

ABSTRACT

Title of dissertation: DEVELOPMENT OF THE
MARYLAND TILTROTOR RIG (MTR)
AND WHIRL FLUTTER
STABILITY TESTING

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Tiltrotor aircraft encounter an aeroelastic instability called whirl flutter at high speeds. Whirl flutter is caused by the complex interaction between the aerodynamics and dynamics of the rotating proprotor blades, hub, and the wing. Current tiltrotors are limited to about 280 kt in cruise. While many computational analyses have been performed to assess potential improvements in whirl flutter stability, few have been validated by test data. There is a scarcity of publicly available test data along with documented model properties. A new tiltrotor rig is developed in this work to address this gap. The new rig, henceforth called the Maryland Tiltrotor Rig (MTR), is a semi-span, floor-mounted, optionally-powered rig with a static rotor tilt mechanism, capable of testing 3-bladed proprotors of up to 4.75-ft diameter in the Glenn L. Martin Wind Tunnel (7.75- by 11-ft section with 200 kt maximum speed). The objective is to experimentally characterize the parameters that affect the onset of whirl flutter which is vital to validating computational models and analyses.

The MTR supports interchangeable hubs (gimballed and hingeless), interchangeable blades (straight and swept tip), and interchangeable wing spars, to allow a systematic variation of components important for tiltrotor flutter and loads. The vision for this rig is to conduct research towards flutter-free tiltrotors capable of achieving 400 kt and higher speeds in cruise. This dissertation lays the groundwork toward that vision by describing the test and evaluation of a baseline gimballed hub model. The features, controls, instrumentation, data acquisition, and all supporting equipment of the rig are described. A simple whirl flutter analysis model is developed, verified, and used for pre-test stability prediction of the MTR. The damping measurement methods are detailed. The first whirl flutter tests of the MTR were carried out at the Naval Surface Warfare Center-Carderock Division wind tunnel between 26 October - 2 November, 2021. Frequency and damping data were measured for four parametric configurations of wing on versus wing off, gimbal free versus gimbal locked, freewheel versus powered rotor, and straight versus swept-tip blades. The tests were conducted up to 100 kt windspeed, restricted only by the tunnel precautionary measures. Since the baseline model is loosely a 1/5.26-scale XV-15, 100 kt translates to a full-scale speed of 230 kt. It was observed that the baseline rig was stable up to 100 kt with an average wing damping lower than 1% critical in beamwise and 1.5% critical in chordwise motion. The effect of wing aerodynamics was insignificant up to 100 kt. Locking the gimbal affected mostly the chord mode and increased its damping significantly. Powering the rotor also affected mostly the chord mode and increased its damping significantly. The swept-tip blades showed interesting trends near 100 kt but higher speeds are needed for definitive conclusions.

Overall, the MTR allowed the controlled variation of parameters that are important for fundamental understanding and analysis validation, but are impossible to carry out on an actual aircraft.

DEVELOPMENT OF THE
MARYLAND TILTROTOR RIG (MTR)
AND WHIRL FLUTTER STABILITY TESTING

by

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Chapter 1: Introduction

This chapter introduces the background of tiltrotor aircraft and the primary features that differentiate tiltrotors from traditional helicopters and turboprop aircraft, which are summarized in Table 1.1. The concept of whirl flutter is also presented. Then, the history of full-scale and model-scale experimental tiltrotor research is summarized. An in-house-built, small-scale, tiltrotor model is also shown as a task performed in preparation for the present research. The scope and contributions of the current research are presented, and lastly, an outline of the dissertation is provided.

1.1 Background and Motivation

Tiltrotors combine the vertical takeoff and landing capability of helicopters and the high-speed forward flight of conventional fixed-wing planes. These platforms can be seen in Fig. 1.1.

Of all tiltrotor vehicles, only the Bell-Boeing V-22 Osprey is in operational service. The Bell V-280 Valor is a technology demonstrator and Bell's offering for the US Army's Future Long Range Assault Aircraft (FLRAA) program. The Leonardo's AW-609 is aiming to be the first commercial tiltrotor and is currently

Table 1.1: Comparison of features between helicopters, traditional turbo-prop planes, and tiltrotors.

Helicopter	Turboprop Plane	Tiltrotor
VTOL	CTOL	VTOL
Flexible/flapping blades	Rigid blades	Flexible/flapping blades
Low twist blades	High twist blades	High twist blades
Cyclic controls	N/A	Cyclic controls
Large rotors for hover	Small propellers	Large proporsors for hover

High Speed Phenomenon

Retreating blade stall	Propeller whirl flutter	Tiltrotor whirl flutter
------------------------	-------------------------	-------------------------



(a) MV-22 Osprey tiltrotor.



(b) V-280 Valor tiltrotor.



(c) AW-609 tiltrotor.



(d) Joby S4 tiltrotor.

Figure 1.1: Modern day tiltrotors in both military and commercial spaces.

undergoing flight certification requirements. Joby's Urban Air Mobility (UAM) vehicle is also undergoing certification tests at the time of writing this thesis.

The appeal of tiltrotors starts with the elimination of runways and the requirements for vast real estate. For UAM, vertiports, essentially an assembly of helipads, reduce the landing zone footprint and can be placed on rooftops or above parking garages. In terms of performance, tiltrotors are much faster and have greater range than traditional helicopter at the price of lower payload.

In broad terms, all aircraft have to overcome weight and drag forces to fly forward by using lift and propulsive thrust, respectively. Traditional helicopters do so by using one device for both lift and thrust: the main rotor. A tiltrotor can decouple these forces in forward flight by having a wing carry the lift and the proprotor providing the thrust. This means engine power is primarily concentrated on overcoming the drag component only. This concept is shown in Fig. 1.2(b).

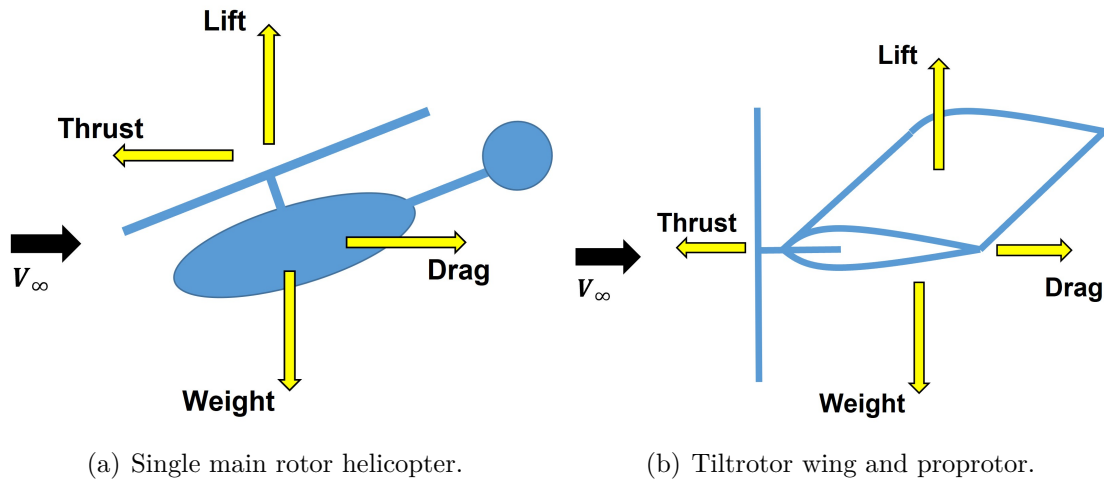


Figure 1.2: Diagrams of primary forces for traditional helicopters and tiltrotors in forward flight.

Helicopters encounter a fundamental limitation in speed due to an asymmetric edgewise flow along the rotor disk shown in Fig. 1.3(a). On the advancing side of the rotor which is the upper part of the diagram, the windspeed seen by the blade tip is the speed of the blade tip plus the speed of the oncoming wind. The sum of these velocities can reach transonic speeds, close to the speed of sound, where compressibility effects, such as shockwaves, occur. On the retreating side of the rotor, which is the bottom half of the diagram, the windspeed seen by the blade is the speed of the blade subtracted the oncoming wind. The faster helicopter is, the slower the windspeed on the retreating side, and this can cause dynamic stall near the tips and a reverse flow region forms on the inboard portion of the blade. These asymmetric effects create large vibrations and ultimately limit the speed of an edgewise rotor.

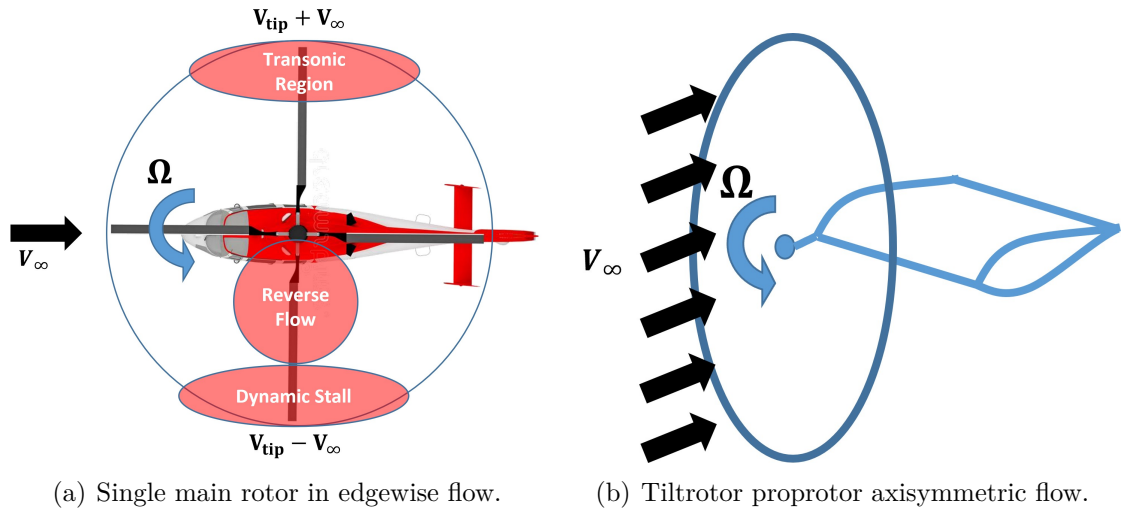


Figure 1.3: Diagrams of flow for rotor in edgewise flight and proprotor in axial flight.

Tiltrotors in forward flight see an axisymmetric flow, shown in Fig. 1.3(b). meaning the flow by one blade on one side of the disk is the same everywhere around the disk. This condition reduces the number of adverse flow effects seen by the rotor.

However, tiltrotors cannot reach the same speeds as conventional fixed wing airplanes because the heavy nacelles and large flexible proprotors at the wing tips bring with them an aeroelastic instability called whirl flutter at high speeds. This phenomenon is far more severe from propeller whirl flutter because of the larger and more flexible rotors required for helicopter mode hover and low-speed edgewise flight.

The first recorded case of whirl flutter occurred on a Lockheed L-188 C Electra II turboprop aircraft, shown in Fig. 1.4, owned by Braniff Airways in 1959 [1]. Less than a year later, another Lockheed L-188 C Electra II owned by Northwest Airlines crashed due to the same phenomenon [2].



Figure 1.4: Lockheed L-188 C Electra II turboprop aircraft.

This caused Lockheed to initiate the Lockheed Electra Action Program (LEAP) to investigate the problems and find solutions. It was found that a propeller whirl mode was exacerbated by weakened mounting of the propeller nacelles to the wing which caused larger than designed deformations and subsequent structural failure of engine and wing structures. A model diagram [3] of this propeller whirl mode is shown in Fig. 1.5.

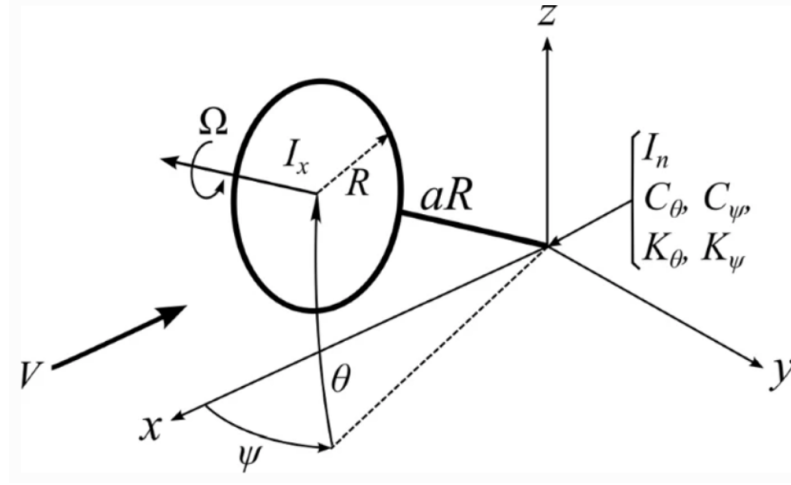


Figure 1.5: Propeller whirl mode model.

This model has two degrees of freedom in pitch, θ , and yaw, ψ . Ω is the propeller RPM, R is the radius of the propeller disk, aR is the length of the rotor shaft, I_x is the propeller mass moment of inertia about the rotational axis. The support structure is represented at the origin with stiffness, K , and damping, C , constants in pitch and yaw and I_n denotes the structural mass moment of inertia. This model assumes the propeller to be rigid. Only the pitch or yaw modes can go unstable and there is no coupling with other parameters. Through this simple model, it was determined that stiffening the support structure and adding additional

damping components, in other words, increasing K and C , were enough to alleviate the instability.

For tiltrotors, the whirl mode is much more complicated. To model a tiltrotor rotor and wing [4, 5], at least nine degrees of freedom are required. Figure 1.6 shows the simplest model for a tiltrotor.

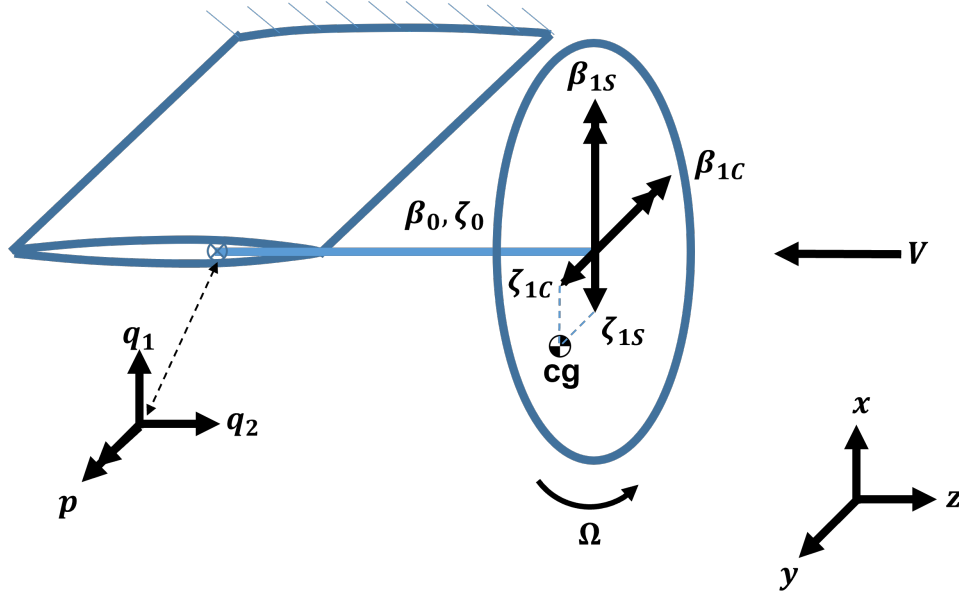


Figure 1.6: Tiltrotor whirl flutter model.

Three modes for the wing: q_1 beam bending, q_2 chord bending, and p torsion. Six rotor modes because the rotor has flexible blades with a gimballed hub, so the rotor can flap and lag and the rigid assumption is not valid. The variables β_0 , β_{1C} , β_{1S} denote the collective flap, longitudinal flap, and lateral flap motions, respectively. The variables ζ_0 , ζ_{1C} , and ζ_{1S} denote the collective lag, longitudinal lag, and lateral lag motions, respectively. The flapping motion occurs out of plane of the hub, whereas the lagging motion occurs in plane. The flapping motion affects the rotor in-plane forces generated as a result of wing perturbation and has a primary

influence in tiltrotor whirl flutter. Tiltrotor whirl flutter derives its name from propeller whirl flutter but the mechanism is far more complicated. As a result, the solution is also more drastic which in turn limits tiltrotor cruise performance below a propeller aircraft.

The current solution to delay whirl flutter on tiltrotors is to use very thick wings, around twice as thick as a propeller aircraft; for example, V-22 wing is 23% thickness-to-chord as compared to a typical 14% for a turboprop. This is the manifestation of increasing the stiffness K of the support structure. However, this creates a significant amount of drag and, ultimately, limits tiltrotors from achieving high speeds. Current tiltrotors are limited to about 280 kt whereas their turboprop airplane counterparts can achieve up to 400 kt and higher. To realize higher speeds, tiltrotor wings must be made thinner without adverse impact on whirl flutter.

Many numerical studies have been conducted to investigate methods to alleviate tiltrotor whirl flutter. Advanced concepts have been explored such as composite tailoring [6, 7], tip anhedral [8, 9], and sweep with inertial tuning [10, 11]; many of which have shown to increase flutter speed. However, none have been experimentally verified.

Therefore, the University of Maryland developed a new laboratory and a new generic tiltrotor model called the Maryland Tiltrotor Rig (MTR) to obtain fundamental understanding of tiltrotor whirl flutter and to explore alternate configurations with interchangeable hubs, wing spars, and advanced geometry blades. The motivation is to eliminate the whirl flutter boundary so tiltrotors can finally achieve speeds up to 400 kt and higher.

1.2 Summary of Previous Work

Tiltrotor tests began with Bell’s XV-3 aircraft in the 1950s. Full-scale tests on the XV-3, and its successor, the XV-15, continued into the late 1970s and model-scale tests occurred from the 1970s to the late 1990s [12]. Since then, there has been a gap in tiltrotor tests, however, new full-scale and model-scale rigs have been developed in the 2010s. A review of previous tiltrotor experiments is presented in this section, separated by full-scale and model-scale tests. Details of a small-scale, tiltrotor model built in-house for risk reduction are also provided.

1.2.1 Full-Scale Tests

Full-scale tiltrotor tests began with the experimental Bell XV-3 tiltrotor at the NASA Ames National Full-Scale Aerodynamics Complex (NFAC) 40- by 80-foot wind tunnel in 1957-1958 and again in the 60s, shown in Fig. 1.7.

The XV-3 had experienced an unstable rotor-pylon backward whirl mode at high advance ratios. So the pitch-flap coupling delta-3, control modifications, and flapping restraints were investigated to prevent the instability [13, 14]. It was found that the damping of the low frequency whirl mode increased with swashplate stabilization, pylon stiffness, and hub restraint.

Full-scale semi-span models built by Bell, Model 300 [15] (25-ft diameter gimbaled hub), and Boeing, Model 222 [16] (26-ft diameter hingeless hub), were tested in the NFAC 40- by 80-ft tunnel in the late 60s and early 70s, shown in Fig. 1.8.



Figure 1.7: Bell XV-3 in NASA Ames 40- by 80-ft wind tunnel.



(a) Bell 25 ft. gimbaled hub proprotor.



(b) Boeing 26 ft. hingeless hub proprotor.

Figure 1.8: Full-scale semi-span models by Bell and Boeing, respectively, in the NFAC 40- by 80-ft wind tunnel.

Properties and data were both documented and available to the public. The dataset, however, is limited. No parametric evaluations of model features were performed.

These led to full-scale tests of the XV-15 Tilt Rotor Research Aircraft (TRRA) in the NFAC 40- by 80-ft tunnel in 1978 [17], shown in Fig. 1.9.



Figure 1.9: Bell XV-15 in NASA Ames 40- by 80-ft wind tunnel.

The XV-15 test focused on lift characteristics with nacelle angles, flap effectiveness with a flap tab, autorotation capability, yaw characteristics with an H-tail, drag measurements, and flow separation visualizations. However, no aeroelastic flutter data was provided.

In the 2010s, a new Tiltrotor Test Rig (TTR) was developed by U.S. Army and NASA Ames. Its first test was in 2017 in the NFAC 40- by 80-ft wind tunnel [18]. Figure 1.10 shows the TTR with a three-bladed proprotor in the test section. While

the state-of-the-art TTR will certainly inform future proprotor designs, it is not a flutter model.



Figure 1.10: NASA Tiltrotor Test Rig (TTR) in NFAC 40- by 80-ft wind tunnel.

1.2.2 Model-Scale Tests

There is also a rich history of small-scale tiltrotor tests. While full-scale tests were conducted to represent real aircraft flight, small-scale models were used for fundamental understanding and to produce empirical databases for validation of the analyses being developed.

In the late 60s through the 70s, Boeing Vertol developed a $1/4.622$ Froude-scale, two 5.5 ft diameter props, full-span (6.78 ft rotor center to rotor center), powered, hingeless tiltrotor model[19, 20] and tested it in the Boeing V/STOL wind tunnel (20- by 20-ft test section), shown in Fig. 1.11. The model was based on

a conceptual tiltrotor designated Model 150/160 in competition with Bell's XV-15 tiltrotor.

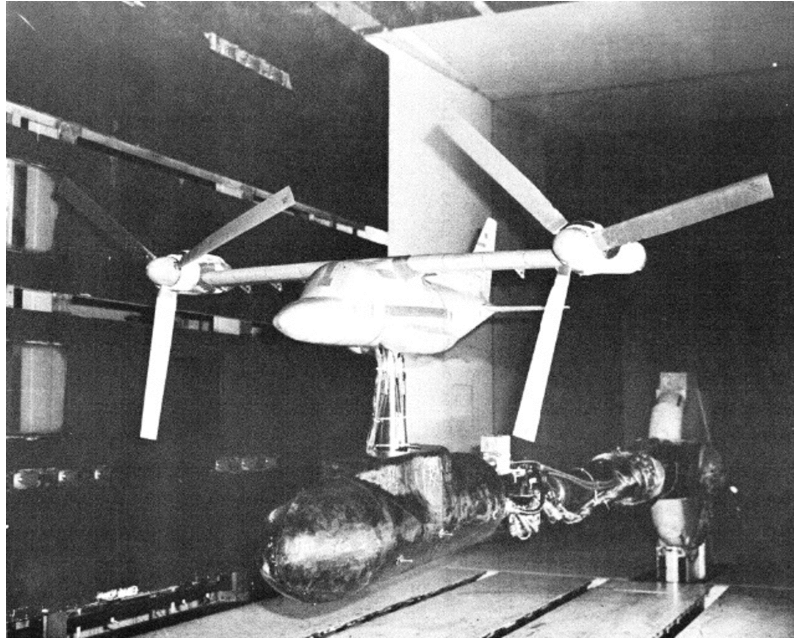


Figure 1.11: Boeing Vertol 1/4.622 M222 model in Boeing V/STOL wind tunnel (BVWT).

This model was used to obtain performance, stability, and loads on the blades. No flutter data were obtained but blade modal frequencies and damping variation with RPM were acquired.

Boeing Vertol Company also built two 2.8-ft diameter Froude scale models, designated M301 and M222, and tested at MIT in the Wright Brothers wind tunnel (elliptical test section: 7.5 ft minor and 10 ft major axes) in 1975 [21, 22], shown in Fig. 1.12.

While they were dynamically scaled, these two models were used to investigate gust response and alleviation methods. No flutter data were recorded.

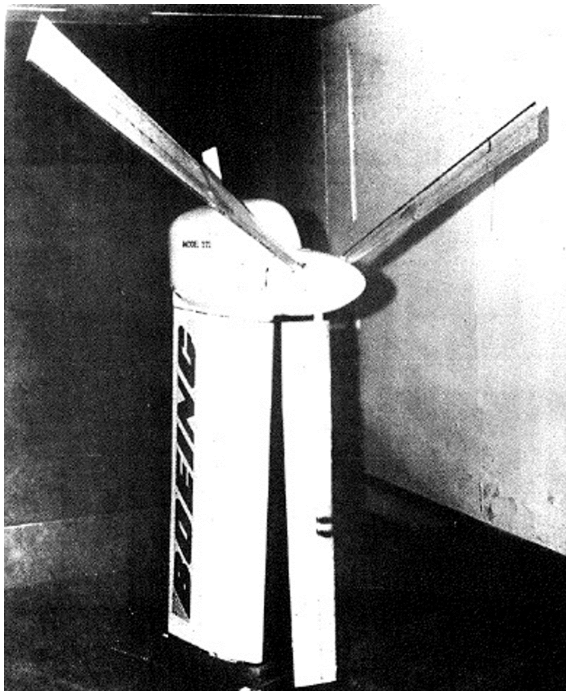


Figure 1.12: Boeing Vertol 2.8-ft diameter Froude-scale model in MIT Wright Brothers wind tunnel.

Bell built a series of 1/5th scale V-22 models and carried out a series of comprehensive tests at NASA Langley's Transonic Dynamics Tunnel (TDT) from 1983-1987 [23] as part of the V-22 development program, shown in Fig. 1.13.

The right-hand rotor of this model later evolved into the Wing and Rotor Aeroelastic Test System (WRATS) [24, 25], shown in Fig. 1.14(a). The WRATS tests investigated the effect of control system stiffness and pitch-flap coupling. A four-bladed, semi-articulated, soft-in-plane rotor system was also tested for aeroelastic stability, shown in Fig. 1.14(b). While these models published some flutter data, the model properties are not available to the public.

Novel concepts were also tested at a small-scale such as the Variable Diameter Tilt Rotor (VDTR) developed by Sikorsky in 1993 which had a 8.2-ft diameter in

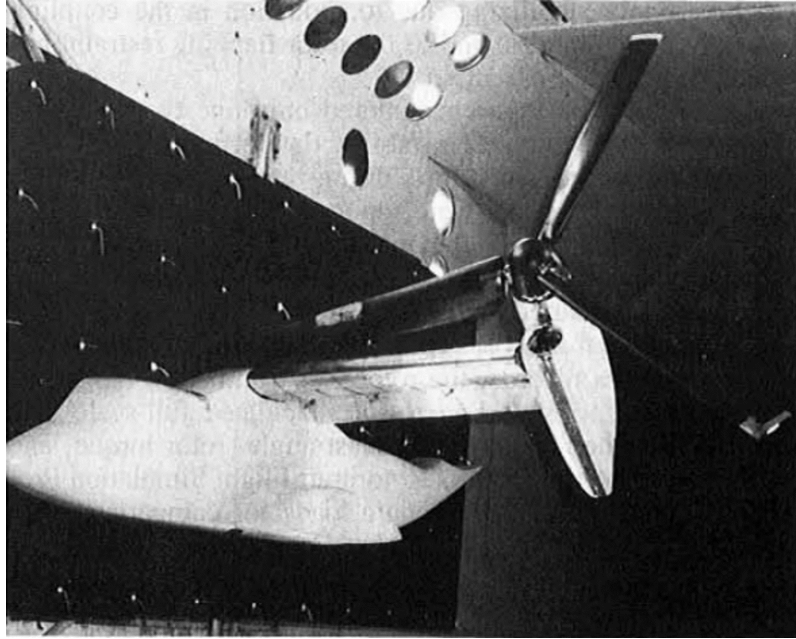


Figure 1.13: Bell 1/5th scale V-22 semi-span model at NASA Langley's Transonic Dynamics Tunnel.



(a) 3-bladed, gimballed hub WRATS model.



(b) 4-bladed, semi-articulated WRATS model.

Figure 1.14: Wing and Rotor Aeroelastic Test System in NASA Langley's Transonic Dynamics Tunnel (TDT).

hover and reduced 5.4-ft diameter in cruise [26], shown in Fig. 1.15. Neither model properties nor flutter data are available to the public.

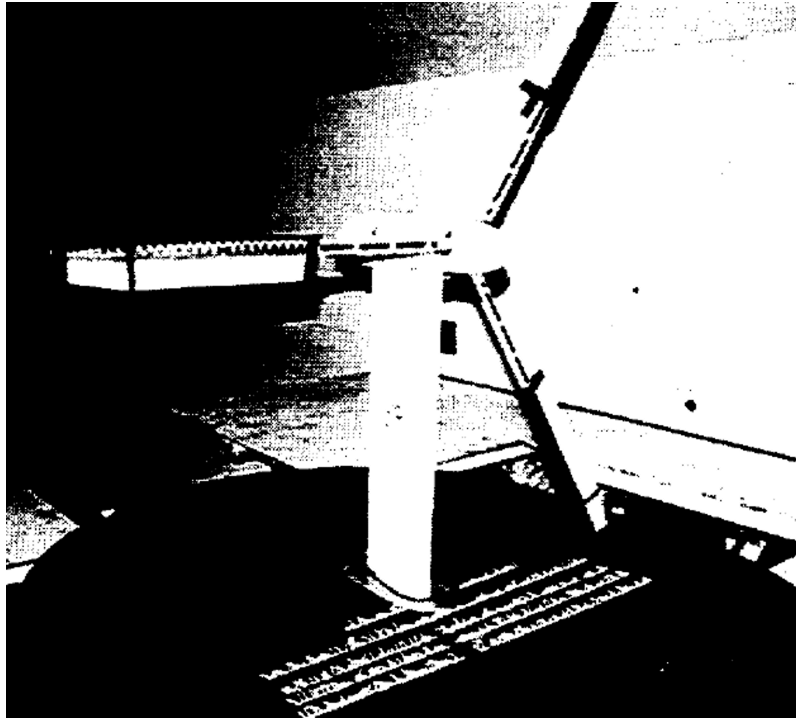


Figure 1.15: Sikorsky's Variable Diameter Tilt Rotor Model.

Two 1/4 scale V-22 models — one isolated-rotor (sting mounted) and another full-span rotor-airframe — were designed and fabricated with powered rotor and conversion mechanism during the late 1990s at NASA Ames [27]. These were the Tiltrotor Aeroacoustic Models (TRAM) shown in Fig. 1.16. None of these models were tested for flutter.

For almost all of these successful models, the tests were in support of the development of the next iteration of the tiltrotor which culminated in the V-22. Model properties and data sets were either extremely limited or were not released in the public domain. Almost all models were also single-point design configurations: mostly straight blades with gimballed hub. Thus, there is a lack of parametric flutter



Figure 1.16: Tiltrotor Aeroacoustic Model.

data that are not specific to a particular aircraft but compares a variety of wings, blades, and hubs systematically.

In parallel with the present effort, the U.S. Army has developed a new model-scale tiltrotor rig, called the Tilt Rotor Aeroelastic Stability Testbed (TRAST) [28], shown in Fig. 1.17.



Figure 1.17: TiltRotor Aeroelastic Stability Testbed (TRAST) in NASA Transonic Dynamics Tunnel.

TRAST focuses on the variation of pylon pitch spring and yaw spring stiffnesses as well as pitch-flap coupling, delta-3. As of this thesis, TRAST has had one wind tunnel test and results are waiting to be published.

1.3 Model Scaling

For scaling of models, there is a need to satisfy similarity requirements [29], so that results at the small-scale are representative of full-scale models.

For aeroelastic scaling, six basic similarity parameters must be matched between model and full-scale: Mach number, M (ratio of inertia and fluid elastic forces), advance ratio, μ (ratio of forward speed and rotor tip speed ΩR), Lock number, γ (ratio of aerodynamic and inertial forces), Froude number, Fr (ratio of inertia and gravity forces), Strouhal number, St (related to oscillatory frequency of a periodic motion), and Reynolds number, Re (ratio of inertia and viscous forces). Additionally, model configuration should be similar to the full-scale system meaning all important degrees-of-freedom must be included.

If all six of the similarity parameters are equal to full-scale then the model is aeroelastically scaled; meaning the frequencies of oscillations with air on are identical to their full-scale counter part, and all measurements on the model can be interpreted to full-scale behavior. This, however is never achieved. In a conventional wind tunnel with air as the test medium, up to four out of the six parameters can be simulated. Advance ratio and Lock number must always be simulated. Strouhal number can be simulated for rotors. If compressibility effects are important Mach number must

be simulated. Otherwise Froude number must be simulated. Reynolds number is not prioritized over Mach or Froude number, so it is typically not simulated. With the use of variable-pressure or heavy gas as a test medium, such as in the NASA Langley Transonic Dynamics Tunnel (TDT), both Mach number, Froude number, and Reynolds number can be simulated for some selected flight conditions and model sizes.

These parameters for similitude are shown in Table 1.2.

Table 1.2: **Similarity parameters for model testing.**

Parameter	Expression
Advance Ratio μ	$\frac{V}{\Omega R}$
Lock Number γ	$\frac{\rho a c R^4}{I_b}$
Strouhal Number St	$\frac{\omega c}{V}$
Mach Number M	$\frac{V}{a}$
Froude Number Fr	$\frac{V^2}{gl}$
Reynolds Number Re	$\frac{\rho V c}{\mu_\nu}$

For the parameters with characteristic lengths in their expression, a scale factor is defined as s , where model scale = $s \times$ full-scale length. Therefore $s = \frac{l_M}{l_{FS}}$. By matching the similarity parameters, a scale factor can be determined for a properly scaled model.

To achieve a Mach-scale model, where compressibility effects must be accounted for,

$$\frac{M_M}{M_{FS}} = \frac{\frac{V_M}{a_M}}{\frac{V_{FS}}{a_{FS}}} = 1 \quad (1.1)$$

Then,

$$\frac{V_M}{V_{FS}} = \frac{a_M}{a_{FS}} \quad (1.2)$$

Where subscripts M denotes the model-scale and FS denotes full-scale. Speed of sound, a , changes with altitude and the ratio of model windspeed to full-scale windspeed, $\frac{V_M}{V_{FS}}$ is directly proportional to the ratio of model test medium speed of sound and full-scale speed of sound, $\frac{a_M}{a_{FS}}$. So if the full-scale vehicle's operating regime is at sea level, and the model is being tested at sea level, then the tunnel speed for the model must be at the same speed as the full scale vehicle's test speed.

To match Strouhal number at the same Mach number,

$$\frac{St_M}{St_{FS}} = \frac{\frac{\omega_M c_M}{V_M}}{\frac{\omega_{FS} c_{FS}}{V_{FS}}} = \frac{c_M V_{FS} \omega_M}{c_{FS} V_M \omega_{FS}} = \frac{c_M a_{FS} \omega_M}{c_{FS} a_M \omega_{FS}} = s \frac{a_{FS} \omega_M}{a_M \omega_{FS}} = 1 \quad (1.3)$$

Then,

$$\frac{\omega_M}{\omega_{FS}} = \frac{1}{s} \frac{a_M}{a_{FS}} \quad (1.4)$$

So if a full-scale vehicle's operating regime is at sea level, and the Mach-scaled model is being tested at sea level, then the model frequency of oscillation must be higher than the full scale by the inverse of the scale factor.

Pertaining to rotors, Strouhal number can be evaluated with $V = \Omega R$ and $l = R$.

$$St = \frac{\omega l}{V} = \frac{\omega R}{\Omega R} = \frac{\omega}{\Omega} \quad (1.5)$$

Thus the frequencies to be kept the same are the non-dimensional frequencies relative the rotor speed, i.e., frequencies in /rev. This is particularly important for tiltrotors:

the wing-pylon frequencies of the model must be kept the same relative to the rotor speed of the model.

To match Reynolds number and Mach number,

$$\frac{Re_M}{Re_{FS}} = \frac{\frac{\rho_M V_M c_M}{\mu_{\nu_M}}}{\frac{\rho_{FS} V_{FS} c_{FS}}{\mu_{\nu_{FS}}}} = \frac{c_M V_M \rho_M \mu_{\nu_{FS}}}{c_{FS} V_{FS} \rho_{FS} \mu_{\nu_M}} = s \frac{a_M \rho_M \mu_{\nu_{FS}}}{a_{FS} \rho_{FS} \mu_{\nu_M}} = 1 \quad (1.6)$$

Then,

$$\frac{a_M \rho_M \mu_{\nu_{FS}}}{a_{FS} \rho_{FS} \mu_{\nu_M}} = \frac{1}{s} \quad (1.7)$$

So, if full scale vehicle's operating regime is at sea level, and the model is being tested at sea level, then in order to match Re , the scale factor must be 1. If the model can be tested in heavy gas, such as R-134a used in NASA TDT, then it is possible to obtain a smaller model. Heavy gas properties for R-134a as compared to air are shown in Table 1.3.

Table 1.3: **Properties of heavy gas compared to air.**

Property	Unit	R-134a	Air
Molar mass	kg/kmol	102.03	28.97
Speed of sound, a	m/s	165	340
Density, ρ	kg/m ³	5.22	1.225
Viscosity, μ_{ν}	Pa-s	1.076×10^{-5}	1.81×10^{-5}

However, even with heavy gas, only 1/3 scale models can be achieved when Mach number is matched; still quite large.

To develop a Froude-scale model, Froude number must be matched.

$$\frac{Fr_M}{Fr_{FS}} = \frac{\frac{V_M^2}{g_M l_M}}{\frac{V_{FS}^2}{g_{FS} l_{FS}}} = \frac{V_M^2 l_{FS} g_{FS}}{V_{FS}^2 l_M g_M} = 1 \quad (1.8)$$

Then,

$$\frac{V_M}{V_{FS}} = \sqrt{s \frac{g_M}{g_{FS}}} \quad (1.9)$$

where g is the gravitation acceleration at altitude. In order to match Mach number with this relationship:

$$\frac{M_M}{M_{FS}} = \frac{\sqrt{s \frac{g_M}{g_{FS}}}}{\frac{a_M}{a_{FS}}} \quad (1.10)$$

Since, on Earth, $g_M \approx g_{FS}$, the only way to match Mach and Froude is to have $s = (\frac{a_M}{a_{FS}})^2$. Using R-134a, this can be achieved with a 1/4-1/5 scale model.

When Froude is matched, then Reynolds number scales as:

$$\frac{Re_M}{Re_{FS}} = \frac{s^{3/2} \sqrt{\frac{g_M}{g_{FS}} \frac{\rho_M}{\rho_{FS}}}}{\frac{a_M}{a_{FS}}} \quad (1.11)$$

So when Froude is matched, Mach and Reynolds cannot be matched. If Froude number is simulated, Mach number is lower by $\sqrt{s}$, and Re number by $s^{3/2}$, when atmospheric conditions are the same. The Mach number similitude can be maintained with heavy gas but only for a particular $s \approx 1/5$. The Re number similitude cannot be achieved. Recall, with heavy gas, Re number and Mach number similitude requires $s \approx 1/3$, and for this size scale Froude number similitude cannot be

achieved. Typically, Re is not simulated, but every attempt is made to keep it near 1 million.

1.4 Damping Measurement

There are various approaches to measure damping of a physical system. Three common methods are described in this section: the textbook logarithmic decrement, Prony method, and the moving-block method. The moving-block method was selected in post-processing to obtain the frequencies and damping of the wing beam and wing chord modes.

The MTR was deliberately designed with high bandwidth electric actuation to introduce perturbation at a specified frequency. Thus, the frequencies of response need not be identified or separated from a transient response; the response is already at the specified frequency. Thus, the moving block method was used to extract damping. This method is also robust to measurement noise. Damping was also extracted by the other methods for a few selected points. It confirmed that the selection of methods made no difference in the results.

1.4.1 Logarithmic Decrement

The Logarithmic Decrement is applicable to a single frequency signal of the form:

$$x = Ae^{st} \tag{1.12}$$

Where, $s = \sigma \pm j\omega$, $\sigma = -\zeta\omega_n$, and $\omega = \omega_n\sqrt{1-\zeta^2}$. It is also known the signal period, T , is related to the frequency ω by the relation $T = \frac{2\pi}{\omega}$ or $T = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$. Then the natural frequency of the signal can found:

$$\omega_n = \frac{2\pi/T}{\sqrt{1-\zeta^2}} \quad (1.13)$$

To obtain damping ratio, ζ , first select a starting peak on the decay envelope: $x_t = Ae^{\sigma t}$. Then pick another peak after nT periods: $x_{t+nT} = Ae^{\sigma(t+nT)}$, where n is the number of periods the second peak is away from the first peak. Divide the amplitude of the first peak by the second peak:

$$\frac{x_t}{x_{t+nT}} = e^{-\sigma nT} \quad (1.14)$$

Then solve for the exponential term:

$$-\sigma T = \frac{1}{n} \ln \left(\frac{x_t}{x_{t+nT}} \right) \quad (1.15)$$

Where the left hand side is $-\sigma T = \zeta\omega_n T = \zeta\omega_n \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$. The right hand side is the logarithmic decrement, δ , the natural log of the ratio of amplitudes of successive peaks.

$$\delta = \frac{1}{n} \ln \left(\frac{x_t}{x_{t+nT}} \right) \quad (1.16)$$

Solving for ζ :

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}} \quad (1.17)$$

The log dec method works well when the decay envelope is easy to identify, which is typically the case for a single frequency signal with no noise. For a signal with multiple frequencies, the log dec method may be used in conjunction with a band-pass filter to isolate the frequency of interest. For noisy signals, applying a moving average would help mitigate the noise.

The raw signal must be manipulated before log dec can practically be applied. The mean of the signal must be ascertained reliably, and removed from the raw signal. The log dec method only applies to the perturbation signal. If the signal has a large positive offset, the log dec method will show an incorrect low damping ratio, ζ . If the signal has a large negative offset, the log dec will show an incorrect negative damping ratio, ζ . Thus, it is important, obviously, that only the perturbation be taken.

If the offset varies with time such that peaks start positive but successive peaks are negative, the log dec, δ , will result in incorrect complex numbers. These are issues that do not arise in theoretical analysis, but are unavoidable in experimental data.

Figure 1.18 shows the raw signal for run N1.78 chord mode trial (see Section 7.4 for naming convention). Figure 1.19 is the same signal with the mean removed. Figure 1.20 shows the amplitudes of each peak found. The log dec, δ , is calculated for each pair of successive peaks such that n is 1 for each calculation, i.e. peaks

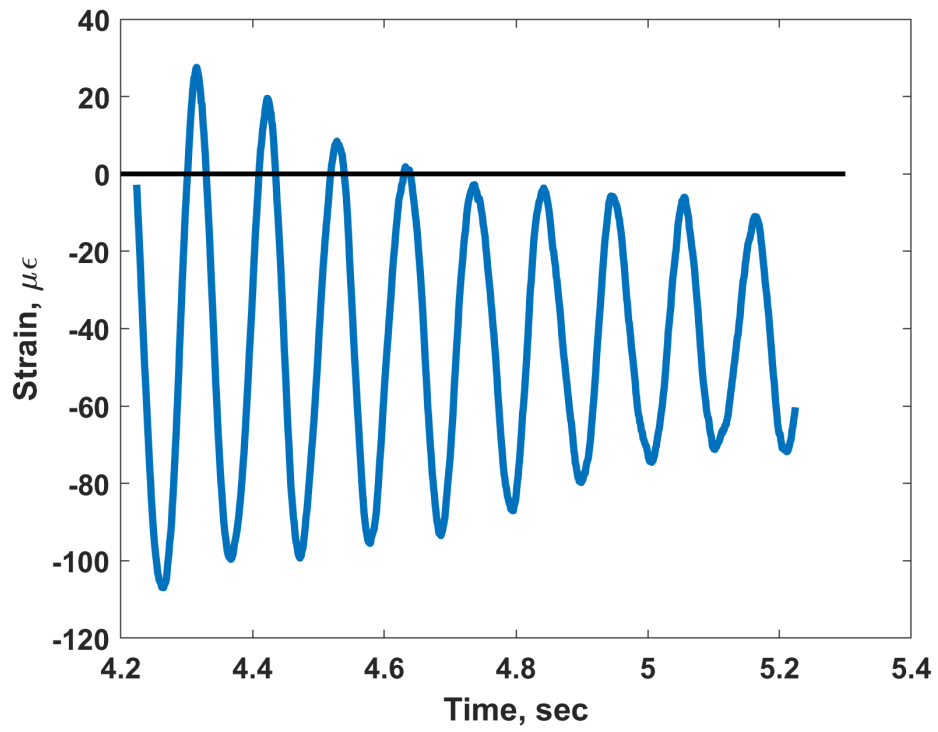


Figure 1.18: N1.78 chord mode response, 1 second duration, 1000 samples.

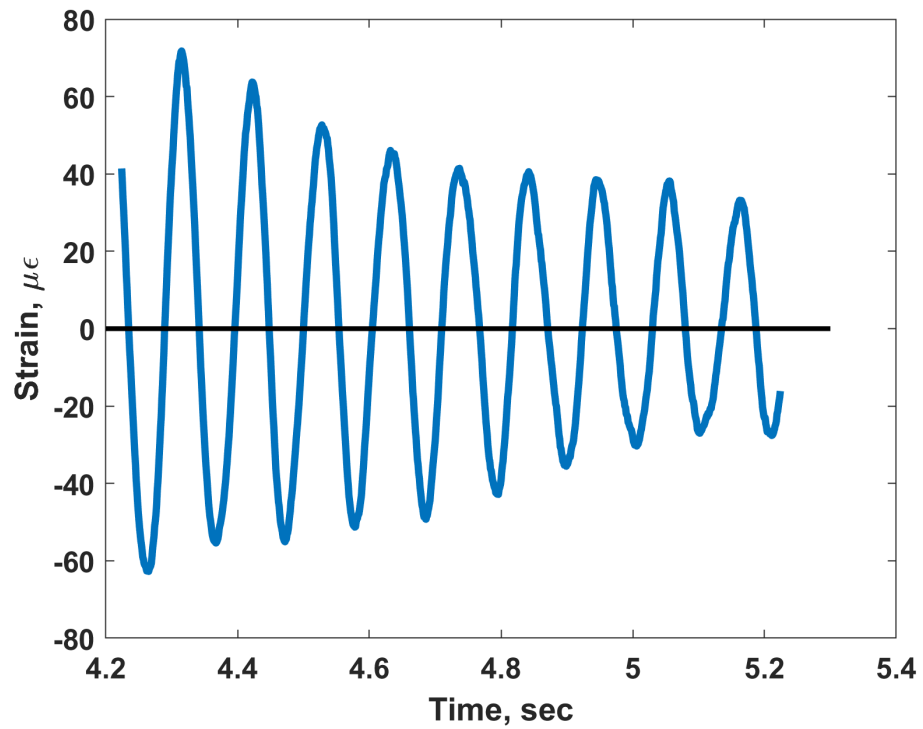


Figure 1.19: N1.78 mean removed from original signal, y-axis centered at zero.

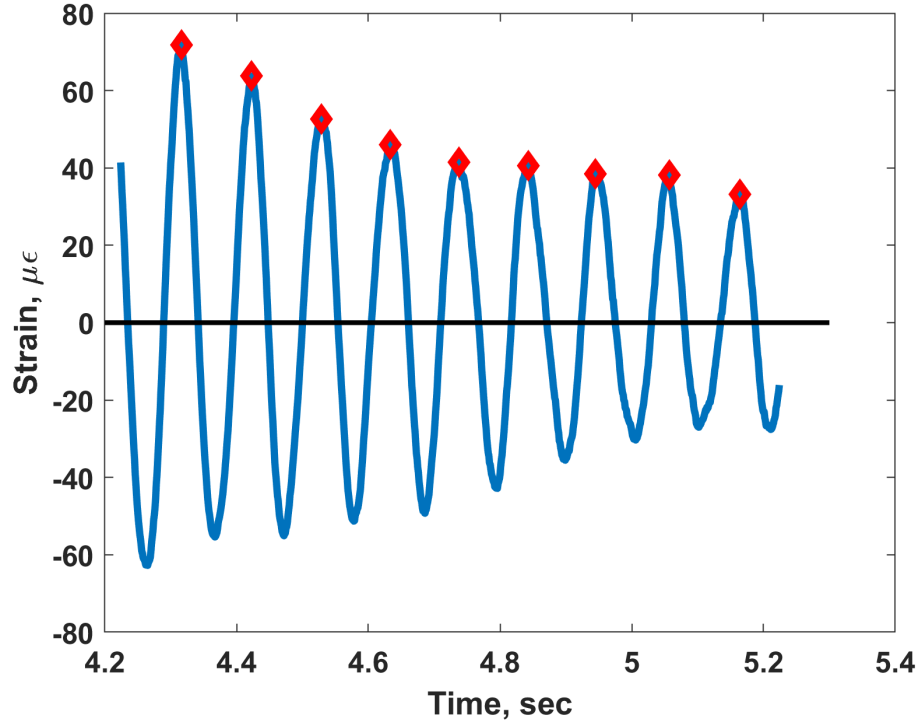


Figure 1.20: N1.78 peaks of signal determined.

1-2, peaks 2-3, peaks 3-4, etc. Then the mean of the damping ratios is calculated to obtain the final damping ratio of the response which in this case is 1.53%. Once ζ is known, natural frequency, ω_n , is identified from Eq. (1.13), where T is the time between peaks.

1.4.2 Prony Method

When there are multiple frequencies in a signal, Prony method can be used to decompose them into superpositions of damped sinusoids. It is essentially the Fourier transform equivalent for damped oscillations. It is irrelevant for MTR as the response frequency is already known but covered here for completeness and comparison.

First, start with a signal, say, with three frequencies for the sake of illustration.

$$x = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_2 e^{s_2 t} \quad (1.18)$$

Then pick seven equidistant points in the signal, where $7 = 2 \times 3$ frequencies + 1. Let them be related linearly through a relation of the following form

$$x_t P_0 + x_{t+1} P_1 + x_{t+2} P_2 + x_{t+3} P_3 + \dots + x_{t+6} P_6 = 0 \quad (1.19)$$

where x_{t+n} are the known signal values, and P_n are some unknown coefficients.

Then, it follows

$$\begin{aligned} & (A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_2 e^{s_2 t}) P_0 \\ & + (A_1 e^{s_1(t+1)} + A_2 e^{s_2(t+1)} + A_2 e^{s_2(t+1)}) P_1 \\ & + (A_1 e^{s_1(t+2)} + A_2 e^{s_2(t+2)} + A_2 e^{s_2(t+2)}) P_2 \\ & + \dots \\ & + (A_1 e^{s_1(t+6)} + A_2 e^{s_2(t+6)} + A_2 e^{s_2(t+6)}) P_6 \\ & = 0 \end{aligned} \quad (1.20)$$

This is rewritten as

$$\begin{aligned}
& A_1 e^{s_1 t} (P_0 + P_1 e^{s_1} + P_2 e^{2s_1} + P_3 e^{3s_1} + \dots + P_6 e^{6s_1}) \\
& + A_1 e^{s_2 t} (P_0 + P_1 e^{s_2} + P_2 e^{2s_2} + P_3 e^{3s_2} + \dots + P_6 e^{6s_2}) \\
& + A_1 e^{s_3 t} (P_0 + P_1 e^{s_3} + P_2 e^{2s_3} + P_3 e^{3s_3} + \dots + P_6 e^{6s_3}) \\
& = 0
\end{aligned} \tag{1.21}$$

For nontrivial $A_1 e^{s_1 t}$, $A_2 e^{s_2 t}$, $A_3 e^{s_3 t}$, the expressions in parentheses in Eq. (1.21) must be zero.

This implies e^{s_1} , e^{s_2} , e^{s_3} are simply the roots of the sixth order polynomial $P_0 + P_1 x + P_2 x^2 + P_3 x^3 + \dots + P_6 x^6 = 0$.

This allows 6 roots, and in general, 3 complex conjugate pairs. Only the coefficients $P_0, P_1, \dots, P_6$ are left to be determined.

Note, one of the coefficients, say P_6 , can be set to 1 or -1, without any loss of generality. Because:

$$P_0 + P_1 x + P_2 x^2 + P_3 x^3 + \dots + P_6 x^6 = 0 \tag{1.22}$$

and

$$\frac{1}{P_6} (P_0 + P_1 x + P_2 x^2 + P_3 x^3 + \dots + P_6 x^6) = 0 \tag{1.23}$$

have the same roots.

Going back to Eq. (1.19), by setting $P_6 = -1$, it is rewritten as:

$$x_t P_0 + x_{t+1} P_1 + x_{t+2} P_2 + x_{t+3} P_3 + \dots + x_{t+5} P_5 = x_{t+6} \quad (1.24)$$

This form is widely used. To calculate the 6 unknown coefficients, it only remains to find 6 equations. These are found by applying the same relation for x_{t+5} , $x_{t+4}, \dots, x_{t+1}$. The assumption is Eq. (1.19) is valid for any seven consecutive points on the signal. This is valid for any signal that is a result of an underlying partial differential equation. It is indeed the basis for any finite difference approximation.

In matrix form

$$\begin{bmatrix} x_t & x_{t-1} & x_{t-2} & x_{t-3} & x_{t-4} & x_{t-5} \\ x_{t+1} & x_t & x_{t-1} & x_{t-2} & x_{t-3} & x_{t-4} \\ x_{t+2} & x_{t+1} & x_t & x_{t-1} & x_{t-2} & x_{t-3} \\ x_{t+3} & x_{t+2} & x_{t+1} & x_t & x_{t-1} & x_{t-2} \\ x_{t+4} & x_{t+3} & x_{t+2} & x_{t+1} & x_t & x_{t-1} \\ x_{t+5} & x_{t+4} & x_{t+3} & x_{t+2} & x_{t+1} & x_t \end{bmatrix} \begin{bmatrix} P_5 \\ P_4 \\ P_3 \\ P_2 \\ P_1 \\ P_0 \end{bmatrix} = \begin{bmatrix} x_{t+1} \\ x_{t+2} \\ x_{t+3} \\ x_{t+4} \\ x_{t+5} \\ x_{t+6} \end{bmatrix} \quad (1.25)$$

Then solve for the coefficients $P_0, P_1, P_2, P_3, P_4, P_5$.

This has the form:

$$AP = x \quad (1.26)$$

Where A is a Toeplitz matrix if it is square, P is the prediction coefficients vector and x is the observation vector.

If A is square then $P = A^{-1}x$. However, in test data, more than 7 observations points are desired. Then A is not square, and there are more equations than the prediction coefficients. In this case, solve in the least square sense:

$$\begin{aligned} A^T A P &= A^T x \\ P &= (A^T A)^{-1} A^T x \end{aligned} \tag{1.27}$$

Which minimizes the error norm

$$||x - AP||_2 \tag{1.28}$$

Finally, $P_0, P_1, P_2, P_3, P_4, P_5$ are obtained.

Now that the coefficients are known, solve for the roots of Eq. (1.22) with $P_6 = -1$. This has the following form.

$$x^6 - P_5x^5 - P_4x^4 - P_3x^3 - P_2x^2 - P_1x - P_0 = 0 \tag{1.29}$$

From root x , find s from $x = e^s$. The root s is complex. The imaginary part of s is unique only up to multiples of $2\pi k$, where k is an integer. This is because $e^s = e^{s \pm j2\pi k}$. So care must be taken to extract the frequency, but there is no ambiguity in σ .

$$\begin{aligned} \sigma &= \text{Re}(s) \\ \omega &= \text{Im}(s) \pm 2\pi k \end{aligned} \tag{1.30}$$

where k can be zero or any integer.

The damping ratio and natural frequency, ζ and ω_n , can now be found. For a complex conjugate pair $s_1 = \sigma_1 \pm j\omega_1$:

$$s_{11} = \sigma_1 + j\omega_1 \quad (1.31)$$

$$s_{21} = \sigma_1 - j\omega_1$$

Then $A_1 e^{s_1 t}$ becomes $A_{11} e^{s_{11} t} + A_{21} e^{s_{21} t}$. Find A_{11} and A_{21} separately, because if s_{11} and s_{21} are complex conjugates, then A_{11} and A_{21} must also be complex conjugates. For 3 pairs of complex conjugates:

$$\begin{bmatrix} e^{s_{11}t} & e^{s_{21}t} & e^{s_{12}t} & e^{s_{22}t} & e^{s_{13}t} & e^{s_{23}t} \\ e^{s_{11}(t-\Delta t)} & e^{s_{21}(t-\Delta t)} & e^{s_{12}(t-\Delta t)} & e^{s_{22}(t-\Delta t)} & e^{s_{13}(t-\Delta t)} & e^{s_{23}(t-\Delta t)} \\ e^{s_{11}(t-2\Delta t)} & e^{s_{21}(t-2\Delta t)} & e^{s_{12}(t-2\Delta t)} & e^{s_{22}(t-2\Delta t)} & e^{s_{13}(t-2\Delta t)} & e^{s_{23}(t-2\Delta t)} \\ e^{s_{11}(t-3\Delta t)} & e^{s_{21}(t-3\Delta t)} & e^{s_{12}(t-3\Delta t)} & e^{s_{22}(t-3\Delta t)} & e^{s_{13}(t-3\Delta t)} & e^{s_{23}(t-3\Delta t)} \\ e^{s_{11}(t-4\Delta t)} & e^{s_{21}(t-4\Delta t)} & e^{s_{12}(t-4\Delta t)} & e^{s_{22}(t-4\Delta t)} & e^{s_{13}(t-4\Delta t)} & e^{s_{23}(t-4\Delta t)} \\ e^{s_{11}(t-5\Delta t)} & e^{s_{21}(t-5\Delta t)} & e^{s_{12}(t-5\Delta t)} & e^{s_{22}(t-5\Delta t)} & e^{s_{13}(t-5\Delta t)} & e^{s_{23}(t-5\Delta t)} \end{bmatrix} \begin{bmatrix} A_{11} \\ A_{21} \\ A_{12} \\ A_{22} \\ A_{13} \\ A_{23} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \quad (1.32)$$

Let

$$\begin{aligned} A_{11} &= \frac{1}{2} C_1 e^{j\phi_1} \\ A_{21} &= \frac{1}{2} C_1 e^{-j\phi_1} \end{aligned} \quad (1.33)$$

Then

$$A_1 e^{s_1 t} = C_1 \cos(\omega_1 t + \phi_1) \quad (1.34)$$

where C_1 is the magnitude and ϕ_1 is the phase content of the first frequency. C_1 and ϕ_1 can be found from A_{11} and A_{21} . Thus, finally the signal is fully decomposed.

To apply Prony method numerically, the following steps are carried out.

1. Create a Toeplitz matrix, T , representation of the data, where the first column is the damped signal and each subsequent column is the signal shifted down one sample. Like so:

$$T = \begin{bmatrix} x_0 & 0 & 0 & \dots & 0 \\ x_1 & x_0 & 0 & \dots & 0 \\ x_2 & x_1 & x_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \dots \\ x_{N-1} & x_{N-2} & x_{N-3} & \dots & x_0 \end{bmatrix} \quad (1.35)$$

Here it is assumed the signal starts at x_0 . Prior values are all zero. T is an $N \times N$ square matrix where N is the number of samples in the signal.

2. Choose a model number, M , which represents the number of polynomial coefficients $P_0, P_1, P_2, \dots, P_M$. M is typically around 30%-40% of the number of samples, N .
3. Select portions of the the Toeplitz matrix using the model number, M , such that $x=T(M:N-1,1)$, and $A=T(M:N-1,2:M)$. Note that this procedure starts

at an index of zero for the first element, whereas Matlab convention for indexing starts at 1. Adjust Matlab code accordingly.

4. Solve for the coefficients vector P in the form $AP = x$. Matlab operation $P = [-1; A \backslash x]$ provides the least squares solution. Recall from Eq. (1.24), P_M is set to -1 and is the first element in the coefficient array. If P_M is set to 1, then $P = [1; -A \backslash x]$ is the coefficient array.
5. Determine the roots of the coefficient array P . The roots are in the form $e^{(s\Delta t)}$, so take the natural log of the roots then multiply by the sampling frequency of the signal, f , to obtain s .
6. Take the absolute value of the imaginary part of s to obtain normal frequencies, ω_n in radians/s.
7. The real part of s is the damping factor $-\zeta\omega_n$, where ζ is the damping ratio. Dividing the real part by the frequencies in the prior step will result in the damping ratios.
8. For a large model number (number of coefficient terms), find the indices of the frequencies of interest and match them to the damping ratios.

If the frequency is already known, the above steps are all that is needed to obtain the damping ratio. If the frequency is not known, sorting the amplitudes in descending order brings the most important modes to the forefront. A numerical code for Prony's method is given in the appendix.

Figure 1.21 shows Prony's method applied to run N1.78 chord mode trial for a model number of 800 in a 1000 sample signal of 1 second duration. The Prony approximation signal is in very good agreement with the original signal. Figure 1.22 shows a sensitivity study for the Prony approximation signal for various model numbers. The results of frequencies and damping ratios extracted from the model numbers are shown in Table 1.4

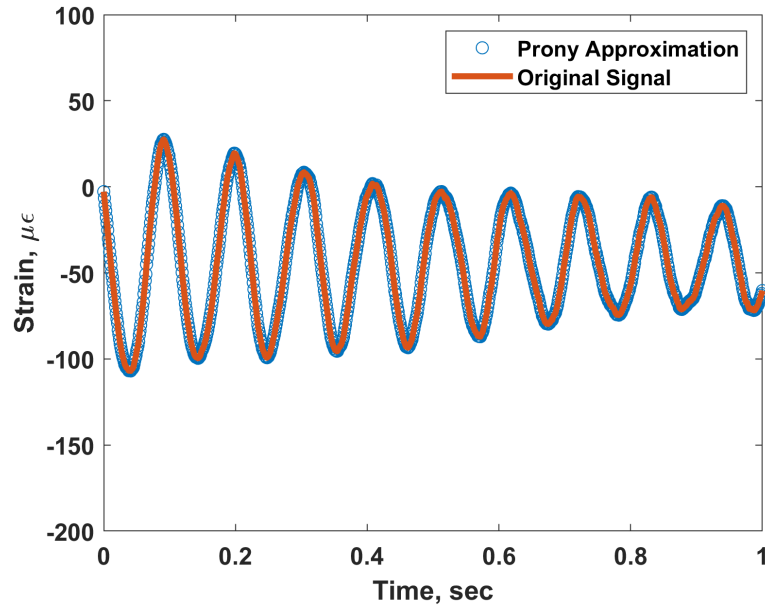


Figure 1.21: Prony method using model number of 400 to produce fitted curve on original signal in run N1.78.

Table 1.4: **Prony-estimated amplitudes, frequencies, and damping for various model numbers on a 1 second duration signal of 1000 samples.**

Model Number	Amplitude ($\mu\epsilon$)	Frequency (Hz)	Damping (% critical)
20	79.40	9.77	16.39
100	71.14	9.45	1.64
200	69.98	9.47	1.58
300	52.13	9.46	1.56
400	69.53	9.47	1.57
500	71.12	9.45	1.55
800	95.67	9.47	1.50

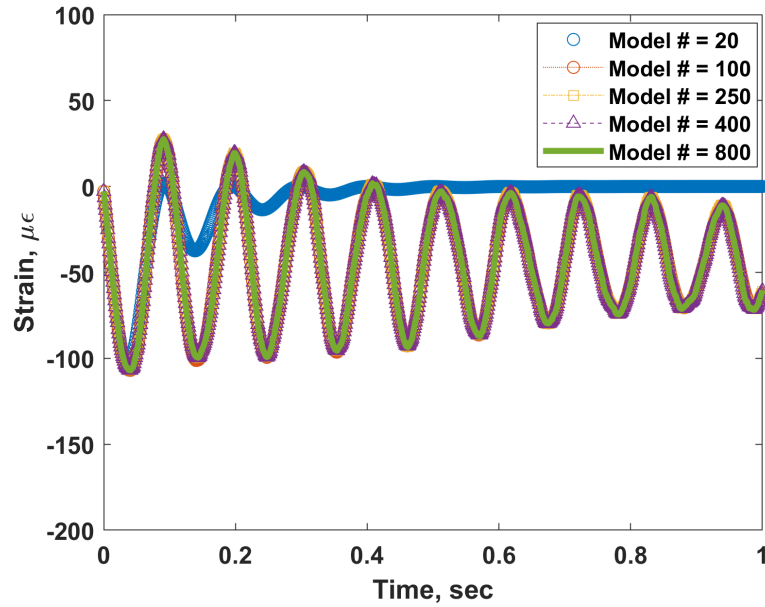


Figure 1.22: Effect of model number on Prony approximation of signal.

Increasing model number (number of coefficient terms, P) improves accuracy, however, model numbers approaching the number of samples in the signal will result in incorrect values of amplitude and damping. This is because the procedure in step 3 uses the number of coefficients to predict $N - M$ terms in the signal, x . If the length of the signal is 1000, and the model number is 999, x becomes a single term, and the coefficients are not unique. on the other hand, low model numbers result in incorrect amplitude, frequency, and damping because there are not enough coefficients to capture the behavior of the signal. Thus, a model number about 30%-40% of the number of samples should be used as a starting point.

It is obvious Prony is an excessive method when the response contains a single specified frequency. A high model number is needed to converge to the same solution as the other methods. However, the method is generic and useful if there are multiple frequencies in the perturbation response.

1.4.3 Moving-Block Method

The Moving-Block method belongs to a broad class of modern block processing techniques of digital signal processing that emerged during the 1960s. The implementation here generally follows Refs. [30–32]. It is used to calculate the frequency and damping of the MTR. Generally, the moving block method is very effective for noisy signals and for low damped signals less than 5% of critical damping. An example for run N1.78 chord mode demonstrates the procedure, shown in Fig. 1.23.

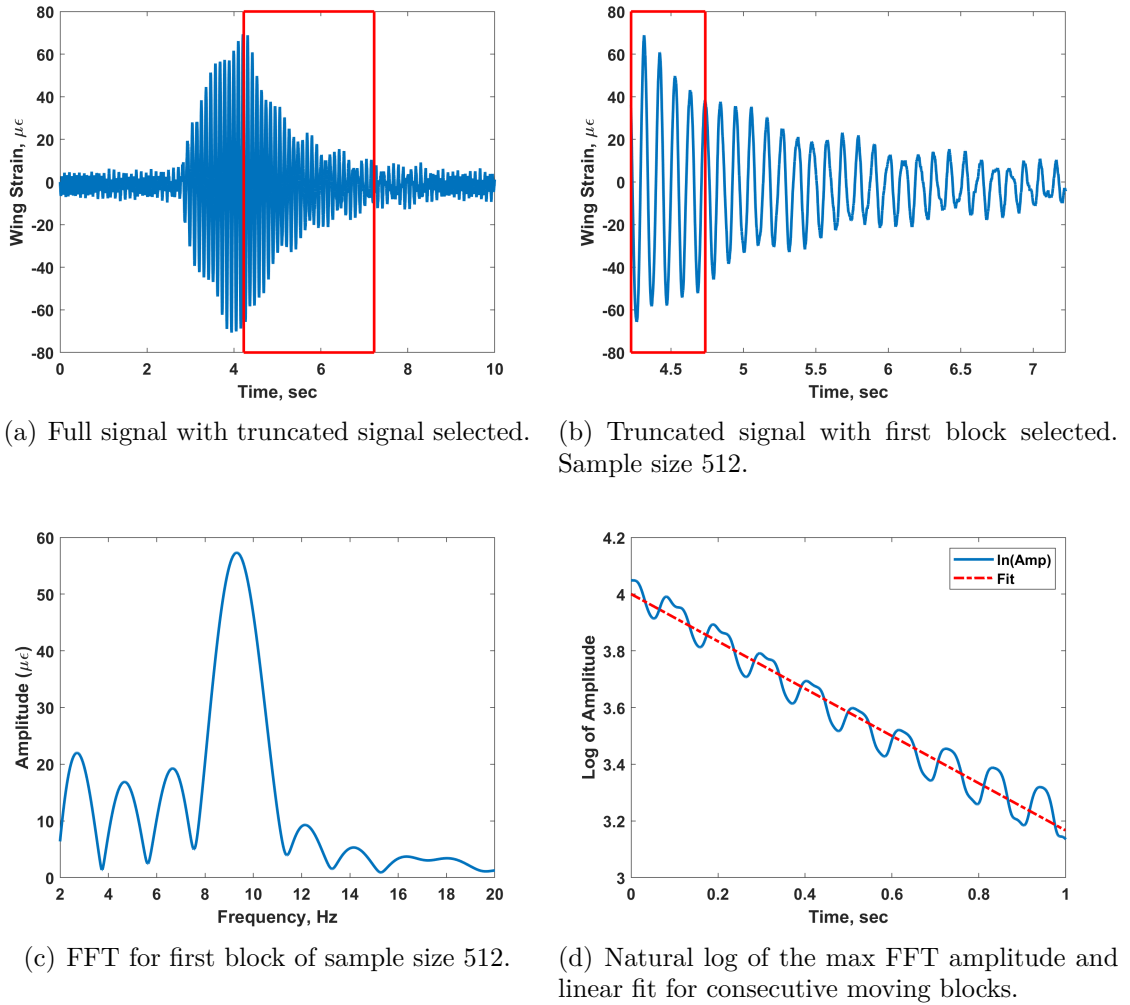


Figure 1.23: Moving-block method applied to run N1.78 chord mode trial.

The raw data at 10 kHz is downsampled to 1 kHz for ease of processing. The downsampled signal from the wing chord strain gauge is shown in Fig. 1.23(a). A region of the decaying signal is selected starting from the end of excitation (4.225 sec) and for a duration of 3 seconds. This region is shown in Fig. 1.23(b). It contains 3000 samples over 3 seconds. A Fast Fourier Transform (FFT) is taken of this signal to determine the key frequency, and equated to ω_n . Then, the signal, denoted as x , is split into moving blocks of 512 samples in the form shown below.

First block:

$$[x_0 \quad x_1 \quad x_2 \quad \dots \quad x_{511}]$$

Second block:

$$[x_1 \quad x_2 \quad x_3 \quad \dots \quad x_{512}]$$

Third block:

$$[x_2 \quad x_3 \quad x_4 \quad \dots \quad x_{513}]$$

And so on until the last block, where N is the length of the signal:

$$[x_{N-512} \quad x_{N-511} \quad x_{N-510} \quad \dots \quad x_{N-1}]$$

These are the so-called moving blocks. On each block, an FFT is performed. The FFT of the first block is shown in Fig. 1.23(c). The FFT is performed with zero padding (appending zeros to the end of the block) which increases the resolution of the FFT bins and, as a result, the peak is more precise with side lobes that appear smooth and continuous. The peak amplitude of the frequency of interest is stored. In this example, the amplitude is 57.3. The next block of 512 samples is treated the same way. This continues for all the blocks in the signal such that an array of $(N-512)$ amplitudes is produced. A plot of the natural log of the amplitude array versus

starting time of the block is generated and the result is an oscillating signal that decreases linearly with time. The slope from a least-squares fit over this oscillating line is equal to $-\zeta\omega_n$ where ω_n is the frequency calculated from an FFT of the three-second signal and ζ is the damping ratio. It is assumed the damped frequency $\omega_d = \omega_n$ because the damping ratio is known to be less than 5%. Figure 1.23(d) shows the results of the natural log and the least-squares fit over a one second time duration. In this example, $\omega_n = 9.5$ Hz and $\zeta = 0.0141$. If the damping is in % critical, it is 1.41%.

The block mentioned above is nothing but a rectangular window, also called a boxcar. A window essentially connects the end of the block with the beginning of the block. The accuracy of the moving-block method can be enhanced by using non-rectangular windows [33], which avoid the jump at the ends of the blocks. A study of windowing functions was performed with several other windows namely Hanning, Hamming, Flat Top, and Blackman-Harris windows. These window functions are shown in Fig. 1.24.

The equations for these windows are shown in Table 1.5.

By multiplying the response signal with one of these windows, then performing an FFT on the output avoids the spurious frequency and amplitude generated by the edges of the rectangular window, called side-lobes seen in Fig. 1.23(c). This is especially true for signals with multiple modes with close-proximity frequencies. The results of the moving block method with the Hanning window are shown in Fig. 1.25. The truncated signal used in this example is the same as Fig. 1.23(b).

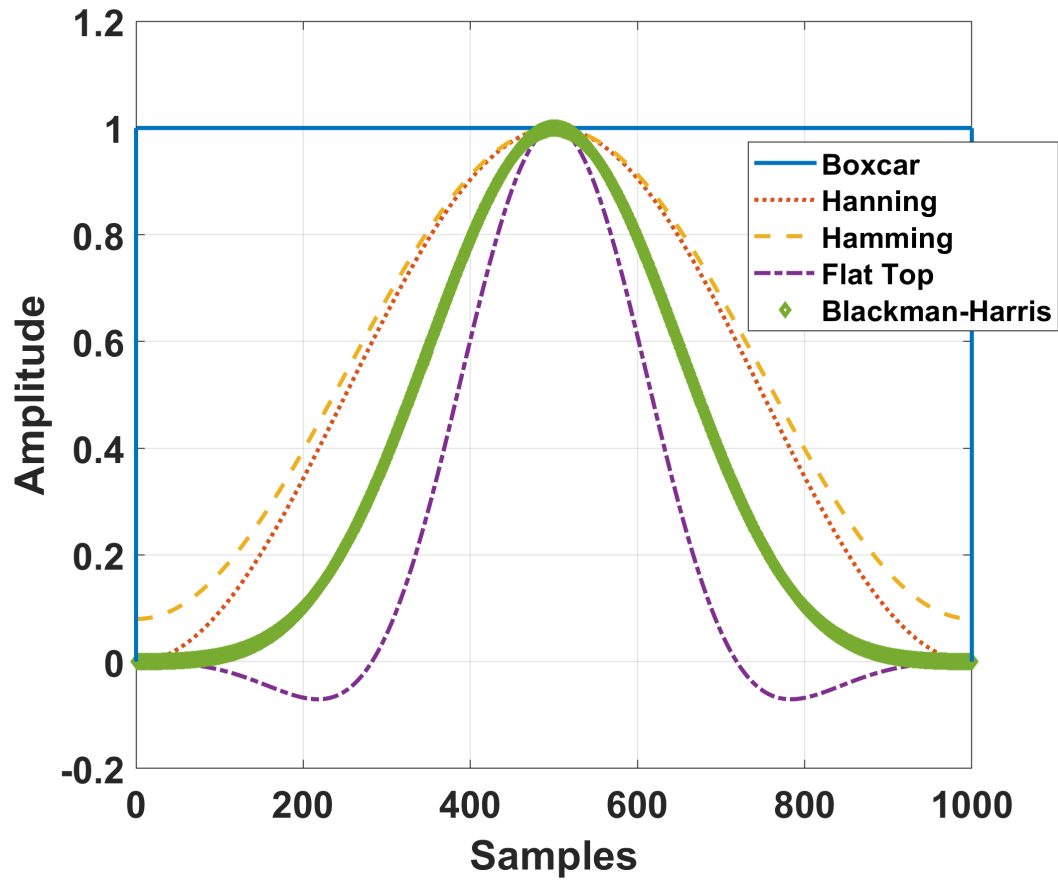
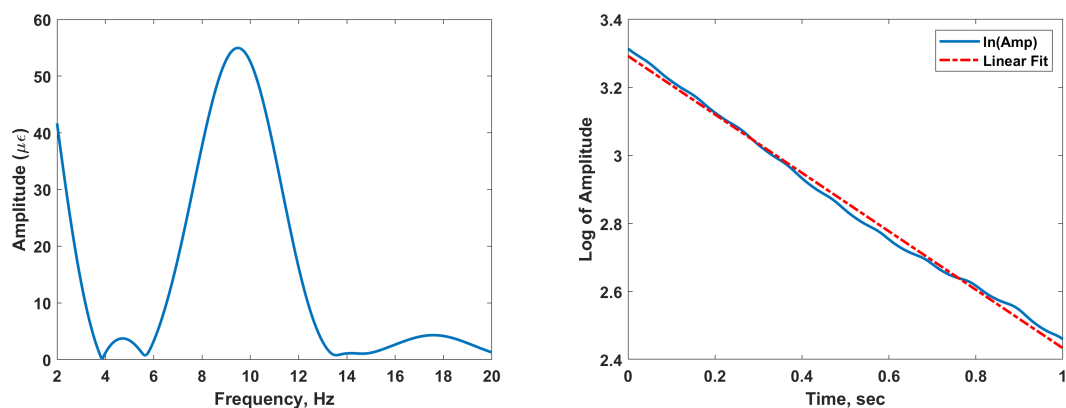


Figure 1.24: Comparison of window functions.



(a) FFT for first block of size 512 with Hanning window applied. (b) Natural log of the FFT amplitudes with linear fit.

Figure 1.25: Moving-block method with Hanning window applied to run N1.78 chord mode. Block sample size of 512.

Table 1.5: **Window function equations; N is length of the response signal minus 1, n is the sample count in the response signal.**

Window	Equation
Boxcar	$w(n) = 1, 0 \leq n \leq N$
Hanning	$w(n) = \frac{1}{2} \left(1 - \cos \left(2\pi \frac{n}{N} \right) \right), 0 \leq n \leq N$
Hamming	$w(n) = 0.54 - 0.46 \cos \left(2\pi \frac{n}{N} \right), 0 \leq n \leq N$
Flat Top	$w(n) = 0.21557895 - 0.41663158 \cos \left(2\pi \frac{n}{N} \right) +$ $0.277263158 \cos \left(4\pi \frac{n}{N} \right) -$ $0.083578947 \cos \left(6\pi \frac{n}{N} \right) + 0.006947368 \cos \left(8\pi \frac{n}{N} \right),$ $0 \leq n \leq N$
Blackman- Harris	$w(n) = 0.35875 - 0.48829 \cos \left(2\pi \frac{n}{N} \right) +$ $0.14128 \cos \left(4\pi \frac{n}{N} \right) - 0.01168 \cos \left(6\pi \frac{n}{N} \right),$ $0 \leq n \leq N$

Table 1.6: **Damping results for run N1.78 for different window functions on block size of 512.**

Window Type	Damping (% critical)
Boxcar	1.40
Hanning	1.44
Hamming	1.44
Flat Top	N/A
Blackman-Harris	1.43

The damping ratios obtained from the various windows are shown in Table 1.6. Damping results are similar between all the window functions with a maximum difference of 3%. The flat top window applied to the 512 sample block suppressed the signal to the point where no peak was detected in the FFT; therefore, larger block sizes are needed for the flat top window to be applied. While non-rectangular windows, generally, result in more accurate results for frequencies and damping, they are most beneficial for separating frequencies in close proximity. For the baseline MTR model, the frequencies are already well separated, but future variations of the model may require these window functions.

Note when using non-rectangular functions, the amplitude of the resulting FFT will not be the same as the rectangular window. FFTs of non-rectangular window functions must be multiplied by correction factors to be compared one-to-one with a rectangular window. A list of correction factors are given in Table 1.7. For the purposes of extracting damping ratio, amplitude corrections are not necessary as the solution relies on the ratio of amplitudes so these factors are canceled out. However, for comparing the FFT amplitudes between different trials or different test conditions, these correction factors are necessary.

Table 1.7: **Correction factors for amplitude for different window functions.**

Window Type	Amplitude Correction Factor
Boxcar	1
Hanning	2
Hamming	1.852
Flat Top	4.639
Blackman-Harris	2.381

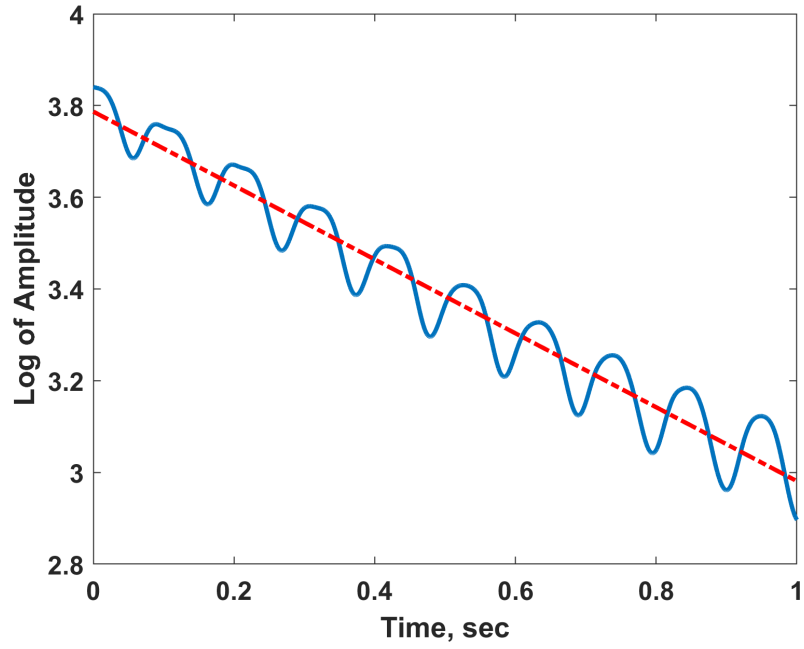
Block size is a key parameter. Too small a block size will not capture the signal frequency peaks accurately, and too large a block size may cause the damping to skew to lower values. Natural systems can also display complex nonlinear damping over time due to phenomenon such as free play and contact. The initial free decay response of the wing, within the first few seconds, normally has the proper aeroelastic damping that is of interest.

Figure 1.26 shows the calculation of the natural log of amplitudes for block sizes of 1024 and 2048 for N1.78 chord mode using the default Boxcar window. The baseline of 512 samples has been shown in Fig. 1.23(d). As can be seen, larger block sizes tend to lower damping values. This is due to higher damping seen at the beginning of the response and over time the damping decreases as the structure reaches a steady state oscillation.

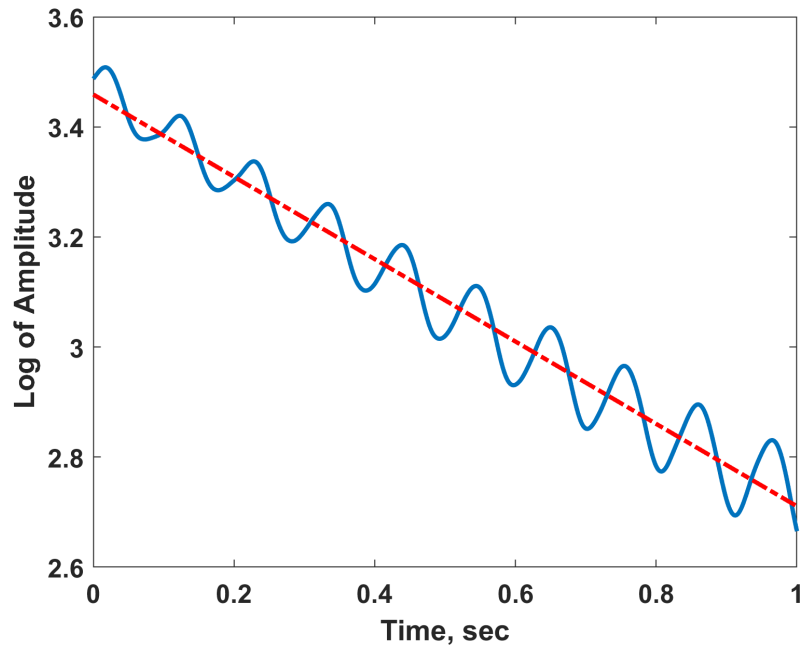
Block sizes of 256 and smaller could not acquire a max amplitude in the FFT between 8 and 11 Hz. So for the N1.78 signal with 9.5 Hz frequency and sampling rate of 1000 samples/second, a block size of 512 samples is the minimum size required. It is recommended practice to choose a block size which incorporates at least 4 cycles of the response signal.

A sensitivity study was conducted to investigate the effect of shifting the blocks by more than 1 sample. Shifting blocks by more than 1 sample speeds up post-processing. Figure 1.27 shows the natural log of the FFT amplitudes plot for each block shift condition. These are applied to run N1.78 with the default boxcar window.

The damping calculated from different block shifts are shown in Table 1.8.



(a) Block size of 1024 samples. 1.35% critical damping.



(b) Block size of 2048 samples. 1.25% critical damping

Figure 1.26: The natural log of amplitude plots with linear fit dotted line for large block sizes. Data from run N1.78 chord mode.

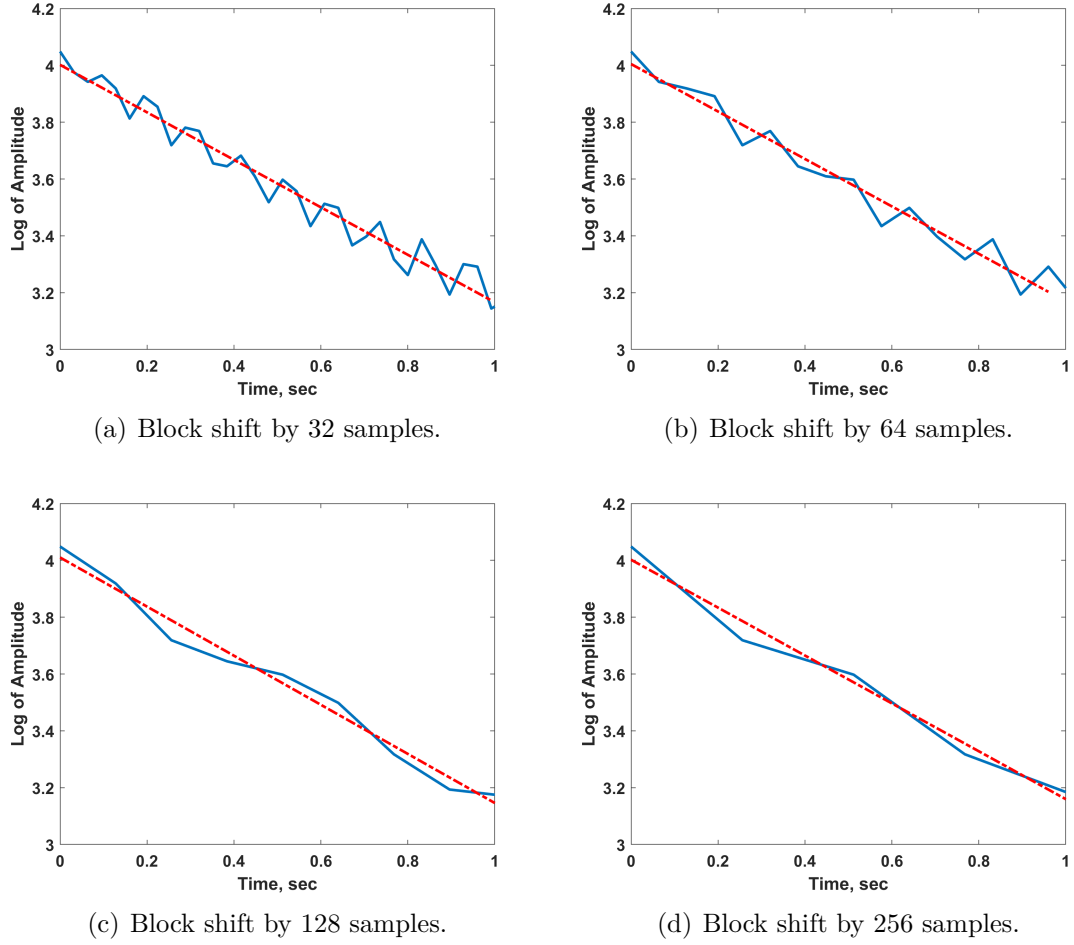


Figure 1.27: The natural log of amplitude plots with linear fit dotted line for varying block shifts. Data from run N1.78 chord mode.

Table 1.8: **Damping results for N1.78 using moving-block method with different block shifts for a block size of 512 samples.**

Block Shift (samples)	Damping (% critical)
1	1.41
32	1.40
64	1.40
128	1.45
256	1.41

The different block shifts for the N1.78 runs shows good agreement. Notice the N1.78 signal is clean from noise, occurs at a single frequency due to the actuation system, and the response does not increase in amplitude as time elapses. Caution must be used because sometimes the response does increase in amplitude seemingly from random disturbances as time elapses. Figure 1.28 from run N1.100 is an example. Here, the beam mode shows sudden, though slight, increases in amplitude at multiple points.

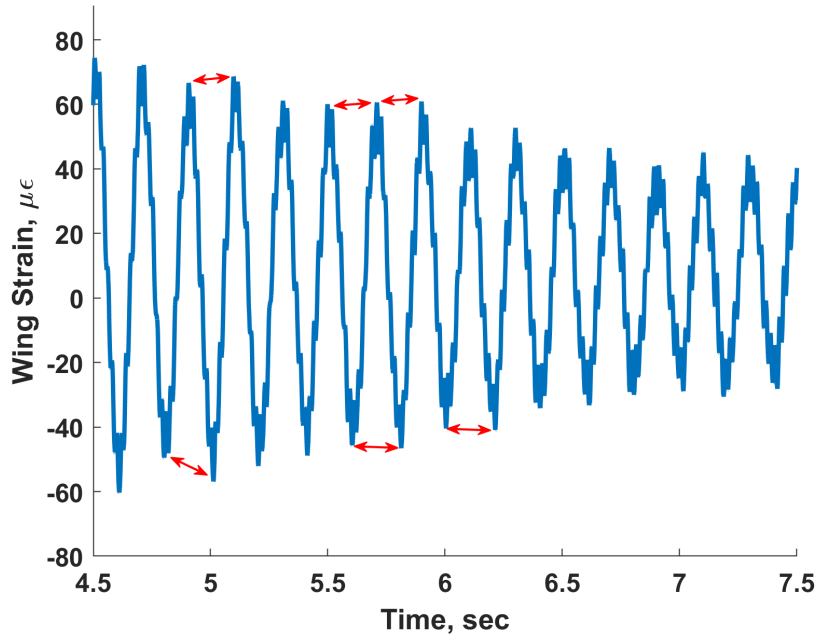


Figure 1.28: Run N1.100 beam mode with markers for where the response increases in amplitude.

When this occurs, for large block shifts, the resolution is lost between the blocks and can skew the results to lower damping. This is shown in Fig. 1.29. The damping results for these block shifts are shown in Table 1.9.

While the large block shifts may result in slightly decreased damping values, they are reasonable as a first pass calculation. Therefore, to save computing

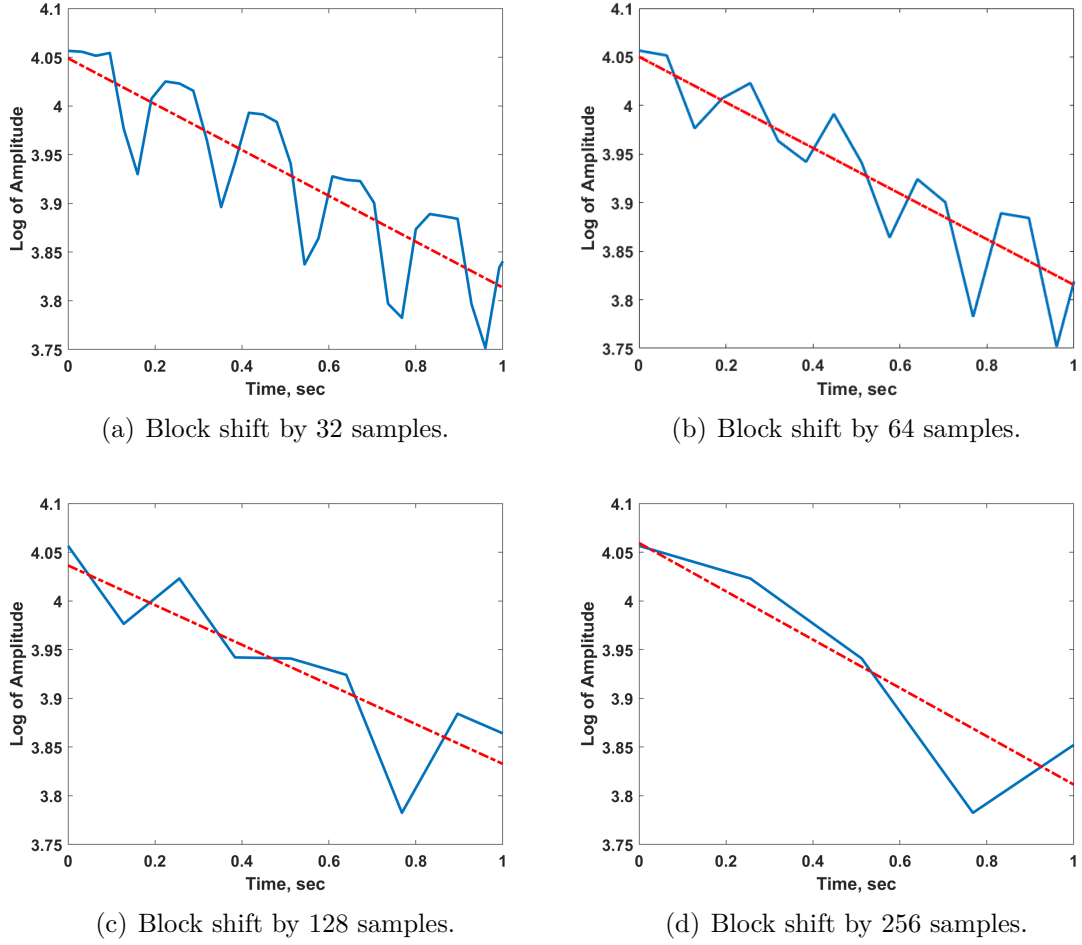


Figure 1.29: The natural log of amplitude plots with linear fit dotted line for varying block shifts. Data from run N1.100 beam mode.

Table 1.9: **Damping results for N1.100 beam mode using moving-block method with different block shifts for a block size of 512 samples.**

Block Shift (samples)	Damping (% critical)
1	0.79
32	0.75
64	0.75
128	0.65
256	0.79

time during wind tunnel tests, large block shifts can be used; however, it is recommended to use single sample shifts during postprocessing to obtain the most accurate, validation-quality results.

1.5 Scope and Contribution of Present Work

The present work has one main objective: to acquire reliable whirl flutter data from the Maryland Tiltrotor Rig (MTR). Much of the experimental work focused on model hardware and software integration, instrument calibrations, power electronics integration, and wind tunnel test setup. Key contributions are listed below:

1. This work obtained the first flutter test data for the Maryland Tiltrotor Rig.

The tests were conducted up to a max speed of 100 kt. Parametric variations of model features were compared for wing on versus wing off, gimbal free versus gimbal locked, powered rotor versus freewheeling rotor, and straight blades versus swept-tip blades. Results showed that wing aerodynamics had minimal effect on damping up to 100 kt, locking the gimbal increased wing chord damping but did not affect beam damping, powered rotor increased chord damping further compared to freewheeling rotor and produced an unexpected peak between 20-30 kt, and swept-tip blades had minimal effect on damping for the baseline configuration of wing on, gimbal free, and freewheeling rotor.

2. This work developed a new calibration stand to calibrate the MTR load cell directly on the rig. The new stand was used to calibrate F_x , F_y , F_z , M_x , and M_y in the laboratory. M_z was calibrated on a separate stand dedicated to

isolated load cell testing. During calibration, it was found that there were two load paths, one of which did not pass through the load cell. Results showed that thrust and torque were minimally affected by the secondary load path.

3. This work evaluated logarithmic decrement, Prony, and moving block methods as candidates for flutter test implementation. The moving block method was used for postprocessing the data and obtaining the final damping values reported in this thesis.
4. This work documented integration of MTR hardware and the data acquisition system using LabVIEW.
5. This work prepared a Test Readiness Review document for the flutter test of the MTR. This document covered the model features, predicted performance, test execution, test matrix, safety criteria and procedures, and personnel roles. This document was needed for access to the NSWC-Carderock wind tunnel and gives a template for what might be needed for future tests at any U.S. Government facility.

1.6 Overview of Dissertation

Chapter 1 introduces the background and motivations of research into tiltrotor whirl flutter. A literature review provides a state-of-art overview on experimental tiltrotor tests, including previous wind tunnel tests and computational analyses. The scope and contribution of the current study are highlighted at the end of chapter.

Chapter 2 describes a 2-ft diameter tiltrotor model that was built for risk-reduction and to explore flutter actuation system methods. The development and fabrication of the model and wind tunnel test in the Glenn L. Martin wind tunnel are detailed.

Chapter 3 shows the design of the Maryland Tiltrotor Rig (MTR) for the Glenn L. Martin wind tunnel. Desired features and capabilities are listed for the rotor, hub, blades, and wing.

Chapter 4 describes the properties and features of the fabricated MTR as well as the electric drive components.

Chapter 5 details the instruments on the MTR and their calibration. The load cell, accelerometer, electric actuators, pitch encoders, shaft torque gauge, pitch link gauges, gimbal hall effect sensors, shaft hall effect sensor, and wing strain gauges are covered. Off-rig instruments for data acquisition are also described.

Chapter 6 describes the methodology to evaluate whirl flutter for a 9 DOF tiltrotor model. Verification results are shown and compared to a Bell 25 ft model. Results for the MTR are also shown and discussed.

Chapter 7 presents the first flutter test results. The test location, conditions, procedures, damping measurement method, and parametric results are shown.

Chapter 8 summarizes the current work, draws key conclusions, and recommends works for future research.

Chapter 2: Flutter System Development Model

A 2-ft diameter, three-bladed, unpowered, gimbaled hub tiltrotor model was developed as a risk-reduction model to gain experience for fabrication of blades, and testing of whirl flutter in the Glenn L. Martin Wind Tunnel (GLMWT). The Flutter System Development (FSD) model was fabricated in-house with only the swashplate and pitch linkages purchased off-the-shelf.

2.1 FSD Description

A picture of the FSD, designed to be 1/12.5-scale XV-15, is shown in Fig. [2.1](#).

The FSD wing is a NACA 0012 airfoil with Rohacell 31 foam, one bi-directional carbon fiber ply for the skin, and 16 unidirectional carbon fiber plies for the spar. The FSD wing is actually a re-purposed rotor blade normally used for the Alfred Gessow Rotorcraft Center's hover stand. The wing properties are shown in Table [2.1](#).



Figure 2.1: 2-ft diameter flutter system development model.

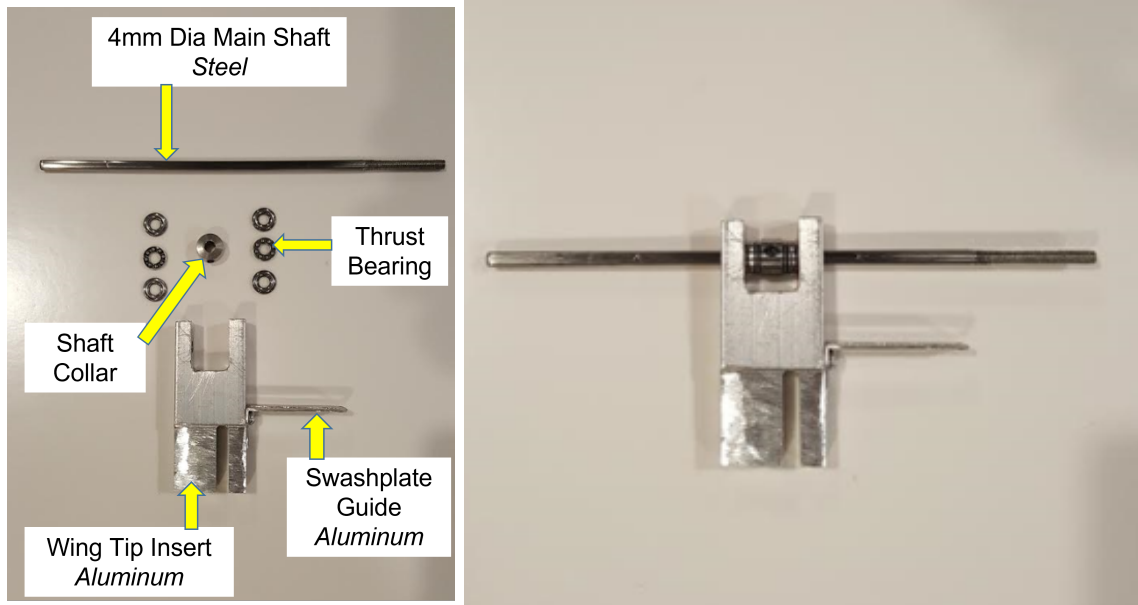
Table 2.1: **FSD wing properties.**

Parameter	Value
Length, in.	28
Chord, in.	3.15
$EI_{beam}, N - m^2$	24.5
$EI_{chord}, N - m^2$	250
$GJ, N - m^2$	23.5

2.2 FSD Pylon and Hub

The pylon assembly of the FSD consisted of a wing tip insert that had three microservo actuators mounted to it, and the main shaft running through the tip.

Pictures of the wing tip insert and its assembly are shown in Fig. 2.2

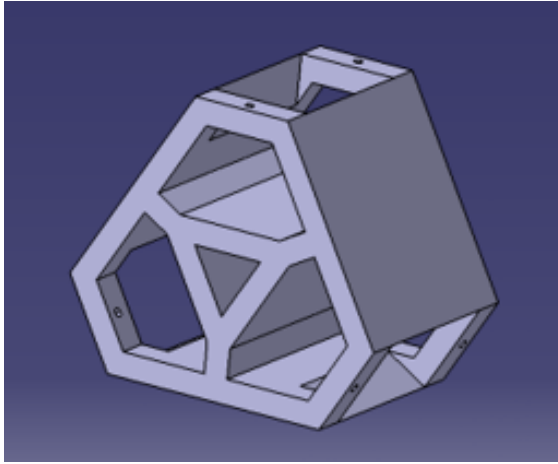


(a) Exploded view of the wing tip components.

(b) Wing tip insert assembled.

Figure 2.2: FSD wing tip assembly.

Adafruit microsensors were used to actuate the swashplate. The servos were mounted to a 3D-printed part that was then fixed to the wing tip insert. A CAD model of the servo cage and the resulting servo assembly is shown in Fig. 2.3.



(a) 3D-printed servo cage, made of ABSplus material.



(b) Servos mounted to cage.

Figure 2.3: FSD actuators assembly.

A CAD model of the FSD hub is shown in Fig. 2.8(a). The hub was fabricated in-house on a manual mill and lathe. The swashplate was the only part bought off-the-shelf from a 3-blade RC helicopter. A list of the purchased parts are shown in Table 2.2. The linkage lengths had to be modified, so long threaded rods were trimmed to the required lengths and used, instead of the rods that came with the linkage set.

The assembly of the FSD actuators and hub to the wing tip insert is shown in Fig. 2.5.



(a) CAD model of the FSD gimballed hub.



(b) Physical hub components.

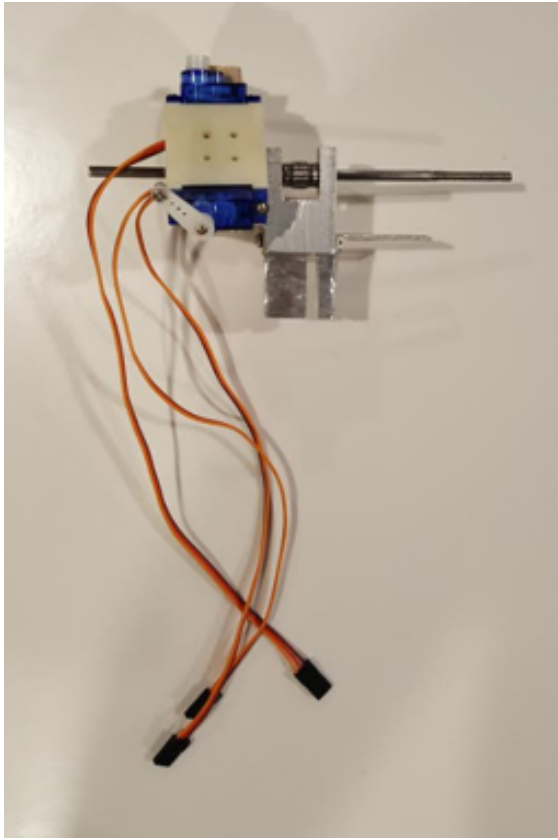


(c) Off-the-shelf RC helicopter swashplate and in-house fabricated linkages.

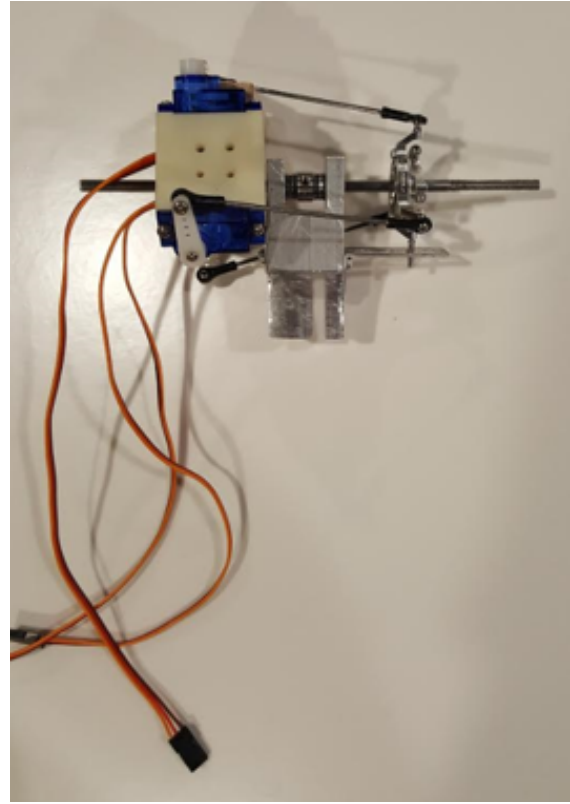
Figure 2.4: FSD rotor model and components.

Table 2.2: **Purchased parts for FSD swashplate.**

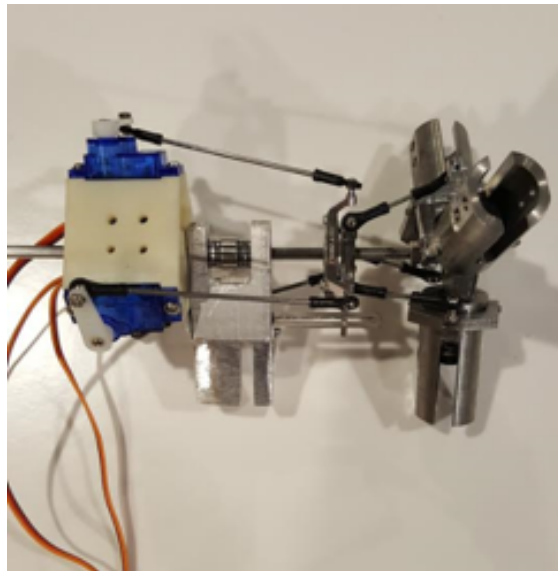
Part #	Description	Price (\$)
OXY3-091-SP	OXY Heli Qube 3-Blade Ultra Swashplate	20.49
OXY3-001-SP	OXY Heli Carbon Steel Main Shaft Set	9.99
OXY3-037	OXY Heli Servo Linkage Set	8.29
OXY3-036	OXY Heli Servo Arm Set	5.59



(a) Wing tip insert with servo cage mounted to rear.



(b) Swashplate connected.



(c) Hub attached.

Figure 2.5: FSD wing tip assembly.

2.3 FSD Blade Fabrication

The FSD had three sets of propotor blades. Straight, -45 deg linear twist, and -45 deg linear twist with swept-tips. The blades were 10.75 inches in length and 1.32 inches in chord. These blades were fabricated in blade molds shown in Fig. 2.6.

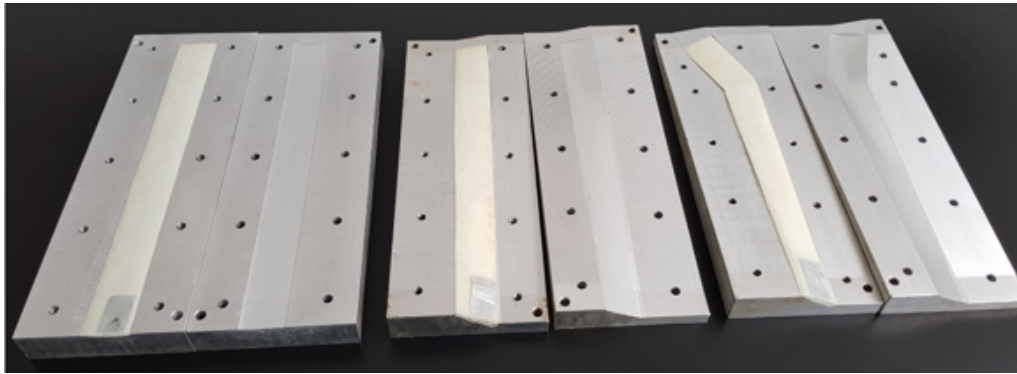


Figure 2.6: FSD blade molds: straight, twisted, and twisted with swept-tips.

Table 2.3 shows the materials used for blade fabrication. At this scale, there was no requirement for adhesive tape, leading edge weights, or a blade spar. Unlike the pre-preg carbon fiber plies for the wing, two layers of fiberglass plies had to be laid on top of each other and then epoxy resin was poured and spread over the area. Extra care needed to be taken to ensure the fiberglass plies did not slide significantly between each other. For root inserts, two small pieces of aluminum sheet metal are used to reinforce the blade root. The sheet metal is pressed into the foam, the foam is wrapped with the fiberglass skin, placed in the mold, then cured with the blade. Once the curing process is complete, the blade is trimmed and holes are drilled into the blade root.

Table 2.3: **FSD blade materials.**

Blade Part	Description	Part #	Vendor
Foam	Rohacell 31 IGF	RF3124	CST
Skin	E-glass Fiberglass Cloth	1080-50	Aircraft Spruce
Adhesive	PR2032/PH3660 Gallon Kit	01-42150	Aircraft Spruce

2.4 FSD Wind Tunnel Test

Figure 2.7 shows a 3D-printed stand made out of 100% filled Onyx on the Markforged Mark 2 printer. The stand was printed in two halves so it could clamp the blade like a clamshell. The stand had holes for the root insert of the wing and holes along the edges to clamp the wing and bolt to the wind tunnel post.

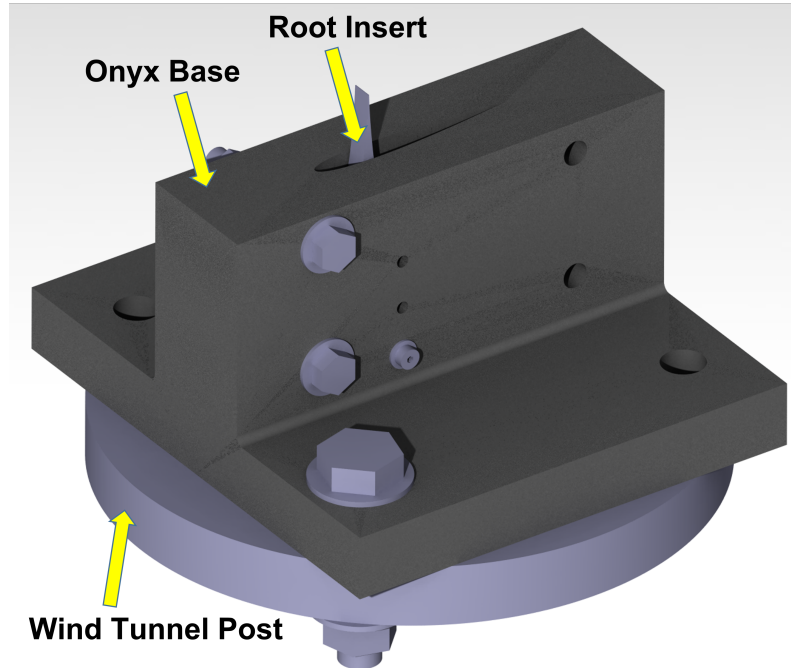


Figure 2.7: FSD base mount, 3D-printed on Markforged Mark 2.

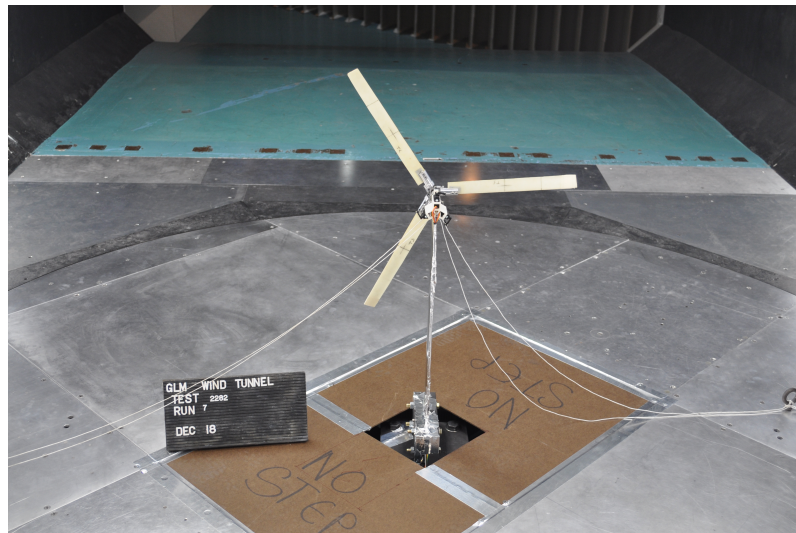
For data acquisition, the FSD used an NI compactDAQ-9174 chassis with NI-9401 Digital I/O module for microservo control, an NI-9237 Strain/Bridge Input

module for wing strains, and an NI-9207 Voltage and Current module for hall effect RPM sensor.



(a) FSD straight blade test up.

(b) FSD swept-tip blade test up.



(c) FSD flutter test setup: back view. Safety string on right, flutter string on left.

Figure 2.8: FSD in GLMWT test setups.

The FSD was tested at the GLMWT in December 2018 for straight-twisted and twisted with swept-tip blades. The test setups are shown in Fig. 2.8. First, the FSD was characterized by mapping the RPM to collective and windspeed. The model was tied down with cables to the wind tunnel floor and turnbuckles used to

tighten the cables. Damping was extracted for wind beam mode only. During the test, the swashplate actuators were not able to excite the rig at the wing frequency, therefore, a string was attached to the FSD wing tip interface and pulled from outside the test-section to impart an impulse. The wing strains were recorded and moving-block method applied to extract the wing damping.

2.5 Chapter Summary and Conclusions

This chapter provided an overview of the 2-ft diameter Flutter System Development (FSD) model. It covered the details of the following.

1. A description of the wing properties and dimensions.
2. A description of the pylon and hub assemblies including off-the-shelf components.
3. A description of the design and fabrication of small-scale blades.
4. A summary of wind tunnel preparation and testing.

The key conclusions are the following.

1. The FSD confirmed that blade fabrication procedures were adequate for the larger scale model and no alternative methods were required.
2. The FSD wind tunnel test revealed that, at the larger scale, actuators must be high frequency to provide flutter actuation for all wing modes.

Chapter 3: Design of the Maryland Tiltrotor Rig

This section describes the design of the Maryland Tiltrotor Rig. The Preliminary Design Review (PDR) occurred in June 2017, and the Critical Design Review (CDR) occurred in October 2017 at Calspan. A summary is published in Ref. [34].

3.1 Features of the New Rig

The new Maryland Tiltrotor Rig (MTR) is characterized by the following special features.

1. A hub with interchangeable gimballed and hingeless options.
2. A drive capable of powered and unpowered operations.
3. The rig is capable of supporting loads of Mach-scale rotors of 4.75-ft diameter.
4. A pylon that can be tilted from 90° to 0° at increments of 5° .

The gimballed hub, representing all current generation tiltrotors, is considered the baseline. The rig is simple but robust, easy to maintain, and easy to expand over time.

A comparison of the general features with other models clarifies its uniqueness. The MTR is different from the unpowered, floor mounted, semi-span, hub

interchangeable models of the 1970s — model scale Boeing M222 and Bell M301, because of the following features:

1. powered rotor
2. pylon tilt
3. straight wing
4. high bandwidth electric actuation of swashplate
5. direct electric drive on the pylon
6. modern instrumentation, including load cell on the pylon.

The MTR is different from the powered, wall mounted, semi-span, gimbaled models — WRATS (Langley TDT), TRAM (NASA Ames) and VDTR (Sikorsky) since then, because of the following features:

1. interchangeable hub
2. motor located in pylon (simulating the engine)
3. floor mounted
4. straight wing
5. load cell on pylon (unlike WRATS and TRAM)
6. flutter rig (TRAM and VDTR measured loads, not flutter)

The extensive 1/5 scale gimballled hub Bell models tested during the 1980s were several generations of development models specific to the V-22 aircraft with restricted properties and datasets.

3.2 Overview of the New Rig

The rig consists of the wing frame, motor drive, rotor shaft, hubs (gimballled and hingeless), swashplate (three-bladed), and major instrumentation hardware (see later). The blades and wing spars can be inserted in and out depending on the nature of investigation. The blades can be Froude- or Mach-scaled; Froude- for whirl flutter and Mach- for loads. The rig is designed to allow Mach-scaled rotors, which means adequate power from the motor, and adequate structure to withstand the higher loads. The detailed design and construction of the rig is carried out by Calspan Systems. The composite blades and wing spars are designed and fabricated in-house at Maryland.

The overall specifications are given in Table [3.1](#).

3.3 Size and Scale

The dimensions of the rig are shown in Figs [3.1](#) and [3.2](#) placed inside the Glenn L. Martin wind tunnel test section.

The wall clearances from the blade tip are determined by the rotor radius and the vertical placement of the hub. Figure [3.3](#) shows the possible tip clearances for a given rotor radius as the hub is moved down (left of curves) or up (right of curves).

Model Parameters	Maximum
Rotor radius R	2.375 ft
Height above ground	3.0875 ft
Rotor speed	2750 RPM
Number of blades	3
Type of hub	Gimballed and hingeless
Pre-cone	2°
Pitch-flap δ_3	-15°, for gimballed only
Thrust	300/ - 70 lbf
Hub Moments	198 ft-lbf
Centrifugal retention per blade	2430 lbf
Weight of blade	0.66 lb
Weight of hub-pylon	65 lb
Power	Electric motor, water cooled 20 HP at 2660 RPM

Table 3.1: **Top level MTR specifications.**

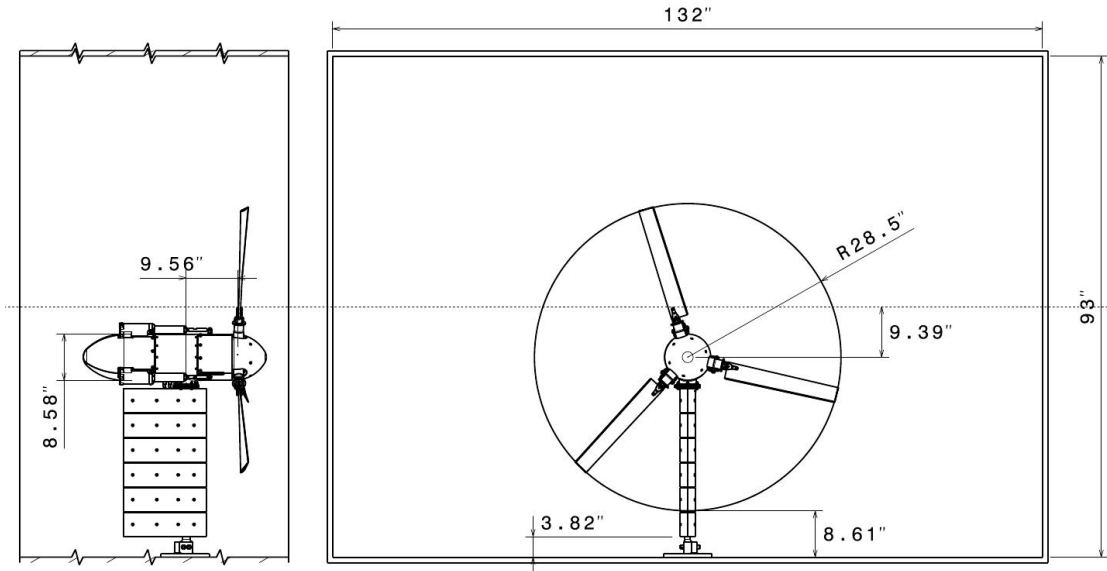


Figure 3.1: The Maryland Tiltrotor Rig (MTR) inside Glenn L. Martin wind tunnel test section (7.75- by 11-ft); dimensions in inches; dashed line is Center Line of tunnel.

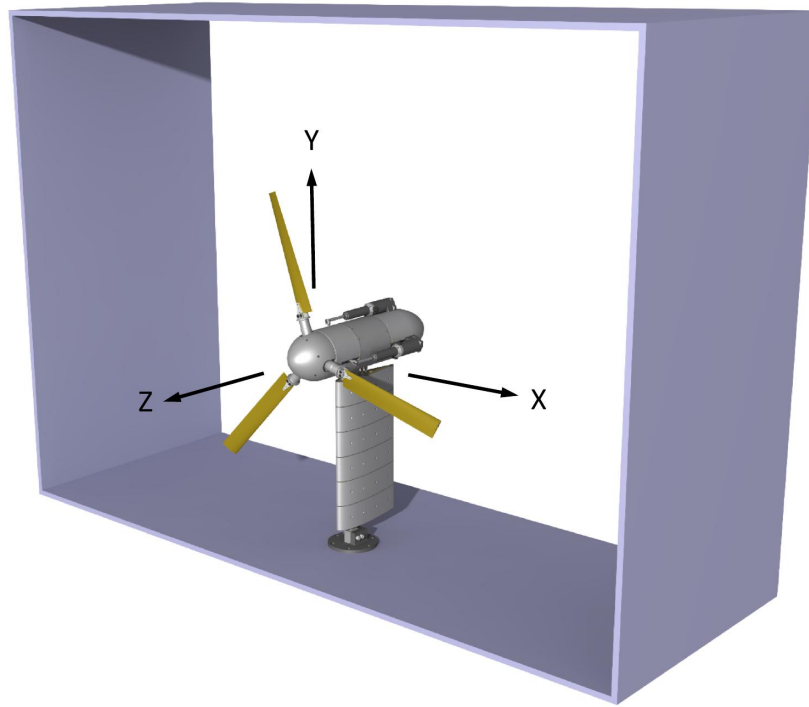


Figure 3.2: **The Maryland Tiltrotor Rig (MTR) inside Glenn L. Martin test section (7.75- by 11-ft); isometric view.**

The V-22 tip clearance from fuselage in cruise is around $0.105 R$. With this clearance at the bottom, a radius of 2.375 ft would leave a clearance of $1.16 R$ at the top. The baseline position is to leave a clearance of $0.3 R$ at the bottom and $0.96 R$ at the top. This still allows a radius of clearance at the top while leaving ample space below for future needs.

The maximum rotor radius is presently set at $R = 2.375$ ft. So the scale factors are:

$1/8.00$ V-22

$1/2.00$ TRAM (which is similar to a $1/4$ scale V-22)

$1/5.26$ full-scale Bell 25-ft dia gimballed rotor tested at Ames 40- by 80-ft (XV-15 rotor)

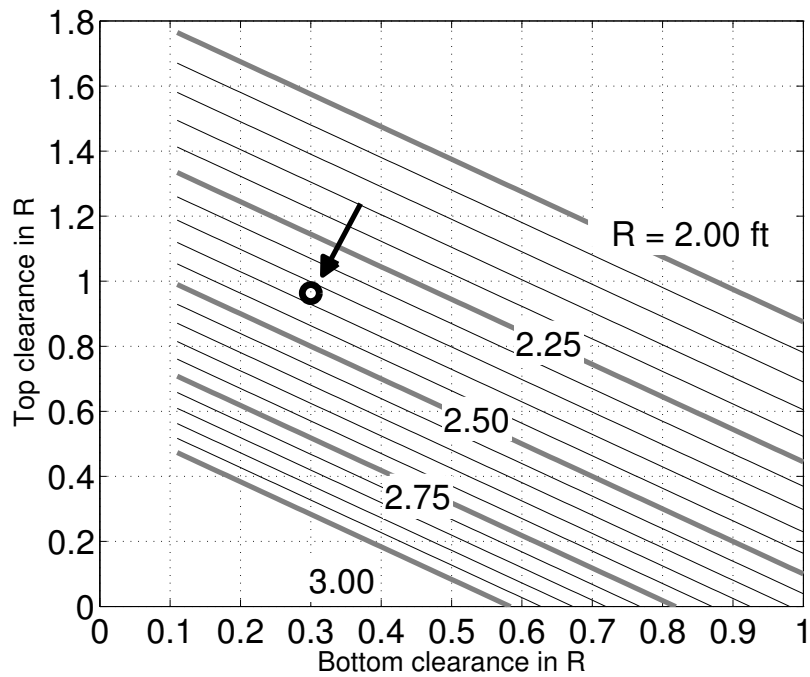


Figure 3.3: **Tip clearances from top and bottom of the wind tunnel as function of rotor radius; the symbol shows the present configuration with $R = 2.375$ ft and clearances of $0.3 R$ and $0.96 R$ from the bottom and top respectively.**

1/5.47 full-scale Boeing 26-ft dia hingeless rotor tested at Ames 40- by 80- ft

(Bo 105-like rotor)

1.69/1 of model Bell M301 gimballed and model Boeing M222 hingeless 2.8-ft

dia

The design is loosely anchored to Bell 25-ft gimballed (XV-15) and Boeing 26-ft hingeless (Bo 105) rotors where possible. The primary departure is in the pylon inertia (weight, C.G., moments of inertia). This is unavoidable because electric motors do not scale as engines, and the pylon includes the load cell and slip ring — components essential for model testing. The increase in inertia can be partially countered through the placement of wing frequencies but expected (and confirmed)

to reduce the flutter frequencies nonetheless. Flutter at low speeds, if encountered, would be considered a positive feature for research purposes as long as the properties are well documented.

The MTR rotor speeds are compared to the Bell (gimballed) and Boeing (hingeless) rotors in Table 3.2.

	Scale Factor	Full-scale RPM	Froude-scale RPM	Mach-scale RPM
Bell full-scale 25-ft dia	1	458	-	-
Bell model M301 2.8-ft dia	1/8.89	-	1366	-
MTR gimballed 4.75-ft dia	1/5.26	-	1050	2409
Boeing full-scale 26-ft dia	1	386	-	-
Boeing model M222 2.8-ft dia	1/9.244	-	1174	-
MTR hingeless	1/5.47	-	903	2111

Table 3.2: **MTR rotor speeds (in revolutions per minute (RPM)) compared to previous Bell (gimballed) and Boeing (hingeless) tests.**

For the MTR, a rotor speed of 2660 RPM will achieve the same tip Mach number $M_T = 0.59$ as V-22 in cruise, and a rotor speed of 3200 RPM will achieve the same tip Mach number $M_T = 0.71$ as V-22 in hover. The latter is deemed too high; a lower tip Mach number $M_T = 0.61$, similar to the TRAM model, is set as the maximum RPM of the MTR. This requires a lower maximum RPM of 2750.

At Froude-scale, flight speeds of 360 and 400 kt of the V-22 will translate into tunnel speeds of 130 and 140 kt respectively, which are well within the 200 kt maximum limit of the Glenn L Martin tunnel.

3.4 Rotor

The maximum rotor radius R is 2.375 ft (0.7239 m). Airfoil profiles are assumed to begin at $0.1R$ for sizing calculations but likely near $0.2 - 0.3R$ for the actual blades. The solidity is $\sigma = 0.1$. The maximum twist is linear -45° : $\theta_t = -33.75^\circ - 45^\circ (x - 0.75)$ where $x = r/R$ and r is the radial dimension. The maximum sweep is 20° (sweep back) over 20% from 0.8 R outboard. At flat pitch, the sectional angle is 29.25° at $0.1R$, 0° at $0.75R$, and -11.25° at the tip. With three blades ($N_b = 3$), and with solidity defined as projected blade area at flat pitch divided by disk area, the mean chord ($\sigma \pi R/N_b$) is 0.0758 R. Taper is zero. A single airfoil is assumed across entire span — the VR-7 with no tab. The maximum RPM is 2750.

These assumptions produce a hover power of 50 HP (37 kW) at a collective angle (θ_{75}) of 16° and thrust of 318 lbf (1.42 kN). This is considered the maximum power hover point. This represents the maximum power of the Maryland hover tower. The tiltrotor rig is not required to supply this power but is designed to withstand loads up to this point so the rotor head can be tested separately on the hover tower in future. The vertical shear at the root of each blade is 106 lbf (472 N). The inplane shear is 23.4 lbf. The centrifugal force at the root requires weight of each blade; the maximum weight considered is 0.66 lb (mass 0.3 kg), which gives 2024 lbf (8.9 kN). For the swept tip blade, a maximum of 20% of the blade mass is assigned to the swept portion, offset from the hub center, inplane, with center of gravity located at 0.9 R (center of the swept portion) along a 20° sweep line. Assuming the

twist to be about local quarter-chord and center of gravity on quarter-chord there is no vertical offset. So the sweep introduces an additional inplane shear of 33 lbf toward the trailing edge.

The bending moment at the root in hover is moment due to aerodynamic force (lift) $M_A = \int r dL$ minus a relief from the centrifugal force in presence of blade flapping. Assuming a spanwise mass distribution m kg/m, and blade flapping β rad, the relief is $\Omega^2 \beta \int m r^2 dr$. The moment at the root is $k_\beta \beta$ where k_β is an equivalent root spring in N-m/rad.

$$\begin{aligned} k_\beta \beta &= \int r dL - \Omega^2 \beta \int m r^2 dr \\ &= M_A - I_\beta \Omega^2 \beta \end{aligned}$$

M_A is 175 ft-lbf at the maximum hover point. I_β is the flapping moment of inertia which for a uniform blade is $I_\beta = \frac{1}{3} m R^3 = 1.2434 \text{ lb/ft}^2$ where $m = 0.66/2.375 = 0.278 \text{ lb/ft}$. The spring is related to the rotating flap frequency ν_β (non-dimensionalized with rotor RPM, in /rev) as $k_\beta = (\nu_\beta^2 - 1) I_\beta \Omega^2$. The blade root bending moment is then related to the flap angle and flap stiffness as in Table 3.3. Greater the allowed flapping angle, lower the moments at the root, but the blade root must be softened to allow flapping, which means a softer spring and flap frequency nearer to 1/rev. If for example, a flapping of 3° is allowed, the net moment at the root is only 7.3 ft-lbf, and the corresponding flap frequency would be 1.022 /rev.

Aero moment M_A ft-lbf	Flap angle β degree	Centrifugal relief due to flapping ft-lbf	Net moment at root ft-lbf	Root spring k_β ft-lbf/rad	Flap frequency ν_β /rev
175	0	0	175	locked	rigid
175	1	56	119	6824	1.77
175	2	112	63	1811	1.25
175	2.25	125.8	49	1254	1.18
175	2.5	139.7	35	808	1.12
175	3	167.7	7.3	140	1.022
175	3.13	175	0	free	1.00

Table 3.3: **Blade root flap moment in hover.**

Aero moment M_A ft-lbf	Flap angle β degree	Centrifugal relief due to flapping ft-lbf	Net moment at root ft-lbf	Root spring k_β ft-lbf/rad	Flap frequency ν_β /rev
175	2	112	63	locked	rigid
175	2.25	125.8	49	11285	2.13
175	2.5	139.7	35	4041	1.50
175	3	167.7	7.3	420	1.064
175	3.13	175	0	free	1.00

Table 3.4: **Blade root flap moment in hover for 2° pre-cone.**

If the blade is pre-coned at an angle, then the same moment can be absorbed by less flexibility at the root. The moment balance with pre-cone is

$$k_\beta (\beta - \beta_P) = M_A - I_\beta \Omega^2 \beta$$

With a pre-cone of $\beta_P = 2^\circ$, the root moment is related to the flap angle and flap stiffness as in Table 3.4. The moment from a 3° flap angle (7.3 ft-lbf) is now absorbed by spring deflection of only $\beta - \beta_P = 1^\circ$, so a stiffer hub can be used. The pre-cone allows higher flap frequencies to be achieved without increasing the root moment. Pre-cone is also an important parameter for whirl flutter. A pre-cone of $\beta_P = 2^\circ$

will be used for the present rig. The maximum root moment is then 63 ft-lbf in flap up direction (considering a rigid hub, but likely lower due to flexibility). The root moment in the lag direction is 32 ft-lbf. The root moment in the pitch direction is a nose down 2 ft-lbf. The nose down moment sizes the pitch link and actuators.

The target flap frequency, non-dimensionalized with rotor RPM (in /rev), is $\nu_\beta = 1.85$ for the gimbaled hub, although frequencies in the range $\nu_\beta = 1.20 - 1.85$ (TRAM to XV-15) are acceptable. This is the collective frequency (with gimbal locked), the cyclic frequency is always 1.0 /rev without a hub spring. The target flap frequency for the hingeless hub is between $\nu_\beta = 1.30 - 1.50$.

3.5 Blades

The blades are interchangeable and many different blade geometries will be tested on the rig. They are likely all carbon fiber composite blades with either rectangular or D-spars. For the baseline blades, the primary flap and lag frequencies were intended to remain within the following range:

$$\begin{aligned} \nu_{\beta 0} &= 1.20 - 1.85 & \nu_\zeta &= 1.20 - 1.80 & \text{gimbaled, with gimbal locked (collective)} \\ \nu_\beta &= 1.20 - 1.50; & \nu_\zeta &= 0.70 - 0.80 & \text{hingeless} \end{aligned} \tag{3.1}$$

However, the lag frequency of the fabricated blades were significantly higher near 5/rev at the Froude-scale RPM of 1050. This was due to limitations of the carbon fiber material and construction of the blade root arm.

Property	Full scale approx	Froude scaling	Value	Mach scaling	Value
$EI_N \text{ Nm}^2$		s^5		s^4	
$0.1 < x < 0.3$	8×10^5		198.7		1046
$0.3 < x < 0.5$	1.5×10^5		37.3		196.1
$0.5 < x < 1.0$	0.5×10^5		12.4		65.4
$EI_C \text{ Nm}^2$		s^5		s^4	
$0.1 < x < 0.3$	7×10^5		173.8		915
$0.3 < x < 0.5$	20×10^5		497		2614
$0.5 < x < 1.0$	12×10^5		298		1567
$m \text{ kg/m}$		s^2		s^2	
$0.0 < x < 0.2$	55		1.99		1.99
$0.2 < x < 1.0$	10		0.36		0.36

Table 3.5: **Nominal stiffness and mass properties for gimballed rotor blades;** EI_N and EI_C are the normal and chord-wise flexural stiffnesses and m the mass per span; $1 \text{ Nm}^2 = 2.42 \text{ lbf-ft}^2$; full-scale values from Bell 25-ft diameter model (XV-15 rotor), $x=r/R$, approximate mean values are taken; $s = \text{model length/full-scale length} = 1/5.26$.

Property	Full scale approx	Froude scaling	Value	Mach scaling	Value
$EI_N \text{ Nm}^2$		s^5		s^4	
$0.0 < x < 0.08$	3×10^5		61.3		335
$0.08 < x < 0.2$	0.5×10^5		10.2		56
$0.2 < x < 1.0$	0.2×10^5		4.1		22.3
$EI_C \text{ Nm}^2$		s^5		s^4	
$0.1 < x < 0.3$	3×10^5		61.3		335
$0.3 < x < 0.5$	3×10^5		61.3		335
$0.5 < x < 1.0$	12×10^5		245		1341
$m \text{ kg/m}$		s^2		s^2	
$0.0 < x < 0.08$	114		3.8		3.8
$0.08 < x < 1.0$	5		0.17		0.17

Table 3.6: **Nominal stiffness and mass properties for hingeless rotor blades;** EI_N and EI_C are the normal and chord-wise flexural stiffnesses and m the mass per span; $1 \text{ Nm}^2 = 2.42 \text{ lbf-ft}^2$; full-scale values from Boeing 26-ft diameter model (Bo 105 rotor), $x=r/R$, approximate mean values are taken; $s = \text{model length/full-scale length} = 1/5.47$.

The gimballed frequencies represent the state of the art range from V-22 and TRAM -like to XV-15 -like proprotors. The hingeless frequencies are expected to produce interesting air resonance instability. The hub can be rigid in flap and still absorb flap loads (first row in Table 3.4) so blades with very high flap frequencies (2/rev and higher) can be investigated.

The nominal targets for blade structural properties are given in Tables 3.5 (gimballed) and 3.6 (hingeless) based on the Bell 25-ft and Boeing 26-ft diameter rotors respectively. The discontinuities near the root are due to hub connections (blade grip, pitch case, hub) and are not expected (or required) to be reproduced. The baseline blades will target average properties from 0.3 R to the tip.

3.6 Hub and Shaft

The hub consists of blade retention, pitch mechanism, and shaft connection. A simple construction with a single load path is desired for both gimballed and hingeless hubs.

The hub contains sensors for motion (gimbal angle and blade root pitch), slip rings for data transmission, a 6-component load cell, a torque transducer, and a tachometer and 1/rev pulser for rotor speed and position.

The hub holds three blades ($N_b = 3$) and allows a configuration change between gimballed and hingeless rotors. The pitch links are on the trailing edge side to provide a negative pitch-flap coupling for the gimballed rotor. A flap up about the gimbal by an angle $\Delta\beta$ would pitch the blades down by an angle $\Delta\theta$, so $\Delta\beta/\Delta\theta$ is

negative and quantified by an angle δ_3 such that $\Delta\beta/\Delta\theta = -\tan\delta_3$. The nominal dimensions are: pitch horn location at 0.019 R, pitch horn radius 0.078 R (behind blade), and a pitch link length of 0.144 R (based on TRAM data). The pitch links are actuated by a conventional swash-plate.

Both hubs have a pre-cone of 2° . The specific features of each hub are given below.

3.6.1 Gimballed hub

A constant velocity joint was desired, but deemed not essential because of the cost and complexity involved. Simplicity was an over-riding factor for this research rig. The gimballed hub is therefore an universal joint, similar to the XV-15 hub — essentially a stiff in-plane hingeless hub sitting on a gimbal. An universal joint will produce a 2/rev excitation from the RPM so the wing-pylon natural frequencies must steer clear of this region. There is no gimbal undersling or torque offset by intent. A small undersling might be a fall-out from the construction.

Gimbal spring is also zero by intent. The full scale Bell 25-ft rotor had a hub spring of 2700 in-lb/ $^\circ$. With scale factor $s = 1/5.26$ the maximum allowable model spring would be $2700 s^2 = 97.6$ in-lb/ $^\circ$.

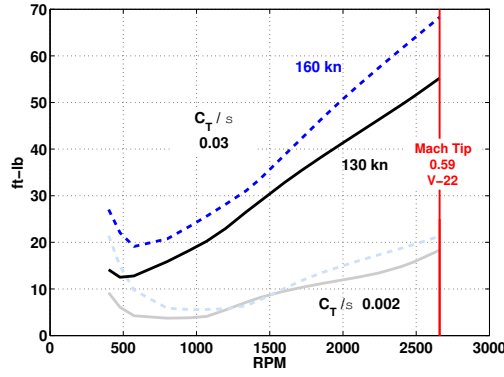
Ideally, pitch-flap coupling should produce the same effect on flap frequency as full-scale, so $\gamma \tan\delta_3$ should remain constant, not δ_3 , where γ is the Lock number. The δ_3 angle also varies with the cosine of collective away from nominal setting. For this rig, the nominal value is set as $\delta_3 = -15^\circ$ at flat pitch $\theta_{75} = 0^\circ$.

3.6.2 Hingeless hub

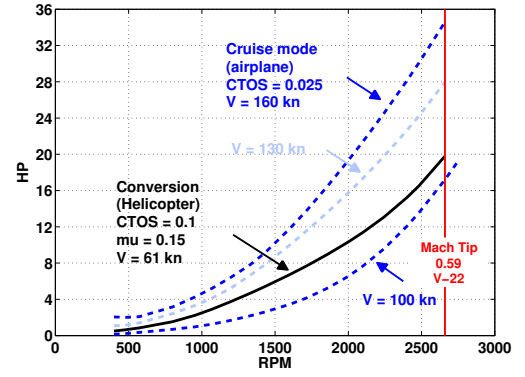
Pitch-flap coupling is not needed for the hingeless hub.

The construction of the hingeless hub is simple by itself. The hub/blade assembly can slide into the shaft as a push fit. Rotor torque can be transmitted to the shaft by a key. The hub can be secured axially with a nut. But the requirement for configuration changeover, from gimballed to hingeless, or vice versa, introduces certain considerations. The changeover would follow a certain step by step procedure. An example of the basic steps would be:

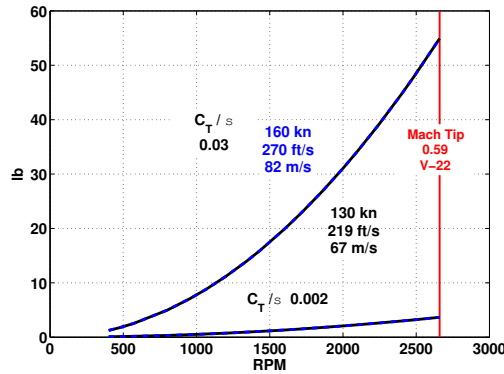
1. Installing or removing instrumentation needed only for the gimballed rotor (gimbal flapping).
2. Installing or removing wing spar ballast weight if needed for the hingeless rotor (the hingeless hub might be lighter and ballast weight might be needed to recover same wing-pylon frequencies).
3. Relocating swashplate scissors.
4. Re-configuring spinner and other shaft end item.
5. Re-calibrating collective and cyclic controls.



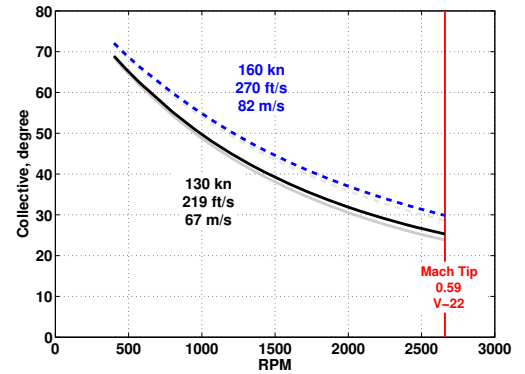
(a) Torque versus rotor speed



(b) Power versus rotor speed



(c) Thrust versus rotor speed



(d) Collective pitch versus rotor speed

Figure 3.4: Predicted MTR performance used for design: torque, power, thrust and collective pitch (at blade root) versus rotor speed; CTOS is blade loading C_T/σ and μ is tip speed ratio μ .

3.7 Drive

The drive supplies the rotor with a maximum of 20 HP at 2660 RPM and can be operated down to at least 900 RPM. In general it can function continuously down to 0 RPM. Three possible options were explored:

1. an electric motor, mounted on the pylon, driving the rotor directly or through a gear-box,

2. an electric motor, located below the rig, with a high powered shaft through the wing, and
3. a hydraulic motor, located outside the rig, with high pressure fluid lines through the wing.

Option 1 offered the best solution.

Option 2 was too complex, costly, less safe and not conducive to frequent spar swaps.

Option 3 was feasible but an electric drive was preferred for its higher bandwidth.

The motor was chosen based on the predicted torque- and power-speed envelopes, Figs 3.4(a) and 3.4(b) respectively, and constraints of pylon weight and dimensions. Hydraulic motors were discarded due to their slow response, but also difficulty in passing high pressure fluid lines through the deforming wing. Electric motors from eleven vendors and pneumatic motors from two vendors were considered. An electric motor with suitable speed and torque control was selected — Plettenberg NOVA 30. This is a water cooled permanent magnet motor operating at 80 – 140 V (nominal 110 V), maximum speed of 5000 RPM, maximum torque of 80 Nm (59 ft-lbf), maximum continuous power of 30 kW (22.4 HP), efficiency of 90% (including controller), diameter of 20.2 cm (7.56 inches), and total weight of 6.8 kg (15 lb) without the controller. The controller (and water tank) will be housed outside the pylon in the tunnel control room.

	V-22	XV-15	Bell 25 ft	Bell M301	Boeing 26 ft	Boeing M222	MTR
	aircraft	aircraft	full scale	1/8.89 Froude	full scale	1/9.24 Froude	nominal
Radius R ft	19	12.5	12.5	1.4	13	1.4	2.375
Cruise RPM	333	517	458	1366	386	1174	1050
Beam per rev	0.53	0.45	0.42	0.38	0.36	0.42	0.35-0.45
Chord per rev	0.80	0.86	0.70	0.66	0.62	0.73	0.60-0.86
Torsion per rev	1.04	1.07	1.30	1.36	1.43	1.58	1.04-1.60

Table 3.7: **Wing-pylon frequencies normalized with cruise RPM.**

Figures 3.4(c) and 3.4(d) show the range of thrust and collective pitch angles (root pitch angles) required for cruise flight. High collectives are required at lower RPM, which is typical of tiltrotors, due to the high blade twist.

3.8 Pylon

The pylon consists of a slip ring, the electric actuators, the electric motor, the load cell (6-component rotating balance on the rotor drive shaft), the shaft, and a set of couplings, adapters, and fairings.

The pylon is integrally mounted on the wing spar through two coupling plates. The plates have holes drilled in a verniered arrangement, every 15° on the wing side and every 20° on the rotor side, providing a 5° resolution of static pylon tilt. The total pylon weight for the gimbaled hub is estimated to be 65 lb (pylon body 8.5, fairings 2.8, three actuators 9, load cell 6, motor 15, slip ring 4.5 and the rotor head without blades 19 lb).

3.9 Wing

The wing is straight (departure from XV-15 or V-22 and similar to V-280), untwisted, and 18% thick with a simple profile (NACA 0018). The state of the art thickness being 20 – 23% (approximately) and the target being 14% this is deemed an appropriate intermediate baseline. The wing has no flaps.

The wing is a critical part of the rig. The coupled wing-pylon natural frequencies (including hub but excluding blades) in bending (beam-wise and chord-wise) and torsion must achieve certain targets to simulate whirl flutter. The wing-pylon natural frequencies for several rigs in previous flutter tests, normalized with rotor RPM, are given in Table 4.3, together with the MTR targets. There is some flexibility in the targets, although attempts will be made to remain near the Bell 25-ft and Boeing 26-ft frequencies. These rotors (and their model-scale versions M301 and M222) have the precedence of being tested on the same wing and might allow historical insights.

The frequencies can be met with a simple spar as in the previous model-scale flutter tests (for example an aluminum box beam from tunnel attachment to pylon coupling plate). The key difference is the powered rotor and hub instrumentation that changes the pylon weight and hence the dimensions. The Bell and Boeing full-scale tests used a thinner 13.5% thick wing; the choice of 18% thickness here is deliberate, it provides ample space for parametric variations of the spar for future needs. The spar is the important piece, the outer profile can always be replaced. The spar attachment to the pylon coupling and to the floor attachment are simple

and robust (bolted) and provide more than 5 Factors of Safety margin in for all operating loads, steady state and oscillatory.

The wing structural damping is also an important parameter. This value will be measured after assembly of the hardware.

Measurement	Sensor	Quantity	Leads	Mounting
Blade pitch angle	Hall sensor	3	12	1 per blade
Gimbal tilt angle	Hall sensor	2	8	1 per blade
Rotor speed and position	1/rev	1		
Rotor torque	Strain gauge	8	8	Two full bridges on the shaft
3-axis forces and moments	Load cell	1		on pylon bulkhead near motor
Vibration	Tri-ax accel	1		pylon frame
Slip ring		1		64 channels/leads, 16 measurements, 4 leads per measurement
Absolute pitch angle	inclinometer	1		tool mounted at blade attachment
Fixed system actuation	integrated			below swashplate

Table 3.8: **MTR on-rig instrumentation**

3.10 Instrumentation

The rotor, pylon, and wing will all be instrumented. The major instrumentation on the rig are listed in Table 3.8. Minor instrumentation like sensors on the

wing, standard equipment like wind tunnel balances, and out of rig equipment like track and balance, VICON, or PIV are left out.

3.11 Chapter Summary and Conclusions

This chapter covered the conceptual design of the Maryland Tiltrotor Rig. It covered the following.

1. An illustration of the overall dimensions of the rig and the clearance as installed in the GLMWT.
2. A description of the architecture of the subassemblies.
3. A description of the load limits on the rotor for hover.
4. A description of the motor requirements and the considerations for motor options.
5. A list of the on-rig instrumentation.
6. A description of the target properties, loosely scaled to the XV-15.

In conclusion, the design of the MTR has matured to the level where fabrication can begin. The following chapter describes the actual rig after fabrication.

Chapter 4: The Maryland Tiltrotor Rig

This chapter describes the Maryland Tiltrotor Rig (MTR). It covers the MTR dimensions, and overviews of the wing assembly, pylon assembly, and rotor assembly.

4.1 Dimensions

The Maryland Tiltrotor Rig (MTR) is a 3-bladed, semi-span, floor-mounted, optionally-powered, tiltrotor rig. The first installation was in November 2019 at the Glenn L. Martin Wind Tunnel (GLMWT). Figure 4.1 shows a photograph of the installation. Figure 4.2 shows the CATIA model of the MTR in its baseline airplane mode cruise configuration. The principal dimensions are shown in this figure. The dimensions are also listed in Table 4.1. Figure 4.3 shows the configuration in all three modes - cruise, transition, and hover.

The baseline rotor and wing spar of the MTR is loosely a Froude-scale model of the Bell XV-15 wing and 25-ft diameter proprotor. With a rotor diameter of 4.75 ft, the MTR is a $1/5.26$ scale model of the full-scale proprotor. The design was constrained to fit within the wind tunnel test section dimensions of the GLMWT (11-ft wide by 7.75-ft tall), as shown in Figure 4.4, while keeping ample margins from the wall.

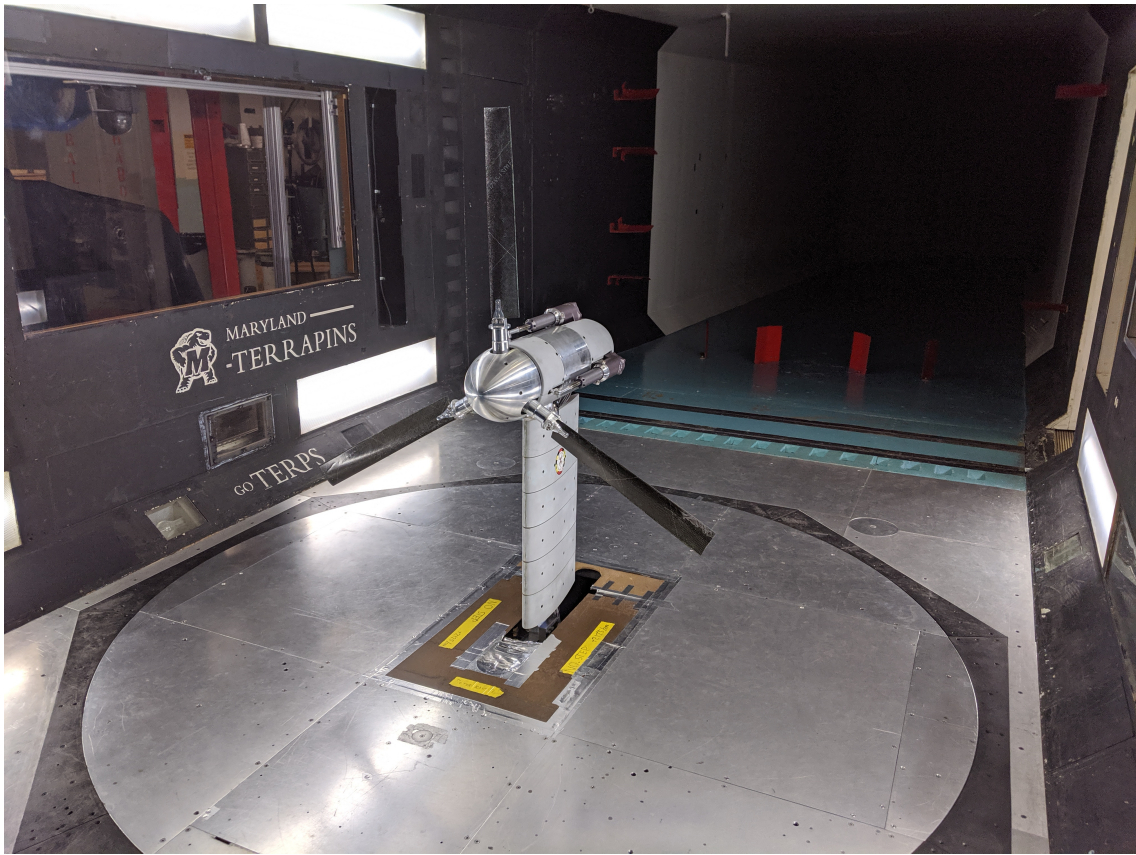


Figure 4.1: Maryland Tiltrotor Rig in the Glenn L. Martin wind tunnel; November 2019.

Table 4.1: MTR Size Overview

Dimension	Value (in)	Value (cm)	x/R
Baseplate to Pylon Center	36.5	92.71	1.28
Tunnel Floor to Pylon Center	34	86.36	1.19
Wing Fairing Span	27.5	69.85	0.96
Wing Chord	15.45	39.24	0.54
Pylon Length	34.02	86.41	1.19
Pylon Diameter	8.58	21.79	0.30
Rotor Radius	28.5	72.39	1.0
Root Cutout	7.55	19.18	0.26

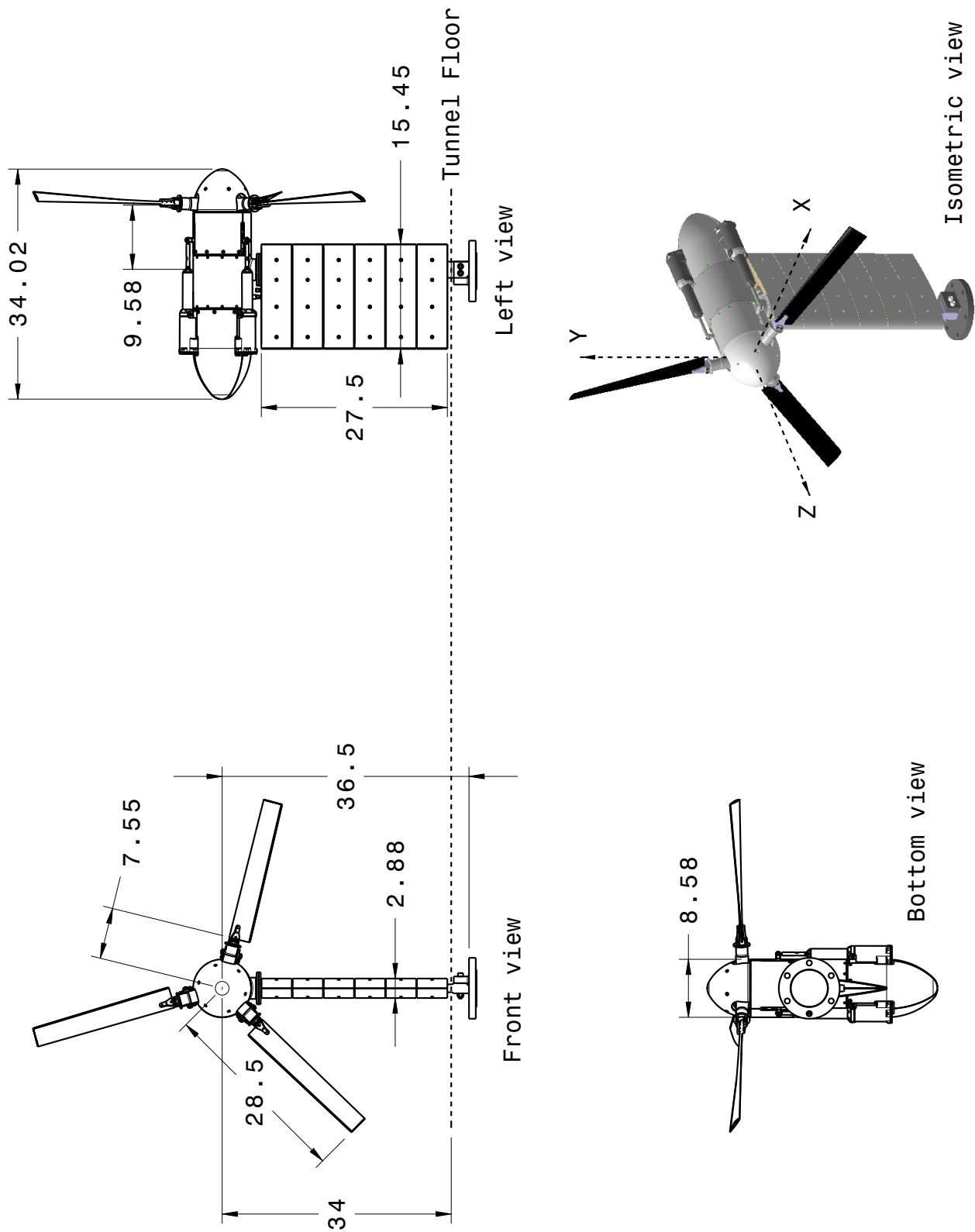


Figure 4.2: Maryland Tiltrotor Rig; all dimensions in inches.

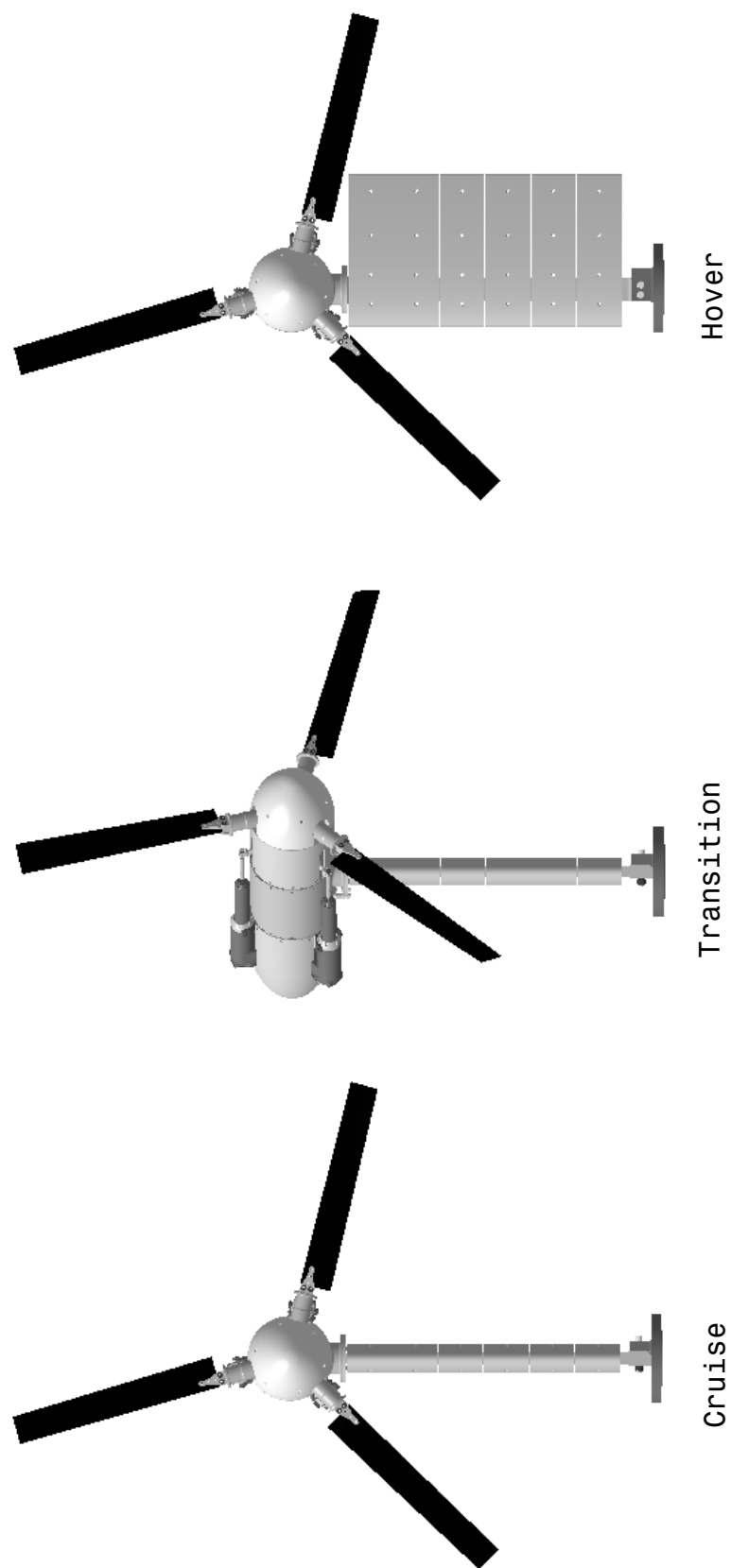


Figure 4.3: Maryland Tiltrotor Rig in cruise, transition, and hover regimes. Freewheel state is same configuration as cruise.

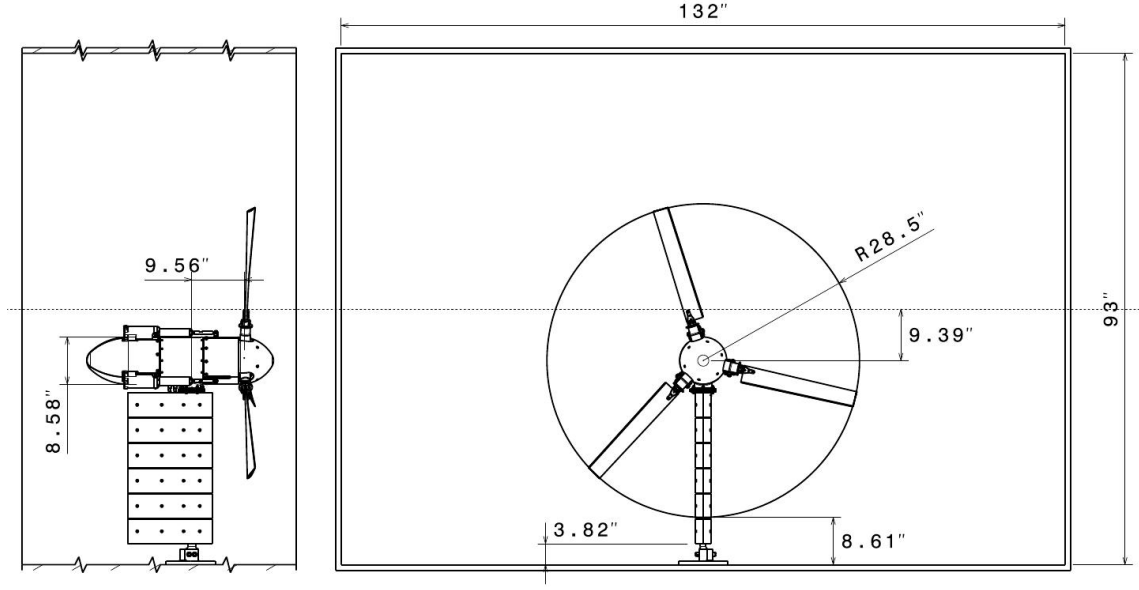


Figure 4.4: MTR designed for the Glenn L. Martin wind tunnel test section.

The MTR is a research rig with interchangeable wing spars, rotor hubs, and rotor blades. The baseline configuration of the MTR has a rectangular, solid aluminum wing spar and an XV-15-like gimbaled hub. This section will describe the major features of the baseline rig starting from the base of the model to the rotor hub.

4.2 Wing Assembly

The wing assembly is shown in Figure 4.5. The wing is straight (a departure from the XV-15 and V-22 but similar to the V-280), untwisted, and has a 18% thickness to chord (t/c) ratio with a NACA 0018 profile. With the current state-of-the-art thickness being 20 – 23% and the ultimate target being 14%, a thickness of 18% was deemed an appropriate intermediate baseline.

The primary requirement for the wing assembly is to allow for interchangeable spars, which necessitated easily removable wing ribs and fairings, shown in Figure 4.6. The wing ribs and fairings were designed to be non-structural, so while they add mass, they minimally affect the stiffness of the wing. The wing also provides a channel for power cables and coolant hoses and instrumentation wires to run to the pylon. The segmented fairings are made with a foam core wrapped in fiberglass skin. Slots are cut into the foam for the ribs and aluminum anchors are placed in locations for screws shown in Figure 4.7. The aluminum anchors in the fiberglass are threaded so screws can keep the two halves of the fairings together. The ribs and fairing fasteners are listed in Table 4.2. The ribs to spar connection uses two long screws with barrel nuts on the other side to clamp the ribs to the spar. Attention must be given to the bored holes on the ribs: one side is bored a little deeper for the barrel nuts to sit in, the other side is slightly shallower for the screw heads.

Table 4.2: **Wing assembly fasteners.**

Connection	Screw Type	Tool Type
Spacer to spar	10-32 3/8" flat cap	1/8" Allen key
Ribs to spar	10-32 2.25" socket cap	5/32" Allen key
	10-32 barrel nut	3/16" Allen key
Ribs to ribs (forward 3 holes)	10-32 5/8" socket cap	5/32" Allen key
Ribs to ribs (trailing edge)	10-32 1/2" socket cap	5/32" Allen key
Fairings to ribs (forward 3 holes)	10-32 3/8" flat cap	1/8" Allen key
Fairings to ribs (trailing edge)	10-32 1/4" flat cap	1/8" Allen key
Fairings to fairings (leading edge)	6-32 3/4" socket cap	7/64" Allen key
Fairings to fairings (trailing edge)	6-32 5/16" socket cap	7/64" Allen key

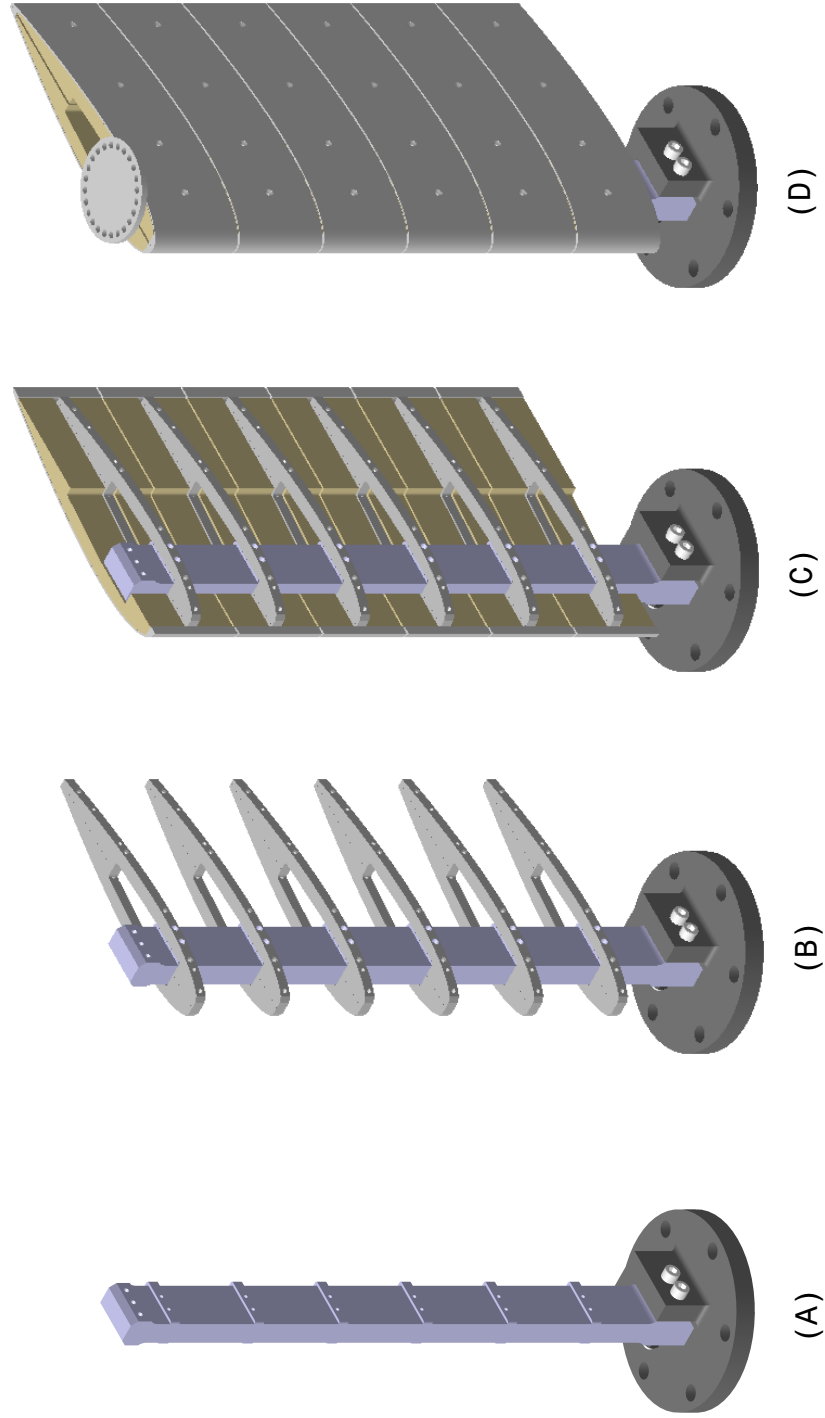


Figure 4.5: Maryland Tiltrotor Rig Wing Assembly: (A) Baseplate and wing spar with spacers (B) Wing ribs attached (C) Segmented fairings attached on right side (D) Fairings completed and coupling plate attached on spar tip.

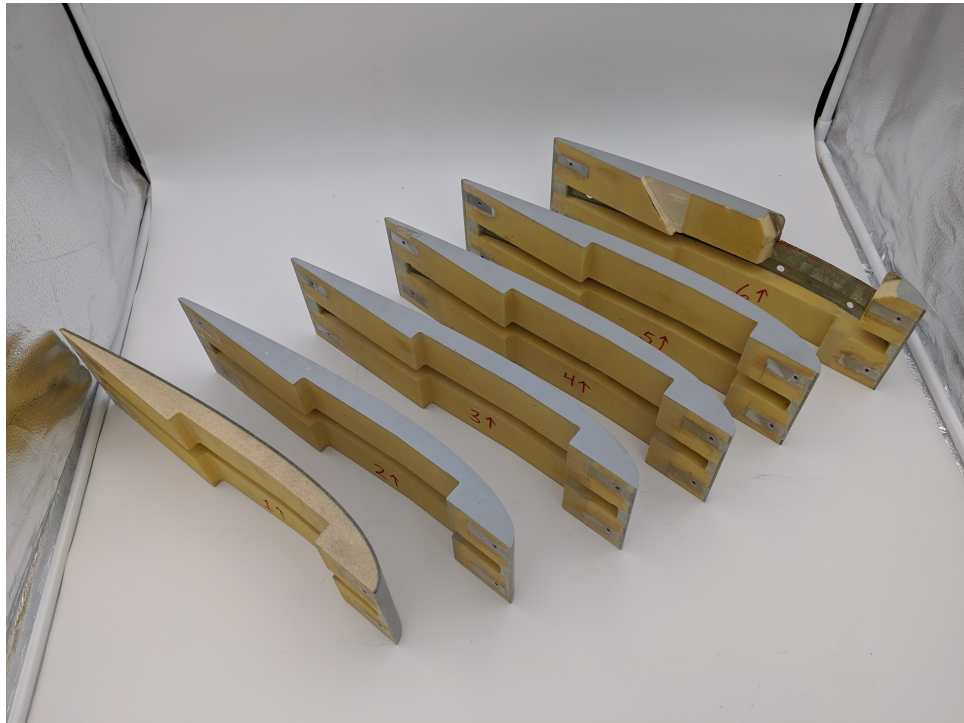


(a) Wing fairings: left to right corresponding to bottom to top of model.

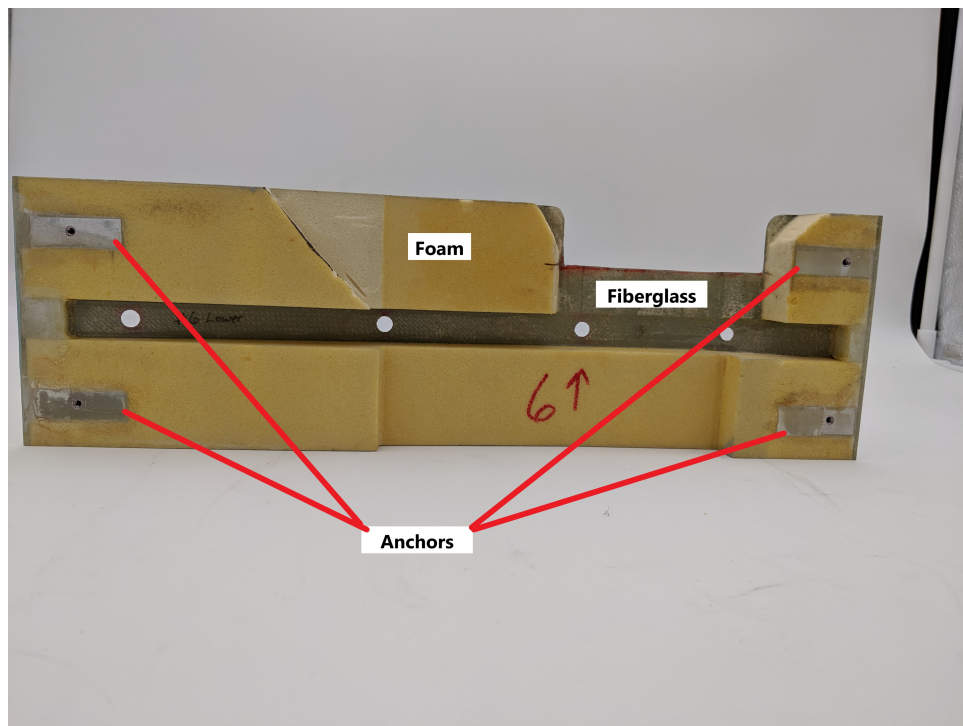


(b) Wing assembly ribs: left to right corresponding to bottom to top of model.

Figure 4.6: Wing assembly fairings and ribs.



(a) Half the wing fairing; the top fairing is cut to make clearance for cables and coupling plates.



(b) Top wing fairing with materials labeled.

Figure 4.7: Wing fairing details.

The baseline wing spar was designed to the Froude-scale frequencies of the XV-15 wing. A comparison of different model frequencies is shown in Table 4.3. The resulting MTR frequencies are somewhat lower than the other models; however, this leaves some clearance for lighter spars in the future. The baseline solid aluminum spar is heavy. Dimensions of the spar can be found in the appendix of Technical Drawings. To facilitate the interchangeability of the spar, spacers are used to bridge the gap between the spar and the wing ribs as shown in Figure 4.8. For future spars with different dimensions, new spacers must be fabricated to match them.

Table 4.3: **Wing-pylon frequencies normalized with cruise RPM; MTR RPM shown is Froude-scale RPM for flutter tests.**

Full-scale	V-22	XV-15	Bell 25 ft Model	Bell M301 Model	MTR
				1/8.89 Froude	1/5.26 Froude
Radius R ft	19	12.5	12.5	1.4	2.375
Cruise RPM	333	517	458	1366	1050
Beam per rev	0.53	0.45	0.42	0.38	0.29
Chord per rev	0.80	0.86	0.70	0.66	0.53
Torsion per rev	1.04	1.07	1.30	1.36	1.06

The MTR steel baseplate, which clamps the wing spar, is bolted to the top of the tunnel post. Two large shoulder bolts hold the wing spar to an L-shaped support on the baseplate.

The spar tip is attached to a coupling plate using three 1/4-inch diameter shoulder bolts. The coupling plate is the connection between the wing spar and the pylon, as shown in Fig. 4.9. Two guide pins on the spar coupling plate slide into

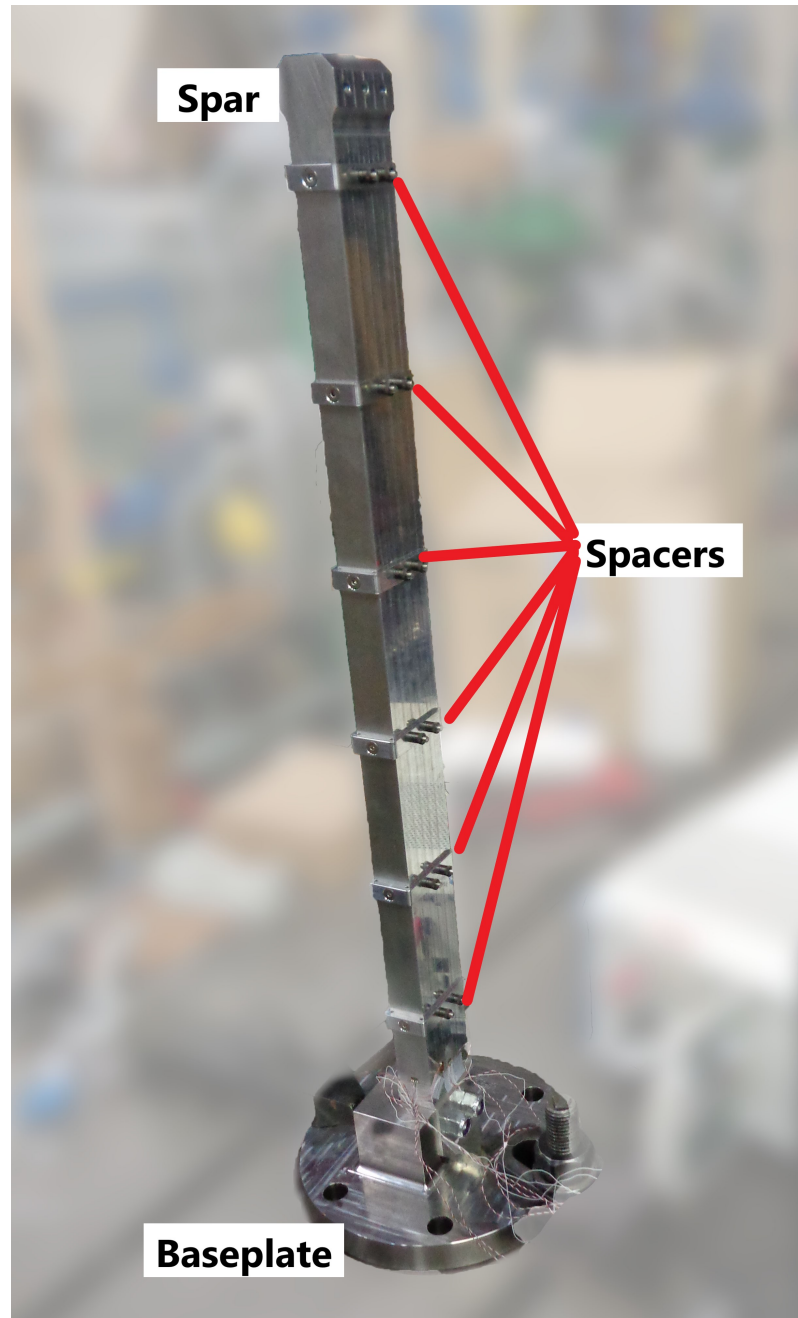


Figure 4.8: Spacers used to fill gap between wing spar and wing ribs.

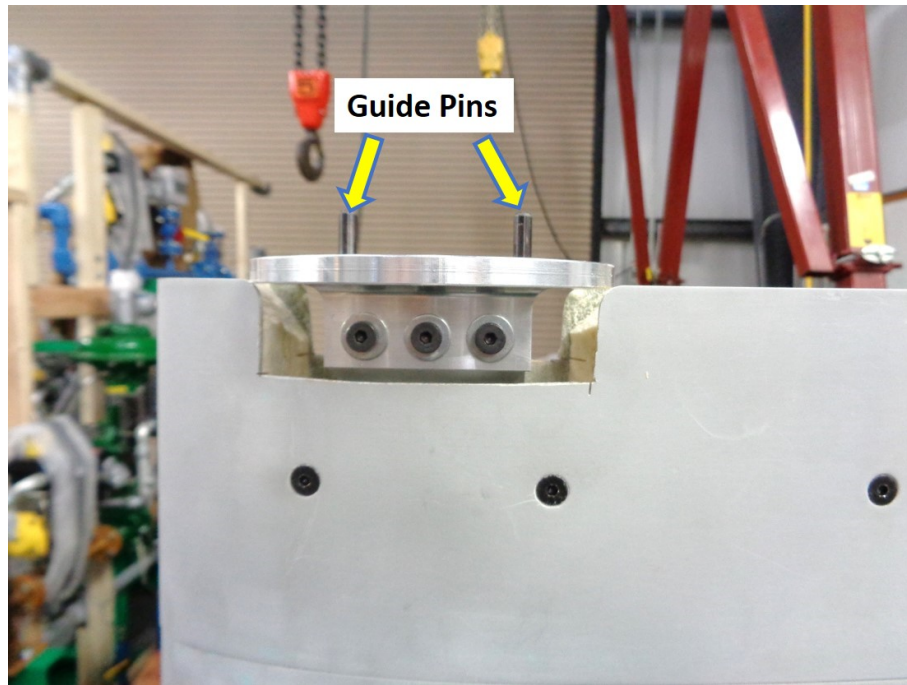
a slot in the pylon-side coupling plate to assist in alignment of the holes, shown in Fig. 4.9(a). The spar-side coupling plate has 24 holes spaced 15° apart while the pylon-side plate has 18 holes spaced 20° apart. This verniered design allows for 5° of static tilt increment of the pylon from 0° to 90° with exactly six shoulder bolts clamping the two plates together in any setting. This coupling plate connection is made deliberately rigid and does not allow for free-play or stiffness adjustments.

4.3 Pylon Assembly

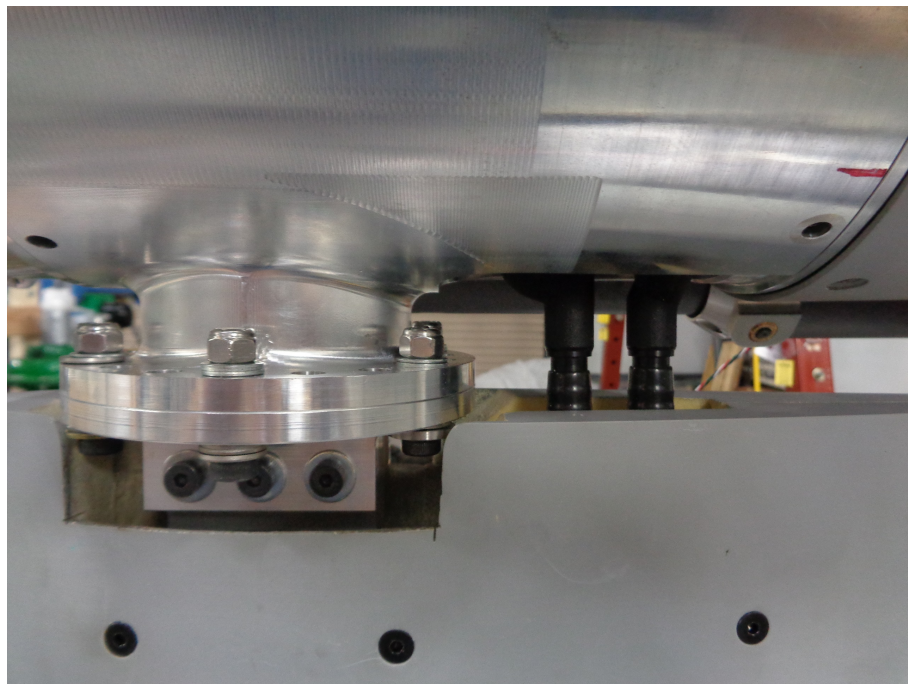
The pylon is a housing that contains the rotor shaft, a water-cooled electric motor, a 6-axis load cell, 3 electric linear actuators, and a 64-channel slip ring. Figure 4.10 shows the pylon mounted to the wing spar. The components can be seen in Figure 4.11.

The motor is water-cooled. The inlet hose is connected to city water supply in the wind tunnel balance room underneath the test section. The outlet hose runs straight down to a drain pipe in the ground on the balance room. The motor properties are shown in Table 4.4.

While the MTR was designed to have all primary load paths go through the load cell, the pylon does have a secondary load path that bypasses the load cell. The load path is described in Figure 4.12. The effect of the secondary load path on load cell calibration is minor, but explored clearly in the Calibration chapter later. Figure 4.12(a) shows the section view of the pylon assembly and labels the major components. Figure 4.12(b) shows the primary load paths that travel from



(a) Spar-side coupling plate bolted to spar.



(b) Pylon-side coupling plate attached in cruise configuration.

Figure 4.9: Wing-pylon connection through coupling plates.

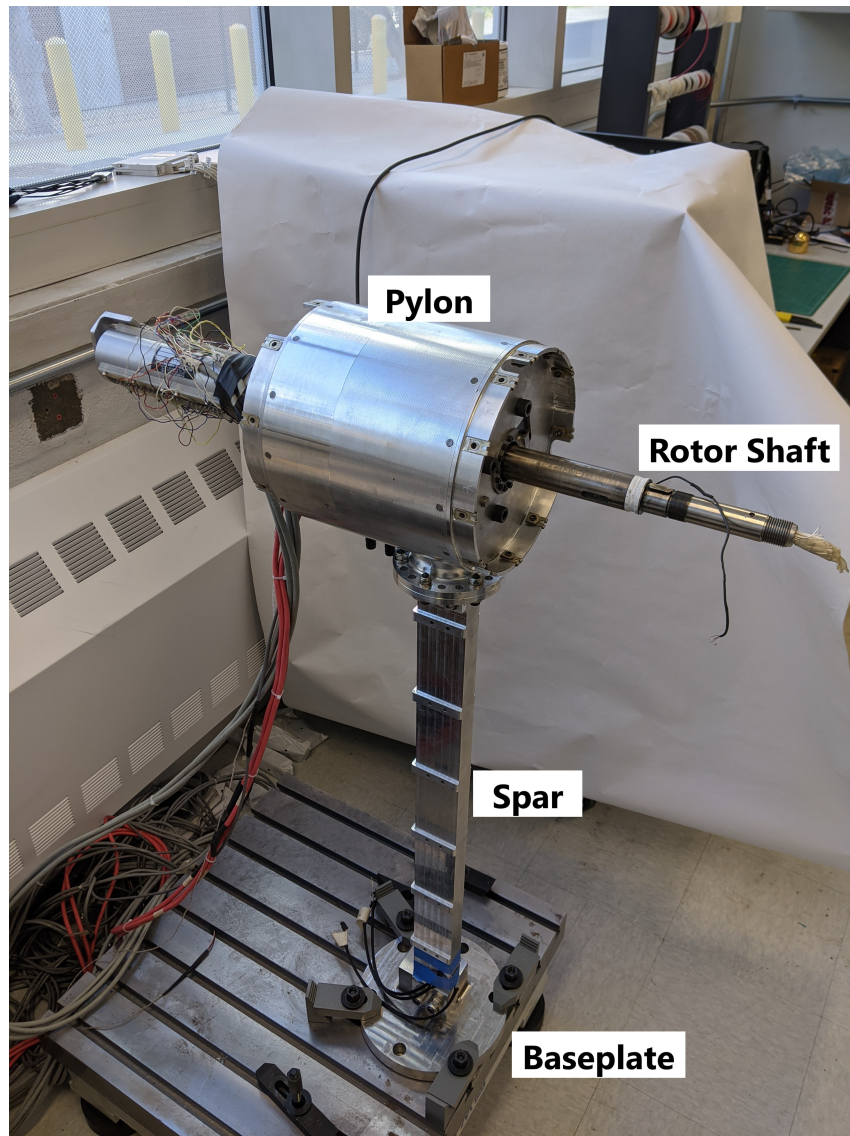
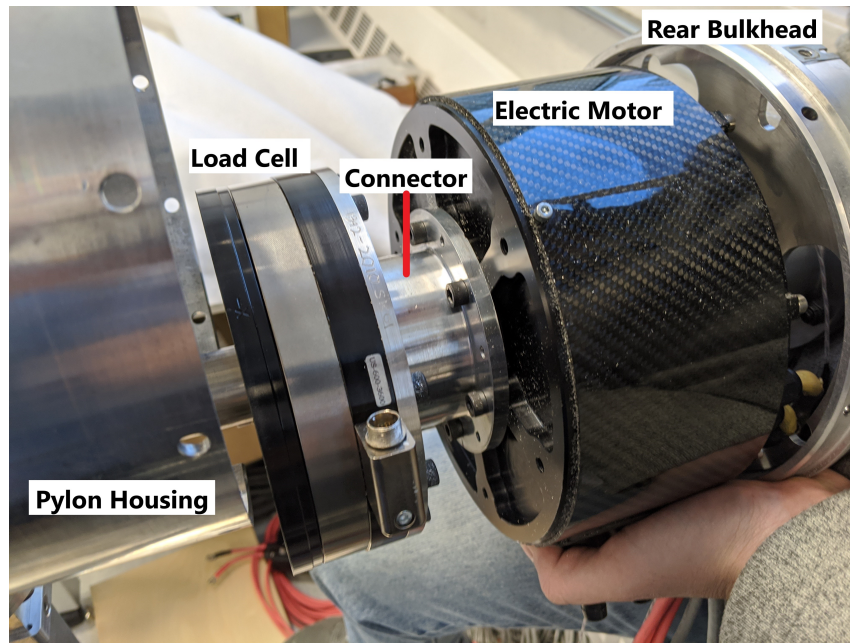
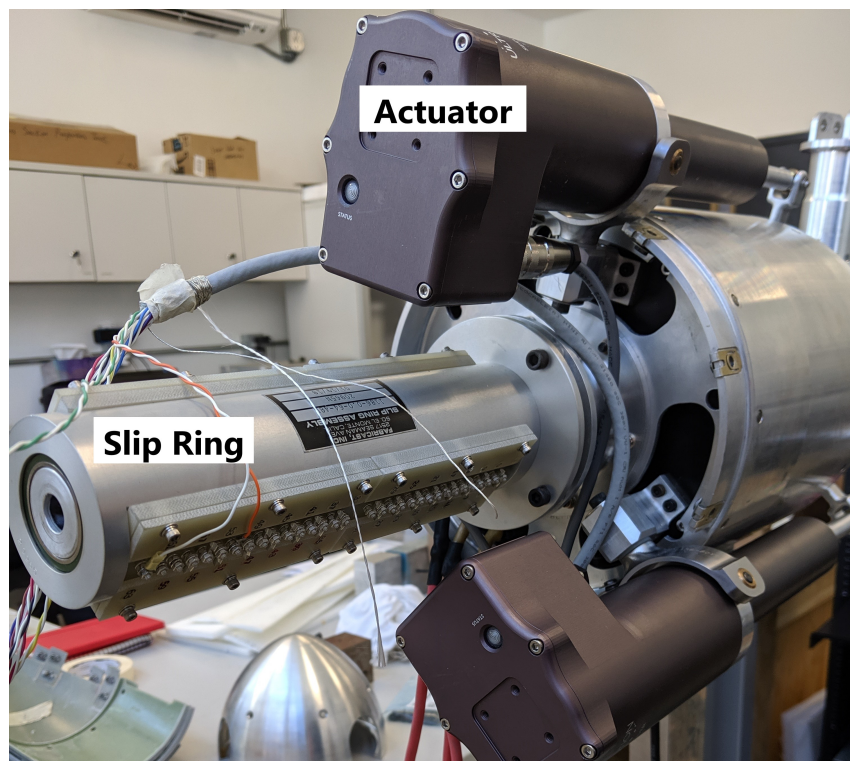


Figure 4.10: Pylon assembly with rotor components removed.



(a) Omega160 load cell and Plettenberg NOVA 30 electric motor.



(b) Fabricast 64-channel slip ring and Ultramotion A2 linear actuators.

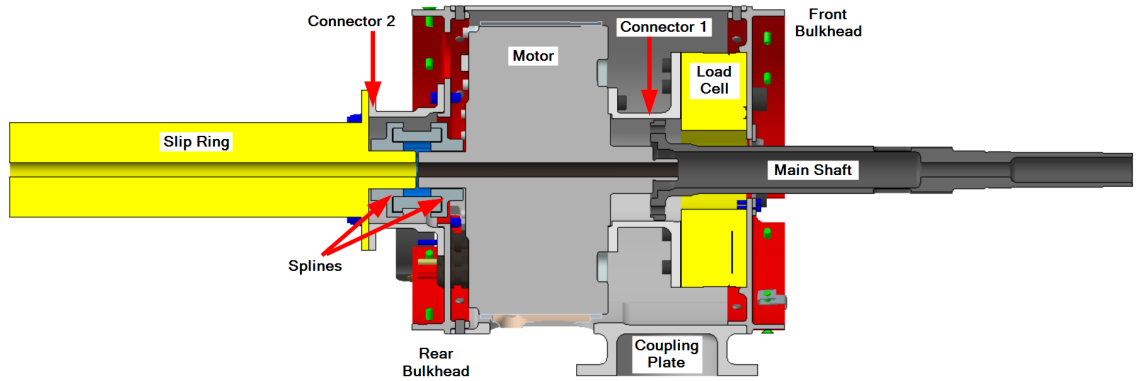
Figure 4.11: Pylon assembly components.

Table 4.4: **Electric motor specifications.**

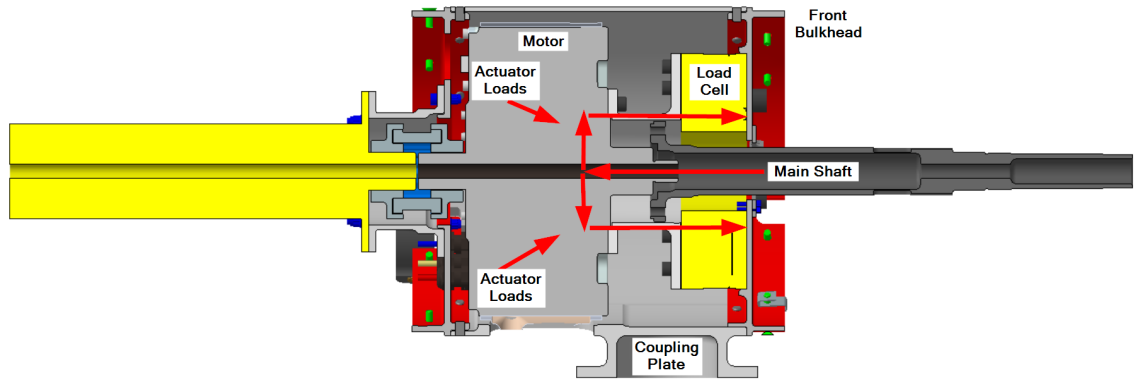
Parameter	Imperial Units	SI Units
Motor weight	15 lb	6.8 kg
Motor diameter	7.8 in	197 mm
Motor length	3.7 in	93 mm
Max power	40 hp	30 kW
Max torque	59 ft-lb	80 N-m
Max RPM	2500 – 5000	
Flow rate minimum	1 gal/min	63.09 cm ³ /s
Pressure range	0.5 – 2 bar 7.25 – 29 psi	50 – 200 kPa

the shaft and from the actuators to the motor. The loads progress into the load cell which is mounted to forward bulkhead, into the outer case and then down into the coupling plate and wing. Figure 4.12(c) shows the secondary load path taking some loads from the actuators and shaft and passing them into the slip ring through a flex coupling. The slip ring is mounted to the rear bulkhead, so the loads pass through the bulkhead and the outer case into the coupling plate and wing from the rear. Figure 4.13 shows the linear actuators mounted directly to the rear side of the electric motor. This was to ensure that all loads and moments affecting the actuators are also captured by the load cell. The motor does not directly touch the rear bulkhead at any location; the only points of contact on the motor are the mounts to the linear actuators, the motor shaft to flex coupling of slip ring, motor shaft to main rotor shaft, and the connector part to the load cell.

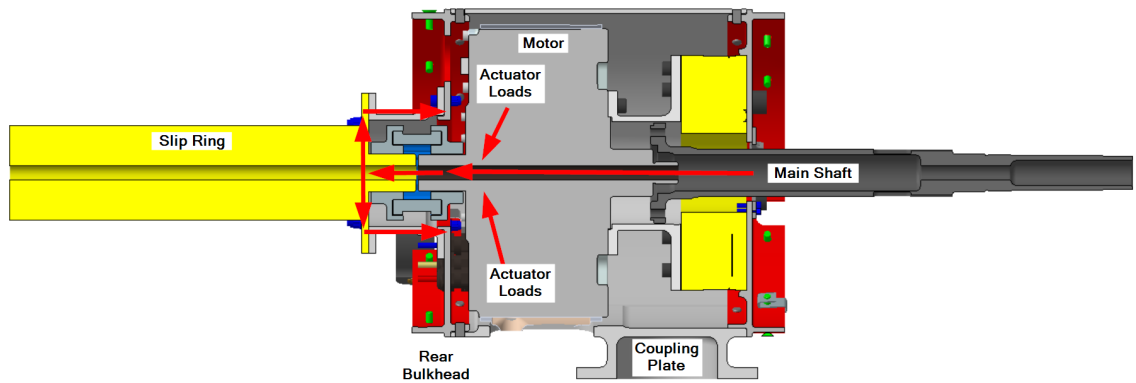
In terms of hub forces, thrust passes through the shaft into the motor and pushes or pulls on the load cell through the connector component, so it is not



(a) Pylon assembly section view of major components.

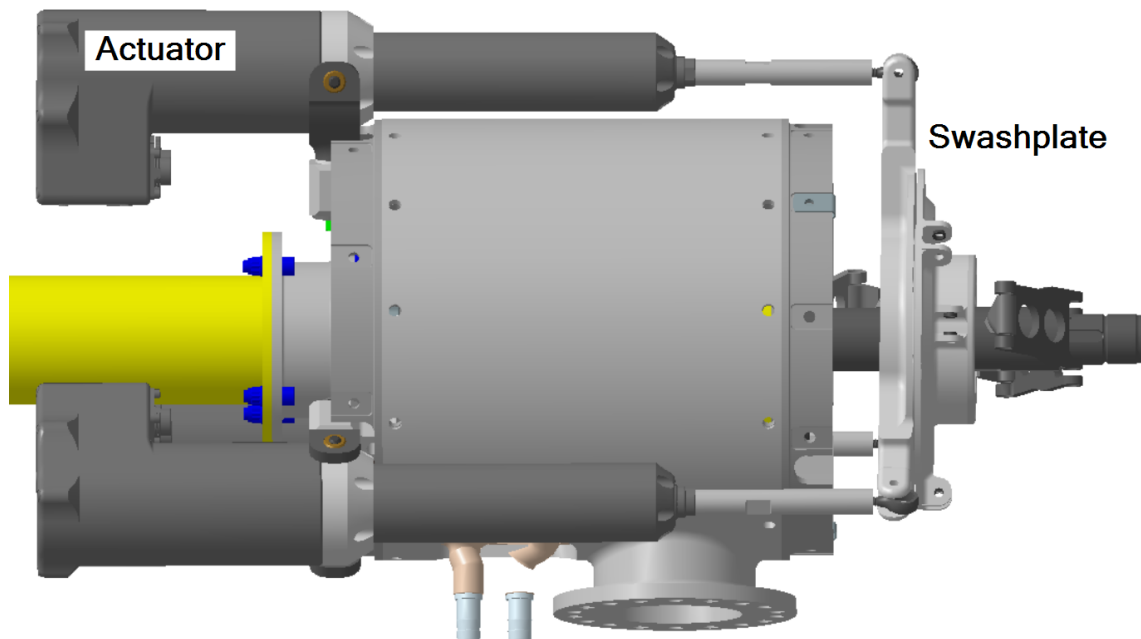


(b) Designed load path.

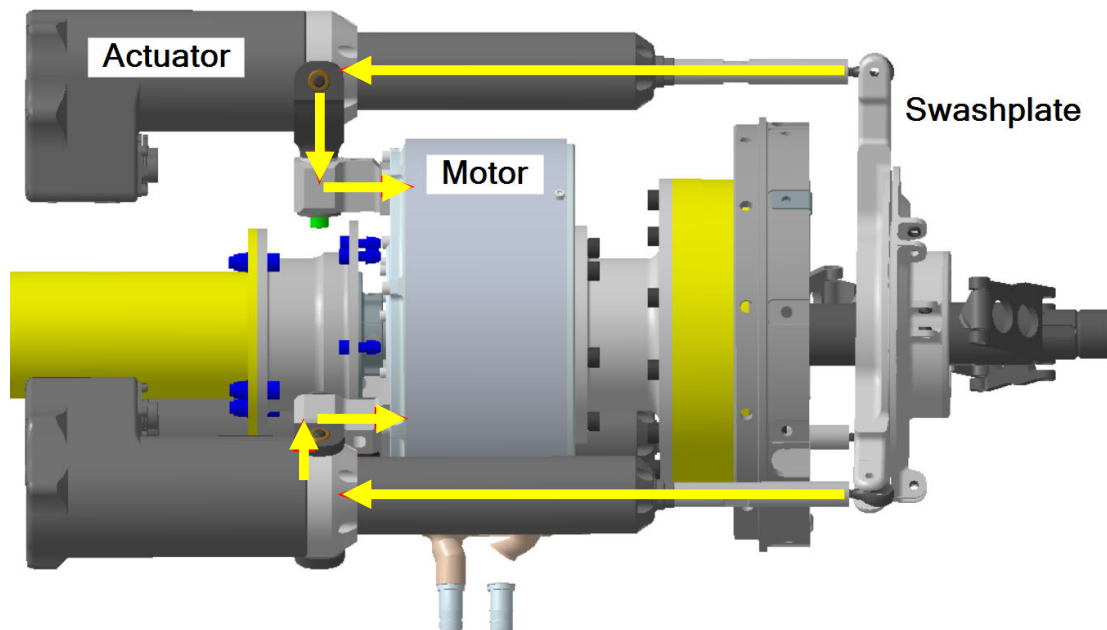


(c) Secondary load path.

Figure 4.12: Pylon assembly load paths.



(a) Linear actuators are attached at two points: the rod-end to swashplate and the mount to the motor.



(b) Load path from swashplate to motor; pylon housing and rear bulkhead removed for clarity.

Figure 4.13: Linear actuators connection and load path.

affected by the secondary load path. For the hub in-plane forces, they travel down the shaft and into the motor which wants to translate with the hub forces. Therefore, the in-plane hub forces do get transferred into the secondary load path and causes inaccurate measurements. For moments, torque is transferred through the shaft and into the motor which causes a torque on the connector and subsequently the load cell, so it is not affected by the secondary load path. For pitch and yaw hub moments, they travel from the shaft into the motor and the connector transfers the moments to the load cell. These moments are also affected by the secondary load path because while the flex coupling should act as a pinned support, in reality, some resistance to rotation is present. For pitch links, the control loads travel into the swashplate, through the actuators, and into the base of the actuators mounted to the rear face of the motor. Control loads are affected by the secondary load path. Only the thrust and torque are relatively unaffected by the secondary load path.

4.4 Electric Drive

The proprotor is driven directly by a Plettenberg NOVA 30 motor shown in Fig. 4.14. This is a three-phase, brushless DC, inrunner motor with water-cooling option. The motor controller, MST 140-200, is delivered with the motor and housed separately. The motor operates at 80 – 140 V (nominal 110 V), has a maximum speed of 5000 RPM, maximum torque of 80 Nm (59 ft-lbf), maximum power of 30 kW (40.2 h.p.) at 5000 RPM, max efficiency of 90% (including controller), diameter of 197 mm (7.76 inches), length of 93 mm (3.7 inches), and weight of 6.8 kg (15 lb). The

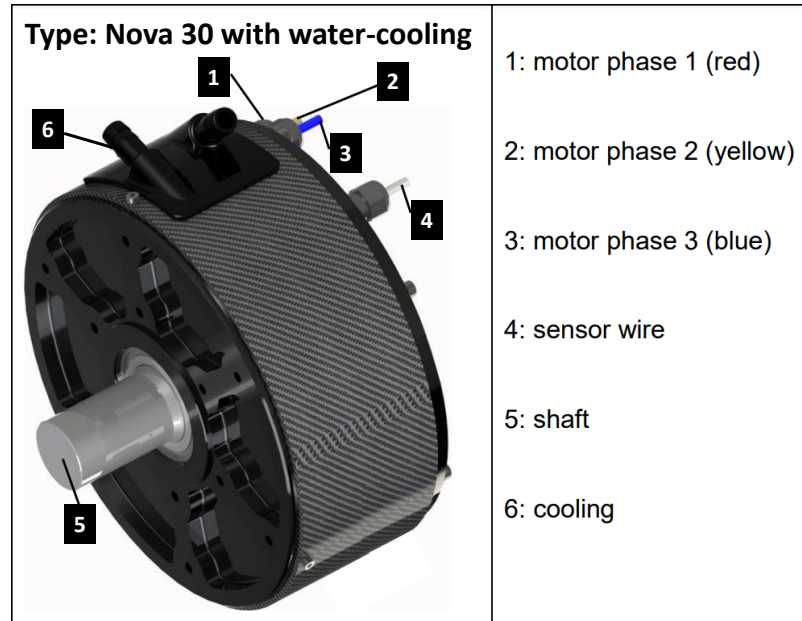


Figure 4.14: Features of Plettenberg NOVA 30 with water cooling.

motor controller is housed outside the pylon in the tunnel control room, connected to a DC power supply. It uses six step commutation, also known as trapezoidal commutation, in connection with three 120° hall sensors for communication. The motor controller allows freewheeling of the motor with no load in freewheeling mode by driving the motor to a zero-current state.

The flow of power from the tunnel to the motor controller is explained in Fig. 4.15, with the components listed in Table 4.5. The physical connections are shown in Fig. 4.16.

The rig draws power from the building. The GLMWT supplies 480 VAC/100A. This is equivalent to 340 VDC/100A, hence 34 kW of power. The MTR is expected to use 15 kW in the powered cruise condition.

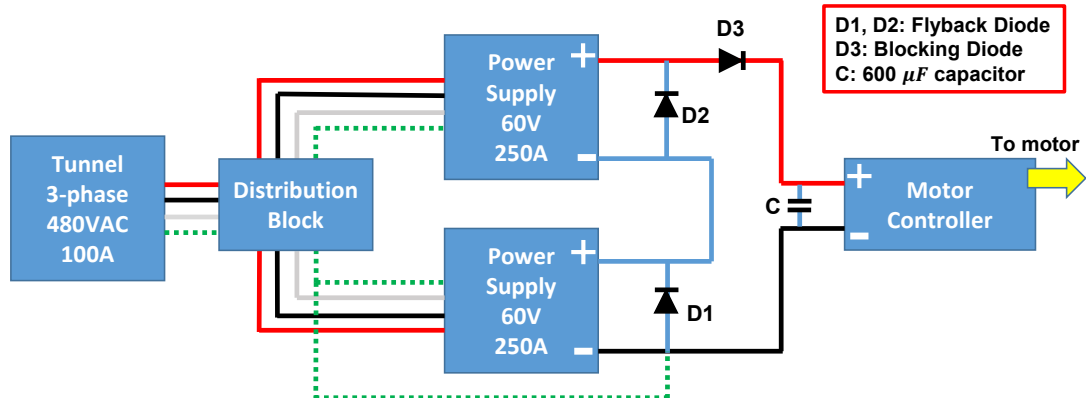


Figure 4.15: Motor power diagram from wind tunnel supply to DC power supplies to motor controller; green dotted wire is ground.

Table 4.5: **Power electronics components.**

Component	Description	Quantity
AC to DC Converter	Sorensen SGX Series 60V/250A	2
Flyback Diode	Vishay UFL130FA60	1
Blocking Diode	Vishay VSKE236/16PBF	1
DC-Link Capacitor	Cornell Dubilier 947D601K901DCRSN	1
Heatsink	C&H Technology CHEHK18-180MM	1

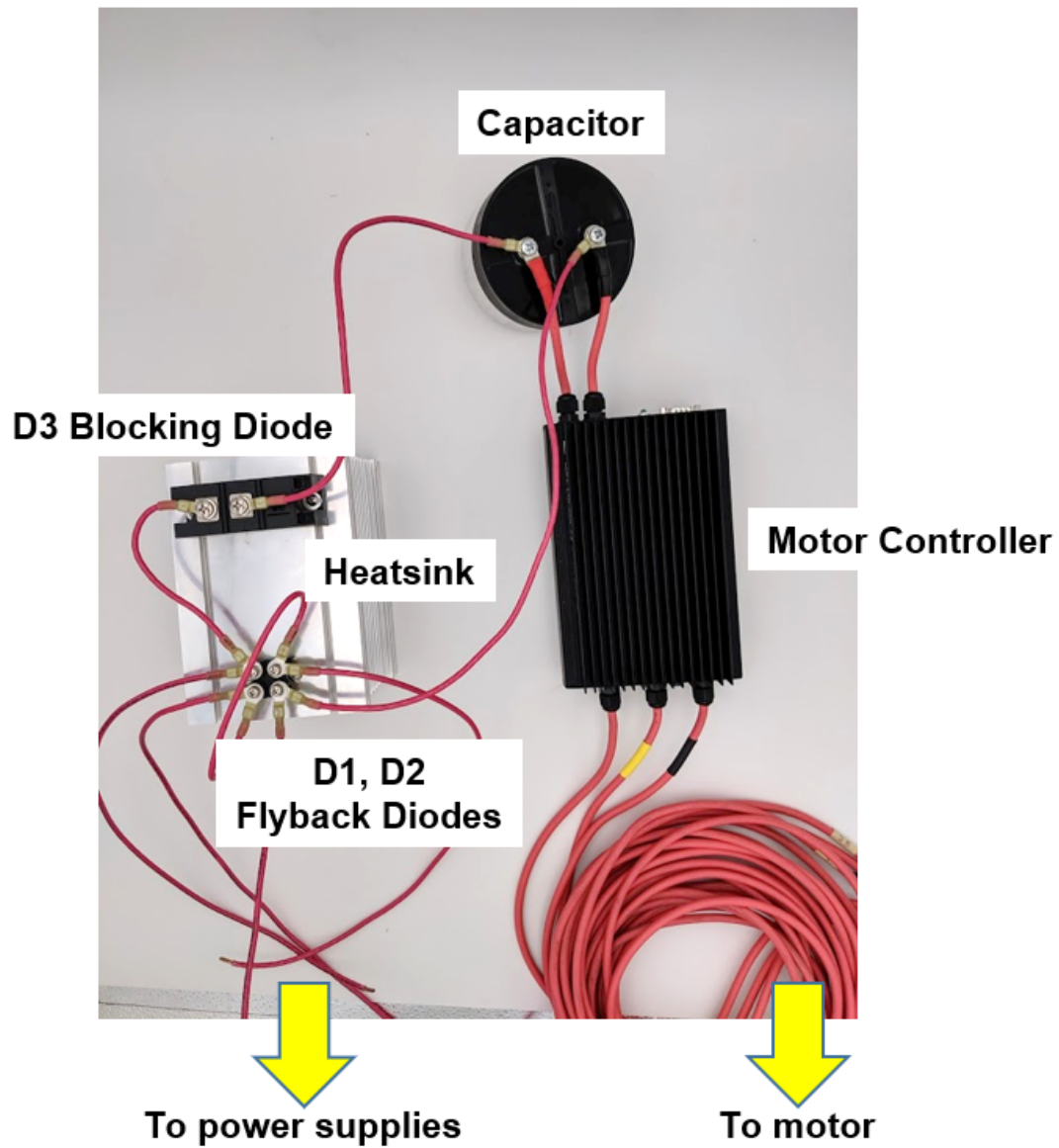


Figure 4.16: Motor power connections from power supplies to motor controller.

From the building power to the rotor, all components are part of the MTR. The building power is first connected to two Sorensen SGX Series 60V/250A DC power supplies to convert AC to DC. From Fig. 4.15, the 3-phase power from the building is run to a distribution block with one phase to each wire and one ground wire. The distribution block splits each wire into two so each power supply gets its own set of power cables. On the output side of the power supplies, they are shown to be connected in series to add the voltage up to 120V/250A. This setup requires flyback diodes, D1 and D2, in case only one power supply is used. The D1 and D2 diodes are two separate diodes but physically placed on the same component. A blocking diode, D3, is implemented as well to protect the power supplies in case of reverse current from the motor in a freewheeling condition. All diodes are mounted to a heat sink with thermal resistance of $0.12\text{ }^{\circ}\text{C/W}$ to assist in power dissipation. This means the heat sink will have a change in temperature of 0.12°C per watt of heat dissipated. A $600\text{ }\mu\text{F}$ capacitor is connected to smooth out any voltage spikes in the power supplies. The capacitor has a rated voltage of 900 VDC and a current of 102 A at an ambient temperature of 55°C . The DC power is connected to the motor controller which uses Six-Step Commutation in connection with three 120° hall sensors for the communication to the motor. The node at the bottom of D1 in Fig. 4.15 is also connected to tunnel ground (green dotted line) which eliminated some noise seen in the sensors on the MTR.

As of this thesis, the cables running from the power supplies to the motor controller are 10 AWG, which limit the maximum current draw to 40 amps, limiting the total power for the motor to 4.8 kW. In Fig. 4.15, all cables to the right of the

power supply and to the left of the capacitor are 10 AWG. Correspondingly, in Fig. 4.16, all cables on the left hand side of the image leading to the capacitor are 10 AWG. By upgrading these cables to 2 AWG, the max current draw would increase to 130 amps, and therefore, the total power would increase to 15.6 kW. The cables would need to be even thicker to allow for higher current and more power. Thicker cables require larger lugs at the cable ends, so the current diodes may need to be replaced with larger diodes due to limited space at the diode terminals. The main focus for these connections is to screw in the cable lugs without any cross-contact with other lugs.

Specifications of the motor controller are provided in Table 4.6.

The motor has a brake that can be switched on in an emergency; it is off by default. The emergency brake switch is a large red button on the graphical user interface which instantly brings the brake voltage to 2V out of a maximum of 5V. To

Table 4.6: **MST140-200 motor controller specifications.**

Feature	Unit	Value
Weight	kg (lb)	1.3 (2.87)
Max Continuous Power	kW (hp)	30 (40.23)
Max Short Term Power	kW (hp)	39 (52.3)
Supply Voltage	V	30-140
Max Continuous Current @ 25°C Ambient Temp	A	220
Max Continuous Current @ 50°C Ambient Temp	A	180
Max Continuous Current @ 75°C Ambient Temp	A	125
Throttle Voltage	V	0-5
Brake Voltage	V	0-5
Accessories Voltage	V	5

prevent damage from a sudden arrest, the brake is not set to the maximum voltage. A physical red button is also wired to the DAQ that will trigger the brake to 2V when pressed as a redundant safety measure.

The motor controller has an RS-232 terminal that outputs motor parameters such as motor temperature, controller temperature, motor voltage and current, and rotational speed. The motor and controller temperatures are monitored throughout the test and testing will pause if the temperatures exceed 140°F or 60°C. High motor temperatures can affect load cell measurements as they are both encased within the pylon without air circulation.

4.5 Rotor Assembly

The rotor assembly consists of the shaft, the hub, blade retention, pitch mechanism, and shaft connections. The design of the MTR rotor allows for different hubs to be installed on the main shaft. The rotor hub is located 34%R ahead of the wing spar which is nominally considered to be the wing elastic axis. Figure 4.17 shows a section view of the gimballed hub model. The components of the baseline gimballed hub are shown in Figure 4.18.

The gimbal is a universal joint, for which the details are explained via CAD in Fig. 4.19. It is inspired by the XV-15 hub. The spider component, in Fig. 4.19(a), is the gimbal that allows the hub to have two unconstrained rotational degrees of freedom relative to the main shaft. A V-22-like constant velocity joint was also pursued during the design phase but proved difficult to achieve within the constraint

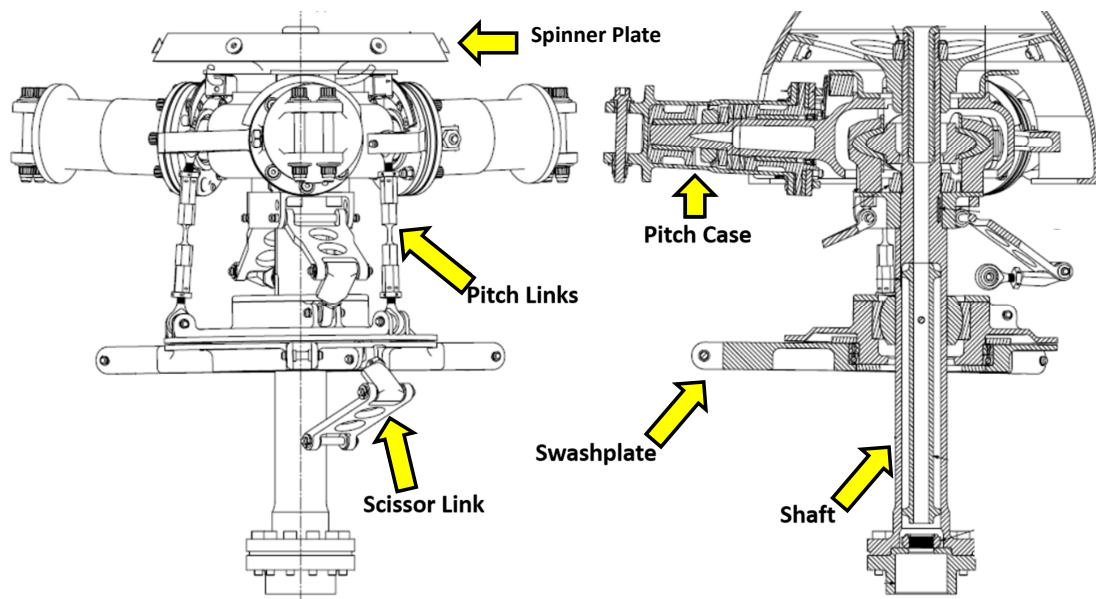


Figure 4.17: MTR gimballed hub section view.

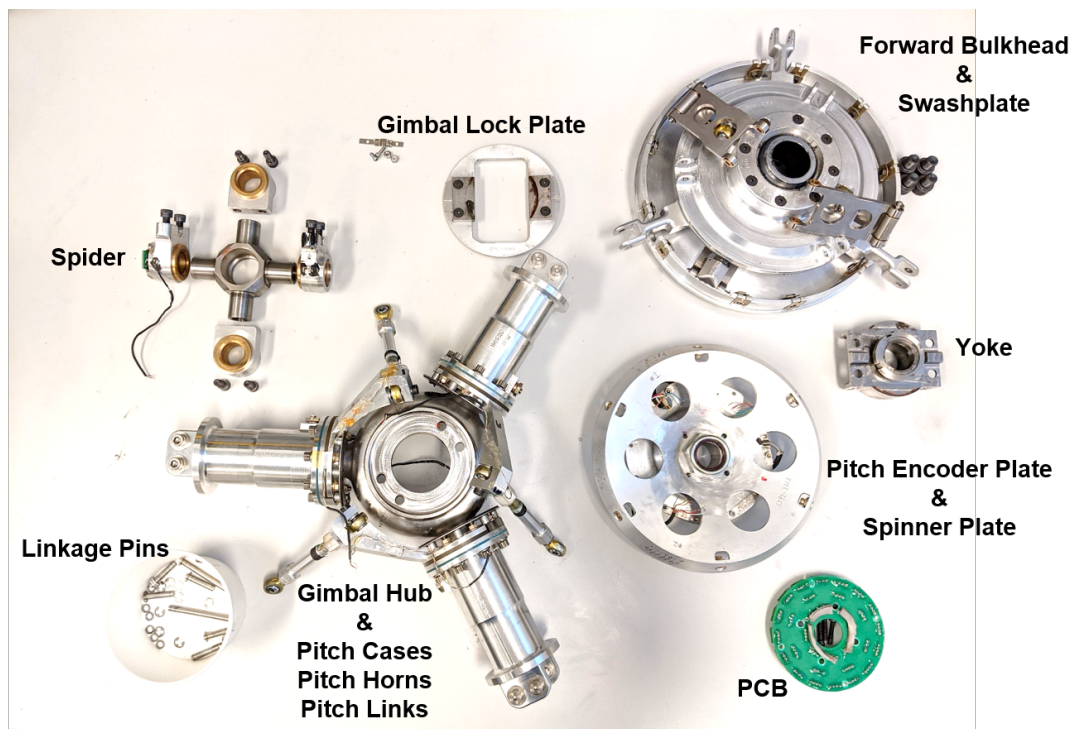
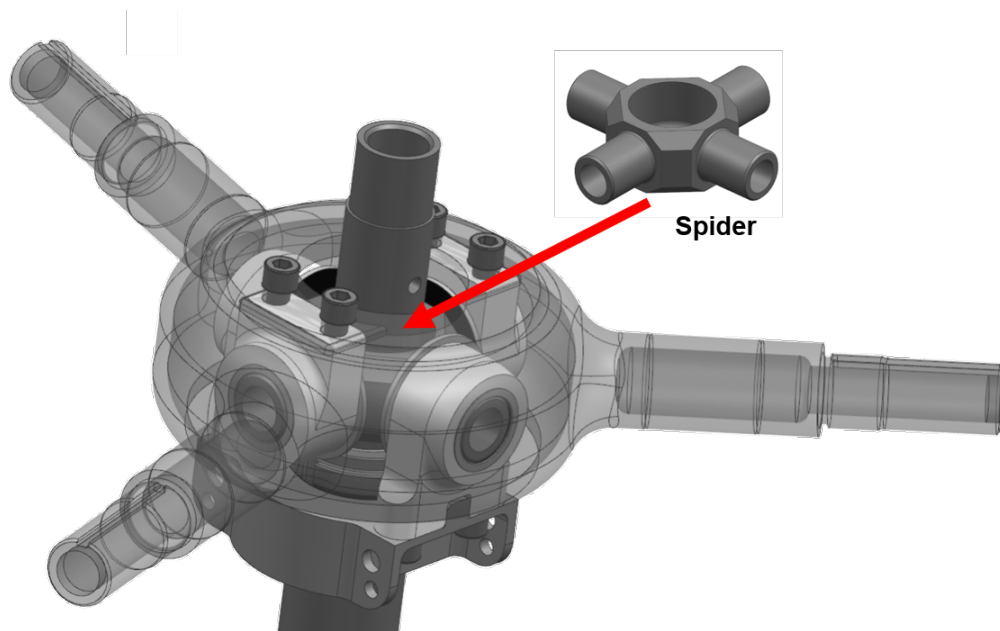


Figure 4.18: Rotor assembly parts.

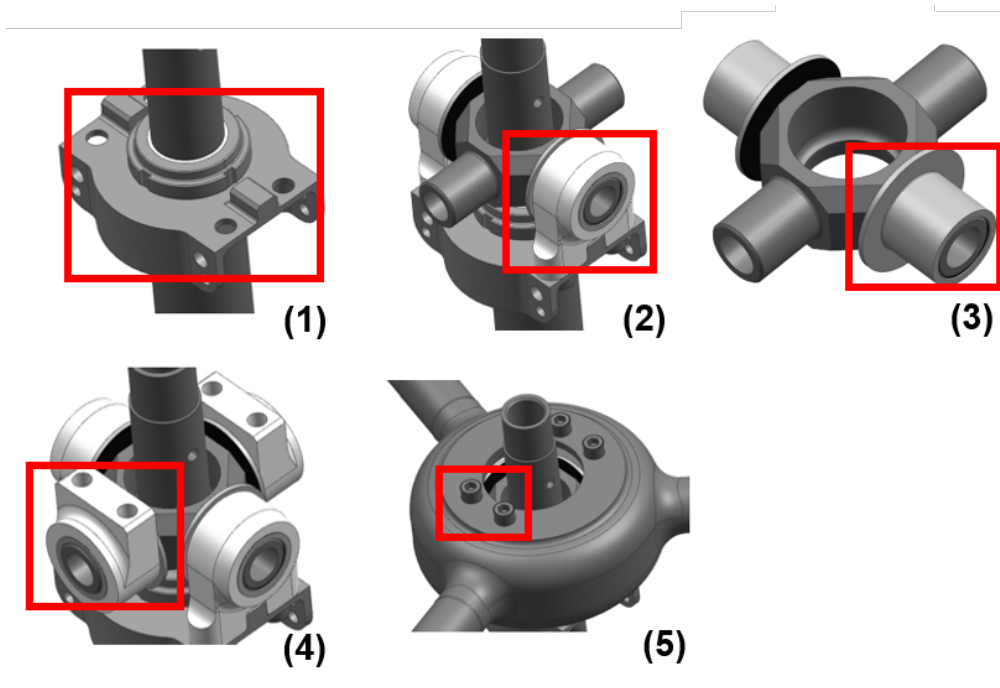
of resources. Of note, a universal joint will produce a 2/rev oscillation in rotor speed if the hub is not perpendicular to the main shaft, and avoiding this 2/rev frequency was important in the rest of the pylon design due to historical reasons. Also, by design, no gimbal spring was provided to zero out moment transmission. Note, however, the shaft was still designed to carry moments due to the interchangeable nature of the hub where a hingeless rotor will be mounted in the future. The hub has a built-in 2° pre-cone angle to relieve hover stresses. Figure 4.19(b) shows how the torque and loads are transferred from and to the shaft.

For a powered condition, as the shaft is rotated by the motor, the torque is passed through the yoke (1). The yoke is locked to shaft by a spanner nut which requires a spanner wrench to tighten. Two bearing housings (2) are bolted to the yoke which transfers torque into the spider component through journal bearings (3). Another pair of bearing housings (4) are slipped onto the spider component and then bolted to the hub housing (5). This is how torque is transferred from the motor to the hub. Loads from the hub travel to the motor in the reverse direction. By design, there is no undersling.

The hub holds three blades ($N_b = 3$) and allows for configuration changes between gimballed and hingeless as long as the number of blades remain the same. The pitch links are on the trailing-edge side of the blades to provide a negative pitch-flap coupling for the gimballed rotor. A flap up about the gimbal by an angle $\Delta\beta$ would pitch the blades up by an angle $\Delta\theta$, so $\Delta\beta/\Delta\theta$ is positive and quantified by an angle δ_3 such that $\Delta\beta/\Delta\theta = -\tan\delta_3$. The MTR has $\delta_3 = -16.5^\circ$ by construction. The pitch links are actuated by a conventional swashplate. Figures 4.20 and 4.21 show



(a) Gimballed hub CAD and spider component.



(b) Description of load path in hub assembly: (1) Through yoke on shaft, (2) Bearing housings on one axis of spider, (3) Journal bearings in housing, (4) Through spider into journal bearing housings on second axis, (5) Through bolts from bearing housings to hub.

Figure 4.19: MTR gimballed hub.

the individual components of the hub and swashplate while Figures 4.22 and 4.23 show the assembled hub. With this construction, the swashplate motion is limited below by the load cell adapter plate seen on the left of Figure 4.23 and by the hub assembly above. The swashplate motion translates to a collective range from -2° to 60° for a total of 62° . The swashplate cyclic range is limited to $\pm 16^\circ$ by the spherical bearing in the swashplate. The gimbal flapping is physically limited to $\pm 8^\circ$ by the spinner plate.

A specially-cut aluminum plate is used to lock the gimbal such that the hub plane remains perpendicular to the rotor shaft during operation. This is shown in Fig. 4.24.

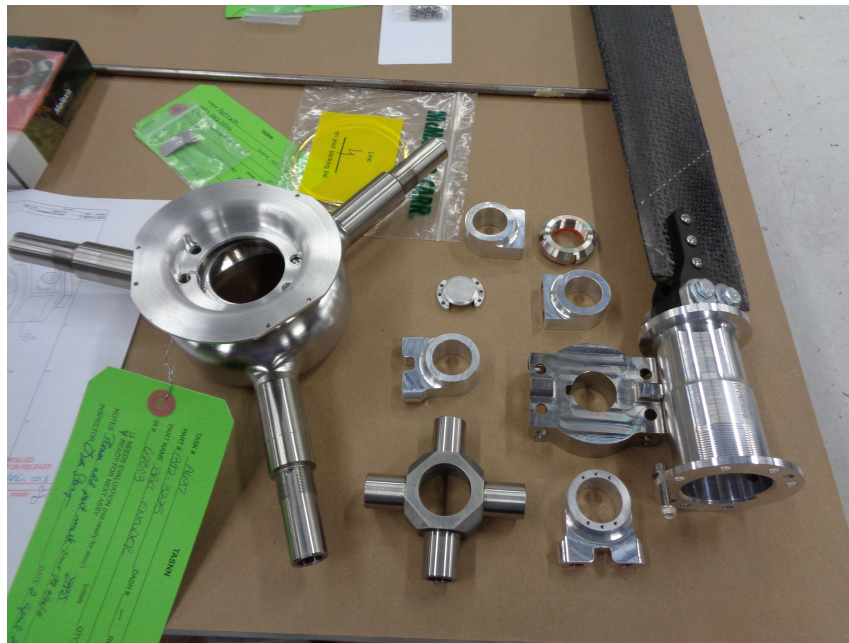


Figure 4.20: Individual components of the gimballed hub.



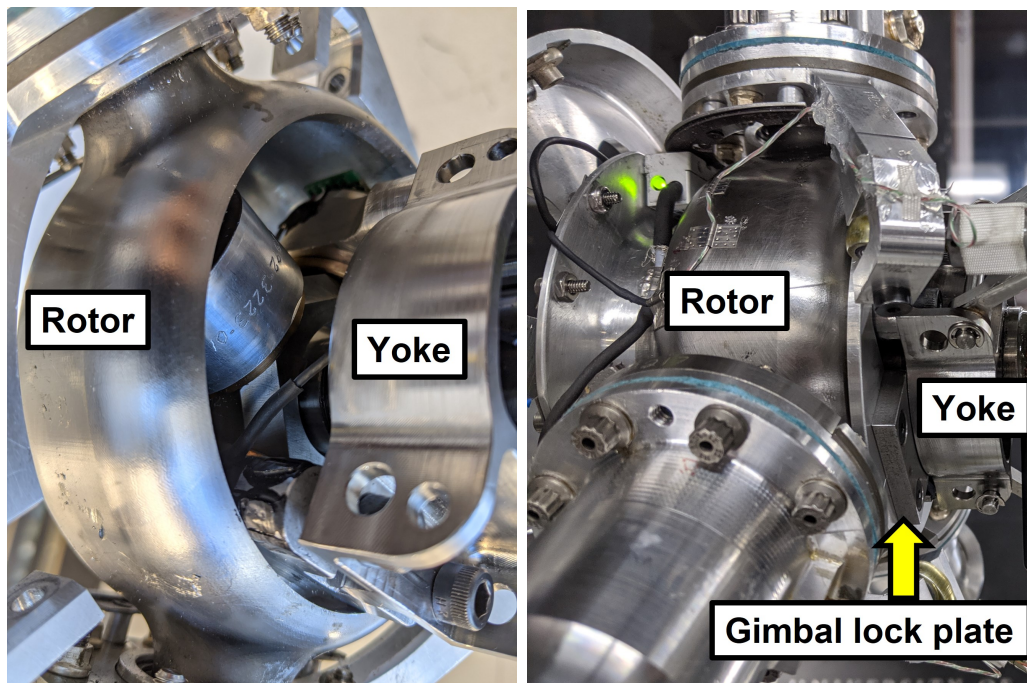
Figure 4.21: Swashplate and yoke mounted on rotor shaft.



Figure 4.22: Pitch case and gimbaled hub component.



Figure 4.23: Rotor assembly mounted to pylon.



(a) Gimbal free.

(b) Gimbal locked.

Figure 4.24: A gimbal lock plate is installed between the yoke and rotor to fix the gimbal.

Table 4.7: **MTR weight breakdown.**

Assembly	Component	Weight (lb)
Wing	Spar	8.95
	Spar Coupling Plate	0.68
	Aluminum Ribs	5.50
	Foam Fairings	8.70
	Wing Total	23.83
Pylon	Slip Ring	4.5
	Linear Actuators	15
	Motor	15
	Load Cell	6
	Body & Fairings	16.36
	Pylon Total	56.84
Rotor	Hub	14.33
	Blades	2.04
	Rotor Total	16.37
Full Rig	Total	97.04

4.6 Weights

Table 4.7 shows the weight breakdown of the MTR. The relatively heavy pylon mass (compared to a scaled XV-15 pylon) is driven by the motor, electric actuators, load cell, and slip ring, which are essential research components non-existent in the actual aircraft, hence impossible to scale. The sub-components in the hub such as the swashplate, hub yoke, spinner, and linkages were not measured individually, however, the pylon and hub assembly together have a total weight of 71.4 lb, and

subtracting the pylon assembly from the total gives the hub weight. The twisted proprotor blades [35] were measured to be about 308 grams or 0.68 lb, including the blade grip. Note how the hub is heavy, driven largely by the gimbal construction.

4.7 Measured Rig Characteristics

This section describes the frequency, damping, and mode shapes of relevant parts and assemblies, measured systematically with gradual build-up.

Frequencies of the interfacing post were obtained using ANSYS and shown in Table 4.8. The goal was to ensure the post was stiff and would not interfere with the frequency and stiffness of the MTR.

Table 4.8: **ANSYS results for interface post frequencies. /rev values shown for Froude and Mach-scale speeds.**

Harmonic	Frequency (Hz)	/rev 1050 RPM	/rev 2300 RPM
1st	315.2	18.0	8.2
2nd	315.5	18.0	8.2
3rd	1129.3	64.5	29.5
4th	1536.3	87.8	40.1
5th	1537.5	87.9	40.1

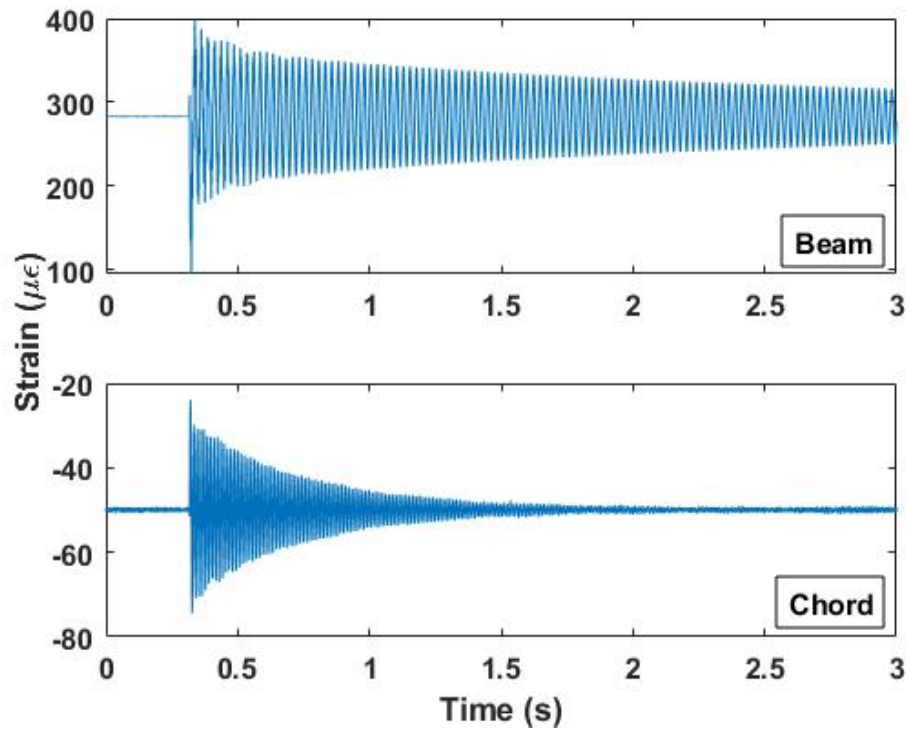
The next step was to bolt the baseline spar to the baseplate. Beam and chord bending responses were acquired. The time histories and frequency responses are shown in Figure 4.25. Data was collected from a set of strain gauges located at the base of the spar. Details of the strain gauges are found in Section 5.12.

Torsion response could not be easily detected with the spar alone, so a single rib was attached near the tip of the spar and used as an arm to excite the torsion mode. This configuration changed the spar inertia; therefore, all frequencies were re-acquired. Figure 4.26 shows the time histories and frequencies of the spar with rib. The beam mode frequency of 35.1 Hz is also detected in chord and torsion gauges. The chord response shows the first chord mode at 66.64 Hz but also detects small amplitudes of frequencies around 160 Hz, 200 Hz, and 250 Hz. The torsion response captures the torsion frequency at 200 Hz and another one at 250 Hz. Predicted frequencies were verified both using simple Rayleigh-Ritz (using shape function $\eta = (x/L)^2$ for beam and chord bending, $\eta = x/L$ for torsion), as well as detailed analysis with ANSYS. Results are shown in Table 4.9.

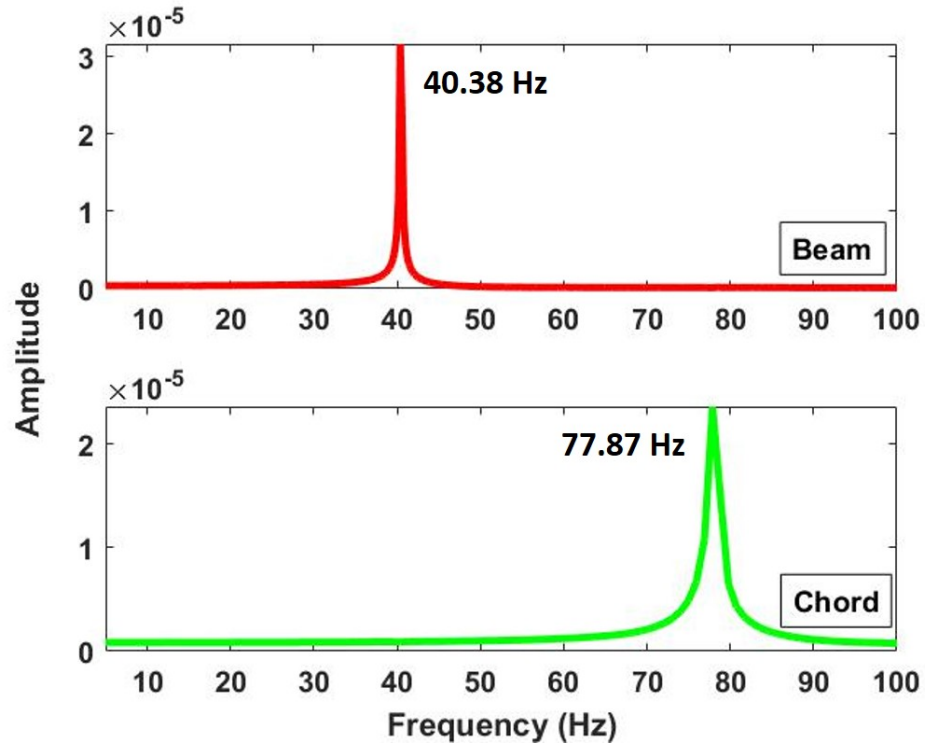
Table 4.9: **Comparison of spar frequencies for first beam, first chord, and first torsion modes.**

	Mode	Measured (Hz)	Analytical (Hz)	ANSYS (Hz)
Baseline Spar	Beam Bending	40.38	49.31	44.46
	Chord Bending	77.87	101.50	87.32
	Torsion	–	817.07	791.29
Spar With Rib	Beam Bending	35.1	40.82	38.53
	Chord Bending	66.64	83.21	74.71
	Torsion	199.90	248.65	223.97

With the rib attached to the spar, the measured frequencies decreased as expected due to additional mass. The beam frequency decreased from 40 Hz to 35 Hz while the chord frequency reduced from 78 Hz to 67 Hz. The torsion frequency,

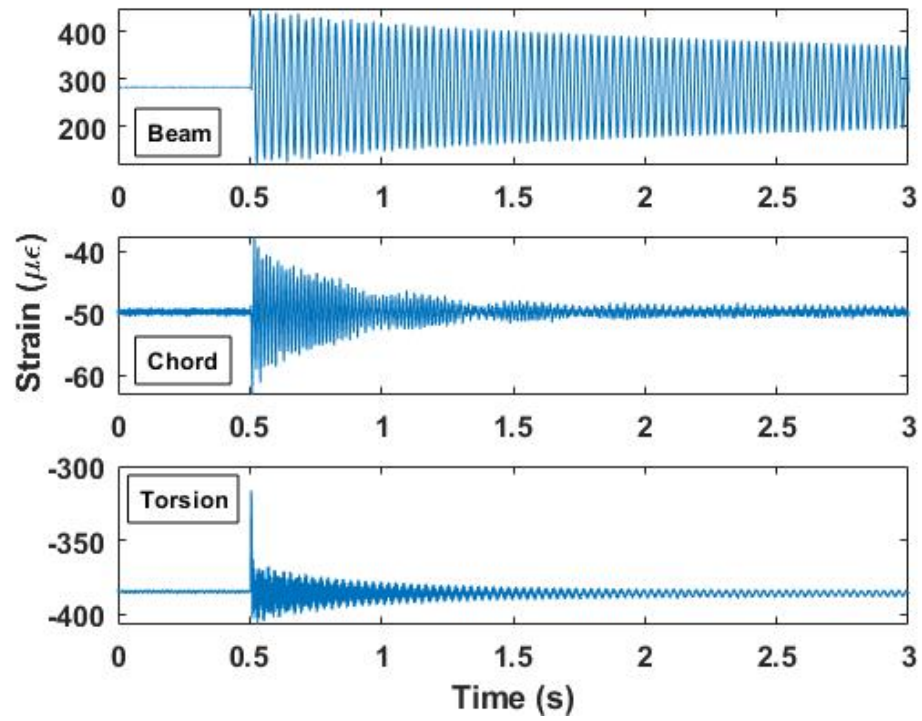


(a) Beam and chord response for spar.

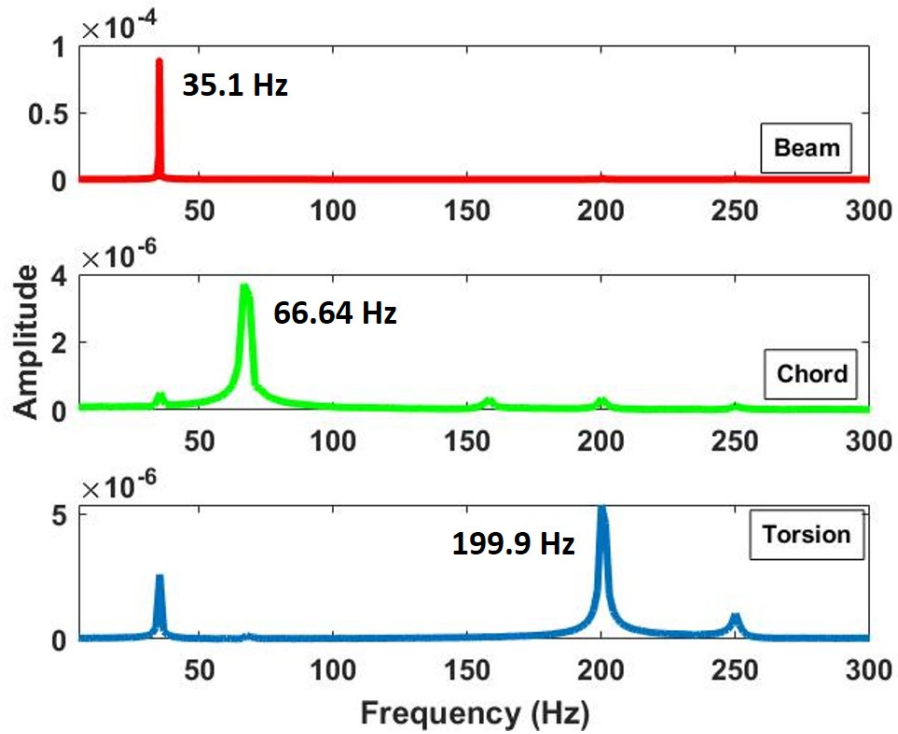


(b) Frequency responses.

Figure 4.25: Results of impact testing for spar without rib.

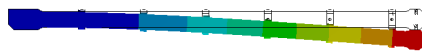


(a) Beam and chord response for spar with rib.

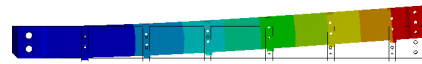


(b) Frequency responses.

Figure 4.26: Results of impact testing for spar with rib.



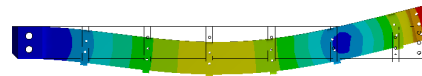
(a) First beam bending mode at 44.46 Hz.



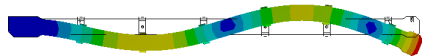
(b) First chord bending mode at 87.32 Hz.



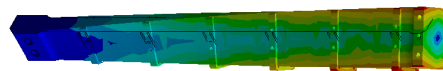
(c) Second beam bending mode at 277.18 Hz.



(d) Second chord bending mode at 533.87 Hz.



(e) Third beam bending mode at 768.76 Hz.



(f) First torsion mode at 791.29 Hz.

Figure 4.27: ANSYS modal analysis of baseline spar; first six modes shown together with the undeformed model wireframe.

predicted to be near 800 Hz for the bare spar, dropped to 200 Hz measured with addition of the rib.

The analytical results and ANSYS predictions have consistently higher frequencies than experimental results. The analytical method assumed a uniform beam without spacers, hence a lower mass, so higher frequencies. ANSYS overpredictions can be attributed to the practical differences in clamping condition of the model and the physical spar and baseplate. With an imperfect clamp, the rig would be expected to be slightly softer (lower frequencies) in all modes, and Table 4.9 shows roughly 10% overprediction by ANSYS across all modes, whereas the analytical method overpredicts the experiment by about 20% or more for each mode. Figures 4.27 and 4.28 show all the mode shapes of the spar through the first torsion mode for the bare spar and spar with rib, respectively.

ANSYS also predicted a rib bending mode shown in Fig. 4.28(c) at about 180 Hz. The rib bending mode is actually captured in the experimental chord response in Fig. 4.26(b) with a frequency of about 160 Hz. A fifth mode was detected in the chord and torsion responses around 250 Hz. ANSYS predicted this to be a coupled beam/torsion mode at 279 Hz.

The overall results were consistent enough to proceed to structural damping measurements.

The damping ratios for the spar alone and spar with rib are shown in Table 4.10. The results show that damping ratios between the spar alone and spar with rib are very similar and quite low. Minimal structural damping is desirable as it allows for measuring small aerodynamic damping later. The moving block method

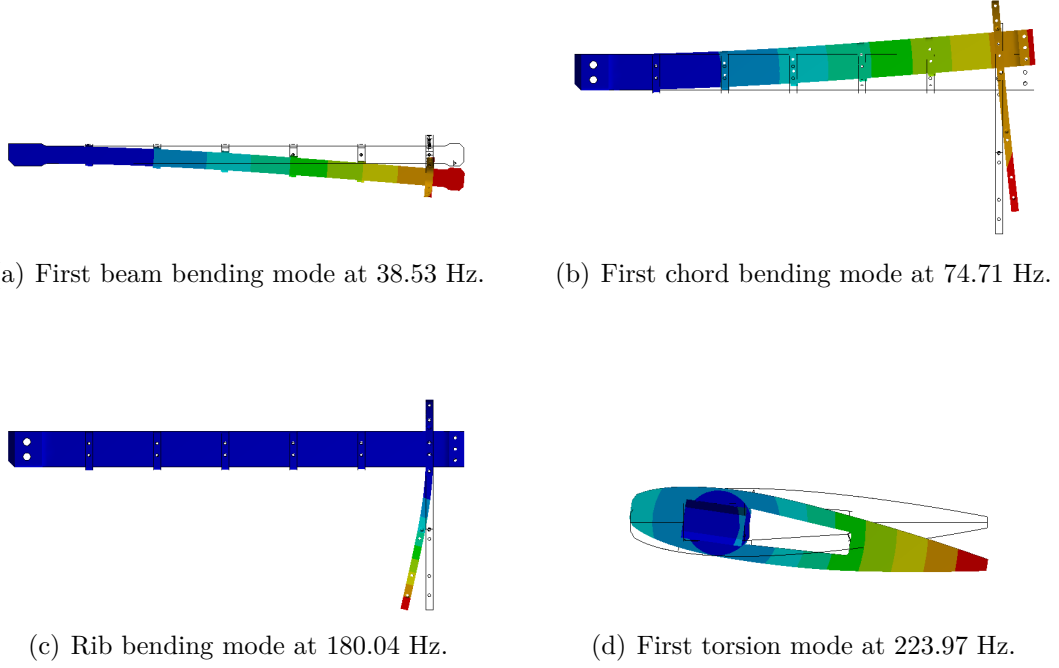


Figure 4.28: ANSYS modal analysis of spar with single rib; first four modes shown together with the undeformed model wireframe.

Table 4.10: Measured damping ratios for spar alone and spar with rib.

Damping Ratio	Spar Alone (%)	Spar with Rib (%)
Beam	0.15	0.23
Chord	0.48	0.51
Torsion	—	0.37

(see Section 1.4.3), is used to calculate damping ratios. The moving block method begins with a block of data at the beginning of the time history of response. Then the same-sized block is shifted in time and the amplitude of the frequency response is calculated again. After a number of blocks, the log of the array of amplitudes is plotted over time. A line fitted through the result gives a slope which is $-\zeta\omega_n$, where ζ is the damping ratio and ω_n is the natural frequency. The method is implemented in MATLAB and ran immediately after a test point is acquired so the damping ratio can be monitored during wind tunnel testing. The moving block method was sufficient since the response contained only one frequency at a time.

The previous results shown were from tests conducted in the tiltrotor lab with the wing spar mounted to the baseplate and clamped to a 500 lb t-slot table. The tests in the lab supported the method of extracting frequency and damping for the rig using strain gauges and the moving block method. The frequencies and damping ratios of the full rig mounted to the interface post and installed in the wind tunnel are shown in Tables 4.11 and 4.12.

Table 4.11: **Measured frequencies and damping ratios for MTR in GLMWT, no wind, unpowered.**

Mode	Frequency (Hz)	Damping Ratio (%)
Beam	5.06	0.53
Chord	9.3	0.54
Torsion	15	0.2

With the wing, pylon, and rotor assembled, but without blades, the heavy masses brought the wing-pylon frequencies much lower, as expected, with the beam

Table 4.12: Measured frequencies and damping ratios for MTR in Navy SWT, no wind, unpowered.

Mode	Frequency (Hz)	Damping Ratio (%)
Beam	5.06	0.4
Chord	9.65	0.57
Torsion	14.4	–

mode around 5 Hz, the chord mode around 9.5 Hz, and the torsion mode around 14.5 Hz. The results between the GLMWT and Navy SWT are very similar for frequency and damping with the exception of the torsion damping in the Navy tunnel. The torsion gauges at the Navy tunnel were damaged, including the back up gauges, so the torsion response was actually taken from the chord strain gauges. The amplitudes were large enough to get an accurate measurement for frequency, but the damping could not be accurately obtained.

4.8 Pylon Center of Gravity and Inertia

Center of gravity (CG) and mass moment of inertia (MOI) properties of the pylon (including the rotor assembly) were measured using a Space Electronics KSR CG/MOI machine at NASA Langley Research Center, shown in Fig. 4.29. The orientations of the pylon are shown in Figs. 4.30 and 4.31.

The machine is a rotating table resting on an air spherical bearing, which eliminates friction, and connected to a vertical torsion rod which allows the table to rotate. It has one moment transducer at a fixed location on the machine. To measure the CG of a test article, four measurements are taken, one at each orthogonal axis

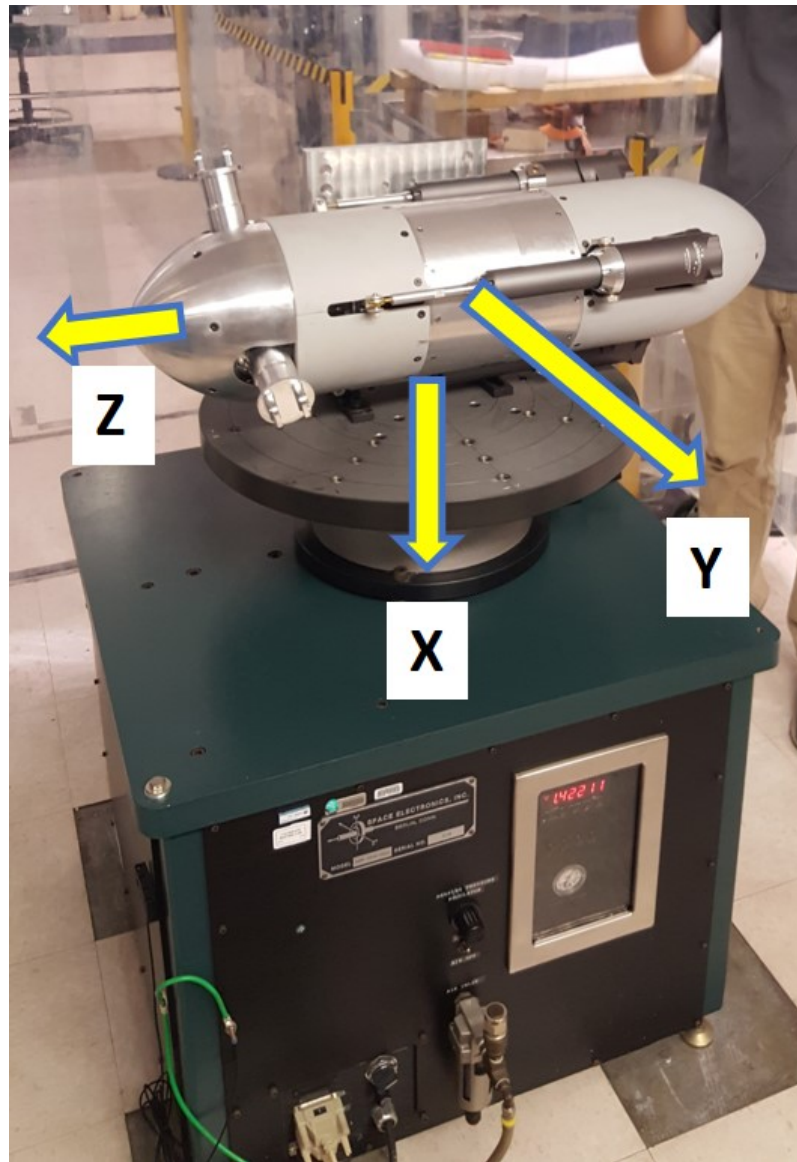


Figure 4.29: KSR CG/MOI machine at NASA Langley; Pylon mounted in yawing configuration.

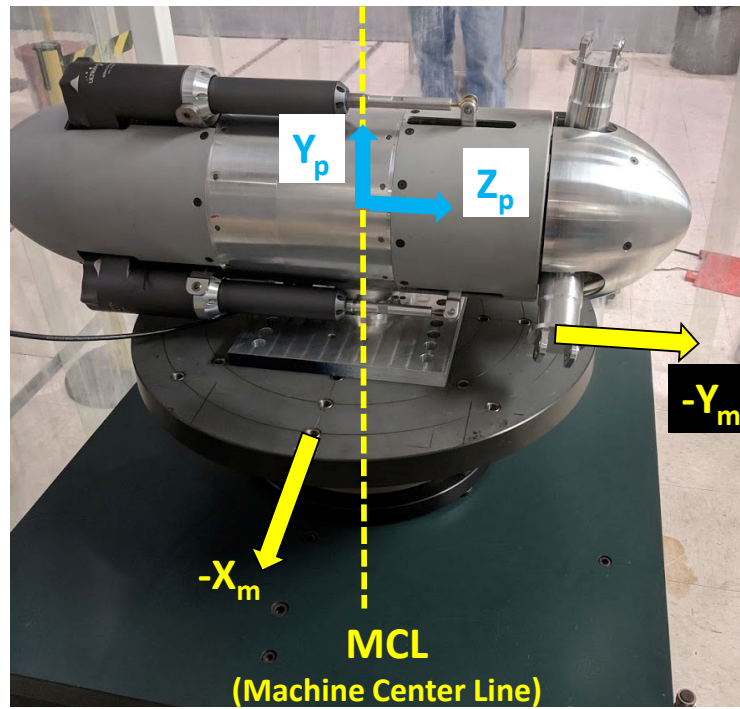


Figure 4.30: Pylon testing in pitch orientation.

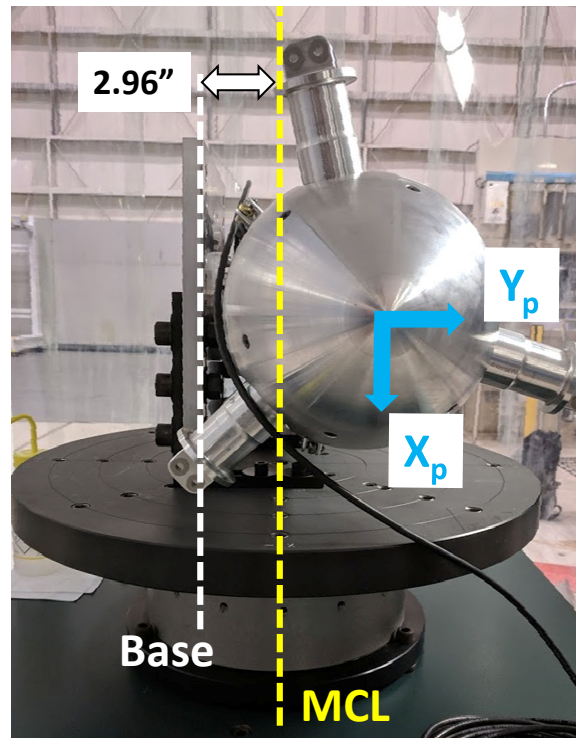


Figure 4.31: Pylon testing in yaw orientation.

as the table rotates. By acquiring both the mass of the pylon and the moments from each of the measurements, the moments divided by mass result in distances and the resultant of these distances is the CG location. This calculated distance is relative to the machine center line (MCL), so it is important to set a reference point on the test article to identify the CG location relative to the test article. The reference point on the pylon was the center of the coupling plate which coincides with the location of the spar.

For inertia measurements, the torsion rod is clamped on the end near the floor, given an initial offset as a starting position, then released. The torsion rod acts as a spring and the entire table becomes a torsional pendulum. The machine is able to measure the period of oscillation T and, from the known spring constant of the torsion rod κ , the moment of inertia is calculated using:

$$I = \kappa T^2$$

where the torsion stiffness of the rod $\kappa = 4686.624 \text{ lb-in}^2/\text{sec}^2$, T is the period of oscillation in seconds, and I is the mass moment of inertia in lb-in^2 . The inertias were converted to kg-m^2 afterward. Results from the Langley tests are shown in Table [4.13](#).

The distance from MCL to pylon base in the yaw configuration is 2.96 ± 0.01 inches. The distance from pylon base to the rotor shaft axis is 5.388 ± 0.002 inches.

Note, the pylon could not simply be placed on the table for testing. An interface plate was required between the machine and the pylon and this was milled

Table 4.13: **Results of CG and MOI tests at NASA Langley.**

Property	Unit	Pitch	Yaw
Part Weight	lb (kg)	71.16 (32.28)	71.26 (32.28)
CG (X) wrt MCL	in (cm)	-0.0223 (-0.06)	1.282 (3.26)
CG (Y) wrt MCL	in (cm)	1.294 (3.29)	-2.370 (-6.02)
Period, T	sec	1.37388	1.4136
I about CG	lb-in ² (kg-m ²)	4284.98 (1.25)	4275.41 (1.25)
I about MCL	lb-in ² (kg-m ²)	4404.15 (1.29)	4791.89 (1.40)

out of aluminum. Steel brackets were used to orient the pylon sideways for yaw inertia. The plate orientations are shown in Fig. 4.32. Figure 4.32(a) shows the plate sitting flush on the machine. The pylon coupling plate is mounted directly over the MCL. Figure 4.32(b) shows the plate mounted vertically. This orientation allows the pylon yaw moment to be measured. The results for the isolated plate are shown in Table 4.14. Table 4.13 already has the tares accounted for in the results.

Table 4.14: **Tare measurements of interface plate at NASA Langley.**

Property	Unit	Pitch	Yaw
Period, T	sec	0.973554	0.987828
I about MCL	lb-in ²	4442.02	4573.23

All measurements from the machine were relative to the axis of rotation on the table. Using parallel axis theorem, the pylon CG and moments of inertia about the wing spar center were obtained. The final values of the extracted properties are shown in Table 4.15.

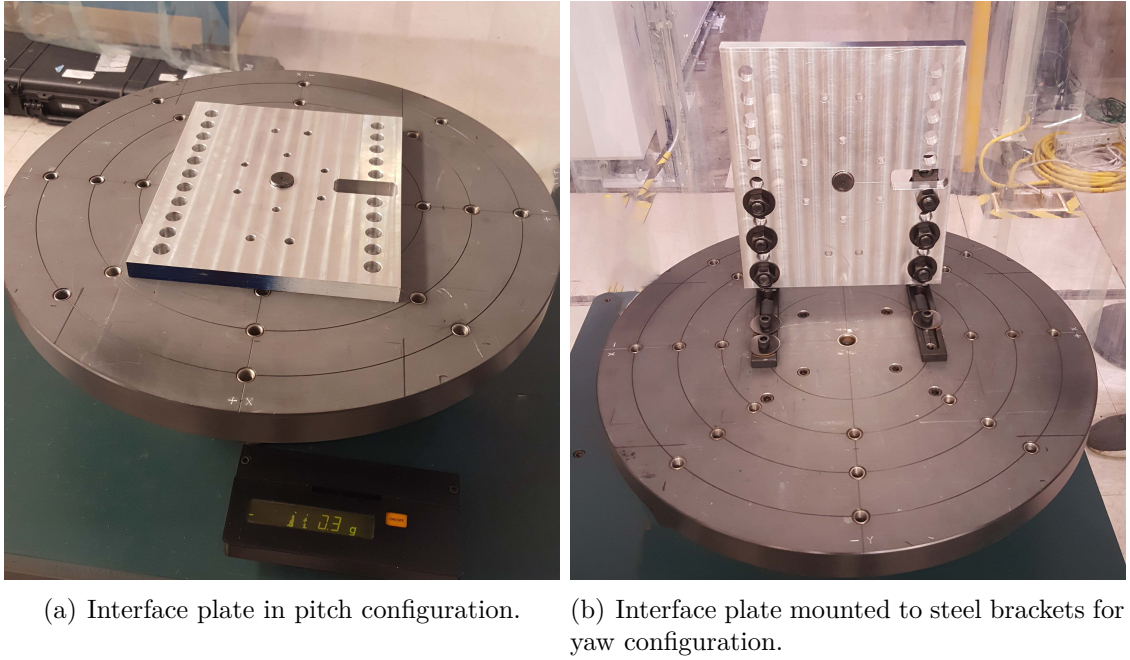


Figure 4.32: Tare runs on the interface plate and brackets.

Table 4.15: **Pylon assembly properties.**

Parameter	Units	Measurement
Mass	kg	32.38
Z_{cg}	cm	-3.27
Y_{cg}	cm	-0.147
X_{cg}	cm	0.06
I_{PX}	kg-m ²	1.286
I_{PY}	kg-m ²	1.289

4.9 Chapter Summary and Conclusions

This chapter described Maryland Tiltrotor Rig and its physical features and key characteristics. It covered the details of the following.

1. A description of the overall rig dimensions. With a rotor diameter of 4.75 ft, the MTR is a $1/5.26$ scale model of a full-scale 25 ft diameter Bell XV-15-like proprotor.
2. A description of the wing assembly and spar design. The MTR wing spar was loosely designed to the Froude-scale frequencies of the XV-15 wing but, with the interchangeability of the spar, there is flexibility built into the design for future excursions.
3. A description of the pylon assembly and load paths. There are two load paths from the rotor to the wing root. The primary load path runs through the load cell and a secondary load path is unmeasured. Rotor thrust and torque are minimally affected by the secondary load path.
4. A description of the electric motor features and power supply components. The electric motor controller power is limited by the cable thickness from the DC power supplies to the motor controller. Increasing the cable thickness to 2 AWG or thicker will allow up to 130 amps at 120 V for 15.6 kW.

5. A description of the rotor assembly and gimbal hub components. The rotor uses a universal joint as the gimbal and has a $\pm 8^\circ$ gimbal flap due to physical constraints.
6. A list of assembly and component weights. The total weight of the rig is less than 100 lb. This does not include the hoses and cables that run from the MTR equipment rack to the pylon.
7. The frequency and damping measurements for spar only. Measured results, analytical calculations, and ANSYS models were in agreement.
8. The frequency and damping measurements for spar with one rib. Measured results, analytical calculations, and ANSYS models were in satisfactory agreement.
9. The frequency and damping measurements for the full rig installed in GLMWT and Navy SWT. The results were in agreement between tunnels with wing beam frequency of about 5 Hz and damping of about 0.5% critical, wing chord frequency of about 9.5 Hz and damping of about 0.5% critical, and wing torsion frequency of about 14.5 Hz and damping of about 0.2% critical. However, the torsion damping at the Navy SWT was not acquired due to broken wing strain gauges.
10. The pylon mass, CG, and MOI measurements obtained from testing at NASA Langley. These properties were used in analysis and predictions.

In conclusion, the MTR is now ready to be instrumented for whirl flutter tests.

Chapter 5: Model Instrumentation and Calibration

This chapter describes the instruments and sensors on the rig and details the calibrations performed to prepare them for testing and data acquisition. A new calibration stand was developed for the rig. A section on the data acquisition system is also presented.

5.1 List of Instruments

The major instrumentations on the rig that are available for all tests are listed in Table 5.1 and shown visually in Fig. 5.1. Off-rig sensors such as track and balance, VICON, or PIV equipment are not included in this list.

A custom-built slip ring is used for acquiring signals in the rotating frame. The slip ring has 64 rings, 4 rings for each signal (2 for power, 2 for signal), which allows for 16 signals to be acquired. There are 10 standard measurements taken in the rotating frame: blade pitch (3), pitch link loads (3), gimbal tilt (2), and torque (2), which leaves 6 additional rotating frame measurements. These additional measurements will be strain gauges on the blades for measuring bending and torsion moments.

Table 5.1: **MTR on-rig instrumentation.**

Measurement	Sensor	Quantity	Slip Ring Leads	Mounting
Blade pitch angle	Magnetic encoder	3	24	1 per blade
Pitch link loads	Strain gauge	3	12	1 per pitch link
Gimbal tilt angle	Hall effect sensor	2	6	1 per tilt axis
Rotor torque	Strain gauge	1	4	On shaft
Blade loads	Strain gauge	4	16	2 full-bridges per blade
Rotor speed and position	Hall effect sensor	1		On shaft
6-axis forces and moments	Load cell	1		On pylon bulkhead directly above coupling plate
Vibration	Tri-ax accelerometer	1		On pylon frame
Slip ring		1		64 channels
Fixed system actuation	Electric actuators	3		Below swashplate
Wing beam/chord/torsion strains	Strain gauge	3		3 full-bridges attached near the wing spar root

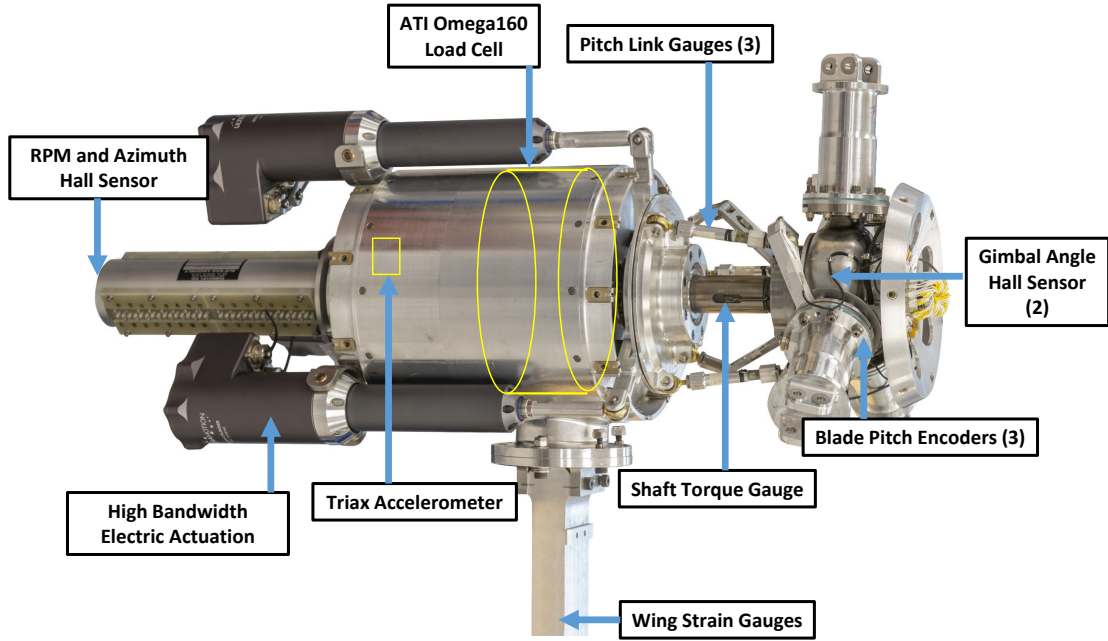


Figure 5.1: On-rig instruments and sensors.

In the non-rotating frame, an ATI Omega160 Transducer is used for loads and torque measurements. The load cell provides forces and moments about all three axes. There is a strain gauge to measure shaft torque on the shaft independent from the load cell. Wing bending and torsion strains are also separate measurements.

5.2 Load Cell Hub Forces and Moments

The ATI Omega160 load cell measurements at the hub were determined to be inaccurate during the first checkout test. ATI provided a calibration matrix for measurements at the load cell surface. A transformation of the calibration matrix could be performed to measure the loads and moments at a point not on the surface of the load cell. For the MTR, the loads and moments were measured at the hub, about 7.5 inches away from the load cell surface. The original, untransformed ATI

calibration matrix is shown in Table 5.2. Each column is dot multiplied to the load cell voltages to obtain the corresponding forces and moments. For example, the first column will output F_x by

$$F_x = -0.96014 V_1 + 0.24773 V_2 + 4.54453 V_3 - 57.60398 V_4 - 0.95628 V_5 + 65.18886 V_6 \quad (5.1)$$

These calibration matrices are unique to each load cell. For the MTR load cell, the official ATI calibration file is FT18654.cal.

Table 5.2: **ATI original calibration matrix.**

	F_x	F_y	F_z	M_x	M_y	M_z
V_1	-0.96014	-0.46179	102.29309	0.56981	297.62589	4.97159
V_2	0.24773	73.28008	-1.26585	60.79710	-3.93646	-144.57342
V_3	4.54453	1.09967	101.56629	-254.00843	-151.74245	5.37580
V_4	-57.60398	-33.16828	0.81479	-29.79184	47.18046	-131.68102
V_5	-0.95628	0.86957	102.35840	256.22731	-147.66212	1.80703
V_6	65.18886	-38.08432	-1.79533	-35.55609	-51.35803	-149.91018

The orientation of the load cell coordinate system is not the same as the pylon/hub coordinate system. Figure 5.2 shows the load cell orientation in the pylon with the load cell axes X' and Y' in dotted red arrows. The Z axis is directed toward the hub along the main shaft. The pylon axes are shown in Fig. 5.3 in dotted green arrows, denoted X_P and Y_P . To measure forces and moments along the pylon axes, the load cell coordinate system must be oriented in the yellow solid

axes show in both figures. If the load cell axes were oriented in the pylon axes, a positive force on the hub along the pylon axis would show up as a negative force from the load cell. This is due to the hub forces traveling to the motor and the motor acting on the rear side of the load cell, shown in Fig. 4.12(b). The load cell is not directly connected to the shaft. The forward surface of the load cell is mounted to the forward bulkhead, and the rear surface is mounted to the connector to the motor. Figures 5.2 and 5.3 display the load cell with the forward bulkhead removed. The transformation is a 17° clockwise rotation of the original axes about the Z-axis such that the Y-axis of the load cell is in the downward direction and the X-axis is to the left. The coordinate system also had to be translated 7.5 inches forward to be located at the center of the hub. After this transformation, the ATI matrix is shown in Table 5.3.

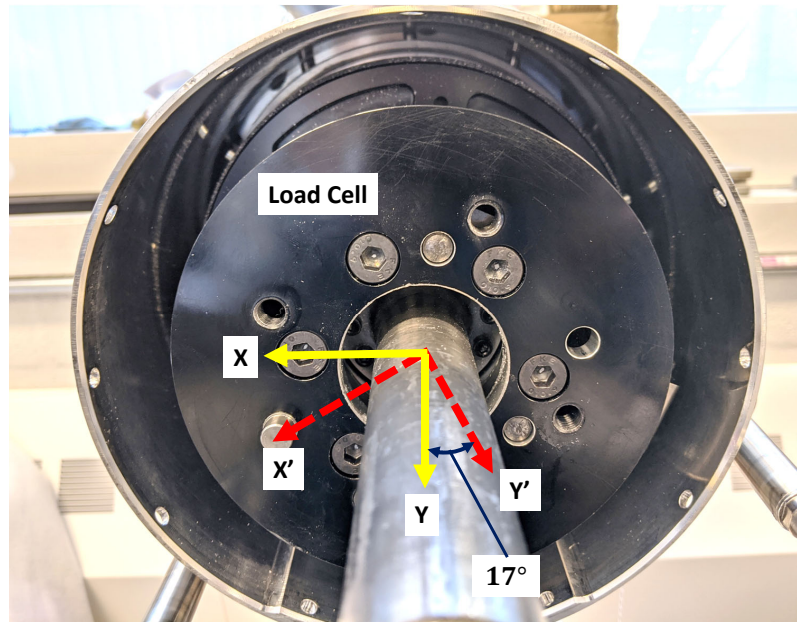


Figure 5.2: Load cell coordinate system orientation: red dotted axes - as installed, yellow solid axes - desired orientation for positive forces in pylon axes.

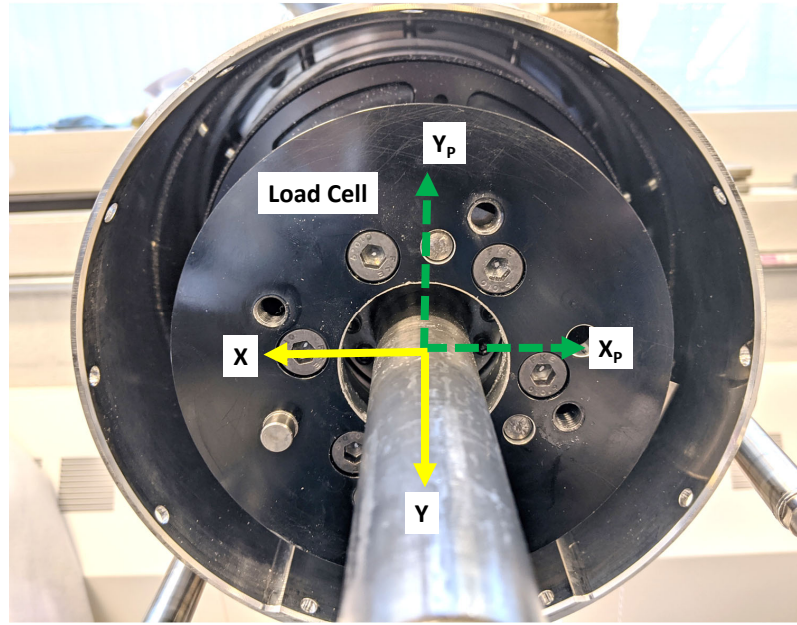


Figure 5.3: Load cell coordinate system orientation: yellow solid axes - desired load cell orientation, green dotted axes - pylon coordinate system.

Table 5.3: ATI calibration matrix transformed to hub location.

	F_x	F_y	F_z	M_x	M_y	M_z
V_1	-0.78317	-0.72233	-102.29308	-91.88996	290.66141	4.97163
V_2	-21.18812	70.15052	1.26584	585.42029	172.92178	-144.57341
V_3	4.02444	2.38032	-101.56631	-180.69186	-249.56026	5.37583
V_4	-45.38854	-48.56046	-0.81478	-406.48776	376.82263	-131.68105
V_5	-1.16874	0.55199	-102.35844	292.34348	-57.53086	1.80705
V_6	73.47520	-17.36083	1.79535	-149.19308	-610.57349	-149.91016

Under loading, a 1×6 array of raw voltages, v , from the load cell is acquired which is then multiplied by the 6×6 calibration matrix to obtain a 1×6 array of six loads, f , containing three forces and three moments. Thus, $v^T C = f^T$.

From the check-out tunnel entry, it was clear the in-plane forces and moments were not accurate, being off by as much as 50% of the load applied. Therefore, calibration tests were conducted to extract a new 6×6 calibration matrix, C . The goal was to obtain the new calibration matrix based on measurements of v and f from loads applied at the hub.

Consider a force, f_r , where $r = 1, 2, \dots, 6$ is an entry of f . This force would produce a voltage vector, v , which, when multiplied with the r^{th} column of the C matrix, produce f_r .

$$v^T c_r = f_r \quad (5.2)$$

Now if six different f_r are imposed and six different voltage vectors v obtained, then putting the voltage vectors in rows would give

$$V c_r = f_{r6} \quad (5.3)$$

Where V is the 6×6 voltage matrix, c_r is the 6×1 calibration vector, and f_{r6} is a 6×1 force vector containing the six different f_r .

In this case, solve for c_r :

$$c_r = V^{-1} f_{r6} \quad (5.4)$$

For proper accuracy, N different f_r should be imposed

$$Vc_r = f_{rN} \quad (5.5)$$

which are N equations but still six unknowns in c_r . Here V is a $N \times 6$ voltage matrix of measurements, c_r is an unknown 6×1 calibration vector, and f_{rN} is the known $N \times 1$ force vector. Each row of V corresponds to 6 voltages in one time sample. The vector f_r is either in units of pounds for loads or inch-pounds for moments. To obtain the calibration vector c_r , multiply by the transpose of V to get square matrices:

$$V^T V c_r = V^T f_r \quad (5.6)$$

Then, solve

$$c_r = (V^T V)^{-1} V^T f_r \quad (5.7)$$

which is the least squares error solution or the Moore-Penrose inverse.

These steps can be invoked numerically by using Matlab's left-matrix-divide operator `\` shown in eq. 5.8.

$$c_r = V \backslash f_r \quad (5.8)$$

This process is repeated for all $r = 1, 2, \dots, 6$ loads to produce the full C matrix of r columns. To obtain c_r accurately, there must be at least six independent rows in V in Eq. (5.4), which is very hard to accomplish in measurement. So, in general, $N \gg 6$ and Eq. (5.8) must be used which results in a least squares solution for C .

5.3 Calibration Setup

The load cell calibration was performed in two ways. For F_x , F_y , F_z forces, and M_x and M_y moments, the load cell was mounted in the MTR pylon to mimic test conditions. Due to the difficulty attaching components to the precise hub location, these forces and moments were applied at the shaft tip, 10 inches away from the load cell surface that is bolted to the forward bulkhead. The M_z moment could not be tested on the rig because the shaft could not be locked to allow for a transfer of torque. The load cell had to be removed from the rig and mounted to a second calibration frame specifically to calibrate the M_z moment. The M_z moment was applied at the hub location relative to the load cell surface, 7.5 inches away.

An 8020 frame was built around the MTR to apply the loads and moments. Figures 5.4, 5.5, and 5.6 show different configurations of the frame for testing in the principal directions.

For the M_z calibration tests, the load cell was removed from the pylon and mounted to a separate test stand, shown in Fig. 5.7. The weights were placed 7.5 inches ahead of the load cell surface and tested at two moment arm locations: 12 inches and 18 inches. The moment applied was not a pure moment so shear forces were accounted for in the X and Y axes in the direction of the desired coordinate transformation. The orientation of the load cell on the calibration frame was with X' axis pointed down and Y' axis pointed to the right. Then for a shear force in the X' direction, the force was split into $X = X' \cos(17^\circ)$ and $Y = X' \sin(17^\circ)$.

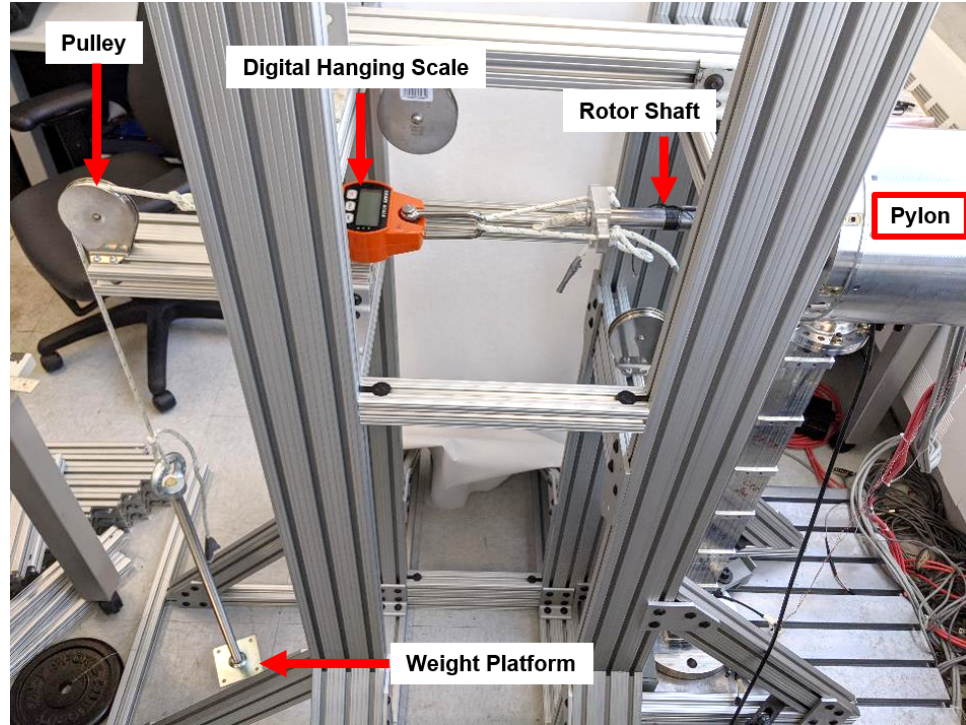


Figure 5.4: Calibration setup for F_z testing.

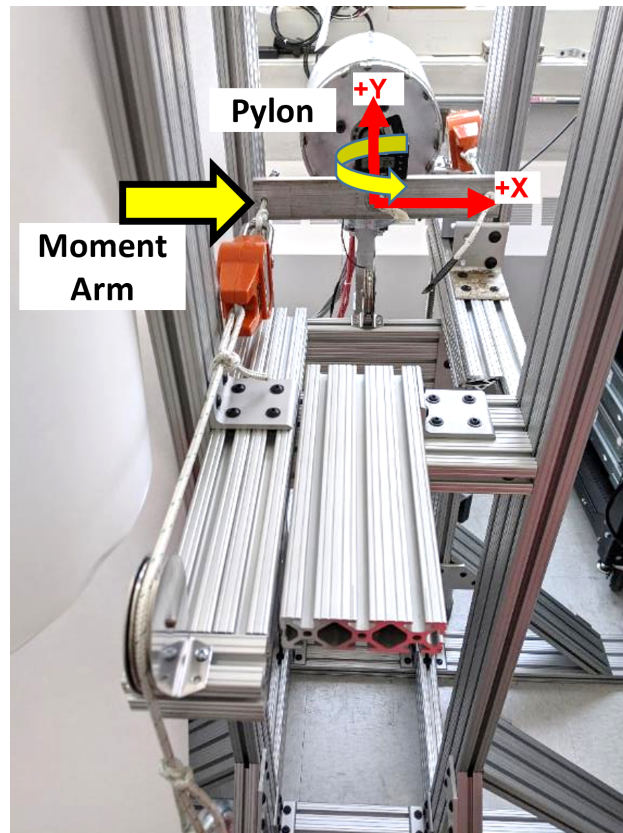


Figure 5.5: Calibration setup for M_y , front view.

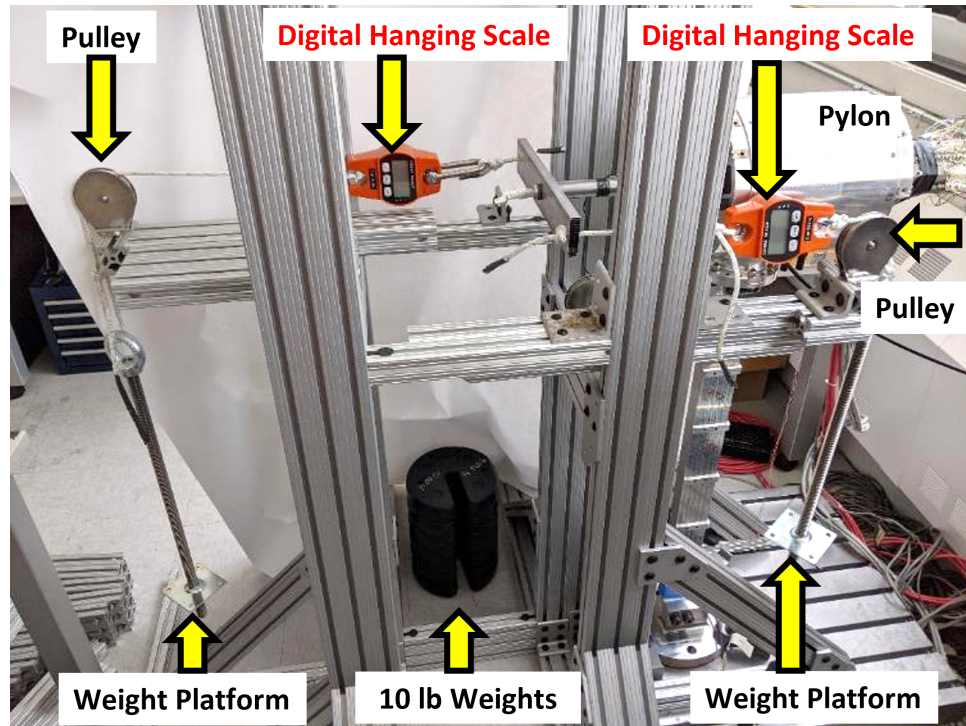


Figure 5.6: Calibration setup for M_y , side view.

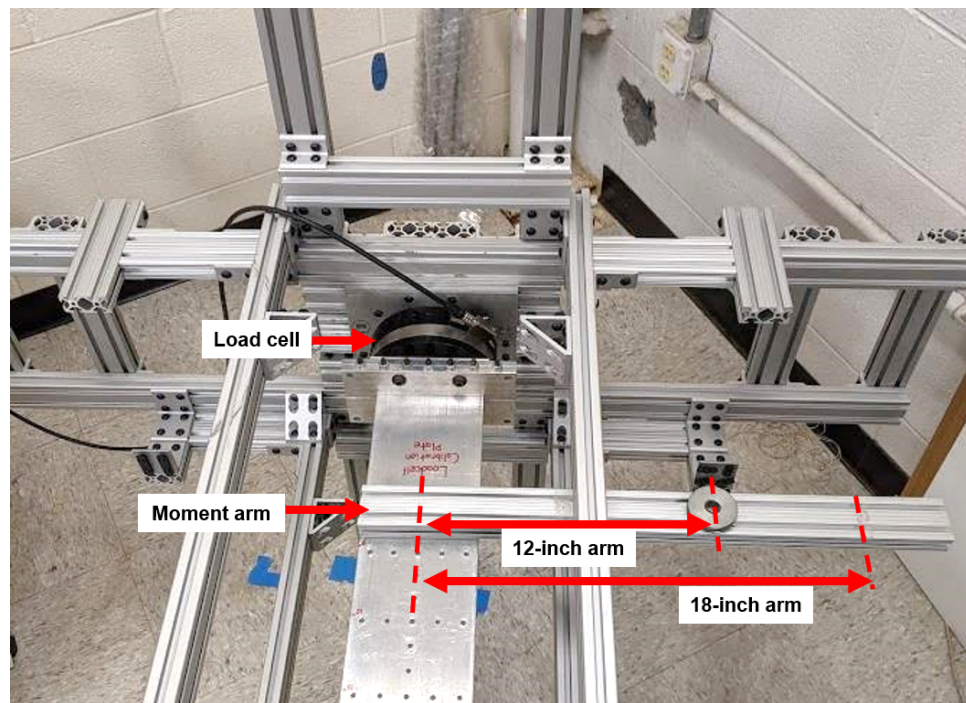


Figure 5.7: Calibration setup for M_z .

5.4 Calibration Results

The final calibration matrix, C , is shown in Table 5.4. Note the calibration matrices shown in this thesis must be multiplied on the right-side of an 1×6 voltage reading to get a 1×6 force and moment vector, in the form $v^T C = f^T$. If the voltage is written as a 6×1 vector, the calibration matrix should be transposed and pre-multiplied on the left-side of the voltage vector, in the form $C^T v = f$. So the matrix C in Table 5.4 should be transposed if it is to be multiplied by a voltage vector to obtain loading f .

Table 5.4: **UMD calibration matrix.**

	F_x	F_y	F_z	M_x	M_y	M_z
V_1	-12.03905	0.25538	-107.84150	-148.84551	402.61671	18.91641
V_2	-21.74887	54.43504	-0.48623	504.46099	191.98466	-163.76769
V_3	13.24228	-7.91720	-103.15160	-356.06921	-439.44723	-3.87865
V_4	-48.07172	-42.26899	0.29361	-432.21865	405.17254	-115.70941
V_5	-3.26362	13.46089	-115.24546	500.00744	37.47586	2.17141
V_6	77.57090	-10.55649	2.78257	-57.56705	-667.76574	-134.70993

Figures 5.8 to 5.13 show all six measurements for an applied force or torque in its axis direction. When F_x is applied, the voltage reading should ideally give zero loads in $F_y, \dots, M_z$. However, this is not the case. Both ATI and UMD matrices show some cross-coupling. Blue denotes the ATI calibration predictions, orange denotes

the UMD calibration predictions, and the black lines are the ideal loads — with zero off-axis readings.

5.5 Triaxial Accelerometer

The accelerometer is epoxied onto the rear bulkhead of the pylon shown in Fig. 5.14. It is a product of Endevco, part number 2270M7A/2771A-10, and it is used in conjunction with a charge amplifier made by PCB Piezoelectronics, part number 422E02. The accelerometer is connected to the charge amplifier through a 3 meter long 10-32 cable. The charge amplifier is connected to the PXIe-4492 module through a 20 ft BNC/BNC cable.

The frequencies measured from the accelerometer were compared with the frequencies measured from the wing strain gauges and are shown in Table 5.5. The results show adequate agreement.

Table 5.5: **Comparison of wing strain gauge FFTs and accelerometer measurements.**

Mode	Strain Frequency Hz	Accelerometer Hz
Wing Beam	5.3	5.3
Wing Chord	9.0	9.8
Wing Torsion	15.0	14.7

5.6 High-Bandwidth Electric Actuators

The swashplate is actuated by high-bandwidth electric actuators. This is a key innovative feature of the rig. The swashplate not only tilts but can have high

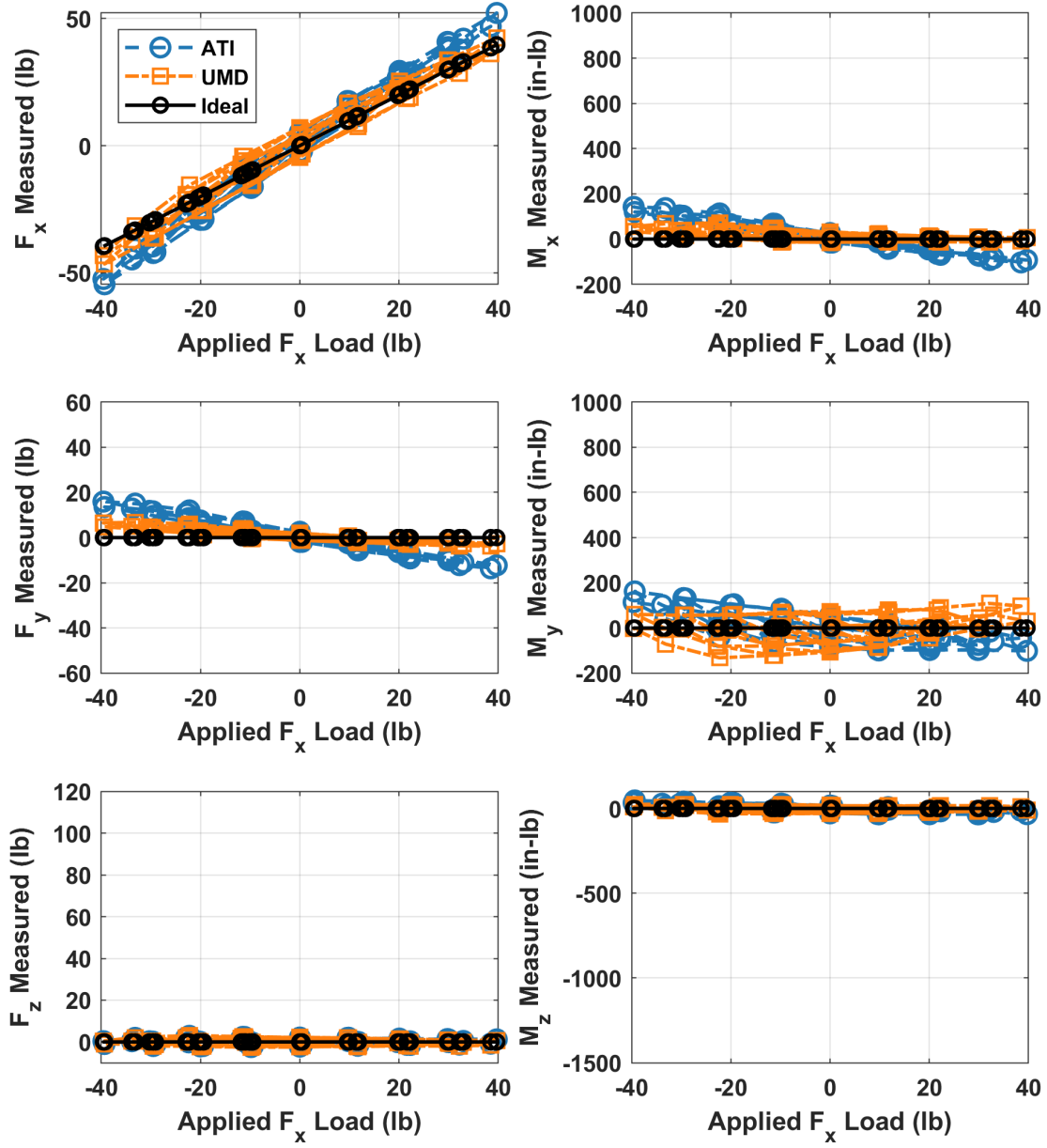


Figure 5.8: All forces and moments measured for an applied force in F_x . Blue - ATI calibration, orange - UMD calibration, black - ideal output.

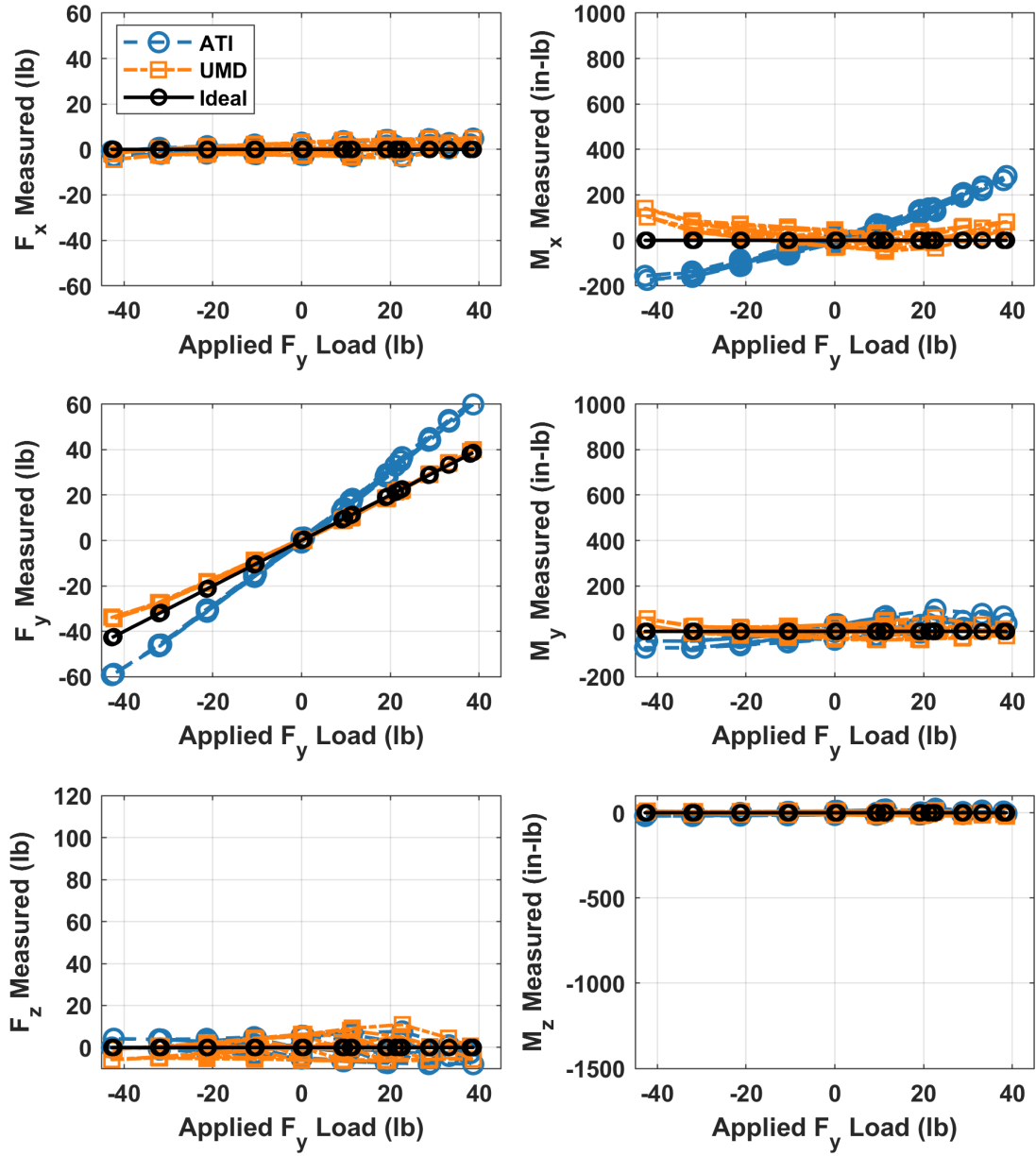


Figure 5.9: All forces and moments measured for an applied force in F_y . Blue - ATI calibration, orange - UMD calibration, black - ideal output.

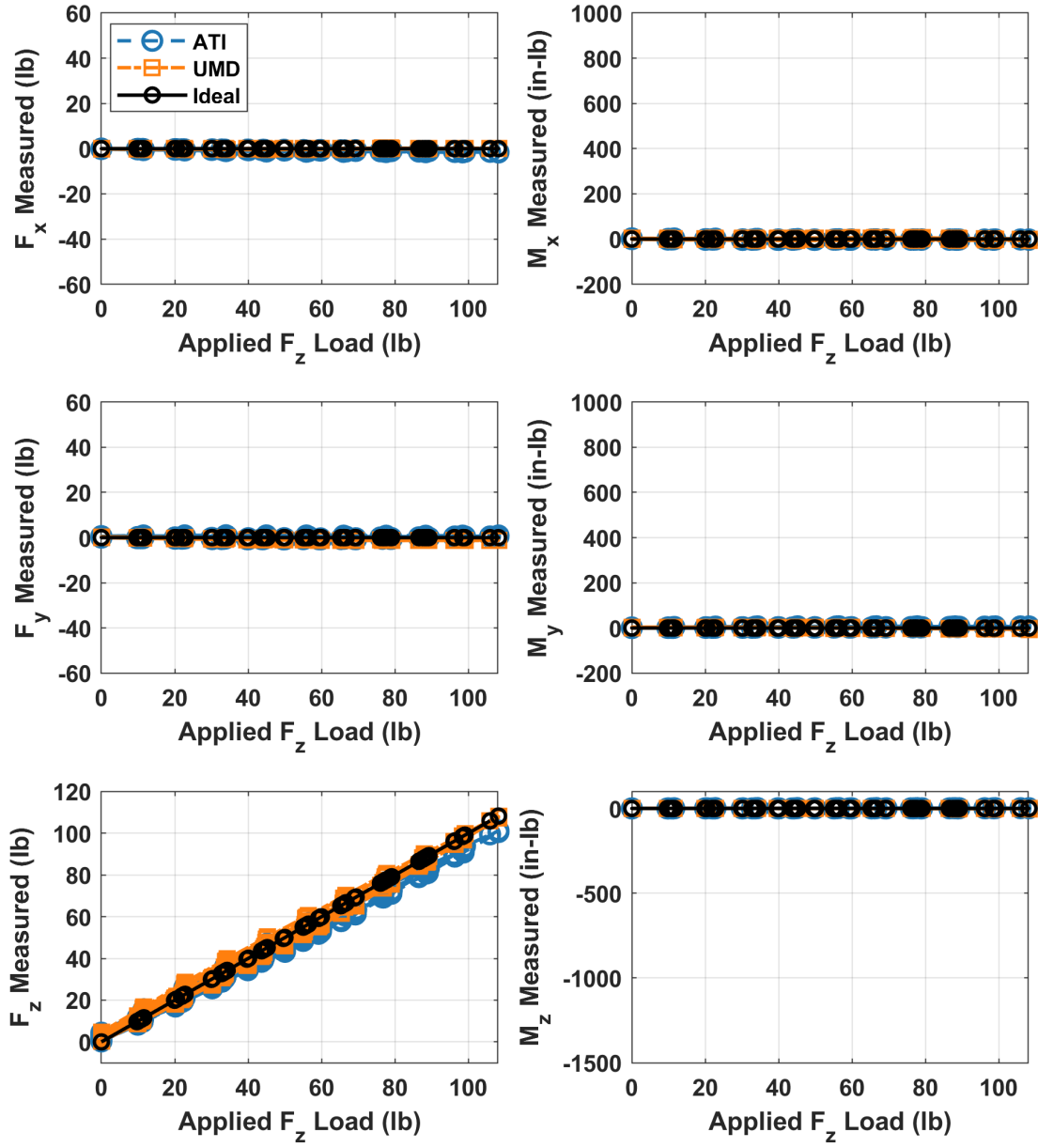


Figure 5.10: All forces and moments measured for an applied force in F_z . Blue - ATI calibration, orange - UMD calibration, black - ideal output.

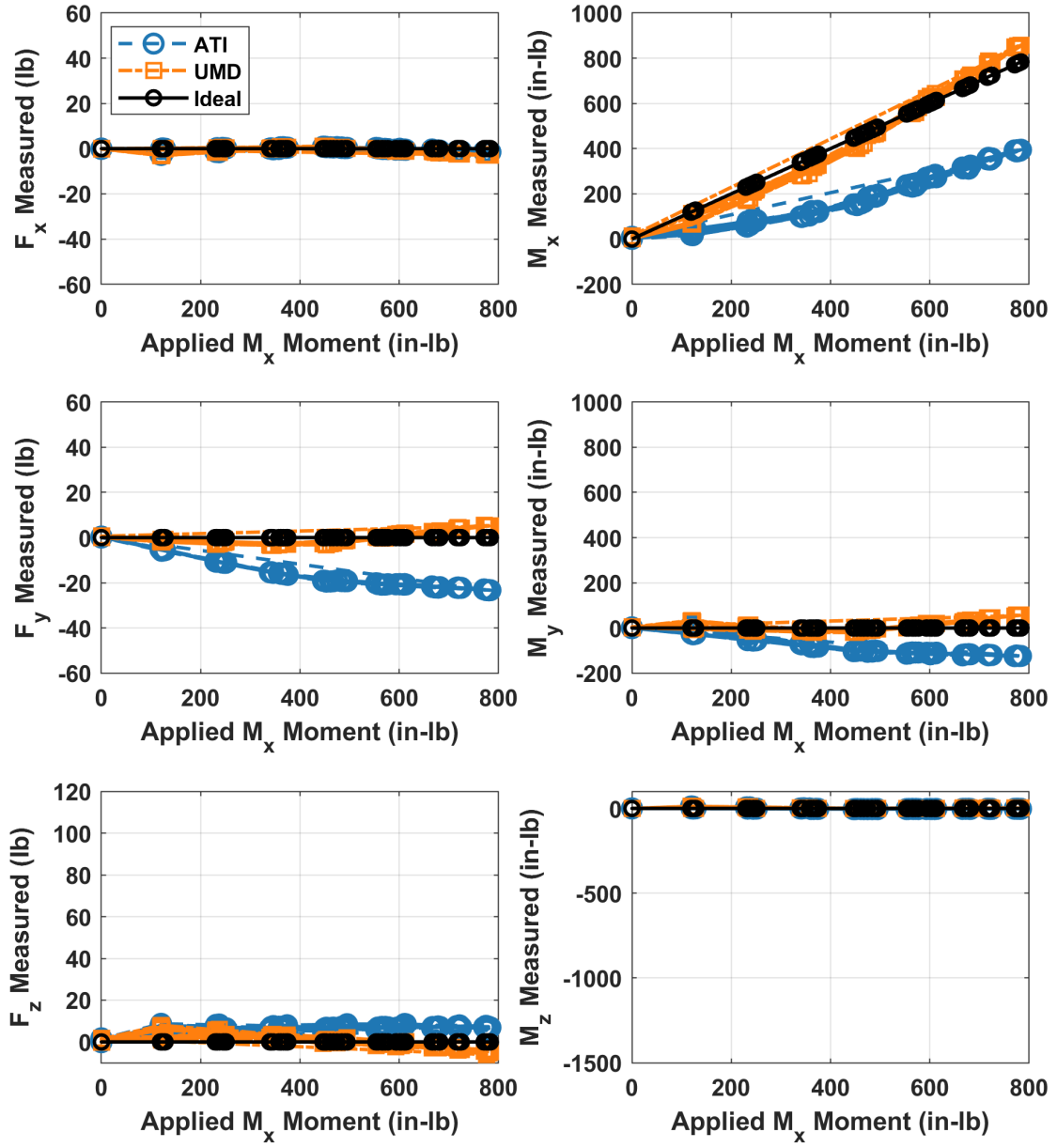


Figure 5.11: All forces and moments measured for a pure moment in M_x . Blue - ATI calibration, orange - UMD calibration, black - ideal output.

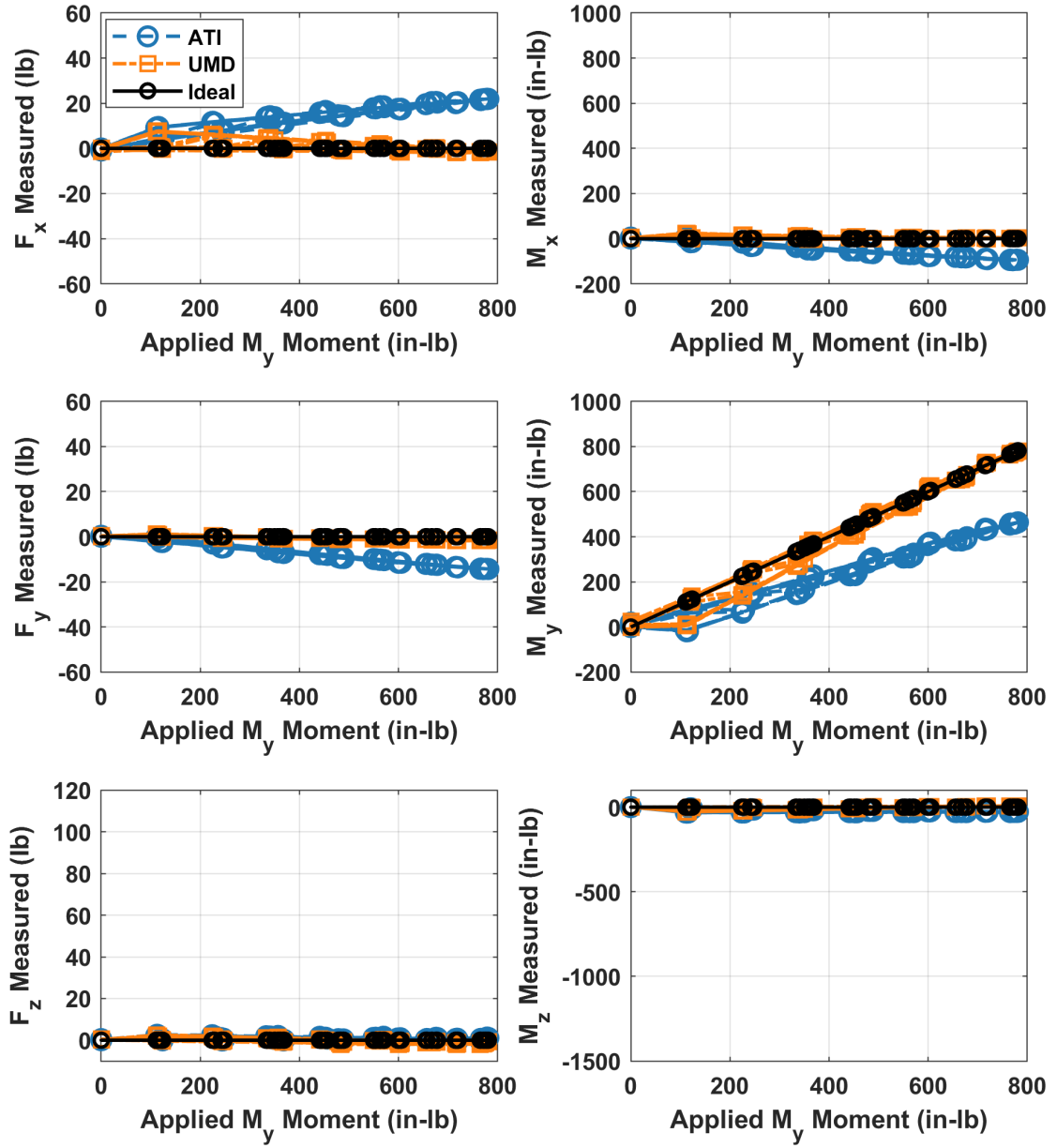


Figure 5.12: All forces and moments measured for a pure moment in M_y . Blue - ATI calibration, orange - UMD calibration, black - ideal output.

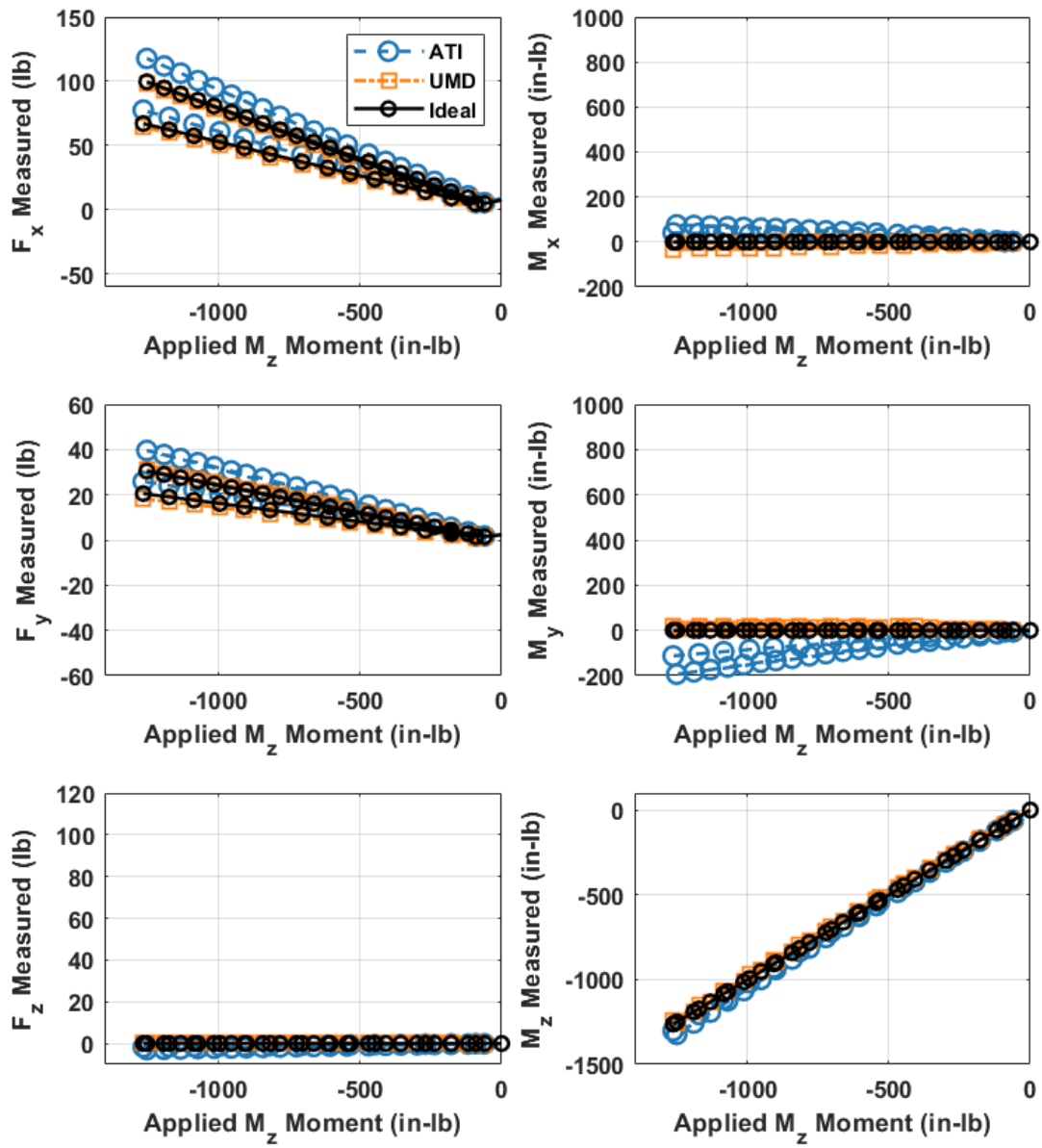


Figure 5.13: All forces and moments measured for an applied moment in M_z . Blue - ATI calibration, orange - UMD calibration, black - ideal output.

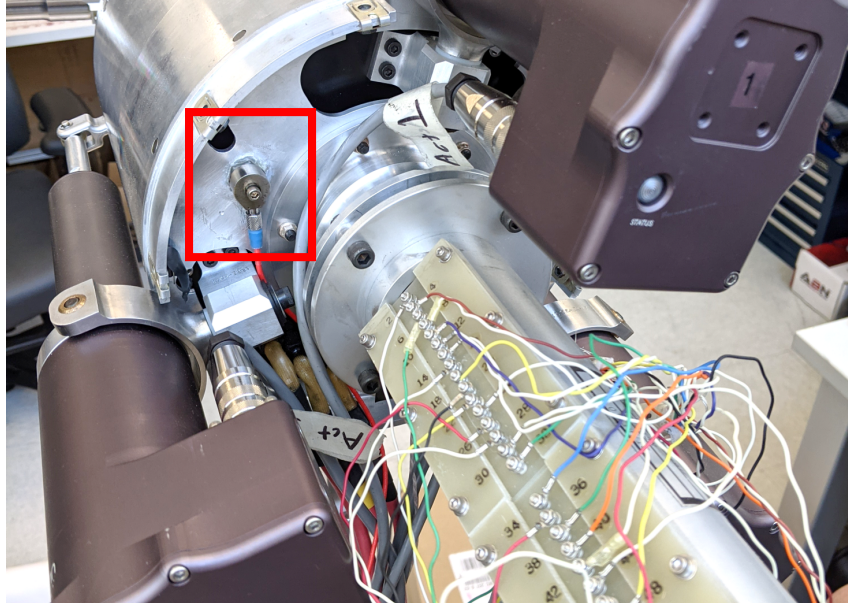


Figure 5.14: Triax accelerometer mounted to rear bulkhead of pylon, shown in red outline.

frequency motions. The three Ultramotion A2 linear actuators seen in Figure 5.15. These actuators perform the conventional functions of trimming the rotor — thrust and flapping (or moments) when the rig is powered and RPM and flapping (or moments) when the rig is in freewheel. They also perform flutter actuation at any specified frequency. During whirl flutter testing, a sinusoidal input from these actuators will be used to excite the fixed-frame wing-pylon frequencies. The actuators have a full stroke of 3.75 inches for an analog input voltage of ± 10 V, as shown in Fig. 5.16, which adequately covers and, in fact, exceeds the allowable travel of the swashplate in the assembly.

The root pitch at zero actuator stroke (zero volts) is 73° . The root pitch at the highest actuator stroke is 17° . Since the pitch link is located at the trailing edge, the highest stroke gives the lowest pitch. For the MTR proprotor blades with -37° linear twist, the root pitches of 17° to 73° translate to θ_{75} of -1° to 55° .



Figure 5.15: High bandwidth electric actuators for high frequency swashplate inputs. Ultramotion A2.

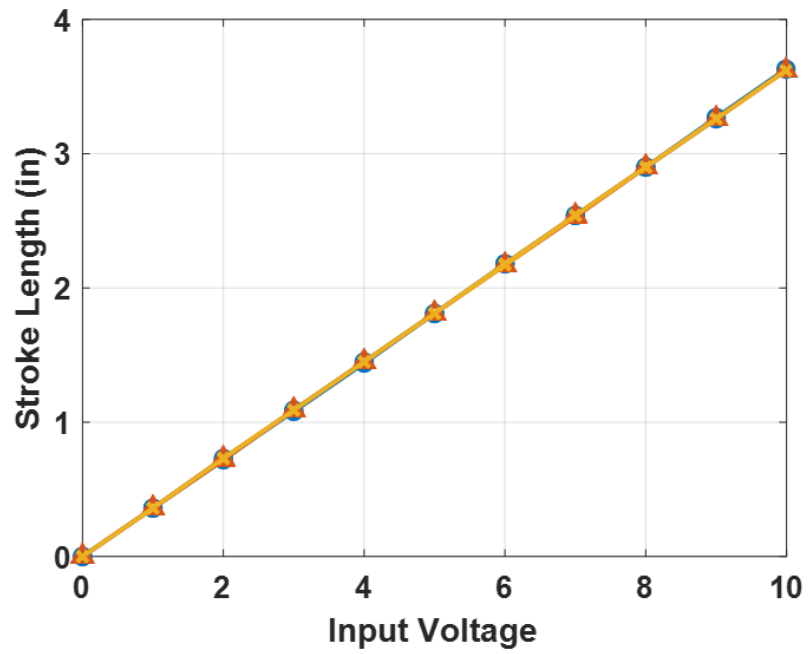


Figure 5.16: Actuators max stroke length.

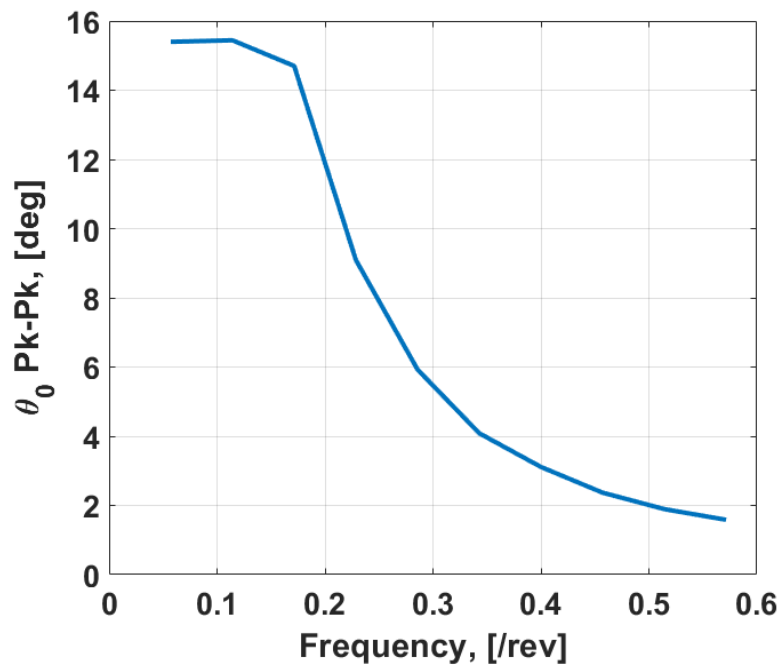


Figure 5.17: Peak to peak collective amplitude due to actuator motion as a excitation frequency increases.

To assess the actuator's ability to actively control and perturb the rotor, the actuators were subjected to a 1 V sinusoidal input at a range of frequencies with no external load applied. Figure 5.17 shows for a 1 V input signal, the amplitude decreases as the excitation frequency increases. This is mainly due to the controller internal to the actuator which limits the rise time of the response. The gains are tunable so the the amplitude can be increase in the future, if desired. At the wing beam bending frequency of 0.3 /rev at 1050 RPM, the actuators provide a collective pitch oscillation of 5.4° peak to peak. At the wing chord bending frequency of 0.53 /rev, peak-to-peak amplitude is about 2° . At wing torsion frequency of 0.83 /rev, the peak-to-peak amplitude is about 0.1° . Later, under loading, the torsion frequency could not be excited.

5.7 Blade Pitch Encoders

The blade pitch encoders on the MTR are contactless high-speed incremental linear magnetic (LM) sensors of rotary motions. They were procured from Rotary and Linear Motion Sensors (RLS). The part number is LM13-IC-10B-E-A-10-F-00. These use quadrature signals to determine the angle based on the relative position of the magnetic track and have a precision of 0.01153° . The encoder and track can be seen in Figs. 5.18 and 5.19.

These linear magnetic (LM) encoders do not allow for detection of a reference mark on the track, meaning once the encoders are turned on, they read the angles relative to the initial angle that is preset in the LabVIEW VI. If the pitch encoders

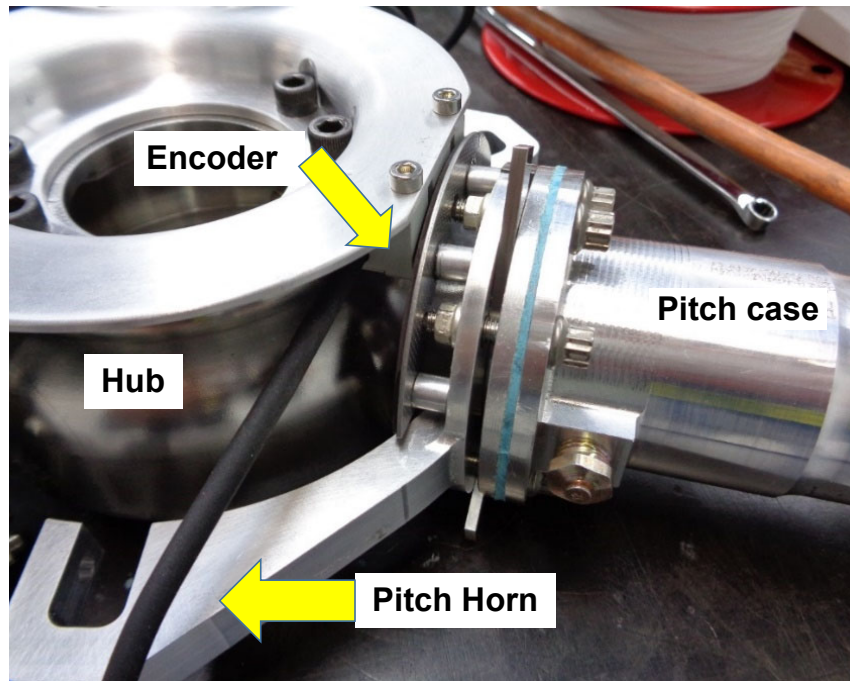


Figure 5.18: Pitch encoder sensor on the hub.

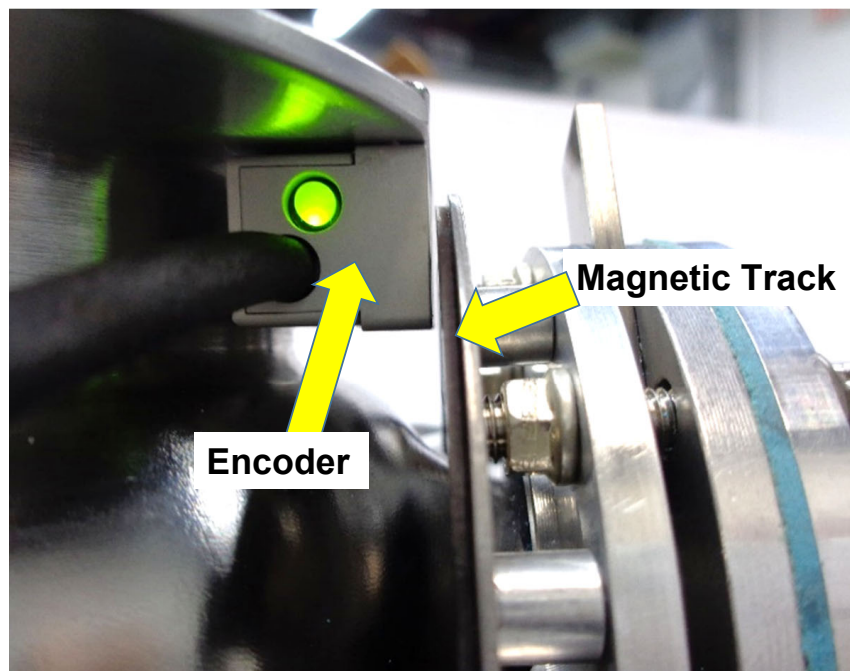


Figure 5.19: Side view of pitch encoder, green light denotes proper working condition with magnetic track within acceptable distance.

lose power at some non-initial position, then regain power, the readings would auto-reset to the initial angle regardless of the magnetic track position. It is recommended that in the future, these encoders should use RLC2IC or RLM printheads which can detect reference marks. Then the user can power on the encoders, manually sweep the collective and the encoders automatically set the correct angle consistently every time.

5.8 Shaft Torque Strain Gauge

To calibrate the shaft torque gauge, the main shaft had to be removed from the MTR and mounted to a fixed frame shown in Fig. 5.20. This allowed for a torque to be applied without the shaft rotating. Yellow arrows in Fig. 5.20(a) shows the load direction about the pulley. The moment arm allows a pure torque to be applied about the shaft; however, as the torque increases the moment arm rotates slightly as seen in Fig. 5.20(b). The result is a shortened moment arm which was accounted for in the calibration.

Figure 5.21 shows the calibration results for the shaft torque strain gauge. The measured strains were very small for the applied torques and the signal to noise ratio is generally low as well. The error bars show twice the standard deviation of the signal which is 95% of data within the bars. The conclusion is the shaft torque gauge is not more accurate than the load cell; however, it can be a secondary verifying signal. After combining all the trials, the relationship between the strains and applied torques was calculated to be $161.5 \text{ in-lb}/\mu\epsilon$.

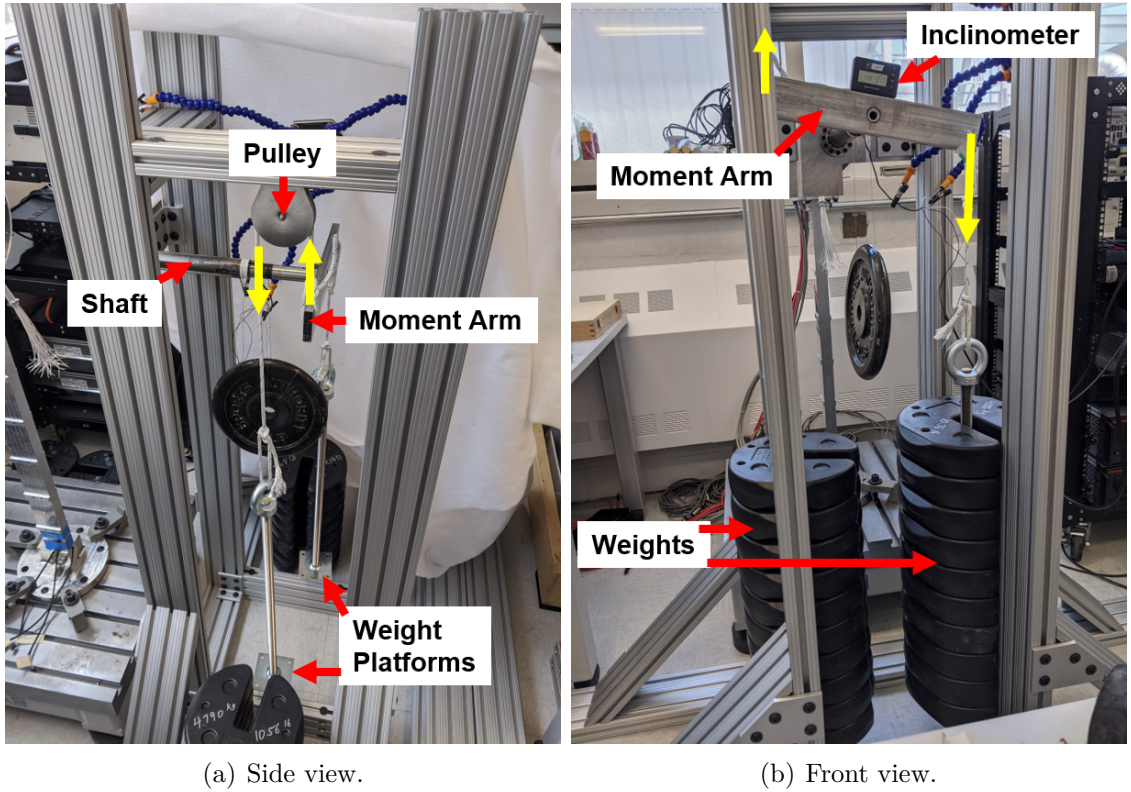


Figure 5.20: Shaft torque gauge calibration frame.

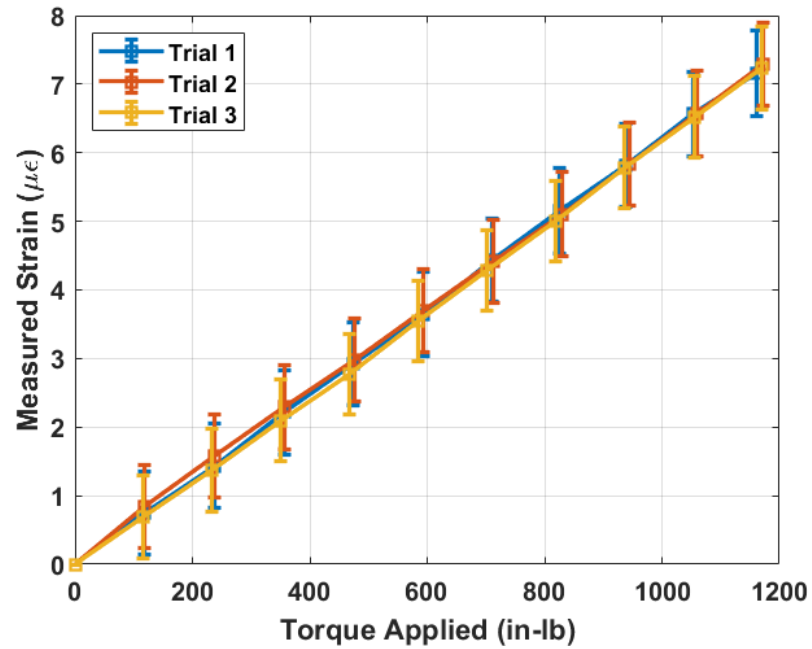


Figure 5.21: Shaft torque gauge calibration results for three trials with errors bars displaying twice the standard deviation.

During testing, the wires on the strain gauge broke at the junction of the shaft and yoke. It was not essential for the whirl flutter test, so repair was postponed. This should be repaired before future tests.

5.9 Pitch Link Strain Gauges

Each pitch link strain gauge was calibrated for tensile loads up to 6 kg (13.23 lb). Tests were conducted with loading and unloading weights to observe for hysteresis. Results are shown in Fig. 5.22.

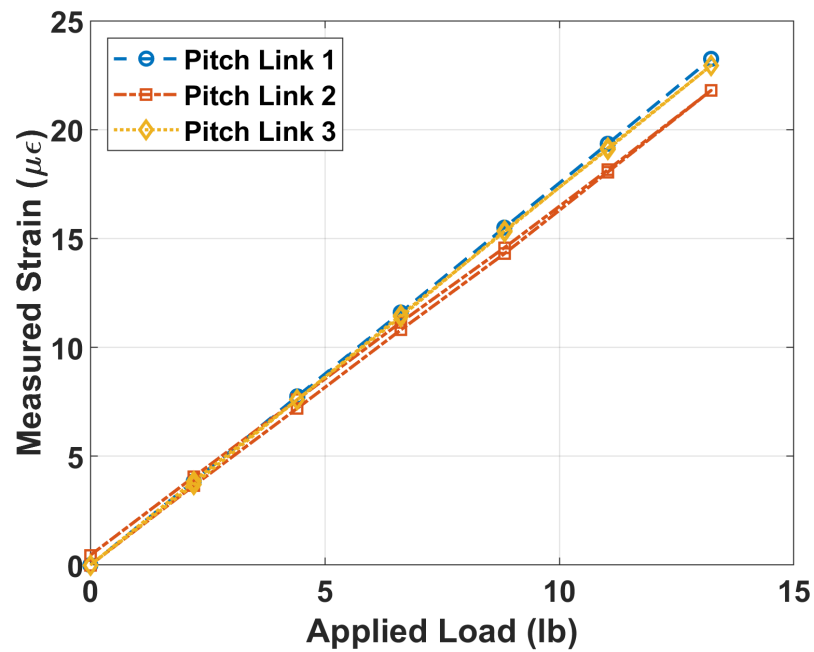


Figure 5.22: Pitch link calibration.

Minor hysteresis was observed for pitch link 2. Fitting a line through the results gives the conversion between strain to pounds. Pitch link 1 factor is 0.5699 lb/μϵ, pitch link 2 factor is 0.6262 lb/μϵ, and pitch link 3 factor is 0.5778 lb/μϵ.

5.10 Gimbal Angle Hall Effect Sensors

The gimbal angle Hall effect sensors are from Melexis Technologies MLX90316-DC. One board is attached to the bearing housing, as shown in Fig. 5.23, and reads a diametrically magnetized magnet installed in the gimbal spider component. This is gimbal sensor 1. The other board is attached on the inner surface of the gimballed hub housing, as shown in Fig. 5.24, also reading a magnet on an orthogonal arm of the spider component. This is gimbal sensor 2.

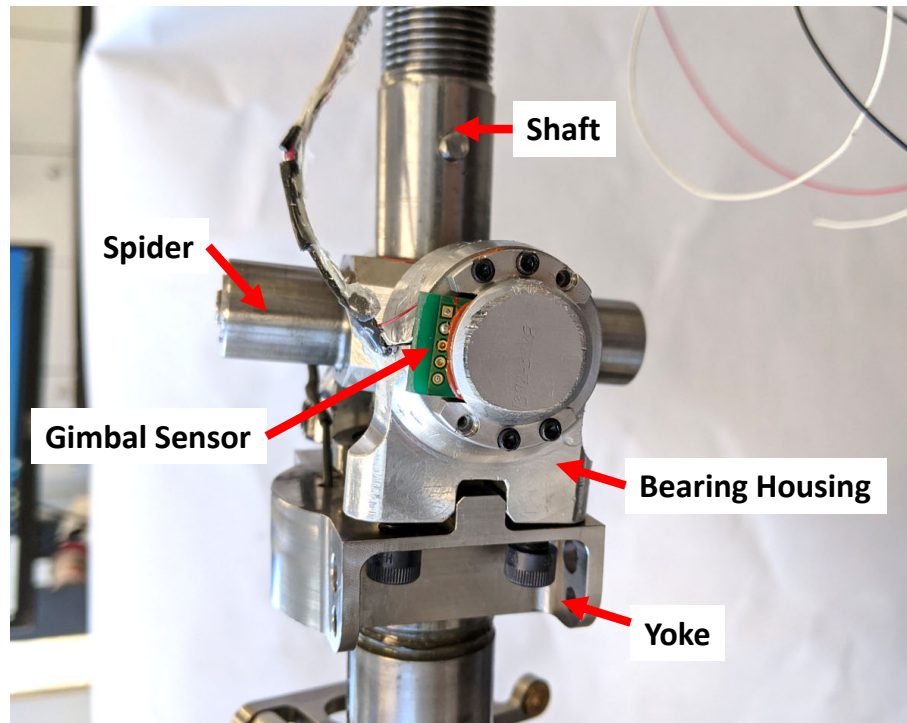


Figure 5.23: Gimbal Hall effect sensor 1 mounted on the bearing housing.

The gimbal sensors were calibrated with the rotor upright, gimbal locked, and with the pitch encoder plate removed, as shown in Fig. 5.25. The hub arms have a 2° pre-cone built-in and there were no flat surfaces perpendicular to the main shaft along the hub arms for pitch gauge placement. Thus, the hub had to be disassembled

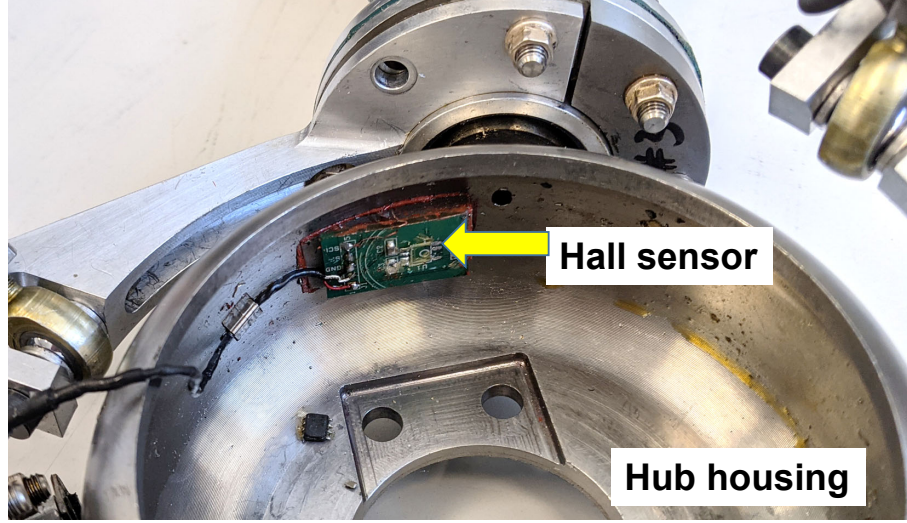


Figure 5.24: Gimbal Hall effect sensor 2 mounted on the inside of the hub.

down to the steel hub housing where the pitch gauge could magnetically adhere to the flat rim on top. The pitch gauge was placed above the location of the gimbal sensor within the hub. Figure 5.26 shows the mapping of the gimbal voltage to the pitch gauge measurements.

For the sensor on the bearing housing, the mapping equation is:

$$D_{g_1} = 107.3262 V_{g_1} - 261.1206 \quad (5.9)$$

For the sensor on the hub housing, the equation is:

$$D_{g_2} = -70.3976 V_{g_2} + 212.2174 \quad (5.10)$$

where V_{g_1} and V_{g_2} are the gimbal sensor voltages and D_{g_1} and D_{g_2} are the output angles in degrees. The mapping equations were found by rotating the gimbal, reading the pitch gauge measurement, and recording the voltage given by the sensors for

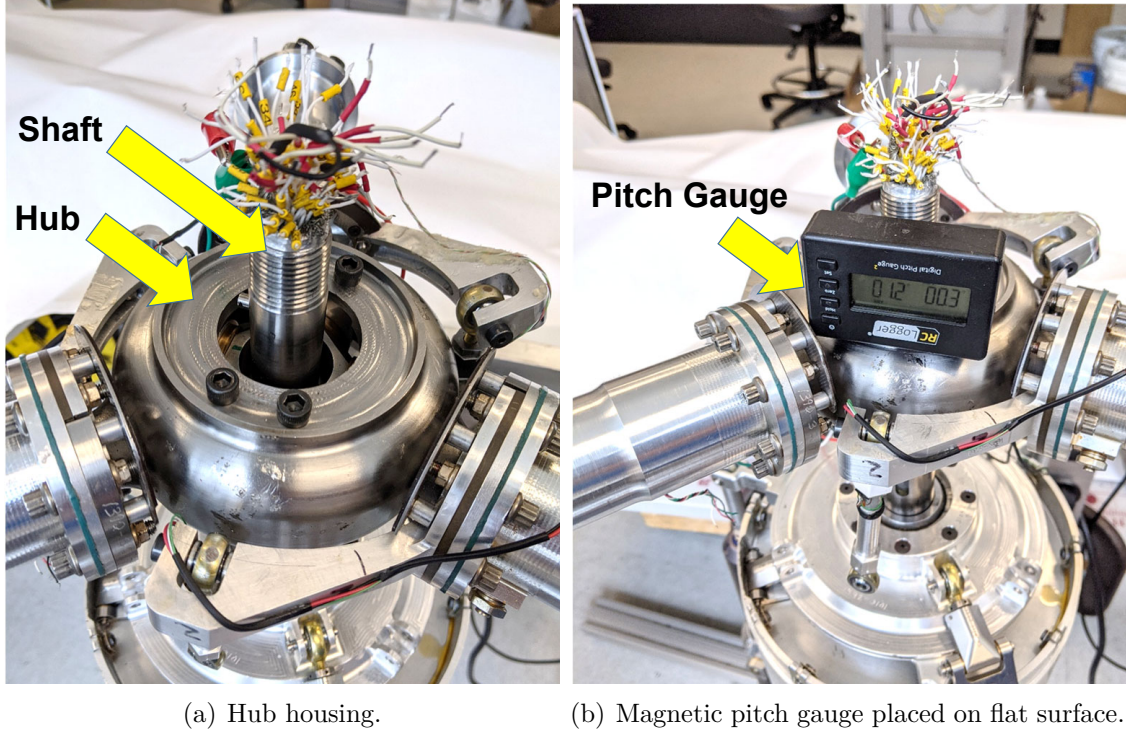


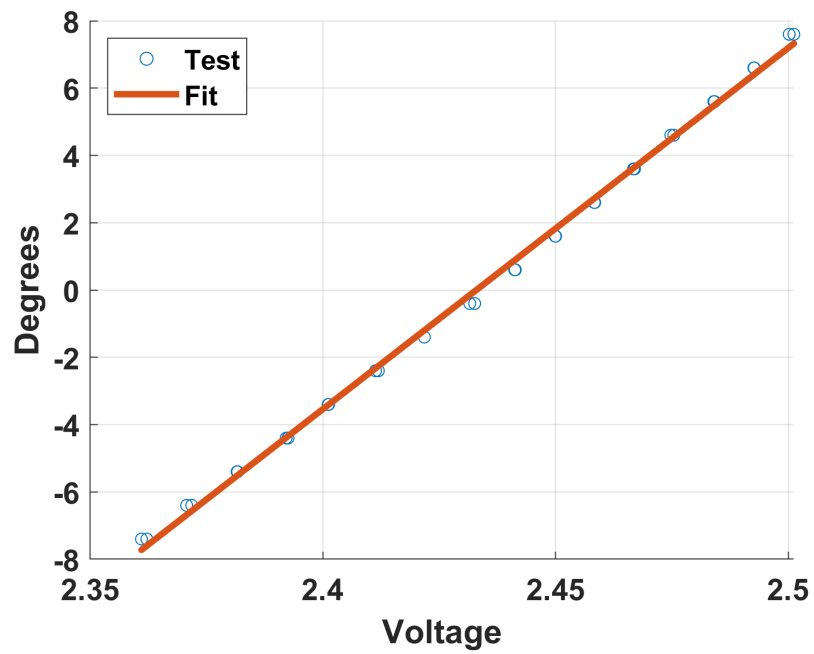
Figure 5.25: Gimbal Hall effect sensor calibration setup.

a minimum of three trials. While the gimbal was free to rotate in any direction, isolating one sensor by rotating along one axis was not difficult.

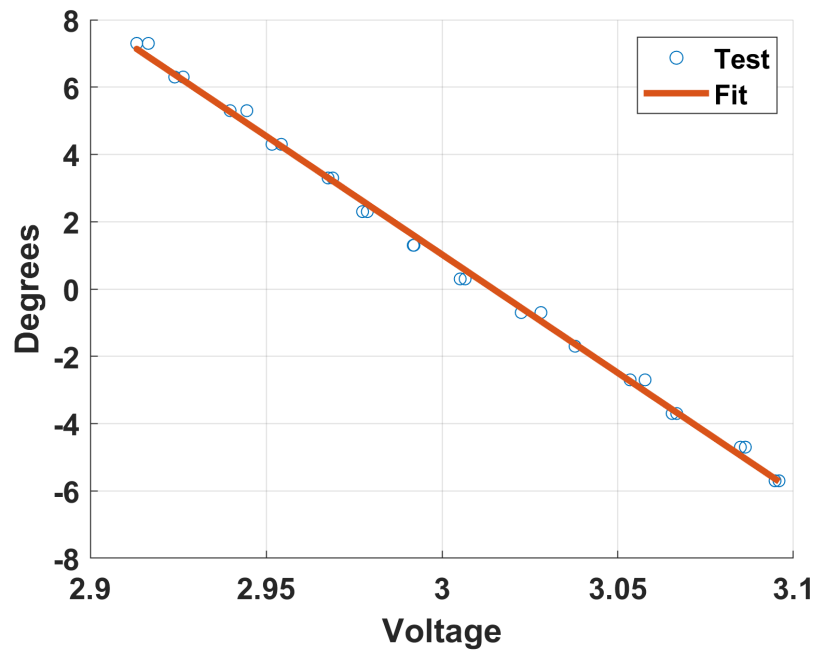
These gimbal angles are converted to fixed frame gimbal angles, β_{1s} and β_{1c} , in conjunction with the azimuth sensor. Lateral flap, β_{1s} , is about the X-axis of the rotor. Longitudinal flap, β_{1c} , is about the negative Y-axis of the rotor. To calculate β_{1s} and β_{1c} , the following relations can be used.

$$\begin{aligned}\beta_{1s} &= D_{g1} \cos(\psi) - D_{g2} \sin(\psi) \\ \beta_{1c} &= -D_{g1} \sin(\psi) - D_{g2} \cos(\psi)\end{aligned}\tag{5.11}$$

where ψ is the rotor azimuth in radians. Zero azimuth is when blade 1 is aligned with the positive X-axis.



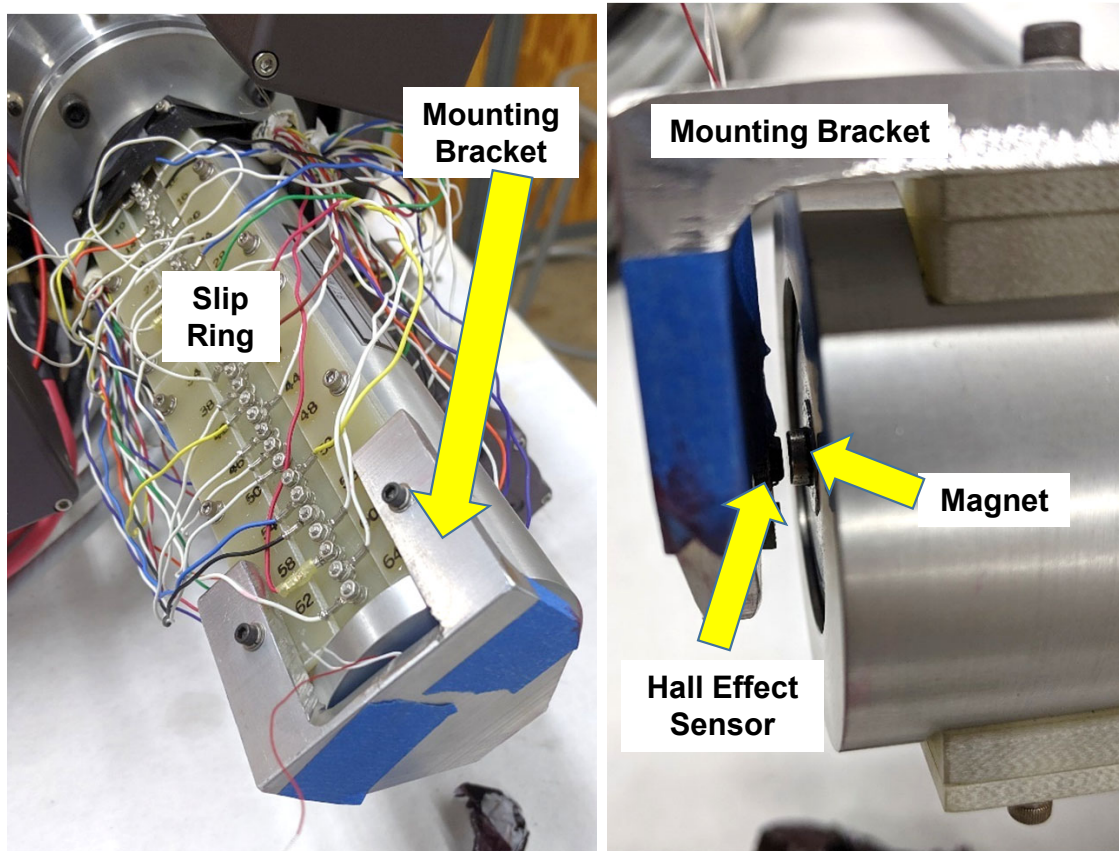
(a) Calibration for gimbal sensor 1 on bearing housing.



(b) Calibration for gimbal sensor 2 on hub housing.

Figure 5.26: Gimbal Hall effect sensor calibrations.

Note, the Hall effect sensors are sensitive to input power. A DC power supply provided a steady 5.00 V power to the sensors during calibration. The same power supply was used for wind tunnel testing.



(a) Rear view of slip ring.

(b) Side view.

Figure 5.27: Azimuth Hall effect sensor.

5.11 Azimuth Hall Effect Sensor

The azimuth Hall effect sensor is the same device as the gimbal Hall effect sensors. A magnet is mounted to the end of the slip ring shaft and the sensor board is mounted to a plate that is attached to the slip ring, as shown in Fig. 5.27.

The azimuth sensor voltage ranges from 0.548V to 4.413V and this is initially mapped to 0° to 360° azimuth. However, the true zero azimuth position is when blade 1 on the hub is aligned in the positive X-axis direction, as shown in Fig. 4.2. This position is -40.117° from the initial mapping. The azimuth sensor generates a sawtooth signal and the RPM is determined from the number of peaks detected per minute.

5.12 Wing Strain Gauges

The wing strain gauges are the most elementary of all sensors yet the most crucial. These sensors will ultimately measure flutter data. The metal foil gauges are attached near the root of the wing spar, as shown in Fig. 5.28.

Two sets of gauges are available with the lowest set located at 4.38% of the wing span for beam and chord, and 6.93% for torsion; the second set is located at 13.72% for beam and chord gauges, and 18.1% for torsion. The wing span is measured from the top surface of the base clamp bracket to the center of the pylon, which is 34.25 inches. Only one set of beam, chord, and torsion gauges are recording during testing, usually the lowest set. The second set of gauges is for redundancy in

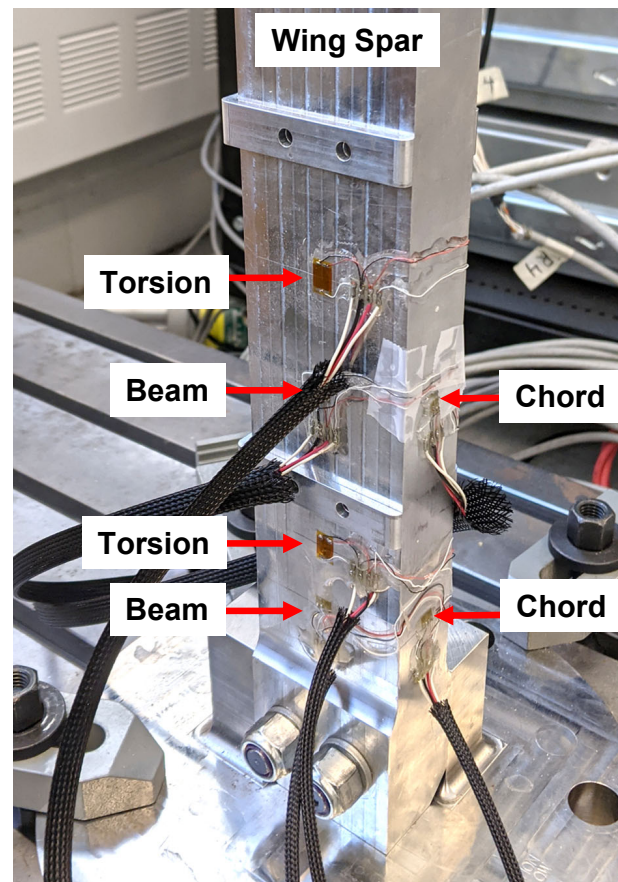


Figure 5.28: Wing strain gauges on the wing spar.

the event the first set of gauges malfunction. These gauges are used for frequency and damping extractions so they were not calibrated for loads.

The beam and chord gauges are double linear strain gauges which are two strain gauges laid parallel to each other. With a double linear strain gauge adhered on opposite faces of the spar, a Wheatstone full-bridge circuit is obtained. The torsion gauges used are dual-grid shear gauges, also adhered to opposite faces of the wing spar. Integrating the strain gauges into the data acquisition system was done by referring to Ref. [36]. A full-bridge circuit provides the highest measurement accuracy and the most resistance to temperature changes compared to quarter- or half-bridge circuits.

5.13 Data Acquisition System

This section details the hardware off-rig and on the equipment rack shown in Fig. 5.29. Two chassis are used for data acquisition: NI PXIe-1082 and NI CompactDAQ-9174. Data is acquired at 10 kHz.

The PXIe-1082 chassis has a collection of installed modules: PXIe-8840 Quad Core Embedded Controller, two PXIe-4331 8-Ch Bridge Analog Input modules, PXIe-4492 Sound and Vibration, PXIe-4322 8-Ch Analog Output, PXIe-8430/8 RS232, and PXIe-6365 Multifunction I/O. The cDAQ-9174 chassis holds two NI-9401 Digital I/O Input modules.

The PXIe-4331 acquires the strain gauge measurements for four blade loads, three pitch link loads, one shaft torque, and three wing strains, with five extra chan-

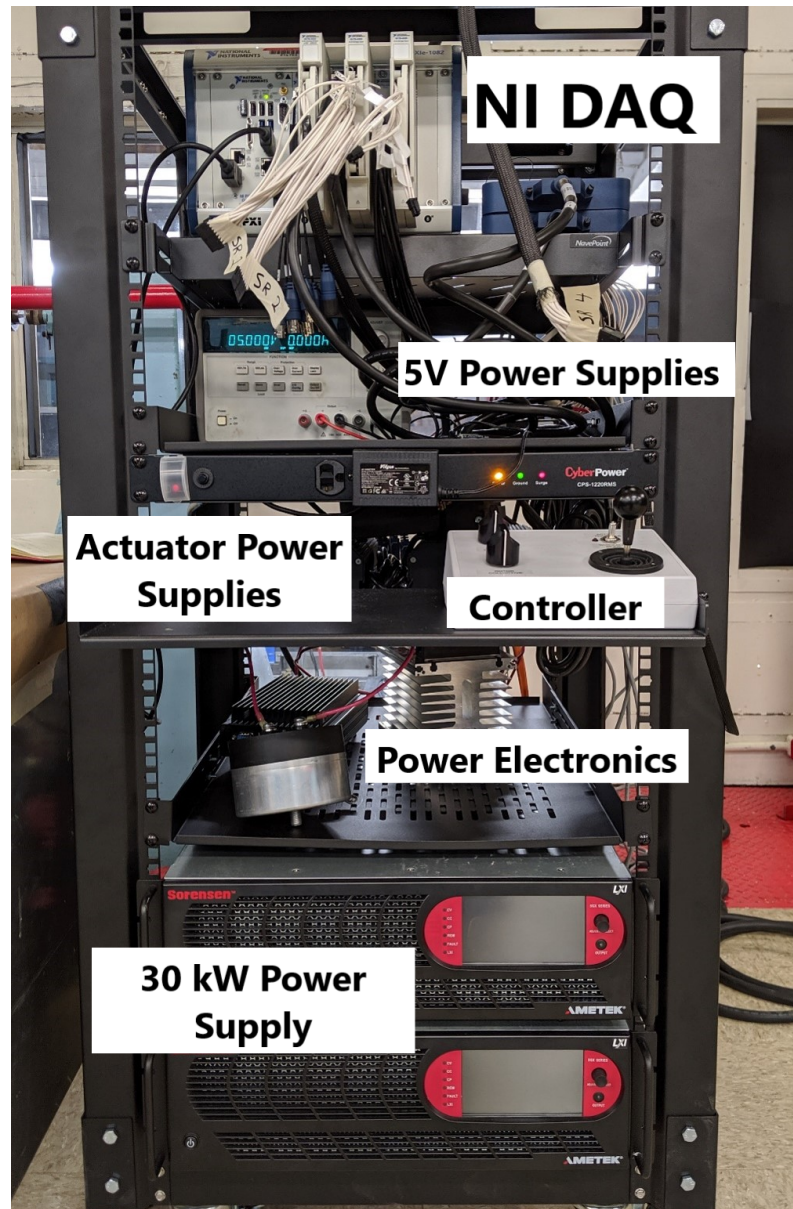


Figure 5.29: MTR systems support rack.

nels available. The PXIe-4492 acquires the tri-axial accelerometer measurements. The PXIe-4322 module provides the voltage to control the electric actuators, motor throttle, and brake. The PXIe-8430/8 module uses four ports to read motor data (1 port) and actuator positions (3 ports). The PXIe-6365 module reads the analog voltage signals of the two gimbal Hall effect sensors, the shaft speed and position Hall effect sensor, and the six load cell measurements. The NI-9401 modules acquire the pitch encoder digital pulse signals.

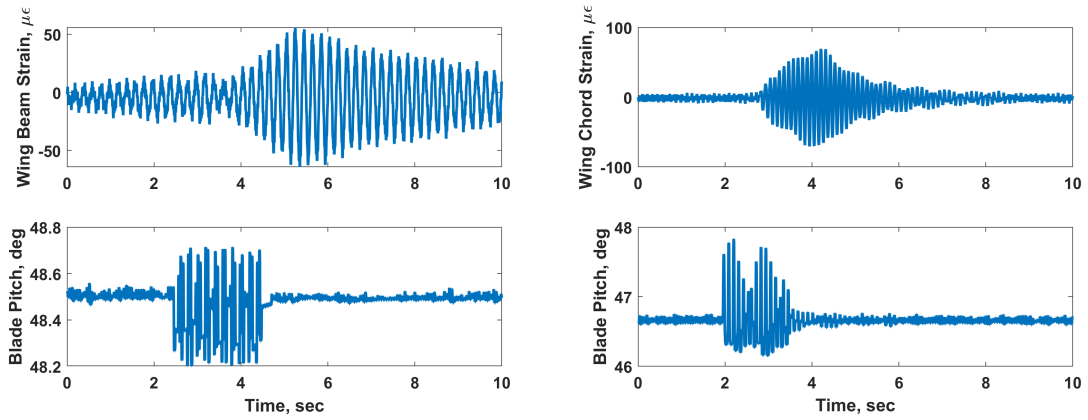
Two regulated DC power supplies are on the shelf below the DAQs to provide 5V power to sensors. One supply powers the two gimbal Hall sensors and the load cell, and the other powers the three pitch encoders.

There are three dedicated power supplies to power each Ultra Motion A2 actuator. Each Ultra Motion unregulated DC power supply can provide 36V/360W continuous output with high burst current, and has an integrated shunt for over-voltage protection.

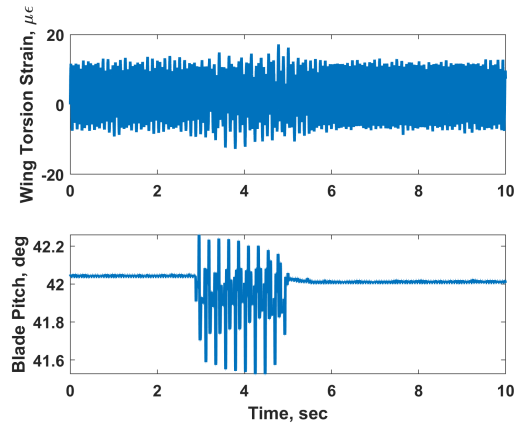
The front of the rack has a shelf that holds the pilot controller. The pilot controller was fabricated in-house. There are two dials, a switch, and a joystick. One dial controls the motor throttle, while the other dial controls the collective angle of the blades. The switch toggles the motor throttle to be active or inactive for the freewheeling condition. The joystick is used as a rate controller for the cyclic motion of the swashplate.

The rotor control system was divided into manual control for trimming and automated control for dynamic excitation. Manual control was performed through the joystick and dials on the physical controller on the equipment rack. The operator

was able to adjust the dial for collective and use the joystick to trim the gimballed hub to zero cyclic first harmonic flapping by monitoring a plot of lateral flapping, β_{1s} , versus longitudinal flapping, β_{1c} . Due to the manual nature of this control, there was difficulty in forcing the gimbal to achieve precisely zero flapping. Therefore, gimbal flapping was trimmed to $\pm 2.5^\circ$ or less. The cyclic control was necessary only for the gimbal free conditions.



(a) Wing beam response due to a 5 Hz longitudinal cyclic excitation. (b) Wing chord response due to a 9.5 Hz collective excitation.



(c) Wing torsion response due to a 14.5 Hz longitudinal cyclic excitation.

Figure 5.30: Flutter excitation of wing modes through swashplate actuation.

Flutter excitation was performed through the LabVIEW interface. The DAQ operator was able to select collective, longitudinal or lateral cyclic, input frequency, input voltage, and the number of cycles. Once the manual control trimmed the rotor, the DAQ operator pressed a start button to activate the excitation of the swashplate according to the parameters. Figure 5.30 shows the wing responses from the swashplate excitation. The excitation is shown as a blade pitch oscillation. Figure 5.30(c) shows the torsion mode was difficult to excite and the response was not substantial enough to obtain an accurate measurement of damping.

5.14 Chapter Summary and Conclusions

This chapter described the instrumentation on the Maryland Tiltrotor Rig, the calibration of the instruments, as well the data acquisition system. It covered the details of the following.

1. The development of a new calibration stand for the load cell. This stand allowed for calibration to be performed while the load cell was mounted in the pylon, creating a test-like environment.
2. An explanation of the load cell calibration results, showing that thrust and torque were the most accurate and most unaffected by the secondary load path present in the pylon.
3. A description of the gimbal Hall effect sensors and azimuth Hall effect sensor. These are used to map the rotating hub to fixed frame β_{1S} and β_{1C} angles.

4. A description of the wing strain gauges used to measure the frequency and damping of the rig. All gauges were full-bridge circuits which provide the highest accuracy and most resistance to temperature changes. There were two sets of full-bridge circuits for each wing mode. However, only one set recorded data during testing, the other set was for redundancy and backup.
5. A description of the data acquisition system. Various NI modules were used to acquire data at 10 kHz for all the rotating and stationary sensors on the MTR.

The chapter concludes the following.

1. The MTR is now fully instrumented and calibrated for testing. In total, it has 22 sensors.
2. The high frequency electric actuators allowed for a range of 56° in blade collective pitch. These are the principal instruments to activating flutter at the wing-pylon frequencies.
3. The shaft torque gauge was not sensitive enough to provide an accurate measurement for torque with a $161.5 \text{ in-lb}/\mu\epsilon$ calibration, but it can be used as a backup measurement device.

Some recommendations for the future are provided.

1. The blade pitch encoders measured the blade root pitch. The actual θ_{75} angle should be directly measured for each new blade set due to fabrication discrepancies in the blade root insert.

2. If an instrument is replaced, rewired, or its power supply is replaced, a recalibration should be performed to ensure accuracy.
3. The shaft torque gauge was left unattached for future students to use as backup load cell torque readings.
4. The rotary magnetic encoders should be upgraded in the future for automatic zero-setting capability.

Chapter 6: Whirl Flutter Analysis

A simplified model was developed to determine the flutter boundary and the behavior of rotor and wing modes as a function of windspeed. This model was meant for pre-test assessment, not detailed validation. This chapter will describe the model methodology, the derivation of which can be found in Johnson [4], and the input parameters for the model.

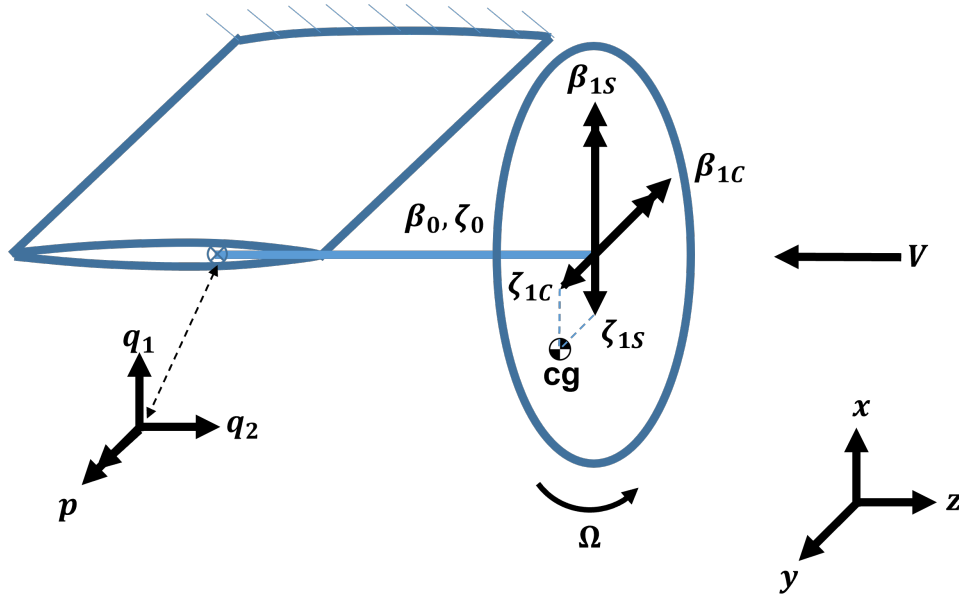


Figure 6.1: Tiltrotor whirl flutter model.

6.1 Methodology

The model consists of a straight wing and gimballed rotor in cruise flight with the shaft always parallel to free stream velocity V , thus the rotor is in pure axial flow. The wing is cantilevered at the wing root. A large, heavy pylon is rigidly attached to the wing tip. The rotor is mounted to the pylon with the hub forward of the wing elastic axis, with the rotor shaft parallel to V . The rotor has three blades, with first mode flap and lag motion for each blade. The rotor hub forces and moments are transmitted through the pylon to the wing tip. Nine degrees of freedom are used to model the wing and rotor, as shown in Fig. 6.1. There are three modes for the wing: q_1 beam bending, q_2 chord bending, and p torsion. There are six rotor modes because the rotor has flexible blades with a gimballed hub. β_0 , β_{1c} , β_{1s} denote the collective flap, longitudinal flap, and lateral flap, respectively. ζ_0 , ζ_{1c} , and ζ_{1s} denotes the collective lag, longitudinal lag, and lateral lag, respectively, which occurs in-plane of the hub. The lag motions move the rotor center of gravity denoted as cg shown in the figure.

The nine equations of motion are in the form:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0} \quad (6.1)$$

where M , C , and K are 9×9 matrices for mass, damping, and stiffness, respectively, and the degree-of-freedom vector is:

$$\mathbf{x} = \begin{bmatrix} \beta_{1c} \\ \beta_{1s} \\ \zeta_{1c} \\ \zeta_{1s} \\ \beta_0 \\ \zeta_0 \\ q_1 \\ q_2 \\ p \end{bmatrix} \quad (6.2)$$

6.2 Rotor Equations

There are three flap equations and three lag equations for a total of six rotor equations. Terms with $*$ are the azimuthal derivative of the parameter. Terms with $^{\sim}$ overhead are non-dimensional lengths, normalized by blade radius R . The three

rotor flap equations in multiblade coordinates are

$$\begin{aligned}
 I_{\beta 0}^* \left(\beta_0^{**} + \nu_{\beta 0}^2 \beta_0 \right) + S_{\beta}^* \bar{z}_P^{**} &= \gamma \bar{M}_{\beta 0} \\
 I_{\beta}^* \left[\beta_{1C}^{**} + 2 \beta_{1S}^* + (\nu_{\beta}^2 - 1) \beta_{1C} \right] + I_{\beta}^* \left(-\alpha_Y^{**} + 2 \alpha_X^* \right) &= \gamma \bar{M}_{\beta 1C}
 \end{aligned} \tag{6.3}$$

$$I_{\beta}^* \left[\beta_{1S}^{**} - 2 \beta_{1C}^* + (\nu_{\beta}^2 - 1) \beta_{1S} \right] + I_{\beta}^* \left(\alpha_X^{**} + 2 \alpha_Y^* \right) = \gamma \bar{M}_{\beta 1S}$$

where the right-hand side perturbation flap moments are

$$\begin{aligned}
 \bar{M}_{\beta 0} &= M_{\theta} (\theta_0 - K_P \beta_0) + \\
 &M_{\dot{\zeta}} \left(\alpha_Z^* - \zeta_0^* \right) + \\
 &M_{\lambda} \bar{z}_P^* + \\
 &M_{\dot{\beta}} \beta_0^*
 \end{aligned} \tag{6.4}$$

$$\begin{aligned}
 \bar{M}_{\beta 1C} &= M_{\theta} (\theta_{1C} - K_P \beta_{1C}) + \\
 &M_{\mu} \left(-\bar{h} \alpha_X^* + \bar{y}_P^* + \lambda \alpha_X \right) + \\
 &M_{\dot{\zeta}} \left(-\zeta_{1C}^* - \zeta_{1S} \right) + \\
 &M_{\dot{\beta}} \left(\beta_{1C}^* + \beta_{1S} - \alpha_Y^* \right)
 \end{aligned} \tag{6.5}$$

$$\begin{aligned}
\overline{M}_{\beta_{1S}} = & M_{\theta} (\theta_{1S} - K_P \beta_{1S}) + \\
& M_{\mu} \left(-\bar{h} \overset{*}{\alpha}_Y - \bar{x}_P \overset{*}{\alpha}_Y + \lambda \alpha_Y \right) + \\
& M_{\dot{\zeta}} \left(-\overset{*}{\zeta}_{1S} + \zeta_{1C} \right) + \\
& M_{\dot{\beta}} \left(\overset{*}{\beta}_{1S} - \beta_{1C} + \overset{*}{\alpha}_X \right)
\end{aligned} \tag{6.6}$$

The three rotor lag equations in multiblade coordinates are

$$\begin{aligned}
\overset{*}{I}_{\zeta 0} \left(\overset{**}{\zeta}_0 + \nu_{\zeta 0}^2 \overset{*}{\zeta}_0 \right) - \overset{*}{I}_{\zeta 0} \overset{**}{\alpha}_Z &= \gamma \overline{M}_{\zeta 0} \\
\overset{*}{I}_{\zeta} \left[\overset{**}{\zeta}_{1C} + 2 \overset{*}{\zeta}_{1S} + (\nu_{\zeta}^2 - 1) \overset{*}{\zeta}_{1C} \right] + \overset{*}{S}_{\zeta} \left(-\bar{y}_P \overset{**}{\alpha}_X + \bar{h} \overset{**}{\alpha}_X \right) &= \gamma \overline{M}_{\zeta 1C} \\
\overset{*}{I}_{\zeta} \left[\overset{**}{\zeta}_{1S} - 2 \overset{*}{\zeta}_{1C} + (\nu_{\zeta}^2 - 1) \overset{*}{\zeta}_{1S} \right] + \overset{*}{S}_{\zeta} \left(\bar{x}_P \overset{**}{\alpha}_Y + \bar{h} \overset{**}{\alpha}_Y \right) &= \gamma \overline{M}_{\zeta 1S}
\end{aligned} \tag{6.7}$$

where the right-hand side perturbation lag moments in fixed coordinates have the same form as the flap moments, only the M -coefficients are changed to Q -coefficients.

$$\begin{aligned}
\overline{M}_{\zeta_0} = & Q_\theta (\theta_0 - K_P \beta_0) + \\
& Q_{\dot{\zeta}} \left(\overset{*}{\alpha}_Z - \overset{*}{\zeta}_0 \right) + \\
& Q_\lambda \bar{z}_P^* + \\
& Q_{\dot{\beta}} \overset{*}{\beta}_0
\end{aligned} \tag{6.8}$$

$$\begin{aligned}
\overline{M}_{\zeta_{1C}} = & Q_\theta (\theta_{1C} - K_P \beta_{1C}) + \\
& Q_\mu \left(-\bar{h} \overset{*}{\alpha}_X + \bar{y}_P^* + \lambda \alpha_X \right) + \\
& Q_{\dot{\zeta}} \left(-\overset{*}{\zeta}_{1C} - \zeta_{1S} \right) + \\
& Q_{\dot{\beta}} \left(\overset{*}{\beta}_{1C} + \beta_{1S} - \overset{*}{\alpha}_Y \right)
\end{aligned} \tag{6.9}$$

$$\begin{aligned}
\overline{M}_{\zeta_{1S}} = & Q_\theta (\theta_{1S} - K_P \beta_{1S}) + \\
& Q_\mu \left(-\bar{h} \overset{*}{\alpha}_Y - \bar{x}_P^* + \lambda \alpha_Y \right) + \\
& Q_{\dot{\zeta}} \left(-\overset{*}{\zeta}_{1S} + \zeta_{1C} \right) + \\
& Q_{\dot{\beta}} \left(\overset{*}{\beta}_{1S} - \beta_{1C} + \overset{*}{\alpha}_X \right)
\end{aligned} \tag{6.10}$$

6.3 Wing Equations

The three wing equations of motion in matrix form are

$$\begin{aligned}
& \begin{bmatrix} I_q^* + m_P^* & 0 & S_w^* \\ 0 & I_q^* + I_{PX}^* \eta'^2 + m_P^* & -S_w^* \delta_{w_2} \\ S_w^* & -S_w^* \delta_{w_2} & I_p^* + I_{PY}^* \end{bmatrix} \begin{pmatrix} q_1^{**} \\ q_2^{**} \\ p^{**} \end{pmatrix} \\
& + \begin{bmatrix} C_{q_1}^* & 0 & 0 \\ 0 & C_{q_2}^* & 0 \\ 0 & 0 & C_p^* \end{bmatrix} \begin{pmatrix} q_1^* \\ q_2^* \\ p^* \end{pmatrix} \\
& + \begin{bmatrix} K_{q_1}^* & 0 & 0 \\ 0 & K_{q_2}^* & 0 \\ C_{pq}^* \eta (\gamma 2C_T / \sigma a) & -C_{pq}^* \eta (\gamma 2C_T / \sigma a) \delta_{w_2} & K_p^* \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ p \end{pmatrix} \\
& = \begin{pmatrix} M_{q_1 \text{aero}}^* \\ M_{q_2 \text{aero}}^* \\ M_{p \text{aero}}^* \end{pmatrix} + \gamma \begin{bmatrix} \eta' \delta_{w_3} (1 - \xi) & -\eta' \delta_{w_2} \\ -\eta' \delta_{w_1} (1 - \xi) & -\eta' \\ -1 & -\delta_{w_1} (1 - \xi) \end{bmatrix} \begin{pmatrix} -2C_{M_Y} / \sigma a \\ 2C_{M_X} / \sigma a \end{pmatrix} \\
& + \gamma \begin{bmatrix} \eta - \eta' \delta_{w_3} \bar{h} (1 - \xi) & -\eta' \delta_{w_2} \bar{h} + \eta \delta_{w_1} (1 - \xi^2) \\ -\eta \delta_{w_2} + \eta' \delta_{w_3} \bar{h} (1 - \xi) & -\eta' \bar{h} + \eta \delta_{w_3} (1 - \xi^2) \\ \bar{h} & \delta_{w_1} \bar{h} (1 - \xi) \end{bmatrix} \begin{pmatrix} 2C_H / \sigma a \\ -2C_Y / \sigma a \end{pmatrix} \\
& + \gamma \begin{bmatrix} 2\eta' \\ -2\eta' \delta_{w_2} \\ 2\delta_{w_3} (1 - \xi) \end{bmatrix} C_Q / \sigma a + \gamma \begin{bmatrix} -2\eta \delta_{w_2} \\ -2\eta \\ -2\xi \delta_{w_1} \eta \end{bmatrix} C_T / \sigma a
\end{aligned} \tag{6.11}$$

where shape functions $\eta(y) = y^2/y_T$ and $\eta'(y) = 2y/y_T$ are evaluated at the wing tip y_T . These result in $\eta(y_T) = y_T$ and $\eta'(y_T) = 2$. Furthermore, η is normalized by R because the input parameters are non-dimensional, so $\eta = \bar{y}_T$. The wing dihedral angle is δ_{w_1} , δ_{w_2} is the wing incidence angle of attack, and δ_{w_3} is the wing sweepback angle (for forward swept wing, δ_{w_3} is negative) in radians. For the MTR, these wing angles are set to zero, which simplifies Eq. (6.11) to

$$\begin{aligned}
& \begin{bmatrix} I_q^* + m_P^* & 0 & S_w^* \\ 0 & I_q^* + I_{PX}^* \eta'^2 + m_P^* & 0 \\ S_w^* & 0 & I_p^* + I_{PY}^* \end{bmatrix} \begin{pmatrix} q_1^{**} \\ q_2^{**} \\ p^{**} \end{pmatrix} \\
& + \begin{bmatrix} C_{q_1}^* & 0 & 0 \\ 0 & C_{q_2}^* & 0 \\ 0 & 0 & C_p^* \end{bmatrix} \begin{pmatrix} q_1^* \\ q_2^* \\ p^* \end{pmatrix} \\
& + \begin{bmatrix} K_{q_1}^* & 0 & 0 \\ 0 & K_{q_2}^* & 0 \\ C_{pq}^* \eta (\gamma 2C_T / \sigma a) & 0 & K_p^* \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ p \end{pmatrix} \\
& = \begin{pmatrix} M_{q_1 \text{aero}}^* \\ M_{q_2 \text{aero}}^* \\ M_{p \text{aero}}^* \end{pmatrix} + \gamma \begin{bmatrix} 0 & 0 \\ 0 & -\eta' \\ -1 & 0 \end{bmatrix} \begin{pmatrix} -2C_{M_Y} / \sigma a \\ 2C_{M_X} / \sigma a \end{pmatrix} \\
& + \gamma \begin{bmatrix} \eta & 0 \\ 0 & -\eta' \bar{h} \\ \bar{h} & 0 \end{bmatrix} \begin{pmatrix} 2C_H / \sigma a \\ -2C_Y / \sigma a \end{pmatrix} \\
& + \gamma \begin{bmatrix} 2\eta' \\ 0 \\ 0 \end{bmatrix} C_Q / \sigma a + \gamma \begin{bmatrix} 0 \\ -2\eta \\ 0 \end{bmatrix} C_T / \sigma a
\end{aligned} \tag{6.12}$$

Equations 6.11 and 6.12 notation for $\eta(y_T)$ and $\eta'(y_T)$ are shortened to η and η' for convenience. The constant $C_{pq}^* \cong 2/3$ is a term that accounts for the bending

motions of the wing creating a torsion moment due to the movement of the rotor thrust. The wing aerodynamic coefficients on the right-hand side are

$$\begin{pmatrix} {}^*M_{q_1\text{aero}} \\ {}^*M_{q_2\text{aero}} \\ {}^*M_{p\text{aero}} \end{pmatrix} = \begin{bmatrix} C_{q_1\dot{q}_1}^* & C_{q_1\dot{q}_2}^* & C_{q_1\dot{p}}^* \\ C_{q_2\dot{q}_1}^* & C_{q_2\dot{q}_2}^* & C_{q_2\dot{p}}^* \\ C_{p\dot{q}_1}^* & C_{p\dot{q}_2}^* & C_{p\dot{p}}^* \end{bmatrix} \begin{pmatrix} {}^*q_1 \\ {}^*q_2 \\ {}^*p \end{pmatrix} + \begin{bmatrix} C_{q_1q_1}^* & C_{q_1q_2}^* & C_{q_1p}^* \\ C_{q_2q_1}^* & C_{q_2q_2}^* & C_{q_2p}^* \\ C_{pq_1}^* & C_{pq_2}^* & C_{pp}^* \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ p \end{pmatrix} \quad (6.13)$$

These coefficients are

$$\begin{aligned}
C_{q_1 \dot{q}_1}^* &= -d_{13} \lambda C_{L_\alpha} e_2 \\
C_{q_1 \dot{q}_2}^* &= -d_{13} \lambda C_{L_O} e_2 \\
C_{q_1 q_1}^* &= -d_{12} \lambda^2 \delta_{w_3} C_{L_\alpha} e_3 \\
C_{q_1 q_2}^* &= -d_{12} \lambda^2 \delta_{w_3} C_{L_O} e_3 \\
C_{q_1 \dot{p}}^* &= d_{22} \frac{1}{2} \lambda \left[\frac{3}{4} + \left(\frac{x_{A_w}}{c_w} \right) \right] C_{L_\alpha} e_4 \\
C_{q_1 p}^* &= d_{12} \lambda^2 C_{L_\alpha} e_4 \\
C_{q_2 \dot{q}_1}^* &= -d_{13} \lambda (C_{D_\alpha} - 2C_{L_O}) e_2 \\
C_{q_2 \dot{q}_2}^* &= -d_{13} \lambda (2C_{D_O} - \delta_{w_2} C_{D_\alpha}) e_2 \\
C_{q_2 q_1}^* &= -d_{12} \lambda^2 \delta_{w_3} (C_{D_\alpha} - 2C_{L_O}) e_3 \\
C_{q_2 q_2}^* &= -d_{12} \lambda^2 \delta_{w_3} (2C_{D_O} - \delta_{w_2} C_{D_\alpha}) e_3 \\
C_{q_2 \dot{p}}^* &= d_{22} \frac{1}{2} \lambda \left\{ \left[\frac{1}{2} + \left(\frac{x_{A_w}}{c_w} \right) \right] (C_{D_\alpha} - 2C_{L_O}) - \frac{1}{4} C_{L_O} \right\} e_4 \\
C_{q_2 p}^* &= d_{12} \lambda^2 (C_{D_\alpha} - C_{L_O}) e_4 \\
C_{p \dot{q}_1}^* &= d_{22} \lambda \left(\frac{x_{A_w}}{c_w} \right) C_{L_\alpha} e_4 \\
C_{p \dot{q}_2}^* &= -d_{22} \lambda^2 C_{m_{ac}} e_4 \\
C_{p q_1}^* &= d_{12} \lambda^2 C_{m_{ac}} f_2 \\
C_{p q_2}^* &= -d_{12} \lambda^2 C_{L_O} f_2 \\
C_{p \dot{p}}^* &= -d_{31} \frac{1}{2} \lambda \left[\frac{1}{4} + \frac{1}{2} \left(\frac{x_{A_w}}{c_w} \right) \right] C_{L_\alpha} f_3 \\
C_{p p}^* &= -d_{21} \lambda^2 \left(\frac{x_{A_w}}{c_w} \right) C_{L_\alpha} f_3
\end{aligned} \tag{6.14}$$

where C_{L_O} and C_{D_O} are the aircraft trim lift and drag coefficients and C_{L_α} and C_{D_α} are their derivatives with respect to angle of attack. For the MTR wing, a NACA 0018 airfoil at 0° incidence angle of attack: $C_{L_O} = 0$, $C_{L_\alpha} = 5.73$, $C_{D_O} = 0.4$, $C_{D_\alpha} = 0$. The distance the aerodynamic center is behind the elastic axis is non-dimensionalized by the chord length as x_{A_w}/c_w , and $C_{m_{ac}}$ is the moment coefficient about the aerodynamic center. The constant $d_{nm} = \bar{c}_w^n \bar{y}_T^m / (\pi \sigma a)$ accounts for the difference in the normalization of the wing and rotor coefficients. Constants e_n and f_n are approximated from integrals of the wing mode shapes and are given as:

$$\begin{aligned}
e_1 &\cong 1/3 \\
e_2 &\cong 1/5 \\
e_3 &\cong 1/2 \\
e_4 &\cong 1/4 \\
f_1 &\cong 1/2 \\
f_2 &\cong 1/12 \\
f_3 &\cong 1/3
\end{aligned} \tag{6.15}$$

For δ_{w_1} , δ_{w_2} , and δ_{w_3} equal to zero, the final wing equations become

$$\begin{aligned}
& \begin{bmatrix} {}^*I_q + {}^*m_P & 0 & {}^*S_w \\ 0 & {}^*I_q + {}^*I_{PX}\eta'^2 + {}^*m_P & 0 \\ {}^*S_w & 0 & {}^*I_p + {}^*I_{PY} \end{bmatrix} \begin{pmatrix} {}^{**}q_1 \\ {}^{**}q_2 \\ {}^{**}p \end{pmatrix} \\
& + \begin{bmatrix} {}^*C_{q_1} - \gamma C_{q_1\dot{q}_1}^* & 0 & -\gamma C_{q_1\dot{p}}^* \\ 0 & {}^*C_{q_2} & 0 \\ 0 & 0 & {}^*C_p - \gamma C_{p\dot{p}}^* \end{bmatrix} \begin{pmatrix} {}^*q_1 \\ {}^*q_2 \\ {}^*p \end{pmatrix} \\
& + \begin{bmatrix} {}^*K_{q_1} & 0 & -\gamma C_{q_1p}^* \\ 0 & {}^*K_{q_2} & 0 \\ {}^*C_{pq}\eta(\gamma 2C_T/\sigma a) & 0 & {}^*K_p \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ p \end{pmatrix} \\
& = \gamma \begin{bmatrix} 0 & 0 \\ 0 & -\eta' \\ -1 & 0 \end{bmatrix} \begin{pmatrix} -2C_{M_Y}/\sigma a \\ 2C_{M_X}/\sigma a \end{pmatrix} \\
& + \gamma \begin{bmatrix} \eta & 0 \\ 0 & -\eta'\bar{h} \\ \bar{h} & 0 \end{bmatrix} \begin{pmatrix} 2C_H/\sigma a \\ -2C_Y/\sigma a \end{pmatrix} \\
& + \gamma \begin{bmatrix} 2\eta' \\ 0 \\ 0 \end{bmatrix} C_Q/\sigma a + \gamma \begin{bmatrix} 0 \\ -2\eta \\ 0 \end{bmatrix} C_T/\sigma a
\end{aligned} \tag{6.16}$$

The rotor force equations are

$$\begin{aligned}
\gamma \frac{C_T}{\sigma a} &= \gamma \frac{C_{Taero}}{\sigma a} - \overset{*}{S}_\beta \overset{**}{\beta}_0 - \overset{*}{M}_b \overset{**}{z}_P \\
\gamma \frac{C_Q}{\sigma a} &= \gamma \frac{C_{Qaero}}{\sigma a} - \overset{**}{\zeta}_0 + \overset{*}{\alpha}_Z \\
\gamma \frac{C_H}{\sigma a} &= \gamma \frac{C_{Haero}}{\sigma a} - \overset{*}{M}_b \left(\overset{**}{\bar{x}}_P + \bar{h} \overset{**}{\alpha}_Y \right) - \frac{1}{2} \overset{*}{S}_\zeta \overset{**}{\zeta}_{1S} \\
\gamma \frac{C_Y}{\sigma a} &= \gamma \frac{C_{Yaero}}{\sigma a} - \overset{*}{M}_b \left(\overset{**}{\bar{y}}_P - \bar{h} \overset{**}{\alpha}_X \right) + \frac{1}{2} \overset{*}{S}_\zeta \overset{**}{\zeta}_{1C}
\end{aligned} \tag{6.17}$$

where

$$\begin{aligned}
\frac{C_{Taero}}{\sigma a} &= \\
&T_\theta (\theta_0 - K_P \beta_0) + \\
&T_{\dot{\zeta}} \left(\overset{*}{\alpha}_Z - \overset{*}{\zeta}_0 \right) + \\
&T_\lambda \overset{*}{\bar{z}}_P + \\
&T_{\dot{\beta}} \overset{*}{\beta}_0
\end{aligned} \tag{6.18}$$

$$\begin{aligned}
\frac{C_{Qaero}}{\sigma a} &= \\
&Q_\theta (\theta_0 - K_P \beta_0) + \\
&Q_{\dot{\zeta}} \left(\overset{*}{\alpha}_Z - \overset{*}{\zeta}_0 \right) + \\
&Q_\lambda \overset{*}{\bar{z}}_P + \\
&Q_{\dot{\beta}} \overset{*}{\beta}_0
\end{aligned} \tag{6.19}$$

$$\begin{aligned}
2 \frac{C_{Haero}}{\sigma a} = & \\
& H_\theta (\theta_{1S} - K_P \beta_{1S}) + \\
& (H_\mu + R_\mu) \left(-\bar{h} \dot{\alpha}_Y^* - \ddot{x}_P^* + \lambda \alpha_Y \right) + \\
& H_{\dot{\zeta}} \left(-\dot{\zeta}_{1S}^* + \dot{\zeta}_{1C} \right) + \\
& H_{\dot{\beta}} \left(\dot{\beta}_{1S}^* - \dot{\beta}_{1C} + \dot{\alpha}_X \right) \\
& - 2 \frac{C_T}{\sigma a} \beta_{1C}
\end{aligned} \tag{6.20}$$

$$\begin{aligned}
2 \frac{C_{Yaero}}{\sigma a} = & \\
& - H_\theta (\theta_{1C} - K_P \beta_{1C}) + \\
& (H_\mu + R_\mu) \left(\bar{h} \dot{\alpha}_X^* - \ddot{y}_P^* - \lambda \alpha_X \right) + \\
& - H_{\dot{\zeta}} \left(-\dot{\zeta}_{1C}^* - \dot{\zeta}_{1S} \right) + \\
& - H_{\dot{\beta}} \left(\dot{\beta}_{1C}^* + \dot{\beta}_{1S} - \dot{\alpha}_Y \right) \\
& - 2 \frac{C_T}{\sigma a} \beta_{1S}
\end{aligned} \tag{6.21}$$

where the inflow ratio $\lambda = V_c/\Omega R + \lambda_i$. The induced inflow, λ_i , is negligible in high inflow conditions so the inflow ratio is simply $\lambda = V_c/\Omega R$. Pitch-flap coupling $K_P = \tan(\delta_3)$ where δ_3 is the angle the pitch link location makes with the flap hinge. The rotor aerodynamic coefficients: blade thrust T , blade flap moment M , blade in-plane force H , blade torque moment Q , and blade radial force R , are approximated by only retaining c_{l_α} terms and neglecting drag c_d terms. The blade radial force is denoted as R_μ to avoid conflict with the blade radius R . The subscripts denote

the contribution due to perturbations in those parameters; advance ratio, μ ; lagging velocity, $\dot{\zeta}$; flapping velocity, $\dot{\beta}$; inflow ratio, λ ; and blade pitch, θ . These coefficients evaluate to

$$\begin{aligned}
M_\mu &= \lambda f_2 & H_\mu &= \lambda^2 f_0 \\
M_{\dot{\zeta}} &= \lambda f_3 & H_{\dot{\zeta}} &= \lambda^2 f_1 \\
M_{\dot{\beta}} &= -f_4 & H_{\dot{\beta}} &= -\lambda f_2 \\
M_\lambda &= -f_3 & H_\lambda &= -\lambda f_1 \\
M_\theta &= g_2 & H_\theta &= \lambda g_0 \\
T_\mu &= \lambda f_1 & Q_\mu &= \lambda^2 f_1 \\
T_{\dot{\zeta}} &= \lambda f_2 & Q_{\dot{\zeta}} &= \lambda^2 f_2 \\
T_{\dot{\beta}} &= -f_3 & Q_{\dot{\beta}} &= -\lambda f_3 \\
T_\lambda &= -f_2 & Q_\lambda &= -\lambda f_2 \\
T_\theta &= g_1 & Q_\theta &= \lambda g_1 \\
R_\mu &= 0
\end{aligned} \tag{6.22}$$

where

$$\begin{aligned}
f_0 &= \frac{1}{2} \ln \left(\frac{1 + \sqrt{1 + \lambda^2}}{\lambda} \right) \\
f_1 &= \frac{1}{2} (\sqrt{1 + \lambda^2} - \lambda) \\
f_2 &= \frac{1}{4} \sqrt{1 + \lambda^2} - \frac{1}{2} \lambda^2 f_0 \\
f_3 &= \frac{1}{6} [\sqrt{1 + \lambda^2} (1 - 2\lambda^2) + 2\lambda^3] \\
f_4 &= \frac{1}{16} \sqrt{1 + \lambda^2} (2 - 3\lambda^2) + \frac{3}{8} \lambda^4 f_0 \\
g_0 &= \frac{1}{4} \sqrt{1 + \lambda^2} + \frac{1}{2} \lambda^2 f_0 \\
g_1 &= \frac{1}{6} [(\sqrt{1 + \lambda^2})^3 - \lambda^3] \\
g_2 &= \frac{1}{16} \sqrt{1 + \lambda^2} (2 + \lambda^2) - \frac{1}{8} \lambda^4 f_0
\end{aligned} \tag{6.23}$$

The hub moment equations are

$$-2 \frac{C_{M_Y}}{\sigma a} = \frac{1}{\gamma} I_{\beta}^* (\nu_{\beta}^2 - 1) \beta_{1C} \tag{6.24}$$

$$2 \frac{C_{M_X}}{\sigma a} = \frac{1}{\gamma} I_{\beta}^* (\nu_{\beta}^2 - 1) \beta_{1S}$$

The relationship between the wing degrees of freedom q_1 , q_2 , and p to the rotor shaft displacement and rotation $(x_P, y_P, z_P, \alpha_x, \alpha_y, \alpha_z)$ is

$$\begin{aligned}
 \alpha_x &= -q_2 \eta'(y_T) \\
 \alpha_y &= p \xi(y_T) \\
 \alpha_z &= -q_1 \eta'(y_T) \\
 x_P &= q_1 \eta(y_T) \\
 y_P &= 0 \\
 z_P &= -q_2 \eta(y_T)
 \end{aligned} \tag{6.25}$$

Then the derivatives are simply

$$\begin{aligned}
 \dot{\alpha}_x &= -\dot{q}_2 \eta'(y_T) \\
 \dot{\alpha}_y &= \dot{p} \xi(y_T) \\
 \dot{\alpha}_z &= -\dot{q}_1 \eta'(y_T) \\
 \dot{x}_P &= \dot{q}_1 \eta(y_T) \\
 \dot{y}_P &= 0 \\
 \dot{z}_P &= -\dot{q}_2 \eta(y_T)
 \end{aligned} \tag{6.26}$$

and

$$\begin{aligned}
\alpha_x^{**} &= -q_2^{**} \eta'(y_T) \\
\alpha_y^{**} &= p^{**} \xi(y_T) \\
\alpha_z^{**} &= -q_1^{**} \eta'(y_T) \\
x_P^{**} &= q_1^{**} \eta(y_T) \\
y_P^{**} &= 0 \\
z_P^{**} &= -q_2^{**} \eta(y_T)
\end{aligned} \tag{6.27}$$

The wing vertical bending q_1 produces α_z (yaw) and x_P (longitudinal) motions of the hub. The wing in-plane bending q_2 produces α_x (roll) and z_P (vertical) motions of the hub. So the wing motion couples the rotor longitudinal and lateral groups: α_y , x_P , z_P , and α_x , α_z , y_P , respectively. Mode shape functions $\xi = y/y_T$ for torsion and $\eta = y^2/y_T$ for bending are used, where $y = y_T$ at the wing tip. Note, all length parameters are non-dimensionalized by the radius R .

6.4 Whirl Flutter Evaluation

The whirl flutter matrices for M, C, and K are shown in Eqs. 6.28, 6.29, and 6.30, respectively. For convenience, all superscripts $*$ and $^-$ are dropped; η is written for $\eta(y_T)$ and η' is written for $\eta'(y_T)$. $\eta(y_T)$ evaluates to y_T and so all y_T in the matrices are written as η . In Johnson [4], the property y_T and the calculation of $\eta(y_T)$, while the same value, are shown as separate parameters. Note, all length properties are non-dimensional, meaning h is h/R , c_w is c_w/R , y_T is y_T/R .

$$M = \begin{bmatrix} I_\beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I_\beta \\ 0 & I_\beta & 0 & 0 & 0 & 0 & 0 & -I_\beta \eta' & 0 \\ 0 & 0 & I_\zeta & 0 & 0 & 0 & 0 & -S_\zeta \eta' h & 0 \\ 0 & 0 & 0 & I_\zeta & 0 & 0 & S_\zeta \eta & 0 & S_\zeta h \\ 0 & 0 & 0 & 0 & I_{\beta_0} & 0 & 0 & -S_{\beta_0} \eta & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{\zeta_0} & I_{\zeta_0} \eta' & 0 & 0 \\ 0 & 0 & 0 & S_\zeta \eta & 0 & 2\eta' & 2M_b \eta^2 + 2\eta'^2 + I_q + m_p & 0 & S_W + 2M_b \eta h \\ 0 & 0 & -S_\zeta \eta' h & 0 & -2S_\beta \eta & 0 & 0 & 2M_b \eta^2 + 2M_b \eta'^2 h^2 + I_{P_X} \eta'^2 + I_q + m_p & 0 \\ 0 & 0 & 0 & S_\zeta h & 0 & 0 & S_W + 2M_b \eta h & 0 & 2M_b h^2 + I_{P_Y} + I_P \end{bmatrix}$$

$-\gamma M_{\beta}$	$2I_{\beta}$	γM_{ξ}	0	0	0	0	0	0	$-2I_{\beta}\eta'$ $-\gamma M_{\mu}\eta'h$	γM_{β}
$-2I_{\beta}$	$-\gamma M_{\beta}$	0	γM_{ξ}	0	0	0	0	$\gamma M_{\mu}\eta$	$\gamma M_{\beta}\eta'$	$2I_{\beta}$ $+\gamma M_{\mu}h$
$-\gamma Q_{\beta}$	0	γQ_{ξ}	$2I_{\xi}$	0	0	0	0	0	$-\gamma Q_{\mu}\eta'h$	γQ_{β}
0	$-\gamma Q_{\beta}$	$-2I_{\xi}$	γQ_{ξ}	0	0	0	0	$\gamma Q_{\mu}\eta$	$\gamma Q_{\beta}\eta'$	$\gamma Q_{\mu}h$
0	0	0	0	$-\gamma M_{\beta}$	γM_{ξ}	0	0	$\gamma M_{\xi}\eta'$	$\gamma M_{\lambda}\eta$	0
0	0	0	0	$-\gamma Q_{\beta}$	γQ_{ξ}	0	0	$\gamma Q_{\xi}\eta'$	$\gamma Q_{\lambda}\eta$	0
0	$-\gamma H_{\beta}\eta$	0	$\gamma H_{\xi}\eta$	$-2\gamma Q_{\beta}\eta'$	$2\gamma Q_{\xi}\eta'$	$\gamma\eta^2(H_{\mu}+R_{\mu})$ $+2\gamma Q_{\xi}\eta'^2$ $+\gamma C_{q_1}-C_{q_1q_1}$	$\gamma H_{\beta}\eta\eta'$ $-\gamma C_{q_1q_2}$ $+2\gamma Q_{\lambda}\eta\eta'$	$\gamma\eta h(H_{\mu}+R_{\mu})$ $-\gamma C_{q_1p}$		
$\gamma H_{\beta}\eta'h$	0	$-\gamma H_{\xi}\eta'h$	0	$2\gamma T_{\beta}\eta$	$-2\gamma T_{\xi}\eta$	$-\gamma C_{q_2q_1}$ $-2\gamma T_{\xi}\eta\eta'$	$-2\gamma T_{\lambda}\eta^2$ $+\gamma\eta'^2h^2(H_{\mu}+R_{\mu})$ $+C_{q_2}-\gamma C_{q_2q_2}$	$-\gamma C_{q_2p}$ $-\gamma H_{\beta}\eta'h$		
0	$-\gamma H_{\beta}h$	0	$\gamma H_{\xi}h$	0	0	$\gamma\eta h(H_{\mu}+R_{\mu})$ $-\gamma C_{pq_1}$	$\gamma H_{\beta}\eta'h$ $-\gamma C_{pq_2}$	$\gamma h^2(H_{\mu}+R_{\mu})$ $+C_p-\gamma C_{pp}$		

C =

$I_\beta(\nu_\beta^2 - 1) + \gamma K_P M_\theta$	$-\gamma M_\beta$	0	γM_ξ	0	0	0	$\gamma M_\mu \eta' \lambda$	0
$\gamma M_{\hat{\beta}}$	$I_\beta(\nu_\beta^2 - 1) + \gamma K_P M_\theta$	$-\gamma M_\xi$	0	0	0	0	0	$-\gamma M_\mu \lambda$
$\gamma K_P Q_\theta$	$-\gamma Q_{\hat{\beta}}$	$I_\xi(\nu_\xi^2 - 1)$	γQ_ξ	0	0	0	$\gamma Q_\mu \eta' \lambda$	0
$\gamma Q_{\hat{\beta}}$	$\gamma K_P Q_\theta$	$-\gamma Q_\xi$	$I_\xi(\nu_\xi^2 - 1)$	0	0	0	0	$-\gamma M_\mu \lambda$
0	0	0	0	$I_{\beta_0} \nu_{\beta_0}^2 + \gamma K_P M_\theta$	0	0	0	0
0	0	0	0	$\gamma K_P Q_\theta$	$I_{\zeta_0} \nu_{\zeta_0}^2$	0	0	0
$2\gamma \eta(H_{\hat{\beta}}/2 + C_T/a\sigma)$	$\gamma K_P H_\theta \eta$	$-\gamma \eta H_\xi$	0	$2\gamma K_P Q_\theta \eta'$	0	K_{q_1} $-\gamma C_{q_1 q_1}$	$-\gamma C_{q_1 q_2}$	$-\gamma C_{q_1 p}$ $-\gamma \eta \lambda(H_\mu + R_\mu)$
$-\gamma K_P H_\theta \eta' h$	$I_\beta \eta'(\nu_\beta^2 - 1) + 2\gamma \eta' h(H_{\hat{\beta}}/2 + C_T/a\sigma)$	0	$-\gamma H_\xi \eta' h$	$-2\gamma K_P T_\theta \eta$	0	$-\gamma C_{q_2 q_1}$	K_{q_2} $-\gamma h \lambda(H_\mu + R_\mu) \eta'^2$ $-\gamma C_{q_2 q_2}$	$-\gamma C_{q_2 p}$
$I_\beta(\nu_\beta^2 - 1) + 2\gamma h(H_{\hat{\beta}}/2 + C_T/a\sigma)$	$\gamma K_P H_\theta h$	$-\gamma H_\xi h$	0	0	0	$-\gamma C_{pq_1}$	$-\gamma C_{pq_2}$	K_p $-\gamma C_{pp}$ $-\gamma \lambda h(H_\mu + R_\mu)$

$K =$

Substituting these matrices into Eq. (6.1) gives the system equation. The right-hand side of the system equation is set to zero to determine the damping behavior from the homogeneous solution. The linear system of equations is solved by obtaining the eigenvalues using the matrix method

$$\dot{\mathbf{x}}^* = \mathbf{A}\mathbf{x} \quad (6.31)$$

where $\mathbf{x}$ is the state vector, and $\dot{\mathbf{x}}$ is the derivative

$$\mathbf{x} = \begin{bmatrix} \beta_{1c} \\ \beta_{1s} \\ \zeta_{1c} \\ \zeta_{1s} \\ \beta_0 \\ \zeta_0 \\ q_1 \\ q_2 \\ p \\ \beta_{1c}^* \\ \beta_{1s}^* \\ \zeta_{1c}^* \\ \zeta_{1s}^* \\ \beta_0^* \\ \zeta_0^* \\ q_1^* \\ q_2^* \\ p^* \end{bmatrix} \quad \dot{\mathbf{x}} = \begin{bmatrix} \beta_{1c}^* \\ \beta_{1s}^* \\ \zeta_{1c}^* \\ \zeta_{1s}^* \\ \beta_0^* \\ \zeta_0^* \\ q_1^* \\ q_2^* \\ \dot{p} \\ \beta_{1c}^{**} \\ \beta_{1s}^{**} \\ \zeta_{1c}^{**} \\ \zeta_{1s}^{**} \\ \beta_0^{**} \\ \zeta_0^{**} \\ q_1^{**} \\ q_2^{**} \\ \dot{p}^{**} \end{bmatrix} \quad (6.32)$$

and $\mathbf{A}$ is an 18×18 square matrix of the form

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \quad (6.33)$$

The eigenvalues of $\mathbf{A}$ are nine pairs of complex conjugates in the form $s = -\zeta\omega_n \pm j\omega_d$. So the damped frequency is the absolute value of the imaginary part and the damping ratio, ζ , can be found from the real part. The natural frequency $\omega_n = \omega_d \sqrt{1 - \zeta^2}$. The eigenvalues of A are obtained for each velocity V and frequencies and damping were calculated.

6.5 Model Properties

MTR properties are listed in Table 6.1 along with the Bell 25-ft diameter gimballed hub model properties used to validate the model and the ratio between the two rigs. The table only includes the parameters necessary to solve the model. Starred parameters are non-dimensionalized by $\frac{N}{\gamma}2I_b$ unless otherwise noted.

Table 6.1: **Bell 25-ft diameter rig and MTR properties used for whirl flutter analyses.**

	Bell	MTR	Ratio MTR/Bell
Rotor			
Rotor radius (m)	3.82	0.7239	0.1895
Continued on next page			

Table 6.1 – continued from previous page

	Bell	MTR	Ratio MTR/Bell
Rotor solidity, σ	0.089	0.0776	0.8719
N_b	3	3	1
Air density, ρ (kg/m ³)	1.225	1.225	1
Lock number, γ	3.83	2.8939	0.7556
Pitch/flap feedback, K_P	-0.268	-0.268	1
Lift curve slope, a	5.7	5.9358	1.041
Rotational speed, Ω (rad/s)	48 (458 RPM)	109.96 (1050 RPM)	2.29
Blade frequencies			
ν_β	$\sqrt{1 + 0.0355(\frac{600}{\Omega R})^2}$	1.0583	
ν_{β_0}	1.85	1.7882	0.967
ν_ζ	$-0.1389(\frac{V}{\Omega R})^3 + 0.6123(\frac{V}{\Omega R})^2$	5.07876	
	$-1.068(\frac{V}{\Omega R}) + 1.88$		
ν_{ζ_0}	0	0	
Blade inertias			
I_b (kg-m ²)	142	0.0552	
M_b^*	6.16	13.7824	2.237
I_β^*	1	1	1
$I_{\beta_0}^*$	0.779	1	1.284
Continued on next page			

Table 6.1 – continued from previous page

	Bell	MTR	Ratio MTR/Bell
I_{ζ}^*	0.67	1	1.5
$I_{\zeta_0}^*$	1 (autorotation)	1	1
S_{ζ}^*	1.035	2.6288	2.54
S_{β}^*	1.212	2.6288	2.17
Wing			
Semispan, y/R	1.333	1.202	0.902
Chord, c/R	0.413	0.542	1.31
Thickness ratio, t/c	13.5%	18%	1.33
Aspect ratio	6.6	4.99	0.756
Mast height, $\bar{h} = h/R$	0.261	0.3362	1.29
Pylon CG, $\bar{z}_{cg} = z_{P_{cg}}/R$	0.05	-0.0452	-0.9
m_P^*	76.9	259.9217	3.38
$I_{P_x}^*$	1.086	15.5314	14.3
$I_{P_y}^*$	1.206	15.5676	12.9
$I_{q_w}^*$	4.03	20.5314	5.1
$I_{p_w}^*$	0.141	2.1377	15.16
S_w^*	2.88	-11.091	-3.85
Torsion mode shape $\xi(y_T)$	0.535	1	1.87

Continued on next page

Table 6.1 – continued from previous page

	Bell	MTR	Ratio MTR/Bell
Bending mode shape $\eta'(y_T)$	1.74	2	1.15
Sweep, δ_{w_3}	-6.5°	0°	
Angle of attack, δ_{w_2}	0°	0°	
Dihedral, δ_{w_1}	0°	0°	
Aerodynamic center, x_{Aw}/c_w	-0.01	0	
Moment coefficient, $C_{m_{ac}}$	-0.005	0	
Full stiffness wing			
K_{q_1} , N/m	9.2 x 10 ⁶	2.62 x 10 ⁴	
K_{q_2} , N/m	2.5 x 10 ⁷	1.159 x 10 ⁵	
K_p , Nm/rad	1.77 x 10 ⁶	1.52 x 10 ⁴	
C_{q_1} , Ns/m	9030 ($\zeta = 1\%$)	7.7330 ($\zeta = 0.53\%$)	
C_{q_2} , Ns/m	27300 ($\zeta = 1.8\%$)	17.7085 ($\zeta = 0.54\%$)	
C_p , Nms/rad	955 ($\zeta = 1.5\%$)	0.6853 ($\zeta = 0.2\%$)	
$K_{q_1}^*$	18.72	26.1719	1.4
$K_{q_2}^*$	50.7	115.7757	2.28
K_p^*	3.595	15.1837	4.22
$C_{q_1}^*$	0.88	0.8494	0.97
$C_{q_2}^*$	2.67	1.9451	0.73

Continued on next page

Table 6.1 – continued from previous page

	Bell	MTR	Ratio
			MTR/Bell
C_p^*	0.093	0.0753	0.81
Resulting frequencies			
q_1	3.2 Hz (0.42/rev)	5 Hz (0.29/rev)	
q_2	5.35 (0.70/rev)	9.3 (0.53/rev)	
p	9.95 (1.30/rev)	15 (0.86/rev)	

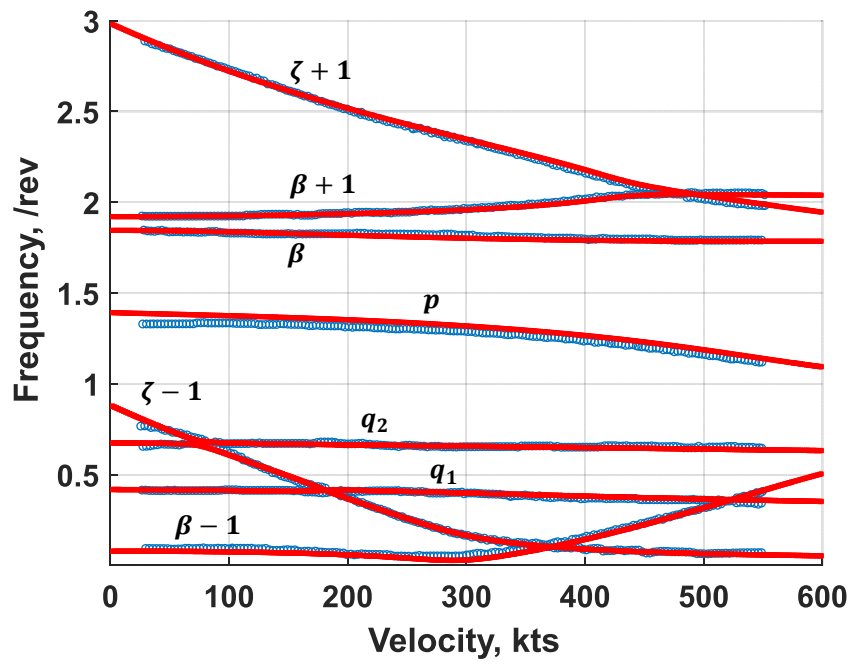
The inflow ratio, λ , is $\frac{V}{\Omega R}$. For Bell’s flap frequency ν_β , $\Omega R = 600$ ft/s. The non-dimensional pylon mass is obtained from $\dot{m}_P^* = m_P y_T^2 / (\frac{N_b}{2} I_b)$; the bending torsion coupling term is obtained from $\dot{S}_w^* = \dot{m}_P^* (z_{P_{cg}}/R)/y_T$. For mode shapes, Johnson [4] uses an approximation with $\xi = y/y_T$ for torsion and $\eta = y^2/y_T$ for bending. These simple shape functions are adequate for basic dynamics. MTR analysis uses the same mode shapes, however it is unclear how Johnson obtained $\xi(y_T) = 0.535$ and $\eta'(y_T) = 1.74$. The wing stiffness and damping terms are non-dimensionalized by $\frac{N_b}{2} I_b \Omega^2$ to obtain $\dot{K}^*$ and $\frac{N_b}{2} I_b \Omega$ to obtain $\dot{C}^*$.

Wing stiffnesses were designed to obtain the target wing frequencies. Wing structural damping terms were measured through RAP tests using the moving-block method. These measurements were taken in the GLMWT in 2019. Later, another set of measurements were taken at the Navy wind tunnel in 2021. The measurements agreed and are documented in Section 4.7.

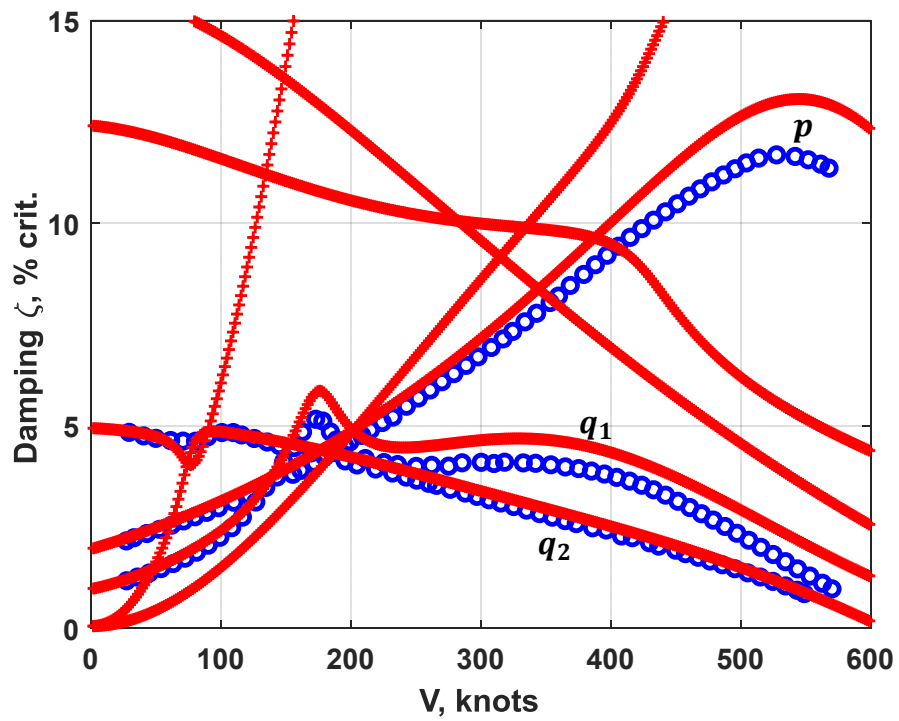
6.6 Validation and Predictions

Figure 6.2 verifies prediction of the present analysis against Johnson for the Bell 25 ft model. The parameters used are in Table 6.1. The frequencies are in agreement. The damping for wing beam q_1 and torsion p are over-predicted although the trends are well captured. Notice in the frequency plot, the crossings of the rotor regressive lag mode with the wing chord and wing beam modes. These frequency crossings cause interesting features in the damping such as the weak trough in wing chord damping around 80 kt and a strong peak in wing beam damping around 180 kt. These frequency crossings are main actors in the stability behavior.

With the code verified, the MTR parameters were substituted. Figure 6.3 shows the predicted frequencies and damping for the MTR. While the MTR will only be tested up to 200 kt, the plot is extended to 600 kt to show the predicted flutter point. The baseline MTR rig will not flutter up to 200 kt. Notice the rotor lag modes are extremely high, do not cross the wing modes below 400 kt, and therefore, the damping shows monotonic behavior until frequencies start to coalesce. Later, during actual testing, data obtained for the chord mode, q_2 , showed a trend that required a higher fidelity analysis to capture, proving the value of the dataset. The wing beam mode, q_1 , was satisfactory and because it was the lowest damped mode up to 60 kt, its satisfactory prediction helped clear the test.

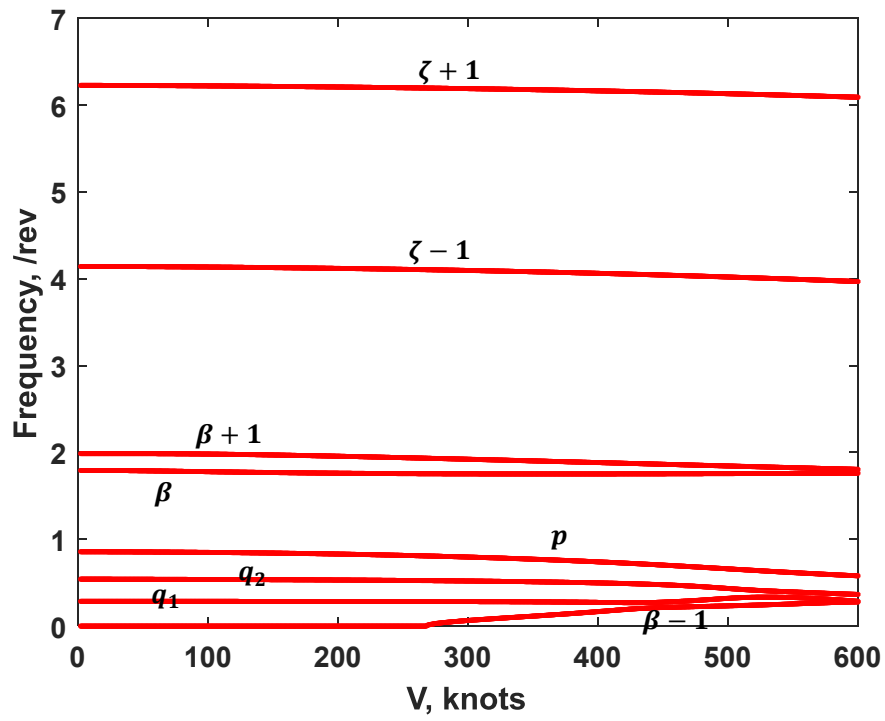


(a) Frequency.

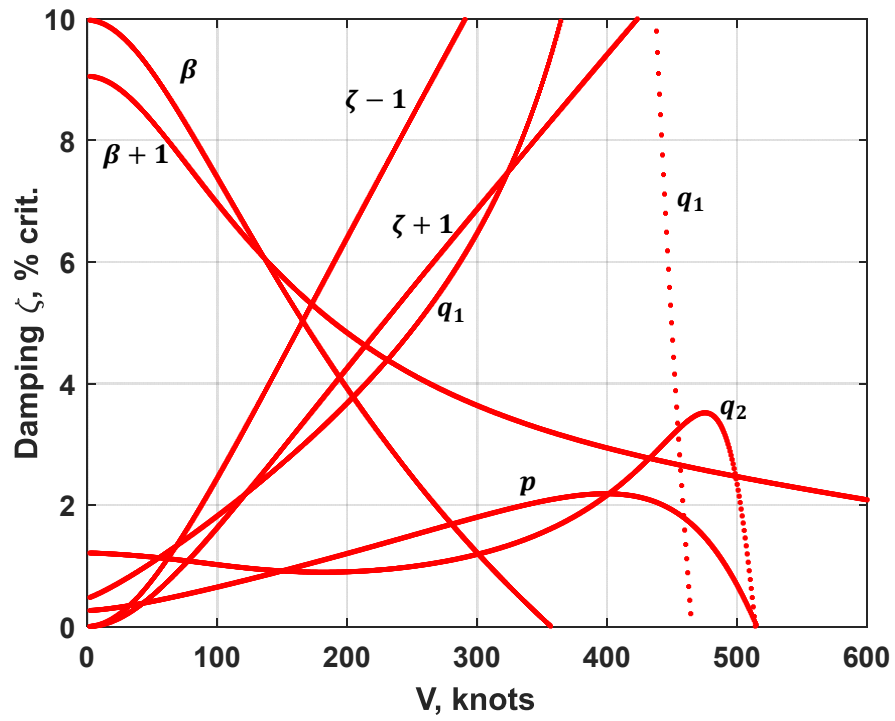


(b) Damping.

Figure 6.2: Validation of analysis with Bell 25-ft model frequencies (/rev) and damping ratio. Red - present analysis; Blue circle - Johnson.



(a) Frequency.



(b) Damping.

Figure 6.3: MTR frequency and damping results from simplified model.

6.7 Comprehensive Analysis Parameters

A new table of properties (Table 6.2) was generated after the 2021 Carderock wind tunnel test. These parameters are more than what can be used in the simplified analysis in the previous section, but will be needed for detailed comprehensive analysis. Many parameters overlap with the simplified model list, however, new properties are the rotor root cutout, pre-cone, gimbal limit, cyclic limit, updated delta-3 angle, blade linear twist, and updated wing stiffnesses.

Table 6.2: **Full-rig MTR properties, compiled after Carderock test in 2021.**

Parameter	Unit	Value
Rotor		
Number of blades		3
Radius	m	0.7239
Root cutout	%R	27
Geometric solidity ratio, σ		0.078
100% RPM (Froude-scale XV-15)		1050
Tip speed	m/s	79.56
Pre-cone	deg	2
Gimbal limit (flap stop)	deg	± 8
Collective root pitch range, θ_0	deg	17 to 73
Cyclic pitch limit	deg	± 16
		Continued

Table 6.2 – continued from previous page

Parameter	Unit	Value
Pitch horn location	%R	2.8
Pitch horn length	cm	6.84
δ_3	deg	-16.5
Blade		
Airfoil section		VR-7
Chord	cm	8
Thickness	% chord	12
Blade linear twist	deg/span	-37
Blade inertia I_b	kg-m ²	0.0552
EI_N	N-m ²	20.1
EI_C	N-m ²	937
GJ	N-m ²	62
Wing/pylon		
Pylon mass (including hub, no blades)	kg	32.28
Airfoil section		NACA 0018
Semi-span/R		1.202
Chord/R		0.542
Mast h/R		0.3362
EI_N	N-m ²	8.8×10^3
Continued		

Table 6.2 – continued from previous page

Parameter	Unit	Value
EI_C	N-m ²	3.51×10^4
GJ	N-m ²	8.5×10^3
ω_B	Hz	5.06
ω_C	Hz	9.65
ω_T	Hz	14.4
ζ_B	% critical	0.4
ζ_C	% critical	0.57
ζ_T	% critical	2

The blade stiffnesses EI_N , EI_C , GJ are the stiffnesses of the blade, however, comprehensive analysis would require variation with span including blade grip, pitch case, and hub. These details can be found in Ref. [37, 38]. For the wing-pylon parameters, the pylon mass, frequency, and damping were all measured. The mass was measured at NASA Langley, and the frequency and damping of the rig were extracted from RAP tests at Carderock. The 2% damping of the torsion mode was actually extracted from the chord mode strain gauge signal because the wing torsion gauges were damaged during the test. This meant the torsion amplitude on the chord gauges was low; it was high enough to capture the torsion frequency accurately, but perhaps too low for a consistent measurement for damping. Despite that, the torsion damping is provided here for completeness. The wing stiffness

values, EI_N , EI_C , and GJ , were tuned to obtain the measured frequencies but were not directly measured through static load tests.

6.8 Chapter Summary and Conclusions

This chapter described the whirl flutter analysis and input parameters used to predict basic trends of the MTR. It covered the following details.

1. A description of a simplified 9 degree of freedom (DOF) flutter model. The wing has three motions: vertical bending, chord bending, and torsion. The rotor has six motions in multiblade coordinates: collective flap, collective lag, longitudinal flap, longitudinal lag, lateral flap, and lateral lag.
2. A comparison of Bell 25 ft diameter rotor and the MTR parameters.
3. A verification of the simplified model predictions using Bell 25-ft diameter model. While the results were in agreement for frequencies, the wing damping values did not fall perfectly on top of the predictions provided by Johnson. However, the major features and trends were captured in damping.
4. The flutter prediction for the baseline MTR configuration - wing on, gimbal free, freewheel rotor, and straight blades. The MTR is not predicted to flutter below 200 kt. The model frequencies are all separated with the blade lag modes very high. Wing beam, q_1 , damping starts around 0.5% damping and quickly increases to 2% by 120 kt and 3.7% by 200 kt. Wing chord, q_2 , damping starts around 1.2% at zero velocity and shows a slightly decreasing trend, dropping

to about 0.9% at 200 kt. Wing torsion, p , damping starts around 0.25% based on the GLMWT test, and steadily rises to 1.2% at 200 kt.

5. An updated list of parameters compiled after the Carderock flutter test for new comprehensive analysis.

This chapter documented a simple flutter model and listed the properties required to calculate flutter. While the model is simple, it provides a quick assessment of gross behavior, which was useful for pre-test clearance.

Chapter 7: Wind Tunnel Testing

The Maryland Tiltrotor Rig (MTR) had a systems checkout test in the Glenn L. Martin Wind Tunnel (GLMWT) in November 2019. Preliminary freewheeling data was also acquired with baseline blades up to 60 kt. The rotor speed in RPM, collective, and windspeed were recorded.

While the MTR was designed for the GLMWT, due to pandemic closures, subsequent tunnel breakdown and overhaul of the drive system, a different test site was needed. In October 2021, the MTR was tested at Naval Surface Warfare Center Carderock Division (NSWCCD) in the Subsonic Wind Tunnel (SWT). This was the first research test of the MTR. Whirl flutter data was acquired with baseline and swept-tip blades up to 100 kt. A complete set of measurements were recorded.

7.1 Glenn L. Martin Wind Tunnel

The GLMWT is a closed-circuit, subsonic wind tunnel with a test section that is 7.75 ft. high, 11 ft. wide, and 13 ft. long. Maximum windspeed is 230 mph (200 kt). A six-component balance is located below the floor with several mounting locations for test articles. The balance is not used for rotor tests, rotor rigs have their own special dedicated balance. Stainless steel floor panels are used to cover up the



Figure 7.1: MTR installed in GLMWT 7.75 by 11-ft subsonic wind tunnel. November 2019.

test section floor and Masonite boards are used to cover any remaining irregularly shaped areas around mounting points. Figure 7.1 shows the MTR installed in the test section.

The MTR steel baseplate is mounted to an interfacing post fabricated especially for the GLMWT. The post dimensions can be found in the appendix of Technical Drawings. The post material is stainless steel 304L. It was fabricated to order by 3DHubs. The bottom flange of the post is mounted to the right pad on the wind tunnel balance. The bolt pattern is specifically designed to interface with the right pad. The wind tunnel balance is supported on jacks and is treated as a fixed support. Grade 8 or higher bolts and washers are used to mount the post to the pad. A picture of the MTR on the post is shown in Fig. 7.2.

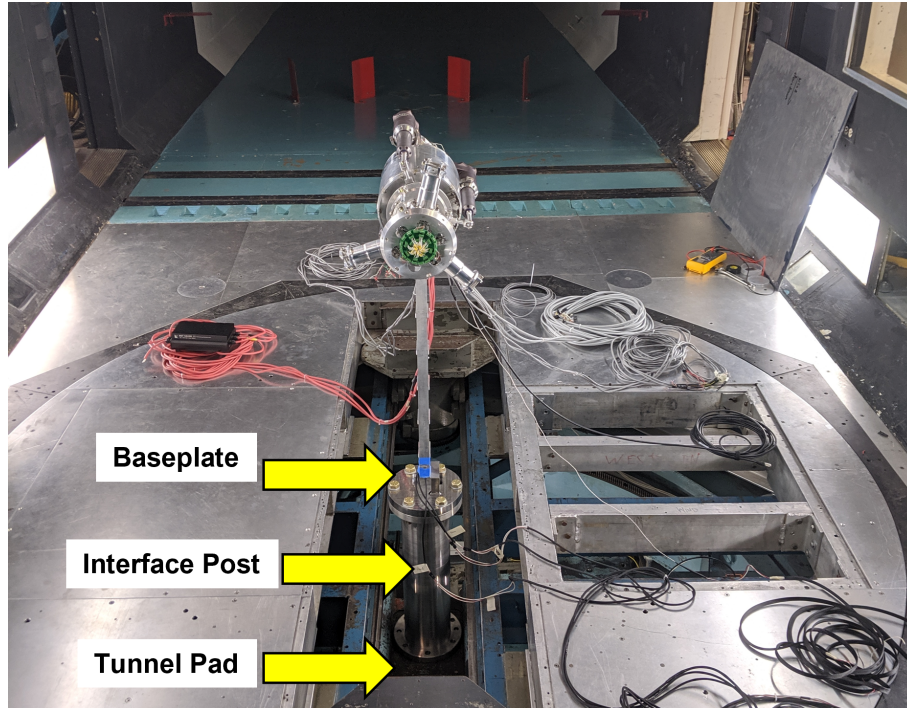
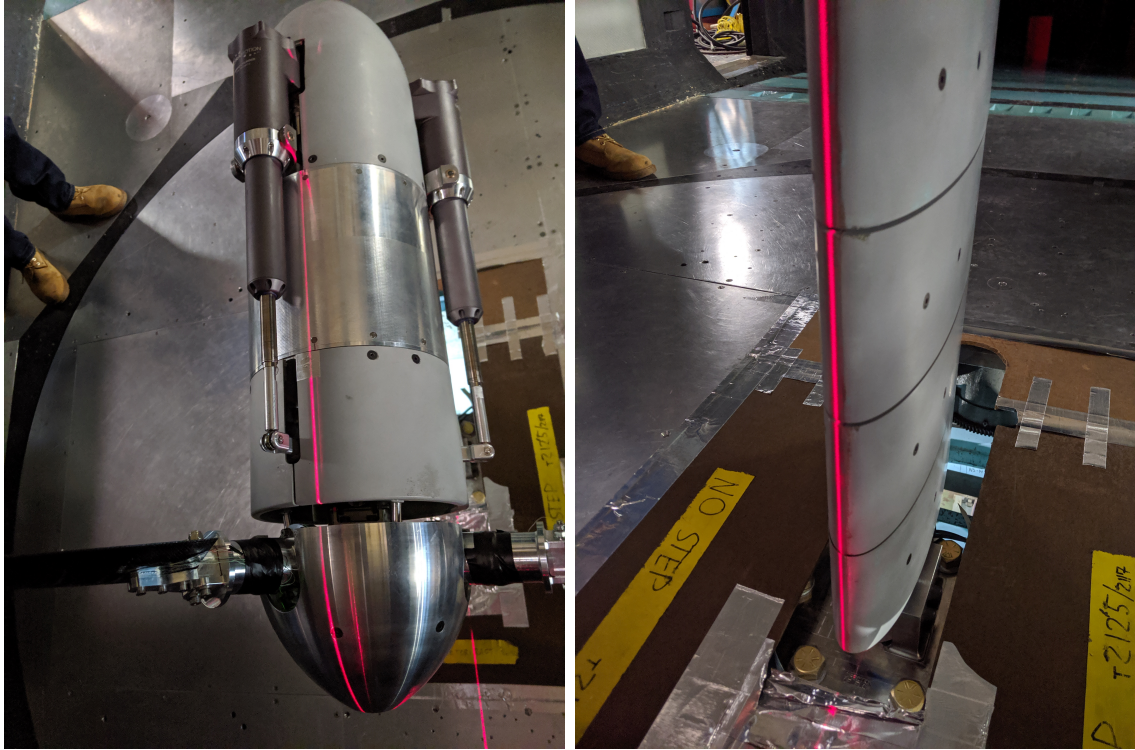


Figure 7.2: MTR mounted to custom post in GLMWT. November 2019.

For alignment of the model, a laser sheet was projected down the centerline of the test section, shown in Fig. 7.3. The wind tunnel balance was rotated until the model was adequately aligned.

The checkout test in November 4-8, 2019, was the first full system test of the MTR. Many issues were discovered with the sensors and hardware integration which were subsequently resolved. For freewheel tests, collective was varied for windspeeds up to 60 kt and RPM was recorded up to 2116. Since this was the very first test, only low collectives were used. The results are shown in Table 7.1.

Even though the rotor speeds easily reached target values for Froude and Mach scales, post test comprehensive analysis showed that the collective range was too low and not representative of tiltrotor cruise. The rotor was in fact operating in the reverse stall regime - a curious and useful regime for analysis validation but of no



(a) GLMWT laser sheet projected onto pylon. (b) GLMWT laser sheet projected on wing.

Figure 7.3: MTR alignment using laser sheet.

Table 7.1: MTR freewheel RPM measurements; average temperature = $63.3^{\circ}F$, average pressure = 29.85 Hg, average density = 0.002343 slugs/ft³

Windspeed (kt)	Collective, 75%R (deg)									
	0	1	2	2.5	3	4	5	6	7	8
25	0					400		591		780
35	350	453	647		696	820	1127	1220	1222	
40	590	780			960	1130	1461	1525	1535	1524
45		772	814	966	1073		1567	1706	1723	1695
50	725	840	1043		1224	1693	1824	1926	1938	1904
55	842	995	1216		1355	1710	2011	2116		
60	845	985	1220		1349	1733	2011			

practical significance. Higher collectives, from 10° to 50° , would be needed later to establish the correct operating regime for all future tests.

7.2 Navy Subsonic Wind Tunnel

The Naval Surface Warfare Center Carderock Division (NSWCCD) subsonic wind tunnel (SWT) is a closed-circuit design with a closed test section measuring 8 ft. high, 10 ft. wide, and 14 ft long. The tunnel wind speed range is 6.8 to 187.5 mph (6 to 163 kt). It has a built-in balance that is suited for six-component force and moment measurements. As with the GLMWT, the MTR uses its own dedicated balance, so the Navy tunnel balance was locked and used as a fixed support. Unlike the GLMWT, where floor plates are placed around the MTR, the SWT uses a continuous circular plate with a 6-inch diameter central hole. This requires the MTR wing spar to be mounted without the pylon or wing assembly, so the circular cover plate can be inserted around the wing spar. Any excess space from the central hole is covered with tape before testing begins. Figure 7.4 shows the MTR installed in the test section.

An interfacing post was also fabricated to connect the MTR baseplate to the SWT balance T-slot table, shown in Fig. 7.5. It was fabricated to order by Xometry. The interface post is 3.5 inches below the tunnel floor which allows the baseplate and clamping bracket to sit below the test section floor. An engineering drawing of this post is found in the appendix.



Figure 7.4: MTR installed in NSWCCD 8- by 10-ft subsonic wind tunnel. October 2021.

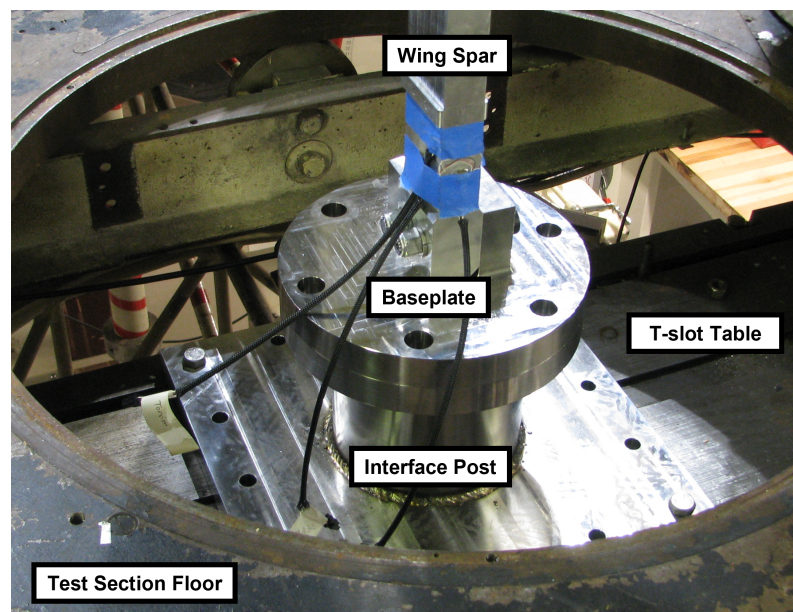


Figure 7.5: Interface post for MTR and T-slot table.

For alignment, the Navy tunnel uses a couple of plumb bobs hung from the ceiling to reference the test section centerline. The bobs point to a centerline drawn on the MTR pylon and the whole model with post is translated along the T-slot table until the line is approximately under the bobs. There is no yaw rotation of the T-slot table, therefore, slight rotations of the post is made by exploiting the tolerance of the bolt holes and slots on the T-slot table.

7.3 Test Conditions

All whirl flutter tests were performed at 1050 RPM. The blades were tracked and balanced near zero collective (θ_{75}). For the freewheeling rotor, at each tunnel speed, the collective was trimmed to achieve 1050 RPM. For example, at 60 kt, the RPM varied with collective as shown in Fig. 7.6. At this speed, a collective of about 26° was required to achieve 1050 RPM.

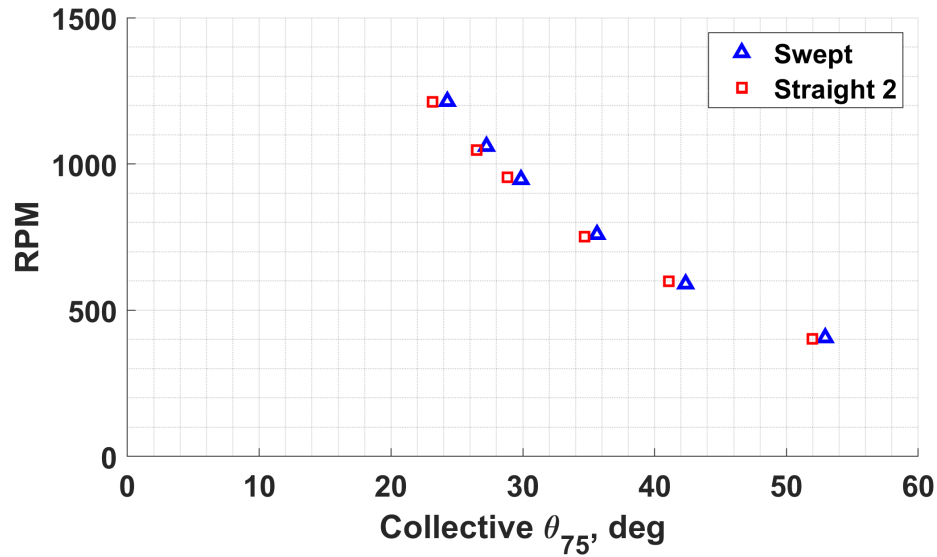


Figure 7.6: RPM variation with collective at windspeed of 60 kt.

A similar plot can be obtained for each speed, with greater collectives needed for the same RPM at higher speeds. The collective needed to maintain 1050 RPM for 30-100 kt is shown in Fig. 7.7. The collective is 10° at 30 kt, 26° at 60 kt, and reaches 40° at 100 kt. This high collective makes tiltrotor trim special and different from helicopter trim and results from high cruise inflow through the rotor. The agreement between blade sets 1 and 2 is excellent, which implies the conclusions made for one set will be applicable to the other. For the freewheeling rotor with gimbal free, the cyclic angles were trimmed concurrently to achieve zero first harmonic flapping. In practice, within $\pm 2.5^\circ$ maximum across all flight conditions was deemed acceptable. Trim is accomplished manually through a cyclic stick. A precise control will require an automatic controller in the future. Note that 30 kt was the minimum speed to allow for adequate cyclic margins to maintain zero gimbal tilt. Below 20 kt, there was no collective setting which could achieve 1050 RPM. Between 20-30 kt, maintaining zero gimbal tilt proved difficult.

Figure 7.8 shows the comparison of the straight blade in freewheel versus powered condition. Only straight blade set 1 was tested in the powered condition due to time constraints. However, the results of the freewheel collective versus speed show that blade set 1 and blade set 2 perform identically. For the powered rotor, the rotor speed was set to the target 1050 RPM and the collective was set to a value predicted by analysis for zero thrust. The predicted value was needed because, at the powered condition, electromagnetic interference (EMI) prevented reliable measurements of the thrust in real-time. It is believed that the EMI was due to the motor and load cell assembled close to each other without insulating material

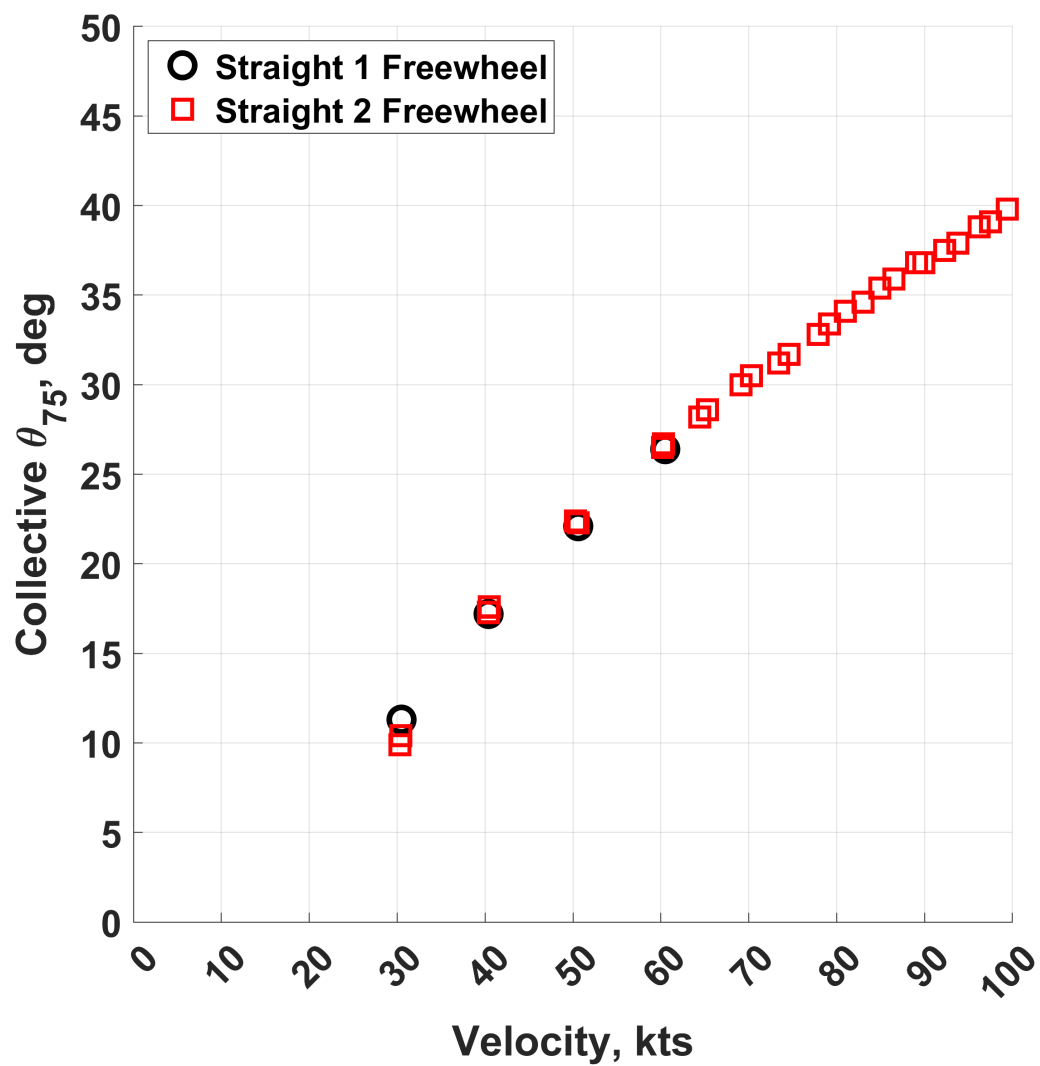


Figure 7.7: Collective variation with speed. Comparison of two straight blade sets; data from Tables 7.2 and 7.3.

between them, but this is under investigation. All powered tests were performed with gimbal locked because the gimbal hall effect sensors were also affected by EMI, which made trimming the gimbal difficult. Some of the noise could be filtered but this being the first test of the MTR, and indeed one of the first flutter tests at SWT, no chances were taken.

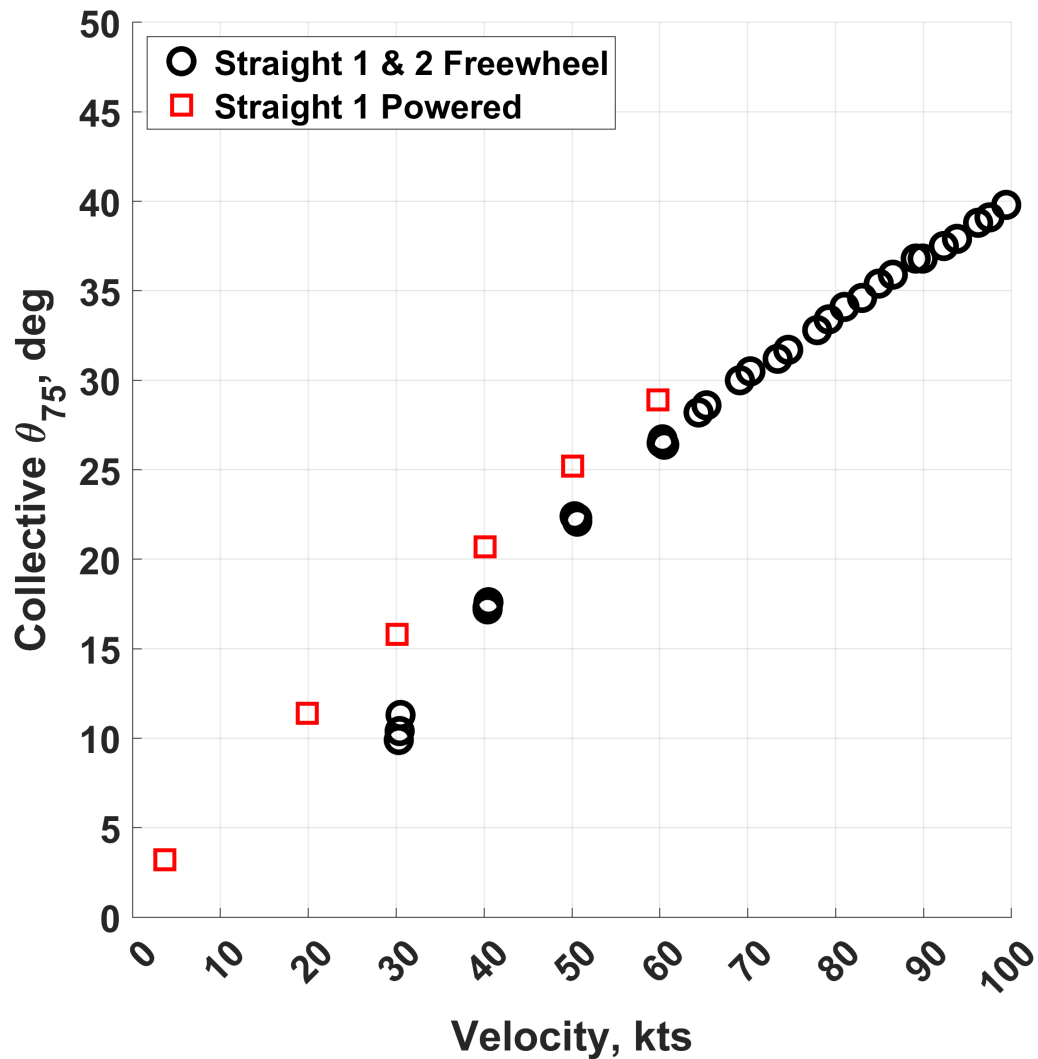


Figure 7.8: Collective variation with speed: powered versus freewheel. Data from Tables 7.2 to 7.4.

Figures 7.9 and 7.10 show the freewheel collective versus speed and the comparison with the powered condition, respectively, for the swept-tip blade set. The collective for the powered condition were set based on prediction as with the straight blade set. The swept-tip blades perform similarly to the straight blades in terms of collective.

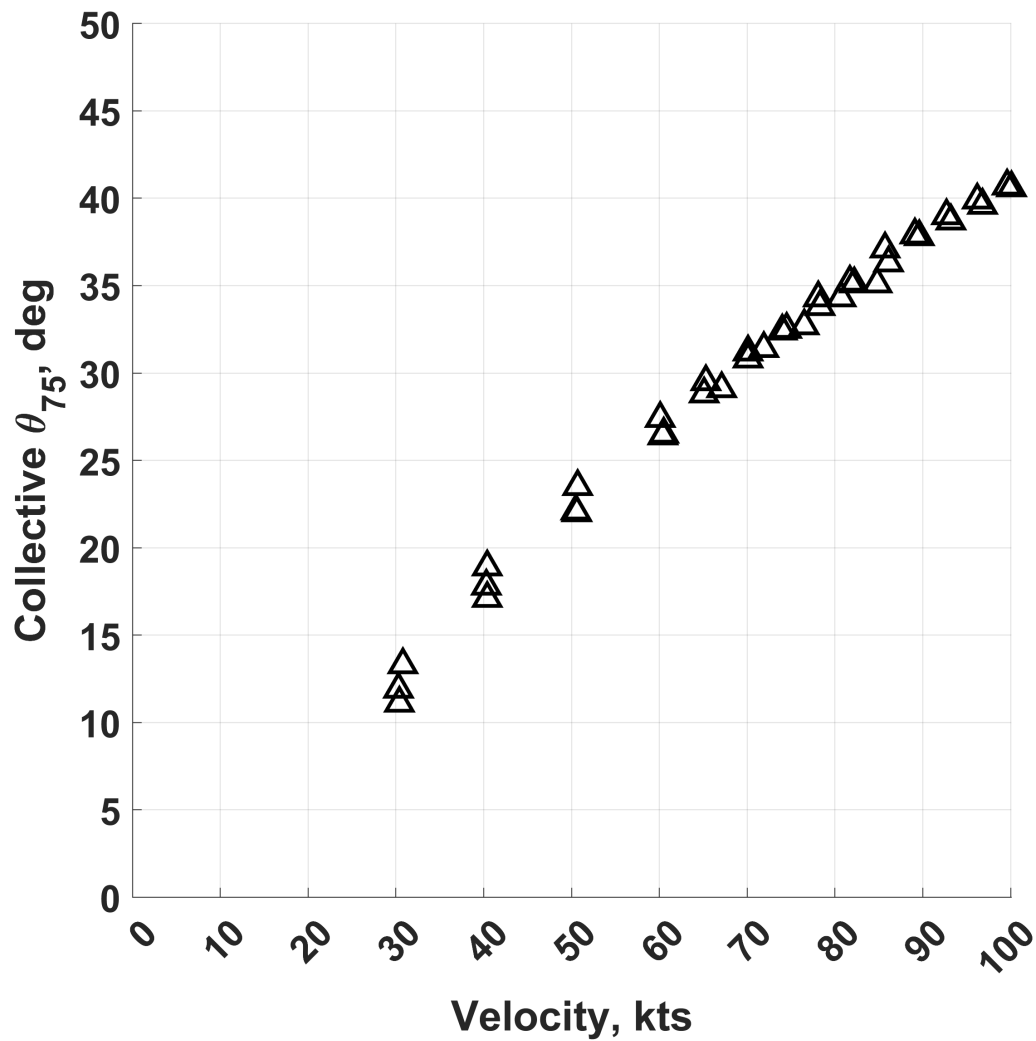


Figure 7.9: Collective variation with speed; swept-tip blades. Data from Table 7.5.

There was a difference in the root pitch angle between the blade sets which was accounted for when reporting the pitch at 75%R θ_{75} . The pitch case angle and

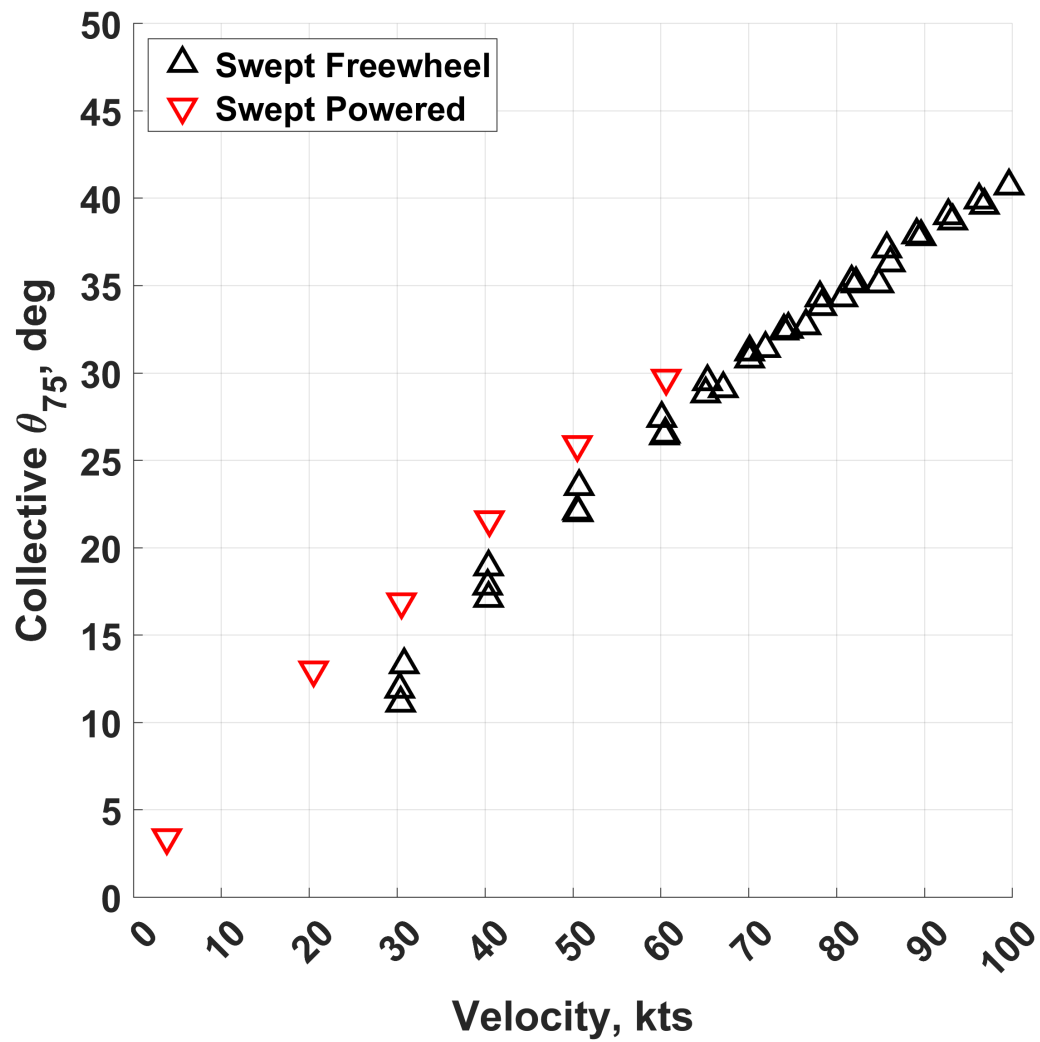


Figure 7.10: Collective variation with speed; powered versus freewheel for swept-tip blades. Data from Tables 7.5 and 7.6.

the blade grip is not naturally the same for all sets. The blade twist is the same for all blades. However, movement of the root insert during fabrication causes the angle they make with the blade grip to vary slightly by a few degrees. Therefore, the blade root installation into the blade grip causes inaccurate predictions of θ_{75} . It is important to physically measure θ_{75} for each installed blade set to avoid any uncertainty. Straight set 1 had a root angle 1.5° higher than predicted. Straight set 2 had a root angle 3° higher, and swept-tip set had a root angle 0.5° higher. The plots in Figs. 7.6 to 7.10 show the accurate θ_{75} after accounting for this offset. The data for those plots are tabulated in Tables 7.2 to 7.6. Table 7.2 is from sweep 3 in Table 7.7. Table 7.3 is from sweep 1 and 2. Table 7.4 is from sweep 4. Table 7.5 is from sweeps 5-7. Table 7.5 is from sweep 8.

Table 7.2: **Straight blade set 1 freewheel collective data.**

V	θ_{75}
kt	deg
30.5	11.3
40.4	17.2
50.6	22.1
60.5	26.4

7.4 Test Procedures

Whirl flutter tests are inherently risky particularly because the wing bending mode can be easily less than 1% damped. The test plan emphasized safety, which determined the sequence of the runs and ultimately limited the maximum speed up to which the tests were permitted. Datapoints were recorded for increasing

Table 7.3: **Straight blade set 2 freewheel collective data.**

V	θ_{75}	V	θ_{75}
kt	deg	kt	deg
30.3	9.9	30.4	10.4
40.5	17.6	40.4	17.3
50.6	22.3	50.3	22.4
60.3	26.7	60.2	26.5
64.4	28.2	65.3	28.6
69.1	30.0	70.3	30.5
73.4	31.2	74.6	31.7
77.9	32.8	79.2	33.4
81.0	34.1	83.0	34.6
84.9	35.4	86.5	35.9
89.1	36.8	89.9	36.8
92.3	37.5	93.8	37.9
96.2	38.8	97.5	39.1
99.4	39.8	100.8	40.1

Table 7.4: **Straight blade set 1 powered collective data.**

V	θ_{75}
kt	deg
3.7	3.2
19.9	11.4
30.1	15.8
40.1	20.7
50.1	25.2
59.8	28.9

Table 7.5: **Swept-tip blades freewheel collective data.**

V	θ_{75}	V	θ_{75}	V	θ_{75}
kt	deg	kt	deg	kt	deg
30.8	13.3	30.3	11.9	30.4	11.1
40.4	18.9	40.3	17.8	40.4	17.1
50.7	23.5	50.6	22.0	50.5	22.1
60.1	27.4	60.4	26.4	60.5	26.5
65.3	29.5	65.1	28.8	67.1	29.1
70.1	31.2	70.1	30.8	71.9	31.4
74.0	32.4	74.5	32.5	76.5	32.7
78.1	34.3	78.3	33.8	80.7	34.3
81.7	35.2	82.2	35.1	84.8	35.1
85.7	37.1	86.1	36.3		
89.1	37.9	89.6	37.8		
92.7	39.0	93.2	38.7		
96.2	39.9	96.8	39.6		
99.6	40.7	100.1	40.6		

Table 7.6: **Swept-tip blades powered collective data.**

V	θ_{75}
kt	deg
3.8	3.4
20.5	13.0
30.5	16.9
40.5	21.6
50.5	25.9
60.6	29.7

windspeeds; there were no decreasing windspeed runs. At each windspeed, repeat trials were taken for each wing mode before moving on to the next speed.

The first step was to acquire the damping of the model with the tunnel off, i.e. at zero velocity. Thus, a powered test had to be performed first. It was not expected predictions would match the data, since validation and refinement of analysis was the point of the test. As a precautionary measure, the go-ahead condition was that the damping measurement must be higher than prediction at least for the lowest damped mode. This tedious process was accomplished by testing during the day and analyzing the data at night to petition a speed increase the next day. Once the measurements were confirmed to be close to prediction, the test was cleared up to 60 kt.

After successful demonstration up to 60 kt, clearance was obtained to proceed to 80 kt and thereafter to 100 kt. The starting configuration was powered, gimbal-locked, wing off with straight blade set 1. Thereafter, clearance was obtained one by one for powered to freewheel, gimbal locked to gimbal free, and wing off to wing on. Furthermore, blade set change-outs were performed three times. All operations, including installation in and out of the tunnel, were accomplished within only five days. In total, 525 test points were collected. Of these, 486 were whirl flutter points. These are tabulated in Table 7.7. Flutter data were collected for the wing beam and wing chord mode. Three trials were performed per mode. The naming convention uses the tunnel, the sweep, and the velocity. For example, N1.78 refers to the test point at Navy tunnel for sweep 1, which was collected at 78 kt. Future tests at

Maryland may use MX.Y convention to refer to test points at the Maryland tunnel for sweep X at speed Y kt.

The data is organized into 8 sweeps in a logical sequence. However, this is not the order in which they were tested. The test sequence is shown in Table 7.8 and was based on clearance, risk assessment, and time windows available. The tunnel speeds and θ_{75} are nominal values. Thus the sweeps in Table 7.7 have different maximum and minimum speeds.

Table 7.8: **Test sequence at Navy SWT.**

Run	Blades	V	RPM	θ_{75}	Gimbal	Mode	Wing
Type		kt		deg			
Clearance obtained up to 60 kt							
Hover	Straight	0	700	2, 4, 6,	Locked	Powered	Off
	Set 1			8, 10			
Hover	Straight	0	1050	2, 4, 6	Locked	Powered	Off
	Set 1						
Flutter	Straight	0, 20, 30,	1050	2, 11, 16,	Locked	Powered	Off
	Set 1	40, 50, 60		20, 24, 28			
Flutter	Straight	30, 40,	1050	10, 16,	Locked	Freewheel	Off
	Set 1	50, 60		21, 25			
Hover	Straight	0	1050	6, 8, 10, 12,	Locked	Powered	Off
	Set 1			14, 16, 18			
Hover	Swept-tip	0	1050	2, 4, 6, 8	Locked	Powered	Off
Flutter	Swept-tip	0, 20, 30,	1050	2, 11, 16,	Locked	Powered	Off
		40, 50, 60		20, 24, 28			

Continued

Table 7.8 – continued from previous page

Run	Blades	V	RPM	θ_{75}	Gimbal	Mode	Wing
Type		kt		deg			
Flutter	Swept-tip	30, 40, 50, 60	1050	10, 16, 21, 25	Locked	Freewheel	Off
		Clearance obtained up to 82 kt					
Flutter	Swept-tip	65, 70, 74,	1050	28, 30, 31,	Locked	Freewheel	Off
		78, 82		33, 34			
Flutter	Swept-tip	30, 40, 50, 60	1050	10, 16, 21, 25	Free	Freewheel	Off
		Clearance obtained up to 100 kt					
Flutter	Swept-tip	65, 70, 74, 78,	1050	28, 30, 31, 33,	Free	Freewheel	Off
		82, 86, 89, 92,		34, 35, 36, 37,			
		96, 100		38, 39			
Flutter	Straight	30, 40, 50, 60,	1050	10, 16, 21, 25,	Free	Freewheel	Off
	Set 2	65, 70, 74, 78,		28, 30, 31, 33,			
		82, 86, 89, 92,		34, 35, 36, 37,			
		96, 100		38, 39			
Flutter	Straight	30, 40, 50, 60,	1050	10, 16, 21, 25,	Free	Freewheel	On
	Set 2	65, 70, 74, 78,		28, 30, 31, 33,			
		82, 86, 89, 92,		34, 35, 36, 37,			
		96, 100		38, 39			
RPM	Straight	60	400, 600,	52, 41, 35, 29,	Free	Freewheel	On
Sweep	Set 2		750, 950,	27, 24			
			1050, 1200				
RPM	Swept-tip	60	400, 600,	52, 41, 35, 29,	Free	Freewheel	On
Sweep			750, 950,	27, 24			
			1050, 1200				

Continued

Table 7.8 – continued from previous page

Run	Blades	V	RPM	θ_{75}	Gimbal	Mode	Wing
Type		kt		deg			
Flutter	Swept-tip	30, 40, 50, 60, 65, 70, 74, 78, 82, 86, 89, 92, 96, 100	1050	10, 16, 21, 25, 28, 30, 31, 33, 34, 35, 36, 37, 38, 39	Free	Freewheel	On

7.5 Flutter: Baseline

The baseline configuration of the MTR consists of straight blades, gimbal free, wing on, and the rotor in freewheel. Wing on refers to the full-up wing with airfoils, ribs, and fairings installed, whereas wing off is the exposed spar alone. This variation allows for investigations into aerodynamic damping and stiffness, which enter only if the airfoil profile is in place. Figure 7.11 shows the frequency and damping of this baseline configuration. The values are listed in Table 7.9.

Table 7.9: Straight blades, gimbal free, wing on, and freewheeling flutter results.

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
30	5.024	0.5	9.523	1.09
30	5.035	0.64	9.502	1.53
Continued				

Table 7.9 – continued from previous page

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
30	5.03	0.35	9.507	1.44
40	5.024	0.6	9.502	1.43
40	5.024	0.64	9.518	1.4
40	5.024	0.67	9.513	1.32
50	5.014	0.62	9.518	1.33
50	5.019	0.8	9.513	1.35
50	5.014	0.6	9.513	1.38
60	5.03	1.1	9.507	1.46
60	5.03	0.9	9.507	1.48
60	5.019	0.78	9.513	1.5
65	5.019	0.78	9.528	1.26
65	5.019	0.89	9.544	1.43
65	5.014	0.87	9.539	1.26
70	5.004	0.98	9.539	1.29
70	5.035	0.77	9.533	1.5
70	5.019	1.13	9.539	1.37
74	5.035	0.94	9.497	1.69
Continued				

Table 7.9 – continued from previous page

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
74	5.03	0.98	9.502	1.38
74	5.024	0.84	9.507	1.49
78	5.035	0.99	9.502	1.47
78	5.03	1	9.507	1.41
78	5.035	1.13	9.513	1.37
82	5.009	1.2	9.497	1.39
82	5.03	0.98	9.492	1.31
82	5.035	1.13	9.486	1.41
85.5	5.024	1.27	9.46	1.44
85.5	5.024	1.23	9.46	1.36
85.5	5.05	1.49	9.46	1.45
89	5.019	0.87	9.46	1.37
89	5.024	1.02	9.455	1.27
89	5.03	1.1	9.455	1.32
92.4	5.03	0.73	9.45	1.38
92.4	5.019	1.09	9.45	1.36
92.4	5.045	0.57	9.455	1.26
Continued				

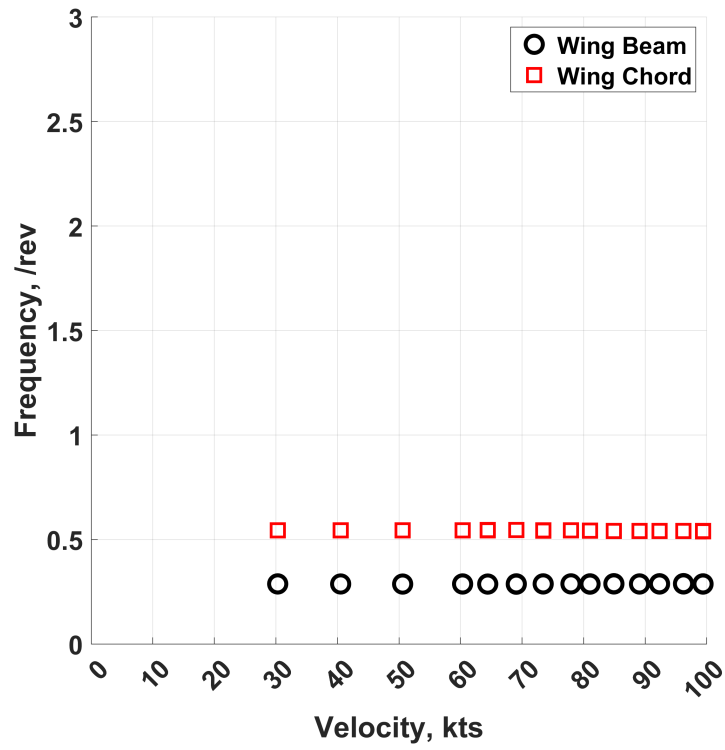
Table 7.9 – continued from previous page

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
95.7	5.056	0.69	9.46	1.28
95.7	5.03	1.11	9.455	1.36
95.7	5.024	0.93	9.455	1.19
99	5.014	0.75	9.45	1.29
99	5.035	0.95	9.445	1.15
99	5.035	1.4	9.445	1.32

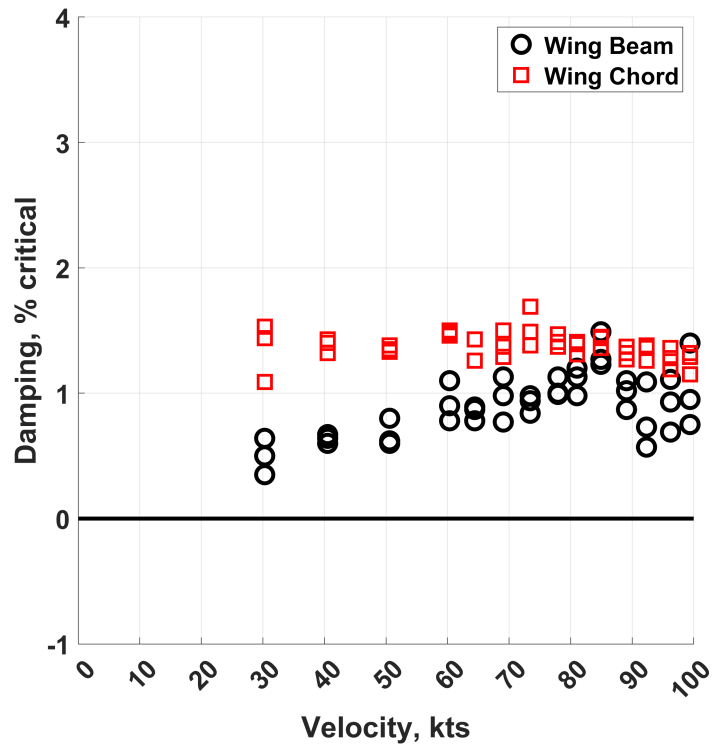
Wing beam damping begins around 0.5% at 30 kt with a scatter of $\pm 0.15\%$ over 3 trials. The damping then steadily increases up to 80 kt and shows a small peak between 80 to 90 kt of about 1.4%. After 90 kt, there is more scatter between the trials, with up to $\pm 0.35\%$ at 100 kt, but shows an increasing trend overall. The wing chord damping starts at 1.3% with a scatter of $\pm 0.2\%$ at 30 kt. It remains relatively steady without any increase in scatter up to 100 kt and features a small peak perhaps around 75 kt. The wing chord damping remains between 1% and 1.5% throughout the sweep.

Table 7.7: **Test conditions at Navy SWT.**

Sweep	V kt	θ_{75} deg	Gimbal	Mode	Wing
Straight blades					
Set 2					
1	30, 40, 50, 60, 65, 70, 74, 78, 82, 86, 89, 92, 96, 100	9.9, 17.6, 22.3, 26.7, 28.2, 30.0, 31.2, 32.8, 34.1, 35.4, 36.8, 37.5, 38.8, 39.8	Free	Freewheel	On
2	30, 40, 50, 60, 65, 70, 74, 78, 82, 86, 89, 92, 96, 100	10.4, 17.3, 22.4, 26.5, 28.6, 30.5, 31.7, 33.4, 34.6, 35.9, 36.8, 37.9, 39.1, 40.1	Free	Freewheel	Off
Set 1					
3	30, 40, 50, 60	11.3, 17.2, 22.1, 26.4	Locked	Freewheel	Off
4	4, 20, 30, 40, 50, 60	3.2, 11.4, 15.8, 20.7, 25.2, 28.9	Locked	Powered	Off
Swept-tip blades					
5	30, 40, 50, 60, 65, 70, 74, 78, 82, 86, 89, 92, 96, 100	13.3, 18.9, 23.5, 27.4, 29.5, 31.2, 32.4, 34.3, 35.2, 37.1, 37.9, 39.0, 39.9, 40.7	Free	Freewheel	On
6	30, 40, 50, 60, 65, 70, 74, 78, 82, 86, 89, 92, 96, 100	11.9, 17.8, 22.0, 26.4, 28.8, 30.8, 32.5, 33.8, 35.1, 36.3, 37.8, 38.7, 39.6, 40.6	Free	Freewheel	Off
7	30, 40, 50, 60, 65, 70, 74, 78, 82	11.1, 17.1, 22.1, 26.5, 29.1, 31.4, 32.7, 34.3, 35.1	Locked	Freewheel	Off
8	4, 20, 30, 40, 50, 60	3.4, 13.0, 16.9, 21.6, 25.9, 29.7	Locked	Powered	Off



(a) Frequency.



(b) Damping.

Figure 7.11: Stability results for baseline configuration at 1050 RPM.

7.6 Flutter: Wing Off

The wing airfoils along with the ribs and fairings are now removed from the baseline. The configuration is straight blades, gimbal free, wing off, and freewheel. So the wing provides structure (exposed spar), but no aerodynamics. Figure 7.12 shows the frequency and damping for the wing off configuration. This is an example of a parametric variation that is not possible in flight but important for validation. It provides insight into the damping and stiffness contribution of the wing aerodynamics. The results are listed in Table 7.10.

Table 7.10: **Straight blades, gimbal free, wing off, and freewheeling flutter results.**

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
30	5.05	0.48	9.46	1.27
30	5.05	0.53	9.492	1.51
30	5.05	0.52	9.492	1.46
40	5.045	0.51	9.429	1.37
40	5.045	0.45	9.434	1.31
40	5.045	0.49	9.434	1.38
50	5.04	0.67	9.445	1.24
50	5.04	0.73	9.44	1.3

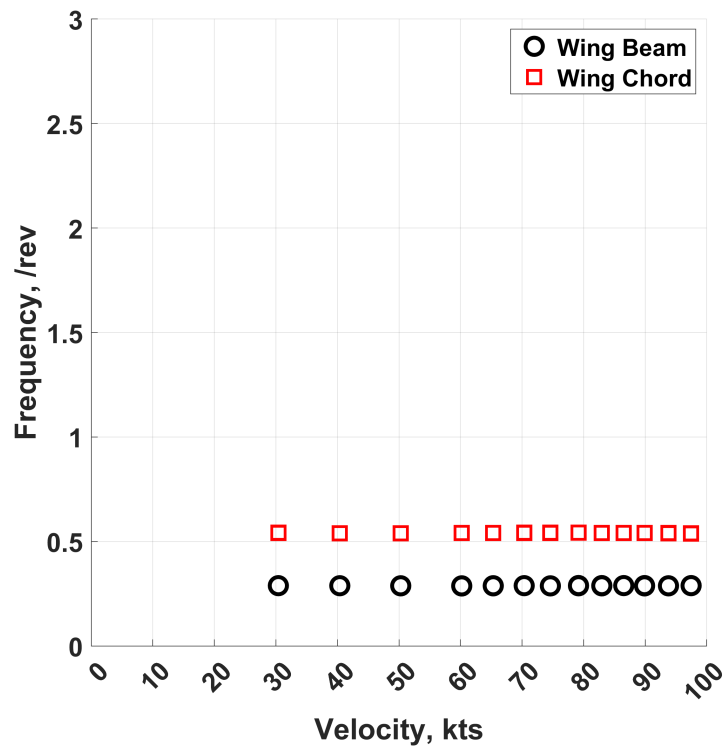
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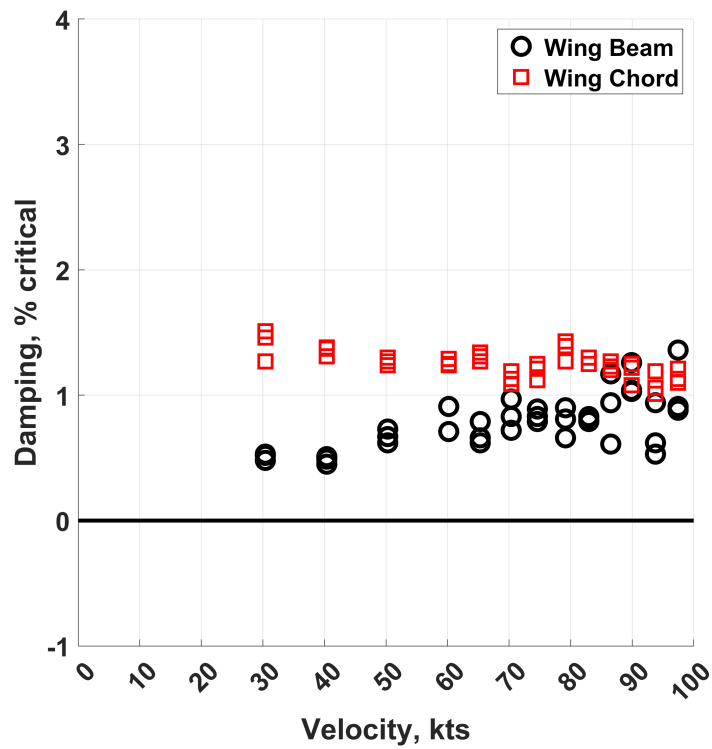
Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
50	5.035	0.62	9.424	1.27
60	5.035	0.71	9.455	1.25
60	5.024	0.91	9.455	1.24
60	5.035	0.71	9.455	1.29
65	5.045	0.79	9.455	1.34
65	5.035	0.66	9.46	1.27
65	5.04	0.62	9.455	1.31
70	5.05	0.83	9.466	1.13
70	5.05	0.72	9.466	1.09
70	5.05	0.97	9.481	1.19
74	5.03	0.79	9.471	1.12
74	5.035	0.89	9.481	1.21
74	5.035	0.83	9.476	1.25
78	5.061	0.9	9.481	1.27
78	5.056	0.66	9.486	1.39
78	5.04	0.81	9.481	1.43
82	5.035	0.79	9.466	1.3
Continued				

Table 7.10 – continued from previous page

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
82	5.045	0.8	9.46	1.25
82	5.056	0.83	9.455	1.25
85.5	5.056	1.17	9.455	1.23
85.5	5.055	0.61	9.455	1.2
85.5	5.056	0.94	9.45	1.27
89	5.05	1.26	9.46	1.22
89	5.03	1.03	9.466	1.25
89	5.056	1.04	9.466	1.08
92.4	5.04	0.53	9.45	1.06
92.4	5.035	0.62	9.44	1.01
92.4	5.04	0.94	9.434	1.19
95.7	5.056	0.91	9.408	1.13
95.7	5.05	0.88	9.434	1.21
95.7	5.056	1.36	9.419	1.1
99	5.087	1.01	9.388	1.1
99	5.082	0.76	9.382	1.12
99	5.061	0.89	9.398	1.22



(a) Frequency.



(b) Damping.

Figure 7.12: Stability results for straight blades, gimbal free, wing off, and rotor in freewheel at 1050 RPM

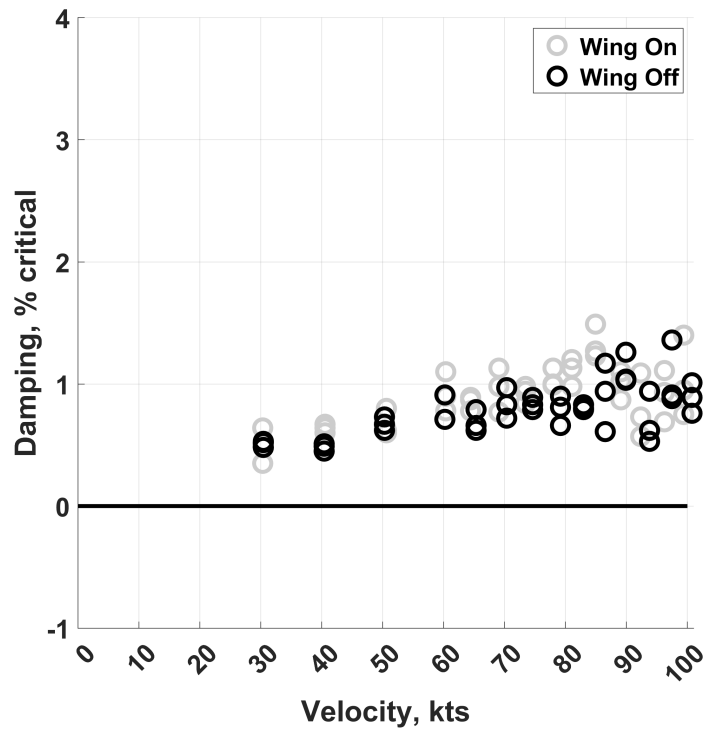
The wing beam damping starts at 0.5% at 30 kt with minimal scatter of $\pm 0.02\%$ over 3 trials. It steadily increases to around 1% at 100 kt with more prominent scatter of $\pm 0.25\%$. The wing chord damping starts at 1.4% with small scatter of $\pm 0.12\%$ at 30 kt. It decreases slightly to around 1.16% at 100 kt with even less scatter of $\pm 0.06\%$.

The damping of the wing beam mode is only marginally reduced without the wing. Around 85 kt, variability between trials appear to increase and it is difficult to discern whether the damping will increase, decrease, or remain level. The chord mode damping starts similar to the wing on configuration and as velocity increases the damping decreases marginally with a slight peak perhaps around 80 kt.

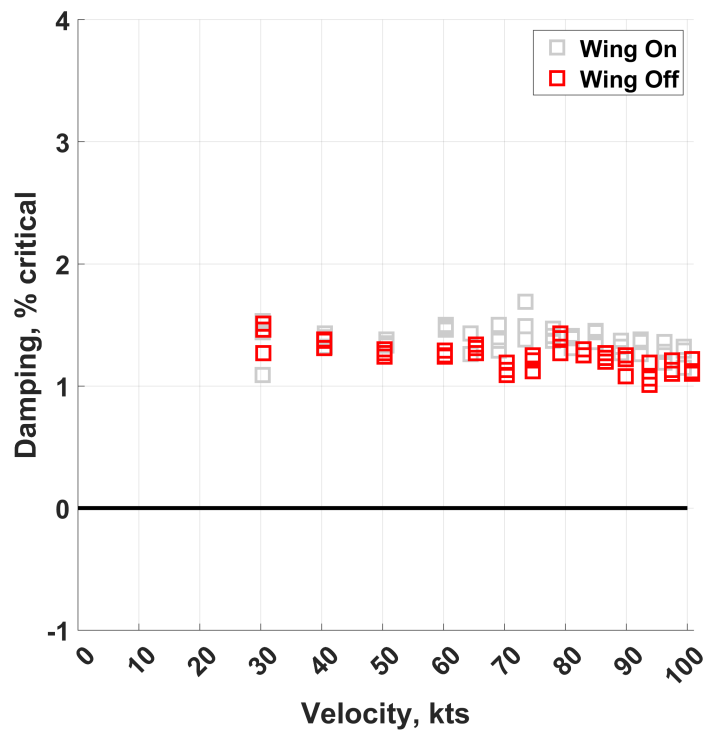
Figure 7.13 shows the wing beam and chord modes for the wing off damping compared to the wing on damping. A key conclusion is that wing aerodynamics has a minimal effect on the model damping, and certainly not important up to 100 kt. There is no sign of interference either. The rotor still dominates and the dynamics is determined by the structure.

7.7 Flutter: Gimbal Locked

The gimbal is now locked. The configuration is straight blades, gimbal locked, wing off, and freewheel. The locked gimbal is an effective method of obtaining, essentially, a very stiff-in-plane hingeless hub. Figure 7.14 shows the frequency and damping for straight blades, gimbal locked, wing off, and rotor in freewheel. The



(a) Beam damping.



(b) Chord damping.

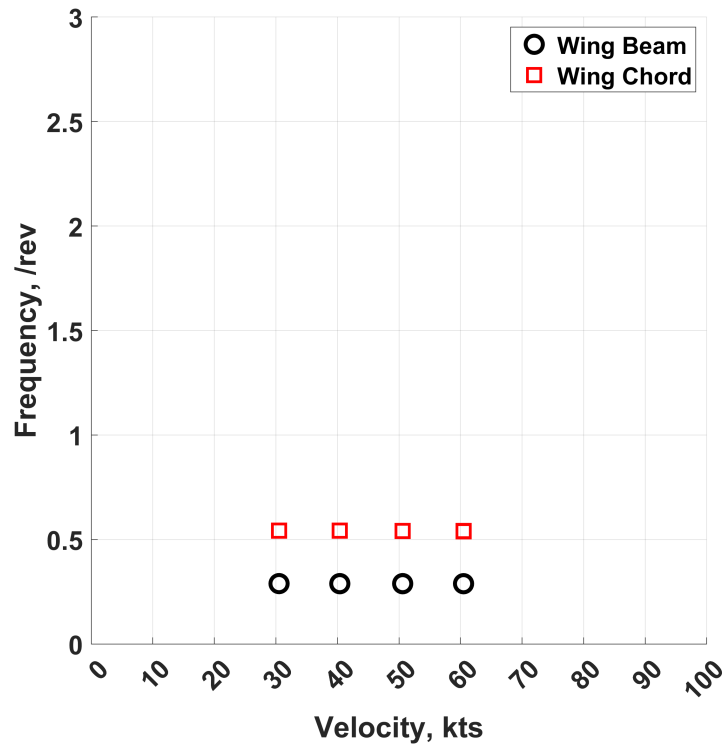
Figure 7.13: Comparison of damping for wing on and wing off configurations.

gimbal locked tests was performed early on, when Navy clearance was available only up to 60 kt; hence, the data stops there. The data is tabulated in Table 7.11.

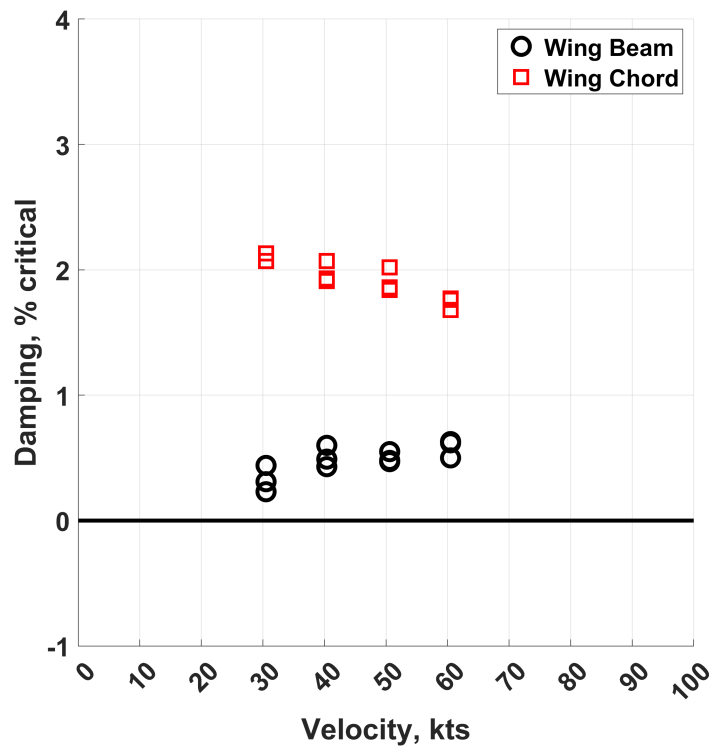
Table 7.11: **Straight blades, gimbal locked, wing off, and freewheeling flutter results.**

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
30	5.057	0.31	9.497	2.13
30	5.055	0.44	9.483	2.07
30	5.054	0.23	9.497	2.13
40	5.057	0.43	9.485	1.93
40	5.057	0.6	9.483	1.91
40	5.045	0.49	9.489	2.07
50	5.057	0.48	9.463	1.86
50	5.043	0.55	9.456	2.02
50	5.051	0.47	9.46	1.84
60	5.047	0.5	9.441	1.68
60	5.043	0.62	9.45	1.76
60	5.051	0.63	9.446	1.77

The wing beam damping starts around 0.33% with scatter of $\pm 0.1\%$ at 30 kt. It steadily increases to 0.57% with scatter of $\pm 0.07\%$ at 60 kt. The wing chord damping starts at 2.1% with scatter of $\pm 0.03\%$ at 30 kt and steadily decreases to 1.73% with



(a) Frequency.



(b) Damping.

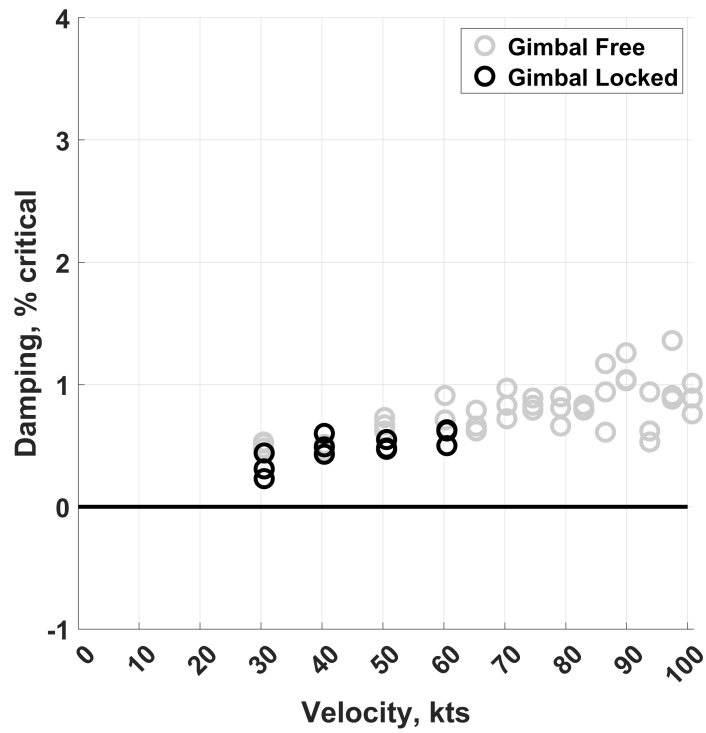
Figure 7.14: Stability results for straight blades, gimbal locked, wing off, and rotor in freewheel at 1050 RPM.

scatter of $\pm 0.04\%$ at 60 kt. Figure 7.15 shows the damping of individual wing modes in the gimbal locked configuration compared to the gimbal free configuration.

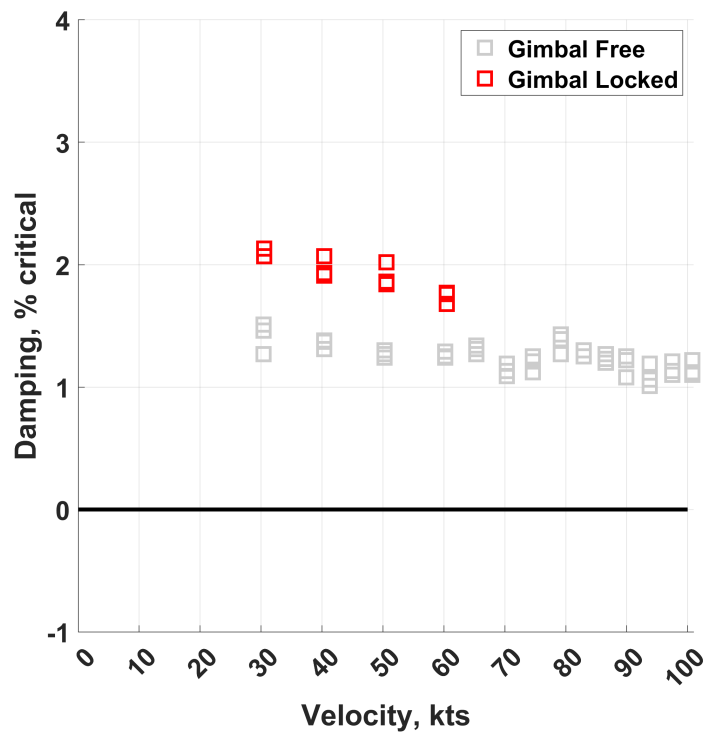
The damping of the wing beam mode is now the lowest of the configurations, although it still increases with speed. The wing chord damping is the highest. It increased significantly from 1.2% for the gimbal free configuration to about 1.8% with gimbal locked at 60 kt. However, there is a steeper decline in wing chord damping relative to the previous configurations.

7.8 Flutter: Powered

Finally, the rotor is powered on. The configuration is straight blades, gimbal locked, wing off, and powered rotor. This means the lower speeds can now be populated. This condition was also performed when clearance was available only up to 60 kt. Figure 7.16 shows the frequency and damping for straight blades, gimbal locked, wing off, and powered rotor. Table 7.12 shows the tabulated data. The powered data shows a small velocity of about 4 kt for a nominal test point at zero velocity due to recirculation in the wind tunnel from the powered rig.

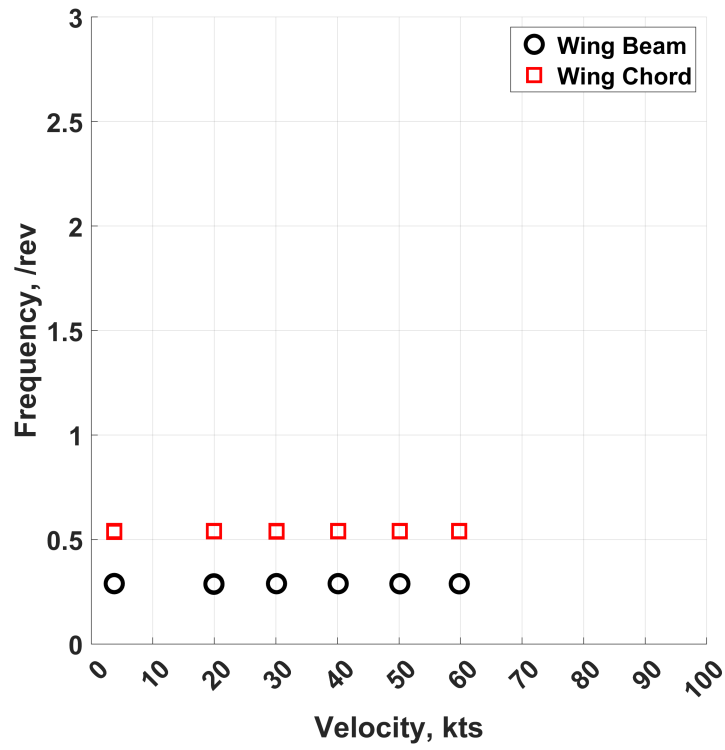


(a) Beam damping.

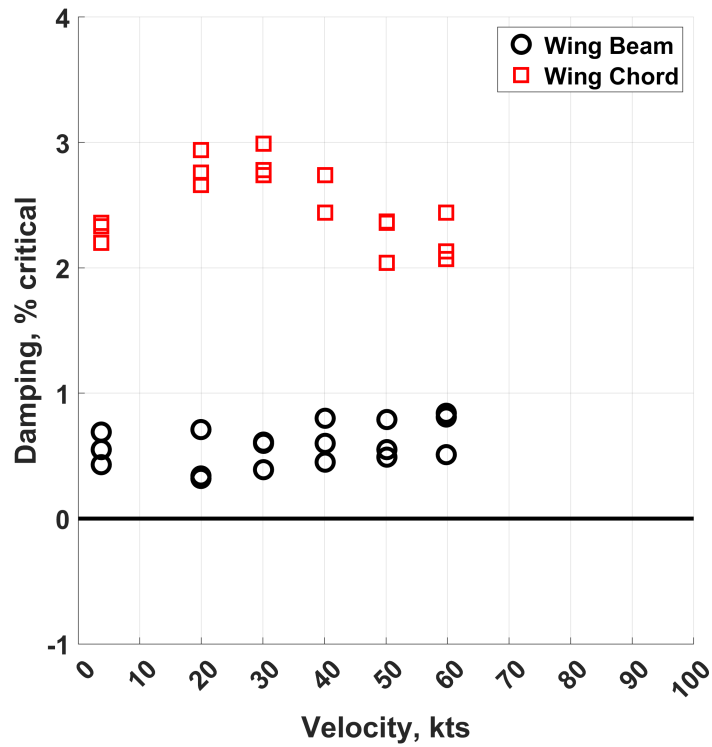


(b) Chord damping.

Figure 7.15: Comparison of damping for gimbal free and gimbal locked configurations.



(a) Frequency.



(b) Damping.

Figure 7.16: Stability results for straight blades, gimbal locked, wing off, and powered rotor at 1050 RPM.

Table 7.12: Straight blades, gimbal locked, wing off, and powered flutter results.

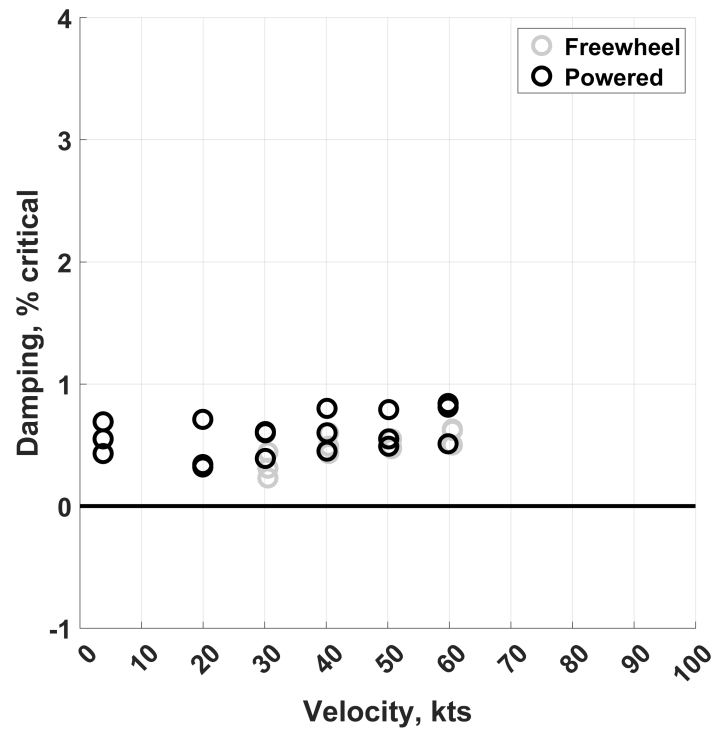
Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
4	5.066	0.55	9.441	2.36
4	5.044	0.43	9.384	2.33
4	5.046	0.69	9.458	2.2
20	5.04	0.34	9.439	2.76
20	4.995	0.71	9.47	2.94
20	5.04	0.32	9.46	2.66
30	5.06	0.6	9.421	2.78
30	5.057	0.61	9.4712	2.99
30	5.044	0.39	9.448	2.74
40	5.046	0.8	9.434	2.74
40	5.042	0.45	9.45	2.44
40	5.061	0.6	9.474	2.44
50	5.054	0.49	9.436	2.04
50	5.04	0.79	9.457	2.36
50	5.034	0.55	9.447	2.37
60	5.05	0.51	9.443	2.07
Continued				

Table 7.12 – continued from previous page

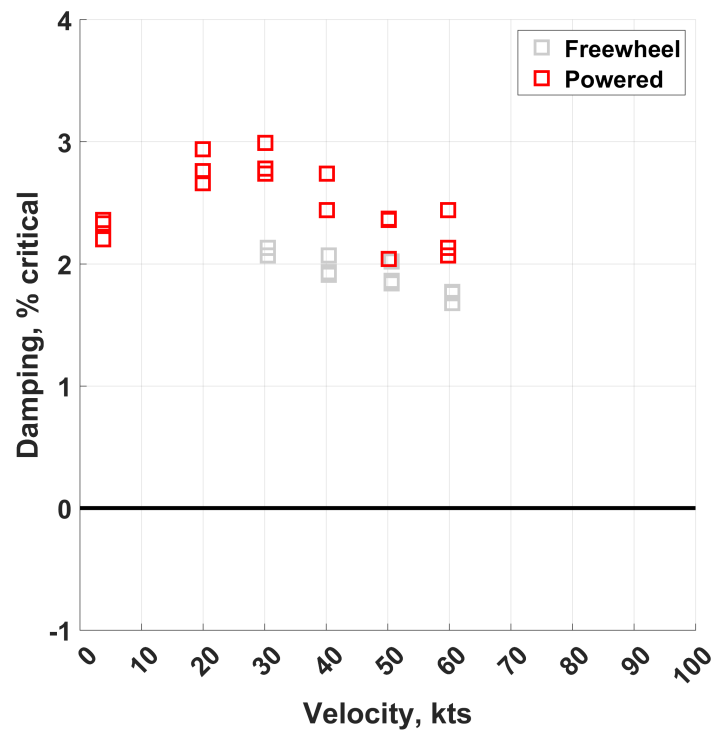
Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
60	5.048	0.84	9.44	2.44
60	5.035	0.81	9.463	2.13

The wing beam damping starts around 0.55% at 4 kt with scatter of $\pm 0.13\%$. Then there seems to be a very slight dip with damping measurements as low as 0.32% at 20 kt. It steadily increases after that to 0.68% with scatter of $\pm 0.17\%$ at 60 kt. The wing chord damping starts 2.28% with scatter of $\pm 0.08\%$ at 4 kt, increases to about 2.86% at 30 kt, then decreases to 2.25% with scatter of $\pm 0.19\%$ at 60 kt.

Figure 7.17 shows the wing mode damping measurements of the powered condition compared with the previous freewheel condition. The wing beam damping is marginally higher than the freewheel condition. However, the wing chord damping shows a peak at the lower speeds that is missed by the freewheel configuration. The peak is significantly higher, showing an increase from 2.1% for freewheel to 2.8% at 30 kt. The chord mode generally stays above 2% critical through 60 kt. The low-speed peak is of academic interest. Though a real aircraft will never be in airplane mode at these low speeds, the results are important for analysis validation. For example, it is not clear why the chord damping behaves the way it does, showing a 30-40% increase in damping in powered mode. Powered rotor introduces



(a) Beam damping.



(b) Chord damping.

Figure 7.17: Comparison of damping for powered and freewheel configurations.

the dynamics of the drive, which in this case is electric, and it is important for the future of electric aviation to understand the mechanism. In this case, the effect on damping happens to be beneficial.

7.9 Flutter: Swept-Tip Blades

Of particular importance are the swept-tip blades. Going back to the baseline, the full configuration is swept-tip blades, gimbal free, wing on, and freewheel rotor. Swept-tip blade is a separate and dedicated companion Ph.D. thesis by Sutherland [39]. Research on the blade design and development, testing, and results are given in Ref. [40]. Figure 7.18 shows the frequency and damping of swept-tip blades configuration. Data is tabulated in Table 7.13. Swept-tips are envisioned to impact stability at high speeds, but the baseline tests were limited to only 100 kt. Nevertheless, some interesting trends are already visible around 100 kt.

Table 7.13: **Swept-tip blades, gimbal free, wing on, and freewheel flutter results.**

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
30	5.03	0.44	9.497	1.49
30	5.024	0.55	9.497	1.41
30	5.03	0.61	9.502	1.42
Continued				

Table 7.13 – continued from previous page

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
40	5.024	0.56	9.502	1.43
40	5.024	0.56	9.486	1.49
40	5.019	0.6	9.507	1.4
50	5.019	0.82	9.518	1.44
50	5.019	0.73	9.513	1.58
50	5.014	0.75	9.518	1.46
60	5.019	0.91	9.513	1.52
60	5.014	0.78	9.507	1.66
60	5.014	0.84	9.507	1.63
65	5.014	0.8	9.486	1.54
65	5.019	0.69	9.497	1.54
65	5.024	1.29	9.486	1.64
70	5.009	1.13	9.518	1.6
70	5.03	1.02	9.492	1.49
70	5.014	0.88	9.513	1.41
74	5.009	1	9.507	1.44
74	5.014	1.1	9.497	1.45
Continued				

Table 7.13 – continued from previous page

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
74	5.014	0.84	9.492	1.42
78	5.024	0.87	9.46	1.44
78	5.024	1.23	9.46	1.35
78	5.024	0.87	9.44	1.36
82	5.024	0.96	9.445	1.48
82	5.019	1.02	9.45	1.46
82	5.03	1.01	9.45	1.46
85.5	5.03	1.14	9.434	1.55
85.5	5.024	1.11	9.434	1.33
85.5	5.05	1.11	9.455	1.42
89	5.03	1.03	9.445	1.47
89	5.03	1.17	9.45	1.47
89	5.03	1.18	9.45	1.32
92.4	5.04	0.87	9.445	1.39
92.4	5.03	1.02	9.46	1.45
92.4	5.045	1.03	9.44	1.35
95.7	5.04	1.11	9.445	1.3
Continued				

Table 7.13 – continued from previous page

Speed, kt	Wing Beam		Wing Chord	
	Frequency	Damping	Frequency	Damping
	(Hz)	(% critical)	(Hz)	(% critical)
95.7	5.014	1.01	9.45	1.35
95.7	5.03	1.36	9.44	1.27
99	5.045	1.31	9.445	1.31
99	5.03	1.05	9.445	1.23
99	5.04	1.5	9.471	1.11

The wing beam damping starts at 0.53% with scatter of $\pm 0.08\%$ at 30 kt. It steadily increases with speed, reaching around 1.28% with scatter of $\pm 0.23\%$ at 100 kt. The wing chord damping starts at 1.45% with scatter of $\pm 0.04\%$ at 30 kt. It remains relatively steady through 70 kt, then slightly decreases to 1.21% with scatter of $\pm 0.1\%$ at 100 kt.

Figure 7.19 shows the comparison of damping measurements between the swept-tip blades and straight blades. Compared to the baseline straight blades, the swept-tip wing beam damping shows less scatter especially at higher speeds. The wing chord damping shows the same magnitude and trend as the straight blades with marginally lower damping at the higher speeds. Overall, swept-tip blades show little effect on the model damping under 100 kt. However, it does not deteriorate the stability of the beam mode. An expansion of the windspeed envelope and RPM envelope is required to fully explore the impact of swept-tip blades.

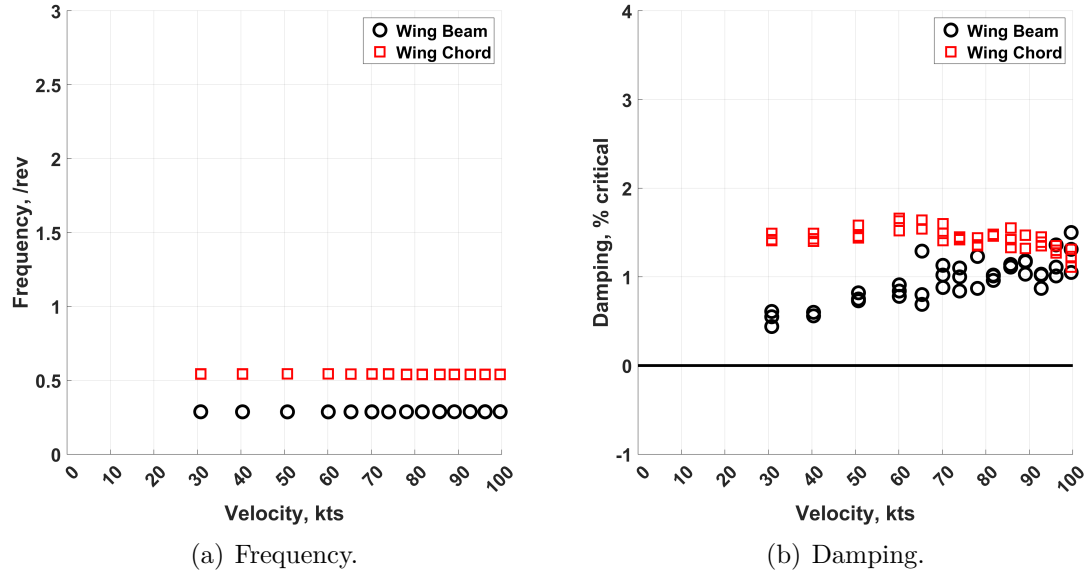
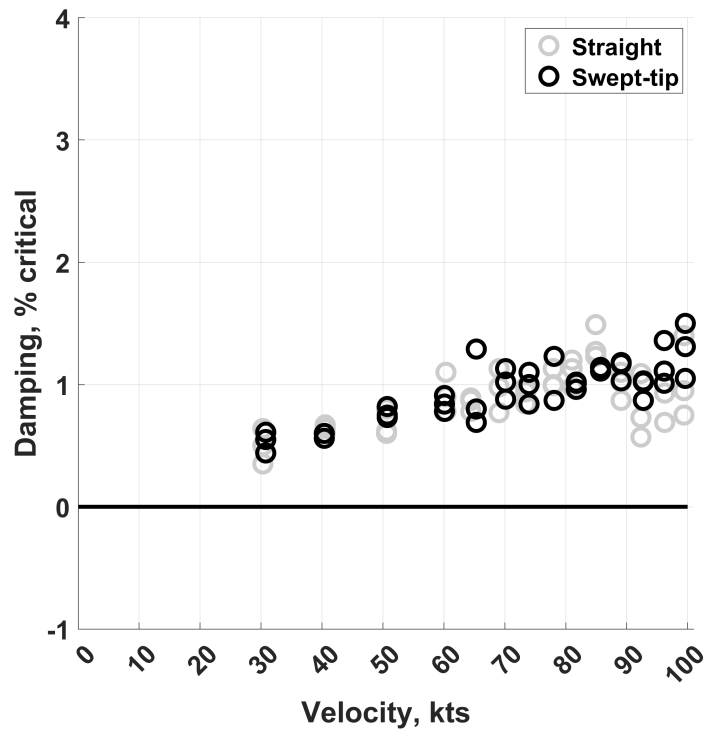


Figure 7.18: Stability results for swept-tip blades, gimbal free, wing on, and rotor in freewheel at 1050 RPM.

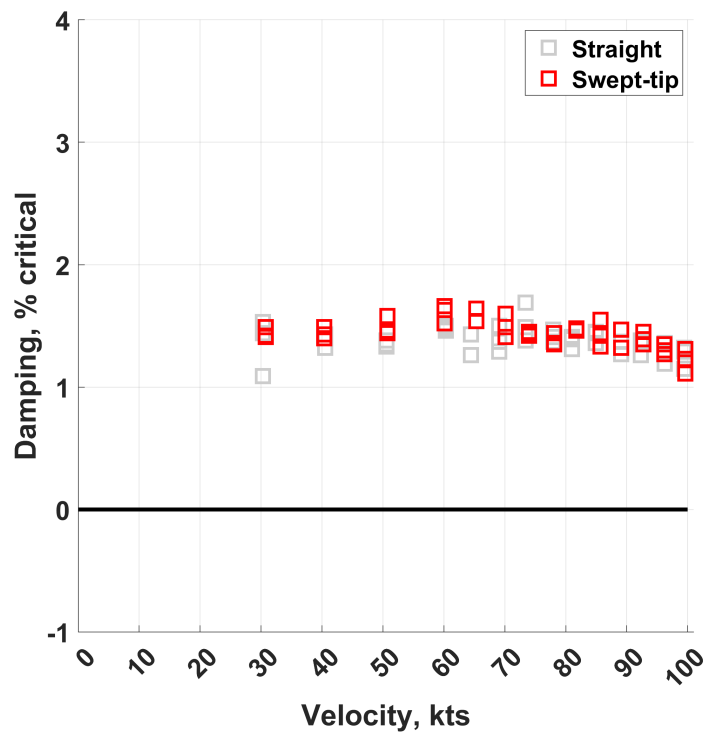
7.10 Flutter Analysis: 9 DOF Model

Using the nine degree of freedom simplified Johnson model[4], the frequency and damping of the MTR were predicted for the baseline configuration of wing on, gimbal free, and freewheeling rotor. Such an analysis appears adequate for a quick and preliminary assessment of basic trends to first order.

This Johnson model consists of rigid flap and lag, linear aerodynamics ($C_l = C_{l_\alpha} \alpha$), assumed wing mode shapes, and variation of lag and flap frequency with collective pitch based on root spring orientation. The Johnson model used measured values of lag and flap frequencies with speed. Here, it is slightly refined to calculate them based on predicted collective θ_{75} at any speed.



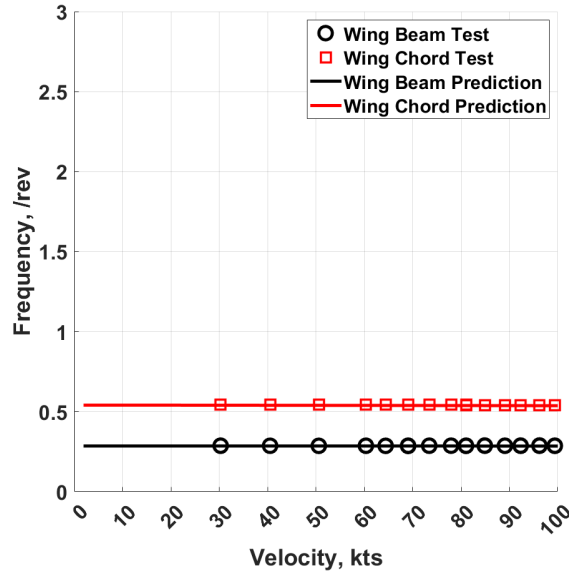
(a) Beam damping.



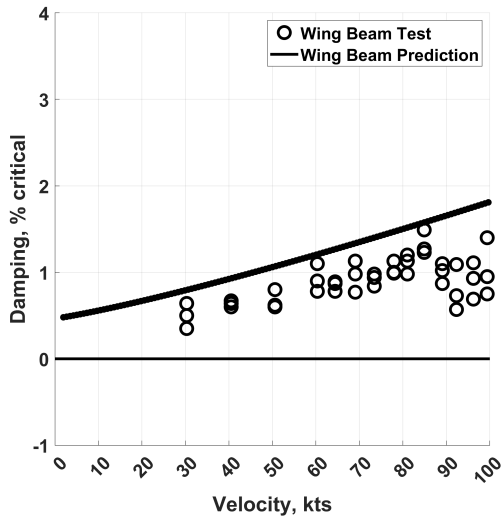
(b) Chord damping.

Figure 7.19: Comparison of damping for swept-tip blades and straight blade baseline configurations.

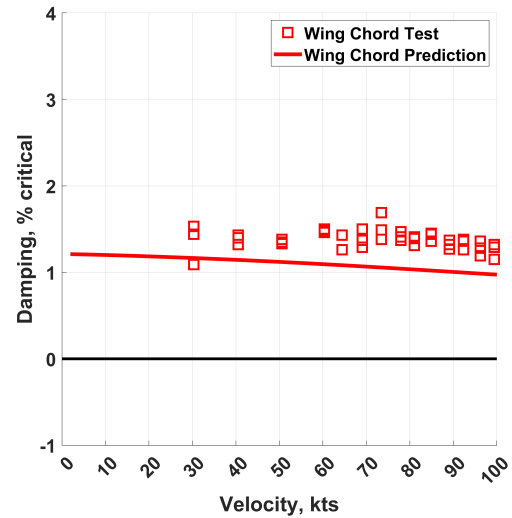
The predictions are overlaid with the test results in Fig. 7.20. The prediction was carried out for initial assessment of data and it confirms the general magnitude and trend of the data.



(a) Beam and chord frequencies.



(b) Beam damping.



(c) Chord damping.

Figure 7.20: Predictions and test data for baseline configuration: straight blades, wing on, gimbal free, and rotor in freewheel at 1050 RPM.

The frequency predictions remain static with velocity and, as expected, in agreement with test data. The damping is more complicated. The wing beam mode damping is overpredicted although the general trend of increase in damping with speed is captured. The chord mode damping is underpredicted but again the general trend of constant to slight reduction with speed is captured. The simplified model does not capture the basic magnitudes let alone details in the data like the peak in the beam mode at 85 kt, or the peak in chord mode at 75 kt. A comprehensive model with airfoil tables, flexible blades, finite element wing, and pylon yaw and pitch stiffnesses would be required to gain further insights. A separate and dedicated companion Ph.D. thesis by Gul [38], deals with the development of such an analysis.

7.11 Chapter Summary and Conclusions

This chapter described the first wind tunnel tests of the MTR. It covered the following details.

1. The details of the first checkout test at the GLMWT where systems were verified for operations and a characterization of the freewheeling RPM to wind-speed and collective was conducted for low collectives only.
2. A description of the installation of the MTR at the Navy subsonic wind tunnel.
3. A description of the flutter test conditions.
4. A description of the flutter test procedures and test requirements.

5. A demonstration of three damping extraction methods applied to run N1.78 data. These methods were logarithmic decrement method, Prony method, and the moving block method.
6. A sensitivity study conducted on four windowing methods used for the moving block method. These were Hanning, Hamming, Flat Top, and Blackman-Harris windows. Block size and sample shifts of the moving blocks were also covered.
7. The flutter results for the baseline configuration: straight blades, free gimbal, wing on, and freewheel rotor were provided.
8. The flutter results for straight blades, free gimbal, wing off, and freewheel rotor were provided.
9. The flutter results for straight blades, locked gimbal, wing off, and freewheel rotor were provided.
10. The flutter results for straight blades, locked gimbal, wing off, and powered rotor were provided.
11. The flutter results for swept-tip blades, free gimbal, wing on, and freewheel rotor were provided.
12. A prediction of the baseline configuration with a simplified 9 DOF model.

Based on these, this chapter reached the following key conclusions.

1. The checkout test was imperative to gain familiarity with the MTR and to resolve any issues with the rig instruments before performing any dynamic tests. It also revealed the correct collective regime where the final test were to be conducted.
2. The design of the MTR allowed it to interface with the Navy subsonic wind tunnel easily.
3. The MTR straight blade sets performed identically, however, direct measurement of θ_{75} must be performed on the blades after installation for precise measurement. The designed blade grip angle and blade twist do not suffice as the twisted blade root inserts can always have a small angular discrepancy from design intent.
4. The baseline configuration of straight blades, free gimbal, wing on, and free-wheel rotor showed low beam damping of 0.5% at 30 kt but increased to a peak of about 1.4% around 80-90 kt. Chord damping was relatively steady between 1%-1.5% up to 100 kt.
5. The straight blades, free gimbal, wing off, and freewheel rotor configuration showed wing aerodynamics was not a major factor in the model stability, at least up to 100 kt. Beam damping was around 0.5% at 30 kt and steadily increased to around 1% at 100 kt. The wing chord damping started around 1.4% at 30 kt and it decreased slightly to around 1.16% at 100 kt.

6. The straight blades, locked gimbal, wing off, and freewheel rotor configuration showed turning the gimbal into a stiff-in-plane hingeless hub had little effect on the wing beam damping, but increased the wing chord damping significantly between 30-60 kt. Beam damping started around 0.33% at 30 kt and steadily increased to 0.57% at 60 kt. Chord damping started around 2.1% at 30 kt and steadily decreased to 1.73% at 60 kt.
7. The straight blades, locked gimbal, wing off, and powered rotor configuration showed the powered rotor had a major and unexpected positive effect on the wing chord damping at low speeds while the wing beam damping was unaffected up to 60 kt. Beam damping started at 0.55% at 4 kt. It dipped slightly to 0.32% at 20 kt, then steadily increased to 0.68% at 60 kt. Chord damping started at 2.28% at 4 kt, increased to about 2.86% at 30 kt, then decreased to 2.25% at 60 kt.
8. For swept-tip blades, free gimbal, wing on, and freewheel rotor configuration, stability of the beam and chord modes were similar to the the straight blade configuration up to 100 kt. Beam damping started at 0.53% at 30 kt. It steadily increased to around 1.28% at 100 kt. Chord damping started at 1.45% at 30 kt. It remained relatively steady through 70 kt, then slightly decreased to 1.21% at 100 kt.
9. A simplified 9 DOF model was used to confirm the basic trends of the stability data. The trends were in agreement, but the model fell short of reliable magnitudes, as expected.

Lastly, a speed of 100 kt for the MTR corresponds to 230 kt at full-scale. This is far short of the envisioned 400 kt objective and future tests should proceed up to 200 kt at the GLMWT.

Chapter 8: Summary and Conclusions

This chapter provides the summary and conclusion of the thesis. A section on future recommendations is also provided.

8.1 Summary

The Maryland Tiltrotor Rig (MTR) was developed by the University of Maryland and fabricated by Calspan Systems. The MTR was designed for testing in the Glenn L. Martin wind tunnel (GLMWT). The rig is a semi-span, floor mounted, 3-bladed, flutter model with gimballed hub and straight blades as the baseline configuration. It was designed to test Froude- and Mach-scaled proprotors of up to 4.75-ft diameter. Compared to full-scale tiltrotors, the MTR is 1/5.26 scale XV-15 and 1/8 scale V-22, but it is not tied to those or any other particular aircraft. It has interchangeable wing spars, hubs, and proprotor blades. Unique features of the rig includes high bandwidth electric actuation, a fully electric direct-drive, and static pylon tilt mechanism.

The MTR wing spar was loosely designed to the Froude-scale frequencies of the XV-15 wing but, with the interchangeability of the spar, there is flexibility in the design of future spars. The MTR non-rotating frequencies and damping were

obtained at both the GLMWT and Navy subsonic wind tunnel (SWT). The results were in agreement between tunnels with wing beam frequency of about 5 Hz and damping of about 0.5% critical, wing chord frequency of about 9.5 Hz and damping of about 0.5% critical, and wing torsion frequency of about 14.5 Hz and damping of about 0.2% critical. However, the torsion damping at the Navy SWT was not accurately acquired due to broken wing strain gauges.

There are two load paths from the rotor to the wing root. The primary load path runs through the load cell and a secondary load path is unmeasured. Rotor thrust and torque are minimally affected by the secondary load path and so flutter tests could commence.

The electric motor controller power is limited by the cable thickness from the DC power supplies to the motor controller. Currently, with 10 AWG cables, the maximum current is 40A. With a power supply voltage of 120V, the max power available for the motor controller is 4.8kW.

Frequency and damping results for the MTR installed in GLMWT and Navy SWT were in agreement between the tunnels. The wing beam frequency was about 5 Hz and damping about 0.5% critical. Wing chord frequency was about 9.5 Hz and damping about 0.5% critical. Wing torsion frequency was about 14.5 Hz and damping about 0.2% critical. However, the torsion damping at the Navy SWT was not accurately acquired due to broken wing strain gauges.

A new calibration stand was developed for the load cell. This stand allowed for calibration to be performed while the load cell was mounted in the pylon, creating a test-like environment. The load cell calibrations showed that thrust and torque

were the most accurate, and most unaffected by the secondary load path present in the pylon.

High-bandwidth electric actuators allowed for a range of up to 56° in blade collective pitch. They also allowed for higher harmonic inputs and were the principal instruments to activating flutter at the wing-pylon frequencies.

Blade pitch encoders measured the blade root pitch. The actual θ_{75} angle should be directly measured for each new blade set due to fabrication discrepancies in the blade root insert.

The shaft torque gauge was not sensitive enough to provide an accurate measurement for torque with a $161.5 \text{ in-lb}/\mu\epsilon$ calibration, but it can be used as a backup measurement device.

The gimbal Hall effect sensors and azimuth Hall effect sensor are used to determine the rotating hub β_{1S} and β_{1C} angles.

The wing strain gauges were attached near the root end of the wing spar and were used to measure the wing response. All gauges were full-bridge circuits which provide the highest accuracy and compensate for temperature changes. There were two sets of full-bridge circuits for each wing mode. However, only one set recorded data during testing, the other set was for redundancy and backup.

A 9 DOF model was developed for simple whirl flutter analysis. It is a semi-span wing with a gimballed rotor in cruise flight. The wing has three motions: vertical bending, chord bending, and torsion. The rotor has six motions: collective flap, collective lag, longitudinal flap, longitudinal lag, lateral flap, and lateral lag. Six rotor equations of motion: three flap and three lag equations, and three wing

equations of motion are used to build the model system. The wing mode shapes were $\eta = y^2/y_T$ for bending and $\xi = y/y_T$ for torsion.

Both the Bell 25 ft rotor parameters and the MTR parameters are used for comparison. The results for Bell 25 ft data was shown for verification. Frequencies were in agreement, but the wing damping values did not fall perfectly on top of the 9 DOF model predictions. However, the major features and trends were captured in damping. The results for the baseline MTR configuration: wing on, gimbal free, freewheel rotor, and straight blades, did not predict any flutter below 200 kt. The model frequencies were all separated with the blade lag modes very high. Wing beam, q_1 , damping started around 0.5% damping and quickly increased to 2% by 120 kt and 3.7% by 200 kt. Wing chord, q_2 , damping started around 1.2% at zero velocity and showed a slight decreasing trend, dropping to about 0.9% at 200 kt. Wing torsion, p , damping started around 0.25% and steadily rose to 1.2% at 200 kt.

Checkout testing at the GLMWT occurred in November 2019. The first flutter test occurred at Navy SWT in October 2021. Test conditions showed that the blades sets used on the MTR performed identically in the SWT. Test procedures were developed to obtain clearance for higher windspeeds at 60 kt and 82 kt. A total of 8 sweeps were completed.

Three damping extraction methods, logarithmic decrement method, Prony method, and the moving block method, were applied to run N1.78 chord mode as a demonstration. The moving block method considered four windowing methods. These were Hanning, Hamming, Flat Top, and Blackman-Harris windows. Block size and sample shifts of the moving blocks were also covered.

Frequency and damping results for the beam and chord modes were shown for the configurations listed below.

1. The baseline configuration consists of straight blades, free gimbal, wing on, and freewheel rotor.
2. The straight blades, free gimbal, wing off, and freewheel rotor configuration which investigated the effect of wing aerodynamics on stability.
3. The straight blades, locked gimbal, wing off, and freewheel rotor configuration which effectively turned the hub into a stiff-in-plane hingeless hub.
4. The straight blades, locked gimbal, wing off, and powered rotor configuration which showed the difference between a freewheeling and powered rotor.
5. The swept-tip blades, free gimbal, wing on, and freewheel rotor configuration which gave a glimpse of the effect of advanced geometry blades on stability.
6. The baseline configuration results were overlaid with a simplified 9 DOF model prediction to determine trends.

8.2 Conclusions

Based on the present research, the following key conclusions were drawn.

1. The MTR nominal wing spar and gimballed hub were fully characterized. The MTR was fully instrumented and calibrated for wind tunnel testing.

2. There were two load paths from the rotor to the wing, but the thrust and torque measurements were minimally affected by the secondary load path. Flutter testing could commence with these primary measurements.
3. The 9 DOF simplified model showed that the MTR blade lag frequencies were too high and the frequencies did not coalesce with other modes. This indicated that it would not achieve a flutter point under 200 kt. While the model produced the gross trends reasonably, a high-fidelity model is obviously desired to accurately capture the finer details of the magnitudes and trends from 0-200 kt. Indeed, it is the objective of this dataset to be used for validation of high-fidelity models.
4. The checkout test in the GLMWT was imperative to gain familiarity with the MTR and to resolve any issues with the rig instruments before performing any detailed dynamic tests. It also revealed the correct collective regime where the flutter test were to be conducted.
5. The MTR straight blade sets performed identically at Navy SWT, however, direct measurement of θ_{75} must be performed on the blades after installation for precise measurement. The designed blade grip angle and blade twist are not sufficient to derive θ_{75} as, during fabrication, the twisted blade root inserts can have a small angular discrepancy from design intent.
6. The baseline configuration of straight blades, free gimbal, wing on, and free-wheel rotor showed low beam damping of 0.5% at 30 kt but increased to a

peak of about 1.4% around 80 to 90 kt. Chord damping was relatively steady between 1% to 1.5% up to 100 kt.

7. Straight blades, free gimbal, wing off, and freewheel rotor configuration showed wing aerodynamics was not a major factor in the model stability, at least up to 100 kt. Beam damping was around 0.5% at 30 kt and steadily increased to around 1% at 100 kt. The wing chord damping started around 1.4% at 30 kt and it decreased slightly to around 1.16% at 100 kt.
8. Straight blades, locked gimbal, wing off, and freewheel rotor configuration showed turning the gimbal into a stiff-in-plane hingeless hub had little effect on the wing beam damping, but increased the wing chord damping significantly between 30-60 kt. Beam damping started around 0.33% at 30 kt and steadily increased to 0.57% at 60 kt. Chord damping started around 2.1% at 30 kt and steadily decreased to 1.73% at 60 kt.
9. Straight blades, locked gimbal, wing off, and powered rotor configuration showed the electric drive had a major and unexpected positive effect on the wing chord damping at low speeds while the wing beam damping was unaffected up to 60 kt. Beam damping started at 0.55% at 4 kt. It dipped slightly to 0.32% at 20 kt, then steadily increased to 0.68% at 60 kt. Chord damping started at 2.28% at 4 kt, increased to about 2.86% at 30 kt, then decreased to 2.25% at 60 kt.

10. For swept-tip blades, free gimbal, wing on, and freewheel rotor configuration, stability of the beam and chord modes were similar to the straight blade configuration up to 100 kt. Beam damping started at 0.53% at 30 kt. It steadily increased to around 1.28% at 100 kt. Chord damping started at 1.45% at 30 kt. It remained relatively steady through 70 kt, then slightly decreased to 1.21% at 100 kt.
11. A simplified nine degree of freedom model was used to confirm the basic trends of the stability data. But, it fell far short of reliable magnitudes as expected.

8.3 Future Work

The University of Maryland, Alfred Gessow Rotorcraft Center, now has a new unique tiltrotor facility. This facility should be used over the next decade to advance tiltrotor technology as well as electric-VTOL technology. First, some necessary model improvements and facility recommendations are considered.

1. Electromagnetic interference (EMI) from the electric motor is intense enough to overwhelm the gimbal sensors and makes it impossible to trim the gimbal during powered operation. It also affects the load cell, so resolving this issue should be the top priority before commencing powered tests.
2. The GLMWT has been unavailable for over two years due to the COVID-19 pandemic and unexpected mechanical issues. The tunnel must be repaired and upgraded as soon as possible. Upgrades should include modern acoustic liners

so acoustic studies can be performed and upgraded electric power sources for eVTOL tests.

3. Carderock SWT facility is an excellent back-up facility with a max windspeed of 168 kt. Should the GLMWT be unavailable for an extended period, this partnership should be leveraged so tests that can continue and data collection can proceed up to 168 kt.
4. NASA Langley Transonic Dynamics Tunnel (TDT) is a facility that would allow for concurrent Froude- and Mach-scale tests using heavy gas medium. Matching these similitude parameters at scale increases the quality of aeroelastic tests of the MTR.

Currently, the hingeless hub is being fabricated by Calspan and will be the focus of studies for the next five years. Recommendations for MTR testing of the next five years pertains to both the gimballed and hingeless hubs.

1. MTR should be tested at least up to 175 kt and perhaps 200 kt if possible.
2. Center of gravity of the pylon should be moved forward to help lower the flutter boundary.
3. Wing pressures and airloads should be measured for interference.
4. Rotor airloads should be measured. The slip ring must be updated to provide more channels for pressure taps.
5. Flight dynamic derivatives should be measured.

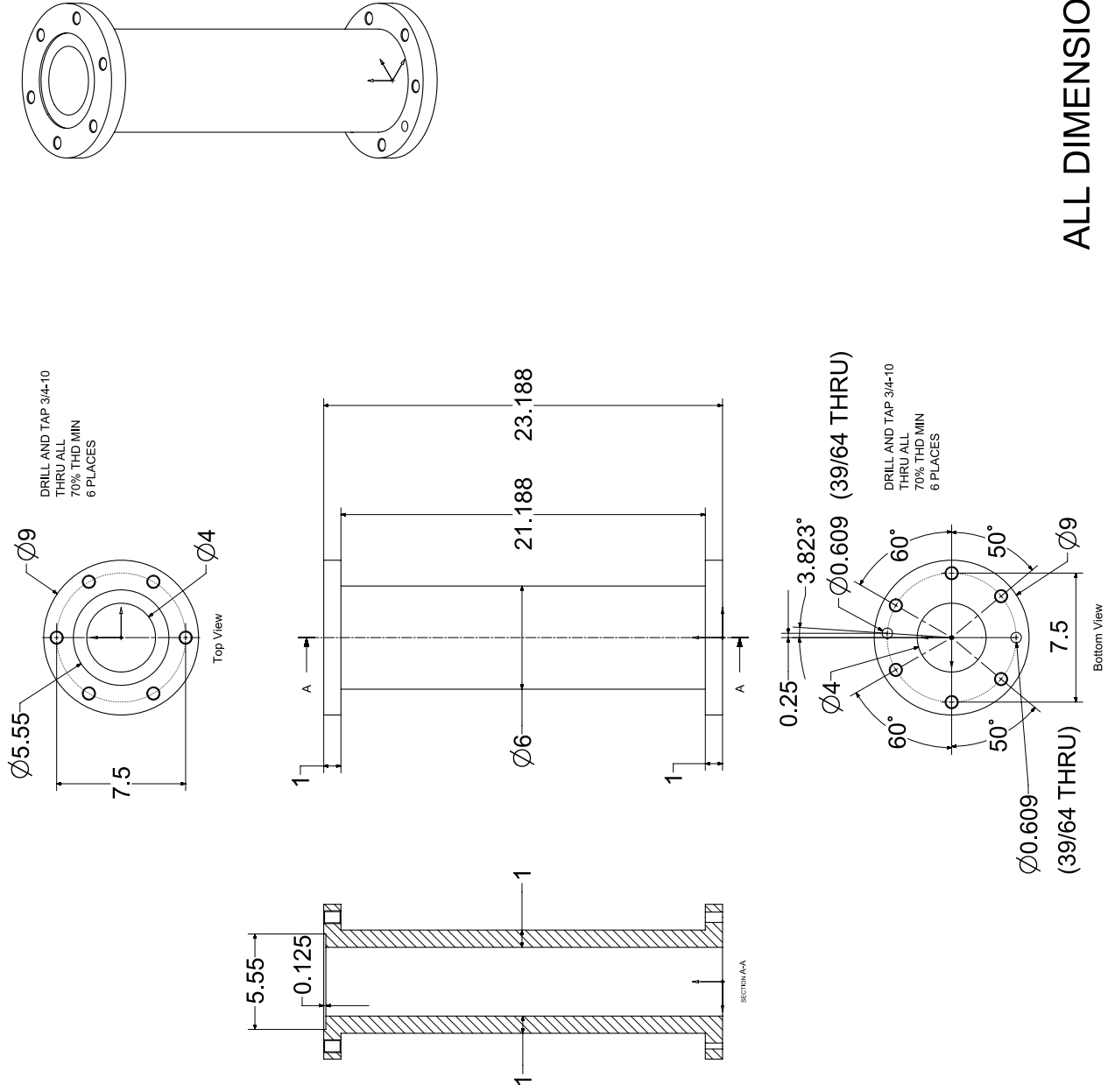
6. Methods to delay or eliminate instabilities should be invented and tested.

It is impossible not to think about what the future may bring beyond the next five years. Future research on the MTR is really unlimited in scope because the flexibility of having interchangeable parts allows for all kinds of studies, small and large. The following recommendations only touch on a few interesting topics.

1. More advanced blade shapes should be developed and tested such as chord variations and composite couplings. Besides their effect on performance and stability, the acoustics should be explored for these advanced blade geometries.
2. With hub interchangeability, a rigid hub with individual blade control (IBC) can be investigated. This would eliminate the need for swashplate actuators, swashplate, and pitch linkages.
3. Adding a shroud to open rotors has always been a promising design and an unrealized vision for VTOL designers. The question of how to mount it on the MTR and how to fabricate it would be a challenging and relevant problem.

Appendices

Technical Drawings



ALL DIMENSIONS IN INCHES

Figure 1: GLM Wind Tunnel Interface Post Drawing

ALL DIMENSIONS IN INCHES

TOLERANCES:	
X	= ±0.01
X.X	= ±0.01
X.XX	= ±0.01
X.XXX	= ±0.005

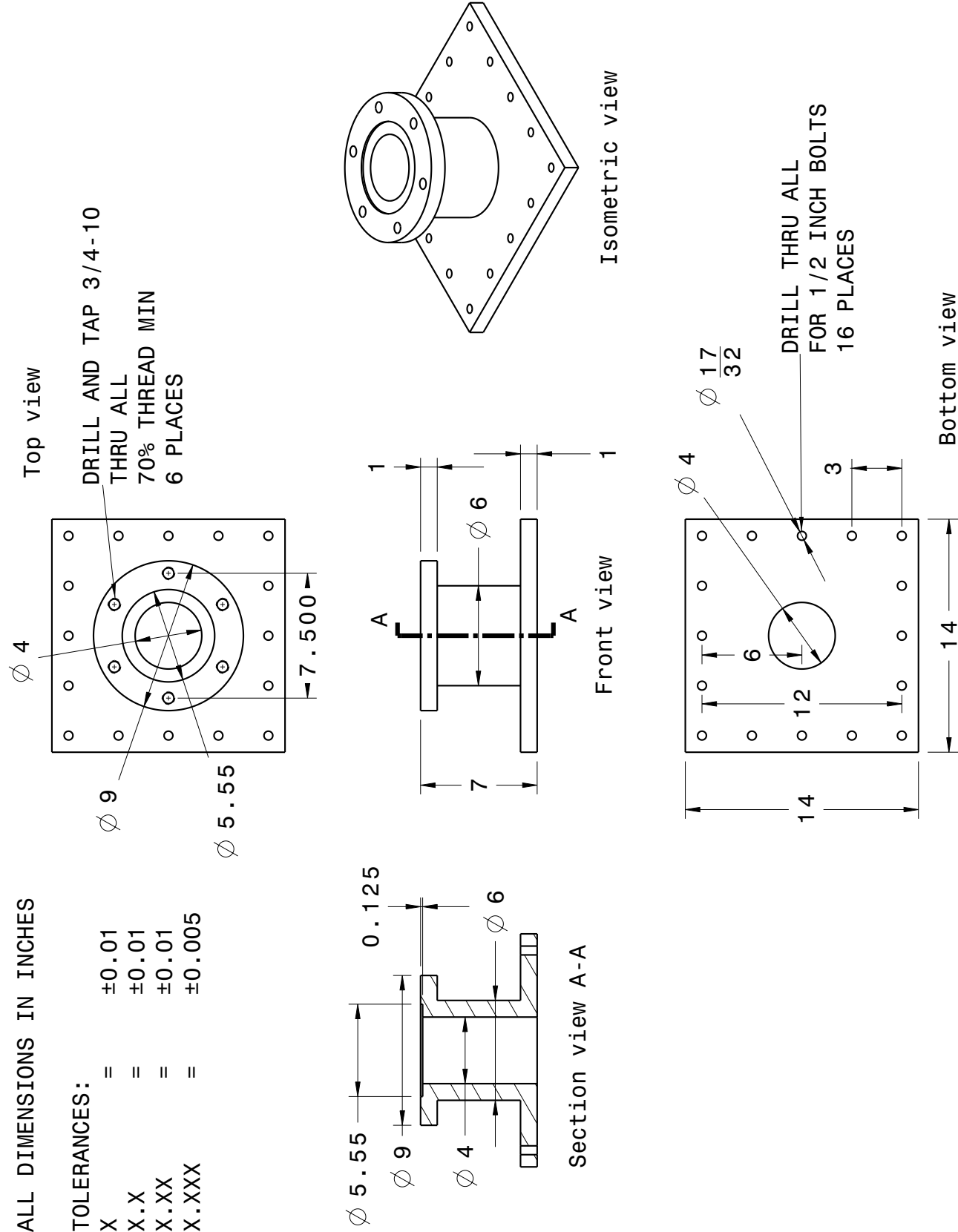


Figure 2: Carderock Wind Tunnel Interface Post Drawing

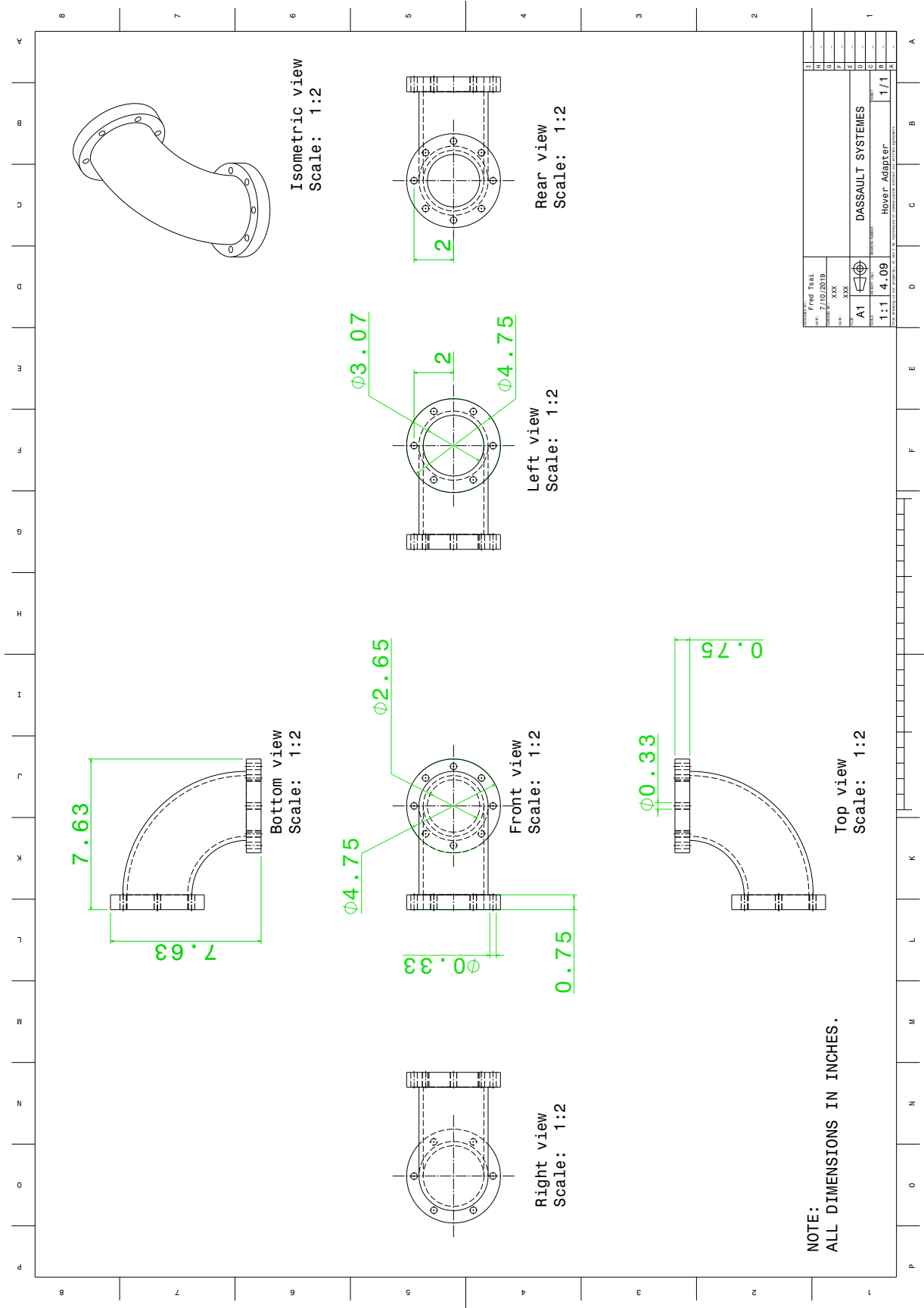


Figure 3: Hover Tower Drawing

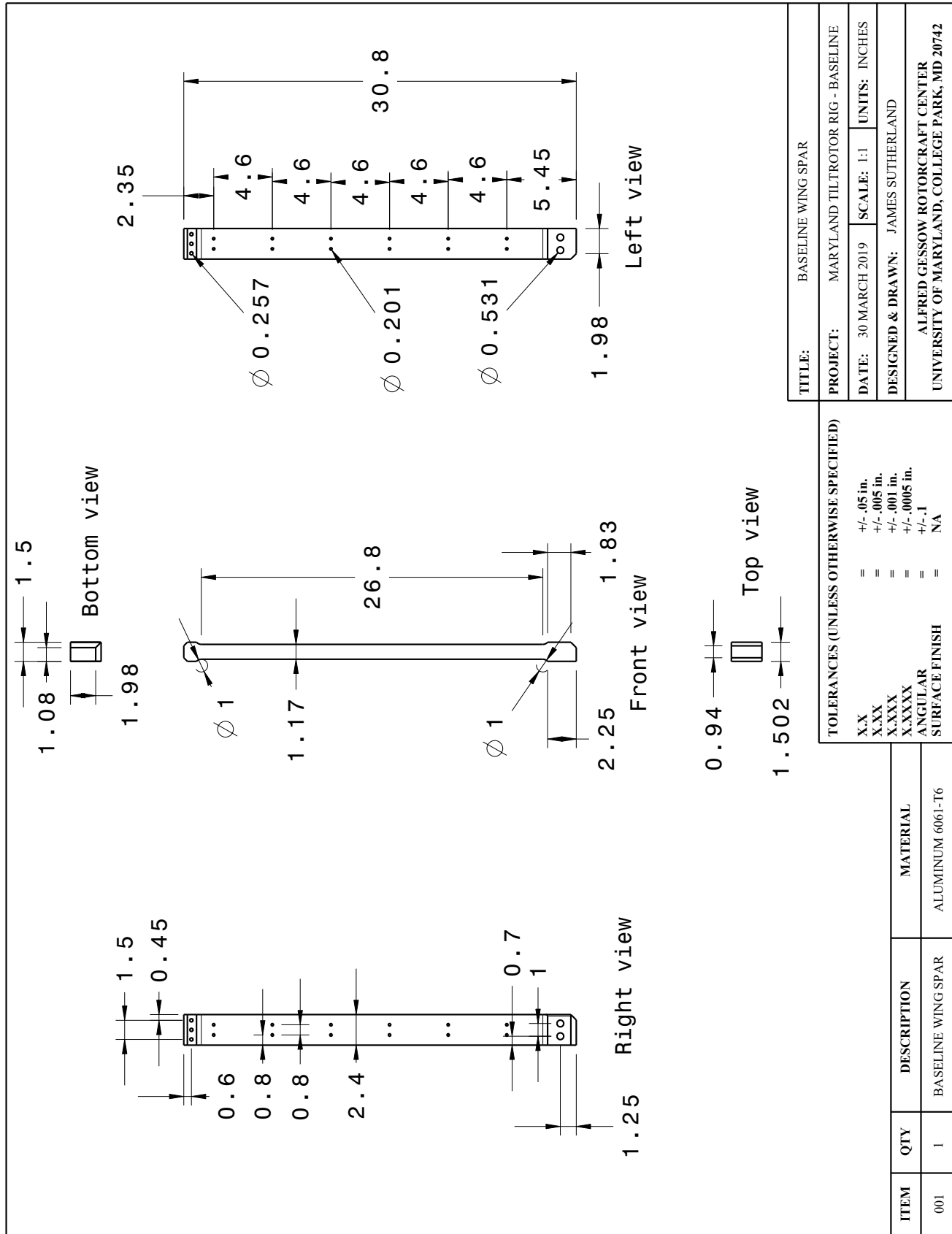


Figure 4: Baseline Wing Spar

Matlab Codes

Code for logarithmic decrement method. The code will take an average of the log decrements of consecutive peaks in the signal.

```
1 %% Test with log dec method
2 signaltime = 1000; % signal duration in samples
3 [Ypk,Xpk,Wpk,Ppk] = findpeaks(s(sig_start:sig_start+signaltime),'
    ↪ MinPeakProminence',20);
4
5 % Plot signal and the peaks
6 figure;
7 ee = plot(Time(sig_start:sig_start+signaltime),s(sig_start:sig_start
    ↪ +signaltime)-mean(s(sig_start:sig_start+signaltime)),Time(Xpk+
    ↪ sig_start),Ypk-mean(s(sig_start:sig_start+signaltime)),'dr');
8 xlabel('Time, sec')
9 ylabel('Strain, \mu\epsilon')
10 set(gca,'FontSize',12,'FontWeight','Bold');
11 set(ee,'Linewidth',3);
12
13 n_peaks = numel(Xpk); % Determine the number of peaks
14
15 %Vector length of interest
16 t_new = Time(Xpk);
```

```

17 y_new = Ypk-mean(s(sig_start:sig_start+signaltime)); % Remove mean
    ↪ of the signal
18
19 %Calculate Logarithmic Decrement, undamped natural frequency,
    ↪ damped natural frequency, damping ratio
20 Log_Dec = zeros(length(n_peaks)); % Initialize
21
22 for nn = 1:n_peaks-1
23     Log_Dec(nn)= log(y_new(nn)/y_new(nn+1)); % computed  $n = 1$ 
        ↪ periods apart
24 end
25
26 Mean_dec = mean(Log_Dec); %Calculate Average Logarithmic Decrement
27
28 % Damping calculation
29 damp_ratio_logdec = 1/sqrt(1+((2*pi/(Mean_dec))^2)); %Assesses
    ↪ Damping Constant

```

Simple code for Prony Method. Signal has sampling frequency of 1000 samp/s.

The signal is a 1 second duration of a damped response from N1.78 from the MTR Carderock test. Model number was chosen based on the 1000 sample signal used.

For signals with more or fewer samples, change the model number accordingly.

```
1 f    = 1000; % samples/sec
2 M    = 400; % model number
3 test_sig = s(sig_start:sig_start+1000);
4 N    = length(test_sig);
5
6 % Perform Prony method
7 T = toeplitz(test_sig,zeros(1,N));
8 x = H((M+1):N,1); % N-M by 1
9 A = H((M+1):N,2:(M+1)); % N-M by N
10 P = [-1;A\x].'; % M+1 by 1
11 p = roots(P);
12
13
14 % Obtain frequency, and damping
15 spoles = log(p)*f;
16 sigma = real(spoles); % Sigma term
17 omega = imag(spoles); % Frequency omega rad/s
18 frequency = abs(omega/(2*pi)); % Hz
```

```
19 damping = (sigma)./(-frequency*2*pi)*100; % in %critical
```

Code for Prony Method using Matlab functions. Signal has sampling frequency of 1000 samp/s. The signal is a 1 second duration of a damped response from N1.78 from the MTR Carderock test. Model number array was chosen based on the 1000 sample signal used. For signals with more or fewer samples, change the model numbers accordingly.

```
1 f    = 1000; % samples/sec
2 M    = [20 100 250 400 800]; % loop through model numbers
3 test_sig = s(sig_start:sig_start+1000);
4
5 figure
6
7 % Perform Prony method for each model number
8 for i = 1:length(M)
9     [inumz,idenz] = prony(test_sig,M(i),M(i));
10    [iapp,tapp] = impz(inumz,idenz,length(test_sig),f); % Used to
        ↪ plot of prony time signal
11    ff    = plot(tapp,iapp,'o');
12    hold on;
13    plot(tapp,test_sig);
14    ylim([-200 100]);
15
```



```

16 [r,p,k] = residuez(inumz,idenz); % Obtain residue, poles, and
    ↪ direct terms. Only r and p needed.

17

18 % Obtain amplitude, frequency, and damping

19 a_list = r;

20 prony_amp = abs(2*a_list); % amplitude

21 spoles = log(p)*f;

22 tau_list = real(spoles); % Sigma term

23 omega_list = imag(spoles); % Frequency omega rad/s

24 frequency = abs(omega_list/(2*pi)); % Hz

25 damping = (tau_list)./(-frequency*2*pi)*100; % in %critical

26

27 % Sort results in descending order in amplitude.

28 [prony_amp_sorted, ISort] = sort(prony_amp);

29 ISort_desc = ISort(end:-1:1);

30 prony_amp_sorted = prony_amp_sorted(end:-1:1);

31 frequency_sorted = frequency(ISort_desc);

32 damping_sorted = damping(ISort_desc);

33

34 % Display first ten results of each model order

35 disp([prony_amp_sorted(1:10) frequency_sorted(1:10)
    ↪ damping_sorted(1:10)]);

```

```
36 end
37
38 % Plot settings
39 xlabel('Time, sec')
40 ylabel('Strain, \mu\epsilon')
41 set(gca,'FontSize',12,'FontWeight','Bold');
42 set(ff,'Linewidth',3);
43 legend('Model # = 20','Model # = 100','Model # = 250','Model # = 400
    ↪ ','Model # = 800')
```

Code for moving block method using a simulated signal.

```
1 close all; clear all; clc;
2
3 %Calculate damping ratio using the moving block method for a simple
   ↪ simulated
4 %signal
5 %1) Use raw transient strain data, determine the frequencies of
   ↪ interest.
6 %2) FFT a window of that data - ideally use 2^n number of samples
   ↪ like 512, 1024, etc.
7 %3) Find the amplitude of the frequencies of interest and store
   ↪ those scalars
8 %4) Shift window down one sample and repeat step 3
9 %5) Plot the log(amplitude) vs time. Time is based on sampling
   ↪ frequency,
10 %if you are sampling at 100 samp/sec then each sample shift is 0.01
   ↪ second.
11 %6) Fit a line through the plot and the damping ratio, zeta, is
   ↪ calculated as
12 % slope = -zeta*omega_n
13
14 %% Create Damped Signal
```

```

15 % Simulated Signal
16 A = 10; % Amplitude 1
17 f = 1000; % sampling frequency samp/sec
18 n = 3000; % multiplier to get full signal length
19 T = 1/f; % sampling interval
20 t = 0:T:n*T; % time array
21 zeta_ini = 0.01; % simulated damping ratio: 1% of critical damping
22 omega1 = 2*pi*5; % 5 Hz natural frequency to rad/s
23
24 % Damped Signal
25 s = A*exp(-zeta_ini(1)*omega1*t).*sin(sqrt(1-zeta_ini(1)^2)*omega1*t
    ↪ );
26
27 %% Add noise - simplified MATLAB function awgn()
28 SNR = 1.5; % ratio of signal to noise
29 sigPower = sum(abs(s).^2)/numel(s);
30 noisePower = sigPower/SNR;
31 noise = sqrt(noisePower)*randn(size(s)); % white noise signal
32 s = s+noise; % final signal plus noise
33
34 %% Plot Damped Signal
35 figure

```

```

36 c = plot(t,s');
37 ylim([-25 20])
38 xlabel('Time, sec')
39 ylabel('Strain, m\epsilon')
40 set(c,'linewidth',2);
41 set(gca,'fontweight','bold','fontsize',12);
42
43 %% Step 1a: Set the index for looking at the damped signal (
    ↪ simulated signal is easy to start at sample 1)
44
45 % Look at signal FFT resolution
46 sig_start = 1; % signal starts at sample 1
47
48 % Calculate the frequency in Hz
49 [sig,ftemp] = dataFFT(s',f); % uncorrected amplitude and energy
50 figure
51 b = plot(ftemp,sig);
52 xlim([0 20])
53 set(b,'linewidth',2)
54 set(gca,'fontweight','bold','fontsize',12)
55 xlabel('Frequency, Hz')
56 ylabel('Amplitude (m\epsilon)')

```

```

57
58 %% Step 1b finding the frequencies of interest
59 lowerfreq = 4; % lower bound when searching for frequency peak
60 upperfreq = 6; % upper bound when searching for frequency peak
61 l1b = find(ftemp>lowerfreq,1); % find the indices of the frequency
    ↪ bands, lower 1st freq bound
62 u1b = find(ftemp>upperfreq,1); % upper 1st frequency bound
63 [pks1,locs1] = findpeaks(sig(l1b:u1b),'SortStr','descend'); % sort
    ↪ by descending order, largest peak first
64 locs1 = l1b+locs1; % the lower bound index + the location index of
    ↪ the largest peak within the lower and upper bounds gives the
    ↪ actual location index
65
66 % These frequencies will be used to calculate damping ratio at the
    ↪ end
67 harmonics = [ftemp(locs1(1)) ftemp(locs1(1)) ftemp(locs1(1))]; %
    ↪ Multiple elements allows for looping, can be changed to have
    ↪ different frequencies when signal has multiple modes.
68
69 %% Step 2 Creating Blocks of the Signal
70 % The size of the block or window is mostly a trial and error
    ↪ exercise

```

```

71 window_size = [2^9 2^10 2^11]; % block sizes per harmonic for f=1000
    ↪ sample rate
72
73 % For each frequency, calculate the damping ratio. For this example
    ↪ there is only 1 harmonic and 3 block sizes so the damping
    ↪ ratio is calculated for each block size.
74 for ind = 1:length(harmonics)
75     num_windows = length(s)-window_size(ind)-1; % Maximum number of
        ↪ sample shifts that can be made in the full signal.
76     time(ind).windowsize = 0:T:(num_windows-1)*T; % time array to
        ↪ match the number of shifts
77
78     % Initialize
79     ini_index = sig_start;
80     amplitude = zeros(1,num_windows);
81
82     % Loop through the moving blocks and record the amplitudes of
        ↪ the interested frequencies
83     for i = 1:num_windows
84         % Creating a block
85         StrainWindow = s(ini_index:ini_index+window_size(ind)-1);
86         [fftstrain, freq] = dataFFT(StrainWindow,f);

```

```

87
88      % % Take a look at the FFT of the first block
89      % if i==1
90      % figure(3)
91      % b = plot(freq,fftstrain);
92      % xlim([0 20])
93      % set(b,'linewidth',2)
94      % set(gca,'fontweight','bold','fontsize',12)
95      % xlabel('Frequency, Hz')
96      % ylabel('Amplitude (m\epsilon)')
97      % hold on
98      % end
99
100     %% Step 3 Storing the Amplitudes of the Frequencies from FFT
101     % Search algorithm to get the frequency band of interest,
102         ↪ and find
103
104     % max peak in that range, can be used for multiple
105         ↪ frequencies
106
107     % if ind == 1
108
109     l1b = find(freq>lowerfreq,1); %find the indices of the
110         ↪ frequency bands, lower 1st freq bound

```



```

106     u1b = find(freq>upperfreq,1); %upper 1st frequency bound
107     [pks,locs] = findpeaks(fftstrain(l1b:u1b),'SortStr','descend'
    ↪ );
108     locs = l1b+locs-1;
109     % elseif ind == 2
110     % l2b = find(freq>8,1); %lower 2nd frequency bound
111     % u2b = find(freq>11,1); %upper 2nd freq bound
112     % [pks,locs] = findpeaks(fftstrain(l2b:u2b),'SortStr','
    ↪ descend');
113     % locs = l2b+locs-1;
114     % elseif ind == 3
115     % l3b = find(freq>16,1); %lower 3rd freq bound
116     % u3b = find(freq>20,1); %upper 3rd freq bound
117     % [pks,locs] = findpeaks(fftstrain(l3b:u3b),'SortStr','
    ↪ descend');
118     % locs = l3b+locs-1;
119     % end
120     amplitude(i) = pks(1); %Store frequency peak value
121
122     %% Step 4 Move block a sample down
123     ini_index = ini_index+1; %Next sample
124

```

```

125     end
126
127     %% Step 5 Take the log of the final amplitudes array and plot it
128     log_amp(ind).windowsize = log(amplitude);
129     figure(4)
130     e = plot(time(ind).windowsize,log_amp(ind).windowsize);%,time,
        ↪ log_amp0)
131     xlabel('Time, sec')
132     ylabel('Log of Amplitude')
133     set(gca,'FontSize',12,'FontWeight','Bold');
134     set(e,'Linewidth',2);
135     hold on
136
137     %% Step 6a Plot the fitted line through the log of amplitude
        ↪ line
138     % Fit a linear line through the result and the slope will allow
        ↪ you to
139     % calculate the damping ratio
140     P = polyfit(time(ind).windowsize,log_amp(ind).windowsize,1);
141     yfit = P(1)*time(ind).windowsize+P(2);
142     hold on;
143     plot(time(ind).windowsize,yfit,'r-.')%,time,yfit0,'b-.');

```

```

144
145     %% Step 6b Calculate the Damping Ratio
146     zeta_movingblock(ind) = -P(1)/(harmonics(ind)*2*pi);
147
148 end
149
150 %% Test fits with lesser time
151 % In some cases, the oscillating line has a higher slope (more
    ↪ damping) in the initial second than the rest of the signal
    ↪ and so this section investigates the damping ratio of shorter
    ↪ signals by performing a linear fit over a shorter duration.
    ↪ This example takes the time used in the largest block size.
152 P1 = polyfit(time(1).windowsize(1:length(time(3).windowsize)),
    ↪ log_amp(1).windowsize(1:length(time(3).windowsize)),1);
153 %yfit1 = P1(1)*time(1).windowsize(1:2000)+P1(2);
154 P2 = polyfit(time(2).windowsize(1:length(time(3).windowsize)),
    ↪ log_amp(2).windowsize(1:length(time(3).windowsize)),1);
155 %yfit2 = P2(1)*time(2).windowsize(1:2000)+P2(2);
156 P3 = polyfit(time(3).windowsize,log_amp(3).windowsize,1);
157 %yfit3 = P3(1)*time(3).windowsize(1:2000)+P3(2);
158 zeta1 = -P1(1)/(harmonics(ind)*2*pi)
159 zeta2 = -P2(1)/(harmonics(ind)*2*pi)

```

```

160 zeta3 = -P3(1)/(harmonics(ind)*2*pi)
161
162 %% FFT function
163 %Calculates the FFT of a signal
164 %Inputs: raw_data = signal array of data
165 %sampling_frequency = scalar denoted in samples/sec
166
167 function [data, freq] = dataFFT(raw_data,sampling_frequency)
168 L = length(raw_data);
169 NFFT = 2^6*L;%2^nextpow2(L); % Zero-padding to get higher
    ↪ resolution in FFT
170 freq = sampling_frequency/2*linspace(0,1,floor(NFFT/2)+1);
171 data = fft(raw_data,NFFT)/L;
172 data = 2*abs(data(1:floor(NFFT/2)+1));
173
174 %% Original FFT function, no zero padding of fft
175 % Y=fft(raw_data);
176 % P2 = abs(Y/L);
177 % data = P2(1:L/2+1);
178 % data(2:end-1) = 2*data(2:end-1);
179 % freq = sampling_frequency*(0:(L/2))/L;
180 end

```

MTR Pin Connections

The wire connection sheet documents the wires and pin connections from the MTR instruments to the DAQ. Follow each of the columns starting from the left to the right of the page. Under System column, PL denotes pitch link. Under Screw Terminal column, the screw terminals at the nose of the rotor are labeled with J#. For example, J5-21 means the pitch link wires connect to the J5-J21 terminals. J5 is the inner terminal that connects to the slip ring wires, and J21 is the outer terminal that connects to the pitch link wires. The pins on terminal J5 and J21 are connected through the printed circuit board directly, so J5 pin 1 is electrically connected to J21 pin 1. Slip Ring Wire # column denotes the slip ring wire which is synonymous with the pins on the slip ring. There are four slip ring cables that run from the slip ring to the DAQ, with 16 wires inside each cable. These wires are also numbered and colored which are documented in columns "Slip Ring Cable End Pin #" and "Slip Ring Cable Wire Color". The slip ring cable wires have 8 colored wires paired with white wires for each color. So to denote the white wires, they are described with the color they are twisted to. For example, "Red" is the fully red wire, and "Red-W" is the fully white wire that is twisted with the red wire. These wires are pinned into connectors, the majority of which are Molex. The connectors have labels on the housing that denote the pin numbers. The last 4 columns show the destination on the DAQ. "Destination on Module" column shows two 4331 modules with (5) and (7) appended; this identifies the modules in slot 5 or slot 7 on the DAQ. "6365-0" and "6365-2" denotes the 6365 module and the slot that the sensors are connected to. There are three slots on the 6365: slot 0, 1 and 2, and only slots 0 and 2 are used. Note it is important to reference the pinout diagram for the 6365 module as

the pins are NOT the same between each slot. Slot 0 has AI/AO and DI/DO pins while slots 1 and 2 are AI pins ONLY. The pitch encoders were separated from other sensors and placed in the cDAQ module because they caused significant noise in the other sensors during pitching motions.

System Cable	System	Wire Function	Wire Color	Screw Terminal	Pin #	Slip Ring Wire #	Slip Ring Cable End Pin #	Slip Ring Cable Wire Color	Connector	Connect or Pin #	Destination Function	Destination on Module	Pin	Pin Label
16-channel Slip Ring Cable SR1	PL 1	SV	Red		1	53	1	Red		1	DAQ	4331 (7)	CH 0 EX+	
	PL 1	GND	Black		2	25	9	Red-W		9	DAQ	4331 (7)	CH 0 EX-	
	PL 1	Signal-	Green	J 5-21	3	61	2	Orange		2	DAQ	4331 (7)	CH 0 AI-	
	PL 1	Signal+	White		4	8	10	Orange-W		10	DAQ	4331 (7)	CH 0 AI+	
	PL 2	SV	Red		1	4	3	Brown		3	DAQ	4331 (7)	CH 1 EX+	
	PL 2	GND	Black	J 10-26	2	42	11	Brown-W		11	DAQ	4331 (7)	CH 1 EX-	
	PL 2	Signal-	Green		3	10	4	Purple		4	DAQ	4331 (7)	CH 1 AI-	
	PL 2	Signal+	White		4	49	12	Purple-W		12	DAQ	4331 (7)	CH 1 AI+	
	PL 3	SV	Red		1	18	5	Yellow	Molex 16-pin	5	DAQ	4331 (7)	CH 2 EX+	
	PL 3	GND	Black	J 8-24	2	12	13	Yellow-W		13	DAQ	4331 (7)	CH 2 EX-	
	PL 3	Signal-	Green		3	17	6	Black		6	DAQ	4331 (7)	CH 2 AI-	
	PL 3	Signal+	White		4	27	14	Black-W		14	DAQ	4331 (7)	CH 2 AI+	
	Shaft	SV	Red		1	32	7	Green		7	DAQ	4331 (7)	CH 3 EX+	
	Shaft	GND	Black	J 1-17	2	14	15	Green-W		15	DAQ	4331 (7)	CH 3 EX-	
	Shaft	Signal-	Green		3	5	8	Blue		8	DAQ	4331 (7)	CH 3 AI-	
	Shaft	Signal+	White		4	24	16	Blue-W		16	DAQ	4331 (7)	CH 3 AI+	
16-channel Slip Ring Cable SR2	Blade 3 Bend	SV	Red		1	51	1	Red		1	DAQ	4331 (7)	CH 4 EX+	
		GND	Black	J 3-19	2	50	9	Red-W		9	DAQ	4331 (7)	CH 4 EX-	
		Signal-	Green		3	40	2	Orange		2	DAQ	4331 (7)	CH 4 AI-	
		Signal+	White		4	33	10	Orange-W		10	DAQ	4331 (7)	CH 4 AI+	
	Blade 1 Tor	SV	Red		1	47	3	Brown		3	DAQ	4331 (7)	CH 5 EX+	
		GND	Black	J 11-27	2	59	11	Brown-W		11	DAQ	4331 (7)	CH 5 EX-	
		Signal-	Green		3	13	4	Purple		4	DAQ	4331 (7)	CH 5 AI-	
		Signal+	White		4	44	12	Purple-W		12	DAQ	4331 (7)	CH 5 AI+	
	Blade 1 Bend	SV	Red		1	48	5	Yellow	Molex 16-pin	5	DAQ	4331 (7)	CH 6 EX+	
		GND	Black	J 13-29	2	15	13	Yellow-W		13	DAQ	4331 (7)	CH 6 EX-	
		Signal-	Green		3	35	6	Black		6	DAQ	4331 (7)	CH 6 AI-	
		Signal+	White		4	30	14	Black-W		14	DAQ	4331 (7)	CH 6 AI+	
16-channel Slip Ring Cable SR3	Pitch Encoder 1	GND	White	J 4-20	1	1	1	Black	Molex 4-pin	1	PWR	5V	Neg	
	Pitch Encoder 2+3	GND	White	J 9-25	1	7	9	Black-W		2	PWR	5V	Neg	
	Pitch Encoder 1	SV	Brown	J 12-28	1	26	2	Red		3	PWR	5V	Pos	
	Pitch Encoder 2+3	SV	Brown	J 2-18	1	16	10	Red-W		4	PWR	5V	Pos	
	Pitch Encoder 1	A-	Yellow	J 12-28	2	29	11	Brown-W		7	cDAQMod2	9401	1	COM
	Pitch Encoder 3	B	Blue	J 4-20	4	63	16	Purple-W		9	cDAQMod1	9401	20	PF14 (ctr3)
	Pitch Encoder 3	B-	Red	J 4-20	2	3	5	Purple		3	cDAQMod1	9401	1	COM
	Pitch Encoder 1	B	Green	J 12-28	3	23	3	Brown		1	cDAQMod2	9401	14	PF10 (ctr0)
	Pitch Encoder 1	A	Blue	J 12-28	4	45	4	Blue		2	cDAQMod2	9401	17	PF12 (ctr0)
	Pitch Encoder 1	B-	Red	J 4-20	3	64	12	Blue-W	Molex 12-pin	8	cDAQMod2	9401	1	COM
	Pitch Encoder 2	A-	Yellow	J 2-18	2	2	14	Green-W		10	cDAQMod2	9401	7	COM
	Pitch Encoder 3	A	Green	J 9-25	2	43	16	Orange-W		12	cDAQMod1	9401	25	PF17 (ctr3)
	Pitch Encoder 3	A-	Yellow	J 9-25	4	58	8	Orange		6	cDAQMod1	9401	1	COM
	Pitch Encoder 2	B	Green	J 2-18	3	21	6	Green		4	cDAQMod2	9401	16	PF11 (ctr1)
	Pitch Encoder 2	A	Blue	J 2-18	4	22	7	Yellow		5	cDAQMod2	9401	19	PF13 (ctr1)
	Pitch Encoder 2	B-	Red	J 9-25	3	60	15	Yellow-W		11	cDAQMod2	9401	7	COM
16-channel Slip Ring Cable SR4	Hall 1	SV	Red	J 7-23	2	41	9	Brown-W	Molex 6-pin	1	PWR	5V	Neg	
				J 15-31	1	46	1	Brown		4	PWR	5V	Pos	
	Hall 1	GND	Black	J 6-22	1	36	2	Red		2	PWR	5V	Pos	
	Hall 2	SV	Green	J 6-22	2	20	3	Black		3	PWR, DAQ	5V	Neg	
	Hall 2	GND	Black	J 16-32	1	57	10	Red-W		5	PWR	5V	Pos	
				J 16-32	2	52	11	Black-W		6	PWR, DAQ	5V	Neg	
	NC - Broken terminal			J 15-31	2	54	12	Orange-W		7				
				J 7-23	1	28	14	Green-W		9				
				J 7-23	4	6	6	Green		3				
				J 15-31	3	62	4	Orange		1				
				J 15-31	4	37	5	Yellow		2				
				J 7-23	3	19	13	Yellow-W	Molex 12-pin	8				
	Hall 1	GND	Black	J 6-22	2	20	3	Black		10	PWR, DAQ	6365-0	58	AI 6+
	Hall 1	Signal	White	J 6-22	3	31	7	Blue		4	DAQ	6365-0	25	AI 6+
	Hall 2	Signal	White	J 16-32	3	56	15	Blue-W		5	DAQ	6365-0	57	AI 7+
	Hall 2	GND	Black	J 16-32	2	52	11	Black-W		11	PWR, DAQ	6365-0	23	AI 7-
				J 6-22	4	55	8	Purple		6	DAQ	6365-0	56	GND
				J 16-32	4	39	16	Purple-W		12	DAQ	6365-0	56	GND
Load Cell Cable	Load Cell	SG0 Output	Brown			9				1	DAQ	6365-0	68	AI 0+
	Load Cell	SG0 Reference	Brown/White			18				9	DAQ	6365-0	34	AI 0-
	Load Cell	SG1 Output	Yellow			8				2	DAQ	6365-0	33	AI 1+
	Load Cell	SG1 Reference	Yellow/White			17				10	DAQ	6365-0	66	AI 1-
	Load Cell	SG2 Output	Green			7				3	DAQ	6365-0	65	AI 2+
	Load Cell	SG2 Reference	Green/White			16				11	DAQ	6365-0	31	AI 2-
	Load Cell	SG3 Output	Blue			6			Molex 16-pin	4	DAQ	6365-0	30	AI 3+
	Load Cell	SG3 Reference	Blue/White			15				12	DAQ	6365-0	6	AI 3-
	Load Cell	SG4 Output	Violet			5				5	DAQ	6365-0	28	AI 4+
	Load Cell	SG4 Reference	Violet/White			14				13	DAQ	6365-0	61	AI 4-
	Load Cell	SG5 Output	Grey			4				6	DAQ	6365-0	60	AI 5+
	Load Cell	SG5 Reference	Grey/White			13				14	DAQ	6365-0	26	AI 5-
	Load Cell	AI GND	Black			22				16	DAQ	6365-0	27	AI GND
	Load Cell	SV	Red			2			Molex 2-pin		NC - 5V supplied from alternate source			
	Load Cell	OV	Red/White			11								
Motor	Motor	Brake Input				1			Molex 12-pin	1	DAQ	4322	AO 0+	
	Motor	Reverse				4				3	DAQ	4322	AO 2+	
	Motor	Throttle Input				6				2	DAQ	4322	AO 1+	
	Motor	Impulse Input				8				4	DAQ	4322	NC	
	Motor	SV				9				5	DAQ	4322	AO 3+	
	Motor	GND				5				7	DAQ	4322	AO 0-4 (-)	
	Motor	GND				2			DB-9	5	DAQ	8430		
	Motor	RxD				5				3	DAQ	8430	Port 4	
Actuator	Actuator 1	ANV	Red								DAQ	4322	AO 5+	
	Actuator 1	GND	Black								DAQ	4322	AO 5-	
	Actuator 2	ANV	Red						Molex 6-pin		DAQ	4322	AO 6+	

note: A and B were swapped in VI because angles were moving wrong directions

note: A and B were swapped in VI because angles were moving wrong directions

Actuator Control	Actuator 2	GND	Black		INDEX 0-pin	DAQ	4322	AO 6-
	Actuator 3	ANV	Red			DAQ	4322	AO 7+
	Actuator 3	GND	Black			DAQ	4322	AO 7-
	Actuator 1	GND	Brown	L	DB-9	DAQ	8430	Port 1
	Actuator 1	RxD	Yellow	M		DAQ	8430	
	Actuator 1	TxD	Green	K		DAQ	8430	
	Actuator 2	GND	Brown	L	DB-9	DAQ	8430	Port 2
	Actuator 2	RxD	Yellow	M		DAQ	8430	
	Actuator 2	TxD	Green	K		DAQ	8430	
	Actuator 3	GND	Brown	L	DB-9	DAQ	8430	Port 3
	Actuator 3	RxD	Yellow	M		DAQ	8430	
	Actuator 3	TxD	Green	K		DAQ	8430	
Control Box	SV		Red		Bare Leads	DAQ	6365-0	14 SV
	GND		Black			DAQ	6365-0	29 AI GND
	Throttle		White			DAQ	6365-2	65
	Collective		White			DAQ	6365-2	68
	Lat Cyc		White			DAQ	6365-2	33
	Long Cyc		White			DAQ	6365-2	32
	E-Stop		Red			DAQ	6365-2	64 AI 92
	E-Stop		Black			DAQ	6365-0	8 SV
Nonrotating Instruments	Shaft Hall	SV	Red			DAQ	6365-0	14 SV
	Shaft Hall	GND	Black			DAQ	6365-2	9 D GND
	Shaft Hall	Signal	White			DAQ	6365-2	52
	Triax Accel				BNC	DAQ	4492	AI0
	Wing Beam	SV	Red		Bare Leads	DAQ	4331 (5)	CH 4 EX+
	Wing Beam	GND	Black			DAQ	4331 (5)	CH 4 EX-
	Wing Beam	Signal-	Green			DAQ	4331 (5)	CH 4 AI-
	Wing Beam	Signal+	White			DAQ	4331 (5)	CH 4 AI+
	Wing Chord	SV	Red		Bare Leads	DAQ	4331 (5)	CH 1 EX+
	Wing Chord	GND	Black			DAQ	4331 (5)	CH 1 EX-
	Wing Chord	Signal-	Green			DAQ	4331 (5)	CH 1 AI-
	Wing Chord	Signal+	White			DAQ	4331 (5)	CH 1 AI+
	Wing Torsion	SV	Red		Bare Leads	DAQ	4331 (5)	CH 2 EX+
	Wing Torsion	GND	Black			DAQ	4331 (5)	CH 2 EX-
	Wing Torsion	Signal-	Green			DAQ	4331 (5)	CH 2 AI-
	Wing Torsion	Signal+	White			DAQ	4331 (5)	CH 2 AI+

LabVIEW Block Diagram

This appendix is a manual for the LabVIEW block diagram and front panel. Pin connections should be deferred to the pin connection sheet as that is the most up to date, however, the layout and overall programming of the block diagrams are correct. This document can be found on the MTR DAQ computer in the MTR folder. Any changes made to the block diagram should be updated in this document.

January 21, 2021

Objectives:

- Act as a basic user manual the Maryland Tiltrotor Rig (MTR) VI
- Describe common functions and important parameters
- Detail how major code blocks function

Overview

Figure 1 shows a snapshot of the VI block diagram is a [VI Snippet](#). There is no way to zoom out to view the entire block diagram, so the best solution is to select all of the diagram by pressing Ctrl+A and create a VI Snippet by selecting “Create VI Snippet from Selection” in the Edit dropdown menu. This document will highlight how sensor datatypes are acquired and what manipulations are performed on the data.

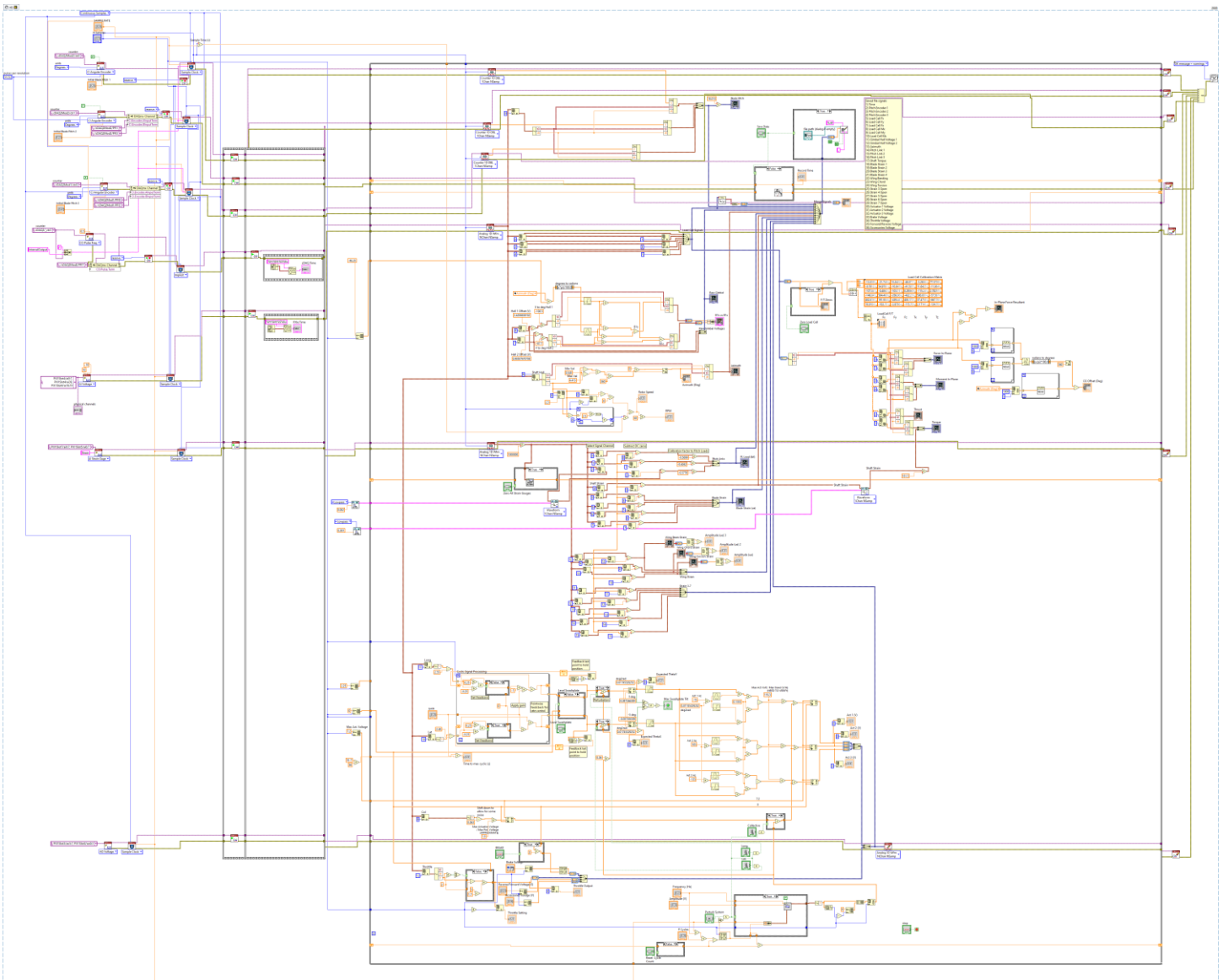


Figure 1 VI snippet of MTR block diagram

The front panel of the VI looks like Figure 2. The separate VI on the far right is for reading the motor serial communication.

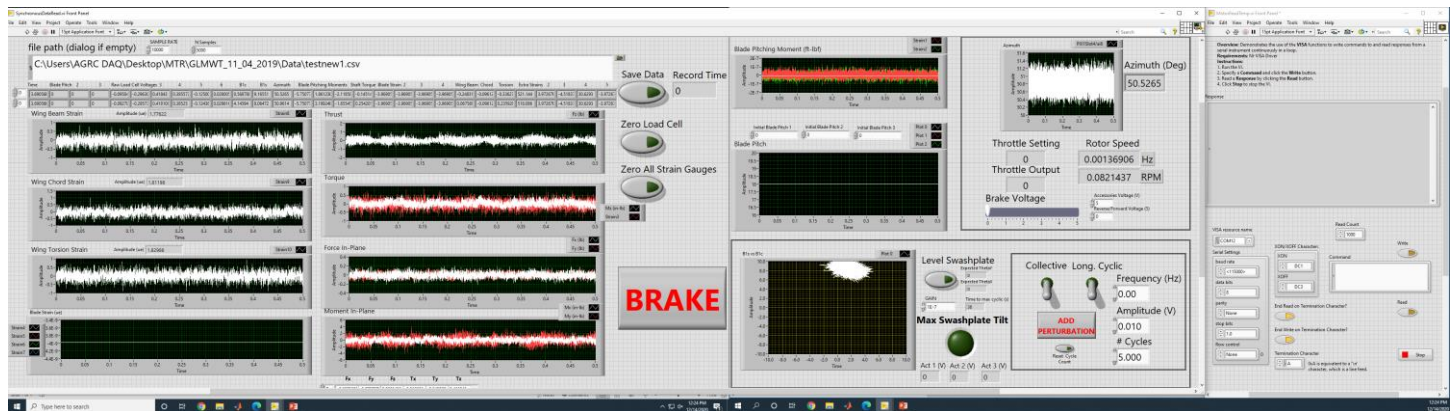


Figure 2 MTR front panel

Using DAQmx

The MTR VI uses DAQmx in LabVIEW, meaning the functions and nodes used are a lower level than the common Express VIs using DAQassistant.

Navigate to the DAQmx palette by right clicking the block diagram, scroll cursor to the Measurement I/O dropdown and select DAQmx – Data Acquisition folder in the upper left. Shown in Figure 3. These are the function nodes that are used to acquire and write data. Only the nodes that are used in the MTR VI will be explained in this document.

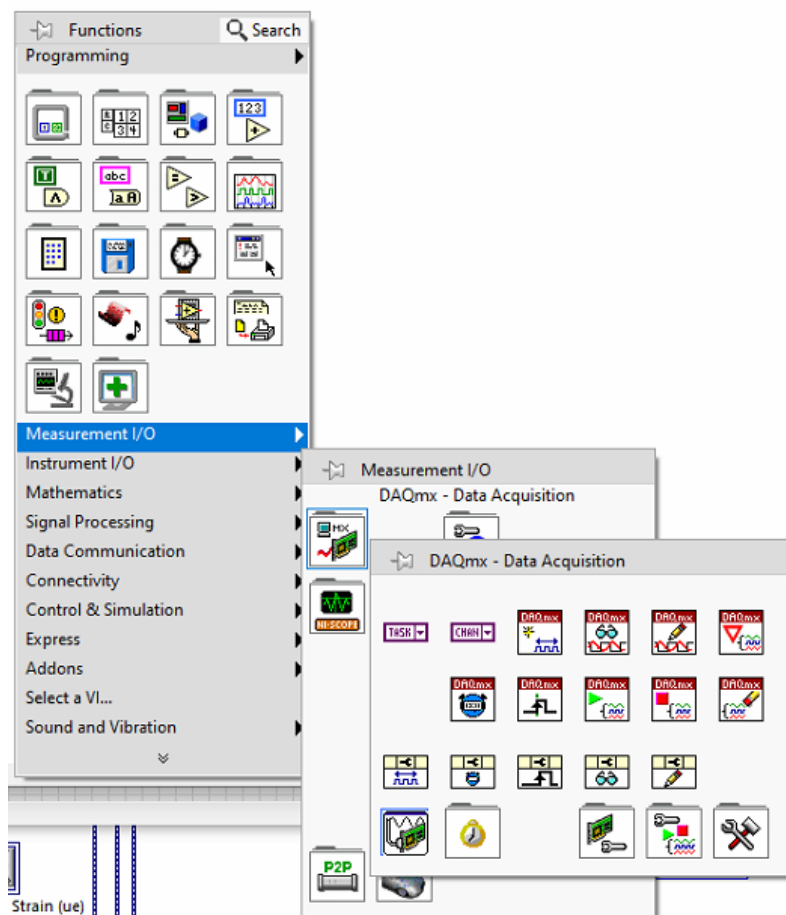


Figure 3 Navigating to DAQmx Palette

Data Acquisition Chassis

Two chassis are used: NI PXIe-1082 and NI CompactDAQ-9174 (cDAQ-9174).

PXIe-1082 Modules:

- PXIe-8840 Quad Core Embedded Controller
- PXIe-4331 8-Ch Bridge Analog Input (x2)
- PXIe-4492 Sound and Vibration
- PXIe-4322 8-Ch Analog Output
- PXIe-6365 Multifunction I/O

cDAQ-9174 Modules:

- NI-9401 Digital I/O Input (x2)

For a list of the sensors and pin connections to the modules, refer to the [Wire Connections.xlsx](#) worksheet.

Sensor Datatypes

The MTR uses:

- Analog Voltage Inputs – Hall effect sensors, load cell
- Analog Voltage Outputs – Actuator controls
- Strain Gage Inputs – Wing and blade strain gages
- Counter Inputs – Pitch encoders
- Serial Communication Inputs – Motor data (separate VI, not in main MTR VI)

Each type of input/output datatype is acquired on the same task.

The counter inputs are exceptions. Each counter-type sensor is on a separate task.

Synchronizing Tasks

MTR VI uses a Flat Sequence Structure to start the tasks. This is so all tasks start at the same time and therefore have the same timestamps during data acquisition. However, due to having a second chassis in the cDAQ-9174, the start task for the cDAQ and the PXIe do not start at exactly the same time.

Each chassis' timebase is separate so they are not synchronized. The only way to bypass this is to import the PXIe 10MHz Timebase available on the backplane into the cDAQ. An attempt was made to do this; however, any signal connection between the PXIe and cDAQ significantly increased the signal noise in the gimbal hall effect sensors. This was deemed unacceptable.

By placing the cDAQ tasks in the same VI as the PXIe tasks, the timestamps from the PXIe tasks can be used for the cDAQ tasks. Since data is acquired at a 10kHz sample rate, the real-time difference between the cDAQ datapoints and corresponding PXIe datapoints is small enough to assume data between the two chassis occurs at the same timestamp.

Setting Up Tasks

The first step in acquiring data is using DAQmx Create Virtual Channel node to declare the type of signal. This section will go through each datatype's setup.

Analog Input Voltages

From the DAQmx palette, place the Create Virtual Channel node into the block diagram. The dropdown menu below the node allows the user to choose the kind of data to be acquired; this node is Analog Input > Voltage. This can be seen in Figure 4.

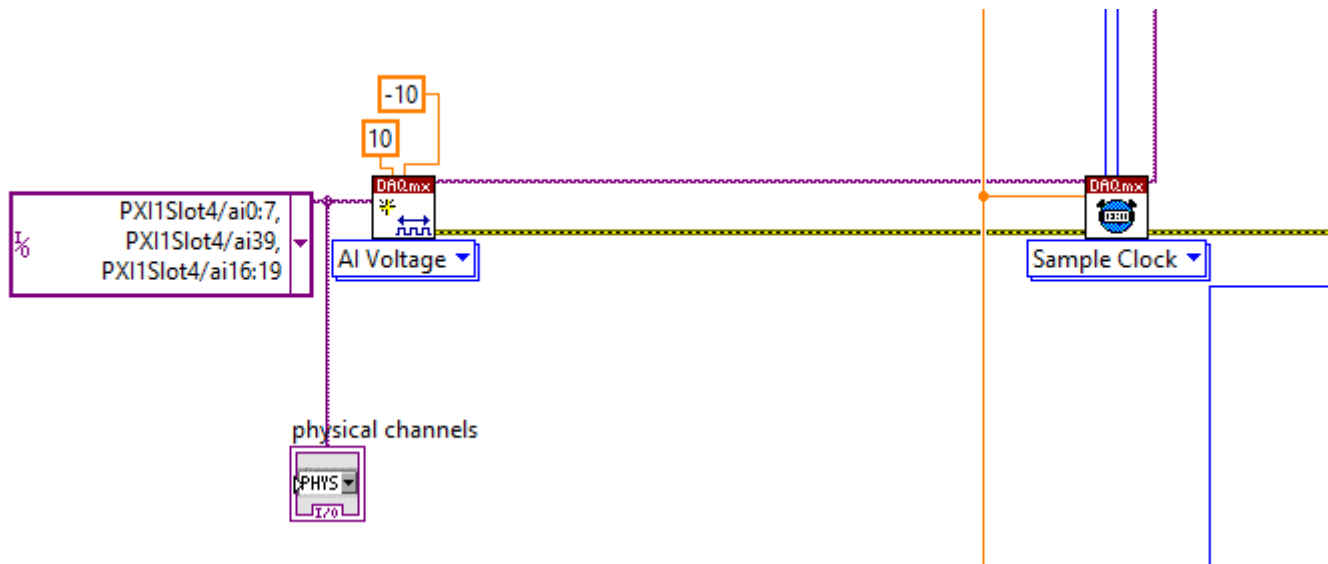


Figure 4 Analog input voltage node

The channel pins are input as a constant, seen as PXI1Slot4/ai0:7, PXI1Slot4/ai39, PXI1Slot4/ai16:19, meaning these are the default pins that the VI will see when the program starts. However, a control is branched off, labeled as “physical channels”, and that allows the user to modify the channels on the front panel. The scalar values 10 and -10 above the block are max and min values for voltage signals. Choosing to make constants, controls, or indicators can be done by right-clicking the terminal of interest and selecting the desired option.

The virtual channel node creates a task and that task terminal, at the upper right corner of the node, must be drawn to the next node, which is the timing function shown on the right side of Figure 4.

Analog Output Voltages

From the DAQmx palette, place the Create Virtual Channel node into the block diagram. The dropdown menu below the node allows the user to choose the kind of data to be acquired; this node is Analog Output > Voltage. This can be seen in Figure 5Figure 4.

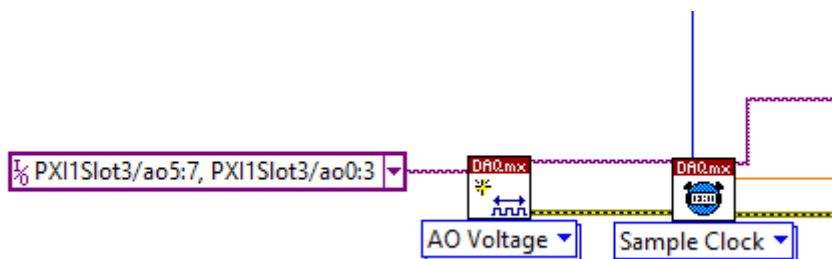


Figure 5 Analog output voltage node

The channels are input as a constant here, but like the Analog Input Voltage Node, another branch for control can be connected to the same terminal.

The virtual channel node creates a task and that task terminal, at the upper right corner of the node, must be drawn to the next node, which is the timing function shown on the right side of Figure 5Figure 4.

This node is used as a means to control the MTR, so the voltages must be written to the module and the physical pins on the module are connected to the electric actuators and the motor controller on the rig. For further explanation jump to the Write Task section.

Strain Gage Inputs

From the DAQmx palette, place the Create Virtual Channel node into the block diagram. The dropdown menu below the node allows the user to choose the kind of data to be acquired; this node is Analog Input > Strain > Strain Gage. This can be seen in Figure 5Figure 4.

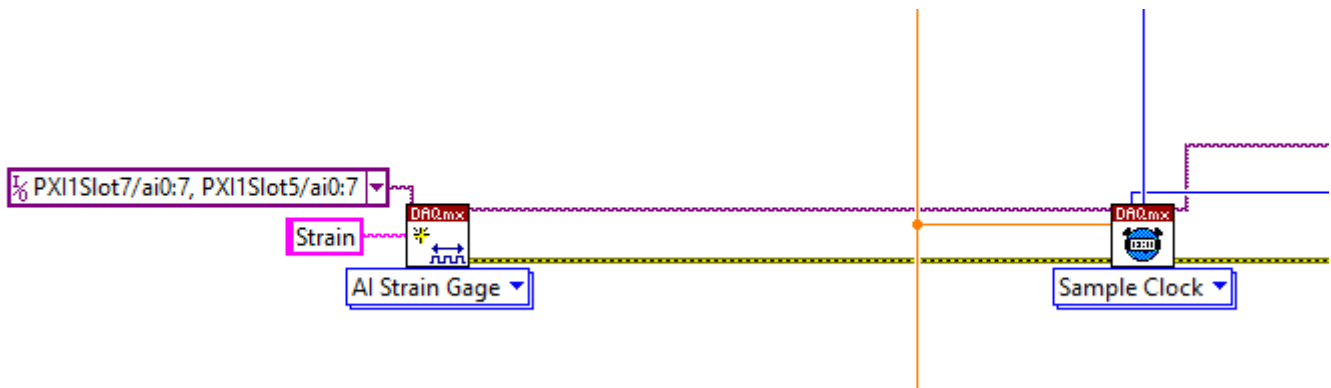


Figure 6 Strain gage input node

The channel pins are input as a constant here, but like the Analog Input Voltage Node, another branch for control can be connected to the same terminal.

The virtual channel node creates a task and that task terminal, at the upper right corner of the node, must be drawn to the next node, which is the timing function shown on the right side of Figure 6Figure 5Figure 4.

Counter Inputs

From the DAQmx palette, place the Create Virtual Channel node into the block diagram. The dropdown menu below the node allows the user to choose the kind of data to be acquired; this node is Counter Input > Position > Angular Encoder. This can be seen in Figure 7Figure 5Figure 4.

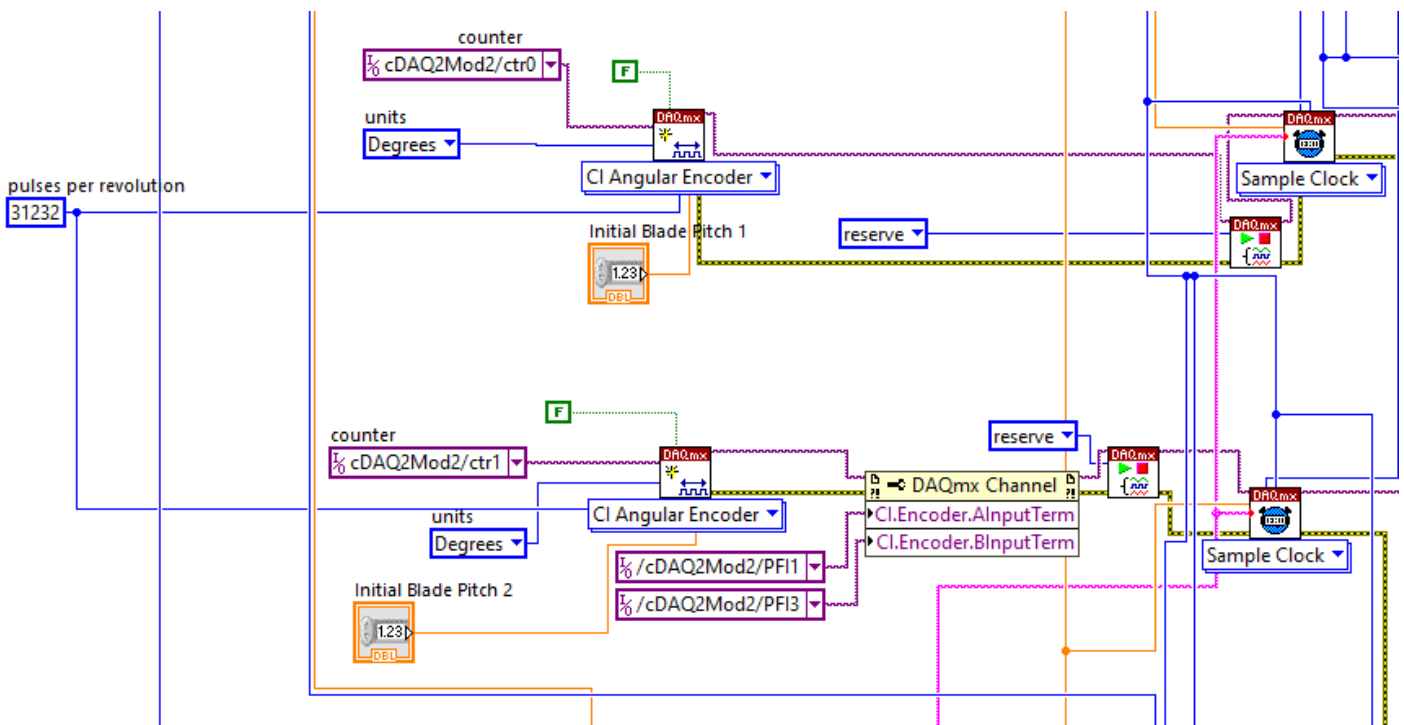


Figure 7 Counter input for pitch encoders

the small arrows to the right side of the box, meaning the node is reading the desired property and not changing anything.

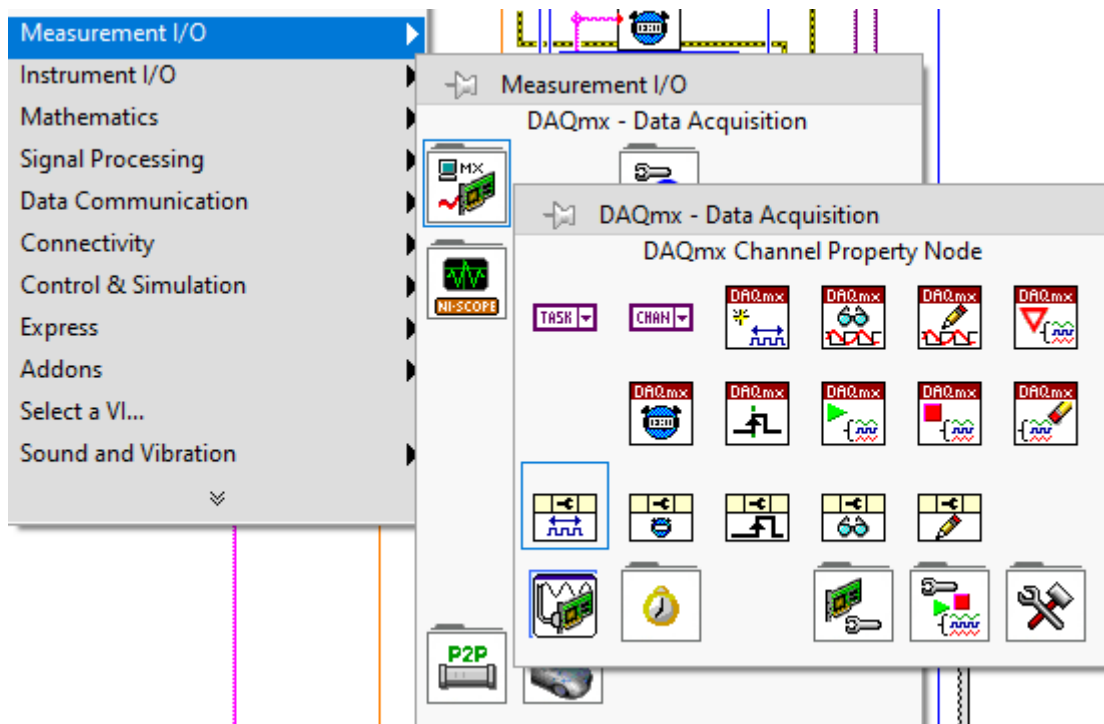


Figure 9 Channel property node

Use the channel property node to define the terminal the signals will go through either as inputs or outputs.

For the counter output task, a terminal is defined for pulse generation, however it is physically unused on the module, it is only declared so no wire will be placed in that terminal.

Control Task Node

The control task is used to reserve the resource, or task. Supposedly this is to avoid using conflicts in using resources. NI has a forum suggestion on how to avoid this [here](#), where all tasks are reserved except one, and that one unreserved task is started first. However, for the MTR VI, all tasks were reserved and that seemed to work just fine.

The control task node can be found by going to the DAQmx palette and navigating to the Advanced Task Options folder shown in Figure 10.

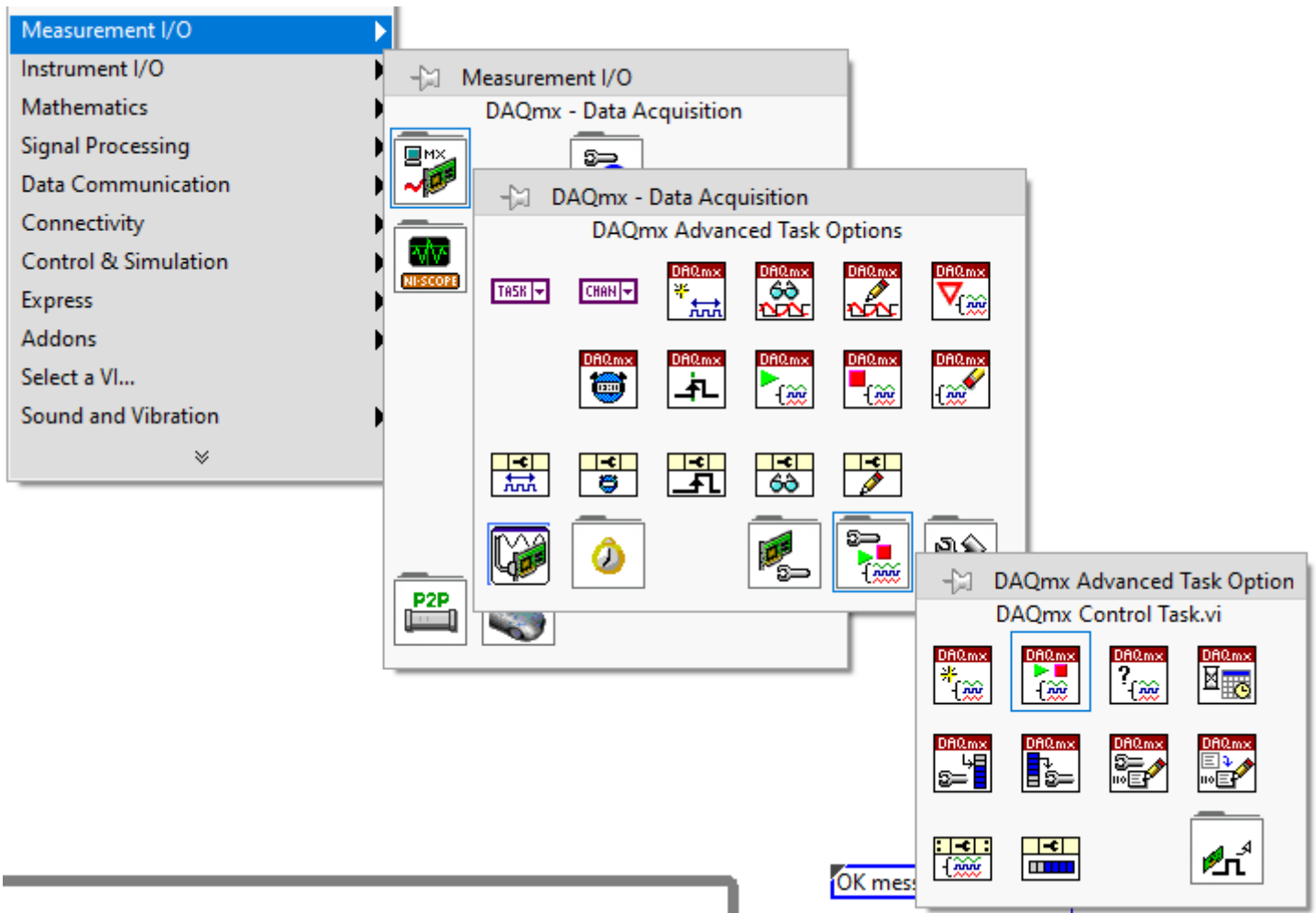


Figure 10 Control Task node

Timing Node

The Timing node is used to get tasks aligned and synchronized for data acquisition. The Timing node can be found in the DAQmx palette shown in Figure 11.

When using one DAQ, for example the PXIe chassis, the user can specify the “Sample Clock” on the Timing node for all tasks and that is all that is needed.

However, with the cDAQ chassis, the clock being used is the generated pulse signal made by the module. This requires some timing manipulation which is explained in the next section.

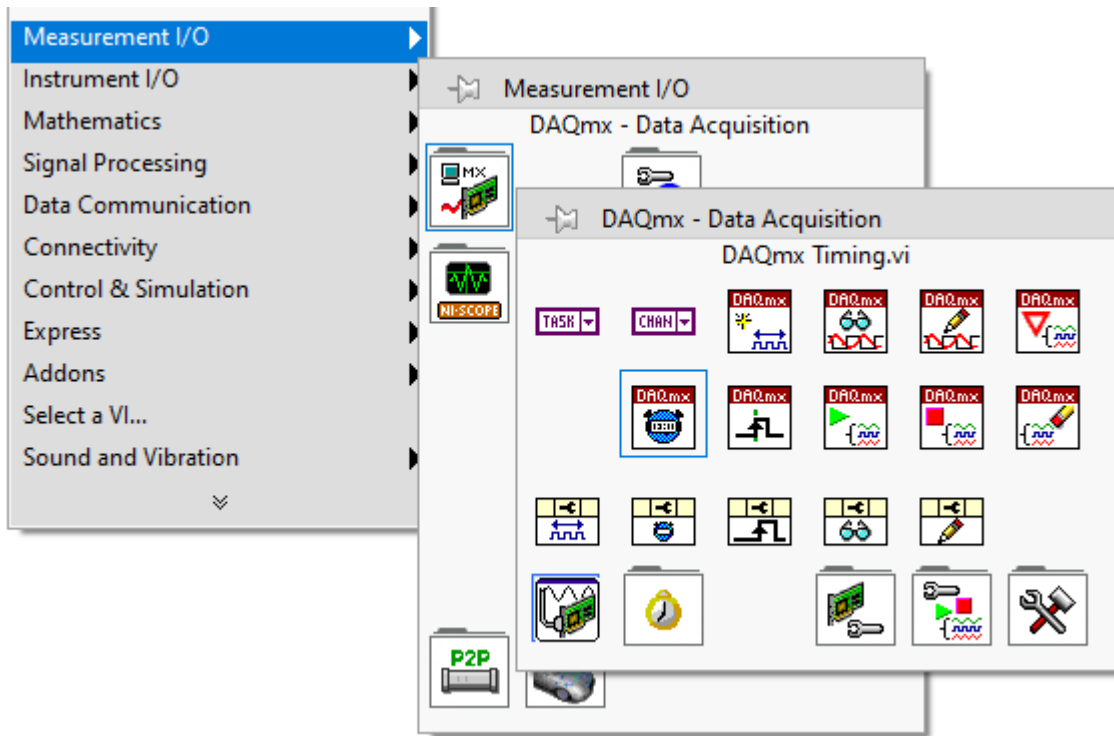


Figure 11 Timing node

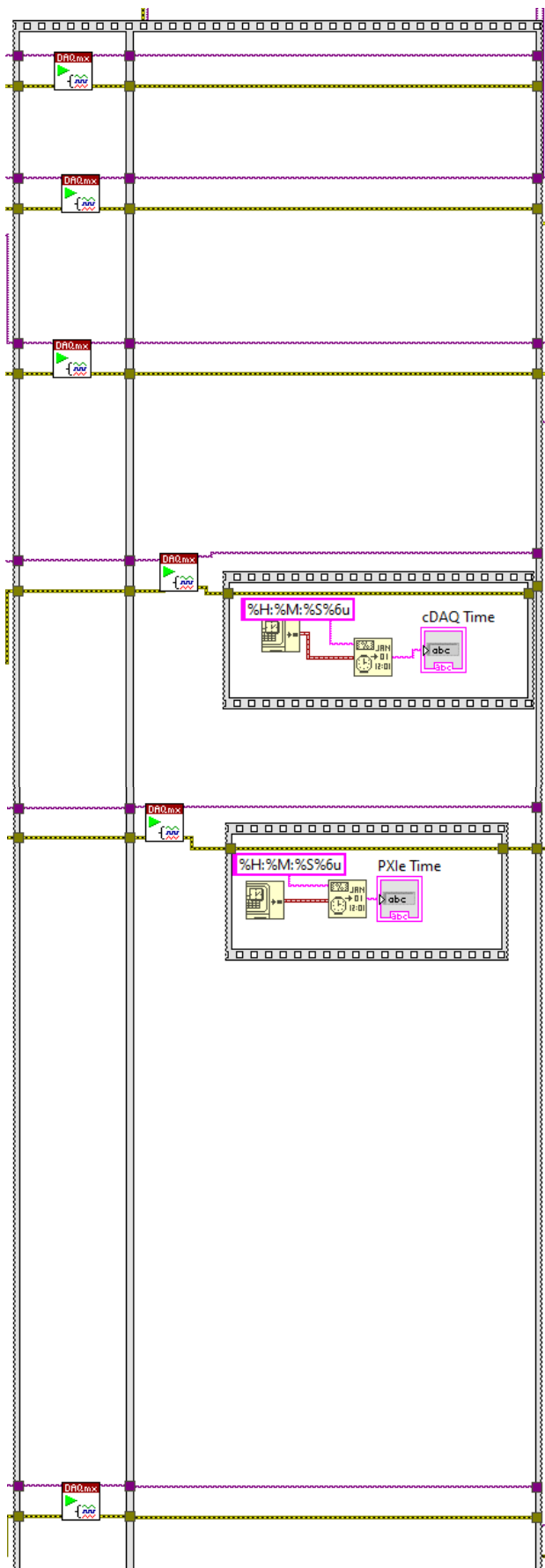
Importing a Generated Clock

Refer to Figure 8 to see the Timing node is using “Implicit (Counter)” as the selected option because the task is not using the chassis’ sample clock, but using its implicit counter as the clock.

The source of the clock generated by the cDAQ is “/cDAQ2/_ctr3InternalOutput”. This string must be concatenated by the Concatenate Strings function shown in Figure 12.

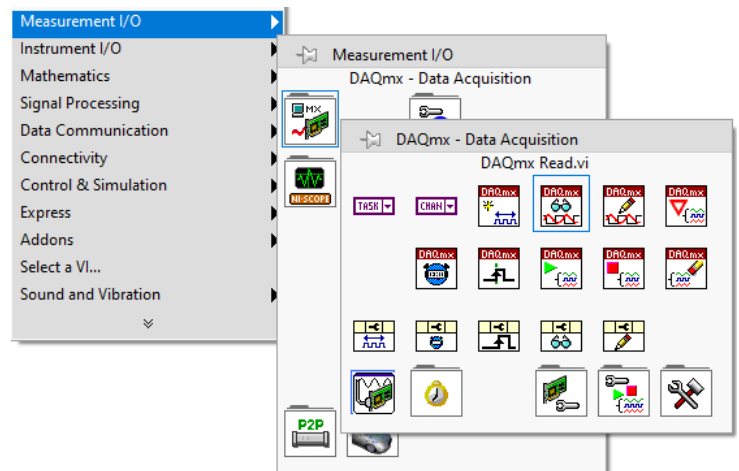
This string must be passed to the other counter input Timing nodes so those counter input tasks recognize the new clock. The line coming from the concatenated string function is pink and must be connected to the source terminal on the Timing nodes of the counter input tasks, shown in Figure 7.

Two constants that are connected to the counter output task are frequency and duty cycle. Duty cycle is the ratio of time that the signal is on vs off and by default is 0.5. For timing purposes, the duty cycle is arbitrary. The frequency is set to be 10kHz, the same frequency as the other tasks being acquired.



Notice the start tasks in the second frame have another small flat sequence structure connected to the error wires of the tasks. This is monitoring the start times of those two start tasks using “Get Date/Time in Seconds” and “Format Date/Time String” functions found in the Timing Palette. The user can observe or save these values if need be.

The MTR VI reads data from all tasks except the analog output voltage task, which writes data.



All Analog Input Data

1. Load Cell Fx
2. Load Cell Fy
3. Load Cell Fz
4. Load Cell Mx
5. Load Cell My
6. Load Cell Mz
7. Gimbal Hall 1
8. Gimbal Hall 2
9. Azimuth Hall
10. Collective
11. Lateral Cyclic

Load Cell Data

The load cell is an ATI Omega 160 load cell rated for:

- Fx, Fy: 600 lb
- Fz: 1500 lb
- Mx, My, Mz: 3600 lbf-in

The output signal from the load cell is a 6x1 array of voltages. This array is multiplied with a calibration matrix to get the final loads and moments.

The Read node obtains waveforms of all the analog input voltages in separate rows of the array. From the dropdown menu below the node select Analog > Multiple Channels > Multiple Samples > 1D Waveform (Samples). The number of samples terminal is connected to a constant of 5000 samples. To separate each row the entire analog input array to manipulate the load cell data, the Index Array function is used. This is shown in Figure 15. Index Array function takes the waveform and splits the row out from it based on the specified index number connected to the index terminal. Since the load cell data consist of the first 6 data rows in the analog input task, the indices range from 0-5. These are then recombined using Merge Signals functions and written to a raw voltage file. In addition, the recombined signals are manipulated to output the actual forces and moments in the front view.



Figure 15 Load cell raw data read

The manipulation portion of the load cell data, shown in Figure 16, includes obtaining the actual loads and moments and calculating the CG offset based on the in-plane forces. These sections will be detailed in the following sections.

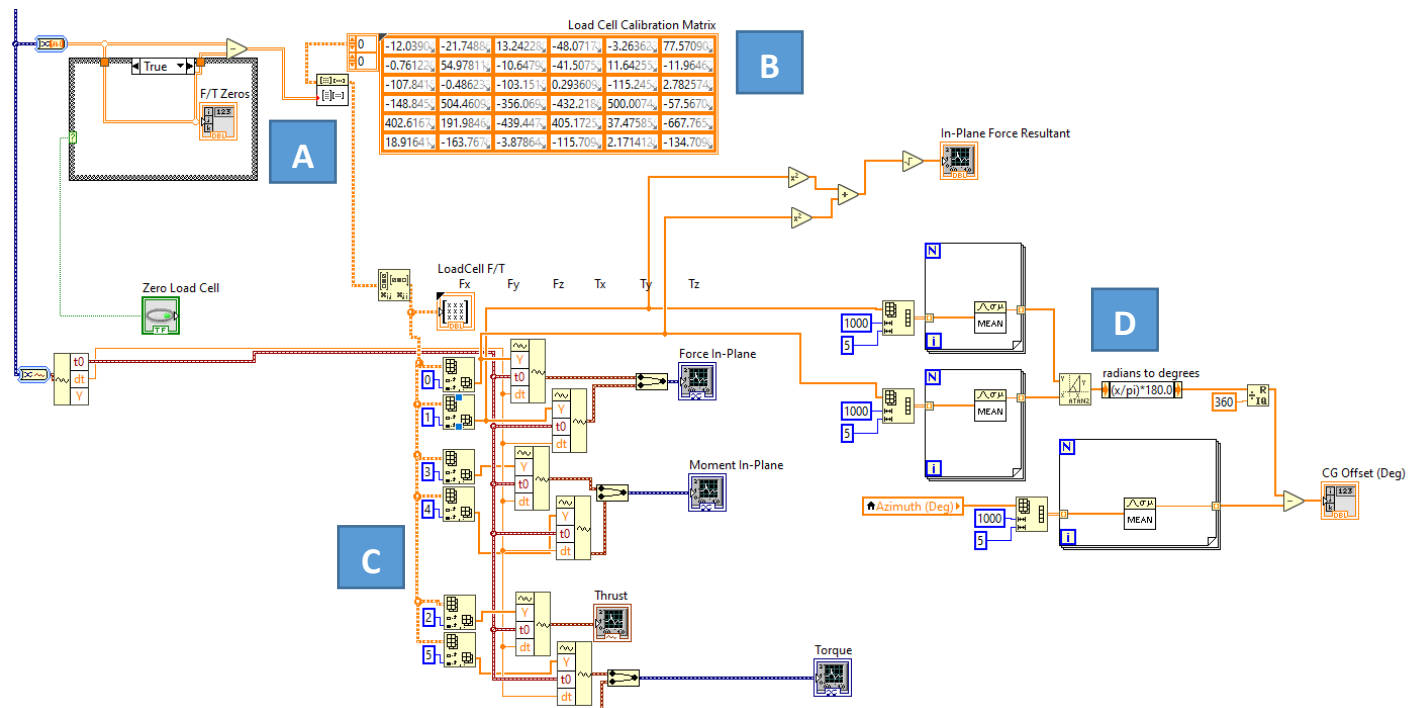


Figure 16 Load cell data manipulation: (A) Zeroing Data, (B) Calibration matrix, (C) Displaying Loads, (D) Calculating CG Offset

A. Zeroing Data

Zeroing the load cell data uses a Case Structure with a Boolean button connected to the frame shown in Figure 17. When the Boolean is True, the current load cell data is stored in an array, F/T Zeros, and subtracted from the original signal. This will always give an output of zero because whatever data is being read is being subtracted at the same time, as long as the case structure stays True. When the case is False, whatever the last stored array for F/T Zeros was, stays stored, and will continually be subtracted from the streaming data. In the False case, the local variable of F/T Zeros is referenced. Local variables are created by right-clicking on the object in the front panel, in this case, the F/T Zeros array, and selecting Create > Local variable.

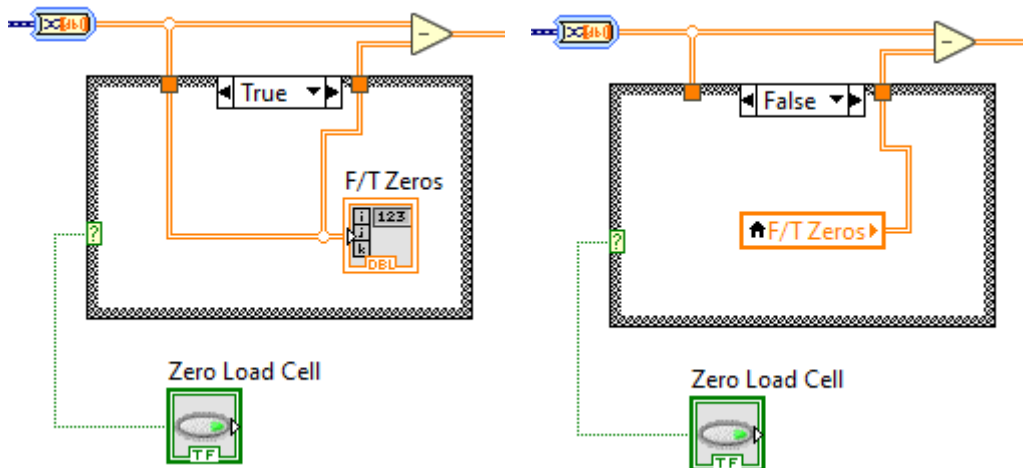


Figure 17 Zeroing data: True stores the data array (left), False pauses storage and subtracts the last stored array from the original signal (right)

B. Calibration Matrix

The calibration matrix is a 6x6 matrix that was determined from calibration tests. Use the Linear Algebra palette to obtain the "A x B" function for matrix multiplication. The calibration matrix is A, and the voltage array is B. The output from the multiplication will be loads and moments in respective rows. The MTR VI displays these in columns by using a Transpose Matrix function found in the Matrix folder within the Linear Algebra palette. These are shown in Figure 18.

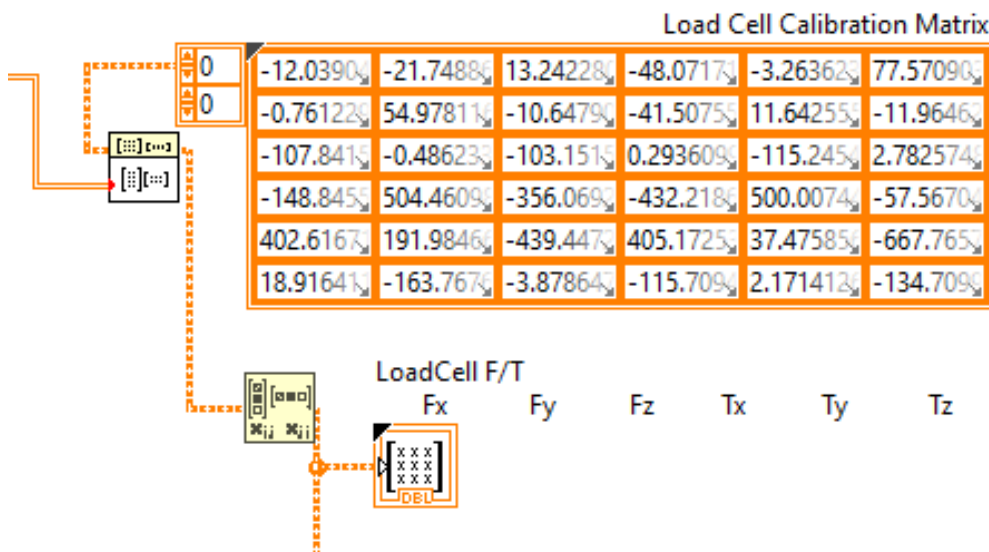


Figure 18 Calibration matrix and final loads and moments

C. Displaying Loads and Moments

When performing the calculation of the loads and moments, the time data of the waveforms gets stripped away. So to properly display the new loads and moments with correct timing, they must be rebuilt into waveforms, shown in Figure 19. Using the raw voltage signal waveform (prior to zeroing or calibration), the waveform is split into its components using “Get Waveform Components” found in the Waveform palette. The components are initial start time t_0 , time increment dt , and signal values Y . By porting the t_0 and dt of the raw voltage signal to the new calibrated loads and moments Y values using the “Build Waveform” function, the new loads and moments can be displayed with the correct timestamps.

With the waveforms rebuilt, the signals can be combined using Merge Signals and shown using graph indicators. The Force In-Plane graph indicator shows F_x and F_y (indices 0 and 1) loads, Moment In-Plane graph indicator shows M_x and M_y (indices 3 and 4) moments, Thrust graph indicator shows F_z (index 2) load, and Torque graph indicator shows M_z (index 5) as well as torque strain gage signal.

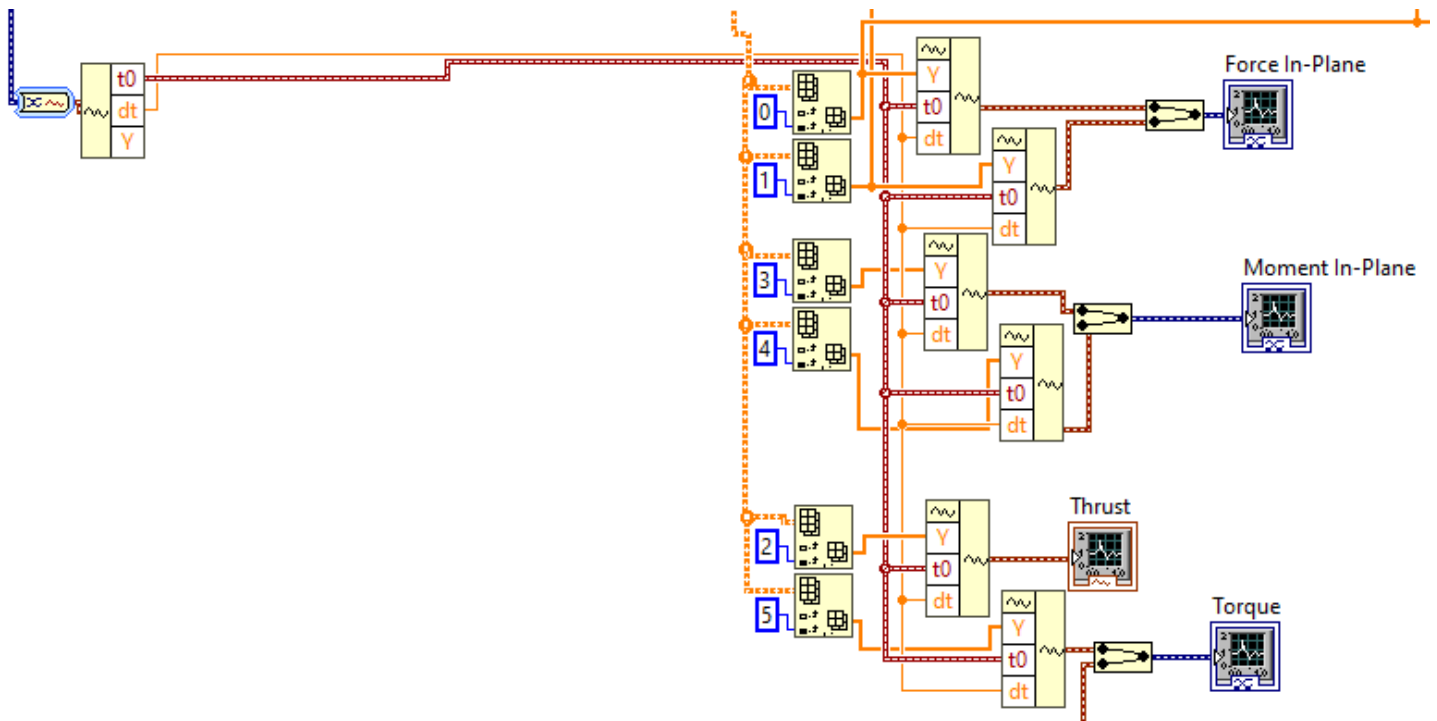


Figure 19 Displaying loads and moments with rebuilt waveforms

D. CG Offset Calculation

The in-plane center of gravity offset is a concern because a large offset will cause a large 1/rev oscillation in the hub which can potentially be damaging to the rig. It is an important task to be able to measure this offset and reduce it by adding weight to the opposing side of the offset.

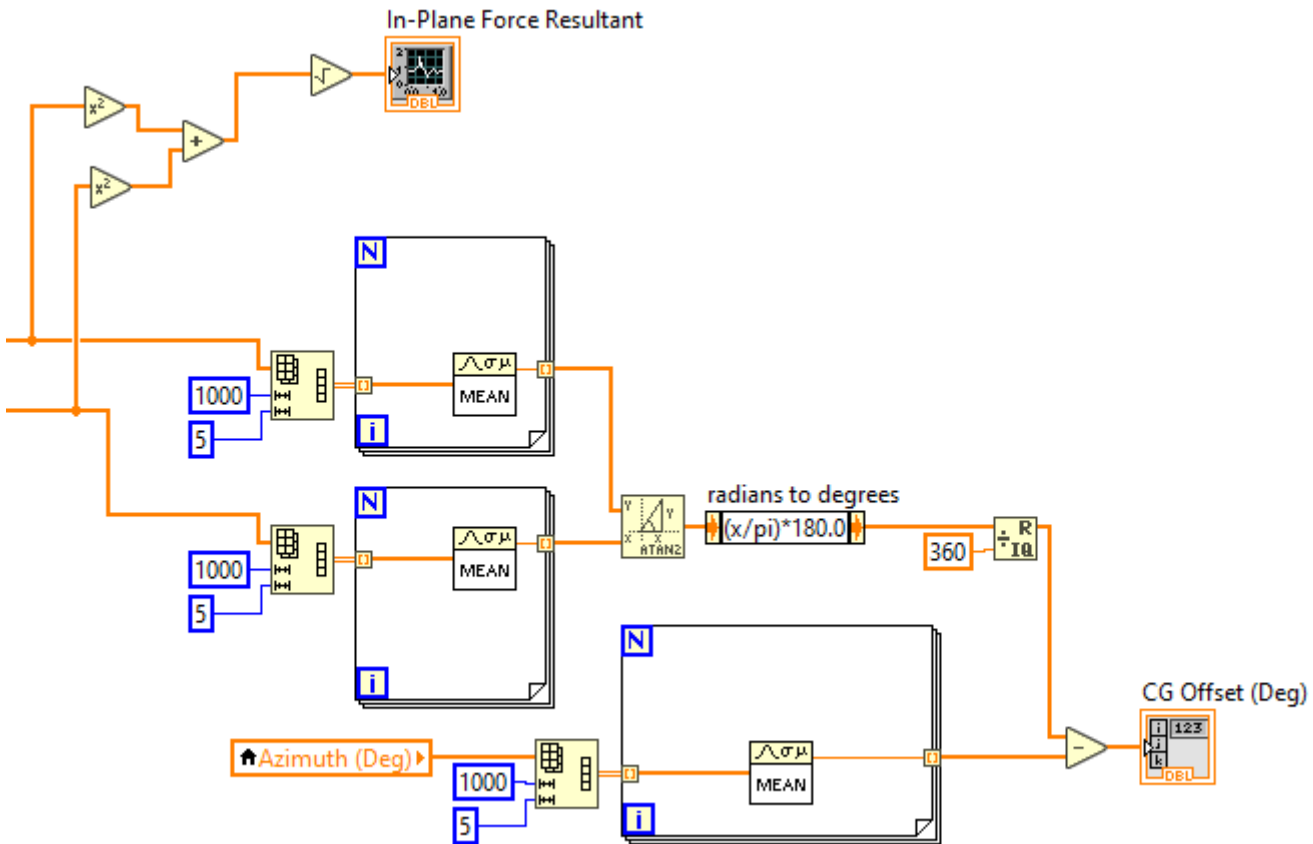


Figure 20 CG Offset Calculation

Following along in Figure 20 from left to right and top to bottom, first the in-plane force resultant is calculated by taking the square root of $F_x^2 + F_y^2$. This is shown in a graphical indicator on the front panel. The resultant will always be positive, but the ideal observation as the rotor is spinning is a zero resultant. If the Observing a large resultant indicates a large CG offset.

The next step is to find out where the CG offset is in relation to the azimuthal position of the shaft. A simple running mean is established for Fx, Fy, and the Azimuth angle signals to reduce the variance in a 10kHz sampling rate. This is done by using "Reshape Array" on the 5000 sample array, changing it to an array of 1000 rows and 5 columns and then using "Mean" function for each row in a For Loop structure. This reduced the data from 5000 samples, to 1000 samples. The outputs of Fx and Fy are fed into "Inverse Tangent (2 Input)" function found in Trigonometric Functions palette, which gives the angles of the signals from $(-\pi: \pi)$. Radians is converted to degrees using an "Expression Node" found in the Numeric palette. To get the range from $(-180:180)$ to $(0:360)$ degrees, the output is fed into the "Quotient and Remainder" function as X. Y is the desired range of 360 degrees. This function is otherwise known as the modulo operation and also found in the Numeric palette.

A local variable of the Shaft Azimuth position is created and also put through a running mean operation.

Subtracting the angle of the shaft azimuth from the angle of the CG offset, the location of the CG offset with respect to the shaft azimuth is obtained. Note the running mean of the shaft will contain a handful of values that are not helpful in this calculation during the jump between 360 degrees and 0 degrees.

The means are taken in this way because the data is a 5000 sample array updated each iteration, and not a continuous stream of data. Some functions work differently on arrays vs. continuous data.

Hall Effect Sensors

Components that use hall effect sensors are the gimbal hub angles and the shaft angle.

Gimbal Hub Angles

ADD AN OFFSET FOR AZIMUTH BETWEEN GIMBALS AND HUB FRAME. There are two hall effect signals for the universal joint in the MTR gimballed hub. This section describes how to get from the hall effect sensor voltages to the B1s vs B1c chart used to trim the rotor to zero flapping. It is assumed the user has performed sensor calibrations to find the relationship between voltage and the position. The two important parameters that are user-inputs are the slope of the line for degrees/voltage, and the zero angle voltage. The LabVIEW code block diagram of this is shown in Figure 21.

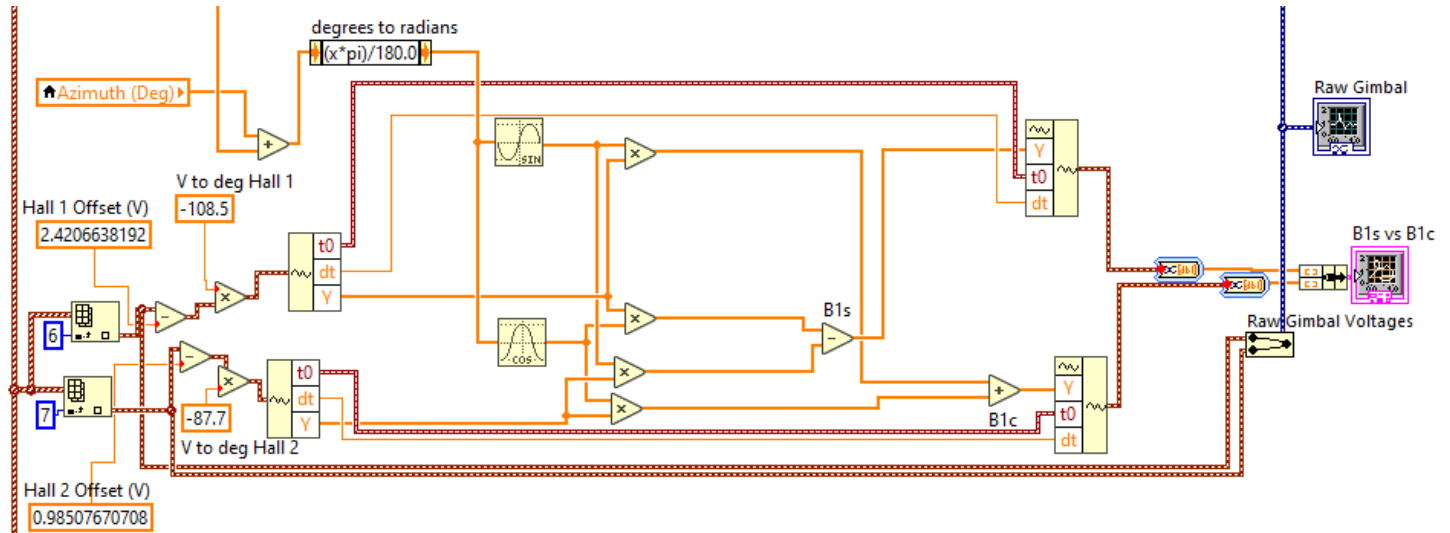


Figure 21 Gimbal hub angles conversion to B1s and B1c chart

Hall sensor 1 is index 6 and hall sensor 2 is index 7 in the list of analog input voltages being read. Following hall sensor 1 wires, the Hall 1 Offset (V) value of 2.42 is the zero angle voltage of that hall effect sensor. That is subtracted from the raw signal, then multiplied by the slope of the degrees/voltage line found from prior calibration tests of -108.5 degrees to get the final gimbal angle in degrees. This is in the rotating frame and must be converted into the fixed frame.

B1s, or lateral flap, is along the Y axis of the rotor. B1c, longitudinal flap, is in the direction of the X axis of the rotor. To calculate B1s and B1c:

$$B1s = \text{gimbal1} * \cos(\psi) - \text{gimbal2} * \sin(\psi)$$

$$B1c = -\text{gimbal1} * \sin(\psi) - \text{gimbal2} * \cos(\psi)$$

Where ψ is the rotor azimuth. ψ is determined by the rotor azimuth, called as a local variable, plus (-46.25) degrees because the hall effect sensors are rotated by that amount about the shaft's zero azimuth point. The rest of the block diagram consists of trig functions found in the Trigonometric Functions palette and arithmetic operators found in the Numeric palette. As with the any manipulation of signals, the waveform time data must be ported to the final reconstructed signals which is done by splitting the raw signal with the Get Waveform Components function and rebuilding the new signal with Build Waveform function.

The final signals are put together using the Bundle function found in the Cluster, Class, and Variant palette. The Bundle function creates clusters and this is wired to display on the front panel through an XY graph indicator.

NOTE: Hall 2 Offset toggles between 0.985 and 3.31 Volts. This phenomenon has been difficult to reproduce so if the user noticed the B1s vs B1c plot is off the chart, most likely the Hall 2 Offset needs to be set to the other value.

Exact values for the zero degree angle voltages are given below for a power supply voltage of 5.002 V:

Gimbal 1: 2.4206638192

Gimbal 2: 0.98507670708 or 3.316209105

Shaft Azimuth and Frequency

The shaft hall sensor is index 8 in the analog input voltage read task.

The signal is used to track the azimuth of the shaft as well as to calculate the rotor speed in Hz and RPM shown in Figure 22.

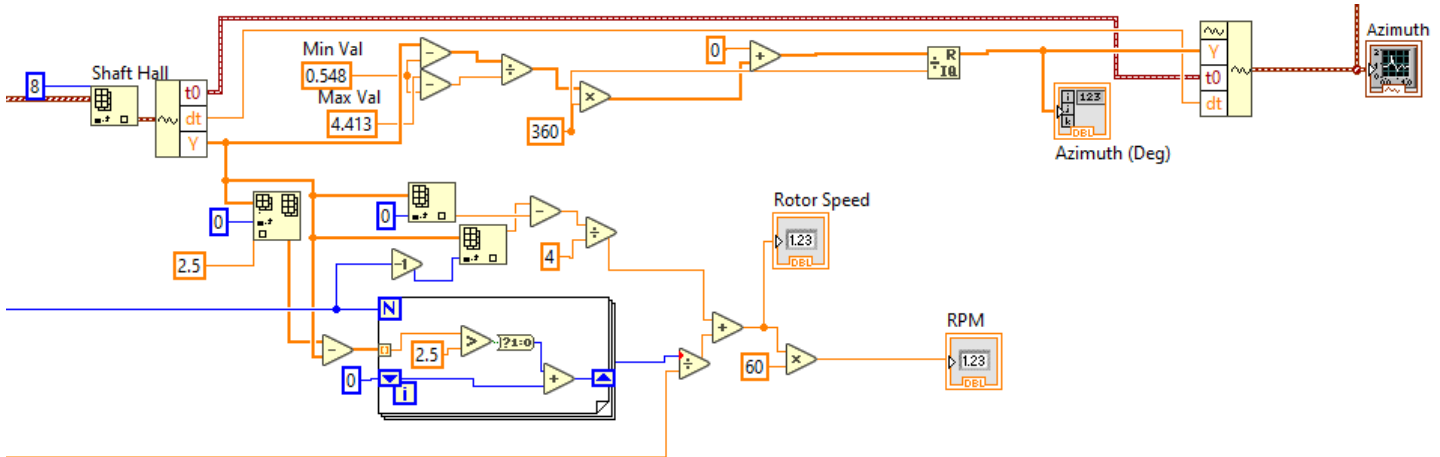


Figure 22 Block diagram for shaft hall sensor data

The raw waveform is split with Get Waveform Components so time data can be ported to the new signal and the Y value component can be manipulated. The minimum and maximum voltage output by the shaft hall sensor are logged and set as the threshold for where the shaft angle jumps from 360 degrees to 0 degrees. The logic is shown below:

$$(raw\ voltage - min\ voltage) / (max\ voltage - min\ voltage) * 360 = \psi\ (degrees)$$

There is an addition operator after the azimuth is converted to degrees to allow for a new shaft position, if desired, i.e. getting the shaft to be zero azimuth at the X-axis. The output is fed into the "Quotient and Remainder" function to ensure the azimuth stays between 0 and 360 degrees. This is connected to an indicator so the scalar value of the azimuth can be observed, then the waveform is reconstructed with the raw time data and wired to a graph indicator.

To calculate the RPM, the discontinuities in the signal from 360 to 0 degrees is first identified by locating the differences in voltage of 2.5 volts.

Using the For Loop structure to iterate through each point in the 5000 sample array, every time a difference in voltage greater than 2.5 volts is detected, a 1 is output, and the shift registers on the border of the For Loop structure stores this value so it is retained as the loop iterates. All the 1 values are added throughout the for loop until the total is determined. This scalar is divided by $\frac{1}{2}$ a second to get frequency in revolutions/second because 5000 samples in a 10kHz sample rate signal is $\frac{1}{2}$ a second. This value is then multiplied by 60 to obtain the frequency in revolutions per minute or RPM.

Controller Voltage Inputs

Collective, Lat Cyclic, Long Cyclic, and Throttle are voltage signals manipulated by the physical controls of the pilot. These get written to the analog voltage output task which control the actuator voltages.

Longitudinal and Lateral Cyclics

The block diagram for cyclic controls are shown in Figure 23. The lateral and longitudinal cyclic signals are indexed as 10 and 11 respectively in the analog voltage data stream.

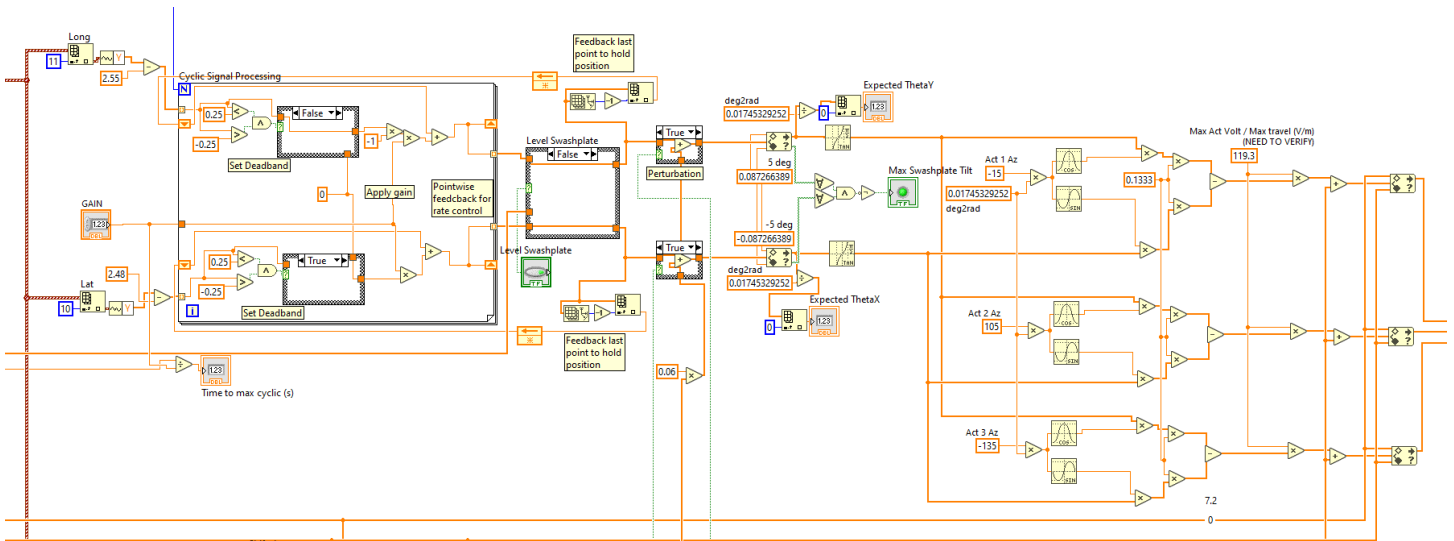


Figure 23 Section of block diagram for longitudinal and lateral cyclics

First step for both these signals is account for a voltage offset by subtracting 2.55V from longitudinal and 2.48V from lateral cyclic data.

Next, the signals are fed into a For Loop structure that sets a deadband range, where any values between -0.25V and 0.25V from the set voltage returns the same set voltage.

The long and lat signals continue into a Case structure for Level Swashplate. This structure, when true, sets the long and lat cyclic feedback signals to zero, seen in Figure 24.

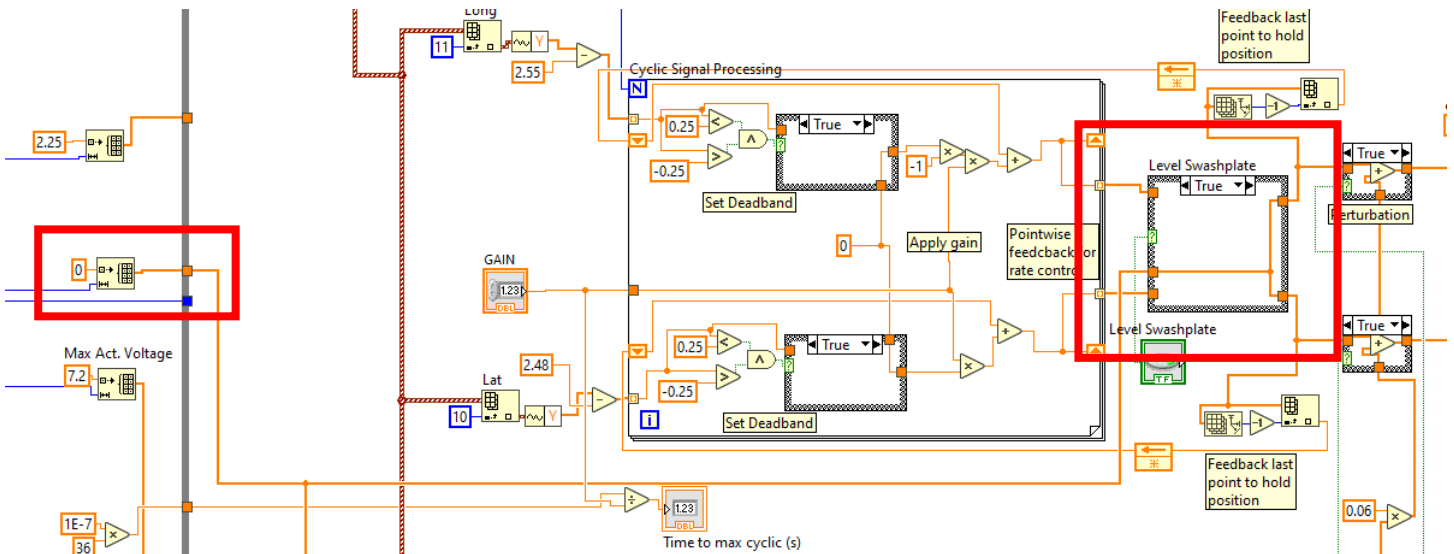


Figure 24 Set Long and Lat signals to zero if Level Swashplate is true.

An array of 5000 zeroes is created outside the While Loop structure using Initialize Array function and fed into the Level Swashplate true case. The signal splits into two, becoming the proxy for the longitudinal and lateral cyclic data signals. The last value of the arrays from the Level Swashplate are stored by using Array Size, Decrement, and Index Array functions. A Feedback Node function, found in the Structures palette, is used to send the value back to the For Loop structure. The Feedback Node is essentially a shift register, storing the previous iteration's values.

When false, the signal is unchanged from the deadband output shown in Figure 25.

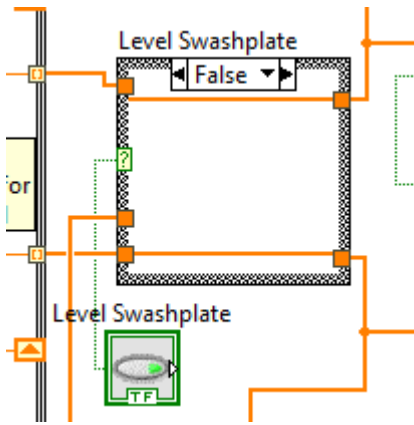


Figure 25 Level Swashplate set to false, signals are unchanged from deadband output.

The next portion is a Case structure for the perturbing the long and lat signals by adding an oscillating signal, which is covered in the Perturbation System section. When false, the perturbation system is bypassed and the signals go into a series of comparisons to determine if they are reached a maximum tilt limit. Then these signals are passed to more mathematical manipulation to determine the actuator voltage that to be written to the actuators, shown in Figure 26.

Using “In Range and Coerce” functions, found in Comparisons palette, the signals are compared to an upper limit of 0.087266389 V and lower limit of -0.087266389 V. If the signals were to exceed the limits, the signals are coerced to stay at that limit. The output labeled “?” is a Boolean array of true if the values are within the range, and false if the values exceed the range. These Boolean arrays are fed into “AND Array Elements” functions which returns true if all elements in the array are true or empty, otherwise it returns false. So if the limit is exceeded by just 1 element within the 5000 sample array, the AND Array Elements function will return a false.

The signals are scalar Booleans at this point and fed into a standard AND function, so if both are true (or both are within range), it returns true, otherwise it is false. This is wired to a NOT function, because the desired output is to flag all the times the Booleans are false to indicate an exceedance in swashplate tilt. This is displayed as a green light indicator on the front panel, lighting when there is an exceedance.

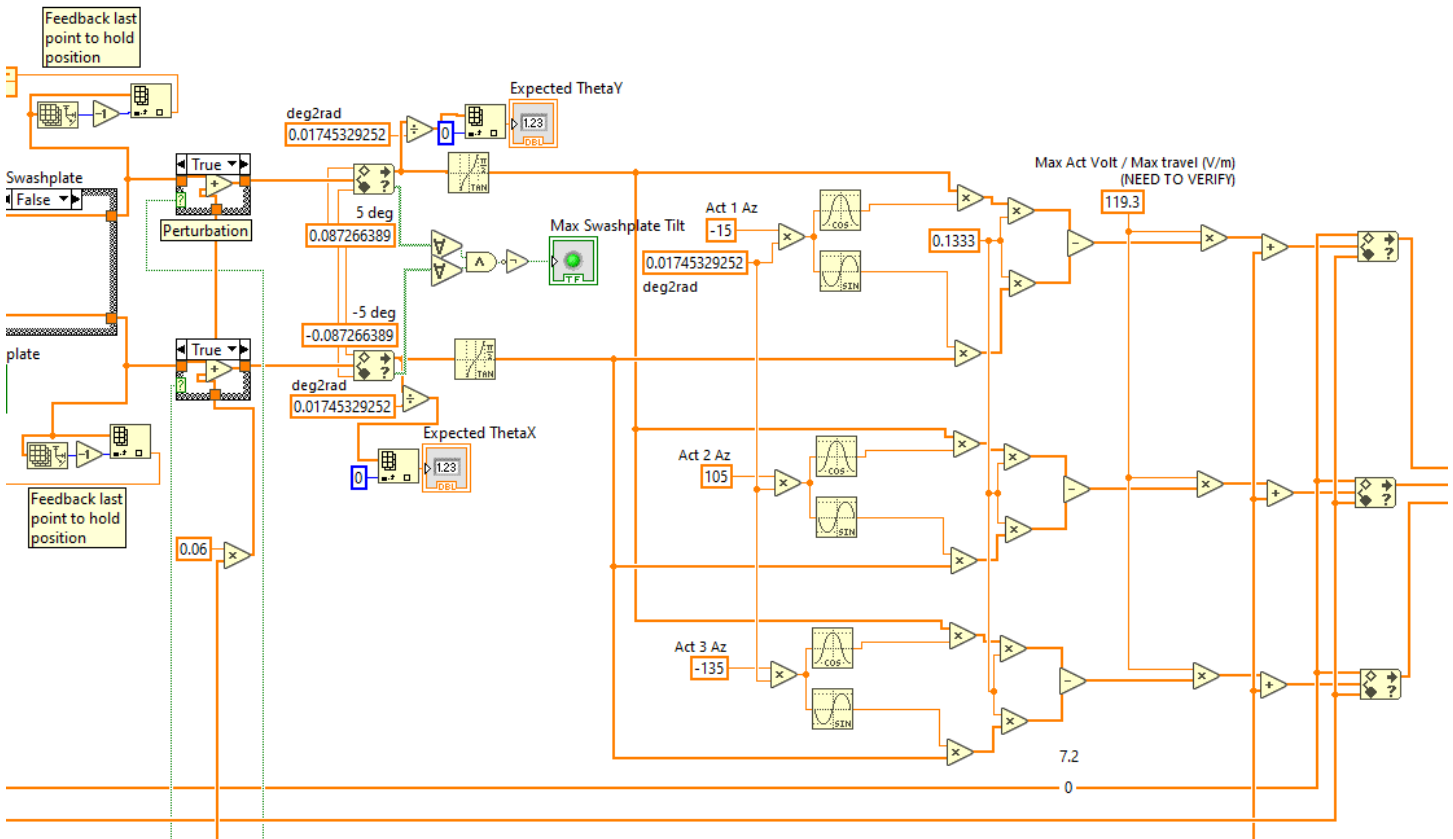


Figure 26 Long and lateral signals manipulations to obtain required actuator voltages.

With the swashplate tilt limits set, the coerced signals are wired to a Tangent function then multiplied by Actuator locations in the body fixed frame, converted to the rotating frame using sines and cosines functions.

Collective

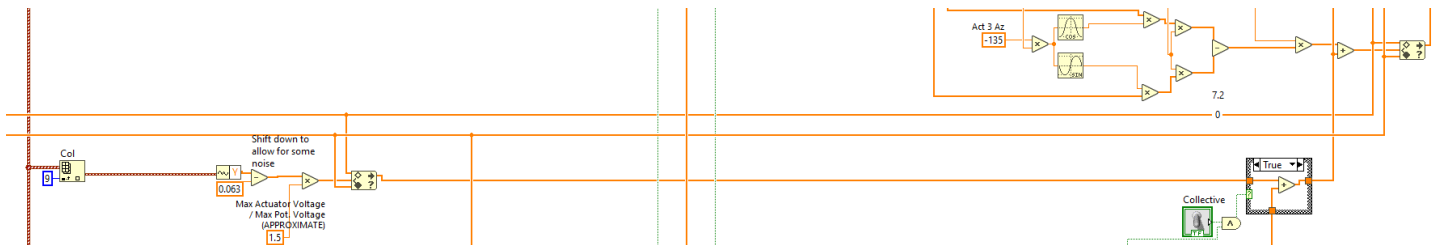


Figure 27 Overlapped at the bottom of the previous figure, collective signal is used to in conjunction with longitudinal and lateral cyclics

Throttle

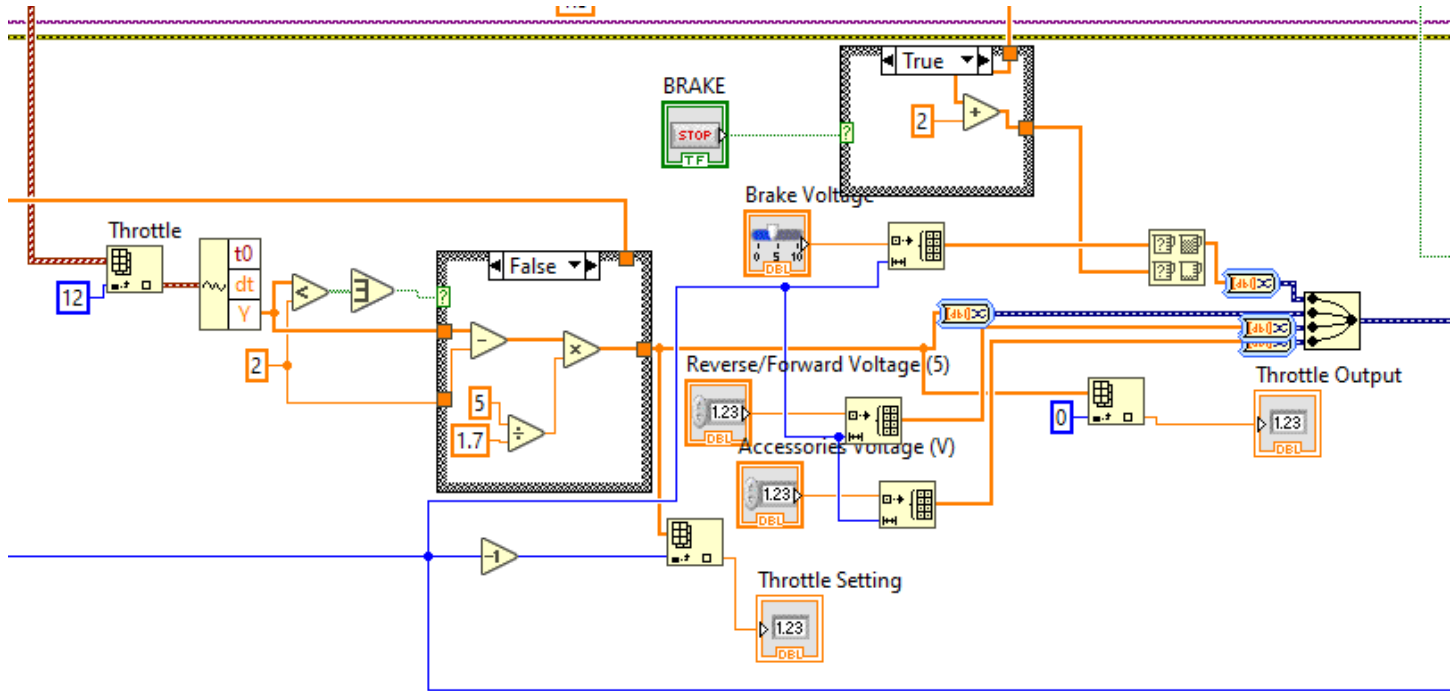


Figure 28 Throttle logic

Perturbation System

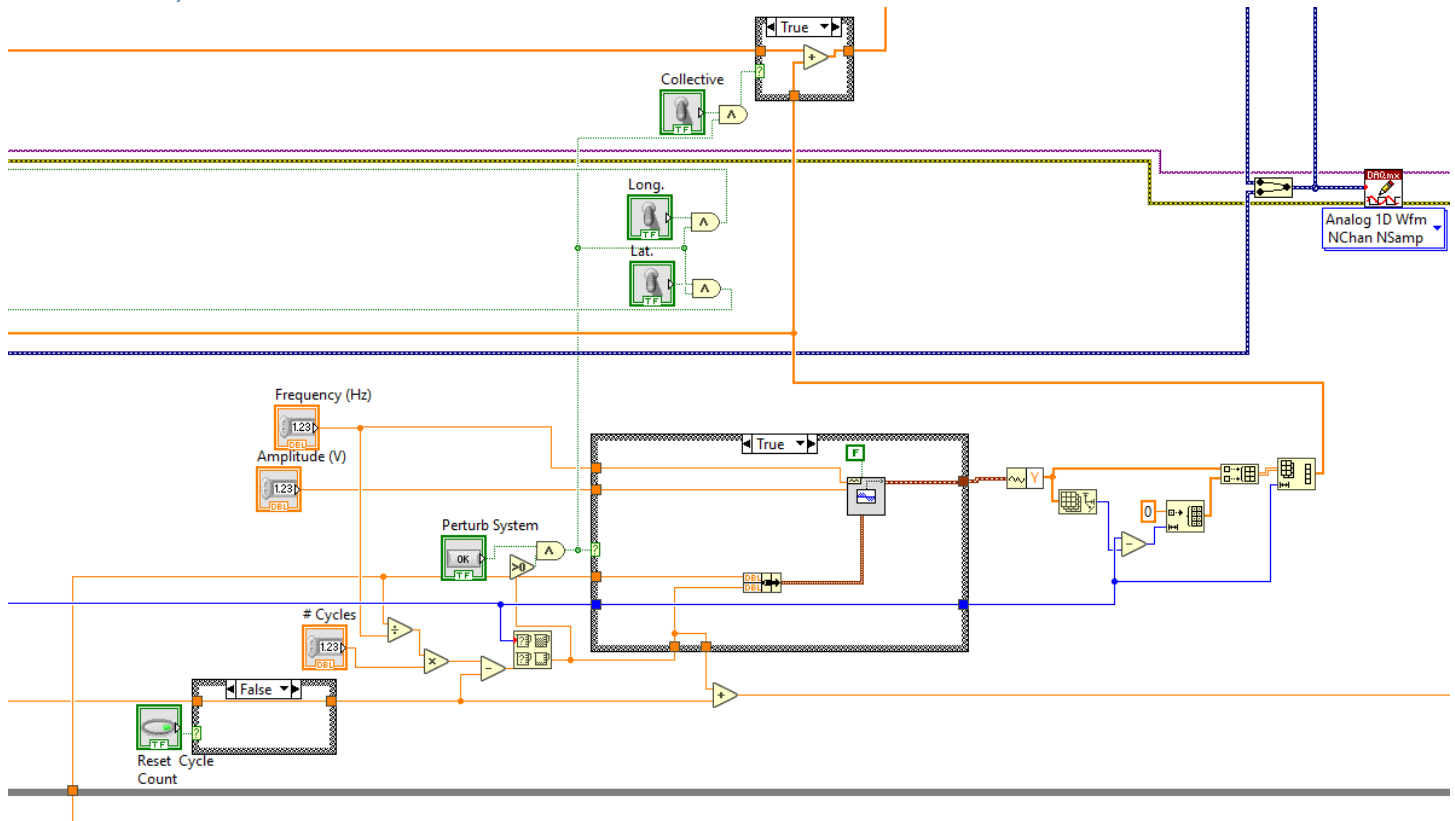


Figure 29 Perturbation logic

Strain Gage Data

Strain gage signals are read from the same task. The type of data is determined from the dropdown menu by selecting Analog > Multiple Channels > Multiple Samples > 1D Waveform (Samples) shown in Figure 30. The signal is immediately multiplied by $1e6$ because the raw data is recorded in strains but the data values are on the order of microstrains. Multiplying by $1e6$ removes all the leading zeroes ahead of the first significant digits. This is important for writing to measurement files because writing to files allots a set amount of characters to be written. If characters being written are leading zeroes, then they reduce the resolution that can be used for significant digits.

A Case structure is used to zero all the strain gages. When True, the raw data is input into a Basic Averaged DC-RMS subVI found in the Waveform Measurements palette. This subVI takes all 5000 samples in each waveform of strain data and outputs a 1D array representing the average value for each waveform. When False, the last stored array is used to zero the raw signals. The output is also drawn to the While Loop border and connected to a Shift Register to retain the array for each loop iteration. This is different from the load cell zeroing method, which does not take an average value, but an instantaneous value.

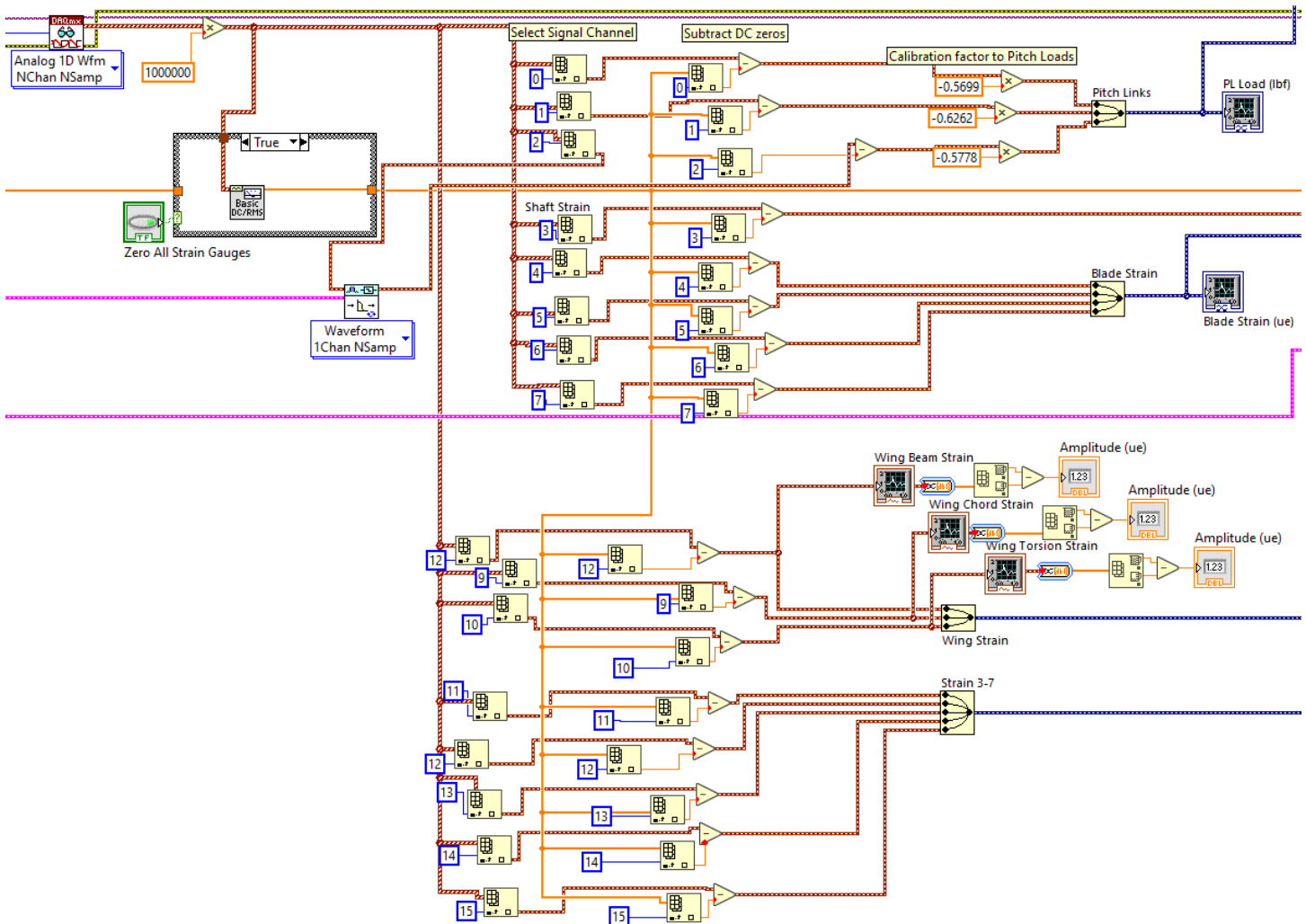


Figure 30 Strain gage Read node

The first 3 signals from the strain gage task are pitch link strains, followed by the shaft torque strain gage, 4 channels for blade strains, 3 wing strains, and 5 open strain channels.

Pitch Link Strains

The pitch link strains are indexed as 0, 1, and 2 in the data stream. These are separated by using the Index Array function and setting the index. The individual pitch link signals are zeroed by subtracting the DC value of each pitch link signal.

This is done by using Index Array function on the Basic Averaged DC-RMS subVI output and setting the respective index of the pitch link signals.

The zeroed pitch link signals are then multiplied by a calibration factor found from calibration tests to obtain the pitch link loads in lbf. These signals are finally compiled together using Merge Signals, plotted on the front panel using a graph indicator, and stored in the measurement file.

Pitch Link 3

Pitch link 3 has a butterworth filter applied to it because there is a large amount of noise in the signal. This was applied through a DFD Filtering node found in the Processing palette and a DFD Butterworth Design subVI found in Advanced IIR Filter Design palette shown in Figure 31.



Figure 31 Butterworth filter for pitch link 3

The Filtering node takes the pitch link 3 raw data as input, the butterworth filter as the filter type, and outputs the filtered signal. The datatype is selected from the dropdown menu of the node: Single Channel > Multiple Samples > Waveform.

The Butterworth filter is placed outside of the While Loop structure and then connected to the Filtering node. This is done due to the array form of the data. If the data was a continuous stream of points, the butterworth filter can be placed inside the while loop.

Notice the order is a Lowpass filter and the low cutoff frequency is $0.002 * (\text{sampling frequency})$ which in the MTR's case is 20 Hz.

Shaft Torque Strain

The shaft torque strain is index 3 in the strain gage data and follows the same manipulation as the pitch link 3, including a butterworth filter because the raw signal is noisy.

The shaft strain is recorded to measurement file before the filter is applied. The filtered signal is shown in graph indicator just to have a second measurement to compare to the load cell Mz component. From Figure 30, the strain gage output is seen to continue off to the right. Figure 32 shows the continuation of the shaft strain gage signal.

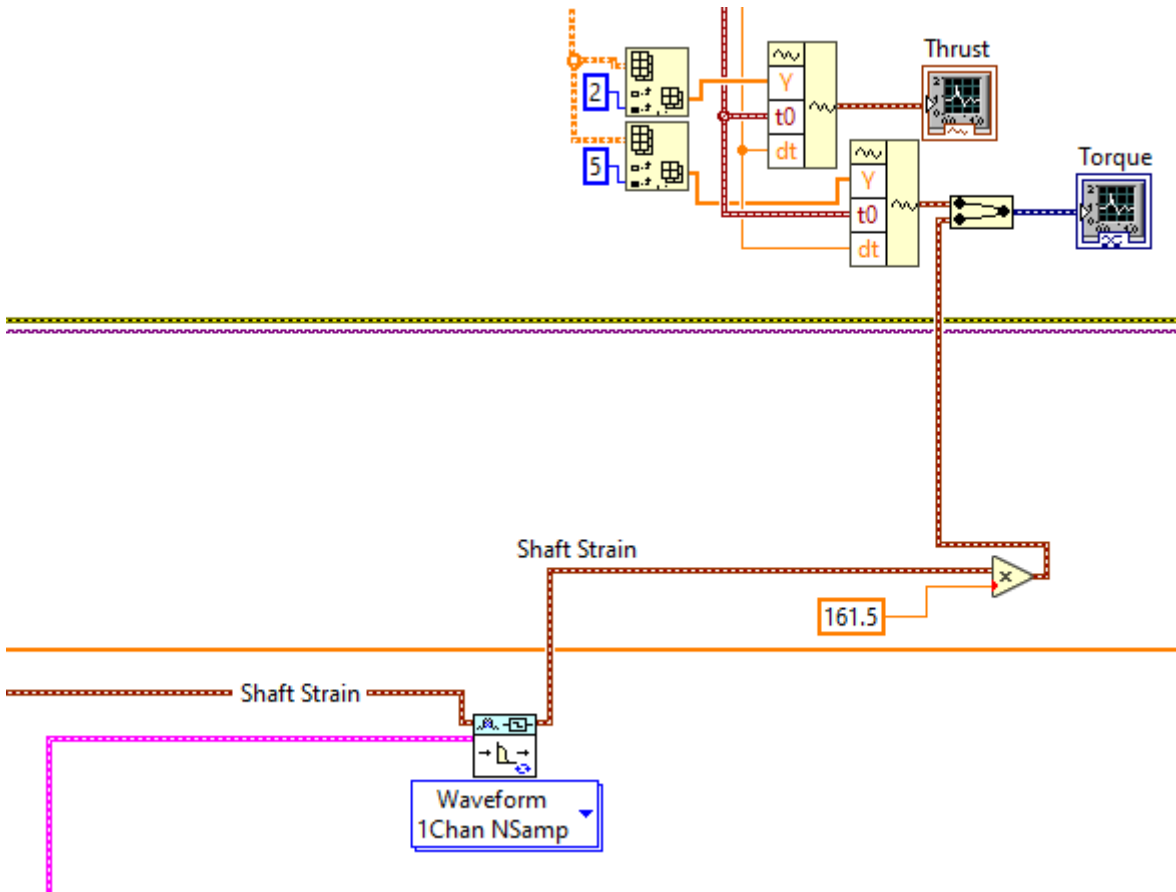


Figure 32 Shaft strain gage continuation

The signal is input into a DFD Filter node and a different butterworth filter is connected to this shown in Figure 33. This butterworth filter subVI is also placed outside the while loop. The filtered signal is multiplied by 161.5, which is a calibration factor from previous calibration tests to obtain the torque in units of in-lb. The result is shown on the same graph indicator as Mz component of the load cell.

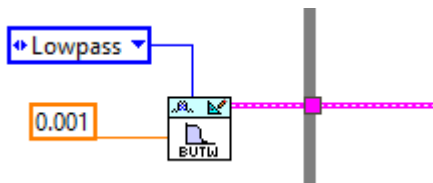


Figure 33 Butterworth filter for shaft strain gage, the pink wire connects to the filter terminal in the Filter node.

The strain gage filter is a lowpass filter with a low cutoff frequency of $0.001 * (\text{sampling frequency})$ or 10Hz.

Wing Strains

The wing strains are indexed as 8, 9 and 10 in the strain gage data, shown in Figure 34. After zeroing the signals, they are all wired to individual graph indicators. The amplitudes are calculated from the signals by using Array Max & Min functions from the Array palette and subtracting the min from the max values. These amplitudes are shown in numeric indicators and the units are in microstrains.

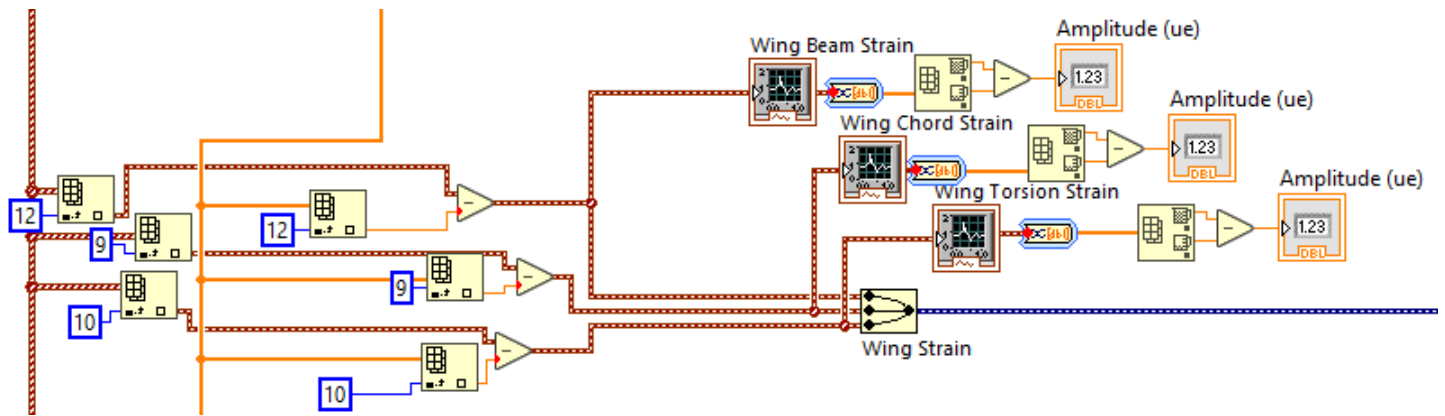


Figure 34 Wing strain data block

The amplitudes are for observation on the front panel, only the zeroed wing strain signals are written to file.

Pitch Encoder Counter Data

The pitch encoders are connected to the cDAQ chassis through the NI 9401 modules and are each on their separate Read nodes shown in Figure 35. These datatypes are selected from the dropdown menu on the node as Counter > Single Channel > Multiple Samples > 1D DBL. Number of samples per channel is 5000.

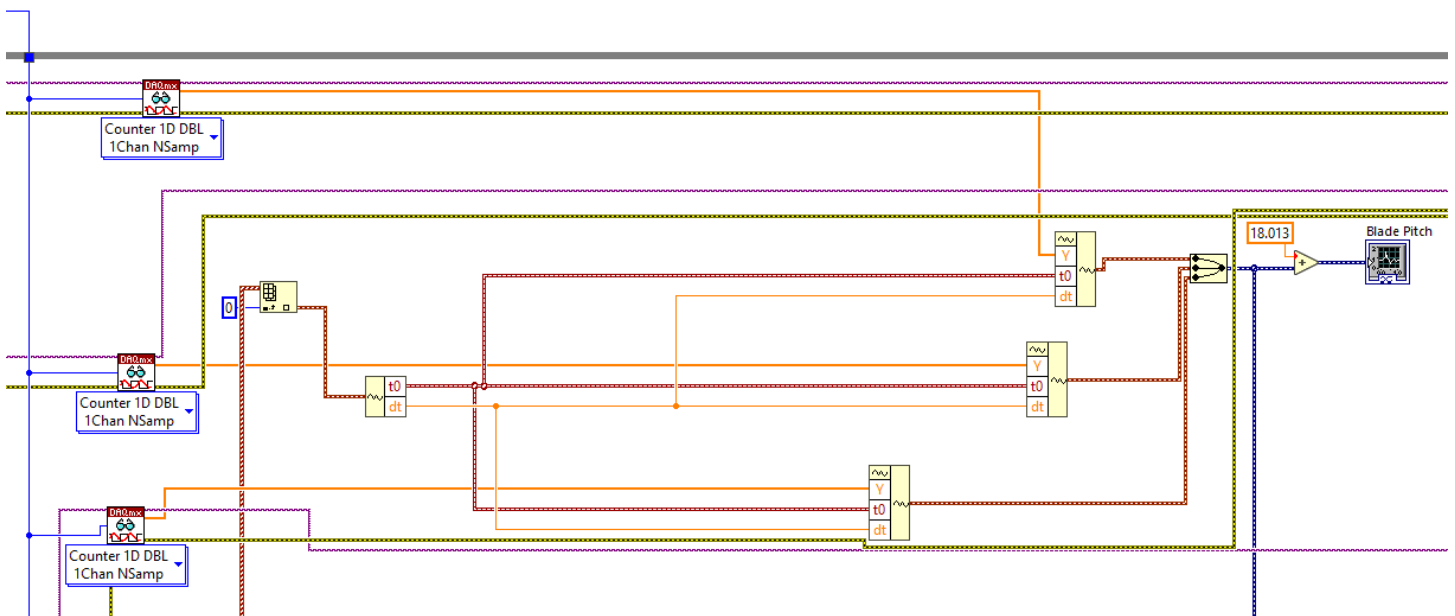


Figure 35 Pitch encoder Read nodes

Because the data is being read from the cDAQ, the time stamps are different from the PXIe timestamps. Therefore, the time data is taken from the Analog Input Voltage Read node and the pitch encoder signals are rebuilt with the PXIe timestamps. This is done using Get Waveform Components function on the analog input data, taking the t0 and dt data and rebuilding the pitch encoder signals using Build Waveform functions. The reconstructed signals are combined using a Merge Signal function and stored in the measurement file. In actuality, the stored data is the blade root pitch angle. The pitch at 75% span of the radius is 18.013 degrees more than the blade root pitch. For observational purposes, the $0.75 \cdot R$ pitch angle is displayed in a graph indicator.

Counter Output Data Task

The cDAQ counter output task line and error lines go directly through the While Loop structure, and out the other side to the Clear Task node. There are no other actions to take with this as it is only used to start the counter inputs at the same time.

Write Task

The data used with a DAQmx Write node are the voltages to the three actuators and the voltages for the motor brake, throttle, reverse/forward direction, and accessories. Once those voltages are acquired and manipulated from the handheld controller, the voltages are written to the Analog Output module on the PXIe. That is why the Write task is simply at the end of all the manipulations for those parameters shown in Figure 36.

The datatype is selected from the dropdown menu through Analog > Multiple Channels > Multiple Samples > 1D Waveform.

This means that the write task is writing 5000 samples to the module every loop which occurs every half second. So the response from the rig is about a half second delayed from when the input is given.

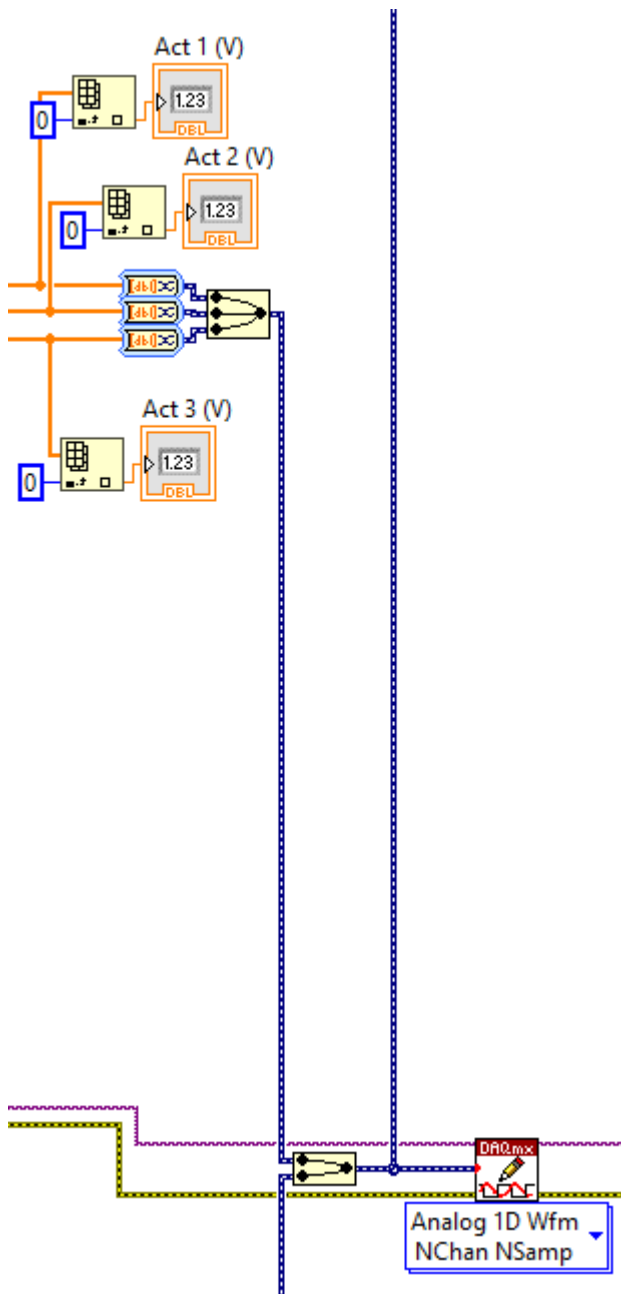


Figure 36 Write node for actuators and motor voltages

The Write node task and errors lines go directly to the While Loop structure and out to Clear Task nodes.

Writing Data to File

Writing the data to file uses the Write Delimited Spreadsheet node found in the File I/O palette and shown in Figure 37. The node takes a file path as a control. The format terminal determines how many digits to include in the data, “%.6f” means 6 digits after the decimal place and the numbers are floats (meaning they have decimal places). The data being written is 2D meaning there are multiple columns and multiple rows. The two Boolean terminals are “append to file?”, and “transpose?”. If append to file is TRUE then each time the node is called, it will append data to the existing file path unless the file path is changed and if transpose is TRUE then the 2D data rows and columns will be flipped like a matrix transpose. The MTR VI has set both to TRUE. At bottom of the node is a terminal for the delimiter which is set to a comma.

The MTR VI does not continually record data. There is a Case structure with a Boolean trigger to determine when the data should be saved. There is also a Case structure for timing the action which allows the user to see the duration of the data is being saved.

All the signals from the Read node and including the Write nodes are saved into the measurement file by using Merge Signal function. The very first signal is a timestamp from one of the analog input voltage signals, and that sets the first column of data to be timestamps.

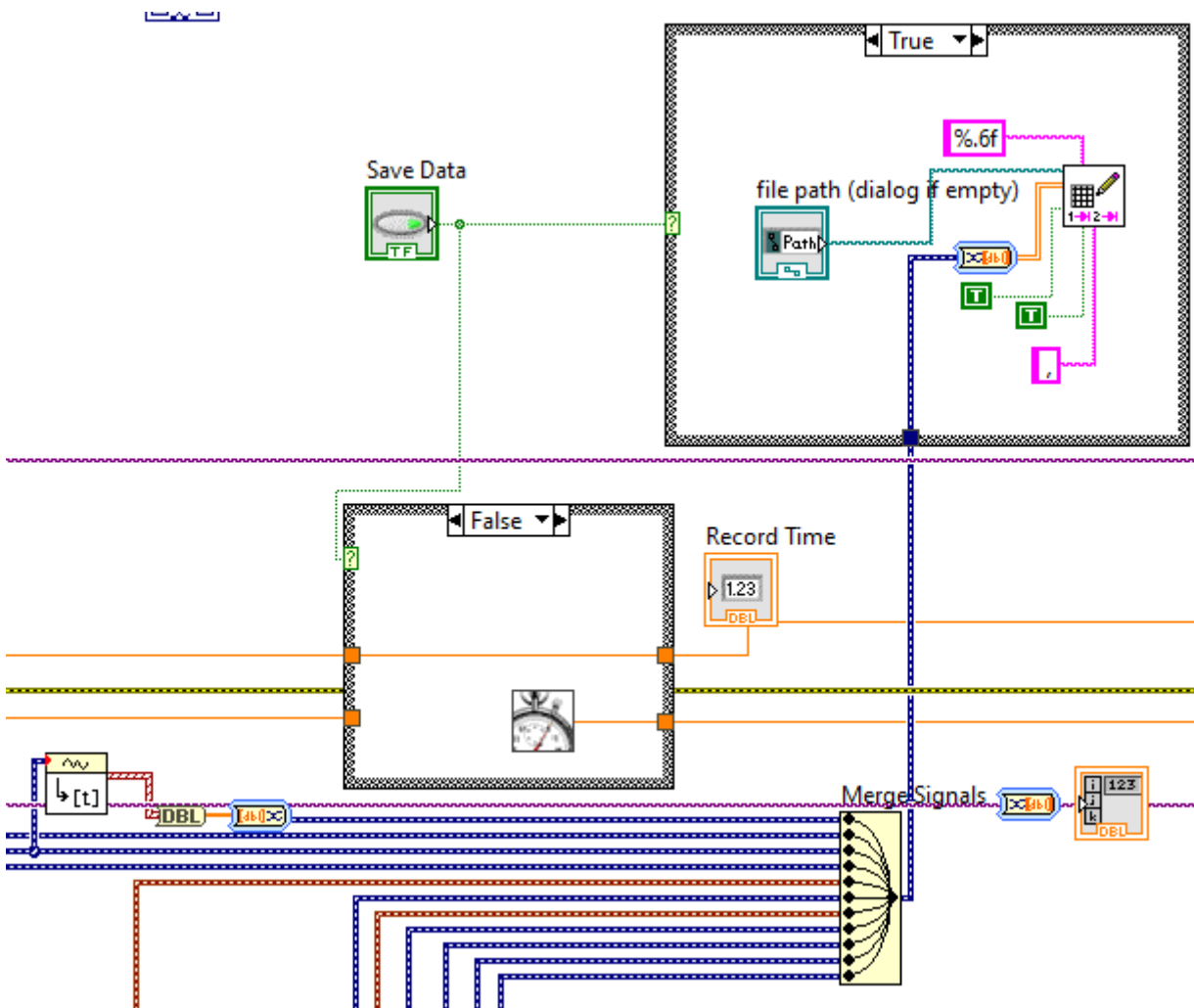


Figure 37 Writing to measurement file using a Write Delimited Spreadsheet node in the upper right hand corner

The list of signals are listed below in order:

1) Time	13) Azimuth	25) Strain 3 Open
---------	-------------	-------------------

2) Pitch Encoder 1	14) Pitch Link 1	26) Strain 4 Open
3) Pitch Encoder 2	15) Pitch Link 2	27) Strain 5 Open
4) Pitch Encoder 3	16) Pitch Link 3	28) Strain 6 Open
5) Load Cell Fx	17) Shaft Torque	29) Strain 7 Open
6) Load Cell Fy	18) Blade Strain 1	30) Actuator 1 Voltage
7) Load Cell Fz	19) Blade Strain 2	31) Actuator 2 Voltage
8) Load Cell Mx	20) Blade Strain 3	32) Actuator 3 Voltage
9) Load Cell My	21) Blade Strain 4	33) Brake Voltage
10) Load Cell Mz	22) Wing Bending	34) Throttle Voltage
11) Gimbal Hall Voltage 1	23) Wing Chord	35) Forward/Reverse Voltage
12) Gimbal Hall Voltage 2	24) Wing Torsion	36) Accessories Voltage

Clearing Tasks and Error Handling

As a matter of standard practice, all task and error wires exiting the While Loop structure should connect to a Clear Task node. The error wires from the Clear Task nodes should be combined using the Merge Errors function found in the Dialog & User Interface palette.

The combined errors go into the Simple Error Handler node which is set to “OK message + warnings” for the type of dialog if an error occurs.

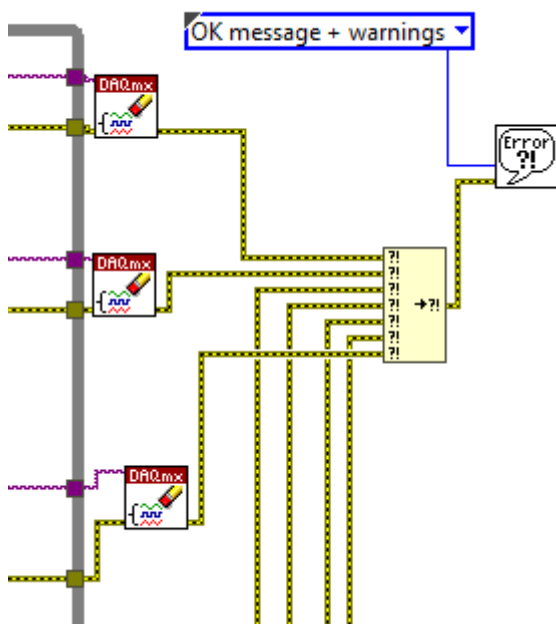


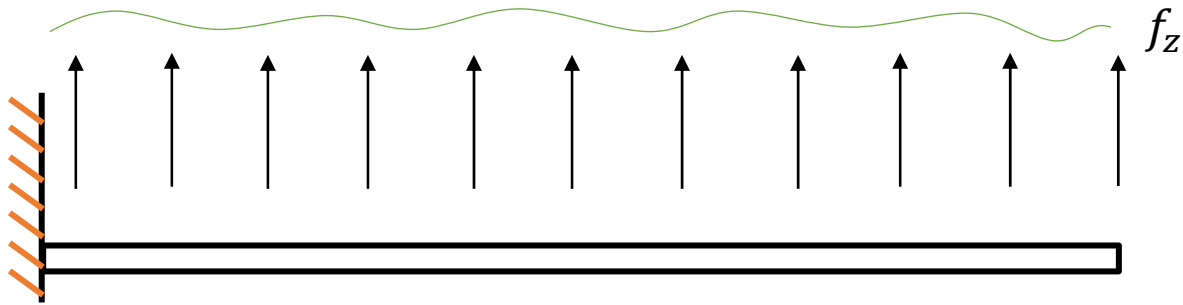
Figure 38 Clear task and collecting errors

Derivations

This appendix covers derivations for beam mode frequencies and moving block method.

Derivations for uncoupled Wing Beam, Chord, and Torsion Frequencies

1) Cantilever Beam w/ distributed load



Equation of motion:

$$m\ddot{w}(x) + (EIw''(x))'' = f_z$$

$$\text{if distributed load } f_z = 0 \text{ then } m\ddot{w}(x) + (EIw''(x))'' = 0$$

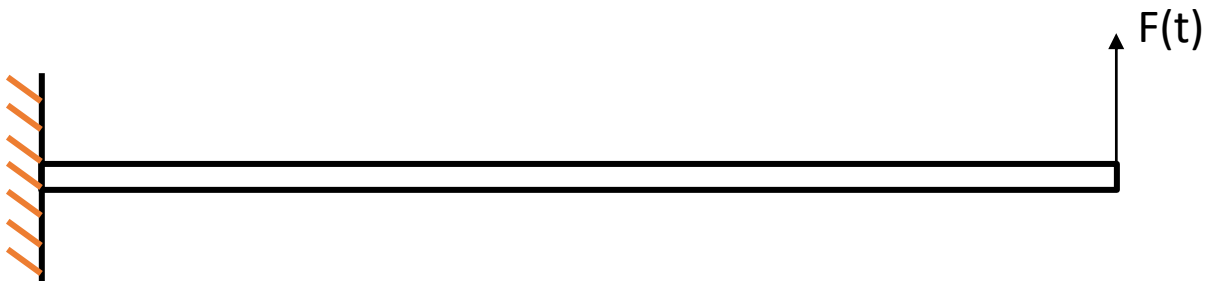
Boundary Conditions:

$$@x = 0, \quad w = 0, w' = 0$$

$$@x = L, \quad \text{shear force } (EIw''(x))' = 0$$

$$\text{and bending moment } EIw'' = 0$$

2) Cantilever beam w/ tip load



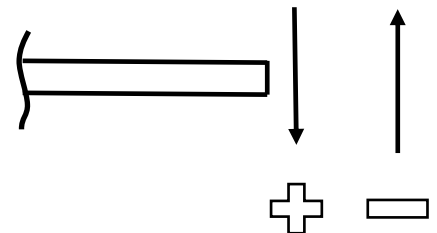
Same equation of motion as 1 without distributed load.

Boundary Conditions:

$$@x = 0, \quad w = 0, w' = 0$$

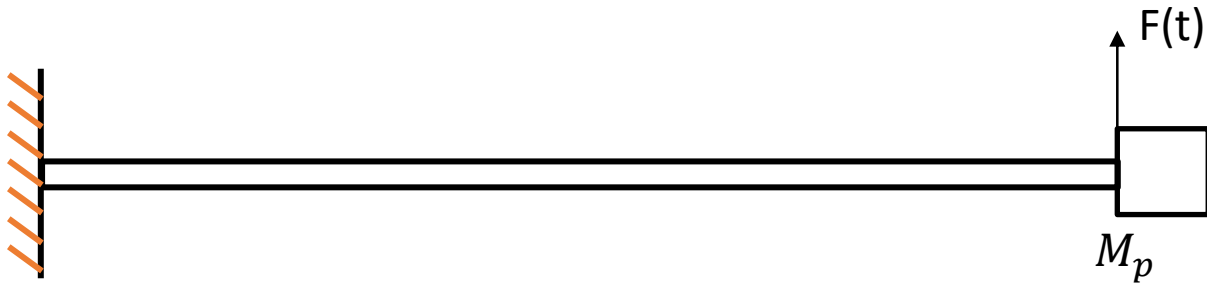
$$@x = L, \quad (EIw''(L))' = -F(t)$$

$$EIw'' = 0$$

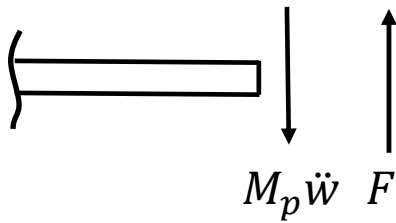


Signs by convention to the right ->>>

3) Cantilever Beam w/ tip load and tip mass



Same EOM as 1 without distributed load



Boundary conditions:

$$@x = 0, \quad w = 0, w' = 0$$

$$@x = L, \quad (EIw''(L))' = M_p \ddot{w}(L) - F(t)$$

$$EIw''(L) = 0$$

Assumed Solution

$$w = \phi q \text{ where } \phi = \text{shape function}$$

Then

$$@x = 0, \quad \phi q = 0, \phi' q = 0$$

$$@x = L, \quad (EI\phi''(L))' q = M_p \phi(L) \ddot{q} - F(t)$$

$$EI\phi''(L)q = 0$$

Sub $w = \phi q$ into EOM, and since it's an approximation, you get some error ϵ :

$$m\phi \ddot{q} + (EI\phi'')'' q = \epsilon$$

Assume error also follows shape function and integrate along the beam:

$$\phi(m\phi \ddot{q} + (EI\phi'')'' q) = \phi \epsilon$$

$$\int_0^L \phi(m\phi \ddot{q} + (EI\phi'')'' q) dx = \int_0^L \phi \epsilon dx$$

Set the error to zero since that is the goal:

$$\int_0^L \phi(m\phi\ddot{q} + (EI\phi'')''q) dx = 0$$

Then:

$$\boxed{\int_0^L \phi m \phi \ddot{q} dx + \int_0^L \phi (EI \phi'')'' q dx = 0}$$

Simplify:

$$\ddot{q} \int_0^L m \phi^2 dx + q \int_0^L \phi (EI \phi'')'' dx = 0$$

For 2nd term in expression, use changing variables rule:

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

So then:

$$q \int_0^L \phi (EI \phi'')'' dx = q[\phi(EI \phi'')]_0^L - q \int_0^L \phi' (EI \phi'')' dx$$

Use changing variables rule again for the highlighted term on right side:

$$-q \int_0^L \phi' (EI \phi'')' dx = -q \left([\phi' EI \phi'']_0^L - \int_0^L \phi'' EI \phi'' dx \right)$$

Combined expression:

$$\ddot{q} \int_0^L m \phi^2 dx + q[\phi(EI \phi'')]_0^L - q \left([\phi' EI \phi'']_0^L - \int_0^L \phi'' EI \phi'' dx \right) = 0$$

The 2nd term becomes:

$$q[\phi(EI \phi'')]_0^L = \phi(L)(M_p \phi(L)\ddot{q} - F)$$

The 3rd term in this equation is zero due to the boundary condition:

$$EI \phi''(L) = 0$$

Then:

$$\ddot{q} \int_0^L m \phi^2 dx + \phi(L)(M_p \phi(L)\ddot{q} - F) + q \int_0^L \phi'' EI \phi'' dx = 0$$

Simplify into Lagrangian form:

$$\left(\int_0^L m \phi^2 dx + M_p \phi^2(L) \right) \ddot{q} + \int_0^L \phi'' EI \phi'' dx q = F \phi(L)$$

$$M = \int_0^L m\phi^2 dx + M_p\phi^2(L)$$

$$K = \int_0^L \phi'' EI \phi'' dx$$

Assume solution q is of form:

$$q = q_s \sin \omega t + q_c \cos \omega t$$

$$\dot{q} = \omega q_s \cos \omega t - \omega q_c \sin \omega t$$

$$\ddot{q} = -\omega^2 q_s \sin \omega t - \omega^2 q_c \cos \omega t = -\omega^2 q$$

Then:

$$M\ddot{q} + Kq = F\phi(L)$$

$$M(-\omega^2 q) + Kq = F\phi(L)$$

Characteristic equation:

$$(K - \omega^2 M)q = F\phi(L)$$

Deflection solution:

$$q = \frac{F\phi(L)}{(K - \omega^2 M)}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{\int_0^L \phi'' EI \phi'' dx}{\int_0^L m\phi^2 dx + M_p\phi^2(L)}}$$

For a shape function:

$$\phi = \left(\frac{x}{L}\right)^2$$

$$\phi' = \frac{2x}{L^2}$$

$$\phi'' = \frac{2}{L^2}$$

$$\phi(L) = 1$$

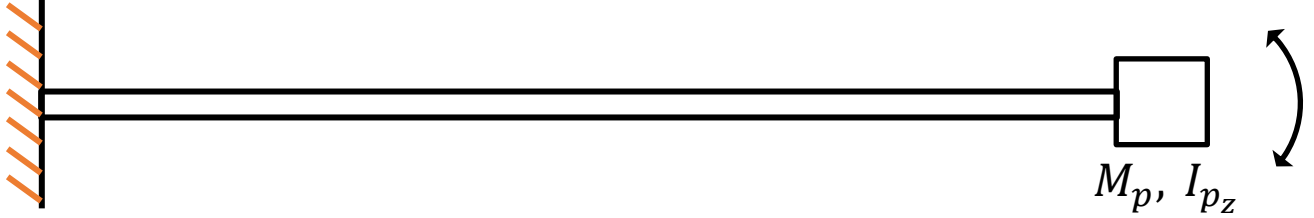
Then:

$$K = EI \int_0^L \phi''^2 dx = \frac{4}{L^4} * L = \frac{4EI}{L^3}$$

$$M = \int_0^L m\phi^2 dx + M_p\phi^2(L) = \frac{mL}{3} + M_p$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{\frac{4EI}{L^3}}{\frac{mL}{3} + M_p}}$$

4) Cantilever beam with tip mass and mass moment of inertia:



Equation of motion, same as the flap EOM, but using v for deflection instead of w .

$$m\ddot{v}(x) + (EIv''(x))'' = 0$$

Assumed solution: $v = \phi q$

Boundary conditions:

$$@x = 0, \quad v = 0, v' = 0$$

$$@x = L, \quad (EIv''(L))' = M_p\ddot{v}(L) = M\phi(L)\ddot{q}$$

$$EIv''(L) = -I_{p_z}\ddot{v}'(L) = -I_{p_z}\phi'(L)\ddot{q},$$

this is negative due to resistance to an applied torque which is positive.

Then going through the same calculus as flap, starting with equation of motion:

$$\ddot{q} \int_0^L m\phi^2 dx + q \int_0^L \phi(EI\phi'')'' dx = 0$$

For 2nd term in expression, use changing variables rule:

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

So then:

$$q \int_0^L \phi(EI\phi'')'' dx = q[\phi(EI\phi'')']_0^L - q \int_0^L \phi'(EI\phi'')' dx$$

Use changing variables rule again for the highlighted term on right side:

$$-q \int_0^L \phi'(EI\phi'')' dx = -q \left([\phi'EI\phi'']_0^L - \int_0^L \phi''EI\phi'' dx \right)$$

Combined expression:

$$\ddot{q} \int_0^L m \phi^2 dx + q [\phi(EI\phi'')]_0^L - q [\phi'EI\phi'']_0^L + q \int_0^L \phi''EI\phi'' dx = 0$$

2nd term becomes:

$$M_p \phi^2(L) \ddot{q}$$

3rd term becomes:

$$I_{p_z} \phi'^2(L) \ddot{q}$$

Then:

$$\ddot{q} \int_0^L m \phi^2 dx + M_p \phi^2(L) \ddot{q} + I_{p_z} \phi'^2(L) \ddot{q} + q \int_0^L \phi''EI\phi'' dx = 0$$

$$\left[\int_0^L m \phi^2 dx + M_p \phi^2(L) + I_{p_z} \phi'^2(L) \right] \ddot{q} + \int_0^L \phi''EI\phi'' dx q = 0$$

$$K = \int_0^L \phi''EI\phi'' dx$$

$$M = \int_0^L m \phi^2 dx + M_p \phi^2(L) + I_{p_z} \phi'^2(L)$$

For a shape function:

$$\phi = \left(\frac{x}{L}\right)^2$$

$$\phi' = \frac{2x}{L^2}$$

$$\phi'' = \frac{2}{L^2}$$

$$\phi(L) = 1$$

$$\phi'(L) = \frac{2}{L}$$

Then:

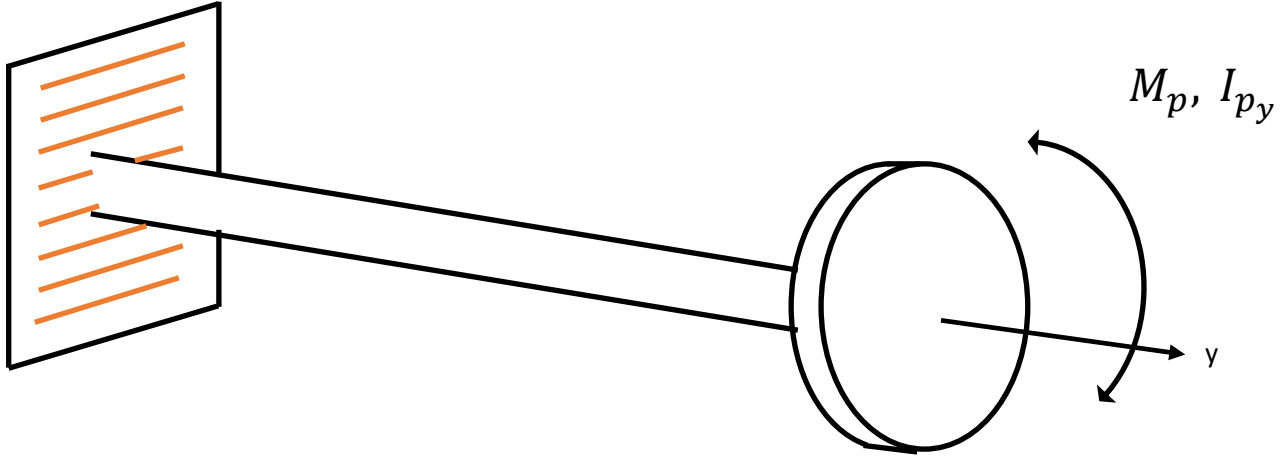
$$K = \frac{4EI}{L^3}$$

$$M = \frac{mL}{3} + M_p + \frac{4}{L^2} I_{p_z}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{\frac{4EI}{L^3}}{\frac{mL}{3} + M_p + \frac{4}{L^2} I_{p_z}}}$$

5) Consider a cantilever beam under torsion.



With an applied tip torque T_{tip} .

Equation of motion:

$$I_f \ddot{\theta} - (GJ\theta')' = 0$$

$$\theta = \phi q$$

Boundary conditions:

$$@x = 0, \quad \theta = 0, \theta' = 0$$

$$@x = L, \quad GJ\theta' = GJ\phi'(L)q = T_{tip} - I_{p_y}\ddot{\theta}(L) = T_{tip} - I_{p_y}\phi(L)\ddot{q}$$

Equation of motion:

$$I_f \phi \ddot{q} - (GJ\phi')' q = 0$$

$$\ddot{q} \int_0^L I_f \phi^2 dx - q \int_0^L \phi (GJ\phi')' dx = 0$$

Changing variables rule on 2nd term:

$$q \int_0^L \phi (GJ\phi')' dx = q[\phi GJ\phi']_0^L - q \int_0^L \phi' GJ\phi' dx$$

Combined equation:

$$\ddot{q} \int_0^L I_f \phi^2 dx - q[\phi GJ\phi']_0^L + q \int_0^L \phi' GJ\phi' dx = 0$$

From B.C, the 2nd term becomes:

$$q[\phi GJ\phi']_0^L = \phi(L) \left(T_{tip} - I_{py}\phi(L)\ddot{q} \right)$$

Then:

$$\ddot{q} \int_0^L I_f \phi^2 dx + I_{py} \phi^2(L) \ddot{q} + q \int_0^L \phi' GJ \phi' dx = \phi(L) T_{tip}$$

Lagrangian form:

$$\left[\int_0^L I_f \phi^2 dx + I_{py} \phi^2(L) \right] \ddot{q} + \int_0^L \phi' GJ \phi' dx q = \phi(L) T_{tip}$$

$$K = \int_0^L \phi' GJ \phi' dx$$

$$M = \int_0^L I_f \phi^2 dx + I_{py} \phi^2(L)$$

Torsion frequency:

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{\int_0^L GJ \phi'^2 dx}{\int_0^L I_f \phi^2 dx + I_{py} \phi^2(L)}}$$

$$\phi = \frac{x}{L}$$

$$\phi' = \frac{1}{L}$$

$$\phi(L) = 1$$

$$K = GJ \int_0^L \frac{1}{L^2} dx = \frac{GJ}{L}$$

$$M = I_f \int_0^L \left(\frac{x}{L}\right)^2 dx + I_{py} = \frac{I_f L}{3} + I_{py}$$

$$\omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{\frac{GJ}{L}}{\frac{I_f L}{3} + I_{py}}}$$

$$\text{where } I_f = \frac{\rho t c^3 L}{12} \text{ and } \rho = m \left(\frac{\text{mass}}{\text{length}} \right) * \frac{1}{A}$$

$$I_f = \frac{m}{tc} * \frac{tc^3 L}{12} = \frac{mc^2 L}{12}$$

Moving Block Method Derivation

Reference: Hammond, C.E. "Determination of Subcritical Damping by Moving-Block/RandomDec Applications"

Starting with a damped sinusoidal signal:

$$f(t) = Ae^{-\zeta\omega_n t} \sin(\omega t + \phi) \quad (1)$$

Where,

$A = \text{amplitude of signal}$

$\zeta = \text{damping ratio}$

$\omega_n = \text{undamped natural frequency}$

$\omega = \omega_n \sqrt{1 - \zeta^2} = \text{damped natural frequency}$

Take a Fourier Transform of the signal:

$$F(\omega) = \int_{\tau}^{T+\tau} f(t) e^{-i\omega t} dt$$

$$F(\omega) = \int_{\tau}^{T+\tau} Ae^{-\zeta\omega_n t} \sin(\omega t + \phi) e^{-i\omega t} dt$$

Combine terms:

$$F(\omega) = \int_{\tau}^{T+\tau} Ae^{-(\zeta\omega_n + i\omega)t} \sin(\omega t + \phi) dt \quad (2)$$

Integration by parts, part 1:

$$\int u dv = uv - \int v du$$

Set:

$$u = e^{-(\zeta\omega_n + i\omega)t}$$

$$\frac{dv}{dt} = \sin(\omega t + \phi)$$

Then:

$$du = -(\zeta\omega_n + i\omega)e^{-(\zeta\omega_n + i\omega)t}$$

$$v = -\frac{1}{\omega} \cos(\omega t + \phi)$$

Apply to Eq. (2):

$$F(\omega) = -\frac{A}{\omega} e^{-(\zeta\omega_n + i\omega)t} \cos(\omega t + \phi) \Big|_{\tau}^{T+\tau} - \frac{A}{\omega} (\zeta\omega_n + i\omega) \int_{\tau}^{T+\tau} e^{-(\zeta\omega_n + i\omega)t} \cos(\omega t + \phi) dt \quad (3)$$

Integration by parts on red highlighted term:

$$u = e^{-(\zeta\omega_n + i\omega)t}$$

$$\frac{dv}{dt} = \cos(\omega t + \phi)$$

Then:

$$du = -(\zeta\omega_n + i\omega)e^{-(\zeta\omega_n + i\omega)t}$$

$$v = \frac{1}{\omega} \sin(\omega t + \phi)$$

Apply to Eq. (3):

$$F(\omega) = -\frac{A}{\omega} e^{-(\zeta\omega_n + i\omega)t} \cos(\omega t + \phi) \Big|_{\tau}^{T+\tau} - \frac{A}{\omega^2} (\zeta\omega_n + i\omega) e^{-(\zeta\omega_n + i\omega)t} \sin(\omega t + \phi) \Big|_{\tau}^{T+\tau} - \frac{1}{\omega^2} (\zeta\omega_n + i\omega)^2 \int_{\tau}^{T+\tau} A e^{-(\zeta\omega_n + i\omega)t} \sin(\omega t + \phi) dt \quad (4)$$

Red highlighted expression is the same expression as Eq. (2), therefore bring to left side, and expand right side:

$$\begin{aligned} F(\omega) \left[1 + \frac{1}{\omega^2} (\zeta\omega_n + i\omega)^2 \right] &= -\frac{A}{\omega} e^{-(\zeta\omega_n + i\omega)t} \cos(\omega t + \phi) \Big|_{\tau}^{T+\tau} \\ &\quad - \frac{A}{\omega^2} (\zeta\omega_n + i\omega) e^{-(\zeta\omega_n + i\omega)t} \sin(\omega t + \phi) \Big|_{\tau}^{T+\tau} \end{aligned}$$

Simplify LHS and expand RHS:

$$\begin{aligned} F(\omega) \left[\left(\frac{\zeta\omega_n}{\omega} \right)^2 + i \frac{2\zeta\omega_n}{\omega} \right] &= -\frac{A}{\omega} e^{-(\zeta\omega_n + i\omega)(T+\tau)} \cos(\omega(T+\tau) + \phi) + \frac{A}{\omega} e^{-(\zeta\omega_n + i\omega)\tau} \cos(\omega\tau + \phi) \\ &\quad - \frac{A}{\omega^2} (\zeta\omega_n + i\omega) e^{-(\zeta\omega_n + i\omega)(T+\tau)} \sin(\omega(T+\tau) + \phi) \\ &\quad + \frac{A}{\omega^2} (\zeta\omega_n + i\omega) e^{-(\zeta\omega_n + i\omega)\tau} \sin(\omega\tau + \phi) \end{aligned}$$

Divide through by $\left[\left(\frac{\zeta\omega_n}{\omega} \right)^2 + i \frac{2\zeta\omega_n}{\omega} \right]$ and collect τ terms and $T + \tau$ terms:

$$\begin{aligned} F(\omega) &= \frac{A}{\omega^2 \left[\left(\frac{\zeta\omega_n}{\omega} \right)^2 + i \frac{2\zeta\omega_n}{\omega} \right]} (\zeta\omega_n + i\omega) e^{-(\zeta\omega_n + i\omega)\tau} \sin(\omega\tau + \phi) \\ &\quad + \frac{A}{\omega \left[\left(\frac{\zeta\omega_n}{\omega} \right)^2 + i \frac{2\zeta\omega_n}{\omega} \right]} e^{-(\zeta\omega_n + i\omega)\tau} \cos(\omega\tau + \phi) \\ &\quad - \frac{A}{\omega^2 \left[\left(\frac{\zeta\omega_n}{\omega} \right)^2 + i \frac{2\zeta\omega_n}{\omega} \right]} (\zeta\omega_n + i\omega) e^{-(\zeta\omega_n + i\omega)(T+\tau)} \sin(\omega(T+\tau) + \phi) \\ &\quad - \frac{A}{\omega \left[\left(\frac{\zeta\omega_n}{\omega} \right)^2 + i \frac{2\zeta\omega_n}{\omega} \right]} e^{-(\zeta\omega_n + i\omega)(T+\tau)} \cos(\omega(T+\tau) + \phi) \end{aligned}$$

Simplified:

$$\begin{aligned}
F(\omega) = & \frac{A}{\zeta\omega_n[\zeta\omega_n + i2\omega]} (\zeta\omega_n + i\omega)e^{-(\zeta\omega_n + i\omega)\tau} \sin(\omega\tau + \phi) \\
& + \frac{A\omega}{\zeta\omega_n[\zeta\omega_n + i2\omega]} e^{-(\zeta\omega_n + i\omega)\tau} \cos(\omega\tau + \phi) \\
& - \frac{A}{\zeta\omega_n[\zeta\omega_n + i2\omega]} (\zeta\omega_n + i\omega)e^{-(\zeta\omega_n + i\omega)(T+\tau)} \sin(\omega(T + \tau) + \phi) \\
& - \frac{A\omega}{\zeta\omega_n[\zeta\omega_n + i2\omega]} e^{-(\zeta\omega_n + i\omega)(T+\tau)} \cos(\omega(T + \tau) + \phi)
\end{aligned}$$

Simplify further:

$$\begin{aligned}
F(\omega) = & \frac{A}{\zeta\omega_n(\zeta\omega_n + i2\omega)} \left[e^{-(\zeta\omega_n + i\omega)\tau} [(\zeta\omega_n + i\omega) \sin(\omega\tau + \phi) + \omega \cos(\omega\tau + \phi)] \right. \\
& \left. - e^{-(\zeta\omega_n + i\omega)(T+\tau)} [(\zeta\omega_n + i\omega) \sin(\omega(T + \tau) + \phi) + \omega \cos(\omega(T + \tau) + \phi)] \right] \quad (5)
\end{aligned}$$

Fourier transform has form:

$$F(\omega) = a + ib$$

And amplitude of transform is:

$$|F(\omega)| = \sqrt{a^2 + b^2}$$

Split Eq. (5) into its real and imaginary components, assuming $\zeta \ll 1, \omega \approx \omega_n$:

$$\begin{aligned}
F(\omega) = & \frac{A}{\zeta\omega_n(\zeta\omega_n + i2\omega_n)} \left[e^{-(\zeta\omega_n + i\omega_n)\tau} [(\zeta\omega_n + i\omega_n) \sin(\omega_n\tau + \phi) + \omega_n \cos(\omega_n\tau + \phi)] \right. \\
& \left. - e^{-(\zeta\omega_n + i\omega_n)(T+\tau)} [(\zeta\omega_n + i\omega_n) \sin(\omega_n(T + \tau) + \phi) + \omega_n \cos(\omega_n(T + \tau) + \phi)] \right] \quad (5.1)
\end{aligned}$$

Starting with green:

$$\frac{A}{\zeta\omega_n(\zeta\omega_n + i2\omega_n)} = \frac{A}{\zeta\omega_n^2(\zeta + i2)} \times \frac{(\zeta - 2i)}{(\zeta - 2i)} = \frac{A(\zeta - 2i)}{\zeta\omega_n^2(\zeta^2 + 4)} = \frac{A\zeta}{\zeta\omega_n^2(\zeta^2 + 4)} - \frac{A2i}{\zeta\omega_n^2(\zeta^2 + 4)}$$

For red and blue, use exponential identity $e^{x+iy} = e^x(\cos y + i \sin y)$:

$$\begin{aligned}
e^{-\zeta\omega_n\tau - i\omega_n\tau} &= e^{-\zeta\omega_n\tau} (\cos(-\omega_n\tau + \phi) + i \sin(-\omega_n\tau + \phi)) = e^{-\zeta\omega_n\tau} (\cos(\omega_n\tau + \phi) - i \sin(\omega_n\tau + \phi)) \\
e^{-\zeta\omega_n(T+\tau) - i\omega_n(T+\tau)} &= e^{-\zeta\omega_n(T+\tau)} (\cos(-\omega_n(T + \tau) + \phi) + i \sin(-\omega_n(T + \tau) + \phi)) \\
&= e^{-\zeta\omega_n(T+\tau)} (\cos(\omega_n(T + \tau) + \phi) - i \sin(\omega_n(T + \tau) + \phi))
\end{aligned}$$

Substitute into Eq. (5.1):

$$\begin{aligned}
F(\omega) = & \left(\frac{A\zeta}{\zeta\omega_n^2(\zeta^2 + 4)} - \frac{A2i}{\zeta\omega_n^2(\zeta^2 + 4)} \right) \left[\left[e^{-\zeta\omega_n\tau} (\cos(\omega_n\tau + \phi) - i \sin(\omega_n\tau + \phi)) \right] [(\zeta\omega_n + i\omega_n) \sin(\omega_n\tau + \phi) \right. \right. \\
& \left. \left. + \omega_n \cos(\omega_n\tau + \phi)] \right. \right. \\
& \left. - \left[e^{-\zeta\omega_n(T+\tau)} (\cos(\omega_n(T + \tau) + \phi) - i \sin(\omega_n(T + \tau) + \phi)) \right] [(\zeta\omega_n + i\omega_n) \sin(\omega_n(T + \tau) + \phi) \right. \right. \\
& \left. \left. + \omega_n \cos(\omega_n(T + \tau) + \phi)] \right] \quad (6)
\end{aligned}$$

Working within the large brackets containing the red and blue terms, expand and simplify:

$$\begin{aligned}
& \left[\left[e^{-\zeta\omega_n\tau} (\cos(\omega_n\tau + \phi) - i \sin(\omega_n\tau + \phi)) \right] [(\zeta\omega_n + i\omega_n) \sin(\omega_n\tau + \phi) + \omega_n \cos(\omega_n\tau + \phi)] \right. \\
& \left. - \left[e^{-\zeta\omega_n(T+\tau)} (\cos(\omega_n(T + \tau) + \phi) - i \sin(\omega_n(T + \tau) + \phi)) \right] [(\zeta\omega_n + i\omega_n) \sin(\omega_n(T + \tau) + \phi) \right. \right. \\
& \left. \left. + \omega_n \cos(\omega_n(T + \tau) + \phi)] \right]
\end{aligned}$$

$$\left[\left[(e^{-\zeta\omega_n\tau} \cos(\omega_n\tau + \phi) - ie^{-\zeta\omega_n\tau} \sin(\omega_n\tau + \phi)) \right] [(\zeta\omega_n + i\omega_n) \sin(\omega_n\tau + \phi) + \omega_n \cos(\omega_n\tau + \phi)] \right. \\ \left. - \left[(e^{-\zeta\omega_n(T+\tau)} \cos(\omega_n(T+\tau) + \phi) - ie^{-\zeta\omega_n(T+\tau)} \sin(\omega_n(T+\tau) + \phi)) \right] [(\zeta\omega_n + i\omega_n) \sin(\omega_n(T+\tau) + \phi) + \omega_n \cos(\omega_n(T+\tau) + \phi)] \right]$$

First half of bracket, multiplying red-bracketed terms:

$$(\zeta\omega_n + i\omega_n)e^{-\zeta\omega_n\tau} \sin(\omega_n\tau + \phi) \cos(\omega_n\tau + \phi) + \omega_n e^{-\zeta\omega_n\tau} \cos^2(\omega_n\tau + \phi) - (\zeta\omega_n + i\omega_n)ie^{-\zeta\omega_n\tau} \sin^2(\omega_n\tau + \phi) \\ - i\omega_n e^{-\zeta\omega_n\tau} \sin(\omega_n\tau + \phi) \cos(\omega_n\tau + \phi)$$

Expand:

$$\zeta\omega_n e^{-\zeta\omega_n\tau} \sin(\omega_n\tau + \phi) \cos(\omega_n\tau + \phi) + i\omega_n e^{-\zeta\omega_n\tau} \sin(\omega_n\tau + \phi) \cos(\omega_n\tau + \phi) + \omega_n e^{-\zeta\omega_n\tau} \cos^2(\omega_n\tau + \phi) \\ - i\zeta\omega_n e^{-\zeta\omega_n\tau} \sin^2(\omega_n\tau + \phi) + \omega_n e^{-\zeta\omega_n\tau} \sin^2(\omega_n\tau + \phi) - i\omega_n e^{-\zeta\omega_n\tau} \sin(\omega_n\tau + \phi) \cos(\omega_n\tau + \phi)$$

Simplify, $\cos^2(x) + \sin^2(x) = 1$, and purple cancels out:

$$\zeta\omega_n e^{-\zeta\omega_n\tau} \sin(\omega_n\tau + \phi) \cos(\omega_n\tau + \phi) + \omega_n e^{-\zeta\omega_n\tau} - i\zeta\omega_n e^{-\zeta\omega_n\tau} \sin^2(\omega_n\tau + \phi)$$

Half angle formula, $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$; double angle formula, $\sin(x) \cos(x) = \frac{1}{2}\sin(2x)$:

$$\frac{\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} \sin(2\omega_n\tau + \phi) + \omega_n e^{-\zeta\omega_n\tau} - \frac{i\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} + \frac{i\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} \cos(2\omega_n\tau + \phi) \quad (6.1)$$

Second half of bracket, multiplying blue-bracketed terms:

$$(\zeta\omega_n + i\omega_n)e^{-\zeta\omega_n(T+\tau)} \cos(\omega_n(T+\tau) + \phi) \sin(\omega_n(T+\tau) + \phi) + \omega_n e^{-\zeta\omega_n(T+\tau)} \cos^2(\omega_n(T+\tau) + \phi) \\ - (\zeta\omega_n + i\omega_n)ie^{-\zeta\omega_n(T+\tau)} \sin^2(\omega_n(T+\tau) + \phi) \\ - \omega_n ie^{-\zeta\omega_n(T+\tau)} \sin(\omega_n(T+\tau) + \phi) \cos(\omega_n(T+\tau) + \phi)$$

Expand:

$$\zeta\omega_n e^{-\zeta\omega_n(T+\tau)} \cos(\omega_n(T+\tau) + \phi) \sin(\omega_n(T+\tau) + \phi) + i\omega_n e^{-\zeta\omega_n(T+\tau)} \cos(\omega_n(T+\tau) + \phi) \sin(\omega_n(T+\tau) + \phi) \\ + \omega_n e^{-\zeta\omega_n(T+\tau)} \cos^2(\omega_n(T+\tau) + \phi) - \zeta\omega_n ie^{-\zeta\omega_n(T+\tau)} \sin^2(\omega_n(T+\tau) + \phi) \\ - i^2\omega_n e^{-\zeta\omega_n(T+\tau)} \sin^2(\omega_n(T+\tau) + \phi) - i\omega_n e^{-\zeta\omega_n(T+\tau)} \sin(\omega_n(T+\tau) + \phi) \cos(\omega_n(T+\tau) + \phi)$$

Simplify, $\cos^2(x) + \sin^2(x) = 1$, purple cancels out:

$$\zeta\omega_n e^{-\zeta\omega_n(T+\tau)} \sin(\omega_n(T+\tau) + \phi) \cos(\omega_n(T+\tau) + \phi) + \omega_n e^{-\zeta\omega_n(T+\tau)} - i\zeta\omega_n e^{-\zeta\omega_n(T+\tau)} \sin^2(\omega_n(T+\tau) + \phi)$$

Half angle formula, $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$; double angle formula, $\sin(x) \cos(x) = \frac{1}{2}\sin(2x)$:

$$\frac{\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} \sin(2\omega_n(T+\tau) + \phi) + \omega_n e^{-\zeta\omega_n(T+\tau)} - \frac{i\zeta\omega_n e^{-\zeta\omega_n(T+\tau)}}{2} + \frac{i\zeta\omega_n e^{-\zeta\omega_n(T+\tau)}}{2} \cos(2\omega_n(T+\tau) + \phi) \quad (6.2)$$

Combine Eq. (6.1) and (6.2) into Eq. (6), remember to subtract Eq. (6.2):

$$F(\omega) = \left(\frac{A\zeta}{\zeta\omega_n^2(\zeta^2 + 4)} - \frac{A2i}{\zeta\omega_n^2(\zeta^2 + 4)} \right) \left(\frac{\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} \sin(2\omega_n\tau + \phi) + \omega_n e^{-\zeta\omega_n\tau} - \frac{i\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} \right. \\ \left. + \frac{i\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} \cos(2\omega_n\tau + \phi) - \frac{\zeta\omega_n e^{-\zeta\omega_n\tau}}{2} \sin(2\omega_n(T+\tau) + \phi) - \omega_n e^{-\zeta\omega_n(T+\tau)} \right. \\ \left. + \frac{i\zeta\omega_n e^{-\zeta\omega_n(T+\tau)}}{2} - \frac{i\zeta\omega_n e^{-\zeta\omega_n(T+\tau)}}{2} \cos(2\omega_n(T+\tau) + \phi) \right) \quad (7)$$

Expand Eq. (7):

$$\begin{aligned}
& \frac{A}{\zeta \omega_n^2 (\zeta^2 + 4)} (\zeta - 2i) \left(\frac{\zeta \omega_n e^{-\zeta \omega_n \tau}}{2} \sin(2\omega_n \tau + \phi) + \omega_n e^{-\zeta \omega_n \tau} - \frac{i\zeta \omega_n e^{-\zeta \omega_n \tau}}{2} + \frac{i\zeta \omega_n e^{-\zeta \omega_n \tau}}{2} \cos(2\omega_n \tau + \phi) \right. \\
& \quad - \frac{\zeta \omega_n e^{-\zeta \omega_n \tau}}{2} \sin(2\omega_n (T + \tau) + \phi) - \omega_n e^{-\zeta \omega_n (T + \tau)} + \frac{i\zeta \omega_n e^{-\zeta \omega_n (T + \tau)}}{2} \\
& \quad \left. - \frac{i\zeta \omega_n e^{-\zeta \omega_n (T + \tau)}}{2} \cos(2\omega_n (T + \tau) + \phi) \right) \\
& \frac{A}{\zeta \omega_n^2 (\zeta^2 + 4)} \left(\frac{\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} \sin(2\omega_n \tau + \phi) + \zeta \omega_n e^{-\zeta \omega_n \tau} - \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} + \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} \cos(2\omega_n \tau + \phi) \right. \\
& \quad - \frac{\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} \sin(2\omega_n (T + \tau) + \phi) - \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} + \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n (T + \tau)}}{2} \\
& \quad - \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n (T + \tau)}}{2} \cos(2\omega_n (T + \tau) + \phi) - i\zeta \omega_n e^{-\zeta \omega_n \tau} \sin(2\omega_n \tau + \phi) - i2\omega_n e^{-\zeta \omega_n \tau} - \zeta \omega_n e^{-\zeta \omega_n \tau} \\
& \quad + \zeta \omega_n e^{-\zeta \omega_n \tau} \cos(2\omega_n \tau + \phi) + i\zeta \omega_n e^{-\zeta \omega_n \tau} \sin(2\omega_n (T + \tau) + \phi) + i2\omega_n e^{-\zeta \omega_n (T + \tau)} \\
& \quad \left. + \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} - \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} \cos(2\omega_n (T + \tau) + \phi) \right)
\end{aligned}$$

Neglect ζ^2 terms assuming they are small, red terms denote imaginary component:

$$\begin{aligned}
& \frac{A}{\zeta \omega_n^2 (\zeta^2 + 4)} \left(\frac{\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} \sin(2\omega_n \tau + \phi) + \zeta \omega_n e^{-\zeta \omega_n \tau} - \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} + \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} \cos(2\omega_n \tau + \phi) \right. \\
& \quad - \frac{\zeta^2 \omega_n e^{-\zeta \omega_n \tau}}{2} \sin(2\omega_n (T + \tau) + \phi) - \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} + \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n (T + \tau)}}{2} \\
& \quad - \frac{i\zeta^2 \omega_n e^{-\zeta \omega_n (T + \tau)}}{2} \cos(2\omega_n (T + \tau) + \phi) - i\zeta \omega_n e^{-\zeta \omega_n \tau} \sin(2\omega_n \tau + \phi) - i2\omega_n e^{-\zeta \omega_n \tau} - \zeta \omega_n e^{-\zeta \omega_n \tau} \\
& \quad + \zeta \omega_n e^{-\zeta \omega_n \tau} \cos(2\omega_n \tau + \phi) + i\zeta \omega_n e^{-\zeta \omega_n \tau} \sin(2\omega_n (T + \tau) + \phi) + i2\omega_n e^{-\zeta \omega_n (T + \tau)} \\
& \quad \left. + \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} - \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} \cos(2\omega_n (T + \tau) + \phi) \right)
\end{aligned}$$

Simplify:

$$\begin{aligned}
& \frac{A}{(2\omega_n)^2 \zeta} \left(\zeta \omega_n e^{-\zeta \omega_n \tau} - \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} - i\zeta \omega_n e^{-\zeta \omega_n \tau} \sin(2\omega_n \tau + \phi) - i2\omega_n e^{-\zeta \omega_n \tau} - \zeta \omega_n e^{-\zeta \omega_n \tau} \right. \\
& \quad + \zeta \omega_n e^{-\zeta \omega_n \tau} \cos(2\omega_n \tau + \phi) + i\zeta \omega_n e^{-\zeta \omega_n \tau} \sin(2\omega_n (T + \tau) + \phi) + i2\omega_n e^{-\zeta \omega_n (T + \tau)} \\
& \quad \left. + \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} - \zeta \omega_n e^{-\zeta \omega_n (T + \tau)} \cos(2\omega_n (T + \tau) + \phi) \right)
\end{aligned}$$

Collect real and imaginary terms:

Real:

$$\frac{A}{4\omega_n} (e^{-\zeta \omega_n \tau} \cos(2\omega_n \tau + \phi) - e^{-\zeta \omega_n (T + \tau)} \cos(2\omega_n (T + \tau) + \phi))$$

Imag:

$$\frac{Ai}{4\omega_n} \left(-e^{-\zeta \omega_n \tau} \sin(2\omega_n \tau + \phi) - \frac{2}{\zeta} e^{-\zeta \omega_n \tau} + e^{-\zeta \omega_n \tau} \sin(2\omega_n (T + \tau) + \phi) + \frac{2}{\zeta} e^{-\zeta \omega_n (T + \tau)} \right)$$

Substitute into amplitude formula:

$$|F(\omega)| = \sqrt{a^2 + b^2}$$

$$|F(\omega)| = \left(\left(\frac{A}{4\omega_n} (e^{-\zeta\omega_n\tau} \cos(2\omega_n\tau + \phi) - e^{-\zeta\omega_n(T+\tau)} \cos(2\omega_n(T+\tau) + \phi)) \right)^2 \right. \\ \left. + \left(\frac{A}{4\omega_n} \left(-e^{-\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) - \frac{2}{\zeta} e^{-\zeta\omega_n\tau} + e^{-\zeta\omega_n\tau} \sin(2\omega_n(T+\tau) + \phi) + \frac{2}{\zeta} e^{-\zeta\omega_n(T+\tau)} \right) \right)^2 \right)^{\frac{1}{2}}$$

$$|F(\omega)| = \frac{A}{4\omega_n} \left(e^{-2\zeta\omega_n\tau} \cos^2(2\omega_n\tau + \phi) - 2e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \cos(2\omega_n\tau + \phi) \cos(2\omega_n(T+\tau) + \phi) \right. \\ \left. + e^{-2\zeta\omega_n(T+\tau)} \cos^2(2\omega_n(T+\tau) + \phi) + e^{-2\zeta\omega_n\tau} \sin^2(2\omega_n\tau + \phi) + \frac{2}{\zeta} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \right. \\ \left. - e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \sin(2\omega_n(T+\tau) + \phi) - \frac{2}{\zeta} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) \right. \\ \left. + \frac{2}{\zeta} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) + \frac{4}{\zeta^2} e^{-2\zeta\omega_n\tau} - \frac{2}{\zeta} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T+\tau) + \phi) - \frac{4}{\zeta^2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \right. \\ \left. - e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \sin(2\omega_n(T+\tau) + \phi) - \frac{2}{\zeta} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T+\tau) + \phi) \right. \\ \left. + e^{-2\zeta\omega_n\tau} \sin^2(2\omega_n(T+\tau) + \phi) + \frac{2}{\zeta} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T+\tau) + \phi) \right. \\ \left. - \frac{2}{\zeta} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) - \frac{4}{\zeta^2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} + \frac{2}{\zeta} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T+\tau) + \phi) \right. \\ \left. + \frac{4}{\zeta^2} e^{-2\zeta\omega_n(T+\tau)} \right)^{\frac{1}{2}}$$

Factor out $\frac{4}{\zeta^2}$ from the square root:

$$|F(\omega)| = \frac{A}{4\omega_n} \left(\frac{2}{\zeta} \right) \left(\frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \cos^2(2\omega_n\tau + \phi) - \frac{\zeta^2}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \cos(2\omega_n\tau + \phi) \cos(2\omega_n(T+\tau) + \phi) \right. \\ \left. + \frac{\zeta^2}{4} e^{-2\zeta\omega_n(T+\tau)} \cos^2(2\omega_n(T+\tau) + \phi) + \frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \sin^2(2\omega_n\tau + \phi) + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \right. \\ \left. - \frac{\zeta}{4} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \sin(2\omega_n(T+\tau) + \phi) - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) \right. \\ \left. + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) + e^{-2\zeta\omega_n\tau} - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T+\tau) + \phi) - e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \right. \\ \left. - \frac{\zeta}{4} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \sin(2\omega_n(T+\tau) + \phi) - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T+\tau) + \phi) \right. \\ \left. + \frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \sin^2(2\omega_n(T+\tau) + \phi) + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T+\tau) + \phi) \right. \\ \left. - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) - e^{-2\zeta\omega_n\tau - \zeta\omega_n T} + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T+\tau) + \phi) \right. \\ \left. + e^{-2\zeta\omega_n(T+\tau)} \right)^{\frac{1}{2}}$$

Eliminate ζ^2 terms:

$$\begin{aligned}
|F(\omega)| = \frac{A}{2\zeta\omega_n} & \left(\frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \cos^2(2\omega_n\tau + \phi) - \frac{\zeta^2}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \cos(2\omega_n\tau + \phi) \cos(2\omega_n(T + \tau) + \phi) \right. \\
& + \frac{\zeta^2}{4} e^{-2\zeta\omega_n(T + \tau)} \cos^2(2\omega_n(T + \tau) + \phi) + \frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \sin^2(2\omega_n\tau + \phi) + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \\
& - \frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \sin(2\omega_n(T + \tau) + \phi) - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) \\
& + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) + e^{-2\zeta\omega_n\tau} - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T + \tau) + \phi) - e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \\
& - \frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) \sin(2\omega_n(T + \tau) + \phi) - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T + \tau) + \phi) \\
& + \frac{\zeta^2}{4} e^{-2\zeta\omega_n\tau} \sin^2(2\omega_n(T + \tau) + \phi) + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) \\
& - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) - e^{-2\zeta\omega_n\tau - \zeta\omega_n T} + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) \\
& \left. + e^{-2\zeta\omega_n(T + \tau)} \right)^{\frac{1}{2}}
\end{aligned}$$

Resulting in:

$$\begin{aligned}
|F(\omega)| = \frac{A}{2\zeta\omega_n} & \left(\frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) + e^{-2\zeta\omega_n\tau} \right. \\
& - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T + \tau) + \phi) - e^{-2\zeta\omega_n\tau - \zeta\omega_n T} - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T + \tau) + \phi) \\
& + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) - \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) - e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \\
& \left. + \frac{\zeta}{2} e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) + e^{-2\zeta\omega_n(T + \tau)} \right)^{\frac{1}{2}}
\end{aligned}$$

Combine like terms, color-coded:

$$\begin{aligned}
|F(\omega)| = \frac{A}{2\zeta\omega_n} & \left(\zeta e^{-2\zeta\omega_n\tau} \sin(2\omega_n\tau + \phi) - \zeta e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n\tau + \phi) + e^{-2\zeta\omega_n\tau} \right. \\
& - \zeta e^{-2\zeta\omega_n\tau} \sin(2\omega_n(T + \tau) + \phi) - 2e^{-2\zeta\omega_n\tau - \zeta\omega_n T} + \zeta e^{-2\zeta\omega_n\tau - \zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) \\
& \left. + e^{-2\zeta\omega_n(T + \tau)} \right)^{\frac{1}{2}}
\end{aligned}$$

Rearranging by $T + \tau$, or τ terms:

$$\begin{aligned}
|F(\omega)| = \frac{A}{2\zeta\omega_n} & \left(e^{-2\zeta\omega_n\tau} + e^{-2\zeta\omega_n(T + \tau)} - 2e^{-2\zeta\omega_n\tau - \zeta\omega_n T} + (e^{-2\zeta\omega_n\tau} - e^{-2\zeta\omega_n\tau - \zeta\omega_n T}) \zeta \sin(2\omega_n\tau + \phi) \right. \\
& \left. + (e^{-2\zeta\omega_n\tau - \zeta\omega_n T} - e^{-2\zeta\omega_n\tau}) \zeta \sin(2\omega_n(T + \tau) + \phi) \right)^{\frac{1}{2}}
\end{aligned}$$

Factor out $e^{-2\zeta\omega_n\tau}$:

$$\begin{aligned}
|F(\omega)| = \frac{Ae^{-\zeta\omega_n\tau}}{2\zeta\omega_n} & \left(1 + e^{-2\zeta\omega_n T} - 2e^{-\zeta\omega_n T} + (1 - e^{-\zeta\omega_n T}) \zeta \sin(2\omega_n\tau + \phi) \right. \\
& \left. - (1 - e^{-\zeta\omega_n T}) \zeta \sin(2\omega_n(T + \tau) + \phi) \right)^{\frac{1}{2}}
\end{aligned} \tag{8}$$

The reference by Hammond gives:

$$|F(\omega)| = \frac{Ae^{-\zeta\omega_n\tau}}{2\zeta\omega_n} \left(1 + e^{-2\zeta\omega_n T} - 2e^{-\zeta\omega_n T} + (1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n\tau + \phi) - e^{-\zeta\omega_n T}(1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n(T + \tau) + \phi) \right)^{\frac{1}{2}}$$

An extra $e^{-\zeta\omega_n T}$ factor is present, but this is not evident in this derivation. Further work to evaluate this discrepancy is needed. Perhaps the initial cancellation of ζ^2 caused terms to be missed.

For convenience, write Eq. (8) as:

$$|F(\omega)| = \frac{Ae^{-\zeta\omega_n\tau}}{2\omega_n} \sqrt{\frac{1 + f(\zeta)}{\zeta^2}} \quad (9)$$

Where,

$$f(\zeta) = e^{-2\zeta\omega_n T} - 2e^{-\zeta\omega_n T} + (1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n\tau + \phi) - (1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n(T + \tau) + \phi)$$

Take the natural log of Eq. (9):

$$\ln|F(\omega)| = -\zeta\omega_n\tau + \ln\left(\frac{A}{2\omega_n}\right) + \frac{1}{2}\ln\left(\frac{1 + f(\zeta)}{\zeta^2}\right) \quad (10)$$

Last term can be expanded in a Maclaurin series, which is a Taylor series centered about zero:

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^n(0)}{n!}x^n + \dots$$

$$f(x) = \frac{1}{2}\ln\left(\frac{1 + f(\zeta)}{\zeta^2}\right)$$

$$f(0) = \frac{1}{2}\ln\left(\frac{1 + f(0)}{0}\right) = \frac{0}{0}$$

Use L'hospital's rule?

$$\lim(\zeta \rightarrow \infty) \frac{f'(\zeta)}{2\zeta} = 0$$

First derivation:

$$f'(\zeta) = -2\omega_n T e^{-2\zeta\omega_n T} + 2\omega_n T e^{-\zeta\omega_n T} + \sin(2\omega_n\tau + \phi) - e^{-\zeta\omega_n T} \sin(2\omega_n\tau + \phi) + \zeta\omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n\tau + \phi) - \sin(2\omega_n(T + \tau) + \phi) + e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) - \zeta\omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi)$$

$$f'(0) = -2\omega_n T + 2\omega_n T + \sin(2\omega_n\tau + \phi) - \sin(2\omega_n\tau + \phi) - \sin(2\omega_n(T + \tau) + \phi) + \sin(2\omega_n(T + \tau) + \phi) = 0$$

Second derivative:

$$f''(\zeta) = 4(\omega_n T)^2 e^{-2\zeta\omega_n T} - 2(\omega_n T)^2 e^{-\zeta\omega_n T} + \omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n\tau + \phi) - e^{-\zeta\omega_n T} \sin(2\omega_n\tau + \phi) + \zeta\omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n\tau + \phi) - \omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) - \omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) + \zeta(\omega_n T)^2 e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi)$$

$$f''(0) = 4(\omega_n T)^2 - 2(\omega_n T)^2 + \omega_n T \sin(2\omega_n\tau + \phi) - \sin(2\omega_n\tau + \phi) - \omega_n T \sin(2\omega_n(T + \tau) + \phi) - \omega_n T \sin(2\omega_n(T + \tau) + \phi)$$

$$f''(0) = 2(\omega_n T)^2 + (\omega_n T - 1) \sin(2\omega_n\tau + \phi) - 2\omega_n T \sin(2\omega_n(T + \tau) + \phi)$$

Third derivative:

$$\begin{aligned}
f'''(\zeta) &= -8(\omega_n T)^3 e^{-2\zeta\omega_n T} + 2(\omega_n T)^3 e^{-\zeta\omega_n T} - (\omega_n T)^2 e^{-\zeta\omega_n T} \sin(2\omega_n \tau + \phi) + \omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n \tau + \phi) \\
&\quad + \omega_n T e^{-\zeta\omega_n T} \sin(2\omega_n \tau + \phi) - \zeta(\omega_n T)^2 e^{-\zeta\omega_n T} \sin(2\omega_n \tau + \phi) \\
&\quad + (\omega_n T)^2 e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) + (\omega_n T)^2 e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) \\
&\quad + (\omega_n T)^2 e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) - \zeta(\omega_n T)^3 e^{-\zeta\omega_n T} \sin(2\omega_n(T + \tau) + \phi) \\
f'''(0) &= -8(\omega_n T)^3 + 2(\omega_n T)^3 - (\omega_n T)^2 \sin(2\omega_n \tau + \phi) + \omega_n T \sin(2\omega_n \tau + \phi) + \omega_n T \sin(2\omega_n \tau + \phi) \\
&\quad + (\omega_n T)^2 \sin(2\omega_n(T + \tau) + \phi) + (\omega_n T)^2 \sin(2\omega_n(T + \tau) + \phi) + (\omega_n T)^2 \sin(2\omega_n(T + \tau) + \phi) \\
f'''(0) &= -6(\omega_n T)^3 + (2\omega_n T - (\omega_n T)^2) \sin(2\omega_n \tau + \phi) + 3(\omega_n T)^2 \sin(2\omega_n(T + \tau) + \phi)
\end{aligned}$$

Substitute into Maclaurin series:

$$\begin{aligned}
f(\zeta) &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots \\
f(0) &= -1 \\
f'(0) &= 0 \\
f''(0) &= 2(\omega_n T)^2 + (\omega_n T - 1) \sin(2\omega_n \tau + \phi) - 2\omega_n T \sin(2\omega_n(T + \tau) + \phi) \\
f'''(0) &= -6(\omega_n T)^3 + (2\omega_n T - (\omega_n T)^2) \sin(2\omega_n \tau + \phi) + 3(\omega_n T)^2 \sin(2\omega_n(T + \tau) + \phi) \\
f(\zeta) &= -1 + \left((\omega_n T)^2 + \frac{\omega_n T - 1}{2} \sin(2\omega_n \tau + \phi) - \omega_n T \sin(2\omega_n(T + \tau) + \phi) \right) \zeta^2 \\
&\quad + \left(-(\omega_n T)^3 + \frac{2\omega_n T - (\omega_n T)^2}{6} \sin(2\omega_n \tau + \phi) + \frac{(\omega_n T)^2}{2} \sin(2\omega_n(T + \tau) + \phi) \right) \zeta^3
\end{aligned}$$

Substitute back into amplitude equation:

$$\begin{aligned}
\ln|F(\omega)| &= -\zeta\omega_n \tau + \ln\left(\frac{A}{2\omega_n}\right) \\
&\quad + \frac{1}{2} \ln\left((\omega_n T)^2 + \frac{\omega_n T - 1}{2} \sin(2\omega_n \tau + \phi) - \omega_n T \sin(2\omega_n(T + \tau) + \phi) \right. \\
&\quad \left. + \zeta\left(-(\omega_n T)^3 + \frac{2\omega_n T - (\omega_n T)^2}{6} \sin(2\omega_n \tau + \phi) + \frac{(\omega_n T)^2}{2} \sin(2\omega_n(T + \tau) + \phi) \right) \right) \\
\ln|F(\omega)| &= -\zeta\omega_n \tau + \ln\left(\frac{A}{2\omega_n}\right) \\
&\quad + \frac{1}{2} \ln\left((\omega_n T)^2 + \frac{\omega_n T - 1}{2} \sin(2\omega_n \tau + \phi) - \omega_n T \sin(2\omega_n(T + \tau) + \phi) \right. \\
&\quad \left. - \zeta\omega_n T \left((\omega_n T)^2 - \left(\frac{2 - \omega_n T}{6}\right) \sin(2\omega_n \tau + \phi) - \frac{\omega_n T}{2} \sin(2\omega_n(T + \tau) + \phi) \right) \right) \tag{11}
\end{aligned}$$

Third term in Hammond's reference, assuming it is the first 4 terms of the series:

$$\begin{aligned}
&\dots + \frac{1}{2} \ln\left((\omega_n T)^2 + \omega_n T (\sin(2\omega_n \tau + \phi) - \sin(2\omega_n(T + \tau) + \phi)) \right. \\
&\quad \left. - \frac{\zeta}{4} \omega_n T \left(\frac{2\omega_n T + \sin(2\omega_n \tau + \phi) - 3 \sin(2\omega_n(T + \tau) + \phi)}{\omega_n T + \sin(2\omega_n \tau + \phi) - \sin(2\omega_n(T + \tau) + \phi)} \right) \right)
\end{aligned}$$

Assuming T is an integral multiple of the basic period of oscillation:

$$T = \frac{2\pi N}{\omega_n}, \text{ for } N = 1, 2, 3, \dots$$

Eq. (11) becomes:

$$\begin{aligned}\ln|F(\omega)| &= -\zeta\omega_n\tau + \ln\left(\frac{A}{2\omega_n}\right) \\ &\quad + \frac{1}{2}\ln\left((2\pi)^2 + \frac{2\pi-1}{2}\sin(2\omega_n\tau + \phi) - 2\pi\sin(2\omega_n(\tau) + \phi)\right. \\ &\quad \left.- \zeta 2\pi\left((2\pi)^2 - \left(\frac{2-2\pi}{6}\right)\sin(2\omega_n\tau + \phi) - \frac{2\pi}{2}\sin(2\omega_n(\tau) + \phi)\right)\right) \\ \ln|F(\omega)| &= -\zeta\omega_n\tau + \ln\left(\frac{A}{2\omega_n}\right) \\ &\quad + \frac{1}{2}\ln\left((2\pi)^2 + \frac{2\pi-1}{2}\sin(2\omega_n\tau + \phi) - 2\pi\sin(2\omega_n\tau + \phi)\right. \\ &\quad \left.- \zeta 2\pi\left((2\pi)^2 - \left(\frac{2-2\pi}{6}\right)\sin(2\omega_n\tau + \phi) - \frac{2\pi}{2}\sin(2\omega_n\tau + \phi)\right)\right)\end{aligned}$$

So if we were to calculate the Fourier amplitude of $A = 10, \zeta = 0.01, \omega_n = 5, \tau = 0, N = 50, T = \frac{2\pi(25)}{\omega_n} = \frac{50\pi}{\omega_n}$

$$|F(\omega)| = \frac{Ae^{-\zeta\omega_n\tau}}{2\omega_n} \sqrt{\frac{1+f(\zeta)}{\zeta^2}}$$

$$f(\zeta) = e^{-2\zeta\omega_n T} - 2e^{-\zeta\omega_n T} + (1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n\tau + \phi) - (1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n(T + \tau) + \phi)$$

$$\begin{aligned}f(0.01) &= e^{-\pi} - 2e^{-\frac{\pi}{2}} - \left(1 - e^{-\frac{\pi}{2}}\right)0.01 \sin(100\pi) \\ &= -0.37254523443775156731949350249822952879202757345290613316...\end{aligned}$$

$$|F(\omega)| = \frac{10}{10} \sqrt{\frac{1+f(0.01)}{0.0001}} = 79.212$$

So if we were to calculate the Fourier amplitude of $A = 10, \zeta = 0.01, \omega_n = 5, \tau = 0, N = 1, T = \frac{2\pi}{\omega_n}$

$$|F(\omega)| = \frac{Ae^{-\zeta\omega_n\tau}}{2\omega_n} \sqrt{\frac{1+f(\zeta)}{\zeta^2}}$$

$$f(\zeta) = e^{-2\zeta\omega_n T} - 2e^{-\zeta\omega_n T} + (1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n\tau + \phi) - (1 - e^{-\zeta\omega_n T})\zeta \sin(2\omega_n(T + \tau) + \phi)$$

$$f(0.01) = e^{-400\pi} - 2e^{-200\pi} - (1 - e^{-200\pi})0.01 \sin(4\pi)$$

$$|F(\omega)| = \frac{10}{10} \sqrt{\frac{1+f(0.01)}{0.0001}} \approx 100$$

Windowing; Multiplication

Multiplying a sine wave by a Hanning Window gives a new signal that contracts at the ends. What is the new equation of the signal?

Sine wave signal:

$$A * \sin \omega t$$

$$A = 10, \omega = 5 \text{ Hz or } 10\pi \frac{\text{rad}}{\text{s}}$$

From $t = 0: 1.9 \text{ s}$

Hanning Window signal:

$$\sin^2\left(\frac{\pi}{1.9}t\right)$$

Multiply:

$$10 \sin(10\pi t) \sin^2\left(\frac{\pi}{1.9}t\right)$$

Trig identity:

$$\sin(a) * \sin(b) = \frac{1}{2}(\cos(a - b) - \cos(a + b))$$

$$\sin(a) * \cos(b) = \frac{1}{2}(\sin(a - b) + \sin(a + b))$$

Then

$$\begin{aligned} & 10 \sin(10\pi t) \sin\left(\frac{\pi}{1.9}t\right) \sin\left(\frac{\pi}{1.9}t\right) \\ & 10 \sin(10\pi t) \frac{1}{2}\left(1 - \cos\left(\frac{2\pi}{1.9}t\right)\right) \\ & 5\left(\sin(10\pi t) - \sin(10\pi t) \cos\left(\frac{2\pi}{1.9}t\right)\right) \\ & 5\left(\sin(10\pi t) - \frac{1}{2}\left(\sin\left(10\pi t - \frac{2\pi}{1.9}t\right) + \sin\left(10\pi t + \frac{2\pi}{1.9}t\right)\right)\right) \\ & -\frac{5}{2}\left(-2 \sin(10\pi t) + \sin\left(10\pi t - \frac{2\pi}{1.9}t\right) + \sin\left(10\pi t + \frac{2\pi}{1.9}t\right)\right) \\ & -\frac{5}{2}\left(\sin\left(\frac{170\pi}{19}t\right) - 2 \sin(10\pi t) + \sin\left(\frac{210\pi}{19}t\right)\right) \end{aligned}$$

New frequencies $\frac{170\pi}{19}$ and $\frac{210\pi}{19}$ are generated. An FFT initially gives you amplitude and phase through complex numbers, which will display these new frequencies in question as zeroes. However, once an absolute value is taken on the FFT values to get magnitude of displacement, these new frequencies are not apparent.

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