Online Supplement to 'Myopic Allocation Policy with Asymptotically Optimal Sampling Rate'

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In this appendix, we test the performance of AOMAP under the unknown variances scenario and compare AOMAP with EI and OCBA. OCBA is referred to a fully sequential OCBA algorithm implemented by the "most starving" sequential rule (Chen and Lee, 2011). We provide five numerical experiments. In all experiments, the prior distribution for deriving AOMAP and EI are chosen as the uninformative prior, i.e. $\mu_i^{(0)} = 0$, $\alpha_i^{(0)} = -1/2$, $\kappa_i^{(0)} = 0$, $\beta_i^{(0)} = 0$, $i = 1, .., 10$, so that the Bayesian statistics match the frequentist statistics (see DeGroot, 2005). 10 initial replications ($n_0 = 10$) are used to estimate sample means and variances in all experiments.

Example 1.

In this example, there are three designs with means $\mu_1 = 0.2$, $\mu_2 = 0.1$, $\mu_3 = 0$, and variances $\sigma_i^2 = 1$, $i = 1, 2, 3$. From Figure 1, we can see AOMAP and OCBA have comparable performance and are better than EI.

Example 2.

In this example, there are three designs with means $\mu_1 = 0.1$, $\mu_2 = \mu_3 = 0$, and variances $\sigma_i^2 = 1$, $i = 1, 2, 3$. The observations in Figure 2 are similar to those in the first example.

Example 3 (Chen et al., 2000).

There are ten designs with means $\mu_i = 10 - i$ and variances $\sigma_i^2 = 6^2$, $i = 1, .., 10$. From Figure 3, we can see that three sampling policies all achieve good performance, while OCBA has a slight edge over AOMAP and EI when the simulation budget is smaller than 1000 replications.

Example 4.

In this example, there are ten designs with means $\mu_1 = 1$, $\mu_i = 0$, $i = 2, .., 10$, and variances $\sigma_i^2 = 6^2$, $i = 1, .., 10$. It is more difficult to differentiate the performance of different designs, based on sample estimates, than in the third example, and the configuration of this example is sometimes called the least favorable configuration in ranking and selection. From Figure 4, we can see that OCBA has an edge over AOMAP and EI, which have

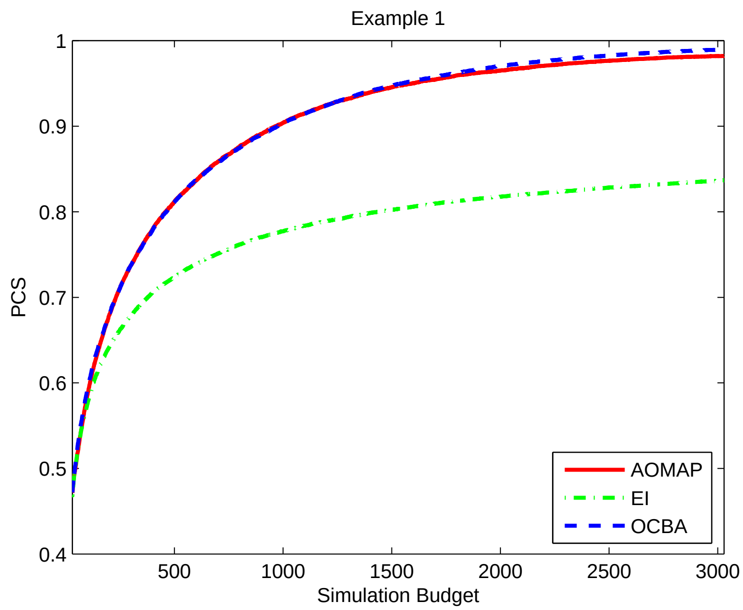

Figure 1: $\mu_1 = 0.2$, $\mu_2 = 0.1$, $\mu_3 = 0$, and $\sigma_i^2 = 1$, $i = 1, 2, 3$. Initial replications $n_0 = 10$. PCSs estimated by 10^5 macro-experiments.

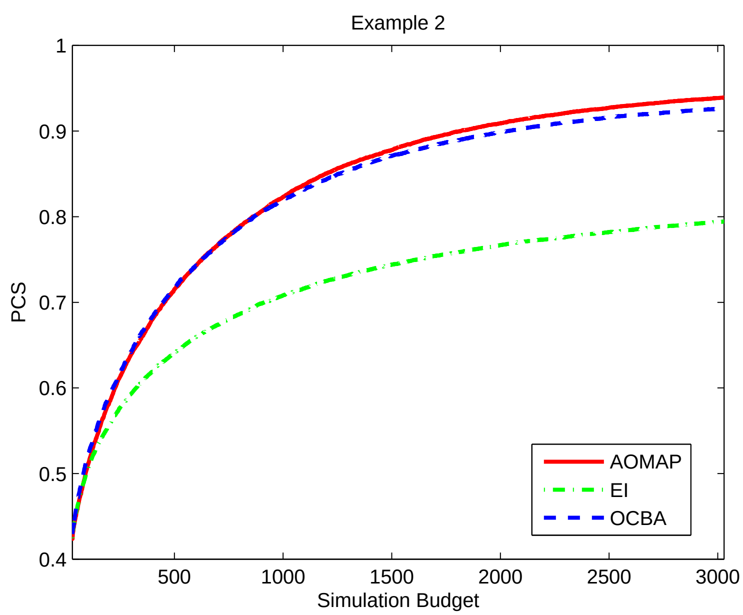

Figure 2: $\mu_1 = 0.1$, $\mu_2 = \mu_3 = 0$, and $\sigma_i^2 = 1$, $i = 1, 2, 3$. Initial replications $n_0 = 10$. PCSs estimated by 10^5 macro-experiments.

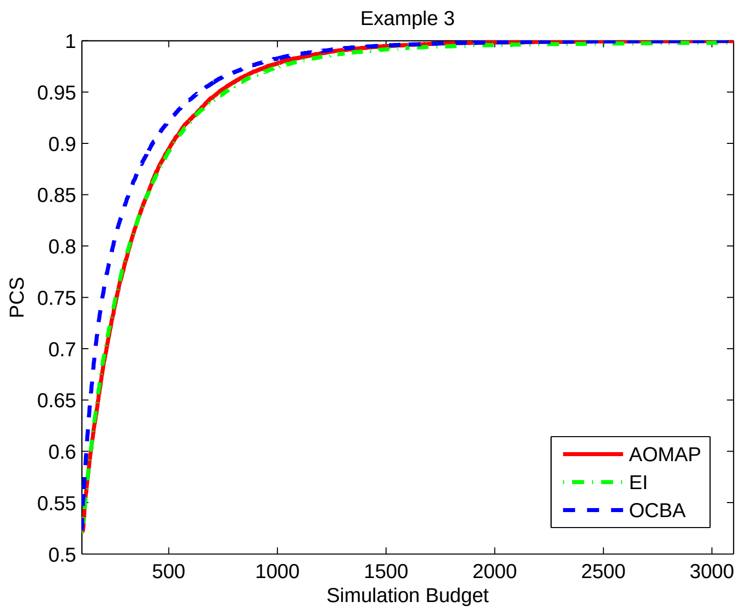

Figure 3: $\mu_i = 10 - i$ and $\sigma_i^2 = 6^2$, $i = 1, .., 10$. Initial replications $n_0 = 10$. PCSs estimated by 10^5 macro-experiments.

comparable performance throughout the experiment, when the simulation budget is smaller than 1000, but is surpassed by the AOMAP and EI when the simulation budget is larger than 1000. OCBA flattens out more quickly than the other two methods. Our conjecture for this phenomenon is that the derivation of the OCBA is based on asymptotic results of PCS, which does not take the learning procedure of unknown parameters into consideration, while the derivations of AOMAP and EI are based on a learning procedure that updates the uncertainty of unknown parameters by a sequential Bayesian mechanism. In this example, the poor performance of OCBA after the simulation budget reaches 1000 might be due to the poor estimates on the unknown parameters.

Example 5.

The configurations are randomly generated from a prior distribution, and the performances of different sampling allocation policies are measured by integrated PCS (IPCS) defined by

$$IPCS = \mathbb{E}\left[\mathbb{E}[\mathbf{1}\{\widehat{i} = i^*\}|\mathcal{E}_T, \theta]\right],$$

where $\widehat{i} = \arg\max_{i=1,..,k} \bar{m}_i^{(T)}$ and $\theta = (\mu_1, .., \mu_k, \sigma_1^2, .., \sigma_k^2)$. In this example, there are ten designs with means and variances generated from a normal-gamma conjugate prior with hyper-parameters $\mu_i^{(0)} = 0$, $\alpha_i^{(0)} = 3$, $\kappa_i^{(0)} = 5$, $\beta_i^{(0)} = 10$, $i = 1, .., 10$. The IPCSs are the average performances of different sampling allocation policies under randomly generated

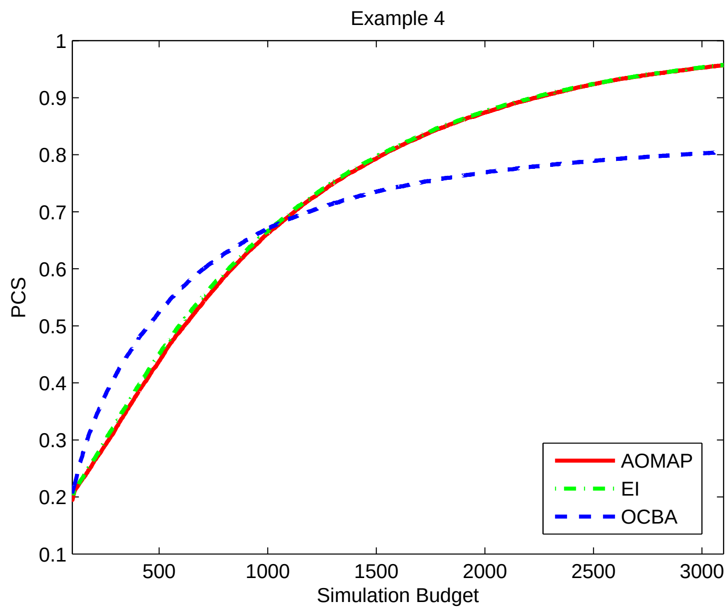

Figure 4: $\mu_1 = 1$, $\mu_i = 0$, $i = 2, .., 10$, and $\sigma_i^2 = 6^2$, $i = 1, .., 10$. Initial replications $n_0 = 10$. PCSs estimated by 10^5 macro-experiments.

configurations. With some calculation, we have for $i = 1, .., 10$,

$$\mathbb{E}[\mu_i] = \mu_i^{(0)} = 0, \quad Var[\mu_i] = \frac{\beta_i^{(0)}}{(\alpha_i^{(0)} - 1)\kappa_i^{(0)}} = 1,$$

$$\mathbb{E}[\sigma_i^2] = \frac{\beta_i^{(0)}}{\alpha_i^{(0)} - 1} = 5, \quad Var(\sigma_i^2) = \frac{(\beta_i^{(0)})^2}{(\alpha_i^{(0)} - 2)(\alpha_i^{(0)} - 1)^2} = 25 \ .$$

Form the statistics above, we know it is relatively difficult to differentiate the performances of different designs, based on sample estimates.

From Figure 5, we can see that AOMAP performs better than EI after the simulation budget reaches 1000, which can be explained by the desirable asymptotic property of AOMAP, whereas OCBA seems to reach a plateau after the simulation budget reaches 200. The poor performance of OCBA can again be explained by insufficiency of information learning for unknown parameters, as in the previous example.

References

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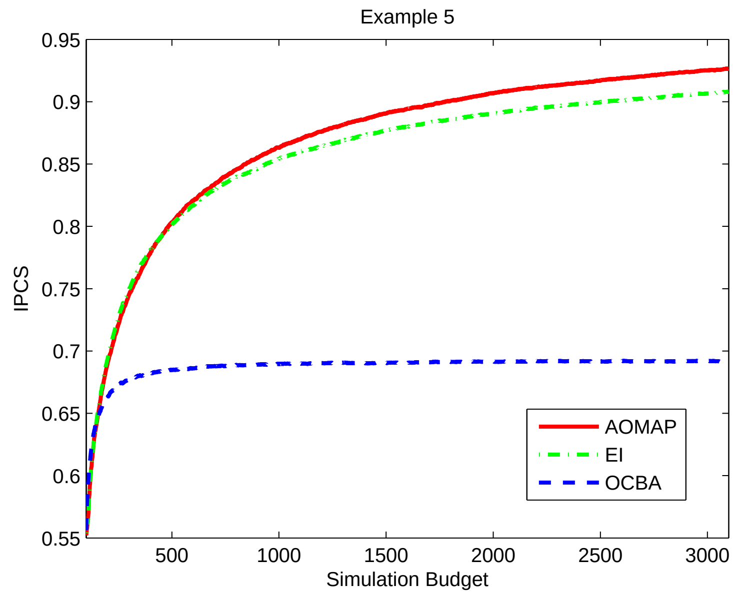

Figure 5: Normal-Gamma conjugate prior distribution, with parameters $\mu_i^{(0)} = 0$, $\alpha_i^{(0)} = 3$, $\kappa_i^{(0)} = 5$, $\beta_i^{(0)} = 10$, $i = 1, .., 10$; IPCS estimated by 10^5 macro-experiments.

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