ABSTRACT

New residential construction is significantly more procyclical in emerging markets than in developed countries, although the correlation between aggregate investment and output is similar across emerging and developed countries. This paper shows that a multi-sector stochastic growth model with a housing production sector can explain this fact. The key feature of the model is that housing demand depends on the cyclical behavior of consumption of tradable goods, which is much more volatile in emerging markets. Therefore, when a positive productivity shock hits the economy, the larger response of consumption of tradable goods implies that it is more attractive for consumers in emerging markets to purchase housing than it is for consumers in developed countries. This paper considers various factors that contribute to the large variability of consumption in emerging markets, and finds that larger trend growth rate shocks in emerging markets than in developed countries are quantitatively important. The reason is that a positive productivity shock signals even higher
productivity in the future with large growth rate shocks, so the current consumption response is large and the return to housing investment is high. While qualitatively the model matches the differences in the cyclicality of new residential construction across emerging markets and developed countries, quantitatively the model underestimates this comovement and the volatilities in housing investment in emerging markets. Furthermore, international interest rate shocks highly correlated with productivity shocks are very important in explaining the large swings in housing investment in emerging markets. Interest rate shocks work through three channels to affect housing investment: the direct 'mortgage rate' effect, the indirect effect through increasing non-housing consumption and the supply effect due to the working capital constraint. Quantitatively, the direct 'mortgage rate' effect is the most important channel. When the housing asset acts as collateral to reduce household's financing costs, it provides an empirically important mechanism to amplify and propagate interest rate shocks over the business cycle. The reason is that housing prices and interest rates reinforce each other to generate more procyclical housing investment and more volatile consumption and output.
ESSAYS ON HOUSING INVESTMENTS IN EMERGING MARKETS

By

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Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park, in partial fulfillment of the requirements for the degree of Doctor of Philosophy 2009

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Dedication

To my parents, my wife and my sons.
Acknowledgements

I am very grateful to Professor Carlos Vegh, Professor Enrique Mendoza and Professor John Shea for many valuable comments on content, structure and grammar of my dissertation. All errors are the author's.
# Table of Contents

Dedication ........................................................................................................................................................................................................................................................................................................................................ ii

Acknowledgements .............................................................................................................................................................................................................................................................................................................. iii

Table of Contents ................................................................................................................................................................................................................................................................................ iv

List of Tables ........................................................................................................................................................................................................................................................................................ vi

List of Figures .................................................................................................................................................................................................................................................................................. vii

Chapter 1: Housing Investments and Housing Prices in Emerging Markets ........................................... 1

1. Housing Investments and Housing Prices in Emerging Markets .......................................................... 1
2. Literature Review .................................................................................................................................. 6

Chapter 2: Housing and Business Cycles in Emerging Markets .............................................................. 10

1. Introduction ..................................................................................................................................... 10
2. Stylized Facts ............................................................................................................................. 16
   2.1 Data ....................................................................................................................................... 16
   2.2 Findings ............................................................................................................................... 18
3. The Model ...................................................................................................................................... 25
   3.1 Representative Household with Endogenous Discount Factor ............................................. 25
   3.2 Firms ....................................................................................................................................... 29
   3.3 Competitive Equilibrium ...................................................................................................... 30
4. Intuition for Model Behavior .......................................................................................................... 31
   4.1 Housing Demand .................................................................................................................. 34
   4.2 Factor Intensity and Housing Supply ...................................................................................... 38
   4.3 The Role of Small Open Economy Assumption .................................................................... 41
   4.4 The Role of Adjustment Costs ............................................................................................. 42
5. Quantitative Analysis ..................................................................................................................... 43
   5.1 Calibration ............................................................................................................................ 43
   5.2 Model Solution and Performance ........................................................................................ 48
   5.3 Impulse Response Function .................................................................................................. 50
   5.4 Sensitivity Analysis .............................................................................................................. 52
6. Conclusion ...................................................................................................................................... 58

Chapter 3: Interest Rate Shocks and Housing Investment Dynamics in Small Open Economies ............... 60

1. Introduction ..................................................................................................................................... 60
2. Stylized Facts ............................................................................................................................. 66
3. A Simple Open Economy Model ................................................................................................. 67
3.1 A Two-period Deterministic Small Open Economy Model ......................... 67
3.2 Closed v.s. Open Economy ......................................................................... 71
3.3 The Role of International Interest Rate Shocks .......................................... 73
4. A Infinite Time Horizon General Equilibrium Model........................................ 75
  4.1 Representative Household with Endogenous Discount Factor ..................... 75
  4.2 Firms ........................................................................................................... 78
  4.3 Competitive Equilibrium ........................................................................... 81
  4.4 Discussion on Theoretical Properties of the Model .................................... 81
5. Quantitative Results ..................................................................................... 84
  5.1 Calibration of the Model .......................................................................... 85
  5.2 Closed v.s. Open Economy ................................................................. 87
  5.3 Real Interest Rate Shocks: Exogenous v.s. Induced Country Premium ....... 89
  5.4 Supply Effect and Direct/Indirect Demand Effect ................................... 89
  5.5 Housing as Collateral to Reduce Financing Cost .................................... 92
6. Conclusion ..................................................................................................... 95
Appendices ......................................................................................................... 97
Bibliography ........................................................................................................ 100
<table>
<thead>
<tr>
<th>Table Number</th>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1A</td>
<td>Stylized Facts: Industrialized Countries</td>
<td>10</td>
</tr>
<tr>
<td>2.1B</td>
<td>Stylized Facts: Emerging Markets</td>
<td>10</td>
</tr>
<tr>
<td>2.2</td>
<td>Significance Test of Difference between Two Groups</td>
<td>11</td>
</tr>
<tr>
<td>2.3A</td>
<td>Housing Price and Stock Price in Korea</td>
<td>14</td>
</tr>
<tr>
<td>2.3B</td>
<td>Housing Price and Stock Price in Canada</td>
<td>14</td>
</tr>
<tr>
<td>2.4A</td>
<td>Baseline Calibration</td>
<td>27</td>
</tr>
<tr>
<td>2.4B</td>
<td>Productivity Shocks</td>
<td>29</td>
</tr>
<tr>
<td>2.4C</td>
<td>Growth Rate Shocks</td>
<td>29</td>
</tr>
<tr>
<td>2.5</td>
<td>Baseline Simulation Result</td>
<td>29</td>
</tr>
<tr>
<td>3.1</td>
<td>Correlation of Housing Investment with Real International Interest Rate</td>
<td>42</td>
</tr>
<tr>
<td>3.2</td>
<td>Comparison between Closed and Open Economy</td>
<td>55</td>
</tr>
<tr>
<td>3.3</td>
<td>Comparison between Exogenous and Induced Country Premium</td>
<td>55</td>
</tr>
<tr>
<td>3.4</td>
<td>Indirect Effect on Demand v.s. Direct Effect on Demand</td>
<td>57</td>
</tr>
<tr>
<td>3.5</td>
<td>Supply Effect: Working Capital Constraint</td>
<td>57</td>
</tr>
<tr>
<td>3.6</td>
<td>Country Premium Dependent on Housing Collateral</td>
<td>59</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Trade Balance-GDP correlation and Housing Construction-GDP Correlation…12

2.2 Housing Construction-GDP Correlation and Relative Volatility of
Consumption…………………………………………………………………………13

2.3 Housing Demand and Housing Supply……………………………………..20

2.4-6 Impulse Response Function………………………………………………31

2.7-14 Sensitivity Analysis………………………………………………………33
Chapter 1: Housing Investments and Housing Prices in Emerging Markets

1. Housing Investment and Housing Prices in Emerging Markets

Housing investment and housing prices are highly volatile and strongly procyclical in emerging markets. Housing investment is almost twice as procyclical in emerging markets as in developed countries. The rental price of housing services, measured by nominal rent deflated by CPI, is almost three times as volatile in emerging markets as in developed countries\(^1\). Available data also show that the housing price is much more volatile in emerging markets than in developed economies. For example, the housing price index in Korea is eight times as volatile as GDP and is roughly as volatile as stock price index. In contrast, in Canada housing prices are much less volatile than stock prices.

The most puzzling fact is that, on average, housing investment is significantly more procyclical in emerging markets than in developed countries, although the correlation between aggregate investment and output is similar across emerging and developed countries. Few attempts have been made to document and compare the statistical properties of housing investment in emerging markets and industrialized countries, and to explain them in an open economy model that incorporates interactions between

\(^1\)Other authors document that housing rent is the most volatile component in CPI during boom-bust cycles in emerging markets. For example, see Mendoza (2000).
domestic investment and international borrowing. And few studies have been done on housing investment dynamics in open economy models, in which the interaction between housing investment and business investment is fundamentally different than in closed economy models and the economies can be subject to external interest rate shocks besides productivity shocks.

This paper shows that a multi-sector stochastic growth model with a housing production sector can explain this fact. The key feature of the model is that housing demand depends on the cyclical behavior of consumption of tradable goods, which is much more volatile in emerging markets. Furthermore, compared to a closed economy model, the open economy model can account for a much larger portion of the observed comovement between housing investment and business investment. And international interest rate shocks highly correlated with productivity shocks are very important in explaining the large swings in housing investment in emerging markets.

The key to understanding these characteristics of the housing market in emerging markets lies in the dynamic properties of the housing demand, which in turn is a function of non-housing consumption and the mortgage rate. Empirical evidence shows that these two factors are important to explain the procyclicality and volatility of housing investment and prices in emerging markets.

Non-housing consumption increases the marginal utility of housing services. Thus, the higher the non-housing consumption, the higher the housing demands. In
emerging markets, output is twice as volatile as in developed countries. Consumption is more volatile than output in emerging markets, whereas consumption is less volatile in developed countries\(^2\). This implies that consumption is much more volatile in both absolute percentage change and relative to output in emerging markets than in developed countries. A larger change in consumption means that it is more attractive for consumers in emerging markets to purchase houses in booms than in developed countries. More procyclical housing demand leads to more procyclical and volatile housing investment.

Housing demand decreases in the mortgage rate. The financing cost of housing is directly determined by the mortgage rate, which in turn is an increasing function of the baseline interest rate in the economy and a decreasing function of the collateral value, i.e. the housing price. Furthermore, housing assets are often used as collateral to reduce financing costs of other consumption through a lower interest rate or easier access to credit. This provides a channel through which fluctuations in housing markets can spread to other sectors in the economy. In emerging markets, interest rates are more volatile and countercyclical than in developed countries. \(^3\). This implies more countercyclical housing financing costs and more procyclical housing demand. Anecdotal evidence also shows that sharp mortgage rate declines and large mortgage credit expansion are associated with economic boom\(^4\). In Mexico, after

\(^2\)See Neumeyer and Perri (2005) and Aguiar and Gopinath (2005) for details

\(^3\)See Chapter II and Neumeyer and Perri (2005) for details

\(^4\)For example, see Guerra's (1997) study on Mexico and Buckly (1991) on Argentina.
1991’s commercial bank liberalization, mortgage interest rate fell and mortgage loans grew fast in the bank sector’s portfolio. As a result, the mortgage debt to total portfolio ratio almost doubles between 1987 and 1994, from 10% to 19%. When the interest rate rose sharply in 1994, mortgage payments also rose sharply. This led to a very high delinquency rate on mortgage debt and the collapse of mortgage markets.

This essay aims to study the empirical characteristics of housing investment and to explain these facts in a modified small open economy business cycle model. Small open economy models are chosen for two reasons. First, most emerging economies are small open economies and are subject to large external shocks. Second, compared to closed economy models, open economy models can account for the observed strong comovements between housing investment and business investment. This is because housing investment can be financed through international borrowing in an open economy even when its rate of return is much lower than business investment. However, business investment crowd out housing investment in a closed economy when the return to business investment is higher.

Chapter II documents the statistical properties of housing investment in emerging markets and contrasts them with those in developed countries. In emerging markets, new housing investment is much more procyclical than that in developed countries, but business investment does not show very different cyclical features between emerging and developed countries. The rest of the chapter studies a multi-sector stochastic growth model with a housing sector to explain those stylized facts. In the model, housing demand depends on the path of tradable consumption, which is much
more volatile in emerging markets. Therefore, when a positive productivity shock hits the economy, it is more attractive for consumers in emerging markets to purchase housing than it is for consumers in developed countries. More procyclical housing demand in turn implies more procyclical housing investment.

Chapter III investigates the role of international interest rate shocks in explaining the volatility and cyclicality of housing investment. It first documents that real interest rates are highly negatively correlated with housing investment, business investment and output in emerging markets. However, real interest rates are only slightly negatively or even positively correlated with output, housing investment and business investment in developed countries. This paper finds that international interest rate shocks highly correlated with productivity shocks are very important in explaining the large swings in housing investment in emerging markets. Interest rate shocks work through three channels to affect housing investment: the direct 'mortgage rate' effect, the indirect effect through increasing non-housing consumption and the supply effect due to working capital constraints. Quantitatively, the direct 'mortgage rate' effect is the most important channel. Furthermore, when the housing asset acts as collateral to reduce household's financing costs, it provides an empirically important mechanism to amplify and propagate interest rate shocks over the business cycles. The reason is that housing prices and interest rates reinforce with each other to generate more procyclical housing investment and more volatile consumption and outputs.
2. Literature Review

There are several strands of literature that study housing investment over the business cycle. The first strand of literature tries to explain housing investment dynamics in a closed economy using real business cycle models. Fisher (1997) studies a modified RBC model with home production and complimentarily in production of business investment and housing investment. The economy is subject to sector specific technology shocks and reallocating resources between production of business investment and housing investment is costly. These assumptions mitigate the tendency to substitute factors of production in one sector for the other one, thus it is optimal to produce housing and business investment at the same time. Therefore, the model can generate a positive correlation between housing investment and output if the sector specific technology shocks are relatively small compared to aggregate technology shocks. This model helps to explain the comovement between housing investment and business investment observed in the data. However, it crucially relies on the existence of a strong aggregate productivity shock that is common to both sectors and it fails to explain the relative volatility of housing investment and business investment.

Davis and Heathcote (2005) studies a multi-sector stochastic growth model with intermediate production sectors. Production of final consumption goods, business investment and housing investment uses construction, manufactures and services as intermediate inputs. This model can generate positive comovement because technology shocks occur in the intermediate goods production. Thus highly correlated...
'productivity' changes in the final good production sector occur due to higher productivity in the intermediate goods production. In addition, technology shocks are labor-augmented, thus more favorable to the more labor intensive construction sector. However, these assumptions are not quite empirically supported, because several papers find slower productivity growth in construction sector than in manufacture sector.

The second strand of literature studies monetary business cycle models and emphasizes the role of housing as collateral assets. Iacoviello (2005) analyzes a monetary business model with nominal debt and borrowing constraints determined by real estate collateral. Housing price shocks generate a positive response in household spending through a more favorable borrowing condition due to higher collateral value. But the paper doesn't study the original source of housing price changes. Jin and Zeng (2004) study a 3-sector economy and emphasize a liquidity channel through which monetary policy affects housing investment. Monacelli (2006) uses a similar setup but studies the optimal monetary policy. Another strand of literature shows the importance of land and structures as collateral assets to borrow from abroad, for example Mendoza (2000).

The third strand of related literature studies business cycles and housing investment in open economies. The earliest paper to study housing investment in an open economy is Matsuyama (1989), which shows the effect of fiscal policy on housing investment and the current account. In his deterministic small open economy model, because of
non-separability of housing consumption and non-housing consumption in the utility function, household's consumption and investment decisions, in particular housing investment, cannot be separated as in a economy with only consumption and business investment. This means that Fisherian separation theorem fails. Thus the housing stock accumulation will be affected by changes in government purchases in a model with residential investment.

Another literature shows that interest rate shocks have large impacts on housing dynamics in a class of closed economy general equilibrium models. For example, Erceg and Levin (2005) shows that the response of housing investment to interest rate shocks is ten times as much as that of nondurable consumption. This has important implications for this essay because fluctuation of the exogenously determined real interest rate in the open economy model could generate large and procyclical movements in housing investment. Since the cost of financing investments with different rate of returns are the key factor that generates comovement between housing investment and business investment in an open economy, shocks to the interest rate in international bond market have critical impact on the dynamics of heterogeneous investments in the theoretical model.

Furthermore, since housing consumption is a type of durable good, this essay is also related to the literature that studies durable good consumption in emerging markets. This strand of literature is aimed at explaining the ERBS phenomenon emphasizes the role of durable goods consumption during different ERBS programs. De
Gregorio, Guidotti, and Vegh (1998) analyzes the 'bunching' purchase behavior of durable goods which drives the initial consumption boom. Buffie and Atolia (2005) combine the policy non-credibility and price stickiness in a model which features both durable and nondurable goods to get a better quantitative match of the key macroeconomic dynamics. However, their focus is limited to the consumption of durable goods (largely tradable durable goods) during ERBS in developing countries.
Chapter 2: Housing and Business Cycles in Emerging Markets

1. Introduction

Highly volatile and procyclical housing investment is one of the key features of emerging markets business cycles, which are often accompanied by large swings in capital flows and the balance of trade. But few attempts have been made to document the statistical properties of housing investment in emerging markets, and to explain them in an open economy model that incorporates interactions between domestic investment and international borrowing. Understanding the business cycle behavior of the housing sector in emerging markets is important not only because housing investment is large\(^5\) and very volatile\(^6\) in national accounts, but also because it contributes to financial and economic crisis. Since banks in emerging markets hold a large amount of loans to the housing sector, large fluctuations of real estate price may cause a substantial increase in bad loans, which in turn may lead to a financial and economic crisis.

\(^5\)Housing investment accounts for approximately 1/3 of gross fixed capital formation. And housing is the largest asset held by most households and accounts for almost one half of the total fixed capital stock. (Estimates are based on OECD National Accounts data and data from central banks.)

\(^6\)Housing investment is much more volatile than non-housing investment. And the fluctuation in housing cost is the most prominent part of the CPI during exchange rate based stabilization episodes in emerging markets. (See De Gregorio, Guidotti, and Vech (1998) and Mendoza (2000).)
This paper has two key objectives. First, it documents key cyclical features of the production and price of housing in small open economies and compares those characteristics between emerging markets and industrialized countries. The second objective is to explain the difference between the two groups of countries with respect to the dynamics of housing production and prices in a multi-sector general equilibrium open economy model.

The empirical analysis shows that the cyclicality of residential construction is drastically different between emerging markets and developed countries. In particular, the residential construction is much more strongly procyclical in emerging markets than in developed countries. On average, the correlation between residential construction and output is 0.63 in emerging markets but it is only 0.25 in developed countries. However, overall investment is only slightly more procyclical in emerging markets than in developed countries. Another empirical finding is that emerging markets tend to have much more countercyclical trade balance than developed countries.\(^7\) In emerging markets, the balance of trade is strongly countercyclical, while it is moderately countercyclical in developed countries.

The large difference between emerging markets and developed countries in cyclicality of new residential construction, in sharp contrast to the little difference in cyclicality of overall investment, is puzzling because residential construction is a

\(^7\)This stylized fact is also documented by Numeyuer and Perri (2005) and Aguiar and Gopinath (2007).
good measure of housing investment.8 Then the question is why the cyclicality is so different for housing investment. Another related interesting question is whether more procyclical housing construction helps to explain that the trade balance/GDP ratio is more countercyclical in emerging markets. Intuitively, this could be true because housing is nontradable by nature. The more procyclical construction means that the nontradable housing sector absorbs more resources during booms and releases resources to tradable sectors during recessions.

The theoretical contribution of this paper is to present a quantitative small open economy model with a construction sector and housing consumption in the utility function to explain the difference in housing construction and the trade balance between emerging markets and developed countries. It is worth knowing whether this modified small open economy RBC model can explain the empirical findings in the first section. The key feature of the model is that housing demand depends on the cyclical behavior of consumption of tradable goods, which is much more volatile in emerging markets. Therefore, when a positive productivity shock hits the economy, the larger response of consumption of tradable goods implies that it is more attractive for consumers in emerging markets to purchase housing than it is for consumers in developed countries. This paper considers various factors that contribute to the large variability of consumption in emerging markets, and finds that larger trend growth

8Residential investment in national accounts is the sum of new construction, improvements, brokers' commissions and some types of equipment that are built into residential structures, such as heating and air-conditioning equipment. However, since new residential construction constitutes over 90 percent of residential investment and is more precisely measured, it is often used as a substitute for residential investment.
rate shocks in emerging markets than in developed countries are quantitatively important. The reason is that a positive productivity shock signals even higher productivity in the future with large growth rate shocks, so the current consumption response is large and the return to housing investment is high.

In examining the role of housing in emerging markets business cycles, it is important to consider housing both as a durable and a nontradable good. It is a durable good, so consumers get utility from the service flow from its stock rather than just the new investment. Thus the intertemporal elasticity of substitution for new purchases of housing is much higher than that of nondurable goods. Furthermore, existing analyzes of the relative price of nontradables in terms of tradables assume that both types of goods are nondurable. However, housing consumption, which constitutes a large proportion of a consumer's expenditure, is a nontradable good by nature. So it is important to see whether the behavior of the key price in a small open economy, i.e. the relative price of nontradables, is different if we introduce durability into the model. In addition, it is nontradable by nature so its supply must be met by demand on the domestic market, which means that new housing construction may have an important impact on external balance of an open economy since resources must be shifted to the housing construction sector. Therefore, the inclusion of housing in modeling a small open economy may be important in understanding business cycles in emerging markets.
It is also important to study a open economy model because, compared to a closed economy model, a strong positive correlation between housing investment and output at business cycle frequencies naturally arises in a small open economy model. In particular, the exogenously determined interest rate reduces substitution among different investments. In an open economy, domestic expenditures do not need to add up to the total output since trade balance can adjust to meet any domestic demand that exceeds domestic production. Therefore, when a favorable productivity shock hits the economy, increased rates of return from investing in both housing and business capital, although different, cause both categories of investment to rise. In contrast, in a closed economy, even a small difference in the rate of return from different investments tends to generate negative comovement among them because of substitution toward more productive investment. So it is interesting to see how housing investment dynamics generated from an open economy model differ from those in a closed economy model.

This paper is related to three strands of literature. First, the recent macroeconomics and finance literature has attempted to empirically document and theoretically explain the link among housing, consumption, asset prices and business cycles in industrialized countries. Davis and Heathcote (2005) document the important difference between business and residential investment dynamics in U.S. business cycles, and explain the observed facts through a multi-sector real business cycle model. Lustig (2004), Davis and Martin (2005) and Jaccard (2007) study asset return and risk premium in models with housing consumption. Fisher (2007) explains why
the housing investment leads other investments by introducing complementarity between the housing and business capital. However, all of this literature confines itself to closed-economy models and thus cannot account for the facts that distinguish industrial countries and emerging markets in the aspect of housing investment.

In the international business cycle literature, the earliest paper to study housing investment in an open economy is Matsuyama (1989), who investigates the effect of fiscal policy on housing investment and the current account. Erceg and Levin (1995) study a model with a construction sector and a distribution sector to explain structures investment dynamics. Another strand of literature shows the importance of land and structures as collateral assets to borrow from abroad, for example Mendoza (2000, 2003 and 2006), and Punzi (2006). However, these papers do not document and explain stylized facts of housing construction over the business cycle in emerging markets.

There is another strand of literature, aimed at explaining the ERBS phenomenon, which emphasizes the role of durable goods consumption patterns during different ERBS programs. De Gregorio, Guidotti, and Vegh (1998) analyze the 'bunching' purchase behavior of durable goods which drives the initial consumption boom. Buffie and Atolia (2005) combine policy non-credibility and price stickiness in a model that features both durable goods and nondurable goods to get a better quantitative match of the key macroeconomic dynamics. However, their focus is
limited to the consumption of durable goods (largely tradable durable goods) during ERBS in developing countries.

The paper is organized as follows. In part 2, I document stylized facts on volatility and cyclicality of housing production and the real housing price in developing countries compared with those in developed countries. In part 3, I outline a multi-sector stochastic growth model with housing production to explain the procyclicality and volatility difference between industrial and emerging markets. Part 4 shows numerical results and part V concludes.

2. Stylized Facts

2.1 Data

The data I use to compute business cycle properties are from the OECD Statistics Compendium and countries' central banks. The sample is quarterly and ranges from 1990 first quarter to 2005 fourth quarter. Specifically, residential construction data on OECD countries is from the OECD Statistics Compendium, Main Economic Indicators. Residential construction data for other countries are from central banks. I use new residential construction data rather than residential investment data for two reasons. First, residential investment data are not available for most emerging economies, while residential construction data are available in a lot of emerging countries and OECD data provides a comparable residential construction index for most OECD countries. In addition, since housing price data are generally of low quality, especially in emerging economies, using a residential construction quantity
index avoids the potential measurement error from deflating nominal residential investment data. Second, because new residential construction constitutes a dominant majority part of (over 90% on average) residential investment, it is often used to represent new housing investment in the housing literature. Empirical evidence shows that depreciation rate of residential capital is much lower than business capital, so ignoring the maintenance part of investment should not be a serious problem in calculating the statistical properties of housing investment. Therefore, although the business cycle properties of new residential construction are not exactly the same as those for the residential investment data reported by most of developed economies, new residential construction is adequate for the purpose of this paper measure since the purpose of this paper.

Investment, GDP and trade balance data are from OECD and IFS. Rental prices of housing are taken from the CPI housing (rent) index data from the OECD Main Economic Indicators or from central banks of respective countries.

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9This is because the residential construction quantity index is calculated based on the construction area or number of rooms.

10The correlation between new residential construction and residential investment at business cycle frequencies is on average high. In the data, it is 0.75.

11Rent data are not directly comparable across countries since some countries only report imputed rents and others only report actual rents. However, to simplify the analysis, the theoretical model in this paper does not differentiate between owning a house and renting it, so both data could be consistent with the model.
2.2 Findings

I use seasonally adjusted per capita new residential construction to represent new housing output (denoted as `ctr' in Table 1) in an economy. I apply the HP-filter with a parameter of 1600 to the series and measure the standard deviation of the filtered series and its correlation with the HP-filtered GDP series (denoted as y in Table 1). Those are shown in the first and third column in Table 1. Similar moments are also calculated for seasonally adjusted constant price series of real investment (denoted as I in Table 1) and the trade balance/GDP ratio (denoted as TB/y in Table 1).

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<td>0.70</td>
<td>5.08</td>
<td>0.80</td>
<td>-</td>
</tr>
<tr>
<td>UK</td>
<td>0.06</td>
<td>0.64</td>
<td>7.54</td>
<td>4.35</td>
<td>-0.53</td>
</tr>
<tr>
<td>US</td>
<td>0.32</td>
<td>0.94</td>
<td>7.99</td>
<td>0.65</td>
<td>-0.51</td>
</tr>
<tr>
<td>Avg</td>
<td>0.25</td>
<td>0.70</td>
<td>7.08</td>
<td>1.54</td>
<td>-0.20</td>
</tr>
</tbody>
</table>
### Table 1B: Stylized Facts: Emerging Markets

<table>
<thead>
<tr>
<th></th>
<th>corr(ctr,y)</th>
<th>corr(1,y)</th>
<th>std(ctr)/std(y)</th>
<th>Std(Rent)</th>
<th>corr(TB/y,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>0.97</td>
<td>0.91</td>
<td>2.96</td>
<td>7.35</td>
<td>-0.70</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.73</td>
<td>0.87</td>
<td>2.42</td>
<td>-</td>
<td>0.01</td>
</tr>
<tr>
<td>Israel</td>
<td>0.37</td>
<td>0.67</td>
<td>4.55</td>
<td>-</td>
<td>-0.01</td>
</tr>
<tr>
<td>Korea</td>
<td>0.47</td>
<td>0.95</td>
<td>10.08</td>
<td>1.52</td>
<td>-0.61</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.87</td>
<td>0.90</td>
<td>2.70</td>
<td>8.67</td>
<td>-0.74</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.42</td>
<td>0.35</td>
<td>10.56</td>
<td>3.14</td>
<td>-0.54</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.88</td>
<td>0.96</td>
<td>3.29</td>
<td>-</td>
<td>-0.83</td>
</tr>
<tr>
<td>Turkey</td>
<td>0.30</td>
<td>0.84</td>
<td>5.40</td>
<td>5.82</td>
<td>-0.69</td>
</tr>
<tr>
<td><strong>Avg</strong></td>
<td><strong>0.63</strong></td>
<td><strong>0.81</strong></td>
<td><strong>5.25</strong></td>
<td><strong>5.30</strong></td>
<td><strong>-0.51</strong></td>
</tr>
</tbody>
</table>

The most interesting fact is that, on average, construction is much more (almost twice) procyclical in emerging markets than in developed countries. In small developed countries, the average correlation between detrended new housing construction and detrended GDP is 0.25, but in emerging markets the average is 0.63. In Table 2, I perform a test of equality of means and an analysis of variance between the two groups and within groups. The null hypothesis (equality of the estimated correlation between ctr and y between the two groups) is rejected at the 1% level, and the between group variance is ten times as large as the within group variance.
Table 2: Significance Test of Difference Between Two Groups

<table>
<thead>
<tr>
<th>Method</th>
<th>df</th>
<th>Value</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-test</td>
<td>15</td>
<td>3.03382</td>
<td>0.0084</td>
</tr>
<tr>
<td>F-statistic</td>
<td>(1, 15)</td>
<td>9.201408</td>
<td>0.0084</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>df</th>
<th>Mean Sq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>1</td>
<td>0.529169</td>
</tr>
<tr>
<td>Within</td>
<td>15</td>
<td>0.057510</td>
</tr>
</tbody>
</table>

In sharp contrast to new housing construction, overall investment is only slightly more procyclical in emerging markets. In developed countries, the average correlation between I and y is 0.70, while in emerging economies it is 0.81.\(^{12}\)

This finding is striking because new housing construction can be considered as a good approximation of housing investment. A natural question is why the difference in cyclicality of housing investment between the two groups is so large. Since per capita data is used to calculate correlations at the business cycle frequency, the effect of low frequency demographic change on the correlation is minimal. Different institutions and different stages of development of the housing mortgage market may be important factors, but before modeling such factors it is worth asking if a frictionless real business cycle model can explain these differences qualitatively and quantitatively. This consideration motivates the model in section III.

\(^{12}\)A panel estimation with fixed effects will not be helpful here since the test of equality of means and variance analysis are enough to show the difference of housing construction dynamics between two groups of countries.
Another fact from Table 1 is that countries with more procyclical new housing construction also tend to have a more countercyclical trade balance. This feature can be easily seen in figure 1. In figure 1, I plot the trade balance-GDP correlation against the housing construction-GDP correlation. The downward sloping line shows the simple regression of the former on the latter. Given that new housing is nontradable good, the negative correlation between the two variables is not surprising, but it points at another question that we will answer in section III: how much does new housing construction contribute to the fact that the trade balance/GDP ratio is more countercyclical in emerging markets?

**Figure 1 : Trade Balance – GDP correlation and Housing Construction – GDP Correlation**

Note: Horizontal axis represents correlation between housing construction and GDP, vertical axis represents correlation between TB/GDP and GDP.
An important fact aside from Table 1 is that countries with more procyclical new housing construction also tend to have higher consumption volatility relative to output volatility. This feature can be easily seen in figure 2. This implies that consumption is much more volatile in both absolute percentage change and relative to output in emerging markets than in developed countries. Actually, consumption is more volatile than output in emerging markets, whereas consumption is less volatile in developed countries\textsuperscript{13}. A larger change in consumption means that it is more attractive for consumers in emerging markets to purchase houses in booms than in developed countries. More procyclical housing demand leads to more procyclical and volatile housing investment. This fact supports the point that a more volatile non-housing consumption in emerging markets is an empirically important factor that helps to explain the strongly procyclical housing investment in emerging markets.

\textsuperscript{13}See Neumeyer and Perri (2005) and Aguiar and Gopinath (2005) for details
Note: Horizontal axis represents correlation between housing construction and GDP, vertical axis represents the relative volatility of consumption to GDP.

Furthermore, in Table 1, the real rents (nominal rents divided by CPI) are almost three times as volatile in emerging markets as in developed countries. This means that the relative price of housing is much more volatile in emerging markets. Two related and maybe more important questions are whether the housing price is more volatile in emerging markets, and how the behavior of the housing price compares to that of equity prices. Although available data does not allow us to draw any robust pattern, I document in Table 3.1 and 3.2 a preliminary but interesting comparison between the cyclical characteristics of the housing price and the stock price index in Korea and Canada. In Korea, the volatility of the housing price index and the stock price index
are roughly the same, and they are roughly eight times more volatile than GDP, while the housing price is more persistent and less procyclical than the stock price index. However, in Canada the housing price is much less volatile than stock prices, although the housing price is more persistent and less procyclical than the stock price index.

Table 3A Housing Price and Stock Price in Korea

<table>
<thead>
<tr>
<th></th>
<th>Real Housing Price</th>
<th>Real KOSPI</th>
<th>Real Rent</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>18.1</td>
<td>20.8</td>
<td>1.51</td>
<td>2.58</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.91</td>
<td>0.79</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>Correlation w/ GDP</td>
<td>0.21</td>
<td>0.55</td>
<td>0.28</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3B Housing Price and Stock Price in Canada

<table>
<thead>
<tr>
<th></th>
<th>Real Housing Price</th>
<th>Real Stock Price</th>
<th>Real Rent</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>2.03</td>
<td>10.12</td>
<td>0.80</td>
<td>1.19</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.39</td>
<td>0.79</td>
<td>0.77</td>
<td>0.90</td>
</tr>
<tr>
<td>Correlation w/ GDP</td>
<td>0.09</td>
<td>0.42</td>
<td>-0.21</td>
<td>1</td>
</tr>
</tbody>
</table>
3. The Model

In this section, I present a modified small open economy real business cycle model. The key departure from the standard two-sector small open economy model is that housing consumption is explicit in the household's utility and a housing construction sector is introduced. Specifically, housing is introduced as a *nontradable durable* good in the household's utility function. The household gets utility from service flows generated by the currently owned housing stock, nontradable goods other than housing services, and tradable goods. Therefore, consumers are smoothing services from the housing stock rather than new housing purchases, which behave more like investment than consumption goods. Another key feature is a construction sector that produces new housing, distinct from the sector that produces tradable goods using capital and labor as inputs. The allocation of capital and labor between these sectors is a key determinant of the dynamics of housing construction over the business cycle.

3.1 Representative Household with Endogenous Discount Factor

The representative household's expected lifetime utility is given by:

$$ U = E_0 \sum_{t=0}^{\infty} \rho_t u(C_t, l_t). $$

(2.1)

Where $\rho_t/\rho_{t-1}$ is the household's time varying subjective discount factor\(^{14}\) which follows the law of motion:

\(^{14}\text{This simplified version of Uzawa preferences is introduced to make the model's steady state independent of initial conditions. In this specification, the discount factor, which the individual household takes as given, is assumed to be a decreasing function of average per capita consumption of}$$
\[ \rho_{t+1} = \rho_t \beta(C_t^a, l_t^a). \] (2.2)

where \( C_t^a \) and \( l_t^a \) are average per capital consumption and hours.

The utility function and endogenous discount factor take the following form\(^{15}\):

\[ u(C_t, l_t) = \frac{(C_t - \nu \Gamma_{t-1} l_t^{\omega}/\omega)^{1-\sigma}}{1-\sigma}. \] (2.3)

\[ \beta(C_t^a, l_t^a) = [1 + C_t^a - \nu \Gamma_{t-1} l_t^a/\omega]^{\beta}. \] (2.4)

where \( \sigma \) is the inverse elasticity of intertemporal substitution, \( 1 - \nu \) is leisure, \( \omega \) determines the elasticity of labor supply and \( \nu \) determines the amount of leisure in steady state. \( C_t^a \) is aggregate consumption (CES aggregator), defined as:

\[ C_t = \left[ (1-s)^{1/q} (c_t^{N (1-\alpha)} c_t^{P_\alpha})^{\alpha} + s^{1/q} (H_t)^{\frac{\alpha-1}{q}} \right]^{\frac{q}{\alpha q-1}}. \] (2.5)

\( c_t^T \) is nondurable tradable consumption. \( H_t \) represents the housing stock owned by the household\(^{16}\); the housing service flow is assumed to be proportional to the

\(^{15}\)The cumulative labor productivity \( \Gamma_{t-1} \) is introduced into the utility function to make steady state leisure consumption a constant share of time endowment.

\(^{16}\)This assumption assumes away a rental housing market. Indeed, there is no difference between owning and renting a house period by period in the model. Since this paper does not model any
housing stock $H_t$ and the proportionality factor is set to one. $s$ is the housing consumption share parameter. $c_{t}^{N}$ is the consumption of a nondurable and nontradable good $c_{t}^{N}$, which is assumed to be a nonproduced endowment. $\eta$ is one of the key parameters in the model. It represents the elasticity of substitution between housing and nonhousing consumption, which include tradable goods and other nontradable goods.

The budget constraint of the representative household is given by:

$$b_{t+1} = R b_t + w_t I_t + u_t^T k_t^T + u_t^H k_t^H + q_t (1 - \delta_H) H_t$$
$$-c_t - I_t - q_t H_{t+1} - \frac{d_t^H}{2} (H_{t+1} - H_t)^2 / H_t + p_t^N y_t^N - p_t^N c_t^N .$$

(2.6)

Capital stock evolves according to:

$$k_{t+1}^T = (1 - \delta_k) k_t^T + I_t^T - \frac{d_t^T}{2} (k_{t+1}^T - k_t^T)^2 / k_t^T .$$

$$k_{t+1}^H = (1 - \delta_k) k_t^H + I_t^H - \frac{d_t^H}{2} (k_{t+1}^H - k_t^H)^2 / k_t^H .$$

(2.7) (2.8) (2.9)

$$I_t = I_t^T + I_t^H .$$

This budget constraint has some nonstandard features due to the availability of housing as another asset and consumption good. First, the household can purchase housing stock $H_t$ at price $q_t$ at the end of period $t - 1$ and consume its service during period $t$. Housing stock depreciates at rate $\delta_H$. The household can sell the financial friction, explicitly modeling a rental market should not change the main results.

17Introducing $c_{t}^{N}$ as an endogenous variable which is produced by using labor and capital does not change the key result on housing investment but require tracking more variables.
undepreciated housing stock at the end of period $t$. Secondly, I assume that changing the housing stock is subject to a convex adjustment cost, which can be justified by the empirical observation that changing the housing stock takes time and the larger the change, the more time and effort are required.\footnote{The introduction of adjustment costs in housing consumption generates realistic housing investment volatility and helps to generate positive comovement between housing investment and output.}

Other parts of the constraint are standard: the representative household supplys labor $l_t$, and rents capital $k^T_t$ and $k^H_t$ to firms in competitive labor and capital markets at prices $w_t$, $u^T_t$ and $u^H_t$, respectively, where the tradable and housing sectors have distinct capital stocks. It also trades its nontradable endowment $y^N_t$ at price $p^N_t$.

In every period, households borrow or lend in the international capital market by trading real bonds $b_t$ at interest rate $r_t$. Finally, households choose the next period's capital stocks $k^T_{t+1}$ and $k^H_{t+1}$, which depreciate at the same rate $\delta_k$ and are subject to convex adjustment costs. There are two reasons why I introduce adjustment costs into the model. Firstly, adjustment costs are commonly used in the open economy RBC literature to generate realistic aggregate investment volatility without changing the model's perfect foresight steady state. Secondly, adjustment costs in reallocating capital across sectors help to induce positive comovement of production across sectors.\footnote{When reallocating capital stock across sectors is costless, the model cannot generate realistic comovement across sectors, but the model can still generate higher procyclicality of housing investment in emerging markets than in developed countries.}
3.2 Firms

Representative firms in a competitive market rent capital and labor from households and produce two types of goods, a durable nontradable good (i.e. housing) and a tradable good. In every period, firms maximize the profits from production of both goods:

\[ y^T_t + q_t y^H_t - \left( u^T_t k^T_t + u^H_t k^H_t \right) - w_t (l^T_t + l^H_t) \]  \quad (2.10)

There are no frictions (transportation cost, distribution cost or markup, etc) in the international tradable goods market, so its price is assumed to be one at all times. The relative price of housing in terms of tradable good is \( q_t \). Firms rent capital from the households at price \( u^T_t \) and \( u^H_t \) in the two sectors and rent labor at the competitive price \( w_t \).

Production technologies take the Cobb-Douglas form:

\[ y^T_t = A^T_t \left( k^T_t \right)^{1-\phi} \left( l^T_t \Gamma_t \right)^\phi \]  \quad (2.11)

\[ y^H_t = A^H_t \left( k^H_t \right)^{\phi} \left( l^H_t \Gamma_t \right)^{1-\phi} \]  \quad (2.12)

Where \( \phi \) and \( \sigma \) are labor intensities in new housing construction and tradable production, respectively. To be consistent with empirical observation on factor intensity in both emerging markets and industrialized countries, \( \phi \) is assumed to be larger than \( \sigma \). \( A^H_t \) and \( A^T_t \) represent total factor productivity and are exogenous.
random shocks in the two sectors, while $\Gamma_t$ is stochastic labor productivity growth, with a mean of one and identical in both sectors.  

3.3 Competitive Equilibrium

A competitive equilibrium in this economy is characterized by the sequences of allocations $\{ b_{t+1}, H_{t+1}, k^T_{t+1}, k^H_{t+1}, l_t, l^T_t, l^H_t, c^T_t, c^H_t, y^T_t, y^H_t, y^M_t, x_t \}$ and sequences of prices $\{ q_t, \rho^N_t, w_t, u^T_t, u^H_t, \lambda_t \}$ that satisfy household and firms optimality conditions described in (2.6)(2.7)(2.8)(2.9)(2.10) and (2.14)(2.15)(2.16)(2.17) subject to budget constraints (2.4)(2.5) and production technology (2.12)(2.13), and the following market clearing conditions:

Capital market clearing:

$$I_t = I^T_t + I^H_t. \quad (2.13)$$

Labor market clearing:

$$l_t = l^T_t + l^H_t. \quad (2.14)$$

Housing market clearing:

$$y^H_t = H_{t+1} - (1 - \delta_H)H_t. \quad (2.15)$$

 Tradable good market clearing:

20 $\Gamma_t$ is assumed to be one in the benchmark model. But it represents labor productivity growth shocks when investigating the model's response to different shocks.
\[ b_{t+1} = R_t b_t + y^T_{t} - c_t - I_t - \frac{d}{2}(k^T_{t+1} - k^T_t)^2 / k^T_t \]

\[ -\frac{d}{2}(k^H_{t+1} - k^H_t)^2 / k^H_t - \frac{d_{nt}}{2}(H^H_{t+1} - H^H_t)^2 / H^H_t \]. \hspace{1cm} (2.16)

Nonhousing nontradable good market clearing:

\[ y^N_t = c^N_t. \] \hspace{1cm} (2.17)

Given those optimality conditions\(^{21}\), the budget constraints (6)(7) and market clearing condition (12)-(16), I solve the model by using a second order approximation around perfect foresight steady state and report quantitative results in section 4.

**4. Intuition for Model Behavior**

The dynamics of new housing construction are determined by the interaction between housing supply and housing demand, both of which are different from supply and demand of nondurable goods and even other durable goods. On the demand side, housing demand dynamics depend in a nonlinear way on the path of tradable and nonhousing nontradable goods. However, the dependence hinges on the magnitude of the elasticity of substitution between housing and aggregate nondurable goods (tradable goods and other nontradable goods). On the supply side, the magnitude of the factor intensity difference between tradable goods and nontradable goods determines the supply elasticity of new housing.

\(^{21}\)See Appendix 1.
In particular, when the economy has a favorable technology shock to both sectors, demand for aggregate consumption rises and so does the demand for housing. However, this increase in demand is mitigated by adjustment costs and an increase in the housing price, which in turn negatively depends on the elasticity of housing supply. In the calibrated model, productivity shocks in tradable sector are larger than in the housing construction sector, there is a strong tendency for labor and capital to be reallocated from less productive housing production to tradable production. This reallocation inventive implies that the housing supply curve actually shifts up in response to the simultaneous productivity shocks hitting both sectors. In developed countries, the small response of consumption to productivity shocks implies that the demand curve shifts up only slightly. Therefore, housing investment does not show strong cyclicality. In sharp contrast, emerging markets feature highly volatile consumption. This large response of consumption to productivity shocks implies a large shift in the housing demand curve, which generates strongly procyclical and volatile housing investment.

The mechanism can be easily seen in the following illustrative diagram. Initial equilibrium residential construction is determined by the intersection of the initial supply and demand curves. Following the productivity shocks to both sectors, the supply curve shifts up. If the demand curve shifts by a small amount to $D'$, residential construction does not increase much and could even decline. However, if
the shift of demand curve is large ($D'$), residential construction increases by a large margin and is thus very procyclical and volatile.

Figure 3. Housing Demand and Housing Supply

To make the mechanism more transparent, the following subsection discusses properties of housing demand and supply in detail. In particular, while this paper uses a second order approximation to solve the model, key properties of the quantitative results can be seen from the log-linearized first order conditions.
4.1 Housing Demand

Housing is different from other consumption goods in some crucial aspects. First, it is a durable good and is much more durable than other nonhousing durables like cars and furniture. Explicitly modeling housing in standard real business cycle model introduces persistence in marginal utility and therefore helps to explain equity premium puzzle which is hard to generate in standard RBC models. (See Davis and Martin (2005) for a detailed review of related literature.) Second, housing provides another way of saving and provides insurance against income shocks. Therefore, new housing purchases can be characterized as investment. Meanwhile, there is a non-negligible adjustment cost associated with housing investment.

However, housing differs from nonhousing investment in several important respects. First, it enters into the consumer's utility function directly. The introduction of durable goods equips the consumer with another way to substitute consumption intertemporally. This is because households get utility from the service flow generated by the stock rather than from new housing construction, so that even large fluctuations in housing investment will not affect marginal utility too much. Second, housing stock can only generate nontradable housing services, and housing stock is nontradable itself. Unlike business investments and other durable goods (cars, furniture, etc), housing is nontradable and its supply elasticity is therefore less than that for tradable investment. Third, the housing depreciation rate is significantly lower than that of other durables and business investment.
Those differences imply different dynamic properties of new housing purchases from that of other investment. In a standard small open economy model, since the rate of return of capital in domestic production must be equal to the world interest rate, the economy's investment decision only affected by household consumption through the channel of labor supply. However, the same argument does not apply to housing investment. As pointed out by Matsuyama (1990), given that there is an income effect on consumer's consumption of housing, housing investment crucially depends on the household's consumption decision. This result is also the key to understanding the difference between new housing purchases in emerging markets and developed economies. This point is easy to see from the demand function for housing.

Assuming zero adjustment costs, combine optimality conditions (24), (25), (26), (27) and (30), and substitute forward to get the following non-bubble solution for the housing price:

\[
q_i = E_i \left\{ \sum_{t = i}^{T} \left( \frac{s(c_{i, t-1}^{N(1-a)} c_{i, t})}{(1-s)H_i} \right)^{\eta} \lambda_{i-t} \lambda_i \rho_{t} \right\}. \tag{2.18}
\]

This expression for the relative housing price is similar to a standard capital price formula. The real housing price is the sum of the discounted stream of 'profits', in this case implicit rents, from owning a unit of housing. But these implicit rents depend on the path of tradable consumption and the intratemporal elasticity of substitution.
In particular, there are two factors that have important effects on housing demand. The first one is the magnitude of responses of consumption to productivity shocks. Larger shocks cause a larger response of tradable consumption $c^T$, which in turn means a higher marginal utility of housing and thus increase in the housing demand. Since emerging markets feature much more volatile and slightly more procyclical consumption than developed countries do, the housing demand is also more volatile and procyclical in emerging markets. Higher housing demand drives the housing price up and attracts more labor and capital to the housing construction sector.

The second key parameter that makes the difference in housing demand between emerging markets and developed countries is the elasticity of substitution $\eta$ between housing and nonhousing consumption. Given the path of tradable consumption, the higher the elasticity of substitution, the lower the degree of dependence of housing demand on the tradable consumption. In the extreme case, when $\eta$ goes to infinity, the housing price formula collapses to a standard Tobin's q expression and housing demand behaves more like business investment. In this case, new housing investment largely depends on the supply side, i.e. the relative magnitude of the productivity shocks in the two sectors and the adjustment costs associated with reallocating capital between these two sectors. In contrast, when $\eta$ is very low, nontradable and tradable consumption are strong complements. In this case the demand for the housing also follows tradable consumption closely and tends to be very procyclical. Quantitative analysis in section IV shows that housing investment is slightly more procyclical.
when $\eta$ increases from the lower range of empirically plausible values to the upper range.

To fix ideas, we consider the properties of a log-linearized version of the model. Assume the following form of new housing demand$^{22}$:

$$
q_{a,t} = E_t \left\{ \sum_{i=1}^{N} \left( s \left( e_i^{\eta (1-\alpha)} c_i^{\alpha} \right) \right)^{1/\eta} \left( 1 - \delta_H \right)^{t-t} \left( \frac{1}{1+r} \right)^{t-t} \right\}.
$$

(2.19)

First, we consider the initial response of the demand to simultaneous productivity shocks in both sectors in an extreme case. Suppose zero housing adjustment costs and housing investment changes only at the time when the shocks hit the economy$^{23}$. In this case, the deviation of housing demand from its steady state value is$^{24}$:

$$
\dot{y}_H = \frac{r + 2\delta_H - \delta_H^2}{r\delta_H + \delta_H} \left( \alpha \hat{c}_t - \eta \hat{q}_H \right).
$$

(2.21)

The most important parameter in the demand function is the elasticity of intratemporal substitution between housing and nonhousing consumption $\eta$. The lower the value of $\eta$, the lower the demand elasticity with respect to price. Another

$^{22}$To simplify the analysis, I assume that the one period discount factor is constant and equal to $1/(1+r)$.

$^{23}$In another extreme case, if housing demand does not change at all, the largest change in housing price upon a shock is given by:

$$
\hat{q}_H = \frac{1}{\eta} \hat{c}_t.
$$

(2.20)

$^{24}$Variables with 'hats' denote log deviations from the steady state of variables without 'hats'.

37
important parameter in determining the quantitative properties of the model is \( \delta_H \), which affects housing demand elasticity and shift of housing demand curve due to increase in nonhousing consumption. A lower housing depreciation rate implies higher volatility and more procyclical housing investment. The intuition is simple: since the housing stock depreciates very slowly, increase in the housing investment upon the productivity shocks lasts for a long time and thus is attractive to undertake.

Further, the response of new housing demand depends on the *endogenous* change in tradable consumption, which in turn depends on the elasticity of intertemporal substitution. A larger response of tradable consumption implies a larger increase in the housing demand, which in turn results in more procyclical housing investment.

4.2 Factor Intensity and Housing Supply

Another important factor that determines the cyclical properties of new housing construction is the difference in factor intensities between the construction and tradable goods sectors. In particular, the smaller the labor intensity difference, the smaller the relative housing price change required to keep labor in the construction sector. To see this, consider the representative firm's optimality conditions, which equate factor returns in the construction and tradable goods sectors. Those conditions hold not only in steady state but also in every period. These conditions can be simplified to get an expression for the housing price in terms of the factor intensity difference:
\[
\ln(q_t) = \frac{\phi}{\theta} \ln(A_t^{F}) - \ln(A_t^{H}) + \Delta. 
\]

(2.22)

where \( \Delta \) is a constant that does not affect the dynamic properties of the model. \( \phi \) and \( \theta \) are labor intensities in new housing construction and tradable production, respectively.

The link between factor intensity difference and housing supply elasticity is more transparent if we assume productivity shock in the housing construction is zero. A positive productivity shock to tradable goods increases the return of its capital stock, so domestic firms will borrow from abroad to invest more. Increased capital/labor ratio and higher productivity raise the wage rate, which has a negative effect on housing construction, since it uses labor more intensively\(^{25}\). The higher the labor intensity in construction relative to tradable goods, the more negative the effect of tradable productivity shocks on housing construction, and the higher the price required to attract labor into the construction sector to increase housing supply.

In other words, the smaller the labor intensity difference, the higher the housing supply elasticity following a shock to the tradable goods sector. In both developed and emerging economies, housing construction is more labor intensive than tradable goods production. However, empirical evidence shows that the labor intensity

\(^{25}\)To be consistent with empirical observation on factor intensity in both emerging markets and industrialized countries, \( \phi \) is assumed to be larger than \( \theta \).
difference between the two sectors in emerging economies is smaller than that in developed countries.\textsuperscript{26} Therefore, a smaller change in the relative housing price is required to generate large change in the supply of housing following a productivity shock, which helps to explain the fact that new housing construction is more procyclical in emerging countries.

In the first period following productivity shocks to both sectors, capital stocks are predetermined and thus cannot respond instantly to the shock. Therefore, the supply curve of new housing supply is given by\textsuperscript{27}:

\[
\hat{y}_H = a_0\hat{A}_H + a_1[\hat{A}_H - \hat{A}_T] + a_2\hat{L} + a_3\hat{q}_H.
\] (2.23)

After a productivity shock, the dynamic properties of new housing supply positively depends on exogenous productivity shocks to the housing sector $\hat{A}_H$, the difference in productivity shocks $\hat{A}_H - \hat{A}_T$ and the \textit{endogenous} response of total labor supply $\hat{L}$. In the numerical analysis, the first two factors are determined by the estimation of total factor productivity, while the response of labor supply depends on the magnitude of productivity shocks and elasticity of labor supply.

\textsuperscript{26} The result is based on the author's calculation. Burstein, Neves and Rebelo (2001) has a similar result.

\textsuperscript{27} $a_0 = 1$, $a_1 = \frac{\phi}{(1-\phi)\bar{L}_H\bar{L}_T + 1 - \phi}$, $a_2 = \frac{\phi(1-\phi)\bar{L}_H}{(1-\phi)\bar{L}_H\bar{L}_T + 1 - \phi}$, and $a_3 = \frac{\phi}{(1-\phi)\bar{L}_H\bar{L}_T + 1 - \phi}$, where variables with bars denote steady state levels.
4.3 The Role of Small Open Economy Assumption

A crucial feature of the model is the exogenously determined interest rate, which reduces substitution among different investments. Intuitively, strong comovement between disaggregated investments can be easily rationalized in a small open economy, since the fixed interest rate in the international bond market mitigates the competition for limited resources between housing investment and business investment. When favorable productivity shocks hit both sectors, increased rates of return from investing in both housing and business capital, although different, cause both categories of investment to rise, because any investment with a rate of return that is higher than the interest rate on the international bond market will be undertaken. In contrast, in a closed economy, even small differences in the rate of return from different investments tend to generate negative comovement. Since in a closed economy interest rate will rise after the favorable productivity shocks hit the economy, there is a strong tendency of substitution toward more productive investment. Therefore, housing investment dynamics generated from an open economy model differ from those in a closed economy model.

Housing investment in the open economy is more likely to be positive following simultaneous favorable productivity shocks to both sectors (or a shock to tradable sector only) than in the closed economy for two reasons. First, after positive productivity shocks, the marginal return of business capital increases, which in turn pushes up the interest rate in a closed economy. But the interest rate will not rise in
the open economy model. The second effect lies in the response of second period nonhousing consumption. The larger wealth effect in the open economy implies a higher life time consumption and higher demand for housing in all periods following the productivity shocks, while there is an intertemporal substitution effect in the closed economy which raises the future path of consumption. Given the empirically plausible range of intertemporal and intratemporal elasticity parameters, the wealth effect dominates the substitution effect, so that nonhousing consumption rises more in the open economy than in the close economy.

4.4 The Role of Adjustment Costs

Adjustment costs in changing both business and housing capital stocks are important in generating housing and business investment dynamics that are consistent with the data. There are two reasons for this. First, adjustment costs introduce frictions in intratemporal substitution in production between housing and nonhousing sectors. Since productivity shocks in the nonhousing production are much larger than in the housing sector, there is a strong tendency for capital and labor to flow from the housing sector to the nonhousing sector. However, the existence of adjustment costs in changing sectoral capital stocks mitigates the substitution towards the more productive nonhousing sector, thus enabling procyclicality in the housing sector.

Second, adjustment costs in changing the housing stock discourage intertemporal substitution in housing demand. In particular, the intertemporal substitution motive towards future consumption is strong when the housing price has a decreasing path,
which is the case when the positive productivity shocks in the housing sector are smaller than in the tradable sector. But adjustment costs imply that it is better to begin adjusting housing stock immediately rather than waiting until the housing price is lower. In the numerical studies in the next section, I assume a standard quadratic functional form\(^{28}\) and follow the small open economy literature in calibrating the adjustment costs parameter to obtain realistic business investment volatility.\(^{29}\)

5. Quantitative Analysis

5.1 Calibration

I estimate the process for productivity shocks and other 'deep parameters' following standard practice in the literature, with results shown in Table 4\(^{30}\).

\(^{28}\)In unreported work, I also consider the effects of nonconvex adjustment costs on the dynamics of housing demand and find that nonconvex costs increase procyclicality of housing demand when productivity shocks are sufficiently large.

\(^{29}\)In calibrating the adjustment costs for housing, I follow Monacelli (2006) and provide a sensitivity analysis.

\(^{30}\)I perform another version of calibration which makes these parameters in Table 4 equal in emerging markets and developed countries except for the elasticity of substitution between housing and nonhousing consumption \(\eta\), for which I still use the values in Table 4. Simulation results are similar to those shown in Table 5. The key statistics, the correlation between housing construction and output is 0.43, which is close to 0.54 as reported in Table 5.
Empirical estimation of $\eta$ is difficult due to a lack of high quality housing stock data in most emerging markets. In existing empirical works, the elasticity of substitution between housing service and other goods is simply assumed to one (Davis and Heathcote (2005), Iacoviello (2005) and Monacelli (2006)). Lustig and Van Nieuwerburgh (2004), Lustig (2005) and Jaccard (2007) assume that housing and nonhousing services are complements, but Davis and Martin (2005) argue that previous literature underestimates the elasticity of substitution and conclude that $\eta$ should be well above 1.

In this paper, I estimate the elasticity of substitution between housing and nonhousing consumption $\eta$ following recent literature. Ogaki and Rinehart (1998) estimate the
elasticity of substitution between durable and nondurable goods to be significantly larger than one at the 5 percent level. However, their estimation does not include housing as a durable good. In the recent housing literature, Davis and Martin (2006) estimate it to be 1.25 using Ogaki and Rinehart method. I use the cointegration method described in Ogaki and Rinehart (1998). In the benchmark model, $\eta$ is set to 2.0 in developed economies and 1.5 in emerging markets.\(^{31}\)

Two other parameters that are special to this model are calibrated to match the long run average of relevant first moments. In particular, $s$ is set so that the steady state expenditure share on housing is equal to the long-run average 0.18. And $v$ is set to match the average fraction of time spent in leisure ($40/14*7$). $\omega$ is set to 1.5, which implies a labor supply elasticity of 2, which in the middle range of estimation in the literature.

Calibration of the subjective discount parameter $\rho$ follows Mendoza (1991,1995). I choose $\rho$ so that the steady state trade balance to GDP ratio generated by the model is equal to the long run average in the data\(^{32}\).

Labor shares in construction and tradables production are set to match the average shares of labor income in each industry in developed countries and emerging markets.

31 The estimation of $\eta$ is still valid even when the international interest rate shocks are present. This is because it is estimated using rental price of housing service, which is not affected by interest rate change.

32 In the benchmark model, the foreign debt to GDP ratio is set to 0.1 in both groups.
The depreciation rate of capital is set to 0.1 annually and the annual depreciation rate of the housing stock is set to 0.025, which is in the middle range of estimates in the housing literature\textsuperscript{33}.

In calibrating productivity shocks to the tradable goods sector in developing and developed countries, I follow Mendoza (1995) and Stockman and Tesar (1990)\textsuperscript{34}. The estimation of total factor productivity shocks to construction sector follows Burstein, Neves and Rebelo (2001) and results are similar. In particular, the productivity shocks follow AR (1) processes and are positively correlated across sectors in both groups of countries. However, in emerging markets these productivity shocks to both tradable goods and housing sector are almost twice as volatile as that in developed countries. In particular, the productivity shocks follow:

\[
\begin{bmatrix}
\log(A^T_{t+1}) \\
\log(A^H_{t+1}) 
\end{bmatrix} = \Omega \begin{bmatrix}
\log(A^T_t) \\
\log(A^H_t)
\end{bmatrix} + \begin{bmatrix}
\epsilon^T_{t+1} \\
\epsilon^H_{t+1}
\end{bmatrix}.
\]

Parameter values of autoregressive matrix $\Omega$ and variance-covariance matrix of $\epsilon^T_t$ are shown in Table 4.

\textsuperscript{33}For example, see Davis and Heathcote(2005) and Monacelli(2006).

\textsuperscript{34}The estimation of total factor productivity shocks to construction sector in emerging countries follows Burstein, Neves and Rebelo (2001) and results are similar.
I also consider the effects of growth rate shocks following Aguiar and Gopinath (2007). In particular, I assume there is a shock to the growth rate of labor productivity in both sectors simultaneously.

\[
\Gamma_t = g_t \Gamma_{t-1} \quad \text{.} \tag{2.25}
\]

\[
\log(g_t) = \rho_g \log(g_{t-1}) + \varepsilon_t^g \quad . \quad \text{ } (2.26)
\]

where \( s_t \) represents the labor productivity growth shock homogenous in both sectors and \( \Gamma_t \) denotes cumulative product of \( s_t \). In emerging markets, the volatility of the growth rate shock is assumed to be three times as large as that of developed countries. The persistence is assumed to be equal across countries and across sectors.
### Table 4C: Growth Rate Shocks

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<tr>
<td>Standard Deviation</td>
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</table>

5.2 Model Solution and Performance

I solve the model using a second order approximation to the policy function as described in Schmitt-Grohe and Uribe (2004). They show that this method is very accurate in a simple asset pricing model and thus is appropriate here.\(^{35}\) The baseline simulation results are summarized in Table 5.

### Table 5: Baseline Simulation Result

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<td>Corr(LY)</td>
<td>0.85</td>
<td>0.81</td>
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<td>Corr(TB/Y,Y)</td>
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<td>Std(Real Rent)</td>
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<tr>
<td>Std(ctr)/Std(Y)</td>
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<tr>
<td>Std(I)/Std(Y)</td>
<td>3.25</td>
<td>3.41</td>
</tr>
</tbody>
</table>

The main results from simulating the model are as follows:

\(^{35}\)For more discussion of application of the method in asset pricing model, see Schmitt-Grohe and Uribe (2004).
First, the calibrated model replicates the fact that new housing construction is more procyclical in emerging markets than in developed countries. In the data, the correlation of the new housing construction with GDP is 0.25 and 0.63 in developed and emerging markets, respectively, while in the model, these correlations are 0.33 and 0.59.

Second, the calibrated model replicates the fact that new housing construction is more volatile in emerging markets than in developed countries. However, it underestimates the relative volatility of new housing construction to GDP. In the data, the standard deviation of new housing construction is 9.23 and 5.25 times of the standard deviation of GDP in developed and emerging markets, respectively, while in the model, these ratios are 4.2 and 3.55.

Third, the calibrated model replicates the fact that the trade balance/GDP ratio is more countercyclical in emerging markets than in developed countries. In the model, these correlations are -0.43 and -0.10.

Fourth, the calibrated model replicates the fact that the real rental price of housing services (measured by Rent/CPI) is more volatile in emerging markets than in developed countries. However, the model overestimates the price volatility in developed countries and underestimates it in emerging markets.
Fifth, the housing prices generated by the model are very persistent and less correlated with GDP than the equity prices represented by Tobin's Q. However, the model can not generate enough volatility in either price. In addition, the real exchange rate follows the dynamics of rent closely and it is quite persistent and procyclical. (The model implied autocorrelation is 0.77 in developed countries and 0.89 in emerging economies, while their correlations with GDP are approximately 0.60 and 0.66, respectively.) These figures are higher than those observed in the data.

5.3 Impulse Response Function

Figure 5 shows impulse response functions of model variables to one percent productivity shocks in the tradable and housing sectors. This figure uses parameter values calibrated for developed economies.
Nonhousing investment responds sharply and immediately to productivity shocks to both sectors simultaneously since productivity follows a persistent AR(1). Tradable consumption also increases due to the increase in income and the decrease in leisure. The increase of nonhousing investment and tradable consumption generates a countercyclical trade balance. By contrast, housing consumption and new housing construction respond more slowly due to the rise in the wage rate that has a negative effect on the more labor intensive housing production. Therefore, housing consumption and housing price are very persistent, which is consistent with the data. Figure 6 compares impulse response functions of housing investment to one percent productivity shock to both sectors simultaneously and to tradable good sector only. When there is only positive productivity shock to the tradable good production sector, the housing investment decreases immediately and rises slowly to its steady state level. The housing investment shows this dynamics because the negative effect of the increase in the wage rate on the housing supply dominates the positive effect of the rise of tradable consumption on the housing demand.
5.4 Sensitivity Analysis

Since I do not calibrate the full model from micro level data, but instead borrow some parameters from previous literature, I perform sensitivity analysis with respect to some key parameters. The sensitivity analysis also clarifies the mechanisms in the model by showing the dependence of results on different parameters. In particular, the results vary with respect to $\alpha$, $\gamma$, and the factor density difference $\phi / \phi'$, as suggested by the discussion in section III. But the main predictions that new housing production is more procyclical in emerging markets and contributes to countercyclical trade are robust.

The procyclicality of housing investment increases with the volatility of productivity shock, as argued in section III. When the standard deviation of the productivity shocks takes the benchmark value of the developed country calibration, the correlation between housing investment and output is 0.33. While if this standard
deviation is doubled, the correlation becomes 0.5, which accounts for more than 1/2 of the benchmark difference between developed countries and emerging markets.

Figure 7: Sensitivity Analysis: Standard Deviation of Productivity Shocks

Note: Vertical axis shows variable’s correlation with GDP.

Over the empirically plausible range of \( \eta \), the cyclical properties of housing investment are sensitive to \( \eta \). The correlation of housing investment with output takes values from 0.28 to 0.39. When \( \eta \) is extremely large or small, there are large changes in the procyclicality of housing investment. In addition to the discussion in section III, when \( \eta \) is very small, the relatively stable durable housing stock actually decreases the volatility of tradable consumption and thus lowers the return to housing investment.
The persistence of productivity shocks has a large effect on the dynamics of housing investment. This is expected since the more persistent the productivity shock, the larger the response of the tradable consumption and housing demand. The benchmark persistence is 0.50. When productivity shocks follow random walk, housing investment and business investment tend to show the same degree of procyclicality.
The cyclicality of housing investment is very sensitive to growth rate shocks. If households think that high productivity today signals even higher productivity tomorrow, their consumption responds more than output does, thus creating very high demand for housing. This point is similar to the effect of the persistence of productivity shocks on the dynamics of housing investment.

Figure 10: Sensitivity Analysis: Growth Rate Shocks

Note: Vertical axis shows variable’s correlation with GDP.

The procyclicality of housing investment declines only slightly when the depreciation rate of housing increases from 0.01 to 0.05, which is the empirically plausible range of estimates found in the housing literature. However, it is obvious that when the housing depreciates by 100% and becomes a nondurable good, new housing production becomes as procyclical as tradable consumption.
The cyclicality of housing investment is not very sensitive to the labor intensity difference between the tradable and housing sectors. But housing investment is less procyclical when the labor intensity of housing increases, as predicted by section III.

The cyclicality of housing investment is not very sensitive to the intertemporal elasticity of substitution. But housing investment is more procyclical when
intertemporal elasticity of substitution is very high. This result is to be expected since the tradable consumption response is larger when households are more willing to substitute consumption intertemporally. Therefore, the demand for housing is larger and housing construction is more procyclical.

The cyclicality of housing investment is not very sensitive to the labor supply elasticity when \( \omega \) takes value between 1.5 and 1.6. But housing investment becomes more procyclical when the labor supply elasticity (given by \( \frac{1}{\omega - 1} \)) is very high. This is to be expected since the tradable consumption response is larger when households supply more labor in response to higher productivity. Therefore, the demand for housing is larger and housing construction is more procyclical.
6. Conclusion

This paper studies a multi-sector stochastic growth model with a housing sector and shows that it can explain differences in the cyclical behavior of housing quantities, housing prices and the trade balance between developed countries and emerging markets. These differences are due to the special properties of housing investment. New housing demand depends on the path of tradable consumption. Since consumption is more volatile in emerging markets than in developed countries, the demand for housing increases more emerging markets than in developed countries following productivity shocks. Although the model can explain most of the observed cyclical properties in emerging markets, there are some important aspects that the
model fails to capture and demand further research. For example, the housing price volatility implied by the model is too low in both emerging countries and developed countries. Furthermore, the model does not explicitly model some factors that are important in understanding the role of housing in business cycles, such as housing’s role as collateral and as insurance against rent risk. Including those factors in the model may help to explain the volatility of housing prices and housing investment fluctuations in emerging markets and thus is a promising future line of research.
Chapter 3: Interest Rate Shocks and Housing Investment

Dynamics in Small Open Economies

1. Introduction

Large variability and strong procyclicality of housing prices and investments are key features of emerging markets business cycles. These features are difficult to explain in a real business cycle model with a housing sector. However, large fluctuations in international interest rates, which are often associated with boom-bust cycles in emerging markets, may help to explain these facts because asset prices and investment crucially depend on the real interest rate, which is the cost of financing a housing purchase. Furthermore, housing assets are often used as collateral to reduce financing cost of other consumption in terms of a lower interest rate or an easier access to credit. This paper aims to investigate the role of international interest rate shocks in explaining the volatility and cyclicality of housing investment.

This paper first documents the strong and negative correlation between real interest rates and housing investment in emerging markets. Data show that real interest rates are strongly countercyclical and housing investment are strongly procyclical in emerging markets. By contrast, real interest rates and housing investment are slightly procyclical or acyclical in developed countries. This observed relationship between
interest rates and housing investment may help solve the procyclicality puzzle in housing investment as described in Fisher (2005) and Qi (2007).

The difficulty for the canonical closed economy RBC model to account for the positive comovement between housing investment and business investment over the business cycles lies in the strong tendency to substitute toward investment with higher rate of returns. In a closed economy general equilibrium model (without international real interest rate shocks), the rise of the real interest rate after a total factor productivity shock discourages housing investment since the letter has a lower rate of return than business investment. Thus, investments of different categories tend to commove negatively. Previous studies show that this co-movement problem can be partly solved by incorporating certain features of housing investment in the closed RBC model. In particular, McGrattan, Rogerson and Wright (1995) study the role of complementarities between nonmarketed services generated by home capital and goods provided on the market. Fisher (1997) analyzes the role of complementarity between housing investment and business investment in generating comovement between disaggregated investment categories. Davis and Heathcote (2005) studies a model in which comovement results from interdependence among different sectors.

However, few studies have been done on housing investment dynamics in open economy models, in which the interaction between housing investment and business investment is fundamentally different than in closed economy models and the economies can be subject to external interest rate shocks besides productivity shocks.
In particular, it is important to see whether the responses of different categories of investment to international real interest rate shocks in the model are consistent with the data. The first reason is that interest rate shocks have been shown to have crucial impacts on housing dynamics in previous closed economy general equilibrium models. For example, Erceg and Levin (2005) show that the response of housing investment to interest rate shocks is ten times as much as that of nondurable consumption. The second reason is that the exogenously determined real interest rate lies at the heart of the open economy model which has the ability to generate positive comovement. Therefore, shocks to the interest rate in the international bond market have a critical impact on the dynamics of heterogeneous investments in the theoretical model. In addition, interest rate shock is an import factor in driving business cycles in emerging markets. (Nuemeyer and Perri (2004) and Uribe and Yue (2006).) Since opportunities to finance investments with different rates of return is the key factor that generates positive comovement between housing investment and business investment in an open economy.

This paper has two goals. The first goal is to get a better understanding of the role of interest rate dynamics in driving the difference in housing and business investments dynamics between open economy models and closed economy models. The crucial feature of the open economy model I study in this paper is the exogenously determined interest rate which reduces substitution among different investments. Conceptually, strong comovement between disaggregated investments can be easily rationalized in a small open economy, since by assumption the fixed interest rate in
the international bond market mitigates the competition for limited resources between housing investment and business investment.

Specifically, in an open economy, domestic expenditures do not need to add up to the total output since the trade balance can absorb any domestic demand that exceeds domestic production. Therefore, when a favorable productivity shock hits the economy, the increased rates of return from investing in housing and business capital, although different, cause both categories of investment to rise. In contrast, in a closed economy, even small differences in the rates of return from different investments tend to generate negative comovement among them because of substitution toward more productive investment. So it is interesting to see how housing investment dynamics generated from an open economy model differ from those in a closed economy model.

The second major goal of the paper is to understand the role of international real interest rate shocks in affecting housing investment dynamics in a small open economy. In particular, I study the response of housing investment, business investment and output to total factor productivity shocks, pure exogenous international interest rate shocks and induced international interest rate shocks.

To investigate the effect of international interest rate shocks on housing investment in a modified open economy model, I introduce international interest rate shocks and
working capital constraints to generate output fluctuations.\textsuperscript{36} This type of constraints will generate asymmetry in the economy since construction sector is more labor intensive than tradable production sectors and thus faces more severe working capital constraints.\textsuperscript{37}

The working capital constraint implies that actual labor input cost depends on the interest rate, and that interest rate shocks generate asymmetric responses of sectorial production. In particular, when interest rate decreases, the more labor intensive construction sector expands more than the tradable sector. Thus countercyclical interest rate shocks in emerging markets will generate more procyclical housing construction.

An important finding of this paper is that positive comovement between housing investment and nonhousing investment is indeed easier to generate in the open economy model. Opening up the economy substantially enhances the ability of the model to account for the strong comovement in the data without relying on strong complementarities either among different investments or between housing services and nonhousing consumption. In particular, compared with a closed economy, the

\textsuperscript{36}Liquidity effect of interest rate shocks on business cycles through the working capital constraint channel is introduced by Christiano and Eichenbaum (1992) and applied in small open economy to study international interest rate shocks and country premium shocks in emerging markets by Nuemeyer and Perri (2004) and Uribe and Yue (2006).

\textsuperscript{37}Another important input in construction sector, land, is also assumed to be subject to working capital constraint since it needs to be purchased before production and thus further contribute to the asymmetric sensitivity to interest shocks.
open economy model suggested in this paper can account for over 25% more comovement between housing investment and business investment.

The paper also finds that induced country premia in international real interest rates is very important in explaining the large swing in housing investment in emerging markets observed in the data. When the interest rate shock is purely exogenous, it has a very small effect on housing dynamics over business cycles. In contrast, an interest rate shock that is highly correlated with productivity shocks contributes more than 1/3 of the observed degree of procyclicality of housing investment in emerging markets. Further, when the country premium is allowed to depend on the value of housing assets (because housing capital can be used as collateral to reduce financing cost), this dependence becomes an important mechanism to amplify and propagate interest shocks over the business cycle and has strong effect on housing investment and housing price dynamics.

The rest of the paper is organized as follows. Section 2 discusses a simple two period small open economy model to fix idea. Section 3 presents the general equilibrium model and quantitative results and compares the open economy model with a closed economy model. Section 4 concludes.
2. Stylized Facts

To motivate the model in section II, I document business cycle statistics, with special attention paid to housing investment and business investment, and real interest rate in four small emerging countries and in four small developed countries\textsuperscript{38}. Housing investment, business investment and GDP data are based on the OECD quarterly database and data compiled by central banks. Real interest data are based on quarterly data provided by Neumeyer and Perri (2005), who calculate these rates from OECD MEI and EMBI data.

The main finding is that real interest rates are highly negatively correlated with housing investment, business investment and output in emerging markets. However, the correlations between the real interest rates and output, and between housing and business investments are only slightly negative or even positive in developed countries. These results are shown in Table 1. In the first column, residential investments and real interest rate in the emerging markets sample have significantly negative comovements. The average correlation coefficient is -0.69 in emerging countries and only -0.01 in developed countries. In emerging markets, the negative correlation is even more significant than that between aggregate investment and the real interest rate. The second column shows that the correlation between real interest rate and GDP is uniformly negative (average is -0.59) in emerging markets and uniformly positive in developed countries. These findings are consistent with those

\textsuperscript{38}Data availability on both interest rates and housing interest rates determines the sample size. However, these countries are often chosen to represent small open developed and emerging markets.
found in Neumeyer and Perri (2005) and Uribe and Yue (2004). In addition, the last two columns show that shares of both housing investment and aggregate investment also have similar correlation patterns with real interest rates in emerging markets and developed countries.

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<td><strong>Residential Investment Share in GDP</strong></td>
<td><strong>Residential Investment Share in GDP</strong></td>
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<tr>
<td>Argentina</td>
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</tbody>
</table>

3. A Simple Open Economy Model

3.1 A Two-period Deterministic Small Open Economy Model

In this section, I present a very simple two-period small open economy model. Since investment dynamics in multiple period models depend on discounted sum of expected returns and thus do not have closed form solution, it is illustrative to consider dynamics of housing investment and business investment in a simple two
period deterministic model. Although very stylized, the model shows some key mechanisms generating realistic dynamics in the fully specified and calibrated model.

The crucial feature of the model is that the exogenously determined interest rate reduces substitution among different investments. Thus the business investment will not crowd out the housing investment.

The economy is populated by a representative consumer whose maximizes lifetime utility given by:

$$U = u(c_t, h_t) + \beta u(c_{t+1}, h_{t+1}).$$

(3.1)

Consumer chooses optimal $c_t$, $b_t$, $b_{t+1}$ and $h_t$ in the first and second period. $\beta$ is subjective discount factor and is set to be equal to $1 + r$. Momentary utility is of the following form:

$$u(c, h) = \left[ (1-s)^{\frac{1}{\eta}} (c)^{\frac{\eta}{\eta-1}} + s^{\frac{1}{\eta}} (h)^{\frac{\eta}{\eta-1}} \right]^{\frac{\eta-1}{\eta}}.$$

(3.2)

$c$ is non-housing consumption and $h$ represents housing consumption. $\eta$ is the elasticity of substitution between housing and non-housing consumption and is share of housing consumption in the utility. The economy has access to frictionless international goods market and can trade one period bond $b_t$ at interest rate $r$.

Suppose that there is a single good produced by business (non-housing) capital $k$ and can be used as consumption $c_t$, business investment $b_{t+1}$ and housing investment $h_{t+1}$. At the beginning of the first period, the economy is endowed with a predetermined amount of international bond $b_1$, business capital $k_1$ and housing
capital $h_1$. At the end of second period, the economy ends and the consumer is assumed to be able to convert any capital and housing stock left to consumption goods. Specifically, the economy's first and second period budget constraint is given by:

$$b_2 = y_1 - c_1 - bi_1 - hi_1,$$  \hfill (3.3)

$$b_3 = (1 + r)b_2 + y_2 - c_2 - bi_2 - hi_2.$$  \hfill (3.4)

Since the economy ends in the second period, optimal level of $b_3$, $k_3$ and $h_3$ are zero. Substitute it into the consumer's lifetime utility maximization problem:

$$\max \{u(c_1, h_1) + \beta u[(1 + r)(A_1 F(k_1) - c_1 - bi_1 - hi_1) + A_2 F(k_1 + bi_1) + k_1 + bi_1 + h_1 + hi_1, h_1 + hi_1])\}.$$  

First order conditions are:

$$u_c(c_1, h_1) = \beta (1 + r)u_c(c_2, h_2),$$  \hfill (3.5)

$$\frac{u_s(c_2, h_1)}{u_c(c_2, h_2)} = r,$$ \hfill (3.6)

$$A_2 F'(k_1 + bi_1) = r.$$  \hfill (3.7)
Equation (3.6) represents the consumer's consumption smoothing behavior. Equation (3.7) shows the consumer's optimal choice between housing and non-housing consumption. In this simple economy, the cost of housing consumption is just the opportunity cost from business investment. This is because the consumer can consume housing service in the second period and then consume the housing stock at the end of second period. So the relative price between housing and non-housing consumption in the second period is just $r$.

Given the specific function form assumed above:

\[
\left[ \frac{sc_2}{(1 - s)h_2} \right]^{1/\eta} = r.
\]

Thus, housing investment is given by:

\[
h_i = \left( \frac{s}{1 - s} \right)^{1 - \eta} \cdot r^{-\eta} \cdot c_2 - h_i.
\]

So given $h_1$ and $c_2$, the housing investment decreases with real interest rate $r$, because housing investment increases when the cost of financing housing investment is lower. In addition, this effect is stronger when the elasticity of substitution is higher between housing and non-housing consumption.\(^{39}\)

\(^{39}\)However, when the elasticity of substitution is low, $c_2$ 's response to the change of $r$ is also low.
Assuming no adjustment cost, the optimal capital stock equalizes marginal return on capital and cost of capital:

\[ A_1 F'(k_1 + bi_i) = r. \]

And the business investment is given by:

\[ bi_i = F^{-1}(r / A_1) - k_i. \] (3.9)

3.2 Closed v.s. Open Economy

By assumption, access to the international bond market mitigates the competition for limited resources between housing investment and business investment. Therefore strong comovement between housing investment and business investment can be easily generated in a small open economy. In particular, following a positive productivity shock, rates of return from investments in both housing and business capital rise accordingly. Higher returns cause both categories of investment to rise, because any investment with a rate of return that is higher than the interest rate on the international bond market will be undertaken. These results can be easily seen in equation (3.9)' and (3.11)'.

this will decrease the housing investment. So the overall effect depends on which is effect is larger.
In contrast, in a closed economy, even small differences in the rate of return from
different investments tend to generate negative comovement among them. After a
positive productivity shock, return from business investment is higher, which in turn
causes interest rate to rise. So there is a strong tendency of substitution toward more
productive business investment. In other words, business investment crowds out
housing investment in this case. Thus housing investment dynamics in a closed
economy model differ from that in an open economy model. In the closed economy,
the equalization of returns between business investment and housing investment
implies\(^{40}\):

$$h_i^A = \left( \frac{s}{1-s} \right) \left( r^d \right)^\eta c_z^d - h_i^A.$$  \hspace{1cm} (3.10)

And the interest rate in autarky is given by:

$$r^d = A_t F'(k_i^d + b_i^d).$$  \hspace{1cm} (3.11)

Housing investment in the open economy is more likely to be positive than that in the
closed economy for two reasons. First, after a positive productivity shock, the real
interest rate in autarky \( r^d = A_t F'(k_i^d + b_i^d) \) is greater than the real interest rate in
the open economy, thus the rise in the interest rate discourages housing investment in
a closed economy. But it does not exist in the open economy model. The second
effect lies in the response of second period non-housing consumption. Larger wealth

\(^{40}\) Variables with a superscript ‘A’ represent corresponding variables in autarky or closed economy.
effect in the open economy implies higher life time consumption and a higher $c^2$, while there is a substitution effect in the closed economy which raises $c^2$. However, given the empirically plausible range of intertemporal and intratemporal elasticity parameters, the wealth effect dominates the substitution effect.

3.3 The Role of International Interest Rate Shocks

Since the opportunity to finance various investments provided by the access to international bond market is the key difference between the open economy model and the closed economy model in determining housing investment dynamics, any change in the international interest rate may have a large effect on the dynamics of housing and business investments. In this part, I will compare the housing investment's responses to purely exogenous international interest rate and to induced international interest rates. And in the next section, numerical results are presented.

**Purely Exogenous Interest Rate Shocks** When the international interest rate is purely exogenous and uncorrelated with domestic factors such like GDP or TFP, housing investment will only be lightly positively correlated with GDP. The reason is that while the housing investment increases when international interest rate goes down, the GDP changes little in response to interest rate shocks. This is because the increase in business investment driven by lower interest rate is only a small proportion of capital stock that determines output. Even when the working capital constraint is
added, purely exogenous international interest rate shocks still cannot generate enough volatility in output. This point has been discussed in Nuemeyer and Perri (2004) and Oviedo (2005). In contrast, when a favorable productivity shock hits the economy, the output and business investment increase, but the response of housing investment is not that large if the consumption does not increase by a large margin. This has been shown in Qi (2007). So the overall effect of purely exogenous international interest shocks can only explain a small part of the strong positive correlation between housing investment and output.

Yet the housing investment has significantly negative correlation with interest rate. This can be seen from (9). The housing investment decreases with real interest rate $r^*$, because housing investment increases when the cost of financing housing investment is lower. In addition, this effect is stronger when the elasticity of substitution is higher between housing and non-housing consumption.

**Interest Rate Shocks Induced by Productivity Shocks** If the interest shocks and productivity shocks are negatively correlated, then housing investment is more procyclical. This is because the interest rate shock and productivity shocks reinforce with each other. For example, if the international interest rate faced by a certain country (or more precisely the country premium) decreases with the level of GDP in a boom, then the housing investment will increase due to lower financing cost and higher wealth. This is the direct effect or implicit mortgage rate effect of an interest rate shock.
There is another indirect effect of interest rate shocks on housing investment through consumption. In equation (3.9), when $r$ decreases, $c_2$ increases because of both interest rate shocks (both substitution effect\(^{41}\) and wealth effect) and productivity shocks (wealth effect), so the overall effect on housing investment is stronger. Thus there is a strong tendency for housing investment to commove negatively with GDP.

4. A Infinite Time Horizon General Equilibrium Model

In this section, I study a fully-specified general equilibrium model similar to Qi (2007). Key departures are assumption of working capital constraint in production in both sectors and introduction of international interest rate shocks. In the extended model, I also introduce a banking sector to motivate an induced country premium which depends on either productivity shocks or housing asset values.

4.1 Representative Household with Endogenous Discount Factor

The representative household's expected lifetime utility is given by:

$$U = E_0 \sum_{t=0}^{\infty} \rho_t u(C_t, l_t),$$

(3.12)

where $\rho_t/\rho_{t-1}$ is the household's time varying subjective discount factor\(^{42}\) which

\(^{41}\)The substitution effect in this simple two period model actually causes $c_2$ to decline. However, in more general multiple period settings, persistent lower interest rates tilt consumption toward present period. Thus the substitution effect also increases housing demand.

\(^{42}\)This simplified version of Uzawa preferences is introduced to make the model's steady state
follows the law of motion:

\[ \rho_{t+1} = \rho_t \beta(C^A_t, l^A_t), \quad (3.13) \]

where \( C^T_t \) and \( l^A_t \) are average per capital consumption and hours.

The utility function and endogenous discount factor take the following GHH form:

\[ u(C_t, l_t) = \frac{(C_t - \nu l_t^{\alpha / \omega})^{1-\sigma}}{1-\sigma}, \quad (3.14) \]

\[ \beta(C^A_t, l^A_t) = [1 + C^A_t - \nu l^{A,\omega} / \omega]^{\beta}, \quad (3.15) \]

where \( \sigma \) is the inverse elasticity of intertemporal substitution, \( 1 - l_t \) is leisure, \( \omega \) determines the elasticity of labor supply and \( \nu \) determines the amount of leisure in steady state. \( C_t \) is aggregate consumption (CES aggregator), defined as:

\[ C_t = \left[ (1 - s)^{1 / \eta} (c_t^{N_t} t^{(1-\alpha) / \eta})^{\eta} + s^{1 / \eta} (H_t)^{\eta + \zeta} \right]^{\eta / \zeta}. \quad (3.16) \]

\( c^T_t \) is nondurable tradable consumption. \( H_t \) represents the housing stock owned by independent of initial conditions. In this specification, the discount factor, which the individual household takes as given, is assumed to be a decreasing function of average per capita consumption of goods and leisure, i.e. households become more impatient the more of goods and leisure the average household consume.
the household\textsuperscript{43}; the housing service flow is assumed to be proportional to the housing stock $H_t$ and the proportionality factor is set to one. $s$ is the housing consumption share parameter. $c_t^N$ is the consumption of a nondurable and nontradable good $c_t^N$, which is assumed to be a nonproduced endowment.\textsuperscript{44} $\eta$ is one of the key parameters in the model. It represents the elasticity of substitution between housing and nonhousing consumption, which include tradable goods and other nontradable goods.

The budget constraint of this representative household has some new features due to the availability of housing as another asset and consumption good. First, the household can purchase housing asset $H_t$ at price $q_t$. Housing depreciates at rate $\delta_H$. The household gets utility from its housing service flow and can sell (buy) undepreciated housing stock in the next period. Secondly, I assume that changing the housing stock is costly. In particular, it is subject to a convex adjustment cost, which can be justified by the empirical observation that substantial transaction costs are required in buying and selling a house and that moving itself is costly in both financial terms and psychological terms.

Other parts of the constraint are standard: households supply labor $l_t$, capital $k_t^T$ and $k_t^{H'}$ to firms in competitive labor and capital markets at prices $w_t$, $u_t^T$ and

\textsuperscript{43}This assumption assumes away a rental housing market. Indeed, there is no difference between owning and renting a house period by period in the model. Since this paper does not model any financial friction, explicitly modeling a rental market should not change the main results.

\textsuperscript{44}Introducing $c_t^N$ as an endogenous variable which is produced by using labor and capital does not change the key result on housing investment but require tracking more variables.
respectively, and they sell their nontradable endowment \( y^N_t \) at price \( p_t \).

In every period, households borrow or lend in the international capital market by trading real bond \( b_t \) at interest rate \( r \). In addition, there is a lump sum government transfer \( \tau_t \). Finally, households choose the next period's capital stocks \( k^T_{t+1} \) and \( k^H_{t+1} \), which depreciate at the same rate \( \delta_k \).

\[
\begin{align*}
    b_{t+1} &= R_b b_t + w_t l_t + u^T_t k^T_t + u^H_t k^H_t + q_t (1 - \delta_H) H_t - c_t - \\
    &\quad I_t - q_t H_{t+1} - \frac{d_t}{2} (H_{t+1} - H_t)^2 / H_t + p_t^N y^N_t - p_t^N c_t^N. \\
    k^T_{t+1} &= (1 - \delta_k) k^T_t + I^T_t - \frac{d_k}{2} (k^T_{t+1} - k^T_t)^2 / k^T_t. \\
    k^H_{t+1} &= (1 - \delta_k) k^H_t + I^H_t - \frac{d_k}{2} (k^H_{t+1} - k^H_t)^2 / k^H_t. \\
    I_t &= I^T_t + I^H_t.
\end{align*}
\]

### 4.2 Firms

Representative firms in a competitive market rent capital and labor from households and produce two types of goods, a durable nontradable good (i.e. housing) and tradable good. It can freely reallocate labor across the two sectors, but adjustments of capital level in either sector are subject to convex cost. There are two reasons why I introduce adjustment cost into the model. Firstly, this practice follows the open economy RBC literature to generate realistic aggregate investment volatility without changing the model's perfect foresight steady state. Secondly, adjustment cost in reallocating capital across sectors is to induce a certain degree of comovement of
production. There is no friction (transportation cost, distribution cost or markup, etc) in the international tradable goods market so its price is assumed to be one at all times.

To study the potential effect interest rate shocks on housing and other investment, I consider an international interest rate shock and introduce working capital constraint to generate output fluctuation. And this type of constraint will generate asymmetry in the economy since construction sector is more labor intensive than tradable production sectors and thus is facing more severe working capital constraints.45

In particular, firm's total labor cost at time $t$ is given by:

$$
w_t(l^T_t + l^H_t) \cdot [1 + \kappa(R_t - 1)],
$$  \hspace{1cm} (3.21)

where $w_t(l^T_t + l^H_t) \cdot \kappa(R_t - 1)$ represents cost of working capital and $\kappa$ is the fraction of wage bill that is paid before production. As a result, actual labor input cost depends on interest rate and it generates asymmetric response of sectorial production. In particular, when interest rate decreases, more labor intensive construction sector expands more than tradable sector. And thus countercyclical interest rate shocks in emerging market will generate more procyclical housing construction.

45Another important input in construction sector, land, is also assumed to be subject to working capital constraint since it needs to be purchased before production and thus further contribute to the asymmetric sensitivity to interest shocks.
Denote the relative price of housing in terms of tradable good as \( q_t \). In every period, firms maximize expected profits from production of both goods:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \rho_t \frac{\lambda_t}{\lambda_0} \pi_t \right\},
\]

where

\[
\pi_t = y_t^T + q_t y_t^H - u_t^T k_t^T - u_t^H k_t^H - w_t (l_t^T + l_t^H) \cdot [1 + \kappa (R_t - 1)].
\]

Since firms are owned by households, firms' discount factor is \( \rho_t^{\lambda_t/\lambda_0} \), where \( \lambda_t \) is household's marginal utility from one unit of life time income at time \( t \). Firms rent capital from the household at price \( u_t^H \) and \( u_t^T \) in the two sectors and rent labor at competitive price \( \omega_t \).

Production technologies take the Cobb-Douglas form:

\[
y_t^T = A_t^T (k_t^T)^{\theta_t} (l_t^T)^{\phi_t},
\]

\[
y_t^H = A_t^H (k_t^H)^{\theta_t} (l_t^H)^{\phi_t},
\]

where \( \phi_t \) and \( \theta_t \) are labor intensity in new house construction and tradable production, respectively. To be consistent with empirical observation on factor intensity in both emerging markets and industrialized countries \( \phi_t \) is assumed to be larger than \( \theta_t \). \( A_t^H \) and \( A_t^T \) represent total factor productivity and are exogenous random shocks in the two sectors.
4.3 Competitive Equilibrium

A competitive equilibrium in this economy is characterized by the sequences of allocations \( \{ b_t, H_t, k_{t+1}, l_t, l^H_t, c^T_t, c^Y_t, y^T_t, y^Y_t, \} \) and sequences of prices \( \{ q_t, w_t, u^T_t, u^H_t, \lambda_t, \} \) that satisfy household and firms optimality conditions described in appendix A subject to budget constraints and production technology, and the market clearing conditions.

4.4 Discussion on Theoretical Properties of the Model

Interest rate shocks are very important in driving housing investment dynamics in business cycles. Basic mechanism is the same as the simple model in the second section, but in this fully-specified model with GHH utility function form and working capital constraint, interest rate shocks work through household's demand for housing and firm's supply of housing. And the supply effect and demand effect work in the same direction to generate strongly procyclical housing investment.

On the demand side, the demand function is very complicated and depends on the path of nonhousing consumption. General form of new housing demand can be derived from the expression for the housing price\(^{46}\)

\[
q^H_t = E_t \left\{ \sum_{i,\ell=1}^{\infty} \frac{s(c_i^{N(1-\rho)\ell^{\alpha}})}{(1-s)H}\left(1-\delta_i\right)^{\ell-t-1}\frac{\lambda_i}{\lambda_t}\rho_t \right\}. \tag{3.26}
\]

\(^{46}\)To simplify the analysis, I assume that the discount factor is constant and equal to \(1/(1 + r)\) in steady state.
To fix idea, let's consider the initial response of the housing investment to interest shock in the log-linearized version of the formula above:

\[
\hat{y}_{t}^{H} = \frac{1}{\delta_{H}} \left( \alpha \cdot \hat{C}_{t+1}^c - \eta \hat{R}q \cdot \hat{R} - \eta \hat{R}q \cdot \hat{q}_{t}^{H} + (1 - \delta_{H}) \hat{q} \cdot \hat{q}_{t}^{H} \right) \quad (3.27)
\]

**Direct Effect on Demand** The interest rate change $\hat{R}$ negatively affects the second term in the housing investment equation. This is because the interest rate $\hat{R}$ is in effect the implicit mortgage financing rate for the household in this model. When the increase in household's holding of housing asset is greater their current income nets consumption and change in holding of business capital, it needs to finance the purchase by borrowing from international market. Thus, the lower the mortgage financing interest rate $\hat{R}$, the higher the demand for housing. This is the direct effect of interest rate shocks on housing demand.

**Indirect Effect on Demand** The response of new housing demand positively depends on endogenous change in nonhousing consumption $\hat{C}_{t+1}^c$. The increase in nonhousing consumption in response to decrease in $\hat{R}$ depends on the elasticity of intertemporal substitution (substitution effect), net debt position (wealth effect)\(^{47}\) and labor supply elasticity. Specifically, a higher response of consumption implied by GHH specification of utility function will generate consumption which is more volatile than

\(^{47}\text{The wealth effect of interest rate works in the same direction as substitution effect in the quantitative experiment}\)
output. (For further discussion, see Nuemeyer and Perri (2004) and Uribe and Yue (2006)). Thus, strong response of non-housing consumption in turn generates larger change in housing demand and a more procyclical housing investment.

**Effect on Housing Supply** Another important part in the housing investment equation is the change in housing price which in turn hinges on the supply in the housing market. In particular, when an interest rate shock hits the economy, housing construction sector responds differently than tradable good sector because of difference in labor intensity.

Since capital stocks are predetermined and thus cannot respond instantly to the shock, the supply function of new housing supply at time $t$ is given by:

$$
\hat{y}^H_t = -a_t \left( \frac{1}{\omega - 1} \right) \left( \frac{\kappa \bar{R}}{1 + \kappa (\bar{R} - 1)} \right) \hat{R}_t + a_t \hat{q}^H_t .
$$

Therefore, the dynamic properties of new housing supply negatively depend on the interest shock. This is because of the working capital constraint assumed in the model. When interest rate is lower, the cost of working capital also becomes lower. This induced lower labor cost by interest shocks creates more labor demand and the effect more prominent in the more labor intensive construction sector. Therefore,

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48 With productivity shocks, the new housing supply also positively depends on exogenous productivity shocks in housing sector $\hat{A}_H$, the difference in productivity shocks $\hat{A}_H - \hat{A}_T$. 

83
interest rate plays an important role in determining dynamics of new housing supply through changing effective of marginal cost of labor due to working capital constraint. In the numerical analysis, the first item is contribution to housing investment change by endogenously determined labor supply change, which in turn increases in the magnitude of interest shocks, working capital constraint parameter $\kappa$ and the elasticity of labor supply.

These three effects of interest rate shocks on housing investment reinforce each other to determine the overall negative correlation between interest rate shocks and housing investment. But the magnitude still depends on parameter values. In the next section, I will evaluate both the overall and individual contribution of these effects to the housing investment dynamics.

5. Quantitative Results

$^1$With productivity shocks, the new housing supply also positively depends on exogenous productivity shocks in housing sector $\hat{A}_H$, the difference in productivity shocks $\hat{A}_H - \hat{A}_T$. 
5.1 Calibration of the Model

In calibrating the real interest rate shocks process, I closely follow Nuemeyer and Perri (2004) and Uribe and Yue (2006). In particular, the real international interest rate faced by an emerging market country \( R_t \) is decomposed to country risk \( R_t^P \) and a benchmark real interest rate \( R_t^\ast \). \( R_t^P \) is the risk premium on a sovereign emerging economy and \( R_t^\ast \) represents the prevalent risk preference required by international investors. Using the data set described in the second section, emerging economies (Argentina) are subject to the following AR(1) international interest rate shocks:

\[
\log(R_t^\ast) = \psi^\ast \log(R_{t-1}^\ast) + \varepsilon^\ast_t, \\
\text{and } \psi^\ast = 0.80, \varepsilon^\ast_t = N(0, 0.006). \tag{3.29}
\]

Specification of country premium is more complicated. Therefore, we inspect two artificial scenarios and conduct numerical experiment according. In the first scenario, \( R_t^P \) is assumed to be totally independent of \( R_t^\ast \) and estimated accordingly:

\[
\log(R_t^P) = \psi^P \log(R_{t-1}^P) + \varepsilon^P_t, \\
\text{and } \psi^P = 0.76, \varepsilon^P_t = N(0, 0.023). \tag{3.30}
\]

And in the second scenario, \( R_t^P \) is assumed to be induced by productivity shocks. Specifically,

\[
\log(R_t^P) = -\psi^P \log(A^T_{t-1}) + \varepsilon^P_{A_t}, \tag{3.31}
\]
where $\psi^{\tau}$ is set to 0.5 in the benchmark case and $\nu^{\tau}$ is set to match the volatility of $\kappa$. This is due to the high standard error when I estimate the correlation between the country premium and productivity shocks. Using Argentina data as described in section II, the correlation is -0.36 with a standard deviation of 0.4. This result is in line with the estimate of -0.4 in Aguiar and Gopinath (2007).

Labor shares in construction and tradable good production are set to match the average shares of wage income in each industry in developed countries and emerging markets. However, since the total labor cost include both wage payments and working-capital cost, $\phi$ and $\theta$ are set to match wage income share multiply by $[1 + \kappa(R_t - 1)]$.

Other parameters are calibrated in the same way as Qi (2007). I use the cointegration method described in Ogaki and Reinhart (1998). In the benchmark model, $\eta$ is set to 2.0 in developed economies and 1.5 in emerging markets.

Two other parameters that are special to this model are calibrated to match the long run average of relevant first moments. In particular, $s$ is set so that the steady state expenditure share on housing is equal to the long-run average 0.18. And $v$ is set to match the average fraction of time spent in leisure (40/14*7). $\sigma$ is set to 1.5, which implies a labor supply elasticity of 2, which in the middle range of estimation in the literature.
Calibration of the subjective discount parameter $\beta$ follows Mendoza (1991, 1995). I choose $\beta$ so that the steady state trade balance to GDP ratio generated by the model is equal to the long run average in the data\textsuperscript{49}.

The depreciation rate of capital is set to 0.1 annually and the annual depreciation rate of the housing stock is set to 0.025, which is in the middle range of estimates in the housing literature\textsuperscript{50}.

In calibrating productivity shocks to the tradable goods sector and housing construction sector in developing countries, I set the autoregressive parameter to 0.95 and choose the volatilities to match the volatility of GDP in Argentina (4.2), keeping the tradable sector 1.6 times more volatile than the housing construction sector.

5.2 Closed v.s. Open Economy

In the first numerical experiment, I compare the statistical properties of business investment and housing investment in closed economy and open economy which subject to productivity shocks to both sectors in the economy only. In a closed economy\textsuperscript{51}, the business investment is strongly procyclical, but the correlation between housing investment and output is only slightly positive. In contrast, in an

\textsuperscript{49}In the benchmark model, the foreign debt to GDP ratio is set to 0.1 in both groups.

\textsuperscript{50}For example, see Davis and Heathcote(2005) and Monacelli(2006).

\textsuperscript{51}In a closed economy, the net export is forced to be constant at the same level in the steady state of an open economy, which is the interest payment of the steady state debt.
open economy housing investment is modestly procyclical, although it is still much less than that in the data. At the same time, business investment is still strongly procyclical and net export is slightly countercyclical. The result is summarized in Table 1.

This sharp difference is due to the crowding-out effect of business investment on housing investment in a closed economy. Specifically, in an open economy, domestic expenditures do not need to add up to the total output and trade balance can absorb any domestic demand that exceed domestic production. Therefore, when a favorable productivity shock hits the economy, increased rate of return from investing in both housing and business capital, although deferent, cause both categories of investment to rise. In contrast, in a closed economy, even small differences in the rate of return from different investments tend to generate negative comovement among them because of substitution toward more productive investment.

<table>
<thead>
<tr>
<th></th>
<th>Closed Economy</th>
<th>Open Economy</th>
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<tbody>
<tr>
<td></td>
<td>Standard Deviation (Relative to Output)</td>
<td>Correlation with Output</td>
</tr>
<tr>
<td>Consumption</td>
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<td>0.98</td>
</tr>
<tr>
<td>Business Investment</td>
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</tr>
<tr>
<td>Housing Construction</td>
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<td>0.09</td>
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<tr>
<td>Net Export</td>
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</tr>
</tbody>
</table>
5.3 Real Interest Rate Shocks: Exogenous v.s. Induced Country Premium

In table 3, I compare key statistics of the model when it is subject to purely exogenous interest rate shocks and when the country premium is correlated with productivity shocks. In the benchmark case, I assume working capital constraint parameter $\kappa$ to be 0.5 and the correlation between interest rate shocks and productivity shocks $-\rho^{\kappa}$ to be 0.5. I find that a purely exogenous interest rate shock itself can generate modestly procyclical housing investment (correlation coefficient is 0.46), but it cannot generate strongly procyclical nonhousing investment (correlation coefficient is 0.30), which is much less than in the data.\(^{52}\) However, in the numerical experiment where interest rate shocks is correlated with productivity shocks, the model can generate a strong procyclical housing construction which is closer to data (0.61) and procyclical housing prices.

<table>
<thead>
<tr>
<th>Table 3. Comparison between Exogenous v.s. Induced Country Premium</th>
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<tbody>
<tr>
<td>Exogenous Country Premium</td>
</tr>
<tr>
<td>$\text{corr}(\xi, \chi) = 0.5$</td>
</tr>
<tr>
<td>Standard Deviation (Relative to Output)</td>
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<tr>
<td>Consumption</td>
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<tr>
<td>Business Investment</td>
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<tr>
<td>Housing Investment</td>
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<tr>
<td>Net Export</td>
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</table>

5.4 Supply Effect and Direct/Indirect Demand Effect

To evaluate the three channels through which interest rate shocks affect housing investment dynamics in the business cycle, I conduct several numerical experiments

\(^{52}\)Capital adjustment cost and housing adjustment cost parameters are adjusted upward to match observed volatility of both types of investments.
and compare those results with the benchmark case, i.e. working capital constraint parameter \( \kappa \) to be 0.5 and the correlation between interest rate shocks and productivity shocks \( \psi_{\beta} \) to be 0.5.

Table 4 shows the comparison between predictions in benchmark model and in a model where the indirect effect of interest rate on housing dynamics through response of tradable good consumption is eliminated by setting the volatility of tradable good consumption to a constant value. (The constant is 0.88 relative to GDP volatility. It is the value of consumption volatility in the open economy model without interest shocks in Table 2.)

When the indirect effect is shut-down (or more precisely, minimized), the housing investment is less procyclical than that in the full model (on average 0.10 in absolute value). And this relationship holds for any level of the correlation between interest rate shocks and productivity shocks \( \psi_{\beta} \).

This confirms the intuition discussed in part II and part III. The response of tradable consumption is a very important factor in influencing housing demand and thus housing investment dynamics in the emerging markets' business cycle.

Table 5 shows business cycle statistics in models where supply effect of interest rate on housing dynamics changes. (The working capital constraint parameter assumes value of 0.1, 0.2, 0.5 and 1.) The correlation between housing investment and output
only slightly increases when the working capital constraint is more stringent. This confirms our theoretical analysis of the supply effect due to labor intensity difference between housing construction and tradable good production, but the effect is quantitatively very small.

<table>
<thead>
<tr>
<th>Table 4. Indirect Effect on Demand vs. Direct Effect on Demand</th>
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<tbody>
<tr>
<td>Constant Consumption Volatility</td>
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<td></td>
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<tr>
<td></td>
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<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Business Investment</td>
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<tr>
<td>Housing Investment</td>
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<tr>
<td>Net Export</td>
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<tr>
<td>Consumption</td>
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<tr>
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<tr>
<td>Housing Investment</td>
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<tr>
<td>Net Export</td>
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<tr>
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<tr>
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<tr>
<td>Business Investment</td>
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<tr>
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<tr>
<td>Net Export</td>
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<td>----------------</td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>K</strong>&lt;sup&gt;+&lt;/sup&gt; = 0.5</td>
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<tr>
<td></td>
</tr>
<tr>
<td><strong>K</strong>&lt;sup&gt;+&lt;/sup&gt; = 0.2</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>K</strong>&lt;sup&gt;+&lt;/sup&gt; = 0.5</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>K</strong>&lt;sup&gt;+&lt;/sup&gt; = 1</td>
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5.5 Housing as Collateral to Reduce Financing Cost

A growing literature studies housing as an important collateral asset in amplifying and propagating shocks in business cycle. Following Kiyotaki and Moore (1997), the first class of model, such like Iacoviello (2005) and Monacelli (2006), studies a constant interest rate and always binding borrowing constraints which depend on amount of housing collateral in a closed economy monetary model.
The second class of model which includes Mendoza (2000, 2005), Lustig (2004) and Uribe (2006) studies occasionally binding borrowing constraints in the presence of land, fixed capital and housing as collateral assets. However, this class of model tends to generate a relative flat interest rate during normal time but drastically higher interest rate at crisis. Uribe (2006) argues that a constant domestic interest rate when borrowing constraint is not binding and a sudden increase in interest rate when the constraint is binding is an undesirable feature of this class of model to study effects of interest rate shocks and borrowing behavior at business cycle frequency.

The third class of model emphasizes the role of collateral asset to reduce financing cost in the presence of information asymmetry and moral hazard problem is well discussed in the literature. (For example Besanko and Thakor(1987)). This type of model generates more realistic interest rates that are negatively correlated with the value of collateral assets. And in reality, the debt-to-housing value ratio, known as LTV (loan to value ratio) is a key factor that determines level of interest rate at which consumers can borrow from banks. The risk of a borrower's default on a loan is positively related to the loan-to-value and the loss for banks in the even of default is negatively correlated with loan-to-value ratio.

In the simple extension of the benchmark model discussed here, I follow Uribe and Yue (2006) and assumes a financing cost function which is positively depend on level of f debt and negatively on housing value.
This can be rationalized by decentralizing the economy into competitive commercial banks which conduct borrowing from international market at rate $R^w_t$ and lending to domestic borrowers at rate $R_t$.

Operational financing cost positively depends on debt level/collateral ration. Denote debt and collateral at time $t$ by $d_t$ and $q_tH_t$ and in steady state by $d^*$ and $q^*H^*$.

\[
\max \left\{ R_t \left( d_t - \chi \left( \frac{d_t}{q_tH_t} \right) \right) - R^w_t d_t \right\},
\]

(3.32)

\[
\chi(d^* / q^*H^*) = 0, \chi' \left( \frac{d^*}{q^*H^*} \right) \in (0,1).
\]

\[
F.O.C. \ R_t = \frac{R^w_t}{1 - \chi \left( \frac{d^*}{q^*H^*} \right)}.
\]

$\chi$ is set to make the volatility of in the model match the volatility of interest rate in the data. It is very small (0.0016). Introducing of the financing cost function into the model improves the model's performance in predicting very procyclical housing construction in emerging markets. Therefore, housing construction is more procyclical in the model with financing cost (0.83) than the benchmark model (0.61). The intuition behind this result is simple: when a favorable shock (positive productivity shock or interest rate shock) hits the economy, housing price increases because household demands more housing and non-housing consumption. The rise in housing collateral value further decreases the bank's financing cost in the following
period and results in lower interest rate, which induces further increase in consumption and housing investment.

Therefore, when housing asset can be used as collateral to reduce the friction in the intermediation between international and domestic finance market, international real interest rate shock will be amplified and propagated over the business cycles.

<table>
<thead>
<tr>
<th>Table 6. Country Premium Dependent on Housing Collateral</th>
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<tbody>
<tr>
<td>Country Premium Dependent on Housing Collateral</td>
</tr>
<tr>
<td>corr((\hat{\zeta}, \hat{x})) = 0.5</td>
</tr>
<tr>
<td>(K^* = 5)</td>
</tr>
<tr>
<td>Standard Deviation (Relative to Output)</td>
</tr>
<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Business Investment</td>
</tr>
<tr>
<td>Housing Investment</td>
</tr>
<tr>
<td>Net Export</td>
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</tbody>
</table>

6. Conclusion

This paper shows that a modified small open economy general equilibrium model subject to international interest rate shocks can help to explain the comovement between housing investment and output. I find that, compared to a closed economy model, the open economy model suggested in this paper can account a sizable proportion of the comovement between housing investment and business investment. The paper also finds that induced interest shocks are very important in explaining the
large swing in housing investment in emerging markets observed in the data. Interest rate shocks help generate strongly procyclical housing investment in emerging markets. In addition, as collateral to reduce finance cost, housing assets provide an important mechanism to amplify and propagate interest shocks over the business cycles. However, this may result in over investment in housing and can increase the default probability of household and finance intermediation industry. But the set-up in the model is not able to endogenize this mechanism, which should be more rigorously studied in future research.
Appendices

A. Competitive Equilibrium:

A competitive equilibrium in this economy is characterized by the sequences of allocations \{ \mathbf{b}_t, \mathbf{H}_t, \mathbf{k}^T_{t+1}, \mathbf{k}^H_{t+1}, \mathbf{l}_t, \mathbf{H}^T_t, \mathbf{H}^H_t, \mathbf{c}^T_t, \mathbf{c}^H_t, \mathbf{y}^T_t, \mathbf{y}^H_t, \mathbf{y}^N_t, \mathbf{r}_t \} and sequences of prices \{ \mathbf{q}_t, \mathbf{w}_t, \mathbf{u}^T_t, \mathbf{u}^H_t, \mathbf{u}^N_t \} that satisfy household and firms optimality conditions described in (2.6)(2.7)(2.8)(2.9)(2.10) and (2.14)(2.15)(2.16)(2.17) subject to budget constraints (2.4)(2.5) and production technology (2.12)(2.13), and the following market clearing conditions.

Capital market clearing:

\[
I_t = I^T_t + I^H_t \quad k_t = k^T_t + k^H_t
\]

Labor market clearing:

\[
l_t = l^T_t + l^H_t
\]

Housing market clearing:

\[
y^H_t = H_t - (1 - \delta_H)H_{t-1}
\]

 Tradable good market clearing:

\[
b_t = R_h b_{t-1} + y^T_t - c_t - I_t - \frac{d_h}{2} (k^T_t - k^T_{t-1})^2/k^T_{t-1} - \frac{d_k}{2} (k^H_t - k^H_{t-1})^2/k - \frac{d_p}{2} (H_t - H_{t-1})^2/H_{t-1}
\]

Nonhousing nontradable good market clearing:

\[
y^N_t = c^N_t
\]

B: Optimality conditions:
Denote $\lambda_t$ and $\mu_t$ as Lagrangian multiplier associated with constraints (2.2) and (2.4).

Denote $\lambda_t$ as Lagrangian multiplier associated with constraints (2.6).

\[
uc(C_t, l_t)C_{ct} = \lambda_t
\]

\[
u_c(C_t, l_t)C_{ct} = \lambda_t p_t^N
\]

\[
u_t(C_t, l_t) = -\lambda_t w_t
\]

\[
\frac{\beta(C_t^A, l_t^A)E_t[u_c(C_{i+1}^A, l_{i+1}^A)C_{i+1}]}{\beta(C_t^A, l_t^A)E_t[q_{i+1}(1-\delta_m)A_{i+1} + d_A(H_{i+1}/H_n - 1)/2]}
\]

\[
\lambda_t[1 + d_k(k_{i+1}^T - k_{i+1}^H)] = \frac{\beta(C_t^A, l_t^A)E_t[\lambda_{i+1}[u_{i+1}^T + 1 - \alpha_k + d_k(k_{i+2}^T/k_{i+1}^T - 1)/2]]}{\lambda_t[1 + d_k(k_{i+1}^H - k_{i+1}^H)] = \frac{\beta(C_t^A, l_t^A)E_t[\lambda_{i+1}[u_{i+1}^H + 1 - \alpha_k + d_k(k_{i+2}^H/k_{i+1}^H - 1)/2]]}{\lambda_t = \frac{\beta(C_t^A, l_t^A)E_t[R_{i+1} \lambda_{i+1}]}{\lambda_t[1 + d_k(k_{i+1}^H - k_{i+1}^H)]}}
\]

Competitive firms rent capital and labor from the households and use them to produce two types of goods, a durable nontradable good (housing) and another tradable good.

The relative price of housing is given by $q$. In every period, firms maximize the profits from production of both goods:

\[
y_t^T + q_t y_t^H - (u_t^T k_t^T + u_t^H k_t^H) - w_t(l_t^T + l_t^H) \cdot [1 + \kappa(R_t - 1)]
\]

With production technologies given by:

\[
y_t^T = A_t^T(k_t^T)^{(1-\theta)}(l_t^T)^\theta
\]
\[ y^H_t = A^H_t (k^H_t)^{(1-\phi)} (l^H_t)^\phi \]

The representative firms' optimality conditions are:

\[ \theta A_t^T (k_t^T)^{(1-\theta)} (l_t^T)^{\theta-1} = w_t [1 + \kappa (R_t - 1)] \]

\[ q_t \phi A_t^H (k_t^H)^{(1-\phi)} (l_t^H)^{\phi-1} = w_t [1 + \kappa (R_t - 1)] \]

\[ (1 - \theta) A_t^T (k_t^T)^{(-\theta)} (l_t^T)^{\theta} = u_t^T \]

\[ q_t (1 - \phi) A_t^H (k_t^H)^{(-\phi)} (l_t^H)^{\phi} = u_t^H \]

Given those optimality conditions, the budget constraints (2.4)(2.5) and market clearing condition (2.10)(2.11)(2.12)(2.13), I solve the model by using second order approximation around perfect foresight steady state and report quantitative results in section 4.
Bibliography


Ostry, Jonathan D. and Carmen M. Reinhart (1992), Private Saving and Terms of Trade Shocks, IMF Staff Papers, 39, 495-517.


