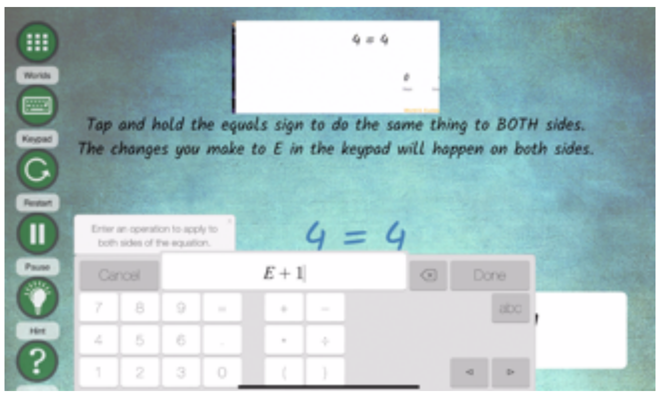


Figure 5.67: Carl's Strategy on Puzzle 8.1

Demi solved Puzzle 8.6 as her final puzzle (Figure 5.68). In Puzzle 8.6, the given equation is $14 = 62 - c$ and the goal is $c + 14 = 62$. To solve Puzzle 8.6 and earn all three clovers, players need to add c in front of E, not after E. However, Demi added c after E (Figure 5.68), which caused extra steps for her because she needed to change the position of c later. After adding c to the equation, she canceled out the c's on the right side by tapping the plus sign between $-c$ and c, then again tapping the plus sign between 62 and 0 to get rid of 0. She finished the puzzle by dragging c in front of 14 on the left side of the equation. She solved the puzzle with four steps and got two clovers.

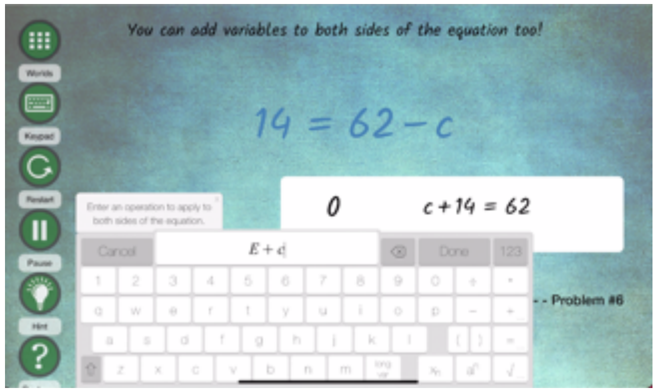


Figure 5.68: Demi's Strategy on Puzzle 8.6

Evan solved Puzzle 7.3 as his final puzzle (Figure 5.69). In Puzzle 7.3, the given equation is $\frac{39}{2+1} - 2\cdot(4+1)$ and the goal is $-7+1+9$. He solved the puzzle in two attempts. In the first attempt, he received informative feedback by tapping the Hint icon (Figure 5.69a). He followed the Hint: "You can start with $2 + 1$ or $(4 + 1)$". Attempting to follow the hint's advice, he got two different keypad errors. First, he tried to decompose -10 into $1 + 9$ (Figure 5.70-Left) and second he tried to decompose 13 as -7 and -6 (Figure 5.70-Right). These examples show that he did not pay attention to the negative and positive properties of 10 and 13, respectively. He then restarted the puzzle and followed these steps:

1. Tap on the plus sign between 4 and 1 made 5
2. Tap on the multiplication sign between 2 and 5 made 10
3. Tap on the plus sign between 2 and 1 made 3
4. Tap on the division sign between 39 and 3 made 13
5. Tap on the keypad icon, tap 13, the Keyboard open, tap $-7\ -6$, tap done [keypad error]
   Delete $-7\ -6$, tap $20 - 7$, tap done
6. Drag 20 onto -10 made 10
7. Tap on the keypad icon, tap 10, the Keyboard open, tap $1 + 9$, tap done. Got three clovers as a reward.

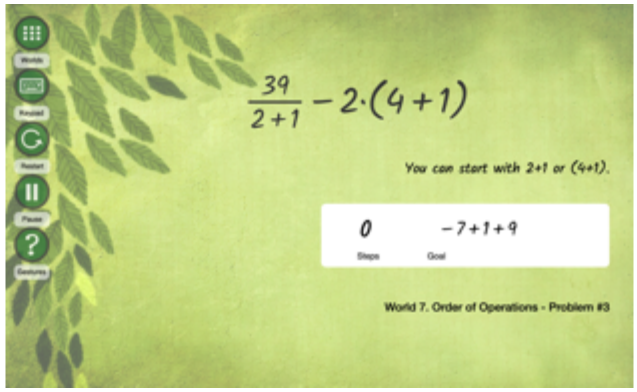


Figure 5.69: Evan – Puzzle 7.3 Hint

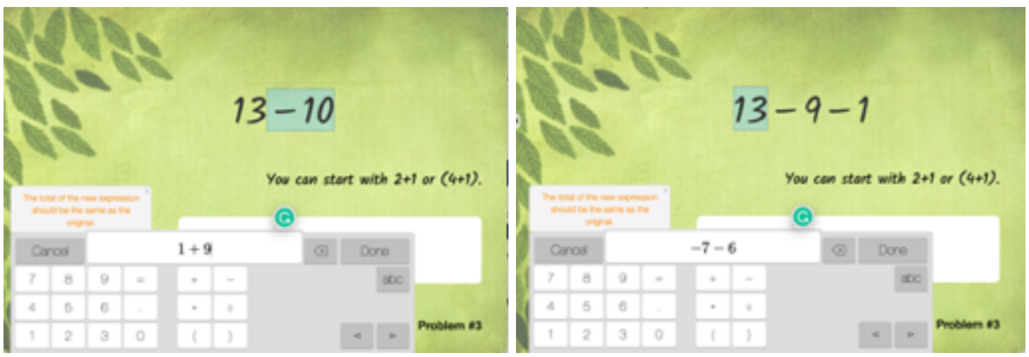


Figure 5.70: Evan – Puzzle 7.3 Keypad Errors

Fin solved Puzzle 12.7 as his final puzzle (Figure 5.71). In Puzzle 12.7, the given equation is $22 = 14 + 10 - 2$ and the goal is $\frac{22+2}{6} = 4$. Puzzle 12.7 can be solved in six steps, which Fin accomplished:

1. Drag -2 from right to left side made $22 + 2 = 14 + 10 - 2 + 2$
2. Tap on the plus sign between 14 and 10 made $22 + 2 = 24 - 2 + 2$
3. Tap on the minus sign between 24 and 2 made $22 + 2 = 22 + 2$
4. Tap on the plus sign between 22 and 2 made $22 + 2 = 24$
5. Tap and hold the equal sign, the Keyboard opens, tap $\div$ 6, tap done made $\frac{22+2}{6} = \frac{24}{6}$
6. Tap on the division sign between 24 and 6 made $\frac{22+2}{6} = 4$

   Got three clovers as a reward.

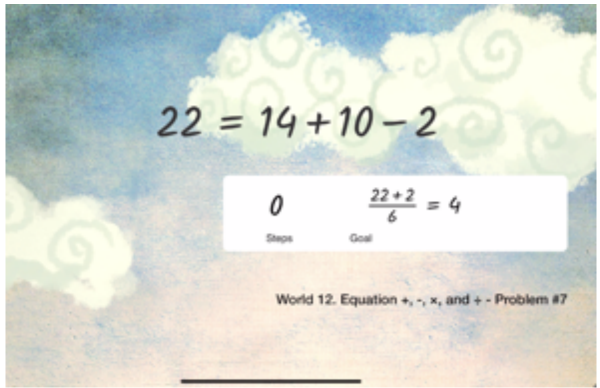


Figure 5.71: Fin - Puzzle 12.7

Gina solved Puzzle 4.10 as her final puzzle (Figure 5.72). Puzzle 4.10 is the first puzzle introducing the feature of substituting a negative number with an equivalent expression. Since this is the first puzzle to introduce this new rule of the game, it displays an animated solution. Gina followed the animated solution by first tapping the keypad icon to open the keyboard, then tap the delete icon, then tapping $-5-3$, and then done on the keyboard. So, she solved the puzzle with one step and got three clovers.

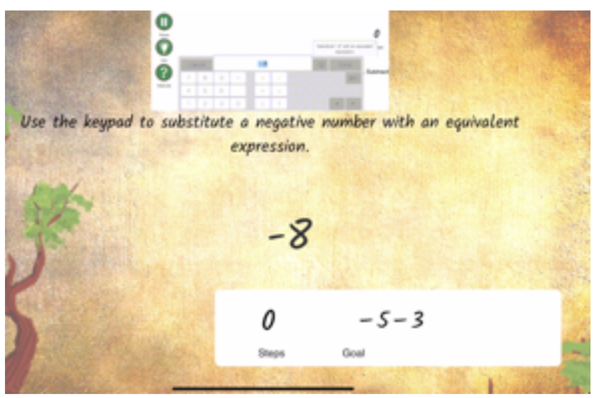


Figure 5.72: Gin - Puzzle 4.10

In summary, Table 5.4 illustrates the performances of the participants. For Q13 and Q15, except for Gina, participants solved them with three clovers. For Q14, except for Ava and Gina, participants solved Q14 with three clovers. While comparing their pre- and post-interview scores (Table 5.5), Bella, Demi, Evan, and Gina showed improvement in Q14, however, Ava did not. Demi and Gina showed improvement in Q15.

Table 5.4: Post-interview Q13-Q14-Q15-Q16 Scores

Name	Q13 - 1.9		Q14 - 2.14		Q15 – 3.14		Q16		
	GP*	PI^	GP	PI	GP	PI	Puzzle	GP	PI
Ava	3	3	1	1	3	3	9.10	-	1
Bella	2	3	3	3	3	3	12.2	-	3
Carl	2	3	3	3	2	3	8.1	-	3
Demi	3	2&3	3	3	3	3	8.6	-	2
Evan	2	3	2	3	3	3	7.3	-	3
Fin	2	3	3	3	3	3	12.7	-	3
Gina	2	2	2	2	2	2	4.10	-	3

Note: *GP: Game Play, ^PI: Post Interview

Table 5.5: Pre-interview Q8-Q9-Q10 Codes

Name	Q8	Q9	Q10
Ava	Perfect/3	Perfect/3	Perfect/3
Bella	Perfect/3	Large Error/1	Perfect/3
Carl	Perfect/3	Perfect/3	Perfect/3
Demi	Perfect/3	Large Error/1	Large Error/1
Evan	Perfect/3	Large Error/1	Perfect/3
Fin	Perfect/3	Perfect/3	Perfect/3
Gina	Small Error/2	Large Error/1	Large Error/1

Discussion

The purpose of this study was to present theoretical justifications and illustrative examples of students' digital and physical gestures while they played the game and expressed their mathematical understanding. For instance, *pointing gestures* demonstrated the particular objects in the game they were mentioning, *representational gestures* (dynamic and static) indicated their mental reflection of both the game and mathematics, *metaphoric gestures* showed their body-based metaphors of the topic, and *feedback gestures* conveyed their emotional response to the environment.

These gestures illustrate that mathematical understanding is embodied in several significant senses in mathematical discourse. This study provides evidence of learner-generated pointing gestures and affirms that mathematical understanding is rooted in the physical and virtual environment that contains mathematical expressions. Evidence from the study indicate that representational gestures show that mathematical understanding includes reflective actions on mathematical objects. Additionally, metaphoric gestures show the understanding of conceptual metaphors of mathematical topics. Finally, evidence from feedback gestures shows understanding of what learners' feelings are when they understand or talk about mathematical topics.

Limitations

I designed this study as a third study of the dissertation research. The overarching aim of the research is to explore how digital gestures connect to students' mathematical understanding when playing FH2T. However, there are some limitations of the research and this study. First, the game, FH2T, is designed for middle school students' mathematical learning, however, undergraduate students were participants in this study. They brought their own previous gameplay experiences and mathematical understandings to the study. Additionally, the playing time was limited, and students did not complete all of the worlds in FH2T. The research time was limited too, as the pre-and post-interview had been done on the same day.

Since this study observes the gameplay of a small sample of participants, I could not generalize the findings. However, this study emphasizes how every player is unique. Each participant had a different gameplay experience when playing FH2T. Some of the participants wanted to get all clovers or to solve every puzzle in each world. Some of them were satisfied with earning only one clover and skipped the last puzzles of each world. There were also different perspectives on the game's failure (keypad errors, shaking errors, and snapping errors), and feedback (formative, summative, and informative) system; some players cared about it while others did not. There were also instances where the participants manifested their thinking and emotions through gestures.

Future Studies

In this study, the participants' replies to the supplemental questions in the play-aloud session and their general opinions of gameplay questions in the post-interview session were not analyzed. Their responses will be analyzed in detail in future studies. Another interesting direction for research would be to observe how students reflect their emotions through gestures while they are in mathematical learning environments.

Conclusion

In this research, I explored four ways participants used gestures when they explained what they were doing while playing the game: *pointing gestures* to show the specific variables they were mentioning, *representational gestures* (dynamic and static) to show their mental reflection, *metaphoric gestures* to show body-based metaphors of the topic, and *feedback gestures* to show their emotional reaction.

In this work, I hope to further initiatives to create strategies that are empirically supported for enhancing students' mathematical gameplay experiences. At the same time, I seek to improve the understanding of the nature of mathematical understanding, specifically how it develops throughout life and how video games may support it. I argue that integrating knowledge of embodied mathematical cognitive processes and behavior in the actual physical, social, and virtual interactions facilitates mathematical understanding.

Chapter 6: Discussion and Conclusion

In this chapter, I discuss the research questions, results, and resulting scholarly contributions of three studies: 1) The Game Interaction, 2) The Quantitative Gesture, and 3) The Student Observation Study. After discussing the three studies individually, I then discuss the broader implications that the fields of Human-Computer Interaction (HCI) and Learning Sciences (LS) should consider. Throughout, I highlight potential future research that my work has identified.

Study 1 – The Game Interaction Study

The Game Interaction Study's research questions focused on how the game FH2T aligns with Game-Based Learning's structures, how failure and feedback manifest in mathematical play within the game, and whether the learning outcomes of the game meet the Common Core State Standards for mathematics required by the state of Maryland. The study's findings demonstrated that FH2T effectively integrates all elements of the Game-Based Learning model, including instructional content, feedback systems, and learning outcomes. In addition, the game's failure and feedback system was found to be well-designed, enabling players to learn from their mistakes while solving algebraic equations. Overall, the study suggests that FH2T has the potential to be an effective tool for teaching mathematics in a game-based learning environment.

My approach to the Game Interaction Study was informed by the interaction analysis method. In short, Jordan and Henderson (1995) defined interaction analysis as “an interdisciplinary method for the empirical investigation of the interaction of human beings with each other and with objects in their environment” (p. 39). Jordan and Henderson's (1995) definition includes both human-to-human and human-to-object interactions, but they do not specifically describe the object of investigation. Therefore, I contend that the object can be anything, technological or non-technological. For example, calculators or an abacus used for calculation might be the subject of an interaction analysis from a mathematics education perspective. From the HCI perspective, Suchman (2007) claims that human-machine interaction covers both technical and popular aspects of computers regardless of their design and their utilization. She also points out that we should take the idea of human-computer interaction seriously as a form of interaction in the same way that we understand interactions between people. Lastly, interaction analysis is used to analyze how thoughts or understandings change through or after an interaction with tools, and Kirsh (2013) noted that "interacting with tools changes the way we think and perceive" (p. 1). In the Game Interaction study, "*I*" investigated FH2T, an educational game, "*I"* interacted with FH2T and interpreted the game from my perspective. Hence, in the study, I am both a participant and a researcher at the same time. While the above researchers did not specifically delimit interaction analysis to only include researchers observing *other people's* interactions with the tools, that is how interaction analysis has been more generally taken up. However, I posit that first-person interaction with tools could be in alignment with other interaction analysis research, and further research should investigate this expansion.

As for the contribution of this study to the field, I offer insightful information on the potential of game-based learning through FH2T as a powerful tool for mathematics instruction. The findings of the study suggest that a well-designed feedback system of educational games can help students learn from their errors and improve their understanding. Although the designers (Hulse et al., 2019; Chan et al., 2021) designed the game and used it in another state, and Common Core Standards may vary from state to state, researchers and teachers in Maryland can use FH2T by showing this work to the authorities as evidence of appropriate fit. Moreover, in addition to other game-based learning research (e.g., de Feritas, 2006; Kiili, 2005; Prensky, 2003; Squire et al., 2005; Van Eck, 2006), this study's findings contribute to the understanding of how game-based learning model can be used to teach mathematics and highlight the potential of such a model as a means of raising students' mathematical engagement and learning.

Study 2 – The Quantitative Gesture Study

The Quantitative Gesture Study's research questions focused on how digital gestures can impact students' engagement and motivation to continue playing the game, FH2T. By using visual learning analytics and an embodied mathematical cognition perspective, this study helps to reveal the relationship between digital gestures and students' choice to continue playing. The study's findings indicate that digital gestures that lead to errors can have a significant influence on students' decisions to continue playing.

The study emphasizes how carefully game designers to pay attention the design of digital gestures within the educational game, for example, to help students stay in the flow (Csikszentmihalyi, 1990), not getting too frustrated when trying to solve the puzzles or too bored by only having easy puzzles. This knowledge can be used by developers to produce games with digital gestures that are more user-friendly and error-proof, which could ultimately result in higher levels of student engagement and retention. In addition to other FH2T studies (Chan et al., 2022; Lee et al., 2022), the study contributes to the emerging field of visual learning analytics through the game by illustrating how digital gestures can be used as a source of data to analyze students' learning behaviors and decision-making processes.

Study 3 – The Student Observation Study

The Student Observation Study's research questions focused on the learner's mathematical gameplay experience in playing FH2T, how failure and feedback manifested in mathematical play within the game through the learner's perspective, and how physical gestures and digital gestures relate to explaining mathematical gameplay. The study on student observation revealed that learners utilized various types of physical gestures while explaining their actions in the game. For instance, they used *pointing gestures* to indicate specific variables, *representational gestures* (dynamic and static) to express their thoughts, *metaphoric gestures* to illustrate body-based metaphors, and *feedback gestures* to show their emotional reactions.

One of the contributions of this study to the field is to present a unique approach to exploring the embodied mathematical cognition that occurs during gameplay by recognizing and categorizing the student's gestures. As I mentioned above, in addition to Alibali and Nathan's (2012) statement that some features of mathematical thinking are embodied and shown by *pointing, representational, and metaphoric* gestures. I included emotional feedback gestures (e.g., thinking, positive and negative emotion gestures) because learning mathematics can involve emotional aspects. The identification of specific types of gestures used by learners to express mathematical concepts can inform the design of educational games that promote deeper learning and better engagement with mathematical concepts. Based on each student's unique needs and preferences, this study can be used to help personalize their learning experiences and interventions.

In addition to the previous FH2T papers (Chan et al., 2021; Hulse et al., 2019; Lee et al., 2021; Ottmar et al., 2015), this study provides insights into the learner's mathematical gameplay experience in playing FH2T. Moreover, in this study, failure and feedback manifest in mathematical play from the learner's perspective. Through this study, understanding how learners experience failure and feedback in mathematical play can help educators develop effective feedback strategies that support learners' mathematical development, I will discuss this further in implications.

Overarching Study

The overarching study of these three studies research questions focused on how digital gestures connect to students' mathematical understanding when playing FH2T. Having the "Gesture" icon and transferring each mathematical rule to digital gestures suggest that FH2T has been designed to help students improve their mathematical understanding through digital gestures. Students were simultaneously explaining their thoughts and describing their gameplay decision-making out loud, while also using physical and digital gestures to communicate. Physical and digital gestures help students visualize mathematical concepts and make them more tangible, leading to a better understanding of mathematics. In the game, each digital gesture provides immediate feedback, allowing students to quickly identify and correct mistakes. Additionally, the game makes mathematical concepts more engaging, which can increase students' motivation and interest in mathematics.

Regarding this study's overall contributions to the field, it provides useful information about the connections among digital gestures, embodied cognition, educational game, and mathematical learning. By comprehending these connections, teachers and researchers can develop new approaches for mathematics education that make better use of both digital and physical gestures to increase students' comprehension of and interest in mathematical concepts. This study can be a valuable tool for researchers and teachers seeking to deepen their understanding of mathematical understanding and promote effective learning strategies in educational games.

Implications for the Game-Based Learning in Mathematics Education

In this section, I discuss two major implications below, both organized around a revised version of the Garris et al. (2002) Game-Based Learning (GBL) Model. In the GBL model, the key component is the *game cycle* which is an iterative process that covers repeated user behavior-system feedback-user judgment loops. When integrating GBL into the classroom, the teacher's feedback, the teacher's judgment, and the teacher's behavior should be added to the model. Through the development of a teacher dashboard, we could develop the students' gameplay and learning experience. I begin by describing how adding a teacher dashboard to FH2T could improve teachers' ability to identify struggling students in class. I then recommend specific design components for any educational game, framed around the different points of information and assessment (collectively called 'feedback' in Chapter 3): formative, summative, and informative.

Teacher Dashboard

FH2T is a self-paced, interactive game that supports mathematics learning. One primary difference between implementing and evaluating learning through playing FH2T in a laboratory setting (e.g., Chapter 5 Student Observation Study) and in a classroom setting is that while I was attending to a single student playing the game, teachers must attend to 30 or more. Consequently, one of the implications of my work is a better understanding of which digital patterns may inform teachers of a student's need for support.

In a traditional classroom, the teacher delivers instruction through lectures, discussions, and other forms of direct instruction while students listen, take notes, and complete assignments. On the other hand, in a classroom using FH2T, students are playing the game through touchscreen devices while they are looking at the game and tapping the variables on the screen to solve the puzzles. These students tend to be highly engaged and motivated to learn through the game (Chan et al., 2021; Hulse et al., 2019; Lee et al., 2021; Ottmar et al., 2015). Some students might become competitive and strive to earn all clovers and beat their classmates, while others might work collaboratively with their peers to solve puzzles and explain solutions through talking and gestures. Additionally, students develop a growth mindset, as they learn to view challenges and mistakes as opportunities for learning and they may also be more willing to take risks and made mistakes. Using the game in the classroom can create a positive learning environment where students feel safe to try new things, without fear of failure or judgment (Gee, 2005; Hattie & Timperley, 2007; Williams-Pierce, 2019).

The FH2T backend data collection process can capture and analyze the complexity of an entire class playing, so a teacher dashboard that is updating live could in some ways replace the traditional classroom indicators of learning that teachers typically attend to. Through the dashboard, the teacher would be able to see an overview of their class, including each student's progress and performance on each puzzle. Moreover, like other technology-mediated classroom tools (e.g., Walkoe et al. 2017), the teacher dashboard of FH2T would help teachers' attend to the important parts of students' thinking. In this section, I talk about some of the implications of my research for the design of a potential teacher dashboard.

The game analytics process that has already been developed can create visualizations such as Indivisualizer, Measure Chart, Treemap, and Sankey Diagram, and the teacher dashboard could have simplified versions of these representations to support them in understanding how individual students and the class as a whole are performing. Teachers could view detailed data on game performance for individuals and the progress of the class as a whole. Below, I describe some potential teacher dashboard designs and simplified visualizations to support the productive use of FH2T in classrooms.

To begin with, the teacher could click on the student's name on the dashboard to view their progress in the game. For instance, an initial individual student view could provide overall descriptive statistics, or a quantitative sense of the student's progress, including: the student's latest puzzle, the errors they made on previous puzzles, the clovers they earned on each puzzle, the time spent on each puzzle, and whether they tapped the hint or gesture button for informative feedback. The objective and learning goal of each puzzle would also be included in the dashboard, so the teacher could better understand each student's performance. They could reassign specific puzzles or levels to resolve to help students achieve their learning objectives. Then clicking on an individual puzzle for the student, a version of the Indivisualizer could show all the student's actions between the start state and the goal state. For instance, the number of steps, the errors, their reset and replay numbers, etc. In the Quantitative Gesture study, I categorized students' performance as *Good*, *Good+Error, Moderate,* and *Developing*, and this categorization could be automated by designers and integrated into the dashboard. Teachers could understand the student's performance by checking for patterns of Moderate and Developing to pinpoint potential struggles, or by using the categories to match up students with different patterns to support peer learning.

The teacher could also click on each puzzle to view the whole class's visualizations, for example, a simplified version of the Measure Chart. The Measure Chart would include data for each puzzle for the whole class on completion rates, number of steps taken, time spent on the puzzle, errors made by students, and use of hints. The Measure Chart interaction view could be improved by future designers, to reveal various ways in which students make specific errors and to make it easier for teachers to see the errors students are making. For instance, when teachers click on a section like "errors" – "keypad errors", "shaking errors", "snapping errors" – the interface may show a list of students who made that type of error in that puzzle, or the section of "use of hint," they can see the list of students who used hints in that puzzle. With this information, teachers can bring the specific puzzle into classroom discussion, and they can talk more about strategies that students can use to improve their learning.

A second visualization that can be incorporated into a teacher's dashboard is a simplified version of the Treemap, which includes data on the percentage of students who took each first step for each puzzle for the whole class. In the Quantitative Gesture study, I calculated the Pearson Correlation coefficient between the first step and the performance of the player. For example, when teachers click on a section, they could see the list of students who took that first step and the following performance of the student in the puzzle. The relation between the first step and the mean number of the steps could be calculated and used to categorize the students. For extra classroom discussion, the teacher can bring the specific puzzles from particular categorizations and use them to foster student discussions. Furthermore, dashboard developers could use the whole user's data from all students who played or are playing the game, and teachers could see how many of their students took a specific first step and could compare it to the rest of the data. Hence, teachers could get a sense of how their students might be struggling with puzzles compared to other students.

Similarly, the Sankey Diagram (Flow Diagram), displays the flow of the strategies for the whole class for a single puzzle, which helps to visualize the different strategies of students into one graph. The vertical bars show the students' mathematical problem-solving steps or their choice of mathematical expressions. The thickness of the path illustrates the number of students who took that path. In the Quantitative Gesture study, I categorized gestures in the Sankey Diagram as *productive* (helping students solve the puzzle in the best number of steps) and *nonproductive* (extra steps that are not necessary for solving the puzzle). This categorization could be incorporated by designers and integrated with a simplified version of the Sankey Diagram into the dashboard, so teachers could understand the category by clicking the vertical bars. Moreover, another feature, when teachers click on any line in the flow, they could see the name of the students who took that step.

In summary, the Measure Chart, Treemap, and Sankey Diagrams could be simplified to provide teachers with an overview of all the students in a class, for each puzzle in FH2T. This could also be directly connected to the Indivisualizers by the designers. Through the useful design of linkages between these graphics, the teacher could better understand how the whole class and each student are performing, and compare the students with others and compare the students' performance puzzle by puzzle. In the GBL model, the debriefing process provides a connection between the game cycle and the accomplishment of learning outcomes (Garris et al., 2002), and the teacher dashboard could help teachers gain valuable insights into the game cycle of students and help teachers debrief students' progress. Then, teachers could guide students through discussion of their experiences with the game and help them to connect their learning in the game to real-world applications. Hence, developing a teacher dashboard of FH2T could be an influential evaluation tool for assessing students, enhancing their learning, tracking their progress, and providing personalized support to students who need it.

Feedback System

System feedback is an essential element of the Game-Based Learning model's process stage (Garris et al., 2002). Players receive immediate feedback from the game on their performance, which helps them to identify the point of success and failure. FH2T provides three types of feedback – formative, summative, and informative (as outlined in the System Feedback section in Chapter 3). In this section, I discuss the implications of these feedback types for designers in more detail, as well as how the manifestation of these feedback types in gameplay could connect to the teacher dashboard. The game system gives on-time feedback, but some students also need in-person feedback.

Formative feedback is given by the game during the puzzle-solving. Through formative feedback, players can follow their actions' reflections on the game screen. After feedback, they can adjust their behavior. For example, an existing type of FH2T's formative feedback is changing the step number to red on the screen (which means they are over the minimum number of steps), so after that feedback, students can decide to reset the puzzle or to keep solving it. In addition, through the development of a teacher dashboard, teachers could see the real-time action of students, and offer formative feedback in-person as guidance to any student who appears to need it. By splitting the screen of the dashboard into multiple smaller screens, teachers could track all students' progress in the game while sitting at their desks. As a broader recommendation, game designers should always consider giving formative feedback during gameplay in any game to help players spot their mistakes and fix them right away.

Summative feedback, on the other hand, is given by the game after puzzle-solving. Symbolic rewards such as collecting clovers, as well as making the next puzzle available, are forms of summative feedback in FH2T. Each clover also has meaning (3 clovers: Best Solution; 2 clovers: Completed with missing the best steps number; 1 clover: Just Completed), so students can assess their performance through clovers and decide whether to replay the puzzle or not. In the literature, for open-online courses, Canessa and Pisani (2013) suggested that students re-watch the same lessons they received in their own classroom online at their own place and pace. Similarly, students have their own accounts for the game, and they could keep playing the game outside of the classroom. Moreover, if they want, they can replay the puzzles and worlds for practicing and getting speed on a mathematical operation. If the teacher dashboard provided a summary of each student's development and performance, teachers could better identify any concepts in which students might benefit from extra help or instruction. Teachers could use these representations of summative assessments to assess their student's understanding of the mathematical concepts covered in the game. Then teachers could use this information to reward students or go over again the concept to support their understanding. More broadly, game designers should offer summative feedback at the end of each puzzle or level to help players understand how well they performed and where they need to improve. This feedback could take the form of a scoreboard or report that details how the player or group of players performed in relation to particular goals and objectives.

Informative feedback is given on-demand by the game system to players seeking extra information. For example, in FH2T, informative feedback can be accessed by tapping the Hint icon to get a hint or by tapping the Gesture icon to revisit all the gesture-related rules needed to solve the puzzle. When students need in-person informative feedback related to solving the puzzle, teachers could provide informative feedback to give each student individualized feedback through the teacher dashboard by tapping a newly designed "teacher" button or in-person by raising their hands and asking a question if they are in the classroom. Asking for informative feedback from the teacher also gives information on which mathematical concepts students struggling with. In the literature, Smith and Ferguson (2005) also recommended real-time chat opportunities with instructors for e-learning in mathematics courses. In game-based learning, also real-time chat or getting informative feedback from the teacher would help the learning. Game designers should also offer informative feedback for each puzzle in their game to assist players.

Overall, FH2T supports learning and offers a fun and interesting way to practice algebra by incorporating formative, summative, and informative feedback. Teachers may be better able to support student learning, assess student progress, and offer specialized support to specific students who may be having difficulties by using these feedback types. Moreover, game designers should try to balance these three types of feedback to make games that are fun, challenging, and aids in the player's skill development.

Implications for the Embodied Mathematical Cognition

Research on embodied mathematical cognition can be used to make useful linkages to educational practice in any educational environment (e.g., mathematics classrooms, mathematical video games, mathematics museums, or summer camps). An embodied view offers a framework for analyzing how learners behave and interact in the educational environment. The embodied perspective also has the potential to anticipate the outcomes of particular learning practices. For instance, learning that involves engaging in actions may result in learners being better able to express themselves in gestures to re-invoke those actions.

Teachers' or educators' implementations are another important bridge between research and practice. An embodied perspective supports teachers to focus on learners' gestures. Many examples of gestures and their meanings have been presented in this work. For instance, *pointing gestures* help educators to follow how students are working with each variable. *Representational gestures* help educators to understand the students' mental understanding of the concept. *Metaphoric gestures* help educators to find out the student's conceptual understanding of the topic. If educators are able to be aware of any of these gestures while teaching, they may prevent misconceptions from arising. Since mathematical understanding is also emotional, feedback gestures may inform educators about how students reflect their emotions in a mathematical learning environment. If educators are able to be aware of any negative emotions, they may want to change the situation in the environment to help students create a positive perception of mathematics.

Educational game designers also can use embodied mathematical cognition to create games that are more engaging and effective for teaching. They can integrate conceptual metaphors of the topic into their game. They can incorporate physical interaction into the game mechanics and design games for virtual reality tools too, as learning can be embodied, distributed, and dynamic in powerful ways through virtual reality tools (Walkington et al., 2021). They can create games that encourage players to take embodied action like measuring or manipulating objects in the game environment. They can also use haptic feedback to provide players with developing a better understanding of mathematical learning.

Future Studies

This study covers three studies, and each study can be extended in different ways. For the extension of the first study, further research could focus on first-person interaction with the tools as a theoretical extension of the interaction analysis methodology. Regarding the students' observation study, the participants were from the United States, which provides results from only one country. Mathematical understanding and embodied cognition, however, appear to be universal, therefore, it would be interesting to research how students from other countries reflect their understanding with gestures while playing FH2T and/or any other mathematical educational game. Moreover, touchscreen devices were used in the study, and future research projects might compare playing with desktop or laptop computers with touchscreen devices.

While designing a teacher dashboard, the user interface should be intuitive and user-friendly for teachers. Future studies should involve, first, understanding the needs of teachers for a game dashboard, then analyzing usability tests with teachers to identify meaningful or overwhelming points of the dashboard, and testing different data visualization formats to determine which are more effective and useful for teachers.

Conclusion

Integrating video games in an educational environment is a complex venture. This dissertation covers three studies (The Game Interaction, The Quantitative Gesture, and The Student Observation studies) to investigate an educational touchscreen video game, FH2T, from three different perspectives (e.g., from the researcher, middle school students, and undergraduate students) and three different methodologies (e.g., inspired by interaction analysis, visual learning analytics, and qualitative analysis). The results show that students used both digital (e.g., dragging, tapping) and physical gestures (e.g., pointing, representational, metaphoric, and feedback) to express their mathematical understanding when playing the game, FH2T, which covers all elements of the Game-Based learning model.

Appendices

Appendix 3.1: Icons on the World 1 Screen

Figure A3.1.1

Outline of World 1 (Left-Started and Right-Finished)

In this Appendix, the icons are described, and their duties are explained. The icon on the left corner (Figure A3.1.1-Left-A) is a “Menu” icon. When players tap the Menu icon, they return to the main page (Figure 3.3). The top banner (Figure A3.1.1-Left-B) covers information about the world such as the number and the name of the world. On the left, the small icon is World 1’s icon which is visualized by a “growing seed” (Figure A3.1.1-Left-C). Next to the world’s icon, the number and the topic of the world are written (Figure A3.1.1-Left-D); this is the first world, and the subject is addition. On the right section of the banner, there is a number, 0/54, and the icon of a three-leaf clover (Figure A3.1.1-Left-E). The number – 0 – shows how many clovers were collected, and the other number – 54 – shows the total number of clovers that can be collected in this world. At the beginning of the game, 0 (zero) out of a possible 54 clovers have been collected (Figure A3.1.1-Left-E). The process for collecting clovers is explained in the System Feedback section. Another icon on the right corner

(Figure A3.1.1-Left-F) is the “Pause” icon visualized by two vertical lines; this icon allows the player to take a break in the world. When the player clicks the Pause icon, a pop-up message appears on the screen (see Figure A3.1.2). Players need to click the “Resume Game” icon to turn back to the game. The lock icons (Figure A3.1.1-Left-G) are opened one by one when the player solves the puzzle and passes to another puzzle. There are also stars on the right corner of some puzzles (Figure A3.1.1-Left-H) that introduce the player/learner to a new rule. For example, the first rule is given in World 1-1 (see Figure A3.1.3) by using both animated graphics (visual) and text (textual). Rules are listed in Appendix 3.3-Rules of the Game).

Figure A3.1.2

Game Paused! - Pop-up Message

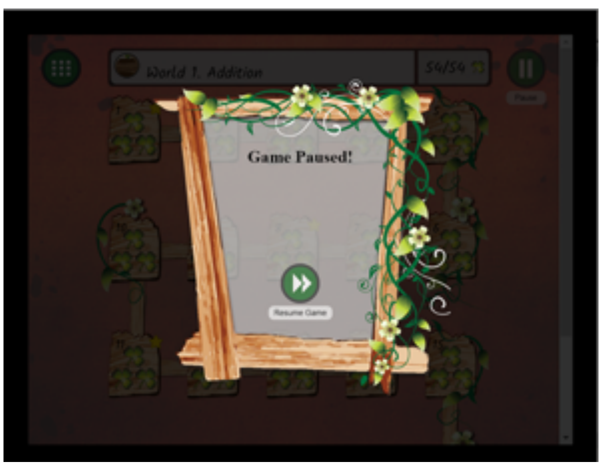
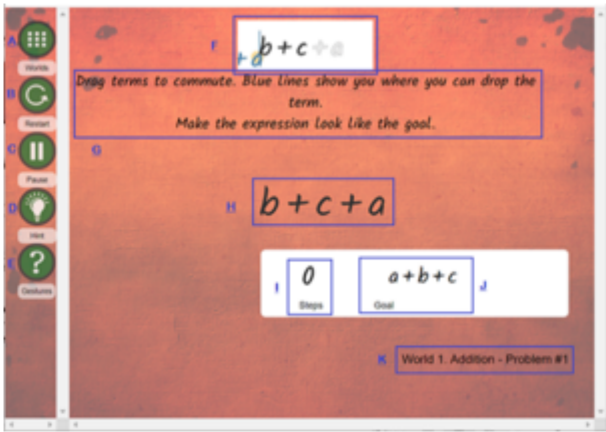


Figure A3.1.3

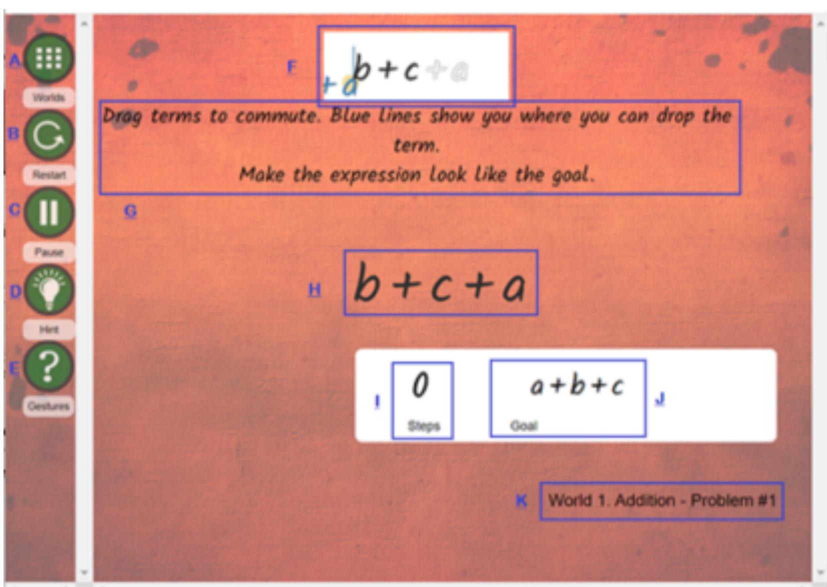
World 1-1



Appendix 3.2: Icons on the Puzzle Screen.

Figure A3.2.1

World 1-1



In this Appendix, the icons are described, and their functions are explained. On the left, the first icon is the “Worlds” icon, which is visualized by nine small boxes (Figure A3.2.1-A). This “Worlds” icon allows the player turns to the “Main

Page" which shows all the available Worlds on the tree (see Figure 3.3). The second icon is the "Restart" icon visualized by a curved arrow (Figure A3.2.1-B), allowing players to start over the puzzle. When the player makes a mistake and wants to undo the last step, but s/he could not undo just one step. By tapping this restart icon, players restart the puzzle from the beginning. The third icon is the "Pause" icon figured by two vertical lines (Figure A3.2.1-C). The pause icon allows players to take a break from the puzzle, and a pop-up message appears on the screen (Figure A3.2.2). After pausing, the player must click the "Resume Game" icon to turn back to the puzzle. The fourth icon is the "Hint" icon visualized by a lightbulb (Figure A3.2.1-D). After clicking this icon, a small hint about how to solve the problem appears in a written format above the white "Goal" box (see Figure A3.2.3- Hint). The fifth icon is the "Gestures" icon figured by a question icon (Figure A3.2.1-E). The Gesture icon gives the players clues to which gesture (rule) was used to solve the puzzle in a listed format with a pop-up message. Figure A3.2.4-Left shows the gesture in World 1 Puzzle 1 (Other gestures are listed in -Appendix 3.4- the table of Gestures and Descriptions). Just one gesture is needed to solve this puzzle – "Commute Terms." When the player clicked the "Commute Terms," a new pop-up message explains it in detail with animated graphics and text format, as shown in Figure A3.2.4-Right. In this paragraph, the icons on the puzzle screen are explained. There are also animated graphics and text on the screen.

In this paragraph, the animated graphics, and text in Figure A3.2.1 are explained. Figure A3.2.1-F (blue box) shows the new rule in the puzzle by animated graphics, and Figure A3.2.1-G (blue box) explains the rule in the text format. Figure

A3.2.1-H is the starting form of the puzzle. Figure A3.2.1-I shows how many steps have been done until now. This number turns red when the player exceeds the best solution steps number. Figure A3.2.1-J is the goal form of the puzzle; as mentioned above, the game aims to practice algebraic equations by reaching the goal form of the problem from the starting form with the best steps. And finally, Figure A3.2.1-K shows that in which puzzle the player is playing.

Figure A3.2.2

Game Paused! - Pop-up Message

Figure A3.2.3

Hint

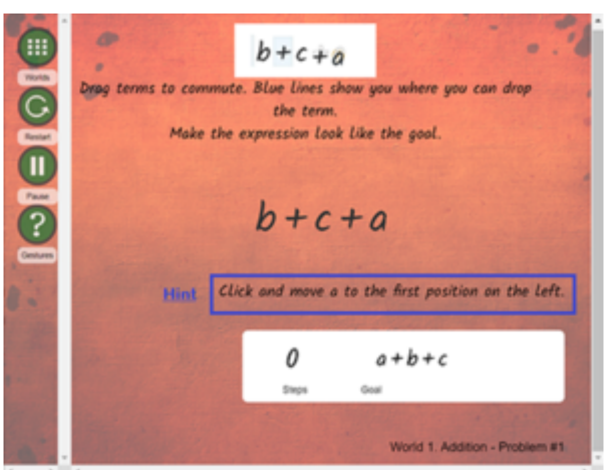


Figure A3.2.4

Gesture (Left- "Commute Terms," Right- "Commute Terms" in detail)

Appendix 3.3: Table of the Rules of the Game

World #	Puzzle #	Rule	New Gesture as a Hint
1	1	Drag terms to commute. Blue lines show you where you can drop the term. Make the expression look like the goal.	Commute Terms
1	3	You can add by tapping on a '+' sign or by dragging one number on top of the other.	Perform Operations by Tapping or Dragging
1	8	The keypad lets you substitute a number with an equivalent.	Substitute Numbers Using the Keypad
1	11	This keypad decomposes a number into two addends. Use the keypad twice to decompose a number into 3 addends.	-
2	1	Drag a number to commute! Does the order of multiplication matter?	-
2	3	You can multiply by tapping the dot (i.e., multiplication sign) or dragging one number on top of the other."	-
2	10	"Use the keypad to factor. You can use the keypad twice.	-
3	1	If the terms shake, you are trying to do something that is not mathematically possible.	Error Shaking
3	9	To move 2y, select the 2, then drag it down until it is joined by the y, then move 2y as one object.	Select Multiple Terms to Commute by Pulling
3	11	Use the keypad and highlight the whole term '2y', then substitute with y + y.	-
4	1	Tap to subtract!	-
4	3	Use the keypad to substitute 3 with a subtraction expression.	-
4	4	Subtracting a negative number has the same result as adding the positive of the number.	Subtracting Negative Numbers
4	8	Since 6-4 is equivalent to 6+(-4), you can commute 6 and -4.	Commuting Negative Numbers
4	10	Use the keypad to substitute a negative number with an equivalent expression.	-
5		NO NEW RULE	
6	1	Tap the division bar to divide	Dividing Numbers by Tapping Bar
6	2	Dragging one number on top of another pulls out the largest common factor.	Finding the Largest Common Factor by Dragging
6	7	Tap the division bar or drag a number across it to factor the numerator and denominator.	Dividing Numbers by Dragging
6	9	Drag to factor each fraction.	Finding Common Factors and Reducing Fractions
6	11	Tap the multiplication sign to bring the 10 to the numerator.	Multiply Numerators
6	13	You can drag a factor out of a fraction.	Dragging to Separate Numerators
7		NO NEW RULE	

8	1	Tap and hold the equals sign to do the same thing to BOTH sides. The changes you make to E in the keypad will happen on both sides.	Performing Operations on Both Sides of the Equations
9	1	A new way to work with equations - dragging. Drag the 2 across the equals sign to add its opposite to both sides. Notice the sign change!	Performing Inverse Operations on Both Sides of the Equations by Dragging
	2	Drag the -3 across the equals sign to add 3 to both sides. Notice the sign changes!	-
10	1	Tap and hold on any bracket to grab the parentheses term. Then drag to commute as one object.	Commuting Parentheses
	2	Drag a number into parentheses to distribute. This multiplies each term in the parentheses by that number.	Distributing a Single Term
	6	Tap the parentheses or subtraction symbols to distribute and clear the parentheses.	Distributing an Operation Sign
	9	If you distribute one expression across another, it multiplies the expression by each term in the parentheses.	Distributing Multiple Terms
11	1	Apply the distributive property to factor an entire expression. Drag the terms that share a common factor on top of each other to factor out the greatest common factor (GCF).	Factoring Terms
	5	Factor one number, then drag a common factor to the other number to factor the whole expression.	Finding Greatest Common Factors
	10	An expression in parentheses can be factored out by clicking and dragging a bracket.	Factoring Multiple Terms
12	1	Tap and hold the equals sign to do the same thing to both sides.	-
	2	NO TEXT- Just Animated graphic	-
	8	To simplify the right hand side, move the denominator towards the numerator. When a line shows up under each term, let go and the result is a sum of two fractions.	Moving to Separate Numerators
	13	The video only shows you how to get rid of the denominator by multiplying both sides by (x+3). You'll have to do the rest!	-
13	1	Drag a number from the denominator until you see a blue line to factor out the unit fraction.	Break a Fraction into 2 Terms
	7	Dragging the 2 across the equals sign is a gesture shortcut to getting 6 alone. You won't have to do the gestures of multiplying both sides by two and clearing (2/2) from the left.	Inverse Operations Shortcut by Dragging Across the Equals Sign
	8	Divide both sides by 3 by dragging a 3 across the equals sign. Division is the inverse to multiplication.	-
14	12	Tap the equals sign to flip the equation.	-

Appendix 3.4: Table of the Gestures and Descriptions

	Gesture	Description
1	Commute Terms	Drag terms to commute. Blue lines show you where you can drop the term.
2	Perform Operations by Tapping or Dragging	You can perform operations by tapping on an operation sign or by dragging one number on top of the other.
3	Substitute Numbers Using the Keypad	The keypad lets you substitute a number with an equivalent expression.
4	Error Shaking	If the terms shake, you are trying to do something that is not mathematically possible.
5	Select Multiple Terms to Commute by Pulling	To move 2y, select the 2, then drag it down until it is joined by the y, then move 2y as one object.
6	Subtracting Negative Numbers	Subtracting a negative number has the same result as adding the positive of the number.
7	Commuting Negative Numbers	Since 6-4 is equivalent to 6+(-4), you can commute the -4
8	Dividing Numbers by Tapping Bar	Tap the division Bar to divide.
9	Finding the Largest Common Factor by Dragging	Dragging one number on top of another pulls out the largest common factor
10	Dividing Numbers by Dragging	When there are several factors, tapping the division bar doesn't work. Drag to simplify.
11	Finding Common Factors and Reducing Fractions	Drag to factor each fraction.
12	Multiply Numerators	Tap the multiplication sign to bring the 10 to the numerator.
13	Dragging to Separate Numerators	If two terms are multiplied in the numerator, you can drag terms out from the fraction.
14	Performing Operations on Both Sides of the Equations	Tap and hold the equals sign to do the same thing to BOTH sides. E represents the Expression on both sides.
15	Performing Inverse Operations on Both Sides of the Equations by Dragging	A new way to work with equations - dragging. Drag the 2 across the equals sign to add its opposite to both sides. Notice the sign change!
16	Commuting Parentheses	You can tap and hold on any bracket to grab the parentheses term. Then drag to commute as one object.
17	Distributing a Single Term	You can drag a number into parentheses to distribute. This multiplies each term in the parentheses by that number.
18	Distributing an Operation Sign	You can tap the parentheses to distribute the operation sign.
19	Distributing Multiple Terms	If you distribute one expression across another, it multiplies the expression by each term in the parentheses.
20	Factoring Terms	Apply the distributive property to factor an entire expression. Drag the terms that share a common factor on top of each other to factor out the greatest common factor (GCF).
21	Finding Greatest Common Factors	Factor one number, then drag a common factor to the other number to factor the whole expression.

22	Factoring Multiple Terms	A group of terms in parentheses can be factored out as well by grabbing and dragging a bracket.
23	Moving to Separate Numerators	Move the denominator on the right towards the numerator. When a line shows up under each term, let go and the result is a sum of two fractions.
24	Break a Fraction into 2 Terms	Drag a number from the denominator until you see a blue line to factor out the unit fraction.
25	Inverse Operations Shortcut by Dragging Across the Equals Sign	Dragging the 2 across the equals sign is a gesture shortcut to getting 6 alone. You won't have to do the gestures of multiplying both sides by two and clearing (2/2) from the left.

Appendix 3.5: Students' demographic information (Chan et al., 2021)

		All (N=475)		FH2T (n=227)		Problem Set (n=248)	
		n	%	*n*	%	*n*	%
Gender	male	261	54.9	127	55.9	134	54.0
	female	214	45.1	100	44.1	114	46.0
Race	White	165	34.7	81	35.7	84	33.9
	Asian	260	54.7	120	52.9	140	56.5
	Hispanic	23	4.8	10	4.4	13	5.2
	African American	10	2.1	6	2.6	4	1.6
	Native American	5	1.1	1	0.4	4	1.6
	Pacific Islander	1	0.2	1	0.4	0	0.0
	Multi-racial	11	2.3	8	3.5	3	1.2
Grade	Sixth	453	95.4	217	95.6	236	95.2
	Seventh	22	4.6	10	4.4	12	4.8
Class	Advanced	400	84.2	192	84.6	208	83.9
	On-Level	34	7.2	14	6.2	20	8.1
	Support	41	8.6	21	9.3	20	8.1
Student Achievement Level	Above grade	248	52.2	119	52.4	129	52.0
	Not above grade	227	47.8	108	47.6	119	48.0
Pretest scores (M , SD)		3.80	1.61	3.86	1.60	3.75	1.62

Appendix 4.1: Players' Status

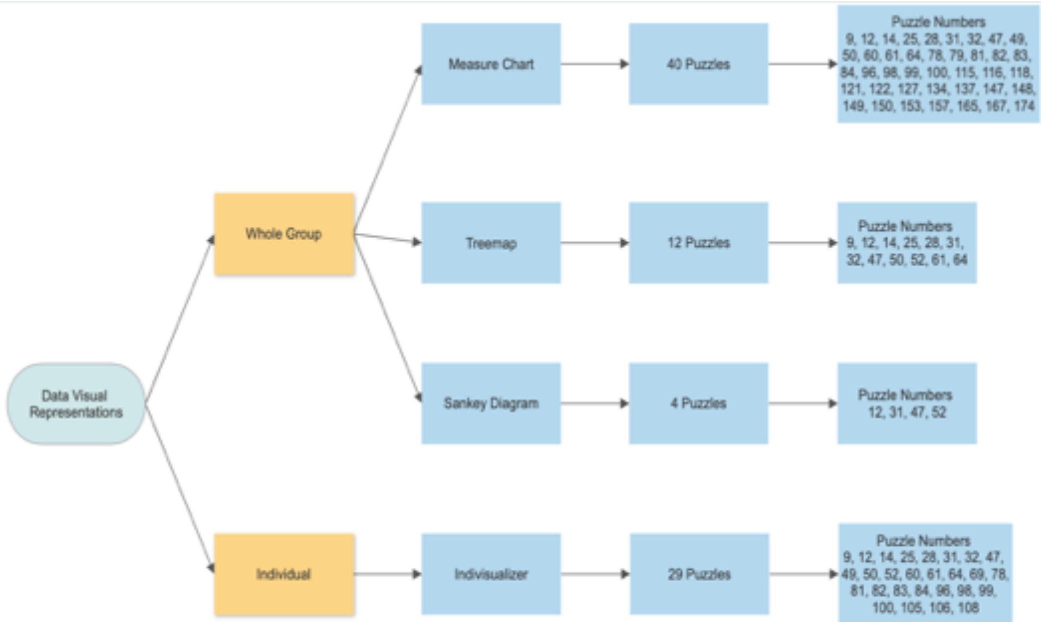
	P.1	P.2	P.3	P.4	P.5	P.6	P.7	P.8	P.9	P.10	P.11	P.12	P.13	P.14	P.15
9	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
12	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
14	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
25	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
28	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
31	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
32	S	S	S	S	S	S	S	S	S	S	S	S	S	S	S
47	S	S	S	NA	S	S	S	S	S	S	S	S	S	S	S
49	S	S	S	NA	S	S	S	S	S	S	S	NS	S	S	S
50	S	S	S	NA	S	S	S	S	S	S	S	NA	S	S	S
52	NA	S	NA	NA	NA	NA	S	NA	S	NA	NA	NA	NA	NS	NA
60	S	S	S	NA	S	S	S	S	S	S	S	NA	S	NA	S
61	S	S	S	NA	S	S	S	S	S	S	S	NA	S	NA	S
64	S	S	S	NA	S	S	S	S	S	S	S	NA	S	NA	S
69	NA	NA	NA	NA	NA	NA	NA	NA	S	NA	S	NA	NA	NA	NA
78	S	S	NA	NA	S	S	S	S	S	S	S	NA	S	NA	NA
79	S	S	NA	NA	S	S	S	S	S	S	S	NA	S	NA	NA
81	S	S	NA	NA	S	S	S	S	S	S	S	NA	S	NA	NA
82	S	S	NA	NA	S	S	S	S	S	S	S	NA	S	NA	NA
83	S	S	NA	NA	S	S	S	S	S	S	S	NA	S	NA	NA
84	S	S	NA	NA	S	S	S	S	S	S	S	NA	S	NA	NA
89	NA	NA	NA	NA	NA	NA	NA	NA	S	NA	NA	NA	NA	NA	NA
96	S	S	NA	NA	S	S	S	S	S	S	NA	NA	S	NA	NA
98	S	S	NA	NA	S	S	S	S	S	S	NA	NA	S	NA	NA
99	NA	S	NA	NA	S	S	S	S	S	S	NA	NA	S	NA	NA
100	NA	S	NA	NA	S	S	S	S	S	S	NA	NA	NS	NA	NA
105	NA	NA	NA	NA	NA	NA	NA	NA	S	NA	NA	NA	NA	NA	NA
106	NA	NA	NA	NA	NA	NA	NA	NA	S	NA	NA	NA	NA	NA	NA
108	NA	NA	NA	NA	NA	NA	NA	NA	S	NA	NA	NA	NA	NA	NA
115	NA	S	NA	NA	S	S	S	S	NA	S	NA	NA	NA	NA	NA
116	NA	S	NA	NA	S	S	S	S	NA	S	NA	NA	NA	NA	NA
118	NA	S	NA	NA	S	S	S	S	NA	S	NA	NA	NA	NA	NA
121	NA	S	NA	NA	S	S	S	S	NA	S	NA	NA	NA	NA	NA
122	NA	S	NA	NA	S	S	S	S	NA	S	NA	NA	NA	NA	NA
125	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
127	NA	S	NA	NA	S	S	S	S	NA	S	NA	NA	NA	NA	NA
134	NA	S	NA	NA	NA	S	NA	S	NA	S	NA	NA	NA	NA	NA
137	NA	S	NA	NA	NA	NS	NA	NS	NA	S	NA	NA	NA	NA	NA
141	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
147	NA	S	NA	NA	NA	NA	NA	NA	NA	S	NA	NA	NA	NA	NA
148	NA	S	NA	NA	NA	NA	NA	NA	NA	NS	NA	NA	NA	NA	NA
149	NA	S	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
150	NA	S	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
153	NA	S	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
157	NA	S	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
165	NA	S	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
167	NA	S	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
174	NA	S	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA
180	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

Coding: (S) means solved the puzzle, (NA) means did not attempt to solve, (NS) means started to solve but not finished.

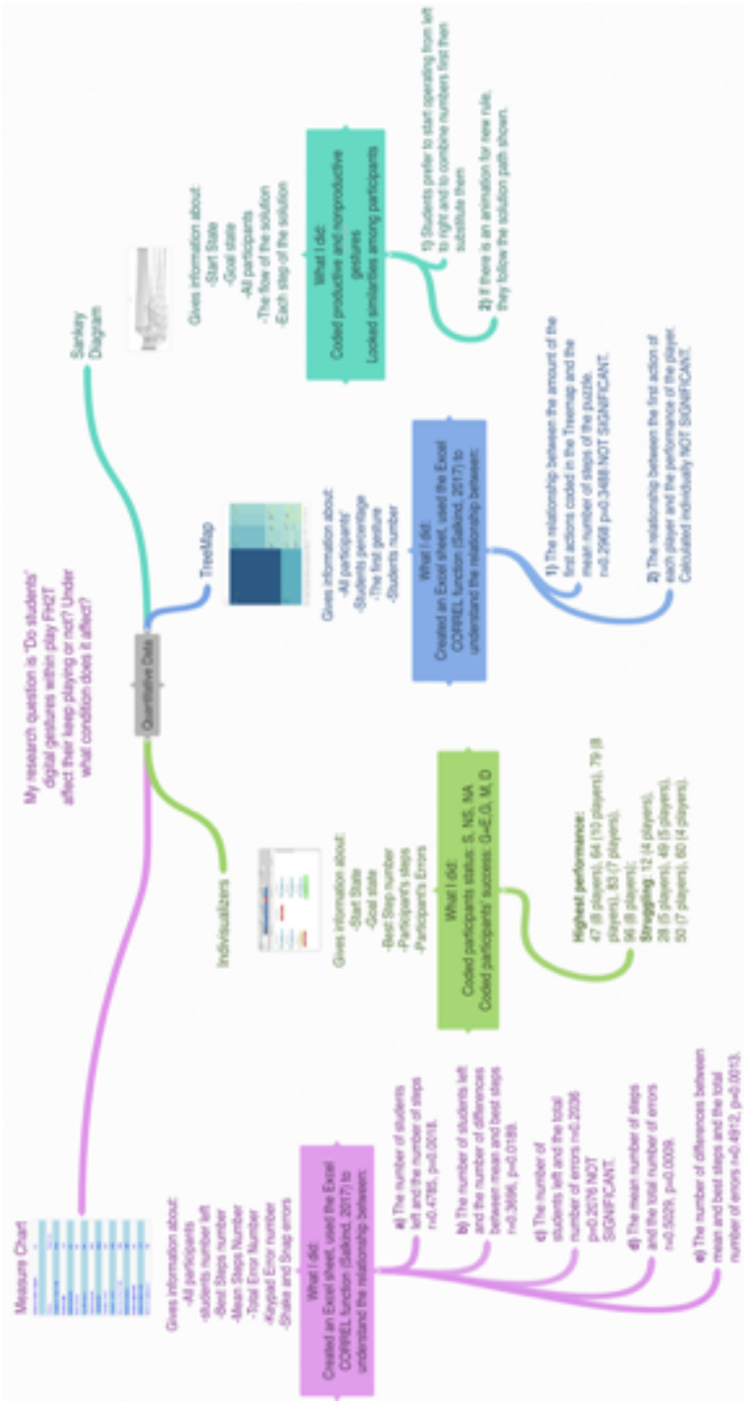
Appendix 4.2: Performance of Players

	P.1	P.2	P.3	P.4	P.5	P.6	P.7	P.8	P.9	P.10	P.11	P.12	P.13	P.14	P.15
9	G+E	M	M	M	M	G	D	G	M	M	M	M	D	G+E	D
12	G	G+E	G+E	D	M	G+E	D	M	M	G+E	G	M	D	M	D
14	G	G	M	M	D	M	D	M	M	G	D	M	M	G	G
25	M	D	M	G	M	M	D	M	M	G+E	M	G+E	G	G+E	G
28	G+E	G+E	D	G+E	G+E	D	G	D	G+E	G	G	G+E	D	D	M
31	M	G	M	M	G+E	M	G+E	D	G	M	D	M	M	G	M
32	M	G+E	D	M	M	M	D	G	M	G	M	M	M	M	G+E
47	G+E	G+E	G	NA	G+E	M	G+E	M	G	G	G	G	G	G	G
49	G	G	G	NA	D	D	G	D	G+E	D	D	NS	G+E	G	G+E
50	D	G	D	NA	G+E	M	D	M	D	D	M	NA	D	M	D
52	NA	G+E	NA	NA	NA	NA	D	NA	G+E	NA	NA	NA	NA	NS	NA
60	D	M	M	NA	D	D	M	M	M	G+E	M	NA	D	NA	M
61	G	M	D	NA	D	M	M	M	G	M	M	NA	M	NA	G
64	G	G	G	NA	G+E	G	M	G	G	G	G	NA	G	NA	G
69	NA	NA	NA	NA	NA	NA	NA	NA	D	NA	D	NA	NA	NA	NA
78	D	M	NA	NA	M	G	G	G	G+E	G	D	NA	M	NA	NA
79	G	G+E	NA	NA	G	G	G	G	G	G	G	NA	D	NA	NA
81	D	D	NA	NA	G	G	D	G	G	G	G+E	NA	G	NA	NA
82	M	M	NA	NA	G	G+E	D	M	D	G	G+E	NA	D	NA	NA
83	G	G	NA	NA	G+E	G	G	G	G+E	G	D	NA	G	NA	NA
84	G+E	D	NA	NA	G	D	G	D	G+E	G+E	M	NA	G	NA	NA
89	NA	NA	NA	NA	NA	NA	NA	NA	G+E	NA	NA	NA	NA	NA	NA
96	G	G	NA	NA	G+E	G	G	G	G	G	NA	NA	G	NA	NA
98	NS	M	NA	NA	M	D	G	M	D	G+E	NA	NA	D	NA	NA
99	NA	M	NA	NA	M	M	G+E	M	D	G	NA	NA	D	NA	NA
100	NA	M	NA	NA	M	M	D	M	D	M	NA	NA	NS	NA	NA
105	NA	NA	NA	NA	NA	NA	NA	NA	G	NA	NA	NA	NA	NA	NA
106	NA	NA	NA	NA	NA	NA	NA	NA	D	NA	NA	NA	NA	NA	NA
108	NA	NA	NA	NA	NA	NA	NA	NA	D	NA	NA	NA	NA	NA	NA

Appendix 4.3: Flow Chart



Appendix 4.4: Concept Map



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