ABSTRACT

Title of dissertation: ν SEESAW USES: UV INSENSITIVE SUPERSYMMETRY BREAKING WITHOUT TACHYONS

Nicholas Setzer, Doctor of Philosophy, 2008

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This document contains a systematic analysis of supersymmetric left-right models in the context of anomaly mediated supersymmetry breaking starting with the high-scale, left-right theory and ending with the supersymmetry-scale theory. It is shown that the combination of supersymmetric left-right models and anomaly mediated supersymmetry breaking retains the attractive features of anomaly mediation while simultaneously providing a solution to the tachyonic slepton problem of the minimal supersymmetric standard model with anomaly mediated supersymmetry breaking.

The supersymmetric left-right theory introduces new yukawa couplings that permit positive slepton mass-squares while retaining the ultra violet insensitivity of anomaly mediated supersymmetry breaking as well its economy. The new couplings are introduced by independent considerations of explaining neutrino oscillation experiments through the seesaw mechanism, and survive below the seesaw scale from an accidental symmetry of the potential. Furthermore, the seesaw mechanism is implemented in such a way that R-parity is a natural residual symmetry—leading to a stable, weakly-interacting particle to explain dark matter.

The resulting mass spectrum is detailed, both qualitatively and quantitatively, providing comparisons with other popular supersymmetry breaking scenarios. It is demonstrated that the model contains gaugino masses that are much closer in size than other schemes, as well as the possibility of a mild squark-slepton mass degeneracy. The issue of higgsino masses is also explored, and attention is paid to the dark matter composition. The model is shown to have a viable dark matter candidate that evades current direct detection bounds but will be probed by future planned experiments.

The low-energy consequences of the model are analyzed, and the matter of electroweak symmetry breaking is expounded. It is shown that the problem of a higgsino mass below the LEP II bound in the next-to minimal supersymmetric standard model with anomaly mediated supersymmetry breaking is easily avoided by this theory. Finally, prospects for confirmation of this theory at the LHC are investigated, as well as potential signatures in lepton flavor violation experiments.

ν SEESAW USES: UV INSENSITIVE SUPERSYMMETRY BREAKING WITHOUT TACHYONS

by

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Preface

The material contained within these pages is a compilation of work previously published in the papers [1, 2, 3] with my collaborators R. N. Mohapatra and S. Spinner.

Dedication

To science, knowledge, and critical thinking.

Acknowledgments

This section is to thank all the people who assisted in my graduate career. It will likely only be read by those who intend to find their name present, yet since such a list must necessarily fail to be comprehensive, I must apologize in advance if you did not get to see your name in print here.

Obviously my advisor, Rabindra Mohapatra, was essential in my achieving a Ph.D.—as were the other members of the Elementary Particle Group at Maryland: Dr. Jogesh Pati, who assisted my career with helpful comments, courses, and letters; Dr. Markus Luty, who provided great insight into quantum field theory and also provided letters of recommendation; Dr. Zacharia Chacko, with whom I have had quite fruitful physics discussions; Dr. Kastaubh Agashe, who presented his insight into extra dimensions; and finally Drs. K. Abazajian, S. James Gates, O. W. Greenburg, X. Ji, and Y. S. Kim.

Additionally, several faculty members outside of the EP group were instrumental in my graduate career: J. Robert Dorfman, who provided an entertaining and informative course in quantum mechanics as well as an excellent class in statistical mechanics; Dr. Alex Dragt, from whom I had the privilege of garnering his superb knowledge of classical mechanics and group theory; Dr. Edward Ott, who provided a wonderful course in electrodynamics; Dr. William Dorland, who provided my first research project and graciously concocted projects to support me over the summers; Dr. Theodore Jacobson, whose physical insight and illuminating teaching style both amazes and inspires me; and lastly, Dr. Michael Coplan, who nursed me through the graduate laboratory course and patiently addressed my definitively naive questions.

A number of elementary particle physics' postdocs have also aided my career: Dr. Manuel Toharia and Dr. Takemichi Okui both deserve a thanks for their conversations and advice.

Naturally, my collaborator and friend, Sogee Spinner, has assisted in innumerable ways—without him I would not have made it here nor learned the many things he taught me. Additionally, my fellow particle graduate students, Aleksandr Azatov, Ken Hsieh, Christopher Allen Krenke, Parul Rastogi, Haibo Yu, and Lijun Zhu deserve mention.

Numerous other friends have also been instrumental in retaining the little sanity that I have throughout my time here: Kate Despain, who was always ready to contradict what I said based on principle; Wayne Witzel, who has shared my craziness and also invented the universe; Meem Mahmud, who is uniquely qualified to eternally distract me and who has made use of this ability often; Jenna Samra, with whom I had a number of nonsensical conversations; Amethyst Tigarus III, who was always willing to be adorable with both sight and sound; and Anaiya, who greeted me every morning and has an insatiable appetite for attention.

The support staff of the physics department, from whom I have made many requests for assistance, have been extremely kind, helpful, and encouraging during my time here.

Finally, all my students from whom I have learned as well as taught deserve thanks.

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List of Abbreviations

AMSB	anomaly mediated supersymmetry breaking
BSM	beyond the standard model
CISLR	conformal invariant supersymmetric left-right
CKM	Cabbibo-Kobayashi-Maskawa
EWSB	electroweak symmetry breaking
GAYMS	gauge-anomaly-yukawa mediated supersymmetry breaking
GMSB	gauge mediated supersymmetry breaking
GUT	grand unified theory
IR	infrared
LEP	large electron positron collider
LSP	lightest supersymmetric particle
LHC	large hadron collider
mAMSB	minimal anomaly mediated supersymmetry breaking
mGMSB	minimal gauge mediated supersymmetry breaking
MSSM	minimal supersymmetric standard model
mSUGRA	minimal supergravity
NLSP	next-to lightest supersymmetric particle
NMSSM	next-to minimal supersymmetric standard model
RGE	renormalization group equation
SM	standard model
SSB	spontaneous symmetry breaking
SUGRA	supergravity
SUSY	supersymmetry
SUSYLR	supersymmetric left-right
UV	ultra violet
VEV	vacuum expectation value

Chapter 1

Introduction

1.1 The Standard Model of Particle Physics

The current understanding of all of particle physics is contained within the standard model (SM)[6, 7, 8, 9, 10, 11]. The theory is defined by the symmetries that it obeys and the transformations of the particle content under those symmetries. Once specified, the lagrangian that contains every possible renormalizable interaction that respects those symmetries is written and the theory is formally complete.

1.1.1 Particle Content

The SM is laid out in Table 1.1. The $SU(3)^c \times SU(2)_L \times U(1)_Y$ gauge symmetry permits the lagrangian

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm KE} + \mathcal{L}_{\rm yuk} - V_{\rm H} \tag{1.1}$$

where

	$SU(3)^c$	$\times$	$SU(2)_L$	$\times$	$U(1)_Y$
Q	3		2		$+\frac{1}{3}$
u_R	$\bar{3}$		1		$+\frac{4}{3}$
d_R	$\bar{3}$		1		$-\frac{2}{3}$
L	1		2		-1
e_R	1		1		-2
Φ	1		2		$+1$

Table 1.1: The Standard Model. The particles are listed by their representations of the gauge groups except for $U(1)_Y$ where the hypercharge is given.

$$\mathcal{L}_{\mathrm{KE}} = \mathbb{i}\bar{\Psi}_Q \gamma^\mu \mathrm{D}_\mu \Psi_Q + \mathbb{i}\bar{\Psi}_{u_R} \gamma^\mu \mathrm{D}_\mu \Psi_{u_R} + \mathbb{i}\bar{\Psi}_{d_R} \gamma^\mu \mathrm{D}_\mu \Psi_{d_R} + \mathbb{i}\bar{\Psi}_L \gamma^\mu \mathrm{D}_\mu \Psi_L + \mathbb{i}\bar{\Psi}_{e_R} \gamma^\mu \mathrm{D}_\mu \Psi_{e_R} + (\mathrm{D}_\mu \Phi)^\dagger \mathrm{D}^\mu \Phi - \frac{1}{4} G^A_{\mu\nu} G^{\mu\nu}_A - \frac{1}{4} W^A_{\mu\nu} W^{\mu\nu}_A - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{1.2}$$

$$\mathcal{L}_{\mathrm{yuk}} = y_u \bar{\Psi}_Q \tilde{\Phi} \Psi_{u_R} + y_d \bar{\Psi}_Q \Phi \Psi_{d_R} + y_e \bar{\Psi}_L \Phi \Psi_{e_R} + \mathrm{h.c.} \tag{1.3}$$

$$V_{\mathrm{H}} = -m^2 |\Phi|^2 + \lambda |\Phi|^4 \tag{1.4}$$

and the fields are defined by[1]

$$\Psi_L \equiv \begin{pmatrix} \Psi_{\nu_e} \\ \Psi_{e_L} \end{pmatrix} \qquad \Psi_{e_{\substack{L\\R}}} \equiv (1 \mp \gamma_5)\, \Psi_e \qquad \Psi_Q \equiv \begin{pmatrix} \Psi_{u_L} \\ \Psi_{d_L} \end{pmatrix}, \text{ etc.} \tag{1.5}$$

$$\tilde{\Phi} \equiv \mathbb{i}\tau_2 \Phi^* \tag{1.6}$$

$$\begin{aligned} G^A_{\mu\nu} &\equiv \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_3 f_3^{ABC} G^B_\mu G^C_\nu \\ W^A_{\mu\nu} &\equiv \partial_\mu W^A_\nu - \partial_\nu W^A_\mu + g_L \epsilon^{ABC} W^B_\mu W^C_\nu \\ B_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \tag{1.7}$$

There are three copies of the fields Q, u_R, d_R, L, and e_R while there is only one Φ field. Each copy of the field is called a generation or family, and in Eqs. (1.2)–(1.3) this family index has been suppressed. Furthermore, the indices corresponding to $SU(3)^c \times SU(2)_L$ have also been omitted from those equations.

The theory as written has a total of 18 parameters—the three gauge couplings g_3, g_L, g_Y; the higgs-sector mass and self-coupling m^2, λ; and the 13 degrees of freedom in the yukawa couplings y_u, y_d, y_e[2]—though these are not the ones typically

[1] Appendix A defines all the notation conventions of this document, including definitions omitted here.

[2] In family space the yukawa couplings are 3×3 complex matrices, thus overall there are 54 degrees of freedom among them. The $SU(3)_Q \times SU(3)_{u_R} \times SU(3)_{d_R} \times SU(3)_L \times SU(3)_{e_R} \times U(1)_\Phi$ symmetry can be used to eliminate 41 of them, leaving 13 free. These can be chosen to be the three leptonic yukawa couplings, the three up sector quark yukawa couplings, and the 7 parameters of y_d. The latter may be subdivided into the three down generation masses, one CP phase, and three mixing angles.

measured by experimentalists. In point, nature has demonstrated that the only respected symmetries are $SU(3)^c$ and $U(1)_{em}$, while the weak force is short-ranged and governed by the massive vector bosons $W^{\pm}$, Z. Furthermore, the leptons have distinct and measurable masses which are absent in Eq. (1.1). All these 'discrepancies', however, are well understood in the SM through spontaneous symmetry breaking (SSB) and the Higgs Mechanism.

1.1.2 Spontaneous Symmetry Breaking and The Higgs Mechanism

Spontaneous symmetry breaking is the idea that the ground state of a system contains only a subset of the symmetries respected by the underlying theory. Thus, the theory actually has more symmetry than realized by the observed lowest-energy state. This idea is not unique to particle physics or the SM, but is prevalent in many areas of physics; for example, ferromagnetism.

In the SM, SSB is achieved through the spin-0 higgs boson, Φ. The idea is that the higgs field acquires a non-zero classical background, called a vacuum expectation value (VEV), and the quantum theory must be written as perturbations around this classical background. The theory still maintains the full symmetry, however the ground state—the one in which the VEV of Φ is non-zero—breaks this symmetry, and thus it is not seen in nature.

Writing the higgs field doublet as

$$\Phi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(\Phi_1 + \mathbb{i}\Phi_2) \end{pmatrix} \tag{1.8}$$

and taking $\Phi^+ = 0$ to preserve electric charge conservation, yields the potential V_{H} depicted in Figure 1.1. As the figure demonstrates, $\Phi_1 = \Phi_2 = 0$ is not a stable point, and so the fields prefer a non-zero classical value. With the non-zero classical background being defined as $\langle\Phi\rangle$, and choosing

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{1.9}$$

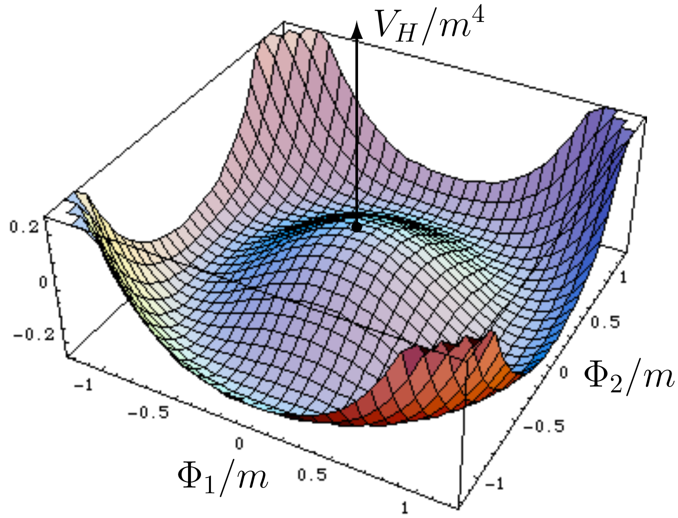


Figure 1.1: The standard model higgs potential for $\lambda = 0.5$, $m^2 > 0$. The point $\Phi_1 = \Phi_2 = 0$ is clearly unstable so the vacuum state has one or both non-zero (thus breaking the symmetry).

the shift to the true vacuum state, $\Phi \to \Phi + \langle\Phi\rangle$, yields the $SU(3)^c \times U(1)_{em}$ theory seen by experimentalists. The $SU(2)_L$ gauge bosons $W^{\pm}_{\mu} \equiv \frac{\eta_{\mu\nu}}{\sqrt{2}}(W_1^{\nu} \mp \mathbb{i} W_2^{\nu})$ acquire a mass, $M_W = g_L v/2$, with the longitudinal degree of freedom coming from the charged Φ. The neutral W_3^{μ} and B^{μ} mix

$$\mathcal{L} \supset \frac{v^2}{8} \begin{pmatrix} W_3^{\mu} & B^{\mu} \end{pmatrix} \begin{pmatrix} g_L^2 & -g_L g_Y \\ -g_L g_Y & g_Y^2 \end{pmatrix} \begin{pmatrix} W_3^{\mu} \\ B^{\mu} \end{pmatrix} \tag{1.10}$$

to yield the massless photon A^{μ} and the massive Z^{μ} (whose longitudinal component is the CP-odd neutral Φ):

$$\begin{pmatrix} Z^{\mu} \\ A^{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_3^{\mu} \\ B^{\mu} \end{pmatrix} \tag{1.11}$$

$$M_Z^2 = \frac{1}{4}\left(g_L^2 + g_Y^2\right) v^2 \tag{1.12}$$

with θ_W the weak mixing angle, $\tan\theta_W = g_Y/g_L$.

The acquisition of mass for the gauge fields through the consumption of a scalar field (one that by the Goldstone Theorem would otherwise be massless) is known as the Higgs Mechanism[7, 8, 11], and explains why the weak force is observed as short-ranged. A further consequence of this mechanism is the existence of the CP-even neutral component of Φ, with a mass $m_h = m\sqrt{2}$. It is therefore predicted by the SM that experimentalists will see a fundamental scalar (somewhere before the TeV scale) which can be identified as this higgs boson.

Finally, the shift to the true vacuum gives the fermions of the theory a mass through their yukawa couplings to the higgs:

$$m_e = \frac{1}{\sqrt{2}} y_e v \qquad m_u = \frac{1}{\sqrt{2}} y_u v \qquad m_d = \frac{1}{\sqrt{2}} y_d v \tag{1.13}$$

with the neutrino remaining massless.

The resulting theoretical picture can then be summarized as in Figure 1.2: $SU(3)^c \times SU(2)_L \times U(1)_Y$ is broken by the vacuum state since $\langle \Phi \rangle \neq 0$. This leaves $SU(3)^c$ intact and produces a massless photon, as well as three massive gauge bosons whose longitudinal modes come from the higgs field. The remaining higgs degree of freedom obtains a non-zero, positive mass and should be seen by experiment. Furthermore, the quarks, electron, muon, and tau pick up masses from the yukawa couplings to the higgs while the neutrino remains massless.

1.1.3 Additional Symmetries

In addition to the gauge symmetries, Eq. (1.1) has several global $U(1)$ symmetries that are an accidental byproduct of the form of the lagrangian[12]. These symmetries are baryon number B, electron lepton number L_e, muon lepton number L_μ, and tau lepton number L_τ; the corresponding charges of the particles are detailed in Table 1.2. All of these symmetries are anomalous—that is, if one attempted to make them local symmetries the symmetry would be violated at loop level. For example, the symmetry $B - L$, where $L \equiv L_e + L_\mu + L_\tau$, has a non-zero loop diagram when two $SU(2)_L$ gauge bosons enter, one hypothetical $B - L$ gauge boson leaves, and fermions run in the loop. This diagram, which is proportional

$SU(3)^c \times SU(2)_L \times U(1)_Y$

$\downarrow$ ground state breaks $SU(2)_L \times U(1)_Y$ through $\langle\Phi\rangle \neq 0$

$SU(3)^c \times U(1)_{em}$

- $W^{\pm}$, Z massive gauge bosons.
- 3 Higgs degrees of freedom consumed by massive gauge fields.
- One massive higgs boson.
- Massive quarks, electron, muon, tau.
- Massless neutrinos.

Figure 1.2: The standard model and theoretical understanding of why the weak force is short ranged and only electromagnetic charge is conserved

	Q	u_R	d_R	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}$	e_R	$\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}$	μ_R	$\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}$	τ_R
B	$+\frac{1}{3}$	$+\frac{1}{3}$	$+\frac{1}{3}$	0	0	0	0	0	0
L_e	0	0	0	+1	+1	0	0	0	0
L_μ	0	0	0	0	0	+1	+1	0	0
L_τ	0	0	0	0	0	0	0	+1	+1

Table 1.2: Accidental symmetries in the standard model. B is baryon number which is the only accidental symmetry of the quarks due their mixing with the CKM matrix[4, 5]. $L \equiv L_e + L_\mu + L_\tau$ is lepton number and in the standard model it is conserved for each generation separately.

to $\mathrm{Tr}(B-L) = 1$, is interesting since the addition of a fermionic, three-generation singlet with $B-L=-1$ would precisely cancel this quantity leaving $B-L$ anomaly free; however, as it stands the SM only has $B-L$ and the aforementioned symmetries as global symmetries.

In addition to the global $U(1)$ symmetries, the standard model also has an important $U(1)$ symmetry that is explicitly broken. It is called the chiral symmetry and can be seen by taking $y_u = 0$, $y_d = 0$, $y_e = 0$. The resultant lagrangian is then invariant under $U(1)_L \times U(1)_R$ where $(\Psi_Q, \Psi_L) \rightarrow \mathrm{e}^{\mathrm{i}\gamma_5\theta_L}(\Psi_Q, \Psi_L)$

and $(\Psi_{u_R}, \Psi_{d_R}, \Psi_{e_R}) \rightarrow \mathbb{e}^{\mathbb{i}\gamma_5\theta_R}(\Psi_{u_R}, \Psi_{d_R}, \Psi_{e_R})$. The yukawa couplings of Eq. (1.3) clearly break this symmetry to $U(1)_{L+R}$ since they mix, for example, L and e_R. As the yukawa couplings explicitly break this symmetry, all quantum corrections must be proportional to these yukawa couplings since the symmetry must be restored as $y_i \rightarrow 0$.

1.2 Neutrinos

As stated the SM has massless neutrinos, but experiments have detected neutrino oscillations[13, 14, 15, 16, 17, 18, 19, 20]—neutrinos changing from one flavor to another—which indicates that they have a non-zero mass whose eigenstate is not a flavor eigenstate[21, 22]. If the neutrino is assumed to be like other standard model particles, then it has a dirac mass originating from a yukawa coupling of the form

$$y_\nu \bar{\Psi}_L \tilde{\Phi} \Psi_{\nu_R} \tag{1.14}$$

which means that the right-handed neutrino, ν_R, must be introduced. Furthermore there is an upper bound on the mass of the neutrino from direct searches the strictest of which is $m_{\nu_e} < 2.0$ eV. Thus Eq. (1.14) implies that

$$y_\nu \sim \frac{2 \text{ eV}}{174 \text{ GeV}} \sim 10^{-8} \sim 10^{-6} y_e \ll y_e \tag{1.15}$$

so that it would appear the neutrino's yukawa coupling is at least six orders of magnitude less than its $SU(2)_L$ partner. As this seems 'unnatural' (or at least in need of an explanation) when considering that the up and down quarks have yukawa couplings within 2 orders of magnitude of each other, there is incentive to postulate that y_ν is actually around y_e but that neutrinos get a small mass through another means such as the seesaw mechanism[23, 24, 25, 26, 22].

The seesaw mechanism exploits the singlet nature of ν_R: because it doesn't transform under the SM gauge group, it can have a mass term

$$M_R \Psi_{\nu_R}^T \mathbb{C} \Psi_{\nu_R} \tag{1.16}$$

called a majorana mass. The left- and right-handed neutrinos then have a mass matrix

$$\mathbf{M}_\nu = \begin{pmatrix} 0 & y_\nu \langle \Phi \rangle \\ y_\nu^\dagger \langle \Phi \rangle & M_R \end{pmatrix} \tag{1.17}$$

which, assuming $M_R \gg \langle \Phi \rangle$, has the heavy eigenvalues M_R and the light eigenvalues

$$m_\nu = -\langle \Phi \rangle^2 y_\nu^\dagger M_R^{-1} y_\nu. \tag{1.18}$$

The yukawa coupling for the neutrino is then permitted to be around y_e due to the suppression of $\langle \Phi \rangle / M_R$. Using the experimental upper limit for neutrino mass, 10^3 GeV $\lesssim M_R \lesssim 10^{15}$ GeV (using $y_\nu = y_e \sim 10^{-5}$ for the lower limit; $y_\nu = \sqrt{4\pi}$ for the upper limit).

At this point it may be argued that not much has been gained: the original small mass of the neutrino has been explained at the expense of introducing a new mass scale M_R that is just as inexplicable. Additionally, the presence of the majorana mass term of Eq. (1.16) means the nature of the neutrino is fundamentally different than that of the other SM particles. It may well be simpler to just assume this character from the start and add the non-renormalizable term

$$\mathcal{L} \supset \frac{\lambda_\nu}{\Lambda} (\Psi_L \Phi)^2 \tag{1.19}$$

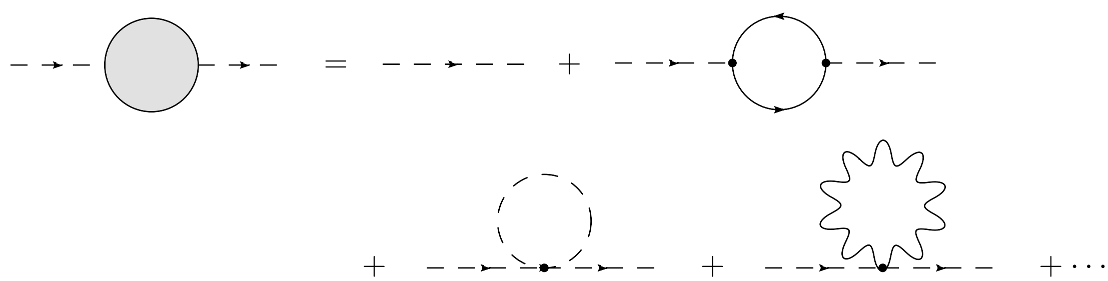
to the theory. Of course this reintroduces an inexplicable new scale $\Lambda \ll M_{\text{Pl}}$ with the same range as M_R.

It therefore appears that if the small neutrino mass is to be explained, a new, unexpected scale must be introduced. If this scale is the Λ of Eq. (1.19), then the new particles have a mass around Λ and will not likely be found in any foreseeable collider (there may be other low-scale implications, however). On the other hand, if this new scale is M_R, then a right-handed neutrino is a necessity of the theory—meaning $U(1)_{B-L}$ becomes anomaly-free (see Section 1.1.3) and may then be gauged.

The new scale M_R is then associated with the scale at which $B - L$ breaks, making this 'unexpected' mass a physical mass of the theory (as opposed to Λ where there is no immediately obvious physical significance).

1.3 Supersymmetry

The SM has a major theoretical issue distinct from its failure to explain neutrino masses and has been a major driving force for physics beyond the standard model (BSM). The problem has many names, among which are "the gauge-hierachy problem", "the hierarchy problem", and "the Planck-weak hierarchy problem", but they all reflect the fact that the higgs mass is susceptible to large corrections from loops. The inverse propagator of the higgs is



$$= p^2 - m^2 + \frac{\Lambda^2}{16\pi^2}\Big(-\sum_i C_F^i y_i^2 + C_\Phi \lambda + \sum_A C_g^A g_A^2\Big) + \cdots \tag{1.20}$$

and if the SM is the full theory until gravity becomes strong, then $\Lambda = M_{\rm Pl} \gg M_W$ so that the higgs field prefers a VEV near the planck scale. Clearly this situation is unacceptable since precision electroweak data places $\langle \Phi \rangle = 174$ GeV $\ll M_{\rm Pl}$. Given the differing signs in Eq. (1.20), it is possible that some terms cancel, but for $\Lambda = M_{\rm Pl}$ the expression in parentheses would need to cancel to the 30th decimal place to keep m^2 at the electroweak scale. Furthermore, the couplings involved in Eq. (1.20) are unrelated to each other and logically there is no reason for them to conspire to cancel to 1 in 10^{-30}—unless there is a symmetry behind the cancellation.

The argument just given may seem a bit suspicious as neutrino oscillations have demonstrated that there is new physics at or above 1 TeV, so it may be more appropriate to use $\Lambda = 1$ TeV. Yet if this physics is just the introduction of the right-handed neutrino, the problem will persist: the new physics will need to fundamentally alter the character of the higgs scalar above a TeV (something the mere addition of right-handed neutrinos does not do).

It is also worth noting that the SM fermions do not have this issue due to the chiral symmetry discussed in Section 1.1.3. Thus if it were possible to associate some chiral symmetry with the scalar higgs field, its mass would also be protected. Supersymmetry (SUSY) does just this by imposing a symmetry between bosons and fermions—grafting the fermion's chiral symmetry onto the scalar bosons in the process[27].

As SUSY is a symmetry between bosons and fermions, it is best described when the fields are placed in multiplets containing both types of particles (called supermultiplets)[28]. This may be accomplished with the introduction of the anticommuting grassmann variables θ_α, $\bar{\theta}^{\dot{\alpha}}$, and the spacetime coordinate $y^\mu \equiv x^\mu + \mathbb{i}\theta\sigma^\mu\bar{\theta}$. With these tools the matter content is contained in a chiral multiplet given by

$$\Phi(y) = \underline{\Phi}(y) + \sqrt{2}\theta\psi_\Phi(y) + F_\Phi(y)\,\theta^2, \tag{1.21}$$

while the gauge fields are expressed in terms of real supermultiplets as

$$\mathcal{V}^A(y) = -\theta\sigma^\mu\bar{\theta}V_\mu^A(y) + \mathbb{i}\theta^2\bar{\theta}\mathrm{v}^{A\dagger}(y) - \mathbb{i}\bar{\theta}^2\theta\mathrm{v}^A(y) + \frac{1}{2}\theta^2\bar{\theta}^2\big(D^A(y) - \mathbb{i}\partial^\mu V_\mu^A(y)\big)\,. \tag{1.22}$$

The fermionic components ψ_Φ, v^A of Eqs. (1.21) and (1.22) are weyl spinors; that is, they are two component objects. The component $\underline{\Phi}$ is a complex scalar field and V_μ^A is the usual vector field. The components F_Φ and D^A are auxiliary fields in that they may be eliminated by the equations of motion—they are necessary for accounting purposes as the number of bosonic degrees of freedom must equal the number of fermionic degrees of freedom in a theory that is symmetric under their exchange.

The lagrangian of a SUSY theory is determined by two functions: the kähler potential, $\mathcal{K}$, and the superpotential, $\mathcal{W}$, as follows

$$\mathcal{L} = \frac{1}{2}\int d^4\theta\ \mathcal{K} + \int d^2\theta\ \mathcal{W} + \text{h.c.} \tag{1.23}$$

The kähler potential is a real or vector superfield since $\mathcal{K}^\dagger = \mathcal{K}$; the superpotential is a chiral superfield. The gauge field kinetic terms enter through $\mathcal{W}$ from the superderivatives

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} + 2\mathbb{i}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^\mu} \tag{1.24}$$

$$\overline{\mathcal{D}}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \tag{1.25}$$

which are used to construct the superstrength

$$\mathscr{V}_\alpha = -\frac{1}{4}\overline{\mathcal{D}}^2 \mathbb{e}^{-2\mathcal{V}^A T_A}\mathcal{D}_\alpha \mathbb{e}^{2\mathcal{V}^G T_G} \tag{1.26}$$

and this appears in the superpotential as $\mathscr{V}^\alpha\mathscr{V}_\alpha$.

1.3.1 Minimal Supersymmetric Standard Model

Of particular interest to a SUSY theory of nature is the minimal supersymmetric standard model (MSSM)—the SM extended to include SUSY with the minimal particle content. The MSSM fields are given in Table 1.3. The MSSM potentials are taken to be

$$\begin{aligned}\mathcal{K} = {} & Q^\dagger \mathbb{e}^{\mathcal{G}^A\lambda_A + \mathcal{W}^A\tau_A + \frac{1}{3}\mathcal{B}}Q + u^{c\dagger}\mathbb{e}^{\mathcal{G}^A\lambda_A - \frac{4}{3}\mathcal{B}}u^c + d^{c\dagger}\mathbb{e}^{\mathcal{G}^A\lambda_A + \frac{2}{3}\mathcal{B}}d^c \\ & + L^\dagger \mathbb{e}^{\mathcal{W}^A\tau_A - \mathcal{B}}L + e^{c\dagger}\mathbb{e}^{2\mathcal{B}}e^c \\ & + H_u^\dagger \mathbb{e}^{\mathcal{W}^A\tau_A + \mathcal{B}}H_u + H_d^\dagger \mathbb{e}^{\mathcal{W}^A\tau_A - \mathcal{B}}H_d \end{aligned} \tag{1.27}$$

$$\begin{aligned}\mathcal{W} = {} & y_u Q H_u u^c + y_d Q H_d d^c + y_e L H_d e^c + \mu H_u H_d \\ & + \frac{1}{4g_Y^2}\mathscr{B}^\alpha\mathscr{B}_\alpha + \frac{1}{8g_L^2}\,\text{Tr}(\mathscr{W}^\alpha\mathscr{W}_\alpha) + \frac{1}{12g_3^2}\,\text{Tr}(\mathscr{G}^\alpha\mathscr{G}_\alpha)\,. \end{aligned} \tag{1.28}$$

	$SU(3)^c$	$\times$	$SU(2)_L$	$\times$	$U(1)_Y$
Q	3		2		$+\frac{1}{3}$
u^c	$\bar{3}$		1		$-\frac{4}{3}$
d^c	$\bar{3}$		1		$+\frac{2}{3}$
L	1		2		-1
e^c	1		1		$+2$
H_u	1		2		$+1$
H_d	1		2		-1

Table 1.3: The Minimal Supersymmetric Standard Model. The particles are listed by their representations of the gauge groups except for $U(1)_Y$ where the hypercharge is given.

The MSSM is defined such that it conserves R-parity; that is, the gauge invariant interactions

$$\mathcal{W}_{\not{R}} = \frac{1}{2}\lambda^{ijk} L_i L_j e^c_k + (\lambda')^{ijk} L_i Q_j d^c_k + \frac{1}{2}(\lambda'')^{ijk} u^c_i d^c_j d^c_k + (\mu')^i H_u L_i \tag{1.29}$$

are forbidden by the discrete symmetry

$$\begin{aligned} (Q, u^c, d^c, L, e^c) &\to -(Q, u^c, d^c, L, e^c) \\ (H_u, H_d) &\to (H_u, H_d)\,. \end{aligned} \tag{1.30}$$

The transformations of Eq. (1.30) actually define matter parity which is trivially related to R-parity through the particle's spin:

$$P_R = (-1)^{3(B-L)+2s} = (-1)^{2s} P_M \tag{1.31}$$

While R-parity prevents rapid proton decay, it also makes the lightest supersymmetric particle (LSP) stable as it would have $P_R = -1$ meaning its decay to SM particles (with $P_R = +1$) would violate R-parity. The fact that the LSP is stable is useful as it provides a candidate for the non-baryonic dark matter of the universe. The conservation of R-parity is therefore an attractive feature of SUSY models. Unfortunately the MSSM has R-parity added by hand; however, since it's related to $B-L$, it is possible to have it as a remnant symmetry if $B-L$ is gauged[29, 30, 31].

As outlined the MSSM with R-parity[3] provides the minimal model that contains supersymmetry; however, as a scalar particle otherwise identical to the electron has not yet been found, SUSY must be a broken symmetry. Since SUSY was introduced to protect the higgs mass, it is important to add SUSY breaking terms that do not ruin this feature. The terms that keep the higgs mass protected but violate SUSY are known as soft SUSY breaking terms and for the MSSM they are given as

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = {}& \frac{1}{2}\big(M_3 \mathrm{g}_A^\alpha \mathrm{g}_\alpha^A + M_L \mathrm{w}_A^\alpha \mathrm{w}_\alpha^A + M_Y \mathrm{b}^2 + \text{h.c.}\big) \\
&+ \big(a_u \underline{Q}\,\underline{H}_u \underline{u}^c + a_d \underline{Q}\,\underline{H}_d \underline{d}^c + a_e \underline{L}\,\underline{H}_d \underline{e}^c + \text{h.c.}\big) \\
&+ m_Q^2 \underline{Q}^\dagger \underline{Q} + m_{u^c}^2 \underline{u}^{c\dagger} \underline{u}^c + m_{d^c}^2 \underline{d}^{c\dagger} \underline{d}^c + m_L^2 \underline{L}^\dagger \underline{L} + m_{e^c}^2 \underline{e}^{c\dagger} \underline{e}^c \\
&+ m_{H_u}^2 \underline{H}_u^\dagger \underline{H}_u + m_{H_d}^2 \underline{H}_d^\dagger \underline{H}_d + (b \underline{H}_u \underline{H}_d + \text{h.c.})
\end{aligned} \tag{1.32}$$

Sadly, Eq. (1.32) adds 105 free parameters to the theory[27], most of which lead to effects that are ruled out by experiment (large CP violation, flavor changing neutral currents, etc[32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51]). Due to this large number of degrees of freedom, as well as the notion that SUSY should be broken in a spontaneous fashion (i.e. similar to the way electroweak is broken to electromagnetism), an organizing principle has been sought to relate these parameters and generate SUSY breaking in a way that naturally yields small effects to known experimental limits. One highly attractive method that does just that is found in the context of theories incorporating gravity and is known as anomaly mediated supersymmetry breaking[52, 53].

1.4 Gravity

As the reader may have noticed, the SM model does not include gravitational interactions. At the energies of current and conceivable future experiments this is easily justified since gravity is entirely insignificant compared to the electroweak

[3]This label is redundant as the model was defined to contain R-parity, but sometimes things need to be explicit.

and strong interactions of particles. Yet a complete theory of nature would include gravity as part of its description, and the early universe presents a regime where both particle interactions and gravity are equally important. Thus, it is worth considering particle theories in the presence of gravity.

One very compelling aspect of supersymmetry is that gauging it, or making it a local symmetry, requires the existence of gravity. This can be seen as follows: since SUSY is a symmetry between fermions and bosons, its parameters, ξ^α, carry spinor, or fermionic, indices. Following the typical procedure to gauge this symmetry, $\xi^\alpha \rightarrow \xi^\alpha(x^\mu)$, introducing variations of the lagrangian proportional to $\partial_\mu \xi^\alpha$ that must be cancelled by an additional gauge field. Because the parameter ξ carries fermionic indices, the new gauge field required to make the action invariant will have to carry both a spacetime index μ and fermionic index α. Such an object, a spinorial 4-vector, describes a spin-3/2 particle. Now because a new fermionic field has been added, invariance under SUSY will mandate its bosonic partner, a spin-2 field, be added. It can then be shown that this spin-2 particle couples to the energy-momentum tensor of the matter fields and is therefore identifiable as the graviton, or the particle mediating gravitational interactions.

While making SUSY local introduces a graviton coupling to matter, its kinetic term still needs to be added to the theory. Since the theory is gravity, it is expected that the kinetic energy should take the form dictated by general relativity. To obtain this expression, it is therefore necessary to briefly discuss the classical theory of gravity.

1.4.1 Classical Gravity

To motivate the general theory of relativity, it is helpful to formulate the newtonian field theory of gravity. First consider a test mass m, in the presence of a mass M, located at distance r from M. The newtonian theory states that m feels a force

$$\vec{F} = -\frac{G_N m M}{r^2}\hat{r}. \tag{1.33}$$

Defining the gravitational field $\vec{g}$ by

$$m\vec{g} \equiv \vec{F} \tag{1.34}$$

yields

$$\vec{g} = -\frac{G_N M}{r^2}\hat{r}. \tag{1.35}$$

Gauss's law relates the mass M to the flux of this field as

$$\int dA\, \vec{g}\cdot\hat{n} = \int d\Omega\, r^2\vec{g}\cdot\hat{r} = -4\pi G_N M. \tag{1.36}$$

The use of the divergence theorem then gives a differential equation for $\vec{g}$:

$$\nabla\cdot\vec{g} = -4\pi G_N \rho \tag{1.37}$$

with ρ the mass density of M. Finally, taking $\vec{g} \equiv -\nabla\Phi$, yields

$$\nabla^2\Phi = 4\pi G_N \rho. \tag{1.38}$$

Eq. (1.38) represents the newtonian field equation when gravity is sourced by a mass density ρ. Making this expression consistent with special relativity modifies it to

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{8\pi G_N}{c^2}T_{\alpha\beta}, \tag{1.39}$$

where

$$
\begin{aligned}
R^{\alpha}{}_{\mu\beta\nu} &= \partial_\beta\Gamma^{\alpha}{}_{\mu\nu} - \partial_\nu\Gamma^{\alpha}{}_{\mu\beta} + \Gamma^{\rho}{}_{\mu\nu}\Gamma^{\alpha}{}_{\rho\beta} - \Gamma^{\rho}{}_{\mu\beta}\Gamma^{\alpha}{}_{\rho\nu} && (1.40)\\
R_{\mu\nu} &= R^{\alpha}{}_{\mu\alpha\nu} && (1.41)\\
R &= g^{\mu\nu}R_{\mu\nu} && (1.42)\\
\Gamma^{\alpha}{}_{\mu\nu} &= \frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}) && (1.43)\\
\det g &= \frac{1}{4!}\epsilon^{\alpha\beta\mu\nu}\epsilon^{\alpha'\beta'\mu'\nu'}g_{\alpha\alpha'}g_{\beta\beta'}g_{\mu\mu'}g_{\nu\nu'}. && (1.44)
\end{aligned}
$$

and $g_{\mu\nu}$ contains the Φ of Eq. (1.38).

In a manner analogous to an electric field existing independent of the charge, the Einstein equation Eq. (1.39) permits gravitation effects without the presence of matter/energy: taking $T_{\alpha\beta} = 0$ yields

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 0. \tag{1.45}$$

Eq. (1.45) describes what is called 'pure gravity'—gravity without any sources—and is essentially the field equation of gravity. Given this status, it is expected that Eq. (1.45) may be obtained, through variational techniques, from some lagrangian. Indeed this is true, and

$$\mathcal{L}_{\mathrm{GRAV}} = -\frac{c^3}{16\pi G_N} R \sqrt{-\det g} \tag{1.46}$$

yields Eq. (1.45) as its Euler-Lagrange equations when varied with respect to $g_{\alpha\beta}$.

Because Eq. (1.45) describes gravity without sources, it represents the 'free-field' lagrangian for a gravitational theory. Due to this, $\mathcal{L}_{\mathrm{GRAV}}$ is precisely the kinetic energy term for the graviton (since the graviton is massless, the free-field theory only contains kinetic energy terms). Recalling that the ultimate particle theory is desired to be supersymmetric, Eq. (1.45) must be extended to capture both the graviton and its spin-3/2 partner, the gravitino. The resulting theory is called supergravity (SUGRA), and is most easily formulated in the context of superconformal invariance, discussed in Section 2.3.

Chapter 2

Anomaly Mediated Supersymmetry Breaking

Anomaly mediated supersymmetry breaking (AMSB) is the idea that the anomaly of superconformal invariance generates SUSY breaking terms in the visible sector when SUSY breaking in the hidden sector gives the auxiliary component of a SUGRA multiplet a VEV[52, 53, 54]. As SUGRA fields will always couple to the hidden sector, this AMSB contribution is always present, though it is not always the dominant contribution. If it is assumed that AMSB is the only significant source of SUSY breaking, then the form of the breaking is completely determined by superconformal invariance. In this case AMSB may be naively thought of as 'placing conformal compensators', which then leads to the 'AMSB rule' for the introduction of SUSY breaking.

2.1 Supercouplings

When discussing the breaking of supersymmetry it is often helpful to treat the SUSY parameters of the theory as superfields themselves[55, 56]. The rationale behind this is that a constant superfield—that is, one that is independent of superspacetime (θ, $\bar{\theta}$, x^μ)—does not break supersymmetry because the generators of SUSY involve superspacetime derivatives:

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - \mathbb{i}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu} \qquad \bar{Q}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + \mathbb{i}\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial x^\mu}. \tag{2.1}$$

Therefore, when operated upon by a generator, a constant superfield yields zero and thus preserves SUSY. Due to this fact, any SUSY parameter may be viewed as a constant superfield. Consequently, extending the SUSY parameters to have superspace (θ, $\bar{\theta}$) dependence will introduce supersymmetry breaking as the coupling will no longer be invariant under the action of the generator.

The parameters can not be extended in an arbitrary fashion, however, because they may always be re-written as a 'dummy' superfield times a complex number. Therefore the superspace extended parameters, or supercouplings for short, must obey the same rules as the superfields themselves. Essentially this means that parameters in the superpotential must be chiral supercouplings while those in the kähler potential must be real supercouplings.

Treating the parameters as supercouplings provides a powerful tool for analyzing the structure of SUSY breaking as well as loop effects. These ideas are best demonstrated in a toy model first written by Wess and Zumino:

$$\mathcal{K}_{\rm WZ} = \mathcal{Z}\Phi^\dagger \left(1 + \frac{\Box}{\Lambda^2}\right)\Phi \tag{2.2}$$

$$\mathcal{W}_{\rm WZ} = \frac{1}{2}\mathcal{M}\Phi^2 + \frac{1}{3!}\lambda\Phi^3 \tag{2.3}$$

where $\mathcal{Z}$ is the wavefunction renormalization constant made into a real supercoupling, Λ is a real supercoupling cut-off, and λ, $\mathcal{M}$ are chiral supercouplings.

Using the Wess-Zumino model, it is straightforward to show the general property that the superpotential receives no loop corrections. Consider, for example, the one-loop correction to the superpotential term $\lambda\Phi^3$. If such a correction were present, the divergent correction would necessarily take the form

$$\Delta_\lambda \mathcal{W}_{\rm WZ}^{\rm DIVRG} = \left(\frac{c\lambda^3}{16\pi^2}\ln\frac{\mu}{\Lambda}\right)\Phi^3 \tag{2.4}$$

which is the typical correction to a yukawa coupling. Yet a term such as Eq. (2.4) can not appear in $\mathcal{W}_{\rm WZ}$ because of the real supercoupling Λ: the superpotential may only contain chiral superfields and supercouplings. Because any divergent correction to the superpotential would contain a Λ, it can be concluded that no divergent correction to the superpotential is possible.

The exclusion of divergent corrections to the superpotential does not eliminate

	$U(1)$	$U(1)_R$
Φ	$+1$	0
$\mathcal{M}$	-2	2
λ	-3	2

Table 2.1: The transformational properties of the superfields and supercouplings in the Wess-Zumino model. For the $U(1)$ the Φ is chosen to have its conventional mass dimension as its charge, the supercouplings $\mathcal{M}$, λ are then chosen to keep Eq. (2.3) invariant. For the $U(1)_R$, Φ may not transform if it is desired to have both $M\Phi^2$ and $\lambda\Phi^3$; therefore, the supercouplings must be chosen to transform to keep $d^2\theta\,\mathcal{W}$ invariant.

the possibility of finite corrections; however, these too must vanish. To see how this must be the case, it is useful to also consider a $U(1) \times U(1)_R$ symmetry with the transformations given by Table 2.1. Any finite corrections to $\mathcal{W}_{\mathrm{WZ}}$ can only depend on λ or $\mathcal{M}$ as these are the only chiral supercouplings; additionally, the corrections must preserve the $U(1) \times U(1)_R$ symmetry so they must come in the ratio of $\lambda/\mathcal{M}$:

$$\Delta W_{\mathrm{WZ}}^{\mathrm{FINITE}} = c_0 \frac{\mathcal{M}^2}{\lambda}\Phi + c_1 \frac{\lambda^2}{\mathcal{M}}\Phi^4 + \cdots \tag{2.5}$$

As either $\lambda \to 0$ or $\mathcal{M} \to 0$ these terms are divergent and therefore not finite corrections. As the limit of either coupling being independently zero is relevant as a special case of a constant superfield, these terms must be forbidden.

The previous argument establishes that the superpotential, and therefore its parameters, do not receive quantum corrections. This is true to all orders in perturbation theory, as demonstrated by the generality of the argument. The couplings in the kähler potential, on the other hand, will receive radiative corrections because they may acquire dependence on the real supercoupling Λ. Thus the only renormalization in a non-gauge SUSY theory is the wavefunction constant.

Inclusion of gauge groups is fairly straightforward as the introduction of gauge fields does not disrupt the previous argument (which indeed must be valid for $g \to 0$); therefore, the only new features the gauge groups add are their couplings. These gauge couplings may also be made supercouplings, called τ_G, and will receive divergent contributions causing them to 'run'.

2.2 Transmitting Radiatively

Section 2.1 discussed making parameters of a SUSY theory into supercouplings which would then give rise to SUSY breaking terms. The idea was also used to establish that the only couplings that are renormalized in a SUSY theory are the wavefunction and the gauge couplings. Therefore, if the theory transmits the breaking of supersymmetry through quantum corrections, the SUSY breaking is entirely captured by the supercouplings $\mathcal{Z}_i{}^j$ and τ_G. For such a theory, the supercouplings may be expanded in θ,

$$\ln \mathcal{Z}_i{}^j = \ln Z_i{}^j + A_i{}^j\theta^2 + \left(A^\dagger\right)_i{}^j\bar{\theta}^2 - \left(m^2\right)_i{}^j\theta^2\bar{\theta}^2 \tag{2.6}$$

$$\tau_G = \frac{1}{2}\varpi_G - \frac{\mathbb{i}\Theta}{4\pi} - M_G\varpi_G\theta^2, \tag{2.7}$$

permitting the identification of the components: $(m^2)_i{}^j$ is the scalar mass-squared, Θ is the vacuum polarization angle, M_G is the gaugino mass, and the $A_i{}^j$ contribute to the SUSY breaking terms as follows[1]:

$$\begin{aligned} -\ell^i &= L^m A_m{}^i \\ -b^{ij} &= \mu^{im}A_m{}^j + (i \leftrightarrow j) \\ -a^{ijk} &= Y^{ijm}A_m{}^k + (i \leftrightarrow k) + (j \leftrightarrow k) \\ -z^{ijk\ell} &= \lambda^{ijkm}A_m{}^\ell + (i \leftrightarrow \ell) + (j \leftrightarrow \ell) + (k \leftrightarrow \ell) \\ &\vdots \end{aligned} \tag{2.8}$$

The usefulness in this expansion is that the theory that radiatively transmits SUSY breaking will yield expressions for the components of $\mathcal{Z}_i{}^j$ and τ_G directly. This then leads to immediate expressions for the SUSY breaking terms. In the case of AMSB, this is the 'AMSB rule' which details how to make $Z_i{}^j$ and ϖ_G supercouplings.

[1] See Appendix A for the definitions of the SUSY breaking terms

2.3 Supergravity

To justify the existence of the 'AMSB rule', it is first necessary to discuss certain aspects of supergravity, as this is the origin of AMSB[52]. The basic ideas are also present in Einstein gravity, so this will be the launching point.

The lagrangian for pure gravity (see Section 1.4) is

$$\mathcal{L}_{\text{GRAV}} = -\frac{c^3}{16\pi G_N} R\sqrt{-\det g} \tag{2.9}$$

with $g_{\mu\nu}$ the metric as determined by the Einstein equation, R the Ricci scalar, and

$$R = g^{\mu\nu} R^{\alpha}{}_{\mu\alpha\nu} \tag{2.10}$$

$$R^{\alpha}{}_{\mu\alpha\nu} = \partial_\alpha \Gamma^{\alpha}{}_{\mu\nu} - \partial_\nu \Gamma^{\alpha}{}_{\mu\alpha} + \Gamma^{\beta}{}_{\mu\nu}\Gamma^{\alpha}{}_{\beta\alpha} - \Gamma^{\beta}{}_{\mu\alpha}\Gamma^{\alpha}{}_{\beta\nu} \tag{2.11}$$

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}) \tag{2.12}$$

$$\det g = \frac{1}{4!}\epsilon^{\alpha\beta\mu\nu}\epsilon^{\alpha'\beta'\mu'\nu'} g_{\alpha\alpha'} g_{\beta\beta'} g_{\mu\mu'} g_{\nu\nu'}. \tag{2.13}$$

The scale transformation $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$ (Ω^2 a constant) leaves $\Gamma^{\alpha}{}_{\mu\nu}$ invariant since it involves the metric and the inverse. Thus $R^{\alpha}{}_{\mu\alpha\nu}$ is also invariant, so that the Ricci scalar only transforms due to the presence of the inverse metric. Under this scaling, then,

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} \qquad g^{\mu\nu} \to \frac{1}{\Omega^2} g^{\mu\nu} \qquad \det g \to \Omega^8 \det g \tag{2.14}$$

$$\Gamma^{\alpha}{}_{\mu\nu} \to \Gamma^{\alpha}{}_{\mu\nu} \qquad R^{\alpha}{}_{\mu\alpha\nu} \to R^{\alpha}{}_{\mu\alpha\nu} \qquad R \to \Omega^2 R \tag{2.15}$$

which does not leave Eq. (2.9) invariant. Yet the two theories yield identical euler-lagrange equations hinting they are the same physical description. To make this symmetry manifest in the lagrangian, the constant of Eq. (2.9) can be made to transform non-trivially:

$$\frac{c^3}{8\pi G_N} \to \frac{1}{\Omega^2}\frac{c^3}{8\pi G_N}. \tag{2.16}$$

The fact that the constant transforms under this scaling hints that it may be better thought of as an expectation value of some scalar field, η, as then the 'true' theory is based upon the lagrangian

$$\mathcal{L}_{\text{SIGRAV}} = -\frac{1}{2}\eta R\sqrt{-\det g} \tag{2.17}$$

with the field η transforming, $\eta \to (1/\Omega^2)\eta$. The symmetry is broken when η acquires a VEV,

$$\langle\eta\rangle = \frac{c^3}{8\pi G_N}, \tag{2.18}$$

yielding Eq. (2.9).

With the introduction of the scalar field η, this scaling symmetry can now be made local; that is, $\Omega^2 \to \Omega^2(x)$ (clearly a constant can not depend on spacetime, by definition). In this context the character of η is revealed: it acts in a manner analogous to the gauge field A^μ—compensating for the spacetime transformation of the metric 'field' by scaling. Given this, it is appropriate to label η a gauge field.

Yet η also represents the gauge freedom, since it may be eliminated by suitably choosing $\Omega^2(x)$, viz.

$$\Omega^2(x) = \eta(x). \tag{2.19}$$

In this sense it differs from a typical gauge field because it itself defines the gauge. The natural consequence of this property is that the physical parameters can not depend on $\eta(x)$, but this is easily seen to be true as an invariant 'tilded' metric, $\tilde{g}_{\mu\nu}$, may be defined,

$$\tilde{g}_{\mu\nu} \equiv \eta(x) g_{\mu\nu}, \tag{2.20}$$

and the theory can then be completely written in terms of $\tilde{g}_{\mu\nu}$.

The final result of the above is that, with the assistance of η, pure Einstein gravity has been made scale invariant. This symmetry actually implies an extended set of transformations called conformal invariance[57, 58], giving $\eta(x)$ the name "conformal compensator".

It should be emphasized that there is no new physical content to this rewriting of the theory: no measured parameter can depend on η and it may always be eliminated from the theory to yield the usual Einstein theory. The advantage to introducing η is in its simplification of calculations. This situation is analogous to that of supersymmetry where the auxiliary fields F and D are introduced—these additional fields are not required by the theory but allow calculations to be performed without invoking the equations of motion. Because η may always be absorbed into the metric, in this sense η is an auxiliary component of the metric 'field'.

The idea of supergravity is to extend the conformally invariant Einstein theory to be supersymmetric. A SUSY theory will require the scalar field η be part of a chiral multiplet labeled ϕ which is also dubbed the "conformal compensator". Furthermore, the conformal transformations will necessarily operate on the full chiral multiplet, leading to an even larger symmetry known as superconformal invariance.

The field ϕ, just like η, is not a physical degree of freedom. It is more analogous to an auxiliary component of the supergravity multiplet that may be easily eliminated to yield the usual SUGRA theory. In this scenario, though, the comparison with the F and D fields is even more appropriate since the standard auxiliary fields break SUSY by acquiring a non-zero VEV. Similarly, the field ϕ may acquire non-zero higher components leading to SUSY breaking, and it is here that there are interesting effects.

It is well established that SUSY breaking must occur in a hidden sector that doesn't couple directly to the visible sector[27]; though there is no known way to prevent a gravitational interaction. Because of this fact, the conformal compensator itself can act as a messenger of SUSY breaking—gravity demands it couple to both the hidden and visible sectors.

Alternatively, the scenario may be pictured in the following manner: the hidden sector breaks SUSY in some manner, perhaps through an O'Raifeartaigh fashion. Because the conformal compensator couples to this SUSY breaking, it picks up a non-zero F-component,

$$\frac{\partial}{\partial\theta^2}\phi \sim \langle F_{\text{hidden}} \rangle \,. \tag{2.21}$$

Choosing the gauge where $\phi = M_{\text{Pl}}$, this may be re-expressed as

$$\phi = M_{\text{Pl}}\big(1 + F_\phi \theta^2\big) \,. \tag{2.22}$$

where F_ϕ has been defined as $\langle F_{\text{hidden}} \rangle / M_{\text{Pl}}$. The non-zero F-component of ϕ then appears in the visible sector due to the conformal compensator's coupling with these fields.

The transmitting of SUSY breaking through this means is unavoidable in any SUGRA model containing conformal invariance; yet because

$$F_\phi \sim \frac{\langle F_{\text{hidden}} \rangle}{M_{\text{Pl}}} \tag{2.23}$$

it can easily be subdominant to other contributions[52]. Alternatively, if the hidden sector is 'sufficiently hidden'[52], then F_ϕ can be the dominant source of SUSY breaking. Hence, the supposition of AMSB is that however SUSY is ultimately broken, the conformal compensator's F-term is the dominant source transmitting this breaking.

2.4 Placing Conformal Compensators

Though critical for understanding the mechanism of transmitting SUSY breaking, the SUGRA origin of the conformal compensator can be ignored if the presence of ϕ is taken as read. In this simplified mindset the form of the supersymmetry breaking is determined strictly by the appearance of ϕ, which is dictated by the superconformal invariance of the SUGRA theory. Fortunately, superconformal invariance contains weyl scale transformations and a $U(1)_R$ symmetry which are

	d_W	R
θ	$-\frac{1}{2}$	$+1$
$\bar{\theta}$	$-\frac{1}{2}$	-1
$d\theta$	$+\frac{1}{2}$	-1
$d\bar{\theta}$	$+\frac{1}{2}$	$+1$

Table 2.2: Weyl weight and R charges of superspace coordinates

	d_W	R
$\mathcal{K}$	$+2$	0
$\mathcal{W}$	$+3$	$+2$

Table 2.3: Derived weyl weight and R charge assignments for the kähler and super potentials

sufficient to determine how the conformal compensator shows up in the lagrangian. The transformations of ϕ under these symmetries are given by its weyl weight, $d_W(\phi) = +1$, and R charge, $+2/3$[56, 55].

To see how to place ϕ's, consider a general supersymmetric theory given by the lagrangian

$$\mathcal{L} = \frac{1}{2}\int d^4\theta\, \mathcal{K} + \int d^2\theta\, \mathcal{W} + \int d^2\theta \sum_G \frac{\varpi_G}{16\pi C_G} \mathrm{Tr}[(\mathscr{V}_G)^\alpha (\mathscr{V}_G)_\alpha] + \text{h.c.} \tag{2.24}$$

where

$$\mathcal{K} = Z_i{}^j \Phi^i \exp\left[2\mathcal{V}_G^A \left(T_{\mathcal{A}}^G\right)_A\right]_j{}^k \Phi_k + \cdots \tag{2.25}$$

$$\mathcal{W} = L^i\Phi_i + \frac{1}{2!}\mu^{ij}\Phi_i\Phi_j + \frac{1}{3!}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{4!}\frac{\lambda^{ijk\ell}}{\Lambda}\Phi_i\Phi_j\Phi_k\Phi_\ell + \cdots \tag{2.26}$$

The charges of the superspace coordinates θ, $\bar{\theta}$—given in Table 2.2—determine the necessary charges of the kähler and super potentials to keep the lagrangian invariant; their appropriate charges are shown in Table 2.3. If we define "tilded" fields so that $d_W(\tilde{\Phi}_i) = d_W(\widetilde{\mathscr{V}}_G^A) = R(\tilde{\Phi}_i) = R(\widetilde{\mathscr{V}}_G^A) = 0$, then we may write

$$\mathcal{W} = \widetilde{\mathcal{W}} X_{\mathcal{W}} \qquad\qquad \mathcal{K} = \widetilde{\mathcal{K}} X_{\mathcal{K}} \tag{2.27}$$

where the "tilded" potentials are functions of only the "tilded" fields. Since the "tilded" fields have no charges, the resulting potentials don't either; hence all the transformational weights belong to the X_n:

$$d_W(X_\mathcal{K}) = +2 \qquad\qquad d_W(X_\mathcal{W}) = +3 \tag{2.28}$$
$$R(X_\mathcal{K}) = 0 \qquad\qquad R(X_\mathcal{W}) = +2 \tag{2.29}$$

Now because the X_n carry charges, they can only depend on the conformal compensator ϕ (we've already removed any other fields' dependence into the potentials). Therefore invariance necessitates

$$X_\mathcal{K} = \phi^\dagger \phi \qquad\qquad X_\mathcal{W} = \phi^3 \tag{2.30}$$

We can now write the most general superconformal invariant lagrangian. It is given by

$$\mathcal{L} = \frac{1}{2}\int d^4\theta\ \phi^\dagger\phi\widetilde{\mathcal{K}} + \int d^2\theta\ \phi^3\widetilde{\mathcal{W}} + \text{h.c.} \tag{2.31}$$

This picture explicitly demonstrates the ϕ couplings as required by superconformal invariance at a cost of using non-canonically normalized fields. It is possible to return to the usual fields by defining

$$\Phi_i = \phi\tilde{\Phi}_i \qquad\qquad \mathcal{D}_\alpha = \frac{\phi^\dagger}{\phi^{1/2}}\widetilde{\mathcal{D}}_\alpha \qquad\qquad \tilde{\mathscr{V}}_G^A = \phi^{3/2}\widetilde{\mathscr{V}}_G^A \tag{2.32}$$

with the last equation being a consequence of the second. It is then clear that the lagrangian of Eq. (2.31), combined with the field redefinitions Eq. (2.32), leads to a lagrangian

$$\mathcal{K} = Z_i{}^j \Phi^i \exp\left(2\mathcal{V}_G^A T_A^G\right)_j{}^k \Phi_k + \cdots \tag{2.33}$$
$$\mathcal{W} = L^i\phi^2\Phi_i + \frac{1}{2!}\mu^{ij}\phi\Phi_i\Phi_j + \frac{1}{3!}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{4!}\frac{\lambda^{ijk\ell}}{\Lambda\phi}\Phi_i\Phi_j\Phi_k\Phi_\ell + \cdots \tag{2.34}$$

The dimensionful terms of Eq. (2.34) explicitly break the conformal invariance, and setting them to zero restores the conformal symmetry at tree level; however, when quantum corrections are included a mass parameter, μ, will be introduced.

The theory will then need to be regularized, which can be done with a cutoff Λ. This type of regulator is a convenient choice since the form of Eq. (2.34) establishes that Λ must come paired with a ϕ should it give rise to non-renormalizable terms[2]. The functional dependence of various parameters on μ will always be of the form μ/Λ, so that the renormalized parameters will become supercouplings by the rule

$$\begin{aligned} \mu &\rightarrow \frac{\mu}{\phi} \qquad \text{(Chiral Superfields)} \\ \mu &\rightarrow \frac{\mu}{|\phi|} \qquad \text{(Real Superfields)} \end{aligned} \tag{2.35}$$

Eq. (2.35) is the 'AMSB rule' for determining the SUSY breaking parameters. To derive expressions for the supersymmetry breaking terms, a form for ${Z_i}^j$ that can be 'promoted' using the 'AMSB rule' is required. Formally solving the beta function,

$$\frac{d \ln {Z_i}^j}{d \ln \mu} = {\gamma_i}^j, \tag{2.36}$$

yields just such an expression:

$$\ln {Z_i}^j(\ln \mu) = \ln {Z_i}^j(\ln \Lambda) - \int_{\ln \mu}^{\ln \Lambda} dt\, {\gamma_i}^j. \tag{2.37}$$

The supercoupling is given as

$$\ln {\mathcal{Z}_i}^j(\ln \mu) = \ln {Z_i}^j(\ln \Lambda) - \int_{\ln \frac{\mu}{|\phi|}}^{\ln \Lambda} dt\, {\gamma_i}^j \tag{2.38}$$

and the components, which yield the SUSY breaking expressions, may be gotten by taking derivatives with respect to θ^2 and $\bar{\theta}^2$.

So, for example, the ${A_i}^j$ term is gotten by

[2]The result that the ultra violet (UV) cutoff gets paired with a ϕ is independent of whether or not it yields non-renormalizable terms; it is merely a useful illustration here

$$\frac{d}{d\theta^2}\ln \mathcal{Z}_i{}^j(\ln\mu)\bigg|_{\theta=\bar\theta=0} = \left[\frac{d}{d\theta^2}\int_{\ln\Lambda}^{\ln\frac{\mu}{|\phi|}} dt\,\gamma_i{}^j\right]_{\theta=\bar\theta=0} \tag{2.39}$$

$$= \gamma_i{}^j(\ln\mu)\left[\frac{d}{d\theta^2}\ln\frac{\mu}{|\phi|}\right]_{\theta=\bar\theta=0} - \gamma_i{}^j(\ln\Lambda)\left[\frac{d}{d\theta^2}\ln\Lambda\right]_{\theta=\bar\theta=0}$$

$$+\left[\int_{\ln\Lambda}^{\ln\frac{\mu}{|\phi|}} dt\,\frac{\partial}{\partial\theta^2}\gamma_i{}^j\right]_{\theta=\bar\theta=0} \tag{2.40}$$

$$= -\frac{1}{2}\gamma_i{}^j(\ln\mu)\,F_\phi. \tag{2.41}$$

The last line follows because neither the anomalous dimension, $\gamma_i{}^j$, or Λ have explicit θ, $\bar\theta$ dependence ($\gamma_i{}^j$ due to it being a physical quantity, Λ due to the UV physics being held fixed).

The scalar masses may be obtained in a similar manner, though there is a subtle complexity as it requires the evaluation of

$$\frac{d}{d\bar\theta^2}\left[\gamma_i{}^j(\ln\mu/|\phi|\,)\right]. \tag{2.42}$$

While the anomalous dimension has no *explicit* ϕ dependence, it does contain implicit dependence through the gauge and yukawa couplings of which it is a function; that is,

$$\left[\frac{d}{d\bar\theta^2}\left[\gamma_i{}^j\left(\ln\frac{\mu}{|\phi|}\right)\right]\right]_{\theta=\bar\theta=0} = \sum_G\frac{\partial\gamma_i{}^j}{\partial\varpi_G}\left[\frac{d}{d\bar\theta^2}\varpi_G\left(\ln\frac{\mu}{|\phi|}\right)\right]_{\theta=\bar\theta=0}$$
$$+\frac{\partial\gamma_i{}^j}{\partial Y^{\ell mn}}\left[\frac{d}{d\bar\theta^2}Y^{\ell mn}\left(\ln\frac{\mu}{|\phi|}\right)\right]_{\theta=\bar\theta=0}$$
$$+\frac{\partial\gamma_i{}^j}{\partial Y_{\ell mn}}\left[\frac{d}{d\bar\theta^2}Y_{\ell mn}\left(\ln\frac{\mu}{|\phi|}\right)\right]_{\theta=\bar\theta=0} \tag{2.43}$$

The gauge and yukawa couplings' implicit dependence on $\bar\theta$ can be obtained in the same fashion as the wavefunction renormalization constant; i.e. formally solving their RGEs.

The same techniques may be applied to the gauge supercoupling, yielding the full set of AMSB expressions:

	$U(1)$
X	-2
$\overline{X}$	$+2$
Ψ	$+1$
$\overline{\Psi}$	-1
S	0

Table 2.4: The $U(1)$ charges and particle content of a toy model that demonstrates decoupling of thresholds in AMSB

$$\left(m^2\right)_i{}^j = -\frac{1}{4}|F_\phi|^2\left[\frac{1}{2}\frac{\partial\gamma_i{}^j}{\partial\varpi_G}\beta_{\varpi_G} + \frac{\partial\gamma_i{}^j}{\partial Y^{\ell mn}}\beta_Y^{\ell mn} + \text{h.c.}\right] \tag{2.44}$$

$$a^{ijk} = -\beta_Y^{ijk}F_\phi \tag{2.45}$$

$$M_G = -\frac{\beta_{\varpi_G}}{2\varpi_G}F_\phi. \tag{2.46}$$

Eqs. (2.44)–(2.46) are powerful expressions because they are the solutions to the RGE at any scale μ. This statement is valid both above and below a threshold, M, provided that the threshold does not introduce any new SUSY breaking comparable to F_ϕ; that is, any effects of the theory above M vanish in the theory below M[59, 60].

2.5 Thresholds

To witness the insensitivity of AMSB to thresholds $M \gg F_\phi$, it is helpful to consider a toy model having the particle content given in Table 2.4 and the superpotential given by

$$\mathcal{W}_{\text{DCPL}} = fX\Psi\Psi + yS\left(X\overline{X} - M^2\phi^2\right). \tag{2.47}$$

In the SUSY limit the scalar components of X and $\overline{X}$ acquire a VEV equal to M thus introducing a threshold. At this point all the fields except $\overline{\Psi}$ gain a mass of $M \gg F_\phi$. Including the effects of SUSY breaking due to AMSB, it is a mere algebraic feat to show

$$\langle \underline{X} \rangle = M + \mathcal{O}\left(\frac{F_\phi^2}{M}\right) + \mathcal{O}\left(\frac{m_{\text{an}}^2}{M}\right) \tag{2.48}$$

$$\langle \underline{\overline{X}} \rangle = M + \mathcal{O}\left(\frac{F_\phi^2}{M}\right) - \mathcal{O}\left(\frac{m_{\text{an}}^2}{M}\right) \tag{2.49}$$

$$\langle F_X \rangle = M F_\phi \tag{2.50}$$

$$\langle F_{\overline{X}} \rangle = M F_\phi \tag{2.51}$$

$$\langle \underline{S} \rangle = -\frac{F_\phi^\dagger}{y} \tag{2.52}$$

$$\langle D \rangle = \frac{1}{2g}\left(m_{\overline{X}}^2 - m_X^2\right) = -\frac{|F_\phi|^2}{4g}\frac{\partial \gamma_X^+}{\partial f}\beta_f \tag{2.53}$$

with the D-term acquiring a VEV because X has the f coupling and $\overline{X}$ does not; hence, the AMSB expression for their scalar masses are different.

Above the threshold M, $\overline{\Psi}$ has a scalar mass given by AMSB

$$\left(m_{\overline{\Psi}}^2\right)^+ = -\frac{5}{4} g^4 m_{\text{an}}^2 \tag{2.54}$$

while below M there is no gauge group and $\mathcal{W}^- = 0$ so that AMSB predicts

$$\left(m_{\overline{\Psi}}^2\right)^- = 0 \tag{2.55}$$

The fact that AMSB predicts $\overline{\Psi}$'s scalar mass to be zero below M raises two questions: how the contribution of the gauge group given by Eq. (2.54) disappeared, and why the D-term VEV—acquired at the threshold—vanished. Both issues are resolved by noting that the Ψ's act as messengers giving a gauge mediated supersymmetry breaking (GMSB) contribution at the threshold M. This is because the lagrangian from Eq. (2.47) contains the term

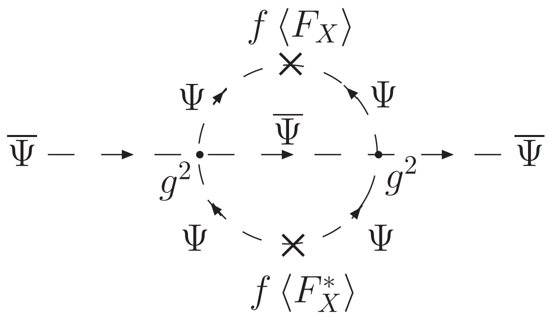
$$\int d^2\theta \, \mathcal{W}_{\text{DCPL}} \supset f \langle F_X \rangle \, \underline{\Psi \Psi} = f M F_\phi \underline{\Psi \Psi} = M_\Psi F_\phi \underline{\Psi \Psi} \tag{2.56}$$

which appears in loops.

For example, the scalar $\overline{\Psi}$ couples to the Ψ's through the D-term potential

$$V_D = \frac{1}{2} g^2 \left[\left(|\underline{X}|^2 - |\overline{\underline{X}}|^2 \right)^2 + |\underline{\Psi}|^2 \left(|\underline{X}|^2 - |\overline{\underline{X}}|^2 \right) \right.$$
$$\left. - |\underline{\Psi}|^2 \left(|\underline{X}|^2 - |\overline{\underline{X}}|^2 \right) + \frac{1}{4} \left(|\underline{\Psi}|^2 - |\overline{\underline{\Psi}}|^2 \right)^2 \right] \quad (2.57)$$

leading to the diagram

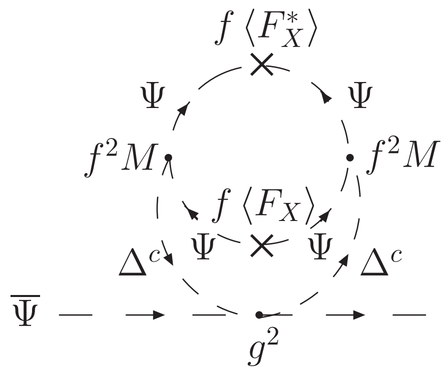


$$\sim \frac{g^4 f^2 |F_X|^2}{(16\pi^2)^2 M_\Psi^2} = g^4 \frac{|F_\phi|^2}{(16\pi^2)^2} \quad (2.58)$$

which is exactly the same structure and size as the AMSB contribution above the threshold. In fact, Eq. (2.58), along with the other diagrams involving gauge fields, yields

$$\left(m_{\overline{\Psi}}^2 \right)^- = \left(m_{\overline{\Psi}}^2 \right)^+ + \left(m_{\overline{\Psi}}^2 \right)_{\text{GMSB}} = 0 \quad (2.59)$$

The GMSB diagrams such as Eq. (2.58) cancel the higher-scale AMSB contributions to $\overline{\Psi}$'s scalar mass; however, they do not remove the D-term portion acquired at the threshold. Rather, this term's cancellation can be seen as a result of the D VEV actually being zero below the threshold—GMSB diagrams like Eq. (2.58), with $\overline{\Psi}$ replaced by X, $\overline{X}$ cause the scalar masses of these fields to be zero below M resulting in the VEV of D vanishing. Alternatively, the cancellation can be seen directly through contributions such as



$$\sim \frac{g^2 f^6 |F_X|^2 M^2}{(16\pi^2)^2 M_\Psi^2 M^2} = g^2 f^4 \frac{|F_\phi|^2}{(16\pi^2)^2} \tag{2.60}$$

which cancel the D-term contribution to the scalar $\overline{\Psi}$.

The result of the argument just given is that AMSB decouples any effects of an intermediate threshold and the resulting scalar masses depend solely on the low-scale physics. There is, as mentioned, a caveat to this statement which is that the threshold must not give any additional SUSY breaking that is comparable to F_ϕ. This condition may be re-expressed as stating that the messengers Ψ must have their mass appear as $M\phi$ in the superpotential; or, alternatively, that the VEV of F_X must be MF_ϕ. If these three equivalent conditions are violated, then the threshold does not decouple and the AMSB expressions are no longer valid below this scale.

2.6 Minimal Supersymmetric Standard Model and Anomaly Mediation

The naive application of AMSB to the MSSM is an unmitigated disaster: electroweak symmetry is not broken by the higgs fields, but electric charge conservation is violated[61, 62, 63, 64, 65, 66, 67, 68, 69, 70]. The latter problem can be seen directly from application of the AMSB formulae to the sleptons; for example, the right-handed selectron's mass is given as

$$m_{e^c}^2 = -22 m_{\rm an}^2 g_Y^4 < 0. \tag{2.61}$$

The negative mass-square is a result of the negligible yukawa couplings for the electron yielding a strictly gauge group dependent mass. Since $U(1)_Y$ is asymptotically enslaved, the beta function is positive, and so the mass-square must be negative. This means that any scalar without yukawa couplings and with only IR-free gauge charges will have a negative mass squared—hence in the MSSM both chiralities of selectron and muon (and likely the stau) will be tachyonic.

While electrically charged scalars obtain the wrong sign scalar mass-squared (thus acquiring a VEV and breaking electric charge), the unified electroweak symmetry is not broken by the higgs fields. The culprit here is the μ term which provides tree-level conformal symmetry breaking.

In the MSSM, the conformal compensator appears as

$$\mathcal{W} = y_u Q H_u u^c + y_d Q H_d d^c + y_e L H_d e^c + \mu\phi H_u H_d. \tag{2.62}$$

The resulting scalar potential for the neutral higgs fields is then

$$\begin{aligned} V_{\mathrm{NH}} = &\left(m_{H_u}^2 + |\mu|^2\right)\left|\underline{H}_u^0\right|^2 + \left(m_{H_d}^2 + |\mu|^2\right)\left|\underline{H}_d^0\right|^2 \\ &- F_\phi \mu \underline{H}_u^0 \underline{H}_d^0 - F_\phi^\dagger \mu^* \underline{H}_u^{0*} \underline{H}_d^{0*} + \frac{1}{8}\left(g_L^2 + g_Y^2\right)\left(\left|\underline{H}_u^0\right|^2 - \left|\underline{H}_d^0\right|^2\right)^2. \end{aligned} \tag{2.63}$$

From the potential Eq. (2.63), two constraints may be obtained. The first is the requirement that the potential as given be bounded from below. While it is typically argued that this is to ensure positive energy states, that is not the case as there may be non-renormalizable terms yielding higher powers of the higgs fields. Such terms, with the correct sign, would eventually turn the potential positive; however, in doing so they would also push the higgs VEV to a value well beyond the one required for electroweak symmetry breaking. Thus, to ensure the correct higgs VEV the potential of Eq. (2.63) must be bounded from below. The most stringent constraint will come when $\langle \underline{H}_u^0 \rangle = \langle \underline{H}_d^0 \rangle$, leading to the condition

$$m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2 \geq \left| F_\phi \mu + F_\phi^\dagger \mu^* \right| \geq 2|F_\phi \mu|. \tag{2.64}$$

Eq. (2.64) can not be satisfied for $F_\phi \sim 10$ TeV unless $\mu \sim F_\phi$, which implies the scalar masses $m_{H_u}^2 \sim m_{H_d}^2 \sim m_{\text{an}}^2 \ll |F_\phi|^2$ are completely irrelevant to the constraint:

$$m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2 \approx 2|\mu|^2 \geq 2|F_\phi \mu| \tag{2.65}$$

The second requirement of Eq. (2.63) is the instability condition for $SU(2)_L \times U(1)_Y$, which ensures that the gauge group is broken. The instability manifests itself as a negative eigenvalue for the higgs mass matrix,

$$\begin{pmatrix} m_{H_u}^2 + |\mu|^2 & -F_\phi^\dagger \mu^* \\ -F_\phi \mu & m_{H_d}^2 + |\mu|^2 \end{pmatrix}, \tag{2.66}$$

which may be accomplished by requiring the determinant to be negative. This yields

$$\left(m_{H_u}^2 + |\mu|^2\right)\left(m_{H_d}^2 + |\mu|^2\right) \approx |\mu|^4 < |F_\phi \mu|^2 . \tag{2.67}$$

Clearly both Eq. (2.65) and Eq. (2.67) can not be satisfied simultaneously, and so the higgs fields do not acquire a VEV.

2.7 Next-to Minimal Supersymmetric Standard Model and Anomaly Mediation

Since the MSSM already has the μ problem without AMSB, it may be argued that AMSB just exaggerates the issue and so the explanation to the MSSM μ problem should solve the troubles of electroweak symmetry breaking when SUSY is broken with AMSB. Unfortunately, this is not the case for the minimal model, the next-to minimal supersymmetric standard model (NMSSM)[71]. The best way to understand the underlying problem is through a toy model. Consider a superpotential given by

$$W_{\text{toy}} = \frac{1}{3}\kappa N^3 \tag{2.68}$$

where N is a singlet field with no gauge symmetries. The resulting scalar potential, including SUSY breaking, is

$$V_{\rm toy} = |\kappa|^2 \, |\underline{N}|^4 + \frac{1}{3}\big(a_\kappa \underline{N}^3 + a_\kappa^* \underline{N}^{*3}\big) + m_N^2 |\underline{N}|^2 \,. \tag{2.69}$$

Taking account for the complex phases by letting

$$\underline{N} = |\underline{N}| \, \mathbb{e}^{\mathbb{i}\delta_N} \qquad \kappa = |\kappa| \, \mathbb{e}^{\mathbb{i}\delta_\kappa} \qquad a_\kappa = |a_\kappa| \, \mathbb{e}^{\mathbb{i}\delta_{a_\kappa}} , \tag{2.70}$$

the minimization condition for the phase δ_N is

$$\sin(3\delta_N + \delta_{a_\kappa}) = 0. \tag{2.71}$$

The resulting minimum condition for $|\underline{N}|$,

$$\begin{aligned} 0 &= 2|\kappa|^2 \, |\underline{N}|^2 + |a_\kappa| \, |\underline{N}| \cos(3\delta_N + \delta_{a_\kappa}) + m_N^2 \\ &= 2|\kappa|^2 \, |\underline{N}|^2 + |a_\kappa| \, |\underline{N}| + m_N^2, \end{aligned} \tag{2.72}$$

is then independent of any phases leaving $\langle \underline{N} \rangle$ real. The solution to Eq. (2.72) states

$$\langle \underline{N} \rangle = \frac{-|a_\kappa| \pm \sqrt{|a_\kappa|^2 - 8|\kappa|^2 \, m_N^2}}{2|\kappa|^2} \tag{2.73}$$

where the soft couplings a_κ and m_N are determined by AMSB via Eqs. (2.44) and (2.45):

$$\begin{aligned} a_\kappa &= \frac{F_\phi}{16\pi^2} 6\kappa^3 \\ m_N^2 &= \frac{|F_\phi|^2}{(16\pi^2)^2} 12\kappa^4. \end{aligned} \tag{2.74}$$

Substituting these into Eq. (2.73) gives

$$\langle \underline{N} \rangle = \frac{|F_\phi|}{16\pi^2} \frac{|\kappa|}{4} \big(-6 \pm \sqrt{-60}\big) \tag{2.75}$$

yielding a contradiction: $\langle \underline{N} \rangle$ must be real, but the large negative under the radical demonstrates this can not be so.

The source of the problem can be identified by examining the potential of N. To expose the difficulty, it is helpful to define

$$x \equiv \frac{\kappa\langle N\rangle}{m_{\rm an}} \tag{2.76}$$

and re-write Eq. (2.69) as

$$\frac{\langle V_{\rm toy}\rangle}{4m_{\rm an}^4} = \frac{1}{4\kappa^2}x^4 + x^3 + 3\kappa^2 x^2 \tag{2.77}$$

where the AMSB expressions of Eq. (2.74) have been substituted. For the potential to have a non-trivial minimum it is necessary that the cubic term dominate for some value of x (since this term is the only one that provides a negative contribution to the potential); however, for large κ, the x^2 term will always be larger than the cubic term. Meanwhile, for small κ the quartic term will dominate the expression. Therefore, if there is any chance for the x^3 term to create a minimum other than zero, it must be that $\kappa \simeq 1$. This leaves the potential as

$$\frac{\langle V_{\rm toy}\rangle}{4m_{\rm an}^4} = \frac{1}{4}x^4 + x^3 + 3x^2 \tag{2.78}$$

where it now becomes clear that neither large x, $x \sim 1$, nor small x will have the cubic term dominate the expression—leaving the only minimum as the trivial one. Thus, the heart of the problem is that AMSB predicts the cubic term's coefficient such that it will always be weaker than either the quartic or quadratic regardless of the parameter regime.

The same problem carries over to the full NMSSM, as pointed out in [71]. In this model, the additional coupling of N to H_u and H_d does not alter the relative strengths of N's quartic, cubic, or quadratic terms, but it does add a linear term to the potential, $a_\lambda v_u v_d N$. The induced linear term shifts the trivial minimum away from zero, but keeps it small. The minimization condition for N can then be approximated as

$$\widetilde{\mu}_N^2\langle N\rangle - \frac{1}{2\sqrt{2}}a_\lambda v^2 \sin 2\beta = 0 \tag{2.79}$$

with $\widetilde{\mu}_N^2 \simeq m_{\rm an}^2$ being essentially the AMSB predicted soft SUSY breaking mass for N. The maximum value occurs when $\sin 2\beta = 1$ so we have that

$$\langle N \rangle \lesssim \frac{a_\lambda v^2}{2\widetilde{\mu}_N^2\sqrt{2}} \simeq \frac{1}{2\sqrt{2}}\frac{v^2}{m_{\text{an}}} \simeq 22 \text{ Gev} \tag{2.80}$$

The small $\langle N \rangle$ then results in a chargino mass which falls below the LEP II bound of about 94 GeV.

Chapter 3

Mixing the Left-Right Model, Supersymmetry, and Anomaly Mediation

This chapter introduces the supersymmetric left-right model and takes the theory "through the scales" ending at the SUSY scale, below which it is the standard electroweak theory. It also argues the presence of "light" $SU(2)_L$ triplets and doubly-charged $SU(2)_L$ singlet fields, whose phenomenology has been the subject of many papers[72, 73, 74, 75, 76, 77].

3.1 The Left-Right Model

The SUSYLR model is defined by Table 3.1. It contains right-handed $B-L = \pm 2$ triplets to break $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$. Because these triplets have $B-L = \pm 2$, Eq. (1.31) implies that R-parity will be a residual symmetry after the breaking—thus, the model naturally enforces R-parity conservation yielding a stable LSP.

Parity has also been enforced in Table 3.1, so that the theory necessarily contains left-handed triplets. While the seesaw mechanism may be achieved with only $SU(2)_R$ higgs fields, demanding parity introduces seesaw like couplings for the left-handed sleptons which, combined with the right-handed seesaw couplings, provides both chiralities of sleptons positive mass-squares.

The fully parity symmetric superpotential is

$$W_{\text{SUSYLR}} = W_{\text{Y}} + W_{\text{H}} + W_{\text{GSPNR}} + W_{\text{GSVNR}} \tag{3.1}$$

where

Fields	$SU(3)^c$	$\times$ $SU(2)_L$	$\times$ $SU(2)_R$	$\times$ $U(1)_{B-L}$
Q	3	2	1	$+\frac{1}{3}$
Q^c	$\bar{3}$	1	2	$-\frac{1}{3}$
L	1	2	1	-1
L^c	1	1	2	$+1$
Φ_a	1	2	2	0
Δ	1	3	1	$+2$
$\bar{\Delta}$	1	3	1	-2
Δ^c	1	1	3	-2
$\bar{\Delta}^c$	1	1	3	$+2$
S	1	1	1	0
N	1	1	1	0

Table 3.1: Assignment of the fermion and Higgs fields' representations of the left-right symmetry group (except for $U(1)_{B-L}$ where the charge under that group is given.)

$$W_{\rm Y} = \mathbb{i} y_Q^a Q^T \tau_2 \Phi_a Q^c + \mathbb{i} y_L^a L^T \tau_2 \Phi_a L^c + \mathbb{i} f_c L^{cT} \tau_2 \Delta^c L^c + \mathbb{i} f L^T \tau_2 \Delta L \tag{3.2}$$

$$\begin{aligned} W_{\rm H} = {} & (M_\Delta \phi - \lambda_S S) \left[\mathrm{Tr}\big(\Delta^c \bar{\Delta}^c\big) + \mathrm{Tr}\big(\Delta \bar{\Delta}\big) \right] + M_S^2 \phi^2 S + \frac{1}{2}\mu_S \phi S^2 + \frac{1}{3}\kappa_S S^3 \\ & + \lambda_N^{ab} N \,\mathrm{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) + \frac{1}{3}\kappa_N N^3 \end{aligned} \tag{3.3}$$

$$\begin{aligned} W_{\rm GSPNR} = {} & \frac{\lambda_A}{M_X \phi} \mathrm{Tr}^2\big(\Delta\bar{\Delta}\big) + \frac{\lambda_A^c}{M_X \phi} \mathrm{Tr}^2\big(\Delta^c \bar{\Delta}^c\big) \\ & + \frac{\lambda_B}{M_X \phi} \mathrm{Tr}(\Delta\Delta)\,\mathrm{Tr}\big(\bar{\Delta}\bar{\Delta}\big) + \frac{\lambda_B^c}{M_X \phi} \mathrm{Tr}(\Delta^c \Delta^c)\,\mathrm{Tr}\big(\bar{\Delta}^c \bar{\Delta}^c\big) \\ & + \frac{\lambda_C}{M_X \phi} \mathrm{Tr}\big(\Delta\bar{\Delta}\big)\,\mathrm{Tr}\big(\Delta^c \bar{\Delta}^c\big) \\ & + \frac{\lambda_{\mathbf{S}}}{M_X \phi} \mathrm{Tr}\big(\Delta\bar{\Delta}\big) S^2 + \frac{\lambda_{\mathbf{S}}^c}{M_X \phi} \mathrm{Tr}\big(\Delta^c \bar{\Delta}^c\big) S^2 + \cdots \end{aligned} \tag{3.4}$$

$$\begin{aligned} W_{\rm GSVNR} = {} & \frac{\lambda_D}{M_{\rm Pl} \phi} \mathrm{Tr}(\Delta\Delta)\,\mathrm{Tr}(\Delta^c \Delta^c) + \frac{\bar{\lambda}_D}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\bar{\Delta}\bar{\Delta}\big)\,\mathrm{Tr}\big(\bar{\Delta}^c \bar{\Delta}^c\big) \\ & + \frac{(\lambda_\sigma)^{ab}}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\Delta\bar{\Delta}\big)\,\mathrm{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) + \frac{(\lambda_\sigma^c)^{ab}}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\Delta^c \bar{\Delta}^c\big)\,\mathrm{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) \\ & + \frac{2\lambda_\alpha \epsilon^{ab}}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\Delta \Phi_a \tau_2 \Phi_b^T \tau_2 \bar{\Delta}\big) + \frac{2\lambda_\alpha^c \epsilon^{ab}}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\Delta^c \tau_2 \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c\big) \\ & + \frac{\lambda_{\mathbf{N}}}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\Delta\bar{\Delta}\big) N^2 + \frac{\lambda_{\mathbf{N}}^c}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\Delta^c \bar{\Delta}^c\big) N^2 \\ & + \frac{\lambda_{\mathbf{s}}}{M_{\rm Pl} \phi} \mathrm{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) S^2 + \frac{\lambda_{\rm M}}{M_{\rm Pl} \phi} S^2 N^2 + \cdots \end{aligned} \tag{3.5}$$

The fields transform under parity as

$$\begin{array}{lll} Q \leftrightarrow -\mathbb{i}\tau_2 Q^{c\,*} & L \leftrightarrow -\mathbb{i}\tau_2 L^{c\,*} & \Phi_a \rightarrow \Phi_a^\dagger \\ \Delta \leftrightarrow \Delta^{c\,\dagger} & \bar{\Delta} \leftrightarrow \bar{\Delta}^{c\,\dagger} & (S, N) \rightarrow (S^*, N^*) \end{array} \tag{3.6}$$

which demands that the couplings be related:

$$\begin{array}{llll} y_Q^a = \left(y_Q^a\right)^\dagger & y_L^a = (y_L^a)^\dagger & f = f_c^* & M_\Delta = M_\Delta^* \\ \lambda_S = \lambda_S^* & M_S^2 = \left(M_S^2\right)^* & \mu_S = \mu_S^* & \kappa_S = \kappa_S^* \\ & \lambda_N = \lambda_N^\dagger & \kappa_N = \kappa_N^*. & \end{array} \tag{3.7}$$

The superpotential of Eq. (3.1) has also had a discrete $\mathbb{Z}_3$ symmetry imposed where the fields transform

$$\begin{gathered} (Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N) \rightarrow \mathbb{e}^{2\mathbb{i}\pi/3}(Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N), \\ (\bar{\Delta}, \bar{\Delta}^c) \rightarrow \mathbb{e}^{4\mathbb{i}\pi/3}(\bar{\Delta}, \bar{\Delta}^c) \end{gathered} \tag{3.8}$$

and S invariant. This symmetry is necessary to keep one singlet light below the right-handed scale since it forbids terms such as

$$W_{\mathbb{Z}_3} = \kappa_{12} S N^2 + \kappa_{21} S^2 N + \lambda_N^c N \,\mathrm{Tr}\big(\Delta^c \bar{\Delta}^c\big) \tag{3.9}$$

which would generate a large, $\mathcal{O}(v_R)$, SUSY mass for N. This symmetry is not gauged, therefore it is global symmetry which is susceptible to violation due to gravitational effects[1]. Due to these considerations, Eq. (3.1) contains the planck suppressed, $\mathbb{Z}_3$ violating, non-renormalizable terms of Eq. (3.5).

The $\mathbb{Z}_3$ violating non-renormalizable terms are not, however, the only ones possible: Eq. (3.4) displays terms which conserve the discrete symmetry, but are nonetheless gauge invariant. As these terms do not violate the $\mathbb{Z}_3$, they quite possibly originate from the next new scale of physics which is conceivably below M_{Pl}. To allow this possibility the terms in Eq. (3.4) are suppressed by M_X and not M_{Pl}.

[1]For example, if a particle charged under this symmetry falls into a blackhole, there is no way to ascertain the amount of this charge the blackhole contains. This can be contrasted with a gauged symmetry where Gauss's law may be utilized to determine the charge enclosed

3.2 Vacuum Structure

The potential generated by Eq. (3.1) is

$$V = V_F + V_D + V_{\rm SB} + V_{\rm SBNR} \tag{3.10}$$

with the F and D term potentials given by

$$\begin{aligned} V_F &= \mathrm{Tr}\big(F_\Delta^\dagger F_\Delta + F_{\bar\Delta}^\dagger F_{\bar\Delta}\big) + F_N^* F_N + \mathrm{Tr}\big(F_{\Phi_a}^\dagger F_{\Phi_a}\big) \\ &\quad + \mathrm{Tr}\big(F_{\Delta^c}^\dagger F_{\Delta^c} + F_{\bar\Delta^c}^\dagger F_{\bar\Delta^c}\big) + F_S^* F_S \qquad (3.11) \\ V_D &= \frac{1}{2}\sum_G D_R^G D_R^G + \sum_G D_L^G D_L^G + D_{BL}^2. \qquad (3.12) \end{aligned}$$

The F and D term expressions, along with the SUSY breaking potentials, may be found in Appendix B.1.

The potential Eq. (3.10) is clearly intractable; however, not all the terms are important at all scales. If it is assumed that $M_\Delta \sim M_S \sim \mu_S \gg F_\phi$, then the the breaking of $SU(2)_R \times U(1)_{B-L}$ can be considered in the SUSY limit. Furthermore, provided $M_X \gg M_\Delta \sim M_S \sim \mu_S$, all the non-renormalizable terms are insignificant. In these limits (and taking the VEV of the sneutrino to be zero), then

$$\begin{aligned} -F_{\Delta^c}^\dagger &= (M_\Delta - \lambda_S \underline{S})\,\underline{\bar\Delta}^c \qquad (3.13) \\ -F_{\bar\Delta^c}^\dagger &= (M_\Delta - \lambda_S \underline{S})\,\underline{\Delta}^c \qquad (3.14) \\ -F_S^* &= -\lambda_S\big[\mathrm{Tr}\big(\underline{\Delta}^c\underline{\bar\Delta}^c\big) + \mathrm{Tr}\big(\underline{\Delta}\underline{\bar\Delta}\big)\big] + M_S^2 + \mu_S\underline{S} + \kappa_S\underline{S}^2 \qquad (3.15) \end{aligned}$$

Evidently Eqs. (3.13) and (3.14) give $\underline{S}$ a VEV

$$\langle \underline{S} \rangle = \frac{M_\Delta}{\lambda_S} \tag{3.16}$$

Eq. (3.15) then implies that $\underline{\Delta}^c$, $\underline{\bar\Delta}^c$ acquire a VEV of

$$\langle \underline{\Delta}^c \rangle \langle \underline{\bar\Delta}^c \rangle = \frac{M_S^2}{\lambda_S} + \left(\frac{\mu_S}{\lambda_S} + \frac{\kappa_S M_\Delta}{\lambda_S^2}\right)\frac{M_\Delta}{\lambda_S} \tag{3.17}$$

where it becomes clear that $M_\Delta \sim \mu_S \sim M_S \sim v_R$—taking v_R as the characteristic right-handed breaking scale.

Inclusion of the non-renormalizable terms shifts the right-handed scale by $\mathcal{O}(v_R^2/M_X)$ which is small compared to v_R. It is important to emphasize that the non-renormalizable terms alter the VEV while preserving SUSY—the ultimate expression for the VEV should decouple the SUSY preserving and SUSY violating parts. This distinction is pertinent as it is entirely conceivable (and it turns out desirable) to have

$$\frac{v_R^2}{M_X} \sim F_\phi \tag{3.18}$$

which results in the non-renormalizable terms yielding a contribution of the same order of magnitude of the SUSY breaking ones. Even though the terms are of the same size, the previous argument guarantees that non-renormalizable contributions never break SUSY[2]—an important feature as it ensures AMSB provides the only source of SUSY breaking.

While the SUSY limit implies $SU(2)_R \times U(1)_{B-L}$ is broken, it does not specify which components of the $SU(2)$ triplets get a VEV. This implies the existence of a continuously connected set of degenerate vacua[78]. Indeed, the degenerate vacua can be deduced from the form of the superpotential of Eq. (3.1): by asserting that the sneutrino has a zero VEV, the yukawa couplings of Eq. (3.2) play no role in determining the vacuum structure; hence, they can be neglected for this discussion. Furthermore, setting the gauge couplings to zero and taking $M_X, M_{\text{Pl}} \to \infty$ unveils a complexified $U(6)$ symmetry for the $SU(2)$ triplets. The $U(6)$ is revealed by defining two new fields

$$\mathbb{\Delta} \equiv (\Delta, \bar{\Delta}^c) \qquad\qquad \bar{\mathbb{\Delta}} \equiv (\bar{\Delta}, \Delta^c) \tag{3.19}$$

which are complex 6 vectors. The terms in Eq. (3.3) can then be written in terms of the quantity $\text{Tr}\left(\mathbb{\Delta}\bar{\mathbb{\Delta}}\right)$, resulting in the freedom to rotate between the 6 components of $\mathbb{\Delta}$ and $\bar{\mathbb{\Delta}}$.

When $SU(2)_R \times U(1)_{B-L}$ breaks, Δ^c and $\bar{\Delta}^c$ acquire a VEV which can be

[2]It should also be emphasized, albeit in smaller text, that this is not generically true for fields with *zero* vacuum expectation value in the SUSY limit and non-zero VEV when SUSY is broken. This is because the zero point of the potential may be highly unstable and susceptible to inducing large VEVs

rotated into one component of $\mathbb{\Delta}$ and $\bar{\mathbb{\Delta}}$; hence the complexified $U(6)$ is broken to a complexified $U(5)$. This SSB yields 22 massless real degrees of freedom. The superhiggs mechanism ensures that 6 are eaten or become massive, leaving 16. These 16 degrees of freedom then parameterize the degenerate vacua.

In terms of goldstone particles, the 16 degrees of freedom lie in the $SU(2)_L$ triplets Δ, $\bar{\Delta}$ and the doubly-charged right-handed particles Δ^{c--}, $\bar{\Delta}^{c++}$. If it is demanded that $SU(2)_L$ be preserved, then the $U(6)$ is broken only by the right-handed triplets. The result is that the vacuum degeneracy of interest is parameterized through the four real degrees of freedom in Δ^{c--} and $\bar{\Delta}^{c++}$. This may be expressed as

$$\langle \Delta^c \rangle = \frac{v_R}{\sqrt{2}} \mathrm{e}^{\mathrm{i}\theta^c} \begin{pmatrix} 0 & \cos\alpha_{CV} \\ \mathrm{e}^{\mathrm{i}\delta} \sin\alpha_{CV} & 0 \end{pmatrix} \tag{3.20}$$

$$\langle \bar{\Delta}^c \rangle = \frac{\bar{v}_R}{\sqrt{2}} \mathrm{e}^{-\mathrm{i}\theta^c} \begin{pmatrix} 0 & \mathrm{e}^{\mathrm{i}\bar{\delta}} \sin\alpha_{CV} \\ \cos\alpha_{CV} & 0 \end{pmatrix} \tag{3.21}$$

where $\alpha_{CV} \neq 0$ represents a vacuum that violates charge conservation.

The vacuum degeneracy is explicitly broken when gauge couplings are included, but because of the choice of $SU(2)_L$ preservation, only by the $SU(2)_R$ D-terms. Their VEV contributes a term to the potential that is dependent upon α_{CV}:

$$\langle V \rangle_{\mathrm{VDC}} = +\frac{1}{8} g_R^2 \left(\bar{v}_R^2 - v_R^2\right)^2 \cos^2 2\alpha_{CV} \tag{3.22}$$

For the charge conserving potential, $\alpha_{CV} = 0$ and $\langle V \rangle_{\mathrm{VDC}}$ is at its maximal value; thus, the charge violating vacuum is the lower, favored state[78].

Naturally, a dynamically-favored, charge-violating vacuum implies that a viable theory must include additional terms that explicitly break the $U(6)$. One option is to break R-parity thus making the seesaw yukawa couplings relevant to the vacuum structure[78]. Another option is to include additional particle content with couplings that explicitly break this symmetry[79, 80]. Alternatively, it is possible to use the given theory if terms generated from a higher scale of physics—that is, non-renormalizable terms—exist that explicitly break the symmetry[81, 80, 82]. Such terms are present in Eq. (3.4), namely λ_B, λ_B^c, λ_D, $\bar{\lambda}_D$, and they are higher in magnitude than the D-term contributions. This is because the contribution to the potential from the D-term is proportional to the square of the difference of the VEVs

$$\langle V\rangle_{\text{VDC}}^{D} \sim \left(\bar{v}_R^2 - v_R^2\right)^2 \sim M_{\text{SUSY}}^4 \tag{3.23}$$

with the last expression following from the fact that the D-term VEV comes from the difference in the soft SUSY breaking masses. The non-renormalizable terms, however, are

$$\langle V\rangle_{\text{VDC}}^{\text{NR}} \sim \frac{v_R^4}{M_X}\left(\bar{v}_R^2 - v_R^2\right) \sim M_X M_{\text{SUSY}}^3 \gg M_{\text{SUSY}}^4 \tag{3.24}$$

assuming $v_R^2 \sim M_X M_{\text{SUSY}}$. It is therefore quite easy to achieve a contribution to the potential from these non-renormalizable terms that places the charge conserving vacuum lower than that of the charge violating one. In fact, it is important to realize that because the doubly-charged particles are the goldstone bosons of the $U(6)$, and this symmetry is responsible for the vacuum degeneracy, requiring the mass-squares of these goldstone bosons to be positive is exactly the same condition as requiring the charge conserving vacuum to be lower than the charge violating one. Hence ensuring one will imply the other.

3.3 The Theory Between v_R and F_ϕ

Once $SU(2)_R$ breaks, the effective theory will contain the doubly-charged Δ^{c--}, $\bar{\Delta}^{c++}$, a pair of left-handed triplets, and the particle content of the NMSSM with an extra set of higgs doublets. This theory shall be called the NMSSM++ as it connotates the presence of doubly-charged particles as well as the notion that it is 'incrementally more than' the NMSSM.

The doubly-charged particles and the left-handed triplets survive below v_R because of the $U(6)$ symmetry discussed in Section 3.2. The non-renormalizable terms give them a SUSY mass of

$$\mu_{\Delta,\bar{\Delta}} \sim \mu_{DC} \sim \frac{v_R^2}{M_X} \tag{3.25}$$

which is precisely the mass of the fermionic components. The scalars, however, will receive a bilinear term due to the explicit breaking of superconformal invariance, resulting in a mass matrix of the form

$$\mathbf{M}_{DC,\Delta,\bar{\Delta}} \sim \frac{v_R^2}{M_X} \begin{pmatrix} \frac{v_R^2}{M_X} & -F_\phi \\ -F_\phi^\dagger & \frac{v_R^2}{M_X} \end{pmatrix}. \tag{3.26}$$

The eigenvalues of $\mathbf{M}_{DC,\Delta,\bar{\Delta}}$ need to be positive, which will be accomplished by requiring

$$\frac{v_R^2}{M_X} \gtrsim F_\phi. \tag{3.27}$$

As it is desired that these particles survive to the F_ϕ scale, the condition

$$\frac{v_R^2}{M_X} = F_\phi(1+\epsilon_\Delta) \tag{3.28}$$

is imposed, resulting in Eq. (3.26) having the eigenvalues of $2|F_\phi|^2$ and $\epsilon_\Delta|F_\phi|^2$.

In addition to providing a mass for the $U(6)$ goldstones, the non-renormalizable terms of Eq. (3.1) also provide important contributions to the electroweak breaking higgses. For instance, the term

$$\frac{\lambda^c_{\mathbf{N}}}{M_{\mathrm{Pl}}\phi} \mathrm{Tr}\left(\Delta^c\bar{\Delta}^c\right) N^2 + \frac{\lambda_{\mathrm{M}}}{M_{\mathrm{Pl}}\phi} S^2 N^2 \tag{3.29}$$

generates a SUSY mass term for N when Δ^c, $\bar{\Delta}^c$, and S get a VEV. Taking this term in the superpotential to be written as $\mu_N \phi N^2$, the mass is

$$\mu_N \equiv \frac{\lambda^c_{\mathbf{N}}}{M_{\mathrm{Pl}}} \langle\underline{\Delta}^c\rangle \left\langle\underline{\bar{\Delta}}^c\right\rangle + \frac{\lambda_{\mathrm{M}}}{M_{\mathrm{Pl}}} \langle\underline{S}\rangle^2 \simeq \frac{v_R^2}{M_{\mathrm{Pl}}}. \tag{3.30}$$

Once again the explicit superconformal breaking yields a bilinear term of

$$-\mu_N F_\phi \underline{N}^2 \equiv -b_N \underline{N}^2 \tag{3.31}$$

with b_N given as

$$b_N = \mu_N F_\phi \simeq \frac{v_R^2}{M_{\text{Pl}}} F_\phi. \tag{3.32}$$

If b_N is to be of the expected order of M_{SUSY}^2, then it must be that

$$v_R^2 \sim \frac{M_{\text{Pl}} M_{\text{SUSY}}^2}{F_\phi} \sim \left(10^{10}\text{–}10^{11}\ \text{GeV}\right)^2 \tag{3.33}$$

and this results in a range for M_X:

$$M_X \sim \frac{v_R^2}{F_\phi} \sim \left(10^{15}\text{–}10^{18}\ \text{GeV}\right). \tag{3.34}$$

Finally, the non-renormalizable terms will also split the masses between the two higgs doublets as the terms

$$\frac{(\lambda_\sigma^c)^{ab}}{M_{\text{Pl}}\phi} \operatorname{Tr}\left(\Delta^c \bar{\Delta}^c\right) \operatorname{Tr}\left(\Phi_a \tau_2 \Phi_b^T \tau_2\right) + \frac{2\lambda_\alpha^c \epsilon^{ab}}{M_{\text{Pl}}\phi} \operatorname{Tr}\left(\Delta^c \tau_2 \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c\right) \tag{3.35}$$

generate a SUSY mass for the Φ's with the second term providing an asymmetry between Φ_1 and Φ_2. This asymmetry will allow a larger mass, say of order F_ϕ, for one set of doublets while the remaining pair stays light.

Taking all these elements under consideration, the resulting theory between v_R and F_ϕ may be written. The superpotential is given as

$$\begin{aligned} W_{\text{NMSSM++}} = {} & \mathbb{i} y_u^a Q^T \tau_2 H_{u_a} u^c + \mathbb{i} y_d^a Q^T \tau_2 H_{d_a} d^c + \mathbb{i} y_L^a L^T \tau_2 H_{d_a} e^c \\ & + f_c e^c \Delta^{c--} e^c + \mathbb{i} f L^T \tau_2 \Delta L \\ & + \mathbb{i} \lambda^{ab} N H_{u_a}^T \tau_2 H_{d_b} + \mathbb{i} \mu^{ab} \phi H_{u_a}^T \tau_2 H_{d_b} + \frac{1}{2}\mu_N \phi N^2 + \frac{1}{3}\kappa N^3 \\ & + \mu_{DC} \phi \Delta^{c--} \bar{\Delta}^{c++} + \mu_\Delta \phi \operatorname{Tr}\left(\Delta \bar{\Delta}\right) \end{aligned} \tag{3.36}$$

where the $SU(2)_L$ doublets H_{u_a}, H_{d_a} come from the bidoublets Φ_a, and the yukawa couplings obey

$$y_Q^a = y_u^a + y_d^a \tag{3.37}$$

at the scale v_R.

The significant new features of the NMSSM++ are the yukawa couplings f

and f_c in Eq. (3.36), which survive to the low-scale theory due to the complexified $U(6)$. Since these are new leptonic couplings, they introduce new terms in the AMSB expression for both the right- and left-handed sleptons. The new yukawa couplings may then be selected to turn both chiralities of sleptons positive.

To make this explicit, the slepton masses are

$$\begin{aligned} m_{e^c}^2 = \frac{1}{2}\frac{|F_\phi|^2}{(16\pi^2)^2} \Bigg[& 8f_c^\dagger (y_L^a)^T (y_L^a)^* f_c + 12 (y_L^a)^\dagger f f^\dagger y_L^a \\ & + 8 f_c^\dagger f_c \Big[(y_L^a)^\dagger y_L^a + 4 f_c^\dagger f_c + \mathrm{Tr}\big(f_c^\dagger f_c\big) \Big] + 4 (y_L^a)^\dagger y_L^a \Big[(y_L^n)^\dagger y_L^n + 2 f_c^\dagger f_c \Big] \\ & + 2 (y_L^a)^\dagger y_L^n \Big[2 (y_L^n)^\dagger y_L^a + \mathrm{Tr}\Big(3 (y_d^n)^\dagger y_d^a + (y_L^n)^\dagger y_L^a \Big) + 4 (\lambda_N^{mn})^* \lambda_N^{ma} \Big] \\ & - 2 g_Y^2 \Big(24 f_c^\dagger f_c + 3 (y_L^a)^\dagger y_L^a + 26 g_Y^2 \Big) - 6 g_L^2 (y_L^a)^\dagger y_L^a + \text{h.c.} \Bigg] \end{aligned} \tag{3.38}$$

$$\begin{aligned} m_L^2 = \frac{1}{2}\frac{|F_\phi|^2}{(16\pi^2)^2} \Bigg[& 6 f (y_L^a)^T (y_L^a)^* f^\dagger + 4 y_L^a f_c^\dagger f_c (y_L^a)^\dagger \\ & + 6 \Big[(y_L^a)^\dagger y_L^a + 12 f f^\dagger + 2\,\mathrm{Tr}\big(f^\dagger f\big) \Big] f f^\dagger + 2 \Big[y_L^n (y_L^n)^\dagger + 3 f f^\dagger \Big] y_L^a (y_L^a)^\dagger \\ & + y_L^n (y_L^a)^\dagger \Big[2 y_L^a (y_L^n)^\dagger + \mathrm{Tr}\Big(3 (y_d^n)^\dagger y_d^a + (y_L^n)^\dagger y_L^a \Big) + 4 (\lambda_N^{mn})^* \lambda_N^{ma} \Big] \\ & - g_Y^2 \Big(18 f f^\dagger + 3 y_L^a (y_L^a)^\dagger + 13 g_Y^2 \Big) - 3 g_L^2 \Big(14 f f^\dagger + y_L^a (y_L^a)^\dagger + 3 g_L^2 \Big) + \text{h.c.} \Bigg] \end{aligned} \tag{3.39}$$

Lepton flavor violating experiments constrain the f's off-diagonal elements[83] so severely that they may be taken to be zero. The f's are then diagonal in flavor space,

$$f = \mathrm{diag}(f_1, f_2, f_3)\,, \qquad\qquad f_c = \mathrm{diag}(f_{c1}, f_{c2}, f_{c3})\,, \tag{3.40}$$

which, along with the usual neglecting of first and second generation yukawa couplings, simplifies Eqs. (3.38) and (3.39) to

$$m_{e^c}^2 = m_{\text{an}}^2\big[40f_{c1}^4 + 8f_{c1}^2\big(f_{c2}^2 + f_{c3}^2\big) - 48f_{c1}^2g_Y^2 - 52g_Y^4\big] \tag{3.41}$$
$$m_{L_1}^2 = m_{\text{an}}^2\big[84f_1^4 + 12f_1^2\big(f_2^2 + f_3^2\big) - 6f_1^2\big(3g_Y^2 + 7g_L^2\big) - 13g_Y^4 - 9g_L^4\big] \tag{3.42}$$

$$m_{\mu^c}^2 = m_{\text{an}}^2\big[40f_{c2}^4 + 8f_{c2}^2\big(f_{c1}^2 + f_{c3}^2\big) - 48f_{c2}^2g_Y^2 - 52g_Y^4\big] \tag{3.43}$$
$$m_{L_2}^2 = m_{\text{an}}^2\big[84f_2^4 + 12f_2^2\big(f_1^2 + f_3^2\big) - 6f_2^2\big(3g_Y^2 + 7g_L^2\big) - 13g_Y^4 - 9g_L^4\big] \tag{3.44}$$

$$\begin{aligned} m_{\tau^c}^2 = m_{\text{an}}^2\,\big[&40f_{c3}^4 + 10(y_\tau^a y_\tau^a)^2 + 8f_{c3}^2\big(f_{c1}^2 + f_{c2}^2\big) + 6(y_\tau^a y_b^a)^2 + 12y_\tau^a y_\tau^a\big(2f_{c3}^2 + f_3^2\big) \\ &+ 8\lambda^{nm}y_\tau^m\lambda^{np}y_\tau^p - 48f_{c3}^2g_Y^2 - 6y_\tau^a y_\tau^a\big(g_Y^2 + g_L^2\big) - 52g_Y^4\big] \end{aligned} \tag{3.45}$$
$$\begin{aligned} m_{L_3}^2 = m_{\text{an}}^2\,\big[&84f_3^4 + 5(y_\tau^a y_\tau^a)^2 + 3(y_\tau^a y_b^a)^2 + 12f_3^2\big(f_1^2 + f_2^2\big) + 2y_\tau^a y_\tau^a\big(9f_3^2 + 2f_{c3}^2\big) \\ &+ \lambda^{nm}y_\tau^m\lambda^{np}y_\tau^p - 6f_3^2\big(3g_Y^2 + 7g_L^2\big) - 3y_\tau^a y_\tau^a\big(g_Y^2 + g_L^2\big) - 13g_Y^4 - 9g_L^4\big] \end{aligned} \tag{3.46}$$

Eqs. (3.41)–(3.46) demonstrate that the sleptons may be made positive by requiring

$$f_1(F_\phi) \simeq f_2(F_\phi) \simeq f_{c1}(F_\phi) \simeq f_{c2}(F_\phi) \gtrsim 0.6 \tag{3.47}$$

with the 0.6 coming from the detailed analysis of Section 3.5.2.

The f and f_c's as well as the (left and right) doubly-charged masses are experimentally constrained from muonium-antimuonium oscillations[84] which occurs through the tree-level exchange of a doubly-charge particle. The constraint imposes the condition

$$\frac{f_{c1}f_{c2}}{4\sqrt{2}m_{DC}^2} \approx \frac{f_1 f_2}{4\sqrt{2}m_{\Delta,\bar{\Delta}}^2} < 3\times 10^{-3}G_F. \tag{3.48}$$

For the minimum f values of Eq. (3.47), this implies a lower bound on the masses of the doubly-charged fields of

$$m_{DC}, m_\Delta \geq 2\ \text{TeV} \tag{3.49}$$

or the ϵ_Δ of Eq. (3.28) is around 1/100. An exciting result of this limit is that masses near this lower bound are accessible at the LHC and may therefore be found in the near future.

3.4 The Theory Between F_ϕ and M_{SUSY}

At F_ϕ the doubly-charged particles and $SU(2)_L$ triplets need to be integrated out; we also choose, at this scale, to integrate out one set of higgs doublets (which is permissible due to asymmetric μ^{ab}—see Section 3.3). The remaining particle content below F_ϕ is that of the NMSSM, though the actual theory contains additional couplings originating from the higher-scale non-renormalizable terms. To distinguish this model as having additional (though natural) couplings, but still having the NMSSM particle content, the theory below F_ϕ is referred to as the $\tilde{\text{N}}$MSSM.

The superpotential for the $\tilde{\text{N}}$MSSM is

$$\begin{aligned} W_{\tilde{\text{N}}\text{MSSM}} &= \mathbb{i} y_u Q^T \tau_2 H_u u^c + \mathbb{i} y_d Q^T \tau_2 H_d d^c + \mathbb{i} y_e L^T \tau_2 H_d e^c \\ &\quad + \mathbb{i}\lambda N H_u^T \tau_2 H_d + \frac{1}{2}\mu_N N^2 + \frac{1}{3}\kappa N^3 \end{aligned} \tag{3.50}$$

with the SUSY breaking potential

$$\begin{aligned} V_{\text{SB}}^{\tilde{\text{N}}\text{MSSM}} &= m_Q^2 \underline{Q}^\dagger \underline{Q} + m_{u^c}^2 \underline{u}^{c\dagger} \underline{u}^c + m_{d^c}^2 \underline{d}^{c\dagger} \underline{d}^c + m_L^2 \underline{L}^\dagger \underline{L} + m_{e^c}^2 \underline{e}^{c\dagger} \underline{e}^c \\ &\quad + m_{H_u}^2 \underline{H}_u^\dagger \underline{H}_u + m_{H_d}^2 \underline{H}_d^\dagger \underline{H}_d + m_N^2 \underline{N}^* \underline{N} \\ &\quad + \left[\mathbb{i} a_u \underline{Q}^T \tau_2 \underline{H}_u \underline{u^c} + \mathbb{i} a_d \underline{Q}^T \tau_2 \underline{H}_d \underline{d^c} + \mathbb{i} a_e \underline{L}^T \tau_2 \underline{H}_d \underline{e^c} + \text{h.c.}\right] \\ &\quad + \left[\mathbb{i} a_\lambda \underline{N H_u^T} \tau_2 \underline{H}_d - \frac{1}{2} b_N \underline{N}^2 + \frac{1}{3} a_\kappa \underline{N}^3 + \text{h.c.}\right] \\ &\quad - \frac{1}{2}\Big(M_3 \mathrm{g}_A^\alpha \mathrm{g}_\alpha^A + M_L (\mathrm{w}_L)_A^\alpha (\mathrm{w}_L)_\alpha^A + M_Y \mathrm{b}_Y^2 + \text{h.c.}\Big). \end{aligned} \tag{3.51}$$

The resulting higgs-sector potential is

$$V_{\tilde{\text{N}}\text{MSSM}} = V_F + V_D + V_{\text{HSB}} \tag{3.52}$$

$$V_F = |\lambda|^2 |\underline{N}|^2 \left(|\underline{H}_u|^2 + |\underline{H}_d|^2\right) + \left|\mathbb{i}\lambda \underline{H}_u^T \tau_2 \underline{H}_d + \mu_N \underline{N} + \kappa \underline{N}^2\right|^2 \tag{3.53}$$

$$V_D = \frac{1}{8}\left(g_Y^2 + g_L^2\right)\left(|\underline{H}_u|^2 - |\underline{H}_d|^2\right)^2 + \frac{1}{2} g_L^2 \left|\underline{H}_u^\dagger \underline{H}_d\right|^2 \tag{3.54}$$

$$\begin{aligned} V_{\text{HSB}} &= m_{H_u}^2 \underline{H}_u^\dagger \underline{H}_u + m_{H_d}^2 \underline{H}_d^\dagger \underline{H}_d + m_N^2 \underline{N}^* \underline{N} \\ &\quad + \left[\mathbb{i} a_\lambda \underline{N H_u^T} \tau_2 \underline{H}_d - \frac{1}{2} b_N \underline{N}^2 + \frac{1}{3} a_\kappa \underline{N}^3 + \text{h.c.}\right] \end{aligned} \tag{3.55}$$

The extra terms μ_N and b_N present in Eq. (3.52) are fortuitous since Section 2.7 demonstrated that the NMSSM with AMSB produces a very small singlet VEV leading to a tiny higgsino mass[71]. As discussed in Section 3.3, the NMSSM++ naturally generates a SUSY breaking bilinear term for N, which carries over to the $\tilde{\text{N}}$MSSM. This term is conceivably of the correct size and magnitude to force the net mass-square of N negative and cure the small singlet VEV problem. To see how this works it is useful to define the tilded parameters

$$\tilde{a}_\lambda \equiv a_\lambda + \lambda\mu_N \tag{3.56}$$
$$\tilde{a}_\kappa \equiv a_\kappa + 3\kappa\mu_N \tag{3.57}$$
$$\tilde{m}_N^2 \equiv m_N^2 + \mu_N^2 - b_N \tag{3.58}$$

which encapsulate the deviation of the $\tilde{\text{N}}$MSSM from the NMSSM.

Taking the VEVs of the fields as

$$\langle \underline{H}_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} \qquad \langle \underline{H}_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \qquad \langle \underline{N} \rangle = \frac{n}{\sqrt{2}} \tag{3.59}$$

as well as defining

$$v_u = v \sin\beta \qquad\qquad v_d = v\cos\beta \tag{3.60}$$

the minimization conditions of Eq. (3.52) may be written:

$$m_{H_u}^2 - \frac{1}{8}\left(g_L^2 + g_Y^2\right) v^2 \cos 2\beta + \frac{1}{2}\lambda^2\left(n^2 + v^2\cos^2\beta\right) - \frac{n}{\sqrt{2}}\left(\tilde{a}_\lambda + \frac{\lambda\kappa n}{\sqrt{2}}\right)\cot\beta = 0 \tag{3.61}$$

$$m_{H_d}^2 + \frac{1}{8}\left(g_L^2 + g_Y^2\right) v^2 \cos 2\beta + \frac{1}{2}\lambda^2\left(n^2 + v^2\sin^2\beta\right) - \frac{n}{\sqrt{2}}\left(\tilde{a}_\lambda + \frac{\lambda\kappa n}{\sqrt{2}}\right)\tan\beta = 0 \tag{3.62}$$

$$\tilde{m}_N^2 + \kappa^2 n^2 + \frac{1}{2}\lambda^2 v^2 + \frac{n\tilde{a}_\kappa}{\sqrt{2}} - \frac{1}{2}v^2\left(\frac{\tilde{a}_\lambda}{n\sqrt{2}} + \lambda\kappa\right)\sin 2\beta = 0 \tag{3.63}$$

Notice that Eq. (3.63) has Eq. (3.58) in lieu of the typical m_N^2. Using Eq. (3.32) this may be re-written:

$$\begin{aligned} \tilde{m}_N^2 &= m_N^2 + \mu_N^2 - \mu_N F_\phi \\ &\approx m_N^2 - \mu_N F_\phi \\ &\simeq \left(\frac{\lambda^4}{(16\pi^2)^2} F_\phi - \mu_N\right) F_\phi \end{aligned} \tag{3.64}$$

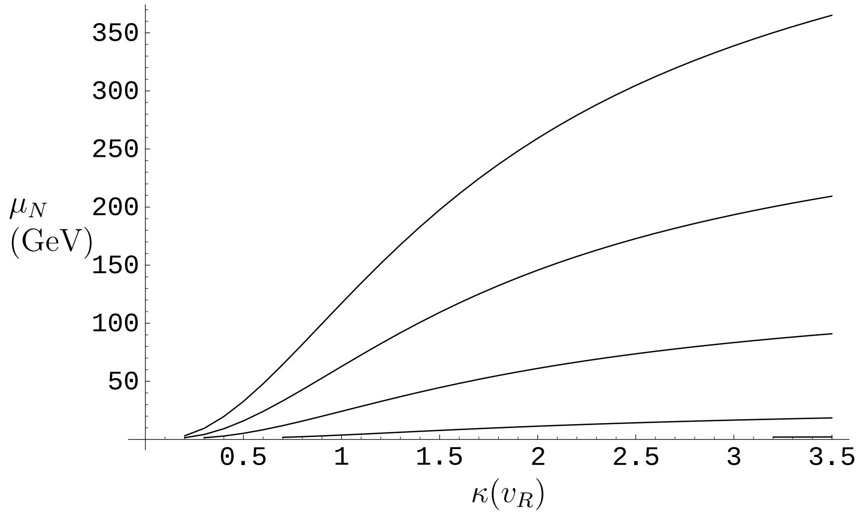


Figure 3.1: Constant n contours in the μ_N–$\kappa(v_R)$ plane where the curves, from top to bottom, correspond to $n = -10000, -7500, -5000, -2500$ and -1000 GeV. A constant value of $\tan\beta = 3.25$ has been assumed with $F_\phi = 33$ TeV and $\lambda(v_R) = 0.5$.

where the second line follows from the fact that $\mu_N \sim \mathcal{O}\left(\frac{M_{\rm SUSY}^2}{F_\phi}\right) \sim \mathcal{O}\left(\frac{F_\phi}{(16\pi^2)^2}\right)$ and therefore the μ_N^2 term is negligible compared to the the other terms. The last line substitutes the AMSB expression for the scalar mass-squared assuming it is dominated by the λ contribution. It is clear from Eq. (3.64) that the λ^4 suppression makes it relatively easy to adjust μ_N to turn $\tilde{m}_N^2$ negative. Thus, the singlet N can achieve a VEV of the necessary size to evade the experimental higgsino mass bound.

Given that constraints from perturbativity limit $\lambda(M_{\rm SUSY}) \lesssim 0.5$ and that $\mu = \frac{\lambda n}{\sqrt{2}}$, it is only necessary for $n \gtrsim 300$ GeV to achieve chargino masses above the LEP II bound. Such a scenario is easily done in the $\tilde{\rm N}$MSSM, as show in Figure 3.1. In the figure, constant n contours are plotted in the μ_N–$\kappa(v_R)$ plane treating the VEVs of the Higgs doublets as constant background values with $\tan\beta = 3.25$, $F_\phi = 33$ TeV, and $\lambda(v_R) = 0.5$. The ample parameter space demonstrates that the additional terms inherent in the model easily provide a means to resolve the conflict between AMSB and the NMSSM.

Perhaps unsurprisingly, the resulting mass spectrum of the $\tilde{\rm N}$MSSM is quite similar to the NMSSM and most results can be obtained by the simple substitution of the appropriate variables with their tilded form. Typically μ_N is small compared to M_{SUSY} so that the untilded variables are usually very good approximations to the tilded ones.

For example, the neutral higgses mass matrix is

$$\mathbf{M}_S^2 = \begin{pmatrix} \mathbf{M}_{\text{HN}}^2 & \mu_{\text{HSN}}^2 \\ (\mu_{\text{HSN}}^2)^T & \text{M}_{\text{SN}}^2 \end{pmatrix} \tag{3.65}$$

with

$$\mathbf{M}_{\text{HN}}^2 = \begin{pmatrix} \frac{v_u^2}{4}(g_Y^2+g_L^2)+\frac{nv_d}{\sqrt{2}v_u}\left(\tilde{a}_\lambda+\frac{\lambda\kappa n}{\sqrt{2}}\right) & \frac{v_dv_u}{4}(4\lambda^2+g_Y^2+g_L^2)-\frac{n}{\sqrt{2}}\left(\tilde{a}_\lambda+\frac{\lambda\kappa n}{\sqrt{2}}\right) \\ \frac{v_dv_u}{4}(4\lambda^2+g_Y^2+g_L^2)-\frac{n}{\sqrt{2}}\left(\tilde{a}_\lambda+\frac{\lambda\kappa n}{\sqrt{2}}\right) & \frac{v_d^2}{4}(g_Y^2+g_L^2)+\frac{nv_u}{\sqrt{2}v_d}\left(\tilde{a}_\lambda+\frac{\lambda\kappa n}{\sqrt{2}}\right) \end{pmatrix} \tag{3.66}$$

$$\left(\mu_{\text{HSN}}^2\right)^T = \left(\lambda^2 n v_u - \frac{v_d}{\sqrt{2}}\tilde{a}_\lambda - \lambda\kappa v_d n \quad \lambda^2 n v_d - \frac{v_u}{\sqrt{2}}\tilde{a}_\lambda - \lambda\kappa v_u n\right) \tag{3.67}$$

$$\text{M}_{\text{SN}}^2 = 2n^2\kappa^2 + \frac{n}{\sqrt{2}}\tilde{a}_\kappa + \frac{v_u v_d}{\sqrt{2}n}\tilde{a}_\lambda \tag{3.68}$$

which is identical to the NMSSM matrix if μ_N is neglected. This is also true of the charged higgs where

$$M_{\text{C}}^2 = \frac{1}{4}\left(g_L^2 - 2\lambda^2\right) + \frac{n}{\sqrt{2}}\left(\tilde{a}_\lambda + \frac{\lambda\kappa n}{\sqrt{2}}\right)\csc 2\beta \tag{3.69}$$

The pseudoscalar mass matrix, however, does get altered because it picks up a contribution from the SUSY breaking b_N term. Generically b_N is significant in size for any reasonable higgsino mass, so this term ensures that the heaviest pseudoscalar is mostly singlet. This can be seen from the mass matrix (after rotating out the zero-mode),

$$\mathbf{M}_{\text{P}}^2 = \begin{pmatrix} \frac{n}{2\sqrt{2}}\left(\tilde{a}_\lambda+\frac{\lambda\kappa n}{\sqrt{2}}\right)\csc 2\beta & \frac{v}{\sqrt{2}}\left(a_\lambda-\lambda\mu_N-\sqrt{2}\lambda\kappa n\right) \\ \frac{v}{\sqrt{2}}\left(a_\lambda-\lambda\mu_N-\sqrt{2}\lambda\kappa n\right) & \frac{v_uv_d}{n\sqrt{2}}\left(\tilde{a}_\lambda+2\lambda\kappa n\sqrt{2}\right)-3\tilde{a}_\kappa\frac{n}{\sqrt{2}}+2b_N+8\kappa\mu_N\frac{n}{\sqrt{2}} \end{pmatrix}, \tag{3.70}$$

for when the $(2,2)$ entry is dominated by b_N: $\operatorname{Tr}\mathbf{M}_{\text{P}}^2 \approx 2b_N$.

As the superpartners to the scalar higgs only get mass from the superpotential, their mass matrices remain unchanged from the NMSSM. For completeness, the charged higgsino mass matrix is

$$\mathbf{M}_{\chi^\pm} = \begin{pmatrix} 0 & \mu_{\chi^\pm}^T \\ \mu_{\chi^\pm} & 0 \end{pmatrix} \qquad \mu_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & \mu \end{pmatrix} \tag{3.71}$$

in the basis $\left(\widetilde{W}^+, \tilde{H}_u^+, \widetilde{W}^-, \tilde{H}_d^-\right)$; while the neutralino mass matrix is

$$\mathbf{M}_{\chi^0} = \begin{pmatrix} M_1 & 0 & M_Z \mathrm{s}_\beta \mathrm{s}_{\theta_W} & -M_Z \mathrm{c}_\beta \mathrm{s}_{\theta_W} & 0 \\ 0 & M_2 & -M_Z \mathrm{s}_\beta \mathrm{c}_{\theta_W} & M_Z \mathrm{c}_\beta \mathrm{c}_{\theta_W} & 0 \\ M_Z \mathrm{s}_\beta \mathrm{s}_{\theta_W} & -M_Z \mathrm{s}_\beta \mathrm{c}_{\theta_W} & 0 & -\frac{\lambda}{\sqrt{2}} n & -\frac{\lambda}{\sqrt{2}} v_d \\ -M_Z \mathrm{c}_\beta \mathrm{s}_{\theta_W} & M_Z \mathrm{c}_\beta \mathrm{c}_{\theta_W} & -\frac{\lambda}{\sqrt{2}} n & 0 & -\frac{\lambda}{\sqrt{2}} v_u \\ 0 & 0 & -\frac{\lambda}{\sqrt{2}} v_d & -\frac{\lambda}{\sqrt{2}} v_u & \sqrt{2}\kappa n + \mu_N \end{pmatrix} \tag{3.72}$$

in the basis $\left(\tilde{B}, \widetilde{W}, \widetilde{H}_u, \widetilde{H}_d, \tilde{N}\right)$.

3.5 Phenomenological Consequences

Running Scheme: The numerical values of this section are based on the parameter running scheme as follows:

- The gauge coupling values are run from the electroweak scale M_Z to $M_{\mathrm{SUSY}} = 1$ TeV using the one-loop SM RGEs. They are then run from M_{SUSY} to F_ϕ using the NMSSM one-loop RGEs. Finally, the gauge couplings are run from F_ϕ to the right-handed scale $v_R = 2 \times 10^{11}$ GeV using the NMSSM++ one-loop RGEs.

- The yukawa couplings are specified at v_R using the 'only third-generation' approximation for y_Q^a and y_L^a. The seesaw couplings are equal at v_R by parity, assumed to be diagonal at this scale due to lepton flavor violation constraints, and approximated as

$$f = \mathrm{diag}(f_1, f_1, f_3) \tag{3.73}$$

mSUGRA $\alpha_Y = \alpha_L$ at $M_{\text{GUT}} \sim 10^{16}$ GeV	m_0 $m_{1/2}$ A_0 $\tan\beta$ $\text{sgn}(\mu)$	universal scalar mass at M_{GUT} universal gaugino mass at M_{GUT} universal trilinear A-term value at M_{GUT} ratio of higgs VEVs sign of the higgs's SUSY mass term
mGMSB	Λ M_{mess} N_5 $\tan\beta$ $\text{sgn}(\mu)$	the scale of SUSY breaking mass of the messengers number of pairs of messengers (assumed to come in complete $SU(5)$ multiplets) ratio of higgs VEVs sign of the higgs's SUSY mass term
mAMSB [53] MSSM+AMSB	m_0^2 F_ϕ $\tan\beta$ $\text{sgn}(\mu)$	universal constant added to scalar mass-squares to avert tachyonic sleptons the scale of SUSY breaking ratio of higgs VEVs sign of the higgs's SUSY mass term

Table 3.2: Popular SUSY breaking schemes.

for simplicity. These yukawa couplings are then run down from v_R to M_{SUSY} [1, 85] with matching at the various thresholds.

- As AMSB is valid until F_ϕ, the SUSY breaking terms need not be run but can be evaluated directly at F_ϕ using the AMSB expressions. From F_ϕ to M_{SUSY} the SUSY breaking parameters are run using the RGEs of the NMSSM [86].

The numerical values are expectedly parameter-choice dependent, so they are compared with the other popular SUSY breaking scenarios of minimal supergravity (mSUGRA), minimal gauge mediated supersymmetry breaking (mGMSB) and minimal anomaly mediated supersymmetry breaking (mAMSB). The m* models are defined by their inputs and assumptions as shown in Table 3.2

3.5.1 The Spectrum Qualitatively

The overall spectrum of SUSYLR+AMSB is shown in Figure 3.2 and Figure 3.3 with the m* models for comparison. The m* models' parameters were chosen so

$\tan\beta = 3.25$		$\text{sgn}(\mu) = +1$	
SUSYLR	mAMSB	mSUGRA	mGMSB
$F_\phi = 33$ TeV $f_1(v_R) = 3.5$ $f_3(v_R) = 3.5$	$m_0 = 645$ GeV $F_\phi = 33$ TeV	$m_0 = 209$ GeV $m_{1/2} = -300$ GeV $A_0 = 265$ GeV	$\Lambda = 99$ TeV $M_{\text{mess}} = 792$ TeV $N_5 = 1$

Table 3.3: The input parameters for Figure 3.2 and Figure 3.3. Each model has $\tan\beta = 3.25$ and $\text{sgn}(\mu) = +1$.

that the gluino mass matched with SUSYLR+AMSB and were calculated using ISAJET[87] through the online tool SUPERSIM. The specific parameters used are given in Table 3.3

Figure 3.3 shows a very striking feature for SUSYLR+AMSB: the sleptons and the squarks are very close in mass. This result is highly dependent on the seesaw couplings (for example, taking $f_1(v_R) = f_3(v_R) = 1.4$ yields sleptons roughly 200 GeV less; the squarks unchanged) but is a possibility that is difficult to achieve in other models.

3.5.2 Sleptons

As the sleptons are made positive by f and f_c, it should come as no surprise that their masses rely heavily upon these couplings' values—therefore making it logical to analyze these couplings when discussing slepton masses. It is convenient to establish a range the f's may take, and an upper bound may be obtained by requiring the theory be perturbative at the v_R scale. This constraint yields $f_{\text{max}}(v_R) = \sqrt{4\pi} \approx 3.5$. A lower bound can be gotten by requiring positive slepton masses, and this will be derived shortly—for the present it suffices to take $f_{\text{min}}(v_R) = 0$.

Figure 3.4 shows the running of f_{c1} as a function of energy for the two extremes of $f_{c3}(v_R)$. It is immediately apparent from the figure that there is a fixed point for $f_{c1}(M_{\text{SUSY}})$ around 0.6 for any value of $f_{c1}(v_R) \gtrsim 1$, though the value is clearly influenced by f_{c3}. The behavior demonstrated in Figure 3.4 actually shows the

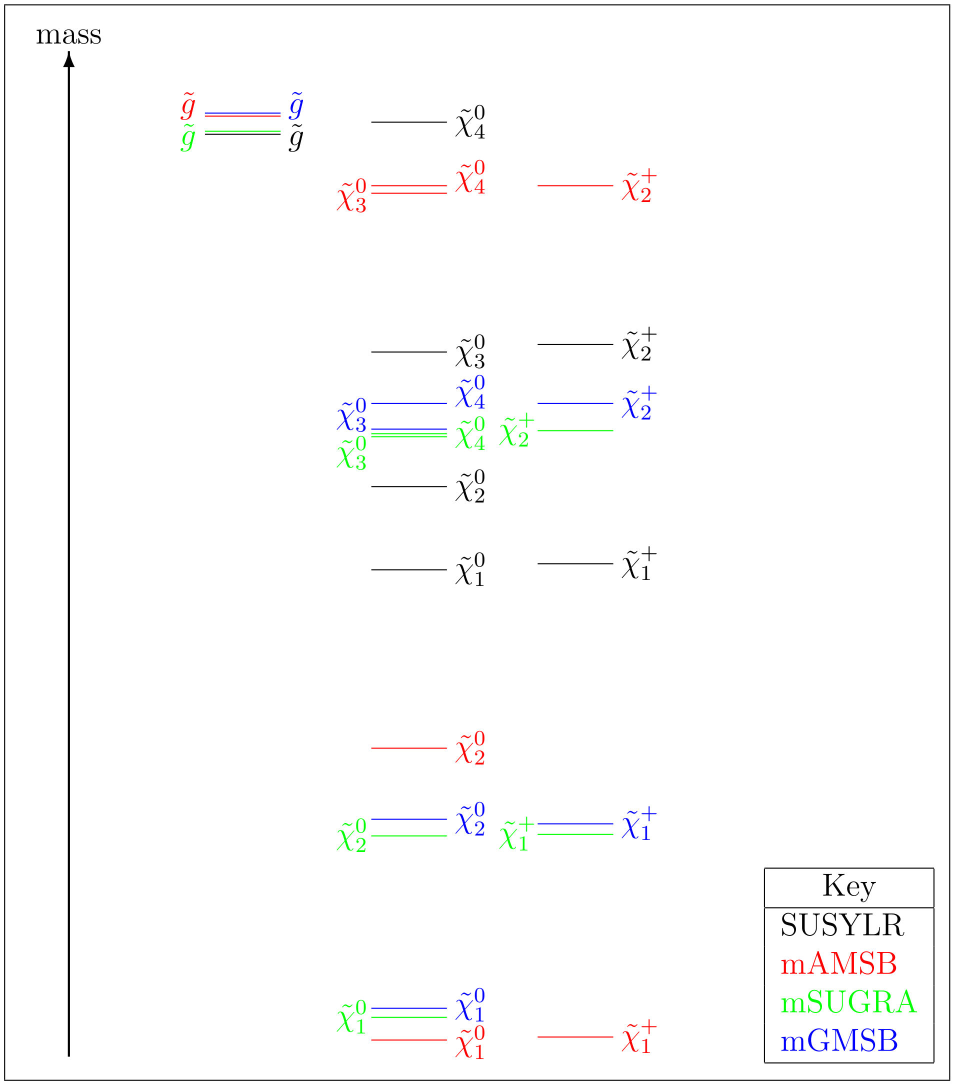


Figure 3.2: The relative masses of the gluino, neutralinos, and charginos in SUSYLR ($F_\phi = 33$ TeV, $f_1(v_R) = f_3(v_R) = 3.5$), mAMSB ($F_\phi = 33$ TeV, $m_0 = 645$ GeV) mSUGRA ($m_0 = 209$ GeV, $m_{1/2} = -300$ GeV, $A_0 = 265$ GeV) mGMSB ($\Lambda = 99$ TeV, $M_{\rm mess} = 792$ TeV, $N_5 = 1$) for $\tan\beta = 3.25$ and $\operatorname{sgn}\mu = +1$.

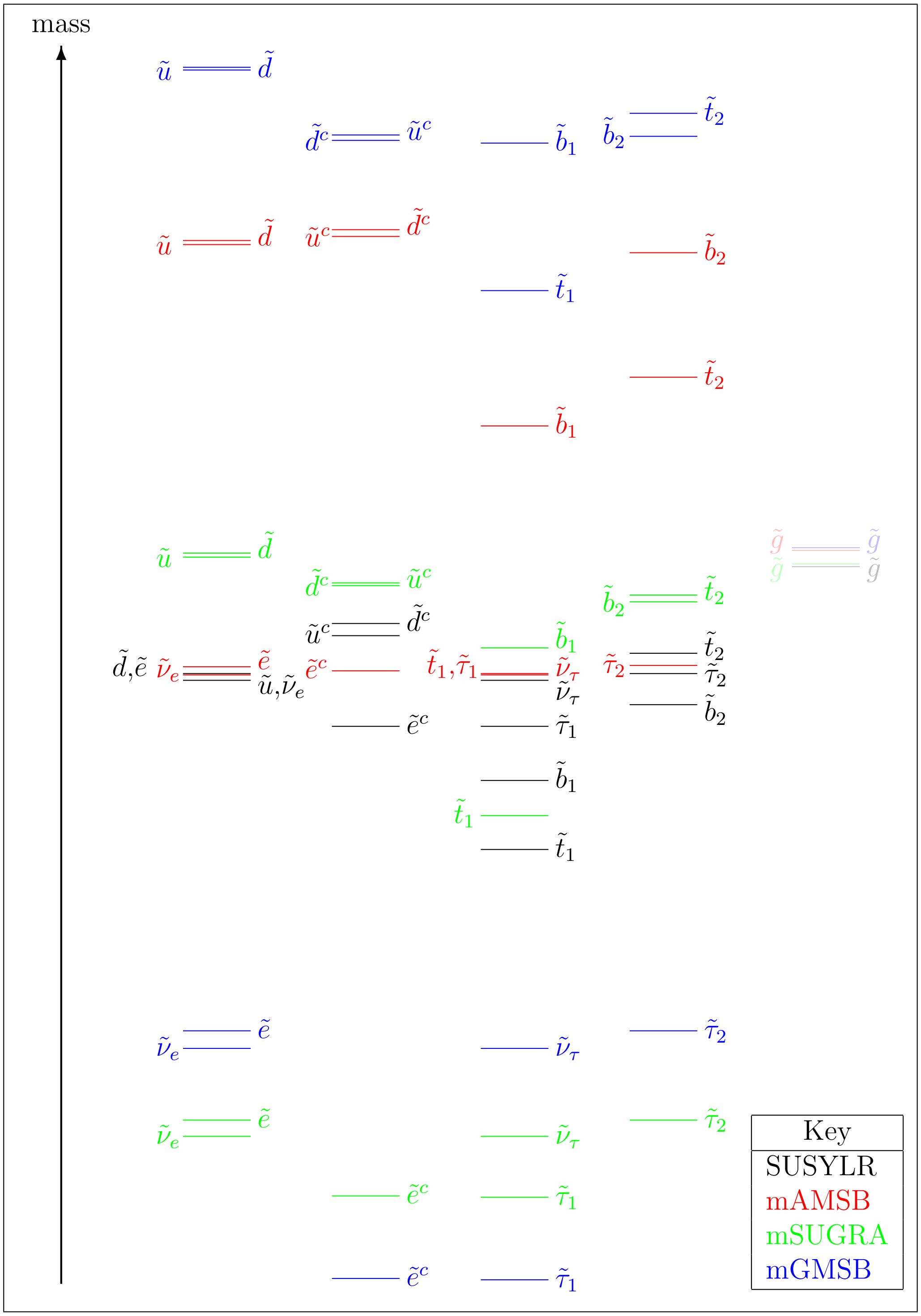


Figure 3.3: The relative masses of the first generation left-handed, first generation right-handed, lightest third generation, and heaviest third generation sfermions in SUSYLR, mAMSB, mSUGRA, mGMSB, for the parameters as defined in Table 3.3. The final column consists of gluino masses for comparison with Figure 3.2

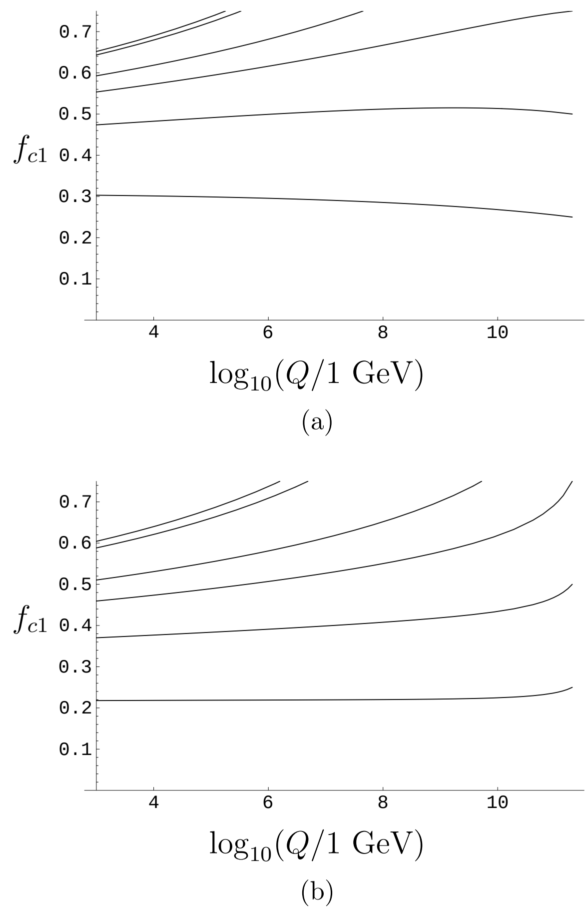


Figure 3.4: Plots of f_{c1} verses the log of the energy scale. The lines correspond, in ascending order, to $f_{c1}(v_R)$ values of 0.25, 0.5, 0.75, 1, 2.25 and 3.5 for (a) $f_{c3}(v_R) = 0$ and (b) $f_{c3}(v_R) = 3.5$.

qualitative feature of all the seesaw couplings; each of f_1, f_3, f_{c1}, f_{c3} tend to a fix point as quantified in Table 3.4. For initial values of $f_1(v_R) = f_{c1}(v_R)$ and $f_3(v_R) = f_{c3}(v_R)$ greater than 1.5, these values are correct up to 2%. The higher fixed point value for the right-handed f_c's is a result of slower running caused by the broken $SU(2)_R$ symmetry.

The fixed-point behavior implies an upper bound for the slepton masses, as can be seen in Figure 3.5. The plot displays the dependence of the selectron masses on the initial value of $f_1(v_R) = f_{c1}(v_R) = f_3(v_R) = f_{c3}(v_R) \equiv f$. Below $f \approx 0.5$ the selectron mass-squares are negative. At $f \approx 0.5$, the seesaw couplings begin

	f_3	f_1	f_{c3}	f_{c1}
Fixed Point Value	0.64	0.64	0.67	0.67

Table 3.4: Fixed point values of the seesaw couplings at F_ϕ assuming initial values are above 1.5.

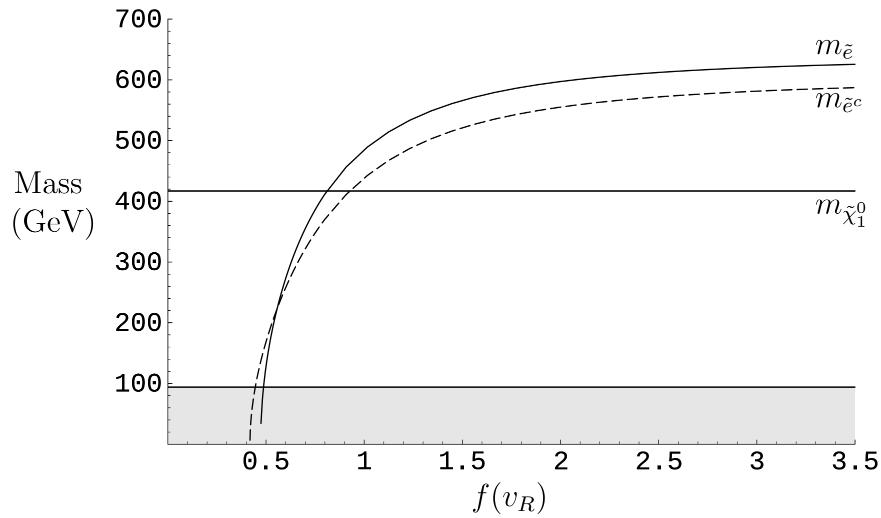


Figure 3.5: Plot of $m_{\tilde{e}^c}$ (dashed) and $m_{\tilde{e}}$ as a function of $f_1(v_R) = f_{c1}(v_R) = f_3(v_R) = f_{c3}(v_R) \equiv f$ for $F_\phi = 33$ TeV. The greyed-out region has been excluded by LEP II. The line around 417 GeV is the mass of the lightest neutralino.

dominating the scalar mass expression and there is a steep rise from their quartic dependence. The ascent levels off, however, around $f \sim 1$ where the asymptotic limit is due to the fixed-point behavior.

The masses of the other sleptons follow the behavior of Figure 3.5, and, as can be seen in the plot, this results in a mild degeneracy between the left- and right-handed slepton masses. The mild degeneracy at first seems a bit contrary to Eqs. (3.41) and (3.42), where the f_1^4 factor is twice as large for left-handed sleptons as it is for the right-handed sleptons; however, this term is limited in size by the fixed-point of f_1 and the negative $SU(2)_L$ contribution happens to be a little less than half of this value. The accidental cancelation between these terms then yields the degeneracy.

The similarities of left- and right-handed slepton masses is an interesting situation phenomologically since it numerically falls in between mSUGRA/mGMSB (with large splittings) and mAMSB (with a high degeneracy). In mSUGRA, left-handed slepton masses get large positive contributions from M_L as they run from the UV, where in mGMSB the boundary conditions dictate the left-handed to right-handed mass ratio to be $\alpha_2 : \alpha_1$. For these two theories then, the left-handed sleptons are always heavier than the right-handed ones. Meanwhile, in mAMSB both sectors get the same (and dominant) contribution from m_0 so they are highly degenerate. Furthermore, accidental cancellations in the anomaly-induced splittings related to the gauge and D-term contributions[62, 88] result in splittings dominated by loop-level effects which are quite small[62].

To demonstrate just how this may be important, mAMSB predicts a mass splitting $\Delta_e = m^2_{\tilde{e}_L} - m^2_{\tilde{e}_R} \sim 751 \text{ GeV}^2$[62, 88] for $\tan\beta = 3.25$ and $F_\phi = 33$ TeV (note that the value of m_0^2 is irrelevant for the mass difference). The percent difference, defined as

$$\%\text{ difference} \equiv \frac{\Delta_e}{(m_{\tilde{e}_L} + m_{\tilde{e}_R})^2}, \tag{3.74}$$

is highly dependent on the masses of the selectrons; but for selectron masses around 450 GeV, the percent difference is less than 1%. The situation in SUSYLR+AMSB is quite different as Figure 3.6 shows: the difference can rise as high as 5%. While a hadron collider would not be able to see this, a lepton collider can achieve a roughly 2% resolution of slepton masses from the end-point lepton distribution of the selectron decays[89]. Therefore such a distinction is feasible, and measurements of mild mass differences around $3-5\%$ will single out this model from mSUGRA and mGMSB while potentially discriminating it from mAMSB (though this will be highly dependent on the values of the seesaw couplings).

Finally, the slepton masses provide an interesting bound when it is demanded that the theory have a viable dark matter candidate. Figure 3.7 shows constant

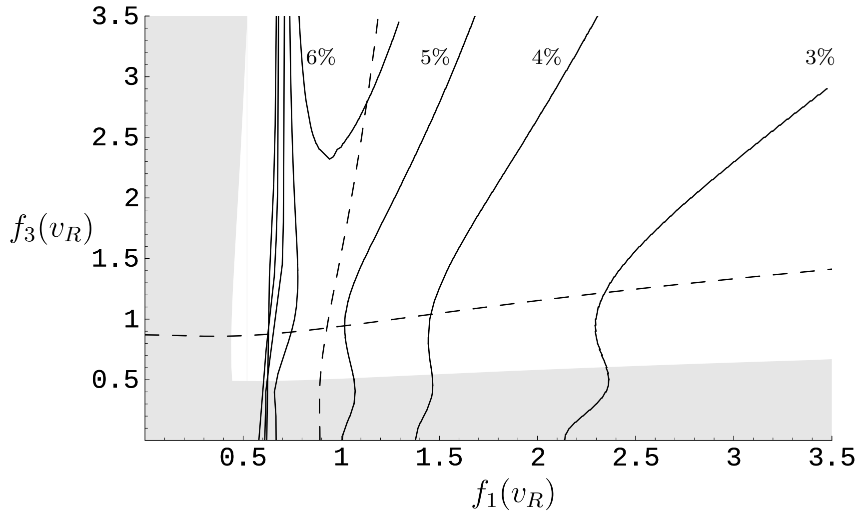


Figure 3.6: Contours of constant $\frac{m^2_{\tilde{e}}-m^2_{\tilde{e}^c}}{(m_{\tilde{e}}+m_{\tilde{e}^c})^2} \times 100\%$ in the $f_3(v_R)$–$f_1(v_R)$ plane. The unlabeled contours on the left side of the plot, from left to right, correspond to $2\%, 3\%, 4\%$ and 5%. The dashed vertical (horizontal) contour corresponds to a $\tilde{\tau}_1$ ($\tilde{e}^c$)constant contour of mass equal to that of the LSP (417 GeV). The shaded region is excluded by LEP II bounds of 81.9 GeV (94 GeV) on the mass of $\tilde{\tau}_1$ ($\tilde{e}^c$).

mass contours for the right-handed selectron in the $f_{c3}(v_R)$–$f_{c1}(v_R)$ plane with the shaded region excluded by LEP II. The dashed lines, on the other hand, define the region with a viable dark matter as they represent $m_{\tilde{\tau}_1} = m_{\tilde{\chi}^0_1}$ (vertical) and $m_{\tilde{e}^c} = m_{\tilde{\chi}^0_1}$ (horizontal). Clearly, insisting the theory explain dark matter yields a much more stringent bound than the experimental constraints. What makes this more interesting is that, as Figure 3.7 indicates, the seesaw couplings need to be within 10% of their fixed-point value, $f_{c1}(F_\phi) \sim f_{c3}(F_\phi) \sim f_1(F_\phi) \sim f_3(F_\phi) \sim 0.6$ if the LSP is the lightest neutralino. Therefore, a successful explanation of dark matter leaves the seesaw couplings larger than about 0.5. Noting that the lightest neutralino has a mass of approximately the wino (see Section 3.5.4), and using the expression of the selectron mass from AMSB, it is straightforward to place a bound on $f(F_\phi) \equiv f_1(F_\phi) = f_3(F_\phi)$ of:

$$f(F_\phi) \gtrsim 0.58. \tag{3.75}$$

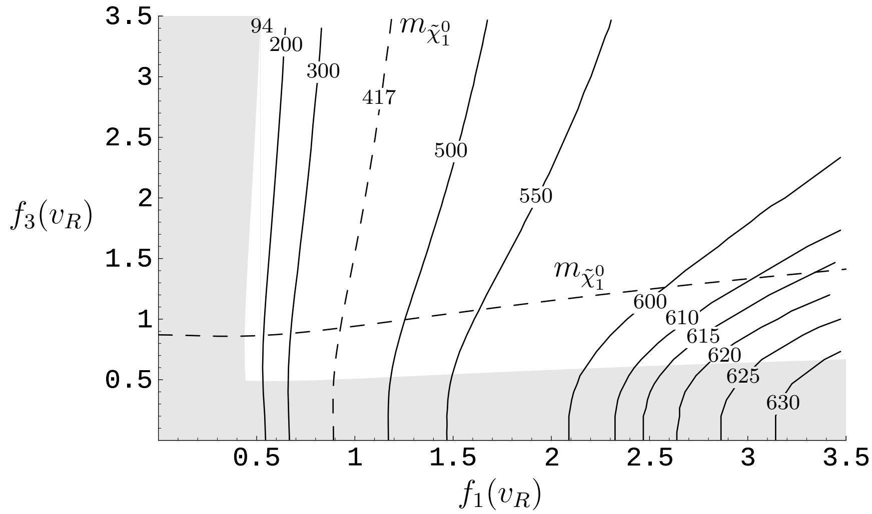


Figure 3.7: Mass contours for the right-handed selectron mass, $m_{\tilde{e}}$ in the $f_3(v_R)$–$f_1(v_R)$ plane at $F_\phi = 33$ TeV. The horizontal and vertical shaded areas are ruled out due to LEP II bounds on the lightest stau ($m_{\tilde{\tau}_1} > 81.9$ GeV) and selectron ($m_{\tilde{e}} > 94$ GeV) masses respectively. The dashed vertical contour is $m_{\tilde{e}} = m_{\tilde{\chi}_1^0} = 417$ GeV indicating the point where the LSP is neutralino. The dashed horizontal curve corresponds to $m_{\tilde{\tau}_1} = m_{\tilde{\chi}_1^0}$. The fixed point behavior of f_1 is apparent after $f_1 \sim 1$ at which point the contours curve more drastically.

3.5.3 Squarks

The main feature of SUSYLR+AMSB is a natural means to avoid tachyonic sleptons; however, the squark masses also have a couple of noteworthy features. First, as in mAMSB, the squark masses decrease with energy because the $SU(2)_L \times U(1)_Y$ gauge couplings become larger and hence their negative contribution increases in magnitude. Therefore, it is conceivable that the negative contribution equals the positive contribution and the squark masses become zero at a certain energy scale, M_{q0}[90]. If this does happen, the squark mass-squares would be negative for energies above M_{q0}, resulting in a vacuum that breaks $SU(3)^c$. This scenario has been considered in mAMSB[90], where it was determined that the squark mass squares do turn negative before M_{Pl}, but at a high, $M_{q0} \sim M_{\mathrm{GUT}} \gg F_\phi$ energy scale. The issue must be reconsidered here because the new particle content alters the slope of the $SU(2)_L \times U(1)_Y$ gauge couplings, resulting in the couplings running faster (for the hypercharge coupling, the effect is quite large). Despite this faster running, the squark masses still turn negative at a high scale, $M_{q0} \sim 10^9$ GeV $\gg F_\phi$, though slightly before the v_R scale. Typically the effect of these negative mass-squares would be expected to turn up from physics at temperatures near M_{q0}, where the vacuum would be capable of breaking color; however, at such temperatures the vacuum of the theory is also affected by temperature corrections. Consequently the mass-square term of the squarks picks up a temperature dependence of the form $\mu^2(T)_{\tilde{q}} \simeq (-M^2_{\text{AMSB}} + \lambda T^2)$. The first term only grows logarithmically with temperature, whereas the second term grows quadratically. The coefficient λ is positive, and even though it is 'small', the mass-square remains positive definite because T is so large. Thus the temperature effects alone ensure the color gauge symmetry remains intact in the early universe.

It is also worth noting that the right-handed squarks are slightly heavier than left-handed squarks because of the negative $SU(2)_L$ contribution. This is different from mSUGRA and mGMSB (though the result is essentially the same in mAMSB) where all gauge couplings yield positive contributions. In these theories, then, the left-handed squarks are always heavier (this can be seen in Figure 3.3).

3.5.4 Bosinos and the Lightest Supersymmetric Particle

Because all superpartners eventually decay into the LSP, its makeup is an important part of SUSY collider phenomenology and dark matter prospects; therefore, understanding that makeup is vital. Cosmological constraints rule out a charged or colored LSP as dark matter[91], limiting the choices to the sneutrino or lightest neutralino. The former, in typical models, makes a poor dark matter candidate as its relic abundances are too light and much of its mass range is ruled out by direct detection [92, 93]. The responsibility of dark matter therefore falls upon an

LSP being the lightest neutralino—a typical candidate in common SUSY scenarios (except in mGMSB where the LSP is always the gravitino, so the neutralino is the next-to lightest supersymmetric particle [94]).

Due to R-parity conservation, the lightest neutralino will be some mixture of the $B - L = 0$ fields: the neutral wino, the bino, the neutral higgsinos, and the singlino. In the SUSYLR+AMSB model, the singlet VEV, n, is typically large compared to the other elements of Eq. (3.72), so the singlino decouples and becomes the heaviest neutralino.

The relative gaugino composition of the lightest neutralino is fairly easily calculated and relatively independent of the point in parameter space: it follows from the gaugino mass ratio which, in AMSB models, depends on both the gauge couplings and the α^{-1} beta functions b. For the NMSSM++ the latter quantity is more important since this is where the effects of the light triplets and doubly-charged higgses are felt the most severely (as demonstrated by Table 3.5). The gaugino mass

	b_Y	b_L	b_3
MSSM	$\frac{33}{5}$	1	-3
NMSSM++	$\frac{78}{5}$	6	-3

Table 3.5: Values of the α^{-1} beta function in the MSSM and NMSSM++.

ratio of the NMSSM++ is $M_3 : M_2 : M_1 \sim 1.3 : 1 : 1.3$. This is a striking ratio due to its close proximity of the gaugino masses, unlike other popular scenarios (see Table 3.6). The nearly-degenerate gaugino masses then implies that the NMSSM++ has a light neutralino with a large wino component and a non-negligible bino component. This is in stark contrast to mAMSB where the ratio implies the gaugino composition is all wino. It also contrasts the mSUGRA and mGMSB where the lightest neutralino is always mostly bino.

Furthermore, in the NMSSM++ the mass of the LSP may be larger because

	$M_3 : M_2 : M_1$
NMSSM++	$1.3 : 1 : 1.3$
mAMSB	$8 : 1 : 3.5$
mSUGRA	$3 : 1 : 0.3$
mGMSB	$3 : 1 : 0.3$

Table 3.6: The gaugino mass ratios for four SUSY breaking scenarios, including the low-energy SUSYLR+AMSB (NMSSM++)

of the gaugino contribution. This is because the gaugino mass terms are bounded by

$$M_Y \lesssim 1350 \text{ GeV} \qquad\qquad M_L \lesssim 980 \text{ GeV}. \tag{3.76}$$

which comes from the limit $F_\phi \lesssim 63$ TeV. This is important in terms of available parameter space: in mAMSB, $M_L \lesssim 200$ GeV [88], and experiment has ruled out much of its parameter space; for the NMSSM++ this is no longer the case.

The higgsino composition, on the other hand, is not independent of other parameters and is therefore not as predictable. Numerical results indicate, however, that the μ value is typically slightly smaller than M_L, resulting in an LSP that is mostly higgsino, significantly wino, and non-negligibly bino. The next lightest neutralino, $\tilde{\chi}_2$, is then all higgsino, while $\tilde{\chi}^0_3$ is mostly wino and significantly higgsino (the composition is complementary to the lightest neutralino $\tilde{\chi}^0_1$). Finally, $\tilde{\chi}^0_4$ is mostly bino (as already established, $\tilde{\chi}^0_5$ is all singlino).

Another important particle in decays is the chargino, which is a mixture of wino and higgsino. As already noted, the μ term is typically smaller than M_L, which implies, from Eq. (3.71), that the lightest chargino, $\tilde{\chi}^+_1$, is mostly higgsino with some wino; therefore, the heaviest chargino, $\tilde{\chi}^+_2$ is mostly wino with some higgsino—a similar composition to $\tilde{\chi}^0_3$, and complementary to $\tilde{\chi}^0_1$.

In addition to the composition, the mass difference between the charginos and neutralinos is also important since the charginos will decay to the LSP. As the neutral higgsino and wino form isospin doublets and triplets with the appropriate charginos, there is potential for a very small mass difference between the lightest neutralino and the lightest chargino. This is very pronounced in mAMSB where the mass difference of the lightest neutralino and chargino is on the order of 100s of MeVs, including leading radiative corrections. Analytical approximations for the difference have been given in [62, 95, 88] for large μ; however, such approximations are not as useful here since $\mu \sim M_Y \sim M_L$.

To obtain an expression for the mass difference of the lightest neutralino and chargino relevant to NMSSM++, it is necessary to consider the two mass matrices given in Eqs. (3.72) and (3.71). Comparison of the two reveals that the neutralino matrix has a mixing dependent on $\tan\beta$ that is absent in the chargino matrix. As $\tan\beta \to 1$, the extra mixing parameter of the neutralinos goes to zero; hence, for $\tan\beta = 1$ the mass difference is minimal. The eigenvalues of the two matrices may then be expanded for $\tan\beta = 1$ using the approximation $M_Y \sim M_L > \mu \gg M_Z$. This yields, to first order:

$$\Delta_{\tilde{\chi}_1} \equiv m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 2\sin^2\theta_W \frac{M_Z^2}{M_Y}. \tag{3.77}$$

The neglected second order term is positive definite so that $\Delta_{\tilde{\chi}_1}$ truly represents the minimal value for the mass splitting. As a check of the expression, $\Delta_{\tilde{\chi}_1} \to 0$ as $\tan\theta_W \to 0$; since $\tan\theta_W \to 0$ restores the custodial $SU(2)$, this is the expected behavior. Additionally, $M_Y \to \infty$ restores the custodial $SU(2)$ when $\tan\beta = 1$, and indeed $\Delta_{\tilde{\chi}_1} \to 0$.

Since Eq. (3.77) depends on $1/M_Y$, while all other parameters are known, its minimum occurs when M_Y is maximized; this is fortunate because Eq. (3.76) gives an upper bound for M_Y. Application of this limit yields

$$\Delta_{\tilde{\chi}_1}^{\min} > 1.4 \text{ GeV} \tag{3.78}$$

Evidently, this is larger than the mAMSB value of a few 100s of MeV. For a quan-

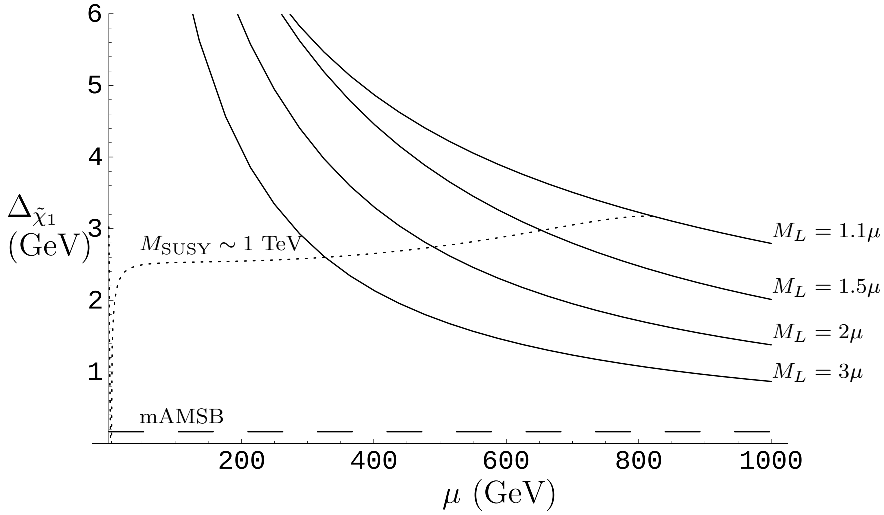


Figure 3.8: Mass difference of the lightest chargino and neutralino as a function of μ for $\lambda = 0.26$, $\tan\beta = 3.25$, and the singlino mass term, $2\left(\mu_N + \frac{1}{\sqrt{2}}n\kappa\right) = 2M_Y$. From top to bottom, $M_L = 1.1\mu$, 1.5μ, 2μ and 3μ. The line at 165 MeV is the asymptotic value for large M_L in mAMSB, while the dotted curve represents where squark masses are about a TeV—below this curve, the Higgs mass would be considered fine-tuned to some extent.

titative comparison, numerical examples of the NMSSM++ mass differences are displayed in Figure 3.8 as a function of $\mu \equiv \frac{1}{\sqrt{2}}\lambda n$.

3.5.5 Dark matter

Due to the preservation of R-parity, the LSP is stable and therefore is a potential dark matter candidate. Section 3.5.4 gives its composition, which is predominantly higgsino, some wino, and a tiny but non-negligible bino (around 1%). The annihilation rate for a mostly higgsino/wino LSP is far too large for its relic density at freeze out to be sufficiently high enough to explain the observed $\Omega_m \approx 0.20$. This issue has already been resolved, however, in mAMSB with a wino LSP[96]. In that case the abundance of LSP at freeze out is not the source of dark matter, rather the late decay of the gravitino—which occurs after LSP freeze out—generates the LSP in a non-equilibrium environment. The generation of these non-thermal LSPs is sufficient to achieve the appropriate dark matter abundance[96].

In the SUSYLR+AMSB model, the same mechanism will also allow an acceptable dark matter abundance to be obtained. This is because the argument given in [96] depends on three properties which are not specific to the wino:

1. the LSP mass,
2. the LSP interactions with the gravitino, and
3. the LSP annihilation rate.

For SUSYLR+AMSB the LSP mass is very similar to those of [96]. Furthermore, as the mass is the same, the interaction with the gravitino is equivalent since this is purely gravitational (i.e. only mass dependent). Finally, the annihilation rate for the higgsino/wino combination is similar to that of the wino because both take place through t-channel chargino exchange and the higgsino couplings to the chargino with the same strength, α_L.

Another consideration for dark matter is the direct detection limit given by nucleus recoil experiments. The current bounds are set by CDMS Soudan and EDELWEISS, and the dark matter of the $\tilde{\text{N}}$MSSM evades these bounds. Perhaps more importantly, however, is that the projected sensitivity of the currently-planned, near-future experiments will probe the mass and cross section of this model's dark matter. Thus the model—at least as far as its dark matter candidate—may be ruled out or confirmed shortly.

3.5.6 Collider Signatures

At the LHC the studied SUSY signals are done in the mSUGRA framework. The pertinent signals are chargino decays yielding dilepton or trilepton signals[97, 90, 98, 27]. The processes for these events are

$$pp \to \tilde{q}_L\tilde{q}_L \qquad \tilde{q}_L \to q\tilde{\chi}_1^+ \qquad \tilde{\chi}_1^+ \to \ell^+\nu\tilde{\chi}_1^0 \tag{3.79}$$

$$pp \to \tilde{q}_L\tilde{q}_L \qquad \begin{cases} \tilde{q}_L \to q\tilde{\chi}_1^+ & \tilde{\chi}_1^+ \to \ell^+\nu\tilde{\chi}_1^0 \\ \tilde{q}_L \to q\tilde{\chi}_2^0 & \tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0 \end{cases} \tag{3.80}$$

where there is a great deal of "schematicity". In this scenario the right-handed squark plays no role because it decays directly to the LSP due to the LSP being mostly bino.

However, in the mAMSB scenario the LSP is mostly wino and the lightest chargino is also purely wino. Therefore the left-handed squarks do not utilize the process Eq. (3.80), but rather decay directly to the LSP instead. The direct decay to the LSP has no chance of detection, because it manifests as missing energy plus a jet, which has far too large a background. The decay chain Eq. (3.79) is then the only one available for mAMSB, but this decay is dominantly $\tilde{\chi}_1^+ \to \pi^+\tilde{\chi}_1^0$ and not leptons. Due to the small mass difference of $\tilde{\chi}_1^+$ and $\tilde{\chi}_1^0$ the resulting pion is far too soft to be detected. Alternatively the right-handed squarks do favor decays to heavier neutralinos since they are mostly bino. The process,

$$pp \to \tilde{q}^c\tilde{q}^c \quad \tilde{q}^c \to q^c\tilde{\chi}_2^0 \quad \tilde{\chi}_2^0 \to \ell^+\ell^-\tilde{\chi}_1^0, \tag{3.81}$$

however, does not yield same-sign dileptons or trilepton signals.

Interestingly enough, the use of SUSYLR with AMSB permits the restoration of the dilepton and trilepton channels. This is because in certain parameter regimes the left-handed squarks will favor decays to $\tilde{\chi}_3^0$ or $\tilde{\chi}_2^+$ due to their higher wino content. These particles may then decay leptonically:

$$pp \to \tilde{q}_L\tilde{q}_L \qquad \tilde{q}_L \to q\tilde{\chi}_2^+ \quad \tilde{\chi}_2^+ \to \ell^+\nu\tilde{\chi}_1^0$$

$$pp \to \tilde{q}_L\tilde{q}_L \quad \begin{cases} \tilde{q}_L \to q\tilde{\chi}_2^+ & \tilde{\chi}_2^+ \to \ell^+\nu\tilde{\chi}_1^0 \\ \tilde{q}_L \to q\tilde{\chi}_3^0 & \tilde{\chi}_3^0 \to \ell^+\ell^-\tilde{\chi}_1^0 \end{cases} \tag{3.82}$$

For other parts of the parameter space, the situation is similar to the mAMSB decays; however, the larger mass difference yields additional potential for detection if it is large enough to produce a τ or a hard μ. Sadly, this advantage is muted by a faster chargino decay which eliminates chances of long-lived charged tracks and no muon chamber activity. Regardless, similar situations have been analyzed and found to be manageable for both lepton colliders[99] and the Tevatron [95, 100].

3.5.7 The Higgs Boson

The lightest higgs boson—the particle that would appear as the SM higgs and hence the particle that prompted the introduction of SUSY—plays an important role in limiting the SUSY breaking parameter, F_ϕ, providing both an upper and lower bound.

As is well established, the lightest higgs mass at tree-level is at most M_Z, meaning the full one-loop radiative corrections are responsible for producing a mass in excess of the LEP II bound (provided it wasn't missed through invisible decays). The most important contribution is due to top and stop loops, and the expressions may be found in [101]. As the top mass is known, it falls on the stop mass to provide a sufficiently large contribution to evade the LEP II limit. This constraint yields a lower bound for the stop mass, $m_{\tilde{t}} \gtrsim 600$ GeV, which immediately implies

$$F_\phi \gtrsim 33. \tag{3.83}$$

Simultaneously, the same corrections to the lightest higgs mass are expected to be of the same order as this higgs's mass if the theory is not to be considered "fine-tuned". That is, if it is expected that SUSY solves the gauge-hierachy problem, the squark masses' contribution to the lightest higgs mass must not conveniently cancel to yield the correct mass of the higgs. This naturalness argument states that the squarks masses must be less than about 1 TeV to avoid any 'fine-tuning'. This translates into an upper bound for F_ϕ,

$$F_\phi \lesssim 63 \text{ TeV}. \tag{3.84}$$

3.5.8 Exotics

While the $SU(2)_L$ triplets and right-handed doubly-charged particles must have a SUSY mass around $F_\phi \sim 33$ TeV to resolve the issue of tachyonic sleptons, this does not eliminate the possibility of their being seen at the LHC. Or rather, it does ensure that the corresponding fermions will not be seen at the LHC (whose center of mass energy, as a reminder, is 14 TeV), but not the scalar particles.

As argued, the scalar triplets and doubly-charged particles split in mass due to SUSY breaking (see Eq. (3.26)), yielding a lighter mass of $\sqrt{\epsilon_\Delta |F_\phi|} \simeq 1.5$ TeV. Such a mass would be accessible to the LHC, and may then be seen through its striking four lepton decays[102].

In addition to the LHC, the upcoming muonium-antimuonium oscillation experiments would also be sensitive enough to detect an $\mathcal{O}(1 \text{ TeV})$ doubly-charged particle since their couplings to the first and second generation leptons must be large in this model.

Chapter 4

Conclusion

This thesis considered the supersymmetric left-right model in the context of anomaly mediated supersymmetry breaking. The model was motivated by phenomenological considerations to explain neutrino oscillations and to provide a viable dark matter candidate. The bottom-up approach was taken to yield a minimal extension of the MSSM that incorporated this phenomenology through the seesaw mechanism and automatic R-parity conservation. This minimal extension achieved these goals through the addition of $B-L=\pm 2$ triplets. These new particles, which were motivated through independent reasons, were then shown to solve the tachyonic slepton problem of AMSB in the context of the MSSM through the introduction of the seesaw couplings.

The resulting combined model, SUSYLR+AMSB, retained the characteristic UV insensitivity and powerful predictability inherent in AMSB while retaining influence from the new particle content. The new high-scale couplings were shown to survive the UV insensitivity "washout" due to an accidental symmetry of the model that was broken solely by non-renormalizable operators. This explicit breaking resulted in low mass values for the left-handed triplets and right-handed doubly-charged particles despite an $\mathcal{O}(10^{11}$ GeV) scale of $B-L$ breaking.

The low energy consequences of the theory were then explored, taking into consideration experimental constraints on the triplets and doubly-charged couplings. It was shown that the particles satisfy the lepton flavor violation limits and the most stringent bound of muonium-antimuonium oscillation. Furthermore, the limits of future planned experiments such as PRISM[103] will probe a majority of the parameter space related to these new particles thus providing a means of falsifying the model. Additionally, it was demonstrated that these exotic particles were capable of having masses within the reach of the LHC and so there are multiple means of discovery.

This thesis proceeded to investigate the viability of EWSB and the size of the $B\mu$ term. The model was revealed to contain a natural solution to the EWSB problem of combining the NMSSM with AMSB. The $B\mu$ term in the theory was demonstrably sizable to permit a large singlet VEV and still satisfy EWSB constraints.

Finally, the low-energy features were considered, and the general features of the superpartners's mass spectrum were given. The same spectrum was compared to other SUSY breaking scenarios, including the mAMSB, and shown to significantly differ. The slepton masses were then explored to determine constraints and dependence on the seesaw parameters. Subsequently, the higgsino and gaugino masses, which is related to the dark matter composition, were investigated and bounds were obtained on the gaugino parameters. The scenario of relic abundance for dark matter was addressed, where it was pointed out that the model can use late-stage gravitino decay to obtain the appropriate abundance. Furthermore the direct detection cross-section was explained to be within experimental limits, but within reach of near future experimental sensitivities.

Appendix A

Notation Conventions

In this appendix we summarize our notational conventions.

A.1 Explanation of Symbols

$[A, B]$	commutator of A and B, defined in Eq. (A.5)
$\{A, B\}$	anti-commutator of A and B, defined in Eq. (A.6)
A^μ	photon vector field
α_G	'fine-structure' constant for the group G
B_μ	hypercharge or $B - L$ vector field
$B_{\mu\nu}$	hypercharge or $B - L$ field strength
$\mathcal{B}$	hypercharge or $B - L$ vector superfield
$\mathscr{B}^\alpha$	field strength for hypercharge or $B - L$ vector superfield
C_G	quadratic casimir invariant/dynkin index for the group G
C_R^G	quadratic casimir invariant in representation R of the group G
$\mathbb{C}$	charge conjugation acting on a dirac field
D_μ	gauge covariant derivative
$\mathcal{D}_\alpha$	superderivative
$\overline{\mathcal{D}}_{\dot{\alpha}}$	superderivative conjugate
∂_μ	partial derivative with respect to spacetime
δ	a small (compared to one) number
δ_a^b	kronecker delta
δ^{AB}	kronecker delta
$\delta(x)$	the dirac delta function

$\mathbb{e}$ base of natural logarithm, see Section A.2

ϵ a small (compared to one) number

$\epsilon^{a_1 a_2 \cdots a_n}$ totally antisymmetric levi-civita tensor density

$\eta_{\mu\nu}$ minkowski (flat) spacetime metric tensor, defined in Eq. (A.3)

f_G^{ABC} structure constants for the group G

F_ϕ $\langle F \rangle$ of the conformal compensator divided by $M_{\rm Pl}$

G_μ^A gluon vector field

$G_{\mu\nu}^A$ gluon field strength

$\mathcal{G}^A$ gluon vector superfield

$\mathscr{G}^\alpha$ field strength for gluon vector superfield

G_N Newton's gravitational constant, see Section A.2

g_3 gauge coupling constant for $SU(3)^c$

g_{BL} gauge coupling constant for $SU(2)_{B-L}$

g_G gauge coupling constant for the group G

g_L gauge coupling constant for $SU(2)_L$

g_R gauge coupling constant for $SU(2)_R$

g_Y gauge coupling constant for the hypercharge $U(1)$

$\bar{g}_Y$ GUT normalized gauge coupling constant for the hypercharge $U(1)$

$g_{\mu\nu}$ metric tensor

${\Gamma^\alpha}_{\mu\nu}$ affine connection

γ_μ dirac matrix

${\gamma_i}^j$ anomalous dimension

$\mathbb{i}$ square-root of minus one, see Section A.2

$\mathcal{K}$ kähler potential

$\mathcal{L}$ lagrangian density, referred to as lagrangian

λ_A Gell-Mann lambda matrices

$m_{\rm an}$	$F_\phi/16\pi^2$
M_G	the gaugino mass for the group G
$M_{\rm GUT}$	the grand unification scale
$M_{\rm Pl}$	the planck mass or reduced planck mass
$M_{\rm SUSY}$	the SUSY scale
M_X	the next higher scale of new physics
M_Z	the mass of the Z boson
M_W	the mass of the W boson
$\mathbb{P}_{L,R}$	projection operator acting on dirac field
$P_{L,R}$	projection operator relating dirac and weyl fields
$\bar{P}_{L,R}$	projection operator relating dirac and weyl fields
∂_μ	partial derivative with respect to spacetime
Φ	scalar field or higgs boson
Φ_i	generic chiral superfield or a higgs bidoublet superfield
$\underline{\Phi}_i$	the scalar component of the superfield Φ_i
ϕ	conformal compensator
ϖ_G	multiplicative inverse of α_G
Ψ	dirac field (four component spinor)
ψ	weyl field (two component spinor)
R	ricci scalar
$R(\Phi_i)$	R charge of Φ_i or representation of Φ_i for a group G
$R_{\mu\nu}$	ricci tensor
$R^\alpha{}_{\mu\beta\nu}$	riemann tensor
S_R^G	dynkin index in representation R of the group G
σ^i	pauli matrices
σ^μ	the identity and σ^i

T^G	generator for the group G
$\left(T_R^G\right)_A$	the A^{th} generator for the group G in representation R
$\left(T_R^G\right)^A$	the A^{th} generator for the group G in representation R
τ_2	second pauli matrix
τ_A	pauli matrices
τ_G	gauge supercoupling for the group G
Θ	vacuum polarization angle
θ^α	grassmann variable
$\bar{\theta}^{\dot{\alpha}}$	grassmann variable conjugate
V_μ^A	generic vector field with group index A
$\mathcal{V}_G$	vector superfield for the group G
$\mathscr{V}_G$	field strength for vector superfield $\mathcal{V}_G$
W_μ^A	$SU(2)_L$, $SU(2)_R$ vector field
$W_{\mu\nu}^A$	$SU(2)_L$, $SU(2)_R$ field strength
$\mathcal{W}^A$	$SU(2)_L$, $SU(2)_R$ vector superfield
$\mathscr{W}^\alpha$	field strength for $SU(2)_L$, $SU(2)_R$ vector superfield
$\mathcal{W}$	superpotential
Z^μ	standard model Z boson
$Z_i{}^j$	wavefunction renormalization constant
$\mathcal{Z}_i{}^j$	wavefunction renormalization superconstant
$\mathbb{Z}$	set of integers
$\zeta_i{}^j$	anomalous mass dimension

A.2 Physical and Mathematical Constants

G_N	6.67×10^{-11} m^3/kg·s^2
h	6.626×10^{-34} J·s
$\hbar$	1.054×10^{-34} J·s $= 6.582 \times 10^{-22}$ MeV·s
c	299 792 458 m/s
$\mathbb{e}$	$\lim_{x\to 0}(1+x)^{1/x} = 2.718\cdots$
$\mathbb{i}$	$\sqrt{-1}$
π	$3.141\,59\cdots$
γ_E	$\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \ln n\right) = 0.577\cdots$

A.3 Field Theory

Field theory takes quantum mechanics and makes it consistent with special relativity. As such it treats space and time on equal footing: taking both as parameters of the theory and postulating that the particle or field (which depends upon spacetime) is an operator. Because space and time are on equal footing, the theory's space and time dependence is most easily written in terms of the contravariant four-vector

$$x^\mu \equiv (t, \vec{x}) \tag{A.1}$$

with $\mu \in \{0, 1, 2, 3\}$ and defining time as the zeroth component. Lengths are determined by a metric, $g_{\mu\nu}$, and given as

$$ds^2 = \sum_\mu \sum_\nu g_{\mu\nu} dx^\mu dx^\nu \tag{A.2}$$

Since field theory[1] deals with flat spacetime, the metric $g_{\mu\nu}$ is the minkowski metric, given the symbol $\eta_{\mu\nu}$, and defined by

[1] at least the field theory considered in this text

$$\eta_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{A.3}$$

Invariant products of four-vectors involve the metric through

$$\sum_\mu \sum_\nu \eta_{\mu\nu} x^\mu x^\nu$$

and it is convenient to define the covariant four-vector

$$x_\mu \equiv \sum_\nu \eta_{\mu\nu} x^\nu. \tag{A.4}$$

The invariant products of four-vectors may then be expressed as

$$\sum_\mu x^\mu x_\mu = \sum_\mu x_\mu x^\mu = x^\mu x_\mu,$$

where the last equal, because the combination is always a subscript/superscript pair, contains an implicit sum—this is the Einstein summation convention which states that any subscript/superscript pair with the same index is assumed to be summed over.

The language of field theory is also the language of quantum mechanics, so that the commutator,

$$[A, B] \equiv AB - BA, \tag{A.5}$$

appears and the fields themselves are chosen to have non-vanishing commutation relations. Furthermore, since the theory will require fermions, the anti-commutator,

$$\{A, B\} \equiv AB + BA, \tag{A.6}$$

is introduced so that the fermionic fields have non-vanishing anticommutation relations.

A.3.1 Scalar Fields

A real, spin zero field Φ of mass m has the 'free' (i.e. non-interacting) lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} m^2 \Phi^2 \tag{A.7}$$

yielding the klein-gordon equation

$$\partial^\mu \partial_\mu \Phi - m^2 \Phi = 0 \tag{A.8}$$

from applying the variational principle.

The field Φ and its conjugate momentum

$$\Pi \equiv \frac{\delta \mathcal{L}}{\delta \partial_0 \Phi} = \partial_0 \Phi \tag{A.9}$$

obey the canonical equal-time commutation relations

$$\begin{aligned} &[\Pi(\vec{x}, t), \Phi(\vec{x}\,', t)] = -\mathbb{i}\delta^3(\vec{x} - \vec{x}\,') \\ &[\Pi(\vec{x}, t), \Pi(\vec{x}\,', t)] = 0 \\ &[\Phi(\vec{x}, t), \Phi(\vec{x}\,', t)] = 0 \end{aligned} \tag{A.10}$$

A.3.2 Fermion Fields

A spin one-half field Ψ of mass m has the 'free' (i.e. non-interacting) lagrangian

$$\mathcal{L} = \mathbb{i}\overline{\Psi}\gamma^\mu \partial_\mu \Psi - m\overline{\Psi}\Psi \tag{A.11}$$

yielding the dirac equation

$$\mathbb{i}\gamma^\mu \partial_\mu \Psi - m\Psi = 0 \tag{A.12}$$

from applying the variational principle.

The lagrangian Eq. (A.11) involves

$$\overline{\Psi} \equiv \Psi^\dagger \gamma^0, \tag{A.13}$$

and the objects γ^μ which, along with an additional object γ_5, obey

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \{\gamma^\mu, \gamma^5\} = 0 \qquad (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \tag{A.14}$$

where

$$\gamma^5 = \gamma_5 = \mathbb{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{\mathbb{i}}{4!}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu = \gamma_5^\dagger. \tag{A.15}$$

The smallest realization of the γ's in $3+1$ spacetime dimensions are 4×4 matrices, making Ψ a four-component object. To make this explicit Ψ is given a subscript index a that runs from 1 to 4. The conjugate of Ψ_a is then defined as

$$\Psi^a \equiv \Psi_a^* \tag{A.16}$$

leading to $\overline{\Psi}$ carrying a superscript index.

The field Ψ_a and its conjugate momentum

$$\Pi^a \equiv \frac{\delta\mathcal{L}}{\delta\partial_0\Psi_a} = \Psi^a \tag{A.17}$$

obey the canonical equal-time anticommutation relations

$$\begin{aligned}\{\Psi^a(\vec{x},t), \Psi_b(\vec{x}\,',t)\} &= \delta^3(\vec{x}-\vec{x}\,')\delta^a_b \\ \left\{\Pi^a(\vec{x},t), \Pi^b(\vec{x}\,',t)\right\} &= 0 \\ \{\Psi_a(\vec{x},t), \Psi_b(\vec{x}\,',t)\} &= 0.\end{aligned} \tag{A.18}$$

A.3.2.1 Dirac vs. Weyl Spinors

The four component dirac spinor Ψ_a defined in Section A.3.2 may be split into two, two-component objects called weyl spinors. Let this two-component weyl spinor be denoted ψ^α, where α runs over 1 and 2.

The products of dirac spinors may be explicitly written using indices:

$$\overline{\Psi}\gamma^\mu\Psi \leftrightarrow \overline{\Psi}^a(\gamma^\mu)_a{}^b\Psi_b \tag{A.19}$$

$$\Psi^T\mathbb{C}\Psi \leftrightarrow \Psi_a\mathbb{C}^{ab}\Psi_b \tag{A.20}$$

$$\overline{\Psi}\mathbb{C}\overline{\Psi}^T \leftrightarrow \overline{\Psi}^a\mathbb{C}_{ab}\overline{\Psi}^b. \tag{A.21}$$

The projection operators on dirac fields can then be defined as

$$(\mathbb{P}_L)_a{}^b = \frac{1}{2}(1-\gamma_5)_a{}^b \qquad (\mathbb{P}_R)_a{}^b = \frac{1}{2}(1+\gamma_5)_a{}^b. \tag{A.22}$$

To relate the dirac and weyl spinors, four hybrid projection operators must also be defined. These satisfy

$$\begin{aligned} \left(\bar{P}_L\right)_a{}^\alpha (P_L)_\alpha{}^b &= (\mathbb{P}_L)_a{}^b & \left(\bar{P}_R\right)_{a\dot{\alpha}} (P_R)^{\dot{\alpha}b} &= (\mathbb{P}_R)_a{}^b \\ (P_L)_\alpha{}^a \left(\bar{P}_L\right)_a{}^\beta &= \delta_\alpha^\beta & (P_R)^{\dot{\alpha}a} \left(\bar{P}_R\right)_{a\dot{\beta}} &= \delta_{\dot{\beta}}^{\dot{\alpha}} \end{aligned} \tag{A.23}$$

Given these projection operators,

$$\begin{aligned} (P_L)_\alpha{}^a \Psi_a &= \psi_{L\alpha} & \overline{\Psi}^a \left(\bar{P}_L\right)_a{}^\alpha &= \psi_R^\alpha \\ (P_R)^{\dot{\alpha}a} \Psi_a &= \psi_R^{\dagger\dot{\alpha}} & \overline{\Psi}^a \left(\bar{P}_R\right)_{a\dot{\alpha}} &= \psi_{L\dot{\alpha}}^\dagger \end{aligned} \tag{A.24}$$

In addition, the canonical anticommutation relations Eq. (A.18) imply the existence of

$$\psi_L^\alpha = \epsilon^{\alpha\beta}\psi_{L\beta} \qquad \psi_{R\alpha} = \epsilon_{\alpha\beta}\psi_R^\beta \tag{A.25}$$

$$\psi_{R\dot{\alpha}}^\dagger = \epsilon_{\dot{\alpha}\dot{\beta}}\psi_R^{\dagger\dot{\beta}} \qquad \psi_L^{\dagger\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\psi_{L\dot{\beta}}^\dagger. \tag{A.26}$$

With these hybrid projection operators, quantities involving the dirac fields and a $\mathbb{P}_{L,R}$ may then be expressed in terms of the weyl spinors. For example

$$\overline{\Psi}\mathbb{P}_L\gamma^\mu\Psi = \overline{\Psi}\mathbb{P}_L\gamma^\mu\mathbb{P}_R\Psi \leftrightarrow \overline{\Psi}^a\left(\bar{P}_L\right)_a{}^\alpha (P_L)_\alpha{}^b (\gamma^\mu)_b{}^c \left(\bar{P}_R\right)_{c\dot{\alpha}} (P_R)^{\dot{\alpha}d}\Psi_d \tag{A.27}$$

$$\overline{\Psi}\mathbb{P}_R\gamma^\mu\Psi = \overline{\Psi}\mathbb{P}_R\gamma^\mu\mathbb{P}_L\Psi \leftrightarrow \overline{\Psi}^a\left(\bar{P}_R\right)_{a\dot{\alpha}} (P_R)^{\dot{\alpha}b} (\gamma^\mu)_b{}^c \left(\bar{P}_L\right)_c{}^\alpha (P_L)_\alpha{}^d\Psi_d \tag{A.28}$$

The gamma matrices, and consequently the charge conjugation matrix, become

$$(P_L)_\alpha{}^b (\gamma^\mu)_b{}^c \left(\bar{P}_R\right)_{c\dot{\alpha}} = \sigma^\mu_{\alpha\dot{\alpha}} \qquad (P_R)^{\dot{\alpha}b} (\gamma^\mu)_b{}^c \left(\bar{P}_L\right)_c{}^\alpha = \bar{\sigma}^{\mu\dot{\alpha}\alpha} \tag{A.29}$$

$$\begin{aligned} \mathbb{C}^{ab}\left(\bar{P}_L\right)_a{}^\alpha \left(\bar{P}_L\right)_b{}^\beta &= \epsilon^{\alpha\beta} & (P_L)_\alpha{}^a (P_L)_\beta{}^b \mathbb{C}_{ab} &= \epsilon_{\alpha\beta} \\ \mathbb{C}^{ab}\left(\bar{P}_R\right)_{a\dot{\alpha}} \left(\bar{P}_R\right)_{b\dot{\beta}} &= \epsilon_{\dot{\alpha}\dot{\beta}} & (P_R)^{\dot{\alpha}a} (P_R)^{\dot{\beta}b} \mathbb{C}_{ab} &= \epsilon^{\dot{\alpha}\dot{\beta}} \end{aligned} \tag{A.30}$$

with

$$\epsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \epsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{A.31}$$

$$\epsilon^{\alpha\lambda}\epsilon_{\lambda\beta} = \delta^{\alpha}_{\beta} \quad \epsilon^{\dot{\alpha}\dot{\lambda}}\epsilon_{\dot{\lambda}\dot{\beta}} = \delta^{\dot{\alpha}}_{\dot{\beta}} \tag{A.32}$$

$$\sigma^0_{\alpha\dot{\alpha}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma^1_{\alpha\dot{\alpha}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2_{\alpha\dot{\alpha}} = \begin{pmatrix} 0 & -\mathbb{i} \\ \mathbb{i} & 0 \end{pmatrix} \quad \sigma^3_{\alpha\dot{\alpha}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.33}$$

$$(\bar{\sigma}^{\mu})^{\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^{\mu}_{\beta\dot{\beta}} \tag{A.34}$$

Using these results a dictionary can be constructed relating dirac and weyl spinors.

A.3.2.2 Weyl-Dirac Spinor Dictionary

Choosing the convention that

$$\psi_R\psi_L \equiv \psi^{\alpha}_R\psi_{L\alpha} \qquad \psi^{\dagger}_L\psi^{\dagger}_R \equiv \psi^{\dagger}_{L\dot{\alpha}}\psi^{\dagger\dot{\alpha}}_R \tag{A.35}$$

the dictionary takes the form

$$\begin{aligned}
\overline{\Psi}_1\mathbb{P}_L\Psi_2 &= \psi_{1R}\psi_{2L} \\
\overline{\Psi}_1\mathbb{P}_R\Psi_2 &= \psi^{\dagger}_{1L}\psi^{\dagger}_{2R} \\
\Psi_1^T\mathbb{C}\mathbb{P}_L\Psi_2 &= -\psi_{1L}\psi_{2L} \\
\Psi_1^T\mathbb{C}\mathbb{P}_R\Psi_2 &= -\psi^{\dagger}_{1R}\psi^{\dagger}_{2R} \\
\overline{\Psi}_1\mathbb{P}_L\mathbb{C}\overline{\Psi}_2^T &= \psi_{1R}\psi_{2R} \\
\overline{\Psi}_1\mathbb{P}_R\mathbb{C}\overline{\Psi}_2^T &= \psi^{\dagger}_{1L}\psi^{\dagger}_{2L} \\
\overline{\Psi}_1\gamma^{\mu}\mathbb{P}_L\Psi_2 &= \psi^{\dagger}_{1L}\bar{\sigma}^{\mu}\psi_{2L} \\
\overline{\Psi}_1\gamma^{\mu}\mathbb{P}_R\Psi_2 &= \psi_{1R}\sigma^{\mu}\psi^{\dagger}_{2R}
\end{aligned} \tag{A.36}$$

A.4 Gauge Groups

Let G be a gauge group of dimension $d(G)$ having the gauge coupling constant g_G, the structure constants f^{ABC}_G, the fundamental symmetric constants d^{ABC}_G, and the generators T_R^G for a representation R. The 'fine structure' constant of G is given by

$$\alpha_G \equiv \frac{g_G^2}{4\pi} \tag{A.37}$$

and its multiplicative inverse is denoted

$$\varpi_G \equiv \frac{1}{\alpha_G} = \frac{4\pi}{g_G^2}. \tag{A.38}$$

The generators of G obey

$$\left[\left(T_R^G\right)^A, \left(T_R^G\right)^B\right] = \mathbb{i}\sum_C f_G^{ABC}\left(T_R^G\right)^C \tag{A.39}$$

and also define the quadratic casimir invariant C_R^G and dynkin index S_R^G through

$$C_R^G \delta_i{}^j = \sum_A \left[\left(T_R^G\right)^A \left(T_R^G\right)^A\right]_i{}^j \tag{A.40}$$

$$S_R^G \delta^{AB} = \mathrm{Tr}\left[\left(T_R^G\right)^A \left(T_R^G\right)^B\right]. \tag{A.41}$$

In the **fundamental representation**, denoted by a subscript zero, the generators may be taken as $n \times n$ matrices obeying

$$\left[\left(T_0^G\right)^A, \left(T_0^G\right)^B\right] = \mathbb{i}\sum_C f_G^{ABC}\left(T_0^G\right)^C \tag{A.42}$$

$$\left\{\left(T_0^G\right)^A, \left(T_0^G\right)^B\right\} = \frac{1}{n}\delta^{AB} + \sum_C d_G^{ABC}\left(T_0^G\right)^C \tag{A.43}$$

with the normalization convention

$$\mathrm{Tr}\left[\left(T_0^G\right)^A \left(T_0^G\right)^B\right] = S_0^G \delta^{AB} = \frac{1}{2}\delta^{AB}. \tag{A.44}$$

In the **adjoint representation**, denoted by a subscript $\mathcal{A}$, the structure constants are the generators,

$$\left[\left(T_{\mathcal{A}}^G\right)^A\right]_{BC} = \mathbb{i} f^{ABC} = \frac{1}{S_0^G}\,\mathrm{Tr}\left(\left[\left(T_0^G\right)^A, \left(T_0^G\right)^B\right]\left(T_0^G\right)^C\right) \tag{A.45}$$

and the quadratic casimir invariant is equal to the dynkin index

$$C_G \delta^{AB} \equiv C_{\mathcal{A}}^G \delta^{AB} = S_{\mathcal{A}}^G \delta^{AB} = \sum_{C,D} f_G^{ACD} f_G^{BCD} \tag{A.46}$$

A.4.1 Gauge Covariant Derivative

Let the gauge group G be a product of n simple groups and p $U(1)$ groups,

$$G = G_1 \times G_2 \times \cdots \times G_n \times U_1(1) \times U_2(1) \times \cdots \times U_p(1). \tag{A.47}$$

with the respective gauge fields

$$\left(V_\mu^{G_1}, V_\mu^{G_2}, \ldots, V_\mu^{G_n}, B_\mu^1, B_\mu^2, \ldots, B_\mu^p\right) \tag{A.48}$$

The gauge covariant derivative is then given by

$$\begin{aligned} {}_{(a_1,a_2,\ldots,a_{n+p})}\mathrm{D}_\mu^{(b_1,b_2,\ldots,b_{n+p})} = {} & \left[\prod_{j=1}^{n+p} \delta_{a_j}^{b_j}\right] \partial_\mu - \mathbb{i} \sum_{j=1}^{n} \left[\prod_{k\neq j}^{n+p} \delta_{a_k}^{b_k}\right] \sum_{A_j=1}^{d(G_j)} g_j \left[\left(T_R^{G_j}\right)^{A_j}\right]_{a_j}^{b^j} \left(V_\mu^{G_j}\right)^{A_j} \\ & - \mathbb{i} \sum_{j=n+1}^{n+p} \left[\prod_{k\neq j}^{n+p} \delta_{a_k}^{b_k}\right] g_j \left(T_R^{U_{j-n}(1)}\right)_{a_j}^{b_j} B_\mu^{j-n} \end{aligned} \tag{A.49}$$

The above expression explicitly separates the group into products of simple or $U(1)$ groups; however, it may just as easily be considered that G has the generators $\mathbf{T}^A$ where

$$\mathbf{T}^A = \begin{cases} T_{G_1}^A & \text{for } 1 \leq A \leq d(G_1) \\ T_{G_2}^{A-d(G_1)} & \text{for } 1 \leq A - d(G_1) \leq d(G_2) \\ \vdots & \\ T_{G_n}^{A-\sum_{j=1}^{n-1} d(G_j)} & \text{for } 1 \leq A - \sum_{j=1}^{n-1} d(G_j) \leq d(G_n) \\ T_{U_1(1)} & \text{for } A = \sum_{j=1}^{n} d(G_j) + 1 \\ T_{U_2(1)} & \text{for } A = \sum_{j=1}^{n} d(G_j) + 2 \\ \vdots & \\ T_{U_p(1)} & \text{for } A = \sum_{j=1}^{n} d(G_j) + p \end{cases}, \tag{A.50}$$

the gauge field $\mathbf{V}_\mu^A$ defined as

$$\mathbf{V}_\mu^A = \begin{cases} \left(V_\mu^{G_1}\right)^A & \text{for } 1 \leq A \leq d(G_1) \\ \left(V_\mu^{G_2}\right)^{A-d(G_1)} & \text{for } 1 \leq A - d(G_1) \leq d(G_2) \\ \vdots & \\ \left(V_\mu^{G_n}\right)^{A-\sum_{j=1}^{n-1} d(G_j)} & \text{for } 1 \leq A - \sum_{j=1}^{n-1} d(G_j) \leq d(G_n) \\ B_\mu^1 & \text{for } A = \sum_{j=1}^{n} d(G_j) + 1 \\ B_\mu^1 & \text{for } A = \sum_{j=1}^{n} d(G_j) + 2 \\ \vdots & \\ B_\mu^p & \text{for } A = \sum_{j=1}^{n} d(G_j) + p \end{cases} , \tag{A.51}$$

and the gauge couplings $\mathbf{g}_A$ given by

$$\mathbf{g}_A = \begin{cases} g_1 & \text{for } 1 \leq A \leq d(G_1) \\ g_2 & \text{for } 1 \leq A - d(G_1) \leq d(G_2) \\ \vdots & \\ g_n & \text{for } 1 \leq A - \sum_{j=1}^{n-1} d(G_j) \leq d(G_n) \\ g_{n+1} & \text{for } A = \sum_{j=1}^{n} d(G_j) + 1 \\ g_{n+1} & \text{for } A = \sum_{j=1}^{n} d(G_j) + 2 \\ \vdots & \\ g_{n+p} & \text{for } A = \sum_{j=1}^{n} d(G_j) + p \end{cases} . \tag{A.52}$$

The gauge covariant derivative may then be expressed more simply as

$${}_i\mathrm{D}_\mu^j = \delta_i^j \partial_\mu - \mathbb{i} \sum_{A=1}^{d(G)} \mathbf{g}_A \left(\mathbf{T}^A\right)_i^j \mathbf{V}_\mu^A \tag{A.53}$$

with $d(G) = \sum_{j=1}^{n} d(G_j) + p$.

A.4.2 The Standard Model Gauge Group

The SM gauge group, $SU(3)^c \times SU(2)_L \times U(1)_Y$, has the gauge couplings g_3, g_L, g_Y, and the gauge fields G, W, and B, respectively.

The value for the SM gauge couplings evaluated at M_Z are

$$\begin{aligned}
\varpi_3 &= 8.50 \pm 0.15 \\
\varpi_L &= 29.61 \pm 0.05 \\
\varpi_Y &= 35.98 \pm 0.03 \\
\overline{\varpi}_Y &= 59.97 \pm 0.05 \\
\varpi_{em} &= 128 \\
g_3 &= 1.22 \pm 0.02 \\
g_L &= 0.6514 \pm 0.0006 \\
g_Y &= 0.5910 \pm 0.0004 \\
\bar{g}_Y &= 0.4616 \pm 0.0002 \\
g_{em} &= e = 0.31
\end{aligned}$$

In the fundamental representation, the generators of $SU(3)^c$ are the Gell-Mann λ matrices divided by two, the generators of $SU(2)_L$ are the pauli matrices divided by two, and the generators of $U(1)_Y$ are, by the convention used here, half of the particle's hypercharge.

The quadratic casimir invariant for a field in the fundamental representation of $SU(3)^c$ is 4/3 and of $SU(2)_L$ is 3/4.

The structure constants of $SU(3)^c$, denoted f_3^{ABC}, have the non-zero values

$$f_3^{123} = 1 \tag{A.54}$$

$$f_3^{147} = f_3^{246} = f_3^{257} = f_3^{345} = \frac{1}{2} \tag{A.55}$$

$$f_3^{156} = f_3^{367} = -\frac{1}{2} \tag{A.56}$$

$$f_3^{458} = f_3^{678} = \frac{\sqrt{3}}{2}; \tag{A.57}$$

the structure constants of $SU(2)_L$, denoted f_2^{ABC}, are the levi-civita tensor density,

$$f_2^{ABC} = \epsilon^{ABC}; \tag{A.58}$$

the structure constants of $U(1)_Y$ are all zero.

The fundamental symmetric constants of $SU(3)^c$, denoted d_3^{ABC}, have non-zero entries

$$d_3^{118} = d_3^{228} = d_3^{338} = \frac{1}{\sqrt{3}} \tag{A.59}$$

$$d_3^{146} = d_3^{157} = d_3^{256} = d_3^{344} = d_3^{355} = \frac{1}{2} \tag{A.60}$$

$$d_3^{247} = d_3^{366} = d_3^{377} = -\frac{1}{2} \tag{A.61}$$

$$d_3^{448} = d_3^{558} = d_3^{668} = d_3^{778} = -\frac{1}{2\sqrt{3}} \tag{A.62}$$

$$d_3^{888} = -\frac{1}{\sqrt{3}}; \tag{A.63}$$

the fundamental symmetric constants of $SU(2)_L$ are all zero.

The gauge covariant derivative for the SM gauge group is

$$\begin{aligned} {}_{(a_3,a_2,a_1)}\mathrm{D}_\mu^{(b_3,b_2,b_1)} = {} & \delta_{a_3}^{b_3}\delta_{a_2}^{b_2}\delta_{a_1}^{b_1}\partial_\mu - \mathbb{i}g_3(T_A)_{a_3}^{b_3}\, G_\mu^A\delta_{a_2}^{b_2}\delta_{a_1}^{b_1} \\ & - \mathbb{i}g_L\delta_{a_3}^{b_3}(T_A)_{a_2}^{b_2}\, W_\mu^A\delta_{a_1}^{b_1} - \mathbb{i}g_Y\delta_{a_3}^{b_3}\delta_{a_2}^{b_2}(T)_{a_1}^{b_1}\, B_\mu \end{aligned} \tag{A.64}$$

A.5 Supersymmetry

Supersymmetry is a symmetry between bosonic degrees of freedom and fermionic degrees of freedom; one means of treating these on equal footing is the introduction of superspacetime coordinates—extending the bosonic spacetime parameters x^μ to include fermionic counterparts.

A.5.1 Superspacetime Coordinates

The fermionic counterpart to x^μ is the four component spinor Θ. Just like the spinors of Section A.3.2, Θ may be broken down into two two-component objects:

$$\begin{aligned} (P_L)_\alpha{}^a\Theta_a &= \theta_\alpha & \qquad \bar{\Theta}^a\big(\bar{P}_L\big)_a{}^\alpha &= \theta^\alpha \\ (P_R)^{\dot{\alpha}a}\,\Theta_a &= \bar{\theta}^{\dot{\alpha}} & \qquad \bar{\Theta}^a\big(\bar{P}_R\big)_{a\dot{\alpha}} &= \bar{\theta}_{\dot{\alpha}}. \end{aligned} \tag{A.65}$$

where, because $P_L\Theta$ is related to $\bar{\Theta}\bar{P}_L$, the four component spinor Θ has only 4 degrees of freedom instead of the usual 8 (as desired to match the degrees of freedom in x^μ). In fact, any four component spinor that has $P_L\Psi$ related to $\overline{\Psi}\bar{P}_L$ is given the special name of majorana spinor.

The coordinates θ, $\bar{\theta}$ obey the anticommutation relations

$$\{\theta_\alpha, \theta_\beta\} = 0 \qquad \{\bar{\theta}_{\dot{\alpha}}, \theta_\alpha\} = 0 \qquad \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0 \tag{A.66}$$

and as in Section A.3.2,

$$\theta_\alpha = \epsilon_{\alpha\beta}\theta^\beta \qquad \theta^\alpha = \epsilon^{\alpha\beta}\theta_\beta \qquad \bar{\theta}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\beta}} \qquad \bar{\theta}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\theta}_{\dot{\beta}}. \tag{A.67}$$

Once again, the convention for products is

$$\theta^2 = \theta^\alpha\theta_\alpha = \epsilon^{\alpha\beta}\theta_\beta\theta_\alpha \qquad \bar{\theta}^2 = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\beta}}\bar{\theta}^{\dot{\alpha}} \tag{A.68}$$

Differentiation with respect to the fermionic spacetime coordinates is taken as

$$\frac{d}{d\theta_\alpha}\theta_\beta = \delta^\alpha_\beta \qquad \frac{d}{d\bar{\theta}^{\dot{\alpha}}}\bar{\theta}^{\dot{\beta}} = \delta^{\dot{\beta}}_{\dot{\alpha}} \tag{A.69}$$

and integration of a function f is defined by

$$\int d\theta_\alpha \; f \equiv \frac{d}{d\theta^\alpha} f \qquad \int d\bar{\theta}^{\dot{\alpha}} \; f \equiv \frac{d}{d\bar{\theta}_{\dot{\alpha}}} f \tag{A.70}$$

A grassmann volume is also defined as

$$d^4\theta \equiv d^2\theta d^2\bar{\theta} \tag{A.71}$$

where the $d^2\theta$ and $d^2\bar{\theta}$ represent products of the differentials, or

$$d^2\theta \equiv d\theta^\alpha d\theta_\alpha = \epsilon^{\alpha\beta}d\theta_\beta d\theta_\alpha \qquad d^2\bar{\theta} \equiv d\bar{\theta}_{\dot{\alpha}}d\bar{\theta}^{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}d\bar{\theta}^{\dot{\beta}}d\bar{\theta}^{\dot{\alpha}} \tag{A.72}$$

Given the fermionic components θ^α, $\bar{\theta}^{\dot{\alpha}}$, a superspacetime vector, z^A, may be constructed

$$z^A \equiv \left(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}\right). \tag{A.73}$$

The components of z^A may be rotated into each other: a rotation from the basis z to z' is given as

$$z'^A = R^A{}_B z^B \tag{A.74}$$

$$R^A{}_B = \begin{pmatrix} R^\nu{}_\mu & R^\nu{}_\alpha & R^\nu{}_{\dot\alpha} \\ R^\beta{}_\mu & R^\beta{}_\alpha & R^\beta{}_{\dot\alpha} \\ R^{\dot\beta}{}_\mu & R^{\dot\beta}{}_\alpha & R^{\dot\beta}{}_{\dot\alpha} \end{pmatrix} \tag{A.75}$$

and

$$\frac{\partial}{\partial z^A} = \frac{\partial z'^B}{\partial z^A}\frac{\partial}{\partial z'^B} = \left[\frac{\partial}{\partial z^A} R^B{}_C z^C\right]\frac{\partial}{\partial z'^B} \tag{A.76}$$

The standard transformation of superspacetime coordinates from x^μ to y^μ is from the rotation matrix

$$\tilde{R}^A{}_B = \begin{pmatrix} \delta^\nu{}_\mu & \frac{1}{2}\mathbb{i}\sigma^\nu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} & -\frac{1}{2}\mathbb{i}\theta^\alpha\sigma^\nu_{\alpha\dot\alpha} \\ 0 & \delta^\beta{}_\alpha & 0 \\ 0 & 0 & \delta^{\dot\beta}{}_{\dot\alpha} \end{pmatrix} \tag{A.77}$$

yielding the transformations

$$\frac{\partial}{\partial x^\mu} \to \frac{\partial}{\partial y^\mu} \tag{A.78}$$

$$\frac{\partial}{\partial \theta^\alpha} \to \frac{\partial}{\partial \theta^\alpha} - \mathbb{i}\sigma^\nu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\frac{\partial}{\partial y^\nu} \tag{A.79}$$

$$\frac{\partial}{\partial \bar\theta^{\dot\alpha}} \to \frac{\partial}{\partial \bar\theta^{\dot\alpha}} + \mathbb{i}\theta^\alpha\sigma^\nu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\nu} \tag{A.80}$$

A.5.2 Supersymmetric Models

A generic lagrangian involving the chiral superfields Φ_i and based upon the gauge groups G—with corresponding vector superfields $\mathcal{V}_G$ and field strengths $\mathscr{V}_G$—is

$$\mathcal{L} = \frac{1}{2}\int d^4\theta\, \mathcal{K} + \int d^2\theta\, \mathcal{W} + \int d^2\theta\, \sum_G \frac{\tau_G}{8\pi C_G}\,\mathrm{Tr}[(\mathscr{V}_G)^\alpha\,(\mathscr{V}_G)_\alpha] + \text{h.c.} \tag{A.81}$$

where

$$\mathcal{K} = \mathcal{Z}_i{}^j \Phi^i \exp\left(2\mathcal{V}_G^A T_A^G\right)_j{}^k \Phi_k + \cdots \tag{A.82}$$

$$\mathcal{W} = L^i \Phi_i + \frac{1}{2!}\mu^{ij}\Phi_i\Phi_j + \frac{1}{3!}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{4!}\frac{\lambda^{ijk\ell}}{\Lambda}\Phi_i\Phi_j\Phi_k\Phi_\ell + \cdots \tag{A.83}$$

$$\ln \mathcal{Z}_i{}^j = \ln Z_i{}^j + A_i{}^j\theta^2 + \left(A^\dagger\right)_i{}^j\bar{\theta}^2 - \left(m^2\right)_i{}^j\theta^2\bar{\theta}^2 \tag{A.84}$$

$Z_i{}^j$ wavefunction renormalization constant

$$\left(A^\dagger\right)_i{}^j = \left(A_i{}^j\right)^* = A^i{}_j \tag{A.85}$$

$$\Phi^i = \Phi_i^* \tag{A.86}$$

$$\tau_G = \frac{1}{2}\varpi_G - \frac{\mathbb{i}\Theta}{4\pi} - M_G\varpi_G\theta^2 \tag{A.87}$$

$$\varpi_G = \frac{1}{\alpha_G} = \frac{4\pi}{g_G^2} \tag{A.88}$$

g_G the gauge coupling constant for the group G

Θ the vacuum polarization angle

M_G the gaugino mass for the group G

C_G quadratic casimir invariant for the group G

The signs of the SUSY breaking terms are defined by specifying the SUSY breaking potential, which is taken to be

$$V_{\text{SB}} = \frac{1}{2}\left(m^2\right)_i{}^j \underline{\Phi}^i\underline{\Phi}_j + \ell^i\underline{\Phi}_i + \frac{1}{2!}b^{ij}\underline{\Phi}_i\underline{\Phi}_j + \frac{1}{3!}a^{ijk}\underline{\Phi}_i\underline{\Phi}_j\underline{\Phi}_k + \frac{1}{4!}z^{ijk\ell}\underline{\Phi}_i\underline{\Phi}_j\underline{\Phi}_k\underline{\Phi}_\ell + \text{h.c.} \tag{A.89}$$

The anomalous dimensions, $\gamma_i{}^j$; and β-functions, β_L^i, β_μ^{ij}, β_Y^{ijk}, β_{ϖ_G} at a given energy scale μ are

$$16\pi^2\gamma_i{}^j = 16\pi^2\frac{d\ln Z_i{}^j}{d\ln\mu} = 16\pi\sum_G \frac{C^G_{R(\Phi_i)}}{\varpi_G}\delta_i{}^j - Y_{ipq}Y^{jpq} \tag{A.90}$$

$$\beta_L^i = \frac{dL^i}{d\ln\mu} = -\frac{1}{2}L^j\gamma_j{}^i \tag{A.91}$$

$$\beta_\mu^{ij} = \frac{d\mu^{ij}}{d\ln\mu} = -\frac{1}{2}\mu^{ip}\gamma_p{}^j + (i\leftrightarrow j) \tag{A.92}$$

$$\beta_Y^{ijk} = \frac{dY^{ijk}}{d\ln\mu} = -\frac{1}{2}Y^{ijp}\gamma_p{}^k + (i\leftrightarrow k) + (j\leftrightarrow k) \tag{A.93}$$

$$\beta_{\varpi_G} = \frac{d\varpi_G}{d\ln\mu} = -\frac{1}{2\pi}\left[\sum_{\{\Phi_i\}} S^G_{R(\Phi_i)} - 3C_G\right] \tag{A.94}$$

with the last equal sign being valid to one-loop. The value $C^G_{R(\Phi_i)}$ is the quadratic Casimir invariant in the representation R (which is the representation of the field Φ_i) of the group G, while $S^G_{R(\Phi_i)}$ is the dynkin index—the definitions of these objects may be found in Appendix A.4.

Appendix B

SUSYLR+AMSB: Briefing

This appendix provides an overview of the model given in the text, providing a complete picture of the physics starting at the high scale v_R and coming down to the electroweak scale. Schematically, the model is pictured in Figure B.1, starting

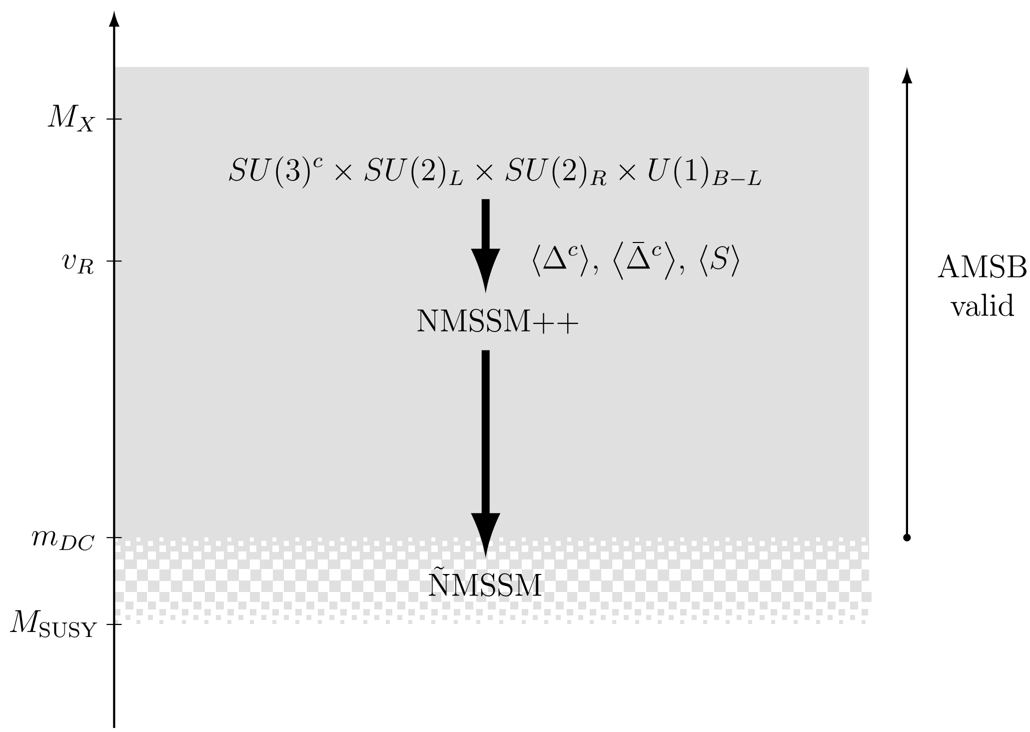


Figure B.1: A schematic of the SUSYLR+AMSB model showing the complete picture through all the energy scales.

above the v_R scale as a parity-conserving SUSYLR model with AMSB generating the SUSY breaking. The theory then breaks down to the NMSSM++ below v_R, maintaining the anomaly mediated supersymmetry breaking through the threshold decoupling. The AMSB form is valid until $m_{DC} \sim F_\phi$, below which the theory is the $\tilde{\text{N}}$MSSM. This remains the theory until M_{SUSY}, where the superpartners are integrated out leaving the standard electroweak model.

B.1 Above v_R

B.1.1 Particles

B.1.1.1 Leptons

$$L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{B.1}$$

$$L_1^c = \begin{pmatrix} e^c \\ -\nu_e^c \end{pmatrix} \qquad L_2^c = \begin{pmatrix} \mu^c \\ -\nu_\mu^c \end{pmatrix} \qquad L_3^c = \begin{pmatrix} \tau^c \\ -\nu_\tau^c \end{pmatrix} \tag{B.2}$$

B.1.1.2 Quarks

$$Q_{1K} = \begin{pmatrix} u_K \\ d_K \end{pmatrix} \qquad Q_{2K} = \begin{pmatrix} c_K \\ s_K \end{pmatrix} \qquad Q_{3K} = \begin{pmatrix} t_K \\ b_K \end{pmatrix} \tag{B.3}$$

$$Q_{1K}^c = \begin{pmatrix} d_K^c \\ -u_K^c \end{pmatrix} \qquad Q_{2K}^c = \begin{pmatrix} s_K^c \\ -c_K^c \end{pmatrix} \qquad Q_{3K}^c = \begin{pmatrix} b_K^c \\ -t_K^c \end{pmatrix} \tag{B.4}$$

K is the color index, running over r,g,b for Q and $\bar{r}$,$\bar{g}$,$\bar{b}$ for Q^c

B.1.1.3 Higgs

$$\Phi_a = \begin{pmatrix} \Phi_{da}^0 & \Phi_{ua}^+ \\ \Phi_{da}^- & \Phi_{ua}^0 \end{pmatrix} \tag{B.5}$$

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \qquad \bar{\Delta} = \begin{pmatrix} \frac{\bar{\Delta}^-}{\sqrt{2}} & \bar{\Delta}^0 \\ \bar{\Delta}^{--} & -\frac{\bar{\Delta}^-}{\sqrt{2}} \end{pmatrix} \tag{B.6}$$

$$\Delta^c = \begin{pmatrix} \frac{\Delta^{c-}}{\sqrt{2}} & \Delta^{c\,0} \\ \Delta^{c--} & -\frac{\Delta^{c-}}{\sqrt{2}} \end{pmatrix} \qquad \bar{\Delta}^c = \begin{pmatrix} \frac{\bar{\Delta}^{c+}}{\sqrt{2}} & \bar{\Delta}^{c++} \\ \bar{\Delta}^{c\,0} & -\frac{\bar{\Delta}^{c+}}{\sqrt{2}} \end{pmatrix} \tag{B.7}$$

B.1.2 Symmetries

B.1.2.1 Gauge Group

Fields	$SU(3)^c$	$\times$ $SU(2)_L$	$\times$ $SU(2)_R$	$\times$ $U(1)_{B-L}$
Q	3	2	1	$+\frac{1}{3}$
Q^c	$\bar{3}$	1	2	$-\frac{1}{3}$
L	1	2	1	-1
L^c	1	1	2	$+1$
Φ_a	1	2	2	0
Δ	1	3	1	$+2$
$\bar{\Delta}$	1	3	1	-2
Δ^c	1	1	3	-2
$\bar{\Delta}^c$	1	1	3	$+2$
S	1	1	1	0
N	1	1	1	0

B.1.2.2 $SU(2)$

$$\begin{array}{llll} Q \to U_L Q & Q^c \to U_R Q^c & L \to U_L L & L^c \to U_R L^c \\ \Delta \to U_L \Delta U_L^\dagger & \bar{\Delta} \to U_L \bar{\Delta} U_L^\dagger & \Delta^c \to U_R \Delta^c U_R^\dagger & \bar{\Delta}^c \to U_R \bar{\Delta}^c U_R^\dagger \\ \Phi_a \to U_L \Phi_a U_R^\dagger & S \to S & N \to N & \end{array} \tag{B.8}$$

B.1.2.3 Parity

$$\begin{array}{lll} Q \leftrightarrow -\mathbb{i}\tau_2\, Q^{c\,*} & L \leftrightarrow -\mathbb{i}\tau_2\, L^{c\,*} & \Phi_a \to \Phi_a^\dagger \\ \Delta \leftrightarrow \Delta^{c\,\dagger} & \bar{\Delta} \leftrightarrow \bar{\Delta}^{c\,\dagger} & S, N \to S^*, N^* \end{array} \tag{B.9}$$

B.1.2.4 Discrete $\mathbb{Z}_3$

$$(Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N) \to \mathbb{e}^{2\mathbb{i}\pi/3}(Q, Q^c, L, L^c, \Delta, \Delta^c, \Phi_a, N) \tag{B.10}$$

$$(\bar{\Delta}, \bar{\Delta}^c) \to \mathbb{e}^{4\mathbb{i}\pi/3}(\bar{\Delta}, \bar{\Delta}^c) \tag{B.11}$$

$$S \to S \tag{B.12}$$

B.1.3 Superpotential

$$W_{\text{SUSYLR}} = W_{\text{Y}} + W_{\text{H}} + W_{\text{GSPNR}} + W_{\text{GSVNR}} \tag{B.13}$$

$$W_{\text{Y}} = \mathbb{i} y_Q^a Q^T \tau_2 \Phi_a Q^c + \mathbb{i} y_L^a L^T \tau_2 \Phi_a L^c + \mathbb{i} f_c L^{cT} \tau_2 \Delta^c L^c + \mathbb{i} f L^T \tau_2 \Delta L \tag{B.14}$$

$$\begin{aligned} W_{\text{H}} &= (M_\Delta \phi - \lambda_S S) \left[\operatorname{Tr}\big(\Delta^c \bar{\Delta}^c\big) + \operatorname{Tr}\big(\Delta \bar{\Delta}\big) \right] + M_S^2 \phi^2 S + \frac{1}{2} \mu_S \phi S^2 + \frac{1}{3} \kappa_S S^3 \\ &\quad + \lambda_N^{ab} N \operatorname{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) + \frac{1}{3} \kappa_N N^3 \end{aligned} \tag{B.15}$$

$$\begin{aligned} W_{\text{GSPNR}} &= \frac{\lambda_A}{M_X \phi} \operatorname{Tr}^2\big(\Delta \bar{\Delta}\big) + \frac{\lambda_A^c}{M_X \phi} \operatorname{Tr}^2\big(\Delta^c \bar{\Delta}^c\big) \\ &\quad + \frac{\lambda_B}{M_X \phi} \operatorname{Tr}(\Delta \Delta) \operatorname{Tr}\big(\bar{\Delta} \bar{\Delta}\big) + \frac{\lambda_B^c}{M_X \phi} \operatorname{Tr}(\Delta^c \Delta^c) \operatorname{Tr}\big(\bar{\Delta}^c \bar{\Delta}^c\big) \\ &\quad + \frac{\lambda_C}{M_X \phi} \operatorname{Tr}\big(\Delta \bar{\Delta}\big) \operatorname{Tr}\big(\Delta^c \bar{\Delta}^c\big) \\ &\quad + \frac{\lambda_{\mathbf{S}}}{M_X \phi} \operatorname{Tr}\big(\Delta \bar{\Delta}\big) S^2 + \frac{\lambda_{\mathbf{S}}^c}{M_X \phi} \operatorname{Tr}\big(\Delta^c \bar{\Delta}^c\big) S^2 + \cdots \end{aligned} \tag{B.16}$$

$$\begin{aligned} W_{\text{GSVNR}} &= \frac{\lambda_D}{M_{\text{Pl}} \phi} \operatorname{Tr}(\Delta \Delta) \operatorname{Tr}(\Delta^c \Delta^c) + \frac{\bar{\lambda}_D}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\bar{\Delta} \bar{\Delta}\big) \operatorname{Tr}\big(\bar{\Delta}^c \bar{\Delta}^c\big) \\ &\quad + \frac{(\lambda_\sigma)^{ab}}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\Delta \bar{\Delta}\big) \operatorname{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) + \frac{(\lambda_\sigma^c)^{ab}}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\Delta^c \bar{\Delta}^c\big) \operatorname{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) \\ &\quad + \frac{2 \lambda_\alpha \epsilon^{ab}}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\Delta \Phi_a \tau_2 \Phi_b^T \tau_2 \bar{\Delta}\big) + \frac{2 \lambda_\alpha^c \epsilon^{ab}}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\Delta^c \tau_2 \Phi_a^T \tau_2 \Phi_b \bar{\Delta}^c\big) \\ &\quad + \frac{\lambda_{\mathbf{N}}}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\Delta \bar{\Delta}\big) N^2 + \frac{\lambda_{\mathbf{N}}^c}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\Delta^c \bar{\Delta}^c\big) N^2 \\ &\quad + \frac{\lambda_{\mathbf{s}}}{M_{\text{Pl}} \phi} \operatorname{Tr}\big(\Phi_a^T \tau_2 \Phi_b \tau_2\big) S^2 + \frac{\lambda_{\text{M}}}{M_{\text{Pl}} \phi} S^2 N^2 + \cdots \end{aligned} \tag{B.17}$$

B.1.4 Potential

$$V = V_F + V_D + V_{\rm SB} + V_{\rm SBNR} \tag{B.18}$$

$$\begin{aligned} V_F = {} & |M_\Delta - \lambda_S \underline{S}|^2 \,\mathrm{Tr}\Big[\big|\underline{\bar{\Delta}}^c\big|^2 + |\underline{\Delta}^c|^2 + \big|\underline{\bar{\Delta}}\big|^2 + |\underline{\Delta}|^2\Big] \\ & + \Big|-\lambda_S\big[\mathrm{Tr}\big(\underline{\Delta}^c\underline{\bar{\Delta}}^c\big) + \mathrm{Tr}\big(\underline{\Delta}\underline{\bar{\Delta}}\big)\big] + M_S^2 + \mu_S\underline{S} + \kappa_S\underline{S}^2\Big|^2 \end{aligned} \tag{B.19}$$

$$V_D = \frac{1}{2}\sum_G D_R^G D_R^G + \sum_G D_L^G D_L^G + D_{BL}^2 \tag{B.20}$$

$$\begin{aligned} V_{\rm SB} = {} & \frac{1}{2}\Big(M_3 \mathrm{g}_A^\alpha \mathrm{g}_\alpha^A + M_L (\mathrm{w}_L)_A^\alpha\,(\mathrm{w}_L)_\alpha^A + M_R (\mathrm{w}_R)_A^\alpha\,(\mathrm{w}_R)_\alpha^A + M_{BL}\mathrm{b}^2 + \mathrm{h.c.}\Big) \\ & + m_Q^2 \underline{Q}^\dagger\underline{Q} + m_{Q^c}^2\,\underline{Q}^{c\,\dagger}\,\underline{Q}^c + m_L^2\underline{L}^\dagger\underline{L} + m_{L^c}^2\,\underline{L}^{c\,\dagger}\,\underline{L}^c \\ & + m_\Delta^2\,\mathrm{Tr}|\underline{\Delta}|^2 + m_{\bar{\Delta}}^2\,\mathrm{Tr}\big|\underline{\bar{\Delta}}\big|^2 + m_N^2|\underline{N}|^2 + \big(m_\Phi^2\big)_{ab}\,\underline{\Phi}_a^\dagger\underline{\Phi}_b \\ & + m_{\Delta^c}^2\,\mathrm{Tr}|\underline{\Delta}^c|^2 + m_{\bar{\Delta}^c}^2\,\mathrm{Tr}\big|\underline{\bar{\Delta}}^c\big|^2 + m_S^2|\underline{S}|^2 \\ & + \Big[-2M_S^2F_\phi\underline{S} - M_\Delta F_\phi\big[\mathrm{Tr}\big(\underline{\Delta}^c\underline{\bar{\Delta}}^c\big) + \mathrm{Tr}\big(\underline{\Delta}\underline{\bar{\Delta}}\big)\big] - \frac{1}{2}\mu_S F_\phi\underline{S}^2 + \mathrm{h.c.}\Big] \\ & + \Big[a_Q^b\underline{Q}\underline{\Phi}_b\underline{Q}^c + a_L^b\underline{L}\underline{\Phi}_b\underline{L}^c + a_{f_c}\underline{L}^c\underline{\Delta}^c\underline{L}^c + a_f\underline{L}\underline{\Delta}\underline{L} \\ & \qquad + \frac{1}{3}a_{\kappa_S}\underline{S}^3 + a_{\lambda_N}^{ab}\,\underline{N}\,\mathrm{Tr}\big(\underline{\Phi}_a^T\tau_2\underline{\Phi}_b\tau_2\big) + \frac{1}{3}a_{\kappa_N}\underline{N}^3\Big] \end{aligned} \tag{B.21}$$

$$\begin{aligned} V_{\rm SBNR} = {} & \Big[\frac{\lambda_A F_\phi}{M_X}\,\mathrm{Tr}^2\big(\underline{\Delta}\underline{\bar{\Delta}}\big) + \frac{\lambda_A^c F_\phi}{M_X}\,\mathrm{Tr}^2\big(\underline{\Delta}^c\underline{\bar{\Delta}}^c\big) \\ & + \frac{\lambda_B F_\phi}{M_X}\,\mathrm{Tr}(\underline{\Delta}\underline{\Delta})\,\mathrm{Tr}\big(\underline{\bar{\Delta}}\underline{\bar{\Delta}}\big) + \frac{\lambda_B^c F_\phi}{M_X}\,\mathrm{Tr}(\underline{\Delta}^c\underline{\Delta}^c)\,\mathrm{Tr}\big(\underline{\bar{\Delta}}^c\underline{\bar{\Delta}}^c\big) \\ & + \frac{\lambda_C F_\phi}{M_X}\,\mathrm{Tr}\big(\underline{\Delta}\underline{\bar{\Delta}}\big)\,\mathrm{Tr}\big(\underline{\Delta}^c\underline{\bar{\Delta}}^c\big) \\ & + \frac{\lambda_{\mathbf{S}} F_\phi}{M_X}\,\mathrm{Tr}\big(\underline{\Delta}\underline{\bar{\Delta}}\big)\,\underline{S}^2 + \frac{\lambda_{\mathbf{S}}^c F_\phi}{M_X}\,\mathrm{Tr}\big(\underline{\Delta}^c\underline{\bar{\Delta}}^c\big)\,\underline{S}^2 \\ & + \frac{\lambda_D F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}(\underline{\Delta}\underline{\Delta})\,\mathrm{Tr}(\underline{\Delta}^c\underline{\Delta}^c) + \frac{\bar{\lambda}_D F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\bar{\Delta}}\underline{\bar{\Delta}}\big)\,\mathrm{Tr}\big(\underline{\bar{\Delta}}^c\underline{\bar{\Delta}}^c\big) \\ & + \frac{(\lambda_\sigma)^{ab}\,F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\Delta}\underline{\bar{\Delta}}\big)\,\mathrm{Tr}\big(\underline{\Phi}_a^T\tau_2\underline{\Phi}_b\tau_2\big) + \frac{(\lambda_\sigma^c)^{ab}\,F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\Delta}^c\underline{\bar{\Delta}}^c\big)\,\mathrm{Tr}\big(\underline{\Phi}_a^T\tau_2\underline{\Phi}_b\tau_2\big) \\ & + \frac{2\lambda_\alpha\epsilon^{ab}F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\Delta}\underline{\Phi}_a\tau_2\underline{\Phi}_b^T\tau_2\underline{\bar{\Delta}}\big) + \frac{2\lambda_\alpha^c\epsilon^{ab}F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\Delta}^c\tau_2\underline{\Phi}_a^T\tau_2\underline{\Phi}_b\underline{\bar{\Delta}}^c\big) \\ & + \frac{\lambda_{\mathbf{N}} F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\Delta}\underline{\bar{\Delta}}\big)\,\underline{N}^2 + \frac{\lambda_{\mathbf{N}}^c F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\Delta}^c\underline{\bar{\Delta}}^c\big)\,\underline{N}^2 \\ & + \frac{\lambda_{\mathbf{s}} F_\phi}{M_{\rm Pl}}\,\mathrm{Tr}\big(\underline{\Phi}_a^T\tau_2\underline{\Phi}_b\tau_2\big)\,\underline{S}^2 + \frac{\lambda_{\rm M} F_\phi}{M_{\rm Pl}}\underline{S}^2\,\underline{N}^2 + \mathrm{h.c.}\Big] \end{aligned} \tag{B.22}$$

B.1.5 F-terms

$$
\begin{aligned}
-F^{\dagger}_{\Delta^c} = {} & \frac{1}{2} f_c \big[\tau_1 \big(\underline{L}^{cT}\, \tau_3 \underline{L}^c - \tau_3\, \underline{L}^{cT}\, \underline{L}^c\big) + \mathbb{i}\tau_3\, \underline{L}^{cT}\, \tau_2 (1+\tau_3)\, \underline{L}^c - \mathbb{i}\, \underline{L}^{cT}\, \tau_2 \underline{L}^c\big] \\
& + (M_\Delta - \lambda_S \underline{S})\, \underline{\bar{\Delta}}^c + \frac{2\lambda^c_A}{M_X} \operatorname{Tr}\big(\underline{\Delta}^c \underline{\bar{\Delta}}^c\big)\, \underline{\bar{\Delta}}^c + \frac{2\lambda^c_B}{M_X} \operatorname{Tr}\big(\underline{\bar{\Delta}}^c \underline{\bar{\Delta}}^c\big)\, \underline{\Delta}^c \\
& + \frac{\lambda_C}{M_X} \operatorname{Tr}\big(\underline{\Delta\bar{\Delta}}\big)\, \underline{\bar{\Delta}}^c + \frac{\lambda^c_{\mathbf{S}}}{M_X} \underline{\bar{\Delta}}^c \underline{S}^2 + \frac{2\lambda_D}{M_{\mathrm{Pl}}} \operatorname{Tr}(\underline{\Delta\Delta})\, \underline{\Delta}^c + \frac{\lambda^c_{\mathbf{N}}}{M_{\mathrm{Pl}}} \underline{\bar{\Delta}}^c \underline{N}^2 \\
& + \frac{(\lambda^c_\sigma)^{ab}}{M_{\mathrm{Pl}}} \underline{\bar{\Delta}}^c \operatorname{Tr}\big(\underline{\Phi}^T_a \tau_2 \underline{\Phi}_b \tau_2\big) + \frac{2\lambda^c_\alpha \epsilon^{ab}}{M_{\mathrm{Pl}}} \left[\tau_2 \underline{\Phi}^T_a \tau_2 \underline{\Phi}_b \underline{\bar{\Delta}}^c - \frac{1}{2} \operatorname{Tr}\big(\tau_2 \underline{\Phi}^T_a \tau_2 \underline{\Phi}_b \underline{\bar{\Delta}}^c\big)\right]
\end{aligned} \tag{B.23}
$$

$$
\begin{aligned}
-F^{\dagger}_{\bar{\Delta}^c} = {} & (M_\Delta - \lambda_S \underline{S})\, \underline{\Delta}^c + \frac{2\lambda^c_A}{M_X} \operatorname{Tr}\big(\underline{\Delta}^c \underline{\bar{\Delta}}^c\big)\, \underline{\Delta}^c + \frac{2\lambda^c_B}{M_X} \operatorname{Tr}(\underline{\Delta}^c \underline{\Delta}^c)\, \underline{\bar{\Delta}}^c \\
& + \frac{\lambda_C}{M_X} \operatorname{Tr}\big(\underline{\Delta\bar{\Delta}}\big)\, \underline{\Delta}^c + \frac{\lambda^c_{\mathbf{S}}}{M_X} \underline{\Delta}^c \underline{S}^2 + \frac{2\bar{\lambda}_D}{M_{\mathrm{Pl}}} \operatorname{Tr}\big(\underline{\bar{\Delta}\bar{\Delta}}\big)\, \underline{\bar{\Delta}}^c + \frac{\lambda^c_{\mathbf{N}}}{M_{\mathrm{Pl}}} \underline{\Delta}^c \underline{N}^2 \\
& + \frac{(\lambda^c_\sigma)^{ab}}{M_{\mathrm{Pl}}} \underline{\Delta}^c \operatorname{Tr}\big(\underline{\Phi}^T_a \tau_2 \underline{\Phi}_b \tau_2\big) + \frac{2\lambda^c_\alpha \epsilon^{ab}}{M_{\mathrm{Pl}}} \left[\underline{\Delta}^c \tau_2 \underline{\Phi}^T_a \tau_2 \underline{\Phi}_b - \frac{1}{2} \operatorname{Tr}\big(\underline{\Delta}^c \tau_2 \underline{\Phi}^T_a \tau_2 \underline{\Phi}_b\big)\right]
\end{aligned} \tag{B.24}
$$

$$
\begin{aligned}
-F^{\dagger}_{\Delta} = {} & \frac{1}{2} f \big[\tau_1 \big(\underline{L}^T \tau_3 \underline{L} - \tau_3 \underline{L}^T \underline{L}\big) + \mathbb{i}\tau_3 \underline{L}^T \tau_2 (1+\tau_3)\, \underline{L} - \mathbb{i}\underline{L}^T \tau_2 \underline{L}\big] \\
& + (M_\Delta - \lambda_S \underline{S})\, \underline{\bar{\Delta}} + \frac{2\lambda_A}{M_X} \operatorname{Tr}\big(\underline{\Delta\bar{\Delta}}\big)\, \underline{\bar{\Delta}} + \frac{2\lambda_B}{M_X} \operatorname{Tr}\big(\underline{\bar{\Delta}\bar{\Delta}}\big)\, \underline{\Delta} \\
& + \frac{\lambda_C}{M_X} \operatorname{Tr}\big(\underline{\Delta}^c \underline{\bar{\Delta}}^c\big)\, \underline{\bar{\Delta}} + \frac{\lambda_{\mathbf{S}}}{M_X} \underline{\bar{\Delta} S}^2 + \frac{2\lambda_D}{M_{\mathrm{Pl}}} \operatorname{Tr}(\underline{\Delta}^c \underline{\Delta}^c)\, \underline{\Delta} + \frac{\lambda_{\mathbf{N}}}{M_{\mathrm{Pl}}} \underline{\bar{\Delta} N}^2 \\
& + \frac{(\lambda_\sigma)^{ab}}{M_{\mathrm{Pl}}} \underline{\bar{\Delta}} \operatorname{Tr}\big(\underline{\Phi}^T_a \tau_2 \underline{\Phi}_b \tau_2\big) + \frac{2\lambda_\alpha \epsilon^{ab}}{M_{\mathrm{Pl}}} \left[\underline{\Phi}_a \tau_2 \underline{\Phi}^T_b \tau_2 \underline{\bar{\Delta}} - \frac{1}{2} \operatorname{Tr}\big(\underline{\Phi}_a \tau_2 \underline{\Phi}^T_b \tau_2 \underline{\bar{\Delta}}\big)\right]
\end{aligned} \tag{B.25}
$$

$$
\begin{aligned}
-F^{\dagger}_{\bar{\Delta}} = {} & (M_\Delta - \lambda_S \underline{S})\, \underline{\Delta} + \frac{2\lambda_A}{M_X} \operatorname{Tr}\big(\underline{\Delta\bar{\Delta}}\big)\, \underline{\Delta} + \frac{2\lambda_B}{M_X} \operatorname{Tr}(\underline{\Delta\Delta})\, \underline{\bar{\Delta}} \\
& + \frac{\lambda_C}{M_X} \operatorname{Tr}\big(\underline{\Delta}^c \underline{\bar{\Delta}}^c\big)\, \underline{\Delta} + \frac{\lambda_{\mathbf{S}}}{M_X} \underline{\Delta S}^2 + \frac{2\bar{\lambda}_D}{M_{\mathrm{Pl}}} \operatorname{Tr}\big(\underline{\bar{\Delta}}^c \underline{\bar{\Delta}}^c\big)\, \underline{\bar{\Delta}} + \frac{\lambda_{\mathbf{N}}}{M_{\mathrm{Pl}}} \underline{\Delta N}^2 \\
& + \frac{(\lambda_\sigma)^{ab}}{M_{\mathrm{Pl}}} \underline{\Delta} \operatorname{Tr}\big(\underline{\Phi}^T_a \tau_2 \underline{\Phi}_b \tau_2\big) + \frac{2\lambda_\alpha \epsilon^{ab}}{M_{\mathrm{Pl}}} \left[\underline{\Delta\Phi}_a \tau_2 \underline{\Phi}^T_b \tau_2 - \frac{1}{2} \operatorname{Tr}\big(\underline{\Delta\Phi}_a \tau_2 \underline{\Phi}^T_b \tau_2\big)\right]
\end{aligned} \tag{B.26}
$$

$$
\begin{aligned}
-F^*_S = {} & -\lambda_S \big[\operatorname{Tr}\big(\underline{\Delta}^c \underline{\bar{\Delta}}^c\big) + \operatorname{Tr}\big(\underline{\Delta\bar{\Delta}}\big)\big] + M^2_S + \mu_S \underline{S} + \kappa_S \underline{S}^2 \\
& + \frac{2\lambda_{\mathbf{S}}}{M_X} \operatorname{Tr}\big(\underline{\Delta\bar{\Delta}}\big)\, \underline{S} + \frac{2\lambda^c_{\mathbf{S}}}{M_X} \operatorname{Tr}\big(\underline{\Delta}^c \underline{\bar{\Delta}}^c\big)\, \underline{S} + \frac{2\lambda_{\mathbf{s}}}{M_{\mathrm{Pl}}} \operatorname{Tr}\big(\underline{\Phi}^T_a \tau_2 \underline{\Phi}_b \tau_2\big)\, \underline{S} + \frac{2\lambda_{\mathrm{M}}}{M_{\mathrm{Pl}}} \underline{SN}^2
\end{aligned} \tag{B.27}
$$

B.1.6 D-terms

$$D_L^G = -\frac{1}{2} g_L \,\mathrm{Tr}\Big(2\underline{\Delta}^{c\,\dagger}\,\tau^G \underline{\Delta}^c + 2\,\underline{\bar{\Delta}}^{c\,\dagger}\,\tau^G \underline{\bar{\Delta}}^c - \underline{\Phi}_a \tau^G \underline{\Phi}_a^\dagger \Big) \tag{B.28}$$

$$D_R^G = -\frac{1}{2} g_R \,\mathrm{Tr}\Big(2\underline{\Delta}^\dagger \tau^G \underline{\Delta} + 2\underline{\bar{\Delta}}^\dagger \tau^G \underline{\bar{\Delta}} + \underline{\Phi}_a^\dagger \tau^G \underline{\Phi}_a \Big) \tag{B.29}$$

$$D_{BL} = -g_{BL} \,\mathrm{Tr}\Big(\underline{\Delta}^{c\,\dagger}\, \underline{\Delta}^c - \underline{\bar{\Delta}}^{c\,\dagger}\, \underline{\bar{\Delta}}^c \Big) \tag{B.30}$$

B.2 NMSSM++: $v_R \longrightarrow F_\phi$

B.2.1 Particles

B.2.1.1 Leptons

$$L_1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad L_2 = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad L_3 = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{B.31}$$

B.2.1.2 Quarks

$$Q_{1K} = \begin{pmatrix} u_K \\ d_K \end{pmatrix} \qquad Q_{2K} = \begin{pmatrix} c_K \\ s_K \end{pmatrix} \qquad Q_{3K} = \begin{pmatrix} t_K \\ b_K \end{pmatrix} \tag{B.32}$$

K is the color index, running over r,g,b

B.2.1.3 Higgs

$$H_{u_a} = \begin{pmatrix} H_{ua}^+ \\ H_{ua}^0 \end{pmatrix} \qquad H_{d_a} = \begin{pmatrix} H_{da}^0 \\ H_{da}^- \end{pmatrix} \tag{B.33}$$

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \qquad \bar{\Delta} = \begin{pmatrix} \frac{\bar{\Delta}^-}{\sqrt{2}} & \bar{\Delta}^0 \\ \bar{\Delta}^{--} & -\frac{\bar{\Delta}^-}{\sqrt{2}} \end{pmatrix} \tag{B.34}$$

B.2.2 Symmetries

B.2.2.1 Gauge Group

Fields	$SU(3)^c$	$\times$	$SU(2)_L$	$\times$	$U(1)_Y$
Q	3		2		$+\frac{1}{3}$
u^c	$\bar{3}$		1		$-\frac{4}{3}$
d^c	$\bar{3}$		1		$+\frac{2}{3}$
L	1		2		-1
e^c	1		1		$+2$
H_{ua}	1		2		$+1$
H_{da}	1		2		-1
Δ	1		3		$+2$
$\bar{\Delta}$	1		3		-2
Δ^{c--}	1		1		-4
$\bar{\Delta}^{c++}$	1		1		$+4$
N	1		1		0

B.2.3 Superpotential

$$\begin{aligned} W_{\text{NMSSM++}} = &\, \mathbb{i} y_u^a Q^T \tau_2 H_{ua} u^c + \mathbb{i} y_d^a Q^T \tau_2 H_{da} d^c + \mathbb{i} y_L^a L^T \tau_2 H_{da} e^c \\ &+ f_c e^c \Delta^{c--} e^c + \mathbb{i} f L^T \tau_2 \Delta L \\ &+ \mathbb{i} \lambda^{ab} N H_{ua}^T \tau_2 H_{db} + \mathbb{i} \mu^{ab} \phi H_{ua}^T \tau_2 H_{db} + \frac{1}{2} \mu_N \phi N^2 + \frac{1}{3} \kappa N^3 \\ &+ \mu_{DC} \phi \Delta^{c--} \bar{\Delta}^{c++} + \mu_\Delta \phi \operatorname{Tr}\big(\Delta \bar{\Delta}\big) \end{aligned} \tag{B.35}$$

B.2.4 F-terms

$$-F_\Delta^\dagger = \frac{1}{2} f \big[\tau_1 \big(\underline{L}^T \tau_3 \underline{L} - \tau_3 \underline{L}^T \underline{L}\big) + \mathbb{i} \tau_3 \underline{L}^T \tau_2 (1 + \tau_3)\, \underline{L} - \mathbb{i} \underline{L}^T \tau_2 \underline{L}\big] + \mu_\Delta \underline{\bar{\Delta}} \tag{B.36}$$

$$-F_{\bar{\Delta}}^\dagger = \mu_\Delta \underline{\bar{\Delta}} \tag{B.37}$$

$$-F_{H_{ua}}^\dagger = \mathbb{i} y_u^a \underline{Q}^T \tau_2 \underline{u}^c - \mathbb{i} \lambda^{ab} \underline{N H}_{db}^T \tau_2 - \mathbb{i} \mu^{ab} \underline{H}_{db}^T \tau_2 \tag{B.38}$$

$$-F_{H_{da}}^\dagger = \mathbb{i} y_d^a \underline{Q}^T \tau_2 \underline{d}^c + \mathbb{i} y_L^a \underline{L}^T \tau_2 \underline{e}^c + \mathbb{i} \lambda^{ab} \underline{N H}_{ua}^T \tau_2 + \mathbb{i} \mu^{ab} \underline{H}_{ua}^T \tau_2 \tag{B.39}$$

$$-F_N^* = \mathbb{i} \lambda^{ab} \underline{H}_{ua}^T \tau_2 \underline{H}_{db} + \mu_N \underline{N} + \kappa \underline{N}^2 \tag{B.40}$$

B.2.5 D-terms

$$D_L^G = -\frac{1}{2} g_L \left[\underline{H}_{ua}^\dagger \tau^G \underline{H}_{ua} + \underline{H}_{da}^\dagger \tau^G \underline{H}_{da} \right] \tag{B.41}$$

$$D_Y = -\frac{1}{2} g_Y \left[\underline{H}_{ua}^\dagger \underline{H}_{ua} - \underline{H}_{da}^\dagger \underline{H}_{da} \right] \tag{B.42}$$

B.2.6 Anomalous Dimensions

B.2.6.1 Leptons

$$\gamma_{L_3} = -\frac{1}{8\pi^2} \left(y_\tau^{a*} y_\tau^a + 6|f_3|^2 - \frac{3}{2} g_L^2 - \frac{1}{2} g_Y^2 \right) \tag{B.43}$$

$$\gamma_{L_1} = -\frac{1}{8\pi^2} \left(6|f_1|^2 - \frac{3}{2} g_L^2 - \frac{1}{2} g_Y^2 \right) \tag{B.44}$$

$$\gamma_{\tau^c} = -\frac{1}{8\pi^2} \left(2 y_\tau^{a*} y_\tau^a + 4|f_{c3}|^2 - 2 g_Y^2 \right) \tag{B.45}$$

$$\gamma_{e^c} = -\frac{1}{8\pi^2} \left(4|f_{c1}|^2 - 2 g_Y^2 \right) \tag{B.46}$$

B.2.6.2 Quarks

$$\gamma_{Q_3} = -\frac{1}{8\pi^2} \left(y_t^{a*} y_t^a + y_b^{a*} y_b^a - \frac{8}{3} g_3^2 - \frac{3}{2} g_L^2 - \frac{1}{18} g_Y^2 \right) \tag{B.47}$$

$$\gamma_{Q_1} = -\frac{1}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{3}{2} g_L^2 - \frac{1}{18} g_Y^2 \right) \tag{B.48}$$

$$\gamma_{t^c} = -\frac{1}{8\pi^2} \left(2 y_t^{a*} y_t^a - \frac{8}{3} g_3^2 - \frac{8}{9} g_Y^2 \right) \tag{B.49}$$

$$\gamma_{u^c} = -\frac{1}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{8}{9} g_Y^2 \right) \tag{B.50}$$

$$\gamma_{b^c} = -\frac{1}{8\pi^2} \left(2 y_b^{a*} y_b^a - \frac{8}{3} g_3^2 - \frac{2}{9} g_Y^2 \right) \tag{B.51}$$

$$\gamma_{d^c} = -\frac{1}{8\pi^2} \left(-\frac{8}{3} g_3^2 - \frac{2}{9} g_Y^2 \right) \tag{B.52}$$

B.2.6.3 Higgs

$$\gamma_N = -\frac{1}{8\pi^2}\big(2|\kappa|^2 + 2\lambda^{ab*}\lambda^{ab}\big) \tag{B.53}$$

$$\gamma_{H_u}^{ab} = -\frac{1}{8\pi^2}\left(3y_t^{a*}y_t^b + \lambda^{ac}\lambda^{bc} - \delta^{ab}\left(\frac{3}{2}g_L^2 + \frac{1}{2}g_Y^2\right)\right) \tag{B.54}$$

$$\gamma_{H_d}^{ab} = -\frac{1}{8\pi^2}\left(3y_b^{a*}y_b^b + y_\tau^{a*}y_\tau^b + \lambda^{ca*}\lambda^{cb} - \delta^{ab}\left(\frac{3}{2}g_L^2 + \frac{1}{2}g_Y^2\right)\right) \tag{B.55}$$

$$\gamma_\Delta = -\frac{1}{8\pi^2}\big(2|f_3|^2 + 2|f_2|^2 + 2|f_1|^2 - 4g_L^2 - 2g_Y^2\big) \tag{B.56}$$

$$\gamma_{\bar{\Delta}} = -\frac{1}{8\pi^2}\big(-4g_L^2 - 2g_Y^2\big) \tag{B.57}$$

$$\gamma_{\Delta^{c--}} = -\frac{1}{8\pi^2}\big(2|f_c3|^2 + 2|f_c2|^2 + 2|f_c1|^2 - 8g_Y^2\big) \tag{B.58}$$

$$\gamma_{\bar{\Delta}^{c--}} = -\frac{1}{8\pi^2}\big(-8g_Y^2\big) \tag{B.59}$$

B.2.7 AMSB Scalar Masses

B.2.7.1 Leptons

$$m_{e^c}^2 = m_{\text{an}}^2\big[40f_{c1}^4 + 8f_{c1}^2\big(f_{c2}^2 + f_{c3}^2\big) - 48f_{c1}^2g_Y^2 - 52g_Y^4\big] \tag{B.60}$$

$$m_{L_1}^2 = m_{\text{an}}^2\big[84f_1^4 + 12f_1^2\big(f_2^2 + f_3^2\big) - 6f_1^2\big(3g_Y^2 + 7g_L^2\big) - 13g_Y^4 - 9g_L^4\big] \tag{B.61}$$

$$m_{\mu^c}^2 = m_{\text{an}}^2\big[40f_{c2}^4 + 8f_{c2}^2\big(f_{c1}^2 + f_{c3}^2\big) - 48f_{c2}^2g_Y^2 - 52g_Y^4\big] \tag{B.62}$$

$$m_{L_2}^2 = m_{\text{an}}^2\big[84f_2^4 + 12f_2^2\big(f_1^2 + f_3^2\big) - 6f_2^2\big(3g_Y^2 + 7g_L^2\big) - 13g_Y^4 - 9g_L^4\big] \tag{B.63}$$

$$\begin{aligned} m_{\tau^c}^2 = m_{\text{an}}^2\,\big[&40f_{c3}^4 + 10(y_\tau^a y_\tau^a)^2 + 8f_{c3}^2\big(f_{c1}^2 + f_{c2}^2\big) + 6(y_\tau^a y_b^a)^2 + 12y_\tau^a y_\tau^a\big(2f_{c3}^2 + f_3^2\big) \\ &+ 8\lambda^{nm}y_\tau^m\lambda^{np}y_\tau^p - 48f_{c3}^2g_Y^2 - 6y_\tau^a y_\tau^a\big(g_Y^2 + g_L^2\big) - 52g_Y^4\big] \end{aligned} \tag{B.64}$$

$$\begin{aligned} m_{L_3}^2 = m_{\text{an}}^2\,\big[&84f_3^4 + 5(y_\tau^a y_\tau^a)^2 + 3(y_\tau^a y_b^a)^2 + 12f_3^2\big(f_1^2 + f_2^2\big) + 2y_\tau^a y_\tau^a\big(9f_3^2 + 2f_{c3}^2\big) \\ &+ \lambda^{nm}y_\tau^m\lambda^{np}y_\tau^p - 6f_3^2\big(3g_Y^2 + 7g_L^2\big) - 3y_\tau^a y_\tau^a\big(g_Y^2 + g_L^2\big) - 13g_Y^4 - 9g_L^4\big] \end{aligned} \tag{B.65}$$

B.2.7.2 Quarks

$$\begin{aligned}
m_{Q_3}^2 = \frac{1}{9}|m_{\rm an}|^2 \big[&54(y_t^m y_t^m)^2 + 54(y_b^m y_b^m)^2 + 9y_b^m y_b^m(2y_t^n y_t^n + y_\tau^n y_\tau^n) + 9(y_b^m y_\tau^m)^2 \\
&+ 9\lambda^{nm} y_b^m \lambda^{np} y_b^p + 9y_t^m \lambda^{mp} y_t^n \lambda^{np} - y_t^m y_t^m\big(13g_Y^2 + 27g_L^2 + 24g_3^2\big) \\
&- y_b^m y_b^m\big(7g_Y^2 + 27g_L^2 + 48g_3^2\big) - 13g_Y^4 - 81g_L^4 + 72g_3^4\big] \qquad \text{(B.66)}\\
m_{t^c}^2 = \frac{1}{9}|m_{\rm an}|^2 \big[&108(y_t^m y_t^m)^2 + 18y_t^m y_t^m y_b^n y_b^n + 18y_t^m \lambda^{mp} y_t^n \lambda^{np} \\
&- 2y_t^m y_t^m\big(13g_Y^2 + 27g_L^2 + 24g_3^2\big) - 208g_Y^4 + 72g_3^4\big] \qquad \text{(B.67)}\\
m_{b^c}^2 = \frac{1}{9}|m_{\rm an}|^2 \big[&108(y_b^m y_b^m)^2 + 18(y_b^m y_\tau^m)^2 + 18y_b^m y_b^m(y_t^n y_t^n + y_\tau^n y_\tau^n) \\
&+ 18\lambda^{nm} y_b^m \lambda^{np} y_b^p - 2y_b^m y_b^m\big(7g_Y^2 + 27g_L^2 + 48g_3^2\big) - 52g_Y^4 + 72g_3^4\big] \qquad \text{(B.68)}
\end{aligned}$$

B.3 $\tilde{\rm N}$MSSM: $F_\phi \to M_{\rm SUSY}$

B.3.1 Particles

B.3.1.1 Leptons

$$L_1 = \begin{pmatrix}\nu_e \\ e\end{pmatrix} \qquad L_2 = \begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix} \qquad L_3 = \begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix} \qquad \text{(B.69)}$$

B.3.1.2 Quarks

$$Q_{1K} = \begin{pmatrix}u_K \\ d_K\end{pmatrix} \qquad Q_{2K} = \begin{pmatrix}c_K \\ s_K\end{pmatrix} \qquad Q_{3K} = \begin{pmatrix}t_K \\ b_K\end{pmatrix} \qquad \text{(B.70)}$$

K is the color index, running over r,g,b

B.3.1.3 Higgs

$$H_u = \begin{pmatrix}H_u^+ \\ H_u^0\end{pmatrix} \qquad H_d = \begin{pmatrix}H_d^0 \\ H_d^-\end{pmatrix} \qquad \text{(B.71)}$$

B.3.2 Symmetries

B.3.2.1 Gauge Group

Fields	$SU(3)^c$	$\times$	$SU(2)_L$	$\times$	$U(1)_Y$
Q	3		2		$+\frac{1}{3}$
u^c	$\bar{3}$		1		$-\frac{4}{3}$
d^c	$\bar{3}$		1		$+\frac{2}{3}$
L	1		2		-1
e^c	1		1		$+2$
H_u	1		2		$+1$
H_d	1		2		-1
N	1		1		0

B.3.3 Superpotential

$$W_{\tilde{\text{N}}\text{MSSM}} = \mathbb{i} y_u Q^T \tau_2 H_u u^c + \mathbb{i} y_d Q^T \tau_2 H_d d^c + \mathbb{i} y_e L^T \tau_2 H_d e^c + \mathbb{i}\lambda N H_u^T \tau_2 H_d + \frac{1}{2}\mu_N N^2 + \frac{1}{3}\kappa N^3 \tag{B.72}$$

B.3.4 Potential

$$V_{\tilde{\text{N}}\text{MSSM}} = V_F + V_D + V_{\text{SB}} \tag{B.73}$$

$$V_F = |\lambda|^2 |\underline{N}|^2 \left(|\underline{H}_u|^2 + |\underline{H}_d|^2\right) + \left|\mathbb{i}\lambda \underline{H}_u^T \tau_2 \underline{H}_d + \mu_N \underline{N} + \kappa \underline{N}^2\right|^2 \tag{B.74}$$

$$V_D = \frac{1}{8}\left(g_Y^2 + g_L^2\right)\left(|\underline{H}_u|^2 - |\underline{H}_d|^2\right)^2 + \frac{1}{2} g_L^2 \left|\underline{H}_u^\dagger \underline{H}_d\right|^2 \tag{B.75}$$

$$V_{\text{SB}} \quad + m_{H_u}^2 \underline{H}_u^\dagger \underline{H}_u + m_{H_d}^2 \underline{H}_d^\dagger \underline{H}_d + m_N^2 \underline{N}^* \underline{N} + \left[\mathbb{i} a_\lambda \underline{N H_u^T} \tau_2 \underline{H}_d - \frac{1}{2} b_N \underline{N}^2 + \frac{1}{3} a_\kappa \underline{N}^3 + \text{h.c.}\right] \tag{B.76}$$

B.3.5 F-terms

$$-F_{H_u}^\dagger = \mathbb{i} y_u \underline{Q}^T \tau_2 \underline{u}^c - \mathbb{i}\lambda \underline{N H_d^T} \tau_2 \tag{B.77}$$

$$-F_{H_d}^\dagger = \mathbb{i} y_d \underline{Q}^T \tau_2 \underline{d}^c + \mathbb{i} y_e \underline{L}^T \tau_2 \underline{e}^c + \mathbb{i}\lambda \underline{N H_u^T} \tau_2 \tag{B.78}$$

$$-F_N^* = \mathbb{i}\lambda \underline{H}_u^T \tau_2 \underline{H}_d + \mu_N \underline{N} + \kappa \underline{N}^2 \tag{B.79}$$

B.3.6 D-terms

$$D_L^G = -\frac{1}{2} g_L \left[\underline{H}_u^\dagger \tau^G \underline{H}_u + \underline{H}_d^\dagger \tau^G \underline{H}_d \right] \tag{B.80}$$

$$D_Y = -\frac{1}{2} g_Y \left[\underline{H}_u^\dagger \underline{H}_u - \underline{H}_d^\dagger \underline{H}_d \right] \tag{B.81}$$

Bibliography

[1] N. Setzer and S. Spinner, Phys. Rev. **D71**, 115010 (2005), hep-ph/0503244.

[2] R. N. Mohapatra, N. Setzer, and S. Spinner, (2007), arXiv:0707.0020 [hep-ph].

[3] R. N. Mohapatra, N. Setzer, and S. Spinner, (2008), arXiv:0802.1208 [hep-ph].

[4] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[5] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[6] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[7] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964).

[8] P. W. Higgs, Phys. Lett. **12**, 132 (1964).

[9] A. Salam and J. C. Ward, Phys. Lett. **13**, 168 (1964).

[10] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).

[11] P. W. Higgs, Phys. Rev. **145**, 1156 (1966).

[12] A. Fayyazuddin and M. Riazuddin, *A modern introduction to particle physics* (World Scientific, Singapore, 1992), Includes examples and exercises.

[13] A. A. Aguilar-Arevalo, FERMILAB-THESIS-2008-01.

[14] Super-Kamiokande, Y. Ashie *et al.*, Phys. Rev. Lett. **93**, 101801 (2004), hep-ex/0404034.

[15] Super-Kamiokande, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998), hep-ex/9807003.

[16] N. Ferrari, Astrophys. Lett. Commun. **33**, 11 (1996).

[17] MiniBooNE, M. H. Shaevitz, Prepared for 12th International Workshop on Neutrinos Telescopes: Twenty Years after the Supernova 1987A Neutrino Bursts Discovery, Venice, Italy, 6-9 Mar 2007.

[18] SNO, N. Oblath, AIP Conf. Proc. **947**, 249 (2007).

[19] L. Ranjan and S. K. Vempati, (2007), 0711.2378.

[20] Super-Kamiokande, J. P. Cravens *et al.*, (2008), 0803.4312.

[21] G. Altarelli, (2007), arXiv:0711.0161 [hep-ph].

[22] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[23] P. Minkowski, Phys. Lett. **B67**, 421 (1977).

[24] M. Gell-Mann, P. Ramond, and R. Slansky, Print-80-0576 (CERN).

[25] T. Yanagida, In Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 13-14 Feb 1979.

[26] S. L. Glashow, NATO Adv. Study Inst. Ser. B Phys. **59**, 687 (1979).

[27] S. P. Martin, (1997), hep-ph/9709356.

[28] J. A. Bagger and J. Wess, *Supersymmetry and supergravity* (Johns Hopkins Univ., Baltimore, MD, 1990).

[29] R. N. Mohapatra, Phys. Rev. **D34**, 3457 (1986).

[30] A. Font, L. E. Ibanez, and F. Quevedo, Phys. Lett. **B228**, 79 (1989).

[31] S. P. Martin, Phys. Rev. **D46**, 2769 (1992), hep-ph/9207218.

[32] J. R. Ellis and D. V. Nanopoulos, Phys. Lett. **B110**, 44 (1982).

[33] R. Barbieri and R. Gatto, Phys. Lett. **B110**, 211 (1982).

[34] J. R. Ellis, S. Ferrara, and D. V. Nanopoulos, Phys. Lett. **B114**, 231 (1982).

[35] W. Buchmuller and D. Wyler, Phys. Lett. **B121**, 321 (1983).

[36] B. A. Campbell, Phys. Rev. **D28**, 209 (1983).

[37] M. J. Duncan, Nucl. Phys. **B221**, 285 (1983).

[38] J. F. Donoghue, H. P. Nilles, and D. Wyler, Phys. Lett. **B128**, 55 (1983).

[39] J. Polchinski and M. B. Wise, Phys. Lett. **B125**, 393 (1983).

[40] D. V. Nanopoulos and M. Srednicki, Phys. Lett. **B128**, 61 (1983).

[41] J. M. Gerard, W. Grimus, A. Raychaudhuri, and G. Zoupanos, Phys. Lett. **B140**, 349 (1984).

[42] L. J. Hall, V. A. Kostelecky, and S. Raby, Nucl. Phys. **B267**, 415 (1986).

[43] F. Gabbiani and A. Masiero, Phys. Lett. **B209**, 289 (1988).

[44] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. **B353**, 591 (1991).

[45] R. Barbieri and G. F. Giudice, Phys. Lett. **B309**, 86 (1993), hep-ph/9303270.

[46] S. Dimopoulos and D. W. Sutter, Nucl. Phys. **B452**, 496 (1995), hep-ph/9504415.

[47] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996), hep-ph/9604387.

[48] A. Masiero, S. K. Vempati, and O. Vives.

[49] J. J. Liu, C. S. Li, L. L. Yang, and L. G. Jin, Phys. Lett. **B599**, 92 (2004), hep-ph/0406155.

[50] S. Bejar, J. Guasch, and J. Sola, Nucl. Phys. Proc. Suppl. **157**, 147 (2006), hep-ph/0601191.

[51] J. Cao, G. Eilam, K.-i. Hikasa, and J. M. Yang, Phys. Rev. **D74**, 031701 (2006), hep-ph/0604163.

[52] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999), hep-th/9810155.

[53] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, JHEP **12**, 027 (1998), hep-ph/9810442.

[54] M. A. Luty, (2005), hep-th/0509029.

[55] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Nucl. Phys. **B212**, 413 (1983).

[56] W. Siegel and J. Gates, S. James, Nucl. Phys. **B147**, 77 (1979).

[57] A. A. Belavin, A. M. Polyakov, and A. B. Zamolodchikov, Nucl. Phys. **B241**, 333 (1984).

[58] P. H. Ginsparg, (1988), hep-th/9108028.

[59] M. Dine and N. Seiberg, JHEP **03**, 040 (2007), hep-th/0701023.

[60] E. Boyda, H. Murayama, and A. Pierce, Phys. Rev. **D65**, 085028 (2002), hep-ph/0107255.

[61] Z. Chacko, M. A. Luty, I. Maksymyk, and E. Ponton, JHEP **04**, 001 (2000), hep-ph/9905390.

[62] T. Gherghetta, G. F. Giudice, and J. D. Wells, Nucl. Phys. **B559**, 27 (1999), hep-ph/9904378.

[63] E. Katz, Y. Shadmi, and Y. Shirman, JHEP **08**, 015 (1999), hep-ph/9906296.

[64] A. Pomarol and R. Rattazzi, JHEP **05**, 013 (1999), hep-ph/9903448.

[65] N. Arkani-Hamed, D. E. Kaplan, H. Murayama, and Y. Nomura, JHEP **02**, 041 (2001), hep-ph/0012103.

[66] B. C. Allanach and A. Dedes, JHEP **06**, 017 (2000), hep-ph/0003222.

[67] I. Jack and D. R. T. Jones, Phys. Lett. **B482**, 167 (2000), hep-ph/0003081.

[68] M. Carena, K. Huitu, and T. Kobayashi, Nucl. Phys. **B592**, 164 (2001), hep-ph/0003187.

[69] M. Ibe, R. Kitano, H. Murayama, and T. Yanagida, Phys. Rev. **D70**, 075012 (2004), hep-ph/0403198.

[70] N. Okada, Phys. Rev. **D65**, 115009 (2002), hep-ph/0202219.

[71] R. Kitano, G. D. Kribs, and H. Murayama, Phys. Rev. **D70**, 035001 (2004), hep-ph/0402215.

[72] M. Dine, N. Seiberg, and S. Thomas, (2007), arXiv:0707.0005 [hep-ph].

[73] T. Han, B. Mukhopadhyaya, Z. Si, and K. Wang, Phys. Rev. **D76**, 075013 (2007), arXiv:0706.0441 [hep-ph].

[74] M. Frank, K. Huitu, and S. K. Rai, Phys. Rev. **D77**, 015006 (2008), arXiv:0710.2415 [hep-ph].

[75] M. Raidal and P. M. Zerwas, Eur. Phys. J. **C8**, 479 (1999), hep-ph/9811443.

[76] G. Barenboim, K. Huitu, J. Maalampi, and M. Raidal, Phys. Lett. **B394**, 132 (1997), hep-ph/9611362.

[77] A. G. Akeroyd, M. Aoki, and H. Sugiyama, (2007), arXiv:0712.4019 [hep-ph].

[78] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. **D48**, 4352 (1993), hep-ph/9306290.

[79] C. S. Aulakh, A. Melfo, A. Rasin, and G. Senjanovic, Phys. Rev. **D58**, 115007 (1998), hep-ph/9712551.

[80] C. S. Aulakh, A. Melfo, and G. Senjanovic, Phys. Rev. **D57**, 4174 (1998), hep-ph/9707256.

[81] R. N. Mohapatra and A. Rasin, Phys. Rev. **D54**, 5835 (1996), hep-ph/9604445.

[82] Z. Chacko and R. N. Mohapatra, Phys. Rev. **D58**, 015003 (1998), hep-ph/9712359.

[83] SINDRUM, U. Bellgardt *et al.*, Nucl. Phys. **B299**, 1 (1988).

[84] L. Willmann *et al.*, Phys. Rev. Lett. **82**, 49 (1999), hep-ex/9807011.

[85] S. P. Martin and M. T. Vaughn, Phys. Rev. **D50**, 2282 (1994), hep-ph/9311340.

[86] S. F. King and P. L. White, Phys. Rev. **D52**, 4183 (1995), hep-ph/9505326.

[87] F. E. Paige, S. D. Protopopescu, H. Baer, and X. Tata, (2003), hep-ph/0312045.

[88] J. L. Feng and T. Moroi, Phys. Rev. **D61**, 095004 (2000), hep-ph/9907319.

[89] M. N. Danielson *et al.*, Prepared for 1996 DPF / DPB Summer Study on New Directions for High-Energy Physics (Snowmass 96), Snowmass, Colorado, 25 Jun - 12 Jul 1996.

[90] F. E. Paige and J. D. Wells, (1999), hep-ph/0001249.

[91] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Nucl. Phys. **B238**, 453 (1984).

[92] D. O. Caldwell *et al.*, Phys. Rev. Lett. **61**, 510 (1988).

[93] C. Arina and N. Fornengo, JHEP **11**, 029 (2007), arXiv:0709.4477 [hep-ph].

[94] G. F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999), hep-ph/9801271.

[95] J. L. Feng, T. Moroi, L. Randall, M. Strassler, and S.-f. Su, Phys. Rev. Lett. **83**, 1731 (1999), hep-ph/9904250.

[96] T. Moroi and L. Randall, Nucl. Phys. **B570**, 455 (2000), hep-ph/9906527.

[97] H. Baer, J. K. Mizukoshi, and X. Tata, Phys. Lett. **B488**, 367 (2000), hep-ph/0007073.

[98] I. Hinchliffe, F. E. Paige, M. D. Shapiro, J. Soderqvist, and W. Yao, Phys. Rev. **D55**, 5520 (1997), hep-ph/9610544.

[99] C. H. Chen, M. Drees, and J. F. Gunion, Phys. Rev. Lett. **76**, 2002 (1996), hep-ph/9512230.

[100] J. F. Gunion and S. Mrenna, Phys. Rev. **D62**, 015002 (2000), hep-ph/9906270.

[101] U. Ellwanger, Phys. Lett. **B303**, 271 (1993), hep-ph/9302224.

[102] B. Dutta and R. N. Mohapatra, Phys. Rev. **D59**, 015018 (1999), hep-ph/9804277.

[103] M. Aoki, Nucl. Instrum. Meth. **A503**, 258 (2003).