
#### Abstract

Title of Dissertation: "WHY BE AVERAGE WHEN YOU COULD BE EXTRAORDINARY?": A CASE STUDY OF AN EXEMPLARY AFRICAN AMERICAN MATH TEACHER


Tarik Assebe Buli, Doctor of Philosophy, 2022
Dissertation directed by:
Assistant Professor, Imani Goffney, Department of Teaching and Learning, Policy and Leadership

It is well documented that as an educational system we subject students from particular racial and socioeconomic backgrounds to unequal schooling experiences. Some researchers attribute the unequal schooling experiences and outcomes of minoritized students of color to their limited access to educational resources, like skilled teachers and quality curriculum. Other researchers identify that even in highly resourced American schools, African American students are specifically subjected to oppressive learning conditions. Given this context of schooling for African American students, this study explores how an African American 8th-grade mathematics teacher, Ms. Collier, may be a protective factor in her students' education. Specifically, I use qualitative case study methods to examine how Ms. Collier's instructional practice relates to historical conceptualizations of African American teachers of African American students, and how her mathematics instruction socially positions her students as learners of mathematics. For this case study, I conducted classroom observations in two differently tracked mathematics classes, as well as semi-structured interviews with Ms. Collier and her students in both classes. I
pay particular attention to how she enacts a historically situated practice of care for her students, through how she facilitates whole class discussions and maintains high expectations for her students. I then consider how her instructional practice positions the students as learners of mathematics and compare how the students are positioned in her honors and on-level classes. The findings of this study suggest that Ms. Collier's instructional practices are rooted in a historical legacy of African American teachers resisting antiblack, deficit characterizations of Black students. Instead, Ms. Collier cares for her students by supporting them in their pursuit of mathematics learning in multifaceted and nuanced ways. Her care manifests in her teaching practice by cultivating a classroom culture that centers student belonging. She does this by allowing students to experience a range of emotions, like nervousness and joy, all the while still perceiving and treating them as mathematically competent. She also makes considerable demands of her students, including that they publicly participate in problem solving during whole class discussions, even when they do not know the answer. The classroom interactions reveal that all of Ms. Collier's students, across both tracked classes, are positioned as mathematically competent. However, there are some distinctions in how the students are positioned across the two classes. Whereas the students in the on-level class are positioned as capable of making sense of and persisting in mathematical problem solving, in the honors class the students are positioned as capable of making mathematical connections and solving problems independently. Despite these differences, all of Ms. Collier's students, across both tracked classes, are positioned as human.

# "WHY BE AVERAGE WHEN YOU COULD BE EXTRAORDINARY?": A CASE STUDY OF AN EXEMPLARY AFRICAN AMERICAN MATH TEACHER 

by

Tarik Assebe Buli

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Advisory Committee:
Professor Imani Goffney, Co-Chair
Professor Andrew Brantlinger, Co-Chair
Professor Daniel Chazan
Professor Kimberly Griffin
Professor Danny Martin
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## Chapter 1: Introduction

In recent years, as unequal mathematics education outcomes persist along racial and socioeconomic lines, there has been increasing urgency to center equity within mathematics education research (Bartell et. al., 2017; NCTM, 2014; Schoenfeld, 2002). Research agendas that attend to equity focus on many different facets of mathematics education including students' opportunities to learn (Esmonde, 2009b; Gresalfi \& Cobb, 2006; Tate, 1995), student identity (Larnell, 2016; McGee, 2013), detracking schools and enacting reform-oriented curriculum (Boaler \& Staples, 2008; Stein et al., 1996), classroom discourse and language use (Gutiérrez, 2002; Khisty, 1995; Walshaw \& Anthony, 2008), students' out of school mathematical knowledge (Carraher et al., 1985; Nasir, 2007; Taylor, 2009) and complex instruction (Cohan \& Lotan, 1997; Esmonde, 2009a; Featherstone et al., 2011). While these bodies of research have been instrumental in highlighting the rich mathematical capacity of students from historically excluded backgrounds and the unjust circumstances in which they experience mathematics instruction, it still remains the case that unequal mathematics education outcomes persist (Martin, 2019; Martin et. al., 2019).

There have been many policy and research agendas to respond to what has historically been considered the underperformance of African American, Latinx, Native, and lower-income students (Nasir \& Hand, 2006). Over the past several decades, the explanations for the disparities in schooling outcomes have taken different shapes. Nasir and Hand (2006) documented how the mainstream educational narratives about historically marginalized students in school performance have evolved. To start, they refer to the work of Richards (1997), Jensen (1969), and Cole and Bruner (1971) to explain the historically pervasive deficit perspective which attributed racial differences in school achievement to the heritability of intellectual capacity and
cultural deficits; these narratives maintained that Black students' underperformance was due to the heritability of IQ scores and/or the consequences of slavery and poverty leading African American families to be disorganized and lack adequate cognitive stimulation. In fact, the narrative about the culture of poverty, which asserts that the values of those living in poverty contribute to the perpetuation of their poverty, was used to design and found social programs like Head Start with the mission of saving children from their impoverished home communities (Lewis, 1969, cited in Nasir \& Hand, 2006).

Transitioning away from a deficit perspective, educational discourses which attempted to explain disparities in racial and socioeconomic schooling outcomes began to take a more neutral position of cultural difference. Nasir and Hand (2006) asserted that this line of research considers how race plays out in schooling environments for Black, Latinx, Native American, Asian, and White students; they find that when "students behave in ways that differ from the norms and expectations of their schooling institutions, both learning and school achievement suffer" (p. 452). In response, research agendas and pedagogical interventions in the form of multicultural education (Banks, 2004; Nieto \& Bode, 2008; Sleeter, 2001), culturally relevant pedagogy (CRP) (Ladson-Billings, 1995), culturally sustaining pedagogy (Paris, 2012), and funds of knowledge (Civil, 2002), have sought to respond to the effects of differences in culture on schooling experiences and outcomes.

While acknowledging that these cultural difference perspectives help elucidate how culture plays out in the local environment where learning takes place, Nasir and Hand (2006) also offered critical commentary on research and pedagogy based on cultural difference. First, they argued that these approaches may target specific racial or ethnic groups, leading to "issues of applicability in current multicultural schools and society" (Nasir \& Hand, 2006, p. 452). That
is, because, for example, funds of knowledge and CRP draws upon the specific cultures of the learners involved, as schools become increasingly more multicultural tailoring instruction to particular students and their cultures may become highly complex. Additionally, although cultural difference scholarship works to empower students from marginalized communities, the focus on individual groups and classrooms may obscure the macrodynamics which contribute to their marginalization in the first place.

In this vein, there are other research agendas with implications for equity in mathematics education which examine how broader societal forces manifest in schools to reproduce inequality. For example, Lareau (2011) considered the role of social capital in reproducing societal stratification. Taking a neutral perspective on issues of race and class, she found that high-income parents raise their children with a sense of entitlement and familiarity with middleclass norms, allowing the child to develop the necessary skills and dispositions to navigate schools and other social institutions, while lower-income parents allow their children to develop more naturally causing the child to experience a sense of constraint when navigating these same institutions. These differences in parenting styles transmit differential advantages to children in the form of cultural capital across economic lines; these differences in social capital are responsible for the hierarchies of achievement in schools and the reproduction of societal stratification.

Taking a more critical perspective, other bodies of research contend that unequal raceand economic-based schooling outcomes are a purposeful consequence of how people with power structure access to resources (Bantlinger, 2013; Ladson-Billings \& Tate, 1995). For example, Anyon (1980) compared mathematics and reading classrooms in schools in five financially resourced communities to show that school experience differs by social class. She
found that differences in the curricular and pedagogical practices at each school developed within students' specific relationships with the production of work and authority; that is, students coming from low-income households received rote, procedural instruction that would prepare them for careers which earn low wages, while students from higher-earning households received conceptual, higher order instruction preparing them to earn high wages by sharpening their reasoning skills. She concluded that by design the different schooling experiences across social class lead to the reproduction of societal stratification. Brantlinger (2003) found that middleclass parents, specifically mothers who have the power to shape schooling structures for their children, present themselves as liberals who espouse progressive social ideologies, but ultimately work to ensure their children's success in schools to improve their competitiveness in the job market. Therefore, despite their claims to support integrated and inclusive education, they work to ensure their children attend school with other students from middle-class homes who teach with direct and systematic instruction so their children may best compete against their peers and excel over children who attend schools in lower-income communities.

As stated at the outset, centering equitable learning experiences and outcomes for students from marginalized backgrounds is urgent in mathematics education research agendas. As described above, there are different perspectives concerning the root of how socioeconomic inequality is produced in and reproduced by schools and mathematics education as part of that. Nonetheless, it is well documented that the United States' educational system subjects Black, Latinx, and Indigenous students, or students from low-income backgrounds, to unequal schooling experiences, which in turn constrains their opportunities to learn mathematics. Given this context, my dissertation study was designed to explore how broader societal systems of
inequality, particularly along the lines of race, affect mathematics teaching and learning in a highly qualified African American teachers' classroom.

## Goals and Outcomes

This dissertation study was designed to investigate how the salience of race in mathematics may present itself in a mathematics class led by an exemplary African American mathematics teacher. Towards this end, I used case study methodology to investigate mathematics teaching in two differently tracked classrooms in a charter school serving primarily African American students from low-income households. In studying the mathematics teaching in these two classrooms, I paid particular attention to the teacher's perspective of teaching mathematics to her predominately Black students, her practice in cultivating a safe and demanding learning environment for students, and how her perspectives and practice position the students as learners of mathematics. The teacher's name, Ms. Collier, is a pseudonym, as are all names and places in this study.

In considering these components of mathematics teaching and learning, I pursued the following research questions:

1. How does an exemplary ${ }^{1}$ African American mathematics teacher's perspective and practice relate to historically situated conceptions of Black teachers and their work with Black students?
2. How do observed classroom interactions across the unit position the students as mathematics learners?
[^0]
## Theoretical Framework

In studying the schooling experiences of students from marginalized communities, my baseline is that the structure of schools and the communities they serve lead to the privileging of some students, namely White middle-income students, and the oppression of others, namely Black, Latinx, Indigenous, and low-income students. Thus, my theoretical framework is designed to make explicit the generally hidden inner-workings of power, privilege, and oppression at play in mathematics classrooms. Based on a review of critical literature, I understand that there are three constructs which are central to elucidating these broader societal constructs at play within the classroom, which will undergird this study: 1) mathematics education as a racialized environment; 2) the historical and contemporary role of African American teachers in their work with African American students; and 3) the social positioning of African American students as learners in mathematics. Using these three dimensions in tandem, I sought to make sense of how a highly qualified Black math teacher works to empower her students, who are predominantly Black, in learning a subject which often contributes to the marginalization of Black learners. I then considered how the teacher's efforts in educating her students position them as mathematics learners.

Mathematics Education as a Racialized Environment.

Martin (2009) asserted that mathematics learning and participation is a racialized form of experience in which:
socially constructed meanings for race in society emerge as highly salient in structuring (1) the way that mathematical experiences and opportunities to learn unfold and are interpreted and (2) the manner in which mathematics literacy and competency are framed, including who is perceived to be mathematically literate and who is not. (p. 323)

This is in direct contrast to mainstream educational narratives which claim that mathematics is an "objective science" (Jackson, 2009, p. 176) whereby those who are smart and capable of complex thoughts will experience mathematics success (Gholson \& Wilkes, 2017). Instead, Martin (2009b) explained that it is not intelligence and cognitive capacity that determines mathematics success, rather it is race that structures who has access to learn mathematical content and who is considered mathematically competent. More recently, Martin (2019) specified that race in mathematics is not only salient in structuring individual interactions, but also in shaping "norms, practices, policies, and relations of power" (pp. 461-462). He explained that this racialization, particularly for Black students, is a manifestation of antiblackness in mathematics education.

To define antiblackness, Martin (2019) referred to Dumas and ross (2016) and the Black Liberation Collective (2017), to explain that antiblackness is not just racism against Black people, but that it "refers to a broader antagonistic relationship between blackness and (the possibility of) humanity" (Dumas \& ross, 2016, p. 429). It involves "an interlocking paradigm of institutions, attitudes, practices and behaviors that work to dehumanize and oppress Black people in order to... to benefit and maintain white supremacy" (Black Liberation Collective, 2017). As a result,
antiblackness marks an irreconcilability between the Black and any sense of social or cultural regard. The aim of theorizing antiblackness is not to offer solutions to racial inequality, but to come to a deeper understanding of the Black condition within a context of utter contempt for, and acceptance of violence against the Black. (Dumas, 2016, p. 13) Because the racialized nature of mathematics classrooms is at the center of my study, it is important to express that race is not biological, but rather it is a social construct. The meaning of
race is constructed fully out of social relationships and is expressed through dynamics of power, in terms of history, culture, economics, and representation (Gillborn et al., 2018).

Acknowledging this reality is not meant to portray communities of color as "passive recipients of racist behavior" (Martin, 2009, p. 300), but instead to honestly represent the reality of racism in individuals' and communities' lived experiences and how they use their agency to navigate this unjust terrain.

Additionally, throughout this work, I refer to Black people both as Black and as African American. I chose to use both terms because in the literature that I cite, particularly the works written by Black researchers, the authors use both terms to refer to this group. Therefore, I felt it was appropriate to use both terms in my own writing. For clarity, when I use Black/African American in the context of my study and participants, I am referring to American Descendants of Slavery (ADOS). In the case of Black participants who are immigrants or the descendants of recent immigrants, I make this distinction clear by hyphenating their country of origin with American. For example, as my parents are Ethiopian and Eritrean immigrants, I would specify that I am Ethiopian- and Eritrean-American. If I am not sure where their specific country of origin is, then I identify them as (first or second generation) African immigrants.

## The Historical and Contemporary Role of Black Teachers

As has been discussed, African American students face unequal schooling experiences in their education broadly, and in mathematics specifically. Understanding the manifestation and reproduction of inequality in the mathematics experiences of African American students requires a study of mathematics teaching and learning that takes into account the "dynamics of microand macro-social, political, and historical forces" (Clark et al., 2013, p.11).

In terms of historical forces, Joseph et al. (2021) called for a critical-historical framework to address the mathematics experiences of Black teachers and students, highlighting that a historical perspective is necessary to make sense of present inequality. They explained:

CritHistory for mathematics education aids researchers in studying, addressing, and making sense of longstanding inequities in mathematics education by focusing specific attention not simply on the existence of structural injustices but also on the long histories of social, moral, and economic oppression against marginalized populations that have sustained those injustices. (p. 480)

Understanding history to make sense of the present requires rejecting ahistoricism, which means "exposing systemic patterns of disregard for Black learners in mathematics across time" (Joseph et. al., 2021, p. 480). As a corollary, by examining and tracing the contexts in which Black students experienced historical patterns of disregard, we may also find the systems of support which buoyed Black learners in those very contexts.

With this framing in mind, this study is based on a historical literature review on the education of Black students over time, paying particular attention to relationships between historical roots of inequality and its present-day manifestation. Drawing from the work of Clark et al. (2013) and Martin (2000), I considered the confluence of multilevel forces such as:
local mathematics education policy (standards, curriculum, assessment); collective school and community expectations of student achievement, success, disengagement, and failure; institutional structures that sort communities and students, intentionally or otherwise, by race (segregation, tracking, special education labeling); and the subtle or overt framing of their students of color as 'full of potential' or 'problems.' (Clark et al., 2013, p. 4)

These forces were discussed through time to convey a detailed image of the historical context of education for Black students and to highlight the agency and self-determination of the Black community to pursue the education they are owed, despite the oppressive conditions they faced.

In tracing relationships between historical and contemporary inequality for Black students, I also considered the protective factors (Spencer, 2006) that supported Black students in the face of antiblackness, and how those protective factors have persisted through time in the face of shifting social and political landscapes. Furthermore, as this study examined the work of a Black teacher, I paid particular attention to the role of Black teachers, their work with Black students, and how they may act as protective factors in their students' education.

## Positioning Theory

Given that this study investigated the work of a Black teacher in supporting her students, I analyzed how the instructional environment that she cultivated socially positions Black students as learners of mathematics. Specifically, I considered how her facilitation of classroom discussions positioned the students in both her on-level and honors sections as learners of mathematics, and compare the students' socially positioning across the two sections. In order to conduct this analysis, I used positioning theory.

Positioning theory refers to the creation of personal stories between members of a conversation (Harre \& van Langenhove, 1999). It has been used as a theory to analyze how power is expressed in classroom learning, by examining teacher and student discourse to understand how different participants are rendered powerless and powerful (e.g., Anderson, 2009; Pinnow \& Chval, 2015). A position is a metaphorical concept that refers to the speaker's rights and responsibilities that they are required, or allowed, to carry out in conversation; the speaker's position can be identified by considering how their spoken contributions are heard by
others (Harre \& van Langenhove, 1999; Pinnow \& Chval, 2015). Positioning theory is based on the principle that not all members in a conversation have equal power, status, and access to the rights and responsibilities that may be meaningful in the specific interaction. Each interlocutor has a specific position which grants them the power to engage in specific modes of discourse, such as issuing orders, assigning grades, and remembering past events (Kayi-Aydar \& Miller, 2018).

Within education, "positioning refers to the discursive processes (often utterances by students and teachers) that place participants into particular roles and relationships, including relations of power (more or less powerful), competence (competent or incompetent) and moral standing (the right to explain or the duty to listen)" (Turner et al., 2012, p. 203). Positioning theory can be used to analyze classroom interactions to reveal how: rights, duties, obligations, and opportunities are distributed among students and teachers in classroom discourse. In paying attention to the dynamic and contingent nature of positioning, scholars are better able to understand how particular forms of knowledge are (de)legitimated. These studies further make it possible to understand who can gain access to classroom talk, what cultural resources are being used and what kinds of learning opportunities are constructed. (Kayi-Aydar \& Miller, 2018, p. 88)

Given the racialized way in which rights, duties, obligations, and opportunities are distributed among students within the same classroom, positioning theory offers a way to understand the manifestation of power, which "is made visible in the ways in which people are positioned in relation to one another, in the ways in which they are given or denied access to particular physical and social spaces or kinds of actions, and in the ways in which issues of status and hierarchy are constructed and challenged" (Esmonde \& Langer-Osuna, 2013, p. 290). Anderson
(2009) elaborates that different individuals occupy different social positions in their classrooms based on their broader community memberships (e.g., race and gender). As a result, during face-to-face interaction, presumptions about these individuals based on their backgrounds limit the possible actions they may take and how these actions are interpreted.

Therefore, when studying students from historically excluded groups who are often rendered less powerful in classroom dynamics, researchers may use positioning theory to understand how learning and broader community membership make "particular forms of engagement in mathematical activities possible" (Esmonde \& Langer-Osuna, 2013 p. 292). In this dissertation study, I use positioning theory to analyze how Black students are positioned as mathematics learners by an experienced and well-regarded Black mathematics teacher. In doing so, I will draw from the three main tenets of positioning theory: positions, acts, and storylines. As stated above, positions are the symbolic rights, duties, and responsibilities that people are allowed, or obligated, to carry out during social interactions (Pinnow \& Chval, 2015). Acts are people's actions like speech, gestures, and gaze, which people use to position themselves and others (Pinnow \& Chval, 2015). The third concept, storylines, broadly refer to the content of what is being said in the social interaction, however, it presents itself in two different forms. One form of a storyline frames the content and context of the conversation being analyzed (Wood, 2013). It refers specifically to the content of the discourse between participants. The other form of a storyline refers to narratives that develop over time in communities and communicates meaning about ourselves and others (Pinnow \& Chval, 2015). Both conceptions of a storyline are relevant in analyzing how Black students are positioned as mathematics learners. While the first construct helps frame the content of the mathematical discourse in which the students are
participating, the second construct help elucidate how broader racialized narratives about Black students as mathematics learners manifest in the classroom.

## Chapter 2: Literature Review

In this chapter, I present a literature review to situate this dissertation study. I start the review by exploring the role of mathematics in education and how Black students experience mathematics learning. After establishing the unequal mathematics experiences and outcomes that Black students are often subjected to, the review transitions to provide a historical overview of education for Black people during the pre-Brown era. In the review, I explore how ADOS achieved access to education in this country, how Black teachers have supported Black students, and how the antiblack social context of the United States has affected education for Black students through time.

## Mathematics Teaching and Learning

Mathematics is often constructed as an "objective science" (Jackson, 2009, p. 176) whereby those who demonstrate mathematics competence are considered smart and capable of complex thoughts (Gholson \& Wilkes, 2017). For this reason, mathematics serves a unique role in schools, and society at large, often functioning as a gatekeeper; students must demonstrate appropriate levels of mathematics proficiency to compete for entrance to competitive colleges and universities and access lucrative STEM careers (U.S. Department of Education, 2008). In this way, mathematics success is a pathway for students' wellbeing by opening doors and increasing prospects for future income and social status.

## Mathematics Standards

Given the power of mathematics success in creating opportunities for students, there have been multiple national efforts to guarantee that "all" students in the United States have access to meaningful mathematics education. Towards this end, the National Council of Teachers of Mathematics (NCTM) released their Curriculum and Evaluation Standards for School

Mathematics (1989) and Principles and Standards for School Mathematics (2000) to guide the content, teaching, and instructional materials for K-12 curricula. The Principles and Standards called for "the development of a core curriculum that prepares students with the mathematical background for quantitative literacy, for the workplace, and for study at the college level" (Schoenfeld, 2002, p. 10). Similarly, the Common Core State Standards for Mathematics (CCSSM) and their Standards for Mathematical Practice (SMP), were adopted by 41 states and Washington, D.C. to outline the mathematical content that all students should learn for success in entry-level careers, college courses, and workforce training programs (National Governors Association Center for Best Practices \& Council of Chief State School Officers, 2010).

Mathematics Domain Knowledge. The K-12 mathematics curriculum has been handed down historically from Western institutions, influenced by theories of learning and development, and is currently regulated by a range of constituencies including mathematicians, mathematics educators and administrators in universities, professional organizations like the NCTM, testing and textbook companies, and state and local policymakers (Brantlinger, 2022; Dowling, 1998; Popkewitz, 2004). The established guiding national standards, as outlined by NCTM's Principles and Standards and the CCSSM and their Standards for Mathematical Practices, can be considered a marker for what we in this country consider to be mathematical domain knowledge. Furthermore, given the power of mathematics as a gatekeeper, these national standards represent the requisite domain knowledge expected of students for them to access post-secondary mathematics or high-earning STEM careers.

Due to the power of mathematics education and related credentials in shaping students' career trajectories, educators like Bob Moses (Moses \& Cobb, 2001) and Gloria Ladson-Billings (Ladson-Billings, 2006, 2007) have identified that equipping and denying Black students with
mathematical domain knowledge is a civil rights issue. For example, Moses (Moses \& Cobb, 2001), civil rights leader and mathematics educator who created The Algebra Project, framed access to high-quality algebra instruction as a civil rights issue analogous to voting rights. He explained that due to our current technological age, people in our society need mathematics literacy for economic access and full citizenship. From Moses' perspective, to equip students with mathematics literacy means to educate students such that they may engage with standard, college preparatory curriculum in their school. Moses and Cobb (2001) detailed what this means in terms of the college preparatory math sequence:
... whatever is out there [students] engage it. In their school system, whatever is in place as the standard college prep curriculum, you want them to engage that. It's important, however, that whatever else is coming in to supplement or replace that curriculum has to be a bona fide college preparation. It can't be something that is put in place to continue a tradition of separate tracks for some students. (p. 15)

This understanding of mathematics literacy, as the capacity to engage with a standard, college preparatory mathematics curriculum, is not trivial. As discussed in Chapter 1, there exists research explaining how racism and classism constrain learning experiences and outcomes for many students. This research highlights how schools that educate primarily low-income students of color lack qualified teachers, adequate curricula, advanced courses, and material resources, like computers and facilities (Darling-Hammond, 2007; Martin et al., 2017; Morton \& RiegleCrumb, 2019). In fact, schools which are comprised primarily of students of color are so frequently associated with a lack of educational resources that Milner (2012) developed the term urban characteristic to refer to schools that may be rural or suburban, but are composed primarily of students of color and experience challenges similar to big-city schools, such as a
lack of resources and qualified teachers. Furthermore, as over $60 \%$ of Black and Latinx students attend schools where they are in the majority (McFarland et. al., 2017; Morton \& Riegle-Crumb, 2020), this research attends to the schooling experiences of many Black students.

## Black Students' Experiences as Mathematics Learners

In addition to the curricular and resource disparities that Black students may face in their mathematics education, they may also face educational inequality unique to them due to the devaluation of Black identity in society and its reflection in schools (Martin, 2012; Perry et al., 2003). One way this inequality manifests in schools is in how Black students are socially positioned as mathematics learners. Within mathematics education literature, storylines about students from minoritized backgrounds, particularly African American, Native American, and Latinx students, communicate that they are amongst the lowest performers on mathematics achievement tests, especially compared to White and Asian American students (Martin, 2012). These storylines contribute to the normalization of African American students' intellectual inferiority (Gutiérrez, 2008; Martin, 2012), shaping how these students are socially positioned as mathematics learners.

Since Martin's (2000) groundbreaking work exploring the histories, experiences, and contextual forces affecting African Americans students' mathematics success, there have been an increasing number of studies documenting the racialized learning experiences for Black students in math environments (Gholson \& Wilkes, 2017; Jackson, 2009; Larnell, 2016; McGee, 2015; Spencer, 2006; Stinson, 2013). For example, in their study on the mathematics identity development of high achieving mathematics and engineering students, McGee and Martin (2011) found that the students experience ubiquitous forms of racism "revolv[ing] around Blacks having a lack of innate ability (due to their limited intelligence) to perform at a high level in
mathematics" (p. 1363). These storylines about intellectual inferiority have immense detrimental consequences on the learning experiences and outcomes of Black students. The consequences include justifying the enactment of a remedial mathematics curriculum in a predominately Black, low-income school despite having no empirical data on the students' mathematical knowledge (Jackson, 2009); teachers referring Black children for mental health disorder diagnoses when they are in fact academically gifted (Berry, 2005); and forcing Black students to bear the burdensome and cumulative weight of learning mathematics content while facing continuous onslaughts against their intellectual worth (Larnell, 2016; McGee, 2013; McGee \& Martin, 2011). Berry (2005) determined that these aggregated instances of erasing Black students' intellectual abilities are "a mechanism that perpetuates institutional racism" (p. 55) by justifying their exclusion from intellectually rich opportunities, which they deserve.

## Black Students' Positioning as Mathematics Learners

In Chapter 1, I discussed the main tenets of positioning theory, positions, acts, and storylines, and how they can be used as a framework to analyze power and differential learning opportunities for marginalized students. Now, in the literature review, I will highlight some of the empirical literature which uses positioning theory to analyze these processes at play in classroom interactions. Some of the empirical studies which use positioning theory consider how interactions between teachers and students construct students as either proficient or failures (Anderson, 2009; Martin-Beltran, 2010). Other studies consider how students from marginalized backgrounds negotiate and wield power in interactions with their White peers (Esmonde \& Langer-Osuna, 2013; Pinnow \& Chval; 2015). Still, others discuss how the same individual student can be positioned differently by others in their interactions, even when the individual enacts the same strategies to negotiate power (Wood, 2013).

Esmonde and Langer-Osuna (2013) used positioning theory to analyze power negotiations between two African American female students and a White male student in their cooperative learning small group in a high school math classroom. In their small group, Riley, the White male, was positioned as the mathematical expert and interacted with his peers like a teacher, by asking them guiding questions to help support their mathematical learning. However, during small group work time the African American females, Dawn and Shayenne, would oftentimes discuss romantic relationships with a boy in their class; only when Shayenne initiated a conversation about the mathematics task could Riley be positioned as their mathematical leader. Esmonde and Langer-Osuna (2013) concluded that although in most classrooms Dawn and Shayenne would be positioned as disruptive and off-task, they were in fact shifting the nature of the mathematical conversation so they could participate more equitably:

The two girls often invoked figured worlds where they held more privileged positions where they were popular and socially sophisticated, and Riley was not... Dawn and Shayenne routinely silenced Riley and were able to effectively decide when the group would engage in mathematical discussion. When Riley attempted to begin mathematical discussions, the two girls sometimes participated, sometimes did not. By contrast, when Shayenne or Dawn attempted to begin mathematical discussions, Riley always went along. In this way, Dawn and Sheyenne's actions tempered Riley's dominance. (p. 303) While Esmonde and Langer-Osuna (2013) showed how African American students can control the content of conversations to distribute power more equitably between themselves and their peers, Pinnow and Chval (2015) followed one student's progression in developing interactional competence to resist being positioned as incompetent. They used interactional competence, how all students are able to use resources mutually and reciprocally in a particular discursive practice,
to analyze how Roberto, an emergent bilingual Latinx student, was able to enter the interactional space of his mathematics classroom. They found that initially, when engaging in an interaction with his classmate Lance, Roberto was frequently positioned on the defensive, having to protest, argue, and defend his rights to participate equally in their interactions. However, over time, Roberto demonstrated improved interactional competence through his use of language and gestures, which allowed him to successfully resist being positioned as an unequal participant in his interactions with another peer, Sam.

Wood (2013) also looked at shifts in how a single student was positioned in his math class. She analyzed the interactions between a small group of fourth-grade students, two African American boys named Jakeel and Daren and a White girl named Rebecca, to examine how Jakeel was differently positioned in his class and what storylines emerged from their interactions. She found that the teacher and Daren both treated Jakeel as competent, by positioning him as an explainer who could use mathematical reasoning and as a mathematical student who could understand the explanations of others.

However, Wood (2013) found that Jakeel was positioned much differently in his interactions with his peer Rebecca. When interacting with Rebecca, she positioned him as mathematically incompetent when she refused to discuss his ideas and instead focused on his physical activity, by telling him when to write, what to write, when to cut, and where to look. Rebecca also positioned Jakeel as inferior when she asked him "What did you just do, boy?" (p. 794). Wood (2013) explained that Rebecca's "tone, question, and use of the pejorative boy, along with her white skin color and Jakeel's darker complexion" (p. 794) alongside her emphasis on controlling Jakeel's physical actions, suggest the storyline of subservience where Rebecca was communicating herself to be in a dominant position over Jakeel in connection with their racial
backgrounds. After analyzing these different interactions, Wood (2013) concluded that how an individual is positioned in a moment of time is sensitive to the context, storylines, and identities of the participants involved. Specifically, in terms of Jakeel's interactions with Rebecca, Wood (2013) wrote that this analysis "also extends Martin's [2000] findings by offering a specific example of how storylines crafted in particular moments knit together larger scale contexts (like race and oppression) and smaller scale contexts" (p. 804).

Central to all of these studies is that how individuals are positioned relies not only on how the individuals express themselves, but also on how their expressions are taken up by others. For example, we see this with Jakeel, whose enactment of the same mathematical practice of explanation was received differently depending on the other person in the interaction. With his teacher, his explanations meant that he was mathematically competent, but with his peer Rebecca, his explanations remained unheard and were unable to deter her efforts to control his body. Using positioning theory as a framework to analyze interactions can shed light on how students like Jakeel make meaning of and navigate these tensions, and what the development of patterns in how Jakeel is positioned means for his wellbeing and his trajectory as a mathematics learner.

## The Education Debt

What has resulted in the United States is a context of education for Black students in which they often are positioned as incompetent and inferior, they may experience curricular and resource disparities, and their intellectual capacity as Black mathematics learners may be called into question. Ladson-Billings $(2006,2007)$ has framed that this unjust learning context for African American students over time has resulted in an accumulated denial of a fair education for them. What results is that this accumulated injustice has led to Black students being owed an
education debt, representing a significant shift in terminology from what is often referred to as an achievement gap. Referencing her communications with economist Robert Haverman, LadsonBillings (2006) explained that the education debt is "the foregone schooling resources that we could have (should have) been investing in (primarily) low-income kids, which deficit leads to a variety of social problems (e.g., crime, low productivity, low wages, low labor force participation)" (p. 5). Therefore, rather than putting the onus of the unequal schooling outcomes on the students themselves, their families, or their communities, she documented that the current state of their schooling is a logical consequence of the "historical, economic, sociopolitical, and moral decisions and policies that characterize our society" (p. 5). She prompted us to recognize the problem as an education debt which is owed to Black students to remind us "that we have accumulated this problem as a result of centuries of neglect and denial of education to entire groups of students" (Ladson-Billing, 2007, p. 321).

The Education of Black Students: A Retrospective
Given Ladson-Billings' $(2006,2007)$ framing of educational inequality as an education debt which has accumulated through centuries worth of neglect for Black students, in this next section of the literature review, I take a historical look at how we got here. To be clear, this historical overview is not meant to be exhaustive, rather I intend to highlight the work of African American educational researchers to situate contemporary educational inequality in a historical context. My goal is threefold: 1) explore how education broadly, and mathematics education specifically, has functioned as an antiblack system; 2) highlight the protective factors (Spencer, 2006), particularly Black teachers, that have supported Black students in the face of an antiblack educational context; and 3) set the stage for my analysis in Chapter 4.

## The Fight for Universal Education for Black Students in the South

I begin by leveraging Anderson's (1988) historical analysis to show how achieving public education for Black students was an act of self-determination by Black people, surrounded by legally sanctioned racial discrimination. First, gaining access to universal education for Black children was an uphill battle. Anderson (1988) detailed the history of education for Black children in the South to show how attaining universal education was an act of self-determination that Black people advocated for and achieved, despite legally sanctioned racial discrimination. Anderson (1988) laid out the historical context in which the descendants of people who had been enslaved pushed for universal education for Black Southern children from the late 1890s to mid1910s, and were met with significant resistance from White southerners and dehumanizing ideologies from White northerners.

To establish the political context of the time, the Plessy v. Ferguson ruling (1896) legally sanctioned racial segregation by allowing "separate but equal" to characterize United States' institutions, including schools. This ruling not only established the political landscape of the time, but represented the social landscape as well. Part of the moral justification of the Plessy ruling rested on "theories of heritability of intelligence, explicitly indicating that the intellectual capacities of African Americans were inferior to those of White European Americans" (Clark et. al., 2013, p. 14). A direct remnant from its predecessor, this rationale was also a justification for slavery in the United States.

It is worth recognizing here that this narrative about the intellect of Black people has persisted through time and remains a contemporary issue Black students face in their educational experiences. For example, McGee's work on stereotype management (McGee, 2013; McGee \& Martin, 2011) unearthed how persistent stereotypes about the intellectual inferiority of Black
people pervade Black students' experiences in mathematics environments and has damaging consequences on their behavior and participation in mathematics. There is a cumulative cost that Black students pay in having to disprove their intellectual inferiority in mathematics. Having to achieve against the backdrop of multiple and compounded stereotypes takes a heavy psychological and emotional toll on both individuals and society. As Anderson (1998) and others argued, this pervasive narrative that affects Black students' mathematical experiences today shaped the initial debates about public schooling for Black children from the late 1800s to the early 1900s.

1890-1910: The Initial Education Debate between White Northerners and White

## Southerners

During the initial education debates between 1890 and 1910, generally, White Northerners advocated for universal public education while White Southerners argued against it. However, Anderson (1988) made clear that while these two groups were on opposing sides of the debate, both rooted their ideas in the inherent intellectual inferiority of African Americans. Anderson (1988) explained that while White Southerners vehemently opposed universal education for Black children, seeing it as a gateway to suffrage for Black people, White Northern philanthropists advocated for it. In order to get White Southerners on board, the Northerners argued that education could be used as a means of a subtle form of social control. The Northerners argued that in the South:
white dominance could best be maintained by educating blacks to accept or internalize the idea that white southerners had some legitimate right to rule over them... universal schooling could influence black children to accept the values ... [and] legitimacy of the South's racial caste system and submit willingly to its order. (Anderson, 1988, p. 98)

While White Northerners viewed public schooling as a means to indoctrinate Black people to submit to the racist caste system of the South, White Southerners vehemently opposed universal education, arguing that even a technical, industrial education, would potentially lead to universal suffrage. This posed a significant problem to the Southern White agenda as in certain states, like South Carolina, this would mean the Black majority would have control of the state. Therefore, Southern Whites stressed maintaining "organized structures of [racial] domination and subordination" (Anderson, 1988, p. 99). The Southern Whites "knew that blacks as a class had never submitted willingly to racist oppression or acknowledged the legitimacy of whites to rule over them" (Anderson, 1988, p. 99).

Recall that antiblackness in part refers to interlocking systems of institutions and attitudes that maintain White supremacy and oppress Black people. The debates among White people around public schools for Black children provide an opportunity to understand how the United States worked to maintain its racial caste system. While the narrative about the intellectual inferiority of Black people was circulating to justify disparate access to resources for Black people, White Southerners simultaneously knew that granting Black people access to education would potentially lead to suffrage for Black people, which "meant, theoretically, black control of the state" (Anderson, 1988, p. 99). From this perspective, it seems that denying Black people access to education has nothing to do with false constructions of their intellectual inferiority. White Southerners knew that Black people were self-interested, and with equal civil rights would exert their political power by changing political outcomes. This false narrative about their intellectual inferiority was circulated to justify the oppression of Black learners and simultaneously to maintain White power, by controlling the state.

## 1900-1935: Universal Education Established for and by Black People

During the early 20th century, there were shifting contexts in United States society that facilitated White Southerners' acceptance of universal education for Black children. This stemmed in part from the Great Migration, which began in 1914, with significant numbers of African Americans moving to the North and Midwest for economic opportunity (Anderson, 1988; Clark, 2013). In 1917, The U.S. Department of Labor investigated the exodus of African Americans out of the South and recommended establishing a viable system of Black schooling "as a means 'to keep the Negroes in the South and make them satisfied with their lot'" (Anderson, 1988, p. 150). To be clear, this rationale evidences antiblackness by shifting one social system, public education, in order to accommodate the larger scale oppressive system of Black labor in the South. After the Great Migration, public schools for Black children did become more readily available, with the majority of African Americans having access to public elementary school by the mid-1930s.

Although the migration of African Americans applied pressure on the White Southerners to improve social conditions, Anderson (1988) made clear that public education for Black children was accomplished through "black southerners' enduring beliefs in universal schooling and their collective social actions to achieve it" (p. 153). Anderson depicted the "collective social actions" of African Americans to achieve universal schooling by highlighting the case of the Rosenwald schools.

The Rosenwald Schools: Funded Primarily by Black Citizens. The Rosenwald Schools were a school building program in the rural south which led to the creation of 4,977 schools, serving 663,615 Black students, that cost $\$ 28,408,520$. Because the schools were named after Chicago philanthropist Julius Rosenwald and his Rosenwald Fund, popular opinion at that time was that Rosenwald funded the development of the schools. However, in actuality the fund paid for only about $15 \%$ of the cost of the schools, with the remaining funds coming primarily from rural Black citizens, either through their private contributions or from public tax funds collected from Black taxpayers. Worth noting here is that the public funds used for the schools came primarily from Black citizens; for the most part, White citizens' taxes did not go towards Black schools. Furthermore, although Black schools could only draw upon the taxes paid by Black citizens, the White schools used the taxes paid by both White and Black citizens. That is, the school funds collected through taxes from private Black citizens were reallocated to building schoolhouses for White children.

While this misappropriation of public funds occurred, Black advocacy for building their own schools was met with the accusation "that black education was a burden on the white taxpayer" (Anderson, 1988, p. 154), which Booker T. Washington corrected by clarifying that "the money is actually being taken from colored people and given to white schools" (Anderson, 1988, p. 154). While it was believed that Black people were undeservedly receiving handouts at the expense of white taxpayers, in fact, Black people were not receiving what they were owed, and were compensating for the difference through their own private contributions. This financial difference represents part of what Ladson Billings (2006) calls the Education Debt, which is not only representative of the differences in funds but also in the resulting educative experiences for

Black students, due to the racist systems their educations are embedded in. In this next section, I examine the teachers' experiences teaching in this segregated environment.

Teachers of Black Children in the South. According to Siddle Walker (2001), the research about African American teachers in the South prior to desegregation tends to characterize them as either caring role models for their students or as victims of oppressive circumstances. Oral histories of African American teachers tend to portray them as caring adults who embody parent-like behaviors with their students, by providing students with resources to meet their needs and motivating students to excel. On the other hand, mainstream accounts about the experiences of Black Southern teachers pre-Brown focus on restricted job opportunities, lower certification levels, and difficult working environments, painting said teachers as "struggling, poorly educated missionary figures who helped as best they could with racial uplift, despite daily difficulties of their jobs" (p. 753). Siddle Walker cautioned against essentializing African American teachers, on either end of this spectrum. She warned that each independently constructed perspective causes:
the history of African American education to present portraits of caring teachers where difficulties are on the periphery of their world, or portraits of difficulties that lack detailed discussions of teachers' agency within those settings. Although neither portrait is completely inaccurate, both are incomplete. (p. 753)

In this section of the literature review, I work to characterize both the conditions in which Black teachers taught and how they have used their agency to best educate their students in deeply unjust and unequal circumstances. This dissertation study focuses on the work of a veteran Black teacher, Ms. Collier, and how her pedagogy supports her Black students. In
exploring the literature about the historical practices of Black teachers working with Black students, my purpose is to situate how Ms. Collier's pedagogy is rooted in a historical legacy.

## African American Teacher Preparation

Of the pre-Brown era, ranging from the 1920s to the 1940s, Siddle Walker (2001) wrote that "the general assumption is that African American teachers had one form of preparation and that is that they were poorly trained" (p. 763). However, there was not one single system of preparation for Black teachers that existed during pre-Brown segregation. Furthermore, both White teachers and Black teachers suffered from poor teacher preparation in the South, making teacher preparation a general education issue in the South rather than an African American issue (2001). For example, from 1924-1925 the average African American teacher's education was four years of high school, while the average White teacher was one and a half years of college (2001). Nearly a decade later, in 1933-1934, this had increased to two years of college for Black teachers and four years of college for White teachers (2001).

While the above characterizes the general scope of African American teacher preparation at the time, there are counterexamples to this experience. For example, in urban areas, like Dunbar High School in Washington D.C., several teachers held advanced degrees in liberal arts, medicine, law, and mathematics (Siddle Walker, 2001). Because of the informal and formal mentoring relationships that exist between teachers, the presence of these highly educated teachers likely improved the knowledge of their colleagues, although this is not reflected in certification levels (Siddle Walker, 2001; Foster, 1997).

In mathematics specifically, Clark et. al. (2013) highlighted the case of Anna Julia Cooper to show the impact of highly educated teachers on students' learning opportunities. Anna Cooper obtained her bachelors in mathematics in 1884 and her master's degree in mathematics in

1887 and worked as a teacher and principal at the M Street School in Washington D.C. (which would later become Dunbar High School). As a principal, Ms. Cooper designed M Street School to enact rigorous coursework, including advanced mathematics, with Black students to prepare them for college. This instructional focus led to tensions with the White D.C. School Board that preferred a technical education for Black students, and eventually forced Ms. Cooper to resign.

Therefore, contrary to dominant narratives about African American teachers' poor preparation, there were highly educated Black teachers and these teachers worked to equip their students to similarly become highly educated. As in the case of Ms. Cooper, in the late 1800s, the White School Board found this unacceptable and forced her to resign.

## Contemporary Understandings of Teacher Quality

Martin et al. (2017) revealed how underqualified teachers in predominantly Black schools are an operation of structural racism. They cited the work of Hill and Lubienski (2007) that explains how schools composed of primarily minoritized students of color are taught by teachers with slightly less mathematical knowledge for teaching than the students in wealthier schools. Martin and colleagues (2017) then highlighted that teachers tend to teach in schools near the ones they themselves attended as a student. What results is a cycle whereby schools which failed to adequately prepare students with mathematical knowledge suffer the consequences when these students later return to teach.

Drawing from Tate (2008) and Bonilla-Silva (2010), Martin and colleagues (2017) framed teacher quality as an intergenerational resource to demonstrate how teacher quality is a result of structural racism. They explained:

Highlighting these particular studies ... situate[s] teacher quality as an intergenerational resource (Tate, 2008). This understanding of teacher quality potentially demonstrates the
operation of "racism without racists" (Bonilla-Silva, 2010). That is, the intergenerational pattern of teacher inequality requires no involvement by 'racist' individuals. The repeated "underfunding" of particular schools with regard to the resource of teacher quality is an intergenerational phenomenon. Thus ... provid[ing] important insight into the structural operation of racism. (Martin et. al., 2017, p. 615)

Worth highlighting here is that contemporarily "the intergenerational pattern of teacher inequality requires no involvement by 'racist' individuals" (Martin et. al., 2017, p. 615). In the present day, existing bodies of research about African American students may highlight that they attend urban schools that lack qualified teachers, adequate curricula, advanced courses, and material resources, like computers and facilities (Darling-Hammond, 2007; Lewis \& Diamond, 2015; Martin et al., 2017; Morton \& Riegle-Crumb, 2019). The present ubiquity of underfunded schools with limited resources helps mask that this educational context for African American students was in fact by design. Although in the present day these patterns may unfold without actions by racist individuals, they are grounded in an antiblack foundation that fired teachers like Ms. Cooper who dared to teach their African American students high-level mathematics.

Dr. Horace Tate, former African American teacher and principal during the 1940s and 1950s, explained that White teachers and the Georgia State Board of Education "didn't want us to be able to read so well" (Siddle Walker, 2001, p. 761). They wanted Black students to be mannerable, but not self-sufficient (Siddle Walker, 2001). Teachers like Ms. Cooper, who taught to the contrary so that their students engaged in rigorous, college preparatory coursework, were fired. Ms. Cooper's case reveals the cyclical nature of teacher quality and its impact on student learning, which is framed as an instantiation of structural racism in modern times. We see the present-day consequences whereby schools composed of primarily African American students
face an intergenerational lack of educational resources, like qualified teachers and advanced curricula.

## Black Teacher Associations

In addition to differences in education levels, there were also disparities in pay between Black and White teachers. While White teachers' salaries were steadily increasing between 1900 and 1930, Black teachers' salaries periodically declined during this period. In fact, their salaries were closer in 1900 than in 1930. Furthermore, without voting rights, African American teachers had limited opportunities to address this inequality, relying upon their collective strength through teacher associations to advocate for better pay.

In order to combat the inequality they faced without suffrage, Black teachers turned towards organizations through Black Teacher Associations (Siddle Walker, 2001). They joined professional associations to confer with one another and embrace unified ideas about teaching Black students that were grounded in beliefs as educators and their understanding of the African American community. These organizations began in the 1880s, with each state having some African American teacher organization by 1900 (Siddle Walker, 2001). In these professional organizations, the teachers discussed pressing issues they faced, such as student enrollment, teacher pay, teacher qualifications, facilities, student attendance, and curriculum (Siddle Walker, 2001).

Siddle Walker (2001) conducted and analyzed interviews with Dr. Horace Tate, the last executive director of the Georgia Teachers and Education Association (GTEA), to better understand the role and function of African American teachers' associations during the 1950s. Tate explained that beyond advocating for improved teacher qualifications, resources, and pay, central to understanding African American teacher associations is understanding their difference
in orientation compared to White teachers associations. Tate characterized that the White Teacher Association was always in support of the State Board of Education and their stance on educating Black children. For these groups, the purpose of education for Black people was so that they could be "trained to read and write only at a level that they could fill the roles White employers needed for the menial positions they offered... [They] want Blacks to be mannerable, but not get to the point of being self-sufficient" (Siddle, Walker, 2001, p. 761).

The perspectives of the State Board of Education and the White Teachers Association are in direct contrast to the GTEA. In fact, the GTEA functioned in direct opposition to the State Board, with an overt agenda centered around resisting White American attitudes about the purpose of education for Black students. They worked to uplift a wholly different perspective about the role of African Americans in society and prospective futures for Black children. The GTEA believed that for African American children "the sky was the limit and that you become educated to be the best-to be as good as anybody in the world" and the organization "intentionally operated to facilitate that goal and to reject the White view of what African American education should be" (Siddle Walker, 2001, p. 761).

While Tate characterized White organizations' perspectives about the education of Black children during the mid-1900s, these orientations about the education of Black children have persisted through time. For example, Oakes (2005) found that partially on the basis of teacher and counselor race- and class-based bias, historically minoritized students were being tracked into lower-level mathematics coursework that taught students basic computation skills and arithmetic grade after grade. After their 12 years of schooling, these students were never exposed to college preparatory mathematics and instead were being groomed for careers as "unskilled labor." Jackson (2009) found that a predominately African American school in a low-income
community emphasized remedial and procedural mathematics, despite framing the school as a college preparatory middle school to families. All this to say, the purpose and function of education for Black students remains a contemporary issue. Although today we have shifted our language to leverage equity-oriented phrases like mathematics for all, in practice, schooling for many Black students remains minimally educative and consequently prepares them for the type of "unskilled labor" that Tate describes above. Martin (2019) explained that leveraging these equity-oriented phrases while simultaneously maintaining the status quo, in terms of African American students' experiences and outcomes, is an act of violence and dehumanization toward Black students.

Given these narratives surrounding the purpose and function of education for Black students, the GTEA's mission was to resist oppressive societal contexts for Black students and instead recognize the full capacity of Black children. This mission did manifest in the teaching practices of Black teachers at the time. In the next section, I discuss literature about Black teacher practices in the South during segregation, considering how the teachers worked to educate and capacitate Black children despite the unjust social landscape of the time.

## Black Teachers' Principles and Practice

Although Siddle Walker (2001) importantly cautioned against characterizing Black teachers in the pre-Brown era exclusively as caring, parent-like role models for their students, it is the case that Black teachers at the time played a wholly unique role in the lives of Black students. Stemming from the foundation of education for Black students in the United States, Black educators worked to improve opportunities for newly freed people who had been enslaved. Anchored in their collective Black identity, these teachers "taught in segregated schools to prepare Black children for freedom, respectability, independence, and self-reliance" (Irvine,

2003, p. 122). Although not in the affectionate, altruistic sense of the word, researchers (Clark et. al., 2013; Foster, 1997; Irvine, 2003; Joseph et. al., 2021; Siddle Walker, 2001) have characterized this instructional style as the practice of care.

To start, Black teachers caring for their students was historically rooted in the mission of supporting students to fulfill the individual and collective aspirations of the Black community (Foster, 1997; Irvine, 2003; Siddle Walker, 2001). Both the teacher and student shared a common interest in the student's education, whereby the student's successes and failures were gains and losses to the Black community. The teachers "view[ed] themselves as ethnically responsible for preparing these youth for future leadership and for making contributions to this unique mission, namely the liberation and enhancement of the quality of life for Black people" (Adair, 1984 as cited in Clark et. al., 2013, p. 206).

Rather than altruism, Black teachers' care for their students is self-interested, whereby the education and betterment of their students improve the quality of life of their own community. This mission to improve the quality of life for the community was all the more urgent due to the social context of the time. The teachers were preparing students to compete in a desegregated world that did not yet exist (Siddle Walker, 2001). For the teachers, this meant that they needed to teach their students to be better than White students if they were to succeed (2001).

With all of this weight and purpose surrounding what it meant for Black teachers to care for their Black students, their care manifested in a teaching practice that centered around high expectations (Foster, 1997; Irvine, 2003; Siddle Walker, 2001). Irvine (2003) explained that for these teachers, their instructional practice is characterized by a "tough-minded, no-nonsense, structured, disciplined classroom environment for young people who[] ... not only can learn but must learn" (Irvine, 2003, p. 43). Furthermore, their expectations did not end with learning
content, but also involved their students' moral development and capacity to maintain productive relationships (Foster, 1997; Siddle Walker, 2001). In certain circumstances, when students would not learn the content, teachers would lecture them about the weight of their academic success in terms of their personal future, their family's future, and the African American community's future (Clark et. al., 2013; Foster, 1997; Irvine, 2003; Siddle Walker, 2001).

From the students' perspectives, Siddle Walker (1996) wrote that the term caring comes from how the students most often described their teachers. Irvine (2003) explained that: students did not equate caring with being nice or friendly. None of them felt that they had been silenced or demonstrated any resentment toward their teachers. Caring for these students meant firm, fair discipline, high standards and expectations, and an unwillingness on the part of the teachers to let students 'slide by' (p. 43). While the teachers maintained these high expectations for their students, they embodied them in their own practice as well. They "committed themselves to the task of teaching African American children -- however many additional hours of work this took -- and to teaching them well" (Siddle Walker, 2001, p. 770). The teachers also worked with the students' families, including by helping with transportation and providing supplies as needed (Siddle Walker, 2001).

It is worth recognizing here how antiblackness is relevant in this practice of care by Black teachers with Black students. As cited in Gholson and Wilkes (2017), Shah (2013) poignantly expressed how high expectations and the necessity of success for Black students is the result of striving for equality in an antiblack society:

For many students, learning math is about solving equations and graphing parabolas-it is just another course requirement. However, for students from persistently marginalized racial backgrounds, the stakes are much higher. Certainly, mathematics can act as a
material gatekeeper, obstructing access to future economic opportunities and full civic participation (Moses \& Cobb, 2001). But learning mathematics can also be about identity and personhood. Living in a world where intelligence has become the primary marker of personhood, and where for five hundred years certain racial groups have been considered under evolved and intellectually deficient, mathematics can represent an opportunity for a student to reclaim cognitive status by showing that she or he is "smart" and can think complex thoughts. Mathematics offers a chance to show the social world that you are a full human being. (Shah, 2013, pp. 30-31) (Gholson \& Wilkes, 2017, pp. 230-231).

In characterizing Black teachers' practice of care, I heed Clark et al.'s (2009) caution against the "underconceptualization and underanalysis of variation in African American life in educational research ... lead[ing] to unsophisticated characterizations of African Americans" (p. 42). Research exists that shows evidence to the contrary of the harmonious, community-based relationships described above. For example, Fairclough (2007, as cited in Clark et. al., 2013) explained that "it would be false, then to depict [African American] teachers as community leaders who enjoyed unalloyed support from parents and pupils" (p. 19). Clark et al. (2013) elaborated that within African American communities:
like all communities, social and power hierarchies based on ancestry, skin tone, wealth, and geographical origins were well entrenched in the African American community during slavery, Reconstruction, and segregation. In some instances, the African American teacher was seen as an outsider, particularly those trained in the North and teaching in the South, and treated with suspicion and disdain (p. 7).

However, Clark et. al. (2013) ultimately determined that:
despite these complexities, there is a strong narrative in the literature describing the African American teacher under segregation as a figure engaged in a noble, uplifting service and perceived by many to be a critical agent in African American children's educational and social development and in the African American community's progress (p. 7).

Acknowledging the heterogeneity in Black teachers' experiences during Segregation, I do agree with Clark et al. (2013) that, on the whole, there was a uniquely uplifting nature to their pedagogy for Black students. Given this historical perspective, I shift now to consider how contemporary Black teachers are constructed in the research literature and in what ways, if any, their practice may relate to this historical practice of care.

## Black Teachers' Contemporary Practice of Care

Contemporary research about Black teachers working with Black students evidences this historical legacy of care. For example, Black teachers continue to hold high expectations for their Black students, which in turn leads their students to perform better (Black Teacher Project, 2022). This is in contrast to White teachers who may buy into deficit narratives about their Black students and underestimate their students' mathematics potential, oftentimes based on behavioral compliance or stereotypes about intellectual inferiority (Berry, 2005; Lim, 2008; McGee, 2013). Black teachers, on the other hand, have been characterized as having high expectations for their Black students and bringing knowledge of self, advocacy, and love into their classrooms as powerful tools to shape their students' educational experience (Kohli, 2018).

This type of instructional practice has been characterized as a warm demander pedagogy (Kleinfeld, 1975) whereby teachers combine personal warmth with active demandingness. It refers to teachers who make intellectual demands of their students, while ensuring their students
understand that the demands stem from the teacher's care for their students (Bondy et. al., 2012: Irvine, 2003; Kleinfeld, 1975; Milner, 2006; Ware, 2006). Black teachers enact a warm demander pedagogy by 1) genuinely believing in their students' capacity to succeed, responding to students' needs, and reusing to give up on them (the warmth) and 2) being deeply committed to their students' success, and unwilling to tolerate anything less than their students' best effort (the demand) (Bondy et. al., 2012; Ware, 2006).

To understand how this Black teacher's contemporary practice of care may manifest in practice, Clark et al. (2009) rendered portraits of two African American teachers, Madison Morgan and Floyd Lee, in their work with predominantly African American students. Morgan, a woman in her early 40s with 13 years of teaching experience, excelled in high school mathematics yet was forced to enroll in remedial mathematics in college, which she believed was partly due to her segregated schooling experiences. As a result, in her instruction, a central goal was to provide her African American students with authentic opportunities to engage in mathematical thinking, problem-solving, and sense-making. In fact, when structuring her Algebra 1 course for the third year she decided to eliminate the content focused on reviewing basic skills like operations with integers, choosing instead to "immerse the students into activities that required them to find patterns and make generalizations from the first day of school (Clark et. al., 2009, p. 55). When explaining why she established this instructional focus, Morgan "spoke passionately about how and why her instructional choices are in a large part driven by her dedication to providing her African American students in particular meaningful, challenging, high-quality mathematical experiences and opportunities so that they, in turn, can develop strong mathematics identities" (Clark et. al., 2009, p. 56). Much like the pre-Brown
teachers, Morgan designed her pedagogy to equip her African American students to access and succeed in their postsecondary education and career.

Floyd Lee, on the other hand, was a 24-year-old Black male in his first two years of teaching. Lee's instructional approach involved:
long speeches peppered with circumstances, language, and cultural referents very familiar to his students, Floyd explicitly advised his students to avoid actions and distractions -teenage sexual activity and pregnancy, alcohol and drug use, glorification of music and sports figures -- that resulted in the very heavy consequence of doing poorly in school. (Clark et. al., 2009, p. 56)

In Lee's pedagogical approach, there are reminders of the pre-Brown teachers' lectures to their students about the weight of academic success in terms of the students' futures, their families' futures, and the shared African American community's future. Lee's purpose was to convey to his students his similarity with them and his success in mathematics and in his life. His message was that his students could achieve similar success if they made good choices. However, differently than Morgan, Lee "did not consider his messages specifically tailored for his Black students" (Clark et. al., 2009, p. 57). Rather he "felt that all students needed to hear what he had to say because all youth -- Black, Latino, white--- were susceptible to the same negative consequences of poor choices and were in the process of managing these choices" (Clark et. al., 2009, p. 57). While Lee was cognizant of racial inequality and external forces that affected Black students' mathematics success, he emphasized the students' 'free will to make choices that best position them to succeed mathematically" (Clark et. al., 2009, p. 57).

Morgan and Lee provide two distinct cases of how care can manifest in Black teachers' instructional practice with Black students. Morgan's pedagogy resembles Black teachers'
historical practice of preparing students for success in college and careers, as was evidenced in Ms. Cooper's practice and the work of Tate and the GTEA. Lee's pedagogy resembles the historical teachers who impressed on students the importance of their education in opening up their life opportunities and their agency to accomplish these successes, although he viewed this as an issue relevant to all youth rather than Black youth specifically.

Theorizing Black Mathematics Teachers' Contemporary Practice of Care and its Historical Roots

The historical work of Black teachers caring for their Black students during the preBrown era can be characterized as resistance. They resisted mainstream American narratives that constructed Black students as intellectually inferior and undeserving of college-preparatory learning experiences. They resisted the limited and menial career opportunities designated for their students by providing them with an education aimed to improve their students' life opportunities. As part of this endeavor, the teachers accepted nothing less than their students' best, maintaining high expectations that their students master the content, as their students' future life opportunities depended on it. This practice of care was rooted in the incredibly high stakes of educating Black children in an antiblack society, whereby acquiring an education was a means not only to access higher paying careers, but also to establish personhood in a society that used false narratives of intellectually inferiority to deny humanity (Shah, 2022). Given this societal reality, historically Black teachers caring for Black students meant ensuring that their students learned for the sake of their life opportunities.

In the present day, Black teachers caring for their students can still be characterized as resistance. Today, mainstream mathematics education is considered a site of physical, symbolic, and epistemological violence against Black children (Martin, 2019; Martin et al., 2019). In terms
of symbolic violence, Martin et al. (2019) outlined how the "disproportionate curricular tracking that results in [Black learners'] warehousing in racially segregated classrooms" is an act of symbolic antiblack violence because it creates "a situation that maintains mathematics as a status symbol and institutional gatekeeper while simultaneously privileging White access to worthwhile mathematics" (pp. 40-41). Therefore, in the tradition of the historical pre-Brown teachers who cared for their students by expecting them to master the content, contemporary Black teachers also care for their students through their "insistence on high levels of academic achievement" (Ladson Billings, 2002, p. 119) and their resulting efforts to cultivate learning environments in which they may demand success from their students.

Ladson Billings (2002) outlined exactly what is required of contemporary teachers so that they may demand that their students succeed. She explained that in order to demand success from their students, teachers must "help them appreciate the power and fulfillment" of the content while also "preserv[ing] each student's sense of self" (p. 118). In terms of preserving students' sense of self during the pursuit of content mastery, Martin (2019) theorized how this may look in mathematics education. He described this approach as refusal in our current system of mathematics education, whereby "the pursuit of mathematics knowledge within this system should not be for the sole purpose of being accepted into anti-Black and white supremacist spaces" (Martin, 2019, p. 471). Rather, he explained that for mathematics education to acknowledge and value Black humanity must start with the foundation of Black learners' brilliance. Starting from this perspective "is a definitive statement of Black humanity, one that can only be denied in the context of antiblackness. It is an act of reclaiming the identities of Black children from anti-Black violence" (Martin, 2019, p. 472).

One final facet of contemporary Black teachers' care as resistance is language use. Martin et. al. (2019) referred to Ferguson (2000) to define Black English vernacular, also referred to as African American Vernacular English (AAVE), as a ""full-blown language with a grammar and syntax of its own that emanates from and reflects the historical and lived experiences of Americans of African descent'" (p. 40). Building from Martin and colleagues' work (2019), Ortiz (2022) has written that "part of the violence rendered against [Black] students in schooling environments is the coercion to abandon any iteration of [Black] language" (p. 3). When Black learners "are subsequently forced to abandon this language in school contexts represents 'a violent and painful assault on their very sense of self and on those with whom they most closely identify' [Ferguson, 2000] (p. 207)" (Martin et.al., 2019, p. 40). Facing this schooling context, Black teachers' care as resistance must then involve "allow[ing] and foster[ing] a receptive environment" that "acknowledge[s] the importance of Black Language in mathematics discourse" (Ortiz, 2022, p. 8). The work of a caring teacher must simultaneously involve affirming students' use of Black language in mathematics discourse alongside demanding that students "acquire the more technical and formal language of the discipline (Schleppegrell, 2007)" (Ortiz, 2022). Black teachers' caring involves resisting the dehumanizing structure of mainstream mathematics education which invalidates Black students and their home ways of communicating while also teaching students the formal language of mathematics so that they may fully master the content.

## Chapter 3: Methodology

This study was designed to investigate how an African American mathematics teacher facilitated instruction and positioned her students in a predominantly African American eighthgrade math classroom. I conducted a qualitative case study to examine how the teacher facilitated whole class discussions in her honors and on-level eighth-grade math classes. I used observations of classroom instruction, interviews with the teacher and students, and an examination of the written curriculum materials to understand how the teacher's instructional practice was situated in the historical work of Black teachers working with Black students. While the observational data provides evidence of what occurred during the classroom discussions, the teacher interview data provide evidence of the teacher's rationale behind her instructional decision making and the student interview data provides evidence of how the teacher's practices were experienced by the students. I use the written curriculum materials to understand the instructional goals of the classroom activities and how they aligned and deviated from the curricular expectations. Using all of these sources of data, I analyzed how Ms. Collier's contemporary practice as an African American math teacher of primarily African American students relates to the work of African American teachers during the pre-Brown era who supported their students in a deeply unjust and inhumane society. After considering the historical roots of Ms. Collier's practice, I then analyzed how her students were socially positioned as mathematics learners, particularly given the elevated status of mathematics education as a gatekeeper in K-12 schooling. In conducting this analysis, I answered the following research questions:

1) How does an exemplary African American mathematics teacher's perspective and practice relate to historically situated conceptions of African American teachers and their work with African American students?
2) How do observed classroom interactions across the unit position the students as mathematics learners?

In this next section, I provide a brief rationale for my research methods of a case study design and elaborate upon the study context, data sources, data collection, and analysis.

## Research Methods Rationale

In order to better understand a math teacher's perspectives and practice in two differently tracked math classrooms, I used case study methods. Case study is an ideal methodology for when the researcher wants to provide an in-depth understanding of clearly identifiable cases within a bounded system (Creswell, 2007). Given that the participants in this study were primarily observed in their mathematics classrooms, their mathematics classroom is the bounded system in which the case is identified. The case I investigate is the Black students in the two mathematics classrooms who were participating ${ }^{2}$ in this study. By gathering in-depth data from multiple sources of information, like classroom observations, interviews with participants, audio and visual recordings, and curriculum materials (Creswell, 2007), I developed an understanding of how the teacher's background, as an exemplary and experienced African American math teacher, related to how she enacted her teaching practice.

Merriam (1998) explained that conducting a descriptive case study allows for a "rich, 'thick' description of the phenomenon under study" (p.29). She explained that descriptive case studies allow for holistic representations of the case over time by using descriptive language to

[^1]elicit images and analyze situations. She explained that these case studies include vivid materials like quotations and interviews, obtain information from a wide variety of sources, and illustrate the complexities of a situation. By conducting a descriptive case study and analyzing two differently tracked math classes, I can help the field better understand how racialization manifests even in settings led by an exemplary Black mathematics teacher who seeks to act as a protective factor in her Black students' education. In terms of the unequal schooling experiences and outcomes of Black students, conducting this case study can help provide background and explain how such a teacher acts as a protective force in her students' educations, but also how her supportive actions are constrained by societal and institutional limitations.

## Study Design

Prior to entering research sites and working with participants, Madison (2005) recommended that it is vital that the researcher identify and reflect upon their own positionality. The actions of conducting research, doing fieldwork, facilitating interviews, and collecting and interpreting data, are all mediated by our subjectivity, socio-political perspectives, and possibly also ideological blinders. Therefore, by contextualizing our positionality we make it accessible, transparent, and open to evaluation. After addressing my positionality, I review the research site and participants, data collection methods, and data analysis process.

## Researcher Positionality

My interest in researching relationships between race and mathematics teaching and learning is largely due to my own developmental experiences in conjunction with my teaching experience. My parents are Ethiopian and Eritrean immigrants who emigrated to this country highly educated, which enabled them to transition from low-income to middle-income during my adolescence. During this time, I attended public schools in a diverse, middle-class, suburban
community. Although I tended to be one of only a few Black students in my classes, and I usually came from a lower-income household than my friends and peers, I was still able to excel academically in my K-12 education.

When I began my teaching career, I taught 6th-grade mathematics at George Washington Middle School (GWMS) a Title-1 middle school in Louisville, KY. All of the students at GWMS received free lunch and the student demographics were nearly one half Black/African-American students, around one quarter emergent bilingual students, and around one quarter White students. When I first started teaching, I was shocked by how my students experienced middle school and how different it was from my own experiences. For example, the students I taught had to be escorted by their teachers from class to class as they walked on a red line because administrators believed this would limit fighting in the hallway. Students were required to wear uniforms which included tucking shirts into their pants and received disciplinary consequences including suspension for having their shirts untucked. The school did not provide teachers or students with a mathematics textbook and half of their math classes were expected to be completed on a computer because that was supposed to better meet their "needs," as over half of the 6th-grade students were performing $2+$ years below grade level, according to standardized exam results.

During my time at Washington Middle School, I felt strongly that my students were not receiving a just and equitable education. I believed that many of the rules the school was enforcing were based on compliance rather than supporting their learning. I felt the school did not have adequate systems in place to give students the attention needed to learn critical 6thgrade mathematics procedures and concepts. In this majority Black, predominately low-income school it was obvious to me that racism and class bias were reducing my students' opportunities to learn mathematics.

However, it was not until I began graduate school that I developed a lens to recognize how pervasive and salient race was in my own education. During my K-12 education, I knew I was subjected to racialized comments from my peers. These mostly took the form of comments, thinly disguised as jokes, which communicated that my racial background, personality, and/or academic standing were at odds. While I recognized these comments as annoying and unfair, I thought they were relatively meaningless in the grand scheme of my education and my life. However, once I began graduate school and started to learn about Critical Race Theory (LadsonBillings \& Tate, 1995) and other perspectives that study how racialization manifests for Black students in mathematics education (Larnell, 2016; Martin, 2012; McGee, 2013; Stinson, 2013), I came to realize how significantly the racialized environment in which I learned mathematics shaped both my participation and performance.

One such body of research stems from Dr. Ebony McGee, who researches stereotype management for high achieving Black students (2013, 2015). She found that one strategy that Black students use to mitigate the stereotypes that surround them is to prove the stereotypes wrong. The problem with this strategy is the huge personal cost and energy the student must expend to prove the stereotype wrong, and how this cost is cumulative and increasingly more expensive as the student progresses to each subsequent course they take in mathematics. After a certain point, if the student is fed up with having to learn mathematics while also bearing the burden of combating these damaging narratives that surround them, they dissociate from the mathematics environment. I saw myself and my experiences in this line of research; despite earning no lower than a B in my courses, I changed my undergraduate major from mathematics to philosophy at the University of Virginia because I found the environment to be so harmful.

Seeing myself in this work pushed me to want to study the pervasive and subtle ways racialization in mathematics manifests for African American students and teachers. It also led me to wonder what my education might have been like if it did not take place exclusively in predominately White environments which were rife with stereotypes about my intellectual capacity. I am now deeply interested in better understanding how to structure learning opportunities for Black students that is free of this racial harm. Towards this end, I am pursuing this study to better understand how being a math student in a predominantly Black classroom led by an exemplary Black teacher may protect students as they pursue mathematics learning, which is considered a racialized endeavor. Given this racialized context, I wonder in what ways their teacher supports and protects them.

## Research Site

This study was conducted in a large, urban city in the Mid-Atlantic. The city has a history of redlining and in recent years has faced gentrification as the long-established African American community is priced out as high earning professionals from throughout the country resettle to it. The education system is comprised of public schools, independently governed charter schools, as well as elite private schools. To investigate how the salience of race presents itself in a predominately Black math class, this study investigates mathematics teaching and learning in two differently tracked eighth-grade math classrooms in a charter school led by the same teacher. One class is an honors math class while the other class is on level ${ }^{3}$. More detailed information on the school is provided below.

[^2]Hummingbird Academy (HA) was one of the first charter schools to open in the city. It is a Title I school that educates about 400 students from grades Pre-K through eighth and it selfdescribes as a school that has strong relationships with families, a secure learning environment, and rigorous academic instruction to empower students for college entrance and success. The school district's school report card provides demographics for the 2018-2019 school year, including that students' racial demographics were 90\% Black, $6 \%$ Latinx, $1 \%$ Asian, $1 \%$ Multiracial, and 1\% White. Additionally, the report card reports that 10\% of students are English Language Learners (ELL) and 16\% of students receive Special Education services. Fifty-six percent of students are identified as "at-risk," meaning they are experiencing homelessness, are in foster care, are eligible for food stamps or welfare, or are one year or more older than the expected age for their grade level. HA is identified as a Tier-1 charter school, meaning based on a ranking system for the school district's charter schools, it is high performing on indicators including student progress, student achievement, and school environment. HA used virtual instruction due to the Covid pandemic for the 2020-2021 school year.

## Participants

The focal participants in this study are an African American eighth-grade mathematics teacher and the consenting students in her honors and on-level classes. These participants were observed, videotaped, and interviewed. To conduct this study, I received IRB approval, principal approval and written permission, and consent from all participants as well as the students' parents/guardians, to participate in the study.

Teacher Participant. The teacher participant is Ms. Collier. She is an African American woman who teaches 8th-grade mathematics at Hummingbird Academy. At the time of the study, she had 16 years of teaching experience, all within the city's public schools and charter schools,
and had been teaching at HA for the past six years. Throughout the course of her 16-year career, she taught in all three grade bands, elementary, middle, and high school, and served as the lead of the mathematics department. At her present school, there is an independent math coach whom she works with and will likely succeed as the next math coach within the coming few years. She has a bachelor's degree in mathematics with a minor in secondary education from a Historically Black College or University (HBCU). When describing her mathematics instruction, she highlighted using exploratory problems, giving all students an access point to her lessons, and making connections between content strands. Her instruction has been described by others as highly rigorous, ambitious, and reform-oriented.

Ms. Collier as Exemplary. In this dissertation study, I highlight Ms. Collier as an exemplary mathematics teacher. Here I explain why I identify her as an exemplary teacher we should learn from. To start, Ms. Collier has won awards recognizing her teaching over the course of her career including Teacher of the Quarter (twice) and the Fishman Prize Honor Roll, a national award recognizing the top 100 teachers in public schools with at least $40 \%$ of students eligible for free and reduced lunch. Beyond these awards, Ms. Collier's identity and background also signify her as a teacher worth learning from. She described herself as a Black woman who grew up in an urban environment on the south side of Chicago. She always attended predominately Black schools and was enrolled in gifted programs, attending magnet schools in the Chicago Public Schools district. Her experiences as a Black student in predominantly Black schools affected where she chose to teach. She explained:

I wanted to teach Black students. Very specifically, I wanted to teach people who look
like me. And I wanted to do that because I had the opportunity to have a mixture of
teachers. ... I've had the gamut of different cultures of people teaching me. And in each of those cultures, I've had an effective teacher.

After being a student who experienced instruction by diverse teachers, diverse teachers who were effective, Ms. Collier explained that this led her to want "to be a positive representation of a Black person teaching math." She elaborated that she wanted HA's predominately Black "population of students to see people that look like them educated and being able to educate them."

Ms. Collier's identity and background, as a Black teacher who experienced a publicschool education in a similar demographic context to the school in which she taught, shaped her rationale for wanting to teach predominantly Black students and her perspective that they deserve to be taught by effective Black teachers. This identity and perspective that Ms. Collier brings to her instruction points toward research perspectives (e.g., Brantlinger, 2018; Haberman 1995) that highlight that closeness to the community in which one teaches, particularly in terms of one's own lived experiences, makes for the best teachers in racially segregated urban schooling contexts. By virtue of her own experiences as a Black student in an urban community and how it had led her to prioritize her own students' experiences as Black mathematics learners, Ms. Collier offers an example of an effective mathematics teacher we could learn from.

Student Participants. In addition to Ms. Collier, the students in her class were also participants. There were 13 students total in the on-level class and 15 students total in the honors class, all of whom were invited to participate in this study. Of the 13 students in the on-level class, 10 students were African American, two students were mixed, and one student was Latinx. Of the 15 students in the honors class, four were African American, five were the children of African immigrants, three were Latinx, and three were mixed. The students were invited to
participate in this study that would involve videotaped classroom observations, during which I would take field notes, and the possibility of being invited for an interview or focus group, for which they would receive a gift card as compensation. They were given informed assent forms to sign and informed consent forms to have their parents sign and submit electronically. Nine of the students and their parents in the on-level class consented and 11 of the students and their parents in the honors class consented to participate. The consenting students, their class sections, and their racial background are reported in Table 1 below. In addition to observing all these students during the classroom lessons, I individually interviewed eight students total, four from each section, and had three focus groups with them. All the students I interviewed were African American; five were female and three were male. The students who did not consent to participate were not discussed in my observational field notes or analysis and did not participate in focus groups or interviews. Because the lessons occurred on Zoom, and Ms. Collier recorded all her lessons already as a classroom resource, these students did appear on the video recordings but were not included as part of my observational notes or analysis.

## Table 1

The Consenting Student Participants, their Class Section, and Race

| Participants in the On-Level Section |  |
| :--- | :--- |
| Student Name | Racial Background |
| Naomi | Black |
| Marcus | Black |
| Kobe | Black |
| Kanaan | Black |
| Nyla | Black |


| Participants in the Honors Section |  |
| :--- | :--- |
| Student Name | Racial Background |
| Emily | African Immigrant |
| Rebka | African Immigrant |
| Camille | African Immigrant |
| Sandra | Latinx |
| MiKayla | Black \& Latinx |


| Kianna | Black \& Latinx |
| :--- | :--- |
| Janae | Black |
| Jonathan | Latinx |
| Andrew | Black |


| Malachi | African Immigrant |
| :--- | :--- |
| Monique | Black |
| Diego | Latinx |
| Carlos | Latinx |
| Jordan | Black \& White |
| Dante | Black |

## Data Collection

Data collection for this case study included classroom observations, field notes, the written curriculum, semi-structured interviews with both teachers and students, focus groups with students, and audiovisual recordings of observations, interviews, and focus groups.

## Classroom Observations

The initial method for data collection was field notes collected as part of classroom observations. The primary focus for classroom observations was the teachers' enactment of mathematics tasks, students' implementation of mathematics tasks, and how the teacher showed high expectations and the historical practice of care with her students. Classroom observations for the study took place in February 2021 and lasted for the duration of one curricular unit, the Functions and Volume unit from the Illustrative Math (IM) curriculum.

Classroom observations occurred during the Covid-19 pandemic, so instruction occurred virtually through Zoom during this time. Because of this, I joined the Zoom lessons to conduct the observations. Observations took place on each day of the unit, which was three times per week in each classroom. The length of the observation was a full class period in each track, which lasted an hour and fifteen minutes. With appropriate permissions from IRB, school administrators, teachers, students, and parents, all observations were audio and video recorded
and I took fieldnotes. Particular videotaped/audiotaped episodes of classroom lessons were transcribed for data analysis, as described below.

In addition to observing the classroom lessons, I also had access to the teacher's guide version of the curriculum, the teacher's pacing guide that she developed with the department chair, and the tasks the teacher selected to use with students.

## The Written Curriculum

Ms. Collier taught from the Illustrative Math curriculum, as did the rest of the math teachers at HA. IM is a problem-based math curriculum that is designed to align with the common core process standards and content standards. The curriculum is designed to have students participate in high cognitive demand activities like "making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers... interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding" (Illustrative Mathematics, 2021). Therefore, given education research that highlights that Black students are often denied access to high-quality advanced curricula, Ms. Collier's use of the IM curriculum provided an opportunity to understand Black students' experiences engaging with a problem-based, reform-oriented curriculum. Observations for this study occurred for the duration of the Functions and Volume unit. The Functions and Volume unit involved introducing students to the concept of a function, that each allowable input has exactly one output, and understanding different representations of functions such as tables, equations, and graphs.

## Field Notes

During each class observation, I took ethnographic field notes (Madison, 2005). The field notes tracked the pacing of the lesson, the speaker, and the task in which the class is engaged. In addition to these logistical details of the observed lesson, I also used the field notes to attend to the historical practices of Black teachers that were present in Ms. Collier's work. Recall here that the historical work of Black teachers caring for their Black students involved a stern, structured learning environment which demanded that students learn. In this vein, the field notes on the classroom observations focused on: 1) Ms. Collier enacting a practice of care with her students; 2) efforts Ms. Collier took to enable students to engage with the problem-based IM curriculum; and 3) similarities and differences in instruction across the honors and on-level sections.

## Interviews

Both the teacher and focal students participated in interviews. There were a total of 12 interviews, five with Ms. Collier and seven with students. With the exception of Ms. Collier's first interview, all interviews began a few weeks after observations started to allow for a level of familiarity between the participants and myself, and to allow the interview questions to more naturally evolve after I had spent time with the participants, the context, and the classroom culture (Madison, 2005). All interviews were audio-recorded and I took notes during each interview on comments of interest or worthy of further questioning.

Teacher Interviews. Of the five interviews I conducted with Ms. Collier, two were very brief ( $\sim 15$ minutes) informational interviews to establish the scope of the study and to understand her classroom structure, teaching background, and student background. The remaining three interviews lasted between 45 minutes to an hour and were designed to better understand the presence of social forces and narratives behind her professional decision making. These forces
include socio-historical social forces about Black students as mathematics learners and the utility of mathematics learning in students' educational trajectories and institutional forces around tracking in the school. The interviews were semi-structured and topical to understand the teacher's lesson planning, instruction, instructional adaptations (if there were any), reflections on the classroom lesson, her perceptions of her students, and her perceptions of the utility of mathematics in her students' lives. This last point was about understanding what Ms. Collier saw as the purpose of school mathematics for her students, and how this perspective manifested in the way she taught, if at all. It helped shed light on broader social narratives that might have impacted how she enacted the curriculum, and in turn shaped students' opportunities to learn.

The teacher interviews shed light upon Ms. Collier's perception of the school, her students, her role as an instructor, and the rationale for their instructional decision making. Sample interview questions for the semi-structured teacher interview are included below, with the full interview protocol for a single interview included in Appendix A.

1. Tell me about your upbringing. Where did you grow up and go to school? What kind of community did you and your family belong to?
2. Can you tell me a bit about the classroom where you teach? What types of students do you teach? How would you characterize the students that you teach? (Do you teach culturally, racially, or linguistically diverse students?)
3. Why did you decide to teach at Hummingbird Academy? I believe throughout your career you've been in this school district, why did you choose this teaching environment?
4. How does the fact that the students and staff are predominately Black impact student learning?
a. How does it affect your instruction?
5. What do you think makes the difference between students who have strong math understandings and those who don't? What does someone need to be good at mathematics?
6. How do you think that being a diverse student (use the types of diversity that they've mentioned in their classroom) has an influence on a students' mathematics learning? Does it make a difference in the child's education? In what ways? Now or in the future?
7. When I observed you the other day, [brief description of event] occurred. Did you intend for this to happen? Can you help me understand your thoughts as it was occurring and as you responded?

Student Interviews and Focus Groups. I conducted semi-structured topical individual interviews with seven students. Each interview lasted 30 minutes and each student was interviewed individually once. In addition to the individual interviews, all seven of these students plus one additional student participated in a focus group. The two focus groups had four students each from the same classroom section and lasted 30 minutes. The additional student was supposed to be individually interviewed also, but was unable to make the interview. All of the students interviewed were Black.

Both the semi-structured interviews and the focus groups were designed to understand how the students made sense of their schooling experiences. In both kinds of interviews, I asked students about their perceptions of their learning environment, the instruction they received, and their interactions with others in their classroom. I also asked them about their rationale for how they participated in class and how they interacted with other students and the teacher. The
student interviews and focus groups helped shed light upon how the students made sense of their opportunities to learn mathematics as well as what resources they could access, outside of their observed classroom lessons, which enabled them to learn mathematics in the classroom.

Sample interview questions for the students are included below, with the full individual interview protocol included in Appendix B, and the focus group protocol in Appendix C:

1. How would you describe yourself as a student?
2. Who are the top students in your class?
3. What role does mathematics play in your life? How important is it to you to learn and succeed in math class?
4. Are you good at math?
5. What do you think makes the difference between students who are good at math and students who aren't? What does someone need to be good at mathematics?
6. How are your parents involved in your math education/learning?
7. Describe a typical day in Ms. Collier's class. How do you feel when you are in class? How do you feel about what you are learning in class?
8. Why did your parents choose to send you to this school? How do you feel about that?
9. Do you think your race, and how you see yourself, has anything to do with your math class?
10. In what ways do you think this school and the students are different or the same from students in other schools?

## Data Analysis

I used the qualitative analysis software NVivo to transcribe all of the interviews and focus group data, which were audio recorded. Additionally, selected segments from the observational classroom data were also transcribed. I paid for a transcription service for the classroom observation data, due to difficulty managing the video data on the qualitative software; issues arose when using the video data leading to the software frequently crashing. I selected which segments of classroom data to transcribe using the analysis methods detailed below. After receiving the observation transcriptions, I then reviewed the transcriptions while watching the observation videos to check for accuracy and to add multimodal information, such as gestures and tone, into the transcript.

The data coding process involved grouping together categories that existed in the data (Madison, 2005). This process involved a few different phases which I elaborate on below. In these phases, I coded my data deductively, according to the theoretical framework used to guide my study, and inductively, to capture the content of the data based on what I saw and heard during observations and interviews.

## Analyzing the Field Notes

My first pass at analyzing the data centered on the field notes. After taking the field notes during the observations, I then began coding the field while the observational data was still being collected. At this stage, I used my theoretical framework to deductively code the field notes I was collecting during each classroom observation. I used the theoretical framework of racialization in math education and the historical work of Black teachers to code my field notes for: 1) Ms. Collier's expectations about student success and engagement; 2) Ms. Collier coaching students to engage with the mathematics; 3) classroom community; 4) discipline practices; and 5)
similarities and differences between the honors and on-level class. After each observation, I coded my field notes for the presence of each of these categories. Then, after every two to three lesson observations, I wrote analytical memos.

In the analytical memos, I detailed patterns, trends, and emergent themes I noticed in the data, typical and atypical occurrences during the observations, connections between the data and preexisting research literature, and areas from the observations to follow up on during the teacher interviews either for clarification or to understand her experience of the event (Saldana, 2016). I also detailed my personal reactions to the observations, including interactions during the lessons that I found surprising or humorous, and moments I found tense and was grappling with how to treat in terms of analyzing and reporting for a broader audience.

I then used the field notes, initial codes, and analytical memos to select which segments from the classroom observations to transcribe. I describe this process after I discuss how I coded the tasks from each lesson, as these two bodies of data were considered in tandem to select segments of classroom observations for transcription.

## Coding Interviews

After conducting all the classroom observations and interviews, I then began to code the interview data. I started with the teacher interview data and began by coding it deductively, using the literature on the historical work of Black teachers as a framework. Using this framework, the deductive codes I used were about the teacher's identity and background, how the teacher cared for her students, how she maintained high expectations for her students, her personal motivations as a teacher, her knowledge of her students, her personal relationships with students, and constraints she experienced while teaching. I also inductively coded the teacher interviews based on what arose from our conversations. The inductive codes that emerged were about virtual
instruction due to Covid, emotions she experienced while teaching, and her perspectives about teaching the on-level and honors sections.

When analyzing the student interviews, my analysis primarily centered on how the students experienced Ms. Collier's class and their broader perceptions of mathematics. The codes that emerged from the student interviews involved their perceptions of the utility of math, participation in Ms. Collier's class, mathematics support outside of school, and Ms. Collier as a teacher. The full coding scheme, from the observations, teacher interviews, and student interviews are included in Appendix D.

## Coding Written Tasks

After I had completed all the classroom observations and began preliminary analysis using my field notes, I then transitioned to analyzing the written curriculum. I analyzed the cognitive demand of the written curriculum in order to understand what mathematics content knowledge the IM curriculum intended to impart to students. I understood this analysis to be important and relevant to understanding the work of a Black teacher with Black students because a central component of the historical work of Black teachers was equipping students to learn the socially-legitimated knowledge necessary for students' success post-schooling. In contemporary mathematics education research, cognitive demand can be representative of the conceptual mathematics knowledge students should learn in their K-12 (Jackson et. al., 2013) For this reason, I analyzed the cognitive demand of the written curriculum to develop an understanding of how the curriculum was designed, to better inform my understanding of how Ms. Collier was enabling students to take up mathematics domain knowledge, in the tradition of historical Black teachers.

I started this analysis by coding the tasks as they were written in the curriculum. Ms.
Collier shared her pacing guide with me, that she created with her math coach to organize what tasks were used on which days. In the pacing guide, Ms. Collier detailed the calendar date, the day in the unit, the lesson activities (pulled from the written curriculum), and student homework. Using this pacing guide as a starting point, I created a table which included the date, the lesson objectives (as outlined in the curriculum), the standards alignment (as outlined in the curriculum), and each task from the curriculum.

Given that the IM curriculum is intentionally designed to engage students with high cognitive demand mathematics, I coded the task potential of each task from Ms. Collier's pacing guide. I deductively coded the task potential using a coding scheme developed by Jackson et al. (2013), which leverages the work of Stein and Lane (1996) and Boston's (2012) Instructional Quality Assessment. Drawing from these bodies of work, Jackson et. al (2013) defined the task potential as the cognitive demand of the task as it appears in the curricular materials. I used the scoring framework in Table 2 below to code the potential of each task.

## Table 2

Task Potential Coding Scheme from Jackson et. al. (2013)

| Code | Definition from Study | Score | Description |
| :--- | :--- | :--- | :--- |
|  |  | 0 | Task is not mathematical in nature |
| Task <br> Potential | Cognitive demand of the <br> task as it appears in the <br> curricular materials | 2 | Low Cognitive Demand: students are to memorize or <br> reproduce facts |
|  |  | Low Cognitive Demand: students are asked to apply a <br> standard procedure to solve a relatively routine problem |  |

I used this coding scheme to code all the tasks that Ms. Collier planned to use during the Functions and Volume unit, which were 26 tasks in total. IM characterized these tasks either as warm ups or lesson activities. There was one other type of lesson activity in the curriculum which I did not code, and this was the lesson synthesis. The lesson synthesis typically took the form of a whole class discussion, so I reserved these parts of the lesson to be considered from the observational data, rather than the task potential of the written curriculum.

After I scored the 26 tasks according to the coding scheme in Table 2, I initially found the following: 1) three of the tasks were low cognitive demand, asking students to apply a standard procedure to a routine problem; 2) 12 of the tasks were high cognitive demand, asking students to engage in complex thinking; and 3) 11 of the tasks were high cognitive demand, asking students to solve a nonroutine problem and provide evidence of their reasoning. After conducting this analysis, I was a bit unsure of how I scored some of the items, with my concerns typically involving distinguishing between a score of " 3 " and "4." Primarily, I wondered how explicitly the task needed to ask for student justification in order for it to earn a four, or whether prompts which implicitly may invite student reasoning would also qualify as a four.

To build confidence in my scoring, I asked one of my dissertation co-chairs to independently code the eight items I was uncertain about. After he coded his eight, I found that we agreed on five of the items and disagreed on three. We discussed the reasons for our disagreements, which primarily centered on my literal interpretation of the coding scheme, whereby any non-routine math task which asked for justification qualified as a four, whereas he expected items to be more complex in quality, perhaps requiring an open middle, to earn a score of four. We ultimately determined since I was using the coding scheme, using the literal
explanations was sufficient, and all items that were nonroutine and asked for student explanation earned a four. After undergoing this process, the final breakdown of the 26 tasks from the curriculum was: two tasks were low cognitive demand and earned a " 2 "; 13 tasks were high cognitive demand and earned a " 3 "; and 11 tasks were high cognitive demand and asked for student explanation and earned a "4." The breakdown of the task potential in each lesson is shown in Table 3 below. There is an extended version of Table 3, included in Appendix E, which includes a brief summary of each task. After coding each of these tasks, my next step in the analysis was to determine which tasks I should select to focus my analysis during the classroom observations.

## Table 3

Task Potential of Each Task from Ms. Collier's Unit 5 Pacing Guide

| Week 1 | Lesson 0 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson Activities | Previous Unit Assessment Review Day |  |  |  |  |  |
| Week 2 | Lesson 1 |  | Lesson 2 |  | Lesson 3 |  |
| Lesson Activities | Warm Up | $\begin{array}{\|l\|l} \hline \text { Task } \\ \text { Potential: } 4 . \\ \hline \end{array}$ | Warm Up | Task Potential: 4. | Warm-Up | Task <br> Potential: 3 |
|  | Activity 2 | Task <br> Potential: 3. | Activity 2 | Task Potential: 4. | Activity 2 | Task <br> Potential: 2 |
|  | $\begin{array}{\|l} \text { Activity } 3 \\ \hline \text { Lesson } \\ \text { Synthesis } \\ \hline \end{array}$ | Task <br> Potential: 2. | Activity 3 | Task Potential: 3. | Activity 3 <br> Lesson <br> Synthesis | Task <br> Potential: 4. |
|  |  | Whole class discussion | Activity 4 | Task Potential: 4. |  | Whole Class Discussion |
|  |  |  | Lesson Synthesis | Whole class discussion |  |  |
| Week 3 | Lesson 4 |  | Lesson 5 |  | Lesson 6 |  |
| Lesson Activities | Warm Up | Task <br> Potential: 3. | Warm Up | Task Potential: 3. | Warm-Up | $\begin{array}{\|l\|} \hline \text { Task } \\ \text { Potential: } 3 . \\ \hline \end{array}$ |
|  | Activity 2 | Task <br> Potential: 3. | Activity 2 | Task Potential: 3. | Activity 2 | Task <br> Potential: 3. |
|  |  | Task <br> Potential: 4. | Activity 3 | Task Potential: 3. | Activity 3 | Task <br> Potential: 3. |
|  | Activity 3 |  |  |  | Activity 4 | Cut from the planning guide |
|  | $\begin{array}{\|l\|} \hline \text { Lesson } \\ \text { Synthesis } \end{array}$ | Whole class discussion | $\begin{array}{\|l\|} \hline \text { Lesson } \\ \text { Synthesis } \\ \hline \end{array}$ | Follow up questions about graph from final task | Lesson Synthesis | Whole Class Discussion |
| Week 4 | Lesson 7 |  | Lesson 8 |  | Lesson 9 |  |
| Lesson Activities | Warm Up | Task <br> Potential: 4. | Warm Up | Task Potential: 4. | Unit Assessment Review Day |  |
|  | Activity 2 | Task <br> Potential: 3. | Activity 2 | Task Potential: 4. |  |  |
|  | Activity 3 | Task <br> Potential: 4. | Activity 3 | Task Potential: 3. |  |  |
|  | Activity 4 | Task <br> Potential: 4. | Lesson Synthesis | Whole class discussion |  |  |
|  | Lesson Synthesis | Whole Class Discussion |  |  |  |  |

## Selecting Focal Episodes from Classroom Observations

My next phase of analysis involved selecting which segments from the observations to transcribe for closer analysis. In order to select these segments, I considered the potential of the task during that part of the lesson, which scored the cognitive demand of the written curriculum, in tandem with my field notes and analytical memos from the live observations, which considered how racialization may manifest in this classroom from a lens of student engagement and success, student uptake of mathematics content, discipline practices, and differences between the honors and on-level classes. I used these two sources of analysis to select which segments of the observational data I wanted to analyze more closely. For my purposes, this meant watching the recordings of the observational data, transcribing this data, and then more closely analyzing how Ms. Collier's practice was rooted in the legacies of Black teachers and what her instruction meant in terms of her students' positioning as math learners. My process for selecting which segments of data to transcribe and further analyze is described below.

First, I decided to further analyze Lesson 2 in its entirety. I decided to focus on Lesson 2 because three of the four tasks in the written curriculum for the lesson were scored a " 4 " for task potential, meaning they presented nonroutine math problems and elicited student explanations. The remaining task was also high cognitive demand, but it was scored a " 3 " because it did not explicitly invite student explanation. It was important for me to capture an entire lesson with the highest level of cognitive demand tasks because it provided an opportunity for me to understand how Ms. Collier enabled her students to engage with ambitious, socially legitimated mathematics content. This is important in terms of situating Ms. Collier's practice in the legacy of Black teachers because part of their work was to equip their students with the highest standard of knowledge available. There was only one other lesson (Lesson 9) which had an equal number of
tasks with the highest cognitive demand potential. Ultimately, I chose to focus on Lesson Two rather than Lesson Nine because it came at the beginning of the unit, and so offered a nice backdrop to compare the students' development over the course of the unit against. Therefore, I transcribed Lesson Two in its entirety, in both the honors and on-level sections, to focus on for further analysis.

After selecting Lesson Two to transcribe and analyze in its entirety, I then began to select segments which I noted in my field notes had evidence of racialization in the math classroom, particularly from a lens of how the teacher's instruction related to the historical work of Black teachers. In my field notes, this primarily manifested through Ms. Collier building deep relationships with students, showing high expectations for students, and caring for students, both by emotionally supporting them and by sternly demanding their effort and engagement. Using this process, I selected a total of 10 segments from the classroom observations to transcribe, with each vignette including either the teacher's facilitation of a task in its entirety or a portion of a task which captured the entire exchange between the teacher and a student around their problemsolving strategy (Turner et. al., 2012). I selected these episodes because they highlighted the teacher's caring for her students in a way that connected to the literature of the historical work of Black teachers educating Black students, in the ways described above.

Of these 10 selected segments, six were from the on-level math class and four were from the honors math class. Of these six episodes from the on-level math class, based on the written curriculum, one episode featured a task with low cognitive demand task potential (it asked students to name the digit in a particular place value), while the remaining five episodes featured tasks with high cognitive demand potential. I chose the remaining four episodes from the honors math class to align with the on-level math class. That is, for each episode I transcribed from the
on-level class, I transcribed the analogous episode featuring the same task from in the honors class so that I could make comparisons about the classroom instruction across both sections. As a result, of the four episodes I selected to transcribe from the honors class, one featured a task with low cognitive demand potential and the remaining three episodes featured tasks with high cognitive demand potential.

There were two episodes from the on-level class in which I did not transcribe an analogous episode from the honors class. This happened because I was focusing exclusively on an interaction with a particular student, Andrew, in which Ms. Collier showed particularly high expectations while he was developing in learning a topic. There was no such interaction I was comparing him against in the honors class.

Table 4 below shows the distribution of analyzed observational segments across the unit in the on-level and honors classes. For each week that I observed the classroom, I closely analyzed at least one episode from the lessons. As mentioned above, episodes 1 and 2 below occurred only during the on-level sections, but for each other episode, 3 through10, there were analogous episodes that centered on the same task in the honors and on-level sections.

## Table 4

Distribution of Episodes Across the Unit which Evidenced Ms. Collier's Tapping into the Legacy of Black Teachers

| Episode | Class <br> Section | Task Potential | Lesson (\# and Title) | Week of <br> Observation |
| :--- | :--- | :--- | :--- | :--- |
| 1 | On Level | High Cognitive <br> Demand | Lesson 0: Previous Unit Review Day | Week 1 |
| 2 | On Level | High Cognitive <br> Demand | Lesson 1: Inputs and Outputs | Week 1 |
| 3 | On Level | Low Cognitive <br> Demand | Lesson 2: Introduction to Functions | Week 2 |
| 4 | On Level | High Cognitive <br> Demand | Lesson 5: More Graphs of Functions | Week 3 |


| 5 | On Level | High Cognitive <br> Demand | Lesson 7: Connecting Representations <br> of Functions | Week 4 |
| :--- | :--- | :--- | :--- | :--- |
| 6 | On Level | High Cognitive <br> Demand | Lesson 8: Linear Functions | Week 4 |
| 7 | Honors | Low Cognitive <br> Demand | Lesson 1: Inputs and Outputs | Week 2 |
| 8 | Honors | High Cognitive <br> Demand | Lesson 5: More Graphs of Functions | Week 3 |
| 9 | Honors | High Cognitive <br> Demand | Lesson 7: Connecting Representations <br> of Functions | Week 4 |
| 10 | Honors | High Cognitive <br> Demand | Lesson 8: Linear Functions | Week 4 |

## Analysis of Focal Segments to Create Vignettes

After selecting the 10 observational segments to transcribe in addition to the complete transcriptions of Lesson Two in the on-level and honors classes, I then analyzed each transcript to determine which observations to focus on to create the vignettes to represent the data. In conducting this analysis, my purpose was to determine which segments best represented Ms. Collier's tapping into the work of Black teachers. To determine this, I wrote analytic memos which broadly summarized what occurred during the observed segment and in what instances Ms. Collier's work seems to have historical roots. These instances typically involved Ms. Collier supporting students to overcome difficulty during the learning process and experience success; Ms. Collier helping students navigate their emotions; Ms. Collier sternly demanding students' participation; Ms. Collier warmly building relationships with students; and Ms. Collier embracing Black culture during instruction.

After writing analytic memos about each segment, I was able to characterize what facets of Ms. Collier's instructions were rooted in a historical legacy. With this understanding, I then selected which segments from the transcribed episodes to develop into full vignettes to represent the data. In selecting these segments to highlight as vignettes, my purpose was twofold: to select segments that were fairly representative of the entire unit and to select segments that highlighted
the nuanced and multifaceted nature of how Ms. Collier cares for her students. That is, I could have chosen different segments to focus on for the vignettes and the details would have been different, but the overall results in terms of Ms. Collier's care for her students and its historical roots would have been the same. In selecting which segments to turn into vignettes and how many vignettes to create, my intention was also to highlight the breadth of ways Ms. Collier showed her care for her students to be sure it was all represented in the results.

My final analysis focused on how the students were positioned as mathematics learners. In order to conduct this analysis, I drew from the interview data to understand what storylines about mathematics Ms. Collier perceived may affect teaching and learning in her classroom. Using her perception as a starting point, I then used positioning theory to analyze how these storylines may have presented themselves in her classroom and how her students were positioned as a result. In conducting this analysis, I worked to consider what rights were available to her students, what duties were expected of them, and how these rights and obligations related to racialized narratives about Black mathematics learners. I present the results of this analysis in Chapter Five.

## Presenting Data: Organizing Data to Present Results

To answer the research questions, I use vignettes (Erickson, 1986) to render rich, thick descriptions (Emerson et. al., 2011) of the classroom observations. My intention here is to represent, as fully as possible, what classroom instruction looked, sounded, and felt like during the Functions and Volume unit that I observed. While rendering the vignettes of the unit to fully display what occurred during the observations, my purpose is also, explicitly, to situate Ms. Collier's instruction in the historical work of Black teachers and Black students. Thus, in creating the vignettes, I also draw connections between what was seen and heard during the
observations with what has historically been described as Black teachers' practice of care. In order to represent what was felt by those in the classroom, I use interview data alongside the observational data, to capture the teacher's motivations behind her instructional decision making and students' perceptions of their classroom experiences. I also discuss how the students are positioned through Ms. Collier's instruction, and what storylines exist about the students in Ms. Collier's mathematics class. I also compare and contrast the positions available to the students in the on-level and honors sections.

Given that I observed two different class periods, I showcase the vignettes from each section, to compare instruction across the two classes. However, in instances when the content of the observation is very similar across both sections, during the start of class, for example, I mention these similarities during the vignette, and highlight the episode from just one section, because it is representative of both sections.

## Chapter 4: Ms. Collier's Practice and the Historical Work of Black Teachers

In this first results chapter, I answer how Ms. Collier's teaching practice and work with her Black students relate to historically situated conceptions of Black educators' teaching practice and work with Black students. I show how Ms. Collier's care for her students is related to the historical work of Black teachers whose care for their students was rooted in resistance. Historically, Black teachers cared for their students by fighting against legally sanctioned racism and segregation which limited their community's and their students' life opportunities. Although anti-black racism has evolved, it still persists and Ms. Collier resisted deficit characterizations of her Black students and constraining societal circumstances which limit their life opportunities. Specifically in mathematics education, a content which attempts to "seamlessly ... undervalue and misrepresent Black children's intellectual capacities" (Martin et. al., 2019, p. 43) to deny their personhood, Ms. Collier resisted these dominant narratives by caring for her students and prioritizing their humanity as they learned mathematics. This care manifested in a multifaceted and nuanced way, depending on the situation Ms. Collier and the students were engaged in. Ms. Collier's care typically manifested in a mathematics teaching practice which: 1) celebrated the shared racial background between her and her Black students; 2) created a safe classroom culture which enabled students to engage in the vulnerable process of learning, doing, and talking about mathematics, and 3) maintained high teaching expectations that all students master the content. In this vein, I highlight episodes from across the Functions and Volume unit in Ms. Collier's honors and on-level eighth-grade math class that provide evidence of Ms. Collier's caring for her Black students in the three ways outlined above. Although the primary purpose of this work is to understand how Ms. Collier's practice as a Black mathematics teacher is rooted in the legacy of historical Black teachers, Ms. Collier also exemplified teaching practices that are consistent with
contemporary research on equity and justice in mathematics education. Therefore, as relevant, I also highlight when she embodied contemporary understandings of equitable mathematics instruction.

## Vignette One: Ms. Collier Welcomes Students to Class

The first vignette, the beginning of Lesson Two, portrays how Ms. Collier cultivated a safe classroom culture for students. As will be shown throughout the remaining vignettes in this chapter, Ms. Collier made considerable demands of her students, demands including publicly sharing their developing, unrefined understanding and publicly persisting in problem solving even though they felt nervous or did not know the answer. For Ms. Collier to make these demands of her students and have them fulfill them, she created a safe, communal classroom environment which allowed students to be willing to participate in the potentially vulnerable work of learning mathematics. Part of how Ms. Collier developed this safe learning environment was how she welcomed students to class. Ms. Collier started class by fostering positive relationships with students. In developing these positive relationships, Ms. Collier started each lesson by greeting students, asking them about their lives outside of the classroom, and preparing them to dive into the content of the lesson. As she did this, she also celebrated the shared racial background between herself and her Black students by speaking in Black English vernacular. The vignette below demonstrates the way Ms. Collier welcomed students to class and expressed care for her them by showing she valued them as people, not just as math learners.

## The Start of Second Period: On-Level Math

This vignette represents the start of class during Lesson 2: Introduction to Functions. For this classroom observation, I logged into Ms. Collier's personal meeting room on Zoom at 12:20, the class's scheduled start time. I was able to observe the students trickle in over the next five
minutes. Upon my arrival, Ms. Collier greeted me warmly. She looked to be in her early- to mid30s wearing a black t-shirt with a white graphic on the front, only partially visible in the camera's line of sight. She wore aviator glasses with a bold silver frame as well as a black and gray headset with an attached microphone.

Ms. Collier logged into the Zoom class sessions from her home, where there is a brown dresser visible behind her, with about a dozen picture frames clustered on top. She sat facing the camera, with a whiteboard easel behind her that had a pad of chart paper leaning against it. To her right, there was a sheet of chart paper posted on the wall that was filled with notes in different colored ink; it was only slightly visible on the camera and unable to be read.

The first student to join the class was Andrew, who had his name on the screen written as Andrew, he/him. Ms. Collier greeted him by saying "Hey Andrew" which he responded to with a two-handed wave in the camera. He then gave a peace sign with each hand and smiled, all while his audio remains muted. She smiled, telling him, "I like the pronouns in the name, let me add mine to show my support" after which she changed her name on the video call to read $M s$. Collier she/her/hers telling him, "Boom, there you go. That's for you, Andy. That's for you. Those are my preferred pronouns. She, her, hers." As students continued to join class, Ms. Collier greeted each student by name. Once three students had arrived, she prompted them to make sure they had their notebooks ready for class. As students continued arriving, she complimented a few of them, for example telling Naomi she liked her hair, which was styled in bright blue shoulder length crochet braids.

When Janae arrived, Ms. Collier greeted her saying, "Hey Janae. Janae, how was the birthday, pretty good? Did you get some good stuff?" Another student, Nyla, wished Janae a happy belated birthday in the chat as Ms. Collier announced to the class:

It was Janae's birthday yesterday. Shout out to all the February babies, we are here! We are large and in charge. If you were born in the month of February, you are the greatest of the greatest. We were meant to be great, that's why they gave us the least amount of days, ya dig?

As she said this, a few students in the class smiled and Janae did a few fist pumps in the camera. The students knew Ms. Collier is also a February birthday because the week prior she shared at the start of class that she would be celebrating her own birthday by traveling to Tampa Bay to watch the Super Bowl. Just as she was joking with the class about the excellence of February birthdays, she joked with the class also about how the Tampa Bay Buccaneers are the greatest of all football teams. After proclaiming how "great" those with February birthdays were, she tempered her pride by conceding, "but all the other months are good too, all the other months are good too."

This back and forth with her students was typical of how Ms. Collier started class. That is, during the first three to five minutes of every class she waited patiently for students to arrive, and greeted each by name once they logged in. She typically asked how they were doing and reminded them about materials they would need to use in the current class session. She also often used this time to joke and celebrate with the students, as she did with Janae and the February birthdays. Even if there was no birthday to celebrate, Ms. Collier still joked about more mundane events, such as weekly reminders to students about the joy of "Friday Eve," or Friday itself, "the best day of the week."

Once nine students had joined (Nyla, Janae, Marcus, Andrew, Kobe, Kianna, Naomi, Kanaan, and Jonathan) she reminded them they would need their notebooks, asking them to confirm they had them. The students either held their notebooks up or left their camera's field of
view, presumably to retrieve their notebooks. Jonathan, the only student with his camera off, appeared to privately message Ms. Collier about having his notebook, leading her to respond aloud "thanks Jonathan for the confirmation." Each student got an individual reminder, by name, to make sure they had their notebooks open and ready. Three minutes into the class period, Ms. Collier oriented students toward the instructional goal of the class, told them the day's learning objective, and then launched into the warm-up.

Before analyzing how Ms. Collier started class in the on-level section, I briefly highlight how Ms. Collier started class in the honors section. She started class in both sections in much the same way but it is important to still highlight both sections because research on tracking has shown that not only is the mathematics taught across the tracks different, but also the teacherstudent interactions and level of rapport vary across the tracks (Oakes, 2005). Therefore, I transition briefly to convey how Ms. Collier welcomed students to the honors class to demonstrate that Ms. Collier used this time to foster positive relationships with students in both tracked classes.

## The Start of Third Period: Honors Math

Much like in the on-level class, Ms. Collier started the honors math class by greeting the students as they trickled in. After she asked the students how they were doing, Emily, one of the students, responded "Good, and you?" Ms. Collier told her "I'm all right. Can't complain. Can't complain. Chilling, chilling, like a villain, villain." Meanwhile, another student, Jordan, was seen eating an apple in what appeared to be his bedroom. There was a corner bookshelf visible behind him that had trinkets and action figures displayed and six anime posters were hanging on the walls. As he ate, Ms. Collier asked him "Is that apple good, bro? Is it good?" to which Jordan
smiled and nodded his head affirmatively. Ms. Collier responded "Okay. You over there savoring that joint," causing Jordan to smile again.

Just as in second period, Ms. Collier started third period by giving students three to five minutes to arrive, greeting and joking with them. After nearly five minutes, Ms. Collier transitioned them to the start of the lesson by saying "Today we're going to talk about what a function is and how we use function language to explain or to say that something is or is not a function."

## Starting Class by Cultivating a Safe Environment for Students

Part of how Ms. Collier cared for her students was through her cultivation of a positive, communal learning environment. When she welcomed students to the virtual classroom, she dedicated the first few minutes of class entirely to fostering relationships with her students. In the episode highlighted above, Ms. Collier affirmed Andrew's public naming of his gender identity and named her own in solidarity. Ms. Collier also checked in on Janae's birthday, asking her how it was, what she received, and highlighting their shared birthday month of February, eliciting a fist pump from Janae. In the honors math class, she commented on the mundane act of Jordan's eating an apple, which still caused him to smile.

Ms. Collier purposefully had personal conversations with students, conversations that were not directly centered on mathematics content. She explained that building relationships with students that extends beyond their mathematics is about making them feel important and cared for:

All of my relationship building is never forced, it just kind of happens right? It always sort of goes back to, I make an effort to get to know my students. I don't go you know 'oh you're here, I know your name, that's all I need to know and whether or not you
understand math.' No, I want to know who they are because I want to be able to make them all feel important.

Although purposeful, Ms. Collier called her relationship building with students "organic;" that is, a lot less structured than the remainder of the mathematics lesson. For Ms. Collier, the start of class was not dedicated exclusively to checking who is present and accounted for, recording attendance, and then diving immediately into the curriculum. Rather, the start of class was dedicated to welcoming students into the learning environment by affirming that their identities extend beyond the classroom and the content. She showed that she cared about them by asking about their lives outside of the classroom.

Ms. Collier elaborated on how these conversations show the students that she cared about them. Explaining her intentions when communicating with students she shared:

I want you to know I care about you. So, I say things that show that I care about you. It's deeper than just, oh you know how to do a math problem. No, I listened to you when you shared with me who your favorite artist was, I listened. So, I might throw that in there on a Tuesday, just to show you that I remember something about you. So that you feel important today. They all deserve to feel important -- they all deserve to feel seen and heard. So, I want to provide that for them.

Ms. Collier did not perceive that her work was only to ensure that students "know how to do a math problem." In her practice as a math teacher, she perceived that her work entailed caring for students by listening to and remembering their personal interests to involve them in the classroom, just as she did with Janae's birthday. When she shared about Janae's birthday with the class, this led other students to wish Janae a happy birthday and caused Janae to smile and fist
pump at the celebration. In Ms. Collier's own words, this communicated to students that she cared about them and that they were important.

For the students to know that Ms. Collier cared about them as whole children who extend beyond her math classroom helped cultivate a safe classroom climate for students so that they could engage in the potentially vulnerable work of learning math content. Expecting students to engage in the learning process requires them to share their developing, unrefined understandings with their peers in small group conversations, with their teachers in one-on-one conversations, and sometimes in front of the entire room in whole class discussions. This is a considerable ask for many students. For some students who are early in their process of learning the content, it may be risky for them to share their developing, unrefined understandings in public. Of course, this is not the case with all students, such as those whose ways of knowing and communicating are already aligned with the mathematics curriculum or for those who are already proficient in the content and share from a position of proficiency rather than developing knowledge. But for those students whose understanding of the content is still developing, asking them to share their thoughts with others in the classroom is asking quite a bit. As I highlight in subsequent episodes from throughout this unit, Ms. Collier consistently asked this of her students. For Ms. Collier to request this type of engagement from her students, and for students to be willing to fulfill this demand, cultivating a classroom environment in which students felt safe sharing their vulnerability in the learning process was required. One important way that Ms. Collier did this was by showing them she cared about their interests and lives outside of her classroom.

However, beyond caring for students in service of mathematics learning, part of Ms. Collier's practice was recognizing that all students, in her words, "deserve to feel important -they all deserve to feel seen and heard." This seems to extend beyond care in service of
mathematics education to care in service of the humanity of her students. Ms. Collier's priority was affirming the humanity of her students by showing them that she saw and heard them, simply because they deserved to be seen and heard - not to help them solve math problems, not to raise their test scores, but because they deserved to be cared for.

This is in the tradition of the pre-Brown teachers who were noble and uplifting figures in their students' educational and social development. Ms. Collier's practice resists the antiblack norms in mathematics education which dehumanize Black students by rejecting their culture, their language, their intelligence, and their lived experience. In fact, as Ms. Collier engaged with the students about their lives outside of math class, she did so while using Black English vernacular ${ }^{4}$. Black mathematics students are often expected to abandon their community-based ways of knowing and being as mathematics learning environments often privilege White, middle-class ways of communicating. Expecting Black students to abandon their home ways of communicating is an act of violence against Black students that diminishes not only their way of speaking but also their sense of self and personhood. That Ms. Collier not only accepted Black English vernacular as a legitimate way of communicating in her math classroom, but also used it herself because it is her own home-based way of speaking represents a significant resistance to the linguistic violence committed against Black students. It also represents a significant affirmation of her students' personhood by socially legitimating Black English vernacular as a valuable way of communicating. Ms. Collier's status as a mathematics teacher, as a person who socializes students into the field of mathematics, is significant in reifying Black English vernacular as a language acceptable for doers of mathematics.

[^3]In the language of contemporary mathematics education research, how Ms. Collier welcomed students to class embodies rehumanizing mathematics teaching. That is, rather than asking students to leave their identities at the door, which can be experienced as dehumanizing by students (Goffney, 2018; Gutiérrez, 2018), Ms. Collier invited and celebrated students' personal identities in the mathematics classroom by making space for their personal lives, interests, and home-based ways of communicating. In doing so, Ms. Collier disrupts the connection between mathematics and domination and compliance, and instead develops deeper connections with community, joy, and belonging (Gutiérrez, 2018). As humans, one of our basic human needs is belonging (Goffney, 2018) and by dedicating the beginning of each lesson to building relationships with and between students, she demonstrated that they belong in this learning environment, enacting a rehumanizing pedagogy.

In drawing from a rehumanizing mathematics perspective, worth recognizing here is that part of what Gutiérrez (2018) calls for is a reconstruction of the mathematics curriculum. She calls for a rehumanizing mathematics curriculum that reflects students' culture and history by, for example, acknowledging ethnomathematics and broadening beyond the general scope of the K12 mathematics curriculum. However, Ms. Collier teaches very closely to the standards-based Illustrative Mathematics curriculum, leaving little room for ethnomathematics in her class. This complicates how Ms. Collier takes up rehumanizing teaching practices, as shown through her taking up the practice of windows and mirrors. Gutiérrez (2018) describes learning mathematics as a window, which allows students to see a new way of viewing the world, and as a mirror, which allows students to come to see themselves in the curriculum. In terms of being a mirror, Ms. Collier herself is a Black woman who grew up in a similar school district to her students, also speaks using Black English vernacular, as that was her own home-based way of
communicating, and is now an educator teaching her students mathematics. These facets of Ms. Collier's identity provide a powerful representation for students so that they may also see themselves as people capable of mathematics success. However, Ms. Collier does not teach from a curriculum that uses these facets of Ms. Collier's and her students' shared identity in service of mathematics instruction, instead, these are parts of her own identity that show up while she teaches. Therefore, it may not be that Ms. Collier takes up the practice of windows and mirrors to the full extent of the rehumanizing mathematics theory. However, highlighting the connections between the theory and Ms. Collier's pedagogy is still important to understand how the theory manifests in practice, given teachers who care deeply for their students, and their students' success, but simultaneously face constraints, for example, by teaching from standards-based curricula established in their schools.

## Vignette Two: Ms. Collier Maintains High Expectations

This second vignette occurred in the on-level math class during the first lesson that I observed, which was a review day for the previous unit Systems of Linear Equations. The vignette centers around Ms. Collier facilitating a whole class discussion during which the class is debriefing Task 1, a task about simplifying a hanger equation used to model the solutions of simple equations. The purpose of sharing this episode is to consider how Ms. Collier's facilitation of the discussion enabled Andrew, the focal student during the discussion, to publicly succeed with mathematics. Specifically, I consider how Ms. Collier maintained high expectations that Andrew was capable of solving the task and cultivated an environment in which Andrew felt supported to persevere and succeed in solving the task. Her instructional style described below, of working with one individual student during the whole class discussion until that individual successfully completed the problem, is typical of how she facilitated whole class discussions.

How this positions the focal student as a learner of mathematics will be discussed in the next chapter.

## Setting up the Task

The class was working on Task 1 that states: "A diamond has a mass of 1.5 grams and a circle has a mass of 3 grams. What is the mass of a square?" Next to the prompt there was a visual of a balanced hanger, which has six squares and one diamond hanging on the left side, and four squares, two diamonds, and one circle hanging on the right side, see Figure 1 below for the task. So far in the lesson, Ms. Collier had instructed students to copy the diagram and identify the key information. She set a timer [of five minutes?] for them to complete this task. As students were working, Ms. Collier checked which students were present, asking "who did we lose," referring to students dropping off the Zoom call, and monitoring the arrival of new students.

Andrew indicated he was done to which Ms. Collier responded, "All right, Andrew beat the timer. You got the key information?" Andrew shook his head no, and Ms. Collier responded, "Oh you just drew your diagram, ah ah ahh ahhh, you fought half the battle. Not bad. You still beat the timer on what I asked you to do first, so that's great."

They then debriefed as a class by identifying the following as key information: a diamond has a mass of 1.5 grams; a circle has a mass of 3 grams; and they want to know the mass of a square. Ms. Collier's notes after this discussion are represented in Figure 1 below. After establishing this key information with the class, Ms. Collier instructed the students to simplify the hanger. She gave them independent work time to remove as many things as they could from the hanger while keeping it balanced.

Ms. Collier then facilitated a whole class discussion to debrief how students simplified the hanger equation. One student, Naomi, had identified that one diamond could be removed
from the right side and the left side of the hanger and it would still be balanced. Ms. Collier then asked the class if any squares could be removed from each side of the hanger. Andrew nodded yes and elaborated that two squares can be removed from each side of the hanger. The resulting conversation between Andrew and Ms. Collier is transcribed below. In it, Andrew persevered in explaining and refining his reasoning, publicly in front of the whole class during the discussion, and Ms. Collier supported Andrew in refining his understanding until he was able to provide a mathematical justification for his answer. I consider how Ms. Collier cultivated an environment in which Andrew felt safe to engage in this vulnerable work of learning and supported him to reach a mathematically justified solution.

## Figure 1

Task 1 Used During Vignette Two with Notes from the Whole Class Discussion

1) A diamond has a mass of 1.5 grams and a circle has a mass of 3 grams. What is the mass of a square?

$$
\begin{aligned}
& \text { - diamond: mass }=1.5 \mathrm{~g} \\
& \text { - circle }: \text { mass }-3 \mathrm{~g} \\
& \text { mass of square : } x
\end{aligned}
$$



## Andrew Perseveres and Succeeds in Solving the Problem

The transcription below occurred during the whole class discussion right after Andrew had identified that two squares could be removed from each side of the hanger and it would still be balanced. Ms. Collier had crossed off the two squares on each side, and the diamonds Naomi identified earlier, as shown in Figure 1 above. The conversation between Andrew and Ms. Collier proceeded as follows. In the transcription, I used italics to indicate non-verbal processes that occurred to add meaning to the interaction.

## Table 5

Transcription Andrew \& Ms. Collier During Whole Class Discussion
$1 \quad$ Ms. Collier $\quad$ Can I take any more?
[13 second pause]
Just tell me what you're thinking?
3
4

5
6
7
8
9 Andrew Yes.
10 Ms. Collier Could I take two more?
11 Andrew No, actually wait. Yes. what?!
12 from both sides still?

Andrew Yes.

Andrew No side?
Andrew Three.

Ms. Collier Okay. Andrew, calm down, Andrew? Andrew? Can you relax for me please? Andrew has buried his head in his hands and repeatedly dips his head up and down. I'm not sure what's happening, but if you could just take one deep breath, just one five second deep breath for me real quick. Can you do that? Let it out. I want you to look at me and I want you to ignore all that noise behind you because I can hear that too. That's probably distracting. (A TV can be heard in Andrew's background whenever he unmutes). I want you to get out of your own head. Okay? You with me? All right. Now, listen to me. Listen to the sound of my voice, sir. For every one I have on the left side, if I have one on the right side, I can take it. Okay? But I cannot take more than I have on both sides. Okay? So, I want you to look, do I have enough to take one square

Ms. Collier Okay, cool. So, let's do that. [She crosses out one red square from each side]. Do I have enough to take another square from both sides?

Ms. Collier Okay. So, let's look. Is there... How many squares are still on the left

32

Ms. Collier And how many squares on the right side?
Andrew One.
Ms. Collier So, could I take one from both sides?
Andrew Oh
Ms. Collier Do they both have one that I could take?
Andrew Oh, yes.
Ms. Collier So, then I can take another one?
Andrew Yes.
Ms. Collier Yes. Can I take another square from both sides?
Andrew No.
Ms. Collier Why not?
Andrew Because there's no more squares on the right.
Ms. Collier Beautiful. Now, see there. Now looky there. Look, give yourself a pat on the back. Since I can't give it to you myself, give yourself one for me. That was beautiful, sir. You made it through. All right?

Andrew Andrew smiles and pats himself on the back.

To situate the interactions depicted in the transcript above in the historic work of Black teachers, I consider it from a lens of how Ms. Collier was caring for Andrew by expecting him to persevere and succeed in simplifying the hanger. Understanding how Ms. Collier could expect Andrew's success requires recognizing how she cultivated an environment in which Andrew could publicly struggle through the problem in the midst of the whole class discussion. That Ms. Collier cultivated an environment that safely permitted her students to publicly struggle, alongside her expectation that they were capable of solving problems and providing justification
for their thinking, represents one part of how Ms. Collier cared for her students by maintaining high expectations for them and providing appropriate supports to enable their success.

To begin this analysis, I highlight evidence that makes it clear that Andrew was engaged in a mathematical struggle. First, in Lines 1-3 after Ms. Collier initially asked if any more squares could be removed, she waited 13 seconds for a response from Andrew. When Andrew was unable to respond after this fairly long wait time, she continued to probe, encouraging him to "just tell me what you're thinking." Then, after Ms. Collier scaffolded the line of reasoning Andrew should apply to simplify the hanger (Lines 5-10), Andrew quickly changed his answer from "no," to "yes," to exclaiming "what?!" His quickly changing responses that culminated in his outright question of "what?!" makes clear that Andrew was unsure of how to proceed in solving the problem. His nonverbal gestures, like burying his head in his hands and dipping his head down, communicate that he was uncomfortable participating in this whole class discussion when he did not know the correct answers. Despite his uncertainty around the answer and his discomfort participating in the discussion, he persevered.

In terms of Ms. Collier supporting Andrew to persevere in the whole class discussion, consider how she encouraged him in Lines 12-24. First, she acknowledged his discomfort and provided him strategies to cope with his nerves, asking him to take a deep breath, focus on looking at and listening to her, and ignore the distractions in his environment. Notably, Ms. Collier's comments about the TV noise in the background were not to discipline Andrew for having the TV on during class, but rather gentle reassurances for Andrew to focus beyond the distractions of learning in the virtual classroom. Only after recognizing and providing strategies for Andrew to focus despite the considerable distraction of learning at home and participating in
a whole class discussion while being uncertain about how to find the answer did Ms. Collier reengage Andrew in solving the simplification problem.

When Ms. Collier reengaged Andrew in solving the problem, she did so by offering suggestions and guidance on how to productively think about the problem without reducing the rigor of the task. That is, when Ms. Collier told Andrew in Lines 21-23 "For every one I have on the left side if I have one on the right side, I can take it. Okay? But I cannot take more than I have on both sides. Okay?" she was providing explicit scaffolds by orienting Andrew in what to consider in order to solve the problem, without reducing the demand by answering these questions for him. Wilson et. al. (2019) identified this instructional practice as coaching, which they define as explicitly intervening to support a student to participate mathematically without reducing the rigor of the task. Coaching students provides them with the scaffolds necessary to engage in conceptual mathematics.

As the conversation between Ms. Collier and Andrew continued, this is exactly what happened. Ms. Collier was coaching Andrew by continually asking him questions that oriented him toward the relevant features of the hanger to consider to simplify (Lines 21-23, 27-40). Although this takes repeated effort from Ms. Collier, eventually Andrew did seem to refine his understanding, explaining that they were done simplifying the hanger "because there are no more squares on the right" in Line 43.

When Andrew was able to provide an explanation for why the simplification was complete, Ms. Collier praised him in Lines 44-46 by using humor, calling his reasoning beautiful, and acknowledging the pressure of what he just endured. Her praise and affirmation elicited a smile from Andrew as he patted himself on the back per her request.

## Ms. Collier's Facilitation of the Discussion and the Historical Practice of Care

To situate Ms. Collier's facilitation of the whole class discussion in the historical practice of Black teachers caring for Black students, recall that this practice includes both the teacher's demanding success from their students and the teacher's maintaining the conditions that support their students in achieving success. Ms. Collier shows evidence of both of these facets of care in her public discussion with Andrew.

Ms. Collier cultivated an environment that supported Andrew's success by creating space for him to experience a full range of emotions. In this vignette, Andrew expressed confusion, nervousness, and joy, all on the very public stage of a whole class discussion. Despite these farranging emotions, none of them were at odds with Andrew's capacity to engage in mathematical thinking and reason through the simplification problem. Rather, given Andrew's uncertainty in how to simplify the hanger problem, Ms. Collier trusted he was capable of applying the appropriate mathematical reasoning to think through it and used questioning to scaffold her instruction to support him in doing so. This is not trivial; allowing Black students to experience confusion and nervousness while simultaneously expecting them to engage in mathematical reasoning is an act of resistance against the dominant narratives about Black mathematics learners which construct them as mathematically incompetent - even illiterate (Martin et. al., 2019). Contemporary mathematics education researchers (e.g., Gutiérrez, 2018; Wilson, 2022) identify that attending to students' emotions is a rehumanizing teaching practice. That is, while the pursuit of school mathematics might encourage students to ignore their emotions and senses in order to succeed (Gutiérrez, 2018), Ms. Collier enacted a rehumanizing practice by creating the time and space to help Andrew experience and process them.

## Other Students' Experiences as the Focal Student

Although Andrew was the focal student during the whole class discussion in this vignette, I observed nearly all of the students in Ms. Collier's class as having been in Andrew's position before. Ms. Collier's facilitation of classroom discussion during virtual instruction typically consisted of calling on individual students to answer different parts of the problem, like Andrew working on simplifying the squares on the hanger which was contained within the larger problem of finding the mass of a square. Once she selected a student to answer part of the problem, she continued working with that individual until they successfully answered the problem and explained their understanding.

Two students, Kianna and Marcus, shared their perspectives about being the focal student in a whole class discussion. Discussing her experience being the focal student in a whole class discussion, Kianna explained:

It helps... Because if you don't know what you're doing and you participate and you say the wrong answer, [Ms. Collier will] help you get to the right answer. But sometimes [Ms. Collier would] urge me to participate and I would not know what I'm doing, but she'll help me figure it out and it'll be pretty easy. So participating, for me, helps me understand what's going on.

Marcus shared a similar perspective about participating as the focal student, despite not being sure about the answer:

I think for me, if I don't know it, I still try to raise my hand up and slowly but surely get my way to the finish line of the problem. And I do. And for me, as soon as the equation is over, I get it right off the top of my head. I know what else to do, how to do it, and how to
complete all my work. Even though I get chills, I get hairs on the back of my neck, I still know what to do.

Consider both Kianna's and Marcus' perspectives from a lens of Ms. Collier demanding her students' success and providing appropriate support so they achieve it. From Andrew's experience in the discussion and Kianna's and Marcus' comments, it is clear that being the focal student in a classroom discussion can be a high-pressure situation. Kianna explained that she sometimes did not volunteer but was urged by Ms. Collier to share in the discussion, even when she did not know the answer. Marcus shared that even if he did not know the answer, he still was willing to volunteer to participate as the focal student, despite the physical discomfort he experienced. For each of these students, publicly engaging in a discussion when they did not know the answer, was a worthwhile risk despite how nervous and uncomfortable it made them. They trusted that Ms. Collier would support them in thinking through the problem and that they would succeed at the task. Furthermore, they knew that engaging in these discussions with Ms. Collier, in front of the class, supported their learning; once they made it to the end of their problem, they had a better sense of how to solve the next one.

This is exactly what happened with Andrew. Immediately after the class completed the task discussed in Vignette Two, they moved onto Task Two which is shown in Figure 2 below. Ms. Collier put the students into breakout rooms to solve Task 2 and circulated between the groups.

## Figure 2

## Task Two Used During Vignette Two when Andrew Succeeds

Here is a balanced hanger diagram. Each triangle weighs 2.5 pounds, each circle weighs 3 pounds. What is the mass of a square? (Lesson 2)


Once the students returned from their breakout rooms, Ms. Collier called on Andrew to tell the class how he simplified the hanger. Their subsequent discussion is transcribed in Table 6 below.

## Table 6

Transcription of Discussion about Task Two: Andrew Succeeds

| 48 | Andrew | Okay. So, when I was crossing out the shapes on the hanger, I wanted |
| :--- | :--- | :--- |
| 49 |  | the hanger to have balance. So, what I crossed out was two squares on |
| 50 | the left and two squares on the right. And right after I crossed those out, |  |
| 51 | I crossed out one triangle on the left and one triangle on the right. So |  |
| 52 | then after that, I crossed out two circles on the left and then two circles |  |
| 53 | on the right. So, the remaining shapes on the hanger from the left is two |  |
| 54 | squares and one triangle, and the remaining shape on the right is a <br> 55 | circle. |
| 56 | Ms. Collier | Ms. Collier crosses out each of the shapes that Andrew identified on the <br> hanger. |
| 57 |  |  |

During the discussion in Task One, it took Ms. Collier asking Andrew 14 questions to scaffold the task before Andrew could explain when the hanger was simplified. In this task, which
immediately followed Task One, Ms. Collier called on Andrew to simplify the hanger, which he answered in Lines 48-55 without any additional questioning or scaffolding from her. This is not to say Andrew solved this problem entirely independently, there were opportunities for him to collaborate with other students and Ms. Collier in the breakout room. Still, during the whole class discussion that he was able to explain in his own words his process for simplifying the shapes on the hanger to maintain balance demonstrates a considerable progression in his learning.

## Vignette Three: Ms. Collier Maintains High Expectations: Unwillingness to Let Students

 Slide ByThis third vignette occurred during the lesson entitled, Inputs and Outputs, in the on-level math class and shows another instance of Ms. Collier's maintaining high expectations for students during whole class discussions. Different than Vignette Two, which explored the warm and supportive way Ms. Collier cared for Andrew through her instruction, this third vignette shows how Ms. Collier cared for students by cultivating a very structured classroom environment in which students had no option other than to learn. My purpose in sharing this third vignette, centered on a second male student named Kobe, is to help nuance the multifaceted nature of a Black teacher caring and maintaining high expectations for her Black students. That is, while teachers caring for their students can take many shapes and forms, a central aspect of teacher care is the genuine belief that their students are capable of learning the content, providing the appropriate support for the students to learn, and then maintaining high expectations that their students follow through and learn the content. This vignette highlights that because of the warmth and support that were evident in Ms. Collier's instruction in Vignettes One and Two, she was able to make the demands of Kobe that are shown in Vignette Three. Specifically, I consider
how Ms. Collier was able to demand Kobe's sustained intellectual effort through the course of the discussion, a demand which Kobe fulfilled.

## Setting up Task Three, On-Level Math

Students were completing an input-output table with the rule "name the digit in the tenths place." Right before this interaction, they had reviewed rows where the input was $3 / 4$ and 2.35 with Ms. Collier emphasizing that students need only name the digit in the tenths place without any calculations to solve this problem. The task and the classroom's collaborative work from the discussion to this point are shown in Figure 3 below.

## Figure 3

## Task Three, with Notes from the Whole Class Discussion



Next, Ms. Collier called on Andrew to share the output when the input is 42, asking him, "What digit is in the tenths place of 42, Andrew?" and Andrew answered, "the four." The discussion continued with Ms. Collier asking questions until Andrew identified that the four is in the tens place but the problem is asking about the tenths. When Ms. Collier continued her questioning by asking "where is the tenths place located, Andrew?," 12 seconds passed before Andrew answered, "I don't know." Ms. Collier then called on Janae who identified that there was
a zero in the tenths place. Ms. Collier rewrote the input 42 as 42.0 then turned her attention back to Andrew and explained:

It's zero Andrew ... [42 is] not a decimal. In order to see the tenths place, you have to make the number a decimal. If you don't see any decimal points, there's a whole bunch of zeros back there. Every place value behind the decimal is a zero. So, they ask me for the tenths, the hundredths, the thousandths, and I don't see any decimals, I don't see a decimal point, every digit behind there is a zero. Do you understand? [Andrew nods his head yes.] Because I don't have any tenths, there are no tenths represented in this number. There's just tens and ones.

Through this explanation, Ms. Collier was trying to support Andrew and the rest of the class in recognizing that whole numbers which are expressed without decimals can be modified to an equivalent number with a visible decimal point. For the final row of the input-output table, Ms. Collier called on Kobe to provide an input and identify the output, whose contribution to the input-output table would rely on this same idea. Their discussion is transcribed in Table 7 below.

## Table 7

## Transcription, Debrief of Task Three with Focal Student Kobe

58 Ms. Collier: All right, somebody give me another input. Kobe, give an input.
$59 \quad 17$ seconds pass
60
Kobe, give me an input
4 more seconds pass
62 Kobe: I don't know.
63 Ms. Collier: So Kobe, you know how I know you haven't been paying attention,

65 Kobe: I don't know what number to pick. It's like...
66 Ms. Collier: Any number you want to pick. What do you mean?
67 Kobe:
Oh, five?

68 Ms. Collier: Okay. Five's your input, what's your output?
69 Kobe: Six?

Kobe: Yes. Wait, no, no, no.
Ms. Collier: You said five, and I'm going to leave it at five. We're not changing the
Ms. Collier. $\quad$ number, now. Your input is five. So, if your input is five, the tenths place of your... If I was to write your number as a decimal, what would it say?
Ms. Collier: So, you're telling me that your number is 5.1 , because I didn't see that. You said five.

Five in a decimal, it'd be 5.0.

94 Ms. Collier: It'd be 5.0 , so what is the digit in your tenths place?
95 Kobe: $\quad$ The tenths place is 5.0.
96 Ms. Collier: The digit in the tenths place. We already told you that the five is in the

97 ones place. I want only the digit in the tenths place. Kobe, the digit in the tenths place, one digit, just one digit.

Kobe: One?
Ms. Collier: Do you see a one in your number?
Kobe: No, it's a zero.
Ms. Collier: What digit is in the tenths place, Kobe, the five or the zero?
Kobe: The five?
Ms. Collier: Kobe, we told you what place value the five was in.
Kobe: Oh, no, the zero.
Ms. Collier: The zero, Kobe. What place value is in the tenths? What digits in the tenths place, Kobe?

Kobe: The zero.
Ms. Collier: The zero. And you want to know why I'm disappointed in you, sir, and I'ma go ahead and let you know I'm disappointed in you, because I just explained that to somebody right here. Look at where I'm circling. Ms. Collier circles the row with 42 as the input and 0 as the output. We literally had that conversation, and because you had your head down and you over there watching TV, you missed it. But you have to learn too, so I can't just leave you behind.

## "I Can't Just Leave You Behind"

To situate this excerpt of a whole class discussion in the historical practice of Black teachers, recall that their caring for their Black students involved a teaching practice that was characterized as structured, disciplined, tough-minded, and unwilling to let students slide by. The historical context of pre-Brown segregated schooling is relevant to situating this practice of care
because its foundation was to provide an education for people who had been enslaved in order to improve the opportunities available to them. Given these incredibly high stakes, whereby an education improved students' opportunities for liberation, and uplifting their shared community, teachers cared for their students by maintaining that they can and must learn. In the present day, remnants of this foundation are still evident in the work of Ms. Collier. Although the sociopolitical climate of our society has certainly transformed since the pre-Brown era, it remains the case that we live in an antiblack society. To this day, education provides an opportunity for both economic access and personhood for Black students. Given these incredibly high stakes, in this analysis I focus on: 1) how Ms. Collier demanded Kobe's sustained intellectual effort; 2) why Ms. Collier demanded this level of effort from Kobe; and 3) the broader classroom climate that allowed Ms. Collier to demand Kobe's effort in this way.

Ms. Collier continuously and effortfully demanded that Kobe intellectually engage in the task despite the amount of time and attention that became dedicated to solving this single row on the input-output table. In terms of time, the discussion between Ms. Collier and Talia lasted 45 seconds to identify the output when the input is 2.35 and the discussion between Ms. Collier, Andrew, and Janae lasted just over two minutes to identify and explain the output when the input is 42. However, the discussion between Ms. Collier and Kobe lasted just under four minutes (three minutes and 52 seconds) despite just having reviewed how to identify the output when the input is a whole number with Andrew minutes before. During these four minutes, Ms. Collier adamantly demanded that Kobe continue to think through this problem and Kobe did exactly that. I highlight evidence that demonstrates Ms. Collier's demands for Kobe's sustained effort.

First, Ms. Collier gave Kobe 17 seconds of wait time, followed by another four seconds to identify an input of his choosing. This is considerably longer than the 10 seconds of wait time
teachers are typically encouraged to allow for students to process their thoughts during classroom discourse (Yeh et. al., 2017). After Kobe repeatedly responded, "I don't know" (lines 62,65 ), Ms. Collier named and provided evidence that he was not paying attention (lines 63-64, 66). As the discussion continued, Kobe's responses were disconnected from the preceding whole class discussion between Ms. Collier and Andrew, and seemed to evidence that Kobe did not understand the task, as evidenced by Kobe's trying to select any output of his choosing (line 69), misidentifying the place value of the digits in 5 (lines $74,76,95,99,103$ ), and misunderstanding the relationship between the rule, the input, and the output (lines $83,86,89$ ). In response to each of Kobe's comments, Ms. Collier repeatedly reoriented him towards the relationship between the input, rule, and the output (lines 70-71, 84-85, 87-88), and corrected his misunderstanding of place value (lines $77-79,81,96-97,104$ ) and pressed him to identify a specific place value given the input of five (lines $81-82,90-92,94,96-98,102,106-107$ ). After all of these turns in conversation, Kobe was finally able to identify that zero is the digit in the tenths place given the input five (lines 105,108 ) leading Ms. Collier to openly lecture him about her disappointment in him due to his inattentiveness during the preceding discussion with Andrew and her unwillingness to leave him behind (lines 109-115).

Ms. Collier demanded Kobe's sustained effort by dedicating nearly four minutes of the class session to working through this input-output problem with him and by continually, repeatedly, and sometimes sternly focusing his attention on the relationships between the input, output, and rule. When he expressed confusion at getting to pick any number as the input, Ms. Collier rejected this as an excuse, asking him "what do you mean?" (line 66) and naming that he was not paying attention. As Kobe continued to share his responses, Ms. Collier continued to respond sternly to his disconnected responses, for example by telling him, "there is no such thing
as the zero place value, Kobe, that is not a thing" (line 77) or "we already told you that the five is in the ones place. I want only the digit in the tenths place. Kobe, the digit in the tenths place, one digit, just one digit" (lines 96-97). Although this discussion between Kobe and Ms. Collier can be interpreted in myriad ways, I argue that Ms. Collier's sternness and open disappointment in Kobe are rooted in the historical work of Black teachers who cared for their students through their unwillingness to let them slide by, and through Ms. Collier's foundational belief that her students are intellectually sophisticated and must act accordingly.

To situate Ms. Collier's practice as an unwillingness to let Kobe slide by, consider how she demands sustained effort from him. After Kobe paused for a long time before answering her (lines 59,61 ) and repeatedly told her he did not know (lines 62,65 ), Ms. Collier refused to accept this as an answer from him and continued pressing him for not paying attention and orienting him in understanding the relationships between inputs, outputs, and place value so that he could make sense of the problem. Consider Ms. Collier's refusal to let Kobe opt out of the discussion from Ladson-Billing's (2002) description of teachers granting African American students permission to fail. Ladson Billings (2002) cautioned that student resistance is a challenge that teachers must face, warning that students in their youth "must not be allowed to determine [their] own demise. [They] are dependent upon caring adults to act in responsible ways" (p. 120) and recognize the impending danger of allowing students to opt out of their own learning.

In her public discussion with Kobe, Ms. Collier provided a genuine classroom example of what it might look like for a teacher to deny her students permission to fail and to instead demand that her students engage in sustained learning, despite student resistance. Part of what makes clear that Ms. Collier was demanding success, rather than contributing to the hyper-
discipline of Black children in schools, is that although her words were sometimes stern, they were always rooted in the content and implicit is Ms. Collier's expectation that Kobe has the capacity to solve this problem and must act accordingly. For example, when Kobe resisted engaging in the discussion by waiting over 20 seconds to respond and repeatedly saying he did not know, Ms. Collier identified that he had not been paying attention and continued to engage him in solving the problem (lines 58-64). At the end of their discussion (lines 109-115), Ms. Collier expressed her disappointment in his inattentiveness, because she had just reviewed identifying the output when the input was a whole number in the preceding discussion with Andrew. Implicit in her lecture to Kobe is the expectation that Kobe has the capacity to listen to the discussion between Ms. Collier and Andrew, learn from it, and understand relationships between inputs, outputs, and rules, and relationships between whole numbers and decimal place value. Her repeated questions to Kobe about identifying different components of these relationships express that Kobe has the capacity to make sense of these relationships in the problem, and so he must. By demanding Kobe's sustained engagement in the problem, despite his resistance, Ms. Collier's practice is rooted in the historical work of Black teachers who refused to let their students slide by.

## Why Ms. Collier Demands Her Students' Sustained Engagement

This discussion transcribed between Ms. Collier and Kobe was not unique; there were multiple such interactions between Ms. Collier and individual students where she refused to let them off the hook and expected them to continue engaging in the problem until they reached a solution. I asked why she demanded this level of engagement from students and she explained it stemmed from: 1) her expectation that her students have the capacity to solve the problems, and so they must; 2) her understanding that other students in the classroom could benefit from the
clarifying conversation; and 3) weighing the benefit to other students against the cost of class time. In her own words, Ms. Collier explained why she demands this level of sustained engagement from individual students:

So it's a two-fold thing. ... Because number one, I can't leave that kid behind, in my heart I just can't... I know they can do it, so I'm gonna wait. This is actually when I'm really patient, because I'm just going to wait until you figure it out. And the second fold to that is they're not the only person in the room who needs to hear what we're talking about, and I know that, and whoever's listening who heard it and was like "oooh! That totally makes sense now," they also benefited from that.

Ms. Collier demands her students' persistent engagement in solving the tasks during the discussions because she knows they are capable of doing so. In terms of care, for Ms. Collier to care about her students means she in fact does not let them opt out of the problem. For her, allowing students to opt out represents leaving the individual student behind. It is also a disservice to all the students in the room who would benefit from the resulting clarifying discussion between Ms. Collier and the individual student. She cares for her students by believing they are capable of reaching the end of the task and supporting them in doing so. Ms. Collier elaborated on her response by explaining the tensions she must balance in how she facilitates whole class discussions. When comparing the costs of working with one individual student until they successfully solve the problem during a classroom discussion, she explained:

The caveat to that is, God it kills a lot of time. It takes a while. And at some point, I do have to let it go. And that's something that I'm working on. Because it's not beneficial to the group all the time, to let it go on that long. But it's also like, do I leave a child in the
middle of a thought, or do I leave them in the middle of a misconception, and allow them to continue that misconception? And I don't really want to do that either. So, it's just this back and forth I have internally, in the moment, where I have to make a judgment call. And nine times out of ten I'm just going to keep explaining it because I want the kid to get it, but I know that that one time I'm going to be like, you know what, I'll address you later, I'll hit you in the chat, or come meet me in tutoring, or we'll talk about it on Wednesday.

Even with Ms. Collier's 16 years of experience and being considered by other researchers and her school administration as an exemplary teacher, she still faces these tensions or dilemmas in facilitating classroom discussions. She has to weigh how much time to give to individual students during discussions and what that costs the rest of the group. However, if she spends less time with the individual student to benefit the group, then the individual might be left behind without ever learning the content of the lesson. Lampert (2001) similarly expressed navigating these dilemmas of teaching while facilitating whole class discussions, particularly in terms of navigating incorrect responses from students, what it means to have students agree and disagree with one another responses, and the cost of time spent exploring different mathematical ideas from students. For Ms. Collier, in her practice, caring for her students means caring for each individual student and making sure that each individual student receives the time and attention they deserve to make sense of the content. She says nine times out of 10 she is going to stick with the individual student over moving on for the sake of the group. And the one time, she will still make herself available to the student, outside of the discussion.

## Vignette Four: Task Three in the Honors Math Class

Later that day, Ms. Collier used the same exact task in the honors section, but her setup of the task was considerably different than in the on-level class described above. In interviews, Ms. Collier explained to me that, while she typically has to take a much more proactive role in leading students through problem-solving in the on-level class, in the honors class, she is able to have the students work more independently and take on the role of facilitator. As will be discussed in the coming vignette, Ms. Collier attributes this distinction to the particularly strong mathematics foundation that the students have in the honors class. Her role as a facilitator is evident in how she sets up Task Three in the honors class. To set up the task in the honors section, Ms. Collier displayed the task on the slide, identical to the task in Figure Three, but without any outputs filled in. After displaying the slide Ms. Collier instructed the class to: "follow the rule ... this time you have to name the digit in the tenths place, three minutes on the clock."

Consider this task set up in comparison to the on-level section, where Ms. Collier reviewed how to find the output when the input is $3 / 4$ and subsequently led a discussion with the class row by row. That is, in the on-level section the entire task was completed with the whole group, whereby she called on different students to identify the output for each row in the table. During the whole group discussions, she provided explanations about identifying the tenths place in whole numbers and how to use this rule to identify the output with different types of numbers.

In the honors class, her set up was considerably different. She set up the task by projecting the task for students, reading the rule for them, and then setting a three-minute timer for the class to complete the table individually. There was no review of place value or coordinating between the rule, the input, and the output in this section. Ms. Collier instructed
students to send their complete answers to the entire table to her in the private, teacher-only chat. As students privately sent their answers to her, Ms. Collier gave them verbal feedback on their work. During the three minutes as students send their responses to her, she provided them verbal feedback such as, "Emily, that first answer is incorrect, my dear. You might want to think about that again." And, "Good Job Camille. Good Job Mikayla. Malachi, you got to make two of your own." After the three minutes had passed, Ms. Collier then led a very brief whole class discussion in which they debriefed their answers (transcribed in Table 8 below).

Table 8
Transcription of the Whole Class Debrief of Task Three in Honors Section
116 Ms. Collier All right. If the input is 2.35, what's the output?

117 Ms. Collier Rebka, what's the output?
118 Rebka Three?
$\begin{array}{ll}119 & \text { Ms. Collier } \\ 120 & \text { Three, because three is what's in the tenths place. Remember the } \\ 121 & \text { tenths place is the digit right behind the decimal. } \\ \text { How about at 42? What's the output, if the input is } 42 \text { ? Karen? }\end{array}$
Karen Zero.

Ms. Collier It's zero. There is no digit in the tenths place, so it's zero. I'm going to tell you guys about some cool ones I saw in the chat.

Ms. Collier Yeah, go ahead, MiAngel.(In response to MiAngel private messaging Ms. Collier asking to use the restroom).

Ms. Collier I saw one in the chat that said, "This is my input 175.555." And then they said, "Well, my output's five." Right? Pretty simple. And then another one says "my input is 2.22 , my output is two." Right? This rule does not require you to do any calculations. If it says name, or write, you just do that.

## Comparisons between the Honors Section and the On Level Section

Consider the setup of Task Three and discussion here in comparison to the on-level section. In the on-level section, Ms. Collier repeatedly explained how to find the output when the input was a whole number after both Kobe and Andrew expressed that they did not know how to answer this question. In the honors class, this explanation from Ms. Collier took a different form. First, Ms. Collier had students independently complete the entire table and then used the chat to gauge student responses and to provide them with individual feedback. After using the chat in this way, the resulting discussion was a very brief review of student answers, the entire discussion lasting just over a minute (one minute and five seconds). As shown in Table 8, student responses were one-word answers in which they identified the output (lines 118, 122) with Ms. Collier providing brief reminders about the location of the tenths place in the different inputs (lines 119-120, 123-123) and relaying student answers from the chat for the final two rows where students chose their own inputs (Lines 127-131).

Given the discussion transcribed in Table 8, specifically the students' single word answers and Ms. Collier's longer explanations, a possible interpretation of this data could be critical of Ms. Collier's facilitation of discourse. Consider these data in light of reform-oriented approaches to mathematics instruction which emphasizes that for rigorous classroom discussions students should "provide thorough explanations of why particular strategies [are] valid and [make] connections between strategies and the underlying mathematical ideas" (Wilson et. al., 2019). Reform-oriented mathematics instruction approaches expect that this type of classroom discussion, where students generate the explanation and connections between mathematics ideas, will foster students' conceptual mathematics knowledge. Worth understanding here is why Ms.

Collier, a veteran teacher with 16 years of experience, who has been shown to care deeply about her students and their mathematics learning, would choose to facilitate the discussion in the way she did.

Ms. Collier explained some of the features of the honors class that may have allowed her to facilitate the discussion in the way she did. Describing the students in the honors level class, she explained "your basis is there, your foundation is very, very strong. And so, I'm able to work with that because I don't have to go back and go, how do we convert a mixed number into an improper fraction? I don't have to talk about multiplying decimals," or in this case, extensively review how to identify decimal place value in whole numbers. Because, in this class, the students' "foundation is strong" she was able to provide a more cursory explanation about the place value.

Furthermore, Ms. Collier's claims that the students in the honors class had a strong foundation were not based on assumptions because of their track placement, but on her extensive use of the chat feature which allowed her to see each students' response to the task to provide individual feedback accordingly. Describing her use of the chat function as a tool for formative assessment, she explained, "it works out in my favor where it's like 'saw your answer, saw you answer' so immediately I can respond, boom, boom, that's wrong, I know the answer that's not it, boom look again, you got the sign wrong. And so ... I'm able to respond fast." By having the students in the honors section work on this task independently and then submit their answers through the chat Ms. Collier was able to very quickly gauge her students' understanding of the task, and drive instruction accordingly. By seeing the students' individual responses, she was able to determine that they did not need the same level of discussion around the tenths place as
the students in the on-level section. This allowed her to provide the cursory explanations seen in Table 8 (lines 119-121, 123-124) and move on to the next task.

Worth highlighting here is that given the brevity of this discussion and the accuracy of students' initial responses, missing from the honor's class discussion were the analogous elements of care that were observed when Ms. Collier refused to let Kobe slide by, or when Ms. Collier publicly supported Andrew's engagement in the vulnerable act of learning and struggling. Because of the nature of the task setup and the types of student responses in the honors class, the way Ms. Collier cared for her honors class looked different than the on-level course. This is not to say she did not care about the students in the honors course; she did in fact care for them by recognizing the foundation they bring to the content and tailoring her instruction accordingly.

However, during comparatively more infrequent instances, the students in the honor's math class did become confused and shared incorrect answers during the classroom discussions. The distinction between the nature of the confusion in the two sections is that generally, in the honors class, this confusion occurred when students were learning new content, whereas in the on-level class this confusion occurred throughout the lesson and oftentimes required clarification on how to perform computations. Ms. Collier captured this distinction in her comment about needing to review multiplying decimals, converting improper fractions, or identifying decimal place value in the on-level section but not the honors section. In the final vignette, I show what happened when the students in the honors section became confused during whole class discussions and how Ms. Collier cared for the students in this class.

Vignette Five: Confusion in the Honors Class
This final vignette centers on a classroom discussion during which the students in the honors class become confused about how to solve a task. In the vignette, Ms. Collier was able to
provide mathematical explanations and elicit responses from students that evidence their making sense of mathematical relationships. The purpose of this vignette is to highlight how, despite the students' initial confusion, Ms. Collier's facilitation of the discussion bridges students' intuitive understandings of function rules and the complex algebraic relationships at play in the problem. In facilitating this discussion, Ms. Collier never watered down the curriculum and instead uplifted students to grapple with the mathematical concepts.

The class was working on Task Four and the instructions are shown in Figure 4 below. Ms. Collier had instructed the class that she would give them a scenario and they were going to respond either yes or no to the prompt, and share their responses with her and the rest of the class in the everybody chat. She clarified what yes and no meant in this particular problem context by explaining:

If you think you can answer the question with one unique answer, with just one unique answer, you're going to hit me with a yes in the chat. If you feel like there's more than one answer to that same question, you're going to say no. If you say yes, draw an inputoutput diagram on your paper. Okay? So, if you say yes, draw an input-output diagram. If you say no, you have to be able to give examples of two different outputs. So, what are two different ways that could play out that are possible for that same input? Okay? She instructed students that after they determined whether the answer to the question was yes or no:

Then you're going to be going through 8-10 minutes' worth of work on these questions. So, it's not quiet time because we're not in a classroom. Unfortunately, I can't do it the same way, but you will have like time to think about it, time to draw your diagrams if diagrams are applicable, and then we'll wrap it up with a class discussion. Okay?

Using this model that Ms. Collier set up, the class had already discussed problems one and two displayed in Figure 5 below which asked them to determine if a question was answerable given the context, and if so to create an input-output diagram, which takes the form input $\rightarrow$ rule $\rightarrow$ output. In response to question one, they had identified that yes, they do know the person's height in inches and created the following input-output diagram: 5.5 feet $\rightarrow$ multiply by $12 \rightarrow 66$ inches. For problem two, the class determined that yes, they do know the square of five and created the following input-output diagram: $5 \rightarrow$ multiply x by itself $\rightarrow 25$.

## Figure 4

## Task Four

Say yes or no for each question. If yes, draw an
input-output diagram. If no, give examples of two
different outputs that are possible for the same input.

## 1) A person is 5.5 feet tall. Do you know their height in inches?

2) A number is 5 . Do you know its square?

## 3) The square of a number is 16 . Do you know the number?

As Ms. Collier concluded the discussion around question two, she asked a student, Jorge, whether there could possibly be any output other than 25 when multiplying 5 by 5 , to which Jorge responded no. Ms. Collier then synthesized the relationships between inputs, outputs, rules, and functions by telling the class, "that's how you know this is a function...That's how you know, okay? That's how you know, hey, that's a function. If I put in something, I'm only going to get back exactly one output... So, you cannot have an input having two or more separate outputs. All right?" She then supported the class by recognizing the complexity of the content they were learning and encouraging them that it may take time to become comfortable with the material:

This is a lot to take in. I remember going through it in my 8th-grade algebra class and being like, 'What? Inputs? Outputs? Who goes where?' It gets confusing. So, we're going to take a few days to process this. So, if you're struggling with this, don't stress. It's okay. We're going to get through it. We're going to keep practicing and we're going to keep working on it, okay? But it takes a while, and it takes using the language inputs, outputs, this is the rule, to really get comfortable with it, okay?

After encouraging the class by recognizing the complexity of the ideas they were working on, and giving them permission to feel confused as they got comfortable with the new concepts and language, she then led the discussion around question three: The square of a number is 16 . Do you know the number? She starts by prompting students to hold up their notebooks to the camera to show her their input-output diagram. The resulting discussion that ensued is transcribed in

Table 9 below.

## Table 9

## Transcript of Whole Class Discussion about Question 3 from Task Four

| 132 | Ms. Collier | All right. So, Camille, I'm going to ask you a question. |
| :--- | :--- | :--- |
| 133 |  | Camille's input output diagram shows: $16 \rightarrow$ divide by $4 \rightarrow 4$ |
| 134 |  | What if my number was 25, would your rule work? |
| 135 | Mikayla, what if my number was $25 ?$ |  |
| 136 | Mikayla's input output diagram shows: $16 \rightarrow \div 4 \rightarrow 4$ |  |
| 137 | Emily, what if my number was 25? |  |
| 138 | Emily's input output diagram shows: $16 \rightarrow$ divide by $4 \rightarrow 4$ |  |
| 139 | Jorge, I asked you what was the number? |  |
| 140 | Jorge's input output diagram shows: $16 \rightarrow x$ by itself $\rightarrow 256$ |  |
| 141 | If the square is 16, what is the number? Not what is the square of the |  |
| 142 | number, if it's 16? Read the question, sir. |  |
|  |  |  |
| 143 | Ms. Collier: | Again, feel free to hold up your notebooks. That's given me a lot of |
| 144 |  | important information. Mikayla come closer. You got to bring that closer. |
| 145 | I can't see that. |  |
| 146 | Mikayla's input output diagram shows: $16 \rightarrow$ times by multiple $\rightarrow 4$ |  |
| 147 | Times by multiple? Girl. Mmph. Well multiples of 16 are 16 and bigger. |  |

What does that say? Least denominator? What does that say? Emily, what does that say?
Emily's input output diagram shows: $16 \rightarrow$ least common factor $\rightarrow 4$
Emily Least common factor, Emily giggles.
Ms. Collier: Okay. We're just making stuff up. This is getting real interesting, the stuff y'all coming up with. Okay. So, somebody tell me what you thought the rule was. Share with me your input/output diagram. What'd you come up with? What'd you come up with? Jordan.

Jordan: Okay. So, the input was 4.
Ms. Collier: That's what they gave you?
Jordan: Oh, no.
Ms. Collier: Then that's not the input. I literally said the input is what I give you.
Jordan: Oh, the input is 16 .
Ms. Collier: Okay. The input is 16 .
Jordan: And then you uh, uh, uhhhh
Ms. Collier: Uh, uh, uhhh, It's my favorite noise. (students laugh on screen) So let's think about it this way. If you were squaring a number, what would you do? Turn your mic on. Don't be tapping out. Turn that microphone on.

Jordan: Multiply by itself.
Ms. Collier: You would multiply by itself. Right?
Jordan: Mm-hmm.
Ms. Collier: So, if I'm looking for the opposite, what would I do?
Jordan: Divide by itself?
Ms. Collier: Divide by itself, right? But I don't know what itself is technically, right? Like, that's kinda odd. That's weird. So, the opposite of a square, you can't say divide by itself when you don't know what itself is. Okay? Does anybody know the terminology for the inverse of a square? The inverse of squaring a number is what, Sandra?

$$
\begin{array}{ll}
177 & \text { Ms. Collier: } \\
178 & \text { Square root. You have to square root the number. Square root the number. } \\
179 & \text { And for everybody who's going, "Uh, square root, what that mean? } \\
180 & \text { What's that?" You've heard about it before. You've just forgotten. I'm } \\
181 & \text { going to remind you. When you're thinking about a square root, you're } \\
182 & \text { simply asking yourself, "Self, what number times itself makes the number } \\
183 & \text { they gave me?" What number times itself will give you 16? Because a lot } \\
184 & \text { of you got to this conclusion, but you found it with the wrong rule. What } \\
\text { number times itself gives you 16? Carlos. }
\end{array}
$$

Carlos: 4.
4. Is that my only option? Thumbs up or down, that's my only option. 4 is Ms. Collier: my only option.
of the 12 students in class, 5 students give a thumbs up, one student alternates between a thumbs up and thumbs down, and the others do not vote
Dante, you look like you don't believe in your own thumb, man. Dante is gesturing a thumbs up, but his thumb is pressed against his forehead masking most of his face.
Here's my vote. Look at my vote. (Ms. Collier shows two thumbs down and vigorously pumps them downwards) No, no, no. There's one other number that can make 16 when you square it. There's one other number that can make 16 when you square it, and you gots to think back to the warmup. Mikayla.

Mikayla: -4.
Ms. Collier: -4. She remembered. She recalled. Ohh, Rebka, yes. (Rebka can be seen mouthing the "Ohhh" in her screen). Yes. Remember, the negative version of a number will result in the same square. The negative version of the same number will result in the same square, because a negative times a negative is a positive. So, when I'm squaring a number, the sign technically doesn't matter because I know it's going to go away anyway. It's going to become a positive. Does that make sense? That means this is not a function. That means that the number is not a function of its square. A number is not a function of its square because I can come up with more than one answer, more than one output for that specific input. For

[^4]| 210 |  | example, if the input was 36, what are my possible outputs? If I say, if the <br> 211 |
| :--- | :--- | :--- |
| 212 | square of a number is 36, do you know the number? What are the possible |  |
| outputs, Dante? |  |  |

## How Ms. Collier Cares for the Honors Class

Evident during this discussion are remnants of Ms. Collier's high expectations that we have seen her hold for the students in the on-level class. These remnants show up in the way that she names that her students' responses are incorrect (Lines 147, 152-153, 159) and her refusal to let students opt out. For example, when determining the rule to get the output four from the input 16, Mikayla offered "times by multiple" (Line 146), and Emily offered "least common factor" (Lines 150, 151). Ms. Collier made clear that both of these responses were not reasonable given the problem context (Lines 147, 152-153), affectionately calling Mikayla "girl" during her correction and poking fun at Emily's response that students are now "just making stuff up" (Line 152). The nature of Ms. Collier's corrections to Mikayla and Emily implies that at a minimum she expects them to recognize that their responses are mathematically inappropriate.

There are other traces of the high expectations that were shown with the on-level group, including her refusal to leave students behind. These traces are exemplified through her interactions with Jordan, during which she refused to let him opt out of the problem by telling
him "turn your mic on. don't be tapping out. Turn that microphone on" (line 165), strongly correcting his error in identifying the input (line 159), and poking fun at his uncertainty by repeating the confused noises he makes and telling him "Uh, uh, uhh, It's my favorite noise" (line 163). Her use of humor poking fun at her students' responses, refusal to let students opt out, and strong reactions to students' incorrect answers demonstrate the classroom environment she has cultivated that is characterized both by warmth and humor and also an expectation that students are capable of mathematics reasoning, and so they must act accordingly. These characteristics have been highlighted in the on-level class as well.

## The Students' Experiences of Ms. Collier's Instruction

The students enjoyed Ms. Collier's style of instruction and received it positively. Both Emily and Mikayla, students whose responses Ms. Collier joked about (Lines 147, 152-153), described their feelings about her class, characterizing it as fun. Emily explained, "Okay, so [Ms. Collier's] class is my last period. I always get happy for her class. I'm like 'we finally got math for last, because that's going to be the fun class." Mikayla shared similar sentiments, "[Ms. Collier's] class, she's really like fun ... She's like very energetic. So, if you're like 'Uhh I don't want to go to school' she'll change your mood real quick and she'll make sure you're involved in the class." Both students who were on the receiving end of her jokes found her class to be fun, with Emily looking forward to it at the end of her day and Mikayla feeling that Ms. Collier's energy can change her whole mood and involvement in the class.

The energy, jokes, and fun were not the only thing that students appreciated about Ms. Collier's math class. Mikayla, Emily, and Camille each shared that what they liked about Ms. Collier's class was how she used students' participation in the discussions to check for student understanding and tailored her instruction accordingly. For example, Mikayla explained that the
whole purpose of participating in discussions is "so Ms. Collier knows what you know and what you don't know. And then if you don't know something, and you say it wrong, you have that 'oh' moment where you realize. She'll explain and you'll learn what you're doing." Mikayla then reflected on how she felt when she was wrong during class discussions, explaining "it's like dang, I got it wrong. But at the end it's like, oh, that's how you do it." Camille similarly shared that she felt comfortable sharing during whole class discussions because "you know [Ms. Collier] won't judge you, she'll just help with the answer."

This "oh" moment that Mikayla mentioned, when students recognize they have made an error during the discussion, happened three times during the transcribed discussion in Table XX. The moment happened twice when Jordan recognized he used the wrong input (lines 150,160 ) and once when Rebka realized that -4 is also a square root of 16 (lines 200, 201). Interestingly, Rebka's "oh' moment of realization happened while she was not even the focal student during the discussion; she had this moment of realization while listening to Ms. Collier and Mikayla discussing the square roots of 16 .

Similarly, Emily characterized how Ms. Collier used student participation to facilitate their learning:
it's like she has this secret way to know if people don't know, don't understand what she's teaching, or if they do understand... she wants everybody to learn, she doesn't want people to just sit in the zoom and just sit down. They're gonna think they know what they're doing, but not really know. She calls on people to know if they understand or not. Camille expressed a similar sentiment when she highlighted the importance that students participate. Camille cautioned that "if you don't participate, she doesn't know what's wrong and that's very unhelpful to her and you." Each of these three students, Mikayla, Emily, and Camille,
experienced being wrong during the discussion in Table 9. However, for each of them, they experienced making an error during a discussion as a learning opportunity during which they trusted that Ms. Collier to use their misunderstanding to help them realize what they had done wrong so they could understand how to do it correctly. Although their feelings about being wrong in the public space differed, with Camille feeling comfortable knowing she would not be judged while Mikayla felt like "dang, I got it wrong," they each felt it is necessary to share both what they knew and what they did not know, so that Ms. Collier could know how much they understood, and instruct them accordingly.

The students elaborated that what they most valued about Ms. Collier's instruction was how she explained the mathematics to them. Mikayla explained that the importance of students participating, even if they did not know the answer, was because "[Ms. Collier will] make sure you understand... she'll support you until you understand." Emily expressed that through Ms. Collier's explanations she was able to deeply understand the content, "the way [Ms. Collier] explains it, I feel like I could accomplish it, because it seems like something that I will know for the rest of my life. Like the way she would explain it, if somebody outside of school asked me how to do this, I would know because of the way that she taught me." Camille detailed specifically what Ms. Collier did during her explanations that so supported students' learning: "I like how [Ms. Collier] just explains things. She's inching you towards the answer, but you don't know that she's inching you towards the answer and by the end of it, she really helped you but didn't give you the answer, so I like that." Camille added that Ms. Collier "explains it in simple terms, and then, later on, she helps you like get into the big words, like the mathematical words, so you know the words and everything."

Each of these layers that Camille detailed about how Ms. Collier facilitated discussions are evident in the discussion transcribed in Table 9. When she asked Camille, Mikayla, and Emily, if their rule would work if the input was 25 (lines $134,135,137$ ) she was inching them towards recognizing that their rule must be generalizable to all inputs, without explicitly telling them so. When she prompted Jordan to use his understanding of how to square a number (lines 164-166) to propose a rule for identifying the number whose square is 16 (Lines 169,170 ) she inched him towards recognizing the inverse relationship between the square and square root, without telling him directly; he reasoned through it himself. Additionally, when Jordan offered "divide by itself" (line 170) as the potential rule, Ms. Collier then guided their discussion to have the students transition from the "simple terms" to the "mathematical words," as Camille described. Ms. Collier acknowledged that "divide by itself" sounds "kinda odd," it's "weird," and then she asked the students specifically if they "know the terminology for the inverse of a square" (lines 171-175), which Sandra identified as "the square root" (line 176).

## Ms. Collier's Mathematical Explanations

Ms. Collier's use of explanations and student responses are related to the historical work of Black teachers with Black students. Specifically, I anchor her teaching in the historical work of Black teachers who cared for their students by resisting deficit characterizations of Black children. These teachers cared for their students by cultivating intellectually rigorous learning environments for them despite dominant narratives that Black students had no need for college preparatory mathematics in their present or future trajectory. In Ms. Collier's practice, I would like to highlight how her facilitation of this discussion refuses to water down the mathematics content the students are engaged with, and instead supported them in bridging their intuitive understandings with the eighth-grade mathematics ideas of the curriculum.

She started by asking the class to identify which numbers when multiplied by themselves would equal 16 (lines 180 - 184). After Carlos and nearly half of the class recognized only the positive solution (lines $185-190$ ), she identified that there was more than one number that when multiplied by itself equals 16 , and urged students to think back to the warmup ${ }^{6}$ (lines $194-198$ ). After orienting students to think back to this warmup, Mikayla recognized that -4 also multiplies by itself to equal 16 (lines 199). At this point, Ms. Collier then led the explanation of why the fact that both -4 and 4 square to equal 16 means that a number is not a function of its square (lines 200-209). She explained that because "the negative version of a number will result in the same square" (lines 201-202) as the positive version, a "number is not a function of its square because I can come up with more than one ... output for that specific input" (lines 208-209).

Here, Ms. Collier is drawing from the Illustrative Mathematics curriculum which puts forth that students should respond to this task by determining that a number is not a function of its square because knowing its square does not determine the number. Ms. Collier communicated these ideas to students by explaining this relationship is not a function because there is more than one output per input (206-209). She then asked the student to apply this idea by identifying the number which has a square of 36 (lines 209-212), which Dante immediately recognized as "6 and -6" (line 213) and, which is therefore not a function (lines 215).

The relationships and use of language students are required to synthesize for this task are very complex. The task required students to name a rule using technical mathematical language, the square root, which they were not yet comfortable with. Ms. Collier, who has taught many of these students in previous years reminded the students that "you've heard about [square root]

[^5]before. You've just forgotten" (Line 179). Then, it required students to recognize that the square numbers have both a positive and a negative square root, also an idea that they had just recently become reacquainted with during this lesson's warmup. Finally, the students had to tie together that because the input, or square number, has two outputs, or square roots, the input does not determine the output and therefore this relationship is not a function. While Ms. Collier carried most of the burden in defining and explaining the connections between these ideas, it is the students who identified key language and relationships along the way. Furthermore, it is precisely Ms. Collier's explanations that the students said they most valued about her mathematics instruction. Through her explanations, she was able to support students in refamiliarizing themselves with technical language and applying it to new concepts such as which relationships are represented by functions. She supported students in mastering the curriculum-based content without watering it down for them or providing the answers for them.

In contemporary reform-oriented mathematics education research, Walshaw and Anthony (2008) called this discourse practice Articulating Thinking and used it to refer to teacher practices that synthesize students' individual contributions to develop mathematical thinking. This practice involves bridging students' intuitive understandings with mathematical domain knowledge and is associated with desirable outcomes for historically marginalized students. Importantly, desirable outcomes refer to not only academic outcomes, but also student outcomes related to affect, communication, and participation. Ms. Collier enacted this practice other times during this classroom discussion, and did so in service of orienting students towards the multiple new concepts at play that they must synthesize to determine whether a number is a function of its square.

In closing, I point toward the immense complexity of the work Ms. Collier was doing by facilitating this whole class discussion online. When Ms. Collier checked her students' work and provided them feedback on their input-output diagrams written in their notebooks (Lines 132142). the entire duration of these interactions lasted only 20 seconds. That is, within those 20 seconds, Ms. Collier was able to read in the Zoom gallery view what four students ( $1 / 3$ of the class) had handwritten in their notebooks and held up to their laptop cameras. The students' diagrams that they had drawn were written in the lines of their notebooks as notes, they were not large diagrams drawn to be viewed by an audience. Additionally, as the students held up their notebooks, their cameras went in and out of focus and the students had to move the notebooks slightly in and out of position so it was very difficult to decipher what was written. I found their work quite difficult to read when watching the recorded videos to transcribe their interactions, and that is with the ability to rewind and pause the recordings. That Ms. Collier could see the students' writing in the moment, decipher what they had written, and offer them immediate feedback points toward her considerable skill in facilitating this discussion over Zoom in the virtual learning environment.

## The On-Level Section

In closing this results chapter, I briefly point toward how this task unfolded in the onlevel class. When discussing the same task in the on-level class, students similarly shared an initial rule of divide by 4 (however, they primarily used the chat rather than showing their notebooks). Ms. Collier then led them to use their knowledge of how to square a number to propose a new general rule for this relationship. With this guidance, a student, Naomi, answered "divide by itself" just like Jordan did in the honors section. In this section, Kianna introduced the language square root, which Ms. Collier then described as "so if we were to square root the
number, basically what we are asking ourselves, is 'hey, hmm self, hey self, what number did I multiply by itself to make $16 ?$ ?" Naomi identified the number four in response to this question, leading Ms. Collier to ask if there were any other options. She called on Naomi, Nyla, Marcus, and Jonathan who each said there were no other possible square roots of 16 . Before Jonathan shared, the last of the students to respond, Ms. Collier told him, "Jonathan. I bet you Jonathan knows because he was the person I talked to in the warmup. Jonathan, don't let me down J."

After he identified "it's just four" Ms. Collier herself went back through the slides they had already reviewed and circled -3 and 3 from the warmup, the same task in the honors class that asked the students to square a list of numbers. After circling these numbers, Ms. Collier asked "what happens when we square these two numbers?" to which Naomi responded, "it made positive nine." Ms. Collier then returned back to the original slide they were working on, and asked Naomi "what's my other option?" for the square root of 16 , to which Naomi responded "4." After Naomi provided this response, Ms. Collier concluded the discussion of this task, telling students:

Now, do you see how we have two different outputs for the same input? Does everybody see that? If you can come up with two or more outputs for the same input, you do not have a function. Let me lock that in there now. This is not a function. Okay? Not a function. And when you see the formal definition, you'll understand why, but I want people to lock that in now. Number three is not a function. Because I have two different outputs for one single input. That's a no, no. Okay?

After providing this conclusion, Ms. Collier moved on to the next task.
Although there are many similarities in how Ms. Collier facilitated the discussion of this task between the two sections, there are also some differences. In the next chapter, I discuss what
these similarities and differences in Ms. Collier's facilitation of the discussion mean for the students as mathematics learners. Specifically, I consider how Ms. Collier's facilitation of the discussion positions the students as learners of mathematics.

## Chapter 5: The Students' Positioning as Mathematics Learners

In this second part of my results, I explore how frequently observed patterns in the classroom interactions across the Functions and Volume unit positioned the students as mathematics learners. To answer this question, I analyzed the classroom interactions from a lens of how Ms. Collier's facilitation of the classroom discussions and cultivation of a warm and safe learning environment positioned students to engage in mathematics learning. Before considering the data, I first establish what positions and storylines are typically made available to Black mathematics learners, to better situate Ms. Collier's pedagogy and its impact on positioning Black mathematics students.

Martin $(2009,2012,2019)$ has shown that within education settings there exists a racial hierarchy of mathematics ability in which Black students are continually positioned at the bottom. The result is an unconscious normalization of the low achievement of Black students (Gutiérrez, 2008) that contributes to larger societal discourses about the meaning of Blackness in society, leading some to question "if, not how, Black children can learn mathematics" (Martin, 2012, p. 48). The resulting storylines about Black students that pervade mathematics education settings center on intellectual inferiority. Given this backdrop about mathematics education as a whole, I now turn towards how Black students were positioned in Ms. Collier's eighth-grade mathematics class, and what role, if any, do larger societal discourses about Black students in mathematics play in how her students were positioned.

In conducting this analysis, I draw from positioning theory and its constructs of positions and storylines as defined in Chapter 1. Specifically, in considering how students are positioned, I consider what "rights and duties they acquire, assume, or have imposed on them" (TaitMcCutcheon \& Loveridge, 2016). Tait-McCutcheon and Loveridge (2016) elaborate that "to
position someone means to collaboratively establish what their rights and duties are and determine what they are allowed or obliged, and not allowed or obliged, to do" (p. 329). Storylines provide the frame around the event and context in which the act of positioning occurred, and so they frame the people involved and the positions available for everyone (Davies \& Harré, 1999; Wood, 2013). In framing the event, context, and people involved in the positioning, storylines "can invoke existing narratives known to participants or introduce new narratives that may not be known to others" (Zangori \& Pinnow, 2020, p. 629).

To determine how the students in Ms. Collier's class were positioned as math learners, I consider what rights and duties were allowed or expected of them through the whole class discussions analyzed in Chapter 4. I also consider what storylines framed the positions. It is worth mentioning here what Wagner and Herbel-Eisenmann (2009) considered the contingent nature of positions and storylines. This contingency refers to the idea that positions and storylines are subjective and open to interpretation. That is, different people can interpret a single situation with different storylines. Because people bring their individual perspectives to their interactions, different storylines can emerge based on these unique perspectives, even given the same event. Furthermore, because each individual is entitled to the perspective that they bring, there is no way to establish a single correct storyline or position. Because individuals' perspectives differ, so too do their interpretations and positionings of a single event, and no single understanding can be established as the correct one. I mention this to say that the results below are $m y$ interpretations of the positions and storylines made available to the students in Ms. Collier's class. That is not to say they are the only positions and storylines available to students in the class, or even the singularly correct ones. It is simply to say that given the perspectives and
frameworks that I bring to this work, these results represent my understanding of the positions afforded to students and the storylines in which they occurred.

Given the inherent subjectivity of this analysis, I leverage the words of the participants as much as possible to ground where my interpretations come from. In this vein, I start these results by highlighting what Ms. Collier identified as the narratives that frame how she delivered the content in her classroom and what demands she made of her students.

## The Narratives Ms. Collier Perceives about the Mathematics Education of her Students

Ms. Collier elaborated on her rationale for why she shows such high expectations for her students. She explained that part of the reason she cares for her students through high expectations is because of the narratives that people hold about Black students in society. She explained that her expectations are "rooted in the fact that I just think people don't expect [my students] to sound so sophisticated and I want them to sound sophisticated, I want my children to sound sophisticated, that's how I feel, that's where I'm at, my expectations are through the roof." When asked to elaborate on what she meant, she explicated the relationship between narratives about her students and how she responds to it as a teacher:

I think that there's the stereotype put on Black children, especially those that live in poverty, that they're going to, you know, talk using urban vernacular all the time. They don't know how to use standard English, poor writing, poor language, poor grammar. The expectations are low, across the board, most of the time. So, for me, it's like, I get a kick and a rise out of proving people wrong, it's my favorite thing to do. What I've learned about children is they also enjoy that. They really, really like to be right and really, really like to show off. It's all about me, it's me me me. So, if you wanna do a me, me, me, do it
in something that's going to make you a better person in what you're doing. So, if what you're doing is practicing math, then you need to be as good as you can be at it. Other researchers have discussed how Black students navigate stereotypes in mathematics environments, particularly $\operatorname{McGee}(2013,2015)$ and her research on stereotype management. McGee and Martin (2011) have found that one way Black students manage pervasive stereotypes about their inferiority in mathematics is by proving the stereotypes wrong by performing behaviors acceptable in White middle-class academic culture, although they do not actually identify with them. Ms. Collier perceived the same narrative that Black students speak only in a stereotypical manner, lack the capacity to effectively communicate in standard English, and so are subjected to low expectations. Similar to those who enact stereotype management, this influenced her instruction by preparing her students to prove the stereotypes wrong. However, contrary to proving the stereotype wrong by performing behaviors that conform to White middleclass culture, Ms. Collier expected her students to resist the stereotypes by being "as good as you can be" at doing math. Ms. Collier did not perform White behaviors in service of her students doing math. Next, I consider how Ms. Collier's focus on having students perform at their personal best in mathematics while not simultaneously prioritizing dominant, White-centric culture, creates storylines about what it means to be both a Black student and a mathematics student.

## The Storylines Available in Ms. Collier's Mathematics Classes

As shown through Vignettes 1-5, Ms. Collier's instructional priority was about maintaining high expectations that students perform at their personal best level. In maintaining these expectations, Ms. Collier did not reify White cultural norms in service of her instruction. Instead, there were elements of Black culture, particularly through language, that were used in
service of the mathematics instruction. For example, in Vignette One, Ms. Collier greeted and welcomed students to class by asking them about their lives and personal interests. In doing so, Ms. Collier communicated to her students using Black English vernacular, like when she pridefully explained those born in the month of February were meant for greatness, "that's why they gave us the least amount of days, ya dig?" She not only communicated like this during the welcome to class, but also while instructing about mathematics. For example, when she responded to Mikayla's incorrect response by saying, "Times by multiple? Girl" or when she summarized their whole class discussion by explaining "the minute that I have one input and more than one output, it's a no go on the function. It's a no go. Okay? All right. Let's keep it pushing." That Ms. Collier did not contribute to the disparagement of Black English vernacular, but used it herself while simultaneously instructing students in mathematics, represents a significant shift from the narratives about the value of Black culture in mathematics education, and presents a new storyline about what it means to be a Black mathematics student in Ms. Collier's class. The emerging storyline that frames Ms. Collier's math class communicates that there is no tension between being a Black student and being a mathematics student. That is, Ms. Collier's classroom is framed by the storyline that Black culture is valuable and useful in the pursuit of mathematics education, and furthermore that Black students can openly enact their cultural identity while learning mathematics, and there is no tension between these relationships. These storylines disrupt the narrative that Black students are intellectually inferior and allow the breadth of positions available to Ms. Collier's students to be far-ranging, positive constructions of mathematics students.

## The Positions Available in Ms. Collier's Class

Given that storylines frame the context around which the position occurs, I now turn to consider what positions were made available to Ms. Collier's students as Black mathematics learners when they learned in an environment that valued their racial identity in the pursuit of mathematics learning.

To start, consider the interactions described in Vignette 2, and how they position Andrew. In Chapter 4, I used this vignette to demonstrate how Ms. Collier facilitated the classroom discussion in such a way that allowed Andrew to safely struggle through the simplification problem until he was able to determine and explain when the equation was simplified. Through Ms. Collier's interactions with Andrew, Andrew was positioned as a mathematical student who had the capacity to apply mathematical reasoning, make sense of the simplification problem, and provide an explanation for when the problem was simplified. While solving the problem, as Andrew became uncertain about how to reason through it, Ms. Collier scaffolded the instruction for him, continuing to support him as the focal student in the discussion while he publicly struggled through the problem. That Andrew maintained the role of the focal student despite his uncertainty about how to simplify the hanger positioned him as a student who had the right to experience confusion and nervousness while simultaneously persevering in solving the problem. That is, through Ms. Collier's interactions with Andrew, his confusion never eroded his capacity as a mathematical thinker; instead, Andrew was positioned as a student who had the right to experience a range of emotions and distractions while engaging in virtual learning in the midst of the pandemic, and still be capable of reasoning and explaining his mathematical thinking. Furthermore, after experiencing Ms. Collier's support and scaffolded instruction during Task 1, in Task 2 Andrew provided a thorough explanation about how to simplify a hanger, without any
scaffolding from Ms. Collier as he shared. His independent, robust explanation of how to simplify the hanger during the whole class discussion of Task 2 positioned him as a competent problem solver who could provide a refined explanation for his mathematical approach.

The interactions between Kobe and Ms. Collier, during Vignette 3, showed a different set of positions available to the students in Ms. Collier's class. The third vignette examined how Ms. Collier maintained high expectations for Kobe through her unwillingness to let him slide by. This manifested in the whole class discussion through Ms. Collier's refusal to let Kobe opt out of the discussion, her sometimes stern responses to Kobe's incorrect answers, and her open disappointment in his inattentiveness. I now consider how these interactions positioned Kobe as a learner of mathematics.

A central component of Ms. Collier's disappointment in Kobe was that he was not paying attention, which she explicitly told him repeatedly (see Chapter 4). By expressing this disappointment in him, Ms. Collier was positioning Kobe as a student who had the duty to actively listen to and learn from whole class discussions, even when he was not the focal student in the discussion. Because he did not fulfill his duty and instead was watching TV, Ms. Collier then became disappointed. Still, despite Kobe's inattentiveness and consequential confusion in completing the row in the input-output table, Ms. Collier refused to let Kobe opt out of the problem. Instead, Ms. Collier positioned Kobe as a student who was obligated to continue intellectually engaging in the problem even though he had been inattentive and did not know how to approach the problem.

Worth recognizing here is that given Kobe's inattentiveness and inability to recognize how to approach this problem, never did Ms. Collier position Kobe as mathematically incompetent. That is, Ms. Collier's criticism in Kobe stemmed from his not paying attention to
the parts of the lesson that she had just reviewed, never was it rooted in the fact that he was mathematically incompetent for not knowing decimal place value. She told him this when she explained that she was disappointed in him because she had just reviewed decimal place value with Andrew. This is further evidenced by how she interacted with Andrew in the discussion that preceded her conversations with Kobe. When Andrew did not know how to recognize the tenths place in the whole number, Ms. Collier did not express disappointment in him. Instead, she turned toward another student, Janae, to identify the solution, and then proceeded to explain how to recognize the decimal place value to Andrew. In the preceding discussion, Andrew was again being positioned as a student who had the right to experience confusion in solving the task and deserving mathematical explanations on how to solve the problem.

That Kobe then received a sterner lecture from Ms. Collier in the subsequent discussion points towards her expectation that students must learn from the whole class discussions with other students, not that they must know the content coming into the lesson. She was not disappointed in Kobe for not knowing decimal place value; she was disappointed in him for not paying attention and applying the reasoning she shared with Andrew earlier. Therefore, rather than positioning Kobe as incompetent for not knowing decimal place value, she positioned him as a student who had the duty to listen to and learn from his peers. Her disappointment in him stemmed from his failure to pay attention, not his failure to know the content.

Ms. Collier explained why she felt this level of disappointment from her students: At the heart of my frustration in I feel like I'm not getting 100 percent effort from kids, and I know that they're better than what they're giving me. That's my frustration. That they are better than what I am getting. And I've seen their best. That's the thing. And like I said, I've taught these kids. So, I've seen your best.

Again, worth pointing out here is that Ms. Collier's frustration and disappointment in her students did not stem from any perspective of her students as lacking or incompetent, rather it stemmed from her expectation that they had a duty to give their best effort. This is important because many of the narratives about Black students in mathematics education emphasize what they do not know and cannot do, for example, by emphasizing their standardized test scores compared to White and Asian students to normalize that they are mathematically illiterate (Martin, 2012). However, in Ms. Collier's class which was shaped by a storyline of accepting and welcoming Black racial identity in pursuit of mathematics learning, an eighth-grade mathematics student struggling with identifying decimal place value was never framed as an issue of mathematical incompetence and her students were never positioned as mathematical illiterate. Instead, these issues were framed as an issue of attention and effort with her students being positioned as having a duty to give their best effort in class and pay attention. When the students did not fulfill this obligation, Ms. Collier then expressed her disappointment in them and sometimes sternly corrected their inaccurate responses.

In terms of the occasionally stern ways Ms. Collier responded to Kobe, Ms. Collier recognized that this could be interpreted as harmful to students. She elaborated on why it was that she spoke to students in this manner, and why it was neither intended nor received as harmful:

I think that if you were just an outsider looking in, and if you had no context on me and you've never really watched my class, it might seem like my whole purpose is to intimidate people's children. It might seem like I'm the biggest bully of life. But my students know, and they never feel bullied because they know that ... I'm coming from a
place of love. Everything that I do and I say... you know I love you. So, you know that I'm going to have to talk to you harshly sometimes.

Referring to the background "context" needed to make sense of how it was that Ms. Collier could be stern with students without being intimidating or bullying, Ms. Collier explained that the foundation which undergirded all of her interactions with students was love. It is why she could show frustration with Kobe and it was not intimidating. It is also why she could demand his sustained effort, which he fulfilled, in a virtual environment when it was very easy for students to exit the Zoom classroom at their will. This foundation of love, which is evident in how she started class by building relationships with students as shown in Vignette 1 or warmly supporting Andrew as he persevered in a problem he was struggling with in Vignette 2, afforded Ms. Collier the ability to position her students as owing her their attention and best effort, and receiving her criticism when they did not fulfill this obligation.

## The Positions Available to the Students in Honors Math Class

Vignettes Four and Five both center on the honors class and how Ms. Collier facilitated discussions with that group. Starting with the fourth vignette in Chapter Four, I used this to describe the very different way Ms. Collier enacted Task Three as compared to in the on-level class described in Vignette Three. I now consider how the instructional choices she made while enacting Task Three positioned the students in the honors class; in the next section, I consider how the positions available to the students in the honors class compared with those available to the students in the on-level section.

Ms. Collier's set up of Task Three and her facilitation of the whole class debrief positioned the students in the honors math class as competent problem solvers with the capacity to reason through the input-output table independently. This position was made available to her
students through her choice to have students complete the entire table independently, rather than row-by-row in a whole group discussion. Because she did not lead the students in solving each row with the whole group, each student had the opportunity to think through the individual input on their own and make sense of how to identify the tenths place with the different types of numbers, including fractions, decimals, and whole numbers, without any support from Ms. Collier. Furthermore, the students submitted their answers to her in the private chat which allowed her to provide verbal, individual feedback such as "Emily, that first answer is incorrect, my dear. You might want to think about that again" (see Chapter 4). Using the chat to receive student answers and provide general feedback to each student positioned the students in the room as capable of independently recognizing what specific error they made and how to fix it.

Vignette Five centered on how Ms. Collier handled student confusion in the honors class, by bridging between students' intuitive understandings of the concepts presented in the Illustrative Math curriculum. In facilitating the discussion in Table 9, Ms. Collier guided students to infer how to refine their ideas, without explicitly telling them their errors and how to fix them. This occurred when she asked three students questions so they would recognize their rule must be generalizable without explicitly telling them so and when she elicited comments from students that allowed them to recognize the inverse relationship between the square and the square root. By handling student confusion in such a way that asked students orienting questions and elicited the nature of the mathematical relationships from them, without explicitly telling them, Ms. Collier positioned the students in the honors class as competent problem solvers who were capable of making mathematical connections (Turner et. al., 2012) without explicitly being told by the teacher. In this final section of the chapter, I compare the positions available to the students in the honors and on-level sections.

## A Comparison of the Positions Available to Students in the On-Level and Honors Sections

The most obvious distinction in how Ms. Collier's students are positioned is through the naming of the two class sections. That the students' math classes are publicly named and identified as "honors" and "on-level" positions students in such a way that suggests a hierarchical difference between the students in each class. Oakes (2005) detailed how the public nature of tracking leads to the stigmatization of students in lower tracks, negatively affecting their self-esteem. Although Ms. Collier teaches all of her students from a foundation of love and positions all of her students as mathematically competent, the students in the on-level class may still experience the stigma of being in the lower track.

Another lens of comparing the positions available to the students in the on-level class versus the honors class considers the support structures Ms. Collier provided to her students. For example, in Vignette Two which focused on Andrew in the on-level class, the positions made available to him include his capacity to make sense of and persist in mathematical problem solving through Ms. Collier's scaffolded instruction and emotional support. After Ms. Collier's thorough support in Task 1, Andrew was positioned as a competent problem solver who could provide a robust and refined explanation in Task 2. Similarly, in Kobe's interactions in Vignette Three in the on-level class, he was positioned as a student who had a duty to pay attention to and learn from classroom discussions, and continue intellectually engaging in the problem until he succeeded. Each of these positions described in the on-level class show how Ms. Collier supported her students to persevere and succeed in mathematics problems during whole class discussions.

While the positions available in the honors section similarly point toward her students' capacity for mathematics success, the positions have a different orientation in terms of the type
of support Ms. Collier provided students in their learning. In the honors class, in Vignette Four, the students were positioned as competent problem solvers with the capacity to solve the problem independently. Similarly, in Vignette Five, Ms. Collier positioned her students as competent problem solvers capable of making mathematical connections by eliciting descriptions of mathematical relationships from them.

In Ms. Collier's own words, she described how the nature of the instruction she provides to students differs across the two sections:
[The on-level] class becomes the Ms. Collier show, right? I would love to hear more of their voices but it becomes a teacher talk, teacher centered environment whereas in [the honors class] I'm able to be more of a facilitator in their learning.

Vignettes Three and Five provide evidence of these differences in the instructional environment. In Vignette Three, Ms. Collier spent just under four minutes directing Kobe on how to solve just one row in the input-output table. During this vignette, Ms. Collier asked him specific questions designed to gather information from him and orient him to focus on key elements of the problem context (Boaler \& Brodie, 2004). These questions were a vehicle for Ms. Collier to correct Kobe's misunderstanding of place value and input-output relationships. That the discussion in this vignette primarily centered on Ms. Collier's leading Kobe through making sense of completing the input-output table may point towards the direct instruction, teacher-centered environment that Ms. Collier described as characterizing the on-level class. These questions also point toward the type of support I described in the on-level section which involved a very scaffolded instructional approach.

On the other hand, in Vignette Five, when the students in the honors class become confused about whether they could identify a unique number with the description "the square of
$16, " \mathrm{Ms}$. Collier still oriented them on how to answer the questions, but she did so by asking different types of questions. In orienting students to make sense of the problem, she asked them to check if their rule generalizes to work with other inputs and she asked students how they might do the opposite of squaring a number. Once students proposed a rule to describe this inverse relationship, she asked them for the correct mathematical language to describe it. Among other things, these questions Ms. Collier asked the students in the honors class encouraged them to explore mathematical relationships, such as between inputs, outputs, and generalizable rules as well as between squares, square roots, and functions. Ms. Collier's questions allowed students to refine their rules, identify and apply mathematical language, and apply their understandings of square numbers and functional relationships to a new input. That students were able to take up and apply these new relationships through the discussion illustrates Ms. Collier's self-described role as a facilitator in students' learning in the honors class.

Ms. Collier's role as a facilitator in this discussion also encapsulates the distinct type of support Ms. Collier provided the students in the honors class. That is, while one facet of Ms. Collier's support for students in the on-level class involved her scaffolding instruction for them so they were able to succeed in the curricular tasks, in the honors class her support involved facilitating students' learning so they were able to explore and apply the underlying links between mathematical relationships (Boaler \& Brodie, 2004). I explore what these different types of support may mean in terms of the students' mathematics education in the Discussion Chapter.

## Similarities in Students' Positions Between the On-Level and Honors Sections

Although there are differences in how the students were positioned in the on-level and honors classes, there are also similarities in how Ms. Collier positioned all of her students. The
way Ms. Collier built relationships with students, supported them in succeeding in the content, demanded their sustained effort, and accepted and enacted Black culture, particularly through language, all positioned her students as human. That is, by cultivating a classroom where each of her students, who are predominantly Black students, were cared for simply because they deserved to be cared for, not only in service of learning mathematics, communicated to the students that they belonged as members of the classroom community and also as learners of mathematics with the capacity to engage in and learn the content. This classroom culture is the antithesis of the antiblackness in mainstream mathematics education that constructs blackness as irreconcilable with humanity. That Ms. Collier's students had the freedom to be confused, to speak Black English vernacular, to experience stress and joy, to not pay attention and still deserve to learn, and to be supported as they made sense of mathematical relationships disrupted the antiblackness evident in educational environments and instead positioned her students as human.

That Ms. Collier positions her students as human is all the more important because there is no neutral. For Black students, they are either in a humane learning environment, where they feel affirmed, seen, and valued, or they are not. In the humane learning environment, they can just worry about learning mathematics without having to worry about the racism that pervades the context. The opposite is that students have to be guarded, and they have to spend mental and emotional energy navigating that. Ms. Collier provides an example of what it may be like when a teacher can lift this burden for them, within the confines of her classroom.

## Chapter 6: Discussion

The purpose of this study was to examine the practice of a veteran mathematics teacher, to understand how her teaching practices were rooted in the historical legacy of Black teachers, and to understand the impact of her teaching practices on positioning students. The findings show that Ms. Collier tapped into historical legacies of Black teachers by deeply caring for her students in nuanced and multifaceted ways. Her care for her students manifested in a teaching practice that supported them in navigating the distractions of learning in the virtual environment while also demanding that they made sense of and understood mathematical relationships. It manifested in a teaching practice that allowed students to experience a range of emotions, from nervousness to confusion to joy, all the while being perceived and treated as competent learners with the capacity to persist and succeed in solving mathematics problems. Each of these facets of how Ms. Collier cared for her students were grounded in her firm belief that her students were capable of learning mathematics, and maintaining high expectations that they do so. Finally, her care manifested in a teaching practice that provided an environment where students belonged and were cared for, because of their intrinsic value as people, not because of their potential as mathematics learners.

Although there were slight differences in the positions available to the students in the onlevel class compared to the honors class, all of Ms. Collier's students were positioned as human. Rehumanizing mathematics classes is an urgent need in mathematics education today, given that mathematics learning environments are often experienced as dehumanizing for Black students (Goffney, 2018; Gutiérrez, 2018). That Ms. Collier, and the historical Black teachers whose practice she tapped into, enacted practices that are consistent with findings of mathematics education research today highlights the historical and contemporary brilliance of the Black
community. In this discussion chapter, I overview what can be learned from this study of Ms. Collier's practice in terms of connections with other research, implications about teachers, and implications of tracking.

## Connections with Other Research

While I situated Ms. Collier's practice in a historical practice of Black teachers caring for their Black students, there exists other research and language which helps make sense of her pedagogy. This research and language, including fictive kinship and othermothering, refers to the very deep community ties between Black teachers and Black students, which allows their relationship to develop a sense of kinship. Foster (1994) explained that fictive kinship refers to the connectedness within Black communities, allowing the teacher-student relationship to reflect a strong sense of mutual obligation that enables non-kin, including teachers, to take on parental social roles in students' lives. One such form of this relationship includes othermothering, which refers, again, to the connectedness between Black teachers and the community, as well as the teachers' use of kinship terms when referring to their students, and their subversion of the school curriculum by equipping Black students to use their education to enrich their lives, challenge the status quo, and oppose ignorance (Dixson, 2003). Frank et. al. (2018) explained that this sense of kinship can be expressed in multiple ways, including through very nurturing relationships and through tough love that maintains high expectations. This sense of kinship cultivated a classroom environment that made space for students' mathematical risk-taking by protecting students from mainstream educational narratives which construct Black students as intellectually inferior (Frank et. al., 2018).

Although I emphasized Ms. Collier's practice as a practice of care, this sense of kinship was present in her class. Worth making explicit here is where the mutual obligation and deep
community ties between Black teachers and Black students stems from. This communal sense of obligation stems from what is at stake in the education of Black students; Black children must be taught so they have they have the knowledge base to support and further Black people's collective cause and survival in an antiblack U.S. society. With these immensely high stakes in the lives of both Black teachers and students, Black teachers like Ms. Collier are able to take on parent-like roles allowing them to make demands of their students, as well as to nurture and care for them in their pursuit of learning.

## Connections With Mainstream Mathematics Education Research

In understanding Ms. Collier's practice as an exemplary mathematics teacher, the way Ms. Collier taught mathematics also largely aligns with recommendations from current mathematics education reformers (Boaler \& Greeno, 2000; NCTM, 2014; National Research Council: Commission on Behavioral and Social Sciences and Education, 2004) To start, Ms. Collier taught from the Illustrative Mathematics curriculum, a problem-based curriculum aligned with the CCSSM. Most of the problems Ms. Collier taught throughout the unit were high cognitive demand problems, that expected students to engage in complex thinking to solve nonroutine problems (see Table 3 in Chapter 3). In using these problems during instruction, mathematics education reformers recommend that teachers and students co-construct knowledge whereby students actively engage and grapple with the concepts at play in the problems. In this learning environment, the teacher operates as a facilitator, gently guiding students towards the relevant ideas to consider as they navigate the problems. For the most part, this type of learning environment characterizes how Ms. Collier facilitated instruction in the honors math class. As Ms. Collier herself noted, in the on-level class she took a more active role in leading students in how to think through the problems, with much scaffolding. Ms. Collier attributed the different
instructional environments in each section to differences in the students' mathematical foundations.

The broader tensions about tracking in Ms. Collier's classes will be discussed in the Implications about Tracking section, but here I consider Ms. Collier's comments about her role as either a facilitator or direct instructor as a consequence of her students' mathematical foundations. To understand how Ms. Collier's perceptions of and responses to differences in students' mathematical foundations relate to reform mathematics education research, I draw from Horn (2007) who explored similar tensions that teachers navigate in a school with detracked mathematics classes. In Horn's (2007) study, when discussing the mismatches between students in the heterogenous classes, a teacher distinguished between the fast and slow kids in her class and wondered how she might be able to mitigate their differences during instruction. The solutions offered by the other teachers in the discussion involved disrupting status issues, for example by problematizing the framing of students as fast and slow, and adapting the curriculum to use group-worthy problems, which would encourage the kids identified as "fast" to slow down and consider how other students approach the problem. Broadly, these solutions about disrupting status issues and using complex tasks that allow for multiple entry points and promote collaboration are consistent with what reform-oriented mathematics education research would recommend (e.g., Cohen \& Lotan, 1997; Featherstone et. al., 2011).

However, considering these solutions in terms of what Ms. Collier described as different mathematical foundations seems to still leave something amiss. Although, the school in Horn's (2007) study is detracked and Ms. Collier discussed these issues in terms of two differently tracked classes, the central issue in both cases is about mismatches between students and between students and the content. Recall that in this dissertation study, this issue was originally
discussed in terms of Kobe in the on-level class, and his difficulty identifying the tenths place in a whole number, leading to explanations and lectures from Ms. Collier. This was juxtaposed against the students in the honors class who solved this same problem (and more) independently and with a very limited, cursory discussion of the task before moving on to the next one. Given these occurrences, detracking the classes and disrupting the framing of the students in the two sections might do well to equalize the hierarchical nature of the two sections. Additionally, allowing for heterogenous groups might allow students like Kobe to learn from his peers, although the status issues may present themselves in the small peer groups (Esmonde \& LangerOsuna, 2013). Still, Ms. Collier pointed toward a very real issue of practice when teaching students rigorous mathematics curriculum, namely how best to enable students with different "foundations" to engage with the content. While there is truth to the interconnected, processoriented structure of doing mathematics (Horn, 2007), Kobe's not knowing place value posed a very real challenge in being able to solve the input-output table. Ms. Collier's decision to use direct instruction in such situations is worth taking seriously when grappling with how to best teach students mathematics.

## Implications about Teachers

A focal aspect of this study is the particular way Ms. Collier constructed a learning environment based on caring relationships and high expectations that allowed her to make demands of her students that ultimately supported their success. In highlighting these facets of Ms. Collier's teaching, I show how her teaching practices are rooted in the historical work of Black teachers, and how she positioned all of her students to engage with the problem-based curriculum. Although these are the frameworks that I applied in understanding and interpreting Ms. Collier's classroom, Ms. Collier offered her own explanation for the rationale behind her
instructional decision making. In her own words, Ms. Collier explained why she makes such high demands from her students:

It goes back to knowing these kids and knowing what they're capable of and their maximum potential. I ask my children all the time, why be average when you could be extraordinary? I don't want you to be basic, I don't what you to do the bare minimum, I want you to go above and beyond because you're capable of doing so. And if you're going to go above and beyond in mathematics... you want to use the language that people use because that is the expectation.

For Ms. Collier, her high expectations stemmed from her foundational belief in her students and their intellectual capacity. She did not push her students to give their best effort to pass mathematical exams or compete for access to lucrative careers. Rather, she expected her students to give their best, to be extraordinary, simply because they were capable of doing so. Although Ms. Collier explained her high expectations in terms of her own personal belief in her students, her perspective is still closely related to the pre-Brown African American teachers who maintained high expectations for their African American students because they believed the sky was the limit for them (Siddle Walker, 2001).

What then might teacher education and other teachers have to learn from Ms. Collier and her perspectives? It has been theorized that mathematics success and intelligence represent an opportunity for personhood for Black students (Shah, 2022), whereby Black students perceived as intelligent in mathematics may be granted a reprieve from the antiblack context of mainstream mathematics education in the United States. However, what is significant, and perhaps most impactful, about Ms. Collier's instruction is that in her classroom, intelligence is not a
prerequisite for humanity, rather her students' humanity is the foundation upon which she builds her pedagogy.

Mathematics teacher education might learn from Ms. Collier how to foreground the humanity of Black students and allow expectations of mathematics performance to be the consequence of their humanity, rather than the prerequisite. What occurs in mainstream education is the reverse. In mainstream education, consistent attention to achievement gap research highlights lower standardized test scores for Black students compared to White and Asian students. This hyperfocus on test scores perpetuates deficit narratives that Black students lack the capacity to understand and succeed in mathematics, constructing the students as inherently unintelligent (Martin, 2012). These dehumanizing narratives about Black students as intellectually inferior become normalized, reducing Black students' status as humans.

To resist these antiblack norms in mathematics education would require the reversal of this cycle. Learning from Ms. Collier, it requires cultivating learning environments where the humanity of Black students is centered. In Ms. Collier's classroom, centering the humanity of Black students meant building relationships with the students which showed she cared about their interests and lives outside the classroom; socially legitimating Black identity and culture (through language) as valuable in the teaching and learning of mathematics; providing students with strategies to cope with and overcome discomfort; and allowing students to experience a range of emotions like confusion, nervousness, and joy. After Ms. Collier structured her classroom to support Black students and their humanity in these ways, she was then able to make demands of her students to support their mathematics learning.

## Implications of Tracking

Tracking in schools is often used as a system against Black students in which they are disproportionately tracked into lower-level courses and denied access to the type of mathematics often necessary for post-secondary success (Clark et. al., 2013; Martin et. al., 2019; Oakes, 2005). Given the function of mathematics education as a gatekeeper, in the lower tracks Black students are often exposed to a remedial, rote curriculum which does not educate them with the knowledge and skills needed to pass the mathematical gates. In terms of Ms. Collier's on-level and honors classes, it is worth considering how the differently tracked classes in Hummingbird Academy differentially privileged and disadvantaged her students, who are predominately Black.

As discussed in Chapter 5, there were differences in how students were positioned in the different sections. Namely, the students in the on-level class were positioned as students with the capacity to persist and succeed in mathematics problems through Ms. Collier's scaffolded support, while the students in the honors class were positioned as competent problems with the capacity to independently problem solve and make mathematical connections. The students' positioning across the sections points towards the differential learning experiences available to students in different tracked classes already established in the research literature. However, aside from the variation in how Ms. Collier facilitated the discussions and thus positioned her students, there are also important similarities across the two sections which offer insight into the consequences of tracking in this particular case study.

In both the on-level and honors sections, Ms. Collier taught from Illustrative Math, which is described as a rigorous, problem-based curriculum that is fully aligned with the CCSSM (Illustrative Mathematics, 2021). In both sections, students engaged in the same problems and Ms. Collier supported both groups to experience success in solving the problems, although there
were variations in the type of support as discussed in Chapter 5. That the students in the on-level class had access to and learned from the same problem-based curriculum as the students in the honors class presents a new understanding of how tracking manifests in schools than what has previously been discussed in the literature (e.g., Oakes, 2005). Ms. Collier's usage of the same curriculum for both classes, and support of students to engage with and succeed in both classes, also evidences the high expectations she maintained for her students, specifically in terms of expecting the on-level students to engage with the same curriculum as the honors students.

Given that the students in both sections engaged with the same curriculum, a relevant question then is what would it take for the students in the on-level course to engage with the curriculum with similar types of support as the students in the honors class? And relatedly, why is it that the same teacher, with the capacity to facilitate whole class discussions in a way that fosters students' independent mathematical thinking, supports the students in the different sections in distinct ways? Recall that this is a teacher who loves all of her students, cares for them as mathematics learners and more broadly as people, and cultivates an environment in which she builds meaningful relationships with them. With the identity and perspectives that she brings to her teaching practice, why then is it that there are differences in how the students were positioned in the two sections, and what would it take for her to be able to facilitate the discussions the same way in both classes.

In response to these questions, mathematics educations reformers (e.g., Boaler \& Staples, 2008) would likely advocate for detracking, arguing that heterogenous learning environments offer students the opportunity to learn from and with their peers, which in turn can provide students with positive learning experiences and improved outcomes. Although students in both Ms. Collier's on-level and honors classes reflected positively on how Ms. Collier supported their
learning and their enjoyment of her class, it is the case that the students in the different sections were positioned differently as mathematics learners. Specifically, the students' positions in the honors class, as mathematics students capable of problem solving and making mathematical connections independently, would likely advantage them in terms of succeeding in such a way that allowed them to pass the mathematical gates of K-12 schooling. Therefore, advocates for detracking would likely argue that in heterogenous classes, students currently in the on-level class would potentially have the opportunity to learn from and with the students currently in the honors class, thereby disrupting the advantages available to the students in the honors only section and equalizing the opportunities available to the students in the on-level section.

Although dated, Noddings (1994) offered a different and relevant perspective on these questions in terms of the different types of instruction in the on-level and honors classes and how (or whether) to equalize them. She began her argument by explaining that the mainstream educational response to equity is to work to ensure that all students succeed in mathematics so they may access high-paying careers and overcome societal inequality. She contended, however, that "the solution to poverty cannot be found in teaching everyone algebra and geometry. The solution lies more obviously in moral education than mathematical education" (p. 91). She questions that by applying such immense pressure to students to succeed in mathematics to improve their life chances, which of their talents will go unnoticed and underdeveloped at the expense of learning mathematics? Therefore, she did not argue for status-based and hierarchical school tracking, but rather different tracks available to students based on their interests. For such an educational structure to work, it would rely on a society in which all students and workers are granted respect and fair compensation for their contributions, rather than reserving these basic
dignities only for those capable of achieving the most selective occupations and status. This, of course, is not the society we live in.

This brings me to distinguish then between the purpose and the function of schooling. Noddings (1994) offered a vision for what the purpose of schooling could theoretically be, in a community where all people deserved and were granted respect and compensation for their work. In such a community, the distinctions in how Ms. Collier's students were positioned across the two sections might mean less, particularly given that the students in both of Ms. Collier's classes were respected, cared for, and supported to succeed in engaging with the curriculum. However, given the function of schooling in the society that we live in, the distinctions in the positions available to the students across the sections may disadvantage the students in the on-level section when in terms of competing to access post-secondary opportunities.

## Limitations

A significant limitation of this work was conducting it during the Covid-19 pandemic and collecting all data virtually. This posed a limitation in terms of being able to carry out the initial study that I designed. The study I initially proposed involved examining two cases on the learning experiences of African American students, in two differently financially resourced schools serving different student demographics. The purpose of this study was to examine how the salience of race in mathematics instruction varied across social class. Due to difficulty conducting and recording virtual observations in the second school, serving the more affluent students, I had to reimagine and redesign this study to examine the case of an African American mathematics teacher in a charter school, and how she served her students.

Although different than the original study I proposed, focusing only on one school site, comprised of predominately African American students and staff, afforded me the opportunity to
examine and understand in what ways learning mathematics in this environment supports students. Additionally, this avoided concerns of my initial study which would draw comparisons across predominately White and Black environments. Although certainly not my original aim, making comparisons across these environments could mask some of the assets to be found in the predominately Black school, which may not have been as apparent using a theoretical lens which draws comparisons rather than focuses on one site alone.

## Future Research

In concluding this study, potential themes arise which may be further explored in future research, particularly in terms of the relationship between the very practical nature of classroom teaching that was studied and the theoretical analysis conducted to understand the practice. When considering Ms. Collier's pedagogy from the lens of what Black teachers have historically done to support Black students, Ms. Collier's work readily aligns with the research perspective, as shown throughout Chapter Four. That is, much of Ms. Collier's instruction, including the way she cared for students by supporting them, demanding their effort, and maintaining high expectations that they engaged with the curriculum, is deeply aligned with how Black teachers have cared for Black students historically. When using the perspectives and practices of historical Black teachers as the framework, there is considerable alignment with Ms. Collier's pedagogy.

However, other theories were also considered to make sense of Ms. Collier's pedagogy. For example, Ms. Collier's practice was connected to the theory of rehumanizing mathematics education, which showed some alignment between Ms. Collier and the theory, but also some constraints on the extent to which Ms. Collier enacted the theory, particularly through her use of the reform-based Illustrative Math curriculum. Additionally, earlier in this discussion chapter,
similarities were drawn between how Ms. Collier facilitates instruction and the recommendations of a reform mathematics education approach. While Ms. Collier does enact many of the recommendations of math education reformers, a central facet of reform instruction is the whole class discussion (Wilson et. al., 2019) and specifically students' robust contributions to the discussions in which they provide the mathematical explanations and build upon each other's contributions. However, the robust explanations in this study were typically provided by Ms. Collier, with students actively making sense of the explanations she provided. My point here is that when considering Ms. Collier's pedagogy from the perspective of these theoretical lenses, there are disconnects between the theory and what occurred in Ms. Collier's practice.

These disconnects between theory and practice are worth further attention in future research. These theories, particularly in terms of rehumanizing mathematics education, provide a thoughtful and transformative perspective on how to reimagine mathematics education to humanely serve Black, Latinx, and Indigenous students. Ms. Collier also provides an example of how to humanely teach mathematics to predominately Black students. Continuing to explore and grapple with how theories designed to reimagine mathematics may manifest in practice, and what educators in the field are doing that that we might learn from, offers a rich research area to learn from with powerful potential to serve Black and other historically marginalized students.

## Appendices

## Appendix A

## Teacher Interview Protocol

1. Why did you decide to teach at Hummingbird Academy? I believe throughout your career you've been in [school district], why did you choose this teaching environment?
a. How has teaching math changed after the pandemic started? Were you given resources from the school/district to help teach online? Were the students?
2. What kind of relationship do you have with students and their parents?
a. When I chatted with Kianna, she mentioned that you called her \& her sister daughter, and that you've called other students "son" in the past. I've also heard you broadly refer to your students as "my children." How did you come to call them that?
b. Kianna \& Mikayla both referenced their different experiences with math, \& how Kianna prefers reading while Mikayla prefers math. Are there differences in their experiences as students that you have noticed?
i. Kianna mentioned that she used to stay at your house after school. Have you done this with other students? Do other teachers at HA do this with students?
3. While I've been in your class, I've noticed that nearly all of the students are African American, \& that staff that I've seen are also African American. Is it the case that the staff is predominately Black?
a. Do you think that the fact that the school \& staff is predominantly Black has any impact on student experiences? Student learning?
b. You mentioned that for high school you went to one of the top 2 magnet schools in Chicago. Was your learning environment similar to your students? In what ways? How were they different?
4. Describe what a successful mathematics student in your class is like. What makes them successful?
5. What do you expect for your students in terms of your class? What do you want them to get out of your class?
6. In what ways do you think being successful in your math class matters for your students? Do you think it has any importance for your students in terms of after 8th grade, like going to high school? Or in terms of career when they graduate high school?
7. Who do you want your students to be in terms of math and in life?
a. Do you think that learning math plays any particular role in your students \& their future learning? Does it open any doors?
8. What do you see as your role in your students' lives?
9. In a previous interview, we were discussing your students' mathematical language use. You were referring to your students \& you said: "I don't want you to do the bare minimum, I want you to go above and beyond because you're capable of doing so. And if you're going to go above and beyond in mathematics, then you want to sound like a mathematician. You want to use the language that people use."
a. And when you explained why this was important to you, you explained that: "It's rooted in the fact that I just think people don't expect them to sound so sophisticated and I want them to sound sophisticated. I want my children to sound sophisticated. That's just how I feel. My expectations are through the roof.
b. Could you talk to me a little bit about that idea? Specifically, that you don't think people expect your students to sound so sophisticated $\&$ what you do as a teacher to respond to it.
c. Do you think that for your students being able to use mathematical language \& "sound sophisticated" will have an impact on their future experiences as students? Maybe in terms of high school or college?
10. We're getting ready to wrap up, but in the end here I wanted to ask a little about what I've heard from students in the past week of interviews.
a. Asked specific questions based on the student interviews for clarification from Ms. Collier's perspective.

## Appendix B

## Student Interview Protocol

## Guiding Questions

## Script:

So, I wanted to take a minute to introduce myself. My name is Tarik Buli \& I am a graduate student at UMD. I'm working on a study that focuses on the opportunities that students in [school district] have to learn math.

So, I spent a few weeks in February observing in Ms. Collier's class to better understand how she teaches math to you guys. Thank you so much for agreeing to be in this interview, I wanted to ask about your experiences learning math!

1. So first, I'd like to start by getting to know a little more about you, so could you please tell me a little about yourself - like what grade are you in, how are you liking your school, and how are you like as a student?
a. If I were a new student, and you were giving me advice about what it's like at your school, what would you tell me? Would you have any advice for me?
b. In what ways do you think this school and the students are different or the same from students in other schools?
c. Why did you \& your parents choose for you to go to this school? How do you feel about that?
d. How are your parents involved in your math?
e. Do you have experiences, outside of school, that have helped you participate in math? If so, what are those experiences and how do they help you in math class?
2. So now I'd like to hear more about your experience in Ms. Collier's math class- how's it going? What's it like for you so far? What do you like, what do you dislike about your experience so far?
a. Describe a typical day in Ms. Collier's class. How do you feel when you are in class? How do you feel about what you are learning in class?
b. What if I was a new student in Ms. Collier's class, would you give me any advice about what it's like and how to do well?
c. (Follow-up) How do you like the way that Ms. Collier goes through problems in class? How do you like the way that she calls on students to participate?
d. (Follow-up) Do you usually feel like this about your other math classes? How did you feel when you were in other schools or had other teachers?
3. Thinking more specifically about your experience in the classes so far - is there anything you've noticed while in class, such as a particular moment that stands out in your mind? Do you have any favorite memories of your class? Maybe a story you've told to your friends or your family about classes so far? Or just something interesting, or a moment you were proud of?
a. (Sometimes follow-up) What do you think of your classwork? How about the homework, quizzes, and exams? How have they been so far?
b. (Sometimes follow-up) How have you been studying for class? On your own? With any help or other resources?
c. I also want to ask about participation in class. How do you feel about participating in class?
i. Do you feel comfortable participating in class?
ii. If you had to categorize your classmates how would you do it? Who are the top students in your class?
iii. Are there any students in your class that stand out to you?
4. We're just about done, but I want to ask a few more broad questions about your experiences before we finish.
a. How has learning math changed after the pandemic started? What do you think about online instruction?
b. How has your math class changed from previous years to this year? Did you have Ms. Collier in previous years?
c. What does learning math mean to you?
d. As an 8th-grader, what do you think about your education in the future? Are you making any plans for high school?
i. How about after high school?
ii. Does learning math play a role in any of these plans?
5. In general, how would you describe yourself as a math student?
a. (Follow-up) Have you always felt this way, or has it changed in different grades or with different teachers?
b. (Follow-up) How would you describe your comfort or confidence with math?
c. Is there anything else you'd like to tell me or think I should know in terms of your experience in the course so far?
6. In terms of demographic information, would you tell me your age, and the race or races with which you identify, and any countries?

## Appendix C

## Focus Group Protocol

Focus Group: $\qquad$ Pseudonyms: $\qquad$ Date: $\qquad$

Start Time: $\qquad$ Ending Time: $\qquad$
Prompt: I am going to ask the group a series of questions one at a time. For these questions, I want you to reflect on your experiences in Ms. Collier's class. There are no right or wrong answers. This focus group will help me to better understand your personal perspectives and experiences.

This focus group will last about 30 minutes. I will record this focus group for accuracy. The recording will be transcribed (your words are typed out). I will use your words as data. I will not share your recording with anyone from the school. Please feel free to be open and honest.
[Turn on audio recording device and test it.]
Thank you for agreeing to participate in this focus group. I'm going to ask the group some questions. Please answer them aloud to the group. Feel free to talk to or respond to another student or you can direct your comments to me. Try to speak loudly so everyone can hear you and so that your thoughts will be captured on the recording. Let's go ahead and start now.

1. I wanted to start by asking about what you think of online instruction. Some of you had a chance to tell me in our previous interviews, but I wasn't able to hear from everyone. So if you think back to $7^{\text {th }}$ grade, and when the pandemic first started a year ago, what was it like to learn online?
a. How about when you got to 8th-grade?
2. How do you feel about going back to in person? What are your plans to return?
3. As 8th-graders, how do you feel about next year? Do you have plans for high school?
a. Does math play a role in any of those plans?
b. How about after high school, when you graduate? Does your education play a role? Math specifically?
4. Why are some students good at math but not others?
5. What makes students successful in your math class?
b. What makes some students unsuccessful?
6. We talked a lot about participation in our interviews. What does it mean to participate?
a. If you're taking notes, but not talking, does that count? If you fill in answers on Desmos?

## Appendix D

## Coding Schemes

| Observational Data Codes | Teacher Interview Codes | Student Interview Codes |
| :---: | :---: | :---: |
| - Coaching students mathematically <br> - Teacher intervenes to support <br> - Change in student's participation | - Knowing Students Background <br> - Personal Relationships <br> - Societal Positioning | - Participation |
| - Classroom community <br> - Joking <br> - Teacher explaining decision making <br> - Productive learning environment | - Teacher Background <br> - Instructional Goals <br> - Personal Characteristics <br> - Personal Motivations | - Feelings about Ms. Collier |
| - Framing students as competent <br> - High expectations <br> - Discipline <br> - Students receiving discipline | - Teaching math content <br> - On-level <br> - Honors <br> - Comparisons of tracks <br> - Constraints <br> - Curriculum | - Feelings about Math <br> - Math Support <br> - Feelings about other math learning experiences |
|  | - High expectations <br> - Care |  |

## Appendix E

Task Summary \& Task Potential of Each Task from Ms. Collier's Unit 5 Pacing Guide

| Week 1 |  | Lesson 0 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lesson Activities | Previous Unit Assessment Review Day |  |  |  |  |  |
| Week 2 |  | Lesson 1 |  | Lesson 2 |  | esson 3 |
| Lesson Activities | Unit 5, Lesson 1, Warm Up | Task Potential: <br> 4. Students are expected to see why an expression that involves dividing by 0 can't be evaluated \& asked to explain their answer. | Unit 5, Lesson 2, Warm Up | Task Potential: <br> 4. Scaffolded group of questions; final question asks students to explain why a list of squares has less numbers that the original list of inputs | Unit 5, Lesson 3, Warm-Up | Task Potential: 3. Students are given an input \& told to find the rule \& output. |
|  | Unit 5, Lesson 1, Activity 2 | Task Potential: <br> 3. Students are asked to figure our different input/output "rules" \& then change roles | Unit 5, Lesson 2, Activity 2 | Task Potential: <br> 4. Students are presented a series of questions like, "A person is 60 inches tall. Do you know their height in feet?" In cases where the answer is yes, students draw an input-output diagram with the rule in the box. In cases where the answer is no, they give examples of an input with two or more outputs. | Unit 5, Lesson 3, Activity 2 | Task Potential: <br> 2. Task provides students with a descpription then asks students to match it with a function diagram, write an equation, find the output with input of 5, and name the independent and dependent variables of each equation. Each question is procedural in nature, either identifying or matching. |
|  | Unit 5, Lesson 1, Activity 3 | Task Potential: <br> 2. Students are given a rule (and sometimes the input) and need to determine the output \& fill it into an input/output table | Unit 5, Lesson 2, Activity 3 | Task Potential 3: Using the questions from activity two, students are to use mathematical language to represent the scenario and identify whether | Unit 5, Lesson 3, Activity 3 | Task Potential 4: Students have to determine whether relationships are or aren't functions \& explain why. They are also provided with an |


|  |  |  |  | the dependent variable is a function of the independent variable. |  | equation and told to find solutions for a variable. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lesson Synthesis | Whole class discussion | Unit 5, Lesson 2, Activity 4 | Task Potential 4: The goal of this task is for students to explain how two different rules can describe the same function and that two functions are the same if and only if all of their input-output pairs are the same. | Lesson Synthesis | Whole Class Discussion |
|  |  |  | Lesson Synthesis | Whole class discussion |  |  |
| Week 3 |  | Lesson 4 |  | Lesson 5 |  | esson 6 |
| Lesson Activities | Unit 5, Lesson 4, Warm Up | Task Potential <br> 3: Notice/wonder about a non-linear graph; sample responses include observations \& thoughts but doesn't require justification or further explanation | Unit 5, Lesson 5, Warm Up | Task Potential: 3. Students identify which graph doesn't belong. | Unit 5, Lesson 7, Warm-Up | Task Potential: <br> 3. Students must make connections across three different representations. |
|  | Unit 5, Lesson 4, Activity 2 | Task Potential: <br> 3. Matching equations and scenarios with graphs. | Unit 5, Lesson 5, Activity 2 | Task Potential: <br> 3. Students make connections between: input/output on a graph; which variable is a function of the other; how to find greater change between two pairs of points. | Unit 5, Lesson 7, Activity 2 | Task Potential: <br> 3. Students <br> must make <br> connections <br> between <br> langauge <br> (outputs and <br> functions along <br> with <br> temperatures <br>  <br> students must compare temperature changes. |
|  | Unit 5, Lesson 4, Activity 3 | Task Potential: <br> 4. Students are given a graph and table and asked questions about the representations, e.g., "is time a function | Unit 5, Lesson 5, Activity 3 | Task Potential: <br> 3. Students have to qualitatively tell the story of changes in a graph. | Unit 5, Lesson 7, Activity 3 | Task Potential: <br> 3. Students are asked to compare and estimate across different |


|  |  | of distance. Explain how you know." |  |  |  | representations (an equation \& a graph). |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Unit 5, Lesson 7, Activity 4 | This activity is in the written curriculum but cut from the planning guide |
|  | Lesson Synthesis | Whole class discussion | Lesson Synthesis | Follow up questions about graph from final task | Lesson Synthesis | Whole Class Discussion |
| Week 4 |  | Lesson 7 |  | Lesson 8 |  | Lesson 9 |
| Lesson Activities | Lesson 8, Warm Up | Task Potential <br> 4: Students need to make connections between ordering components of graphs (slope \& y-intercept) from least to greatest. | Lesson 9, <br> Warm Up | Task Potential: <br> 4. Given three input/output pairs (in a candle burning context) students must identify and explain if the given relationship is a function \& linear |  |  |
|  | Lesson 8, Activity 2 | Task Potential: <br> 3. Given a unit conversion students need to create an equation, graph, identify variables \& input output pairs. | Lesson 9, Activity 2 | Task Potential: <br> 4. Given three input/output pairs (in a shadow over time context) students must identify and explain if the given relationship is a function \& linear |  |  |
|  | Lesson 8, Activity 3 | Task Potential: <br> 4. Students need to create an equation given a scenario. Students need to connect qualitative descriptions to the appropriate context \& explain solution. | Lesson 9, Activity 3 | Task Potential: 3. Students sketch a linear model on a data set of discrete points. <br> Students need to make claims about appropriateness of the model over different domains of the data. |  |  |
|  | Lesson 8, Activity 4 | Task Potential: 3. Students are comparing across different | Lesson Synthesis | Whole class discussion |  |  |


|  |  | representations (an <br> equation and a graph) <br> and asked to explain <br> how they know. |  |  |
| :--- | :--- | :--- | :--- | :--- |

## References

Adair, A. V. (1984). Desegregation: The illusion of Black progress. University Press of America.
Anderson, J. D. (1988). The Education of Blacks in the South, 1860-1935. University of North Carolina Press.

Anderson, K. (2009). Applying positioning theory to the analysis of classroom interactions: Mediating micro-identities, macro-kinds, and ideologies of knowing. Linguistics and Education, 20, 291-310.

Anyon, J. (1980). Social class and the hidden curriculum of work. Journal of Education, 162(1), 67-92.

Banks, J. (2004). Teaching for social justice, diversity, and citizenship in a global world. The Educational Forum, 68, 289-298.

Bartell, T., Wager, A., Edwards, A., Battey, D., Foote, M., \& Spencer, J. (2017). Toward a framework for research linking equitable teaching with the standards for mathematical practice. Journal for Research in Mathematics Education, 48(1), 7-21.

Black Liberation Collective. (2017). http://www.blackliberationcollective.org/our- beliefs/
Berry, R. (2005). Voices of success: Descriptive portraits of two successful African American . Journal of African American Studies, 8(4), 46-62.

Black Teacher Project. (2022). Every child deserves a Black teacher. https://www.blackteacherproject.org/vision

Boaler, J., \& Brodie, K. (2004). The importance, nature and impact of teacher questions. Proceedings of the twenty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 2, 774-782.

Boaler, J., \& Greeno, J. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaeler (Ed.), Multiple Perspectives on Mathematics Teaching and Learning. (Vol. 1, pp. 171-200). Ablex Publishing.

Boaler, J., \& Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. Teachers College Record, 110(3), 608-645.

Bondy, E., Ross, D. D., Hambacher, E., \& Acosta, M. (2013). Becoming warm demanders: Perspectives and practices of first year teachers. Urban Education, 48(3), 420-450.

Bonilla-Silva, E. (2010). Racism without racists: Color-blind racism \& racial inequality in contemporary America. Rowman \& Littlefield.

Boston, M. (2012). Assessing the quality of mathematics instruction. Elementary School Journal, 113(1), 76-104.

Brantlinger, A. (2018). Humanizing and dehumanizing trends in a critical mathematics classroom. In C. Knipping, H. Straehler-Pohl, \& U. Gellert, Inside the mathematics classroom: Sociological perspectives on participation, inclusion, and enhancement. (pp. 247-268). Springer.

Brantlinger, A. (2022). Critical and Vocational Mathematics: Authentic Problems for Students. Journal for Research in Mathematics Education, 53(2), 154-172.

Brantlinger, E. (2013). Dividing classes: How the middle-class negotiates and rationalizes school advantage. Routledge.

Carraher, T. C. (1985). Mathematics in the street and in schools. British Journal of Developmental Psychology, 3(1), 21-29.

Civil, M. (2002). Culture and mathematics: A community approach. Journal of Intercultural Studies, 23(2), 133-148.

Clark, L. M., Frank, T. J., \& Davis, J. (2013). Conceptualizing the African American mathematics teacher as a key figure in the African American education historical narrative. Teachers College Record, 115(2), 1-29.

Clark, L. M., Johnson, W., \& Chazan, D. (2009). Researching African American mathematics teachers of African American students: Conceptual and methodological considerations. In D. Martin (Ed.), Mathematics teaching, learning, and liberation in the lives of Black children (pp. 49-72). Routledge.

Cohen, E., \& Lotan, R. (1997). Working for equity in heterogenous classrooms: Sociological theory in practice. Teachers College Press.

Cole, M., \& Bruner, J. (1971). Cultural differences and inferences about psychological processes. American Psychologist, 26(10), 867-876.

Creswell, J. (2007). Qualitative inquiry and research design: Choosing among five approaches. Sage Publications.

Darling-Hammond, L. (2007). The flat earth and education: How America's commitment to equity will determine our future. Educational Researcher, 36(6), 318-334.

Davies, B., \& Harré, R. (1999). Positioning and personhood. In R. Harré \& L. van Langenhove (Eds.), Positioning theory (pp. 32-52). Blackwell.

Dixson, A. D. (2003). "Let's do this!" Black women teachers' politics and pedagogy. Urban Education, 38(3), 217-235.

Dumas, M. J., \& ross, k. m. (2016). Be real Black for me: Imagining Blackcrit in education. Urban Education, 51(4), 415-442.

Emerson, R. M., Fretz, R. I., \& Shaw, L. L. (2011). Writing ethnographic fieldnotes. University of Chicago Press.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), Handbook of research on teaching (3rd edition, pp. 119-161). Macmillan.

Esmonde, I. (2009). Ideas and identities: Supporting equity in cooperative mathematics learning. Review of Educational Research, 79(2), 1008-1043.

Esmonde, I. (2009). Mathematics learning in groups: Analyzing equity in two cooperative activity structures. The Journal of the Learning Sciences, 18(2), 247-284.

Esmonde, I., \& Langer-Osuna, J. (2013). Power in numbers: Student participation in mathematical discussions in heterogenous spaces. Journal for Research in Mathematics Education, 44(1), 288-315.

Fairclough, A. (2007). A class of their own: Black teachers in the segregated south. Cambridge, MA: Harvard University Press.

Featherstone, H., Crespo, S., Jilk, L., Oslund, J., Parks, A. N., \& Wood, M. B. (2011). Smarter together! Collaboration and equity in the elementary math classroom. National Council of Teachers of Mathematics.

Ferguson, A. A. (2000). Bad boys: Public schools in the making of Black masculinity. University of Michigan Press.

Foster, M. (1994). Effective Black teachers: A literature review. In E. Hollins, J. King, W. Hayman (Eds.), Teaching diverse populations: Formulating a knowledge base, pp. 225241. State University of New York Press.

Foster, M. (1997). Black teachers on teaching. The New Press.
Gholson, M. L., Bullock, E. C., \& Alexander, N. N. (2012). On the brilliance of Black children: A response to a clarion call. Journal of Urban Mathematics Education, 5(1), 1-7.

Gholson, M., \& Wilkes, C. (2017). (Mis)Taken identities: Reclaiming identities of the "Collective Black" in mathematics education research through and excercise in Black specificity. Review of Research in Education, 41(1), 228-252.

Gillborn, D., Warmington, P., \& Demack, S. (2018). Quant:Crit: education, policy, 'Big data' and principles for a critical race theory of statistics. Race Ethincity and Education, 21(2), 158-179.

Goffney, I. (2018). Where do we go from here? Next steps in rehumanizing mathematics for Black, Indigenous, and Latinx Students. In I. Goffney, R. Gutiérrez, \& M. Boston (Eds.), Rehumanizing Mathematics for Black, Indigenous, and Latinx Students (pp. 159-170). National Council of Teachers of Mathematics.

Gresalfi, M., \& Cobb, P. (2006). Cultivating students' discipline-specific dispositions as a critical goal for pedagogy and equity. Pedagogies: An International Journal , 1(1), 49-57.

Gutierrez, R. (2002). Beyond essentialism: The complexity of language in teaching mathematics to Latina/o students. American Educational Research Journal, 39(4), 1047-1088.

Gutierrez, R. (2008). A "gap-gazing" fetish in mathematics education? Problematizing research on the achievement gap. Journal for Research in Mathematics Education, 39(4), 357-364.

Gutiérrez, R. (2018). Introduction: The need to rehumanize mathematics. In I. Goffney, R. Gutiérrez, \& M. Boston (Eds.), Rehumanizing mathematics for Black, Indigenous, and Latinx students. (pp. 1-10). National Council of Teachers of Mathematics.

Haberman, M. (1995). Selecting 'star' teachers for children and youth in urban poverty. Phi Delta Kappan, 76(10), 777-781.

Haberman, M. (2000). Urban schools: Day camps or custodial centers?. Phi Delta Kappan, 82(3), 203-208.

Harre, R., \& van Langenhove, L. (1999). The dynamics of social ppisodes. In R. Harre, \& L. van Langenhove, Positioning theory: Moral contexts of intentional action (pp. 1-13). Blackwell Publishers.

Hill, H., \& Lubienski, S. (2007). Teachers' mathematics knowledge for teaching and schools context: A study of California teachers. Educational Policy, 21(5), 747-768.

Horn, L. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in mathematics teachers' conversations. Journal of the Learning Sciences, 16(1), 37-79.

Illustrative Mathematics. (2021). Learn Math for Life: https://illustrativemathematics.org/mathcurriculum/

Irvine, J. J. (2003). Educating teachers for diversity: Seeing with a cultural eye (Vol. 15). Teachers College Press.

Jackson, K. (2009). The social construction of youth and mathematics: The case of a fifth-grade classroom. In D. Martin (Ed.), Mathematics teaching, learning, and liberation in the lives of Black children (pp. 175-199). Routledge.

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., \& Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. Journal for Research in Mathematics Education, 44(4), 646-682.

Jensen, A. (1969). How much can we boost IQ and scholastic achievement? Harvard Educational Review, 39(1), 1-123.

Frank, T. J., Khalil, D., Scares, B., Odoms, S. (2018).Listening to and learning with Black teachers of mathematics. In I. Goffney, R. Gutiérrez, \& M. Boston (Eds.), Rehumanizing

Mathematics for Black, Indigenous, and Latinx Students (pp. 147-158). National Council of Teachers of Mathematics.

Joseph, N. M., Frank, T. J., \& Elliott, T. Y. (2021). A call for a critical-historical framework in addressing the mathematical experiences of Black teachers and students. Journal For Research in Mathematics Education, 52(4), 476-490.

Kayi-Aydar, H., \& Miller, E. R. (2018). Positioning in classroom discourse studies: A state-of-the-art review. Classroom Discourse, 9(2), 79-94.

Khisty, L. (1995). Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In W. G. Secada, E. Fennema, \& L. B. Adajian, New Directions for Equity in Mathematics Education (pp. 279-297). Cambridge University Press.

Kleinfeld, J. (1975). Effective teachers of Eskimo and Indian students. The School Review, 83(2), 301-344.

Kohli, R. (2018). Behind school doors: The impact of hostile racial climates on urban teachers of color. Urban Education, 53(3), 307-333.

Ladson-Bililngs, G. (2007). Pushing past the achievement gap: An essay on the language of deficit. Journal of Negro Education, 76(3), 316-323.

Ladson-Billings, G. (1995). But that's just good teaching! The case for culturally relevant pedagogy. Theory Into Practice, 34(3), 159-165.

Ladson-Billings, G. (2002). I ain't writin'nuttin': Permissions to fail and demands to succeed in urban classrooms. In L. Delpit, \& J. K. Dowdy (Eds.), The skin that we speak: Thoughts on language and culture in the classroom. (pp. 107-120). New Press.

Ladson-Billings, G. (2006). From the achievement gap to the education debt: Understanding achievement in US schools. Educational Researcher, 35(7), 3-12.

Ladson-Billings, G., \& Tate, W. F. (1995). Toward a critical race theory of education. Teachers College Record. 97(1), 47-68

Lampert, M. (2001). Teaching problems and the problems of teaching. Yale University Press.
Lareau, A. (2011). Unequal childhoods: Class, race, and family life. University of California Press.

Larnell, G. (2016). More than just skill: Examining mathematics identities, racialized narratives, and remediation among black undergraduates. Journal for Research in Mathematics Education, 47(3), 233-269.

Leonard, J., \& Martin, D. (2013). The brilliance of Black children in mathematics. Information Age Publishing.

Lewis, A. E., \& Diamond, J. B. (2015). Despite the best intentions: How racial inequality thrives in good schools. Oxford University Press.

Lewis, O. (1969). Review of C. Valentine's, culture and poverty: A critique and counter proposals. Current Anthropology, 10, 189-192.

Lim, J. H. (2008). The road not taken: Two African-American girls' experiences with school mathematics. Race Ethnicity and Education, 11(3), 303-317.

Madison, D. (2005). Critical ethnography: Method, ethics \& performance. Sage.
Martin, D. (2009). Researching race in mathematics education. Teachers College Record, 111(2), 295-338.

Martin, D. B. (2000). Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influence, and individual agency. Routledge.

Martin, D. B. (2012). Learning mathematics while Black. Educational Foundations. 26(1-2), 4766.

Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. Race Ethnicity and Education, 22(4), 459-478.

Martin, D. B., Anderson, C., \& Shah, N. (2017). Race and mathematics education. In J. Cai (Ed.), Compendium for research in mathematics education, (pp. 607-636). National Council of Teachers of Mathematics.

Martin, D. B., Price, P. G., \& Moore, R. (2019). Refusing systemic violence against Black children: Toward a liberatory mathematics education. In J. Davis , \& C. Jett (Eds.), Critical race theory in mathematics education. (pp. 32-55). Routledge.

McFarland, J., Hussar, B., de Brey, C., Snyder, T., Wang, X., Wilkinson-Flicker, S., Gebrekristos, S., Zhang, J., Rathbun, A., Barmer, A., Bullock Mann, F., Hinz, S. (2017). The condition of education U.S. Department of Education. National Center for Education Statistics.

McGee, E. (2013). Threatened and placed at Risk: High achieving African American males in urban high schools. Urban Review, 45(4), 448-471.

McGee, E. (2015). Robust and fragile mathematical identities: A framework for exploring racialized experiences and high achievement among black college students. Journal for Research in Mathematics Education, 46(5), 599-625.

McGee, E., \& Martin, D. (2011). "You would not believe what I have to go through to prove my intellectual value!" Stereotype management among academically successful Black mathematics and engineering students. American Educational Research Journal, 48(6), 1347-1389.

Merriam, S. (1998). Qualitative research and case study applications in education. Wiley.

Milner, R. (2006). The Promise of Black Teachers' Success with Black Students. Educational Foundations, 20, 89-104.

Milner, R. (2012). But what is urban education? Urban Education, 47(3), 556-561.
Morton, K., \& Riegle-Crumb, C. (2019). Who gets in? Examining inequality in eighth-grade algebra. Journal forResearch in Mathematics Education, 50(5), 529-554.

Morton, K., \& Riegle-Crumb, C. (2020). Is school racial/ethnic composition associated with content coverage in algebra. Educational Researcher, 49(6), 441-447.

Moses, B., \& Cobb, C. (2001). Radical equations: Math literacy and civil rights. Beacon.
Nasir, N. S. (2007). Identity, goals, and learning: The case of basketball mathematics. In N. Nasir, P. Cobb (Eds.), Improving access to mathematics: Diversity and equity in the classroom, (pp. 132-145). Teachers College Press

Nasir, N., \& Hand, V. (2006). Exploring sociocultural perspectives on race, culture, and learning. Review of Educational Research, 76(4), 449-475.

Nasir, N., Hand, V., \& Taylor, E. (2008). Culture and mathematics in school: Boundaries between "cultural" and "domain" knowledge in the mathematics classsroom and beyond. Review of Research in Education, 32(1), 187-240.

National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Author.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Author.

National Governors Association Center for Best Practices \& Council of Chief State School Officers. (2010). Common core state standards for mathematics. www.corestandards.org/assets/CCSSI_Math\ Standards.pdf

National Research Council: Commission on Behavioral and Social Sciences and Education. (2004). How People Learn: Brain, Mind, Experience, and School (Expanded Ed.) (J.D. Bransford, A. L. Brown, R. R. Cocking, M. S. M. Donovan, J. D. Bransford, \& J. W. Pellegrino (eds.)). National Academy Press.

Nieto, S., \& Bode, P. (2008). Affirming diversity: The sociopolitical context of multicultural education (Fifth ed.). Pearson.

Noddings, N. (1994). Does everybody count? Reflections on reforms in school mathematics. Journal of Mathematical Behavior, 13(1), 89-104.

Oakes, J. (1985). Keeping track: How schools structure inequality (2nd ed.). Yale University Press.

Ortiz, N. (2022). Let our rejoicing rise without bound: A focus on Black language and mathematics learning. International Journal of Qualitative Studies in Education, 1-13.

Paris, D. (2012). Culturally sustaining pedagogy: A needed change in stance, terminology, and practice. Educational Researcher, 41(3), 93-97.

Perry, T., Steele, C., \& Hilliard, A. G. (2003). Young, gifted, and Black: Promoting high achievement among African-American sudents. Beacon Press.

Pinnow, R., \& Chval, K. (2015). "How much you wanna bet?": Examining the role of positioning in the development of L2 learner interactional competencies in the content classroom. Linguistics and Education, 30, 1-11.

Popkewitz, T. (2004). The Alchemy of the Mathematics Curriculum: Inscriptions and the Fabrication of the Child. American Educational Research Journal, 41(1), 3-34.

Richards, G. (1997). Race, racism, and psychology. Routledge.
Saldaña, J. (2016). The coding manual for qualitative researchers (3rd ed.). Sage.
Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. Reading \& Writing Quarterly, 23(2), 139-159.

Schoenfeld, A. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. Educational Researcher, 31(1), 13-25.

Shah, N. (2013). Racial discourse in mathematics and its impact on student learning, identity, and participation. (Unpublished doctoral dissertation). University of California at Berkeley, California.

Shah, N. (2022, April 29). Mathematics education under the permanence of racism. Center for Math Education Colloquium.

Siddle Walker, V. (2001). African American teaching in the South: 1940-1960. American Educational Research Journal, 38(4), 751-779.

Sleeter, C. (2001). Preparing teachers for culturally diverse schools: Research and the overwhelming presence of whiteness. Journal of Teacher Education, 52(2), 94-106.

Spencer, M. (2006). Phenomenology and ecological systems theory: Development of diverse groups. In Handbook of child psychology.

Stein, M. K., Grover, B., \& Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33(2), 455-488.

Stein, M., \& Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. Educational Research and Evaluation, 2(1), 50-80.

Stinson, D. (2013). Negotiating the "White male myth": African american male students and success in school mathematics. Journal for Research in Mathematics Education, 44(1), 69-99.

Tait-McCutcheon, S. (2016). Examining equity of opportunities for learning mathematics through positioning theory. Mathematics Education Research Journal, 28(2), 327-348.

Tate, W. (2008). The political economy of teacher quality in school mathematics: African American males, opportunity structures, politics and method. American Behavioral Scientist, 51(7), 953-971.

Tate, W. F. (1995). School mathematics and African American Students: Thinking seriously about opportunity-to-learn. Educational Administration Quarterly, 31(3), 424-448.

Taylor, E. (2009). The purchasing practice of low-income students: The relationship to mathematical development. Journal of Learning Sciences, 18(3), 370-415.

Turner, E., Dominguez, H., Maldonado, L., \& Emspon, S. (2012). English learners' participation in mathematical discussion: Shifting positionings and dynamic identities. Journal for Research in Mathematics Education, 44(1), 199-234.
U.S. Department of Education. (2008). Foundations of success: Final report of the National Mathematics Advisory Panel. U.S. Government Printing Office.

Varela, C. R., \& Harré, R. (1996). Conflicting varieties of realism: causal powers and the problems of social structure. Journal for the Theory of Social Behaviour, 26(3), 313-325.

Wagner, D., \& Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. Educational Studies in Mathematics, 72(1), 1-15.

Walshaw, M., \& Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. Review of Educational Research, 78(3), 516-551.

Ware, F. (2006). Warm demander pedagogy: Culturally responsive teaching that supports a culture of achievement for African American students. Urban Education, 41(4), 27456.

Wilson, J., Nazemi, M., Jackson, K., \& Wilhelm, A. (2019). Investigating teaching in conceptually oriented mathematics classrooms characterized by African American student success. Journal for Research in Mathematics Education, 50(4), 362-400.

Wood, M. B. (2013). Mathematical micro-identities: Moment-to-moment positioning and learning in a fourther-grade classroom. Journal for Research in Mathematics Education, 44(5), 775-808.

Yeh, C., Ellis, M. W., \& Hurtado, C. K. (2017). Reimagining the mathematics classroom: Creating and sustaining productive learning environments, $K-6$. National Council of Teachers of Mathematics.

Zangori, L., \& Pinnow, R. (2020). Positioning participation in the NGSS era: What counts as success? Journal of Research in Science Teaching, 57, 623-648.


[^0]:    ${ }^{1}$ The reasons I describe Ms. Collier as exemplary will be discussed in Chapter 3: Methods, in the participants section when I elaborate on Ms. Collier's background

[^1]:    ${ }^{2}$ An explanation of how the number of Black student participants relates to the total student population in the classes will be elaborated in the Participants section of this chapter.

[^2]:    ${ }^{3}$ Throughout this dissertation I refer to the students in each class section as being in the on-level or honors class. I do this because I am using the language used by the school to classify each of these classes. At the same time, I understand there are significant connotations associated with each of these labels, and that sorting and classifying students in this way reifies hierarchies associated with mathematics and can cause harm to students. In distinguishing between these two classes as being honors and on-level my purpose is to understand and shed light upon the instruction in the two tracked sections.

[^3]:    ${ }^{4}$ Black English vernacular refers to a full and complete language system, including grammar and syntax, which stems from and reflects the contemporary and historical lived experiences of ADOS. Schooling environments often force Black students to abandon this language, subjecting them to symbolic violence (Martin et. al., 2019)

[^4]:    ${ }^{5}$ Because we generally use the term "square root" to refer to principal square roots, or positive square roots, it is worth clarifying here that all positive real numbers do have two square roots, one which is positive and one which is negative. In common usage we use $\sqrt{x}$ to refer to nonnegative square roots and define the square root function as the principal square root function, whereby $\sqrt{ } x$ is the positive version of the inverse function of $f(x)=$ $x^{2}$ for $x \geq 0$ (Weisstein, n.d.; Spanier \& Oldham, 2008). In task 2.2, the purpose of the assignment was for students to recognize that without restricting to only the positive solutions, inversing the square of a number will lead to two possible numbers, and so it is not a function.

[^5]:    ${ }^{6}$ During the warm up, students were given a list of both positive and negative numbers to square $(1,-3,1 / 2$, $3,2,1 / 4,-0.5$ ), and had to explain why there was a different number of original numbers than unique squares of the numbers.

