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Supported by the National Science Foundation Engineering Research Center Program (NSFD CD 8803012), Industry and the University

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TR 92-58

Dycord+ '92
College Park, MD, 1992

On the Stability of Nonlinear Quadratic Dynamic Matrix Control

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Abstract: The extension of Quadratic Dynamic Matrix Control (QDMC) to nonlinear process models is an attractive option for industrial implementation. Although a nonlinear model is utilized, one has to solve only a single Quadratic Program on-line. In this paper, we present the stability properties for the global asymptotic stability of the closed-loop system under NLQDMC law. The conservativeness of these properties is examined by following the steps of the proofs when this algorithm is applied to a simple example. We also demonstrate the application of the nonlinear version of QDMC to processes for which the sign of the system gain changes around the operating point.

Keywords: Model Predictive Control, Quadratic Dynamic Matrix Control, Stability, Nonlinear Control.

1 Introduction

The application of Quadratic Dynamic Matrix Control (QDMC) to processes which can be assumed linear has been mostly successful in the industrial environment, for multivariable systems with input and output constraints (García and Prett, 1986). García (1984) proposed an extension of linear QDMC to nonlinear processes (abbreviated to NLQDMC from here onwards). Gattu and Zafiriou (1992) extended this formulation to open-loop unstable systems, by incorporating a Kalman filter. The requirement of solving only one QP on-line at each sampling time makes this algorithm an attractive option for industrial implementation.

Model Predictive Control (MPC) algorithms utilizing nonlinear models, based on either Nonlinear Programming techniques or linearization approaches, seem very promising; not much progress has been made, however, in understanding the stability and performance properties. Li *et al.* (1990) and Li and Biegler (1989) presented stability conditions for Newton-Type controllers. Peterson *et al.* (1992) presented sufficient conditions for stability of a nonlinear MPC algorithm. However, there are no numerical results available that illustrate how conservative these stability conditions are. One goal of this paper is to present stability properties which guarantee the global asymptotic stability of the closed-loop system under NLQDMC law. Later we quantify the conservativeness of these conditions with the aid of an example. We also demonstrate the application of NLQDMC to processes where the sign of the system gain changes around the operating point. The paper is structured as follows. Section two presents an overview of the NLQDMC algorithm with state estimation. Section three demonstrates the application of NLQDMC to a process with a sign change in the gain near the operating point. Finally in section four, we present the stability results and illustrate the conservativeness of these results with the aid of an example.

2 State Estimation NLQDMC

García (1984) proposed an extension of linear QDMC to nonlinear processes. Gattu and Zafiriou (1992) extended this algorithm to open-loop unstable processes, by incorporating a Kalman filter for state estimation. For a special choice of tuning parameters, state estimation NLQDMC reduces to García's algorithm. For a detailed description the reader is referred to Gattu and Zafiriou (1992).

For the general case of Multi-Input Multi-Output (MIMO) systems, consider process and measurement models of the form

$$\dot{x} = f(x, u) + w \tag{1}$$

$$y = h(x) + v \tag{2}$$

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where x is the state vector, y is the output vector, u is the vector of manipulated variables, and $w \sim (0,Q)$ and $v \sim (0,R)$ are white noise processes uncorrelated with each other. Q and R are covariance matrices associated with process and measurement noise respectively. It is assumed that $Q \approx \sigma_w^2 I$ and $R \approx \sigma_v^2 I$, where σ_w^2 and σ_v^2 are scalar variances. Define $\sigma = \sigma_w/\sigma_v$ and let $\sigma_v^2 = 1$. The ratio of σ_w^2 to σ_v^2 determines the value of the Kalman filter gain. Intuitively, σ is the ratio between statistical measures of the uncertainty in the state and the uncertainty in a measurement. Therefore, σ can be used as a tuning parameter for stability and robustness in the presence of model-plant mismatch, external disturbances and measurement noise.

Algorithm:

Known at sampling instant $k+1$:

$y(k+1)$ the plant measurement, $\hat{x}(k+1|k)$ the estimate of state vector at $k+1$ based on information at k, and $u(k)$ the manipulated variable.

Effect of future manipulated variables

Step 1: Linearize the nonlinear model $\dot{x} = f(x,u)$ at $\hat{x}(k+1|k)$ and $u(k)$ to obtain

$$\begin{aligned} \dot{\hat{x}} &= A_k\hat{x} + B_k u \\ y &= C_k\hat{x} \end{aligned} \tag{3}$$

where

$$\begin{aligned} A_k &= (\frac{\partial f}{\partial x})|_{x=\hat{x}(k+1|k),u=u(k)} \\ B_k &= (\frac{\partial f}{\partial u})|_{x=\hat{x}(k+1|k),u=u(k)} \\ C_k &= (\frac{\partial h}{\partial x})|_{x=\hat{x}(k+1|k)} \end{aligned}$$

Step 2: Discretize (3) to obtain

$$\begin{aligned} \hat{x}_{j+1} &= \Phi_k\hat{x}_j + \Gamma_k u_j \\ y_j &= C_k\hat{x}_j \end{aligned} \tag{4}$$

where Φ_k and Γ_k are discrete state space matrices (e.g., Aström and Wittenmark, 1984), obtained from A_k, B_k and the sampling time.

Step 3: Compute the step response coefficients $S_{i,k+1}$ $(i = 1,2,\ldots,P)$ where P is the prediction horizon. Step response coefficients are used only to represent the effect of future manipulated variables. Therefore, only P step response coefficients are required. For more details on computation issues the reader is referred to Gattu and Zafiriou (1992). The contribution of the future manipulated variables to the predicted output values at $k+\ell+1$ is represented as $\sum_{i=1}^{\ell} S_{i,k+1}\Delta u(k+\ell+1-i)$ $(\ell = 1,2,\ldots,P)$, where Δu is the change in manipulated variables, defined as $\Delta u(k+1) \triangleq u(k+1) - u(k)$.

Computation of filter gain

Step 4: Compute the steady state Kalman gain using the recursive relation (Aström and Wittenmark, 1984):

$$\begin{aligned} P_{(j+1)k} &= \Phi_k P_{jk}\Phi_k^T + Q - \Phi_k P_{jk} C_k^T \\ &\quad (C_k P_{jk} C_k^T + R)^{-1} C_k P_{jk}\Phi_k^T \qquad (5) \\ K_k &= \Phi_k P_{\infty k} C_k^T [C_k P_{\infty k} C_k^T + R]^{-1} \qquad (6) \end{aligned}$$

where P_{jk} is the state covariance at iteration j for the model obtained by linearization at sampling point $k+1$. $P_{\infty k}$ is the steady state value of state covariance for that model.

Effect of past manipulated variables

Step 5: The effect of past inputs on future output predictions, $y^\star(k+2), y^\star(k+3), \ldots, y^\star(k+P+1)$ is computed as follows. Here the superscript '$\star$' indicates that input values in the future are kept constant and equal to $u(k)$.

- Set $\hat{x}^\star(k+1|k) = \hat{x}(k+1|k)$.
- Define $d(k+1|k+1) \triangleq y(k+1) - h(\hat{x}(k+1|k))$.
- In the absence of measurement information in the future, it is assumed that $d(k+i|k) = d(k+1|k+1)$ for $i = 2,3,\ldots,P+1$.
- For $i = 1,2,\ldots,P$, successively integrate $\dot{x} = f(x,u)$ over one sampling time from $\hat{x}^\star(k+i|k)$, with $u(k+i) = u(k)$ and then add $K_k d(k+1|k+1)$ to obtain $\hat{x}^\star(k+i+1|k)$. Addition of $K_k d$ provides correction to the state. We can then write

$$y^\star(k+i) = h(\hat{x}^\star(k+i|k))(i = 2,\ldots,P+1) \tag{7}$$

Output Prediction

Step 6: The predicted output is computed as the sum of the effect of past and future manipulated variables and the future predicted disturbances.

$$\begin{aligned} \hat{y}(k+\ell) &= y^\star(k+\ell) + \sum_{i=1}^{\ell} S_{i,k+1}\Delta u(k+\ell-i) \\ &\quad + d(k+1|k+1) \ (\ell = 2,\ldots,P+1) \end{aligned} \tag{8}$$

Define,

$$\begin{aligned} \hat{Y}(k+2) &\triangleq [\hat{y}(k+2),\ldots,\hat{y}(k+P+1)]^T \\ Y^\star(k+2) &\triangleq [y^\star(k+2),\ldots,y^\star(k+P+1)]^T \\ \mathcal{D}(k+1) &\triangleq [d(k+1),\ldots,d(k+1)]^T \\ \Delta U(k+1) &\triangleq [\Delta u(k+1),\ldots,\Delta u(k+M)]^T \\ \mathcal{Y}_r &\triangleq [y_r, y_r,\ldots,y_r]^T \\ \mathcal{S} &\triangleq \begin{bmatrix} S_1 & 0 & \ldots & 0 \\ S_2 & S_1 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ S_M & S_{M-1} & \ldots & S_1 \\ \vdots & \vdots & & \vdots \\ S_P & S_{P-1} & \ldots & S_{P-M+1} \end{bmatrix} \end{aligned}$$

where y_r is the reference setpoint and M is the number of future moves to be computed. It is assumed that $M \leq P$ and $u(k+M) = u(k+M+1) = \ldots = u(k+P)$. Then the predicted output can be written in compact form as

$$\hat{Y}(k+2) = Y^{\star}(k+2) + S\Delta U(k+1) + \mathcal{D}(k+1) \quad (9)$$

Optimization

$$\min_{\Delta U(k+1)} \tfrac{1}{2}\|\Gamma(\hat{Y}(k+2) - \mathcal{Y}_r)\|^2 + \tfrac{1}{2}\|\Lambda\Delta U(k+1)\|^2 \quad (10)$$

where $\|\bullet\|^2$ is defined by $\|x\|^2 = x^T x$. Γ and Λ are diagonal weight matrices.

Step 7: The M future manipulated variables are computed, but only the first move is implemented (García and Morshedi, 1986). The closed form of control law for unconstrained systems is given by

$$\begin{aligned}\Delta u(k+1) &= e^T(S^T\Gamma^T\Gamma S + \Lambda^T\Lambda)^{-1}S^T\Gamma^T\Gamma \\ &\quad (\mathcal{Y}_r - Y^{\star}(k+2) - \mathcal{D}(k+1)) \quad (11)\end{aligned}$$

where $e^T = [1, 0, \ldots, 0]$

Estimation of state

Step 8: Integrate $\dot{x} = f(x,u)$ from $\hat{x}(k+1|k)$ and $u(k+1)$ over one sampling time and add $K_k d$ to obtain $\hat{x}(k+2|k+1)$.

The above steps are repeated at each sampling time. When K_k is 0.0, which results to σ equal to 0.0, this formulation is equivalent to García's (1984) algorithm.

3 Illustration

Gattu and Zafiriou (1992) applied the state estimation NLQDMC to various examples. The incorporation of state estimation in NLQDMC results in better disturbance rejection and allows to deal with open-loop unstable processes. The performance of the algorithm is comparable with the algorithms which utilize nonlinear programming techniques. The robustness characteristics of NLQDMC were demonstrated by application to an industrial challenge problem presented by Chylla and Haase (1990).

Intuitively, one would expect that a challenging application for NLQDMC would be a process for which the sign of the linearized model changes around the operating point. Here, we demonstrate the applicability of the algorithm to such a process.

The example problem is taken from Economou *et al.* (1986). The process consists of an ideal continuous stirred tank reactor, where the following reversible exothermic reaction takes place:

$$A \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} R$$

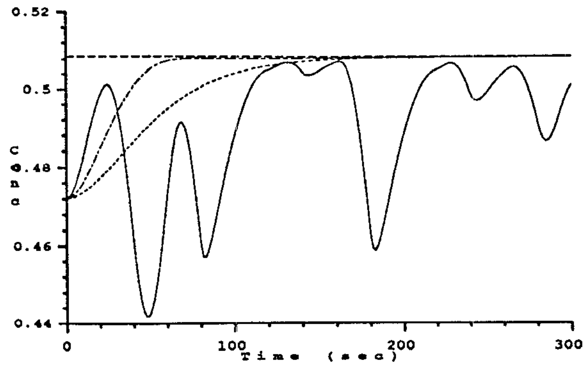

Figure 1: Concentration vs. Time; Solid line $M = 5, P = 5$ and $\Lambda = 0.0$; Alternate dots and dashes $M = 5, P = 5$ and $\Lambda = 0.0004$; Dotted line $M = 1, P = 10$ and $\Lambda = 0.0$

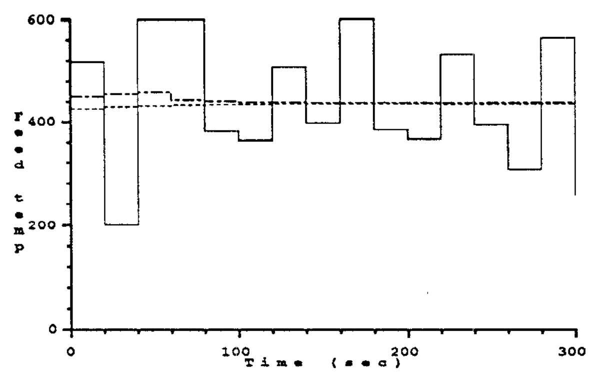

Figure 2: Feed Temperature vs. Time; Solid line $M = 5, P = 5$ and $\Lambda = 0.0$; Alternate dots and dashes $M = 5, P = 5$ and $\Lambda = 0.0004$; Dotted line $M = 1, P = 10$ and $\Lambda = 0.0$

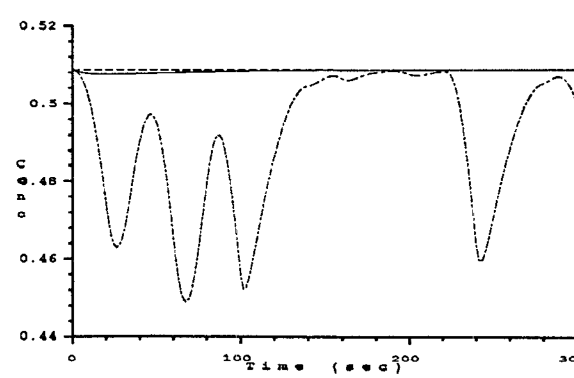

Figure 3: Concentration vs. Time; Solid line $M = 5, P = 5$ and $\Lambda = 0.00005$; Alternate dots and dashes $M = 5, P = 5$ and $\Lambda = 0.0$

The process is described by the differential equations

$$
\begin{aligned}
\frac{dA_0}{dt} &= \frac{1}{\tau}(A_i - A_0) - k_1 A_0 + k_{-1} R_0 \\
\frac{dR_0}{dt} &= \frac{1}{\tau}(R_i - R_0) + k_1 A_0 - k_{-1} R_0 \\
\frac{dT_0}{dt} &= \frac{-\Delta H_r}{\rho C_p}(k_1 A_0 - k_{-1} R_0) + \frac{1}{\tau}(T_i - T_0) \\
y &= R_0
\end{aligned}
$$

where $k_1 = C_1 exp(-Q_1/RT_0)$ and $k_{-1} = C_{-1} exp(-Q_{-1}/RT_0)$ and the parameters and steady state operating conditions used in this example are shown in Table I.

Table I: Parameter values

τ	$= 60\ s$
C_1	$= 5.0e{+}03\ s^{-1}$
C_{-1}	$= 1.0e{+}06\ s^{-1}$
Q_1	$= 10000\ cal\ mol^{-1}$
Q_{-1}	$= 15000\ cal\ mol^{-1}$
R	$= 1.987\ cal\ mol^{-1}\ K^{-1}$
$-\Delta H_r$	$= 5000\ cal\ mol^{-1}$
ρ	$= 1.0\ kg/L$
C_p	$= 1000\ cal\ kg^{-1}\ K^{-1}$
A_i	$= 1.0\ mol/L$
R_i	$= 0.0\ mol/L$
A_0	$= 0.4913$
R_0	$= 0.5087$
T_i	$= 435.9\ K$
T_0	$= 438.47\ K$

The reactor equilibrium curve of conversion as a function of feed temperature has a well defined maximum. The control objective is to operate the reactor at these steady state conditions. The model has a gain very close to zero at the steady state. The inlet feed stream temperature T_i and the desired product concentration R_0 are chosen as manipulated and controlled variables respectively.

Figs. 1 and 2 give the response of the reactor for a setpoint change from an initial condition $A_0 = 0.5279, R_0 = 0.4721, T_0 = 412.0$ and $T_i = 409.64$ to $R_0 = 0.5087$. We assume that there is no model-plant mismatch. The lower and upper bounds on T_i are kept at $200K$ and $600K$ respectively. A sampling time of 20 sec is used in simulations. The simulations demonstrate that, for $P = M$ and $\Lambda = 0.0$, the controller inverts the model and the response becomes unstable. The system can be stabilized by choosing $P > M$ or a very small nonzero value for the weight on the change in manipulated variables. Fig 3 gives the response of the reactor for a step disturbance of 0.02 units in the feed concentration A_i in a system running at the steady state setpoint. The responses are similar to those for setpoint change.

4 Stability Analysis

As a simple case, we consider the analysis of NLQDMC algorithm without state estimation. The approach taken here is similar to the approach taken by Li *et al.* (1990), for establishing global property for Newton-Type controllers. We show that, for open-loop globally asymptotically stable systems, by choosing $M = 1$, and a large sampling time and weights on change in manipulated variables, global asymptotic stability of the closed-loop system under NLQDMC is guaranteed. Later in this section, we quantify the conservativeness of the this result with the aid of a simple example.

It is assumed that the inputs are piecewise constant functions ($u(t) = u(k); t_k \leq t < t_k + T$). For the discrete system with sampling time T, the solution of $\dot{x} = f(x, u)$ at $k+1$ is defined as

$$\chi(k) \triangleq x(k+1) = \chi(T; x(k), u(k)) \tag{12}$$

$$\phi_{ki} \triangleq \frac{\partial \chi(iT; x(k), u(k))}{\partial x(k)} \tag{13}$$

Theorem 1 *If an unconstrained open-loop system is globally asymptotically stable, then the global asymptotic stability of the closed-loop system under NLQDMC law is guaranteed, by choosing $M = 1$, and a sufficiently large sampling time and weight Λ.*

Proof: Assume that the setpoint is constant in the future and there is no model error or disturbances. For simplicity we assume $\Gamma = I$ and $\Lambda = \lambda I$. $< \cdot\,, \cdot >$ is defined as $< x, y >= x^T y$. In the following development the notation $\Delta u(k+1)$ and Δu_{k+1} is equivalent and is used interchangeably. For $M = 1$, the objective function J is

$$J(\Delta u(k+1)) = \frac{1}{2}||\hat{Y}(k+2) - \mathcal{Y}_r||^2 + \frac{\lambda^2}{2}||\Delta u(k+1)||^2 \tag{14}$$

$$J(0) = \frac{1}{2}||Y^*(k+2) - \mathcal{Y}_r||^2 \tag{15}$$

For an unconstrained system, at optimal solution:

$$J(\Delta u(k+1)) \leq J(0) \tag{16}$$

$$||\hat{Y}(k+2) - \mathcal{Y}_r||^2 + \lambda^2 ||\Delta u(k+1)||^2 \leq ||Y^*(k+2) - \mathcal{Y}_r||^2 \tag{17}$$

$$
\begin{aligned}
Y(k+2) &\triangleq [y(k+2), y(k+3), \ldots, y(k+P+1)]^T \\
\mathcal{Y}(k+2) &\triangleq [y(k+2), y(k+2), \ldots y(k+2)]^T \\
O(\Phi_k) &\triangleq [O(\phi_{k1}), O(\phi_{k2}), \ldots, O(\phi_{kP})]^T \\
O(\bar{\Phi}_{k+1}) &\triangleq [0, O(\phi_{(k+1)1}), \ldots, O(\phi_{(k+1)P-1})]^T
\end{aligned}
$$

The effect of past inputs on the predicted output at $k+2$ is related to the output of nonlinear model at $k+1$ as

$$Y^*(k+2) = \mathcal{Y}(k+1) + O(\Phi_k) \tag{18}$$

$$
\begin{aligned}
\Rightarrow ||Y^*(k+2) - \mathcal{Y}_r||^2 & \\
= \quad & ||\mathcal{Y}(k+1) - \mathcal{Y}_r + O(\Phi_k)||^2 \\
= \quad & ||\mathcal{Y}(k+1) - \mathcal{Y}_r||^2 + ||O(\Phi_k)||^2 \\
& +2 < \mathcal{Y}(k+1) - \mathcal{Y}_r, O(\Phi_k) > \\
\leq \quad & ||\mathcal{Y}(k+1) - \mathcal{Y}_r||^2 + ||O(\Phi_k)||^2 + \\
& 2||\mathcal{Y}(k+1) - \mathcal{Y}_r||||O(\Phi_k)|| \qquad (19)
\end{aligned}
$$

The output of the nonlinear model at $k+2$ is related to the predicted output at $k+2$ as

$$Y(k+2) = \hat{Y}(k+2) + O(\Delta u_{k+1}^2) \qquad (20)$$

$$Y(k+2) = \mathcal{Y}(k+2) + O(\tilde{\Phi}_{k+1}) \qquad (21)$$

$$
\begin{aligned}
||\mathcal{Y}(k+2) - \mathcal{Y}_r||^2 & \\
= \quad & ||\hat{Y}(k+2) - \mathcal{Y}_r + O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})||^2 \\
= \quad & ||\hat{Y}(k+2) - \mathcal{Y}_r||^2 + ||O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})||^2 \\
& +2 < \hat{Y}(k+2) - \mathcal{Y}_r, O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1}) > \\
\leq \quad & 2||\hat{Y}(k+2) - \mathcal{Y}_r||||O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})|| \\
& +||\hat{Y}(k+2) - \mathcal{Y}_r||^2 \\
& +||O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})||^2 \qquad (22)
\end{aligned}
$$

From (17),(19) and (22) we get

$$
\begin{aligned}
& ||\mathcal{Y}(k+2) - \mathcal{Y}_r||^2 - ||O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})||^2 \\
& -2||\hat{Y}(k+2) - \mathcal{Y}_r||||O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})|| \\
& \qquad +\lambda^2||\Delta u(k+1)||^2 \\
& \leq ||\hat{Y}(k+2) - \mathcal{Y}_r||^2 + \lambda^2||\Delta u(k+1)||^2 \\
& \leq ||Y^*(k+2) - \mathcal{Y}_r||^2 \\
& \leq ||\mathcal{Y}(k+1) - \mathcal{Y}_r||^2 + ||O(\Phi_k)||^2 + \\
& \qquad 2||\mathcal{Y}(k+1) - \mathcal{Y}_r||||O(\Phi_k)|| \qquad (23)
\end{aligned}
$$

Choose λ and T such that

$$
\begin{aligned}
\lambda^2||\Delta u(k+1)||^2 & \\
\geq \quad & 2||\mathcal{Y}(k+1) - \mathcal{Y}_r||||O(\Phi_k)|| \\
& +2||\hat{Y}(k+2) - \mathcal{Y}_r||||O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})|| \\
& +||O(\Phi_k)||^2 + \frac{||\mathcal{Y}(k+1) - \mathcal{Y}_r||^2}{\lambda^3} \\
& +||O(\Delta u_{k+1}^2) - O(\tilde{\Phi}_{k+1})||^2 \qquad (24)
\end{aligned}
$$

This can be accomplished because as $T \to \infty$, the first and third terms on the right hand side go to zero, the fifth term goes to zero as fast as $\frac{1}{\lambda^8}$ and the second term goes to zero as fast as $\frac{1}{\lambda^4}$. The fourth term goes to zero as fast as $\frac{1}{\lambda^3}$, whereas the term on the left hand side of the inequality goes to zero as fast as $\frac{1}{\lambda^2}$. So there exist a λ and T such that (24) is satisfied.

Then from (23)

$$||\mathcal{Y}(k+2) - \mathcal{Y}_r||^2 \leq \theta||\mathcal{Y}(k+1) - \mathcal{Y}_r||^2 \qquad (25)$$

where $\theta = (1 - \frac{1}{\lambda^3})$.

$$\Rightarrow ||y(k+2) - y_r||^2 \leq \theta||y(k+1) - y_r||^2 \qquad (26)$$

with $0 < \theta < 1$.

From (26), for large k, $y(k)$ tends to y_r. For large sampling time, from (18), $Y^*(k+1)$ tends to $\mathcal{Y}(k)$. Hence from (11), for large k, the value of $\Delta u(k)$ tends to zero. And for large T and k, it can be shown that $u(k)$ converges to a constant value u_r. For open-loop globally stable system

$$
\begin{aligned}
\lim_{T\to\infty} f(x(k+1), u(k)) &= 0 \qquad (27) \\
\lim_{T\to\infty} f(\lim_{k\to\infty} x(k), u_r) &= 0 \qquad (28)
\end{aligned}
$$

which implies $\lim_{k\to\infty} x(k) = x_r$ where x_r is the unique solution to $f(x, u_r) = 0$. Hence, the closed-loop system under NLQDMC law is globally asymptotically stable.

Example (CSTR)

The example problem is taken from Limqueco and Kantor (1990). The modeling equations are described as

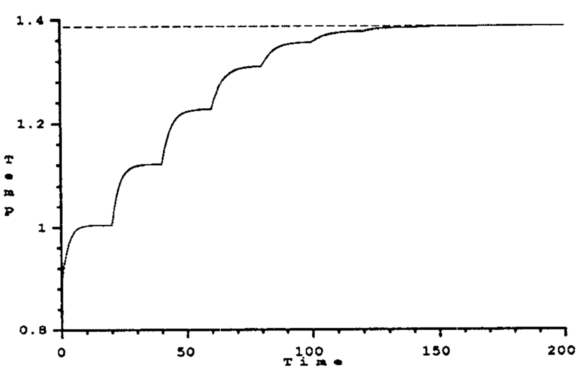

Figure 4: Dimensionless temperature vs. Time; T = 20 and λ = 3.2

$$
\begin{aligned}
\frac{dx_1}{dt} &= -\phi x_1 K + q(x_{1f} - x_1) \qquad (29) \\
\frac{dx_2}{dt} &= \beta\phi x_1 K - (q+\alpha)x_2 + \alpha u + q x_{2f} \qquad (30) \\
K &= exp\frac{x_2}{1 + x_2/\gamma} \qquad (31) \\
y &= x_2 \qquad (32)
\end{aligned}
$$

where x_1 is the dimensionless concentration, x_2 is the dimensionless temperature and u is the dimensionless cooling jacket temperature. The parameter values are $\phi = 0.072$, $\gamma = 20.0$, $x_{1f} = 1.0$, $x_{2f} = 0.0$, $q = 1.0$, $\alpha = 0.3$, $u = 0.0$ and $\beta = 8.0$. For these parameter values there are three possible steady states. So, this example violates the assumption of open-loop global asymptotic stability of the stability theorem. However, it serves the purpose of demonstrating the conservativeness by making a small setpoint change around the stable steady state setpoint. We examine the conservativeness of the stability conditions by making a setpoint change of 0.5 units in the output from the stable steady state $x^T = [0.8560, 0.8859]$.

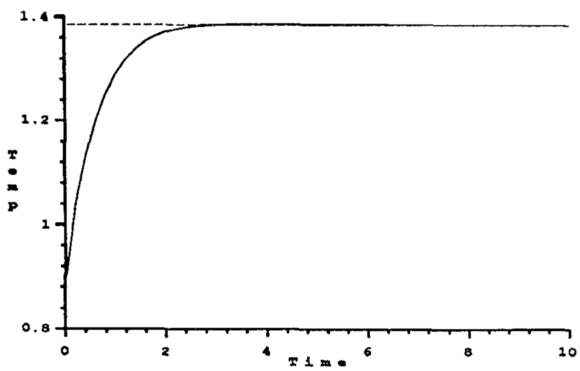

Figure 5: Dimensionless temperature vs. Time; T = 0.2 and λ = 0.0

The tuning parameter values of $P = 5$ and $M = 1$ are used in the simulations. To satisfy the inequality in (24) at every sampling time, the values of $T \geq 20.0$ and $\lambda \geq 3.2$ are required. In other words, for convergence towards setpoint a sampling time of at least 20.0 units and a value of 3.2 or greater for weight in change in manipulated variable are required. The simulations are shown in Fig. 4. However, the simulations in Fig. 5 show that convergence towards setpoint can be achieved by a value of $T = 0.2$ and $\lambda = 0.0$. Therefore, the stability conditions established are extremely conservative. Also, there is no procedure for off-line analysis to compute the values of T and λ, as (24) depends on the values of manipulated variables and the measurement at each sampling time.

5 Conclusions

In this paper, we have applied the nonlinear QDMC algorithm to a process where the sign of the system gain changes around the operating point. We have demonstrated that by choosing the proper tuning parameter values, such processes can be stabilized. Both setpoint changes and disturbance rejection were considered. In the second part of the paper, we have presented the stability conditions that guarantee the global asymptotic stability of the closed-loop system under NLQDMC law. we have examined the stability conditions by following the steps of the proof when NLQDMC algorithm is applied to a simple example. It was quantitatively demonstrated that, as expected, the stability conditions are quite conservative. Moreover, there is no procedure for an off-line analysis to compute values of sampling time and weights on change in manipulated variables that stabilize the system. Further research is required to reduce the conservativeness of the stability conditions.

Acknowledgements

Support for this project was provided by the National Science Foundation (Presidential Young Investigator grant CTS-9057292), the Systems Research Center and an unrestricted grant from BP America.

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