ABSTRACT

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This thesis is concerned with the role of product quality in explaining observed price and trade patterns. The first chapter introduces the topic, summarizes the main findings of the dissertation and contrasts them to other results in the literature. The second chapter develops a tractable general equilibrium model that includes quality differentiation among heterogeneous firms. The theory explicitly demonstrates how heterogeneity in a single exogenous parameter, productivity, can produce dispersion in product quality and price. The framework predicts that relatively productive firms will choose to produce high quality varieties. This finding accords well with the observation that the unit value of exported varieties increases with exporter’s income, capital- and skill- abundance. The model is used to analyze how international trade policy and quality differentiation interact to shape patterns of production and trade flows. In particular, the model predicts a positive relationship between product quality and export status at the firm level and that trade liberalization decreases the average quality of a country’s exports.
The third chapter evaluates the importance of vertical product differentiation in explaining price and export status patterns observed in microdata on U.S. manufacturing plants. The main difficulty in exploring the impact of vertical product differentiation is that product quality is not directly observable. The analysis tackles the problem from two angles. First, the chapter develops a novel empirical strategy to obtain a proxy for quality, which is then used to evaluate important conditional correlations. The results show that both quality and productivity are important determinants of price and export status pattern. Second, the simulated method of moments is used to obtain structural estimates of the parameters of the model and to assess the importance of quality differentiation. The estimates suggest that quality differentiation plays an important role in explaining the variation in price, size and export status across U.S. manufacturing plants.

The fourth chapter briefly concludes by summarizing the main findings and suggesting avenues for future research. Overall the analysis presented in this dissertation implies that vertical product differentiation, or quality, plays an important role in explaining dispersion in producer output price and export status.
PRODUCT DIFFERENTIATION IN INTERNATIONAL TRADE

By

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Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park, in partial fulfillment of the requirements for the degree of Doctor of Philosophy

2009

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Ring the bells that still can ring.
Forget your perfect offering.
There is a crack in everything.
That’s how the light gets in.

Leonard Cohen
Dedication

A mes parents, Jeannine et Maurice Gervais.
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Chapter 1: Introduction

The rigorous analysis of U.S. import data by Schott (2004) reveals a striking pattern: within some narrowly defined product categories, firms located in high income, capital-rich and skill-abundant countries export relatively high priced units to the U.S.\(^1\) This observation has at least two important implications: First, assuming that firms in wealthy countries are on average more productive, it must be the case that relatively productive firms produce varieties for which the consumer’s willingness to pay is higher. Second, for differentiated products, the unit value reflects not only the efficiency of the production process but also the product’s \emph{quality}. These considerations suggest that studying the relationship between firm productivity and product quality would lead to an improved understanding of international trade flows.

The potential for quality differentiation forces us to rethink the impact of trade liberalization on both industrialized and developing countries. Quality upgrading is an important margin producers in developed countries can exploit to resist low-wage import competition. For instance, the entry of low cost producers, such as Chinese and Indian firms, could lead to a reallocation of resources towards high quality varieties in technologically advanced countries. Moreover, since worker relocation is likely to be easier within than across industries, within-industry specialization reduces the predicted welfare loss associated with the short-run adjustment that usually follows a trade liberalization episode.

\(^1\) Other recent papers look at price in aggregate trade data. See for instance, Hummels and Klenow (2005), Faruq (2006), Helble and Obuko (2007), and Johnson (2008).
The motives for quality upgrading are important for issues beyond international trade. The analysis of vertical differentiation brings to the fore an important weakness of a widely used productivity estimation procedure. Typically, in the absence of producer-level price information, output revenues and input expenditures are deflated by sector-level price indices and productivity estimates are defined as the residual in a regression of log deflated output revenues on log deflated input expenditures at the producer-level. If variation in product quality leads to price dispersion, however, such a procedure will lead to systematic biases in the productivity estimates. Moreover, Foster, Haltiwanger and Syverson (2008) point to the difficulty of obtaining accurate productivity estimates in the presence of vertical product differentiation even when microdata information on output quantity is available.\(^2\) In general, high input price plants have low quantity total factor productivity values because their input expenditures per units of output are larger than those of their industry counterparts.\(^3\)

Should we therefore conclude that these plants are less productive if their output is of superior quality and can fetch a higher unit value on the market? Not necessarily. If producing a high quality product requires relatively costly high quality inputs then the quantity productivity estimates understate the true productivity of the firm. The same is true if the quantity of inputs required to produce one unit of output is increasing in quality. In general both inputs and outputs should be computed on a quality adjusted basis in order to make accurate inferences on plant productivity. Understanding the

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\(^2\) Their work explores the separate contributions of idiosyncratic technology and demand to plant survival and productivity growth. The analysis uses a subset of 11 homogenous products to minimize differences in quality across plants.

\(^3\) Quantity TFP reflects producer’s average cost per unit (i.e. dispersion in physical efficiency and factor input prices). In Foster et al. (2008) and the current study it is obtained by computing the difference between the log quantity produced and the log of a constant returns to scale Cobb-Douglas input index.
impact of product quality on production cost, output price and revenue could lead to
the development of improved measures of productivity.

The importance of quality in explaining trade flows was first emphasized by Linder (1961), who argued that consumers in rich countries spend relatively more on high quality goods than consumers in poor countries. In that case, closeness to demand provides richer countries with a comparative advantage in the production of high-quality goods. In the late 1980s several economists formalized the demand side relationship between trade and quality in general equilibrium models. The main prediction that bilateral trade should be decreasing in the countries’ dissimilarity, generally measured by income per capita difference, received some empirical support. However, it is not clear that consumers in rich countries purchase higher quality goods exclusively because they have different preferences. Holding preferences fixed, the distribution of product quality may not be independent of the distribution of firm productivity. Unfortunately, despite the wealth of evidence about the importance of vertical differentiation, the literature still lacks a general framework to think about the supply-side factors affecting differences in product quality across countries.

The main objective of chapter 2 is to fill this gap by introducing vertical differentiation in a heterogeneous firms framework. The model focuses on the supply side implications linking firms’ productivity to quality choice and can be used to

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5 Hallak (2006) estimates destination-country income effects and find evidence supporting the hypothesis that richer countries have relatively high demand for high quality. Choi et al. (2006) report that the pairs of importers whose income distributions look more similar have more export partners in common and more similar import price distributions.
answer many important questions that have, so far, escaped the scrutiny of economists. For instance, how do vertical differentiation and trade policies interact to shape patterns of production and international trade within an industry? How does trade liberalization affect the average quality of output in a country? Is there a systematic relationship between firm level productivity and product quality?

The basic set-up of the model is borrowed from Melitz (2003) and is extended to allow for multiple market segments each characterized by a specific level of quality. In the model quality is defined as variation in demand due to voluntary actions by the firm and unexplained by changes in price. The core of the model is relatively simple. After learning their productivity, firms simultaneously choose the price and quality of the goods they produce. If the firm invests in an advanced technology and incurs relatively high fixed and variable production costs, consumers classify the output as high quality. As a result, the firm obtains a favorable demand shift and can charge a relatively high unit price for its output. The model leads to endogenous sorting of firms across market segments and predicts that, in equilibrium, the most productive firms choose to produce high quality varieties. Intuitively, since within each market segment the increase in firm-level profit is limited by the decreasing marginal utility of consumers, the gain from a price reduction as firm productivity increases eventually becomes less important than the gain from switching to a higher quality variety. As a result, highly productive firms choose to acquire an expensive technology and produce high quality products.

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6 For example, the increase in demand caused by an increase advertising expenditure is interpreted as quality.
Introducing endogenous product quality decisions provides a number of new results and helps reconcile the theory with the observed facts. For instance, in the extended model an increase in average firm level productivity increases the average quality of output such that, assuming firms in rich countries are generally more productive than firms in developing countries, the average quality of output will be positively correlated with the country’s income. Moreover, the extended model predicts a positive relationship between trade cost and average export quality – a result akin to the “shipping the good apples out” paradigm described by Alchian and Allen (1964). Both predictions are opposite to a benchmark model without endogenous quality and are consistent with empirical observations.

The ability of the model to replicate aggregate trade flow patterns hinges on assumptions about the structure of cost in the industry. Essentially, the framework assumes that quality is costly to produce, which leads to a positive relationship between quality and productivity. The obvious next step is to evaluate the empirical relevance of this assumption. This is one of the main objectives of chapter 3. Since the model is built to replicate aggregate patterns, the analysis must go beyond country level data and focus on data at the production unit level to obtain meaningful tests of the theory. Fortunately, the framework provides many important testable predictions. For instance, the model predicts positive relationships between unit price and production cost, between productivity and quality and between export status and quality at the producer level.

The main difficulty in exploring the impact of vertical product differentiation is that quality is not directly observable in general. Recent papers in the trade literature
use the average unit value, an estimate of price, to make inferences about the role of product quality in determining export patterns. However, this strategy potentially leads to biased results. First, many factors besides variation in quality can lead to price dispersion. For instance, Syverson (2004) shows that variation in regional demand and competition are important sources of price heterogeneity. Second, price dispersion does not necessarily capture the full extent of quality variation. The model in chapter 2 clearly demonstrates that in the presence of vertical product differentiation, productivity affects price through two distinct channels. On the one hand, productivity leads to a decrease in marginal production cost, thereby decreasing the equilibrium price. On the other hand, productivity increases the quality of output, which raises the marginal production cost and the equilibrium price. The overall impact of productivity on price and, as a result, the relationship between price and quality, depend on the underlying parameters of the model. For instance, in those industries where price and quality are only weakly positively related, an increase in product quality will not be reflected in price but rather in the quantity demanded. It thus seems important to move away from unit value and to take into account the separate roles of productivity and other factors affecting price dispersion when studying product quality.

In the model quality is defined as non random variation in demand unexplained by changes in price. These demand “residuals” are estimated for U.S. manufacturing plants producing in 125 five-digit standard industrial classification (SIC) industries

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7 In particular, the recent studies of Hallak and Sivadasan (2008) and Kugler and Verhoogen (2008) show that the correlation between average unit value and export status is positive. Manova and Zhang (2009) use unit value to study the impact of trade cost on export price.
8 Since this variation demand does not arise because of the firm’s action it is does not enter the definition of quality used in this study.
over the period 1972-1997 using the residuals from producer level regressions of quantity on price controlling for regional demand and plant age. The estimated demand residuals are positively correlated with advertising and new technology expenditures and marginal production cost at the producer level. These results suggest that demand residuals are not random but rather arise from deliberate activities on the part of the plant aimed at increasing the consumer’s valuation of its output.

Using the demand residuals as a measure of quality, the study obtains the following producer-level results: (i) Quality is positively correlated with unit cost and price on average; (ii) Productivity is negatively correlated with unit cost and price on average; (iii) Productivity and quality are positively correlated on average; (iv) Quality is an important determinant of the plant’s export status. All of these findings are consistent with the model and suggests that vertical product differentiation plays an important role in explaining plant-level price and export status patterns.

The second part of the empirical analysis uses the simulated method of moment (SMM) to obtain estimates for the structural parameters and evaluate the ability of the model to reproduced observed facts. The basic question is the following. Suppose that the best possible values – in a sense to be made precise – for the structural parameters are selected, how well can the model reproduce patterns observed in the data, such as the distribution of revenue across producers or the share of exporters in the industry? If the model captures the essential behavioral characteristics of producers, the simulated moments should be similar to the actual moments. Overall the model is able to replicate important features of the data such as the standard deviation in price and revenue and the distribution of industry revenue across plant. Further, the
estimated parameter values provide evidence of the importance of vertical product differentiation.

To summarize, the dissertation uses theoretical, empirical and computational methods to study the role of product differentiation in shaping price dispersion and trade patterns. Overall the thesis demonstrates that the scope for quality differentiation has an important effect on the behavior of producer and the characteristics of the industry and should not be ignored.
Chapter 2: Theoretical Results

2.1 Introduction

This chapter develops a theoretical model that incorporates quality differentiation into a heterogeneous firm framework. The model is used to analyze the impact of the quality margin on the relationship between firm level productivity, price and export status. The basic set-up is borrowed from Melitz (2003) and is extended to allow for multiple market segments each characterized by a specific level of quality. The extended model is based on two reasonable assumptions: (i) holding quality fixed, an increase in productivity decreases unit production cost, and (ii) holding productivity fixed, an increase in product quality increases production cost. The quality of the output is chosen endogenously by the firms and depends on the technology employed in its production. Expensive technologies are associated with superior product quality and, as a result, higher demand conditional on price. The model leads to endogenous sorting across product quality and predicts that, in equilibrium, high productivity firms choose to produce high quality goods. The intuition for this result is simple. On the one hand, the marginal gain from increasing sales by lowering price is limited by the decreasing marginal utility of consumers. On the other hand, the marginal cost of production is decreasing in productivity, thereby increasing the gains from quality upgrading.

Introducing endogenous product quality decisions provides a number of new results and helps reconcile the theory with the observed facts. In the benchmark
model without quality, high prices are charged by firms with low productivity. Since a fixed cost has to be paid in order to sell in foreign markets, these firms are unlikely to export. This implies that countries populated by relatively productive firms will export low unit value varieties, a prediction which runs against the observed trade patterns. In particular, Schott (2004) presents strong empirical evidence that unit value of U.S. imports is increasing in the exporter’s income, and capital and skill abundance. In the extended model, varieties can be vertically differentiated at some cost such that higher prices reflect, at least in part, higher quality. When firms are allowed to climb this quality ladder, the relationship between productivity and price is no longer monotonic: the unit price is decreasing in productivity within a given market segment but it is increasing in quality across segments. Since producing high quality varieties is relatively costly, only the most productive firms are able to supply them profitably to the market. As a result, in the extended model, an increase in average firm level productivity increases the average quality of output such that, assuming firms in rich countries are generally more productive than firms in developing countries, the average quality of output will be positively correlated with the country’s income. Importantly, this result is not driven by consumer preferences but by changes in the firms’ productivity distribution. The model is therefore a supply-side explanation for the pattern of unit-value in trade flows described by Schott (2004).

Moreover, the benchmark model without quality predicts that an increase in trade costs forces the marginally profitable exporters out of the foreign market, thereby decreasing the average unit value of exported varieties. Again, this prediction is
inconsistent with observed characteristics of trade flows. Baldwin and Harrigan (2007), in their study of product-level data on bilateral U.S. exports, report that “distance has a very large positive effect on unit values.” In the extended model, trade liberalization decreases the average quality of a country’s exports. This happens because trade liberalization leads to tougher competition by raising the productivity threshold above which firms decide to upgrade the quality of their product and increases the share of exporting firms. Together these results imply that a larger fraction of exporting firms produce low quality varieties in equilibrium. Therefore, the extended model predicts a positive relationship between trade cost and quality – a result akin to the “shipping the good apples out” paradigm described by Alchian and Allen (1964). An important corollary of this result is that trade liberalization, by increasing imports of high quality goods, leads to an increase in the average quality of consumption. This happens because the average quality of exported goods is relatively high compared to the average quality of overall production.

The rest of the chapter proceeds as follows. The next section introduces quality differentiation among heterogeneous firms in a closed economy setting. The equilibrium is then analyzed in detail in section 2.3. In section 2.4, the model is extended to a multi-country trading world. Sections 2.5 and 2.6 analyze the impact of trade and trade liberalization respectively while section 2.7 concludes. Derivation of major results and proofs of propositions are presented in appendix A at the end of the dissertation.

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9 See Hummels and Skiba (2004) for a recent empirical evaluation of this conjecture.
2.2 Closed Economy Model

Consider an economy composed of a measure $L$ of identical infinitely lived consumers each endowed with one unit of labor per period. Consumers have no taste for leisure and inelastically supply their labor to the market at the prevailing wage rate. Therefore, in each period, the labor supply is equal to $L$.

2.2.1 Preferences and Demand

Consumers derive their utility from the consumption of varieties produced in a single industry. The industry is interpreted as consisting of a narrowly defined product class that addresses specific needs and admits a fair amount of differentiation (e.g. the automobile industry). It is composed of multiple vertically differentiated market segments characterized by a unique level of quality – the precise meaning of which will be discussed at length below – within which producers can develop horizontally differentiated varieties.\(^\text{10}\) In equilibrium, a measure $X \equiv \{X(\omega, p)\}_{\omega \in N}$ of commodities, defined on the set of quality (or market segments, $N$) and price ($p$) is available for consumption. The number of segments as well as the segment-specific quality levels ($\omega_i$) are assumed to be constant over time and exogenously determined. For simplicity, the analysis considers the case of two market segments; which are called high quality ($\omega_h$) and low quality ($\omega_l$).

\(^{10}\) The terms horizontal and vertical have a different meaning here than in the multinational firms literature. In the current context, the firm’s output within a specific product category is differentiated along two dimensions: horizontal and vertical. For instance, consider the auto industry. Autos exhibit vertical differentiation (the Honda Civic and Bentley Continental are not directed at the same consumer base) and horizontal differentiation within segment (the Toyota Tercel competes for the same consumers as the Honda Civic).
Preferences over the differentiated varieties are additively separable with weights defined by the quality of the commodity. This implies that all varieties of the same quality and trading at the same price are consumed at the same rate. The composite good Q is a version of the Dixit and Stiglitz (1977) aggregator extended to allow for substitution between quantity and quality:

$$Q = \left[ \int_{x \in X} \omega(x)^{1-\rho} q(x)^\rho dx \right]^{1/\rho}, \quad (1)$$

where $q(x)$ and $\omega(x)$ represent the consumption and the quality of variety $x$. Since all consumers are identical and there is no asset accumulation, there is no borrowing and lending. The optimal level of consumption of each commodity is chosen to minimize the cost of acquiring the aggregate $Q$, which implies that:

$$q(x) = RP^{\varepsilon-\rho} \omega(x)p(x)^{-\varepsilon}, \quad (2)$$

where $R = PQ$ denotes aggregate expenditure, $\varepsilon \equiv 1/(1-\rho) > 1$ is the price elasticity of substitution between varieties and $P$ is the ideal aggregate quality-adjusted price index, which is defined as:

$$P = \left( \sum_{i \in \{o,H\}} \Psi_i \right)^{1-\varepsilon} \text{ with } \Psi_i \equiv \omega_i \int_{x \in X_i} p(x)^{1-\varepsilon} dx. \quad (3)$$

Note that $\Psi_i$ is negatively related to a weighted sum of varieties’ prices and positively related to the segment’s quality. It will be interpreted as the segment’s price-adjusted quality index.

The representation of preferences given in (1) lends itself to the interpretation of multiple segments within a single industry. Indeed, consumers see varieties of different quality as substitutes and, although they have a taste for diversity, would be
fine with consuming, say, only high quality varieties. The share of total expenditure on each of the segments is endogenous and the revenue in each segment can be expressed as follows:

\[ R_i = \int_{x \in X_i} p(x)q(x)dx = \frac{i}{\Psi_o + \Psi_H} R, \]

where \( X_i = \{X(\omega, p)\} \) is the mass of varieties of quality \( i \) available for consumption. Since the distribution of demand for varieties across segments depends on the relative competitiveness of each segment this characterization of preferences introduces an additional adjustment margin that allows consumers to influence the types of goods produced in equilibrium. When the segment’s price-adjusted quality index (\( \Psi_i \)) is relatively high, the share of expenditure that goes to that particular segment will also be relatively high.11 Finally, all else equal, the preferences defined in (1) imply that it takes a smaller mass of high quality varieties (\( X_H \)) than low quality varieties (\( X_o \)) to attain a given level of utility. Hence, intuitively, consumers are willing to sacrifice diversity to obtain quality.

The optimal demand schedule, defined in (2), reveals that the quantity demanded of each variety is decreasing in the price of the variety (\( p \)) and increasing in industry size (\( R \)) and aggregate price (\( P \)) – all standard results in the Dixit and Stiglitz (1977) taste for variety model. One difference, however, is the presence of the quality index (\( \omega \)), which acts as a demand shifter. Conditional on price and industry characteristics (\( P, Q \) and \( \varepsilon \)), the quantity demanded is increasing in the quality of the commodity if and only if \( \omega_o < \omega_H \). This assumption will be maintained for the rest of the chapter.

11 In limiting cases, when \( \Psi_i/\Psi_j \) goes to zero, only varieties of quality \( j \) will be available in equilibrium.
2.2.2 Technology and Firm Behavior

The quality of the output depends on the technology used in its production.\textsuperscript{12} For convenience, assume that only two technologies are available: a basic or “low” technology that can be acquired at low fixed cost \((f_o)\) to produce varieties of low quality \((\omega_o)\), and an advanced or “high” technology that can be acquired at high fixed cost \((f_H > f_o)\) to produce varieties of high quality \((\omega_H > \omega_o)\).\textsuperscript{13} In order to obtain tractable results, mild assumptions are imposed on these technologies: First, the characteristics of both technologies are common knowledge to all potential entrants in the industry. Second, both technologies are available for purchase to all firms. Thus, ex-ante, each firm can potentially enter either market segment. Third, conditional on technology choice, the quality of production is independent of other firm-level characteristics. Hence, it is possible to define a one-to-one mapping between the set of technologies and the set of product qualities. Fourth, the general form of the total cost function is the same for both technologies and is given by:

\textsuperscript{12} A number of “technology adoption” models have recently been developed in an international trade context. For instance, in a recent empirical study, Bustos (2007) extends Melitz’s model to allow firms to choose between a high fixed cost, low marginal cost technology and a low fixed cost, high marginal cost technology, and uses the framework to evaluate the impact of trade on technology upgrading and demand for skilled labor. Yeaple (2005) combines technology adoption and labor force heterogeneity to generate an endogenous distribution of firm productivity. These models differ from the current study on two important dimensions. First, in these studies the choice of technology has an impact on the production cost, but has no direct influence on the quality of the output and the consumer’s willingness to pay. Second, in the current study a high fixed cost does not lead to a low marginal cost. Both costs are increasing in quality. This assumption follows the industrial organization literature starting with Spence (1976).

\textsuperscript{13} When multiple technologies can be used to produce the same quality some complications arise. For instance, consider the case when two different technologies can be used to produce varieties of the same quality. To be relevant, the technology with the higher fixed cost must be associated with a lower marginal cost. If this is the case, each technology will be perceived as the most profitable by a certain range of firms. Those with low productivity will choose the low fixed cost, high marginal cost technology, while those with high productivity will choose the high fixed cost, low marginal cost technology.
\[
\Gamma_i(\varphi) = \left( f_i + \frac{c_i}{\varphi} \right) w, \quad \text{with } f_o < f_H \text{ and } c_o < c_H, \tag{4}
\]

where the subscript \(i\) indexes the technology, or equivalently the quality of production, \(\varphi\) is a measure of firm-level productivity, and \(w\) is the common wage rate hereafter normalized to one. This formulation implies that, within each segment \(i\), all firms share the same labor overhead cost \((f_i)\), but the variable cost \((c_i/\varphi)\) is decreasing in firm-level productivity \((\varphi)\). This captures the idea that the acquisition cost of each technology is the same for all firms but that, as a result of efficient management, firms operated by high ability entrepreneurs will be better able to exploit the technology and achieve lower marginal costs relative to firms managed by entrepreneurs of lesser ability. Finally, both the fixed and constant marginal cost of production are assumed to increase in quality such that producing quality is costly.

Firms are assumed to be single-plant, single-product profit maximizers. As a result, they will set marginal cost equal to marginal revenue. This leads to the following conditional pricing rule:

\[
p_i(\varphi) = \frac{c_i}{\rho \varphi}. \tag{5}
\]

Thus, mill-pricing with a constant mark-up over marginal cost is optimal for all firms. In standard Melitz type models firms have heterogeneous productivity but quality is homogeneous across firms and marginal costs are normalized to one (i.e. \(c_i = 1\)).\(^{14}\) As a result prices are given by \(p(\varphi) = 1/(\rho \varphi)\) and more productive firms always charge lower prices. In contrast to the benchmark formulation, the extended model allows the

\(^{14}\text{In the Melitz model higher productivity can also be interpreted as producing a higher quality variety at equal cost. The current framework explicitly accounts for both dimensions.}\)
schedule of unit prices to be increasing across market segments. Therefore, as long as more productive firms produce higher quality varieties and the effect of quality upgrading dominates the direct influence of productivity on price, more productive firms will charge higher prices per unit.

Using the pricing rule \(5\) and the optimal demand schedule defined in \(2\), the firm’s revenue as a function of productivity and quality can be expressed as:

\[
    r_i(\varphi) = R(\varphi \rho P)^{\varepsilon - 1} \Omega_i, \quad \text{where } \Omega_i \equiv \omega_i c_i^{1-\varepsilon}.
\]

Hence, for any given level of productivity \(\varphi\), revenue is increasing in the aggregate expenditure \(R\) and the aggregate price index \(P\). By definition, the firm’s segment specific profit is the difference between its revenue and production costs, and can be expressed as:

\[
    \pi_i(\varphi) = r_i(\varphi) - \Gamma_i(\varphi) = \frac{r_i(\varphi)}{\varepsilon} - f_i,
\]

where the last equality uses equations \((4)-(6)\). Firms will produce if and only if profits are non-negative. Since profits are increasing in productivity there exists a productivity threshold, \(\varphi_i\), above which firms find it profitable to produce a variety of quality \(i\). Specifically, let \(\varphi_i\) satisfy \(\pi_i(\varphi_i) = 0\), so that from \((6)\) and \((7)\):

\[
    \varphi_i = \frac{1}{pP} \left( \frac{\varepsilon}{R} \right) \left( \frac{f_i}{\Omega_i} \right)^{1-\varepsilon}.
\]

This equation indicates, in particular, that the profitability cutoff \(\varphi_i\) is increasing in fixed costs \(f_i\) and decreasing in the segment specific component of revenue \(\Omega_i\).

Examples of the profit functions defined in \((7)\) are depicted in Figure 1 in \((\varphi^{\varepsilon - 1}, \pi)\) space. So far, nothing precludes the possibility that the productivity cutoff

\[17\]
for the high segment (ϕ_H) is lower than that of the low segment (ϕ_o) as illustrated by the curves \( \{π_o, π'_H\} \). Similarly, it could be the case that all firms prefer to produce a low quality variety as illustrated by the \( \{π_o, π'_o\} \) case. In both cases every incumbent prefers the same market segment such that all varieties produced and consumed in equilibrium are of the same quality.

![Figure 1: Profit Functions and Productivity Cutoffs](image)

To make the model interesting, conditions that rule out such specialization in one market segment are required. The first step is to find the productivity level \( ϕ_{olH} \) at which a firm is indifferent between producing a low or a high quality variety. Formally, let the transition productivity cutoff \( ϕ_{olH} \) satisfy \( π_o(ϕ_{olH}) = π_H(ϕ_{olH}) \) so that from (7) and (8):

\[
ϕ_{olH} = Δ \cdot ϕ_o, \quad \text{with} \quad Δ ≡ \left( \frac{H - f_o}{f_o} \cdot \frac{Ω_o}{Ω_H - Ω_o} \right)^{\frac{1}{χ - 1}}. \tag{9}
\]
This equation clearly shows that the productivity of the marginal firm in the high segment ($\phi_{oh}$) is proportional to the productivity of the marginal firm in the low segment ($\phi_o$). Furthermore, the proportionality factor, $\Delta$, is exogenously fixed by the model’s parameters and, as one would expect, is increasing in the percentage change in fixed cost from upgrading from the low to the high quality segment, and decreasing in the associated percentage change in revenue.

By definition both qualities are produced in equilibrium if and only if $\Delta > 1$. This requires that two conditions are met: (i) the percentage difference in fixed cost between high and low quality is greater than the percentage difference in revenue, or equivalently $\Omega_H/\Omega_o < f_H/f_o$; (ii) conditional on productivity, the revenue earned in the high segment is greater than that earned in the low segment, so that $1 < \Omega_H/\Omega_o$.

If condition (i) is not satisfied, $\Delta < 1$ and every firm finds it optimal to produce in the high segment since the extra revenue earned from upgrading more than covers the extra fixed cost associated with the higher technology. If condition (ii) is not satisfied, the transition productivity cutoff $\phi_{oh}$ is negative. In that case, the revenue earned in the low segment is higher for every firm and it is never optimal to upgrade to the high technology. Revenue is increasing in quality only if marginal costs are sufficiently insensitive to quality upgrading; formally this requires that $(c_H/c_o)^{\epsilon-1} < \omega_H/\omega_o$.

Henceforth, conditions (i) and (ii) are assumed to be satisfied such that:

Assumption 1. $1 < \Omega_H/\Omega_o < f_H/f_o$. 

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The presence of fixed costs implies that firms will choose to produce a unique variety, different from the varieties produced by all other firms in the same segment. Moreover, since firms are profit maximizers, they will produce in segment \( j \) only if segment \( j \) provides them with the highest conditional profit, formally if \( \{ j : \pi_j(\phi) \geq \pi_i(\phi), \text{ for } i, j \in \{ o, H \} \} \), and if their revenue at least covers the cost associated with production in that segment, \( \pi_j(\phi) \geq 0 \). When varieties of both qualities are produced in equilibrium, firm behavior can be described as follows: exit if \( \phi < \phi_o \), produce a low quality variety if \( \phi \in [\phi_o, \phi_{oh}] \), and produce a high quality variety if \( \phi \geq \phi_{oh} \). This corresponds to the \( \{ \pi_o, \pi_H \} \) case illustrate in Figure 1.

This section explained how vertical differentiation introduces a new adjustment margin available to the firm. Since within each segment the increase in firm-level profit is limited by the decreasing marginal utility of consumers, as the firm becomes more productive the gain from increasing sales by decreasing price becomes less important than the gain from switching to a higher quality market segment. As a result, highly productive firms will choose to acquire an expensive technology and produce high quality products.

2.2.3 Quality

The core assumption of the above framework is that firms can choose the position of their demand curve in the quantity-price space. By investing in an expensive technology and paying more per unit produced, firms effectively purchase a positive demand shift which, for simplicity, is called quality. In the current context, quality should therefore be interpreted as a comprehensive vector of variables, other than
price, that have a direct influence on demand and that can be controlled (or at least
influenced) by the firm.\textsuperscript{15} These factors can be classified in two broad categories. The
first includes intangible characteristics, such as the consumer’s perception of the
product, brand recognition, after sale service, warranty, reliability, or availability. The
second includes tangible characteristics, such as better design or materials which
increase the performance and durability of the product. Both types of characteristics
increase the service flow obtained from the product, thereby raising the consumer’s
willingness to pay.

2.2.4 Industry Equilibrium

This subsection characterizes the economy’s equilibrium. Entry is assumed to be
costly as product development and production start up costs must be disbursed. The
entry cost is the same for all potential entrants and is denoted \( f_e \). Prior to entering the
industry the firm does not know its productivity. Thus, the value of the investment
opportunity is learned only once the fixed entry cost is sunk and the firm learns its
productivity, \( \varphi \), which is assumed to be a random draw from the distribution \( G(\varphi) \) on
support \( \Phi \subseteq [1, \infty) \). Once the firm learns its productivity, it can decide to exit the
industry immediately or develop and produce a variety in its preferred market
segment.

Since profits are increasing in productivity and firms stay in the industry only if
profits are non-negative, free entry determines a productivity threshold below which
firms will decide to exit the industry. Given the assumption on technology, less

\textsuperscript{15} Firms may be able to perfectly control their expenditures on advertising but they cannot perfectly
control their impact on the consumers’ willingness to pay.
profitable firms will choose to produce low quality varieties. The equilibrium profitability threshold is therefore equal to the cutoff for the low segment \((\varphi_o)\). The zero-profit condition that determines this threshold is given by:

\[
\pi_o(\varphi_o) = 0 \iff r_o(\varphi_o) = \varepsilon f_o.
\]  

(10)

Firms that draw an ability below the profitability threshold will exit the industry. Those drawing ability above will engage in profitable production.

Each period producing firms face a probability \(\delta\) of being hit by an exogenous shock that will force them to exit the industry. Hence, the value of the firm is zero if it draws a productivity below the profitability threshold and exits, and equal to the stream of future profits discounted by the probability of exit if it draws an ability above the cutoff value and produces. Since profit is the same in every period, the value of the firm, conditional on its productivity, can be expressed as:

\[
V(\varphi) = \max \left\{ 0, \sum_{t=0}^{\infty} (1-\delta)^t \pi(\varphi) \right\} = \max \left\{ 0, \frac{\pi_u(\varphi)}{\delta}, \frac{\pi_h(\varphi)}{\delta} \right\}
\]

where \(t\) is the time index.

The ex-post probability density function for productivity, \(\mu(\varphi)\), is conditional on successful entry and is truncated at the profitability cutoff \((\varphi_o)\). To obtain closed form solutions some structure needs to be put on the productivity distribution. It is therefore assumed that productivity is distributed Pareto.\(^{16}\) Therefore, the ex-ante cumulative distribution function is given by \(G(\varphi) = 1 - \varphi^{-\sigma}\) where \(\sigma > \max\{2, \varepsilon - 1\}\) is

---

\(^{16}\) The Pareto distribution is tractable and provides a reasonable approximation to the actual productivity distribution; see Cabral and Mata (2003) for evidence. It is therefore widely used in the literature; see for instance Helpman, Melitz, and Yeaple (2004) and Helpman, Melitz, and Rubinstein (2007). By definition of the Pareto distribution, an increase in the shape parameter \(\sigma\) decreases both the mean and the variance of the productivity and \(\sigma > 2\) is required to ensure a finite variance. The assumption that \(\sigma > \varepsilon - 1\) is required to ensure a well behaved equilibrium.
a parameter that affects the shape of the distribution. Under these assumptions, the conditional ex-post distribution is given by:

$$
\mu(\varphi) = \begin{cases} 
\sigma \varphi_o \varphi^{-(1+\sigma)} & \text{if } \varphi > \varphi_o \\
0 & \text{otherwise}
\end{cases}
$$

while the probability of successful entry in the industry is given by

$$
\zeta_e \equiv 1 - G(\varphi_o) = \varphi_o^{-\sigma}.
$$

There exists an unbounded set of potential entrants in the industry. Firms will attempt entry in the industry as long as the expected value from entry is greater than the sunk entry cost $f_e$. Since the characteristics of the ex-ante distribution of productivity $G(\varphi)$ are assumed to be common knowledge, the expected value of entry ($V^E_e$) is identical for all potential entrants and is given by the product of the average incumbent’s value ($\bar{\pi}/\delta$) and the probability of successful entry $\zeta_e$. As a result, the free entry condition can be written:

$$
V^E_e \equiv \frac{\zeta_e}{\delta} \bar{\pi} = f_e, \quad \text{with } \bar{\pi} = \int_{\varphi_o}^{\varphi_e} \pi(\varphi) \mu(\varphi) d\varphi.
$$

(11)

The specific properties of the Pareto distribution imply that the relative output of high quality varieties produced in equilibrium is unaffected by the value the productivity cutoff.\(^{17}\) As a result, the average revenue of producing firms is not affected by the cutoff either and can be expressed as a function of preferences, technology and distribution parameters alone; see the appendix for details:

\(^{17}\) Any continuous slice of the Pareto is itself a Pareto with the same shape parameter. It is this uncommon property that makes the Pareto so attractive.
This equation shows that the average revenue is increasing in the threshold revenue \( \epsilon f_o \) and the percentage increase in revenue associated with quality upgrading.

Further, average revenue is decreasing in \( \Delta \), the ratio of threshold productivities for high and low quality production, since a higher \( \Delta \) reduces the fraction of firms producing high quality varieties. Using (7), the average profit is given by:

\[
\bar{\pi} = \frac{1}{\epsilon} \int_{\varphi_o}^\infty \pi(\varphi) \mu(\varphi) d\varphi = \frac{\bar{r}}{\epsilon} - \bar{r}^i, \tag{13}
\]

where \( \bar{r} = (1 - \Delta^{-\sigma}) f_o + \Delta^{-\sigma} f_H \) represent the average fixed production cost and \( \Delta \) is defined as in (9). Taking (13) into account, the free entry condition (11) can be expressed as a function of only one endogenous variable, the profitability threshold \( (\varphi_o) \):

\[
V^E \equiv E(\pi | G(\varphi), \delta, \varphi_o) = \frac{\bar{\pi}(\varphi_o)}{\delta \varphi_o^\sigma} = f_c \tag{14}
\]

As illustrated in Figure 2, the expected value of entry is monotonically decreasing in the profitability threshold. Thus, the free entry condition alone pins down the equilibrium value of the threshold as a function of the parameters of the model. From (14), this threshold can be expressed as:

\[
\varphi_o^* = \left( \frac{\bar{\pi}}{\delta f_c} \right)^{\frac{1}{\sigma}} = \left( \left[ \frac{\sigma}{1 + \sigma - \epsilon} \right] \Lambda \left( 1 - \Delta^{-\sigma} \right) f_o + \Delta^{-\sigma} f_H \right) (\delta f^c)^{1/\sigma}, \tag{15}
\]

where \( \Delta \) and \( \Lambda \) are defined in (9) and (12) respectively. It can be shown that:
Proposition 1. \textit{There exists a unique closed economy equilibrium.}

\textit{Proof:} See appendix. \(\square\)

The equilibrium profitability threshold, \(\varphi_0^*\), is increasing in average profit and decreasing in the probability of exit (\(\delta\)) and the fixed entry cost (\(f_e\)). A decrease in the probability of exit increases the expected value of entry which, all else equal, increases the mass of entrants in the industry. From (3) this increase leads to a reduction in the price index and, as a result, a decrease in firm-level revenue. This decrease in profitability forces the less productive firms to exit the industry, thereby increasing the equilibrium threshold productivity and decreasing the expected value of entry, which returns to its equilibrium value of (\(f_e\)).

![Figure 2: The Equilibrium Productivity Threshold](image)

The equilibrium mass of producing firms in the industry (M) can be obtained by dividing the total revenue (R) by the average revenue (\(\bar{r}\)) defined in (12) and can be
expressed as:

$$M = \left( \frac{1 + \sigma - \varepsilon}{\sigma} \right) \frac{R}{\Lambda \varepsilon f_o}, \quad (16)$$

where $\Lambda$ is defined in (12). It follows that the equilibrium mass of incumbents is fixed and proportional to the size of the industry ($R$). The equilibrium threshold and mass of incumbents can be used to obtain an expression for the equilibrium price index defined in (3); see the appendix for details:

$$P = \frac{1}{\rho \varphi_o} \left( \frac{\varepsilon f_o}{R} \right)^{\frac{1}{\varepsilon - 1}} \left( \frac{\Lambda}{\theta} \right)^{\frac{1}{\varepsilon - 1}} \quad (17)$$

where $\theta \equiv (1 - \Lambda^{1+\varepsilon})\Omega_o + \Lambda^{1+\varepsilon} \Omega_{\delta}$, and $\Lambda$, $\Lambda$, and $\varphi_o$ are defined in (9), (12) and (15) respectively. This expression makes clear that the price index is increasing in the markup ($1/\rho$) but decreasing in the productivity threshold ($\varphi_o$) and industry size ($R$).

Intuitively, when the markup is high all varieties are more expensive whereas an increase in productivity will decrease the (quality adjusted) price of varieties. An increase in industry size ($R$) will increase the number of varieties available for consumption which, because of consumers’ taste for variety, decreases the price index.

By definition, in a stationary equilibrium, every aggregate variable must remain constant over time. This requires a mass of new entrants in each period, such that the mass of successful entrants exactly replaces the mass of incumbents hit by the exogenous shock and forced to exit. Formally, this aggregate stability condition requires $\zeta \varepsilon M_e = \delta M$. Note that the equilibrium productivity distribution will not be
affected by this dynamic entry/exit process, since the successful entrants and failing incumbents have the same productivity distribution.

Finally, the labor used for investment purposes by entrants must be reflected in the accounting for aggregate labor and affects the aggregate labor available for production. In equilibrium it must be the case that \( L = L_e + L_p \), where \( L_p \) is the aggregate labor used by incumbents for production purposes and \( L_e \) denotes aggregate labor used for investment purposes by prospective entrants. Aggregate payments to production workers must match the difference between aggregate revenue and profit in every segment. Normalizing the wage rate to one without loss of generality, this implies that \( L_p = R - \Pi \), where \( R \) and \( \Pi \) denote aggregate revenue and profit, respectively. Moreover, aggregate payments to investment workers must satisfy \( L_e = M_e f_e \). Using the aggregate stability condition \( (\zeta_M = \delta M) \), and the free entry condition \( (\pi = \delta f_e \phi^e) \) this implies that \( L_e = M_e f_e = M\bar{\pi} = \Pi \). Then revenue must equal total payments to labor since \( R = L_p + L_e = L_p + \Pi = L \), so that revenue is exogenously fixed by the size of the country.

The characterization of the unique stationary equilibrium in the closed economy is now complete. The next section analyzes this equilibrium, contrasts its implications with existing models and highlights the model’s novel predictions related to product quality and endogenous technological choice.
2.3. Analysis of Equilibrium

In this section, a number of important properties of the closed economy equilibrium are explored. To emphasize the novel implications of the additional adjustment margin (quality), assumption 1 is maintained throughout the section. Moreover, some of the results depend on the relationship between the elasticity of substitution ($\varepsilon$) and the shape parameter of the productivity distribution ($\sigma$). To obtain well behaved results the following additional assumption is required:

Assumption 2. $\sigma > \varepsilon$.

Appendix A contains an extensive discussion of this assumption. When it fails to hold the model generates unintuitive results. For instance, the relative output of the high to the low quality segment output is increasing in the transition threshold productivity.

2.3.1 Sorting and Optimal Output

As shown in Figure 1, in equilibrium lower productivity firms choose to produce low quality varieties, while higher productivity firms produce high quality varieties. Intuitively, since higher productivity firms face lower marginal costs they can charge lower prices and sell a larger number of units. This allows them to overcome both “barriers” to quality: the increase in the fixed cost of technology (f) and the increase in marginal production cost (c).

Proposition 2. There is endogenous sorting of firms across quality such that higher productivity firms choose to produce high quality varieties.

Proof: Direct from assumption 1. $\Box$
As indicated by equation (16), the mass of firms is proportional to the size of the industry and average revenue. The proportionality factor is function of the preferences, technology and distribution parameters. In particular,

Proposition 3. Any change in parameters that increases the relative profitability of the high quality segment (an increase in $\Lambda$ defined in (12)) will reduce the equilibrium mass of firms.

Proof: See appendix.

This suggests that, in countries where taste for quality is pronounced or the relative cost of producing high quality varieties is low, the number of firms will be small but firms will be relatively large. In terms of the model, an increase in the fraction of high quality varieties will increase the average revenue. Holding the aggregate revenue fixed, the mass of incumbents must go down to maintain the equilibrium. Intuitively, since quality and variety are substitutable, consumers will be satisfied with less variety when the average quality is higher.

From the optimal demand (2), the pricing rule (5), and the equilibrium condition (10), it can be shown that the equilibrium firm level output is given as follows; see the appendix for details:

$$q_o(\varphi) = \frac{\rho e f_o}{c_o \varphi_o^{e-1}} \varphi^e \quad \text{and} \quad q_H(\varphi) = \left(\frac{\Omega_H}{\Omega_o}\right) \frac{\rho e f_o}{c_o \varphi_o^{e-1}} \varphi^e,$$

(18)

where $\varphi_o$ is defined (15). Therefore, output is increasing in the markup ($1/\rho$) and decreasing in the productivity threshold ($\varphi_o$). Intuitively, an increase in markup
reduces the number of producing firms thereby increasing the demand for the incumbent’s output. An increase in the productivity threshold (\( \varphi_o \)) increases competition in the industry by inducing a decrease in the price index (P). The relative price of variety is higher, so that the demand is lower.

Figure 3 illustrates the evolution of firm level output as a function of productivity. Two important features should be noted. First, since all else equal high quality firms face higher demand, there is a discontinuity in output at the margin between segments, which is at the transition productivity \( \varphi_{oH} \). From (18), the ratio of high quality to low quality output at \( \varphi_{oH} \) is given by \( 1 < \Omega_H/\Omega_o \), where the inequality follows from assumption 1. Second, a change in productivity has a greater impact on output for firms in the high quality segment. In other words the slope of the quantity schedule is greater in the high segment. This happens because a small decrease in price associated with an increase in productivity will increase the demand relatively more for firms producing high quality varieties. It follows that:

> Proposition 4. Plant size, as measured by units of output or revenue, is unambiguously increasing in firm productivity (\( \varphi \)).

> *Proof:* See appendix. □

Finally, note that combining proposition 2 and 4 implies that quality and firm size (as measured either by output or revenue) are positively correlated.\(^{18}\)

\(^{18}\)This is potentially a counter-intuitive prediction of the model that does not fit some industries. For instance, at the very upper echelon of the auto industry, Ferrari produces a very small quantity of cars.
Using (18) it is possible to obtain expressions for aggregate output within each segment as functions of the parameters of the model and the endogenous productivity threshold; see the appendix for details:

\[
O_o \equiv M \int_{\phi_o}^{\Lambda \phi_o} q_o(\phi) \mu(\phi) d\phi = (1 - \Delta^{\varepsilon-\sigma}) \left(1 + \sigma - \varepsilon \right) \left(1 - \alpha L \right) \frac{\rho \phi_o}{c_o},
\]

\[
O_H \equiv M \int_{\phi_H}^{\infty} q_H(\phi) \mu(\phi) d\phi = \Delta^{\varepsilon-\sigma} \left(\frac{\Omega_H}{\Omega_o} \right) \left(1 + \sigma - \varepsilon \right) \left(1 - \alpha L \right) \frac{\rho \phi_o}{c_o},
\]

where \(O_o\) and \(O_H\) denote the total output of the high and low segments respectively and \(\Lambda\) is defined in (12). Both expressions in (19) are increasing in the country size \(L\) and decreasing in the maximum price of a low quality variety \((c_o/\rho \phi_o)\), so that large, low cost countries will produce more units of differentiated goods. Using (18), it can be shown that the equilibrium ratio of high to low quality output is given by:

\[
\frac{O_H}{O_o} = \left(\frac{1}{\Delta^{\varepsilon-\sigma} - 1}\right) \frac{\Omega_H}{\Omega_o}
\]

Figure 3: Equilibrium Firm-level Output Schedule
A few important points are worth noting: First, the ratio is independent of the threshold productivity. This is due to the specific properties of the Pareto distribution and would not be the case in general. Second, country size (\(L\)) as no impact on the average quality of output. Third, the relative output depends positively on the relative revenue \(\Omega_H/\Omega_o\) and, through \(\Delta\), negatively on relative fixed cost \(f_H/f_o\). Therefore, as expected, an increase in the relative profitability of a segment increases the relative output of that segment. Finally, the ratio depends negatively on the shape parameter:

Proposition 5. All else equal, an industry characterized by a high firm-level mean and variance of productivity (low \(\sigma\)) will produce relatively more high quality varieties.

Proof: Direct from (20). □

This happens because a reduction in the shape parameter transfers mass from below the average to above the average. This increases the share of firms that find the high quality segment more profitable and raises the relative output of high quality goods.

2.3.2 Industry Price Schedule

From the optimal pricing rule (5), it is clear that within each market segment, firm-level price is decreasing in productivity. Therefore, the highest (lowest) price charged for a quality \(j\) commodity will be the optimal price set by the least (most) productive firm using technology \(j\). Further, because \(c_H\) is strictly greater than \(c_o\) there is a discontinuity in prices at the margin between segments, which is at the threshold
productivity $\varphi_{oh}$. Thus, a key feature of the industry’s price schedule, illustrated in Figure 4, is its nonlinearity. An important implication of this nonlinearity is that it introduces the possibility of positive correlation between firm-level price and productivity.

![Figure 4: Industry Price Schedule](image)

The appendix shows that the correlation between price and productivity is given by:

$$
\text{corr}[p(\varphi), \varphi] = \left[ \beta_3 - \left( \frac{\varphi_1}{\varphi_2 - 1} \right) \beta_2 \right] \left[ \frac{\sigma}{(\sigma-1)^2(\sigma-2)} \left( \frac{\sigma}{2+\sigma} \beta_1^2 - \left( \frac{\sigma}{1+\sigma} \right)^2 \beta_2^2 \right) \right]^{-1/2}, \quad (21)
$$

where $\beta_i, i \in \{1, 2, 3\}$ are different weighted averages of production costs. Several interesting features of this result are worth highlighting. First, the correlation between firm-level price and productivity is not a function of the profitability threshold. Again this is due to the specific properties of the Pareto distribution. Second, the covariance
is a nonlinear function the shape parameter of the productivity distribution \( \sigma \). Unfortunately, given the complexity of equation (21), it is not possible to obtain simple, meaningful conditions under which the sign of the correlation is determinate. Nevertheless, it is possible to show that the derivative of the price-productivity correlation with respect to the shape parameter \( \sigma \) is positive when evaluated at the point where the correlation is zero. Therefore:

Proposition 6. For any technology and taste parameters that satisfy the model’s assumptions, there exists a range of the shape parameter \( \sigma \) such that the correlation between firm-level price and productivity is positive.

Proof: See appendix. □

Consider a given set of model parameters, and assume that the distribution of firms is such that the price-productivity correlation is zero. In that case, a small reduction in dispersion (i.e. a higher \( \sigma \)), will put more mass around the transition cutoff productivity \( \phi_{\text{cut}} \) and the correlation will be positive.

2.3.3 Welfare

Welfare effects can be evaluated by looking at the behavior of the inverse of the equilibrium aggregate price index. From (17) and the fact that \( R = L \), aggregate welfare in equilibrium is given by:

\[
W \equiv P^{-1} = \rho \phi_a \left[ \frac{(1 - \alpha)L}{\epsilon \Gamma_o} \right]^{\frac{1}{c-1}} \left( \frac{\theta}{\Lambda} \right)^{\frac{1}{c-1}},
\]
where $\Lambda$, $\varphi_o$, and $\theta$ are defined in (12), (15) and (17) respectively. Therefore, welfare is increasing in country size ($L$) and the productivity threshold ($\varphi_o$). This happens because more varieties are available for consumption in large countries and consumers value variety. All else equal, an increase in threshold productivity implies an increase in average productivity, which leads to lower prices and higher welfare. By extension, since the ex post mean and variance of productivity are increasing in the threshold, this implies that welfare will be higher when the mean and the dispersion of productivity are high (low $\sigma$).

This concludes the analysis of the unique closed economy equilibrium. In the next section the model is extended to include multiple countries. The set-up will then be used to study the impact of trade on the characteristics of the industry.

### 2.4 The Open Economy Model

This section extends the framework developed in the last section to obtain a model of a trading world composed of $n+1$ identical countries of the type previously described. When all countries are identical, they all share the same aggregate variables. Since it greatly simplifies the analysis this assumption is maintained for the remainder of the chapter. It is important to note, however, that since the number of countries is variable, the size of the domestic country relative to the rest of the world is left unrestricted.
2.4.1 Costless Trade

If there are no trade costs, the consumers’ love for variety implies that firms will divide their sales between domestic and foreign markets based on the size of their country relative to the world economy. Since all countries are identical, trade will be balanced, as each country will send the same fraction of each variety to each of the other countries. Thus, firms are not affected by costless trade but consumers enjoy greater welfare as they gain access to greater product variety. Moreover,

Proposition 7. When trading partners are symmetric, the equilibrium profitability threshold is unaffected by a move from autarky to free trade.

Proof: See appendix. □

Intuitively, as the economy is opened to costless trade, all firms in the industry – irrespective of their productivity – experience increased demand for their products in export markets and reduced demand in domestic market. In the case of symmetric countries, the gain from trade associated with foreign market access is just equal to the loss from trade due to the entry of foreign firms in the domestic market.

2.4.2 Costly Trade

The assumption of costless trade does not accord well with empirical observations; see Roberts and Tybout (1997) for instance. In order to sell their products in foreign markets, firms must build and maintain relations with foreign distributors. Moreover, firms generally face tariff barriers and pay freight costs to send their products to foreign markets. These trade impediments are assumed to take the form of a fixed
export cost \( f_x \) that must be paid every period by exporting firms, and a constant melting-iceberg cost per-unit shipped to foreign countries. Precisely, if \( \tau > 1 \) units are shipped to the foreign country, only one unit arrives. These costs are assumed common to every market segment so that arbitrary difference in trade costs do not drive any of the results.

The increase in marginal cost will be reflected by a proportional increase in price such that the pricing rule for exported varieties is:

\[
p_i^*(\phi) = \tau p_i(\phi),
\]

where \( p_i(\phi) \), defined in (5), and \( p_i^*(\phi) \) respectively denote the domestic and foreign price of a domestically produced variety. Using this result in the optimal demand defined in (2), it follows that the additional revenue from export to any foreign market is equal to:

\[
r_i^*(\phi) = \tau^{1-\epsilon} r_i(\phi),
\]

where \( r_i(\phi) \), the domestic revenue, is defined in (6). Similarly, the additional profit from exports is given by:

\[
\pi_i^*(\phi) = \frac{\tau^{1-\epsilon} r_i(\phi)}{\epsilon} - f_x.
\]

This implies that the total profit of a domestic producer is dependent on its exporting status as follow: \( \pi_i(\phi) = \pi_i^d(\phi) + \max\{0, n \pi_i^*(\phi)\} \), where the profit from domestic sales, \( \pi_i^d(\phi) \), is defined in (7). Note that in the open economy the ideal price index, defined in (3), must include imported varieties.\(^{19}\)

\(^{19}\) There is a slight abuse of notation. Here \( \omega(\phi) \) denotes the optimal choice of quality for a firm with productivity \( \phi \).
\[ P = \left\{ \frac{M}{\xi_x} \int_{\omega(\varphi)}^{\infty} \omega(\varphi) \, p(\varphi)^{1-\varepsilon} \, \mu(\varphi) \, d\varphi + nM_x \int_{\Delta_x, \xi_x}^{1} \omega(\varphi) \left[ \tau p(\varphi) \right]^{1-\varepsilon} \mu(\varphi) \, d\varphi \right\}^{\frac{1}{1-\varepsilon}}, \quad (24) \]

where \( M \) defines the mass of domestic incumbents while \( M_x \equiv \xi_x M \) denotes the mass of exporting firms, and where \( \xi_x \equiv \frac{1 - G(\varphi^*_x)}{1 - G(\varphi_o)} \) denotes the probability of exporting conditional on producing. The total mass of firms competing in any country, or equivalently the total mass of varieties available for consumption in any country, is thus given by \( M_T = (1 + n\xi_x)M \).

Consumers’ love for variety and the presence of fixed export costs ensure that no firm will export without also producing for its domestic market. Also since trade barriers are symmetric across countries, if a firm finds exporting to one of the foreign markets profitable, it will export to all countries. Thus, each firm now faces four different options: (i) produce a low quality variety and sell exclusively in the domestic market; (ii) produce a low quality variety and export; (iii) produce a high quality variety and sell exclusively in the domestic market; (iv) produce a high quality variety and export. Define the export productivity threshold \( \varphi^*_i \) as the minimum level of productivity required to be profitable in the export market, conditional on producing in segment \( i \). That is, \( \varphi^*_i \) satisfies \( \pi^*_i(\varphi^*_i) = 0 \). From the profit functions defined in (23) the cutoffs are equal to:

\[ \varphi^*_i = \frac{\tau}{\rho P \left( \frac{\varepsilon}{R} \right)^{\frac{1}{\varepsilon - 1}} (\frac{f_x}{\Omega_i})^{\frac{1}{\varepsilon - 1}}}. \quad (25) \]

These thresholds are decreasing in the market size (\( R \)), the aggregate price index (\( P \)) and the quality (through \( \Omega \)) but increasing in fixed and variable trade costs (\( f_x \) and \( \tau \)).
However, the number of trading partners (n) has no impact on the export productivity threshold. Further by definition of the thresholds and assumption 1 it follows that $\varphi^*/\varphi^o = (\Omega_H/\Omega_o)^{1/(e-1)} > 1$. Therefore, if a low quality firm finds it profitable to export, every high quality firm will find it profitable to export. Further, since the ratio of the export to the domestic cutoff is given by $\varphi^x/\varphi = \tau(f_x/f_i)^{1/(e-1)}$, partitioning of firms by export status within a segment can occur only if $\tau^{e-1}f_x > f_i$, a condition more likely to hold in the low segment given that $f_o < f_H$ by assumption. These results imply that there can be selection of firms along exporting status in at most one market segment. The analysis focuses on the equilibrium where both high and low quality varieties are exported. Formally, this requires that:

Assumption 3. $1 < \Delta_x < \Delta$, where $\Delta_x \equiv \tau^{(e-1)} \left( \frac{f_x}{f_o} \right)^{1/(e-1)}$.

Under this assumption the productivity cutoffs are ordered as follows: $\varphi_o < \varphi^x < \varphi_{oH}$.

Together, assumptions 1 and 3 ensure that there is sorting across market segments and selection along exporting status. The behavior of firms conditional on productivity in this type of equilibrium is illustrated in Figure 5. As productivity increases, firms expand their potential consumer base by exporting their production to foreign markets before investing in a more expensive technology that enables them to produce high quality varieties.

![Figure 5: Exporters and Non-exporters](image-url)
The exogenous factors affecting the entry/exit process and the productivity distribution are the same as in the closed economy. In particular, producing firms face a probability $\delta$ of being hit by an exogenous shock that will force them to exit the industry. Therefore, the value of the firm in a costly trade world is zero if the firm exits and equal to the discounted sum of profits if it produces. Hence the value of the firm conditional on its productivity can be expressed as:

$$V(\phi) = \max\left\{0, \frac{\pi(\phi)}{\delta}\right\}, \quad \text{with} \quad \pi(\phi) = \max\{\pi_o(\phi), \pi_o(\phi) + \pi^e_o(\phi), \pi_h(\phi) + \pi^e_h(\phi)\}$$

where $\pi_i(\phi)$ is defined in (7) and $\pi^e_i(\phi)$ is defined in (23). As in the closed economy firms will enter the industry until the expected value of entry is equal to the cost of entry. This implies that the costly trade free entry condition is given by:

$$V^E = \frac{\zeta n_x}{\delta} = f_e$$

with $\bar{\pi} = \int_{\phi_e}^{\infty} \pi(\phi) \mu(\phi) \, d\phi$, $\bar{\pi}_x = \int_{\phi_x}^{\infty} \pi_x(\phi) \mu_x(\phi) \, d\phi$

and where $\mu_x(\phi) \equiv (\xi / \xi_x) \mu(\phi)$ denotes the probability density function of productivity conditional on exporting. The costly trade economy expected value of entry is equal to the closed economy expected value, defined in equation (11), plus the expected additional discounted profit realized in the export market ($\zeta n_x / \delta$).

As in the closed economy equilibrium, it is possible to express the average profit earned in the export market as a function of the model’s parameters alone; see the appendix for details:

$$\pi_x = \left[\frac{\sigma}{1 + \sigma - \varepsilon}\lambda_x - 1\right] f_x$$

(27)
where $\Lambda_x = 1 + \frac{1}{\Omega_n - \Omega_o}(\Delta_x / \Delta)^{1+\sigma}$, and $\Delta$ and $\Delta_x$ are defined in (9) and assumption 3 respectively. Thus average export profit is increasing in the fixed trade cost and in the ratio of the marginal exporter’s productivity to the marginal entrant’s productivity ($\Delta_x$). Taking this result into account, the free entry condition (26) can be expressed as a function of only one endogenous variable, the equilibrium profitability threshold ($\phi_o$):

$$\frac{\bar{\pi} + n \Delta_x \pi_x}{\delta \phi_o} = f_e.$$  

(28)

As in the closed economy, the free entry condition alone pins down the equilibrium value of the threshold as a function of the parameters of the model. It can be shown that:

**Proposition 8.** There exists a unique costly trade open economy equilibrium.

**Proof:** See appendix. □

It is also possible to express the average revenue of producing firms as a function of the model’s parameters alone; see the appendix for details:

$$\bar{r} = \int_{\phi_o}^{\infty} r(\phi) \mu(\phi) d\phi = \left(\frac{\sigma}{1+\sigma-\varepsilon}\right) (\Lambda + n\tau^{1+\varepsilon} \Delta_x^{1-\sigma}) \phi_o f_o.$$  

(29)

where $\Lambda$ is defined in (12) and $\Lambda_x$ is defined in (27). Thus the average revenue is the sum of domestic revenue, defined in (12), and the product of a function of trade impediments and average revenue from export. Since all countries are identical, trade is balanced and the share of the total revenue in each country is equal to the income in
each country. Thus, as in the closed economy, the equilibrium mass of incumbents can be obtained by dividing total revenue by average revenue:

\[
M_T = \frac{R}{r} = \left(\frac{1+\sigma - \varepsilon}{\sigma}\right) \frac{(1-\alpha)L}{(\Lambda + n\tau^{1-\varepsilon} \Delta^{1-\sigma}_x \Lambda_x) f_o}. \tag{30}
\]

Finally, the equilibrium profitability threshold and mass of incumbents can be used in (24) to solve for the equilibrium price index. This yields:

\[
P = \frac{1}{\rho \phi_o} \left[ \left(\frac{\Lambda + n\tau^{1-\varepsilon} \Delta^{1-\sigma}_x \Lambda_x}{\theta_x} \right) \frac{\varepsilon f_o}{(1-\alpha)L} \right]^\frac{1}{\varepsilon-1}, \tag{31}
\]

where \( \theta_x \equiv \theta + n\xi_x \tau^{1-\varepsilon} [\Omega_\sigma (\Delta^{1-\sigma}_x - \Delta^{1-\sigma}) + \Omega_{\mu} \Delta^{1-\sigma}] \). As in the closed economy, in a stationary equilibrium, the aggregate variables must remain constant over time. This requires a mass \( M_e \) of new entrants in each period, such that the mass of successful entrants \( \zeta_e M_e \) exactly replaces the mass of incumbents \( \delta M \) hit by the exogenous shock and forced to exit. This aggregate stability condition requires: \( \zeta_e M_e = \delta M \).

This completes the characterization of the unique costly trade open economy equilibrium. The following two sections use this framework to study the impact of trade and trade liberalization.

2.5 The Impact of Trade

The open economy model developed in the previous section can be used to study the effects of changes in trade policy in the presence of export costs, firm heterogeneity, and quality differentiation. This section studies the effects of trade by comparing the closed and open economy equilibrium. The subsequent section analyses the effect of
incremental changes in trade impediments once the economy is open. Since both analyses compare different steady states, they should be interpreted as the long run economic impact of trade and trade liberalization. Incorporating decisions about endogenous product quality introduces an additional adjustment margin along which firms can respond to trade liberalization. Examination of this new channel yields a number of novel implications.

An important message of the open economy free entry condition (26) is that, since the cost of entry in the industry \( (f_e) \) is unchanged by trade, the equilibrium expected value of the firm will be the same as in the closed economy. Hence, whenever \( \pi_x \) is positive, the equilibrium profitability threshold is greater in the costly trade equilibrium than in the closed economy. This implies that moving from autarky to costly trade increases the average productivity of firms in the industry. This occurs for two related reasons. First, trade offers new profit opportunities for the more productive incumbents that decide to enter the export markets. Second, trade raises the expected value of entry, thereby increasing the mass of successful entrants. This leads to an increase in demand for labor. Since the supply of labor is fixed the wage rate goes up and the marginally profitable firms are forced to exit the industry.

Since \( \Delta \) is unaffected by costly trade, the increase in the profitability threshold \( (\varphi_o) \) leads to a proportional increase in the transition productivity cutoff \( (\varphi_{ot}) \). The adjustment can take two forms. First, if switching between segments is costless the pre-trade marginally profitable firms in the high segment go down the quality ladder and start producing a low quality variety. Second, if firms cannot change segments they exit and are replaced by new entrants with the same productivity level that
produce a low quality variety. This last scenario implies that trade induces exit of some relatively productive firms that were marginally profitable in the high quality segment in addition to the marginally profitable low quality firms.

Trade also has an impact on the average quality of output. The appendix shows that the relative output of high quality varieties under costly trade is given by:

\[
\frac{O_H}{O_o} = \frac{(1 + n \tau^{-\varepsilon})\Delta^{e-\sigma}}{1 - \Delta^{e-\sigma} + n \tau^{-\varepsilon}(\Delta_x^{e-\sigma} - \Delta^{e-\sigma})} \left(\frac{\Omega_H}{\Omega_o}\right),
\]

(32)

where \(O_i\) represents total output of quality \(i\). Note that if the variable trade cost is prohibitive (\(\tau \to \infty\)) or if all firms export (\(\Delta_x = 1\)) this ratio is the same as the closed economy ratio defined in (20). However, as long as both qualities are exported (\(\Delta_x < \Delta\)) and transport costs are non-trivial (\(\tau > 1\)), the relative output of high quality varieties is greater under costly trade than in the closed economy.

Further, comparing equations (12) and (29) reveals that the average revenue per producing firm is generally greater under costly trade than in autarky. When trade is allowed, the more productive firms enter the export market and increase their production. Since the optimal price (net of trade cost) is unaffected by trade the increase in output leads to an increase in revenue. Hence, since the revenue of the most productive firms goes up while low productivity, low revenue firms exit the market, average revenue increases. Moreover, since the total revenue in each market (\(R\)) is unaffected by costly trade, the increase in average revenue implies that the mass of incumbents is lower under costly trade than under autarky. Meanwhile, the ratio of high to low quality incumbents is given by:
\[
\frac{M_{il}}{M_o} = \frac{1}{\Delta \sigma \epsilon - 1}.
\]  
(33)

Therefore while costly trade increases the relative output of high quality varieties, it has no effect on the relative number of firms in each segment. Finally, using (16) and (30), and taking into account the fact that the mass of producers \( M_r \) is equal to \((1 + n^x)M\), it can be shown that the decrease in the number of incumbents due to a more competitive job market is more important than the increase due to the entry of exporters. Therefore, the overall mass of firms competing in each country is lower under costly trade than under autarky. Intuitively this happens because a small number of exporters enter the foreign market relative to the number of incumbents forced to exit.

The effects of a move from autarky to costly trade are summarized in the following proposition:

Proposition 9: As long as there is selection in the export market and varieties of both qualities are exported, a move from autarky to costly trade:

(i) increases average revenue, profit, and productivity per producing firm.

(ii) reduces the mass of incumbents producing in each country (both overall and within each segment).

(iii) reduces the overall mass of firms competing in each country or, equivalently, reduces the overall mass of varieties available for consumption in each country.

(iv) leads to exit and firm-level quality downgrading.
(v) increases the industry-level relative output of the high quality segment.

(vi) increases the average quality of varieties available for consumption in each country.

Proof. See appendix. □

Another interesting point to note is that, since only a fraction of firms export, the average quality of exported varieties is not the same as that of varieties produced for the domestic market. It can be shown that the share of high quality varieties in the total quantity of exports is higher than the share of high quality varieties sold domestically. Formally:

\[
\frac{O^D_H}{O^D_o + O^D_H} < \frac{O^X_H}{O^X_o + O^X_H},
\]

where \( O^k_j \) denotes the total domestic output of quality \( j \) varieties sold in market \( k \), where \( k \in \{D,X\} \). Therefore:

Proposition 10: Under costly trade, when there is selection in the export market, the average quality of exported products is higher than the average quality of varieties produced for domestic sale.

Proof: See appendix. □

This implies that trade leads to an increase in the average quality of production and consumption in each market.
Finally, the effect of costly trade on welfare can be evaluated by looking at the change in the price index defined in (31). Costly trade affects welfare through multiple channels. On the one hand, following the introduction of costly trade, the mass of varieties available in each market will decrease. On the other hand, consumers gain access to higher quality products on average. Since these effects work in opposite directions, the overall impact of costly trade on welfare is ambiguous.

2.6 The Impact of Trade Liberalization

While studying the transition from autarky to trade is a useful analytical exercise and provides some benchmark results, more realistic comparative statics are obtained by studying the impact of trade liberalization on the margin. In the current context the latter can take three basic forms: a decrease in the iceberg cost ($\tau$), a decrease in the fixed export cost ($f_x$), or an increase in the number of trading partners ($n$). This section study these cases simultaneously and use the term trade liberalization to refer to any of these events.\(^{20}\)

An important message of the open economy free entry condition (28) is that, since the cost of entry in the industry ($f_c$) is unaffected by trade costs, the equilibrium expected value of the firm is unaffected by trade liberalization. From (27), trade liberalization increases average export profit and, from (13), has no impact on domestic average profit. From (28), these changes increase the expected value of entry for any value of the productivity threshold. Since ($f_c$) is unchanged, this implies

\(^{20}\) The comparative statics exercises assume that both the ex ante and ex post equilibrium are such that varieties of both qualities are produced and exported.
that trade liberalization will increase the equilibrium productivity cutoff, thereby inducing marginally profitable firms to exit.\textsuperscript{21} The impact of trade liberalization on the equilibrium productivity threshold is illustrated in Figure 6.

![Figure 6: The Impact of Trade Liberalization](image)

Since from (9) $\Delta$ is unaffected by trade liberalization, the increase in the equilibrium threshold productivity implies that the transition productivity cutoff ($\phi_{oH}$) will also increase. Therefore trade liberalization, by making competition tougher in every market, induces some firms to move down the quality ladder. Trade liberalization also affects the export productivity threshold ($\phi_{oE}^*$). On the one hand, it can be shown that a decrease in trade costs (either variable or fixed) decreases the threshold productivity above which firms become exporters.\textsuperscript{22} Since these changes reduce trade barriers, it becomes relatively easier and more profitable to export such

\textsuperscript{21} The impact of changes in the fixed trade costs are complex and signing derivatives requires an additional assumption. A sufficient (but not necessary) condition for a decrease in fixed trade cost to increase the threshold is that the elasticity of substitution ($\varepsilon$) is greater than 2.

\textsuperscript{22} See previous footnote.
that less productive firms decide to enter the export markets. On the other hand, an increase in the number of trading partners (n) will increase the profitability threshold above which firms can profitably export. This happens because, while such a change has no effect on trade costs, it increases the demand for labor by continuing exporters. This increase in competition for workers in the labor market forces marginally profitable firms to exit the export market.

Using (32) it can be shown that an increase in the number of trading partners will increase the share of high quality varieties in total output – varieties produced for domestic sale plus varieties produced for export. Recall that, from proposition 10, the average quality of exports is higher than the average quality of aggregate output. Therefore, an increase in the number of trading partners, which as will be demonstrated below increases the share of production for export, also increases the average quality of aggregate output. Further, again from (32), an increase in fixed trade costs will increase the average quality of output. Intuitively, this happens because low quality exporters exit the foreign market, thereby reducing the output of the low quality segment. Finally, the impact of a change in variable trade costs depends on the relationship between trade impediments, the demand elasticity and the shape parameter of the productivity distribution. However, it is possible to show that whenever trade impediments are large enough, an increase in the iceberg transport cost leads to an increase in the relative output of the high quality incumbents. The share is increasing if and only if

\[ \sigma < \epsilon \left[ (\Delta^{x_e - x} + n^x - x) / (1 + n^x - x) \right] \]

where the term in square bracket is increasing in transport cost, fixed trade costs and decreasing in the number of trading partners.
mechanism by which transport costs affect quality is similar to that of changes in fixed trade costs.

The model also delivers some important predictions for the impact of trade liberalization on the average quality of exported varieties. The relative volume of high quality to low quality exports is given by; see appendix for detail:

\[
\frac{O^x_H}{O^x_o} = \frac{\Lambda^\varepsilon}{\Delta^\varepsilon - \Delta^\sigma} \left( \frac{\Omega^x_H}{\Omega^x_o} \right),
\]

(35)

where \( O^x_i \) represents total number of units of quality \( i \) produced for export. Changes in trade impediments will affect this ratio through their impact on \( \Delta_x \), which governs the selection of firms into export status. Since this threshold is independent of the number of trading partners \( n \), changes in \( n \) will have no effect on the quality composition of exports. However, a decrease in the iceberg cost \( (\tau) \) or in the fixed trade cost \( (f_x) \) will decrease \( \Delta_x \) and induce a decrease in the average quality of exports. Hence, there is a positive relationship between average quality and trade costs \( (\tau \text{ and } f_x) \). This is akin to the well-known Alchian-Allen “shipping the good apples out” paradigm.

The fraction of firm that exports is given by the probability of exporting conditional on producing, which in equilibrium is given by \( \zeta_x = \Delta_x^{-\sigma} \). Therefore the number of trading partners \( n \) has no effect on the share of exporting firms. However, a decrease in the iceberg cost \( (\tau) \) or fixed trade costs \( (f_x) \) will decrease \( \Delta_x \) and increase the share of exporting firms. Hence there is a negative relationship between the share of exporters and trade costs \( (\tau \text{ and } f_x) \). Further, it can be shown that the
share of exporting firms’ output that is produced for export rather than the domestic market is given by:

$$s^x = \frac{n\tau^{-x}}{1 + n\tau^{-x}}.$$  \hspace{1cm} (36)

Therefore the share of exporters’ output ($s^x$) for export is completely determined by the elasticity of substitution ($\varepsilon$), the number of trading partners ($n$) and the iceberg trade cost ($\tau$). As one would expect, this share is increasing in the number of trading partners and decreasing in variable trade costs and the demand elasticity. Since all markets are symmetric, the opening of a new market will increase the share of production for export. A decrease in the iceberg cost lowers the relative price of foreign varieties, making them more attractive to consumers. Finally a decrease in elasticity raises the demand for imported goods because consumers are less sensitive to the increase in price due to transport costs.

From (30), it can be shown that the number of domestic incumbents producing in each market will go down as a result of trade liberalization. When trade is liberalized, competition in each market increases due to the entry of additional foreign varieties. This forces the least productive incumbents to exit the industry. It can further be shown that a decrease in iceberg or fixed costs will lead to a decrease in the overall mass of firms competing in each market. This happens because the mass of entering exporters is smaller than the mass of low productivity firms that exit. The impact of an increase in the number of trading partners ($n$) on the overall mass of firms competing in each market is ambiguous. An increase in $n$ implies an increase in the mass of high productivity firms. All else equal, this tends to increase the number of firms competing in each market since more firms find exporting profitable. However,
the entry of high productivity firms in each market increases the toughness of competition which leads to exit of less productive firms and reduces the number of firms competing in the market.

Since quality is generally not observable a growing number of empirical studies use unit value to make inferences about product quality.\textsuperscript{24} The average export price can be expressed as:

\[
\bar{p}_x(\phi_o) = \left( \frac{\sigma}{1 + \sigma} \right) \frac{\tau c_o}{\rho \Delta x \phi_o} \left[ 1 + \left( \frac{c_H}{c_o} - 1 \right) \left( \frac{\Delta x}{\Delta} \right)^{1+\alpha} \right].
\] (37)

Therefore, an increase in the number of trading partners will increase the profitability threshold (\(\phi_o\)), thus reducing the average export price. A decrease in fixed trade costs has two offsetting effects, increasing the profitability threshold and decreasing the export threshold (\(\Delta x\)); the overall impact is ambiguous. The effect of a reduction in variable trade costs is even more complex. First, it leads to a proportional reduction in export prices for all firms, thereby decreasing the average price. Second, it increases the profitability threshold thus increasing the average productivity and decreasing the average price. Third, it decreases trade barriers (\(\Delta x\)), which leads to an increase in the fraction of low quality exporters and the average price. When the first two effects dominate, there is a positive association between average export prices and variable trade costs. In the end the sign of the relationship depends in a complex way on the value of the parameters of the model.

\textsuperscript{24} This includes among others Johnson (2008), Hallak and Sivadasan (2008), Kugler and Verhoogen (2008) and Manova and Zhang (2009).
The effects of trade liberalization are summarized in the following proposition. The proposition focuses on a decrease in variable trade costs, since this is the parameter most readily affected by policy.

Proposition 11. *Trade liberalization, in the form of a decrease in unit export costs* \((\tau)\), will:

(i) increase average revenue and profit per firm.

(ii) Increase average industry productivity.

(iii) decrease the mass of domestic incumbents and the overall mass of firms competing in each market or, equivalently, decrease the total mass of products available for consumption in each market.

(iv) lead to exit and firm-level quality downgrading.

(v) increase the share of exporting firms and decrease the average productivity of exporting firms.

(vi) increase the share of exporters’ output sent to foreign markets.

(vii) decrease the average quality of exports.

*Proof:* See appendix. □

Finally, the effect of trade liberalization on welfare can be assessed by looking at comparative statics on the inverse of the ideal price index defined in (31). Precisely:

\[
W = \rho \varphi_o \left[ \frac{(1 - \alpha)L}{\varepsilon f_o} \right]^{\frac{1}{\varepsilon - 1}} \left( \frac{\theta_x}{\Lambda + \pi \varepsilon f \Delta x \Delta x^{-1 - \alpha} \Lambda x} \right) \frac{1}{\varepsilon - 1},
\]
where $\Lambda$, $\varphi$, $\Delta$, $\Lambda_{x}$, and $\theta_{x}$ are defined in (12), (15), assumption 3, (27), and (31) respectively. Trade liberalization affects welfare through multiple channels. First it increases the average productivity of firms in the industry which raises welfare by decreasing the general level of prices. Second, since the average quality of imported varieties is higher than the average quality of domestic production, trade liberalization increases the average quality of output. However, by increasing competition in the industry, trade liberalization decreases the overall mass of varieties available for consumption. Since these effects work in opposite directions, the overall impact of trade liberalization on welfare is ambiguous. Formally, these effects can be evaluated by taking derivatives of $W$ with respect to $n$, $\tau$ and $f_{x}$. However, due to the complexity of the resulting equations general conditions are not easy to interpret.

2.7 Conclusion

This chapter develops a general equilibrium model that includes quality differentiation among heterogeneous firms. The model is used to analyze how international trade policy and quality differentiation interact to shape patterns of production and trade flows. Introducing endogenous product quality decisions yields important new implications and helps reconcile the theory with the observed facts. First, conversely to the benchmark Melitz (2003) model, the framework predicts that relatively productive firms will choose to produce high quality varieties that, under certain conditions, they sell at relatively high prices. This finding accords well with the observation that the unit value of exported varieties increases in the income, capital- and skill- abundance of the exporting country. Second, contrary to the
benchmark case, the model with quality predicts that trade liberalization decreases the average price of a country’s exports. This prediction is akin to the “shipping the good apples out” paradigm described by Alchian and Allen (1964) and is consistent with observed characteristics of trade flows.
Chapter 3: Empirical Results

3.1 Introduction

The main objective of this chapter is to explore the role of vertical product differentiation in explaining the price and export status patterns observed in micro data for a representative set of US manufacturing plants. The chapter begins by developing a tractable model of endogenous quality that explicitly demonstrates how heterogeneity in a single exogenous parameter, productivity, can produce dispersion in quality, price, and export status. The framework is a generalization of the previous chapter’s model in which producers can choose from a continuous range of technologies. Removing the high/low dichotomy results in a one-to-one mapping of productivity into quality and makes the model better suited for the empirical investigation.

The main difficulty in exploring the impact of vertical product differentiation is that quality is not directly observable in general. Recent papers in the trade literature use the average unit value, an estimate of price, to make inferences about the role of product quality in determining export patterns. However, there are many reasons to believe this strategy potentially leads to biased results. First, many factors besides

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25 The research in this chapter was conducted while the author was Special Sworn Status researcher of the U.S. Census Bureau at the Center for Economic Study Census Research Data Center in Washington. Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.

26 As mentioned earlier, the recent studies of Hallak and Sivadasan (2008) and Kugler and Verhoogen (2008) show that the correlation between average unit value and export status is positive, while Manova and Zhang (2009) use the unit value to study the impact of trade cost on export price.
variation in quality can lead to price dispersion. For instance, Syverson (2004) shows
that variation in regional demand and competition are important sources of price
heterogeneity. Second, price dispersion does not necessarily capture the full extent of
quality variation. The theory presented in this chapter clearly demonstrates how, in
the presence of vertical product differentiation, the firm’s productivity affects output
price through two distinct channels. On the one hand, productivity leads to a decrease
in marginal cost, which lowers the equilibrium price. On the other hand, productivity
increases the optimal quality, which raises marginal production cost and the
equilibrium price. The overall impact of productivity on price and, as a result, the
relationship between price and quality depend on the underlying parameters of the
model. In those industries where price and quality are only weakly positively related,
an increase in product quality will not be reflected in price but rather in the quantity
demanded. It thus seems important to move away from unit value and to take into
account the separate roles of productivity and other factors affecting price dispersion
when studying product quality.

In the theoretical model, quality is defined as variation in demand unexplained by
variation in price and due to producer behavior. These demand “residuals” are
estimated for U.S. manufacturing plants producing in 125 five-digit standard
industrial classification (SIC) industries over the period 1972-1997. The estimated
demand residuals are positively correlated with advertising, new technology
expenditure and unit production cost at the plant level. These results suggest that
demand residuals are not random but instead arise from deliberate activities on the

\[\text{27 These industries are listed in Table III. This is not the first paper to look at the impact of demand}
\text{shocks in studying plant behavior. See Melitz (2000), Eslava et al. (2004), Foster, Haltiwanger and}
\text{Syverson (2008a, 2008b), and De Leocker (2009).}\]
part of the plant aimed at increasing the consumer’s valuation of its output.\textsuperscript{28} Using the demand residuals as a measure of quality, the analysis produces the following producer-level results: (i) Quality is positively correlated with unit cost and price on average; (ii) Productivity is negatively correlated with unit cost and price on average; (iii) Productivity and quality are positively correlated on average; (iv) Quality, in addition to productivity, is an important determinant of the plant’s export status. All of these findings are consistent with the quality-extended model and imply that vertical product differentiation plays an important role in explaining plant-level price and export status patterns.

The demand residuals that serve as proxy for quality are likely to contain more than just information about product quality. Therefore, while the empirical findings are compelling evidence of a link between producer behavior and consumer demand it is not clear that the estimated correlations are due exclusively to vertical product differentiation. This implies that the magnitudes of the estimated coefficients may be biased and overstate the actual importance of quality. The last part of the analysis tackles the issue by using the simulated method of moments (SMM) to estimate the model’s structural parameters in order to evaluate the ability of the model to reproduce observed facts and assess the importance of quality differentiation. The numerical results show that marginal production costs are concave in quality while fixed production costs are convex in quality. Since the parameters underlying these findings are left unrestricted in the estimation these results support the idea that plants

\textsuperscript{28} In a contemporaneous study Foster et. al. (2008b) exploit time series variation in demand residuals (defined differently) to show that they are related to the age and accumulated sales of the plants. The two works are complementary. Foster et. al. (2008b) study the dynamic evolution of demand and explain time series variation. The current analysis concentrates on the determinants of time invariant heterogeneity in demand and explains cross sectional variation only.
use quality as a means to compete in the market. Further, the results suggest that using demand residuals as proxy for quality is a reasonable empirical strategy. Evaluated at the optimal parameter values the correlation between the theoretically accurate measure of quality and the constructed proxy estimated from the simulated data is above 0.9.

Another advantage of structural estimation is that it allows for counterfactual experiments. Once the estimates for the parameters are known the model can be used to evaluate the effect of exogenous shocks. The current analysis concentrates on three different types of trade liberalization: (i) A ten percent decrease in variable trade costs; (ii) A ten percent decrease in fixed trade costs; (iii) A ten percent decrease in both variable and fixed trade costs. The results show that trade is an important determinant of productivity in the industry.

Overall this chapter provides substantial empirical evidence that the ability to produce varieties of high quality confers an important competitive advantage to the firm and influences many aspects of its behavior. The rest of the chapter is structured as follows. The next section introduces quality differentiation among heterogeneous firms and analyzes the unique open economy equilibrium. Section 3 describes the dataset and explains the sample construction. Section 4 develops the econometric methodology and explores the relationship between productivity, quality, price, cost and export status. Section 5 presents the structural estimation and computational analysis of the model. Conclusions are presented in section 6. Derivation of major results and theoretical proofs, and a description of the computational algorithm used
to estimate the model using SMM can be found in appendix B at the end of the dissertation.

3.2 Theory

Consider an economy composed of a measure $L$ of infinitely lived consumers each endowed with one unit of labor per period. Consumers have no taste for leisure and inelastically supply their labor to the market at the prevailing wage rate, which is normalized to one without loss of generality.

3.2.1 Preferences

As in the previous chapter, preferences over the differentiated commodities are additively separable with weights defined by the quality of the commodity. Letting $q(x)$ and $\omega(x)$ represent the consumption level and quality of variety $x$, preferences are given by:

$$Q = \left[ \int_{x \in X} \omega(x)^{\frac{1}{\rho}} q(x)^{\rho} \, dx \right]^{\frac{1}{1-\rho}}. \tag{1}$$

The optimal consumption of each commodity is chosen to minimize the cost of acquiring the aggregate $Q$, so that the optimal demand for variety $x$ is:

$$q(x) = \omega(x)Q \left( \frac{P}{p(x)} \right)^{\rho} \text{ with } P = \left[ \int_{x \in X} \omega(x)p(x)^{\rho} \, dx \right]^{\frac{1}{1-\rho}}, \tag{2}$$

where $p(x)$ represents the price of variety $x$, $\varepsilon = 1/(1-\rho) > 1$ is the elasticity of substitution between varieties, and the aggregator $P$ is the quality-adjusted ideal price index and represents the price of the aggregate consumption bundle $Q$. 

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3.2.2 Production

Production entails both fixed and marginal costs. By assumption quality is costly to produce, so that production costs are increasing in product quality. The total cost function depends on the firm’s productivity ($\phi$) and the quality of its output ($\omega$) and is assumed to take the following form:

$$\Gamma(\omega, \phi) = \left[ F(\omega) + \frac{\omega^n}{\phi} q \right] w$$

where $F(\omega) = f_c + \omega^\gamma$, with $f_c$, $\eta$, $\gamma > 0$, \hspace{1cm} (3)

and $w$ is the common wage rate hereafter normalized to one. The fixed cost, $F(\omega)$, is increasing in quality and bounded below by $f_c$, which represents the fixed cost incurred by a firm producing a variety of “zero” quality.\(^\text{29}\) Conditional on productivity, the marginal cost ($\omega^n$) is increasing in the quality of output. Intuitively this assumes that the production of higher quality units requires more resources (labor).\(^\text{30}\) The total cost function implies that, holding quality fixed, all producers share the same labor overhead cost, but that the variable cost is decreasing in productivity. This captures the idea that, conditional on quality, the maintenance cost of each technology is the same for all firms but that, as a result of efficient management, firms operated by high ability entrepreneurs will be able to better exploit their resources to achieve lower marginal costs than firms managed by entrepreneurs of lesser ability. Implicitly the model assumes that firms are different in their productive efficiency but not in their ability to produce higher quality varieties.

\(^\text{29}\) It is never optimal for a firm to produce a variety of quality zero in equilibrium. This formulation includes the Melitz (2003) production function as a special case, in which $\omega$ equals one for all firms and $1 + f_c$ equals $f$.

\(^\text{30}\) In practice, the increase in production costs could also be due to the use of a different, more costly bundle of inputs (e.g. skilled labor or better materials).
3.2.3 International Trade

The world is composed of two identical countries.\textsuperscript{31} In order to sell their products in foreign markets, plants must build and maintain relations with foreign distributors, as discussed in Roberts and Tybout (1997). In addition, plants face tariffs and pay freight costs to send their products to foreign markets. These trade impediments take the form of a fixed export cost ($f_x$) that must be paid every period by exporting plants, and a constant melting-iceberg cost per-unit shipped to foreign countries ($\tau$). Precisely, if $\tau > 1$ units are shipped to the foreign country, only one unit arrives. These costs are assumed to be constant with respect to quality to prevent arbitrary differences in trade costs from driving the results.

3.2.4 Profit Maximization

Producers are single-plant, single-product profit maximizers. Entry is assumed to be costly as production start up costs must be incurred. The entry cost is the same for all potential entrants and is denoted $f_e$. The value of the investment opportunity is learned only once the fixed entry cost is sunk and the entrepreneur learns its productivity ($\varphi$), which is assumed to be a random draw from the distribution $G(\varphi)$ on support $\Phi \subseteq [1, \infty)$. After learning its productivity, the entrepreneur can decide to exit the industry immediately or build a plant and produce a variety. If the entrepreneur stays in the industry he makes three inter-related choices simultaneously. He chooses the quality and unit price of the plant’s output and

\textsuperscript{31} When countries are identical, they share the same aggregate variables, which greatly simplify the analysis. Extending the model to include N countries is straightforward but keeping an eye on the empirical analysis provides no additional insight.
whether or not to enter the foreign market, in order to maximize the plant’s profit function:

\[ \pi(\omega, \varphi) = pq - \Gamma(\omega, \varphi) + I_x (\tilde{p} \tilde{q} - f_x) \]  
(4)

where \( \Gamma(\omega, \varphi) \) is defined in (3), \( I_x \) is an indicator variable equal to 1 if some output is exported and 0 otherwise, and \( \tilde{p} \) and \( \tilde{q} \) represent the price and demand of a domestic variety sold in the foreign market.

An interesting feature of this problem is that the optimal choice of quality depends not only on the productivity of the firm but also on its exporting status. This implies that the mapping from productivity to quality is discontinuous at the marginal exporter’s productivity level. The solution is obtained in three steps: (i) Solve for the optimal price conditional on quality; (ii) Solve for the optimal quality conditional on export status; (iii) Solve for the threshold productivity level above which firms decide to enter the foreign market. Together these three results provide the equilibrium mapping between productivity, quality, price, quantity, revenue and export status.

Profit maximization implies that firms will set marginal cost equal to marginal revenue. This leads to the following pricing rules:

\[ p(\omega, \varphi) = \frac{\omega^n}{\rho \varphi}, \quad \text{and} \quad \tilde{p}(\omega, \varphi) = \tau p(\omega, \varphi) \]  
(5)

where \( p(\omega, \varphi) \) and \( \tilde{p}(\omega, \varphi) \) respectively denote the domestic and foreign price of a domestically produced variety conditional on the firm’s productivity and quality. These equations highlight the importance of the interaction between quality and productivity in determining the output price. While the price is increasing in the product’s quality it is decreasing in the plant’s productivity. Note that the curvature of
the price function in the quality space is governed by the quality elasticity of marginal production costs ($\eta$). This is due to the markup nature of the pricing rule: any change in production cost results in a proportional change in price.

Substituting (3) and (5) into (4) and using the fact that, from (2) and (5), $\bar{p}q = \tau^{1-\varepsilon}pq$, conditional profits for non-exporters ($\pi_d$) and exporters ($\pi_x$) can respectively be expressed as:

$\pi_d(\omega, \phi) = \Lambda(\omega, \phi) - f_c$  
where $\Lambda(\omega, \phi) \equiv \varepsilon^{-1} R (Pp)^{\varepsilon-1} \omega^{1-\eta(\varepsilon-1)} \phi^{\varepsilon-1} - \omega^\gamma$, and

$\pi_x(\omega, \phi) = (1 + \tau^{1-\varepsilon})^\beta \Lambda(\omega, \phi) - f_c - f_x$,  
where $\beta \equiv [\gamma + \eta(\varepsilon - 1) - 1]^{-1}$.  

(6)

Taking the first order conditions with respect to quality ($\omega$) yields the following optimal quality choice as a function of export status:

$\omega_d(\phi) = A\phi^{A(\varepsilon-1)}$,  
$\omega_x(\phi) = (1 + \tau^{1-\varepsilon})^\beta A\phi^{A(\varepsilon-1)}$,  
where $A \equiv \left[ \frac{1 - \eta(\varepsilon - 1)}{\gamma} \left( \frac{R}{\varepsilon} \right) (Pp)^{\varepsilon-1} \right]^\beta$.  

(7)

Quality is positive if the constant $A$, common to all producers in the market, is positive.$^{32}$ This requires that:  

Assumption 1: $1 > \eta(\varepsilon - 1)$.  

This condition is assumed to hold for the rest of the chapter. It requires that the quality elasticity of cost ($\eta$) and the price elasticity of demand ($\varepsilon$) are small. If this is not the case the increase in revenue from upgrading quality is less than the increase in cost and, as a result, upgrading quality is never optimal. This happens because consumers are very sensitive to price increases and put relatively more weight on

$^{32}$ The aggregate expenditure ($R$) is exogenous and equal to the size of the population ($L$), because the wage rate is normalized to one. Therefore, the constant $A$ depends on only one endogenous variable, the price index $P$.  

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price (or quantity) rather than quality – recall that \( \epsilon \) becomes large as \( \rho \) goes to 1, which from the preferences defined in (1) implies that quality has no importance. Further, whenever the iceberg transport cost is not trivial, equation (7) implies that there is a discontinuity in the function mapping productivity to optimal quality at the export margin and that exporters produce higher quality varieties. This happens because access to a larger market increases the return to quality upgrading. This point will be discussed further below.

Finally, define \( \Delta \equiv \pi_x - \pi_d \) as the difference between exporter and non-exporter profits. By definition, a profit maximizing firm will enter the foreign market if and only if \( \Delta \geq 0 \). Substituting (7) into \( \Lambda(\omega, \varphi) \), defined in (6), results in:

\[
\Lambda(\varphi) = \frac{A^Y}{\beta[1-\eta(\epsilon-1)]} \varphi^{(\beta-1)} (1-\epsilon-\gamma),
\]

where \( \beta \) and \( A \) are defined in (6) and (7) respectively. Hence, again from (6), the difference between exporter and non-exporter profits can be expressed as:

\[
\Delta(\varphi) = \frac{(1+\tau^{1-\epsilon})^{\beta}(\tau^{1-\epsilon}) - 1}{1-\eta(\epsilon-1)} \left[ \frac{A^Y}{\beta} \right] \varphi^{(\beta-1)} - f_x.
\]

The threshold productivity above which firms enter the export market is defined as the productivity level, \( \varphi_x \), such that \( \Delta(\varphi_x) = 0 \) and is given by:

\[
\varphi_x = \left[ \frac{1-\eta(\epsilon-1)}{(1+\tau^{1-\epsilon})^{\beta}-1} \left( \frac{\beta f_x}{A^Y} \right) \right]^{\frac{1}{\beta(\epsilon-1)}}.
\]

Note that this threshold implicitly depends on the distribution of productivity in the industry through \( A \), which depends in part on \( P \).
3.2.5 Equilibrium

Since profits are increasing in productivity and firms stay in the industry only if their profits are non-negative, free entry determines a profitability threshold (\( \phi_o \)) below which non-exporters will decide to exit the industry. The zero-profit condition that determines this profitability threshold is given by:

\[
\phi_o \equiv \pi_d (\phi_o) = 0 \iff \frac{A^\gamma}{\beta [1 - \eta (\epsilon - 1)]} \phi_o^{\beta (\epsilon - 1)} - f_c = 0, \tag{11}
\]

where the last expression follows from (6) and (8). Finally, combining (10) and (11) it follows that the productivity profitability threshold for export (\( \phi_x \)) is function of the profitability threshold for non-exporters and can be expressed as:

\[
\phi_x = \kappa \phi_o \quad \text{where} \quad \kappa \equiv \left\{ \frac{f_x}{f_x [(l + \tau^{-\epsilon})^{\beta \gamma} - 1]} \right\}^{\frac{1}{\beta \gamma (\epsilon - 1)}}. \tag{12}
\]

If \( \kappa < 1 \) the export threshold is lower than the non-exporters profitability threshold and all producers export. To make the model interesting \( \kappa \) is assumed to be greater than one. This requires that:

Assumption 2: \( f_x [(l + \tau^{-\epsilon})^{\beta \gamma} - 1]^{-1} > f_c \).

Intuitively, the export costs (\( f_x \) and \( \tau \)) have to be large relative to the lower bound on fixed cost (\( f_c \)). Under this assumption, plants drawing productivity below the profitability threshold, defined in (11), will exit the industry. Those drawing productivity above will engage in profitable production. In addition, if the
productivity is above the export profitability threshold, defined in (12), the plant will export.

Each period, incumbents face a probability $\delta \in (0,1)$ of being hit by an exogenous shock that will force them to exit the industry. Hence, the value of entry is zero if the entrant draws a productivity below the profitability threshold and exits, and equal to the stream of future profits discounted by the probability of exit if it draws an ability above the cutoff value and produces. Since profit is the same in every period, the value of entry, conditional on the productivity draw, can be expressed as:

$$V(\phi) = \sum_{t=0}^{\infty} (1-\delta)^t \pi(\phi) = \frac{\pi(\phi)}{\delta}, \text{ where } \pi(\phi) = \max\{0, \pi_d(\phi), \pi_x(\phi)\}$$

and $t$ is the time index. The ex-post probability density function for productivity, $\mu(\phi)$ is conditional on successful entry and is truncated at the profitability threshold ($\phi_o$). To obtain tractable closed form solutions and to make progress towards an empirical and computational model, more structure needs to be put on the distribution of productivity. Following the literature, productivity is assumed to be Pareto distributed.\(^{33}\) The ex-ante cumulative distribution function of productivity is thus given by $G(\phi) = 1 - \phi^{-\sigma}$ where $\sigma$ is a parameter that affects the shape of the distribution. Some restrictions on this parameter are required.

Assumption 3: $\sigma > \max\{2, \beta \gamma (c-1)\}$.

\(^{33}\) As explained earlier, in addition to being tractable, the Pareto distribution provides a reasonable approximation of the empirical productivity distribution; see for instance Cabral and Mata (2003).
By definition of the Pareto distribution, $\sigma > 2$ is required to ensure a finite variance. The assumption that $\sigma > \beta \gamma (\epsilon - 1)$ is required to ensure a well defined equilibrium; see the appendix for details. Under these conditions, the conditional ex-post distribution is given by:

$$
\mu(\phi) = \begin{cases} 
\sigma \phi_{o}^{\sigma} \phi^{-(1+\alpha)} & \text{if } \phi > \phi_{o} \\
0 & \text{otherwise},
\end{cases}
$$

while the ex-ante probability of successful entry in the industry is given by $1 - G(\phi_{o}) = \phi_{o}^{-\sigma}$.

There exists an unbounded set of potential entrants in the industry. Plants will attempt entry in the industry as long as the expected value from entry is greater then the sunk entry cost $f_{e}$. The characteristics of the ex-ante distribution of productivity $G(\phi)$ are assumed to be common knowledge such that the expected value of entry is identical for all potential entrants and given by the product of the average incumbent’s value and the ex-ante probability of successful entry. Therefore, the free entry condition can be written:

$$
V^{E} \equiv E(\pi | G(\phi), \delta, \phi_{o}) = \frac{\bar{\pi}(\phi_{o})}{\delta \phi_{o}^{\sigma}} = f_{e}, \text{ where } \bar{\pi}(\phi_{o}) \equiv \int_{\phi_{o}}^{\infty} \pi(\phi) \mu(\phi) \, d\phi \quad (13)
$$

is the average profit in the industry conditional on the profitability threshold. Equation (13) clearly shows that the free entry condition depends on only one endogenous parameter, the profitability threshold ($\phi_{o}$). This condition alone is sufficient to pin down the unique equilibrium value of the threshold ($\phi_{o}^{*}$) as a function of the parameters of the model. In the appendix it is shown that:
\[
\varphi_0^* = \left[ \frac{\beta \gamma (\epsilon - 1) \left( f_e + f_s \kappa^{-\sigma} \right)}{\sigma - \beta \gamma (\epsilon - 1) \delta f_e} \right]^{\frac{1}{\sigma}}.
\]

(14)

The profitability threshold is always positive since from assumption 3, \( \beta \gamma (\epsilon - 1) < \sigma \).

However the assumptions on the parameters are not sufficient to guarantee that \( \varphi_0^* \) is greater than one – the lower bound on productivity. It is therefore possible to have an equilibrium in which every entrant stays in the industry regardless of their productivity.

Since countries are identical, trade is balanced and revenue is the same in each country. The equilibrium mass of producers in the industry (\( M \)) can be obtained by dividing aggregate expenditure (\( R = L \)) by the average firm-level revenue (\( \bar{r} \)) – see the appendix. The equilibrium threshold and mass of incumbents can be used to obtain an expression for the equilibrium price index defined in (2); again see the appendix. By definition, in a stationary equilibrium, every aggregate variable must remain constant over time. This requires a mass of new entrants (\( M_e \)) in each period, such that the mass of successful entrants (\( \zeta_e M_e \)) exactly replaces the mass of incumbents (\( \delta M \)) hit by the exogenous shock and forced to exit. This aggregate stability condition requires \( \zeta_e M_e = \delta M \). Finally, it can be shown that:

Proposition 1. \textit{There exists a unique equilibrium.}

\textit{Proof:} See appendix. \( \square \)
This completes the characterization of the unique costly trade open economy equilibrium.

3.2.6 Analysis of Equilibrium

This section explores the theoretical implications of quality differentiation. The analysis focuses on predictions that are testable given the available data. The main results are summarized in three propositions that will be examined empirically.

From (7), productivity and quality are positively related in equilibrium as long as

\[
\beta = \left[ \gamma + \eta(\epsilon - 1) - 1 \right]^{-1} > 0,
\]

which requires that \((1 - \gamma)/\eta < \epsilon - 1\). Intuitively this condition holds if the marginal production cost increases fast enough in quality relative to the fixed cost. In this case, the function mapping productivity into profits associated with a high quality variety will intersect the lower quality profit functions from below and the model leads to endogenous quality sorting of producers by productivity, such that in equilibrium quality and productivity are positively correlated.

Proposition 2. If the marginal production cost increases fast enough in quality relative to the fixed cost \((\beta > 0)\), product quality and plant productivity are positively correlated.

As mentioned earlier, there is a discontinuity at the margin between exporter and non-exporters in the function mapping productivity to optimal quality as defined in (7). The percentage difference in quality between exporters and non-exporters conditional on productivity is approximately equal to \(\beta \ln(1 + \tau^{1-\epsilon})\). Therefore the jump in quality
depends on the demand and technology parameters as well as the iceberg trade cost. A decrease in trade costs increases the share of the foreign market domestic firms will be able to capture, thereby increasing the gains from quality upgrading for exporters.

The model predicts that productivity has two opposite effects on the equilibrium price. From (7), the optimal choice of quality is increasing in productivity. From the pricing rule (5), price is increasing in quality but decreasing in productivity. Therefore to obtain the relationship between price and productivity, the effect of productivity on quality must be taken into account. Replacing with (7) in (5) provides the equilibrium domestic price schedule as a function of productivity and export status only:

\[
p(\varphi) = \frac{A^\eta}{\rho} \varphi^{\eta(\varepsilon-1)-1} \quad \text{and} \quad p_x(\varphi) = (1 + \tau^{1-\varepsilon})^{\eta} p(\varphi).
\]

where \( p_x(\varphi) \) is the home price of a domestically produced variety manufactured by an exporter. These equations have two important implications. First, price and productivity are negatively correlated only if \( \beta(\varepsilon-1)\eta < 1 \) which, if quality and productivity are positively related \( (\beta > 0) \) is equivalent to \( \gamma > 1 \). Therefore price is negatively correlated with productivity only if the fixed production cost is convex in quality.\(^{34}\) Second, in the presence of transport costs, exporters will charge higher prices. The size of the discontinuity in price at the exporting margin is given by \( (1 + \tau^{1-\varepsilon})^{\eta} \), which is decreasing in \( \tau \). As mentioned earlier, a decrease in trade costs increases the gains from quality upgrading, which leads to higher prices.

\(^{34}\) This is a necessary but not a sufficient condition. Recall that at the export margin there is a jump in quality and, as a result, in price. This could lead to a positive correlation despite the convexity of the fixed cost. However, if the convexity is strong enough the correlation between price and productivity will be negative.
Proposition 3. *Holding quality fixed, price is decreasing in productivity, while holding productivity fixed, price is increasing in quality. Further, price and productivity are negatively correlated only if the fixed production cost is convex in quality.*

From (12), the model predicts that only the most productive firms will engage in international trade. Therefore, since productivity and quality are positively related, exporting firms will produce relatively high quality varieties. Further, from (2) and (5), exporters will be larger in terms of revenue and output. Finally, firms engage in international trade only if the *extra* profit from entering the foreign market \( \pi_x \) is greater than zero. From (6) and (7) it can be shown that:

\[
\tilde{\pi}_x(\phi) = R (Pp)^{\epsilon-1} \omega_x (\phi)^{1-\eta(\epsilon-1)} \phi^{\epsilon-1} - f_x ,
\]

where \( \omega_x(\phi) \) is defined in (7). This implies that the effect of productivity on the probability of export can be decomposed into two components: a direct or price effect \( (\phi^{\epsilon-1}) \) and an indirect or quality effect \( (\omega^{1-\eta(\epsilon-1)}) \). These results are summarized in the following proposition:

Proposition 4. *Exporting firms are more productive, larger (in terms of revenue and output) and produce higher quality varieties than firms that sell their output in the domestic market only.*

This concludes the analysis of the unique open economy equilibrium.
3.3 Data

The data set is derived from the Census of Manufactures (CM) – a component of the US Census Bureau’s Economic Census. The CM is conducted every five years and the analysis uses information from 1972 to 1997. The unit of observation for the analysis is a plant/product/year combination. The CM covers all manufacturing plants with one or more paid employees. However very small plants – known as administrative records – are exempt from filling out Census forms. In those cases, the plant’s information is imputed. These establishments represent a very small share of overall U.S. manufacturing output and are removed from the sample. The CM contains plant level data on payroll, employment, book values of equipment and structures, the cost of materials and energy, and export value and plant-by-product level data on the value of shipments. In addition, for a subset of products, the CM collects information on shipments in physical units. Since the empirical analysis requires data on price and quantity, the sample is limited to those products for which such information is available.\(^{35}\)

For the empirical analysis, products are defined as five-digit standard industrial classification (SIC) product classes.\(^{36}\) The 1987 SIC code segments manufacturing output into 459 four-digit industry and 1446 five-digit product classes according to its end use.\(^{37}\) Table I provides a sense of the relative level of detail between five-digit product classes, four-digit industries and two-digit major sector. The table lists the

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\(^{35}\) The subset varies over time. For instance, much of the apparel major group (SIC 23) was dropped in 1982. Plant-by-product balancing codes, receipt for contract work, resale, and miscellaneous receipts are removed since these observations are unrelated to production.

\(^{36}\) Plants are required to report quantity produced at the seven-digit SIC level. For those plants in the sample that produce more than one seven-digit product within their primary five-digit product class the quantities are aggregated.

\(^{37}\) Additional details can be obtained from the Numerical List of Manufactured and Mineral Products published by the U.S. Census Bureau (1996).

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products contained in SIC industry 2051, “Bread, Cake and Related Products”, which is one of the industries in SIC major sector 20, “Food and Kindred Products”. The industry contains six products which, although related in end use, differ in terms of material inputs and production technologies. Therefore, using the much finer five-digit classification removes a lot of undesired horizontal differentiation from the analysis.

Table I: Product Categories in SIC 2051

<table>
<thead>
<tr>
<th>SIC</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Food and Kindred Products</td>
</tr>
<tr>
<td>2051</td>
<td>Bread, Cake and Related Products</td>
</tr>
<tr>
<td>20511</td>
<td>Bread: White, Wheat and Rye</td>
</tr>
<tr>
<td>20512</td>
<td>Rolls, Bread-Type</td>
</tr>
<tr>
<td>20513</td>
<td>Sweet Yeast Goods</td>
</tr>
<tr>
<td>20514</td>
<td>Soft Cakes</td>
</tr>
<tr>
<td>20515</td>
<td>Pies</td>
</tr>
<tr>
<td>20518</td>
<td>Pastries</td>
</tr>
</tbody>
</table>


Minor revisions to SIC categories are made in each census year and major revisions were made in 1977 and 1987. These changes make it difficult to keep track of products over time while ensuring that the product’s definition remains the same. Therefore, to ensure uniformity of products the analysis is limited to codes that appear in every year.

The average unit value of output, a proxy for price, is defined as the ratio of the nominal product shipment value and quantity produced. In order to limit large
reporting errors, observations with an output price above 5 times or lower than one fifth of the product’s median price are removed from the sample.

In the CM factor inputs are reported not separately by product but rather at the plant level. To reduce measurement problems in computing productivity measures and to increase the accuracy of the production cost measures the sample includes only the primary product of specialized plants. A multi-product plant is considered to be specialized if the primary product accounts for at least 50 percent of the total nominal value of plant shipments.

Further, to ensure that there is enough variation to estimate plant fixed effects, the sample is limited to product classes for which there are at least 25 specialized observations that satisfy all the above criteria in each year. Finally, a few product classes with heterogeneous units of measurement for quantity are removed from the sample.

Together, these rules lead to a sample of 107,115 observations distributed across 125 five-digit SIC product classes. The sample contains about 4.5 percent of the total plant/year observations in the CM and about 6 percent of the five-digit SIC codes are represented in the sample. Table II provides basic statistics for the sample at the two-digit SIC major sector level. Although most sectors are represented in the sample, some are more important than others. In particular the “food and kindred product” sector (SIC 20) accounts for 25 percent of observations and about half of the revenue and export value in the sample. The “lumber and wood products” (SIC 24) and the “stone, clay, and glass products” (SIC 32) sectors also account for relatively large

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38 Using a minimum of 50 plants leads to almost identical results.
<table>
<thead>
<tr>
<th>SIC2</th>
<th>Name</th>
<th>Total Number of Products</th>
<th>Sample Share of Plants</th>
<th>Sample Share of Revenue</th>
<th>Sample Share of Export</th>
<th>St. Dev. of Log Price</th>
<th>Revenue Share (%) of Advertising</th>
<th>Revenue Share (%) of Software</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Food and Kindred Products</td>
<td>46</td>
<td>0.25</td>
<td>0.43</td>
<td>0.53</td>
<td>0.41</td>
<td>0.437</td>
<td>0.038</td>
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<tr>
<td>22</td>
<td>Textile Mill Products</td>
<td>13</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
<td>0.52</td>
<td>0.076</td>
<td>0.054</td>
</tr>
<tr>
<td>23</td>
<td>Apparel and Other Textile Products</td>
<td>3</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.56</td>
<td>0.548</td>
<td>0.104</td>
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<td>24</td>
<td>Lumber and Wood Products</td>
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<td>0.10</td>
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<td>0.41</td>
<td>0.141</td>
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<td>25</td>
<td>Furniture and Fixtures</td>
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<td>0.03</td>
<td>0.02</td>
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<td>Paper and Allied Products</td>
<td>6</td>
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<td>0.07</td>
<td>0.02</td>
<td>0.36</td>
<td>0.023</td>
<td>0.043</td>
</tr>
<tr>
<td>28</td>
<td>Chemicals and Allied Products</td>
<td>4</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.60</td>
<td>0.071</td>
<td>0.054</td>
</tr>
<tr>
<td>29</td>
<td>Petroleum and Coal Products</td>
<td>2</td>
<td>0.01</td>
<td>0.15</td>
<td>0.08</td>
<td>0.43</td>
<td>0.135</td>
<td>0.040</td>
</tr>
<tr>
<td>31</td>
<td>Leather and Leather Products</td>
<td>6</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.58</td>
<td>0.706</td>
<td>0.364</td>
</tr>
<tr>
<td>32</td>
<td>Stone, Clay, and Glass Products</td>
<td>5</td>
<td>0.18</td>
<td>0.05</td>
<td>0.01</td>
<td>0.25</td>
<td>0.060</td>
<td>0.029</td>
</tr>
<tr>
<td>33</td>
<td>Primary Metal Industries</td>
<td>8</td>
<td>0.07</td>
<td>0.07</td>
<td>0.11</td>
<td>0.47</td>
<td>0.058</td>
<td>0.070</td>
</tr>
<tr>
<td>34</td>
<td>Fabricated Metal Products</td>
<td>1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.50</td>
<td>0.032</td>
<td>0.079</td>
</tr>
<tr>
<td>35</td>
<td>Industrial Machinery and Equipment</td>
<td>2</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.69</td>
<td>0.366</td>
<td>0.223</td>
</tr>
<tr>
<td>37</td>
<td>Transportation Equipment</td>
<td>1</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.70</td>
<td>0.836</td>
<td>0.027</td>
</tr>
<tr>
<td>39</td>
<td>Miscellaneous Manufacturing</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.43</td>
<td>0.066</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Sample 125 107,115 0.42 0.322 0.085

*Notes:* This table shows the number of product and plants/year observations by SIC2 sectors, as well as each sector’s share of total real revenue and export in the sample (pooled across all years). The table also shows the standard deviation of log price and the average revenue share of advertising and new software expenditure at the plant-level in each sector. Product-year fixed effects are removed from price before computing the standard deviations.
fractions of observations. As can be seen from Table III, which presents the complete list of five-digit SIC products included in the analysis, this uneven representation across sectors is mostly due to the distribution of sample products across two-digit SIC sectors rather than the distribution of plants across five-digit SIC product classes. This characteristic of the sample reflects the Census Bureau’s decision to collect product level physical quantity information in some sectors and not in others. To get a sense of how representative of the CM universe this sample is, the share of revenue, export and number of plant across two-digit major are calculated and compared to those in the sample. On the one hand, the Food and Kindred Products (SIC 20), the Lumber and Wood Products (SIC 24) and the Stone, Clay, and Glass Products (SIC 32) each account for much larger fraction of plant/year observations, revenue and export revenue in the sample than in the CM. On the other hand Printing and Publishing (SIC 27), Fabricated Metal Products (SIC 34) and Industrial Machinery and Equipment (SIC 35) are all underrepresented in the sample. Therefore it is not clear that the conclusions of this study can be applied to the whole manufacturing sector.

An important message of Table II is the substantial variation in price within each product class. On average, after removing product/year fixed effects, the standard deviation of mean log price is about 0.42. Moreover, there is considerable heterogeneity in this measure of dispersion across two-digit SIC major sectors. The dispersion estimates range from 0.06 for the “stone, clay, and glass products” sector (SIC 32) to 0.49 in the “transportation equipment” sector (SIC 37). In general the

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39 One notable exception is the “ready-mixed concrete” product class (SIC 32730) which comprises 14,414 plants/year, or 13.5 percent of the observations.
Table III: Product Characteristics

<table>
<thead>
<tr>
<th>SIC5</th>
<th>Name</th>
<th>Plant/year</th>
<th>Share</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>20111</td>
<td>Beef, Not Canned or Made Into Sausage (NCOMIS)</td>
<td>1,458</td>
<td>1.4</td>
<td>-4.33 (0.34)**</td>
</tr>
<tr>
<td>20114</td>
<td>Pork, NCOMIS</td>
<td>360</td>
<td>0.3</td>
<td>-2.01 (0.26)**</td>
</tr>
<tr>
<td>20117</td>
<td>Sausages and Similar Products, Not Canned (NC)</td>
<td>250</td>
<td>0.2</td>
<td>-3.04 (0.52)**</td>
</tr>
<tr>
<td>20136</td>
<td>Pork, Processed or Cured, Including Frozen (NCOMIS)</td>
<td>508</td>
<td>0.5</td>
<td>-2.91 (0.32)**</td>
</tr>
<tr>
<td>20137</td>
<td>Sausage and Similar Products (NC)</td>
<td>1,366</td>
<td>1.3</td>
<td>-2.54 (0.17)**</td>
</tr>
<tr>
<td>20151</td>
<td>Young Chickens (Usually Under 20 Weeks Of Age) Whole or Parts</td>
<td>1,011</td>
<td>0.9</td>
<td>-3.01 (0.29)**</td>
</tr>
<tr>
<td>20153</td>
<td>Turkeys (Including Frozen, Whole or Parts)</td>
<td>256</td>
<td>0.2</td>
<td>-3.83 (0.94)**</td>
</tr>
<tr>
<td>20159</td>
<td>Liquid, Dried, and Frozen Eggs</td>
<td>213</td>
<td>0.2</td>
<td>-2.74 (0.42)**</td>
</tr>
<tr>
<td>20223</td>
<td>Natural Cheese, Except Cottage Cheese</td>
<td>1,877</td>
<td>1.8</td>
<td>-4.95 (0.38)**</td>
</tr>
<tr>
<td>20235</td>
<td>Dry Milk Products and Mixtures</td>
<td>346</td>
<td>0.3</td>
<td>-3.52 (0.71)**</td>
</tr>
<tr>
<td>20240</td>
<td>Ice Cream and Ices</td>
<td>1,221</td>
<td>1.1</td>
<td>-2.36 (0.15)**</td>
</tr>
<tr>
<td>20331</td>
<td>Canned Fruits, Except Baby Foods</td>
<td>384</td>
<td>0.4</td>
<td>-3.03 (0.42)**</td>
</tr>
<tr>
<td>20332</td>
<td>Canned Vegetables, Except Hominy and Mushrooms</td>
<td>979</td>
<td>0.9</td>
<td>-3.44 (0.33)**</td>
</tr>
<tr>
<td>2033A</td>
<td>Canned Fruit Juices, Nectars, and Concentrates</td>
<td>250</td>
<td>0.2</td>
<td>-3.16 (0.63)**</td>
</tr>
<tr>
<td>20343</td>
<td>Dried and Dehydrated Fruits and Vegetables (Including Freeze-Dried)</td>
<td>382</td>
<td>0.4</td>
<td>-2.58 (0.29)**</td>
</tr>
<tr>
<td>20352</td>
<td>Pickles and Other Pickled Products</td>
<td>416</td>
<td>0.4</td>
<td>-1.50 (0.12)**</td>
</tr>
<tr>
<td>20354</td>
<td>Mayonnaise, Salad Dressings, and Sandwich Spreads</td>
<td>319</td>
<td>0.3</td>
<td>-1.96 (0.19)**</td>
</tr>
<tr>
<td>20372</td>
<td>Frozen Vegetables</td>
<td>640</td>
<td>0.6</td>
<td>-3.01 (0.29)**</td>
</tr>
<tr>
<td>20382</td>
<td>Frozen Dinners</td>
<td>608</td>
<td>0.6</td>
<td>-2.12 (0.17)**</td>
</tr>
<tr>
<td>20384</td>
<td>Frozen Specialties</td>
<td>310</td>
<td>0.3</td>
<td>-1.29 (0.18)**</td>
</tr>
<tr>
<td>20411</td>
<td>Wheat Flour, Except Flour Mixes</td>
<td>900</td>
<td>0.8</td>
<td>-3.06 (0.28)**</td>
</tr>
<tr>
<td>20440</td>
<td>Milled Rice and Byproducts</td>
<td>217</td>
<td>0.2</td>
<td>-2.03 (0.29)**</td>
</tr>
<tr>
<td>20473</td>
<td>Dog Food</td>
<td>524</td>
<td>0.5</td>
<td>-2.38 (0.22)**</td>
</tr>
<tr>
<td>20481</td>
<td>Chicken and Turkey Feed, Supplements, Concentrates, and Premixes</td>
<td>1,032</td>
<td>1.0</td>
<td>-3.75 (0.34)**</td>
</tr>
<tr>
<td>20482</td>
<td>Dairy Cattle Feed, Complete</td>
<td>574</td>
<td>0.5</td>
<td>-5.20 (1.27)**</td>
</tr>
<tr>
<td>20485</td>
<td>Swine Feed Supplements, Concentrates, and Premixes</td>
<td>318</td>
<td>0.3</td>
<td>-2.77 (0.51)**</td>
</tr>
<tr>
<td>20487</td>
<td>Beef Cattle Feed Supplements, Concentrates, and Premixes</td>
<td>274</td>
<td>0.3</td>
<td>-4.63 (1.13)**</td>
</tr>
<tr>
<td>20511</td>
<td>Bread: White, Wheat, and Rye (Including Frozen)</td>
<td>2,112</td>
<td>2.0</td>
<td>-2.01 (0.11)**</td>
</tr>
<tr>
<td>20514</td>
<td>Soft Cakes</td>
<td>336</td>
<td>0.3</td>
<td>-1.45 (0.19)**</td>
</tr>
<tr>
<td>20521</td>
<td>Crackers, Pretzels, Biscuits, and Related Products</td>
<td>394</td>
<td>0.4</td>
<td>-2.81 (0.33)**</td>
</tr>
<tr>
<td>20610</td>
<td>Sugarcane Mill Products and Byproducts</td>
<td>245</td>
<td>0.2</td>
<td>-1.64 (0.15)**</td>
</tr>
<tr>
<td>20630</td>
<td>Refined Beet Sugar and Byproducts</td>
<td>265</td>
<td>0.3</td>
<td>-2.07 (0.17)**</td>
</tr>
<tr>
<td>20680</td>
<td>Nuts and Seeds (Salted, Roasted, Cooked, or Blanched)</td>
<td>394</td>
<td>0.4</td>
<td>-1.91 (0.22)**</td>
</tr>
<tr>
<td>20771</td>
<td>Grease and Inedible Tallow</td>
<td>550</td>
<td>0.5</td>
<td>-2.52 (0.30)**</td>
</tr>
<tr>
<td>20772</td>
<td>Feed and Fertilizer Byproducts</td>
<td>430</td>
<td>0.4</td>
<td>-1.83 (0.20)**</td>
</tr>
<tr>
<td>20791</td>
<td>Shortening and Cooking Oils</td>
<td>314</td>
<td>0.3</td>
<td>-1.73 (0.23)**</td>
</tr>
<tr>
<td>20821</td>
<td>Canned Beer and Ale Case Goods</td>
<td>213</td>
<td>0.2</td>
<td>-10.1 (4.87)**</td>
</tr>
<tr>
<td>20840</td>
<td>Wines, Brandy, and Brandy Spirits</td>
<td>816</td>
<td>0.8</td>
<td>-1.41 (0.07)**</td>
</tr>
<tr>
<td>20853</td>
<td>Bottled Liquor, Except Brandy</td>
<td>293</td>
<td>0.3</td>
<td>-1.50 (0.18)**</td>
</tr>
<tr>
<td>20923</td>
<td>Frozen Fish</td>
<td>386</td>
<td>0.4</td>
<td>-3.19 (0.54)**</td>
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<tr>
<td>20925</td>
<td>Frozen Shellfish</td>
<td>393</td>
<td>0.4</td>
<td>-3.27 (0.53)**</td>
</tr>
<tr>
<td>20951</td>
<td>Roasted Coffee</td>
<td>539</td>
<td>0.5</td>
<td>-3.52 (0.38)**</td>
</tr>
<tr>
<td>20961</td>
<td>Potato Chips and Sticks, Plain and Flavored</td>
<td>832</td>
<td>0.8</td>
<td>-2.73 (0.24)**</td>
</tr>
</tbody>
</table>
Table III: Product Characteristics (continued)

<table>
<thead>
<tr>
<th>SIC5</th>
<th>Name</th>
<th>Plant/year</th>
<th>Share</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>20970</td>
<td>Manufactured Ice</td>
<td>1,045</td>
<td>1.0</td>
<td>-1.96 (0.10)***</td>
</tr>
<tr>
<td>20980</td>
<td>Macaroni, Spaghetti, and Egg Noodle Products</td>
<td>420</td>
<td>0.4</td>
<td>-2.21 (0.27)***</td>
</tr>
<tr>
<td>20996</td>
<td>Vinegar and Cider</td>
<td>185</td>
<td>0.2</td>
<td>-1.98 (0.25)***</td>
</tr>
<tr>
<td>2221J</td>
<td>Finished Manmade Fiber and Silk Broad woven Fabrics</td>
<td>197</td>
<td>0.2</td>
<td>-1.35 (0.15)***</td>
</tr>
<tr>
<td>22516</td>
<td>Women’s and Misses’ Finished Panty Hose, Including Tights</td>
<td>394</td>
<td>0.4</td>
<td>-1.77 (0.22)***</td>
</tr>
<tr>
<td>22522</td>
<td>Men’s Finished Seamless Hosiery (Sizes 10 and Up)</td>
<td>591</td>
<td>0.6</td>
<td>-1.73 (0.15)***</td>
</tr>
<tr>
<td>22573</td>
<td>Finished Wef (Circular) Knit Fabrics, Except Hosiery</td>
<td>552</td>
<td>0.5</td>
<td>-1.79 (0.18)***</td>
</tr>
<tr>
<td>22581</td>
<td>Warp Knit Fabrics Greige Goods</td>
<td>249</td>
<td>0.2</td>
<td>-1.97 (0.30)***</td>
</tr>
<tr>
<td>22617</td>
<td>Finished Cotton Broad woven Fabrics (Not Finished in Weaving Mills)</td>
<td>276</td>
<td>0.3</td>
<td>-1.16 (0.14)***</td>
</tr>
<tr>
<td>22629</td>
<td>Finishing of Manmade Fiber and Silk Broad Woven Fabrics</td>
<td>533</td>
<td>0.5</td>
<td>-1.43 (0.10)***</td>
</tr>
<tr>
<td>22690</td>
<td>Finished Yarn, Raw Stock, and Narrow Fabrics</td>
<td>345</td>
<td>0.3</td>
<td>-1.55 (0.14)***</td>
</tr>
<tr>
<td>22811</td>
<td>Carded Cotton Yarns</td>
<td>515</td>
<td>0.5</td>
<td>-2.34 (0.31)***</td>
</tr>
<tr>
<td>22814</td>
<td>Spun Noncellulosic Fiber and Silk Yarns</td>
<td>903</td>
<td>0.8</td>
<td>-3.13 (0.26)***</td>
</tr>
<tr>
<td>22825</td>
<td>Textured, Crimped, or Bulked Filament Yarns, Including Stretch Yarn</td>
<td>323</td>
<td>0.3</td>
<td>-1.59 (0.19)***</td>
</tr>
<tr>
<td>22971</td>
<td>Non Woven Fabrics</td>
<td>266</td>
<td>0.3</td>
<td>-1.47 (0.12)***</td>
</tr>
<tr>
<td>22982</td>
<td>Soft Fiber Cordage and Twine (Except Cotton)</td>
<td>280</td>
<td>0.3</td>
<td>-1.34 (0.13)***</td>
</tr>
<tr>
<td>23230</td>
<td>Men’s and Boys’ Neckwear</td>
<td>345</td>
<td>0.3</td>
<td>-1.82 (0.20)***</td>
</tr>
<tr>
<td>23532</td>
<td>Cloth Hats and Caps</td>
<td>503</td>
<td>0.5</td>
<td>-1.84 (0.17)***</td>
</tr>
<tr>
<td>23871</td>
<td>Leather Belts</td>
<td>307</td>
<td>0.3</td>
<td>-1.35 (0.10)***</td>
</tr>
<tr>
<td>24111</td>
<td>Softwood Logs, Bolts, and Timber</td>
<td>2,215</td>
<td>2.1</td>
<td>-4.05 (0.53)***</td>
</tr>
<tr>
<td>24113</td>
<td>Pulpwood</td>
<td>840</td>
<td>0.8</td>
<td>-3.99 (1.14)***</td>
</tr>
<tr>
<td>24211</td>
<td>Hardwood Lumber, Rough and Dressed, Except Siding</td>
<td>3,808</td>
<td>3.6</td>
<td>-2.41 (0.14)***</td>
</tr>
<tr>
<td>24212</td>
<td>Softwood Lumber, Rough and Dressed, Except Siding</td>
<td>4,707</td>
<td>4.4</td>
<td>-3.75 (0.20)***</td>
</tr>
<tr>
<td>24217</td>
<td>Softwood Cut Stock</td>
<td>334</td>
<td>0.3</td>
<td>-2.12 (0.32)***</td>
</tr>
<tr>
<td>24261</td>
<td>Hardwood Flooring</td>
<td>265</td>
<td>0.3</td>
<td>-1.53 (0.20)***</td>
</tr>
<tr>
<td>24262</td>
<td>Hardwood Dimension Stock, Furniture Parts, and Vehicle Stock</td>
<td>982</td>
<td>0.9</td>
<td>-1.35 (0.08)***</td>
</tr>
<tr>
<td>24266</td>
<td>Wood Furniture Frames For Household Furniture</td>
<td>629</td>
<td>0.6</td>
<td>-2.30 (0.19)***</td>
</tr>
<tr>
<td>24311</td>
<td>Wood Window Units</td>
<td>376</td>
<td>0.4</td>
<td>-1.60 (0.19)***</td>
</tr>
<tr>
<td>24314</td>
<td>Wood Doors, Interior And Exterior</td>
<td>679</td>
<td>0.6</td>
<td>-1.56 (0.10)***</td>
</tr>
<tr>
<td>24341</td>
<td>Wood Kitchen Cabinets and Cabinetetwork, Stock Line</td>
<td>1,127</td>
<td>1.1</td>
<td>-2.08 (0.18)***</td>
</tr>
<tr>
<td>24351</td>
<td>Hardwood Plywood</td>
<td>387</td>
<td>0.4</td>
<td>-1.41 (0.11)***</td>
</tr>
<tr>
<td>24354</td>
<td>Hardwood Veneer, Not Reinforced or Backed</td>
<td>470</td>
<td>0.4</td>
<td>-1.39 (0.12)***</td>
</tr>
<tr>
<td>24364</td>
<td>Softwood Veneer, Not Reinforced or Backed</td>
<td>266</td>
<td>0.3</td>
<td>-1.61 (0.15)***</td>
</tr>
<tr>
<td>24365</td>
<td>Softwood Plywood, Rough, Including Touch Sanded</td>
<td>405</td>
<td>0.4</td>
<td>-7.55 (2.11)***</td>
</tr>
<tr>
<td>24390</td>
<td>Fabricated Structural Wood Products</td>
<td>704</td>
<td>0.7</td>
<td>-2.08 (0.26)***</td>
</tr>
<tr>
<td>24511</td>
<td>Manufactured (Mobile) Homes (35 Feet or More In Length)</td>
<td>1,825</td>
<td>1.7</td>
<td>-5.02 (0.30)***</td>
</tr>
<tr>
<td>24522</td>
<td>Precut Packages for Stationary Buildings (Complete Units)</td>
<td>370</td>
<td>0.4</td>
<td>-1.85 (0.30)***</td>
</tr>
<tr>
<td>24524</td>
<td>Stationary Buildings Shipped in Three-Dimensional Assemblies</td>
<td>417</td>
<td>0.4</td>
<td>-2.22 (0.26)***</td>
</tr>
<tr>
<td>24931</td>
<td>Particleboard, Produced at this Location</td>
<td>322</td>
<td>0.3</td>
<td>-2.04 (0.27)***</td>
</tr>
<tr>
<td>25112</td>
<td>Wood Living Room, Library, Family Room, and Den Furniture</td>
<td>680</td>
<td>0.6</td>
<td>-1.58 (0.14)***</td>
</tr>
<tr>
<td>25113</td>
<td>Wood Dining Room And Kitchen Furniture, Except Kitchen Cabinets</td>
<td>608</td>
<td>0.6</td>
<td>-1.99 (0.15)***</td>
</tr>
<tr>
<td>25115</td>
<td>Wood Bedroom Furniture</td>
<td>864</td>
<td>0.8</td>
<td>-1.48 (0.09)***</td>
</tr>
<tr>
<td>25120</td>
<td>Upholstered Wood Household Furniture</td>
<td>2,736</td>
<td>2.6</td>
<td>-1.65 (0.05)***</td>
</tr>
</tbody>
</table>
Table III: Product Characteristics (continued)

<table>
<thead>
<tr>
<th>SIC5</th>
<th>Name</th>
<th>Plant/year</th>
<th>Share</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>25145</td>
<td>Metal Household Dining Room and Kitchen Furniture</td>
<td>212</td>
<td>0.2</td>
<td>-1.14 (0.13)***</td>
</tr>
<tr>
<td>25147</td>
<td>Other Metal Household Furniture</td>
<td>201</td>
<td>0.2</td>
<td>-1.75 (0.25)***</td>
</tr>
<tr>
<td>25151</td>
<td>Innerspring Mattresses, Excluding Crib-Size</td>
<td>1,181</td>
<td>1.1</td>
<td>-1.91 (0.15)***</td>
</tr>
<tr>
<td>26530</td>
<td>Corrugated And Solid Fiber Boxes, Including Pallets</td>
<td>5,796</td>
<td>5.4</td>
<td>-2.47 (0.07)***</td>
</tr>
<tr>
<td>26552</td>
<td>Fiber Cans, Tubes, and Similar Fiber Products</td>
<td>884</td>
<td>0.8</td>
<td>-1.33 (0.06)***</td>
</tr>
<tr>
<td>26570</td>
<td>Folding Paperboard Boxes, Packaging, and Packaging Components</td>
<td>1,549</td>
<td>1.5</td>
<td>-1.85 (0.10)***</td>
</tr>
<tr>
<td>26741</td>
<td>Grocers’ Bags and Sacks and Variety and Shopping Bags</td>
<td>721</td>
<td>0.7</td>
<td>-1.60 (0.11)***</td>
</tr>
<tr>
<td>26742</td>
<td>Shipping Sacks and Multiwall Bags, All Materials Except Textiles</td>
<td>384</td>
<td>0.4</td>
<td>-1.46 (0.12)***</td>
</tr>
<tr>
<td>26753</td>
<td>Pasted, Lined, Laminated, or Surface-Coated Paperboard</td>
<td>215</td>
<td>0.2</td>
<td>-4.46 (0.95)***</td>
</tr>
<tr>
<td>26752</td>
<td>Surfactants, Finishing Agents, and Assistants</td>
<td>203</td>
<td>0.2</td>
<td>-2.14 (0.48)***</td>
</tr>
<tr>
<td>26754</td>
<td>Lithographic and Offset Inks</td>
<td>1,055</td>
<td>1.0</td>
<td>-1.61 (0.09)***</td>
</tr>
<tr>
<td>28932</td>
<td>Metal Powders, Paste, and Flakes</td>
<td>316</td>
<td>0.3</td>
<td>-1.18 (0.09)***</td>
</tr>
<tr>
<td>30823</td>
<td>Hot Impression Die Impact, Press, and Upset Steel Forgings</td>
<td>716</td>
<td>0.7</td>
<td>-2.03 (0.11)***</td>
</tr>
<tr>
<td>35733</td>
<td>Commercial Refrigerators and Related Equipment</td>
<td>476</td>
<td>0.4</td>
<td>-1.41 (0.11)***</td>
</tr>
<tr>
<td>37322</td>
<td>Outboard Motorboats, Including Commercial and Military</td>
<td>604</td>
<td>0.6</td>
<td>-1.99 (0.19)***</td>
</tr>
<tr>
<td>39951</td>
<td>Metal Caskets and Coffins, Lined and Trimmed, Adult Sizes</td>
<td>424</td>
<td>0.4</td>
<td>-2.37 (0.27)***</td>
</tr>
</tbody>
</table>

| Full Sample | 107,115 | 100 | -2.46 (0.02)*** |

Notes: This table shows the number and share of plants by product in the sample (pooled across all years). This table also shows the results of estimating demand curves by 2SLS separately for each product. All regressions include year fixed effects. Standard errors, clustered by plant, are in parenthesis. The symbols *, ** and *** indicate statistical significance above 10, 5 and 1 percent respectively. Note that SIC 20117 and 20137 do have the same name.
ranking of sectors in terms of price dispersion corresponds to prior beliefs. Primary resource sectors (stone, paper, food, lumber and petroleum) have lower price variance than finished goods sectors (apparel, leather products, chemicals, industrial machinery and transportation equipment).

3.4 Empirics

This section confronts the model with the data. In the theoretical model firm productivity affects outcome variables (such as price and export status) through two distinct channels: directly through cost and indirectly through quality. Much of the empirical analysis that follows tries to quantity the relative importance of these two components in an attempt to understand how producers use the margins of cost and quality to maximize their profits.

The section starts by formulating an empirical strategy to obtain estimates for product quality at the plant level. It then explores the properties of these estimates – in particular the relationship of quality to advertising, new technology expenditure and production cost. The analysis then turns to the estimation of some important conditional covariances in order to quantify the relationships between productivity, price, quality, and export status. Overall the results suggest that both quality differentiation and cost are important channels through which firms exploit their productivity advantage, and that quality contributes significantly to observed variation in price and export status in plant-level data.
3.4.1 Estimating Product Quality

In theory the price and quality of a firm’s output are both determined by its productivity. However, from the point of view of the consumer, price and quality are two distinct product attributes. Equation (2) shows how the representative consumer combines these two signals to determine its optimal demand for a particular variety and suggests an empirical strategy to identify product quality. Adding a multiplicative error term to the demand function (2) and taking logs yields:

\[
\ln q_{jt} = \lambda_i + \nu_j - \varepsilon \ln p_{jt} + e_{jt},
\]

where \( j \) and \( t \) index plant and time respectively. The first term, \( \lambda_i \equiv (\varepsilon - 1) \ln P_i + \ln R_i \), is a time varying effect common to all plants producing varieties of the same product. The second term, \( \nu_j \), is a plant fixed effect, which captures the time invariant component of demand unexplained by price (\( p \)) and aggregate factors (\( \lambda \)). The plant fixed effect is equal to the log of the product’s quality (i.e. \( \nu_j \equiv \ln \omega_j \)). Finally, random shocks unexplained by the theory are represented by the error term \( e \). Equation (16) implies that demand is log linear in quality and price and that demand is increasing in quality and decreasing in price. From (16), it follows that an estimate for product quality can be obtained by including producer fixed effects in a regression of quantity demanded on price and controlling for aggregate factors.

If plants respond to positive random demand shocks (\( e \)) by raising their prices then estimating (16) using ordinary least squares leads to biased estimates of the price elasticity (\( \varepsilon \)) and, as a result, of the plant’s average output quality (\( \nu \)) – see Wooldridge (2002). Following Foster, Haltiwanger, and Syverson (2008a) a 2SLS
procedure using the quantity total factor productivity (TFP) as an instrument for price is used to estimate (16) consistently. In essence, TFP is the variation in physical quantity produced unexplained by variation in inputs. It is constructed from the typical constant returns to scale index form:

$$\text{TFP}_{jt} = \ln q_{jt} - \psi_K \ln K_{jt} - \psi_L \ln L_{jt} - \psi_E \ln E_{jt} - \psi_M \ln M_{jt},$$

where $q$, $K$, $L$, $E$ and $M$ represent establishment-level output quantities, capital stocks, labor hours, energy and materials inputs, and $\psi_j$ for $j \in \{K, L, E, M\}$ are the factor elasticities for the corresponding inputs. In the presence of price variation due to quality differentiation quantity TFP is a more accurate measure of physical productivity than revenue total factor productivity (RTFP), which uses nominal revenue deflated by a product-level price index as a measure of output. For instance, if price is increasing in quality, RTFP will overestimate the physical productivity of plants producing high quality varieties and underestimate the physical productivity of plants producing low quality varieties. Therefore regressing plants’ prices on RTFP in the first stage will produce biased estimates of the fitted prices used in the second stage. This potentially results in biased estimates of price elasticities and plant-level product quality. For the remainder of the analysis, productivity refers to quantity total factor productivity.

Labor, materials, and energy cost shares are computed from reported expenditures in the CM. The real capital stock is the sum of plants’ reported book values for their

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40 The empirical work of Baily, Hulten and Campbell (1992) and Olley and Pakes (1996) on plant-level production function estimation supports the assumption of constant returns to scale. Note that the index formulation is a departure from the theory: the model as a single factor of production and assumes that production costs are increasing in quality. If the heterogeneity in quality is not controlled for the productivity estimates will be biased. Developing a productivity estimation procedure robust to variation in quality is an important avenue for future work.
structures and equipment, stocks deflated to 1987 levels using sector-specific deflators from the Bureau of Economic Analysis. Labor inputs are measured as plants’ reported production-worker hours multiplied by the ratio of total payroll to production workers’ payroll. The real cost of labor is obtained by multiplying the hours worked by the real wage. Real materials and energy inputs are plants’ reported expenditures on each deflated using the corresponding four-digit SIC input price indices from the NBER Productivity Database. For multi-product plants the inputs are scaled down using the primary product’s share of the plant’s nominal shipments. The input elasticities, $\psi_j$, are estimated using five-digit SIC average cost shares over the sample. The cost of capital is constructed by multiplying the real capital stock by the capital rental rate for the plant’s respective two-digit industry. These rental rates are from unpublished data constructed and used by the Bureau of Labor Statistics in computing their Multifactor Productivity series.\textsuperscript{41}

For the 2SLS procedure to produce reliable estimates for product quality four important identifying assumptions must hold. First, productivity should exhibit “moderate” persistence over time. On the one hand, it should be persistent enough to be uncorrelated with any short-run plant-specific demand shocks that affect prices (e). On the other hand, it should vary enough over time so that it is not perfectly collinear with the time invariant demand residual (v).\textsuperscript{42} Second, changes in TFP should not cause within-plant fluctuations in product quality. In terms of the model this would be true if the cost of adjusting quality is prohibitive, so that plants must make a once and for all choice of product quality based on the permanent (time-invariant) component

\textsuperscript{41} See Foster, Haltiwanger, and Syverson (2008a) for additional details.
\textsuperscript{42} In the full sample, regressing productivity and price on their own lag yields estimated coefficients equal to 0.73 and 0.48 respectively.
of productivity. Third, fluctuations in TFP must cause within-plant variations in price and quantities. Together the last two assumptions require that time-varying productivity shocks affect price only through the direct cost channel and leave quality unchanged. In that case short-run fluctuations in price and productivity are negatively correlated. Fourth, fluctuations in measured TFP must reflect genuine productivity variations, rather than unmeasured cyclical variation in capital utilization driven by demand shocks.\textsuperscript{43} Burnside, Eichenbaum and Rebello (1996) and Basu, Fernald and Rebelo (2004) present empirical evidence that factor utilization is procyclical and affects measured productivity in two-digit SIC manufacturing industries. If this is the case capital stock is not an accurate measure of capital utilization and will lead to biased estimates of productivity.\textsuperscript{44}

Equation (16) is estimated separately for each of the 125 five-digit products using a 2SLS procedure that instruments price with TFP. The estimated demand elasticities ($\varepsilon$) are reported in the last column of Table III. In all cases, the elasticity is negative and statistically significant. The instrument’s strength can be evaluated from first stage statistics. In all cases the first stage F statistic is large and the estimated coefficient for TFP is statistically significant. This suggests that the variation in TFP has some explanatory power over price. Finally, the IV and (unreported) OLS estimates differ substantially. About 75 percent of the IV estimates are larger in absolute value than the OLS estimates, suggesting a positive correlation between the exogenous random demand shocks ($e$) and prices ($p$). Further, the OLS results are not

\textsuperscript{43} Since labor is measured in hours worked and not employment, it is less likely that unmeasured fluctuations in labor utilization could bias TFP estimates.

\textsuperscript{44} The author suggests using energy usage to proxy for capital utilization. Using this measure of productivity does not affect the results. This is likely to be due to the low capital cost share in the sample.
very well behaved: 6 estimated elasticities are positive and 72 are between zero and minus one. Overall these results support the use of the 2SLS estimation procedure.

The standard deviation across plants of the estimated time invariant demand residuals ($v$) is quite large (1.56). This suggests an important role for factors other than price in explaining the dispersion in output across plants manufacturing the same product. According to the theory these demand residuals contain information on product quality. To a certain extent this can be verified empirically by looking at the characteristics of the demand residuals. In a first exercise, for each of the 125 products, the observations are split into two categories according to the value of their demand residuals. Plants with residuals above the median are called high quality and the others low. Table IV presents differences in mean log price, cost, output, revenue and productivity between the high and low group, normalized by each variable’s standard deviation in the pooled sample. Formally, the statistic is defined as $(Z_H - Z_L) / \text{stdev}(z)$ where $\bar{Z}_i$ is the mean of the log of variable $Z$ in group $i$ and $Z \equiv Z_H \cup Z_L$.

<table>
<thead>
<tr>
<th>Table IV: High vs. Low Demand Residuals Plants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>0.52</td>
</tr>
</tbody>
</table>

Notes: This table shows the normalized differences between the means of high and low demand residual plants for the full sample (107,115 observations). Unit production costs are measured as the sum of capital rental payments and depreciation, payroll, and energy and material expenditures. All variables are in logs. Product-by-year fixed effects are removed before computing the statistics.

The overall message of Table IV is clear: plants classified as high quality producers are different from those classified as low quality producers. On average,
plants with high demand residuals charge higher prices, have higher unit production costs, are larger in terms of quantity and revenue, and are more productive. These results have at least two important implications. First, on average, plants with large estimated demand residuals face higher unit cost and charge relatively high prices despite being more productive. Second, these same plants enjoy relatively large market shares despite charging relatively high prices. These observations are consistent with the hypothesis that producers use quality differentiation in addition to price to compete in the industry.

Profit-maximizing plants will incur greater costs in an attempt to increase quality only if such investments increase the consumer’s willingness to pay. Thus, the plausibility of the quality estimates can be evaluated by computing their correlation with different indicators of the plant’s investment in quality. Advertising is generally an effective way to convey information to the consumer and to increase the perceived (or intangible) quality of the product, while new technology expenditure can be targeted at product development and can increase the (or tangible) quality by introducing new designs. The CM contains plant-level information on advertising expenditure and software and data processing services purchased from other companies for Census years 1992 and 1997, and information on new computer expenditure for Census year 1992 for all plants, and for years 1977, 1982 and 1987 on a subset of observations. The expenditures are divided by plant revenue to remove the impact of plant size.\footnote{This measure is similar in spirit to that used in Kugler and Verhoogen (2008) but is constructed using information from the sample and is defined at a more disaggregated level (SIC5).} The average revenue share of advertising and software expenditures by two-digit SIC sector are reported in Table II. These shares are small
(less than half a percent on average) but nevertheless exhibit significant variation across sectors. As can be seen from Table V, plants that invest a larger fraction of their revenue in advertising and software generally have larger demand residuals. The point estimates suggest a high return to advertising and new technology expenditures. A ten percent increase in the revenue share of advertising, software and computer expenditures are respectively associated with a 2.4 percent increase, a 3.6 percent increase, and 0.9 percent in the demand residual.

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log Advertising</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log Software</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log Computer</td>
<td>0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample Size</td>
<td>32,083</td>
<td>32,083</td>
<td>23,255</td>
</tr>
<tr>
<td>R²</td>
<td>0.06</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>SE of reg.</td>
<td>1.50</td>
<td>1.50</td>
<td>1.30</td>
</tr>
</tbody>
</table>

Table V: Investment in Quality I

Notes: This table reports the results from regressing a plant’s time invariant demand residual on advertising, software and computer expenditures, expressed as log shares of plant revenues. Standard errors clustered by plants are in parenthesis. All regressions include product-by-year fixed effects. The symbols *, ** and *** indicate statistical significance at 10, 5 and 1 percent respectively.

While it is reassuring that the demand residuals are positively related to investment in new software, the interpretation of the relationship between these expenditures and quality is not as clear as for advertising. For instance, new software and computers could be acquired in an effort to increase productivity, and higher
productivity could in turn motivate firms to produce higher quality varieties. To control for that possibility, measures of productivity are included in regressions of the revenue share of expenditure on new software and computer on the demand residuals. Before presenting the results it is important to point out that these estimating equations contain generated regressors. As a results, inference based on the usual OLS standard errors will be invalid since it ignores the sampling variation due to the estimation of these variables – see Wooldridge (2002). Therefore, for the remainder of the analysis, bootstrap standard errors are reported whenever an estimating equation includes one or more generated regressors. As it happens, the difference between the bootstrapped and the usual OLS estimated standard errors clustered by plant is negligible in the current analysis, and using clustered standard errors would not change the significance of any of the coefficients.

The results from regressing new technology expenditures on the demand residuals controlling for plant productivity are reported in Table VI. The partial correlation between software expenditure and quality remains positive and significant. Overall these results suggest that producers devote resources to increase the demand for their product.

Finally, unit cost patterns provide additional evidence that firms voluntarily invest in product quality. From the cost function, defined in (3), the marginal cost of production (c) depends on productivity and quality as follows:

---

46 For a basic introduction to the bootstrap see Horowitz (2000). In the current context, the bootstrap is an appealing alternative to the use of asymptotic theory since it does not require a closed form solution for the variance-covariance matrix, which is difficult to obtain and evaluate in the current context.
\[ c(\omega, \varphi) = \frac{\omega^{\varphi}}{\varphi}. \]

Adding a multiplicative error term and taking logs yields the following estimating equation:

\[ \ln c_{jt} = \xi_0 + \xi_1 \hat{v}_{jt} + \xi_2 \ln \hat{\phi}_{jt} + e_{jt}, \tag{17} \]

where \( j \) and \( t \) index plant and time respectively. The second term, \( \hat{v} \), is the estimated demand residual, while \( \hat{\phi} \) is the estimated plant productivity and \( e \) represents other unspecified factors affecting production costs. If the estimated demand residuals capture product quality, then according to the theory they should be positively correlated with the unit cost of production once productivity is controlled for – in other words \( \xi_1 = \eta > 0 \). The regression also includes a full set of product-by-year fixed effects (\( \xi_0 \)) to control for product level and aggregate exogenous shocks uncorrelated with quality that could influence the production cost.

<table>
<thead>
<tr>
<th>Table VI: Investment in Quality II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>Dependent: log Software</td>
</tr>
<tr>
<td>Demand residuals</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>log TFP</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Sample Size</td>
</tr>
<tr>
<td>( R^2 )</td>
</tr>
<tr>
<td>SE of reg.</td>
</tr>
</tbody>
</table>

Notes: Bootstrap standard errors are in parenthesis. All regressions include product-by-year fixed effects. The symbols *, ** and *** indicate statistical significance at 10, 5 and 1 percent respectively.
Total production costs include capital rental payment and depreciation, payroll, and energy and material expenditures. All of these cost components are likely to be measured with error in the data. Importantly, productivity is also constructed from the same badly measured input information, as a result there is correlated measurement error on both sides of the estimating equation. Since the sign of the bias depends on many unknowns, very little can be said, except that the bias goes to zero as measurement errors become small; see the appendix for a detailed discussion. For the analysis, variable unit costs are defined as material expenditure over physical quantity produced. This choice reduces the correlation between measurement errors and is closely related to variable production costs.\footnote{Including labor and energy costs does not change the qualitative properties of the results. Capital costs are excluded because they are more closely related to fixed costs.}

The results from estimating (17) are presented in Table VII. The first two columns look at the separate effect of the demand residuals and TFP on production costs. In column (1) only the demand residual is included in the regression. The estimated effect of the demand residual on cost is positive and significant but small. As can be seen in column (2), productivity has a negative and significant impact on unit production costs. Column (3) presents the results of a specification including both productivity and the demand residual. As expected, the qualitative properties of the estimated coefficient are unchanged but the magnitudes are larger in absolute value. This happens because the productivity and quality effects work in opposite direction. The point estimates reveal that effect of the demand residual on unit cost is large, positive, and statistically significant. These correlations are consistent with the cost function defined in equation (16) and suggest that unit production costs are increasing
in quality but decreasing in productivity. The estimates imply that the marginal cost function is concave in quality since the quality elasticity of cost, $\eta$, is estimated to be between 0 and 1. Finally, the variations in the demand residuals and productivity have much more explanatory power over the variation in unit costs when both are included. According to the R-squares, together changes in quality and productivity explain 42 percent of the variation in unit costs while alone quality explains only 5 percent and productivity 21 percent.

The relative importance for unit production cost of productivity and quality should vary across products according to their potential for differentiation. For instance, for very homogenous products such as “hardwood plywood” (24351), the indirect effect of productivity through quality might be almost inoperative compared to a product such as “ice cream” (20240) for which brand recognition plays an

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand Residual</td>
<td>0.07</td>
<td>0.17</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.002)***</td>
<td>(0.004)***</td>
<td></td>
</tr>
<tr>
<td>log TFP</td>
<td>-0.20</td>
<td>-0.30</td>
<td>-0.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)***</td>
<td>(0.002)***</td>
<td>(0.002)***</td>
<td></td>
</tr>
<tr>
<td>Demand residuals</td>
<td></td>
<td></td>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td>x Scope for Differentiation</td>
<td></td>
<td></td>
<td></td>
<td>(0.004)***</td>
</tr>
<tr>
<td>Sample Size</td>
<td>107,115</td>
<td>107,115</td>
<td>107,115</td>
<td>107,115</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.05</td>
<td>0.21</td>
<td>0.42</td>
<td>0.70</td>
</tr>
<tr>
<td>SE of reg.</td>
<td>0.51</td>
<td>0.46</td>
<td>0.39</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the unit cost of production, defined as real material expenditures per unit of output. All regressions include a full set of product-by-year fixed effects. Bootstrap standard errors are in parenthesis. The symbols *, ** and *** indicate statistical significance at 10, 5 and 1 percent respectively.
important role in consumer’s decision. In general, when the scope for vertical differentiation is limited, the direct effect of productivity on cost should be more important than its indirect effect through quality. In terms of the model, variation in product quality across producers implies variation in fixed production costs. Therefore, within-product dispersion in advertising expenditure across producers should be a good indicator of the potential for vertical differentiation in a particular product class. Intuitively, an industry in which dispersion in advertising expenditure across plants is high is likely characterized by high vertical product differentiation. Column (4) tests this idea by including an interaction of the demand residual with the within-product standard deviation of the revenue share of advertising expenditure across plants. As can be seen from column (4), the impact of the demand residual on costs is more important for products with a higher potential for differentiation. While more differentiated goods may have higher unit costs, the increase in the elasticity of unit cost with respect to quantity produced provides additional support for the quality interpretation.

Overall the results presented in this section are compelling evidence that the time invariant demand residuals contain some information about the product attributes or at the very least about the consumer’s willingness to pay for the product. For simplicity, the demand residuals will be referred to as quality for the rest of the paper.

3.4.2 Quality, Productivity, and Price

This section investigates the link between product quality and producer characteristics using the demand residuals as a proxy for quality. One of the central predictions of
the model is the positive correlation between firm productivity and product quality. Intuitively, the return to quality upgrading is increasing in the plant’s productivity since higher productivity plants face relatively lower marginal costs and, as a result, can charge lower prices and sell more units. The association between quality and productivity can be evaluated formally by estimating the following equation:

\[ \hat{v}_j = \xi_0 + \xi_1 \ln \hat{\phi}_j + e_{j\mu}, \] (18)

where \( \hat{v} \) is the estimated product quality, \( \hat{\phi} \) is the estimated plant productivity, \( \xi_0 \) denotes a full set of product-by-year fixed effects, and \( e \) represents unspecified factors affecting quality. As predicted, an increase in productivity leads to a positive and statistically significant increase in product quality. This result implies that the component of the plant’s output unexplained by its level of inputs (the productivity measure) is positively related to the component of demand unexplained by its price (the quality measure).

According to the pricing rule, defined in equation (5), productivity affects price through two interrelated channels: cost and quality. On the one hand, the plant forwards productivity gains to the consumer. On the other hand, the increase in production costs associated with producing high quality goods will be reflected by a proportional increase in price. Adding a multiplicative error term to the pricing rule and taking logs provides the following regression equation:

\[ \ln p_{j\mu} = \xi_0 + \xi_1 \hat{v}_j + \xi_2 \ln \hat{\phi}_j + e_{j\mu}, \] (19)

where \( p \) is the unit price of output, \( \hat{v} \) is the estimated demand residual, which serves as a proxy for product quality, \( \hat{\phi} \) is the estimated plant productivity, and \( e \) represents unspecified factors affecting price. The regression also includes product-by-year fixed
effect (\(\xi_0\)) to control for time-varying and time-invariant factors affecting all producers of a given product.

The results from estimating regression equation (19) are presented in Table VIII. The first two columns report the results of regressing price on fixed effects and one either quality or TFP. As expected, the quality elasticity of price is positive and statistically significant while the impact of productivity is negative and significant. Column (3) reports the results for a specification including quality and productivity simultaneously. The estimated effects are statistically significant and larger in absolute value when both quality and productivity are included in the regression. This happens because the two effects of productivity on price work in opposite directions. Finally, as can be seen from column (4), the effect of productivity through quality is more important for products with a higher potential for differentiation, measured as above using the within-product dispersion in advertising expenditure across plants.

According to the theoretical model, the coefficients on quality in equations (17) and (19) should be equal, since the quality elasticity of unit production cost and the quality elasticity of price are both equal to \(\eta\). Comparing the estimates presented in Table VII and VIII reveals that these are indeed of similar magnitudes. The fact that the impacts of the demand residual on cost and price are positive and of similar magnitudes provides strong evidence that a mechanism of the type described in the quality model is at work: Producers can influence consumer’s willingness to pay at some cost, and maximize their profit by balancing the gain in demand associated with quality upgrading to the decrease in demand due to the associated price increase.
## Table VIII: Quality, Productivity, and Price

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log Quality</td>
<td>0.10</td>
<td>0.19</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.001)***</td>
<td>(0.003)***</td>
<td></td>
</tr>
<tr>
<td>log TFP</td>
<td>-0.18</td>
<td>-0.29</td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.001)***</td>
<td>(0.002)***</td>
<td></td>
</tr>
<tr>
<td>Log Quality x Scope for Differentiation</td>
<td>0.04</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.003)***</td>
</tr>
<tr>
<td>Sample size</td>
<td>107,115</td>
<td>107,115</td>
<td>107,115</td>
<td>107,115</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.13</td>
<td>0.24</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>SE of reg.</td>
<td>0.39</td>
<td>0.37</td>
<td>0.25</td>
<td>0.20</td>
</tr>
</tbody>
</table>

**Notes:** This table reports the results of regressing price on the time invariant demand residual and productivity. All regressions include product-by-year fixed effects. Bootstrap standard errors are in parenthesis. The symbols *, ** and *** indicate statistical significance at 10, 5 and 1 percent respectively.

As argued in the introduction, using unit values to make inferences about product quality could lead to invalid conclusions since a high unit value could arise from high quality or low productivity. To give a sense of whether price is an accurate indicator of quality, observations are divided into four groups using product-level medians as a separation point: high price and high quality, high price and low quality, low price and high quality and low price and low quality. If price is a good indicator of quality the observations would mostly fall into the high/high and low/low categories. Pooling all the observations together, these groups account for only 27 percent of the observations each. Using finer bins (e.g. quartiles or deciles) to classify the observations reveals that price is not a good indicator of quality even in the extreme parts of the distribution. Therefore, while price does contain information on vertical product differentiation, inferences on quality based only on unit value should be interpreted carefully.
3.4.3 Product Quality and Export Status

This section explores the relationship between productivity, quality and export status. It begins by presenting descriptive statistics comparing exporting to non-exporting plants. It then develops and estimates a Probit model that is used to evaluate the separate impact of productivity and quality on the probability of export. The analysis presented in this section uses only a subset of the data since the CM contains plant-level information on export only for years 1987 onward. Plants that report positive export revenue are classified as exporters.48

To begin, the observations are divided into two categories according to their export status. Exporters and non-exporters are then compared using the differences in log means of price, output, cost, revenue and productivity across groups normalized by the standard deviation of the variable. Formally, the statistic is defined as 
\[(\bar{Z}_X - \bar{Z}_{NX}) / \text{se}(z)\] where \(\bar{z}_i\) is the mean of the log of variable Z in group i and \(z \equiv z_X \cup z_{NX}\). The statistics presented in Table IX confirm well-known results: exporters are larger in terms of output and revenue and are more productive on average; see for instance Bernard and Jensen (1999). Less well known facts are that exporters have slightly higher unit production costs and charge higher prices on average. These findings are consistent with the theoretical model. From equation (7), exporters produce varieties of higher quality. Therefore, whenever the direct effect of productivity on cost is smaller than its indirect effect through quality, costs and prices will be higher for exporters.

48 Since export information is available at the plant level only, for multi-product plants it is impossible to know for sure if the plant’s export revenue comes from the product of interest or another product. However, since the sample includes only multi-product plants for which at least 50 percent of their revenue comes from their primary product, it seems highly likely that it generates at least part of the export revenue.
### Table IX: Exporting vs. Non Exporting Plants

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Cost</th>
<th>Quantity</th>
<th>Revenue</th>
<th>TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.06</td>
<td>0.05</td>
<td>0.43</td>
<td>0.48</td>
<td>0.34</td>
</tr>
</tbody>
</table>

*Notes:* This table shows the normalized differences between the means of exporting and non exporting plants. All variables are in logs. Product-by-year fixed effects are removed before computing the statistics.

As the theoretical model makes clear, firms enter the export market only if the extra profit from exporting is positive. From equation (15) it is possible to define the following related variable:

$$T_{jt} = \frac{R_t (P_t \rho)^{\epsilon-1}}{f_x} \omega_{x,j}^{1-\eta(\epsilon-1)} \eta_{jt}^{\epsilon-1},$$

where $\omega_x$ is defined in (7). The variable $T_{jt}$ measures the ratio of variable export profits to the fixed export costs for plant $j$ in year $t$. By definition, positive exports are observed if and only if $T > 1$. Adding a multiplicative error term and taking logs yields:

$$\ln T_{jt} = \xi_t + \xi_1 \ln \omega_{jt} + \xi_2 \ln \eta_{jt} + e_{jt}, \quad \text{where } e_{jt} \sim N(0, \sigma_e^2).$$

where $\xi_t \equiv R_t (P_t \rho)^{\epsilon-1}/f_x$, $\xi_1 \equiv 1 - \eta(\epsilon-1)$, and $\xi_2 \equiv \epsilon-1$. Although $\ln T_{jt}$ is unobserved, the presence of trade flows is observed. Define the indicator variable $X$ to equal 1 when the plant exports and 0 when it does not. Let $\chi_{jt}$ be the probability that plant $j$ exports at time $t$, conditional on the observed variables and define the following Probit equation from (20):

$$\chi_{jt} \equiv \Pr\{X = 1 | \text{observed variables}\} = \Phi(\xi_t + \xi_1 \ln \omega_{jt} + \xi_2 \ln \eta_{jt} + e_{jt}),$$

where $\Phi$ is the cumulative distribution function of the standard normal distribution.
where $\Phi(\cdot)$ is the cdf of the unit-normal distribution, and every hat coefficient represents the original coefficient divided by $\sigma_i^2$. This transformation ensures that the error is distributed standard normal.

Results from estimating Probit equation (21) are presented in Table X. The first two columns each include one regressor in addition to a full set of product/year fixed effects. As predicted quality has a large, positive and significant impact on the probability of exporting. Column (3) includes quality and productivity in the regression simultaneously. As expected, the impact of quality is now smaller than before, as part of the overall impact of TFP is now captured through its indirect effect on quality. Finally, from column (4), the impact of quality on export status is not affected by the scope for vertical product differentiation.

Table X: Quality, Productivity, and Export Status

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log Quality</td>
<td>0.23</td>
<td>0.19</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)***</td>
<td>(0.008)***</td>
<td>(0.01)***</td>
<td></td>
</tr>
<tr>
<td>log TFP</td>
<td>0.19</td>
<td>0.09</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)***</td>
<td>(0.008)***</td>
<td>(0.008)***</td>
<td></td>
</tr>
<tr>
<td>log Quality x Scope for Differentiation</td>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample Size</td>
<td>49,639</td>
<td>49,639</td>
<td>49,639</td>
<td>49,639</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-19027</td>
<td>-19448</td>
<td>-18950</td>
<td>-18920</td>
</tr>
</tbody>
</table>

Notes: This table presents the results of Probit regressions. The dependent variable is a binary variable equal to 1 if the plant is classified as an exporter and 0 otherwise. All regressions include product-by-year fixed effects. Bootstrap standard errors are in parenthesis. The symbols *, ** and *** indicate statistical significance at 10, 5 and 1 percent respectively.
3.4.4 Robustness

While in theory the demand residuals are estimates of product quality, in practice they are likely to include information on other factors that could also influence the demand for a particular variety. This section controls for some of these factors and evaluates the impact on the estimates effects of product quality.

First, it is possible that some markets areas are regional in nature such that firms in certain locations face different conditions than firms in other. For instance, Syverson (2004) describes how differences in the density of demand affect the distribution of plant productivity in markets characterized by regional segmentation. If plants compete in different markets it is likely that the demand residuals will capture some regional characteristics and introduce a bias in the estimation of quality. To control for this possibility a set of regional fixed effect are included in the regressions. Regions are defined according to the Bureau of Economic Analysis’ definition of Labor Market Areas. This measure of geography is superior to political division such as State or Counties since it is developed from commuting patterns. It therefore better captures the economic interactions between groups of producers and consumers. Results are presented in Table XI. For space consideration only a subset of the estimated coefficients are presented. Overall removing regional variation from the demand residuals does not affect the results. Quality remains positively related to advertising expenditure, unit production cost, productivity, price and export probability.

Second, because it takes time for consumers to learn about new products, older vintage varieties might have an advantage over newly introduced ones. In fact, Foster,
**Table XI: Robustness I – Regional Variation in Demand**

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log Advertising</td>
<td>0.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.006)***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log Quality</td>
<td>0.13</td>
<td>0.19</td>
<td>0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)***</td>
<td>(0.001)***</td>
<td>(0.008)***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log TFP</td>
<td>-0.35</td>
<td>0.58</td>
<td>-0.29</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)***</td>
<td>(0.006)***</td>
<td>(0.002)***</td>
<td>(0.008)***</td>
<td></td>
</tr>
<tr>
<td>Sample size</td>
<td>30,444</td>
<td>105,409</td>
<td>105,409</td>
<td>105,409</td>
<td>47,960</td>
</tr>
<tr>
<td>R²</td>
<td>0.08</td>
<td>0.64</td>
<td>0.21</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Log Likelihood</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-17810</td>
</tr>
</tbody>
</table>

Notes: All regressions include regional fixed effects. Regions are defined according to the Bureau of Economic Analysis’ definition of Labor Market Areas. The dependent variable for each regression is listed at the top of each column. All regressions include a full set of product/year fixed effects. Standard errors are in parenthesis. The symbols *, ** and *** indicate statistical significance above 10, 5, and 1 percent respectively.

Haltiwanger and Syverson (2008b) find empirical support for this conjecture using U.S Census micro data on manufacturing plants. The model does not account for the accumulation of quality capital such as brand recognition or consumer habit but rather concentrates on the contemporaneous relationship between production costs and demand. Therefore regressions a measure of plant age is included in the regressions to partial out the fraction of residual demand explained by learning and reputation. Since plant age cannot be measured accurately in the sample, observations are divided into three categories: plants appearing for the first time in a Census are classified as young, plants that appeared in at least two but not more than four Censuses are classified as medium, and plants that appear in more than four Censuses are classified.
Results for regressions including plant age dummies are presented in Table XII. While the effect of age on demand is generally statistically significant, controlling for plant age variation does not affect the results. Quality remains positively related to advertising expenditure, unit production cost, productivity, price and export probability.

<table>
<thead>
<tr>
<th>Table XII: Robustness II – Learning and Reputation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>Estimation:</td>
</tr>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>log Advertising</td>
</tr>
<tr>
<td>log Quality</td>
</tr>
<tr>
<td>log TFP</td>
</tr>
<tr>
<td>Young</td>
</tr>
<tr>
<td>Medium</td>
</tr>
<tr>
<td>Old</td>
</tr>
<tr>
<td>Sample size</td>
</tr>
<tr>
<td>R²</td>
</tr>
<tr>
<td>Log Likelihood</td>
</tr>
</tbody>
</table>

Notes: The dependent variable for each regression is listed at the top of each column. All regressions include product/year fixed effects. Standard errors are in parenthesis. The medium aged dummy is excluded to remove collinearity with the constant. The symbols *, ** and *** indicate statistical significance above 10, 5, and 1 percent respectively.

49 This classification is based on the plant’s age which is computed from the entire Census of manufactured sample, not the reduced sample used for the empirical analysis.
Finally, although five-digit SIC product category provides narrow definitions of industries, there still remains horizontal differentiation of an unwanted type. For instance the product class “wood bedroom furniture” (SIC 25115) comprises beds, dressers and night tables. In that case comparing the output and average unit value across plants leads to incorrect inferences whenever there is heterogeneity across plants in the bundle of furniture produced. To account for the impact of horizontal differentiation a new sample that contains only products classified as not horizontally differentiated by Rauch (1999) is constructed. Regressions are re-estimated in that sample. The results are presented in Table XIII. Overall results are robust to these changes in sample. Quality remains positively related to advertising expenditure, unit production cost, productivity, price and export probability.

<table>
<thead>
<tr>
<th>Table XIII: Robustness III – Removing Horizontal Differentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent: log Quality</td>
</tr>
<tr>
<td>Estimation: OLS</td>
</tr>
<tr>
<td>Variables</td>
</tr>
<tr>
<td>log Advertising</td>
</tr>
<tr>
<td>(0.002)***</td>
</tr>
<tr>
<td>log Quality</td>
</tr>
<tr>
<td>(0.003)***</td>
</tr>
<tr>
<td>log TFP</td>
</tr>
<tr>
<td>(0.003)***</td>
</tr>
<tr>
<td>Sample size</td>
</tr>
<tr>
<td>R²</td>
</tr>
</tbody>
</table>

Notes: The dependent variable for each regression as well as the estimation procedure is listed at the top of each column. All regressions include product/year fixed effects. Standard errors are in parenthesis. The symbols *, ** and *** indicate statistical significance above 10, 5, and 1 percent respectively.
Overall, the results presented in this section indicate that the conclusions are robust to the inclusion of additional factors in the analysis. Quality remains positively related to advertising expenditure, unit production cost, productivity, price and export probability even when regional variation in demand and plant age are controlled for or when the sample is and hold even in very homogenously defined product classes.

3.5 Structural Estimation

The last section presented correlations between productivity, quality, price, and export status consistent with the theory. However, as mentioned previously, the demand residuals that serve as proxy for quality are likely to contain more than just information about product quality. Therefore, while the empirical findings are compelling evidence of a link between producer behavior and consumer demand it is not clear that this relationship is due exclusively to vertical product differentiation. Therefore the estimated conditional covariance presented in the last section may overstate the importance of quality. As an alternative, this section uses the simulated method of moments (SMM) to estimate the model’s structural parameters, evaluate the ability of the model to reproduce observed facts and assess the importance of vertical product differentiation. If the model captures the essential characteristics of the industry the simulated moments should be close to the data moments. In that sense SMM provides an interesting test of the model. Further, the importance of quality differentiation can be evaluated by looking at the estimated values for the structural parameters of the model. Since these are left unrestricted in the estimation
procedure, the quality elasticity of fixed and marginal costs should be close to zero if there is no quality differentiation.

Another reason to estimate the model structurally using SMM is that this allows for counterfactual experiments. Once the estimates for the parameters are known, the model can be used to evaluate the effect of exogenous shocks on industry characteristics such as average price and the productivity distribution. The analysis focus on the effect of three different types of trade liberalization: (i) A ten percent decrease in the variable trade cost; (ii) A ten percent decrease in the fixed trade cost; (iii) A ten percent decrease in both variable and fixed trade costs. Overall the results show that trade costs are important determinants of plant behavior.

3.5.1 Calibration

The model is governed by 10 parameters that can be divided into 5 categories: demand (ε and L), productivity (σ), entry/exit (f_e, δ), technology (f_c, γ, and η) and trade costs (τ, and f_x). Some of the parameters can be fixed a priori or calibrated to match data moments directly. This reduces the parameter space to a manageable size for the estimation. The remainder of the subsection explains the choice of calibrated values for the parameters, which are summarized in Table XIV.

The total revenue (R) is equal to the product of the wage rate and the size of the population (L) and is normalized to 1 without loss of generality. The probability of exit (δ) is calibrated to 0.13 to match the sample’s estimated annual exit rate.\textsuperscript{50} The

\textsuperscript{50} The exit rate is computed by looking at fraction of plants in year t that exit between t and t + 5, the next Census year, then adjusting to an annual basis.
elasticity of substitution ($\varepsilon$) is set to 2.46. The theory shows that the producer’s share of revenue from exports is a function of the elasticity of substitution and the iceberg trade cost as follows:

$$S_x = \frac{\tilde{p}_x \tilde{q}_x}{\tilde{p}_x q_x + \tilde{p}_x \tilde{q}_x} = \frac{\tau^{1-\varepsilon}}{1 + \tau^{1-\varepsilon}}.$$ 

In theory this share is constant across producers, but in practice there is a lot of dispersion across exporters. Hence, the revenue-weighted share of revenue from export among all exporters is used to estimate $S_x$. This share is equal to 0.35 in the full sample. Taking into account the calibrated value for the elasticity of substitution, this implies an iceberg transport cost parameter ($\tau$) of 1.53. According to this estimate, the price of a domestic variety will be 53 percent higher when sold in the foreign market. This is a relatively high but not totally unreasonable increment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon$</td>
<td>Elasticity of substitution</td>
<td>2.46</td>
<td>Data</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Iceberg trade Cost</td>
<td>1.53</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Probability of Exit</td>
<td>0.13</td>
<td>Data</td>
</tr>
<tr>
<td>$f_e$</td>
<td>Fixed Entry Cost</td>
<td>N</td>
<td>Normalization</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Lower bound of fixed production cost</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$L$</td>
<td>Population size</td>
<td>1</td>
<td>Normalization</td>
</tr>
</tbody>
</table>

*Notes: This table presents the values for parameters that are selected a priori or calibrated directly to match data moments. The elasticity of substitution, the iceberg trade cost and the probability of exit are estimated from the full sample (107,115 observations). The constant $N$ is equal to the size of the random sample used to approximate the distribution of productivity.*

---

51 This value is obtained by estimating equation (16) in the full sample following the procedure used to generate the parameter estimates reported in Table III augmented to include a set of product-by-year fixed effects.
Finally, the model does not allow the identification of all fixed costs parameter. Hence, the fixed entry cost \( (f_e) \) and the lower bound on the fixed production \( (f_c) \) are normalized.\(^52\) To understand this result, first note that the fixed entry cost \( (f_e) \) only appears in equation (14), characterizing the productivity threshold \( (\phi_o) \), while the fixed production cost enters both in (14) and in equation (12) characterizing the export threshold \( (\kappa) \). Therefore any change in the entry cost can be offset by a proportional change in the fixed production and export cost such that the productivity threshold is unchanged. Also the export threshold is a function only of the ratio of the fixed export to the fixed production cost. Therefore for the current purposes there are redundancies in the parameters. The lower bound of the fixed production cost is normalized to 1. Since the fixed entry cost controls the size of the industry its value should be proportional to number of firms used in the computational sample. If this is not the case only a small fraction of the entrants produce in equilibrium.\(^53\) For simplicity, the fixed entry cost \( (f_e) \) is set to \( N \), the number of random draws used to approximate the productivity distribution.

The remaining four parameters \( (\sigma, f_x, \gamma, \text{ and } \eta) \) capture the core mechanisms of the model and are estimated using SMM. The shape parameter of the productivity distribution \( (\sigma) \) measures the dispersion in productivity across producers in the industry. The fixed export cost controls the partitioning of plants along export status.

\(^{52}\) The innocuous nature of calibrating these fixed costs can be verified graphically and numerically. Plots of the objective criterion evaluated at the optimal parameter values as a function of each of the fixed cost are U shaped. Further, the numerical derivatives of the objective criterion are almost identical to zero when evaluated at the optimal parameter values. These are evidence that the objective is minimized at the calibrated values.

\(^{53}\) In other words, when the fixed entry cost is small only a small fraction of the random draws used to approximate the distribution of productivity will survive to affect the equilibrium. When this happens the ex post distribution of productivity does not have the properties of a Pareto distribution.
Finally, the quality elasticities of fixed and variable cost ($\gamma$ and $\eta$) govern the relationship between quality, production cost and price.

3.5.2 Moments

At least one moment condition per estimated parameter is required for the system to be identified. There is a large set of moments to choose from in the data and the econometric theory does not provide a clear guide in the choice of an optimal set of moments to use. Therefore moments are selected based on two criteria. First, they should capture the essential characteristics of the industry that the model tries to explain. Second, they should provide enough information to identify the structural parameters of the model. In other words, variations in parameter values should result in different values for the simulated moments.

The first three moments are the differences in log mean price, quantity, and revenue between high and low quality variety producers, normalized by each variable’s standard deviation. Since the theoretical model is built to explain the relationship between the distributions of price, revenue, and quality, these statistics capture important features of the data and should inform the structural parameters of the model. The distribution of output is very asymmetrical in the data. To more fully capture this property, the shares of revenue accounted for by the top 10 percent, the top 20 percent and the top 50 percent of producers are included as additional moments in the estimation process. Finally, the share of exporting plants in the sample is also included since it is likely to convey useful information about the fixed export cost. The procedure therefore uses 7 moment conditions to estimate four parameters.
It is important to recall that quality is not directly observed in the data. The empirical analysis estimated a proxy for quality by including a producer fixed effect in a 2SLS regression of quantity on price using productivity as an instrument. For consistency, the same procedure is used to construct a proxy for quality in the simulated data. There is one important difference, however. In the data productivity is a Solow residual, defined as the log difference between quantity produced and an index of inputs implied by a constant returns to scale Cobb-Douglas production function. In the model this residual would be exactly equal to productivity since there is only one input. Therefore, in the simulated data, productivity is simply defined as a random draw from a Pareto distribution with shape parameter $\sigma$.

The data moments are computed from the full sample and summarized in Table XV. As mentioned earlier plants classified as high quality producers are different from plants classified as low quality producers. On average high quality plants charge prices half a standard deviation above low quality plants and have quantity and revenue more than one standard deviation above low quality producers. The asymmetry of the revenue distribution across producers is evidenced by the next three moments. About 67 percent of the overall revenue is accounted for by the largest 10 percent of producers while the top half of producers account for 96 percent of overall revenue. Finally, about 19 percent of plants in the sample are classified as exporters.

3.5.3 Estimation
The SMM estimation procedure is briefly described here; more details can be found in the appendix:
1. Calculate the vector of moments with the actual data, $M(\theta)$.

2. For a given vector of parameters $\hat{\theta}$, simulate the model using $N$ realizations of random draws to approximate the productivity distribution and generate artificial data.

3. Use the artificial data to calculate the vector of simulated moments, $\hat{M}(\hat{\theta})$.

4. Compare the vectors of moments from the actual and simulated data and search over the parameter space $\Theta$ for the solution to the following minimization problem:\(^{54}\)

$$\hat{\theta} = \underset{\theta}{\text{arg min}} \ [M(\theta) - M(\hat{\theta})]' W [M(\theta) - M(\hat{\theta})],$$  \hspace{1cm} (22)

where the matrix $W$ provides the weight given to each of the moments in the optimization procedure.

To obtain an efficient estimator for $\theta$ the weights are inversely proportional to the standard deviation of the data moments, so that more precisely estimated moments are given more weight in the estimation.\(^ {55}\)

Even with the most powerful of computers there is a constraint on the number of draws that can be used to approximate the productivity distribution. This implies that simulation error is always present to some degree in the simulated method of moment estimates. The estimation procedure takes this into account and is repeated ten times using different starting values and sets of random draws.\(^ {56}\) In each simulation ten thousand random draws are used to approximate the distribution of productivity

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\(^{54}\) The optimization algorithm is the simulated annealing method described in Goffe et al. (1994).

\(^{55}\) See Adda and Cooper (2003) for a precise definition of $W$. Since the variance-covariance matrix of the moments is unobserved it must be estimated. See appendix for details.

\(^{56}\) The seed for the random number generator is different for each set of random draws.
across plants. Each SMM estimation leads to a vector of parameter $\hat{\theta}_b$ that solves equation (22).

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Simulated Mean</th>
<th>Simulated Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation difference in log mean price</td>
<td>0.52</td>
<td>0.54</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Standard deviation difference in log mean quantity</td>
<td>1.12</td>
<td>1.19</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Standard deviation difference in log mean revenue</td>
<td>1.28</td>
<td>1.39</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Share of revenue top 10% percent</td>
<td>0.67</td>
<td>0.22</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Share of revenue top 20% percent</td>
<td>0.81</td>
<td>0.35</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Share of revenue top 50% percent</td>
<td>0.96</td>
<td>0.63</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Share of exporters</td>
<td>0.19</td>
<td>0.46</td>
<td>(0.009)</td>
</tr>
</tbody>
</table>

Notes: This table shows the moments computed from actual data and the mean and standard deviation of the simulated moments. The data moments are computed from the full sample of 107,115 observations. The model moments are averages across ten independent estimations using SMM.

The means and standard deviations of the simulated moments evaluated at the optimal values are presented in Table XV. The small standard deviations indicate that the estimates are robust to changes in random draws and starting values. The model is able to reproduce the dispersion is size and output price across manufacturing plants observed in the data. The simulated normalized differences in log mean price, quantity and revenue are all very close to their actual values. This suggests that the model captures factors causing quality differences across plants, a comforting finding given that the theoretical model is built towards explaining such differences. Further, the simulated distribution of revenue across the top deciles of producers has a shape

57 To mimic the empirical procedure the distribution is truncated using a five median rule. Productivities above five times and below one fifth of the median productivity are dropped for the estimation procedure. This condition is binding only when the shape parameter of the productivity distribution ($\sigma$) is low. At the estimated optimal values of the parameters it is not binding so that all draws are included in the computation of the simulated moments.
similar to the actual distribution in that the biggest producers in the simulated data account for a disproportionate fraction of total revenue. However, the magnitudes of the simulated shares of output for the largest producers are lower than their actual values. One likely culprit is the assumption that the world is composed of only two symmetric countries. If instead the world was composed of several destinations, the difference between exporters and non-exporters would be amplified, as exporters’ share of output sold abroad would increase thereby increasing there relative size. Since exporters are the biggest producers in the economy, this would increase the share of revenue accounted for by the top deciles of producers. Further, if each foreign destination was characterized by different fixed and variable trade costs, there would be multiple export thresholds, so that only the most efficient of exporters would export to the toughest destinations. This would generate additional variation in revenue and increase the relative size of very high productivity plants. Finally, the model has difficulty in matching the share of exporters in the sample. In the data only 19 percent of plants export, while in the simulated data 46 percent of plants export. Since exporters are more productive and quality is increasing in productivity, this implies that all exporters in the simulated data are classified as high quality plants. The high simulated value for the share of exporters reveals that the model uses the discontinuity in the mapping from productivity to quality at the export margin to generate variation in normalized log means across high and low quality producers. Despite its limitations, the simple general equilibrium model including vertical product differentiation captures some important features of the data. The criterion function (22) evaluated at the optimum is equal to 2.04.
The means and standard deviations of the estimated parameters underlying the simulated moments are reported in Table XVI. The small standard deviations indicate that the estimates are robust to changes in random draws and starting values. This suggests that the estimates of the structural parameters attain the global minimum of the objective function (22). The quality elasticities of unit cost ($\eta$) and fixed production costs ($\gamma$) are estimated at 0.1 and 1.29 respectively. The point estimates imply that a ten percent increase in quality leads to 1 percent increase in marginal production costs and a 13 percent increase in fixed production costs. From proposition (3), the convexity of the mapping from quality to fixed production costs implies that price and productivity are negatively correlated in equilibrium, a finding consistent with the data. Both quality elasticity estimates are statistically significantly different than zero at conventional levels. Since these parameters are left unrestricted in the estimation procedure, this is strong evidence that vertical product differentiation contributes significantly in explaining the variation in producer characteristics. Finally, it interesting to note that the structural estimate of the quality elasticity of marginal production costs ($\eta$) is not far from the OLS estimates presented in Tables VII and VIII respectively.

The shape parameter of the Pareto distribution is estimated at 12.8, a value equal to the upper bound of the range of estimates presented in the studies of Eaton and Kortum (2002) and Eaton, Kortum, and Kramaz (2008). Since a high value of

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58 Eaton et al. (2008) find a value of 2.5 by calibrating a model using French micro data. Eaton and Kortum (2002) presents three separate estimates of 3.6, 8.3 and 12.8 obtained using different approaches in aggregate data and Bernard et al. (2003) find a value of 3.6 by calibrating a model using U.S. Census micro data. The last two studies use the Fréchet extreme value distribution instead of the Pareto. However, as argued in the appendix to chapter 2, the two distributions are related so that the estimates are comparable.
the shape parameter of the Pareto distribution implies less dispersion in productivity, the relatively high estimated value is consistent with the idea that quality acts as a multiplier on the effect of productivity – it compounds the comparative advantage effect of productivity, so that a smaller amount of dispersion in random productivity is needed to generate the observed degree of inequality in revenue and output. Further, the average fixed cost of production can be calculated as \( f_e + \bar{v}^0 \). Evaluated at the optimal values of the structural parameters this average is equal to 80.6, or about 19 percent of average revenue in the industry. Finally the fixed export cost is estimated at 47.3, so that in the simulated data the fixed export cost is equal to about 59 percent of the average fixed production cost and 10 percent of the average revenue.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Mean</th>
<th>Std.Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>Quality elasticity of unit production costs</td>
<td>0.10</td>
<td>(0.02)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Quality elasticity of fixed production costs</td>
<td>1.29</td>
<td>(0.01)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Shape parameter of Pareto distribution</td>
<td>12.8</td>
<td>(1.59)</td>
</tr>
<tr>
<td>( f_x )</td>
<td>Fixed trade cost</td>
<td>47.3</td>
<td>(3.15)</td>
</tr>
</tbody>
</table>

Notes: This table presents the means and standard deviations across the ten simulations of the parameter estimates obtained using SMM.

The theoretically accurate measure of product quality, defined in equation (7), can be calculated in the simulated data. This allows for a comparison in the simulated data between the proxy for quality and the “true” measure of quality based on the theoretical model. Evaluated at the optimal parameter values the correlation between these two measures is about 0.97. However, it is important to
note that this is likely to be a biased estimate of the correlation between the two measures in the actual data, because the actual and simulated procedures differ in certain respects. First, in the actual data, quality is estimated by including a producer fixed effect in a regression of quantity on price. In the simulated data only one observation per plant is available, so quality is defined as the residual from the same regression omitting producer fixed effects. Second, as explained earlier, the “true” plant productivity – defined from the random draw – is observed in the simulated data whereas it needs to be estimated in the actual data. Third, in the simulation there are no plant factors other than price and quality affecting demand, while in the real world there are many. Finally, there is no measurement error in the model. Together, these considerations imply that the procedure might identify the quality component more accurately in the simulated data than in actual data. Nevertheless, the high estimated correlation suggests that using demand residuals as a proxy for quality is a reasonable empirical strategy.

3.5.4 Counterfactuals

The estimated model can be used to perform counterfactual experiments. Of particular interest is the impact of trade liberalization on the distributions of price, quality and productivity across manufacturing plants. The analysis here considers three different types of liberalization: a 10 percent decrease in variable trade cost (τ), a 10 percent decrease in fixed trade costs (f_x), and a 10 percent decrease in both variable and fixed trade costs. Throughout the analysis, the random draws used to approximate the productivity distribution are held fixed. This ensures that the
measured changes are due to general-equilibrium effects and not changes in the underlying heterogeneity.

The effects of trade liberalization on productivity are summarized in Table XVII. In all cases, a decrease in trade costs lowers the productivity threshold above which plants decide to enter the foreign market. For instance, a ten percent decrease in the iceberg trade cost reduces the minimum productivity required to enter the export market profitably by about 2 percent and increases the number of exporters by about 5.5 percent. Because exporters expand their production in response to new profit opportunities, the demand for labor goes up, forcing marginally profitable plants out of the industry. Therefore the number of producers in equilibrium goes down. In particular, a ten percent decrease in the iceberg trade cost decreases the number of incumbents by about 12 percent. Responses are qualitatively similar for a 10 percent decline in the fixed trade cost.

<table>
<thead>
<tr>
<th>Table XVII: Counterfactual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Export threshold ($\phi_x$)</td>
</tr>
<tr>
<td>Number of exporters</td>
</tr>
<tr>
<td>Number of producers</td>
</tr>
<tr>
<td>Mean Price</td>
</tr>
<tr>
<td>Mean Quantity</td>
</tr>
<tr>
<td>Mean Quality</td>
</tr>
</tbody>
</table>

Notes: This table presents the effect of three types of trade liberalization. The mean price is computed by dividing total revenue by total quantity consumed, which is equal to the quantity produced minus the output used to pay the variable export cost. The mean quality is the average quality of a unit of output.

Trade liberalization also has a significant impact on the characteristics of the typical consumption basket. In all cases, trade liberalization increases the price of the
average unit purchased. For example, a 10 percent decrease in transport costs leads to a 4 percent increase in price. The impact on quality varies across the type of trade liberalization. While a ten percent decrease in variable trade costs raises the quality of the average unit consumed by about 4 percent, a ten percent decrease in fixed export costs has almost no effect on quality. From equation (7), a decrease in transport cost increases the optimal quality conditional on productivity. Therefore, as transport cost goes down more plants enter the export market and the quality of varieties produced by exporters goes up. This implies that the average quality of consumption goes up. However, fixed export costs affect quality only indirectly – through their impact on the price index \( (P) \) included in the constant \( A \). A decrease in fixed exports cost increases the share of exporting firms, but the share of output for export and the quality of exported varieties is almost unchanged. The overall impact on quality is therefore negligible.

Finally, trade decreases the overall consumption of differentiated product in equilibrium. This happens for two reasons. First, when trade impediments are lower the share of production for export increases. This implies that more output is lost in transportation. Second, following trade liberalization, the typical consumer substitutes higher quality imported varieties for low quality domestic varieties. Therefore the consumption basket becomes smaller but contains better products.

Overall the changes in price, quality and quantity consumed increase consumer welfare. The estimated impact is small however (less then one percent). The rise in welfare associated with a ten percent decrease in transport cost is due to the increase in the average quality of consumed varieties. In the case of a decrease in fixed export
cost welfare increase because of the increase in the number of varieties available for consumption.

3.6. Conclusion

This chapter makes three significant contributions. First, it develops a tractable general equilibrium model that includes vertical product differentiation in a heterogeneous producer framework. The theory clearly demonstrates how productivity affects price and export status through two distinct channels: directly by reducing unit production costs and indirectly by increasing quality. Second, using the theory as a guide, the chapter develops a novel empirical strategy to obtain a proxy for quality and uses it to evaluate the importance of vertical product differentiation in explaining observed price and export patterns. The empirical findings are consistent with the model: (i) On average, quality is positively correlated with unit cost and price; (ii) On average, productivity is negatively correlated with unit production costs and output price and positively correlated with quality; (iii) Quality, in addition to productivity, is an important determinant of the plant’s export status. Finally, the chapter uses the simulated method of moments to obtain additional evidence of the presence and importance of product quality differentiation. According to the structural estimates of the model’s parameters, marginal production costs are concave in quality while fixed production costs are convex in quality. Since the parameters are left unrestricted in the estimation, these findings support the idea that plants use quality as a mean to compete in the market. Further, the results suggest that using demand residuals as of proxy for quality is a reasonable empirical strategy. Evaluated
at the optimal parameter values the correlation between the theoretically accurate measure of quality and the constructed proxy estimated from the simulated data is above 0.9.

Overall the results presented in this chapter provide strong support to the idea that within industry vertical product differentiation is important to explaining variation in price and export status patterns observed in micro data on U.S. manufacturing plants.
Chapter 4: Conclusion

The dissertation uses theoretical, empirical and computational methods to study the role of product differentiation in shaping price dispersion and trade patterns observed in microdata on U.S manufacturing plants. Overall the thesis demonstrates that the potential for product quality differentiation has an important effect on the behavior of producer and the characteristics of the industry and should not be ignored.

While the analysis is arguably an important contribution, it is clearly not exhaustive. Much remains to be done. For instance, evaluating the reduction in the welfare cost of trade liberalization would be an important avenue for future research. It is often argued that the long-run gains associated with trade liberalization can only be obtained at the expense of costly short-run adjustments. In particular, the loss in welfare due to worker reallocation is frequently cited by policy makers as a major hurdle to import tariff reduction. In the presence of vertical differentiation, quality upgrading is an important margin producers in developed countries can exploit to resist low-wage import competition. Since worker relocation is likely to be easier within than across industries, within-industry specialization reduces the predicted welfare loss associated with the short-run adjustments. Quantitative estimates for the effect of quality on the short term adjustment cost associated with trade liberalization would be useful.

Further, in industries characterized by vertical differentiation it is possible that firms manufacture an array of products of different quality within the same industry. In that case, the firm’s export bundle could changes across foreign markets in
response to differences in trade costs. Therefore, developing a model with multi-
quality firms could help explain the variation across export destinations in producer
level prices observed in U.S. microdata.
Appendices

A. Appendix to Chapter 2

A.1 The Closed Economy

The zero-profit condition that determines the profitability threshold is given by 
\[ \pi_o(\varphi_o) = 0. \]
From (7) this implies that: 
\[ r_o(\varphi_o) = \varepsilon f_o. \]
Further, from (6), the ratio of revenue functions for firms with different productivity can be expressed as 
\[ \frac{r_o(\varphi)}{r_o(\varphi')} = \left( \frac{\varphi}{\varphi'_o} \right)^{\varepsilon - 1}, \]
if they produce varieties of the same quality and 
\[ \frac{r_h(\varphi)}{r_o(\varphi')} = \left( \frac{\Omega_h}{\Omega_o} \right) \left( \frac{\varphi}{\varphi'_o} \right)^{\varepsilon - 1} \]
if one produces a high quality variety while the other produces a low quality variety. Replacing \( \varphi' = \varphi_o \) it follows that:
\[ r_o(\varphi) = \varepsilon f_o \left( \frac{\varphi}{\varphi_o} \right)^{\varepsilon - 1} \quad \text{and} \quad r_h(\varphi) = \varepsilon f_o \left( \frac{\Omega_h}{\Omega_o} \right) \left( \frac{\varphi}{\varphi_o} \right)^{\varepsilon - 1}. \]
These relations can be used to express average revenue as a function of the parameters of the model and the equilibrium threshold:
\[ \bar{r} = \int r(\varphi)\mu(\varphi)d\varphi = \int r_o(\varphi)\mu(\varphi)d\varphi + \int r_h(\varphi)\mu(\varphi)d\varphi \]
\[ = \left( \frac{\sigma}{1 + \sigma - \varepsilon} \right) \left[ 1 + \left( \frac{\Omega_h - \Omega_o}{\Omega_o} \right)^{\Delta^{\varepsilon - 1 - \sigma}} \right] \varepsilon f_o. \]
From the profit functions defined in (7) it must be the case that 
\[ \bar{\pi} = \bar{r}/\varepsilon - \bar{f}, \]
where the average fixed production cost (\( \bar{f} \)) is given by:
\[ \bar{f} = f_o \int_{\Delta \varphi_o} \varphi^{-\sigma} d\varphi + f_h \int_{\Delta \varphi_o} \varphi^{-\sigma} d\varphi = (1 - \Delta^{-\sigma}) f_o + \Delta^{-\sigma} f_h. \]
From (3), the aggregate price index is given by:

\[
P = \left( \sum_{x \in X} \omega \int_{x \in X} p(x)^{1-\varepsilon} \, dx \right)^{\frac{1}{1-\varepsilon}} = M^{\frac{1}{1-\varepsilon}} \left( \int_{\phi_{o}} \omega_{o} p_{o}(\phi)^{1-\varepsilon} \mu(\phi) d\phi + \int_{\Delta_{\phi_{o}}} \omega_{H} p_{H}(\phi)^{1-\varepsilon} \mu(\phi) d\phi \right)^{\frac{1}{1-\varepsilon}}
\]

\[
= \frac{M^{\frac{1}{1-\varepsilon}}}{\rho \phi_{o}} \left( \frac{\sigma}{\varepsilon - 1 - \sigma} \left( c_{o}^{1-\varepsilon}(\Delta^{1-\sigma} - 1) - c_{H}^{1-\varepsilon} \Delta^{1-\sigma} \right) \right)^{\frac{1}{1-\varepsilon}} .
\]

Substituting for the equilibrium mass using (16) yields (17).

By definition of the revenue function, in equilibrium it must be the case that:

\[
r_{o}(\phi_{o}) \equiv p_{o}(\phi_{o}) q(\phi_{o}) = ef_{o},
\]

where the equality follows from (10). This implies that the optimal quantity of low variety produced by the marginal entrant is given by:

\[
q_{o}(\phi_{o}) = ef_{o} \rho \phi_{o} / c_{o} .
\]

From the optimal demand (2), and the pricing rule (5), it can be shown that:

\[
q_{o}(\phi)/q_{o}(\phi^{e}) = (\phi/\phi^{e})^{e} \quad \text{and} \quad q_{H}(\phi)/q_{o}(\phi^{e}) = (\phi^{e}/\phi^{e})^{e}(\Omega_{H}/\Omega_{o}).
\]

Replacing \( \phi^{e} = \phi_{o} \) equation (18) is obtained.

A.2 A Discussion of Assumption 2

This subsection discusses the empirical validity of assumption 2 which state that \( \sigma > \varepsilon \). On the one hand, a number of studies provide estimates for the elasticity of substitution (\( \varepsilon \)) in a CES demand context. Feenstra (1994) considers the annual U.S imports of six manufactured products and presents estimates around 2 in absolute value. Bernard et al. (2003) calibrate a model to fit U. S. plants and macro trade data, report an estimated elasticity of 3.8. Broda and Weinstein (2006) using data on the US report that for the period between 1990 and 2001 report an average elasticity was
around 5 for four-digit (SITC) Rauch-differentiated goods. Finally, using data on U.S. manufacturing plants chapter 3 obtains an estimate of 2.5 in absolute value. On the other hand, estimates for the productivity distribution are not as abundant. Eaton et al. (2008) obtain a value of 2.46 by calibrating a model using micro data on French firms. Eaton and Kortum (2002) assume that productivity follows a Fréchet distribution and calibrate a model to fit bilateral trade data on 19 OECD countries. Using three different approaches they obtain estimates of 3.60, 8.28 and 12.9. The next paragraph argues that these estimates can serve as indicator for the Pareto distribution’s shape parameter (σ). Overall these empirical estimates do not provide strong evidence against assumption 2 (σ > ε ) since the average shape parameter of the productivity distribution (σ) is greater than the average elasticity of substitution (ε).

Holding the distribution fixed, a larger sample of random variables is likely to have a larger maximum. In fact it can be formally demonstrated that the probability density function of the maximum tends to a limiting form: an extreme value distribution. For instance, the cumulative distribution function (cdf) for the Pareto is:

\[ \Pr\{\varphi \leq \overline{\varphi}\} = G(\overline{\varphi}) = 1 - \overline{\varphi}^{-\sigma} \text{ for } \overline{\varphi} > 1, \ \sigma > 0. \]

By definition, the cdf for the maximum, \( \varphi_{\text{max}}(m) \), is the probability that all of the m random draws are less than \( \overline{\varphi} \), so:

\[ \Pr\{\varphi_{\text{max}} \leq \overline{\varphi}\} = G_{\text{max}}(\overline{\varphi}) = [G(\overline{\varphi})]^m. \]

Define the centralized variable: \( \varphi_c = \varphi_{\text{max}}/a(m) \), where \( a(m) = G^{-1}[1 - (1/m)] = m^{1/\sigma} \) is a scale factor that provides the relationship between the supremum of the distribution and the sample maximum as a function of the sample size (m). Hence, the limiting distribution of the cdf for the scaled variable
$\varphi_c$ will be the Fréchet distribution since: $\lim_{m \to +\infty} G_m(\varphi_c) = \lim_{m \to +\infty} [1 - (\bar{\varphi}^{-\sigma} / m)]^m = \exp(-\bar{\varphi}^{-\sigma})$. Importantly, the Pareto and Fréchet distribution are function of the same unique shape parameter ($\sigma$).

A.3 The Correlation between Price and Productivity

By definition $\text{cov}[p(\varphi), \varphi] = E\{[p(\varphi) - \bar{p}] \cdot (\varphi - \bar{\varphi})\} = E[p(\varphi)\varphi] - \bar{p}\bar{\varphi}$ where $E$ denotes the expectation operator and $\text{corr}[p(\varphi), \varphi] = \text{cov}[p(\varphi), \varphi] / \sqrt{\text{var}[p(\varphi)]\text{var}(\varphi)}$. From the properties of the Pareto distribution it follows that the expected value of the product of the price and productivity is $E[p(\varphi)\varphi] = \int_{\varphi_o}^{\infty} \varphi p(\varphi) \mu(\varphi) d\varphi = \beta_3 / \rho$, where the constant $\beta_3 = c_o (1 - \Delta^{-\sigma}) + c_h \Delta^{-\sigma}$, while the average price and the average productivity are given respectively by $\bar{p} = \int_{\varphi_o}^{\infty} p(\varphi) \mu(\varphi) d\varphi = \sigma \beta_2 / \rho c_o (1 + \sigma)$ and $\bar{\varphi} = \int_{\varphi_o}^{\infty} \varphi \mu(\varphi) d\varphi = [\sigma / (\sigma - 1)] \varphi_o$ where $\beta_2 = (1 - \Delta^{-1-\sigma}) c_o + c_h \Delta^{-1-\sigma}$. Similarly, the variance of productivity is $\text{var}(\varphi) = \int_{\varphi_o}^{\infty} (\varphi - \bar{\varphi})^2 \mu(\varphi) d\varphi = \sigma \sigma_2^2 / [\sigma (-1)^2 (\sigma - 2)]$ while the variance of unit price is given by $\text{var}(p) = \int_{\varphi_o}^{\infty} (p - \bar{p})^2 \mu(\varphi) d\varphi = [\sigma / (2 + \sigma)] (\beta_1 / \rho c_o)^2 - [\sigma / (1 + \sigma)]^2 (\beta_2 / \rho c_o)^2$ where $\beta_1 = c_o^2 (1 - \Delta^{-2-\sigma}) + c_h^2 \Delta^{-2-\sigma}$. Equation (21) is obtained by combining these results. Note that since by assumption $c_o < c_h$ and by definition $\Delta^{-2-\sigma} < \Delta^{-1-\sigma} < \Delta^{-\sigma}$, $\forall \Delta > 1$ it must be the case that $0 < \beta_1 < \beta_2 < \beta_3$. 

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A.4. The Open Economy

By definition, the average extra profit earned in the export market, the average (overall) revenue and the average export price can be expressed as:

\[
\bar{\pi}_x = \int_{\Phi_{\Delta_0}}^{\infty} \pi^x(\varphi)\mu_x(\varphi)\,d\varphi = \int_{\Phi_{\Delta_0}}^{\infty} \left[ \frac{\tau^{1-\varepsilon}r_o(\varphi)}{\varepsilon} - f_x \right] \mu_x(\varphi)\,d\varphi + \int_{\Phi_{\Delta_0}}^{\infty} \left[ \frac{\tau^{1-\varepsilon}r_H(\varphi)}{\varepsilon} - f_x \right] \mu_x(\varphi)\,d\varphi,
\]

\[
\bar{r} = \int_{\Phi_{\Delta_0}}^{\infty} r(\varphi)\mu(\varphi)\,d\varphi
\]

\[
\bar{p}_x(\varphi) = \int_{\Phi_{\Delta_0}}^{\infty} p_x(\varphi)\mu_x(\varphi)\,d\varphi = \int_{\Phi_{\Delta_0}}^{\infty} p_o^x(\varphi)\mu(\varphi)\,d\varphi + \int_{\Phi_{\Delta_0}}^{\infty} p_H^x(\varphi)\mu(\varphi)\,d\varphi.
\]

Taking into account the productivity distribution characteristics, equation (27), (29) and (37) follow. Since \( r_i^x(\varphi) = \tau^{1-\varepsilon} r_i(\varphi) \) it must be the case that \( p_o^x(\varphi)q_o^x(\varphi) = \tau^{1-\varepsilon} p_o(\varphi)q_o(\varphi) \) which implies that \( q_o^x(\varphi) = \varepsilon pf_o\varphi^{1-\varepsilon}c_o^{1-\varepsilon} \). Using this result and the fact that the firm-level production of high quality is given by \( q_H^x(\varphi) = (\Omega_H/\Omega_o)q_o^x(\varphi) \) it follows that the segment specific output for domestic and foreign sales are given by:

\[
O_o^D \equiv M \int_{\Phi_{\Delta_0}}^{\infty} q_o(\varphi)\mu(\varphi)\,d\varphi, \quad O_H^D \equiv M \int_{\Phi_{\Delta_0}}^{\infty} q_H(\varphi)\mu(\varphi)\,d\varphi,
\]

\[
O_o^X \equiv M \int_{\Phi_{\Delta_0}}^{\infty} n_o^x(\varphi)\mu(\varphi)\,d\varphi, \quad O_H^X \equiv M \int_{\Phi_{\Delta_0}}^{\infty} n_H^x(\varphi)\mu(\varphi)\,d\varphi.
\]

Using these results equation (32) and (35) are easily obtained. Finally note that \( q_i^x(\varphi) = \tau^{-\varepsilon} q_i(\varphi) \), such that simplifying \( s_i^x(\varphi) \equiv nq_i^x(\varphi)/[q_i(\varphi) + nq_i^x(\varphi)] \) leads to equation (36).
A.5 Proofs of Propositions

Proof of proposition 1. From (14) it is easy to show that \( \frac{\partial V^E}{\partial \phi} < 0 \), \( \lim_{\phi \to 0} V^E = \infty \) and \( \lim_{\phi \to \infty} V^E = 0 \). These results imply the expected value of entry monotonically goes from infinity to zero as the profitability threshold goes from zero to infinity. Since the equilibrium expected value of entry is fixed by the entry cost \( f_e \), there exists a unique value of the profitability threshold such that the free entry condition (14) is satisfied. □

Proof of proposition 3. The relative profitability of the high segment is increasing in \( \Omega_H/\Omega_o \) – a positive function of \( \omega_H/\omega_o \) and a negative function of \( c_H/c_o \) – and decreasing in \( f_H/f_o \). Since, from (9) and (12), \( \Lambda \) is increasing in \( \Omega_H/\Omega_o \) and decreasing in \( f_H/f_o \), the mass of incumbent (M), defined in (16), is negatively related to the relative profitability of the high quality segment. □

Proof of proposition 4. First, from (18), it follows that \( \frac{\partial q_i(\phi)}{\partial \phi} > 0 \) for \( i \in \{o, H\} \). Second, at the transition cutoff it is the case that \( q_o(\phi_{oH}) < q_H(\phi_{oH}) \) which implies that the quantity produced is increasing across segment. Together these results imply that there is a positive association between firm-level output and productivity. The same reasoning can be applied to (6) such that there is a positive association between firm-level revenue and productivity. □

Proof of proposition 6. From (21) if \( \text{corr}(p, \phi) = 0 \) then \( A \equiv \beta_1 - \sigma^2/(\sigma^2 - 1)\beta_2 = 0 \). The derivative of A with respect to \( \sigma \) will therefore have the same sign as the price/productivity correlation evaluated at the point where \( \text{corr}(p, \phi) = 0 \). It can be shown that \( (\partial A/\partial \sigma)|_{\text{corr}(p, \phi)=0} = (c_H - c_o)\Delta^{-\sigma} \ln \Delta \{1 + 2\sigma/[\Delta(\sigma^2 - 1)^2]\} \) which is positive.
since by assumption $\Delta > 1$ and $c_H > c_o$. It is important to note that $\lim_{\sigma \to \infty} \beta_2 = \lim_{\sigma \to \infty} \beta_3$
such that $A$ is always negative in the limiting case. □

*Proof of proposition 7.* The free entry condition (14) uniquely pins down $\phi_o$ as a
function of model parameters. Since a move from autarky to costless trade does not
change any of the parameters the profitability threshold is unaffected by this change.
The export remains the same since $\Delta$, defined in (9), and the profitability threshold
$\phi_o$ are unaffected by costless trade. Since the profitability and transition thresholds
are unchanged every other variables remains the same. □

*Proof of proposition 8.* From (26) it is easy to show that $\partial V^E / \partial \phi_o < 0,
\lim_{\phi_o \to 0} V^E = \infty$ and $\lim_{\phi_o \to \infty} V^E = 0$. Therefore the expected value of entry
monotonically goes from infinity to zero as the profitability threshold goes from zero
to infinity. Since the equilibrium expected value of entry is fixed by the entry cost $f_x$, there
exists a unique value of the profitability threshold such that the free entry
condition (28) is satisfied. □

*Proof of proposition 9.* Part (i): From (12) and (29) it follows that
$$\bar{r}_{\text{Costly Trade}} = \bar{r}_{\text{Autarky}} + B$$ where $B \equiv [\sigma/(1 + \sigma - \epsilon)]n\epsilon^{x-\epsilon} \Lambda_x e f_o$ is greater than zero
whenever trade occurs. Hence the average revenue is greater under costly trade than
under autarky. By definition, the costly trade open economy average profit is given
by $\bar{\pi} + \xi x \bar{\pi} > \bar{\pi}$. Further, from (28), the open economy profitability threshold can be
expressed as $\phi_{o}^{\epsilon} \equiv (\bar{\pi} + \Delta_x^{-\epsilon} \bar{\pi}^{x}) / \delta f_x$ and by definition of the Pareto distribution the
average productivity – which is given by $[\sigma/(\sigma-1)]\phi_o$ – is increasing in the
profitability threshold. Hence, as long as some firms are exporting such that $\bar{\pi}^x > 0$, 
the profitability threshold as well as the average productivity is greater in the costly trade open economy. □

Part (ii): Part (i) established that, as long as some firms are exporting, the profitability threshold is higher in the costly trade open economy. This implies that firms with productivity between the autarky threshold and the costly trade open economy threshold will exit the industry. Also since the ratio \( \Delta \) is unaffected by trade the transition productivity cutoff \( \phi_{oh} \equiv \Delta \phi \) will also be greater under costly trade. This implies that firms with productivity between the autarky transition threshold and the costly trade open economy transition threshold will move down the quality ladder and start producing a low quality variety. □

Part (iii): When all countries are identical the aggregate revenue is the same under costly trade and autarky such that the increase in average revenue implies that the mass of incumbent must go down in costly trade – Recall that \( M = R / \tilde{r} \). Since from (33), the relative number of firms in each segment is unaffected by trade, costly trade reduces the number of firm in each segment as well as overall. □

Part (iv): From (16), (30) and the fact that \( M_T = (1 + n\xi_x)M_{CostlyTrade} \), it follows that \( M_T / M_{Autarky} = (1 + n\xi_x) / [1 + n\tau^{1-\varepsilon}(\Lambda_x / \Lambda)] \). This implies that \( M_T < M_{Autarky} \) if and only if \( \Lambda < \tau^{1-\varepsilon} \Lambda_x \Delta_x^\sigma \). Note that by assumption \( \sigma > \varepsilon - 1 \), \( \tau^{1-\varepsilon} \Delta_x^{-1} = f_x / f_o > 1 \), and \( \Delta_x > 1 \). Therefore, it must be the case that:

\[
\begin{align*}
\tau^{1-\varepsilon} \Lambda_x \Delta_x^\sigma &= \tau^{1-\varepsilon} \Delta_x^{-1} + \tau^{1-\varepsilon} \Delta_x^\sigma \left( \frac{\Omega_H - \Omega_o}{\Omega_o} \right) \Delta^{\varepsilon-1-\sigma} > \tau^{1-\varepsilon} \Delta_x^{-1} \left[ 1 + \left( \frac{\Omega_H - \Omega_o}{\Omega_o} \right) \Delta^{\varepsilon-1-\sigma} \right] \\
&> 1 + \left( \frac{\Omega_H - \Omega_o}{\Omega_o} \right) \Delta^{\varepsilon-1-\sigma}
\end{align*}
\]
It follows that $\Lambda < \tau^{1-\varepsilon} \Lambda_x^{\sigma}$ such that the mass of firms competing in each market is lower in costly trade than in autarky. □

**Part (v):** From (20) and (32) it follows that:

$$
\frac{(O_H/O_o)_{Costly\ Trade}}{(O_H/O_o)_{Autarky}} = \frac{(1 + n \tau^{-\varepsilon})(1 - \Delta^{1-\sigma})}{(1 + n \tau^{-\varepsilon})(1 - \Delta^{1-\sigma}) - n \tau^{-\varepsilon}(1 - \Delta^{\varepsilon-\sigma})} = \frac{1 + n \tau^{-\varepsilon}}{1 + n \tau^{-\varepsilon} B} > 1,
$$

with $B \equiv (\Delta_x^{1-\sigma} - \Delta^{1-\sigma})/(1 - \Delta^{1-\sigma}) > 1$, where inequality follows since $\Delta_x > 1$. Hence, the ratio of high to low quality output is higher under costly trade than under autarky. □

**Part (vi):** From part (iv) the average quality of the domestic production is higher. From proposition 10, the average quality of exports is greater than the average quality of varieties produced for domestic sales. Since consumption is the same in every country and equal to a mix of domestic and imported varieties, it must be the case that the quality of consumption if greater than the quality of varieties produced for domestic sale. □

**Proof of proposition 10.** From (20) and (35), it follows that:

$$
\frac{O_o^D}{O_H^D} = \left(\frac{\Omega_o}{\Omega_H}\right)\frac{1 - \Delta^{1-\sigma}}{\Delta^{1-\sigma}} \quad \text{and} \quad \frac{O_o^X}{O_H^X} = \left(\frac{\Omega_o}{\Omega_H}\right)\frac{\Delta^{1-\sigma} - \Delta^{\varepsilon-\sigma}}{\Delta^{\varepsilon-\sigma}}.
$$

Since, by assumption $\Delta_x > 1$ and $\sigma - \varepsilon > 0$ it follows that $\Delta_x^{1-\sigma} < 1$ which implies that $O_o^X/O_H^X < O_o^D/O_H^D$ which is equivalent to equation (34). □

A.6 The impact of Trade Liberalization

From (28), the equilibrium condition can be expressed as:

$$
j(\varphi_o) + n(\varphi_o, \Lambda_x) = \delta f_e, \quad (A.1)
$$
where \( j(\phi_o) \equiv \xi_o \pi = \phi_o^{-\sigma} \pi \) and \( j_x(\phi_o, \Lambda_x) \equiv \xi_x \xi_x \pi_x = \Delta_x^{-\sigma} \phi_o^{-\sigma} \pi_x \). \( \text{(A.2)} \)

Note that by definition: \( j_x(\phi_o, \Lambda_x) / j(\phi_o) = \Delta_x^{-\sigma} (\pi_x / \pi) > 0 \). Further, from (A.2) and the fact that \( \partial \pi_x / \partial \phi_o = \partial \Lambda_x / \partial \phi_o = 0 \), it follows that:

\[
\dot{j}(\phi_o) = -\sigma j(\phi_o) / \phi_o < 0 \quad \text{and} \quad \dot{j}_x(\phi_o, \Lambda_x) = -\sigma j_x(\phi_o, \Lambda_x) / \phi_o < 0.
\] \( \text{(A.3)} \)

Also note that:

\[
\frac{\partial j_x(\phi_o, \Lambda_x)}{\partial \tau} = -(\varepsilon - 1) \left( \frac{\sigma}{1 + \sigma - \varepsilon} \right) \frac{1}{k_x} \Lambda_x j_x(\phi_o, \Lambda_x) < 0, \\
\frac{\partial j_x(\phi_o, \Lambda_x)}{\partial f_x} = \frac{j_x(\phi_o, \Lambda_x)}{k_x f_x} \left[ (2 - \varepsilon) \left( \frac{\sigma}{1 + \sigma - \varepsilon} \right) \Lambda_x - 1 \right] < 0 \quad \text{if} \quad \varepsilon > 2, \\
\frac{\partial j_x(\phi_o, \Lambda_x)}{\partial n} = 0.
\] \( \text{(A.4)} \)

Finally, it will be useful to define \( k_x \equiv \left( \frac{\sigma}{1 + \sigma - \varepsilon} \right) \Lambda_x - 1 \) where \( \Lambda_x \) is defined in (27) such that \( \pi_x = k_x f_x \).

From (A.3), it follows that:

\[
\frac{d\phi_o}{d\tau} = -\frac{n}{\dot{j}(\phi_o) + n j_x(\phi_o, \Lambda_x)} \frac{\partial j_x(\phi_o, \Lambda_x)}{\partial \tau} < 0, \\
\frac{d\phi_o}{df_x} = -\frac{n}{\dot{j}(\phi_o) + n j_x(\phi_o, \Lambda_x)} \frac{\partial j_x(\phi_o, \Lambda_x)}{\partial f_x} < 0 \quad \text{if} \quad \varepsilon > 2, \\
\frac{d\phi_o}{dn} = -\frac{j_x(\phi_o, \Lambda_x)}{\dot{j}(\phi_o) + n j_x(\phi_o, \Lambda_x)} > 0,
\] \( \text{(A.5)} \)

where the inequalities follows from (A.3) and (A.4). Recall that, by definition

\( \phi_o^* = \Delta_x \phi_o \), hence:
\[
\frac{\partial \phi_x}{\partial \tau} = \Delta_x \frac{\partial \phi_o}{\partial \tau} + \varphi_o \frac{\partial \Delta_x}{\partial \tau} = \Delta_x \frac{\partial \phi_o}{\partial \tau} (1 - D_1) > 0,
\]

\[
\frac{d \phi_o^x}{d f_x} = \Delta_x \frac{\partial \phi_o}{\partial f_x} + \varphi_o \frac{\partial \Delta_x}{\partial f_x} = \Delta_x \frac{\partial \phi_o}{\partial f_x} (1 - D_2) > 0 \quad \text{if } \epsilon > 2, \tag{A.6}
\]

\[
\frac{d \phi_o^x}{d n} = \Delta_x \frac{\partial \phi_o}{\partial n} > 0,
\]

where:

\[
D_1 \equiv \left(1 + \sigma - \epsilon \right) \left[ \frac{j(\phi_o)}{nj_x(\phi_o, \Lambda_x)} + 1 \right] \frac{k_x}{\Lambda_x} \quad \text{and}
\]

\[
D_2 \equiv \sigma k_x \left[ \frac{j(\phi_o)}{nj_x(\phi_o, \Lambda_x)} \right] \left[ 1 + (\epsilon - 2) \left( \frac{\sigma}{1 + \sigma - \epsilon} \right) \Lambda_x \right]^{-1}.
\]

The inequalities follows from (A.5) and the fact that \( D_1 \) and \( D_2 \) are both greater than 1. This last statement can be shown as follow. For \( D_1 \):

\[
D_1 > 1 \iff \left(1 + \sigma - \epsilon \right) \left[ \frac{j(\phi_o)}{nj_x(\phi_o, \Lambda_x)} + 1 \right] \frac{k_x}{\Lambda_x} > 1 \iff \frac{j(\phi_o)}{nj_x(\phi_o, \Lambda_x)} > \left( \frac{\epsilon - 1}{1 + \sigma - \epsilon} \right) \Lambda_x \frac{k_x}{\Lambda_x} - 1
\]

This is true since \( \frac{j(\phi_o)}{nj_x(\phi_o, \Lambda_x)} = \frac{\pi}{n \Delta_x \varphi_x} > 0 \) and since \( \frac{k_x}{\Lambda_x} = \left( \frac{\sigma}{1 + \sigma - \epsilon} \right) - \frac{1}{\Lambda_x} \) is equivalent to:

\[
\left( \frac{\epsilon - 1}{1 + \sigma - \epsilon} \right) \Lambda_x \frac{k_x}{\Lambda_x} - 1 < 0 \iff \epsilon - 1 < \sigma \text{ where the last inequality is true by assumption.}
\]

For \( D_2 \), the inequality follows since the assumption \( \epsilon > 2 \) implies that

\[
\left( \frac{1 + \sigma}{\sigma} \right) \left( \frac{1 + \sigma - \epsilon}{2 + \sigma - \epsilon} \right) < 1 < \Lambda_x \text{ which, in turn implies that } 1 + (\epsilon - 2) \left( \frac{\sigma}{1 + \sigma - \epsilon} \right) \Lambda_x < \sigma k_x.
\]

**Proof of proposition 11. Part(i):** The first statement follows from taking the derivative of equation (29):
\[
\frac{\partial \bar{\pi}}{\partial \tau} = -\left( \frac{\sigma}{1 + \sigma - \varepsilon} \right) \varepsilon f_o [(\varepsilon - 1) n \tau \Delta^{\varepsilon - 1} \Lambda + (1 + \sigma - \varepsilon) \Delta^{\varepsilon - 1} \tau^{-1}] < 0 ,
\]

\[
\frac{\partial \bar{\pi}}{\partial f_x} = -\sigma \varepsilon f_o \Delta^{\varepsilon - 1} < 0 ,
\]

\[
\frac{\partial \bar{\pi}}{\partial n} = \left( \frac{\sigma}{1 + \sigma - \varepsilon} \right) (\tau^{1 - \varepsilon} \Delta^{\varepsilon - 1} \Lambda) \varepsilon f_o > 0 .
\]

Since the domestic profit is independent of trade variables, the derivative of average profit is the same as the derivative of \[n \Delta^{\varepsilon - 1} \bar{\pi} = n \Delta^{\varepsilon - 1} \left( \frac{\sigma}{1 + \sigma - \varepsilon} \Lambda - 1 \right) f_x .\]

Therefore:

\[
\frac{\partial (n \Delta^{\varepsilon - 1} \bar{\pi})}{\partial n} = \Delta^{\varepsilon - 1} \bar{\pi} > 0 ,
\]

\[
\frac{\partial (n \Delta^{\varepsilon - 1} \bar{\pi})}{\partial \tau} = -(\varepsilon - 1) \left( \frac{\sigma}{1 + \sigma - \varepsilon} \right) \frac{n \Delta^{\varepsilon - 1} f_x \Lambda}{\tau} < 0 ,
\]

\[
\frac{\partial (n \Delta^{\varepsilon - 1} \bar{\pi})}{\partial f_x} = \sigma n \Delta^{\varepsilon - 1} \left( \frac{1}{\sigma} - \left[ \frac{2(1 + \sigma) - \varepsilon}{1 + \sigma - \varepsilon} \right] \Lambda \right) < 0 ,
\]

where the last inequality follows since \(1 < \sigma, \left[ \frac{2(1 + \sigma) - \varepsilon}{1 + \sigma - \varepsilon} \right] > 1\) and \(\Lambda > 0\).

\[\Box\]

\[\text{Part (ii):}\] Follows directly from (A.5).

\[\Box\]

\[\text{Part (iii):}\] From (16) it follows that:

\[
\frac{\partial M}{\partial n} = -\left( \frac{1 + \sigma - \varepsilon}{\sigma} \right) \frac{(1 - \alpha) L}{\varepsilon f_o} (n \tau^{1 - \varepsilon} \Delta^{\varepsilon - 1} \Lambda)^{-2} \frac{\partial (n \tau^{1 - \varepsilon} \Delta^{\varepsilon - 1} \Lambda)}{\partial n} < 0 ,
\]

\[
\frac{\partial M}{\partial \tau} = -\left( \frac{1 + \sigma - \varepsilon}{\sigma} \right) \frac{(1 - \alpha) L}{\varepsilon f_o} (n \tau^{1 - \varepsilon} \Delta^{\varepsilon - 1} \Lambda)^{-2} \frac{\partial (n \tau^{1 - \varepsilon} \Delta^{\varepsilon - 1} \Lambda)}{\partial \tau} > 0 ,
\]
\[ \frac{\partial M}{\partial f_x} = - \left( 1 + \sigma - \varepsilon \right) \frac{(1 - \alpha) L}{\varepsilon f_o} \left( n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \right) - 2 \frac{\partial \left( n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \right)}{\partial f_x} < 0, \]

where the inequalities follow from the fact that:

\[ \frac{\partial \left( n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \right)}{\partial n} = \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda > 0, \]

\[ \frac{\partial \left( n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \right)}{\partial \tau} = -n \tau^{\varepsilon} \left[ \sigma \Delta^{c - \varepsilon} + (\varepsilon - 1) \left( \frac{\Omega_{H} - \Omega_{O}}{\Omega_{o}} \right) \Delta^{c - \varepsilon} \right] < 0, \]

\[ \frac{\partial \left( n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \right)}{\partial f_x} = -(1 + \sigma - \varepsilon) \frac{n \tau^{l - \varepsilon} \Delta^{c - \varepsilon}}{f_x} < 0. \]

By definition, \( M_T = (1 + n \xi) M \) such that:

\[ \frac{\partial M_T}{\partial n} = \left( 1 + \sigma - \varepsilon \right) \frac{(1 - \alpha) L}{\varepsilon f_o} \left[ \frac{n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \left( n \Delta^{c - \varepsilon} - 1 \right) - \Lambda}{(\Lambda + n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda) \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda} \right], \]

\[ \frac{\partial M_T}{\partial \tau} = -\sigma \frac{\Delta^{c - \varepsilon}}{\tau} \left( \frac{1 + \sigma - \varepsilon}{\sigma} \right) \frac{(1 - \alpha) L}{\varepsilon f_o (\Lambda + n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda)} \]

\[ - \left( 1 + \sigma - \varepsilon \right) \frac{(1 - \alpha) L}{\varepsilon f_o (\Lambda + n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda)} \frac{\partial \left( n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \right)}{\partial \tau} < 0, \]

\[ \frac{\partial M_T}{\partial f_x} = -\sigma \frac{\Delta^{c - \varepsilon}}{f_x} \left( \frac{1 + \sigma - \varepsilon}{\sigma} \right) \frac{(1 - \alpha) L}{\varepsilon f_o (\Lambda + n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda)} \]

\[ - \left( 1 + \sigma - \varepsilon \right) \frac{(1 - \alpha) L}{\varepsilon f_o (\Lambda + n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda)} \frac{\partial \left( n \tau^{l - \varepsilon} \Delta^{c - \varepsilon} \Lambda \right)}{\partial f_x} < 0. \]

Note that the sign of \( \partial M_T / \partial n \) is ambiguous. □

**Part (iv):** Follows directly from (A.5) and the fact that \( \Delta \) is unaffected by trade liberalization. □

**Part (v):** The equilibrium share of exporting firms is simply \( \xi = \Delta^{c - \varepsilon} \). Hence, since \( \partial \xi / \partial \tau = -\sigma \Delta^{c - \varepsilon} / \tau < 0 \), a decrease in iceberg cost increases the share of exporting firm increase. Further, from (A.6) a decrease in iceberg cost decreases the
productivity threshold above which firms decide to export thereby decreasing the average productivity of exporting firms. □

*Part (vi)*: From (36), it follows that $\frac{\partial s^x}{\partial \tau} = -\varepsilon n \tau^{-\varepsilon-1} / (1 + n \tau^{-\varepsilon})^2 < 0$. □

*Part (vii)*: Since $\frac{\partial \Delta_{x^\sigma}}{\partial \tau} = -\frac{(\sigma - \varepsilon) \Delta_{x^\sigma}}{\tau} < 0$, it follows from (35) that $\frac{\partial (O_{n^*}^x / O_{o^*}^x)}{\partial \tau} > 0$. □
B. Appendix to Chapter 3

B.1 Equilibrium

This appendix provides an overview of the computation required to solve for the unique equilibrium of the economy. From (11), domestic profits can be expressed as
\[ \pi_d(\phi) = B\phi^\beta - f_c \]
where \( B \equiv \{\beta[1-\eta(\varepsilon-1)]\}^{-1}A^\gamma \), so that by definition of \( \varphi_o \) it follows that \( B\varphi_o^\beta = f_c \). Then using the fact that:
\[
\frac{B\varphi^{\beta \varepsilon-1}}{B\varphi_o^{\beta \varepsilon-1}} = \left( \frac{\varphi}{\varphi_o} \right)^{\beta \varepsilon-1} \Rightarrow B\varphi^{\beta \varepsilon-1} = \left( \frac{\varphi}{\varphi_o} \right)^{\beta \varepsilon-1}
\]
the profit from domestic sales can expressed as:
\[ \pi_d(\phi) = B\varphi^{\beta \varepsilon-1} - f_c = \left[ \frac{\varphi}{\varphi_o} \right]^{\beta \varepsilon-1} - 1 f_c. \]

Similarly, from (9) the extra profit from exporting can be expressed as
\[ \Delta(\phi) = [(1+\tau^{\varepsilon-1})^{\beta \varepsilon-1}]B\varphi_x^{\beta \varepsilon-1} - f_x, \]
so that by definition of the export productivity threshold, it follows that \( \varphi_x \), \( [(1+\tau^{\varepsilon-1})^{\beta \varepsilon-1}]B\varphi_x^{\beta \varepsilon-1} = f_x \). Then it must be the case that:
\[
\frac{[(1+\tau^{\varepsilon-1})^{\beta \varepsilon-1}]B\varphi_x^{\beta \varepsilon-1}}{[(1+\tau^{\varepsilon-1})^{\beta \varepsilon-1}]B\varphi_x^{\beta \varepsilon-1}} = \left( \frac{\varphi}{\varphi_x} \right)^{\beta \varepsilon-1} \Rightarrow \Delta(\phi) = \left[ \frac{\varphi}{\varphi_x} \right]^{\beta \varepsilon-1} - 1 f_x.
\]

Further, by definition of \( \Delta \), the (overall) profit of an exporting plant can be written as
\[ \pi_x(\phi) = \pi_d(\phi) + \Delta(\phi), \]
so that:
\[ \pi_x(\varphi) = \left( \frac{\varphi}{\varphi_o} \right)^{\beta \gamma (e-1)} f_c + \left[ \frac{\varphi}{\varphi_x} \right]^{\beta \gamma (e-1)} f_x \]

Using this result and the fact that \( \varphi_x = \kappa \varphi_o \), the average profit can be expressed as a function of only one endogenous variable, the profitability threshold \( \varphi_o \):

\[
\overline{\pi}(\varphi_o) = \int_{\varphi_o}^{\infty} \pi(\varphi) \mu(\varphi) \, d\varphi = \int_{\varphi_o}^{\infty} \pi_x(\varphi) \mu(\varphi) \, d\varphi + \int_{\varphi_o}^{\infty} \pi_x(\varphi) \mu(\varphi) \, d\varphi \\
= \int_{\varphi_o}^{\infty} \left( \frac{\varphi}{\varphi_o} \right)^{\beta \gamma (e-1)} f_c \sigma \varphi_o \varphi^{-(1+\sigma)} \, d\varphi + \int_{\varphi_o}^{\infty} \left( \frac{\varphi}{\kappa \varphi_o} \right)^{\beta \gamma (e-1)} f_x \sigma \varphi_o \varphi^{-(1+\sigma)} \, d\varphi
\]

Which implies that the expected value of entry defined in (13) can be written as:

\[
V^E = \frac{\sigma}{\delta} \left[ f_c \int_{\varphi_o}^{\infty} \left( \frac{\varphi}{\varphi_o} \right)^{\beta \gamma (e-1)} \varphi^{-(1+\sigma)} \, d\varphi + f_x \int_{\varphi_o}^{\infty} \left( \frac{\varphi}{\kappa \varphi_o} \right)^{\beta \gamma (e-1)} \varphi^{-(1+\sigma)} \, d\varphi \right]
\]

It is easy to show that \( \frac{\partial V^E}{\partial \varphi_o} < 0 \), \( \lim_{\varphi_o \to 0} V^E = \infty \) and \( \lim_{\varphi_o \to \infty} V^E = 0 \). This implies that there exist a unique \( \varphi_o \) such that \( V^E(\varphi_o) = f_c \). The threshold as a function of the parameter can be obtained by solving the integral on the right hand side of the equation. The result is given in equation (14) in the text.

To complete the solution it remains only to obtain the endogenous price index \( P \).

From (6), (7) and the definition of the threshold \( \varphi_o \), it follows that:

\[
A = \{ \alpha f_c [1 - \eta(\epsilon - 1)] \varphi_o^{\beta \gamma (1-\epsilon)} \}^{1/\gamma},
\]

Using this result in (7), taking into account the definition of the threshold \( \varphi_o \) given in (14), it is possible to obtain the equilibrium value for the price index as:
\[
\begin{aligned}
P & = \frac{1}{\rho} \left[ \frac{\sigma \gamma}{1 - \eta (\varepsilon - 1)} \left\{ \beta f_e [1 - \eta (\varepsilon - 1)] \right\}^{1/\beta_1} \left[ \frac{\sigma - \beta \gamma (\varepsilon - 1)}{1 - \beta L} \right]^{1/\beta_1} \delta f_e \right]^{1/\sigma},
\end{aligned}
\]

where \( \kappa \) is defined in (11). Using the solution for the aggregate price index \( P \) it is possible to solve for every other endogenous variables such as price \( (p) \), quantity \( (q) \), revenue \( (r) \), and quality \( (\omega) \). Note that this solution algorithm does not require the knowledge of the mass of incumbent \( (M) \) in order to compute \( P \). To obtain \( M \) first compute the average revenue in the industry \( (\bar{r}) \) then since the wage rate is equal to one it must be the case that \( M = (1 - \beta) L / \bar{r} \).

### B.2 Measurement Error

Suppose that the econometrician wishes to estimate the following regression model:

\[
Y = X \xi + \epsilon \quad \text{with } E(\epsilon^{\prime} X) = 0.
\]

Unfortunately, both the dependent and independent variable are measured with error so that the econometrician observes \( Y_\ast \) and \( X_\ast \) instead. Formally, assume that the following holds:

\[
\begin{aligned}
Y_\ast & = Y + e_Y \quad \text{with } E(e_Y^{\prime} Y) = 0, \ E(e_Y^{\prime} X) = 0 \ \text{and} \\
X_\ast & = X + e_X \quad \text{with } E(e_X^{\prime} X) = 0, \ E(e_X^{\prime} Y) = 0.
\end{aligned}
\]

In addition, while the measurement errors, \( e_Y \) and \( e_X \), are uncorrelated with the error term \( \epsilon \), they are correlated amongst themselves. Formally assume that:

\[
E(e_Y^{\prime} e_Y) = 0, \ E(e_X^{\prime} e_X) = 0, \ \text{and } E(e_Y^{\prime} e_X) = \Sigma_{\gamma X} \neq 0.
\]

Taking into account these assumptions, the estimated regression model can be expressed as follow:
\[ Y_s = X_s \xi + \tilde{e} \text{ with } \tilde{e} \equiv e + e_v - e_x \xi. \]

Using this result, the OLS estimator is defined as follow:
\[ \hat{\xi} = (X'_eX_e)^{-1}X'_eY_e = \xi + (X'_eX_e)^{-1}X'_e\tilde{e}. \]

The probability limit of this estimator is given by:
\[ \text{p} \lim (\hat{\xi}) = \xi + [N^{-1}\text{p} \lim (X'_eX_e)]^{-1} \cdot N^{-1}\text{p} \lim (X'_e\tilde{e}) = \xi + (\Xi_{XX} + \Sigma_{XX}^{-1})(\Sigma_{XY} - \Sigma_{XX} \xi) \]

where \( \Sigma_{XX} = \text{E}(e'_X e_X) \) and \( \Xi_{XX} \equiv \text{p} \lim (N^{-1}X'X) < \infty \). Therefore the sign and magnitude of the bias depends on the unknown variance of measurement error in the independent variables, \( \Sigma_{XX} \), and the correlation between the measurement errors in the dependent and independent variables, \( \Sigma_{XY} \). Since these are unknowable in practice very little can be said about the properties of the estimator in the current context except that that bias goes to zero as the measurement errors become small.

### B.3 Solution Algorithm

This appendix develops the algorithm used to solve the model computationally. The procedure consists of three major steps.

1. Obtain the productivity threshold above which producers decide to stay in the industry. Given a vector of parameters this is easily done using (14). Using this threshold it is possible to obtain the ex-post distribution of producer from the ex-ante random vector of entrant generated from the productivity distribution. It is also possible to obtain \( \varphi_x \), the export productivity threshold as well as the number of producers and exporters in the sample.
2. Use the equilibrium productivity threshold to compute the value of \( A \) as follow – see appendix B.1:

\[
A = \left\{ \beta f_c [1 - \eta (\varepsilon - 1)] \phi _o ^{\beta (1 - \varepsilon)} \right\} ^{1/y}.
\]

Given \( A \), it is possible to obtain the equilibrium value for the price index and a related constant \( D \) which is going to be useful in future computations. These are respectively defined as:

\[
P = \left[ \frac{\varepsilon \rho ^{1 - \varepsilon}}{1 - \eta (\varepsilon - 1)} \frac{A ^{1/\beta}}{R} \right] ^{1/(1 - \varepsilon)}, \quad \text{and} \quad D \equiv R P ^{1 - \varepsilon} = \frac{\varepsilon \rho ^{1 - \varepsilon}}{1 - \eta (\varepsilon - 1)} A ^{1/\beta}.
\]

3. Obtain equilibrium values for the variables. Once \( A \) is known the equilibrium values for the variables are easily obtained as follow. From (7) quality is given be:

\[
\omega_d (\rho) = A \phi ^{\beta (1 - \varepsilon)}, \quad \text{and} \quad \omega_x (\rho) = (1 + \tau ^{1 - \varepsilon})^\beta A \phi ^{\beta (1 - \varepsilon)}.
\]

Then from (5), the domestic and export price of a domestic variety is given by:

\[
p_d (\rho) = \frac{\omega_d (\rho) \eta}{\rho \phi} = A ^{\eta} \frac{\phi ^{\beta (1 - \varepsilon) - 1}}{\rho}, \quad \text{and} \quad p_x (\rho) = (1 + \tau ^{1 - \varepsilon})^\beta p_d (\rho).
\]

From the optimal demand function, defined in (2):

\[
q_d (\rho) = \omega_d (\rho) R P ^{1 - \varepsilon} p_d (\rho) ^{- \varepsilon} = D \rho ^{\varepsilon} A ^{1 - \eta \varepsilon} \phi ^{\beta (1 - \varepsilon) (1 - \eta \varepsilon) + \varepsilon}, \quad \text{and} \quad q_x (\rho) = (1 + \tau ^{1 - \varepsilon}) (1 + \tau ^{1 - \varepsilon} )^\beta (1 - \eta \varepsilon) q_d (\rho).
\]

The share of exporting firms is computed by dividing the number of exporters by the number of producers. The industry’s share of revenue from exporting is obtained by dividing the total revenue from export by the total revenue in the industry.
B.4 Weighting Matrix for SMM Estimation

Given the nature of the moments used in the estimation, the variance-covariance matrix of the moments cannot be computed from the data. For example, it is not possible to compute the variance of the share of exporting firm since it is only observed once. A bootstrapped procedure must therefore be used to obtain an estimate for the weighting matrix \( W \). First, using the identity matrix as an estimate for \( W \), it is possible to obtain consistent estimates for the vector of structural parameters \( \theta \). These estimates can then be used to generate samples of artificial data (100 in this case) from which an estimate for the variance-covariance matrix of the simulated moments can be computed. The estimated matrix is shown in Table XVIII.

<table>
<thead>
<tr>
<th></th>
<th>Stat(p)</th>
<th>Stat(r)</th>
<th>Stat(q)</th>
<th>Top10r</th>
<th>Top20r</th>
<th>Top50r</th>
<th>Exp/Prod</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.002</td>
<td>0.005</td>
<td>0.004</td>
<td>0.003</td>
<td>0.003</td>
<td>-0.002</td>
<td>-0.002</td>
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<tr>
<td>Stat(r)</td>
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<td>0.084</td>
<td>0.096</td>
<td>0.099</td>
<td>0.070</td>
<td>-0.059</td>
</tr>
<tr>
<td>Stat(q)</td>
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<td>0.084</td>
<td>0.063</td>
<td>0.067</td>
<td>0.069</td>
<td>0.049</td>
<td>-0.041</td>
</tr>
<tr>
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<td>0.096</td>
<td>0.067</td>
<td>0.196</td>
<td>0.201</td>
<td>0.141</td>
<td>-0.118</td>
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<tr>
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<td>0.201</td>
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<td>0.145</td>
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<tr>
<td>Top50r</td>
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<td>0.049</td>
<td>0.141</td>
<td>0.145</td>
<td>0.102</td>
<td>-0.085</td>
</tr>
<tr>
<td>Exp/Prod</td>
<td>-0.002</td>
<td>-0.059</td>
<td>-0.041</td>
<td>-0.118</td>
<td>-0.121</td>
<td>-0.085</td>
<td>0.071</td>
</tr>
</tbody>
</table>

Note: This table shows the variance-covariance matrix of the simulated moments which is used as an estimate for the optimal weighting matrix \( W \).

Using \( \hat{W} \) efficient estimates for the structural parameters can be obtained. Another possibility would be to obtain a bootstrapped estimate by creating sample from random draws with replacement from the actual data.
Bibliography


