ABSTRACT

Title of Dissertation: A Theoretical and Empirical Study of Computing Earnings Per Share
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In this dissertation, I propose a theoretic foundation to compute earnings per share (EPS) for firms with both common shares and dilutive securities outstanding. I derive a new EPS measure, market EPS, which is defined as the expectation of the future earnings per share. From the view of investors, market EPS naturally captures EPS information in stock prices. It is compared to basic EPS and diluted EPS, which are suggested in the dual presentation under the current U.S. rule. The comparisons show that market EPS is below the range defined by basic EPS and diluted EPS as long as the expected future abnormal earnings is zero. This indicates a weakness behind the thinking of the current rule.

I also find that the diluted EPS by the treasury stock method overstates market EPS more than that by the if-converted method. In addition, given all conditions the same, the upward bias of diluted EPS of growth firms is smaller than that of non-growth firms.
To support the proposed theory, I conduct an empirical study using a dataset containing 3130 firm-year employee stock option plans from 1997 to 2006. The results show that diluted EPS under the rule is, on average, larger than market EPS by 1%. Furthermore, the bias is larger for firms that are heavy users of employee stock options and for firms that have higher earnings volatility.
A Theoretical and Empirical Study of Computing Earnings Per Share

By

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Dedication

To Kaiyan, Haibin, and my parents
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Chapter 1: Introduction

*Earnings per share* (EPS) is one of the most widely used accounting numbers. It presents a company’s current and future potential debt and provides stockholders with information on the portion of earnings belong to each share. One important issue in the study of EPS is the potential future conversion from option-like securities to common stocks. Historically, the main controversy is the extent of such conversion to be counted in the computation of the number of common stock. Many previous studies focus on the calculation of earnings per share. However, few of them explicitly analyze the usefulness of the EPS information to investors.

In this dissertation, from the view of investors, I develop a theoretical model for computing EPS. This dissertation also provides theoretical and empirical evidence that diluted EPS overestimates the earnings per share information incorporated in stock prices. Intuitively, at any time between issue of dilutive securities and maturity date, investors estimate the probability of future conversions or exercises of dilutive securities. They incorporate this information into stock prices. In this dissertation I argue that the ideal EPS must reflect the expectations of future conversions or exercises and show that both the if-converted method and the treasury stock method fail to capture this information.

This study starts with the classical valuation model to analyze the earnings per share information incorporated in stock prices. Then it proposes a new EPS measure, *market EPS (MEPS)*, which is defined as the expectation of the future earnings per
share. Market EPS is compatible with the information captured in stock price and therefore provides useful information for investors.

Based on the theoretical analysis, I assess the current U.S. rule about the EPS disclosure. The current U.S. rule requires dual presentation of basic EPS and diluted EPS for firms with complex capital structures. Basic EPS is simply calculated as earnings available to common shareholders divided by weighted-average common shares outstanding. This measure does not consider the effect of potential dilutive securities. On the other hand, diluted EPS assumes maximum dilution and never exceeds basic EPS. For diluted EPS, the rule suggests two methods to account for the effects of potential dilution: the if-converted method and the treasury stock method. However, the rule does not provide any instruction for investors on how to use basic EPS and dilutive EPS measures. SFAS No. 128, *Earnings per Share* claims, “Presenting undiluted and diluted EPS data would give users the most factually supportable range of EPS possibilities” (SFAS No. 128 Page 29).

The proposed Market EPS is compared to basic EPS and diluted EPS. The comparisons show that, under both the if-converted method and the treasury stock method, the diluted EPS is greater than or equal to the market EPS, as long as no future abnormal earnings are expected. This result indicates that market EPS is below the dilution range defined by basic EPS and diluted EPS. Since this range is supposed to provide the range of possible dilution, the fact that market EPS is below the range reflects that the objective of the range is not accomplished. Moreover, I show that, under both the if-converted method and the treasury stock method, the difference between diluted EPS and market EPS increases with the volatility of the future
abnormal earnings. That is, since investors discount more current EPS when higher volatility of future earnings exists, the higher earnings volatility means the lower market EPS and therefore the more bias in diluted EPS.

Another important finding is that given all conditions the same, the diluted EPS by the treasury stock method is higher than or equal to that by the if-converted method. Intuitively, the reason comes from ignoring the possibility of future stock price increases in the treasury stock method. However, when stock prices go up in future, more dilutive securities will be exercised. The current earnings are expected to be more diluted. Therefore, compared to the if-converted method, the treasury stock method understates more dilution effects of dilutive securities.

I also examine EPS for growth firms whose earnings are assumed to grow in the future instead of following a random walk process. Given all the conditions the same, investors expect better outcomes from growth firms than from non-growth firms. Therefore, the expected future EPS is higher for growth firms than for non-growth firms. Furthermore, the upward bias in diluted EPS by the rule is smaller for growth firms than for non-growth firms.

My empirical analysis indicates that diluted EPS under the rule overstates the expected future EPS. In a sample of 495 firms over the period of 1997-2006, dilute EPS, on average, is larger than market EPS by 1%. For intensive users of options, the difference between diluted EPS and market EPS averages as much as 2.7% of diluted EPS. Moreover, my theoretical model predicts that option intensity and earnings volatility positively relate to the bias in diluted EPS. The empirical evidence supports the predictions.
In related research, Core et al. (2002) propose a measure of diluted EPS based on option valuation model. They divide a firm’s value to common shareholders and the option holders. Casson and Mckenzie (2007) propose a benchmark model to calculate basic EPS and diluted EPS in the theory of contingent claims. In their model, the basic EPS is defined as the changes in a firm’s net asset over a reporting period, and the diluted EPS as the change in the value of the claims of each common share on the firm’s net asset.

These papers introduce the option prices into the theoretical model of calculating EPS. The drawback of this method is that option valuation model involves assumptions like the future stock prices, the employee exercise behaviors, and expected dividend policy. The discretion of these assumptions affects the reliability of EPS measures.

Different from the option valuation model, market EPS is built on the classical valuation model and clearly defined as the expectation of future earnings per share. Because it is compatible with the stock price, market EPS provide useful information for investors.

In summary, this dissertation spells out a clear theory of the EPS measure. The contribution is three-fold. First, it shows that market EPS is below the range of basic EPS and diluted EPS. Since basic EPS and diluted EPS are supposed to provide a range of EPS possibilities, this observation indicates a weakness of thinking behind the current U.S. rule. By pointing out this problem, this dissertation helps investors to have a better understanding about the rule. Second, I prove that the diluted EPS by the treasury stock method overstates market EPS to a greater extent than EPS by the
if-converted method. This analysis provides insights for rule makers developing a diluted EPS measure. Third, an empirical study is conducted using 3130 firm-year data, the results support the proposed theory.

The dissertation is organized as follows: In the next chapter, the history of U.S. rules and international rules on earnings per share is reviewed. The relevant literature on earnings per share is also summarized. Chapter 3 develops a theoretical model for firms with complex capital structure and defines the market EPS. After that, in Chapter 4, the current U.S. rule on EPS is evaluated. Diluted EPS by if-converted method and by treasury stock method are compared to market EPS. Chapter 5 describes the samples and gives the empirical results. Finally, Chapter 6 presents the conclusion.
Chapter 2: Literature Review

This chapter begins by introducing the history of U.S. accounting rules on EPS. Then, it describes the international accounting rules on EPS. U.S. rules on EPS and international rules on EPS are generally the same. All these rules adopt the dual representation method for EPS presentation and use the if-converted and the treasury stock methods for EPS calculation. Finally, the chapter concludes with a discussion on the relevant studies on EPS.

History of U.S. Rule on EPS

Currently, dual presentation of basic EPS and diluted EPS is the major approach for EPS representation in the US and many other countries.

The U.S. rule has several revisions for the computation of EPS. During the 1950s, earnings per share presentations became quite common. In Accounting Research Bulletin (ARB) No. 49, issued in 1961, the Committee on Accounting Procedure (CAP) suggested general guides to be used in computing EPS. It was proposed that EPS should be used to designate the amount applicable to each share of common stock or other residual security outstanding.

The Accounting Principles Board (APB) first addressed the subject of EPS in APB Opinion No. 9, Reporting the Results of Operations (AICPA, 1966). In the opinion, APB strongly encouraged companies to disclose EPS amounts in the income statement for income before extraordinary items, for extraordinary items, and for net
income, instead of reporting a single EPS figure. It also stated that if a potential dilution in EPS occurred, pro forma EPS should be reported showing what the earnings would be if the conversions or other stock issuances took place.

APB Opinion No. 15, *Earnings per Share* (AICPA, 1969) was the first official accounting pronouncement to require presentation of EPS figures and provide detailed information on how to compute EPS. Firms with significant contingently issuable common shares were required to report both the “primary” EPS and the “fully diluted” EPS. Primary EPS is intended to reflect the expected dilution in current shareholder interests by determining the common stock equivalency in the denominator of EPS calculation. However, Opinion 15 received strong criticism from users and academics because of the arbitrary methods by which common stock equivalents were determined.

In 1997, FASB issued Statement of Financial Accounting Standards (SFAS) No. 128, *Earnings per Share*, which is the current rule for instructing the EPS disclosure in U.S. SFAS No. 128 simplifies the standards for computing earnings per share in APB Opinion No. 15 and makes them comparable to international EPS standards. The statement replaces the presentation of primary EPS with the basic EPS and requires dual presentation of basic and diluted EPS on the face of the income statement for all firms with potential common stock outstanding.

Basic EPS is computed by dividing income available to common stockholders by the weighted-average number of common shares outstanding for the period. Diluted EPS reflects the potential dilution that could occur if securities or other contracts to issue common stock were exercised or converted into common stock or
resulted in the issuance of common stock that then shared in the earnings of the entity. The objective of both basic EPS and diluted EPS is to measure the performance of an entity over the reporting period.

*International Rules on EPS*

In October 1993 the International Accounting Standards Committee (IASC) issued a draft statement “Earnings Per Share”. The following year Financial Accounting Standards Board (FASB) added to its technical agenda a project on earnings per share to be pursued concurrently with IASC, with the object of improving and simplifying US GAAP on the matter and issuing a standard that would be compatible with an international standard. The exposure drafts, published early in 1996, showed that the FASB and IASC had reached agreement on many of the main points of principle.

IASC issued IAS 33 *Earnings per share* in 2003. It claims that the objective of the standard is to improve performance comparisons between different entities in the same reporting period and between different reporting periods for the same entity. Like SFAS No.128 *Earnings per share*, the standard also adopts the dual presentation method, basic earnings per share and diluted earnings per share. Dilution is defined as a reduction in earnings per share or an increase in loss per share resulting from the assumption that potential ordinary shares conversed to ordinary shares. To calculate diluted earnings per share, an entity is required to adjust profit or loss attributable to ordinary equity holders.
Consistent with SFAS No. 128, IAS 33 adopts the dual presentation method: basic earnings per share and diluted earnings per share. It does not permit future projections and adopts the historical view for diluted earnings per share. IAS 33 claims that the diluted earnings per share figure is an additional past performance measure, consistent with that of basic earnings per share and also giving effect to all dilutive potential ordinary shares. The main difference between SFAS No. 128 and IAS 33 is the disclosure for loss firms. Under SFAS No. 128, loss firms are not required to disclose diluted EPS, while under IAS 33, loss firms need to disclose diluted EPS if there is an increase in loss per share from the hypothetical conversion of potential common shares.

In 2001, Canadian Institute of Chartered Accountants (CICA) approved the EPS standard harmonizing Canadian EPS reporting with that of the FASB and the IASC. The Treasury stock method is adopted to calculate diluted earnings per share for stock options rights. The firm’s current stock price is used in the dilutive adjustment.

Before 2001, the former Canadian Institute of Chartered Accountants (CICA) Handbook section 3500 intended fully diluted EPS to be forward-looking. It claimed that the purpose of the presentation of fully diluted EPS is to show the potential dilution on a prospective basis. The pre-2001 Canadian GAAP adopted the *imputed earnings method* to calculate diluted earnings per share for stock options or rights. Under the imputed earnings method, fully diluted EPS assumes that all convertibles, options, and rights that could be converted or exercised within the next 10 years were outstanding as common shares during the current year. These hypothetical shares are
added to the denominator in the fully diluted EPS calculation. The numerator of the
calculation is then increased by the dividends or interest that would be saved if
convertible securities were converted into common shares, and by the estimated new
income that would be earned on the assumed reinvestment of the cash inflow from the
exercise of options or rights. The latter calculation involves multiplying the cash to be
received by an “appropriate” rate of return, which is often interpreted in practice as an
average historical rate of return on equity.

The main difference between pre-2001 Canadian GAAP and SFAS No. 128 is
that the pre-2001 Canadian diluted EPS is calculated without reference to the firm’s
stock price, while the American diluted EPS is based on the firm’s stock price by
treasury stock method.

FASB denied the imputed earnings method in SFAS 128. The board said that
imputed earnings method “requires an arbitrary assumption about the appropriate rate
of earnings, it overstates dilution because it treats anti-dilutive potential common
shares as if they were dilutive, and it gives the same effect to all options and warrants
regardless of the current market price” (p32).

In UK, before 1998, imputed earnings method was adopted by the Accounting
Standards Committee (ASC) in SSAP 3 Earnings per share. To calculate the fully
diluted EPS, it was assumed that the proceeds from the exercise of options are
invested in government securities.
In 1998, ASC issued FRS 14, *Earnings Per Share*. This standard adopts the similar text of the International Accounting Standards Committee’s standard IAS 33, *Earnings Per Share*. In 2003, ASC issued FRS 22, *Earnings Per Share*, which supersedes FRS 14. FRS 22 revised some presentation requirements and provides more guidance on detail problems. The dual presentation principle and the basic calculation methods are still consistent with FASB and IASC.

Many other countries have adopted IAS 33 *Earnings per share*. For example, In Hong Kong, there is no major textual difference between HKAS 33, *Earnings per share* and IAS 33 *Earnings per share*. In Australia, AASB 133, *Earnings per share* is equivalent to IAS 33 issued by IASB. New Zealand, NZIAS 33 is equivalent to IAS 33.

In general, the International Accounting Standard Committee’s standard on EPS is mostly compatible with the FASB’s standard on EPS. They are similar at the dual presentation principle and the calculation of basic EPS and diluted EPS. They differ on the disclosure requirement for loss firms. Many countries issued earnings per share standard compatible with IAS 33 Earning Per Share, including UK, Canada, Hong Kong, etc.

*Literature on EPS*

The studies on EPS can be roughly classified to three classes. The first class focuses on the primary EPS before the issue of SFAS No. 128. The second class
examines the relationship of stock prices and earnings per share. The third class suggests option valuation model to calculate earnings per share. Among these studies, the third class is the most relevant to this dissertation. The following paragraph first briefly discusses the first and the second classes. After that it focuses on the discussion on the third class, i.e., the option valuation model to calculate EPS.

First, APB Opinion 15 requires firms with complex capital structure to report the primary EPS, which incorporates “expected” dilution. The criteria to determine the possible future dilution had received substantial criticism. Before 1990s, many papers discussed the computation of primary EPS. Frank and Weygandt (1970), Arnold and Humann (1973), and Givoly and Palmon (1981) empirically investigated the criteria for common stock equivalent. Frankfurter and Horwitz (1972) investigated the criteria by simulation. Shank (1971) examined the relationship between stock prices and the primary EPS. Curry (1971) suggested alternative way to disclose convertible bonds. Barlev (1983) examined the modified treasury stocks method theoretically. In general, they all concluded that the criteria of common stock equivalent could not provide good prediction of future conversion.

Second, several studies examine the relationship between market and dilution. Generally, these studies show that the market can see through the dilution effect of potential common shares. Some research directly examines the information content of diluted EPS. Jennings, et al. (1997) empirically compared the extent to which basic, primary, and fully diluted EPS explain variation in stock prices using a sample of
firms from 1989 to 1995. They found that fully diluted EPS explains more variation in stock prices than both basic and primary EPS.

On the other hand, some studies focus on the relationship between stock price and dilution. Huson, et al. (2001) used a sample of firms from 1970 to 1995 and documented that the stock return response to changes in accounting income is smaller for firms with more shares reserved for conversion. They also showed that the return earnings response is smaller for firms with higher recent returns, where recent returns proxy for the extent to which options and convertible securities are in the money. Garvey and Milbourn (2002) examined whether the stock prices incorporate the potential dilution of employee stock options. They found that the stock market tends to undervalue the costs of employee stock options. Li and Wong (2005) showed that stock price estimate under a warrant-pricing approach is lower than that under the traditional valuation method and is also closer to the actual stock price.

Third, many papers recommend using the option valuation model to construct an alternative EPS measure to primary EPS or diluted EPS by accounting rules. Vigeland (1982) suggested assessing the probability of future conversion or exercising from the option prices model. Wiseman (1990) presented a current value method for EPS dilution calculation. The basic EPS is adjusted by the changes in the fair values of warrant holders’ claims on a firm’s net assets. The clean surplus characteristic of this adjustment provides the link between EPS and the value of common shares for a firm with a capital structure including warrants. Jerris (1992) developed an EPS measure based on the future conversion probability using the
option price model. Empirically, he showed that this measure is more correlated with security returns than primary EPS is.

Graham et al. (1962) proposed a method for calculating dilution that effectively apportions earnings between current common shareholders and holders of warrants on the basis of the fair values of the respective financial instruments at the end of the reporting period. This principle is also used by King (1984) in his measure of model EPS for convertibles. Core et al. (2002) used a similar method to construct a measure of diluted EPS based on option valuation model. They divided the firm’s value to common shareholders and the option holders based on the stock prices and the option prices. Options issued by a firm are assumed to be equivalent to the number of additional common shares with the same aggregate fair value as that of the option issue, and the denominator in the EPS calculation increased by this amount. Therefore, the denominator in the EPS calculation is the sum of current shares and the number of additional shares with equivalent value to the options.

Core et al. (2002) showed that the treasury stock method systematically overstates the earnings per share when compared to the options-diluted EPS. Using firm-wide data on 731 employee stock option plans over the period 1994-1997, their proposed measure suggests that economic dilution from options is, on average, 100 percent greater than dilution in reported diluted EPS using the FASB treasury-stock method. For intensive users of stock options, such as high-growth firms, the difference between their estimate of economic dilution and the FASB treasury-stock method dilution is as much as 8 percent of weighted average common shares outstanding.
Casson and McKenzie (2007) proposed a benchmark model to calculate the basic EPS and the diluted EPS in the theory of contingent claims. In their model, the basic EPS is defined as the changes in a firm’s net asset over a reporting period, and the diluted EPS as the change in the value of the claims of each common share on the firm’s net asset. Simulations are used to compare the benchmark with the diluted EPS under the treasury stock method, imputed earnings method, option-diluted method, and holding loss/gain method. They concluded that the treasury stock method performs worst among the four approaches, while the imputed earnings approach provides a reasonable approximation to the benchmark.

These papers introduce the option prices in the theoretical model of EPS. The option valuation model involves assumptions like the future stock prices, the employee exercise behaviors, and the expected dividend policy. The discretion of these assumptions affects the reliability of EPS measures under the option valuation model.

Different from the EPS measure based on option-value model, my work is built on the classical valuation model. From the view of investors, this dissertation proposes an EPS measure that naturally captures the EPS information in stock prices. Compatible with stock prices, this measure provides the most useful information for investors.
Chapter summary

This chapter first reviews the history of U.S rule on EPS. The current U.S. rule adopts the dual presentation, basic EPS and diluted EPS. The if-converted method and treasury stock method are the two main methods to calculate diluted EPS.

Second, the international rules on EPS are summarized. Generally, IASC issued EPS standards compatible with U.S. rule. Many countries, including U.K., Canada, Australia etc., have adopted IASC’s standards on EPS.

Third, studies on EPS issue are reviewed. Some studies show evidence that the market can see through the dilution effect of potential common shares to some extent. For calculation of EPS, many studies suggest an option valuation model. The drawback of this method is that it brings the discretion assumptions of the option valuation model into the calculation of EPS.
Chapter 3: Theoretical Model of Market EPS

In this chapter, a theoretical model for computing EPS is developed for a firm with both common stocks and dilutive securities outstanding. From the view of investors, the earnings per share information incorporated in the stock prices is analyzed. The market EPS is defined as the expectation of future earnings per share incorporated in prices.

The following section first shows that the problem of EPS calculation mainly comes from the uncertainty of future conversions of potential common shares. Then, the model settings for a firm with both common shares and warrants outstanding are described. After that, the stock price model is used to develop a new EPS measure, i.e., market EPS, which reflects the investors’ expectation of the future earnings per share. In the forth subchapter, several properties of market EPS are examined.

The Problem

As one of the most important accounting numbers, earnings per share (EPS) is defined as “the amount of earnings attributable to each share of common stock” (SFAS 128, Earnings per Share, Appendix E Page 75). EPS aims to demonstrate a company’s current and future debt and provide stockholders with information about the portion of earnings that belongs to each share of common stock. Therefore, an earnings per share number and forecasts of future earnings per share provide key information for fundamental analysis, equity valuation, and performance evaluation.
Despite the simple definition of EPS, the existence of dilutive securities causes the computation of EPS to be a problem. For example, in my sample, options outstanding average 8.31 percent of shares outstanding for the period from 1996 to 2006. These potential common shares raise the question of how to measure this potential dilution in the calculation of EPS.

The difficulty of computing EPS mainly comes from the uncertainty of future conversions or exercises of dilutive securities and how the uncertainty affects EPS. On the one hand, what is the appropriate portion of earnings belonging to the shareholder, the numerator in EPS calculation? On the other hand, how many shares should be used to divide the earnings, the denominator in EPS calculation?

Targeting this problem, this dissertation studies the effects of dilutive securities on the calculation of EPS from the view of investors. Investors expect the possibilities of future conversion of dilutive securities and incorporate this expectation in the stock price valuation. Motivated by this observation, I propose a new EPS measure that captures the expectation of future EPS and therefore provides useful information for investors.

The theoretical analysis begins with the valuation model that stock prices equal to the sum of discounted expectation of future EPS.

There are three classes of potential common stock as identified in SFAS No. 128: (1) convertible securities, including convertible debt and convertible preferred shares; (2) employee stock options, warrants, and rights etc.; and (3) contingent stock agreements, which refer to the contracts that may be settled with the stock or in cash, and the contingently issuable shares, whose issuance is contingent upon the
satisfaction of certain conditions. The third class falls out of the scope of my study, since the effects of these agreements on EPS are determined by specific conditions on contracts or special agreement. This study focuses on convertible securities and options or warrants. Furthermore, convertible securities could be separated as a normal debt or preferred share and a warrant. I will focus on warrants representing dilutive securities in the following analysis.

Model Settings

My model focuses on the relationships among earnings, number of shares and prices. Since the future conversion of potential common shares is determined by the stock price, the future number of shares is determined by the stock price. Meanwhile, the information contained in the earnings per share affect the stock price. To examine the relationships among earnings, number of shares and stock prices, I start with the classical valuation model in which a firm’s stock price is equal to the present value of expected cash flows.

Consider a firm has a capital structure with $N$ shares common stock outstanding and $n$ warrants with exercise price $X$. Two periods are considered. From now on, without lost of generality, let the reporting date be $t=1$, and the maturity date for the warrants be $t=2$. During period 1 the firm realizes the earnings, $E_1$.

The timeline of the model is shown in Figure 1. During period 1, the firm realizes the earnings $E_1$. The maturity date of the warrants is at the end of period 2. The problem is at reporting date what is the most useful EPS for investors.
Supposing the present value of expected EPS accruing to the common shareholders is equal to the present value of cash flow, we have the following relationship:

\[
P_t = \sum_{\tau=1}^{\infty} \frac{(1 + r)^{-\tau}}{\tau} E_{t+\tau} \left[ \frac{E_{t+\tau}}{N_{t+\tau}} \right]
\]

(1)

where

\( P_t \): the prices of common stock at time \( t \);
\( r \): the rate of return;
\( E_t \): the income available to common shareholders from the period \((t-1, t)\);
\( N_t \): the number of common stock outstanding at time \( t \);
\( EXP[.]. \): the expectation value at time \( t \).

To derive the association between prices and EPS, several assumptions are made as follows.

- **Time-series behavior of earnings**: This assumption supposes that earnings follow a random walk process. That is, \( E_{t+1} = E_t + \varepsilon_{t+1} \), where \( \varepsilon_t \sim N(0, \sigma^2) \) follows a normal distribution and \( \varepsilon_1, \varepsilon_2 \ldots \varepsilon_t \ i.i.d \).

- **Constant rate of return**: This assumption assumes that the rate of return for discounting expected future cash flow is constant over time.

- **Risk neutral**: This assumption says that the investors are assumed to be risk neutral.

The first assumption, that earnings follow a random walk process, is a major assumption in the model. Based on this assumption, I develop the relationship among current EPS, future EPS and stock prices. The limitation of this assumption is that it
assumes no expected future earnings growth. Nonetheless, in Chapter 4 I relax this assumption and examine the EPS for growth firms.

The second assumption, on constant rate of return, simplifies the theoretical analysis without losing generality.

The Market EPS

From the perspective of investors, a useful EPS measure should capture both the current and the future earnings per share information in stock prices. However, the current dual presentation disclosure does not provide a direct solution (cf. Chapter 4). Targeting this problem, I start with the expression of stock prices and examine what EPS measure compatible with the stock prices.

First, at the maturity date, there are two states. The warrant holders compare the stock price to the exercise price and choose whether to exercise the warrants or not. Therefore, the stock prices are equal to the minimum of future possible values. Suppose that, in period 2, the firm realizes earnings available to common shareholders, $E_2$. Denote the stock price at time $t=2$ as $P_2$, we have

$$P_2 = \min \left\{ \frac{1}{r} \frac{E_2}{N}, \frac{1}{r} \frac{E_2 + nXr}{N+n} \right\} = \begin{cases} \frac{1}{r} \frac{E_2}{N} & \text{when } \frac{E_2}{N} < Xr \\ \frac{1}{r} \frac{E_2 + nXr}{N+n} & \text{when } \frac{E_2}{N} \geq Xr \end{cases}$$

At the maturity date, if the EPS is greater than or equal to the returns from assets $X$, i.e., $\frac{E_2}{N} \geq Xr$, warrant holders will exercise the warrants. The future earnings are expected to increase by the return of the proceeds from issuing new stocks, i.e., $nXr$. 

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Correspondingly, the number of common stocks is expected to become \( N + n \).

Otherwise, if \( \frac{E_2}{N} < Xr \), the warrant holders will not exercise the warrants. The expected future EPS remains \( \frac{E_2}{N} \).

The price at the maturity date, \( P_2 \), increases with the realized earnings, \( E_2 \). However, as \( E_2 \) increases, apparently more warrants will be exercised. The increase in prices is attenuated by the increase in the number of common shares outstanding. As a result, the current common stockholders’ benefit from the firm’s good outcomes is diluted.

At the reporting date, \( t=1 \), stock prices are determined by the possible future outcomes. Although the exercises of securities are potential, these securities have an equity value. Therefore they have an equity cost to common shareholders at any point in time from the issuing date to the maturity date. At the issuing date, shareholders exchange some residual value in the firm for lower coupon payment by convertible securities or for services by employee’s stock options. Since these securities broaden equity ownership, this information should be included in the earnings per share.

Therefore, at the reporting date \( t=1 \), the investors form the expectations of future outcomes. Since earnings is assumed to follow a random walk process, that is, \( E_2 = E_1 + \varepsilon_2 \), the stock price is expressed as follows:

\[
P_1 = \int_{-\infty}^{\infty} \frac{1}{N} \frac{E_1 + \varepsilon_2}{r} f(\varepsilon_2) d\varepsilon_2 + \int_{NX-E_1}^{\infty} \frac{1}{N+n} \frac{E_1 + \varepsilon_2 + nXr}{r} f(\varepsilon_2) d\varepsilon_2
\]

In equation (3), \( f(\varepsilon_2) \) denotes the density function of \( \varepsilon_2 \) (normal distribution). The first term on the right-hand side of the equality represents the expected value of
common stocks over the range $\epsilon_2 < NXr - E_1$, which is the range the warrant holders will not exercise the warrants. The second term shows the expected value of common stocks over the range $\epsilon_2 \geq NXr - E_1$, which is the range the warrant holders will exercise the warrants.

The stock price can be rewritten as:

$$P_1 = \frac{1}{r} \left[ \int_{-\infty}^{NXr - E_1} \frac{E_1 + \epsilon_2}{N} f(\epsilon_2) d\epsilon_2 + \int_{NXr - E_1}^{+\infty} \frac{E_1 + \epsilon_2 + nXr}{N + n} f(\epsilon_2) d\epsilon_2 \right]$$

In the above equation, the term in the parenthesis represents investors’ expectation of future EPS. Compatible with the stock price, the expectation of future EPS also includes two parts. The first part represents the expected future earnings per share when the warrant holders will not exercise the warrants. The second part shows the expected future earnings per share when the warrant holders will exercise the warrants.

I define *market EPS* as the expectation of the future earnings per share incorporated in stock prices. Therefore at the reporting date, market EPS (denoted as $MEPS$) is expressed as:

$$MEPS = \int_{-\infty}^{NXr - E_1} \frac{E_1 + \epsilon_2}{N} f(\epsilon_2) d\epsilon_2 + \int_{NXr - E_1}^{+\infty} \frac{E_1 + \epsilon_2 + nXr}{N + n} f(\epsilon_2) d\epsilon_2$$  \hspace{1cm} (4)

Market EPS provide very useful information for investors. First, $MEPS$ is compatible with the stock prices. Furthermore, the way it is constructed is similar to the way the stock prices are constructed. Since the warrant holders have the right to share the future good outcomes of the firm with the common stock holders, the use of dilutive securities impairs the benefit of resources obtained from issuing these
securities. Therefore, MEPS discounts the current earnings with the probability of future dilutions. Second, MEPS represent the EPS information incorporated in the stock prices. Based on equation (3) and (4), the stock price, $P_t$, can be expressed as

$$P_t = \frac{1}{r} MEPS$$  \hspace{1cm} (5)

Equation (5) shows that the stock price is equal to the discounted MEPS. At the reporting date, for valuation purpose, investors need to estimate the expectation of future earnings, and the future number of shares. Therefore, MEPS provides important information to evaluate the stock price.

**Properties of MEPS**

In this section, I examine the factors that affect the magnitude of MEPS. As we know, the future dilution is determined by the firm’s future performance and the characteristic of warrants and. Correspondingly, MEPS is also determined by the uncertainty of future earnings and the characteristic of warrants.

Since the distribution of future abnormal earnings determines the future potential dilution, the variance of future abnormal earnings is examined first. Meanwhile, Exercise prices and the number of warrants are two major characteristics of warrants, therefore, the exercise prices and the number of warrants are examined later. As a result, I examine three factors affecting the magnitude of MEPS, including the variance of future abnormal earnings, the exercise prices and the number of warrants.
The firm’s future performance determines whether the warrants will be exercised or not. Property 1 shows how the variance of future earnings affects MEPS, the expectation of future earnings per share.

**Property 1.** All conditions being equal, the more volatility the future earnings, the lower will be the MEPS. That is,

$$\frac{\partial MEPS}{\partial \sigma^2} < 0.$$ 

The proof is given in the Appendix.

Intuitively, property 1 says that as the volatility of future earnings increases, the probability of extreme values for future earnings increases, which causes the increasing of the probability of dilution by exercise of warrants as well.

The features of outstanding warrants also affect MEPS. Property 2 and 3 show how the exercise prices and the number of warrants affect MEPS respectively.

**Property 2.** Given the distribution of $\varepsilon_2$, MEPS is affected by the number of warrants $n$ and the exercise price $X$. When $n$ is a constant, MEPS increases with $X$. That is,

$$\frac{\partial MEPS}{\partial X} > 0.$$ 

This property of MEPS is achieved by taking derivative of MEPS with regard to $X$. The increase in the exercise prices of warrants has two effects on the expectation of future earnings. First, the higher $X$, the lower is the probability of exercises at the maturity date. Therefore, the increase in $X$ implies the lower probability of dilution of ownership in future. On the other hand, when the warrants are in the money, the higher exercise prices bring more proceeds from issuing new stocks. As the exercise
price of warrants approaches to infinite, the warrants will never be exercised. So the MEPS incorporated in stock prices is the earnings divided by the number of outstanding shares. That is, \( \lim_{X \to \infty} MEPS = \frac{E}{N} \).

**Property 3.** Regarding to the relationship between MEPS and the number of warrants \( n \), we have, given \( X \), the exercise price of warrants, we have

\[
\frac{\partial MEPS}{\partial n} < 0.
\]

This property of MEPS is achieved by taking derivative of MEPS with regard to the number of warrants \( n \).

The intuition of the property is, for firms with intensive conversion securities, the possible dilution of ownership in the future is large. Current shareholders expect more sharing of future good outcome by the warrant holders. Consequently, the expected EPS incorporated in stock prices is small. For example, the stock prices are more diluted for companies with intensive using of employee stock options.

Figure 2 illustrates an example of how MEPS varies with the standard deviation of \( \varepsilon_2 \) for a typical firm with a capital structure of common stock and warrants. The firm is assumed to have 100 common stocks outstanding and 50 warrants with an exercise price of $60 at the reporting date. The risk-free rate of return is assumed to be 10%. The earnings available to common shareholders are assumed to be $1000 for the reporting period. The figure shows clearly that MEPS decreases when \( \sigma \) increases.

Figure 3 illustrates that MEPS increases with the exercise prices of the warrants, \( X \). The deviation of \( \varepsilon_2 \) is assumed to be 500. All the other parameters are the same as those in Figure 2 except \( X \). Figure 4 illustrates that MEPS decreases with the number...
of warrants, \( n \). The deviation of \( \varepsilon_2 \) is assumed to be 500. All the other parameters are the same as those in Figure 2 except \( n \).

Chapter summary

In this chapter, I develop a theoretical model for calculating EPS. Market EPS is defined as the expectation of future EPS from the view of investors. The calculation of market EPS is compatible with stock prices. In addition, market EPS increases with the exercise prices of warrants. On the other hand, market EPS decreases with the number of warrants outstanding and the earnings volatility.
Chapter 4: Assessment on the Current U.S. EPS Rule

This chapter assesses the current U.S. rule on EPS by using the proposed market EPS. First a summary of the characteristics of the current U.S. rule suggests the dual presentation of basic EPS and diluted EPS. After that, the relationship between market EPS and diluted EPS is studied, under both the if-converted method and the treasury stock method. The conclusion is that market EPS is below the range of basic EPS and diluted EPS. Then the market EPS for growth firms is discussed. Finally, a summary of this chapter is given.

Introduction of Current U.S. Rule

The current rule in US, SFAS No. 128, Earnings per Share, was issued in 1997. It can be characterized as the following.

First, dual presentation: Dual presentation suggests that a firm computes and discloses two EPS numbers. The first one is the “basic EPS” that ignores the presence of dilutive securities. The second one is the “diluted EPS” that is based on an assumption of maximum dilution. Dilutive securities refer to any outstanding securities that have been issued by the firm and may turn into common shares sometime in the future.

Second, the if-converted method and the treasury stock method: These are two approaches for calculating the diluted EPS. The if-converted method is employed for convertible bonds and preferred stocks, while the treasure stock method is used for
employee stock options, warrants or rights. Both methods assume that all dilutive securities outstanding at the end of the fiscal year are converted or exercised at the beginning of the fiscal period (or at the date of issuance if issued during the year). They differ in the adjustment to the numerator (i.e., earnings) and the denominator (i.e., number of common shares) in EPS calculation. Under the if-converted method, both the numerator and the denominator are adjusted for the effects of hypothetical conversions of dilutive securities. The interests or preferred dividends are added back to the numerator, and the increased number of shares is included in the denominator. In contrast, the treasure stock method assumes the proceeds from exercise be used to purchase common stock and only the incremental shares be included in the denominator when computing the diluted EPS.

Third, no anti-dilution: No anti-dilution means that, if the assumed conversion or exercise of a security increases the EPS number, the security is excluded when computing the diluted EPS. As a result, the diluted EPS will never exceed the basic EPS after applying this principle.

* Diluted EPS by the If-Converted Method*

Calculation of basic EPS is straightforward. Basic EPS (denoted by $BEPS$) is equal to the earnings available to the common stockholders divided by the number of outstanding common stocks, i.e.

$$BEPS = \frac{E_1}{N}$$
The calculation of diluted EPS by the if-converted method (denoted by $DEPS_{IF}$) contains two steps. First, an EPS measure by the if-converted method (denoted by $EPS_{IF}$) is calculated. The if-converted method assumes the conversions or exercises of dilutive securities at the beginning of the reporting period, or the date of issue if later. It adjusts the impacts of exercises of warrants on both the earnings and the number of common shares. Applying the if-converted method to the calculation of $EPS_{IF}$, we have

$$EPS_{IF} = \frac{E_i + nXr}{N + n},$$

where $nX$ is the proceeds from the exercise of warrants, $r$ is the rate of return, $nXr$ refers to the annual increase in earnings from the proceeds of issuing new common stocks.

Second, no anti-dilution principle is applied to get $DEPS_{IF}$. It requires that only the dilutive securities decreasing basic EPS are included when computing the diluted EPS. Therefore, $DEPS_{IF}$ equals to the minimum of basic EPS and the EPS calculated by the if-converted method. That is,

$$DEPS_{IF} = \min(BEPS, EPS_{IF}) = \min(\frac{E_i}{N}, \frac{E_i + nXr}{N + n}).$$

(6)

In summary, BEPS is the EPS without considering the dilutive securities, while $EPS_{IF}$ is the EPS when the dilutive securities conversed or exercised. $DEPS_{IF}$ is the result of conservatism employed widely in the U.S. accounting rules. No increase in BEPS is recognized.

In the following paragraphs, I analyze the relationship between $DEPS_{IF}$ and $MEPS$. The rule requires the firms with complex capital structure to disclose both
basic EPS and diluted EPS. The purpose of this dual presentation is to provide investors with a range of possible dilution. However, the rule is silent about how to use basic EPS and diluted EPS. By comparing $DEPS_{IF}$ and $MEPS$, I show what the difference between these two measures is and furthermore, what factor determines the magnitude of the difference. This analysis helps investors to better understand basic EPS and diluted EPS.

The difference between $DEPS_{IF}$ and $MEPS$ lies in the different ways of dealing with the uncertainty about future exercise of warrants. $DEPS_{IF}$ assumes the warrants exercised by ignoring the uncertainty of future exercise. In comparison, $MEPS$ incorporates the uncertainty of future exercise by considering the expectation of future EPS. Since the distribution of future abnormal earnings, $\varepsilon_2$, determines future exercise of warrants, the variances of $\varepsilon_2$ are examined to show the relationship between $DEPS_{IF}$ and $MEPS$.

First, a special case is examined when no uncertainty about future exercise exists. If future earnings are constant, whether the warrants will be exercised or not is known to everyone at the reporting date.

**Proposition 1:** At the reporting date, given $\sigma^2 = 0$, then $DEPS_{IF} = MEPS$.

Moreover, if $\frac{E_t}{N} \leq Xr$, then $DEPS_{IF} = MEPS = BEPS$; otherwise if $\frac{E_t}{N} > Xr$, then $DEPS_{IF} = MEPS = EPS_{IF}$.

**Proof:** If $\sigma^2 = 0$, future earnings forms a constant sequence $(E_1, E_1, \ldots)$. When $\frac{E_t}{N} > Xr$, warrant holders will exercise the warrants at the end of next period and share earnings with current stockholders. The expected future EPS is
therefore \( \frac{E_i + nXr}{N + n} \). Otherwise, when \( \frac{E_i}{N} \leq Xr \), warrant holders will not exercise the warrants. The expected future EPS remains \( \frac{E_i}{N} \). Consequently, at the reporting date, \( \text{MEPS} = \min(\frac{E_i}{N}, \frac{E_i + nXr}{N + n}) \), which equals to \( \text{DEPS}_{IF} \). Q.E.D.

Proposition 1 shows that \( \text{DEPS}_{IF} \) is equivalent to \( \text{MEPS} \) when future earnings are constant. The intuition is that if the future earnings are constant, investors can determine whether the warrants will be exercised or not. There is no uncertainty about both the future earnings and the number of shares. Thus, the market EPS is the minimum of future possible outcomes, which equals the diluted EPS by if-converted method.

Next, more general situations about future earnings are analyzed. Proposition 1 shows an extreme case that has no uncertainty about the future earnings. In more realistic situations, however, there is volatility about future earnings. In this case, we have the following relationship between \( \text{DEPS}_{IF} \) and \( \text{MEPS} \).

**Proposition 2.** At the reporting date, assume \( \varepsilon_2 \sim N(0, \sigma^2) \), for any \( \sigma^2 > 0 \), then \( \text{DEPS}_{IF} > \text{MEPS} \).

The proof of Proposition 2 is given in the Appendix. Proposition 2 indicates that, as long as the expectation of future abnormal earnings is zero, the market EPS incorporated in the stock prices is less than both \( \text{BEPS} \) and \( \text{DEPS}_{IF} \). The intuition can be explained by the following examples.

First, to compare \( \text{BEPS} \) and \( \text{MEPS} \), suppose there are two firms A and W, both having \( N \) shares of common stocks. Both of them realize \( E_1 \) in period 1 and have no
expected future abnormal earnings. The only difference is that A has no dilutive securities while W has. When future cash flow is bad, firm A and firm W have the same future EPS. However, when future cash flow is good, for firm W, the warrant holders share the good outcome with current common shareholders. This is not the case for firm A. Therefore, the future EPS of firm W is lower than that of firm A. Consequently, at the reporting date, the expectation of future EPS of firm W is lower than that of firm A. That is, \( MEPS < \frac{E_1}{N} = BEPS \).

On the other hand, to compare \( EPS_{IF} \) and \( MEPS \), suppose there is a firm B with \( N+n \) shares of common stocks and no dilutive securities. Firm B realizes \( E_{1+n}Xr \) in period 1. The future abnormal earnings is also expected to zero. Let us compare the possible future EPS of firm B and firm W. When future cash flow is good, firm B and firm W have the same future EPS. When future cash flow is bad, for firm W, no warrant is exercised and the bad outcome is distributed among \( N \) shares. While for firm B, the bad outcome is undertaken by \( N+n \) shares. In this case, the future EPS of firm W is lower than that of firm B. Therefore, at the reporting date, the expectation of future EPS of firm W is lower than that of firm B, i.e., \( MEPS < \frac{E_{1+n}Xr}{N + n} = EPS_{IF} \).

Proposition 2 indicates two facts about \( DEPS_{IF} \). First, \( MEPS \) is out of the range of basic EPS and diluted EPS suggested by the rule. The diluted EPS by the if-converted method overstates the EPS information incorporated in stock prices as long as the expectation of future abnormal earnings is zero. The misspecification arises because the diluted EPS inadequately addresses the sharing of equity value with warrant holders when calculating earnings per share.
Second, the misspecification of $DEPS_{IF}$ is irrelevant to the magnitudes of basic EPS and the exercise prices. The probability of future exercise of warrants does not affect the fact that $DEPS_{IF}$ overstates the market EPS captured in stock prices.

Following Proposition 2, it is natural to study the difference between $DEPS_{IF}$ and $MEPS$. Since the difference is determined by the distribution of future abnormal earnings, I study how it is affected by the variance of future abnormal earnings. This leads to the following proposition.

**Proposition 3.** Given $\varepsilon_2 \sim N(0, \sigma^2)$, the difference between $DEPS_{IF}$ and $MEPS$, that is, $|DEPS_{IF} - MEPS|$, increases with $\sigma^2$, which is the variance of $\varepsilon_2$.

**Proof:** From Property 1 of $MEPS$, we know that given $\varepsilon_1 \sim N(0, \sigma^2)$, 

\[
\frac{\partial MEPS}{\partial \sigma^2} < 0.
\]

$DEPS_{IF}$ is constant and greater or equal to $MEPS$. Therefore, the difference between $DEPS_{IF}$ and $MEPS$, $|DEPS_{IF} - MEPS|$ increases with $\sigma^2$.

Q.E.D

This proposition indicates that the misspecification of $DEPS_{IF}$ is exacerbated by the increases in variances of unexpected earnings. The intuition is that common stockholders discount more current EPS when higher volatility of earnings exists.

Figure 5 compares three EPS measures at the reporting date: $BEPS$, $DEPS_{IF}$ and $MEPS$ when $\frac{E}{N} > Xr$. From the figure, it is clear that both $BEPS$ and $DEPS_{IF}$ are always equal to or larger than $MEPS$ regardless of different variances of future abnormal earnings. In particular, when the variance of $\varepsilon_2$ equals to 0, which means there is no uncertainty in future earnings, we have $DEPS_{IF} = MEPS$. As the variance
of $\epsilon$ increases, the difference between $DEPS_{IF}$ and $MEPS$ increases accordingly. The upward bias of $DEPS_{IF}$ in the expectation of future earnings per share increases.

* Diluted EPS by the Treasury Stock Method *

The treasury stock method is widely adopted by the general accounting principle makers. It has been adopted in APB Opinion 15 for calculating the primary EPS for warrants and options. In SFAS No. 128, FASB retained the use of the treasury stock method from Opinion 15 because of its relative simplicity and lack of subjectivity, and its adoption by IASC.

Similar to the calculation of $DEPS_{IF}$, the calculation of diluted EPS by the treasury stock method (denoted by $DEPS_{TR}$) also has two steps. First, an EPS measure by the treasury stock method (denoted by $EPS_{TR}$) is calculated. The treasury Stock method assumes the exercises of option or warrants at the beginning of the reporting period, or the date of issue if later. Its difference from the if-converted method lies in the assumption that the proceeds from issuing new stocks are used to purchase the firm’s existing common stocks. As a result, the treasury stock method avoids the adjustment in the numerator of EPS. $EPS_{TR}$ is expressed as:

$$EPS_{TR} = \frac{E_1}{N + \left(\frac{P_1 - X}{P_1}\right)n}$$

Second, the principle of “no anti-dilution” must also be applied to get $DEPS_{TR}$. The warrants are included in the diluted EPS calculation only if $P_1 > X$. Otherwise,
the warrants will increase the basic EPS. Therefore, $DEPS_{TR}$ is defined as the minimum of the Basic EPS and the EPS calculated by treasury stock method. That is,

$$DEPS_{TR} = \min(BEPS, EPS_{TR}) = \min \left( \frac{E_1}{N}, \frac{E_1}{N + \left( \frac{P_1 - X}{P_1} \right)n} \right) \quad (7)$$

An interesting property of $DEPS_{TR}$ is, given $\varepsilon_2 \sim N(0, \sigma^2)$, $DEPS_{TR}$ decreases as $P_1$ increases. That is,

$$\frac{\partial DEPS_{TR}}{\partial P_1} \leq 0 .$$

This property directly comes from taking derivative of $DEPS_{TR}$ relative to $P_1$. The intuition of this negative relation is that the better future outcomes the market expects, the more likely the future exercise of warrants incurs, and therefore the lower $DEPS_{TR}$.

Now I compare the market EPS to the two diluted EPS measures, one by the treasury stock method and the other by the if-converted method. The analysis shows that like $DEPS_{IF}$, $DEPS_{TR}$ also overstates the EPS information in prices. Furthermore, $DEPS_{IF}$ includes all dilutive warrants as long as the hypothesis exercise of the warrants decreases basic EPS. However, $DEPS_{TR}$ ignores the warrants that are currently out of money, but may become in the money in the future. Therefore, This neglect of $DEPS_{TR}$ makes the misspecification of $DEPS_{TR}$ larger than that of $DEPS_{IF}$.

Following the negative relation between $DEPS_{TR}$ and $P_1$, we immediately have the relationship between $DEPS_{TR}$ and $MEPS$. 

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**Proposition 4.** Given \( \varepsilon_2 \sim N(0, \sigma^2) \), the \( \text{DEPS}_{TR} \) measure decrease with \( \text{MEPS} \).

That is, \( \frac{\partial \text{DEPS}_{TR}}{\partial \text{MEPS}} \leq 0 \).

**Proof:** We know \( P_1 = \frac{1}{r} \text{MEPS} \). In addition, \( \frac{\partial \text{DEPS}_{TR}}{\partial P_1} \leq 0 \) from the above analysis. This implies that \( \frac{\partial \text{DEPS}_{TR}}{\partial \text{MEPS}} \leq 0 \). Q.E.D.

Proposition 4 indicates a negative association between \( \text{DEPS}_{TR} \) and \( \text{MEPS} \). The better the market expectation of future EPS, the lower is the diluted EPS by the treasury stock method. This negative association makes \( \text{DEPS}_{TR} \) less an accurate estimator of the future EPS information incorporated in stock prices. Similar conclusions are drawn by other researchers.

The difference among \( \text{DEPS}_{TR} \), \( \text{DEPS}_{IF} \), and \( \text{MEPS} \) is determined by the distribution of future earnings. Thus, to compare these measures, I start with a special case where future earnings are constant.

**Proposition 5.** At the reporting date, given \( \varepsilon_2 \sim N(0, \sigma^2) \), if \( \sigma^2 = 0 \), then 

\( \text{DEPS}_{TR} = \text{DEPS}_{IF} = \text{MEPS} \).

**Proof:** The proof is straightforward. If there is no uncertainty about future earnings, share holders know whether or not the warrants will be exercised at the reporting date. Therefore there is no uncertainty about the number of future common shares. If \( \frac{E_1}{N} \geq Xr \), shareholders expect the warrants will be exercised. Substitute \( P_1 = \frac{1}{r} \frac{E_1 + nXr}{N + n} \) to equation (7), we have \( \text{DEPS}_{TR} \) equal to \( \text{DEPS}_{IF} \) and \( \text{MEPS} \). If
\[
\frac{E_i}{N} < Xr, \text{ the warrants are expected not to be exercised. We have}
\]

\[
DEPS_{TR} = DEPS_{IF} = MEPS = \frac{E_i}{N}.
\]

Q.E.D.

Under the assumption of no uncertainty about future earnings, the treasury stock method is equivalent to the if-converted method. The common shareholders’ wealth is not affected by the use of the proceeds from the exercise of warrants. FASB’s concern about the reasonability of hypothetical purchase of treasury stock is not an issue. However, this equivalence of \( DEPS_{TR} \) and \( DEPS_{IF} \) holds only under the extreme case, that is, future earnings are certain.

Proposition 5 shows a special case about future earnings. In general, the future earnings are uncertain, i.e., \( \sigma^2 \neq 0 \). In these cases, Proposition 5 usually does not hold, and the distribution of future abnormal earnings determines the relationship among \( DEPS_{TR}, DEPS_{IF}, \) and \( MEPS \), as described by the following propositions.

**Proposition 6.** At the reporting date, given \( \varepsilon_2 \sim N(0, \sigma^2) \), if \( \sigma^2 > 0 \), then \( DEPS_{TR} \geq DEPS_{IF} > MEPS \). Moreover, the difference between \( DEPS_{TR} \) and \( MEPS \), i.e. \( |DEPS_{TR} - MEPS| \), increases with \( \sigma^2 \); and the difference between \( DEPS_{TR} \) and \( DEPS_{IF} \), i.e. \( |DEPS_{TR} - DEPS_{IF}| \), also increases with \( \sigma^2 \).

The proof is given in the Appendix.

This proposition indicates that the diluted EPS by the treasury stock method also overstate market EPS. Moreover, the misspecification of the treasury stock method is more severe than the if-converted method.
FASB mentioned the concerns that the treasury stock method may understate the dilution in EPS. It says, “the treasury stock method understates potential dilution because it gives no dilutive effect to options and warrants whose exercise prices exceed current common stock prices and, therefore, are anti-dilutive under the treasury stock method but may be dilutive sometime in the future” (SFAS No. 128, page33). Core et al. (2002) also indicate that the EPS measure by the treasury stock method is greater than that by the option-based method, as long as the option price is greater than the option’s intrinsic value, which is the stock price minus exercise price. Since options have time value, the option price is always greater than the difference between the stock price and the exercise price. Consequently, the treasury stock method always overstates the earnings per share compared to the option-based method. Their empirical results also weakly support the hypothesis that the treasury stock method systematically underestimates the dilutive effect of employee stock options.

Figure 5 illustrates the $DEPS_{TR}$ along with $BEPS$, $DEPS_{IF}$ and $MEPS$. It shows that $DEPS_{TR}$ increases with the variance of $\varepsilon_2$, i.e. $\sigma^2$. In addition, for any $\sigma^2>0$, $DEPS_{TR}$ is greater than $DEPS_{IF}$.

A special case is loss firms. In SFAS No.128, for a conservative purpose, the loss firm is not required to disclose diluted EPS. The rule claims that when an entity has a loss, including potential common shares in the denominator of a diluted per share computation always results in an antidilutive per share amount. Therefore, diluted EPS is not disclosed by a loss firms.
The theoretical analysis above show that market EPS is always less than or equal to basic EPS. As defined in the rule, basic EPS ignores the presentation of potential future common shares, the overstatement of basic EPS for the market expectation of future EPS is more than that of diluted EPS. Therefore, for loss firms, the upward bias in the disclosed basic EPS is more severe than that for profit firms.

*The growth firms*

In this section, I relax the assumption of time-series property of earnings and focus on the dilutive effects of potential common shares on growth firms. First, I present market EPS for growth firms. Second, diluted EPS by the if-converted method for growth firms are discussed. Finally, diluted EPS by the treasury stock method for growth firms are presented.

The previous discussion focuses on non-growth firms. The above conclusions are based on an essential assumption that earnings follow a random walk process. That means the expectation of future abnormal earnings is zero.

To model the time series property of earnings for growth firms, I assume the earnings of growth firms follow the relationship:

\[(i) \quad E'_{t+1} = gE'_t + \varepsilon'_{t+1}\]
\[(ii) \quad g > 1\]

where:

\(E'_t\) is the earnings for the period \(t+1\);

\(g\) is the growth parameter and greater than 1 by definition;

\(\varepsilon'_{t+1}\) is the abnormal earnings for the period \(t+1\), \(\varepsilon'_{t+1} \sim N(0, \sigma^2)\).
To derive the market EPS for a growth firm, the information incorporated in stock prices is analyzed. At the maturity date $t=2$, the stock price of growth firms has two scenarios. The warrant holders compare the stock price to the exercise price to decide whether to exercise the warrants. When we denote the stock price of growth firms at time $t=2$ as $P'_2$, we have

$$P'_2 = \begin{cases} \frac{g}{1 + r - g} \frac{E'_2}{N} & \text{when } \frac{g}{1 + r - g} \frac{E'_2 + nXr}{N + n} < X \\ \frac{g}{1 + r - g} \frac{E'_2 + nXr}{N + n} & \text{when } \frac{g}{1 + r - g} \frac{E'_2 + nXr}{N + n} \geq X \end{cases}.$$  

The difference between the stock price of growth firms and that of non-growth firms is the discounting rate of future earnings per share. For growth firms, the future growth of earnings is included in the discounting rate for future earnings per share, while this is not true for non-growth companies.

When $\frac{g}{1 + r - g} \frac{E'_2 + nXr}{N + n} \geq X$, the warrants holders exercise the warrants, the future earnings increase by the return of the proceeds from issuing new common stocks, that is, $nXr$. Correspondingly, the number of shares increases to $N+n$. Otherwise, the warrants holders will not exercise the warrants, the future EPS remains to be $\frac{E'_2}{N}$.

At the reporting date $t=1$, investors evaluate the future outcomes from growth firms. The growth model of earnings is used to estimate the future earnings. Furthermore, since there are two scenarios at $t=2$, the stock price at $t=1$, denoted as $P'_1$, is composed by two parts:
\[
P_1' = \int_{-\infty}^{\frac{1}{1+r-g} \left( N + n \right) X - n X r - g E_1'} \frac{g E_1' + \varepsilon_2'}{N} f(\varepsilon_2')d\varepsilon_2' + \int_{\frac{1}{1+r-g} \left( N + n \right) X - n X r - g E_1'}^{\infty} \frac{g E_1' + \varepsilon_2' + n X r}{N + n} f(\varepsilon_2')d\varepsilon_2',
\]

where \( a = \frac{1 + r - g}{g} (N + n) X - n X r - g E_1' \).

The first term on the right-hand side of the equality represents the expected value of common stocks over the range \( \varepsilon_2 < a \), which is the range the warrant holders will not exercise warrants at \( t=2 \). Meanwhile, the probability of the exercise scenario is also evaluated. The second term shows the expected value of common stock over the range \( \varepsilon_2 \geq a \), which is the range the warrant holders will exercise the warrants.

By the definition of growth firms, the future earnings increase each year. The market EPS of growth firms is defined as the expected EPS at maturity date. This definition is different from that for non-growth firms. Since the future earnings of non-growth firms are assumed to be a random walk process, the abnormal earnings is zero mean, the expected future earnings is constant for non-growth firms.

Like the stock price \( P_1' \) at \( t=1 \), the market EPS at \( t=1 \) for growth firms also includes two parts:

\[
MEPS' = \int_{-\infty}^{\frac{1 + r - g}{g} (N + n) X - n X r - g E_1'} \frac{g E_1' + \varepsilon_2'}{N} f(\varepsilon_2')d\varepsilon_2' + \int_{\frac{1}{1+r-g} \left( N + n \right) X - n X r - g E_1'}^{\infty} \frac{g E_1' + \varepsilon_2' + n X r}{N + n} f(\varepsilon_2')d\varepsilon_2',
\]

where \( a = \frac{1 + r - g}{g} (N + n) X - n X r - g E_1' \) as above.

The first term on the right-hand side of the equality represents the expected EPS at the maturity date when the warrants are not exercised. That is, if the future earnings at the maturity date are not high enough, the warrant holders choose to not exercise...
the warrants and the number of common shares remains no change. The second term, on the other hand, shows the expectation of future EPS when the future earnings are good and the warrants are exercised.

The market EPS for growth firms has a construction similar to the market EPS for non-growth firms. Comparing $MEPS'$ to $MEPS$, we can see that: Given all the conditions the same, when $E_i \geq 0, MEPS' > MEPS$. (The proof is given in Appendix D.)

The intuition behind the idea that the market EPS for growth firms are always greater than that of non-growth firms is quite direct. There are two reasons. First, investors expect future earnings growth for growth firms not for non-growth firms. Second, the critical value of future abnormal earnings, $e_{t+1}$, is lower for growth firms than for non-growth firms. Because of the expected future earnings growth, small earnings may be good enough for warrant holders to exercise the warrants.

Figure 6 shows the simulation of market EPS for growth firms. I assume the growth parameter, $g$, to be 1.02. Other parameters are consistent with the simulation assumptions for non-growth firms in Chapter 3. From figure 6, we can see that as the earnings volatility increases, the market EPS for growth firms decreases. This property is the same as the market EPS for non-growth firms.

Next, I discuss the diluted EPS by the if-converted method and by the treasury stock method for growth firms. Diluted EPS is compared to market EPS of growth firms. Moreover, the difference of comparison results between growth firms and non-growth firms are discussed.
First, the if-converted method assumes that the warrants are exercised at the beginning of the reporting period. The return from the proceeds is added to the earnings and the number of common shares is increased by the new shares issued. The if-converted method is not affected by the growth property of earnings. Therefore, diluted EPS by the if-converted method for growth firms and non-growth firms are the same. Diluted EPS by the if-converted method for growth firms is denoted as $DEPS_{IF}^\prime$ and can be calculated by the following formula:

$$DEPS_{IF}^\prime = \min \left( \frac{E_1}{N}, \frac{E_1 + nXr}{N + n} \right).$$

From Proposition 1 and 2, we know that $DEPS_{IF}$ is always greater or equal to $MEPS$. At the same time, given all the conditions the same, $MEPS^\prime > MEPS$. Therefore, the difference between diluted EPS by if-converted method and the market EPS of growth firms is less than the difference of non-growth firms. In other words, the overstatement of diluted EPS by if-converted method of growth firms is smaller than that of non-growth firms.

Now, I discuss the diluted EPS by the treasury stock method for growth firms. The treasury stock method also assumes that the warrants are exercised at the beginning of the reporting period. Furthermore, the treasury stock method assumes that the proceeds from the hypothetical exercise of warrants are used to purchase the firms’ existing shares. The diluted EPS by the treasury stock method for growth firms is denoted as $DEPS_{TR}^\prime$.
\[ DEPS_{TR}^\prime = \min \left\{ \frac{E_1^\prime}{N}, \frac{E_1^\prime}{N + \left( \frac{P_1^\prime - X}{P_1^\prime} \right)n} \right\}. \]

Since given all the conditions the same, \( MEPS > MEPS_\prime \), therefore, the stock price for growth firms is higher than the price for non-growth firms. We know that the current share price determines the denominator of diluted EPS by the treasury stock method. Because under the treasury stock method, higher stock prices mean a higher number of shares in the denominator and therefore more dilution in EPS. Accordingly, the diluted EPS by the treasury stock method of growth firms is smaller than that of non-growth firms.

The overstatement of diluted EPS by the treasury stock method of growth firms is smaller than that of non-growth firms. The reason comes from two factors. Given all the conditions the same, on the one hand, the diluted EPS by the treasury stock method of growth firms is smaller than that of non-growth firms. On the other hand, the market EPS of growth firms is larger than that of non-growth firms. Therefore, the overstatement of diluted EPS by the treasury stock method of growth firms is smaller than that of non-growth firms.

Figure 6 shows the relationship among market EPS, diluted EPS by the if-converted method and diluted EPS by the treasury stock method for growth firms. Generally, these EPS measures of growth firms have the similar pattern with those of non-growth firms. Comparing Figure 5 to Figure 6, we can see that the overstatement of diluted EPS measures of growth firms is smaller than that of non-growth firms.
Chapter Summary

In this chapter, comparing current US rules on EPS to market EPS, several conclusions are drawn about the current EPS rule.

First, market EPS is below the range defined by basic EPS and diluted EPS as long as the expected future abnormal earnings is zero. The analysis shows that both the if-converted method and the treasury stock method overstate the market EPS. The difference between the market EPS and the diluted EPS increases with the variance of future abnormal earnings.

Second, the misspecification of diluted EPS under treasury stock method is larger than that under the if-converted method. There is a negative relation between the diluted EPS by the treasury stock method and the market EPS. Intuitively, the treasury stock method ignores the possibility of options becoming in the money in the future. Therefore, the diluted EPS under treasury stock method is greater than that under if-converted method.

Third, the misspecification of diluted EPS of growth firms is smaller than that of non-growth firms. Given the same current earnings, the market EPS of growth firms is larger than that of non-growth firms. That is, investors expect higher future earnings per share from growth firms than from non-growth firms.
Chapter 5: Empirical Tests and Results

This chapter shows the empirical test of market EPS. The first section explains the sample selection procedure and provide descriptive statistics on the sample firms’ characteristics and employee stock option plans. Then, the second section explains how market EPS is calculated and provides descriptive statistics on the difference between diluted EPS and market EPS (denoted by DIFF) for the sample firms. Finally, the factors affecting DIFF, including option intensity, price-to-strike ratio, and the earnings volatility are examined.

Sample Selection and Descriptive Statistics

My sample is based on 495 Compustat firms in an S&P 500 industrial index. The sample period covers the ten years 1997 through 2006. The first year of the sample is 1997 because it is the first year in which SFAS No. 128 became effective and required disclosure of both diluted EPS and basic EPS.

Financial data and the employee stock options data are from Compustat. The risk-free interest rate and price are obtained from CRSP. The firm-years with missing data from the CRSP or Compustat are removed. Also removed are firm-years with losses because the current accounting rule does not require disclosing diluted EPS for firms with losses. Also eliminated are the most extreme 1 percent earnings volatility observations. The resulting sample consists of 495 firms and 3130 firm-years of observations for fiscal years from 1997 to 2006.
Employee stock options are presented as the potential common shares to estimate the market EPS. The dilutive effects of convertible securities and other potential common share arrangements are ignored because of data unavailability. Moreover, employee stock options are a significant part of potential common shares. In my sample, options outstanding average 8.31 percent of shares outstanding for the period from 1996 to 2006.

Panel A of Table 1 summarizes descriptive statistics about the sample firms’ information. The earnings average $1,107 million, with a median of $409 million. The average number of shares outstanding is 534 million. The average price is $48. Panel B in table 1 reports statistics for the employee stock options. On average, the firms have 43 million employee stock options outstanding, or 8.31 percent of the common shares outstanding. The option plan sizes vary from 0 percent to 53 percent of the common shares outstanding. The average price-to-strike ratio of options is 1.56, indicating the options are substantially in the money.

*Difference Between Diluted EPS and Market EPS*

In this section, I present tests of the theoretical prediction that diluted EPS from the rule overstates the market expectation of future EPS. First I explain how to measure market EPS. Then I examine the difference between diluted EPS and market EPS.

My measure of the market EPS reflects the investors’ expectation of the future EPS. To estimate market EPS, we need the information about characteristics of firms and employee stock options. From equation (4), it is clear that market EPS is
determined by the number of shares outstanding, current earnings, rate of return, the number of shares under options, the exercise prices of options and the variance of abnormal earnings.

The numbers of shares outstanding and current earnings are from Compustat. The figure for the number of shares under option is from the Compustat and represents the number of options at the end of year. The weighted average exercise prices at the end of the year are used in the computation of market EPS. The distribution of the abnormal earnings is assumed to be a normal distribution. The mean of the abnormal earnings for each firm-year is assumed to be zero. The estimation of market EPS requires information about the variance of the firm’s abnormal earnings, $\sigma^2$. Twenty year earnings changes, from 1987 to 2006, are used to estimate the variances of the firm’s abnormal earnings. The yield rates of ten year treasury bonds are used as the proxy of average risk free rate.

The theoretical analysis shows that diluted EPS by both the if-converted method and the treasury stock method overestimate market EPS. The following paragraph examines the difference between the disclosed diluted EPS and the estimated market EPS.

Table 2 compares the two EPS measures, diluted EPS and market EPS. Diluted EPS averages $2.34 while market EPS average $2.31. On average, the difference between diluted EPS and market EPS is $0.02 with a range of $-1.58$ to $4.06$. On average, diluted EPS is statistically larger than market EPS by 1% of diluted EPS (untabulated). Furthermore, 65% of diluted EPS is higher than the corresponding market EPS (untabulated). The results in Table 2 provide evidence in support of the
prediction that diluted EPS under the rule overstates the expected future EPS captured by the market.

The diluted EPS is from Compustat. According to SFAS No. 128, convertible securities, warrants and options, and potential common shares arrangements are all included in the calculation of diluted EPS as long as they decrease the value of diluted EPS. However, the estimated market EPS is calculated only from the employee stock options, due to the restriction of data availability. The estimated market EPS ignores the dilution effects of convertible securities and other potential common shares arrangements. Therefore, the results that show that diluted EPS is on average higher than market EPS are conservative.

*The Factors Affecting the Difference Between Diluted EPS and Market EPS*

In the above section, the results show that diluted EPS is statistically larger than market EPS. The further question is what factors affecting the magnitude of the overstatement of diluted EPS. From Chapter 3, we know that market EPS is affected by three factors: the number of options, the exercise prices, and earnings volatilities. Therefore, this section examines how these three factors affect the difference between diluted EPS and market EPS in three corresponding terms: option intensity, price-to-strike ratio, and earnings volatilities.

The first factor is the intensity of firms’ use of employee stock options. The analysis in Chapter 3 shows that market EPS decreases with option intensity. More potential common shares dilute the expectation of future EPS. Therefore, the average of the difference between diluted EPS and market EPS (Denoted by $DIFF$) is
expected to increase with option intensity. The ratio of the number of options outstanding over the number of stock outstanding is used as the proxy for option intensity.

The second factor is the price-to-strike ratio, which indicates the extent of the option in the money. Since market EPS increases with exercise prices of options, the firm-years with a higher price-to-strike ratio are expected to have a larger $DIFF$. The average price and the weighted average exercise price of options are used to calculate the price-to-strike ratio.

The third factor is the earnings volatility. Propositions 3 and 6 indicate that the difference between diluted EPS and market EPS increases with the variance of earnings. The intuition is that investors expect lower market EPS for the firms with higher risk about future earnings. Therefore, I predict that the firm-years with higher earnings volatility will have larger $DIFF$. I use twenty years earnings data to estimate the earnings variance.

To examine the effects of the three factors on the bias of diluted EPS, $DIFF$ changes by ranking firms on variables: option intensity, price-to-strike ratio, and earnings volatilities respectively were estimated. Based on each factor, the sample was partitioned into quintiles and the mean of the difference between diluted EPS and market EPS for each portfolio was estimated.

Table 3 reports the means of $DIFF$ using the pooled sample of 3130 firm-years from 1997 to 2006. The average $DIFF$ for each option-intensity quintile (first column), for each price-to-strike quintile (second column), and then for each earnings volatility quintile (third column) was then estimated.
The results in the first column of table 3 show the expected positive relation between option intensity and the average \( DIFF \). The average \( DIFF \) for the most intensive stock option user firms is 0.06, compared to -0.04 for the least intensive users. The average \( DIFF \) monotonically increases with option intensity. This result is consistent with the prediction of the positive relation between option intensity and \( DIFF \). Moreover, diluted EPS and market EPS are statistically different for all portfolios. The results for portfolios ranked on price-to-strike ratio do not show any difference among \( DIFF \) for five portfolios. The third column shows that the result on earnings volatility is generally consistent with the predicted positive relation. The average \( DIFF \) for firms with the highest earnings volatility is 0.04. Diluted EPS and market EPS are statistically different for most portfolios ranked on earnings volatility.

**Chapter Summary**

This chapter tests the prediction that diluted EPS overstate market EPS with the sample firms from 1997 to 2006. The results provide evidence that diluted EPS under the rule is on average 1% larger than market EPS, the expected future EPS captured by market. In addition, it also examines the factors affecting the bias in diluted EPS. My empirical results support that option intensity and earnings volatility positively relate to the difference between diluted EPS and market EPS.
Chapter 6: Conclusion

This dissertation develops a theoretical model for computing the earnings per share from the perspective of investors. A benchmark EPS, *market EPS*, is defined as the expectation of future earnings per share.

Market EPS is compared to basic EPS and diluted EPS that are suggested by the current U.S. rule. Through the comparison, I find that market EPS is below the range defined by basic EPS and diluted EPS as long as the expectation of future abnormal earnings is zero. Furthermore, the difference between the market EPS and the diluted EPS increases with the variance of unexpected future earnings. In addition, two methods to calculate diluted EPS suggested by the rule, the if-converted method and the treasury stock method, are evaluated by market EPS. The analysis shows that the treasury stock method overstates market expected future EPS more than the if-converted method does.

I also examine how earnings growth affects diluted EPS and market EPS. Given all conditions the same, the upward bias of diluted EPS of growth firms is smaller than that of non-growth firms.

Empirical results indicate that diluted EPS under the rule is, on average, larger than market EPS by 1% of diluted EPS. For intensive users of options, the difference between diluted EPS and market EPS is as great as 2.7% of diluted EPS. Furthermore, the evidence suggests that option intensity and earning volatility positively relate to the difference between diluted EPS and market EPS.
This dissertation provides insights for accounting policy setters concerned with developing EPS measures useful for investors. The dual presentation suggested by the current U.S. rule is supposed to provide a range of possible EPS values. However, this study shows the theoretical and empirical evidence that the market’s expectation of future EPS is lower than the range defined by basic EPS and diluted EPS. These findings point out a weakness of the thinking behind of the rule.

The results in this study are useful for financial analysis and accounting rule makers, however, this study has limitations on theoretical and empirical implication. The theoretical model to derive the market EPS is based on the assumption of the time series property of earnings. The uncertainty of future earnings questions the reliability of market EPS. On the other hand, in the empirical tests, market EPS is estimated from the employee stock option and does not catch all possible dilution.

In future study, the natural question is how the suggested EPS theory helps to improve the EPS rule? Therefore, the more implications on EPS rule making could be explored. Examples include the better understandings of the current rule, the possible improvement on the current rule and the more efficient use of the EPS numbers.
Appendix A: Proof of Property 1

Proof of Property 1, i.e., \( \frac{\partial MEPS}{\partial \sigma^2} < 0 \).

The definition of MEPS is:

\[
MEPS = \int_{-\infty}^{\infty} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 + \int_{N\varepsilon - E_1}^{\infty} \frac{E_1 + \varepsilon_2 + nXr}{N + n} f(\varepsilon_2) d\varepsilon_2 \tag{1}
\]

The expression of MEPS can be simplified as:

\[
MEPS = \frac{E_1 + nXr}{N + n} + \frac{n}{N(N + n)} \int_{-\infty}^{\infty} (\varepsilon_2 - a) f(\varepsilon_2) d\varepsilon_2 \tag{2}
\]

Let \( a = N\varepsilon - E_1 \)

So \( \frac{\partial MEPS}{\partial \sigma^2} = \frac{\partial Z(\sigma^2)}{\partial \sigma^2} \), where \( Z(\sigma^2) = \int_{-\infty}^{a} (\varepsilon_2 - a) f(\varepsilon_2) d\varepsilon_2 \)

Suppose \( \varepsilon_{i1} \sim N(0, \sigma_1^2) \), \( \varepsilon_{i2} \sim N(0, \sigma_2^2) \) and \( \sigma_1^2 < \sigma_2^2 \). Let \( f_1(\varepsilon_{i1}) \) denotes the density functions \( \varepsilon_{i1} \) and \( f_1(\varepsilon_{i2}) \) for \( \varepsilon_{i2} \). So \( Z_1 \) and \( Z_2 \) are expressed as:

\[
Z_1 = \int_{-\infty}^{a} (\varepsilon_2 - a) f_1(\varepsilon_2) d\varepsilon_2 \tag{3}
\]

\[
Z_2 = \int_{-\infty}^{a} (\varepsilon_2 - a) f_2(\varepsilon_2) d\varepsilon_2 \tag{4}
\]

First, let us consider the case when \( a < 0 \).

Since \( \varepsilon_{i1} \) and \( \varepsilon_{i2} \) are normal distributions, we have

\[
\int_{-\infty}^{a} f_1(\varepsilon_2) d\varepsilon_2 - \int_{-\infty}^{a} f_2(\varepsilon_2) d\varepsilon_2 < 0 \tag{5}
\]
Divide the range $a < 0$ into two parts by $c$ such that, $c < 0$, and $f_1(c) = f_2(c)$. It is clear that

$$\begin{cases} f_1(\varepsilon_2) < f_2(\varepsilon_2) & \text{when } \varepsilon_2 < c \\ f_1(\varepsilon_2) \geq f_2(\varepsilon_2) & \text{when } \varepsilon_2 \geq c \end{cases}$$

Now we have two situations:

(1) When $a \leq c$, for any $\varepsilon_2 \in (-\infty, a)$, $f_1(\varepsilon_2) < f_2(\varepsilon_2)$, so $Z_1 > Z_2$.

(2) When $0 > a > c$, Split the integral to $(-\infty, c]$ and $(c, a]$,

$$\int_{-\infty}^{c} [f_1(\varepsilon_2) - f_2(\varepsilon_2)]d\varepsilon_2 + \int_{c}^{a} [f_1(\varepsilon_2) - f_2(\varepsilon_2)]d\varepsilon_2 < 0 \quad (6)$$

since

$$\int_{-\infty}^{c} (\varepsilon_2 - a)[f_1(\varepsilon_2) - f_2(\varepsilon_2)]d\varepsilon_2 > \int_{-\infty}^{c} (c - a)[f_1(\varepsilon_2) - f_2(\varepsilon_2)]d\varepsilon_2 \quad (7)$$

$$\int_{c}^{a} (\varepsilon_2 - a)[f_1(\varepsilon_2) - f_2(\varepsilon_2)]d\varepsilon_2 > \int_{c}^{a} (c - a)[f_1(\varepsilon_2) - f_2(\varepsilon_2)]d\varepsilon_2 \quad (8)$$

Combining inequalities (6), (7) and (8), we have

$$\int_{-\infty}^{a} (\varepsilon_2 - a)f_1(\varepsilon_2)d\varepsilon_2 - \int_{-\infty}^{a} (\varepsilon_2 - a)f_2(\varepsilon_2)d\varepsilon_2 > 0 \quad (9)$$

that is, $Z_1 > Z_2$.

Therefore, when $a < 0$, $Z_1 > Z_2$.

$$\frac{\partial \text{MEPS}}{\partial \sigma^2} = \frac{\partial Z(\sigma^2)}{\partial \sigma^2} < 0$$

For the case when $a \geq 0$, we also get $\frac{\partial \text{MEPS}}{\partial \sigma^2} < 0$ due to the symmetry of normal distribution.
This proves that \( \frac{\partial \text{MEPS}}{\partial \sigma^2} < 0 \) for any \( a \). Q.E.D
Appendix B: Proof of Proposition 2

Proof of Proposition 2.

At the reporting date, MEPS is expressed as:

\[
MEPS = \int_{-\infty}^{\infty} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 + \int_{N \varepsilon - E_1}^{\infty} \frac{E_1 + nXr + \varepsilon_2}{N + n} f(\varepsilon_2) d\varepsilon_2
\]  

(10)

Let \( a = N X r - E_1 \)

We divide the proof to two cases, \( a < 0 \) and \( a \geq 0 \).

Case 1: When \( a < 0 \), we have

\[
\frac{E_1 + nXr}{N + n} \leq \frac{E_1}{N}, \text{ and } DEPS_{IF} = \frac{E_1 + nXr}{N + n}.
\]

In the first term on the right-hand side of equation (10), \( \frac{E_1 + \varepsilon_2}{N} \leq \frac{E_1 + nXr + \varepsilon_2}{N + n} \) over the range \( \varepsilon_2 < a \). Then,

\[
\int_{-\infty}^{a} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 \leq \int_{-\infty}^{a} \frac{E_1 + nXr + \varepsilon_2}{N + n} f(\varepsilon_2) d\varepsilon_2
\]  

(11)

Putting equation (10) and inequality (11) together, we have

\[
MEPS \leq \int_{-\infty}^{\infty} \frac{E_1 + nXr + \varepsilon_2}{N + n} f(\varepsilon_2) d\varepsilon_2 = DEPS_{IF}.
\]

Case 2: When \( a \geq 0 \), \( DEPS_{IF} = \frac{E_1}{N} \).

Similar to case (1), \( MEPS \leq \int_{-\infty}^{\infty} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 = DEPS_{IF} \). Q.E.D
Appendix C: Proof of Proposition 6

Proof of Proposition 6.

The proof includes three portions.

(1) $DEPS_{TR} \geq DEPS_{IF} > MEPS$

From Proposition 4, we know $\frac{\partial DEPS_{TR}}{\partial MEPS} \leq 0$. From Property 1, we also know $\frac{\partial MEPS}{\partial \sigma^2} < 0$. Therefore, $\frac{\partial DEPS_{TR}}{\partial \sigma^2} \geq 0$.

Proposition 5 shows that when $\sigma^2 = 0$, $DEPS_{TR} = DEPS_{IF}$.

Therefore, for any $\sigma^2 > 0$, $DEPS_{TR} \geq DEPS_{IF}$

From Proposition 2, we know for any $\sigma^2 > 0$, $DEPS_{IF} > MEPS$

Combining these two inequalities, we have $DEPS_{TR} \geq DEPS_{IF} > MEPS$.

(2) $|DEPS_{TR} - MEPS|$, increases with $\sigma^2$.

From the above analysis, we know $\frac{\partial DEPS_{TR}}{\partial \sigma^2} \geq 0$ and $\frac{\partial MEPS}{\partial \sigma^2} < 0$.

Since $DEPS_{TR} > MEPS$, we have $|DEPS_{TR} - MEPS|$, increases with $\sigma^2$.

(3) $|DEPS_{TR} - DEPS_{IF}|$, also increases with $\sigma^2$.

From the above analysis, we know $\frac{\partial DEPS_{TR}}{\partial \sigma^2} \geq 0$ and $DEPS_{TR} \geq DEPS_{IF}$.

Therefore, $|DEPS_{TR} - DEPS_{IF}|$, also increases with $\sigma^2$. Q.E.D
Appendix D: Proof of MEPS’ vs. MEPS

Proof of “Given all the conditions the same, when \( E_1 \geq 0 \), MEPS’ > MEPS”

As we know, MEPS is expressed as:

\[
MEPS = \int_{-\infty}^{\infty} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 + \int_{\infty}^{\infty} \frac{E_1 + nXr + \varepsilon_2}{N + n} f(\varepsilon_2) d\varepsilon_2
\]

MEPS’ is expressed as:

\[
MEPS' = \int_{-\infty}^{\infty} \frac{gE_1' + \varepsilon_2'}{N} f(\varepsilon_2') d\varepsilon_2' + \int_{\infty}^{\infty} \frac{gE_1' + \varepsilon_2' + nXr}{N + n} f(\varepsilon_2') d\varepsilon_2',
\]

where 
\[
a = \frac{1 + r - g}{g} (N + n)X - nXr - gE_1'
\]

All the conditions the same, so \( E_1 = E_1' \) and \( \varepsilon_1 = \varepsilon_1' \).

Let 
\[
S = \int_{-\infty}^{\infty} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 + \int_{\infty}^{\infty} \frac{E_1 + nXr + \varepsilon_2}{N + n} f(\varepsilon_2) d\varepsilon_2
\]

By the definition of g, \( g > 1 \), so \( a < N X r - E_1 \), furthermore, \( S > MEPS \).

When \( E_1 \geq 0 \), we have \( MEPS' \geq S \)

Therefore, when \( E_1 \geq 0 \), \( MEPS' \geq MEPS \). Q.E.D.
Table 1: Descriptive statistics on 3130 firm-year observations from 1997-2006a

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Firm characteristics</strong></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Earnings, $million</td>
<td>$1,107.24</td>
<td>$2,341.50</td>
<td>$0.60</td>
<td>$191.90</td>
<td>$409.00</td>
<td>$980.00</td>
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<tr>
<td>Number of shares outstanding, million</td>
<td>534.00</td>
<td>1,055.35</td>
<td>8.65</td>
<td>117.73</td>
<td>220.96</td>
<td>439.00</td>
<td>10,862.00</td>
</tr>
<tr>
<td>Average Stock Price</td>
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<td>$29.60</td>
<td>$1.38</td>
<td>$29.98</td>
<td>$43.26</td>
<td>$59.39</td>
<td>$509.75</td>
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<tr>
<td><strong>Panel B: ESO portfolio</strong></td>
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<td></td>
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<tr>
<td>Number of options outstanding, million</td>
<td>43.21</td>
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<td>7.50</td>
<td>15.29</td>
<td>37.49</td>
<td>1549.00</td>
</tr>
<tr>
<td>Number of options / shares outstanding</td>
<td>8.31%</td>
<td>5.03%</td>
<td>0.00%</td>
<td>4.68%</td>
<td>7.48%</td>
<td>10.80%</td>
<td>53.11%</td>
</tr>
<tr>
<td>Price-to-strike ratio of options (excludes firms with no options)b</td>
<td>1.56</td>
<td>1.04</td>
<td>0.31</td>
<td>1.07</td>
<td>1.35</td>
<td>1.70</td>
<td>21.85</td>
</tr>
</tbody>
</table>

a. I obtain financial data and option details from Computat and stock price data from CRSP. The sample firm-years with losses are excluded. I remove the most extreme 1 percent earnings volatility observations.
b. The twelve-month average price and the weighted average exercise prices of options are used to calculate the price-to-strike ratio.
Table 2: Descriptive statistics on EPS measures: Diluted EPS, Basic EPS, and Market EPS

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diluted EPS, $</td>
<td>2.34</td>
<td>1.99</td>
<td>0.01</td>
<td>1.11</td>
<td>1.82</td>
<td>2.93</td>
<td>26.85</td>
</tr>
<tr>
<td>Basic EPS, $</td>
<td>2.40</td>
<td>2.05</td>
<td>0.01</td>
<td>1.14</td>
<td>1.87</td>
<td>2.98</td>
<td>27.49</td>
</tr>
<tr>
<td>Market EPS, $</td>
<td>2.31</td>
<td>1.99</td>
<td>-0.80</td>
<td>1.10</td>
<td>1.81</td>
<td>2.91</td>
<td>26.34</td>
</tr>
<tr>
<td>Difference between diluted EPS</td>
<td>0.02</td>
<td>0.21</td>
<td>-1.58</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.05</td>
<td>4.06</td>
</tr>
<tr>
<td>and market EPS, $</td>
<td></td>
<td></td>
<td>(t=6.37)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The sample contains 495 firms and 3,130 firm-year observations from 1997 to 2006. Diluted EPS and Basic EPS come from the compustat. Market EPS is calculated by equation (4):

\[
MEPS_t = \int_{-\infty}^{\infty} \frac{E_i + e_{t+1}}{N} f(e_{t+1})de_{t+1} + \int_{N\epsilon - E_i}^{+\infty} \frac{E_i + e_{t+1} + nXr}{N + n} f(e_{t+1})de_{t+1}
\]

\(E_i\) is the earnings at time \(t\). \(N\) is the number of common shares outstanding at time \(t\).
\(n\) is the number of options outstanding at time \(t\).
\(X\) is the exercise prices of options, estimated as the weighted average exercise price of options.
\(r\) is the risk free interest rate, estimated as the yield of ten year treasury bonds.
\(e_{t+1}\) is the abnormal earnings at time \(t+1\) and \(e_i \sim N(0, \sigma^2)\)
\(\sigma^2\) is estimated by twenty years earnings data.
\(f(e_{t+1})\) is the distribution function of \(e_{t+1}\).
Table 3: Variation in the Mean of the Difference between Diluted EPS and Market EPS:
Mean Varies by Option Intensity, by Price-to-Strike Ratio and by Earnings Volatility

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Ranked on Option Intensity</th>
<th>Ranked on Price-to-Strike Ratio</th>
<th>Ranked on Earnings Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio 1</td>
<td>-0.04 (t=-4.59)</td>
<td>0.02 (t=2.81)</td>
<td>0.00 (t=-0.44)</td>
</tr>
<tr>
<td>Portfolio 2</td>
<td>0.02 (t=2.27)</td>
<td>0.02 (t=1.83)</td>
<td>0.02 (t=4.59)</td>
</tr>
<tr>
<td>Portfolio 3</td>
<td>0.04 (t=4.63)</td>
<td>0.03 (t=3.25)</td>
<td>0.01 (t=1.81)</td>
</tr>
<tr>
<td>Portfolio 4</td>
<td>0.04 (t=6.32)</td>
<td>0.02 (t=5.45)</td>
<td>0.05 (t=4.19)</td>
</tr>
<tr>
<td>Portfolio 5</td>
<td>0.06 (t=6.42)</td>
<td>0.02 (t=4.10)</td>
<td>0.04 (t=3.50)</td>
</tr>
</tbody>
</table>

a. The sample contains 3130 firm-year observations from 1997 to 2006. Diluted EPS comes from the Compustat. Market EPS is calculated by equation (4):

\[
MEPS_t = \int_{-\infty}^{\infty} \frac{E_t + \epsilon_{t+1}}{N} f(\epsilon_{t+1}) d\epsilon_{t+1} + \int_{N\epsilon_t-E_t}^{+\infty} \frac{E_t + \epsilon_{t+1} + nXr}{N+n} f(\epsilon_{t+1}) d\epsilon_{t+1}
\]

\(E_t\) is the earnings at time t. N is the number of common shares outstanding at time t.
\(n\) is the number of options outstanding at time t.
\(X\) is the exercise prices of options, estimated as the weighted average exercise price of options.
\(r\) is the risk free interest rate, estimated as the yield of ten year treasury bonds.
\(\epsilon_{t+1}\) is the abnormal earnings at time t+1 and \(\epsilon_t \sim N(0, \sigma^2)\)
\(\sigma^2\) is estimated by twenty years earnings data.
\(f(\epsilon_{t+1})\) is the distribution function of \(\epsilon_{t+1}\).

b. Option intensity is calculated as the number of options outstanding divided by the number of common shares outstanding.

c. Price-to-strike ratio is calculated as the average share price divided by the weighted average exercise price of options.

d. Earnings volatility is estimated from twenty years earnings data.
Figure 1 The Timeline of Computing Earnings Per Share

Reporting date: $t=1$

Maturity date: $t=2$

$E_1$

$\text{EPS}=\ ?$
Figure 2 MEPS vs. $\sigma$

The plots are based on a firm with 100 outstanding common stocks and 50 warrants with exercise price of $60. Earnings realized in period 1 are $1000. The risk-free rate of return is 10%. The X-axis is the standard deviation of unexpected earnings in period 2, $\varepsilon_2$. MEPS are calculated based on the following expressions:

$$MEPS = \int_{-\infty}^{N \varepsilon_1 - E_1} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 + \int_{N \varepsilon_1 - E_1}^{+\infty} \frac{E_1 + \varepsilon_2 + nXr}{N + n} f(\varepsilon_2) d\varepsilon_2$$
The plots are based on a firm with 100 outstanding common stocks and 50 warrants. Earnings realized in period 1 are $1000. The variance for future abnormal earnings is 500. The risk-free rate of return is 10%. The X-axis is the exercise prices of the warrants. MEPS are calculated based on the following expressions:

\[
MEPS = \int_{-\infty}^{\infty} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 + \int_{N X - E_1}^{+\infty} \frac{E_1 + \varepsilon_2 + nXr}{N + n} f(\varepsilon_2) d\varepsilon_2
\]
The plots are based on a firm with 100 outstanding common stocks and the warrants with exercise price of $60. Earnings realized in period 1 are $1000. The variance for future abnormal earnings is 500. The risk-free rate of return is 10%. The X-axis is the number of the warrants. MEPS are calculated based on the following expressions:

\[
MEPS = \int_{-\infty}^{\xi_1-E_1} \frac{E_1 + \xi_2}{N} f(\xi_2) d\xi_2 + \int_{\xi_1-E_1}^{+\infty} \frac{E_1 + \xi_2 + nXr}{N + n} f(\xi_2) d\xi_2
\]
Figure 5 Comparison of EPS measures based on: (1) BEPS, (2) DEPS\textsubscript{IF}, (3) DEPS\textsubscript{TR}, (4) MEPS.

Comparison of EPS measures based on: (1) BEPS, (2) DEPS\textsubscript{IF}, (3) DEPS\textsubscript{TR}, (4) MEPS

The plots are based on a firm with 100 outstanding common stocks and 50 warrants with exercise price of $60. Earnings realized in period 1 are $1000. The risk-free rate of return is 10%. The X-axis is the standard deviation of unexpected earnings in period 2, $\varepsilon_2$.

BEPS, DEPS\textsubscript{IF}, DEPS\textsubscript{TR} and MEPS are calculated based on the following expressions:

\[
BEPS = \frac{E_1}{N}
\]

\[
DEPS_{IF} = \min \left( \frac{E_1}{N}, \frac{E_1 + nXr}{N + n} \right)
\]
\[ DEPS_{TR} = \min \left\{ \frac{E_1}{N}, \frac{E_1}{N + \left( \frac{P_1 - X}{P_1} \right)n} \right\} \]

\[ MEPS = \int_{-\infty}^{\infty} \frac{E_1 + \varepsilon_2}{N} f(\varepsilon_2) d\varepsilon_2 + \int_{N + r - E_1}^{\infty} \frac{E_1 + \varepsilon_2 + nXr}{N + n} f(\varepsilon_2) d\varepsilon_2 \]
Comparison of EPS measures for growth firms based on: (1) BEPS’, (2) DEPS’\textsubscript{IF}, (3) DEPS’\textsubscript{TR}, (4) MEPS’

The plots are based on a firm with 100 outstanding common stocks and 50 warrants with exercise price of $60. Earnings realized in period 1 are $1000. The growth rate of earnings is 1.02. The risk-free rate of return is 10%. The X-axis is the standard deviation of unexpected earnings in period 2, \(\varepsilon_2\). BEPS’, DEPS’\textsubscript{IF}, DEPS’\textsubscript{TR} and MEPS’ are calculated based on the following expressions:

\[
BEPS = \frac{E'_1}{N}
\]

\[
DEPS'_{IF} = \min \left( \frac{E'_1}{N}, \frac{E'_1 + nXr}{N + n} \right)
\]
\[
DEPS_{TR} = \min \left\{ \frac{E'}{N}, \frac{E'}{N + \left(\frac{P_1 - X}{P_1}\right)n} \right\}
\]

\[
MEPS' = \int_{-\infty}^{a} \frac{gE_1' + \varepsilon_2'}{N} f(\varepsilon_2') d\varepsilon_2' + \int_{a}^{\infty} \frac{gE_1' + \varepsilon_2' + nXr}{N + n} f(\varepsilon_2') d\varepsilon_2',
\]

where \( a = \frac{1 + r - g}{g} (N + n)X - nXr - gE_1' \)
Bibliography


Hong Kong Institute of Certified Public Accountants, 2004. HKAS 33: Earnings Per Share.


