

ABSTRACT

Title of Dissertation: **EFFECTS OF VIBRATIONAL NONEQUILIBRIUM
ON THE ACOUSTIC NOISE RADIATED
BY A COMPRESSIBLE BOUNDARY LAYER**

Graeme I. Gillespie
Doctor of Philosophy, 2023

Dissertation Directed by: **Professor Stuart Laurence
Department of Aerospace Engineering**

Turbulence-generated acoustic noise is of critical concern in the nozzle flows of conventional high-speed wind tunnels, where the disturbance environment encountered by models in the freestream is substantially stronger than that experienced in atmospheric flight and leads to much reduced transition Reynolds numbers. To obtain more accurate comparisons of experimental, computational, and free-flight data, a new control mechanism is needed to reduce freestream disturbance levels. Therefore, the aim of the present work is to investigate the ability of vibrational nonequilibrium processes to attenuate acoustic radiation emitted by turbulent boundary layers in high-speed facilities.

Predicting the attenuation from vibrational nonequilibrium processes remains a challenge, and there exist limited experimental data for model validation, particularly at elevated temperatures. To better understand the absorption properties of various gas mixtures, a heated acoustic chamber is developed to measure the attenuation of CO_2 , N_2O , and mixtures of CO_2/He , CO_2/N_2 ,

and $\text{N}_2\text{O}/\text{He}$ at temperatures up to 529 K. In mixtures of CO_2/He at room temperature, an increase in helium is found to decrease the peak attenuation modestly, but increase the peak attenuation frequency. At higher temperatures, the peak attenuation increased substantially, but as the helium fraction increased, the rate of increase in peak attenuation drops and the values asymptote at lower temperatures. These results illustrate that varying the fraction of helium in mixtures of CO_2/He can shift the attenuation to a desired frequency range, providing a method to control acoustic radiation.

The effects of vibrational nonequilibrium processes on turbulence-generated acoustic noise are investigated in a Mach-2.8 shock-tunnel facility at the University of Maryland. CO_2 , N_2 , He , and He/CO_2 mixtures are injected into the lower boundary layer of the flow through a porous plate located in the upstream region of the test section. A four-point Focused Laser Differential Interferometer (FLDI) positioned above the turbulent boundary layer is used to obtain freestream fluctuation measurements assumed to be representative of entropic fluctuations propagating along streamlines and acoustic disturbances along Mach lines. Compared to a boundary layer of pure air, the injection of 30%, 35%, and 40% He/CO_2 mixtures resulted in reduced fluctuation powers correlated along a Mach line in the frequency range of 200 – 800 kHz. Minimal reductions in fluctuation power were measured along corresponding streamlines; therefore, it could be concluded that the vibrationally active gas species in the boundary layer primarily affected acoustic radiation and not entropic disturbances.

As measurements are affected by noise radiated from the boundary layers on all four walls of the facility, a mathematical disturbance model is created to examine the sensitivity of the measured attenuation to acoustic disturbances propagating from the lower boundary layer only. Disturbances are modeled as Gaussian wave packets propagating along Mach lines from the four test

section walls and along streamlines. Modeling the acoustic disturbances from the lower boundary layer with a 15 – 30% amplitude reduction resulted in amplitude spectral densities and cross power spectral densities that agreed well with the FLDI measurements. Thus, the injection of a vibrationally active gas into a turbulent boundary layer has the potential to significantly reduce acoustic-disturbance amplitudes in the freestream, greatly expanding the utility of conventional high-speed facilities to study flows in which transition plays an important role.

EFFECTS OF VIBRATIONAL NONEQUILIBRIUM ON THE
ACOUSTIC NOISE RADIATED BY A COMPRESSIBLE BOUNDARY
LAYER

by

Graeme I. Gillespie

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2023

Advisory Committee:
Professor Stuart Laurence, Chair/Advisor
Professor Kenneth Yu
Professor Christopher Cadou
Professor James Baeder
Professor Johan Larsson

© Copyright by
Graeme I. Gillespie
2023

Acknowledgments

I would first like to thank my advisor, Professor Stuart Laurence, for providing me with the opportunity to conduct research in the fascinating field of hypersonics. I greatly appreciate his invaluable guidance and advice throughout my time at UMD. This body of work would not have been possible without his unwavering support and patience. I would also like to thank Professors Kenneth Yu, Christopher Cadou, James Baeder, and Johan Larsson for their insightful advice and perspective.

Thank you to all my colleagues in the High-Speed Aerodynamics and Propulsion Lab, past and present. These experiments are the product of their tremendous support and selfless assistance in the lab. I feel very fortunate to have had the opportunity to work with you. I am also tremendously grateful to Bruce Rowley and Jack Touart for their work in the construction of the test articles used throughout this project.

To my friends back in Canada, thank you for the visits, texts, phone calls, and fantasy football leagues. Your friendship always made me feel close to home.

None of this would have been possible without my incredible family. To my parents, Ian and Dahlia, there are not enough words to express how grateful I am for the lifetime of encouragement and loving support you have provided me. Finally, thank you to Abbey for being by my side and making this journey so memorable.

This project was funded by the National Science Foundation (Program Manager Ronald Joslin) under grant number 1751965.

Table of Contents

Acknowledgements	ii
Table of Contents	iii
List of Tables	v
List of Figures	vi
Nomenclature	x
Chapter 1: Introduction	1
1.1 Motivation	1
1.2 Noise in High-Speed Facilities	2
1.3 Attenuation of Acoustic Disturbances	8
1.4 Focused Laser Differential Interferometry	13
1.5 Scope of Current Work	16
Chapter 2: Acoustic Attenuation in a Vibrationally Active Gas: Theory and Measurements	19
2.1 Acoustic Attenuation Model	19
2.1.1 Vibrational Relaxation	20
2.1.1.1 Transition Probabilities	20
2.1.1.2 Relaxational Absorption	24
2.1.1.3 Dependence on Frequency and Pressure	28
2.1.2 Classical and Diffusional Attenuation	29
2.2 Heated Acoustic Chamber	31
2.2.1 Acoustic Chamber Design	31
2.2.2 Data Analysis	35
2.3 Attenuation Coefficient Measurements	38
2.3.1 Carbon Dioxide and Nitrogen	39
2.3.2 Carbon Dioxide and Helium	40
2.3.3 Nitrous Oxide and Helium	42
2.3.4 Summary	43
Chapter 3: Attenuation of Acoustic Radiation in a High-Speed Facility	45
3.1 Shock-Tunnel Facility	45
3.2 Nozzle Characterization	50

3.3	Injection Conditions	51
3.4	Schlieren	57
3.5	FLDI Setup	59
3.5.1	Four-Point FLDI	59
3.5.2	FLDI Beam Measurements	61
3.6	Analysis of FLDI Data	65
3.6.1	Instrument Model	65
3.6.2	Depth of Focus	70
3.6.3	Convection Velocity	73
3.7	Measurements of Attenuated Acoustic Radiation	75
3.7.1	Time Trace	75
3.7.2	Convection Velocity	76
3.7.3	Fluctuation Spectra	77
3.7.4	Comparison Between Acoustic Chamber and FLDI Results	85
Chapter 4:	Disturbance Model	91
4.1	Framework	91
4.2	Results	95
Chapter 5:	Conclusion	100
5.1	Summary	100
5.2	Contributions	104
5.3	Future Work	106
Appendix A:	Multi-Point FLDI in a Mach-6 Freestream	109
A.1	Facility and Test Conditions	109
A.2	FLDI Instrument	110
A.2.1	FLDI Setup	110
A.2.2	Influence of Nozzle Shear Layers	112
A.2.3	Disturbance Velocity Analysis	114
A.3	Results and Discussion	116
A.3.1	Disturbance Velocity	117
A.3.2	Density-Based Turbulence Intensity	118
A.3.3	Acoustic Noise Contribution	122
A.4	Summary	126
Appendix B:	Acoustic Reflection in Helium	128
B.1	General Acoustic Transmission	128
B.2	Density and Sound-Speed Profiles	130
B.3	Reflection Coefficient	134
B.4	Results	137
Bibliography		142

List of Tables

2.1	The vibrational modes (ν), degeneracies (g), steric factors (P_ν), Lennard-Jones potential depth (ϵ) (Hirschfelder et al., 1964), collisional diameter (r_0) (Hirschfelder et al., 1964), and vibrational amplitude coefficients ($\bar{A}^2$) (Lambert and Lambert, 1977) for gases used in the vibrational relaxation model of Dain and Lueptow (2001) and Zhang et al. (2013).	27
3.1	HyperTERP test conditions.	50
3.2	Injection conditions.	53
3.3	Relative mass flow rate through the porous plate for each injection condition.	57
3.4	Mean streamwise convection velocities for each injection condition.	78
A.1	Mach-6 HyperTERP test conditions.	110
A.2	Estimate of shear-layer thickness for each condition.	114
A.3	Mean convection velocities measured with FLDI signal pair 1-3.	118

List of Figures

1.1	Freestream disturbances in high-speed wind tunnels from Schneider (2008). . . .	4
1.2	Plane acoustic waves radiated by eddy sources of a turbulent boundary layer from Schilden and Schröder (2019).	6
1.3	Numerical schlieren image in a streamwise wall-normal plane of a double wall Mach-5.86 configuration from Duan et al. (2015).	7
1.4	Attenuation per wavelength for mixtures of CO ₂ and CH ₄ with air (symbols: □, 92 kHz; ○, 149 kHz; △, 215 kHz; ◇, 1 MHz) from Petculescu et al. (2006). Curves represent model predictions.	11
1.5	Effect of CO ₂ mole fraction in the test gas on transition Reynolds number for flows over a 5 degree half-angle cone from Leyva et al. (2009).	13
2.1	Lennard-Jones potential function.	24
2.2	Cross section schematic of heated acoustic chamber for attenuation measurements.	33
2.3	Photograph of acoustic chamber with details highlighted.	33
2.4	Direction of electronic signal used in the operation of the acoustic chamber. . . .	34
2.5	Transmitted pulse consisting of 8 cycles of a sine wave at 90 kHz (black), and the resulting output signal (blue). The initial output response (red) is the signal used in the data analysis.	36
2.6	Power of the output signal at 90 kHz as a function of distance between the rods, normalized by wavelength, for CO ₂ at 1 atm and 293 K.	37
2.7	Natural logarithm of the normalized output signal amplitude at 90 kHz as a function of distance between the rods, normalized by wavelength, for CO ₂ at 293 K and various pressures. For each pressure, a line of best fit is shown in red.	37
2.8	Attenuation spectra of CO ₂ and mixtures of CO ₂ with 20%, 40% and 60% N ₂ at 293 K, plotted with the theoretical attenuation model (dashed lines).	40
2.9	Attenuation spectra of CO ₂ and mixtures of CO ₂ with 10%, 20%, and 40% He at 293 K. The theoretical attenuation model for CO ₂ at 293 K is plotted as a dashed line.	41
2.10	Attenuation spectra of CO ₂ and mixtures of CO ₂ with 20% and 40% He at 293-529 K. The theoretical attenuation model for CO ₂ at 293 K is plotted as a dashed line.	42
2.11	Attenuation spectra of N ₂ O and mixtures of N ₂ O with 10%, 25% and 40% He at 293 K, plotted with the theoretical attenuation model for N ₂ O (dashed line). . . .	43

3.1	Schematic of HyperTERP shock tunnel with direct-connect test section.	46
3.2	Schematic of the nozzle and direct-connect test section with porous plate injector.	47
3.3	Schlieren image of the upstream flowfield used to measure the angle of a Mach wave with a sub-pixel edge detection algorithm.	51
3.4	Mach number during the steady test time computed from an analysis of Mach angles.	52
3.5	Typical reservoir pressure trace (black) and injector plenum pressure (blue) for the case of CO ₂ injection at 4.18 bar. The steady test time occurs between between two dashed lines.	53
3.6	The ratio of total mass through the porous plate, calculated with the Forchheimer equation, to mass loss in the Ludwieg tube for CO ₂ , N ₂ , and He at various Ludwieg-tube reservoir pressures.	56
3.7	No injection (left), injection of He at 4.04 bar (middle), and injection of He at 7.28 bar (right). A dotted red line indicates the y -position at which the weak shock meets the trailing edge of the upstream window.	59
3.8	The location of the FLDI focal point in relation to a schlieren image of flow features in the downstream window from the injection of CO ₂ at 3.89 bar. The x -axis is relative to the leading edge of the upstream window.	60
3.9	Schematic of the four-point Focused Laser Differential Interferometer.	61
3.10	Beam spacing of the four-point Focused Laser Differential Interferometer, with $\Delta x_1 = 0.187 \pm 0.008$ mm, $\Delta x_2 = 1.070 \pm 0.008$ mm, $\Delta y = 0.365 \pm 0.008$ mm, and $\mu = 20.82 \pm 0.12^\circ$	62
3.11	Beam intensity profiles at (left) $z - z_0 = 1$ mm and (right) $z - z_0 = 4$ mm, centered on FLDI point 1.	64
3.12	Measurements of the beam radius, ω , along the z -axis with a line of best fit ($R^2 = 0.9991$) to determine the radius at the focal point, ω_0	64
3.13	(a) Variation of the product of FLDI transfer functions $H_{\Delta x} H_{\omega}$ with wavenumber and distance z and (b) full width at half maximum (FWHM) of the FLDI transfer functions (depth of focus) as a function of wavenumber for $\lambda_0 = 532$ nm, $\omega_0 = 2.5$ μ m, and $\Delta x_1 = 187$ μ m.	71
3.14	Schematic of test section in relation to FLDI beams.	72
3.15	Estimated ratio of FLDI signal from the core flow, relative to the sidewall boundary layers.	73
3.16	Time trace for a shot without injection. (Top) Reservoir pressure; (Middle) voltage output from FLDI signals with moving mean indicated in red, offset for clarity; (Bottom) spectrogram of FLDI point 1.	76
3.17	Amplitude spectral density of phase fluctuations from FLDI points 1-4 without injection.	80
3.18	Amplitude spectral density of phase fluctuations from FLDI point 1 for various injection conditions.	80
3.19	Magnitude-squared coherence of FLDI signals along a Mach line (pairs 1-2 and 3-4) and a streamline (pairs 1-3 and 2-4). For comparison, the coherence of FLDI signal pair 1-4 is also plotted.	81

3.20	Mean cross power spectral densities of phase fluctuations along a Mach line (a, c, e) and streamline (b, d, f) for injection of 30%, 35%, and 40% He/CO ₂ mixtures compared to the case of no injection. Markers indicate the frequencies at which the means are unequal with 95% confidence intervals.	84
3.21	Mean cross power spectral densities of phase fluctuations along a Mach line (a, c, e) and streamline (b, d, f) for injection of CO ₂ , He, and N ₂ compared to the case of no injection. Markers indicate the frequencies at which the means are unequal with 95% confidence intervals.	86
3.22	Attenuation spectra of CO ₂ and mixtures of CO ₂ with 10%, 20% and 40% He at 293 K with theoretical attenuation of CO ₂ scaled to match experimental results.	88
3.23	Theoretical acoustic pressure ratio across a 1 cm thick gas layer at 0.51 bar, (a) 293 K and (b) 373 K.	89
4.1	Coordinate system for a generalized plane wave with wavevector $\vec{k}$	92
4.2	Arbitrary single-frequency acoustic pulse train propagating in the xy-plane at an instant in time.	93
4.3	Amplitude spectral density of modeled fluctuations at FLDI points 1, 2, and 3 without any attenuation and FLDI point 2 with a 30% amplitude reduction for acoustic disturbances from the bottom wall. The range of experimentally determined fluctuation amplitudes from various injection conditions (Figure 3.18) is shown in gray.	97
4.4	Mean cross power spectral densities of modeled fluctuations along a Mach line and streamline with no amplitude reductions compared to FLDI results with no injection.	97
4.5	Mean cross power spectral densities of modeled fluctuations along a Mach line (a, c, e) and streamline (b, d, f) with 0%, 15%, 30%, and 45% reduced amplitudes for acoustic disturbances from the bottom wall. In each case, the disturbance model is compared to average FLDI results with no injection, and injection of 30%, 35%, and 40% He/CO ₂ mixtures.	99
A.1	(a) Variation of the product of FLDI transfer functions $H_{\Delta x}H_{\omega}$ with wavenumber and distance z and (b) full width at half maximum (FWHM) of the FLDI transfer functions (depth of focus) as a function of wavenumber for $\lambda_0 = 532$ nm, $\omega_0 = 8.0$ μ m, and $\Delta x_1 = 465$ μ m.	113
A.2	Estimated ratio of FLDI signal from the core flow, relative to the nozzle shear layers.	115
A.3	Time trace for shot E1. (Top) Reservoir pressure; (Middle) voltage output from FLDI signals, offset for clarity; (Bottom) spectrogram of FLDI point 1.	116
A.4	Frequency-dependent convection velocities measured with FLDI signal pair 1-3.	119
A.5	Amplitude spectral density of (a) phase fluctuations from FLDI points 1, 2 and 3 of shot E1 and (b) voltage signal from FLDI point 1 at each condition, prior to application of Equation 3.21.	120
A.6	Density fluctuation amplitude spectra from shots A3, B1, C1, D1, and E1.	121
A.7	RMS density fluctuations for wavenumber band (a) $k_x = [100 - 3000]$ m ⁻¹ and (b) $k_x = [2100 - 3000]$ m ⁻¹	123

A.8	Magnitude-squared coherence of FLDI pairs 1-2 (Mach line) and 1-3 (streamline) from shots A3, B1, C1, D1, and E1.	124
A.9	Schematic of the FLDI in relation to the nozzle and angle ϑ used to estimate the portion of the total acoustic field measured with signal pair 1-2.	125
A.10	Ratio of the RMS fluctuations of signal pair 1-2 (Mach line) to signal pair 1-3 (streamline) from the respective CPSDs in the wavenumber band of $k_x = [2100 - 3000] \text{ m}^{-1}$	126
B.1	(a) Density variation and (b) sound-speed variation in three-layer model from Robins (1990, 1991).	131
B.2	Sinusoidal variation in molar ratio of helium throughout the boundary layer. . . .	138
B.3	(a) Density variation and (b) sound-speed variation in profiles from Robins (1990, 1991) and a representative boundary layer with $\bar{X}_{He} = 0.3$	139
B.4	Power reflection coefficients for various boundary-layer profiles with a thickness of 1 cm and an acoustic incidence angle of (a) 0° and (b) $90^\circ - \mu = 69.2^\circ$	140
B.5	Power reflection coefficients for boundary-layer profiles of various heights and an acoustic incidence angle of (a) 0° and (b) $90^\circ - \mu = 69.2^\circ$	141

Nomenclature

Acronyms

AEDC	Arnold Engineering Development Complex
ASD	Amplitude Spectral Density
BAM6QT	Boeing/Air Force Office of Scientific Research Mach 6 Quiet Tunnel
CPSD	Cross Power Spectral Density
DNS	Direct Numerical Simulation
FLDI	Focused Laser Differential Interferometry
FWHM	Full Width at Half Maximum
HWA	Hot-Wire-Anemometry
HyperTERP	Hypersonic Tunnel for Educational and Research Purposes
IMH	Isochoric Molar Heat
LDI	Laser Differential Interferometry
NIST	National Institute of Standards and Technology
PQFLT	Purdue Mach 4 Quiet Flow Ludwig Tube
PSD	Power Spectral Density
PSUSWT	Penn State Supersonic Wind Tunnel
RMS	Root Mean Square
SNR	Signal-to-Noise Ratio
SSH	Schwartz, Slawsky and Herzfeld

Variables

A^2	Cartesian displacement of atom
$A_{i,t}$	Cross-sectional area of acoustic wave
$A(f)$	Amplitude of density fluctuations
a_∞	Freestream sound speed
C_{ab}	Sutherland constant of mixture
C_v^{eff}	Effective isochoric molar heat of mixture
C_v^∞	External isochoric molar heat of mixture
C_j^{vib}	Vibrational isochoric molar heat of mode j
C_p	Isobaric specific heat
c	Sound speed
D	Mutual diffusion coefficient OR photodetector OR total fluctuation amplitude
E	Energy
f	Frequency OR final state of molecule OR focal length
g_j	Degeneracy of vibrational mode j
$g(z)$	Non-dimensional amplitude of density fluctuations

G_1	Pulse train of Gaussian curves
G_2	Gaussian curve oriented perpendicular to propagation
h_P	Planck's constant
h	Transition layer thickness
H	FLDI transfer function
H_ν	Hankel function
I_0	Intensity distribution
k	Wavenumber
k_B	Boltzmann constant
K	Thermal conductivity OR Gladstone-Dale constant
L	Thickness of porous plate
M_∞	Mach number
M_i	Molar mass of species i
m	Mass
$\dot{m}$	Mass flow rate
Δm_L	Change in mass inside Ludwieg tube
Δm_p	Mass through porous plate
n	Index of refraction OR plane wave amplitude
P	Pressure OR transition probability OR number of wave packets
P_a	Acoustic pressure
P_f	Power of output signal at acoustic frequency f
P_p	Plenum pressure
$P_{s,12/13}$	Correlated RMS fluctuations
P_0	Equilibrium pressure OR stagnation pressure
P_∞	Freestream pressure
P_ν	Steric factor
R	Gas constant OR reflection coefficient OR cross-correlation
R_Π	Power reflection coefficient
Re_∞	Freestream Reynolds number
r_c	Distance of closest approach
r_0	Distance for zero potential value
r	Molecular separation OR radial component OR density ratio
S	Surface area of porous plate OR cross power spectral density
T	Temperature OR transmission coefficient OR steady test time
T_0	Equilibrium temperature OR stagnation temperature
T_Π	Power transmission coefficient
T_j^{vib}	Temperature of vibrational mode j
t	Time
U_s	Acoustic source velocity
U_∞	Freestream velocity
$\langle U_c \rangle$	Mean streamwise convection velocity
$U_x(f)$	Frequency-dependent convection velocity
V	Volume of Ludwieg tube OR FLDI output voltage OR Robins model variable
V_ν	Vibrational factor
V_{LJ}	Lennard-Jones potential function

V_{exp}	Shifted exponential fit
v_0^*	Transition-favorable incident velocity
x	Streamwise coordinate
X_i	Molar ratio of species i
Δx_1	Intra-pair spacing of FLDI beams
Δx_2	Inter-pair spacing of FLDI signals
Y_i	Mass fraction of species i
y	Vertical coordinate
Z_c	Collision number
Z	Acoustic impedance
z	Spanwise coordinate OR acoustic propagation direction
z_0	FLDI focal plane OR initial separation distance
α	Acoustic attenuation coefficient
α^*	Decay parameter of exponential potential function
α_c	Friction-dependent permeability coefficient
β_c	Inertia-dependent permeability coefficient
γ	Ratio of specific heats OR coherence
δ	Boundary-layer thickness
ϵ	Splitting angle of Wollaston prism
ϵ_{ab}	Potential depth
ζ	Polar angle of disturbance propagation
η	Azimuthal angle of disturbance propagation
θ	Beam divergence angle OR polar coordinate OR wave packet phase OR incidence angle OR grazing angle
θ_j^{vib}	Characteristic vibrational temperature of mode j
ϑ	Translation of Gaussian curve OR angular coordinate in nozzle
λ	Wavelength
μ	Mach angle OR shear viscosity OR reduced mass of collision pair
ν	Vibrational mode
ρ	Density
ρ_p	Plenum density
ρ_0	Equilibrium density
ρ_∞	Freestream density
σ	Standard deviation
τ_c	Time between molecular collisions
τ	Vibrational relaxation time OR time delay between FLDI signals
ϕ_I	Acoustic wavefront inclination angle
ϕ_{ij}	Wilke's coefficients
$\Delta\phi$	Phase shift between FLDI beams
ϕ_s	Phase of the cross spectrum
ψ	Disturbance phase
ω	Radial frequency OR Gaussian beam radius
ω_0	Gaussian beam radius at focal point

Chapter 1: Introduction

1.1 Motivation

The complex flow physics encountered in high-speed flight include turbulent boundary layers, shear layers, laminar-turbulent transition, shock-wave/boundary-layer interactions, fluid-structure interactions, heat transfer, nonequilibrium chemistry, and more. The breadth of these complexities requires detailed models to better understand their effects on flight performance. While theoretical modeling has provided insight into some underlying mechanisms, practical vehicle design requires both theoretical models and empirical approximations. Despite advancements in computational power and efficiency, numerical simulation can still be incapable of capturing the flow physics necessary to represent the dynamics of the full vehicle system. Furthermore, high-speed flight testing is expensive, has limited onboard diagnostics, and involves extensive planning and collaboration among universities and military organizations. As a result of these challenges, ground-test facilities are heavily relied on to study high-speed flow phenomena.

Although the methods by which high-speed flow is created in wind tunnels can vary depending on facility type, the resulting flow will always be affected by its generation process. In particular, the gas processes used to create the reservoir conditions, and nozzles used to accelerate the flow invariably introduce disturbances into the flowfield. These disturbances can have significant impacts on the aerodynamic behavior of the test model in the freestream flow. Since each

ground-test facility is inherently unique, this makes it difficult to compare experimental results obtained from various wind tunnels. Additionally, comparisons to computational and free-flight data are challenging. Thus, the accurate comparison of experimental, computational and free-flight data requires advances in our ability to reduce freestream disturbance levels in ground-test facilities.

1.2 Noise in High-Speed Facilities

The noise environment encountered by models in the freestream flow of conventional high-speed wind tunnels is substantially stronger than that experienced in atmospheric flight and leads to much reduced transition Reynolds numbers. The processes by which boundary layer transition occurs is discussed by [Morkovin \(1994\)](#) and [Reshotko \(1994\)](#). As the disturbance amplitudes in the freestream increase, receptivity mechanisms of laminar boundary layers lead to the growth of instabilities, resulting in breakdown and transition to turbulence. Laminar-turbulent transition has dramatic effects on heat transfer, skin friction, and flow separation ([Pate and Schueler, 1969](#); [Schneider, 2001](#)). Therefore, accurate predictions of the transition location is required to reduce uncertainties involved in vehicle design and prevent over-engineered thermal protection systems.

The effects of freestream noise on the transition location complicates the use of conventional tunnels for studying hypersonic flow problems in which boundary-layer transition is important. This has led to the development of “quiet” tunnels, which provide low noise levels comparable to flight ([Schneider, 2008](#)). The first supersonic and hypersonic quiet tunnels were developed from the late 1960s to the mid 1990s at the NASA Langley Research Center. A summary of their developmental history is presented by [Schneider \(2008\)](#). Building on these achievements,

a low-Reynolds number Mach-4 quiet Ludwig tube (PQFLT) and Mach-6 quiet Ludwig tube (BAM6QT) were developed at Purdue University, which serve as the primary sources of quiet-tunnel data to date. These successful facilities feature axisymmetric nozzles used to eliminate crossflow and corner flows, long nozzles with small curvature to dampen centrifugal (Görtler) instabilities, highly-polished throats to reduce roughness effects, and bleed slots upstream of the throat to remove streamwise vorticity and limit boundary-layer transition along the nozzle walls. While great progress has been made in the development of high-speed quiet tunnels, substantial research and funding is still needed to obtain transition measurements under fully quiet conditions. Although existing quiet flow facilities are well-suited for stability and transition research at limited Reynolds numbers, conventional tunnels are still widely employed due to their accessibility, relatively inexpensive development, and range of operating Reynolds numbers.

Investigations into freestream disturbances in high-speed tunnels date back to [Kovasznay \(1953\)](#), who showed that small fluctuations in a viscous compressible flow can be described by three disturbance fields: entropy inhomogeneities, vortical perturbations, and acoustic waves. [Morkovin \(1957, 1959\)](#) expanded on the origin of the disturbance fields and discussed the relative importance of each, which can vary widely depending on the facility type. Shown in Figure 1.1 from [Schneider \(2008\)](#) are the various sources of freestream fluctuations. In reflected-shock tunnels, entropy disturbances are known to originate in the reservoir and travel in the streamwise direction at the mean flow velocity, while acoustic disturbances are radiated from the turbulent boundary layers on the nozzle walls and propagate into the freestream along characteristic Mach lines ([Morkovin, 1959](#)). Flaws in the nozzle wall can also lead to shivering Mach waves, which are standing waves that are modulated to “shiver” by passing eddies within the turbulent boundary layer ([Williams and Maidanik, 1965](#)). Vorticity fluctuations originate upstream of the nozzle and

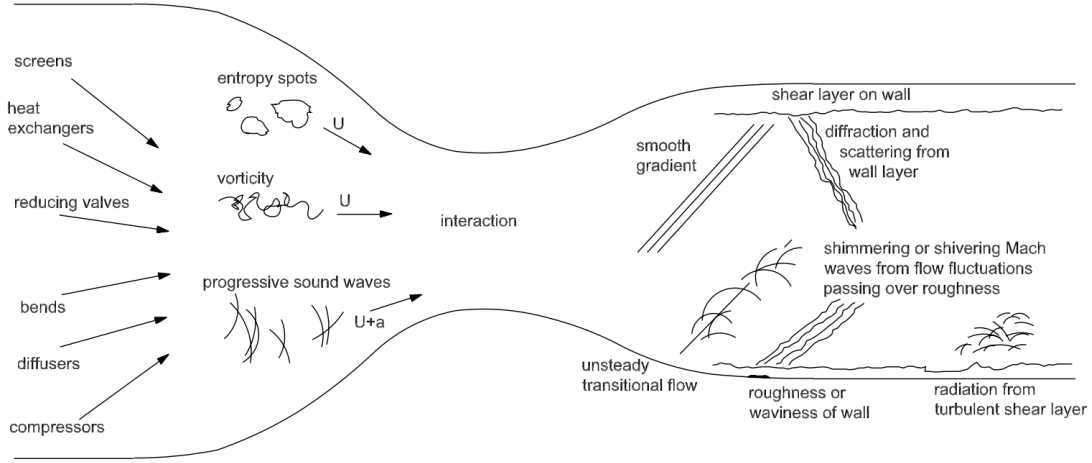


Figure 1.1: Freestream disturbances in high-speed wind tunnels from [Schneider \(2008\)](#).

result from the variation of the solenoidal component of the velocity field. In high-speed facilities, however, vortical fluctuation levels are often very low due to the large velocity ratio across the nozzle ([Morkovin, 1959](#)), leaving entropic and acoustic disturbances as the primary contributions to freestream turbulence intensity.

The origin of acoustic noise in compressible shear and boundary-layer flows has been investigated theoretically ([Phillips, 1960](#); [Pao, 1973](#)), experimentally ([Pate and Schueler, 1969](#); [Laufer, 1964](#)), and numerically ([Duan et al., 2014, 2015, 2016](#); [Zhang and Duan, 2016](#)). The current understanding of the origin of acoustic noise is based on the eddy Mach-wave concept of [Phillips \(1960\)](#), whereby radiation is produced from structures within the boundary layer moving supersonically with respect to the freestream. As the Mach number is increased, an increasing fraction of turbulent eddies acquire a relative supersonic velocity, leading to an increase in radiation intensity at higher Mach numbers. Hot-wire-anemometry (HWA) measurements by [Laufer and Vrebalovich \(1960\)](#) and [Laufer \(1961, 1964\)](#) indicated that acoustic disturbances are likely to dominate the disturbance environment at Mach numbers of 2.5 or above. [Laufer \(1961\)](#) also showed that radiated acoustic noise had a preferred orientation in the laboratory reference frame

that was different from the Mach angle. This incidence angle of radiated waves is linked to the source velocity of the moving eddy in a turbulent boundary layer. In the frame of reference of the moving source, the wavefront is a Mach wave. The relation between the inclination angle, ϕ_I , and source velocity, U_s , is shown in Figure 1.2 from Schilden and Schröder (2019). As detailed by Schilden and Schröder (2019), the two-dimensional wavevector of the resulting wavefront is

$$(k_x, k_y) = \frac{\omega}{U_\infty \cos \phi_I \pm a_\infty} (\cos \phi_I, \sin \phi_I), \quad (1.1)$$

where ω is the angular frequency, U_∞ is the freestream velocity, a_∞ is the speed of sound in the freestream, and the positive and negative sign denotes the fast and slow acoustic waves, respectively. The phase velocity of the radiated acoustic wave along the wall-parallel coordinate is equal to the source velocity, that is $U_s = \omega/k_x$. For slow acoustic waves, the source velocity must be within the interval $0 \leq U_s \leq U_\infty - a$, and the inclination angle is related to the source velocity by

$$\cos \phi_I = \left(1 - \frac{U_s}{U_\infty}\right)^{-1} M_\infty^{-1}, \quad (1.2)$$

where $M_\infty = U_\infty/a_\infty$ is the Mach number. The lower bound of the source velocity ($U_s \rightarrow 0$) gives $\phi_I = \cos^{-1}(1/M_\infty)$, i.e., a Mach wave, while the upper bound ($U_s \rightarrow U_\infty - a$) represents a slow acoustic wave without inclination ($\phi_I = 0$). The Mach number dependence of the non-dimensional source velocity, U_s/U_∞ , can be estimated from a linear fit of the experimental and

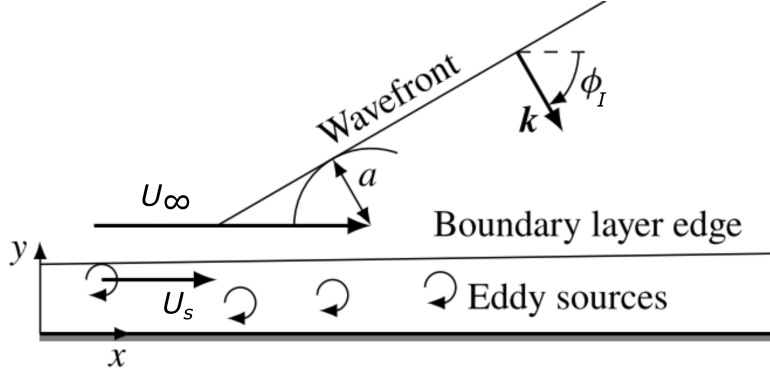


Figure 1.2: Plane acoustic waves radiated by eddy sources of a turbulent boundary layer from [Schilden and Schröder \(2019\)](#).

numerical data by [Laufer \(1961\)](#) and [Duan and Choudhari \(2013\)](#), which gives

$$\frac{U_s}{U_\infty} = 0.0784M_\infty + 0.1861 \quad (1.3)$$

in the range $2 \leq M_\infty \leq 6$.

Seen in Figure 1.3 from [Duan et al. \(2015\)](#) is a numerical schlieren image of acoustic radiation from turbulent boundary layers along channel walls in Mach-5.86 flow. [Duan et al. \(2015\)](#) showed that the waves radiated from either channel wall hardly change their preferred orientations in the over-lapping region, and the resulting fluctuation intensity is the summation of the two sources. In the spanwise direction, the waves are not truly planar, with a finite spanwise extent attributed to the acoustic sources within the turbulent boundary layer also being of finite size ([Duan et al., 2016](#)).

Acoustic disturbances in the freestream are particularly relevant to the boundary-layer transition process of slender, smooth bodies at small incidence. For these configurations, the laminar boundary-layer receptivity to freestream disturbances leads to the growth of a dominant “second-mode” instability, defined by [Mack \(1975\)](#). The second-mode instability is acoustic in nature,

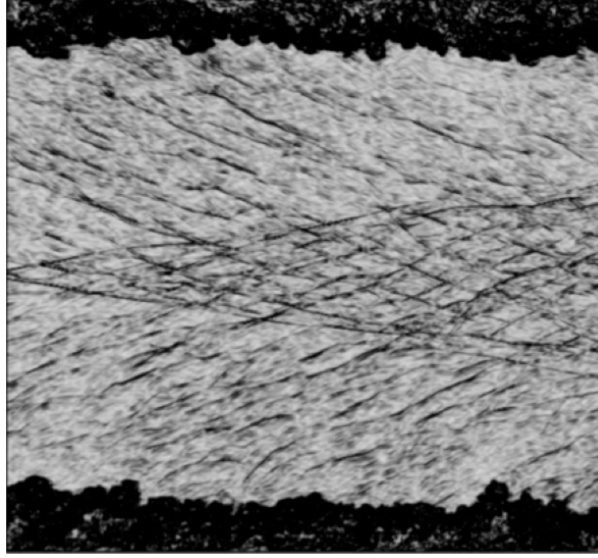


Figure 1.3: Numerical schlieren image in a streamwise wall-normal plane of a double wall Mach-5.86 configuration from [Duan et al. \(2015\)](#).

primarily characterized by fluctuations in density, and the most unstable second-mode waves (i.e., possessing the highest amplification rates) are two-dimensional. A number of experimental studies including those from [Demetriades \(1960, 1974, 1977\)](#), [Kendall \(1975\)](#), [Stetson et al. \(1983\)](#), and [Stetson and Kimmel \(1992\)](#) confirmed the existence of second-mode disturbances and revealed their “rope-like” structure with fundamental frequencies approximately proportional to $1/(2\delta)$, where δ is the boundary-layer thickness. For circular cones, these frequencies are typically on the order of 100 – 400 kHz ([Laurence et al., 2016](#); [Butler and Laurence, 2022](#); [Kennedy et al., 2022](#)). From the perspective of the thicker, upstream radiating boundary layer, the critical freestream disturbances for transition are at relatively large wavenumbers. Additionally, direct numerical simulation (DNS) studies by [Duan et al. \(2014, 2016\)](#) have shown conclusively that the dominant source of acoustic radiation within a high-speed turbulent boundary layer is the near-wall region. Therefore, for boundary-layer transition problems, the most relevant high-frequency acoustic disturbances will have propagated many wavelengths through the upstream radiating

boundary layer to the freestream. Mechanisms capable of attenuating the acoustic disturbances within the upstream turbulent boundary layer thus may play a critical role in the reduction of freestream-disturbance amplitudes.

1.3 Attenuation of Acoustic Disturbances

The absorption and dispersion of acoustic waves in gases results from several mechanisms. These include viscous dissipation and thermal conductivity, diffusion in mixtures, and excitation-relaxation processes between internal (e.g., vibrational) and external (translational and rotational) degrees of freedom. The attenuation related to viscous dissipation and heat conduction is based on the classical formulation of [Stokes \(1845\)](#) and [Kirchhoff \(1868\)](#). Classical attenuation is well understood for ideal gas mixtures with a known viscosity and conductivity. Attenuation due to diffusion occurs when the propagation of an acoustic wave causes local fluctuations in concentration as a result of the varying masses of the mixture components. The decrease in entropy in the arrangement of molecules results in an energy reduction in the acoustic wave. If the appropriate gas constants are known, the attenuation from diffusion can be reliably calculated ([Bhatia, 1985](#)).

Predicting the molecular relaxation processes and their effect on attenuation in polyatomic gases and mixtures, however, remains a challenge. In the excitation-relaxation mechanism, the propagating acoustic wave excites vibrational energy states of the gas molecules, and the subsequent transfer of energy between internal and external modes occurs with a relaxation time that is dependent on the collisional dynamics, vibrational modes available, and gas constants, but is significantly longer than that required to achieve translational/rotational equilibrium. If the period of the acoustic wave is of the same order as the vibrational relaxation time, oscillations in the

thermodynamic state of the gas remain out of the phase with the acoustic disturbance, resulting in significant attenuation and dispersion of the propagating waves ([Clarke et al., 1964](#)). This occurs because the acoustic wave speed in a real gas does not propagate at the well known isentropic sound speed for all frequencies, but rather propagates at a speed dependent on the internal energy modes. If the acoustic frequency is significantly lower than the relaxation rate of the gas, the translational and vibrational modes of the gas have time to respond to the disturbance and remain in equilibrium. In this case, the wave will propagate at the equilibrium speed of sound. At high frequencies, the gas is unable to adjust to the change in state, and the wave will propagate at the frozen sound speed. Between these regimes, the relaxing gas acts as a dispersive medium and the propagation of the acoustic wave results in energy transfer between modes. This interaction between an acoustic wave and a vibrationally active gas species was investigated numerically by [Wagnild \(2012\)](#).

Attenuation rates in air between 3000 K and 5000 K can be larger than 0.1 per wavelength ([Fujii and Hornung, 2003](#)). At lower temperatures, the vibrational and dissociational modes of air constituents freeze out; however, the vibrational modes of carbon dioxide and other polyatomic gases remain substantially excited even at room temperature, leading to significant attenuation of sound waves. Therefore, CO₂ and mixtures of CO₂ with other polyatomic gases are of particular interest for applications of acoustic damping at low and moderate temperatures.

The theory of vibrational energy transfer in diatomic gases was first presented by [Zener \(1931\)](#), [Landau \(1932\)](#) and [Landau and Teller \(1936\)](#). Later, the theory was applied to polyatomic gases by [Kneser \(1933\)](#), [Schwartz et al. \(1952\)](#) (SSH), and [Tanczos \(1956\)](#), who utilized a quantum-mechanical theory to model the probability of molecular transition for a given collision between two molecules. The complex molecular relaxation processes can lead to coupled

mechanisms of vibrational-translational (V-T) and vibrational-vibrational (V-V) energy transfer between different energy levels, producing several effective relaxation frequencies. In multi-component gas mixtures, the relaxation frequencies also depend on the concentrations of the components. Although some theories on decoupling the multimode vibrational relaxation energy exchange have been proposed by [Bauer \(1965\)](#), [Bauer et al. \(1972\)](#), [Shields \(1970\)](#), and [Bass et al. \(1972\)](#), they cannot be used to analyze multi-component mixtures. More recently, [Dain and Lueptow \(2001\)](#) developed a model for molecular relaxation in ternary mixtures of polyatomic gases based on the coupled relaxation equations given by [Schwartz et al. \(1952\)](#) and [Landau and Teller \(1936\)](#). [Zhang et al. \(2013\)](#) later extended the theory of [Dain and Lueptow \(2001\)](#) in order to decouple the V-V energy exchange, providing an analytical model that can lead to multiple peaks of sound absorption in a multi-component gas mixture at room temperature.

Despite this development in theoretical modeling of the vibrational relaxation processes for certain gas mixtures, there remains limited experimental data for comparison and validation, particularly at elevated temperature. Typically, experimental measurements of attenuation coefficients are obtained by sending acoustic waves through a known test gas at a specified temperature and pressure. By varying the distance between an emitter/receiver pair of ultrasonic transducers, the voltage output from the receiving transducer can be related to the attenuation coefficient of the gas. Various experimental setups are detailed in the works of [Fricke \(1940\)](#), [Winter and Hill \(1967\)](#), [Wang and Springer \(1973\)](#), [Shields and Bass \(1977\)](#), [Zuckerwar and Griffin \(1980\)](#), [Ejakov et al. \(2003\)](#) and [Petculescu et al. \(2006\)](#). The most recent comprehensive measurements of acoustic attenuation are from the work of [Petculescu et al. \(2006\)](#), which included mixtures of $\text{C}_2\text{H}_4/\text{N}_2$, CO_2/air , and CH_4/air at room temperature, and [Ejakov et al. \(2003\)](#), which featured O_2/N_2 , CH_4/N_2 , and CO_2/N_2 at room temperature. Seen in Figure 1.4 from [Petculescu et al.](#)

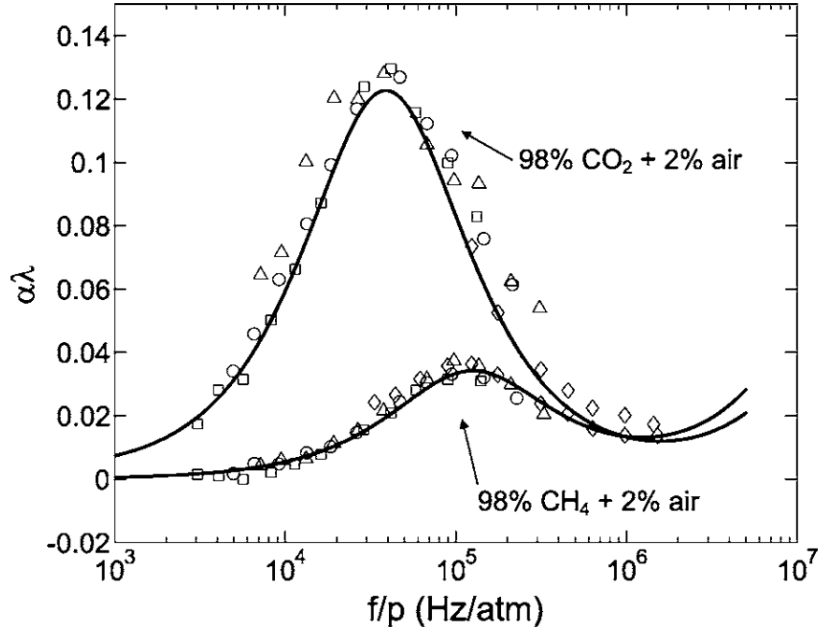


Figure 1.4: Attenuation per wavelength for mixtures of CO₂ and CH₄ with air (symbols: □, 92 kHz; ○, 149 kHz; △, 215 kHz; ◇, 1 MHz) from [Petculescu et al. \(2006\)](#). Curves represent model predictions.

(2006) are measurements of attenuation per wavelength of CO₂ and CH₄ with air, where the symbols represent the various acoustic frequencies at which the transducers operated at. At room temperature, CO₂ with 2% air has a substantial attenuation of approximately 0.13 per wavelength at a frequency of 40 kHz/atm. It is also evident that the measured attenuation is significant over a considerable range of f/P values. Attenuation measurements at higher temperatures for select gas mixtures have been reported by [Bass \(1973\)](#) and [Bass et al. \(1974\)](#) (CO₂, CO₂/O₂, H₂O); [Wang and Springer \(1973\)](#) (CH₄, Ar, C₂H₂, C₂H₄, C₃H₆); [Lewis and Shields \(1967\)](#) and [Shields \(1967, 1959\)](#) (SO₂, SO₂/Ar, CO₂, CO₂/He); and [Winter and Hill \(1967\)](#) (H₂, D, O₂, N₂). Outside of these works from the 1950s-1970s, few other high-temperature attenuation results are available.

High-temperature acoustic attenuation measurements are particularly important for potential applications in high-speed flows. At the high temperatures of typical stagnation regions in

high-speed flows, the gases can exhibit significant nonequilibrium effects, resulting in molecular excitation-relaxation processes and attenuation of acoustic radiation in the flow. Investigations of the effects of thermodynamic nonequilibrium on turbulence-related phenomena are rare. In the low-speed turbulent-channel-flow experiments of [Fuller et al. \(2014\)](#), N_2 was vibrationally excited using a radiofrequency plasma generation system; the plasma induced nonequilibrium, thermal heating, and flow acceleration resulted in significantly reduced peak turbulence intensities and Reynolds shear stresses. Numerical simulations of canonical turbulence-in-box ([Neville et al., 2014](#)) and shear flows ([Neville et al., 2015](#)) have also indicated that thermodynamic nonequilibrium may have a significant effect on the fluid dilatation and temperature fluctuations in hot turbulent flows. Attenuation from vibrational relaxation in CO_2 has been shown by [Fujii and Hornung \(2003\)](#), [Leyva et al. \(2009\)](#), and [Jewell et al. \(2013\)](#) to delay laminar-turbulent transition on a 5-degree half-angle cone in the T5 shock tunnel at the California Institute of Technology (CalTech): this was attributed to damping of the second-mode disturbances which are primarily acoustic in nature and were assumed to be responsible for transition at these conditions. Presented in Figure 1.5 are the results from [Leyva et al. \(2009\)](#), who reported that the transition Reynolds number more than doubled when the freestream gas was composed of 40% N_2 with CO_2 compared to 100% N_2 . [Fujii and Hornung \(2003\)](#) also showed that the sound absorption rates of CO_2 were of the same order as the second-mode amplification rates in the range of frequencies where the acoustic disturbances were most amplified. For the case of air, the sound absorption rates were greatest at frequencies much lower than those at which the acoustic amplification rates became significant; thus, transition delay due to nonequilibrium processes would not be expected. Similarly, a computational study by [Elliott et al. \(2019\)](#) found that CO_2 was effective in reducing the amplification rates of disturbances in the boundary layer on a 10-degree

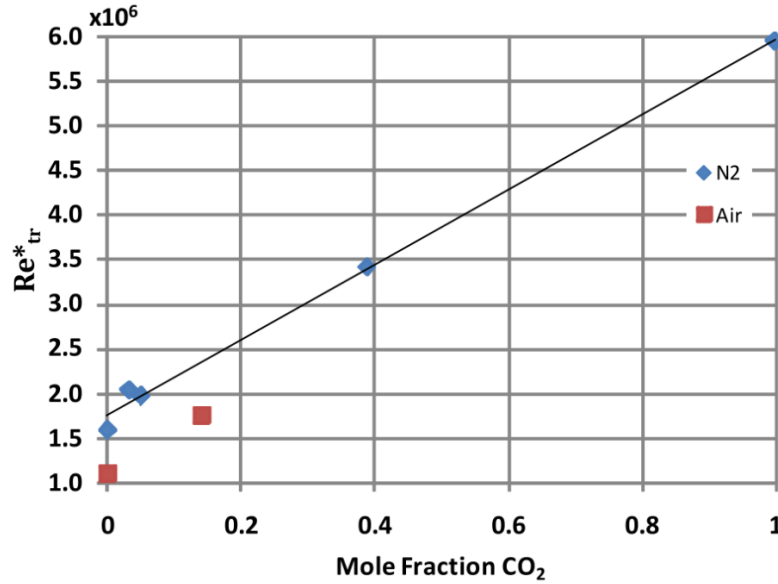


Figure 1.5: Effect of CO₂ mole fraction in the test gas on transition Reynolds number for flows over a 5 degree half-angle cone from [Leyva et al. \(2009\)](#).

half-angle cone in high- and low-enthalpy flow in the same T5 facility. While these studies highlight some aspects of the effects of nonequilibrium processes on acoustic instabilities within a boundary layer, the effects of nonequilibrium on turbulence-generated acoustic radiation have not been reported to date.

1.4 Focused Laser Differential Interferometry

Measurement of acoustic radiation in high-speed facilities is challenging. Pitot-pressure measurements suffer from low bandwidth and the creation of bow shocks that modulate and filter flow features of interest ([Mahesh and Lee, 1995](#)). Additionally, Pitot measurements are dependent on the probe geometry and disturbance propagation directions, which complicates the analysis and increases uncertainty ([Stainback and Wagner, 1972](#); [Bounitch et al., 2011](#); [Schilden and Schröder, 2019](#)). Flush-mounted surface transducers meanwhile provide poor spatial resolution and are generally unsuitable for measuring the noise level in a hypersonic freestream. Hot-wire

anemometry (HWA) also has a limited bandwidth (generally $\ll 1$ MHz) and are often unable to withstand the harsh environment of a hypersonic tunnel. Despite measurements of freestream pressure fluctuations tracing back to those of [Laufer \(1964\)](#), there remains limited experimental freestream characterization that is suitable for developing reliable transition models ([Duan et al., 2016](#)); this has led to the application of direct numerical simulations (DNS) to estimate facility noise by numerous authors, notably [Duan and Choudhari \(2013\)](#), [Duan et al. \(2014\)](#), [Duan et al. \(2016\)](#), [Zhang et al. \(2018\)](#), and [Gloerfelt and Berland \(2013\)](#).

A promising diagnostic called Focused Laser Differential Interferometry (FLDI) provides an alternative method for high-sensitivity, high-bandwidth measurements of tunnel noise ([Parziale, 2013](#)). The technique was invented by Smeets and George in the 1970s ([Smeets and George, 1971](#); [Smeets, 1972](#); [Smeets and George, 1973, 1975](#); [Smeets, 1977](#)) and is a non-intrusive, non-imaging, polarizing interferometer that is sensitive to phase differences between two closely-spaced probe volumes. The differences in phase are the result of varying densities at each FLDI probe; therefore, the measured voltage fluctuations are proportional to the density gradient between the two focal volumes. FLDI is capable of high frequency response - generally above 10 MHz ([Parziale et al., 2012](#)) - with high spatial resolution (on the order of micrometers) and a high signal-to-noise ratio. This is particularly important for the measurement of high-frequency, weak acoustic disturbances that contribute to the second-mode instability mechanism. Modified from its original form, Laser Differential Interferometry (LDI), the FLDI is arranged such that the beam radii are large and overlapping away from the focal plane. This focusing arrangement of the FLDI allows unwanted signal to be rejected away from the flow features of interest at the instrument's focus. Detailed explanations of the working principle of FLDI are given by [Schmidt and Shepherd \(2015\)](#), [Parziale et al. \(2014\)](#), [Settles and Fulghum \(2016\)](#), [Ful-](#)

ghum (2014) and Ceruzzi et al. (2021b). More recently, work by Hameed et al. (2020), Ceruzzi and Cadou (2019), Ceruzzi et al. (2020), Jewell et al. (2019), and Bathel et al. (2020, 2021) has extended the instrument to provide a non-intrusive measurement of velocity by means of a two-point FLDI system; further advancements to multi-point line and linear array FLDI have been made by Weisberger et al. (2021) and Gragston et al. (2021).

The relationship between the FLDI response and measured density gradients has been investigated by several authors, most notably by Fulghum (2014), Schmidt and Shepherd (2015), Settles and Fulghum (2016), Lawson (2021), and Ceruzzi (2022). Fulghum (2014) presents a comprehensive analysis of the use of transfer functions to model how the instrument filters fluctuations away from the focus. Schmidt and Shepherd (2015) utilized similar analytical methods and conducted a ray tracing simulation to interpret the FLDI signal. Lawson (2021) further builds upon this work and validates the ray tracing method, while also introducing a detailed analytical transfer function that incorporates disturbances convecting at an arbitrary angle. Most recently, Ceruzzi (2022) presented a simple transfer function method that accounts for velocity variation in space and frequency.

Within the last 10 years, FLDI has been used numerous times to investigate disturbances in high-speed facilities. Freestream turbulence spectra from single-point FLDI have been obtained by Fulghum (2014) and Settles and Fulghum (2016) in the Penn State Supersonic Wind Tunnel and Arnold Engineering Development Complex (AEDC) Hypervelocity Wind Tunnel 9. In the T5 shock tunnel at Caltech, Parziale et al. (2014) quantified freestream density perturbations and turbulence intensities. Measurements of second-mode instabilities within the boundary layer on a cone surface have been made by Benitez et al. (2020) in the BAM6QT facility, Bathel et al. (2020) in the 20-Inch Mach 6 Wind Tunnel at NASA Langley, and Hameed et al. (2022) in the

T5 shock tunnel. [Ceruzzi et al. \(2020, 2021a\)](#) used a two-point FLDI to probe the turbulent boundary layer within the AEDC’s calibration tunnel (M3CT) at Mach 2.6 and Hypervelocity Wind Tunnel 9 at Mach 18, respectively. [Houpt and Leonov \(2021\)](#) also employed a cylindrical FLDI to probe disturbances close to the surface of flat walls in the blowdown wind tunnel SBR-50 at the University of Notre Dame.

The renewed interest in FLDI has led to a fairly extensive amount of turbulence intensity studies in high-speed facilities; however, it has not yet been utilized to isolate acoustic radiation in the freestream. By orientating a multi-point FLDI along the Mach angle, fluctuations correlated between the probes separated along a Mach line would consist primarily of acoustic radiation. Therefore, comparisons could be made between entropic disturbances propagating along streamlines and acoustic disturbances propagating along Mach lines.

1.5 Scope of Current Work

This study seeks to investigate the effect of vibrational nonequilibrium processes on acoustic radiation from a compressible turbulent boundary layer. The aim of the present work is to provide a physical basis for a new control mechanism to reduce high-frequency (> 100 kHz) freestream noise levels in high-speed facilities. This would potentially allow for a “quasi-quiet” capability in conventional hypersonic wind tunnels, greatly expanding their utility for studying flows in which transition plays an important role. If such a control mechanism could be applied to conventional tunnels, it would allow for a wider range of flow conditions to be achieved than in true quiet tunnels, and at a potentially much reduced development cost.

This investigation consists of the following steps: (i) identify gases and gas mixtures with

desirable acoustic absorption characteristics at temperatures and pressures relevant to high-speed flows; (ii) develop a multi-point FLDI capable of obtaining reliable measurements of high-frequency acoustic disturbances; (iii) measure acoustic radiation emitted by boundary-layer flows composed of various gas mixtures and compare with air flows; (iv) create a mathematical model of the disturbance environment and compare theoretical results with the freestream fluctuations measured experimentally.

First, an acoustic chamber is designed to study the attenuation properties of various gas mixtures at elevated temperatures. By measuring the the attenuation spectra of previously unmeasured gas mixtures at a variety of temperatures and pressures, investigations into the effect of vibrational nonequilibrium on turbulence-generated noise can be better informed. In particular, acoustic attenuation coefficients are measured in CO_2 , N_2O , and mixtures consisting of CO_2/N_2 , CO_2/He , and $\text{N}_2\text{O}/\text{He}$ at temperatures up to 529 K. Next, a four-point FLDI is developed to measure the acoustic radiation in high-speed flows. Transfer functions for interpreting the FLDI signal are applied to estimate the instrument's sensitivity to disturbance wavenumber, as well as the influence of the sidewall boundary layers in corrupting the FLDI signal. In a Mach-2.8 shock-tunnel facility at the University of Maryland, the gas mixtures of interest (as identified from the acoustic chamber measurements) are then introduced into the turbulent boundary layer along the bottom wall of the test section. The injection of the gas mixtures occurs through a porous plate, the process of which is controlled by a Ludwig tube and solenoid valve system. High-speed schlieren visualization is used to examine the effect of injection on the external flowfield, as well as determine where the FLDI focal plane should be located. The four-point FLDI is subsequently positioned above the turbulent boundary layer, downstream of the injection location, and simultaneous measurements are made of freestream disturbances propagating along streamlines and

Mach lines. Through an analysis of the fluctuations correlated along a streamline and Mach line, the relative strengths of entropic and acoustic disturbances are compared for the various injection conditions. Since the correlations of fluctuations along a streamline and Mach line are not completely independent of disturbances propagating in other directions, a mathematical model of freestream disturbances is used to isolate the reductions in acoustic fluctuation amplitudes that occurred only as a result of the vibrationally active gas species in the lower boundary layer.

In Chapter 2, a theoretical model of acoustic attenuation from vibrational nonequilibrium processes is presented and the design of a high-temperature acoustic chamber is described. The measurements of attenuation coefficients obtained from the acoustic chamber are then discussed. In Chapter 3, the attenuation of acoustic radiation emitted by a turbulent boundary in a high-speed facility is investigated. The experimental methodologies of the shock-tunnel tests are detailed, the schlieren and FLDI diagnostics are characterized, and the acoustic attenuation measurements are discussed. Chapter 4 presents the mathematical framework and results of the theoretical disturbance model. Finally, in Chapter 5, the conclusions and major contributions of the work are presented, along with recommendations for future research.

Chapter 2: Acoustic Attenuation in a Vibrationally Active Gas: Theory and Measurements

To better understand how vibrationally active gas mixtures could affect acoustic radiation and turbulence intensities in high-speed flows, measurements of attenuation coefficients are made at a variety of pressures and temperatures inside a heated acoustic chamber. The resulting coefficients are compared to previous results in literature and a theoretical model of vibrational relaxation.

2.1 Acoustic Attenuation Model

Acoustic attenuation in gas mixtures can be attributed to vibrational relaxation processes, classical attenuation, and diffusional attenuation. The total sound absorption coefficient, α , of a gas medium can be expressed as the summation of these contributions ([Ejakov et al., 2003](#)), that is

$$\alpha = \alpha_{\text{vib}} + \alpha_{\text{class}} + \alpha_{\text{diff}}. \quad (2.1)$$

In the following, we consider the first and latter two effects separately.

2.1.1 Vibrational Relaxation

In a gaseous medium, the molecules exchange energy via kinetic collisions. Excitation-relaxation processes transfer energy between internal (e.g., vibrational) and external (translational and rotational) degrees of freedom, and/or among internal degrees of freedom. When a sound wave travels through the medium, excited molecules do not exchange vibrational energy infinitely fast with the translational degrees of freedom associated with the temperature fluctuations. As a result, the medium becomes dispersive, and the specific heat of the relaxing gas becomes complex-valued and frequency dependent ([Petculescu and Lueptow, 2005](#)). The complex process in which the molecules change state upon collision requires decoupling the various energy exchanges that include: (1) vibrational - translational (V-T) energy exchange between vibrational and translational degrees of freedom for one type of molecule; (2) V-T energy exchange for vibrational and translational degrees of freedom between different types of molecules; (3) vibrational-vibrational (V-V) energy exchange between vibrational modes within a given type of molecule; and (4) V-V energy exchange between vibrational modes of different types of molecules ([Dain and Lueptow, 2001](#)).

2.1.1.1 Transition Probabilities

Vibrational relaxation theory is built upon the calculation of transition probabilities. These are based on the quantum-mechanical treatment of the inelastic scattering of molecules of species a , initially in vibrational state i_a , from a second species b , initially in state i_b . After the collision, molecules a and b are in the final state f_a and f_b , respectively ([Petculescu and Lueptow, 2005](#)). There exist two ways in which the exchange of translational and vibrational energy occurs. First,

there are processes whereby only one collision partner changes vibrational state. For example, the probability of molecule a losing one quantum ($i_a = 1 \rightarrow f_a = 0$) while the vibrational state of molecule b is unaltered ($i_b = 0 \rightarrow f_b = 0$) would be described by $P_{00}^{01}(a, b)$. This is the V-T process. Second, collisions may exist in which both partners gain/lose one quantum each. This V-V process is described by the probability $P_{10}^{01}(a, b)$ (Petculescu and Lueptow, 2005). Based on the SSH theory (Schwartz et al., 1952), the calculation of the probability that a pair of molecules will undergo such transition is given by Tanczos (1956) as

$$P_{i_b \rightarrow f_b}^{i_a \rightarrow f_a}(a, b) = \frac{1.364}{1 + \frac{C_{ab}}{T}} P_{\nu_a} P_{\nu_b} \left(\frac{r_c}{r_0} \right)^2 V_{\nu_a} V_{\nu_b} 8 \left(\frac{\pi}{3} \right)^{\frac{1}{2}} \left(\frac{8\pi^3 \mu_{ab} \Delta E_{ab}}{\alpha^{*2} h_p^2} \right)^2 \zeta^{\frac{1}{2}} \exp \left(-3\zeta + \frac{\Delta E_{ab}}{2k_B T} + \frac{\epsilon_{ab}}{k_B T} \right), \quad (2.2)$$

with

$$\zeta = \frac{\mu_{ab} \nu_0^{*2}}{2k_B T} = \left(\frac{\Delta E_{ab}^2 \mu_{ab} \pi^2}{2\alpha^{*2} h_p^2 k_B T} \right)^{1/3}, \quad (2.3)$$

$$\Delta E_{ab} = h_p \nu_a (i_a - f_a) + h_p \nu_b (i_b - f_b), \quad (2.4)$$

$$\mu_{ab} = \frac{m_a m_b}{m_a + m_b}. \quad (2.5)$$

Here, C_{ab} is the Sutherland constant of the mixture, r_c is the distance of closest approach between the molecules, r_0 is the distance for zero potential value, μ_{ab} is the reduced mass of the collision pair, ΔE_{ab} denotes the net total change in translational energy involved during a collision, ν is the vibrational mode, $h_p = 6.62607 \times 10^{-34} \text{ m}^2\text{kg/s}$ is Planck's constant, and $k_B = 1.3807 \times 10^{-23} \text{ J/K}$ is the Boltzmann constant. The interaction potential is described by the properties α^* , ϵ_{ab} , and ν_0^* . P_ν is a steric factor that is required to account for the fact that

the molecules are not spherically symmetrical and that some collision orientations will be more effective than others. This is usually taken as 1/3 for diatomic molecules and for longitudinal vibrations of linear polyatomic molecules, and 2/3 for non-linear polyatomic molecules and for bending modes of linear molecules (Lambert and Lambert, 1977). For larger molecules, however, there is a considerable amount of arbitrariness in this choice (Miklavc and Ficher, 1976; Doi et al., 1973; Dickens and Linnett, 1957). V_v is the vibrational factor, and represents the coupling of the initial and final states of the oscillator under the influence of the perturbation produced by the collision (Lambert and Lambert, 1977). Vibrational factors are defined for zero and single quantum jumps as

$$V_v^{i-i} = 1, \quad (2.6)$$

$$V_v^{(i+1)-i} = V_v^{i-(i+1)} = \frac{\alpha^* \bar{A}^2 (i+1) h_P}{8\pi^2 \nu}. \quad (2.7)$$

The vibrational amplitude coefficient, A^2 , is the Cartesian displacement of an atom for unit change of the normal coordinate of a given vibration (Lambert and Lambert, 1977). The values of A^2 for the surface atoms of a molecule are then averaged and summed over the number of surface atoms to give $\bar{A}^2$. For a diatomic molecule XY, the amplitude coefficient is dependent only on the masses of the atoms X and Y:

$$\bar{A}^2 = A_X^2 + A_Y^2 = \frac{m_Y}{m_X(m_X + m_Y)} + \frac{m_X}{m_Y(m_Y + m_X)}. \quad (2.8)$$

For polyatomic molecules, $\bar{A}^2$ must be obtained from a normal coordinate analysis of the molecular vibrations (Wilson et al., 1980). The resulting amplitude coefficients for certain polyatomic

molecules are listed in Table 3.1 of [Lambert and Lambert \(1977\)](#).

The interaction potential of the collision is approximated by the Lennard-Jones (LJ) function

$$V_{\text{LJ}}(r) = 4\epsilon_{ab} \left[(r_0/r)^{12} - (r_0/r)^6 \right], \quad (2.9)$$

where r is the molecular separation and ϵ_{ab} is the pairwise potential depth ([Jones, 1924](#)). For a two-species mixture, $\epsilon_{ab} \equiv (\epsilon_a \epsilon_b)^{1/2}$ and $r_0 \equiv (r_{0a} + r_{0b})/2$. Presented in Figure 2.1, the Lennard-Jones function describes the repulsive interactions that result from overlapping electron clouds at short distances, and attractive interactions due to temporary dipoles (Van der Waals forces). The use of the Lennard-Jones function, however, can lead to mathematically intractable integrals when calculating the matrix elements of the transition probabilities. Therefore, SSH ([Schwartz et al., 1952](#)) introduced a shifted exponential potential function that has both positive (repulsion) and negative (attraction) values. SSH fit the magnitude and slope of the shifted exponential function to the Lennard-Jones potential at the point of closest approach, r_c , for the transition-favorable incident velocity v_0^* . The shifted exponential fit is given by

$$V_{\text{exp}}(r) = (\epsilon_{ab} + E^*) e^{\alpha^*(r_c - r)} - \epsilon_{ab}, \quad (2.10)$$

with

$$E^* = \frac{1}{2} \mu_{ab} v_0^{*2}, \quad (2.11)$$

where α^* is the decay parameter of the exponential function ([Petculescu and Lueptow, 2005](#)). At the classical turning point, r_c , all the kinetic energy of the incident molecules, E^* , is converted into potential energy. At the hard-sphere collision diameter, r_0 , $V_{\text{LJ}}(r_0) = V_{\text{exp}}(r_0) = 0$. Using these

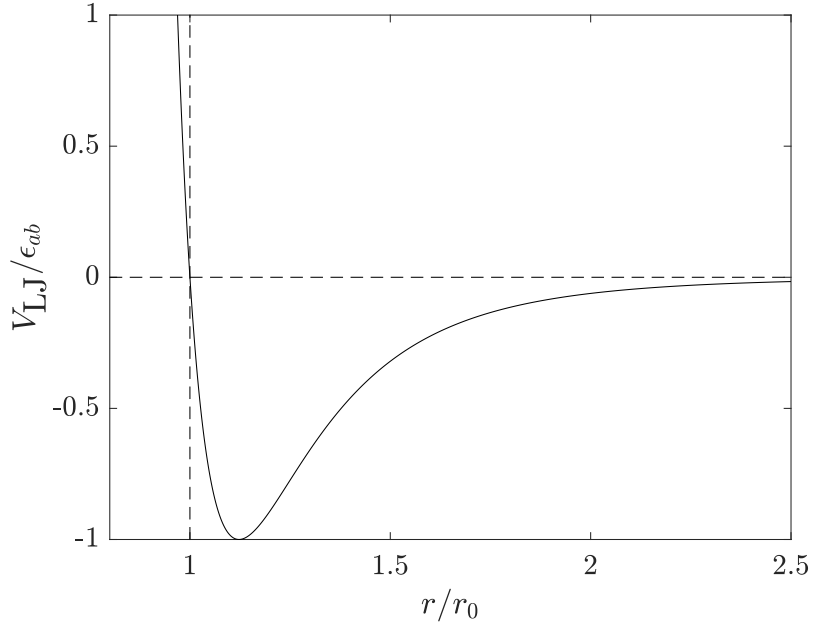


Figure 2.1: Lennard-Jones potential function.

two fitting points, the two potential functions are matched iteratively ([Petculescu and Lueptow, 2005](#)). Starting with arbitrary values of r_c and α^* , both parameters are varied until $V_{\text{exp}}(r_0) = V_{\text{LJ}}(r_0) = 0$ and $V_{\text{exp}}(r_c) = V_{\text{LJ}}(r_c) = E^*$. From Equation 2.3 and Equation 2.11, it can be seen that $E^* = \zeta k_B T$. Therefore, for a given ΔE_{ab} , α^* , and r_c , the values of $V_{\text{LJ}}(r_c)$ and $V_{\text{exp}}(r_0)$ can be iteratively calculated until α^* , r_c and v_0^* are found.

2.1.1.2 Relaxational Absorption

In a gas mixture, the total change of thermodynamic energy caused by the sound propagation is the sum of the energy changes of all molecular degrees of freedom. For a gas mixture with

a total of N vibrational modes and W kinds of molecules, the change in thermodynamic energy is

$$\begin{aligned}
 dE &= C_V^{\text{eff}} dT \\
 &= C_{\text{mix}}^{\infty} dT + \sum_{j=1}^N X_j C_j^{\text{vib}} dT_j^{\text{vib}} \\
 &= \left(C_{\text{mix}}^{\infty} + \sum_{j=1}^N X_j C_j^{\text{vib}} dT_j^{\text{vib}} / dT \right) dT,
 \end{aligned} \tag{2.12}$$

where $C_{\text{mix}}^{\infty} = \sum_{l=1}^W X_l C_l^{\infty}$ is the external isochoric molar heat (IMH) of the mixture, C_l^{∞} is the external IMH of molecule l , X_l is the mole fraction of molecule l , T_j^{vib} is the instantaneous temperature of vibrational mode j , C_j^{vib} is the vibrational IMH of mode j , and X_j is the mole fraction of mode j . If molecule l contains N_l vibrational modes, these N_l modes share the same mole fraction as X_l (Zhang et al., 2013). For a linear molecule, $C_l^{\infty} = 5R/2$; for a nonlinear molecule, $C_l^{\infty} = 3R$. The vibrational IMH of mode j , C_j^{vib} , is given by the Planck-Einstein function for a harmonic oscillator:

$$C_j^{\text{vib}} = g_j R \left(\frac{\theta_j^{\text{vib}}}{T_0} \right) \frac{\exp(\theta_j^{\text{vib}}/T_0)}{(\exp(\theta_j^{\text{vib}}/T_0) - 1)^2}, \tag{2.13}$$

with

$$\theta_j^{\text{vib}} = \frac{h_P \nu_j}{k_B}. \tag{2.14}$$

Here, T_0 is the equilibrium temperature, θ_j^{vib} is the characteristic vibrational temperature, ν_j is the characteristic frequency for mode j , and g_j is the degeneracy of mode j . From Equation 2.12, the

effective IMH of the mixture is

$$C_V^{\text{eff}} = C_{\text{mix}}^{\infty} + \sum_{j=1}^N X_j C_j^{\text{vib}} y_j, \quad (2.15)$$

with

$$y_j = dT_j^{\text{vib}}/dT, \quad (2.16)$$

where y_j is the ratio between the temperature fluctuation of vibrational mode j and that of the external degree of freedom. As the result of V-V coupling, y_j is related to the energy relaxation of all vibrational modes in the mixture. Therefore, Equation 2.15 is the coupled expression of effective IMH. Decoupling y_j subsequently decouples the multimode relaxation in a gas mixture, the process of which is described by Zhang et al. (2013). In the derivation presented by Zhang et al., the transition probabilities used in the calculation of the V-T and V-V transition rates are those presented in Section 2.1.1.1.

Upon decoupling the multimode relaxation, Zhang et al. (2013) shows that the relationship between the effective IMH and the sound relaxational absorption by the effective thermodynamic sound speed, c_{eff} , is given by

$$c_{\text{eff}} = \sqrt{\frac{P_0 (C_V^{\text{eff}} + R)}{\rho_0 C_V^{\text{eff}}}}, \quad (2.17)$$

where P_0 and ρ_0 are the equilibrium pressure and density, respectively. The sound propagation in an excitable gas can be characterized in terms of a complex-valued effective wave number, k_{eff} . The real and imaginary parts of k_{eff} then yield the frequency-dependent phase speed, c , and

relaxational absorption, α_{vib} , given by

$$k_{\text{eff}} = \frac{\omega}{c} - i\alpha_{\text{vib}} = \frac{\omega}{c_{\text{eff}}}, \quad (2.18)$$

and

$$\alpha_{\text{vib}} = \frac{-2\pi\text{Im}(c_{\text{eff}}^{-1})}{\lambda\text{Re}(c_{\text{eff}}^{-1})}, \quad (2.19)$$

where α_{vib} is the acoustic attenuation due to vibrational relaxation processes.

The gas properties and Lennard-Jones parameters used in the vibrational relaxation model are listed in Table 2.1 for the relevant gases. The vibrational amplitude coefficient, $\bar{A}^2$, for CO_2 was obtained from [Ejakov et al. \(2003\)](#), who adjusted the value to best fit the experimental results. Similarly, the vibrational amplitude coefficient for N_2O was adjusted to fit the experimental attenuation spectra of the present work. The steric factor, P_ν , for the vibrational mode $\nu = 667 \text{ cm}^{-1}$ of CO_2 was taken as 1/2, as was done by [Dickens and Linnett \(1957\)](#) for similar modes of SO_2 .

Table 2.1: The vibrational modes (ν), degeneracies (g), steric factors (P_ν), Lennard-Jones potential depth (ϵ) ([Hirschfelder et al., 1964](#)), collisional diameter (r_0) ([Hirschfelder et al., 1964](#)), and vibrational amplitude coefficients ($\bar{A}^2$) ([Lambert and Lambert, 1977](#)) for gases used in the vibrational relaxation model of [Dain and Lueptow \(2001\)](#) and [Zhang et al. \(2013\)](#).

Gas	$\nu (\text{cm}^{-1})$	g	P_ν	$\epsilon/k_B (\text{K})$	$r_0 (\text{\AA})$	$\bar{A}^2 (\text{amu}^{-1})$
CO_2	667	2	1/2	190	3.996	0.05
	1388	1	1/3			
	2349	1	1/3			
N_2O	589	2	2/3	220	3.879	0.03
	1285	1	1/3			
	2224	1	1/3			
N_2	2331	1	1/3	91.5	3.681	0.0714

2.1.1.3 Dependence on Frequency and Pressure

It is important to discuss the dependence of attenuation on the relaxation time τ , acoustic frequency f , and pressure P . The propagation of an acoustic wave through a gaseous medium results in a sudden increase in the energy of a fluid element. This leads to a redistribution of energy amongst the various translational degrees of freedom, as well as the exchange of energy between the external and internal degrees of freedom. The time constant that characterizes the former process can be described by the time between molecular collisions, τ_c . The energy transfer between internal and external degrees of freedom, however, takes significantly longer and can be characterized by a relaxation time τ (Bhatia, 1985). Let Z_c be the average number of collisions required to reduce the deficit of internal energy to e^{-1} of its value if the translational energy is suddenly increased (as described by Bhatia (1985)). Therefore,

$$\tau = Z_c \tau_c, \quad (2.20)$$

where Z_c is known as the collision number, and Z_c^{-1} is the average probability that the required amount of energy is transferred in a single collision, known as the collision efficiency.

The average time between successive collisions, τ_c , is inversely proportional to the density of the fluid, hence for an ideal gas at constant temperature $\tau_c \propto P^{-1}$. The collision efficiency Z_c^{-1} , however, depends only on the relative velocity and distance of closest approach of the colliding molecules, and is independent of P . Thus, from Equation 2.20,

$$\tau P = \text{constant}, \quad (2.21)$$

and the combination $f\tau$ may be increased by either increasing f or decreasing P . For this reason, the attenuation at a given temperature depends only on the ratio f/P and not on the individual values of f and P . This was first noted by [Richards and Reid \(1934\)](#), and is commonly used in experimental investigations since it is typically easier to vary pressure than frequency.

2.1.2 Classical and Diffusional Attenuation

The attenuation related to viscous dissipation and irreversible heat conduction is based on the classical formulation of [Stokes \(1845\)](#) and [Kirchhoff \(1868\)](#). The classical attenuation, α_{class} , is expressed as

$$\alpha_{\text{class}} = \frac{2\pi^2 f}{\rho c^2} \left(\frac{4}{3} \mu + \frac{(\gamma - 1) K}{C_p} \right), \quad (2.22)$$

where f is the frequency of the acoustic wave, ρ is the density, c is the sound speed, μ is the shear viscosity, γ is the ratio of specific heats, K is the thermal conductivity, and C_p is the specific heat at constant pressure. For a pure gas species, the shear viscosity was calculated using Sutherland's law. For a mixture, the shear viscosity is calculated using

$$\mu_{\text{mix}} = \sum_i \frac{X_i \mu_i}{X_i + \sum_{j \neq i} (X_i \phi_{ij})}, \quad (2.23)$$

with

$$\phi_{ij} = \frac{\left(1 + \left(\frac{\mu_i}{\mu_j} \right)^{\frac{1}{2}} \left(\frac{M_i}{M_j} \right)^{\frac{1}{4}} \right)^2}{\frac{4}{\sqrt{2}} \left(1 + \frac{M_i}{M_j} \right)^{\frac{1}{2}}}. \quad (2.24)$$

Here, X_i is the molar ratio of species i , M_i is the molar mass of species i , and ϕ_{ij} are Wilke's coefficients (Davidson, 1993). Similarly, from Mason and Saxena (1958), the thermal conductivity of the mixture can be expressed as

$$K_{\text{mix}} = \sum_i \frac{X_i K_i}{X_i + \sum_{j \neq i} (X_i \phi_{ij})}. \quad (2.25)$$

Diffusional attenuation is the result of local fluctuations in concentration of a gas mixture caused by temperature and pressure fluctuations associated with a propagating acoustic wave. The attenuation due to diffusion is detailed by Bhatia (1985) with the mutual diffusion coefficients, D_{12} , from Lide (2004). From Bhatia (1985), the attenuation due to diffusion in a mixture of two species is expressed as

$$\alpha_{\text{diff}} = 2\pi^2 f D_{12} \frac{\rho}{P} X_1 X_2 \left(\frac{M_1 - M_2}{M_{\text{mix}}} \right)^2, \quad (2.26)$$

where M_{mix} is the molar mass of the mixture.

As depicted in Equation 2.1, the total attenuation of a gas mixture, α , is the linear summation of the contributions from vibrational relaxation processes α_{vib} , classical attenuation α_{class} , and diffusional attenuation α_{diff} . Although the primary peak in attenuation is the result of vibrational relaxation processes (as seen in Figure 1.4), attenuation from classical and diffusional effects can be substantial above $f/P \approx 1 \text{ MHz/atm}$ (Ejakov et al., 2003).

2.2 Heated Acoustic Chamber

A heated acoustic chamber was designed to experimentally determine the attenuation coefficients of various gas mixtures at elevated temperatures. By measuring the attenuation spectra of previously unmeasured gas mixtures at a variety of temperatures and pressures, investigations into the effect of vibrational nonequilibrium on turbulence-generated noise can be better informed.

2.2.1 Acoustic Chamber Design

The acoustic chamber design was based on similar facilities from [Winter and Hill \(1967\)](#) and [Wang and Springer \(1973\)](#). Seen in Figure 2.2, the heated chamber consists of two cylindrical Watlow ceramic fiber radiative heaters surrounding a quartz tube. The 650 W cylindrical heaters have a 76.2 mm inner diameter, 177.8 mm outer diameter and a total heated length of 304.8 mm. The heaters are wrapped in 25.4 mm thick, 128 kg/m³ dense ceramic fiber insulation, and encased in a 12.7 mm thick aluminum tube. Inside the 67 mm diameter quartz tube, two quartz rods with a diameter of 22 mm and length of 635 mm are used as acoustic buffer rods to transmit and receive ultrasonic waves through the test gas. This was accomplished with a pair of Steminc SMFM21971MH transducers epoxied to the rod ends. The quartz rods are supported by optical posts on breadboards, and are secured with shaft collars clamped around a protective neoprene rubber. The distance between the quartz rods is varied with a linear translation stage, controlled by a programmable servo motor (Thorlabs KDC101). As will be discussed in Section 2.2.2, this allows the attenuation coefficient to be measured using the differential path method ([Winter and Hill, 1967](#); [Wang and Springer, 1973](#)). The true distance traveled by the translation stage is

measured with a linear variance indicator secured to the stage's edge. The percentage error in the indicated position of the stage was found to be approximately -1.1% , and the corrected distance travelled is used in the data analysis. A photograph of the acoustic chamber setup with details highlighted is seen in Figure 2.3.

A high-temperature thermocouple probe provides the control and high limit inputs into a PID controller. The thermocouple probe used is capable of measuring temperatures up to 1600 K with a response time of 0.55 s. The PID variables are tuned manually depending on the gas mixture, setpoint temperature, and pressure. Using an NI DAQ module and LabVIEW program, a second high-temperature thermocouple probe provides continuous measurements of the chamber temperature. Pressure measurements are made with an Omega transducer located along the gas supply inlet tubing, sufficiently far from the high temperatures of the test gas. For each mixture and pressure, the temperature throughout the tests had a standard deviation less than 4 K. As a result of the temperature variations and slight change in chamber volume throughout the tests, the pressure varied by approximately $\pm 0.5\%$. For all attenuation coefficients presented Section 2.3, an uncertainty of $\pm 7\%$ is conservatively estimated from the standard deviation of repeated measurements in the acoustic chamber.

The direction of the electronic signal used in the operation of the acoustic chamber is illustrated in Figure 2.4. First, a computer signals the translation stage to move a distance of 0.25 mm. After moving, the translation stage sends a trigger signal to a Tektronix AFG1022 waveform generator, which outputs a pulsed sine wave to an Apex PA08 amplifier. The amplifier then sends the resulting $64 V_{PP}$ sine wave signal to the transmitting transducer, causing the piezoelectric material to oscillate and transmit an ultrasonic wave. The ultrasonic wave propagates down the quartz buffer rod, through the test gas, and down the second quartz buffer rod, resulting in a voltage

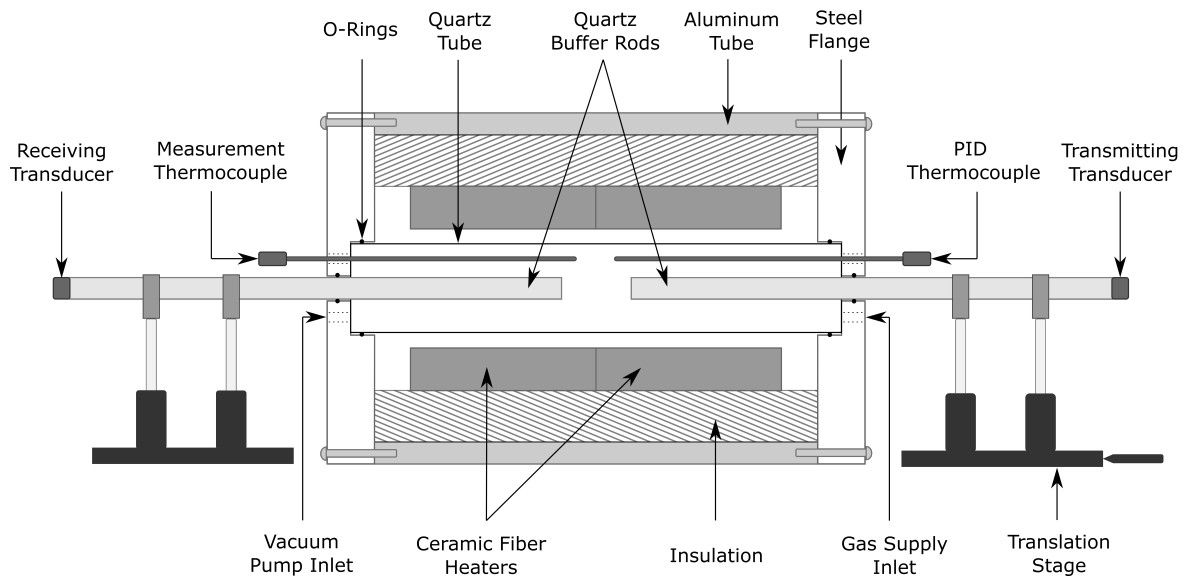


Figure 2.2: Cross section schematic of heated acoustic chamber for attenuation measurements.

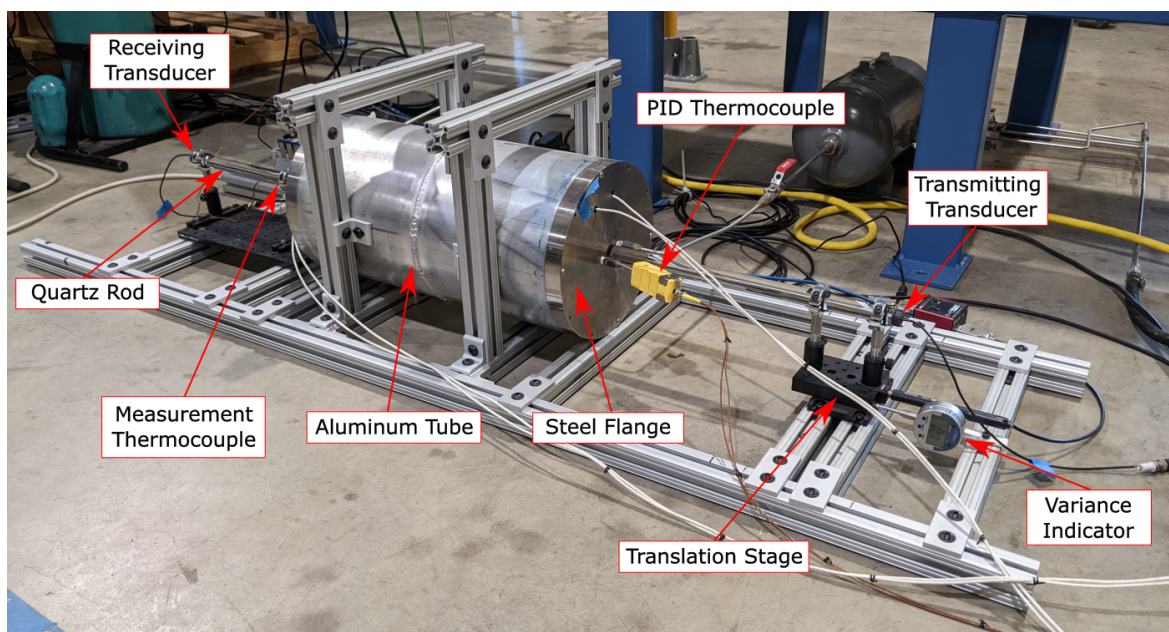


Figure 2.3: Photograph of acoustic chamber with details highlighted.

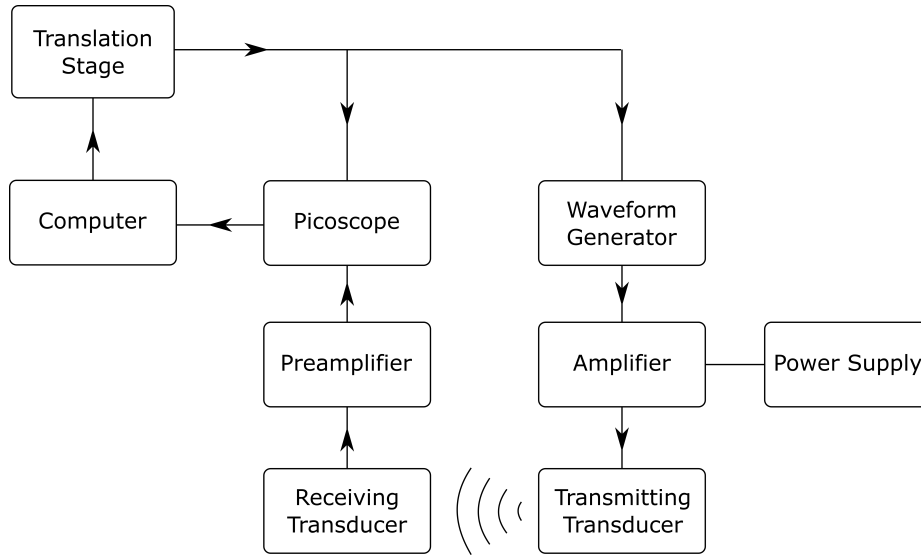


Figure 2.4: Direction of electronic signal used in the operation of the acoustic chamber.

response from the receiving transducer. The output voltage from the receiving transducer travels through a Stanford SR560 preamplifier and is sampled at 20.83 MHz with a Picoscope 5444D. The Picoscope is also triggered by the initial translation stage signal, with a delay for the ultrasonic propagation accounted for. The output voltage signal is then recorded on the computer and the process is repeated 100 times, for a total test time of approximately 8 minutes.

Since the attenuation at a given temperature depends only on the ratio of frequency/pressure (detailed in Section 2.1.1.3), either the acoustic frequency or chamber pressure can be varied. It was determined that the transducers have the strongest sub-harmonics at 50 kHz and 90 kHz; therefore, measurements were made with these acoustic frequencies at chamber pressures varying from 1–0.75 atm at 50 kHz, and 1–0.2 atm at 90 kHz. Transmitted signals consisted of 8 cycles at 90 kHz and 5 cycles at 50 kHz. At pressures lower than 0.2 atm, the signal-to-noise ratio (SNR) of the output voltage was too low for consistent measurements to be made. Additionally, the chamber was designed for vacuum pressures only, so pressures above 1 atm could not be tested. The rods are initially separated by approximately 20 mm, which was found to be sufficiently far

as to not elicit a standing wave response, while still providing a high SNR.

2.2.2 Data Analysis

The attenuation coefficient of a gas can be found experimentally with the differential path method (Winter and Hill, 1967; Wang and Springer, 1973). The acoustic pressure, P_a , decreases with distance, z , from the emitter according to

$$P_a = P_{a,0}e^{-\alpha z}, \quad (2.27)$$

where $P_{a,0}$ is the amplitude of the acoustic wave at the emitter, and α is the attenuation coefficient (Ejakov et al., 2003). By varying the distance between the emitting and receiving transducers, the attenuation can be found from the slope of the natural logarithm of voltage amplitude (proportional to acoustic pressure (Ejakov et al., 2003)) as a function of separation distance. Equation 2.27 is valid for plane waves, but a correction may be applied to account for the diffraction and divergence of acoustic waves emitted from a transducer of finite size (Ejakov et al., 2003; Wang and Springer, 1973). A correction factor computed by Pinkerton (1949) was used in the works of Ejakov et al. (2003) and Petculescu et al. (2006), but is not applicable in the case where the distance between the emitter and receiver is similar to the size of the emitter, as in this experimental setup. A similar correction was computed by Wang and Springer (1973), who determined that its application had an effect of less than 10% on the attenuation coefficient. In the current study, the measured acoustic amplitudes were found to closely match the form of Equation 2.27; therefore, no diffraction corrections were applied in order to simplify the analysis.

An example of a 90 kHz transmitted sine wave and subsequent output signal is seen in Fig-

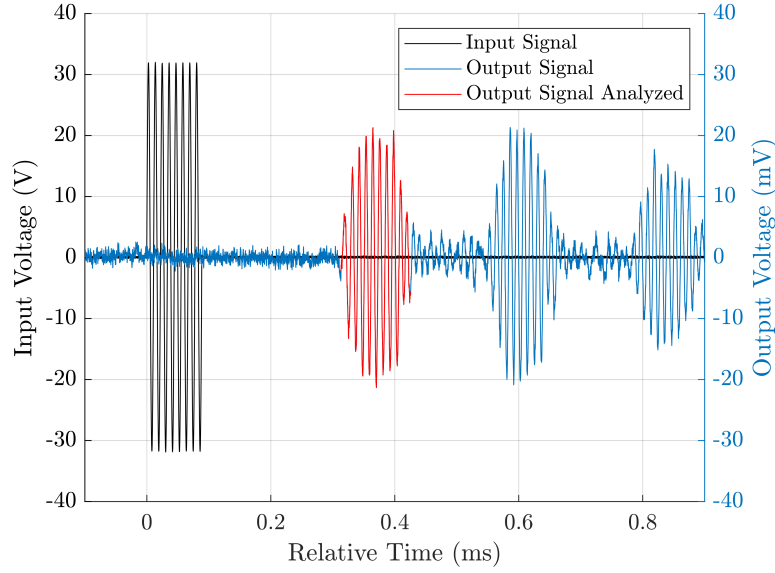


Figure 2.5: Transmitted pulse consisting of 8 cycles of a sine wave at 90 kHz (black), and the resulting output signal (blue). The initial output response (red) is the signal used in the data analysis.

Figure 2.5 for CO_2 at 1 atm and 293 K. Only the initial output signal (outlined in red in Figure 2.5) is used in the data analysis, while the reflected signals are ignored. The power of the output signal at the acoustic frequency, P_f , is computed with a power spectral density (PSD). The corresponding amplitude is then plotted as a function of separation distance between the rods, and the attenuation is found from the slope of the natural logarithm of the amplitude. Presented in Figure 2.6 is the power of the output signal as a function of distance between the rods, normalized by wavelength, for CO_2 at 1 atm and 293 K, where z_0 is the initial separation distance. In Figure 2.7, the natural logarithm of the signal amplitude, normalized by the amplitude at the initial separation distance, is plotted as a function of normalized distance between the rods for CO_2 at 293 K and various pressures. In each case, a line of best fit is plotted in red. For the 90 kHz signal shown here, decreasing the pressure below 1 atm results in a decrease in the attenuation coefficient.

The attenuation coefficient is typically plotted as the attenuation per wavelength, $\alpha\lambda$, versus

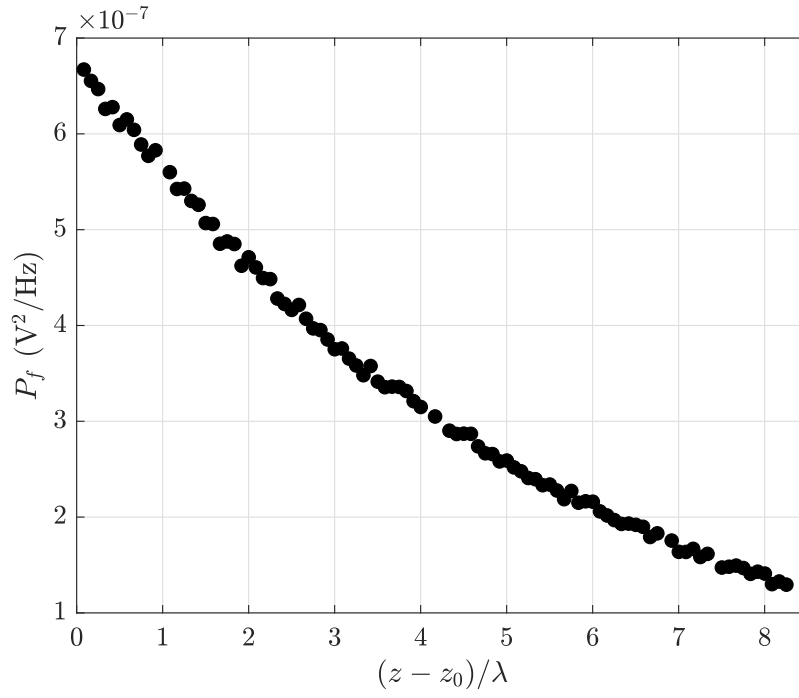


Figure 2.6: Power of the output signal at 90 kHz as a function of distance between the rods, normalized by wavelength, for CO₂ at 1 atm and 293 K.

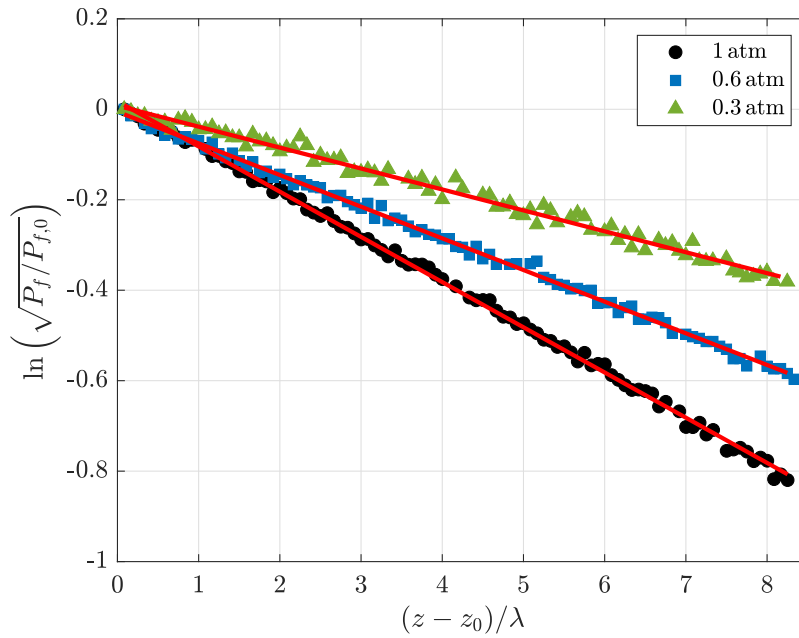


Figure 2.7: Natural logarithm of the normalized output signal amplitude at 90 kHz as a function of distance between the rods, normalized by wavelength, for CO₂ at 293 K and various pressures. For each pressure, a line of best fit is shown in red.

f/P . The wavelength of the acoustic wave is calculated from the adiabatic sound speed in the gas mixture (Zhang et al., 2020), given by

$$\lambda = \frac{\sqrt{\gamma_{\text{mix}} R_{\text{mix}} T}}{f}, \quad (2.28)$$

with

$$\gamma_{\text{mix}} = \frac{\sum_i Y_i C_{p_i}(T)}{\sum_i Y_i C_{p_i}(T) - R_{\text{mix}}}, \quad (2.29)$$

$$R_{\text{mix}} = \frac{R}{\sum_i X_i M_i}, \quad (2.30)$$

where γ_{mix} is the ratio of specific heats for the mixture, R_{mix} is the gas constant of the mixture, Y_i is the mass fraction of species i , X_i is the molar fraction of species i , and C_{p_i} is the isobaric molar heat of species i . The temperature-dependent isobaric molar heats of each species were obtained from the National Institute of Standards and Technology (NIST) Chemistry WebBook (Chase Jr and Tables, 1998).

2.3 Attenuation Coefficient Measurements

The attenuation coefficients of various gas mixtures at elevated temperatures were measured with an acoustic chamber, as detailed in Section 2.2. In this section, attenuation spectra are presented for mixtures of CO_2/N_2 and $\text{N}_2\text{O}/\text{He}$ at room temperature, and CO_2/He at temperatures up to 529 K. The experimental results of CO_2/N_2 , CO_2 , and N_2O at room temperature are compared with the attenuation model, which includes vibrational relaxation theory, classical

attenuation, and diffusional attenuation.

2.3.1 Carbon Dioxide and Nitrogen

The attenuation spectra for CO₂ and mixtures of CO₂ with 20%, 40% and 60% N₂ at 293 K are seen in Figure 2.8. Measurements were made in mixtures of CO₂/N₂ at room temperature in order to compare the present methodology to previous results in literature. For all mixtures, the theoretical attenuation model underpredicts the measured attenuation coefficients, but matches the peak absorption frequencies reasonably well; this underprediction was also noted by Ejakov et al. (2003), who stated that the discrepancy could be the result the linear structure of the CO₂ molecule not being adequately accounted for in the collisional dynamics model. Although measurements were unable to be made at values of $f/P < 50$ kHz/atm, the peak absorption measured for CO₂ ($\alpha\lambda = 0.137$ at $f/P = 50$ kHz/atm) is slightly higher than previously recorded in literature. With varying experimental setups, Leonard (1940), Fricke (1940) and Angona (1953) found the peak attenuation to be $\alpha\lambda = 0.125$ at $f/P = 30$ kHz/atm, $\alpha\lambda = 0.115$ at $f/P = 20$ kHz/atm, and $\alpha\lambda = 0.13$ at $f/P = 35$ kHz/atm, respectively. The increase of N₂ fraction in the mixture decreased the peak attenuation without significantly affecting the peak attenuation frequency. The maximum attenuation measured for the N₂ mixtures agrees well with peak attenuation measurements by Ejakov et al. (2003), who reported $\alpha\lambda$ values of approximately 0.103, 0.090 and 0.065 for 20%, 40% and 60% N₂, respectively. Compared to the maximum $\alpha\lambda$ values measured in the present work, these correspond to percent differences of 15.63%, -0.56%, and 0.01%.

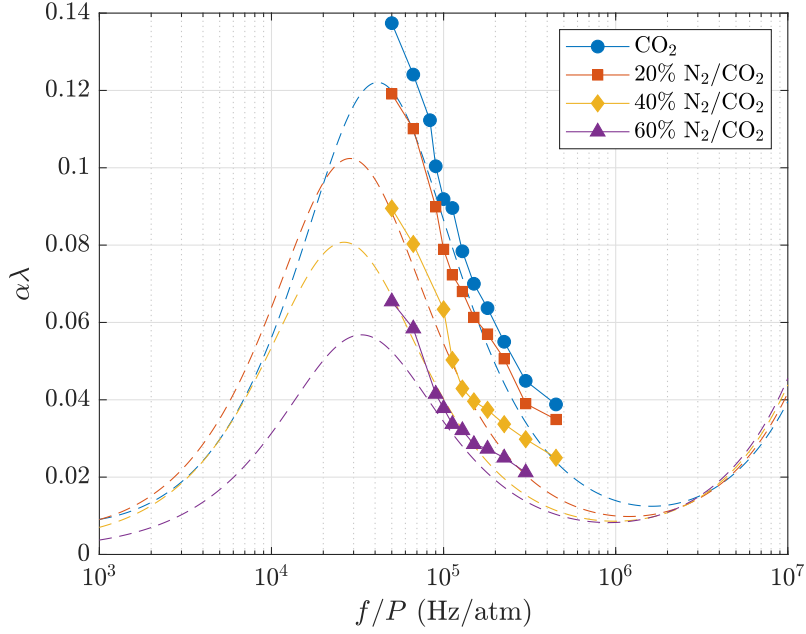


Figure 2.8: Attenuation spectra of CO₂ and mixtures of CO₂ with 20%, 40% and 60% N₂ at 293 K, plotted with the theoretical attenuation model (dashed lines).

2.3.2 Carbon Dioxide and Helium

The attenuation spectra for mixtures of CO₂ with 10%, 20%, and 40% He at 293 K are seen in Figure 2.9. Increasing the percentage of He in CO₂ results in an increase in the peak attenuation frequency, and a slight decrease in the peak attenuation. The increase in peak attenuation frequency is attributed to the light He molecules acting as an efficient collision partner with CO₂, resulting in lower relaxation times for the mixture (Lewis and Shields, 1967). The decrease in peak absorption can be attributed to the decrease of CO₂ in the mixture. Although there is limited literature to compare to, the peak attenuation of 10% He/CO₂ ($\alpha\lambda = 0.133$ at $f/P = 100$ kHz/atm) agrees well with the work of Lewis and Shields (1967), who reported a peak attenuation of $\alpha\lambda = 0.13$ at $f/P = 100$ kHz/atm for a mixture of 11% He/CO₂ at 298 K.

The attenuation spectra for pure CO₂ and mixtures of CO₂ with 20% and 40% He at 293-529 K are presented in Figure 2.10. For pure CO₂, a significant increase in attenuation is seen

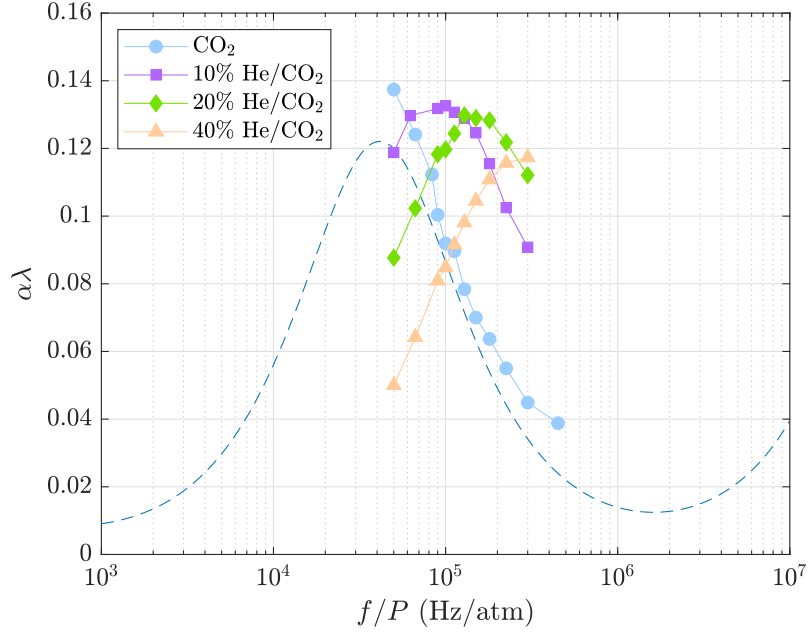


Figure 2.9: Attenuation spectra of CO₂ and mixtures of CO₂ with 10%, 20%, and 40% He at 293 K. The theoretical attenuation model for CO₂ at 293 K is plotted as a dashed line.

as the temperature is changed from 293 K to 373 K. As the temperature is increased further, the rate of increase of the peak attenuation drops until asymptoting at $\alpha\lambda \approx 0.255$ at 528 K. The peak attenuation frequency also increases to $f/P = 90$ kHz/atm. Other high-temperature attenuation results of CO₂ are limited to the work of [Shields \(1959\)](#) and [Bass \(1973\)](#) who found peak attenuation values of $\alpha\lambda = 0.22$ at $f/P = 90$ kHz/atm and 578 K, and $\alpha\lambda = 0.21$ at $f/P = 100$ kHz/atm and 600 K, respectively. Similar trends are seen for mixtures of 20% He in CO₂ at high temperatures, though the increase in peak absorption is not as significant. As the temperature is increased to 529 K, the peak attenuation reaches a value of $\alpha\lambda = 0.20$ at $f/P = 300$ kHz/atm. For mixtures of 40% He in CO₂, the attenuation appears to asymptote at lower temperatures, with a negligible change upon increasing the temperature from 423 K to 479 K.

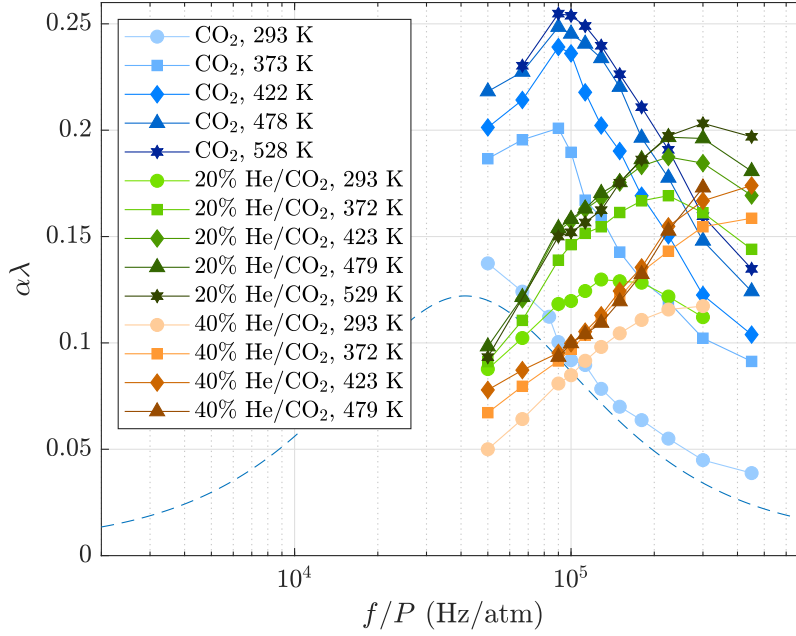


Figure 2.10: Attenuation spectra of CO₂ and mixtures of CO₂ with 20% and 40% He at 293-529 K. The theoretical attenuation model for CO₂ at 293 K is plotted as a dashed line.

2.3.3 Nitrous Oxide and Helium

The attenuation spectra for N₂O and mixtures of N₂O with 10%, 25% and 40% He at 293 K are seen in Figure 2.11. Similar to the results of CO₂, the theoretical attenuation model underpredicts the measured attenuation of N₂O. The peak attenuation of N₂O was measured to be $\alpha\lambda = 0.152$ at $f/P = 225$ kHz/atm, which agrees well with the measurements by Fricke (1940), who reported a peak attenuation of $\alpha\lambda = 0.148$ at $f/P = 157$ kHz/atm. Increasing the amount of helium in the mixture resulted in a decrease in the peak attenuation and increase in the peak attenuation frequency. Similar to the CO₂/He mixtures, this is likely caused by light He molecules acting as an efficient collision partner with N₂O. To the authors' knowledge, these are the first reported attenuation measurements of N₂O/He mixtures. High-temperature measurements were not made due to the potential for exothermic decomposition reactions of N₂O at temperatures above 423 K.

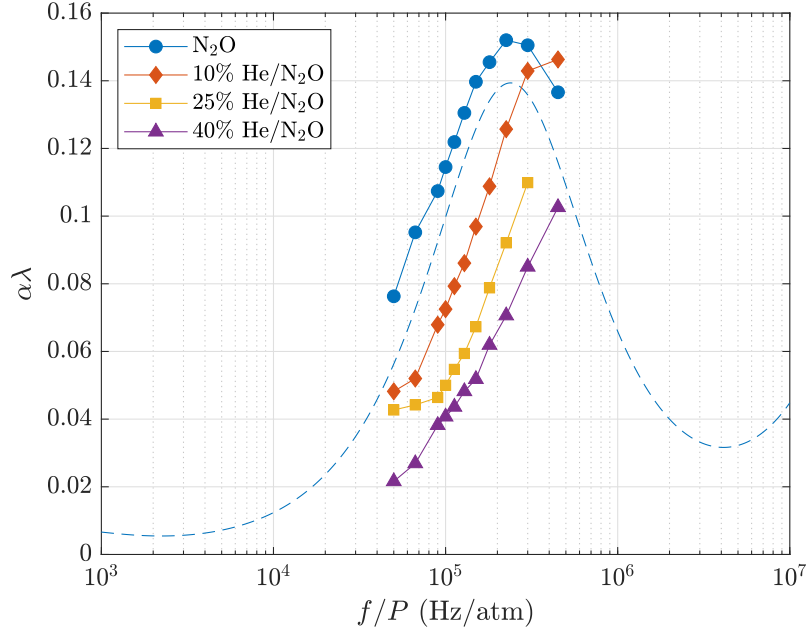


Figure 2.11: Attenuation spectra of N_2O and mixtures of N_2O with 10%, 25% and 40% He at 293 K, plotted with the theoretical attenuation model for N_2O (dashed line).

2.3.4 Summary

The effect of vibrational nonequilibrium processes on acoustic attenuation in polyatomic gases and mixtures is challenging to predict. Additionally, there exist limited measurements of acoustic attenuation to validate and compare to vibrational relaxation models. Therefore, a high-temperature acoustic chamber was developed to measure the acoustic attenuation in CO_2 , N_2O , and mixtures consisting of CO_2/N_2 , CO_2/He , and N_2O/He . The vibrational relaxation model of [Dain and Lueptow \(2001\)](#) and [Zhang et al. \(2013\)](#) was found to underpredict the peak attenuation of CO_2 and N_2O at 293 K, but matched the peak attenuation frequencies well. The peak attenuation of N_2O was slightly higher than that of CO_2 , and occurred at a higher frequency of $f/P = 225 \text{ kHz/atm}$, compared to $f/P = 40 \text{ kHz/atm}$ for CO_2 . This agrees well with the N_2O measurements by [Fricke \(1940\)](#). Increasing the percentage of N_2 in CO_2 at 293 K decreased the peak attenuation, but did not significantly change the peak attenuation frequency, as was also

found by [Ejakov et al. \(2003\)](#). In mixtures of CO₂/He at 293 K, an increase in He was found to decrease the peak attenuation, while increasing the peak attenuation frequency. Similar trends were found for mixtures of N₂O/He at 293 K. As the temperature was increased to 528 K in CO₂, the peak attenuation asymptoted to a value of $\alpha\lambda \approx 0.255$ at $f/P = 90$ kHz/atm. For mixtures of 20% He/CO₂ and 40% He/CO₂, the rate of increase in peak attenuation with temperature was less significant, and the values asymptoted at lower temperatures.

These high-temperature attenuation measurements provide insight into how CO₂ and N₂O gas mixtures could affect acoustic radiation and turbulence intensities in high-speed flows. In particular, varying the fraction of helium in mixtures of CO₂/He can shift the attenuation to a desired frequency range, providing a potential method to control the acoustic radiation. By measuring the attenuation spectra of previously unmeasured gas mixtures at a variety of temperatures and pressures, future investigations into the effect of vibrational nonequilibrium on turbulence-generated noise can be better informed.

Chapter 3: Attenuation of Acoustic Radiation in a High-Speed Facility

The gas mixtures of interest, as identified by the acoustic chamber measurements, are injected into a turbulent boundary layer in the test section of a high-speed facility, and the effects of vibrational nonequilibrium processes on acoustic radiation emitted by this compressible boundary layer are investigated.

3.1 Shock-Tunnel Facility

Injection experiments were conducted in the HyperTERP facility at the University of Maryland ([Butler and Laurence, 2019](#)). HyperTERP is a reflected-shock tunnel capable of operating at a variety of freestream enthalpy and unit Reynolds numbers. In its original configuration, HyperTERP is operated with a Mach-6 free-jet nozzle, with typical stagnation pressures of 10-25 bar and stagnation temperatures of up to 1600 K. In this configuration, HyperTERP has been utilized for studies of hypersonic boundary-layer transition ([Butler and Laurence, 2021c, 2022](#); [Paquin et al., 2022b,a](#); [Hameed et al., 2022](#)), optical free-flight tracking ([Whalen and Laurence, 2021](#); [Starshak and Laurence, 2021](#)), and shock-wave/boundary-layer interactions ([Maszkiewicz et al., 2022](#)).

Presented in Figure [3.1](#) is a schematic view of the facility as configured for the present work. The reflected-shock tunnel consists of a high pressure “driver” section and a low pressure

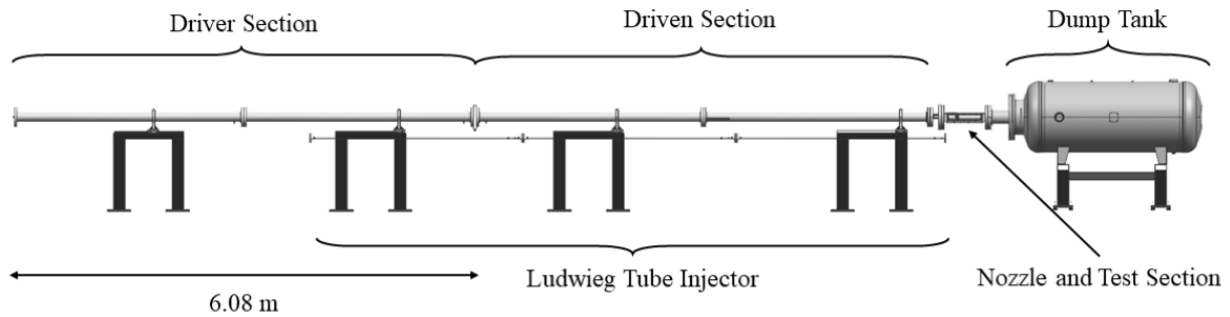


Figure 3.1: Schematic of HyperTERP shock tunnel with direct-connect test section.

“driven” section that are separated by a solenoid-actuated diaphragm assembly. The driver gas is a mixture of helium and air, and the desired composition is achieved according to Dalton’s Law of Partial Pressures. The driven section contains the test gas (air) and is isolated from the nozzle by a secondary diaphragm. The primary diaphragm assembly consists of two sheets of polyester film (thickness of 250 – 500 μm) positioned on either side of a small pressurized cavity. To initiate a shot, a solenoid valve is triggered and the small volume of gas within the cavity is exhausted into a larger vacuum tank. This creates a large pressure differential across the upstream diaphragm which causes the material to burst ([Butler, 2021](#)). This brings the driver and driven gases into contact, causing a shock wave to propagate downstream and an expansion wave to travel upstream. The incident shock is then reflected off the end-wall of the driven tube and bursts the secondary diaphragm. This results in the formation of a high-pressure, high-temperature reservoir condition which is then expanded through the nozzle. A steady-state test time is achieved up until the expansion wave is reflected from the driver tube end wall and arrives in the reservoir. Tailoring of the tunnel is achieved by selecting the driver and driven gas compositions and pressures such that the contact surface becomes stationary after the shock-wave/contact-surface interaction. When achieved, this produces constant reservoir conditions until the arrival of the expansion wave.

The propagation speed of the incident shock wave is measured by four PCB 113B26 pres-

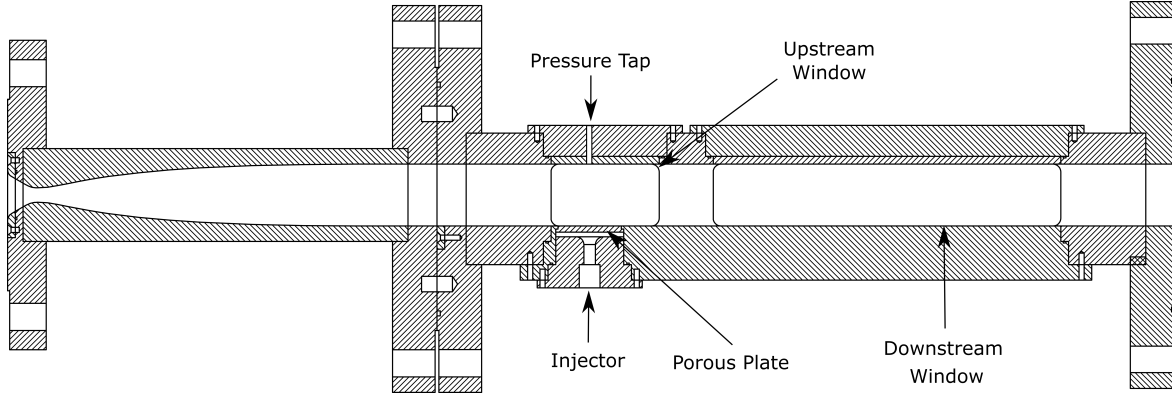


Figure 3.2: Schematic of the nozzle and direct-connect test section with porous plate injector.

sure transducers sampled at 100 kHz. The sensors are located at three different stations along the driven-tube wall, including two sensors located 8.9 cm upstream of the nozzle that serve as the reservoir measurement. The signals are conditioned using a PCB 482C05 signal conditioner.

For the present work, HyperTERP was operated with a newly developed Mach-2.8 nozzle and direct-connect test section. The planar contoured nozzle was designed with the algorithm of [Starshak et al. \(2018\)](#), and the characterization of its Mach number is detailed in Section 3.2. The test section, shown in Figure 3.2, has a cross section of $50.8 \text{ mm} \times 50.8 \text{ mm}$ and a total length of 606.5 mm. Optical access into the test section is provided by 12.7 mm thick, NBK7-glass windows with dimensions of $88.9 \text{ mm} \times 50.8 \text{ mm}$ and $285.8 \text{ mm} \times 50.8 \text{ mm}$ in the upstream and downstream regions, respectively.

Introduction of the gas mixtures into the lower boundary layer of the flow was by means of a Ludwig-tube-based injection system and porous plate. The Ludwig tube consisted of a 8.31 m long, 25.4 mm diameter pipe connected to a ASCO 8210P093 fast-acting valve which controlled the flow into the injector assembly in the test section. Located in the upstream region of the test section, the injector assembly was a smooth channel that opened up to a small plenum underneath a porous plate; approximately 4 mm separated the porous plate from the top of the

injector housing. The porous plate consisted of a sintered stainless steel material with a PORAL grade of 5 (as defined by the manufacturer) and dimensions of 41.9 mm × 52.1 mm × 5 mm. The leading edge of the porous plate was a distance of 123 mm from the nozzle exit. At the location of the porous plate, the boundary layers along the walls were in a well-developed turbulent state.

The Ludwieg tube is held at a desired pressure until the fast-acting solenoid valve is opened. After opening, the pressure at the exit of the Ludwieg tube is nearly constant through the time taken for the initial expansion fan to traverse the length of the tube and back again. Additional expansion-wave cycles occur until the pressure ratio across the solenoid valve is insufficient to maintain the flow. During each cycle, the stagnation pressure is approximately constant, but a drop in stagnation pressure occurs between cycles. Thus, if the timing between the opening of the solenoid valve and bursting of the shock-tunnel's primary diaphragm is controlled, a constant-pressure injection condition can be achieved over the steady test time. This will be discussed further in Section [3.3](#).

Two Kulite XCQ-093-250A pressure transducers are mounted upstream of the solenoid valve, with an additional two transducers installed in the injector plenum, underneath the porous plate; the signals are recorded at 100 kHz. Mean surface pressure measurements are provided by a Kulite XCQ-062-1.7BARA transducer instrumented along the spanwise centerline at a distance of 57.2 mm from the leading edge of the upstream window. The transducer was mounted behind taps of 0.5 mm diameter and 0.89 mm depth. Kulite sensors have a tendency to drift, and need to be frequently recalibrated. Therefore, a two-point calibration was conducted at the start of each day of testing; the ambient pressure was set to 1.01 bar, while full vacuum as achieved by the vacuum pump was set to 0 bar. This assumes that the linearity of the transducer was consistent. In the same facility, [Pee \(2021\)](#) conducted an error analysis of the Kulites by comparing them to

a calibrated, high-accuracy TERPS sensor (accuracy of ± 10 Pa), and it was determined that the Kulites had an error of less than 1% over a pressure range of 0-1.01 bar.

The mean reservoir and freestream conditions in the test section are summarized in Table 3.1. For all parameters, the shot-to-shot standard deviation was greater than the standard deviation from flow unsteadiness throughout the steady test time for a given shot. The reservoir and freestream pressure is measured directly, while all other parameters are calculated indirectly. To calculate these parameters, thermally perfect shock relations across the measured incident and reflected shocks provides the temperature jump caused by the incident shock. Assuming that the reflection of the incident shock results in a stationary contact surface, the initial reservoir conditions may be calculated. The gas in this theoretical state is then isentropically expanded or compressed (with a calorically imperfect gas model) to the instantaneous pressure recorded by the PCB sensors, yielding the instantaneous reservoir temperature (Butler, 2021). Finally, the gas is expanded to the freestream conditions assuming a thermally perfect gas model with freezing at the nozzle throat. The standard deviations listed in Table 3.1 for T_∞ , ρ_∞ , U_∞ , and Re_∞ were computed with error propagation from the Mach number uncertainty; the resulting standard deviations were considerably greater than those from shot-to-shot variation. The Mach number and characterization of the nozzle will be discussed in the following section. During the steady test time (approximately 5 ms), the reservoir-pressure unsteadiness is typically 2%; shot-to-shot variation in the mean pressure is on the order of 1.3%, with systematic uncertainty (calibration and nonlinearity) estimated as 1.6%. The uncertainty in the reservoir temperature from the shock-speed measurement is 0.4% (Butler and Laurence, 2021c).

Table 3.1: HyperTERP test conditions.

P_0 (bar)	13.6 ± 0.2
T_0 (K)	697 ± 4
P_∞ (bar)	0.51 ± 0.01
T_∞ (K)	272 ± 7
ρ_∞ (kg/m ³)	0.62 ± 0.04
U_∞ (m/s)	932 ± 30
Re_∞ ($\times 10^6$ m ⁻¹)	34 ± 2
M_∞	2.82 ± 0.06

3.2 Nozzle Characterization

The Mach number of the nozzle was verified through an analysis of schlieren images and freestream pressure data. The high-speed schlieren was recorded at 62 kHz; a description of the setup will be detailed in Section 3.4. From schlieren visualization through the upstream window of the test section, a sub-pixel edge detection algorithm by [Laurence and Karl \(2010\)](#) was used to measure the angle of a Mach wave throughout the steady test time. Figure 3.3 shows a schlieren image of the flowfield above the injector, with a linear regression analysis on a series of pixels that were identified by the algorithm to represent a Mach wave. Also seen in this image are weak shocks that have been generated by small discontinuities at the edges of the porous plate insert. The angle of a well defined Mach wave was found for a series of 100 images during the steady test time, and the resulting Mach number was computed using $M_\infty = 1/\sin\mu$, as plotted in Figure 3.4. This process was repeated for subsequent series of 100 images throughout the steady test time, and the computed Mach number was consistently found to be 2.84 ± 0.05 .

Alternatively, the Mach number of the flow can be calculated from reservoir and freestream pressure measurements. For an isentropic expansion through the nozzle, the Mach number can

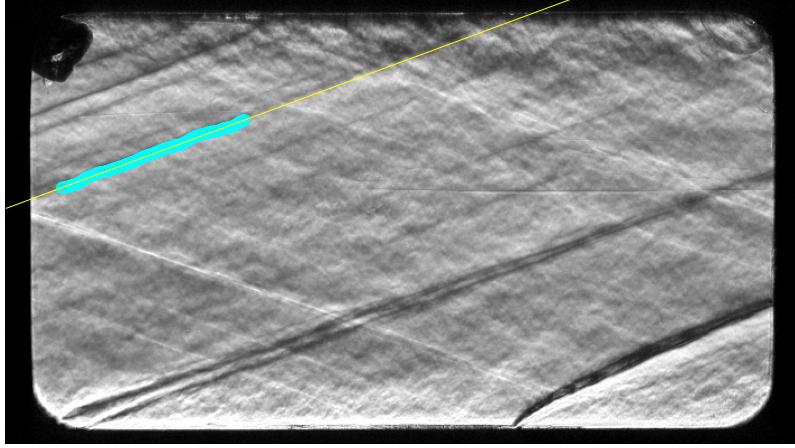


Figure 3.3: Schlieren image of the upstream flowfield used to measure the angle of a Mach wave with a sub-pixel edge detection algorithm.

be estimated with

$$\frac{P_{\infty}}{P_0} = \left(1 + \frac{\gamma - 1}{2} M_{\infty}^2\right)^{-\frac{\gamma}{\gamma - 1}}, \quad (3.1)$$

where $\gamma = 1.4$ for air. For a given shot, the total pressure measured throughout the test time was scaled with Equation 3.1 to match the freestream pressure measurements. It was determined that best agreement occurred when $M_{\infty} = 2.80$, which agrees with the Mach number from the schlieren analysis. Considering the results from both calculation methods, the Mach number of the flow was taken to be $M_{\infty} = 2.82 \pm 0.06$. This is sufficiently high that frozen Mach radiation from the turbulent boundary layers should be the most significant contribution to the freestream noise (Laufer, 1961).

3.3 Injection Conditions

Mixtures of CO_2/He are of particular interest for the injection experiments, since the addition of He in CO_2 has been previously shown by Lewis and Shields (1967) to increase the frequency of peak attenuation in CO_2 . Therefore, mixtures of CO_2/He could potentially be used

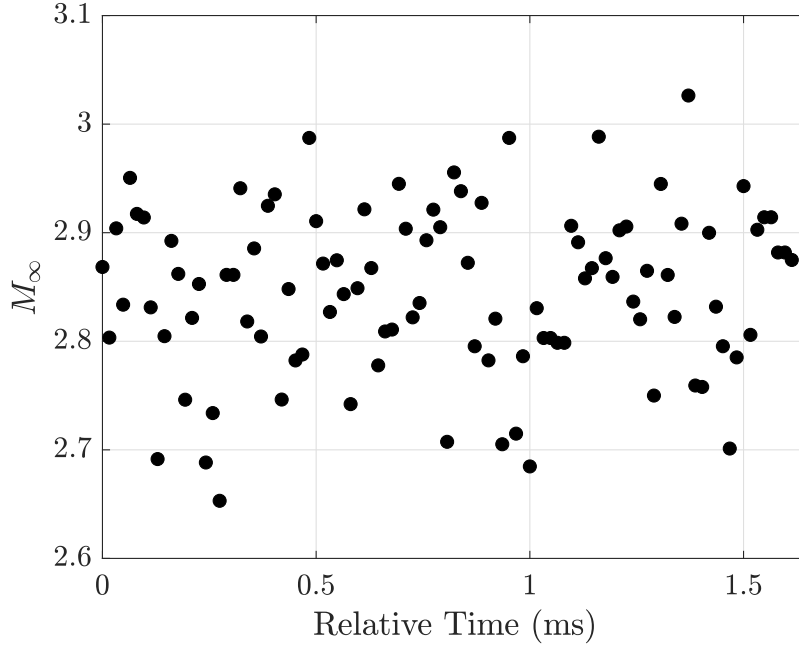


Figure 3.4: Mach number during the steady test time computed from an analysis of Mach angles.

to adjust the attenuation to certain desired frequency ranges. For this reason, the injection experiments focused on these mixtures. The test campaign consisted of multiple shots at various injection conditions, summarized in Table 3.2, where P_p is the mean pressure in the injector plenum during the steady test time. In addition to CO_2/He mixtures, injection of N_2 is used to ensure that the injection process itself is not responsible for any measured effects, and the injection of pure He is used to examine the effects of mean sound-speed gradients in the boundary layer that would also be present (to a lesser degree) in CO_2/He mixtures. In each case, the injection timing was adjusted with a variable delay circuit such that the steady plenum pressure of the injector coincided with the steady reservoir pressure in the shock tunnel. The design of the variable delay circuit is detailed by Pee (2021). A typical time trace of the reservoir pressure and injector plenum pressure is shown in Figure 3.5 for CO_2 injection at 4.18 bar, where the steady test time occurs from 7-12 ms. For all injection conditions, the plenum pressure had a standard deviation of less than 0.8% during the steady test time.

Table 3.2: Injection conditions.

Injection Gas	P_p (bar, $\pm 0.8\%$)	Number of Shots
None	N/A	12
50% He/CO ₂	4.82	2
45% He/CO ₂	3.98	3
40% He/CO ₂	4.73	4
40% He/CO ₂	3.99	3
40% He/CO ₂	3.60	3
35% He/CO ₂	4.14	3
35% He/CO ₂	3.89	3
30% He/CO ₂	3.87	3
30% He/CO ₂	3.48	3
25% He/CO ₂	4.14	3
25% He/CO ₂	3.86	3
CO ₂	4.18	3
CO ₂	3.89	6
He	4.04	3
N ₂	4.23	3
N ₂	3.95	3

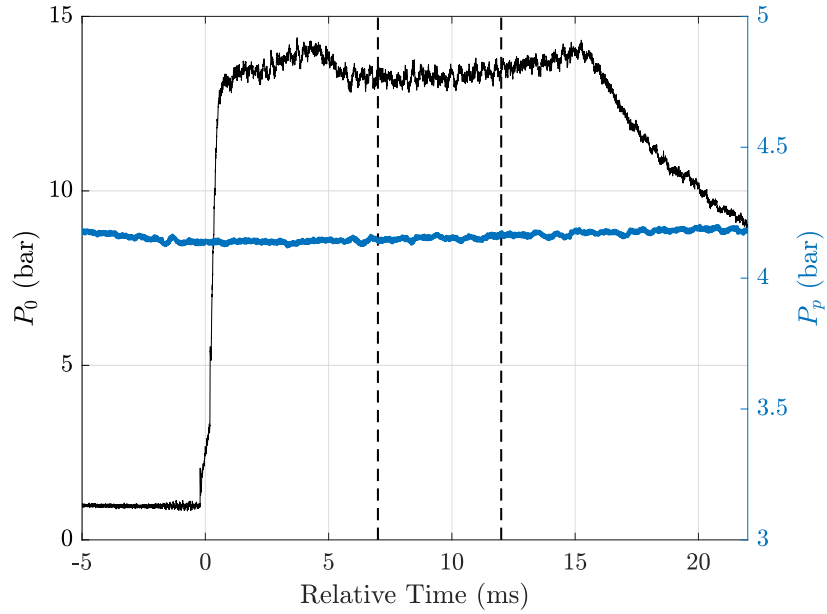


Figure 3.5: Typical reservoir pressure trace (black) and injector plenum pressure (blue) for the case of CO₂ injection at 4.18 bar. The steady test time occurs between between two dashed lines.

The plenum pressure can be used to calculate the mass flow rate through the plate for a given gas mixture. As detailed by [Moreira et al. \(2004\)](#), the flow of a compressible fluid through a porous medium is described by the Forchheimer equation:

$$\frac{P_p^2 - P_\infty^2}{2P_p L} = \alpha_c \mu \frac{\dot{m}}{\rho_p S} + \beta_c \frac{\dot{m}^2}{\rho_p S^2}, \quad (3.2)$$

where P_p is the plenum pressure underneath the porous plate, P_∞ is the pressure above the plate, L is the thickness of the plate, α_c is a friction-dependent permeability coefficient, β_c is an inertia-dependent permeability coefficient, μ is the dynamic viscosity, $\dot{m}$ is the mass flow rate, ρ_p is the density in the plenum, and S is the surface area of the plate. For the porous plate used in the present experiments, $\alpha_c = 111 \times 10^{10} \text{ m}^{-2}$, $\beta_c = 9 \times 10^5 \text{ m}^{-1}$, $L = 5 \text{ mm}$, and $S = 2.18 \times 10^{-3} \text{ m}^2$. Coefficients α_c and β_c were provided by the manufacturer, PORAL. Rearranging Equation 3.2, the mass flow rate can be expressed as

$$\dot{m} = -\frac{\alpha_c \mu S}{2\beta_c} + \left[\left(\frac{\alpha_c \mu S}{2\beta_c} \right)^2 + \frac{\rho_p S^2}{2\beta_c P_p L} (P_p^2 - P_\infty^2) \right]^{1/2}. \quad (3.3)$$

From Equation 3.3, the total mass that flows through the plate can be determined from the integration of the mass flow rate throughout the injection process. The accuracy of Equation 3.2 for the present experiments can be therefore be determined by comparing the total mass through the plate to the mass loss from the Ludwig-tube reservoir, assuming there are no leaks around the plate. Using the ideal gas law, the change in mass inside the Ludwig tube is given by

$$\Delta m_L = \frac{V}{RT} (P_{L,i} - P_{L,f}), \quad (3.4)$$

where V is the volume of the Ludwig tube, T is the temperature, $P_{L,i}$ is the pressure in the tube prior to injection, and $P_{L,f}$ is the pressure in the tube upon reaching equilibrium after injection.

The ratio of the total mass through the plate, Δm_P , to the mass loss from the Ludwig tube, Δm_L , was found for CO_2 , N_2 , and He for a range of injection pressures. In each case, the final pressure in the Ludwig tube was recorded 10 s after injection to ensure equilibrium conditions were reached. The change in mass ratio (i.e., the measured value divided by the Forchheimer prediction) is presented in Figure 3.6 for various Ludwig-tube reservoir pressures. It can be seen that the Forchheimer equation seems to overpredict the total mass through the porous plate for CO_2 , and underpredict the total mass for He. For the injection test matrix in Table 3.2, the Ludwig-tube reservoir pressures used to obtain the listed plenum pressures ranged from a low of $P_{L,i} = 4.95$ bar for He, up to $P_{L,i} = 6.40$ bar for various He/ CO_2 mixtures. Therefore, from Figure 3.6, we conclude that the Forchheimer equation (3.2) is accurate to within approximately $\pm 18\%$ for the injection conditions of the present work.

The mass flow rate through the porous plate for the injection conditions in Table 3.2 can be compared to the mass flow rate of the lower boundary layer in the test section. From schlieren visualization, the boundary-layer height at the location of the porous plate was determined to be approximately 5 mm without any injection occurring. To approximate the temperature and velocity profiles within the boundary layer, the coupled non-linear partial differential equations for a compressible laminar boundary layer were solved as outlined by White and Majdalani (2006) (which follows the transformation approach of Illingworth (1950)). The similarity solution was calculated with a cold wall temperature of 293 K (due to the short test time), a Prandtl number of 0.75, and a power-law approximation for the viscosity-temperature relation of a gas ($\mu \propto T^n$ with $n \approx 2/3$). This gives a peak temperature of 372 K in the boundary layer. The density profile is

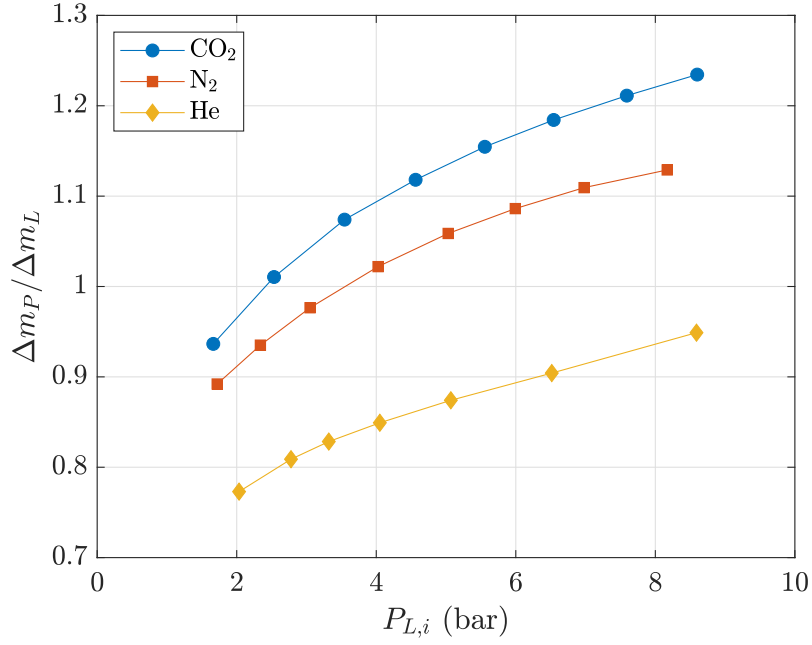


Figure 3.6: The ratio of total mass through the porous plate, calculated with the Forchheimer equation, to mass loss in the Ludwig tube for CO₂, N₂, and He at various Ludwig-tube reservoir pressures.

found with the ideal gas law, assuming a constant pressure across the boundary layer. The mass flow rate in the boundary layer is then computed with

$$\dot{m}_{BL} = W \int_0^\delta \rho(y) U(y) dy, \quad (3.5)$$

where $W = 50.8$ mm is the width of the flow region; this gives $\dot{m}_{BL} \approx 0.096$ kg/s.

The mass flow rate through the porous plate was determined with Equation 3.3, where ρ_p is computed from the ideal gas law at plenum conditions. For a mixture, the coefficient of viscosity was calculated using Equation 2.23. The injection experiments were conducted at room temperature, and there was no temperature gradient across the porous plate under the assumption of a cold wall with a temperature of 293 K; therefore, the viscosity was taken at room temperature for each mixture component. Results of these calculations are presented in Table 3.3, where

Injection Gas	P_p (bar, $\pm 0.8\%$)	$\dot{m}/\dot{m}_{BL}$
50% He/CO ₂	4.82	0.196
45% He/CO ₂	3.98	0.155
40% He/CO ₂	4.73	0.216
40% He/CO ₂	3.99	0.166
40% He/CO ₂	3.60	0.140
35% He/CO ₂	4.14	0.187
35% He/CO ₂	3.89	0.169
30% He/CO ₂	3.87	0.178
30% He/CO ₂	3.48	0.150
25% He/CO ₂	4.14	0.209
25% He/CO ₂	3.86	0.188
CO ₂	4.18	0.263
CO ₂	3.89	0.236
He	4.04	0.026
N ₂	4.23	0.173
N ₂	3.95	0.155

Table 3.3: Relative mass flow rate through the porous plate for each injection condition.

the ratio of the mean injection mass flow rates to the mass flow rate of the undisturbed lower boundary layer is provided in each condition. We see that the injected mass flux ranges from approximately 15 – 26% of the boundary-layer mass flux for He/CO₂, CO₂, and N₂ injection conditions, and is considerably less for He injection (2.6%). Further downstream, near the FLDI measurement location, the boundary-layer height without injection is roughly twice the size of that at the location of the porous plate; therefore, the injected mass flux ratios downstream are roughly half of those presented in Table 3.3.

3.4 Schlieren

A standard Z-type schlieren with a horizontal knife edge was employed to visualize the effect of injection on the external flowfield. The light source was a Cavilux HF laser with pulses of 20 ns duration. The Cavilux HF is a pulsed-diode laser that emits 640 nm light from a 1.5 mm

diameter pupil at the exit of a fiber-optic cable. This 280 W laser source provides monochromatic and low-coherence light, making it well-suited for high-speed imaging. The light was collimated through the test section and refocused on the other side by a pair of spherical mirrors with a diameter of 152.4 mm and focal length of 1524 mm. At the focal point, a knife-edge cutoff was placed horizontally to visualize the density gradients normal to the bottom wall of the test section. A Phantom v2640 with a Tamron 16-300mm F/3.5-6.3 lens was used to capture the images at a frame rate of 62 kHz and a resolution of 896×400 pixels. The schlieren was first used for visualization through the upstream window under quiescent conditions to ensure that the injection process through the porous plate was uniform and no leaks occurred around the plate's perimeter. The injection was then imaged in flow-on experiments for various gas mixtures and plenum pressures to determine the maximum pressure that could be injected such that the external flowfield was not significantly affected. Examples of flowfields are seen in Figure 3.7 for the cases of no injection, injection of He at 4.04 bar, and injection of He at 7.28 bar. It should be noted that there is a crack in the top left corner of the window, obstructing a small region of the flowfield. With no injection, weak shocks are generated by small discontinuities on either side of the porous plate insert. The weak shock at the leading edge of the plate is slightly steepened by approximately 7° for He injection at 4.04 bar, while it is significantly affected at 7.28 bar injection. If the weak shock generated at the leading edge of the porous plate is steepened by greater than 40% (approximately 8°), then the plenum pressure is deemed to be too high. Therefore, 4.04 bar was chosen as the maximum plenum pressure for He injection. This criterion was used for all injection conditions.

The flowfield in the downstream window was imaged for a variety of injection gases and pressures in order to locate an area above the lower boundary layer in which to position the FLDI

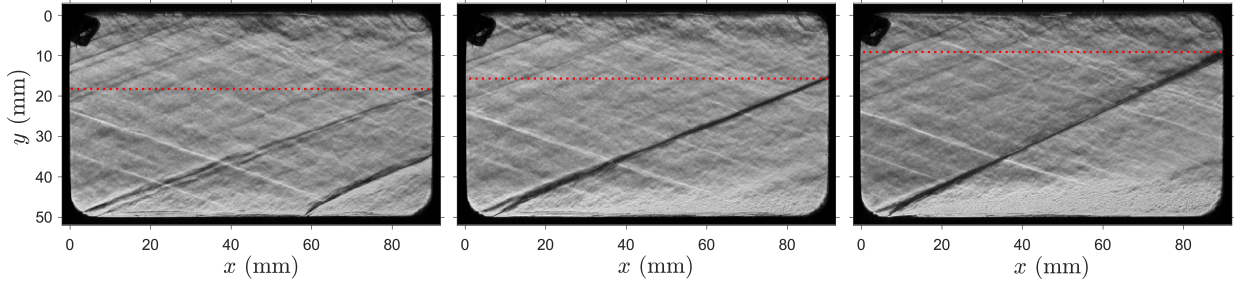


Figure 3.7: No injection (left), injection of He at 4.04 bar (middle), and injection of He at 7.28 bar (right). A dotted red line indicates the y -position at which the weak shock meets the trailing edge of the upstream window.

focal points. As a result of the narrow flowpath, weak shocks generated by small discontinuities along the test section are reflected downstream. The focal points of the FLDI should be positioned in a region of undisturbed flow, since fluctuating weak shocks would overwhelm the signal from freestream disturbances. To ensure that the spatial resolution of the schlieren imaging was consistent for each shot, an image was taken of the downstream window with a grid placed over the glass prior to each shot. The grid was made of clear acrylic and featured small holes separated by 12.7 mm. Therefore, the location of flow features could be accurately compared across different shots. A consistent area of clean flow for all injection conditions was identified 93.5 mm from the trailing edge of the downstream window, and 19.4 mm above the bottom wall, as shown in Figure 3.8 for CO_2 injection at 3.89 bar.

3.5 FLDI Setup

3.5.1 Four-Point FLDI

A four-point FLDI was used for the simultaneous measurement of fluctuations assumed to be representative of entropic disturbances propagating in the streamwise direction and acoustic radiation propagating along Mach lines. Typically, acoustic radiation is interpreted as a wave-

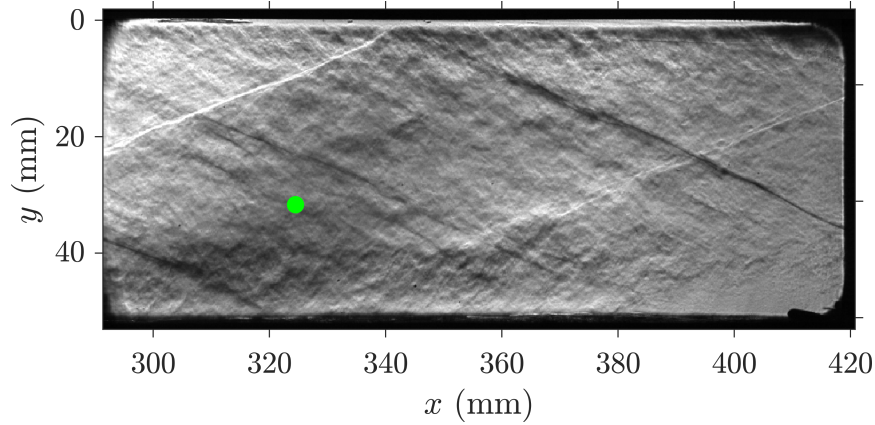


Figure 3.8: The location of the FLDI focal point in relation to a schlieren image of flow features in the downstream window from the injection of CO_2 at 3.89 bar. The x -axis is relative to the leading edge of the upstream window.

front propagating in the streamwise direction at the sound source velocity, with an inclination angle linked to the source velocity (Laufer, 1961). This wavefront is the result of a superposition of acoustic waves that originate within the boundary layer at varying streamwise locations, and is a Mach wave in the frame of reference of the moving source. Note, however, that this interpretation relies on the assumption of a “frozen” disturbance source moving at constant velocity within the boundary layer. A more general description (relying only on the assumption of weak disturbances) treats the radiation at a given point in the flowfield as an arbitrary superposition of acoustic signals propagating along Mach lines of the freestream flow from a locus of upstream positions: hence the choice of the Mach line (rather than the velocity-dependent inclination angle) in the present work.

Compared to a single point FLDI, the four-point system consists of two additional Wollaston prisms and polarizers, as well as three additional photodetectors. A schematic of the four-point FLDI is shown in Figure 3.9, with the flow in the $+x$ -direction: here, LS is the laser, C_1 is a plano-convex lens, H is a half-wave plate, LP is a linear polarizer, W is a Wollaston prism, C_2 is an achromatic doublet lens, T are the test section windows, C_3 is a plano-convex lens, C_4

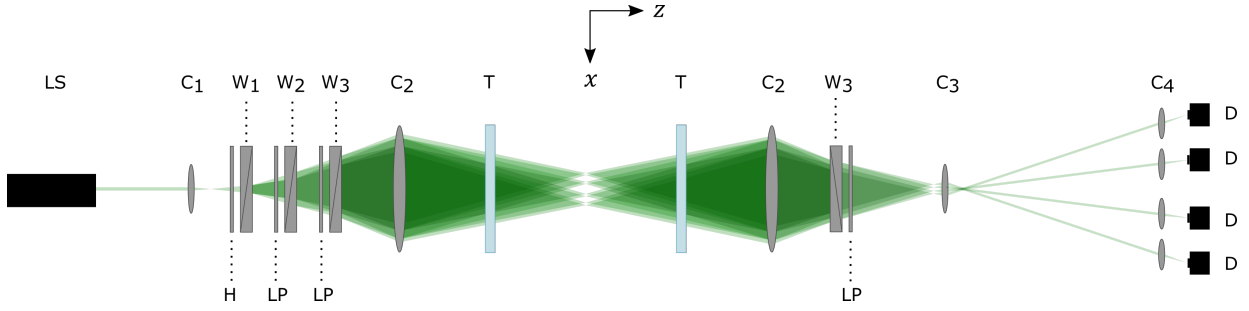


Figure 3.9: Schematic of the four-point Focused Laser Differential Interferometer.

is a bi-convex lens, and D is a photodetector. The half-wave plate and linear polarizers were model number WPH05M-532 and LPVISE200-A from Thorlabs, respectively. The FLDI signals were recorded by four DET36A2 biased Si photodetectors terminated with $50\ \Omega$ resistors. The signals were then digitized by a 14-bit Picoscope 5444D and sampled at 25 MHz. For the laser, a Cobolt Samba was used to emit 532 nm light at a power output of 400 mW. The Cobolt Samba is a diode pumped laser that is well-suited for interference measurements due to its ultra narrow spectral linewidth (± 0.3 nm), long coherence length (> 100 m), and low noise output ($< 0.1\%$ RMS). The specifications of the lenses used are the following: C_1 had a focal length of 25 mm, C_2 was a 50.8 mm-diameter lens of focal length 80 mm, C_3 had a focal length of 25 mm, and C_4 had a focal length of 40 mm. The Wollaston prisms were custom made by United Crystals to produce splitting angles of 75 arcmin, 52 arcmin and 8 arcmin for W_1 , W_2 , and W_3 , respectively. This multi-point FLDI methodology was first tested and validated in a Mach-6 freestream, as detailed in Appendix A.

3.5.2 FLDI Beam Measurements

The orientation of the beam spacings on the focal plane of the multi-point FLDI is seen in Figure 3.10, where Δx_1 and Δx_2 represent the intra- and inter-pair spacing, respectively. The

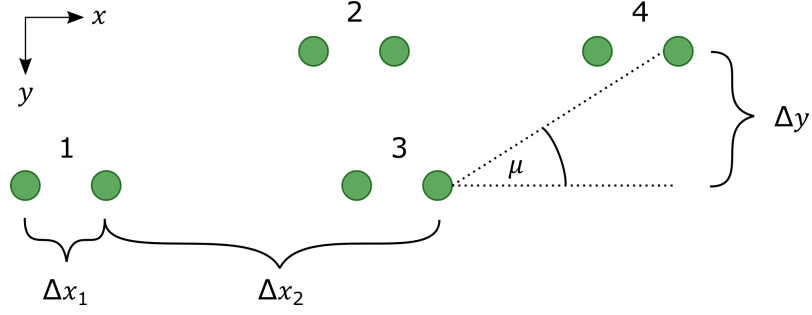


Figure 3.10: Beam spacing of the four-point Focused Laser Differential Interferometer, with $\Delta x_1 = 0.187 \pm 0.008$ mm, $\Delta x_2 = 1.070 \pm 0.008$ mm, $\Delta y = 0.365 \pm 0.008$ mm, and $\mu = 20.82 \pm 0.12^\circ$.

beam spacings were measured with a LBP2-HR-VIS2 beam profiler at the focal plane. To set the intra-pair spacing, Δx_1 , the Wollaston prism W_3 is positioned a focal-length distance away from the field lens, C_2 , such that

$$\Delta x_1 = 2f_{\text{FL},C_2} \tan\left(\frac{\epsilon_1}{2}\right), \quad (3.6)$$

where ϵ_1 is the splitting angle of prism W_1 . From Equation 3.6, $\Delta x_1 = 187 \mu\text{m}$, and the FLDI was adjusted until this separation was attained ($\pm 8 \mu\text{m}$). As seen in Figure 3.10, Wollaston prism W_2 was used to split the beams along a Mach line, where μ is intended to be the Mach angle at Mach 2.82 (i.e., 20.77°). Using the beam profiler, the spacings were measured to be $\Delta x_2 = 1.070 \pm 0.008$ mm and $\Delta y = 0.365 \pm 0.008$ mm, giving $\mu = 20.82 \pm 0.12^\circ$. The beam pairs were arranged such that the spacing between points 1-2 and 3-4 was approximately equal to Δx_2 .

An important parameter used when determining the influence of the sidewall boundary layers on the measured signal is the Gaussian beam radius, ω . Defined as the radius at which the beam irradiance reaches $1/e^2$ of its peak, ω can be found at any point along the z -axis with

$$\omega^2(z) = \omega_0^2 \left[1 + \left(\frac{\lambda_0 z}{\pi \omega_0^2} \right)^2 \right], \quad (3.7)$$

where ω_0 is the radius at the focal point, and λ_0 is the wavelength of the light (Siegman, 1986).

In the far field, the beam divergence angle is given by

$$\theta = \frac{\lambda_0}{\pi\omega_0}. \quad (3.8)$$

In order to accurately measure the Gaussian beam radius at the focal plane of the system, ω_0 , an estimate of the beam divergence angle must be computed from the slope of measured beam radii along the optical path, i.e., $\theta = \text{atan}(\Delta\omega/\Delta z)$. Since it is not possible to translate the beam profiler along the optical path inside the narrow test section, the FLDI was designed such that each half could be independently removed from the rail without altering any of the optical components. This allowed the beam radii to be measured on a benchtop, prior to installation around the test section. Using small optical windows to replicate the test section windows, the beam profiler was translated along the optical path with a manual translation stage. Images were taken at increments of 1 mm from $z - z_0 = [0 - 8]$ mm, where z_0 is the plane where the measured beam radius was the smallest. Beam intensity profiles of FLDI point 1 are seen in Figure 3.11 at two locations. Gaussian curves were fit to each beam intensity profile, and the beam radius was found from $\omega = 2\sigma$, where σ is the standard deviation of the Gaussian distribution. From the slope of the beam radius as a function of $z - z_0$, the beam waist was calculated with Equation 3.8 to be $\omega_0 = 2.5 \pm 0.5 \mu\text{m}$. This differed slightly from the measured beam waist of $5.3 \mu\text{m}$, which is likely due to the spatial limitation of the beam profiler resulting from its pixel size of $3.69 \mu\text{m} \times 3.69 \mu\text{m}$. Measurements of the beam radius along the z -axis are plotted in Figure 3.12 with the fitted linear regression between $z - z_0 = [3 - 8]$ mm.

Ideally, the instrument would also have an f-number as small as the physical constraints

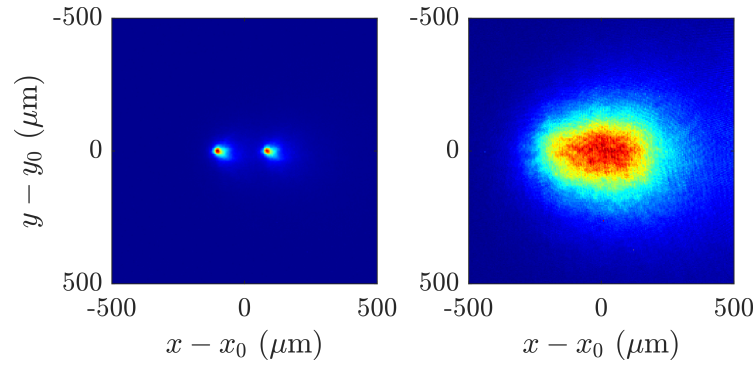


Figure 3.11: Beam intensity profiles at (left) $z - z_0 = 1$ mm and (right) $z - z_0 = 4$ mm, centered on FLDI point 1.

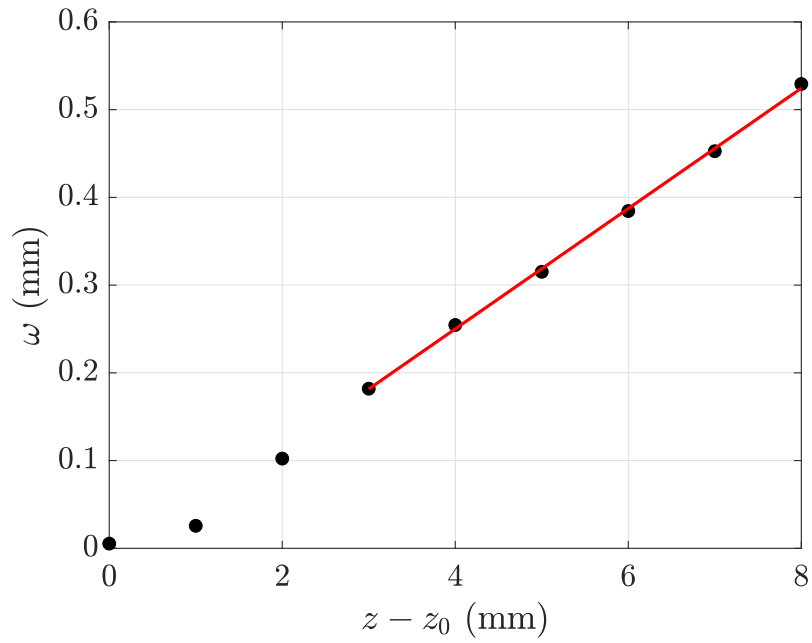


Figure 3.12: Measurements of the beam radius, ω , along the z -axis with a line of best fit ($R^2 = 0.9991$) to determine the radius at the focal point, ω_0 .

of the setup allowed, thus increasing its ability to filter flow structures away from the focus. In the case of FLDI, the f-number is defined as the distance from the field lens to the focal point divided by the beam diameter (2ω) at the field lens. The measured f-number of the four-point system was $f/7.2$. For comparison, the f-number of the single point FLDI used by [Ceruzzi et al. \(2021b\)](#) in a benchtop turbulent jet was $f/18.1$, and that of a two-point FLDI used in AEDC Hypervelocity Tunnel 9 (T9) was $f/35$. [Fulghum \(2014\)](#) also reported single point FLDI f-numbers of $f/9.8$ and $f/14.4$ in the Penn State supersonic wind tunnel (PSUSWT) and T9, respectively. In order to decrease the f-number of an FLDI, a shorter focal length objective lens, C_1 , can be used, but this often necessitates a smaller lens diameter, which can cause an Airy-disc diffraction pattern in the beams and result in poor constructive and destructive interference once recombined. Alternatively, the objective lens can be placed further from the field lens; however, due to the number of components needed for the four-point system, long optical rails and large Wollaston prism apertures are needed, restricting this possibility.

3.6 Analysis of FLDI Data

3.6.1 Instrument Model

The FLDI beams propagate through a region of interest, where index of refraction gradients create a phase shift, $\Delta\phi$, between the beams. The relationship between the output voltage and measured phase change was modified from [Parziale \(2013\)](#) by [Lawson et al. \(2019\)](#) to be

$$V = V_1 \cos(\Delta\phi - \Delta\phi_0) + V_2, \quad (3.9)$$

with $V_1 = (V_{\max} - V_{\min})/2$ and $V_2 = (V_{\max} + V_{\min})/2$. $V_{\max}$ and $V_{\min}$ are determined by observing a full interference curve of the photodetector output, which is obtained by translating the final Wollaston prism in the direction the beams are split. Prior to each shot, the final W_3 prism is translated along the x -axis using a manual translation stage and the maximum and minimum voltage of each FLDI signal is recorded. The baseline phase shift, $\Delta\phi_0$, is determined from the baseline voltage, V_0 , obtained when no density gradient is present. Equation 3.9 is a more general equation than that used by Parziale (2013) in that it allows for deviations from a perfect interference-fringe pattern, for which the photodetector would register zero voltage at total deconstructive interference and $2V_0$ at total constructive interference.

As suggested by Schmidt and Shepherd (2015) and Born and Wolf (1980), for two beams traversing paths s_1 and s_2 , respectively, the phase difference at the detector face, D , resulting from index of refraction field $n(\mathbf{x})$ can be modeled by paraxial ray-tracing according to

$$\Delta\phi(t) = \frac{2\pi}{\lambda_0} \int \int_D I_0 \left[\int_S^D n(\mathbf{x}_1, t) ds_1 - \int_S^D n(\mathbf{x}_2, t) ds_2 \right] d\xi d\eta, \quad (3.10)$$

where $\mathbf{x}_i(s_i) = (x_i, y_i, z_i)$, S is the location of the light source, ξ and η are spatial coordinates on the detector plane, and $I_0(\xi, \eta)$ is the normalized intensity on the plane when no phase change is present. For a Gaussian beam propagating along the z -axis, the intensity distribution is modeled as

$$I_0(r, \theta) = \frac{2}{\pi\omega^2} \exp\left(-\frac{2r^2}{\omega^2}\right), \quad (3.11)$$

where r and θ are polar coordinates in the xy -plane (Siegman, 1986), related to the Cartesian

coordinates by

$$x = r \cos \theta, \quad (3.12)$$

$$y = r \sin \theta. \quad (3.13)$$

The relationship between density and index of refraction is given by the Gladstone-Dale relation:

$$n = 1 + K\rho, \quad (3.14)$$

where K is the Gladstone-Dale constant, equal to $2.2705 \times 10^{-4} \text{ m}^3/\text{kg}$ for air at 288 K and $\lambda_0 = 532 \text{ nm}$ (Fomin, 1998).

Equation 3.10 can be simplified by modeling the density field as a collection of plane sinusoidal waves convecting at a streamwise velocity $\langle U_c \rangle$, whose magnitude varies with frequency f along the z -axis such that

$$\rho(x, y, z, t) = g(z) \int_{-\infty}^{\infty} A(f) e^{i(k_x x - 2\pi f t + \psi)} df. \quad (3.15)$$

Here, k_x is the disturbance wavenumber oriented parallel to the flow direction, $\psi = \psi(f)$ is the disturbance phase, $A(f)$ is the average amplitude of the density fluctuation at frequency f , and $g(z)$ models the frequency-averaged disturbance amplitude variation with z (Ceruzzi and Cadou, 2022). The wavenumber is related to the disturbance frequency, f , by

$$k_x = \frac{2\pi f}{\langle U_c \rangle}. \quad (3.16)$$

Equation 3.15 assumes one-dimensional disturbances correlated in the y and z directions. While this may not be true for tunnel noise, it allows for significant analytical simplifications of the governing equation and was recently demonstrated as a reasonable model for the FLDI's response to a turbulent jet (Ceruzzi and Cadou, 2022).

Using the density fluctuation model given by Equations 3.15 & 3.16, incorporating Equations 3.7, 3.11 & 3.14, and assuming that phases of the disturbances $\psi(f)$ are uncorrelated (Lawson, 2021), the Fourier transform of Equation 3.10 (as derived by Ceruzzi and Cadou (2022)), becomes

$$\mathcal{F}[\Delta\phi] = \frac{4\pi K}{\lambda_0} A(f) \int_{-L_i}^{L_i} |g(z)H_{\Delta x}H_{\omega}| dz, \quad (3.17)$$

where L_i is the distance from the field lens C_2 to the focal plane, and

$$H_{\Delta x} = \sin\left(\frac{k_x \Delta x_1}{2}\right), \quad (3.18)$$

$$H_{\omega} = \exp\left(-\frac{\omega^2 k_x^2}{8}\right). \quad (3.19)$$

Here, $H_{\Delta x}$ and H_{ω} are transfer functions which describe the signal modulation induced by the finite difference performed along the x -axis and the spatial averaging caused by beams with finite radius, respectively. The form of $H_{\Delta x}$ illustrates that the signal is maximized for wavenumber $k_x = \pi/\Delta x_1$, while that of H_{ω} describes how the relative size of the local beam radius and wavenumber leads to signal attenuation when either becomes large. As the beam radius grows with $|z|$, it results in a reduced sensitivity away from the focus ($z = 0$). These transfer functions have been derived in previous works with varying terminologies (Parziale et al., 2014; Fulghum, 2014; Schmidt

and Shepherd, 2015; Lawson et al., 2020) and have recently been validated for acoustic waves by Lawson et al. (2019), and for some turbulent flow fields by Ceruzzi et al. (2021b).

The left-hand-side of Equation 3.17 is the Fourier transform of the measured signal, and the right-hand-side is the response of the instrument, defined by instrument parameters λ_0 , Δx_1 , and ω_0 , and flow parameters $g(z)$, $\langle U_c \rangle$, and $A(f)$. Combining the transfer functions $H_{\Delta x} H_\omega$, Ceruzzi and Cadou (2022) define the FLDI “sensitivity function” as

$$H_{\text{FLDI}} = \int_{-L_i}^{L_i} |g(z) H_{\Delta x} H_\omega| dz. \quad (3.20)$$

Therefore, rearranging Equation 3.17 gives an expression for the average density-fluctuation spectrum in terms of the measured phase difference, instrument parameters, flow parameters, and frequency:

$$A(f) = \frac{\lambda_0}{4\pi K} \frac{\mathcal{F}[\Delta\phi]}{H_{\text{FLDI}}(\lambda_0, \omega_0, \Delta x_1, g, \langle U_c \rangle, f)}. \quad (3.21)$$

This result holds for plane sinusoidal waves convecting in the x -direction. In the case of a four-point FLDI that is simultaneously measuring disturbances convecting along streamlines and Mach lines, this result will not account for the decreased wavenumber k_x resulting from the inclined propagation of the acoustic radiation. A more general model of the flow field that incorporates disturbances convecting at an arbitrary angle is explored by Lawson (2021) as well as Hameed and Parziale (2022), however, evaluating the analytic expression of the resulting transfer function is non-trivial and beyond the scope of this work.

3.6.2 Depth of Focus

The sensitivity of the FLDI instrument as a function of wavenumber along the optical path (z -axis) is plotted in Figure 3.13a. The combined effects of $H_{\Delta x}$ and H_{ω} on the response of the FLDI illustrate that sensitivity is maximum for disturbances with $k_x = \pi/\Delta x_1$ (16800 m^{-1}) at the focus ($z = 0$). This wavenumber represents the spatial Nyquist limit of the instrument. Ceruzzi and Cadou (2022) define the instruments' depth of focus as the full width at half maximum (FWHM) of the sensitivity for a given disturbance wavenumber. This can be expressed analytically as

$$\text{FWHM} = \frac{2 \sqrt{8 \ln 2} \pi \omega_0}{\lambda_0 k_x}. \quad (3.22)$$

The FWHM is plotted as a function of wavenumber in Figure 3.13b. As the wavenumber increases past $k_x = \pi/\Delta x_1$, the depth of focus decreases slightly, i.e., the spatial resolution of the instrument increases. As the wavenumber decreases below $k_x = \pi/\Delta x_1$, the instrument's depth of focus rises rapidly, and the overall response decreases. This illustrates the FLDI's ability to resolve large wavenumber disturbances near the focal plane of the system, while small wavenumber disturbances are measured across a greater spatial extent with a decreased sensitivity.

Prior to computing the density fluctuation spectrum, the influence of the sidewall boundary layers on the FLDI signal must first be examined. Although the FLDI sensitivity decreases with distance from the focal plane, disturbances far from the focus may still overwhelm signals from disturbances near the focus as a result of their relative strength (Schmidt and Shepherd, 2015; Settles and Fulghum, 2016). To compare the signal strength of the core flow to that of the sidewall

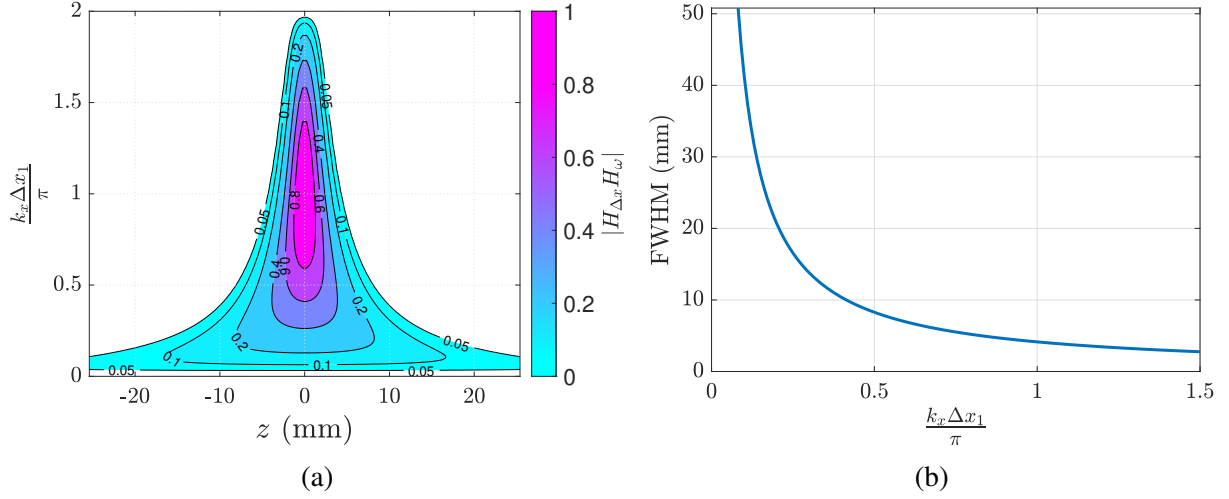


Figure 3.13: (a) Variation of the product of FLDI transfer functions $H_{\Delta x} H_{\omega}$ with wavenumber and distance z and (b) full width at half maximum (FWHM) of the FLDI transfer functions (depth of focus) as a function of wavenumber for $\lambda_0 = 532$ nm, $\omega_0 = 2.5$ μ m, and $\Delta x_1 = 187$ μ m.

boundary layers, the ratio of FLDI sensitivity functions (Equation 3.20) in the two regions is computed. Detailed by [Ceruzzi \(2022\)](#), this ratio is expressed as

$$\frac{H_m}{H_r} = \frac{\int_{-L+\delta_{BL}}^{L-\delta_{BL}} |g(z) H_{\Delta x} H_{\omega}| dz}{2 \int_{L-\delta_{BL}}^L |g(z) H_{\Delta x} H_{\omega}| dz}, \quad (3.23)$$

where H_m represents sensitivity to the region of the flow field one wants to measure, H_r represents sensitivity to the regions of flow one wishes to reject, δ_{BL} is the boundary-layer thickness, and L is the half-width of the flow region, equal to 25.4 mm. H_m/H_r is an estimated measure of the proportion of FLDI signal that arises from the core flow. A schematic of the flow region in relation to the FLDI is seen in Figure 3.14, where T is a test section window, and C_2 is the field lens.

[Zhang et al. \(2018\)](#) found that the magnitudes of density-based turbulence intensity across all wavenumbers in a Mach-2.5 boundary layer DNS were on the orders of 8–10%. In a summary of the AEDC 16-foot supersonic tunnel (PWT 16S) at Mach 2, 2.5, and 3, [Lafferty and Norris](#)

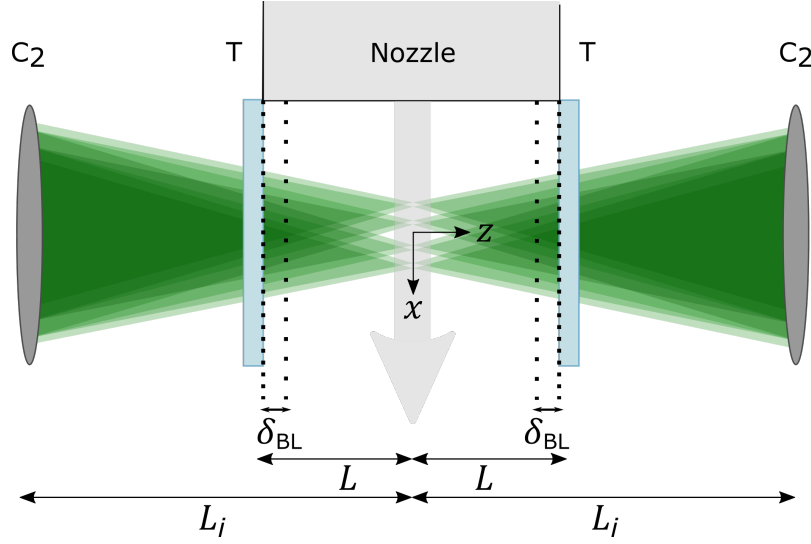


Figure 3.14: Schematic of test section in relation to FLDI beams.

(2007) found that freestream Pitot-probe RMS fluctuations were typically on the order of 0.7%, 1.5%, and 3%, respectively. In the AEDC VKF Tunnel A at Mach 3, Lafferty and Norris (2007) report a freestream RMS fluctuation of around 2 – 4%. Therefore, disturbances in the core flow are assumed to have uniform magnitude ($g = 1$), while disturbances in the sidewall boundary layers are assumed to have a magnitude 3 times greater, i.e.,

$$g(z) = \begin{cases} 1 & \text{for } |z| < L - \delta_{BL} \\ 3 & \text{for } L - \delta_{BL} < |z| < L \\ 0 & \text{otherwise.} \end{cases} \quad (3.24)$$

The boundary layer along the top wall of the test section has a thickness of approximately 1 cm, as depicted by schlieren images of the flowfield in the vicinity of the FLDI focal points (Section 3.4). It can be assumed that this boundary layer is also representative of the boundary layer along the windows.

The solution to Equation 3.23 is presented in Figure 3.15, along with the spatial Nyquist limit of the instrument ($k_x = \pi/\Delta x_1$). The H_m/H_r ratio rises rapidly for $k_x > 2000 \text{ m}^{-1}$, and

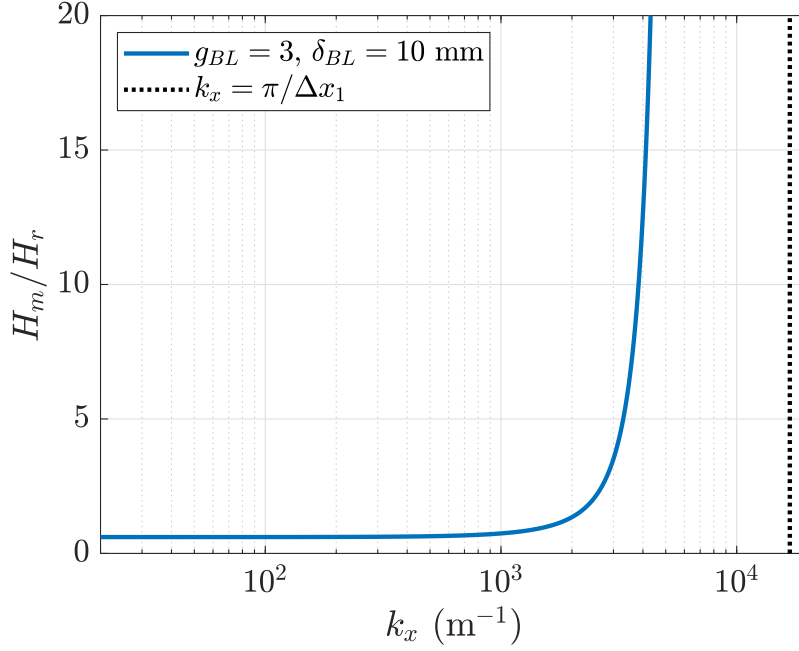


Figure 3.15: Estimated ratio of FLDI signal from the core flow, relative to the sidewall boundary layers.

exceeds 5 at $k_x \approx 3200 \text{ m}^{-1}$. This represents a highly conservative cutoff wavenumber; therefore, disturbances with $k_x > 3200 \text{ m}^{-1}$ can be attributed to the core flow. While there is some arbitrariness to defining the cutoff wavenumber for when $H_m/H_r = 5$, it is a conservative estimate and has been shown by [Ceruzzi \(2022\)](#) to agree well with trends in convection velocity and fluctuation spectra measured by a two-point FLDI in the University of Tennessee Space Institute’s Mach-4 Ludwig tube (UTM4) and the AEDC T9 facility.

3.6.3 Convection Velocity

Similar to a two-point FLDI, signal pair 1-3 and 2-4 in the four-point system (Figure 3.10) can be used to measure streamwise convection speeds. If the distance between the beam pairs is small relative to the wavelength of the disturbance, it is reasonable to assume that the turbulent structures are convected from one measurement volume to the other effectively unchanged.

Calculating the maximum cross-correlation of the signal pair gives the time delay, Δt , associated with this convection. The normalized cross-correlation between FLDI signal pairs 1-3 or 2-4 is given by

$$R_{13,24}(\Delta t) = \int_0^T \frac{\Delta\phi_{1,2}(t)\Delta\phi_{3,4}(t + \Delta t)}{\sqrt{|\Delta\phi_{1,2}|^2} \sqrt{|\Delta\phi_{3,4}|^2}} dt, \quad (3.25)$$

where T is the sample period. Therefore, the time-averaged convective speed in the x -direction is given by

$$\langle U_c \rangle = \frac{\Delta x_2}{\Delta t (\max(R_{13,24}))}. \quad (3.26)$$

In order to achieve a higher temporal resolution in the measured time delays, the maximum was determined from a 4th-order polynomial fit to the peak of the cross-correlation.

By using a multi-Wollaston prism configuration for the four-point FLDI, the beam paths through the measurement volume ($-L$ to $+L$ in Figure 3.14) are slightly non-parallel. Therefore, the streamwise separation distance, Δx_2 , varies along the optical axis. From paraxial ray-tracing of the four-point FLDI setup, it was determined that Δx_2 would decrease linearly along the optical axis. If we make the reasonable assumption of a symmetric flowfield, then the larger Δx_2 for $z < 0$ would be offset by the smaller Δx_2 for $z > 0$ when the mean velocities are computed. Thus, for the current configuration, the non-parallel beam paths are not expected to bias the resulting velocity measurements.

3.7 Measurements of Attenuated Acoustic Radiation

3.7.1 Time Trace

For all 63 shots detailed in Table 3.2, the data analysis was based on 5 ms of test time during which the reservoir conditions remained approximately constant. A sample time trace is presented in Figure 3.16 for a shot without injection, together with corresponding voltage outputs from the FLDI signals and calculated short-time spectrogram from FLDI point 1. The spectrogram was computed with Hanning windows of length 625 μs and 50% overlap. Examining the FLDI voltage signals, it is evident that there are low-frequency oscillations in the mean voltage. These oscillations have a frequency of approximately 370 Hz, and begin prior to the flow arrival at 0 ms. Therefore, they are likely caused by vibrations in the test section windows that are the result of the shock tunnel's primary diaphragm being burst. Unlike the Mach-6 configuration of HyperTERP that features a free-jet nozzle in a sliding seal, the Mach-2.8 nozzle and test section are rigidly attached to the shock-tunnel, thus transmitting the vibrations to the windows. The FLDI was floated on two pneumatically isolated optical tables, so it is unlikely that the vibrations were shaking the interferometer itself. Since the FLDI is sensitive to fluctuations in the density gradient relative to a mean value, it is important to account for these low-frequency vibrations in the data analysis. To do so, the mean voltage was calculated with a sliding window of length 240 μs over the steady test time from 10 – 15 ms, indicated by red lines in the voltage signals of Figure 3.16. Therefore, when converting from voltage to phase difference in Equation 3.9, V_0 varies with time. Throughout the steady test time, the spectrogram indicates that there are no obvious large scale trends in noise, an observation that held generally. Although Figure 3.16

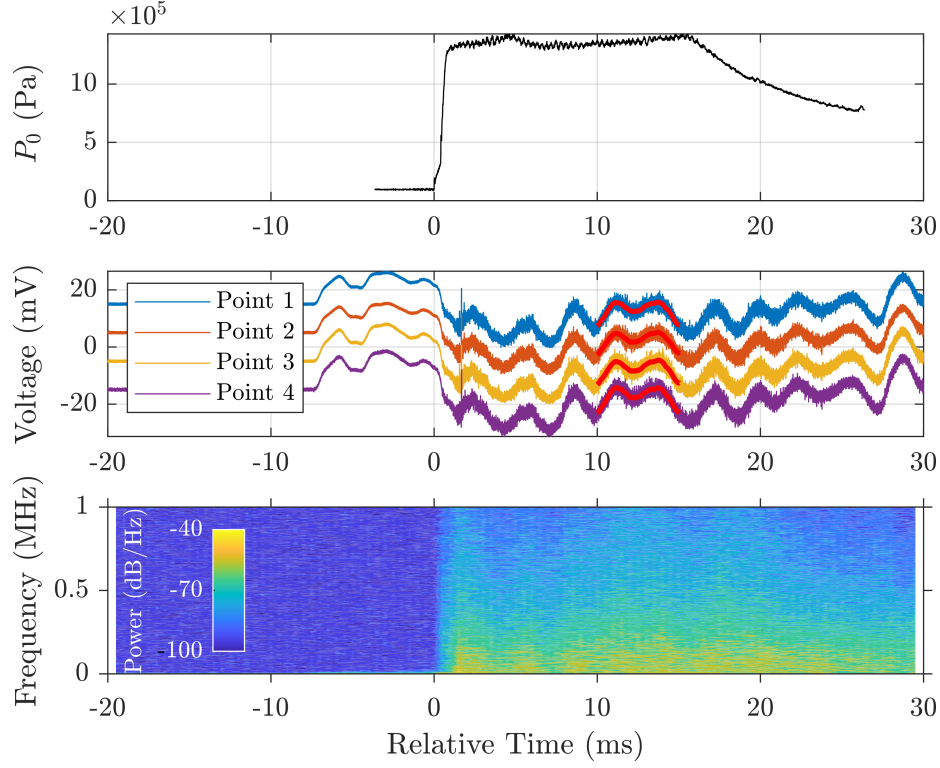


Figure 3.16: Time trace for a shot without injection. (Top) Reservoir pressure; (Middle) voltage output from FLDI signals with moving mean indicated in red, offset for clarity; (Bottom) spectrogram of FLDI point 1.

represents a shot without injection, similar findings were made for all injection conditions in Table 3.2.

3.7.2 Convection Velocity

Using the procedure described in Section 3.6.3, the streamwise convection velocities were analyzed with a cross-correlation of the streamwise FLDI signal pairs 1-3 and 2-4, computed over consecutive 1 ms intervals throughout the 5 ms test time. The resulting mean convection velocities, $\langle U_c \rangle$, and their standard deviations throughout the test time are seen in Table 3.4. For each injection condition, the reported convection velocity is the mean from signal pairs 1-3 and 2-4, and the standard deviation is computed from the square root of the mean variances from

each shot. It is evident that there are no significant differences in measured convection velocity for each injection condition, which indicates that the injection process is not affecting the external flowfield. Across all shots, the measured convection velocity is $\langle U_c \rangle = 627 \pm 31$ m/s. This is below the theoretical freestream velocity of 932 m/s, with an average percent difference of -33% . The measured FLDI convection velocities are expected to be lower than the theoretical freestream velocity, since the FLDI is unable to reject all of the low wavenumber eddies convecting in the shear layer along the optical path, and inclined frozen Mach radiation originating upstream in the turbulent boundary layers will not be convecting at the local flow velocity. Note, for example, that [Laufer \(1961\)](#) conducted freestream HWA studies at Mach 1.6 – 5, and found that acoustic radiation from turbulent boundary layers had a source velocity ranging between 40 – 50% of the freestream velocity. The convection velocities measured here are similar to those in [Ceruzzi \(2022\)](#), who used two-point FLDI to measure freestream convection velocities in a Mach-18 flow and found mean propagation speeds ranging between 52 – 84% of the freestream.

3.7.3 Fluctuation Spectra

For each injection condition, the FLDI voltage signals were converted to phase fluctuations as described in Section [3.6.1](#). The power spectral density (PSD) of the phase fluctuations was then calculated using Welch’s method with Hanning windows of length $278 \mu\text{s}$ and 50% overlap, and the amplitude spectral density (ASD) was computed from the square root of the PSD. Presented in Figure [3.17](#) are the four ASDs, together with the measurement noise floor, for a shot without injection. We see that the ASDs are essentially identical for each of the four FLDI points; similar consistency was found for all shots. A straight line with slope -1.75 (which would correspond to

Table 3.4: Mean streamwise convection velocities for each injection condition.

Injection Gas	P_p (bar, $\pm 0.8\%$)	$\langle U_c \rangle \pm \sigma$ (m/s)
None	N/A	618 ± 21
50% He/CO ₂	4.82	628 ± 25
45% He/CO ₂	3.98	623 ± 21
40% He/CO ₂	4.73	626 ± 39
40% He/CO ₂	3.99	627 ± 17
40% He/CO ₂	3.60	625 ± 17
35% He/CO ₂	4.14	631 ± 36
35% He/CO ₂	3.89	630 ± 32
30% He/CO ₂	3.87	624 ± 22
30% He/CO ₂	3.48	624 ± 24
25% He/CO ₂	4.14	629 ± 28
25% He/CO ₂	3.86	628 ± 30
CO ₂	4.18	628 ± 40
CO ₂	3.89	628 ± 50
He	4.04	637 ± 33
N ₂	4.23	621 ± 23
N ₂	3.95	627 ± 45

-3.5 for the PSD) is also plotted for reference, which is a freestream spectra roll-off that has been reported in pressure data by numerous authors, as summarized by [Duan et al. \(2019\)](#); it is seen that it also provides a reasonable match to the high-frequency data here. The ASDs of the phase fluctuations from FLDI point 1 are presented in Figure 3.18 for various injection conditions. When compared to the case without injection, no obvious attenuation in the disturbance fluctuations is seen for the injection shots. Although this might lead one to conclude that there is no attenuation present, we note that the ASDs at a single FLDI point incorporate noise contributions propagating from the bottom, top, and sidewall boundary layers, as well as entropic disturbances propagating in the streamwise direction. Therefore, the acoustic fluctuations propagating from the bottom boundary layer must be isolated to analyze the relevant acoustic attenuation. Importantly, however, the disturbance fluctuation amplitudes of the injection conditions are very similar to those without injection, indicating that the injection process is not having a significant impact

on the freestream flowfield. Thus, any measured changes in the acoustic disturbances will not be the result of the injection process itself.

In order to isolate the contributions to the overall noise from the lower-wall boundary layer and freestream entropic disturbances, the coherence was computed for the Mach-line and stream-line signal pairs. The magnitude-squared coherence for a representative signal pair a - b is defined as

$$\gamma^2(f) = \frac{|S_{ab}(f)|^2}{S_{aa}(f)S_{bb}(f)}, \quad (3.27)$$

where S_{ab} is the cross power spectral density (CPSD) of signal pair a - b , and S_{aa} and S_{bb} are the auto power spectral densities (note that Figure 3.17 demonstrates that the denominator of Equation 3.27 is essentially identical for both signal pairs). The CPSD is defined as

$$S_{ab}(f) = \sum_{m=-\infty}^{\infty} R_{ab}(m)e^{-j2\pi fm}, \quad (3.28)$$

where S_{ab} represents the distribution of power per unit frequency for the correlated signal pair. The coherence was calculated using Welch's method with Hanning windows of length 100 μ s and 50% overlap. For the calculation to be physically representative, we must account for the different times taken for the relevant disturbances to propagate between probe pairs (owing to both differing distances and propagation speeds). For streamline pair 1-3 or 2-4, this delay was specified as $\Delta t_{13,24} = \Delta x_2/U_\infty$, while for Mach-line pair 1-2 or 3-4, the delay was

$$\Delta t_{12,34} = \frac{\Delta y/\sin\mu}{\sqrt{U_\infty^2 - a_\infty^2}}, \quad (3.29)$$

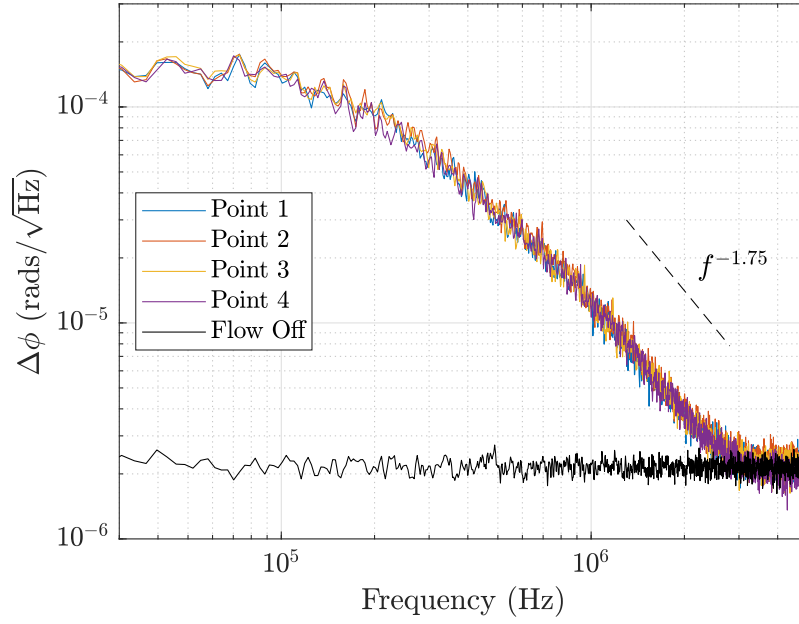


Figure 3.17: Amplitude spectral density of phase fluctuations from FLDI points 1-4 without injection.

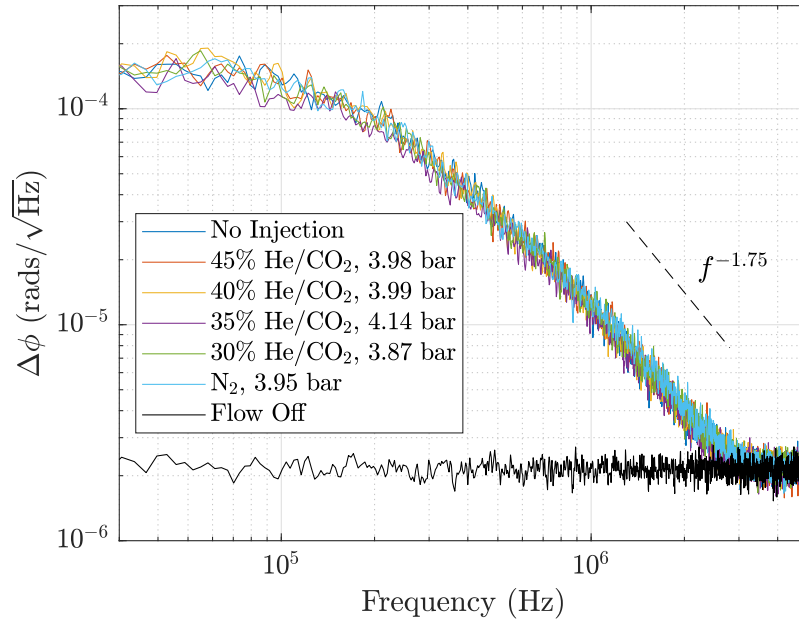


Figure 3.18: Amplitude spectral density of phase fluctuations from FLDI point 1 for various injection conditions.

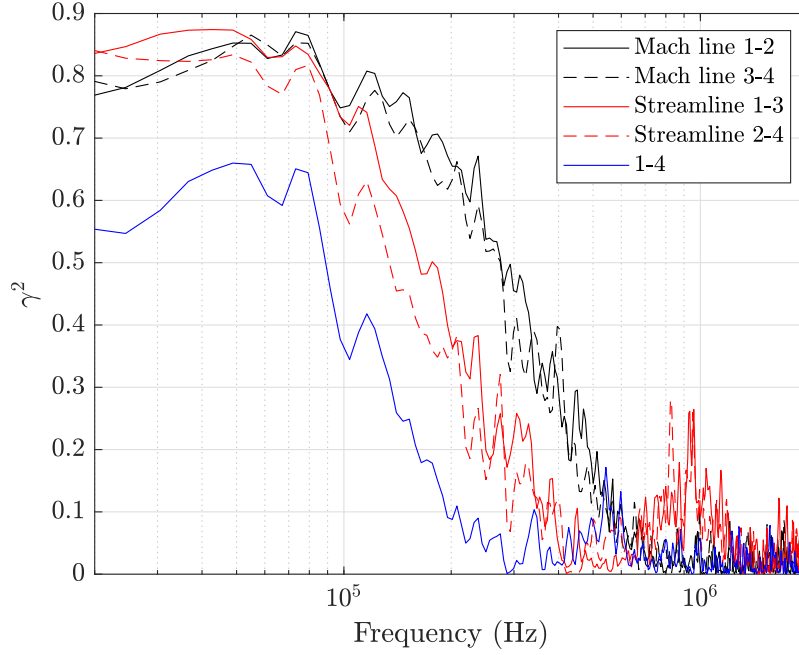


Figure 3.19: Magnitude-squared coherence of FLDI signals along a Mach line (pairs 1-2 and 3-4) and a streamline (pairs 1-3 and 2-4). For comparison, the coherence of FLDI signal pair 1-4 is also plotted.

where $\Delta y = 0.365$ mm (Figure 3.10) and a_∞ is the theoretical freestream sound speed.

The coherence of a representative shot without injection is presented in Figure 3.19. Consistent for all shots, the coherence of the Mach-line pair (1-2 or 3-4) is substantially higher than that of the streamline pair (1-3 or 3-4) over a wide range of frequencies, with the curves generally converging at low frequencies. At frequencies greater than 1 MHz, the coherence has reached its noise floor. The coherence of the FLDI signal pair 1-4 is also plotted for comparison, assuming an acoustic propagation speed of $\sqrt{U_\infty^2 - a_\infty^2}$. The decreased coherence magnitudes for signal pair 1-4 illustrates that freestream disturbances are primarily convecting along the streamline or Mach-line direction; however, further investigation could be done if the four-point FLDI was extended to a multi-line FLDI system, as detailed by Weisberger et al. (2021). This would allow for a study of the coherence between upstream and downstream signals to determine the separation angle that results in the maximum correlation (though we would expect this to occur along a

Mach line), as well as the sensitivity of the coherence to the separation angle.

While the coherence provides an estimate of the normalized cross power spectral density, only the relative magnitudes of the fluctuating signals are obtained. A characterization of the relative fluctuation powers of the correlated signals requires the CPSDs themselves to be compared. The CPSD was calculated for Mach-line and streamline signal pairs using Welch's method with Hanning windows of length $33\ \mu\text{s}$ and 50% overlap. Upon calculating the CPSD for each Mach-line pair and injection condition, Welch's t-test was used to determine if there was a statistically significant difference between the CPSD with and without injection at any particular frequency. Welch's t-test assumes the data sets have normal distributions, unequal variances and unequal sample sizes, which are valid assumptions for the CPSD measurements. The t-test was computed with MATLAB's built-in *ttest2* function with a 95% confidence interval.

Ideally, the correlated signals from FLDI pair 1-2 or 3-4 would consist uniquely of acoustic disturbances and that of pair 1-3 or 2-4 of entropic disturbances; however, in reality, these signals will not be completely independent due to the close proximity of the measurement points. Nevertheless, if we make the reasonable assumption that the relative dependence is consistent across all shots, the relation between the correlated fluctuations obtained from a Mach-line and streamline signal pair will illustrate the relative contributions of acoustic and entropic noise when compared to other shots.

Relative to shots without injection, the injection conditions that resulted in the most significant attenuation from the correlated Mach-line signals were 30% He/CO₂ at 3.87 bar, 35% He/CO₂ at 4.14 bar and 40% He/CO₂ at 3.99 bar. The mean CPSDs of these mixtures compared to the no injection case are seen in Figure 3.20 for the Mach-line pair (3.20a, 3.20c, 3.20e) and streamline pair (3.20b, 3.20d, 3.20f), respectively. For all shots, there were minimal differences

between the Mach-line signal pairs 1-2 and 3-4, and streamline signal pairs 1-3 and 2-4; therefore, the signal pairs 1-2 and 1-3 were typically used for analysis.

In Figure 3.20, the mean CPSDs are plotted with the standard deviation from all shots at that condition. Markers indicate the frequencies at which the means are unequal with 95% confidence intervals, as determined by Welch's t-test. For a given injection condition, it can be seen that the correlated signals along the Mach line have higher fluctuation powers than those along a streamline. This is also seen in the coherence magnitudes of Figure 3.19. Considering there are contributions from all four walls, this indicates that acoustic disturbances dominate the noise environment. For 30% He/CO₂ in Figure 3.20a, a clear reduction in the Mach-line CPSD was measured relative to the case without injection, particularly at frequencies above 300 kHz. In Figure 3.20b, however, no significant reductions are measured for fluctuations correlated along the corresponding streamline. This indicates that the vibrationally active gas species in the boundary layer is affecting acoustic radiation but not entropic disturbances, as expected. Similar results are found for 35% He/CO₂ (Figure 3.20c and 3.20d) and 40% He/CO₂ (Figure 3.20e and 3.20f), where a statistically significant attenuation is measured for the Mach-line fluctuations, with lesser or zero attenuation for the streamline fluctuations. Injection of 45% and 50% He/CO₂ resulted in a mean Mach-line CPSD with smaller fluctuation powers than the no-injection case; however, these differences were not statistically significant. A similar result occurred with the injection of 25% He/CO₂, but the difference was smaller still.

Turning to the reference injection cases, the mean CPSDs for injections of pure CO₂, He and N₂ are presented in Figure 3.21. Markers indicate the frequencies at which the mean CPSD from a given injection condition is statistically unequal to the mean CPSD without injection, where blue and red markers represent a decrease and increase in correlated fluctuation power,

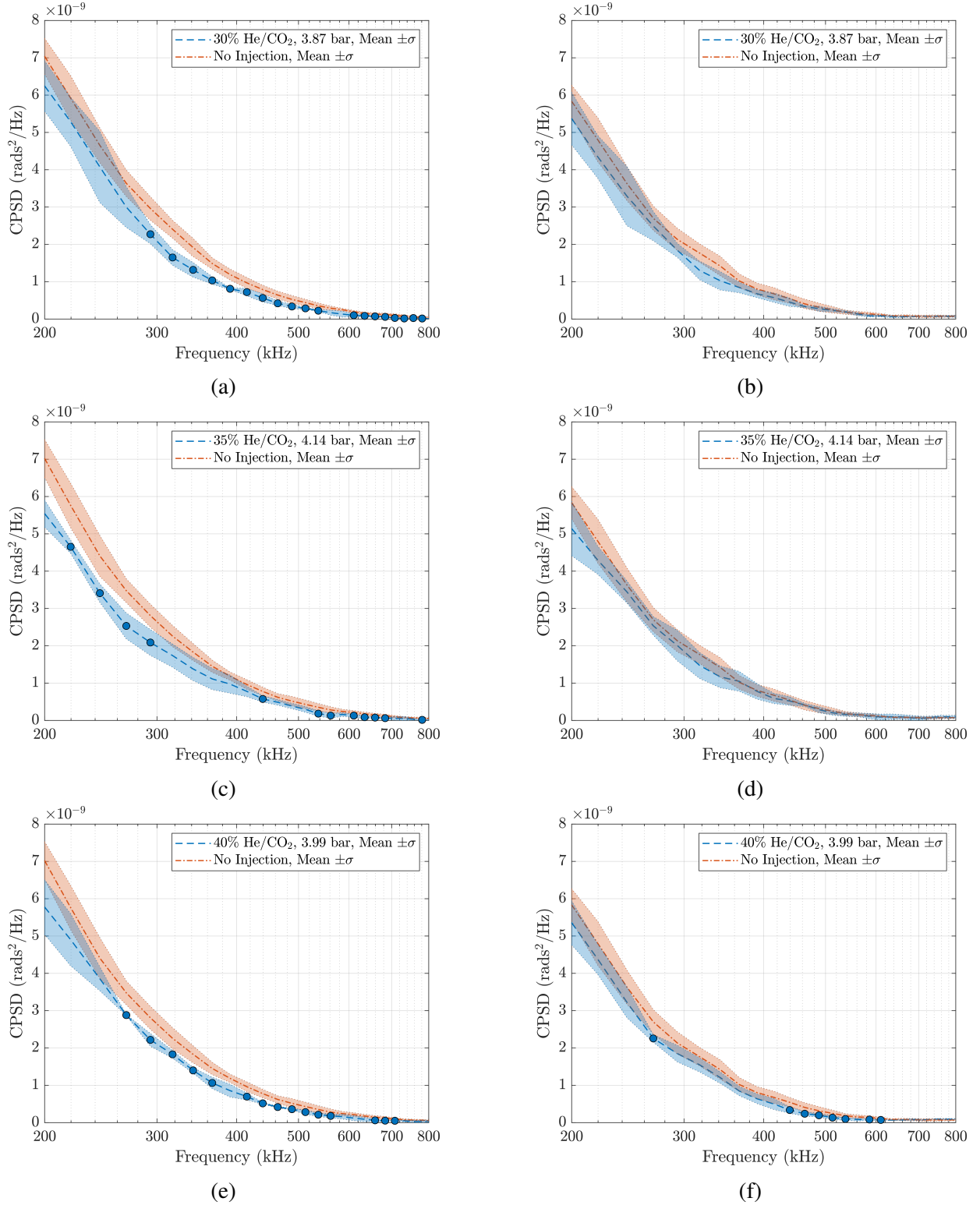
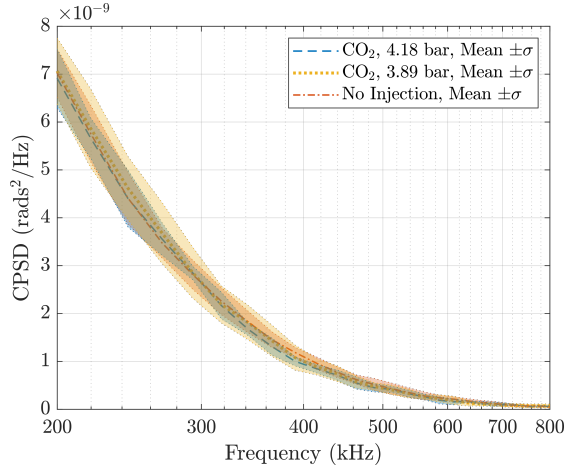


Figure 3.20: Mean cross power spectral densities of phase fluctuations along a Mach line (a, c, e) and streamline (b, d, f) for injection of 30%, 35%, and 40% He/CO₂ mixtures compared to the case of no injection. Markers indicate the frequencies at which the means are unequal with 95% confidence intervals.

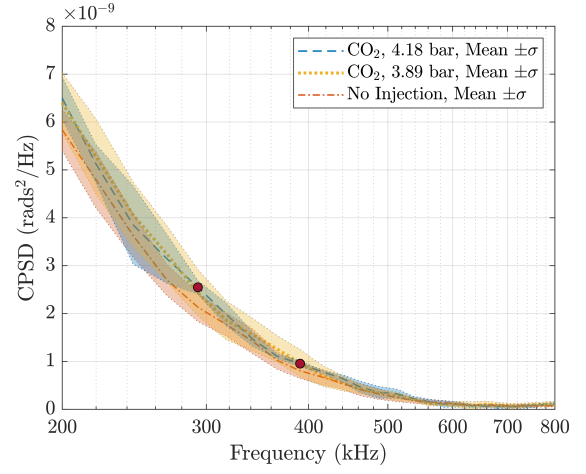
respectively. For the case of CO_2 and N_2 , circular and square markers represent the higher and lower injection pressures, respectively. In Figure 3.21a, no attenuation in acoustic fluctuations is measured for the injection of pure CO_2 . Similarly, no attenuation in streamwise disturbances is measured in Figure 3.21b, with instead a slight increase in fluctuation power occurring for CO_2 injection at 4.18 bar. A discussion as to why attenuation is observed along a Mach line for He/CO_2 mixtures but not pure CO_2 is presented in Section 3.7.4. For He injection in Figure 3.21c and 3.21d, a small reduction in acoustic fluctuation powers is measured at high frequencies, while no reductions are measured for the entropic disturbances. Although we would not expect any vibrational nonequilibrium processes to occur for a monatomic gas, it is possible that the large density gradient in the boundary layer created by the light helium molecules could lead to trapping of acoustic waves. That is, the amplitude reduction of freestream acoustic disturbances could be caused by the reflection of acoustic waves, not absorption. Such effects would be less pronounced in the mixtures, since the fraction of He is lower. This is further investigated in Appendix B. For N_2 injection in Figure 3.21e and 3.21f, essentially no attenuation was measured in the Mach-line CPSDs, with a slight increase in fluctuation powers for streamline correlations at the higher injection pressure. The lack of acoustic attenuation for the injection of N_2 further indicates that the injection process itself is not responsible for the attenuation measured in He/CO_2 mixtures (Figure 3.20).

3.7.4 Comparison Between Acoustic Chamber and FLDI Results

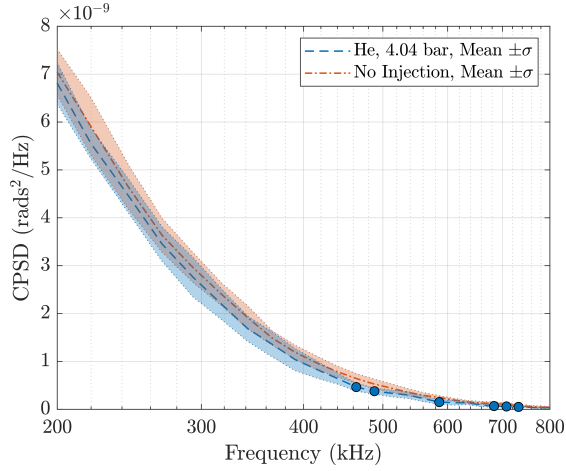
Since attenuation of acoustic fluctuations was measured for 30%, 35%, and 40% He/CO_2 mixtures, but no reductions were observed in pure CO_2 , this may be explained by the shifted



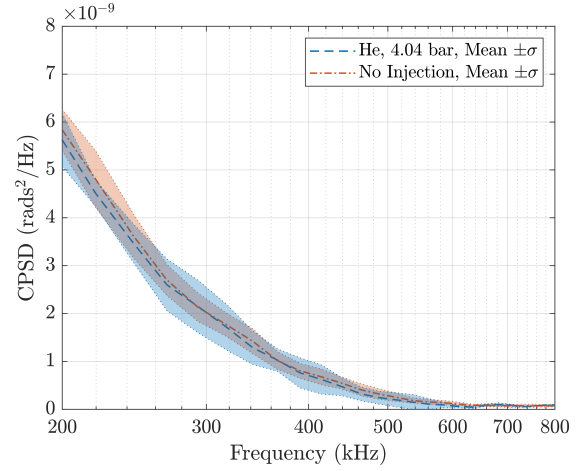
(a)



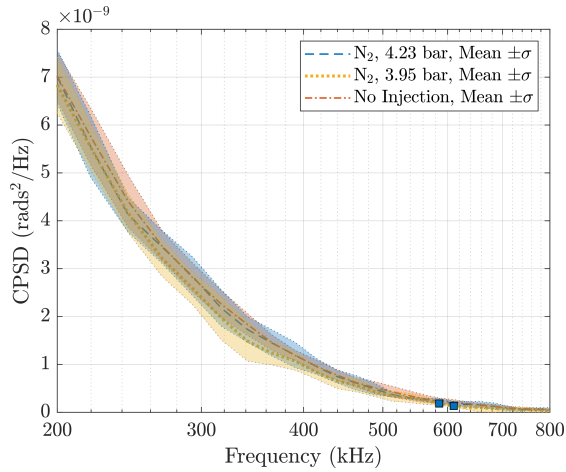
(b)



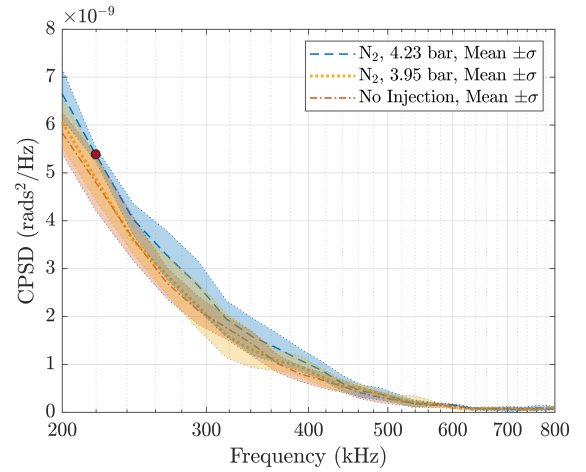
(c)



(d)



(e)



(f)

Figure 3.21: Mean cross power spectral densities of phase fluctuations along a Mach line (a, c, e) and streamline (b, d, f) for injection of CO_2 , He, and N_2 compared to the case of no injection. Markers indicate the frequencies at which the means are unequal with 95% confidence intervals.

attenuation spectra of He/CO₂ mixtures. To better understand the effect of increasing He fraction in CO₂, the attenuation spectra obtained from the acoustic chamber (Figure 2.9) are re-examined here. In Figure 3.22 we show the attenuation spectra of CO₂ and He/CO₂ mixtures at 293 K, with the theoretical attenuation model of CO₂ (Dain and Lueptow, 2001; Zhang et al., 2013) scaled and shifted in each case to match the experimental measurements. This scaled theoretical model allows for the attenuation coefficient measurements to be extrapolated to a wider range of f/P values. The scaled model can then be used to estimate the attenuation from acoustic wave propagation through a gas mixture of a given thickness and pressure. From Equation (2.27), the ratio of acoustic pressures across a gas mixture of height δ is given by

$$\frac{P_{a,2}}{P_{a,1}} = e^{-\alpha\delta}. \quad (3.30)$$

The boundary-layer height is determined from schlieren images of the flowfield in the vicinity of the FLDI focal points, which depicts $\delta \approx 1$ cm. The attenuation spectra can then be plotted as the ratio of acoustic pressures as a function of frequency by substituting in δ and the freestream pressure. Seen in Figure 3.23a, an increase in helium fraction results in a large increase in attenuation in the frequency range of 200 – 800 kHz. This frequency range is also where significant attenuation was measured from fluctuations correlated along a Mach line in 30%, 35%, and 40% He/CO₂ mixtures (Figure 3.20a, 3.20c, 3.20e), while no attenuation was measured for pure CO₂ (Figure 3.21a). Since the temperature in the boundary layer is expected to range from approximately 272 – 372 K, the theoretical attenuation model was also scaled and shifted to match attenuation spectra obtained at 373 K (Figure 2.10). The resulting acoustic pressure ratios for CO₂, 20% He/CO₂, and 40% He/CO₂ are seen in Figure 3.23b. Similar to the attenuation spectra at

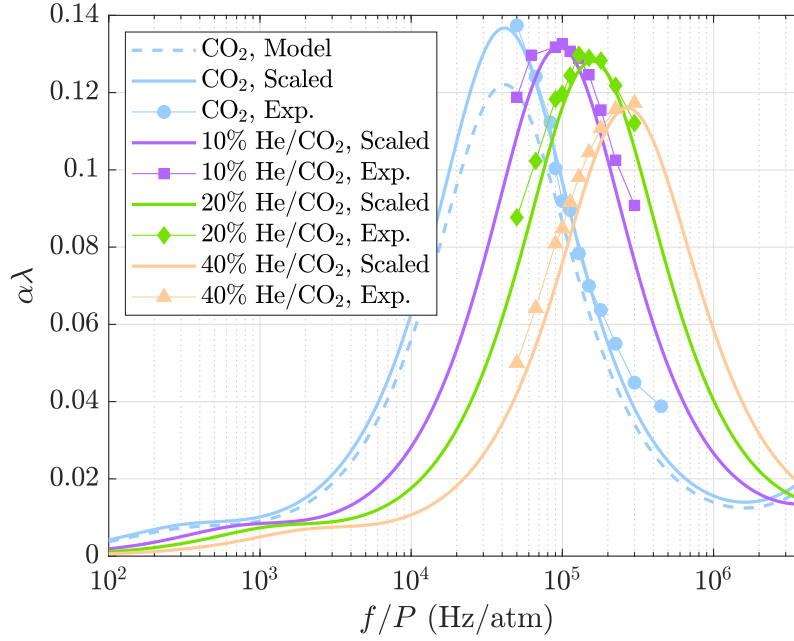


Figure 3.22: Attenuation spectra of CO₂ and mixtures of CO₂ with 10%, 20% and 40% He at 293 K with theoretical attenuation of CO₂ scaled to match experimental results.

293 K, an increase in helium fraction at 373 K results in a considerable increase in attenuation in the frequency range of 200 – 800 kHz. Therefore, the trends in measured attenuation coefficients at the expected boundary-layer temperatures are consistent with the findings of the shock-tunnel experiments.

In comparing Figure 3.20a, 3.20c, and 3.20e to Figure 3.21a, however, it should be kept in mind that the measured attenuation from a correlated Mach-line signal may also be affected by the the FLDI instrument itself. As discussed in Section 3.6.2, the low-frequency components of the FLDI signal may be corrupted by sidewall boundary layers, whereas high-frequency fluctuations are more representative of disturbances in the core flow. Therefore, by shifting the attenuation spectra to higher frequencies, the FLDI is more sensitive to freestream acoustic disturbances affected by the vibrationally active gas species. To examine this, we must revisit the discussion of cutoff wavenumber, which was conservatively chosen to be $k_x \approx 3200 \text{ m}^{-1}$. Using the measured

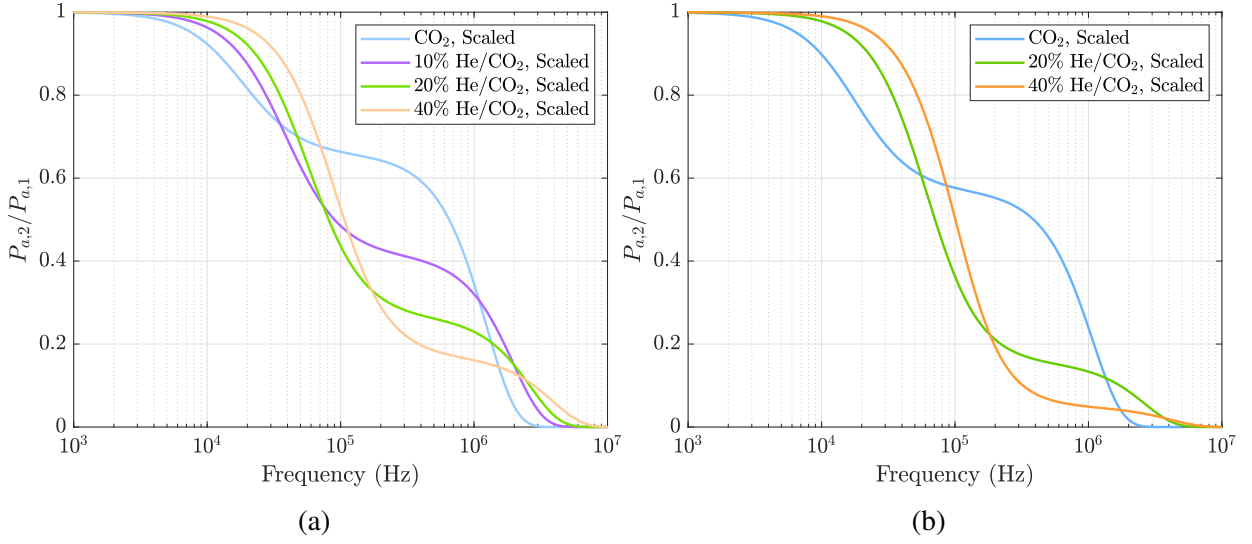


Figure 3.23: Theoretical acoustic pressure ratio across a 1 cm thick gas layer at 0.51 bar, (a) 293 K and (b) 373 K.

convection velocity of $\langle U_c \rangle = 627$ m/s, the frequency associated with this cutoff wavenumber is $f_c = \frac{k_x \langle U_c \rangle}{2\pi} \approx 319$ kHz. Disturbances above this frequency can be attributed to the core flow and are not corrupted by disturbances in the sidewall boundary layers. In Figure 3.20, the measured reductions in correlated fluctuation amplitudes along the Mach line typically occurred at frequencies of 300 kHz and higher, which is consistent with this estimated cutoff frequency.

We additionally note that Figure 3.23 assumes all acoustic waves are propagating through the vibrationally active gas species, whereas, because of the relatively close probe spacing, the correlated measurements along a Mach line are not completely independent of acoustic disturbances propagating from the top and sidewall boundary layers, and entropic disturbances propagating in the streamwise direction. Thus, the attenuation measured from correlated Mach-line signals is not as substantial as what is predicted by the theoretical attenuation spectra and it is difficult to quantify precisely the degree of attenuation within the lower boundary layer. This motivates the development of a mathematical disturbance model to provide greater insight into

the sensitivity of the measured attenuation to disturbances from the lower boundary lower only.

Chapter 4: Disturbance Model

4.1 Framework

A theoretical model was developed to replicate the dynamics of the disturbance environment in the freestream flow incorporating both entropic and acoustic disturbances propagating through the FLDI focal points. The methodology employed is as follows. First, the model parameters are tuned to match the experimental FLDI results for the case of no injection. The amplitudes of the acoustic disturbances from the lower boundary layer in the model are then reduced until the corresponding CPSD curves match those from the FLDI results for the injection of 30%, 35%, and 40% He/CO₂. Although the disturbance generation process is not directly modeled, the resulting disturbance field is consistent with the known physics of the problem, and we expect the comparison of reference and reduced disturbance fields to be physically meaningful.

The coordinate system of the model is shown in Figure 4.1, where the propagation direction is described by angles ζ and η oriented in perpendicular planes. The disturbances are modeled as Gaussian wave packets propagating in five different directions: entropic disturbances propagating in the streamwise direction, and acoustic disturbances propagating along Mach lines from the four walls of the test section. For each disturbance, a Gaussian width is prescribed in the direction perpendicular to its propagation, within the same plane as the wave packet. Therefore, for a wavevector propagating in the xy -plane ($\eta = 0$), the amplitude and phase will be constant along

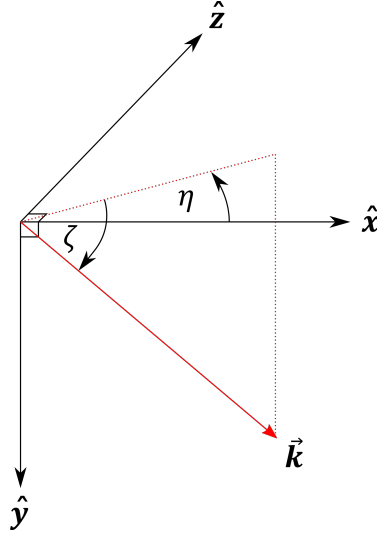


Figure 4.1: Coordinate system for a generalized plane wave with wavevector $\vec{k}$.

the z -axis at a given instant in time. An arbitrary acoustic disturbance pulse train propagating in the xy -plane is illustrated in Figure 4.2 for a single frequency at an instant in time.

While the disturbances are modeled in three-dimensional space, the simulated FLDI signals are obtained at the focal points only, i.e., the FLDI is not modeled as an integrated signal along the optical axis (z). This is acceptable for our purposes since the model will be tuned to match the experimental signals, and we are not attempting to recreate the measurement environment in its entirety. The composite freestream-disturbance signal is therefore recorded at locations corresponding to FLDI points 1, 2, and 3 (Figure 3.10) along the centerline of the test section; FLDI point 4 is redundant.

The wave packets are created by multiplying three components together: planar waves, a pulse train of Gaussian curves, and a Gaussian curve oriented perpendicular to the propagation axis. A generalized time-varying plane wave propagating in 3D space can be modeled by

$$n_{ij}(\vec{x}, t) = A_{ij} \cos(\vec{k}_{ij} \cdot \vec{x} - 2\pi f_j t + \psi_{ij}), \quad (4.1)$$

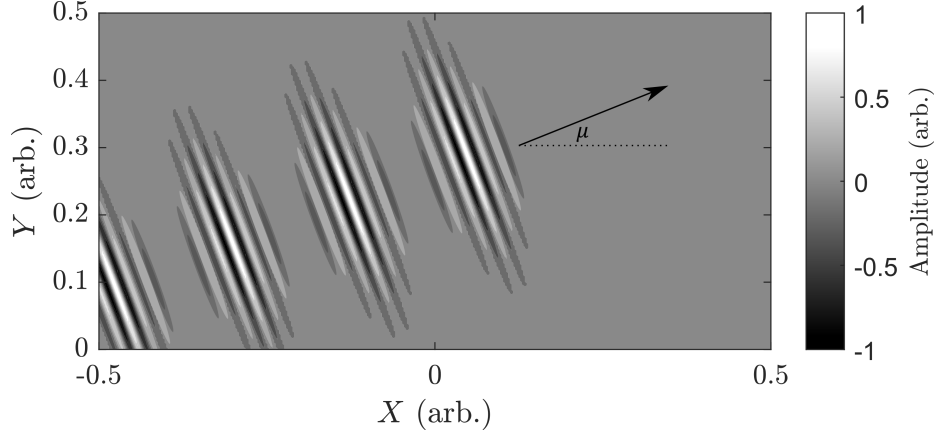


Figure 4.2: Arbitrary single-frequency acoustic pulse train propagating in the xy -plane at an instant in time.

where indices i and j represent the propagation direction and frequency, respectively, A_{ij} is the amplitude, $\vec{k}_{ij}$ is the wavevector, ψ_{ij} is a randomized phase, and $\vec{x} = (x, y, z)$. The phases ψ_{ij} are drawn randomly from the uniform distribution $[0, 2\pi)$. The wavevector is related to the disturbance frequency, f_j , by

$$k_{ij} = \frac{2\pi f_j}{U_i}, \quad (4.2)$$

where $\vec{k}_{ij} \parallel \vec{U}_i$. The propagation velocity is U_∞ for entropic disturbances, and $U_\infty \cos \mu$ for acoustic disturbances. From the coordinate system in Figure 4.1, the propagation angles are defined as

$$(\zeta, \eta) = \begin{cases} (\pm\mu, 0) & \text{for acoustic disturbances from top/bottom walls} \\ (0, \pm\mu) & \text{for acoustic disturbances from sidewalls} \\ (0, 0) & \text{for entropic disturbances.} \end{cases} \quad (4.3)$$

A pulse train of Gaussian envelopes is then created as

$$G_{1,ij}(\vec{x}, t) = \sum_{p=1}^{P_i+1} \exp \left[-\frac{\left[(\hat{k}_{ij} \cdot \vec{x} + s_{ip}) + \theta_{ij} - U_i t \right]^2}{2\sigma_{ij}^2} \right], \quad (4.4)$$

with

$$s_{ip} = \frac{U_i T}{P_i} (p - 1). \quad (4.5)$$

Here, s_{ip} is the distance between wave packets along the propagation axis, P_i is the number of wave packets that propagate during the steady test time T , θ_{ij} is the randomized phase of the wave packet, and σ_{ij} is the width of the wave packet, which is linearly proportional to the wavelength of the disturbance. The pulse train propagates at the same velocity as the disturbance, i.e., the group and phase velocity are equal.

The Gaussian curve used to define the extent of the wave packet in the direction perpendicular to propagation is computed from

$$G_{2,ij}(\vec{x}, t) = \exp \left[-\frac{\left[(\hat{k}_{ij}^\top \cdot \vec{x}) \pm \vartheta_i \right]^2}{2\epsilon_{ij}^2} \right], \quad (4.6)$$

where $\hat{k}_{ij}^\top$ lies within the xy -plane for entropic disturbances and acoustic disturbances from the top and bottom walls, and xz -plane for acoustic disturbances from the sidewalls. ϵ_{ij} is the width of the Gaussian, which is linearly proportional to the wavelength of the disturbance, and ϑ_i is a translation relative to FLDI point 1 ($\vec{x}_1 = (0, 0, 0)$ in the model) used for disturbances propagating towards FLDI point 2 or 3.

Combining Equations 4.1, 4.4, and 4.6, the overall disturbance environment is given by

$$D(\vec{x}, t) = \frac{1}{\sqrt{N}} \sum_{i=1}^M \sum_{j=1}^N G_{2,ij} G_{1,ij} n_{ij}, \quad (4.7)$$

where N and M are the number of frequency components and propagation directions, respectively. A scaling factor of $1/\sqrt{N}$ is used to keep the root-mean-square (RMS) constant as N increases for a fixed bandwidth (Lawson, 2021). The disturbance environment, $D(\vec{x}, t)$, is computed for the case of wave packets centered on FLDI point 1, 2, or 3. The total fluctuation signal is then obtained from the summation of these three cases, that is,

$$D_{\text{tot}}(\vec{x}, t) = D_{\text{pt.1}}(\vec{x}, t) + D_{\text{pt.2}}(\vec{x}, t) + D_{\text{pt.3}}(\vec{x}, t). \quad (4.8)$$

Finally, to match the electronic noise floor of the FLDI, Gaussian white noise was added.

4.2 Results

The model was first tuned to match the experimental results for the case of no injection. The amplitudes A_{ij} , wave packet width σ_{ij} , and Gaussian width ϵ_{ij} were varied manually until the modeled signal provided a reasonable match to the ASD and CPSD results from the FLDI. All acoustic disturbances were treated identically. The model was computed with $N = 800$, a bandwidth of $f = [1 \times 10^4, 3 \times 10^6]$ Hz, and a sample rate of 6.77 MHz. Since the CPSD is dependent on phase correlations, it was found that the modeled CPSD varied slightly for each computation as a result of the randomized phases. Therefore, the disturbance environment was iterated 60 times until the mean CPSDs reached convergence. For the ASD analysis, only one

iteration was needed.

The PSD of the modeled fluctuations was calculated using Welch's method with Hanning windows of length $278\ \mu\text{s}$ and 50% overlap, consistent with the parameters used in the experimental data analysis. The ASD was computed from the square root of the PSD. Presented in Figure 4.3 is the ASD obtained at the location of FLDI points 1, 2 and 3 without any attenuation applied to the disturbances, as well as at FLDI point 2 with a 30% amplitude reduction for acoustic disturbances from the bottom wall. Compared to the experimental results in Figure 3.18, the model matches the roll-off slope and high-frequency fluctuation amplitudes well. The model, however, has slightly higher fluctuation amplitudes around 300 kHz. It can also be seen that the ASD is essentially identical at each FLDI point, both with and without attenuation applied. This was noted in the experimental results, and further indicates that a CPSD analysis is needed to assess the attenuation effects on the fluctuations.

The CPSD was calculated for modeled Mach-line and streamline signal pairs using Welch's method with Hanning windows of length $33\ \mu\text{s}$ and 50% overlap, which again matches the parameters used in the experimental analysis. In Figure 4.4, the Mach-line and streamline CPSDs for modeled fluctuations without any amplitude reductions are compared to the mean experimental CPSDs from the case of no injection. For the experimental Mach-line CPSD without injection, the model with no amplitude reductions matches very well across all frequencies. The modeled CPSD along a streamline without amplitude reductions provides a reasonable match to the experimental results without injection, with a slight discrepancy around 400 – 600 kHz.

In Figure 4.5, the CPSDs are presented for modeled fluctuations in cases for which the amplitudes of the acoustic disturbances from the bottom wall have been reduced by 0%, 15%, 30%, and 45%. The modeled CPSDs are compared to the mean experimental CPSDs along a

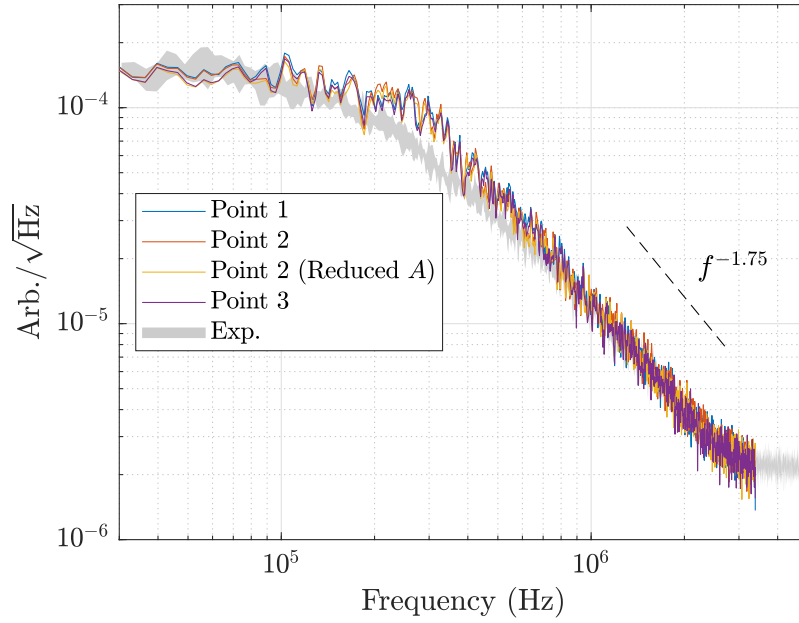


Figure 4.3: Amplitude spectral density of modeled fluctuations at FLDI points 1, 2, and 3 without any attenuation and FLDI point 2 with a 30% amplitude reduction for acoustic disturbances from the bottom wall. The range of experimentally determined fluctuation amplitudes from various injection conditions (Figure 3.18) is shown in gray.

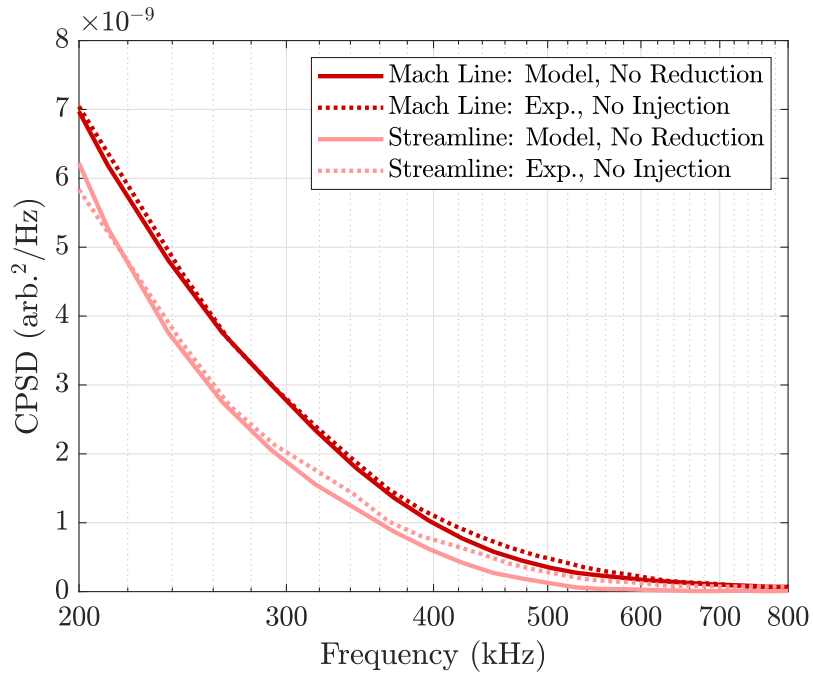


Figure 4.4: Mean cross power spectral densities of modeled fluctuations along a Mach line and streamline with no amplitude reductions compared to FLDI results with no injection.

Mach line and streamline from the case of no injection, and injection of 30% (4.5a, 4.5b), 35% (4.5c, 4.5d), and 40% (4.5e, 4.5f) He/CO₂. Comparing the modeled Mach-line CPSD to the injection of 30% He/CO₂, it appears that an amplitude reduction of approximately 15 – 30% matches the experimental results, depending on the frequency. For the injection of 35% He/CO₂, the modeled Mach-line CPSD is consistent with a 30% amplitude reduction until approximately 400 kHz, and for the injection of 40% He/CO₂, an amplitude reduction of 15 – 30% provides good agreement. In Figures 4.5b, 4.5d, and 4.5f, it can be seen that the modeled CPSD along a streamline is fairly insensitive to the amplitude reductions for acoustic disturbances from the bottom wall, with at most small differences between cases without amplitude reductions and with the maximum 45% reduction. These results are also consistent with the 15 – 30% reduction that was found to match the Mach-line data for these experiments.

The mathematical disturbance model indicates that the presence of a vibrationally active gas species in the lower boundary layer resulted in a 15 – 30% reduction in measured acoustic-disturbance amplitudes. The model also illustrates that reduced acoustic-disturbance amplitudes from the lower boundary have a minimal effect on the fluctuations correlated along a streamline, which is consistent with the experimental measurements. These significant results show that this simple theoretical model provides a framework for analyzing correlated fluctuations which contain unknown dependencies on the direction of disturbance propagation. Although the model relies on empirical inputs, it nonetheless offers valuable insight into the sensitivity of these correlated measurements to entropic and acoustic-disturbance amplitudes.

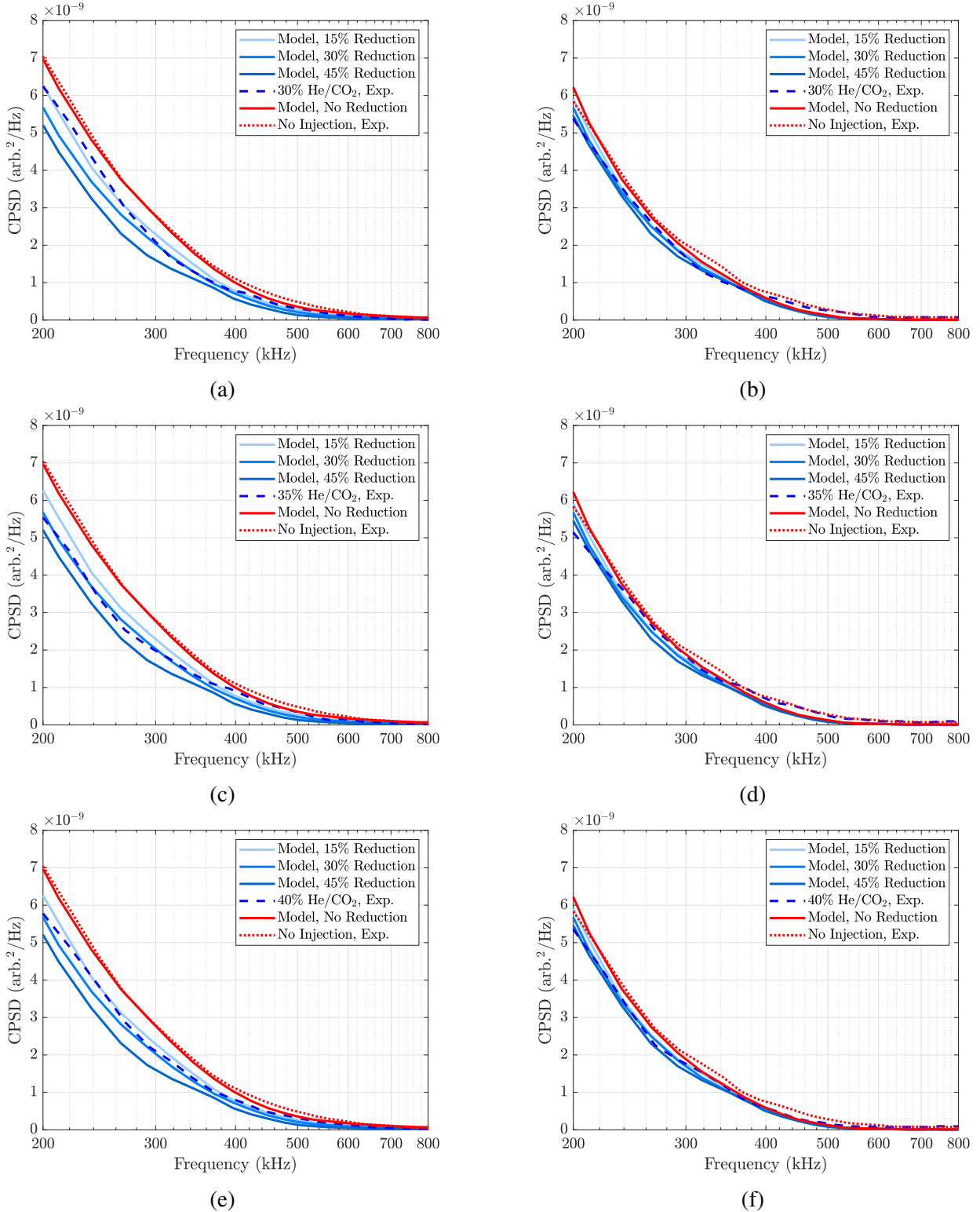


Figure 4.5: Mean cross power spectral densities of modeled fluctuations along a Mach line (a, c, e) and streamline (b, d, f) with 0%, 15%, 30%, and 45% reduced amplitudes for acoustic disturbances from the bottom wall. In each case, the disturbance model is compared to average FLDI results with no injection, and injection of 30%, 35%, and 40% He/CO₂ mixtures.

Chapter 5: Conclusion

5.1 Summary

The present work investigates the effects of vibrational nonequilibrium processes on acoustic radiation from a compressible turbulent boundary layer. In conventional (non-quiet) high-speed wind tunnels, increased disturbance amplitudes in the freestream lead to the growth of instabilities in laminar boundary layers, resulting in early breakdown and transition to turbulence. Since the noise environment in conventional high-speed wind tunnels is substantially stronger than that experienced in atmospheric flight, comparisons between ground-test and free-flight data are difficult to make. Additionally, each high-speed facility has widely varying freestream turbulence intensities, making it challenging to compare experimental results obtained from different tunnels. At high Mach numbers, it is known that acoustic disturbances typically dominate the noise environment; therefore, mechanisms capable of attenuating acoustic disturbances within an upstream turbulent boundary layer can play a critical role in the reduction of freestream-disturbance amplitudes.

Vibrational nonequilibrium processes can lead to significant acoustic attenuation in polyatomic gases and mixtures. Predicting the resulting attenuation, however, remains a challenge, and there exist limited experimental data for comparison and validation. High-temperature measurements of attenuation coefficients are particularly rare, and are needed to inform future inves-

tigations into the effects of vibrational nonequilibrium on turbulence-generated noise. Therefore, a high-temperature acoustic chamber was developed to measure the acoustic attenuation in CO_2 , N_2O , and mixtures consisting of CO_2/N_2 , CO_2/He , and $\text{N}_2\text{O}/\text{He}$. Using the differential path method, the acoustic attenuation was measured by transmitting and receiving ultrasonic pulses through quartz buffer rods separated by a varying distance inside a heated quartz tube. In mixtures of CO_2/N_2 at room temperature, increasing the percentage of N_2 decreased the peak attenuation, but the peak attenuation frequency remained unchanged. In mixtures of CO_2/He at room temperature, however, an increase in helium was found to decrease the peak attenuation modestly, but increase the peak attenuation frequency. This trend was also observed at temperatures up to 528 K. This is attributed to the light He molecules acting as an efficient collision partner with CO_2 , resulting in lower relaxation times for the mixture. As the temperature was increased to 528 K in CO_2 and mixtures of CO_2/He , the peak attenuation increased substantially, but the rate of increase was less significant as the percentage of helium increased. The results of the acoustic chamber measurements show that varying the fraction of helium in mixtures of CO_2/He can shift the attenuation to a desired frequency range, providing a method to control the acoustic radiation in high-speed facilities.

The acoustic disturbances radiated by a turbulent boundary layer were investigated experimentally in a Mach-2.8 reflected-shock tunnel facility. The gas mixtures of interest were introduced into the lower boundary layer of the flow through a porous plate located in the upstream region of the test section. Using schlieren imaging, the effect of injection on the external flowfield was visualized, and the maximum injection pressure was determined to occur when weak shocks in the vicinity of the porous plate were no longer significantly steepened by the injected mass flow.

A four-point FLDI was developed to measure the high-frequency disturbances in the freestream. The FLDI beams were split such that there were beam pairs oriented along streamlines and Mach lines. This provides simultaneous measurements of entropic and acoustic disturbances propagating in these two characteristic directions. The FLDI sensitivity transfer function developed by [Ceruzzi and Cadou \(2022\)](#) was applied to analyze the influence of the sidewall boundary layers on the core flow signal, and a cutoff wavenumber of $k_x \approx 3200 \text{ m}^{-1}$ was conservatively estimated as the point at which the sidewall boundary layers are no longer corrupting the FLDI signal from the core flow. Using the cross-correlation of the streamwise-separated FLDI beam pairs, the mean convection velocity was found to be $627 \pm 31 \text{ m/s}$. Compared to the theoretical freestream velocity, the measured convection velocity has a percent difference of -33% . This is to be expected, since the inclined frozen Mach radiation will not be convecting at the local flow velocity.

The focal points of the FLDI were positioned just above the lower boundary layer and sufficiently far downstream of the injector location to ensure sufficient mixing in the boundary layer. Amplitude spectral densities of the FLDI signals showed negligible differences between the cases with and without injection. Since the ASDs at a single FLDI point incorporate disturbances propagating from all four sidewalls, correlated fluctuations between two FLDI signals were computed to isolate the contributions to the overall noise from the lower-wall boundary layer and freestream entropic disturbances. Correlated fluctuations of FLDI signal pairs separated along a Mach line thus were analyzed using a cross power spectral density. Baselines for comparison were provided both by similar correlations along a streamline (i.e., corresponding to entropic disturbances) and reference, no-injection experiments. Mixtures of 30%, 35%, and 40% He/CO₂ were found to have the most significant attenuation in Mach-line-correlated fluctuations. As minimal reduc-

tions in fluctuation power were measured along corresponding streamlines, it could be concluded that the vibrationally active gas species in the boundary layer primarily affected acoustic radiation and not entropic disturbances. Additional control cases with N_2 and He were run to examine the influence of the injection itself and the density gradients within the boundary layer introduced by a light gas; minimal attenuation effects were seen in either case.

Injection of pure CO_2 did not result in any significant attenuation. From the attenuation spectra measured in the acoustic chamber, it was determined that the addition of He to CO_2 results in a large increase in expected attenuation in the frequency range of 200 – 800 kHz, which was consistent with the frequency range where significant attenuation was measured in the He/ CO_2 mixtures. This frequency range is also where the FLDI instrument was most sensitive, as indicated by a wavenumber sensitivity analysis which conservatively estimated that disturbances above 319 kHz can be attributed to the core flow, and are unaffected by sidewall boundary layers. For pure CO_2 , however, the attenuation coefficients at these higher frequencies are too low to have a measurable effect on the acoustic-disturbance amplitudes.

Since the correlated Mach-line fluctuations were not completely independent of other disturbances (i.e., acoustic noise propagating from the top and sidewall boundary layers and entropic disturbances) the measured decrease in CPSD was not necessarily a reliable quantitative indicator of the attenuation from the lower boundary layer. Therefore, a mathematical disturbance model was developed to isolate the effects of attenuation from this lower boundary layer alone. The disturbances were modeled as Gaussian wave packets propagating along Mach lines from the four test section walls and along streamlines. The model parameters were tuned to results from no-injection experiments, and the contribution from the lower boundary layer was then systematically reduced. An amplitude reduction of 15 – 30% was found to be consistent with the CPSD

attenuation measured for He/CO₂ mixtures in experiments.

5.2 Contributions

This investigation made a number of contributions to the understanding of vibrational nonequilibrium effects on acoustic disturbances. The design of a high-temperature acoustic chamber built upon previous chamber designs ([Winter and Hill, 1967](#); [Wang and Springer, 1973](#)), and provides the opportunity for future attenuation coefficient measurements of any gas composition. The attenuation coefficient measurements of CO₂/He mixtures at room temperature confirmed previous findings in literature: the addition of helium shifts the attenuation to higher acoustic frequencies. This effect was also measured at elevated temperatures, providing a more comprehensive set of experimental data that can be used for validation of vibrational relaxation models. While there are some limited results in literature of CO₂ attenuation coefficients at high temperatures, the results of the present work provide greater insight into how the coefficients vary over a wide range of temperatures. The acoustic chamber results also provide the first reported measurements of N₂O/He mixtures, which indicate that the addition of helium has a similar effect with N₂O to that with CO₂.

Another significant contribution of the present work was in the development of a four-point FLDI. The novel application of splitting the FLDI beams along streamlines and Mach lines allows for simultaneous measurements of disturbances propagating in these two characteristic directions. By examining the correlated fluctuations along these propagation directions, the relative fluctuation powers of entropic and acoustic disturbances can be compared. Additionally, the application of the transfer function analysis by [Ceruzzi and Cadou \(2022\)](#) allows for the discussion

of FLDI results to take into account the wavenumber dependence of the instrument. The sensitivity of the instrument to disturbances away from its focal plane can have a significant effect on the measured signal, and the influence of sidewall boundary layers or shear layers is often neglected in published FLDI studies.

To the authors' knowledge, the results presented here provide the first reported measurements of nonequilibrium effects on turbulence-generated acoustic noise, and in particular, demonstration of the potential for damping of such noise. Additionally, the disturbance modeling provides a framework for analyzing data in such experiments. This significant result expands upon previous investigations into the effect of nonequilibrium processes on acoustic instabilities within a boundary layer ([Fujii and Hornung, 2003](#); [Leyva et al., 2009](#); [Jewell et al., 2013](#); [Elliott et al., 2019](#)). The present work represents a critical first step towards developing a "quasi-quiet" capability in conventional high-speed wind tunnels, whereby existing facilities could be modified to incorporate the injection of absorbing gas species along the nozzle walls. This would reduce the freestream turbulence intensity of the flow, without changing the composition of the freestream gas (i.e., without using He/CO₂ as the test gas to attenuate acoustic disturbances). Additionally, by modifying existing high-speed tunnels, the expensive development and limited operating conditions of quiet tunnels can be eliminated. Operating conventional high-speed facilities with reduced noise levels would greatly expand their utility for studying flows in which transition plays an important role, and comparisons with computational and free-flight data could be made with greater accuracy.

5.3 Future Work

Additional work is required to analyze the degree of dependence between the FLDI signals from the Mach-line and streamline pairs, for example, by varying the distance between the measurement pairs. The present analysis assumes that the degree of dependence is constant across all shots; therefore, the correlated fluctuation powers are only discussed relative to other injection conditions. In addition to the dependence between the correlated signals, the proportion of freestream turbulence intensity that is measured by the correlated Mach-line signal pair is also not known. Although it has been shown numerically that the freestream turbulence intensity is the summation of disturbances propagating from each wall ([Duan et al., 2015](#)), in practice this is complicated by the integrated measurement performed by the FLDI along its optical axis. That is, since the FLDI signal is not a perfect point measurement at the focal plane, the contributions to the overall noise environment may not be a linear summation from each of the four walls. Therefore, a sophisticated model that simulates the integrated FLDI signal through the test section could provide an estimate of the proportion of total acoustic radiation that is measured by the FLDI Mach-line signal pair. Stronger conclusions could then be made regarding the freestream fluctuations that result from the independent acoustic and entropic sources. The four-point FLDI could also be extended to a multi-line FLDI system, as detailed by [Weisberger et al. \(2021\)](#). This would allow for a study of the coherence between upstream and downstream signals to determine the sensitivity of the coherence to the separation angle. Finally, a further improvement to the current FLDI analysis would be the application of a transfer function that includes disturbances convecting in directions other than parallel to the flow.

One limitation of this investigation was the test facility, which was characterized by a brief

test time and generally greater shot-to-shot variation than non-shock-driven facilities. This variation led to a relatively large degree of scatter in the attenuation results, making it difficult to draw firm conclusions from any single run. If a greater degree of attenuation was obtained by injecting the vibrationally active gas species into boundary layers along multiple walls, the effect of shot-to-shot variation could be diminished; however, this substantially increases the complexity of the injection system and may restrict optical access near the injection location. If this study were to be extended to include porous injection into all four sidewall boundary layers, uncertainties associated with the sensitivity of the Mach-line signal to acoustic disturbances propagating from a single boundary layer could be neglected. Additionally, a porous plate with a larger surface area could be used to inject a greater percentage of gas without disturbing the boundary layer. To eventually achieve a “quasi-quiet” capability in a conventional high-speed tunnel, the vibrationally active gas should be injected radially along the surface of the nozzle itself, such that the injection system does not interfere with the test section and model.

In addition to the short test time, the tunnel in the present work featured a narrow test section which facilitated the downstream reflection of weak shocks generated from discontinuities along the flowpath. A wider test section would produce a cleaner flow with less influence from sidewall boundary layers; however, this necessitates that the FLDI beams are sufficiently expanded to maintain a small f -number. If a small f -number can be achieved with a larger test section, the FLDI would likely measure a greater degree of acoustic attenuation at lower frequencies and be less affected by shot-to-shot variation.

The mathematical disturbance model presented in this work could be expanded to include a detailed DNS study of the fluctuation environment. A DNS study of the freestream disturbances would allow for a direct comparison to the experimental results, without the need to anchor

the numerical model to empirical measurements. The DNS could be analyzed with an FLDI model that simulates the integrated signal from the time-varying density fields, similar to the model detailed by [Benitez et al. \(2022\)](#). A comprehensive comparison between the experimental and numerical FLDI results could then be discussed, and further insight could be made into the FLDI's ability to measure weak acoustic disturbances propagating in multiple directions.

The present work was conducted at a single Mach number and total temperature. Further investigation could be done at higher Mach numbers, where the relative contribution of acoustic disturbances to the overall disturbance environment is even greater. In this case, correlated fluctuations along a Mach line would be less affected by streamwise disturbances. An increase in the Mach number and/or total temperature of the facility would result in higher temperatures within the boundary layer; therefore, from the measurements of attenuation coefficients, it would be expected that a greater degree of attenuation would be obtained from the injection CO_2 and He/CO_2 mixtures. A test matrix with varying total temperatures could examine the degree at which this trend is observed in a high-speed tunnel.

Appendix A: Multi-Point FLDI in a Mach-6 Freestream

A three-point FLDI instrument was implemented to investigate the freestream disturbance environment in a Mach-6 shock tunnel at a variety of unit Reynolds numbers and enthalpies. These experiments were conducted to test and validate the multi-point FLDI system prior to its application in the Mach-2.8 injection experiments.

A.1 Facility and Test Conditions

Experiments were conducted in the same HyperTERP facility detailed in Section 3.1; however, in this case, the facility was configured with a Mach-6 axisymmetric contoured nozzle. This free-jet nozzle has an exit diameter of 0.22 m and is connected to a 0.33 m-diameter test section. Optical access into the test section is provided by 15.24 cm diameter, 1.27 cm thick NBK7-glass windows. The FLDI focal region for this investigation was located approximately 108 mm from the exit plane of the nozzle, and 11 mm below the centerline.

The mean reservoir and freestream conditions are summarized in Table A.1 for each shot. The test campaign consisted of three shots at each of the five conditions employed to allow an assessment of the repeatability of results. Conditions A, B, and C had a total specific enthalpy that was held approximately constant at 0.89 MJ/kg, while conditions D and E had lower enthalpies of approximately 0.61 MJ/kg and 0.57 MJ/kg, respectively. Based on previous Pitot measurements

Table A.1: Mach-6 HyperTERP test conditions.

Condition	M_∞	Shot	$P_0 (\times 10^5 \text{ Pa})$	$T_0 \text{ (K)}$	$T_\infty \text{ (K)}$	$\rho_\infty \text{ (kg/m}^3\text{)}$	$U_\infty \text{ (m/s)}$	$Re_\infty (\times 10^6 \text{ m}^{-1})$
A	5.95	1	9.84	888	110	0.0196	1262	3.25
		2	9.77	891	110	0.0195	1263	3.21
		3	9.85	892	110	0.0196	1265	3.23
B	5.95	1	13.9	902	112	0.0272	1271	4.47
		2	13.5	890	110	0.0269	1263	4.44
		3	13.4	890	110	0.0267	1263	4.40
C	5.95	1	16.3	908	112	0.0318	1276	5.20
		2	16.3	907	112	0.0317	1275	5.19
		3	16.1	908	112	0.0315	1276	5.15
D	6.10	1	18.2	613	75	0.0534	1043	10.9
		2	18.3	613	75	0.0535	1044	10.9
		3	18.2	612	75	0.0533	1043	10.9
E	6.10	1	21.3	568	70	0.0674	1004	14.4
		2	21.4	570	70	0.0676	1005	14.4
		3	21.5	570	70	0.0678	1005	14.4

in the tunnel, the Mach number of conditions A, B, and C is approximately 5.95 ([Butler and Laurence, 2021a](#)), and that of conditions D and E is 6.10. Conditions A, B, and C have been used extensively in transition experiments conducted by [Butler and Laurence \(2021a,b\)](#), while conditions D and E were employed by [Maszkiewicz et al. \(2022\)](#) for studies of transitional, fin-induced shock-wave/boundary-layer interactions.

A.2 FLDI Instrument

A.2.1 FLDI Setup

A three-point FLDI was used for the simultaneous measurement of fluctuations assumed to be representative of entropic disturbances propagating in the streamwise direction and acoustic radiation propagating along Mach lines. The FLDI setup is similar to the description given in [Section 3.5.1](#), with the primary differences being that C_2 is a 100 mm-diameter lens of focal

length 200 mm, and a knife-edge mirror was used to split two of the beams perpendicular to the optical axis and into their respective photodetectors. Additional details are provided by Gillespie et al. (2022).

The orientation of the beam spacings on the focal plane of the multi-point FLDI is seen in Figure 3.10, where Δx_1 and Δx_2 represent the intra- and inter-pair spacing, respectively. It should be noted, however, that the FLDI point labels in the work of Gillespie et al. (2022) are different than those used here; point 2 and point 3 are interchanged. The beam spacings were measured by placing a LBP2-HR-VIS2 beam profiler at the center of the test section. From Equation 3.6, $\Delta x_1 = 465 \mu\text{m}$, and the FLDI was adjusted until this separation was attained ($\pm 8 \mu\text{m}$). As seen in Figure 3.10, Wollaston prism W_2 was used to split the beams along a Mach line, where μ is intended to be the Mach angle at Mach 6 (i.e., 9.59°). Using the beam profiler, the spacings were measured to be $\Delta x_2 = 1.583 \pm 0.008 \text{ mm}$ and $\Delta y = 0.360 \pm 0.008 \text{ mm}$, giving $\mu = 9.69 \pm 0.12^\circ$.

In this setup, the spacing of the optical components resulted in a phase difference between the four signals when no density gradient is present. Since the FLDI is most sensitive to density fluctuations when the mean signal is halfway between the maximum and minimum interference, the system should be set up within this linear regime to avoid phase ambiguity. Due to the phase difference between the four signals, however, there was no longer a position of the final W_3 prism that satisfied this for all four interference fringe patterns. Therefore, in order to ensure that all signals had a baseline voltage within the linear portion of their fringe pattern, only FLDI signals 1, 2, and 3 of Figure 3.10 were recorded.

The radius of the Gaussian beam at the focal plane, ω_0 , was computed with the procedure described in Section 3.5.2. From the slope of the beam radius as a function of $z - z_0$, the beam waist was calculated with Equation 3.8 to be $\omega_0 = 8.0 \pm 0.5 \mu\text{m}$, and the measured f-number of

the FLDI was $f/24$. Further details of this calculation can be found in the work of [Gillespie et al. \(2022\)](#).

A.2.2 Influence of Nozzle Shear Layers

The FLDI is analyzed with the same instrument model presented in Section 3.6.1. The sensitivity of the FLDI instrument employed in this study as a function of wavenumber along the optical path (z -axis) is plotted in Figure A.1a. The combined effects of $H_{\Delta x}$ and H_{ω} on the response of the FLDI illustrate that sensitivity is maximum for disturbances with $k_x = \pi/\Delta x_1$ (6756 m^{-1}) at the focus ($z = 0$). The instruments' depth of focus is defined as the full width at half maximum (FWHM) of the sensitivity for a given disturbance wavenumber (Equation 3.22) ([Ceruzzi and Cadou, 2022](#)). The FWHM is plotted as a function of wavenumber in Figure A.1b. As the wavenumber increases past $k_x = \pi/\Delta x_1$, the depth of focus decreases slightly, i.e., the spatial resolution of the instrument increases. As the wavenumber decreases below $k_x = \pi/\Delta x_1$, the instrument's depth of focus rises rapidly, and the overall response decreases. This illustrates the FLDI's ability to resolve large wavenumber disturbances near the focal plane of the system, while small wavenumber disturbances are measured across a greater spatial extent with a decreased sensitivity.

To compare the signal strength of the core flow to that of the nozzle shear layers, the ratio of FLDI sensitivity functions (Equation 3.20) in the two regions is computed with Equation 3.23. In this study, δ_{BL} is replaced with δ_{SL} , the shear-layer thickness, and L is the half-width of the flow region, equal to the nozzle radius of 0.11 m. [Zhang et al. \(2018\)](#) found that the magnitudes of density based turbulence intensity across all wavenumbers in a Mach-6 boundary layer DNS

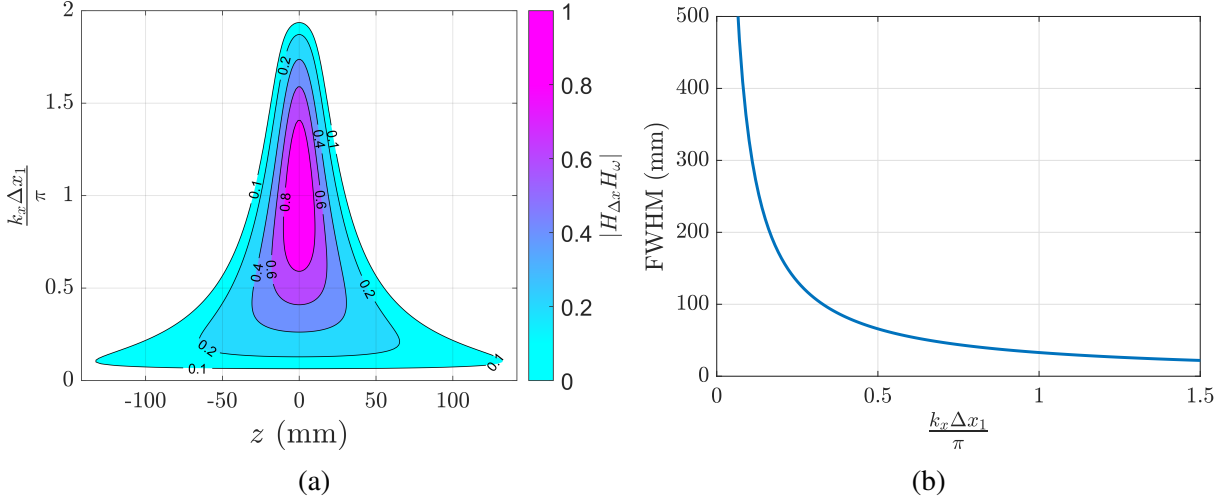


Figure A.1: (a) Variation of the product of FLDI transfer functions $H_{\Delta x} H_{\omega}$ with wavenumber and distance z and (b) full width at half maximum (FWHM) of the FLDI transfer functions (depth of focus) as a function of wavenumber for $\lambda_0 = 532$ nm, $\omega_0 = 8.0$ μ m, and $\Delta x_1 = 465$ μ m.

were on the orders of 20–25%. In a summary of various facilities at Mach 6, [Lafferty and Norris \(2007\)](#) found that freestream Pitot probe RMS fluctuations were typically on the order of a 1–4%. Therefore, disturbances in the core flow are assumed to have uniform magnitude ($g = 1$), while disturbances in the shear layers are assumed to have a magnitude 8 times greater, i.e.,

$$g(z) = \begin{cases} 1 & \text{for } |z| < L - \delta_{\text{SL}} \\ 8 & \text{for } L - \delta_{\text{SL}} < |z| < L \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A.1})$$

To estimate the shear-layer thickness, previous simulations of the mean nozzle flow from VULCAN CFD software were used to determine the radius of the core flow ([Starshak et al., 2018](#)). Using condition B, the core flow region was found to extend approximately 89 mm from the nozzle centerline with a Mach number variation of $< 1\%$ at the exit plane; this resulted in $\delta_{\text{SL}} \approx 21$ mm for condition B. To estimate the shear-layer thickness for the other conditions, the incompressible turbulent boundary-layer scaling of $\delta \propto Re_{\infty}^{-1/7}$ was used and the resulting values are seen in Table A.2.

Table A.2: Estimate of shear-layer thickness for each condition.

Condition	$\overline{Re}_\infty (\times 10^6 \text{ m}^{-1})$	$\delta_{\text{SL}} (\text{mm})$
A	3.23	22.0
B	4.44	21.0
C	5.18	20.5
D	10.9	18.5
E	14.4	17.7

The solution to Equation 3.23 is presented in Figure A.2. The H_m/H_r ratio rises rapidly for $k_x > 1000 \text{ m}^{-1}$, and exceeds 5 at $k_x \approx 2100 \text{ m}^{-1}$. This represents a highly conservative cutoff wavenumber; therefore, disturbances with $k_x > 2100 \text{ m}^{-1}$ can be attributed to the core flow. The FLDI results are thus computed for the core flow using Equation 3.21 with $g = 1$. It is also seen in Figure A.2 that there is minimal difference in the cutoff wavenumber among the different conditions. Although the estimate of shear-layer thickness neglects shear-layer growth, the insensitivity of the cutoff wavenumber to shear-layer thickness illustrates that its impact would be negligible. This insensitivity is the result of the large core flow and relatively small variations in shear-layer thickness for this flow field, which is generally the case for hypersonic wind tunnels.

A.2.3 Disturbance Velocity Analysis

In addition to the measurement of mean convection velocities (detailed in Section 3.6.3), the frequency-dependent convection velocities are also of interest. To measure this dispersion relation, the cross spectrum can be utilized (Poggie et al., 2000; Borg et al., 2015). The cross power spectral density for signal pair 1-3 is defined as

$$S_{13}(f) = \sum_{m=-\infty}^{\infty} R_{13}(m) e^{-j2\pi f m}, \quad (\text{A.2})$$

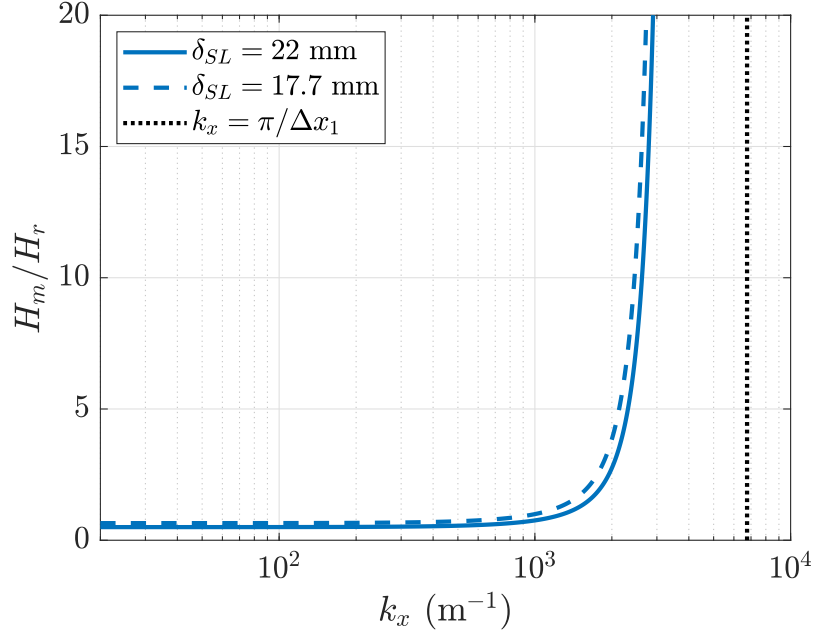


Figure A.2: Estimated ratio of FLDI signal from the core flow, relative to the nozzle shear layers.

where S_{13} is the distribution of power per unit frequency for the correlated signal pair 1-3. The phase of the cross spectrum can then be defined as

$$\phi_s(f) = \arctan \left[\frac{\text{Im}[S_{13}(f)]}{\text{Re}[S_{13}(f)]} \right]. \quad (\text{A.3})$$

The phase function is related to the time delay between the two signals according to

$$\tau(f) = \frac{\phi_s(f)}{2\pi f}, \quad (\text{A.4})$$

which allows the frequency-dependent convection velocities to be calculated as

$$U_x(f) = \frac{\Delta x_2}{\tau(f)}. \quad (\text{A.5})$$

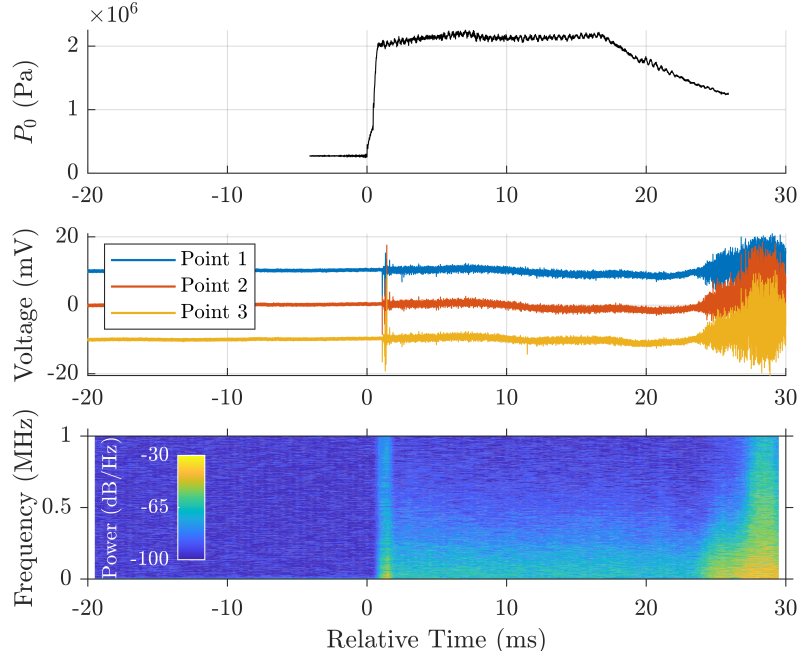


Figure A.3: Time trace for shot E1. (Top) Reservoir pressure; (Middle) voltage output from FLDI signals, offset for clarity; (Bottom) spectrogram of FLDI point 1.

A.3 Results and Discussion

For all shots listed in Table A.1, the data analysis was based on 6 ms of test time during which the reservoir conditions remained approximately constant. A sample time trace is presented in Figure A.3 for shot E1, together with corresponding voltage outputs from the FLDI signals and calculated short-time spectrogram from point 1. The spectrogram was computed with Hanning windows of length 120 μ s and 50% overlap. In this case, the steady test time was taken to occur from 10 – 16 ms. At around 24 ms, tunnel unstart leads to large fluctuations in the FLDI signal, but the spectrogram indicates that there are no obvious large scale trends in noise throughout the steady test time, an observation that held generally.

A.3.1 Disturbance Velocity

Using the procedure described in Section 3.6.3, the streamwise convection velocities were analyzed with a cross-correlation of the streamwise FLDI signal pair 1-3, computed over consecutive 1 ms intervals throughout the 6 ms test time. The resulting mean convection velocities, $\langle U_c \rangle$, and their standard deviations throughout the test time, $\sigma_{\langle U_c \rangle}$, are seen in Table A.3. In all shots, the measured convection velocities were below the theoretical freestream velocities, with an average percent difference of $-8.7 \pm 3.3\%$. The magnitude of the percent difference was found to increase slightly with Reynolds number, with a maximum of -12.7% in shot E3, and minimum of -0.9% in shot A3. As discussed in Section 3.7.2, the measured FLDI convection velocities are expected to be lower than the theoretical freestream velocities, since the FLDI is unable to reject all of the low wavenumber eddies convecting in the shear layer along the optical path, and inclined frozen Mach radiation originating upstream in the turbulent nozzle boundary layer will not be convecting at the local flow velocity.

Frequency-dependent convection velocities were measured as outlined in Section A.2.3. The cross power spectral density was computed using Welch's method with Hanning windows of length $40 \mu\text{s}$ and 50% overlap. The resulting frequency-dependent convection velocities are plotted in Figure A.4 together with the average theoretical freestream velocity for the relevant condition, and mean convection velocities, $\langle U_c \rangle$, for each shot. For conditions A, B, and C, the general trend is one of gradually increasing velocity with frequency; however, for conditions D and E, a localized dip in velocity is observed to occur in the region 175–350 kHz, with a slight increase in velocity at higher frequencies. In the previous discussion of the shear-layer influence on the FLDI signal in Section A.2.2, it was determined that the majority of the FLDI's signal above

Table A.3: Mean convection velocities measured with FLDI signal pair 1-3.

Condition	Shot	U_∞ (m/s)	$\langle U_c \rangle$ (m/s)	$\sigma_{\langle U_c \rangle}$ (m/s)	Percent Difference (%)
A	1	1262	1202	47	-4.8
	2	1263	1180	87	-6.6
	3	1265	1254	56	-0.9
B	1	1271	1135	71	-10.8
	2	1263	1140	83	-9.7
	3	1263	1172	96	-7.2
C	1	1276	1192	66	-6.6
	2	1275	1139	39	-10.7
	3	1276	1156	43	-9.4
D	1	1043	943	71	-9.6
	2	1044	974	69	-6.7
	3	1043	911	45	-12.6
E	1	1004	879	72	-12.4
	2	1005	901	97	-10.4
	3	1005	877	25	-12.7

$k_x = 2100 \text{ m}^{-1}$ can be attributed to the core flow. From Equation 3.16, this cutoff wavenumber corresponds to frequencies of approximately 392 kHz for conditions A, B and C, 315 kHz for condition D, and 296 kHz for condition E. Therefore, it is likely that the strong shear-layer disturbances present at these latter high-Reynolds-number conditions are corrupting the measured freestream convection velocities below this cutoff wavenumber. Additionally, the nozzle is operating off-design for conditions D and E, which may be resulting in larger fluctuations in the shear layers.

A.3.2 Density-Based Turbulence Intensity

For the following analysis, shots A3, B1, C1, D1 and E1 are used for comparison unless otherwise specified. Shot A3 was chosen because an increased amount of intermittency was observed among the other condition A shots, as described by Gillespie et al. (2022). In each case,

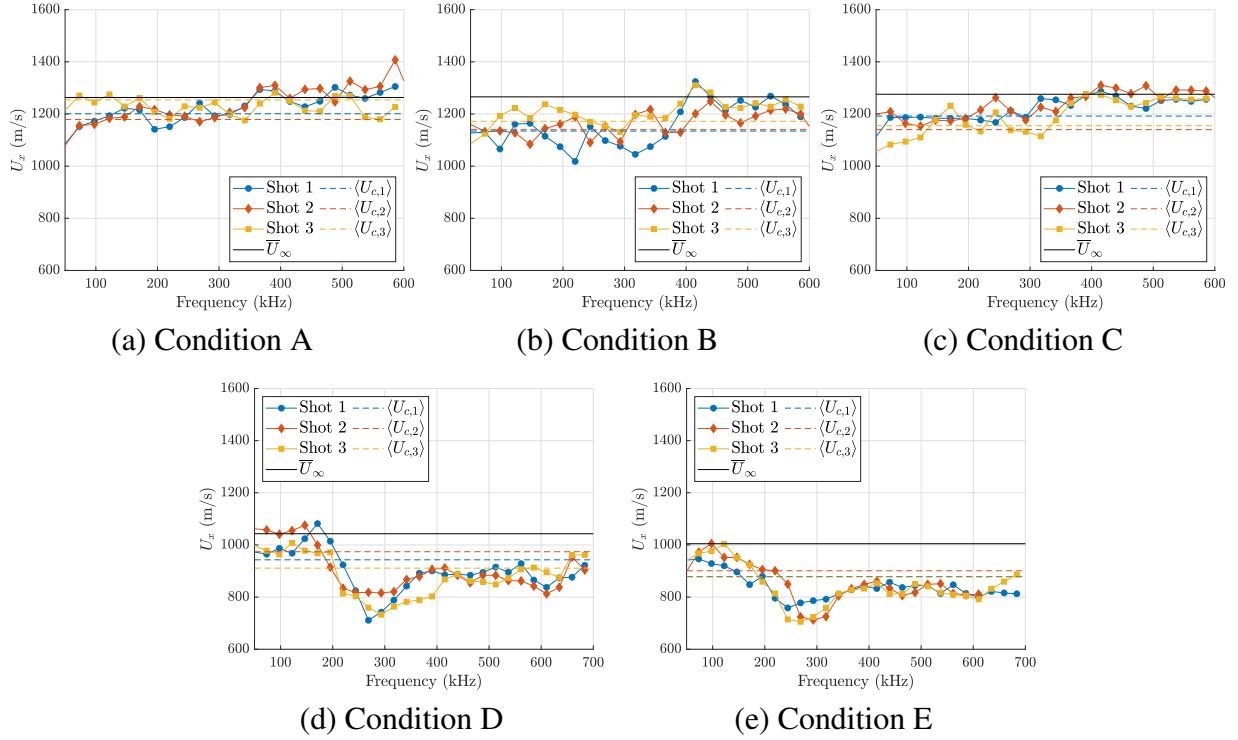


Figure A.4: Frequency-dependent convection velocities measured with FLDI signal pair 1-3.

voltage signals were converted to phase fluctuations using Equation 3.9. The power spectral density (PSD) of the phase fluctuations was then calculated using Welch's method with Hanning windows of length $300 \mu\text{s}$ and 50% overlap, and the amplitude spectral density (ASD) was computed from the square root of the PSD. As presented in Figure A.5a for shot E1, the ASDs were essentially identical for each of the FLDI points 1, 2 and 3; similar consistency was found for all shots. Also seen in Figure A.5b is the ASD of the raw voltage signal of FLDI point 1 for each condition. We note a high signal-to-noise ratio at all conditions, with an increase in the low-frequency content as the freestream Reynolds number (i.e., density) is increased. A straight line with slope -1.75 (which would correspond to -3.5 for the PSD) is also plotted for reference, which is a freestream spectra roll-off that has been reported in pressure data by numerous authors, as summarized by Duan et al. (2019); we see that it also provides a reasonable match to

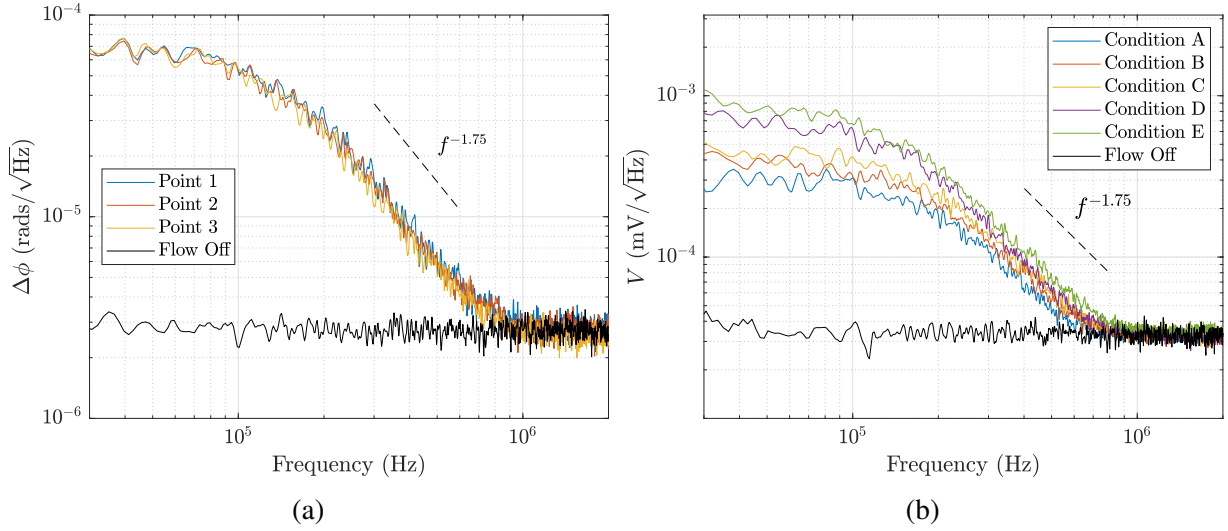


Figure A.5: Amplitude spectral density of (a) phase fluctuations from FLDI points 1, 2 and 3 of shot E1 and (b) voltage signal from FLDI point 1 at each condition, prior to application of Equation 3.21.

the high-frequency data here.

The phase fluctuation spectra, computed in frequency space, were converted to wavenumber space using Equation 3.16, with $\langle U_c \rangle$ taken as the mean convection velocity measured for a given shot (Table A.3). Figure A.6 shows the density fluctuations calculated from Equation 3.21 for FLDI point 1, in each case normalized by the freestream density. A factor of $\sqrt{\langle U_c \rangle / 2\pi}$ was applied to convert from units of $\text{Hz}^{-1/2}$ to $\text{m}^{1/2}$ for consistency in the wavenumber space. These spectra are consistent over a large wavenumber range, but it is evident that they diverge at high wavenumbers. This can be attributed to the different signal-to-noise ratios between conditions, which causes the signals to reach their noise floor at varying frequencies (see Figure A.5b). To illustrate this, the vertical blue dotted line shows the wavenumber at which the noise floor is reached by condition A ($k_x = 2\pi f_{\text{nf}} / \langle U_c \rangle$). A roll-off slope of $k_x^{-1.8}$ was found for the density fluctuation spectra, which is similar to the roll-off of $f^{-1.75}$ seen in the phase fluctuation and raw voltage spectra in Figure A.5. This indicates that the sensitivity function (Equation 3.20) has

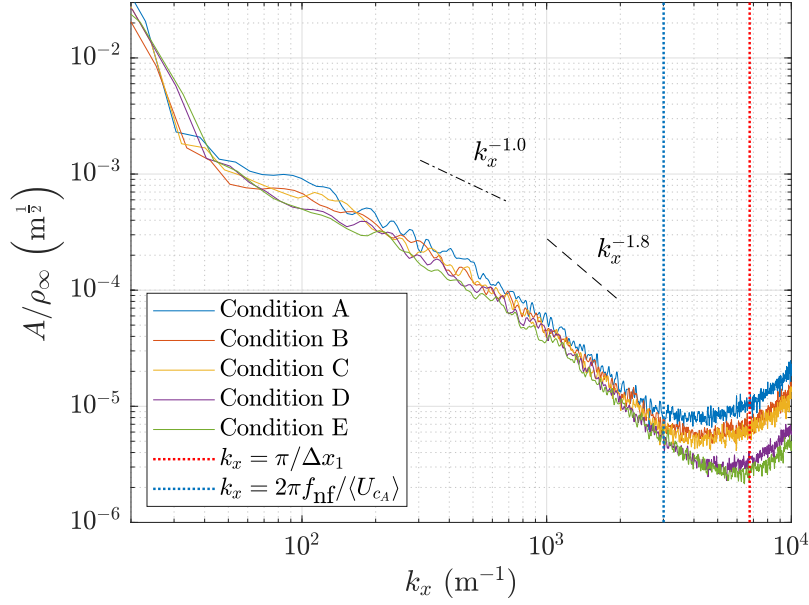


Figure A.6: Density fluctuation amplitude spectra from shots A3, B1, C1, D1, and E1.

only a small wavenumber dependence for $1000 \text{ m}^{-1} < k_x < 3000 \text{ m}^{-1}$. [Ceruzzi \(2022\)](#) also found a relatively constant sensitivity function over a given wavenumber band, which results from the competing effects of $H_{\Delta x}$ and H_ω .

The RMS density fluctuations over certain wavenumber bands of interest were calculated in frequency space using

$$(A/\rho_\infty)_{\text{RMS}} = \sqrt{\int_{f_1}^{f_2} (A/\rho_\infty)^2 df}. \quad (\text{A.6})$$

RMS fluctuations were computed over wavenumber bands of $k_x = [100 - 3000] \text{ m}^{-1}$ and $k_x = [2100 - 3000] \text{ m}^{-1}$, using Equation 3.16 to convert to frequencies. The fluctuations over the wider range represent a broadband density-based turbulence intensity; however, the discussion of Section A.2.2 showed that a significant fraction of this turbulence intensity can be attributed to the shear layers. Therefore, such a broadband intensity measurement may not accurately represent the fluctuations present in the core flow. This motivates the calculation of the RMS fluctuations

over the wavenumber band of $k_x = [2100 - 3000] \text{ m}^{-1}$, better representing an estimate of this core flow turbulence intensity. This range corresponds to frequencies of approximately $f = [392 - 560] \text{ kHz}$ for conditions A, B, and C, $f = [315 - 450] \text{ kHz}$ for condition D, and $f = [296 - 423] \text{ kHz}$ for condition E. Likewise, a wavenumber of $k_x = 100 \text{ m}^{-1}$ corresponds to frequencies of approximately $f = 18.7 \text{ kHz}$ for conditions A, B, and C, $f = 15 \text{ kHz}$ for condition D, and $f = 14.1 \text{ kHz}$ for condition E. For comparison, the expected frequency range of second-mode disturbances in conditions A, B, and C, is $170 - 330 \text{ kHz}$, as found by [Butler and Laurence \(2021b,a\)](#).

Figure [A.7](#) depicts a clear trend of decreasing fluctuation strength with increasing Reynolds number for both wavenumber bands; this is consistent with Pitot measurements in numerous other high-speed facilities, as summarized by [Lafferty and Norris \(2007\)](#) and [Chou et al. \(2018\)](#). Direct comparisons of the absolute magnitudes of disturbance intensities documented here to earlier measurements, however, is complicated by several factors, primarily the differing frequency ranges of the instruments and assumptions regarding the respective transfer functions. Recently, [Lawson et al. \(2022\)](#) found good agreement between fluctuation spectra obtained from Pitot probes ($f = [0.1 - 40] \text{ kHz}$) and FLDI ($f = [30 - 200] \text{ kHz}$) in Mach-4 and Mach-5 freestream flows: if a broadband turbulence intensity is required, some combination of the two would generally be necessary.

A.3.3 Acoustic Noise Contribution

Although the density-based turbulence intensity in the core flow provides a characterization of the overall noise environment, it does not enable any insight into the relative contributions of

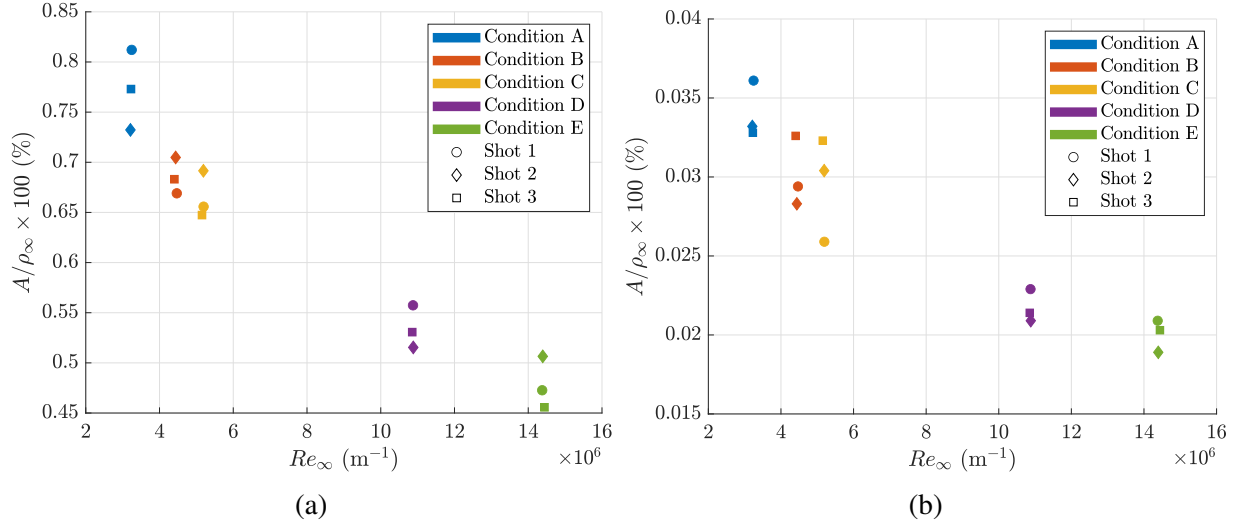


Figure A.7: RMS density fluctuations for wavenumber band (a) $k_x = [100 - 3000] \text{ m}^{-1}$ and (b) $k_x = [2100 - 3000] \text{ m}^{-1}$.

the acoustic and entropic disturbances. This can instead be provided by examining the coherence of the FLDI signal pairs along the streamline (1-3) and Mach line (1-2). The magnitude-squared coherence is computed as described in Section 3.7.3. The coherence of a representative shot at each condition is presented in Figure A.8. Consistent in all cases, the coherence of the Mach-line pair (1-2) is substantially higher than that of the streamline pair (1-3) over a wide range of wavenumbers, with the curves generally converging at low wavenumbers and then close to the noise floor. It should further be noted that the influence of the shear layers can be expected to enhance the streamwise coherence at low wavenumbers.

While the coherence provides an estimate of the normalized cross power spectral density, only the relative magnitudes of the fluctuating signals are obtained. A characterization of the relative fluctuation powers of the correlated signals requires the CPSDs themselves to be compared. Ideally, the correlated signals from FLDI pair 1-3 would consist uniquely of entropic disturbances and that of pair 1-2 of acoustic disturbances; however, in reality, these signals will not be completely independent due to the close proximity of the measurement points. Nevertheless, if we

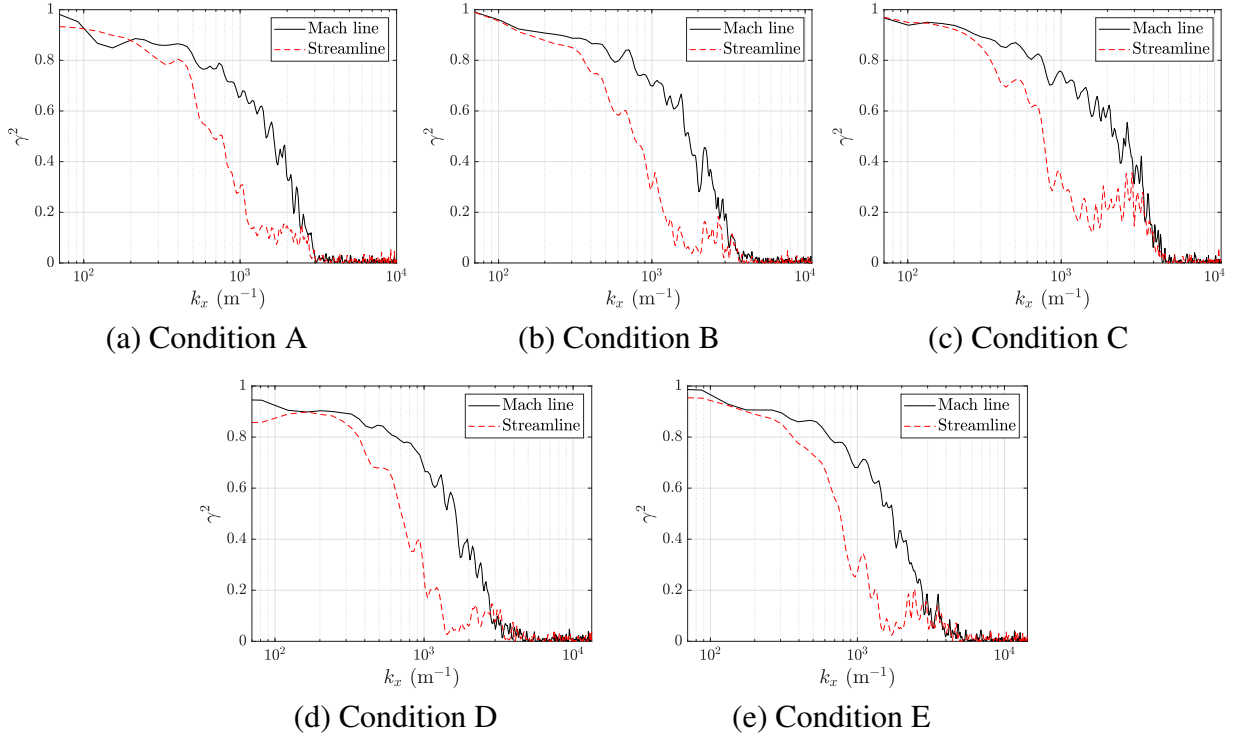


Figure A.8: Magnitude-squared coherence of FLDI pairs 1-2 (Mach line) and 1-3 (streamline) from shots A3, B1, C1, D1, and E1.

make the reasonable assumption that the relative dependence is consistent across all shots, the relation between the RMS fluctuations obtained from signal pair 1-2 and 1-3 will illustrate the relative contributions of acoustic and entropic noise when compared to other shots.

The DNS studies of [Duan et al. \(2015\)](#) and [Huang and Duan \(2017\)](#) have demonstrated that the freestream acoustic noise intensity results from the linear summation of all radiating surfaces. Since the Mach-line signal pair (1-2) will only measure acoustic radiation from a portion of the nozzle boundary layer (i.e., the region below the measurement points), an estimate of the proportion of the measured acoustic intensity to the total intensity is needed. A simple but physically plausible estimate is given by a $\cos\vartheta$ dependence, where ϑ is measured from the $+y$ -axis (i.e., vertically down) in the yz -plane of the axisymmetric nozzle (see Figure A.9). Thus, in the case of the FLDI focal region being on the centerline of the tunnel ($Y = 0$ in Figure A.9), the portion

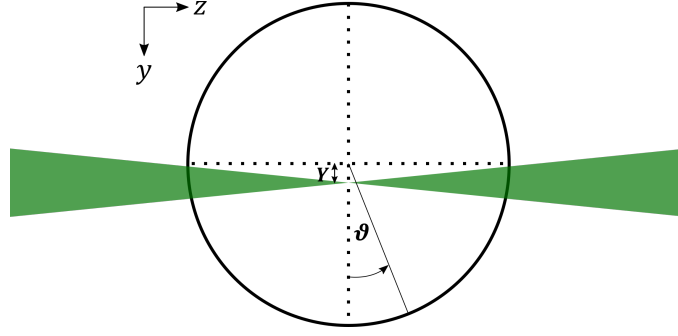


Figure A.9: Schematic of the FLDI in relation to the nozzle and angle ϑ used to estimate the portion of the total acoustic field measured with signal pair 1-2.

of the total acoustic field that is measured with the signal pair 1-2 is

$$\text{Acoustic Fraction} = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} \cos\vartheta d\vartheta = \frac{1}{\pi}. \quad (\text{A.7})$$

To account for the FLDI focal region being located approximately 11 mm below the tunnel centerline, the integral was performed on $\cos\left(\vartheta \frac{\pi}{2.94}\right)$, from $\vartheta = [-1.47 - 1.47]$ rads. Therefore, an acoustic contribution factor of 3.35 (as opposed to π) is appropriate.

The RMS fluctuations of the correlated Mach-line and streamline signal pairs were computed from the square root of the CPSD over the wavenumber band of $k_x = [2100 - 3000] \text{ m}^{-1}$ to isolate the part of the signal uncontaminated by the nozzle shear layer. The ratio of the RMS fluctuations, $P_{s,12}/P_{s,13}$, are presented in Figure A.10. No clear trend with Reynolds number is seen, but in each case we see that the ratio lies above unity, meaning that even this partial acoustic contribution is stronger than the entropic component. When multiplied by the acoustic contribution factor of 3.35, the ratio of RMS fluctuations range from 3.91 – 4.96. Therefore, it is evident that the acoustic radiation dominates the disturbance environment in all cases. To examine the sensitivity of $P_{s,12}/P_{s,13}$ to the cutoff wavenumber, the ratios were also computed over the wavenumber band of $k_x = [1100 - 3000] \text{ m}^{-1}$. A similar trend with Reynolds number was found,

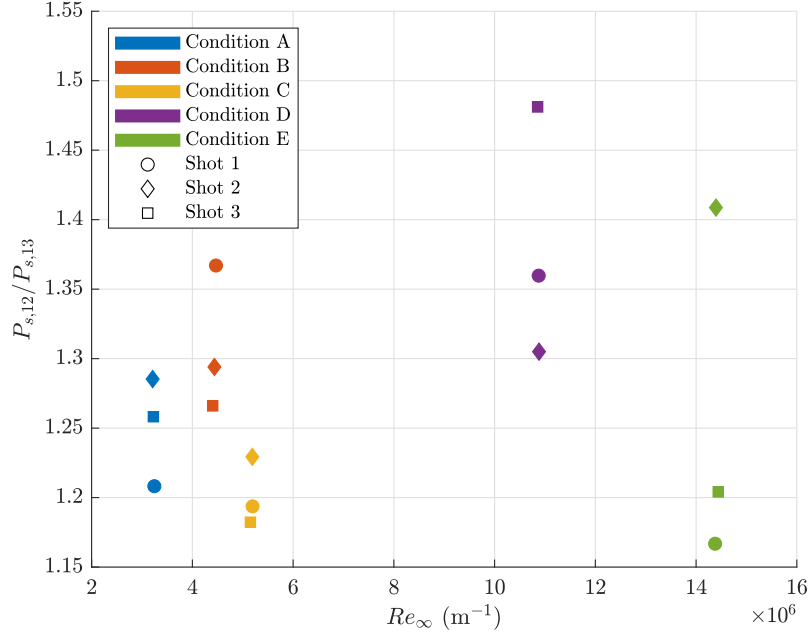


Figure A.10: Ratio of the RMS fluctuations of signal pair 1-2 (Mach line) to signal pair 1-3 (streamline) from the respective CPSDs in the wavenumber band of $k_x = [2100 - 3000] m^{-1}$.

and when multiplied by the acoustic contribution factor, $P_{s,12}/P_{s,13}$ ranged from 4.10 – 5.52. This indicates that the measured proportion of acoustic noise in the freestream is fairly insensitive to the cutoff wavenumber.

A.4 Summary

A three-point FLDI system was implemented in a Mach-6 reflected-shock tunnel for simultaneous measurements of freestream disturbances convecting along streamlines and Mach lines at various unit Reynolds numbers and enthalpies. Using the streamwise-separated FLDI beam pairs, the mean convection velocities were lower than theoretical freestream values, with an average percent difference of around 9%. The frequency-dependent convection velocities showed a general trend of increasing with frequency; however, a dip in velocity was found to occur around 175 – 350 kHz for the highest Reynolds number (lowest enthalpy) conditions, D and E. The FLDI

sensitivity transfer function developed by [Ceruzzi and Cadou \(2022\)](#) was applied to analyze the influence of the shear layer on the core flow signal, and a cutoff wavenumber of $k_x = 2100 \text{ m}^{-1}$ was estimated as the point at which the nozzle shear layers are no longer corrupting the FLDI signal from the core flow. The convection velocities measured below this wavenumber were suspected to be heavily influenced by the strong shear-layer disturbances in conditions D and E, leading to the anomalous behavior. Broadband and core-flow density-based turbulence intensities were examined: in both cases, the RMS density fluctuations decreased with Reynolds number.

The relative contributions of the acoustic and entropic disturbances to the overall noise environment were also studied. Through a coherence analysis of the FLDI signal pairs separated along a streamline and a Mach-line, the three-point FLDI instrument was demonstrated to be effective in simultaneously measuring the entropic disturbances and acoustic radiation. Using the cross power spectral densities, the ratio of RMS fluctuations of the correlated Mach-line pair to that of the streamline pair was calculated and exceeded unity in all cases. No clear trend of this ratio with Reynolds number was found. A simple model was developed to estimate of the fraction of the total acoustic field measured by the FLDI Mach-line pair; based on this model, the acoustic radiation was determined to dominate the disturbance environment at all conditions.

Appendix B: Acoustic Reflection in Helium

When an acoustic wave encounters the plane interface of a second, homogeneous medium, reflected and transmitted waves are generated. If there is an abrupt change in the density and speed of sound at the boundary of the two media, the amplitude of the reflected wave is given by the well-known Rayleigh reflection coefficient. If, however, the transition between the two media is of non-zero thickness and features a continuously varying density and sound speed, then the reflection of waves at the interface becomes considerably more complex. This problem was investigated analytically and numerically by [Robins \(1990, 1991\)](#), who studied the transmission of an acoustic plane wave through a stratified fluid layer of finite thickness. The analytical solutions presented by [Robins \(1990, 1991\)](#) can be applied to the case of an acoustic wave propagating through a He/air mixture of finite thickness, representative of a boundary layer. The effect of density and sound-speed gradients in this representative boundary layer would also be present in He/CO₂ mixtures to a lesser degree. Therefore, the resulting attenuation effects in the boundary layer can be investigated independently from the effects of vibrational nonequilibrium processes.

B.1 General Acoustic Transmission

The reflection and transmission of acoustic waves should first be discussed for the simple case of a boundary separating fluids of constant density and sound speed. The pressure amplitude

ratios of the reflected and transmitted waves to those of the incident wave depend on the incidence angle and characteristic acoustic impedances in the two media (Kinsler et al., 2000). The acoustic impedance of a medium is defined as $Z = \rho c$, where ρ is the equilibrium density of the fluid and c is the speed of sound. If the complex pressure amplitude of the incident wave is P_i , that of the reflected wave P_r , and that of the transmitted wave P_t , then the transmission and reflection coefficients are defined as $T = P_t/P_i$ and $R = P_r/P_i$, respectively. For an incident wave in medium “1” and transmitted wave in medium “2”, it can be shown that

$$R = \frac{Z_2/\cos \theta_t - Z_1/\cos \theta_i}{Z_2/\cos \theta_t + Z_1/\cos \theta_i}, \quad (\text{B.1})$$

where θ_i is the angle of incidence, θ_t is the angle of the transmitted wave, and Snell’s law gives

$$\frac{\sin \theta_i}{c_1} = \frac{\sin \theta_t}{c_2}. \quad (\text{B.2})$$

Since the average intensity of an acoustic wave is $I = P^2/(2\rho c)$ (Kinsler et al., 2000), the intensity transmission and reflection coefficients are defined by

$$T_I = I_t/I_i = \frac{Z_1}{Z_2}|T|^2, \quad (\text{B.3})$$

$$R_I = I_r/I_i = |R|^2. \quad (\text{B.4})$$

For a general case where the cross-sectional area of the incident wave, A_i , is not equal to the cross-sectional area of the transmitted wave, A_t , the power transmission and reflection coefficients are

defined by

$$T_{\Pi} = \frac{A_t Z_1}{A_i Z_2} |T|^2, \quad (\text{B.5})$$

$$R_{\Pi} = |R|^2. \quad (\text{B.6})$$

Typically the cross-sectional area is sufficiently large compared to the wavelength, and $A_t = A_i$ for most cases ([Kinsler et al., 2000](#)). Assuming no losses during wave propagation, conservation of energy gives

$$R_{\Pi} + T_{\Pi} = 1. \quad (\text{B.7})$$

B.2 Density and Sound-Speed Profiles

For a fluid with a continuously varying acoustic impedance, the transmission of an acoustic wave depends on the density and sound-speed profiles, thickness of the transition layer, incidence angle, and acoustic frequency. The following derivation of the acoustic transmission has been summarized from [Robins \(1990, 1991\)](#). For consistency with the notation of Robins, the acoustic wave is propagating in the $-z$ -direction, and the reflection coefficients are derived for a representative density and sound-speed profile as shown in [Figure B.1](#). In the following derivation, the subscripts “0”, “1”, and “2” denote properties in the upper, transition, and lower media, respectively.

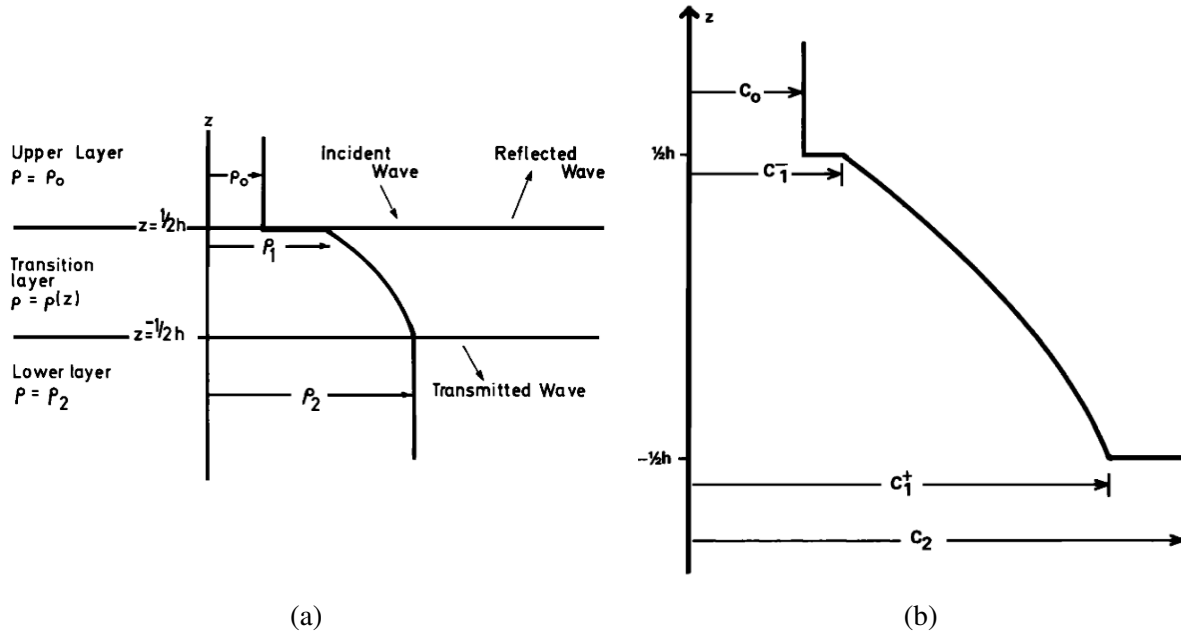


Figure B.1: (a) Density variation and (b) sound-speed variation in three-layer model from [Robins \(1990, 1991\)](#).

The wave equation governing acoustic propagation in a medium of varying density is

$$(\nabla^2 + k^2)p = \frac{1}{\rho} \nabla \rho \cdot \nabla p, \quad (\text{B.8})$$

where $k = \omega/c$ is the wavenumber. Assuming that the density depends only on the vertical coordinate, z , Equation B.8 becomes

$$(\nabla^2 + k^2)p = \frac{1}{\rho} \frac{d\rho}{dz} \frac{\partial p}{\partial z}. \quad (\text{B.9})$$

In the upper medium ($z > h/2$), an incident plane wave is described by

$$p = e^{ik_0(x \cos \theta_0 - z \sin \theta_0)}, \quad (\text{B.10})$$

where $k_0 = \omega/c_0$, and θ_0 is the grazing angle, defined as the complement to the incidence angle. Reflection from the transition layer results in an upward (+z) traveling wave of amplitude R , giving the total pressure in the upper medium:

$$p_0 = e^{ik_0 x \cos \theta_0} \left(e^{-ik_0 z \sin \theta_0} + R e^{ik_0 z \sin \theta_0} \right). \quad (\text{B.11})$$

The transmitted wave in the lower medium ($z < -h/2$) is described by

$$p_2 = T e^{ik_2(x \cos \theta_2 - z \sin \theta_2)}, \quad (\text{B.12})$$

where $k_2 = \omega/c_2$ and the angle θ_2 is determined from Snell's law,

$$k_2 \cos \theta_2 = k_0 \cos \theta_0. \quad (\text{B.13})$$

The pressure in the transition layer can be described by

$$p = e^{ik_0 x \cos \theta_0} p_1(z). \quad (\text{B.14})$$

Substituting Equation B.14 into Equation B.9 and defining a new variable $q(z) = \rho^{-1/2} p_1(z)$, the wave equation can be expressed as

$$\frac{d^2 q}{dz^2} + q \left[\frac{\omega^2}{c^2} - k_0^2 \cos^2 \theta_0 + \left(\frac{1}{2\rho} \frac{d^2 \rho}{dz^2} - \frac{3}{4} \left(\frac{d\rho}{\rho dz} \right)^2 \right) \right] = 0. \quad (\text{B.15})$$

Robins (1990) shows that a density profile which permits simple solutions to Equation B.15

is given by

$$\rho = \frac{B_r e^{\alpha_r z}}{(e^{\alpha_r z} + a_r)^2}, \quad (\text{B.16})$$

where B_r , a_r and α_r are constants. Note that α_r here is not to be confused with the attenuation coefficient used in other chapters. For the case of $d\rho/dz = 0$ at $z = -h/2$, it can be shown that

$$a_r = e^{-\alpha_r h/2} = \frac{1 - (1 - r)^{1/2}}{r^{1/2}}, \quad (\text{B.17})$$

where $r = \rho_1/\rho_2$. Therefore, for a given density ratio r and layer thickness h , parameters a_r and α_r can be calculated. Substituting Equation B.16 into B.15 gives

$$\frac{d^2 q}{dz^2} + q \left(\frac{\omega^2}{c^2} - k_0^2 \cos^2 \theta_0 - \frac{\alpha_r^2}{4} \right) = 0. \quad (\text{B.18})$$

Solutions to Equation B.18 are presented by Robins (1991) for three sound-speed profiles: constant sound speed, linear k^2 sound speed, and an inverse-square sound speed. Ideally, the modeled sound-speed profile would be tuned to closely match a boundary-layer profile; therefore, the inverse-square profile is chosen since it contains the most parameters that can be manually varied. This profile is described by the equation

$$\frac{1}{c^2} = \frac{b_r^2}{c_0^2} + \left(\frac{1}{c_1^2} - \frac{b_r^2}{c_0^2} \right) \frac{1}{(1 - \gamma_r z)^2}, \quad (\text{B.19})$$

where b_r and γ_r are constants, c_0 is the initial sound speed ($z > h/2$), and c_1 is sound speed in the middle of the transition layer ($z = 0$). By defining a new variable $\zeta = 1 - \gamma_r z$, the solution to

Equation B.18 is now given by

$$q = \zeta^{1/2} \left[C'' H_v^{(1)}(\beta_r \zeta) + D'' H_v^{(2)}(\beta_r \zeta) \right], \quad (\text{B.20})$$

where C'' and D'' are arbitrary constants, $H_v^{(1)}$ and $H_v^{(2)}$ are Hankel functions of the first and second kind, respectively, and parameters β_r and ν_r are defined by

$$\beta_r^2 = \frac{1}{\gamma_r^2} \left(k_0^2 b^2 - k_0^2 \cos^2 \theta_0 - \frac{\alpha_r^2}{4} \right), \quad (\text{B.21})$$

$$\nu_r^2 = \frac{1}{4} - \frac{\omega^2}{\gamma_r^2} \left(\frac{1}{c_1^2} - \frac{b_r^2}{c_0^2} \right). \quad (\text{B.22})$$

B.3 Reflection Coefficient

For the density and sound-speed profiles described by Equations B.16 and B.19, respectively, computing the reflection coefficient involves the cross product of Hankel functions. The Hankel function notation used by Robins (1991) is

$$H_-^{(1)} = H_v^{(1)} [\beta_r (1 - \gamma_r h/2)], \quad (\text{B.23})$$

$$H_+^{(2)} = H_v^{(2)} [\beta_r (1 + \gamma_r h/2)], \quad (\text{B.24})$$

where the subscript “+” represents values at the bottom of the transition layer, and “−” represents values at the top. As derived by Robins (1991), the reflection coefficient is then found from

$$Re^{ik_0 h \sin \theta_0} = -N_r/D_r, \quad (\text{B.25})$$

with

$$\begin{aligned}
N_r = & s_r(\beta_r\gamma_r h)^2 \left[H_+^{(1)'} H_-^{(2)'} - H_+^{(2)'} H_-^{(1)'} \right] \\
& + s_r(\beta_r\gamma_r h)(\lambda_r + Q) \left[H_-^{(1)'} H_+^{(2)} - H_-^{(2)'} H_+^{(1)} \right] \\
& + (\beta_r\gamma_r h)(s_r\tau_r + V) \left[H_-^{(1)} H_+^{(2)'} - H_-^{(2)} H_+^{(1)'} \right] \\
& + (\lambda_r + Q)(s_r\tau_r + V) \left[H_+^{(1)} H_-^{(2)} - H_+^{(2)} H_-^{(1)} \right],
\end{aligned} \tag{B.26}$$

$$\begin{aligned}
D_r = & s_r(\beta_r\gamma_r h)^2 \left[H_+^{(1)'} H_-^{(2)'} - H_+^{(2)'} H_-^{(1)'} \right] \\
& + s_r(\beta_r\gamma_r h)(\lambda_r + Q) \left[H_-^{(1)'} H_+^{(2)} - H_-^{(2)'} H_+^{(1)} \right] \\
& + (\beta_r\gamma_r h)(s_r\tau_r + W) \left[H_-^{(1)} H_+^{(2)'} - H_-^{(2)} H_+^{(1)'} \right] \\
& + (\lambda_r + Q)(s_r\tau_r + W) \left[H_+^{(1)} H_-^{(2)} - H_+^{(2)} H_-^{(1)} \right],
\end{aligned} \tag{B.27}$$

$$\lambda_r = -\frac{\gamma_r h}{2 + \gamma_r h}, \tag{B.28}$$

$$\tau_r = -\frac{\gamma_r h}{2 - \gamma_r h}, \tag{B.29}$$

$$Q = f_r\alpha_r h + it_r k_2 h \sin \theta_2, \tag{B.30}$$

$$V = g_r s_r \alpha_r h + ik_0 h \sin \theta_0, \tag{B.31}$$

$$W = g_r s_r \alpha_r h - ik_0 h \sin \theta_0, \tag{B.32}$$

$$f_r = \frac{a_r - e^{-\alpha_r h/2}}{2(a_r + e^{-\alpha_r h/2})}, \tag{B.33}$$

$$g_r = \frac{a_r - e^{\alpha_r h/2}}{2(a_r + e^{\alpha_r h/2})}, \tag{B.34}$$

$$s_r = \frac{\rho_0}{\rho_{1,z=h/2}}, \tag{B.35}$$

$$t_r = \frac{\rho_{1,z=-h/2}}{\rho_2}. \tag{B.36}$$

In the low-frequency limit ($k_0 \rightarrow 0$), the reflection coefficient is a function only of the

properties of the upper and lower media. That is, it is independent of the properties of the transition layer, whose thickness is negligible compared to the acoustic wavelength. In this case, the reflection coefficient is given by

$$R_{\text{lim}} = \frac{\rho_2 c_2 \sin \theta_0 - \rho_0 c_0 \sin \theta_2}{\rho_2 c_2 \sin \theta_0 + \rho_0 c_0 \sin \theta_2}, \quad (\text{B.37})$$

which is equivalent to the general Rayleigh reflection coefficient in Equation B.1 for a given incidence angle. In the high-frequency limit ($k_0 \rightarrow \infty$), the reflection coefficient oscillates between two values, R_1 and R_2 , given by

$$R_1 = \left(\frac{\rho_1^+ \sin \theta_0 \sin \theta_2}{\rho_2 c_0 c_2} - \frac{\rho_0 \sin \theta_1^- \sin \theta_1^+}{\rho_1^- c_1^- c_1^+} \right) \left(\frac{\rho_1^+ \sin \theta_0 \sin \theta_2}{\rho_2 c_0 c_2} + \frac{\rho_0 \sin \theta_1^- \sin \theta_1^+}{\rho_1^- c_1^- c_1^+} \right)^{-1}, \quad (\text{B.38})$$

$$R_2 = \left(\frac{\rho_2 \sin \theta_0 \sin \theta_1^+}{\rho_1^+ c_0 c_1^+} - \frac{\rho_0 \sin \theta_2 \sin \theta_1^-}{\rho_1^- c_2 c_1^-} \right) \left(\frac{\rho_2 \sin \theta_0 \sin \theta_1^+}{\rho_1^+ c_0 c_1^+} + \frac{\rho_0 \sin \theta_2 \sin \theta_1^-}{\rho_1^- c_2 c_1^-} \right)^{-1}, \quad (\text{B.39})$$

where the angles θ_1^+ and θ_1^- are defined by

$$\frac{\cos \theta_1^+}{c_1^+} = \frac{\cos \theta_1^-}{c_1^-} = \frac{\cos \theta_0}{c_0}. \quad (\text{B.40})$$

In this high-frequency limit, the reflection coefficient is independent of the shape of the density or sound-speed profile, but depends on the values of ρ and c at the top and bottom of the transition layer.

B.4 Results

The constants in the density and sound-speed profiles presented in Equation B.16 and B.19, respectively, are manually varied to match a boundary-layer profile with a given concentration of helium and air. The boundary layer is modeled with a sinusoidal profile of temperature as a simple approximation to a boundary layer with an adiabatic wall, and it is assumed that the temperature profile is not affected by the helium injection. This temperature profile is given by

$$T(y) = \frac{T_w - T_\infty}{2} \sin\left(\frac{\pi y}{\delta} + \frac{\pi}{2}\right) + \frac{T_w + T_\infty}{2}, \quad (\text{B.41})$$

where $T_w = 293 \text{ K}$ is the wall temperature. Although a cold wall temperature profile is a better representation of the boundary layer in the tunnel (as detailed in Section 3.3), the adiabatic wall profile allows for an easier comparison with the monotonic Robins profiles. The density variation is computed with the ideal gas law and scaled to match a value of 0.62 kg/m^3 in the freestream (Table 3.1), assuming a constant pressure across the boundary layer. The molar ratio of helium, X_{He} , throughout the boundary layer is also approximated with a sinusoidal profile, as described by

$$X_{He}(y) = \bar{X}_{He} \sin\left(\frac{\pi y}{\delta} + \frac{\pi}{2}\right) + \bar{X}_{He}, \quad (\text{B.42})$$

where $\bar{X}_{He}$ is the mean molar ratio in the boundary layer. The variation in molar ratio of helium is presented in Figure B.2 for the case of $\bar{X}_{He} = [0.1 - 0.4]$. This range of concentrations represent a realistic helium variation in the boundary layer, where the molar ratio directly at the

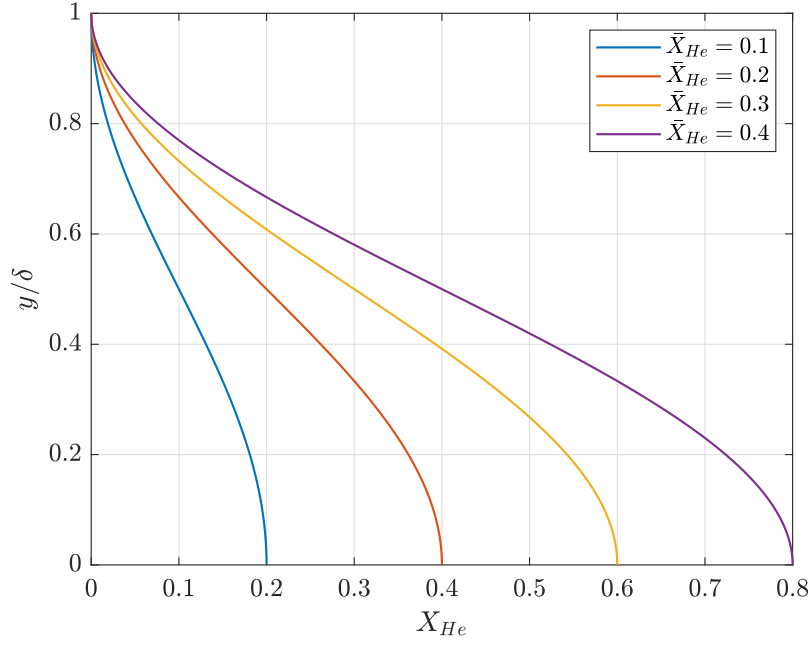


Figure B.2: Sinusoidal variation in molar ratio of helium throughout the boundary layer.

wall is a maximum of 80%. In the experimental setup, it is unlikely that the concentration at the wall is 100% helium since the gas is injected into a turbulent boundary layer upstream of the measurement location, resulting in a well-mixed boundary layer at the measurement location.

To match the boundary-layer profile, the density and sound-speed profiles of [Robins \(1990, 1991\)](#) are computed without any discontinuities. That is, $\rho_0 = \rho_1^-$ and $\rho_1^+ = \rho_2$ in [Figure B.1a](#), and $c_0 = c_1^-$ and $c_1^+ = c_2$ in [Figure B.1b](#). In this case, the reflection coefficient in the high-frequency limit is $R_1 = R_2 = 0$. In [Figure B.3](#), the profiles from Robins are compared to the sinusoidal boundary-layer profile for the case of $\bar{X}_{He} = 0.3$. Although there are some discrepancies between the boundary layer and Robins profiles, they match well enough to provide a reasonable estimate of the reflection coefficient.

The power reflection coefficients for various boundary-layer profiles with a thickness of 1 cm are presented in [Figure B.4](#) for an acoustic incidence angle of 0° and 69.2° (equal to $90^\circ - \mu$, where μ is the Mach angle). These incidence angles represent the two bounds of acoustic propa-

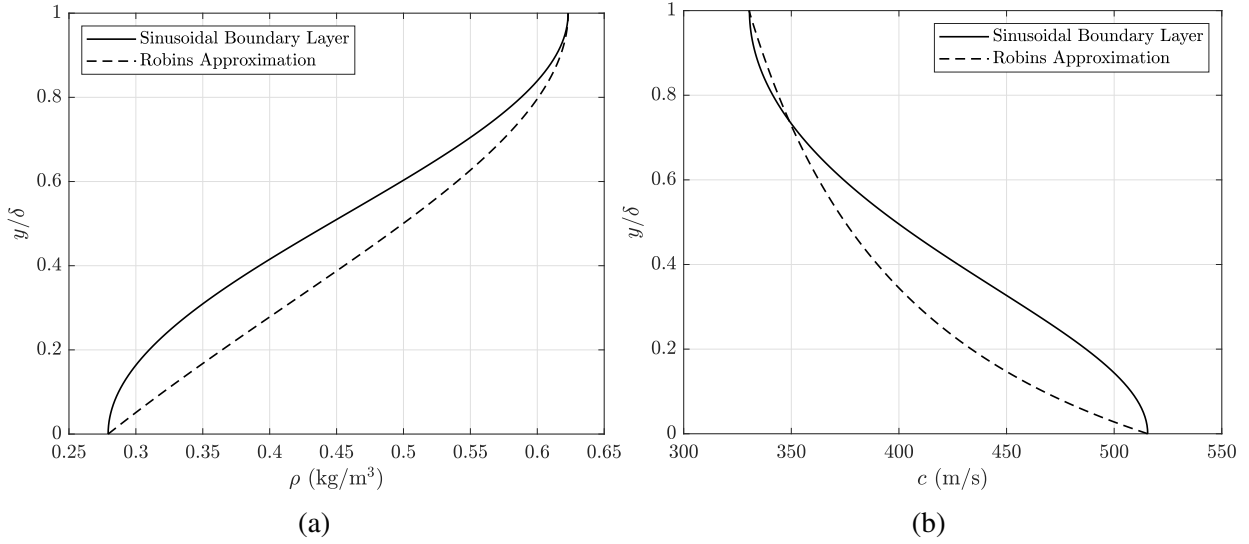


Figure B.3: (a) Density variation and (b) sound-speed variation in profiles from Robins (1990, 1991) and a representative boundary layer with $\bar{X}_{He} = 0.3$.

gation direction. In reality, the propagation angle of the acoustic waves will vary throughout the boundary layer. For normal incidence in Figure B.4a, the power reflection coefficients increase with helium fraction. This is to be expected, since a greater concentration of helium leads to stronger gradients in acoustic impedance. At low frequencies, there is a moderate amount of acoustic reflection; however, minimal reflection occurs at frequencies above 30 kHz. Since the power transmission coefficient can be computed from $T_{\Pi} = 1 - |R|^2$ (Equation B.7), it is evident that the power of the transmitted wave is nearly unaffected above 30 kHz. In Figure B.4b, the large incidence angle results in increased reflectance at high frequencies, with a peak around 30 kHz. Although the reflectance here is greater than the 0° incidence case, it is still not very significant at frequencies greater than 100 kHz. At these higher frequencies, $T_{\Pi} > 0.97$ for $\bar{X}_{He} = 0.4$. Therefore, although helium injection into the boundary layer can create a strong gradient in acoustic impedance, it does not result in significant acoustic reflection at high frequencies.

The effects of boundary-layer height on acoustic reflection are also examined. Presented in

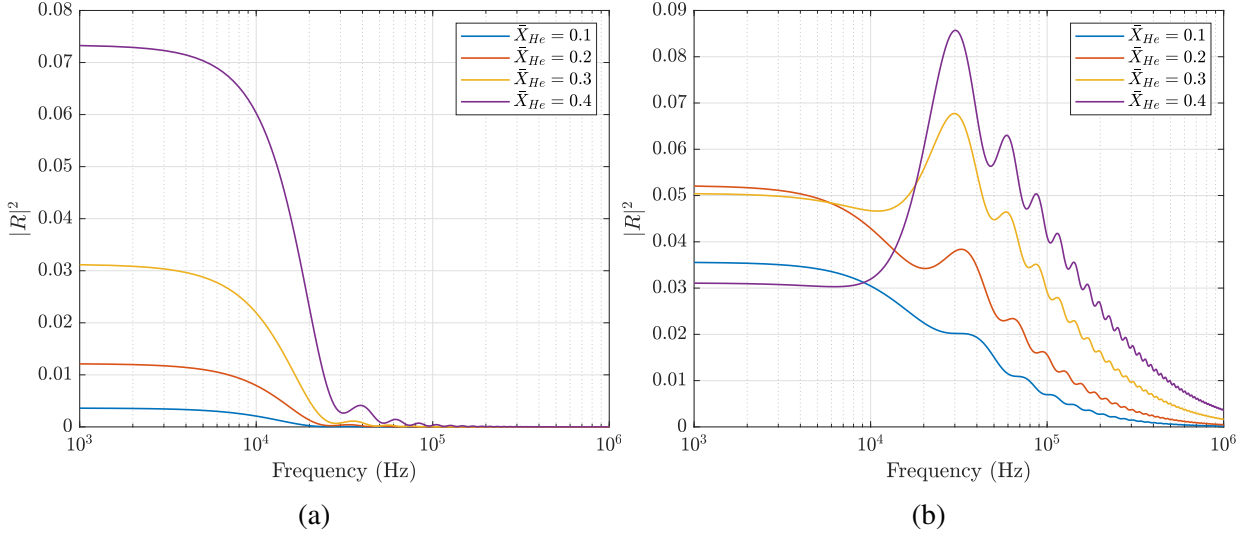


Figure B.4: Power reflection coefficients for various boundary-layer profiles with a thickness of 1 cm and an acoustic incidence angle of (a) 0° and (b) $90^\circ - \mu = 69.2^\circ$.

Figure B.5 are the power reflection coefficients for boundary-layer profiles of various heights with $\bar{X}_{He} = 0.3$. Also plotted is the low-frequency limit of the reflection coefficient (Equation B.37). For incidence angles of 0° (Figure B.5a) and 69.2° (Figure B.5b), an increase in the boundary-layer height shifts the power reflection coefficients to lower frequencies. Since a larger boundary layer decreases the gradient in acoustic impedance, this results in lower reflection coefficients at high frequencies. To maximize the acoustic reflection in a boundary layer composed of helium and air, the boundary layer thickness should be small and the helium concentration should be large at the wall. In practice, this would be difficult to achieve, since the injection of helium into a boundary layer promotes the growth and transition of the boundary layer, increasing turbulent mixing and reducing the gradient of helium concentration at the wall.

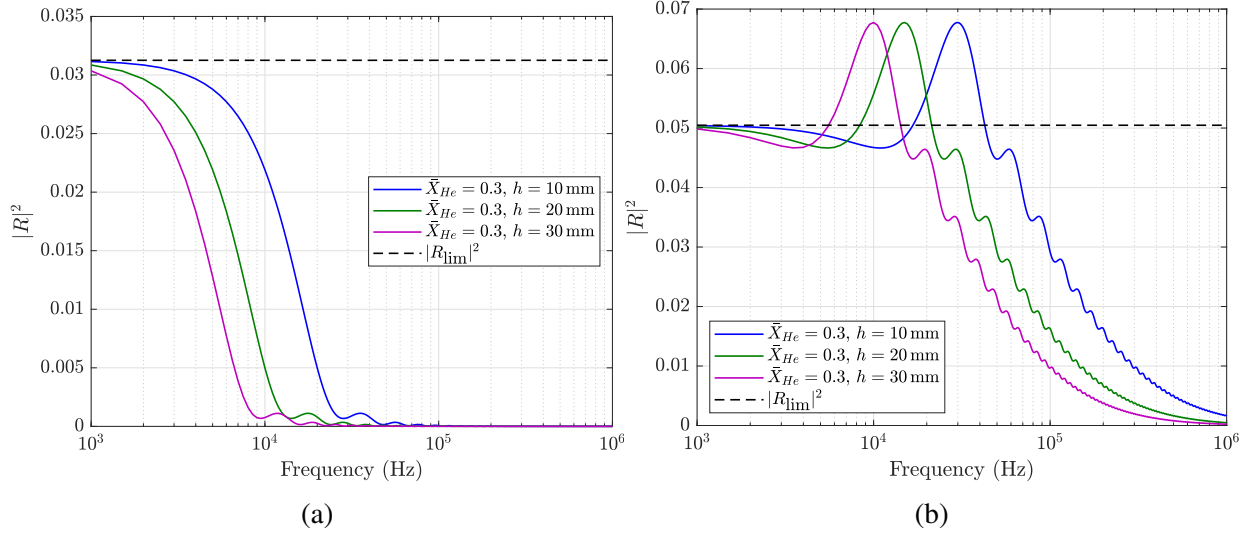


Figure B.5: Power reflection coefficients for boundary-layer profiles of various heights and an acoustic incidence angle of (a) 0° and (b) $90^\circ - \mu = 69.2^\circ$.

Bibliography

- F. A. Angona. The absorption of sound in gas mixtures. *The Journal of the Acoustical Society of America*, 25(6):1116–1122, 1953.
- H. Bass. Vibrational relaxation in CO₂/O₂ mixtures. *The Journal of Chemical Physics*, 58(11):4783–4786, 1973.
- H. Bass, H.-J. Bauer, and L. Evans. Atmospheric absorption of sound: Analytical expressions. *The Journal of the Acoustical Society of America*, 52(3B):821–825, 1972.
- H. E. Bass, J. R. Olson, and R. C. Amme. Vibrational relaxation in H₂O vapor in the temperature range 373–946 K. *The Journal of the Acoustical Society of America*, 56(5):1455–1460, 1974.
- B. F. Bathel, J. M. Weisberger, G. C. Herring, R. A. King, S. B. Jones, R. E. Kennedy, and S. J. Laurence. Two-point, parallel-beam focused laser differential interferometry with a nomarski prism. *Applied Optics*, 59(2):244–252, 2020.
- B. F. Bathel, J. M. Weisberger, G. C. Herring, R. Jagannathan, C. T. Johansen, S. B. Jones, and A. A. Cavone. Analysis of the amplitude response of a two-point and a two-line/multi-point focused laser differential interferometer. In *AIAA Scitech 2021 Forum-0598*, 2021.
- H.-J. Bauer. Phenomenological theory of the relaxation phenomena in gases. In *Physical acoustics*, volume 2, pages 47–131. Elsevier, 1965.
- H.-J. Bauer, F. D. Shields, and H. Bass. Multimode vibrational relaxation in polyatomic molecules. *The Journal of Chemical Physics*, 57(11):4624–4628, 1972.
- E. K. Benitez, J. S. Jewell, and S. P. Schneider. Focused laser differential interferometry for hypersonic flow instability measurements with contoured tunnel windows. In *AIAA Scitech 2020 Forum-1282*, 2020.
- E. K. Benitez, M. P. Borg, J. L. Hill, M. T. Aultman, L. Duan, C. L. Running, and J. S. Jewell. Quantitative focused laser differential interferometry with hypersonic turbulent boundary layers. *Applied Optics*, 61(31):9203–9216, 2022.
- A. B. Bhatia. *Ultrasonic absorption: an introduction to the theory of sound absorption and dispersion in gases, liquids, and solids*. Courier Corporation, 1985.

- M. P. Borg, R. L. Kimmel, and S. Stanfield. Traveling crossflow instability for the HIFiRE-5 elliptic cone. *Journal of Spacecraft and Rockets*, 52(3):664–673, 2015.
- M. Born and E. Wolf. Basic properties of the electromagnetic field. *Principles of Optics*, 44: 1–70, 1980.
- A. Bounitch, D. Lewis, and J. Lafferty. Improved measurements of “tunnel noise” pressure fluctuations in the AEDC hypervelocity wind tunnel no. 9. In *49th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition-1200*, 2011.
- C. Butler. *Response of hypersonic boundary-layer disturbances to compression and expansion corners*. PhD thesis, University of Maryland, College Park, 2021.
- C. Butler and S. J. Laurence. HyperTERP: A newly commissioned hypersonic shock tunnel at the University of Maryland. In *AIAA Aviation 2019 Forum-2860*, 2019.
- C. S. Butler and S. J. Laurence. Interaction of second-mode wave packets with an axisymmetric expansion corner. *Experiments in Fluids*, 62(7), Jun 2021a. ISSN 1432-1114. doi: 10.1007/s00348-021-03235-2. URL <https://doi.org/10.1007/s00348-021-03235-2>.
- C. S. Butler and S. J. Laurence. Interaction of second-mode disturbances with an incipiently separated compression-corner flow. *Journal of Fluid Mechanics*, 913, 2021b.
- C. S. Butler and S. J. Laurence. Interaction of second-mode wave packets with an axisymmetric expansion corner. *Experiments in Fluids*, 62(7):1–17, 2021c.
- C. S. Butler and S. J. Laurence. Transitional hypersonic flow over slender cone/flare geometries. *Journal of Fluid Mechanics*, 949, 2022.
- A. Ceruzzi. *Development of Two-Point Focused Laser Differential Interferometry for Applications in High-Speed Wind Tunnels*. PhD thesis, University of Maryland, 2022.
- A. Ceruzzi and C. P. Cadou. Simultaneous velocity and density gradient measurements using two-point focused laser differential interferometry. In *AIAA Scitech 2019 Forum-2295*, 2019.
- A. Ceruzzi, B. Callis, D. Weber, and C. P. Cadou. Application of focused laser differential interferometry (FLDI) in a supersonic boundary layer. In *AIAA Scitech 2020 Forum-1973*, 2020.
- A. Ceruzzi, B. McManamen, and C. P. Cadou. Demonstration of two-point focused laser differential interferometry (2pFLDI) in a Mach 18 flow. In *AIAA Scitech 2021 Forum-0983*, 2021a.
- A. Ceruzzi, C. Neisess, B. McManamen, and C. P. Cadou. Investigation of focused laser differential interferometry (FLDI) sensitivity function. In *AIAA Scitech 2021 Forum-1299*, 2021b.
- A. P. Ceruzzi and C. P. Cadou. Interpreting single-point and two-point focused laser differential interferometry in a turbulent jet. *Experiments in Fluids*, 63(7):1–25, 2022.

- M. W. Chase Jr and N.-J. T. Tables. Data reported in NIST standard reference database 69, June 2005 release: NIST Chemistry WebBook. *J. Phys. Chem. Ref. Data, Monograph*, 9:1–1951, 1998.
- A. Chou, A. Leidy, R. A. King, B. F. Bathel, and G. Herring. Measurements of freestream fluctuations in the NASA Langley 20-inch Mach 6 tunnel. In *2018 Fluid Dynamics Conference-3073*, 2018.
- J. F. Clarke, M. McChesney, and L. Talbot. The dynamics of real gases. *Physics Today*, 17(11), 1964.
- Y. Dain and R. M. Lueptow. Acoustic attenuation in three-component gas mixtures—theory. *The Journal of the Acoustical Society of America*, 109(5):1955–1964, 2001.
- T. A. Davidson. *A simple and accurate method for calculating viscosity of gaseous mixtures*. US Department of the Interior, Bureau of Mines, 1993.
- A. Demetriades. An experiment on the stability of hypersonic laminar boundary layers. *Journal of Fluid Mechanics*, 7(3):385–396, 1960.
- A. Demetriades. Hypersonic viscous flow over a slender cone. III-Laminar instability and transition. In *7th Fluid and Plasma Dynamics Conference-535*, 1974.
- A. Demetriades. Laminar boundary layer stability measurements at Mach 7 including wall temperature effects. Technical report, Ford Aerospace and Communications Corp Newport Beach CA Aeronautic Div, 1977.
- P. Dickens and J. W. Linnett. Calculation of vibrational relaxation times in gaseous sulphur dioxide. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, 243(1232):84–93, 1957.
- H. Doi, Y. Kono, and K. Higashi. Vibrational relaxation for UF₆, WF₆, MoF₆, and SF₆. *The Journal of the Acoustical Society of America*, 54(5):1267–1270, 1973.
- L. Duan and M. Choudhari. Numerical study of pressure fluctuations due to a Mach 6 turbulent boundary layer. In *51st AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition-532*, 2013.
- L. Duan, M. M. Choudhari, and M. Wu. Numerical study of acoustic radiation due to a supersonic turbulent boundary layer. *Journal of Fluid Mechanics*, 746:165–192, 2014.
- L. Duan, M. M. Choudhari, and C. Zhang. Acoustic radiation from high-speed turbulent boundary layers in a tunnel-like environment. In *53rd AIAA Aerospace Sciences Meeting-0836*, 2015.
- L. Duan, M. M. Choudhari, and C. Zhang. Pressure fluctuations induced by a hypersonic turbulent boundary layer. *Journal of Fluid Mechanics*, 804:578–607, 2016.

- L. Duan, M. M. Choudhari, A. Chou, F. Munoz, R. Radespiel, T. Schilden, W. Schröder, E. C. Marineau, K. M. Casper, R. S. Chaudhry, et al. Characterization of freestream disturbances in conventional hypersonic wind tunnels. *Journal of Spacecraft and Rockets*, 56(2):357–368, 2019.
- S. G. Ejakov, S. Phillips, Y. Dain, R. M. Lueptow, and J. H. Visser. Acoustic attenuation in gas mixtures with nitrogen: experimental data and calculations. *The Journal of the Acoustical Society of America*, 113(4):1871–1879, 2003.
- O. S. Elliott, R. Greendyke, J. S. Jewell, and J. R. Komives. Effect of CO₂ concentration in the hypersonic boundary layer on second mode disturbances. In *AIAA Aviation 2019 Forum-2851*, 2019.
- N. A. Fomin. *Speckle Photography for Fluid Mechanics Measurements*. Springer Science & Business Media, 1998.
- E. F. Fricke. The absorption of sound in five triatomic gases. *The Journal of the Acoustical Society of America*, 12(2):245–254, 1940.
- K. Fujii and H. G. Hornung. Experimental investigation of high-enthalpy effects on attachment-line boundary-layer transition. *AIAA Journal*, 41(7):1282–1291, 2003.
- M. R. Fulghum. *Turbulence measurements in high-speed wind tunnels using focusing laser differential interferometry*. PhD thesis, The Pennsylvania State University, 2014.
- T. J. Fuller, A. G. Hsu, R. Sanchez-Gonzalez, J. C. Dean, S. W. North, and R. D. Bowersox. Radiofrequency plasma stabilization of a low-Reynolds-number channel flow. *Journal of Fluid Mechanics*, 748:663–691, 2014.
- G. I. Gillespie, A. P. Ceruzzi, and S. J. Laurence. A multi-point focused laser differential interferometer for characterizing freestream disturbances in hypersonic wind tunnels. *Experiments in Fluids*, 63(11):1–17, 2022.
- X. Gloerfelt and J. Berland. Turbulent boundary layer noise: direct radiation at Mach number 0.5. *Journal of Fluid Mechanics*, 723:318–351, 2013.
- M. Gragston, T. Price, K. Davenport, Z. Zhang, and J. D. Schmisser. Linear array focused-laser differential interferometry for single-shot multi-point flow disturbance measurements. *Optics Letters*, 46(1):154–157, 2021.
- A. Hameed and N. J. Parziale. Focused Laser Differential Interferometric Investigation of Turbulent Jet Spectra. *Journal of Spacecraft and Rockets*, pages 1–9, 2022. doi: 10.2514/1.A35292.
- A. Hameed, N. J. Parziale, L. A. Paquin, C. Butler, and S. J. Laurence. Hypersonic slender-cone boundary-layer instability in the UMD HyperTERP shock tunnel. In *AIAA Scitech 2020 Forum-0362*, 2020.
- A. Hameed, D. Shekhtman, N. Parziale, L. A. Paquin, S. Skinner, S. J. Laurence, W. M. Yu, and J. M. Austin. Hypersonic boundary-layer instability on a highly cooled cone. Part I: Q-FLDI measurement and instability calculations. In *AIAA Scitech 2022 Forum-0734*, 2022.

- J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird. Molecular theory of gases and liquids. *Molecular theory of gases and liquids*, 1964.
- A. Houpt and S. Leonov. Cylindrical focused laser differential interferometer. *AIAA Journal*, 59(4):1142–1150, 2021.
- J. Huang and L. Duan. Direct numerical simulation of acoustic noise generation from the nozzle wall of a hypersonic wind tunnel. In *47th AIAA Fluid Dynamics Conference-3631*, 2017.
- C. Illingworth. Some solutions of the equations of flow of a viscous compressible fluid. In *Mathematical Proceedings of the Cambridge Philosophical Society*, volume 46, pages 469–478. Cambridge University Press, 1950.
- J. Jewell, R. Wagnild, I. Leyva, G. Candler, and J. Shepherd. Transition within a hypervelocity boundary layer on a 5-degree half-angle cone in air/CO₂ mixtures. In *51st AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition-523*, 2013.
- J. S. Jewell, A. Hameed, N. J. Parziale, and S. P. Gogineni. Disturbance speed measurements in a circular jet via double focused laser differential interferometry. In *AIAA Scitech 2019 Forum-2293*, 2019.
- J. E. Jones. On the determination of molecular fields.—II. From the equation of state of a gas. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 106(738):463–477, 1924.
- J. M. Kendall. Wind tunnel experiments relating to supersonic and hypersonic boundary-layer transition. *AIAA Journal*, 13(3):290–299, 1975.
- R. E. Kennedy, J. S. Jewell, P. Paredes, and S. J. Laurence. Characterization of instability mechanisms on sharp and blunt slender cones at Mach 6. *Journal of Fluid Mechanics*, 936, 2022.
- L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders. *Fundamentals of acoustics*. John Wiley & Sons, 2000.
- G. Kirchhoff. Ueber den einfluss der wärmeleitung in einem gase auf die schallbewegung. *Annalen der Physik*, 210(6):177–193, 1868.
- H. O. Kneser. The interpretation of the anomalous sound-absorption in air and oxygen in terms of molecular collisions. *The Journal of the Acoustical Society of America*, 5(2):122–126, 1933.
- L. S. Kovasznay. Turbulence in supersonic flow. *Journal of the Aeronautical Sciences*, 20(10):657–674, 1953.
- J. Lafferty and J. Norris. Measurements of fluctuating pitot pressure, “tunnel noise,” in the AEDC hypervelocity wind tunnel no. 9. In *2007 US Air Force T&E Days-1678*. 2007.
- J. D. Lambert and J. D. Lambert. *Vibrational and rotational relaxation in gases*. Number 1 in International Series of Monographs on Chemistry. Oxford University Press, 1977.
- L. Landau and E. Teller. Zur theorie der schalldispersion. *Phys. Z. Sowjetunion*, 10(1), 1936.

- L. D. Landau. Zur theorie der energieubertragung II. *Z. Sowjetunion*, 2:46–51, 1932.
- J. Laufer. Aerodynamic noise in supersonic wind tunnels. *Journal of the Aerospace Sciences*, 28(9):685–692, 1961.
- J. Laufer. Some statistical properties of the pressure field radiated by a turbulent boundary layer. *The Physics of Fluids*, 7(8):1191–1197, 1964.
- J. Laufer and T. Vrebalovich. Stability and transition of a supersonic laminar boundary layer on an insulated flat plate. *Journal of Fluid Mechanics*, 9(2):257–299, 1960.
- S. Laurence, A. Wagner, and K. Hannemann. Experimental study of second-mode instability growth and breakdown in a hypersonic boundary layer using high-speed schlieren visualization. *Journal of Fluid Mechanics*, 797:471–503, 2016.
- S. J. Laurence and S. Karl. An improved visualization-based force-measurement technique for short-duration hypersonic facilities. *Experiments in Fluids*, 48(6):949–965, 2010.
- J. Lawson, M. Neet, I. Grossman, and J. Austin. Static and dynamic characterization of a focused laser differential interferometer. *Experiments in Fluids*, 61(8):1–11, 2020.
- J. M. Lawson. *Focused Laser Differential Interferometry*. PhD thesis, California Institute of Technology, 2021.
- J. M. Lawson, M. C. Neet, I. J. Grossman, and J. M. Austin. Characterization of a focused laser differential interferometer. In *AIAA Scitech 2019 Forum-2296*, 2019.
- J. M. Lawson, M. C. Neet, J. W. Hofferth, and J. M. Austin. Supersonic freestream density fluctuations from focused laser differential interferometry and pitot-probe measurements. *AIAA Journal*, pages 1–14, 2022.
- R. W. Leonard. The absorption of sound in carbon dioxide. *The Journal of the Acoustical Society of America*, 12(2):241–244, 1940.
- J. Lewis and F. Shields. Vibrational relaxation in carbon dioxide/helium mixtures. *The Journal of the Acoustical Society of America*, 41(1):100–102, 1967.
- I. Leyva, S. Laurence, A. Beierholm, H. Hornung, R. Wagnild, and G. Candler. Transition delay in hypervelocity boundary layers by means of CO₂/acoustic instability interactions. In *47th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition-1287*, 2009.
- D. R. Lide. *CRC handbook of chemistry and physics*, volume 85. CRC press, 2004.
- L. M. Mack. Linear stability theory and the problem of supersonic boundary-layer transition. *AIAA Journal*, 13(3):278–289, 1975.
- K. Mahesh and S. Lee. The interaction of an isotropic field of acoustic waves with a shock wave. *J. Fluid Mech*, 300:383–407, 1995.

- E. Mason and S. Saxena. Approximate formula for the thermal conductivity of gas mixtures. *The Physics of Fluids*, 1(5):361–369, 1958.
- S. A. Maszkiewicz, G. I. Gillespie, and S. J. Laurence. Experimental investigation of transitional, sharp-fin-induced shock-wave/boundary-layer interactions at Mach 6. In *AIAA SciTech 2022 Forum-1818*, 2022.
- A. Miklavc and S. Ficher. On the theory of collision-induced vibrational transitions in polyatomic molecules: The mode-matching model. *Chemical Physics Letters*, 44(2):209–213, 1976.
- E. Moreira, M. Innocentini, and J. Coury. Permeability of ceramic foams to compressible and incompressible flow. *Journal of the European Ceramic Society*, 24(10-11):3209–3218, 2004.
- M. Morkovin. Transition in open flow systems-a reassessment. *Bull. Am. Phys. Soc.*, 39, 1994.
- M. V. Morkovin. On transition experiments at moderate supersonic speeds. *Journal of the Aeronautical Sciences*, 24(7):480–486, 1957.
- M. V. Morkovin. On supersonic wind tunnels with low free-stream disturbances. *Journal of Applied Mechanics*, 26(3):319–324, 1959.
- A. G. Neville, I. Nompelis, P. K. Subbareddy, and G. V. Candler. Effect of thermal non-equilibrium on compressible isotropic turbulence. In *7th AIAA Theoretical Fluid Mechanics Conference-3204*, 2014.
- A. G. Neville, I. Nompelis, P. K. Subbareddy, and G. V. Candler. Thermal non-equilibrium effects in turbulent compressible shear flows. In *45th AIAA Fluid Dynamics Conference-3218*, 2015.
- S. Pao. Aerodynamic noise emission from turbulent shear layers. *Journal of Fluid Mechanics*, 59(3):451–479, 1973.
- L. A. Paquin, S. Skinner, S. J. Laurence, A. Hameed, D. Shekhtman, N. J. Parziale, W. M. Yu, and J. M. Austin. Hypersonic boundary-layer instability on a highly cooled cone. Part II: Schlieren analysis of boundary-layer disturbances. In *AIAA Scitech 2022 Forum-0947*, 2022a.
- L. A. Paquin, S. N. Skinner, and S. J. Laurence. Hypersonic boundary-layer disturbances on a cooled, slender cone at Mach 6. *Journal of Spacecraft and Rockets*, pages 1–12, 2022b.
- N. Parziale, J. Shepherd, and H. Hornung. Reflected shock tunnel noise measurement by focused differential interferometry. In *42nd AIAA Fluid Dynamics Conference and Exhibit-3261*, 2012.
- N. Parziale, J. Shepherd, and H. Hornung. Free-stream density perturbations in a reflected-shock tunnel. *Experiments in Fluids*, 55:1–10, 2014.
- N. J. Parziale. *Slender-body hypervelocity boundary-layer instability*. PhD thesis, California Institute of Technology, 2013.
- S. R. Pate and C. Schueler. Radiated aerodynamic noise effects on boundary-layer transition in supersonic and hypersonic wind tunnels. *AIAA Journal*, 7(3):450–457, 1969.

- X. Y. Pee. *Unsteady Shock Propagation in a Constant-Area Thermally Choked Flow: Numerical and Experimental Investigations*. PhD thesis, University of Maryland, College Park, 2021.
- A. Petculescu, B. Hall, R. Fraenzle, S. Phillips, and R. M. Lueptow. A prototype acoustic gas sensor based on attenuation. *The Journal of the Acoustical Society of America*, 120(4):1779–1782, 2006.
- A. G. Petculescu and R. M. Lueptow. Fine-tuning molecular acoustic models: sensitivity of the predicted attenuation to the Lennard–Jones parameters. *The Journal of the Acoustical Society of America*, 117(1):175–184, 2005.
- O. M. Phillips. On the generation of sound by supersonic turbulent shear layers. *Journal of Fluid Mechanics*, 9(1):1–28, 1960.
- J. M. M. Pinkerton. On the pulse method of measuring ultrasonic absorption in liquids. *Proceedings of the Physical Society. Section B*, 62(5):286, 1949.
- J. Poggie, R. L. Kimmel, and S. N. Schwoerke. Traveling instability waves in a Mach 8 flow over an elliptic cone. *AIAA Journal*, 38(2):251–258, 2000.
- E. Reshotko. Boundary layer instability, transition and control. In *32nd Aerospace Sciences Meeting and Exhibit*, 1994.
- W. T. Richards and J. A. Reid. Acoustical studies. III. the rates of excitation of vibrational energy in carbon dioxide, carbon disulfide and sulfur dioxide. *The Journal of Chemical Physics*, 2(4):193–205, 1934.
- A. J. Robins. Reflection of plane acoustic waves from a layer of varying density. *The Journal of the Acoustical Society of America*, 87(4):1546–1552, 1990.
- A. J. Robins. Reflection of a plane wave from a fluid layer with continuously varying density and sound speed. *The Journal of the Acoustical Society of America*, 89(4):1686–1696, 1991.
- T. Schilden and W. Schröder. Inclined slow acoustic waves incident to stagnation point probes in supersonic flow. *Journal of Fluid Mechanics*, 866:567–597, 2019.
- B. E. Schmidt and J. Shepherd. Analysis of focused laser differential interferometry. *Applied Optics*, 54(28):8459–8472, 2015.
- S. P. Schneider. Effects of high-speed tunnel noise on laminar-turbulent transition. *Journal of Spacecraft and Rockets*, 38(3):323–333, 2001.
- S. P. Schneider. Development of hypersonic quiet tunnels. *Journal of Spacecraft and Rockets*, 45(4):641–664, 2008.
- R. Schwartz, Z. Slawsky, and K. Herzfeld. Calculation of vibrational relaxation times in gases. *The Journal of Chemical Physics*, 20(10):1591–1599, 1952.

- G. S. Settles and M. R. Fulghum. The focusing laser differential interferometer, an instrument for localized turbulence measurements in refractive flows. *Journal of Fluids Engineering*, 138(10), 2016.
- F. D. Shields. Measurements of thermal relaxation in CO₂ extended to 300C. *The Journal of the Acoustical Society of America*, 31(2):248–249, 1959.
- F. D. Shields. Vibrational relaxation in pure SO₂ and SO₂/Ar mixtures. *The Journal of Chemical Physics*, 46(3):1063–1069, 1967.
- F. D. Shields. On obtaining transition rates from sound absorption and dispersion curves. *The Journal of the Acoustical Society of America*, 47(5B):1262–1268, 1970.
- F. D. Shields and H. E. Bass. Atmospheric absorption of high frequency noise and application to fractional-octave bands. Technical report, NASA, 1977.
- A. E. Siegman. Lasers university science books. *Mill Valley, CA*, 1986.
- G. Smeets. Laser interferometer for high sensitivity measurements on transient phase objects. *IEEE Transactions on Aerospace and Electronic Systems*, AES-8(2):186–190, 1972.
- G. Smeets. Flow diagnostics by laser interferometry. *IEEE Transactions on Aerospace and Electronic Systems*, AES-13(2):82–90, 1977.
- G. Smeets and A. George. Gas-dynamic investigations in a shock tube using a highly sensitive interferometer. Technical report, Institut Saint-Louis, 1971.
- G. Smeets and A. George. Laser interferometer with phase compensation. *NASA STI/Recon Technical Report N*, 77, 1973.
- G. Smeets and A. George. Laser-differential interferometer applications in gas dynamics. Technical report, Institut Saint-Louis, 1975.
- P. Stainback and R. Wagner. A comparison of disturbance levels measured in hypersonic tunnels using a hot-wire anemometer and a pitot pressure probe. In *7th Aerodynamic Testing Conference-1003*, 1972.
- W. C. Starshak and S. J. Laurence. Computer-graphics-based optical tracking for hypersonic free-flight experiments. *AIAA Journal*, 59(12):4955–4968, 2021.
- W. C. Starshak, C. Butler, and S. J. Laurence. Method of characteristics design of high-temperature wind tunnel nozzles with vibrational relaxation. In *2018 Aerodynamic Measurement Technology and Ground Testing Conference-3565*, 2018.
- K. Stetson and R. Kimmel. On hypersonic boundary-layer stability. In *30th Aerospace Sciences Meeting and Exhibit-737*, 1992.
- K. Stetson, E. Thompson, J. Donaldson, and L. Siler. Laminar boundary layer stability experiments on a cone at Mach 8. I-sharp cone. In *16th Fluid and Plasma Dynamics Conference-1761*, 1983.

- G. G. Stokes. On the theories of the internal friction of fluids in motion, and of the equilibrium and motion of elastic solids. 1845.
- F. I. Tanczos. Calculation of vibrational relaxation times of the chloromethanes. *The Journal of Chemical Physics*, 25(3):439–447, 1956.
- R. M. Wagnild. High enthalpy effects on two boundary layer disturbances in supersonic and hypersonic flow. Technical report, Minnesota University, 2012.
- J. C. Wang and G. S. Springer. Vibrational relaxation times in some hydrocarbons in the range 300–900 K. *The Journal of Chemical Physics*, 59(12):6556–6562, 1973.
- J. M. Weisberger, B. F. Bathel, G. C. Herring, G. M. Buck, S. B. Jones, and A. A. Cavone. Two-line focused laser differential interferometry of a flat plate boundary layer at Mach 6. In *AIAA SciTech 2021 Forum-0601*, 2021.
- T. J. Whalen and S. J. Laurence. Experiments on the separation of sphere clusters in hypersonic flow. *Experiments in Fluids*, 62(4):1–19, 2021.
- F. M. White and J. Majdalani. *Viscous fluid flow*, volume 3. McGraw-Hill New York, 2006.
- J. F. Williams and G. Maidanik. The Mach wave field radiated by supersonic turbulent shear flows. *Journal of Fluid Mechanics*, 21(4):641–657, 1965.
- E. B. Wilson, J. C. Decius, and P. C. Cross. *Molecular vibrations: the theory of infrared and Raman vibrational spectra*. Courier Corporation, 1980.
- T. G. Winter and G. L. Hill. High-temperature ultrasonic measurements of rotational relaxation in hydrogen, deuterium, nitrogen, and oxygen. *The Journal of the Acoustical Society of America*, 42(4):848–858, 1967.
- C. Zener. Interchange of translational, rotational and vibrational energy in molecular collisions. *Physical Review*, 37(5), 1931.
- C. Zhang and L. Duan. Acoustic radiation from a Mach 14 turbulent boundary layer. In *54th AIAA Aerospace Sciences Meeting-0048*, 2016.
- C. Zhang, L. Duan, and M. M. Choudhari. Direct numerical simulation database for supersonic and hypersonic turbulent boundary layers. *AIAA Journal*, 56(11):4297–4311, 2018.
- K.-S. Zhang, S. Wang, M. Zhu, Y. Ding, and Y. Hu. Decoupling multimode vibrational relaxations in multi-component gas mixtures: analysis of sound relaxational absorption spectra. *Chinese Physics B*, 22(1), 2013.
- X. Zhang, S. Wang, and M. Zhu. Locating the inflection point of frequency-dependent velocity dispersion by acoustic relaxation to identify gas mixtures. *Measurement Science and Technology*, 31(11), 2020.
- A. J. Zuckerwar and W. A. Griffin. Resonant tube for measurement of sound absorption in gases at low frequency/pressure ratios. *The Journal of the Acoustical Society of America*, 68(1): 218–226, 1980.