ABSTRACT

Title of Dissertation: **GAINS FROM CONTRACTING FOR US HOG GROWERS**

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Vertical coordination through contracts between farmers and other stages of the agro food chain have been of growing importance in US agriculture. Production contract arrangements between contractors and individual growers have been one of the major vehicles of this emerging system of vertical coordination. Despite the unprecedented success of production contracts as claimed by many through risk reduction, income stabilization, use of improved managerial inputs, and know-how transfer from contractors to growers, contract growers dissatisfied with existing contract payments complain that contractors are extracting too much of contract benefits while growers gain only small, or even negative, returns from contract production. Thus, measuring growers’ gains from contracting, and understanding what determines the returns to contracting, is important for evaluating the policy issues associated with contracting in agriculture. This study examines hog growers’ gains from contracting and explores the distribution of the gains from contracting among contract hog growers.

The purpose of this dissertation is threefold. The first purpose is to review the major issues that have been examined in the literature on principal-agent theory, with special attention to the issues that are important in the agricultural sector in general and hog production in particular. Some further extensions of the basic theories are developed to enable solving the empirical puzzles. Some implications for agents’ gains from contracting in both static and dynamic settings are derived. Related discussion shows how hog contracts relate to standard principal-agent theories. The main finding is that for the most plausible information structure, that is, when growers have partial but better knowledge of their ability than contractors, some low ability growers with below average productivity receive negative gains from contracting on average. This conclusion holds even when renegotiation-proof long-term contracts are in place for each ability distribution. In contrast, none of the growers receives negative gains from contracting when they have complete knowledge of their ability before signing the contract.

The second purpose is an explicit theoretical modeling of hog contracts to theoretically analyze optimal incentive structures for hog contracts. A principal-agent model allowing reservation profit to vary with ability is developed to explore whether some contract growers receive negative gains from contracting on average. The results of this theoretical development suggest a rich set of alternative conditions where negative
average gains from contracting are possible for growers with below average productivity of any particular ability level discernible by the contractor. These losses are likely to be repeated under long-term contracting when ability is a permanent random draw for the grower that is different than expected. Even low-ability growers with above average productivity can experience an ex post loss from contracting.

The third purpose of this dissertation is to test the main theoretical findings on contract growers’ gains from contracting using revealed preference data from the well-known Agricultural Resource Management Survey (ARMS) for 2004. In order to do this, contract growers’ gains from contracting are measured using standard impact evaluation methods. By going beyond typical estimation of how contracting affects average growers’ profits, estimates are developed to show how high-profit growers are affected differently from low-profit growers, and whether some growers are worse off with contracting. The results are especially relevant for policy analysis regarding hog contracting because it shows what share of contract growers lose from contracting and identifies their characteristics. The impact distribution of contract growers’ gains is also explored using quantile regression. The estimated growers’ gains from contracting are then used to evaluate theoretical predictions of the hog model.

The main empirical findings of this research can be summarized as follows. First, both risk reduction and limited credit are important motivations for hog contracting. Second, the sorting effect is positive, implying that contract growers tend (because of the effect of unobservables) to choose contracting because of a comparative advantage in doing so. A positive selection bias is estimated, which tends to give contract growers a comparative disadvantage from independent operation. Third, high ability growers earn higher profits on average than low ability growers as predicted by the hog contracting model. Fourth, the mean effect of contracting for contract growers (ATET) is positive for all contract growers. However, when contract growers are divided into quartiles by size, the ATET is positive only for the lower three quartiles whereas it is negative for the highest. Fifth, the ATET decrease over quantiles of the profit distribution for contract growers and the ATNT decreases over quantiles of the profit distribution for independent growers. Sixth, one third of the contract growers receive negative gains from contracting. Below average productivity growers lose from contracting as predicted by the hog contracting model. Seventh, the mean effect of contracting for independent growers (ATNT) is negative. Eighth, the ATET exceeds the ATNT, meaning that independent growers would gain less than contract growers had they contracted. Ninth, contract and independent growers are different with respect to the productivity of the variable factors of production but unilateral technological superiority of one group to the other is not found. Finally, the results suggest that small growers will be forced either to exit the hog business or expand operations regardless of their contracting status.
GAINS FROM CONTRACTING FOR US HOG GROWERS

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DEDICATION

This dissertation is dedicated to my parents
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Chapter 1:

Introduction

Vertical coordination through contracts between farmers and other stages of the agro food chain have been of growing importance in US agriculture. Production contract arrangements between contractors, often referred to as integrators, and individual growers have been one of the major vehicles of this emerging system of vertical coordination. Even though the number of farms using contracts in US agriculture grew slowly from 6 percent in 1969 to 11 percent in 2001, the increase in the share of the value of production under contracts grew from 12 percent in 1969, to 28 percent in 1991 and 36 percent in 2001 (MacDonald et al, 2004).

Although most of the value of the contracted production was produced under marketing contracts, the share of contracted value under production contracts was remarkable. The share of the value of production under production contracts went up from 10.6 percent in 1996-97 to 17.5 percent in 2003 (MacDonald and Korb, 2006). The share of poultry and eggs produced under production contracts was 92.3 percent in 2001-2002 (MacDonald and Korb, 2006). The share of hogs produced under production contracts had reached 50.4 percent by 2003 (MacDonald and Korb, 2006). This growing share of the value of production under production contracts in agriculture has raised growing concern from various quarters about the impact of contracting on the parties, particularly on the growers.

Some have argued that production contracts have benefited growers by reducing risk and stabilizing income with low capital requirements (Rhodes, 1989; Rhodes and Grimes, 1992; Johnson and Foster, 1994; Knoeber and Thurman, 1995; USDA, 1996;
Martin, 1997; Vukina and Foster, 1998). Some have pointed out that contract production appears to have aided expanding broiler and hog operations by providing the capital necessary for operations of unprecedented size (Kliebenstein and Lawrence, 1995). In addition, contracting appears to raise farm productivity by promoting growers' use of improved managerial inputs, and by transferring know-how from contractors to growers rapidly (Just, Mitra, and Netanyahu, 2005). Others argue that contracting raises farm productivity through technology adoption facilitated by growers’ easy access to credit (McBride and Key, 2003).

Despite the unprecedented success of production contracts as claimed by many, an increasing number of dissatisfied contract growers are complaining about specific features of the contracts in place in recent years (Vukina, 2003). Many contract-growers, dissatisfied with existing contract payments, complain that contractors are extracting too much of contract benefits while growers gain only small, or even negative, returns from contract production (Kolmer et al. 1963; Aho, 1988; Morison, 1996a and 1996b; Guebert, 1996; Russell, 1996; Lipton, 1997). Concerns have been raised about the bargaining power disparity in contractual arrangements that goes against the growers’ interest. Concerns have also been raised about the risk shifting implication of large asset-specific investments by contract growers in broiler and hog operations. The loss of transparency in transactions because of the confidentiality clauses that limit potential contract growers from evaluating and negotiating contract terms raises concerns (Iowa Department of Justice, 2001). State and federal lawmakers are taking such concerns seriously and moving forward to exert concerted efforts to place some legal constraints on the type of contracts that both the parties can sign (Vukina, 2003).
Nonetheless, few investigative attempts have been made to confirm or reject such claims by contract growers or to quantify the distribution of growers’ gains from contracting. Measuring growers’ gains from contracting and understanding what determines the returns to contracting is important for evaluating the policy issues associated with contracting in agriculture. This study examines hog growers’ gains from contracting and explores the distribution of gains from contracting among contract growers.

1.1 Background

The typical production contract is an agreement between a contractor (also called an integrator) and a farmer detailing specific farmer and contractor responsibilities for production inputs and practices, as well as a mechanism for determining payment. According to many livestock production contracts, the grower cares for the animals, and usually provides land, labor, housing, utilities, and other operating expenses, such as repairs and maintenance. The contractor provides feed, veterinary supplies and services, and young animals. Expenses for fuel and litter can be shared or paid by either party, depending on the nature of the contract. Typically, the contractor also owns and operates hatcheries, feed mills, and a processing plant, and provides transportation of feed and live animals (Tsoulouhas and Vukina, 2001; Knoeber, 1989). Contractors rather than farmers often retain ownership of the commodity during the production process and marketing of the products.

According to Tsoulouhas and Vukina (2001), nearly all livestock production contracts have a fairly similar payment structure, taking the form of a two-part piece rate
tournament or a fixed performance standard. A two-part piece rate tournament consists of a fixed base payment per pound of live meat produced and a variable bonus payment based on the grower’s performance relative to other growers. Performance is largely driven by the effectiveness with which growers convert feed to live meat. Often, the performance is measured by the so-called settlement cost, which is obtained by combining feed with other contractor’s costs (animals to be grown, medication, etc) divided by the total pounds of live weight produced. The relative performance is determined by comparing the individual grower’s performance with the group average for a given flock of animals in the same area. For a feed-conversion ratio below average (that is, for above-average performance) the grower receives a positive amount over the base payment and for a feed-conversion ratio above average (that is, for below average performance) the grower receives a penalty (Tsoulouhas and Vukina, 2001; Knoeber, 1989).

A second type of grower remuneration approach is a two-part piece rate based on a fixed performance standard. It consists of a fixed base payment per pound of live meat produced and a variable bonus payment based on the grower’s performance compared to a predetermined feed conversion standard. In this case, the benchmark is not determined by a contest among the growers as in a tournament. Instead, the benchmark is a predetermined technological constant. Another version of the fixed performance standard is a discrete scheme where, for a given weight of the finished animal, the contract design specifies different bonus payments for each different feed conversion interval (bracket) Tsoulouhas and Vukina, 2001).
Two more variations of the payment scheme are sometimes used: (i) a version where, along with a base payment per live weight, the bonus payment is paid per head of the delivered animal, and (ii) a version where there is no direct base payment but the entire payment per pound of live weight delivered varies with the bracket in which the individual grower’s feed conversion lies (Tsoulouhas and Vukina, 2001). Two-part piece-rate tournaments are used by almost all broiler contractors whereas they are almost nonexistent in the hog industry. On the other hand, fixed performance standards dominate hog production contracts, but they are almost nonexistent in the broiler industry.

The poultry industry is one of the first agricultural sectors to use production contracts widely. The share of poultry and eggs produced under production contract is 87.2 percent in 2003 (MacDonald and Korb, 2006), with the reminder mainly raised at processor-owned facilities. Beginning in the 1950s, the poultry industry experienced a remarkable change from a “backyard” family owned industry to a specialized hatchery and broiler operation. Now it produces more than 900 million birds for meat per year (Madison and Harvey, 1997). There has been a substantial increase in productivity and decrease in the real price of broilers during this expansion phase (Lasley, 1983). Today, the broiler industry is one of the most competitive and tightly coordinated sub-sectors in the U.S. food and agricultural sector (Schrader, 1981). This industry is often cited as an eventual model of the organization that may portray most of U.S. farming in near future (Perry, Banker, and Green, 1999).

The hog industry appears to be following a path similar to broilers as it moves toward a vertical organization with widespread use of production contracts. Seventy-five years ago nearly every farm raised some hogs. Hog production has changed incredibly
from that state in the last quarter century (Rhodes, 1998). Since 1920, the number of farms in the U.S. has fallen dramatically. At the same time, the percentage of U.S. farms with hogs has also been falling dramatically. Most farms discontinuing hog production had fewer than 100 head in inventory.

At the same time the number of farms with hogs has been declining, the concentration of hog production on remaining farms has been increasing dramatically in recent years. Total inventory on farms with at least 2,000 head in inventory rose from 16.6 million head in 1992 to 28.6 million head in 1996. Farms with at least 2,000 pigs in inventory accounted for 51 percent of total U.S. swine inventory in 1996 although they had represented only 3.1 percent of all farms having at least one pig (Zering, 1998).

Production contracts are widely used in these rapidly expanding hog operations. The share of hog production under contract rose remarkably from only 5 percent in 1992 to 40 percent in 1998. But the aggregate data conceal sharp and striking changes that occurred in specialized hog operations. Production contracts grew from 8 percent in 1992 to 83 percent in 1998 on specialized feeder pig operations. Among specialized hog feeding operations, production contracts grew from 22 percent in 1992 to 62 percent in 1998. By comparison, the growth of contracting among farrow-to-finish operations was less impressive during that period (McBride and Key, 2003).1 In addition, hog growers realized an unprecedented growth in average farm size with contractual arrangements in place.2

1 By comparison, the share of cattle produced under production contracts has grown only from 11.1 percent in 1996-97 to 25.4 percent in 2003 (MacDonald and Korb, 2006).
2 Average hog sales and contract removals per farm increased 174 percent between 1992 and 1998, from 945 to 2,589. There was an extraordinary growth in the average size of specialized hog operations during that period. Feeder pig operations increased their sales and removals by an average of 400 percent. Hog finishing operations showed an average increase of 240 percent in sales and removals. In contrast, farrow-
In view of this dramatic growth in contract production, a fundamental question attracting much research is what motivates almost all broiler growers and half of hog growers to participate in contract production? Why have farmers and their buyers shifted to contracts from spot markets? And what are the implications for farm profits?

1.2 Motivation for Contracting

Two wide-ranging explanations – risk-sharing and transactions cost – have been used to explain the choice between spot markets and contracts. The risk-sharing approach considers contracts as a means of reducing price and production risks faced by farmers and shifting them to the contractors who are more able to bear the risks. The transactions cost approach highlights the costs of using spot markets to organize transactions and considers contracts as a means to reduce those costs. Transaction costs arise because of the conflicting interests between the parties (the contractor and the growers) when the grower's efforts cannot be easily monitored by the contractor and where the output is not influenced by the grower's effort alone but by factors beyond the control of the grower. Examples of such transactions costs include costs associated with negotiation, supervision, and enforcement of spot transactions.

The most important reason cited by hog growers for choosing contract farming is risk reduction (Rhodes, 1989; Rhodes and Grimes, 1992; Johnson and Foster, 1994; Kliebenstein and Lawrence, 1995; Lawrence and Grimes, 2001). This strain of literature, which emphasizes the reduction of farmers’ economic risks through contracting with

to-finish operations showed a smaller expansion of only 40 percent in their sales and removals during that period. (McBride and Key, 2003).
contractors, appears to be aligned with the risk sharing approach. But this approach has an inherent moral hazard problem associated with the extent of risk that can be efficiently shifted to contractors. Risk-reducing contracts eliminate growers’ incentives to carry out standard management practices, and can therefore result in higher total costs through lower effort (Knoeber, 2000). Thus, the transactions cost of providing incentives limit the extent of risk that can be efficiently shifted to contractors.

Turning to the transactions cost explanation, Williamson’s (1985) approach, which explains vertical coordination by its lower transaction costs compared to market exchange, has become the conventional wisdom. Contracts can increase efficiency in organizing production, making the adoption of large-scale and specialized techniques easy. Thus, contracts can reduce transaction costs through lower costs or higher product quality (Knoeber, 2000; Lawrence, Schroeder, and Hayenga, 2001; Hueth and Hennessy, 2002). Large farms, which are handling rapidly growing shares of agricultural production, use contracts much more than other farms (Hoppe and Korb, 2002).

Large farms make large investments to exploit the benefit of economies of size and scale in operations. But these large investments are often asset- and site-specific in nature. The specificity arises when assets are much less useful, and hence less valuable, in any other use than the one for which they were initially designed. When production requires investing in an asset that is specialized to a particular trading partner, any deal made prior to investing in the specialized asset may not be enforceable once the investment is made. The non-investing party may have an incentive to use his newly created bargaining power by demanding more favorable terms (MacDonald et al, 2004).
For example, large broiler and hog farms make large asset and site-specific investments in chicken and hog facilities, respectively, to exploit the benefit of economies of size and scale in these operations. But these site and asset specific investments create the so-called “hold-up” problem discussed by *Klein, Crawford and Alchian* (1978). Contracts can mitigate this hold-up problem because farmers can be guaranteed of a compensation scheme before making an investment, although typical contracts do not cover the full economic life of the facilities. In fact, in some cases, processors may help farmers finance investments directly through the contractual arrangements (*MacDonald et al*, 2004).³

Production contracts that require both parties to invest in assets specialized to the other further help to alleviate the hold-up problem. However, this may not be the case with broilers and hogs even though growers invest in feeding facilities and contractors invest in breeding facilities, feed mills, and processing plants. The weakness in applying this argument to broiler and hog production is that specific investments from both sides alone may not cancel out the opportunistic incentives and consequences. Rather, the extent of the stakes that each party has in the other’s specific investment must be weighed in drawing such a conclusion.

The role of transaction costs reduction using livestock production contracts is reviewed by *Knoeber* (1989), *Frank and Henderson* (1992), *Barry, Sonka and Lajili* (1992) and *Sporleader* (1992). Contracting is believed to lower the transaction costs associated with growers’ uncertainty; resolve the common problem of asymmetric information between growers and contractors about product quality; and improve

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³ Another reason growers enter contract farming is to obtain credit for financing the investment needed for building facilities (*Kliebenstein and Lawrence*, 1995).
coordination of product delivery (McBride and Key, 2003; Knoeber and Thurman, 1995). An efficient contract should solve these problems with the lowest transaction costs.4

Finally, a resource-providing contract, which is a better alternative for the purpose of providing the contractor a consistent supply of quality meat, both reduces the hold-up for growers by requiring less investment and relaxes the grower’s credit constraint, freeing funds to use factor inputs at a more efficient level (Hueth and Hennessy, 2002).

Regardless of motivation, contracting is expected to add value in some way if growers and contractors are to go to the trouble of setting up contractual relationships. But the question is: What is the grower’s gain from contracting? Additional claims have been that growers benefit from having an assured market, a higher price, and access to a wider range of production inputs (USDA, 1996). Others claim that contracts benefit growers by offering opportunities to earn income with low capital requirements, by easing cash flow constraints, and by allowing enterprise diversification on the farm (Vukina and Foster, 1998).

Based on broiler data, Knoeber and Thurman (1995) estimate that a substantial amount of risk is shifted from growers to contractors through contracting. Martin (1994) has argued that the extent of risk shifting is not as prominent in swine production as for broilers. However, Martin (1997) argues that the contractor provides most of the variable

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4 However, it has been claimed that transaction costs reduction by contracting is not the most important reason for contract farming in hog operations. The increased returns from being a leader in reducing production costs have been the main incentive for contract farming. It has been argued that most hog operations have been induced to contract more by high returns on equity in hog production than the small savings attainable in transaction costs (Rhodes, 1993).
inputs and guarantees a payment to the hog grower. So considerable risk associated with input and output price variability is shifted from the grower to the contractor.

1.3 Complaints about Gains

Despite the risk shifting properties often claimed for contracts, many contract growers, dissatisfied with existing payment mechanisms complain about their gains from contracting. In recent years, the number of contract growers complaining about the features of the contracts has been increasing (Vukina, 2003). Growers complain that contractors receive large contract benefits while the growers gain only small, or even negative, returns from contract production (Kolmer et al. 1963; Aho, 1988; Morison, 1996a and 1996b; Guebert, 1996; Russell, 1996; Lipton, 1997).

Since both broiler and hog production involves large asset- and site-specific investments in chicken and hog plants, respectively, concerns have been raised about the possibility that contractors are extracting quasi-rents from contract growers. Large specific investments may reduce the bargaining power of contract growers, making growers vulnerable to changes in contract terms (Shelanski and Klein, 1995). Even though risk reduction is the primary motivation for contracting, contracts do not fully insulate producers from economic risks because of the need to maintain incentives. As a result, both hog and poultry producers face considerable production and quality risk (Hueth and Hennessy, 2002). This may be another reason for grower discontent.\footnote{Tsoulouhas and Vukina (2001) consider poultry growers complaints that tournament schemes are unfair because the set of growers in a group continually changes. For a given set of production outcomes, group composition can substantially affect payment outcomes. Thus, tournament schemes may be replacing...}
1.4 Legal Limitations

Few investigative attempts have been made to confirm or reject the above claims by contract growers or to examine the how growers’ gains from contracting vary. Nevertheless, state and federal lawmakers’ moves to introduce legislation for growers’ protection is further fueling growers’ expression of discontent. Out of grower discontent, some states have already considered legislation to protect growers (Vukina, 1997; Lewin, 1998; Hamilton and Andrews, 1992). On the federal level, policy makers are also taking such concerns seriously and moving toward necessary steps to protect contract growers. Concerns regarding the implications of reorganization and the increasing use of production contracts in certain sectors have led to calls for legislation to protect producers in these sectors from unfair business practices. With this pressure from grower circles to adopt more concrete regulatory measures to protect them, empirical analysis that analyzes the impacts of contracting on growers profits and growers’ vulnerabilities to loss by contracting is sorely needed.

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6 Contractors in some Southern states have blocked legislative proposals regulating broiler contracts. One of those attempts failed in North Carolina in 1993 when an attempt was made to introduce a bill prohibiting payments to a grower based on relative performance (Vukina, 1997). Various forms of legislation aimed at regulating contracts were passed in Minnesota, Wisconsin, and Kansas in the early 1990s (Lewin). Iowa was the first state to adopt anti-vertical integration legislation for livestock packing firms. The legislation was amended in 1988 to prohibit contracting by packers. According to Hamilton and Andrews (1992), eight states – Iowa, Kansas, Minnesota, Nebraska, North Dakota, Oklahoma, South Dakota, and Wisconsin – have passed anti-corporate farming legislation. They also mention adoption of some form of legislation regulating production contracts in agriculture in Iowa, Kansas, and Minnesota.
1.5 Research Objectives

The closely related literature on regulation has criticized government regulation on the grounds that any regulatory action targeting distribution will interfere with the ability of economic parties to achieve efficient outcomes. Certainly, if new public policy in this area is to be informed, research must be conducted on the economic impact of contractor practices and procedures on contract growers. Such research should be conducted before regulatory intervention, particularly given that regulatory intervention is largely irreversible once implemented.

In order to measure the impact of contracting on contract growers, research is needed to determine what the returns to farming would have been had each grower chosen not to contract. This information is not directly observable because contract growers do not produce independently at the same time. To overcome this problem of missing data, impact evaluation methods use the mean returns of independent growers as the counterfactual for the mean returns of contract growers (although without sophisticated methods, these simple analyses can be quite misleading).

Since the broiler and hog industries have the potential to become role models for rest of the US agriculture, and because of the importance of contracting in these industries, they are ideal candidates for examining the incentives underlying contract design and calculating the contracting gains for contract growers from contract production. Unfortunately, at this point in time, data on broiler operations are available only on contract broiler growers. Data on independent broiler growers is almost nonexistent because the share of broilers produced under contract exceeds 90 percent, with the rest mostly raised at processor-owned facilities. Although the new larger hog
growers are overwhelmingly engaged in contract production, unlike broiler industry, the 
hog industry still has a large core of independent hog producers who sell on the open 
market. Thus, hog contracting seems to offer the only good opportunity for this analysis.

The dissertation uses the *Agricultural Resource Management Survey Phase III, 
ARMS III V4) data to examine the impact of contracting on contract hog growers.7 These 
data present a typical impact evaluation problem which requires obtaining credible 
estimates of the counterfactual returns that would have been realized had contract 
growers not participated in contracting. Conventional approaches to impact evaluation 
problems assume that the impact of participation is the same for every grower.8 Such 
approaches do not account for heterogeneity in responses to participation. Hog growers 
are heterogeneous because they operate in different regions with different backgrounds, 
differing innate abilities, different farm sizes, different levels of risk aversion, use 
different levels of inputs, etc. Assuming a common impact of contracting is not sensible 
when growers are heterogeneous.

In recent years, statistical techniques have been developed to estimate models in 
which the impact of participation differs across participants. One implication of 
heterogeneity is that it may cause self-selection bias by affecting participation in 
contracting.9 A major goal of this dissertation is to measure the impact of contracting

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7 Because the greatest mix of behavior is found in the feeder pig-to-finish category, and to maintain 
maximum comparability by choosing a single type of operation, this study focuses only on feeder pig-to-
finish producers.

8 Participation will be used interchangeably to refer to contract hog growing, and nonparticipation will refer 
to independent growing.

9 Contracting may have different effects on different participants. If no one can predict in advance who will 
gain more and who will gain less or who will lose, the variation in impacts will have no effect on who 
initially participates in contracting. In this case, the typical self-selection problem may be reduced. But this 
is not the case for hog growers. Hog growers face predetermined standards, and know the production
when the impact is heterogeneous. For this purpose, sample selection bias is corrected in a parametric fashion using two-step estimation procedures introduced by Heckman (1979).

While systematic heterogeneity in the impact of contracting is recognized, the mean impact measures do not tell the whole story about contracting effects. Measuring dispersion of contracting effects is required to characterize completely the consequences of contracting and to understand the discontent among contract growers. With heterogeneous impacts, the mean impact of contracting may be large and positive despite unfavorable outcomes for many contract growers. The second major research goal of this dissertation is to explore the differential effects of contracting on contract growers using quantile regression techniques. These techniques allow investigation of contracting effects at various quantiles of the conditional profit distribution. Hence, quantile treatment effects at various quantiles of the conditional profit distribution are estimated to perform an in-depth examination of the effect of contracting, and to examine the inter-quantile differences of contracting impacts (Heckman, 1979; Ichimura, 1993; Newey, 1991; and Buchinsky, 1998).

1.6 Organization of the Dissertation

The outline of this dissertation is as follows. The next four chapters review the major issues that have been examined in the literature on principal-agent theory, with special attention to the issues that are important in the agricultural sector in general and environment associated with production uncertainty. They participate in contract production based on subjective expectations of their own productivity. In this case, the grower-specific component of the impact may affect participation in contract production. As a result, based on the prior expected outcomes (or productivities), those participating in contract production may be systematically different from nonparticipants. This systematic difference between the two groups may cause self-selection bias.
hog production in particular. These chapters review some empirical studies of incentives in the agricultural sector, examining how these studies relate to the basic theoretical predictions. Then some further development and extensions of the basic theories are discussed to enable solving the empirical puzzles. Some implications for agents’ gains from contracting are also derived.

Chapter 2 begins with a brief review of the general theory of incentives by discussing the frictions that lie at the heart of incentive problems. The principal's optimal responses to these frictions are explored, taking as given the characteristics of the agents with whom the principal interacts in a non-repeated setting. Since Prendergast (1999), Gibbons (1987), Sappington (1991), Hart and Holmstrom (1987) and Laffont and Maskin (1982) offer excellent recent surveys of principal agent theories, I present only a brief and selective review focusing on the aspects that are particularly relevant for hog contracting. Then in chapter 3, heterogeneity in the characteristics of agents and principals is introduced in a multiple-agent setting and theoretical predictions of agents’ gains from contracting are derived.

Chapter 4 explores the principal's optimal responses to heterogeneous agents in a setting of repeated contracting, and examines whether the theoretical predictions of chapter 3 hold in a repeated or dynamic setting. The question of how the theoretical predictions fit hog contracting is addressed in chapter 5. In this chapter, I show how hog contracts relate to standard principal-agent theories, and refine theoretical predictions of agents’ gains from contracting applicable to hog contracting parties, and especially to contract growers. Chapter 6 models hog contracts explicitly with separating contract parameters and explores whether some growers are left with negative gains from
contracting on average with this separation. Possibilities are also explored for uniform contracts based on payment parameters.

Chapter 7 reviews the econometric methods that provide all the necessary techniques to estimate not only the average effects of contracting but how contracting affects individual growers in the overall distribution of growers. Finally, chapter 8 describes the data used for estimation, provides a summary analysis of the variables used in this study. Then results are presented and discussed regarding the estimated contracting effects on revenues and costs employing two-step estimation methods and related quantile regression techniques. The chapter ends with a summary of the results.
Chapter 2:

The Elementary Theory of Incentives

2.1 Introduction

In the last few decades, the principal-agent model has received considerable recognition and attention as an important analytical device in the study of incentive schemes and contracts among economic agents. Whenever there are gains to specialization there is likely a relationship in which one party (agent) acts on behalf of another (principal) because of comparative advantage. If the agent could costlessly be induced to internalize the principal’s objectives, there would be no reason to study agency theory. This problem becomes interesting only when objectives of the parties cannot be automatically aligned with each other. In principal-agent theory, the central concern is how the principal can best induce the agent to perform as the principal would prefer, taking into account the difficulties in monitoring the agent's activities.

Economic relationships in which one party (the principal) wishes to affect the actions of another (the agent) by means of incentives are ubiquitous. Examples abound including workers supplying labor to a firm, managers acting on behalf of owners, doctors serving patients, lawyers advising clients, the government taxing its citizens to provide government services and a regulator controlling firms. All of these examples are substantial problems in their own right (Sappington, 1991).

Under certain circumstances, it is possible for a principal to induce agents to behave exactly as the principal would if the principal shared the agents' skills and knowledge. By describing these circumstances, it becomes possible to pinpoint the sources of friction between principal and agent that typically preclude this ideal
arrangement. Section 2.2 of this chapter introduces the basic moral hazard model, where the agent chooses his effort before he observes random states of nature, and discusses various aspects of it. Under the same information structure, section 2.3 discusses a simple closed-form agency model in which linear schemes are optimal. Section 2.4 explores the limited liability contracts that arise from a specific information structure where the agent observes random states of nature before he chooses his effort. Section 2.5 explains precontractual asymmetric information contracts that arise from another class of information structure where the agent observes random states of nature even before he accepts or signs the contract. Finally, section 2.6 describes the results of some of the empirical studies that test the basic theoretical predictions.

2.2. The Basic Moral Hazard Model

To set the stage, consider the classic example of the principal-agent relationship between a worker (agent) and a firm (principal) where the agent works for the principal. Suppose that there is only one firm and one agent. There are two widely used formulations of the agency problem – the state-space formulation and the parameterized distribution formulation – each with its own merits.

To outline the general structure of the hidden action model with the state-space formulation, suppose the agent makes an effort $e$, unobservable by the principal or any third party, to produce a verifiable output $\pi$. The effort $e$ affects the probability distribution of the output. Let $\theta$ represent a state of nature drawn from a distribution $G(\theta)$, with a density $g(\theta)$. The agent’s effort and the state of nature jointly determine the verifiable output $\pi = \pi(e, \theta)$. For the nonobservability of agential effort to have any
consequence, the agent’s effort must not be perfectly deducible from observable $\pi$.\(^{10}\) Higher realizations of either the productivity parameter, $\theta$, or the agent’s effort, $e$, both increase the agent’s expected output. The state-space formulation of the agency problem was developed by *Wilson* (1969), *Spence and Zeckhauser* (1971), and *Ross* (1973). Its main advantage is that the technology is presented in what appears to be the most natural terms.

The parameterized distribution formulation provides an equivalent way of representing the principal-agent problem that yields more economic insights. Assume that the firm’s profit $\pi$ is stochastically related to $e$. By the choice of $e$, the agent effectively chooses a distribution over $\pi$, which can be derived from $G(\theta)$ via the technology $\pi(e,\theta)$. That is, a technology represented by $\pi(e,\theta)$ given the distribution of states, $G(\theta)$, generates the distribution of output, $F(\pi | e)$, with corresponding density $f(\pi | e)$ where $f(\pi | e) > 0$ for all $e \in E$ and all $\pi \in [\underline{\pi}, \overline{\pi}]$. Thus, any potential realization of $\pi$ can arise following any given effort choice by the agent. This parameterized distribution formulation was pioneered by *Mirrlees* (1974, 1976) and further explored by *Holmstrom* (1979). In the discussion that follows, the principal’s problem is stated mathematically with a parameterized distribution and then the state-space approach is used to find a closed form solution.

Although the principal cannot observe the efforts of the agent, she can potentially overcome the unobservability problem through a set of signals that are correlated with the agent's effort. She can then condition the agent’s payments for exerted effort on the set of

---

\(^{10}\) For example, if $\pi = e\theta$, then performance is proportional to the farmer’s effort and to the amount of rainfall, $\theta$. If performance does not vary with $\theta$, the magnitude of the agent’s effort can be inferred perfectly from $\pi$, making the incentive problem a trivial one. To consider the interesting and realistic case, I assume, as is standard, that although the principal’s profits are affected by $e$, they are not fully determined by it.
signals that are correlated with the agent's effort. One such signal of effort is output $\pi$. The principal’s problem is to construct a reward scheme $w(\pi)$ that takes output into account to determine payments to the agent. The price of output is assumed to be 1. The principal’s profit (output minus wage costs), is $\pi - w(\pi)$. The agent is assumed to be an expected utility maximizer with a Bernoulli utility function $u(w,e)$ over his wage $w$ and effort level $e$. This agent’s utility function is assumed to satisfy $u_w(w,e) > 0$, $u_e(w,e) < 0$, and $u_{ww}(w,e) \leq 0$ for all $(w,e)$ where subscripts denote partial derivatives). That is, the agent prefers more income to less, is weakly risk averse over income lotteries, and disikes a high level of effort. The agent and the principal agree on the distribution $G(\theta)$, the technology $\pi(e,\theta)$, and the utility and cost functions.

Suppose the agent’s utility is additively separable in the form $u(w,e) = u(w(\pi)) - c(e)$. The principal is endowed with all of the bargaining power in this simple setting, and thus she can make a "take-it-or-leave-it" offer to the agent. An offer here specifies the agent’s wage payment as a function of observed output $\pi$. The principal's problem is to devise a payment scheme $w(\pi)$ to maximize her expected profit, $\int (\pi - w(\pi)) f(\pi | e) d\pi$. The principal is assumed to be risk neutral throughout unless explicitly assumed otherwise. The agent will accept the contract offered by the principal if and only if the terms of the contract provide the agent with a level of expected utility that exceeds his reservation utility level, $\bar{u}$. This reservation level is assumed known to both the principal and the agent.

The timing of interaction between the principal and the agent in this simple setting is the following. First, the principal designs the terms of the contract and then offers the contract to the agent. Next, the agent decides whether to accept or reject the
contract. If the agent rejects the contract, the relationship is terminated. In this case the
principal receives a payoff of zero. It is assumed throughout that the principal is better off
making the agent an offer that he will accept. If the agent accepts the contract, he begins
his "employment" and decides how much effort to exert. Then the state of nature, \( \theta \),
ocurs. Finally, the agent's output is observed, and the payment is made to the agent as
promised in the contract.

2.2.1 Contract design with observable effort

I first develop as a point of reference for later discussion the trivial case where
effort is observable. When the effort, \( e \), is directly observable or verifiable, \( w \) and \( e \) can
be used jointly to achieve a Pareto optimal or first-best outcome. The optimal contract for
the principal if effort is observable is to solve the following problem (for notational
simplicity, the lower and upper limits of integration, \( \pi \) and \( \bar{\pi} \), are suppressed):

\[
\begin{align*}
\text{Max} & \int_{\pi, w(\pi)} (\pi - w(\pi)) f(\pi | e) d\pi \\
\text{s.t.} & (i) \int v(w(\pi)) f(\pi | e) d\pi - c(e) \geq \bar{u}.
\end{align*}
\]

Constraint (i) of (2.1) is known as the Individual Rationality (IR) constraint. It requires
that the agent must receive an expected utility of at least \( \bar{u} \) in order to accept the contract
that the principal offers.

In this problem, the principal first decides on the effort she wants the agent to
implement. Then she picks the least cost incentive scheme \( w(\pi) \) that induces the agent to
exert that effort. Thus, it is convenient to think of this problem in two stages (Mas-Colell,
Whinston, and Green, 1995; pp. 477-88). First, for each choice of \( e \), what is the cost
minimizing compensation scheme \( w(\pi) \) is determined that will make that \( e \) the agent’s best choice? Second, among the cost minimizing \( w(\pi) \)’s for different effort levels, the profit maximizing \( e \) is chosen? Given that the contract specifies effort level \( e \), choosing \( w(\pi) \) to maximize \( \int (\pi - w(\pi)) f(\pi | e) d\pi = \int \pi f(\pi | e) d\pi - \int w(\pi) f(\pi | e) d\pi \) is equivalent to minimizing the principal’s expected compensation costs, \( \int w(\pi) f(\pi | e) d\pi \), so formally the optimal incentive scheme for implementing \( e \) must solve

\[
\text{Min}_{w(\pi)} \int w(\pi) f(\pi | e) d\pi
\]

s.t. (i) \( \int v(w(\pi)) f(\pi | e) d\pi - c(e) \geq \bar{u} \).

Constraint (i) of (2.2) always binds at a solution to this problem; otherwise, the principal can reduce \( w(\pi) \) while still inducing the agent to accept the contract. Where \( \gamma \) is the Lagrangian multiplier for the constraint, the agent’s compensation scheme \( w(\pi) \) at the solution to problem (2.2) must satisfy the first-order condition

\[- f(\pi | e) + \gamma v'(w(\pi)) f(\pi | e) = 0 \]

which can also be expressed as

\[1 / v'(w(\pi)) = \gamma .\] (2.3)

If the agent is strictly risk averse [so that \( v'(w) \) is strictly decreasing in \( w \)], the implication of condition (2.3) is that the optimal compensation scheme \( w(\pi) \) is a constant.

For observable effort, there is no problem with providing incentives. Thus, the risk-neutral principal should fully insure the risk-averse agent against any risk in his wage. Hence, given the contract’s specification of \( e \), the principal offers a fixed wage \( w^* \) such that the agent receives exactly his reservation utility level, \( \bar{u} \), that is, \( v(w^*) - c(e) = \bar{u} \), which can also be expressed as
\[ w_e^* = v^{-1}(\bar{u} + c(e)). \tag{2.4} \]

For a risk neutral agent, constraint (i) of (2.2) is replaced by
\[ \int w(\pi)f(\pi | e)d\pi - c(e) \geq \bar{u}. \]
First order condition (2.3) thus becomes \( \gamma = 1 \). The implication of this condition is that no restriction other than the agent’s IR constraint is required for optimal risk sharing. This condition is necessarily satisfied for any compensation scheme such that the wage payment \( w_e^* \) satisfies
\[ E(w_e^*) = \bar{u} + c(e). \tag{2.5} \]
A fixed wage scheme is merely one of many possible optimal compensation schemes that satisfy (2.5). Any other compensation scheme that gives the agent an expected wage payment equal to his reservation wage plus effort cost is also optimal.

Now consider the optimal choice of \( e \). The principal optimally specifies the effort level \( e \) that maximizes expected output less wage payment,
\[ \int \pi f(\pi | e)d\pi - v^{-1}(\bar{u} + c(e)) \]
(or \[ \int \pi f(\pi | e)d\pi - c(e) - \bar{u} \] for risk neutral case).

2.2.2 Contract design with unobservable effort

More realistically, effort is not verifiable. In this case, the agent will in general not find the first-best effort level to be optimal because he dislikes effort. This fact causes a conflict between the interests of the principal and the agent. This conflict results from the unobservability of the agent’s effort and creates a moral hazard whereby the agent shirks in his effort to the detriment of the principal. An important assumption in this model is that the agent observes the state of nature, \( \theta \), after he chooses his effort, \( e \). The case, where the agent observes \( \theta \) before he chooses his effort \( e \) is discussed in section 2.4.
The case, where the agent observes $\theta$ before he accepts or signs the contract is discussed in section 2.5.

Another important assumption is that the principal never observes $\theta$. She only observes output, $\pi$. If the state of nature, $\theta$, is directly observable or verifiable, then $\pi$ and $\theta$ can be used jointly to induce the first-best effort $e$. Then there would be no moral hazard problem provided that output is observed without error as in the case of section 2.2.1. At this point, output $\pi$ is assumed to be measured without error. This assumption will be relaxed later to explore its potential impact on the contract parameters. Since the agent’s effort is not observable the contract cannot specify it in an effective manner because there is simply no way to verify whether the agent has exerted the contracted effort. In this situation, the principal must redesign $w(\pi)$ in a way that indirectly gives the agent the incentive to choose the desired effort that would be contracted if effort were observable.

An incentive to induce high effort can be provided only at the cost of having the agent bear part of the loss resulting from lower effort. This must be accomplished while maintaining the IR constraint whereby the agent must receive an expected utility of at least $\bar{u}$ if he is to accept the offered contract. Since the agent’s effort is unobservable, however, the principal also faces a second constraint that the agent must desire to choose effort $e$ when facing the offered incentive scheme (Mas-Colell, Whinston, and Green, 1995; pp. 477-88). This means it is not in the interest of the agent to deviate from the optimal effort. The optimal contract for the principal thus solves the problem,

$$\max_{e,w(\pi)} \int (\pi - w(\pi)) f(\pi | e) d\pi$$

s.t. (i) $\int u(w(\pi)) f(\pi | e) d\pi - c(e) \geq \bar{u}$

\[ (2.6) \]
(ii) \[ e \in \arg \max_{\tilde{e}} \int u(w(\pi)) f(\pi | \tilde{e}) d\pi - c(\tilde{e}). \]

The added constraint (ii) of (2.6) is known as the Incentive Compatibility (IC) constraint. It requires that under compensation scheme \( w(\pi) \) the agent’s optimal effort choice is the optimized effort desired by the principal. Following the two-stage solution described in the introduction of section 2.2 and as demonstrated by *Mas-Colell, Whinston, and Green* (1995; pp. 477-88), the optimal incentive scheme must solve

\[
\begin{align*}
\text{Min}_{\overline{w(\pi)}} & \quad \int w(\pi) f(\pi | e) d\pi \\
\text{s.t.} & \quad \int u(w(\pi)) f(\pi | e) d\pi - c(e) \geq \bar{u} \\
& \quad e \in \arg \max_{\tilde{e}} \int u(w(\pi)) f(\pi | \tilde{e}) d\pi - c(\tilde{e}).
\end{align*}
\]

(2.7)

For this parameterized formulation, I restrict attention to the case where the agent has only two possible effort choices. Most of the general insights of moral hazard models can be conveyed in the simple setting where the agent has only two effort levels from which to choose. In section 2.2.5, the model is generalized using a first-order approach for continuous effort choice.

For the case with two possible effort choices, let \( e_H \) denote high effort and \( e_L \) denote low effort, \( e_H > e_L \). Suppose the distribution of \( \pi \) conditional on \( e_H \) stochastically dominates the distribution conditional on \( e_L \) in a first-order sense; that is, the distribution functions \( F(\pi | e_L) \) and \( F(\pi | e_H) \) satisfy \( F(\pi | e_H) \leq F(\pi | e_L) \) at all \( \pi \in [\bar{\pi}, \overline{\pi}] \), with strict inequality on some open set \( \Pi \subset [\bar{\pi}, \overline{\pi}] \). This implies that the level of expected output when the agent chooses \( e_H \) is larger than that from \( e_L \),

\[ \int \bar{\pi} F(\pi | e_H) d\pi > \int \bar{\pi} F(\pi | e_L) d\pi. \]
If the principal wants to implement \( e_L \), she optimally offers the agent the fixed wage payment \( w_e^* = u^{-1}(\bar{u} + c(e_L)) \), the same payment she would offer if contractually specifying effort \( e_L \) when effort is observable. If the optimal choice is \( e_L \), then the incentive problem is solved because of \( c(e_H) > c(e_L) \). In this case, the agent’s wage payment is unaffected by his effort, and so he will choose the effort level that involves the lowest cost, namely \( e_L \). Doing so, he earns exactly \( \bar{u} \). The more interesting case arises when the principal wishes to implement the effort level \( e_H \), because some risk-sharing benefits have to be sacrificed in order to provide the agent a sufficient incentive to expend high effort. In this case, constraint (ii) of (2.7) can be written as

\[
(ii_H) \quad \int u(w(\pi)) f(\pi | e_H) d\pi - c(e_H) \geq \int u(w(\pi)) f(\pi | e_L) d\pi - c(e_L).
\]

Letting \( \gamma \geq 0 \) and \( \mu \geq 0 \) be the Lagrangian multipliers for constraints (i) and (ii_H), respectively, \( w(\pi) \) must satisfy the following Kuhn-Tucker first-order condition at every \( \pi \in [\underline{\pi}, \bar{\pi}] \):

\[
-f(\pi | e_H) + \gamma u'(w(\pi)) f(\pi | e_H) + \mu (f(\pi | e_H) - f(\pi | e_L)) u'(w(\pi)) = 0
\]

or, equivalently,

\[
1/u'(w(\pi)) = \gamma + \mu (1 - f(\pi | e_L)/f(\pi | e_H)).
\]


---

11 The optimal incentive scheme would not change materially by assuming that the principal is risk-averse; only the left-hand side of equation (2.8) would change to \( v'(\pi \cdot w(\pi))/u'(w(\pi)) \).

12 Throughout this dissertation, to avoid excessive use of parentheses and brackets, I assume that any expression of the form \( a/b + c/d \) means \( (a/b) + (c/d) \).
In any solution to problem (2.7), where \( e = e_H \), both \( \gamma \) and \( \mu \) are strictly positive (Mas-Colell, Whinston, and Green, 1995; pp. 477-88). With \( \mu \) positive, \( w(\pi) \) will vary with the output \( \pi \), trading off some risk-sharing benefits for incentive provisions. More precisely, as implied by (2.8), it will vary with the likelihood ratio \( f(\pi | e_L)/f(\pi | e_H) \). Thus, I next explore conditions on the likelihood ratio that shape the optimal compensation scheme, \( w(\pi) \).

### 2.2.3 The shape of the optimal compensation scheme

The likelihood ratio is a concept familiar from statistical inference. It reflects how strongly \( \pi \) signals that the true distribution from which the sample was drawn is \( f(\pi | e_L) \) rather than \( f(\pi | e_H) \). A high likelihood ratio evidence in favor of \( e_L \) and against \( e_H \); a value of one is the intermediate case in which nothing new is learned from the sample, because it implies the two distributions are equally likely.

Given that both \( \gamma \) and \( \mu \) are strictly positive, condition (2.8) can be used to derive some useful insights into the shape of the optimal compensation scheme, \( w(\pi) \). Consider, for example, the fixed wage payment \( \hat{w} \) such that \( 1/\nu'(\hat{w}) = \gamma \). According to condition (2.8), \( w(\pi) > \hat{w} \) if \( f(\pi | e_L)/f(\pi | e_H) < 1 \) and \( w(\pi) < \hat{w} \) if \( f(\pi | e_L)/f(\pi | e_H) > 1 \). Thus, the optimal compensation scheme pays more than \( \hat{w} \) for outputs that are statistically more likely to occur under \( e_H \) than under \( e_L \), as determined by having a likelihood ratio \( f(\pi | e_L)/f(\pi | e_H) \) less than 1. Similarly, it offers less compensation for outputs that are relatively more likely when \( e_L \) is chosen. However, in an optimal incentive scheme, compensation is not necessarily monotonically increasing in outputs.
As is clear from examination of condition (2.8), for the optimal compensation scheme to be monotonically increasing in \( \pi \), the likelihood ratio \( f(\pi | e_L)/f(\pi | e_H) \) must be decreasing in \( \pi \). That is, as \( \pi \) increases, the likelihood of getting output level \( \pi \) if effort is \( e_H \) relative to the likelihood if effort is \( e_L \) must increase. This property, known as the monotone likelihood ratio property (MLRP) following Milgrom (1981), is not implied by first-order stochastic dominance. First-order stochastic dominance ensures that an increase of effort is good for the principal in a very strong sense, namely, that any principal with a utility function increasing in \( \pi \) favors a higher effort level. However, to reward the agent with a payment increasing in \( \pi \) a higher output level must be clearly evidence that the agent has made a higher effort. The MRLP provides this additional information. It states that a higher effort level increases the likelihood of a high output level more than the likelihood of a low output level. Because, from (2.8), the sharing rule is monotone in the likelihood ratio, the MRLP assures a monotone sharing rule. The same conclusion follows for the continuous effort case as long as MLRP holds.

2.2.4 Optimal effort

Given the variability that is optimally introduced into \( w(\pi) \), the expected value of the agent’s wage payment must be enough greater than his (fixed) wage payment in the reservation case to compensate for any risk bearing, \( w_{e_H}^* \geq u^{-1}(\bar{u} + c(e_H)) \). In choosing which effort level to induce, the principal compares the incremental change in expected output from the two effort levels, \( \int \pi F(\pi | e_H) d\pi \) and \( \int \pi F(\pi | e_L) d\pi \), with the difference in expected wage payments in the contracts that optimally implement each.
From the preceding analysis, the wage payment for $e_L$ is exactly the same as when effort is observable, whereas the expected wage payment when the principal implements $e_H$ under nonobservability is strictly larger than his payment in the observable case described in section 2.2.1. Thus, nonobservability in this model raises the cost of implementing $e_H$ and does not change the cost of implementing $e_L$. This fact means that nonobservability of effort can lead to implementation of an inefficiently low level of effort. When $e_L$ would be the optimal effort level if effort were observable, then it is also optimal when effort is unobservable. In contrast, when $e_H$ would be optimal if effort were observable, then one of two things may happen: it may be optimal to implement $e_H$ using an incentive scheme or, alternatively, the risk-bearing costs may be high enough that the principal decides that it is better to simply implement $e_L$. In either case, nonobservability causes a welfare loss to the principal (the agent’s expected utility is $\bar{u}$ in either case), but the latter case also incurs a joint or social loss compared to observable effort.

2.2.5 The continuous effort case

Consider next the continuous effort case in contrast to the case with two effort levels. Economically, not much is different but it is important to understand why.

Consider the common case where the agent’s effort is a continuous variable. The agent’s IC constraint (ii) in this case is problematic. A standard practice is to replace it with the more manageable restriction representing the first-order condition as

$$(ii') \int u(w(\pi))f_{c}(\pi | e)d\pi - c'(e) = 0$$

where $f_{c}(\pi | e)$ is the partial derivative of $f(\pi | e)$ with respect to $e$. Relaxing (ii) in this way is called the first-order approach in the literature. It is easy to proceed to a
characterization of the optimal scheme, provided the relaxation in (ii’) is appropriate. In this case, equation (2.8) becomes

\[ \frac{1}{u'(w(\pi))} = \gamma + \mu \frac{f_e(\pi | e)}{f(\pi | e)}. \]

Here, \( f_e(\pi | e)/f(\pi | e) \) is the continuous counterpart of the likelihood ratio. Thus, when this characterization is correct, the same qualitative insights on \( w(\pi) \) are obtained as from the simple case with two effort levels.

With \( \mu \) positive, \( w(\pi) \) will vary with the output \( \pi \), trading off some risk-sharing benefits for incentive provisions. In particular, \( w(\pi) \) will vary with the likelihood ratio \( f_e(\pi | e)/f(\pi | e) \) so as to assure the risk-averse agent of an expected utility level of \( \bar{u} \) that compensates him for the risk he is bearing. The fact that the unobservability of effort leads only to downward distortions in the agent’s effort is a special feature of the two-effort-level specification. With many possible effort choices, unobservability may alter the level of effort induced in an optimal contract from its level under full observability, but the direction of the bias can be upward as well as downward depending on skewness in the distribution (Mas-Colell, Whinston, and Green, 1995; pp. 477-88).

2.2.6 Validity of the first-order approach

The first-order approach does not always work because it can identify a scheme that in the end does not satisfy the global IC constraint (ii). If first-order conditions do not fully identify global optima for the agent, then the solution to the principal’s problem replacing the IC constraint by the first-order conditions of the agent’s problem may not maximize the agent’s utility. As a result, the identified maximum of the principal’s problem may not be attainable by the principal. Mirrlees (1975) was the first to recognize
this dilemma. Subsequently, *Grossman and Hart* (1983) and *Rogerson* (1985) worked out conditions that ensure the validity of the first-order approach.

The necessary conditions to substitute the agent’s first-order condition for the agent’s IC constraint are satisfied if the solution to the agent’s first-order condition is unique and the agent’s optimization problem is concave. Sufficient conditions are met by the MLRP together with convexity of the conditional distribution function condition (CDFC) The CDFC requires that the distribution function $F(\pi | e)$ is convex in $e$, i.e., $F(\pi | \zeta e + (1 - \zeta)e') \leq \zeta F(\pi | e) + (1 - \zeta)F(\pi | e') \forall e, e' \in E$ and $\zeta \in [0,1]$. These two conditions essentially guarantee that the agent’s optimization problem is concave, and hence, that the first-order conditions fully identify the global optimum for the agent. However, the CDFC and MLRP together are very restrictive conditions. For instance, none of the well-known distribution functions satisfy both of these conditions simultaneously.

### 2.2.7 Linearity of $w(\pi)$ in $\pi$

An interesting issue is whether all the stated conditions can be met with a compensation scheme such that $w(\pi)$ is linear in $\pi$. Condition (2.8) suggests that the optimal contract is not likely to take a simple (e.g., linear) form. The optimal shape of $w(\pi)$ is a function of the informational content of various output levels (through the likelihood ratio), which is unlikely to vary with $\pi$ in a simple manner in many problems. The problem is that the connection between $\pi$ as a physical output and as statistical information is tenuous. In fact, the physical properties of $\pi$ are rather irrelevant for the solution. All that matters is the distribution of the posterior (or likelihood ratio) as a
function of the agent’s effort. In other words, all that matters is the signaling value of $\pi$. Thus, cardinality of $\pi$ is not required to have the same information content. Because the information content of $\pi$ determines the shape of the optimal incentive scheme, determining natural economic assumptions that connect the agent’s reward in any particular way to the physical measure of $\pi$ is difficult.

As a result, one problem with the basic agency model is its sensitivity to distributional assumptions. It manifests itself in an optimal sharing rule that is complex, responding to the slightest changes in the informational content of the output $\pi$. Such fine-tuning appears unrealistic. In the real world, incentive schemes show variety, but not to the degree predicted by the basic theory (Bolton, and Dewatripont, 2005). Linear or piece-wise linear schemes, for instance, are used frequently and across a wide range of environments. Their popularity is hardly explained by shared properties of the information technology, as the basic model would have it. Other technological or organizational features excluded from the simple model may be responsible for regularities in shapes observed empirically.

Also, without specifying more about the various functions in the above formulation, very little can be said about the solution (see Grossman and Hart, 1983). As a result, empirical work has often been based linearity of $w(\pi)$ in the agency problem as found in Holmstrom and Milgrom (1987).

2.3 Holmstrom and Milgrom’s Linear Scheme

Holmstrom and Milgrom (1987) have proposed a simple closed-form agency model in which linear schemes are optimal because the agent is assumed to have a rather
rich effort space. This special case assumes normally distributed output, negative exponential utility, and a linear incentive scheme. Output, $\pi$, is assumed to be equal to effort plus noise, $\pi = e + \theta$, where $\theta$ is normally distributed with zero-mean and variance $\sigma_\pi^2$. In other words, the agent controls the mean of a normally distributed output. The distribution of $\theta$ is common knowledge unless otherwise indicated.

The agent has risk preferences following constant absolute risk-aversion (CARA), which requires $-u''/u' = \eta$ for $\eta$ constant. Solving this differential equation, CARA implies a negative exponential utility function of the form $u(w(\pi), e) = -e^{-\eta[w(\pi) - c(e)]}$, aside from inconsequential affine transformations, where $\eta > 0$ is the agent’s coefficient of absolute risk aversion. If the agent is risk neutral, then $u'$ is constant so (without solving a differential equation) the utility function can simply be represented as $u(w(\pi)), e) = w(\pi) - c(e)$. Similarly, the principal’s utility can be characterized by $v(w(\pi), \pi) = -e^{-\tau[w(\pi)]}$ where $\tau > 0$ is the principal’s coefficient of absolute risk aversion. Except where indicated otherwise, however, the principal is assumed to be risk neutral, in which case the principal’s utility function can be represented as $v(w(\pi), \pi) = \pi - w(\pi)$. For simplicity of illustration, the effort cost function is assumed to be quadratic and given explicitly by $c(e) = ce^2/2$. In contrast with formulations thus far, effort cost here is measured in monetary units. Suppose that the principal and agent can write only linear contracts of the form $w(\pi) = t + s\pi$ where $t$ is the base salary and $s$ is the marginal reward or bonus per unit of $\pi$ produced.
2.3.1 Risk neutrality and observability

A useful reference point is the case of optimal contracting when effort is observable and the agent is risk neutral. If the agent’s effort is observable, then the contracting problem is relatively straightforward. The contract would simply specify the exact action to be taken by the agent and the compensation (wage payment) that the principal is to provide in return. The principal’s objective is to maximize \( E(\pi) - E(w(\pi)) \)

where \( E(\pi) = E(e + \theta) = e \) and \( E(w(\pi)) = E(t + s\pi) = t + sE(\pi) = t + se \). Thus, \( E(\pi) - E(w(\pi)) = (1-s)e - t \). Hence, the optimal contract for the principal solves

\[
\max_{e,t,s} (1-s)e - t \tag{2.9}
\]

s.t. (i) \( t + se - ce^2 / 2 \geq 0 \).

Constraint (i) always binds at a solution. Otherwise, the principal could lower the agent’s wages without the agent rejecting the contract.\(^\text{13}\) Therefore, an equivalent problem is

\[
\max_{e,t,s} (1-s)e - t \tag{2.10}
\]

s.t. (i) \( t + se - ce^2 / 2 = 0 \).

Upon substitution for \( t \) from constraint (i) of (2.10), this problem reduces to maximization of \( e - ce^2 / 2 \) with respect to \( e \). The first-order condition with respect to \( e \) is solved by \( e^* = 1/c \). Thus, the principal offers a wage payment \( E(w(\pi)) = t + se^* \) such that \( t + se^* = c(e^*)^2 / 2 = 1/(2c) \). The principal’s profit is \( E(\pi) - E(w(\pi)) = 1/c - 1/(2c) = 1/(2c) \).

In this case, a fixed wage payment \( t^* = 1/(2c) \) with output share \( s = 0 \) is merely one of

\(^\text{13}\) Most of this literature assumes that when the agent is indifferent among efforts or actions, e.g., between accepting or rejecting a contract, the agent will choose the action most preferred by the principal. This method of "breaking ties" resolves a technical open-set problem of limited economic interest (\textit{Sappington}, 1991)
2.3.2 Risk neutrality and unobservability

When the agent’s effort is not observable, however, the contract can no longer specify the effort level because there is no way to verify whether the agent has fulfilled his obligations. The agent may exert effort less than $1/c$ while getting paid $1/(2c)$ for an agreed effort level $1/c$. In this circumstance, the principal must design the agent’s compensation scheme in a way that indirectly gives the agent an incentive to choose the contracted effort level.

For the case of unobservable effort where the risk-bearing concern is absent, the principal can achieve the same outcome as when effort is observable. Specifically, there is a contract the principal can offer that gives her the same payoff as when effort is observable. This contract must therefore be an optimal contract for the principal because the principal can never do better when effort is not observable than when it is. Consider the problem

$$\max_{e,t,s} (1-s)e-t \quad (2.11)$$

s.t. (i) $t + se - ce^2 / 2 \geq 0$

(ii) $e \in \arg\max_e (t + se - ce^2 / 2)$.

The second constraint of (2.11) is the IC constraint, which ensures that the principal’s optimal effort choice will also be the maximizing choice of the agent. Hence, the agent will have no incentive to deviate from the optimal effort. The first-order condition of the
agent’s problem, \( \max_e (t + se - ce^2 / 2) \), implies \( e = s / c \). Thus, the principal’s problem becomes

\[
\text{Max}_{t,s} (1-s)s / c - t
\]

s.t. (i) \( t + s^2 / (2c) \geq 0 \).

Again, constraint (i) in (2.12) will hold as an equality at the optimum. Thus, solving for \( t = -s^2 / (2c) \) and substituting for \( t \) into the maximand transforms the constrained problem into the unconstrained problem of maximizing \( s/c - s^2 / (2c) \) with respect to \( s \). The first-order condition requires \( 1/c - s/c = 0 \), and thus \( s^* = 1 \). The complete solution is thus \( e^* = 1/c \), and \( t^* = -1/(2c) \).

This contract induces the first-best effort level \( e^* \) as under full observability. With \( s^* = 1 \), the agent receives the full \( \pi \) and pays a fixed fee, \( 1/(2c) \), to the principal. The agent receives \( \{E(\pi) - t \mid e = e^* \} = e^* - 1/(2c) = 1/(c) - 1/(2c) = 1/(2c) \). This is exactly his reservation wage 0 net of his effort cost \( 1/(2c) \). Thus, the optimal fixed fee is set to extract the entire surplus of the agent. With the compensation scheme \( s^* = 1 \) and \( t^* = -1/(2c) \), both the principal and the agent receive exactly the same payoff as when effort is observable. The only difference is that instead of the principal choosing \( e \), the agent chooses \( e \).

The basic idea behind this result is that, if the agent is risk neutral, then the problem of risk sharing disappears. When the agent is risk neutral, the principal can ensure her most preferred arrangement with a simple contract that promises a payment \( w(\pi) \) to the agent equal to the profit less some fixed payment \( 1/(2c) \) that can be interpreted as a "franchise fee" or fixed fee for the right to work for the principal. This franchise fee is set equal to the expected net profit from efficient operation. Since the
agent pays that fee regardless of the exerted effort, any effort less than the efficient effort, \(1/c\), has no impact on the principal’s payoff. Rather, the agent is punished for inefficient operation.

In summary, this result implies that making the agent the residual claimant for the firm’s profit is an optimal response to the moral hazard problem if the agent is risk neutral. In other words, the principal, in effect, sells the property rights over the firm to the agent. As usual, a proper allocation of property rights is sufficient to induce efficiency. After "buying" the "franchise," the agent's goals are perfectly aligned with the principal's initial goals. Therefore, the agent acts as the principal would if she shared the agent's superior information and expertise.

A critical assumption of this result is that the agent has enough wealth to pay the fixed fee, \(1/(2c)\), in any state. Also noteworthy is that this fixed fee contract would be strictly preferred by the principal even in the case of observable effort if she were risk-averse rather than risk neutral.

2.3.3 Assumptions behind the simple results

The simple solution of what might, at first, appear to be a nontrivial incentive problem relies heavily on some special features of the canonical model (Sappington, 1991). These special features are what create frictions in the principal-agent relationship, and thus necessitate the use of a broader set of tools and institutions. The first feature is the assumption that the agent is risk neutral. Under a fixed fee contract, the agent bears all the risk associated with output stochasticity. Since the agent is risk neutral, he does not care about randomness in the output he produces. In general, whenever the agent is
risk-averse, he has to be paid for bearing this randomness and, accordingly, some sharing
of the risk between the principal and the agent will be optimal. For example, optimal risk
sharing would not have a risk-averse agent bear the entire burden of a poor output due to
extreme bad luck (when $\theta$ turns out to be unusually small).

The second feature is the assumption of the agent’s full commitment to the fixed
fee contract. That means the agent can be bound costlessly to carry out the terms of any
contract he accepts. A critical case is where the agent observes such an unfavorable
production environment that the best he can do is earn an expected net profit below his
reservation utility. The canonical model assumes the agent is unable to breach or
renegotiate the contract even though he knows an unfavorable state has occurred. The
same applies to the principal's commitment. The payment schedule announced by the
principal cannot be changed after the output is observed. This fact assures that the agent
will not be "held up" by the principal after costly effort has been exerted. In practice, a
worker's commitment abilities are not perfect. Also, labor laws prohibit slavery, so an
employee cannot credibly promise to serve his employer indefinitely. The commitment
ability of a principal is often limited in practice too. This assumption is relaxed in section
2.4 to determine its potential impact on the contract.

The third feature is the assumption of precontractual symmetric beliefs about the
potential states of nature represented by $\theta$. If the principal and the agent do not share the
same beliefs about $\theta$, they might not agree on the value of the fixed fee to buy or sell the
firm, rendering inapplicable the convenient separation of incentive issues (that motivate
the agent to choose an efficient level of effort) from distribution issues (that determine
how profit is divided). However, as long as precontractual beliefs are symmetric, the
fixed fee contract maximizes total profit. Symmetric beliefs imply that both parties are able to anticipate fully all possible contingencies that might arise during their relationship. This assumption is relaxed in section 2.5.

Finally, in the simplest setting described above, because all contracting frictions can be costlessly avoided with a fixed fee contract, the principal will not pay to obtain information on the working environment or the magnitude of the agent's efforts. When frictions are caused by precontractual asymmetries of information, risk aversion, limited commitment abilities, or problems in measuring the agent's effort, the principal generally will benefit from redesigning the simplest franchise fee contract in several ways. These complications are discussed next.

2.3.4 Contracting with agent risk aversion

This section considers implications of relaxing the strong assumption that agents are risk neutral. Again, a useful starting point is the optimal contracting problem where effort is observable.

2.3.4.1 Contracting with effort observability

The optimal contract for the principal when the agent is risk averse and effort is observable solves

\[
\text{Max}_{ed\delta} (1-s) e - t \quad (2.13)
\]

s.t. (i) \( t + se - ce^2/2 - \eta s^2 \sigma^2 /2 \geq 0 \).

This problem is similar to (2.9) except that the right hand side of the IR constraint in (2.13) represents the certainty equivalent income rather than the expected wage. Again,
the constraint in (2.13) always binds at the optimal solution. Otherwise, the principal could lower the agent’s wages without causing him to reject the contract. Substitution of $t = -[se - ce^2/2 - \eta s^2 \sigma_x^2/2]$ into (2.13) converts the problem to maximization of $e - ce^2/2 - \eta s^2 \sigma_x^2/2$ with respect to $e$ and $s$. First-order conditions with respect to $e$ and $s$ yield $e^* = 1/c$ and $s^* = 0$, which imply $t^* = - (s^* e^* - c(e^*)^2/2 - \eta (s^*)^2 \sigma_x^2/2) = 1/(2c)$. The agent receives $1/(2c)$ which is 0 net of effort cost and the principal receives $1/c - 1/(2c) = 1/(2c)$.

The implication is that the optimal compensation scheme $w(\pi)$ is a constant. That is, the principal provides the agent with a fixed-wage payment. This is an optimal risk-sharing result. Given that the contract explicitly dictates the agent’s effort choice and that providing incentives is not a problem, the risk-neutral principal fully insures the risk-averse agent against any risk. Therefore, as long as effort is observable, a first-best solution is obtained regardless of the agent’s risk preferences.

2.3.4.2 Contract design with unobservable effort

When the agent’s efforts are not observable, the contract cannot specify effort effectively because the agent’s effort cannot be verified. In this circumstance, the principal must design the agent’s compensation scheme to give an indirect incentive to take the correct action (the action that would be contracted if his actions were observable). When the effort is unobservable, incentives for high effort can be provided only at the cost of imposing risk on the agent. The optimal incentive scheme for implementing a specific effort level $e$ minimizes the principal’s expected wage payment subject to two constraints. As before, the agent must receive a certainty equivalent
income net of effort cost of at least 0 if he is to accept the contract. The agent’s certainty equivalent is

\[
CE(w(\pi)) = E(w(\pi)) - \eta V(w(\pi))/2
\]

\[
= E(t + s\pi) - \eta V(t + s\pi)/2 = t + se - \eta s^2 V(\pi)/2
\]

\[
= t + se - \eta s^2 \sigma^2/2.
\]

When the agent’s effort is unobservable, however, the principal also faces a second constraint that the agent must desire to choose the optimal effort when facing the incentive scheme. The principal’s problem is

\[
\begin{align*}
\text{Max}_{e,t,s} & \quad (1 - s)e - t \\
\text{s.t.} \quad & (i) \quad t + se - \eta s^2 \sigma^2/2 - ce^2/2 \geq 0 \text{ and} \\
& (ii) \quad e \in \arg\max_e t + se - \eta s^2 \sigma^2/2 - ce^2/2.
\end{align*}
\]

The IC constraint (ii) of (2.14) insures that the agent’s optimal effort choice is \(e\) under compensation scheme \(w(\pi) = t + s\pi\). In other words, constraint (ii) assures that the incentive scheme is consistent with the effort the principal wants the agent to choose.

Solving constraint (ii) yields \(ce = s\). This equation implies that, for any level of \(e\), if the marginal cost of effort, \(ce\), is set equal to the variable compensation component, \(s\), then exerting any lesser effort is not a maximizing strategy for the agent. Therefore, this constraint induces the agent to exert the effort level intended by the principal. Using a variant of this equation, \(e = s/c\), the principal’s problem becomes

\[
\begin{align*}
\text{Max}_{t,s} & \quad s/c - t + s^2/c \\
\text{s.t.} \quad & (i) \quad t + s^2/(2c) - \eta s^2 \sigma^2/2 \geq \bar{w}
\end{align*}
\]
Converting constraint (i) of (2.15) to an equality, because no alternative action can be optimal for the principal, substitution of the constraint obtains the problem

$$\text{Max } s/c - s^2/(2c) - \eta s^2 \sigma^2 / 2 - w$$

with respect to $s$. The first-order condition with respect to $s$ yields $s^* = 1/(1 + c\eta \sigma^2)$. Substitution for $s^*$ into $e = s/c$ thus implies $e^* = s^*/c = 1/(c + \eta c^2 \sigma^2)$. Further substituting $s^*$ and $e^*$ into $t + s^2/(2c) - \eta s^2 \sigma^2 / 2 = \bar{w}$ then obtains $t^* = \bar{w} - (1 - \eta c \sigma^2) / (2c + 2c^2 \sigma^2)^2$. Expected net profit is

$$\pi_e = E(\pi) - E(w) = E(\pi) - [t + sE(\pi)]$$

$$= (1 - s)E(\pi) - t = (1 - s)e - t,$$

or after substituting for $s^*$, $t^*$ and $e^*$, $\pi_e^* = 1/[2c(1 + \eta c \sigma^2)] - \bar{w}$. From these expressions, both effort ($e^*$) and the variable compensation rate ($s^*$) decrease when $c$ (cost of effort), $\eta$ (degree of risk aversion), or $\sigma^2$ (randomness of output) increase.

Comparing to the case of nonstochastic technology, a fixed rental contract ($s^* = 1$) is optimal when $\sigma^2 = 0$, although the principal can also offer a fixed fee contract ($s^* = 0$) as well. With no output randomness, the principal can infer the effort from the output without error, which also permits the principal to pay based on the effort. Since the risk premium is zero when $\sigma^2 = 0$, the agent faces no risk in his payment even if his payment is based on output or equivalent sharing contracts are used. That is why the first-best effort level, $e^* = 1/c$, is implemented when $\sigma^2 = 0$ and moral hazard is not a problem.

When the technology becomes stochastic ($\sigma^2 > 0$) the moral hazard problem becomes an issue. The only optimal solution is a risk-sharing contract. For $0 < s^* < 1$, the agent gets only a fraction of the output of his effort (or bears only a fraction of the loss of
output for reduced effort) at the margin. This fraction is smaller (the incentive for effort is weaker) the larger is the variance of the error ($\sigma^2_x$) with which the observable output indicates the underlying effort. Incentives can be sharper when the agent is less risk-averse. That is, a low $\eta$ implies a high $s$. In the extreme case of a risk-neutral agent ($\eta = 0$), the optimal share is $s^* = 1$ implying that the principal sells the firm to the agent for a fixed fee just as in the risk neutral case of section 2.3.2.

More generally, this formulation emphasizes the tradeoff between the agent's risk and incentive. The main prediction of this simple model is that a principal operating in more risky environments offers her agents compensation schemes in which incentives are less intense, i.e., the larger the variance, $\sigma^2_x$, the smaller is the share ($s^*$). Also, for $\sigma^2_x > 0$, the resulting effort level is less than the first-best level $e^* = 1/c$. Thus, effort is less the larger is the variance of the error ($\sigma^2_x$). On the other hand, the resulting effort is larger when the agent is less risk-averse (low $\eta$ implies high $e^*$) to the point that a risk-neutral agent ($\eta = 0$) exerts the first-best effort, $e^* = 1/c$.

These results raise the question of why the principal should not force the agent to bear the entire risk associated with production. In other words, why bias the optimal effort downward from the first-best effort? The answer is that the agent’s required risk premium for bearing all the risk becomes excessive requiring the principal to offer excessive fixed compensation to satisfy the agent’s IC constraint. Thus, while the agent generally receives greater compensation for higher realized output, the agent's incremental output is less than the value to the principal of that additional output. In this sense, the agent is not the sole residual claimant in the relationship, as in the case of a fixed fee contract. This occurs because the agent's goals are no longer perfectly aligned
with the principal's goals. Since the agent no longer benefits as much from outstanding performance, his effort incentive is diminished, which is reflected in lower effort. Formal details along these lines are provided in Stiglitz (1974, 1975), Harris and Raviv (1979), Holmstrom (1979), and Shavell (1979). Also, Grossman and Hart (1983) and Laffont and Tirole (1986), among others.

Alternatively, these results raise the question of why the principal should not reduce the risk premium by paying a fixed wage to the agent. The answer is that a fixed wage provides no incentive for effort to the agent. Risk sharing between the principal and the agent acts as a form of insurance for the agent. By increasing the effort incentive, the agent’s risk premium increases but more output enables the principal to offer sufficient fixed compensation to bear it.

2.3.5 Payments based on multiple signals

Thus far output, $\pi = e + \theta$, has been used as the sole signal of effort. Further generalization can admit multiple signals as are relevant for hog contracting in chapter 5. Consider the case where an additional objective signal about the agent's effort level is available. An objective signal is one that can be verified for contractual purposes. For illustrative purposes, suppose one more objective signal $y$ is available. Signal $y$ is measured as $y = e + \epsilon$, where $\epsilon \sim N(0, \sigma_y^2)$ where $\sigma_y^2$ is the variance in measurement error of the signal $y$. Similarly, let $\sigma_\pi^2$ be the variance in measurement error of the signal $\pi$.

Following Holmstrom and Milgrom (1987), the optimal contract relating wages to these observed signals is assumed to be linear and given by $w(\pi, y) = t + s_\pi \pi + s_y y$ where $t$ is the
agent's base pay and \( s_\pi \) is the piece rate on signal \( \pi \) previously denoted by \( s \), and \( s_y \) is the piece rate on signal \( y \).

Solving the principal’s problem with respect to multiple signals obtains the relative weights of the piece rates \( s_\pi \) and \( s_y \). The principal maximizes expected output, where expected output is given by the effort of the agent \( e \). All random variables are assumed to be mutually uncorrelated. Repeating the same maximization as in section 2.3.4.2 with one additional argument in the payment scheme, \( w(\pi, y) = t + s_\pi \pi + s_y y \), the optimal effort \( e^* \) is given by \( e^* = (s_\pi + s_y)/c \) (compared to the single signal result, \( e^* = s_\pi/c \) in section 2.3.4.2). The first-best effort level, \( 1/c \), occurs when \( s_\pi + s_y = 1 \). Optimizing over the choice of payment scheme, the principal chooses piece rates of \( s_\pi = \sigma^2_\pi/(\sigma^2_\pi + \sigma^2_y + \eta \sigma^2_\pi \sigma^2_y) \), and \( s_y = \sigma^2_y/(\sigma^2_\pi + \sigma^2_y + \eta \sigma^2_\pi \sigma^2_y) \) (Prendergast, 1999).

This model further illustrates the trade-offs between incentives and risk. For a risk neutral agent (\( \eta = 0 \)), \( s_\pi + s_y = 1 \). Thus, the first-best level of effort \( 1/c \) is exerted. However, if \( \eta > 0 \) with measurement error in both signals (\( \sigma^2_\pi > 0 \) and \( \sigma^2_y > 0 \)), effort is less than the first-best level, \( s_\pi + s_y < 1 \). Since higher variance implies higher measurement error, a particular signal's weight is decreasing in its variance, so noisy signals receive less weight. However, the weight attached to any signal is increasing in the noisiness of the other, although total incentives, \( s_\pi + s_y \), are decreasing in the noisiness of any signal.

An interesting question is when will the principal choose to base the agent's payment on both signals rather than one? Perhaps the most important observation of the early contributions to agency theory (Holmstrom, 1979) is what has become known as the informativeness principle, which implies that any measure of performance that (on
the margin) reveals information on the effort level chosen by the agent should be included in the payment scheme. This means, whenever two signals together provide more information about the agent's effort than does the agent's output alone, the agent's compensation under the optimal contract will be based on both signals. In effect, use of an additional imperfect signal does not impose additional risk on the agent because the weight of the first decreases just as a balanced portfolio attains less risk than an unbalanced one.

2.3.6 Criticisms of linear contracts

The linear contract model is not without criticism. A valid criticism is that the true first best can be approximated arbitrarily closely by step-function schemes that offer first-best risk-sharing (a fixed wage) for almost all outputs except extremely bad ones for which a severe punishment can be applied. To see this point, suppose the support of $\theta$ is $[-\hat{\theta}, +\hat{\theta}]$ where $0 < \hat{\theta} < \infty$. Suppose for simplicity that $\theta$ is uniformly distributed on this interval. Through his effort choice, the agent can then change the support of $\pi$. Under this specification, the agent’s moral hazard problem disappears altogether and the first best can always be achieved.

To see this, consider $e^*$, the first-best effort, and $w^*$, the first-best transfer, associated with the problem in (2.9) in section 2.3.1. With a bounded support, the principal can rule out certain output realizations, provided the agent chooses $e^*$. The lower and upper bound for output are thus $[e^* - \hat{\theta}]$ and $[e^* + \hat{\theta}]$, respectively, given that the agent has exerted effort $e^*$. Any output realization smaller than $[e^* - \hat{\theta}]$ results from an effort smaller than $e^*$. Thus, by punishing the agent very severely for outputs outside
Alternatively, outputs are not perfectly informative of the agent’s effort if $\theta$ has an unbounded support. However, outputs may be arbitrarily informative even when $\theta$ has an unbounded support (Mirrlees, 1975). This is the case, for example, when $\theta$ is normally distributed. The normal distribution has a likelihood ratio $f(\pi | e)/f(\pi | e)$ that can take any value between negative and positive infinity. Mirrlees shows that this information can be used to approximate the first best arbitrarily closely. That is, the principal can choose $\pi$ such that, for all $\pi < \pi^*$, the transfer to the agent of $w(\pi)$ is very low (a form of extreme punishment), but for $\pi \geq \pi^*$ the transfer is fixed at $w(\pi) = w^* + \varepsilon$, slightly higher than the first-best wage level, $w^*$. Under such a compensation scheme, the agent faces a negligible risk of getting punished when he chooses $e^*$, and his IR constraint is satisfied by the fixed wage $w^* + \varepsilon$ with $\varepsilon$ positive but arbitrarily small.

2.3.7 Support for linear contracts

While the results of the last subsection raise concerns about the linear-CARA-normal distribution formulation, Holmstrom and Milgrom (1987) have identified conditions under which linear contracts are optimal. Beyond assuming CARA preferences, they consider a dynamic model where effort is chosen in continuous time by the agent. For instance, consider a dynamic context where the agent is paid, say, at the end of the week, and assume he can observe his own output during the week so that he can adjust his effort, say labor input, as a function of the realized path of output. Then
step-functions will induce a path of effort that will be both erratic and, on average, low. Generally, the agent will bide his time to see if there is any need to work at all once the realized output reaches the critical level.

For example, if $\pi = \pi$ in the case of a bounded support, then the agent will not exert more effort because additional costly effort will bring no extra benefit. In contrast, a linear scheme that offers the same current incentive no matter what the output history will lead to a uniform choice of effort. This suggests that the optimality of a step-function incentive scheme is highly sensitive to the assumption that the agent chooses his effort only once. *Laffont and Tirole* (1986) illustrate this result in the case of a risk-neutral agent, where numerous schemes will be first best. They consider a linear scheme with unitary slope as well as the step-function scheme. If the agent receives noisy information about the technology before choosing his effort, the linear scheme is uniformly optimal.

More generally, the specifications of preferences and measurement errors used to this point are also far from innocuous. First, effort is specified as one-dimensional. A more general setting would allow the agent to carry out multiple activities. Second, the efforts of the agent can affect only the mean of the distribution of output rather than higher moments of the distribution. More generally, agents may be able to affect the riskiness of various performance measures. Finally, the exponential specification of preferences ignores wealth effects. The combination of normal errors and absence of wealth effects are critical to optimality of linear contracts. In general the sharing rule will not be linear. See *Holmstrom* (1979) and *Grossman and Hart* (1983) for details. Nevertheless, the simplicity and tractability of the linear structure has led to its widespread practical use.
2.4 Limited Liability

The model in section 2.2 assumes that the principal and agent make an agreement at a point in time when they share symmetric beliefs about the probability distribution of a random state of nature, \( \theta \). It also assumes that the realization of \( \theta \) is subsequently observed by the risk-neutral agent alone after choosing his unobservable level of effort. This specification follows the information asymmetry considered by *Grossman and Hart* (1983), *Holmstrom* (1979) and *Shavell* (1979) where the agent observes the true state of nature after he chooses his effort. Their precontractual information structure is also similar to that analyzed in section 2.2 because the principal and the agent share identical beliefs about the true state of nature when they reach an agreement to govern their future interaction. Further insights about contracts can be found by relaxing these assumptions.

The case where the realization of \( \theta \) is subsequently observed by the risk-neutral agent (alone) before choosing his effort is remarkable. This type of information asymmetry is considered by *Harris and Raviv* (1979), *Green and Stokey* (1980), and *Sappington* (1980). However, *Harris and Raviv* differ from the others by considering precontractual information asymmetry explicitly.

*Harris and Raviv* show that the self-interested principal can and will design a contract that induces an output in every state of nature that is Pareto efficient. They show that under the conditions of asymmetric information considered here (and more general conditions including risk aversion on the part of the principal) that the principal’s expected profit maximizing contract in the absence of liability restrictions is a first-best contract of the form \( w(\pi) = \pi - k^* \) where \( k^* \) is the expected net profit from efficient operation in excess of that required for the IR constraint of the agent. Specifically, \( k^* = \)


\( \pi^* - c(e^*) \)

2.4.1 Contracting with ex ante agent information

An example can serve to explain Harris and Raviv’s results in the case of a discrete random variable, \( \theta \). For simplicity, I will use a slightly different functional specification. Following Sappington (1983), suppose the explicit functional form of the cost of effort is \( c = (\pi_i / \theta_i)^2 / 2 \) where larger \( \theta \) reflects a better production environment and, hence, lower cost compared to lower \( \theta \).

When the agent has ex ante information (observes \( \theta \) before choosing effort), he will generally have more control over output that with ex post information. With ex post information, the principal finds the profit maximizing effort level given the distribution of \( \theta \). Then he sells the firm to the risk-neutral agent at the price of the net profit from that effort. The agent falls short of his reservation utility (0) if he gives less effort. Hence, he does not have any incentive to shirk. But with ex ante information, the agent has better information before choosing effort than the principal. The principal can pay depending on both effort and the state of nature if she knows the state of nature. In absence of such information, she can make a payment contingent on the output, \( \pi \). The principal’s problem is thus

\[
\begin{align*}
\text{Max } & \sum_{i=1}^{n} p_i (\pi_i - w_i) \\
\text{s.t. } & \sum_{i=1}^{n} p_i (w_i - (\pi_i / \theta_i)^2 / 2) \geq 0, 
\end{align*}
\] (2.16)
where \( w_i = w(\pi_i) \) and \( p_i \) is the probability that \( \theta = \theta_i \) is realized. Again, the constraint in (2.16) always binds at a solution to this problem. Therefore, the problem is

\[
\begin{align*}
\text{Max } & \sum_{i=1}^{n} p_i (\pi_i - w_i) \\
\text{s.t. } & \sum_{i=1}^{n} p_i (w_i - (\pi_i / \theta_i)^2 / 2) = 0.
\end{align*}
\]  

Consider the case with \( n = 2 \) where larger \( \theta = \theta_2 \) reflects conditions of higher productivity compared to \( \theta = \theta_1 \). Then (2.17) can be written as

\[
\begin{align*}
\text{Max } & p_1(\pi_1 - w_1) + p_2(\pi_2 - w_2) \\
\text{s.t. } & p_1(w_1 - (\pi_1 / \theta_1)^2 / 2) + p_2(w_2 - (\pi_2 / \theta_2)^2 / 2) = 0.
\end{align*}
\]  

The constraint in (2.18) implies that the expected payment to the agent is equal to expected cost, i.e., \( p_1 w_1 + p_2 w_2 = \frac{[p_1(\pi_1 / \theta_1)^2 + p_2(\pi_2 / \theta_2)^2]}{2} \). Substitution for the expected payment, \( E(w) = p_1 w_1 + p_2 w_2 \), in the principal’s problem yields the unconstrained maximization problem

\[
\begin{align*}
\text{Max } & p_1 \pi_1 + p_2 \pi_2 - \frac{[p_1(\pi_1 / \theta_1)^2 + p_2(\pi_2 / \theta_2)^2]}{2}.
\end{align*}
\]  

First order conditions imply \( \pi_1^* = \theta_1^2 \) and \( \pi_2^* = \theta_2^2 \). Suppose \( \theta_1 = 2 \) and \( \theta_2 = 4 \) so that \( \pi_1^* = 4 \) and \( \pi_2^* = 16 \). Then \( p_1 = p_2 = 1/2 \) implies \( E(w) = \frac{[p_1(\pi_1 / \theta_1)^2 + p_2(\pi_2 / \theta_2)^2]}{2} = 5 \) and expected output is \( E(\pi) = p_1 \pi_1^* + p_2 \pi_2^* = 10 \).

Expected net profit, \( k^* \), is 5. Irrespective of the state of nature the agent pays 5 to the principal and in return retains the entire (efficient) output that he chooses to produce.

When \( \theta_1 = 2 \) occurs, he receives a net payment of \( w_1 = 4 - 5 = -1 \) and his utility is \( w_1 - (\pi_1 / \theta_1)^2 / 2 = -1 - 2 = -3 \). When \( \theta_2 = 4 \) occurs, his payment is \( w_2 = 16 - 5 = 11 \) and
2.4.2 Potential problems with fixed fee contracts

The results in section 2.4.1 assume that the agent has sufficient wealth to pay the fixed fee if the bad state occurs. If the fixed fee is paid ex post, the wealth constraint cannot bind if the good state occurs. But if the fixed fee is paid in advance, wealth can be insufficient in both good and bad states.

Although the contract in section 2.4.1 promises the risk-neutral agent his reservation expected utility on average, when the production environment turns out to be less favorable than expected (when \( \theta = \theta_1 \) is realized), the agent can do no better under this contract than suffer a loss in utility \(-3\) below his reservation utility achieved in autarky. In such states, the agent prefers to breach the contract. Hence, such a contract is necessarily optimal for the principal only when institutions exist that guarantee the agent will not breach the contract after observing the state of nature no matter how debilitating compliance may be for the agent.

One such “institution” is simply the requirement that the agent pays the lump sum component at the time the contract is signed. This institution may not be feasible; however, when the agent’s total wealth is less than the required lump sum and he cannot acquire income insurance. Another such “institution” could be an asset specific investment required by the agent before observing the state of nature.

\[ w_2 - \left( \pi_2 / \theta_2 \right)^2 / 2 \]
For one or more reasons, such an institution may not exist in reality. Therefore, if the principal offers a fixed fee contract in an environment where the agent's maximum loss liability is limited by his option to rescind the contractual obligation, the agent will exert effort and remain in the principal's employ only for higher realizations of $\theta$. For example, in the example of section 2.4.1, the agent would receive 11 leaving 5 to the principal when $\theta = \theta_2$. If the agent is able to breach when $\theta = \theta_1$ occurs, then the principal’s expected profit is $(1/2)(0) + (1/2)(5) = 5/2$, whereas the agent’s expected utility is $(1/2)(0) + (1/2)(11−8) = 3/2$. Total expected surplus, $5/2 + 3/2 = 4$, is less than 5 obtained in the case of section 2.4.1 for the case where no breach can occur. This happens because expected surplus when $\theta = \theta_1$ is 0 whereas it was $(1/2)(4) − (1/2)(2) = 1$ when no breach was possible. As a result, allowing a breach is not an efficient arrangement from either the perspectives of the principal or society as a whole.

Therefore, if the principal must respect the agent's right to abrogate the terms of the original contract, then the principal will find it advantageous to alter the terms of the contract offered to the agent. In particular, the optimal contract will generally induce positive output ($\pi_i > 0$) by the agent even for less favorable realizations of the environment (i.e., when $\theta = \theta_1$). But this expanded output will not be induced simply by lowering the fixed fee ($k^*$). To do so would grant too much of the net profit to the agent in favorable states, thus raising the agent's expected utility above his reservation utility. Instead, the principal will implement a sharing of the total realized net profit. By promising the agent a fraction of the net profit associated with his effort, the agent can be induced to deliver productive effort, albeit less than the efficient level of effort in unfavorable states. Consequently, limited liability restrictions, like risk aversion on the
2.4.3 First-best limited liability contracts

Sappington (1983) examines the properties of the contract that emerges between a principal and a risk-neutral agent when limits are imposed on the maximum loss that the agent can be forced to bear as a consequence of contracting with the principal. Contracts which incorporate such limits on the ex post liability of the agent are called limited liability contracts. For purpose of illustration and analytic convenience, Sappington initially focuses on the special case of limited (zero) liability contracts in which the agent has the legal right to disassociate himself from the principal without penalty after observing $\theta$. The force of such arrangements can be to ensure that, after becoming informed about the production environment, the agent never expects to receive less than his reservation utility level $0.14$.

It is convenient to see what happens when the first-best limited (zero) liability contract is offered to the agent as a benchmark case. Continuing with the two-state problem of section 2.4.1, the principal’s problem if effort is observable is

$$\begin{align*}
\text{Max} & \quad p_1 (\pi_i - w_i) + p_2 (\pi_2 - w_2) \\
\text{s.t.} & \quad (i) \quad w_i - \left(\frac{\pi_i}{\theta_1}\right)^2 / 2 \geq 0 \\
& \quad (ii) \quad w_2 - \left(\frac{\pi_2}{\theta_2}\right)^2 / 2 \geq 0
\end{align*}$$

---

14 Asset specific investments can be one such bondage that can result in the agent receiving less than his reservation utility part of the time.
Constraints (i) and (ii) of (2.19) will hold with strict equality at the principal’s optimum since otherwise the principal can increase her profit by lowering \( w_1 \) or \( w_2 \) while still satisfying the zero limited liability constraints. Constraint (iii) of (2.19) is redundant because the agent’s wage is non-negative in each of the states under the limited liability constraints. Hence, it can be dropped. The solution to this problem is \( \pi_1^* = 4, \pi_2^* = 16, w_1 = 2, \) and \( w_2 = 8 \) for \( \theta = \theta_1, \theta = \theta_2, \) and \( p_1 = p_2 = 1/2. \) As a result, the agent is no longer the residual claimant. The principal’s expected profit is \( p_1(\pi_1^* - w_1) + p_2(\pi_2^* - w_2) = 5 \) if the agent produces 4 when \( \theta = \theta_1 \) and 16 when \( \theta = \theta_2. \) The agent’s utility is 0 in either case. Hence, his expected utility is 0 just matching his reservation utility.

### 2.4.4 Emergence of the adverse selection problem

The problem of section 2.4.3 does not apply when the principal cannot observe what state occurs. For example, the agent’s utility is

\[
(w_i - (\pi_i^* / \theta_i)^2 / 2) = w_i - (\pi_i^* / \theta_i)^2 / 2 + (\pi_i^* / \theta_i)^2 / 2 - (\pi_i^* / \theta_i)^2 / 2 = (\theta_i^2 - \theta_i^2)(\pi_i^* )^2 / (2\theta_i^3 \theta_i^2) = 3/2
\]

when he produces 4 even when \( \theta = \theta_2 = 4 \) occurs. The term \( (\theta_i^2 - \theta_i^2)(\pi_i^* )^2 / (2\theta_i^3 \theta_i^2) \) is known as information rent in the incentive literature. The agent’s utility is

\[
(w_2 - (\pi_2^* / \theta_2)^2 / 2) = -24 \text{ when he produces 16 and } \theta_i = 2. \text{ Thus, the agent has an incentive to produce 4 when } \theta = \theta_2. \text{ His expected utility from doing so is}
\]

\[
p_1(w_i - (\pi_i^* / \theta_i)^2 / 2) + p_2(w_1 - (\pi_1^* / \theta_1)^2 / 2) = 3/4. \text{ The principal’s expected profit is then}
\]

\[
p_1(\pi_1^* - w_i) + p_2(\pi_2^* - w_1) = 2 \text{ since the agent produces 4 irrespective of the state that
occurs. The moral hazard problem is thus transformed into an adverse selection problem because of the limited liability constraint. Therefore, the principal must design the contract in such a way that the agent does not have an incentive to lie about what state occurs.

The easiest way to avoid this problem is to offer the agent \((w_2 - (\pi_2^* / \theta_2)^2 / 2) = (\theta_2^2 - \theta_1^2)(\pi_1^*)^2 / (2\theta_1^2 \theta_2^2)\). That is \(w_2 = (\pi_2^* / \theta_2)^2 / 2 + (\theta_2^2 - \theta_1^2)(\pi_1^*)^2 / (2\theta_1^2 \theta_2^2) = 8 + 3/2 = 19/2\) when \(\theta = \theta_2\). Then the agent has no incentive to produce 4 when \(\theta = \theta_2\). The agent’s expected utility is the same, \(p_1 (w_1 - (\pi_1^* / \theta_1)^2 / 2) + p_2 (w_2 - (\pi_2^* / \theta_2)^2 / 2) = 3/4\). The principal’s expected profit is thus \(p_1 (\pi_1^* - w_1) + p_2 (\pi_2^* - w_2) = 17/4\). This profit is also less than 5, the full-information optimum of section 2.4.1. As will be shown in the following section, this is not the best solution for the principal. The principal can distort \(\pi_1^*\) to reduce the information rent \((\theta_2^2 - \theta_1^2)(\pi_1^*)^2 / (2\theta_1^2 \theta_2^2)\) and receive an expected profit larger than 17/4.

2.4.5 Second-best limited liability contracts

Another question is whether the principal can improve her profit by distorting the first-best contract of section 2.4.3. The answer is yes. In order to do so, the principal solves the problem

\[
\begin{align*}
\text{Max} &\quad p_1 (\pi_1 - w_1) + p_2 (\pi_2 - w_2) \\
\text{s.t.} &\quad (i) \quad w_1 - (\pi_1 / \theta_1)^2 / 2 \geq 0 \\
&\quad (ii) \quad w_2 - (\pi_2 / \theta_2)^2 / 2 \geq 0
\end{align*}
\]
Constraint (i) holds as an equality at the optimum as before. Constraint (iii) can be ignored for the same reason as in section 2.4.3. The added constraint (iv) removes the incentive for the agent to lie about what state occurs. The problem in (2.20) is no more a direct moral hazard problem. The principal designs the compensation scheme $w(\pi)$ in such a way that revealing the true state of nature will be optimal for the agent. The principal no longer provides the agent direct incentives for effort. The moral hazard model is thus turned into an adverse selection or asymmetric information model.

Constraint (iv), along with (i) and (ii), induces the agent to accept the contract without lying about what state occurs. Constraint (iv) will hold as an equality whenever the agent faces an incentive to lie about the state under a first-best contract. Substituting constraint (i), stated as an equality, into constraint (iv), also stated as an equality, obtains

$$w_2 = \frac{(\pi_2 / \theta_2)^2}{2} + (\theta_2^2 - \theta_1^2)\pi_1^2 / (2\theta_1^2 \theta_2^2).$$

Whereas the term $(\theta_2^2 - \theta_1^2)\pi_1^2 / (2\theta_1^2 \theta_2^2)$ in this $w_2$ equation is the information rent that the agent earns for misreporting the state of nature, the term $(\theta_2^2 - \theta_1^2)\pi_1^2 / (2\theta_1^2 \theta_2^2)$ represents the information rent obtained per unit of output $\pi_1$ produced from mimicking $\theta = \theta_1$ when $\theta = \theta_2$ occurs. Given $\theta_1$ and $\theta_2$, the principal can lower both the unit rent and the total rent by reducing $\pi_1$. Substitution of $w_1$ and $w_2$ implied by constraints (i) and (iv), stated as equalities, into (2.20) yields the unconstrained limited liability maximization problem,

$$\text{Max}_{\pi_1, \pi_2} p_1(\pi_1 - (\pi_1 / \theta_1)^2 / 2) + p_2(\pi_2 - (\pi_2 / \theta_2)^2 / 2 - (\theta_2^2 - \theta_1^2)\pi_1^2 / (2\theta_1^2 \theta_2^2)),$$
which has maximizing outputs \( \pi_1^L = \theta_1^2 / [1 + p_2 / p_1(1 - \theta_1^2 / \theta_2^2)] < \pi_1^* \) and \( \pi_2^L = \theta_2^2 = \pi_2^* \) in states \( \theta_1 \) and \( \theta_2 \), respectively, where the \( L \) superscript denotes the limited liability case.

Thus, no distortion occurs in the output \( \pi_2^L \) in the good state (\( \theta = \theta_2 \)) compared to the good-state first-best output \( \pi_2^* \). But the output \( \pi_1^L \) in the low-state (\( \theta = \theta_1 \)) is less than the low-state first-best output \( \pi_1^* \). For \( \theta_1 = 2 \), \( \theta_2 = 4 \), and \( p_1 = p_2 = 1/2 \), the explicit solution to this limited liability problem is \( \pi_1^L = 16/7 \), \( \pi_2^L = 16 \), \( w_1 = 32/49 \), and \( w_2 = 416/49 \).

The principal’s expected profit is \( p_1(\pi_1^L - (\pi_1^L / \theta_1)^2 / 2) + p_2(\pi_2^L - (\pi_2^L / \theta_2)^2 / 2 - (\theta_2^2 - \theta_1^2)(\pi_1^L)^2 / (2 \theta_1^2 \theta_2^2) = 416/49 \) which is larger than 17/4 obtained in section 2.4.4, even though it is smaller than first-best profit of 5 obtained in section 2.4.3.

The agent’s expected utility is \( p_1(w_1 - (\pi_1^L / \theta_1)^2 / 2) + p_2(w_2 - (\pi_2^L / \theta_2)^2 / 2 = 12/49 \) which is smaller than the 3/4 obtained in section 2.4.4, even though it is larger than the reservation utility 0 obtained in the first-best case described in section 2.4.3. The expected utility in this case is known as the agent’s limited liability rent in the contracting literature. Compared to the case without limited liability, this rent is the additional payment the principal must incur because of the conjunction of moral hazard and limited liability.

### 2.4.6 Wealth and limited liability rents

An important generalization for the application in this dissertation is the case where the agent must undertake a significant investment, such as in plant and equipment, to facilitate production. Suppose the agent is required to have an asset holding represented by \( \varpi \) in order to participate in the contract offered by the principal. Then the
liability limit implies that the net transfer of the agent, taking into account his own asset holding $A$, must be no smaller than $-A$. If bad state occurs the agent will be paid $w_1$ while his effort cost will be $(\pi_1 / \theta_1)^2 / 2$. He will have a deficit of $(\pi_1 / \theta_1)^2 / 2 - w_1$ in covering his effort cost. He will have to use $A$ of his own assets to cover this shortage. As a result, his net transfer in the bad state is $-A$ if $(\pi_1 / \theta_1)^2 / 2 - w_1 > A$. Limited liability clauses may protect the agent by allowing bankruptcy in any state with a net transfer smaller than $-A$. In this case, the agent avoids bankruptcy even when he has to pay the optimal penalty (no larger than $A$) to the principal if a bad state occurs. The formulation in (2.20) has a zero liability limit meaning that only non-negative net transfers are feasible. Therefore, the model in (2.20) represents a contractual environment in which the agent is not required to own any assets at the time of contracting with the principal.

Limited liability rent is increasing in the liability limit. As the agent is endowed with more assets, the conflict between moral hazard and limited liability diminishes and eventually disappears. For example, if the liability limit is $-3$ for the formulation in (2.20) so that $w_1 - (\pi_1 / \theta_1)^2 / 2 \geq -3$ and $w_2 - (\pi_2 / \theta_2)^2 / 2 \geq -3$, then the agent will earn higher rents compared with the zero liability limit. A liability limit of $-3$ means that the principal cannot force the agent to bear any loss larger than 3 in any state. Within this limit, the agent will honor a fixed fee contract even if he observes a bad state after signing the contract. Otherwise, he can breach the contract with a penalty equal to 3 after observing that the bad state has occurred.

A limited liability constraint on ex post rents may reduce the efficiency of ex ante contracting. If the limited liability constraint on the net transfer in the bad state is stringent enough, the principal must reduce the bad state’s output to meet the limited
liability constraint. The bad state output is lowered to 16/7 in (2.20) from 4 in (2.19). By doing so, the limited liability rent is reduced from 3/4 to 12/49. As the limited liability constraint is further tightened, the principal must relax the agent’s ex ante individual rationality constraint. More precisely, with a limited liability constraint, the optimal contract behaves as if the agent had an infinite risk aversion below a wealth of \(-A\). As a result, the principal faces an incentive to select wealthy agents if multiple agents compete for the contract.

Thus, designing the contract in such a way that the agent is compensated for producing an inefficiently small output in the bad state, the principal reduces the magnitude of the payment needed to induce a higher level of output in good state. However, the principal weighs the expected benefits of setting \(\pi_1^L\) below \(\pi_1^*\) against the costs of inefficiency (costs which are borne if \(\theta = \theta_1\) is realized). As \(\theta = \theta_2\) becomes more likely and \(\theta = \theta_1\) is less likely (i.e., as \(p_2/p_1\) becomes larger), \(\pi_1^L\) is set further below \(\pi_1^*\) in the contract most preferred by the principal. Similarly, as the ratio of \(\theta_1^2/\theta_2^2\) goes down, \(\pi_1^L\) is set further below \(\pi_1^*\). Also, because the benefits associated with inducing an inefficient output in any state are realized only when good state of nature occurs, the principal has no incentive to induce an inefficient output in good state of nature. When the technology is more general, the distribution of \(\theta\) may be such that in some states the expected benefits of elevating \(\pi_2^L\) above \(\pi_1^L\) outweigh the expected costs. Under such circumstances, \(\pi_2^L\) and \(\pi_1^L\) will coincide and the limited liability contract offered to the agent will be a “pooling” contract in the terminology of Stiglitz (1977).

Finally, it should be emphasized why the foregoing concerns are relevant only in the presence of limited liability restrictions. Absent any floor on the payoff to the risk-
neutral agent, any rent that the agent may gain when the principal expands $\pi^*_1$ to its efficient level $\pi^*_1$ can be effectively negated by demanding that the agent pay a larger lump sum payment in order to contract at all. Consequently, only when limited liability constraints are binding are the maximization of social efficiency and private utility not coincidental.

Throughout the foregoing analysis the agent is assumed to be risk-neutral. If the agent is risk averse, the qualitative results described here are unlikely to change (Harris and Raviv, 1979). Instead, there would be an additional reason for the principal to choose something other than a first-best contract to take advantage of the risk-sharing opportunities. Properties of such contracts are discussed, for example, by Holmstrom (1979) and Shavell (1979).

2.5 Precontractual Information Asymmetry

To this point, the principal and the agent have been assumed to have symmetric precontractual beliefs about the distribution of $\theta$. I have considered cases (i) where the agent first signs the contract, then exerts effort, and then finally observes $\theta$, and (ii) where the agent first signs the contract, then observes $\theta$, and then finally exerts effort. This section turns to case (iii) where the agent first observes $\theta$, then accepts or rejects the contract offered by the principal, and then exerts effort if the offer is accepted. In cases (i) and (ii), the principal and the risk-neutral agent share the same beliefs on $\theta$ before signing the contract, but this is not so in case (iii).

As described in the section 2.4, when the principal and the risk-neutral agent share the same beliefs before signing the contract, the optimal contract will have the
principal receive a lump sum payment \( (k^*) \) and the agent receive, \( w(\pi) = \pi - k^* \), the difference between the value of the output produced and the lump sum payment. Such is the case whether or not the agent receives perfect information after signing the contract (see, e.g., Harris and Raviv, 1979, Holmstrom, 1979, and Shavell, 1979). Because such fixed fee contracts induce the agent to value output ex post exactly as does the principal, the contract ensures that the agent supplies the level of effort that is Pareto efficient conditional upon his private information. Only in case (ii), when the agent is protected by limited liability clauses, does a fixed fee contract fail to achieve Pareto efficiency.

2.5.1 Problems created by precontractual information asymmetry

In case (iii), in the absence of symmetric precontractual beliefs on \( \theta \), the principal and agent will not necessarily agree upon whether any particular contract (of the limited liability variety or otherwise) provides a level of expected utility for the agent that exceeds his reservation level. An analysis of this complication and related ones can be found in Sappington (1980).

The exact details of the optimal sharing arrangement and the number of distinct contracts the principal offers will depend on a number of factors, including the nature of the agent's precontractual information and whether he subsequently acquires better information. Precontractual information \( \theta \) in this case can represent the productivity or innate ability of the agent, which may be known only to the agent. To illustrate, suppose that at the time a contract is signed, the agent has better knowledge than the principal about likely productivity and the agent's information on productivity is perfect. Obviously, there is no opportunity for the agent to acquire better information after a
contract is agreed upon in this case. When the agent's initial information about the productivity is superior to the principal's information, the fixed fee contract creates a problem similar to that of limited liability protection in section 2.4.

To see the connection, redefine the information structure of section 2.4 to introduce precontractual information asymmetry. The agent in this case knows $\theta$ even before signing the contract. Suppose the principal offers a basic fixed fee contract with a fixed fee, $k^* = 5$, to an agent who has very accurate information about his productivity. This fixed fee contract is characterized by the principal’s problem in (2.18). The agent will reject this fixed fee contract if he knows that his productivity is low (i.e., $\theta = \theta_1$) because he can do no better under this contract than suffer a loss in utility, $-3$ below his reservation utility level, 0.

The agent will accept the contract only when he knows $\theta = \theta_2$. In this state, the agent receives 11 leaving a fixed fee of 5 to the principal. The principal’s expected benefit is $(1/2)(0) + (1/2)(5) = 5/2$, whereas the agent’s expected utility is $(1/2)(0) + (1/2)(11 - 8) = 3/2$. This is exactly the same outcome as when the agent is protected by a limited liability clause but discovers the level of $\theta$ only after signing the contract. In this case, the agent rejects the contract when it is offered in the bad state. In the limited liability case, since the contract is already signed, the agent breaches the contract when he finds that the inferior state of nature has occurred. Anticipating this behavior, the principal will again modify the contract by inducing some output from the agent even in the inferior state, without granting the agent the entire realized profit. The upshot of such arrangements is that the principal must ensure that the agent never expects to receive less than his reservation utility level, 0, after signing a contract.
2.5.2 Precontractual asymmetric information contracts

To ensure that the agent never expects to receive less than his reservation utility level after signing the contract, the principal’s problem becomes

\[
\max_{\pi_1, \pi_2, w_1, w_2} \pi_1 (\pi_1 - w_1) + \pi_2 (\pi_2 - w_2) \tag{2.21}
\]

s.t.

(i) \( w_1 - (\pi_1 / \theta_1)^2 / 2 \geq 0 \),

(ii) \( w_2 - (\pi_2 / \theta_2)^2 / 2 \geq 0 \),

(iii) \( \pi_1 (w_1 - (\pi_1 / \theta_1)^2 / 2) + \pi_2 (w_2 - (\pi_2 / \theta_2)^2 / 2 \geq 0 \)

(iv) \( w_2 - (\pi_2 / \theta_2)^2 / 2 \geq (w_1 - (\pi_1 / \theta_1)^2 / 2) \).

Constraints (i) and (ii) ensure that the agent receives at least his reservation utility level, 0, and, hence, accepts the contract irrespective of his precontractual information on \( \theta \).

Constraint (3) is the participation constraint that is trivially satisfied once the first two constraints are satisfied. Constraint (iv) induces the agent not to lie about what information he has on \( \theta \). Thus, the problem in (2.21) is the same problem the principal solves in the case of limited liability. As a result, the principal faces the same qualitative tradeoffs when the agent's initial information about the productive environment is superior to the principal's information as she faces when the agent is protected by limited liability covenants.

Thus, the solution of the precontractual asymmetry information case is exactly the same, implying

\[
\pi_1^p = \theta_1^2 / [1 + p_1 \theta_2^2 - \theta_1^2 / \theta_2^2] = \pi_1^L < \pi_1^*
\]

\[
\pi_2^p = \theta_2^2 = \pi_2^L = \pi_2^*
\]
where the $P$ superscript refers to the precontractual asymmetric information case. There is no distortion in the good-state output $\pi_2^*$ when $\theta = \theta_2$ compared to the good-state first-best output $\pi_2^*$. But the bad-state output $\pi_1^*$ is less than the bad-state first-best output $\pi_1^*$ when $\theta = \theta_1$. As in section 2.4.5, the moral hazard model is thus turned into an adverse selection model.

Applications of the asymmetric precontractual information model of this section include problems of optimal taxation (e.g., Mirrlees, 1971), price discrimination (e.g., Goldman et al, 1984; Roberts, 1979; and Spence, 1977) and labor contracts (e.g., Azariadis, 1983, and Grossman and Hart, 1981). In these literatures, the agent (representing the taxpayer, the consumer, or the worker) knows his type (i.e., his earning ability, his reservation price, or the productivity of his labor), while the principal (representing the government, a monopolist, or an employer) does not. There are analogous models in the standard principal-agent literature (e.g., Sappington, 1983). The distinguishing feature of these models is that only extreme types of the agents will be given efficient incentives. The optimal contract will induce other types of agents to realize outputs that are inefficient ex post.

So far I have discussed contract form when the agent's initial information about the productive environment is superior to the principal's information in the case where the agent’s information is assumed to be perfect. However, the results are similar for the case where the agent’s information is superior but imperfect at the contracting stage of the principal-agent relationship.
2.6 Theoretical Predictions at Odds with Empirical Results

Whether risk aversion, limited liability restrictions, or precontractual asymmetric information complicate the basic model of section 2.3, similar qualitative effects emerge. The most important effect is that a fixed fee contract imposes too much risk on the agent or delivers too great a share of the realized profit to the agent so that the principal resorts to a "sharing" contract. Because the agent's compensation is less sensitive to his output under a sharing arrangement than under a fixed fee contract, the agent exerts less effort. This reduced effort results in loss for the principal relative to the benchmark optimality of the first-best model.

Standard moral hazard models stress this trade-off between incentives and risk-sharing in the determination of contractual forms. Fixed fee contracts are relatively efficient from the incentives viewpoint because the agent is both the main decision maker and the residual claimant. However, they also generate an inefficient allocation of risk, in which all the risk is borne by the agent who is generally presumed to be more risk averse. When uncertainty is small, risk sharing matters less and fixed fee contracts are more likely to be adopted. On the other hand, in a very uncertain environment risk sharing is paramount and a sharing contract is the natural contractual form. This prediction can be readily tested using data on existing contracts provided that a proxy for the level of risk is available.

This section discusses briefly the correspondence of these results with empirical observations, first in the share cropping problem (which is the most common practical problem in the literature and inspired the early work), and then in the context of the partial application in this dissertation to hog contracting.
2.6.1 Evidence from share cropping

Several inconsistencies between theoretical predictions and observed facts have been found for share cropping. The theory predicts that more risky crops are more likely to be grown under sharecropping contracts in agriculture. For instance, if some crops are known to be more risky than others, the theory predicts that these crops are more likely to be associated with sharecropping contracts given that available growers have common risk-averse preferences.

A number of papers have tested this prediction by regressing contract choice on crop riskiness. Higgs (1973) finds weak evidence consistent with this implication for corn and cotton in the early twentieth century in the southern United States. Rao (1971) finds opposing evidence for Indian farms. More recently, Allen and Lueck (1992, 1999) for farms and Leffler and Rucker (1991) for timberland find no relation between risk and the nature of contracts chosen.

2.6.2 Evidence from hog contracting

While several factors are likely to influence decisions regarding hog contracting, the most important reason cited by hog growers for choosing contract farming is risk reduction (Rhodes, 1989; Rhodes and Grimes, 1992; Johnson and Foster, 1994; Kliebenstein and Lawrence, 1995; Lawrence and Grimes, 2001). In their hog producers’ survey, Lawrence and Grimes (2001) found that producers cited financial risk reduction as the key advantage of production contracts. Martinez, Smith, and Zering (1998) analyzed the motivating factors behind increasing use of contracts and vertical integration in the hog industry and concluded that risk reduction is the most important reason for
contracting. The studies that emphasize the reduction of farmers’ economic risks view contracts as a vehicle to shift risks to integrators, which appeals to the risk sharing approach to modeling contracting.

If wealth is taken as a proxy for risk-aversion, then poorer growers are more likely to be more risk averse (under decreasing absolute risk aversion) and thus more likely to be under a sharing contract for a given production uncertainty. But the facts from contract hog growing do not support this theoretical prediction if risk sharing is the main motivation for contracting. If risk aversion is the main motivation, then more small-scale hog producers should be contract growers. But, in fact, more large-scale producers are contract growers. Furthermore, this is a growing trend (MacDonald et al., 2004). Thus, risk reduction does not appear to be the main motivation for contracting in hog production.

However, as in share cropping, this simple casual observation of facts does not confirm that the theoretical predictions are altogether wrong. As explained in section 2.4 and 2.5, sharing contracts emerge not solely because agents are risk-averse. Risk neutral agents may have wealth constraints that lead to sharing contracts because of necessary limited liability protection. Even when wealth constraints are not binding, precontractual information asymmetry may lead to sharing contracts. A more plausible motivation for hog contracting may be found from limited liability and wealth issues or from precontractual asymmetric information issues. Thus, this inconsistency of theoretical predictions and observed facts does not necessarily imply that the theory is irrelevant. Rather, it suggests that generalizations in the theory may be necessary to better match the conditions of observed facts. These possibilities are explored in chapter 3.
Chapter 3:

Gains from Contracting in a Model with Heterogeneous Agents

3.1 Introduction

The fundamental model described in chapter 2 has an obvious weakness for application in problems where principals face many potential agents. It disregards endogeneity in matching agents to principals. In other words, the theoretical predictions described in chapter 2 hold only when the characteristics of the principal and the agent are given. Such a model is appropriate only if the agents facing different contracts do not differ by relevant characteristics.15

This chapter examines contracting when agents facing different contracts differ in their characteristics. Heterogeneity in agents’ characteristics is incorporated into the basic model explored in detail. Because the agents are heterogeneous, an important issue is whether all the agents gain from contracting, or whether some agents can find only negative gains from contracting. The models presented in the literature do not consider the possibility that part of the agents may lose while others gain. Part of the literature ignores this possibility by assuming agents are homogeneous in characteristics. When agents are homogeneous, the same contract can be offered to all agents and no agent will receive less than his reservation utility under the conditions of chapter 2. For this reason the models of chapter 2 cannot explain negative gains among only a share of contracting agents.

15 Of course, such a model is also appropriate only with a single principal. The case with multiple principals (who possibly differ in characteristics) is not considered here for reasons given in the application to hog contracting in chapter 5.
Another reason the possibility of negative gains for a share of contracting agents is not considered is that, even when heterogeneity is assumed say in agents’ ability; agents are assumed to have complete knowledge of their ability. Results in this chapter (sections 3.3.2 and 3.4.2) show that when the agents know their ability perfectly they never receive less than their reservation utilities on average. But when agents have no knowledge of their ability or have partial knowledge of their ability, then the potential for some agents to lose from contracting becomes relevant. I explore this issue in detail in this chapter. I adapt existing models for different information structures and determine whether agents gain or lose from contracting.

Section 3.2 describes how standard theoretical predictions are modified or reversed once agential heterogeneity and transaction costs are introduced. The transaction costs considered in this chapter are incentive cost required to induce a given effort from an agent. They include negotiation, supervision, monitoring, and enforcement costs as well as costs associated with asset specific investment. Section 3.3 presents the formal theoretical model with agential heterogeneity and the theoretical predictions regarding agents’ gains from contracting under different information structures. Section 3.4 examines contracts under an alternative form of technology and ensures that the results in section 3.3 are not driven by the specific functional form of the technology that is assumed. Section 3.5 discusses the issue of whether agents know their ability or productivity parameters perfectly before they enter a contract and examines contracting when agents know their ability only imperfectly before they enter a contract.
3.2 Agential Heterogeneity: Basic Theoretical Issues

Intuition suggests that endogeneity occurs in the matching of agents to contracts in reality. In chapter 2, theoretical predictions of contract choice were derived for a given principal with particular characteristics (degree of risk aversion, $\tau = 0$) and a given agent with particular characteristics (degree of risk aversion, $\eta \geq 0$, randomness of output $\sigma_{\pi}^2 > 0$, and cost of effort $c$). Optimal solutions given all these characteristics then determined the optimal contract form (e.g., the share of output to be given to the agent as a function of these characteristics.

For example, the optimal contract form, $s^* = 1/(1+c\eta \sigma_{\pi}^2)$, in section 2.3.4.2 of chapter 2, implies that, if risk effects are an important determinant of contract choice, then risky crops (i.e., those with large $\sigma_{\pi}^2$) will more likely to be associated with production contracts (i.e., $0 < s^* < 1$) than with fixed fee contracts (i.e., $s^* = 1$). Allen, and Lueck (1992) examine whether the inherent riskiness of a crop affects the type of contract used for that crop. Empirically, they do not find this correlation and conclude that risk sharing is not an important determinant of contract choice. In this chapter, I suggest that one reason a correlation of risk and share cropping is not found may be that heterogeneity of agent’s characteristics is missing in their study.

Suppose agents have heterogeneous degrees of risk aversion. For simplicity, assume that a fraction of the agents are risk neutral while the rest are risk averse. Different agents will then be drawn to different crops. Efficiency suggests that risk neutral agents should specialize in the more risky crops. But risk neutral agents are also best suited to fixed fee contracts since risk sharing is not an issue for them. Thus, given heterogeneous risk aversion, fixed fee contracts are conceivably associated with more
risky crops so the standard prediction is reversed (Ackerberg, and Botticini, 2002). This extreme example has an empirical implication that is exactly the reverse of Allen and Lueck’s (1992) arguments where fixed-fee contracts are found on the risky crops. The problem here is that while the “crop riskiness” may be exogenous to the landlord who owns the land, it is endogenous through principal-agent matching to the tenant.

With two types of agents characterized by high and low risk aversion, both types prefer a sharing contract, all else equal. However, the preferred piece rate, \( s^* = \frac{1}{1+c\eta \sigma^2} \), varies negatively with the degree of risk aversion. But in many cases the more risk-averse agents may not enter sharing contracts at all. This might occur because other differences between low and high risk-averse agents are not incorporated into the contracts.

On the other hand, a higher piece rate with lower base pay may discourage the more risk-averse agents from entering the contract. If this is the case, more low cost and, hence, less risk averse agents are likely to be contract growers. As a result, heterogeneity among growers can drive some seemingly peculiar contractual arrangements reversing the standard theoretical predictions. Also if low cost growers are risk neutral, they will prefer fixed fee contracts unless restricted by wealth constraints, all else equal, but a fixed fee contract may not be offered by contractors who cannot separately distinguish risk preferences. So agents may choose sharing contracts even though they are risk neutral, which is contrary to the standard theoretical prediction.
3.2.1 Multiple agents and matching

Preserving the risk sharing role of the payment scheme, Wright (2004) provides another reason why a positive relationship between risk and incentives might be observed. He argues that crucial factors are the existence of agents with different degrees of risk aversion and competition between principals for these agents. It is less costly for principals to induce a given effort from a less risk-averse agent than from a more risk-averse agent because less has to be paid to compensate the less risk-averse agent for the risk he bears. Hence, identical principals prefer to select agents with low risk aversion, all else equal, by offering contracts with a high piece rate that discourages high risk-averse agents from contracting. This kind of selection is not considered in the standard model of chapter 2 because only one agent (or, implicitly, many similar agents) are considered. Selection preferences are considered in this chapter in the case of multiple heterogeneous agents.

Also, provision of incentives to a high risk agent is costly when \( \sigma_{\pi}^2 \) is large. Under certain conditions, principals who operate in a riskier environment are prepared to pay more for the services of less risk-averse agents than principals operating in a less risky environment. Therefore, competition ensures that less risk-averse agents are hired by principals operating in riskier environments. This result carries over to the case of asymmetric information about agent types. Such matching of less risk-averse agents to riskier principals was hypothesized in an empirical paper by Ackerberg, and Botticini (2002) and further argued to be reasonable by Prendergast (2002).

Although, all else equal, the more risky the environment the less intense are the incentives, it is also true that the less risk averse the agent the more intense are the
incentives. If the latter effect dominates the former, then principals in riskier environments will offer compensation schemes with more intense incentives than those offered by principals in less risky environments. As a result, an observed positive relationship between risk and incentives is consistent with agency theory. With this approach, *Ackerberg, and Botticini* (2002) find support for agency theory after controlling for endogenous matching.

These examples mainly consider matching based on risk and risk aversion. But many other characteristics can also lead to matching between heterogeneous principals and heterogeneous agents. For example, principals with higher ability to monitor or measure output might prefer low-share, high-rent contracts and thus match up with agents with more risk aversion, more credit constraints, or a higher cost of effort (who would also prefer low-share or wage contracts, all else equal). This type of matching minimizes transaction costs associated with risk aversion.

### 3.2.2 Transaction costs in principal-agent problems

One of the simplifications of standard neoclassical marginal analysis is the assumption that supervision, monitoring, inspection, and enforcement are costless in spot markets. However, spot market transactions involve significant costs. These costs can be reduced or avoided if firms choose to internalize certain activities. Thus, it might be efficient for disparate activities to be combined within a given firm. The transaction costs approach of *Coase* (1937) was a first step in seeking to understand why this happens. As elaborated later by *Coase* (1960), transaction costs can be minimized by expanding a firm to internalize externalities otherwise imposed on it by the actions of others.
However, activities intended to avoid transaction costs carry their own costs because spot market attributes of clarity and focus are often obscured or absent in dealings within a firm. Internal costing and pricing can be problematic, so the management of these internalized activities can become as costly as their purchase or sale. Whenever transaction costs of internal (e.g., vertically integrated) organizations are smaller than those of spot market, vertical integration is preferred. A production contract is an intermediate form between the cases of vertical integration and spot markets. Thus, transaction costs provide a useful perspective for examining the choice among spot markets, production contracts, and vertical integration as devices to organize production (Williamson 1975, 1979; Alchian and Demsetz 1972; Joskow, 1987).

In a principal-agent setting, choices are made between fixed rents (i.e., markets), fixed wages (i.e., internal organization), and production contracts. Transaction costs arise because of the conflicting interests between parties (the principal and agent) when the agent's efforts cannot be easily monitored by the principal and where the output is not influenced by the agent's effort alone but by factors beyond the control of the agent.

Stated succinctly, with transaction costs and a risk averse agent, the production contract is chosen because it distributes the variance of the output among the contracting parties. But if savings in transaction costs from alternative contracts outweigh risk sharing benefits, then alternative contracts would be preferred. Thus, in the choice of contracts, a trade-off exists between risk aversion and transaction costs associated with different types of contracts. The choice of contractual arrangement is made so as to maximize the gain from sharing risk subject to the constraint of transaction costs. While the constraint of transaction costs was not absent in the theoretical analyses of chapter 2,
it was assumed identical for all agents, thus ignoring the potential impact of its heterogeneity on contract choice.

When transaction costs associated with various contracts vary across agents and agents are heterogeneous in risk aversion, more low risk-averse than high risk-averse agents are likely to be observed in production contracts because of their relative efficiency in transactional costs. Inducing a given effort from a low risk-averse agent is less costly for the principal than from a high risk-averse agent because less is required to compensate the low risk-averse agent for the risk he bears. Thus, the transaction costs of effort from a low risk-averse agent are lower. Accordingly, given heterogeneous transaction costs and risk aversion, production contracts conceivably can be associated with less risk-averse agents, thus reversing the standard prediction. In the extreme, agents may choose production contracts even though they are risk neutral, which is contrary to the standard theoretical prediction.

3.2.2.1 Production contracts versus spot markets and vertical integration

The choice of contract depends on how transaction costs associated with negotiation, supervision, and enforcement as well as risk vary across contracts. In considering the three contract forms—wage, share (of production), and fixed rent—negotiation costs are unlikely to vary across contracts. On the other hand, supervision costs are likely to vary significantly by contract type. For example, supervision costs likely decrease as effort is more closely linked with payment. Since a tenant with fixed rent receives all the benefit from extra effort, incentives under fixed rents are superior to
those of production contracts, and hence, less supervision is required than for production contracts.

In fact, supervision costs are least under a fixed-rent contract because the principal receives her fixed fee regardless of the length and intensity of the agent’s effort, provided that the payment is made before the agent exerts effort or that the effort is certain to yield a sufficient output to the principal. For the similar reason, supervision costs are greatest under wage contracts because the payment is based on the amount of effort an agent spends rather than on his output. Since an agent under a production contract receives only a share of his marginal product, supervision costs of production contracts less than for wage contracts but greater than for fixed rent contracts. However, this simple ordering of contracts might not hold if other factors such as underreporting of output are considered.

In addition to supervising agents’ effort, the principal has an incentive to monitor the use of any other inputs that she supplies to the production process. Careless or excessive use of the principal’s work stock or equipment would result in the depreciation of such assets. The more inputs and the more valuable are the inputs that the principal supplies, the greater is the incentive to monitor. However, the marginal cost of monitoring inputs decreases as the number of inputs supplied increases. For example, the marginal cost of supervising agents’ effort may be small if the principal already monitors the use of her work stock by the agent.

Along with supervision and monitoring costs, enforcement costs also vary across contracts. Enforcement cost ensures that an agent honors the length of the contract, i.e., that he does not breach the contract before it expires. The greater the opportunity cost of
breaking a contract, the less likely a breach will occur. For a competitive labor market, an agent’s expected wage is larger under a fixed-rent contract than a production contract because fixed rent requires lower costs of supervision than production contracts. The agent receives a risk premium for bearing more risk. Since opportunity costs increase with the expectation of a larger ex post payment, a fixed rent contract is more likely to dominate. Similarly, a production contract gives the agent a higher expected payment than a wage contract because the opportunity cost (reflected by the expected ex post payment) of leaving the contract exceeds that of a wage earner who is paid on the basis of time, bears no risk, and requires considerable supervision.

Another source of transaction costs is also worth explaining. Transaction costs of markets may be high if there is opportunistic behavior by any of the transacting parties. The principal and agent might have the ability and the incentive to be opportunistic in misrepresenting their type and other transaction related information, for example, by hiding intentions on contract renewal. The principal can be opportunistic and take advantage of an agent’s immobility or asset specific investment. Such specific investment may inflict high transaction costs on the parties. This, in turn, would play a role in influencing the type of contract.

3.2.2.2 Asset specificity and the holdup problem

Research since Coase’s (1937) seminal paper on the nature of the firm has inspired many modifications in economics, but all contributors thus far apparently agree upon the fact that asset-specific investment is one of the main contributing factors to a high level of transaction costs as defined by Williamson (1975, 1985). Williamson
(1975) and *Klein, Crawford, and Alchian* (1978) reinvigorated the transactions costs approach by studying asset specificity and investments. Specificity arises when assets are much less useful and, hence, less valuable in any other use. *Williamson* (1979) defined asset specificity as the magnitude of economic costs associated with redeploying an asset to its best alternative uses and by best alternative users. An extension of the specificity notion is that of relation-specific investment, where an investment is more valuable in one business relationship than in alternative relationships.

Where the principal or the agent makes sizable investments in assets specific to an ongoing supply relationship, spot markets are unlikely. Although relation-specific investments facilitate total value added, the incentive structure for such investments can be weak. When production requires an agent to invest in an asset that is specialized to a particular principal, any deal made prior to investing in the specialized asset may not be enforceable once the investment is made. Thus, agents become vulnerable to being held up by the principal for shares of the quasirent once specific investments have been made.

The quasirent of an asset is the difference between the ex ante best alternative return on the capital invested in an asset and the ex post return it must receive to prevent alternative use of the capital invested in the asset (*Klein, Crawford, and Alchian*, 1978). Thus, relation-specific investments create a divergence between the rate of return that an agent must expect in order to be induced into the relationship and the rate of return below which the agent will exit the relationship.

A holdup involves opportunistic behavior of one or both of the parties in a transaction, the intent of which is to capture quasirents arising in relation-specific investments or other specialized assets. Once the contract is signed and the assets are
deployed one of the parties may threaten to pull out of the arrangement, thereby reducing the value of the specific assets unless a greater share of the quasirents of joint production find their way into the threat-maker’s pockets. Thus, the non-investing party can seek to extract quasirents by use of the market power arising from the difficulty of the relation-specific asset holder in exiting the contract and entering another relationship.

Wary of such opportunism, investing parties—especially parties with low bargaining power—might choose to invest less in such assets than is socially optimal. Also, they may choose less specific, i.e., less specialized and less productive, technology. One way to mitigate this hold-up problem is to vertically integrate the agent and the principal. If they pool their capital into a single enterprise for which profits are jointly shared, then the incentives for quasirent-seeking activities are attenuated. Because such integrated enterprises would choose more productive specialized technology, they would perform better than the contractual alternative.

Another way to address the holdup problem is to sign a long-term contract before the irreversible, asset-specific investment is made. As Williamson (1975) suggests, this type of solution, which is based on vertical integration, depends as much on bounded rationality as it does on opportunism. Opportunism serves little when information is perfect. In a world of certainty and unrestricted cognitive ability, enforceable long-term contracts could be written that preempt unproductive ex post rent-seeking behavior and thus obviate vertical integration. Thus, to prevent opportunism and ensure adequate investment in relation-specific assets, either long-term contracting designed to discourage opportunism or vertical integration, which eliminates the incentive for opportunism, is the preferred approach for organizing production (Williamson, 1979).
This discussion of transaction costs based on relation-specific assets offers both insight into the organization of production and a puzzle. While relation-specific transaction costs provide a convincing explanation for the lack of spot markets, it seems unable to explain why one or the other (production contracts or vertical integration) is preferred. The choice between production contracts and vertical integration depends largely on the anticipated need to adapt to a changing technology or uncertain future and other related characteristics of the technology and the parties. As more change is anticipated or the future is more uncertain, contracts must either become more complex or less complete. Complexity adds to the cost of writing and enforcing a contract, and incomplete contracts retain some incentive for opportunism. The result is that vertical integration becomes the likely choice to organize production (Williamson, 1979).

However, in certain cases, asset-specific investments by agents establish supremacy of production contracts over vertical integration. This supremacy can result from lower supervision, monitoring, and enforcement costs with contracts than with vertical integration. An important function of an agent’s investment in specific assets is the inducement for self-selection by high-ability agents. Requiring agents to invest in specific assets acts as an entry fee and can be used in conjunction with an adjustment of the piece rate to discourage low-ability agents from signing contracts while simultaneously attracting high-ability agents. High-ability agents will remain highly motivated because they will find their investment more profitable than low-ability agents, and thus will not require much supervision and monitoring.

Because monitoring maintenance of specific assets is difficult, an additional benefit of contracts arises from providing proper maintenance incentives to agents. The
performance of a hired agent managing a principal-provided asset will not be assured as
in the case where the same asset is owned by the agent unless the agent’s performance is
supervised or/and monitored (Alchian and Demsetz, 1972). Production contracts
requiring agents to provide these assets (or a fraction of these assets) provide proper
incentives for maintenance without any need for expensive supervision or/and
monitoring.

Also, when agents do not provide these assets, discontinuance is not onerous to
the agent, and thus enforcement costs are larger. Agent provision of assets creates a bond
that assures better performance and makes the relationship a long-term one. Another
advantage is that agent provision of these assets increases the financial leverage of the
principal due to the substitution of agent capital for that of the principal. Reasons for
preferring long-term contracts to vertical integration are discussed further in chapter 5 in
the context of hog contracts. Since long-term contracts are preferred to vertical
integration in the hog sector, the theoretical analysis in this chapter focuses only
production contracts.

Because the preferred approach is long-term contracting, the imperfect
information about agents’ productive capabilities or efficiencies becomes relevant. This
can occur because the productivity of an agent depends on the asset-specific investments
or because the agent has no experience with some technology-imbedded inputs supplied
by the principal. Thus, it is not possible for an agent to know his productive capability
before signing the contract. As will be shown in this chapter, if agents do not know their
abilities either completely or partially, some low-ability agents earn less than their
reservation utilities from operation under contracts. But in the case of large asset-specific
investments, agents earning less than their ex ante reservation utility may have no alternative but to continue to produce in order to recover at least part of the fixed investments.

Turning to hog contracting, in particular, competition among principals by agents is not practical (as explained in chapter 5). Therefore, I limit discussion to the single principal case. An interesting question is whether a principal will offer a single sharing contract with the intent that more able agents will self select it. Also, I explore the important issue of whether all the agents gain from contracting, or whether some of them are left with only negative gains from contracting. This issue has not been explored previously in the literature.

3.3 Contracting with Heterogeneous Agents

Suppose an agent’s output is given by \( \pi = e + \alpha + \theta \) where expected output is the sum of the effort of the agent \( e \) and his ability \( \alpha \). As in chapter 2, \( \theta \) is the state of nature where \( \theta \sim N(0, \sigma^2_\pi) \). But suppose now following Prendergast (1999) that the principal faces multiple heterogeneous agents where agents’ heterogeneity is captured in ability \( \alpha \) where \( \alpha \sim N(0, \sigma^2_\alpha) \). Assume that all the random variables are uncorrelated with one another. Also assume that the certainty equivalent of \( \bar{u} \) is 0 where \( \bar{u} \) is the reservation utility of an agent.

Two cases are discussed: one where agents have no knowledge of their ability (section 3.3.1) and one where they have complete knowledge of their ability (section 3.3.2). The intermediate case when agents have partial knowledge of their ability is addressed later in section 3.5.
3.3.1 The case where agents do not know their ability

This section assumes that abilities are unknown to both the agents and the principal. But the ex ante distribution of abilities is known to all contracting parties. Therefore, contracting takes place under symmetric information. In this case, symmetric distributional knowledge of $\alpha$ drives some of the interesting theoretical predictions about agents’ ex post gains from contracting. Again, the principal and agent are assumed to write a linear contracts of the form $w(\pi) = t + s\pi$ where the base pay is $t$ and the piece rate (i.e., the marginal payment or bonus) is $s$ per unit of $\pi$ produced.

3.3.1.1 Risk neutrality

Again, the risk neutral case can serve as a useful benchmark. As described in section 2.2 of chapter 2, solving the problem as if the agent himself is the principal and is maximizing his own profit and then transferring the expected profit to the principal is one way to find the solution for the risk neutral case. Profit from an agent with ability $\alpha$ is $\Pi = \pi - w(\pi) = e + \alpha + \theta - t - s(e + \alpha + \theta)$ because of $\pi = e + \alpha + \theta$ and $w(\pi) = t + s\pi$.

Expected profit over all $\alpha$ is $E(\Pi) = (1 - s)e - t$. Surplus for an agent with ability $\alpha$ is $S = w(\pi) - C(e) = t + s\pi - ce^2/2 = t + s(e + \alpha + \theta) - ce^2/2$ because $C(e) = ce^2/2$. Thus, expected surplus over all $\alpha$ is $E(S) = t + se - ce^2/2$. The principal’s problem is

$$\max_{e,t,s} (1 - s)e - t$$

s.t. 
(i) \quad t + se - ce^2/2 = 0,
(ii) \quad e \in \arg\max_{e} t + se - ce^2/2.
From constraint (i) of (3.1), \( t = -(se - ce^2/2) \). The IC constraint (ii) of (3.1) implies \( e = s/c \). Substituting \( t \) and \( e \) into the objective function obtains the unconstrained problem of the principal,

\[
\max_s \ (s/c)(1-s/2).
\]

The first-order condition with respect to \( s \) is \((1-s)/c = 0\), which implies \( s^* = 1 \). Similarly, \( e^* = 1/c \) and \( t^* = -1/(2c) \). The fixed fee that an agent has to pay the principal to own the firm is \( k^* = -t^* = 1/(2c) \). This is the maximized expected profit that the agent can earn if he owns the firm. When the agent owns the firm he maximizes expected profit \( E(II) = e - ce^2/2 \) with respect to \( e \). Maximizing \( E(II) \) with respect to \( e \) yields \( e^* = 1/c \) and, thus, maximized expected profit, \( E(II^*) = e^* (1 - ce^*/2) = 1/(2c) \).

The fixed fee, \( k^* \), is same for all the agents irrespective of their ability \( \alpha \). Since the reservation wage is assumed to be zero, an agent’s expected gain from contracting is \( E(II^*) - k^* = 0 \). Again, this assumes agents accept contracts if their ex ante expected gain from contracting is nonnegative (i.e., \( E(II^*) - k^* \geq 0 \)).

### 3.3.1.2 A risk neutral agent’s gain from contracting

This model raises the issue that some agents are left with negative gains from contracting on average (over all states of nature). Consider the case of a given agent \( j \) with ability \( \alpha_j \). Maximized surplus for this agent is \( S^* = t^* + s^* (e^* + \alpha_j + \theta) - c(e^*)^2/2 = \alpha_j + \theta \). Thus, agent \( j \)'s conditional gain from contracting (conditional on both \( \alpha \) and \( \theta \)) is \( S^* = \alpha_j + \theta \) and agent \( j \)'s unconditional expected gain is \( E(S^* | \alpha_j) = \alpha_j \). This means that the reservation wage of zero is met in expectations only for an agent with ability \( \alpha = 0 \). Some agents will have expected gains above their reservation wage and some will be below.
When a high ability agent observes an unfavorable production environment (i.e., a low $\theta$ indicating a bad state), he can still enjoy a non-negative $S^*$ if the realized $\theta$ is not extremely low. On the other hand, when a low ability agent observes a favorable production environment (i.e., a high $\theta$ indicating a good state), he can still receive a negative $S^*$ if the realized $\theta$ is not extremely high. Thus, even though agents accept ex ante contracts depending on their expected gain without knowledge of their $\alpha$, they may, in fact, have a negative expected gain that for some will occur for most or all possible states of $\theta$.

When contracting relationships between the principal and the agents are repeated, the agents observe more than one realization of these random variables. If $\theta$ can be observed separately from $\alpha$, then both the agent and principal can know $\alpha$ after one round of contracting. Alternatively, if $\theta$ cannot be observed separately but only as combined with $\alpha$ in a sum, $\alpha + \theta$, then $\alpha$ can be determined only with error. From the agent’s standpoint, however, if his ability is fixed over time then using $E(\alpha)$ as his expected ability is not reasonable after one round of contracting.

With each additional round of contracting $\theta$’s will occur both below and above $E(\theta)$ but observations on $\alpha_j + \theta$ will obey the law of averages enabling increasingly accurate assessment of $\alpha_j + E(\theta)$. Thus, after many rounds of contracting, the agents ex post expected gain from contracting, which will serve as the ex ante expectation for future contracting, will become the true conditional expectation, $E(S^* | \alpha_j) = \alpha_j$. Ex post, agent $j$ realizes this gain on average. For any strictly positive realization of $\alpha_j$, agent $j$’s $E(S^* | \alpha_j)$ is positive while for any negative realization of $\alpha$ agent $j$’s $E(S^* | \alpha_j)$ is negative. Thus, any agent with realized $\alpha$ greater than its mean has a positive gain from contracting.
on average, and any agent with realized $\alpha$ less than its mean has a negative gain from contracting on average.

3.3.1.3 Risk aversion

Now consider the case where agents are risk-averse with absolute risk aversion $\eta$. Where utility follows the negative exponential form assumed in chapter 2, the agent’s certainty equivalent is $CE(S) = E(S) - \eta V(S)/2 = t + se - \eta s^2 \left( \sigma_x^2 + \sigma_a^2 \right)/2 - ce^2/2$, given that $S = t + s(e + \alpha + \theta) - ce^2/2$. The principal’s problem is to solve

$$\begin{align*}
\max_{e,t,s} & \quad (1-s)e - t \\
\text{s.t.} & \quad t + se - \eta s^2 \left( \sigma_x^2 + \sigma_a^2 \right)/2 - ce^2/2 \geq 0 \\
& \quad e \in \arg\max_e \quad t + se - \eta s^2 \left( \sigma_x^2 + \sigma_a^2 \right)/2 - ce^2/2
\end{align*}$$

Constraint (ii) of (3.2) yields the simple result, $ce = s$. What this equation implies is that if the marginal cost of effort, $ce$, is set equal to the piece rate, $s$, for any level of $e$, then exerting any lesser effort by the agent is not a maximizing strategy for him. Therefore, this constraint induces the agent to exert the effort level that is optimal for the principal. Knowing that $e = s/c$, the principal solves

$$\begin{align*}
\max_{t,s} & \quad s/c - t + s^2/c \\
\text{s.t.} & \quad t + s^2/(2c) - \eta s^2 \left( \sigma_x^2 + \sigma_a^2 \right)/2 = \bar{w}.
\end{align*}$$

With further substitution into (3.3), the principal solves the unconstrained problem

$$\begin{align*}
\max_s & \quad s/c - s^2/(2c) + \eta s^2 \left( \sigma_x^2 + \sigma_a^2 \right)/2 + \bar{w}
\end{align*}$$
The first-order condition of (3.4) with respect to $s$ requires $1/c - s/c - \eta s (\sigma_\pi^2 + \sigma_a^2) = 0$.

Further manipulation yields $s^* = 1/[1 + \eta c(\sigma_\pi^2 + \sigma_a^2)]$. The complete solution to this problem yields optimal agent effort $e^* = 1/[c + \eta c^2(\sigma_\pi^2 + \sigma_a^2)]$, optimal base pay $i^* = [1 - \eta c(\sigma_\pi^2 + \sigma_a^2)]/[2c + 2\eta c^2(\sigma_\pi^2 + \sigma_a^2)]^2$, and optimal expected output, $\pi_c^* = 1/[2c + 2\eta c^2(\sigma_\pi^2 + \sigma_a^2)]$.

This simple model illustrates how the tradeoff between risk and incentives are modified by the introduction of heterogeneous agents with risk aversion. A high degree of risk aversion mutes incentives even more than with homogeneity. Even with no randomness in the state of nature (i.e., $\sigma_\pi^2 = 0$), a fixed fee contract is not optimal. That is, even if $\sigma_\pi^2 = 0$, effort cannot be inferred from the realized output with certainty because even with a nonstochastic $\theta$, one part of the agent’s output is explained by the ability variable $\alpha$ that is not observable. A low output may be due to either low ability or low effort from the agent. As a result, an agent with low ability making normal effort will be paid less for the lower output just as if he exerted low effort with normal ability. Thus, he will not be compensated enough to reach his reservation utility. Sensing this problem, agents will exert less effort.

As a result, the effort level, already distorted downward by risk aversion to $e^* = 1/(c + \eta c^2\sigma_\pi^2)$ from its first-best level $e^* = 1/c$ is further distorted downward to $e^* = 1/(c + \eta c^2(\sigma_\pi^2 + \sigma_a^2))$ because of heterogeneity in agents’ ability. Also, the piece rate $s^* = 1/(1 + \eta c\sigma_\pi^2)$ is distorted downward to $s^* = 1/(1 + \eta c(\sigma_\pi^2 + \sigma_a^2))$, which reduces the punishment for lower outputs resulting from lower ability. However, some agents fall short of their reservation utilities unless the piece rate is set to zero.
3.3.1.4 A risk-averse agent’s gain from contracting

In this model, some risk-averse agents receive a negative gain from contracting on average ex post. The conditional certainty equivalent surplus (conditioned on \( \alpha \)) is

\[
CE(S \mid \alpha) = t + se + sa - \eta s^2 \sigma^2 / 2 - ce^2 / 2 \quad \text{because} \quad E(w(\pi) - ce^2/2 \mid \alpha) = t + sE(\pi \mid \alpha) - ce^2/2 = t + se + sa - ce^2/2 \quad \text{and} \quad V(w(\pi) - ce^2/2 \mid \alpha) =
\]

\[
V(t + s(\pi \mid \alpha)) = V(s(e + \alpha + \theta)) = s^2 \sigma^2. \quad \text{After maximization,} \quad CE(S^* \mid \alpha) = t^* + s^* e^* + s^* a - \eta(s^*)^2 \sigma^2 / 2 - c(e^*)^2 / 2. \quad \text{From condition (i) in (3.2),} \quad CE(S^*) = t^* + s^* e^* - \eta(s^*)^2 (\sigma_x^2 + \sigma_a^2) / 2
\]

\[- c(e^*)^2 / 2 = 0. \quad \text{Thus, after maximization,} \quad CE(S^* \mid \alpha) = s^* a + \eta(s^*)^2 \sigma_a^2 / 2. \quad \text{This is the ex post gain from contracting on average for an agent with realized ability} \ \alpha. \quad \text{This is negative if} \ \alpha < \hat{\alpha} \quad \text{where} \ \hat{\alpha} = - \eta s^* \sigma_a^2 / 2.
\]

Therefore, only those agents whose ability is \( \alpha = \hat{\alpha} \) gain exactly 0 on average from contracting. Agents with \( \alpha > \hat{\alpha} \) have a positive average gain from contracting while those with \( \alpha < \hat{\alpha} \) have a negative average gain from contracting. Another interesting result is that, all else equal, the higher the piece rate \( s^* \) the smaller is the threshold ability \( \hat{\alpha} \). Thus, by offering a higher piece rate the principal can ensure positive ex post gains to a larger pool of agents, and vice versa.

Also, with a higher piece rate, agents with \( \alpha > \hat{\alpha} \) will have a larger positive gain from contracting. In an optimal solution, however, the principal will try to shrink the gain of agents. For this reason, whenever agents differ substantially in their abilities, the optimal piece rate \( s^* \) is smaller. This is clear because \( s^* = 1/(1 + \eta c(\sigma_x^2 + \sigma_a^2)) \) is decreasing in the variance of ability \( \sigma_a^2 \). As a result, the smaller is the piece rate \( s^* \), the larger is the threshold ability \( \hat{\alpha} \). Thus, by offering a smaller piece rate, the principal
imposes negative ex post gains on a larger pool of agents. Compared with the risk neutral case, the qualitative result that agents with below (above) average abilities have conditional expected gains from contracting that are negative (positive) is unaltered except that the threshold $\alpha$ moves down from 0 to $\hat{\alpha}$.

### 3.3.2 The case where agents know their ability

Assume now that abilities have a positive mean and are known privately only to the agents. Consider a definition of the critical value of ability $\alpha = \alpha^C$ that leaves an agent with exactly zero conditional expected gain from contracting. For the linear contract where $w(\pi) = t + s\pi$ and $E(w(\pi) \mid \alpha^C) = t + sE(\pi \mid \alpha^C) = t + s(E + \alpha^C + \theta) = t + s(E + \alpha^C)$, only those agents whose ability is equal to $\alpha^C$ will earn exactly $\bar{w}$ implying

$$t + s(\alpha^C + e^*) - \eta s^2 \sigma_\pi^2 / 2 - C(e^*) = 0.$$ Rearranging obtains (Prendergast, 1999)

$$\alpha^C = \eta s \sigma_\pi^2 / 2 - t / s + C(e^*) / s - e^*.$$

Thus, agents with $\alpha < \alpha^C$ have a negative expected gain from contracting. When agents know their ability before signing contracts, only agents with $\alpha \geq \alpha^C$ will elect to sign contracts. This outcome demonstrates how compensation contracts can have selection effects such that higher piece rates are relatively more attractive to more able agents. By offering greater pay for performance, firms can hire a better distribution of agents because the more able benefit more from these contracts than the less able (Prendergast, 1999).

The primary focus of the agency literature has been on how contracts induce certain behaviors from a given set of agents. However, contracts can also play a central role in recruiting agents that better serve the principal’s objectives. Often, an important
component of the principal's task is to select the best agent or agents. The procurer of an item wants to select the least-cost supplier. Banks seek to identify the most reliable loan applicants. In general, principals seek to design contracts not only to induce certain behaviors but also to influence the type of agents that they hire. Lazear (1986) investigated selection effects in a multiple agent setting with tournaments. According to his finding, the average quality of worker is increased by a firm shifting from a fixed wage scheme to one with piece rates.

Accordingly, this may change the design of the optimal contract because contracts now fulfill a dual role of both inducing effort and aiding the selection of better agents. As a result, in order to select certain types of agents, the principal may distort the effort decisions from the otherwise optimal choices derived above. This selection effect of contracting is an important one when agents know their ability but the principal does not. However, if both the parties are unaware of agents’ types, such as at the beginning of a contracting problem, then this selection issue is not relevant. But when agents are unaware of their type, some low ability agents do not get paid enough to match their reservation utilities.

3.4 Exploring Different Specifications

Even though some of the agents are left with less than their reservation utilities irrespective of their risk preferences in sections 3.3.1 and 3.3.2, optimal effort is not distorted when agents are risk neutral because of the additive specification of the technology. Even with a multiplicative specification of the technology, such as \( \pi = ea + \theta \), agents with low ability are left with negative expected gains from contracting.
However, this specification introduces a distortion in optimal effort choices, and thus reduces the principal’s surplus.

3.4.1 The case where agents do not know their ability

To consider the implications of the multiplicative specification of the technology, a useful starting point is the optimal contracting problem when ability is known both to the principal and the agent.

3.4.1.1 The first-best case

Suppose the principal can design the contract and calculate the surplus conditional on known ability $\alpha$ of the agent. Then she calculates her total expected surplus over all the agents (over all $\alpha$’s). For a given $\alpha$, $\pi = e\alpha + \theta$, and taking expectations over $\theta$ implies $E(\pi) = e\alpha$, $w(\pi) = t + s\pi = t + s(e\alpha + \theta)$, $E(w(\pi)) = t + se\alpha$ and $E(\pi) - E(w(\pi)) = e\alpha - t - se\alpha$. The principal solves

$$\max_{e,\pi} e\alpha - t - se\alpha$$

s.t. (i) \[ t + se\alpha - ce^2/2 \geq 0, \]

where for simplicity the agent’s reservation wage $\bar{w}$ is normalized to zero. In the optimal solution, (i) of (3.5) holds as equality, i.e., $t + se\alpha = ce^2/2$. Substitution into the objective function of (3.5) yields the unconstrained optimization problem $\max_{e} e\alpha - ce^2/2$. The associated first-order condition requires $e^* = a/c$. The principal thus receives $E(\pi) - E(w(\pi)) = a^2/c - a^2/(2c) = a^2/(2c)$ from an agent with ability $a$. The agent with ability $a$ receives $E(w(\pi)) = t + se\alpha = a^2/(2c)$. The agent’s effort cost is $C(e^*) = c(e^*)^2/2 = a^2/(2c)$. Thus, the agent’s expected gain from contracting is $E(w(\pi)) - C(e^*) = a^2/(2c) - a^2/(2c)$.
= 0. None of the agent receive a negative expected gain from contracting in this case. The agent’s effort is increasing in \( \alpha \), and hence, the agent’s payment must be increasing in \( \alpha \) to cover effort cost.

3.4.1.2 The second-best case when both effort and ability are unobservable

Suppose now that the principal does not know either \( \alpha \) or \( e \) whereas the agent knows only \( e \). Then \( E(\pi) = eE(\alpha), E(w(\pi)) = t + seE(\alpha), \) and \( E(\pi) - E(w(\pi)) = eE(\alpha) - t - seE(\alpha). \) The principal now solves

\[
\text{Max}_{e,t,s} eE(\alpha) - t - seE(\alpha)
\]  

s.t. (i) \( t + seE(\alpha) - ce^2/2 \geq 0 \)

(ii) \( e \in \arg \max_e t + seE(\alpha) - ce^2/2. \)

Solving \( e \in \arg \max_e t + seE(\alpha) - ce^2/2 \) implies \( e = sE(\alpha)/c \). Substituting this value of \( e \) into the objective function and constraint (i) of (3.6) and using further substitution yields the unconstrained optimization problem \( \text{Max}_s sE(\alpha)/c - s^2E(\alpha)/(2c) \). The first-order condition requires \( s^* = 1 \) and \( e^* = E(\alpha)/c \) and implies \( t^* = -(s^* e^* E(\alpha) - c(e^*)^2)/2 = - (E(\alpha))^2/(2c) \).

In this case, the agent owns the output and pays \( (E(\alpha))^2/(2c) \) to the principal for the right of ownership. As a result, from the principal’s perspective, selling the project to the agent solves the moral hazard problem that arises from the unobservability of effort. Optimal effort for all agents is \( e^* = E(\alpha)/c \). From this expression, agents with above average ability will exert less effort than in the first-best case whereas agents with below average ability will exert more effort than in the first-best case.
Since he owns the project, the expected earning of agent $j$ is $ae^* = aE(a)/c$ and his expected gain from contracting (i.e., his expected earning net of effort cost and the principal’s fee) is $E(a)[a - E(a)]/c$. Thus, $E(a)[a - E(a)]/c > 0$ for $a > E(a)$ and $E(a)[a - E(a)]/c < 0$ for $a < E(a)$. This implies expected gains from contracting are negative for agents with below average ability, while they are positive for agents with above average ability (and zero for agents with average ability).

Because $V(a) = E(a^2) - (E(a))^2$, a positive variance of $a$ implies $E(a^2) > (E(a))^2$ and, hence, $E(a^2)/(2c) > (E(a))^2/(2c)$. Thus, the principal earns more when she observes $a$ along with the agent as in the first-best case. In this case, it is in the interest of the principal to induce different efforts for different abilities rather than the same effort depending on $E(a)$. This result contrasts with the linear specification, $\pi = e + a + \theta$, in section 3.3.1 and 3.3.2, where the optimal effort was not dependent on $a$. But the multiplicative case confirms that the result with negative expected gains for agents with below average abilities is not driven by linearity of the agent’s output.

3.4.2 The case where agents know their ability

In this section, the multiplicative ability model of the section 3.4.1 is adapted for expositional purposes to the case with two types of abilities. Suppose the ability types of agents are either $\alpha_1$ or $\alpha_2$ (where $\alpha_2 > \alpha_1 > 0$) with probabilities $p_1$ and $p_2$, respectively ($p_2 = 1 - p_1$). Suppose at the beginning of the contractual relationship that agents know their abilities and the principal does not. But the principal knows that the agents know their abilities. Therefore, the principal can design the contract in such a way that the agents
reveal their type to the principal through their choices of contracts. In this case, the principal solves the following problem:

\[
\begin{align*}
\text{Max} & \quad p_1(e_1, \alpha_1 - t_1 - s_1, \alpha_1) + p_2(e_2, \alpha_2 - t_2 - s_2, \alpha_2) \\
\text{s.t.} & \quad (\text{ia}) \quad t_1 + s_1, \alpha_1 - c e_1^2 / 2 \geq 0 \\
& \quad (\text{ib}) \quad t_2 + s_2, \alpha_2 - c e_2^2 / 2 \geq 0 \\
& \quad (\text{ii}) \quad t_2 + s_2, \alpha_2 - c e_2^2 / 2 \geq t_1 + s_1, \alpha_1 - c e_1^2 / 2 \\
& \quad (\text{iii}) \quad e_1 \in \arg \max_{\alpha_1} t_1 + s_1, \alpha_1 - c e_1^2 / 2 \\
& \quad (\text{iv}) \quad e_2 \in \arg \max_{\alpha_2} t_2 + s_2, \alpha_2 - c e_2^2 / 2,
\end{align*}
\]  

(3.7)

where \( t_j, s_j, \) and \( e_j \) are the base pay, bonus pay and effort for agents with ability \( \alpha_j \), respectively.

Constraint (ii) of (3.7) guarantees that high ability agents will not choose the contract offered for low ability agents. For low ability agents, the principal can set \( t_1 + s_1, \alpha_1 - c e_1^2 / 2 = 0 \). Thus, low ability agent’s expected gain from contracting is zero.

From (ii) we obtain,

\[
t_2 + s_2, \alpha_2 - c e_2^2 / 2 \geq t_1 + s_1, \alpha_1 - c e_1^2 / 2 = t_1 + s_1, \alpha_1 - c e_1^2 / 2 + s_1, (\alpha_2 - \alpha_1)
= s_1, (\alpha_2 - \alpha_1).
\]

That is, the principal has to offer at least a rent of \( s_1, (\alpha_2 - \alpha_1) \) to each high ability agent. Otherwise, the high ability agent will choose the contract \((t_1, s_1, e_1)\) intended for the low ability agent. Thus, high ability agent’s expected gain from contracting is \( s_1, (\alpha_2 - \alpha_1) \) which is positive if positive effort is exerted by low ability agents.
Solving constraint (iii) and (iv) of (3.7) for both $e_1$ and $e_2$ obtains $e_1 = s_1 \alpha_1 / c$ and $e_2 = s_2 e_2 / c$. Substituting $e_1$ and $e_2$ into the objective function and constraints (ia), (ib), and (ii) of (3.7), and further substitution from these constraints into the objective function of (3.7) yields the unconstrained principal’s problem,

$$
\max_{s_1, s_2} \left[ p_1(s_1 \alpha_1^2 / c - s_1^2 \alpha_1^2 / (2c)) + p_2(s_2 \alpha_2^2 / c - s_2^2 \alpha_2^2 / (2c)) - s_1^2 \alpha_1(\alpha_2 - \alpha_1) / c \right].
$$

(3.8)

The first-order conditions of (3.8) with respect to $s_1$ and $s_2$ yield $s_1^* = 1/[1 + 2(p_2/p_1)(\alpha_2 - \alpha_1)/\alpha_1]$ and $s_2^* = 1$. Thus, $e_1^* = \alpha_1 / [c + 2c(p_2/p_1)((\alpha_2 - \alpha_1)/\alpha_1)]$, $e_2^* = \alpha_2 / c$, $t_1 = -s_1^* e_1^* a_1 + c (e_2^*)^2 / 2$, and $t_2 = -s_2^* e_2^* a_2 + c (e_2^*)^2 / 2 + s_1^* e_1^* (\alpha_2 - \alpha_1)$. This is a separating contract because effort and transfer are different for each ability type. In section 3.4.1, the contract is not a separating contract because effort, $e^* = E(\alpha) / c$, and transfer, $t^* = -E(\alpha)^2 / (2c)$, are the same for every agent irrespective of their ability. The principal is not able to offer separating contracts in that case because agents do not know their abilities.

Obviously, under agential heterogeneity, a sharing contract emerges for all but the highest-ability agents even in the case of risk neutral agents. This is the same result found in sections 2.3.5 and 2.4.2 with limited liability and pre-contractual information asymmetry models, respectively. In this separating case, none of the agents are left with a negative expected gain from contracting. Only agents with high ability receive a positive expected gain from contracting while agents with low ability have a zero expected gain from contracting.

The case of the multiplicative specification under risk aversion is not examined here because it reveals no qualitative differences compared to the additive specification ($\pi = e + \alpha + \theta$) except that the threshold ability level moves downward from 0. Low ability agents receive a negative ex post gain from contracting on average if they do not
know their ability ex ante as described in section 3.3.1. Thus, negative ex post gains from contracting on average for low-ability agents are the consequences of their lack of knowledge of their abilities.

As this and previous sections make clear, a central question is whether agents know their abilities before entering a contract. Section 3.5 discusses the results when abilities are known partially. Intuitively, partial knowledge of abilities is derived from past performance.

3.5 The Case Where Agents Know Their Ability Imperfectly

When two parties engage in a contracting relationship, often they have partial but incomplete knowledge about abilities. Uncertainty about a particular agent’s ability is represented by a subjective distribution, which may be different for the agent than for the principal. Both the principal and the agent can gain additional information with each period of production, but the clarity of additional information depends on explicit observability of the state of nature, which follows a distribution assumed to be common knowledge. (Laffont, and Martimort, 2002). The initial contract may be characterized as an agent choosing whether to make a costly investment in technology. As a result, the agent has to decide whether to accept or reject a contract before knowing his ability to use the technology efficiently.

Likely, the assumption that agents or employees have no information about abilities is extreme. Likewise, the assumption that agents know their abilities perfectly before commencing production is extreme. In this section, I consider whether agents with low ability receive negative ex post gains from contracting on average when they have
partial information about their abilities. The extent of partial information is critical in determining what share of agents receive negative gains from contracting.

The risk neutral and risk averse cases in section 3.3.1 suggest no qualitative differences regarding low-ability agents’ negative gains from contracting on average. The only difference that occurs under risk aversion is in the threshold ability level, which is smaller in case of risk-averse agents. Obviously, since agents get only a fraction of the profit they generate, they get only a fraction of the benefits (or losses) associated with their realized abilities. Therefore, this section focuses on the risk neutral contracting case.

3.5.1 The imperfect information model

Following *Harris and Townsend* (1981) and *Sappington* (1984), suppose the random ability $\alpha$ can follow any one of $D$ possible distributions rather than a single distribution with $n$ states of $\alpha$. Consider the information structure where the agent knows the actual distribution at the time the contract is signed but the principal has only a nondegenerate prior defined over these $D$ distributions. Thus, the agent’s information is better but imperfect, and the principal is aware of this fact. Thus, the principal’s problem is to solve

$$\begin{align*}
\text{Max}_{e,t,s} & \sum_{d=1}^{D} \phi^d \int [e\alpha - t(\alpha) - s(\alpha)e\alpha] f^d(\alpha) d\alpha \\
\text{s.t.} & (i) \int [t(\alpha) + s(\alpha)e\alpha - ce^2/2] f^d(\alpha) d\alpha \geq 0 \ \forall d = 1,...,D, \\
& (ii) \int [t(\alpha) + se\alpha - ce^2/2] f^d(\alpha)d\alpha \geq \int [t(\alpha) + se\alpha - ce^2/2] f''(\alpha)d\alpha \ \forall d = 1,...,D, \\
& (iii) e \in \arg \max_{e} \int [t(\alpha) + s(\alpha)e\alpha - ce^2/2] f^d(\alpha)d\alpha \ \forall d = 1,...,D,
\end{align*}$$

(3.9)
where $\phi^d$ is the probability that the actual distribution is $f^d(\alpha)$, $s(\alpha)$ is the share of output that goes to the agent with ability $\alpha$ under distribution $f^d(\alpha)$, and $t(\alpha)$ is the base payment to the agent with ability $\alpha$ for exerting effort $e$ under distribution $f^d(\alpha)$, $d = 1, \ldots, D$.

The individual rationality constraints in (i) of (3.9) guarantee that any contract selected by the agent achieves an expected utility that (weakly) exceeds his reservation level, which is normalized to zero for simplicity. The self-selection among contracts constraint (ii) of (3.9) guarantees that the agent selects contract $[t(\alpha), s(\alpha)]$ when $f^d(\alpha)$ is his actual distribution of $\alpha$. The self-selection-within-contracts constraint (iii) of (3.9) ensure that the agent chooses the maximizing effort $e$ under the contract $[t(\alpha), s(\alpha)]$. If $D = 1$, then (3.9) reduces to the typical principal-agent model in which the principal and agent share the same pre-contractual beliefs (where a single distribution is applicable).

*Sappington* (1984), building on an idea suggested by *Harris and Townsend* (1981), has shown that the optimal strategy for the principal is to design at most $D$ distinct contracts from which the agent is permitted to make a binding choice. The rationale is that, if the contracts are designed appropriately, then agents can be induced to use their private information under some distributions to select contracts that the principal prefers to the single contract that the principal would design in the absence of structural information about the distribution of $\alpha$.

However, Sappington’s distributions are considered over the random states $\theta$ rather than abilities $\alpha$. His model does not focus on agents’ gains from contracting and has no heterogeneity in agents’ characteristics. I adapt his model to the case of heterogeneity in agents’ characteristics, and determine whether agents gain or lose from contracting when they have partial knowledge of their ability.
3.5.2 The discrete case

For simplicity, I solve the discrete version of this problem where \( D = 2 \) and \( n = 2 \).

Suppose ability distribution 1 (denoted by \( P \)) has probability \( p_1 \) of ability \( \alpha_{11} \) and probability \( p_2 \) of ability \( \alpha_{12} \) \((p_2 = 1 - p_1)\) while distribution 2 (denoted by \( Q \)) has probability \( q_1 \) of ability \( \alpha_{21} \) and probability \( q_2 \) of ability \( \alpha_{22} \) \((q_2 = 1 - q_1)\). Also assume that \( E(\alpha_1) < E(\alpha_2) \) where \( E(\alpha_1) = p_1\alpha_{11} + p_2\alpha_{12} \) and \( E(\alpha_2) = q_1\alpha_{21} + q_2\alpha_{22} \). Let \( r_1 \) and \( r_2 \) represent the principal’s subjective probability that the agent is from distributions \( P \) and \( Q \), respectively, whereas the agent knows whether his ability distribution is \( P \) or \( Q \). The principal solves

\[
\begin{align*}
\text{Max} & \quad E \left[ r_1 \left( p_1 (e, \alpha_{11} + \theta - t_1 - s_1 e, \alpha_{11} - s_1 \theta) + p_2 (e, \alpha_{12} + \theta - t_1 - s_1 e, \alpha_{12} - s_1 \theta) \right) + \\
& \quad r_2 \left( q_1 (e, \alpha_{21} + \theta - t_2 - s_2 e, \alpha_{21} - s_2 \theta) + q_2 (e, \alpha_{22} + \theta - t_2 - s_2 e, \alpha_{22} - s_2 \theta) \right) \right] \\
\text{s.t.} & \quad (i.a) \quad E \left[ p_1 (t_1 + s_1 e, \alpha_{11} - c e_1^2 / 2) + p_2 (t_1 + s_1 e, \alpha_{12} - c e_1^2 / 2) + s_1 \theta \right] \geq 0 \\
& \quad (i.b) \quad E \left[ q_1 (t_2 + s_2 e, \alpha_{21} - c e_2^2 / 2) + q_2 (t_2 + s_2 e, \alpha_{22} - c e_2^2 / 2) + s_2 \theta \right] \geq 0 \\
& \quad (ii) \quad E \left[ q_1 (t_2 + s_2 e, \alpha_{21} - c e_2^2 / 2) + q_2 (t_2 + s_2 e, \alpha_{22} - c e_2^2 / 2) + s_2 \theta \right] \geq 0 \\
& \quad E \left[ q_1 (t_1 + s_1 e, \alpha_{11} - c e_1^2 / 2) + q_2 (t_1 + s_1 e, \alpha_{12} - c e_1^2 / 2) + s_1 \theta \right] \geq 0 \\
& \quad (iii) \quad e_1 \in \arg \max_{e_1} E \left[ \left[ p_1 (t_1 + s_1 e, \alpha_{11} - c e_1^2 / 2) + p_2 (t_1 + s_1 e, \alpha_{12} - c e_1^2 / 2) + s_1 \theta \right] \right] \\
& \quad (iv) \quad e_2 \in \arg \max_{e_2} E \left[ \left[ q_1 (t_2 + s_2 e, \alpha_{21} - c e_2^2 / 2) + q_2 (t_2 + s_2 e, \alpha_{22} - c e_2^2 / 2) + s_2 \theta \right] \right].
\end{align*}
\]

Because \( E(\theta) = 0 \), problem (3.10) boils down to

\[
\begin{align*}
\text{Max} & \quad r_1 \left[ p_1 (e, \alpha_{11} - t_1 - s_1 e, \alpha_{11}) + p_2 (e, \alpha_{12} - t_1 - s_1 e, \alpha_{12}) \right] + \\
& \quad r_2 \left[ q_1 (e, \alpha_{21} - t_2 - s_2 e, \alpha_{21}) + q_2 (e, \alpha_{22} - t_2 - s_2 e, \alpha_{22}) \right] \\
& \quad r_1 \left[ p_1 (e, \alpha_{11} - t_1 - s_1 e, \alpha_{11}) + p_2 (e, \alpha_{12} - t_1 - s_1 e, \alpha_{12}) \right] + \\
& \quad r_2 \left[ q_1 (e, \alpha_{21} - t_2 - s_2 e, \alpha_{21}) + q_2 (e, \alpha_{22} - t_2 - s_2 e, \alpha_{22}) \right] \quad (3.11)
\end{align*}
\]
s.t. (ia) \[ p_1(t_1 + s_1 e_1 \alpha_{11} - ce_1^2 / 2) + p_2(t_1 + s_1 e_1 \alpha_{12} - ce_1^2 / 2) \geq 0 \]

(ib) \[ q_1(t_2 + s_2 e_2 \alpha_{21} - ce_2^2 / 2) + q_2(t_2 + s_2 e_2 \alpha_{22} - ce_2^2 / 2) \geq 0 \]

(ii) \[ q_1(t_2 + s_2 e_2 \alpha_{21} - ce_2^2 / 2) + q_2(t_2 + s_2 e_2 \alpha_{22} - ce_2^2 / 2) \geq q_1(t_1 + s_1 e_1 \alpha_{21} - ce_1^2 / 2) + q_2(t_1 + s_1 e_1 \alpha_{22} - ce_1^2 / 2) \]

(iii) \[ e_1 \in \arg\max_{e_1} p_1(t_1 + s_1 e_1 \alpha_{11} - ce_1^2 / 2) + p_2(t_1 + s_1 e_1 \alpha_{12} - ce_1^2) / 2 \]

(iv) \[ e_2 \in \arg\max_{e_2} q_1(t_2 + s_2 e_2 \alpha_{21} - ce_2^2 / 2) + q_2(t_2 + s_2 e_2 \alpha_{22} - ce_2^2 / 2) \]

Since agents do not know their exact ability, the principal cannot induce them to reveal it. But the principal can induce them to choose the contract that is intended for their distribution. As a result, the principal induces only one effort for each distribution, say, \( e_1 \) for \( P \) and \( e_2 \) for \( Q \). This is done by choosing a transfer and piece rate for each distribution type, say, \( t_1 \) and \( s_1 \) for \( P \), and \( t_2 \) and \( s_2 \) for \( Q \), respectively.

Constraint (ia) of (3.11) is the individual rationality constraint for agents from distribution \( P \). Constraint (ib) of (3.11) is individual rationality constraint for agents from distribution \( Q \). The self-selection-between-contracts constraint (ii) of (3.11) ensures that agents with the high ability distribution \( Q \) will not choose the contract intended for agents with the low ability distribution. Constraint (ia) is redundant when both (ia) and (ii) are satisfied. The self-selection–within-contracts constraints (iii) and (iv) of (3.11) require that agents with both distributions choose the effort that is profit maximizing for them. At the optimum, constraint (ii) will hold as an equality for the same reason that constraint (ia) will hold as an equality. Thus, problem (3.11) can be written as

\[
Max_{t_1, t_2, s_1, s_2, e_1, e_2} r_1[e_1 E(\alpha_1) - t_1 - s_1 e_1 E(\alpha_1)] + r_2[e_2 E(\alpha_2) - t_2 - s_2 e_2 E(\alpha_2)]
\] (3.12)
s.t. (i) \[ t_1 + s_1 e_1 E(\alpha_1) - ce_1^2 / 2 = 0 \]

(ii) \[ t_2 + s_2 e_2 E(\alpha_2) - ce_2^2 / 2 = t_1 + s_1 e_1 E(\alpha_2) - ce_1^2 / 2 \]

(iii) \[ e_1 \in \arg \max_{\alpha_1} t_1 + s_1 e_1 E(\alpha_1) - ce_1^2 / 2 \]

(iv) \[ e_2 \in \arg \max_{\alpha_2} t_2 + s_2 e_2 E(\alpha_2) - ce_2^2 / 2 . \]

Solving constraints (iii) and (iv) of (3.12) yields \( e_1 = s_1 E(\alpha_1) / c \) and \( e_2 = s_2 E(\alpha_2) / c \), respectively. Constraint (ii) of (3.12) implies

\[
t_2 + s_2 e_2 E(\alpha_2) - ce_2^2 / 2 = t_1 + s_1 e_1 E(\alpha_2) - ce_1^2 / 2 + s_1 e_1 E(\alpha_1) - s_1 e_1 E(\alpha_1)
= s_1 e_1 (E(\alpha_2) - E(\alpha_1)).
\]

Substituting for \( t_1, t_2, e_1, \) and \( e_2 \) from the constraints into the objective function obtains the unconstrained problem

\[
Max \ r_1 [s_1 (E(\alpha_1))^2 / c - s_1^2 (E(\alpha_1))^2 / (2c)] + r_2 [s_2 (E(\alpha_2))^2 / c

- s_2^2 (E(\alpha_2))^2 / (2c) - s_2^2 (E(\alpha_1) - E(\alpha_1))]
\]

(3.13)

First-order conditions of (3.13) with respect to \( s_1 \) and \( s_2 \) yield

\[
s_1^* = 1 / [1 + 2(r_2 / r_1)(E(\alpha_2) - E(\alpha_1)) / E(\alpha_1)]
\]

\[
s_2^* = 1.
\]

Thus,

\[
e_1^* = E(\alpha_1) / [c + 2 c(r_2 / r_1)(E(\alpha_2) - E(\alpha_1)) / E(\alpha_1)]
\]

\[
e_2^* = E(\alpha_2) / c.
\]
3.5.3 Gains from contracting

Now consider whether some agents are left with negative gains from contracting. The fixed fee for an agent whose ability is from the distribution $P$ is $t_1 = -s_1^*e_1^*E(\alpha_1) + c(e_1^*)^2/2$. If the agent’s ability is $\alpha_{11}$, then his surplus is $s_1^*e_1^*\alpha_{11} + s_1^*\theta - c(e_1^*)^2/2$ and his gain from contracting is

$$s_1^*e_1^*\alpha_{11} + s_1^*\theta - c(e_1^*)^2/2 = s_1^*\theta + s_1^*(\alpha_{11} - E(\alpha_1)).$$

His ex post gain from contracting on average is $s_1^*(\alpha_{11} - E(\alpha_1)) < 0$. This happens because ability $\alpha_{11}$ is worse than expected and constraint (ia) sets the expected gain from contracting to zero for expected ability. Alternatively, if the agent’s ability is $\alpha_{12}$ then his gain from contracting is $s_1^*\theta + s_1^*(\alpha_{12} - E(\alpha_1))$. His ex post gain from contracting on average is $s_1^*(\alpha_{12} - E(\alpha_1)) > 0$ for similar reasoning.

The gain from contracting for an agent with high ability expectations (distribution $Q$) is $s_1^*e_1^*(E(\alpha_2) - E(\alpha_1))$. If the ability distributions satisfy $E(\alpha_1) < \alpha_{21} < \alpha_{22}$, then all agents with high expected ability receive a positive ex post gain from contracting on average because $s_1^*e_1^*(\alpha_{22} - E(\alpha_1)) > s_1^*e_1^*(\alpha_{21} - E(\alpha_1)) > 0$. Agents with highest ability receive the highest ex post average gain, $s_1^*e_1^*(\alpha_{22} - E(\alpha_1))$. However, for ability distributions such that $\alpha_{21} < E(\alpha_1)$, agents with high ability expectations but low ex post ability will receive negative ex post gains from contracting on average.
3.6 Conclusions

In this chapter, heterogeneity in agents’ characteristics is incorporated into the basic model of chapter 2. Some important standard theoretical predictions are reversed once agential heterogeneity is introduced. As a result, an observed positive relationship between risk and incentives is consistent with agency theory. The important issue of whether all agents gain from contracting under heterogeneity is explored in detail. Three cases have been discussed depending on whether agents have no knowledge of their ability, complete knowledge of their ability, or partial knowledge of their ability.

For the case when the agents are risk neutral and have no knowledge of their ability, agents with realized ability greater than their expectations receive positive gains from contracting on average and agents with realized ability less than their expectations receive negative gains from contracting on average. When agents are risk averse under this information structure, essentially the same qualitative results apply except that the threshold where low ability agents receive negative ex post gains from contracting on average is lower than the ex ante expected ability level. For the intermediate case when agents have partial knowledge of their ability, some low ability agents receive negative gains from contracting on average. However, none of the agents receives negative gains from contracting when they have complete knowledge of their ability before signing the contract.

These results raise the concern about what actions low ability agents can take when they realize their ability is lower than expected and, consequently, their gain from contracting will be negative on average. Do they simply abandon the contract after one period or do they try to negotiate a revised contract with the principal? If for some reason
the principal will not or cannot make significant changes in the contract terms after first period, such agents may terminate the contract unless binding terms prevent doing so, or if they have sunk investment costs in capital that has no attractive alternative use.

To explore the implications of negative ex post gains from contracting (i.e., earning less than reservation utilities), the principal-agent relationship must be examined in a repeated contracting framework. The relevant question is whether the insights from one-period models are altered in a multi-period setting. This, of course, requires explicit modeling of dynamic contracts. This issue is addressed in the next chapter.
Chapter 4:

Intertemporal Aspects of Agential Heterogeneity

4.1 Introduction

Most principal-agent relationships develop over a period of repeated contracting during which the agent exerts efforts several times and the principal observes output and other effort-related signals several times. Both the principal and the agents have several opportunities to update their contractual information. This generates new possibilities for incentive schemes. Thus, an important question is whether the insights from studying one-period models are significantly changed with multi-period contracting. An important issue relates to low ability agents who receive negative ex post gains from contracting on average in early periods. Do they leave the contract or renegotiate the contract terms with the principal? To explore this negative gain issue in detail, this chapter considers the principal-agent relationship in a two-period model.

Section 4.2 briefly explains the implications of long-term commitments in a two-period setting. Section 4.3 explores the dynamic setting where the principal and agent commit to a long-term contract in which contract terms are not changed in response to better information throughout the ongoing relationship. Section 4.4 explores dynamic contracts when the commitments of the principal and agent are limited and the contract can be renegotiated if either or both of them want to do so. Section 4.5 compares derived predictions for the agent’s gain from contracting between the static models described in chapter 3 and dynamic models described in this chapter. Section 4.6 considers contractual issues when the agent must make an irreversible asset-specific investment before the initial contract is signed.
4.2 The Commitment Issue

When interaction between the principal and agent is repeated, friction caused by the principal's limited intertemporal commitment becomes important. To illustrate this friction, suppose the productive environment (for example, the productivity or innate ability of the agent) is the same in each period, and is known to both the agent and the principal at the beginning of the repeated contracting relationship. For simplicity, suppose also that the amount of output produced is a deterministic function of the agent's effort and ability. With a deterministic production function, the moral hazard problem vanishes. Whenever ability and output is known to the principal, effort can be deduced with certainty. However, this cannot happen when the principal has imperfect information on ability. In this situation, the agent faces a temptation not to reveal his information on ability to the principal. Thus, the moral hazard model is turned into an adverse selection or asymmetric information model.

In this setting, the agent will realize that if the principal can determine the true ability, then she will be in a position to extract all the rents from him from that point forward. Recognizing this fact, the agent will be reluctant to let his performance reveal his true ability. Therefore, truthful revelation by high-quality agents will no longer be obtained in equilibrium. Thus, in a two-period adverse selection problem, if the principal cannot commit herself to a contract for both periods, the agent will not necessarily reveal his information on ability in the first period.

Alternatively, the principal may choose to offer a sufficiently high rent in the first period to induce revelation of ability. A high-quality agent hides his information on ability in the first period to obtain rent in the second period. But the agent will reveal his
information to the principal if she offers him a rent equivalent to his two-period discounted rents. But the discounted rent may be so high that she chooses to offer instead a common or pooling contract regardless of type (Laffont and Tirole, 1993). Thus, the agent may be asked simply to produce the same minimal harvest regardless of the ability, thereby ensuring that the principal cannot infer ability from output (Weitzman, 1980; Freixas, Guesnerie, Tirole, 1985; Sappington, 1986; Baron and Besanko, 1987; Laffont and Tirole, 1988).

Realizing the fact that superior performance (i.e., high output) will be rewarded by "ratcheting up" future targets, agents have limited incentive to perform up to their potential. This under performance (i.e., low output) of an agent is a fundamental problem of no-commitment or limited commitment dynamic contracts. Thus, the remainder of this chapter explores these dynamic aspects of contracting assuming different commitments of the principal under various information structures.

4.3 Dynamics under Full Commitment

The purpose of this section is to compare the optimal long-term contract with its static counterpart and derive further insights regarding low ability agents’ potential negative gain from contracting. This is done by considering simple repetition of the heterogeneous agent model described in section 3.3 of chapter 3.

Assume that both the principal and agent commit to a two-period contract and have the ability to fulfill it. Assuming both agents fulfill their commitments, the principal and agent then abide by the same mechanism regardless of what information is gained from the first round of contracting. This assumption is important because endogenous
changes in the information structure may arise in a repeated relationship, which could facilitate valuable renegotiation as time passes.

4.3.1 The case where the agent does not know his ability

Consider the simplest case where the ability variable $\alpha$ is the same in both periods. As described in section 3.3 of chapter 3, the agent’s output is given by $\pi_t = e_t + \alpha + \theta_t$ where expected output is given by the sum of the effort of the agent $e_t$ and his ability $\alpha$. As in chapter 3, $\theta_t$ is normally distributed with zero mean and variance $\sigma_t^2$. Agents’ heterogeneity is captured in the ability $\alpha$, which is assumed to be unknown initially to both agents and the principal. However, a prior distribution over $\alpha$, $\alpha \sim N(0, \sigma_\alpha^2)$, is common ex ante knowledge to all contracting parties (the case where it is privately known is considered in section 4.3.2 of this chapter). Therefore, contracting takes place under symmetric information.

All random variables are assumed uncorrelated with one another. The reservation utility of an agent is normalized to zero for simplicity. Hence, the reservation certainty equivalent is zero.

4.3.1.1 The risk-neutral case

The case of risk neutrality provides a useful benchmark. As described in section 2.2 of chapter 2, solving the problem as if the agent owns the project and maximizes his own surplus, and then transferring the expected surplus to the principal, provides the solution for the risk neutral case. For period 1, profit from an agent is

$$\Pi_1 = \pi_1 - w(\pi_1) = e_1 + \alpha + \theta_1 - t_1 - s_1(e_1 + \alpha + \theta_1)$$
and expected profit is

$$E(\Pi_1) = e_1 - t_1 - s_1 e_1.$$  

For period 2, profit from the agent is

$$\Pi_2 = \pi_2 - w(\pi_2) = e_2 + \alpha + \theta_2 - t_2 - s_2(e_2 + \alpha + \theta_2)$$

and expected profit is

$$E(\Pi_2) = e_2 - t_2 - s_2 e_2.$$  

The principal’s objective is to maximize expected discounted profit,

$$E(\Pi) = E(\Pi_1) + \delta E(\Pi_2)$$

$$= e_1 - t_1 - s_1 e_1 + \delta(e_2 - t_2 - s_2 e_2)$$

$$= e_1 + \delta e_2 - t_1 - \delta t_2 - s_1 e_1 - \delta s_2 e_2$$ where $\delta$ is a discount factor, $\delta \geq 0$.

Where the agent has the same discount factor as the principal, his objective is to maximize discounted surplus

$$S = S_1 + \delta S_2 = t_1 + s_1(e_1 + \alpha + \theta_1) - c e_1^2 / 2 + \delta(t_2 + s_2(e_2 + \alpha + \theta_2) - c e_2^2 / 2)$$

$$= t_1 + \delta t_2 + s_1 e_1 + \delta s_2 e_2 - c e_1^2 / 2 - \delta c e_2^2 / 2 + s_1(\alpha + \theta_1) + \delta s_2(\alpha + \theta_2)$$

where first-period surplus is

$$S_1 = w(\pi_1) - C(e_1) = t_1 + s_1 \pi_1 - c e_1^2 / 2$$

$$= t_1 + s_1(e_1 + \alpha + \theta_1) - c e_1^2 / 2$$

and second-period surplus is

$$S_2 = w(\pi_2) - C(e_2)$$

$$= t_2 + s_2 \pi_2 - c e_2^2 / 2$$

$$= t_2 + s_2(e_2 + \alpha + \theta_2) - c e_2^2 / 2.$$
Here, $t_t$ and $s_t$ are the base payment and piece rate for period $t$, respectively, and $C(e_t) = ce_t^2/2$ is the cost of effort $e_t$ in period $t$. But the agent does not know his ability $\alpha$ and states of nature $\theta$. Thus, he maximizes expected discounted surplus

$$E(S) = t_1 + \delta t_2 + s_1 e_1 + \delta s_2 e_2 - ce_1^2/2 - c e_2^2/2.$$  

The principal’s problem is

$$\max_{e_t, s_t, t_t, s_t} e_t + \delta e_2 - t_1 - \delta t_2 - s_t e_1 - \delta s_2 e_2$$

subject to

(i) $t_1 + \delta t_2 + s_t e_1 + \delta s_2 e_2 - ce_1^2/2 - c e_2^2/2 \geq 0$

(ii) $e_1, e_2 \in \arg \max \left[ t_1 + \delta t_2 + s_t e_1 + \delta s_2 e_2 - ce_1^2/2 - c e_2^2/2 \right].$

The incentive compatibility condition (ii) of (4.1) implies $e_1 = s_1/c$ and $e_2 = s_2/c$.

Constraint (i) of (4.1) holds as equality at the optimum because, otherwise, the principal can lower the payment and still get the agent to agree to the contract. Thus,

$$t_1 + \delta t_2 = -(s_1 e_1 - c e_1^2/2 + \delta s_2 e_2 - c e_2^2/2) = -(s_1^2 + \delta s_2^2)/(2c).$$

Substituting for $t_1 + \delta t_2$ and the effort levels in the objective function obtains the unconstrained problem of the principal,

$$\max_{s_t, s_t} s_t/c + \delta s_2/c - s_1^2/(2c) - c e_2^2/(2c).$$

The first-order condition with respect to $s_t$ is $(1-s_t)/c = 0$, which implies $s_1^* = 1$.

Similarly, $s_2^* = 1$, $e_1^* = 1/c$, $e_2^* = 1/c$, and $t_1 + \delta t_2 = -(1 + \delta)/(2c)$. Thus, where agents are risk neutral and abilities are constant over time, the optimal long-term contract with full two-period commitment simply imposes the optimal static contract described in section 3.3.1 of chapter 3 on each period.
The agent with realized ability $\alpha$ receives maximized discounted surplus

$$S^* = t_1^* + \delta t_2^* + s_1^* e_1^* + \delta s_2^* e_2^* - c (e_1^*)^2 / 2 - \delta c (e_2^*)^2 / 2 + s_1^* (\alpha + \theta_1) + \delta s_2^* (\alpha + \theta_2)$$

$$= -(1 - \delta)/(2c) + 1/c + \delta/c - 1/(2c) - \delta/(2c) + (1 + \delta)\alpha + (\theta_1 + \delta\theta_2)$$

$$= (1 + \delta)\alpha + (\theta_1 + \delta\theta_2).$$

On average an agent with ability $\alpha$ receives discounted surplus $E(S^* | \alpha) = (1 + \delta)\alpha$. Since the reservation wage is assumed to be zero, this is the ex post gain from contracting on average for an agent with realized ability $\alpha$. Thus, any agent with ability $\alpha$ smaller than the mean (which is zero) receives a negative ex post gain from contracting on average in this two-period, full-commitment contract. In effect, the optimal two-period contract implements the same effort levels and the same intertemporal ex post gain from contracting on average as the optimal static contract in section 3.3.1 of chapter 3 repeated twice.

However, some indeterminacy remains concerning the intertemporal distribution of these gains. This indeterminacy is resolved when the principal decides how much of total transfer, $t_1^* + \delta t_2^* = (1 + \delta)/(2c)$, she collects in the first period. For instance, when the principal collects $t_1^* = 1/(2c)$ in the first period and, hence, $t_2^* = 1/(2c)$ in the second period, the agent gains $\alpha$ in each period making total discounted gain equal to $(1 + \delta)\alpha$.

4.3.1.2 The risk-averse case

For the risk-averse case, the agent’s objective function is

$$EU(S) = E[-\exp\{-\eta(t_1 + \delta t_2 + s_1 e_1 + \delta s_2 e_2 - c (e_1^*)^2 + \delta (e_2^*)^2 / 2 + s_1 (\alpha + \theta_1) + \delta s_2 (\alpha + \theta_2))\}]$$
and intertemporal utility is assumed to be multiplicative. Although this is not an
innocuous assumption, it is widely used in the relevant literature (Laffont and
Martimort, 2002). The certainty equivalent of $EU(S)$ is

$$CE(S) = E(S) - \eta V(S)/2 = t_1 + \delta t_2 + s_1 e_1 + \delta s_2 e_2 - \frac{c(e_1^2 + \delta e_2^2)}{2}$$

$$-\eta[s_1^2(\sigma_1^2 + \sigma_a^2) + \delta^2 s_2^2(\sigma_2^2 + \sigma_a^2) + 2\delta s_1 s_2 \sigma_a^2]/2.$$

The principal solves

$$\max_{e_1, e_2, t_1, t_2, s_1, s_2} [e_1 + \delta e_2 - t_1 - \delta t_2 - s_1 e_1 - \delta s_2 e_2]$$

s.t. (i) $t_1 + \delta t_2 + s_1 e_1 + \delta s_2 e_2 - \frac{c(e_1^2 + \delta e_2^2)}{2}$

$$-\eta(s_1^2(\sigma_1^2 + \sigma_a^2) + \delta^2 s_2^2(\sigma_2^2 + \sigma_a^2) + 2\delta s_1 s_2 \sigma_a^2)/2 = 0,$$

(ii) $e_1, e_2 \in \arg\max t_1 + \delta t_2 + s_1 e_1 + \delta s_2 e_2 - \frac{c(e_1^2 + \delta e_2^2)}{2}$

$$-\eta(s_1^2(\sigma_1^2 + \sigma_a^2) + \delta^2 s_2^2(\sigma_2^2 + \sigma_a^2) + 2\delta s_1 s_2 \sigma_a^2)/2$$

where constraint (i) of (4.2) holds as equality because, otherwise, the principal can lower
the payment while the agent is still induced to accept the contract. The incentive
compatibility condition (ii) of (4.2) requires $e_1 = s_1/c$ and $e_2 = s_2/c$. Substituting these
values into (i) of (4.2) yields

$$(t_1 + \delta t_2) = -[s_1^2/(2c) + \delta s_2^2/(2c) - \eta(s_1^2(\sigma_1^2 + \sigma_a^2) + \delta^2 s_2^2(\sigma_2^2 + \sigma_a^2) + 2\delta s_1 s_2 \sigma_a^2)/2].$$

Substituting for $e_1, e_2,$ and $t_1 + \delta t_2$ in the objective function thus obtains

$$\max_{s_1, s_2} c/s_1 + \delta s_2/c - s_1^2/(2c) - \delta s_2^2/(2c)$$

$$-\eta[s_1^2(\sigma_1^2 + \sigma_a^2) + \delta^2 s_2^2(\sigma_2^2 + \sigma_a^2) + 2\delta s_1 s_2 \sigma_a^2]/2.$$

The first-order conditions with respect to $s_1$ and $s_2$, respectively, require

$$(1 - s_1)/c - \eta s_1(\sigma_1^2 + \sigma_a^2) - \delta \eta s_2 \sigma_a^2 = 0.$$
δ(1 – s_2)/c – δ^2 ηs_2(σ_2^2 + σ_α^2) – δηs_2σ_α^2 = 0.

Solving these two first-order conditions simultaneously yields maximizing values

\[ s_1^{SB} = (1 + δc σ_2^2) / (1 + cΩ) \]
\[ s_2^{SB} = (1 + ηc σ_2^2) / (1 + cΩ) \]
\[ e_1^{SB} = (1 + δηc σ_2^2) / (c + η^2Ω) \]
\[ e_2^{SB} = (1 + ηc σ_2^2) / (c + η^2Ω) \]

where

\[ Ω = σ_2^2 + σ_α^2 + δ(σ_2^2 + σ_α^2) + δηc(σ_1^2 σ_2^2 + σ_2^2 σ_α^2 + σ_α^2 σ_2^2). \]

Similarly, substitution of \( s_1^{SB} \) and \( s_2^{SB} \) obtains

\[ (t_1^{SB} + δt_2^{SB}) = -(s_1^{SB})^2 / (2c) – δ(s_2^{SB})^2 / (2c) \]
\[ + η[((s_1^{SB})^2(σ_1^2 + σ_α^2) + δ^2(s_2^{SB})^2(σ_2^2 + σ_α^2)+ 2δs_1^{SB}s_2^{SB}(s_2^{SB})^2)]/2. \]

Thus, with risk-averse agents for whom abilities remain constant over contracting periods, the optimal long-term contract with full two-period commitment simply applies the optimal static contract to both periods.

At the optimum, contraint (i) of (4.2) requires

\[ t_1^{SB} + δt_2^{SB} + s_1^{SB} e_1^{SB} + δs_2^{SB} e_2^{SB} – c[(e_1^{SB})^2 – δ(e_2^{SB})^2]/2 \]
\[ – η((s_1^{SB})^2(σ_1^2 + σ_α^2) + δ^2(s_2^{SB})^2(σ_2^2 + σ_α^2)+ 2δs_1^{SB}s_2^{SB}σ_α^2)]/2 = 0. \]

Using this expression, certainty equivalent income of an agent with ability \( α \) is

\[ CE(S | α) = t_1^{SB} + δt_2^{SB} + s_1^{SB} e_1^{SB} + s_2^{SB} α + δs_2^{SB} e_2^{SB} + δs_2^{SB} α – c[(e_1^{SB})^2 – δ(e_2^{SB})^2]/2 \]
\[ – η((s_1^{SB})^2σ_1^2 + δ^2(s_2^{SB})^2σ_2^2)]/2 \]
\[ = (s_1^{SB} + δs_2^{SB})α + ησ_α^2[(s_1^{SB})^2 + δ^2(s_2^{SB})^2 + 2δs_1^{SB}s_2^{SB}]/2. \]
This is the ex post gain from contracting on average for an agent with realized ability $\alpha$. This is negative if $\alpha < \hat{\alpha}$ where $\hat{\alpha} = -\eta\sigma^2_{\alpha}(s_1^{SS} + \delta s_2^{SS})/2$. Therefore, agents with realized ability $\alpha < \hat{\alpha}$ receive a negative gain from contracting on average in this intertemporal case.

The dynamic nature of the optimal contract with full commitment was first analyzed by Roberts (1983) and Baron and Besanko (1984). The applicability of the revelation principal in a dynamic context was demonstrated by Myerson (1986) and, at a more abstract level, by Laffont and Tirole (1988). These studies do not consider the issue of individual agents’ gains from contracting. I adapt these models for heterogeneity in agents’ characteristics, and determine whether individual agents gain or lose from contracting when they have complete, partial, or no knowledge of their ability when the principal-agent relationship is repeated twice. My purpose is to show that the results derived in the static cases are unaltered in this dynamic setting. Low ability agents who receive negative gains from contracting on average in the one-period case also receive negative gains from contracting on average in a dynamic setting.

Again, compared with the risk neutral case, no qualitative differences for low ability agents emerge. Regardless of risk preferences, some low ability agents receive a negative gain from contracting on average. The only difference is in the critical ability level which is smaller under risk aversion because $\hat{\alpha} < 0$. Therefore, for the rest of this chapter, I focus only on the risk neutral case of the intertemporal contracting problem. My intent is to avoid complexities without losing qualitative insight. Also, because risk neutrality will be assumed, there is no loss in generality by assuming a nonstochastic technology, i.e., omitting $\theta$ from the problem.
4.3.2 The case where the agent knows his ability

Consider next the case where agents know their abilities before signing a contract. In sections 3.3 and 3.4 of chapter 3, the additive specification of the technology led to the same optimal level of effort irrespective of agents’ abilities whereas the multiplicative specification induces a different effort for agents of different abilities. For the static case of the additive specification ($\pi = e + \alpha + \theta$), the principal does not have an incentive to separate agents by ability because the marginal benefit of effort is same (equal to 1) for all agents. As a result, they are induced to exert the same level of effort $1/c$. The principal thus receives the first-best expected surplus with fixed fee contracts. High ability agents receive positive ex post gains from contracting at the expense of low ability agents receiving negative ex post gains.

For the multiplicative specification of $\pi = ea + \theta$, the marginal benefit of effort (which is $a$) increases in ability $a$. Given that the cost of effort ($ce^2/2$) does not vary across agents, the principal has an incentive to induce higher effort from agents with higher abilities. As discussed in section 3.5 of chapter 3, omission of $\theta$ from the functional form $\pi = ea + \theta$ creates a slight difference in how the gain from contracting is interpreted. Thus, to represent incentives to the principal for separating agents of different abilities, the multiplicative specification of the technology ($\pi_t = e_o\alpha$) will be used for the rest of this chapter.

Suppose at the beginning of the first period that the agent knows his ability and the principal does not. But the principal knows that the agent knows his ability. Therefore, the principal may design a contract such that the agent will reveal his ability through his contracting choice. However, the agent may be unwilling to reveal his ability
if the contracting relationship will last more than one period. To illustrate, consider a
two-period problem where each of two agents have different abilities, \( \alpha_1 \) and \( \alpha_2 \), with \( \alpha_1 < \alpha_2 \). The principal’s problem is

\[
\max \sum_{t=1}^{2} \delta^{t-1} \left\{ p_1[(e_t \alpha_1 - ce_t^2 / 2) - U_1] + p_2[(e_t \alpha_2 - ce_t^2 / 2) - U_2] \right\}
\]

s.t. (ia) \( U_1 \equiv \sum_{t=1}^{2} \delta^{t-1} (t_t + s_t e_t \alpha_1 - ce_t^2 / 2) \geq 0 \),

(ib) \( U_2 \equiv \sum_{t=1}^{2} \delta^{t-1} (t_t + s_t e_t \alpha_2 - ce_t^2 / 2) \geq 0 \)

(ii) \( U_2 \geq \sum_{t=1}^{2} \delta^{t-1} (t_t + s_t e_t \alpha_2 - ce_t^2 / 2) = U_1 + \sum_{t=1}^{2} \delta^{t-1} s_t e_t (\alpha_2 - \alpha_1) \)

(iii) \( e_{11}, e_{12} \in \arg \max_{e_{11}, e_{12}} U_1 \)

(iv) \( e_{21}, e_{22} \in \arg \max_{e_{21}, e_{22}} U_2 \)

where \( p_j \) is the probability that the agent’s ability is \( \alpha_j \), \( e_t \) is effort of the agent with ability \( \alpha_j \) in period \( t \), and \( t_t \) and \( s_t \) are the base payment and piece rates for the agent with ability \( \alpha_j \) in period \( t \), respectively. Conditions (iii) and (iv) of (4.3) imply \( e_{11} = s_{11} \alpha_1 / c, e_{12} = s_{12} \alpha_1 / c, e_{21} = s_{21} \alpha_2 / c, \) and \( e_{22} = s_{22} \alpha_2 / c \). Substituting these values into the objective function of (4.3), solving constraints (ia) and (ii) of (4.3) as equalities, noting that constraint (ib) of (4.3) is redundant, and making further substitutions, the principal’s unconstrained problem is

\[
\max \sum_{t=1}^{2} \delta^{t-1} \left\{ p_1[(s_t \alpha_1^2 - s_{1t}^2 \alpha_1^2 / 2)] + p_2[s_2 \alpha_2^2 - s_{2t}^2 \alpha_2^2 / 2 - s_{1t}^2 \alpha_4 (\alpha_2 - \alpha_1) / c] \right\}
\]

(4.4)

For the agent with ability \( \alpha_2 \), first-order conditions of (4.4) require \( s_{21}^* = s_{22}^* = 1 \) and

\( e_{21}^* = e_{22}^* = \alpha_2 / c \). For the agent with ability \( \alpha_1 \), first-order conditions require \( s_{11}^B = s_{12}^B = 0 \), and
1/(1 + 2p_2/p_1(\alpha_2 - \alpha_1)/\alpha_1) and e_{11}^{sp} = e_{12}^{sp} = \alpha_1/[c(1 + 2p_2/p_1(\alpha_2 - \alpha_1)/\alpha_1)]. Again, the optimal long-term contract with full commitment for two periods simply repeats the optimal static contract described in section 3.3.2 of chapter 3.

A sharing contract is thus used for all but the agent with highest ability even in this risk neutral agent case. In this separating case, similar to that in section 3.4.2, none of the agents are left with less than their reservation wages. However, only high ability agents earn positive rents while low ability agents earn exactly their reservation wage. Furthermore, when the ability differential, \( \alpha_2 - \alpha_1 \), is large compared to the low ability, i.e., when \((\alpha_2 - \alpha_1)/\alpha_1\) is large, and the proportion of low ability agents \(p_1\) is small, then the principal may set one sharing contract that targets high ability agents while excluding low ability agents from contracting. If \((\alpha_2 - \alpha_1)/\alpha_1\) is small and the proportion \(p_1\) of low ability agents is large, then the principal may set one sharing contract targeting low ability agents while excluding high ability agents from contracting. Further, for certain combinations of \((\alpha_2 - \alpha_1)/\alpha_1\) and \(p_1\), the principal may offer a single pooling contract. In this case, low ability agents who have negative gains from contracting will select not to enter into the contract.

4.3.3 The case where the agent knows his ability imperfectly

This section extends the static model in section 3.5 of chapter 3, where the agent knows his ability imperfectly, to the two-period case. Thus, the agent does not know his ability perfectly as in section 4.3.1, but he has more specific knowledge about his ability than does the principle, unlike the case in section 4.3.2. As in section 3.6 of chapter 3, suppose an agent’s random ability \(\alpha\) may follow any one of \(D\) possible distributions rather
than one distribution with \( n \) states. When a contract between a principal and agent is
signed, the principal has a nondegenerate prior defined over these \( D \) distributions
whereas the agent knows the actual distribution. Thus, the agent’s information is better
but imperfect, and the principal is aware of this fact. Again, for simplicity, I consider
only the case where \( D = 2 \) and \( n = 2 \).

Assume ability distribution 1 (denoted by \( P \)) has probability \( p_1 \) of ability \( \alpha_{11} \) and
probability \( p_2 \) of ability \( \alpha_{12} \) while distribution 2 (denoted by \( Q \)) has probability \( q_1 \) of
ability \( \alpha_{21} \) and probability \( q_2 \) of ability \( \alpha_{22} \). Also assume that \( E(\alpha_1) < E(\alpha_2) \) where \( E(\alpha_i) =
\alpha_{11}p_1 + \alpha_{12}p_2 \) and \( E(\alpha_2) = q_1\alpha_{21} + q_2\alpha_{22} \). Let \( r_1 \) and \( r_2 \) be the principal’s subjective
probability that the agent has ability distribution \( P \) and \( Q \), respectively. In this
information structure, any agent whose distribution is \( P \) knows his ability is either \( \alpha_{11} \) or
\( \alpha_{12} \) and any agent whose distribution is \( Q \) knows his ability is either \( \alpha_{21} \) or \( \alpha_{22} \). But the
principal, who does not know agents’ actual distributions, knows only that an agent’s
ability is any of four possibilities (i.e., \( \alpha_{11}, \alpha_{12}, \alpha_{21}, \) or \( \alpha_{22} \)). The principal solves

\[
\begin{align*}
\text{Max } r_1 & \left[ \sum_{i=1}^{2} \sum_{j=1}^{2} \delta^{i-1} p_j (e_{ij}, \alpha_{ij} - t_{ij} - s_{ij} \alpha_{ij} - U_1) \right] \\
& + r_2 \left[ \sum_{i=1}^{2} \sum_{j=1}^{2} \delta^{i-1} q_j (e_{ij}, \alpha_{ij} - t_{ij} - s_{ij} \alpha_{ij} - U_2) \right] \\
\text{s.t. } (ia) & \quad U_1 \equiv \sum_{i=1}^{2} \sum_{j=1}^{2} \delta^{i-1} p_j (t_{ij} + s_{ij} \alpha_{ij} - c e_{ij}^2 / 2) \geq 0 \\
\text{(ib) } & \quad U_2 \equiv \sum_{i=1}^{2} \sum_{j=1}^{2} \delta^{i-1} q_j (t_{ij} + s_{ij} \alpha_{ij} - c e_{ij}^2 / 2) \geq 0 \\
\text{(ii) } & \quad U_2 \geq \sum_{i=1}^{2} \sum_{j=1}^{2} \delta^{i-1} q_j (t_{ij} + s_{ij} \alpha_{ij} - c e_{ij}^2 / 2)
\end{align*}
\]
(iii) \[ e_{11}, e_{12} \in \arg \max_{e_{11}, e_{12}} U_1 \]

(iv) \[ e_{21}, e_{22} \in \arg \max_{e_{21}, e_{22}} U_2 \]

where \( e_{1t} \) is effort of an agent with distribution \( P \) in time \( t \), \( e_{2t} \) is effort of an agent with distribution \( Q \) at time \( t \), \( t_{1t} \) and \( t_{2t} \) are base payments at time \( t \) for agents with distributions \( P \) and \( Q \), respectively, and \( s_{1t} \) and \( s_{2t} \) are piece rates at time \( t \) for agents with distributions \( P \) and \( Q \), respectively. Constraint (ii) of (4.5) is necessary so that an agent with distribution \( Q \) will not mimic one with distribution \( P \).

After some manipulation, the problem in (4.5) can be rewritten as

\[
\begin{align*}
\max_{\alpha_1, \alpha_2} \quad & \sum_{t=1}^{2} \delta^{-1} \left\{ r_1 [(e_{1t} E(\alpha_1) - t_{1t} - s_{1t} e_{1t} E(\alpha_1)) - U_1] + r_2 [(e_{2t} E(\alpha_2) - t_{2t} - s_{2t} e_{2t} E(\alpha_2)) - U_2] \right\} \\
\text{s.t.} \quad & U_1 \equiv \sum_{t=1}^{2} \delta^{-1} (t_{1t} + s_{1t} e_{1t} E(\alpha_1) - c e_{1t}^2 / 2) \geq 0 \\
& U_2 \equiv \sum_{t=1}^{2} \delta^{-1} (t_{2t} + s_{2t} e_{2t} E(\alpha_2) - c e_{2t}^2 / 2) \geq 0 \\
& U_2 \geq U_1 + \sum_{t=1}^{2} \delta^{-1} s_{1t} e_{1t} (E(\alpha_2) - E(\alpha_1)) \\
& e_{11}, e_{12} \in \arg \max_{e_{11}, e_{12}} U_1 \\
& e_{21}, e_{22} \in \arg \max_{e_{21}, e_{22}} U_2.
\end{align*}
\] (4.6)

Problem (4.6) is similar to problem (4.3) except that \( r_1 \) here is the probability that the agent is from distribution \( P \) whereas in (4.3) \( p_1 \) is the probability that agent’s ability is \( \alpha_1 \). Also, \( \alpha_1 \) and \( \alpha_2 \) of problem (4.3) are replaced by \( E(\alpha_1) \) and \( E(\alpha_2) \), respectively. Whereas agents know their ability exactly in (4.3), in this case they know their expected ability exactly. And whereas the principal knows only the probability distribution of agents’
abilities in (4.3), in this case the principal has a probability associated with each
distribution of agent abilities.

As a result, the solution of problem (4.3) can be adapted to obtain the solution of
problem (4.6). This obtains

\[ s_{21}^* = s_{22}^* = 1 \]
\[ e_{21}^* = e_{22}^* = E(\alpha_2)/c \]
\[ s^S_{11} = s^S_{12} = 1/(1 + 2r_2/r_1(E(\alpha_2) - E(\alpha_1))/E(\alpha_1)) \]
\[ e^S_{11} = e^S_{12} = E(\alpha_1)/[c(1 + 2r_2/r_1(E(\alpha_2) - E(\alpha_1))/E(\alpha_1))]. \]

Again, the optimal long-term contract with full two-period commitment simply repeats
the optimal static contract of section 3.5 of chapter 3.

In the optimal solution, constraint (ia) of (4.6) holds as an equality so that

\[ t^S_{11} + s^S_{11} e^S_{11} E(\alpha_1) - c(e^S_{11})^2/2 + \delta(t^S_{12} + s^S_{12} e^S_{12} E(\alpha_1) - c(e^S_{12})^2/2) = 0. \]

Thus, the expected transfer to an agent with distribution \( P \) is equal to

\[ t^S_{11} + \delta t^S_{12} = -[s^S_{11} e^S_{11} E(\alpha_1) - c(e^S_{11})^2/2 + \delta(s^S_{12} e^S_{12} E(\alpha_1) - c(e^S_{12})^2/2)]. \]

Agents with abilities \( \alpha_{11} \) and \( \alpha_{12} \) receive the same transfer \( T^* = t^S_{11} + \delta t^S_{12} \). This negative
payment is the fixed fee that the agents pay to the principal. However, the surpluses they
receive are not the same even though they pay the same fixed fee. The surplus received
by an agent with ability \( \alpha_{11} \) is

\[ S^* = [(s^S_{11} e^S_{11} \alpha_{11} - c(e^S_{11})^2/2) + \delta(s^S_{12} e^S_{12} \alpha_{11} - c(e^S_{12})^2/2)]. \]

His net surplus is \( S^* + T^* = s^S_{11} e^S_{11} (\alpha_{11} - E(\alpha_1)) + \delta s^S_{12} e^S_{12} (\alpha_{11} - E(\alpha_1)), \) which is negative
because \( \alpha_{11} < E(\alpha_1) \). This is his ex post gain from contracting with realized ability \( \alpha_{11} \).

Similarly, the ex post gain from contracting for an agent with realized ability \( \alpha_{12} \) is
$s_{11}^{SB} e_{11}^{SB} (a_{12} - E(a_1)) + \delta s_{12}^{SB} e_{12}^{SB} (a_{12} - E(a_1))$, which is positive because $a_{12} > E(a_1)$. Thus, agents with the lowest realized ability receive negative gains from contracting.

Net surplus for an agent drawn from distribution $Q$ is equal to $U_2$. From constraint (ii) of (4.6), $U_2 \geq U_1 + s_{11} e_{11} (E(a_2) - E(a_1)) + \delta s_{12} e_{12} (E(a_2) - E(a_1))$. This constraint holds as equality in the optimal solution because, otherwise, the principal can increase the fixed fee $t_{21}^{SB} + \delta t_{22}^{SB}$ without violating the incentive compatibility constraint. The net surplus of an agent drawn from distribution $Q$ whose realized ability is $a_{22}$ is $s_{11}^{SB} e_{11}^{SB} (a_{22} - E(a_1)) + \delta s_{12}^{SB} e_{12}^{SB} (a_{22} - E(a_1)) > 0$. The net surplus of an agent drawn from distribution $Q$ whose ability is $a_{21}$ is $s_{11}^{SB} e_{11}^{SB} (a_{21} - E(a_1)) + \delta s_{12}^{SB} e_{12}^{SB} (a_{21} - E(a_1))$. The sign of this expression depends on the sign of the term $a_{21} - E(a_1)$. For $a_{21} < E(a_1)$, agents with realized ability $a_{21}$ receive negative gains from contracting.

Furthermore, if the ability differential, $E(a_2) - E(a_1)$, is very large compared to $E(a_1)$, i.e., when $(E(a_2) - E(a_1))/E(a_1)$ is large, and the probability $r_1$ is small, then the principal may set one sharing contract targeting only agents from distribution $Q$ while excluding agents from distribution $P$. But if $(E(a_2) - E(a_1))/E(a_1)$ is small and $r_1$ is large, then the principal may set one sharing contract targeting only agents from distribution $P$ while excluding agents from distribution $Q$. However, for certain combinations of $(E(a_2) - E(a_1))/E(a_1)$ and $r_1$, the principal may find it more profitable to offer a single pooling contract. In this case again, agents with low ability from both distributions will receive negative gains from contracting if $a_{21} < a_{12}$. 
4.4 Dynamics under Limited Commitment

A credible optimal dynamic contract in a multi-period setting is characterized by a full commitment from both contractual partners to stick to the agreed contract, ruling out any renegotiation of their initial agreement. The assumption that economic agents have the ability to commit to non-renegotiation is an extreme assumption about the perfection of the judicial system. In practice, it is impractical to expect fulfillment of a commitment not to renegotiate when gains are possible from doing so. Starting with Dewatripont’s (1989) paper, the literature has considered the implications of this institutional “imperfection” that corresponds to the inability to commit to non-renegotiation.

4.4.1 The ability to renegotiate and limited commitment

A primary reason for limited commitment arises when the principal can renegotiate the contract offered to the agent during the course of actions. Renegotiation is a voluntary act that may benefit both the principal and the agent. This is in contrast to a breach of contract, which can hurt one of the contracting parties. The possibility of renegotiation can be viewed as the ability of the contracting partners to achieve a Pareto improving trade if any become feasible during the course of actions.

For instance, consider the case described in section 4.3.2 where the agents know their abilities but the principal does not. The principal designs the contract in such a way that the agents truthfully reveal their abilities by choosing the contracts designed for each possible ability type. In the first period, agents with high ability (i.e., $\alpha_2$) reveal their ability to the principal by selecting the first-best contract $(s_{21}^*, e_{21}^*)$ while agents with low ability (i.e., $\alpha_1$) reveal their ability by choosing contract $(s_{11}^{SB}, e_{11}^{SB})$. Once agents reveal
their abilities, the principal may propose renegotiation to avoid the allocative inefficiency he has imposed on inefficient agents’ efforts because $s_{11}^{SR} < s_{11}^*$ and $e_{11}^{SR} < e_{11}^*$.

The gain from this renegotiation comes from raising the second period’s allocative efficiency for the inefficient type and thus moving effort from the second best level $e_{12}^{SB} = \alpha_1/[c + 2cp_2/p_1(\alpha_2 – \alpha_1)/\alpha_1]$ to the first-best level $e_{12}^* = \alpha_1/c$ by increasing the piece rate from second best $s_{12}^{SB} = 1/(1 + 2p_2/p_1(\alpha_2 – \alpha_1)/\alpha_1)$ to the first best $s_{12}^* = 1$. To induce low-ability agents to renegotiate, the principal must share these gains from trade with the low-ability agent by offering him at least the same utility level as without renegotiation (i.e., $U_1 = 0$).

A high-ability agent can expect that, once the inefficient agents reveal their types to the principal in first period, the principal will revise the inefficient agents’ contracts in the second period, offering the first-best contract instead. Since a high-ability agent’s rent, $U_2 = U_1 + s_{11}^{SB} e_{11}^{SB} (\alpha_2 – \alpha_1) + \delta s_{12}^{SB} e_{12}^{SB} (\alpha_2 – \alpha_1)$, is increasing in $s_{12}$ and $e_{12}$, increasing $s_{12}$ and $e_{12}$ tighten the incentive compatibility constraint associated with high-ability agents. A high-ability agent knows that if he truthfully reveals his type he will receive $s_{11}^{SB} e_{11}^{SB} (\alpha_2 – \alpha_1) + \delta s_{12}^{SB} e_{12}^{SB} (\alpha_2 – \alpha_1)$ as rent. But if he lies, he can receive $s_{11}^{SB} e_{11}^{SB} (\alpha_2 – \alpha_1) + \delta s_{12}^{SB} e_{12}^{SB} (\alpha_2 – \alpha_1)$, which is larger than the former. Thus, a high-ability agent is better off hiding his type so that he can obtain more rent. Therefore, truthful revelation by high-ability agents is no longer obtained in equilibrium. Thus, there is a fundamental trade-off between raising second period efficiency and tightening incentives when renegotiation is possible.
4.4.2 Renegotiation-proof separating contracts

Moving away from full commitment contracts raises other numerous issues, such as how to model the renegotiation game, how agents update their beliefs dynamically, and how to design and characterize executable contracts. The nature of the difficulty due to imperfect commitments in repeated contracting models with adverse selection is discussed in this section. The discussion assumes that the principal cannot commit to non-renegotiation. The agent is assumed to know that any information he reveals in the first period will be fully utilized by the principal in the second period if renegotiation is feasible. But the principal is assumed to have all the bargaining power at the renegotiation stage, which takes place before the second-period output is realized.

Since the first-period contract fully separates agents by ability, the second-period outputs are efficient for agents of both abilities and are thus given by $s_{12}^*$ (inducing $e_{12}^*$) and $s_{22}^*$ (inducing $e_{22}^*$) for respective agent abilities $\alpha_1$ and $\alpha_2$. This requires an intertemporal incentive constraint for agents of high ability, which must be satisfied to induce information revelation in the first period, $U_2 \geq U_1 + s_1 e_1 (\alpha_2 - \alpha_1) + \delta s_{12}^* e_{12}^* (\alpha_2 - \alpha_1)$.

If this constraint is not satisfied, then high ability agents will mimic low ability agents, and thus separation will not be possible. The principal must offer the low-ability agent at least the same utility level as before renegotiation, which is $U_1 = 0$.

With such a separating contract, high-ability agents earn the highest possible rent of $\delta s_{12}^* e_{12}^* (\alpha_2 - \alpha_1)$ in the second period, and hence, they have no incentive to renegotiate the terms of the second-period contract. Given this initial commitment, coupled with the fact that the principal is fully informed of agents’ abilities at the renegotiation stage, the principal cannot further raise the second-period ex post efficiency, because it is already
maximized by \( e_{12}^* \) and \( e_{22}^* \). Hence, this type of long-term separating contract is renegotiation proof.

Therefore, the principal solves the following problem for the renegotiation-proof and separating (RPS) equilibrium contract:

\[
\text{Max}_{t_1, t_2, s_1, s_2, e_1^*, e_2} \quad p_1\left[e_1^* \alpha_1 + \delta e_{12}^* \alpha_1 - c(e_1^2 + \delta(e_{12}^*)^2)/2 - U_1\right] + p_2\left[e_2^* \alpha_2 + \delta e_{22}^* \alpha_2 - c(e_2^2 + \delta(e_{22}^*)^2)/2 - U_2\right]
\]

s.t. (ia) \( U_1 \equiv t_1 + s_1 e_1 \alpha_1 - c e_1^2 / 2 + \delta(t_{12}^* + s_{12}^* e_{12}^* \alpha_1 - c(e_{12}^*)^2 / 2) \geq 0 \)

(ib) \( U_2 \equiv t_2 + s_2 e_2 \alpha_2 - c e_2^2 / 2 + \delta(t_{22}^* + s_{22}^* e_{22}^* \alpha_2 - c(e_{22}^*)^2 / 2) \geq 0 \)

(ii) \( U_2 \geq U_1 + s_1 e_1 (\alpha_2 - \alpha_1) + \delta s_{12}^* e_{12}^* (\alpha_2 - \alpha_1) \)

(iii) \( e_1 \in \arg\max_{e_1} U_1 \)

(iv) \( e_2 \in \arg\max_{e_2} U_2 \).

Problem (4.7) solves for the first-period equilibrium and then sets the high-ability agent’s discounted second-period rent, \( \delta s_{12}^* e_{12}^* (\alpha_2 - \alpha_1) \), at least as high as the rent received with full commitment, \( \delta s_{12}^{SB} e_{12}^{SB} (\alpha_2 - \alpha_1) \). The RPS solution thus yields

\[
\begin{align*}
S_{2}^{RPS} &= s_2^* = 1 \\
S_{22}^{RPS} &= s_{22}^* = 1 \\
S_{1}^{RPS} &= s_1^{SB} = 1/(1 + 2p_2/p_1(\alpha_2 - \alpha_1)/\alpha_1) \\
S_{12}^{RPS} &= s_{12}^* = 1.
\end{align*}
\]

The only remaining distortion is in the low-ability agent’s first-period contract. None of the agents receive less than their reservation wages for both periods combined.
4.4.3 Renegotiation-proof pooling contracts

Suppose instead that agents in period 1 choose the same behavior irrespective of their abilities. In this case, the principal learns nothing from the first-period contract. The continuation contract for period 2 is thus equal to the optimal static contract, conditional on the prior beliefs represented by \( p_1 \) and \( q_1 \). In this case, the second-period contracts are defined by \( s^{RPP}_{22} = s^*_2 = 1 \) and \( s^{RPP}_{12} = s^{SB}_{12} = 1/(1 + 2p_2/p_1(\alpha_2 - \alpha_1)/\alpha_1) \). The principal thus offers a single contract in the first period which induces full pooling between both abilities. The intertemporal incentive constraint of the high-ability agent is \( U_2 \geq U_1 + se_1(\alpha_2 - \alpha_1) + \delta s^{SB}_{12} e^{SB}_{12} (\alpha_2 - \alpha_1) \). The principal’s problem, which consists of finding the best long-term contract that induces full pooling in the first period, is

\[
\begin{align*}
\max_{t_1, t_2, e_1, e_2} & \quad p_1 [e_1 \alpha_1 + \delta e^{SB}_{12} \alpha_1 - c(e_1^2 + \delta (e^{SB}_{12})^2)/2 - U_1 ] \\
& \quad + p_2 [e_2 \alpha_2 + \delta e^{SB}_{22} \alpha_2 - c(e_2^2 - \delta (e^{SB}_{22})^2)/2 - U_2 ] \\
\text{s.t.} & \quad U_1 \equiv t + se_1 \alpha_1 - ce_1^2 / 2 + \delta (t^{SB}_{12} + s^{SB}_{12} e^{SB}_{12} \alpha_1 - c(e^{SB}_{12})^2 / 2) \geq 0, \\
& \quad U_2 \equiv t + se_2 \alpha_2 - ce_2^2 / 2 + \delta (t^{SB}_{22} + s^{SB}_{22} e^{SB}_{22} \alpha_2 - c(e^{SB}_{22})^2 / 2) \geq 0, \\
& \quad U_2 \geq U_1 + se_1 (\alpha_2 - \alpha_1) + \delta s^{SB}_{12} e^{SB}_{12} (\alpha_2 - \alpha_1), \\
& \quad e_1 \in \arg\max_{e_1} U_1, \\
& \quad e_2 \in \arg\max_{e_2} U_2.
\end{align*}
\]

Constraints (iii) and (iv) of (4.8) imply that \( e_1 = s\alpha_1/c \) and \( e_2 = s\alpha_2/c \). To consider the effort level chosen in this case, let \( p_{\alpha}(t,s,e) \) denote the principal’s posterior (updated) probability after first-period performance that the agent’s ability is \( \alpha_1 \). Perfect Bayesian equilibrium requires strategies and beliefs to be consistent, i.e., that the strategies are
optimal given the beliefs and that the beliefs are derived from strategies using Bayes rule.

The optimal first-period action of the low-ability agent in dynamic equilibrium can only be the optimal one-period action because, regardless of beliefs, \( p_u \), the principal will not allow the low-ability agent to receive a strictly positive rent in the second period. Thus, a low-ability agent simply maximizes his first-period rent, which leads to optimality at \( s^* = 1 \) and \( e_1^* = \alpha_1/c \).

Because \( e_1^* \) belongs to the support of the high-ability agent’s effort, Bayes rule and the fact that \( e_1^* \) is optimal for low-ability agents implies that the principal’s posterior beliefs, \( p_u \), must be zero unless \( e = e_1^* \). However, if agents of both abilities choose \( e_1^* \), then the pooling equilibrium is obtained and the principal’s posterior belief is \( p_u(t^*, s^*, e^*_1) = p_1 \). Then the principal’s second-period maximization is done with the initial prior represented by \( p_1 \), which leads to a second-period static solution with \( s^*_{22} = 1 \), and \( s_{12}^{SB} = 1/(1 + 2p_2/p_1(\alpha_2 - \alpha_1)/\alpha_1) \). Therefore, the complete solution with a renegotiation proof pooling (RPP) contract is \( s^{RPP} = 1 \), \( s_{22}^{RPP} = 1 \), and \( s_{12}^{RPP} = s_{12}^{SB} = 1/(1 + 2p_2/p_1(\alpha_2 - \alpha_1)/\alpha_1) \).

Comparing the principal’s surplus between the RPS and RPP cases, there is a critical value, \( \delta_0 \), of the discount factor \( \delta \) such that RPS (RPP) is preferred if \( \delta < (>) \delta_0 \). However, this problem is aggravated to some extent if moral hazard enters the problem. Moral hazard makes the critical value of \( \delta_0 \) smaller and thus makes a separating equilibrium less likely. For example, if the production process is stochastic, then the information on ability and output does not allow the principal to infer effort conclusively. As a result, both moral hazard and adverse selection enter the problem. The dynamic
problem changes because greater effort in the first period allows the principal to infer abilities better and thus offer less rent in the second period (Meyer and Vickers, 1997).

As a result, agents may distort effort more and, sensing this potential problem, the principal may resort to pooling contracts instead.

When a pooling contract is offered, agents with low ability receive negative gains from contracting. But if low-ability agents know their abilities, they will not contract with the principal. In the more general case where agent abilities vary continuously and agents know their abilities imperfectly before signing contracts, some agents with low ability will not sign contracts while others with low ability will sign contracts because of imperfect knowledge of their ability and then be left with negative gains from contracting ex post. These agents may not be able to cover their losses from contracts through future renegotiations.

4.4.4 Renegotiation for the imperfect information case

Renegotiation matters can be even worse when the agents know their type imperfectly. In this case, complete separation of agents by their abilities is not possible, although separation of agents according to their ability distributions is possible. However, because an additional discounted future rent has to be paid to separate the agents according to distributions $P$ and $Q$, separation of agents according to their ability may be too costly for the principal. In this case, the loss from contracting may be even greater for agents compared to the full commitment case.

As described in section 4.3.3, agents from distribution $Q$ reveal their abilities to the principal by selecting the first-best contract, which yields $s_{21}^* = 1$ and $e_{21}^* = E(\alpha_2)/c$. 
while agents from distribution $P$ reveal their abilities by choosing the second best contract, which yields $s_{11}^{SB} = 1/(1 + 2r_2/r_1(E(\alpha_2) - E(\alpha_1))/E(\alpha_1))$ and $e_{11}^{SB} = E(\alpha_1)/(c + 2cr_2/r_1(E(\alpha_2) - E(\alpha_1))/E(\alpha_1))]$. Once agents reveal their abilities to the principal by contract selection, the principal may propose renegotiation to reduce the allocative inefficiency imposed on low-ability agents because $s_{12}^{SB} < s_{12}^{*}$ and $e_{12}^{SB} < e_{12}^{*}$. Allocative efficiency in the second period is reduced for agents from distribution $P$ by increasing the piece rate from $s_{12}^{SB}$ to $s_{12}^{*} = 1$, which changes effort from the second best $e_{12}^{SB}$ to the first best $e_{12}^{*} = E(\alpha_1)/c$. To share these gains with agents that have the $P$ distribution, the principal must offer them at least the same utility level as before renegotiation (i.e., $U_1 = 0$).

Since agents with ability distribution $Q$ receive rent

$$U_2 = U_1 + s_{11}e_{11}(E(\alpha_2) - E(\alpha_1)) + \delta s_{12}e_{12}(E(\alpha_2) - E(\alpha_1))$$

$$= s_{11}e_{11}(E(\alpha_2) - E(\alpha_1)) + \delta s_{12}e_{12}(E(\alpha_2) - E(\alpha_1)),$$

which is increasing in $s_{12}$ and $e_{12}$, this action tightens the incentive compatibility constraint for agents with ability distribution $Q$. A high-ability ($Q$ distribution) agent knows that if he truthfully reveals his high-ability distribution, then he receives rent

$$s_{11}^{SB}e_{11}^{SB}(E(\alpha_2) - E(\alpha_1)) + \delta s_{12}^{SB}e_{12}^{SB}(E(\alpha_2) - E(\alpha_1)),$$

but if he lies he can receive the greater rent $s_{11}^{SB}e_{11}^{SB}(E(\alpha_2) - E(\alpha_1)) + \delta s_{12}^{SB}e_{12}^{SB}(E(\alpha_2) - E(\alpha_1))$. Obviously, $s_{12}^{SB}e_{12}^{SB} < s_{12}^{*}e_{12}^{*}$. Thus, a high-ability agent receives this greater increment in second-period rent without giving up any rent by the deceitful action in the first period.

His first period expected rent is $s_{11}^{SB}e_{11}^{SB}(E(\alpha_2) - E(\alpha_1))$. He can earn this rent by either truthful or deceitful action. Thus, he is indifferent between telling the truth and lying. Standard literature assumes that he breaks the tie in favor of telling the truth.
However, in this case breaking the tie in favor of lying gives him more rent in the second period. So, he will lie because lying has no first-period cost. Because agents with ability distribution $Q$ may prefer to hide their ability knowledge to earn more rent later, truthful revelation of ability distributions is no longer obtained in equilibrium.

In this case, if the principal wants to separate agents she has to pay rent

$$s_{11}^{SB} e_{11}^{SB} (E(a_2) - E(a_1)) + \delta s_{12}^{*} e_{12}^{*} (E(a_2) - E(a_1))$$

for agents with distribution $Q$. If $a_{12} < a_{21}$, then both ability types $a_{21}$ and $a_{22}$ from distribution $Q$ earn positive rent. However, if $a_{21} < E(a_1)$, then $a_{21} - E(a_1)$ is negative and

$$s_{11}^{SB} e_{11}^{SB} (a_{21} - E(a_1)) + \delta s_{12}^{*} e_{12}^{*} (E(a_2) - E(a_1)) < 0.$$ 

Thus, the sign of the rent of the low-ability agents from distribution $Q$ depends on the relationship of the distributions $P$ and $Q$. For agents from the $P$ distribution, agents with ability $a_{12}$ earn positive rent whereas agents with ability $a_{11}$ earn negative rent.

### 4.5 Asset Specificity and the Holdup Problem

A number of papers have tested the main predictions from the transactions cost literature developed by Williamson (1975, 1985, and 1996). One of the best-known results from the transactions cost literature is that contracts with a longer duration are likely when relationship-specific investments matter more (so as to reduce holdup problems). Asset specificity and the holdup problem are discussed in a one-period setting in section 3.2.2.2 of chapter 3. This section extends the holdup problem to illustrate why inadequate investment may occur in relation-specific assets in a dynamic setting.

Suppose the agent’s productivity of effort is determined by his nonreversible asset-specific investment. In this case, the principal cannot commit to reward the agent for his effort because she does not know the productivity of effort before the investment
is in place. When the principal offers the contract, the agent’s investment has already been sunk. The principal thus loses the role as Stackelberg leader in the design of incentives for effort, and so the Nash equilibrium\footnote{In a Nash equilibrium, each player’s strategy choice is a best response to the strategies actually played by his/her rivals (Nash, 1951). Nash equilibrium applies to simultaneous-move games, in which all players move only once and at the same time.} between the principal and the agent must be examined. The principal offers a contract anticipating an effort productivity associated with a particular choice of investment by the agent. Anticipating the contract he will receive from the principal, the agent chooses an investment level, and, hence, indirectly an effort productivity to maximize his utility (Laffont and Martimort, 2002).

For each value of the effort productivity chosen by the agent at \( t = 0 \), the principal can implement the first-best contract at period \( t = 1 \) since she has all the bargaining power (Laffont and Martimort, 2002). Thus, as shown in numerous cases in chapters 2 and 3, and in this chapter, the agent obtains his reservation utility with a first-best contract aside from the sunk cost of investment. In the principal-agent models considered thus far, the principal moves first, announcing a payment schedule, and the agent decides on effort to maximize his utility after observing the payment schedule. The agent then adjusts his effort to changes in payment schedule in succeeding contracting periods. But in this case, once an effort productivity is chosen with an investment, the agent cannot reverse it depending on the offer he receives from the principal at \( t = 1 \). Anticipating this, the agent will undertake the minimum required investment at \( t = 0 \). This is how the holdup problem leads to a Nash equilibrium with underinvestment in specific assets (Laffont and Martimort, 2002).

However, if the agent determines his distribution of effort productivity by an investment, then the agent is not assured of his investment cost in contracting periods
beginning at $t = 1$. Even with a minimum investment of zero, the agent will be left with less than his reservation utility if his realized productivity falls below his expectations. In this case, the agent will be left with the same utility level in future contracting periods unless both the principal and the agent renegotiate the contract.

As discussed in section 4.4, the principal and the agents may be able to renegotiate initial contracts to improve efficiency in mutually beneficial ways. As section 4.4 of this chapter shows for renegotiation-proof separating equilibrium, raising the efficiency or payment of the low-ability agents tightens the incentive constraints of high-ability agents. Thus, a fundamental trade-off arises between raising low-ability agents’ ex post efficiency and tightening incentives for high-ability agents when renegotiation is possible. This trade-off may discourage the principal from renegotiating the initial long-term contract. Furthermore, because the low ability agent is merely unable to recover his fixed cost, renegotiation may not alter marginal behavior and thus may not have benefits for the principal. In this case, the principal may not have an incentive to renegotiate.

4.6 Conclusion

Results in this chapter have shown that, when agents do not know their abilities, full-commitment, long-term contracts have the same problems as short-term contracts as far as possibilities that agents can face negative ex post gains from contracting on average. Alternatively, when the agents know their abilities, they earn at least their reservation wages irrespective of abilities (although some may do so by choosing not to contract). These conclusions are not qualitatively different from those obtained in the static cases with similar information structure. But, again, the assumption that an agent
knows his ability exactly is extreme. The assumption that agents know their abilities imperfectly is more practical.

When agents do not know their abilities, the long-term contract is simply a repetition of the static contract. Without knowledge of abilities, the pooling contract is thus optimal for the first period. In this case, the principal has the same prior at the beginning of the second period as she had at the beginning of the first period. However, the agent then knows his ability perfectly (assuming no measurement error) so the principal can offer a separating contract that attains efficiency for agents with high ability and inefficiency for agents with low ability.

If the agent knows his ability imperfectly, then a qualitatively similar pooling equilibrium occurs in the first period. But the principal can induce separation of agents according to their ability distributions while pooling agents within ability distributions. As a result, for each distribution, some low-ability agents will receive less than their reservation wages. Also, because agents do not know their abilities perfectly, a fraction of low ability agents receive negative gains from contracting even when renegotiation-proof long-term contracts are in place for each ability distribution.

The likelihood that long-term contracts will be renegotiated as information becomes available raises the question of why parties should not sign one-period contracts before moving to long-term relationships. In the theoretical simplicity of this chapter, the advantage of a one-period contract is that the agent can learn his ability and then sign a long-term contract that permits efficiency. However, if the agent makes an irreversible asset-specific investment before signing the contract, then he losses his bargaining power. He may be more likely to ensure returns on his investment and effort with a long-term
commitment from the principal before investment. Nevertheless, even with asset-specific investment, parties can renegotiate the contract beneficially for both sides after information on abilities becomes more apparent, provided that both sides agree.

Based on the numerous theoretical results of the general principal-agent models developed in this and previous chapters, chapter 5 turns to discussion of the relevance and applicability of this model to hog contracting.
Chapter 5:

Contract Theory Applied to Hog Contracts

5.1 Introduction

To see how the general theory of incentives sketched thus far relates to practices in US hog sector contracting, this chapter reviews features of hog contracting, and considers how they differ from the principals and agents for which the theory was initially developed. Many differences between the actors of the theoretical models and hog contractors and contractees are differences of degree, rather than of kind. Also, the hog sector is large, complex, and diverse; many of the theoretical predictions reviewed and derived apply only partially to hog contracting and perhaps none of them apply precisely. However, by modifying the hog contract slightly to place it in the standard contract format, conclusions regarding agents’ gains from contracting appear to be quite relevant to contract hog growers.

Section 5.2 begins by discussing and reshaping hog contract features in detail while comparing the hog compensation scheme with the standard linear one of contract theory. Section 5.3 discusses the reasons why the contracting parties in the hog sector prefer production contracts or vertical integration to the spot market and whether the reasons for this preference are similar to those in standard contract theory. This section also discusses asset-specific investments and their implications for hog contracting parties. Section 5.4 explains why production contracts are preferred to vertical integration in the hog sector. Section 5.5 explains how the holdup problem in this sector is solved using long-term contracts.
Finally, section 5.6 concludes that, because of imperfect information on grower productivity and pooling contracts for different grower types, some contract growers earn less than their reservation utilities.

5.2 Hog Contract Features

5.2.1 Description of the hog contract payment scheme

A hog production (grow-out) contract is an agreement between a contractor and a grower that binds the grower to specific production practices. The contractor is often called an integrator, characterized as a large conglomerate or corporate organization that contracts with many growers to produce hogs. Integrators typically market hogs through marketing contracts or other arrangements with slaughter plants. Input suppliers and packers are other distinct types of contractors that use contract production to vertically integrate business activities, such as feed or hog processing. Growers can also be contractors that employ other producers as growers in order to expand or specialize in their hog operations (Tsoulouhas and Vukina, 2001).

According to contractual arrangements, the grower cares for piglets to be grown out, and usually provides land and housing facilities, utilities, labor, and other operating expenses, such as repairs and maintenance. The contractor provides piglets to be grown to slaughter weight, feed, veterinary supplies and services. Expenses for fuel and litter can be shared or paid by either party, depending on the nature of the contract (Tsoulouhas and Vukina, 2001). Most hog finishing contracts have fairly similar payment structures, taking the form of a fixed performance standard,

\[ R_{it} = bQ_{it} + \beta(s - C_{it}/Q_{it})Q_{it}, \]  \hspace{1cm} (5.1)
where $R_{it}$ is the total payment to the $i$th contract grower for the $t$th batch, $i$ indexes growers, $t$ indexes batches, $b$ is the base payment per pound of live meat produced, $Q_{it}$ is the amount of hog weight produced (in pounds), $\beta$ is a bonus factor measuring the intensity by which the fixed standard influences the total payment the grower receives, $s$ represents a fixed feed-conversion ratio, and $C_{it}$ is the amount of feed used by grower $i$ to produce $Q_{it}$.

Performance is determined largely by the feed-conversion ratio, which is measured in pounds of feed used to produce a pound of live weight. Frequently, performance is measured by the so-called settlement cost which is obtained by combining feed cost with other contractor’s costs (piglets, medication, etc) divided by the total pounds of live meat produced. For a feed-conversion ratio below standard $s$ the grower receives a positive amount over the base payment, and for a feed-conversion ratio above standard $s$ he receives a penalty.

5.2.2 Payment based on multiple signals

One of the important features of a hog contract is that the growers’ payment depends on multiple signals of growers’ effort. Equation (5.1) can be rewritten as

$$R_{it} = \gamma Q_{it} - \beta C_{it},$$

(5.2)

where $\gamma = (b + \beta s)$. For this contract the grower’s payment is not based on his effort directly. The agent’s payment in standard contracting models is also not based on the agent’s effort directly, but rather on the output produced by the agent. Output is a signal of the agent’s effort. For hog contracts, in addition to output, the grower’s payment depends on the feed provided by the contractor. The format of the hog payment scheme
in equation (5.2) corresponds to the standard payment scheme as discussed in section 2.2.9.3 of chapter 2 in the context of multiple signals.

Section 2.2.9.3 of chapter 2 explains how additional signals are helpful and how the weights on additional signals should be determined. The informativeness principle described in that section states that any measure of performance that (on the margin) reveals information on the effort level chosen by the agent should be included in the payment scheme. That means, whenever two signals together provide more information about the agent's effort than does the agent's output alone, the agent's compensation under the optimal contract will be based on both the signals.

It is important to understand why hog payment schemes use two signals of grower’s effort rather than one, i.e., why one of the production inputs in addition to output is used to determine a hog grower’s payment. Using both total output $Q_{it}$ and total feed use $C_{it}$ is not beyond the theoretical insight as described in section 2.2.9.3. Hog output, $Q_{it}$ is a function of both the grower’s inputs and contractor provided feed $C_{it}$. The grower’s effort in the hog contracting problem can be regarded conceptually as grower provided inputs (including labor) in addition to contractor provided feed that jointly determine the weight gain. Thus, the same amount of output $Q_{it}$ could have been produced with a different combination of grower’s inputs and contractor’s feed $C_{it}$. Any given output $Q_{it}$ produced with a relatively smaller amount of contractor’s feed signals a larger amount of grower’s input use. Alternatively, the same output $Q_{it}$ produced with a relatively larger amount of contractor’s feed signals a smaller amount of grower’s input use. As a result, total output $Q_{it}$ alone may not be a sufficient statistic for the grower’s input. Since contractor provided input $C_{it}$ is observable with virtually no cost, contractor
provided input $C_{it}$ is used as one of the signals to measure grower’s inputs. Using contractor provided feed $C_{it}$ along with $Q_{it}$ thus provides better information on the grower’s input use.

5.2.3 The base payment

Apart from multiple signals, another feature of the hog contract payment in (5.2) is that the intercept with respect to output is zero. Hog contracts with a zero base payment are reasonable because the opportunity cost of not producing any output is zero unless there is some unforeseen disaster that severely affects the output. Thus, under a reasonable assumption of constant returns to scale the linear scheme with scaling by output quantity seems reasonable for a typical production environment where zero output implies zero effort and zero feed use and, hence, deserves zero payment.

Further manipulation of equation (5.2) reveals that one base pay is built into the payment scheme for unit production. To see this, equation (5.2) can be written as

$$r_{it} = \frac{R_{it}}{Q_{it}} = \gamma - \beta \left( \frac{C_{it}}{Q_{it}} \right) = \gamma - \beta c_{it} ,$$

where the grower payment for per pound of live meat produced is $r_{it}$ and $c_{it}$ is contractor supplied feed used for per pound of live meat produced. The unit payment, $r_{it}$, makes the payment a function of two signals of the grower’s input, pounds produced and feed used to produce it. The feed use per pound of live meat produced, $c_{it}$, which combines the information from both signals, is used to determine the unit payment, $r_{it}$. This $r_{it}$ can be compared with the piece rate $s$ in earlier models. This reveals the similarity in the linear payment structure, $w = t + s\pi$, described in the theoretical model of section 2.2.9.2 in chapter 2, and the linear payment structure $R_{it} = \gamma Q_{it} - \beta C_{it}$. 
Based on the fact that hog contracts use a similar (linear) payment scheme as the standard theory, is it valid to conclude that the standard theoretical predictions derived in last two chapters apply to the contract hog growers? To answer this question requires knowing whether these similar linear contracts have resulted from the reasons described in the standard theory of chapters 2, 3, and 4. Otherwise, the similarities may be a coincidence.

If dissimilar reasons are behind the motivation for contracts in hog sector, then applying the standard theoretical predictions of agent’s gains from contracting to conclude that hog growers gain from contracting may not be valid. In order to see whether similar motivations are behind production contracts in the hog industry, the potential reasons for both the parties to enter the hog contracts must be examined.

5.3 Reasons for Hog Contracting

A major deficiency of the discussion thus far is that it deals mainly with a given structural relationship where a grower is contracting with a contractor. It can, in itself, say little about the evolving structure of the relationship, specifically, about why the grower is using the contractor technology, inputs, and production practices through a production contract in his own production facilities rather than remaining an independent grower. Why certain tasks are given to the grower, why the two parties do not interact through the marketplace, and why they do not vertically integrate remains unclear.

Section 3.2 of chapter 3 suggests that transactions cost along with risk determine the optimal form of contracts. As discussed in that section, the size of the transactions cost of using spot markets, production contracts, and vertical integration explains why
one of them dominates the others. Of course, risk cannot be ignored because it affects the transaction costs. Since production contracts dominate others at least in part of the hog sector, transactions cost might be lower when production takes place under production contracts. The theoretical results of section 3.2 of chapter 3 suggest that transactions cost resulting from asset specific investment may be the prime motivating factor for contracting in hog sector. This specific component of transactions cost also explains why hog production contracts are long-term contracts. This section considers transactions cost in the hog sector related to bearing of risk, consistent quality demands of consumers, and asset-specific investment.

5.3.1 Risk reduction

One strain of the hog contracting literature considers risk reduction as one of the main reasons why growers are motivated to contract production (Rhodes, 1989; Rhodes and Grimes, 1992; Johnson and Foster, 1994; Kliebenstein and Lawrence, 1995; Lawrence and Grimes, 2001). This point of view emphasizes the stochastic nature of prices and production that growers face and views production contracts as a vehicle to shift (or share) these risks. According to this view, the important benefit of production contracts is the reduction of risk-bearing costs. Transferring these uncertainties and risks onto risk-averse growers is costly. Optimality requires removing risks from the risk-averse growers. Since this approach emphasizes the reduction of growers’ economic risks by viewing contracts as the vehicle to shift risks to the contractors, this is known as the risk-sharing approach.
Emphasizing the benefits of shifting risk presumes that impediments prevent direct risk-shifting opportunities for hog growers, such as trading risks in insurance markets. Production contracts are viewed as a second-best technique to reallocate these risks of hog operations. In their hog producers’ survey, Lawrence and Grimes (2001) found growers citing financial risks reduction as the key advantage of production contracts. However, one problem of the risk-sharing approach is its failure to recognize that there is a limit on how much risk production contracts can shift from the growers to the contractors. For example, Johnson and Foster (1994) ignore any incentive effects and evaluate the desirability of hog production contracts to growers in terms of the risks that growers must bear. They find that a broad choice of contract terms allow hog growers with different degrees of risk aversion to find suitable tradeoffs between risk and expected returns.

The interests of the growers and contractors diverge, and thus it might be impossible to construct an incentive mechanism that simultaneously reduces grower’s risks and maximizes total potential surplus when the inputs and/or outputs of the growers are not observable by the contractor. Growers’ efforts cannot be easily monitored by contractors because output is influenced not only by the growers’ effort but also by factors beyond their control. Because of the nonobservability of growers’ efforts, complete removal of growers’ risks mutes their incentives to exert high effort.

This risk incentive trade-off is discussed in section 2.2.9.2 of chapter 2. Transaction costs, i.e., costs of providing incentives, arise because of this conflicting interest of the parties. Because of this trade-off, common incentive structures exclude fixed wage contracts and include a payment scheme consisting of a small base fee and a
large piece rate implying high incentives. Thus, part of the risk must be passed on to growers to provide incentives. Thus, complete risk shifting from the growers is not possible.

However, even if partial risk reduction is the prime motivation for growers to enter a contract, and the benefits from reduction of the risk premium is the prime motivation for the contractor, then a contractor would prefer more risk-averse rather than less risk-averse hog growers. Equations (2.4) and (2.5) of chapter 2 show that the principal pays more to the risk neutral agent, \( \bar{u} + c(e) \), than she pays to a risk-averse agent, \( v^{-1}(\bar{u} + c(e)) \), for the same task. Because effort observability eliminates the incentive problem in that case, the principal acts essentially as an insurer. Due to the curvature of the \( v \) function, the risk-averse agent’s payment is lower as agent’s risk aversion is higher. Thus, the principal’s profit increases as the agent’s risk aversion increases. If the contractor acts as an insurer, then benefits from shifting higher risk premiums should motivate her to contract with more risk-averse hog growers.

However, that is not what is observed in hog operations. Because absolute risk aversion is believed to be decreasing and because contracting hog growers tend to be large while independent growers tend to be small, the risk motivation appears to be inconsistent with the facts. If wealth is taken as a proxy for risk-aversion, then small-scale growers are more likely to be more risk averse (under decreasing absolute risk aversion) and thus more likely to be under a production contract for a given production risk. But small-scale growers are mostly independent hog operators. In fact, more large-scale producers are contract growers.
For example, contract hog finishing operations had an average of more than 5,000 hogs removed in 1998, compared with an average of about 1,500 head sold from independent operations. The distribution of hog finishing farms by typology shows that 67 percent of contract operations were among the large farm groups, while 64 percent of independent operations were in the small farm categories (McBride and Key, 2003). Furthermore, this is a growing trend in the hog sector (MacDonald et al, 2004). Martin (1994) estimates the extent of risk shifting afforded by production contracts for hog production. But she does not find the evidence of substantial risk shifting in her study. Since this is the case, risk reduction does not appear to be the main underlying motivation for contracting in hog production.

A more reasonable conclusion is that risk reduction is one of the important motivations for growers’ to enter into contracts. But whether growers enter into contracts solely because of risk-sharing inducements offered by contractors may not explain why contracting occurs. In fact, a payment scheme with high incentives, and thus consisting of a low base pay and a high piece rate, may discourage more risk-averse hog growers from entering production contracts, particularly when the risk involves risk associated with unknown grower abilities that cannot be shifted by contracting. As discussed in detail in section 3.2 of chapter 3, when the transactions cost (of providing incentives) associated with various contracts vary across growers and growers are heterogeneous in risk aversion, more low risk-averse growers are likely to be in production contracts than high risk-averse growers because of their relative transactional efficiency. It is less costly for contractors to induce a given effort from a low risk-averse grower than from a high risk-averse one because less has to be paid to compensate the low risk-averse grower for the
risk he bears. If this is true, many more hog growers with low transactions cost (less risk-aversion) are likely to be contract growers.

Given heterogeneous transactions cost and risk aversion, production contracts are likely to be more prevalent among less risk-averse growers. Nevertheless, the growers who enter contracts may cite risk-sharing as the major reason for doing so because, from their perspective, risk-sharing is the inducement that persuades them to enter an agreement that reduces transactions cost for the contractor. Thus, instead of risk aversion as the primary underlying motivation for contracting, the predominance of contracting depends on how transactions cost (i.e., negotiation, supervision, and enforcement costs) vary across contracts and growers. However, even if only less risk-averse hog growers self select to contract, a considerable amount of risk shifting from hog growers to contractors may be observed.

Presuming that risk shifting is not the primary motivation for contracting, production contracts are often analyzed under risk neutrality, explicitly ignoring any risk-bearing effects. Once the parties enter a contract, however, contract parameters are determined partly by the risk bearing costs due to mechanisms necessary to preserve incentives. For example, in the context of broiler contracting, Knoeber and Thurman (1995) state that risk reduction is the end result of contracting, not the cause of it. Similarly, I conclude that risk sharing does not explain why hog contracting occurs. Therefore, understanding the role of other transactions cost is necessary to explain why both parties prefer production contracts to other schemes.
5.3.2 Quality consistency and efficiency

Consumers today are concerned primarily about quality hogs at a low cost. They are demanding leaner pork, which requires different genetics. They do not want the variability of quality that is inevitable when hogs of various breeds are produced under a variety of production processes and the resulting pork products are intermingled in the marketing system. On the question of quality, consumers want consistent quality but they do not necessarily want the same quality. They want to know when they buy the center-cut pork chop today that it will be pretty much the same as the center-cut pork chop they bought last week. People have different tastes and preferences, and so all consumers do not want the same quality. Some want their pork chops to be larger and others smaller. Some want theirs to be leaner and others well-marbled. Some want theirs darker in color and others lighter. Some like their chops firm and others want theirs to be moister, and so on.

Smaller family hog operations simply cannot provide consumers with the consistent quality of pork they demand at as low a cost as the large-scale contractor can. Of course, some large family hog farms are as efficient as hog contractors. But corporate operations by contractors are far more efficient than the average family hog operation (McBride and Key, 2003; MacDonald et al, 2004). The large-scale contractor who operates through production contracts or vertically integrated operations can provide consistent quality meat with variety through basic genetic selection, uniform feed supply, climate controlled facilities, drugs and hormones, and controlling animals for same age, weight, and size. Further, contractors, who are mostly processors, can operate their processing units and slaughter houses efficiently ensuring a steady flow of slaughter hogs
to them with the help of production contracts or company-owned hog operations (i.e., vertical integration).

The important benefit of these production arrangements is the reduction of transactions cost and production cost afforded by specialization, easier access to capital, quality control, and the incentives that contracts provide for effort and information revelation. For example, under production contracts, tying a grower’s payment more closely to production outcomes provides an incentive to improve these production outcomes. The focus is thus on how contract design affects the incentives of growers (and contractors) and ultimately the transactions cost. As described in section 5.3.1, the grower’s aversion to risk limits the extent to which incentives can be provided. Risk is treated as a cost of providing better incentives entailing lower transaction costs. In this context, higher risk and risk aversion implies higher transaction costs. Risk at a very high level might call for company-owned hog operations as an alternative to production contracts. Thus, the transaction cost of a consistent supply of quality pork and steady flow of animals to slaughter houses are lower under these production arrangements than would prevail with spot market arrangements.

5.3.3 Asset specificity and the holdup problem

As explained in section 3.2 of chapter 3, transactions cost of spot (or auction) markets may be large if opportunist behavior is undertaken by any of the transacting parties. Where the principal or the agent makes sizable relation-specific investments in assets specific to an ongoing supply relationship, spot markets are less attractive. For an agent, when production requires investing in an asset that is specialized to a particular
principal, any deal made prior to investment may not be enforceable once the investment is undertaken, at least when competing contractors are not available to the agent. Specific investments make agents vulnerable to being held up by the principal for a share of the associated quasirent. That is, opportunistic behavior by the principal can take advantage of agents' immobility or asset-specific investments. These possibilities may inflict high transactions cost on the parties, which would influence the type of contract. The remainder of this section considers asset-specific investment in the hog sector the implications it has for growers and contractors in the hog sector.

5.3.3.1 Asset specificity in the hog sector

The hog industry has been moving toward more specialized hog production and processing operations for over 60 years and the trend appears to have accelerated in the 1990s (Hurt, 1994). Modern hog facilities used in the larger hog operations are equipped with state-of-the-art technology dedicated only to pork production. These new technologies require large investments that are specific to hog production.

Williamson (1979) delineates several types of asset specificity that are relevant to the hog sector. Site specificity is one. Site specificity involves location-specific advantages favoring a particular relationship that might be due to some unique cost-reducing or revenue-enhancing characteristics of the location. The costs of transporting feed and feeder pigs and mature hogs have led growers and contractors to locate near one another. For example, hog contractors might concentrate production near a feed mill or packing or processing plant to reduce transportation costs and stress on hogs from transportation (Hennessy and Lawrence, 1999). Thus, contractors indirectly restrict the
location of contract growers to within a certain distance of feed mill and processing facilities. As a result, most growers have only one or perhaps a few contractors with whom they can contract. Thus, contractors likely have the monopsony power (or a close approximation) attributed to the principal in typical principal agent problem within a given geographical area. That is, growers may have limited if any opportunity to contract with other contractors. In this case, site specificity translates into market power.

Similarly, specialized processing facilities depend upon purchases by nearby growers. These, too, are site-specific assets. Using Williamson’s (1985) categorization, both the hog facilities provided by growers within a certain distance of the contractor, and the feed mills and processing facilities provided by contractors within a certain distance of growers, are relation-specific assets because of their site specificity (Hennessy and Lawrence, 1999).

Another type is physical asset specificity involves the specification of asset form to accord with the needs of other parties to a transaction. Examples of specific assets include the specialized equipment required for hog production, such as manure storage facilities, or equipment for manure handling, barn ventilation, or equipment to deliver feed and water to hogs. Contracts require that grower facilities are constructed to conform to the contractor’s requirements. The contractor may require, for example, a one-thousand-head modern confinement building that meets her building standards. She may require growers to construct specific, highly automated finishing barns including ventilation systems beyond what growers had contemplated; specified roads on the grower’s farm to provide access for feed and live-haul semi-trucks; adequate loading and unloading areas; an automatic-switch stand-by generator capable of running the water,
feed, lighting, and other equipment; a small portable scale for weighing pigs (to her specifications) in each facility, and so on (Swinton and Martin, 1997).

In brief, the contractor’s specifications must be followed for land, buildings, equipment, water, power, fuel, electricity, and other facilities because the contractor may consider them necessary to properly care for and raise pigs to marketable age. As a result, the value of these facilities depends upon an ongoing supply relationship with the particular contractor. These facilities are valuable assets within a contract with the particular contractor, whereas outside the contract they may be of limited value or require modification before a contract with another contractor could be undertaken.

Human asset specificity, which involves the acquisition of skills and information that facilitate a particular firm-to-firm relationship, is also important. Although difficult to quantify, this type of specificity undoubtedly exists in the hog industry. Opportunities for legal disputes abound in contractual arrangements. Parties to an ongoing business relationship might establish from past performance that the other party is honest, reasonable, and fair (Klein and Leffler, 1981; Shapiro, 1983, and Winfree and McCluskey, 2005). This might also discourage the grower from shopping around for alternative contractors (Hennessy and Lawrence, 1999).

However, all of the significant management decisions—selection of facilities design, genetic stock, health program, breeding dates, when to place on feed, feeding system, when to price, when to deliver—are made by the contractor rather than the grower. Growers follow a company-structured feed and management plan. Even though considerable animal raising skills are required to implement a company management plan
profitably, these skills are not be company specific and, hence, growers may have little human capital that is relation-specific.

5.3.3.2 The holdup problem in hog contracting

Because of all the specificities mentioned above, the growers’ assets are sources of potentially appropriable quasirents in the sense that they have low salvage value outside the bilateral contractual relationship. This constitutes a holdup problem that can manifest itself in two ways. First, according to Williamson (1985), appropriable quasirents affect the level of investments. Often, because of economies of scale, there is only one local hog contractor, and transporting the hog product to an alternative market is costly. Thus, a hog producer who makes a costly investment in a specific asset is vulnerable to holdup. Growers vulnerable to holdup will be reluctant to invest in specific assets. Being aware of the possibility that they may be held up by contractors, growers will cautiously invest in specific assets even if they do so. These investments are likely suboptimal compared to the situation where contractors and growers vertically integrate. The magnitude of the underinvestment problem may vary with factors determining the salvage value of the investment, which in turn affects the magnitude of quasirents.

Second, after housing facilities have been constructed, the contractors may exploit their advantageous bargaining position by frequently requesting upgrades and technological improvements as conditions for contract renewal. Growers may be held up because physical specificity could effectively reduce the growers’ compensation without causing additional moral hazard problems. That is, when a contract involves physical asset specificity, the fear of contract termination can induce the agent to exert high effort
without the need for efficient compensation (Lewin, 1998; Vukina, and Leegomonchai, 2006). The problems of suboptimal investment and opportunistic behavior would seem to make the parties favor vertical integration as an alternative.

In addition, anticipation of a changeable or uncertain future should tend toward vertically integrated production. As inputs (e.g., animals to be grown, feed) become increasingly more productive, contracts are likely to require constant renegotiation and so become subject to opportunism. Anticipating this problem, hog producers might be discouraged from contract production. Beyond this, there is considerable risk in hog production. If this risk is borne by growers, contracting costs need not be large, but grower risk-bearing costs will be. To reduce these risk-bearing costs by shifting them to the contractors (a cheaper risk bearer), however, would seem to require complex contingent contracts. Again, the costs of such complex contracting would seem to favor vertical integration. But reality is puzzling. Vertical integration is rare, and contract production dominates the hog sector.

5.4 Vertical Integration versus Production Contracts

The discussion of section 5.3 under a transactions cost framework offers both insight into the organization of hog production and a puzzle. As predicted by the transactions cost framework, the importance of relation-specific assets provided by growers and the contractor explain why spot markets are not an economically efficient device for organizing hog production. However, while transactions cost provides a convincing explanation for the replacement of spot markets in hog production, it remains to explain why production contracts are used to grow hogs rather than vertically
integrated company farms. This section considers how contracting provides positive advantages relative to vertical integration.

5.4.1 Capital constraints

As demonstrated in chapters 2, 3 and 4, other factors beyond risks motivate agents to enter into contracting. Even with no concern for risks or agential effort, a wealth constraint or asymmetric information may motivate the design of contracts. Section 2.3 of chapter 2 explains how a wealth constraint can impose a sharing contract on the parties even if the agents are risk neutral. One such constraint in the hog sector is the capital constraint, i.e., capital required to finance the investment in hog facilities (Kliebenstein and Lawrence, 1995).

A resource-providing contract that reduces the grower’s initial investment is a better alternative for a grower compared to private operation with spot markets. But an arrangement where growers provide capital reduces the capital requirement for the contractor for the purpose of acquiring a consistent supply of quality meat compared to financing company-owned farms in a vertically integrated operation. Financing a single standard (e.g., one-thousand-head) modern confinement building, which costs roughly $150,000 to $200,000, is difficult for most hog growers (Swinton and Martin, 1997), but would require a staggering investment for a contractor financing many farms. Potential lenders often require a production/marketing contract to ensure stable revenue. By providing a resource-providing production contract to growers, the contractor can relax binding credit constraints for growers without taxing credit constraints faced by the contractor.
5.4.2 Moral hazard and performance

Another important feature of hog contracts is the requirement that growers must provide their own time as effort but also capital in the form of hog facilities. Hog facilities, i.e., capital specific to the contracting relationship between growers and the contractor, act as a bond to assure growers’ effort and performance. Upon the failure of a grower to comply with any of the terms of a production contract agreement, the contractor typically has the right at her option, without legal process, to (i) reduce payments to the grower, (ii) take immediate possession of the delivered pigs and raise them to maturity on the land and with the facilities of the grower, or (iii) to remove pigs from the possession of the grower and raise them to maturity elsewhere. Growers agree in the contract that any expense and cost incurred by contractors in raising the pigs to maturity will be paid by the growers. Likewise, upon failure of a grower to comply with any of the terms of the agreed contract, the contractor has the right to terminate the contract (Swinton and Martin, 1997).

Production contracts typically state that contracting with a grower will be discontinued if performance is consistently well below average. If a grower does not provide hog houses, discontinuance would not be onerous to the grower, which would thus provide little incentive to the grower to perform. But this is not the case if growers provide capital in the form of hog houses and other hog facilities. Discontinuance would imply a loss of investment. Thus grower capital provides an incentive to meet some minimum level of performance. Most important, since the skills necessary to raise hogs are widely available and techniques (a feed and management plan) and advice are
provided by the company, there is little relation-specific human capital that might also serve this purpose.

The absence of relation-specific human capital explains why a monetary performance bond from a hired grower cannot be used instead of grower-provided capital to induce performance, i.e., why a hired grower managing a contractor-owned hog operation would not assure performance. Performance bonds are commonly used in other sectors and industries to assure the performance of hired managers (see, for example, Lazear, 1979). The use of hog houses, however, has two desirable characteristics not associated with performance bonds.

First, if the bond accrues to the contractor in the event that a grower does not perform, the contractor may have an incentive to obstruct grower performance. This would be the case if the performance bond is greater than the loss to the contractor from poor performance. This is a moral hazard problem where the contractor behaves opportunistically. Grower-provided hog houses are not transferred to the contractor if performance fails as is a performance bond. Rather, the value simply dissipates. Thus, an incentive for grower performance is provided without simultaneously creating an incentive for the contractor to obstruct grower performance.

Second, since hog houses are long-lived assets, ending the contractual relationship between grower and contractor for any reason (poor performance or not) imposes a cost on the grower which reduces enforcement costs. Enforcement cost is discussed in section 3.2.2.1 of chapter 3. Grower provision of hog houses not only bonds performance but also ensures that the contractual relation will be a long-term relationship. According to
Knoeber (1989), this is one of the important reasons why contractors in the broiler industry use chicken houses rather than any other monetary bond.

An additional effect of grower-provided capital is the provision of proper maintenance incentives to growers. If it is difficult to monitor maintenance, hired managers using facilities provided by the contractor on company farms will have too little incentive to maintain them (Alchian and Demsetz, 1972). Careless or excessive use of these facilities would result in the depreciation. Thus, in addition to supervising growers’ efforts, the contractor would have to monitor the use of hog facilities along with other inputs that she supplies to the production process. Requiring growers to provide hog facilities provides proper incentives for maintenance without any need for expensive monitoring, and thus saves monitoring costs. Monitoring cost is also discussed in section 3.2.2.1 of chapter 3. In summary, grower provision of hog facilities creates a bond that assures grower performance, better maintenance, and a long-term contracting relationship.

5.4.3 Selection of high-ability growers

As described in section 2.4 of chapter 2, where agents differ in ability and other characteristics, choosing agents to match productive circumstances is important. When agents know their ability but the principal does not, the principal can design contracts to induce agents to self-select into more efficient arrangements. In hog operations, large facilities employ large amounts of capital and high-ability growers are required to reap the full potential benefits of large facilities. Thus, self-selection of high-ability growers is
important for the contractor. One important dimension of grower ability is the ability to adapt quickly to rapid technical change.

Technical change in hog production includes improved genetics and advances in nutrition, housing and handling equipment, veterinary and medical services, and management practices that improve the efficiency of operations. For example, farrowing and weaning performance improved substantially between 1992 and 1998. The number of pigs farrowed and weaned per litter increased by 8 and 12 percent, respectively, over the 1992-98 period. Labor and feed efficiency gains for that period were also substantial. Labor efficiency on hog farms was nearly 60 percent higher in 1998 than in 1992.

Technical change in hog production also contributed to a decline in real production costs during that period. Average operating and ownership costs per hundred weight (cwt) of gain, expressed in 1998 dollars, were about 16 percent lower in 1998 than in 1992 among all U.S. hog growers (McBride and Key, 2003).

Thus, ongoing technical change provides a reason to prefer high-ability growers over low-ability growers. An important function of the requirement that growers provide hog facilities (especially hog houses) is the inducement for self-selection by high-ability growers. Requiring growers to provide hog houses acts as an entry fee and can be used in conjunction with an adjustment of the piece rate (payment per pound of live meat produced) to discourage low-ability growers from signing contracts while simultaneously attracting high-ability growers. With proper contract parameters, high-ability growers find the investment profitable while those with low-ability do not. Also, high-ability growers are more capable of making large asset- and site-specific investments in hog facilities to
exploit the benefit of economies of size and scale in these operations. So the contractors can extract higher profits from high-ability growers with larger operations.

The contractor’s screening out of low-ability growers through requiring a larger entry fee (larger hog facilities) may be one reason why larger hog operations are observed for contract hog growers. A similar reasoning apparently explains the form of the contract used for broiler growers. Knoeber (1989) notes that broiler companies claim that an important reason for the use of contract growers is the refusal of high-quality growers to work for wages. That is, contract production selects for high-quality growers. He proves that hired managers indeed perform less well than contract growers in broiler production.

In summary, credit and capital sharing, performance assurance, and self-selection of high-ability growers explains the preference for production contracts compared to vertical integration. Even though these added benefits from production contracts solve the puzzle of why production contracts are chosen over vertically integrated company farms, they do not explain how the likely holdup problem is solved by these contracts.

5.5 A Solution to the Holdup Problem

This section addresses the holdup problem in hog contracting. Before developing a solution, however, its existence and nature requires discussion.

5.5.1 Arguments for and against holdup in hog contracting

Arguments have been advanced both supporting and rejecting the existence of a significant holdup problem in bilateral hog contracting. The empirical testing of transactions cost theory suggests that the direct evidence of one party being held up by
the other is rather rare. This is because the parties in transactions are aware of such problems and have already adopted suitable institutional arrangements to address the problem of expropriation in advance. Without those mechanisms, the parties would be reluctant to invest, or their investment level would be suboptimal. As Joskow (1987) shows, coal mines eventually sign long-term contracts or vertically integrate with electricity firms in order to avoid the holdup problem. The empirical evidence of holdup in franchising contracts, which are similar to livestock production contracts organizationally, appears to be quite rare as well (Beales and Muris 1995). However, by using the cross-sectional national survey of broiler growers, Vukina and Leegomonchai (2006) show moderate empirical support for the presence of holdup in broiler industry production contracts.

For hog operations, insignificance of holdup may not be the case because a suboptimal level of asset-specific investment in hog facilities is mitigated by the fact that contractors enforce investment of growers by other means. This enforcement comes through contractors’ screening out of low-ability growers, which in effect requires a larger entry fee (i.e., larger hog facilities). The contractor may require, for example, a one-thousand-head modern confinement building that meets her building standards. If they decide to contract, they have to build a one-thousand-head modern confinement facility.

However, holdup may take place in some non-apparent ways. When contracts are up for renewal, which implicitly happens whenever a new batch of feeder pigs is delivered to the operation, the bargaining power of the grower can be substantially diminished, depending on the degree of asset specificity. The contractor may exploit this
situation by not changing the nominal payment to growers even if the period has experienced significant cost inflation. Alternatively, the contractor may require frequent upgrades of facilities and equipment without necessarily making adequate provisions in the contract that will secure the grower’s market rate of return on this additional investment. After several rounds on contracting in this mode, the grower’s capital may not appear to be suboptimal as would otherwise evidence a holdup problem.

Some suggest that the holdup problem can be symmetric. Production contracts that require both parties to invest in assets specialized to the other (or an exchange of hostages as described in standard theory by Williamson, 1985), as is the case where growers invest in hog facilities and contractors invest in breeding facilities, feed mills, and processing plants, help to alleviate the holdup problem. This role for livestock production contracts is emphasized by Knoeber (1989), Frank and Henderson (1992), Barry, Sonka and Lajili (1992) and Sporleader (1992). But the weakness in applying this argument to hog production is that specific investments from both sides alone do not cancel out the opportunistic intents and consequences.

Rather, the extent of the stakes that each party has in the other’s specific investment must be weighed in drawing such a conclusion. For example, when a contractor is contracting with hundreds of growers in the vicinity, opportunistic behavior by the contractor and a specific grower may not be offsetting. The contractor can run her processing unit at almost full capacity with one less grower, whereas terminating the contract may be disastrous for a single grower. Thus, resisting opportunistic behavior by the contractor may require uniting a large number of the growers, which also has its
costs. Furthermore, clauses in typical contract agreements prevent such a concerted effort of the growers.

Some argue that physical asset specificity of hog grower facilities may not be as significant as others claim, i.e., hog facilities may have substantial salvage value outside the bilateral contractual relationship, contrary to the discussion in section 5.3.3.1. However, the holdup problem may not be solved even in this case. Site specificity may still exist whereby the contractor can exploit growers locally. As a result, a simple production contract cannot solve the holdup problem.

5.5.2 Long-term contracts as solutions to holdup

As explained in section 5.3, the main arguments for extensive bilateral contractual arrangements in hog operations are based on transactions cost and production risk. Asset-specific investments and risk are the main contributing factors to a high level of transactions cost as defined by Williamson (1975, 1985), all of which explain requirements of alternative institutional arrangements as opposed to spot markets. Given the problem of suboptimal investment associated with holdup, the parties in a hog transaction might be attracted to vertical integration. But as argued in section 5.4, lower supervision, monitoring, and enforcement costs seem to explain a preference for production contracts compared to vertical integration. However, it has been widely recognized that simple short-term contracts do not solve the problem. Only long-term production contracts can minimize transactions cost for two parties engaging in a commitment involving significant specific assets if vertical integration is not feasible. Empirical research on the energy sector lends support to asset specificity theory (Joskow,
and suggests that substantial efficiency gains from asset-specific investments might be prime motives behind long-term contractual relationships.

Short-term contracts are for a single period, where a period is defined as the length of time within which the grower performs his tasks, the outcome is realized, and he receives payment. In the context of hog production, one period typically corresponds to a calendar year. Long-term contracts are agreements that last more than one period. Long-term contracts that incorporate requirement clauses, fee indexation, liquidated damages, arbitration, and other provisions are possible means of overcoming the holdup problem without vertical integration. In fact, these provisions are observed in hog contracts as, at least in part, a solution to the holdup problem. But other reasons may motivate growers to prefer long-term contracts. For example, asset specific financing by lenders may require long-term production/marketing contracts to ensure stable revenue compared to the instability of frequent switching or renegotiation of contracts.

5.5.3 Length of hog production contracts

Given the importance of long-term contracts, remaining issues are optimal contract duration and provisions for contract renegotiation. What is the duration of the production contract that solves the holdup problem of hog growers? Hog production contracts are generally written with five to twelve years duration and often require an advance notice of termination, usually by about six months. Provisions often exist to extend the initial terms for an additional time period subject to mutual consent (an evergreen clause) or to give the contractor the right of first refusal (i.e., the right to match a competing offer). In addition, the contracts sometimes provide for renegotiation of
terms if new technologies or regulations arise, thus reducing the de facto contract length
(Hennessy and Lawrence, 1999; Swinton and Martin, 1997).

Perhaps the most important problem associated with long-term contracts is that unforeseen circumstances can arise over time. The initial motivation to enter a contract may depend on the temporal assessment of contract costs and benefits (Hennessy and Lawrence, 1999). Also, parties may prefer to be locked into reasonable contract terms. However, as time passes, conditions underlying written contracts change. It may become clear to one party, or even to both, that gains would be higher outside the contract. So an opportunity to terminate the contract might be welcome. In these cases, an incentive to renegotiate the contract exists as well. Thus, a trade-off typically exists between welfare gains from risk-sharing associated with a longer-term contract and flexibility gains arising from shorter-term contracts (Hennessy and Lawrence, 1999). In the context of this trade-off, Short-term production contracts are also observed in hog production.

The 2003 ARMS questionnaire included questions on the prices and fees growers received under contracts, the process used to determine prices and fees, and contract terms. Contract terms included the length of time covered by contracts as well as the quantities and the set of production tasks growers commit under contract. From this survey data, duration of production contracts for hogs is shown in Table 1. While only 37 percent of contract hog producers reported that they had a contract of at least 5 years duration, these operations accounted for more than half (56 percent) of contract hog production. Thus, long-term contracting appears to dominate for large-scale hog producers.
Table 1: Hog Contract Length

<table>
<thead>
<tr>
<th>Length of contract</th>
<th>Percent of Contracts</th>
<th>Percent of Contract Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>No length specified</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short term: 12 months or less</td>
<td>27.9</td>
<td>21.1</td>
</tr>
<tr>
<td>Medium term: 13-59 months</td>
<td>5.1</td>
<td>3.5</td>
</tr>
<tr>
<td>Long term: 60 months or more</td>
<td>36.9</td>
<td>56.0</td>
</tr>
</tbody>
</table>


5.6 Gains from Contracting

As demonstrated in chapter 3, when a principal contracts with a group of heterogeneous agents, the imperfect information of an agent’s productive capability or productive efficiency is extremely important. Whenever agents do not know their abilities (whether perfectly or imperfectly), some low-ability agents have negative average ex post gains from contracting. In a multi-period setting, these agents can be expected either to renegotiate contract terms with the principal or terminate the contract. As demonstrated in chapter 4, however, this does not happen when agents make considerable investments in assets that are specific to the ongoing relationship with the principal. The question addressed in this section is whether these results apply to the case of contract hog growers in US agriculture.

17 The ARMS questionnaire asked respondents to state the length of their contract, in months, and to report zero for those contracts that did not specify a length.
The reasons for contracting in the hog sector are similar to those described and identified in the theoretical survey and derivations in chapters 2 and 3. That is, the transactions cost associated with asset-specific investments in hog facilities have motivated parties representing the majority of the US hog sector to enter into long-term contractual relationships. The linear contract payment schemes that contractors use for paying contract hog growers are similar to those identified in the theoretical principal agent literature. To measure contract growers’ gains from contracting and to conclude whether some contract growers earn less than their reservation utilities requires modeling hog contracts explicitly.
Chapter 6:

An Explicit Model of Separation in Hog Contracting

6.1 Modeling Hog Contracts

Thus far, attempts have made to understand the factors that might explain contract hog growers’ gains from contracting based on the standard models of the contract literature presented in chapter 3. This chapter models hog contracts explicitly with separating contract parameters and they might apply to hog contractors. Results show that some growers are left with negative gains from contracting on average with this separation. Possibilities are also explored for uniform contracts based on payment parameters.

6.2 The Model

Suppose following Tsoulouhas and Vikina (2001) and GIPSA (2007) that the total payment to hog grower $i$ is

$$R_i = b_i Q_i + \beta_i (s_i - C_i/Q_i)Q_i$$

where $b_i$ is the base payment per pound of live meat produced, $Q_i = n_i q_i$ is the total weight gain, $q_i$ is the weight gain per animal, $n_i$ is the size of facility in numbers of hogs required by the contractor, $\beta_i$ is a bonus factor measuring the intensity by which a fixed feed-conversion ratio standard $s_i$ influences the total payment the grower receives ($\beta_i > 0$), and $C_i$ is the amount of feed used by grower $i$ to produce $Q_i$. Thus, the total payment equation becomes

$$R_i = b_i n_i q_i + \beta_i s_i n_i q_i - \beta_i n_i C_i$$
\[ n_i q_i (\gamma_i - \beta_i c / q_i) \]

where \( \gamma_i = b_i + \beta_i s_i \) and \( c \) is cost per animal (all animals are assumed to receive the same inputs under the contractor’s requirements so \( c \) is constant). Suppose the contractor can offer a three-parameter contract \((\gamma_i, \beta_i, n_i)\) where \( n_i \) also measures the facility size agreed upon with the grower. Also suppose weight gain depends on the grower’s effort following \( q_i = \lambda_i e_i c + \theta_i \), where \( e_i \) is grower effort, for which the grower incurs a cost of \( n_i e_i^2 / 2 \), \( \lambda_i \) represents grower ability \((\lambda_i > 0)\), and \( \theta_i \) is a random production shock with \( \theta_i \sim N(0, \sigma^2) \). This specification parallels the one explored in section 3.4 of chapter 3.

The total payment to grower \( i \) is

\[ R_i = n_i q_i (\gamma_i - \beta_i c / q_i) = n_i \gamma_i e_i c - n_i \beta_i c . \]

Thus, the expected total payment to grower \( i \) is

\[ E(R_i) = n_i \gamma_i E(q_i) - n_i \beta_i c = n_i \gamma_i e_i c - n_i \beta_i c . \]

Effort \( e_i \) represents all grower-provided inputs such as labor including labor used to raise feeder pigs, dispose of dead animals, and manage manure as well as other variable inputs such as fuel, lubricant, and electricity.

Growers provide a hog facility of size \( n_i \), which is observable by the contractor. Let \( \alpha n_i^2 / 2 \) represent the annualized cost of building a facility of size \( n_i \), which generates an increasing marginal facility cost proportional to \( n_i, \alpha > 0 \). Suppose that the growers are risk averse and that grower \( i \) has absolute risk aversion parameter \( \phi_i > 0 \). The price of hogs is assumed to be normalized at 1.

The income of grower \( i \) is thus

\[ \pi_i = R_i - n_i e_i^2 / 2 - \alpha n_i^2 / 2 = n_i \gamma_i (\lambda_i e_i c + \theta_i) - n_i \beta_i c - n_i e_i^2 / 2 - \alpha n_i^2 / 2 . \]
which generates expected income

\[ E(\pi_i) = n_i\gamma_i\lambda_i c - n_i\beta_i c - n_i e_i^2/2 - \alpha n_i^2/2 \]

and certainty equivalent income

\[ CE_i = n_i\gamma_i\lambda_i c - n_i\beta_i c - n_i e_i^2/2 - \alpha n_i^2/2 - \phi_i\gamma_i^2 n_i^2\sigma_i^2/2. \]

Surplus for the contractor from contracting with grower \( i \) is

\[ S_i = n_i(\lambda_i c + \theta_i) - R_i = n_i(1 - \gamma_i)(\lambda_i c + \theta_i) + n_i\beta_i c, \]

which yields expected surplus

\[ E(S_i) = n_i(1 - \gamma_i)\lambda_i c + n_i\beta_i c \]

Expected surplus also serves as the certainty equivalent for the contractor since she is risk neutral.

### 6.3 The Case Where the Contractor and Growers Know Ability

Establishing the first-best solution provides a useful benchmark. To examine contractor behavior when grower effort is observable, suppose reservation utilities possibly differ across growers depending on grower ability as represented by \( R(\lambda_i) \). That is, if a grower chooses not to contract, the sensible alternative for a grower with exceptional ability will be to grow hogs as an independent grower, in which case he makes a greater profit than does an average independent grower. In this case, the contractor’s problem is

\[
\text{Max}_{e, n_i, \beta_i, \gamma_i > 0} E(\sum_i S_i) = \sum_i [n_i(1 - \gamma_i)\lambda_i c + n_i\beta_i c]
\]

s.t. \( CE_i \geq R(\lambda_i) \) for all \( i \)
(ii) \( e_i \in \arg \max_{e} CE_i \) for all \( i \).

In this problem, contacting with many growers reduces to separate contracting problems with individual growers because all surplus can be extracted from each grower to the point where constraint (i) holds with equality. The individual problems are

\[
\begin{align*}
\text{Max} & \quad E(S_i) = n_i(1 - \gamma_i)\lambda_i e_i c + n_i \beta_i c \\
\text{s.t.} & \quad (i) \quad CE_i \geq R(\lambda_i), \\
& \quad (ii) \quad e_i \in \arg \max_{e} CE_i.
\end{align*}
\]

The first-order condition that satisfies (ii) implies \( e_i = \gamma_i \lambda_i c > 0 \) if \( \gamma_i > 0 \).

Substituting this implication of the second constraint, the problem reduces to

\[
\begin{align*}
\text{Max} & \quad E(S_i) = n_i(1 - \gamma_i)\lambda_i^2 c^2 + n_i \beta_i c \\
\text{s.t.} & \quad (i) \quad CE_i = n_i \gamma_i^2 \lambda_i^2 c^2/2 - n_i \beta_i c - \alpha n_i^2/2 - \phi \gamma_i^2 n_i^2 \sigma_0^2/2 \geq R(\lambda_i). \\
& \quad (ii) \quad e_i \in \arg \max_{e} CE_i.
\end{align*}
\]

(6.1)

Constraint (i) always binds at a solution to this problem; otherwise, the contractor can reduce the growers’ payment while still inducing the grower to accept the contract.

Equality in constraint (i) implies \( \beta_i = [n_i \gamma_i^2 \lambda_i^2 c^2 / 2 - \alpha n_i^2 / 2 - \phi \gamma_i^2 n_i^2 \sigma_0^2 / 2 - R(\lambda_i)]/(n_i c) \)

so that upon substitution the problem reduces to

\[
\begin{align*}
\text{Max} & \quad L = n_i(1 - \gamma_i)\gamma_i^2 \lambda_i^2 c^2 + n_i \gamma_i^2 \lambda_i^2 c^2/2 - \alpha n_i^2/2 - \phi \gamma_i^2 n_i^2 \sigma_0^2/2 - R(\lambda_i) \\
& \quad \text{except that a contract would not be offered if } \beta_i \leq 0. \text{ This problem has first-order conditions}
\end{align*}
\]

\[
L_n = (1 - \gamma_i)\gamma_i^2 \lambda_i^2 c^2 + \gamma_i^2 \lambda_i^2 c^2/2 - \alpha n_i - \phi \gamma_i^2 n_i \sigma_0^2 = 0
\]

= \((1 - \gamma_i/2)\gamma_i^2 \lambda_i^2 c^2 - \alpha n_i - \phi \gamma_i^2 n_i \sigma_0^2 = 0\)
\[ L_{\gamma} = (1 - 2\gamma_i) n_i \lambda_i^2 c^2 + n_i \gamma_i \lambda_i^2 c^2 - \phi_i \gamma_i n_i^2 \sigma_0^2 \]
\[ = (1 - \gamma_i) n_i \lambda_i^2 c^2 - \phi_i \gamma_i n_i^2 \sigma_0^2 = 0 \]

assuming that the reservation utility does not depend on either \( n_i \) or \( \gamma_i \). This is plausible since these are contract parameters that are inapplicable if the grower chooses not to contract.

Eliminating the solution with \( n_i = 0 \), these conditions can be rearranged as

\[ n_i = \frac{(1 - \gamma_i/2) \gamma_i \lambda_i^2 c^2}{\alpha + \phi_i \gamma_i^2 \sigma_0^2} \quad (6.2) \]

\[ \gamma_i = \frac{\lambda_i^2 c^2}{\lambda_i^2 c^2 + \phi_i n_i \sigma_0^2}. \quad (6.3) \]

Equation (6.3) verifies that \( \gamma_i \leq 1 \) for an interior solution with \( n_i > 0 \) while the former verifies that \( n_i > 0 \) if \( \gamma_i \leq 1 \). From (6.1), \( (n_i \gamma_i^2/2)(\lambda_i^2 c^2 - \phi_i n_i \sigma_0^2) \geq R(\lambda_i) + n_i \beta c + \alpha n_i^2/2 \),

which implies \( \lambda_i^2 c^2 > \phi_i n_i \sigma_0^2 \). Substituting this in (6.3) implies that \( \gamma_i > 0.5 \), which together with \( \gamma_i \leq 1 \) (from above) bounds \( \gamma_i \) to the interval \( 0.5 < \gamma_i \leq 1 \). Within this bound, \( \partial n_i / \partial \gamma_i > 0 \) in (6.2) and \( \partial \gamma_i / \partial n_i < 0 \) in (6.3) so the solution is unique.

Second-order conditions require

\[ L_{mn} = -\alpha - \phi_i \gamma_i^2 \sigma_0^2 < 0, \quad (6.4) \]

\[ L_{\gamma \gamma} = -n_i \lambda_i^2 c^2 - \phi_i n_i \sigma_0^2 < 0, \quad (6.5) \]

\[ D = L_{mn} L_{\gamma \gamma} - L_{m \gamma}^2 > 0 \quad (6.6) \]

where

\[ L_{m \gamma} = L_{\gamma m} = (1 - \gamma_i) \lambda_i^2 c^2 - 2\gamma_i \phi_i n_i \sigma_0^2 = -\gamma_i \phi_i n_i \sigma_0^2 < 0 \quad (6.7) \]
where the last equality in (6.7) follows upon substituting from (6.3) or the second first-order condition. The first conditions in (6.4) and (6.5) obviously hold. To examine condition (6.6), substitution of (6.4), (6.5), and (6.7) yield
\[
D = (\alpha + \phi \gamma_i^2 \sigma_0^2)(n_i \lambda_i^2 c^2 + \phi n_i^2 \sigma_0^2) - (-\gamma_i \phi n_i \sigma_0^2)^2
\]
\[
= \alpha(n_i \lambda_i^2 c^2 + \phi n_i^2 \sigma_0^2) + n_i \lambda_i^2 c^2 \phi \gamma_i^2 \sigma_0^2 > 0.
\]
Thus, second-order conditions hold unambiguously.

However, equations (6.2) and (6.3) do not yield explicit solutions for either \(n_i\) or \(\gamma_i\). To understand the marginal effects of grower ability or risk aversion on these contract parameters requires comparative static analysis. Marginal effects can be found from
\[
L_{\phi_i} d n_i + L_{\gamma_i} d \gamma_i = -L_{\phi_i} d \phi_i - L_{\lambda_i} d \lambda_i
\]
and
\[
L_{\phi_i} d n_i + L_{\gamma_i} d \gamma_i = -L_{\phi_i} d \phi_i - L_{\lambda_i} d \lambda_i,
\]
which implies
\[
\begin{bmatrix}
\frac{d n_i}{d \phi_i} \\
\frac{d \gamma_i}{d \phi_i}
\end{bmatrix} = -\begin{bmatrix}
L_{nn} & L_{ny} \\
L_{yn} & L_{yy}
\end{bmatrix}^{-1} \begin{bmatrix}
L_{\phi n} \\
L_{\phi y}
\end{bmatrix} = -\frac{1}{D} \begin{bmatrix}
L_{\gamma n} & -L_{\gamma y} \\
-L_{\gamma n} & L_{nn}
\end{bmatrix} \begin{bmatrix}
L_{\phi n} \\
L_{\phi y}
\end{bmatrix}
\]
if \(d \lambda_i = 0\), and
\[
\begin{bmatrix}
\frac{d n_i}{d \lambda_i} \\
\frac{d \gamma_i}{d \lambda_i}
\end{bmatrix} = -\begin{bmatrix}
L_{nn} & L_{ny} \\
L_{yn} & L_{yy}
\end{bmatrix}^{-1} \begin{bmatrix}
L_{n\lambda} \\
L_{\gamma \lambda}
\end{bmatrix} = -\frac{1}{D} \begin{bmatrix}
L_{\gamma n} & -L_{\gamma y} \\
-L_{\gamma n} & L_{nn}
\end{bmatrix} \begin{bmatrix}
L_{n\lambda} \\
L_{\gamma \lambda}
\end{bmatrix}
\]
if \(d \phi_i = 0\). Thus,
\[
\frac{d n_i}{d \phi_i} = -(1/D)(L_{\gamma n} L_{\phi y} - L_{\gamma y} L_{\phi n}) \quad \text{(6.8)}
\]
\[
\frac{d \gamma_i}{d \phi_i} = -(1/D)(L_{nn} L_{\phi y} - L_{yn} L_{\phi n}) \quad \text{(6.9)}
\]
\[
\frac{d n_i}{d \lambda_i} = -(1/D)(L_{\gamma n} L_{n\lambda} - L_{\gamma y} L_{\gamma \lambda}) \quad \text{(6.10)}
\]
where

\[ L_{n\phi} = -\gamma_i^2 n_i \sigma_\phi^2 < 0 \]  
\[ L_{n\lambda} = (2 - \gamma_i) \gamma_i \lambda c^2 > 0 \]  
\[ L_{n\psi} = -\gamma_i n_i^2 \sigma_\phi^2 < 0 \]  
\[ L_{n\delta} = 2(1 - \gamma_i) n_i \lambda c^2 > 0. \]

Under risk neutrality \((\phi_i = 0)\), equation (6.3) implies that \(\gamma_i = 1\) so that \(L_{n\psi} = 0\). In this case, the second right hand terms vanish in (6.8)-(6.11) so that (6.4), (6.5), and (6.12)-(6.15) imply \(dn_i/d\phi_i < 0, \ d\gamma_i/d\phi_i < 0, \ dn_i/d\lambda_i > 0, \) and \(d\gamma_i/d\lambda_i > 0\). Relaxing risk neutrality, the second right hand terms would add to these effects if \(L_{n\psi} > 0\). However, \(L_{n\psi} < 0\) from (6.7) so each of these effects is attenuated under risk aversion.

To show that these effects are not reversed under risk aversion, substitute equations (6.5), (6.7), (6.12) and (6.14) into (6.8) reveals that

\[ dn_i/d\phi_i = -(1/D)((n_i \lambda_i^2 c^2 + \phi_i n_i^2 \sigma_\phi^2)(\gamma_i^2 n_i \sigma_\phi^2) - \gamma_i \phi_i n_i \sigma_\phi^2 (\gamma_i n_i^2 \sigma_\phi^2)) \]  
\[ = -(1/D)n_i \lambda_i^2 c^2 \gamma_i^2 n_i \sigma_\phi^2 < 0. \]

Thus, a contractor will unambiguously offer a contract with a smaller facility size to a more risk averse grower. This is a plausible action undertaken to balance the efficiency loss associated with risk aversion among growers.

Substituting equations (6.4), (6.7), (6.12), and (6.14) into (6.9) yields

\[ d\gamma_i/d\psi_i = -(1/D)[(\alpha + \phi_i \gamma_i^2 \sigma_\phi^2)\gamma_i n_i^2 \sigma_\phi^2 - \phi_i \gamma_i n_i \sigma_\phi^2 \gamma_i^2 n_i \sigma_\phi^2] \]  
\[ = -(1/D)\alpha \gamma_i n_i^2 \sigma_\phi^2 < 0. \]
Thus, a contractor will offer a lower incentive to grower with higher risk aversion. This is plausible because the incentive does not yield as much payoff in this case.

Substituting equations (6.5), (6.7), (6.13), and (6.15) into (6.10) verifies that

\[
\frac{dn}{d\lambda} = \frac{1}{D}((n\lambda^2c^2 + \phi n^2\sigma_o^2)((2 - \gamma_i)\gamma_c \lambda^2c^2) - (\gamma_i\phi ni\sigma_o^2)(2(1 - \gamma_i)n\lambda^2c^2)) \\
= \frac{1}{D}((2 - \gamma_i)n\lambda^2c^2\gamma_c \lambda^2c^2 + (2 - \gamma_i)\phi n^2\sigma_o^2\gamma_c \lambda^2c^2 - 2(1 - \gamma_i)\gamma_i\phi n_i\gamma_o^2n\lambda^2c^2) \\
= \frac{1}{D}((2 - \gamma_i)n\lambda^2c^2\gamma_c \lambda^2c^2 + \gamma_i\phi n^2\sigma_o^2(\gamma_i\lambda^2c^2)) > 0.
\]

Thus, a contractor will offer a contract for a larger facility size to a grower with greater ability. This is plausible because greater ability will tend to make up for the higher risk inefficiency incurred with a larger facility size.

Substituting equations (6.4), (6.7), (6.13), and (6.15) into (6.11) shows that

\[
\frac{d\gamma_c}{d\lambda} = \frac{1}{D}[(2(1 - \gamma_i)n\lambda^2c^2)(\alpha + \phi\gamma_i^2\sigma_o^2) - ((2 - \gamma_i)\gamma_i\lambda^2c^2)(\gamma_i\phi ni\sigma_o^2)] \\
= \frac{1}{D}\lambda^2c^2[2(1 - \gamma_i)(\alpha n_i + \phi\gamma_i^2n\sigma_o^2) - (2 - \gamma_i)\gamma_i^2\phi n_i\sigma_o^2] \\
= \frac{1}{D}\lambda^2c^2[2(1 - \gamma_i)(1 - \gamma_i/2)\gamma_i\lambda^2c^2 - (2 - \gamma_i)(1 - \gamma_i)\gamma_i\lambda^2c^2] \\
= \frac{1}{D}\gamma_i\lambda^2c^2(1 - \gamma_i)(2 - \gamma_i) - (2 - \gamma_i) = 0
\]

where the third equality follows by substituting \( \alpha n_i + \phi\gamma_i^2n\sigma_o^2 = (1 - \gamma_i/2)\gamma_i\lambda^2c^2 \) and \( \gamma_i\phi ni\sigma_o^2 = (1 - \gamma_i)\lambda^2c^2 \), which follow from (6.2) and (6.3), respectively. Thus, the incentive offered by the contractor does not vary with ability. This is plausible because, holding other factors constant, the contractor’s payoff is constant at the margin when productivity is proportional to ability.
6.4 The Case of Imperfect and Asymmetric Information

The most plausible information structure is where none of the parties know ability exactly but the growers have better information. This section examines the implications of this information structure regarding the growers’ gains from contracting.

6.4.1 The imperfect information set up

Suppose ability $\lambda_i$ can follow either one of 2 possible distributions rather than a single distribution. Suppose ability distribution 1 (denoted by $P$) has probability $p_1$ of ability $\lambda_{11}$ and probability $p_2$ of ability $\lambda_{12}$ ($p_2 = 1 - p_1$) while distribution 2 (denoted by $Q$) has probability $q_1$ of ability $\lambda_{21}$ and probability $q_2$ of ability $\lambda_{22}$ ($q_2 = 1 - q_1$) where $\lambda_{ij} > 0$ for $i,j = 1,2$. Also assume that $\lambda_1 < \lambda_2$ where $\lambda_1 = p_1\lambda_{11} + p_2\lambda_{12}$ and $\lambda_2 = q_1\lambda_{21} + q_2\lambda_{22}$.

Consider the information structure where the growers know the actual distribution at the time the contract is signed but the contractor has a nondegenerate prior defined over these 2 distributions. Thus, the grower’s information is better but imperfect, and the contractor is aware of this fact.

Let $r_1$ and $r_2$ represent the contractor’s subjective probability that the grower is from distributions $P$ and $Q$, respectively. This information structure was examined for a simpler stylized problem in section 3.5 of chapter 3. Since growers do not know their exact abilities, the contractor cannot induce them to reveal their exact abilities. But the contractor can induce them to choose a contract intended for their distribution. As a result, the contractor induces only one effort for each distribution, say, $e_1$ for $P$ and $e_2$ for $Q$ distribution growers. This is done by choosing payment parameters $\beta_i$, $\gamma_i$ and facility
size \( n_1 \) for \( P \) and payment parameters \( \beta_1, \gamma_2 \) and facility size \( n_2 \) for \( Q \) distribution growers.

For a grower from distribution \( P \), expected income is

\[
E(\pi_1) = n_1 \gamma_1 \lambda_1 e_1 c - n_1 \beta_1 c - n_1 e_1^2/2 - \alpha n_1^2/2 ,
\]

and certainty equivalent income is

\[
CE_1 = n_1 \gamma_1 \lambda_1 e_1 c - n_1 \beta_1 c - n_1 e_1^2/2 - \alpha n_1^2/2 - \phi_1 \gamma_1^2 n_1^2 \sigma_0^2/2 ,
\]

where \( \phi_1 = p_1 \phi_1 + p_2 \phi_2 \) is average absolute risk aversion among growers with the \( P \) distribution, \( \phi_1, \phi_{11}, \phi_{12} > 0 \). The respective expected and certainty equivalent incomes of a grower from distribution \( Q \) are

\[
E(\pi_2) = n_2 \gamma_2 \lambda_2 e_2 c - n_2 \beta_2 c - n_2 e_2^2/2 - \alpha n_1^2/2
\]

\[
CE_2 = n_2 \gamma_2 \lambda_2 e_2 c - n_2 \beta_2 c - n_2 e_2^2/2 - \alpha n_1^2/2 - \phi_2 \gamma_2^2 n_2^2 \sigma_0^2/2 ,
\]

where \( \phi_2 = q_1 \phi_{21} + q_2 \phi_{22} \) is average absolute risk aversion for a grower with the \( Q \) distribution, \( \phi_2, \phi_{21}, \phi_{22} > 0 \). An assumption that high ability growers have lower absolute risk aversion \( (\phi_1 > \phi_2) \) may be plausible, but a weaker condition, \( \phi_1 > r_2 \phi_2 \), suffices for most results here and is assumed henceforth.

The contractor solves the problem

\[
\max_{\beta_1, \gamma_1, \phi_1, n_1>0} E(S_i) = r_1[p_1 n_1 \lambda_1 e_1 c(1-\gamma_1) + p_2 n_1 \lambda_1 e_1 c(1-\gamma_1) + n_1 \beta_1 c] + r_2[q_1 n_2 \lambda_2 e_2 c(1-\gamma_2) + q_2 n_2 \lambda_2 e_2 c(1-\gamma_2) + n_2 \beta_2 c] \\
\text{s.t.} \sum_{i=1}^2 p_i(\gamma_1 \lambda_1 n_1 e_1 c - n_1 \beta_1 c - n_1 e_1^2/2 - \alpha n_1^2/2 - \phi_1 \gamma_1^2 n_1^2 \sigma_0^2/2) \geq ER_i)
\]

18 Although risk aversion is not regarded as changing because a grower realizes ex post that he has above or below average ability, the risk aversion levels of groups with above or below average productivity may differ a priori.
where \( ER_1 \) is the reservation utility of growers with the \( P \) distribution and \( ER_2 \) is the reservation utility of growers with the \( Q \) distribution. After collecting terms, this problem can be written as

\[
\text{Max } r_j[(1-\gamma_j)\lambda_j n_j e_j c + n_j \beta_j c] + r_j[(1-\gamma_j)\lambda_j n_j e_j c + n_j \beta_j c]
\]

s.t. (i) \[
\gamma_j \lambda_j n_j e_j c - n_j \beta_j c - n_j e_j^2/2 - \alpha n_j^2/2 - \phi_j \gamma_j n_j^2 \sigma_j^2/2 - ER_j \geq 0, \ j = 1,2,
\]

(ii) \[
\gamma_j \lambda_j n_j e_j c - n_j \beta_j c - n_j e_j^2/2 - \alpha n_j^2/2 - \phi_j \gamma_j n_j^2 \sigma_j^2/2 - ER_j \geq \gamma_j \lambda_j n_j e_j c - n_j \beta_j c - n_j e_j^2/2 - \alpha n_j^2/2 - \phi_j \gamma_j n_j^2 \sigma_j^2/2 - ER_j
\]

(iii) \[
e_j \in \arg \max \gamma_j \lambda_j n_j e_j c - n_j \beta_j c - n_j e_j^2/2 - \alpha n_j^2/2 - \phi_j \gamma_j n_j^2 \sigma_j^2/2, \ j = 1,2.
\]

Individual rationality constraint (ia) always binds at a solution to this problem for the similar reason described in last section. The self-selection-between-contracts constraint (ii) guarantees that the growers from ability distribution \( Q \) will not choose the contract offered for the growers of ability distribution \( P \). The self-selection–within-contracts constraints (iiia) and (iiib) require that growers with both distributions choose the effort that is profit maximizing for them.
However, unlike the standard contracting problems of chapter 3 that have zero reservation values, this problem can further simplify in two different ways. If the reservation utility for high-ability growers is not too much higher than for low-ability growers, then constraint (ii) will be binding as when reservation utilities are zero. But if the reservation utility for high-ability growers is much higher, then constraint (ib) is binding while constraint (ii) is slack. In this case, the contracting problems simplifies to the case where contracting is considered separately with each group because neither group will prefer to enter the contract offered to the other group. The case appears to be less interesting compared to the standard problem, and has straightforward comparative static results, so henceforth constraint (ib) is assumed to be nonbinding. Constraint (ii) holds as an equality whenever the growers of ability distribution \( Q \) face incentives to lie about their ability distribution under a first-best contract. In this case, constraint (ib) is redundant when both (ia) and (ii) are satisfied. Substituting the implications of these constraints, the problem simplifies to

\[
\text{Max}_{\beta, \gamma, \alpha, \phi, \sigma} r_1[(1-\gamma_1)\bar{\lambda}_1 n_1 e_1 c + n_1 \beta_1 c] + r_2[(1-\gamma_2)\bar{\lambda}_2 n_2 e_2 c + n_2 \beta_2 c]
\]

s.t. (ia) \( \gamma_1\bar{\lambda}_1 n_1 e_1 c - n_1 \beta_1 c - n_1 e_1^2/2 - \alpha n_1^2/2 - \phi_1 \gamma_1 n_1^2 \sigma_1^2/2 - ER_1 = 0 \)

(ii) \( \gamma_2\bar{\lambda}_2 n_2 e_2 c - n_2 \beta_2 c - n_2 e_2^2/2 - \alpha n_2^2/2 - \phi_2 \gamma_2 n_2^2 \sigma_2^2/2 = \gamma_1\bar{\lambda}_1 n_1 e_1 c - n_1 \beta_1 c - n_1 e_1^2/2 - \alpha n_1^2/2 - \phi_1 \gamma_1 n_1^2 \sigma_1^2/2 \)

(iii) \( e_j \in \arg \max_{e} \gamma_j\bar{\lambda}_j n_j e_j c - n_j \beta_j c - n_j e_j^2/2 - \alpha n_j^2/2 - \phi_j \gamma_j n_j^2 \sigma_j^2/2, j = 1,2. \)

Solving the constraints in (iii) yields \( \gamma_j\bar{\lambda}_j n_j c - n_j e_j = 0 \), or equivalently,

\( e_j = \gamma_j\bar{\lambda}_j c > 0, \) if \( \gamma_j > 0, j = 1,2. \) After substitution for \( e_j \), constraints (ia) and (ii) can be solved for \( \beta_1 \) and \( \beta_2 \) to obtain
\[
\beta_1 = (n_1 y_1^2 \lambda_1^2 c^2 / 2 - \alpha n_1^2 / 2 - \phi y_1^2 n_1^2 \sigma_\theta^2 / 2 - ER_i) / (n_1 c)
\]
\[
\beta_2 = [n_2 y_2^2 \lambda_2^2 c^2 / 2 - \alpha n_2^2 / 2 - \phi y_2^2 n_2^2 \sigma_\theta^2 / 2 \\
+ n_2 y_1^2 c^2 \lambda_1 (\lambda_1 - \lambda_2) - ER_i + (\phi_1 - \phi_2) y_1^2 n_1^2 \sigma_\theta^2 / 2] / (n_2 c)
\]
(if either \(\beta_j\) is negative, then a separating contract is not offered to induce participation from both groups). Substituting for \(e_1, e_2, \beta_1\) and \(\beta_2\) as implied by the constraints into the objective function obtains the unconstrained problem

\[
\text{Max } \sum_{n_1, n_2, \gamma_1, \gamma_2 > 0} n_1 y_1^2 \lambda_1^2 c^2 (1 - \gamma_1 / 2) - r_1 \alpha n_1^2 / 2 + r_2 n_2 y_2^2 \lambda_2^2 c^2 (1 - \gamma_2 / 2) - r_2 \alpha n_2^2 / 2 \\
- r_2 \phi_2 y_2^2 n_2^2 \sigma_\theta^2 / 2 - r_2 n_1 y_1^2 \lambda_1 c^2 (\lambda_2 - \lambda_1) - (\phi_1 - r_2 \phi_2) y_1^2 n_1^2 \sigma_\theta^2 / 2 - ER_i
\]

First-order conditions with respect to \(n_1, n_2, \gamma_1\) and \(\gamma_2\) yield

\[
L_n = n_1 y_1^2 \lambda_1^2 c^2 (1 - \gamma_1 / 2) - r_1 \alpha n_1^2 - r_2 y_2^2 \lambda_2 c^2 (\lambda_2 - \lambda_1) - (\phi_1 - r_2 \phi_2) y_1^2 n_1^2 \sigma_\theta^2 = 0
\]
\[
L_{n_2} = r_1 y_2^2 \lambda_2^2 c^2 (1 - \gamma_2 / 2) - r_2 \alpha n_2^2 - r_2 \phi_2 y_2^2 n_2^2 \sigma_\theta^2 = 0
\]
\[
L_{\gamma_1} = n_1 y_1^2 \lambda_1^2 c^2 (1 - \gamma_1) - 2 r_2 n_1 y_1^2 c^2 (\lambda_2 - \lambda_1) - (\phi_1 - r_2 \phi_2) y_1^2 n_1^2 \sigma_\theta^2 = 0
\]
\[
L_{\gamma_2} = r_2 n_2 y_2^2 \lambda_2^2 c^2 (1 - \gamma_2) - r_2 \phi_2 y_2^2 n_2^2 \sigma_\theta^2 = 0
\]

These conditions can be rearranged as

\[
n_1 = \frac{[r_1 \lambda_1 (1 - \gamma_1 / 2) - r_2 y_1^2 (\lambda_2 - \lambda_1)] y_1 \lambda_1 c^2}{r_1 \alpha + (\phi_1 - r_2 \phi_2) y_1^2 \lambda_1 c^2}
\]
\[
n_2 = \frac{y_2 \lambda_2^2 c^3 (1 - \gamma_2 / 2)}{\alpha + \phi_2 y_2^2 \lambda_2^2 c^2 \sigma_\theta}
\]
\[
\gamma_1 = \frac{r_1 \lambda_1^2 c^2}{r_1 \lambda_1^2 c^2 + 2 r_2 c^2 \lambda_1 (\lambda_2 - \lambda_1) + (\phi_1 - r_2 \phi_2) n_1 \sigma_\theta^2}
\]
\[
\gamma_2 = \frac{\lambda_2^2 c^2}{\lambda_2^2 c^2 + \phi_2 n_2 \sigma_\theta^2}
\]
Equation (6.17) yields an internal solution \( n_2 > 0 \) if \( \gamma_2 \leq 2 \) and (6.19) assures an internal solution with \( \gamma_2 \leq 1 \). Also, upon substitution of \( e_2 = \gamma_2 \overline{\lambda}_2 c \), constraint (ib) implies that \( \overline{\lambda}_2^2 c^2 > \overline{\phi}_2 n_2 \sigma_\theta^2 \) verifying that \( 0.5 \leq \gamma_2 \leq 1 \). To verify that (6.18) yields an internal solution for \( \gamma_1 \), using the assumptions \( \overline{\lambda}_2 > \overline{\lambda}_1 \) and \( \overline{\phi}_1 > r_2 \overline{\phi}_2 \) reveals that \( 0 \leq \gamma_1 \leq 1 \). The latter assumption also shows that the denominator of (6.16) is positive. Removing the term associated with risk aversion from the denominator of (6.18), which is positive, shows that \( \gamma_1 \leq r_1 \overline{\lambda}_1 / [r_1 \overline{\lambda}_1 + 2r_2 (\overline{\lambda}_2 - \overline{\lambda}_1)] \), which can be rearranged to show that
\[ r_1 \overline{\lambda}_1 (1 - \gamma_1) - 2r_2 \gamma_1 (\overline{\lambda}_2 - \overline{\lambda}_1) \geq 0. \]
Thus, the numerator of (6.16) must be positive because the term in brackets in the numerator of (6.16) exceeds this by \( r_1 \overline{\lambda}_1 \). Thus, (6.16)-(6.19) provide an internal solution with \( n_1, n_2, \gamma_1, \gamma_2 > 0 \) in the interesting case with \( \beta_1, \beta_2 > 0 \).

### 6.4.2 Comparative static effects

As in the first-best solution of Section 6.3, equations (6.16)-(6.19) do not yield explicit solutions. To derive the marginal effects of ability and risk aversion on contract parameters, note that

\[
L_{n_1} \frac{dn_1}{d\gamma_1} + L_{n_2} \frac{dn_2}{d\gamma_1} + L_{\gamma_1} \frac{d\gamma_1}{d\gamma_1} + L_{\gamma_2} \frac{d\gamma_2}{d\gamma_1} = -L_{n_1} \frac{d\overline{\phi}_1}{d\gamma_1} - L_{n_2} \frac{d\overline{\phi}_2}{d\gamma_1} - L_{\gamma_1} \frac{d\overline{\lambda}_1}{d\gamma_1} - L_{\gamma_2} \frac{d\overline{\lambda}_2}{d\gamma_1}
\]

\[
L_{n_1} \frac{dn_1}{d\gamma_2} + L_{n_2} \frac{dn_2}{d\gamma_2} + L_{\gamma_1} \frac{d\gamma_1}{d\gamma_2} + L_{\gamma_2} \frac{d\gamma_2}{d\gamma_2} = -L_{n_1} \frac{d\overline{\phi}_1}{d\gamma_2} - L_{n_2} \frac{d\overline{\phi}_2}{d\gamma_2} - L_{\gamma_1} \frac{d\overline{\lambda}_1}{d\gamma_2} - L_{\gamma_2} \frac{d\overline{\lambda}_2}{d\gamma_2}
\]

\[
L_{n_1} \frac{dn_1}{d\gamma_3} + L_{n_2} \frac{dn_2}{d\gamma_3} + L_{\gamma_1} \frac{d\gamma_1}{d\gamma_3} + L_{\gamma_2} \frac{d\gamma_2}{d\gamma_3} = -L_{n_1} \frac{d\overline{\phi}_1}{d\gamma_3} - L_{n_2} \frac{d\overline{\phi}_2}{d\gamma_3} - L_{\gamma_1} \frac{d\overline{\lambda}_1}{d\gamma_3} - L_{\gamma_2} \frac{d\overline{\lambda}_2}{d\gamma_3}
\]

where \( L_{n_1}^{(\gamma_1)} = L_{n_2}^{(\gamma_1)} = 0, \quad L_{\gamma_1}^{(\gamma_1)} = L_{\gamma_2}^{(\gamma_1)} = 0, \quad L_{n_1}^{(\gamma_2)} = L_{n_2}^{(\gamma_2)} = 0, \quad L_{\gamma_1}^{(\gamma_2)} = L_{\gamma_2}^{(\gamma_2)} = 0, \quad L_{n_1}^{(\gamma_3)} = 0, \quad L_{n_2}^{(\gamma_3)} = 0, \quad L_{\gamma_1}^{(\gamma_3)} = 0, \quad L_{\gamma_2}^{(\gamma_3)} = 0 \), and
\[ L_{n_1} = -\alpha r_1 - (\phi_1 - r_2 \phi_2) g_1^2 \sigma^2 < 0 \] (6.20)

\[ L_{n_2} = L_{n_2} = r_1 \bar{\lambda}_1 c^2 - r_1 \bar{\gamma}_1 \bar{\lambda}_1^2 c^2 - 2 r_2 \bar{\gamma}_1 \bar{\lambda}_1 c^2 (\bar{\lambda}_2 - \bar{\lambda}_1) - 2(\bar{\phi}_1 - r_2 \bar{\phi}_2) g_1 n_1 \sigma^2 \]

\[ = -(\bar{\phi}_1 - r_2 \bar{\phi}_2) g_1 n_1 \sigma^2 < 0 \text{ by (6.18)} \] (6.21)

\[ L_{n_3} = -\gamma_1^2 n_1 \sigma^2 < 0 \] (6.22)

\[ L_{n_4} = r_2 \gamma_1^2 n_1 \sigma^2 > 0 \] (6.23)

\[ L_{n_5} = \gamma_1 c^3 \left[ r_1 \bar{\lambda}_1 (2 - \gamma_1) - r_2 \gamma_1 (\bar{\lambda}_2 - 2 \bar{\lambda}_1) \right] > 0 \text{ by (6.16)}^{19} \] (6.24)

\[ L_{n_6} = -r_2 \gamma_1^2 \bar{\lambda}_1 c^2 < 0 \] (6.25)

\[ L_{n_7} = -\alpha r_2 - r_2 \bar{\phi}_2 \gamma_2^2 \sigma^2 < 0 \] (6.26)

\[ L_{n_8} = L_{n_8} = r_2 \bar{\lambda}_2 c^2 (1 - \gamma_2) - 2 r_2 \gamma_2 \bar{\phi}_2 n_2 \sigma^2 = -r_2 \gamma_2 \bar{\phi}_2 n_2 \sigma^2 < 0 \text{ by (6.19)} \] (6.27)

\[ L_{n_9} = -r_2 \gamma_2^2 n_2 \sigma^2 < 0 \] (6.28)

\[ L_{n_{10}} = r_2 \gamma_2 \bar{\lambda}_2 c^2 (2 - \gamma_2) > 0 \] (6.29)

\[ L_{n_{11}} = -r_1 n_1 \bar{\lambda}_1 c^2 - 2 r_2 n_1 c^2 \bar{\lambda}_1 (\bar{\lambda}_2 - \bar{\lambda}_1) - (\bar{\phi}_1 - r_2 \bar{\phi}_2) n_1 \sigma^2 \]

\[ = -r_1 n_1 \bar{\lambda}_1^2 c^2 / \gamma_1 < 0 \text{ by (6.18)} \] (6.30)

\[ L_{n_{12}} = -\gamma_1 n_1^2 \sigma^2 < 0 \] (6.31)

\[ L_{n_{13}} = r_2 \gamma_1^2 n_1 \sigma^2 > 0 \] (6.32)

\[ L_{n_{14}} = [r_1 \bar{\lambda}_1 (1 - \gamma_1) - r_2 \gamma_1 (\bar{\lambda}_2 - 2 \bar{\lambda}_1)] 2 n_1 c^2 \]

\[ = [r_2 \gamma_1 \bar{\lambda}_2 c^2 + (\bar{\phi}_1 - r_2 \bar{\phi}_2) n_1 \gamma_1^2 \sigma^2 / \bar{\lambda}_1] 2 n_1 > 0 \text{ by (6.18)} \] (6.33)

\[ L_{n_{15}} = -2 r_2 n_1 \gamma_1 \bar{\lambda}_1 c^2 < 0 \] (6.34)

\[ ^{19} \] The term in brackets here exceeds the term in brackets in the numerator of (6.16), which is positive, by the amount \( r_2 \gamma_1 \bar{\lambda}_2 \).
\[ L_{j,j} = -r_2 n_2 (\lambda_2 c^2 + \phi_2 n_2 \sigma_2^3) < 0 \] 
(6.35)

\[ L_{j,\phi} = -r_2 \gamma_2 n_2^2 \sigma_2^2 < 0 \] 
(6.36)

\[ L_{j,\delta} = 2r_2 n_2 \lambda_2 c^2 (1 - \gamma_2) > 0 . \] 
(6.37)

In matricial form, these results can be written as

\[
\begin{bmatrix}
    \frac{dn_1}{d\gamma_1} \\
    \frac{dn_2}{d\gamma_1} \\
    \frac{dn_1}{d\gamma_2} \\
    \frac{dn_2}{d\gamma_2}
\end{bmatrix}
\begin{bmatrix}
    L_{n,n} & 0 & L_{n,n} & 0 \\
    0 & L_{n,n} & 0 & L_{n,n} \\
    L_{n,n} & 0 & L_{n,n} & 0 \\
    0 & L_{n,n} & 0 & L_{n,n}
\end{bmatrix}
\begin{bmatrix}
    \frac{dn_1}{d\gamma_1} \\
    \frac{dn_2}{d\gamma_1} \\
    \frac{dn_1}{d\gamma_2} \\
    \frac{dn_2}{d\gamma_2}
\end{bmatrix}
= 
\begin{bmatrix}
    L_{n,\phi} & L_{n,\phi} & L_{n,\phi} & L_{n,\phi} \\
    0 & L_{n,\phi} & 0 & L_{n,\phi} \\
    L_{n,\phi} & L_{n,\phi} & L_{n,\phi} & L_{n,\phi} \\
    0 & L_{n,\phi} & 0 & L_{n,\phi}
\end{bmatrix}
\begin{bmatrix}
    \frac{d\phi_1}{d\gamma_1} \\
    \frac{d\phi_2}{d\gamma_1} \\
    \frac{d\phi_1}{d\gamma_2} \\
    \frac{d\phi_2}{d\gamma_2}
\end{bmatrix}
\]

where

\[
A^{-1} = \frac{1}{|A|} \begin{bmatrix}
    L_{\gamma,\gamma} Z_2 & 0 & -L_{n,\gamma} Z_2 & 0 \\
    0 & L_{\gamma,\gamma} Z_1 & 0 & -L_{n,\gamma} Z_1 \\
    -L_{n,\gamma} Z_2 & 0 & L_{n,\gamma} Z_2 & 0 \\
    0 & -L_{n,\gamma} Z_1 & 0 & L_{n,\gamma} Z_1
\end{bmatrix},
\]

and combining (6.20), (6.26), (6.30), (6.35), \( Z_j > 0, j = 1,2, \)
and \( |A| > 0. \) To verify that \( Z_j > 0, j = 1,2, \) which implies \( |A| > 0, \) combining (6.20), (6.21),
and (6.30) implies

\[
Z_1 = (\alpha_1 + (\phi_1 - r_2 \phi_2) \gamma_1 \sigma_1^3)(r_1 n_1 \lambda_1 c^2 + 2r_1 n_1 c^2 (\lambda_2 - \lambda_1) + (\phi_1 - r_2 \phi_2) n_1^2 \sigma_2^3) \\
- (\phi_1 - r_2 \phi_2) \gamma_1 n_1 \sigma_2^3 \\
= \alpha_1 (r_1 n_1 \lambda_1 c^2 + 2r_1 n_1 c^2 (\lambda_2 - \lambda_1) + (\phi_1 - r_2 \phi_2) n_1^2 \sigma_2^3) \\
+ (\phi_1 - r_2 \phi_2) \gamma_1 n_1 \sigma_2^3 (r_1 n_1 \lambda_1 c^2 + 2r_1 n_1 c^2 (\lambda_2 - \lambda_1)) > 0
\]

and combining (6.26), (6.27), and (6.35) implies
Thus, all second-order conditions hold unambiguously.

Where \( d\bar{\phi}_2 = 0 \), \( d\bar{\lambda}_1 = 0 \), and \( d\bar{\lambda}_2 = 0 \), these results imply

\[
\begin{bmatrix}
\frac{dn_1}{d\bar{\phi}_1} \\
\frac{dn_2}{d\bar{\phi}_1} \\
\frac{d\gamma_1}{d\bar{\phi}_1} \\
\frac{d\gamma_2}{d\bar{\phi}_1}
\end{bmatrix} = -A^{-1}
\begin{bmatrix}
L_{n,\bar{\rho}} \\
0 \\
L_{n,\gamma_1,\bar{\rho}} \\
0
\end{bmatrix}
= -\frac{1}{|A|}
\begin{bmatrix}
Z_2(L_{n,\gamma_1,\bar{\rho}}L_{n,\bar{\rho}} - L_{n,\gamma_1}L_{n,\bar{\rho}}) \\
0 \\
Z_2(L_{n,n,\gamma_1,\bar{\rho}}L_{n,\gamma_1}L_{n,\bar{\rho}} - L_{n,n,\gamma_1}L_{n,\bar{\rho}}) \\
0
\end{bmatrix}
\]

which upon substituting (6.21), (6.22), (6.30), and (6.31) yields

\[
dn_1/d\bar{\phi}_1 = -(1/|A|)Z_2(L_{n,\gamma_1,\bar{\rho}}L_{n,\bar{\rho}} - L_{n,\gamma_1}L_{n,\bar{\rho}})
\]

\[
= -(1/|A|)Z_2[(n,\bar{\gamma}_1,\bar{\lambda}_2c^2/\gamma_1)(\gamma_1^2n_1\sigma_\phi^2) - (\bar{\phi}_1 - r_2\bar{\phi}_2)\gamma_1n_1\sigma_\phi^2(\gamma_1n_1^2\sigma_\phi^2)]
\]

\[
= -(1/|A|)Z_2\gamma_1n_1^2\sigma_\phi^2[\gamma_1,\bar{\gamma}_1,\bar{\lambda}_2c^2 + 2r_2\gamma_1c^2\bar{\lambda}_2(\bar{\lambda}_2 - \bar{\lambda}_1)] < 0
\]

(6.38)

and upon substituting (6.20), (6.21), (6.22), and (6.31) yields

\[
d\gamma_1/d\bar{\phi}_1 = -(1/|A|)Z_2(L_{n,n,\gamma_1,\bar{\rho}}L_{n,\gamma_1}L_{n,\bar{\rho}})
\]

\[
= -(1/|A|)Z_2(\alpha r + (\bar{\phi}_1 - r_2\bar{\phi}_2)\gamma_1^2\sigma_\phi^2)\gamma_1n_1^2\sigma_\phi^2
\]

\[
- (\bar{\phi}_1 - r_2\bar{\phi}_2)\gamma_1n_1\sigma_\phi^2(\gamma_1n_1^2\sigma_\phi^2)
\]

\[
= -(1/|A|)Z_2\alpha r\gamma_1n_1^2\sigma_\phi^2 < 0
\]

where the latter equality of (6.38) follows by substitution of (6.18). Thus, as in the first-best case and with the same intuition, a contractor will unambiguously reduce the facility size and incentive offered to low-ability growers as their average risk aversion is higher because the risk premium and related inefficiency is higher.
Turning to the effects of high-ability grower characteristics on the contracts offered to low-ability growers, a similar approach using (6.21), (6.23), (6.30), and (6.32) finds

\[
dn_i/\phi_2 = -(1/|A|)Z_2(L_{\gamma_1\lambda_1\lambda_1} - L_{\gamma_2\lambda_1})
\]

\[
= (1/|A|)Z_2[(n_i\lambda_1^2c^2/\gamma_1)(r_2\gamma_1^2n_i\sigma_\theta^2) - ((\vec{\phi}_1 - r_2\phi_2)\gamma_1n_i\sigma_\theta^2)(r_2\gamma_1^2n_i\sigma_\theta^2)]
\]

\[
= (1/|A|)Z_2\gamma_1n_1^2\sigma_\theta^2[r_1\lambda_1^2c^2 + 2r_2\gamma_1c^2\lambda_1\lambda_2] > 0
\]

where the latter equality follows from (6.18). Thus, the facility size offered by the contractor to low-ability growers is greater when high-ability growers have higher average risk aversion. This occurs because the opportunity cost of offering a larger facility size to low-ability growers declines when the high ability growers have higher average risk aversion and thus more inefficiency associated with their risk premiums.

Use of (6.20), (6.21), (6.23), and (6.32) obtains

\[
d\gamma_1/\phi_2 = -(1/|A|)Z_2(L_{\gamma_1\lambda_1\lambda_1} - L_{\gamma_2\lambda_1})
\]

\[
= (1/|A|)Z_2[(\gamma_1\alpha + (\vec{\phi}_1 - r_2\phi_2)\gamma_1^2\sigma_\theta^2)(r_2\gamma_1^2n_i\sigma_\theta^2) - ((\vec{\phi}_1 - r_2\phi_2)\gamma_1n_i\sigma_\theta^2)(r_2\gamma_1^2n_i\sigma_\theta^2)]
\]

\[
= (1/|A|)Z_2\alpha r_1n_1^2\sigma_\theta^2 > 0.
\]

Thus, the incentive offered by the contractor to low-ability growers is greater when high-ability growers have higher average risk aversion. This occurs because the opportunity cost of offering a higher incentive to low-ability growers declines when the high ability growers have higher average risk aversion and thus more inefficiency associated with their risk premiums.

Likewise, (6.21), (6.25), (6.30), and (6.34) yields
\[
d n_i/d\bar{\lambda}_2 = -(1/|A|)Z_2(L_{x_{i1}}L_{x_{i2}} - L_{x_{i1}}L_{x_{i2}}) \\
= -(1/|A|)Z_2[(n_1r_1\bar{\lambda}_1^2c^2/\gamma_1)(r_2\gamma_1^2\bar{\lambda}_1c^2) - ((\bar{\phi}_1 - r_2\bar{\phi}_2)\gamma_1\sigma_\phi^2(2r_2n_1\gamma_1\bar{\lambda}_1c^2))] \\
= -(1/|A|)Z_2[r_2\gamma_1n_1\bar{\lambda}_1c^2(\gamma_1\bar{\lambda}_1^2c^2 - 2\gamma_1\sigma_\phi^2(\bar{\phi}_1 - r_2\bar{\phi}_2))]
\]

which implies that \( d n_i/d\bar{\lambda}_2 > (=)(<) 0 \) as \( r_1\bar{\lambda}_1^2c^2 > (=)(<)2\gamma_1n_1\sigma_\phi^2(\bar{\phi}_1 - r_2\bar{\phi}_2) \). Thus, the marginal effect of average ability of high-ability growers on the facility size offered by the contractor to low-ability growers is ambiguous. If the average ability of low-ability growers is high (low) relative to the difference in average risk aversion between the two groups, then the marginal effect of the ability of high-ability growers on the facility size offered to low-ability growers is positive (negative). Intuitively, the contractor is faced with balancing the inefficiency associated with risk aversion and the efficiency associated with productivity between the two groups. If the ability of high-ability growers increases and the risk aversion of high ability growers is low, then expected surplus increases sufficiently relative to the inefficiency of risk that some additional risk from low-ability growers can be optimally absorbed.

Also, (6.20), (6.21), (6.25), and (6.34) shows that

\[
d\gamma_1/d\bar{\lambda}_2 = -(1/|A|)Z_2(L_{x_{i1}}L_{x_{i2}} - L_{x_{i1}}L_{x_{i2}}) \\
= -(1/|A|)Z_2[(\alpha r_1 + (\bar{\phi}_1 - r_2\bar{\phi}_2)\gamma_1^2\bar{\lambda}_1^2c^2)(2r_2n_1\gamma_1\bar{\lambda}_1c^2) \\
- ((\bar{\phi}_1 - r_2\bar{\phi}_2)\gamma_1n_1\sigma_\phi^2(r_2\gamma_1\bar{\lambda}_1c^2))]
= -(1/|A|)Z_2r_2\gamma_1n_1\bar{\lambda}_1c^2[2\alpha r_1 + (\bar{\phi}_1 - r_2\bar{\phi}_2)r_2\gamma_1^2\sigma_\phi^2] < 0.
\]

Thus, the incentive offered by the contractor to low-ability growers is reduced when high-ability growers have greater average ability. This occurs because the opportunity cost of offering incentive to low-ability growers increases when the high-ability growers have greater average ability.
As far as the effects of characteristics of the low-ability group on contracts offered to the high-ability group, the story is quite different. Neither the average ability nor the average risk aversion of low-ability growers has any effect on either the facility size or incentive offered to high-ability growers by the contractor, $dn_z/d\overline{\lambda}_1 = 0$, $d\gamma_z/d\overline{\phi}_1 = 0$, and $d\gamma_z/d\overline{\phi}_1 = 0$. Intuitively, this appears to be explained by the residual position held by low-ability growers in the contractor’s problem.

Turning to the effects of characteristics of high-ability growers on the contracts offered to them, using (6.27), (6.28), (6.35), and (6.36) yields

\[
dn_z/d\overline{\phi}_2 = -(1/|A|)Z_1(L_{\overline{\gamma}_z\overline{\lambda}_1} - L_{\overline{\gamma}_z\overline{\phi}_1}) \nonumber \\
= -(1/|A|)Z_1[(r_{\overline{\gamma}_z}\overline{\lambda}_1^2 c^2 + \overline{\phi}_1 n_2 \sigma_\theta^2)(r_{\overline{\gamma}_z} n_2 \sigma_\theta^2) - (r_{\overline{\gamma}_z}\overline{\phi}_1 n_2 \sigma_\theta^2)(r_{\overline{\gamma}_z} n_2 \sigma_\theta^2)] 
= -(1/|A|)Z_1 r_{\overline{\gamma}_z} n_2 \gamma_z^2 \overline{\lambda}_1^2 c^2 \sigma_\theta^2 < 0.
\]

Thus, as in the first-best case and with the same intuition, a contractor will reduce the facility size offered to high ability growers as they have higher average risk aversion because the risk premium and related inefficiency is higher.

Also, using (6.26), (6.27), (6.28), and (6.36) obtains

\[
d\gamma_z/d\overline{\phi}_2 = -(1/|A|)Z_1(L_{n_2\overline{\gamma}_z} - L_{n_2\overline{\phi}_1}) \nonumber \\
= -(1/|A|)Z_1[(\alpha_{\overline{\gamma}_z} + r_{\overline{\gamma}_z}\overline{\phi}_2 \gamma_z^2 \sigma_\theta^2)(r_{\overline{\gamma}_z} n_2 \sigma_\theta^2) - (r_{\overline{\gamma}_z}\overline{\phi}_2 n_2 \sigma_\theta^2)(r_{\overline{\gamma}_z} n_2 \sigma_\theta^2)] 
= -(1/|A|)Z_1 \alpha_{\overline{\gamma}_z} r_{\overline{\gamma}_z} n_2 \gamma_z^2 \sigma_\theta^2 < 0.
\]

Thus, as in the first-best case and with the same intuition, a contractor will unambiguously reduce the incentive offered to high-ability growers as their average risk aversion is higher because the risk premium and related inefficiency is higher.

Use of (6.27), (6.29), (6.35), and (6.37) shows that
\[
\frac{dn_z}{d\bar{\alpha}_2} = -(1/|A|)Z_1(L_{\gamma z} - L_{n_z} - L_{n_2} - L_{n_z})
\]
\[
= (1/|A|)Z_1\left((r_{n_z}(\bar{\alpha}_2^2 c + \bar{\phi}_2 n_2 \sigma_\phi^2))(r_{\gamma z}^2 \bar{\alpha}_2^2 c^2 (2 - \gamma_z))
\right.
\]
\[
-(r_{\gamma z} \bar{\phi}_2 n_2 \sigma_\phi^2)(2r_{n_2} \bar{\alpha}_2 c^2 (1 - \gamma_z))\]
\[
= (1/|A|)Z_1 r_{n_z}^2 \gamma_z n_2 c^2 \bar{\alpha}_2^2 c^2 (2 - \gamma_z) + r_{n_2} \bar{\phi}_2 \sigma_\phi^2 > 0.
\]

Thus, much as in the first-best case and with the same intuition, a contractor will offer a contract for a larger facility size to high-ability growers as their average ability is greater because the risk inefficiency of a larger facility size is thereby reduced.

Also, (6.26), (6.27), (6.29), and (6.37) yields
\[
\frac{d\gamma_2}{d\bar{\alpha}_2} = -(1/|A|)Z_1(L_{n_2} - L_{n_z} - L_{n_2} - L_{n_z})
\]
\[
= (1/|A|)Z_1\left((r_{n_2} + r_{\bar{\phi}_2 \gamma_2^2 \sigma_\phi^2})(2r_{n_2} \bar{\alpha}_2 c^2 (1 - \gamma_z))
\right.
\]
\[
-(r_{\gamma z} \bar{\phi}_2 n_2 \sigma_\phi^2)(2r_{n_2} \bar{\alpha}_2 c^2 (2 - \gamma_z))\]

Substituting \( r_{n_2} + r_{\bar{\phi}_2 \gamma_2^2 \sigma_\phi^2} = r_{\gamma z} \bar{\alpha}_2 c^2 (1 - \gamma_z) / n_z \), which follows from (6.17), into the first term in brackets, and \( r_{\gamma z} \bar{\phi}_2 n_2 \sigma_\phi^2 = \bar{\alpha}_2 c^2 (1 - \gamma_z) \), which follows from (6.19), into the second term in brackets then obtains
\[
\frac{d\gamma_2}{d\bar{\alpha}_2} = (1/|A|)Z_1 r_{n_2}^2 \gamma z \bar{\alpha}_2 c^2 \bar{\alpha}_2 c^2 (1 - \gamma_z)(2 - \gamma_z)\]
\[
= 0.
\]

Thus, much as in the first-best case and by the same intuition, the incentive offered by the contractor to growers of high ability does not vary with their average ability, i.e., the contractor’s payoff is constant at the margin when productivity is proportional to ability.

The most interesting results in this analysis have to do with the effects of the average ability of low-ability growers on the contracts offered to them. Using (6.21), (6.24), (6.30), and (6.33), after substituting from (6.16) and considerable manipulation finds that
\[
\frac{dn_i/d\lambda_i}{n} = \frac{1}{|A|} Z_2 \left[ (L_{x_i} L_{n_i} - L_{n_i} L_{x_i}) \right] \\
= \frac{1}{|A|} Z_2 \left\{ n_i r_i \lambda_i c^2/\gamma_i \right\} \left[ r_i \lambda_i (2 - \gamma_i) - r_2 \gamma_i (\lambda_i - 2 \lambda_i) \right] \\
- \left( (\bar{\phi}_i - r_2) n_i c^2 \gamma_i n_i \sigma_\phi^2 \right) \left[ r_i \lambda_i (1 - \gamma_i) - r_2 \gamma_i (\lambda_i - 2 \lambda_i) \right] \\
= \frac{1}{|A|} Z_2 \left\{ n_i r_i \lambda_i c^2 \left[ r_i \lambda_i (2 - \gamma_i) + r_2 \gamma_i (2\lambda_i - \lambda_i) \right] \\
- 2 \gamma_i n_i \sigma_\phi^2 \left( \bar{\phi}_i - r_2 \right) \left[ r_i \lambda_i (1 - \gamma_i) + r_2 \gamma_i (2\lambda_i - \lambda_i) \right] \right\}.
\]

This result reveals that the average ability of low-ability growers has an ambiguous effect on the facility size in contracts offered to them by contractors. The terms in brackets in both the first and second right hand terms are likely positive (the case where low-ability growers have less than half the productivity of high-ability growers is unlikely). But these two terms are of opposite signs. If the likelihood of low-ability growers is high compared to the likelihood of high ability growers (\(r_1\) is large) and costs or average ability of low-ability growers is large relative to the risk \(\sigma_\phi^2\) and the difference in risk among groups, then the first term dominates so that \(dn_i/d\lambda_i > 0\), i.e., an increase in the average ability of low ability growers causes the contractor to offer them contracts with larger facility size. On the other hand, if risk \(\sigma_\phi^2\) and the difference in risk aversion among the two groups of growers dominates, then \(dn_i/d\lambda_i < 0\) so that greater average ability among low-ability growers causes contractors to offer them contracts with smaller facility size. This is a disturbing result in that low-ability growers appear to be penalized for improving their ability, for example, by investment.

Finally, use of (6.20), (6.21), (6.24), and (6.33) yields
\[
\frac{d\gamma_1}{d\lambda_1} = -(1/|A|)Z_2(L_{n_{\gamma_1}}L_{\gamma_1\gamma_1} - L_{n_{\gamma_1}}\gamma_1)
\]
\[
= (1/|A|)Z_2 \left( (\alpha r_1 + (\phi_1 - r_2 \phi_2)\gamma_1^2\sigma_\phi^2)(\gamma_1 r_1(1 - \gamma_1) - r_2\gamma_1(2\lambda_1 - 2\lambda_1)) \right)
\]
\[
= (1/|A|)Z_2 n_2c^2 \left( 2\alpha r_1 \gamma_1 r_1(2\lambda_1 - 2\lambda_1) \right)
\]

This result reveals that the average ability of low-ability growers has an ambiguous effect on the incentive offered to them. If the optimal incentive for low-ability growers \(\gamma_1\) is small and the annualized investment per animal \(\alpha\) is high, then the first right hand term in braces is positive and dominates. This effect also tends to dominate if growers are more likely to be high-ability growers than low-ability growers, \(r_1 < r_2\). On the other hand, if growers are more likely to be low-ability growers than high-ability growers, then \(r_1 > r_2\) so that the term in brackets in the last line is negative. Thus, if the annualized investment \(\alpha\) is low, then this term dominates so that the contractor responds to an increase in the ability of low-ability growers by lowering their incentive. This is the most interesting and disturbing result in this analysis. The fact that, say, investment in ability may reduces the incentive for low-ability growers implies that they may actually have a disincentive to invest.

Incidentally, because \(\lambda_1 = p_1\lambda_{11} + p_2\lambda_{12}, \lambda_2 = q_1\lambda_{21} + q_2\lambda_{22}, \phi_1 = p_1\phi_{11} + p_2\phi_{12}, \phi_2 = q_1\phi_{21} + q_2\phi_{22}\), comparative static results with respect to each of the individual abilities and risk parameters follow by the chain rule and yield identical qualitative results as those obtained for the effects of average ability and average risk aversion in each case.
6.4.3 Gains from contracting for low-ability growers

Now consider whether some growers are left with negative gains from contracting on average. Gains from contracting can be measured either ex ante or ex post. The contracting problem assures that all growers receive their ex ante reservation utilities in expectations. However, what ultimately matters is ex post gains from contracting. For low-ability growers, the ex ante gains compare expected gains to the reservation utility $ER_1$, which is anticipated before the grower’s actual ability is realized. The ex post gains measure the actual realized gains from contracting, which are determined by whether the grower’s realized ability, $\lambda_{11}$ or $\lambda_{12}$. This ex post realization of ability may also change the grower’s assessment of his reservation utility, e.g., his estimate of how much he could earn as an independent grower. For this purpose, denote the ex post reservation utility by $R_{1j}$ in the case where realized ability is $\lambda_{1j}, j = 1,2$.

The gain for a grower with ability $\lambda_{1j}$, after substituting $e_i = \gamma_i x_i c$, is thus

$$G_{1j} = CE_{1j} - R_{1j}$$

$$= \gamma_i^2 \lambda_{1j} x_i c^2 - n_i \beta_i c - n_i \gamma_i^2 \lambda_{1j}^2 c^2/2 - \alpha n_i^2/2 - \phi_i^2 n_i^2 \sigma_i^2/2 - R_{1j}, j = 1,2.$$ 

Substituting for $\beta_1$ as implied by constraint (ia) of section 6.4.1 thus obtains

$$G_{1j} = \gamma_i^2 n_i \lambda_{1j} c^2 (\lambda_{1j} - \lambda_i) - (\phi_{1j} - \phi_i) \gamma_i^2 n_i^2 \sigma_i^2/2 - R_{1j} + ER_1$$

$$= \gamma_i^2 n_i \lambda_{1j} c^2 (\lambda_{1j} - \lambda_i) - (\phi_{1j} - \phi_i) \gamma_i^2 n_i^2 \sigma_i^2/2 - \Delta_{1j} (\lambda_{1j} - \lambda_i)$$

$$= (\gamma_i^2 n_i \lambda_{1j} c^2 - \Delta_{1j}) (\lambda_{1j} - \lambda_i) - (\phi_{1j} - \phi_i) \gamma_i^2 n_i^2 \sigma_i^2/2, j = 1,2,$$

where $\Delta_{1j} = (R_{1j} - ER_1)/(\lambda_{1j} - \lambda_i), j = 1,2$, is the average increment in reservation utility per unit of ability that applies in evaluating ex post reservation utility for low-ability growers. For example, suppose the reservation utility representing the foregone benefit
from operating as an independent hog grower is proportional to ability. Then $\Delta_{1j}$
measures the maringal effect of ability on the ex post assessment of the reservation
utility. Thus, $\Delta_{1j}$ compares to $\gamma_{1}^{2}n_{1}\bar{\lambda}_{1}c^{2}$, which is the marginal increment in returns per
unit of ability under contracting.

The net effect of the change in ability on rent is the difference in these two,
$\gamma_{1}^{2}n_{1}\bar{\lambda}_{1}c^{2} - \Delta_{1j}$, multiplied by the deviation in realized ability from expected ability,$$
\lambda_{1j} - \bar{\lambda}_{1}. \text{ The terms } \gamma_{1}^{2}n_{j}\bar{\lambda}_{j}c^{2} - \Delta_{1j}, j = 1, 2, \text{ can plausibly be either positive or negative}
depending on whether the marginal benefits of ability are greater as a contract grower or
an independent grower (assuming the reservation utility represents operation as an
independent grower). The term $\gamma_{1}^{2}n_{1}\bar{\lambda}_{1}c^{2} - \Delta_{1j}$ represents the related difference in the
risk premium of low-ability growers with lower than average realized ability from the
average risk premium for low-ability growers. (If risk aversion is assumed to be the same
across growers drawn from the same distribution, then this term vanishes.) Thus, growers
with ability $\lambda_{1j}$ receive a positive gain on average ($G_{1j} > 0$) if

$$
\gamma_{1}^{2}n_{j}\bar{\lambda}_{j}c^{2} - \Delta_{1j} \geq 0, \quad j = 1, 2.
$$

Obviously, the term $\gamma_{1}^{2}n_{1}\bar{\lambda}_{1}c^{2} - \Delta_{1j}$ affects both the gains from contracting and the
threshold ability level where the grower just breaks even from contracting.

Suppose $\lambda_{11} < \bar{\lambda}_{1} < \lambda_{12}$ so that a low-ability grower who realizes less than expected
ability is represented by the case with $j = 1$. If $\gamma_{1}^{2}n_{1}\bar{\lambda}_{1}c^{2} - \Delta_{11} > 0$, a low-ability grower
who realizes less than expected ability ($\lambda_{11} < \bar{\lambda}_{1}$) receives a positive gain on average if
the degree risk of aversion and ability are positively correlated and the variation in risk
aversion is large, i.e., the term \((\phi_1 - \bar{\phi})\gamma^2 n^2 \sigma^2 / [2(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1)]\) is absolutely large. If either of these conditions do not hold, he receives a negative gain on average if \(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1 > 0\). If \(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1 < 0\), then a grower with ability \(\lambda_{11}\) receives a positive gain on average if the degree of risk aversion and ability are positively correlated or the variation in risk aversion represented by \((\phi_1 - \bar{\phi})\gamma^2 n^2 \sigma^2 / [2(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1)]\) is absolutely small, or receives a negative gain on average if the degree risk of aversion and ability are positively correlated and the variation in risk aversion represented by
\[
(\phi_1 - \bar{\phi})\gamma^2 n^2 \sigma^2 / [2(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1)]
\]
is large. If a low-ability grower with lower than average realized ability \((\lambda_{11} < \bar{\lambda}_1)\) is risk neutral, then \(G_{11} \sim 0\) as \(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1 \sim 0\).

Alternatively, consider a low-ability grower who realizes greater than expected ability \((j = 2)\). For \(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1 > 0\), a grower with ability \(\lambda_{12}\) where \(\lambda_{12} > \bar{\lambda}_1\) receives a positive gain on average if the degree risk of aversion and ability are negatively correlated or the variation in risk aversion represented by
\[
(\phi_1 - \bar{\phi})\gamma^2 n^2 \sigma^2 / [2(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1)]
\]
is small, and receives a negative gain on average if the degree of risk aversion and ability are positively correlated and the variation in risk aversion represented by \((\phi_1 - \bar{\phi})\gamma^2 n^2 \sigma^2 / [2(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1)]\) is large. If \(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1 < 0\), then a grower with ability \(\lambda_{12}\) receives a positive gain on average if the degree risk of aversion and ability are negatively correlated and the variation in risk aversion represented by \((\phi_1 - \bar{\phi})\gamma^2 n^2 \sigma^2 / [2(\gamma^2 n^2 \lambda^2 c^2 - \Delta_1)]\) is absolutely large, or receives a negative gain if the degree risk of aversion and ability are positively correlated.
or the variation in risk aversion with ability represented by

\[ (\phi_1 - \bar{\phi})\gamma_1^2 n_1^2 \sigma_0^2/[2(\gamma_1^2 n_1 \bar{\lambda} c^2 - \Delta_{12})] \] is absolutely small. If a low-ability grower with higher than average realized ability \((\lambda_{12} > \bar{\lambda})\) is risk neutral, then

\[ G_{12} > (\leq) 0 \text{ as } \gamma_1^2 n_1 \bar{\lambda} c^2 - \Delta_{12} > (\leq) 0. \]

For the risk neutral case, a grower with ability \(\lambda_{11}\) receives a negative (positive) gain from contracting on average as \(\gamma_1^2 n_1 \bar{\lambda} c^2 > \Delta_{11}\) \((\gamma_1^2 n_1 \bar{\lambda} c^2 < \Delta_{11})\) whereas a grower with ability \(\lambda_{12}\) receives positive (negative) gain from contracting on average as \(\gamma_1^2 n_1 \bar{\lambda} c^2 > \Delta_{12}\) \((\gamma_1^2 n_1 \bar{\lambda} c^2 < \Delta_{12})\). Thus, it is not possible for both grower types to gain or lose at the same time if the ex post reservation utilities do not differ.

The results of this section suggest a rich set of alternative conditions where one group or the other among low-ability growers can experience an ex post loss from contracting that is likely to be repeated under the same conditions due to realizing an ability different than expected.

### 6.4.4 Gains from contracting for high-ability growers

Defining the gains from contracting similarly for high ability growers as

\[ G_{2j} = CE_{2j} - R_{2j}, \; j = 1, 2, \] and after substituting \(e_2 = \gamma_2 \bar{\lambda}_2 c\), the ex post gain for a high-ability grower with ex post ability \(\lambda_{2j}\) is

\[ G_{2j} = \gamma_2^2 \lambda_{2j} \bar{\lambda}_2 n_2 c^2 - n_2 \beta_2 c - n_2 \gamma_2^2 \bar{\lambda}_2 c^2/2 - \alpha n_2^2/2 - \phi_2 \gamma_2^2 n_2^2 \sigma_0^2/2 - R_{2j} \]

where \(R_{2j}\) is the ex post reservation utility for a high-ability grower with realized ability \(\lambda_{2j}\), \(j = 1, 2\). Substituting for \(\beta_2\) as implied by constraint (ii) of section 6.4.1 thus obtains
\[ G_{2j} = \gamma_2^2 \lambda_2 n_2 c^2 (\lambda_{2j} - \lambda_2) - (\phi_{2j} - \bar{\phi}_2) \gamma_2^2 n_2^2 \sigma_\theta^2 / 2 + n_1 \gamma_1^2 c^2 \lambda_1 (\lambda_2 - \lambda_1) \]

\[ - (\bar{\phi}_2 - \bar{\phi}_1) \gamma_1^2 n_1^2 \sigma_\theta^2 / 2 + ER - R_{2j} \]

\[ = \gamma_2^2 \lambda_2 n_2 c^2 (\lambda_{2j} - \lambda_2) - (\phi_{2j} - \bar{\phi}_2) \gamma_2^2 n_2^2 \sigma_\theta^2 / 2 + n_1 \gamma_1^2 c^2 \lambda_1 (\lambda_2 - \lambda_1) \]

\[ - (\bar{\phi}_2 - \bar{\phi}_1) \gamma_1^2 n_1^2 \sigma_\theta^2 / 2 + \Delta_{2j}^* (\lambda_1 - \lambda_{2j}), \quad j = 1, 2, \]

where \( \Delta_{2j}^* = (R_{2j} - ER_i) / (\lambda_{2j} - \lambda_i), \quad j = 1, 2, \) is the average increment in reservation utility per unit of ability that applies in comparing the ex post reservation utility for high-ability growers to the ex ante reservation utility of low-ability growers.

Before proceeding, however, the additional constraint (ia) must be borne in mind for the purpose of determining whether some high-ability growers lose from contracting.

After substituting \( e_2 = \gamma_2^* \lambda_2 c, \) constraint (ia) of section 6.4.1 implies

\[ n_2 \gamma_2^* \lambda_2 c^2 / 2 - n_2 \beta_2 c - \alpha n_2^2 / 2 - \bar{\phi}_2 \gamma_2^* n_2^2 \sigma_\theta^2 / 2 - ER \geq 0, \]

and, after substituting for \( \beta_2 \) as implied by constraint (ii) of section 6.4.1, implies

\[ n_1 \gamma_1^* c^2 \lambda_1 (\lambda_2 - \lambda_1) - (\bar{\phi}_2 - \bar{\phi}_1) \gamma_1^* n_1^2 \sigma_\theta^2 / 2 \geq ER - ER_i. \]

Substituting this constraint into (6.39) implies

\[ G_{2j} \geq \gamma_2^2 \lambda_2 n_2 c^2 (\lambda_{2j} - \lambda_2) - (\phi_{2j} - \bar{\phi}_2) \gamma_2^2 n_2^2 \sigma_\theta^2 / 2 + ER - ER_i + ER_i = R_{2j} \]

\[ \geq \gamma_2^2 \lambda_2 n_2 c^2 (\lambda_{2j} - \lambda_2) - (\phi_{2j} - \bar{\phi}_2) \gamma_2^2 n_2^2 \sigma_\theta^2 / 2 + \Delta_{2j} (\lambda_2 - \lambda_2j), \quad j = 1, 2, \]

where \( \Delta_{2j} = (R_{2j} - ER_i) / (\lambda_{2j} - \lambda_i), \quad j = 1, 2, \) is the average increment in reservation utility per unit of ability that applies in evaluating ex post reservation utility for high-ability growers. Combining these results,

\[ G_{2j} = (\gamma_2^2 \lambda_2 n_2 c^2 - \Delta_{2j})(\lambda_{2j} - \lambda_2) - (\phi_{2j} - \bar{\phi}_2) \gamma_2^2 n_2^2 \sigma_\theta^2 / 2 - (\Delta_{2j}^* - \Delta_{2j})(\lambda_{2j} - \lambda_2) \]

\[ + (n_1 \gamma_1^2 c^2 \lambda_1^* - \lambda_2)(\lambda_2 - \lambda_1) - (\bar{\phi}_2 - \bar{\phi}_1) \gamma_1^2 n_1^2 \sigma_\theta^2 / 2 \]

\[ \geq (\gamma_2^2 \lambda_2 n_2 c^2 - \Delta_{2j})(\lambda_{2j} - \lambda_2) - (\phi_{2j} - \bar{\phi}_2) \gamma_2^2 n_2^2 \sigma_\theta^2 / 2, \quad j = 1, 2, \]
which implies that

\[
(n_{j} \gamma_{i}^{2} c^{2} \lambda_{i} - \Delta_{j}^{*})(\lambda_{2} - \lambda_{1}) - (\phi_{j} - \bar{\phi}_{j}) \gamma_{j}^{2} n_{j}^{2} \sigma_{\theta}^{2}/2 - (\Delta_{j}^{*} - \Delta_{j})(\lambda_{2j} - \lambda_{2}) \geq 0, \ j = 1, 2.
\]

Thus, high ability growers can experience ex post losses on average only if

\[
(\gamma_{2}^{2} \lambda_{2j} n_{2j} c^{2} - \Delta_{j})(\lambda_{2j} - \lambda_{2}) - (\phi_{j} - \bar{\phi}_{j}) \gamma_{j}^{2} n_{j}^{2} \sigma_{\theta}^{2}/2 \leq 0, \ j = 1, 2. \tag{6.41}
\]

Now suppose \( \lambda_{21} < \lambda_{2} < \lambda_{22} \) so that a high-ability grower who realizes less than expected ability is the case with \( j = 1 \) and a high-ability grower who realizes greater than expected ability is the case with \( j = 2 \). Consider first the case of risk neutrality where (6.41) reduces to \( (\gamma_{2}^{2} \lambda_{2j} n_{2j} c^{2} - \Delta_{j})(\lambda_{2j} - \lambda_{2}) \). This implies that a high-ability grower who realizes greater than expected ability will never lose from contracting. However, the gain for a high-ability grower who realizes less than expected ability from (6.40) is

\[
G_{2j} = (\gamma_{2}^{2} \lambda_{2j} n_{2j} c^{2} - \Delta_{j})(\lambda_{2j} - \lambda_{2}) + (n_{j} \gamma_{i}^{2} c^{2} \lambda_{i} - \Delta_{j}^{*})(\lambda_{2} - \lambda_{1}) - (\Delta_{j}^{*} - \Delta_{j})(\lambda_{2j} - \lambda_{2}),
\]

which can be negative because \( (\gamma_{2}^{2} \lambda_{2j} n_{2j} c^{2} - \Delta_{j})(\lambda_{2j} - \lambda_{2}) \) is negative.

A loss is incurred by a high-ability grower who realizes less than expected ability if the latter two terms, \( (n_{j} \gamma_{i}^{2} c^{2} \lambda_{i} - \Delta_{j}^{*})(\lambda_{2} - \lambda_{1}) \) and \( (\Delta_{j}^{*} - \Delta_{j})(\lambda_{2j} - \lambda_{2}) \), are both small or offset one another. Both terms are zero if the marginal benefit of ability in determining the ex post reservation utility is constant so that \( \Delta_{2j}^{*} = \Delta_{2j} \), and is equal to the marginal benefit of ability to a low-ability grower under contracting, \( n_{j} \gamma_{i}^{2} c^{2} \lambda_{i} = \Delta_{2j}^{*} \). The two terms offset one another if the ability of high-ability growers who realize less than expected ability is the same as expected by low ability growers, \( \lambda_{2j} = \lambda_{1} \), and the marginal benefit of ability in determining ex post reservation utility for high ability growers is equal to the
marginal benefit of ability under contracting for low-ability growers, $\Delta_{2j} = n_i \gamma_i^2 c^2 \bar{\lambda}_1$.

Both of these are plausible although special cases.

But these loss possibilities also suggest that some cases are possible where a high-ability grower who realizes less than expected ability can experience a loss even though his ability exceeds the expected ability of low-ability growers who do not incur a loss on average. This can occur in the first of the two cases of the previous paragraph if $\lambda_{2j} > \bar{\lambda}_1$ and either $\Delta_{2j}^* > \Delta_{2j}$ while $n_i \gamma_i^2 c^2 \bar{\lambda}_1 = \Delta_{2j}^*$, or $n_i \gamma_i^2 c^2 \bar{\lambda}_1 < \Delta_{2j}^*$ while $\Delta_{2j} = \Delta_{2j}$. It can occur in the second of the two cases if $\lambda_{2j}$ is slightly greater than $\bar{\lambda}_1$ but $\Delta_{2j}$ is substantially greater than $n_i \gamma_i^2 c^2 \bar{\lambda}_1$.

Turning to the risk averse case, assume for the purposes of this discussion that high-ability growers have less absolute risk aversion than low-ability growers, as is plausible $(\bar{\phi}_2 < \bar{\phi})$. Then the risk aversion terms detract from gains if the risk aversion of high-ability growers does not differ between those who have lower and higher than expected realized ability. However, from (6.41), high-ability growers who realize greater than expected ability still gain from contracting except in the seemingly implausible case where the risk aversion of such growers is considerably higher than average risk aversion among all high-ability growers, $(\gamma_i^2 \bar{\lambda}_2 n_2 c^2 - \Delta_{22})(\lambda_{22} - \bar{\lambda}_2) < (\phi_{22} - \bar{\phi}_2)\gamma_i^2 n_2^2 \sigma_\phi^2 / 2$. For high-ability growers who realize less than expected ability, the risk terms make an ex post loss more likely if $(\phi_{2j} - \bar{\phi}_2)\gamma_i^2 n_2^2 + (\bar{\phi}_2 - \bar{\phi})\gamma_i^2 n_1^2 > 0$. Thus, a loss is more likely under risk aversion unless

$$\phi_{2j} > \bar{\phi} \frac{\gamma_i^2 n_1^2}{\gamma_i^2 n_2^2} + \bar{\phi} \left[ 1 - \frac{\gamma_i^2 n_1^2}{\gamma_i^2 n_2^2} \right].$$
In the plausible case where \( \gamma_2^2 n_2^2 > \gamma_1^2 n_1^2 \), this means risk terms will make loss more likely unless the risk aversion of high-ability growers with less than expected realized ability have is higher than average risk aversion among low-ability growers.

### 6.4.5 Comparison with the predictions of standard contracting theory

The conclusions of this chapter differ from the conclusions derived under a similar information structure in section 3.5 of chapter 3. In that section, the gains from contracting are negative for low-ability growers (from the \( P \) distribution) who realize lower than expected ability, and are positive for such agents who realize higher than expected ability. In the model of this chapter, the gains can be positive or negative for either realized ability depending on the ex post re-assessment of the reservation utility after ability is realized. If ex post reservation utilities do not depend on realized ability, then conclusions are more similar to section 3.5 of chapter 3. In this case,

\[
G_{ij} = (\lambda_{ij} - \lambda_i)\gamma_i^2 n_i \lambda_i c^2 - (\phi_{ij} - \phi_i)\gamma_i^2 n_i \sigma_\phi^2 / 2, \quad j = 1,2,
\]

because \( \Delta_{ij} = 0 \) for \( i, j = 1,2 \). Thus, low-ability growers with less than expected ability receive negative gains from contracting unless they have considerably less risk aversion than the average low-ability grower. Similarly, low-ability growers with greater than expected ability receive positive gains from contracting unless they have considerably greater risk aversion than the average low-ability grower.

If ex post reservation utilities do not depend on realized ability, then the gains from contracting for high-ability growers are
Reservation utilities disappear in this case only if both low- and high-ability growers have the same reservation utility, although they disappear from the lower bound associated with the constraint in (ib) in any case. Otherwise, this difference detracts from the gains of high ability growers assuming high-ability growers have higher reservation utilities than low-ability growers. Nevertheless, for the risk neutral case, the lower bound again shows that high-ability growers who realize greater than expected ability still gain from contracting except in the seemingly implausible case where the risk aversion of such growers is considerably higher than high-ability growers with less than expected ability, \( (\gamma_2^2 \lambda_2 n_2 c^2 - \Delta_{22}) (\lambda_{22} - \bar{\lambda}_2) < (\phi_{22} - \bar{\phi}_2) \gamma_2^2 n_2^2 \sigma^2 / 2 \).

Comparing to standard contracting theory in the case of a high-ability grower who realizes less than expected ability, a loss is more likely both because reservation values are not the same for both groups and because the risk aversion of growers with lower than expected realized ability may be more risk averse.

Comparing to the results of section 3.5 of chapter 3, the results of this model are rich because both ability and effort affect the comparisons. The results also emphasize the role of risk aversion whereby qualitative results can be reversed depending on how much risk aversion differs among groups. Thus, use of risk neutral models may not only overstate or understate some results. They may err even in the qualitative implications. The results are further complicated if reservation utilities are affected by ex post realizations of ability. Ex post evaluation of the benefits of contracting seems to be the
most practical assumption considering that the most likely alternative to operating as a contract grower is operating as an independent grower. The wide variety of possibilities thus reveals many cases where more than one group can lose from contracting ex post even though they are induced to contract by expectations ex ante.

6.5 The Case Where Growers Know Their Ability Perfectly

The most widely considered information structure in the literature is where one of the parties has perfect information and the other does not. This simplified information structure is considered in this section. Suppose growers have either of two abilities, \( \lambda_1 \) or \( \lambda_2 \) (where \( \lambda_2 > \lambda_1 > 0 \)), with probabilities \( r_1 \) and \( r_2 \) (\( r_2 = 1 - r_1 \)), respectively. Suppose at the beginning of the contractual relationship that growers know their abilities and the contractor does not. But the contractor knows that the growers know their abilities and also knows the alternative ability levels and probability of each. Thus, the contractor can design the contract so that growers reveal their ability to the contractor by their contract choices.

This corresponds to the case of sections 6.3 and 6.4 where \( \lambda_1 = \lambda = \lambda_{11} = \lambda_{12} \), \( \lambda_2 = \lambda_{21} = \lambda_{22} \), \( \phi_1 = \phi = \phi_{11} = \phi_{12} \), and \( \phi_2 = \phi_2 = \phi_{21} = \phi_{22} \). The only difference is in interpretation whereby the contractor in this case induces each grower to choose the contract intended for his exact ability rather than his ability distribution.

The solutions for this problem appear exactly as in (6.16)-(6.19) except that overbars are eliminated. Because growers do not realize unexpected ex post variations in ability, their realized abilities do not divide them into groups by risk aversion levels nor are ex post reservation utilities modified from ex ante ones. Substituting
\lambda_1 = \bar{\lambda}_1 = \lambda_{11} = \lambda_{12} \quad \text{and} \quad \phi_1 = \bar{\phi}_1 = \phi_{11} = \phi_{12} \quad \text{in to} \quad G_{11} \text{ or } G_{12} \quad \text{above reveals that} \quad \text{gains from contracting for low ability growers are zero} \quad (G_1 = 0). \quad \text{Similar substitution in (6.40) shows that}

\begin{equation}
G_2 = \gamma^2 n_i \bar{\lambda}_i c^2 (\lambda_2 - \lambda_i) - (\phi_2 - \phi_i) \gamma^2 n_i \sigma_\phi^2 / 2 + ER_1 - ER_2 \geq 0
\end{equation}

because both terms that determine a non-zero lower bound on \( G_2 \) depend on probabilistic differences in ability and risk aversion within the high-ability group.

In this information structure, no agents earn negative gains from contracting on average because they know their abilities perfectly and thus accept only contracts that guarantee their reservation utilities on average. However, assuming hog growers know their abilities (and thus their ex post reservation utilities) perfectly before they sign the contracts is extreme and implausible. Relaxing this assumption reveals that some contract growers will have negative gains from contracting despite the asymmetric information advantage they hold compared to the contractor.

### 6.6 Uniform Parameters and Dynamic Considerations

Hog contracts in common use tend to have uniform parameters regarding payments per pound of gain even though they vary across growers by other parameters such as facility size. Thus, some further discussion about applicability of the model of this chapter is warranted.

One obvious reason why separation may not be advantageous for the contractor is that the limited information available to the contractor may offer little separation between the ability distributions of low-ability and high-ability growers. If the average risk
aversion levels are approximately equal \( (\bar{\phi}_1 = \bar{\phi}_2) \) and the average abilities are approximately equal \( (\bar{\lambda}_1 = \bar{\lambda}_2) \), then (6.16) and (6.17) imply that

\[
 n_1 = \frac{r_1 \bar{\lambda}_1 (1 - \gamma_1/2) - r_2 \gamma_1 (\bar{\lambda}_2 - \bar{\lambda}_1) \gamma_1^{\frac{1}{2}} \sigma_\phi}{\alpha + (\bar{\phi}_1 - r_2 \phi_2) \gamma_1^{\frac{1}{2}} \sigma_\phi^2} \frac{\gamma_1 \bar{\lambda}_2 \sigma_\lambda^2 (1 - \gamma_2/2) \alpha + \phi_2 \gamma_2 \sigma_\lambda^2}{\gamma_2 \bar{\lambda}_2 \sigma_\lambda^2 (1 - \gamma_2/2)} = n_2
\]

and (6.18) and (6.19) imply that

\[
 \gamma_1 = \frac{r_1 \bar{\lambda}_1 \sigma_\lambda^2}{2 r_2 \sigma_\lambda^2 (\bar{\lambda}_2 - \bar{\lambda}_1) + (\bar{\phi}_1 - r_2 \bar{\phi}_2) n_i \sigma_\phi^2} \frac{\bar{\lambda}_2 \sigma_\lambda^2}{\bar{\lambda}_2 \sigma_\lambda^2 + \sigma_\phi^2 n_i \sigma_\phi^2} = \gamma_2.
\]

This could explain the optimality of no separation. But this would not explain separation by facility size without separation by the incentive payment.

Another possibility is that the ability and risk aversion differences in (6.18) roughly offset one another. If \( 2 \sigma_\lambda^2 (\bar{\lambda}_2 - \bar{\lambda}_1) (\bar{\phi}_2 - \bar{\phi}_1) n_i \sigma_\phi^2 \) is substituted into the denominator of (6.18), then (6.18) and (6.19) imply that \( \gamma_1 \neq \gamma_2 \), as in observed contracts. But this condition does not prevent large differences in the facility sizes, \( n_1 \) and \( n_2 \), offered growers of different ability distributions as an optimal separating contract. Thus, this condition rationalizes observed hog contracts. In reality, this condition may only hold approximately. Alternatively, it may be that contractors’ knowledge is simply not sufficient to differentiate from this condition among growers.

Several additional reasons that extend beyond the scope of the model developed in this chapter may also explain a preference for uniform payment parameters among contractors. For example, contracts with uniform incentive parameters are easier to implement. More substantively, given that information about growers’ type can be used to their disadvantage in future periods, growers may be reluctant to reveal their ability unless substantial rent or long-term guarantees are offered. This reluctance also would
not apply to facility size because that is readily observable and contractable. This
dynamic aspect of contracting was discussed in section 4.4 of chapter 4 in the context of
standard principal agent theory. Such generalizations become too complex for analysis in
the context of the model of this chapter, but similar concerns obviously apply. For all
these reasons, strong separation in contracts with respect to $\gamma$ and $\beta$ may not add much to
contractor profits even though separation with respect to facility size is advantageous.\footnote{An additional possible reason is that separating growers is impossible if growers do not have some information about their ability. But this appears highly unlikely in the hog contracting problem.}

6.7 Conclusions

The assumption that growers have no information about their abilities is extreme. Likewise, the assumption that growers know their abilities perfectly before commencing production is extreme. The practical assumption is the intermediate case where a grower has partial knowledge of his ability or productivity. Grower productivity depends on on asset-specific investments, including the size and features of the hog facility and growers’ innate and acquired ability. Contractor-supplied, technology-imbedded inputs such as the genetic traits of feeder pigs, the feed mix, prescribed management practices add to productivity and explain why earnings under contracting can exceed reservation utilities. Additionally, growers undertake initial irreversible investment in hog facilities upon signing a contract that may improve productivity over the case of independent operations.

Contractor specifications may lead to certain expectations regarding both the mean and variance of productivity based on performance statistics among other growers who use the contractor’s specifications. But growers do not know their own ability or how their own ability will complement the contractor’s specifications at the time the
contract is signed. Thus, a grower’s productivity at that time of contracting may be represented by a random productivity parameter that can take different possible values corresponding to unanticipated deviations in grower productivity from the average. Because the distribution is determined by the contractor’s technology, the probability distribution may be regarded as common knowledge.

Thus, when choosing whether to sign a contract and incur the irreversible investment, growers are uninformed about their ex post realization of productivity. Not until the end of a production period are they able to observe output and their productivity. Further, because of random variation in production, many production periods may be required to develop a reliable estimate of their individual productivity.

On the other hand, the contractor has the opportunity to observe the productivity realizations of many growers at the same time. So she is likely to know the distribution of productivity among potential growers. But setting contract parameters targeted to the average grower will not maximize contractor profit, as demonstrated in section 3.4.1 of chapter 3. By experience, she may be able to use information about observable characteristics of growers to develop partial information about potential growers’ productivity.

The model of this chapter incorporates each of these characteristics of the hog contracting problem. Results show that whether contract parameters are targeted to the average grower or to groups of growers based on imperfect information, some growers may incur negative ex post gains. Ex post assessments of gains from contracting may be further affected as improved information on ability is obtained through experience by the grower. While these issues introduce a host of questions about dynamic aspects of hog
contracting, these are beyond the scope of this dissertation. It suffices to say that signing a long-term production contract without knowing productivity parameters places growers at great risk, but undertaking large asset-specific investments without a long-term commitment from the contractor also places growers at great risk. Thus, if productivity is known imperfectly, then some low-ability growers can be expected to receive negative ex post gains from contracting under longer-term contracting just as under short-term contracting.
Chapter 7:

Identification and Estimation of Contract Gains

7.1 Introduction

Measuring contract growers’ gains from contracting first requires knowing their reservation wages, i.e., the returns to hog farming that growers would have earned from operating independently. Then these reservation wages can be compared with what they have received as contract growers with the differences attributed to contracting. However, reservation wages are not observable because contract growers do not operate independently at the same time. To overcome this missing data problem, impact evaluation methods use the mean returns of a control group (independent growers in this case) as the counterfactual for the mean returns of the treatment group (contract growers in this case) (Heckman and Robb, 1985, 1986; Heckman, 1992; Heckman, Smith and Clements, 1997; Heckman and Vytlacil, 2001; and Heckman, 2001). Most of the empirical work in this evaluation literature focuses on group impact measures such as mean returns and, in particular, on the mean effect of treatment on those who receive treatment (Heckman, Lalonde, and Smith, 1999).

While estimating how contracting affects growers’ profits on average yields straightforward interpretations, measuring the mean impact alone misses much of what is crucial for policy purposes. The conceptual models in previous chapters reveal that distributional issues are important because some growers may find themselves worse off after irreversible contracts are signed. Thus, a critical issue is how contracting affects contract growers’ profits differently. For example, while contracting may not matter for average profit, it may adversely affect growers on the low end of the conditional profit
distribution. In short, focusing on distribution addresses not only the question of whether contracting matters, but also the question of for whom contracting matters.

This contract evaluation problem is formally introduced in this chapter in an econometric context. Different impact measures are discussed and their identification strategies are outlined in detail. The implicit assumptions underlying the different estimators are also presented. To introduce this evaluation problem formally in an econometric context, a benchmark model of economic choice is presented in section 7.2. This benchmark model is adapted from *Heckman and Navarro-Lozano* (2003) and *Heckman, Lalonde, and Smith* (1999). Section 7.3 discusses selection bias based on observables and outlines their identification strategies. This section also discusses various estimation methods that are used to estimate the mean effect of contracting on contract growers, solving the potential selection bias problem based on observables.

Section 7.4 discusses selection bias based on unobservables and outlines their identification strategies. This section also discusses various methods that are used to estimate the mean effect of contracting on contract growers, solving the selection bias problem based on unobservables. Section 7.5 explores quantile treatment effects and explains the quantile regression method as an estimation method for quantile treatment effects.

7.2 The Econometric Model and Mean Impact Measures

7.2.1 The econometric model

Let $Y_0$ and $Y_1$ denote the profit of an independent grower and a contract grower, respectively. These profits can be expressed as a function of a vector of conditioning
variables, \(X_i\). Let the conditional expectations of the profit variables be given by \(\alpha_0 + X_i \beta_0\) and \(\alpha_1 + X_i \beta_1\), respectively, where \(\alpha_0\) and \(\alpha_1\) are unknown scalars, and \(\beta_0\) and \(\beta_1\) are unknown vectors of parameters. That is, suppose profits can be expressed as

\[
Y_{0i} = \alpha_0 + X_i \beta_0 + U_{0i} \quad (7.1a)
\]
\[
Y_{1i} = \alpha_1 + X_i \beta_1 + U_{1i} \quad (7.1b)
\]

where \(E(U_{0i} | X_i) = 0\) and \(E(U_{1i} | X_i) = 0\). These \(U\)'s are assumed to be known ex ante to the grower but unknown to the econometrician. The gain to grower \(i\) of moving from independent production to contract production is denoted by \(\Delta_i = Y_{1i} - Y_{0i}\).

If both \(Y_{0i}\) and \(Y_{1i}\) can be observed for the same grower at the same time, the gain from contracting, \(\Delta_i\), would be observable. However, the profit for each grower is observed in only one state or the other. The no-contracting profit is not observed for the contract grower nor is the contracting profit observed for the independent grower. To overcome this missing data problem, most empirical work in the evaluation literature focuses on the mean effect of contracting and, in particular, estimates the mean effect of contracting on those who contract. By dealing with aggregates rather than individual growers, estimates of group impact measures are sometimes possible even though measuring the impacts of contracting on any particular grower is impossible.

To see this point more formally, consider the switching regression model with two regimes denoted by “1” and “0” introduced by Quandt (1972). The observed profit \(Y_i\) is given by

\[
Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i} \quad (7.2)
\]

where \(Y_{1i}\) is observed if \(D_i = 1\) and \(Y_{0i}\) is observed if \(D_i = 0\) and \(Y_{0i}\) and \(Y_{1i}\) are defined by
equations (7.1a) and (7.1b). The potential profit actually realized depends on the decision made by growers of whether to contract or not. A model in this form is typically completed by adding a binary participation decision equation for \( D_i \) as 
\[
D_i^* = Z_i \gamma + U_{2i}
\]
where \( D_i = 1 \) iff \( D_i^* > 0 \) and \( D_i = 0 \), otherwise. The variables in \( Z_i \) may overlap with those in \( X_i \), but at least one component of \( Z_i \) is assumed to be unique as a nontrivial determinant of \( D_i \). That is, \( D_i \) is assumed to have at least one independent source of variation. Further interpretation of \( D_i \) is provided in section 7.2.2.

This model of potential profits is variously attributed to Fisher (1951), Neyman (1935), Roy (1951), Quandt (1972, 1988) and Rubin (1974). The linear regression representations in (7.1a) and (7.1b) imply
\[
Y_i = \alpha_0 + X_i \beta_0 + D_i[\alpha + X_i(\beta_i - \beta_0)] + \{U_{oi} + D_i(U_{ii} - U_{oi})\}
\]
(7.2a)
where \( \alpha = \alpha_1 - \alpha_0 \). Adding and subtracting \( D_iE(U_{1i} - U_{0i} | X_i, D_i = 1) \) on the right hand side of (7.2a) gives
\[
Y_i = \alpha_0 + X_i \beta_0 + D_i[\alpha + X_i(\beta_i - \beta_0) + E(U_{ii} - U_{oi} | X_i, D_i = 1)]
+ \{U_{oi} + D_i((U_{ii} - U_{oi}) - E(U_{ii} - U_{oi} | X_i, D_i = 1))\}.
\]
(7.2b)

### 7.2.2 Mean impact measures

The impact measure most commonly considered in the program evaluation literature is average treatment effect (ATE), which is the expected effect of contracting on a randomly drawn grower from the population with characteristics \( X_i \):
\[
ATE(X) = E(Y_{ii} - Y_{oi} | X_i) = \alpha + X_i(\beta_i - \beta_0)
\]
(7.3)
In terms of the switching regression model, this measure is the effect of \( D_i \) in the deterministicic component of equation (7.2a), where the term in braces is the error.
Integrating $ATE(X_i)$ over the support of $X_i$ yields $ATE = \int ATE(X_i) dF(X_i) \approx \hat{\alpha} + \bar{X}(\hat{\beta}_i - \hat{\beta}_0)$ where $\bar{X}$ is the average value of $X_i$ among the population of growers.

However, this measure has been criticized as not being especially relevant for policy purposes. Because it averages across the entire population, it includes agents who would never choose to contract.

In practice most non-experimental and experimental studies do not estimate $ATE$ (Heckman and Navarro-Lozano, 2003; Heckman, Lalonde, and Smith, 1999). Instead, most non-experimental studies estimate the average treatment effect on the treated ($ATET$), which is the mean effect for those who actually contract. This measure conditions on contracting as follows:

$$ATET(X) = E(Y_i - Y_0 | X_i, D_i = 1) = \alpha + X_i (\hat{\beta}_i - \hat{\beta}_0) + E(U_{i1} - U_{0i} | X_i, D_i = 1)$$ (7.4)

It is the coefficient on $D_i$ in the deterministic component of the regression equation (7.2b). Integrating $ATET(X_i)$ over the domain of $X_i | D_i = 1$ yields $ATET$. Measure $ATET$ combines structural parameters in $\alpha + X_i (\hat{\beta}_i - \hat{\beta}_0)$ with the means of the unobservables $E(U_{i1} - U_{0i} | X_i, D_i = 1)$. It measures the average gain in profits for growers who choose to contract compared to what they would have experienced as independent growers. It computes the average gain in terms of both observables and unobservables. Inclusion of the unobservables, $E(U_{i1} - U_{0i} | X_i, D_i = 1)$, makes this measure look nonstandard compared to the constant effect model as represented by (7.6). Most econometric activity is devoted to separating $\beta_0$ and $\beta_1$ from the effects of the regressors on $U_0$ and $U_1$.

In general, the mean of the composite error term is not zero because $E[U_{0i} + D_i(U_{1i} - U_{0i})] = E(U_{1i} - U_{0i} | D_i = 1)Pr(D_i = 1)$. If $U_{1i} - U_{0i}$, or variables statistically dependent upon it, helps determine $D_i$, then $E(U_{1i} - U_{0i} | D_i = 1) \neq 0$. Intuitively, a grower
is more likely to have gains from contracting if \( E(U_{1i} - U_{0i} \mid D_i = 1) > 0 \). Selection bias arises when the contract indicator \( D_i \) is correlated with the errors \((U_{0i}, U_{1i})\) in the profit equations. This correlation could be induced by incorrectly omitted observable variables that partly determine \( D_i \) and \( Y_i \). Then the omitted variable component of the regressor error will be correlated with \( D_i \), which is the case of selection on observables. Another source of selection consists of unobserved factors that partly determine both \( D_i \) and \( Y_i \). This is the case of selection on unobservables.

The two impact measures are the same, i.e., \( ATE(X_i) = ATET(X_i) \), if the unobservables are common across the two states. This happens when the unobservable errors in the profit equations vary across hog operations but are not affected by whether the operation is a contract grower, i.e., when \( U_{0i} = U_{1i} = U_i \). From (7.2b), the following regression model is obtained when \( U_{0i} = U_{1i} = U_i \):

\[
Y_i = \alpha_0 + X_i \beta_0 + D_i [\alpha + X_i (\beta_1 - \beta_0)] + U_i .
\]

(7.5)

A more common but restrictive specification has \( \alpha + X_i (\beta_1 - \beta_0) = \alpha \) in which the contracting group has an additional intercept reflected in \( \alpha \), but the slope coefficients of the regressors, \( X_i \), are unaffected by contracting. When the slope coefficients are the same in both regimes, i.e., \( \beta_0 = \beta_1 = \beta \), (7.5) boils down to

\[
Y_i = \alpha_0 + X_i \beta + D_i \alpha + U_i .
\]

(7.6)

Model (7.6) dominates the conventional evaluation literature. The conventional econometric evaluation literature focuses on \( \alpha \), and more rarely on \( \alpha + X_i (\beta_1 - \beta_0) \) where \( \beta_0 \neq \beta_1 \) and selection bias arises from the correlation between \( D_i \) and \( U_i \). Selection bias arises when some component of the contract decision is relevant to the profit
determination process. That is, when some of the determinants of the contract decision also determine profit.

Another impact measure often considered in non-experimental studies is the *average treatment effect on the non-treated (ATNT)*, which is the mean effect for those who do not receive treatment. This measure conditions on contracting as follows:

\[
ATNT(X) = E(Y_{i1} - Y_{0i} | X_i, D_i = 0) = \alpha + X_i(\beta_1 - \beta_0) + E(U_{i1} - U_{0i} | X_i, D_i = 0).
\]

Integrating \( ATNT(X_i) \) over the domain of \( X_i | D_i = 0 \) yields \( ATNT \). This measure combines structural parameters in \( \alpha + X_i(\beta_1 - \beta_0) \) with the means of the unobservables \( E(U_{i1} - U_{0i} | X_i, D_i = 0) \) to measure the average gain in profits for independent growers had they chosen to contract. This is the difference in average profits for independent growers between what they would have experienced as contract growers and what they actually make as independent growers. This measure becomes important when policy makers intend to induce more independent growers to contract.

### 7.3 Identification and Estimation of Mean Gains

#### 7.3.1 Selection based on observables

When \( D_i \) and \((Y_{0i}, Y_{1i})\) are correlated, an assumption is needed to identify contracting effects. *Rosenbaum and Rubin* (1983) introduced the following assumption, which they called *ignorability of treatment* (given observed covariates \( X_i \)):

**Assumption 1**: Conditional on \( X_i, D_i \) and \((Y_{0i}, Y_{1i})\) are independent.

This assumption is also known as the *conditional independence assumption* (CIA). It implies

\[
F(Y_{ji} | X_i, D_i = 1) = F(Y_{ji} | X_i, D_i = 0) = F(Y_{ji} | X_i), j = 0,1,
\]
which further implies

\[ F(U_{ji} \mid X_i, D_i = 1) = F(U_{ji} \mid X_i, D_i = 0) = F(U_{ji} \mid X_i), \ j = 0, 1. \]

For many purposes, it suffices to assume ignorability in a *conditional mean independence* (CMA) sense:

**Assumption 1a:** \( E(Y_{ji} \mid X_i, D_i) = E(Y_{ji} \mid X_i), \ j = 0, 1. \)

Obviously, CMA is weaker than CIA. The idea underlying this assumption is that if one can observe enough information (contained in \( X_i \)) that determines contract choice, then \((Y_{0i}, Y_{1i})\) might be mean independent of \(D_i\), conditional on \(X_i\). Loosely speaking, even though \((Y_{0i}, Y_{1i})\) and \(D_i\) might be correlated, they are uncorrelated once the effects of the \(X_i\)s are removed. Thus, if the relationship between the contract decision and profit is purely through the observables, rather than unobservables \((U_{0i}, U_{1i})\), then selection can be controlled by including the appropriate conditioning variables in the profit equation. If valid, CMA implies no omitted variable bias once \(X_i\) is included in the regression. Thus, sample selection bias will not arise purely because of the differences in observable characteristics between the contract and independent growers.

### 7.3.1.1 Identification with no heterogeneity

The average difference between profits for contract growers and independent growers, which can be estimated from (7.6) as the coefficient on \(D_i\) after regression of \(Y_i\) on \((D_i, X_i)\), is:

\[
E(Y_1\mid X_i, D_i = 1) - E(Y_1\mid X_i, D_i = 0) = \alpha + E(U_i\mid X_i, D_i = 1) - E(U_i\mid X_i, D_i = 0)
\]

\[
= \alpha + E(U_i\mid X_i) - E(U_i\mid X_i)
\]

\[
= \alpha
\]
The second-line right-hand side follows from CMA. When CMA holds, the mean difference in the pre-treatment unobservables between those who contract and those who do not, \( E(U_i \mid X_i, D_i = 1) - E(U_i \mid X_i, D_i = 0) \), disappears. Entering all the observables that affect both profit and contract decision into the profit equation (7.6) solves the selection bias problem based on observables. In this case, a simple comparison of the average profits of contract growers and independent growers estimates the effect of contracting on contract growers.

While familiar, the framework of (7.6) is a special case. Potential profits \((Y_{0i}, Y_{1i})\) differ only by a constant \((Y_{0i} - Y_{1i} = \alpha)\). The grower with the best \(Y_{1i}\) also has the best \(Y_{0i}\) and all growers gain or lose the same amount in switching from independent to contract operation. Even though growers may be heterogeneous, their gains from contracting are not.

In the more general case with \(\beta_0 \neq \beta_1\), represented by equation (7.5), \( E(Y_i \mid X_i, D_i = 1) - E(Y_i \mid X_i, D_i = 0) = \alpha + X_i(\beta_1 - \beta_0) \) is obtained. In this case, all growers with the same \(X_i\) receive the same gains in switching from independent to contract operation. Absence of heterogeneity in response to treatments is a strong assumption (Heckman, Smith, and Clements, 1997).

7.3.1.2 Identification with heterogeneity

If the unobservable errors in the profit equations are affected by whether the operation is a contract grower or not, that is, if \(U_{0i} \neq U_{1i}\), then equation (7.2a) implies

\[
E(Y_i \mid X_i, D_i = 1) = \alpha_i + X_i\beta_i + E(U_{0i} \mid X_i, D_i = 1) + E(U_{1i} - U_{0i} \mid X_i, D_i = 1)
\]

\[
E(Y_i \mid X_i, D_i = 0) = \alpha_0 + X_i\beta_0 + E(U_{0i} \mid X_i, D_i = 0).
\]
The average difference between profits for contract growers and independent growers, which can be estimated from (7.2a) as the coefficient on $D_i$ after regression of $Y_i$ on $(D_i,X_i)$, is:

$$E(Y_i | X_i, D_i = 1) - E(Y_i | X_i, D_i = 0)$$

$$= \alpha + X_i(\beta_1 - \beta_0) + E(U_{1i} - U_{0i} | X_i, D_i = 1)$$

$$+ E(U_{0i} | X_i, D_i = 1) - E(U_{0i} | X_i, D_i = 0)$$

$$= \alpha + X_i(\beta_1 - \beta_0) + E(U_{1i} - U_{0i} | X_i) + E(U_{0i} | X_i) - E(U_{0i} | X_i)$$

$$= \alpha + X_i(\beta_1 - \beta_0) + E(U_{1i} - U_{0i} | X_i).$$

The third-line right-hand side follows from CMA.

In addition to CMA, suppose that $E(U_{1i} | X_i) = E(U_{0i} | X_i)$. This condition could arise if growers who choose to contract either do not know or do not act upon either $U_{1i} - U_{0i}$ or information dependent on $(U_{1i} - U_{0i})$ in making their decision to contract. This may generate ex post heterogeneity. But if this information is not used ex ante, then

$$E(Y_i | X_i, D_i = 1) - E(Y_i | X_i, D_i = 0) = \alpha + X_i(\beta_1 - \beta_0)$$

Thus, $ATE(X_i) = ATET(X_i) = \alpha + X_i(\beta_1 - \beta_0)$. However, the unconditional contracting effect is generally not equal to this because $ATE$ is the expected value of $ATE(X_i)$ across the entire population (i.e., over both contract and independent growers) whereas $ATET$ is the expected value of $ATET(X_i)$ in the contracting subpopulation. Mathematically, $ATE = E[ATE(X_i)]$ and $ATET = E[ATE(X_i)|D_i = 1]$. This compares with $ATE(X_i) = ATE = ATET(X_i) = ATET = \alpha$ when $\beta_0 = \beta_1$. That is, the unconditional equality of these two measures follows from the restriction that the slope coefficients are the same in both regimes.
7.3.2 Estimation with selection based on observables

Estimation of $ATE(X_i)$ requires consistent estimation of conditional expectations $E(Y_i \mid X_i, D_i = 1)$ and $E(Y_i \mid X_i, D_i = 0)$, which depend on observables. Since a random sample of observations on $(Y_i, X_i, D_i)$ from the population of hog growers is available, these conditional expectations are parametrically and nonparametrically identified and, hence, can be estimated consistently. But complexity arises when the impact measures $ATE(X_i)$ and $ATET(X_i)$ are to be identified from conditional expectations. Parametric and nonparametric estimation of these conditional expectations based on selection on observables, and thus estimation of $ATE(X_i)$ and $ATET(X_i)$, are discussed in this section.

A number of cross section methods have been proposed in the evaluation literature to deal with selection bias in treatment effect models. Two broad groups of methods that yield unbiased estimators when only observable factors affect the decision to contract are ordinary least squares (OLS) and propensity scoring methods.

7.3.2.1 The ordinary least squares method

Consider estimation of the simplest contracting effect model of section 7.2 which is represented by equation (7.6) as $Y_i = \alpha_0 + X_i \beta + D_i \alpha + U_i$. Under usual regularity conditions and assuming that selection is based on observables, that is $E(U_i \mid X_i, D_i) = 0$, $\alpha$ can be estimated consistently by OLS. In the more general case with $\beta_0 \neq \beta_1$ represented by equation (7.5), the estimated equation is $Y_i = \alpha_0 + X_i \beta_0 + D_i [\alpha + X_i (\beta_1 - \beta_0)] + U_i$. In this case, OLS can be used separately on observations with $D_i = 0$ and $D_i = 1$ to obtain $\hat{E}(Y_i \mid X_i, D_i = 0) = \hat{\alpha}_0 + X_i \hat{\beta}_0$ and $\hat{E}(Y_i \mid X_i, D_i = 1) = \hat{\alpha}_1 + X_i \hat{\beta}_1$. Thus,
\[
\text{ATE}(X_i) = \text{ATET}(X_i) = \hat{E}(Y_i | X_i, D_i = 1) - \hat{E}(Y_i | X_i, D_i = 0) \\
= (\hat{\alpha}_1 - \hat{\alpha}_0) + X_i(\hat{\beta}_1 - \hat{\beta}_0) = \hat{\alpha} + X_i(\hat{\beta}_1 - \hat{\beta}_0).
\]

Alternatively, integration over the distribution of \(X_i\) yields unconditional estimates as
\[
\text{ATE} = \int \text{ATE}(X_i) dF(X_i) \approx \frac{1}{N} \sum_{k=1}^{N} \text{ATE}(X_i) = \hat{\alpha} + \bar{X}(\hat{\beta}_1 - \hat{\beta}_0)
\]
\[
\text{ATET} = \int \text{ATET}(X_i) dF(X_i | D_i = 1) \approx \frac{1}{N_1} \sum_{k=1}^{N_1} D_i \text{ATET}(X_i)
\]

where \(N\) is sample size and \(N_1\) is the number of growers with \(D_i = 1\) in the sample. In both of these OLS regressions, \(X_i\) is assumed to include all of the variables that affect both the contract decision and profit.

### 7.3.2.2 Matching estimators based on common support

In the more recent evaluation literature, researchers have focused on nonparametric matching estimators. When these estimators are applied to a rich data set, they have been found to perform well in replicating the results of benchmark experiments (Heckman, Ichimura and Todd, 1997). The advantage of matching estimation over traditional regression analysis is that it does not make the functional form assumption that regression requires. Also, it highlights the support problem in a way that regression does not. Stated simply, matching makes plain whether or not comparable independent operations are available for each contract operation. In this way, it avoids identifying effects solely by projections into regions where no data points are observed (Wooldridge, 2002, pp.603-644).

To illustrate the support problem, suppose only one binary independent variable, \(X_i\), determines profits, and that every grower in the population with \(X_i = 1\) contracts.
Then, while \( E(Y_i \mid X_i = 1, D_i = 1) \) can be estimated with a random sample from the relevant population, \( E(Y_i \mid X_i = 1, D_i = 0) \) cannot because no data are available on the subpopulation with \( X_i = 1 \) and \( D_i = 0 \). Therefore, \( ATE(X_i) \) is not identified at \( X_i = 1 \). If some hog growers with \( X_i = 0 \) contract while others do not, \( ATE(X_i) \) is identified at \( X_i = 0 \). Explicitly, \( ATE = P(X_i = 0) \cdot ATE(X_i = 0) + P(X_i = 1) \cdot ATE(X_i = 1) \). Since \( ATE(X_i = 1) \) cannot be identified, the unconditional \( ATE \) cannot estimated. In effect, \( ATE \) can be estimated only over the grower population with \( X_i = 0 \), which means the population of interest has to be redefined.

Although this example is extreme, it illustrates consequences that arise in more plausible settings. Suppose that \( X_i \) is a vector of binary indicators for the size of hog operations. For most size intervals, the probability of contracting is strictly between zero and one, which means observations are available for both contract and independent hog growers. If the contracting probability is zero or one at some size level, however, then observations with these size(s) have to be excluded from the hog grower population to avoid the identification problem. This kind of exclusion of observations from the grower population will be used later when the results of impact measures obtained by propensity score matching methods are presented.

7.3.2.3 Propensity score matching

Matching is a non-parametric or semi-parametric analogue to regression that is used for the evaluation of binary treatments. It uses non-parametric regression methods to construct counterfactuals under an assumption of selection on observables. Intuitively, matching contrasts the profits of contract growers with those of the “comparable”
independent growers. Differences in profits between the two groups are attributed to contracting. While the parametric regression approaches in sections 7.2, 7.3.1, and 7.3.2.1 make functional form assumptions about $E(U_{1i}|X_i)$ and $E(U_{0i}|X_i)$, $ATE(X_i)$ and $ATET(X_i)$ can both be estimated alternatively by modeling $p(X_i)$, the probability of contracting. This probability is also known as the propensity score given the covariates.

Formally, the propensity score is defined by Rosenbaum and Rubin (1983) as the conditional probability of contracting given pre-contracting characteristics $X_i$, 

$$p(X_i) \equiv P(D_i = 1 | X_i) = E\{D_i | X_i\}$$  \hspace{1cm} (7.7)

They shown that if the contracting decision is random within cells defined by $X_i$ then it is also random within cells defined by the values of the one-dimensional propensity score, $p(X_i)$. As a result, if the propensity score $p(X_i)$ is known, the $ATET$ can be estimated as 

$$ATET = E(Y_{1i} - Y_{0i} | D_i = 1)$$

$$= E\{E[Y_{1i} - Y_{0i} | D_i = 1, p(X_i)]\}$$

$$= E\{E[Y_{1i} | D_i = 1, p(X_i)] - E[Y_{0i} | D_i = 0, p(X_i)] | D_i = 1\}$$  \hspace{1cm} (7.8)

where the outer expectation is over the distribution of $p(X_i) | D_i = 1$. Formally, the following assumption is needed in addition to CIA to derive (7.8) from (7.7).

Assumption 2: $0 < p(X_i) < 1$ for all $X_i$.

An equivalent representation of CIA is $Pr(D_i = 1|Y_{0i}, Y_{1i}, X_i) = Pr(D_i = 1|X_i)$.

Rosenbaum and Rubin (1983) establish that conditional on $p(X_i)$, $(Y_{0i}, Y_{1i})$ and $D_i$ are uncorrelated when both of these assumptions hold. They call Assumptions 1 and 2 together strong ignorability of treatment. As explained in section 7.3.1, CIA is a restriction that implies the choice of whether to contract is purely random for similar growers. Loosely speaking, even though $(Y_{0i}, Y_{1i})$ and $D_i$ might be correlated, they are
uncorrelated once the effects of the $X_i$s are removed. Assumption 2 is the common support condition for identification. This condition guarantees that matches can be made for all values of $X_i$. If all the hog growers with a given covariate $X_i$ choose contracting, then no observations will be available for similar growers who choose to remain independent growers, and vice versa. Assumption 2 is used to avoid this problem.

The matching approach is motivated by the following thought experiment. Suppose a propensity score, $p(X_i)$, is chosen at random from the grower population. Then, two growers sharing the chosen propensity score are selected from the population, where one contracts and the other does not. Under CIA, the expected difference in the observed profits for these growers is

$$E[Y_i | D_i = 1, p(X_i)] - E[Y_i | D_i = 0, p(X_i)] = E[Y_{1i} - Y_{0i} | p(X_i)],$$

which is the ATE conditional on $p(X_i)$. By iterated expectations, averaging across the distribution of propensity scores gives $ATE = E(Y_{1i} - Y_{0i})$ (Wooldridge, 2002: pp.603-644).

An estimation strategy requires estimating the propensity scores, estimating the profit differences for pairs matched on the basis of the estimated propensity scores, and then averaging over all such pairs. However, the probability of observing two growers with exactly the same value of the propensity score is in principle zero since $p(X_i)$ is a continuous variable. Because finding identical predicted probabilities is often unlikely when there are many covariates or the covariates are continuous variables, contract and independent growers with similar scores are matched and compared instead. Effectively, growers with similar propensity scores are considered a match. Various definitions of ‘similar’ yield various propensity score matching estimators.
7.3.2.4 Popular matching methods

A number of different matching methods have evolved in the literature (see 
*Heckman, Ichimura and Todd*, 1997, 1998; *Heckman, Ichimura, Smith and Todd*, 
1998; *Becker and Ichino*, 2002). Four of the most widely used are nearest-neighbor 
matching, kernel matching, and bias-adjusted matching. Since the widely-used impact 
measure is \( ATET \), all of the matching estimators will be explained in terms of impact 
measure \( ATET \).

**Nearest-neighbor matching.** The nearest-neighbor matching method matches a 
contract grower with the independent grower who has the nearest propensity score. 
Neighbors can be chosen with or without replacement, where “with replacement” means 
that a given independent grower can be a best match for more than one contract grower. 
Although not necessary, the nearest-neighbor method is usually applied with 
replacement. Once each contract grower is matched with an independent grower, the 
difference between the profits of these growers is computed. The \( ATET \) is then obtained 
by averaging these differences.

For nearest-neighbor matching, the comparison group for each contract grower 
indexed by \( i \), \( A_j(p(X_i)) \), is defined as 
\[ A_j(p(X_i)) = \{ j \mid \min_j | p(X_i) - p(X_j) | \} \], which is a 
singleton set unless there are multiple nearest neighbors that generate the same difference 
in propensity scores. More generally, nearest-neighbor estimators can be defined such 
that a fixed number of nearest neighbors are selected for each contracting grower 
regardless of ties in differences in propensity scores. This variant of nearest-neighbor 
matching is known as *simple average nearest-neighbor* matching. For simple average 
nearest-neighbor estimation one must decide first how many neighbors to use. Once the
number of neighbors is decided, that number of neighbors must be selected by their proximity to the contract grower $i$. Once each contract grower is matched with a set of independent growers, the difference between the profit of the contract grower and the average profit of the matched independent growers is computed. The $ATET$ is then obtained by averaging these differences over all contract growers.

Specifically, let $w(i, j)$ be the weight placed on the $j$th independent grower in constructing the counterfactual for the $i$th contract grower. The weights of nearest neighbors must satisfy $\sum_j w(i, j) = 1$, and $0 \leq w(i, j) \leq 1$ for all $i$. Let $N_0$ and $N_1$ be the number of independent and contract growers in the sample, respectively. Also, let $N_m^i$ denote the number of independent growers included in the matched set $A_j(p(X_i))$.

Obviously, $N_m^i = 1$ if the nearest neighbor is a singleton. The typical weighting scheme for the nearest-neighbor matching estimator with simple averaging is $w^{NN}(i, j) = (N_m^i)^{-1}$ if $j \in A_j(p(X_i))$, and 0 otherwise. Then the weighted comparison group mean for contract grower $i$, which is the counterfactual profit $\hat{Y}_i^c = (\hat{Y}_0^c | X_i, D_i = 1)$, is given by

$$\hat{Y}_i^c (w^{NN}) = \sum_{j=1}^{N_m^i} w^{NN}(i, j) Y_{0j}$$

and the estimated treatment effect for contract grower $i$ is $Y_{1i} - \hat{Y}_i^c (w^{NN})$. The associated nearest-neighbor matching estimator for $ATET$ can be written in the form

$$ATET^{NN} = \frac{1}{N_1} \sum_{i=1}^{N_1} [Y_{1i} - \hat{Y}_i^c (w^{NN})] = \frac{1}{N_1} \sum_{i=1}^{N_1} [Y_{1i} - \sum_{j=1}^{N_m^i} w^{NN}(i, j) Y_{0j}]$$.

All contract growers have a match with the nearest-neighbor method. However, some of these matches may be poor because the nearest neighbor for some contract growers may have a very different propensity score and, nevertheless, contribute to the
estimation of the contracting effect regardless of this difference. This problem gets worse with the simple average nearest-neighbor method. The kernel and local linear regression matching methods attempt to address this problem.

**Kernel matching.** With kernel matching, the weights are inversely proportional to the distance between the propensity scores of contract and independent growers. This weighting is achieved through a kernel function. Kernel matching uses the entire sample of independent growers defines weights

\[ w^K(i, j) = \frac{K[(p(X_i) - p(X_j))/h]}{\sum_{j=1}^{N_i} K[(p(X_i) - p(X_j))/h]} \]

where \( K \) is a kernel function and \( h \) is a bandwidth or smoothing parameter analogous to the choice of the number of neighbors included in a nearest-neighbor approach or the size of the radius in a radius matching approach. Kernel functions are usually chosen to satisfy \( \int K(s)ds = 1 \) and \( \int K(s)sds = 0 \). Like before, the kernel matching estimator for \( ATET \) can be written in the form

\[ ATET^K = \frac{1}{N_1} \sum_{i=1}^{N_1} [Y_{i1} - \hat{Y}_{i1}^c(w^K)] = \frac{1}{N_1} \sum_{i=1}^{N_1} [Y_{i1} - \sum_{j=1}^{N_0} w^K(i, j)Y_{0j}] . \]

Thus, different matching estimators of \( ATET \) are generated by varying the way the weights \( w^K(i,j) \) are constructed.

**Bias-adjusted matching.** The simple matching estimator will be biased in finite samples when the matching is not exact (Abadie et al., 2001). Abadie and Imbens (2002) show that matching discrepancies result from differences in covariates between matched
units. Thus, their matches bias matching estimators. In practice, some of this bias can be removed after matching. The bias-adjusted matching estimator adjusts the difference within the matches for the differences in their covariate values.

*Abadie and Imbens* (2002) consider a bias-corrected matching estimator where the difference within the matches is regression-adjusted for the difference in covariate values. The adjustment is based on an estimate of the regression function for the matched controls $\mu_0(x) = E(Y_0|X = x)$ using only the data in the matched sample. Following *Rubin* (1973) and *Abadie and Imbens* (2002), *Abadie et. al.* (2001) approximate the regression function by a linear function estimated by using least squares on the matched observations. Given the estimated regression function, the counterfactual profit is calculated as:

$$
\hat{Y}^c_{i} (w^{BC}) = \sum_{j=1}^{N_0} w^{BC}(i, j)[Y_{0j} + \mu_0(x_i) - \mu_0(x_j)].
$$

Therefore, the bias corrected matching estimator for $ATET$ can be written in the form

$$
ATET^{BC} = \frac{1}{N_1} \sum_{i=1}^{N_1} [Y_{ii} - \hat{Y}^c_{i} (w^{BC})] = \frac{1}{N_1} \sum_{i=1}^{N_1} [Y_{ii} - \sum_{j=1}^{N_0} w^{BC}(i, j)[Y_{0j} + \mu_0(x_i) - \mu_0(x_j)]].
$$

The bias-corrected matching estimator provides a link between matching and regression estimators, highlighting advantages and disadvantages of both.

### 7.3.3 Selection based on unobservables

If the ignorability-of-treatment assumption (Assumptions 1 or 1a) fails, the identification strategy for $ATE(X_i)$ and $ATET(X_i)$ in previous sections breaks down. These assumptions fail when the unobservable characteristics $U_{2i}$ affecting the contract decision are correlated with the unobservable characteristics $(U_{0i}, U_{1i})$ affecting the profits. This
correlation generates a relationship between the contract decision and the profit-determination process through unobservables. In this case, controlling for the observable characteristics, \( X_i \), when explaining profits is insufficient because some additional process influences the profit, namely, the process determining whether a grower contracts.

If these unobservable characteristics are correlated with the observables then failure to include an estimate of the unobservables creates omitted variable bias leading to incorrect inference regarding the impact of the observables on profits. Even if they are uncorrelated with the observables, failure to include an estimate of them may lead to an incorrect estimate of the intercept coefficients, which is a component of the impact of contracting. In this situation, a simple comparison of the average profits of contract growers and independent growers does not estimate the effect of contracting on contract growers.

7.3.3.1 Identification with no heterogeneity

Consider first the standard case represented by (7.6) where \( U_{0i} = U_{1i} = U_i \). The average difference between profits for contract growers and independent growers, which can be estimated from (7.6) as the coefficient on \( D_i \) after regression of \( Y_i \) on \( (D_i, X_i) \), is:

\[
E(Y_i \mid X_i, D_i = 1) - E(Y_i \mid X_i, D_i = 0)
= \alpha + E(U_i \mid X_i, D_i = 1) - E(U_i \mid X_i, D_i = 0)
= \alpha + E(U_i \mid U_{2i} > -Z_i \gamma) - E(U_i \mid U_{2i} \leq -Z_i \gamma).
\]
Here, $ATE(X_i) = ATET(X_i) = \alpha$ is not identified from the average difference between profits for contract growers and independent growers. When the slope coefficients are not the same in regimes as represented by (7.5), the same difference is

$$E(Y_i \mid X_i, D_i = 1) - E(Y_i \mid X_i, D_i = 0)$$

$$= \alpha + X_i(\beta_1 - \beta_0) + \underbrace{E(U_i \mid U_{2i} > -Z_i \gamma) - E(U_i \mid U_{2i} \leq -Z_i \gamma)}_{\text{Selection Bias}}$$

In this case, $ATE(X_i) = ATET(X_i) = \alpha + X_i(\beta_1 - \beta_0)$ is not identified either. The selection bias $E(U_i \mid U_{2i} > -Z_i \gamma) - E(U_i \mid U_{2i} \leq -Z_i \gamma)$ is the mean difference in the no-contracting unobservables between those who contract and those who do not. In other words, it is the difference in unobservables between what (profit) contract growers would have realized if they were independent growers and what independent growers realized.

In order to identify the impact measures, it is important to quantify the selection bias term using the correlation between $U_i$ and $U_{2i}$, and then include this term in the equation as an independent variable. In this way, the omitted variable bias problem due to unobservables can be solved. Heckman’s two-step estimation procedure can be applied for this purpose. Alternatively, a two stage least squares estimation method can be used to estimate $\alpha$, $\beta_0$ and $\beta_1$ consistently provided that instruments are available that are correlated with $D_i$ but not with $U_i = U_{0i} = U_{1i}$.

### 7.3.3.2 Identification with heterogeneity

If $U_{0i} \neq U_{1i}$, then the average difference between profits for contract growers and independent growers, which can be estimated from (7.2a) as the coefficient on $D_i$ after regression of $Y_i$ on $(D_i, X_i)$, is:
\[ E(Y_i \mid X_i, D_i = 1) - E(Y_i \mid X_i, D_i = 0) \\
= \alpha + X_i(\beta_1 - \beta_0) + E(U_{1i} - U_{0i} \mid X_i, D_i = 1) \\
+ E(U_{0i} \mid X_i, D_i = 1) - E(U_{0i} \mid X_i, D_i = 0) \\
= \alpha + X_i(\beta_1 - \beta_0) + E(U_{1i} - U_{0i} \mid D_i = 1) \\
\text{ATE}(X_i) \\
\text{SortingEffect} \\
+ E(U_{0i} \mid D_i = 1) - E(U_{0i} \mid D_i = 0) \\
\text{SelectionBias} \\
= \alpha + X_i(\beta_1 - \beta_0) + E(U_{1i} - U_{0i} \mid U_{2i} > -Z_i\gamma) \\
\text{ATE}(X_i) \\
\text{SortingEffect} \\
+ E(U_{0i} \mid U_{2i} > -Z_i\gamma) - E(U_{0i} \mid U_{2i} \leq -Z_i\gamma) \\
\text{SelectionBias} \\
\]

The first right hand side follows from the fact that the errors \((U_{0i}, U_{1i}, U_{2i})\) are independent of \((Z_i, X_i)\). Since \(ATET(X_i)\) and \(ATE(X_i)\) differ by a sorting effect, \(ATE(X_i) \neq ATET(X_i)\) unless the sorting effect is zero. The sorting effect, \(E(U_{1i} - U_{0i} \mid D_i = 1)\), is the mean gain from unobservables for growers who choose to contract. In order to identify \(ATET(X_i)\), the selection bias term must be quantified and included in the equation as an independent variable. In order to identify \(ATE(X_i)\), in addition to the selection bias term, the sorting effect term must be quantified and included in the equation as an independent variable. In this case, obtaining and using a set of instruments correlated with \(D_i\) but not with \((U_{0i}, U_{1i})\) does not solve the identification problem. Hence, instrumental variables (IV) do not provide a consistent estimator of the mean return to contracting in the presence of heterogeneity and selection bias. Heckman’s two-step estimation method obtains consistent estimates of \(ATE\) and \(ATET\) by solving the omitted variables bias associated with unobservables.
7.3.4 Heckman’s two-step estimation method

The most widely-used methods for estimating sample selection models allow for selection on unobservables and are based on Heckman’s two-step method. This estimation procedure fully exploits the correlation between \((U_0, U_1)\) and \(U_2\). To obtain consistent estimates of the impact measures, this procedure quantifies the selection bias term using the correlation between \((U_0, U_1)\) and \(U_2\) and includes it in the profit equation as an independent variable.

7.3.4.1 The simple case with no heterogeneity

First, the standard case represented by (7.6) will be considered maintaining the assumption that any unobservables have the same effect on profits irrespective of contracting status, i.e., \(U_0 = U_1 = U_i\). The average difference between profits for contract growers and independent growers, which can be estimated from (7.6), as the coefficient on \(D_i\) after regression of \(Y_i\) on \((D_i, X_i)\), is:

\[
E(Y_i | X_i, D_i = 1) - E(Y_i | X_i, D_i = 0) = \alpha + E(U_i | X_i, D_i = 1) - E(U_i | X_i, D_i = 0)
\]

\[
= \alpha + E(U_i | U_{2i} > -Z_i \gamma) - E(U_i | U_{2i} \leq -Z_i \gamma)
\]

\[
= \alpha + \sigma_u \rho \frac{\phi(-Z_i \gamma)}{1 - \Phi(-Z_i \gamma)} - \sigma_u \rho \frac{\phi(-Z_i \gamma)}{\Phi(-Z_i \gamma)}
\]

\[
= \alpha + \lambda \frac{\phi(Z_i \gamma)}{\Phi(Z_i \gamma)[1 - \Phi(Z_i \gamma)]}
\]

where \(\sigma_u\) is the standard deviation of \(U_i\), \(\rho\) is the correlation between \(U_i\) and \(U_{2i}\), and \(\lambda = \rho \sigma_u\). The terms \(\phi(-Z_i \gamma)/[1 - \Phi(-Z_i \gamma)]\) and \(\phi(-Z_i \gamma)/\Phi(-Z_i \gamma)\) are known as the inverse
Mills ratios in the literature. Regression of \( Y_i \) on \((D_i, X_i)\) and
\[
\phi(Z_i \gamma) / [\Phi(-Z_i \gamma)(1 - \Phi(-Z_i \gamma))] \quad \text{gives the coefficient on } D_i \text{ as an unbiased estimate of } \\
ATET = ATE = \alpha.
\]

7.3.4.2 Heterogeneity in observables

The methods of section 7.3.4.1 extend readily to the case where the effects of observable characteristics to vary with contract status \((\beta_0 \neq \beta_1)\). The average difference between the profits of contract and independent growers, which can be estimated from (7.5) as the coefficient on \( D_i \) after regression of \( Y_i \) on \((D_i, X_i)\), is:
\[
E(Y_i | X_i, D_i = 1) - E(Y_i | X_i, D_i = 0) = \alpha + X_i(\beta_1 - \beta_0) + \lambda \frac{\phi(Z_i \gamma)}{\Phi(-Z_i \gamma)(1 - \Phi(-Z_i \gamma))}. \tag{7.10}
\]

In this case, estimation of a probit model is required to obtain estimates of the correction terms for both contracting and independent operations separately. Then the correction terms are included as regressors in the second stage model of profits separately for each group. Conditional expectations \( E(Y_i | X_i, D_i = 0) \) and \( E(Y_i | X_i, D_i = 1) \) can then be written as
\[
E(Y_i | X_i, D_i = 0) = \alpha_0 + X_i \beta_0 + \lambda \frac{\phi(-Z_i \gamma)}{\Phi(-Z_i \gamma)}
\]
\[
E(Y_i | X_i, D_i = 1) = \alpha_1 + X_i \beta_1 + \lambda \frac{\phi(-Z_i \gamma)}{1 - \Phi(-Z_i \gamma)}.
\]

Estimation of \( E(Y_i | X_i, D_i = 0) \) and \( E(Y_i | X_i, D_i = 1) \) yields estimators \( \hat{\alpha}_0, \hat{\alpha}_1, \hat{\beta}_0 \), and \( \hat{\beta}_1 \). Thus, \( ATE(X_i) = ATET(X_i) = \hat{\alpha}_1 - \hat{\alpha}_0 + X_i(\hat{\beta}_1 - \hat{\beta}_0) = \hat{\alpha} + X_i(\hat{\beta}_1 - \hat{\beta}_0) \).

Alternatively, integration over the distribution of \( X_i \) yields unconditional estimates
\[ ATE = \int ATE(X_i) dF(X_i) \approx \frac{1}{N} \sum_{k=1}^{n} ATE(X_i) = \hat{\alpha} + \bar{X} (\hat{\beta}_1 - \hat{\beta}_o) \]

\[ ATET = \int ATET(X_i) dF(X_i | D_i = 1) \approx \frac{1}{N_1} \sum_{k=1}^{n} D_i ATET(X_i) \]

where \(N\) is sample size and \(N_1\) is the number of growers with \(D_i = 1\) in the sample.

### 7.3.4.3 Heterogeneity in both observables and unobservables

The selection correction two-step estimator in the section 7.3.4.2 allows for correlation between the errors in the selection model and the profit model. However, it requires that unobservable factors affecting profits must have the same effect irrespective of contracting status even though observable factors are have different effects depending on contracting status. This rules out the possibility that the effect of contracting may differ with the unobserved aspects of the profit of a hog operation. Even when \(U_{0i} \neq U_{1i}\) and the idiosyncratic gain, \(U_{1i} - U_{0i}\), is observed by the individual grower when making the contract decision, the two-step estimation procedure can still apply with added complications.

Allowing the effect of observable characteristics to vary between contracting and non-contracting regimes, the expected profit for hog operations conditional on the choice of contracting status are:

\[
E(Y_{ii} | X_i, D_i = 1) = \alpha_i + X_i \beta_i + \sigma_{1z} \sigma_{2z} \frac{\phi(-Z_i \gamma)}{1 - \Phi(-Z_i \gamma)} = \alpha_i + X_i \beta_i + \lambda_i \frac{\phi(Z_i \gamma)}{\Phi(Z_i \gamma)}
\]

\[
E(Y_{0i} | X_i, D_i = 1) = \alpha_0 + X_i \beta_0 + \sigma_{02} \sigma_{02} \frac{\phi(-Z_i \gamma)}{1 - \Phi(-Z_i \gamma)} = \alpha_0 + X_i \beta_0 + \lambda_0 \frac{\phi(Z_i \gamma)}{\Phi(Z_i \gamma)}
\]

\[
E(Y_{0i} | X_i, D_i = 0) = \alpha_0 + X_i \beta_0 + \sigma_{02} \sigma_{02} \frac{\phi(-Z_i \gamma)}{\Phi(-Z_i \gamma)} = \alpha_0 + X_i \beta_0 + \lambda_0 \frac{\phi(Z_i \gamma)}{1 - \Phi(Z_i \gamma)}
\]
\[
E(Y_{ii} | X_i, D_i = 0) = \alpha_i + X_i\beta_i + \sigma_{i2}\rho_{j2}\frac{\phi(-Z_i\gamma_i)}{\Phi(-Z_i\gamma_i)} = \alpha_i + X_i\beta_i + \lambda_i\frac{\phi(Z_i\gamma_i)}{1-\Phi(Z_i\gamma_i)}
\]

where \(\rho_{j2} = Corr(U_{ji}, U_{2i}), j = 0, 1\), and \(\phi(\cdot)\) and \(\Phi(\cdot)\) are the standard normal density and distribution functions, respectively. These expressions are derived under the assumption of jointly normally distributed errors,

\[
\begin{bmatrix}
U_2 \\
U_1 \\
U_0
\end{bmatrix}
\sim N
\left(0, \begin{bmatrix}
1 & \sigma_{12} & \sigma_{02} \\
\sigma_{12} & \sigma_{11} & \sigma_{10} \\
\sigma_{02} & \sigma_{10} & \sigma_{00}
\end{bmatrix}\right),
\]

where the variance parameter in the selection equation is normalized to unity without loss of generality.

Impact measures can be consistently estimated through a two-step procedure as follows (Heckman et al., 2001; and Aakvik et al., 2003):

1. Estimate \(\hat{\gamma}\) from a probit model on the selection equation.

2. Compute the appropriate selection correction terms evaluated at \(\hat{\gamma}\),

i.e. \(\phi(Z_i\gamma_i)/\Phi(Z_i\gamma_i)\) when \(D_i = 1\), and \(\phi(Z_i\gamma_i)/(1-\Phi(Z_i\gamma_i))\) when \(D_i = 0\).

3. Fit profit regressions for contract and independent growers, conditioning on observable covariates that directly effect profit, \(X_i\), and the selection correction term, i.e.,

\[
\hat{Y}_{ii} = \hat{\alpha}_i + X_i\hat{\beta}_i + \hat{\lambda}_i\phi(Z_i\gamma_i)/[\Phi(Z_i\gamma_i)]
\]

\[
\hat{Y}_{oi} = \hat{\alpha}_0 + X_i\hat{\beta}_0 + \hat{\lambda}_0\phi(Z_i\gamma_i)/[1-\Phi(Z_i\gamma_i)].
\]

4. Use the estimated coefficients from the independent and contract profit regressions to predict the counterfactual profit \(\hat{Y}_{ii}^c\) for contract grower \(i\) and \(\hat{Y}_{oi}^c\) for independent grower \(i\), respectively, i.e.,
\[ \hat{Y}_{ci} = \hat{\alpha}_0 + X_i \hat{\beta}_0 + \hat{\lambda}_0 \phi(Z_i, \hat{\gamma}) / \Phi(Z_i, \hat{\gamma}) \]

\[ \hat{Y}_{0i} = \hat{\alpha}_0 + X_i \hat{\beta}_1 + \hat{\lambda}_0 \phi(Z_i, \hat{\gamma}) / (1 - \Phi(Z_i, \hat{\gamma})) \]

where \( \hat{\alpha}_0, \hat{\beta}_0 \) and \( \hat{\lambda}_0 \) (\( \hat{\alpha}_1, \hat{\beta}_1 \) and \( \hat{\lambda}_1 \)) are used instead of \( \hat{\alpha}_1, \hat{\beta}_1 \) and \( \hat{\lambda}_1 \) (\( \hat{\alpha}_0, \hat{\beta}_0 \) and \( \hat{\lambda}_0 \)) to predict the counterfactual profit that would occur without (with) contracting.

5. Estimate the average effect of contracting on a randomly selected grower as

\[ ATE(X_i) = \hat{\alpha}_0 + X_i (\hat{\beta}_1 - \hat{\beta}_0) = \hat{\alpha} + X (\hat{\beta}_1 - \hat{\beta}_0) \]

\[ ATE = \hat{\alpha} + X (\hat{\beta}_1 - \hat{\beta}_0) , \]

estimate the average effect of contracting on contract growers as

\[ ATET(X_i, Z_i) = \hat{\alpha}_0 + X_i (\hat{\beta}_1 - \hat{\beta}_0) + (\hat{\lambda}_1 - \hat{\lambda}_0) \phi(Z_i, \hat{\gamma}) / \Phi(Z_i, \hat{\gamma}) , \]

and estimate the average effect of contracting on independent growers as

\[ ATNT(X_i, Z_i) = \hat{\alpha}_0 + X_i (\hat{\beta}_1 - \hat{\beta}_0) + (\hat{\lambda}_1 - \hat{\lambda}_0) \frac{\phi(Z_i, \hat{\gamma})}{1 - \Phi(Z_i, \hat{\gamma})} . \]

Alternatively, integration over the distribution of \((Z_i, X_i)\) yields the unconditional estimate

\[ ATET = \int ATET(X_i, Z_i) dF(X_i, Z_i | D_i = 1) \approx \frac{1}{n_1} \sum_{i=1}^{n_1} D_i ATET(X_i, Z_i) \]

\[ ATNT = \int ATNT(X_i, Z_i) dF(X_i, Z_i | D_i = 0) \approx \frac{1}{n_0} \sum_{i=1}^{n_0} (1 - D_i) ATNT(X_i, Z_i) . \]
7.4 Quantile Treatment Effects

Contracting effects may be heterogeneous and varying along with the profit distribution. The presence of heterogeneity in contracting effects is important for evaluating contract gains and understanding the pattern of grower discontent with contracting. Policymakers are often interested in the distributional consequences of contracting. A measure of interest in the presence of heterogeneous effects is the quantile treatment effect (QTE). As originally defined by Lehmann (1974) and Doksum (1974), QTE corresponds, for any fixed percentile, to the horizontal distance between two cumulative distribution functions. To define QTE as a treatment effect at the grower level, a useful assumption is that the a grower’s rank in the distribution given the contracting decision does not depend on the decision to contract, following the line of argument of Doksum (1974), and Lehmann (1974). This assumption is known as the rank invariance assumption.

7.4.1 The model in quantile form

Rewriting (7.1a) in the quantile regression form considered by Koenker and Bassett (1978) gives

\[ Y_{0i} = \alpha_0^0 + X_i \beta_0^0 + U_{0i}^0 \]  
\[ Y_{1i} = \alpha_1^0 + X_i \beta_1^0 + U_{1i}^0, \]

where \( U_{0i}^0 = \alpha_0^0 - \alpha_0^0 + X_i (\beta_0^0 - \beta_0^0) + U_{0i} \) and \( U_{1i}^0 = \alpha_1^0 - \alpha_1^0 + X_i (\beta_1^0 - \beta_1^0) + U_{1i} \) and the conditional quantile of \( Y_{0i} \) and \( Y_{1i} \) (conditional on \( X_i \)) satisfies

\[ \text{Quant}_\theta(Y_{0i} \mid X_i) = \alpha_0^0 + X_i \beta_0^0 \]
and

$$Quant_{\theta}(Y_{it} | X_i) = \alpha_{i}^{\theta} + X_i \beta_{i}^{\theta}$$

for $0 \leq \theta \leq 1$, so that $Quant_{\theta}(U_{0i} | X_i) = 0$ and $Quant_{\theta}(U_{1i} | X_i) = 0$. In the presence of the selection mechanism, the conditional quantile of observed profit is given by

$$Quant_{\theta}(Y_{0i} | X_i, D_i = 0) = \alpha_{0}^{\theta} + X_i \beta_{0}^{\theta} + Quant_{\theta}(U_{0i} | X_i, D_i = 0)$$

$$Quant_{\theta}(Y_{1i} | X_i, D_i = 1) = \alpha_{1}^{\theta} + X_i \beta_{1}^{\theta} + Quant_{\theta}(U_{1i} | X_i, D_i = 1).$$

Thus, observed profit can be written as

$$Y_i = \alpha_{0}^{\theta} + X_i \beta_{0}^{\theta} + D_i [\alpha^{\theta} + X_i (\beta_{0}^{\theta} - \beta_{0}^{\theta}) + [U_{0i}^{\theta} + D_i (U_{1i}^{\theta} - U_{0i}^{\theta})]]. \quad (7.13)$$

If the slope coefficients of the regressors are unaffected by contracting status (i.e., if $\beta_0 = \beta_1$), then

$$Y_i = \alpha_{0}^{\theta} + X_i \beta_{0}^{\theta} + D_i \alpha^{\theta} + [U_{0i}^{\theta} + D_i (U_{1i}^{\theta} - U_{0i}^{\theta})]. \quad (7.14)$$

### 7.4.2 Quantile regression to estimate quantile treatment effects

The QTE is estimated using quantile regression, which estimates the effect of explanatory variables on the dependent variable at different points of the dependent variable’s conditional distribution. Quantile regression was initially introduced as a robust regression technique to allow for estimation where the typical assumption of normality of the error term may not be satisfied (Koenker and Bassett, 1978). It has also been used to estimate models with censoring (Powell, 1984, 1986; Buchinsky, 1994, 1995). Most recently, quantile regressions have been used simply to get information about points in the distribution of the dependent variable other than the conditional mean (Buchinsky, 1994, 1995). Quantile regression is used here to examine whether the effects
of contracting differ across the quantiles in the conditional distribution of profit effects of contracting.

Consistent estimates of conditional quantiles Quant\(\theta\)(\(Y_{0i}\mid X_i, D_i = 0\)) and Quant\(\theta\)(\(Y_{1i}\mid X_i, D_i = 1\)) are required to estimate QTE. The quantile difference between profits for contract and independent growers, which can be estimated from (7.14) as the coefficient on \(D_i\) after quantile regression of \(Y_i\) on \((D_i, X_i)\), is

\[
QTE = \theta + \text{Quant}_\theta (U_{0i} - U_{0i} \mid X_i, D_i = 1) - \text{Quant}_\theta (U_{0i} \mid X_i, D_i = 0)
\]

where \(\alpha = \alpha_1 - \alpha_0\). If CIA holds, i.e., if \(D_i\) and \((Y_{0i}, Y_{1i})\) are independent conditional on \(X_i\), then the selection bias term disappears. Thus, QTE is identified as

\[
QTE = \alpha + \text{Quant}_\theta (U_{0i} - U_{0i} \mid X_i, D_i = 1).
\]

However, if CMA holds instead of CIA (i.e., if \(E(Y_{ji} \mid X_i, D_i) = E(Y_{ji} \mid X_i)\) for \(j = 0,1\)), then the selection bias term does not necessarily disappear. However, if CMA is replaced by conditional quantile independence, \(\text{Quant}_\theta (Y_{0i} \mid X_i, D_i) = \text{Quant}_\theta (Y_{0i} \mid X_i)\), then the selection bias term disappears, and thus QTE is identified.

Alternatively, if selection is based on unobservables, the selection bias term can be quantified using the correlation between \((U_{0i}^\theta, U_{1i}^\theta)\) and \(U_{2i}\) and included in the equation as an independent variable. Thus, omitted variable bias through unobservables can be solved. In the more general case with \(\beta_0 \neq \beta_1\), which is represented by equation (7.13),
quantile regressions can be done separately on observations with $D_i = 0$ and $D_i = 1$ to obtain

$$\text{Quant}_\theta(Y_{ii} \mid X_i, D_i = 1) - \text{Quant}_\theta(Y_{ii} \mid X_i, D_i = 0)$$

$$= \alpha^\theta + X_i(\beta^\theta - \beta^\theta_0) + \text{Quant}_\theta(U^\theta_{ii} \mid X_i, D_i = 1)$$

$$- \text{Quant}_\theta(U^\theta_{ii} \mid X_i, D_i = 0)$$

$$= \alpha^\theta + X_i(\beta^\theta - \beta^\theta_0) + \text{Quant}_\theta(U^\theta_{ii} - U^\theta_{ii} \mid X_i, D_i = 1)$$

$$+ \text{Quant}_\theta(U^\theta_{ii} \mid X_i, D_i = 1) - \text{Quant}_\theta(U^\theta_{ii} \mid X_i, D_i = 0).$$

Thus, the selection bias term disappears when the CIA holds. Thus,

$$\text{QTE} = \alpha^\theta + X_i(\beta^\theta - \beta^\theta_0) + \text{Quant}_\theta(U^\theta_{ii} - U^\theta_{ii} \mid X_i, D_i = 1).$$

Alternatively, when selection is based on unobservables, a two-step estimation procedure can be used. In first step, the selection bias term is quantified using the correlation between $(U^\theta_{ii}, U^\theta_{ii})$ and $U_{2i}$ where the selection bias is given by

$$\text{Quant}_\theta(U^\theta_{ii} \mid X_i, D_i = 1) - \text{Quant}_\theta(U^\theta_{ii} \mid X_i, D_i = 0)$$

$$= \text{Quant}_\theta(U^\theta_{ii} \mid U_{2i} > -Z_i \gamma') - \text{Quant}_\theta(U^\theta_{ii} \mid U_{2i} \leq -Z_i \gamma').$$

In second step, this term is included in the equation as an independent variable. Thus, omitted variable bias through unobservables can be corrected while estimating $\text{QTE}$.

This chapter discusses various estimation methods that are used to estimate the mean and quantile effects of contracting on independent and contract growers, solving the potential selection bias problem based on observables and unobservables. The econometric methods reviewed in this chapter provide the necessary techniques to estimate not only the average effects of contracting but how contracting affects individual growers in the overall distribution of growers in the following chapter.
Chapter 8:

Estimation of the Profit Effects of Hog Contracting

8.1 Introduction

This chapter uses the estimation methodology of chapter 7 to estimate the profit effects of hog contracting. This is done using revealed preference data from the well-known Agricultural Resource Management Survey (ARMS) for 2004. The major difficulty in implementing a nonexperimental evaluation strategy is choosing among the wide variety of estimation methods available in the literature. This choice is important given the substantial evidence that impact measures are often highly sensitive to the estimators chosen (Lalonde, 1986). As described in chapter 7, a number of cross section methods have been proposed in the evaluation literature that deals with selection bias in treatment effect models. Two broad groups of methods allowing potential selection bias in estimating the effect of contracting on contract growers are examined here. The cross section methods that rely on the assumption that only observable factors affect the decision to be a contract grower are OLS and propensity scoring methods. The widely used cross-section method that attempts to allow for selection based on unobservables is Heckman’s two-step method.

However, each of these treatment effect methods focuses on average treatment effects. Such standard methodologies may miss how contracts affect gains differently at different points on the conditional profit distribution. For example, while contracting may have positive gains on average, the gains from contracting may not be positive at all points of the conditional profit distribution. Measuring effects on average gains may obscure zero effects at some points of the distribution and negative effects at others. That
is, while estimating how contracting affects average growers’ profits, useful information would also indicate how high-profit growers are affected differently from low-profit growers and whether some growers are worse off with contracting. This is especially relevant for measuring hog contract gains if this analysis is to be relevant for policy advice. Section 8.2 describes the available dataset and provides summary statistics of the variables used in this study. Section 8.3 presents estimation results for the first step of Heckman’s two-step estimation method. Section 8.4 reports estimation results for revenues and costs obtained with Heckman’s two-step estimation method. Section 8.5 estimates gains from contracting for contract and independent hog growers and explores the incidence of negative gains from contracting. Section 8.6 presents quantile regression results. Finally, section 8.7 summarizes conclusions.

8.2 Data Description

8.2.1 ARMS hog data

To estimate the effect of contracting on contract hog growers, I employ data from the Agricultural Resource Management Survey Phase III, Hogs Production Practices and Costs and Returns Report, Version 4, for 2004 (hereafter ARMS III V4) data. This survey conducted by USDA collected information from a cross section of U.S. hog operations chosen from a list of farm operations maintained by the National Agricultural Statistics Service (NASS). This particular version of the ARMS incorporated questions to obtain information on measures of farm size and financial characteristics, production costs, production facilities and practices, business arrangements, and farm operator
characteristics. Additional information on the other control variables such as state- and county-level characteristics is obtained from the 2002 US Agricultural Census.

The ARMS III V4 target population was farms with 25 or more hogs on the operation during 2004 and included 1,414 hog growers from 19 states (AR, CO, GA, IL, IN, IA, KS, KY, MI, MN, MO, NE, NC, OH, OK, PA, SD, VA, and WY). From the survey, NASS produced 1232 usable observations, of which 34 were deleted because of extensive missing data. As shown in Table 2, out of the final 1198 observations, 331 operations are farrow-to-finish, 72 are farrow-to-feeder pig, 478 are feeder pig-to-finish, 96 are farrow-to-weanling, 83 are weanling-to-feeder pig, and 138 are mixed producers.

<table>
<thead>
<tr>
<th>Table 2. Observed types of hog production and contracting operations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Operation</strong></td>
</tr>
<tr>
<td>Contract</td>
</tr>
<tr>
<td>Independent</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Only 1 farrow-to-finish producer used production contracts and only 3 weanling-to-feeder pig producers operated independently. Out of 72 farrow-to-feeder pig producers 19 used production contracts and out of 96 farrow-to-weanling producers only 19 operated independently. Only 31 out of 138 mixed producers are contract producers. Because the greatest mix of behavior is found in the feeder pig-to-finish category, and to maintain maximum comparability by choosing a single type of operation, this study focuses only on the 478 feeder pig-to-finish producers.

Of the 478 feeder pig-to-finish producers, 169 are independent producers and 309 are contract producers. Of these 309 contract producers, 245 of them use production contracts exclusively, 58 of them use both production and marketing contracts, 4 of them
use both production contracts and cash or open market sales, and 2 of them use all of these marketing arrangements. Two of the contract growers use homegrown feed, and thus are excluded. To estimate gains from production contracts precisely, these 66 producers are dropped to eliminate the complications of other confounding factors, and the remaining 243 contract producers are used for this study. Thus, of the 412 feeder pig-to-finish producers used in this analysis, 169 are independent producers and 243 are contract producers.

From these data, 24 observations for small growers (producing less than 275 hogs in 2004) were discarded for purposes of estimation for a variety of reasons. First, conditions differ widely in relative terms among these observations. Second, only two observations among this group are contract growers so reliability of comparisons for this group is limited. Third, an analysis of outliers in the data revealed that 13 of the 22 observations on independent growers in this group had distinct outliers (more than 3.25 standard deviations from the mean), thus challenging the reliability of these observations. Fourth, although some less distinct outliers existed in the data for other size groups, statistical tests revealed no significant differences in behavior whereas data for small growers including these extreme outliers caused statistically significant differences in estimated behavior (for example, rejecting expected utility maximization) and sufficient data for analysis of small growers are not available otherwise. However, 8 more observations producing at least 275 hogs in 2004 were also discarded because they contain distinct outliers (one or more of the factor input variables for an observation were more than 3.25 standard deviations from the mean on a per head basis). Four of these were contract observations.
8.2.2 Variables included

The survey data available for this study include over a thousand variables. The endogenous variable of interest in this analysis is the profit per animal of a grower, which is the difference between total revenue received and total production cost incurred by the grower. In the survey data, revenue for contract hog growers is measured as total payments received from contract production while revenue for independent growers is measured by cash or open market sales plus any additional receipts from marketing contracts after deducting marketing expenses.

Production cost involves both operating and ownership costs. Operating cost consists of costs for labor; feed; feeder pigs; fuel and lubrication (which includes electricity); veterinary and medical services; custom services; bedding and litter; marketing; and operating interest. Ownership costs include the annualized cost of maintaining the capital investment in hog facilities and equipment (capital recovery costs). Overhead costs consist of general overhead, non real estate property taxes and insurance, maintenance and repair costs, and land costs.

Table 3 presents these cost components as shares of total costs for the contract and independent operations used in this analysis. The major costs for contract operations are feeder pig costs at 41.0 percent followed closely by feed costs at 37.2 percent. For independent operations, feed costs are most important at 38.4 percent followed closely by feeder pig costs as 31.7 percent.
Table 3: Input cost as a share of total operation cost

<table>
<thead>
<tr>
<th>Variables</th>
<th>Percent of independent operation cost</th>
<th>Percent of contract operation cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (Standard deviation)</td>
<td>Mean (Standard deviation)</td>
</tr>
<tr>
<td>Labor</td>
<td>9.5% (6.4%)</td>
<td>5.7% (4.2%)</td>
</tr>
<tr>
<td>Feed</td>
<td>38.4% (10.0%)</td>
<td>37.2% (12.6%)</td>
</tr>
<tr>
<td>Feeder pigs</td>
<td>31.7% (11.3%)</td>
<td>41.0% (12.4%)</td>
</tr>
<tr>
<td>Fuel &amp; lubricant</td>
<td>1.6% (1.6%)</td>
<td>1.1% (0.8%)</td>
</tr>
<tr>
<td>Veterinary &amp; medical</td>
<td>1.1% (1.3%)</td>
<td>0.7% (0.8%)</td>
</tr>
<tr>
<td>Custom services</td>
<td>0.4% (1.0%)</td>
<td>0.7% (1.1%)</td>
</tr>
<tr>
<td>Bedding &amp; litter</td>
<td>0.1% (0.2%)</td>
<td>0.0% (0.2%)</td>
</tr>
<tr>
<td>Marketing expense</td>
<td>0.7% (0.6%)</td>
<td>1.1% (0.3%)</td>
</tr>
<tr>
<td>Operating interest</td>
<td>0.6% (0.1%)</td>
<td>0.7% (0.1%)</td>
</tr>
<tr>
<td>Total operating cost</td>
<td>4.5% (4.3%)</td>
<td></td>
</tr>
<tr>
<td>General overhead</td>
<td>1.9% (2.3%)</td>
<td>1.8% (1.5%)</td>
</tr>
<tr>
<td>Taxes &amp; insurance</td>
<td>1.0% (0.9%)</td>
<td>0.6% (0.7%)</td>
</tr>
<tr>
<td>Maintenance &amp; repairs</td>
<td>1.4% (1.4%)</td>
<td>1.0% (1.3%)</td>
</tr>
<tr>
<td>Land</td>
<td>0.1% (0.3%)</td>
<td>0.0% (0.1%)</td>
</tr>
<tr>
<td>Total overhead cost</td>
<td>4.4% (3.4%)</td>
<td></td>
</tr>
<tr>
<td>Capital recovery cost</td>
<td>11.5% (6.3%)</td>
<td>8.5% (4.5%)</td>
</tr>
</tbody>
</table>

Because this study focuses on identifying the benefits to growers from contracting, the proportion of these costs under contracting that are borne by the growers is critical. Obviously, independent growers bear the total costs of independent operation. The data show for all observations that contractors bear 100 percent of the cost of feeder pigs whereas contract growers bear 100 percent of the cost of labor, general overhead, taxes and insurance, repairs, land, and capital recovery. Also, in all but three exceptions, contractors bear 100 percent of feed costs. For these exceptions, two contract growers paid 100 percent of feed costs and another paid 10 percent of feed costs. Because these growers accounted for such a small share of the sample and too few observations are available to generate reliable estimates of how they differ, these three observations were discarded from the sample.
For fuel and lubricant, veterinary and medical, custom supplies, and bedding and litter expenses, contract growers’ cost shares ranged from a minimum of zero to a maximum of 100 percent with averages of 93 percent, 9 percent, 39 percent, and 37 percent, respectively. Because these four categories of expenses amount to a total of only 2.5 percent of total expenses for contract operations, separate modeling of these shares is regarded as unnecessary. Thus, these four categories of costs are combined into a single category of “other” costs for empirical purposes, although an accurate measure of the cost share for this category of costs is included in the model.

Of the remaining costs, marketing costs are deducted from revenues for purposes of comparability of contract and independent growers. Costs of items that do not contribute to productivity are not included in the production model but are included in the cost model. These variables are interest, general overhead, taxes and insurance, and repairs and maintenance.

Thus, the following variables are defined for the production model.

**Operator Characteristics**

\[ revenue = \text{revenue per animal less marketing expenses (in dollars)} \]

\[ contract = \text{contracting indicator (1 if the operator is a contract grower; 0 otherwise)} \]

\[ op\_exp = \text{years in the hog business} \]

\[ op\_educ = \text{education indicator (1 if operator has 4 years of college or a higher level of education; 0 otherwise)} \]

\[ op\_ocup = \text{occupation indicator (1 if the operator’s main occupation is farming; 0 otherwise)} \]
special = specialization indicator (1 if the operator is specialized in livestock; 0 otherwise)

**Farm Characteristics**

assets = assets per animal (in thousand dollars)

debt = debt per animal (in thousand dollars)

labor = labor cost per animal (in dollars)

capital = capital recovery cost per animal (in dollars)

feed = feed cost per animal (in dollars)

pigs = feeder pig cost per animal (in dollars)

other = other cost per animal (fuel & lubricant, veterinary and medicine, custom supplies, and bedding and litter in dollars)

n = number of animals (in thousands)

**Regional Characteristics**

rn = regional indicator for northern states (1 if MI, MN, PA, SD; 0 otherwise)

rs = regional indicator for southern states (1 if AR, GA, MO, KY; 0 otherwise)

re = regional indicator for eastern states (1 if NC, VA; 0 otherwise)

rw = regional indicator for western states (1 if CO, KS, OK, NE, WY; 0 otherwise)

rm = regional indicator for mid-western states (1 if IL, IN, IA, OH; 0 otherwise)

**Others**

ncr = county average net cash returns per farm (in thousands of dollars)

---

21 No observations are from states other than those included in one of these regions.
8.2.3 Descriptive statistics

Summary statistics of the contract and independent operations given in Table 4 suggest that contract operations have mainly appealed to less educated entrants to the hog industry. But the educational differences between the two groups are not statistically significant. The difference in hog experience between the two groups is statistically significant. More than 49 percent of contract feeder pig producers had been producing hogs less than 10 years in 2004, while 66 percent of independent producers had been in business 10 years or more. The difference in main occupation of the two groups is also statistically significant. Farming is the main occupation for 93 percent of independent growers but only 71 percent of the contract growers. The specialization in livestock production is significantly different between the two groups with a $p$-value of 0.090.

The average total asset value per animal for independent operations is $1,023, which is almost four times the average total asset value per animal for contract operations, $289. The average debt per animal for independent operations is $157, which is over three times the average debt per animal for contract operations, $47. Obviously, from means of assets and debts for both types of operations in Table 4, operations with a high borrowing capacity tend to be independent operators rather than contract operators.
### Table 4. Summary statistics for feeder pig-to-finishing operations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Independent operations</th>
<th>Contract operations</th>
<th>t-ratio&lt;sup&gt;b&lt;/sup&gt;</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard deviation</td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>contract</td>
<td>0.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>revenue</td>
<td>127.818</td>
<td>32.140</td>
<td>130.309</td>
<td>19.654</td>
</tr>
<tr>
<td>op_educ</td>
<td>0.252</td>
<td>0.436</td>
<td>0.244</td>
<td>0.430</td>
</tr>
<tr>
<td>op_exp</td>
<td>18.014</td>
<td>12.071</td>
<td>10.731</td>
<td>5.961</td>
</tr>
<tr>
<td>op_ocup</td>
<td>0.930</td>
<td>0.256</td>
<td>0.714</td>
<td>0.453</td>
</tr>
<tr>
<td>special</td>
<td>0.671</td>
<td>0.471</td>
<td>0.752</td>
<td>0.433</td>
</tr>
<tr>
<td>assets</td>
<td>1.023</td>
<td>1.364</td>
<td>0.289</td>
<td>0.463</td>
</tr>
<tr>
<td>debt</td>
<td>0.157</td>
<td>0.258</td>
<td>0.047</td>
<td>0.074</td>
</tr>
<tr>
<td>labor</td>
<td>13.929</td>
<td>13.894</td>
<td>6.485</td>
<td>6.902</td>
</tr>
<tr>
<td>capital</td>
<td>15.236</td>
<td>9.294</td>
<td>9.177</td>
<td>5.670</td>
</tr>
<tr>
<td>feed</td>
<td>50.819</td>
<td>19.304</td>
<td>42.229</td>
<td>18.774</td>
</tr>
<tr>
<td>pigs</td>
<td>40.967</td>
<td>16.062</td>
<td>44.957</td>
<td>14.883</td>
</tr>
<tr>
<td>other</td>
<td>4.228</td>
<td>3.521</td>
<td>2.654</td>
<td>1.901</td>
</tr>
<tr>
<td>n</td>
<td>4.290</td>
<td>5.339</td>
<td>8.814</td>
<td>8.888</td>
</tr>
<tr>
<td>rn</td>
<td>0.259</td>
<td>0.439</td>
<td>0.222</td>
<td>0.417</td>
</tr>
<tr>
<td>re</td>
<td>0.000</td>
<td>0.000</td>
<td>0.363</td>
<td>0.482</td>
</tr>
<tr>
<td>rs</td>
<td>0.091</td>
<td>0.288</td>
<td>0.107</td>
<td>0.310</td>
</tr>
<tr>
<td>rw</td>
<td>0.154</td>
<td>0.362</td>
<td>0.068</td>
<td>0.253</td>
</tr>
<tr>
<td>rm</td>
<td>0.497</td>
<td>0.502</td>
<td>0.239</td>
<td>0.428</td>
</tr>
<tr>
<td>ncr</td>
<td>2.943</td>
<td>1.509</td>
<td>4.577</td>
<td>3.418</td>
</tr>
</tbody>
</table>

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<sup>a</sup>The variables revenue, asset, debt, and ncr are measured in thousands of dollars, labor, capital, feed, pigs, and other are measured in dollars, and n is measured in thousands of animals. All other variables are indicator variables as defined in the text. See the text for variable definitions.

<sup>b</sup>The t-statistics are for the hypothesis that the mean of a variable is the same for independent and contract operations. The p-values give the probabilities of more extreme t-ratios under the hypothesis of a zero mean. Asterisks (*) denote non-central t-tests, which were used when F-tests rejected the hypothesis of equal variances between the two groups at the 5 percent level.

The average labor and capital recovery costs per animal are $13.93 and $15.24 for independent operations and $6.49 and $9.18 for contract operations, respectively. Independent operations incur significantly more labor and capital cost per animal compared with their contracting counterparts. Similar differences are observed for feed and other inputs. However, feeder pig cost is significantly more for contract operations.
The size differential between contract and independent operations is also apparent. Independent operations average less than half the size of contract operations, a difference that is highly significant. Most contract feeder pig farms are highly specialized industrial-scale operations with an average of more than 8,814 head sold per year compared to an average of about 4,290 head sold from independent operations.

8.3 Two-step Estimation

As emphasized in section 8.1, choosing among the wide variety of estimation methods available in the literature is crucial. As described in section 7.3.2 of chapter 7, before applying OLS and a variety of propensity score matching methods, an assessment of the plausibility of the CIA for this hog data is required. In order to satisfy the CIA, all variables that jointly influence the contract decision and profit must be included as right hand side variables. This requires that all factors affecting the decision to become a contract grower are included as covariates in the profit equation and in the contract decision equation that eventually provides propensity scores for matching. However, propensity score matching methods require that errors in the contract decision equation are uncorrelated with those in the profit equation. If this assumption is valid and all the variables in contract decision equation are included in the profit equation, then the CIA is satisfied and the coefficient on the contracting indicator variable in the OLS regression of profit identifies the effect of contracting on contract growers, and matching methods work like a randomized experiment.

The dataset, described in section 8.2, contains a rich set of variables. Thus, arguments supporting the CIA are plausible if contract and independent operators are
similar in characteristics and production practices. The comparisons in Table 4, however, show that contract growers differ significantly from their independent counterparts in many aspects including experience, occupation, assets, debt, feed expense, operation scale, and regional and county characteristics. However, unobservable differences in entrepreneurial skill and quality of feed and other inputs and genetic quality of animals may also exist.

*Key and McBride* (2003) argue that contractor-provided goods and services, such as veterinary care, feed, and especially the genetic quality of the animals, are superior to those available to an independent producer. These yield healthier animals and produce more weight gain from an equal amount of feed, labor, and capital. If these unobservable factors also affect the contract decision, then the CIA will not hold and CIA-based methods will not produce unbiased estimates of the contracting effect.

Thus, I begin with Heckman’s two-step estimation method, which does not require the CIA. As described in section 7.3.4 of chapter 7, this estimator relies simply on the assumption that the errors in the selection model and the profit model are jointly normally distributed.

8.3.1 First-step estimation

In the two-step procedure, the inverse Mill’s ratio (*imr*) is calculated in the first step and then used as an independent variable to estimate the profit equation in the second step. The inverse Mill’s ratio used in the two-step method is estimated using a probit model where the dependent variable, *contract*, is an indicator variable that takes the value 1 if the grower participates in contracting and zero otherwise. The independent
variables are chosen to capture the operator and farm level characteristics, differences in operation size, regional characteristics, and county-level variation. Table 5 gives the results of the probit estimation that are used in the following second step estimation of profit. The results suggest that the model significantly and correctly predicts 83 percent of operators’ choices.

### Table 5: Probit model estimates of the contracting probability

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated coefficient</th>
<th>Standard error</th>
<th>z-statistics</th>
<th>p-values</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>0.026</td>
<td>0.018</td>
<td>1.44</td>
<td>0.149</td>
<td>-0.01 - 0.06</td>
</tr>
<tr>
<td>labor</td>
<td>-0.583</td>
<td>0.230</td>
<td>-2.54</td>
<td>0.011</td>
<td>-1.03 - -0.13</td>
</tr>
<tr>
<td>assets</td>
<td>-0.444</td>
<td>0.150</td>
<td>-2.97</td>
<td>0.003</td>
<td>-0.74 - -0.15</td>
</tr>
<tr>
<td>debt</td>
<td>-1.068</td>
<td>0.883</td>
<td>-1.21</td>
<td>0.227</td>
<td>-2.80 - 0.66</td>
</tr>
<tr>
<td>op_educ</td>
<td>-0.281</td>
<td>0.192</td>
<td>-1.46</td>
<td>0.143</td>
<td>-0.66 - 0.10</td>
</tr>
<tr>
<td>op_ocup</td>
<td>-1.014</td>
<td>0.273</td>
<td>-3.72</td>
<td>&lt;0.001</td>
<td>-1.55 - -0.48</td>
</tr>
<tr>
<td>op_exp</td>
<td>-0.050</td>
<td>0.009</td>
<td>-5.25</td>
<td>&lt;0.001</td>
<td>-0.07 - -0.03</td>
</tr>
<tr>
<td>special</td>
<td>-0.724</td>
<td>0.212</td>
<td>-3.42</td>
<td>0.001</td>
<td>-1.14 - -0.31</td>
</tr>
<tr>
<td>rw</td>
<td>-1.372</td>
<td>0.317</td>
<td>-4.32</td>
<td>&lt;0.001</td>
<td>-1.99 - -0.75</td>
</tr>
<tr>
<td>rn</td>
<td>-0.949</td>
<td>0.276</td>
<td>-3.44</td>
<td>0.001</td>
<td>-1.49 - -0.41</td>
</tr>
<tr>
<td>rm</td>
<td>-1.169</td>
<td>0.238</td>
<td>-4.92</td>
<td>&lt;0.001</td>
<td>-1.64 - -0.70</td>
</tr>
<tr>
<td>re</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A - N/A</td>
</tr>
<tr>
<td>ncr</td>
<td>0.094</td>
<td>0.036</td>
<td>2.64</td>
<td>0.008</td>
<td>0.02 - 0.16</td>
</tr>
<tr>
<td>constant</td>
<td>3.353</td>
<td>0.487</td>
<td>6.89</td>
<td>&lt;0.001</td>
<td>2.40 - 4.31</td>
</tr>
</tbody>
</table>

The p-values give the smallest significance level at which the hypothesis of a zero coefficient is rejected. The dependent variable is the indicator variable for contracting, contract. The results are produced with 377 observations obtaining a pseudo $R^2$ of 0.4187, a log likelihood of -145.45, and a likelihood ratio chi-square statistic of 125.89, corresponding to a p-value less than 0.0001.

For this probit estimation n represents number of animals in thousands.

No independent growers were observed in the east, i.e., “N/A” means not applicable.

A legitimate question for this type of model is whether all of the right hand side variables are exogenous. The variable most likely to be endogenously influenced by the decision to contract is labor. That is, growers who contract could require less labor as a result of contractor-provided inputs, rather than the probit conclusion that those who have less labor available are more likely to contract. Similarly, the exogeneity question might also be raised for assets and debt although these are long-term variables less likely to be
influenced by an annual decision to contract. Thus, Smith-Blundell tests of exogeneity were performed for labor, assets and debt using instruments such as the interest rate on debt, the unpaid labor wage rate, the paid labor wage rate, and total land (not land used for hog production).

A Smith-Blundell test of exogeneity for labor produces a chi-square statistic of 0.37 with 1 degree of freedom, which has a $p$-value of 0.5424, and thus is far from rejecting exogeneity. The hypothesis of joint exogeneity of labor, assets, and debt is not rejected at the 5 percent level by a chi-square statistic of 7.19 with 3 degrees of freedom. While this test has a $p$-value of 0.066, which is close to rejection at the 5 percent level, it suffices to support estimation without endogeneity corrections, particularly given the strength of the test for exogeneity of labor and the heuristic strength of the arguments that assets and debt are long term accumulation variables and thus are not likely to be influenced by short-term contracting decisions.

Table 5 shows that the most significant characteristics affecting the decision to contract are specialization both in farming ($op\_ocup$) and livestock ($special$), operator experience in the hog business ($op\_exp$), and, to a lesser extent, operator education ($op\_educ$). Each makes contracting less likely, and all except education are highly significant beyond the 1 percent level. These results appear plausible and in harmony with the simple statistics in Table 4 where the difference between contract and independent growers is highly significant for all of these except education.

Intuitively, a more experienced, educated, and specialized full-time farmer is more likely to possess the know-how that a contractor can provide to a less experienced, less educated, and part-time grower. The lower level of specialization among independent
growers in Table 4 therefore appears to be a spurious relationship rather than indicative of a causal relationship. Thus, an important explanation of the choice to contract appears to be the comparative disadvantage that some growers have in know-how.

Regional indicators also play a highly significant role in determining contract decisions. Regional indicators \( rw, rn, \) and \( rm \) (the effect of \( rs \) is included in the constant term) are each significant beyond the 1 percent level, all with negative coefficients, suggesting that contracting is significantly more likely in the south. As described by Key and McBride (2003), regional differences such as in climate, technologies, factor quality, or prices are unobservable. These indicators may reflect such differences. The significant negative signs on these three regions is in harmony with the regional concentrations of contracting as reflected in Table 4, which shows that contracting is more likely in the south but less likely in the west, north, and midwest.

Table 5 also reveals that assets decrease the likelihood of contracting with significance beyond the 1 percent level. The negative sign of the coefficient is expected because, if total assets serve as a proxy for risk-aversion, then poorer growers are more likely to be more risk averse (under decreasing absolute risk aversion) and thus more likely to operate under a sharing contract. Thus, risk reduction appears to be one of the motivations for contracting in hog production.

The debt coefficient is also negative although insignificant. Debt can have several effects. On the one hand, it reduces net worth, which would increase wealth and thus reduce risk aversion. On the other hand, debt causes greater financial vulnerability thus suggesting that a more risk-averse operator would tend to carry less debt. Debt servicing (paying interest and principal) requires more stable income generation, which is more
likely with a sharing contract. The latter explanation is consistent with the summary statistics in Table 4, which show that contract operations carry less debt.

To sort out the separate roles of debt and wealth, the latter of which is likely a better proxy for risk aversion, the component of the probit equation in Table 5 represented by $-0.444 \text{ assets} - 1.068 \text{ debt}$ can be rewritten as $-0.444 \text{ net worth} - 1.512 \text{ debt}$ where $\text{net worth} = \text{assets} - \text{debt}$. Thus, more net worth clearly tends to independent operation following the risk aversion argument, whereas the additional negative debt effect may represent the lack of credit availability for contract growers not represented elsewhere in the model. But it may also represent a risk aversion effect not captured by net worth.

The results of Table 5 also show that the size of the hog operation affects the choice to contract. Size, as measured by the number of animals ($n$), is significant only at the 15 percent level, but the positive coefficient implies that the probability of contracting rises with the operation size.

Table 5 further shows that operations with higher per animal labor input are less likely to contract. This is also expected because, as described in section 8.2.3, independent operations incur more than twice as much labor cost per animal compared to contract operations. Intuitively, because contract operations are significantly less labor intensive than independent operations, households with more alternative employment opportunities are likely to prefer contracting to independent operations. This intuition is consistent with the higher observed specialization in farming among independent growers.
Finally, Table 5 suggests that operators living in a county with higher net cash returns per farm \((ncr)\) are more likely to be contract growers. This result may reflect the tendency of contractors to operate in counties that have characteristics favorable for hog production.

8.3.2 Theoretical relevance of selection results

An important suggestion from these results is that risk reduction matters for hog growers and is one of the motivations for contracting. As described in section 5.3.1 of chapter 5, one strain of the hog contracting literature considers risk reduction as one of the main reasons why growers are motivated to contract production. The results of the theoretical model in sections 6.4.3 and 6.4.4 of chapter 6 show how risk can be important in measuring growers’ gains from contracting, and, thus, the relevance of risk in explaining the decision to contract.

As indicated by the literature reviewed in section 2.6.2 of chapter 2 and section 5.3.1 of chapter 5, the relevance of risk for hog contracting is disputed. This dispute is based on the argument that, if wealth is taken as a proxy for risk-aversion, then poorer growers are more likely to be more risk averse (under decreasing absolute risk aversion). Thus, they are more likely to operate under a production sharing contract, contrary to the observed pattern where large growers are more likely to contract. However, this argument only applies for a given level of risk.

A more careful analysis of the facts from contract hog growing reveals the fallacy of this argument. The argument is flawed because the size of an operation in animal numbers is used as a proxy for wealth. The results here show that contract growers tend
to have less assets and less net worth than independent growers for a given size of operation, as measured by number of animals. This is the more relevant comparison when a grower with a given size of facility is deciding whether to contract or not.

Finally, as demonstrated in chapters 2 through 5, other factors beyond risk motivate agents to contract. One such factor is the capital constraint (Kliebenstein and Lawrence, 1995). The probit results suggest that the capital constraint matters. More assets relax the capital constraint whereas poorly capitalized growers may be required by lenders to have contracts to get what credit they can. Less debt for a given amount of assets may reflect this motivation for contracting. Further, this intuition explains the additional intensity of the debt variable compared to net worth. For example, Section 2.3 of chapter 2 explains how a wealth constraint can impose a sharing contract on the parties even if the agents are risk neutral. Thus, the implicit significant negative coefficient on net worth may represent both a capital-constraint and risk-aversion.

Aside from statistical significance, the economic importance of individual factors in Table 5 is also of interest. To measure economic importance, the estimated coefficients in Table 5 can be converted into an effect of the variable on the probability of contracting. Evaluating all other variables at their means across both contract and independent observations, the change in the predicted probability due the average difference in labor between contract and independent growers is –0.21. Similar calculations for \( n, \text{assets}, \text{debt}, \text{op}_\text{exp}, \) and \( ncr \) yield estimated effects of .01, –0.16, 0.39, –0.02, and 0.03 respectively. Thus, debt appears to have the most important economic effect among the economic variables, suggesting that credit availability plays a strong role in the decision to contract. This ranking of economic importance is somewhat
different than for statistical significance in Table 5 in which debt has the lowest
significance.

Similarly, each of the estimated coefficients of indicator variables can be
converted into a discrete change in probability associated with the indicator variable.
Again, evaluating all other variables at their means across both contract and independent
observations, the changes in the predicted probability due to a discrete change from zero
to 1 in \textit{op\_educ}, \textit{op\_ocup}, \textit{special}, \textit{rw}, \textit{rn}, and \textit{rm} are $-0.11$, $-0.30$, $-0.24$, $-0.50$, 
$-0.36$, and $-0.43$, respectively. Thus, regional location is the most economically
important determinant in the decision to contract with contracting least likely in West
followed by the Midwest and North, while contracting is most likely in the South. These
results may be largely driven by contractor location. Among the other indicator variables,
occupational specialization is the most economically important variable affecting the
contracting decision.

\section*{8.4 Second-Step Estimation of Revenues and Costs}

Depending on whether the effects of observable and unobservable characteristics
on profit differ between contract and independent operations, three variants of the two-
step estimator are discussed in section 7.3.4 of chapter 7. In the standard and most
restrictive case of section 7.3.4.1, the effects of observable and unobservable
characteristics on profit must be the same for contract and independent growers so that
the contracting effect is the shift term associated with the intercept. If the effects of
observable characteristics on profit differ with contract status as in section 7.3.4.2, then
the contracting effect also depends on the slope coefficients of the profit equation. If both
observable and unobservable characteristics differ by contract status, then the approach of section 7.3.4.3 is required. For the empirical analysis, I start with this most general case and then test whether the other variants are applicable in section 8.4.2.

8.4.1 Second-step estimation of revenues

The regression specification for this study cannot follow standard profit function approaches that require positive profits because of many profit observations have negative values. For this reason, the profit equation is estimated here by decomposing profit into revenues and costs. In general, decomposition allows more accurate estimation (a general principle from simultaneous equation estimation). For this purpose, second-step estimation is applied to estimation of revenues and variable costs separately. Gains from contracting are then calculated in section 8.5 by subtracting both variable costs and overhead costs (viewed as fixed) from revenue. This allows estimation of how the difference in profit between contract and independent operation depends on various individual grower characteristics and circumstances.

8.4.1.1 The production model for independent growers

Suppose the aggregate weight gain of the herd of an independent grower can be represented as

\[ Q = AK^g X^l_j X^f_j X^p_j X^o \epsilon, \]

where \( K \) represents unobserved fixed physical capital and \( X^l_j, X^f_j, X^p_j \), and \( X^o \) represent unobserved variable input quantities of labor, feed, feeder pigs and other inputs (fuel & lubricant, veterinary & medicine, custom supplies, and bedding & litter), respectively.
is a disturbance representing stochastic variation of production among growers, and $A, \phi, \text{ and } \eta, \ i = l, f, p, o$, are unknown coefficients. Thus, the aggregate revenue for the herd can be represented as

$$R = pQ = pAK^\phi X_l^\eta X_f^\eta X_p^\eta X_o^\eta e^\epsilon,$$

(8.1)

where $p$ is the mean output price among growers and $\epsilon$ is redefined to incorporate random variation in the output price among growers.

Variable cost is represented by

$$V = C_l + C_f + C_p + C_o = w_lX_l + w_fX_f + w_pX_p + w_oX_o,$$

where

$$C_i = w_iX_i, \ i = l, f, p, o,$$

(8.2a)

are the variable costs of labor, feed, feeder pigs, and other inputs, respectively, and the $w_i$’s are the respective input prices. Short-run profit of an independent grower is thus

$$\pi = R - V = pAK^\phi X_l^\eta X_f^\eta X_p^\eta X_o^\eta e^\epsilon - \sum_i w_iX_i$$

Capital recovery cost of the grower is represented by

$$C_k = rK$$

(8.2b)

where $r$ is an unobserved rate of recovery on fixed physical capital that must be maintained for profitable operation. Thus, net profit is

$$\Pi = R - V - C_k - O = pAK^\phi X_l^\eta X_f^\eta X_p^\eta X_o^\eta e^\epsilon - \sum_i w_iX_i - rK - O$$

(8.3)

where $O$ represents the overhead cost of items that do not contribute to grower productivity, e.g., taxes, insurance, maintenance and repair, etc.

With a utility function $U(\Pi)$ satisfying $U'(\Pi) > 0$ and $U''(\Pi) < 0$, the expected utility maximization problem is
\[
\max_{X_l, X_f, X_o} \text{EU}(pAK^\theta X_l^{\eta_l} X_f^{\eta_f} X_o^{\eta_o} e^\varepsilon - \sum w_i X_i - rK - O),
\]
which has first-order conditions
\[
E(U' \cdot (\eta_i pAK^\theta X_l^{\eta_l} X_f^{\eta_f} X_o^{\eta_o} X_i^{-1} e^\varepsilon - w_i)) = E(U' \cdot \eta_i p(\bar{Q}/X_i)e^\varepsilon - w_i))
\]
\[
= \eta_i p(\bar{Q}/X_i)E(U' \cdot e^\varepsilon) - w_i E(U') = 0,
\]
\[i = l, f, p, o,\]
where \(\bar{Q} = AK^\theta X_l^{\eta_l} X_f^{\eta_f} X_o^{\eta_o}.\) These conditions can be represented by
\[
\frac{\eta_i p\bar{Q}}{w_i X_l} = \frac{\eta_f p\bar{Q}}{w_f X_f} = \frac{\eta_p p\bar{Q}}{w_p X_p} = \frac{\eta_o p\bar{Q}}{w_o X_o} = \frac{E(U'(\Pi))}{E(U'(\Pi)e^\varepsilon)}.
\]
Multiplying through by \(e^\varepsilon\) and dropping the unobservable right-hand expression,
these conditions can be expressed as
\[
\frac{R}{C_l} = \frac{R}{C_f} = \frac{R}{C_p} = \frac{R}{C_o},
\]
which yield conditions on cost shares, \(S_i = C_i/R,\ i = l, f, p, o,\) in the form
\[
\eta_i S_i = \eta_i S_i, \ i = p, f, o.
\]
For econometric purposes, errors in optimization can be added to these equations
obtaining
\[
\eta_i S_i = \eta_i S_i + \delta_i, \ i = f, p, o,
\]
where each \(\delta_i\) is a disturbance with mean zero.

In the survey data available for this study, data on capital \(K,\) feeder pig quantity \(X_f,\) and the quantity of other inputs \(X_o,\) are not available. Because some of the input quantities are not observable in this study, the empirical approach must depart from typical empirical production studies that are based on their observability. Rather, the comparable observable variables are the costs, \(C_k, C_l, C_f, C_p,\) and \(C_o.\) Substitution of (8.2a)-(8.2b) into the revenue expression (8.1) yields
\[ R = pr^{-\theta} w_i^{-\eta_i} w_p^{-\eta_p} w_o^{-\eta_o} A C_i^{\eta_i} C_p^{\eta_p} C_o^{\eta_o} e^e. \]

Because all observations in the survey data are for the same year, a reasonable assumption is that all the hog growers in the same region face essentially the same prices aside from unobservable random differences that can be further incorporated into \( \epsilon \). If the prices of output, variable inputs, and the rates of interest, taxes, and depreciation are constant within regions after adjustment for operator and farm characteristics, then

\[ pr^{-\theta} w_i^{-\eta_i} w_p^{-\eta_p} w_o^{-\eta_o} A \]

can be replaced by a function of variables representing these factors.

Suppose this function is of the form \( e^{z} \) where \( Z \) is a vector of indicator variables reflecting region, operator and farm characteristics. Then

\[ pr^{-\theta} w_i^{-\eta_i} w_p^{-\eta_p} w_o^{-\eta_o} A = e^{z} \]

so that the estimated revenue equation can be expressed as

\[ R = C_i^{\eta_i} C_p^{\eta_p} C_o^{\eta_o} e^z e^e \]

and the net profit equation in (8.3) becomes

\[ \Pi = C_i^{\eta_i} C_p^{\eta_p} C_o^{\eta_o} e^z e^e - \sum C_i - C_k - O. \]

With constant returns to scale, (8.5) can be written as

\[ R / n = (C_i / n)^{\phi} (C_i / n)^{\eta_i} (C_p / n)^{\eta_p} (C_o / n)^{\eta_o} e^{\nu} e^e \]

where \( n \) is number of animals. For non-constant returns to scale, the function can be written as

\[ R / n = (C_i / n)^{\phi} (C_i / n)^{\eta_i} (C_p / n)^{\eta_p} (C_o / n)^{\eta_o} n^{\zeta} e^{\nu} e^e \]

where \( \zeta = \phi + \eta_i + \eta_p + \eta_o - 1 \) and thus \( \zeta = 0 \) under constant returns to scale.
8.4.1.2 *Estimation of the production model for independent growers*

The system of estimable equations thus consists of (8.6) and the three equations in (8.4). In log form, equation (8.6) can be expressed linearly as

\[
\ln(R/n) = \phi \ln(C_i/n) + \sum_{i=l,p,f,o} \eta_i \ln(C_i/n) + \zeta \ln n + \nu Z + \varepsilon. \tag{8.7}
\]

A linear system of equations is then obtained by combining (8.7) and the three equations in (8.4),

\[
\begin{bmatrix}
\ln(R/n) \\
0 \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
\ln(C_l/n) & \ln(C_i/n) & \ln(C_p/n) & \ln(C_o/n) & \ln n & Z
\end{bmatrix}
\begin{bmatrix}
\phi \\
\eta_l \\
\eta_f \\
\eta_p \\
\eta_o \\
\zeta \\
\nu
\end{bmatrix} +
\begin{bmatrix}
\varepsilon \\
\delta_f \\
\delta_p \\
\delta_o
\end{bmatrix}, \tag{8.8}
\]

where each entry, aside from the column vector of parameters, represents a column of observations across all independent growers.

The system in (8.8) is estimated allowing different variances of disturbances for \( \varepsilon, \delta_f, \delta_p, \) and \( \delta_o, \) as well as possible covariances among the \( \delta_i \)’s because they contain information about common first-order conditions. The vector \( Z \) contains indicator variables reflecting operator education (\( op\_educ \)), operator occupation (\( op\_ocup \)), and region (\( re, rm, rn, \) and \( rw \)) as well as a location-specific variable reflecting local farming opportunities (\( ncr \)). To correct possible selection bias, an inverse Mill’s ratio (\( imr \)) based on results in Table 5 is also added to the system as a right-hand side variable in the first equation.

Because the optimization conditions in (8.4) represent a conjectural hypothesis, the joint applicability of the structure represented by (8.7) and the optimization conditions in (8.4) is first tested by testing applicability of the cross-equation parameter
constraints in the system in (8.8). That is, the model is first estimated without cross-equation parameter constraints where $S_i$ is associated with an alternative multiplicative parameter, say, $\eta_f^*, \eta_p^*$, and $\eta_o^*$, respectively, in each of the latter three equations.

Applicability of the cross-equation parameter constraints $(\eta_f = \eta_f^*, \eta_p = \eta_p^*, \eta_o = \eta_o^*)$ is not rejected at the 5 percent level with an $F$-statistic of 2.59, which has a p-value of 0.0518 with 3 and 555 degrees of freedom. Thus, the model in equations (8.1)-(8.8) appears to capture both the technology and behavior of independent hog growers reasonably well.\(^{22}\)

Another legitimate question for this type of model is whether all of the right hand side variables are exogenous. If right hand side endogeniety is present, it most likely comes through labor, feed, feeder pigs, and other costs. To consider this possibility, exogeneity of labor, feed, pig, and other costs was tested with a Hausman test using the labor wage rate, total land (not land used for hog production), operator age, assets and debt as instruments. The chi-square statistic is 18.17, which has a $p$-value of 0.1513 with 13 degrees of freedom. On this basis, the restricted model in (8.8) is estimated without endogeneity corrections for independent growers with results as reported in Table 6.

\(^{22}\) Another approach is to divide the sample into size classes. By dividing the sample into only two size classes, the $F$-tests for applicability of (8.8) have $p$-values far from rejection in each separate size class. By comparison, while the test statistic is close to rejection when combining all independent observations, the convenience of a coherent single representation for subsequent analysis is far more preferable and makes the results far more comprehensible.
### Table 6: Independent grower revenue per head (restricted case)\(^a\)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated coefficient</th>
<th>Standard error</th>
<th>(t)-ratios</th>
<th>(p)-values (^b)</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln(n))</td>
<td>0.177</td>
<td>0.015</td>
<td>11.79</td>
<td>&lt; 0.001</td>
<td>0.15 - 0.21</td>
</tr>
<tr>
<td>(\ln(labor))</td>
<td>0.118</td>
<td>0.018</td>
<td>6.46</td>
<td>&lt; 0.001</td>
<td>0.08 - 0.15</td>
</tr>
<tr>
<td>(\ln(capital))</td>
<td>0.191</td>
<td>0.024</td>
<td>7.86</td>
<td>&lt; 0.001</td>
<td>0.14 - 0.24</td>
</tr>
<tr>
<td>(\ln(feed))</td>
<td>0.334</td>
<td>0.025</td>
<td>13.46</td>
<td>&lt; 0.001</td>
<td>0.29 - 0.38</td>
</tr>
<tr>
<td>(\ln(pigs))</td>
<td>0.294</td>
<td>0.022</td>
<td>13.11</td>
<td>&lt; 0.001</td>
<td>0.25 - 0.34</td>
</tr>
<tr>
<td>(\ln(other))</td>
<td>0.013</td>
<td>0.020</td>
<td>0.64</td>
<td>0.524</td>
<td>-0.03 - 0.05</td>
</tr>
<tr>
<td>(op_educ)</td>
<td>-0.120</td>
<td>0.029</td>
<td>-4.11</td>
<td>&lt; 0.001</td>
<td>-0.18 - -0.06</td>
</tr>
<tr>
<td>(op_ocup)</td>
<td>0.222</td>
<td>0.058</td>
<td>3.81</td>
<td>&lt; 0.001</td>
<td>0.11 - 0.34</td>
</tr>
<tr>
<td>(ncr)</td>
<td>0.028</td>
<td>0.010</td>
<td>2.88</td>
<td>0.004</td>
<td>0.01 - 0.05</td>
</tr>
<tr>
<td>(re)^c</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A - N/A</td>
</tr>
<tr>
<td>(rm)</td>
<td>-0.052</td>
<td>0.055</td>
<td>-0.93</td>
<td>0.352</td>
<td>-0.16 - 0.06</td>
</tr>
<tr>
<td>(rn)</td>
<td>-0.084</td>
<td>0.054</td>
<td>-1.56</td>
<td>0.118</td>
<td>-0.19 - 0.02</td>
</tr>
<tr>
<td>(rw)</td>
<td>0.059</td>
<td>0.058</td>
<td>1.01</td>
<td>0.312</td>
<td>-0.06 - 0.17</td>
</tr>
<tr>
<td>(imr)</td>
<td>-0.084</td>
<td>0.034</td>
<td>-2.46</td>
<td>0.014</td>
<td>-0.15 - -0.02</td>
</tr>
<tr>
<td>(constant)</td>
<td>-0.005</td>
<td>0.009</td>
<td>-0.56</td>
<td>0.577</td>
<td>-0.02 - 0.01</td>
</tr>
</tbody>
</table>

\(^a\) These results were produced with 143 (after stacking 572) observations obtaining an \(F\)-statistic of 9346.34, \(R^2\) of 0.9954 and adjusted \(R^2\) of 0.9953, which corresponds to a \(p\)-value less than 0.0001. The dependent variable is the log of grower revenue per animal.

\(^b\) The \(p\)-values give probabilities of more extreme \(t\)-ratios for the hypothesis of a zero coefficient.

\(^c\) No observations in this region were available for this regression, i.e., "N/A" means not applicable.

The results show that revenue per animal increases with the size of hog operation and with each of the specific factor inputs (labor, capital, feed, and feeder pigs) as implied by production theory. Each of these effects is statistically significant beyond a 1 percent level. The effect of \(other\) inputs is also positive but not significant. Specialization of the operator in farming has a positive impact on revenue per animal with significance beyond 1 percent, as is plausible. Operator education, however, has a negative estimated effect which seems counter intuitive. However, this variable could reflect the intensity of off-farm labor beyond the effects that can be represented by the dichotomous variable \(op\_ocup\) and, thus, represent reduced specialization for more educated growers that work off-farm more intensively.
The positive and highly significant coefficient on county net cash returns per farm \((ncr)\) implies that growers in a county with more lucrative alternatives have higher revenue per animal, as is plausible with normal competitive forces. The regional indicators \((rw, rn, rm, re)\) are not highly significant individually. Which regional indicators are quantitatively important depends on which region is arbitrarily included in the constant term. However, an \(F\)-test for their joint significance yields an \(F\)-statistic of 4.490 with a \(p\)-value of 0.004. These differences likely reflect factors such as climate and unobservable input and output price differences among regions. Finally, the inverse Mills ratio \((imr)\) has a highly significant negative impact on revenue per animal, implying that unobservables affect both the contract decision and grower revenue.

8.4.1.3 The production model for contract growers

The revenue of a contract grower is the total payment received from a contractor. As described in section 6.2 of chapter 6, the total payment to a contract grower is

\[
R^c = bQ - \beta(C_f/Q - s)Q = \gamma Q - \beta C_f
\]

where \(\gamma = b + \beta s\), \(b\) and \(\beta\) are the base and incentive parameters of the per-pound-of-gain component of the payment, \(c\) is feed cost per animal, and \(s\) is the standard to which feed conversion ratios are compared. Thus, contract grower revenue is

\[
R^c = \gamma AK^\gamma X_i^\beta X_p^\gamma X_f^\beta X_s^\beta e^\epsilon - \beta C_f.
\]

In the model of equations (8.2a)-(8.2b),

\[
R^c = \gamma r^{-\phi} w_i^{-\eta_i} w_p^{-\eta_p} w_f^{-\eta_f} w_o^{-\eta_o} AC_k^\phi C_i^\eta_i C_p^\eta_p C_f^\eta_f C_o^\eta_o e^\epsilon - \beta C_f
\]

Thus, the contracting grower’s revenue is of the same form as the independent grower’s revenue after substituting \(\gamma\) for \(p\) and adding the incentive term for feed efficiency.
By substituting $\gamma r^{-\delta} w^{-\eta_i} w_p^{-\eta_j} w_o^{-\eta_k} A = (\gamma / p) e^{\nu Z}$, contract grower revenue $R^c$ can be rewritten as

$$R^c = (\gamma / p) C_k^c C_p^c C_f^c C_o^e e^{\nu Z} e^c - \beta C_f.$$ 

The variable cost equation is also modified from the independent grower case to represent the fact that the contractor bears only a portion of some of the variable costs including the entire feed cost. The variable cost incurred by the grower is thus

$$V = w_i X_i + \psi_p w_p X_p + \psi_o w_o X_o = C_i + \psi_p C_p + \psi_o C_o$$

where $\psi_i$ is the share of input $i$ provided by the grower (with the contractor providing the rest). These shares are observable and vary by grower according to individual grower contract parameters in the survey data. Accordingly, net profit is

$$\Pi = (\gamma / p) C_k^c C_p^c C_f^c C_o^e e^{\nu Z} e^c - C_i - \beta C_f - \psi_p C_p - \psi_o C_o - C_k - O.$$ 

The expected utility maximization problem is

$$\text{Max } E(U(\gamma A K^\phi X_i^y X_p^y X_f^y X_o^y e^e)$$

$$- w_i X_i - \psi_p w_p X_p - \beta w_f X_f - \psi_o w_o X_o - rK - O),$$

which has first-order conditions

$$E(U' \cdot (\eta_i \gamma A K^\phi X_i^y X_p^y X_f^y X_o^y e^e - w_i) = E(U' \cdot \eta_i \gamma (\bar{Q} / X_i) e^e - w_i) = \eta_i \gamma (\bar{Q} / X_i) E(U' e^e) - w_i E(U') = 0,$$

$$E(U' \cdot (\eta_j \gamma A K^\phi X_i^y X_p^y X_f^y X_o^y e^e - \beta w_f) = E(U' \cdot \eta_i \gamma (\bar{Q} / X_f) e^e - \beta w_f) = \eta_j \gamma (\bar{Q} / X_f) E(U' e^e) - \beta w_f E(U') = 0,$$

and
These first-order conditions can be represented as

\[
\eta_i \gamma (Q/X_i)^e = E(U' \cdot \eta_i \gamma (Q/X_i) + \psi_i, w_i) = \eta_i \gamma (Q/X_i) E(U' \cdot e^e) - \psi_i, w_i, i = p, o.
\]

Multiplying through by \((p/\gamma)e^e\) and dropping the unobservable right-hand expression, these conditions can be expressed as

\[
\frac{\eta_i \gamma Q}{w_i X_i} = \frac{\eta_i \gamma Q}{\beta w_f X_f} = \frac{\eta_i \gamma Q}{\rho w_p X_p} = \frac{\eta_i \gamma Q}{\omega w_o X_o} = \frac{E(U'(\Pi))}{E(U'(\Pi)e^e)}.
\]

which yield conditions in terms of cost shares in the form

\[
\beta \eta_i S_j = \eta_i S_i
\]

and

\[
\eta_i \psi_i S_i = \eta_i S_i, i = p, o.
\]

For econometric purposes, errors in optimization can be added to these equations obtaining

\[
\beta \eta_i S_j = \eta_i S_j + \delta_j \quad (8.10a)
\]

and

\[
\eta_i \psi_i S_i = \eta_i S_i + \delta_i, i = p, o. \quad (8.10b)
\]

where each \(\delta_i\) is a disturbance with mean zero.

If \(\beta\) is observable, then the revenue equation can be linearized by defining a pseudo total revenue variable \(R^*\) for the contract grower as

\[
R^* = \Pi + C_i + \beta C_f + \psi_p C_p + \psi_o C_o + C_k + O,
\]
and representing the revenue equation as

\[ R^* = (\gamma/p)C^*_kC^n_lC^n_pC^n_fC^n_o e^{\nu Z} e^\epsilon. \]

For non-constant returns to scale, this function can be written as

\[ R^*/n = (\gamma/p)(C^*_k/n)^\phi (C^n_l/n)^\nu (C^n_p/n)^\psi (C^n_f/n)^\eta (C^n_o/n)^\theta e^{\nu Z} e^\epsilon. \]

This equation can be linearized as

\[ \ln(R^*/n) = \tau + \phi \ln(C^*_k/n) + \nu \ln(C^n_l/n) + \psi \ln(C^n_p/n) + \eta \ln(C^n_f/n) + \theta \ln(C^n_o/n) + \epsilon \ln n + \nu Z + \epsilon. \quad (8.11) \]

where \( \tau \) is a coefficient to be estimated representing \( \ln(\gamma/p) \) and \( \Delta \) is an indicator variable for contracting equal to 1 for a contract grower and zero otherwise.

This pseudo total revenue \( R^* \) differs from the actual total revenue from contract operations, \( \overline{R} = (p/\gamma)(R^c + \beta C_f), \) because it is evaluated at \( \gamma \) rather than \( p \) and because it adds back in the feed efficiency incentive otherwise subtracted from the grower’s payment. If \( \beta \) is unobservable, then this equation provides a way to estimate \( \beta \) before estimating (8.11). This suggests regressing \( R^c \) on \( \overline{R} \) and \( C_f \) with no constant term following the equation

\[ R^c = (\gamma/p)\overline{R} - \beta C_f, \quad (8.12) \]

where after adding a random disturbance for econometric purposes. This regression provides estimates of both the ratio \( \gamma/p \) and \( \beta \). The results of estimation of (8.12) are presented in Table 7.
Table 7: Estimation of contract parameters for contract growers\(^{a}\)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated coefficient</th>
<th>Standard error</th>
<th>(t)-ratios</th>
<th>(p)-values (^{b})</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total revenue</td>
<td>0.111</td>
<td>0.011</td>
<td>9.67</td>
<td>0.000</td>
<td>0.09 0.13</td>
</tr>
<tr>
<td>Feed cost</td>
<td>-0.039</td>
<td>0.033</td>
<td>-1.20</td>
<td>0.232</td>
<td>-0.10 0.03</td>
</tr>
</tbody>
</table>

\(^{a}\) These results were produced with 234 observations obtaining an \(F\)-statistic of 230.22, an \(R^2\) of 0.6650 and adjusted \(R^2\) of 0.6621, which corresponds to a \(p\)-value less than 0.001.

The dependent variable is contract grower revenue.

\(^{b}\) The \(p\)-values give probabilities of more extreme \(t\)-ratios for the hypothesis of a zero coefficient.

The survey data shows that contractors in 2004 received an average of about $130.31 per animal whereas contract growers received an average of about $13.08 per animal. This corresponds to growers receiving about 10.04 percent of total receipts on average. Thus, 11.1 percent in the regression appears highly plausible after correcting for the feed incentive.

8.4.1.4 Estimation of the production model for contract growers

Using equations (8.10a) and (8.10b), additional information is available to identify the key parameters just as in the independent grower case. Combining (8.11) with the 3 equations in (8.10a) and (8.10b) obtains the linear regression system

\[
\begin{bmatrix}
\ln(\hat{R}/n) \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
\Delta \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
\ln(C/n) \\
\ln(C_f/n) \\
\ln(C_p/n) \\
\ln(C_o/n) \\
\ln(n)
\end{bmatrix} + \begin{bmatrix}
\tau \\
\phi \\
\eta_i \\
\eta_f \\
\eta_p \\
\eta_o
\end{bmatrix} + \begin{bmatrix}
\epsilon \\
\delta_f \\
\delta_p \\
\delta_o
\end{bmatrix} (8.13)
\]

where all entries, other than in the parameter vector, represent vectors over all contract growers. The system in (8.13) is estimated allowing different variances of disturbances.
for \( \varepsilon, \delta_f, \delta_p, \) and \( \delta_o \), as well as covariances among the \( \delta_i \)'s because they contain information about common first-order conditions.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated coefficient</th>
<th>Standard error</th>
<th>( t )-ratios</th>
<th>( p )-values</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln(n) )</td>
<td>0.112</td>
<td>0.012</td>
<td>9.57</td>
<td>(&lt; 0.001 )</td>
<td>0.09</td>
</tr>
<tr>
<td>( \ln(labor) )</td>
<td>0.081</td>
<td>0.021</td>
<td>3.91</td>
<td>(&lt; 0.001 )</td>
<td>0.04</td>
</tr>
<tr>
<td>( \ln(capital) )</td>
<td>0.228</td>
<td>0.029</td>
<td>7.95</td>
<td>(&lt; 0.001 )</td>
<td>0.17</td>
</tr>
<tr>
<td>( \ln(feed) )</td>
<td>0.177</td>
<td>0.025</td>
<td>7.22</td>
<td>(&lt; 0.001 )</td>
<td>0.13</td>
</tr>
<tr>
<td>( \ln(pigs) )</td>
<td>0.149</td>
<td>0.025</td>
<td>5.89</td>
<td>(&lt; 0.001 )</td>
<td>0.10</td>
</tr>
<tr>
<td>( \ln(other) )</td>
<td>0.099</td>
<td>0.028</td>
<td>3.54</td>
<td>(&lt; 0.001 )</td>
<td>0.04</td>
</tr>
<tr>
<td>( op_{educ} )</td>
<td>0.022</td>
<td>0.032</td>
<td>0.70</td>
<td>0.485</td>
<td>0.09</td>
</tr>
<tr>
<td>( op_{ocup} )</td>
<td>0.057</td>
<td>0.034</td>
<td>1.67</td>
<td>0.095</td>
<td>-0.01</td>
</tr>
<tr>
<td>( ncr )</td>
<td>0.010</td>
<td>0.005</td>
<td>1.93</td>
<td>0.054</td>
<td>0.00</td>
</tr>
<tr>
<td>( re )</td>
<td>-0.443</td>
<td>0.060</td>
<td>-7.42</td>
<td>(&lt; 0.001 )</td>
<td>-0.56</td>
</tr>
<tr>
<td>( rm )</td>
<td>-0.150</td>
<td>0.057</td>
<td>-2.62</td>
<td>0.009</td>
<td>-0.26</td>
</tr>
<tr>
<td>( rm )</td>
<td>-0.220</td>
<td>0.056</td>
<td>-3.93</td>
<td>(&lt; 0.001 )</td>
<td>-0.33</td>
</tr>
<tr>
<td>( rv )</td>
<td>-0.314</td>
<td>0.079</td>
<td>-4.00</td>
<td>(&lt; 0.001 )</td>
<td>-0.47</td>
</tr>
<tr>
<td>( imr )</td>
<td>-0.120</td>
<td>0.064</td>
<td>-1.87</td>
<td>0.061</td>
<td>-0.24</td>
</tr>
<tr>
<td>( constant )</td>
<td>-0.015</td>
<td>0.009</td>
<td>-1.68</td>
<td>0.093</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

\( ^a \) These results were produced with 234 (after stacking 931 with 5 missing) observations obtaining an \( F \)-statistic of 2135.65, an \( R^2 \) of 0.9703, and an adjusted \( R^2 \) of 0.9698, which corresponds to a \( p \)-value less than 0.0001. The dependent variable is the log of pseudo grower revenue per animal.

\( ^b \) The \( p \)-values give probabilities of more extreme \( t \)-ratios for the hypothesis of a zero coefficient.

\( ^c \) No observations in this region were available for this regression, i.e., "N/A" means not applicable.

To test joint applicability of the system in (8.13), the system was estimated without imposing cross-equation parameter constraints as in the case of independent growers. Where \( \eta_f^*, \eta_p^*, \) and \( \eta_o^* \) replace \( \eta_f, \eta_p, \) and \( \eta_o, \) respectively, in each of the latter three equations of (8.13), a test of the applicability of cross-equation parameter constraints \( (\eta_f = \eta_f^*, \eta_p = \eta_p^*, \eta_o = \eta_o^*) \) produces an \( F \)-statistic of 0.66 with 3 and 914 degrees of freedom corresponding to a \( p \)-value of 0.5146. This lends strong support for both the structural assumption representing technology and the assumed optimization.
behavior. For contract growers, the exogeneity of labor, feed, pig, and other costs is also not rejected at a 5 percent level with a chi-square statistic of 21.99, which corresponds to a p-value of 0.0787 with 14 degrees of freedom (using the same instruments as for the exogeneity test related to Table 6). Although this exogeneity would be rejected at the 10 percent level, use of the same estimation technique is desirable in these conditions to avoid introducing differences due to different estimation techniques. Thus, the results of the restricted model in Table 8 are well supported for contract growers.

The results in Table 8 are remarkably comparable to Table 6. They show that revenue per animal increases with the size of the hog operation and with each of the factor inputs (labor, capital, feed, feeder pigs, and other inputs) as is consistent with production theory. Each of these effects are statistically significant beyond the 1 percent level. The lower elasticity on labor and higher elasticity on capital and other inputs compared to those for independent growers in Table 6 seem plausible for the more sophisticated technology represented by contracting, although the lower elasticity on feeder pigs is surprising.

Specialization of the operator in farming also has a positive impact on revenue per animal although quantitatively less than for independent growers and with less significance. Operator education, however, has a positive sign compared to a negative sign for independent growers although without statistical significance. The positive result in Table 8 seems intuitively reasonable for contract farmers who deal with more legal restrictions and rigorous specifications.

The sign on county net cash returns per farm (ncr) is positive but smaller in magnitude and less significant than for independent growers. This could be due to
contractors being located only in the best areas so local conditions do not differentiate grower returns as much. The regional indicators \((rw, rn, rm, re)\) are all negative and highly significant implying the south (included in the constant term) is the most profitable region. Statistical significance in each case is beyond the 1 percent level. These differences likely reflect factors such as climate and unobservable input and output price differences among regions as well as contractor location.

Finally, the inverse Mills ratio \((imr)\) has a highly significant negative impact on revenue per animal implying that unobservables affect both the contract decision and grower revenue.

8.4.2 Testing commonality of production elasticities

Because contractors claim to offer growers improved technology in feed rations, genetic quality of pigs, and other services, an interesting hypothesis is whether the technology of contract growers differs from independent growers. For this purpose, the equation systems in (8.8) and (8.13) can be stacked to test for uniformity of production elasticities between independent and contract growers. The most interesting hypothesis is whether productivity of the key inputs remains the same when switching to contract production. When properly specified to allow for different variances of disturbances between the independent and contracting cases, estimated coefficients and standard errors are identical to Tables 5 and 7. So further results of estimation need not be reported for the case where independent and contract growers are combined.

With the combined model, the results of various tests of productivity differences between the two groups are reported in Table 9.
## Table 9. Tests of coefficient differences between independent and contract growers

<table>
<thead>
<tr>
<th>Hypothesis test of no change in coefficients</th>
<th>F-Statistic</th>
<th>Degrees of Freedom</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln((n))</td>
<td>8.62</td>
<td>1, 1474</td>
<td>0.003</td>
</tr>
<tr>
<td>ln((labor))</td>
<td>2.07</td>
<td>1, 1474</td>
<td>0.151</td>
</tr>
<tr>
<td>ln((capital))</td>
<td>0.76</td>
<td>1, 1474</td>
<td>0.384</td>
</tr>
<tr>
<td>ln((feed))</td>
<td>16.87</td>
<td>1, 1474</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>ln((pigs))</td>
<td>16.06</td>
<td>1, 1474</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>ln((other))</td>
<td>5.41</td>
<td>1, 1474</td>
<td>0.020</td>
</tr>
<tr>
<td>All of the above</td>
<td>40.82</td>
<td>5, 1474</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>op_educ</td>
<td>8.88</td>
<td>1, 1474</td>
<td>0.003</td>
</tr>
<tr>
<td>op_ocup</td>
<td>5.46</td>
<td>1, 1474</td>
<td>0.020</td>
</tr>
<tr>
<td>ncr</td>
<td>2.01</td>
<td>1, 1474</td>
<td>0.157</td>
</tr>
<tr>
<td>re</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>rm</td>
<td>1.90</td>
<td>1, 1474</td>
<td>0.168</td>
</tr>
<tr>
<td>rn</td>
<td>3.01</td>
<td>1, 1474</td>
<td>0.083</td>
</tr>
<tr>
<td>rw</td>
<td>16.88</td>
<td>1, 1474</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>All regional indicators</td>
<td>22.05</td>
<td>4, 1474</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>imr</td>
<td>0.09</td>
<td>1, 1474</td>
<td>0.768</td>
</tr>
<tr>
<td>constant</td>
<td>0.37</td>
<td>1, 1474</td>
<td>0.542</td>
</tr>
</tbody>
</table>

Table 9 rejects equality of the farm size elasticity of revenue between the two groups beyond the 1 percent level, implying larger returns to scale for the independent growers. This suggests the motivation for rejecting the seemingly more sophisticated technology represented by contract growers. However, the higher capital elasticity for contract growers compared to independent growers suggests the motivation for the use of the more sophisticated technology represented by contract growers who have limited capital, although equality of the capital elasticity between the two groups is not rejected at the 10 percent level. Table 9 also does not reject equality of the labor elasticity of revenue between the two groups at the 10 percent level.

The lower elasticities of feed and feeder pigs for contract growers compared to independent growers, which are significantly different, are surprising and seem to refute the claimed superiority of contractor supplied feed and feeder pigs as claimed by many. A possible explanation for these estimated differences is that the quality and quantity of
these inputs varies less for contract growers because of contract specifications so these variables explain less of the variability of revenue.

Table 9 rejects equality of the other inputs elasticity of revenue between the two groups at the 2 percent level. The higher elasticity of other inputs (i.e., fuel & lubricant, veterinary & medicine, custom supplies, and bedding & litter) for contract growers compared to independent growers also reflects the higher quality of inputs specified by grower contracts and be the most important manifestation of the superior technology under contracting. This elasticity is over seven times greater for contract growers than independent growers.

The results in Table 9 reject the equality of the combined factor productivity of size, labor, capital, feed, feeder pigs, and other variable inputs beyond the 1 percent level. This verifies that the technology of contract growers indeed differs from independent growers.

Results also reject equality of the coefficients on \( op_{educ} \) and \( op_{ocup} \) between the two groups at the 1 and 2 percent levels, respectively even though equality is not rejected for \( ncr \) at the 10 percent level. Differences in the regional effects for contract and independent growers are not significant at the 5 percent level except for the West region \( (rw) \). (Note that the constant terms that represent the South are also not significantly different.) This suggests that growers in the rural South, who tend to have fewer alternative economic opportunities and lower skill levels, benefit relatively more with contract operations than independent operations compared with other regions, where alternative economic activities are more plentiful. Finally, the coefficient of the inverse Mill’s ratio \( (imr) \) is not significantly different between the two groups.
Thus, while several of the coefficients of variables determining revenue are not significantly different between contract and independent growers, others are. A Chow test for equivalence of all coefficients other than the constant term between the two groups of growers yields an $F$-statistic of 139.97 which has a p-value of less than 0.001 with 14 and 1474 degrees of freedom. This result further underscores that differences in the technology and its dependence on grower attributes are introduced by contracting and contractor provided inputs.

Additionally, rejection of equality of models according to this Chow test rejects the first two variants of two-step methods discussed in section 7.3.4 of chapter 7 for estimating revenues of the growers. Thus, I apply the most general two-step estimator for revenue estimation. This means the separately estimated revenues of independent and contract growers, presented in Tables 6 and Table 8, respectively, are used for further analysis.

8.4.3 The second-step estimation of costs

To estimate costs for both independent and contract growers, a cost function approach can be used. However, the cost functions for both independent and contract growers must be specified in a manner compatible with the production specification.

8.4.3.1 The cost function for independent growers

Although the production problem assumes expected utility maximization, the inputs do not affect risk and input prices can be reasonably assumed known at the time purchases are made. Thus, cost minimization applies so that the cost function is the same
as for the standard expected profit maximization case with the production function

\[ Q = AK^\phi X_i^n X_p^{\eta_p} X_f^{\eta_f} X_o^{\eta_o} e^\varepsilon \]

where \( E(\varepsilon) = 0 \). Suppose \( \varepsilon = \zeta + \nu \) where \( \zeta \) is the part of \( \varepsilon \) that is known to the grower when input decisions are made, e.g., representing deviations from regional input prices specific to the grower, and \( \nu \) is the part of \( \varepsilon \) that is unknown to the grower, e.g., random biological performance of hogs during the growing season, with assumed to have zero expectations. Then output expected by the grower given input choices is

\[ \bar{Q} = AK^\phi X_i^n X_p^{\eta_p} X_f^{\eta_f} X_o^{\eta_o} E(\nu')e^\zeta. \]

The cost function that minimizes cost with this production function is

\[
c(w | \bar{Q}, K) = \{ \min_{X_i, X_p, X_f, X_o} w_i X_i + w_p X_p + w_f X_f + w_o X_o | \bar{Q} = AK^\phi X_i^n X_p^{\eta_p} X_f^{\eta_f} X_o^{\eta_o} E(\nu')e^\zeta \} = \eta \left[ \bar{Q} K^{-\phi} A^{-1} w_i^{\eta_i} w_p^{\eta_p} w_f^{\eta_f} w_o^{\eta_o} \right]^{1/\eta}.
\]

where \( w = (w_i, w_p, w_f, w_o) \) and \( \eta = \eta_i + \eta_p + \eta_f + \eta_o \) (see Just, Hueth, and Schmitz 2004, p. 82).

Recall that

\[
p r^{-\phi} w_i^{-\eta_i} w_p^{-\eta_p} w_f^{-\eta_f} w_o^{-\eta_o} A = e^{\nu Z}, \]

which implies

\[
A^{-1} w_i^{\eta_i} w_p^{\eta_p} w_f^{\eta_f} w_o^{\eta_o} = pr^{-\phi} e^{-\nu Z}. \]

Upon substituting this expression, using (8.2b), and treating \( V \) as an observation on \( c(w | \bar{Q}, K) \), the cost equation becomes

\[
V = \eta \left[ p \bar{Q} C^{-\phi} e^{-\nu Z} \eta_i^{-\eta_i} \eta_p^{-\eta_p} \eta_f^{-\eta_f} \eta_o^{-\eta_o} \right]^{1/\eta},
\]

or in per animal terms, recalling that \( \zeta = \phi + \eta - 1 \),

\[
V / n = \eta \left[ \left(p \bar{Q} / n \right) \left(C^{\phi} / n\right)^{-\phi} e^{-\nu Z} n^{-\zeta} \eta_i^{-\eta_i} \eta_p^{-\eta_p} \eta_f^{-\eta_f} \eta_o^{-\eta_o} \right]^{1/\eta},
\]

or in logarithmic per animal form,

\[
\ln(V / n) = \eta^* (\ln(R/n) - \phi \ln(C^\phi / n) - \zeta \ln(n - \nu Z)) + \nu \]

(8.14)
where \( R = pQe^{\nu} \) and \( \eta^* = \ln \eta + (1/\eta) \ln(\eta_i^{\eta_i} \eta_p^{\eta_p} \eta_f^{\eta_f} \eta_o^{\eta_o}). \)

### 8.4.3.2 Estimation of the cost model for independent growers

Equation (8.14) could be incorporated into the system in (8.8) and estimated jointly with cross-equation parameter constraints. However, the cross-equation constraints would be highly nonlinear in a system that already has substantial structure. Thus, a practical approach is to estimate this equation separately by a regression of per animal variable cost on a constant term, per animal revenue, per animal capital cost, number of animals (all in log form) and the same characteristic variables included in the production model (\( op\_educ, \) \( op\_ocup, \) \( ncr, \) \( re, \) \( rm, \) \( rn, \) \( rw \)).

On this basis, the cost model for independent growers is estimated in Table 10. Because the exogeneity of revenue may be a questionable in this regression, the exogeneity of revenue was tested using a Hausman test. The chi-square statistic is 1.46, which has a \( p \)-value of 0.9990 with 10 degrees of freedom, suggesting no hint of endogeneity. Thus, the results are presented without endogeneity corrections.
The results show that growers who gain greater revenue per head generally incur more variable cost per head. The elasticity of revenue is 0.43 and is statistically significant beyond the 1 percent level. This is as expected because, for given fixed factors of production, higher revenue per head can generally be obtained from more variable factor input use at rational production levels. The results also show that variable cost decreases with the size of hog operation. The elasticity of size is –0.040. This implies that cost returns to scale in hog production are increasing, which adds to the revenue returns to scale implied by the highly significant positive elasticity on \( n \) in the revenue regression for independent growers. However, the elasticity of size in Table 10 is statistically insignificant at the 10 percent level.

Capital, perhaps surprisingly, has a positive elasticity although without statistical significance. This result, however, is not implausible given the much larger and highly

### Table 10: Variable cost per head for independent growers

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated coefficient</th>
<th>Standard error</th>
<th>( t )-ratios</th>
<th>( p )-values</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln(revenue) )</td>
<td>0.431</td>
<td>0.073</td>
<td>5.93</td>
<td>&lt;0.001</td>
<td>0.29</td>
</tr>
<tr>
<td>( \ln(n) )</td>
<td>-0.040</td>
<td>0.028</td>
<td>-1.40</td>
<td>0.165</td>
<td>-0.10</td>
</tr>
<tr>
<td>( \ln(capital) )</td>
<td>0.005</td>
<td>0.044</td>
<td>0.12</td>
<td>0.906</td>
<td>-0.08</td>
</tr>
<tr>
<td>( op_educ )</td>
<td>0.102</td>
<td>0.048</td>
<td>2.14</td>
<td>0.034</td>
<td>0.01</td>
</tr>
<tr>
<td>( op_ocup )</td>
<td>-0.196</td>
<td>0.098</td>
<td>-1.99</td>
<td>0.048</td>
<td>-0.39</td>
</tr>
<tr>
<td>( ncr )</td>
<td>-0.014</td>
<td>0.016</td>
<td>-0.87</td>
<td>0.386</td>
<td>-0.05</td>
</tr>
<tr>
<td>( re )</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>( rm )</td>
<td>-0.052</td>
<td>0.093</td>
<td>-0.57</td>
<td>0.573</td>
<td>-0.24</td>
</tr>
<tr>
<td>( rn )</td>
<td>0.017</td>
<td>0.090</td>
<td>0.18</td>
<td>0.854</td>
<td>-0.16</td>
</tr>
<tr>
<td>( rw )</td>
<td>-0.161</td>
<td>0.097</td>
<td>-1.65</td>
<td>0.102</td>
<td>-0.35</td>
</tr>
<tr>
<td>( imr )</td>
<td>0.192</td>
<td>0.056</td>
<td>3.43</td>
<td>0.001</td>
<td>0.08</td>
</tr>
<tr>
<td>( constant )</td>
<td>3.260</td>
<td>0.426</td>
<td>7.66</td>
<td>&lt;0.001</td>
<td>2.42</td>
</tr>
</tbody>
</table>

\( a \) These results were produced with 143 observations obtaining an \( F \)-statistic of 8.48, \( R^2 \) of 0.3912 and adjusted \( R^2 \) of 0.3451, which corresponds to a \( p \)-value less than 0.0001. The dependent variable is total variable cost per animal.

\( b \) The \( p \)-values give probabilities of more extreme \( t \)-ratios for the hypothesis of a zero coefficient.

\( c \) No observations in this region were available for this regression, i.e., "N/A" means not applicable.
significant positive elasticity for capital in the independent grower revenue equation. The implication is that capital increases net returns because it increases revenue by more than it increases variable costs. For example, capital may increase the ability to use more variable inputs effectively. Operator education, surprisingly, has a positive estimated effect on variable cost. This effect seems counter intuitive given that education has a negative effect on independent grower revenue. As described for revenue in section 8.4.1.2, this variable could reflect the intensity of off-farm labor that is not reflected in the indicator variable used for that purpose. Thus, it may reflect reduced specialization for more educated growers who work off-farm more intensively. Specialization of the operator in farming has a negative impact on variable cost per animal, with significance beyond 5 percent, as is plausible under cost economies of scope.

The negative coefficient on $ncr$ is plausible because higher county returns to farming likely occur in locations where factor prices are less. Regional indicators (i.e., $rw$, $rn$, and $rm$) are jointly significant with an $F$-statistic of 2.49 with 3 and 131 degrees of freedom, corresponding to a $p$-value of 0.0630. This is expected because these differences likely reflect factors such as climate and unobservable input price differences among regions. Finally, the inverse Mill’s ratio ($imr$) has a positive impact on variable cost, which implies that unobservables that affect the contract decision have a net positive impact on growers’ variable cost. Without this term, growers’ variable cost would have been underestimated. Thus, with the explanation for education, all the estimated coefficients appear plausible in all cases for independent growers.
8.4.3.3 The cost function for contract growers

Comparing the optimization problems of independent and contract growers, the argument of the utility function changes from \( pAK^{\phi}X_i^{\eta_i}X_f^{\eta_f}X_o^{\eta_o}e^{\xi} - \sum_i w_iX_i - rK - O \) to \( \gamma AK^{\phi}X_i^{\eta_i}X_f^{\eta_f}X_o^{\eta_o}e^{\xi} - w_iX_i - \psi w_pX_p - \beta w_fX_f - \psi w_oX_o - rK - O \). The only changes are to substitute \( \gamma \) for \( p \), \( \beta w_f \) for \( w_f \), and \( \psi w_i \) for \( w_i \) for \( i = p,o \). The same is true of the respective first-order conditions and their implications. Thus, the same changes apply for the cost function where costs are now characterized by including the feed efficiency incentive, \( \beta C_f \), with cost rather than revenue. Thus, the cost minimization problem for contract growers is

\[
c(w|\bar{Q},K) = \{ \min_{X_i,X_f,X_o} w_iX_i + \psi w_pX_p + \beta w_fX_f + \psi w_oX_o | \bar{Q} = AK^{\phi}X_i^{\eta_i}X_f^{\eta_f}X_o^{\eta_o}E(e^{\xi}) \}
\]

\[
= \eta \left[ \bar{Q}K^{-\phi}A^{-1}\beta^{\eta_p}\psi^{\eta_o}w_i^{\eta_i}w_p^{\eta_p}w_f^{\eta_f}w_o^{\eta_o}\eta_i^{-\eta_p}\eta_o^{-\eta_o} \right]^{1/\eta}
\]

where \( V^* = C_i + \psi C_p + \beta C_f + \psi C_o \) is regarded as an observation on this cost function.

Substituting \( A^{-1}w_i^{\eta_i}w_p^{\eta_p}w_f^{\eta_f}w_o^{\eta_o} = pr^{-\phi}e^{-\nu Z} \) and using (8.2b), this cost function becomes

\[
V^* = \eta \left[ p\bar{Q}\mathcal{C}_k^{-\phi}e^{-\nu Z}A^{-1}\beta^{\eta_p}\psi^{\eta_o}\eta_i^{-\eta_p}\eta_o^{-\eta_o} \right]^{1/\eta}
\]

or in per animal terms, recalling that \( \zeta = \phi + \eta - 1 \),

\[23\] The feed efficiency incentive term, \( \beta C_f \), must be incorporated into the cost to obtain the correct results by analogy with the independent grower case. Also, note that the revenue variable appropriate for this representation of the cost equation for the contract grower is total revenue to the contractor and contract grower combined, which does not include the feed efficiency incentive term.
\[ V^*/n = \eta \left[ \frac{p\bar{Q}}{n}(C_k/n)^{-\phi} e^{-\nu Z} n^{-\zeta} \beta^{\eta_p} \psi_\phi^{\eta_o} \eta_i^{-\eta_p} \eta_f^{-\eta_o} \eta_o^{-\eta_i} \right]^{1/\eta}, \]

or in logarithmic per animal form,

\[ \ln(V^*/n) = \eta_c^* + \left(1/\eta\right) \left[ \ln(R/n) - \phi \ln(C_k/n) - \zeta \ln n - \nu Z \right] + \nu \tag{8.15} \]

where, as for independent growers, \( R = p\bar{Q}e^\nu \). The only notable differences in this estimated equation from the independent grower case is the inclusion of the feed efficiency incentive, the weighting of input costs by the contract grower shares in the left hand side, and the different constant term, \( \eta_c^* = \ln \eta + \left(1/\eta\right) \ln(\beta^{\eta_p} \psi_\phi^{\eta_o} \eta_i^{-\eta_p} \eta_f^{-\eta_o} \eta_o^{-\eta_i}) \), on the right hand side.

### 8.4.3.4 Estimation of the cost model for contract growers

The cost model for contract growers in (8.15) is estimated in Table 11. As in the case for independent growers, because the exogeneity of revenue may be a questionable the exogeneity of revenue was tested using a Hausman test. The chi-square statistic is 0.75, which has a \( p \)-value of 1.0000 with 11 degrees of freedom, suggesting no hint of endogeneity. Thus, the model is estimated without endogeneity corrections.
<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated coefficient</th>
<th>Standard error</th>
<th>t-ratios</th>
<th>p-values</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln(revenue) )</td>
<td>0.071</td>
<td>0.151</td>
<td>0.47</td>
<td>0.639</td>
<td>-0.23</td>
</tr>
<tr>
<td>( \ln(n) )</td>
<td>-0.216</td>
<td>0.031</td>
<td>-6.95</td>
<td>&lt;0.000</td>
<td>-0.28</td>
</tr>
<tr>
<td>( \ln(capital) )</td>
<td>0.125</td>
<td>0.053</td>
<td>2.35</td>
<td>0.020</td>
<td>0.02</td>
</tr>
<tr>
<td>op_educ</td>
<td>0.068</td>
<td>0.056</td>
<td>1.22</td>
<td>0.224</td>
<td>-0.04</td>
</tr>
<tr>
<td>op_ocup</td>
<td>-0.111</td>
<td>0.060</td>
<td>-1.86</td>
<td>0.065</td>
<td>-0.23</td>
</tr>
<tr>
<td>ncr</td>
<td>-0.004</td>
<td>0.009</td>
<td>-0.45</td>
<td>0.655</td>
<td>-0.02</td>
</tr>
<tr>
<td>re</td>
<td>0.034</td>
<td>0.096</td>
<td>0.35</td>
<td>0.724</td>
<td>-0.16</td>
</tr>
<tr>
<td>rm</td>
<td>-0.296</td>
<td>0.098</td>
<td>-3.04</td>
<td>0.003</td>
<td>-0.49</td>
</tr>
<tr>
<td>rn</td>
<td>-0.392</td>
<td>0.093</td>
<td>-4.22</td>
<td>&lt;0.000</td>
<td>-0.58</td>
</tr>
<tr>
<td>rw</td>
<td>-0.258</td>
<td>0.130</td>
<td>-1.98</td>
<td>0.049</td>
<td>-0.51</td>
</tr>
<tr>
<td>imr</td>
<td>0.530</td>
<td>0.107</td>
<td>4.96</td>
<td>&lt;0.000</td>
<td>0.32</td>
</tr>
<tr>
<td>constant</td>
<td>3.548</td>
<td>0.753</td>
<td>4.71</td>
<td>&lt;0.000</td>
<td>2.06</td>
</tr>
</tbody>
</table>

These results were produced with 234 observations obtaining an \( F \)-statistic of 16.61, \( R^2 \) of 0.4514 and adjusted \( R^2 \) of 0.4242, corresponding to a \( p \)-value less than 0.0001. The dependent variable is \( V^* \) as defined in the text, which is the variable cost per head incurred by the grower adjusted for the feed incentive.

The results in Table 11 are remarkably comparable to Table 10 although some important differences are significant and plausible. The results show that growers who incur more variable cost per head tend to generate more revenue per head. However, this effect is statistically insignificant for contract growers, probably owing to the constraints on the technology imposed by contractors. Nevertheless, an \( F \)-statistic of 5.19, which has a \( p \)-value of 0.0233 with 1 and 354 degrees of freedom, does not reject equality of the revenue elasticity between the two groups at the 1 percent level.

An \( F \)-statistic of 16.38, which has a \( p \)-value of less than 0.001 with 1 and 354 degrees of freedom, rejects equality of the size elasticity between the two groups beyond the 1 percent level. The higher absolute elasticity on size for contract growers compared to independent growers seems plausible for the more sophisticated technology represented by contracting because larger operations realize proportionately greater
benefits from better technology. However, as evident from the revenue regressions, the magnitude of returns to scale for revenue is larger for independent growers than for contract growers. These results jointly imply that contract growers’ returns to scale result more from cost savings than for independent growers, which is plausible if contractors’ specifications are cost efficient.

The high positive elasticity for capital may be surprising, but is consistent with a contractor-imposed strategy of using capital to facilitate more intensive use of variable inputs. On the other hand, the substitution of capital for variable inputs is limited for contractors because contract growers follow contractor-provided production practices whereby certain input quantities are largely dictated. Further, contract growers have greater incentives to use variable inputs for which they only bear a portion of the costs under grower contracts, whereas independent growers pay for all of their inputs. Finally, contract growers have greater maintenance incentives because they can do so partly with contractor-provided inputs. Thus, the net effect of capital on variable cost is likely to be large and positive for contract growers. Nevertheless, an $F$-statistic of 2.65, which has a $p$-value of 0.1044 with 1 and 354 degrees of freedom, does not reject equality of the capital elasticities of cost for contract and independent growers at the 10 percent level.

Operator education has a lower impact on variable cost compared to independent growers, although without statistical significance. As argued in section 8.3.1, a comparative disadvantage in know-how likely induces some growers to contract. These contract growers follow contractor prescribed production practices, which thus reduces the dependence of production practices on education. However, an $F$-statistic of 0.15, which has a $p$-value of 0.7029 with 1 and 354 degrees of freedom, does not reject
equality of the coefficient on operator education for contract and independent growers at the 10 percent level.

Specialization of the operator in farming has a negative impact on variable cost per animal although quantitatively less than for independent growers and with more significance. Again, a quantitatively smaller impact is likely due to using contractor-specified technology. However, an $F$-statistic of 0.52, which has a p-value of 0.4727 with 1 and 354 degrees of freedom, does not reject equality of the coefficient on operator specialization for contract and independent growers at the 10 percent level.

The sign on $n_{cr}$ is negative but smaller in absolute magnitude and less significant than for independent growers. This may be due to contractors providing inputs procured with mass buying power, thus making growers less dependent on local market conditions within their county. However, an $F$-statistic of 0.11, which has a p-value of 0.7417 with 1 and 354 degrees of freedom, does not reject equality of the coefficient on $n_{cr}$ for contract and independent growers at the 10 percent level.

Regional indicators (i.e., $r_{w}$, $r_{n}$, $r_{m}$ and $r_{e}$) are jointly significant with an $F$-statistic of 8.54 with 4 and 222 degrees of freedom, corresponding to a $p$-value of less than 0.001. Highly significant negative coefficients on $r_{m}$, $r_{n}$, and $r_{w}$ imply lower variable cost in the Midwest, North and West regions compared with the South (reflected by the intercept).

Finally, the inverse Mills ratio ($im_{r}$) has a highly significant positive impact on variable cost per animal implying that unobservables affect both the contract decision and grower variable cost. An $F$-statistic of 8.69, which has a p-value of 0.0034 with 1 and 354 degrees of freedom, rejects equality of the coefficient on $im_{r}$ for contract and independent
growers beyond the 1 percent level. This result implies that, without this term, contract growers’ variable cost would have been underestimated more compared with independent growers. It also implies that unobservables that explain the contracting decision differ between contract and independent growers.

While several of the coefficients of variables determining cost are not significantly different between contract and independent growers, others are. A Chow test for equivalence of all coefficients other than the constant term between the two groups of growers yields an $F$-statistic of 7.97 which has a $p$-value of less than 0.001 with 11 and 354 degrees of freedom. This result implies that contract growers as a group differ from independent growers in the effect their attributes have on costs or in other aspects of productivity due to contractor provided inputs. This Chow test also rejects the first two variants of the two-step method discussed in section 7.3.4 of chapter 7 for estimating variable cost per head. Therefore, I apply the most general two-step estimator for variable cost estimation. That is, the separately estimated variable cost functions for independent and contract growers, as reported in Table 10 and Table 11, respectively, are used for further analysis.

8.5 Gains from Contracting

I now turn to use of the impact measures ($ATE$, $ATET$, and $ATNT$) discussed in section 6.4.3 of chapter 6 to analyze the ex post gains from contracting. These measures are used to estimate the actual realized gains from contracting, which depend on the grower’s realized ability. The ex post realization of ability changes the grower’s assessment of his reservation profit, e.g., his estimate of how much he could earn as an
independent grower. The actual realized gain from contracting of a grower is calculated as the difference in his profit and assessment of his reservation profit. This difference in profit and assessment of reservation profit is computed for each of the contract growers. Different impact measures are then obtained by averaging these differences over different sets of growers.

Based on the regression results in Tables 5, 6, 8, 10, and 11, I predict profit and counterfactual profit (i.e., assessment of reservation profit) for a grower with arbitrary characteristics under both independent and contract operations. These predictions are applied to construct Table 12. Specifically, Table 12 is constructed following steps 1 through 5 of section 7.3.4.3 of chapter 7. Table 12 presents a comparison among various impact measures of contracting for the contract and independent growers as a whole and for different size groups.

8.5.1 Calculating gains for contract growers

Steps 1 and 2 (computing the appropriate selection correction terms, i.e., inverse Mills ratios) are performed on the basis of the probit regression results reported in Table 5. Step 3 (obtaining the predicted profits for independent and contract growers) is performed on the basis of the regression results presented in Tables 6, 8, 10, and 11. Step 4 (obtaining the counterfactual profits for contract and independent growers using the estimated coefficients from the independent and contract profit regressions, respectively)

---

24 Profits are adjusted for differences in operating capital costs between the two groups. In the above framework, operating capital has a role in determining profit but not production. Since $C_i$ represents the expenditure on productive variable inputs, the operating capital requirement is $OC = \sum_i r_i C_i$ where $r_i$ represents the interest rate on ith variable costs for the period of time from expenditure to receiving production revenue in each production cycle. This cost does not appear in the production function because it contributes nothing to production, but would be an additional cost subtracted in the variable cost function.
is performed based on the same sets of tables. Finally, Step 5 (estimation of impact measures $ATE$, $ATET$, and $ATNT$) is performed combining the results obtained from steps 3 and 4. Steps 1 through 4 facilitate construction of counterfactual profits for each of the contract growers, and thus allow investigation of ex post gains from contracting for subgroups of contract growers. Thus, these steps permit investigating the possibility of negative gains from contracting for a fraction of contract growers, which is a major point of focus of this dissertation.

The second column of Table 12 presents a comparison among various impact measures of contracting for contract and independent growers as a whole. The average gain from contracting for a randomly selected grower is $11.01, as given by the $ATE$. The average gain from contracting for those who contract is $20.50, as given by the $ATET$. The average gain from contracting for those who operate independently is -$4.51, as given by $ATNT$. As expected, and as confirmation of the plausibility of empirical results, contract growers not only gain more from contracting than those who do not contract, but contract growers have absolute gains from contracting on average and independent growers would experience absolute losses from contracting on average. These result are explained by the estimated sorting effect and selection bias.
Table 12: Two-step estimates of mean effects of contracting

<table>
<thead>
<tr>
<th>Size</th>
<th>All</th>
<th>n ≤ 3,000</th>
<th>3,000 &lt; n ≤ 6,000</th>
<th>6,000 &lt; n ≤ 11,000</th>
<th>n &gt; 11,000</th>
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</thead>
<tbody>
<tr>
<td>OLS</td>
<td>23.11</td>
<td>54.58</td>
<td>34.00</td>
<td>17.51</td>
<td>-16.87</td>
</tr>
<tr>
<td>ATE</td>
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<td>26.34</td>
<td>13.92</td>
<td>5.43</td>
<td>-18.86</td>
</tr>
<tr>
<td>ATET</td>
<td>20.50</td>
<td>52.48</td>
<td>31.22</td>
<td>13.92</td>
<td>-16.74</td>
</tr>
<tr>
<td>ATNT</td>
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<td>7.21</td>
<td>-20.70</td>
<td>-15.09</td>
<td>-34.21</td>
</tr>
<tr>
<td>Selection Bias&lt;sup&gt;a&lt;/sup&gt;</td>
<td>2.61</td>
<td>2.10</td>
<td>2.77</td>
<td>3.59</td>
<td>-0.13</td>
</tr>
<tr>
<td>Sorting Effect&lt;sup&gt;b&lt;/sup&gt;</td>
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<td>26.14</td>
<td>17.31</td>
<td>8.49</td>
<td>2.12</td>
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<tr>
<td>Total Bias&lt;sup&gt;c&lt;/sup&gt;</td>
<td>12.10</td>
<td>28.25</td>
<td>20.08</td>
<td>12.08</td>
<td>1.99</td>
</tr>
</tbody>
</table>

<sup>a</sup> Selection bias = OLS – ATET.
<sup>b</sup> Sorting effect = ATET – ATE.
<sup>c</sup> Total bias = OLS – ATE.

Recall that the average difference in profit for contact growers and independent growers is decomposed following section 7.3.3.2 of chapter 7 into the ATE, the sorting effect, and the selection bias where ATE and the sorting effect combined are the ATET. The selection bias measures how much of the difference in estimated profits is due to the difference in unobservables between contract and independent growers and, thus, would not be an effect of switching contract status by a given (average) grower. The estimated selection bias reported in Table 12 is $2.61. A positive selection bias means that the average grower who chose independent operation would have earned $2.61 greater profit per head by contracting than the average grower who chose to contract because independent growers have different unobservables on average. While this effect alone suggests a counterintuitive comparative disadvantage for contract growers from contracting, this effect likely cannot be captured by any particular grower. The more interesting effect is the sorting effect.

The total bias is the sum of sorting and selection bias. The positive total bias of $12.10 means that the OLS estimate of the contract effect associated with observable variables would be biased upward by that amount. The sorting effect is the mean gain for
growers who choose to contract due to unobservables, i.e., from variables other than those used in the regressions. The estimated sorting effect is $9.49. Not surprisingly, the sorting effect is large and positive on average. The sorting effect is roughly of the same order of magnitude as the \( ATE \), which reflects the difference in profits from contracting due to the observable variables for which estimated coefficients are estimated in Tables 6, 8, 10, and 11. Thus, growers who choose to contract gain greater profits from contracting both because of the observables as reflected by the \( ATE \) ($11.01 on average) and because of the effect of unobservable variables as reflected by the sorting effect ($9.49 on average).

The decision to contract depends on the \( ATET \), which is combines of both these effects. The difference due to observables \( (ATE) \) is presumably available to growers who chose independent operation by switching to the technology provided by contractors, but the sorting effect may not be available to those who chose independent operation because it represents a difference in unobservables that are not controllable (at least as characterized by the regression analysis). The results thus evidence purposive sorting into groups of contracting and independent growers on the basis of a comparative advantage in gains from contracting, which differs between the two groups of growers (even after switching technologies) by the sorting effect (Heckman and Li, 2004). These results imply that the principle of comparative advantage provides an important explanation for contracting in the hog industry.

The results in the second column of Table 12, however, only reveal part of the story. Columns 3, 4, 5, and 6 provide the breakdown of impact estimates for four approximately equal-sized groups of contract growers \((n \leq 3,000, 3,000 < n \leq 6,000, \ldots)\).
The ATEs are $26.34, $13.92, $5.43 and –$18.86 for the first, second, third and fourth quartiles of contract growers, respectively. Thus, size has a strong decreasing effect on the estimated impact of contracting due to observables (ATE). Similarly, size has a strong decreasing effect on the estimated ATET with a negative ATET of –$16.74 for the highest quartile. Further, size has a strong decreasing effect on the OLS estimate of gains from contracting. Thus, irrespective of the estimation method (OLS, ATE, or ATET), larger contract growers receive on average smaller gains from contracting, with negative estimated gains for the highest quartile. Negative gains for the largest quartile are puzzling because larger shares of growers with larger size choose to contract.

The sorting effects are $26.14, $17.31, $8.49, and $2.12 for the respective quartiles. These results suggest unobservables tend to contribute to gains from contracting for growers in all size classes, even though this effect declines with size. The selection biases are $2.10, $2.77, $3.59, and –$0.13 for the respective quartiles. Curiously, size tends to increase the selection bias, except for the highest quartile, although the variation is small. The negative estimate for the fourth quartile implies that large growers who chose to contract would experience a smaller profit effect of unobservables than large growers who chose not to contract. The positive total biases for all the quartiles means that OLS estimates of contracting effects would be biased upward for all quartiles.

The estimates of ATNT are $7.21, –$20.70, –$15.09, and –$34.21 for the respective quartiles. The effect of size on the estimates of ATNT is generally negative except for a slight increase between the second and third quartile. This means larger
independent growers have greater disincentives to contract. Since ATET exceeds ATNT for all sizes, growers in all size classes who choose to contract have, on average, greater benefits from contracting than those who do not choose to contract. Thus, the results have a plausible consistency across size classes. But results are not fully as expected in all size classes. For example, in the smallest quartile, even though the gains for growers who chose to contract are larger, the growers who chose not to contract could also have had positive gains from contracting. This result is surprising given that smaller shares of small size growers choose to contract. This apparent anomaly is explained below.

8.5.2 Negative gains: theoretical predictions versus empirical results

This section presents the rather unique empirical results of this dissertation that identify individual growers who experience losses from contracting and their characteristics. The important issue of whether all growers gain from contracting under heterogeneity is explored theoretically in detail in chapter 3, 4, and 6. The main theoretical prediction derived in section 3.5 of chapter 3 and section 4.3 of chapter 4 is that for the most plausible information structure, that is, when growers have partial knowledge of their ability, some low-ability growers with below average productivity receive negative gains from contracting on average. This implies that the average gain of high-ability growers is expected to be larger than that of their low-ability counterparts. However, when ex post assessment of reservation profit varies with ability as assumed in the theoretical modeling of hog contracts in section 6.3 of chapter 6, a negative average gain from contracting is possible for a contract grower of below average productivity of any ability. Even low-ability growers with above average productivity are can experience
an ex post loss from contracting in this case. Thus, the average gain of high-ability growers may be no greater than their low-ability counterparts.

This section explores this negative and relative gain issue. This section also considers the theoretical implication that the contractor gives an incentive to high-ability growers to use their higher ability with greater effort and larger facilities than these of low-ability growers. According to this theoretical result, high-ability growers are expected to earn higher profits than low-ability growers. Specifically, three questions are answered in this section: (i) whether high-ability contract growers earn higher profits on average than low-ability growers, (ii) whether losses from contracting are observed for some contract growers of all ability levels or only for the low-ability levels, and (iii) whether contract growers of both below and above average productivity or only below average productivity lose from contracting. Answers to the latter two questions hinge on growers’ ex post assessments of reservation profits. The first two of these questions answer the stated puzzle in the previous section.

8.5.2.1 Consistency across abilities

For the purposes of this discussion, operation size is regarded as a proxy for ability as suggested by the theoretical model of chapter 6. If operation size is used as a proxy for ability then large size growers are expected to gain more from contracting. This suggests that large shares of the large sizes would choose to contract compared to smaller sizes, as observed. However, the results of the previous section show that the average gain is larger among smaller size growers. This is inconsistent with the theoretical predictions of chapters 3 and 4 unless ability is negatively proxied by size (that is,
growers with larger size have less ability). However, these theoretical predictions fail to consider that average reservation profit as well as average actual profit of larger growers is larger than for smaller growers. This resolves the seemingly counterintuitive implication that growers with larger size have less ability (as well as the more implausible implication that a contractor induces lower ability growers to construct larger facilities).

The underlying assumption in chapters 3 and 4 is that the assessment of reservation profits does not vary across realized abilities, which is relaxed in the theoretical model adapted to the hog industry in chapter 6. For the relaxed assumption that the reservation profits vary with realized ability as in chapter 6, the theory predicts a wide variety of possibilities revealing many cases where more than one group can lose from contracting ex post. In particular, larger gains for smaller growers and negative gains for larger size growers are not precluded.

The empirical results show that a large number of contract growers experience negative gains from contracting. Of 234 contract growers, 76 experience negative gains from contracting, which is 33 percent of all contract growers. Similarly, 80 independent growers, which is 56 percent of all independent growers, would have experienced negative gains from contracting had they contracted. Negative gains for independent growers are reasonable due to self selection, but why do so many contract growers choose to contract if they lose from contracting? Also, what are the characteristics of those who lose from contracting? Do they tend to be the smaller or larger contract growers? How do these results relate to the comparative advantage argument for contracting? Are these losses behind the recent trend toward independent hog operations?
These questions can be answered by analyzing the counterfactual profits (the reservation profits) of contract growers as presented in Table 13. This table is constructed following the 5 steps used for construction of Table 12. The average profits (per animal) are $−15.83, $−7.97, $−4.60 and $−3.37 for first, second, third, and fourth quartiles, respectively. Thus, the profits of contract growers are increasing in size. This confirms the theoretical prediction that high-ability growers earn higher profits than low-ability growers where size is taken as a proxy for ability. These results also explain, in part, why large shares of the large sizes choose to contract when the gains to them are on average smaller compared with their smaller counterparts. But this is a comparison of profits across growers of different abilities, not gains from contracting for growers of different abilities.

| Table 13: Two-step estimates of revenues, costs, and profits of contract growers |
|-----------------|---------------|---------------|---------------|---------------|
| Size            | $n \leq 3,000$ | $3,000 < n \leq 6,000$ | $6,000 < n \leq 11,000$ | $n > 11,000$ |
| Revenues        | 14.09         | 14.42         | 15.00         | 14.91         |
| Costs           | 29.92         | 22.39         | 19.60         | 18.28         |
| Counterfactual revenues | 93.87 | 104.15 | 116.86 | 139.39 |
| Counterfactual costs | 162.18 | 143.35 | 135.39 | 126.03 |
| Counterfactual profits | $-68.31$ | $-39.19$ | $-18.53$ | 13.37 |
| ATET            | 52.48         | 31.22         | 13.92         | $-16.74$      |

The further breakdown in Table 13 offers insight explaining the higher average profit for contract growers of large sizes. The average revenue per animal is $14.09, $14.42, $15.00 and $14.91 for the respective quartiles. Thus, larger average profits of


25 The yearly average live-weight prices of hog were $47.41, $49.45, $39.58, $43.56, $52.51, $50.05, 47.26, and $47.09 for the years 2000 through 2007, respectively. Thus, estimates of negative profits when fully accounting for family labor and imputed capital cost are not due to unusually low prices. In fact, the 2004 average price was the highest in this period, implying that growers are operating with less than market wages for family labor and/or less that market rates of return on capital.
larger contract growers do not result from larger revenues. Rather, they result mainly from smaller average costs per animal given by $29.92, $22.39, $19.60 and $18.28 for the respective quartiles. The average counterfactual profits (i.e., reservation profits) are –$68.31, –$39.19, –$18.53 and $13.37 for the respective quartiles. Thus, even though the average profit of large contract growers is larger than that of small contract growers, their smaller average gain from contracting is due to their larger average reservation profits.

If the reservation profit were uniform across all growers, e.g., at a level between –$4.60 and –$3.37, then only contract growers in the fourth quartile would have positive average gains from contracting. Or at a uniform reservation profit level between –$7.97 and –$4.60 contract growers from both the third and fourth quartiles would have positive average gains from contracting, and so on. Or a uniform reservation profit greater than –$3.37 would imply negative average gains from contracting for all contract growers. Regardless of the level of any uniform reservation profit, negative average gains would be implied for the smaller growers if any. Assuming size is a proxy for ability, these results are thus consistent with the theoretical predictions of chapters 3 and 4 whereby low-ability contract growers receiving negative gains from contracting, i.e., where the reservation profit is the same across all growers.

Interestingly, both actual profits and reservation profits of the first and second quartiles of contract growers are highly negative. Apparently, these operations would either have to expand to operate more efficiently or quit hog operations since they are losing money on average. For the larger two quartiles in contrast, breakeven profit is possible if only paid labor costs are considered. The unpaid family labor costs are $10.76, $5.48, $3.98 and $2.28 for the respective quartiles. These results offer an explanation for
why small and medium size growers are either expanding their hog operations or closing down.26 These results also explain why large hog growers have preferred independent operations to contract operations recently. Large reservation profits for contract growers in the fourth quartile explain the preference for independent operations.

Given this explanation, a breakdown of reservation profits by size is useful. Table 13 shows that the average reservation revenues are increasing in size while the average reservation costs are decreasing in size. Thus, while the variation in contract growers’ actual profits across sizes occur mostly from variations in costs, variations in contract growers’ reservation profits across sizes are due to both variations in costs and revenues. Thus, grower size is critical. Small size growers are being forced out of the hog business regardless of contracting status unless they expand their operation size. These results are explained by the estimated returns for scale. Both of the per animal variable cost regressions in section 8.4.3 estimate a negative coefficient for the size variable ln(n).

Similarly, both of the revenue per animal regressions in section 8.4.1 estimate a positive coefficient on the size variable ln(n).

Thinning spot market may explain why both actual profits and reservation profits are small for smaller growers. Thin spot markets make spot market transactions costlier for all market participants. Further, spot market transactions are even more costly for smaller participants who cannot reap scale benefits. Thus, small growers make small

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26 Between 1994 and 1999, the number of U.S. hog farms fell from over 200,000 to less than 100,000 showing a decrease of more than 50 percent, while the hog inventory remained relatively stable. During the same six-year period, farms with at least 2,000 head increased their share of total swine inventory from 37 percent to 81 percent (Key, 2004). Similarly, rapid growth occurred among very large operations. Operations producing at least 50,000 head increased their share of total hogs marketed from 17 percent in 1994 to 37 percent in 1997 to 51 percent in 2000 (Lawrence and Grimes, 2001; Key, 2004). According to McBride and Key (2003), extraordinary growth occurred in the average size of specialized hog operations between 1992 and 1998. Hog finishing operations showed an average increase of 240 percent in sales and removals during that period.
profits as independent growers, which means their reservations profits are small. Consequently, contractors will pay them less or compel them to carry larger costs than their large size counterparts. Thus, they also earn smaller profits if they contract.\textsuperscript{27}

These conclusions of this section are confirmed by the qualitative survey responses. In response to the survey question “How many more years do you expect this operation will be producing hogs?”, 25 percent (54 contract, 40 independent) growers mentioned 5 years or less while 8 percent did not respond to this question. Not surprisingly, 61 percent of these 94 growers who indicated a likelihood of quitting the hog business in the next 5 years are in the first quartile of growers (which is 46 percent of the growers in this quartile) while the corresponding share is 20 percent for second quartile growers (which is only 23 percent of this quartile). The corresponding shares for the third and fourth quartiles are 11 and 8 percent, respectively. These figures definitely suggest continuing disappearance of small-scale hog operations irrespective of their contracting status.

8.5.2.2 Consistency within abilities

The discussion thus far explains why larger growers have smaller gains from contracting than smaller growers based on differences in ability and its implications for

\textsuperscript{27} Roberts and Key (2005) explore a similar line of reasoning for negative gains by developing a model that shows how introducing the opportunity to contract can lower welfare for some, and perhaps all, contracting parties. They consider a situation where processors can obtain inputs from suppliers (farmers) using either a spot market or contractual arrangements, and where spot market transaction costs depend on the volume of trade in the spot market. Contracting parties may lose when more contracting causes higher transaction costs for spot market participants. At the margin, firms and input suppliers gain from signing contracts. However, contracting raises spot-market transaction costs for those who do not sign contracts, which provides a greater incentive for others to sign contracts, ultimately inducing more contracting than optimal. Their model demonstrates why structural or organizational change may be rapid and why the private minimization of transaction costs may not lead to optimal institutional arrangements.
variation in reservation profits. This section turns to analysis of negative average gains from contracting for contract growers of a given ability. This discussion relates to the theoretical implication that a contract grower loses from contracting if his realized productivity is below the average productivity of the growers of that ability.

In section 3.5 of chapter 3, the gains from contracting are negative for low-ability agents who realize lower than expected ability and positive for such agents who realize higher than expected ability. The conclusions of chapter 6 derived under a similar information structure differ from the conclusions of chapter 3. The results of section 6.4.3 of chapter 6 suggest a rich set of alternative conditions where one group or the other among low-ability growers can experience an ex post loss from contracting that is likely to be repeated under the same conditions due to realizing an ability different than expected.

For contract growers of a given average ability (reflected by size), Table 14 presents average realized profits for contract growers who gain from contracting and for contract growers who lose from contracting, compared to the average profit of all contract growers of that ability. For example, the first row presents the average profit of growers for whom their realized profit is greater than their reservation profit.

| Table 14: Profit of contract growers who gain and lose from contracting |
|--------------------------------------------------|-----------------|-----------------|-----------------|-----------------|
| Size                                             | \( n \leq 3,000 \) | \( 3,000 < n \leq 6,000 \) | \( 6,000 < n \leq 11,000 \) | \( n > 11,000 \) |
| Growers who gain                                 | -15.54          | -7.82           | -4.16           | -3.02           |
| Growers who lose                                 | -19.82          | -8.50           | -5.39           | -3.56           |

For the first quartile, the average realized profit of 4 of 60 contract growers who lose from contracting was –$19.42, considerably below the average of –$15.83, while the
average realized profit for contract growers who gain was –$15.54, only slightly above the average. Similarly, for the second quartile, 13 of 58 contract growers who lose by contracting had an average realized profit of –$8.50, considerably less than the average profit of –$7.97, while contract growers who gain had an average realized profit of –$7.82, only slightly above the average. For the third quartile, 21 of 58 contract growers who lose from contracting had an average realized profit of –$5.39 compared to an average realized profit of –$4.60 for those who gain. Finally, for the fourth quartiles, 38 of 58 contract growers who lose have an average realized profit of –$3.56 compared to an average realized profit of –$3.37 for those who gain.

Thus, for all quartiles (i.e., all abilities), growers with below expected realized abilities experience negative gains from contracting, which validates theoretical predictions of both chapters 3 and 6. However, both the difference in realized profits between gainers and losers and the negative magnitude of realized profits decreases with size (i.e., with average ex ante ability). Interestingly, profits increase in size (i.e., ability) but gains decrease in size as implied by the increasing share of contract growers in each quartile who lose from contracting (6.8%, 22.4%, 36.2%, and 65.5% of growers in the respective quartiles). This happens because reservation profits increase (in ability) faster than profits.

8.5.2.3 Appropriateness of other methods

In general, the two-step results described thus far suggest that selection based on unobservable characteristics is substantial. The results also suggest purposive sorting into groups of contracting and independent growers on the basis of comparative advantage or
gains from contracting. The inverse Mill’s ratio, $imr$, is highly statistically significant in three of the four regressions used for calculating gains from contracting and is near 5-percent significance in the fourth. This means the CIA discussed in chapter 6 is not satisfied because selection is based on unobservables.

Further, impact heterogeneity can be another reason for violation of the CIA. With heterogeneity in how the observables affect revenues as in section 8.4.2, contracting effects are not simply the differences in intercepts between contracting and independent grower regressions at a given size. A similar argument applies to the contracting effects of costs. Matching methods assume the difference in the intercepts in these regressions are the contracting effects on revenues and costs. Thus, heterogeneity makes the predicted revenues and costs dependent on the contracting status, violating the CIA because the same observables give different levels of predicted profits depending on the contracting status.

For these reasons, propensity score matching methods have been shown to be inappropriate and highly likely to produce biased impact estimates for the purposes of this dissertation. Note that the CIA-based OLS estimate was obtained and its implications are reported in Table 12, showing that OLS impact estimates have significant biases.

8.6 The Quantile Effects of Contracting

Thus far the results obtained with Heckman’s two-step estimation procedure analyze the mean effects of contracting on contract and independent growers. Using the quantile regression method, this section explores further how contracting affects contract growers’ gains from contracting differently at different points along the conditional profit
distribution. Two-step results signal that gains from contracting decrease over profit quantiles. This result is generated from the fact that profits increase in size and gains decrease in size as presented in Table 13. However, the quantile regression method has several other virtues for my analysis. The quantile regression estimator gives less weight to outlier data points of the dependent variable than least squares methods, which weakens the impact such data points might have on the results. Also, by allowing the parameter estimates for the marginal effects of the explanatory variables to differ across quantiles of the dependent variable, robustness to potential heteroskedasticity is achieved. Further, when the error terms are non-normal, quantile regression estimators may be more efficient than least squares estimators. The main advantage, however, is the semi-parametric nature of the approach, which relaxes the restriction that parameters must be constant across the entire distribution of the dependent variables (Koenker and Bassett, 1978; Buchinsky, 1994, 1995).

8.6.1 Treatment effects at different quantiles

As discussed in section 8.4.2, test results rejecting pooled two-step regression invalidate the first two-variants of two-step estimation procedures for revenues so only the third variant is used here. Thus, I apply separate regressions for independent and contract growers both in revenue and cost estimation. The quantile regression results at the 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, and 0.9 conditional quantiles are presented in Tables 15a and 15b for independent and contract growers, respectively. Similarly, the quantile regression results for variable costs per head at the 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, and 0.9
conditional quantiles are presented in Table 16a and 16b for independent and contract growers, respectively.
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</tbody>
</table>

<sup>a</sup> These results were produced with 143 (572 after stacking) observations obtaining pseudo \( R^2 \)'s of 0.8857, 0.9125, 0.9404, 0.9501, 0.9586, 0.9692, and 0.9619 for quantiles 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, and 0.9, respectively. The dependent variable is grower revenue per animal. Standard errors are reported below the coefficients.

<sup>b</sup> No observations in this region were available for this regression, i.e., "N/A" means not applicable.
Table 15b: Quantile regression estimates for contract grower revenue

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients at different quantiles&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>ln(n)</td>
<td>0.046</td>
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<td></td>
<td>0.000</td>
</tr>
<tr>
<td>ln(labor)</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>ln(capital)</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
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<tr>
<td>ln(feed)</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>ln(pigs)</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>ln(other)</td>
<td>0.103</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>op Educ</td>
<td>0.084</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>op Occup</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>ncr</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>re</td>
<td>-0.183</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>rm</td>
<td>-0.293</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>rn</td>
<td>-0.542</td>
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<td></td>
<td>0.000</td>
</tr>
<tr>
<td>rw</td>
<td>-0.628</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>imr</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>constant</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
</tr>
</tbody>
</table>

<sup>a</sup>These results were produced with 234 (931 after stacking, 5 missing) observations obtaining pseudo $R^2$'s of 0.7569, 0.8305, 0.8944, 0.9124, 0.9274, 0.9453, and 0.9281 for quantiles 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, and 0.9, respectively. The dependent variable is grower revenue per animal. Standard errors are reported below the coefficients.
Table 16a: Quantile estimates of variable cost for independent growers

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients at different quantiles&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>ln(revenue)</td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td>0.382</td>
</tr>
<tr>
<td>ln(n)</td>
<td>-0.060</td>
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<tr>
<td></td>
<td>0.168</td>
</tr>
<tr>
<td>ln(capital)</td>
<td>0.003</td>
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<tr>
<td></td>
<td>0.247</td>
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<tr>
<td>op_educ</td>
<td>0.117</td>
</tr>
<tr>
<td></td>
<td>0.235</td>
</tr>
<tr>
<td>op_ocup</td>
<td>-0.274</td>
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<tr>
<td></td>
<td>0.512</td>
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<tr>
<td>special</td>
<td>0.224</td>
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<tr>
<td></td>
<td>0.260</td>
</tr>
<tr>
<td>ncr</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>0.094</td>
</tr>
<tr>
<td>re&lt;sup&gt;b&lt;/sup&gt;</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
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<tr>
<td>rm</td>
<td>-0.140</td>
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<tr>
<td></td>
<td>0.388</td>
</tr>
<tr>
<td>rn</td>
<td>-0.062</td>
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<tr>
<td></td>
<td>0.484</td>
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<tr>
<td>rw</td>
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<td>0.545</td>
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<td>imr</td>
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<td>constant</td>
<td>2.587</td>
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<td></td>
<td>2.475</td>
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</table>

<sup>a</sup> These results were produced with 143 independent observations obtaining pseudo $R^2$'s of 0.2634, 0.2445, 0.2204, 0.2098, 0.2282, 0.3155, and 0.4173 for quantiles 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, and 0.9, respectively. The dependent variable is labor costs per animal. Standard errors are reported below the coefficients.

<sup>b</sup> No observations in this region were available for this regression, i.e., "N/A" means not applicable.
Table 16b: Quantile estimates of variable cost for contract growers

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients at different quantiles&lt;sup&gt;a&lt;/sup&gt;</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
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<tr>
<td>ln(revenue)</td>
<td>0.310</td>
</tr>
<tr>
<td></td>
<td>0.154</td>
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<tr>
<td>ln(n)</td>
<td>-0.196</td>
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<tr>
<td></td>
<td>0.051</td>
</tr>
<tr>
<td>ln(capital)</td>
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<td></td>
<td>0.083</td>
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<td>op_educ</td>
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<td>0.084</td>
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<td>op_ocup</td>
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<td>special</td>
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<td>0.149</td>
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<td>rm</td>
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<td>0.177</td>
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<td>constant</td>
<td>1.860</td>
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<td>0.841</td>
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</tbody>
</table>

<sup>a</sup> These results were produced with 134 contract observations obtaining pseudo $R^2$'s of 0.1884, 0.1956, 0.2383, 0.2560, 0.2767, 0.3411, and 0.3666 for quantiles 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, and 0.9, respectively. The dependent variable is variable costs per animal. Standard errors are reported below the coefficients.
Following the similar 5-step procedure applied in section 8.5 and discussed in section 7.3.4.3 of chapter 7, quantile regression estimates of contracting effects are obtained and reported in Table 17. Steps 1 and 2 (computing the appropriate selection correction terms, i.e., inverse Mill’s ratios) are performed on the basis of the probit regression results reported in Table 5.\textsuperscript{28} Step 3 (obtaining the predicted quantile profits for contract and independent growers) is performed on the basis of the regression results presented in Tables 15a, 15b, 16a, and 16b. Step 4 (obtaining the counterfactual quantile profits for contract and independent growers using the estimated coefficients from the independent and contract profit regressions, respectively) is performed on the basis of the results presented in the same set of tables. Finally, Step 5 (estimation of the impact measures $ATE$, $ATET$, and $ATNT$ at various quantiles) is performed combining the results obtained from steps 3 and 4.

Table 17 presents the impact measures estimated at different quantiles conditional on profits. This table reports only the coefficients on the main parameter of interest to this study—the quantile treatment effect of contracting for growers. For comparison purposes, both estimates at the mean (copied from Table 12) and estimates by quantile regression at various quantiles are presented in Table 17.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>0.1</th>
<th>0.2</th>
<th>0.4</th>
<th>Median</th>
<th>0.6</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ATE$</td>
<td>11.01</td>
<td>44.65</td>
<td>24.53</td>
<td>5.83</td>
<td>1.65</td>
<td>1.39</td>
<td>-50.89</td>
<td>-37.05</td>
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<tr>
<td>$ATET$</td>
<td>20.50</td>
<td>60.86</td>
<td>34.94</td>
<td>11.82</td>
<td>8.20</td>
<td>11.77</td>
<td>-55.45</td>
<td>-27.30</td>
</tr>
<tr>
<td>$ATNT$</td>
<td>-4.51</td>
<td>18.12</td>
<td>7.50</td>
<td>-3.98</td>
<td>-9.07</td>
<td>-15.58</td>
<td>-43.44</td>
<td>-53.02</td>
</tr>
</tbody>
</table>

\textsuperscript{28} Ideally, a probit regression would be performed at each quantile but lack of enough observations to make such estimates meaningful forced reliance on a single probit regression for the purpose of two-step quantile regressions.
The estimate of \( ATET \) is positive for all but the 0.8 and 0.9 quantiles. The estimate at the median is $8.20, which is considerably smaller than the estimate at the mean of $20.50. Estimates at the extreme points of the profit distribution are also quite different than the impact estimate at the median. The \( ATET \) of $60.86 for the 0.1 quantile is substantially larger than the median estimate of $8.20, which is substantially larger than the estimate of –$27.30 at quantile 0.9. The impact estimates of \( ATET \) have an obvious pattern whereby estimated impacts decrease over quantiles except for some apparent noise in the relationship at quantiles 0.6 through 0.9. This implies that the decreasing trend in gains from contracting over profit quantiles is due to an increasing trend in reservation profits over quantiles because the average reservation profit is equal to the average profit minus \( ATET \).

The estimate of \( ATNT \) is negative at all but the 0.1 and 0.2 quantiles. The estimate at the median is -$9.07 which is smaller than the estimate at the mean of -$4.51. Estimates at the extreme points of the profit distribution are also quite different than the impact estimate at the median. The \( ATNT \) estimate of $18.12 for the 0.1 quantile is substantially larger than the median estimate of –$9.07, which is substantially greater than the estimate of –$53.02 at quantile 0.9. The impact estimate of \( ATNT \) also has a clear pattern with estimated impacts decreasing over quantiles. These results imply that the decreasing trend in counterfactual gains from contracting for independent growers over profit quantiles are due to an increasing trend in the profits they earn as independent operations over quantiles.

In general, these results suggest that the effect of contracting differs across the distribution of profits for growers. Contracting has a substantially different effect at the
top and bottom of the profit distribution for growers. The prevailing pattern is a decrease in impact estimates across profit quantiles. Thus, a general conclusion from Table 17 is that contracting has higher benefits not only for smaller growers but also growers toward the lower end of their conditional profit distributions.

8.7 Conclusions

The various models specified in chapter 7 for estimating mean and quantile contracting effects for which empirical analysis supports assumptions have been estimated. The results provide a number of important insights regarding the benefits and costs of contracting depending on both observable and unobservable differences among growers. The results are summarized as follows:

1. Risk reduction and limited credit availability both appear to be important motivations for hog contracting.

2. The technologies employed by contract and independent growers differ with respect to the productivity of individual variable factor inputs.

3. High ability growers earn higher profits on average than low ability growers as predicted by hog contracting theory.

4. The average effect of contracting for contract growers (ATET) is positive for all contract growers as a group, but when contract growers are considered by size quartiles, the ATET is positive for lower three quartiles whereas it is negative for the highest quartile.
5. A positive selection bias is estimated, which tends (because of the effect of unobservables) to give contract growers a comparative disadvantage from independent operation.

6. The sorting effect is positive, implying that contract growers tend (because of the effect of unobservables) to choose contracting because of a comparative advantage in doing so.

7. The mean effect of contracting for independent growers (ATNT) is negative, suggesting that their choice not to contract is rational.

8. The ATET exceeds the ATNT, meaning that independent growers would gain less than contract growers had they contracted.

9. Some 33 percent contract growers receive negative gains from contracting.

10. Losses from contracting are explained by below average productivity, which may not have been anticipated by growers at the time they committed to investments in the hog business.

11. The contracting effect for contract growers, measured by ATET, tends to decrease over quantiles of the profit distribution implying that gains from contracting are smaller for higher-profit growers.

12. The contracting effects for independent growers, measured by ATNT, also tend to decrease over quantiles of the profit distribution and are negative for all but the lowest quantiles.

13. The results suggest that small growers will be forced either to exit the hog business or else expand operations regardless of their contracting status.
The results thus offer a considerably complex explanation of contracting effects that highlight heterogeneity of effects among growers. The regression results on which these conclusions are based are highly statistically significant, are remarkably consistent with economic theory given the number of parameters that are estimated, and offer a remarkably coherent explanation that supports confidence in the conclusions.
Bibliography:


Ichimura H. “Semiparametric least squares (SLS) and weighted SLS estimation of single index models.” Journal of Econometrics 58, 1993: 71-120.


