

## ABSTRACT

Title of dissertation: THE UPSIDE OF MINIMAL LEFT-RIGHT  
SUPERSYMMETRIC SEESAW IN  
DEFLECTED ANOMALY MEDIATION

Sogee Spinner, Doctor of Philosophy, 2008

Dissertation directed by: Professor Rabindra Mohapatra  
Department of Physics

The state of the standard model of particle physics is reviewed focusing on two of its major issues: the hierarchy problem and its inconsistency with observed neutrino masses. Supersymmetry, an elegant solution to the former, and the seesaw mechanism in left-right models, a natural solution to the latter, are then introduced. The work then focuses on a specific supersymmetric left-right models, which has an additional discrete symmetry allowing a prediction of the seesaw scale at around  $10^{11}$  GeV—consistent with neutrino oscillation data. It also solves the  $\mu$  problem and guarantees automatic  $R$ -parity conservation and a pair of light doubly-charged Higgses which can be searched for at the LHC.

This model has interesting properties in the context of anomaly mediated supersymmetry breaking (AMSB). After a brief introduction to this topic, it is shown that this model is an instance of the Pomarol Rattazzi model of deflected AMSB. The tachyonic slepton problem of AMSB is solved in a combination of two ways: the right-handed sleptons are saved by their couplings to the low energy

doubly-charged fields while the left-handed sleptons receive positive contributions from the partially decoupled  $D$ -terms. The resulting phenomenology is similar to that of minimal AMSB due to the gaugino spectrum; however, same generation mass differences in the sfermion sector are much larger than that of mAMSB and the right-handed selectron can be as massive as the squarks. Finally, this model also contains a mechanism for solving the EWSB problem of AMSB and a dark matter candidate.

THE UPSIDE OF MINIMAL LEFT-RIGHT SUPERSYMMETRIC  
SEESAW IN DEFLECTED ANOMALY MEDIATION

by

Sogee Spinner

Dissertation submitted to the Faculty of the Graduate School of the  
University of Maryland, College Park in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy  
2008

Advisory Committee:

Professor Rabindra Mohapatra, Chair/Advisor

Professor Michael Coplan, Dean's Representative

Professor Kaustubh Agashe

Professor Zackaria Chacko

Professor Sarah Eno

© Copyright by  
Sogee Spinner  
2008

## Acknowledgments

The task of my physics training has been successfully carried by my adviser, Dr. Rabindra Mohapatra. None of this would have been possible without him, the effort he has put forth towards my education and the guidance he has provided. Specifically, I would like to thank him for instilling in me the importance of physical intuition. He also gave a very illuminating course on field theory which has formed the basis of my knowledge on that subject.

My education and growth were further assisted by other members of the particle group who deserve my gratitude: Dr. Jogesh Pati has been a constant source of encouragement and knowledge, especially through his wonderful particle physics courses; Dr. Markus Luty was always willing to talk through the thornier issues in theoretical arguments and taught a very deep course on field theory; I am very grateful that I had a chance to collaborate with Dr. Kaustubh Agashe, to take his course on extra dimensions and for his interest in my postgraduate opportunities; conversations with Dr. Zackaria Chacko have always been relevant and illuminating and I especially appreciate his honesty and advice; Dr. Sarah Eno has been very helpful with questions concerning the experimental side of our field; Dr. James Gates' course on string theory was a very useful introduction to the subject; and finally Drs.: Kevork Abazajian, Xiangdong Ji, Young Suh Kim, Wally Greenberg, Takemichi Okui and Manuel Toharia have all added to my experience here.

There are a few Maryland professors outside the particle group who have also served as advisers and motivators to whom I owe much thanks. Dr. Michael

Coplan has been a very thoughtful mentor—always willing to listen and offer advice. He taught a very thorough electronics course and served as a great model teacher while I was his teaching assistant. Dr. Bob Dorfman has helped me keep things in perspective and his physics stories have kept me smiling. In addition, his course on quantum mechanics solidified my grasp of that subject. I would also like to thank my undergraduate professors: Drs. Asim Gangopadhyaya, Jeffrey Mallow and Gordon Ramsey who have continued encouraging me throughout graduate school.

The extent of my grasp of physics is due to the large number of exceptional teachers at Maryland. In addition to the professors mentioned above, I would also like to thank Drs.: Alex Dragt for a fantastic course in classical mechanics, Edward Ott for helping me finally understand electrodynamics and Theodore Jacobson for inspirational lectures on general relativity.

Students at Maryland have also helped to enrich my time here. I have had very illuminating physics discussion with my fellow particle physics students: Dr. Haibo Yu, Dr. Ken Hsieh, Dr. Parul Rastogi, Matthew Wetstein, Aleksandr Azatov and Christopher Krenke. I owe a summer of meaningful studying and success on the qualifier to my friends: Kate Despain, Emily Edwards, Lee Elberson and Dr. Rosemary Russ. I have also had the pleasure of collaborating with Lijun Zhu, and thank him for his enthusiasm. Finally, Nicholas Setzer and I have collaborated on many projects together. Our discussions and conversations have contributed immensely to my physics knowledge and my personal life.

I owe much gratitude to the people who have provided moral support for me during graduate school. This includes my family: Shoshana, Yigal, Mika, Snir and

Galli Spinner. It also includes the friends I have made during my time here: Meem Mahmud, Aaron Moss, Anita Roychowdhury, Sonya Weisburd and especially Rachel Maisler. I also owe much gratitude to my grandparents: Yakov and Miryam Greenfield, who supported me both morally and financially through my undergraduate education and made it possible for me to seek out this new academic adventure.

Lastly, I would like to thank the physics support staff and apologize to the people who have accidentally been left out here.

# Table of Contents

|  |     |
|--|-----|
| List of Tables   | vii |
| List of Figures  | ix  |
| List of Abbreviations                                  | x   |
| 1 Introduction   | 1   |
| 1.1 The Standard Model                                 | 1   |
| 1.1.1 The Standard Model Lagrangian                    | 2   |
| 1.1.2 Standard Model Summary and New Physics Wish List | 7   |
| 1.2 Massive Neutrinos and Left-Right models            | 11  |
| 1.2.1 The Seesaw Mechanism In Left-Right Models        | 14  |
| 1.2.2 Motivation and Consequences of Left-Right Models | 15  |
| 1.3 Supersymmetry                                      | 16  |
| 1.3.1 The Minimal Supersymmetric Standard Model        | 19  |
| 1.3.2 Methods of SUSY Breaking                         | 24  |
| 1.3.3 Minimal Supersymmetric Standard Model Details    | 28  |
| 1.3.4 Summary and Issues                               | 30  |
| 1.4 Organization                                       | 31  |
| 2 Predicting the Seesaw Scale                          | 33  |
| 2.1 Motivation   | 33  |
| 2.2 Supersymmetric Left-Right Models                   | 36  |
| 2.3 Theoretical Model And the Seesaw Scale             | 43  |
| 2.3.1 Effective Theory and the $\mu$ Problem           | 49  |
| 2.3.2 Charge Violation Consideration                   | 51  |
| 2.4 Mass Spectrum and Numerical Analysis               | 52  |
| 2.4.1 Mass Spectrum                                    | 52  |
| 2.4.2 Numerics   | 60  |
| 2.4.3 Implications                                     | 61  |
| 2.5 Grand unification prospects                        | 63  |
| 2.6 Conclusion   | 65  |
| 3 Anomaly Mediated Supersymmetry Breaking, A Review    | 66  |
| 3.1 Superconformal Invariance                          | 66  |
| 3.2 Superconformal Invariance Breaking and Soft Terms  | 71  |
| 3.3 Decoupling Thresholds                              | 74  |
| 3.4 Problems and Solutions                             | 75  |
| 3.5 Deflected Anomaly Mediation                        | 77  |
| 3.6 $D$ -Terms   | 81  |



|       |  |     |
|-------|--|-----|
| 4     | Predictive Supersymmetric Left-Right Model and Anomaly Mediation | 86  |
| 4.1   | The Theory   | 86  |
| 4.1.1 | Right-Handed Sleptons  | 90  |
| 4.1.2 | Left-Handed Slepton Masses and $D$ -Terms                        | 91  |
| 4.1.3 | Below the Right-Handed Scale                                     | 93  |
| 4.2   | Phenomenology  | 98  |
| 4.2.1 | Sfermions  | 99  |
| 4.2.2 | Bosinos and The LSP  | 105 |
| 4.2.3 | Doubly-Charged Higgs and Low Energy Phenomenology                | 109 |
| 4.2.4 | Collider Signatures  | 110 |
| 4.2.5 | Dark Matter  | 112 |
| 4.3   | Conclusion   | 112 |
| 5     | Conclusion   | 114 |
| A     | The Predictive Supersymmetric Left-Right Model                   | 116 |
| A.1   | Above the Right-Handed Scale                                     | 116 |
| A.2   | Below The Right-Handed Scale                                     | 118 |
| A.3   | Below $F_\phi$   | 120 |
| B     | Masses   | 124 |
| B.1   | Masses in the Minimal Supersymmetric Standard Model              | 124 |
| B.1.1 | Sfermion Masses  | 124 |
| B.1.2 | Bosino Masses  | 125 |
| B.2   | Type II Singular Seesaw Mechanism                                | 126 |

## List of Tables

|     |  |    |
|-----|--|----|
| 1.1 | Representation assignment for the fermion and Higgs fields of the SM where the superscript $i = 1..3$ represents generation, the prime indicates that these are gauge and not mass eigenstates and the subscript $L$ ( $R$ ) denotes 2 component left-handed (right-handed) fermions. The electric charge is given by $Q = T_3 + \frac{1}{2}Y$ where $T_3$ is the isospin charge and $Y$ the hypercharge . . . . . | 3  |
| 1.2 | Representation assignment for the fermion and Higgs fields of the left-right model where the superscript $i = 1..3$ represents generation. .   | 13 |
| 1.3 | Particle content and representation assignment for the MSSM superfields where the superscript $i = 1..3$ represents generation. The superfields are composed of complex scalars and left-handed Weyl fermions . . . . .  | 20 |
| 1.4 | Physical Higgs degrees of freedom in the MSSM with $\bar{m}^2 \equiv m_{A^0}^2 + M_Z^2$ , $c_\theta$ ( $s_\theta$ ) $\equiv \cos \theta$ ( $\sin \theta$ ) and at tree level $\frac{s_{2\alpha}}{s_{2\beta}} \equiv -\frac{\bar{m}^2}{m_{H^0}^2 - m_{h^0}^2}$ . . . . .  | 29 |
| 2.1 | Representation assignment for the chiral supermultiplets of MSLRM where the superscript $i = 1..3$ represents generation, $a = 1..n$ the number of bidoublets. . . . .   | 37 |
| 2.2 | $D$ -term contributions to soft masses due to the breaking of $SU(2)_R \times U(1)_{B-L}$ . The last two terms corresponding to the electroweak Higgses have already been accounted for in Eq. (2.41) with $D \equiv \frac{1}{4\pi} \frac{m_{\Delta^c} - m_{\bar{\Delta}^c}}{\alpha_{B-L} + \alpha_R}$ . . . . .   | 60 |
| 2.3 | Points in parameter space used to evaluate the Higg masses . . . . .   | 61 |
| 2.4 | The Higgs masses at tree level based on parameters from Table 2.3. The masses are given in GeV. As predicted previously, the doubly charged particles ( $D^{++}$ and $d^{++}$ ) have masses in the electroweak range. . . . .  | 62 |
| 2.5 | Vacuum related parameters based on parameters from Table 2.3. The second column shows $v_R$ and as can be seen is the correct order of the seesaw scale. The last column presents the difference in the ground state energy of the charge violating and the charge conserving vacuum. A positive value in this column indicates that the charge conserving vacuum is the stable one. . . . .                       | 62 |
| 3.1 | Weyl weight and $R$ charges of superspace coordinates . . . . .  | 68 |

|     |   |     |
|-----|---|-----|
| 3.2 | Derived Weyl weight and $R$ charge assignments for the Kähler and Super Potentials . . . . .  | 68  |
| 3.3 | The $U(1)$ charges and particle content of a toy model that demonstrates decoupling/deflection of thresholds in AMSB . . . . .  | 77  |
| 4.1 | Behavior of $\alpha_{B-L}$ and the $U(1)_{B-L}$ and $SU(2)_R$ $D$ -term contributions for different limits of $\alpha_R$ . Remember that $\alpha_1 \sim 0.022$ . . . . .  | 101 |
| 4.2 | Schematic behavior of the mass boundary condition of various fields and the sign of their $D$ -term contributions . . . . .   | 103 |
| 4.3 | Masses of the first generation sfermions at $F_\phi = 36$ TeV , $f = 1$ and $\alpha_R = 0.05$ . A large hierarchy in the selectron can be seen as well as the fact that the selectron can be as massive as the squarks. . . . . | 107 |
| 4.4 | Values for the $b$ parameter in the MSSM and PSLRM. . . . .   | 108 |

## List of Figures

|     |  |     |
|-----|--|-----|
| 4.1 | The mass of the right-handed selectron at the right-handed scale broken up into the AMSB, mixed and GMSB contributions with $F_\phi = 36$ TeV and $\alpha_1 = 0.02$ . The AMSB contribution dominates and is positive for $f > 0.6$ . . . . .  | 92  |
| 4.2 | The squared mass of the left-handed selectron at the right-handed scale with $F_\phi = 36$ TeV and $\alpha_1 = 0.02$ . At $f > 0.85$ , the $D$ -terms cause the square mass to be positive. The boundary condition is independent of $f$ and corresponds to the $f = 0$ part of the curve. . . . .   | 94  |
| 4.3 | The squared mass of the lightest down squark, up squark and selectron at the SUSY scale (1 TeV) with $F_\phi = 36$ TeV and $\alpha_1 = 0.02$ verses $f$ at the right-handed scale. The light shaded region is excluded for squark masses from the Tevatron and the darker shaded region is excluded for slepton masses from LEP II. The dashed line is the mass of the LSP. In order for it to be the dark matter candidate, all SUSY masses must be above it. The allowed parameter space is about $0.8 \leq f \leq 1.2$ . . . . .                    | 100 |
| 4.4 | The squared mass of the lightest down squark, up squark and selectron at the SUSY scale (1 TeV) with $F_\phi = 36$ TeV and $f = 1$ at the right-handed scale verses $\alpha_R$ at the right-handed scale. The light shaded region is excluded for squark masses from the Tevatron and the darker shaded region is excluded for slepton masses from LEP II. The dashed line is the mass of the LSP. In order for it to be the dark matter candidate, all SUSY masses must be above it. The allowed parameter space is $0.75 \leq f \leq 1.22$ . . . . . | 102 |
| 4.5 | The allowed parameters space in the $\alpha_R - f$ plane taking into account the collider limits used in the previous figures. Here $F_\phi = 36$ TeV. . . . .   | 104 |
| 4.6 | The allowed parameters space in the $\alpha_R - f$ plane taking into account the collider limits used in the previous figures and that the wino is the LSP. Here $F_\phi = 36$ TeV. . . . .  | 105 |
| 4.7 | Plot of $m_{\tilde{u}_1}$ and $m_{\tilde{u}_2}$ , the latter being the heavier one for $f = 1$ and $F_\phi = 36$ TeV. The line at about 250 GeV correspond to Tevatron limits as in the plots show in Figures 4.3 and 4.4. . . . .   | 106 |

## List of Abbreviations

|       |   |
|-------|---|
| AMSB  | anomaly mediated supersymmetry breaking         |
| BSM   | beyond the standard model                       |
| CKM   | Cabibbo-Kobayashi-Maskawa                       |
| CP    | charge parity                                   |
| EWSB  | electroweak symmetry breaking                   |
| FCNC  | flavor changing neutral currents                |
| GMSB  | gauge mediated supersymmetry breaking           |
| GUT   | grand unified theory                            |
| LHC   | Large Hadron Collider                           |
| LSP   | lightest supersymmetric particle                |
| mAMSB | minimal anomaly mediated supersymmetry breaking |
| mGMSB | minimal gauge mediated supersymmetry breaking   |
| MSLRM | minimal supersymmetric left-right model         |
| MSSM  | minimal supersymmetric standard model           |
| NMSSM | next-to-minimal supersymmetric standard model   |
| PSLRM | predictive supersymmetric left-right model      |
| QCD   | quantum chromodynamics                          |
| QED   | quantum electrodynamics                         |
| RGE   | renormalization group equation                  |
| SLRM  | supersymmetric left-right model                 |
| SM    | standard model of particle physics              |
| SUGRA | supergravity                                    |
| SUSY  | supersymmetry                                   |
| UV    | ultraviolet                                     |
| VEV   | vacuum expectation value                        |

# Chapter 1

## Introduction

### 1.1 The Standard Model

The most fundamental physical questions are “What are the building blocks of the universe” and “how do those blocks interact with each other?”. Even though this question is quite old, it is the underlying question of the relatively young field of particle physics. The modern answer is summed up in the standard model (SM) [1, 2, 3].

Matter content in the SM can be broken down into two general categories: quarks and leptons. Each category can then be further subdivided into 3 generations with each generation containing 2 different flavors. In the quark sector, the generations in ascending order are: up and down, charm and strange and top and bottom; for the leptons: electron and electron neutrino, muon and muon neutrino and tau and tau neutrino. Each successive generation is a heavier copy of the previous one.

In general these particles interact through four forces: strong, weak, electromagnetic and gravitational; the SM provides a quantum understanding of the first three. Quarks are the only strongly interacting particles. Both quarks and leptons experience the weak force and only the neutrinos do not interact electromagnetically. Everyday experiences are limited to the electromagnetic and gravitational

forces. The strong force plays a role only on the scale of  $10^{-15}\text{m}$  because quarks are confined inside hadrons, *e.g.* protons and neutrons. The weak force is mediated by massive particles and therefore have a characteristic scale associated with the mass of those mediators, also roughly  $10^{-15}\text{m}$ .

The SM states that each of these three forces is mediated via spin 1 bosons called gauge bosons. Gauge boson-matter interaction strengths are proportional to charge. Strong force charge is called color and has three different values: red, green and blue. Mediation of the strong force takes place via eight massless gluons, which also carry color charge. Isospin determines weak force interaction. The weak force carriers are three massive gauge bosons  $W^+$ ,  $W^-$  and  $Z^0$ . Finally the electromagnetic force is mediated by massless, chargeless photons. Particle charges are summed up in the language of group theory in Table 1.1.

### 1.1.1 The Standard Model Lagrangian

Mathematically, the description of the three forces is quite remarkable and links them to a beautiful area of mathematics: group theory. Specifically the SM forces correspond to local (or gauge) symmetries of the Lagrangian. Therefore it is necessary to know the symmetry groups, the particle content and its charge under the groups. The Lagrangian is then just all the operators of dimension four or less that can be built from the particle content which are invariant under local transformations of the symmetry groups. The SM forces are the direct product:  $SU(3)_c \times SU(2)_L \times U(1)_Y$ , with the subscript letters standing for: color, left and

hypercharge. Particle representations under this direct product are summarized in Table 1.1. The electric charge is given by  $Q = T_3 + \frac{1}{2}Y$  where  $T_3$  is the isospin charge and  $Y$  the hypercharge. Here the superscript  $i = 1..3$  denotes generation so

| Fields  | $SU(3)_c \times SU(2)_L \times U(1)_Y$ |
|---------|--|
| $Q_L^i$ | $(3, 2, +\frac{1}{3})$                 |
| $u_R^i$ | $(3, 1, +\frac{4}{3})$                 |
| $d_R^i$ | $(3, 1, -\frac{2}{3})$                 |
| $L_L^i$ | $(1, 2, -1)$                           |
| $e_R^i$ | $(1, 1, -2)$                           |
| $H$     | $(1, 2, -1)$                           |

Table 1.1: Representation assignment for the fermion and Higgs fields of the SM where the superscript  $i = 1..3$  represents generation, the prime indicates that these are gauge and not mass eigenstates and the subscript  $L$  ( $R$ ) denotes 2 component left-handed (right-handed) fermions. The electric charge is given by  $Q = T_3 + \frac{1}{2}Y$  where  $T_3$  is the isospin charge and  $Y$  the hypercharge

that for example:

$$Q_L^3 = \begin{pmatrix} t'_L \\ b'_L \end{pmatrix} \quad L_L^3 = \begin{pmatrix} \nu'_\tau \\ \tau'_L \end{pmatrix} \quad u_R^2 = c'_R \quad (1.1)$$

while  $L$  ( $R$ ) represents two component left-handed (right-handed) fermions with  $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5) \psi$ . Also, color degrees of freedom will not be indicated in the rest of this discussion.



The SM also includes an  $SU(2)_L$  scalar doublet, the Higgs:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad (1.2)$$

with a potential given by:

$$V = -\mu^2|H|^2 + \lambda|H|^4 \quad (1.3)$$

Given a  $\mu^2 > 0$  the trivial vacuum expectation value (VEV) of H leads to an unstable ground state, i.e. for  $\langle 0|H|0 \rangle \equiv \langle H \rangle = 0$ . The stable ground state can be found by minimizing the potential:

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad v = \sqrt{\frac{\mu^2}{\lambda}} \quad (1.4)$$

Since a non-zero VEV for a quantum field is not well-defined, it is necessary to shift  $H^0 \rightarrow H^0 + v$ . This corresponds to a spontaneous breaking of the gauge symmetry and leads to massive gauge bosons as can be seen in the covariant derivative. This is known as the Higgs mechanism [4, 5, 6]. The covariant derivative is defined so that  $D_\mu\psi$  is invariant under the local gauge group. In general the SM covariant derivative is

$$D_\mu = \partial_\mu - ig_2\tau^a W_\mu^a - i\frac{Y}{2}g_1 B_\mu \quad (1.5)$$

where for the Higgs boson  $Y = +1$ . The  $SU(2)_L$  generators are given by  $\tau^a = \frac{\sigma^a}{2}$  where the  $\sigma^a$  are the Pauli spin matrices.

Once the Higgs field is shifted, the Higgs covariant derivative term produces

the following mass terms for the bosons:

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_1 \mp iW_2) \qquad M_W = \frac{g_2 v}{2} \qquad (1.6)$$

$$Z_\mu^0 \equiv \cos \theta_w W_\mu^3 - \sin \theta_w B_\mu \qquad M_Z = \frac{1}{2} v \sqrt{g_1^2 + g_2^2} \qquad (1.7)$$

$$A_\mu \equiv \sin \theta_w W_\mu^3 + \cos \theta_w B_\mu \qquad M_A = 0 \qquad (1.8)$$

$$h \equiv \sqrt{2}(\Re(H^0) - v) \qquad M_h = \sqrt{2\lambda} v \qquad (1.9)$$

where  $\sin \theta_w \equiv g_1 / \sqrt{g_1^2 + g_2^2}$  and  $A_\mu$  represents the massless photon of electromagnetism. Colloquially, it is said the gauge bosons have eaten some of the degrees of freedom of the Higgs to become massive. Three new degrees of freedom exist in the gauge sector corresponding to each gauge boson that has become massive. This corresponds to the three scalar Higgs degrees of freedom which no longer appear in the SM. Furthermore, the masses of the gauge bosons are such that the cross-section for  $W$  pair production does not violate unitarity, as would be true for a theory in which gauge boson masses are naively inserted.

An important quantity in the SM can now be defined

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_w} \qquad (1.10)$$

which is equal to one at tree level and is sensitive to beyond the SM (BSM) physics. Experimental verification of this is one of the indicators of the validity of the standard model.

Fermion couplings to the gauge sector via the covariant derivatives generate charged and neutral currents. The charged current,  $J_\mu^+$ , is characterized by  $V - A$  interactions and couples to  $W^\pm$ . At low energies, where the  $W$  boson is integrated

out, the charged current reduces to the effective Lagrangian responsible for muon and beta decay. The two neutral currents are: the electromagnetic vector current coupled to the photon and the current coupled to the  $Z^0$ . It is important to note that these do not contain flavor changing neutral currents (FCNC). Such processes only arise at loop level with W propagators and are therefore small. Experimental measurements of FCNC are consistent with this framework, another success for the SM.

While fermion mass terms are not gauge invariant Yukawa interaction terms between the fermions and the Higgs are possible. Once the Higgs is shifted by its VEV, fermion mass terms are generated

$$\mathcal{L}_{mass} = \frac{v}{\sqrt{2}} \sum_{ij} (y_{ej}^i \bar{e}'_{Li} e'^j_R + y_{uj}^i \bar{u}'_{Li} u'^j_R + y_{dj}^i \bar{d}'_{Li} d'^j_R + h.c.) \quad (1.11)$$

Neutrinos remain massless because of the absence of right-handed neutrinos, which have not been observed in nature. A given  $3 \times 3$  mass matrix is proportional to the appropriate Yukawa matrices and is not necessarily diagonal. Diagonalizing the mass matrices rotates the fermions from gauge eigenstates to the more physical mass eigenstates. The consequences of this is that while the up-type quark gauge eigenstates can be identified with the mass eigenstate, the down-type quarks are related as follows

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (1.12)$$

where the unprimed fields are mass eigenstates and  $V_{CKM}$  is a  $3 \times 3$  unitary matrix known as the Cabibbo-Kobayashi-Maskawa (CKM) matrix. It is this mixing that

allows for charged flavor changing currents at tree level and ultimately FCNC at loop level.

### 1.1.2 Standard Model Summary and New Physics Wish List

The success of the SM was hinted at in Section 1.1.1 with the  $\rho$  parameter and FCNC but includes more than that. In fact, almost all particle phenomena to date fits within the theoretical framework of the SM. Perhaps the most striking example of this is the discovered of the W and Z bosons at CERN in 1983 with masses of about 80 and 92 GeV respectively. This is consistent with a VEV,  $v = 246.3$  GeV [7, 8]. Furthermore, the SM unifies the weak and electromagnetic forces into the electroweak force, which is a consequences of the spontaneous  $SU(2)_L \times U(1)_Y$  breaking.

The SM is truly satisfying theoretically since it's construction is guided by the basic principle of gauge symmetries. The resulting operators of the SM are not only invariant under the gauge groups but also manifest the accidental symmetries of baryon and lepton number conservation. Furthermore, 't Hooft showed that all operators of dimension four or less in the SM are renormalizable. However, despite all of this phenomenal successes, model builders have been trying to expand on the SM almost since its inception. Most of these have used the principle of gauge symmetries and the Higgs mechanism that have made the SM so successful. The wish list of new physics includes solutions to these issues:

**The Gauge Hierarchy Problem:** Scalar masses are very sensitive to higher

physics scales through quantum corrections. Calculation of such a correction must be proportional to some mass scale squared: the tree level mass of the Higgs or the scale of new physics, which cuts off the loop momentum integral. A quantum correction proportional to the Higgs mass would indicate a symmetry forbidding the Higgs mass: as the Higgs mass went to zero at tree level, quantum corrections would too. However, no symmetry can forbid the term  $m^2|h|^2$ , therefore the quantum corrections to the Higgs mass must be proportional to the scale of new physics. For a Yukawa coupling between the Higgs and the top quark:

$$\mathcal{L}_{yukawa} \supset y_t \bar{t} h t \tag{1.13}$$

a one loop mass correction exists. The contribution from this goes as

$$\delta m_h^2 \sim -\frac{y_t^2}{16\pi^2} \Lambda^2 \tag{1.14}$$

where  $\Lambda$  is the new scale. In the SM, the next scale of physics is possibly the Planck scale,  $\Lambda \sim 10^{18}$  GeV, making this quantum correction quite large.

The physical Higgs mass is given by

$$m_h^2 = m_0^2 + \delta m_h^2 \tag{1.15}$$

where  $m_0^2$  is the bare Higgs mass parameter in the Lagrangian. The physical Higgs mass must be less than 1 TeV in order to solve the unitarity problem of W pair production. Therefore, the sum of  $m_0^2$  and  $\delta m_h^2$  must cancel to one part in  $10^{30}$ . This is an enormous amount of fine-tuning and while it is not an actual theoretical problem, it seems to signal some underlying mechanism at work here. This has been the greatest driving force in BSM model building.

**Neutrino Masses and Oscillations:** Around 2000, oscillations between neutrinos of different gauge eigenstates were experimentally observed. Such oscillations signal massive neutrinos, which are not consistent with the tree level SM Lagrangian. A naive addition of right-handed neutrinos would lead to neutrino Dirac masses comparable to the masses of the other fermions, which are too large (assuming neutrino Yukawa couplings comparable to the Yukawas in the SM).

**Dark Matter:** Observation of galaxy rotation curves [9] show that the velocity of stars does not drop off with radius of the galaxy as expected from Newton's laws. This can be a result of either a modification of Newtonian gravity or a concentration of a large amount of dark matter in the outer regions of the galaxy. Other data exists to confirm the latter hypothesis and shows that dark matter makes up about 23% of the universe. Observations and calculations [10] indicate that dark matter must be: electric and color neutral, non-baryonic, cold (moving at non-relativistic speeds) and stable. The most likely SM dark matter candidate, the neutrino is relativistic and therefore excluded [11].

**Charge Quantization and Higher Symmetries:** One of the consequences of quantum corrections is the running of coupling constants with energy. In the SM, an evolution of the three gauge couplings shows that they almost intersect at about  $10^{16}$  GeV. It is very tantalizing to suppose that at this energy scale, a new gauge symmetry exists which unifies the three forces of the SM into one and whose matter multiplets unify quarks and leptons. Models that assume such symmetries are referred to as grand unified theories (GUTs). The unified group would not include  $U(1)$  factors and therefore its quantum numbers would all be quantized.

Since electric charge would need to be related to these quantum numbers, it too would be quantized instead of depending on the seemingly unphysical hypercharge quantum numbers (which are chosen to cancel triangle gauge anomalies). Other options, such as relating electric charge to more physical quantum numbers or partial unification, also exist.

**Why Electroweak Symmetry Breaking:** While the SM explains how the mechanism of electroweak symmetry breaking (EWSB) takes place, there is no explanation why. Specifically, why is  $\mu^2$  positive in Eq. (1.3) making the Higgs doublet tachyonic thereby destabilizing the trivial vacuum.

**Generations and Hierarchy:** The SM is a very economical model but the inclusion of three similar generations, instead of one, seem to challenge this economy. Furthermore, the mass hierarchy between the different generations is startling. The top to electron mass ratio is on the order of about  $3 \times 10^5$ .

**Strong CP Problem** The QCD Lagrangian allows for a charge parity (CP) violating term

$$\mathcal{L}_\theta = \theta \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} G_{\mu\nu}^a G_{\alpha\beta}^a \quad (1.16)$$

with  $\theta < 10^{-9}$  necessary to agree with the observed electric dipole moment of the neutron. This is unnaturally small and suggests the existence of a symmetry forbidding this term. The most simple solution, an extra  $U(1)_{PQ}$  Peccei-Quinn symmetry leads to an unobserved pseudo-Goldstone boson, the axion.

This thesis focuses on a type of model which addresses at least two of these

issues: supersymmetric left-right models (SLRMs). Supersymmetry (SUSY) is a symmetry between bosons and fermions and provides an elegant solution to the gauge hierarchy problem by canceling quantum contributions to the Higgs mass between fermionic and bosonic loops. It will be discussed in Section 1.3. Under certain circumstances, SUSY also: provides a dark matter candidate, dynamically triggers EWSB and allows for a true intersection of the gauge couplings at around  $10^{16}$  GeV.

Left-right models, discussed in Section 1.2, extend the symmetry of the SM by replacing  $U(1)_Y$  by  $SU(2)_R \times U(1)_{B-L}$ . The Higgs mechanism is then invoked to break  $SU(2)_R \times U(1)_{B-L}$  down to  $U(1)_Y$ . Left-right models are attractive because they put left-handed and right-handed particles on the same footing and have the potential to be invariant under parity. Their particle content naturally includes the right-handed neutrino and can easily facilitate small neutrino masses via the seesaw mechanism. They can also lead to a natural solution to the strong CP problem and relate electric charge to the more physical quantum number: baryon minus lepton number.

## 1.2 Massive Neutrinos and Left-Right models

Neutrinos produced from weak interactions are gauge eigenstates and superpositions of mass eigenstates weighted by a phase factor of  $e^{-iEt}$   $E = \sqrt{m^2 + p^2}$ . If the masses are different, then the probability that a given neutrino with flavor  $i$  will oscillate into a neutrino with flavor  $j \neq i$  is non-zero. Such oscillations were



first proposed by Pontecorvo in the 1960s and experimentally observed in the 2000s. They represent the first empirical evidence for BSM physics.

Neutrino oscillations have been measured for two neutrino sources: solar[12, 13, 14] and atmospheric[15, 16]. Solar electron neutrinos are produced through fusion reactions in the sun but the observed flux through the SNO underground detector was less than that expected from fusion calculations. The result can be explained in terms of neutrino oscillations with a mass difference  $\Delta m_{12}^2 \sim 6 \times 10^{-5} \text{ eV}^2$ . Atmospheric muon neutrinos are produced when cosmic ray interact with the Earth's atmosphere. The flux is isotropic yet the underground Super-Kamiokanda detector found less upwards flux than downwards. Since the upward flux must travel further, it was concluded that the smaller flux is a reflection of oscillations into tau neutrinos. This reflects a mass difference of about  $m_{23} \sim 2.5 \times 10^{-3} \text{ eV}^2$ .

The SM does not allow for tree level masses for the neutrinos to explain these observations. However non-renormalizable operators

$$\mathcal{L}_\nu = c \frac{(LH)^2}{M} \tag{1.17}$$

can exist, where  $M$  is some mass scale. Given that  $10^{-5} \leq c \leq 1$ , as is true for the Yukawa couplings, and the mass bound  $m_{\nu_e} \leq 2 \text{ eV}$  from tritium beta decay then the mass scale for new physics lies in the range

$$10^{10} \text{ GeV} \leq M \leq 10^{15} \text{ GeV} \tag{1.18}$$

although  $c$  and therefore the mass scale can be smaller. A simple extension of the SM, well motivated by neutrino masses, is the left-right symmetric model. The

gauge group is  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  where  $B - L$  is baryon minus lepton number. When the  $g_L = g_R$  is also assumed parity is also a symmetry of the Lagrangian. Left-right theories are a natural framework in which to invoke the seesaw mechanism[17, 18, 19, 20, 21] for small neutrino masses. The particle content and its  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representation is given in Table 1.2. where the

| Fields     | $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ |
|------------|--|
| $Q_L^i$    | $(2, 1, +\frac{1}{3})$                     |
| $Q_R^i$    | $(1, 2, +\frac{1}{3})$                     |
| $L_L^i$    | $(2, 1, -1)$                               |
| $L_R^i$    | $(1, 2, -1)$                               |
| $\Phi_1$   | $(2, 2, 0)$                                |
| $\Delta_L$ | $(3, 1, 2)$                                |
| $\Delta_R$ | $(1, 3, 2)$                                |

Table 1.2: Representation assignment for the fermion and Higgs fields of the left-right model where the superscript  $i = 1..3$  represents generation.

$\Delta_R$  is a right-handed triplet which facilitates  $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$  while the bidoublet  $\Phi$  plays the role of the SM Higgs inducing  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ .

The VEV structure is:

$$\langle \Phi_1 \rangle = \left\langle \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \right\rangle = \frac{1}{\sqrt{2}} e^{i\alpha} \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \quad (1.19)$$

$$\langle \Delta_{L,R} \rangle = \left\langle \left( \begin{array}{cc} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{array} \right)_{L,R} \right\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \quad (1.20)$$

with  $v_L < \kappa' \ll \kappa \ll v_R$ . Note that in general  $v_L$  does not have to be small since terms such as  $\text{Tr} \Delta_L \phi \Delta_R \phi$  must be included in the potential which will source  $\Delta_L$ , however large  $v_L$  will lead to  $\rho \neq 1$ . Therefore  $v_L$  is highly constrained and must be less than a few GeV.

### 1.2.1 The Seesaw Mechanism In Left-Right Models

The seesaw mechanism arises through Yukawa couplings in left-right models

$$\begin{aligned} \mathcal{L}_{Yukawa} \supset & y_{Q_j}^{ia} \bar{Q}_{Li} \Phi_a Q_R^j + y_{L_j}^{ia} \bar{L}_{Li} \Phi_a L_R^j \\ & + i f_{ij} (L_L^{iT} C^{-1} \Delta_L L_L^j + L_R^{iT} C^{-1} \Delta_R L_R^j) + h.c. \end{aligned} \quad (1.21)$$

where  $a = 1, 2$  and  $\Phi_2 \equiv \tau_2 \Phi_1^* \tau_2$ . The terms in the last line have the same coupling due to parity. Once the Higgses are VEVed, Eq. (1.21) produces two types of mass terms for the neutrinos: Dirac with  $m_D \sim y_L \kappa$  and Majorana with  $M_R \sim f v_R$  and  $m_L \sim f v_L$  leading to the mass matrix

$$M_\nu = \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} \quad (1.22)$$

where the elements of this matrix are  $3 \times 3$  in generation space. The eigenvalues are

$$m_\nu = m_L - m_D^T M_R^{-1} m_D = f v_L - \kappa y_L^T \frac{1}{f v_R} \kappa y_L \quad (1.23)$$

$$m_N = M_R = f v_R \quad (1.24)$$

$m_\nu$  is naturally very small given a large  $M_R$  and is the mass of the observed mostly left-handed neutrino.  $N$  is the mostly right-handed, heavy neutrino. Now the right-handed scale  $v_R$  can be identified with the new mass scale  $M$  in Eq. (1.17), the mass of the mostly right-handed neutrino and the seesaw scale. This is the type I seesaw mechanism.

### 1.2.2 Motivation and Consequences of Left-Right Models

Left-right models are motivated by more than just neutrino masses. Theoretically, gauging  $B - L$  is well motivated, since it is the only anomaly free  $U(1)$  symmetry of the Lagrangian, aside from hypercharge, once the right-handed neutrino is included. The upshot of this is that the electric charge is now given by

$$Q_{em} = I_{3L} + I_{3R} + \frac{1}{2}(B - L) \quad (1.25)$$

which is more physically significant than the SM form which depends on hypercharge quantum numbers. Furthermore, parity symmetry is aesthetically appealing since it puts left-handed and right-handed fields on equal footing. It also solves the strong CP problem by forbidding the strong CP violating term linear in  $\theta$  since  $\theta \rightarrow -\theta$  under parity.

Utilization of the seesaw mechanism via left-right models has a variety of interesting experimental consequences. The Majorana mass term for neutrinos violates lepton number by  $\Delta L = 2$ . This ingredient allows for interesting experimental effects such as: neutrinoless double beta decay and lepton flavor violating processes:  $\mu^-, \tau^- \rightarrow e^- e^+ e^-$ , muon-electron conversion and muonium-antimuonium

oscillation— $\mu^+e^- \rightarrow \mu^-e^+$ . Meanwhile, baryon number continues to be a good symmetry below the right-handed scale guaranteeing a stable proton.

Right-handed currents lead to possible CP violation in the weak sector. Limits on new CP violation in various meson systems can be used to put a lower bound on the mass of the right-handed  $W_R$

$$m_{W_R} \sim \frac{1}{2}g_2v_R \gtrsim 2\text{TeV} \quad (1.26)$$

where the exact bound is dependent on model details. This translates into a lower bound on the right-handed scale itself and has implications in the neutrino sector.

### 1.3 Supersymmetry

Many useful reviews have been written on SUSY [22, 23, 24], which can be considered the most pleasing solution to the gauge hierarchy problem. It's success is easy to understand since quantum corrections to a scalar mass stemming from fermionic couplings are negative while those from the scalar couplings are positive. Therefore, if there were some way to relate these couplings to each other in the correct way they will cancel and the gauge hierarchy problem would be solved. This is the case when the Lagrangian is invariant under a symmetry whose generators transform fermions to bosons and vice-versa. Such a symmetry must necessarily have fermionic generators, but its form is highly constrained by the Coleman-Mandula theorem which states that the most general Lie algebra cannot contain such operators. It is possible to side step this theorem by considering so called graded Lie

algebras. The simplest and the one that will be considered here is:

$$\{Q_\alpha, Q_\alpha^\dagger\} = 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \quad (1.27)$$

$$\{Q_\alpha, Q_\beta\} = \{Q_\alpha^\dagger, Q_\beta^\dagger\} = 0 \quad (1.28)$$

$$[Q_\alpha, P_\mu] = [Q_\alpha^\dagger, P_\mu] = 0 \quad (1.29)$$

known as  $\mathcal{N} = 1$  SUSY, where  $\mathcal{N}$  represents the number of fermionic generators, in this case just  $Q$ .  $P^\mu$  is simply the momentum of a supermultiplet and since it commutes with the SUSY generators  $P^2 = m^2$  must be equal for each member of a multiplet. Finally,  $\sigma^\mu \equiv (\mathbb{1}, \vec{\sigma}_{Pauli})$ . It can be proved that a given supermultiplet must have the same number of bosonic and fermionic degrees of freedom.

SUSY can be viewed as an extension of four dimensional space time by four fermionic or Grassman dimensions,  $\theta$  and its charge conjugate  $\bar{\theta}$ . Each have two degrees of freedom and dimension  $-\frac{1}{2}$ . Superfields can then be expanded in terms of these fermionic dimensions. Note that because of the fermionic properties  $\theta^3, \bar{\theta}^3 = 0$ . Two types of superfields can be defined: chiral ( $\Phi$ ) and vector ( $V$ )

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F \quad (1.30)$$

$$V = -\theta\sigma_\mu\bar{\theta}A^\mu + 2i\theta^2\bar{\theta}\lambda^\dagger - 2i\bar{\theta}^2\theta\lambda\theta^2\bar{\theta}^2 D$$

where  $\phi$  is a complex scalar, its superpartner is  $\psi$ , a Weyl fermion,  $F$  is a non-propagating complex scalar needed for off-shell matching of degrees of freedom,  $A^\mu$  is a gauge boson,  $\lambda$  is  $A_\mu$ 's spin half superpartner and  $D$  is non-propagating real scalar. These fields are in the Wess-Zumino gauge.

Integration over Grassman variables is possible and is fairly straight forward

with:

$$\begin{aligned}\int d\theta^2\theta^2 &= \int d\bar{\theta}^2\bar{\theta}^2 = 1 \\ \int d\theta^2 &= \int d\bar{\theta}^2 = 0\end{aligned}\tag{1.31}$$

This type of integration can then be used to pick up certain components of products of superfields, say the  $\theta^2\bar{\theta}^2$  component of  $\Phi^\dagger\Phi$ . This is useful since the highest component of  $\theta$  and  $\bar{\theta}$  in a given product of superfields transforms into a total space-time derivative leaving the action invariant. Therefore, a general SUSY Lagrangian is given by:

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta} \mathcal{K}(\Phi, \Phi^\dagger, V) + \left( \int d^2\theta (\mathcal{W}_\alpha \mathcal{W}^\alpha + W(\Phi)) + h.c. \right)\tag{1.32}$$

$$\mathcal{K}(\Phi, \Phi^\dagger, V) = \Phi^\dagger\Phi + \dots\tag{1.33}$$

$$W(\Phi) = M_{ij}\Phi^i\Phi^j + Y_{ijk}\Phi_i\Phi_j\Phi_k\tag{1.34}$$

$$\mathcal{W}_\alpha = -i\lambda_\alpha + \left( \delta_\alpha^\beta D - \frac{1}{2}i(\sigma^\mu\sigma^\nu)_\alpha^\beta F_{\mu\nu} \right) \theta_\beta + \theta^2\sigma_{\alpha\dot{\alpha}}^\mu\partial_\mu\lambda^{\dot{\alpha}}\tag{1.35}$$

The Kahler Potential,  $K(\Phi, \Phi^\dagger, V)$ , is real and has mass dimensions of two. The superpotential,  $W(\Phi)$ , is dimension three and is holomorphic meaning it is a function of  $\Phi$  and not  $\Phi^\dagger$ . As suggested by Eq. (1.32), the superpotential generates Yukawa and mass terms. The SUSY field strength is contained in  $\mathcal{W}_\alpha\mathcal{W}^\alpha$ .

Auxiliary fields such as  $F$  and  $D$  can be replaced in the Lagrangian using their equations of motion

$$-F_i^* = \frac{\partial W(\phi_j)}{\partial \phi_i}\tag{1.36}$$

$$D_A^a = -g_A\phi^{i*}T^a\phi_i$$

where  $W$  is understood to be a function of the scalar components of the superfields and not the superfields themselves. For the  $D$ -term,  $T^a$  are the generators corresponding to the representation of  $\phi$  and  $A$  stands for a specific gauge group so that for  $D_3^a$  is the  $D$  term for the  $a^{\text{th}}$  generator of  $SU(3)_c$ . The scalar potential in an unbroken SUSY model is then given completely in terms of  $F$ - and  $D$ -terms

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_{a,A} (D_A^a)^2 \quad (1.37)$$

which can be derived from Eq. (1.32). Yukawa interactions between the scalar and fermion components of the different chiral fields are given by:

$$\mathcal{L}_{Yukawa} = -\frac{1}{2} \sum_{ij} \left( \frac{\partial^2 W(\phi_k)}{\partial \phi_i \partial \phi_j} \psi^i \psi^j \right) - \sqrt{2} \sum_{A,a,i} g_A \phi^{i*} T^a \psi_i \lambda^a + h.c. \quad (1.38)$$

where in the first term, the superpotential is again a function of the scalar fields as in Eq. (1.36) and the second term is a sort of superpartner to the D term in Eq. (1.36).

Armed with these tools it is now possible to discuss the minimal supersymmetric standard model (MSSM).

### 1.3.1 The Minimal Supersymmetric Standard Model

The particle content of the MSSM in terms of superfields and representations under the SM gauge group are given in Table 1.3, with  $i = 1..3$  indicating generation. Each superfield is composed of a complex scalar and a two component Weyl fermion, so that MSSM has about two times as many particles as the SM. All Weyl fermions are represented as left-handed fermions, *e.g* the fermion component of  $u^c$  is



the charge conjugate of the right-handed Weyl fermion  $u$ , itself a left-handed field. In terms of nomenclature, SM fermion superpartner are called sfermions: squarks, sleptons, stops, while SM boson superpartners are called bosinos: Higgsino, gluino, wino and bino. Collectively the superpartners of the gauge bosons are called gauginos.

| Fields  | $SU(3)_c \times SU(2)_L \times U(1)_Y$ | Superpartner    |
|---------|--|-----------------|
| $Q^i$   | $(3, 2, +\frac{1}{3})$                 | $\tilde{Q}^i$   |
| $u_i^c$ | $(3, 1, -\frac{4}{3})$                 | $\tilde{u}_i^c$ |
| $d_i^c$ | $(3, 1, +\frac{2}{3})$                 | $\tilde{d}_i^c$ |
| $L^i$   | $(1, 2, -1)$                           | $\tilde{L}^i$   |
| $e_i^c$ | $(1, 1, +2)$                           | $\tilde{e}_i^c$ |
| $H_u$   | $(1, 2, +1)$                           | $\tilde{H}_u$   |
| $H_d$   | $(1, 2, -1)$                           | $\tilde{H}_d$   |
| $g$     | $(8, 1, 0)$                            | $\tilde{g}$     |
| $W$     | $(1, 3, 0)$                            | $\tilde{W}$     |
| $B$     | $(1, 1, 0)$                            | $\tilde{B}$     |

Table 1.3: Particle content and representation assignment for the MSSM superfields where the superscript  $i = 1..3$  represents generation. The superfields are composed of complex scalars and left-handed Weyl fermions

Aside from the doubling of the particle content due to the introduction of superpartners, the MSSM also introduces a new Higgs superfield. This is necessary

for anomaly cancellation due to the Higgsino. The Higgs subscript indicates which couples to the up and which to the down sector.

The most general superpotential based on this particle content can be broken up into three parts

$$W_{MSSM} = y_{ui}^j Q^i H_u u_j^c + y_{di}^j Q^i H_d d_j^c + y_{ei}^j L^i H_d e_j^c + \mu H_u H_d \quad (1.39)$$

$$W_{\mathcal{L}} = \frac{1}{2} \lambda_{ij}^k L^i L^j e_k^c + \lambda_{ij}^{\prime k} L^i Q^j d_k^c + \mu'_i L^i H_u \quad (1.40)$$

$$W_{\mathcal{B}} = \lambda^{ijk} u_i^c d_j^c d_k^c \quad (1.41)$$

where  $W_{\mathcal{L}}$  and  $W_{\mathcal{B}}$  break lepton and baryon number respectively. This is one of the consequences of having extra degrees of freedom in the SUSY model. Such terms are potentially highly disagreeable with experiments, especially the combination of  $\lambda^{ijk}$  and  $\lambda_{ij}^{\prime k}$ , which lead to rapid proton decay. The current lower bound on the proton lifetime is  $1.9 \times 10^{29}$  years. It is important to note though, that even if the proton lifetime is under control,  $W_{\mathcal{L}}$  would lead to LSP decay and therefore no dark matter candidate.

In order to avoid such problems the MSSM is defined to contain a discrete symmetry in addition to the SM gauge groups. This symmetry can be viewed as a symmetry which commutes with SUSY, matter parity, or one which does not,  $R$ -parity. The results are equivalent. Charges under  $R$ -parity are

$$P_R = (-1)^{3(B-L)+2s} \quad (1.42)$$

where  $s$  the spin of the particle. This symmetry forbids  $W_{\mathcal{L}}$  and  $W_{\mathcal{B}}$  while allowing  $W_{MSSM}$  and has interesting phenomenological consequences. One of these is that all

SUSY partners have a charge of  $-1$  while SM fields have a charge of  $+1$ . Since terms in the Lagrangian must have a charge of  $+1$  to be invariant, SUSY particles must appear in even numbers in any given term. SUSY particles must then decay into an odd number of SUSY particles. Decays of the lightest supersymmetric particle (LSP) to other SUSY particles is kinematically forbidden making the LSP stable and a dark matter candidate. This is another pleasing feature of SUSY models.

Despite its benefits, SUSY is incompatible with nature since spin 0 versions of the electrons (or in fact any fundamental scalar fields) have never been observed. The goal then is to build models with broken SUSY in which the superparticles are more massive than their superpartners. SUSY breaking can be achieved in one of two ways:  $D$ -term breaking,  $\langle D \rangle \neq 0$ , or  $F$ -term breaking,  $\langle F \rangle \neq 0$ . Utilization of these techniques with MSSM fields will shift superparticle masses away from the masses of their SM superpartners but these masses still observe the following sum rule

$$STr(m^2) \equiv (-1)^s (2s + 1) Tr(m_s^2) = 0 \quad (1.43)$$

where  $STr$  is the supertrace and  $s$  is the spin. Given the conservation of lepton flavor, it is reasonable to assume that the electron sector decouples so that

$$m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2 = 2m_e^2 \quad (1.44)$$

where  $\tilde{e}_L$  ( $\tilde{e}_R$ ) is the superpartner to the left-handed (right-handed) electron. This is again in violation of empirical data. However this sum rule does not preclude the possibility of SUSY breaking in a hidden sector, not accessible to experiments. SUSY breaking can then be communicated indirectly to the visible sector. The resulting

SUSY Lagrangian must contain only soft parameters (dimension 1 or greater) or the gauge hierarchy problem would be reintroduced. Also, parameters should on the order of a TeV, the SUSY scale  $m_{SUSY}$ , otherwise the fine-tuning problem starts to creep back in.

It is possible to parameterize such a Lagrangian in a general way

$$\begin{aligned}
\mathcal{L}_{Soft} = & -\frac{1}{2} \left( M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{g} \tilde{g} + c.c. \right) \\
& - \left( a_u \tilde{Q} H_u \tilde{u}^c + a_d \tilde{Q} H_d \tilde{d}^c + a_e \tilde{L} H_d e^c + c.c. \right) \\
& - \tilde{Q}^\dagger m_{\tilde{Q}}^2 \tilde{Q} - u^c m_{\tilde{u}^c}^2 \tilde{u}^{c\dagger} - \tilde{d}^c m_{\tilde{d}^c}^2 \tilde{d}^{c\dagger} - \tilde{L}^\dagger m_{\tilde{L}}^2 \tilde{L} \\
& - \tilde{e}^c m_{\tilde{e}^c}^2 \tilde{e}^{c\dagger} - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 - (b H_u H_d + c.c.)
\end{aligned} \tag{1.45}$$

The first line contains mass terms for the gauginos, the second trilinear scalar couplings (dimension one) analogue to the Yukawa couplings in the superpotential, the third and fourth have mass terms for all the scalars while the last term is a bilinear term of dimension two analogue to the Higgs  $\mu$  term in the superpotential. While generation indices have been suppressed, it is important to note that all of the trilinear  $a$ -terms and all of the sfermion and slepton mass terms are  $3 \times 3$  matrices in generation space. Therefore, Eq. (1.45) has 105 free parameters negating the economy of SUSY. Furthermore, many of these parameters are severely constrained by data on FCNC and violation of (CP). In order for SUSY to be theoretically satisfying, there should be a dynamical method of hidden sector SUSY breaking such that the resulting SUSY breaking Lagrangian relies on only a few parameters and which predicts the dangerous parameters to be close to zero. This has been the focus of many SUSY efforts.

### 1.3.2 Methods of SUSY Breaking

There are currently three popular scenarios of hidden sector SUSY breaking:

1. gravity mediated SUSY breaking,
2. gauge mediated SUSY breaking (GMSB)
- and 3. anomaly mediated SUSY breaking (AMSB).

- **Gravity mediation** is based on gravity strength interactions (suppressed by  $M_P$ ) between the hidden and visible sector. Mass terms stem from terms of the form

$$\int d^4\theta \frac{z_j^i}{M_P^2} X^\dagger X Q_i^\dagger Q^j \quad (1.46)$$

in the Kahler potential,  $d^4\theta \equiv d^2\theta\bar{\theta}$ . Here X is a hidden sector field with a SUSY breaking VEV  $\langle F_X \rangle \neq 0$ . Therefore

$$m_{susy} \sim \frac{\langle F_X \rangle}{M_P} \quad (1.47)$$

so that  $\langle F_X \rangle \sim 10^{11}$  GeV. This has potential issues with FCNC and CP violation due to the fact that  $z_j^i$  need not be diagonal in the basis of quark mass eigenstates and could lead to mass terms such as  $m_{s\bar{d}}^2 \neq 0$ . Such a term would lead to the  $K^0 - \bar{K}^0$  mixing and must obey

$$\frac{m_{s\bar{d}}^2}{m_{\tilde{Q}}^2} \lesssim 10^{-3} \left( \frac{m_{\tilde{Q}}}{500 GeV} \right) \quad (1.48)$$

Extra simplifying assumptions can be made with the resulting boundary con-

ditions at  $M_P$  for the soft parameters:

$$M_3 = M_2 = M_1 = m_{1/2} \quad (1.49)$$

$$m_{\tilde{Q}}^2 = m_{\tilde{u}^c}^2 = m_{\tilde{d}^c}^2 = m_{\tilde{L}}^2 = m_{\tilde{e}^c}^2 = m_{\tilde{u}^c}^2 = m_{H_u}^2 = m_{H_d}^2 = m_0^2$$

$$a_u = A_0 y_u, a_d = A_0 y_d, a_e = A_0 y_e$$

This scenario is known as mSUGRA and it is the mostly widely phenomenologically studied SUSY breaking model. The number of free parameters has been decreased from 105 to three.

- **GMSB** proposes that the messenger sector is composed of fields charged under the SM gauge group. These fields couple directly to the hidden sector thereby gaining SUSY breaking masses. Once these fields are integrated out of the Lagrangian, they communicate the SUSY breaking to the visible sfermions (gauginos) via two loop (one loop) diagrams. The important quantity here is the ratio

$$\Lambda = \frac{\langle F_X \rangle}{M_{mess}} \quad (1.50)$$

where  $M_{mess}$  is the mass scale of the messengers and  $\langle F_X \rangle \neq 0$  yields a SUSY violating mass term. The boundary conditions for the soft terms at the messenger scale in the minimal model are

$$M_a = \frac{\alpha_a}{4\pi} \Lambda \quad (1.51)$$

$$m_{\phi_i}^2 = 2\Lambda^2 \left( \left( \frac{\alpha_a}{4\pi} \right)^2 C_a(i) \right) \quad (1.52)$$

$$A = 0 \quad (1.53)$$

where  $C_a(i)$  are the Casimir invariants for the scalar  $\phi$ . Because SM currents do not violate flavor, these mass terms won't either. These terms depend on two free parameters.

The limitations of the GMSB is related to the gravitino mass which must go as

$$m_{3/2} = \frac{\langle F_X \rangle}{M_P} \quad (1.54)$$

The messenger scale must be much less than  $M_P$  where possible flavor violating physics will introduce flavor violation into Eq. (1.51). Comparing Eq. (1.54) to Eq. (1.50) shows that  $m_{3/2} \ll \Lambda \sim 16\pi m_{susy}$ . Therefore, the gravitino will be the LSP and may lead to cosmological problems. Furthermore, it is too relativistic to be a dark matter candidate.

- **AMSB** is theoretically very rich and will be discussed in detail in Chapter 3.

For now, the important thing to note is that SUSY breaking in the visible sector occurs because of the breaking of scale invariance at loop level. Therefore, the SUSY breaking parameters are functions of the low energy beta and gamma functions as well as the mass parameter  $F_\phi$

$$m_{\phi_i}^2 = -\frac{1}{4}|F_\phi|^2 \left( \frac{\partial \gamma_i}{\partial g_a} \beta_{g_a} + \left( \frac{\partial \gamma_i}{\partial y^{jkl}} \beta_{y^{jkl}} + c.c \right) \right) \quad (1.55)$$

$$a^{ijk} = -F_\phi \beta_{y^{ijk}} \quad (1.56)$$

$$M_a = \frac{\alpha_a b_a}{4\pi} F_\phi \quad \text{no sum over } a \quad (1.57)$$

where  $g_a$  represents all the gauge couplings,  $y^{jkl}$  represents all the Yukawa couplings and sums over repeated indices are assumed unless otherwise stated.

Unlike mGSMB and mSUGRA, these relationships apply at any scale—they are not boundary conditions. Given the loop suppression factor,  $F_\phi$  should be of order 30 TeV to produce a viable superparticle spectrum. Note that since the  $\beta$ - and  $\gamma$ - functions are dependent on known low energy parameters, AMSB has only one free parameter  $F_\phi$ , making it very predictable.

A quick test of Eq. (1.55) for the right-handed selectron uncovers a serious problem. The Yukawa couplings of the selectron are negligible, therefore

$$\begin{aligned}
m_{\tilde{e}^c}^2 &= -\frac{1}{4}|F_\phi| \frac{\partial \gamma_{e^c}}{\partial g_1} \beta_{g_1} & (1.58) \\
&= -\left(\frac{|F_\phi|}{16\pi^2}\right)^2 (C_a g_a) (b_a g_a^3) \\
&= -\left(\frac{|F_\phi|}{16\pi^2}\right)^2 \left(\frac{6}{5}g_1\right) \left(\frac{33}{5}g_1^3\right) \\
&= -\left(\frac{|F_\phi|}{16\pi^2}\right)^2 \left(\frac{198}{25}g_1^4\right)
\end{aligned}$$

where the MSSM beta and gamma functions are given in Appendix A.3. The selectron mass squared is therefore negative, the selectron tachyonic and therefore the charge conserving vacuum is unstable—a clear violation of the empirical world. This will be true for all the slepton since their mass expression are dominated by the gauge contributions and because their gauge slope parameter,  $b$  is positive (those gauge groups are UV enslaved). This will not be true for the squarks since  $SU(3)_c$  is asymptotically free and therefore  $b_3 < 0$ .

This situation is difficult to remedy because of the lack of free parameters in AMSB. Still there have been many proposed solutions to this problem, with the most well studied one being minimal AMSB (mAMSB) where a universal



parameter  $m_0$  is added to all the soft mass terms at some higher scale (two free parameters in this case). A model to solve this problem will be proposed in Chapter 4 in the framework of deflected AMSB and based on the work in Chapter 2.

### 1.3.3 Minimal Supersymmetric Standard Model Details

If the MSSM is to have the potential to replace the SM, it is crucial that it break electroweak symmetry. To determine if this happens, it is necessary to examine the scalar potential for the neutral Higgses. This is the sum of the SUSY breaking contributions and the SUSY conserving ones

$$\begin{aligned}
V &= V_F + V_D + V_{soft} & (1.59) \\
&= |F|^2 + \frac{1}{2}(D)^2 + V_{soft} \\
&= (m_{H_u}^2 + \mu^2) |H_u^0|^2 + (m_{H_d}^2 + \mu^2) |H_d^0|^2 - (bH_u^0 H_d^0 + h.c.) \\
&\quad + \frac{1}{8}(g_1^2 + g_2^2) (|H_u^0|^2 - |H_d^0|^2)^2
\end{aligned}$$

In order for electroweak symmetry breaking to occur, it is necessary for a linear combination of Higgses to be tachyonic. This is equivalent to a negative determinant for the Higgs mass matrix

$$(m_{H_u}^2 + \mu^2) (m_{H_d}^2 + \mu^2) - b^2 < 0 \quad (1.60)$$

This results for either large  $b$  or for either negative  $(m_{H_u}^2 + \mu^2)$  or  $(m_{H_d}^2 + \mu^2)$  but not both. As it turns out  $m_{H_u}^2$  is usually driven negative by its renormalization group equation (RGE), which can potentially make  $(m_{H_u}^2 + \mu^2)$  negative. This is

known as radiative EWSB. In this way, the MSSM can provide a solution to the question of why EWSB posed in Section 1.1.2.

Using the definitions

$$\langle H_u^0 \rangle \equiv v_u \quad \langle H_d^0 \rangle \equiv v_d \quad \tan \beta \equiv \frac{v_u}{v_d} \quad v_u^2 + v_d^2 = v^2 \sim (246.3 \text{ GeV})^2 \quad (1.61)$$

the minimization of the potential yields the conditions

$$\mu^2 = -\frac{1}{2}M_Z^2 - \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{1 - \tan^2 \beta} \quad (1.62)$$

$$b = \frac{\sin 2\beta}{2}(2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2) \quad (1.63)$$

As in the SM, EWSB will give mass to the  $W^\pm$  and  $Z^0$  bosons meaning three of the Higgs degrees of freedom must be eaten. Since at the start there were eight, there should be five physical Higgs degrees of freedom left. They are given in Table 1.4. The appearance of the gauge boson masses in the mass expressions in Table 1.4 is

| Field   | Linear Combination  | Mass Squared   |
|---------|---|--|
| $A^0$   | $\sqrt{2}(c_\beta \Im(H_u^0) + s_\beta \Im(H_d^0))$                 | $2b/s_{2\beta}$  |
| $H^\pm$ | $c_\beta H_u^\pm + s_\beta H_d^{\mp*}$                              | $m_{A^0}^2 + M_W^2$  |
| $H^0$   | $\sqrt{2}(s_\alpha(\Re(H_u^0) - v_u) + c_\alpha(\Re(H_d^0) - v_d))$ | $\frac{1}{2}\left(\bar{m}^2 + \sqrt{\bar{m}^4 - 4M_Z^2 m_{A^0}^2 c_{2\beta}^2}\right)$ |
| $h^0$   | $\sqrt{2}(c_\alpha(\Re(H_u^0) - v_u) - s_\alpha(\Re(H_d^0) - v_d))$ | $\frac{1}{2}\left(\bar{m}^2 - \sqrt{\bar{m}^4 - 4M_Z^2 m_{A^0}^2 c_{2\beta}^2}\right)$ |

Table 1.4: Physical Higgs degrees of freedom in the MSSM with  $\bar{m}^2 \equiv m_{A^0}^2 + M_Z^2$ ,

$c_\theta (s_\theta) \equiv \cos \theta (\sin \theta)$  and at tree level  $\frac{s_{2\alpha}}{s_{2\beta}} \equiv -\frac{\bar{m}^2}{m_{H^0}^2 - m_{h^0}^2}$

not surprising since Eq. (1.6) states that post symmetry breaking Higgs masses are related to the quartic coupling times the VEV. In the MSSM, the quartic couplings

are the gauge couplings whose product with the VEV yield the gauge boson masses. This leads to a somewhat uncomfortable situation since the expression for the mass of  $h^0$  in Table 1.4 implies

$$m_{h^0} < |\cos(2\beta)| M_Z < 92 \text{ GeV} \quad (1.64)$$

while current LEP II bounds state that  $m_{h^0} > 114.4 \text{ GeV}$ . Fortunately, since SUSY is broken, there are still quadratic corrections to the Higgs. The most relevant term is

$$\delta m_{h^0} = \frac{3}{4\pi^2} m_t^2 y_t^2 \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \quad (1.65)$$

which can lift the Higgs mass above the LEP II bound.

Sfermions and their masses are discussed in Appendix B.1.1. Their spectrum is very dependent on the SUSY breaking mechanism although in general it is assumed that squarks are heavier than sleptons. Bosino masses are noted in Appendix B.1.2. These are all also dependent on the SUSY breaking scenario. The lightest bosino is usually the bino and the default LSP in most models. The LSP is important in collider phenomenology since SUSY particles will cascade decay to it.

### 1.3.4 Summary and Issues

Despite its highlights: solving the gauge hierarchy problem, radiative EWSB, dark matter and gauge coupling unification at  $10^{16}$ , SUSY also has some issues:

**SUSY Breaking:** What is the correct method of SUSY breaking and specifically, how is the lack of new flavor and CP violation explained in the MSSM.

**The  $\mu$  Problem:** The Higgs mass parameter,  $\mu$ , in Eq. (1.39) must be of order 1 TeV, around the SUSY scale, for correct EWSB, Eq. (1.62). However, mass parameters in a superpotential are expected to coincide with other possible SUSY conserving scales such as the GUT scale,  $10^{16}$  GeV, or the Planck scale,  $10^{18}$  GeV. This coincidence of  $\mu \sim m_{susy}$  is known as the  $\mu$  problem.

**The Little Hierarchy Problem:** At tree level, the Higgs mass has an upper bound of the  $Z$  mass, Eq. (1.64), lower than the LEP II bound of 114.4 GeV. Radiative corrections can push the physical mass above this bound and depend on the log of the stop mass, Eq. (1.65). But the stop mass also tends to increase the value of  $|m_{H_u}|$  by feeding into it's RGE. At the large  $\tan\beta$  limit where the bound in Eq. (1.64) is saturated, Eq. (1.65) becomes:

$$|\mu|^2 - m_{H_u}^2 \sim -\frac{1}{2}M_Z^2 \quad (1.66)$$

Therefore, the fine-tuning in the MSSM is approximately calculated as

$$F \sim \frac{M_Z^2}{2m_{H_u}^2} \sim 2\% \quad (1.67)$$

for a Higgs mass above the LEP II bound. This means that  $\mu$  and  $m_{H_u}$  must cancel up to two parts in one hundred, large enough to cause some worry.

**$R$ -Parity:** The need for an extra discrete symmetry is unsatisfying.

## 1.4 Organization

Of the SM issues mentioned in Section 1.1.2, the most striking theoretical issue is the gauge hierarchy problem while the existence of neutrino masses is the

only empirical particle physics phenomena which the SM fails to explain. It is therefore well motivated to analyze SLRM. However, even in these models, the right-handed scale is arbitrary and at best, a wide range can be guessed at based on neutrino mass bounds Eq. (1.18). Chapter 2 examines a minimal SLRM with a  $Z_4$  discrete symmetry and shows that the right-handed scale is given by the geometric mean of two well motivated scales: the Planck scale and the SUSY scale— $v_R \sim \sqrt{M_P \times m_{SUSY}}$ . It also generates a  $\mu$  term of the right order solving the  $\mu$  problem.

SUSY breaking can proceed through any of the known mechanisms in this type of minimal model. However, the model has interesting consequences in AMSB (AMSB is reviewed in Chapter 3). The shallow potential of this model introduces new SUSY breaking at the right-handed scale and therefore the AMSB trajectories are deflected below that scale. The low energy model has new Yukawa couplings to the right-handed sleptons which can be large enough to save the right-handed sleptons from their tachyonic fate. The left-handed sleptons are saved by partially decoupled  $D$ -terms, which are decoupled completely in AMSB models. If the model is extended by a singlet so that the low energy theory is the next-to-minimal SUSY standard model (NMSSM) instead of the MSSM, then the problem associated with EWSB problem can also be solved. All of these properties can be seen as a consequence of neutrino masses and will be discussed in Chapter 4.

## Chapter 2

### Predicting the Seesaw Scale

#### 2.1 Motivation

One of the simplest ways to understand small neutrino masses is to use the seesaw mechanism where the fact that the right-handed neutrino is a standard model singlet allows a large Majorana mass,  $M_R$ , for it leading to a small effective Majorana mass for the left-handed neutrino given by  $m_\nu \sim -\frac{m_D^2}{M_R} \ll m_{e,u,d}$ . This is discussed in Section 1.2.1 in the context of left-right models. While left-right models are not necessary for implementation of the seesaw mechanism, they do answer the following two questions associated with the seesaw mechanism

- Is there a natural way for the right-handed neutrino to appear in the theory rather than just being added to the standard model by hand?
- How large is the seesaw scale  $M_R$ ? In particular, why is  $M_R \ll M_P$  as required by observations?

The answers to these questions are connected. To answer the second question, one may start with the observation that the Majorana masses of the right-handed neutrinos break the  $B-L$  symmetry, and if  $B-L$  is a gauge symmetry of nature[25], then that will explain why  $M_R \ll M_P$ . The right-handed neutrino is necessary in  $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$  ( $G_{211}$ ) or the left-right symmetric group  $SU(2)_L \times$

$SU(2)_R \times U(1)_{B-L}$  due to anomaly cancellation. So,  $B - L$  naturally explains both the seesaw scale and the presence of  $\nu_R$ .

None of these considerations, however, indicate the magnitude of the seesaw scale,  $M_R$  and experimental considerations only give a very rough range,  $10^{10}\text{GeV} \leq M_R \leq 10^{15}\text{GeV}$ , see Eq. (1.18). The higher value is tantalizingly close to the conventional GUT scale in SUSY theories. As a result, in GUT theories such as  $SO(10)$ , one can identify  $M_R$  with the scale of grand unification. Yet such theories allow many different values for  $M_R$  while remaining consistent with the grand unification of couplings[26, 27, 28]. The choices involved in the symmetry breaking and the choice of Higgs multiplets prevents this connection between  $M_R$  and grand unification from being unique. Nonetheless, simple one or two step symmetry breaking  $SO(10)$  models have provided a compelling class of models for studying the consequences of the seesaw mechanism for neutrino masses and mixings and need to be taken very seriously.

This chapter takes an alternative point of view to the understanding of neutrino masses by making a minimal extension of the SM to the SLRM [29, 30, 31, 32]. It will show that if in addition to this, a discrete  $Z$ -symmetry is added, then the model predicts  $M_R \simeq \sqrt{M_{SUSY}M_P} \sim 10^{11}$  GeV. The reasoning for this is straight forward: the  $Z$ -symmetry prohibits bilinear Higgs terms from the superpotential but allows quartic terms. This would combine with the soft SUSY breaking terms to produce a potential of the form:

$$V = -m_{soft}^2|\phi|^2 - \frac{m_{soft}}{M_P}\phi^4 + \frac{1}{M_P^2}|\phi|^6 \quad (2.1)$$

It then follows that  $\langle\phi\rangle\sim\sqrt{m_{soft}M_P}$ — which is of the right order of magnitude to be the seesaw scale. This is the main result of this chapter and is of interest since it determines the seesaw scale from first principles without the assumption of grand unification.

Before filling in the details of this discussion, a brief introduction to SLRM will be given in Section 2.2. Specifically it will be shown that in the class of models considered here,  $R$ -parity is an automatic symmetry of the superpotential. Section 2.3, will present the minimal  $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$  model where the seesaw scale is predicted as the geometric mean of the weak scale and the Planck scale. It will also analyze the ground state of the theory since including both  $Z$ -symmetry and  $R$ -Parity can be dangerous. The  $Z$ -symmetry restricts the number of parameters in the superpotential and  $R$ -parity allows a stable charge violating vacuum; therefore, it is not obvious *a priori* that this model has a stable, electric charge conserving vacuum. The effective low-energy theory will also be presented. Verifications will be made that it contains the MSSM,  $SU(2)_L\times U(1)_Y$  breaking is possible and that the model provides a solution to the  $\mu$  problem. In Section 2.4 the group theoretical arguments for the low energy extended Higgs spectrum of the model will be discussed. This sector contains TeV scale doubly charged fermions and bosons—confirming the results discussed in Section 2.2—as well as new light states. The mass spectrum of this sector will also be given symbolically and numerically for sample parameters and checks will be made that there are no tachyonic states. GUT prospects are presented in Section 2.5.



## 2.2 Supersymmetric Left-Right Models

As discussed in Section 1.2.2, left-right models include in their gauge group  $U(1)_{B-L}$ . By restating  $R$ -parity charge of Eq. (1.42) in terms of the equivalent matter parity.

$$P_M = (-1)^{3(B-L)} \quad (2.2)$$

it becomes clear that gauged  $B - L$  automatically guarantees all terms are  $R$ -parity conserving. Early analyzes of the vacuum structure of SLRMs showed that spontaneous breaking of parity into a charge conserving vacuum required that  $R$ -parity also be spontaneously broken by  $\langle \tilde{\nu}^c \rangle \neq 0$ . This still excludes rapid proton decay terms from appearing in the superpotential but the LSP is unstable and therefore not a dark matter candidate.

Two solutions were discussed on how to break  $SU(2)_R \times U(1)_{B-L}$  while conserving automatic  $R$ -parity in the low-energy effective theory: adding new  $B - L = 0$  triplets which allow for separate  $U(1)_{B-L}$  and  $SU(2)_R$  breaking scales[33] and SLRM with non-renormalizable terms[32, 34, 33]. Both of these methods only give VEVs to fields which are evenly charged under  $B - L$  so that their VEVs do not spontaneously break  $R$ -parity. The models discussed in the rest of this thesis will fall into the non-renormalizable category and shall be referred to as minimal supersymmetric left-right models (MSLRM).

The chiral supermultiplet content of MSLRM and its charge under  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  is given in Table 2.1. Note that typically, two bi-doublets are necessary to reproduce the CKM matrix however, one of these is usually assumed to

| Fields           | $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ |
|------------------|--|
| $Q^i$            | $(2, 1, +\frac{1}{3})$                     |
| $Q_i^c$          | $(1, 2, -\frac{1}{3})$                     |
| $L^i$            | $(2, 1, -1)$                               |
| $L_i^c$          | $(1, 2, 1)$                                |
| $\Phi_a$         | $(2, 2, 0)$                                |
| $\Delta$         | $(3, 1, 2)$                                |
| $\bar{\Delta}$   | $(3, 1, -2)$                               |
| $\Delta^c$       | $(1, 3, -2)$                               |
| $\bar{\Delta}^c$ | $(1, 3, +2)$                               |
| $S$              | $(1, 1, 0)$                                |
| $B$              | $(1, 1, 0)$                                |
| $W_L$            | $(3, 1, 0)$                                |
| $W_R$            | $(1, 3, 0)$                                |
| $g$              | $(1, 1, 0)$                                |

Table 2.1: Representation assignment for the chiral supermultiplets of MSLRM where the superscript  $i = 1..3$  represents generation,  $a = 1..n$  the number of bidoublets.

decouple below the right-handed scale. This is achieved through some fine-tuning.

The supermultiplets have the following  $SU(2)_L$  and  $SU(2)_R$  transformations

$$Q \rightarrow U_L Q \quad Q^c \rightarrow U_R Q^c \quad L \rightarrow U_L L \quad L^c \rightarrow U_R L^c$$

$$\begin{aligned}
\Delta &\rightarrow U_L \Delta^c U_L^\dagger & \bar{\Delta} &\rightarrow U_L \bar{\Delta} U_L^\dagger & \Phi_a &\rightarrow U_L \Phi_a U_R^\dagger \\
\Delta^c &\rightarrow U_R \Delta^c U_R^\dagger & \bar{\Delta}^c &\rightarrow U_R \bar{\Delta}^c U_R^\dagger
\end{aligned}$$

and therefore may be written in component form as

$$\begin{aligned}
Q &= \begin{pmatrix} u \\ d \end{pmatrix} & L &= \begin{pmatrix} \nu \\ e \end{pmatrix} \\
Q^c &= \begin{pmatrix} d^c \\ -u^c \end{pmatrix} & L^c &= \begin{pmatrix} e^c \\ -\nu^c \end{pmatrix} \\
\Delta &= \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} & \bar{\Delta} &= \begin{pmatrix} \frac{\Delta^-}{\sqrt{2}} & \Delta^0 \\ \Delta^{--} & -\frac{\Delta^-}{\sqrt{2}} \end{pmatrix} \\
\Delta^c &= \begin{pmatrix} \frac{\Delta^{c-}}{\sqrt{2}} & \Delta^{c0} \\ \Delta^{c--} & -\frac{\Delta^{c-}}{\sqrt{2}} \end{pmatrix} & \bar{\Delta}^c &= \begin{pmatrix} \frac{\bar{\Delta}^{c+}}{\sqrt{2}} & \bar{\Delta}^{c++} \\ \bar{\Delta}^{c0} & -\frac{\bar{\Delta}^{c+}}{\sqrt{2}} \end{pmatrix} \\
\Phi &= \begin{pmatrix} \Phi_d^0 & \Phi_u^+ \\ \Phi_d^- & \Phi_u^0 \end{pmatrix}
\end{aligned}$$

These fields form the general superpotential

$$W = W_{Yukawa} + W_{singlet} + W_{Mass} + W_{NR} \quad (2.3)$$

$$W_{Yukawa} = \mathfrak{i} y_Q^a Q \tau_2 \Phi_a Q^c + \mathfrak{i} y_L^a L \tau_2 \Phi_a L^c + \mathfrak{i} f L \tau_2 \Delta L + \mathfrak{i} f^c L^c \tau_2 \Delta^c L^c \quad (2.4)$$

$$W_{singlet} = S(\lambda_\Delta \text{Tr}(\Delta \bar{\Delta}) + \lambda_{\Delta^c} \text{Tr}(\Delta^c \bar{\Delta}^c) - M_R^2) + \lambda^{ab} S \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) \quad (2.5)$$

$$W_{Mass} = \mu_\Delta \text{Tr}(\Delta \bar{\Delta}) + \mu_{\Delta^c} \text{Tr}(\Delta^c \bar{\Delta}^c) + \mu^{ab} \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) \quad (2.6)$$

$$\begin{aligned}
W_{NR} &= \frac{\lambda_A}{M_P} (\text{Tr}(\Delta^c \bar{\Delta}^c))^2 + \frac{\lambda_B}{M_P} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) \\
&+ \frac{\lambda_\alpha^{ab}}{M_P} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \dots
\end{aligned} \quad (2.7)$$

Invariance of the theory under the following parity transformations

$$\begin{array}{ll}
Q \rightarrow -i\tau_2 Q^{c*} & Q^c \rightarrow i\tau_2 Q^* \\
L \rightarrow -i\tau_2 Q^{c*} & L^c \rightarrow i\tau_2 Q^* \\
\Delta \rightarrow \tau_2 \Delta^{c*} \tau_2 & \Delta^c \rightarrow \tau_2 \Delta^* \tau_2 \\
\bar{\Delta} \rightarrow \tau_2 \bar{\Delta}^{c*} \tau_2 & \bar{\Delta}^c \rightarrow \tau_2 \bar{\Delta}^* \tau_2 \\
\Phi \rightarrow \Phi^\dagger & S \rightarrow \pm S^* \\
\theta \leftrightarrow \bar{\theta} & \tilde{W}_L \leftrightarrow \tilde{W}_R^* \\
\tilde{B} \leftrightarrow \tilde{B}^* & \tilde{g} \leftrightarrow \tilde{g}^*
\end{array}$$

implies that

$$\begin{array}{lll}
y_{Q,L} = y_{Q,L}^\dagger & f = f^{c*} & \lambda_\Delta = \pm \lambda_{\Delta^c}^* \\
\lambda^{ab} = \pm \lambda^{ab*} & \mu^{ab} = \mu^{ab*} & \mu^\Delta = \mu_{\Delta^c}^* \\
\lambda_{A,B,\alpha} = \lambda_{A,B,\alpha}^* & M_L = M_R^* & M_{B-L,3} = M_{B-L,3}^* \quad (2.8)
\end{array}$$

where  $\theta$  and  $\bar{\theta}$  are the Grassman dimensions introduced in Section 1.3 and  $M_{B-L,L,R,3}$  are the gaugino mass for  $U(1)_{B-L}$ ,  $SU(2)_L$ ,  $SU(2)_R$ ,  $SU(3)_c$  respectively. Note that the singlet can be either parity odd or even.

MSLRM can now be further subdivided into two classes of theories: those which contain the singlet  $S$  and those that do not. The analysis here will be done without the singlet since this is more minimal and does not rely on the arbitrary  $M_R$ . It will also prove more useful later on. The superpotential is the same as in

Eq. (2.3) but without  $W_{singlet}$ . The triplet  $F$ -terms are

$$\begin{aligned}
-F_{\Delta^{c0}}^* &= \bar{\Delta}^{c0} \left( \mu_{\Delta^c} + \frac{2\lambda_A}{M_P} \text{Tr}(\Delta^c \bar{\Delta}^c) \right) \\
-F_{\bar{\Delta}^{c0}}^* &= \Delta^{c0} \left( \mu_{\Delta^c} + \frac{2\lambda_A}{M_P} \text{Tr}(\Delta^c \bar{\Delta}^c) \right) \\
-F_{\Delta^0}^* &= \bar{\Delta}^0 \left( \mu_{\Delta} + \frac{2\lambda_A}{M_P} \text{Tr}(\Delta \bar{\Delta}) \right) \\
-F_{\bar{\Delta}^0}^* &= \Delta^0 \left( \mu_{\Delta} + \frac{2\lambda_A}{M_P} \text{Tr}(\Delta \bar{\Delta}) \right)
\end{aligned} \tag{2.9}$$

and can be satisfied with the choice

$$\begin{aligned}
\langle \Delta^{c0} \rangle &= \langle \bar{\Delta}^{c0} \rangle = \sqrt{\frac{\mu_{\Delta^c} M_P}{2\lambda_A}} \equiv v_R \\
\langle \Delta^0 \rangle &= \langle \bar{\Delta}^0 \rangle = 0
\end{aligned} \tag{2.10}$$

Note that in general, non-renormalizable operators of the schematic form  $\Delta \Phi^2 \Delta^c$  are allowed and source  $\Delta$  forcing it to have a non-zero VEV but these will be small due to the small size of  $\langle \Phi \rangle$  and the large  $M_P$  suppression. Mass scales associated with the new particle content should be larger than the electroweak scale to avoid experimental bounds. Specifically,  $\mu_{\Delta^c} > 100$  GeV. This then puts a lower bound on the right-handed scale

$$v_R \sim \sqrt{\mu_{\Delta^c} M_P} > 10^{10} \text{ GeV} \tag{2.11}$$

hence justifying the assumption of large  $v_R$  in the MSLRM.

Regardless of the singlet, these models allow for a powerful statement: they always contains light doubly-charged Higgses[32, 34]. This can be argued on the basis of group theory. The particle content only allows triplets to appear in renor-

malizable terms with non-zero VEVs in the following manner

$$\langle W \rangle = f(\langle \text{Tr}(\Delta^c \bar{\Delta}^c) \rangle) \quad (2.12)$$

These types of terms have an extended complexified global symmetry,  $U(3)$  (a  $U(3)$  whose rotational parameters are complex). The right-handed VEV breaks this symmetry to a  $U(2)$ . Therefore, the vacuum is invariant under the four generators of  $U(2)$  and not invariant under  $9 - 4 = 5$  broken generators of  $U(3)$ . This corresponds to five complex massless degrees of freedom (or ten real massless degrees of freedom) as dictated by the Goldstone theorem. Three of these degrees of freedom will be eaten by  $W_R^\pm$  and a linear combination of  $W_{3R}$  and  $B$ . These will be either singularly charged or neutral based on the charges of the now fat gauge bosons. Three of them will also pick up mass from the  $D$ -terms, Eq. (1.36), in a supersymmetric analogue to the eaten fields. This leaves four massless real degrees of freedom, which are the two doubly-charged Higgs bosons. Furthermore, since this argument exists independent of SUSY breaking, the Higgsinos must also obey it.

In total then, there are two massless doubly-charged Higgs bosons and two massless doubly-charged Higgsinos. This is where the non-renormalizable terms become important. The  $\lambda_B$  term in Eq. (2.3) explicitly breaks the  $U(3)$  symmetry and can therefore generate mass of order  $\frac{v_R^2}{M_P}$ . The fact that doubly-charged fields have not yet been observed puts a lower bound of

$$v_R > 10^{10} \text{GeV} \quad (2.13)$$

which is now the bound in the MSLRM and justifies the assumption of large  $v_R$  in both derivations.

The above analysis shows that realistic SLRMs can be built in which  $R$ -parity is an automatic symmetry of the effective superpotential below the right-handed scale and that such models can lead to interesting phenomenology such as light doubly-charged Higgs fields. This means that SLRMs can put SUSY back on the same footing as the SM by producing a low energy theory with accidental tree-level baryon and lepton conserving terms. Aside from being aesthetically pleasing, this also solves a practical issue with the MSSM.

In addition, SLRM with parity has other nice features. For example, the strong CP problem, Section 1.1.2 is even more severe in SUSY and requires

$$\bar{\theta} \equiv \theta \arg \det(M_u M_d) - 3 \arg M_3 < 10^{-9} \quad (2.14)$$

in order to satisfy experimental data on the electric dipole moment of the neutron. However,  $M_3$  is real in SLRM with parity due to Eq. (2.8) and parity demands that the  $\theta \rightarrow -\theta$  and therefore forbids Eq. (1.16). Furthermore, it can be shown that both the up and down type mass matrices are real in SLRM, therefore solving the strong CP problem at tree level. There have been several papers showing how this solution also extends to loop level [35].

Finally, SLRMs have an advantage over their non-SUSY cousins with regards to the seesaw mechanism. As argued in Section 1.2, the non-SUSY left-right models will always have  $\langle \Delta_L \rangle$ , whose size is dependent on the parameters in the scalar potential. However, in SLRMs, the mixing term which sources  $\Delta_L$  is suppressed by  $M_P$  thereby producing a naturally small  $\langle \Delta_L \rangle$ . The seesaw matrix in SLRM is then

given by

$$M_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \quad (2.15)$$

and depends on one less parameter than Eq. (1.22).

### 2.3 Theoretical Model And the Seesaw Scale

As mentioned in Section 2.1, the seesaw scale, and therefore the right-handed scale is arbitrary in SUSY left-right models. The only clue comes from a lower bound of about  $10^{10}$  GeV due to the lower bounds on the mass of light doubly-charged Higgses, as shown Section 2.2. More specifically, in MSLRMs, the seesaw scale is predicted in terms of arbitrary mass parameters in the superpotential, namely  $v_R \sim \sqrt{\mu M_P}$ . Is it possible to change this into a prediction instead of a lower bound?

This would be possible if  $\mu$  was some well motivated mass scale instead of being arbitrary. Aside from  $M_P$ , the only well motivated mass scale in the theory is the SUSY scale. Typically, if  $\mu \sim m_{SUSY}$  then the sum of  $m_{SUSY}$  and  $\mu$  will appear in the lower bound for  $v_R$  instead of just  $\mu$ . Therefore, if the  $\mu$  terms in the superpotential Eq. (2.3) can be forbidden then a prediction of  $v_R \sim \sqrt{M_P m_{SUSY}}$  can be made.

One possibility for restricting the  $\mu$  terms can be borrowed from the next-to-minimal supersymmetric standard model, in which all mass terms are forbidden by a discrete  $Z_3$  symmetry. This is easy to do because of the holomorphic property of



the superpotential which does not allow terms of the form  $m_\phi|\phi|^2$ . In this case, a discrete  $Z_4$  is appropriate with:

$$(\bar{\Delta}^c, \Delta^c, \Phi) \rightarrow e^{i\pi/2}(\bar{\Delta}^c, \Delta^c, \Phi) \quad (2.16)$$

$$(L, L^c, Q, Q^c) \rightarrow e^{-i\pi/4}(L, L^c, Q, Q^c) \quad (2.17)$$

Furthermore parity symmetry is assumed to be broken at a high scale so that the left-handed partner of the  $\Delta^c$  and  $\bar{\Delta}^c$  are not included in the theory. This is done for simplicity and is not necessary. In addition only one bidoublet will be included, again for simplicity and because the mixing attributed to the CKM matrix can be reproduced in SLRMs through SUSY breaking terms. The superpotential is then that given in Eq. (2.3) without  $W_{Mass}$  and  $W_{singlet}$  and is reproduced here for convenience

$$W = W_{Yukawa} + W_{NR} \quad (2.18)$$

$$W_{Yukawa} = iy_Q Q \tau_2 \Phi Q^c + iy_L L \tau_2 \Phi L^c + if^c L^c \tau_2 \Delta^c L^c \quad (2.19)$$

$$W_{NR} = \frac{\lambda_A}{M_P} (\text{Tr}(\Delta^c \bar{\Delta}^c))^2 + \frac{\lambda_B}{M_P} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) \quad (2.20)$$

$$+ \frac{\lambda_\alpha}{M_P} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi^T \tau_2 \Phi \tau_2) + \dots$$

Several comments regarding this superpotential are in order:

- As planned there are no bilinear terms. This was achieved via a  $Z_4$  symmetry but there may be other motivations for this. This model is then dubbed the predictive supersymmetric left-right model (PSLRM) and was first discussed in [36].

- There is no distinct  $\text{Tr}(\Delta^c \tau_2 \Phi^T \tau_2 \Phi \bar{\Delta}^c)$  in the non-supersymmetric theory.

This is due to the the holomorphic property of the superpotential which only allows  $\Phi$  terms in the form  $\Phi^T \tau_2 \Phi$  to satisfy  $SU(2)_L$  invariance. The trans-  
pose then forces another  $\tau_2$  to be involved. This, coupled with the fact that  $\Phi^T \tau_2 \Phi \tau_2 = \mathbb{1} \cdot \det \Phi$ , gives that  $\text{Tr}(\Delta^c \tau_2 \Phi^T \tau_2 \Phi \bar{\Delta}^c) = \frac{1}{2} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi^T \tau_2 \Phi \tau_2)$ .

The Higgs potential can be derived from Eq. (2.18). The  $F$ -terms are

$$-F_{\Delta^{c0}}^* = \frac{2\lambda_A}{M_P} \Delta^{c0} \bar{\Delta}^{c02} + \frac{\lambda_\alpha}{M_P} \bar{\Delta}^{c0} \text{Tr}(\Phi^T \tau_2 \Phi \tau_2) \quad (2.21)$$

$$-F_{\bar{\Delta}^{c0}}^* = \frac{2\lambda_A}{M_P} \Delta^{c02} \bar{\Delta}^{c0} + \frac{\lambda_\alpha}{M_P} \Delta^{c0} \text{Tr}(\Phi^T \tau_2 \Phi \tau_2) \quad (2.22)$$

It is worth nothing that since  $\langle \Delta^{c0} \rangle \sim 10^{10} \sim v_R$  GeV while  $\langle \Phi \rangle \sim 200$  GeV the triplet  $F$ -terms will be non-zero,  $F_{\Delta^c} \sim \frac{v_R^3}{M_P}$ , and therefore will introduce new SUSY breaking. The potential and its components are

$$V(\Phi, \Delta^c, \bar{\Delta}^c) = V_F + V_D + V_{Soft} \quad (2.23)$$

$$\begin{aligned} V_F = & \frac{4\lambda_A^2}{M_P^2} |\text{Tr}(\Delta^c \bar{\Delta}^c)|^2 \left( \text{Tr}|\Delta^c|^2 + \text{Tr}|\bar{\Delta}^c|^2 \right) \\ & + \frac{4\lambda_B^2}{M_P^2} \left[ |\text{Tr}(\bar{\Delta}^c \bar{\Delta}^c)|^2 \text{Tr}|\Delta^c|^2 + |\text{Tr}(\Delta^c \Delta^c)|^2 \text{Tr}|\bar{\Delta}^c|^2 \right] \\ & + \frac{\lambda_\alpha^2}{M_P^2} |\text{Tr}(\Phi^T \tau_2 \Phi \tau_2)|^2 \left( \text{Tr}|\Delta^c|^2 + \text{Tr}|\bar{\Delta}^c|^2 \right) + \frac{4\lambda_\alpha^2}{M_P^2} |\text{Tr}(\Delta^c \bar{\Delta}^c)|^2 \text{Tr}|\Phi|^2 \\ & + \left[ \frac{4\lambda_A \lambda_B}{M_P^2} \text{Tr}(\Delta^c \bar{\Delta}^c) \left( \text{Tr}(\Delta^c \Delta^c)^* \text{Tr}(\bar{\Delta}^{c\dagger} \Delta^c) + \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c)^* \text{Tr}(\bar{\Delta}^c \Delta^{c\dagger}) \right) \right. \\ & + \frac{2\lambda_A \lambda_\alpha}{M_P^2} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi^T \tau_2 \Phi \tau_2)^* \left( \text{Tr}|\Delta^c|^2 + \text{Tr}|\bar{\Delta}^c|^2 \right) \\ & + \frac{2\lambda_B \lambda_\alpha}{M_P^2} \text{Tr}(\Phi^T \tau_2 \Phi \tau_2)^* \left( \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \Delta^{c\dagger}) + \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) \text{Tr}(\bar{\Delta}^{c\dagger} \Delta^c) \right) \\ & \left. + \text{c.c} \right] \quad (2.24) \end{aligned}$$

$$\begin{aligned}
V_D = & \frac{g_R^2}{8} \sum_a [\text{Tr}(2\Delta^{c\dagger}\tau_a\Delta^c + 2\bar{\Delta}^{c\dagger}\tau_a\bar{\Delta}^c - \Phi\tau_a\Phi^\dagger)]^2 + \frac{g_L^2}{8} \sum_a [\text{Tr}(\Phi^\dagger\tau_a\Phi)]^2 \\
& + \frac{g_{BL}^2}{2} [\text{Tr}(\bar{\Delta}^{c\dagger}\bar{\Delta}^c - \Delta^{c\dagger}\Delta^c)]^2
\end{aligned} \tag{2.25}$$

$$\begin{aligned}
V_{Soft} = & -m_{\Delta^c}^2 \text{Tr}(\Delta^{c\dagger}\Delta^c) - m_{\bar{\Delta}^c}^2 \text{Tr}(\bar{\Delta}^{c\dagger}\bar{\Delta}^c) - m_\Phi^2 \text{Tr}(\Phi^\dagger\Phi) \\
& - \frac{Z_A m_{3/2}}{M_P} [(\text{Tr}(\Delta^c\bar{\Delta}^c))^2 + \text{c.c.}] + \frac{Z_B m_{3/2}}{M_P} [\text{Tr}(\Delta^c\Delta^c) \text{Tr}(\bar{\Delta}^c\bar{\Delta}^c) + \text{c.c.}] \\
& - \frac{Z_\alpha m_{3/2}}{M_P} [\text{Tr}(\Delta^c\bar{\Delta}^c) \text{Tr}(\Phi^T\tau_2\Phi\tau_2) + \text{c.c.}]
\end{aligned} \tag{2.26}$$

with  $V_F$  being the  $F$ -term contribution,  $V_D$  the  $D$ -term, and  $V_{Soft}$  the SUSY breaking terms, the most general soft potential given the superpotential. Here mass terms in the form  $m_\phi|\phi|^2$  are allowed since they cannot be forbidden by any symmetry.  $V_{Soft}$  is necessary in this case since in the SUSY limit the minimum of the potential is the trivial one.

The minimization conditions, correct up to electroweak order (i.e. neglecting terms of order  $v_{wk}^3/M_P$  and smaller), are:

$$\begin{aligned}
\frac{1}{\Delta^{c0}} \frac{\partial V}{\partial \Delta^{c0}} = & \frac{1}{2}(g_{BL}^2 + g_R^2)(v_R^2 - \bar{v}_R^2) + \frac{1}{4}g_R^2(\kappa_u^2 - \kappa_d^2) - m_{\Delta^c}^2 \\
& - \frac{Z_A m_{3/2}}{M_P} \bar{v}_R^2 + \frac{\lambda_A^2 \bar{v}_R^2}{M_P^2} (2v_R^2 + \bar{v}_R^2) = 0
\end{aligned} \tag{2.27}$$

$$\begin{aligned}
\frac{1}{\bar{\Delta}^{c0}} \frac{\partial V}{\partial \bar{\Delta}^{c0}} = & -\frac{1}{2}(g_{BL}^2 + g_R^2)(v_R^2 - \bar{v}_R^2) - \frac{1}{4}g_R^2(\kappa_u^2 - \kappa_d^2) - m_{\bar{\Delta}^c}^2 \\
& - \frac{Z_A m_{3/2}}{M_P} v_R^2 + \frac{\lambda_A^2 v_R^2}{M_P^2} (v_R^2 + 2\bar{v}_R^2) = 0
\end{aligned} \tag{2.28}$$

$$\begin{aligned} \frac{1}{\Phi_u^0} \frac{\partial V}{\partial \Phi_u^0} &= \frac{1}{4} g_R^2 (v_R^2 - \bar{v}_R^2) + \frac{1}{8} (g_R^2 + g_L^2) (\kappa_u^2 - \kappa_d^2) - m_\Phi^2 + \frac{\lambda_\alpha^2 v_R^2 \bar{v}_R^2}{M_P^2} \\ &\quad - v_R \bar{v}_R \left[ \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha (v_R^2 + \bar{v}_R^2)}{M_P^2} \right] \frac{\kappa_d}{\kappa_u} = 0 \end{aligned} \quad (2.29)$$

$$\begin{aligned} \frac{1}{\Phi_d^0} \frac{\partial V}{\partial \Phi_d^0} &= -\frac{1}{4} g_R^2 (v_R^2 - \bar{v}_R^2) - \frac{1}{8} (g_R^2 + g_L^2) (\kappa_u^2 - \kappa_d^2) - m_\Phi^2 + \frac{\lambda_\alpha^2 v_R^2 \bar{v}_R^2}{M_P^2} \\ &\quad - v_R \bar{v}_R \left[ \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha (v_R^2 + \bar{v}_R^2)}{M_P^2} \right] \frac{\kappa_u}{\kappa_d} = 0 \end{aligned} \quad (2.30)$$

where we have taken the vacuum expectation values (VEVs) to be the real part of the neutral field; that is

$$v_R \equiv \langle \text{Re } \Delta^{c0} \rangle \quad \bar{v}_R \equiv \langle \text{Re } \bar{\Delta}^{c0} \rangle \quad \kappa_u \equiv \langle \text{Re } \Phi_u^0 \rangle \quad \kappa_d \equiv \langle \text{Re } \Phi_d^0 \rangle \quad (2.31)$$

Considering only Eq. (2.27) for the moment, take

$$v_R = v \sin \theta_R \quad \bar{v}_R = v \cos \theta_R \quad \kappa_u = \kappa \sin \beta \quad \kappa_d = \kappa \cos \beta \quad (2.32)$$

Now, the difference of the squares of  $v_R$  and  $\bar{v}_R$  must be of order  $v_{wk}^2$  (subtracting the two equations in Eq. (2.27) will reveal this), so  $\theta_R$  must be near  $\pi/4$ . Therefore, let

$$\theta_R = \frac{\pi}{4} + \frac{\epsilon}{2} \quad (2.33)$$

and expand to first order in  $\epsilon$  (as we shall see,  $\epsilon \sim v_{wk}/M_P$ —so  $\epsilon$  is quite small).

The sum of the two equations in (2.27) yields a quadratic for  $v^2$ :

$$-(m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2) - \frac{Z_A m_{3/2}}{M_P} v^2 + \frac{3 \lambda_A^2 v^4}{2 M_P^2} = 0 \quad (2.34)$$

and the difference gives an expression for  $\epsilon$ :

$$\epsilon = \frac{v_R^2 - \bar{v}_R^2}{v^2} = \frac{m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2 + \frac{1}{2} g_R^2 \kappa^2 \cos 2\beta}{(g_{BL}^2 + g_R^2) v^2} \quad (2.35)$$

The solution to Eq. (2.34),

$$v^2 = \left( \frac{Z_A m_{3/2} + \sqrt{(Z_A m_{3/2})^2 + 6\lambda_A^2 (m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2)}}{3\lambda_A^2} \right) M_P \quad (2.36)$$

gives the prediction of the right breaking scale. Since  $Z_A \sim \lambda_A \sim 1$  and  $m_{\Delta^c} \sim m_{\bar{\Delta}^c} \sim m_{3/2} \sim v_{wk}$ , we get the result  $v \simeq \sqrt{v_{wk} M_P}$ . This shows that the seesaw scale can be determined in terms of two other commonly assumed and well motivated scales in the theory; *i.e.* the Planck scale in four dimensions and the supersymmetry breaking scale (which is of the order of the weak scale to solve the gauge hierarchy problem). The seesaw scale, then, is  $M_R \simeq v \sim 10^{11}$  GeV and is a realistic mass scale in regards to both neutrino masses and the masses of the doubly-charged fields. From Eq. (2.35),  $\epsilon \lesssim v_{wk}/M_P \sim 10^{-16}$ .

Now turn to Eq. (2.29)—again using Eq. (2.32) and expanding to first order in  $\epsilon$ , their sum yields an expression for  $\sin 2\beta$ :

$$\sin 2\beta = \frac{\left[ \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha v^2}{M_P^2} \right] v^2}{\frac{\lambda_\alpha^2 v^4}{2M_P^2} - 2m_\Phi^2} \quad (2.37)$$

and their difference—after using Eq. (2.35) and Eq. (2.37)—gives

$$\cos 2\beta = \frac{\frac{1}{2} \frac{g_R^2}{g_{BL}^2 + g_R^2} (m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2)}{\frac{1}{4} \left( g_L^2 + \frac{g_{BL}^2 g_R^2}{g_{BL}^2 + g_R^2} \right) \kappa^2 + \frac{\lambda_\alpha^2 v^4}{2M_P^2} - 2m_\Phi^2} \quad (2.38)$$

Both Eq. (2.37) and Eq. (2.38) are consistent with any value of  $\beta$  and constrain the parameter space once a value of  $\beta$  has been specified. It is also easy to make an analogy between them and the usual MSSM results as we now do.

### 2.3.1 Effective Theory and the $\mu$ Problem

We begin our discussion of the effective low energy theory with the relationship between our parameters and those in the MSSM. When  $SU(2)_R \times U(1)_{B-L}$  is broken the  $\text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi^T \tau_2 \Phi \tau_2)$  of Eq. (2.18) will yield a mass term for the  $SU(2)_L$  doublets  $\phi_u$  and  $\phi_d$ ; since these are basically the  $H_u$  and  $H_d$  of the MSSM, this is the usual  $\mu$  term. We then have that

$$|\mu| = \left| \frac{\lambda_\alpha v^2}{2M_P} \right| \sim v_{wk} \quad (2.39)$$

which is of the desired order of magnitude without any extra assumptions and therefore solves the  $\mu$  problem

Similar reasoning yields that the SUSY breaking bilinear term,  $B$ , will have a contribution resulting from the  $Z_\alpha$  term in Eq. (2.26); however, it will also receive a contribution from the  $F$ -term of Eq. (2.24)—specifically the coefficient of  $\lambda_A \lambda_\alpha$ . Together these give

$$b = \left[ \frac{Z_\alpha m_{3/2}}{2M_P} - \frac{\lambda_A \lambda_\alpha v^2}{2M_P^2} \right] v^2 \quad (2.40)$$

Using the expressions for  $b$  and  $\mu$  and examining the minimization conditions given by Eq. (2.29), we can read off  $m_{H_u}^2$  and  $m_{H_d}^2$ :

$$m_{H_u}^2 = -m_\Phi^2 + \frac{1}{4} g_R^2 (v_R^2 - \bar{v}_R^2) \quad (2.41)$$

$$m_{H_d}^2 = -m_\Phi^2 - \frac{1}{4} g_R^2 (v_R^2 - \bar{v}_R^2) \quad (2.42)$$

Here it is noticed that  $m_{H_u}^2 \neq m_{H_d}^2$  despite the apparent symmetry of the superpotential and the soft-breaking mass term. This splitting is due to the  $D$ -terms, which is reflected in the fact that their difference is proportional to  $g_R^2$ . Specifically,

it is the  $D$ -term involving  $\tau_3$  (the ones involving  $\tau_1$  and  $\tau_2$  won't contribute because when the VEVs are placed in for  $\Delta^c$  and  $\bar{\Delta}^c$  these are zero) that gives a positive  $\bar{v}_R^2 - v_R^2$  contribution to  $m_{H_u}^2$  and a negative one to  $m_{H_d}^2$ .

Using Eq. (2.35) these expressions may be recast into the form

$$m_{H_u}^2 = -m_\Phi^2 + \frac{1}{4} \frac{g_R^2}{g_{BL}^2 + g_R^2} (m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2) \quad (2.43)$$

$$m_{H_d}^2 = -m_\Phi^2 - \frac{1}{4} \frac{g_R^2}{g_{BL}^2 + g_R^2} (m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2) \quad (2.44)$$

This form is advantageous because we now have that

$$m_{H_u}^2 - m_{H_d}^2 = \frac{1}{2} \frac{g_R^2}{g_{BL}^2 + g_R^2} (m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2) \quad (2.45)$$

which, with the masses of the left-handed (standard model) particles

$$M_Z^2 = \frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_R^2}{g_{BL}^2 + g_R^2} \right] \kappa^2 \quad (2.46)$$

$$M_W^2 = \frac{1}{4} g_L^2 \kappa^2 \quad (2.47)$$

allows us to write Eq. (2.37) and Eq. (2.38) in the enticing form

$$\sin 2\beta = \frac{2B}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2} \quad (2.48)$$

$$\cos 2\beta = \frac{m_{H_u}^2 - m_{H_d}^2}{m_Z^2 + 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2} \quad (2.49)$$

which can be related to the usual minimization expressions of the MSSM, see Eq. (1.62).

The interesting aspect of this result is that, provided  $m_{\Delta^c}^2 \neq m_{\bar{\Delta}^c}^2$ , there exists a region of the parameter space where  $\tan \beta \equiv \frac{\kappa_u}{\kappa_d} \gg 1$ . This is an important feature because it has been noted that for theories involving a single bidoublet [37, 38]

getting  $\tan\beta > 1$  is difficult. However, it is necessary for realistic quark and lepton masses and mixings. Since this model does not require additional particles to achieve  $\tan\beta \gg 1$  (as opposed to those previously discussed[39, 40]), it is truly a minimal scheme.

### 2.3.2 Charge Violation Consideration

The above model is based on VEVs that are consistent with the charge conserving vacuum. However, it has been noted in earlier works that in SUSYLR models, the  $\Delta^c$  fields may have a VEV that breaks electric charge conservation[30] unless one breaks R-parity. In this model though, the existence of non-renormalizable terms allow for the charge conserving vacuum to have a much lower ground state energy than the charge conserving one for large regions of the parameter space. This ensures that the theory will spontaneously break into the phenomenologically viable vacuum—the charge conserving one.

To see this we can compare the ground state values of the two potentials, the charge violating one (CV) and the charge conserving (CC) one. The VEVs for the CC case have already been discussed, their analogues in the CV case are:

$$\langle\Delta^c\rangle = \begin{pmatrix} 0 & \frac{v_R}{\sqrt{2}} \\ \frac{v_R}{\sqrt{2}} & 0 \end{pmatrix} \quad \langle\bar{\Delta}^c\rangle = \begin{pmatrix} 0 & \frac{\bar{v}_R}{\sqrt{2}} \\ \frac{\bar{v}_R}{\sqrt{2}} & 0 \end{pmatrix}$$

The resulting ground state expressions, to order  $v_R$ , are:

$$\langle V\rangle_{CV} = -\frac{1}{2}v^2(m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2) + \frac{(Z_B - Z_A)m_{3/2}v^4}{2M_P} + \frac{(\lambda_A + \lambda_B)^2 v^6}{M_P^2} \quad (2.50)$$

$$\langle V\rangle_{CC} = -v^2(m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2) - \frac{2Z_A m_{3/2}v^4}{M_P} + \frac{8\lambda_A^2 v^6}{M_P^2} \quad (2.51)$$



Where  $v^2$  for the CC case was given in Eq. (2.36) and  $v^2$  for the CV case is:

$$v^2 = \frac{M_P}{6(\lambda_A + \lambda_B)^2} \left( (Z_A - Z_B) m_{3/2} + \sqrt{((Z_A - Z_B) m_{3/2})^2 + 6(\lambda_A + \lambda_B)^2 (m_{\Delta^c}^2 + m_{\bar{\Delta}^c}^2)} \right) \quad (2.52)$$

The crucial point here is that the CV ground state expression has a dependence on both  $Z_B$  and  $\lambda_B$ , which do not appear in the CC expression. This means that for sufficiently large values of these parameters, the CC ground state will be lower. In the numerical analysis conducted in a later section, this will be taken into account and the difference between the two ground state values will be compared.

## 2.4 Mass Spectrum and Numerical Analysis

### 2.4.1 Mass Spectrum

Once the value of the minimization conditions and the values of the VEVs have been determined, the mass spectrum can be explored to ensure that all the resulting physical Higgs bosons have positive mass squares. This is nontrivial because if too few terms are included in the superpotential, there is no *a priori* guarantee that there is a stable minimum instead of a flat direction or an unstable minimum. In this section only first order in  $\epsilon$  is retained.

Before diving into the algebra, it would very profitable to consider the group theoretical arguments for the mass spectrum, similar to those given in Section 2.2. Here the analysis is somewhat different due to the lack of a mass term, which means that before symmetry breaking there are twelve massless degrees of freedom. Once

the triplets acquire a VEV, the SUSY Higgs mechanism will cause three degrees of freedom to be eaten: a pseudo scalar and a charged field and three to gain mass from the  $D$ -terms: a scalar and a charged field. This leaves six real massless degrees of freedom. Four of these are the doubly charged fields. This leaves two massless degrees of freedom: a scalar and a pseudoscalar since all other degrees of freedom have been accounted for.

The analyzes begins with  $\text{Im } \Delta^{c0}$ ,  $\text{Im } \bar{\Delta}^{c0}$ ,  $\text{Im } \Phi_u^0$ , and  $\text{Im } \Phi_d^0$  (the imaginary components of the neutral fields) since two linear combinations of them are eaten by gauge bosons (so there are two zero modes). The four by four mass matrix resulting after the spontaneous symmetry breaking can be split into two by two matrices for the  $\Delta^c$ 's and the  $\Phi$ 's:

$$V_{mass} \supset \frac{1}{2} \begin{pmatrix} \text{Im } \bar{\Delta}^{c0} & \text{Im } \Delta^{c0} \end{pmatrix} M_{I\Delta^c}^2 \begin{pmatrix} \text{Im } \bar{\Delta}^{c0} \\ \text{Im } \Delta^{c0} \end{pmatrix} \quad (2.53)$$

$$+ \frac{1}{2} \begin{pmatrix} \text{Im } \Phi_u^0 & \text{Im } \Phi_d^0 \end{pmatrix} M_{I\Phi}^2 \begin{pmatrix} \text{Im } \Phi_u^0 \\ \text{Im } \Phi_d^0 \end{pmatrix}$$

where

$$M_{I\Delta^c}^2 = \begin{pmatrix} \frac{Z_A m_{3/2}}{M_P} v^2 & \frac{Z_A m_{3/2}}{M_P} v^2 \\ \frac{Z_A m_{3/2}}{M_P} v^2 & \frac{Z_A m_{3/2}}{M_P} v^2 \end{pmatrix} \quad (2.54)$$

$$M_{I\Phi}^2 = \begin{pmatrix} \frac{1}{2} \frac{\kappa_d}{\kappa_u} \left[ \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha v^2}{M_P^2} \right] v^2 & \frac{1}{2} \left[ \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha v^2}{M_P^2} \right] v^2 \\ \frac{1}{2} \left[ \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha v^2}{M_P^2} \right] v^2 & \frac{1}{2} \frac{\kappa_u}{\kappa_d} \left[ \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha v^2}{M_P^2} \right] v^2 \end{pmatrix} \quad (2.55)$$

The above matrices each have determinant equal to zero, and the remaining

non-zero eigenvalues are, respectively,

$$m_{B^0}^2 = \frac{2Z_A m_{3/2} v^2}{M_P} \quad (2.56)$$

$$m_{A^0}^2 = \frac{\lambda_\alpha^2 v^4}{2M_P^2} - 2m_\Phi^2 \quad (2.57)$$

where the latter value has been simplified using Eq. (2.37). Here we have introduced  $B^0$  as the axial Higgs boson associated with the  $\Delta^c$  fields and  $A^0$  is the usual MSSM axial Higgs boson.

The mass of  $B^0$  will always be positive provided  $Z_A > 0$ , which means that the minus sign in front of the  $Z_A$  term in Eq. (2.26) is crucial for a positive mass-square. The mass of  $A^0$  could easily be positive depending on the value of  $\lambda_\alpha$  and the phase of  $m_\Phi^2$ . Furthermore, this will always be light and agrees with the group theoretical argument at the beginning of this section.

Next we move on to the singly charged fields since they also have two zero masses. The mass matrix for these fields can not be split apart; however, if we write

$$V_{mass} \supset \begin{pmatrix} \Phi_u^{+*} & \Phi_d^- & \bar{\Delta}^{c+*} & \Delta^{c-} \end{pmatrix} M_{SC}^2 \begin{pmatrix} \Phi_u^+ \\ \Phi_d^{-*} \\ \bar{\Delta}^{c+} \\ \Delta^{c-*} \end{pmatrix} \quad (2.58)$$

then  $M_{SC}^2$  can be seen to be three distinct two by two matrices:

$$M_{SC}^2 = \begin{pmatrix} M_{\Phi^\pm}^2 & M_{\Phi\Delta^{c\pm}}^2 \\ (M_{\Phi\Delta^{c\pm}}^2)^\dagger & M_{\Delta^{c\pm}}^2 \end{pmatrix} \quad (2.59)$$

where

$$M_{\Phi^\pm}^2 = \begin{pmatrix} \frac{1}{4}g_R^2\kappa_d^2 + \frac{1}{2}\frac{\kappa_d}{\kappa_u}\Lambda_\alpha v^2 & \frac{1}{4}g_R^2\kappa_u\kappa_d + \frac{1}{2}\Lambda_\alpha v^2 \\ \frac{1}{4}g_R^2\kappa_u\kappa_d + \frac{1}{2}\Lambda_\alpha v^2 & \frac{1}{4}g_R^2\kappa_u^2 + \frac{1}{2}\frac{\kappa_u}{\kappa_d}\Lambda_\alpha v^2 \end{pmatrix} \quad (2.60)$$

$$M_{\Phi\Delta^{c\pm}}^2 = \begin{pmatrix} \frac{1}{4}g_R^2 v\kappa_d & -\frac{1}{4}g_R^2 v\kappa_d \\ \frac{1}{4}g_R^2 v\kappa_u & -\frac{1}{4}g_R^2 v\kappa_u \end{pmatrix} \quad (2.61)$$

$$M_{\Delta^{c\pm}}^2 = \begin{pmatrix} \frac{1}{4}g_R^2 v^2(1 + \epsilon') + \frac{1}{2}\Lambda_A v^2 & -\frac{1}{4}g_R^2 v^2 - \frac{1}{2}\Lambda_A v^2 \\ -\frac{1}{4}g_R^2 v^2 - \frac{1}{2}\Lambda_A v^2 & \frac{1}{4}g_R^2 v^2(1 - \epsilon') + \frac{1}{2}\Lambda_A v^2 \end{pmatrix} \quad (2.62)$$

with

$$\Lambda_\alpha \equiv \frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A \lambda_\alpha v^2}{M_P^2} \quad \Lambda_A \equiv \frac{Z_A m_{3/2}}{M_P} - \frac{\lambda_A^2 v^2}{M_P^2} \quad \epsilon' \equiv \epsilon + \frac{\kappa_u^2 - \kappa_d^2}{v^2} \quad (2.63)$$

Checking the order of magnitude of each of those matrices, it can be seen that

$$\left| (M_{\Phi^\pm}^2)_{ij} \right| \sim \epsilon v^2 \quad \left| (M_{\Phi\Delta^{c\pm}}^2)_{ij} \right| \sim \sqrt{\epsilon} v^2 \quad \left| (M_{\Delta^{c\pm}}^2)_{ij} \right| \sim v^2 \quad (2.64)$$

so,  $M_{SC}^2$  may be written as

$$\begin{pmatrix} \epsilon\Lambda_1 v^2 & \sqrt{\epsilon}\Lambda_2 v^2 \\ \sqrt{\epsilon}\Lambda_2^\dagger v^2 & \Lambda_3 v^2 \end{pmatrix} \quad (2.65)$$

where each element of each  $\Lambda$  matrix is of order one. This matrix structure is exactly that of the neutrino mass matrix in the type II singular seesaw scenario with the associations<sup>1</sup>

$$\delta^2 m_L \rightarrow M_{\Phi^\pm}^2 \quad \delta m_D \rightarrow M_{\Phi\Delta^{c\pm}}^2 \quad M_R \rightarrow M_{\Delta^{c\pm}}^2. \quad (2.66)$$

Since the determinant of  $M_{\Delta^{c\pm}}^2$  is zero, there is only one large eigenvalue given by

$$m_{D+}^2 = \frac{1}{2}g_R^2 v^2 \quad (2.67)$$

---

<sup>1</sup>for a review of the type II singular seesaw mechanism see Appendix B.2

the resulting mass matrix for the lighter fields is then read directly from the Seesaw formula:

$$\begin{pmatrix} \frac{1}{4}g_L^2\kappa_d^2 - \frac{1}{2}\frac{\kappa_d}{\kappa_u}\left[\frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A\lambda_\alpha v^2}{M_P^2}\right]v^2 & \frac{1}{4}g_L^2\kappa_u\kappa_d - \frac{1}{2}\left[\frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A\lambda_\alpha v^2}{M_P^2}\right]v^2 & 0 \\ \frac{1}{4}g_L^2\kappa_u\kappa_d - \frac{1}{2}\left[\frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A\lambda_\alpha v^2}{M_P^2}\right]v^2 & \frac{1}{4}g_L^2\kappa_u^2 - \frac{1}{2}\frac{\kappa_u}{\kappa_d}\left[\frac{Z_\alpha m_{3/2}}{M_P} - \frac{\lambda_A\lambda_\alpha v^2}{M_P^2}\right]v^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.68)$$

Evidently zero is one of the eigenvalues, and the determinant of the remaining two by two is also zero. These correspond to the two modes that are eaten by the charged gauge bosons. The trace of the two by two is then the non-zero eigenvalue, which corresponds to the MSSM charged Higgs Boson  $h^+$ . So, after using Eq. (2.37)

$$m_{h^+}^2 = \frac{1}{4}g_L^2\kappa^2 + \frac{\lambda_\alpha^2 v^4}{2M_P^2} - 2m_\Phi^2 \quad (2.69)$$

Note that the first term of the right-hand side is just  $m_W^2$  and that the last two terms sum to the aforementioned  $m_{A^0}^2$ . We can therefore rewrite Eq. (2.69) as

$$m_{h^+}^2 = m_W^2 + m_{A^0}^2 \quad (2.70)$$

which matches the MSSM result and will be positive if  $m_{A^0}^2$  is.

The remaining charged fields—the doubly charged Higgs bosons—can only consist of  $\Delta^c$  and  $\bar{\Delta}^c$ , so this mass matrix is a two by two. For these fields we have

$$V_{mass} \supset \begin{pmatrix} \bar{\Delta}^{c++*} & \Delta^{c--} \end{pmatrix} M_{\Delta^{c\pm\pm}}^2 \begin{pmatrix} \bar{\Delta}^{c++} \\ \Delta^{c--*} \end{pmatrix} \quad (2.71)$$

where

$$M_{\Delta^{c\pm\pm}}^2 = \begin{pmatrix} g_R^2 v^2 \epsilon - \frac{1}{2} g_R^2 (\kappa_u^2 - \kappa_d^2) & (\Lambda_B - \frac{1}{2} \Lambda_A) v^2 \\ + \frac{1}{2} \Lambda_A v^2 + \frac{\lambda_B (\lambda_A + \lambda_B) v^4}{M_P^2} & \\ (\Lambda_B - \frac{1}{2} \Lambda_A) v^2 & -g_R^2 v^2 \epsilon + \frac{1}{2} g_R^2 (\kappa_u^2 - \kappa_d^2) \\ & + \frac{1}{2} \Lambda_A v^2 + \frac{\lambda_B (\lambda_A + \lambda_B) v^4}{M_P^2} \end{pmatrix} \quad (2.72)$$

with

$$\Lambda_B \equiv \frac{Z_B m_{3/2}}{M_P} + \frac{\lambda_A \lambda_B v^2}{M_P^2}. \quad (2.73)$$

The eigenvalues are

$$m_{D^{++}/d^{++}}^2 = \left( \frac{1}{2} \Lambda_A + \frac{\lambda_B (\lambda_A + \lambda_B) v^2}{M_P^2} \right) v^2 \pm \sqrt{\left( -g_R^2 \epsilon v^2 + \frac{1}{2} g_R^2 (\kappa_u^2 - \kappa_d^2) \right)^2 + \left[ \Lambda_B - \frac{1}{2} \Lambda_A \right]^2 v^4}. \quad (2.74)$$

These eigenvalues are of order  $v_{wk}^2$  as expected from the group theoretical argument. Furthermore, they are positive for sufficiently large  $\lambda_B$  corresponding to charge conserving vacuum for large  $\lambda_B$ .

Finally, we come to the real neutral fields. These fields, like the singly charged, require use of the seesaw mechanism (as discussed in Appendix B.2). If we write

$$V_{mass} \supset \frac{1}{2} \begin{pmatrix} \text{Re } \Phi_u^0 & \text{Re } \Phi_d^0 & \text{Re } \bar{\Delta}^{c0} & \text{Re } \Delta^{c0} \end{pmatrix} M_{RN}^2 \begin{pmatrix} \text{Re } \Phi_u^0 \\ \text{Re } \Phi_d^0 \\ \text{Re } \bar{\Delta}^{c0} \\ \text{Re } \Delta^{c0} \end{pmatrix} \quad (2.75)$$

where

$$M_{RN}^2 = \begin{pmatrix} M_{\Phi\Phi}^2 & M_{\Phi\Delta^c}^2 \\ (M_{\Phi\Delta^c}^2)^\dagger & M_{\Delta^c\bar{\Delta}^c}^2 \end{pmatrix} \quad (2.76)$$

with

$$M_{\Delta^c\bar{\Delta}^c}^2 = \begin{pmatrix} \frac{1}{2}(g_{BL}^2 + g_R^2)v^2(1 - \epsilon) + \frac{\lambda_A^2 v^4}{M_P^2} & -\frac{1}{2}(g_{BL}^2 + g_R^2)v^2 - \Lambda_A v^2 \\ -\frac{1}{2}(g_{BL}^2 + g_R^2)v^2 - \Lambda_A v^2 & \frac{1}{2}(g_{BL}^2 + g_R^2)v^2(1 + \epsilon) + \frac{\lambda_A^2 v^4}{M_P^2} \end{pmatrix} \quad (2.77)$$

$$M_{\Phi\Phi}^2 = \begin{pmatrix} \frac{1}{4}(g_L^2 + g_R^2)\kappa_u^2 + \frac{1}{2}\frac{\kappa_d}{\kappa_u}\Lambda_\alpha v^2 & -\frac{1}{4}(g_L^2 + g_R^2)\kappa_u\kappa_d - \frac{1}{2}\Lambda_\alpha v^2 \\ -\frac{1}{4}(g_L^2 + g_R^2)\kappa_u\kappa_d - \frac{1}{2}\Lambda_\alpha v^2 & \frac{1}{4}(g_L^2 + g_R^2)\kappa_d^2 + \frac{1}{2}\frac{\kappa_u}{\kappa_d}\Lambda_\alpha v^2 \end{pmatrix} \quad (2.78)$$

$$M_{\Phi\Delta^c}^2 = \begin{pmatrix} \frac{1}{2\sqrt{2}}g_R^2 v\kappa_u & -\frac{1}{2\sqrt{2}}g_R^2 v\kappa_u \\ -\frac{1}{2\sqrt{2}}g_R^2 v\kappa_d & \frac{1}{2\sqrt{2}}g_R^2 v\kappa_d \end{pmatrix} \quad (2.79)$$

and make the associations

$$\delta^2 m_L \rightarrow M_{\Phi\Phi}^2 \quad \delta m_D \rightarrow M_{\Phi\Delta^c}^2 \quad M_R \rightarrow M_{\Delta^c\bar{\Delta}^c}^2 \quad (2.80)$$

we have that the single large eigenvalue of  $M_{\Delta^c\bar{\Delta}^c}^2$  is given by

$$m_{D0}^2 = (g_{BL}^2 + g_R^2)v^2 \quad (2.81)$$

then the resulting mass matrix for the lighter fields is

$$\begin{pmatrix} \frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_R^2}{g_{BL}^2 + g_R^2} \right] \kappa_u^2 & -\frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_R^2}{g_{BL}^2 + g_R^2} \right] \kappa_u \kappa_d & 0 \\ + \frac{1}{2} \frac{\kappa_d}{\kappa_u} \Lambda_\alpha v^2 & -\Lambda_\alpha v^2 & 0 \\ -\frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_R^2}{g_{BL}^2 + g_R^2} \right] \kappa_u \kappa_d & \frac{1}{4} \left[ g_L^2 + \frac{g_{BL}^2 g_R^2}{g_{BL}^2 + g_R^2} \right] \kappa_d^2 & 0 \\ -\Lambda_\alpha v^2 & + \frac{1}{2} \frac{\kappa_u}{\kappa_d} \Lambda_\alpha v^2 & 0 \\ 0 & 0 & \left[ \frac{3\lambda_A^2 v^2}{M_P^2} - \frac{Z_A m_{3/2}}{M_P} \right] v^2 \end{pmatrix} \quad (2.82)$$

Clearly one of the eigenvalues can be read off, it is

$$m_{d^0}^2 = \left[ \frac{3\lambda_A^2 v^2}{M_P^2} - \frac{Z_A m_{3/2}}{M_P} \right] v^2 \quad (2.83)$$

of electroweak order and corresponds to the light scalar field in the group theoretical argument.

The remaining two by two matrix has the eigenvalues

$$m_{H^0/h^0}^2 = \frac{1}{2} \left[ m_Z^2 + m_A^2 \pm \sqrt{(m_Z^2 + m_A^2)^2 - 4m_Z^2 m_A^2 \cos 2\beta^2} \right] \quad (2.84)$$

where we have used Eq. (2.57) and Eq. (2.46) to simplify this expression. Note that these also match MSSM expressions.

That completes the Higgs spectrum analysis. The additional fermionic content of the theory is composed of three light fermions: two doubly charged and a neutral one. All of the fermions has a mass in the electroweak range. The neutral one is the superpartner of the  $d^0$ .

Also of note is the fact that sfermion soft masses will receive  $D$ -term contributions,  $\delta m^2$  proportional to the differences in the right-handed VEVs squared and



the charge of that field. This is not a new contribution and is present in general when gauge symmetries are broken. The contributions are listed in Table 2.2 where

$$D \equiv \frac{1}{4\pi} \left( \frac{m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2}{\alpha_{B-L} + \alpha_R} \right) \quad (2.85)$$

| Field         | $\delta m_{B-L}^2$            | $\delta m_R^2$   |
|---------------|-------------------------------|------------------|
| $\tilde{Q}$   | $-\frac{\pi}{3}\alpha_{B-L}D$ | 0                |
| $\tilde{u}^c$ | $\frac{\pi}{3}\alpha_{B-L}D$  | $-\pi\alpha_R D$ |
| $\tilde{d}^c$ | $\frac{\pi}{3}\alpha_{B-L}D$  | $\pi\alpha_R D$  |
| $\tilde{L}$   | $\pi\alpha_{B-L}D$            | 0                |
| $\tilde{e}^c$ | $-\pi\alpha_{B-L}D$           | $\pi\alpha_R D$  |
| $H_u$         | 0                             | $\pi\alpha_R D$  |
| $H_d$         | 0                             | $-\pi\alpha_R D$ |

Table 2.2:  $D$ -term contributions to soft masses due to the breaking of  $SU(2)_R \times U(1)_{B-L}$ . The last two terms corresponding to the electroweak Higgses have already been accounted for in Eq. (2.41) with  $D \equiv \frac{1}{4\pi} \frac{m_{\Delta^c} - m_{\bar{\Delta}^c}}{\alpha_{B-L} + \alpha_R}$ .

## 2.4.2 Numerics

The purpose of this subsection is to validate the above arguments with numerical analysis. Specifically, our purpose is simply to show that the general arguments about the positivity of the Higgs masses can be supported in the parameter space. Other values of interest are also reported including:  $v_R$ ,  $\tan \beta$ , and the difference in

the ground state values of the CC and CV potentials (as mentioned earlier we need  $\langle V \rangle_{CV} - \langle V \rangle_{CC} > 0$ , so this is verified in that last column of Table 2.5).

We will keep six of the dimensionful parameters constant (in GeV)

$$m_{\Delta^c} = 350 \quad m_{\bar{\Delta}^c} = 450 \quad m_{3/2} = 450 \quad \kappa = 250 \quad M_P = 2.44 \times 10^{18}$$

and three of the coupling constants at:

$$g_R = 1.2 \quad g_L = .65 \quad g_1 = .38$$

We vary the remaining according to Table 2.3.

| Case | $\lambda_A$ | $\lambda_B$ | $\lambda_\alpha$ | $Z_A$ | $Z_B$ | $Z_\alpha$ | $m_\Phi^2 (\text{GeV}^2)$ |
|------|-------------|-------------|------------------|-------|-------|------------|---------------------------|
| 1    | 0.9         | 0.8         | 0.99             | 0.65  | 0.3   | 1.29       | $300^2$                   |
| 2    | 0.5         | 0.45        | 0.1              | 0.54  | 0.3   | 0.16       | $-100^2$                  |
| 3    | 0.4         | 0.4         | 0.2              | 0.36  | 0.3   | 0.29       | $100^2$                   |
| 4    | 0.2         | 0.3         | 0.1              | 0.18  | 0.3   | 0.14       | $100^2$                   |
| 5    | 0.9         | 0.85        | 0.2              | 0.54  | 0.3   | 0.25       | $-100^2$                  |

Table 2.3: Points in parameter space used to evaluate the Higg masses

These values yield the following tree level masses for the Higgs Bosons (in GeVs) and the vacuum defining parameters respectively:

### 2.4.3 Implications

The TeV scale theory in this model differs from MSSM in that we have several new particles in the 100 GeV–TeV range. These particles are :  $d^{++}$ ,  $D^{++}$ ,  $d^0$ ,  $\tilde{d}^{++}$ ,

| Case | $D^{++}$ | $d^{++}$ | $D^+$                | $H^+$ | $D^0$                | $d^0$ | $H^0$ | $h^0$ | $B^0$ | $A^0$ |
|------|----------|----------|----------------------|-------|----------------------|-------|-------|-------|-------|-------|
| 1    | 990      | 160      | $3.4 \times 10^{10}$ | 190   | $5.0 \times 10^{10}$ | 920   | 170   | 93    | 620   | 170   |
| 2    | 1100     | 240      | $4.8 \times 10^{10}$ | 190   | $7.1 \times 10^{10}$ | 980   | 170   | 90    | 800   | 170   |
| 3    | 1200     | 210      | $5.2 \times 10^{10}$ | 190   | $7.8 \times 10^{10}$ | 950   | 170   | 90    | 720   | 170   |
| 4    | 1600     | 560      | $7.4 \times 10^{10}$ | 190   | $11 \times 10^{10}$  | 950   | 170   | 93    | 710   | 170   |
| 5    | 990      | 170      | $3.3 \times 10^{10}$ | 190   | $4.9 \times 10^{10}$ | 900   | 170   | 93    | 550   | 170   |

Table 2.4: The Higgs masses at tree level based on parameters from Table 2.3. The masses are given in GeV. As predicted previously, the doubly charged particles ( $D^{++}$  and  $d^{++}$ ) have masses in the electroweak range.

| Case | $v_R$ (GeV)          | $\epsilon$            | $\tan \beta$ | $\langle V \rangle_{CV} - \langle V \rangle_{CC}$ (GeV <sup>4</sup> ) |
|------|----------------------|-----------------------|--------------|---|
| 1    | $2.8 \times 10^{10}$ | $5.0 \times 10^{-17}$ | $\infty$     | $1.7 \times 10^{27}$  |
| 2    | $4.0 \times 10^{10}$ | $2.5 \times 10^{-17}$ | 9.9          | $4.0 \times 10^{27}$  |
| 3    | $4.4 \times 10^{10}$ | $2.1 \times 10^{-17}$ | 9.9          | $5.4 \times 10^{27}$  |
| 4    | $6.1 \times 10^{10}$ | $1.0 \times 10^{-17}$ | 50           | $18 \times 10^{27}$   |
| 5    | $2.8 \times 10^{10}$ | $5.2 \times 10^{-17}$ | 50           | $1.6 \times 10^{27}$  |

Table 2.5: Vacuum related parameters based on parameters from Table 2.3. The second column shows  $v_R$  and as can be seen is the correct order of the seesaw scale. The last column presents the difference in the ground state energy of the charge violating and the charge conserving vacuum. A positive value in this column indicates that the charge conserving vacuum is the stable one.

$\tilde{D}^{++}$  and  $\tilde{d}^0$ . The charged particles lead to spectacular signatures in colliders due to their decay modes:  $d^{++} \rightarrow \ell^+\ell^+$ ,  $\tilde{d}^{++} \rightarrow \ell^+\ell^+\chi_1^0$ . On the other hand, the neutral particles will be hard to produce in the laboratory because of their low coupling values to MSSM matter content. Their dominant decay channel is via  $d^0 \rightarrow \chi_1^0\chi_1^0$  with decay lifetimes of the order  $10^{-10}$  sec for generic values of the parameters. It is worthwhile to mention that  $d^0$  and  $\tilde{d}^0$  would have been present in the early stages of the universe, but would have decayed away before the era of Big Bang nucleosynthesis and therefore do not alter our understanding of this period.

## 2.5 Grand unification prospects

Since the effective TeV scale theory in our model is very different from MSSM (due to the presence of a pair of doubly charged fields), it is interesting to explore whether there is grand unification of couplings. This question was investigated in [41], where it was noted that if there are two pairs of Higgs doublets (corresponding to two bidoublets  $\phi_{1,2}(2, 2, 0)$ ), at the TeV scale, the gauge couplings unify around  $10^{12}$  GeV or so. This raises an interesting point: if there is a grand unified theory at  $10^{12}$  GeV, then this theory must be very different from conventional GUT theories. This is because limits on the proton life time require that the scale of grand unification be  $10^{15}$  GeV. Our GUT theory, should it exist, must conserve baryon number due to the low unification scale.

An example of such a theory is the  $SU(5) \times SU(5)$  model discussed in [42], which embeds the left-right symmetric group we are discussing. We do not discuss

the details of this theory here, but rather indicate the basic features: we envision  $SU(5) \times SU(5)$  to be broken[42] down to  $SU(3)^c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  by a Higgs multiplet belonging to the representation  $\Phi \equiv (\mathbf{5}, \bar{\mathbf{5}})$  with vev as follows:  $\langle \Phi \rangle = \text{diag}(a, a, a, 0, 0)$ . This is then subsequently broken to the standard model.

The fermions in this model belong to the  $(\bar{\mathbf{5}}, \mathbf{1}) \oplus (\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \bar{\mathbf{5}}) \oplus (\mathbf{1}, \mathbf{10})$  representation as follows:

$$F_L = \begin{pmatrix} D^c \\ D^c \\ D^c \\ \nu \\ e \end{pmatrix} \quad T_L = \begin{pmatrix} 0 & U^c & U^c & u & d \\ -U^c & 0 & U^c & u & d \\ -U^c & -U^c & 0 & u & d \\ -u & -u & -u & 0 & E^+ \\ -d & -d & -d & -E^+ & 0 \end{pmatrix} \quad (2.86)$$

and similarly for the right chiral fields.

Implementation of the seesaw mechanism in this model requires the addition of the Higgs representation  $(\mathbf{15}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{15})$  along with their complex conjugate representations. The multiplet  $b(\mathbf{1}, \mathbf{15})$  plays the role of  $\Delta^c$  of the left-right model. When the  $\nu^c \nu^c$  component of  $(\mathbf{1}, \mathbf{15})$  acquires a vev, it gives mass to the right handed neutrino fields triggering the seesaw mechanism. The doubly charged Higgs fields are part of the right handed  $(\mathbf{1}, \mathbf{15})$  Higgs representation. Symmetry breaking and fermion masses in this model are briefly touched on in [42].

## 2.6 Conclusion

Left-right models are well motivated for various reasons including the naturalness of the seesaw mechanism for neutrino masses in SLRM, providing a solution to the strong CP problem and the gauging of  $B - L$ , which allows a relationship between electric charge and the physical  $B - L$  quantum numbers. Certain SUSY versions of these models have the added advantages of automatic  $R$ -parity, a solution to the SUSY strong CP problem, and a seesaw mechanism which depends on fewer parameters and does not allow for a large left-handed triplet VEV. In addition, if the superpotential of the model is assumed to obey an  $Z$ -symmetry, then the  $B - L$  breaking scale (seesaw scale) can be predicted to be around  $10^{11}$  GeV—a phenomenologically acceptable value for this scale. This model also solves the  $\mu$  problem of the MSSM and predicts two TeV scale doubly charged bosons and fermions which couple to like sign dileptons and like sign lepton-slepton respectively. Such particles have been searched for in various existing experiments and will be searched for at the LHC and other future colliders[43]. Additionally, the model predicts unstable neutral bosons and fermions which can not be easily probed by experiment, but which would have been produced in the early universe.

Finally the conclusions of this paper can be equally applied to the group  $SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ , with the mass spectrum being identical except for the lack of light doubly charged particles and a heavy singly charged particle.

## Chapter 3

### Anomaly Mediated Supersymmetry Breaking, A Review

This chapter is a more in depth look into AMSB than the one given in Section 1.3.2. It starts with a brief introduction to superconformally invariant supergravity in Section 3.1. Section 3.2 shows how the soft terms are generated by the loop-level breaking of the superconformal invariance and the independence of the soft terms to thresholds are verified in Section 3.3. The issues with AMSB are quickly discussed along with proposed solutions in Section 3.4 and the deflected AMSB scenario is expanded upon in Section 3.5. The last section, Section 3.6 discusses  $D$ -terms in both deflected and pure AMSB scenarios. The work presented here is not new but simply acts as background for Chapter 4.

#### 3.1 Superconformal Invariance

Utilizing gravity in the mediation of SUSY breaking effects is attractive due to the ubiquity of gravity and because it is unnecessary to appeal to some messenger sector like in GMSB. Gravity mediation discussed in Section 1.3.2, takes advantage of this by coupling heavy fields with large VEVs to the MSSM with an  $M_P$  suppression. Once these are integrated out, the soft Lagrangian is generated. AMSB[44, 45] is a more sophisticated approach in that SUSY breaking is communicated via the light supergravity multiplet itself, which is sourced by the hidden sector. The con-

tributions in AMSB are most easily understood in the context of superconformal invariance of the supergravity Lagrangian. This formalism is analogous to the procedure in which the Einstein Lagrangian is made locally scale invariant by introducing an unphysical scalar field (the conformal compensator), which can be gauged away to recover the original theory. The benefit of this approach is that the superconformal invariance severely restricts the Lagrangian making it relatively easy to write. Once the the gauge freedom is taken away, the left-over theory is the more difficult to postulate supergravity theory.

In SUSY, the superconformal invariance is a product of both the scale invariance and a  $U(1)_R$  symmetry. The conformal compensator now becomes an unphysical chiral supermultiplet, the superconformal compensator. It has a Weyl weight  $d_W(\phi) = +1$ , corresponding to the scale invariance, and a  $U(1)_R$  charge of  $+2/3$ [46, 47]. It is given by

$$\phi = \eta + \sqrt{2}\theta\chi + \theta^2 F_\phi \tag{3.1}$$

where in the original theory,  $F_\phi$  is an auxiliary field in the supergravity supermultiplet. As such, it is analogous to the  $D$ -terms of the MSSM gauge groups. For  $\langle F_\phi \rangle \neq 0$ , SUSY is broken in an analogous fashion to  $D$ -term breaking and is then communicated to the visible sector, which is charged under gravity, of course. Specifically

$$\langle \phi \rangle = 1 + \theta^2 F_\phi \tag{3.2}$$

The form of the SUSY breaking contributions are then very closely related to superconformal invariance and understanding the form of the latter facilitates an



|                 | $d_W$          | $R$  |
|-----------------|----------------|------|
| $\theta$        | $-\frac{1}{2}$ | $+1$ |
| $\bar{\theta}$  | $-\frac{1}{2}$ | $-1$ |
| $d\theta$       | $+\frac{1}{2}$ | $-1$ |
| $d\bar{\theta}$ | $+\frac{1}{2}$ | $+1$ |

Table 3.1: Weyl weight and  $R$  charges of superspace coordinates

|               | $d_W$ | $R$  |
|---------------|-------|------|
| $\mathcal{K}$ | $+2$  | $0$  |
| $\mathcal{W}$ | $+3$  | $+2$ |

Table 3.2: Derived Weyl weight and  $R$  charge assignments for the Kähler and Super Potentials

understanding of the former. This can be done by starting with a general Lagrangian

$$\mathcal{L} = \int d^4\theta \mathcal{K}(D_\alpha, Q, W_\alpha) + \left( \int d^2\theta \mathcal{W}(Q, W_\alpha) + \text{h.c.} \right) \quad (3.3)$$

where  $Q$  collectively represents the matter content and  $\mathcal{W}$  is a sum of the superpotential and  $W_\alpha W^\alpha$ , where the latter contains the superfield strength. Note that the dependence of  $\mathcal{K}$  on  $\bar{D}_{\dot{\alpha}}$ ,  $Q^\dagger$ , etc. has been suppressed

The superspace coordinate charge assignments (See Table 3.1) force the Kähler potential and superpotential to have the charges shown in Table 3.2. Given  $d_W(\tilde{Q}) = d_W(\tilde{W}_\alpha) = R(\tilde{Q}) = R(\tilde{W}_\alpha) = 0$  (with  $\tilde{Q}$  being the matter fields and  $\tilde{W}_\alpha$  the gauge

fields, but not in the canonically normalized form), then

$$\mathcal{W} = \widetilde{\mathcal{W}}X_{\mathcal{W}} \qquad \mathcal{K} = \widetilde{\mathcal{K}}X_{\mathcal{K}} \qquad (3.4)$$

where the “tilded” potentials are functions of only the “tilded” fields. Since the “tilded” fields have no charges, the resulting potentials don’t either; hence all the transformational weights belong to the  $X_n$ :

$$\begin{aligned} d_W(X_{\mathcal{K}}) &= +2 & d_W(X_{\mathcal{W}}) &= +3 \\ R(X_{\mathcal{K}}) &= 0 & R(X_{\mathcal{W}}) &= +2 \end{aligned}$$

Now because the  $X_n$  carry charges, they can only depend on the conformal compensator  $\phi$  (we’ve already removed any other fields’ dependence into the potentials). Therefore invariance necessitates

$$X_{\mathcal{K}} = \phi^\dagger \phi \qquad X_{\mathcal{W}} = \phi^3 \qquad (3.5)$$

We can now write the most general superconformal invariant lagrangian. It is given by

$$\mathcal{L} = \frac{1}{2} \int d^4\theta \phi^\dagger \phi \widetilde{\mathcal{K}}(\tilde{D}_\alpha, \tilde{Q}, \tilde{W}_\alpha) + \int d^2\theta \phi^3 \widetilde{\mathcal{W}}(\tilde{Q}, \tilde{W}_\alpha) + \text{h.c.} \qquad (3.6)$$

This picture explicitly demonstrates the  $\phi$  couplings as required by superconformal invariance. It is possible to return to the usual fields by defining

$$Q = \phi \tilde{Q} \qquad W_\alpha = \phi^{3/2} \tilde{W}_\alpha \qquad (3.7)$$

To illustrate how these definitions return the canonical fields, the potentials must

be rewritten schematically as

$$\tilde{\mathcal{K}} = Z\tilde{Q}^\dagger e^W \tilde{Q} + \dots = Z\tilde{Q}^\dagger \tilde{Q} + \dots \quad (3.8)$$

$$\tilde{\mathcal{W}} = L\tilde{Q} + M\tilde{Q}^2 + Y\tilde{Q}^3 + \frac{\lambda}{\Lambda}\tilde{Q}^4 + \dots + \tilde{W}^\alpha \frac{1}{4g^2}\tilde{W}_\alpha + \dots \quad (3.9)$$

where  $Z$  is the wave-function renormalization for  $Q$  and  $g$  is the gauge coupling associated with  $W_\alpha$ , where the canonical gauge field  $A_\mu$  has been transformed  $A_\mu \rightarrow gA_\mu$ . It is then clear that the Lagrangian of Eq. (3.6), combined with the field redefinitions Eq. (3.7), leads to

$$\begin{aligned} \mathcal{L} = & \int d^4\theta Z [Q^\dagger Q + \dots] \\ & + \left( \int d^2\theta \left[ L\phi^2 Q + M\phi Q^2 + YQ^3 + \frac{\lambda}{\Lambda}\phi Q^4 + \dots + W^\alpha \frac{1}{4g^2}W_\alpha + \dots \right] + \text{h.c.} \right) \end{aligned} \quad (3.10)$$

Terms with dimensionful couplings break tree-level superconformal invariance and therefore introduce a  $\phi$  into the superpotential—something relevant for the MSSM because of the  $\mu$  term. Non-renormalizable terms always contain the pair  $\Lambda\phi$  to some power. As these terms usually result from a threshold, this form will be important when discussing intermediate thresholds. The tree-level superconformal invariance breaking directly translates into tree-level SUSY breaking:

$$\mathcal{L}_{Soft} = 2LF_\phi + MF_\phi - \frac{\lambda F_\phi}{\Lambda} + \dots \quad (3.11)$$

Scalar mass, trilinear- $a$  and gaugino mass terms correspond to the dimensionless couplings  $Z$  and  $\frac{1}{g^2}$ , which do not break the superconformal invariance at tree-level and therefore do not lead to their SUSY breaking counterparts at tree-level. However, a Lagrangian of the form Eq. (1.45) can still be generated since loop-level

calculations force the introduction of a dimensionful parameter,  $\mu$  the renormalization scale, which break the scale invariance.

### 3.2 Superconformal Invariance Breaking and Soft Terms

When evaluating loop order calculations, some type of regulator is required, which can be chosen to be a cutoff  $\Lambda$ . This regulator is convenient to use because it has already been established that such a cutoff must be paired with  $\phi$  should it give rise to non-renormalizable terms of the form in Eq. (3.10) (the ultraviolet (UV) cutoff gets paired with a  $\phi$  independent of whether or not it yields non-renormalizable terms; however, it is a convenient illustration here). Therefore, renormalized quantities ( $Z$  and  $\frac{1}{g^2}$ ) will be functions of the form

$$\ln\left(\frac{\Lambda|\phi|}{\mu}\right) \tag{3.12}$$

From this, it is understood that all of the SUSY breaking can be parameterized by

$$\mu \rightarrow \frac{\mu}{|\phi|} = \ln \mu - \frac{1}{2}(\theta^2 F_\phi + h.c.) \tag{3.13}$$

where the expression to the right of the equal sign is the expansion of the natural logarithm around  $\mu$ . To utilize this, it is necessary to analytically continue  $Z$  and  $\frac{1}{g^2}$  into superspace, making them superfields. A general superfield for the wave function renormalization is

$$Z_i(\mu) \rightarrow \mathcal{Z}_i(\mu) = Z_i(\mu) + (B_i(\mu)\theta^2 + h.c.) + C_i(\mu)\theta^2\bar{\theta}^2 \tag{3.14}$$

where  $i$  represents a specific field  $Q_i$ . Upon substitution into Eq. (3.10), it can be seen that this can be expressed in a more convenient fashion

$$\ln \mathcal{Z}_i(\mu) = \ln Z_i(\mu) + (A_i(\mu) \theta^2 + h.c.) - m_i(\mu)^2 \theta^2 \bar{\theta}^2 \quad (3.15)$$

where  $m_i^2$  is the soft mass for  $Q_i$  and  $A_i$  contributes to trilinear  $a$ -terms, *e.g.* for  $a_t Q H_u t^c$ ,  $a_t = y_t(A_Q + A_{H_u} + A_{t^c})$ . The relationship between the  $\mathcal{Z}$  and  $Z$  is

$$\mathcal{Z}_i(\mu) = Z_i\left(\frac{\mu}{|\phi|}\right) \quad (3.16)$$

and  $\ln Z_i\left(\frac{\mu}{|\phi|}\right)$  can be expanded around  $\ln \mu$  for  $\phi = 1$ . This yields

$$\begin{aligned} \ln \mathcal{Z}_i(\mu) &= \ln Z_i(\mu) + \left. \frac{\partial \ln Z_i(\mu)}{\partial \ln \mu} \right|_{\phi=1} \left( \ln \mu - \frac{1}{2}(\theta^2 F_\phi + h.c.) - \ln \mu \right) \\ &\quad + \frac{1}{2} \left. \frac{\partial^2 \ln Z_i(\mu)}{\partial \ln \mu^2} \right|_{\phi=1} \left( \ln \mu - \frac{1}{2}(\theta^2 F_\phi + h.c.) - \ln \mu \right)^2 \\ &= \ln Z_i(\mu) - \frac{1}{2} \gamma_i(\mu) (\theta^2 F_\phi + h.c.) + \frac{1}{4} \frac{\partial \gamma_i(\mu)}{\partial \ln \mu} |F_\phi|^2 \theta^2 \bar{\theta}^2 \end{aligned} \quad (3.17)$$

where  $\gamma_i \equiv \frac{\partial \ln Z_i(\mu)}{\partial \ln \mu}$  is the anomalous dimension of  $Q_i$  and all higher order terms are zero. Comparing eqns. 3.15 and 3.17 yields

$$A_i(\mu) = -\frac{1}{2} \gamma_i(\mu) F_\phi \quad (3.18)$$

$$m_i^2(\mu) = -\frac{1}{4} \frac{\partial \gamma_i(\mu)}{\partial \ln \mu} |F_\phi|^2 = -\frac{1}{4} |F_\phi|^2 \left[ \beta_{g_a} \frac{\partial \gamma_i}{\partial g_a} + \left( \beta_{y_a} \frac{\partial \gamma_i}{\partial y_a} + h.c. \right) \right] \quad (3.19)$$

where the last expression in Eq. (3.19) is derived using the chain rule and a sum over all gauge couplings,  $g_a$  and Yukawa couplings  $y_a$ , is implied. Also,  $\beta_x \equiv \frac{\partial x}{\partial \ln \mu}$ .

The gauge coupling must be promoted into a chiral superfield because it appears in Eq. (3.10) in the  $d\theta^2$  integral.

$$\frac{1}{2} g_a^{-2}(\mu) \rightarrow R_a(\mu) = \frac{1}{2} g_a^{-2}(\mu) - \frac{i\Theta_a}{16\pi^2} - \frac{M_a}{g_a^2} \theta^2 \quad (3.20)$$

where  $M_a$  is the mass for the gaugino associated with the  $a$  gauge group. Using

$$R(\mu) = \frac{1}{2}g_a^{-2}\left(\frac{\mu}{\theta}\right) \quad (3.21)$$

$$\frac{\mu}{\theta} = \ln \mu - F_\phi \theta^2$$

Expanding  $g_a^{-2}\left(\frac{\mu}{\theta}\right)$  around  $\mu$

$$R_a(\mu) = \frac{1}{2}g_a^{-2} + \frac{1}{2} \left. \frac{\partial g_a^{-2}}{\partial \ln \mu} \right|_{\phi=1} (\ln \mu - F_\phi \theta^2 - \ln \mu) \quad (3.22)$$

$$= \frac{1}{2}g_a^{-2} - \frac{1}{2}\beta_{g_a^{-2}}F_\phi\theta^2$$

Comparing eqns. 3.20 and 3.22 yields

$$M_a = \frac{g_a^2}{2}\beta_{g_a^{-2}}F_\phi = \frac{b_a\alpha_a}{4\pi}F_\phi \quad (3.23)$$

where  $\beta_{g_a^{-2}} = \frac{b_a}{8\pi^2}$  has been used and there is no sum over  $a$ .

Eqns. 3.23, 3.18 and 3.19 are the AMSB expressions for the soft terms. They are renormalization scale invariant since they were derived for an unspecified renormalization scale,  $\mu$ . The name anomaly is employed since the generation of these terms are associated with the anomalous breaking of the superconformal invariance. Note that compared to the tree-level SUSY breaking terms in Eq. (3.11), they are suppressed by  $16\pi^2$  per dimension (this suppressions are contained in the  $\beta$ - and  $\gamma$ -functions, both of which contain a factor of  $\frac{1}{16\pi^2}$ ). This means that  $F_\phi \gtrsim 20$  TeV for soft terms of the right order and therefore, in the MSSM, the tree-level soft term  $b = \mu F_\phi \sim 16\pi^2 m_{susy} \mu$ , is too large for EWSB. This indicates that the MSSM is not compatible with AMSB.

### 3.3 Decoupling Thresholds

The AMSB soft expressions, eqns. 3.23, 3.18 and 3.19, are very interesting because they are independent of thresholds. To understand this, assume there is a threshold  $M$  such that  $\Lambda \gg M \gg F_\phi$  and that this threshold does not introduce any new SUSY breaking effects, as would be possible with light singlets. Once the heavy fields have been integrated out, the leading effects of  $M$  can only appear as logarithms via quantum corrections: no positive powers can exist. It is then possible to make an analogy to the previous situation.  $M$  is a threshold and must always appear with a factor of  $\phi$ . Therefore Eq. (3.12) becomes

$$\ln\left(\frac{\Lambda|\phi|}{\mu}\right) \rightarrow C^+ \ln\left(\frac{\Lambda|\phi|}{M|\phi|}\right) + C^- \left(\frac{M|\phi|}{\mu}\right) \quad (3.24)$$

where  $C^\pm$  are parameters related to physics above/below the threshold  $M$ . The second term on the right-hand side is the new threshold term and the first term on the right-hand-side will figure into the boundary value at  $M$ . Of course, the  $\phi$  dependence cancels in the boundary value term and so the wave function renormalization and gauge couplings have the following dependences:

$$Z_i\left(\frac{\mu}{|\phi|}, M\right), \quad g_a^{-2}\left(\frac{\mu}{|\phi|}, M\right) \quad (3.25)$$

The only difference between these terms and eqns. 3.16 and 3.21 is the presence of the threshold, which has been suppressed in the latter two equations and which has been renamed to  $M$  in the former. Expansions would follow as they do in eqns. 3.17 and 3.22 yielding the same results as eqns. 3.23, 3.18 and 3.19, the AMSB soft term expressions therefore proving the decoupling of the threshold.

This decoupling analysis applies to both mass thresholds, such if there exists a vector-like pair of heavy quarks, or to thresholds generated through some spontaneous symmetry breaking. The condition of no new SUSY violation in the latter case corresponds to a VEV of the superfield

$$\langle X \rangle = M\phi \tag{3.26}$$

As long as this is true, the soft terms will continue on their AMSB trajectories below  $M$ .

### 3.4 Problems and Solutions

The EWSB problem associated with AMSB has been briefly mentioned in Section 3.2, which can be potentially cured by applying AMSB to the NMSSM instead of the MSSM. In the NMSSM, the  $\mu$  term is understood as a VEV of a singlet field,  $\mu = \frac{1}{\sqrt{2}}\lambda\langle N \rangle$ . The Higgs superpotential is

$$W = \lambda N H_u H_d + \frac{1}{3}\kappa N^3 \tag{3.27}$$

and has no dimensionful parameter and therefore no tree-level SUSY breaking.

The tachyonic slepton problem was demonstrated in Eq. (1.58) and exists for all sleptons. Many solutions to this problem have been proposed [48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58] despite the fact that solutions to this problem are hard to come by due to the independence of higher thresholds. Two of these solutions are: new low energy particle content with Yukawa couplings to the leptons and introduction of new SUSY breaking, in addition to AMSB, so that soft masses are deflected from their AMSB trajectories.



Solutions involving new Yukawa couplings have the advantage of retaining the AMSB trajectories. However, they typically lack justification since they are normally just an *ad hoc* addition of fields. There are some exceptions, such as Yukawa couplings due to  $R$ -parity violation in the lepton sector[53]. While this is a pleasing minimal model with no new fields, it does push the value of  $F_\phi$  very high ( $\sim 200$  TeV, exasperating the little hierarchy problem) because of the stringent lepton number constraints. A different approach is to utilize the Yukawa couplings of the naturally light doubly charged Higgses in MSLRMs[59, 58]. This has the advantage of being well-motivated and that typical scales of  $F_\phi$  can be used. However, it requires that the doubly-charged fields have a mass on the order of  $F_\phi$ , which is a bit of a coincident problem since they may be as massive as the GUT scale.

Scenarios in which new SUSY breaking is introduced can be divided into the two types:  $F$ -term[51] and  $D$ -term[54]. The former case is due to some SUSY breaking threshold,  $M$ , in a shallow potential, which produces light singlets. These allow the mass scale  $M$  to appear as a linear term in the Lagrangian below the threshold. For instance

$$\mathcal{K} \supset \int d^4\theta (S^\dagger S + (cM\phi S^\dagger + h.c.) + \dots) \quad (3.28)$$

This would lead to a large SUSY breaking  $F$ -term for the singlet  $S$

$$F_S = -cMF_\phi \quad (3.29)$$

Depending on the value of  $c$ , this new contribution can be significant compared to the AMSB contribution. If this is the case, the AMSB trajectories will be deflected, the threshold will not fully decouple and the sleptons masses can potentially be

|              | $U(1)_X$ |
|--------------|----------|
| $X$          | $-2$     |
| $\bar{X}$    | $+2$     |
| $\Psi$       | $+1$     |
| $\bar{\Psi}$ | $-1$     |
| $S$          | $0$      |

Table 3.3: The  $U(1)$  charges and particle content of a toy model that demonstrates decoupling/deflection of thresholds in AMSB

saved. Further details of this scenario as well as a toy model will be discussed in Section 3.5.

### 3.5 Deflected Anomaly Mediation

Deflected AMSB was introduced by Pomarol and Rattazzi using the superpotential:

$$W_{PR} = fX\Psi\Psi + \frac{\lambda}{\Lambda\phi}(X\bar{X})^2 \quad (3.30)$$

with charges under a  $U(1)_X$  local gauge symmetry given in Table 3.3. The only source of SUSY breaking in this theory is AMSB and is reflected in the appearance of the factor of  $\phi$  in Eq. (3.30). The  $F$ -terms and corresponding  $F$ -potential for the

$X$  superfields assuming that the  $\Psi$  superfields are VEV-less, are given by

$$-F_X^* = \frac{\partial W}{\partial X} = \frac{2\lambda}{\Lambda} x \bar{x}^2 \quad (3.31)$$

$$-F_{\bar{X}}^* = \frac{\partial W}{\partial \bar{X}} = \frac{2\lambda}{\Lambda} x^2 \bar{x} \quad (3.32)$$

$$V_F = \frac{4\lambda^2}{\Lambda^2} x^2 \bar{x}^2 (x^2 + \bar{x}^2) \quad (3.33)$$

where the lower case letters are the scalar components of the corresponding upper case superfields. From Eq. (3.31) it is clear that if the  $x$  fields acquire a VEV it will be SUSY breaking since the  $F$ -terms will be non-zero. In order to investigate if this happens, it is necessary to consider the tree-level SUSY breaking associated with the non-renormalizable term. It contributes

$$V_{Soft} = \frac{\lambda F_\phi}{\Lambda} x^2 \bar{x}^2 \quad (3.34)$$

Defining

$$\langle x \rangle \sim \langle \bar{x} \rangle \sim M \quad (3.35)$$

and minimizing the potential yields

$$M = \sqrt{\frac{-F_\phi \Lambda}{6\lambda}} \quad (3.36)$$

It is now possible to investigate the  $F$ -terms more closely, using Eq. (3.31) and Eq. (3.36)

$$-F_X^* \sim -F_{\bar{X}}^* = \frac{1}{3} F_\phi M = M F_\phi + F_{\mathcal{M}} \quad (3.37)$$

and

$$F_{\mathcal{M}} \equiv r M F_\phi \quad (3.38)$$

$$r \equiv -\frac{2}{3}$$

where the right-hand side of Eq. (3.37) is useful since it separates the two sources of SUSY breaking. The first contribution,  $MF_\phi$  is simply the traditional AMSB contribution, which will lead to a decoupling threshold. It is possible to think of it as the VEV of a two superfields:

$$\langle X_D \rangle = M\phi = M + MF_\phi\theta^2 \quad (3.39)$$

in line with Eq. (3.26). Again this leads to the same decoupling discussed in Section 3.3. The second contribution is new:

$$\langle X_{ND} \rangle = \mathcal{M} = M - \frac{2}{3}MF_\phi\theta^2 \neq \tilde{M}\phi \quad (3.40)$$

where  $\tilde{M}$  is some mass scale. This makes it clear that new SUSY breaking has been introduced and  $r$  is a measure of the additive deviation from the AMSB contribution. Since the decoupling associated with AMSB is related to the contribution in Eq. (3.39), the contribution from Eq. (3.40) will not decouple and will lead to new SUSY breaking contributions proportional to  $r$ .

These contributions will be GMSB. This is because the SUSY violating super-VEV  $\mathcal{M}$  will introduce mass splittings between heavy fermionic and bosonic degrees of freedom in the  $X$  fields related to  $-\frac{2}{3}MF_\phi$ . Once these heavy fields are integrated out, this SUSY breaking will be relayed to the visible sector fields by the messenger  $\Psi$  fields assuming that the visible sector is also charged under  $U(1)_X$ .

Derailment from the AMSB trajectory has its consequences. It is no longer possible to calculate the solutions to the RGEs as functions of the low energy  $\beta$ - and  $\gamma$ -functions, rather it is necessary to find the value at the threshold (which depends on high energy physics) and then evolve it to the necessary scale using RGEs. The

boundary conditions are calculated in a similar fashion to the AMSB expressions given in eqns. 3.23, 3.18 and 3.19 by expanding the supercouplings. However, it is now necessary to expand around the threshold,  $\mathcal{M}$ , instead of an arbitrary  $\mu$  value. The key is an understanding of how SUSY breaking is parameterized in the wave function renormalization and the gauge coupling comparable to Eq. (3.24). Since it has already been established that the SUSY-conserving threshold,  $M$ , is unimportant, it is just necessary to focus on  $\mathcal{M}$  and  $\phi$ . This translates into simply replacing  $M$  by  $\mathcal{M}$  in Eq. (3.24):

$$\ln\left(\frac{\Lambda|\phi|}{\mu}\right) \rightarrow C^+ \ln\left(\frac{\Lambda|\phi|}{\mathcal{M}|\phi|}\right) + C^- \left(\frac{\mathcal{M}|\phi|}{\mu}\right) \quad (3.41)$$

Since all SUSY breaking in a quantity can be picked out by the derivative  $\frac{\partial^2}{\partial\theta^2}$ , understanding Eq. (3.41) allows the identification:

$$\frac{\partial^2}{\partial\theta^2} \rightarrow rF_\phi \frac{\partial}{\partial \ln M} - F_\phi \frac{\partial}{\partial \ln M} \quad (3.42)$$

given Eq. (3.40). The result will therefore depend on parameters associated with the higher scale,  $C^+$  and lower scale  $C^-$  physics and is calculated to be

$$M_a(M) = \frac{\alpha_a}{4\pi} F_\phi [r(b_a^+ - b_a^-) - b_a^-] \quad (3.43)$$

$$A_Q(M) = \frac{1}{2} F_\phi [r(\gamma_Q^+ - \gamma_Q^-) - \gamma_Q^-] \quad (3.44)$$

$$m_Q^2(M) = -\frac{1}{4} |F_\phi|^2 \sum_{g_a} \left[ r^2 \left( \beta_{g_a}^+ \frac{\partial \gamma_Q^+}{\partial g_a} - 2\beta_{g_a}^+ \frac{\partial \gamma_Q^-}{\partial g_a} + \beta_{g_a}^- \frac{\partial \gamma_Q^-}{\partial g_a} \right) - 2r \left( \beta_{g_a}^+ \frac{\partial \gamma_Q^-}{\partial g_a} - \beta_{g_a}^- \frac{\partial \gamma_Q^-}{\partial g_a} \right) + \beta_{g_a}^- \frac{\partial \gamma_Q^-}{\partial g_a} \right] \quad (3.45)$$

where  $Q$  represents some superfield. These equations share some general interesting features. In the limit  $r \rightarrow 0$ , only the last term in each equation remains and agrees

with the AMSB contributions eqns. 3.23, 3.18 and 3.19 as expected. In the limit that  $F_\phi \rightarrow 0$  such that  $rF_\phi$  remains constants, the equations reproduce the GMSB results with  $\Lambda = \frac{F_\phi}{M}$ . The second term in Eq. (3.45) is a cross-term between AMSB and GMSB. Furthermore, the GMSB contributions in each of these equations in some sense indicates the amount of particle content that has been integrated out, as can be seen by the difference in quantities above and below the scale. Therefore, sectors of the theory that do not couple strongly to the heavy fields will remain more or less on their AMSB trajectories. Lastly, since deflected AMSB has introduced a dependence on higher energy physics, it must be assumed that the threshold lies below any sort of flavor physics to conserve the solution to the SUSY flavor problem. This is yet another price to pay for the deflection.

### 3.6 $D$ -Terms

This phenomena of a partially decoupled threshold has implications even for the  $D$ -terms, which have not yet been discussed in the context of AMSB. In general, it is expected that a broken gauge group will lead to  $D$ -term contributions to soft masses (Table 2.2 shows such contributions for MSLRM). However, such contributions are zero in AMSB. Taking a detour to explore a concrete AMSB preserving model will help shed some light on this and the issues discussed earlier. To start, examine a theory with the same particle content as Table 3.3 but without a shallow potential:

$$W_{AMSB} = fX\Psi\Psi + yS(X\bar{X} - M^2\phi^2) \quad (3.46)$$

In the SUSY limit, the scalar components of  $X$  and  $\bar{X}$  acquire a VEV equal to  $M$  thus introducing a threshold. At this point all the fields, except  $\bar{\Psi}$ , gain a mass of  $M \gg F_\phi$ . The VEV structure is

$$\langle x \rangle = M \tag{3.47}$$

$$\langle \bar{x} \rangle = M \tag{3.48}$$

$$\langle F_X \rangle = MF_\phi \tag{3.49}$$

$$\langle F_{\bar{X}} \rangle = MF_\phi \tag{3.50}$$

$$\langle s \rangle = -\frac{F_\phi^\dagger}{y} \tag{3.51}$$

$$\langle D \rangle = \frac{1}{2g}(m_{\bar{X}}^2 - m_X^2) = -\frac{|F_\phi|^2}{4g} \frac{\partial \gamma_X^+}{\partial f} \beta_f \tag{3.52}$$

with the  $D$ -term acquiring a VEV because  $X$  couples to  $\Psi$  and  $\bar{X}$  does not; hence, the AMSB expression for their scalar masses are not equal.

Above the threshold  $M$ ,  $\bar{\Psi}$  has a scalar mass given by AMSB

$$(m_{\bar{\Psi}}^2)^+ = -\frac{5}{4}g^4 \left( \frac{F_\phi}{16\pi^2} \right)^2 \tag{3.53}$$

while below  $M$  there is no gauge group and  $W_{AMSB}^- = 0$  so that AMSB predicts

$$(m_{\bar{\Psi}}^2)^- = 0 \tag{3.54}$$

The fact that AMSB predicts  $\bar{\Psi}$ 's scalar mass to be zero below  $M$  raises two questions: how the contribution of the gauge group given by Eq. (3.53) disappeared, and why the  $D$ -term VEV—acquired at the threshold—vanished. Both questions are resolved by noting that the  $\Psi$ 's act as messengers giving a GMSB contribution at the threshold  $M$ . This is because the lagrangian from Eq. (3.46) contains the

term

$$\int d^2\theta W_{AMSB} \supset f \langle F_X \rangle \Psi \Psi = f M F_\phi \Psi \Psi = M_\Psi F_\phi \Psi \Psi \quad (3.55)$$

which appears in loops.

For example, the scalar  $\bar{\Psi}$  couples to the  $\Psi$ 's through the  $D$ -term potential

$$V_D = \frac{1}{2} g^2 \left[ \left( |X|^2 - |\bar{X}|^2 \right)^2 + |\Psi|^2 \left( |X|^2 - |\bar{X}|^2 \right) - |\Psi|^2 \left( |X|^2 - |\bar{X}|^2 \right) + \frac{1}{4} \left( |\Psi|^2 - |\bar{\Psi}|^2 \right)^2 \right] \quad (3.56)$$

leading to the diagram

$$\bar{\Psi} \rightarrow \frac{1}{g^2} \left[ \begin{array}{c} \Psi \nearrow \\ \Psi \searrow \end{array} \right] \rightarrow \bar{\Psi} \rightarrow \frac{1}{g^2} \left[ \begin{array}{c} \Psi \searrow \\ \Psi \nearrow \end{array} \right] \rightarrow \bar{\Psi} \sim \frac{g^4 f^2 |F_X|^2}{(16\pi^2)^2 M_\Psi^2} = g^4 \frac{|F_\phi|^2}{(16\pi^2)^2} \quad (3.57)$$

which is exactly the same structure and size as the AMSB contribution above the threshold. In fact, Eq. (3.57), along with the other diagrams involving gauge fields, yields

$$(m_\Psi^2)^- = (m_\Psi^2)^+ + (m_\Psi^2)_{\text{GMSB}} = 0 \quad (3.58)$$

The GMSB diagrams such as Eq. (3.57) cancel the higher-scale AMSB contributions to  $\bar{\Psi}$ 's scalar mass; however, they do not remove the  $D$ -term portion acquired at the threshold. Rather, this term's cancellation can be seen as a result of the  $D$  VEV actually being zero below the threshold—GMSB diagrams like Eq. (3.57), with  $\bar{\Psi}$  replaced by  $X$ ,  $\bar{X}$  cause the scalar masses of these fields to be zero below  $M$  resulting in the VEV of  $D$  vanishing.



Now it is possible to turn back to the model with a partially decoupled threshold, specified by Eq. (3.30), and examine the  $D$ -terms. Once again, the splitting executed in Eq. (3.37) is advantageous because then the  $\langle F_X \rangle \Psi \Psi$  term in the lagrangian splits apart as

$$\int d^2\theta W_{PR} \supset f \langle F_X \rangle \Psi \Psi = f M F_\phi \Psi \Psi + f F_{\mathcal{M}} \Psi \Psi \quad (3.59)$$

so that the diagram of Eq. (3.57) cleaves into

$$\begin{aligned} \bar{\Psi} \rightarrow \frac{1}{g^2} \left( \begin{array}{c} \Psi \xrightarrow{f \langle F_X \rangle} \Psi \\ \bar{\Psi} \\ \Psi \xrightarrow{f \langle F_X^* \rangle} \Psi \end{array} \right) \rightarrow \bar{\Psi} = \\ \underbrace{\bar{\Psi} \rightarrow \frac{1}{g^2} \left( \begin{array}{c} \Psi \xrightarrow{f F_{\mathcal{M}}} \Psi \\ \bar{\Psi} \\ \Psi \xrightarrow{f F_{\mathcal{M}}^*} \Psi \end{array} \right) \rightarrow \bar{\Psi}}_{\text{This diagram is from the new SUSY breaking and so it must survive below } M} + \underbrace{\bar{\Psi} \rightarrow \frac{1}{g^2} \left( \begin{array}{c} \Psi \xrightarrow{f M F_\phi} \Psi \\ \bar{\Psi} \\ \Psi \xrightarrow{f M F_\phi^\dagger} \Psi \end{array} \right) \rightarrow \bar{\Psi}}_{\text{This diagram is the GMSB contribution that cancels the higher scale AMSB part so it has no net effect below } M} \end{aligned} \quad (3.60)$$

By defining  $F_{\mathcal{M}}$  the net result of Eq. (3.60) is that the higher-scale AMSB portion (which is canceled below  $M$ ) is factored out and all that remains are the contributions due to  $F_{\mathcal{M}}$ :

$$(m_{\bar{\Psi}}^2)_{\mathcal{M}}^- = \underbrace{(m_{\bar{\Psi}}^2)_{\text{AMSB}}^+ + (m_{\bar{\Psi}}^2)_{\text{GMSB}}}_{\text{this is just } (m_{\bar{\Psi}}^2)_{\text{AMSB}}^- = 0} + (m_{\bar{\Psi}}^2)_{\mathcal{M}} \quad (3.61)$$

This is true for the  $\bar{\Psi}$  field, *but it is also true for the  $X$  and  $\bar{X}$  scalar masses.*

The latter point is important because the VEV of  $D$ , which depends on the difference

of the scalar masses of  $X$  and  $\bar{X}$ , will consequently also be non-zero:

$$\langle D \rangle = \frac{1}{2g} \left[ (m_{\bar{X}}^2)_{\mathcal{M}} - (m_X^2)_{\mathcal{M}} \right] \sim \frac{1}{4g} \left| \frac{F_{\mathcal{M}}}{M} \right|^2 f^4 \quad (3.62)$$

The last expression of Eq. (3.62) follows from the fact that the only difference between  $X$  and  $\bar{X}$  is the coupling  $f$ .

Now, as  $\langle D \rangle \neq 0$ , there is an additional contribution to the  $\bar{\Psi}$  scalar mass from this term:

$$(m_{\bar{\Psi}}^2)^- = (m_{\bar{\Psi}}^2)_{\mathcal{M}} + (m_{\bar{\Psi}}^2)_D \quad (3.63)$$

This new contribution will simply be the old typical non-AMSB contributions multiplied by a factor of  $r^2$ . Therefore, deflected AMSB leads to new contributions both from the boundary conditions and  $D$ -terms.

## Chapter 4

### Predictive Supersymmetric Left-Right Model and Anomaly

#### Mediation

The model discussed in Chapter 2 can be embedded in any SUSY breaking scenario and studies have been done on the effects of the light-doubly charged Higgs on the SUSY spectrum in mSUGRA and GMSB scenarios[41, 60] as well as the GUT prospects for such models. However, MSLRM have a more interesting effect in AMSB models. Section 4.1 points out that PSLRM is an instance of the deflected AMSB scenario and that the slepton masses are made non-tachyonic by a combination of new Yukawa couplings to the doubly-charged fields for the right-handed sleptons and partially decoupled  $D$ -terms for the left-handed ones. Section 4.2 then discusses the phenomenology of this model including slepton and squark masses and the consequences of the LSP in both collider and astro physics.

#### 4.1 The Theory

Comparing  $W_{PR}$  in Eq. (3.30) to the PSLRM superpotential, Eq. (2.18) makes it clear that PSLRM can be viewed as an instance of the Pomarol Rattazzi model. It has the necessary ingredients: a shallow potential that gives rise to light singlets, Section 2.4.1, and  $F$ -terms that introduce new SUSY breaking, Eq. (2.21). Furthermore, it is in line with the conformal invariance at the renormalizable-level. The

superpotential is

$$W_{PSLRM} = W_Y + W_{NR} + W_N \quad (4.1)$$

$$W_Y = \text{i}y_Q^a Q^T \tau_2 \Phi_a Q^c + \text{i}y_L^a L^T \tau_2 \Phi_a L^c + \text{i}f_c L^{cT} \tau_2 \Delta^c L^c \quad (4.2)$$

$$W_N = \lambda^{ab} N \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{3} \kappa N^3 \quad (4.3)$$

$$\begin{aligned} W_{NR} = & \frac{\lambda_A}{M_P \phi} \text{Tr}^2(\Delta^c \bar{\Delta}^c) + \frac{\lambda_B}{M_P \phi} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) \quad (4.4) \\ & + \frac{\lambda_\alpha^{ab}}{M_P \phi} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{\lambda_\beta^{ab}}{M_P \phi} \text{Tr}(\Delta^c \tau_2 \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c) \\ & + \frac{\lambda_N}{M_P \phi} \text{Tr}(\Delta^c \bar{\Delta}^c) N^2 + \dots \end{aligned}$$

where  $a = 1..2$  runs over  $\Phi$  generations. Eq. (4.1) has several differences compared to Eq. (2.18):

- An additional  $\Phi$  has been introduced for a realistic CKM matrix. Using the soft terms for this purpose is no longer an option since they will be constrained by deflected AMSB.
- Non-renormalizable terms that previously were used to solve the  $\mu$  problem, schematically  $\frac{\lambda}{M_P} \Delta^c \bar{\Delta}^c \Phi^2$ , can no longer do so since they would lead to  $b = \mu_{eff} F_\phi$  which is too large for EWSB. This will be expanded on in Section 4.1.3.
- A singlet,  $N$ , has been introduced so that the low energy theory is the NMSSM Eq. (3.27). This allows for a solution to the  $\mu$  problem via the VEV of  $N$  and does not involve terms that break the conformal invariance at the renormalizable level.
- To guarantee that the low energy theory is the NMSSM, an  $R$ -parity symmetry

must also be enforced:

$$(\Delta^c, \Phi, N) \rightarrow -(\Delta^c, \Phi, N) \quad (4.5)$$

$$\bar{\Delta}^c \rightarrow i\bar{\Delta}^c \quad (4.6)$$

$$(Q, Q^c, L, L^c) \rightarrow (Q, Q^c, L, L^c) \quad (4.7)$$

This will keep  $N$  light.

- Regardless of the presence of low energy NMSSM, a problem still exists with EWSB. It and its solution will be briefly addressed in Section 4.1.3.

In spite of these differences, modifications to the vacuum structure of the theory, as discussed in Section 2.3, are due to AMSB, which constrains the SUSY breaking potential

$$\begin{aligned} V_{\text{Soft}} = & m_{\Delta^c}^2 \text{Tr}(\Delta^{c\dagger}\Delta^c) + m_{\bar{\Delta}^c}^2 \text{Tr}(\bar{\Delta}^{c\dagger}\bar{\Delta}^c) + (m_{\Phi}^2)^{ab} \text{Tr}(\Phi_a^\dagger\Phi_b) \\ & + \left( a_\lambda^{ab} N \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{3} a_\kappa N^3 + \text{h.c.} \right) \\ & + \left( \frac{\lambda_A F_\phi}{M_P} \text{Tr}^2(\Delta^c \bar{\Delta}^c) + \frac{\lambda_B F_\phi}{M_P} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) + \dots + \text{h.c.} \right) \end{aligned} \quad (4.8)$$

Comparing Eq. (4.8) to Eq. (2.26) allows the identifications

$$-Z_A = \lambda_A \quad Z_B = \lambda_B \quad m_{3/2} = F_\phi \quad (4.9)$$

$F_\phi \gg m_{\Delta^c}, m_{\bar{\Delta}^c}$ , since the latter two are loop suppressed, the minimum given by Eq. (2.36) simplifies to

$$v^2 = \frac{-2M_P F_\phi}{3\lambda_A} \quad (4.10)$$

so that

$$\langle \Delta^{c0} \rangle = \sqrt{\frac{-M_P F_\phi}{6\lambda_A}} \quad (4.11)$$

where  $\langle \Delta^{c0} \rangle \equiv \sqrt{2}v_R$  and  $v^2 = 2v_R^2$ . As a result, the prediction of the seesaw scale is now slightly modified,  $M_R \sim \sqrt{F_\phi M_P} \sim 10^{11}$ , larger than the previous result. In regards to the Pomarol Rattazzi model, Eq. (4.11) is the same as Eq. (3.36) and the results from Section 3.5 can be used here and

$$r = -\frac{2}{3}. \quad (4.12)$$

However, there is an important new phenomena here: the messengers  $\Psi$  are replaced by the right-handed neutrinos. Unlike the  $\Psi$  fields, the right-handed neutrinos have direct coupling to the light right-handed sleptons through the seesaw term. These couplings will then also mediate SUSY breaking resulting in a mixed Gauge and Yukawa mediated SUSY breaking. Eqns. 3.44 and 3.45 already take these contributions into account. Throughout the rest of chapter, a reference to GMSB also implies the Yukawa mediated contributions.

These Yukawa contributions can save the masses of the right-handed sleptons as in [58, 59]. However, the left-handed contributions will not have a corresponding mechanism. This is why the deflection is important. As argued earlier, this will cause only a partial decoupling of the threshold and will allow for non-zero  $D$ -term contribution to the slepton masses. The important question is: will they be of the right sign and magnitude? The next two subsections will quickly explore both the masses of the right- and left-handed masses to see whether they can be made

positive. If they can, this would be a realistic and well-motivated instance of a Pomarol Rattazzi model.

#### 4.1.1 Right-Handed Sleptons

Now that the theoretical groundwork has been laid out, it is important to investigate the nature of the slepton masses. Specifically, can this scenario provide for non-tachyonic sleptons? For the right-handed sleptons, this can be ascertained from the mass boundary condition, Eq. (3.45). As mentioned in Section 3.5 this equation contains both the AMSB contributions, GMSB-like contributions and a mixture. The AMSB contribution will usually be the largest contributor since the other terms involve differences in  $\beta$ -functions.

The easiest mass to study is that of the right-handed selectron. It has new Yukawa couplings due to the light doubly-charged Higgses:

$$W_{DC} = f_c e^c \Delta^{c--} e^c \quad (4.13)$$

Its mass will depend on the  $f_c$ , which is a  $3 \times 3$  matrix in generation space and on the gauge couplings for  $SU(2)_R, U(1)_{B-L}$  and  $U(1)_Y$ . In order to simplify the analysis, the following assumptions are made:

$$f_{cj}^i = f \delta_j^i \quad \text{and} \quad (4.14)$$

$$\alpha_R = \alpha_{B-L} = 2\alpha_1 \sim 0.044, \quad (4.15)$$

where

$$\alpha_1^{-1} = \alpha_R^{-1} + \alpha_{B-L}^{-1} \quad (4.16)$$

has been used in Eq. (4.15), a result of the right-handed symmetry breaking structure and since  $\alpha_1 \sim 0.022$  is weakly dependent of the parameter space.

Separating Eq. (3.45) into its AMSB, GMSB and mixed contributions yields

$$m_{\tilde{e}c}^2 = m_{AMSB}^2 + m_{mixed}^2 + m_{GMSB}^2 \quad (4.17)$$

$$m_{AMSB}^2 = \frac{1}{144\pi^4} F_\phi^2 \left( \frac{63}{2} f^4 - 108\pi\alpha_1 f^2 - 18\pi^2 b_1 \alpha_1^2 \right) \quad (4.18)$$

$$m_{mixed}^2 = \frac{1}{144\pi^4} F_\phi^2 (12f^4 - 96\pi\alpha_1 f^2 + 24\pi^2 \alpha_1^2 (b_1 - b_R - b_{B-L})) \quad (4.19)$$

$$m_{GMSB}^2 = \frac{1}{144\pi^4} F_\phi^2 (3f^4 + 100\pi f^2 \alpha_1 - 8\pi^2 \alpha_1^2 (b_1 + b_R - b_{B-L})) \quad (4.20)$$

where the  $\gamma$ - and  $\beta$ -functions above the  $v_R$  scale are given in Appendix A.1 and below in Appendix A.2 and all quantities are evaluated at the right-handed scale. Each of these term appears with a positive quartic dependence on  $f$ . As  $f$  is increased, it quickly dominates the negative gauge contributions (given that  $\alpha_1(v_R) \sim 0.02$ ) and yields positive terms. Figure 4.1 indicate that this happens at around  $f \gtrsim 0.6$ . For this figure,  $F_\phi = 36$  TeV, but the result is independent of the overall scale. The largest contribution in this positive regime comes from the AMSB term, which has the largest prefactor to the  $f$  quartic. Finally, the non-tachyonic nature of these masses will be preserved to the SUSY scale since the slepton masses run very slowly. Therefore, this analysis shows that the right-handed slepton masses can be made positive in this scenario.

#### 4.1.2 Left-Handed Slepton Masses and $D$ -Terms

New large Yukawa couplings in the left-handed slepton sector do not exist in this model. Therefore Eq. (3.45) will still give tachyonic left-handed sleptons as the



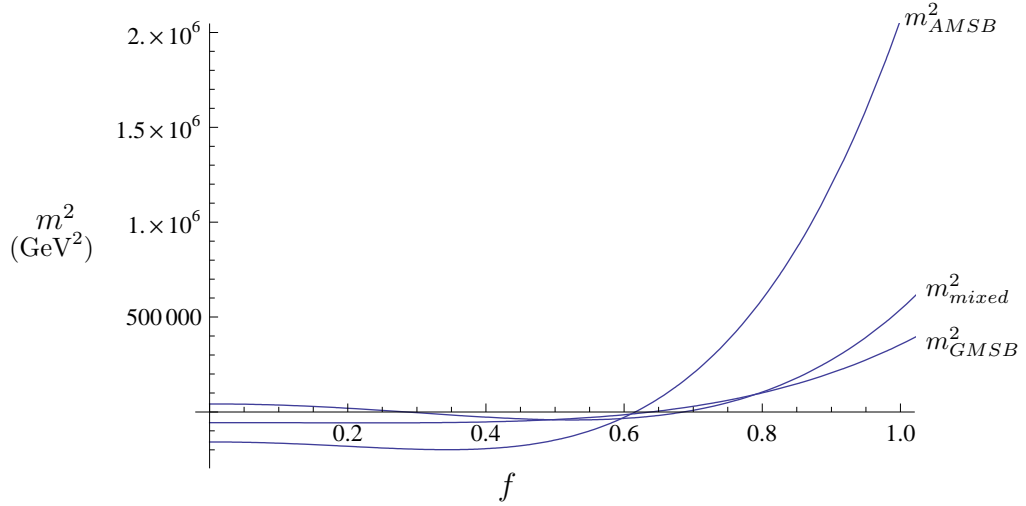


Figure 4.1: The mass of the right-handed selectron at the right-handed scale broken up into the AMSB, mixed and GMSB contributions with  $F_\phi = 36$  TeV and  $\alpha_1 = 0.02$ . The AMSB contribution dominates and is positive for  $f > 0.6$ .

right-handed scale. Since these values do not run much, they will also be tachyonic at the SUSY scale. The only hope for saving these masses are the  $D$ -terms, which only partially decouple as discussed in Section 3.6. The question is, will these  $D$ -terms be of the right sign and magnitude?

Table 2.2 shows that the  $D$ -term contribution to  $m_L^2$  is

$$\delta m_L^2 = \frac{1}{4} \frac{\alpha_{B-L}}{\alpha_R + \alpha_{B-L}} (m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2) \quad (4.21)$$

Clearly, the sign of the contribution depends on the sign of  $m_{\Delta^c}^2 - m_{\bar{\Delta}^c}^2$ . In AMSB, this is predicted based on the couplings of  $\Delta^c$  and  $\bar{\Delta}^c$ . Both have identical gauge couplings, which will vanish from their difference. However,  $\Delta^c$  also has the seesaw Yukawa coupling,  $f$  that is the only term that will survive the difference. Using the

same simplifying assumptions as in Eq. (4.15) in addition to  $y_L = 0$  yields

$$\delta m_{\tilde{L}}^2 = \frac{3}{512\pi^4} r^2 F_\phi^2 (9f^4 - 40\pi\alpha_1 f^2) \quad (4.22)$$

where as mentioned in Section 3.6, the  $D$ -term does not fully decouple because of the deflection from AMSB parameterized by  $r$ . Therefore, the  $D$ -term value below the threshold is suppressed by  $r^2$  but is clearly positive for large enough  $f$ . This is very important since there was no freedom for the sign here based on the seesaw mechanism. It is a fascinating accident that AMSB predicts positive  $D$ -terms contributions for the left-handed sleptons which would otherwise be tachyonic.

The boundary condition is independent of  $f$  and is calculated to be about  $-90000 \text{ GeV}^2$ . The sum of these two contributions at  $F_\phi = 36 \text{ TeV}$  is plotted in Figure 4.2. Here a value of  $f > 0.85$  is needed for positive squared mass, larger than the value needed for right-handed selectrons. Again this is independent of  $F_\phi$ . Note that at the right-handed scale,  $f \lesssim 3.5$  based on perturbativity.

Once again, the value of the squared mass does not run much, therefore, for large enough  $f$ , the left-handed sleptons have also been saved.

### 4.1.3 Below the Right-Handed Scale

Once  $SU(2)_R$  breaks around the seesaw scale of  $10^{11} \text{ GeV}$ , the effective theory contains the NMSSM, an extra set of higgs doublets and the doubly-charged fields and will be referred to as the NMSSM $++$ . The non-renormalizable terms of Eq. (4.1) also influence the form of the lower scale theory and produce some important effects that aid in construction of a realistic low-energy theory. One significant contribution

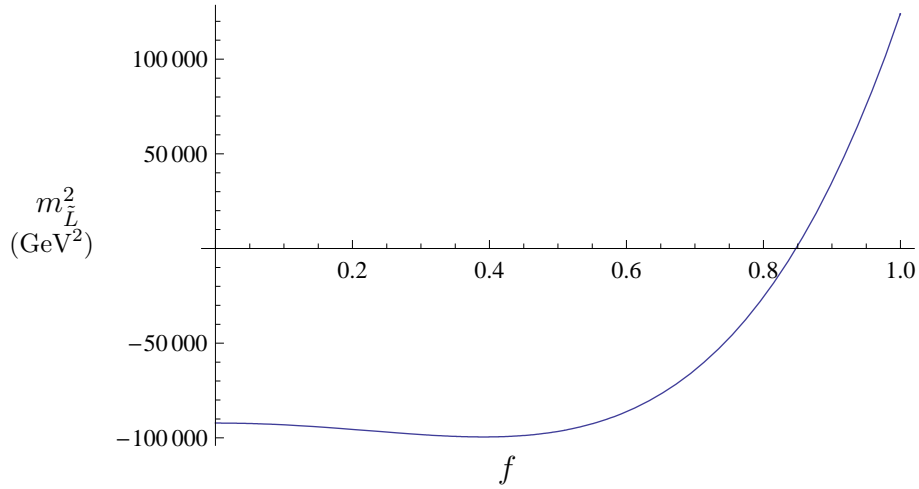


Figure 4.2: The squared mass of the left-handed selectron at the right-handed scale with  $F_\phi = 36$  TeV and  $\alpha_1 = 0.02$ . At  $f > 0.85$ , the  $D$ -terms cause the square mass to be positive. The boundary condition is independent of  $f$  and corresponds to the  $f = 0$  part of the curve.

comes from the higher dimensional operators: the generation of a SUSY mass term for  $N$ . Specifically non-renormalizable term involving  $N$  generate a superpotential term of  $\frac{\mu_N}{\phi} N^2$  when  $\Delta^c$  and  $\bar{\Delta}^c$  get a VEV.

This explicit mass term produces a SUSY breaking bilinear term proportional to  $F_\phi, b_N$  given as

$$b_N = \mu_N F_\phi \simeq \frac{v_R^2}{M_P} F_\phi. \quad (4.23)$$

This term will be shown to play an important role EWSB.

The doubly charged fields will also get an effective  $\mu$  term on the order of  $\frac{v_R^2}{M_P}$ . As will be discussed in Section 4.2.3, this term must be of order  $F_\phi$ , meaning the doubly-charged fields will decouple at this scale. Furthermore, the non-renormalizable terms can also be used to simplify the low-energy theory, though

this is not necessary. Consider the terms involving  $\Phi$  in Eq. (4.1) which yield a low energy mass matrix that is not symmetric between  $\Phi_1$  and  $\Phi_2$  (due to the  $\lambda_\beta$  term). The asymmetry could generate an operator of the form:

$$W \supset iMH_{u2}\tau_2H_{d1} \quad (4.24)$$

without the corresponding  $H_{u1}H_{d2}$  term. This allows a for a possible fine tuning that can lead to a doublet-doublet splitting mechanism at around  $F_\phi$ . The upshot of which is that one doublet set will be heavy ( $H_{u2}$  and  $H_{d1}$  with mass of about  $F_\phi$ , while the other is massless in the limit of  $\langle N \rangle = 0$ ). The light set is then just the regular Higgses of the MSSM/NMSSM and the theory below  $F_\phi$  is the NMSSM with an the additional  $\mu$  term for the singlet.

This  $\mu$  term is important because the low-energy NMSSM cannot achieve a realistic mass spectrum—the singlet  $N$  would get a very small VEV, and the Higgsino would be lighter than allowed by experiment[61]. The origin of this problem is best illustrated with a toy model. Consider a superpotential given by

$$W_{\text{toy}} = \frac{1}{3}\kappa N^3 \quad (4.25)$$

where  $N$  is a singlet field . The resulting scalar potential, including SUSY breaking, is

$$V_{\text{toy}} = \kappa^2|N|^4 + \frac{1}{3}(a_\kappa N^3 + a_\kappa^* N^{*3}) + m_N^2|N|^2. \quad (4.26)$$

Taking account for the complex phases by letting

$$N = |N|e^{i\delta_N} \quad \kappa = |\kappa|e^{i\delta_\kappa} \quad a_\kappa = |a_\kappa|e^{i\delta_{a_\kappa}}, \quad (4.27)$$

the minimization condition for the phase  $\delta_N$  is

$$\sin(3\delta_N + \delta_{a_\kappa}) = 0. \quad (4.28)$$

The resulting minimum condition for  $|N|$ ,

$$\begin{aligned} 0 &= 2|\kappa|^2 |N|^2 + |a_\kappa| |N| \cos(3\delta_N + \delta_{a_\kappa}) + m_N^2 \\ &= 2|\kappa|^2 |N|^2 + |a_\kappa| |N| + m_N^2, \end{aligned} \quad (4.29)$$

is then independent of any phases leaving  $\langle N \rangle$  real. The solution to Eq. (4.29) states

$$\langle N \rangle = \frac{-|a_\kappa| \pm \sqrt{|a_\kappa|^2 - 8|\kappa|^2 m_N^2}}{2|\kappa|^2} \quad (4.30)$$

where the soft couplings  $a_\kappa$  and  $m_N$  are determined by AMSB via Eqs. 3.18 and 3.19 (note that since the singlet does not couple significantly to the messengers, this argument would hold in PSLRM as well):

$$\begin{aligned} a_\kappa &= \frac{F_\phi}{16\pi^2} 6\kappa^3 \\ m_N^2 &= \frac{|F_\phi|^2}{(16\pi^2)^2} 12\kappa^4. \end{aligned} \quad (4.31)$$

Substituting these into Eq. (4.30) gives

$$\langle N \rangle = \frac{|F_\phi| |\kappa|}{16\pi^2 4} (-6 \pm \sqrt{-60}) \quad (4.32)$$

yielding a contradiction:  $\langle N \rangle$  must be real, but the large negative under the radical demonstrates this can not be so.

The same problem carries over to the full NMSSM, as pointed out in [61]. In this model, the additional coupling of  $N$  to  $H_u$  and  $H_d$  adds a linear term to the potential,  $a_\lambda v_u v_d N$ . The induced linear term shifts the trivial minimum away from

zero, but keeps it small. Given this limitation of the NMSSM, it is desirable to explore methods that either alter the relative strengths of the terms or yield a large tadpole term for  $N$ . The former may be done by adding vector-like matter (as in [48]), while the latter was explored in [61] by introducing a linear term for  $N$ . The solution used here is different and is already present in this model: utilizing the  $b_N$  term. This was discussed in [58].

The size of  $b_N$  is quite conveniently around the SUSY breaking scale and will serve for turning the net mass-square of  $N$  negative. To establish this property we now turn to the minimization condition for  $N$ .

$$\tilde{m}_N^2 + \kappa^2 n^2 + \frac{1}{2}\lambda^2 v^2 + \frac{n\tilde{a}_\kappa}{\sqrt{2}} - \frac{1}{2}v^2\left(\frac{\tilde{a}_\lambda}{n\sqrt{2}} + \lambda\kappa\right) \sin 2\beta = 0 \quad (4.33)$$

The tilded variables are introduced to display the deviations from the usual NMSSM. The definitions are

$$\tilde{a}_\lambda \equiv a_\lambda + \lambda\mu_N \quad (4.34)$$

$$\tilde{a}_\kappa \equiv a_\kappa + 3\kappa\mu_N \quad (4.35)$$

$$\tilde{m}_N^2 \equiv m_N^2 + \mu_N^2 - b_N \quad (4.36)$$

The variable  $\tilde{m}_N^2$  can be approximated as

$$\begin{aligned} \tilde{m}_N^2 &= m_N^2 + \mu_N^2 - \mu_N F_\phi \\ &\approx m_N^2 - \mu_N F_\phi \\ &\simeq \left(\frac{\lambda^4}{(16\pi^2)^2} F_\phi - \mu_N\right) F_\phi \end{aligned}$$

The second line follows from the fact that  $\mu_N \sim \mathcal{O}\left(\frac{M_{\text{SUSY}}^2}{F_\phi}\right) \sim \mathcal{O}\left(\frac{F_\phi}{(16\pi^2)^2}\right)$

and therefore the  $\mu_N^2$  term is negligible compared to the the other terms. The last line uses the AMSB expression for the square of the scalar mass, assuming it is dominated by the  $\lambda$  contribution. As can be seen, due to the  $\lambda^4$  suppression, it is relatively easy to adjust  $\mu_N$  to a value to make  $\tilde{m}_N^2$  negative and therefore induce a singlet VEV of the correct size. Given that  $\lambda(M_{\text{SUSY}}) \lesssim 0.5$  (from constraints of perturbativity to the right-handed scale) and that  $\mu = \frac{\lambda n}{\sqrt{2}}$ , it is only necessary for  $n \gtrsim 300$  GeV to achieve chargino masses above the LEP II bound. The resulting spectrum is similar to the NMSSM given in [62].

## 4.2 Phenomenology

In the following sections, the numerical values are based on the parameter running scheme used and some simplifying assumptions. The gauge coupling values from are evolved from the electroweak scale to the right-handed scale taking the  $F_\phi$  threshold into account by decoupling the doubly-charged fields and extra Higgs doublet. Yukawa couplings are then inputs at the right-handed scale: the third generation values for the SM couplings ( $y_Q, y_L$ ) and all three generations of the seesaw couplings ( $f_c$ ). These are evolved down to the SUSY scale using [63, 64]. The seesaw coupling is assumed to be diagonal to agree with lepton flavor violating constraints given in [65]. It is also assume it is proportional to the identity for simplicity. As usual the MSSM Yukawa couplings are assumed to have a non-zero value only in the three-three position in generation space.

### 4.2.1 Sfermions

The arguments given in sections 4.1.1 and 4.1.2 show that this model can solve the tachyonic slepton problem of AMSB by the seesaw Yukawa couplings for the right-handed sleptons and the  $D$ -terms for the left-handed ones. However, the  $D$ -terms can lead to problems of its own. Examining Table 2.2 once more shows that negative  $D$ -term contributions to masses exist for the left-handed squark field due to  $U(1)_{B-L}$ , the right-handed up squark due to  $SU(2)_R$  and the right-handed selectron due to  $U(1)_{B-L}$ . This should be expected since  $D$ -terms are proportional to charge, therefore particles of charge opposite of  $L$  will get negative contributions. The right-handed selectron will be non-tachyonic because it also receives a positive contribution from  $SU(2)_R$   $D$ -terms and because its boundary condition is quite large when  $f > 0.6$ . The right-handed up squark may also be safe because it gets positive contributions from  $U(1)_{B-L}$   $D$ -terms. Therefore, the only field that has purely negative contributions to its squared mass is the left-handed squark field, which brings its mass into conflict with that of the left-handed sleptons: the larger the  $D$ -term the larger the left-handed slepton mass but the smaller the left-handed squark mass.

To study this situation, it is sufficient to examine the lightest first generation masses. The only difference between these and those of the third generation is the lack of Yukawa couplings in the latter which usually help the situation. Figure 4.3 plots the mass of the lightest down squark, up squark and selectron at the SUSY scale (1 TeV) versus  $f$  at the right-handed scale using the same assumptions as in



Section 4.1.1. The lightly shaded region is the excluded region for squark masses based on Tevatron data and the darker region is excluded slepton masses based on LEP II data as well. The dashed line is the mass of the LSP. In order for the LSP to be a dark matter candidate, all SUSY masses must be heavier than the LSP.

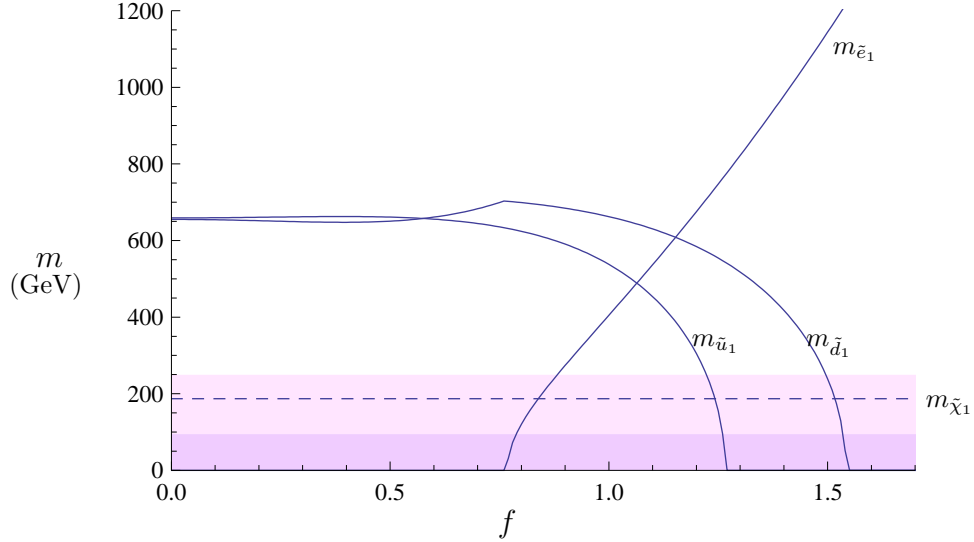


Figure 4.3: The squared mass of the lightest down squark, up squark and selectron at the SUSY scale (1 TeV) with  $F_\phi = 36$  TeV and  $\alpha_1 = 0.02$  versus  $f$  at the right-handed scale. The light shaded region is excluded for squark masses from the Tevatron and the darker shaded region is excluded for slepton masses from LEP II. The dashed line is the mass of the LSP. In order for it to be the dark matter candidate, all SUSY masses must be above it. The allowed parameter space is about  $0.8 \leq f \leq 1.2$ .

The strongest constrain to come from the up squark and the selectron masses although they admit some parameter space? The down squark mass has an interesting sharp increase at around  $f = 0.75$ . Below this, the lightest down squark is right-handed because the low values of  $f$  cause the contributions in Table 2.2

to have the opposite signs of the ones shown, *i.e.* the left- (right-) handed down squark gets a positive (negative) contribution. After this point, the composition of the lightest eigenstates switches to the left-handed down squark which falls as  $-f^4$ . The right-handed down squark has the opposite behavior growing quickly as  $f^4$ . The same behavior can be seen for  $m_{\tilde{u}_1}$ . Meanwhile,  $m_{\tilde{e}_1}$  increases rapidly with  $f$  as argued above.

A similar plot can be constructed for the other parameter which influences these mass terms:  $\alpha_R$  at the right-handed scale ( $v_R \sim 10^{10}$  GeV), Figure 4.4. Here  $f = 1$  at the right-handed scale.

The behavior of the plots in Figure 4.4 is a bit more complex but is worth tabulating here. First it is helpful to keep in mind the behavior of  $D$ -term contributions. Since  $f$  is held constant here, the  $D$ -terms are simply proportional to the ratios of gauge couplings: the  $U(1)_{B-L}$  contribution goes as  $\frac{\alpha_{B-L}}{\alpha_{B-L} + \alpha_R}$  and the  $SU(2)_R$  contribution goes as  $\frac{\alpha_R}{\alpha_{B-L} + \alpha_R}$ . Their behavior in terms of  $\alpha_R$  are shown in Table 4.1. It is also worth recording the schematic behavior of the mass boundary condition at the right-handed scale in terms of the gauge couplings. This is given in Table 4.2 along with the sign of the  $D$ -term contributions.

| $\alpha_R$                      | $\alpha_{B-L}$ | $U(1)_{B-L}$ $D$ -term | $SU(2)_R$ $D$ -term |
|---------------------------------|----------------|------------------------|---------------------|
| $\alpha_R \rightarrow \alpha_1$ | $\infty$       | 1                      | 0                   |
| $\alpha_R \rightarrow \infty$   | 1              | 0                      | 1                   |

Table 4.1: Behavior of  $\alpha_{B-L}$  and the  $U(1)_{B-L}$  and  $SU(2)_R$   $D$ -term contributions for different limits of  $\alpha_R$ . Remember that  $\alpha_1 \sim 0.022$ .

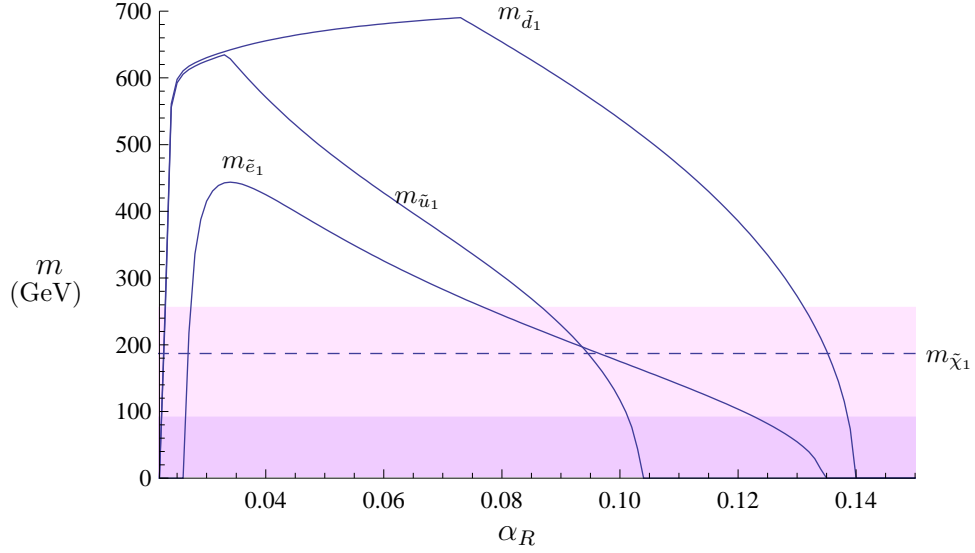


Figure 4.4: The squared mass of the lightest down squark, up squark and selectron at the SUSY scale (1 TeV) with  $F_\phi = 36$  TeV and  $f = 1$  at the right-handed scale verses  $\alpha_R$  at the right-handed scale. The light shaded region is excluded for squark masses from the Tevatron and the darker shaded region is excluded for slepton masses from LEP II. The dashed line is the mass of the LSP. In order for it to be the dark matter candidate, all SUSY masses must be above it. The allowed parameter space is  $0.75 \leq f \leq 1.22$ .

It is possible to systematically step through the rows of Table 4.2 and Table 4.1 these to understand Figure 4.4. The lightest selectron will always be the left-handed slepton in this case because the value of  $f$  pushes the right-handed slepton to be quite massive. At low  $\alpha_R$  close to the value of  $\alpha_1$ , the mass is dominated by the  $-\alpha_{B-L}^2$  contribution which pushes it toward large negative values. At larger values of  $\alpha_1$ , the  $U(1)_{B-L}$   $D$ -term contribution approaches zero asymptotically returning the selectron mass to it's negative value once more.

The content of the lightest squarks actually flip back and forth explaining

| <i>Field</i> | Mass at $v_R$                       | Sign of $B - L$ | Sign of $R$ |
|--------------|-------------------------------------|-----------------|-------------|
| $L_1$        | $\sim -\alpha_{B-L}^2$              | +               | 0           |
| $e^c$        | $\sim -\alpha_{B-L}^2 - \alpha_R^2$ | -               | +           |
| $Q_1$        | $\sim -\alpha_{B-L}^2$              | -               | 0           |
| $u^c$        | $\sim -\alpha_{B-L}^2 - \alpha_R^2$ | +               | -           |
| $d^c$        | $\sim -\alpha_{B-L}^2 - \alpha_R^2$ | +               | +           |

Table 4.2: Schematic behavior of the mass boundary condition of various fields and the sign of their  $D$ -term contributions

the maxima seen in Figure 4.4. For low  $\alpha_R$  both the right-handed and left-handed squarks get large negative contributions from  $\alpha_{B-L}^2$  but the right-handed ones also have negative contributions from  $\alpha_R^2$  hence making the lightest squark right-handed. At moderate values of  $\alpha_R$ , the  $D$ -terms help boost the right-handed mass above the left-handed ones so that the content switches. This process is more prevalent in the down squark since it has two positive contributions from the  $D$ -terms. Finally, as  $\alpha_R$  increases, the boundary conditions dominate once more driving the right-handed squark masses toward zero as  $-\alpha_R^2$  (but not the left-handed squarks). Therefore, in this regime, the lightest eigenstates are once more right-handed. This happens at about  $\alpha_R = 0.07$  for the down squark and  $\alpha_R = 0.03$  for the up squark.

Figures 4.4 and 4.3 hint at the available parameter space but a plot in  $\alpha_R - f$  space is more useful for this purpose. This is given in Figure 4.5 for  $F_\phi = 36\text{TeV}$  taking into account the collider constraints used in the previous two figures and in Figure 4.6 where the the selectron is more massive than the wino making the wino

the LSP. The space is constrained on the left due to tachyonic/light sleptons, to the right because of tachyonic/light squarks, below because  $\alpha_{B-L}$  is non-perturbative and above because of tachyonic squarks. The plot makes an interesting prediction that  $f$  needs to be very close to 1 at the right-handed scale (translating to about 0.6 at the  $F_\phi$  scale). It also severely restricts the  $\alpha_R$  space. Figure 4.6 pushes the value  $f$  up by about 5% so that the selectron is heavier than the wino.

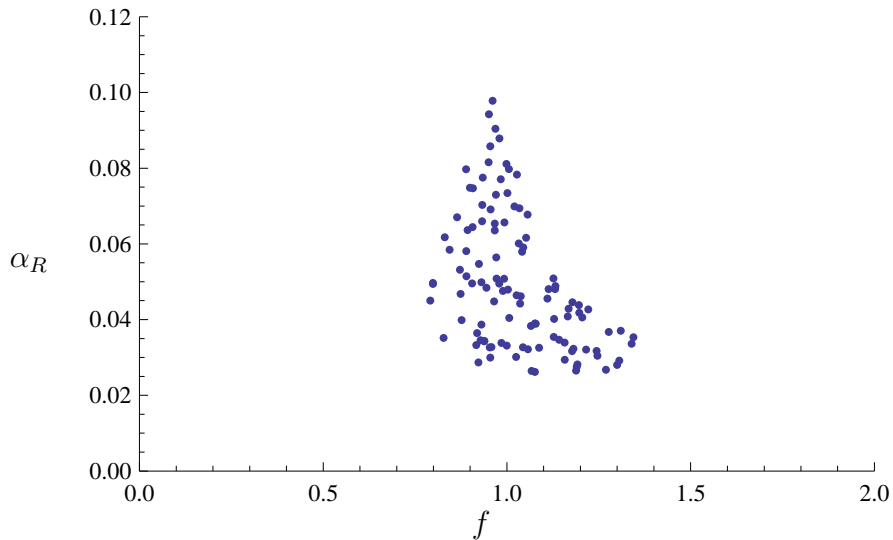


Figure 4.5: The allowed parameters space in the  $\alpha_R - f$  plane taking into account the collider limits used in the previous figures. Here  $F_\phi = 36$  TeV.

The parameter space allows for some interesting situations. Consider Figure 4.7 which shows both up squark eigenvalues as a function of  $\alpha_R$ . For large  $\alpha_R$ , this plot shows that a large hierarchy exists between the two eigenstates. This quite atypical since models such as mSUGRA, GMSB and AMSB all predict same flavor squark masses that are quite degenerate since the contributions to those masses are dominated by  $\alpha_3$  and is independent of handedness. Hierarchies for the down squark is also possible and a hierarchy exists through most of the parameter space for the

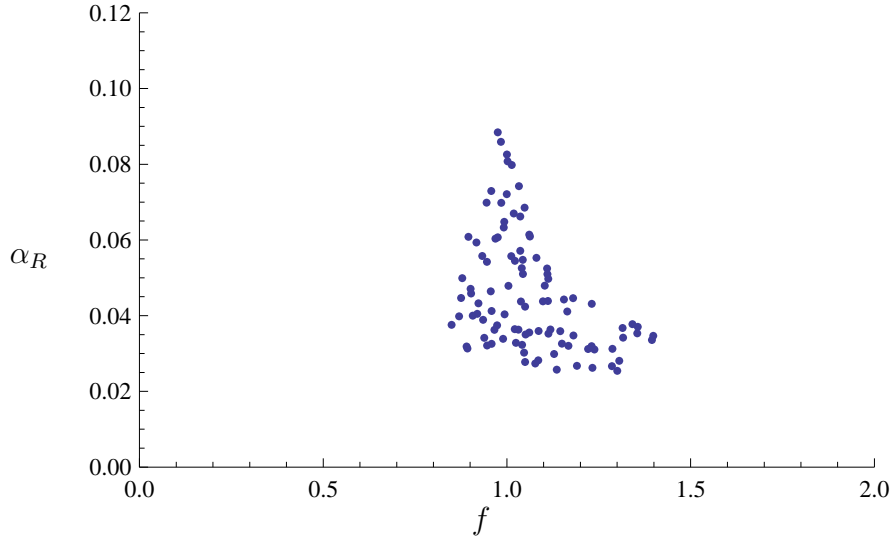


Figure 4.6: The allowed parameters space in the  $\alpha_R - f$  plane taking into account the collider limits used in the previous figures and that the wino is the LSP. Here  $F_\phi = 36$  TeV.

slepton, although this latter trait also exists in GMSB.

Finally, Table 4.3 presents masses corresponding to the center area of the parameter space in Figure 4.5 ( $f = 1$ ,  $\alpha_R = 0.05$ ) with  $F_\phi = 36$  TeV. Only the first generation masses are shown since the other generations introduce a dependence on extra parameters. Masses are at the SUSY scale, 1 TeV. A large hierarchy can be seen in the slepton sector. Furthermore, the down squark is heavier than the up which is a general feature due to its  $D$ -term contributions and  $m_{\tilde{e}_2}$  is comparable to squark masses.

## 4.2.2 Bosinos and The LSP

Because all superpartners eventually decay into the LSP, its makeup is an important part of SUSY collider phenomenology and dark matter prospects and

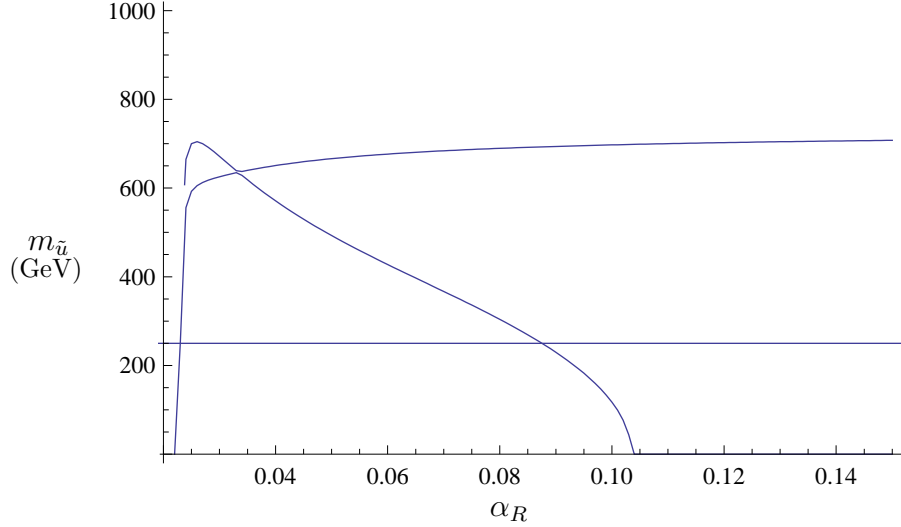


Figure 4.7: Plot of  $m_{\tilde{u}_1}$  and  $m_{\tilde{u}_2}$ , the latter being the heavier one for  $f = 1$  and  $F_\phi = 36$  TeV. The line at about 250 GeV correspond to Tevatron limits as in the plots show in Figures 4.3 and 4.4.

understanding that makeup is an important task. Cosmological constraints rule out a charged or colored LSP [66], hence limiting the choices to the sneutrino or the lightest neutralino. The former, in typical models, makes a poor dark matter candidate (relic abundances are too light; much of its mass range ruled out by direct detection [67, 68]). It is therefore more interesting to consider the lightest neutralino as the LSP, the candidate in common SUSY scenarios (except in mGMSB where it is the next to lightest SUSY particle but has the same collider significance [69]).

The lightest neutralino will be some mixture of the wino, bino and Higgsino. Its gaugino composition follows from the gaugino mass ratio which is easily calculated and relatively independent of the point in parameter space. It is worthwhile to compare this ratio in PSLRM to the mAMSB where  $M_3 : M_2 : M_1 \sim \frac{\alpha_3 b_3}{\alpha_2 b_2} : 1 : \frac{\alpha_1 b_1}{\alpha_2 b_2} \sim 8 : 1 : 3.5$ . In PSLRM the gluino is also is still on its AMSB trajectory how-

| <i>Field</i> | Lightest State (GeV) | Heaviest State (GeV) |
|--------------|----------------------|----------------------|
| $\tilde{e}$  | 370                  | 760                  |
| $\tilde{u}$  | 490                  | 670                  |
| $\tilde{d}$  | 670                  | 810                  |

Table 4.3: Masses of the first generation sfermions at  $F_\phi = 36$  TeV ,  $f = 1$  and  $\alpha_R = 0.05$ . A large hierarchy in the selectron can be seen as well as the fact that the selectron can be as massive as the squarks.

ever,  $b_2 = 2$  instead of 1 because of the extra Higgs doublet below the right-handed scale. This makes the wino about twice as massive as in mAMSB. Furthermore, the bino picks up a similar sized contributions since the doubly-charged particle content almost double  $b_1$ . In addition, there are also boundary terms which add to the AMSB contributions (see Eq. (3.43)) which increases its value. The ratio is then calculated to be 4 : 1 : 3, making the wino the lightest and therefore the dominant gaugino content of the LSP. This is very different from mSUGRA where the ratio is given by 3 : 1 : 0.3 and the bino is the lightest because in mSUGRA the mass ratios depend only the gauge couplings and not on the gauge coupling RGE slopes (see Table 4.4 for  $b$  values in this model below the  $v_R$  scale, compared to the minimal case).

The Higgsino contribution is not independent of other parameters however in general it is significantly larger than  $M_2$  since it must be comparable in size to  $m_{H_u}$  or  $m_{H_d}$  to allow for EWSB, see Eq. (1.62), and  $m_{H_u}$  receives large contributions from the top Yukawa coupling. Therefore, the LSP will be predominantly wino as



in the mAMSB case. It is interesting that this scenario is different from the deflected AMSB model by Pomarol and Rattazzi where the wino gets extra mass contribution from a messenger sector charged under  $SU(2)_L$  and is therefore not the LSP.

|              | $b_1$          | $b_2$ | $b_3$ |
|--------------|----------------|-------|-------|
| MSSM         | $\frac{33}{5}$ | 1     | -3    |
| <i>PSLRM</i> | $\frac{60}{5}$ | 2     | -3    |

Table 4.4: Values for the b parameter in the MSSM and PSLRM.

In this case, it is important to note that Winos form isospin triplets. Therefore when they play the role of the lightest neutralino there is a very small mass difference between the lightest neutralino and the lightest chargino on the order of 100s of MeVs. This value includes leading radiative corrections. Analytically, this mass difference can be approximated at tree-level as (see [49, 70, 71]):

$$\Delta_\chi \equiv m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}_1^0} = \frac{M_W^4 \tan^2 \theta_W}{(M_1 - M_2) \mu^2} \sin^2 2\beta \quad (4.37)$$

which is suppressed for large  $\tan \beta$ , large  $M_1$  and large  $\mu$ . The mass splitting is so small that radiative corrections have to be considered. In the limit  $M_2 \gg M_W$ , this reduces to

$$\Delta_\chi = \frac{\alpha M_W}{2(1 + \cos \theta_w)} \left( 1 - \frac{3}{8 \cos \theta_w} \frac{M_W^2}{M_2^2} + \mathcal{O}\left(\frac{M_W^3}{M_2^3}\right) \right) \quad (4.38)$$

where  $\alpha$ , the fine structure constant, is about  $\frac{1}{128}$  at the relevant scale. This value asymptotes to 165 MeV for large  $M_2$  and reflects the Columb contributions to the self-energy of  $\tilde{W}^\pm$  which does not exist for  $\tilde{W}^0$ .

### 4.2.3 Doubly-Charged Higgs and Low Energy Phenomenology

The doubly-charged superfields have an effective  $\mu$  term below the right-handed scale

$$W_{\text{DC Mass}} = \frac{\mu_{DC}}{\phi} \Delta^{c--} \bar{\Delta}^{c++} \quad (4.39)$$

where  $\mu_{DC} \sim \frac{v_R^2}{M_P} \sim F_\phi$ . This is the mass term for the Higgsinos, so they decouple there. However, the doubly-charged fields also pick up a SUSY breaking  $b_{DC}$

$$V_{\text{Soft DC Mass}} = \mu_{DC} F_\phi \Delta^{c--} \bar{\Delta}^{c++} \quad (4.40)$$

therefore,  $b_{DC} \sim F_\phi^2$ . The mass matrix for the doubly-charged Higgses is:

$$\mathcal{M}_{DC} = \mu_{DC}^2 \begin{pmatrix} 1 & 1 - \epsilon_{DC} \\ 1 - \epsilon_{DC} & 1 \end{pmatrix} \quad (4.41)$$

with  $\epsilon_{DC} = 1 - \frac{F_\phi}{\mu_{DC}}$ . The eigenvalues of this mass matrix are  $m_{DC}^2 = \epsilon_{DC} \mu_{DC}^2$  and  $M_{DC}^2 = 2\mu_{DC}^2$ . Since  $\epsilon_{DC}$  depends on  $\mu_{DC}$ , and  $\mu_{DC}$  on the couplings  $\lambda_A$  and  $\lambda_B$ , it is possible that one doubly-charged Higgs is light and therefore accessible at the LHC. Its presence would also be felt indirectly in upcoming muonium-antimuonium oscillation experiments since the couplings to first and second generation leptons must be large. Current bounds on these couplings and the masses of the doubly-charged field are experimentally constrained from the most recent muonium-antimuonium oscillation data [72] to be

$$\frac{f_{c1} f_{c2}}{4\sqrt{2} m_{DC}^2} \lesssim 3 \times 10^{-3} G_F; \quad (4.42)$$

Given that  $f$  has a quasi-fixed point value of around 0.5, this bound roughly

translates to

$$m_{DC} \gtrsim 1000 \text{ GeV}. \quad (4.43)$$

#### 4.2.4 Collider Signatures

The small size of  $\Delta_{\tilde{\chi}_1}$  from Eq. (4.38) can be problematic at a collider because the soft decay products,  $X$ , in the process  $\chi_1^+ \rightarrow X\chi_1^0$ , cannot be tagged. This is a feature shared by all models with a Wino LSP and in some cases with a Higgsino LSP. Such situations have been analyzed for general (non-AMSB) cases in lepton colliders [73, 74] where it was shown that successful discovery could be made for triggers of photons with energy greater than 10 GeV and vetoing other energetic particles. The most significant background is  $e^+e^- \rightarrow \gamma\nu\nu$  where the neutrinos mimic the missing energy of the LSP. An analysis more specific to AMSB was conducted [49] which discussed the gamma signals as well as possible leptonic signals in different parts of the parameter space.

General analysis for hadron colliders (the Tevatron) been have also done [70, 75, 71]. In [70] it was shown that for the mAMSB parameter space  $\Delta_{\tilde{\chi}} > m_{\pi^+}$  causing the chargino to have a decay length less than 10 cm and usually less than 1 cm. This excludes the possibility of it reaching the muon chamber. It is then argued that the best route is to trigger on hard jets and missing energy and then look for the chargino track in the detector. Because of the similarity of the LSP in this scenario to mAMSB, these analyzes would also apply here.

The reason for the difficulty in detection is that typical SUSY discovery scenar-

ios are based on left-handed squark decays to mostly wino charginos and neutralinos (the wino is not the LSP in this case). These in turn can decay leptonically producing trilepton signals or same sign dilepton signals [76, 22], both of which have potentially manageable backgrounds. However, when the wino is the LSP, no such signals exist since the chargino decays to a soft pion. One must then adopt the triggers mentioned above or investigate other leptonic signals from bino decays. Binons would be produced in the decay of right-handed squarks and would decay leptonically but would not produce the trilepton and same sign dilepton signals. LHC studies of such scenarios in mAMSB still found the reach to be significant [77, 78]. Again, this would apply in PSLRM.

If any of the signals mentioned here are found, discriminating this model from others would have to be based on mass differences. As was shown in Section 4.2.1, there is a potential for a large mass difference in the squark sector not possible in other models. Furthermore, Table 4.3 shows that the heavier selectron can have mass as high as that of the squarks. This is in general true in this model. This is yet another discriminating factor since it is not possible in GMSB, mSUGRA or mAMSB. Such a discovery may have to await the International Linear Collider though. Aside from the sfermion sector, the real smoking gun for this model would be a discovery of the doubly-charged Higgs, if it is light enough. At the LHC production would proceed via sea-quark annihilation into a virtual  $\gamma$  and would lead to the production of a pair of doubly-charged fields. Many phenomenological studies have been conducted on various doubly-charged fields, [79, 80, 81, 82, 83, 84].

### 4.2.5 Dark Matter

As noted in the previous section, the LSP in PSLRM model is a predominantly wino as is true in mAMSB. Conventionally the annihilation rate for such an LSP is too large and its relic density is not sufficient to explain the observation that 23% of the universe is dark matter (the wino self annihilates via a t-channel chargino exchange). This issue has been discussed in [85]. In this paper, the authors show that due to the large mass of the gravitino,  $F_\phi \gtrsim 20$  TeV, it decays in the late stage of the universe: before big bang nucleosynthesis and after the freeze out of the wino. Its wino decay products will then be non-thermal and can exist with sufficient density to make the wino a viable dark matter candidate. Furthermore [85] scanned the parameters and found that such dark matter does evade current bounds on direct detection by CDMS Soudan and EDELWEISS but will be detectable by future experiments. Since the wino sector in PSLRM is identical to that in mAMSB, all of these arguments also apply to the wino here making it a promising dark matter candidate.

### 4.3 Conclusion

PSLRM gives appropriate masses to the neutrinos, predicts the seesaw scale, guarantees  $R$ -parity conservation, predicts the presence of light doubly-charged fields which, through their couplings to the right-handed sleptons, cause those particles to be non-tachyonic and allows for non-tachyonic left-handed sleptons via partially decoupled  $D$ -terms. The best part is that all this is a result of looking at a minimal

seesaw SLR model with an extra  $Z$ -symmetry.

It is interesting to compare this model to [59], which solves the tachyonic slepton problem with light doubly-charged and left-handed triplets fields in the context of MSLRM and pure AMSB. The latter retains the renormalization scale invariance. It also solves the strong CP problem. However, there is a coincidence problem associated with the mass of the light fields. This model loses the possibility of solving the strong CP problem but the coincidence problem is solved by the prediction of the right-handed scale. Furthermore, the GUT potential is better since it does not contain the light left-handed triplets. In this sense, it is also more economical.

## Chapter 5

### Conclusion

This thesis has focused on a specific MSLRM which contains an additional discrete symmetry. In general this leads to a prediction of the right-handed scale of about  $10^{11}$  GeV which is consistent with neutrino oscillation data. It also solves the  $\mu$  problem and introduces light neutral fields in addition to the typical doubly-charged fields associated with MSLRM.

While SUSY breaking can be implemented in a variety of ways in this model, AMSB takes on a special form here. Specifically, this model can be used as a specific instance of the Pomarol Rattazzi model of deflected AMSB. The necessary ingredients for this already exist in the model: a shallow potential leading to light singlets. As such, it is well motivated due to neutrino masses. The right handed neutrinos play the roll of messengers instead of the arbitrary fields serving this purpose. The new SUSY breaking fields are the right-handed triplets which are necessary for the seesaw mechanism and the breaking of  $SU(2)_R$  instead of being arbitrary singlets.

Most importantly, the slepton masses are saved from their non-tachyonic fate by a combination of two mechanisms. The right-handed sleptons are saved due to the extra Yukawa couplings ( $f$ ) to the light doubly-charged fields. This is independent of the deflection. The left-handed sleptons get positive contributions from

the partially decoupled  $D$ -terms which are intimately connected to the deflection of AMSB. Furthermore, the sign of the  $D$ -term contribution is positive in the regime where  $f$  is large enough to save the right-handed sleptons. It is important that there was no freedom here and is quite interesting that the sign works out the way it does. The dependence on  $D$ -terms also adds an interesting constraint to the  $f - \alpha_R$  parameter space since they contribute negatively to the masses of the up squarks.

The model at low energy is the NMSSM which solves the  $\mu$  problem and also contains the means to solve the EWSB problem of AMSB, which exists even in the NMSSM. This solution is achieved through non-renormalizable terms in the superpotential, which allow for an effective  $\mu$  and  $b$  terms for the singlet. The latter can help trigger a VEV for the singlet which would otherwise have been too small leading to an unacceptable chargino spectrum. This mechanism exists *a priori* in this theory, disposing of the need for *ad hoc* colored triplets.

Phenomenologically, the doubly-charged doublets might be visible in collider experiments such as the LHC and in future muonium-antimuonium oscillation experiments. The model also provides a realistic dark matter candidate: the LSP which is mostly wino. Furthermore, its gaugino structure is similar enough to the mASMB case so that previous mAMSB studies can be applied here.

The sum total of these indicate that this is an interesting model and a contender for SUSY BSM physics. Furthermore, it is an appealing addition to the work already done on exploring the remarkable relationship between neutrino masses and AMSB [59, 86, 58].



## Appendix A

### The Predictive Supersymmetric Left-Right Model

This appendix contains relevant technical information for PSLRM including gamma functions and Yukawa beta functions in the different energy regimes of the model.

#### A.1 Above the Right-Handed Scale

The superpotential is reproduced here:

$$W_{PSLRM} = W_Y + W_{NR} + W_N \quad (\text{A.1})$$

$$W_Y = \text{i}y_Q^a Q^T \tau_2 \Phi_a Q^c + \text{i}y_L^a L^T \tau_2 \Phi_a L^c + \text{i}f_c L^{cT} \tau_2 \Delta^c L^c \quad (\text{A.2})$$

$$W_N = \lambda^{ab} N \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{3} \kappa N^3 \quad (\text{A.3})$$

$$\begin{aligned} W_{NR} = & \frac{\lambda_A}{M_P \phi} \text{Tr}^2(\Delta^c \bar{\Delta}^c) + \frac{\lambda_B}{M_P \phi} \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\bar{\Delta}^c \bar{\Delta}^c) \\ & + \frac{\lambda_\alpha^{ab}}{M_P \phi} \text{Tr}(\Delta^c \bar{\Delta}^c) \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{\lambda_\beta^{ab}}{M_P \phi} \text{Tr}(\Delta^c \tau_2 \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c) \\ & + \frac{\lambda_N}{M_P \phi} \text{Tr}(\Delta^c \bar{\Delta}^c) N^2 + \dots \end{aligned}$$

The gauge coupling  $b$  values are:

$$b_3 = -3 \quad b_2 = 2 \quad b_R = 6 \quad b_{B-L} = 10 \quad (\text{A.4})$$

where no GUT based normalization has been imposed on  $b_{B-L}$ . A general gauge coupling RGE is given by

$$\frac{\partial \alpha_A}{\partial t} = \frac{b_A \alpha_A^2}{2\pi} \quad (\text{A.5})$$

no sum over  $A$ . The gamma functions for the particle content are

$$\begin{aligned}
16\pi^2\gamma_{Q_3} &= -4|y_Q^a|^2 + 8\pi\left(\frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_2 + \frac{1}{18}\alpha_{B-L}\right) \\
16\pi^2\gamma_{Q_1} &= 8\pi\left(\frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_2 + \frac{1}{18}\alpha_{B-L}\right) \\
16\pi^2\gamma_{Q_3^c} &= -4|y_Q^a|^2 + 8\pi\left(\frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_R + \frac{1}{18}\alpha_{B-L}\right) \\
16\pi^2\gamma_{Q_1^c} &= 8\pi\left(\frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_R + \frac{1}{18}\alpha_{B-L}\right) \\
16\pi^2\gamma_{L_3} &= -4|y_L^a|^2 + 8\pi\left(\frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_{B-L}\right) \\
16\pi^2\gamma_{L_1} &= 8\pi\left(\frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_{B-L}\right) \\
16\pi^2\gamma_{L_3^c} &= -4|y_L^a|^2 - 12|f_{c3}|^2 + 8\pi\left(\frac{3}{2}\alpha_R + \frac{1}{2}\alpha_{B-L}\right) \\
16\pi^2\gamma_{L_1^c} &= -12|f_{c1}|^2 + 8\pi\left(\frac{3}{2}\alpha_R + \frac{1}{2}\alpha_{B-L}\right) \\
16\pi^2\gamma_{\Phi_a^b} &= -6y_Q^a y_Q^{b*} - 2y_L^a y_L^{b*} - 8\lambda^{ac}\lambda^{cb*} + 8\pi\delta^{ab}\left(\frac{3}{2}\alpha_2 + \frac{3}{2}\alpha_R\right) \\
16\pi^2\gamma_N &= -4\kappa^2 - 16|\lambda^{ab}|^2 \\
16\pi^2\gamma_{\Delta^c} &= -4|f_{c3}|^2 - 4|f_{c2}|^2 - 4|f_{c1}|^2 + 8\pi(4\alpha_R + 2\alpha_{B-L}) \\
16\pi^2\gamma_{\bar{\Delta}^c} &= 8\pi(4\alpha_R + 2\alpha_{B-L}) \tag{A.6}
\end{aligned}$$

under the convention  $\gamma_\phi \equiv \frac{\partial \ln Z_\phi}{\partial \ln \mu}$ . It is assumed that:  $f_c = \text{diag}(f_{c1}, f_{c2}, f_{c3})$  and the remaining  $3 \times 3$  Yukawa matrices are only non-zero for the three-three entry. Second generation  $\gamma$ -functions are the same as first generation ones except for the right-handed leptons where  $f_{c1} \rightarrow f_{c2}$ . The  $\Phi$  generational index,  $a = 1..2$ . The

$\beta$ -functions for the Yukawa couplings are then given by

$$\begin{aligned}
\frac{\partial y_Q^a}{\partial t} &= -\frac{1}{2} \left( y_Q^a (\gamma_{Q_3} + \gamma_{Q_3^c}) + y_Q^b \gamma_{\Phi_b^a} \right) \\
\frac{\partial y_L^a}{\partial t} &= -\frac{1}{2} \left( y_L^a (\gamma_{L_3} + \gamma_{L_3^c}) + y_L^b \gamma_{\Phi_b^a} \right) \\
\frac{\partial \lambda^{ab}}{\partial t} &= -\frac{1}{2} \left( \lambda^{ab} \gamma_N + \lambda^{ac} \gamma_{\Phi_c^b} + \lambda^{cb} \gamma_{\Phi_c^a} \right) \\
\frac{\partial f_{ci}}{\partial t} &= -\frac{1}{2} \left( f_{ci} \gamma_{\Delta^c} + 2f_{cj} \gamma_{L_j^c} \right) \\
\frac{\partial \kappa}{\partial t} &= -\frac{1}{2} (3\kappa \gamma_N)
\end{aligned} \tag{A.7}$$

where  $i, j = 1..3$  represent generation indices.

## A.2 Below The Right-Handed Scale

The effective superpotential between  $v_R$  and  $F_\phi$  is given by:

$$\begin{aligned}
W_{\text{NMSSM}++} &= W_Y + W_N + W_{\text{Mass}} + \dots \\
W_Y &= i y_u^a Q^T \tau_2 H_{ua} u^c + i y_d^a Q^T \tau_2 H_{da} d^c + i y_e^a L^T \tau_2 \Phi_a e^c + f_c e^c \Delta^{c--} e^c \\
W_N &= \lambda^{ab} N \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{3} \kappa N^3 \\
W_{\text{Mass}} &= \frac{\mu_{\Delta^c}}{\phi} \Delta^{c--} \bar{\Delta}^{c++} + \frac{1}{2} \frac{\mu_N}{\phi} N^2 + \dots
\end{aligned} \tag{A.8}$$

with

$$y_u^a = y_d^a = y_Q^a \tag{A.9}$$

$$y_e^a = y_L^a \tag{A.10}$$

at the right-handed scale. The gauge coupling  $b$  values are:

$$b_3 = -3 \quad b_2 = 2 \quad b_1 = 20 \tag{A.11}$$

where no GUT based normalization has been imposed on  $b_1$ . A general gauge coupling RGE is given by

$$\frac{\partial \alpha_A}{\partial t} = \frac{b_A \alpha_A^2}{2\pi} \quad (\text{A.12})$$

no sum over  $A$ . The gamma functions for the particle content are

$$\begin{aligned}
16\pi^2 \gamma_{Q_3} &= -2|y_t^a|^2 - 2|y_b^a|^2 + 8\pi \left( \frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_2 + \frac{1}{18}\alpha_{B-L} \right) \\
16\pi^2 \gamma_{Q_1} &= 8\pi \left( \frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_2 + \frac{1}{18}\alpha_{B-L} \right) \\
16\pi^2 \gamma_{t^c} &= -4|y_t^a|^2 + 8\pi \left( \frac{8}{3}\alpha_3 + \frac{8}{9}\alpha_1 \right) \\
16\pi^2 \gamma_{b^c} &= -4|y_b^a|^2 + 8\pi \left( \frac{8}{3}\alpha_3 + \frac{2}{9}\alpha_1 \right) \\
16\pi^2 \gamma_{u^c} &= 8\pi \left( \frac{8}{3}\alpha_3 + \frac{8}{9}\alpha_1 \right) \\
16\pi^2 \gamma_{d^c} &= +8\pi \left( \frac{8}{3}\alpha_3 + \frac{2}{9}\alpha_1 \right) \\
16\pi^2 \gamma_{L_3} &= -2|y_\tau^a|^2 + 8\pi \left( \frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1 \right) \\
16\pi^2 \gamma_{L_1} &= 8\pi \left( \frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1 \right) \\
16\pi^2 \gamma_{\tau^c} &= -4|y_\tau^a|^2 - 8|f_{c3}|^2 + 8\pi(2\alpha_1) \\
16\pi^2 \gamma_{e^c} &= -8|f_{c1}|^2 + 8\pi(2\alpha_1) \\
16\pi^2 \gamma_{H_{ua}^{ub}} &= -6y_t^a y_t^{b*} - 8\lambda^{ac} \lambda^{cb*} + 8\pi \delta^{ab} \left( \frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1 \right) \\
16\pi^2 \gamma_{H_{da}^{db}} &= -6y_b^a y_b^{b*} - 2y_\tau^a y_\tau^{b*} - 8\lambda^{ac} \lambda^{cb*} + 8\pi \delta^{ab} \left( \frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1 \right) \\
16\pi^2 \gamma_N &= -4\kappa^2 - 16|\lambda^{ab}|^2 \\
16\pi^2 \gamma_{\Delta^{c--}} &= -4|f_{c3}|^2 - 4|f_{c2}|^2 - 4|f_{c1}|^2 + 8\pi(8\alpha_1) \\
16\pi^2 \gamma_{\bar{\Delta}^c} &= +8\pi(8\alpha_1)
\end{aligned} \quad (\text{A.13})$$

under the convention  $\gamma_\phi \equiv \frac{\partial \ln Z_\phi}{\partial \ln \mu}$ . It is assumed that:  $f_c = \text{diag}(f_{c1}, f_{c2}, f_{c3})$ ,  $y_u^a = \text{diag}(0, 0, y_t^a)$ ,  $y_d^a = \text{diag}(0, 0, y_b^a)$  and  $y_e^a = \text{diag}(0, 0, y_\tau^a)$ . Second generation  $\gamma$ -functions are the same as first generation ones except for the right-handed leptons where  $f_{c1} \rightarrow f_{c2}$ . The  $\beta$ -functions for the Yukawa couplings are then given by

$$\begin{aligned}
\frac{\partial y_t^a}{\partial t} &= -\frac{1}{2} \left( y_t^a (\gamma_{Q_3} + \gamma_{t^c}) + y_t^b \gamma_{H_{ub}^{H_{ua}}} \right) \\
\frac{\partial y_b^a}{\partial t} &= -\frac{1}{2} \left( y_b^a (\gamma_{Q_3} + \gamma_{b^c}) + y_b^b \gamma_{H_{db}^{H_{da}}} \right) \\
\frac{\partial y_\tau^a}{\partial t} &= -\frac{1}{2} \left( y_\tau^a (\gamma_{L_3} + \gamma_{\tau^c}) + y_\tau^b \gamma_{H_{db}^{H_{da}}} \right) \\
\frac{\partial \lambda^{ab}}{\partial t} &= -\frac{1}{2} \left( \lambda^{ab} \gamma_N + \lambda^{ac} \gamma_{H_{dc}^{H_{db}}} + \lambda^{cb} \gamma_{H_{uc}^{H_{ua}}} \right) \\
\frac{\partial f_{ci}}{\partial t} &= -\frac{1}{2} \left( f_{ci} \gamma_{\Delta^c} + 2 f_{cj} \gamma_{e_j^c} \right) \\
\frac{\partial \kappa}{\partial t} &= -\frac{1}{2} (3\kappa \gamma_N)
\end{aligned} \tag{A.14}$$

where  $i, j = 1..3$  represent generation indices.

### A.3 Below $F_\phi$

The effective superpotential between  $F_\phi$  and  $v_R$  is the NMSSM with a singlet mass term and is given by:

$$\begin{aligned}
W_{\text{NMSSM}} &= W_Y + W_N + W_{\text{Mass}} \\
W_Y &= \mathfrak{i} y_u Q^T \tau_2 H_u u^c + \mathfrak{i} y_d Q^T \tau_2 H_d d^c + \mathfrak{i} y_e L^T \tau_2 H_d e^c \\
W_N &= \lambda^{ab} N \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \frac{1}{3} \kappa N^3 \\
W_{\text{Mass}} &= \frac{1}{2} \frac{\mu_N}{\phi} N^2
\end{aligned} \tag{A.15}$$

where in general the Yukawa couplings here are some linear combination of the ones above  $F_\phi$  but based on the doublet-doublet splitting described in the Section 4.1.3

and used in Section 4.2

$$\begin{aligned}y_u &= y_u^1 \\y_d &= y_d^2 \\y_e &= y_e^2 \\ \lambda &= 2\lambda_{12}\end{aligned}\tag{A.16}$$

where the superscripts here indicate a  $\Phi$  generation and not exponentials. The gauge coupling  $b$  values are:

$$b_3 = -3 \quad b_2 = 1 \quad b_1 = 11\tag{A.17}$$

where no GUT based normalization has been imposed on  $b_1$ . A general gauge coupling RGE is given by

$$\frac{\partial\alpha_A}{\partial t} = \frac{b_A\alpha_A^2}{2\pi}\tag{A.18}$$

no sum over  $A$ . The gamma functions for the particle content are

$$\begin{aligned}
16\pi^2\gamma_{Q_3} &= -2|y_t|^2 - 2|y_b|^2 + 8\pi\left(\frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_2 + \frac{1}{18}\alpha_{B-L}\right) \\
16\pi^2\gamma_{Q_1} &= 8\pi\left(\frac{8}{3}\alpha_3 + \frac{3}{2}\alpha_2 + \frac{1}{18}\alpha_{B-L}\right) \\
16\pi^2\gamma_{t^c} &= -4|y_t|^2 + 8\pi\left(\frac{8}{3}\alpha_3 + \frac{8}{9}\alpha_1\right) \\
16\pi^2\gamma_{b^c} &= -4|y_b|^2 + 8\pi\left(\frac{8}{3}\alpha_3 + \frac{2}{9}\alpha_1\right) \\
16\pi^2\gamma_{u^c} &= 8\pi\left(\frac{8}{3}\alpha_3 + \frac{8}{9}\alpha_1\right) \\
16\pi^2\gamma_{d^c} &= +8\pi\left(\frac{8}{3}\alpha_3 + \frac{2}{9}\alpha_1\right) \\
16\pi^2\gamma_{L_3} &= -2|y_\tau|^2 + 8\pi\left(\frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1\right) \\
16\pi^2\gamma_{L_1} &= 8\pi\left(\frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1\right) \\
16\pi^2\gamma_{\tau^c} &= -4|y_\tau|^2 + 8\pi(2\alpha_1) \\
16\pi^2\gamma_{e^c} &= +8\pi(2\alpha_1) \\
16\pi^2\gamma_{H_u} &= -6|y_t|^2 - 2|\lambda|^2 + 8\pi\left(\frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1\right) \\
16\pi^2\gamma_{H_d} &= -6|y_b|^2 - 2|y_\tau|^2 - 8\lambda^{ac}\lambda^{cb*} + 8\pi\left(\frac{3}{2}\alpha_2 + \frac{1}{2}\alpha_1\right) \\
16\pi^2\gamma_N &= -4\kappa^2 - 4|\lambda|^2
\end{aligned} \tag{A.19}$$

under the convention  $\gamma_\phi \equiv \frac{\partial \ln Z_\phi}{\partial \ln \mu}$ . It is assumed that:  $y_u = \text{diag}(0, 0, y_t)$ ,  $y_d = \text{diag}(0, 0, y_b)$  and  $y_e = \text{diag}(0, 0, y_\tau)$ . Second generation  $\gamma$ -functions are the same as the first generation ones. The MSSM  $\gamma$ -functions can be calculated by setting  $\lambda$  and  $\kappa$  to zero in Eq. (A.19). The  $\beta$ -functions for the Yukawa couplings are then given

by

$$\begin{aligned}\frac{\partial y_t}{\partial t} &= -\frac{1}{2}(y_t(\gamma_{Q_3} + \gamma_{t^c} + \gamma_{H_u})) \\ \frac{\partial y_b}{\partial t} &= -\frac{1}{2}(y_b(\gamma_{Q_3} + \gamma_{b^c}\gamma_{H_d})) \\ \frac{\partial y_\tau}{\partial t} &= -\frac{1}{2}(y_\tau(\gamma_{L_3} + \gamma_{\tau^c}\gamma_{H_d})) \\ \frac{\partial \lambda}{\partial t} &= -\frac{1}{2}(\lambda(\gamma_N + \gamma_{H_d} + \lambda\gamma_{H_u})) \\ \frac{\partial \kappa}{\partial t} &= -\frac{1}{2}(3\kappa\gamma_N)\end{aligned}\tag{A.20}$$

where  $i, j = 1..3$  represent generation indices. These are also given in [87].



## Appendix B

### Masses

#### B.1 Masses in the Minimal Supersymmetric Standard Model

##### B.1.1 Sfermion Masses

The third generation left-handed sfermions potentially mix with the right-handed ones due to Yukawa couplings although this is usually only substantial in the top sector. The mass matrices are

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_t^2 + D_{\tilde{u}_L} & v(a_t^* \sin \beta - \mu y_t \cos \beta) \\ v(a_t \sin \beta - \mu^* y_t \cos \beta) & m_{t^c}^2 + m_t^2 + D_{\tilde{u}_R} \end{pmatrix} \quad (\text{B.1})$$

$$m_{\tilde{b}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_b^2 + D_{\tilde{d}_L} & v(a_b^* \cos \beta - \mu y_b \sin \beta) \\ v(a_b \cos \beta - \mu^* y_b \sin \beta) & m_{b^c}^2 + m_b^2 + D_{\tilde{d}_R} \end{pmatrix} \quad (\text{B.2})$$

$$m_{\tilde{\tau}}^2 = \begin{pmatrix} m_{L_3}^2 + m_\tau^2 + D_{\tilde{e}_L} & v(a_t a u^* \cos \beta - \mu y_\tau \sin \beta) \\ v(a_t a u \cos \beta - \mu^* y_\tau \sin \beta) & m_{\tau^c}^2 + m_\tau^2 + D_{\tilde{d}_R} \end{pmatrix} \quad (\text{B.3})$$

where  $D_\phi = (T_3(\phi) - Q_{EM}(\phi)) \cos(2\beta) M_Z^2$  represents electroweak  $D$ -term contributions to the mass of scalar  $\phi$ . The masses for the first and second generation sfermions are typically assumed to be equal and are simply the sum of the appropriate mass term and the  $D$ -term contributions  $m_\phi^2 + D_\phi$ . The lightest (heaviest) sfermion of a given flavor is denoted with a subscript 1 (2).

The sfermion mass spectrum is heavily dependent on the soft parameters and therefore the SUSY breaking mechanism. Since the popular models relate soft masses to gauge couplings or because of the effects of gauge couplings in RGEs, it is usually assumed that the squarks are the heaviest sfermions and that mostly left-handed sfermions are heavier than mostly right-handed ones.

### B.1.2 Bosino Masses

Bosinos can be subdivided into neutralinos and charginos fields. There are four neutralinos in the MSSM and their mass matrix in the  $(\tilde{B}, \tilde{W}, \tilde{H}_d^0, \tilde{H}_u^0)$  basis is

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -c_\beta s_w M_z & s_\beta s_w M_z \\ 0 & M_2 & c_\beta c_w M_z & -s_\beta c_w M_z \\ -c_\beta s_w M_z & c_\beta c_w M_z & 0 & -\mu \\ s_\beta s_w M_z & -s_\beta c_w M_z & -\mu & 0 \end{pmatrix} \quad (\text{B.4})$$

where  $c_w$  ( $s_w$ )  $\equiv \cos \theta_w$  ( $\sin^2 \theta_w \sim 0.22$ ). The mass eigenstates are  $\tilde{\chi}_1^0 \dots \tilde{\chi}_4^0$  from lightest. The lightest neutralino usually turns out to be mostly bino (mSUGRA and mGMSB).

The chargino mass matrix in the  $(\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-)$  basis is

$$M_{\tilde{\chi}^\pm} = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \quad X \equiv \begin{pmatrix} M_2 & \sqrt{2} s_\beta M_W \\ \sqrt{2} c_\beta M_W & \mu \end{pmatrix} \quad (\text{B.5})$$

Winos form a nearly degenerate isospin triplet (mass differences are on the order of 100 MeV or so) while the Higgsinos form nearly degenerate isospin triplet

although a larger mass difference exists in this case because of larger mixings,  $\sim 1\text{GeV}$ .

The LSP is usually assumed to be the lightest neutralino since dark matter candidates must be neutral and the sneutrinos do not have the correct properties in minimal models. However, satisfying the bounds is non-trivial since the amount of dark matter depends on how quickly the LSP self annihilates in the early universe. The bino self-annihilates slowly often leading to a universe with too much dark matter (over closed) while the wino and Higgsino self-annihilate too quickly leading to a dark matter abundance that is too low. Some mix is sometimes necessary.

The LSP is also usually important in collider phenomenology since produced particles will cascade decay to the LSP. At the LHC then, a gluino or squark might be pair produced and the resulting cascade decays would include jets, missing energy from the LSP, which does not register on the detector and possibly leptons.

## B.2 Type II Singular Seesaw Mechanism

We start with a mass matrix of the form

$$\mathcal{M} = \begin{pmatrix} \delta^2 m_L & \delta m_D \\ \delta m_D^\dagger & M_R \end{pmatrix} \quad (\text{B.6})$$

where  $\delta$  carries the relative order of magnitude of the elements of each of the three  $(n \times n)$  matrices  $m_L$ ,  $m_D$ , and  $M_R$ —i.e. there is a hierarchy which can be thought of as either  $\left| (M_R)_{ij} \right| \gg |(\delta m_D)_{k\ell}| \gg \left| (\delta^2 m_L)_{pq} \right|$ ; or, alternatively,  $\delta \ll 1$ ,  $\left| (M_R)_{ij} \right| \sim |(\delta m_D)_{k\ell}| \sim \left| (m_L)_{pq} \right| \equiv v$ .

It is not assumed, however, that all the eigenvalues of  $M_R$  are of this high scale  $v$ , so in the limit  $\delta \rightarrow 0$ , it is possible that  $\det M_R = 0$ . Therefore, to exploit this hierarchy it is necessary to extract those smaller eigenvalues. This is done as follows:

First, diagonalize  $M_R$  through an  $(n \times n)$  rotation  $R$  via

$$\mathcal{R} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & R \end{pmatrix} \quad (\text{B.7})$$

so that

$$\mathcal{R} \mathcal{M} \mathcal{R}^T = \begin{pmatrix} \delta^2 m_L & \delta m_D R^T \\ \delta R m_D^\dagger & M_d \end{pmatrix} \quad (\text{B.8})$$

with  $M_d \equiv R M_R R^T$  which is a diagonal matrix. The matrix  $R$  should be chosen so that the first  $k$  eigenvalues of  $M_d$  are the small ones—thus, for  $1 \leq i \leq k$

$$(\delta^2 \mu_R)_{ii} \equiv (M_d)_{ii} = \delta^2 \lambda_i v^2 \quad (\text{B.9})$$

where  $\lambda_i \sim 1$  and  $(\delta^2 \mu_R)_{ij} = 0$  for  $i \neq j$ . The remaining (large) eigenvalues are then placed in a separate matrix:

$$\Delta_R \equiv \text{diag}\left((M_d)_{k+1,k+1}, (M_d)_{k+2,k+2}, \dots, (M_d)_{n,n}\right) \quad (\text{B.10})$$

Also define

$$\delta \mu_1 \equiv \begin{pmatrix} 0 & \text{col}_1(\delta m_D R^T) & \cdots & \text{col}_k(\delta m_D R^T) \\ \text{row}_1(\delta R m_D^\dagger) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \text{row}_k(\delta R m_D^\dagger) & 0 & \cdots & 0 \end{pmatrix} \quad (\text{B.11})$$

$$\delta^2\mu_2 \equiv \begin{pmatrix} \delta^2 m_L & 0 \\ 0 & \delta^2\mu_R \end{pmatrix} \quad (\text{B.12})$$

$$\delta\mu_D \equiv \begin{pmatrix} \text{col}_{k+1}(\delta m_D R^T) & \text{col}_{k+2}(\delta m_D R^T) & \cdots & \text{col}_n(\delta m_D R^T) \\ 0 & 0 & \cdots & 0 \end{pmatrix} \quad (\text{B.13})$$

With those definitions we may write

$$\mathcal{R}\mathcal{M}\mathcal{R}^T = \begin{pmatrix} \delta\mu_1 + \delta^2\mu_2 & \delta\mu_D \\ \delta\mu_D^\dagger & \Delta_R \end{pmatrix} \quad (\text{B.14})$$

Now a matrix  $P$  is chosen so that it block diagonalizes  $\mathcal{R}\mathcal{M}\mathcal{R}^T$ . This  $P$  is implemented through  $\mathcal{P}$  which, to order  $\delta^2$ , is given by

$$\mathcal{P} = \begin{pmatrix} \mathbb{1} - \frac{1}{2}\delta^2 P P^\dagger & -\delta P \\ \delta P^\dagger & \mathbb{1} - \frac{1}{2}\delta^2 P^\dagger P \end{pmatrix} \quad (\text{B.15})$$

The matrix  $P$  is then determined by the requirement

$$\mathcal{P}\mathcal{R}\mathcal{M}\mathcal{R}^T\mathcal{P}^\dagger = \begin{pmatrix} m & 0 \\ 0 & M \end{pmatrix} \quad (\text{B.16})$$

with  $m$  the  $(n+k) \times (n+k)$  mass matrix of interest and  $M = \Delta_R + \mathcal{O}(\delta)$ . The off-block-diagonal condition yields

$$P = \mu_D \Delta_R^{-1} \quad (\text{B.17})$$

and then using that  $P$ , the mass matrix for the light eigenstates can be determined:

$$m = \delta\mu_1 + \delta^2\mu_2 - \delta^2\mu_D \Delta_R^{-1} \mu_D^\dagger \quad (\text{B.18})$$

## Bibliography

- [1] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).
- [2] A. Salam and J. C. Ward, Phys. Lett. **13**, 168 (1964).
- [3] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
- [4] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964).
- [5] P. W. Higgs, Phys. Lett. **12**, 132 (1964).
- [6] P. W. Higgs, Phys. Rev. **145**, 1156 (1966).
- [7] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **96**, 032002 (2006), hep-ph/0510099.
- [8] ALEPH, Phys. Rept. **427**, 257 (2006), hep-ex/0509008.
- [9] V. C. Rubin, N. Thonnard, and J. Ford, W. K., Astrophys. J. **238**, 471 (1980).
- [10] WMAP, D. N. Spergel *et al.*, Astrophys. J. Suppl. **170**, 377 (2007), astro-ph/0603449.
- [11] E. W. Kolb and M. S. Turner, Front. Phys. **69**, 1 (1990).
- [12] N. Ferrari, Astrophys. Lett. Commun. **33**, 11 (1996).
- [13] SNO, N. Oblath, AIP Conf. Proc. **947**, 249 (2007).
- [14] Super-Kamiokande, J. P. Cravens *et al.*, (2008), 0803.4312.
- [15] Super-Kamiokande, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998), hep-ex/9807003.
- [16] Super-Kamiokande, Y. Ashie *et al.*, Phys. Rev. Lett. **93**, 101801 (2004), hep-ex/0404034.
- [17] P. Minkowski, Phys. Lett. **B67**, 421 (1977).
- [18] M. Gell-Mann, P. Ramond, and R. Slansky, Print-80-0576 (CERN).
- [19] T. Yanagida, In Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 13-14 Feb 1979.
- [20] S. L. Glashow, NATO Adv. Study Inst. Ser. B Phys. **59**, 687 (1979).
- [21] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [22] S. P. Martin, (1997), hep-ph/9709356.
- [23] M. Drees, (1996), hep-ph/9611409.

- [24] M. Dine, (1996), hep-ph/9612389.
- [25] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980).
- [26] D. Chang, R. N. Mohapatra, J. Gipson, R. E. Marshak, and M. K. Parida, Phys. Rev. **D31**, 1718 (1985).
- [27] N. G. Deshpande, E. Keith, and T. G. Rizzo, Phys. Rev. Lett. **70**, 3189 (1993), hep-ph/9211310.
- [28] M. Lindner and M. Weiser, Phys. Lett. **B383**, 405 (1996), hep-ph/9605353.
- [29] M. Cvetič and J. C. Pati, Phys. Lett. **B135**, 57 (1984).
- [30] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. **D48**, 4352 (1993), hep-ph/9306290.
- [31] R. M. Francis, M. Frank, and C. S. Kalman, Phys. Rev. **D43**, 2369 (1991).
- [32] C. S. Aulakh, A. Melfo, and G. Senjanovic, Phys. Rev. **D57**, 4174 (1998), hep-ph/9707256.
- [33] C. S. Aulakh, A. Melfo, A. Rasin, and G. Senjanovic, Phys. Rev. **D58**, 115007 (1998), hep-ph/9712551.
- [34] Z. Chacko and R. N. Mohapatra, Phys. Rev. **D58**, 015003 (1998), hep-ph/9712359.
- [35] R. N. Mohapatra and A. Rasin, Nucl. Phys. Proc. Suppl. **52A**, 182 (1997).
- [36] R. N. Mohapatra, N. Setzer, and S. Spinner, Phys. Rev. **D73**, 075001 (2006), hep-ph/0511260.
- [37] K. S. Babu, B. Dutta, and R. N. Mohapatra, Phys. Rev. **D60**, 19 (1999), hep-ph/9812421.
- [38] K. S. Babu, B. Dutta, and R. N. Mohapatra, Phys. Lett. **B458**, 93 (1999), hep-ph/9904366.
- [39] K. S. Babu, B. Dutta, and R. N. Mohapatra, Phys. Rev. Lett. **85**, 5064 (2000), hep-ph/0006329.
- [40] S. Dar, Q. Shafi, and A. Sil, Phys. Lett. **B632**, 517 (2006), hep-ph/0508037.
- [41] B. Dutta and R. N. Mohapatra, Phys. Rev. **D59**, 015018 (1999), hep-ph/9804277.
- [42] R. N. Mohapatra, Phys. Rev. **D54**, 5728 (1996).
- [43] B. Mukhopadhyaya and S. K. Rai, Phys. Lett. **B633**, 519 (2006), hep-ph/0508290.

- [44] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999), hep-th/9810155.
- [45] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, JHEP **12**, 027 (1998), hep-ph/9810442.
- [46] W. Siegel and J. Gates, S. James, Nucl. Phys. **B147**, 77 (1979).
- [47] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Nucl. Phys. **B212**, 413 (1983).
- [48] Z. Chacko, M. A. Luty, I. Maksymyk, and E. Ponton, JHEP **04**, 001 (2000), hep-ph/9905390.
- [49] T. Gherghetta, G. F. Giudice, and J. D. Wells, Nucl. Phys. **B559**, 27 (1999), hep-ph/9904378.
- [50] E. Katz, Y. Shadmi, and Y. Shirman, JHEP **08**, 015 (1999), hep-ph/9906296.
- [51] A. Pomarol and R. Rattazzi, JHEP **05**, 013 (1999), hep-ph/9903448.
- [52] N. Arkani-Hamed, D. E. Kaplan, H. Murayama, and Y. Nomura, JHEP **02**, 041 (2001), hep-ph/0012103.
- [53] B. C. Allanach and A. Dedes, JHEP **06**, 017 (2000), hep-ph/0003222.
- [54] I. Jack and D. R. T. Jones, Phys. Lett. **B482**, 167 (2000), hep-ph/0003081.
- [55] M. Carena, K. Huitu, and T. Kobayashi, Nucl. Phys. **B592**, 164 (2001), hep-ph/0003187.
- [56] M. Ibe, R. Kitano, H. Murayama, and T. Yanagida, Phys. Rev. **D70**, 075012 (2004), hep-ph/0403198.
- [57] N. Okada, Phys. Rev. **D65**, 115009 (2002), hep-ph/0202219.
- [58] R. N. Mohapatra, N. Setzer, and S. Spinner, (2008), 0802.1208.
- [59] R. N. Mohapatra, N. Setzer, and S. Spinner, (2007), arXiv:0707.0020 [hep-ph].
- [60] N. Setzer and S. Spinner, Phys. Rev. **D75**, 117701 (2007), hep-ph/0612318.
- [61] R. Kitano, G. D. Kribs, and H. Murayama, Phys. Rev. **D70**, 035001 (2004), hep-ph/0402215.
- [62] J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski, and F. Zwirner, Phys. Rev. **D39**, 844 (1989).
- [63] N. Setzer and S. Spinner, Phys. Rev. **D71**, 115010 (2005), hep-ph/0503244.
- [64] S. P. Martin and M. T. Vaughn, Phys. Rev. **D50**, 2282 (1994), hep-ph/9311340.
- [65] SINDRUM, U. Bellgardt *et al.*, Nucl. Phys. **B299**, 1 (1988).



- [66] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Nucl. Phys. **B238**, 453 (1984).
- [67] D. O. Caldwell *et al.*, Phys. Rev. Lett. **61**, 510 (1988).
- [68] C. Arina and N. Fornengo, JHEP **11**, 029 (2007), arXiv:0709.4477 [hep-ph].
- [69] G. F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999), hep-ph/9801271.
- [70] J. L. Feng, T. Moroi, L. Randall, M. Strassler, and S.-f. Su, Phys. Rev. Lett. **83**, 1731 (1999), hep-ph/9904250.
- [71] J. L. Feng and T. Moroi, Phys. Rev. **D61**, 095004 (2000), hep-ph/9907319.
- [72] L. Willmann *et al.*, Phys. Rev. Lett. **82**, 49 (1999), hep-ex/9807011.
- [73] C. H. Chen, M. Drees, and J. F. Gunion, Phys. Rev. Lett. **76**, 2002 (1996), hep-ph/9512230.
- [74] S. D. Thomas and J. D. Wells, Phys. Rev. Lett. **81**, 34 (1998), hep-ph/9804359.
- [75] J. F. Gunion and S. Mrenna, Phys. Rev. **D62**, 015002 (2000), hep-ph/9906270.
- [76] I. Hinchliffe, F. E. Paige, M. D. Shapiro, J. Soderqvist, and W. Yao, Phys. Rev. **D55**, 5520 (1997), hep-ph/9610544.
- [77] H. Baer, J. K. Mizukoshi, and X. Tata, Phys. Lett. **B488**, 367 (2000), hep-ph/0007073.
- [78] F. E. Paige and J. D. Wells, (1999), hep-ph/0001249.
- [79] M. Dine, N. Seiberg, and S. Thomas, (2007), arXiv:0707.0005 [hep-ph].
- [80] T. Han, B. Mukhopadhyaya, Z. Si, and K. Wang, Phys. Rev. **D76**, 075013 (2007), arXiv:0706.0441 [hep-ph].
- [81] M. Frank, K. Huitu, and S. K. Rai, Phys. Rev. **D77**, 015006 (2008), arXiv:0710.2415 [hep-ph].
- [82] M. Raidal and P. M. Zerwas, Eur. Phys. J. **C8**, 479 (1999), hep-ph/9811443.
- [83] G. Barenboim, K. Huitu, J. Maalampi, and M. Raidal, Phys. Lett. **B394**, 132 (1997), hep-ph/9611362.
- [84] A. G. Akeroyd, M. Aoki, and H. Sugiyama, (2007), arXiv:0712.4019 [hep-ph].
- [85] T. Moroi and L. Randall, Nucl. Phys. **B570**, 455 (2000), hep-ph/9906527.
- [86] R. N. Mohapatra, N. Okada, and H.-B. Yu, (2007), arXiv:0711.0956 [hep-ph].
- [87] S. F. King and P. L. White, Phys. Rev. **D52**, 4183 (1995), hep-ph/9505326.