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NONCOHERENT HYBRID DS-SFH SPREAD-SPECTRUM MULTIPLE-ACCESS COMMUNICATIONS

by

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ABSTRACT

The performance of noncoherent reception in synchronous and asynchronous hybrid direct-sequence/slow-frequency-hopped spread-spectrum multiple-access communication systems operating through additive white Gaussian noise channels is investigated. Systems with binary and M-ary frequency-shift-keying modulation and noncoherent demodulation, as well as systems with differential-phase-shift keying modulation and differentially coherent demodulation are examined and their probability of error is evaluated for random frequency-hopping patterns and signature sequences.

The multiple-access capability of noncoherent hybrid spread-spectrum is shown to be superior to that of noncoherent purely frequency-hopped spread-spectrum and inferior to that of noncohernt purely direct-sequence spread-spectrum for systems with the same bandwidth expansion. Comparison of hybrid systems with coherent and noncoherent demodulation shows a considerable loss in the performance of the noncoherent systems.

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I. INTRODUCTION

This paper constitutes a companion to our work of [1], which deals with coherent reception in hybrid direct-sequence/slow-frequency hopped (DS/SFH) spread-spectrum multiple-access (SSMA) communications. In [1] we examined the performance of hybrid DS-SFH/SSMA systems employing phase-shift-keying (PSK) or quaternary-phase-shift-keying (QPSK) modulation and coherent demodulation. For these systems the hopping rate was assumed to be considerably smaller (about two orders of magnitude) than the data rate.

In this paper we complete the study of hybrid SSMA systems by considering the performance of noncoherent reception. We deal with the situation in which the receiver can acquire time synchronization with the desired signal but it can not acquire a phase reference, therefore, coherent demodulation is not feasible. This might be the case because of high levels of interference in the channel due to jamming or the presence of a large number of spread-spectrum signals in the same frequency band.

In [1] hopping rates considerably smaller than the data rate have to be implemented for coherent demodulation to be feasible. However, in many cases hopping rates of the same order of magnitude as the data rate are of interest. In these cases noncoherent hopping (i.e., a change in phase, which is practically impossible to track, accompanies the change in the frequency of transmission) prohibits the coherent reception of the desired signal. By employing modulation schemes which allow for noncoherent reception we can implement hybrid DS-SFH/SS systems with hopping rates slightly smaller than or even equal to the data rate. [In this case the hopping rates are only limited by the current technology of the frequency synthesizers.]

Therefore, the modulation schemes of interest are differential-phase- shift keying (DPSK) and frequency-shift-keying (FSK). Both binary FSK and M-ary FSK (MFSK) are considered in this paper. For systems with DPSK modulation differentially-coherent demodulation is employed; for systems with FSK modulation noncoherent demodulation is employed.

Although in this paper we only consider the performance of hybrid DS-SFH/SSMA communication systems with noncoherent reception over additive white Gaussian (AWGN) channels, our analysis can be extended to hybrid SSMA communication over fading (e.g., specular multipath) channels.

The performance measure considered is the *average probability of error* at the output of the receiver. For its evaluation we combine the techniques presented in [1, Section IV] for coherent hybrid SSMA systems and in [2] for noncoherent DS/SSMA systems. In particular, by using the method of [2] we evaluate the conditional error probability given the number of hits from other users and then we average over the distribution of hits. Our results are concerned with hybrid systems employing random frequency- hopping patterns and random signature sequences. However, the analysis can be also applied to systems with deterministic sequences. Both synchronous and asychronous systems are considered.

Besides the average error probability, the *multiple-access capability* of the noncoherent hybrid spread-spectrum systems is also evaluated. This is defined as the maximum number of simultaneously transmitted signals from users in the vicinity of a particular receiver so that the resulting error probability does not exceed a prespecified level.

The paper is organized as follows. In Section II the system models are described. Then, in Section III the average error probabililty is evaluated for random signature sequences and frequency-hopping patterns. Finally, in Section IV several numerical results are presented and conclusions about the performance of noncoherent hybrid SSMA systems are drawn.

II. SYSTEM MODELS

Our model for the **asynchronous** hybrid SSMA system resembles that of [1] where PSK and QPSK modulation schemes with coherent demodulation are employed but it is concerned with DPSK and FSK modulation with noncoherent demodulation. Therefore, it combines the features of both the slow- frequency-hopped SSMA model of [2] and the DPSK or FSK direct-sequence SSMA model of [3].

As shown in Figure 1 the transmitter for the k-th signal ($1 \leq k \leq K$, where K is the number of active users) consists of two parts: the DS/SS modulator (or spreader) and the frequency-hopper. Notice that despite similarities, the over all system model is different from the models of [1] and [2] and in order to define the necessary interference terms which are required for evaluating the performance of the systems we have to present the models in detail.

The models for the direct-sequence modulator are similar to those considered in [3], we repeat them here in order that we introduce the necessary concepts and notation. The output of the k-th DS/SS modulator $c_k(t)$ is different for the DPSK and FSK modulation schemes. For *DPSK modulation* it is given by

$$c_k(t) = 2\sqrt{2P} \;\; b_k(t)\, \Psi(t)\, a_k(t) \cos(2\pi f_c\, t + \theta_k). \tag{1}$$

In (1) f_c is the common frequency carrier, P is the common power of the transmitted signal (this assumption can be relaxed; it will be discussed in Section III), θ_k is the phase angle introduced by the k-th DPSK modulator, and $\Psi(t) = \psi(s)$ for $s = t \ (mod\ T_c)$, where $\psi(s)$ is a chip waveform. The chip waveform has duration T_c and is normalized so that $T_c^{-1} \int_0^{T_c} \psi^2(s)ds = 1$, where T_c^{-1} is the chip rate of the system. The k-th data signal $b_k(t)$, which is a differentially-encoded version of the information signal, is a sequence of rectangular pulses whose l-th pulse has amplitude $b_l^{(k)}$ taking values +1 and -1. The code waveform $a_k(t)$ is a periodic sequence of unit amplitude positive and negative rectangular pulses of duration T_c. The j-th code pulse has amplitude $a_j^{(k)}$. We assume that there are N code pulses in each data pulse ($T = NT_c$) and the period of the signature sequence $(a_j^{(k)})$ is N.

For *binary FSK modulation* $c_k(t)$ is given by

$$c_k(t) = 2\sqrt{2P}\ \Psi(t)\ a_k(t)\cos\{2\pi[f_c + b_k(t)W]t + \theta_k(t)\} \tag{2}$$

In (2) the waveform $\theta_k(t)$ is the phase introduced by the k-th FSK modulator; that is, if the l-th pulse of the information sequence $b_l^{(k)} = m$ for m = +1 or -1, then $\theta_k(t) = \theta_k$, where θ_k is the phase angle which corresponds to the frequency tone $f_c + mW$ (m = -1, 1) and the spacing between the two frequency tones is 2W. We assume that W >> T so that there is negligible interference between the two FSK tones of any signal.

Notice that for *MFSK modulation* we only need to modify the model above in an obvious way to account for the fact that the information sequence $(b_l^{(k)})$ is M-ary rather than binary and thus m takes on values in the set {1,2, ...,M} instead of the set {-1, 1}. Each M-ary symbol has duration $T_s = T\log_2 M$. For a fixed chip rate $T_c^{-1} = NT^{-1}$ a signature sequence of length $N' = N\log_2 M$ is used within each symbol.

As shown in Figure 1 the DS/SS signal $c_k(t)$ is frequency-hopped according to the k-th hopping pattern $f_k(t)$, which is derived from a sequence $(f_j^{(k)})$ of frequencies from a set $S = \{\nu_1, \nu_2, \ldots, \nu_q\}$ of q not necessarily equally spaced frequencies with minimum spacing W'. Let T_h denote the duration of a single hopping interval (dwell time) and $f_j^{(k)}$ denote the frequency used by the k-th sigal during the j-th dwell-time. We assume that $W' >> 2T^{-1} = 2(NT_c)^{-1}$ so that there is no overlapping of the DS/SS signals when hopped to adjacent frequencies. The number of data bits transmitted per hop $N_b = T_h/T$ is a positive integer. The transmitted signal takes the form

$$s_k(t) = \sqrt{2P}\ b_k(t)\ \Psi(t)\ a_k(t)\cos\{2\pi[f_c + f_k(t)]t + \theta_k + \alpha_k(t)\} \tag{3}$$

for a DPSK DS-SFH/SS signal, and

$$s_k(t) = \sqrt{2P}\ \Psi(t)\ a_k(t)\cos\{2\pi[f_c + b_k(t)W + f_k(t)]t + \theta_k(t) + \alpha_k(t)\} \tag{4}$$

for an FSK DS-SFH/SS signal, respectively. In (3) and (4) $\alpha_k(t)$ represents the phase waveform introduced by the k-th frequency-hopper; it takes on the constant value $\alpha_j^{(k)}$ during the j-th dwell time.

As in [1] - [3] it is assumed that K users share the same frequency band and that, because of the lack of a common time reference or the difference in propagation times along the various communciations links within the network, they cannot synchronize their transmissions. Therefore, the signal at the input of a particular receiver is

$$r(t) = \sum_{k=1}^{K} s_k(t-\tau_k) + n(t) \tag{5}$$

where τ_k for $1 \leq k \leq K$ denote the time delays along the communication links between the K transmitters and the particular receiver and n(t) is an AWGN process of two-sided spectral density $\frac{1}{2}N_0$.

The receiver for the i-th signal is shown in Figures 2 and 3 for hybrid systems with DPSK and FSK modulation, respectively. The receiver is assumed capable of acquiring frequency-hopping pattern, signature sequence, and time synchronization with the i-th signal. The first part of the receiver, the frequency dehopper, is common to both systems. The received signal is the input to a bandpass filter (see [3] or [1] for a detailed discussion of the filter characteristics). This filter is followed by the dehopper which introduces a phase waveform $\beta_k(t)$ analogous to that introduced by the frequency hopper [$\beta_j^{(k)}$ now stands for the constant phase introduced during the j-th hopping interval]. The dehopper is followed by a bandpass filter (again see [1] for its specifications) which removes the high frequency components. The output of the filter is

$$r_d(t) = \sum_{k=1}^{K} \sqrt{\tfrac{1}{2}P}\ \delta[f_k(t-\tau_k), f_i(t)]\ b_k(t-\tau_k)\ \Psi(t-\tau_k)\ a_k(t-\tau_k)\ cos\,[2\pi f_c t + \Phi_k(t)]$$

$$+ \hat{n}(t) \tag{6}$$

for DPSK signals, and

$$r_d(t) = \sum_{k=1}^{K} \sqrt{\tfrac{1}{2}P}\ \delta[f_k(t-\tau_k), f_i(t)]\ \Psi(t-\tau_k)\ a_k(t-\tau_k)\ \cos\{2\pi[f_c + b_k(t-\tau_k)W]t + \Phi_k(t)\}$$

$$+ \hat{n}(t) \tag{7}$$

for FSK signals. In (6) and (7) $\hat{n}(t)$ is a bandlimited version of n(t) which can be treated as

WGN with spectral density $N_0/8$ (see the discussion in [1] or [3] for justification). The phase waveform $\Phi_k(t)$ is defined as $\Phi_k(t) = \theta_k - 2\pi[f_c + f_k(t-\tau_k)]\tau_k + \alpha(t-\tau_k) - \beta_i(t)$ for DPSK signals, and as $\Phi_k(t) = \theta_k(t-\tau_k) - 2\pi[f_c + b_k(t-\tau_k)W + f_k(t-\tau_k)]\tau_k + \alpha_k(t-\tau_k) - \beta_i(t)$ for FSK signals. Finally, the Krönecker function δ is defined by $\delta(u,v) = 0$ for $u \neq v$ and $\delta(u,u) = 1$ for all real u and v.

The second part of the receiver consists of the demodulator and it is different (compare Figures 2 and 3) for the hybrid systems which employ DPSK or FSK data modulation. For *DPSK hybrid systems* a possible implementation of the differentially-coherent matched filter [5] receiver is shown in Figure 2 (for a rectangular chip waveform). As discussed in [2] for the corresponding DS/SSMA system, the implementation of Figure 2 requires that $f_c T = 2\pi L$, (L is a positive integer) and was selected for the simplicity of its analysis; a more practical implementation is described in [2]. The outputs of the in-phase components of the two branches of the receiver during the reception of the λ-th data bit (where $\lambda = j_i N_b + n_i$, i.e., for the n_i-th data bit of the j_i-th hop) are

$$Z_c = \int_{\lambda T}^{(\lambda+1)T} r_d(t)\, \Psi(t)\, a_i(t) \cos(2\pi f_c t)\, dt \tag{8a}$$

and

$$Z_{c,d} = \int_{\lambda T}^{(\lambda+1)T} r_d(t-T)\, \Psi(t)\, a_i(t) \cos(2\pi f_c t)\, dt. \tag{8b}$$

In (8a) - (8b) $r_d(t)$ is given by (6) and for simplicity of notation we have suppressed the dependence [through $r_d(t)$] of Z_c and $Z_{c,d}$ on (j_i, n_i) and i. The outputs of the quadrature components can be obtained from (8a) - (8b) by replacing $\cos(\cdot)$ with $\sin(\cdot)$. The receiver forms the statistic $Z_c Z_{c,d} + Z_s Z_{s,d}$ and compares it with a zero threshold. The estimate $\hat{b}_\lambda(i)$ should then be differentially decoded. As it is usual in SSMA systems, we assume that the receiver matched to the i-th signal can acquire time synchronization. We can, therefore, set $\tau_i = 0$ and consider time delays relative to the delay of the i-th signal. We do not require knowledge of the phase but, for DPSK communication, we require that the phase θ_i does not change over the duration of two adjacent data bits. It is assumed that the number of data bits transmitted during each dwell time is strictly larger than 1 ($N_b > 1$) and that there is not going to be a decision on the first data bit of the j_i-th dwell time; it will be used for acquiring the phase reference. Thus, any two adjacent bits of the j_i-th dwell interval, say the n_i-th and (n_i+1)-th (where $0 < n_i < N_b$), will have phase $\theta_i + \alpha_{j_i}^{(i)} - \beta_{j_i}^{(i)}$.

For *binary FSK hybrid systems* the receiver is shown in Figure 3 (for a rectangular chip waveform). During the reception of the λ-th data bit ($\lambda = j_i N_b + n_i$) the outputs of the in-phase components of the two branches are given by

$$Z_{c,m} = \int_{\lambda T}^{(\lambda+1)T} r_d(t)\, \Psi(t)\, a_i(t)\, \cos\left[2\pi(f_c + m\,\Delta)t\right]\, dt \tag{9}$$

for m = 1, -1. The outputs of the quadrature components of the two branches can be obtained from (9) by replacing $\cos(\cdot)$ with $\sin(\cdot)$. The receiver forms the statistics $\tilde{R}_m^2 = Z_{c,m}^2 + Z_{s,m}^2$ m = 1, -1 and outputs the estimate $\hat{\beta}_\lambda^{(i)} = m'$ if $\tilde{R}_{m'}^2 = \max\{\tilde{R}_1^2, \tilde{R}_{-1}^2\}$. Again it is assumed that the receiver is time-synchronous with the i-th signal, but now, we do not impose any restrictions on the phases $\theta_{i,m}$, since the reception is noncoherent. In this case the number of data bits per hop N_b is larger than or equal to 1. Thus, the case of a hopping rate equal to the data rate can be considered.

For *MFSK hybrid systems* the receiver has M branches each with an in- phase and quadrature sub-branch. The outputs of the in-phase branches of the receiver matched to the i-th signal are still given by (9) where we should replace $f_c + m\,\Delta$ by $f_c + (2m-1-M)\Delta$ for m = 1, 2, ..., M and T by the symbol duration $T \log_2 M$.

As in [1] - [4] the phase angles (θ_k or $\theta_{k,m}$, $a_j^{(k)}$ for $k \neq i$, and $\beta_j^{(i)}$), time delays (τ_k), and data streams ($b_n^{(k)}$) are for each k modeled as mutually independent random variables uniformly distributed in $[0,2\pi]$, $[0, N_h T_h]$, and $\{-1,+1\}$, respectively. Random variables characterizing different users are also mutually independent. The quantity N_h is equal to 1 for random frequency-hopping patterns and is equal to the period for periodic deterministic patterns.

In the **synchronous** case $\tau_k = j_k T_h$ is an integer multiple of the dwell time T_h for $1 \leq k \leq K$. In general it might not be that easy for the different users to acquire and maintain time synchronization at the dwell time level, but it will be easier (at least for $N_b > 1$) than synchronizing at the bit level as is necessary for synchronous DS/SSMA systems [2], [6].

III. AVERAGE ERROR PROBABILITY

A. Asynchronous Hybrid DS-SFH/SSMA Systems

To evaluate the error probabilities of the systems under consideration we first need to provide a detailed description of the outputs of the various matched filter receivers. The various components of the interference must be identified and characterized. Although for hybrid DS-SFH/SSMA systems these components resemble the corresponding entities of the DS/SSMA systems described in [2], there are also essential differences which make their accurate desription necessary.

For hybrid systems which employ *DPSK modulation* we may use (8a), (6), and the fact that $f_c >> T^{-1}$ in practical spread-spectrum systems [to ignore the double-frequency terms in (8a)] to express the output of the in-phase component of the upper branch of the demodulator of Figure 2 Z_c as

$$Z_c = D_c + N_c + \sqrt{P/8}T \sum_{k \neq i} I_c^{(k,i)}. \tag{10}$$

In (10) N_c is a zero-mean Gaussian random variable with variance $N_0 T/16$ and the desired signal component D_c is defined by

$$D_c = \sqrt{P/8}T \; b_\lambda^{(i)} \; cos\,[\theta_i + \alpha_{j_i}^{(i)} - \beta_{j_i}^{(i)}]. \tag{11}$$

The term $I_c^{(k,i)}$ denotes the multiple-access interference due to the k-th signal and is defined as follows. Let $j_k = \lfloor \tau_k / T_h \rfloor$ and $n_k = \lfloor (\tau_k - j_k T_h)/T \rfloor$, where $\lfloor u \rfloor$ denotes the integer part of the real number u. Also define for $0 \leq j < N$ and $0 \leq n < N_b$

$$d(j) = \delta(f_{j_i-j}^{(k)}, f_{j_i}^{(i)}) \tag{12}$$

and $L(j,n) = (j_i - j)N_b + n_i - n$. Recall that for the λ-th data bit $\lambda = j_i N_b + n_i$ where $1 \leq n_i < N_b$. Notice that if $d(j_k)=1$, then during the $(j_i - j_k)$-th dwell time of the k-th signal and the j_i-th dwell time of the i-th signal the same frequency is occuppied (a hit occurs in the terminology of [3]). If $d(j_k)=0$, then there is no interference during the j_i-th dwell time of the i-th signal caused from the $(j_i - j_k)$-th dwell time of the k-th signal.

For $0 \leq n_k < n_i$ we can write

$$I_c^{(k,i)} = d(j_k)[e(j_k,n_k) + \hat{e}(j_k,n_k)]\cos[\tilde{\psi}(j_k)]. \tag{13a}$$

This corresponds to a possible full hit during the $(j_i - j_k)$-th dwell time of the k-th signal. For $n_k = n_i$ we have

$$I_c^{(k,i)} = d(j_k+1)e(j_k,n_k)\cos[\tilde{\psi}(j_k+1)] + d(j_k)\hat{e}(j_k,n_k)\cos[\tilde{\psi}(j_k)] \tag{13b}$$

which corresponds to possible partial hits during either the $(j_i - j_k - 1)$-th or the $(j_i - j_k)$-th dwell time of the k-th signal, or to a possible full hit when $f_{j_i-j_k-1}^{(k)} = f_{j_i-j_k}^{(k)} = f_{j_i}^{(i)}$ [i.e., $d(j_k+1) = d(j_k) = 1$]. Finally for $n_i < n_k < N_b$

$$I_c^{(k,i)} = d(j_k+1)[e(j_k,n_k) + \hat{e}(j_k,n_k)]\cos[\tilde{\psi}(j_k+1)], \tag{13c}$$

which corresponds to a possible full hit during the $(j_i - j_k - 1)$-th dwell time of the k-th signal. In (13a) - (13c) the quantities e, $\hat{e}$, and $\tilde{\psi}$ are defined as

$$e(j,n) = b_{L(j,n+1)}^{(k)} R_{k,i}(\tau_k - jT_h - nT)/T, \tag{14a}$$

$$\hat{e}(j,n) = b_{L(j,n)}^{(k)} \hat{R}_{k,i}(\tau_k - jT_h - nT)/T, \tag{14b}$$

and $\tilde{\psi}(j) = \theta_k - 2\pi[f_c + f_{j_i-j}^{(k)}]\tau_k + \alpha_{j_i-j}^{(k)} - \beta_{j_i}^{(i)}$. In (14a) - (14b) $R_{k,i}$ and $\hat{R}_{k,i}$ are the continuous partial crosscorrelation functions [4]. These functions depend on $C_{k,i}$, the discrete aperiodic crosscorrelation function of the sequences $(a_l^{(k)})$ and $(a_l^{(i)})$, and on the autocorrelation functions of the chip waveform R_ψ and $\hat{R}_\psi$. To obtain the output of the lower branch $Z_{c,d}$ we need to replace $b_\lambda^{(i)}$ by $b_{\lambda-1}^{(i)}$ in (11) and n_i by $n_i - 1$ in the definition of the quantity L(j,n). Finally to obtain the outputs of the quardature components Z_s and $Z_{s,d}$ we only need to replace $\cos(\cdot)$ by $-\sin(\cdot)$ in (11) and (13a) - (13c).

For hybrid systems employing *binary FSK modulation* we can use (9) and (7) to show that the outputs of the in-phase components of the two branches of the demodulator of Figure 3 are given by

$$Z_{c,m} = D_{c,m} + N_{c,m} + \sqrt{P/8}\,T \sum_{k \neq i} I_{c,m}^{(k,i)} \tag{15}$$

for m = 1, -1. In (15) $N_{c,m}$ has the same distribution as N_c of (11) and the desired signal component is now given by

$$D_{c,m} = \sqrt{P/8}T\ \delta(b_\lambda^{(i)}, m)\ cos\,[\theta_{i,m} + \alpha_{j_i}^{(i)} - \beta_{j_i}^{(i)}]. \tag{16}$$

The multiple-access interference due to the k-th signal is now given for $0 \leq n_k < n_i$ by

$$I_{c,m}^{(k,i)} = d(j_k)\{g(j_k, n_k)\cos[\psi'(j_k)] + \hat{g}(j_k, n_k)\cos[\psi''(j_k)]\} \tag{17a}$$

and by

$$I_{c,m}^{(k,i)} = d(j_k+1)g(j_k, n_k)\cos[\psi'(j_k)] + d(j_k)\hat{g}(j_k, n_k)\cos[\psi''(j_k)] \tag{17b}$$

for $n_k = n_i$. For $n_i < n_k < N_b$ use (17a) with j_k replaced by j_k+1 in $d(\cdot)$, $\psi'(\cdot)$, and $\psi''(\cdot)$. The quantities g, $\hat{g}$, ψ', and ψ'' are defined as

$$g(j,n) = \delta(b_{L(j,n+1)}^{(k)}, m)\ R_{k,i}(\tau_k - jT_h - nT)/T, \tag{18a}$$

$$\hat{g}(j,n) = \delta(b_{L(j,n)}^{(k)}, m)\ \hat{R}_{k,i}(\tau_k - jT_h - nT)/T, \tag{18b}$$

whereas for $b_{L(j,n+1)}^{(k)} = m'$ and $b_{L(j,n)}^{(k)} = m''$ we have defined

$$\psi'(j) = \theta_{k,m'} - 2\pi[f_c + m'\Delta + f_{j_i-j}^{(k)}]\tau_k + \alpha_{j_i-j} - \beta_{j_i},$$

and similarly for $\psi''(j)$ with m'' replacing m'. An interpretation similar to that given for the full and partial hits in the DPSK hybrid signals case can be given to (17a) - (17b); the quantities g, $\hat{g}$ and the phases ψ', ψ'' are different than e, $\hat{e}$, and $\tilde{\psi}$ since they account for the different modulation schemes (FSK versus DPSK). Finally, the quadrature components $Z_{s,m}$ m = 1, -1 can be obtained from (15), (16) and (17a) - (17b) if we replace $cos(\cdot)$ by $-sin(\cdot)$.

For hybrid systems with *MFSK modulation* we can use the description of the various components of the outputs given for the binary FSK case [eq. (15) - (18)]. The necessary modifications are the following. There are now M branches so that m takes values in {1, 2, ..., M}. The data bit duration T should be replaced by the symbol duration $T_s = T\log_2 M$. Instead of data bits, now $b_\lambda^{(i)}$ or the $b_L^{(k)}$'s denote M-ary symbols. The number of bits per hop N_b should be replaced by $N_s = N_b/\log_2 M$ the number of M-ary symbols per hop (assume that N_b is an integer multiple of $\log_2 M$).

Notice that the expressions for the outputs of the matched filters that we have obtained so far are valid for arbitrary signature sequences and frequency-hopping patterns.

Next we proceed to the evaluation of the average error probability at the output of the receiver of the hybrid SSMA systems. The average should be computed with respect to all the random variables involved. We use a combination of a modified version of the technique employed in Section IV of [1] for coherent hybrid systems and the technique employed in [2] for DS/SSMA systems with noncoherent reception. In particular, we decouple the effect of hits from other users due to frequency-hopping from the multiple- access interference due to the direct-sequence spread-spectrum signals. This is done by first evaluating the conditional probability of error given the number of full hits and the number of partial hits and then averaging with respect to the distribution of the full hits and partial hits. Given that a number of full hits from other users has occurred the hybrid SSMA systems under consideration are equivalent to the DS/SSMA systems with noncoherent reception which we analysed in [2]. For the case of partial hits we need to modify the results of [2] in straightforward way.

Although this technique is applicable to the case when deterministic signature sequences are employed and the case when the powers of the transmitted signals are unequal, the analytical formulas are very complicated and the computational effort becomes gradually prohibitive. Therefore, we analyze hybrid DS-SFH/SSMA systems which employ *random signature sequences* and random frequency-hopping patterns and assume that the powers of the transmitted signals are nearly equal.

First we write the error probability of the hybrid SSMA system $\overline{P}_e$ (which for systems employing random signature sequences and hopping patterns is independent of i) as

$$\overline{P}_e = \sum_{k_f=0}^{K-1} \sum_{k_p=0}^{K-1-k_f} P_h(k_f, k_p) \overline{P}_e(k_f, k_p). \tag{19}$$

In (19) $P_e(k_f, k_p)$ denotes the probability of the occurrence of k_f full hits and k_p partial hits from the other K-1 users; and $\overline{P}_e(k_f, k_p)$ denotes the conditional error probability of the system given that k_f full hits and k_p partial hits occurred. For independent hopping patterns the joint probability of k full hits and k' partial hits is given by

$$P_h(k,k') = \binom{K-1}{k}\binom{K-1-k}{k'} P_f^k \, P_p^{k'} \, (1-P_f-P_p)^{K-1-k-k'} \tag{20}$$

where $0 \leq k < K$, $0 \leq k' < K-k$, and P_f and P_p denote the probability of a full and a partial hit from other users, respectively. These probabilities have been calculated in [3] and [1] and for asynchronous systems, first-order Markov random hopping patterns, and AWGN channels are given by

$$P_f = (1 - N_b^{-1})q^{-1} \tag{21a}$$

and

$$P_p = 2N_b^{-1}q^{-1}. \tag{21b}$$

As discussed in [1] and [3] for moderately large values of q (21a) and (21b) are also close approximations and bounds to the probabilities of hits of the memoryless random hopping patterns and the Reed-Solomon periodic hopping patterns, respectively. Therefore we can safely use (21a) - (21b) for most applications. For systems employing MFSK N_b of (21a) - (21b) should be replaced by $N_s = N_b/\log_2 M$.

To evaluate the conditional error probability $\overline{P}_e(k_f, k_p)$ when k_f full hits occur, notice that the hybrid SSMA systems are equivalent in terms of the performance to the corresponding DS/SSMA systems with k_f+1 users. Thus the technique of [2] is applicable. When partial hits occur a straightforward modification is required. In [2] we approximated the outputs of the matched filters: $Z_c, Z_{c,d}, Z_s, Z_{s,d}$ for DPSK, $Z_{c,1}, Z_{c,-1}$ for binary FSK, $Z_{c,1}, Z_{s,1}, Z_{c,2}, Z_{s,2}, \ldots, Z_{c,M}, Z_{s,M}$ for MFSK with Gaussian random variables having the same second order moments. The resulting approximations were shown in [2] to have satisfactory accuracy. Next, we illustrate the technique and evaluate $\overline{P}_e(k_f, k_p)$ for a hybrid DS-SFH/SSMA system employing binary FSK modulation we will then also cite the results for MFSK and DPSK systems without giving the details.

For an asynchronous DS/SSMA system with K users which employs *binary FSK modulation* and noncoherent demodulation and an AWGN channel the error probability at the output of the receiver matched to the i-th signal can be approximated [2] by

$$P_{e,i}^{G} = v_{i,-1}(v_{i,1} + v_{i,-1})^{-1} exp\,[-\frac{1}{2}(v_{i,1} + v_{i,-1})^{-1}] \tag{22}$$

where the normalized variances $v_{i,m}$ of $Z_{c,m}$ m = 1, -1 are defined by

$$v_{i,m} = (2E_b/N_0)^{-1} + \sum_{k \neq i} \sigma_{k,i;m}^2 . \tag{23}$$

In (23) $E_b = PT$ is the average energy per bit in the absence of multiple-access interference and the quantity $\sigma_{k,i;m}^2 = Var\,\{I_{c,m}^{(k,i)}\}$ is given by

$$\sigma_{k,i;m}^2 = \frac{1}{4} T^{-3} \int_0^T [R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau) + R_{k,i}(\tau)\hat{R}_{k,i}(\tau)]\; d\,\tau. \tag{24}$$

Eq. (22) was obtained from [7, pp. 320-323] after approximating $Z_{c,1}$ and $Z_{c,-1}$ (also $Z_{s,1}$, $Z_{s,-1}$) by Gaussian random variables [2] and normalizing their variances (divide by $PT^2/8$). Then (23) is obtained from (15), whereas (24) is obtained from (17) - (18) assuming that the k-th user causes a full hit [i.e., when in (17a) $d(j_k)$ is equal to 1, or in (17b) both $d(j_k)$ and $d(j_k+1)$ are equal to 1] and averaging with respect to the data bits, the phase angles, and the data delays. Notice that $\sigma_{k,i;m}^2$ and thus $v_{i,m}$ turn out to be independent of m. When the k-th user causes a partial hit [i.e., when in (17b) one but not both of $d(j_k)$ and $d(j_k+1)$ is equal to 1] (24) should be replaced by

$$\sigma_{k,i;m}^2 = \frac{1}{8} T^{-3} \int_0^T [R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau)]\; d\,\tau. \tag{25}$$

Suppose now that the signature sequences are random, that is, for each k $(a_l^{(k)})$ consists of a sequence of mutually independent random variables taking values in $\{+1,-1\}$ with equal probability, and that signature sequences assigned to different users are mutually independent. Averaging the variance $\sigma_{k,i;m}^2$ over the ensemble of random signature sequences of length N we obtain

$$\overline{\sigma}_f^2 = \overline{E}\{\sigma_{k,i;m}^2\} = m_{\psi}/(2N) \tag{26}$$

for the case of a full hit, and

$$\bar{\sigma}_p^2 = \bar{E}\{\sigma_{k,i;m}^2\} = m_\psi/(4N) \tag{27}$$

for the case of a partial hit. In (26) - (27) $\bar{E}$ denotes expectation with respect to the ensemble of random signature sequences of length N and $m_\psi = T_c^{-3}\int_0^{T_c} R_\psi^2(\tau)\,d\tau = T_c^{-3}\int_0^{T_c} \hat{R}_\psi^2(\tau)\,d\tau$. For a rectangular chip waveform $m_\psi = 1/3$; for a sine chip waveform $m_\psi = (15+2\pi^2)/12\pi^2$. To obtain eq. (26) and (27) we used the facts from [2] that

$$\bar{E}\{\int_0^T [R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau)]d\tau\} = 2N^2 m_\psi T_c^3 \tag{28}$$

and

$$\bar{E}\{\int_0^T R_{k,i}(\tau)\hat{R}_{k,i}(\tau)d\tau\} = 0, \tag{29}$$

respectively. Finally, upon substitution from (28) - (29) in (24) and from (29) in (25) we can verify (26) and (27). Both (26) and (27) are independent of k,i, and m, and so is $\bar{E}\{v_{i,m}\}$. Combining these facts in (23) and assuming that k_f users cause full hits and k_p users cause partial hits we obtain

$$\bar{E}\{v_{i,m}\} = (2E_b/N_0)^{-1} + k_f\,\bar{\sigma}_f^2 + k_p\,\bar{\sigma}_p^2, \tag{30}$$

so that (22) reduces to

$$\bar{P}_e(k_f, k_p) = \frac{1}{2}\exp\left\{-\frac{1}{4}\left[\left(\frac{2E_b}{N_0}\right)^{-1} + (k_f + \frac{1}{2}k_p)\frac{m_\psi}{2N}\right]^{-1}\right\}. \tag{31}$$

For *MFSK modulation* we should modify (22) (see [2]) to be

$$\bar{P}_{e,i}^G = \sum_{m=1}^{M-1}\binom{M-1}{m}(-1)^{m+1}v_{i,2}(mv_{i,1} + v_{i,2})^{-1}exp\,[-\frac{1}{2}m\,(mv_{i,1} + v_{i,2})^{-1}] \tag{32}$$

where $v_{i,m}$ m = 1, 2, ..., M are still defined by (23), and $\sigma_{k,i;m}^2$ are now given by

$$\sigma_{k,i;m}^2 = \frac{1}{2}M^{-1}\int_0^{T_s}[R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau) + 2M^{-1}R_{k,i}(\tau)\hat{R}_{k,i}(\tau)]\,d\tau \tag{33}$$

for full hits and by

$$\sigma_{k,i;m}^2 = \frac{1}{4}M^{-1}T_s^{-3}\int_0^{T_s}[R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau)]\,d\tau \tag{34}$$

for partial hits. Recall that $T_s = T\log_2 M$ and we use $N' = N\log_2 M$ chips per symbol. For random signature sequences (33) and (34) reduce to

$$\overline{\sigma}_f^2 = m_\psi/(MN') \tag{35}$$

and

$$\overline{\sigma}_p^2 = m_\psi/(2MN'), \tag{36}$$

so that finally

$$\overline{P}_e(k_f, k_p) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1} \exp\left\{-\frac{m}{2(m+1)} \left[\left(\frac{2E_b \log_2 M}{N_0} \right)^{-1} + (k_f + \frac{1}{2}k_p) \frac{m_\psi}{MN'} \right]^{-1} \right\} \tag{37}$$

Equation (37) gives an expression for the symbol error probability of the MFSK hybrid system. It can also serve as a conservative estimate of the bit error probability.

Finally, for *DPSK modulation* we should modify (22) (see [2]) as

$$P_{e,i}^G = \frac{1}{2}(1-\tilde{v}_i/v_i) exp\,(-\frac{1}{2}v_i^{-1}) \tag{38}$$

where the normalized moments $v_i = Var\,\{Z_{c,d}\}$ and $\tilde{v}_i = Cov\,\{Z_c, Z_{c,d}\}$ are given by

$$v_i = (2E_b/N_0)^{-1} + \sum_{k \neq i} \sigma_{k,i}^2 \tag{39a}$$

and

$$\tilde{v}_i = \sum_{k \neq i} \tilde{\sigma}_{k,i}^2 . \tag{39b}$$

Using (13) and (14) we can show that the quantities $\sigma_{k,i}^2 = Var\,\{I_c^{(k,i)}\} = Var\,\{I_{c,d}^{(k,i)}\}$ and $\tilde{\sigma}_{k,i}^2 = Cov\,\{I_c^{(k,i)}, I_{c,d}^{(k,i)}\}$ take the form

$$\sigma_{k,i}^2 = \frac{1}{2}T^{-3} \int_0^T [R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau)]\, d\tau \tag{40a}$$

$$\tilde{\sigma}_{k,i}^2 = \frac{1}{2}T^{-3} \int_0^T R_{k,i}(\tau)\hat{R}_{k,i}(\tau)\, d\tau \tag{40b}$$

for full hits, and

$$\sigma_{k,i}^2 = \frac{1}{4} T^{-3} \int_0^T [R_{k,i}^2(\tau) + \hat{R}_{k,i}^2(\tau)] \, d\tau \tag{41a}$$

$$\tilde{\sigma}_{k,i}^2 = 0 \tag{41b}$$

for partial hits. For random signature sequences (40) and (41) reduce to

$$\sigma_f^2 = m_\psi / N \tag{42a}$$

$$\tilde{\sigma}_f^2 = 0 \tag{42b}$$

and

$$\sigma_p^2 = m_\psi / (2N) \tag{43a}$$

$$\tilde{\sigma}_p^2 = 0, \tag{43b}$$

respectively. Upon substitution from (42) and (43) into (39) we can show that, if k_f users cause full hits and k_p users cause partial hits, then

$$\bar{E}\{v_i\} = (2E_b / N_0)^{-1} + k_f \, \sigma_f^2 + k_p \, \sigma_p^2 \tag{44a}$$

and

$$\bar{E}\{\tilde{v}_i\} = k_f \, \tilde{\sigma}_f^2 + k_p \tilde{\sigma}_p^2 = 0 \tag{44b}$$

Using these values for v_i and $\tilde{v}_i$ in (38) we obtain the final result

$$\bar{P}_e(k_f, k_p) = \frac{1}{2} \exp\left\{-\frac{1}{2}\left[\left(\frac{2E_b}{N_0}\right)^{-1} + (k_f + \frac{1}{2} k_p) \frac{m_\psi}{N}\right]^{-1}\right\}. \tag{45}$$

Therefore, to evaluate the average error probability of the asynchronous hybrid SSMA systems we use (19), (20), and the expressions (31), (37), or (45) for binary FSK, MFSK, or DPSK modulation schemes, respectively.

B. Synchronous Hybrid DS-SFH/SSMA Systems

In the synchronous case any user k ($k \neq i$) can cause only full hits with probability

$$P_h = q^{-1} \tag{46}$$

for first-order Markov and memoryless random hopping patters [for periodic Reed-Solomon hopping patterns (46) is a tight approximation for moderately large q]. We can then use the following modified form of (19)

$$\bar{P}_e = \sum_{k=0}^{K-1} \binom{K-1}{k} P_h^k (1-P_h)^{K-1-k} \bar{P}_e(k). \tag{47}$$

where P_h is defined by (46) and $\bar{P}_e(k)$, the conditional probability of error given that k full hits from other users occur, has to be computed for the binary FSK, M-ary FSK, and DPSK modulation schemes under consideration.

The computation of $\bar{P}_e(k)$ is considerably easier in the synchronous case. The interference terms $I_c^{(k,i)}$ and $I_{c,m}^{(k,i)}$ of (13) - (14) and (17) - (18) take now the simpler forms

$$I_c^{(k,i)} = d(j_k) b_{-j_k}^{(k)} [\theta_{k,i}(0)/N] \cos[\tilde{\psi}(j_k)] \tag{48}$$

and

$$I_{c,m}^{(k,i)} = d(j_k) \delta(b_{-j_k}, m) [\theta_{k,i}(0)/N] \cos[\psi' (j_k)] \tag{49}$$

where $\tau_k = j_k T_h$ and $\theta_{k,i}$ denotes the even discrete crosscorrelation function [4] of the signature sequences $(a_l^{(k)})$ and $(a_l^{(i)})$ [actually $\theta_{k,i}(0) = C_{k,i}(0)$]. Following the same procedure as for the evaluation of $\bar{P}_e(k_f, k_p)$ in Section III.A above and using the fact [8] that for random signature sequences $\bar{E}\{\theta_{k,i}^2(0)\} = N$ we can show that for *binary FSK hybrid SSMA* systems

$$\bar{P}_e(k) = \frac{1}{2} \exp\left\{ -\frac{1}{4} \left[\left(\frac{2E_b}{N_0} \right)^{-1} + \frac{k}{4N} \right]^{-1} \right\}, \tag{50}$$

for *MFSK hybrid SSMA* systems

$$\bar{P}_e(k) = \sum_{m=1}^{M-1} \binom{M-1}{m} \frac{(-1)^{m+1}}{m+1} \exp\left\{ -\frac{m}{2(m+1)} \left[\left(\frac{2E_b \log_2 M}{N_0} \right)^{-1} + \frac{k}{2MN} \right]^{-1} \right\}, \tag{51}$$

and for *DPSK hybrid SSMA* systems

$$\bar{P}_e(k) = \frac{1}{2} \exp\left\{ -\frac{1}{2} \left[\left(\frac{2E_b}{N_0} \right)^{-1} + \frac{k}{2N} \right]^{-1} \right\}. \tag{52}$$

Combining (47) and (46) with (50), (51), or (52) we find the average error probability of synchronous hybrid SSMA systems employing binary FSK, MFSK, or DPSK modulation, respectively.

IV. NUMERICAL RESULTS AND CONCLUSIONS

All the hybrid SSMA systems considered in this section employ random signature sequences and first-order Markov random hopping patterns. The communication channel is an AWGN channel. The approximation to error probability $\overline{P}_e$ is calculated via the method presented in Section III.

In Figure 4 the average error probability $\overline{P}_e$ is plotted versus the signal-to-noise ratio E_b/N_0 (in dB) for both synchronous and asynchronous systems with K=20,50,100,200,300,500,800, and 1000 simultaneous users employing DPSK modulation and differentially coherent demodulation. The systems use N=31 chips per bit, q=100 frequencies, and $N_b=10$ bits per hop. The asynchronous systems perform better than the synchronous systems. This is due to the extra randomization involved in the former systems and is observed for systems with $N_b>1$. Notice the graceful degradation of the system's performance as the number of simultaneous users increases.

In Figure 5 $\overline{P}_e$ is plotted versus E_b/N_0 for hybrid SSMA systems with K=20,50,100,200,300,500,800,and 1000 simultaneous users employing binary FSK modulation and noncoherent demodulation. The system parameters are again N=31, q=100, and $N_b=10$. The same observations as for Figure 4 are valid. Notice that now all the curves are shifted up and to the right to larger values of the error probabilities.

In Figure 6 the error probability of hybrid SSMA systems with K=100 users and parameters N=31, q=100, and $N_b=10$ is compared for PSK, DPSK, and binary FSK modulation schemes with coherent, differentially-coherent, and noncoherent demodulation, respectively. For asynchronous hybrid SSMA systems with an error probability of 10^{-4}, the system employing DPSK modulation requires almost 2 dB more in signal-to-noise ratio to achieve the same error probability as the system employing PSK modulation. Similarly FSK requires 3 dB more than DPSK. Similar comparisons hold for synchronous systems.

In Table 1 we compare the performance of hybrid SSMA systems with different chip waveforms. The system parameters are N=31, q=100, and $N_b=10$. The number of simultaneous users is K=100. DPSK or binary FSK modulation schemes are employed. The error probability of synchronous systems is independent of the chip waveform. For asynchronous systems the sine chip waveform outperforms the rectangular chip waveform; however, the difference becomes substantial only at high E_b/N_0's. This is due to the fact that the approximations to the average probability of error which are given by (45) and (31) (for DPSK and binary FSK, respectively) depend on the quantitiy m_ψ, which was defined after eq. (27) and takes the values .333 and $(15+2\pi^2)/(12\pi^2)$ = .208 for rectangular and sine chip waveforms respectively.

In Figure 7 we have plotted $\bar{P}_e$ for hybrid SSMA systems which employ 16-ary FSK modulation with noncoherent demodulation. The system parameters are N'=31, q=50, and $N_s=1$. The number of simultaneous users is K=20,50,100,200,300,500,800, and 1000. Several observations are in order. First, notice that for K equal or larger than 200 the synchronous sytems perform better than the asynchronous ones. This is in contrast to the results presented in Figure 5. This is justified by the fact that now one ($N_s=1$) 16-ary symbol per hop is transmitted instead of $N_b=10$ bits per hop; therefore for asynchronous systems only partial hits occur with probability 2/q, which is twice the probability of a (full) hit for the synchronous systems. For a small number of users (see the curves for K=20, 50, and 100) the effect of partial hits is not dominant and as in the cases described in Figure 5 the asynchronous systems perform slightly better than the synchronous systems. Second, although the bandwidth expansion required for this system is $MN'q/\log_2 M=6200$, that is identical to that of the system of Figure 5 the 16-ary FSK modulation provides considerably higher multiple access capability. Thus at E_b/N_0=14 dB Figure 7 indicates that 500 simultaneous users can be accomodated at an error probability of 10^{-4}; at the same values of signal-to-noise ratio and error probability the system of Figure 5 can accomodate 100 users.

In Tables 2 (a), 2 (b), and 2 (c) we compare the performance of several hybrid SSMA systems employing MFSK modulation with noncoherent demodulation. All the systems require the same bandwidth expansion $MN'q/\log_2 M=6200$. Recall that N' denotes the number of chips per M-ary symbol. The number of M-ary symbols transmitted in one hop is $N_s=1$. The number of simultaneous users is K=100 in all cases. In particular, in Table 2 (a) M=2 and the product N'q is held (approximately) fixed. Notice that the system with the largest number of chips per symbol N' performs slightly better than the other systems with fixed N'q. Then, in Table 2 (b) N'=31 and the product Mq/log_2M is held fixed. Now the system with the largest value of M performs considerably better than the other systems with fixed $Mq/\log_2 M$. Finally, in Table 2 (c) q=50 and the product $MN'/\log_2 M$ is held (approximately) fixed. Again in this case the system with the largest value of M performs considerably better than the other systems with fixed $MN'/\log_2 M$. In conclusion, when the hybrid SSMA system should satisfy a bandwidth constraint, it is advantageous to use M-ary FSK modulation with M considerably higher than 2 and higher chip rates.

In Tables 3 (a) and 3 (b) the multiple-access capability of synchronous and asynchronous hybrid DS-SFH/SSMA systems with different data modulation and demodulation schemes is cited. Specifically, the maximum number of simultaneous users that can be accomodated by hybrid systems with PSK, DPSK, and binary FSK modulation with coherent, differentially coherent, and noncoherent demodulation, respectively is tabulated for two different values of the tolerated probabiity of error $P_e = 10^{-3}$ and $P_e = 10^{-5}$ and different values of q and N where the bandwidth qN is held fixed at 700. The other system parameters are $E_b/N_0 = 12$ dB and $N_b = 100$ bits per hop. Notice the considerable loss in the multiple-access capability of the hybrid system from the coherent system to the differentially coherent system and then to the noncoherent system. Also notice that the asynchronous systems perform better than the synchronous systems. This is due to fact that, since N_b is large (of the order of hundreds so that coherent demodulation may be feasible) the probabilities of hits in (21a) - (21b) and (46) are approximately the same. Thus, only the direct-sequence modulation part dominates the difference between the two types of systems and the asynchronous systems, where extra randomization is involved perform better [e.g., for FSK compare eq. (31) and (50); m_ψ is involved in the former, $\frac{1}{2}$ is involved in the latter and $m_\psi < \frac{1}{2}$ for the rectangular and sine waveforms]. The multiple-access capability increases as N increases for fixed product qN. That is, the direct-sequence modulation part of the hybrid system provides more multiple-access capability than the frequency-hopped modulation part of it. The cases q=700, N=1 and q=1, N=700 correspond to the purely frequency-hopped and purely direct-sequence SSMA systems, respectively.

Finally, in Table 4 the multiple-access capability of hybrid DS-FH/SSMA systems which employ MFSK modulation and noncoherent demodulation is tabulated for two different values of the tolerable error probability $P_e = 10^{-3}$ and $P_e = 10^{-5}$ and different values of q, N, and M. The other system parameters are $E_b/N_0 = 12$ dB and $N_s = 1$ M-ary symbols per hop. Several observations are in order here. First, the multiple-access capability increases as N increases for fixed qN and fixed M [as also noted in Tables 3 (a) and 3 (b) above]. Second, the muktiple-access capability increases as M increases for fixed q and N. Third, for fixed product qN, comparison of synchronous to asynchronous hybrid SSMA systems with the same parameters shows that when q is large and N is small the former systems outperform the latter ones. In contrastto this observation, as q decreases and N increases the situation is reversed and the asynchronous systems perform better. This phenomenon is justified as follows. For large q and small N the frequency modulation part of the hybrid system is dominant and since the probabilities of hits for synchronous and asynchronous systems are (for $N_s = 1$) 1/q and 2/q, respectively, the synchronous systems perform better. In contrast, as q decreases and N increases the direct-sequence modulation part of the hybrid systems becomes dominant and thus the asynchronous systems (which involve extra randomization, refer to the discussion at the end of the previous paragraph) perform better.

The comparison of noncoherent hybrid (DS-SFH) SSMA systems to noncoherent purely frequency-hopped (FH) SSMA and noncoherent purely direct-sequence (DS) SSMA systems indicates that, under the assumptions of equal power for the transmitted signals of all users, identical bandwidth expansion, same data modulation and demodulation schemes, and same other system parameters, the multiple-access capability of hybrid systems is superior to that of purely FH systems and inferior to that of purely DS systems. This makes them attractive especially for applications involving error-control coding, since hybrid SSMA systems can be interleaved more efficiently than DS/SSMA systems and have a larger mutiple-access capability than FH/SSMA systems.

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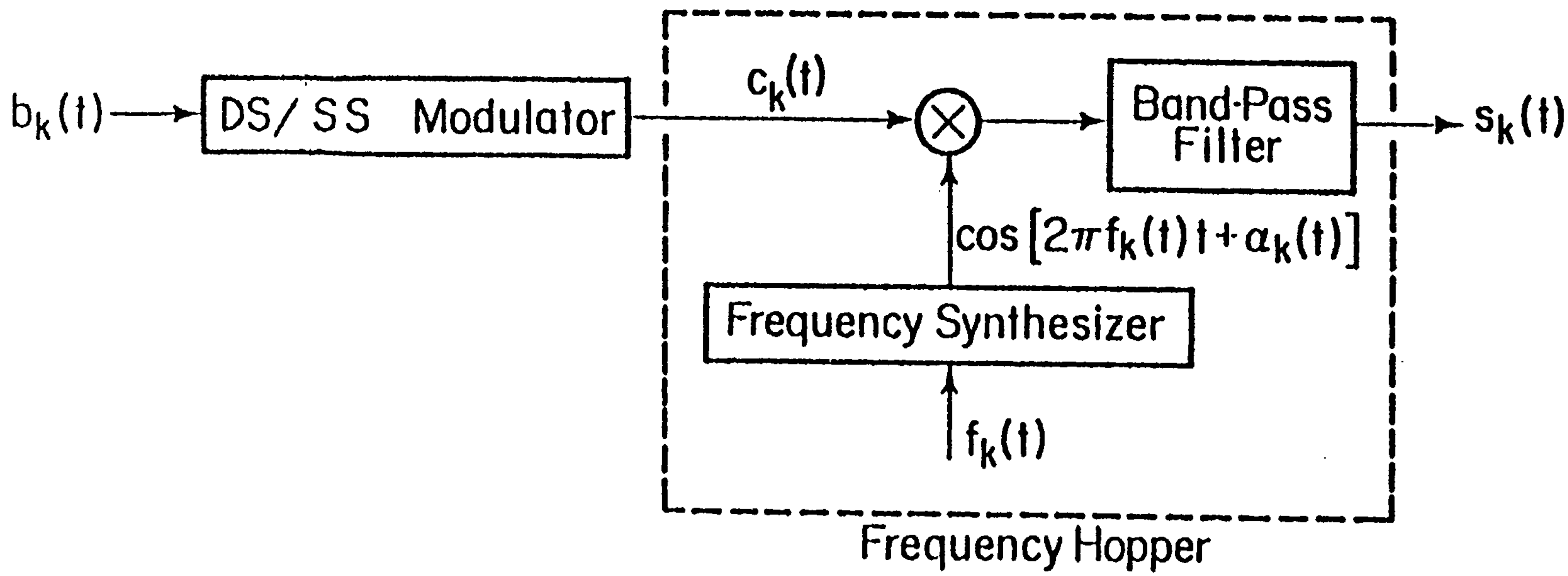

Figure 1. Transmitter for a hybrid DS-SFH/SSMA system.

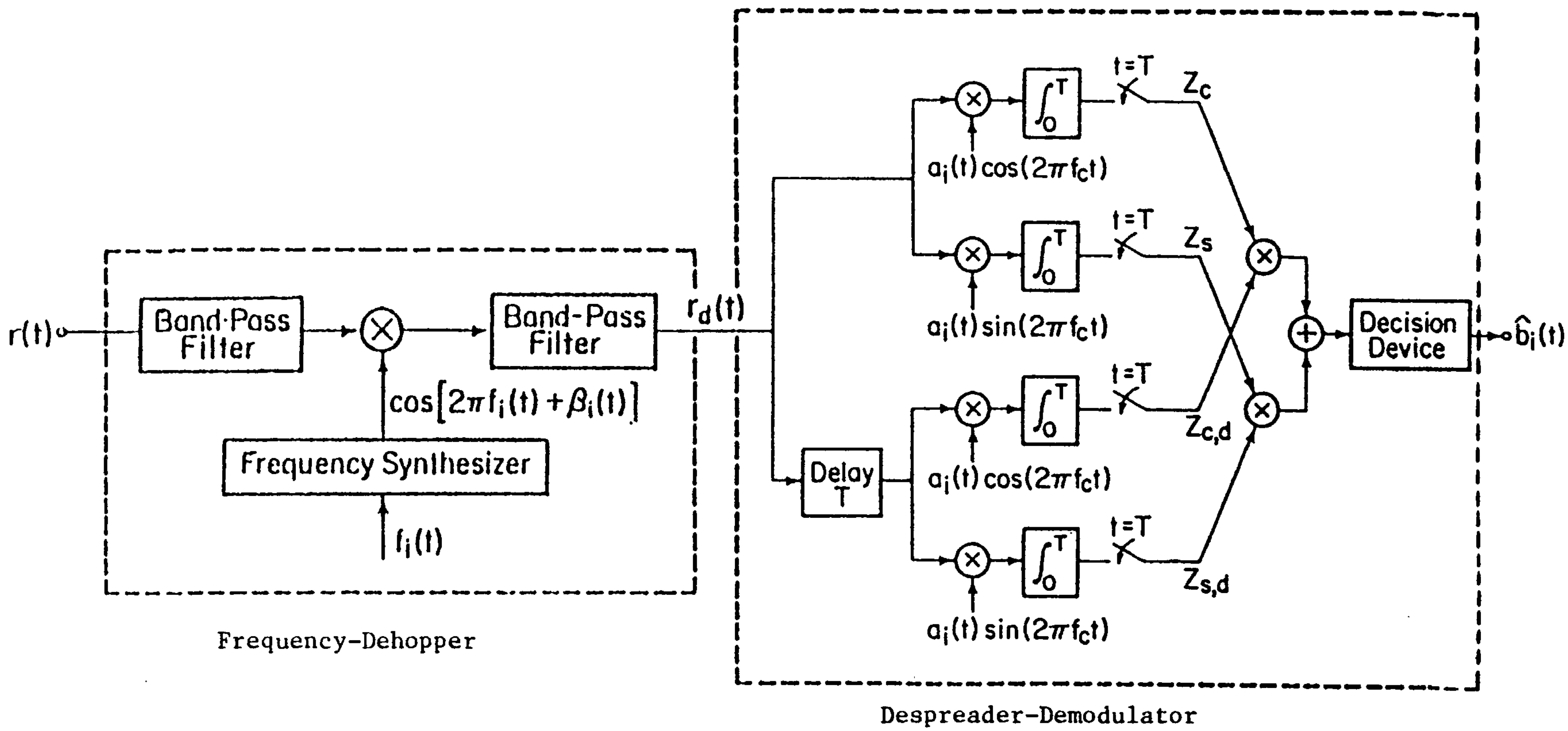

Figure 2. Receiver for a hybrid DS-SFH/SSMA system employing DPSK modulation.

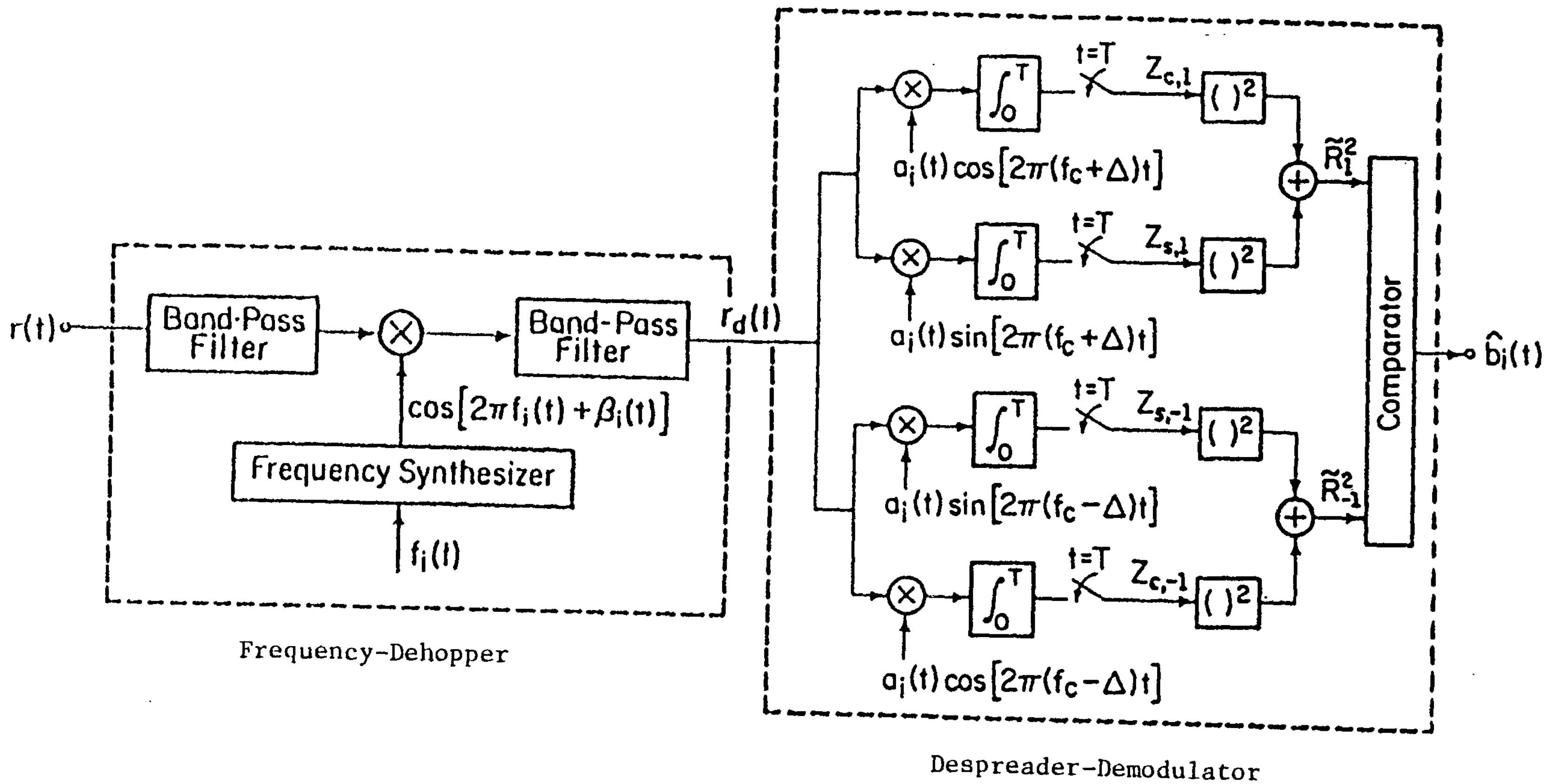

Figure 3. Receiver for a hybrid DS-SFH/SSMA system employing FSK modulation.

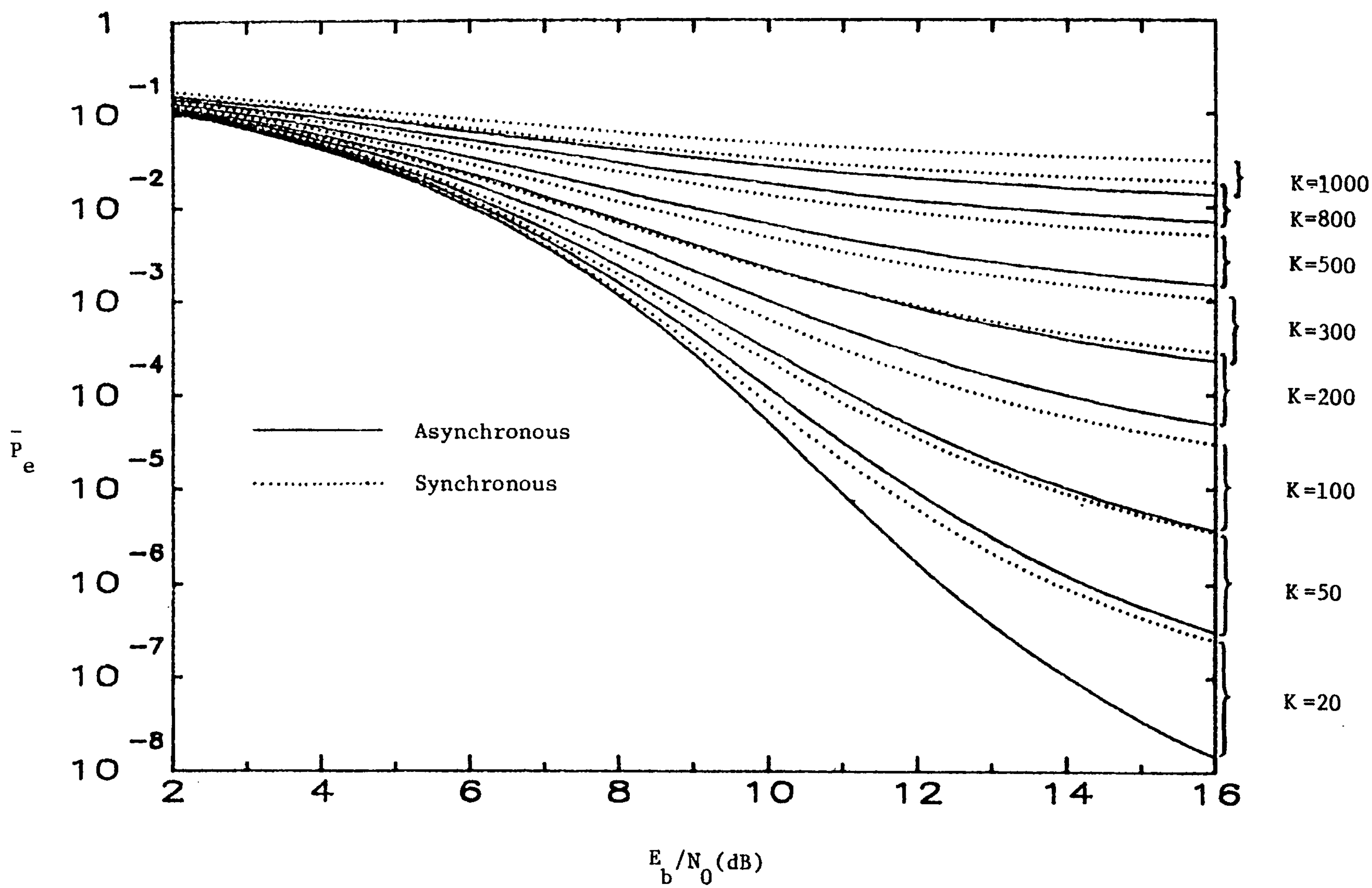

Figure 4. Probability of error for DS-SFH/SSMA systems employing DPSK modulation ($N=31$, $q=100$, $N_b=10$)

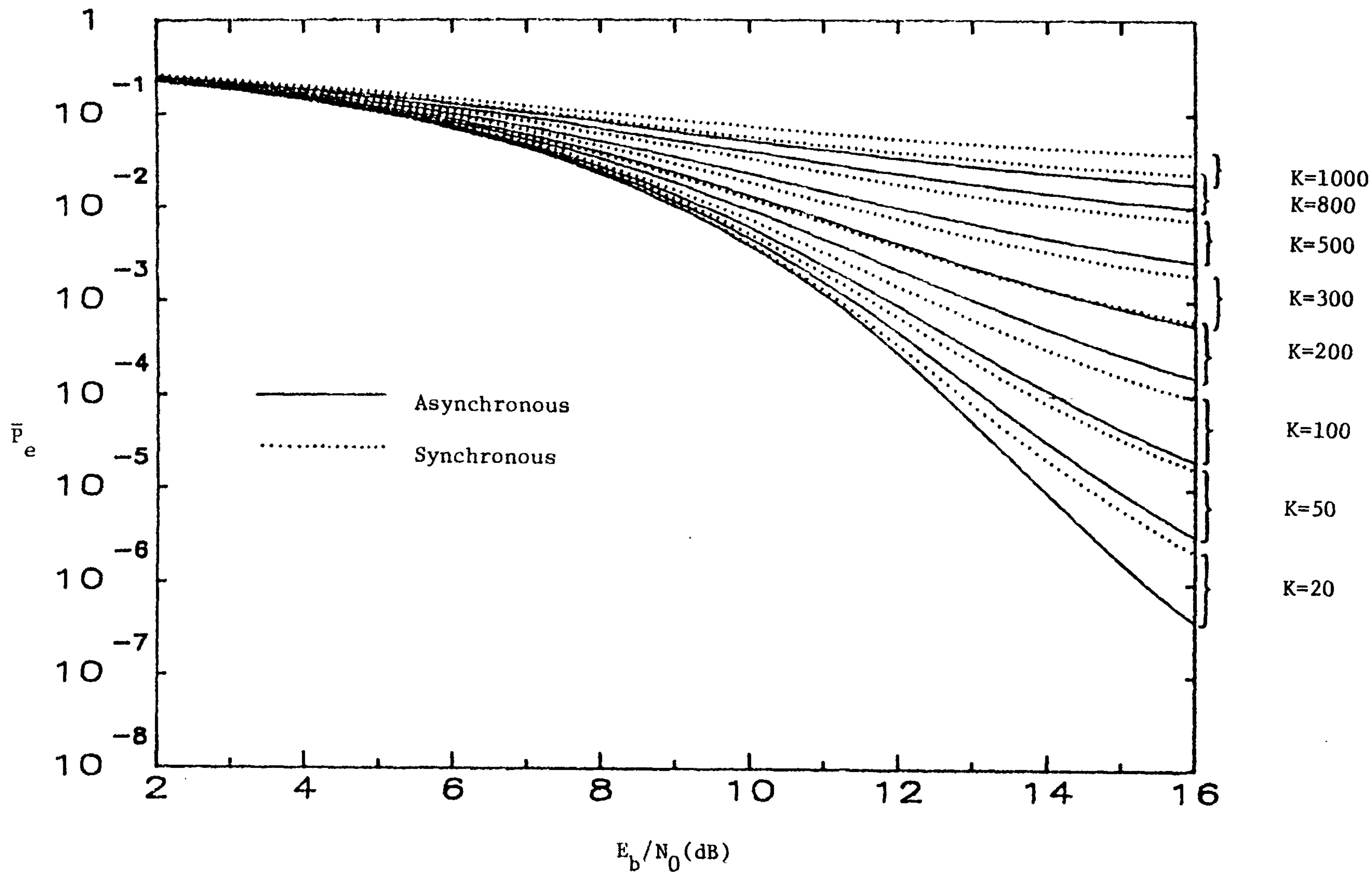

Figure 5. Probability of error for DS-SFH/SSMA systems employing binary FSK modulation (N=31, q=100, N_b=10)

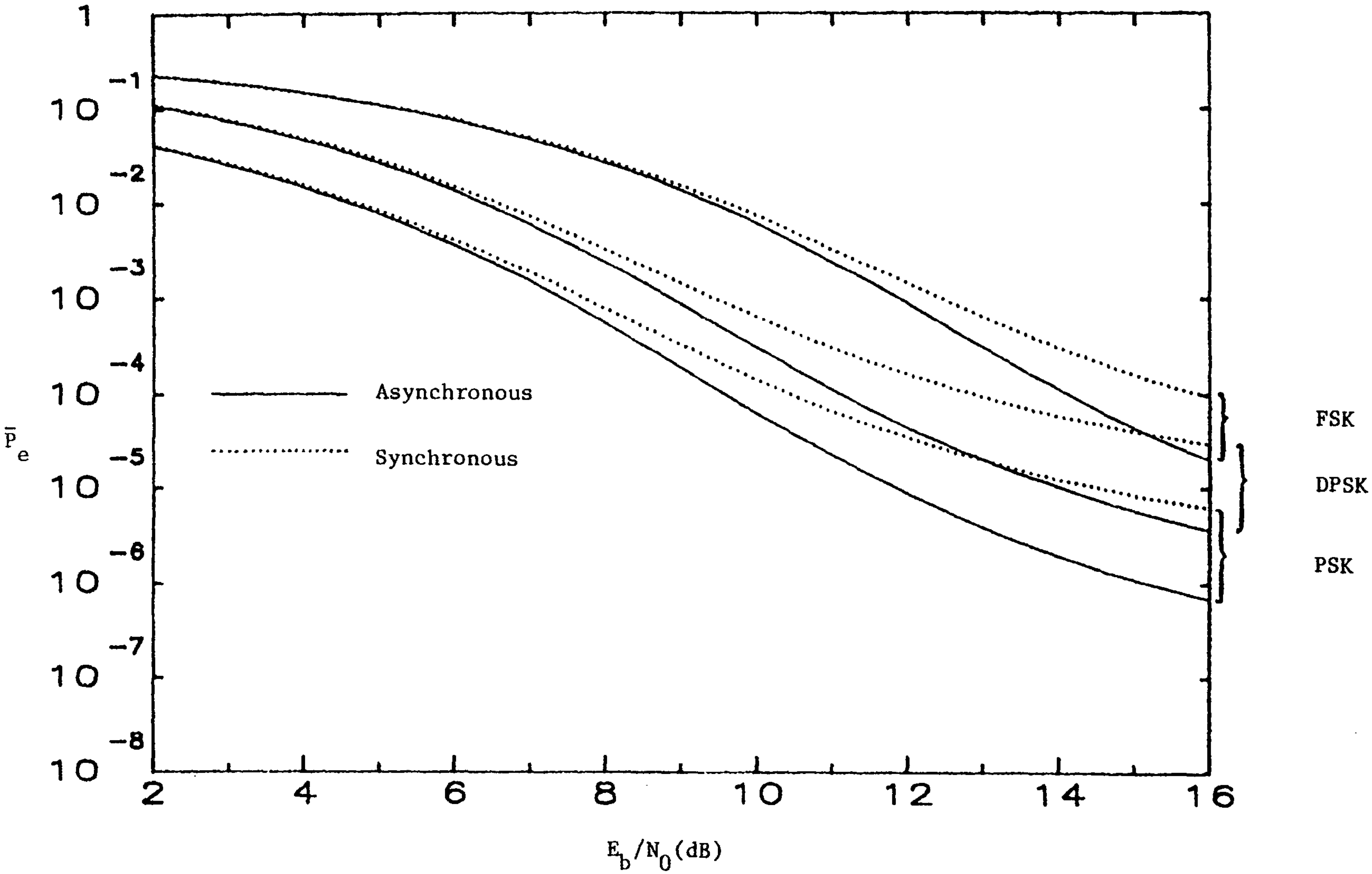

Figure 6. Probability of error for DS-SFH/SSMA systems with different modulation schemes (K=100, N=31, q=100, N_b=10)

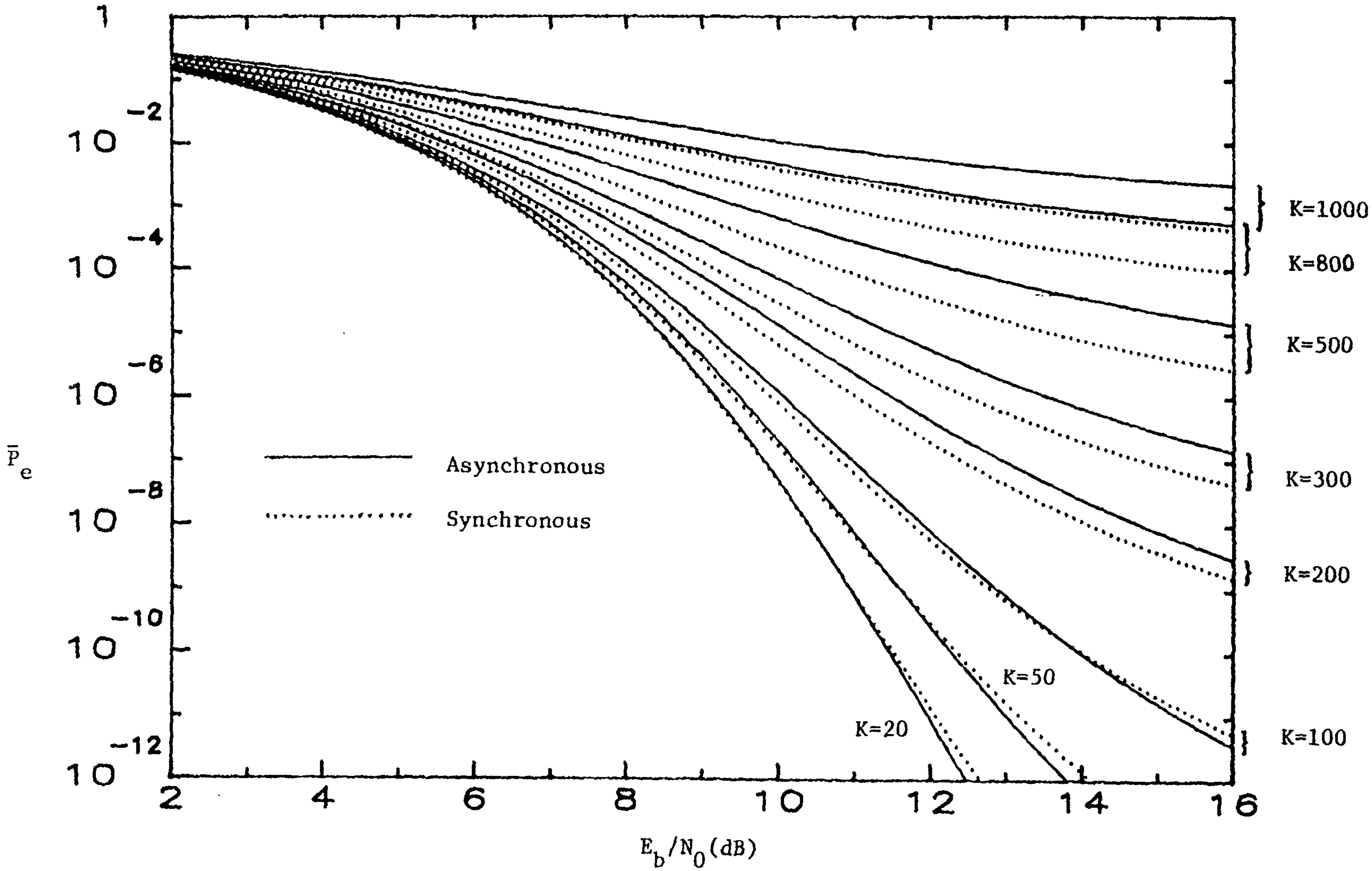

Figure 7. Probability of error for DS-SFH/SSMA systems employing 16-ary FSK modulation ($N'=31$, $q=50$, $N_s=1$)

Table 1

Error probability for DS-SFH/SSMA systems with different chip waveforms (K=100, N=31, q=100, N_b =10)

	Synchronous		Asynchronous					
	DPSK	FSK	DPSK			FSK		
E_b/N_0 (dB)	rect/sine	rect/sine	rect	sine		rect	sine	
8	3.24 ($\times 10^{-3}$)	2.87 ($\times 10^{-2}$)	2.06	1.87	($\times 10^{-2}$)	2.57	2.52	($\times 10^{-2}$)
10	6.31 ($\times 10^{-4}$)	7.31 ($\times 10^{-3}$)	2.28	1.78	($\times 10^{-4}$)	5.51	5.21	($\times 10^{-3}$)
12	1.60 ($\times 10^{-4}$)	1.42 ($\times 10^{-3}$)	2.85	1.75	($\times 10^{-5}$)	7.09	6.06	($\times 10^{-4}$)
14	5.91 ($\times 10^{-5}$)	3.06 ($\times 10^{-4}$)	5.75	2.82	($\times 10^{-6}$)	7.76	5.40	($\times 10^{-5}$)
16	3.00 ($\times 10^{-5}$)	0.94 ($\times 10^{-4}$)	1.88	0.78	($\times 10^{-6}$)	1.21	0.66	($\times 10^{-5}$)

Table 2

Error probability for DS-SFH/SSMA systems with MFSK modulation

(a) K=100, M=2, N_s =1

	N'=31, q=100			N'=63, q=50			N'=127, q=25		
E_b/N_0 (dB)	sync	asyn		sync	asyn		sync	asyn	
10	7.31	5.03	$(\times 10^{-3})$	6.94	5.20	$(\times 10^{-3})$	6.76	5.28	$(\times 10^{-3})$
12	1.42	0.54	$(\times 10^{-3})$	1.12	0.54	$(\times 10^{-3})$	0.99	0.54	$(\times 10^{-3})$
14	3.06	0.38	$(\times 10^{-4})$	1.58	0.29	$(\times 10^{-4})$	1.02	0.25	$(\times 10^{-4})$

(b) K=100, N'=31, N_s =1

	M=2, q=100			M=4, q=100			M=16, q=50		
E_b/N_0 (dB)	sync	asyn		sync	asyn		sync	asyn	
10	7.31	5.03	$(\times 10^{-3})$	4.22	1.65	$(\times 10^{-4})$	7.67	1.21	$(\times 10^{-6})$
12	1.42	0.54	$(\times 10^{-3})$	3.39	0.30	$(\times 10^{-5})$	56.8	0.48	$(\times 10^{-10})$
14	3.06	0.38	$(\times 10^{-4})$	43.3	0.45	$(\times 10^{-7})$	1025	0.13	$(\times 10^{-13})$

(c) K=100, q=50, N_s =1

	M=2, N'=63			M=4, N'=63			M=16, N'=31		
E_b/N_0 (dB)	sync	asyn		sync	asyn		sync	asyn	
10	6.94	5.20	$(\times 10^{-3})$	3.26	1.65	$(\times 10^{-3})$	7.67	1.21	$(\times 10^{-6})$
12	1.12	0.54	$(\times 10^{-3})$	1.41	0.22	$(\times 10^{-5})$	56.8	0.48	$(\times 10^{-10})$
14	1.58	0.29	$(\times 10^{-4})$	7.33	0.13	$(\times 10^{-7})$	1025	0.13	$(\times 10^{-13})$

Table 3

Number of simultaneous users accomodated by hybrid DS-SFH/SSMA systems employing different data modulation and demodulation schemes ($E_b/N_0 = 12$ dB, $N_b = 100$)

(a) At an error probability of 10^{-3}

		Synchronous			Asynchronous		
q	N	PSK	DPSK	FSK	PSK	DPSK	FSK
700	1	9	4	3	15	6	5
100	7	44	18	7	84	40	14
50	14	65	34	12	111	61	22
25	28	81	47	17	129	77	28
10	70	93	59	21	143	91	33
1	700	103	69	25	154	103	37

(b) At an error probability of 10^{-5}

		Synchronous			Asynchronous		
q	N	PSK	DPSK	FSK	PSK	DPSK	FSK
700	1	1	1	1	1	1	1
100	7	2	1	1	5	2	1
50	14	6	2	1	14	6	1
25	28	13	6	1	25	12	1
10	70	23	6	1	38	21	1
1	700	33	21	1	50	31	1

Table 4

Number of simultaneous users accomodated by hybrid DS-SFH/SSMA systems employing MFSK modulation with noncoherent demodulation ($E_b/N_0 = 12$ dB, $N_s = 1$)

			$P_e = 10^{-3}$		$P_e = 10^{-5}$	
q	N	M	Synchronous	Asynchronous	Synchronous	Asynchronous
700	1	2	3	3	1	1
700	1	4	4	4	1	1
700	1	8	10	15	1	1
700	1	16	57	81	1	4
700	1	32	251	271	22	40
700	1	64	790	747	158	205
100	7	2	7	7	1	1
100	7	4	39	40	2	3
100	7	8	122	111	18	23
100	7	16	361	264	86	88
100	7	32	711	576	278	251
100	7	64	1522	1196	729	610
50	14	2	12	11	1	1
50	14	4	61	55	6	8
50	14	8	163	137	40	40
50	14	16	374	301	138	123
50	14	32	799	625	367	306
50	14	64	1632	1255	861	686
25	28	2	17	14	1	1
25	28	4	79	66	13	13
25	28	8	193	154	62	55
25	28	16	415	324	179	148
25	28	32	851	653	429	341
25	28	64	1694	1287	942	730
10	70	2	21	17	1	1
10	70	4	94	74	21	18
10	70	8	215	166	84	68
10	70	16	444	339	213	167
10	70	32	885	671	475	366
10	70	64	1733	1308	998	760
1	700	2	25	37	1	1
1	700	4	105	157	30	44
1	700	8	231	346	102	153
1	700	16	463	695	239	358
1	700	32	907	1361	507	760
1	700	64	1758	2637	1035	1552