
#### Abstract

Title of Thesis:

\title{ RELATING RISKS TO PAY FACTORS FOR HIGHWAY PAVEMENTS THROUGH MONTE CARLO SIMULATION }

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The majority of State Highway Agencies (SHAs) now employ statistical-based specifications for the acceptance of highway materials and pavement construction. The parameters of these statistical acceptance plans are specified based on engineering judgment and may result in a high level of risk to both agency and contractor. In order to appropriately apply such specifications to the pavement construction industry, the associated production quality (i.e., materials and construction variability) needs to be well understood by all parties involved and its potential impacts require to be assessed. To address this objective of this study was to: (i) quantify the risks to the agencies and contractors (i.e., Type I and Type II errors); (ii) examine how the key components in a statistical acceptance plan impact its performance; and, (iii) identify a methodology to balance the risks and pay factors. Risk and pay factor analysis were conducted for both single and multiple quality characteristics through Monte Carlo simulation, and the development of Operating Characteristic, OC, curves. Furthermore, case studies were presented to demonstrate the value of the analyses proposed in this study. The methodology and findings identified in the study can be applied elsewhere to evaluate the acceptance plans and the associated risks pertinent to pavement construction and the production of highway materials.


# RELATING RISKS TO PAY FACTORS FOR HIGHWAY PAVEMENTS THROUGH MONTE CARLO SIMULATION 

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## Chapter 1 Introduction

The most State Highway Agencies (SHAs) are currently using statistical Quality Assurance (QA) acceptance specifications for highway construction and pavement material. These specifications include statistical acceptance plans that monitor whether the construction and material satisfy the quality standards. The following three fundamental items must be included in such a statistical acceptance plan (Burati et al, 2004): (1) the desired quality levels specified by agencies; (2) how to determine (or estimate) the quality level of the production; (3) the consequences for the contractors when the quality level is above or below the desired quality levels. Whether the acceptance plan includes simple pass/fail decisions or pay adjustment provisions, its appropriate development and evaluation are crucial for the acceptance plan to be effective. In order to fully evaluate the statistical acceptance plans, a good understanding of statistics, materials, production quality, and construction variability is required. This study identifies an alternative statistical basis for a statistical acceptance plan and examines how its key components impact the pay adjustments and examines how risks of acceptance can be related to rational and desirable pay schedules.

In the statistical acceptance plans, samples are used to make estimates about the quality of a larger amount of production, and thus risks are involved; there is some probability that the random samples will not represent the quality of the production as a whole, and thus will lead to an incorrect estimate of the production quality. Evaluation and quantification of the risks involved in the statistical acceptance plans are critical to ensure the effectiveness of these acceptance plans. In reality, however, few SHAs have evaluated the risks associated with their statistical acceptance plans.

A number of SHAs also have implemented the pay adjustment provisions in their acceptance plans to encourage the contractor to produce at the desired quality levels during construction. The evaluation of risks and pay factors becomes much more complex when such pay adjustment provisions are included in the acceptance plans. When the acceptance plan is properly developed, it should provide the best insurance that the contractor will be paid a fair price and that the contracting agency will get what was paid for with a reasonable level of risk. In order to address these issues, Monte Carlo simulation analysis is implemented to quantify the potential risks with pay factors to the agencies and contractors.

### 1.1 Literature Review

The evolution of modern QA specifications has taken place over several decades. In current QA specifications, statistical acceptance specifications are simple acceptance procedures which are monitoring methods used to evaluate whether a particular construction process meets the quality standards (Weed, 1996 as cited in Stephen and Joe, 2001). Freeman and Grogan (1998) described the statistical acceptance procedures in detail and proposed methods for developing statistical acceptance plans for pavement construction. Stephen and Joe (2001) conducted a thorough and critical literature review of what statistical acceptance specification are, why they are used and what their advantages and limitations are, and summarized the key components involved in the statistical acceptance plans including (1) type of acceptance plans, (2) acceptance quality characteristics, (3) specification limits, (4) desired quality levels (AQL and RQL), (5) statistical models, (6) risks and (7) pay factors. They also pointed out the acceptance plan performances will be significantly influenced by these components. Two comprehensive project reports FHWA-RD-02-095 (Burati et al, 2003), Optimal Procedures for

Quality Assurance Specifications, and FHWA-HRT-04-046 (Burati et al, 2003), Evaluation of Procedures for Quality Assurance Specifications, provided step-by-step procedures and instructions for developing and evaluating QA specifications and acceptance plans.

A computer simulation tool known as OCPLOT was developed by Weed (1995) and has been widely used (Stephen and Joe. 2001; Burati et al. 2003, 2004, 2005, 2011) to build OC curves and evaluate risks associated with accept/reject acceptance plans as well as pay adjustment acceptance plans. However, the statistical bases and procedures for developing the OC curves were not well studied and explained, and this simulation tool is not able to fully evaluate the risk associated with multiple quality characteristics. Villiers et al. (2003) proposed a method to develop OC curves, which is using the standard error to construct the normal distribution and calculate the probability of acceptance. One significant limitation of this method is that it is only valid when the acceptance limit is $50 \%$ ( Z -score $=0$ ). More recently, Karimi and Goulias (2013) built OC curves for superpave Hot Mix Asphalt (HMA) mixture characteristics (i.e., aggregate passing $0.075 \mathrm{~mm}, 2.36 \mathrm{~mm}, 4.75 \mathrm{~mm}$ sieves, asphalt content) using the procedure followed by Villiers et al. (2003) and developed a simulation tool for expected pay analysis.

Although the methods for evaluating risk associated with acceptance plans were proposed and discussed, none of these studies systematically examined how the key components of the acceptance plan affects its overall performance and associated risks, or provided methodologies and suggestions of how to balance the risks with pay factors based on the principal findings from the OC curves. Especially the evaluation of risks with pay factors has not been well-studied when multiple quality characteristics are used to determine the pay factor for a lot.

### 1.2 Research Objectives

The primary goal of this research was to (1) quantify the potential risks to the SHAs and contractors, (2) identify methods to balance the risks with pay factors associated with pay adjustment acceptance plans for highway materials and pavement construction. In order to accomplish this objective, the following research elements were addressed:
(1) Literature review on existing risk and pay factor analysis for highway material and pavement construction;
(2) Development of OC curves and quantification of risks for accept/reject acceptance plans supported by statistical theory;
(3) Development of OC and EP curves for pay adjustment acceptance plans and systematical examination how the key components of the statistical acceptance plan affect its overall performance, and associated risks with pay factors through Monte Carlo simulation;
(4) Assessment of risks with multiple quality characteristics;
(5) Case studies illustrating the value of the proposed analysis

### 1.3 Organization of Thesis

The first chapter presents an introduction to the study, the literature review on the risk and pay factor analysis associated with statistical acceptance plans for highway material and pavement construction, the research objectives, and the organization of the thesis.

Chapter 2 provides a background, definitions and concepts related to statistical acceptance plans. This chapter also presents the development of OC curves and quantification of risks for accept/reject acceptance plans.

Chapter 3 presents the development of OC and EP curves based on individual and composite quality characteristics for pay adjustment acceptance plans and an evaluation of the influence of its key components (i.e., sample size, pay equation, RQL, AQL, specification limits) on the performance of the acceptance plans and associated risks.

Chapter 4 presents the relations of risks to pay factors and provides case studies to demonstrate the value of the analyses proposed in this study.

Chapter 5 summaries the key findings and conclusions from the study and provide recommendations to balance and risk and pay factors.

## Chapter 2 Operating Characteristic (OC) Curves and Risks

In the statistical acceptance plans, using samples to make an estimation about the quality of the population involves risk; the random samples may not be representative of the quality of the population as a whole if it is obtained and tested improperly, and thus the quality of the population will be incorrectly estimated. Therefore, risk is inherent in the statistical acceptance plans. The risks associated with a particular statistical acceptance plan can be evaluated using Operating Characteristic (OC) curves. This Chapter presents the statistical basis and procedures for developing OC curves and quantifying the risks.

### 2.1 Acceptance Plans Basics

The statistical acceptance plans are acceptance procedures used to determine whether construction or materials should be accepted, rejected or accepted with pay adjustment (Freeman and Grogan, 1998). In order to appropriately apply statistical acceptance plans in pavement construction, it is important to properly implement the key components of an acceptance plan and its associated statistics. The definitions and concepts associated with statistical acceptance plans were identified from the Transportation Research Board (TRB) Transportation Research Circular Number E-C037, "Glossary of Highway Quality Assurance Terms".

### 2.1.1 Acceptance Plan Types

Acceptance plan: also called acceptance sampling plan or statistical acceptance plan. An agreed-upon process for evaluating the acceptability of a lot of material. It includes acceptance plan types, quality measure, quality characteristics, desired quality levels,
specification/acceptance limit(s), lot size and sample size (i.e. number of samples), evaluation of risks, and pay adjustment provisions.

There are two types of acceptance plans: variable acceptance plan and attribute acceptance plan. The analysis in this study is based on the variable acceptance plan which is the most commonly used in pavement material and construction. The variable acceptance plan assumes that the measured characteristics are normally distributed which is true for constructionrelated lot characteristics (Markey et al., 1994; Aurilio and Raymond, 1995; Cadicamo, 1999).

Variable acceptance plan: A statistical acceptance procedure where quality is evaluated by (1) under measuring the numerical magnitude of a quality characteristic for each of the units or samples in the group consideration and (2) computing statistics such as the average and the standard deviation of the group.

Acceptance limit: Also called the rejection limit in accept/reject acceptance plans. In variables acceptance plans, the limiting upper or lower value, placed on a quality measure, that will permit acceptance of a lot. [Unlike specification limits placed on a quality characteristic, an acceptance limit is placed on a quality measure.

Acceptance constant (k): the minimum allowable quality index (Q). [The acceptance constant k is the acceptance limit associated with the quality index measure. In other words, for acceptance, Q must be greater than or equal to k .]

### 2.1.2 Desired Quality Levels and Risks

There are two desired quality levels in an acceptance plan including the Acceptable Quality Level (AQL) and Rejectable Quality Level (RQL). The following definitions of AQL and RQL are provided in $\mathbf{E C - 0 3 7}$.

Acceptable quality level (AQL): for a given quality characteristic, that minimum level of actual quality at which the material or construction can be considered fully acceptable. For example, when quality is based on PWL, the AQL is that actual (not estimated) PWL at which the quality characteristic can just be considered fully acceptable. [Acceptance plans should be designed such that AQL material will receive an EP of $100 \%$.]

Rejectable quality level (RQL): for a given quality characteristic, that maximum level of actual quality at which the material or construction can be considered unacceptable (rejectable). For example, when quality is based on PD, the RQL is that actual (not estimated) PD at which the quality characteristic can just be considered fully rejectable. [Removal and replacement, corrective action, or the assignment of a relatively low pay factor is appropriate when RQL work is detected.]

The selection of appropriate AQL and RQL depends on the judgments using history data, statistics, and experience. In this analysis, how the selection of AQL and RQL impacts the performance of the acceptance plans are examined.

Two types of risks were defined in an OC curve based on the concepts of $A Q L$ and $R Q L$ :

Seller's risk: also called risk of a type I error. The probability that an acceptance plan will erroneously reject acceptance quality level (AQL) material or construction with respect to a single acceptance quality characteristic. It is the risk the contractor or producer takes in having AQL material or construction rejected.

Buyer's risk: also called risk of a type II error. The probability that an acceptance plan will erroneously fully accept (100 percent or greater) rejectable quality level (RQL) material or construction with respect to a single acceptance quality characteristic. It is the risk the highway agency takes in having RQL material or construction fully accepted. [the probability of having RQL material or construction accepted (at any pay) may be considerably greater than the buyer's risk.]

For a well-written acceptance plan, the AQL and RQL must be defined, and the specification limits and acceptance limits must be determined. The selection of acceptance limits is related to the risks to the contractor and agency. Sufficiently restrictive acceptance limits will be effective in controlling quality. The development of reasonable limits relates to the determination of risks. Risk analysis should be conducted based on acceptance limits and sample size. The risks associated with PWL acceptance plans are determined in this study by developing OC and curves using Monte Carlo simulation.

### 2.1.3 Quality Measures

Percent within limit (PWL) or percent defective (PD) have been identified as the most effective measures to consider mean and standard deviation (AASHTO R-9 and R-42). In this study, the PWL is used as the quality measure for analysis and simulation. The PWL is estimated using the quality index, Q . The Q -statistic represents the distance in sample standard deviation units that the sample mean is away from the specification limit. The quality index for a lot corresponding to the specification limits can be calculated based on the following equations:

$$
\begin{equation*}
Q_{L}=\frac{\bar{X}-L S L}{s} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
Q_{U}=\frac{U S L-\bar{X}}{s} \tag{2}
\end{equation*}
$$

Where: $\quad Q_{U}=$ quality index for the upper specification limit.

$$
Q_{L}=\text { quality index for the lower specification limit. }
$$

$U S L=$ upper specification limit.
$L S L=$ lower specification limit.
$\bar{X}=$ the sample mean for the lot.
$s=$ the sample standard deviation for the lot.

Once the quality index, Q , is calculated for the lot, the PWL could be estimated by the use of a PWL table (Specification Conformity Analysis, FHWA Technical Advisory T5080.12, June 23, 1989). $Q_{L}$ is used for a one-sided lower specification limit, and $Q_{U}$ is used for a onesided upper specification limit. For two-sided specification limits, the PWL is estimated by the following equation:

$$
\begin{equation*}
\mathrm{PWL}_{T}=\mathrm{PWL}_{U}+\mathrm{PWL}_{L}-100 \tag{3}
\end{equation*}
$$

Where: $\mathrm{PWL}_{U}=$ percent below the upper specification limit (based on $\mathrm{Q}_{U}$ ). $\mathrm{PWL}_{L}=$ percent above the lower specification limit (based on $\left.\mathrm{Q}_{L}\right)$. $\mathrm{PWL}_{T}=$ percent within the upper and lower specification limits.

### 2.1.4 Acceptance Quality Characteristics

AASHTO R-10 defines an Acceptance Quality Characteristic (AQL) as "A quality characteristic that is measured and used to determine acceptability." The definition of AQC in these guidelines was modified to include only those characteristics most related to pay factor adjustment and ultimately to the quality and performance of the pavements. Commonly used

AQCs for evaluating the quality and performance of concrete pavements including compressive strength, thickness, and smoothness are analyzed in this study. The example data used for the three parameters are taken from the National Cooperative Highway Research Program (NCHRP) Project No.10-79 report and summarized in Table 2.1. The mean values and standard deviations for each characteristic of 10 test results are used as population characteristics.

Table 2.1 Strength, thickness and roughness data from NCHRP Project No.10-79

| Test or Measurements | Strength, (psi) | Thickness, (in) | Roughness, (in/mile) |
| :--- | :--- | :--- | :--- |
| \#1 | 4,691 | 11.5 | 62.13 |
| \#2 | 5,007 | 12.4 | 66.09 |
| \#3 | 4,899 | 12.8 | 75.29 |
| \#4 | 4,590 | 11.4 | 67.87 |
| \#5 | 3,794 | 12.2 | 64.04 |
| \#6 | 3,940 | 12.6 | 55.06 |
| \#7 | 3,772 | 11.3 | 53.01 |
| \#8 | 4,677 | 11.8 | 54.54 |
| \#9 | 4,881 | 12.3 | 48.93 |
| \#10 | 5,111 | 12.5 | 49.94 |
| Mean | 4,536 | 12.1 | 59.69 |
| Std. Dev. | 509.9 | 0.54 | 8.69 |

### 2.1.5 Example Specification for Acceptance of Concrete Pavements

Specification Limits:

1) Compressive strength: - Minimum 3500 psi
2) Thickness -target thickness 12 " (spec limit: $\pm 7 \%$ )
3) Ride Quality: Max $75 \mathrm{in} / \mathrm{mi}$

### 2.2 Functions of OC curves for Variable Sampling Plans

When the calculated quality index, Q , is greater than the acceptance constant k , and the standard deviation is less than the maximum standard deviation, then the entire lot will be accepted. The following relation should be satisfied if a lot is accepted:

$$
\begin{equation*}
\bar{X}+k \sigma \leq U S L \text { or } \bar{X}-k \sigma \geq L S L \tag{4}
\end{equation*}
$$

Assuming that the lower specification limit is specified (single-sided specification limit). If the lot is accepted, the quality index $(\mathrm{Q})$ must be greater or equal to k .

$$
\begin{equation*}
Q_{L}=\frac{\bar{X}-L S L}{\sigma} \geq k \tag{5}
\end{equation*}
$$

Where $\bar{X}$ and $\sigma$ represent the mean and standard deviation of the population

Adding and subtracting $\frac{u}{\sigma}$ to the left side where $u$ represents the means of samples.

$$
\begin{equation*}
Q_{L}=\frac{\bar{X}-u}{\sigma}+\frac{u-L S L}{\sigma} \geq k \text { or } \frac{\bar{X}-u}{\sigma} \geq k-\frac{u-L S L}{\sigma} \tag{6}
\end{equation*}
$$

Multiplying both sides by $\sqrt{n}$ results in the following relation

$$
\begin{equation*}
\frac{\bar{X}-u}{\sigma / \sqrt{n}} \geq\left(k-\frac{u-L S L}{\sigma}\right) \sqrt{n} \tag{7}
\end{equation*}
$$

The probability of acceptance of the lot becomes

$$
\begin{equation*}
P_{a}(p)=P\left\{\frac{\bar{X}-u}{\sigma} \geq k\right\}=P\left\{\frac{\bar{X}-u}{\sigma / \sqrt{n}} \geq\left(k-\frac{u-L S L}{\sigma}\right) \sqrt{n}\right\} \tag{8}
\end{equation*}
$$

Where:

$$
\frac{\bar{X}-u}{\sigma / \sqrt{n}} \sim N(0,1)
$$

Based on the above formulations, the probability of acceptance for different quality levels (PWLs) could be calculated by means of Monte Carlo simulation and thus OC curves can be developed. The relation of probability, $\mathrm{P}_{\mathrm{a}}$, and percent within the specification limit, PWL, is illustrated in Figure 2.1.


Figure 2.1 Probability of acceptance and percent within limit

### 2.3 Construction of Operating Characteristic Curves

In order to build OC curves for these quality characteristics, simulation analysis based on the population characteristics was conducted with MATLAB programming.

### 2.3.1 Simulation Approach

For the three commonly used acceptance quality characteristics for evaluating concrete pavement including compressive strength, thickness, and roughness, the normal distributions were developed using actual standard deviations shown in Table 2.1. Provided the normally distributed characteristics have a mean $\mu$ and standard deviation $\sigma$ with an upper specification limit $U S L$, the quality of the lot based on PWL could be estimated by the use of quality index ( Q )
from Equation (1) \& (2) and cumulative distribution function $F(y)$ for standard normal deviate.
PWL is given by:

$$
\begin{equation*}
P W L=F\left(Q_{U}\right) \tag{9}
\end{equation*}
$$

Where F is the cumulative distribution function for standard normal distribution and $Q_{U}$ is the quality index of the lot.

$$
\begin{gather*}
F(y)=\int_{-\infty}^{y} \frac{1}{\sqrt{2 \pi}} \exp \left(-\frac{1}{2} t^{2}\right) d t \quad t \sim N(0,1)  \tag{10}\\
Q_{U}=\frac{U S L-\mu}{\sigma} \tag{11}
\end{gather*}
$$

The probability of acceptance of the lot can be calculated using equation 8 :

$$
\begin{equation*}
\operatorname{Pa}(\mathrm{p})=F\left(\left(Q_{U}-k\right) \sqrt{n}\right) \tag{12}
\end{equation*}
$$

Where k , acceptance constant, is the minimum allowable quality index corresponding to the acceptance limit and n is the sample size.

By shifting the mean value of the normal distribution based on the quality characteristics and keeping the standard deviation, a wide range of quality levels can be obtained (PWL from $0 \%-100 \%$ ) and the corresponding probability of acceptance can be calculated based on equations 9-12. The simulations and calculations were conducted in MATLAB.


Figure 2.2 Shifting of populations to develop operating characteristic curves

Figure 2.2 illustrates the construction of the OC curves by illustrating two cases. It is assumed the mean value of the normal distribution curve is at the target in case I. The area under the curve within the specification limits is the PWL and the corresponding probability of acceptance could be calculated based on equation 12. For case II, it is assumed that the mean value is away from target exactly on the specification limit. The same standard deviation and sampling frequency were used for developing the normal distribution. In this case, the area under normal distribution within the specification limit (PWL) is $50 \%$. Different values of percent within limit can be obtained by shifting the mean value of the normal distribution curve and keeping the same standard deviation and sampling frequency, and the corresponding probability-of-acceptance values can be calculated based on equations $9 \& 10$.

### 2.3.2 Operating Characteristic Curves

The OC curves for several recommended acceptance plans were developed; the seller's and buyer's risks were determined. The results are compared with risk levels recommended by ASSHTO R-9. The reasonable sample size, acceptance limit and AQL \& RQL corresponding to the recommended risk levels can also be determined.

The OC curves are developed for all the quality characteristics based on different acceptance limits and different sampling frequencies. An AQL of $90 \%$ and RQL of $50 \%$ recommended by the AASHTO Quality Assurance Guide Specifications (AASHTO, 1995) is used in the analysis to interpret the seller's and buyer's risks. For each OC curve, the seller's and buyer's risk are determined. Figure 2.3 shows the OC curves for concrete strength representing population characteristics with different acceptance limits and a sample size of five, while Figures 2.4 and 2.5 show the results for thickness and roughness with different acceptance limits and same sampling frequency.


Figure 2.3 OC curves for compressive strength with different acceptance limits and $\mathrm{n}=5$


Figure 2.4 OC curves for thickness with different acceptance limits and $\mathrm{n}=5$


Figure 2.5 OC curves for roughness with different acceptance limits and $n=5$

It can be observed from Figures 2.3-2.5, the OC curves not only provide seller's risks and buyer's risks but also a very good indication of the risks over a wide range of possible quality level, which enables to evaluate how the acceptance plan actually perform in practice. The $\alpha$ and $\beta$ risks are summarized in Table 2.2. As the acceptance limit increases, the probability of rejecting a good quality material increases so that the seller's risk increases as the increase of acceptance limit. However, the buyer's risk decreases as the increase of the acceptance limit. For example, the probability of rejecting an AQL of $90 \%$ material (buyer's risk) increases from $0.25 \%$ to $1.98 \%$ as increasing the acceptance limit from $45 \%$ to $60 \%$; the probability of accepting an RQL of $50 \%$ material decreases from $59.78 \%$ to $30.63 \%$.

On the other hand, for a certain acceptance limit and sample size, the probability of acceptance is always $50 \%$ when the PWL is right at the acceptance limit. This is because the probability of acceptance is the area on the right side of $k$ (acceptance constant which is the acceptance limit associated with the quality index measure) value under the normal distribution as shown in Figure 2.1. It is illustrated in Figure 2.6 that when the quality level (PWL) is exactly at the acceptance limit (case I \& II), the distance between the mean and specification limit equals the k corresponding to the acceptance limit, and the probability of acceptance will be half area (50\%) of the normal distribution.


Figure 2.6 Illustration of the intersection in OC curves

### 2.3.3 Seller's and Buyer's risks

The seller's risk is the probability of rejecting the production that is exactly at the AQL level of quality, while the buyer's risk is the probability of accepting the production that exactly at RQL quality level. These seller's and buyer's risks were obtained from OC curves and summarized in Table 2.2.

Table 2.2 Buyer's and seller's risks associated with various levels of acceptance PWL limits

| Acceptance sampling plan |  | Acceptance limit (\%) | Seller's risk $(\alpha)$ <br> $@$ AQL=90\% | Buyer's risk $(\beta)$ <br> $@$ RQL=50\% |
| :--- | :--- | :--- | :--- | :--- |
| strength | $\mathrm{n}=5$ <br> Single-sided | 60 | $1.98 \%$ | $30.70 \%$ |
|  | 70 | $6.50 \%$ | $14.72 \%$ |  |
|  | 80 | $18.93 \%$ | $4.63 \%$ |  |
|  | 90 | $49.99 \%$ | $0.52 \%$ |  |
| Roughness | $\mathrm{n}=5$ | 45 | $0.25 \%$ | $59.78 \%$ |
|  | Single-sided | 50 | $0.52 \%$ | $49.98 \%$ |
|  |  | 55 | $1.04 \%$ | $39.96 \%$ |
|  | 60 | $1.98 \%$ | $30.63 \%$ |  |
|  | $\mathrm{n}=5$ | 65 | $3.22 \%$ | $22.18 \%$ |


| Thickness | Double-sided | 75 | $10.21 \%$ | $8.91 \%$ |
| :--- | :--- | :--- | :--- | :--- |
|  |  | 85 | $29.27 \%$ | $1.92 \%$ |
|  |  | 95 | $74.87 \%$ | $0.00 \%$ |

It can be observed in Table 2.2 that the $\alpha$ risk increases with the increase of acceptance limit while the $\beta$ risk decreases as increasing the acceptance limit. For example, the $\alpha$ risk increased from $1.98 \%$ to $6.5 \%$ with increasing the acceptance limit from $60 \%$ to $70 \%$ for the concrete strength population characteristics.

In order to analyze how changing AQL and RQL affect the risks, the AQL and RQL are modified to $95 \%$ and $40 \%$ respectively. Table 2.3 provides the $\alpha$ and $\beta$ risks based on the revised AQL and RQL. From Table 2.2 and 2.3, it can be observed that the $\alpha$ risk can be reduced by increasing the AQL while the $\beta$ risk can be reduced by decreasing the RQL. For example, the $\alpha$ risk was calculated to be $1.98 \%$ for an AQL $90 \%$ and $0.24 \%$ for an AQL of $95 \%$ respectively based on strength population and an acceptance limit of 60 . The $\beta$ risk was calculated to be $30.70 \%$ for an RQL of $50 \%$ and $15.49 \%$ for an RQL of $40 \%$. It can be concluded that the risks can be balanced by modifying the desired quality levels (AQL \& RQL).

Table 2.3 Buyer's and seller's risks associated with various levels of acceptance PWL limit

| Acceptance sampling plan |  | Acceptance limit (\%) | Seller's risk $(\alpha)$ <br> $@$ AQL=95\% | Buyer's risk $(\beta)$ <br> $@$ RQL=40\% |
| :--- | :--- | :--- | :--- | :--- |
| strength | $\mathrm{n}=5$ <br> Single-sided | 60 | $0.24 \%$ | $15.49 \%$ |
|  | 70 | $1.13 \%$ | $5.94 \%$ |  |
|  | 80 | $4.99 \%$ | $1.41 \%$ |  |
|  | 90 | $22.20 \%$ | $0.11 \%$ |  |
| Roughness | $\mathrm{n}=5$ | 45 | $0.02 \%$ | $39.70 \%$ |
|  | 50 | $0.05 \%$ | $30.53 \%$ |  |
|  |  | 55 | $0.12 \%$ | $22.27 \%$ |
|  |  | 60 | $0.24 \%$ | $15.50 \%$ |
| Thickness | Double-sided | 65 | $0.58 \%$ | $10.03 \%$ |
|  | 75 | $2.60 \%$ | $3.14 \%$ |  |
|  | 85 | $11.13 \%$ | $0.49 \%$ |  |
|  | 95 | $49.85 \%$ | $0.00 \%$ |  |

Table 2.4 shows the probability of acceptance in relation to PWL using the concrete strength population with an acceptance limit of $60 \%$ and a sample size of five.

Table 2.4 Probability of acceptance in relation to PWL ( $n=5$, acceptance limit=60\%)

| Lot concrete strength <br> PWL $(\%)$ | Probability of acceptance (\%) |
| :--- | :--- | :--- |
| 100 | 100 |
| 95 | 99.74 |
| 90 AQL | $97.99 \quad \alpha=2.01 \%$ |
| 85 | 93.87 |
| 80 | 88.03 |
| 75 | 79.99 |
| 70 | 70.59 |
| 65 | 60.44 |
| 60 | 50.00 |
| 55 | 39.94 |
| 50 | 30.97 |
| 45 | 22.53 |
| 40 RQL | $15.56 \quad \beta=15.56 \%$ |

### 2.4 The Effects of Sample Size

It should be noted that the sample size has a direct effect on the operating characteristic curves and risk levels associated with the acceptance plans. The sensitivity of the sample size to agency and contractor's risks have to be well understood. In order to evaluate the effect of sample size on OC curves as well as the impact on seller's and buyer's risks, the sample size is varied to construct OC curves for concrete strength. Figure 2.7 shows typical OC curves for six different sampling sizes for concrete strength while the risks are shown in Table 2.5.


Figure 2.7 OC curves with different sampling sizes using strength population (Acceptance limit=70\%)

Table 2.5 Buyer's and seller's risks associated with different sample sizes (Acceptance limit=70\%)

| Sample size | $\alpha @ \mathrm{AQL}=90 \%$ | $\beta$ @ RQL=40\% |
| :--- | :--- | :--- |
| 4 | $6.31 \%$ | $14.76 \%$ |
| 5 | $3.49 \%$ | $12.09 \%$ |
| 6 | $3.06 \%$ | $9.99 \%$ |
| 7 | $2.16 \%$ | $8.31 \%$ |
| 8 | $1.53 \%$ | $6.94 \%$ |
| 9 | $1.09 \%$ | $5.82 \%$ |

Figure 2.7 clearly illustrates how changing the sample size affects the OC curves. The OC curves become steeper as the increase of sample size, and they have a common intersection point at PWL of $70 \%$ (acceptance limit). This means that with the increase of sample size, the probability of acceptance decreases faster as the reduction of the quality (PWL). It can also be observed in Table 2.5 that $\alpha$ and $\beta$ risks can be reduced by increasing the sample size. For
example, the $\alpha$ risk was reduced from $6.31 \%$ to $1.09 \%$ and the $\beta$ risk was reduced from $14.76 \%$ to $5.82 \%$, respectively, as increasing the sample size from 4 to 9 .

### 2.5 Alternative Approach to Build OC Curves with Standard Error

Another method for constructing the OC curve was developed by Viller at all. (2003). In this method, the distribution is shifted to obtain different percent within limits, and the standard error, which is the ratio of the population standard deviation and the square root of the sample size, is used to calculate the probability of acceptance. The OC curves were reproduced using Villier's method with different sample sizes as shown in Figure 2.8.


Figure 2.8 OC curves with different sample sizes based on Viller's approach

Similar to Figure 2.7, the OC curves in Figure 2.8 become steeper as increasing the sample size but with an intersection point at PWL of $50 \%$. As illustrated in Figure 2.6, the
probability of acceptance is always found to be $50 \%$ when the quality level (PWL) is exactly at the acceptance limit. This indicates that the Villier's approach assumes the acceptance limit is $50 \%(\mathrm{k}=0)$ and fails to consider the effect of the acceptant limit (acceptance constant k$)$ on the OC curves. It can be concluded that the Villier's approach is only valid for the acceptance plan with an acceptance limit of $50 \%$. However, in reality, the acceptance limits (i.e., $40 \%, 50 \%$, $60 \%$ ) may vary for different SHAs such that using Villier's approach is not enough fully evaluated the acceptance plans and the risks.

Table 2.6 summarized the $\alpha$ and $\beta$ risks for different sample sizes using Villier's approach. Similar to the results in Table 2.5, increasing sample size reduces both $\alpha$ and $\beta$ risks. For example, as increasing the sample size from 4 to 9 , the $\alpha$ risk was reduced from $0.48 \%$ to $0.01 \%$ for an AQL of $90 \%$ and the $\beta$ risk was reduced from $30.58 \%$ to $22.31 \%$ for an RQL of $40 \%$, respectively.

Table 2.6 Buyer's and seller's risks with different sample sizes based on Villier's approach

| Sample size | $\alpha @$ AQL=90\% | $\beta$ @ RQL=40\% |
| :--- | :--- | :--- |
| 4 | $0.48 \%$ | $30.58 \%$ |
| 5 | $0.19 \%$ | $28.51 \%$ |
| 6 | $0.08 \%$ | $26.70 \%$ |
| 7 | $0.03 \%$ | $25.09 \%$ |
| 8 | $0.01 \%$ | $23.64 \%$ |
| 9 | $0.01 \%$ | $22.31 \%$ |

### 2.6 Acceptance Plan Based on Risk Levels Recommended by AASHTO

As the above analysis, the $\alpha$ and $\beta$ risks can be calculated and the probability of acceptance can be determined for any PWL. Increasing the sample size reduces both $\alpha$ and $\beta$ risks, however, it also increases the inspection or testing costs. For a given sample size,
increasing the acceptance limit means reducing the probability of accepting poor quality materials ( $\alpha$ risk) and increasing the probability of rejecting good quality material ( $\beta$ risk). So the risk must be balanced. Selecting the proper level of $\alpha$ and $\beta$ risks is a matter of judgment. The appropriate levels of alpha and beta risks for highway construction and pavement material suggested by AASHTO R-9, "Acceptance Sampling Plans for Highway Construction," are 1\% for an AQL of $90 \%$ and $5 \%$ for an RQL of $40 \%$, respectively. A variable acceptance can be designed such that the OC curve passes through two points (AQL, $\alpha$ ) and (RQL, $\beta$ ). The required sample size and acceptance limit (critical distance k ) can be obtained by the following calculations.

Based on equation 5, Let those values of sample means, $u$, which produce $P_{1}$ and $P_{2}$ quality be designated as $\mathrm{u}_{1}$ and $\mathrm{u}_{2}$, the corresponding z -scores $\left(\mathrm{Z}_{1}\right.$ and $\left.\mathrm{Z}_{2}\right)$ are as following:

$$
\begin{align*}
& Z_{1}=\frac{u_{1}-L S L}{\sigma / \sqrt{n}}  \tag{13}\\
& Z_{2}=\frac{u_{2}-L S L}{\sigma / \sqrt{n}} \tag{14}
\end{align*}
$$

The following probability statements can be derived based on the concept of $\alpha$ and $\beta$ ( specified $\alpha$ @ AQL and $\beta$ @ RQL) and equation 8

$$
\begin{align*}
& \operatorname{Pr}\left(\frac{X-u}{\sigma / \sqrt{n}} \geq\left(k-Z_{1}\right) \sqrt{n}\right)=1-\alpha  \tag{15}\\
& \operatorname{Pr}\left(\frac{X-u}{\sigma / \sqrt{n}} \geq\left(k-Z_{2}\right) \sqrt{n}\right)=\beta \tag{16}
\end{align*}
$$

Since

$$
X \sim N\left(u, \sigma^{2}\right) \quad \text { (normal distribution) }
$$

Then

$$
\frac{X-u}{\sigma / \sqrt{n}} \sim N(0,1) \quad \text { (standard normal deviates) }
$$

Which means

$$
\begin{align*}
& \left(k-Z_{1}\right) \sqrt{n}=Z_{1-\alpha}  \tag{17}\\
& \left(k-Z_{2}\right) \sqrt{n}=Z_{\beta} \tag{18}
\end{align*}
$$

For the acceptance plan suggested by AASHTO:
$\mathrm{AQL}=90 \%$
$\mathrm{RQL}=40 \%$
$\alpha=0.01$
$\beta=0.05$
Let $\mathrm{P}_{1}=90 \%$ and $\mathrm{P}_{2}=40 \%$, from the Z -table (attached in appendix) the corresponding $\mathrm{Z}_{1}$,
$Z_{2}, Z_{\alpha}$ and $Z_{\beta}$ can be obtained:
$\mathrm{Z}_{1}=1.282 \quad \mathrm{Z}_{2}=-0.254$
$Z_{\alpha}=2.33 \quad Z_{\beta}=1.645$
Based on equation (13) and (14) $n$ and $k$ can be obtained:

$$
\begin{gathered}
n=\left(\frac{Z_{\alpha}+Z_{\beta}}{Z_{1}-Z_{2}}\right)^{2}=6.7 \simeq 7 \\
k=Z_{1}-\frac{Z_{\alpha}}{\sqrt{n}}=Z_{2}+\frac{Z_{\beta}}{\sqrt{n}}=\frac{Z_{\alpha} Z_{2}+Z_{\beta} Z_{1}}{Z_{\alpha}+Z_{\beta}}
\end{gathered}
$$

k can be calculated based on individual $Z_{1}$ and $Z_{\alpha}$, which will give us a risk of $\alpha$ at exactly AQL:

$$
k=Z_{1}-\frac{Z_{\alpha}}{\sqrt{n}}=1.282-\frac{2.33}{\sqrt{7}}=0.4
$$

The acceptance limit based on $\mathrm{k}=0.4$ is $65 \%$. Thus, in order to reach the risk level suggested by AASHTO, the sample size should be 7 and the acceptance limit should set to be $65 \%$. The readers are encouraged to refer to Figure 2.9 which illustrates the above calculations and relationships graphically.


Figure 2.9 Illustration of the effects of sample size (after Duncan, 1952)

Figure 2.9 shows the OC curve for the acceptance plan with a sample size of seven and an acceptance limit of $65 \%$, while Table 2.7 shows the probability of acceptance in relation to PWL for this acceptance plan. The performance of this acceptance plan can be fully evaluated using the OC curve shown in Figure 3.10. The probability of acceptance for any PWL can be
obtained from Figure 2.10. For example, there is an $87.88 \%$ probability that an 80 PWL material will be accepted. It can be observed in Table 2.7 that the $\alpha$ and $\beta$ risks are $0.95 \%$ and $4.31 \%$ respectively (within the risk levels recommended by AASHTO). However, It should be noted that even though the $\alpha$ and $\beta$ risks can be reduced to the risk levels that AASHTO recommended, very few contracting agencies use a sample size of 7 because of economic considerations.


Figure 2.10 OC curve for an acceptance limit of $65 \%$ and $n=7$ using concrete population

Table 2.7 Probability of acceptance in relation to PWL ( $\mathrm{n}=7$, acceptance limit=65.54\%)

| Lot concrete strength <br> PWL (\%) | Probability of acceptance (\%) |
| :--- | :--- |
| 100 | 100 |
| 95 | 99.95 |
| 90 AQL | 99.05 |
| 85 | 95.33 |
| 80 | 87.88 |
| 75 | 76.75 |


| 70 | 62.66 |  |
| :--- | :--- | :--- |
| 65 | 48.47 |  |
| 60 | 34.90 |  |
| 55 | 23.43 |  |
| 50 | 14.49 |  |
| 45 | 8.20 |  |
| 40 RQL | 4.31 | $\beta=0.0431$ |

## Chapter 3 Pay Factor Analysis

Moving away from the accept/reject PWL acceptance plan to the acceptance plan with pay adjustment provision, the evaluation of risks becomes more complex. The OC curves developed in Chapter 2 not only determine the $\alpha$ and $\beta$ risks but also provide an indication of the probabilities of acceptance over a wide range of quality levels (PWLs). However, the evaluation of $\alpha$ and $\beta$ risks are applied to construction or materials for the case of a pass/fail (accept/reject) decision. In order to fully evaluate the risks in the acceptance plans with pay adjustment provision and relate the risks to pay factors, the OC curves for the pay adjustment acceptance plans and expected payment curves are necessary. In this chapter, the OC curves for pay adjustment acceptance plan and expected pay curves are developed by implementing Monte Carlo simulation in MATLAB based on the typical population characteristics from NCHRP Project NO. 10-79. These multiple OC curves along with EP curves could be used to evaluate the risks associated with pay adjustment acceptance plans.

### 3.1 Concepts and Definitions

The TRB glossary provides the following definitions for the OC curve (for acceptance plan with pay adjustment) and expected pay curve:

OC curves for payment adjustment acceptance plan: A graphic representation of an acceptance plan that shows the relationship between the actual quality of a lot and the probability of its acceptance at various payment levels.

EP curve: A graphic representation of an acceptance plan that shows the relation between the actual quality of a lot and its EP (i.e. mathematical pay expectation, or the average pay the contractor can expect to receive over the long run for submitted lots of a given quality.

Figure 3.1 shows the typical OC curves associated with pay factors. It is shown in Figure 3.1 that each curve illustrates the probability of receiving a pay factor equal to or larger than that demonstrated for the line. For example, an AQL quality level material has approximately a $25 \%$ probability of receiving a pay factor equal to or greater than 1.04 and a 64 percent probability of receiving a pay factor equal to or greater than 1.0. Additionally, it also can be observed that this material has a $100 \%$ chance of receiving a pay factor equal to or greater than 0.8 . Similarly, the probabilities of receiving $\geq$ various pay factors (i.e., $0.75,0.9$ ) can also be obtained at any quality level. The $\alpha$ risk, in this case, can be considered as the probability of receiving less than $100 \%$ pay for an AQL quality level, while the $\beta$ risk can be interpreted as the probability of receiving greater than $100 \%$ pay for an RQL material. However, the use of $\alpha$ and $\beta$ to evaluate the risks is simply not enough. For example, a contractor may be also interested in what is the probability of rejection for an AQL material.


Figure 3.1 Typical OC curves for pay adjustment acceptance plan (after Burati et al. 2003)

A typical EP curve is shown in Figure 3.2. The horizontal axis indicates the quality levels (PWL) of the lots while the vertical axis provides a long-run average expected pay factor corresponding to each quality level. As shown in Figure 3.2, as desired, an expected pay of 100 percent is received for an AQL quality level. For quality level better than AQL, an incentive pay factor up to $105 \%$ will be received. RQL quality level receives an expected pay of 24 percent. The $\alpha$ risk, in this case, can be considered as the probability of receiving less than $100 \%$ pay for an AQL quality level, while the $\beta$ risk can be interpreted as the probability of receiving greater than $100 \%$ pay for an RQL material. Using $\alpha$ and $\beta$ risks to evaluate the acceptance plans is simply not sufficient. Both OC and EP curves for the pay adjustment acceptance plans should be developed to fully evaluate the risks involved in.


Figure 3.2 Typical EP curve for pay adjustment acceptance plan (after Burati et al. 2003)

### 3.2 Pay Equations

Three pay equations are applied to develop the OC curves for pay adjustment acceptance plans and expected pay curves. The first pay equation 19 is recommended by AASHTO Quality
assurance guide specification (1996) while the second pay equation 20 was proposed by Burati et al. (2003).

$$
\begin{equation*}
P F=55+0.5 \times P W L \tag{19}
\end{equation*}
$$

$$
\begin{equation*}
P F=10+1 \times P W L \tag{20}
\end{equation*}
$$

The third one is a stepped pay equation recommended by NCHRP 10-79 shown in Table 3.1. It can be observed in equation 13 that the maximum pay factor will be $105 \%$ for PWL of 100 while the minimum pay factor is $50 \%$ for a PWL of 0 . For equation 20 , the maximum pay factor will be $110 \%$ for PWL of 100 while the minimum pay factor is $10 \%$ for a PWL of 0 . However, in practice, there is usually some form of rejection or replacement $(\mathrm{PF}=0)$ if the quality level of a lot is below a certain PWL, such as $50 \%$ or $60 \%$ (RQL). The pay equation 19 and 20 along with different rejection or replacement levels (i.e., $\mathrm{PF}=0$ if $\mathrm{PWL}<50 \%, \mathrm{PF}=0$ if $\mathrm{PWL}<$ $60 \%$ ) were used to conduct pay factor analysis. Figure 3.3 shows the three pay equations graphically with " $\mathrm{PF}=0$ if $\mathrm{PWL}<50 \%$ ". How well these pay equations would perform in practice is examined and evaluated by developing OC and EP curves through Monte Carlo simulation.

Table 3.1 Stepped pay equation (from NCHRP 10-79)

| Estimated PWL | Pay Factor, $\%$ |
| :--- | :--- |
| $98.0-100$ | 105 |
| $94.0-97.9$ | 103 |
| $92.0-93.9$ | 101 |
| $88.0-91.9$ | 100 |
| $84-87.9$ | 98 |
| $82.0-83.9$ | 96 |
| $78.0-81.9$ | 94 |


| $74.0-77.9$ | 92 |
| :--- | :--- |
| $70.0-73.9$ | 90 |
| $66.0-69.9$ | 88 |
| $62.0-65.9$ | 86 |
| $58.0-61.9$ | 84 |
| $54.0-57.9$ | 82 |
| $50.0-53.9$ | 80 |
| $<50.0$ | 0 |



Figure 3.3 Illustration of the three different pay equations

### 3.3 Pay Factor Analysis for Individual Quality Characteristics

Monte Carlo simulations are conducted in Matlab to develop OC curves associated with receiving various pay factors and EP curves. The population distributions (means and standard deviations shown in Table 3.2) are shifted to produce different PWLs. Each PWL represents the quality level of a simulated lot, and the simulated lots have the same standard deviation as the population distribution. The quality indexes associated with each simulated lot are calculated using equation 5 and the PWLs are estimated by using a PWL estimation table (Appendix). Then
the pay equations (19\&20) were used to calculate the pay factors for each simulated lot. The probabilities of receiving $\geq$ various PFs (i.e., $0.7,0.8,0.9,1.0,1.04$ ) for a specific PWL can be determined as follows.

$$
\text { Probability of receiving } \geq \mathrm{PF}=\frac{\text { number of lots } \geq \mathrm{PF}}{\text { Total number of lots }}
$$

Multiple OC curves for several specified payment levels $(0.75,0.8,0.9,1.0,1.04)$ are plotted for concrete strength, thickness, and roughness representing the population distribution.

Table 3.2 Means, standard deviations and specification limits for different quality characteristics

| Quality characteristics | Strength, (psi) | Thickness, (in) | Roughness, (in/mile) |
| :--- | :--- | :--- | :--- |
| Standard deviation | 509.9 | 0.54 | 8.69 |
| Average | 4536 | 12.1 | 59.69 |
| Specification limit | LSL $=3500$ | LSL=11.2, USL=12.8 | USL=75 |

### 3.3.1 OC Curves for Pay Adjustment Acceptance Plans

The OC curves shown in Figure 3.4 were developed using the population standard deviation of concrete strength based on pay equation 19. The probability values of receiving equal or larger than various pay factors (i.e., $0.7,0.8,0.9,1.0$ and 1.04) are shown in Table 4.3. It can be seen in Table 3.3 that the probability of receiving $\mathrm{PF} \geq 1.0$ is $2.83 \%$ for RQL of $50 \%$ quality level while the chance of receiving $\mathrm{PF} \geq 1$ is $60.84 \%$ for AQL of $90 \%$ quality level. This indicates that there is approximately a $40 \%$ probability that a contractor would not receive full payment $(100 \%)$ for an AQL production. The risk may seem to be very high if only the $\mathrm{PF}=1$ curve was considered. However, it is somehow balanced by the fact that there is a more than $40 \%$ chance of receiving a pay factor of 1.04 or greater indicated by the $\mathrm{PF}=1.04$ curve. Similar,
the probability of receiving $\geq$ various PFs (i.e., $0.7,0.8,0.9,1.0$ and 1.04 ) for any quality levels can be estimated using Figure 3.4.

Table 3.3 Probability of receiving $\geq \mathrm{PF}$ based on pay equation 19 and $\mathrm{n}=5$

| PWL | Prob. of receiving $\geq$ PF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0.18 | 0.18 | 0 | 0 | 0 |
| 20 | 3.00 | 3.00 | 0.24 | 0.02 | 0.02 |
| 30 | 13.00 | 13.00 | 2.06 | 0.31 | 0.12 |
| 40 | 29.40 | 29.4 | 6.32 | 1.09 | 0.57 |
| 50 | 50.00 | 50.00 | 14.88 | 2.83 | 1.12 |
| 60 | 70.00 | 70.00 | 28.49 | 7.14 | 3.33 |
| 70 | 86.94 | 86.94 | 50.12 | 15.8 | 8.48 |
| 80 | 96.92 | 96.92 | 75.15 | 32.36 | 18.96 |
| 90 | 99.85 | 99.85 | 94.14 | 60.84 | 40.96 |
| 100 | 100 | 100 | 100 | 100 | 100 |

*10000 simulated lots. $\mathrm{PF}=0$ if $\mathrm{PWL}<50$


Figure 3.4 OC curves using strength population characteristics based on pay equation 19 and $n=5$

In order to analyze the effects of sample size on the probability values of receiving various pay factors, another sample size of 15 was used to develop the OC curves. Increasing the
sample size would reduce the variability associated with the PWLs of the simulated lots. Figure 3.5 shows a histogram of estimated PWL and PF of 10000 simulated lots for an AQL (PWL=90\%) production with a sample size of five while Figure 3.6 shows the same information but using a sample size of fifteen. It should be noted a sample size of 15 is used just for the purpose of illustrating how changing the sample size affects the variability of PWLs.


Figure 3.5 Variability of estimated PWL and PF for an AQL production with n=5


Figure 3.6 Variability of estimated PWL and PF for an AQL production with $\mathrm{n}=15$

Figures 3.5 \& 3.6 clearly shows that sample size directly affects the spread of PWL and PF. It can be observed in Figure 3.5 that the spread of PWL and PWL is very large when a sample size of 5 is used. However, the high and low PWLs tend to balance out to an average PWL of $90 \%$ over a large number (10000 simulated lots) of lots. The standard deviation of the estimated PWL was calculated to be 11.2 for a sample size of 5 , while the standard deviation was calculated to be 5.9 for a sample size of 15 . Even though the variability can be reduced significantly by increasing the sample size, it may not be practical to use large samples size because of economic considerations.

The Probability of receiving $\geq \mathrm{PF}$ using the strength population for a sample size of 15 are summarized in Table 3.4 and plotted in Figure 3.7. Overall, the OC curves shown in Figure $3.7(\mathrm{n}=15)$ are more spread compared to OC curves in Figure $3.4(\mathrm{n}=5)$. It can also be seen that an $\mathrm{AQL}(\mathrm{PWL}=90 \%)$ quality level has an $8.69 \%$ chance of receiving a pay factor equal or larger than 1.04 and a $100 \%$ probability receiving a pay factor equal or larger than 1.04. This means that the distribution of PFs using a sample size 15 is much more centered at $\mathrm{PF}=1.0$ than that using a sample size of 5 .

Table 3.4 Probability of receiving $\geq \mathrm{PF}$ based on pay equation 19 and $\mathrm{n}=15$

| PWL | Prob. of receiving $\geq$ PF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0.03 | 0.03 | 0 | 0 | 0 |
| 30 | 2.67 | 2.67 | 0 | 0 | 0 |
| 40 | 16.62 | 16.62 | 0.3 | 0 | 0 |
| 50 | 49.8 | 49.8 | 3.32 | 0.01 | 0 |
| 60 | 82.87 | 82.87 | 16.7 | 0.18 | 0.01 |
| 70 | 97.92 | 97.92 | 53.29 | 1.54 | 0.03 |
| 80 | 99.96 | 99.96 | 89.5 | 13.08 | 0.59 |
| 90 | 100 | 100 | 99.86 | 60.07 | 8.67 |
| 100 | 100 | 100 | 100 | 100 | 100 |

[^0]

Figure 3.7 OC curves based on pay equation 19 and $n=15$

### 3.3.2 Expected Pay Curves for Different Sample Sizes

The OC curves along with the histogram of PWLs and PFs enable us to evaluate the risk involved in the pay adjustment acceptance plan. However, using such OC curves and histograms is not a simple way to evaluate the overall pay performance of an acceptance plan. The expected payment curves are developed to represent the pay performance. Table 3.5 and Figure 3.8 show the expected pays in relation to PWL for sample sizes of $5 \& 15$ using equation 19. The EP curves illustrate the pay performance by combining all possible pay factors into a single average pay factor in the long run for any quality levels.

Table 3.5 Expected payments in relation to PWL for $\mathrm{n}=5$ \& 15

| PWL (\%) | Average pay factor (\%) |  |
| :--- | :--- | :--- |
|  | $\mathrm{N}=5$ | $\mathrm{~N}=15$ |
| 100 | 105 | 105 |
| 90 | 100 | 100 |
| 80 | 93.0 | 95.0 |
| 70 | 80.0 | 88.5 |


| 60 | 63.0 | 71.0 |
| :--- | :--- | :--- |
| 50 | 43.0 | 42.0 |
| 40 | 25.5 | 13.8 |
| 30 | 11.4 | 2.2 |
| 20 | 2.5 | 0.03 |
| 10 | 0.15 | 0 |
| 0 | 0 | 0 |

*10000 simulated lots. $\mathrm{PF}=0$ if $\mathrm{PWL}<50$


Figure 3.8 EP curves based on pay equation 19 for $\mathrm{n}=5 \& 15$

It can be seen in Figure 3.8 that the EP curve becomes steeper as the increase of sample size. This indicates that as the sample size increases, the expected pay decreases faster as the PWL reduces. The average pay factor in the long run are $100 \%$ for a $90 \%$ AQL material for both sample sizes of five and fifteen. For an $80 \%$ PWL quality, the average pay factor at the long run is $95.0 \%$ for a sample size of 15 and $93.0 \%$ for a sample size of 5 respectively. However, for a poor quality level $(\mathrm{PWL}=40 \%)$, the average pay factor in the long run is $13.8 \%$ for a sample size of 15 and $25.5 \%$ for a sample size of 5 respectively. This is because as the sample size increases, a better estimation of the population characteristic can be obtained.

### 3.3.3 OC Curves Based on Thickness Population Characteristics

The multiple OC curves shown in Figure 4.8 were developed using the population standard deviation of thickness based on pay equation 19. Because the tolerances for thickness are close to each other, the maximum PWL can be achieved by shifting the thickness mean value is $86 \%$ such that the standard deviation was reduced in order to achieve a higher PWL (i.e., $90 \%$ and $100 \%$ ). It can be observed from Figure 4.8 that OC curves obtained by using thickness population characteristics are identical to those obtained by using the strength population if all other parameters remain the same (i.e., sample size, pay equation). This is because the variability of the estimated PWL and PF is not affected by the population distributions. The pay factor simulation analysis developed here is mainly used to evaluate the pay-performance and risk of different acceptance plans. However, how the simulation analysis can be employed by contractors with different production variabilities (standard deviation) and population means will be discussed later in the next Chapter.

Table 3.6 Probability of receiving $\geq \mathrm{PF}$ based on pay equation 19 and $\mathrm{n}=5$

| PWL | Prob. of receiving $\geq \mathrm{PF}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0.17 | 0.17 | 0 | 0 | 0 |
| 20 | 3.2 | 3.2 | 0.26 | 0.02 | 0.02 |
| 30 | 12.89 | 12.89 | 2.06 | 0.31 | 0.12 |
| 40 | 29.4 | 29.4 | 6.32 | 1.09 | 0.57 |
| 50 | 50.2 | 50.2 | 14.88 | 2.83 | 1.12 |
| 60 | 71.1 | 71.1 | 28.36 | 7.18 | 3.32 |
| 70 | 86.94 | 86.94 | 50.12 | 15.8 | 8.48 |
| 80 | 97.8 | 97.8 | 75.15 | 32.36 | 18.96 |
| 90 | 99.71 | 99.71 | 86.82 | 47.04 | 29.09 |
| 100 | 99.93 | 99.93 | 94.52 | 60.96 | 40.52 |

* 10000 simulated lots. $\mathrm{PF}=0$ if $\mathrm{PWL}<50$


Figure 3.9 OC curves using pay equation 19 based on thickness population and $\mathrm{n}=5$

### 3.3.4 OC Curves Using Different Pay Equations

## OC Curves Using Pay Equations 20

Table 3.7 summaries the probability of receiving $\geq \mathrm{PF}$ for various PWL from simulation analysis while Figure 3.10 shows the OC curves for the pay adjustment acceptance plan using pay equation 20. Overall the OC curves in Figure 3.10 became less spread to each other compared to the OC curves in Figure 3.4. The $\mathrm{PF}=1$ curve in Figure 3.10 is very similar to that in Figure 4.3. However, compared to the OC curves using equation 19 (Figure 3.4), the probabilities of receiving a pay factor equal to or larger than 1.04 ( $\mathrm{PF}=1.04$ curve) increase for any given quality levels, while the probabilities of receiving a pay factor that smaller than 0.7 , 0.8 and 0.9 decrease. For example, it is shown in Figure 4.9 that the probability is approximately $50.8 \%$ (compared to $40 \%$ in Figure 3.4) for an AQL material to receive a $104 \%$ pay, while the
probability is approximately $81 \%$ (compared to $90.4 \%$ in Figure 3.4) for an AQL material to receive a $90 \%$ pay. This indicates that using equation 20 increases the spread of PF estimates.

Table 3.7 Probability of receiving $\geq \mathrm{PF}$ based on pay equation 20 and $\mathrm{n}=5$ for strength population

| PWL | Prob. of receiving $\geq \mathrm{PF}$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0.06 | 0.01 | 0 | 0 | 0 |
| 20 | 0.87 | 0.23 | 0.07 | 0.04 | 0.03 |
| 30 | 5.5 | 1.8 | 0.6 | 0.27 | 0.15 |
| 40 | 14.7 | 6.06 | 2.5 | 0.93 | 0.59 |
| 50 | 30.89 | 14.25 | 6.16 | 2.75 | 1.93 |
| 60 | 50.55 | 29.34 | 14.87 | 6.63 | 4.65 |
| 70 | 73.1 | 50.7 | 29.9 | 15.87 | 11.8 |
| 80 | 90.79 | 74.92 | 52.65 | 31.41 | 24.6 |
| 90 | 99.07 | 94.6 | 81.4 | 60.25 | 50.85 |
| 100 | 100 | 100 | 100 | 100 | 100 |

*10000 simulated lots. $\mathrm{PF}=0$ if $\mathrm{PWL}<50, \mathrm{PF}=105$ if $\mathrm{PF}>105$


Figure 3.10 OC curves using pay equation 20 and $n=5$ for strength population

## OC Curves Using Stepped Pay equation

The stepped pay equation (Table 3.1) was used to developed multiple OC curves through simulation analysis. Table 3.8 summaries the probability of receiving $\geq \mathrm{PF}$ for various PWL while Figure 3.11 shows the OC curves for the pay adjustment acceptance plan developed using population characteristics of concrete strength based on the stepped pay equation and a sample size of five. Overall the OC curves shown in Figure 3.11 are very similar to the OC curves in Figure 3.4 except that the probabilities of receiving $100 \%$ pay ( $\mathrm{PF}=1$ curve) for any PWLs in Figure 3.11 are slightly larger than those in Figure 3.4. This means that these pay schedules produce similar expected pays.

Table 3.8 Probability of receiving $\geq \mathrm{PF}$ based on stepped pay equation and $\mathrm{n}=5$ for strength population

| PWL | Prob. of receiving $\geq$ PF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0.43 | 0.43 | 0.02 | 0 | 0 |
| 20 | 3.22 | 3.22 | 0.13 | 0 | 0 |
| 30 | 13.3 | 13.3 | 1.77 | 0.33 | 0.07 |
| 40 | 28.97 | 28.97 | 6.18 | 1.3 | 0.51 |
| 50 | 49.72 | 49.72 | 15.1 | 3.33 | 1.27 |
| 60 | 70.02 | 70.02 | 29 | 8.1 | 3.32 |
| 70 | 87.1 | 87.1 | 51 | 18.08 | 8.46 |
| 80 | 97.02 | 97.02 | 75.21 | 35.64 | 18.91 |
| 90 | 99.85 | 99.85 | 95.69 | 65.09 | 41.06 |
| 100 | 100 | 100 | 100 | 100 | 100 |

*10000 simulated lots. $\mathrm{PF}=0$ if $\mathrm{PWL}<50$


Figure 3.11 OC curves based on stepped pay equation and $\mathrm{n}=5$ for strength population

### 3.3.5 EP curves Based on Different Pay Equations

To fully evaluate and compare the overall pay performance of the three different pay equations, it is necessary to develop EP curves. Table 3.9 summarizes the average pay factors in the long run for various quality levels from simulation analysis, while Figure 3.12 shows the EP curves of the three pay schedules for a sample size of 5 .

It can be seen in Figure 3.12 that the EP curves for pay equation 19 and the stepped pay equation are essentially identical. The expected pay for AQL materials using equation 19 and stepped pay schedule is $100 \%$, however, only $98.08 \%$ payment for an AQL material was obtained using pay 20 . This can be considered that pay equation 20 results in a larger pay risk in the long run for the contractor. One possible way to reduce such risk is to use equation 20 with a maximum incentive payment of $110 \%$ rather than $105 \%$.

Table 3.9 Expected payments in relation to PWL for three pay equations

| PWL (\%) | Average pay factor (\%) |  |  |
| :--- | :--- | :--- | :--- |
|  | $(13)$ | $(14)$ | Stepped |
| 100 | 105.00 | 105.00 | 105.00 |
| 90 | 100.00 | 98.08 | 100.00 |
| 80 | 93.00 | 87.50 | 92.12 |
| 70 | 80.00 | 73.20 | 80.67 |
| 60 | 63.00 | 55.00 | 64.45 |
| 50 | 43.00 | 37.80 | 42.93 |
| 40 | 25.50 | 21.00 | 23.35 |
| 30 | 11.40 | 10.00 | 10.25 |
| 20 | 2.50 | 1.90 | 2.65 |
| 10 | 0.15 | 0.15 | 0.36 |
| 0 | 0.00 | 0.00 | 0.00 |

* 10000 simulated lots. $\mathrm{PF}=0$ if $\mathrm{PWL}<50$

Figure 3.12 Expected pay curves for different pay equations and $\mathrm{n}=5$


### 3.3.6 The Effect of Rejection or Replacement Provisions on OC and EP Curves

The above pay factor analysis considered a lot to be rejected or replaced $(\mathrm{PF}=0)$ if the PWL is less than $50 \%$. However, this level varies from state to state. For example, the North Carolina DOTs specification indicates that a replacement is called if the PWL is less than $60 \%$
for AQCs in the asphalt mixture. So, it is necessary to explore the effects of this rejection and replacement provision on OC and EP curves.

In the following analysis, the pay equation 19 with different rejection or replacement provisions (i.e., $\mathrm{PF}=0$ if $\mathrm{PWL}<40 \%, \mathrm{PF}=0$ if $\mathrm{PWL}<60 \%$ ) are used to develop multiple OC curves and EP curves based on the same sample size of 5 . Table 3.10 summarizes the probability of receiving $\geq P F$ for various $P W L$ while Figure 3.13 shows the corresponding OC curves using equation 19 with $\mathrm{PF}=0$ if $\mathrm{PWL}<40 \%$. Table 3.11 and Figure 3.14 show similar information using the same pay equation but with $\mathrm{PF}=0$ if $\mathrm{PWL}<60 \%$. It is illustrated in Figures 3.13 \& 3.14 that changing rejection or replacement provisions from $\mathrm{PWL}<40 \%$ to $\mathrm{PWL}<60 \%$ has nonsignificant effects on the $\mathrm{PF}=0.9,10$ and 1.04 curves, however, the probabilities of receiving > $\mathrm{PF}=0.8$ and 0.7 for each PWL become smaller. For example, an AQL quality level has approximately a $99.96 \%$ probability of receiving a pay factor $\geq 0.7$ and a $61.5 \%$ chance of receiving a pay factor $\geq 1.0$ using pay equation 19 with $\mathrm{PF}=0$ at $\mathrm{PWL}<40 \%$, while this AQL quality level has approximately a $99.01 \%$ probability of receiving a pay factor $\geq 0.7$ and a $61.5 \%$ chance of receiving a pay factor $\geq 1.0$ using pay equation 13 with $\mathrm{PF}=0$ at $\mathrm{PWL}<60 \%$

Table 3.10 Probability of receiving >PF using pay equation 19 with $\mathrm{PF}=0$ at $\mathrm{PWL}<40 \%$ and $\mathrm{n}=5$

| PWL | Prob. of receiving $\geq$ PF |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 1.46 | 0.21 | 0 | 0 | 0 |
| 20 | 10.59 | 3.09 | 0.25 | 0.03 | 0.01 |
| 30 | 32.09 | 14.25 | 2.29 | 0.27 | 0.11 |
| 40 | 52.1 | 30 | 6.29 | 0.89 | 0.33 |
| 50 | 71.15 | 49.27 | 15.08 | 2.75 | 1.16 |
| 60 | 86.58 | 70.58 | 29.35 | 7.26 | 3.62 |
| 70 | 95.31 | 87.13 | 50.38 | 15.37 | 8.6 |
| 80 | 99.16 | 96.87 | 75.04 | 32.96 | 18.71 |
| 90 | 99.96 | 99.86 | 94.16 | 61.45 | 41.22 |
| 100 | 100 | 100 | 100 | 100 | 100 |



Figure 3.13 OC curves using pay equation 19with $\mathrm{PF}=0$ at $\mathrm{PWL}<40 \%$ and $\mathrm{n}=5$

Table 3.11 Probability of receiving > PF using pay equation 19 with $\mathrm{PF}=0$ at $\mathrm{PWL}<60 \%$ and $\mathrm{n}=5$

| PWL | Prob. of receiving $\geq$ PF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10 | 0.05 | 0.05 | 0.00 | 0.00 | 0.00 |
| 20 | 0.79 | 0.79 | 0.19 | 0.00 | 0.00 |
| 30 | 5.59 | 5.59 | 1.90 | 0.30 | 0.12 |
| 40 | 15.25 | 15.25 | 6.13 | 0.84 | 0.37 |
| 50 | 29.70 | 29.70 | 14.56 | 3.00 | 1.43 |
| 60 | 50.68 | 50.68 | 29.20 | 6.84 | 3.48 |
| 70 | 73.27 | 73.27 | 51.18 | 15.39 | 7.94 |
| 80 | 90.90 | 90.90 | 74.47 | 31.78 | 18.20 |
| 90 | 99.01 | 99.01 | 94.77 | 61.57 | 41.60 |
| 100 | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 |



Figure 3.14 OC curves using pay equation 19 with $\mathrm{PF}=0$ at $\mathrm{PWL}<60 \%$ and $\mathrm{n}=5$

The expected pay in relation to PWLs using pay equation 13 with different rejection and replacement provisions (i.e., $\mathrm{PF}=0$ if $\mathrm{PWL}<40 \%, \mathrm{PF}=0$ if $\mathrm{PWL}<50 \%$ and $\mathrm{PF}=0$ if $\mathrm{PWL}<$ 60\%) are summarized in Table 3.12 and plotted in Figure 3.15. Overall, it is shown in Figure 3.15 that the average pay factor in the long run decreases, as expected, with the increase of $\mathrm{PWL}<40 \%$ to $\mathrm{PWL}<60 \%$. Changing the rejection or replacement provisions from A to C does not have a significant impact on the long-run average expected pay factor for the AQL production. For example, the long-run average expected pay is $99.3 \%$ for an AQL production using equation 13 with provision A, while the long-run average expected pay increases to $100 \%$ for an AQL production using same pay equation with provision C . This also indicates the pay risk for contractor using equation 19 with provision C is higher than that with provision A . However, for a poor-quality level (i.e., $\mathrm{PWL}=50 \%$ ), it also should be noticed that the pay risk for seller using equation 19 with provision C is lower than that with provision A .

Table 3.12 Expected pay in relation to PWL based on pay equation 19 and $n=5$

| PWL (\%) | Average pay factor (\%) |  |  |
| :--- | :--- | :--- | :--- |
|  | $\mathrm{A}:$ <br> $\mathrm{PF}=0$, if PWL $<50 \%$ | $\mathrm{B}:$ <br> $\mathrm{PF}=0$, if $\mathrm{PWL}<40 \%$ | $\mathrm{C}:$ <br> $\mathrm{PF}=0$, if PWL < $60 \%$ |
| 100 | 105.0 | 105.0 | 105.0 |
| 90 | 99.9 | 100.0 | 99.3 |
| 80 | 93.1 | 94.6 | 87.4 |
| 70 | 80.5 | 86.7 | 68.6 |
| 60 | 63.2 | 76.2 | 46.8 |
| 50 | 43.3 | 60.1 | 27.1 |
| 40 | 25.5 | 42.9 | 13.7 |
| 30 | 11.4 | 25.8 | 5.0 |
| 20 | 2.5 | 8.4 | 0.7 |
| 10 | 0.2 | 1.2 | 0.0 |
| 0 | 0.0 | 0.0 | 0.0 |



Figure 3.15 Expected pay curves using pay equation 19 and $\mathrm{n}=5$

### 3.4 Pay Factor Analysis for Multiple Quality Characteristics

The pay factor analyses above are based on a single quality characteristic. However, it is more often that State Highway Agencies (SHAs) use multiple quality characteristics to determine the pay factor for a lot. There are two different ways to calculate the PF associated with multiple quality characteristics. The first approach is using a weighting system to combine individual PWL and calculate a Composite Percent Within Limit (CMPWL) as shown in Equation 21. Then, using Equation 22 or 23 to determine the Composite Pay Factor (CMPF) for each CMPWL. For example, Maryland State Highway applies this method to calculate pay factors for pavements (2008 specification). The more important quality characteristics would be assigned a larger weighting.

$$
\begin{equation*}
C M P W L=0.25 \times P W L_{\text {strenght }}+0.35 \times P W L_{\text {thickness }}+0.4 \times P W L_{\text {roughness }} \tag{21}
\end{equation*}
$$

Where

PWL $_{\text {strength }}=$ percent within specification limit for strength
$\mathrm{PWL}_{\text {roughness }}=$ percent within specification limit for roughness
$\mathrm{PWL}_{\text {thickness }}=$ percent within specification limits for thickness

$$
\begin{equation*}
C M P F=55+0.5 \times C M P W L \tag{22}
\end{equation*}
$$

$$
\begin{equation*}
C M P F=10+C M P W L \tag{23}
\end{equation*}
$$

It should be noted that in order to differentiate the CPF calculated using the pay equation 24, CMPF was used here to represent the composite pay factor calculated based on CMPWL (equation 22\&23).

The second method was recommended by NCHRP 10-79 (Hughes et al. 2011) and widely used by the State Departments of Transportation (DOTs) across the country. This method suggests that the pay factors for individual characteristics should be calculated first using Equation 19 or 20, then a composite pay equation 24 will be used to combine the individual pay factors to calculate the Composite Pay Factor (CPF).

$$
\begin{equation*}
C P F=0.25 \times P F_{\text {strength }}+0.35 \times P F_{\text {thickness }}+0.4 \times P F_{\text {roughness }} \tag{24}
\end{equation*}
$$

Where
$\mathrm{PF}_{\text {strength }}=$ pay factor for strength
$\mathrm{PF}_{\text {roughness }}=$ pay factor for roughness
$\mathrm{PF}_{\text {thickness }}=$ pay factor for thickness

Figure 3.16 shows the flow chart of conducting pay factor analysis for multiple quality characteristics using two different methods. Both of the two methods were used to develop the OC curves for multiple quality characteristics. The CMPWLs were calculated based on the equation 21. It should be noted that a certain CMPWL could be obtained from different combinations of PWLs for each parameter. However, in this analysis, the same PWL for individual characteristics is used to produce a certain level of CMPWL. For example, a CMPWL of $90 \%$ is obtained when every single PWL is at the AQL quality level ( $90 \%$ ). The effects of different combinations of PWL producing the same CMPWL will be discussed in the next Chapter.


Figure 3.16 Flow chart of pay factor analysis for multiple quality characteristics

As in the previous analysis, the population distributions for each quality characteristic (strength, thickness, and roughness) were shifted to produce different PWLs. Equation 21 was used to calculate CMPWL of the estimated lots, and the CMPF was determined by equation 22 or 23 . Table 3.13 summarizes the probability values of receiving $\geq$ CMPF for various CMPWL from simulation using pay equation 22 while Figure 3.17 shows the OC curves based on CMPWL and CMPF. It is shown in Table 3.13 that the probability of receiving CMPF $\geq 1.0$ is $0 \%$ for rejectable quality level (CMPWL=50\%) while the chance of receiving CMPF $\geq 1.0$ is $57.78 \%$ for AQL (CMPWL=90\%) quality product. Overall, the OC curves (Figure 3.17) developed based on multiple quality characteristics are more spread than that developed based on a single quality
characteristic (Figure 3.4) indicating that the dispersion of CMPWL and CMPF estimates are smaller than that of PWL and PF estimates. This can also be demonstrated by the histogram for an AQL population showing the variability of PWL, CMPWL and CMPF in Figure 3.18. The standard deviation (variability) of the estimated PWLs for a single characteristic was approximately 11.0 for a sample size of 5 , while the standard deviation was calculated to be 6.34 for CMPWLs and 3.17 for CMPF, respectively. This is because the large number of individual PWLs were randomly combined leading to a balance out between high and low PWLs such that the CMPWL was more centered to $90 \%$. The histogram for a $50 \%$ PWL population showing the variability of CMPWL and CMPF based on equation 22 and $n=5$ is plotted in Figure 3.19. It can be seen that almost half of the simulated lots were assigned a CMPF of 0 . This is because that the rejection or replace level $(\mathrm{CMPF}=0)$ was set at CMPWL of 50.

Table 3.13 Probability of receiving $\geq$ CMPF using pay equation 22 based on population characteristics and $\mathrm{n}=5$

| CMPWL | Prob. of receiving $\geq$ CMPF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0.15 | 0.15 | 0 | 0 | 0 |
| 30 | 2.72 | 2.72 | 0 | 0 | 0 |
| 40 | 17.02 | 17.02 | 0.18 | 0 | 0 |
| 50 | 50.66 | 50.66 | 2.52 | 0 | 0 |
| 60 | 84.73 | 84.73 | 17.55 | 0.06 | 0 |
| 70 | 98.25 | 98.25 | 54.55 | 1.07 | 0 |
| 80 | 99.92 | 99.92 | 88.48 | 11.47 | 0.32 |
| 90 | 100 | 100 | 99.62 | 57.78 | 7.83 |
| 100 | 100 | 100 | 100 | 100 | 100 |

* 10000 simulated lots, CMPF=0 if CMPWL<50


Figure 3.17 OC curves using pay equation 22 based on population characteristics and $\mathrm{n}=5$


Figure 3.18 AQL population showing the variability of CMPWL and CMPF based on equation 22 and

$$
\mathrm{n}=5 \text { (long-run average } \mathrm{CMPF}=100 \%)
$$



Figure 3.19 Variability of CMPWL and CMPF for population with $50 \%$ PWL based on equation 22 and $\mathrm{n}=5$

The OC curves were then developed using pay equation 23 based on population characteristics. The results were summarized in Table 3.14 and plotted in Figure 3.20. For an AQL production, the same average CMPF of $100 \%$ was obtained in the long run based on pay equations 22 and 23 , however, the probabilities of receiving CMPF $\geq$ various pay factors (i.e., $0.7,0.8,0.9,1.0$ and 1.04) and OC curves are different. For example, the probability of receiving CMPF $\geq 1.04$ was estimated to be $7.83 \%$ based on pay equation 22 while the probability of receiving CMPF $\geq 1.04$ changed dramatically to $31.2 \%$ for an AQL quality based on equation 23. This indicates that if equation 23 was used, the contractor tends to get more incentive pay for an AQL production. Figure 3.21 provides the histogram for an AQL population showing the variability of CMPWL and CMPF based on pay equations 23 and $n=5$. Compared to Figure 3.18, a much larger variability (standard deviation estimated to be 5.5) for CMPF was observed.

Table 3.14 Probability of receiving $\geq$ CMPF using pay equation 23 based on population characteristics and $\mathrm{n}=5$

| CMPWL | Prob. of receiving $\geq$ CMPF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0.15 | 0.01 | 0 | 0 | 0 |
| 40 | 2.75 | 0.2 | 0.01 | 0 | 0 |
| 50 | 16.63 | 2.55 | 0.12 | 0 | 0 |
| 60 | 50.52 | 16.44 | 2.08 | 0.06 | 0.01 |
| 70 | 86.62 | 53.38 | 16.22 | 1.12 | 0.22 |
| 80 | 98.61 | 88.08 | 54.49 | 11.65 | 3.48 |
| 90 | 99.97 | 99.47 | 93.7 | 57.28 | 31.2 |
| 100 | 100 | 100 | 100 | 100 | 100 |

* 10000 simulated lots, CMPF=0 if CMPWL<50


Figure 3.20 OC curves using pay equation 23 based on population characteristics and $\mathrm{n}=5$






Figure 3.21 AQL population showing the variability of CMPWL and CMPF based on pay equation 23 and $\mathrm{n}=5$ (long-run average CMPF $=100 \%$ )

The above pay factors were determined using CMPWL while the following pay factor analysis was based on the composite pay equation 24 , CPF, which combines the pay factors of individual quality characteristics. As shown in Figure 3.16, the PF of the simulated lots for a single quality characteristic can be calculated using equation 19 or 20, and then the CPFs can be determined using pay equation 24 . The OC curves for multiple quality characteristics using equation 19 with a combination of equation 24 were developed. The results were summarized in Table 3.15 and plotted in Figure 3.22. It can be observed from Table 3.15 that the probability values of receiving CPF $\geq 1.04$ and 1.0 are calculated to be approximately $10.91 \%$ and $55.23 \%$, respectively, while the probabilities (Table 3.13) of receiving CMPF $\geq 1.04$ and 1.0 are calculated to be approximately $7.83 \%$ and $57.0 \%$.

Figure 3.23 shows the histogram of CPF for an AQL population using on pay equation 19\&24 and a sample size of five, while Figure 3.24 shows the histogram of CMPWL and CMPF for the same population based on equation 22. It can bee seen that the histogram of CPF is very similar to the histogram of CMPF for an AQL population. For an AQL quality level, the long-run average pay factor is calculated to be $100 \%$ for both CPF and CMPF. This indicates that the two pay factor analysis methods for multiple quality characteristics provide similar results for the AQL population.

Figure 3.25 shows the histogram of PF and CPF for an $80 \%$ CMPWL population using on pay equation 19\&24 and a sample size of five while Figure 3.26 shows the histogram of CMPWL and CMPF for the same population based on equation 22. It can be observed that the histogram of CPF is different from the histogram of CMPF for an 80 CMPWL population. The long-run average CMPF was calculated to be $95.3 \%$ while the long-run average CPF was determined to be $93.1 \%$. As shown in Figure 3.25 , for an 80 PWL population, approximately $3 \%$ of simulated lots have PWL less than $50 \%$ for each individual quality characteristic (i.e., strength, thickness, and roughness). These individual simulated lots were randomly combined to calculate the CMPWL. For example, a $20 \%$ PWL for strength may combine with $100 \%$ PWL of thickness and roughness resulting in a CMWPL $=80 \%$. In this case, the pay factor for the lot will be $95 \%$ if CMPWL were used (equation 22). However, the CPF was determined to be $78.75 \%$ for this lot using equation 24 with $\mathrm{PF}_{\text {strength }}=0$ for $\mathrm{PWL}=20 \%$ and $\mathrm{PF}_{\text {roughness }}=\mathrm{PF}_{\text {thickness }}$ $=105 \%$ for PWL $=100 \%$. This indicates that, for multiple quality characteristics, using CMPWL to calculate the PF for a lot is not appropriate. It fails to apply a cut off (i.e., $\mathrm{PF}=0$ if PWL $<50 \%$ ) for individual quality characteristics such that the pay factor for a lot may be overestimated.

Table 3.15 Probability of receiving $\geq$ CPF using pay equation $19 \& 24$ based on population characteristics and $\mathrm{n}=5$

| CMPWL | Prob. of receiving $\geq$ CPF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0.09 | 0.09 | 0 | 0 | 0 |
| 40 | 1.56 | 1.56 | 0.17 | 0 | 0 |
| 50 | 12.23 | 12.12 | 2.6 | 0.01 | 0 |
| 60 | 41.26 | 40.96 | 19.16 | 0.37 | 0.02 |
| 70 | 70.18 | 69.42 | 48.51 | 2.55 | 0.13 |
| 80 | 92.14 | 91.44 | 83.81 | 14.92 | 1.36 |
| 90 | 99.68 | 99.45 | 99.12 | 55.23 | 10.91 |
| 100 | 100 | 100 | 100 | 100 | 100 |

* $\mathrm{n}=5,10000$ simulated lots, $\mathrm{PF}=0$ if $\mathrm{PWL}<50$


Figure 3.22 OC curves using pay equation 19 \& 24 based on population characteristics and n=5


Figure 3.23 PF and CPF for an AQL population using pay equation $19 \& 24$ and $\mathrm{n}=5$



Figure 3.24 CMPF for an AQL population using pay equation 22 and $\mathrm{n}=5$


Figure 3.25 CMPF for an 80 CMPWL population using pay equation 22 and $n=5$


Figure 3.26 PF and CPF for an 80 CMPWL population using pay equation $19 \& 24$ and $n=5$

The OC curves for multiple quality characteristics using equation 20 with a combination of 24 were developed. The results were summarized in Table 3.16 and plotted in Figure 4.26. Similarly, the probabilities of $>\operatorname{PF}(1.04,1.0,0.9,0.8,0.7)$ are very similar to the results in Figure 3.27 for a 90 CMPWL quality level, however, the probabilities of $>\mathrm{PF}(0.9,0.8,0.7)$ are smaller for CMPWLs that are less than 90.

Table 3.16 Probability of receiving $\geq$ CPF using pay equation $20 \& 24$ based on population characteristics and $\mathrm{n}=5$

| CMPWL | Prob. of receiving $\geq$ CPF |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.7 | 0.8 | 0.9 | 1.0 | 1.04 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0.06 | 0 | 0 | 0 | 0 |
| 40 | 1.22 | 0.23 | 0 | 0 | 0 |
| 50 | 9.76 | 2.99 | 0.32 | 0.01 | 0 |
| 60 | 38 | 18.7 | 4.92 | 0.43 | 0.08 |
| 70 | 68.79 | 48.53 | 18.24 | 2.55 | 0.82 |
| 80 | 92.97 | 84.34 | 53.66 | 14.75 | 5.91 |
| 90 | 99.64 | 99.25 | 93.04 | 58.09 | 34.04 |
| 100 | 100 | 100 | 100 | 100 | 100 |

* $\mathrm{n}=5,10000$ simulated lots, $\mathrm{PF}=0$ if $\mathrm{PWL}<50$


Figure 3.27 OC curves using pay equation 29 \& 24 based on population characteristics and $\mathrm{n}=5$

### 3.5 Sensitivity Analysis

A producer might be able to improve the production control by either reducing the production variability (standard deviation) or improving population means. The objective of this analysis is to determine how changing the standard deviation and mean of a single quality characteristic would affect the average composite pay factor in the long run. Firstly, the means were kept as the same as population means, and only standard deviations were changed to examine how variability would affect the pay factors. Then, the population standard deviations were used, and the population means were changed to evaluate the effects of changing means on composite pay factors. A sample size of 5 was used for different quality characteristics (i.e., strength, thickness and roughness).

The standard deviation of compressive strength was gradually changed while population values were used for all other characteristics. The effects of changing in strength standard deviation on CMPWL and CPF using different pay equations are summarized in Table 3.17 and plotted in Figure 3.28. It can be seen that a CMPWL of $96.63 \%$ can be calculated if the current population characteristics values were used. Reducing the standard deviation by $70 \%$ only resulted in a $0.7 \%$ increase in CPF since the current mean of strength is far away from spec and small weight of strength in calculating CMPWL. It is also shown that an increase in strength variability for three times produces a reduction in CPF of approximately $2.3 \%$ using pay equation 19 and approximately $4 \%$ using pay equation 20 .

Table 3.17 Effects of change in strength variability on CMPWL and CPF

| Strength |  | $\mathrm{PF}=55+0.5^{*}$ PWL |  | $\mathrm{PF}=10+$ PWL |  | Stepped Pay Schedule |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Std | Std/Std(pop) | CMPWL <br> $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ |
| 51 | 0.1 | 98.20 | 104.10 | 0.77 | 104.30 | 0.68 | 104.00 | 0.97 |
| 153 | 0.3 | 98.00 | 104.00 | 0.68 | 104.30 | 0.68 | 104.00 | 0.97 |
| 255 | 0.5 | 97.73 | 103.80 | 0.48 | 104.26 | 0.64 | 103.80 | 0.78 |
| 510 | 1.0 | 96.63 | 103.30 | 0.00 | 103.60 | 0.00 | 103.00 | 0.00 |
| 765 | 1.5 | 95.08 | 102.50 | -0.77 | 102.60 | -0.97 | 102.40 | -0.58 |
| 1020 | 2.0 | 93.80 | 101.90 | -1.36 | 101.40 | -2.12 | 101.70 | -1.26 |
| 1530 | 3.0 | 91.70 | 100.90 | -2.32 | 99.50 | -3.96 | 100.60 | -2.33 |



Figure 3.28 Effects of changing strength variability on CPF

The mean of compressive strength was gradually changed while population values were used for all other characteristics. The effects of changing in strength mean on CMPWL and CPF using different pay equations are summarized in Table 3.18 and plotted in Figure 3.29. Increases in strength mean by $10 \%$ produce an increase in CPF of approximately $0.68 \%$ (based on equation

19\&24) and after that the change of CPF became constant meaning the PWL for strength has achieved $100 \%$, while reductions in strength mean by $10 \%$ produces a reduction in CPF of approximately $11.4 \%$ (based on equation 19\&24) and after that the change of CPF became constant reflecting the PWL for strength has been reduced to $0 \%$.

Table 3.18 Effects of change in strength means on CMPWL and CPF

| Strength |  |  | $\mathrm{PF}=55+0.5 * \mathrm{PWL}$ |  | $\mathrm{PF}=10+\mathrm{PWL}$ |  | Stepped Pay Schedule |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean | Change \% | CMPWL $\%$ | $\begin{aligned} & \text { CPF } \\ & \% \end{aligned}$ | Change <br> CPF, \% | $\begin{aligned} & \mathrm{CPF} \\ & \% \end{aligned}$ | Change CPF, \% | $\begin{array}{\|l\|l\|} \hline \text { CPF } \\ \% \\ \hline \end{array}$ | Change CPF, \% |
| 2268.0 | -50 | 73.10 | 91.50 | -11.42 | 83.00 | -19.96 | 90.70 | -11.94 |
| 2721.6 | -40 | 73.10 | 91.40 | -11.52 | 83.00 | -19.96 | 90.70 | -11.94 |
| 3175.2 | -30 | 73.30 | 91.60 | -11.33 | 83.00 | -19.96 | 90.70 | -11.94 |
| 3628.8 | -20 | 76.90 | 93.30 | -9.68 | 86.70 | -16.39 | 92.20 | -10.49 |
| 4082.4 | -10 | 87.80 | 98.90 | -4.26 | 97.10 | -6.36 | 98.10 | -4.76 |
| 4536.0 | 0 | 96.60 | 103.30 | 0.00 | 103.70 | 0.00 | 103.00 | 0.00 |
| 4989.6 | 10 | 98.10 | 104.00 | 0.68 | 104.30 | 0.58 | 104.10 | 1.07 |
| 5443.2 | 20 | 98.20 | 104.10 | 0.77 | 104.40 | 0.68 | 104.10 | 1.07 |
| 5896.8 | 30 | 98.20 | 104.10 | 0.77 | 104.40 | 0.68 | 104.10 | 1.07 |



Figure 3.29 Effects of changing strength means on CPF

Similar, the effects of changing standard deviation and mean of thickness would affect CMPWL and CPF were examined and the results were summarized in Table 3.19 and plotted in Figure 3.30. Reducing the standard deviation of thickness by $50 \%$ only resulted in a $0.77 \%$ increase in CPF because the PWL of the current thickness population was ready very high. It is also shown that an increase in strength variability for three times produces a reduction in CPF of approximately $6.4 \%$ (pay equation $19 \& 24$ ) reflecting the heavyweight of thickness in the CMPWL equation.

Table 3.19 Effects of change in thickness variability on CMPWL and CPF

| Thickness |  |  | $\mathrm{PF}=55+0.5 * \mathrm{PWL}$ |  | $\mathrm{PF}=10+\mathrm{PWL}$ |  | Stepped Pay Schedule |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Std | Std/Std(pop) | CMPWL $\%$ | $\begin{array}{\|l} \hline \text { CPF } \\ \% \\ \hline \end{array}$ | Change CPF, \% | $\begin{aligned} & \text { CPF } \\ & \% \\ & \hline \end{aligned}$ | Change <br> CPF, \% | $\begin{aligned} & \mathrm{CPF} \\ & \hline \end{aligned}$ | Change CPF, \% |
| 0.11 | 0.20 | 98.40 | 104.20 | 0.87 | 104.53 | 0.80 | 104.20 | 0.87 |
| 0.27 | 0.50 | 98.20 | 104.10 | 0.77 | 104.50 | 0.77 | 104.10 | 0.77 |
| 0.43 | 0.80 | 97.40 | 103.70 | 0.39 | 104.12 | 0.41 | 103.60 | 0.29 |
| 0.54 | 1.00 | 96.60 | 103.30 | 0.00 | 103.70 | 0.00 | 103.30 | 0.00 |
| 0.59 | 1.50 | 93.76 | 101.90 | -1.36 | 101.50 | -2.12 | 101.50 | -1.74 |
| 1.08 | 2.00 | 90.60 | 100.30 | -2.90 | 98.60 | -4.92 | 99.60 | -3.58 |
| 1.62 | 3.00 | 84.10 | 96.70 | -6.39 | 92.80 | -10.51 | 96.00 | -7.07 |



Figure 3.30 Effects of changing thickness variability on CPF

The effects of changing in thickness mean on CMPWL and CPF using different pay equations are summarized in Table 3.20 and plotted in Figure 3.31. Because thickness has a double side specification limit and tolerances are close to each other, a small change in thickness means would produce a significant change in CMPWL and correspondingly CPF. It is can be observed that the maximum achievable CPF (103.6\%) can be obtained by reducing the mean of approximately $1 \%$.

Increasing thickness mean by $10 \%$ produce a reduction in CPF of approximately $7 \%$ (based on equation 19\&24). The change of CPF by shifting thickness mean for other pay equations can also be checked in Table 3.20 and Figure 3.31.

Table 3.20 Effects of change in thickness means on CMPWL and CPF

| Thickness |  |  | PF=55+0.5*PWL |  | PF=10+PWL |  | Stepped Pay Schedule |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean | Change <br> $\%$ | CMPWL <br> $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ |
| 10.9 | -10 | 67.60 | 88.00 | -14.81 | 76.90 | -25.77 | 86.60 | -15.92 |
| 11.1 | -8 | 76.00 | 92.70 | -10.26 | 85.70 | -17.28 | 91.50 | -11.17 |
| 9.7 | -5 | 88.90 | 99.40 | -3.78 | 97.90 | -5.50 | 98.80 | -4.08 |
| 11.5 | -2 | 96.20 | 103.10 | -0.19 | 103.40 | -0.19 | 102.90 | -0.10 |
| 12.1 | 0 | 96.60 | 103.30 | 0.00 | 103.60 | 0.00 | 103.00 | 0.00 |
| 12.3 | 2 | 93.30 | 101.60 | -1.65 | 101.20 | -2.32 | 101.10 | -1.84 |
| 12.7 | 5 | 81.90 | 95.90 | -7.16 | 91.60 | -11.58 | 94.90 | -7.86 |
| 13.1 | 8 | 68.80 | 88.70 | -14.13 | 78.10 | -24.61 | 87.50 | -15.05 |
| 13.3 | 10 | 62.60 | 84.70 | -18.01 | 71.50 | -30.98 | 83.40 | -19.03 |



Figure 3.31 Effects of changing thickness means on CPF

Finally, the effects of changing mean and standard deviation of roughness were evaluated. It is shown in Figure 3.32 the PWL for roughness based on current population characteristic values is $100 \%$ indicating the mean of roughness is much smaller than the specification limit, and thus increasing the standard deviation by $100 \%$ only resulted in an $0.76 \%$ reduction in CPF.

Table 3.21 Effects of change in roughness variability on CMPWL and CPF

| Roughness |  |  | $\mathrm{PF}=55+0.5 * \mathrm{PWL}$ |  | $\mathrm{PF}=10+\mathrm{PWL}$ |  | Stepped Pay Schedule |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Std | Std/Std(pop) | CMPWL \% | $\begin{aligned} & \mathrm{CPF} \\ & \% \end{aligned}$ | Change <br> CPF, \% | $\begin{aligned} & \hline \mathrm{CPF} \\ & \% \end{aligned}$ | Change CPF, \% | CPF | Change CPF, \% |
| 0.87 | 0.10 | 96.60 | 103.30 | 0.01 | 103.60 | 0.00 | 103.00 | 0.00 |
| 4.35 | 0.50 | 96.60 | 103.30 | 0.01 | 103.60 | 0.00 | 103.00 | 0.00 |
| 6.95 | 0.80 | 96.60 | 103.30 | 0.01 | 103.60 | 0.00 | 103.00 | 0.00 |
| 8.69 | 1.00 | 96.60 | 103.29 | 0.00 | 103.60 | 0.00 | 103.00 | 0.00 |
| 10.43 | 1.20 | 96.50 | 103.20 | -0.09 | 103.60 | 0.00 | 103.00 | 0.00 |
| 13.04 | 1.50 | 96.20 | 103.00 | -0.28 | 103.40 | -0.19 | 102.80 | -0.19 |
| 17.38 | 2.00 | 95.30 | 102.50 | -0.76 | 102.20 | -1.35 | 102.20 | -0.78 |
| 26.07 | 3.00 | 92.30 | 100.80 | -2.41 | 99.70 | -3.76 | 100.40 | -2.52 |



Figure 3.32 Effects of changing roughness variability on CPF

The effects of changing in roughness mean on CMPWL and CPF using different pay equations are summarized in Table 3.22 and plotted in Figure 3.33. Reducing in strength means does not produce any change in CPF meaning the PWL for roughness has achieved $100 \%$ based on current population characteristics values. Increases in strength mean by $20 \%$ produce a reduction in CPF of approximately $4.26 \%$.

Table 3.22 Effects of change in roughness means on CMPWL and CPF

| Roughness |  | PF $=55+0.5 *$ PWL |  | PF=10+PWL |  | Stepped Pay Schedule |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Mean | Change <br> $\%$ | CMPWL <br> $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ | CPF <br> $\%$ | Change <br> CPF, $\%$ |
| 47.75 | -20 | 96.60 | 103.30 | 0.00 | 103.60 | 0.00 | 103.00 | 0.00 |
| 53.72 | -10 | 96.60 | 103.40 | 0.10 | 103.60 | 0.00 | 103.00 | 0.00 |
| 59.69 | 0 | 96.60 | 103.30 | 0.00 | 103.60 | 0.00 | 103.00 | 0.00 |
| 65.66 | 10 | 95.90 | 102.90 | -0.39 | 103.00 | -0.58 | 102.60 | -0.39 |
| 71.63 | 20 | 88.70 | 98.90 | -4.26 | 96.20 | -7.14 | 98.40 | -4.47 |
| 77.60 | 30 | 71.20 | 89.00 | -13.84 | 79.00 | -23.75 | 87.70 | -14.85 |
| 83.57 | 40 | 62.67 | 83.60 | -19.07 | 70.60 | -31.85 | 82.40 | -20.00 |
| 89.54 | 50 | 61.67 | 83.00 | -19.65 | 69.00 | -33.40 | 81.90 | -20.49 |
| 107.44 | 80 | 61.70 | 83.00 | -19.65 | 69.00 | -33.40 | 81.50 | -20.87 |



Figure 3.33 Effects of changing roughness means on CPF

## Chapter 4 Relating Risks to Pay Factors

The risk and pay factor analysis necessary for developing a statistical QA specification has been conducted in Chapters 2 and 3. In order to properly apply statistical specifications with pay adjustment provision in the quality assurance (QA) system, it is also necessary to evaluate the risk associated with a certain pay factor to ensure the effectiveness of the pay adjustment acceptance plans. When a pay adjustment acceptance plan is appropriately formulated, the statistical description of desired quality characteristics provides the best insurance that the seller will be paid a fair price and that the buyer will get what was paid for with a reasonable level of risk. The objective of this Chapter is to (i) relate acceptance risks to pay factors, and (ii) present case studies to demonstrate the value of the analyses proposed in this study.

As the analysis in Chapter 3, the pay factor equations (i.e., $19 \& 20$ ) are usually applied to relate the pay factor $(\mathrm{PF})$ to the actual quality (PWL) of a lot in the payment adjustment acceptance plans. In addition, a number of SHAs acceptance specifications also include some form of rejection or replacement $(\mathrm{PF}=0)$ if the quality of the lot is below a certain PWL, such as $50 \%$ or $60 \%$ For example, the South Carolina DOT specifications use the pay equation 19 to determine a pay factor for a lot and call the lot to be removed and replaced ( $\mathrm{PF}=0$ ) if any one of the single quality characteristics have a PWL less than $60 \%$. When such rejection or replacement provision is included, the concepts of $\alpha$ risk can be interpreted as the probability of rejecting an AQL quality material or construction while the $\beta$ risk can be interpreted as the probability of accepting an RQL quality level. The OC curves developed in Chapter 3 provide the relationship between the PWL of a lot and the probability of receiving various payment factors (i.e., $0.7,0.8$, $0.9,1.0,1.04)$ for the lot. However, this is simply not enough when the rejection or replacement provision is included in the pay adjustment acceptance plans. It is also necessary to determine the
probability of rejection (risk of rejection) for various PWLs. So, both of the average pay factors at long run and the probability of rejection for various PWLs were determined to illustrate the effects of such rejection or replacement provisions on the pay factors and the risks for single and multiple quality characteristics.

### 4.1 Relating Risks to Pay Factors for a Single Quality Characteristic

The OC curves developed in Chapter 2 provided a relationship between the PWL of a lot and the probability of its acceptance/rejection while the long-average pay factor based on the PWL of the same lot can be obtained from the pay factor analysis presented in Chapter 3, and thus the relationship between the probability of rejection and pay factors can be built. Table 4.1 summarizes the probabilities of rejection and pay factors for different PWLs based on different pay equations and a sample size of five.

It can be observed from Table 4.1 that the probability of rejection is $0.2 \%$ for an AQL of $90 \%$ using pay equation 19 with an RQL of $50 \%$ ( $\mathrm{PF}=0$ if $\mathrm{PWL}<50 \%$ ), and the average pay factor at the long run for this quality was determined to be $99.89 \%$. The probability of rejection is estimated to be $1.1 \%$ using the same pay equation but with an RQL of $60 \%$ ( $\mathrm{PF}=0$ if PWL < $50 \%$ ), and the average pay factor at the long run decreases from $99.89 \%$ to $99.01 \%$. Table 4.1 also shows that that same pay equation was used in A and B, however, the risks associated with each PWL in A are much larger than those in pay schedule B. This indicates that increasing the RQL (i.e., from $50 \%$ to $60 \%$ ) will increase the acceptance risk and reduce the expected pay for contractors when the same pay equation was used. Similarly, from the results of B and D, it can be demonstrated that changing the pay equation has no effects on the risks associated with each PWL, but it will affect the average pay factors in the long run for each PWL.

Table 4.1 Probability of rejection and pay factor in relation to PWL for different pay equations with $\mathrm{n}=5$

| PWL | $\mathrm{PF}=55+0.5 * \mathrm{PWL}$ |  |  |  | $\mathrm{PF}=10+\mathrm{PWL}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  | B |  | C |  | D |  |
|  | If PWL<50\%, PF=0 |  | If PWL<60\%, PF=0 |  | If PWL<50\%, PF=0 |  | If PWL<60\%, PF=0 |  |
|  | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) |
| 100 | 0.00 | 105.00 | 0.00 | 105.00 | 0.00 | 110.00 | 0.00 | 110.00 |
| 95 | 0.01 | 102.32 | 0.08 | 102.01 | 0.01 | 104.64 | 0.08 | 104.4 |
| 90 | 0.20 | 99.89 | 1.10 | 99.01 | 0.20 | 99.98 | 1.10 | 99.25 |
| 85 | 1.02 | 96.96 | 3.98 | 94.23 | 1.02 | 94.55 | 3.98 | 92.93 |
| 80 | 3.02 | 93.17 | 9.06 | 88.30 | 3.02 | 89.50 | 9.06 | 85.68 |
| 75 | 6.40 | 87.64 | 17.14 | 78.30 | 6.40 | 82.08 | 17.14 | 74.70 |
| 70 | 12.05 | 80.50 | 27.37 | 69.01 | 12.05 | 74.08 | 27.37 | 64.91 |
| 65 | 19.43 | 73.56 | 38.52 | 60.00 | 19.43 | 66.25 | 38.52 | 54.31 |
| 60 | 28.49 | 62.77 | 50.00 | 46.55 | 28.49 | 55.70 | 50.00 | 42.74 |
| 55 | 38.80 | 54.80 | N/A | N/A | 38.80 | 47.08 | N/A | N/A |
| 50 | 50.00 | 43.71 | N/A | N/A | 50.00 | 37.63 | N/A | N/A |

Figure 4.1 shows the relation of the probability of rejection and the average pay factors at the long run for different pay equations and RQL. For a specific risk level, C gives the lowest expected pay while B provides the highest pay factor. For a given pay factor, such as $90 \%$, B produces the highest probability of rejection (9\%) while C offers the lowest probability of rejection (4\%). The relation of risks and pay factors are very similar for A and D.


Figure 4.1 Pay factor and the probability of rejection using different pay equations with $\mathrm{n}=5$

In order to analyze the effects of sample size on the relation of risks and pay factors, sample sizes of 3 and 6 were used to calculate the probability of rejection and pay factor corresponding to different PWLs. Tables 4.2 and 4.3 summarize the probability of rejection and pay factor in relation to PWLs for sample sizes of 6 and 3 respectively. Figures 4.1 through 4.3 show the relation between pay factor and probability of rejection for different sample sizes (i.e., 3,5 and 6 ). It can be seen from Figures 4.1 through 4.3 that the probability of rejection increases as increasing the sample size for a certain pay factor. For a given risk value, the pay factor increases with the increase of sample size. For example, the probability of rejection associated with $100 \%$ payment in plan A was estimated to be $1.15 \%$ for a sample size of three. This probability reduced to $0.08 \%$ as increasing the sample size to six. When a sample size of three is used, plans A, B, and D provide similar expected payment for a given risk level.

Table 4.2 Probability of rejection and pay factor in relation to PWLs for different pay equations with $n=6$

| PWL | $\mathrm{PF}=55+0.5 * \mathrm{PWL}$ |  |  |  | $\mathrm{PF}=10+\mathrm{PWL}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  | B |  | C |  | D |  |
|  | If PWL<50\%, PF=0 |  | If $\mathrm{PWL}<60 \%, \mathrm{PF}=0$ |  | If PWL<50\%, PF=0 |  | If PWL<60\%, $\mathrm{PF}=0$ |  |
|  | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) |
| 100 | 0.00 | 105.00 | 0.00 | 105.00 | 0.00 | 110.00 | 0.00 | 110.00 |
| 95 | 0.00 | 102.50 | 0.03 | 102.35 | 0.00 | 104.60 | 0.03 | 104.74 |
| 90 | 0.08 | 100.01 | 0.58 | 99.54 | 0.08 | 99.79 | 0.58 | 99.60 |
| 85 | 0.59 | 97.47 | 2.70 | 95.20 | 0.59 | 95.01 | 2.70 | 94.05 |
| 80 | 1.90 | 93.94 | 7.37 | 89.40 | 1.90 | 89.55 | 7.37 | 86.20 |
| 75 | 4.81 | 88.52 | 14.76 | 80.78 | 4.81 | 82.59 | 14.76 | 75.97 |
| 70 | 10.02 | 82.15 | 25.00 | 70.43 | 10.02 | 75.07 | 25.00 | 65.13 |
| 65 | 17.20 | 75.20 | 36.88 | 59.26 | 17.20 | 66.84 | 36.88 | 55.38 |
| 60 | 26.54 | 64.65 | 50.00 | 45.67 | 26.54 | 56.14 | 50.00 | 41.47 |
| 55 | 37.74 | 55.48 | N/A | N/A | 37.74 | 47.40 | N/A | N/A |
| 50 | 50.03 | 43.74 | N/A | N/A | 50.03 | 36.80 | N/A | N/A |



Figure 4.2 Probability of rejection and pay factor using different pay equations with $n=6$

Table 4.3 Probability of rejection and pay factor in relation to PWL for different pay equations with $n=3$

| PWL | $\mathrm{PF}=55+0.5 * \mathrm{PWL}$ |  |  |  | $\mathrm{PF}=10+\mathrm{PWL}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  | B |  | C |  | D |  |
|  | If PWL<50\%, PF=0 |  | If PWL<60\%, PF=0 |  | If PWL<50\%, PF=0 |  | If PWL<60\%, PF=0 |  |
|  | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) | Probability of rejection (\%) | Pay factor (\%) |
| 100 | 0.00 | 105.00 | 0.00 | 105.00 | 0.00 | 110.00 | 0.00 | 110.00 |
| 95 | 0.22 | 102.02 | 0.75 | 101.13 | 0.22 | 104.48 | 0.68 | 103.53 |
| 90 | 1.30 | 99.10 | 3.74 | 95.80 | 1.30 | 99.06 | 3.74 | 96.27 |
| 85 | 3.55 | 94.81 | 8.66 | 89.28 | 3.55 | 93.33 | 8.66 | 88.31 |
| 80 | 7.22 | 89.66 | 14.88 | 81.35 | 7.22 | 87.19 | 14.88 | 80.40 |
| 75 | 12.17 | 82.69 | 23.04 | 73.04 | 12.17 | 79.53 | 23.04 | 70.56 |
| 70 | 18.26 | 76.54 | 31.64 | 62.68 | 18.26 | 71.57 | 31.64 | 60.48 |
| 65 | 25.23 | 69.56 | 40.16 | 54.45 | 25.23 | 64.46 | 40.16 | 53.65 |
| 60 | 33.00 | 60.74 | 50.28 | 44.81 | 33.00 | 55.45 | 50.28 | 43.00 |
| 55 | 41.40 | 54.45 | N/A | N/A | 41.40 | 47.46 | N/A | N/A |
| 50 | 50.01 | 45.31 | N/A | N/A | 50.01 | 39.85 | N/A | N/A |



Figure 4.3 Probability of rejection and pay factor using different pay equations with $n=3$

### 4.2 Relating Risks to Composite Pay Factors

As mentioned previously, the majority of state DOTs usually use multiple quality characteristics to determine the pay factor for a lot. Then it becomes important to fully evaluate the risks associated with composite pay factors. However, using multiple quality characteristics may create an issue concerning that different combinations of individual PWL may produce the same composite quality level (CMPWL). For example, two hypothetical lots are shown in Table 5.3. The lot 1 and lot 2 have the same CMPWL of $90 \%$ however, the PWLs of each individual characteristic are different. This obviously represents the boundary combinations, but it illustrates that using multiple quality characteristics introduces risks that are not present when single quality characteristic is used. Figures 4.4 and 4.5 illustrate the range of PWL for individual quality characteristics that produce the same composite quality level (CMPWL). As it is shown in Figure 4.5, the PWL for a single quality characteristic producing a CMPWL of 80 may be as low as 50 . The purpose of these analyses is to address the risks associated with multiple quality characteristics using Monte Carlo simulation and analyze the relation of such risks (probability of rejection) between CMPWL and PF.

Table 4.4 Lots with different combinations of PWL producing CMPWL of $90 \%$

| Quality characteristic | PWL for Lot 1 | PWL for Lot 2 |
| :--- | :--- | :--- |
| Strength | $60 \%$ | $90 \%$ |
| Thickness | $100 \%$ | $90 \%$ |
| Roughness | $100 \%$ | $90 \%$ |
| CMPWL | $90 \%$ | $90 \%$ |
| PF based on CMPWL(CMPF) | $100 \%$ | $100 \%$ |
| Composite pay factor (CPF) | $94.9 \%$ | $100 \%$ |



Figure 4.4 Range of PWL for individual characteristic producing CMPWL $=90 \%$


Figure 4.5 Range of PWL for individual characteristics producing CMPWL=80\%

The different combinations of PWLs for a single quality characteristic producing the same CMPWL can be obtained through simulation. Some boundary combinations are shown in Table 4.5. It has been mentioned in Chapter 3 that there are two different ways to obtain the pay
factors for multiple quality characteristics. The first approach is using CMPWL (equation 22 or 23) to calculate the pay factor while the second method is to calculate the PF (equation 19 0r 20) for each individual quality characteristic and use equation 24 to determine the CPF for a lot. Both methods were used in the following analysis to calculate CPF and CMPF based on CMPWL.

The probability of rejection and PF in relation to various CMPWLs were also summarized in Table 4.5 and plotted in Figure 4.6. As shown in Table 4.5 the probability of rejection ( $\mathrm{p} 1, \mathrm{p} 2$, and p 3 ) for each PWL can be obtained using Monte Carlo Simulation as following:

$$
\text { Probability of rejection }=\frac{\text { Number of lots with } \mathrm{PF}=0}{\text { Total number of simulated lots }}
$$

If the three quality characteristics are assumed to be independent, the probability of rejecting at least one of these quality characteristics can be considered as the risk associated with multiple quality characteristics, and the binomial distribution (equation 25) can be used to determine this probability.

$$
\begin{equation*}
f(k, n, p)=\operatorname{Pr}(k ; n, p)=\operatorname{Pr}(X=k)=\binom{n}{k} p^{k}(1-p)^{n-k} \tag{25}
\end{equation*}
$$

Where

$$
\binom{n}{k}=\frac{n!}{k!(n-k)!}
$$

$n=$ number of trials
$p=$ the probability of success
$1-p=$ probability of failure
$k=k$ successes in n trials

It is assumed that the probabilities of rejection for strength, thickness, and roughness are $\mathrm{p} 1, \mathrm{p} 2$, and p 3 , respectively. Then the probabilities of acceptance can be calculated as 1-p1, 1-p2 and 1-p3. According to the binomial distribution theory, the probability of rejecting or replacing at least one of three independent quality characteristics can be calculated as

$$
\begin{equation*}
C P R=1-\left(1-p_{1}\right) \times\left(1-p_{2}\right) \times\left(1-p_{3}\right) \tag{26}
\end{equation*}
$$

Where
$\mathrm{CPR}=$ composite probability of rejection (the risk of rejection three independent quality characteristics including strength, thickness, and roughness)

P1 = probability of rejection for strength
P2 = probability of rejection for roughness
P3 = probability of rejection for thickness

Then CPR (composite probability of rejection) for each combination of PWL can be calculated by equation 26 . For example, with three quality characteristics, each at the AQL of $90 \%$ PWL producing a $90 \%$ CMPWL, the probability of rejection or replacement for each quality characteristic was determined to be $0.21 \%$. The composite probability of rejection with three quality characteristics (CPR) is calculated to be $0.63 \%$ (equation 27).

$$
\begin{equation*}
C P R=1-(1-0.21 \%) \times(1-0.21 \%) \times(1-0.21 \%)=0.63 \% \tag{27}
\end{equation*}
$$

if the concrete strength is at $60 \%$ PWL while thickness and roughness are at $100 \%$ PWL, the CMPWL also calculated to be $90 \%$ CMPWL, the probability of rejection was calculated to
be $29.5 \%$ for strength and $0 \%$ for both thickness and roughness. The composite probability of rejection with three quality characteristics (CPR) increases dramatically to $28.5 \%$ (equation 28 ).

$$
\begin{equation*}
C P R=1-(1-28.5 \%) \times(1-0) \times(1-0)=28.5 \% \tag{28}
\end{equation*}
$$

The composite probability of rejection with multiple quality characteristics (CPR) for a certain CMPWL can be different depending on the PWL of the individual quality characteristics. The CPRs corresponding to some specific combinations of PWL are summarized in TABLE 5.4. For a certain CMPWL, if all the three quality characteristics have the same PWL as CMPWL (i.e., $\mathrm{PWL}=90$ for strength, roughness and thickness producing $\mathrm{CMPWL}=90$ ), the risk can be minimized. However, the highest CPR is observed when the lowest achievable PWL is observed for one of the three individual quality characteristics. In this case, a CPR of $28.5 \%$ was observed since the PWL for strength equals to $60 \%$.

Table 4.5 Probability of rejection and pay factors for multiple quality characteristics with RQL of $50 \%$

| CMPWL | PWL, \% |  |  | Probability of rejection, \% |  |  | $\begin{aligned} & \text { CPR } \\ & \% \end{aligned}$ | $\begin{aligned} & \mathrm{CPF} \\ & \% \end{aligned}$ | $\begin{aligned} & \text { CMPF } \\ & \% \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Strength | Thickness | Roughness | Strength (p1) | Thickness (p2) | Roughness (p3) |  |  |  |
| 100 | 100 | 100 | 100 | 0 | 0 | 0 | 0 | 105 | 105 |
| 95 | 95 | 95 | 95 | 0.01 | 0.01 | 0.01 | 0.03 | 102.7 |  |
|  | 80 | 100 | 100 | 3.01 | 0 | 0 | 3.01 | 102.1 | 102.6 |
|  | 99 | 99 | 89 | 0 | 0 | 0.31 | 0.31 | 102.4 |  |
|  | 100 | 88 | 98 | 0 | 0.45 | 0 | 0.45 | 102.3 |  |
| 90 | 90 | 90 | 90 | 0.21 | 0.21 | 0.21 | 0.63 | 100.0 |  |
|  | 60 | 100 | 100 | 28.50 | 0 | 0 | 28.50 | 94.9 | 100.0 |
|  | 100 | 100 | 75 | 0 | 0 | 6.67 | 6.67 | 98.0 |  |
|  | 98 | 74 | 99 | 0 | 7.62 | 0 | 7.62 | 98.0 |  |
| 85 | 85 | 85 | 85 | 1.02 | 1.02 | 1.02 | 3.03 | 96.6 |  |
|  | 50 | 94 | 99 | 50 | 0.03 | 0 | 50.02 | 88.5 | 97.0 |
|  | 99 | 59 | 99 | 0 | 30.5 | 0 | 30.50 | 90.2 |  |
|  | 99 | 99 | 64 | 0 | 0 | 21.01 | 21.01 | 92.0 |  |
| 80 | 80 | 80 | 80 | 3.01 | 3.01 | 3.01 | 8.76 | 93.3 |  |
|  | 50 | 90 | 90 | 50 | 0.21 | 0.21 | 50.21 | 86.4 | 94.5 |
|  | 90 | 50 | 100 | 0.21 | 50 | 0 | 50.11 | 82.5 |  |
|  | 100 | 100 | 50 | 0 | 0 | 50 | 50.00 | 80.5 |  |
|  | 75 | 75 | 75 | 6.67 | 6.67 | 6.67 | 18.71 | 88.3 |  |


| 75 | 50 | 70 | 95 | 50 | 11.98 | 0 | 55.99 | 80.6 | 88.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 70 | 50 | 100 | 11.98 | 50 | 0 | 55.99 | 77.6 |  |
|  | 80 | 100 | 50 | 3.01 | 0 | 50 | 51.51 | 77.6 |  |
|  | 70 | 70 | 70 | 11.98 | 11.98 | 11.98 | 31.81 | 81.8 |  |
| 70 | 50 | 50 | 100 | 50 | 50 | 0 | 75.00 | 67.9 | 82.0 |
|  | 60 | 100 | 50 | 28.5 | 0 | 50 | 64.25 | 71.0 |  |
|  | 65 | 65 | 65 | 19.54 | 19.54 | 19.54 | 47.91 | 72.5 |  |
| 65 | 50 | 54 | 84 | 50 | 40.86 | 1.31 | 70.82 | 68.7 | 72.9 |
|  | 54 | 50 | 85 | 40.86 | 50 | 1.02 | 70.73 | 68.9 |  |
|  | 54 | 90 | 50 | 40.86 | 0.21 | 50 | 70.49 | 65.8 |  |
|  | 60 | 60 | 60 | 28.50 | 28.50 | 28.50 | 63.45 | 66.0 |  |
| 60 | 50 | 50 | 75 | 50 | 50 | 6.67 | 76.67 | 62.2 | 66.5 |
|  | 55 | 75 | 50 | 38.65 | 6.67 | 50 | 71.37 | 61.4 |  |
|  | 55 | 55 | 55 | 38.65 | 38.65 | 38.65 | 76.91 | 55.0 |  |
| 55 | 50 | 54 | 59 | 50 | 40.86 | 45.99 | 84.03 | 55.0 | 55.0 |
|  | 54 | 50 | 60 | 40.86 | 50 | 28.5 | 78.86 | 55.3 |  |
| 50 | 56 | 60 | 50 | 36.59 | 28.5 | 50 | 77.33 | 49.8 |  |

$* \mathrm{PF}=0$ if $\mathrm{PWL}<50 \%$, sample size $\mathrm{n}=5$

Figure 4.6 illustrates the range of the composite probability of rejection (CPR) for different CMPWLs. It can be seen from Figure 5.6 that for AQL of $90 \%$ CMPWL the CPR varies from $0.63 \%$ to $28.5 \%$ depending on the PWLs of the individual quality characteristics while, for an $80 \%$ CMPWL, the CPR ranges from $3.03 \%$ to $50 \%$. This clearly indicates that using multiple quality characteristics to determine the pay factor for the lot places much greater risks to the contractor than using a single characteristic.


Figure 4.6 Range of CPR for different CMPWLs ( $\mathrm{PF}=0$ if $\mathrm{PWL}<50 \%$ )

As the analysis in Chapter 3, the pay factors for multiple quality characteristics were calculated using two different methods and summarized in Table 4.5. The CPF is calculated by combining the PFs for the individual characteristics using equation 24 while the CMPF is calculated by using Equation 21 based on CMPWL. There is a unique CMPF corresponding to a CMPWL, while the values of CPF depend on the combinations of PWLs for the individual quality characteristics (Table 4.5).

Figure 4.7 shows the histograms of CMPF calculated based on CMPWL for a population with all three independent quality characteristics at their AQL of 90 PWL while Figure 4.8 shows the histograms of CPF for the same population. In this case, both the long-run average CMPF and CPF were determined to be $100 \%$. However, it can be observed that the variability of

CPF is much smaller than CMPF. The composite probability of rejection (risk of rejection) for this population is calculated to be $0.63 \%$.

Figure 4.9 shows the histograms of CMPF calculated based on CMPWL for a population with strength at 60 PWL while thickness and roughness at 100 PWL, while Figure 4.10 shows the histograms of CPF for the same population. Even though the two populations have the same CMPWL of 90, the distributions and long-run averages CMPF and CPF are different. It is shown in Figure 5.9 that the long-run average CMPF was calculated to be $100 \%$ for this population, however, the average CPF in the long run was estimated to be $94.9 \%$. This is because approximately $28.5 \%$ of simulated lots for strength have an estimated PWL less than $50 \%$ and these lots will be assigned a pay factor of 0 meaning that this 28 percent of lots were rejected. In this case, using CMPWL to calculate the pay for such a population places a greater risk to the contractor than using CPF (Equation 24).



Figure 4.7 PWL and CMPF with all three characteristics at an AQL of 90\% PWL (long-run average

$$
\mathrm{PF}=100 \%)
$$



Figure 4.8 PWL and CPF with all three characteristics at an AQL of 90\% PWL (long-run average $\mathrm{PF}=100 \%$ )


Figure 4.9 PWL and CMPF with strength at 60\% PWL while thickness and roughness at $100 \%$ PWL (long-run average $\mathrm{PF}=100 \%$ )


Figure 4.10 PWL and CPF with strength at $60 \%$ PWL while thickness and roughness at $100 \%$ PWL (long-run average $\mathrm{PF}=94.9 \%$ )

Figure 4.11 illustrates the relation of the composite probability of rejection and CPF while Figure 4.12 shows the relationship between the composite probability of rejection and CMPF. Overall, the CPF and CMPF decreases as the increase of the composite probability of rejection. For a CMPWL of 90 , the CPF varies from $94.9 \%$ to $100 \%$, however, the corresponding risks range $28.5 \%$ to $0.67 \%$. It also can be seen in Figure 4.12, for a PF of $100 \%$ calculated based on CMPWL, the risk can be as high as $28.5 \%$.


Figure 4.11 CPR and CPFs for various CMPWLs ( $\mathrm{PF}=0$ if $\mathrm{PWL}<50 \%$ )


Figure 4.12 The relation of CPR and CMPFs ( $\mathrm{PF}=0$ if $\mathrm{PWL}<50 \%$ )

Similarly, the CPR associated with CPF using the same pay equations with an RQL of $60 \%$ are summarized in Table 4.6 and plotted in Figure 4.13. It is shown in Table 4.6 that the risk for a 90 CMPWL as can be as low as $50 \%$ while the corresponding CPF is estimated to be $89.6 \%$ which is much lower than $100 \%$. This again means increasing RQL will place more risks on contractors as well as reduce the long-run average pay factors. Figure 5.13 illustrated the ranges of CPR for various CMPWL with an RQL of $60 \%$. Compared to Figure 4.13, the range of CPR for CMPWL of 90 becomes wider as the rejection or replace level increases from $50 \%$ to 60\%.

Table 4.6 Probability of rejection and pay factors for multiple quality characteristics with RQL of 60\%

| CMPWL | PWL, \% |  |  | Probability of rejection, \% |  |  | $\begin{aligned} & \text { CPR } \\ & \% \end{aligned}$ | $\begin{aligned} & \mathrm{CPF} \\ & \% \end{aligned}$ | $\begin{aligned} & \text { CMPF } \\ & \% \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Strength | Thickness | Roughness | Strength (p1) | Thickness (p2) | Roughness (p3) |  |  |  |
| 100 | 100 | 100 | 100 | 0 | 0 | 0 | 0 | 105.0 | 105.0 |
| 95 | 95 | 95 | 95 | 0.1 | 0.1 | 0.1 | 0.3 | 103.0 |  |
|  | 80 | 100 | 100 | 9.54 | 0 | 0 | 9.54 | 100.8 | 102.8 |
|  | 99 | 99 | 89 | 0 | 0 | 1.44 | 1.44 | 101.7 |  |
|  | 100 | 88 | 98 | 0 | 1.93 | 0 | 1.93 | 102.0 |  |
| 90 | 90 | 90 | 90 | 1.14 | 1.14 | 1.14 | 3.38 | 99.4 |  |
|  | 60 | 100 | 100 | 50 | 0 | 0 | 50.00 | 89.6 | 99.9 |
|  | 98 | 74 | 99 | 0 | 18.54 | 0 | 18.54 | 94.6 |  |
|  | 100 | 100 | 75 | 0 | 0 | 17.27 | 17.27 | 93.3 |  |
| 85 | 85 | 85 | 85 | 4.08 | 4.08 | 4.08 | 11.75 | 94.2 |  |
|  | 60 | 88 | 98 | 50 | 1.92 | 0 | 50.96 | 88.4 | 94.0 |
|  | 96 | 60 | 100 | 0.04 | 50 | 0 | 50.02 | 84.2 |  |
|  | 99 | 99 | 64 | 0 | 0 | 40.68 | 40.68 | 85.5 |  |
| 80 | 80 | 80 | 80 | 9.54 | 9.54 | 9.54 | 25.98 | 88.2 |  |
|  | 60 | 84 | 89 | 50 | 4.88 | 1.52 | 53.16 | 84.5 | 88.5 |
|  | 76 | 60 | 100 | 15.64 | 50 | 0 | 57.82 | 80.2 |  |
|  | 84 | 100 | 60 | 4.88 | 0 | 50 | 52.44 | 79.6 |  |
| 75 | 75 | 75 | 75 | 17.41 | 17.41 | 17.41 | 43.66 | 79.2 |  |
|  | 60 | 64 | 94 | 50 | 40.68 | 0.16 | 70.39 | 72.9 | 80.0 |
|  | 64 | 60 | 95 | 40.68 | 50 | 0.1 | 70.37 | 71.8 |  |
|  | 64 | 100 | 60 | 40.68 | 0 | 50 | 70.34 | 70.9 |  |
| 70 | 70 | 70 | 70 | 27.24 | 27.24 | 27.24 | 61.48 | 70.0 |  |
|  | 60 | 60 | 85 | 50 | 50 | 3.92 | 75.98 | 66.0 | 70.1 |
|  | 65 | 85 | 60 | 38.58 | 3.92 | 50 | 70.49 | 67.4 |  |
|  | 100 | 60 | 60 | 0 | 50 | 50 | 75.00 | 64.0 |  |
| 65 | 65 | 65 | 65 | 38.58 | 38.58 | 38.58 | 76.83 | 58.0 |  |
|  | 60 | 64 | 69 | 50 | 40.68 | 29.47 | 79.08 | 59.1 | 59.2 |
|  | 80 | 60 | 60 | 9.54 | 50 | 50 | 77.39 | 58.8 |  |
|  | 73 | 65 | 60 | 16.1 | 38.58 | 50 | 74.23 | 58.5 |  |
| 60 | 60 | 60 | 60 | 50 | 50 | 50 | 87.50 | 48.3 | 50.0 |

*PF=0 if PWL < $60 \%$, sample size $\mathrm{n}=5$


Figure 4.13 The relation of CPR and CMPFs ( $\mathrm{PF}=0$ if $\mathrm{PWL}<60 \%$ )


Figure 4.14 CPR and CPFs for various CMPWLs ( $\mathrm{PF}=0$ if $\mathrm{PWL}<60 \%$ )


Figure 4.15 Illustration of the relation of risks and CMPFs ( $\mathrm{PF}=0$ if $\mathrm{PWL}<60 \%$ )

Form the above analysis, it can be concluded that using multiple quality characteristics to calculate the pay factor a lot introduces a new source of risk that is not present when only one characteristic is applied. The use of multiple quality characteristics places much greater risks on contractors than using a single quality characteristic, and such risk increases as increasing the RQL.

### 4.3 Case Studies

Case studies were examined to illustrate how the above analysis can be used for contractors to assess and improve their production quality, and for agencies to formulate proper pay adjustment acceptance plans that provide the best insurance that the seller will be paid a fair price and that buyer will get what was paid with a reasonable level of risk. Figure 4.16 illustrates the procedure of performing risk and expected pay analysis. The simulation tool developed in this study is capable of determining $\alpha$ and $\beta$ associated in an acceptance plan using OC and EP curves. It is also able to determine average pay factors in the long run and risks of rejection for either an individual or multiple quality characteristics for a production.


Figure 4.16 Flow chart of risk and expected pay analysis

### 4.3.1 Effects of Production Population Characteristics

The first set of case studies consider contractors' production quality in relation to an acceptance plan.. In the acceptance plan, it has been specified that pay equation 19 is used to calculate the pay factors with a rejection or replacement $(\mathrm{PF}=0)$ if the PWL is less than $50 \%$. An AQL of $90 \%$ and a sample size of five are also specified in the acceptance plan. Then, population distributions representing the productions from three different contractors were analyzed and presented to demonstrate how risks and pay factors can be assessed for these population distributions. In the examples of Figure 4.17 contractor 1 and 2 have the same average production quality mean. Nevertheless, contractor 2 is able to produce more uniform quality (smaller variance). Contractor 3 has a higher average production quality mean but with a larger variance.

The PWL, PF, and the probability of rejection (risk) for each population distribution were determined through simulation analysis and summarized in Table 4.7. Figures 4.17 through 4.19 illustrate the population distributions in relation to the specs for strength, roughness and thickness. Population \#2 represents an average quality of production with a CMPWL of 84.61\%, and the long-run average expected pay based on CMPWL is $97.40 \%$. Population \#1 represents a better production quality with a lower variability (standard deviation $=2.76$ ) compared to population \#2 (standard deviation=4.14). The CMPWL and CPF for population \#2 are estimated to be $91.24 \%$ and $100.56 \%$ respectively. Population \#3 represents a construction quality with a large variability (standard deviation=5.52). However, in order to maximize incentive payments, contractor 3 has targeted a mean value far away from the specification limit. Even though contractor 3 can receive a CPF of $97.52 \%$ by targeting the mean far away from the specification
limit, the cost to reach the target mean value for contractor 3 will be much higher than that for contractor $1 \& 2$.

Table 5.6 also shows the risks of rejection for the three populations. The composite probability of rejection with three quality characteristics (CPR) is the lowest ( $0.43 \%$ ) for population \#1 because it represents the best production quality among the three populations. The composite probability of rejection (CPR) for populations $2 \& 3$ is estimated to be $4.5 \%$ and $4.3 \%$ respectively.

Table 4.7 Case studies showing risk and PF for different populations

| Quality characteristics | Contractor \# | Population distribution |  | $\begin{aligned} & \text { PWL } \\ & \% \end{aligned}$ | $\begin{aligned} & \mathrm{PF} \\ & \% \end{aligned}$ | Probability of rejection, \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | Standard deviation |  |  |  |
| Strength (MPa) | 1 | 31.0 | 2.76 | 89.72 | 99.75 | 0.23 |
|  | 2 | 31.0 | 4.14 | 80.00 | 92.54 | 3.00 |
|  | 3 | 34.5 | 5.52 | 89.71 | 99.60 | 0.23 |
| Thickness (mm) | 1 | 305 | 7.62 | 90.90 | 100.45 | 0.14 |
|  | 2 | 305 | 8.89 | 85.17 | 97.25 | 1.00 |
|  | 3 | 308 | 10.16 | 78.47 | 92.24 | 3.94 |
| Roughness ( $\mathrm{mm} / \mathrm{m}$ ) | 1 | 1.0 | 0.11 | 92.50 | 101.27 | 0.06 |
|  | 2 | 1.0 | 0.14 | 86.99 | 97.95 | 0.59 |
|  | 3 | 0.95 | 0.17 | 91.69 | 100.77 | 0.10 |
|  |  |  |  | CMPWL | CMPF | CPR |
| Composite quality level (CMPF) | 1 | N/A | N/A | 91.24 | 100.67 | 0.43 |
|  | 2 | N/A | N/A | 84.61 | 97.40 | 4.50 |
|  | 3 | N/A | N/A | 86.62 | 98.25 | 4.30 |
|  |  |  |  |  | CPF | CPR |
| Composite pay factor (CPF) | 1 | N/A | N/A | N/A | 100.56 | 0.43 |
|  | 2 | N/A | N/A | N/A | 96.35 | 4.50 |
|  | 3 | N/A | N/A | N/A | 97.52 | 4.30 |

*CPR: composite probability of rejection


Figure 4.17 Population distribution and spec for strength


Figure 4.18 Population distribution and specs for thickness


Figure 4.19 Population distribution and spec for roughness

### 4.3.2 Changing the Acceptance Plan Parameters

It is often that an agency may want to adjust the acceptance plans in order to balance the risks and PF. The following case studies were used to illustrate how modifying the acceptance plan parameters (i.e., Spec limits/Tolerances, sample size, rejection or replace provision and pay equations) would affect the risks and PF for a given quality level.

## Modifying Specification Limits

As shown in Figure 4.20 the specification limits were shifted to a lower quality level. The PWL, PF and risks were determined for the three populations and summarized in table 5.7. It can be seen in Table 4.8 that the PF can be increased while the risks can be reduced for all populations by moving spec limits away from the means. For example, by reducing the spec limit for strength for approximately $8 \%$, the risk for rejection for population \#2 can be reduced from $3 \%$ to $0.1 \%$. At the same time, the PF for strength was increased from $92.54 \%$ to $101.1 \%$.

However, the increases of PF for the population $1 \& 3$ are approximately $4 \%$ and $3 \%$
respectively. This indicates the changing specification limits will produce different levels of benefits or penalties to these contractors. As shown in Figure 4.20, the contractor 2 benefits the most since now more material is within the specification limits than before while contractor 3 benefits the least from reducing specification limit by $7 \%$ for concrete strength. Thus, how modifying specification limits would affect the PFs and risks for different contractors should be examined using simulation analysis based on the specification limits and contractor's population distributions.

Table 4.8 Effects of changing Specs on PF and risks

| Quality characteristic | Contractor \# | Population distribution |  | Spec | $\begin{aligned} & \text { PWL } \\ & \% \end{aligned}$ | $\begin{aligned} & \text { PF } \\ & \% \end{aligned}$ | Probability of rejection \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | Standard deviation |  |  |  |  |
| Strength <br> (MPa) | 1 | 31 | 2.76 | 27.6 | 89.72 | 99.75 | 0.23 |
|  |  |  |  | 25.5 | 97.89 | 103.98 | 0.00 |
|  | 2 | 31 | 4.14 | 27.6 | 80.00 | 92.54 | 3.00 |
|  |  |  |  | 25.5 | 91.25 | 101.10 | 0.10 |
|  | 3 | 34.5 | 5.52 | 27.6 | 89.71 | 99.60 | 0.23 |
|  |  |  |  | 25.5 | 95.03 | 102.48 | 0.01 |
| Thickness (mm) | 1 | 305 | 7.61 | $\pm 4.2 \%$ | 90.90 | 100.45 | 0.14 |
|  |  |  |  | + 5\% | 95.78 | 102.91 | 0.01 |
|  | 2 | 305 | 8.89 | $\pm 4.2 \%$ | 85.17 | 97.25 | 1.00 |
|  |  |  |  | + 5\% | 91.80 | 100.89 | 0.09 |
|  | 3 | 308 | 10.16 | $\pm 4.2 \%$ | 78.47 | 92.24 | 3.94 |
|  |  |  |  | + 5\% | 86.19 | 97.80 | 0.74 |
| Roughness <br> ( $\mathrm{mm} / \mathrm{m}$ ) | 1 | 1.0 | 0.11 | 1.184 | 92.50 | 101.27 | 0.06 |
|  |  |  |  | 1.26 | 98.52 | 104.23 | 0.00 |
|  | 2 | 1.0 | 0.14 | 1.184 | 86.99 | 97.95 | 0.59 |
|  |  |  |  | 1.26 | 95.34 | 102.65 | 0.01 |
|  | 3 | 1.0 | 0.17 | 1.184 | 91.82 | 100.83 | 0.10 |
|  |  |  |  | 1.26 | 96.74 | 103.35 | 0.00 |
|  |  |  |  |  | CMPWL | CMPF | CPR |
| Composite quality level | 1 | N/A |  |  | 91.24 | 100.67 | 0.40 |
|  |  |  |  |  | 97.97 | 103.7 | 0.00 |
|  | 2 | N/A |  |  | 84.61 | 97.40 | 4.50 |
|  |  |  |  |  | 93.07 | 101.54 | 0.20 |
|  | 3 | N/A |  |  | 86.62 | 98.25 | 4.30 |
|  |  |  |  |  | 92.61 | 101.31 | 0.75 |
|  |  |  |  |  |  | CPF | CPR |
| Composite pay factor (CPF) | 1 | N/A |  |  |  | 100.56 | 0.40 |
|  |  |  |  |  |  | 103.67 | 0.00 |
|  | 2 | N/A |  |  |  | 96.35 | 4.50 |
|  |  |  |  |  |  | 101.51 | 0.20 |


|  | 3 | N/A | 97.52 | 4.30 |
| :--- | :--- | :--- | :--- | :--- |
|  |  | 101.10 | 0.75 |  |



Figure 4.20 Effects of changing specification limits for roughness

## Changing Sample Size

In this analysis, two additional sample sizes ( $\mathrm{n}=2$ and $\mathrm{n}=4$ ) were considered to conduct simulation analysis for populations from all three contractors. The results were summarized in Table 4.9. It can be seen in Table 4.9 that increasing sample size from 3 to 5 has very little effects (the PWLs increase less than $1 \%$ ) on the average PWL for all populations. However, the risks were reduced significantly. For example, the risk of rejection associated with CPF for population \#2 was reduced from $11.2 \%$ to $4.54 \%$ by increasing sample size from 3 to 5 .

The effects of changing sample size on the long-run average PF is not significant when the population has a good quality level ( $\mathrm{PWL}>90$ ), while this effect became more significant as population quality (PWL) drops. As shown in Table 4.9, the average PF at the long run for
thickness of population \#1 increased slightly from $99.33 \%$ to $100.45 \%$ with increasing the sample size from three to five, whereas the PF for thickness of population \#3 increased significantly from $85.95 \%$ to $92.24 \%$. This is because increasing sample size reduces the variability of estimated PWL such that less simulated lots will be assigned a PF of 0 . In statistical terms increasing sample size also provides a better estimate of the "true" population characteristics.

Table 4.9 Effects of sample sizes on PF and risk

| Quality characteristic | Contractor \# | Population distribution |  | Sample size | $\begin{aligned} & \text { PWL } \\ & \% \end{aligned}$ | $\begin{aligned} & \mathrm{PF} \\ & \% \end{aligned}$ | $\begin{aligned} & \text { Risk } \\ & \% \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | Standard deviation |  |  |  |  |
| Strength | 1 | 31 | 2.76 | 3 | 89.36 | 98.63 | 1.54 |
|  |  |  |  | 4 | 89.71 | 99.31 | 0.57 |
|  |  |  |  | 5 | 89.72 | 99.75 | 0.23 |
|  | 2 | 31 | 4.14 | 3 | 80.00 | 89.71 | 7.26 |
|  |  |  |  | 4 | 80.05 | 91.61 | 4.60 |
|  |  |  |  | 5 | 80.11 | 92.54 | 3.00 |
|  | 3 | 34.5 | 5.52 | 3 | 89.51 | 98.70 | 1.48 |
|  |  |  |  | 4 | 89.64 | 99.46 | 0.59 |
|  |  |  |  | 5 | 89.71 | 99.60 | 0.23 |
| Thickness | 1 | 305 | 7.61 | 3 | 90.70 | 99.33 | 1.05 |
|  |  |  |  | 4 | 90.73 | 100.20 | 0.40 |
|  |  |  |  | 5 | 90.90 | 100.45 | 0.14 |
|  | 2 | 305 | 8.89 | 3 | 85.15 | 94.39 | 3.42 |
|  |  |  |  | 4 | 85.15 | 96.51 | 1.86 |
|  |  |  |  | 5 | 85.17 | 97.25 | 1.00 |
|  | 3 | 308 | 10.16 | 3 | 77.83 | 85.95 | 9.18 |
|  |  |  |  | 4 | 77.93 | 89.78 | 6.06 |
|  |  |  |  | 5 | 78.47 | 92.24 | 3.94 |
| Roughness | 1 | 1.0 | 0.11 | 3 | 92.64 | 100.95 | 0.60 |
|  |  |  |  | 4 | 92.32 | 100.96 | 0.22 |
|  |  |  |  | 5 | 92.50 | 101.27 | 0.06 |
|  | 2 | 1.0 | 0.14 | 3 | 86.82 | 96.45 | 2.59 |
|  |  |  |  | 4 | 86.87 | 97.36 | 1.25 |
|  |  |  |  | 5 | 86.99 | 97.95 | 0.59 |
|  | 3 | 1.0 | 0.17 | 3 | 91.71 | 100.11 | 0.81 |
|  |  |  |  | 4 | 91.44 | 100.49 | 0.32 |
|  |  |  |  | 5 | 91.82 | 100.83 | 0.10 |
|  |  |  |  |  | CMPWL | CMPF | CPR |
| Composite quality level (CMPWL) | 1 | N/A |  | 3 | 91.13 | 100.55 | 4.30 |
|  |  |  |  | 4 | 91.11 | 100.66 | 1.19 |
|  |  |  |  | 5 | 91.24 | 100.70 | 0.43 |
|  | 2 | N/A |  | 3 | 84.55 | 97.09 | 12.8 |
|  |  |  |  | 4 | 84.52 | 97.22 | 7.54 |
|  |  |  |  | 5 | 84.61 | 97.40 | 4.54 |


|  | 3 | N/A | 3 | 86.30 | 98.08 | 11.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 4 | 86.26 | 98.12 | 6.91 |
|  |  |  | 5 | 86.62 | 98.25 | 4.26 |
|  |  |  |  |  | CPF | CPR |
| Composite pay <br> factor <br> (CPF) | 1 | N/A | 3 | N/A | 99.68 | 4.30 |
|  |  |  | 4 |  | 100.33 | 1.19 |
|  |  |  | 5 |  | 100.56 | 0.43 |
|  | 2 | N/A | 3 | N/A | 94.06 | 11.2 |
|  |  |  | 4 |  | 95.68 | 7.54 |
|  |  |  | 5 |  | 96.35 | 4.54 |
|  | 3 | N/A | 3 | N/A | 97.52 | 11.2 |
|  |  |  | 4 |  | 96.25 | 6.91 |
|  |  |  | 5 |  | 94.89 | 4.26 |

*PF=55+0.5*PWL, $\mathrm{PF}=0$ if $\mathrm{PWL}<50$

However, it should be noted that increasing the sample size in QC implies a higher cost for QA/QC for any project.

## Changing the Rejection and Replacement Provisions

The effects of changing rejection or replacement provisions on the risks of rejection and PFs were also examined for the three cases of population distributions considered. The rejection or replacement provisions $(\mathrm{PF}=0)$ are usually triggered at a PWL value ( RQL ), such as $40 \%$ or $50 \%$. This PWL value (RQL) was changed from $40 \%$ to $60 \%$ Overall, it is shown in Table 4.10 that changing the RQL from $40 \%$ to $60 \%$ increase the risks associated with both single and multiple quality characteristics. For example, the CPR associated with CPF increases from $1.15 \%$ to $13.1 \%$ as changing the rejection or replace level from $40 \%$ to $60 \%$.

Overall, increasing the RQL from $40 \%$ to $60 \%$ reduces the average PF at the long run, and this effect became more significant as population quality became poorer. For example, the CPF for contractor \#1 (good quality) decreased from $100.36 \%$ to $97.43 \%$ while the CPF for contractor \#3 decreased dramatically from $97.17 \%$ to $89.63 \%$ (relatively poor quality) with increasing the RQL from $40 \%$ to $60 \%$. As shown in Table 4.10, the average PF at the long run for thickness population \#1 decreased slightly from $100.48 \%$ to $99.93 \%$ as the increase of the

RQL from $40 \%$ to $60 \%$, whereas the PF for thickness population \#3 decreased significantly from $93.57 \%$ to $84.45 \%$.

Table 4.10 Effects of rejection or replacement provision on risks and PF

| Quality characteristic | Contractor \# | Population distribution |  | $\begin{aligned} & \text { PF=0 if PWL } \\ & <(\%) \end{aligned}$ | $\begin{array}{\|l} \hline \text { PWL } \\ \% \end{array}$ | $\begin{aligned} & \hline \mathrm{PF} \\ & \% \end{aligned}$ | Probability of rejection \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | Standard deviation |  |  |  |  |
| Strength (MPa) | 1 | 31 | 2.76 | 40 | 89.82 | 99.88 | 0.03 |
|  |  |  |  | 50 | 89.72 | 99.75 | 0.23 |
|  |  |  |  | 60 | 89.67 | 98.79 | 1.20 |
|  | 2 | 31 | 4.14 | 40 | 80.11 | 94.41 | 0.69 |
|  |  |  |  | 50 | 80.11 | 92.54 | 3.00 |
|  |  |  |  | 60 | 80.47 | 88.13 | 8.80 |
|  | 3 | 34.5 | 5.52 | 40 | 89.94 | 99.94 | 0.03 |
|  |  |  |  | 50 | 89.71 | 99.60 | 0.23 |
|  |  |  |  | 60 | 89.74 | 98.85 | 1.20 |
| Thickness (mm) | 1 | 305 | 7.61 | 40 | 90.78 | 100.48 | 0.02 |
|  |  |  |  | 50 | 90.90 | 100.45 | 0.14 |
|  |  |  |  | 60 | 90.94 | 99.93 | 0.77 |
|  | 2 | 305 | 8.89 | 40 | 85.22 | 97.59 | 0.18 |
|  |  |  |  | 50 | 85.17 | 97.25 | 1.00 |
|  |  |  |  | 60 | 85.14 | 94.50 | 3.84 |
|  | 3 | 308 | 10.16 | 40 | 77.79 | 93.57 | 1.11 |
|  |  |  |  | 50 | 78.47 | 92.24 | 3.94 |
|  |  |  |  | 60 | 78.40 | 84.45 | 11.56 |
| Roughness (mm/m) | 1 | 1.0 | 0.11 | 40 | 92.63 | 101.31 | 0.01 |
|  |  |  |  | 50 | 92.50 | 101.27 | 0.06 |
|  |  |  |  | 60 | 92.76 | 101.02 | 0.36 |
|  | 2 | 1.0 | 0.14 | 40 | 87.18 | 98.52 | 0.09 |
|  |  |  |  | 50 | 86.99 | 97.95 | 0.59 |
|  |  |  |  | 60 | 87.20 | 96.29 | 2.39 |
|  | 3 | 1.0 | 0.17 | 40 | 91.71 | 100.84 | 0.01 |
|  |  |  |  | 50 | 91.82 | 100.83 | 0.10 |
|  |  |  |  | 60 | 91.63 | 100.29 | 0.59 |
|  |  |  |  |  | CMPWL | CMPF | CPR |
| Composite quality level | 1 | N/A |  | 40 | 91.28 | 100.64 | 0.06 |
|  |  |  |  | 50 | 91.24 | 100.70 | 0.43 |
|  |  |  |  | 60 | 91.35 | 100.67 | 2.31 |
|  | 2 | N/A |  | 40 | 84.72 | 97.40 | 0.96 |
|  |  |  |  | 50 | 84.61 | 97.40 | 4.54 |
|  |  |  |  | 60 | 84.78 | 97.16 | 15.3 |
|  | 3 | N/A |  | 40 | 86.40 | 98.20 | 1.15 |
|  |  |  |  | 50 | 86.62 | 98.25 | 4.26 |
|  |  |  |  | 60 | 86.52 | 98.18 | 13.1 |
|  |  |  |  |  |  | CPF | CPR |
|  | 1 | N/A |  | 40 | N/A | 100.36 | 0.06 |
|  |  |  |  | 50 |  | 100.56 | 0.43 |


| Composite pay factor |  |  | 60 |  | 97.43 | 2.31 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | N/A | 40 | N/A | 96.20 | 0.96 |
|  |  |  | 50 |  | 96.35 | 4.54 |
|  |  |  | 60 |  | 87.94 | 15.3 |
|  | 3 | N/A | 40 | N/A | 97.17 | 1.15 |
|  |  |  | 50 |  | 94.89 | 4.26 |
|  |  |  | 60 |  | 89.63 | 13.1 |

*Sample size $\mathrm{n}=5$

## The Effects of Pay Equations on Risks and PF

Two pay equations (i.e., A, B shown in Figure 4.21) were used to run the simulation analysis and calculate the PFs and relate those to the risks of rejection for the populations from the three contractors, and the results are summarized in Table 5.10. It can be seen that the average PFs in the long run produced by the two equations are almost the same for the strength population (PWL at AQL of $90 \%$ ) from contractor \#1. If the PWL of the population is less than $90 \%$, the PF equation B produces a smaller average PF at long run than PF equation A . For example, the PF calculated by PF equation A was $92.54 \%$ for strength population $(\mathrm{PWL}=80.11 \%)$ from contractor \#1, whereas the PF calculated by PF equation B was only $88.32 \%$. However, if a population has a PWL larger than AQL of $90 \%$, at long run the average PFs calculated by PF equation B will be larger than that calculated by PF equation A . For example, the PF for roughness population (PWL=91.82\%) from contractor \#3 was calculated to be $101.51 \%$ using PF equation A, while the PF was calculated to be $100.83 \%$ for the same population using PF equation $B$. This indicates that PF equation $B$ tends to provide more incentive payments if a contractor produces quality level higher than AQL of $90 \%$, while this equation produces more payment reduction if the population quality is less than AQL of $90 \%$. Changing the pay equation itself will not affect the risk associated with a quality level if all other parameters remain constant.


Figure 4.21 Illustration of pay equation A and B

Table 4.11 Effects of pay equations on risk and PF

| Quality characteristic | Contractor \# | Population distribution |  | Pay equation | $\begin{aligned} & \text { PWL } \\ & \% \end{aligned}$ | $\begin{aligned} & \mathrm{PF} \\ & \% \end{aligned}$ | $\begin{aligned} & \text { Risk } \\ & \% \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Mean | Standard deviation |  |  |  |  |
| Strength | 1 | 31 | 2.76 | A | 89.72 | 99.75 | 0.23 |
|  |  |  |  | B | 89.70 | 99.63 | 0.23 |
|  | 2 | 31 | 4.14 | A | 80.11 | 92.54 | 3.00 |
|  |  |  |  | B | 80.11 | 88.32 | 3.00 |
|  | 3 | 34.5 | 5.52 | A | 89.71 | 99.60 | 0.23 |
|  |  |  |  | B | 89.46 | 99.73 | 0.23 |
| Thickness | 1 | 305 | 7.61 | A | 90.90 | 100.45 | 0.14 |
|  |  |  |  | B | 90.84 | 100.82 | 0.14 |
|  | 2 | 305 | 8.89 | A | 85.17 | 97.25 | 1.00 |
|  |  |  |  | B | 85.03 | 94.79 | 1.00 |
|  | 3 | 308 | 10.16 | A | 78.47 | 92.24 | 3.94 |
|  |  |  |  | B | 77.83 | 86.19 | 3.94 |
| Roughness | 1 | 1.0 | 0.11 | A | 92.50 | 101.27 | 0.06 |
|  |  |  |  | B | 92.64 | 102.59 | 0.06 |
|  | 2 | 1.0 | 0.14 | A | 86.99 | 97.95 | 0.59 |
|  |  |  |  | B | 87.13 | 96.75 | 0.59 |
|  | 3 | 1.0 | 0.17 | A | 91.82 | 100.83 | 0.10 |
|  |  |  |  | B | 91.58 | 101.51 | 0.10 |
|  |  |  |  |  | CMPWL | CMPF | CPR |
| Composite quality level | 1 | N/A |  | A | 91.24 | 100.70 | 0.43 |
|  |  |  |  | B | 91.27 | 101.23 | 0.43 |
|  | 2 | N/A |  | A | 84.61 | 97.40 | 4.54 |
|  |  |  |  | B | 84.64 | 94.71 | 4.54 |


*Sample size n=5

## Chapter 5 Summary, Conclusions and Recommendations

### 5.1 Summary

In order to properly implement statistical acceptance specifications in the pavement construction, this thesis provides (1) an examination of how the key components of an acceptance plan affect its performance, (2) a detailed quantification of seller's and buyer's risks with Monte Carlo simulation and the development of OC curves, and, (3) an evaluation of the risks associated with a certain pay factor (i.e., single pay factor and composite pay factor).

In chapter 2, the statistical basis related to the statistical acceptance plan was presented to provide a good understanding of the associated statistics. The effects of the key components (i.e., AQL, RQL, and sample size) of a statistical acceptance plan on its performance were examined with the development of OC curves. The OC curves were developed for concrete strength, thickness, and roughness using typical population characteristics from the NCHRP 10-79 study with varying acceptance limits and sample sizes. Then, the seller's and buyer's risks were determined for these populations and compared to the risk levels recommended by AASHTO R9 (1997). The desired quality levels (AQL \& RQL) were modified to determine the risks to the contractor and the agency.

In the pay factor analysis, the OC and EP curves for the pay adjustment acceptance plan were developed by using Monte Carlo simulation in Matlab for both single and multiple quality characteristics. These OC curves along with EP curves were used to evaluate the risks and pay performance associated with the acceptance plans. Three different pay equations were applied to analyze how the pay equation affects the effectiveness of the acceptance plan. The RQL (i.e., $40 \%, 50 \%$, and $60 \%$ ) was modified to examine how it impacts the risks and pay factors to the contractor and agency. The effects of sample size on risks and pay factors were also analyzed.

The risks associated with composite pay factors were evaluated as well. Furthermore, a sensitivity analysis was conducted to determine how changing the standard deviation and mean of a single quality characteristic (i.e., strength, thickness and roughness) affect the average composite pay factors at the long run.

Additionally, the relationship between risk and pay factors were obtained for both single and multiple quality characteristics based on different sample sizes and RQL. The risk associated with a certain pay factor was evaluated. Using multiple quality characteristics to calculate the pay factor for a lot introduces a new source of risk that is not present when only one characteristic is applied. Such risks were calculated based on the binomial distribution theory. Simulation analysis was conducted to analyze how the different combinations of PWL affect the composite probability of rejection, CPR, and the composite pay factors. Finally, case studies were presented to demonstrate how the proposed analysis can be used by contractors to assess and improve their production quality, and for the agencies to formulate effective statistical acceptance plans with rational pay adjustment provisions.

### 5.2 Conclusions \& Recommendations

1. Proper selection in regards to key components of a statistical acceptance plan has a significant influence on its performance and associated risks. This analysis provides a perspective and guidance to SHAs on how to balance risks by modifying acceptance limits, sample sizes, and, AQL and RQL . As the acceptance limit increases, (i.e., from $50 \%$ to $60 \%$ ) the probability of rejecting material increases and thus the seller's risk increases. On the contrary, the buyer's risk decreases as the acceptance limit is increased. The seller's risk can be reduced by increasing the AQL while the buyer's risk can be reduced by decreasing the RQL . Both $\alpha$ and $\beta$ risks can be effectively reduced by increasing sample size. Thus, it is proposed that SHAs quantify the risks to the contractors and agencies associated in their current acceptance specifications through the analysis methods proposed in this study.
2. An acceptance limit of $65 \%$ and a sample size of seven are required for the populations examined in this study to attain the seller's risk of 0.01 at AQL of $90 \%$ and the buyer's risk of 0.05 at RQL of $50 \%$ as recommended by AASHTO R9 (1997). This sample size is larger than the typical sample size of five used by the majority of SHAs.
3. The pay factor analysis has shown that (i) increasing RQL will increase the probability of awarding lower pay to the contractors, as well as reduce the average pay factors at the long run; (ii) Increasing the sample size from five to fifteen has no significant impact on the average pay factor at the long run when an AQL of $90 \%$ is considered. However, this impact becomes more significant when the production quality (PWL) drops. As expected,
increasing sample size provides a better inference of the true population characteristics; (iii) the three alternative pay equations (i.e., continuous pay equations $19 \& 20$, and stepped pay equation) provided the same overall average pay factor (i.e., $100 \%$ pay) at the long run for a production with an AQL of $90 \%$ and when an RQL of $50 \%$ is considered with a sample size of five. The continuous pay equation 19 and the stepped pay one produces similar EP, while pay equation 20 provides relatively lower EP for quality levels lower than AQL of $90 \%$; (iv) Because of the higher weighting factor for roughness in the composite pay equation, the effects of modifying roughness population characteristics (i.e., mean and standard deviation) has a more significant impact on the composite pay factor than the remaining properties.
4. Overall a reduction in expected pay can be observed for both single and multiple quality characteristics as the probability of rejection increases. The risks associated with a certain pay factor can be obtained by building the relationship between the probability of rejection and PF .
5. Using multiple quality characteristics to determine the pay factor for a production lot places much greater risks to the contractor than using a single quality characteristic; the composite probability of rejection associated with three quality characteristics (i.e., strength, roughness and thickness) ranges from $0.65 \%$ to $28.5 \%$ for an AQL of $90 \%$ production depending on the PWL of the individual quality characteristic, and with an RQL of $50 \%$ and a sample size of five. Any SHA that uses multiple quality characteristics to determine the pay factor for a production lot should perform the
simulation analysis presented in this study to fully evaluate the risks and associated pay factors.
6. When calculating the risks associated with composite pay factors, this study considered the binomial distribution theory which assumes that the three quality characteristics are independent to each other. This is valid for the analysis presented herein since strength, thickness, and smoothness are the product of distinct construction operations for pavements. However, SHAs may choose acceptance quality parameters that may be correlated to each other (i.e., strength and modulus for example). In this case, the correlation coefficients should be determined and used in calculating the probability of acceptance and thus risks. Such composite probabilities and risks can be estimated based on the composite probability theory.
7. Based on the production case studies presented herein it can be concluded that for the high-quality level of production ( $\mathrm{PWL}>90 \%$ ), increasing the sample size from 3 to 5 has no significant influence on the average pay factor at the long run. On the opposite, sample size effects are more significant as the production quality drops. Similarly, the effects of RQL on the PF are related to the quality of the population. As expected, modifying the specification limits will also affect risk and rewards to contractors. Thus, the suggested approach and analysis should be performed based on the production quality characteristics observed by each agency in order to define rational and defensible specification limits, pay adjustment equations, and define $A Q L, ~ R Q L$ and lot sample sizes.

## Appendices

## MATLAB codes

## Development of OC curves

```
clear
close all
clc
n=1000; % number of lots
m=5; % sample size
for i=1:n
N=normrnd(5030,270,m,1);
MEAN=mean (N);
STDEV=std(N);
U=5000;
v(i)=(MEAN-U) . / STDEV;
w(i)}=(v(i)-0.255).*sqrt(m)
w_1(i)=(v(i)-0.525).*sqrt(m);
w_2(i)=(v(i)-0.842).*sqrt(m);
w_3(i)=(v(i)-1.281).*sqrt(m);
end
F=zeros(n,1);
P=zeros(n,1);
P_1=zeros(n,1);
P_2=zeros(n,1);
P_3=\operatorname{zeros}(n,1);
for j=1:n
fun=@(x) (1./sqrt(2.*pi))*exp(-x.^2./2);
F(j)=integral(fun,-inf,v(j));
P(j)=integral(fun,-inf,w(j));
P_1(j)=integral(fun,_inf,w_1(j));
P_2(j)=integral(fun, -inf,w_2(j));
P_3(j)=integral(fun,_inf,w_3(j));
end
x=sort(F);
y=sort(P);
y1=sort(P_1);
y2=sort(P_2);
y3=sort(P_3);
figure(1)
plot(x,y,'--r','Linewidth',1.5);
grid on
hold on
plot(x,y1,'-g','linewidth',1.5);
plot(x,y2,':b','linewidth',1.5);
```

```
plot(x,y3,'-.c','linewidth',1.5);
grid on
lgd=legend('60','70','80','90');
title(lgd,'Acceptance limit');
xlabel('PWL');
ylabel('Probability of Acceptance');
xlim([0 1])
ylim([0 1])
% Convert y-axis values to percentage values by multiplication
a=[cellstr(num2str(get(gca,'xtick')'*100))];
b}=[cellstr(num2str(get(gca,'ytick')'*100))]
% Create a vector of '%' signs
pct = char(ones(size(a,1),1)*'%');
% Append the '%' signs after the percentage values
new xticks = [char(a),pct];
new_yticks = [char(b),pct];
% 'Reflect the changes on the plot
set(gca,'xticklabel',new_xticks)
set(gca,'yticklabel',new_yticks)
set ( gca, 'xdir', 'reverse' )
```


## OC curves based on Villier's Approach

```
clear
close all
clc
n=1000; % number of lots
mu=11.588;
sigma=0.9;
SE=zeros(1,5);
Pa_roughness=zeros(n,5);
N_roughness=normrnd(mu,20,n,1);
PWL_roughness=cdf('normal',7.0,N_roughness,sigma); % 7.0 is the
usl
SE(1,1)=sigma/sqrt(4); % standard error with sample size
SE (1,2)=sigma/sqrt(5);
SE (1,3)=sigma/sqrt (6);
SE (1,4)=sigma/sqrt(7);
SE (1,5)=sigma/sqrt(8);
for i=1:5
    Pa_roughness(:,i)=cdf('normal',7.0,N_roughness,SE(1,i));
end
x=sort(PWL_roughness);
```

```
yl=sort(Pa_roughness(:,1));
y2=sort(Pa_roughness(:,2));
y3=sort(Pa_roughness(:,3));
y4=sort(Pa roughness(:,4));
y5=sort(Pa_roughness(:,5));
figure(1)
plot(x,y1,'--r','Linewidth', 2) ;
grid on
hold on
plot(x,y2,'-g','linewidth',1.5);
plot(x,y3,':b','linewidth',1.5);
plot(x,y4,'-.c','linewidth',1.5);
plot(x,y5,'-m','linewidth',1.5);
set( gca, 'xdir', 'reverse' );
legend('n=4','n=5','n=6','n=7','n=8')
xlabel('PWL');
ylabel('Probability of Acceptance');
```


## Pay Factor simulation analysis

```
clear
close all
clc
% import PWL table
filename1 = 'mydata A.xlsx';
sheet = 1;
xlRange = 'A1:P51';
subsetA=xlsread(filename1, sheet, xlRange);
n=10000; % number of lots
m=4; % number of sample sizes
QL=zeros(n, 2);
QU=zeros (n,2);
PL=zeros (n, 2);
PU=zeros (n, 2);
PWL=zeros(n,3);
PF=zeros(n,3);
CPF=zeros(n,1);
CPF1=zeros(n,1);
CMPWL=zeros(n,1);
% population characteristics (mean and std)
mu=[[3935 12.38 67];
sigma=[500 0.49 9.5];
% specification limits
USL=[75,12.8];
LSL=[3500,11.2];
for k=1:n
```

```
    N_strength=normrnd(mu(1,1),sigma(1,1),m+1,1);
    N_thickness=normrnd(mu (1,2),sigma (1, 2),m+1,1);
    N_roughness=normrnd (mu (1, 3), sigma (1, 3),m+1,1);
    N=[N_strength,N_thickness,N_roughness];
    X = \overline{N};
    MEAN=mean(X);
    STDEV=std(X);
    QU (k,1) =chop ((USL (1, 1) -MEAN (1, 3))./STDEV (1, 3) , 3) ;
    QU (k,2)=chop ((USL (1, 2) -MEAN (1, 2))./STDEV (1, 2), 3) ;
    QL (k,1) = chop ((MEAN (1,1) - LSL (1, 1))./STDEV (1, 1) , 3);
    QL (k, 2) = chop ((MEAN (1, 2) - LSL (1, 2) )./STDEV (1, 2) , 3);
for j=1:2
    for i=1:50
        if (QL(k,j)==subsetA(i,m))
        PL(k,j)=subsetA(i,1);
        end
        if ((QL(k,j)>subsetA(i+1,m)) && (QL(k,j)<subsetA(i,m)))
        PL(k,j)=subsetA(i, 1);
        end
        if (QL(k,j)>subsetA(1,m))
        PL (k,j)=100;
        end
        if (-QL(k,j)==subsetA(i,m))
        PL(k,j)=100-subsetA(i,1);
        end
        if ((-QL(k,j)>subsetA(i+1,m)) && (-QL(k,j)<subsetA(i,m)))
        PL(k,j)=100-subsetA(i,1);
        end
        if (-QL(k,j)>subsetA (1,m))
        PL (k,j) =0;
        end
        if (QU(k,j)==subsetA(i,m))
        PU(k,j)=subsetA(i, 1);
        end
        if ((QU(k,j)>subsetA(i+1,m)) && (QU(k,j)<subsetA(i,m)))
        PU(k,j)=subsetA(i,1);
        end
        if (QU(k,j)>subsetA(1,m))
        PU(k,j)=100;
        end
        if (-QU(k,j)==subsetA(i,m))
        PU(k,j)=100-subsetA(i,1);
        end
        if ((-QU(k,j)>subsetA(i+1,m)) && (-QU(k,j)<subsetA(i,m)))
        PU(k,j)=100-subsetA(i,1);
        end
```

```
        if (-QU(k,j)>subsetA (1,m))
        PU(k,j)=0;
        end
        end
    end
        PWL (k,1)=PL (k,1);
        PWL (k,2)=PU(k,1);
        PWL (k, 3)=PU(k, 2) +PL (k, 2) -100;
        CMPWL (k,1)=0.25*PWL (k,1)+0.35*PWL (k, 2) +0.4*PWL (k, 3);
    if (CMPWL(k,1)<=100 && CMPWL(k,1)>=50)
        CPF1 (k,1)=55+0.5* CMPWL (k,1);
    end
    if (CMPWL (k,1)<50)
        CPF1(k,1)=0;
    end
    if (PWL(k,1)<=100 && PWL (k,1)>=50)
        PF (k,1)=55+0.5*PWL (k,1);
    end
    if (PWL(k,1)<50)
        PF}(k,1)=0
    end
    if (PWL (k,2)<=100 && PWL (k,2)>=50)
        PF (k,2)=55+0.5*PWL (k,2);
    end
    if (PWL(k,2)<50)
        PF (k,2)=0;
    end
    if (PWL(k,3)<=100 && PWL (k,3)>=50)
        PF (k,3)=55+0.5* PWL (k,3);
    end
    if (PWL(k,3)<50)
        PF (k,3)=0;
    end
    CPF}(k,1)=0.25*PF(k,1)+0.35*PF(k,2)+0.4*PF(k,3)
end
MEAN_CMPWL=mean(CMPWL);
MEAN_PWL=mean(PWL);
MEAN_PF=mean(PF);
```

```
MEAN_CPF=mean(CPF);
PF_9\overline{5}=prctile(CPF,95);
PF_5=prctile(CPF,5);
figure(1)
subplot(2,2,1)
histogram(PWL(:,1),20)
xlabel('PWL')
ylabel('Number of lots')
title('Strength')
subplot(2,2,2)
histogram(PWL(:,2),20)
xlabel('PWL')
ylabel('Number of lots')
title('Roughness')
subplot(2,2,3)
histogram(PWL(:,3),20)
xlabel('PWL')
ylabel('Number of lots')
title('Thickness')
subplot(2,2,4)
histogram(CMPWL,20,'FaceColor','r')
xlabel('CMPWL')
ylabel('Number of lots')
title('Composite')
figure(2)
histogram(CPF,30,'FaceColor','r')
title('Composite PF')
xlabel('CPF')
ylabel('Number of lots receiving a given pay fator')
figure(3)
histogram(CPF1,30,'FaceColor','r')
title('Composite PF')
xlabel('PF based on CMPWL')
ylabel('Number of lots receiving a given pay fator')
% probability of receving PF > 0.7, 0.8, 0.9, 1.0 and 1.04
PF70=sum(histc(CPF,70:0.1:105))/n*100;
PF80=sum(histc(CPF,80:0.1:105))/n*100;
PF90=sum(histc(CPF,90:0.1:105))/n*100;
PF100=sum(histc(CPF,100:0.1:105))/n*100;
PF104=sum(histc(CPF,104:0.1:105))/n*100;
PF_average=[PF75,PF80,PF90,PF100,PF104];
```


## Different combinations of PWLs

```
clear
close all
clc
CMPWL= [];
count=1;
for ii=50:100
    for jj=50:100
        for kk=50:100
            if (0.25*ii+0.35*jj+0.4*kk == 90)
                CMPWL(count,:)=[ii,jj,kk];
                count=count+1;
            end
        end
    end
end
save('CMPWL.mat', 'CMPWL');
load CMPWL.mat
filename2 = 'CMPWL.mat';
x1 = min(CMPWL(:, 1));
x2 = min(CMPWL(:, 2));
x3 = min(CMPWL(:, 3));
a=CMPWL (CMPWL (:, 1) ==x1, :) ;
b=CMPWL (CMPWL (:, 2)==x2, :);
c=CMPWL (CMPWL (:, 3)==x3,:);
```


## PWL estimation table

| PWL | $n=3$ | $n=4$ | $n=5$ | $n=6$ | $n=7$ | $n=8$ | $n=9$ | $\begin{gathered} n=10 \\ \text { to } 11 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 1.16 | 1.50 | 1.79 | 2.03 | 2.23 | 2.39 | 2.53 | 2.65 |
| 99 | - | 1.47 | 1.67 | 1.80 | 1.89 | 1.95 | 2.00 | 2.04 |
| 98 | 1.15 | 1.44 | 1.60 | 1.70 | 1.76 | 1.81 | 1.84 | 1.86 |
| 97 | - | 1.41 | 1.54 | 1.62 | 1.67 | 1.70 | 1.72 | 1.74 |
| 96 | 1.14 | 1.38 | 1.49 | 1.55 | 1.59 | 1.61 | 1.63 | 1.65 |
| 95 | - | 1.35 | 1.44 | 1.49 | 1.52 | 1.54 | 1.55 | 1.56 |
| 94 | 1.13 | 1.32 | 1.39 | 1.43 | 1.46 | 1.47 | 1.48 | 1.49 |
| 93 | - | 1.29 | 1.35 | 1.38 | 1.40 | 1.41 | 1.42 | 1.43 |
| 92 | 1.12 | 1.26 | 1.31 | 1.33 | 1.35 | 1.36 | 1.36 | 1.37 |
| 91 | 1.11 | 1.23 | 1.27 | 1.29 | 1.30 | 1.30 | 1.31 | 1.31 |
| 90 | 1.10 | 1.20 | 1.23 | 1.24 | 1.25 | 1.25 | 1.26 | 1.26 |
| 89 | 1.09 | 1.17 | 1.19 | 1.20 | 1.20 | 1.21 | 1.21 | 1.21 |
| 88 | 1.07 | 1.14 | 1.15 | 1.16 | 1.16 | 1.16 | 1.16 | 1.17 |
| 87 | 1.06 | 1.11 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 | 1.12 |
| 86 | 1.04 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 | 1.08 |
| 85 | 1.03 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 |
| 84 | 1.01 | 1.02 | 1.01 | 1.01 | 1.00 | 1.00 | 1.00 | 1.00 |
| 83 | 1.00 | 0.99 | 0.98 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 |
| 82 | 0.97 | 0.96 | 0.95 | 0.94 | 0.93 | 0.93 | 0.93 | 0.92 |
| 81 | 0.96 | 0.93 | 0.91 | 0.90 | 0.90 | 0.89 | 0.89 | 0.89 |
| 80 | 0.93 | 0.90 | 0.88 | 0.87 | 0.86 | 0.86 | 0.86 | 0.85 |
| 79 | 0.91 | 0.87 | 0.85 | 0.84 | 0.83 | 0.82 | 0.82 | 0.82 |
| 78 | 0.89 | 0.84 | 0.82 | 0.80 | 0.80 | 0.79 | 0.79 | 0.79 |
| 77 | 0.87 | 0.81 | 0.78 | 0.77 | 0.76 | 0.76 | 0.76 | 0.75 |
| 76 | 0.84 | 0.78 | 0.75 | 0.74 | 0.73 | 0.73 | 0.72 | 0.72 |
| 75 | 0.82 | 0.75 | 0.72 | 0.71 | 0.70 | 0.70 | 0.69 | 0.69 |
| 74 | 0.79 | 0.72 | 0.69 | 0.68 | 0.67 | 0.66 | 0.66 | 0.66 |
| 73 | 0.76 | 0.69 | 0.66 | 0.65 | 0.64 | 0.63 | 0.63 | 0.63 |
| 72 | 0.74 | 0.66 | 0.63 | 0.62 | 0.61 | 0.60 | 0.60 | 0.60 |
| 71 | 0.71 | 0.63 | 0.60 | 0.59 | 0.58 | 0.57 | 0.57 | 0.57 |
| 70 | 0.68 | 0.60 | 0.57 | 0.56 | 0.55 | 0.55 | 0.54 | 0.54 |
| 69 | 0.65 | 0.57 | 0.54 | 0.53 | 0.52 | 0.52 | 0.51 | 0.51 |
| 68 | 0.62 | 0.54 | 0.51 | 0.50 | 0.49 | 0.49 | 0.48 | 0.48 |
| 67 | 0.59 | 0.51 | 0.47 | 0.47 | 0.46 | 0.46 | 0.46 | 0.45 |
| 66 | 0.56 | 0.48 | 0.45 | 0.44 | 0.44 | 0.43 | 0.43 | 0.43 |
| 65 | 0.52 | 0.45 | 0.43 | 0.41 | 0.41 | 0.40 | 0.40 | 0.40 |
| 64 | 0.49 | 0.42 | 0.40 | 0.39 | 0.38 | 0.38 | 0.37 | 0.37 |
| 63 | 0.46 | 0.39 | 0.37 | 0.36 | 0.35 | 0.35 | 0.35 | 0.34 |
| 62 | 0.43 | 0.36 | 0.34 | 0.33 | 0.32 | 0.32 | 0.32 | 0.32 |
| 61 | 0.39 | 0.33 | 0.31 | 0.30 | 0.30 | 0.29 | 0.29 | 0.29 |
| 60 | 0.36 | 0.30 | 0.28 | 0.27 | 0.27 | 0.27 | 0.26 | 0.26 |
| 59 | 0.32 | 0.27 | 0.25 | 0.25 | 0.24 | 0.24 | 0.24 | 0.24 |
| 58 | 0.29 | 0.24 | 0.23 | 0.22 | 0.21 | 0.21 | 0.21 | 0.21 |
| 57 | 0.25 | 0.21 | 0.20 | 0.19 | 0.19 | 0.19 | 0.18 | 0.18 |
| 56 | 0.22 | 0.18 | 0.17 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |
| 55 | 0.18 | 0.15 | 0.14 | 0.14 | 0.13 | 0.13 | 0.13 | 0.13 |
| 54 | 0.14 | 0.12 | 0.11 | 0.11 | 0.11 | 0.11 | 0.10 | 0.10 |
| 53 | 0.11 | 0.09 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 | 0.08 |
| 52 | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| 51 | 0.04 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 |
| 50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

Source: Specification Conformity Analysis, FWHA Technical Advisory T5080.12, June 231989

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[^0]:    *10000 simulated lots. $\mathrm{PF}=0$ if $\mathrm{PWL}<50$

