ABSTRACT

Title of dissertation: FINANCIAL MARKET ACCESS AND INTERNATIONAL RISK SHARING
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In the past three decades the stock of assets and liabilities of developing countries measured as a ratio of GDP has tripled. It is commonly believed that an increase in opportunities for diversifying risk allows more consumption smoothing. However, the data show that volatility of consumption in developing countries has persisted at high levels, showing only an average 11 percent decrease from the 60’s to the 90’s. This paper aims to explain this phenomenon by investigating to what extent domestic financial frictions related to heterogeneous home financial market access can help resolve the quantitative discrepancy between the change in volatility of consumption in the data and that predicted by a model economy that allows for higher degrees of financial integration. We show that in an endowment economy, if only 40 percent of the population has access to financial markets, full access to insuring country risk in international markets would reduce consumption volatility by 24 percent. In a world in which all agents have equal access to financial markets, the predicted impact of integration with world markets would be a much higher drop of 49 percent. The absence of a forward
international market for the nontradable good and the inability of some agents to access a forward market for the tradable good opens a new role for the spot market of tradable and nontradable goods: individuals excluded from financial markets use the goods market to attenuate tradable risk, which is reflected in higher consumption volatility for these agents following international financial integration. In an extended version of the model allowing for production, opening the economy brings even less change in consumption volatility.

Later, we investigate whether limited domestic financial market participation can break the theoretical result found by Backus and Smith (1993) that consumption ratios and the real exchange rate are perfectly correlated for pairs of countries. We consider a two-country world inhabited by individuals with heterogeneous access to financial markets in one country and full access in the other. Both countries are endowed with tradable and nontradable goods. We find that consumption ratios for individuals with access to financial markets are perfectly correlated with the real exchange rate across countries but the aggregate consumption ratio and the real exchange rate might not be perfectly correlated across countries.
FINANCIAL MARKET ACCESS
AND INTERNATIONAL RISK SHARING

by

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Dedication

To my mother, Vera Dutra.
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Chapter 1

Introduction

The uncertainty involved in economic decisions has opened a field of study that investigates how economic agents diversify risk. In international economics, this field has been applied to understand how countries share risk. The advent of technology followed by a process of financial deregulation has led to a marked increase in international asset trade. In this context, with intensification of financial globalization, interest in international risk sharing has increased.

The terms financial globalization and financial integration are used to refer to an increase of both foreign holdings of domestic assets and domestic holdings of foreign assets. Partially, these exchanges are attributed to financial liberalization, an umbrella terminology for a group of policy measures that lower the barriers for financial trade. Not surprisingly, some countries have adopted liberalizing measures in similar periods and the academic literature has focused on events including capital account liberalization, exchange rate policy, domestic banking system and stock market deregulation.\(^1\) Lately, these events have been considered in conjunction, popularizing the use of the expression financial integration.

Given the vast combination of measures that defines the financial openness

\(^1\)See Kaminsky and Schmukler (2008) for a definition of financial liberalization.
of a country, it is a complex task to ascertain the stage of financial liberalization of each economy. Firstly, the uncertainty surrounding the costs and benefits of capital flows has made financial liberalization a learning-by-doing process, with alternating periods of relaxation and tightening of liberalizing policy measures. An example of such policy alteration is discussed in Cardoso and Goldfajn (1998). They build a measure of capital controls for Brazil based on a linear combination of changes in taxes and restrictions on capital flows occurring between 1983 and 1996. The index shows constant shifts from periods of dominant liberalizing measures and periods of stronger regulation. Secondly, even though financial liberalization potentially creates more scope for financial trade, it does not guarantee financial integration per se. The debate over using de jure or de facto measures of financial integration follows from this observation. The measures seem to differ significantly among countries, impacting the results of recent studies (see Kose et al. (2006)).

On the empirical side, a large literature explores the effect of financial liberalization on economic growth. The direct effect of financial liberalization relates to its potential of allowing capital to be allocated to places where returns are higher, fomenting investment, production and growth. Two broad surveys of the literature, Eichengreen (2001) and Kose et al. (2006), find respectively no strong support for capital account liberalization and financial integration impacting growth. The most recent literature has shifted attention to the indirect benefits of financial liberalization. These relate to the spillover effects of financial liberalization, such as the potential development of domestic financial markets,
domestic institutions and the stability of macroeconomic policies. The evidence points to the existence of financial and institutional development threshold levels. Without achieving the required levels, a country seems unlikely to benefit from financial integration (see Kose et al. (2009)).

On the theoretical side, studies on financial integration have aimed at measuring quantitatively the potential welfare gains from financial integration. Gourinchas and Jeanne (2006) do so in a pure neoclassical framework, in order to quantify the gains of capital reallocation from capital-abundant to capital-scarce economies. The results for a calibrated non-OECD economy shows that the gains appear to be rather small, equivalent to a 1 percent permanent increase in domestic consumption. Departing from perfect markets, Mendoza et al. (2007) show that in the presence of different degrees of market incompleteness across countries (modeled as differences in borrowing constraints across countries), there are adverse welfare effects for the less financially developed countries. As in the empirical literature, frictions in the financial markets are suggested as an explanation for the low or negative gains accrued from financial integration. Bai and Zhang (2009) suggest that the problem is due to relatively low capital flows relative to what would be required in a perfect world to allow for higher risk sharing. In their model, capital flows would increase by 6 times if default risk on debt contracts were eliminated.

This dissertation takes two approaches to the study of international risk sharing. The first motivation is based on the observed disparity of consumption volatility between industrialized and developing economies. The fact that the
discrepancy is still in place despite financial integration, leads us to develop a theoretical framework that can help in explaining it. Chapter 2 is devoted to presenting this discussion. We then take a step back and look at a more general anomaly observed in the data in both industrialized and developing economies. As Backus and Smith (1993) have identified, the observed correlation of the real exchange rate and cross country consumption ratios is at odds with a world that shares risk perfectly. In Chapter 3 we present a model that helps in elucidating this puzzle.
Chapter 2

Financial Integration with Heterogeneous Home Financial Market Access

2.1 Introduction

The volatility of consumption in developing countries has remained high for the past four decades. In Table 2.1 we provide a summary of the data for a group of 23 emerging markets. The first column of Table 2.1 uses the estimates of Lane and Milesi-Ferretti (2007) of external assets and liabilities to calculate the ratio of trade of foreign assets to GDP. As the table shows, this ratio has increased by approximately three times from the 70’s to the 90’s. In the second column we present the volatility of consumption growth for the same group of countries.\(^1\)

Despite the significantly increased amount of financial trade, consumption growth volatility has been persistently high for the past four decades. The average standard deviation of annual consumption growth for the entire period is

\(^1\)Consumption volatility is the standard deviation of annual per capita consumption growth over 1960-2000. Real GDP is average adjusted gross domestic product converted to international dollars using Purchasing Parity rates over 1960-2000. In the last column of Table 2.1, we first compute the ratio of volatility of consumption to volatility of output for each country and then present the cross country average. Data is from Heston, Summers and Aten (2006), Penn World Table Version 6.2.
around 4.7 percent, which is significantly higher than the average for more advanced economies. This fact can be seen in Figure 2.1, which presents data on consumption volatility and real GDP per capita for both industrialized and developing economies. For the industrialized economies we consider a group of 21 countries. Note that for industrialized economies, the average level of consumption volatility is 2.2 percent and the degree of dispersion around this number is relatively low across countries. In Table 2.1, the last column shows the ratio of consumption to output growth volatility. Even if we take into account volatility of consumption relative to output, there seems to have been little change in consumption smoothing during the three last decades, when financial integration intensified around the world.

This fact has been pointed out previously by Kose et al. (2003). These authors find that the median of the ratio of private and government consumption volatility to income volatility increased during the 90s for the same group of developing countries listed in Table 2.1. In a survey on the consequences of financial globalization, Kose et al. (2006) conclude “there is no evidence that financial globalization has delivered on the promised benefit of improved international risk sharing and reduced volatility of consumption”.

When looking more closely at emerging markets, one observes that domestic financial markets are poorly developed. Table 2.2 shows the number of deposit accounts per capita for both emerging markets and industrialized economies, using aggregate indicators collected from bank regulators provided by Beck et al. (2007). For the former these figures are much lower, indicating that a significant
Table 2.1: Financial Integration and Consumption Growth Volatility
in Emerging Markets

| Decade | Foreign assets /GDP + Volatility of Volatility of Volatility of consumption/ Consumption output output output |
|--------|---------------------------------|------------------|------------------|------------------|------------------|
|        | Foreign liabilities/GDP         | Volatility of consumption | Volatility of output | Volatility of consumption/ output |
| 60’s    | -                               | 4.95 (0.45)       | 3.78 (0.35)       | 1.49              |
| 70’s    | 0.56                            | 4.70 (0.57)       | 3.77 (0.38)       | 1.23              |
| 80’s    | 1.20                            | 4.60 (0.50)       | 3.93 (0.41)       | 1.28              |
| 90’s    | 1.66                            | 4.45 (0.39)       | 3.71 (0.31)       | 1.28              |

Sources: Lane and Milesi-Ferretti (2007); Penn World Table. Note: Averages for the group of 23 countries listed below; standard errors are reported in parentheses. Countries: Argentina, Brazil, Chile, China, Colombia, Egypt, Hong Kong SAR, India, Indonesia, Israel, Jordan, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Singapore, South Africa, Thailand, Turkey, Venezuela.

part of the population does not benefit from financial development. The evidence on bank accounts suggests that some agents in these economies do not have access to domestic financial markets. Presumably these agents also lack access to international markets as well.

The figures in Table 2.2 should be seen only as suggestive. It is possible that some individuals hold more than one bank account; the ratio of bank accounts per capita in most industrialized economies exceeds one. Also, some less developed economies are seen as tax havens and might attract deposits from foreigners. Although not a perfect measure of financial access, low ratios provide some evidence of low use of the financial system.
In this paper we explicitly take limited financial participation into account. The increase of international capital flows in the past decades is a result of the process of financial liberalization that, as described by in Kaminsky and Schmukler (2008), encompasses the liberalization of a country’s capital account, the domestic financial system and the stock market. Here we choose to model this process considering two extreme cases: first an economy in financial autarky and second an economy with a completely open capital account. We describe a small open economy that gains access to international financial markets and is allowed to trade securities that promise to pay one unit of the tradable good. However, agents within the economy have heterogeneous access to financial markets. Some agents have no access to financial markets irrespective of whether the economy is open to international financial markets or not.
We first illustrate the main mechanism of the model in an endowment economy. As in Lahiri et al. (2007), the fraction of agents with access to domestic financial markets is assumed to be constant and exogenously given. In this paper we abstract from idiosyncratic individual-specific shocks and compute the economy’s response to a country specific endowment shock. We then compute the impact of financial globalization on consumption volatility for different degrees

\(^2\)Townsend and Ueda (2006) explore the implications of having an endogenous rate of domestic financial market participation. Their goal is to evaluate how changes in individual wealth affect participation decisions in the financial markets and to assess the feedback from participation to economic growth.
of domestic financial market development.

In a small open economy with only tradable goods, individuals with access to financial markets would be perfectly able to insure. Since we assume an endowment process with country-specific shocks but no aggregate volatility at the world level, in this case consumption volatility would be zero for individuals with access to financial markets and equal to the endowment volatility for individuals with no access. However, the presence of nontradable goods makes markets incomplete because it is not possible to write international contracts that promise to deliver units of nontradable goods.

The combination of incomplete markets with heterogeneity of agents in their ability to access financial markets creates a new role for the spot market of goods, which is used as a way of channeling insurance for tradable consumption from individuals with financial market access to individuals with no access, in exchange for nontradable goods. In bad times the domestic supply of both tradables and nontradables shrinks and consumers with financial market access will have contracted to receive tradable goods from abroad. This will cause an increase in the domestic relative price of nontradable goods. Individuals with no financial market access benefit from the inflow of tradables from abroad, buying them cheap in the spot market in exchange for the more valuable good, nontradables. Since in good times the opposite occurs (that is, there is a positive trade balance and the price of nontradables goes down), the volatility of consumption of nontradable goods for individuals with no financial market access is greater than in autarky, and furthermore, this volatility increase dominates the drop in
the volatility of consumption for tradable goods, generating an increase in total volatility of consumption for agents that do not participate in financial markets.

In calibrating the model we use the share of the population that holds a bank account in developing countries as a proxy for the share of individuals who participate in financial markets. The average number for our sample is approximately 40 percent. For this degree of domestic financial development, we find that consumption volatility should be expected to decrease by 24 percent when the economy is opened to international financial trade. The consumption volatility of individuals with financial market access decreases with financial integration, which compensates for the increase in volatility experienced by individuals with no access.

Next, we examine a more realistic two sector model with production in order to make the results comparable with a standard small open economy business cycle model. The production economy exhibits two competing forces affecting consumption volatility relative to the endowment economy. First production in the two sectors might be adjusted solely by the reallocation of domestic resources within the small open economy. If there is an excess supply of nontradables relative to tradables, labor and capital can be reallocated from the nontradable sector to the tradable sector, potentially mitigating the relative decline in the nontradable price that induced higher volatility of nontradable consumption among agents without financial market access in the endowment economy. Second, resources are also mobile across countries. In good times, capital will flow from the rest of the world to the small open economy. This increases the volatility of do-
mestic tradable and nontradable output, contributing to an increase of consumption volatility of agents without financial market access, who cannot smooth this extra output risk using state contingent bonds. In a calibrated version of the model, we find that the second effect dominates. Moving from financial autarky to a financially integrated market reduces consumption volatility of agents with financial market access, but it increases consumption volatility of agents without access by roughly the same amount. As a result, integration produces only a 0.5 percent decline in consumption volatility. If all individuals had financial market access, consumption volatility would be expected to drop by 33 percent.

Some previous literature has attempted to explain high consumption volatility in developing countries and its relation to financial integration. In Leblebicioglu (2006) terms of trade shocks can lead to higher consumption volatility under financial integration relative to financial autarky. While all households in this economy have access to international and domestic financial markets, the owners of the non-traded sector cannot borrow abroad; therefore they need to borrow domestically from households to finance investment and production. Frictions arise due to contract enforceability problems that imply borrowing constrains for the non-traded sector. Levchenko (2005) considers an economy in which all agents have access to domestic financial markets but only a fraction of these can participate in the international markets. Because there is limited commitment, opening up to international markets also raises the volatility of consumption when risks are idiosyncratic. As such, frictions in both of these papers arise in the intermediation of funds from abroad to the segment of the domestic sector that has no
access to international markets. These papers do not consider the implications of heterogeneous domestic financial access.

2.2 Endowment Economy

We consider a small open economy inhabited by a large number of infinitely-lived consumers. In each period $t$ the economy experiences one of the finitely many events $s_t$. Denote by $s^t = (s_1, s_2, ..., s_n)$ the history of events up to and including period $t$. The probability at date 0 of any particular history $s^t$ is given by $\pi(s^t)$.

There are two types of goods: a tradable and a non-tradable. There is no production in the economy and all consumers are endowed with the same amount of tradable and non-tradable goods each period. The endowments of tradable and non-tradable goods are given by $y^T_t$ and $y^N_t$ respectively. The relative price of non-tradable goods in terms of tradable goods for every history $s^t$ is denoted by $p^N(s^t)$.

All households have identical preferences which can be described by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c^{iT}(s^t), c^{iN}(s^t)),$$

where $\beta < 1$ is the discount factor; $c^{ij}_t$ denotes consumption of individuals with access to capital markets if $i = 1$ or no access if $i = 2$; and consumption of tradables if $j = T$ or the non-tradable good if $j = N$. We assume that households are heterogeneous in their ability to trade assets. A measure $\chi$ of the individuals have access to capital markets ($i = 1$) while $(1 - \chi)$ are excluded from the financial
markets \((i = 2)\).

2.2.1 Consumers with access to capital markets

During each period, individuals with access to capital markets are allowed to trade one-period state-contingent assets \(B(s^{t+1})\). At each period \(t\), the asset sells at a price of \(Q(s^{t+1}/s^t)\) and promises to pay one unit of tradable good in period \(t+1\) if the history \(s^{t+1}\) is realized. The budget constraint at each state \(s^t\) is

\[
p^N(s^t)c^1N(s^t) + c^1T(s^t) + \sum_{s^{t+1}} Q(s^{t+1}/s^t)B(s^{t+1}) = B(s^t) + p^N(s^t)y^N(s^t) + y^T(s^t),
\]

and the transversality condition is given by

\[
\lim_{j \to \infty} \beta^j \pi(s^{t+j}/s^t) \frac{u_{c1T}(s^{t+j})}{u_{c1T}(s^t)} B(s^{t+j}) = 0.
\]

The problem faced by households of type 1 is then to maximize (2.1) subject to (2.2) and (2.3). Letting \(\lambda^1(s^t)\) denote the Lagrange multiplier on equation (2.2), the first order conditions are given by

\[
\pi(s^t)u_{c1T}(s^t) = \lambda^1(s^t),
\]

\[
\pi(s^t)u_{c1N}(s^t) = \lambda^1(s^t)p^N(s^t),
\]

\[
\lambda(s^t)Q(s^{t+1}/s^t) = \beta\lambda^1(s^{t+1}).
\]

If we combine equations (2.4) and (2.5), we arrive at the intratemporal efficiency condition that

\[
p^N(s^t) = \frac{u_{c1N}(s^t)}{u_{c1T}(s^t)}.
\]
From equations (2.4) and (2.6), we obtain the intertemporal condition

\[ Q(s^{t+1}/s^t) = \beta \pi(s^{t+1}/s^t) \frac{u_{c2T}(s^{t+1})}{u_{c1T}(s^t)}. \] (2.8)

### 2.2.2 Consumers with no access to capital markets

Households excluded from financial markets are able to trade only goods in the spot market. The budget constraint in each state \( s^t \) is

\[ p^N(s^t) c^{2N}(s^t) + c^{2T}(s^t) = p^N(s^t) y^N(s^t) + y^T(s^t). \] (2.9)

Notice that even though these individuals do not have access to the technology that allows them to trade future claims on tradable goods, they are able to trade goods at every point in time with households that have access to this technology.

The problem faced by households of type 2 is then to maximize (2.1) subject to (2.9). Letting \( \lambda^2(s^t) \) denote the Lagrange multiplier on equation (2.9), the first order conditions are given by

\[ \pi(s^t) u_{c2T}(s^t) = \lambda^2(s^t), \] (2.10)

\[ \pi(s^t) u_{c2N}(s^t) = \lambda^2(s^t) p^N(s^t). \] (2.11)

Combining equations (2.10) and (2.11), we arrive at

\[ p^N(s^t) = \frac{u_{c2N}(s^t)}{u_{c2T}(s^t)}. \] (2.12)
2.2.3 Equilibrium

2.2.3.1 Financial Autarky

Initially, the economy has no access to international capital markets (financial autarky). Since except for market access all consumers are homogeneous, the solution here is trivial: in every period each agent consumes its own endowment of the tradable and non-tradable good. Next we suppose the economy is open to international financial markets.

2.2.3.2 Financial Integration

Since this is a small economy, the prices of securities that promise to pay in terms of tradable goods are determined in the international market and in equilibrium are not affected by the decisions of the small country. We abstract from aggregate uncertainty and we assume that the world output is the same in all states of nature. This assumption will translate into actuarially fair securities (for reference, see Obstfeld and Rogoff (1996) and Lahiri et al. (2007)). That is, given $s^t$, for any histories $i$ and $j$ in $t+1$,

$$\frac{Q(s^{t+1}_i / s^t)}{Q(s^{t+1}_j / s^t)} = \frac{\pi(s^{t+1}_i / s^t)}{\pi(s^{t+1}_j / s^t)}. \tag{2.13}$$

Taking the ratio of (2.8) for any two histories $i$ and $j$ in $t+1$, we can conclude that the marginal utility of consumption for tradable goods will be equalized across states of nature for agents with financial market access:

$$u_{c,T}(s^{t+1}_j) = u_{c,T}(s^{t+1}_i).$$
Define $r$ as the risk free interest rate. By arbitrage the return on buying one unit of each state contingent bond must be the same as receiving the risk free interest rate. Therefore

$$(1 + r)^{-1} = \sum_{s^{t+1}} Q(s^{t+1}/s^t).$$

Summing over all states of nature in equation (2.13) and assuming that $\beta^{-1} = 1 + r$, we arrive at

$$\beta \pi(s^{t+1}/s^t) = Q(s^{t+1}/s^t). \tag{2.14}$$

Substituting the condition above into result (2.8), we conclude that for agents with financial market access, the marginal utility of consumption for tradable goods will be equalized over time,

$$u_{c_1T}(s^t) = u_{c_1T}(s^{t+1}).$$

We can therefore restate the first order condition (2.4) as

$$u_{c_1T}(s^t) = \psi, \quad \forall s^t. \tag{2.15}$$

The result that the marginal utility of consumption for tradables is smoothed over time and states is familiar in the context of complete markets. The difference here is due to the presence of nontradable goods. Since securities that promise to pay in terms of nontradable goods cannot be traded internationally, the price of these securities is not equalized across countries and therefore risk sharing is limited in this case. In equilibrium the consumption of nontradables in the domestic economy must be equal to its supply in every period:

$$\chi c_1^N + (1 - \chi)c_2^N = y_t^N. \tag{2.16}$$
After multiplying (2.2) by $\chi$ and (2.9) by $(1-\chi)$, we add up the result and use (2.16) to arrive at

$$
\chi \left[ \sum_{s^{t+1}} Q \left( \frac{s^{t+1}}{s^t} \right) B \left( \frac{s^{t+1}}{s^t} \right) \right] = y^T_t - \chi c^{iT}_t - (1-\chi)c^{2T}_t,
$$

(2.17)
or in terms of the trade balance

$$
\chi t_b_t = y^T_t - \chi c^{iT}_t - (1-\chi)c^{2T}_t.
$$

(2.18)

Let $lnA_t = [lnA^T_t, lnA^N_t]$. For simplicity we assume that output follows the following stochastic process

$$
y^T_t = A^T_t \bar{y}^T_t,
$$

(2.19)

$$
y^N_t = A^N_t \bar{y}^N_t,
$$

(2.20)

where, in matrix notation,

$$
\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim NIID(0, \Sigma_{\epsilon}).
$$

(2.21)

Here $\rho$ denotes a $2 \times 2$ matrix of autoregressive coefficients.

The utility function assumes the following form:

$$
u(c^T_t, c^N_t) = \frac{[c(c^T_t, c^N_t)]^{1-\sigma} - 1}{1-\sigma}
$$

where $c(c^T_t, c^N_t) = [\omega (c^T_t)^{-\eta} + (1-\omega)(c^N_t)^{-\eta}]^{-1/\eta}$, \( i = 1, 2 \).

The parameter $\sigma$ is the coefficient of constant relative risk aversion. The elasticity of substitution between consumption of tradables and non-tradables is given by $\frac{1}{1+\eta}$, and $\omega$ is the weighting factor.
**Definition** Taking \( p^N \) as given, the consumption based price index \( p^c \) is defined as the minimum expenditure \( \chi(c_1^T + p_t^N c_1^N) + (1 - \chi)(c_2^T + p_t^N c_2^N) = z \) such that the aggregate consumption \( c = \chi c(c_1^T, c_1^N) + (1 - \chi)c(c_2^T, c_2^N) = 1 \).

Using the above specification for the CES aggregator in the utility function, the consumption based price index is found to be

\[
p_t^c = \left[ \omega \frac{1}{1 + \eta} + (1 - \omega) \frac{1}{1 + \eta} (p_t^N)^{\frac{1 + \eta}{\eta}} \right]^{\frac{1 + \eta}{\eta}}.
\]  

(2.22)

We can now define the equilibrium for this economy.

**Definition** A competitive equilibrium is a set of processes

\[
\{ c_1^T, c_1^N, c_2^T, c_2^N, p_t^N, \lambda_t, t \} \}_{t=0}^{\infty}
\]

satisfying equations (2.7), (2.9), (2.12), (2.15), (2.16), (2.18), (2.19) and (2.20), given (3.3.1) and \( A_0 \).

We solve the above system of stochastic difference equations using a first order approximation around the steady state.\(^3\)

\(^3\)We solved for a second order approximation as well but since the impact on second moments was small, we report the results using a first order approximation.

\(^4\)We assume that the steady state of the financially integrated economy coincides with the steady state of the economy in financial autarky. Since this assumption makes individuals homogeneous in the steady state, we induce heterogeneity in the steady state by giving a higher fixed endowment to the individual of type one every period. This assumption generates different levels of consumption in steady state for the two types of individuals but keeps the income volatility of the two types the same. Asymmetry of consumption in the steady state guarantees that price effects are first order; otherwise, price effects become second order.
2.2.4 Model Calibration

To obtain a measure of domestic participation in financial markets, we take the average number of bank accounts per capita provided by Beck et al. (2007) and Claessens (2006) whenever data was available for the developing economies listed in Table 2.1. The first column of Table 2.3 presents data on deposit accounts per capita collected through surveys of bank regulators. The second column refers to data on the share of households with bank accounts based on household surveys. The last column displays the estimates obtained in Beck et al. (2007). Regardless of methodology, both the mean and median shares are around 0.4 so we initially set the share of financial market participants $\chi$ equal to 0.4 and then later provide some sensitivity analysis for changes in this parameter.

We parameterize the model above such that the ratio of output of tradables to non-tradables match the data for Mexico. The data ratio and parameters follow Mendoza and Uribe (2001), Mendoza (2005) and Schmitt-Grohé and Uribe (2003). We set $\bar{y}^T/\bar{y}^N = 0.648$. The elasticity of substitution is 0.830 which implies $\eta = 0.204$. The parameters $\omega = 0.342$ and $\sigma = 2$. The rate of time preference equals 0.960.

Since our goal here is to investigate a potential channel that could lead to a change in consumption volatility rather than explaining the level of volatility itself, we will set the endowment shock such that the total consumption volatility in the financial autarky model matches the average consumption volatility observed in the 60’s (see Table 2.1). The period predates the large increase in fi-
### Table 2.3: Share of Individuals with Bank Account

<table>
<thead>
<tr>
<th>Countries</th>
<th>Data of Survey 1 on Countries’ Regulators</th>
<th>Household Data 2</th>
<th>Predictions based 3 on Household Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.37</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>0.63</td>
<td>0.43</td>
<td>0.39</td>
</tr>
<tr>
<td>Chile</td>
<td></td>
<td></td>
<td>0.46</td>
</tr>
<tr>
<td>Colombia</td>
<td>0.61</td>
<td>0.41</td>
<td>0.39</td>
</tr>
<tr>
<td>China</td>
<td></td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td></td>
<td>0.48</td>
<td></td>
</tr>
<tr>
<td>Jordan</td>
<td>0.47</td>
<td></td>
<td>0.37</td>
</tr>
<tr>
<td>Malaysia</td>
<td></td>
<td></td>
<td>0.56</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.31</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>Pakistan</td>
<td>0.19</td>
<td>0.12</td>
<td>0.23</td>
</tr>
<tr>
<td>Peru</td>
<td>0.32</td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.30</td>
<td></td>
<td>0.36</td>
</tr>
<tr>
<td>Singapore</td>
<td></td>
<td></td>
<td>0.84</td>
</tr>
<tr>
<td>South Africa</td>
<td></td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td></td>
<td></td>
<td>0.59</td>
</tr>
<tr>
<td>Turkey</td>
<td></td>
<td></td>
<td>0.54</td>
</tr>
<tr>
<td>Venezuela</td>
<td>0.49</td>
<td></td>
<td>0.29</td>
</tr>
<tr>
<td>average</td>
<td>0.41</td>
<td>0.37</td>
<td>0.41</td>
</tr>
<tr>
<td>median</td>
<td>0.39</td>
<td>0.42</td>
<td>0.39</td>
</tr>
</tbody>
</table>

**Source:** Beck et al. (2007) and Claessens (2006)

1 Deposit account per capita; deposit account includes: checking accounts, savings accounts, time deposits and others. We excluded data on Chile, Singapore, Thailand and Turkey for data being too far off from household data and/or being considered offshore tax havens.

2 Share of household with checking or savings bank accounts.

3 Beck et al. (2007) calculate the predicted share of households with bank accounts by using the coefficients from the regression of number of deposits accounts per 1,000 people and the number of branches per 1,000km²
Table 2.4: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>0.342</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.204</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$r$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\rho_{11}$</td>
<td>0.42</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{21}$</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{22}$</td>
<td>0.42</td>
</tr>
<tr>
<td>$\Sigma_{11}^{11}$</td>
<td>0.045^2</td>
</tr>
<tr>
<td>$\Sigma_{12}^{12}$</td>
<td>0.045^2</td>
</tr>
<tr>
<td>$\Sigma_{21}^{21}$</td>
<td>0.045^2</td>
</tr>
</tbody>
</table>

financial asset trade and therefore will serve as a benchmark. We set $\psi$ such that the marginal utility of consumption of tradables is the same under financial autarky and financial integration in steady state. Table 2.4 summarizes the parameters.

In what follows, we will show the impulse responses of model variables to an endowment shock, which will help us provide some intuition for the model. We then compute second moments and their sensitivity to changes in the share of participation in financial markets $\chi$.

2.2.5 Results

In order to understand the impact of financial integration on the volatility of consumption, we need first to disentangle the impact of shocks on both types of consumers. Figure 2.2 illustrates the impulse response of the economy to a positive production shock of one unit. We assume that outputs of the tradable and non-tradable sectors are perfectly correlated.

According to equation 2.15, we know that after financial integration, indi-
individuals with access to financial markets will smooth marginal utility of consumption of tradable goods perfectly across all dates and states:

\[ u_{c_{1T},c_{1N}}(c_{1T}^T(s^t), c_{1N}^N(s^t)) = \psi, \quad \forall s^t. \]

Whether consumption of tradables and nontradables for individuals with access will covary positively or negatively in this model, depends on the sign of the partial-cross derivative of the utility function. Given the assumptions of CRRA utility over the composite good and the CES aggregator of the two goods, the intertemporal elasticity of substitution and the elasticity of substitution between the two goods are governed respectively by \( \frac{1}{\sigma} \) and \( \frac{1}{1+\eta} \). In fact, one can show that

\[
\text{sign}\{u_{c_{1T},c_{1N}}(.)\} = \text{sign}\left\{\frac{1}{\sigma} - \frac{1}{1+\eta}\right\}. \tag{2.23}
\]

The goods are said to be Edgeworth-Pareto complements if the partial-cross derivative of utility is greater than zero. In this case, the marginal utility of tradable goods is raised by an increase in nontradable consumption. Otherwise, the goods are Edgeworth-Pareto substitutes. Notice that the goods can be gross complements (elasticity of substitution between tradables and nontradables lower than 1) and Edgeworth-Pareto substitutes if \( \frac{1}{\sigma} \) is lower than \( \frac{1}{1+\eta} \).

This is the case in the benchmark model, in which is assumed that the intertemporal elasticity of substitution is equal to 0.5 while the elasticity of substitution between the two goods is 0.83. For individuals with access, an increase in the supply of nontradable goods will imply an increase in consumption of nontradable goods and by condition (2.23) a decrease in the marginal utility of
consumption of tradables. By the concavity of the utility function, a drop in the consumption of the tradable good will be required to increase the marginal utility of consumption of tradables such that the first order condition (2.15) is satisfied.

On the one hand we have that in good states of nature, individuals with financial market access will carry out their promise to pay units of tradable goods to foreigners, and the trade balance will be positive. At the same time, the excess supply of nontradable goods drives down the price of nontradables, which makes it worthwhile for individuals with no access to exchange consumption of tradable goods for cheap nontradable goods. 5

If goods were Edgeworth-complements, which could be achieved by assuming a lower elasticity of substitution between the two goods, the only qualitative difference in the impulse responses relative to Figure 2.2 would be that consumption of tradables for individuals with financial market access would increase. This would imply a lower trade surplus.

Next, we compute the variance-covariance matrix of the state and control variables of the system, letting the degree of financial market participation (measured by the parameter $\chi$) vary between zero (no participation) and one (equal access).

Figure 2.3 presents consumption volatility of different types of goods and individuals under financial autarky and financial integration, computing volatil-

---

5 The relationship between consumption of the two goods for the two types of individuals can be seen in Figure 2. In a first order approximation the relationship is governed by $\hat{c}^{1T} - \hat{c}^{1N} = \hat{c}^{2T} - \hat{c}^{2N}$, where a hat indicates deviations from the steady-state value.
Figure 2.2: Impulse response to a production shock

ity for all possible values of financial market participation. The first, second and third columns respectively refer to the consumption volatility of individuals with access (type 1), no access (type 2) and all individuals. The first, second and third rows respectively display the consumption volatility of tradables, nontradables and all goods. Therefore Figure 2.3(i) represents the aggregate volatility of consumption.

Consider now what happens to individuals of type 1. Since under financial integration these agents gain access to state contingent bonds, consumption volatility is always lower than it would be under financial autarky (figure 2.3(c)).
However it is worth noting that type 1 agents lose the ability to smooth consumption of non-tradable goods as more individuals gain access (Figure 2.3(b)). In the extreme case in which all individuals gain access ($\chi = 1$), Figure 2.3(b) shows us that the volatility of consumption of non-tradable goods is the same as under autarky. At this point, there is no additional gains from trading further contingent bonds in terms of tradable goods to smooth the consumption of nontradables.

As for individuals with no access, total consumption volatility is slightly greater under financial integration than under financial autarky as shown in Figure 2.3(f). This result comes from the fact that consumption volatility of non-
Figure 2.4: From financial autarky to financial integration

tradable goods increases despite the decrease in the consumption volatility of tradable goods. When $\chi = 0$, individuals with no access behave as if they lived in an economy in autarky; they are unable to trade goods with agents that have ability to insure against shocks and therefore consumption volatility is the same under F.I. and F.A.

Figure 2.4 summarizes the results displayed on the third row of Figure 2.3 for the financially integrated economy. We can see that total consumption in the economy becomes less volatile after financial integration occurs. For a degree of market participation of 0.4, the decrease in aggregate consumption volatility is
equal to 24 percent. Going back to the data presented on Table 2.1, the average consumption volatility of developing countries has dropped from 4.95 percent per year in the 60s to 4.45 percent in the 90s, a decline of 11 percent, despite a similar average output volatility for the two decades. This fact can be contrasted with the expected drop of 49 percent that the model would predict assuming that domestic markets are fully developed, given by $\chi = 1$.

In Table 2.5 we assess the sensitivity of the results to changes in the elasticity of substitution between goods. The numbers express the percentage change in consumption volatility from financial autarky to financial integration for different levels of financial market participation and for different levels of the elasticity of substitution between tradables and nontradables.

Table 2.5: Sensitivity analysis for different elasticity of substitution between tradables and nontradables

<table>
<thead>
<tr>
<th>$\chi$</th>
<th>$ES = .17$</th>
<th>$ES = .44$</th>
<th>$ES = .76$</th>
<th>$ES = .83$</th>
<th>$ES = .99$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.05</td>
<td>-0.07</td>
<td>-0.07</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
<tr>
<td>$\chi = .1$</td>
<td>-0.12</td>
<td>-0.20</td>
<td>-0.20</td>
<td>-0.24</td>
<td>-0.26</td>
</tr>
<tr>
<td>$\chi = .4$</td>
<td>-0.15</td>
<td>-0.28</td>
<td>-0.28</td>
<td>-0.38</td>
<td>-0.40</td>
</tr>
<tr>
<td>$\chi = .7$</td>
<td>-0.16</td>
<td>-0.33</td>
<td>-0.33</td>
<td>-0.49</td>
<td>-0.53</td>
</tr>
<tr>
<td>$\chi = 1$</td>
<td>-0.16</td>
<td>-0.33</td>
<td>-0.33</td>
<td>-0.53</td>
<td>-0.53</td>
</tr>
</tbody>
</table>

Note: $ES$ denotes the elasticity of substitution between tradable and nontradable goods where $ES = \frac{1}{\eta + 1}$. Here we compute the change in consumption volatility that occurs when the economy moves from autarky to financial integration.

28
Most existing empirical studies suggest that the elasticity of substitution for developing countries is between .4 and .83. As the complementarity between goods increases, the impact of integration on the volatility of consumption declines because agents prefer to let tradables and nontradables comove positively following endowment shocks, leaving less scope for consumption smoothing. In the appendix we show the impact of integration on disaggregated consumption volatility for different elasticities of substitution.

We next turn to an economy with production.

2.3 Two-Sector Production Economy

We extend our model to incorporate production in a two-sector economy. The aim is to verify if the mechanism we identified in the last section is still in place when allowing the supply side of the economy to adjust. In addition, allowing for production will enable us to establish comparisons between our model and RBC models.

2.3.1 Firms

Suppose that there are two types of goods that can be produced: tradable and nontradables. We assume that factors of production are homogeneous and can be perfectly reallocated between the two sectors. The specification of the tradable and nontradable sectors will have a direct impact on the dynamics of the

---

6see Mendoza (2005) for discussion
relative price of nontradable goods. The assumption that capital is homogeneous in the two sectors, as opposed to assuming that capital is fixed in the nontradable sector (another commonly assumption in the literature), will assure a similar dynamics for both tradable and nontradable output and this will be reflected on the adjustment process of the relative price of nontradable goods. Denote $l^j$ and $k^j$ as labor and capital in sector $j$. Capital goods are considered to be tradable goods.

In equilibrium, this assumption Production in each sector is given by

$$y^T_t = A^T_t (k^T_t)^{\alpha_T} (l^T_t)^{1-\alpha_T},$$  \hspace{1cm} (2.24)

$$y^N_t = A^N_t (k^N_t)^{\alpha_N} (l^N_t)^{1-\alpha_N},$$  \hspace{1cm} (2.25)

where $A^j_t$ is the productivity shock in sector $j$. Markets for both goods are perfectly competitive. A representative firm maximizes aggregate profits $\pi_t = y^T_t + p^N_t y^N_t - w_t l_t - r_t k_t$ subject to (2.24), (2.25) and

$$l^T_t + l^N_t = l_t,$$  \hspace{1cm} (2.26)

$$k^T_t + k^N_t = k_t.$$  \hspace{1cm} (2.27)

The first order conditions for the firm problem are

$$\alpha_T A^T_t (k^T_t)^{\alpha_T - 1} (l^T_t)^{1-\alpha_T} = r_t,$$  \hspace{1cm} (2.28)

$$(1 - \alpha_T) A^T_t (k^T_t)^{\alpha_T} (l^T_t)^{-\alpha_T} = w_t,$$  \hspace{1cm} (2.29)

$$p^N_t \alpha_N A^N_t (k^N_t)^{\alpha_N - 1} (l^N_t)^{1-\alpha_N} = r_t,$$  \hspace{1cm} (2.30)

$$p^N_t (1 - \alpha_N) A^N_t (k^N_t)^{\alpha_N} (l^N_t)^{-\alpha_N} = w_t.$$  \hspace{1cm} (2.31)

Equalization of the marginal products of capital and labor across sectors
leads to:

\[ p_t^N = \frac{A_t^T}{A_t^N} \left( \frac{\alpha^T}{\alpha^N} \right)^{\alpha_T} \left( \frac{1 - \alpha^T}{1 - \alpha^N} \right)^{(1-\alpha_T)} \left( \frac{k_t^N}{k_t^T} \right)^{(\alpha_T - \alpha_N)} \]

The above equilibrium condition helps us understand the behavior of the relative price of the nontradable good. The assumption that firms can freely reallocate capital and labor between sectors translates into a frictionless supply adjustment, in which the gap of the capital factor share across the tradable and nontradable sector governs the impact of the change in the capital-labor ratio on the relative price. A production economy with perfect reallocation can then be seen as a polar opposite case of the endowment economy, in which supply conditions were given exogenously.

2.3.2 Households

Consumers work and are paid a wage \( w_t \) for their labor. Labor generates disutility. Households still differ in their ability to access financial markets. Consumers with access to capital markets own capital and rent it to firms at a rate \( r_t^K \). Capital depreciates at a rate \( \delta \), and gross investment is given by

\[ i_t = k_{t+1} - (1 - \delta)k_t. \tag{2.32} \]

We assume that there are costs for adjusting the capital stock, given by the function \( \Phi(.) = \frac{\Phi(0)}{2}(k_{t+1} - k_t)^2 \), where \( \Phi(0) = \Phi'(0) = 0^7 \).

\(^7\)Here we follow Schmitt-Grohé and Uribe (2003). This formulation ensures that in the steady state there are no costs for adjusting and that the interest rate equals the marginal product of capital minus depreciation.
2.3.2.1 Consumers with access to capital markets

Consumers with access to capital markets maximize utility according to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi (s^t) \ u (c^{1N} (s^t), c^{1T} (s^t), l^1 (s^t))$$

subject to the following budget constraint

$$p (s^t) c^{1N} (s^t) + c^{1T} (s^t) + \sum_{s^{t+1}} Q (s^{t+1} / s^t) B (s^{t+1}) + k (s^{t+1}) - (1 - \delta) k (s^t) =$$

$$B (s^t) + w (s^t) l^1 (s^t) + r^k (s^t) k (s^t) - \frac{\phi}{2} (k (s^{t+1}) - k (s^t))^2, \quad (2.33)$$

and transversality condition:

$$\lim_{j \to \infty} \beta^j \pi (s^{t+j} / s^t) \frac{u_{c^1T} (s^{t+j})}{u_{c^1T} (s^t)} B (s^{t+j}) = 0. \quad (2.34)$$

Letting $\lambda^1 (s^t)$ denote the Lagrange multiplier on equation (2.33), the first order conditions for this problem are:

$$\pi (s^t) u_{c^1T} (s^t) = \lambda^1 (s^t), \quad (2.35)$$

$$\pi (s^t) u_{c^1N} (s^t) = \lambda^1 (s^t) p^N (s^t), \quad (2.36)$$

$$-\pi (s^t) u_{l1} (s^t) = \lambda^1 (s^t) w (s^t), \quad (2.37)$$

$$\lambda^1 (s^t) Q (s^{t+1} / s^t) = \beta \lambda^1 (s^{t+1}), \quad (2.38)$$

$$\lambda^1 (s^t) [1 + \phi (k (s^{t+1}) - k (s^t))] = \beta \lambda^1 (s^{t+1}) [(1 - \delta) + r^K (s^{t+1}) + \phi (k (s^{t+2}) - k (s^{t+1}))]. \quad (2.39)$$

2.3.2.2 Consumers with no access to capital markets

Consumers with no access to capital markets maximize utility according to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi (s^t) \ u (c^{2N} (s^t), c^{2T} (s^t), l^2 (s^t)),$$
subject to
\[ p \left( s^t \right) c^{2N} \left( s^t \right) + c^{2T} \left( s^t \right) = w \left( s^t \right) l^2 \left( s^t \right). \]  
(2.40)

Letting \( \lambda^2 \left( s^t \right) \) denote the Lagrange multiplier corresponding to constraint (2.40), the first order conditions for this problem can be written as
\[ \pi \left( s^t \right) u_{c2T} \left( s^t \right) = \lambda^2 \left( s^t \right), \]  
(2.41)
\[ \pi \left( s^t \right) u_{c2N} \left( s^t \right) = \lambda^2 \left( s^t \right) p^N \left( s^t \right), \]  
(2.42)
\[ -\pi \left( s^t \right) u_{l2} \left( s^t \right) = \lambda^2 \left( s^t \right) w \left( s^t \right). \]  
(2.43)

### 2.3.3 Equilibrium

In equilibrium, we must have that
\[ \chi l^1_t + \left( 1 - \chi \right) l^2_t = l_t, \]  
(2.44)
\[ \chi c^{1T}_t + \left( 1 - \chi \right) c^{2T}_t = c^T_t, \]  
(2.45)
\[ \chi c^{1N}_t + \left( 1 - \chi \right) c^{2N}_t = c^N_t. \]  
(2.46)

Total consumption of nontradables must be met by domestic production:
\[ y^N_t = c^N_t. \]  
(2.47)

Combining (2.33) and (2.40), we arrive at the resource constraint for the economy
\[ \chi \sum_{s^{t+1}} Q \left( s^{t+1} / s^t \right) B \left( s^{t+1} \right) - \chi B \left( s^t \right) = y^T_t - \chi i_t - \chi \phi \left( k_{t+1} - k_t \right)^2 - c^T_t, \]  
(2.48)
which can be rewritten in terms of the trade balance as
\[ \chi tb_t = y^T_t - \chi i_t - \chi \phi \left( k_{t+1} - k_t \right)^2 - c^T_t. \]  
(2.49)

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Assume that technology shocks follows a first order autoregressive process. Let \( \ln A_t = [\ln A_T^T, \ln A_N^T] \). Then

\[
\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, \Sigma_{\epsilon}). \quad (2.50)
\]

A competitive equilibrium is a set of processes

\[
\{c_{ij}^t, l_t^t, l_t^i, y_t^j, K_{t+1}^j, p_t^N, w_t, r_t^k, \lambda_t^i, t_{b_t}\}_{t=0}^\infty
\]

for \( i = 1, 2 \) and \( j = T, N \) satisfying equations (2.24)-(2.32),(2.41)-(2.47) and (2.49) given (2.50), \( A_0 \) and \( k_0 \).

We solve the above system of stochastic difference equations using a first order approximation around the steady state.\(^8\) In small open economies with a single risk-free bond that has an exogenous rate of return, the equilibrium dynamics become nonstationary. In our model, the presence of complete state-contingent claims is enough to induce stationarity.

### 2.3.4 Functional Forms and Model Calibration

The model is calibrated using Brazilian sectoral data and other parameters commonly used in international business cycle studies. The exception is the financial access parameter, which as in the endowment economy model, is set equal to \( \chi = .4 \), the average share of individuals that hold a bank account in developing economies. Notice that this number is very close to the share observed in Brazilian household data (see Table 2.3).

\(^8\)We solved for a second order approximation as in Schmitt-Grohé and Uribe (2004) but since the impact on second moments was small, we report the results using a first order approximation.
The sectoral data is taken from the Brazilian Input-Output matrix published by IBGE (Brazilian Institute of Geography and Statistics). The data are available only for the period of 1990 through 2004. The sectoral data is used to compute the tradable and nontradable macroeconomic aggregates of the economy. The methodology used to classify each sector as tradable or nontradable follows Mendoza and Uribe (2001). The method consists in computing the ratio of total trade to gross production in each sector of the economy. If the ratio lies below 5 percent then the sector is classified as nontradable.

According to this criteria, 47 percent of Brazilian GDP is composed of nontradable goods, or more precisely, the ratio $y^T/y$ equals .5665. Stockman and Tesar (1995), using data for Canada, Germany, Italy, Japan and the US, find an average nontradable sector share of 50 percent. Mendoza and Uribe (2001) finds a 60 percent nontradable share of GDP for the Mexican economy.

Using Brazilian National Account annual data from 1970-2007 from IBGE, we compute the average investment to output ratio $i/y$ and the trade balance to output ratio $tb/y$, which are found to be $i/y = .1937$ and $tb/y = .0137$.

From the Input-Output matrix we compute the tradables sector’s labor share $1 - \alpha^T$ as labor payments divided by value added in the tradable sector. The calculation gives us an estimate of $\alpha^T$ of .66. For the nontradable sector, the same procedure implies an estimate of $\alpha^N$ of .54. As Gollin (2002) and Bernanke and Gurkaynak (2001) suggest, published series on employee compensation may significantly understate total labor compensation in developing economies, generating lower estimates of labor’s share in developing economies than in industrial
countries. They show that when taking into account the inclusion of the income of self-employed workers in national account statistics, labor’s share tends to be similar in developing and industrial economies, fluctuating around the average of .65. We thus adopt Stockman and Tesar (1995)’s estimates of capital shares using OECD data: $\alpha_T = .39$ and $\alpha_N = .44$.\footnote{These are the average labor shares for Canada, France, Germany, Italy, Japan, United Kingdom and United States.}

The utility function assumes the following form:

$$u(c^T_t, c^N_t) = \frac{[c(c^T_t, c^N_t) - h\gamma]^{1-\sigma} - 1}{1 - \sigma}$$

where $c(c^T_t, c^N_t) = [\omega(c^T_t)^{-\eta} + (1 - \omega)(c^N_t)^{-\eta}]^{-1/\eta}$, $i = 1, 2$.

As mentioned before, most existing empirical studies suggest that the elasticity of substitution for developing countries is between .4 and .83. The elasticity of substitution between tradables and nontradables is set to .83, which implies that $\eta = .204$. Later, we will provide a robustness check by setting the elasticity to 0.4, the lower bound of this interval.

The remaining parameters are set as in Mendoza (2001) and Schmitt-Grohé and Uribe (2003). We set $\gamma = 1.455$, $r = .04$, $\phi = .1$ and $\sigma = 2$. Normalizing $A^N$ to 1 and using the data ratios and parameters along with the steady-state conditions implied by the system defined above, we obtain the following values: $\delta = .0355$, $\omega = 0.4231$ and $A^T = 0.7464$.

In order to estimate the stochastic process of the TFP shocks, we ideally would calculate the sectoral Solow residuals. The available sectoral data, how-
ever, includes data from the 1994 Real Plan stabilization episode, which has a large impact on the data. Unfortunately, dropping the period of 1990-1994 leave us with a very short time series. Instead, we adopt the stochastic process for the TFP shocks estimated in Stockman and Tesar (1995).\textsuperscript{10} The table below summarizes the parameter values.

Table 2.6: Benchmark Calibration Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_T$</td>
<td>0.39</td>
<td>$\omega$</td>
<td>0.4231</td>
</tr>
<tr>
<td>$\alpha_N$</td>
<td>0.44</td>
<td>$\eta$</td>
<td>0.204</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.028</td>
<td>$\gamma$</td>
<td>1.455</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.4</td>
<td>$r$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\rho_{11}$</td>
<td>0.154</td>
<td>$\Sigma_{11}$</td>
<td>0.0362\textsuperscript{2}</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>0.04</td>
<td>$\Sigma_{12}$</td>
<td>0.0123\textsuperscript{2}</td>
</tr>
<tr>
<td>$\rho_{21}$</td>
<td>-0.15</td>
<td>$\Sigma_{21}$</td>
<td>0.0123\textsuperscript{2}</td>
</tr>
<tr>
<td>$\rho_{22}$</td>
<td>0.632</td>
<td>$\Sigma_{22}$</td>
<td>0.0199\textsuperscript{2}</td>
</tr>
</tbody>
</table>

2.3.5 Results

Table 2.7 displays the standard deviation of some key variables. Even though we do not aim to match all the moments in the data, Table 2.7 shows that consumption, investment and trade balance volatilities implied by the model appear to be very close to the data. In the top of Table 2.8 we calculate the standard deviations for individual and aggregate consumption under financial autarky and financial integration, for various degrees of financial market participation $\chi$.

Unlike an endowment economy, an open production economy can attract

\textsuperscript{10} Stockman and Tesar use OECD data for Canada, Germany, Italy, Japan and the US.
Table 2.7: Comparing volatilities: model x data

<table>
<thead>
<tr>
<th>Volatility $^1$</th>
<th>Data $^2$</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>$\chi = 0.4$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2.16</td>
<td>4.10</td>
</tr>
<tr>
<td>$y^T$</td>
<td>4.47</td>
<td>6.22</td>
</tr>
<tr>
<td>$y^N$</td>
<td>3.99</td>
<td>2.89</td>
</tr>
<tr>
<td>$c$</td>
<td>2.96</td>
<td>2.62</td>
</tr>
<tr>
<td>$c^T$</td>
<td>2.48</td>
<td>3.41</td>
</tr>
<tr>
<td>$c^N$</td>
<td>9.57</td>
<td>2.89</td>
</tr>
<tr>
<td>$p^N$</td>
<td>4.36</td>
<td>4.17</td>
</tr>
<tr>
<td>$i$</td>
<td>7.64</td>
<td>7.23</td>
</tr>
<tr>
<td>$tb/y$</td>
<td>4.27</td>
<td>2.67</td>
</tr>
</tbody>
</table>

$^1$ Standard Deviation of variable
$^2$ Period of 1995-2003; sources: Central Bank of Brazil and IBGE.

investment from abroad during high productivity states. Periods of prosperity will be associated with an increase in investment and a deficit in the trade balance, producing a countercyclical trade balance.$^{11}$ The higher supply of tradable goods is accompanied by a high demand for tradable capital goods. The high demand for capital goods induces some reallocation of capital and labor from nontradable sectors. Since shocks are not orthogonal under the benchmark calibration, the tradable sector also receives a shock of the size of the covariance between tradable and nontradable sectors.

$^{11}$In the Appendix we provide impulse responses to a one standard deviation shock to the nontradable sector.
the nontradable to the tradable sector. Financial integration thus produces both higher investment volatility and an increase in tradable output volatility relative to nontradables.

On the household side, both types of agents have identical labor supply in all states, due to our preference assumption that income effects do not affect labor supply (Greenwood, Hercowitz and Huffman (1988)). Agents with access to capital markets always have lower consumption volatility than agents without access given their ability to self-insure.

Our main interest is in the change in consumption volatility induced by financial integration. From Table 2.8 we observe that, when the economy opens up, consumption volatility for individuals with no access to financial markets increases by 2 percent, while the consumption volatility for individuals with access to financial markets drops by 5 percent. Total consumption volatility remains practically unchanged (dropping by 0.5 percent). In comparison, a two-sector production economy in which all agents have access to the financial market would see a drop of 33 percent in total consumption volatility following integration.

In a closed economy, any desire to invest during a good state of the world needs to be met by domestic resources. In this context, tradable capital goods become more valuable in good states, driving factor inputs from the nontradable to the tradable sector. A high volatility of factor inputs translates into income and consumption volatility. A financially integrated economy, on the other hand, is able to use foreign resources to invest during good times, which translates into
Table 2.8: Effect of Financial Integration varying Financial Market Participation $\chi$ – Standard deviation (s.d.) in (%)

<table>
<thead>
<tr>
<th>variable</th>
<th>$\chi = .3$</th>
<th>$\chi = .4$</th>
<th>$\chi = .5$</th>
<th>$\chi = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(c_1)$</td>
<td>1.3</td>
<td>1.5</td>
<td>1.7</td>
<td>2.1</td>
</tr>
<tr>
<td>$\sigma(c_2)$</td>
<td>4.1</td>
<td>4.2</td>
<td>4.2</td>
<td>4.3</td>
</tr>
<tr>
<td>$\sigma(c)$</td>
<td>2.7</td>
<td>2.6</td>
<td>2.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Benchmark Calibration – % change in s.d. from F.A. to F.I.

<table>
<thead>
<tr>
<th>variable</th>
<th>$\chi = .3$</th>
<th>$\chi = .4$</th>
<th>$\chi = .5$</th>
<th>$\chi = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(c_1)$</td>
<td>-3.6</td>
<td>-5.0</td>
<td>-8.2</td>
<td>-33.2</td>
</tr>
<tr>
<td>$\sigma(c_2)$</td>
<td>0.7</td>
<td>1.6</td>
<td>1.6</td>
<td>-6.4</td>
</tr>
<tr>
<td>$\sigma(c)$</td>
<td>-0.4</td>
<td>-0.5</td>
<td>-2.4</td>
<td>-32.9</td>
</tr>
</tbody>
</table>

Benchmark Calibration – % change in s.d. from F.A. to F.I.

<table>
<thead>
<tr>
<th>variable</th>
<th>$\chi = .3$</th>
<th>$\chi = .4$</th>
<th>$\chi = .5$</th>
<th>$\chi = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(c_1)/\sigma(y)$</td>
<td>-4.3</td>
<td>-6.4</td>
<td>-9.5</td>
<td>-28.2</td>
</tr>
<tr>
<td>$\sigma(c_2)/\sigma(y)$</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.6</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>-1.1</td>
<td>-2.0</td>
<td>-3.8</td>
<td>-27.9</td>
</tr>
</tbody>
</table>

Elasticity of Substitution $\eta=0.4$ – s.d. under F.I.

<table>
<thead>
<tr>
<th>variable</th>
<th>$\chi = .3$</th>
<th>$\chi = .4$</th>
<th>$\chi = .5$</th>
<th>$\chi = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(c_1)$</td>
<td>1.4</td>
<td>1.5</td>
<td>1.7</td>
<td>2.2</td>
</tr>
<tr>
<td>$\sigma(c_2)$</td>
<td>4.2</td>
<td>4.2</td>
<td>4.2</td>
<td>4.3</td>
</tr>
<tr>
<td>$\sigma(c)$</td>
<td>2.7</td>
<td>2.6</td>
<td>2.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Elasticity of Substitution $\eta=0.4$ – % change in s.d. from F.A. to F.I.

<table>
<thead>
<tr>
<th>variable</th>
<th>$\chi = .3$</th>
<th>$\chi = .4$</th>
<th>$\chi = .5$</th>
<th>$\chi = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(c_1)$</td>
<td>0.4</td>
<td>-4.0</td>
<td>-7.1</td>
<td>-33.1</td>
</tr>
<tr>
<td>$\sigma(c_2)$</td>
<td>3.3</td>
<td>2.3</td>
<td>2.4</td>
<td>-6.4</td>
</tr>
<tr>
<td>$\sigma(c)$</td>
<td>2.8</td>
<td>0.4</td>
<td>-1.4</td>
<td>-32.8</td>
</tr>
</tbody>
</table>
lower factor input volatility and higher investment volatility. However, since access to both international markets and the investment good is not homogeneous, individuals with financial access bear a higher burden in borrowing from abroad to invest and, in aggregate, part of the adjustment still takes place through real factor reallocation across sectors.

Under the benchmark parameterization, consumption volatility remains fairly unchanged despite financial integration. The ratio of consumption to output volatility exhibits a similar pattern; drop by just 2 percent when financial access equals .4 and by 28 percent when financial access equals 1. This finding suggests that financial market participation has an important role in explaining the surprisingly small reduction in consumption volatility found in the data.

In the benchmark calibration, the elasticity of substitution between tradable and nontradables is set to .83. In the bottom part of table 2.8, we show results for an elasticity of substitution of 0.4, the lower bound of existing estimates. A lower elasticity of substitution leads to a lower impact of financial integration on the volatility of consumption for both types of individuals, though the magnitude of the change is very small. Hence, our main results are not very sensitive to changes in the elasticity of substitution between tradables and nontradables.

2.3.6 Alternative Calibration

In the benchmark calibration, we opted to have an estimate of the stochastic process of the TFP shocks by using OECD data. In this section, in order to better
align the model to Brazilian data, we calibrate the parameters of the TFP process to match second moments of Brazilian output data. We use sectoral annual data from 1947-2007 from IBGE and the classification described previously to group sectors into tradable and nontradables. Using a longer time series, we are able to estimate the autoregressive coefficients of a VAR process of tradable and nontradable output. We use these estimates to calibrate the autoregressive coefficients of tradable and nontradable TFP shocks. The coefficients that were not statistically significant at the 5 percent level were set to zero. The result is $\rho_{11} = 0.607$ and $\rho_{22} = 0.685$ while the other coefficients equal zero. Using the same data set, we compute the covariance matrix of tradable and nontradable output. The standard deviation of tradable output is slightly higher than nontradable output and they are equal to 3.68 and 3.49 respectively. The correlation between the two sectors is positive and equal to 0.20. Using the Simulated Method of Moments, we calibrate the parameters of the variance covariance matrix of shocks to match tradable and nontradable output variances and covariance. We find the variance covariance matrix elements of the shocks to be $\Sigma_{\epsilon}^{11} = 0.0180^2$, $\Sigma_{\epsilon}^{12} = \Sigma_{\epsilon}^{21} = -0.00022$ and $\Sigma_{\epsilon}^{22} = 0.0237^2$. Table 2.9 summarizes the parameters of the alternative calibration.

2.3.7 Results

Table 2.10 displays the standard deviation of the same key variables as in Table 2.7. Despite tradable output being more volatile than nontradable output, nontradable consumption implied by the model is more volatile than tradable
consumption as suggested by the data. On the other hand, investment and trade balance volatilities implied by the model are now higher than they appear in the data.

In Table 2.11 we recalculate the standard deviations for individual and aggregate consumption under financial autarky and financial integration. We find that consumption volatility for individuals with access drops by 5.8 percent with financial integration while this variable increases by 7.8 percent for individuals with no access. Overall, aggregate consumption volatility increases by 1.6 percent, close to the prediction of a drop of 0.5 percent in the benchmark calibration.

Stockman and Tesar (1995) have pointed out the importance of the non-tradable sector in explaining aggregate consumption behavior in open economy models. In order to extract the role of the nontradable sector in generating our results, we run the same experiment but considering only one sector (tradables). The choice of parameters is the same as before, and the tradable sector in the one-sector economy corresponds to the tradable sector of the two-sector economy. We
then compute the change in the volatility of consumption that occurs when the economy moves from financial autarky to integration with international financial markets assuming $\chi = 1$. In this case, when the economy gains access to international markets, the domestic economy becomes a complete market economy.\footnote{The small open economy with complete asset markets corresponds exactly to the same economy described in Schmitt-Grohé and Uribe (2003).}
Table 2.11: Effect of Financial Integration for $\chi = 0.4$ – Standard deviation (s.d.) in (%)

<table>
<thead>
<tr>
<th>Alternative Calibration – standard deviation (s.d.) under F.I.</th>
<th>(\chi = 0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma(c_1))</td>
<td>1.6</td>
</tr>
<tr>
<td>(\sigma(c_2))</td>
<td>3.0</td>
</tr>
<tr>
<td>(\sigma(c))</td>
<td>2.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternative Calibration – % change in s.d. from F.A. to F.I.</th>
<th>(\chi = 0.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma(c_1)/\sigma(y))</td>
<td>-12.5</td>
</tr>
<tr>
<td>(\sigma(c_2)/\sigma(y))</td>
<td>0.1</td>
</tr>
<tr>
<td>(\sigma(c)/\sigma(y))</td>
<td>-5.6</td>
</tr>
</tbody>
</table>

In this experiment, integration reduces consumption volatility by 47 percent. From previous results, introducing nontradables but retaining full participation attenuates the decline in volatility to 33 percent, while introducing nontradables and limited participation eliminates the reduction of consumption volatility entirely.
2.4 Conclusion

The analysis above considers an economy that gains access to international financial markets but in which only a fraction of individuals have access to capital markets. Both types of individuals have the desire to smooth consumption intertemporally and to smooth consumption between goods within periods. In an endowment economy, if there is a negative perfectly correlated shock to both goods, individuals with access to capital markets can borrow from abroad to smooth consumption of tradables. At the same time, they will be able to partially smooth their consumption of non-tradables by offering tradable goods to individuals with no access to financial markets.

The greater supply of tradables increases the relative price of the non-tradable good. Consumers excluded from financial markets will then buy cheap tradable goods in order to smooth the consumption of the tradable good intertemporally. However, in doing so they will cut consumption of nontradables in bad times even more than they would under autarky (while increasing consumption of non-tradables relative to autarky in good times). Following financial integration, the consumers excluded from financial markets will exhibit more volatile consumption of the nontradable good and less volatile consumption of the tradable good.

As we would expect from this model, the volatility of total consumption drops under financial integration for any degree of asset market segmentation. But the drop is much less than in a model with universal financial access. Thus, the above exercise helps us to clarify the importance of domestic financial devel-
opment in a country that faces financial integration.

When we move to an economy with production and allow the supply side of the economy to adjust we find that aggregate volatility is not much affected by financial integration, shedding some light on why consumption volatility has remained high in emerging markets despite the higher degrees of financial integration experienced by these economies.
Chapter 3

The Backus-Smith Puzzle Revisited

3.1 Introduction

In the previous chapter we focused on how increased possibilities for risk sharing can impact consumption volatility. Another implication of international risk sharing is related to the behavior of real exchange rates and cross country consumption ratios. In theory, one could argue that if two economies are able to share risk, a relatively higher price of consumption in one of the economies should be associated with a relatively lower consumption. Thus, consumption ratios and the real exchange rate would be monotonically related. Moreover, in a world in which Purchasing Power Parity (PPP) holds, this relation should translate into a high consumption correlation across countries. The fact that in the data consumption ratios across countries and the real exchange rate are either uncorrelated or negatively correlated has been denoted the “Backus-Smith puzzle”.

The literature has investigated if models that allow for market incompleteness can reproduce the observed low consumption correlation across countries and the low correlation between the real exchange rate and ratios of consumption. Cole and Obstfeld (1991) argue that incomplete markets are not enough to account for the lack of risk-sharing across countries. They show that even with lack of asset trade, the terms of trade in goods can adjust to deliver benefits sim-
ilar to those of asset trade, challenging the gains associated with international capital mobility. Corsetti et al. (2008), in a similar setting, show that such a result holds true only for a restricted set of parameters. They find that the sign of the correlation between relative consumption and the real exchange rate depends on the elasticity of substitution across home and foreign goods, as well as on consumption home bias. More precisely, a negative Backus-Smith correlation can occur when a positive supply shock is associated with an improvement in the country’s terms of trade. In the presence of home-bias in a two-country world, where both economies are equally-sized, the domestic economy’s demand for home goods represents an important share of total demand. If income effects dominate substitution effects, a larger supply of domestic goods can only be matched by an equally high demand when the terms of trade appreciate. Even though a terms of trade appreciation is not necessary for a negative Backus-Smith correlation, it is necessary that substitution effects be relatively small.

Chari et al. (2002) also emphasize the need to move away from the assumption of complete markets in order to break free from the proportionality of the real exchange rate and marginal utilities of consumption. They argue that other frictions in the goods and labor markets like sticky prices, sticky wages and shipping costs are not effective in solving what they refer to as the consumption-real exchange rate anomaly. Benigno and Thoenissen (2008) consider a setting similar to Chari et al. (2002), but add a production sector that requires nontradable inputs and allow for an incomplete asset market structure. They show that the consumption-real exchange rate anomaly can be successfully addressed depend-
ing on the structure of shocks that hit both tradable and nontradable sectors. They show that relatively larger shocks to the tradable sector lead to a large and negative consumption-real exchange rate correlation, whereas dominant shocks to the nontradable sector imply a positive correlation.

In order to incorporate asset market incompleteness, it is commonly assumed that the only asset available in the economy is a one-period bond. We opt to explore a different avenue in which there is a full set of state contingent bonds but limited financial market participation. Our set up is close to Backus and Smith (1993), who assume a two-country world with one tradable good and a country specific nontradable good. As Backus and Smith (1993) show, the presence of a nontradable sector does not prevent perfect risk sharing across countries. Even if nontradable risk might cause consumption baskets across countries to behave differently, the real exchange rate will still perfectly comove with consumption ratios.

The key difference in our approach is to add a measure of consumers that do not have access to financial markets but still participate in the goods market. For simplicity, I assume limited and heterogeneous financial market access in only one of the countries. The behavior of aggregate consumption depends on the interaction of three types of consumers. Despite the fact that consumers display identical preferences and there is no idiosyncratic risk, they will be exposed to different aggregate risks and will face distinct possibility choices. Let country $A$ exhibit heterogeneous financial market access. Consumers in $A$ with access will be able to trade goods intra period with consumers with no access within
and can exchange tradables intertemporally with individuals in $B$. Individuals in $A$ with no access to financial markets cannot trade with individuals from country $B$ in any market, because they do not have access to the technology to trade intertemporally, while tradable goods are homogeneous across countries. This single departure from Backus and Smith (1993) allows us to show that a perfect consumption-real exchange rate correlation ceases to hold. The value of this correlation will depend on the structure of output shocks across sectors and countries.

3.2 A Two-Country World

There are two countries in the world economy. The countries are symmetric except for the fact that in country $A$, a fixed share of individuals $1 - \chi$ do not have access to financial markets. The remaining fraction $\chi$ of country $A$’s nationals have access to markets and can trade assets with nationals of country $B$.

3.2.1 Country $A$

Households have identical preferences which can be described by

$$
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_{iT}^A(s^t), c_{iN}^A(s^t)),
$$

where $\beta < 1$ is the discount factor; $c_{ij}^A$ denotes consumption of individuals in country $A$ with access to capital markets if $i = 1$ or no access if $i = 2$; and consumption of tradables if $j = T$ or the non-tradable good if $j = N$. We assume that households are heterogeneous in their ability to trade assets. During each period,
individuals with access to capital markets are allowed to trade a one-period state-contingent asset $B(s^{t+1})$. At each period $t$, the asset sells at a price of $Q(s^{t+1}/s^t)$ and promises to pay one unit of tradable good in period $t + 1$ if the history $s^{t+1}$ is realized. The budget constraint at each state $s^t$ is

$$
p_N^A(s^t) c_{1N}^A(s^t) + c_T^A(s^t) + \sum_{s^{t+1}} Q(s^{t+1}/s^t) B(s^{t+1}) = B(s^t)
$$

and the transversality condition is given by

$$
\lim_{j \to \infty} \beta^j \pi(s^{t+j}/s^t) \frac{u_{c_{1T}^A}(s^{t+j})}{u_{c_{1T}^A}(s^t)} B(s^{t+j}) = 0.
$$

The problem faced by households of type 1 is then to maximize (3.1) subject to (3.2) and (3.3). Letting $\lambda_1^A(s^t)$ denote the Lagrange multiplier for constraint (3.2), the first order conditions are given by

$$
\pi(s^t) u_{c_{1T}^A}(s^t) = \lambda_1^A(s^t),
$$

$$
\pi(s^t) u_{c_{1N}^A}(s^t) = \lambda_1^A(s^t) p_N^A(s^t),
$$

$$
\lambda_1^A(s^t) Q(s^{t+1}/s^t) = \beta \lambda_1^A(s^{t+1}).
$$

If we combine equations (3.4) and (3.5), we arrive at the intratemporal efficiency condition that

$$
p_N^A(s^t) = \frac{u_{c_{1N}^A}(s^t)}{u_{c_{1T}^A}(s^t)}.
$$

From equations (3.4) and (3.6), we obtain the intertemporal condition

$$
Q(s^{t+1}/s^t) = \beta \pi(s^{t+1}/s^t) \frac{u_{c_{1T}^A}(s^{t+1})}{u_{c_{1T}^A}(s^t)}.
$$
Households excluded from financial markets are able to trade only goods in the spot market. The budget constraint in each state $s^t$ is

$$p^A_N (s^t) c^A_{2N} (s^t) + c^A_{2T} (s^t) = p^A_N (s^t) y^A_N (s^t) + y^A_T (s^t). \quad (3.9)$$

Notice that even though these individuals do not have access to the technology that allows them to trade future claims on tradable goods, they are able to trade goods at every point in time with households that have access to this technology.

The problem faced by households of type 2 is then to maximize (3.1) subject to (3.9). Letting $\lambda^A_2 (s^t)$ denote the Lagrange multiplier corresponding to constraint (3.9), the first order conditions are given by

$$\pi (s^t) u^A_{c^A_{2T}} (s^t) = \lambda^A_2 (s^t), \quad (3.10)$$

$$\pi (s^t) u^A_{c^A_{2N}} (s^t) = \lambda^A_2 (s^t) p^A_N (s^t). \quad (3.11)$$

Combining equations (3.10) and (3.11), we obtain

$$p^A_N (s^t) = \frac{u^A_{c^A_{2N}} (s^t)}{u^A_{c^A_{2T}} (s^t)}. \quad (3.12)$$

### 3.2.2 Country B

Households have identical preferences which can be described by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi (s^t) u \left( c^B_T (s^t), c^B_N (s^t) \right), \quad (3.13)$$

where $\beta < 1$ is the discount factor; $c^B_j$ denotes consumption in country $B$ of tradables if $j = T$ or the non-tradable good if $j = N$. The budget constraint at each state $s^t$ is

$$p^B_N (s^t) c^B_N (s^t) + c^B_T (s^t) + \sum_{s^{t+1}} Q \left( s^{t+1} / s^t \right) B (s^{t+1}) = B (s^t)$$
and the transversality condition is given by

\[
\lim_{j \to \infty} \beta^j \pi(s^{t+j}/s^t) \frac{u_{cB}(s^{t+j})}{u_{cF}(s^t)} B(s^{t+j}) = 0.
\]  

(3.15)

The problem faced by all households in country \(B\) is to maximize (3.13) subject to (3.14) and (3.15). Letting \(\lambda^B(s^t)\) denote the Lagrange multiplier on equation (3.14), the first order conditions are given by

\[
\pi(s^t) u_{cB}(s^t) = \lambda^B(s^t),
\]  

(3.16)

\[
\pi(s^t) u_{cN}(s^t) = \lambda^B(s^t) p^B_N(s^t),
\]  

(3.17)

\[
\lambda^B(s^t) Q(s^{t+1}/s^t) = \beta \lambda^B(s^{t+1}).
\]  

(3.18)

If we combine equations (3.16) and (3.17), we arrive at the intratemporal efficiency condition that

\[
p^B_N(s^t) = \frac{u_{cB}(s^t)}{u_{cF}(s^t)}.
\]  

(3.19)

From equations (3.16) and (3.18), we obtain the intertemporal condition

\[
Q(s^{t+1}/s^t) = \beta \pi(s^{t+1}/s^t) \frac{u_{cB}(s^{t+1})}{u_{cF}(s^t)}.
\]  

(3.20)

3.2.3 Equilibrium

Individuals from country \(A\) and \(B\) with access to financial markets trade promises to deliver tradable consumption one period ahead. From equations (3.8) and (3.20), the growth of marginal utility of tradable consumption is equalized across type 1 individuals in country \(A\) and individuals in country \(B\):

\[
\frac{u_{c1F}(s^{t+1})}{u_{c1F}(s^t)} = \frac{u_{c1F}(s^{t+1})}{u_{cB}(s^t)}.
\]
It is also true that the ratio of (3.8) evaluated at $s_{i}^{t+1}$ to (3.8) evaluated at $s_{j}^{t+1}$ is the same as the ratio of (3.20) evaluated at $s_{i}^{t+1}$ to (3.20) evaluated at $s_{j}^{t+1}$:

$$\frac{u_{c_{T}^{A}}(s_{i}^{t+1})}{u_{c_{T}^{B}}(s_{j}^{t+1})} = \frac{u_{c_{T}^{A}}(s_{i}^{t+1})}{u_{c_{T}^{B}}(s_{j}^{t+1})}.$$

Combining the two equations above implies that the marginal utility of tradable consumption scaled by a constant factor will be equalized across countries for individuals that have access to financial markets:

$$\frac{u_{c_{T}^{A}}(s_{i}^{t+1})}{u_{c_{T}^{B}}(s_{j}^{t+1})} = \bar{\psi}, \quad \forall s^{t+1} \quad (3.21)$$

The condition above captures the ability of countries to share risk in a two-country setting. Notice that in the small open economy case, the above condition assumes a stronger form, where the marginal utility of tradable consumption is constant, abstracting from the possibility of any aggregate world risk. From equations (3.10) and (3.16), we find that the same condition (3.21) does not hold for individuals with no access to financial markets; that is, the ratio of marginal utility of tradable consumption between individuals with no access and the individuals in country $B$ is state dependent and given by

$$\frac{u_{c_{T}^{A}}(s_{i}^{t+1})}{u_{c_{T}^{B}}(s_{j}^{t+1})} = \frac{\lambda_{A}^{A}(s_{i}^{t+1})}{\lambda_{B}^{B}(s_{j}^{t+1})}, \quad \forall s^{t+1} \quad (3.22)$$

In equilibrium, the consumption of nontradables must be equal to its supply in every period for both countries:

$$\chi c_{1N}^{A}(s^{t}) + (1 - \chi)c_{2N}^{A}(s^{t}) = y_{N}^{A}(s^{t}), \quad (3.23)$$

$$c_{N}^{B}(s^{t}) = y_{N}^{B}(s^{t}). \quad (3.24)$$
Multiplying (3.2) by $\chi$ and (3.9) by $(1-\chi)$, we add up the result and use (3.23) to arrive at

$$\chi \left[ \sum_{s^t+1} Q \left( \frac{s^{t+1}}{s^t} \right) B \left( s^{t+1} \right) - B \left( s^t \right) \right] = y^A_T(s^t) - \chi c^A_{1T}(s^t) - (1 - \chi)c^A_{2T}(s^t). \quad (3.25)$$

Rewriting the above equation in terms of the trade balance for country $A$ and using (3.14), we can define the external accounts for both countries:

$$\chi tb^A(s^t) = y^A_T(s^t) - \chi c^A_{1T}(s^t) - (1 - \chi)c^A_{2T}(s^t), \quad (3.26)$$

$$tb^B(s^t) = y^B_T(s^t) - c^B_{2T}(s^t). \quad (3.27)$$

Finally, in order to satisfy the world resource constraint, tradable consumption must equal world tradable resources, so that

$$\chi tb^A(s^t) + tb^B(s^t) = 0. \quad (3.28)$$

**Definition** A competitive equilibrium is a set of processes

$$\{c^A_{1t}, c^A_{2t}, c^A_{2t}, c^B_{2t}, c^B_{2t}, p^A_{Nt}, p^B_{Nt}, \lambda^A_{1t}, \lambda^A_{2t}, \lambda^B_{t}, tb^A_t, tb^B_t\}_{t=0}^\infty$$

satisfying equations (3.7), (3.9), (3.12), (3.19), (3.21), (3.23), (3.24), (3.26) and (3.27), given $\{y^A_T, y^A_{Nt}, y^B_T, y^B_{Nt}\}_{t=0}^\infty$ and $A_0$.

We solve the above system of stochastic difference equations using a first order approximation around the steady state.

3.2.4 The Planner’s Problem

In a similar endowment economy with tradables and nontradable goods but in a multi-country setting in which all agents have access to financial markets, Backus and Smith (1993) compute a competitive equilibrium as the solution
to a social planning problem, following Negishi and Debreu (1996) and Mantel (1971). We use a similar approach to compute the social optimum of the economy described above. A planner chooses quantities \( \{c_{A}^1, c_{A}^2, c_{B}^1, c_{B}^2\} \) to maximize

\[
\phi_1 A \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_{1T}^A(s^t), c_{1N}^A(s^t)) + \phi_2 A \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_{2T}^A(s^t), c_{2N}^A(s^t)) + \phi_B \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_{T}^B(s^t), c_{N}^B(s^t)),
\]

subject to the resource constraints

\[
\chi c_{1T}^A(s^t) + (1 - \chi) c_{2T}^B(s^t) + c_{T}^B(s^t) = y_{T}^A(s^t) + y_{T}^B(s^t), \quad (3.29)
\]
\[
\chi c_{1N}^A(s^t) + (1 - \chi) c_{2N}^A(s^t) = y_{N}^A(s^t), \quad (3.30)
\]
\[
c_{N}^B(s^t) = y_{N}^B(s^t). \quad (3.31)
\]

Denote the Lagrange multipliers of the resource constraint for tradable goods as \( \beta^t \pi(s^t) q_T(s^t) \), for nontradables in country A as \( \beta^t \pi(s^t) q_N^A(s^t) \) and nontradables in country B as \( \beta^t \pi(s^t) q_N^B(s^t) \). Define the relative price of nontradables in country \( m \) as \( p_m^N(s^t) \) as the ratio of \( q_N^m(s^t) \) and \( q_T(s^t) \), for \( m = A, B \).

The first order conditions for the problem above are

\[
\phi_1 A u_{c_{1T}^A} (s^t) = q_T(s^t), \quad (3.32)
\]
\[
\phi_1 A u_{c_{1N}^A} (s^t) = q_N^A(s^t), \quad (3.33)
\]
\[
\phi_1 A u_{c_{2T}^A} (s^t) = q_T(s^t), \quad (3.34)
\]
\[
\phi_2 A u_{c_{2N}^A} (s^t) = q_N^A(s^t), \quad (3.35)
\]
\[
\phi_B u_{c_{T}^B} (s^t) = q_T(s^t), \quad (3.36)
\]
\[
\phi_B u_{c_{N}^B} (s^t) = q_N^B(s^t), \quad (3.37)
\]
Equations (3.32), (3.34) and (3.36) reveal that at the social optimum, the ratio of marginal utilities of tradable consumption across agents does not depend on the state of the economy. Going back to the decentralized equilibrium, equation (3.22) shows that marginal utility of tradable consumption is state dependent for individuals of type 2. Despite the fact that individuals of type 1 and 2 can trade tradable and nontradable goods, this trading mechanism is not enough to allow individuals of type 2 to perfectly share risk with the individuals of the foreign country. Therefore, we conclude that the decentralized equilibrium does not attain social efficiency.

3.3 The Backus-Smith correlation

Backus and Smith (1993) found a monotone relation between the real exchange rate (RER) and the relative consumption across countries. In order to express the RER as a function of the consumption ratio, we will assume that utility takes the following form:

$$u(c_T^m, c_N^m) = \left[ c(c_T^m, c_N^m)^{1-\sigma} - 1 \right] \frac{1}{1 - \sigma}$$

where

$$c(c_T^m, c_N^m) = \left[ \omega(c_T^m)^{-\eta} + (1 - \omega)(c_N^m)^{-\eta} \right]^{-1/\eta}, \quad m = A, B.$$ 

The parameter $\sigma$ is the coefficient of constant relative risk aversion. The elasticity of substitution between consumption of tradables and non-tradables is given by $\frac{1}{1+\eta}$, and $\omega$ is the weighting factor. Using the above specification for the CES aggregator in the utility function, the consumption based price index is
Combining (3.4), (3.5), (3.16), (3.17) and (3.38), we can establish a monotone relation between the real exchange rate and the consumption ratio of individuals that have access to the foreign market:

\[ \psi \left( \frac{c_1}{c_B} \right)^\sigma = \frac{p_c^B}{p_c^A}. \]  

Deriving the analogous expression for type 2 consumers by combining (3.10), (3.11), (3.16), (3.17) and (3.38) we arrive at:

\[ \frac{\lambda^B (s^t)}{\lambda^A_2 (s^t)} \left( \frac{c_1^A}{c_B} \right)^\sigma = \frac{p_c^B}{p_c^A}. \]  

Defining \( \frac{p_c^B}{p_c^A} \) as the real exchange rate (RER), we can use the above two equations to find that:

\[ RER = \chi \psi \left( \frac{c_1^A}{c_B} \right)^\sigma + (1 - \chi) \frac{\lambda^B (s^t)}{\lambda^A_2 (s^t)} \left( \frac{c_1^A}{c_B} \right)^\sigma. \]  

Backus and Smith (1993) derive the above equation for \( \chi = 1 \). Hence, if the degree of financial market access is equal to 1, the log of the consumption ratio and the log of the real exchange between two countries are perfectly correlated. As equation (3.41) for the RER suggests, this might not be true in our model. We show in the next section that this correlation will depend on the degree of financial market participation and the stochastic process of the shocks.
3.3.1 Real Exchange Rate and Consumption Ratios

In this section, we aim to show the behavior of the RER-consumption correlation allowing for different sets of assumptions regarding the underlying shocks that hit the economy and the degree of access to financial markets. In a first step, we assume a simple and symmetric structure of shocks for both countries. We later incorporate the structure of shocks estimated in Stockman and Tesar (1995) for an average industrialized economy. Most of the parameter values follow Stockman and Tesar (1995). The nontradable sector corresponds to half of total output. The degree of risk aversion is given by $\frac{1}{\sigma} = 0.5$ and the rate of time discount equals 0.96. The share of tradables $\omega$ is set to 0.5. As Corsetti et al. (2008) claim, the elasticity of substitution estimated in Stockman and Tesar (1995) include both developed and developing economies, so we set the elasticity of substitution to 0.74, which is estimated to a sample of industrialized countries.

Among the factors determining the consumption-RER correlation are the underlying correlations of the shocks that affect both the tradable and nontradable sectors in countries $A$ and $B$. For simplicity we assume that output follows the following stochastic process, where, in matrix notation,

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim NIID(0, \Sigma_\epsilon).$$

Where $\ln A_t = [\ln A_{A,T,t}, \ln A_{A,N,t}, \ln A_{B,T,t}, \ln A_{B,N,t}]$, $\rho$ denotes a $4 \times 4$ matrix of autoregressive coefficients and $\Sigma_\epsilon$ the variance-covariance matrix. The endowment process $y_t = [y_{A,T,t}, y_{A,N,t}, y_{B,T,t}, y_{B,N,t}]$ then follows $y_t = A_t \bar{y}$.

The benchmark assumption will be that all four sectors are uncorrelated, $\rho$.
is a diagonal matrix with all diagonal elements equal to .42 and $\Sigma_{\epsilon}$ is also diagonal with all its elements equal to 0.0129. We will then relax this assumption and allow pairwise correlation across sectors but maintaining the assumption that countries $A$ and $B$ have a symmetric shock structure. In certain cases, perfect correlation across two sectors will imply a constant RER. In order to avoid such cases, we will assume almost perfect positive correlation of .95 and almost perfect negative correlation of -.95. Unless otherwise noted, correlations across any two sectors are assumed to be zero in what follows.

Figure 3.1 shows the consumption-RER correlation as a function of $\chi$, the share of individuals with access in country $A$, for Cases 1 through 3. Case 1 is the benchmark case where all four shocks are assumed to have zero correlation. Case 2 assumes that the tradable sector shocks are almost perfectly positively correlated (correlation of .95) between $A$ and $B$. In this case, trading assets barely brings any gains to individuals participating in the financial markets. The allocations of individuals without access to financial markets in country $A$ are very close to those of individuals with access and hence the RER-consumption correlation is almost perfect as in Backus-Smith.

The solution of the log-linearized system around the steady state allows us to represent the behavior of the RER as a function of the endowments of tradables and nontradables in countries $A$ and $B$, the degree of financial market participation $\chi$, and the other parameters of the model:

$$\hat{RER} = \alpha(1 + \eta)(1 - \chi)(\hat{y}_A^T - \hat{y}_B^T) + (2(1 + \nu \chi) - (1 - \chi)) (\hat{y}_B^N - \hat{y}_A^N) 2\Omega(1 + \nu \chi) - (2\Omega - 1)(1 - \chi).$$
In the above equation the hatted variables denote log deviations from the steady-state. The countries are assumed to be symmetric in steady-state. The parameter \( \alpha = (1-\omega)^{\frac{1}{1+\eta}}\bar{p}_c^\frac{1}{1+\eta}\bar{p}_N^{1+\eta} \), where \( \bar{p}_N \) and \( \bar{p}_c \) are the steady-state values of the relative price of nontrables and the price of the consumption basket respectively. The parameter \( \nu = \frac{1-\omega}{\omega}(\frac{\bar{y}_N}{\bar{y}_T})^{-\eta} \), where \( \bar{y}_N \) and \( \bar{y}_N \) are the steady-state values of tradable and nontradable endowments. Finally, \( \Omega = \frac{\alpha^{1+\eta}}{\sigma} + \frac{1}{1+\nu} \), which depends on the parameter \( \sigma \) that governs risk aversion and the parameter \( 1-\eta \) that governs the elasticity of substitution between tradables and nontrades.

In a extreme scenario where all agents have access to financial markets, the RER would cease to be affected by shocks to the tradable sectors. If we take the limit of \( \hat{RER} \) when \( \chi \) approaches 1, we find that
\[
\hat{RER} = \frac{\alpha(1+\eta)}{\Omega}(\hat{y}_B^N - \hat{y}_A^N).
\]
The other extreme case is financial autarky, in which each country consumes its own endowment of goods every period. At every point in time the ratio of endowments will determine the nontradable relative prices and the price of consumption in each country. In the linearized solution to the RER, we find that shocks to both tradable and nontradable sectors will be equally weighted in their effect to the real exchange rate under autarky. This point can be seen if we take the limit of RER as \( \chi \) approaches 0. The RER approaches
\[
\hat{RER} = \alpha(1+\eta)(\hat{y}_T^A - \hat{y}_T^B + \hat{y}_N^B - \hat{y}_N^A).
\]
Comparing the three expressions above, we see that the lower the degree of financial market participation, the higher will be the effect on the exchange
rate of changes in the relative tradable endowment $\hat{y}_T^A - \hat{y}_T^B$. Case 2 of Figure 3.1, assumes that the tradable sectors are nearly perfect correlated. The effect of the tradable sectors on the exchange rate cancels out and the RER behaves as in the case when $\chi$ approaches 1. Regardless of the actual $\chi$, the result in Figure 3.1 is an almost flat line. In Case 3, the tradable sectors are almost perfectly negatively correlated (correlation of -.95). This is an example where international asset trade brings potential high gains for both economies. As a result, the correlation of RER and the consumption ratio is highly sensitive to $\chi$.

Figure 3.2 shows another three cases: Case 4, Case 5 and Case 6. Case 4 assumes that nontradable shocks across countries are nearly perfect positively correlated, while Case 5 considers negatively correlated nontradable shocks. When
the nontradable sectors are positively correlated, we move further away from the Backus-Smith correlation of 1. Case 6 assumes positively correlated tradable and nontradable sectors within both economies. Notice that regardless of the other parameters of the economy, if both tradable and nontradable shocks are perfectly correlated within each country, the RER will be constant. In Case 7, Figure 3.3, the tradable and nontradable sectors are almost perfectly negatively correlated and the RER-consumption correlation is almost 1 for any degree of financial integration.

Since Case 4 provides us with the lowest Backus-Smith correlation, in Case 8 we explore the combination of Cases 3 and 4, imposing a negative correlation of tradables across countries and positive correlation of nontradables across countries, while in Case 9, we assume the combination of Cases 2 and 4, where each sector is positively correlated with the same sector in the foreign country.

Finally, we consider the case of independent sectors with distinct volatilities. Case 10 in Figure 3.4 assumes that tradables’ standard deviation is twice the standard deviation of nontradables. The RER-consumption correlation becomes very negative for low degrees of access to financial markets. However, RER-consumption correlation becomes positive when nontradable shocks are more volatile than tradable shocks (Case 11).

To further understand the last results, consider a one unit impulse to the endowment of tradable goods in country \( A \), letting the other shocks equal zero. Initially, the supply of tradable goods relative to nontradable goods in country \( A \) will be larger than the supply of tradable goods relative to nontradable goods in
country $B$. Risk sharing between country $A$ and $B$ will lead country $A$ to export tradable goods to country $B$, but as long as the increase in tradable goods is larger in country $A$, the relative price of nontradables in country $A$ will increase by more than the relative price of nontradables in country $B$. The result will be a drop in the RER. Meanwhile, tradable and nontradable consumption will be reallocated across individuals of type 1 and 2, but as long as type 2 is not able to perfectly share risk, the increase in supply of tradables in country $A$ will be larger than in country $B$, and consumption in country $A$ will increase relatively to country $B$. That is, the real exchange rate and relative consumption will be negatively correlated in response to independent shocks to tradables.

Consider now a unit shock to the nontradable sector in country $A$. The
only possibility of country $A$ sharing risk in this case is to export tradable units to country $B$. The endowment of tradables relative to nontradables will drop in country $A$ and rise in country $B$. The relative price of nontradable goods in country $A$ will then drop, while it will increase in country $B$, causing the real exchange rate to go up. Again, as long as the final effect on endowment is higher in country $A$, consumption in country $A$ relatively to country $B$ will increase, and the RER-consumption correlation will be positive.

Benigno and Thoenissen (2008) find a similar result in their work. While they also consider a combination of market incompleteness and inclusion of nontradable goods, access to financial markets is homogeneous in their set up and financial trade is limited by the use of a unique type of asset, a one period bond.
Moreover, each country produces a differentiated intermediary good which is used in the production of the tradable consumption good. Nonetheless, their results show that whenever the source of disturbance arises in the nontradable sector, the real exchange rate and consumption commove positively. A negative comovement is observed when tradable disturbances prevail.

Case 12 assumes the stochastic process estimated in Stockman and Tesar (1995) using annual sectoral data to an average industrialized economy (Canada, Germany, Italy, Japan and US) for 1970-1986. The process is estimated The shocks are positively correlated across countries and sectors. The correlation between tradable shocks across countries is 0.33, between nontradable shocks across countries is 0.14 and across sectors within countries is 0.46. The variance of shocks is about twice as high in tradables relative to nontradables.
Backus and Smith (1993) report a RER-Consumption correlation of .17 using quarterly data for Australia, Canada, France, Germany, Japan, Sweden, United Kingdom and the United States for 1971-1990. When considering Stockman and Tesar shocks, we find that the model would require a very low degree of financial market participation to reproduce such a RER-consumption correlation. Heathcote and Perri (2002) highlight how international business cycle models assuming financial autarky can reproduce the observed cross country correlation in consumption better than model economies that allow for asset trade, while better matching terms of trade volatility observed in the data. Not surprisingly, Stockman and Tesar (1995)’s complete market model, while able to reproduce a handful of international business cycle moments, overstates the cross-country correlation of consumption of traded goods.

3.4 Conclusion

The Backus-Smith puzzle has raised the question of how well standard models can explain observed patterns of international risk sharing. While Backus and Smith (1993) ruled out nontradable risk as a way of breaking the perfect correlation between RER and relative consumption, departures from the complete market setting have been promising. In this paper we show that adding heterogeneous financial access on top of nontradable risk allows us to break away from perfect correlation. Using a simple parametrization of the model, we find that only extremely low access to financial markets allow the model to replicate ob-
served levels of this correlation. The literature has shown that allowing for highly persistent shocks, close to a unit root, generates a higher wedge between complete and incomplete markets, and can induce the RER-consumption correlation to be as low as in the data (see Baxter and Crucini (1995), Corsetti et al. (2008) and Heathcote and Perri (2002)). A possible extension of our model and an alternative to assuming higher persistence of the shocks would be to allow for heterogeneous access in both countries. In this chapter the asymmetric assumption was made for simplicity, in order to highlight the potential role for this financial friction in explaining the Backus-Smith puzzle. Presumably, the presence of heterogeneous access in both countries would increase market incompleteness and would tend to lower the RER-consumption correlation.
Appendix A

Overview

Figure A.1: Disaggregated consumption volatility and ES=0.99
Figure A.2: Disaggregated consumption volatility and ES=0.44
Figure A.3: Disaggregated consumption volatility and ES=0.17
Appendix B

Overview

Figure B.1: Impulse responses to one standard deviation shock to the nontradable sector – log-deviations from the steady state in % (tradable sector is shocked by square root of covariance between tradable and nontradable sectors)
Figure B.2: Impulse responses to one standard deviation shock to the nontradable sector – log-deviations from the steady state in % (tradable sector is shocked by square root of covariance between tradable and nontradable sectors)
Figure B.3: Impulse responses to one standard deviation shock to the nontradable sector – log-deviations from the steady state in % (tradable sector is shocked by square root of covariance between tradable and nontradable sectors)
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