

Distributional Convergence of Inter-Meeting Times Under Generalized Hybrid Random Walk Mobility Model

Richard J. La

The
Institute for
Systems
Research



A. JAMES CLARK
SCHOOL OF ENGINEERING

ISR develops, applies and teaches advanced methodologies of design and analysis to solve complex, hierarchical, heterogeneous and dynamic problems of engineering technology and systems for industry and government.

ISR is a permanent institute of the University of Maryland, within the A. James Clark School of Engineering. It is a graduated National Science Foundation Engineering Research Center.

www.isr.umd.edu

Distributional Convergence of Inter-Meeting Times Under Generalized Hybrid Random Walk Mobility Model

Richard J. La

Department of ECE & ISR

University of Maryland, College Park

Abstract

The performance of a *mobile* wireless network depends on the *time-varying* connectivity of the network as nodes move around. Therefore, there has been a growing interest in the distribution of inter-meeting times between two nodes in mobile wireless networks. We study the distribution of inter-meeting times under the (generalized) Hybrid Random Walk mobility model. We show that when the (conditional) probability that the two nodes can communicate directly with each other given that they are in the same cell is small, the distribution of inter-meeting times can be well approximated using an exponential distribution. In addition, the mean of the inter-meeting times can be estimated using the number of cells in the network and the aforementioned conditional probability of having a communication link when the two nodes are in the same cell. We also show that such an approximation does *not* hold for the Random Walk mobility model.

I. INTRODUCTION

Recently there has been a growing interest in understanding the distribution of inter-meeting times between mobile nodes in wireless networks (e.g., [1], [4], [9], [11], [15]). An inter-meeting time between two nodes refers to the amount of time during which they stay unable to

communicate directly with each other after they lose the “communication link” between them.¹ Since the ability of a (multi-hop) wireless network to transfer information between a pair of nodes in a timely manner depends critically on the (time-varying) network connectivity, understanding the statistical properties of inter-meeting times is of much interest. Such an understanding is even more pressing in Disruption Tolerant Networks (DTNs) that rely on *intermittent* and/or *sparse* connectivity between nodes to forward information.

We summarize a few studies that are most relevant to this paper: Groenevelt et al. [8] studied the distribution of inter-meeting times between two nodes under the popular Random Waypoint (RWP) mobility model and suggested that the distribution can be well approximated by an exponential distribution. Chaintreau et al. [3] examined several sets of traces collected in different settings and reported an interesting observation that the empirical distributions exhibit a power law over a wide range followed by an exponential tail. Karagiannis et al. [10], using different sets of measurements, first illustrated the existence of a similar dichotomy in the empirical distributions of inter-meeting times. Then, they demonstrated that such a dichotomy exists even under a simple Random Walk (RW) mobility model on a circle. An interesting study by Cai and Eun [2] suggests that, in most scenarios where the domain of mobility is bounded, the distribution is expected to have an exponential tail. On the other hand, when the domain is unbounded, a power law can emerge, hinting at the possibility that a bounded domain used for simulation may be a main source of the emergence of an exponential tail in some cases.

The main contributions of this paper can be summarized as follows:

- First, we study the distribution of inter-meeting times under the Hybrid Random Walk (HRW) mobility model, first introduced by Sharma et al. [14]. The HRW mobility model is a generalization of the RW mobility model [5], which also includes the independent and identically distributed (i.i.d.) mobility model used in [12]. In particular, we prove that as the network size

¹We say that there is a communication link between two nodes if their achievable signal-to-noise ratio (SNR) is sufficiently high to allow correct decoding of the signal from each other with a high probability.

increases, the inter-meeting times can be well approximated using exponential random variables (rvs) under the assumption that, given that two nodes are in the same cell, the (conditional) probability that they can communicate directly with each other is small.

- We extend this result in two directions; first, we introduce a generalized HRW mobility model, of which the HRW mobility model is a special case, and demonstrate that a similar approximation of inter-meeting times using exponential rvs, holds under the generalized HRW mobility model. Secondly, we allow *heterogeneous* mobility among the nodes and prove that the same result is true under weak technical conditions, without assuming a large network size. These findings suggest that the distribution of inter-meeting times is not sensitive to the details of nodes' mobility and may resemble an exponential distribution under a set of mild assumptions when the nodes' mobility is independent.

- Finally, we illustrate that, perhaps somewhat surprisingly, the same approximation does *not* hold for the RW mobility model. This is a consequence of the fact that, under the RW mobility model, the earlier assumption that when two nodes are in the same cell the conditional probability of having a communication link between them is small, does not hold.

We emphasize the point that the aim of this paper is *not* to disprove the dichotomy exhibited by empirical distributions of inter-meeting times (e.g., [3], [10]). Instead, our goals are (i) to show that, when running simulation with a certain class of mobility models, including the HRW mobility model, one may expect the distribution of inter-meeting times to resemble an exponential distribution under some conditions and (ii) to provide some insight into the emergence of exponential distributions. Moreover, our findings (Theorems 1 - 3 in Sections IV and VI) suggest that the exponential tail of the distribution may be caused by factors other than a bounded domain of mobility, which is suggested as one of main reasons for the emergence of exponential tail by Cai and Eun [2] as mentioned earlier.

The rest of this paper is organized as follows: Section II describes the RW and HRW mobility models, and Section III defines the inter-meeting times between two nodes. Our first result under

the HRW mobility model is presented in Section IV, followed by a study under the RW mobility model in Section V. Extensions to the generalized HRW mobility model and heterogeneous mobility cases are discussed in Section VI. Simulation results are provided in Section VII.

II. BACKGROUND

In this section we describe two mobility models - the RW mobility model and the HRW mobility model - that are the focus of this paper.

A. Random Walk mobility model

The RW mobility model was used by El Gamal et al. in [5] in the context of studying the scaling laws of the network transport throughput for multi-hop wireless networks. For each fixed $n = 1, 2, \dots$, a unit square area is divided into a discrete torus of size $n \times n$. Each of n^2 rectangular areas is called a *cell*, and each cell is identified by a pair (i, j) , $i, j \in \{0, 1, \dots, n-1\}$, as shown in Fig. 1.

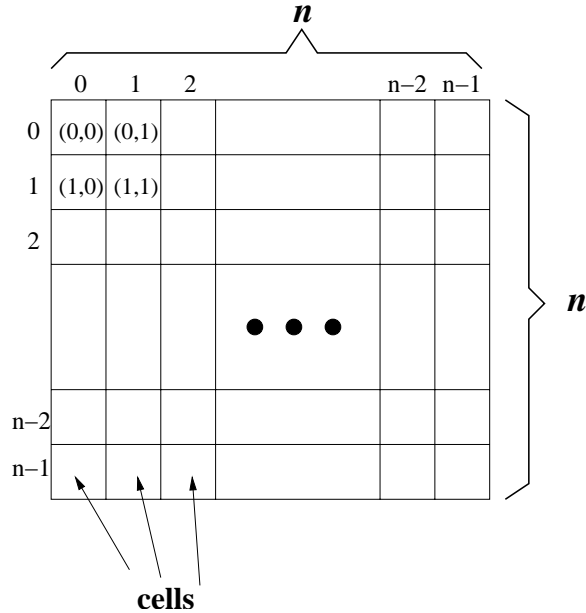


Fig. 1. The RW mobility model.

Time is slotted into contiguous timeslots $t = 0, 1, \dots$. At timeslot $t = 0$, a node is initially placed in one of n^2 cells according to some probability mass function (pmf). After its initial

placement, a node in a cell, say (i, j) , first selects one of the adjacent cells, i.e., cells $(i + 1, j)$, $(i - 1, j)$, $(i, j + 1)$, and $(i, j - 1)$,² with equal probability of $1/4$ independently of the past, and moves to the selected cell at timeslot $t = 1$. The node then repeats this process in every subsequent timeslot.

The location of a node at timeslot $t = 0, 1, \dots$, is denoted by $C^{(n)}(t)$, which indicates the cell where the node lies. From the description of the RW mobility model, it is clear that the discrete-time stochastic process $\{C^{(n)}(t); t = 0, 1, \dots\}$ is a time homogeneous Markov chain with state space $\{(i, j) \mid i, j \in \{0, 1, \dots, n - 1\}\}$.

B. Hybrid Random Walk mobility model

The HRW mobility model can be viewed as a generalization of the RW mobility model in the previous subsection [14]. It is parameterized by $\beta, 0 \leq \beta \leq 1/2$. For each fixed $n = 1, 2, \dots$, the unit square area is first divided into a discrete torus of $n^\beta \times n^\beta$ cells. Each cell is then further divided into $n^{(1-2\beta)/2} \times n^{(1-2\beta)/2}$ subcells. Thus, there are a total of n subcells. A subcell $\ell^{(n)}$ in the unit square area is uniquely identified by a pair $\ell^{(n)} = (\mathbf{c}^{(n)}, \mathbf{s}^{(n)})$, where $\mathbf{c}^{(n)} = (c_1^{(n)}, c_2^{(n)})$ with $c_1^{(n)}, c_2^{(n)} \in \{0, 1, \dots, n^\beta - 1\}$ specifies the cell to which the subcell $\ell^{(n)}$ belongs, and $\mathbf{s}^{(n)} = (s_1^{(n)}, s_2^{(n)})$ with $s_1^{(n)}, s_2^{(n)} \in \{0, 1, \dots, n^{(1-2\beta)/2} - 1\}$ designates the position of the subcell within the cell $\mathbf{c}^{(n)}$.

The location of a node at timeslot $t = 0, 1, \dots$ is given by the *subcell* in which the node lies and is denoted by $L^{(n)}(t) = (C^{(n)}(t), S^{(n)}(t))$. Here, $C^{(n)}(t) = (C_1^{(n)}(t), C_2^{(n)}(t))$ is the cell at which the node is located, and $S^{(n)}(t) = (S_1^{(n)}(t), S_2^{(n)}(t))$ provides the position of the subcell within $C^{(n)}(t)$ where the node resides. The initial location $L^{(n)}(0)$ of the node at timeslot $t = 0$ is selected as follows: First, a cell $C^{(n)}(0)$ is chosen according to some pmf. Then, one of the subcells in the cell $C^{(n)}(0)$ is selected according to the discrete uniform distribution over the set of $n^{1-2\beta}$ subcells in the cell.

²All operations are modulo n operations.

The transition of a node from one subcell at timeslot $t = 0, 1, \dots$, to another subcell at timeslot $t+1$ is described by the following: A node located at subcell $\ell^{(n)}$ at timeslot t first selects one of the adjacent cells with equal probability of $1/4$ (as in the RW mobility model). Then, it chooses one of the subcells in the selected adjacent cell with equal probability of $n^{-(1-2\beta)}$, independently of the past and the selected cell. Hence,

$$\mathbf{L}^{(n)} := \{L^{(n)}(t); t = 0, 1, \dots\} = \{(C^{(n)}(t), S^{(n)}(t)); t = 0, 1, \dots\},$$

which we call the *trajectory* of the node, is a discrete-time stochastic process where $\mathbf{C}^{(n)} := \{C^{(n)}(t); t = 0, 1, \dots\}$ evolves according to the RW mobility model (hence is a time homogeneous Markov chain) and $\mathbf{S}^{(n)} := \{S^{(n)}(t); t = 0, 1, \dots\}$ is a sequence of i.i.d. rvs. The stochastic processes $\mathbf{C}^{(n)}$ and $\mathbf{S}^{(n)}$ are mutually independent because the subcells are selected independently of the past and selected cells as explained earlier.

When $\beta = 0.5$, the HRW mobility model reduces to the usual RW mobility model since there is only one subcell in each cell. On the other hand, when $\beta = 0$, a node moves according to the i.i.d. mobility model used in [12]. This is because there is only one cell consisting of n subcells and the node selects one of the subcells with equal probability n^{-1} in each timeslot, independently of the past.

III. INTER-MEETING TIMES

In this paper we are interested in studying the distribution of inter-meeting times as the network size becomes large. To this end we investigate the asymptotic distribution of inter-meeting times (under appropriate scaling) as the network size increases. This is done by introducing a parametric scenario with a sequence of networks in which the number of subcells increases (e.g., [5], [14]). We assume a discrete-time model where the time is divided into contiguous timeslots $t = 0, 1, \dots$ throughout the rest of the paper.

A. The HRW mobility model under consideration

In the current and following sections we consider the HRW mobility model described in subsection II-B, but in a slightly more general form: For each fixed $n = 1, 2, \dots$, the unit square area is divided into a discrete torus of $h_1(n) \times h_1(n)$ cells. Each cell is then further divided into $h_2(n) \times h_2(n)$ subcells. Both $h_1(n)$ and $h_2(n)$ are assumed to be positive integers. It is clear that the total number of subcells is $(h_1(n) \times h_2(n))^2 =: N(n)$. Let $\mathcal{C}^{(n)} = \{(i, j) \mid i, j \in \{0, 1, \dots, h_1(n) - 1\}\}$ be the set of cells and $\mathcal{S}^{(n)} = \{(a, b) \mid a, b \in \{0, 1, \dots, h_2(n) - 1\}\}$ be the set of subcells in a cell.

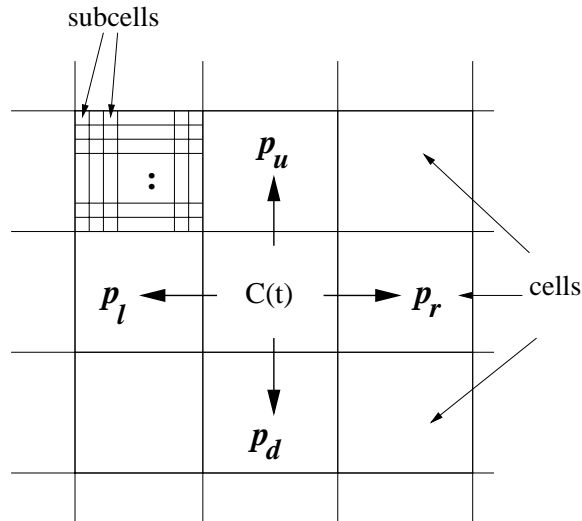


Fig. 2. The HRW mobility model.

A node residing in a subcell at timeslot $t = 0, 1, \dots$, (i) selects one of the four neighboring cells with corresponding probability p_l, p_r, p_u and p_d (as shown in Fig. 2) and (ii) picks one of the subcells in the selected cell according to some pmf $P^{(n)}$ over the set $\mathcal{S}^{(n)}$, independently of the past and the selected cell. Then, the node moves to the chosen subcell at timeslot $t + 1$.³ This process is repeated in each of subsequent timeslots. We assume that the probabilities p_l, p_r, p_u and p_d are strictly positive.

³We assume that the node moves to a new subcell at the *beginning* of each timeslot.

B. Inter-meeting times between two nodes

For each $n = 1, 2, \dots$, we have two nodes $i = 0, 1$, moving according to the HRW mobility model on a discrete torus with $N(n)$ subcells as described in the previous subsection. The location of node i at time $t = 0, 1, \dots$, is given by the subcell $L_i^{(n)}(t) = (C_i^{(n)}(t), S_i^{(n)}(t))$ at which the node is located. As described in subsection II-B, $C_i^{(n)}(t) \in \mathcal{C}^{(n)}$ and $S_i^{(n)}(t) \in \mathcal{S}^{(n)}$ denote the cell and the subcell within $C_i^{(n)}(t)$, respectively, of node i 's location at timeslot t . The trajectory of node $i = 0, 1$, is given by

$$\mathbf{L}_i^{(n)} := \{L_i^{(n)}(t); t = 0, 1, \dots\} = \{(C_i^{(n)}(t), S_i^{(n)}(t)); t = 0, 1, \dots\} .$$

The stochastic processes $\mathbf{L}_i^{(n)}$, $i = 0, 1$, are assumed mutually independent.

Definition 1: We say that two nodes are *in contact* at timeslot t if they are in the same subcell, i.e., $L_0^{(n)}(t) = L_1^{(n)}(t)$. Similarly, two nodes are said to *meet* at timeslot t if (i) $L_0^{(n)}(t-1) \neq L_1^{(n)}(t-1)$ and (ii) $L_0^{(n)}(t) = L_1^{(n)}(t)$.

Throughout the paper we assume that two nodes can communicate directly with each other at timeslot $t = 0, 1, \dots$, if and only if they are in contact during the timeslot t .

Define $\mathbf{U}^{(n)} = \{U^{(n)}(t); t = 0, 1, \dots\}$, where

$$U^{(n)}(t) = \mathbf{1} \{L_0^{(n)}(t) = L_1^{(n)}(t)\} = \begin{cases} 1, & \text{if } L_0^{(n)}(t) = L_1^{(n)}(t) \\ 0, & \text{otherwise} \end{cases} .$$

The rvs $U^{(n)}(t)$, $t = 0, 1, \dots$, are indicator functions of the event that the two nodes are in contact at timeslot t .

Let $\mathcal{M}^{(n)} := \{M^{(n)}(k); k = 0, 1, \dots\}$ be a sequence of non-negative integers defined as follows: (i) $M^{(n)}(0) = 0$, and (ii) for $k \geq 1$,

$$M^{(n)}(k) = \inf\{t \geq M^{(n)}(k-1) + 1 \mid U^{(n)}(t-1) = 0 \text{ and } U^{(n)}(t) = 1\} .$$

Then, $M^{(n)}(k)$, $k \geq 1$, denotes the time at which the two nodes meet for the k -th time, with the first meeting taking place at $M^{(n)}(1) \geq 1$. Thus, we refer to the sequence $\mathcal{M}^{(n)}$ as the *meeting*

times. The *inter-meeting times* are given by $\mathcal{I}^{(n)} = \{I^{(n)}(k); k = 1, 2, \dots\}$ with

$$I^{(n)}(k) := M^{(n)}(k) - (H^{(n)}(k) + 1) ,$$

where

$$H^{(n)}(k) := \max\{0, \sup\{t \leq M^{(n)}(k) - 1 \mid U^{(n)}(t) = 1\}\} \quad (1)$$

is the maximum of zero and the last timeslot during which the two nodes were in contact before their k -th meeting occurs, i.e., $M^{(n)}(k)$.

An example is shown in Fig. 3. In this example, the first three meetings take place at $M^{(n)}(1) = 1$, $M^{(n)}(2) = 4$, and $M^{(n)}(3) = 8$. According to our definition in (1), $H^{(n)}(1) = 0$, $H^{(n)}(2) = 1$, and $H^{(n)}(3) = 6$. Thus, the first three inter-meeting times are equal to $I^{(n)}(1) = 0$, $I^{(n)}(2) = 2$, and $I^{(n)}(3) = 1$. It is clear from the example that the inter-meeting times $I^{(n)}(k), k \geq 2$, indeed refer to the number of timeslots during which the two nodes are not in contact with each other.

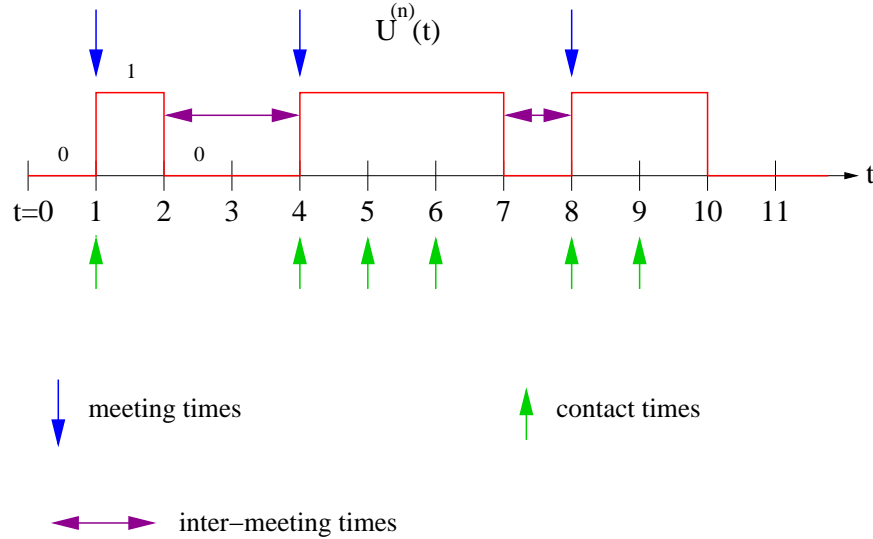


Fig. 3. Plot of indicator functions $U^{(n)}(t), t = 0, 1, 2, \dots$.

Recall from the description of the HRW mobility model that the selection of the next cell and the subcell within the chosen cell is (i) independent of the past and (ii) determined according to the same fixed probabilities (i.e., p_l, p_r, p_u, p_d and $P^{(n)}(\mathbf{s}), \mathbf{s} \in \mathcal{S}^{(n)}$) for all $t = 0, 1, \dots$.

Therefore, as nodes move on a discrete torus, from the viewpoint of the nodes, each time two nodes meet at timeslot t , they start anew from the same conditions they were in the last time they met. This suggests that the number of timeslots that elapse between two consecutive meetings, $M^{(n)}(k+1) - M^{(n)}(k), k \geq 1$, are i.i.d.

Note (from the example in Fig. 3) that the difference $M^{(n)}(k+1) - M^{(n)}(k), k \geq 1$, is the sum of two independent rvs - (i) the number of consecutive timeslots the two nodes spend in contact following their k -th meeting and (ii) the $(k+1)$ -th inter-meeting time $I^{(n)}(k+1)$. This tells us that the inter-meeting times $I^{(n)}(k), k \geq 2$, are i.i.d. rvs, while the distribution of $I^{(n)}(1)$ may be different; $I^{(n)}(1)$ can be zero (as it is in the example), whereas $I^{(n)}(k) \geq 1$ for all $k \geq 2$.

IV. DISTRIBUTIONAL CONVERGENCE OF INTER-MEETING TIMES UNDER THE HRW MOBILITY MODEL

In this section we examine the distribution of the inter-meeting times $I^{(n)}(k), k \geq 2$,⁴ between two nodes under the HRW mobility model. In particular, we are interested in their distribution as the size of the network $N(n)$ becomes large with increasing n .

For each $n = 1, 2, \dots$, define $p^{(n)} := \sum_{\mathbf{s} \in \mathcal{S}^{(n)}} (P^{(n)}(\mathbf{s}))^2 > 0$. Note that this is the probability that the two nodes are in contact conditional on the event that they are in the same cell. In other words, $p^{(n)} = \Pr [L_0^{(n)}(t) = L_1^{(n)}(t) \mid C_0^{(n)}(t) = C_1^{(n)}(t)] = \Pr [S_0^{(n)}(t) = S_1^{(n)}(t)] > 0, t = 0, 1, \dots$. We first introduce the following assumptions on $h_1(n)$ and $p^{(n)}$.

Assumption 1: (i) $\{h_1(n); n = 1, 2, \dots\}$ is a sequence of odd positive integers. (ii) $\lim_{n \rightarrow \infty} p^{(n)} = 0$.

If $h_1(n)$ is an even positive integer, depending on the initial locations of the two nodes, they may never meet.⁵ Thus, in order to ensure that the two nodes will eventually meet with

⁴We do not investigate the distribution of $I^{(n)}(1)$ as it depends on the initial locations of the nodes and, more importantly, does not refer to a real inter-meeting time between two consecutive meetings as mentioned in the previous section.

⁵This is a consequence of the Markov chains $\mathbf{C}_i^{(n)}, i = 0, 1$, being periodic as it takes an even number of timeslots to return to the same cell.

probability one, we assume that $h_1(n)$ is odd. The second assumption implies that the conditional probability that the two nodes are in contact provided that they are in the same cell, decreases to zero as $n \rightarrow \infty$. This necessarily implies that $h_2(n) \rightarrow \infty$ as $n \rightarrow \infty$, i.e., the number of subcells in a cell grows unbounded. For example, this assumption holds if a node selects each subcell with equal probability $1/(h_2(n))^2$ while $h_2(n) \rightarrow \infty$. Note that we do not assume the sequence $\{h_1(n); n = 1, 2, \dots\}$ grows to infinity as $n \rightarrow \infty$.

Theorem 1: Under Assumption 1 we have the following distributional convergence:

$$\lim_{n \rightarrow \infty} \Pr \left[\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}} \leq x \right] = \begin{cases} 1 - e^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases} \quad (2)$$

In other words, the rvs $\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}}$, $n = 1, 2, \dots$, converge in distribution to an exponential rv with parameter one as $n \rightarrow \infty$.

Since $I^{(n)}(k)$, $k \geq 2$, are i.i.d. rvs, Theorem 1 tells us that, for all sufficiently large n , the inter-meeting times can be well approximated using exponential rvs with mean $(h_1(n))^2/p^{(n)}$.

Proof: In order to prove the theorem, we first introduce two other sequences of rvs that are closely related to the meeting times, $M^{(n)}(k)$, and inter-meeting times, $I^{(n)}(k)$. Define $\mathcal{Z}^{(n)} := \{Z^{(n)}(k); k = 0, 1, \dots\}$ to be a sequence of non-negative integers, where (i) $Z^{(n)}(0) = 0$, and (ii) for $k \geq 1$,

$$Z^{(n)}(k) = \inf \left\{ t \geq 0 \mid \sum_{\tau=0}^t U^{(n)}(\tau) \geq k \right\} .$$

Note that at the end of timeslot $Z^{(n)}(k)$ the nodes will have spent k timeslots in contact with each other. We call the rvs $Z^{(n)}(k)$, $k = 1, 2, \dots$, the *contact times* throughout the paper. The sequence of *inter-contact times* is denoted by $\mathcal{X}^{(n)} := \{X^{(n)}(k); k = 1, 2, \dots\}$, where $X^{(n)}(k) := Z^{(n)}(k) - Z^{(n)}(k-1)$ is the time between the $(k-1)$ -th and k -th contacts.

In the earlier example in Fig. 3, the first six *contacts* occur at $Z^{(n)}(1) = 1, Z^{(n)}(2) = 4, Z^{(n)}(3) = 5, Z^{(n)}(4) = 6, Z^{(n)}(5) = 8$, and $Z^{(n)}(6) = 9$. The first six *inter-contact times* are $X^{(n)}(1) = 1, X^{(n)}(2) = 3, X^{(n)}(3) = 1, X^{(n)}(4) = 1, X^{(n)}(5) = 2$, and $X^{(n)}(6) = 1$. Thus,

when two nodes meet and remain in contact for more than one consecutive timeslots, although they meet only once, we assume that they make multiple counts of contact during the period. As a result, while the nodes stay in contact in consecutive timeslots, the inter-meeting times are equal to one.

One can argue that, for a similar reason that the inter-meeting times $I^{(n)}(k), k \geq 2$, are i.i.d. (explained at the end of the previous section), the inter-contact times $X^{(n)}(k), k \geq 2$, are also i.i.d.⁶ In addition, the distribution of the inter-meeting times $I^{(n)}(k), k \geq 2$, is the same as the *conditional* distribution of $X^{(n)}(2) - 1$ given the event $\{X^{(n)}(2) > 1\}$. This can be seen from the earlier example in Fig. 3. Note that inter-meeting times $I^{(n)}(2)$ and $I^{(n)}(3)$ are equal to the first two inter-contact times greater than one, namely $X^{(n)}(2)$ and $X^{(n)}(5)$, minus one, respectively. With a little abuse of notation, we denote this fact by

$$I^{(n)}(2) =_{st} [X^{(n)}(2) - 1 \mid X^{(n)}(2) > 1] , \quad (3)$$

where $=_{st}$ denotes *equality in law*.

We introduce a lemma that is used to complete the proof. The proof of the lemma is provided in the appendix.

Lemma 1: Under Assumption 1 the following distributional convergence holds:

$$\lim_{n \rightarrow \infty} \Pr \left[\frac{X^{(n)}(2)}{(h_1(n))^2/p^{(n)}} \leq x \right] = \begin{cases} 1 - e^{-x} , & x > 0 \\ 0 , & x \leq 0 \end{cases} \quad (4)$$

The lemma states that, under Assumption 1, the (appropriately scaled) inter-contact times converge in distribution to an exponential rv with parameter one.

In light of Lemma 1, in order to complete the proof of the theorem, it suffices to show that, for all $x > 0$,

$$\lim_{n \rightarrow \infty} \Pr \left[\frac{X^{(n)}(2)}{(h_1(n))^2/p^{(n)}} > x \right] = \lim_{n \rightarrow \infty} \Pr \left[\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}} > x \right] .$$

⁶The distribution of $X^{(n)}(1)$ is different from that of $X^{(n)}(k), k \geq 2$. Note that $X^{(n)}(1)$ can be zero, whereas $X^{(n)}(k) \geq 1$ for all $k \geq 2$. In this sense, $X^{(n)}(1)$ is not a real inter-contact time, and we do not concern ourselves with $X^{(n)}(1)$.

For notational simplicity, let $I^{(n)}$ and $X^{(n)}$ denote the rvs with the same distribution as $I^{(n)}(2)$ and $X^{(n)}(2)$, respectively. For any $x > 0$,

$$\begin{aligned} & \Pr \left[\frac{I^{(n)}}{(h_1(n))^2/p^{(n)}} > x \right] \\ &= \Pr \left[\frac{X^{(n)} - 1}{(h_1(n))^2/p^{(n)}} > x \mid X^{(n)} > 1 \right] \\ &= \Pr \left[X^{(n)} > 1 \mid \frac{X^{(n)} - 1}{(h_1(n))^2/p^{(n)}} > x \right] \times \Pr \left[\frac{X^{(n)} - 1}{(h_1(n))^2/p^{(n)}} > x \right] / \Pr [X^{(n)} > 1] , \end{aligned}$$

where the first equality follows from (3), and the second equality is an application of the Bayes' rule [16, p.20].

First, $\Pr [X^{(n)} > 1] = 1 - \Pr [X^{(n)} = 1] \geq 1 - p^{(n)}$ because $\Pr [X^{(n)} = 1] = p^{(n)} \cdot (p_u^2 + p_d^2 + p_l^2 + p_r^2) \leq p^{(n)}$. Since $p^{(n)} \rightarrow 0$ by Assumption 1(ii), $\lim_{n \rightarrow \infty} \Pr [X^{(n)} > 1] = 1$. Secondly, for any $x > 0$,

$$\Pr \left[X^{(n)} > 1 \mid \frac{X^{(n)} - 1}{(h_1(n))^2/p^{(n)}} > x \right] = \Pr \left[X^{(n)} > 1 \mid X^{(n)} > \frac{x \cdot (h_1(n))^2}{p^{(n)}} + 1 \right] = 1 .$$

Therefore,

$$\begin{aligned} \lim_{n \rightarrow \infty} \Pr \left[\frac{I^{(n)}}{(h_1(n))^2/p^{(n)}} > x \right] &= \lim_{n \rightarrow \infty} \Pr \left[\frac{X^{(n)} - 1}{(h_1(n))^2/p^{(n)}} > x \right] \\ &= \lim_{n \rightarrow \infty} \Pr \left[\frac{X^{(n)}}{(h_1(n))^2/p^{(n)}} > x + \frac{p^{(n)}}{(h_1(n))^2} \right] \\ &= \lim_{n \rightarrow \infty} \Pr \left[\frac{X^{(n)}}{(h_1(n))^2/p^{(n)}} > x \right] \\ &= \exp(-x) , \end{aligned}$$

where the third equality follows from the assumption that $p^{(n)} \rightarrow 0$ as $n \rightarrow \infty$ while $h_1(n) \geq 1$, and the last equality is a consequence of Lemma 1. As a result, both inter-contact times and inter-meeting times (appropriately scaled) converge in distribution to an exponential rv with parameter one under Assumption 1. This completes the proof of the theorem. \blacksquare

V. DISTRIBUTION OF INTER-MEETING TIMES UNDER THE RW MOBILITY MODEL

In this section we study the distribution of inter-meeting times under the RW mobility model described in subsection II-A, which is a special case of the HRW mobility model with $h_2(n) =$

1. We show that, unlike in the HRW mobility model under Assumption 1, the (appropriately scaled) inter-meeting times under the RW mobility model do *not* converge in distribution to an exponential rv as $h_1(n) \rightarrow \infty$. As we will explain, this is a consequence of the fact that the RW mobility model does not satisfy Assumption 1(ii) in Section IV.

A. Role of Assumption 1(ii)

We consider the same setting employed in Section IV with the HRW mobility model. Suppose that Assumption 1(ii) does not hold and there exists $\underline{p} > 0$ such that $p^{(n)} \geq \underline{p}$ for all $n \geq 1$. Note that this can happen even when $h_2(n)$ increases unbounded. Clearly, from (3)

$$\Pr [I^{(n)}(2) = 1] = \Pr [X^{(n)}(2) = 2 | X^{(n)}(2) > 1] = \frac{\Pr [X^{(n)}(2) = 2]}{1 - \Pr [X^{(n)}(2) = 1]}.$$

Since $\Pr [X^{(n)}(2) = 1] = p^{(n)} \times (p_u^2 + p_d^2 + p_l^2 + p_r^2) \geq \underline{p} \times (p_u^2 + p_d^2 + p_l^2 + p_r^2)$, we have the following lower bound.

$$\Pr [I^{(n)}(2) = 1] \geq \frac{\Pr [X^{(n)}(2) = 2]}{1 - \underline{p} \cdot (p_u^2 + p_d^2 + p_l^2 + p_r^2)} \quad (5)$$

For every $t = 0, 1, \dots$, let us define the following events:

$$\mathcal{A}^{(n)}(t) = \{C_0^{(n)}(t) = C_1^{(n)}(t)\} \quad \text{and} \quad \mathcal{B}^{(n)}(t) = \{L_0^{(n)}(t) = L_1^{(n)}(t)\} = \{U^{(n)}(t) = 1\}$$

We can rewrite $\Pr [X^{(n)}(2) = 2]$ as

$$\begin{aligned} \Pr [X^{(n)}(2) = 2] &= \Pr [\mathcal{B}^{(n)}(2) \cap (\mathcal{B}^{(n)}(1))^c | \mathcal{B}^{(n)}(0)] \\ &= \Pr [\mathcal{A}^{(n)}(2) \cap (\mathcal{B}^{(n)}(1))^c | \mathcal{B}^{(n)}(0)] \times \Pr [\mathcal{B}^{(n)}(2) | \mathcal{A}^{(n)}(2)], \end{aligned}$$

where \mathcal{B}^c denotes the complement of the event \mathcal{B} , and the second equality follows from the assumption that subcells are chosen independently of the past and selected cells.

Clearly, $\Pr [\mathcal{B}^{(n)}(2) | \mathcal{A}^{(n)}(2)] = \Pr [S_0^{(n)}(2) = S_1^{(n)}(2)] = p^{(n)}$ by the definition of $p^{(n)}$.

Thus, we can lower bound $\Pr [X^{(n)}(2) = 2]$ as follows:

$$\begin{aligned}
\Pr [X^{(n)}(2) = 2] &= p^{(n)} \cdot \Pr [\mathcal{A}^{(n)}(2) \cap (\mathcal{B}^{(n)}(1))^c \mid \mathcal{B}^{(n)}(0)] \\
&= p^{(n)} \cdot \left(\Pr [\mathcal{A}^{(n)}(2) \cap (\mathcal{B}^{(n)}(1))^c \cap \mathcal{A}^{(n)}(1) \mid \mathcal{B}^{(n)}(0)] \right. \\
&\quad \left. + \Pr [\mathcal{A}^{(n)}(2) \cap (\mathcal{B}^{(n)}(1))^c \cap (\mathcal{A}^{(n)}(1))^c \mid \mathcal{B}^{(n)}(0)] \right) \\
&\geq \underline{p} \cdot \Pr [\mathcal{A}^{(n)}(2) \cap (\mathcal{B}^{(n)}(1))^c \cap (\mathcal{A}^{(n)}(1))^c \mid \mathcal{B}^{(n)}(0)] \\
&= \underline{p} \cdot \Pr [\mathcal{A}^{(n)}(2) \cap (\mathcal{A}^{(n)}(1))^c \mid \mathcal{B}^{(n)}(0)] \tag{6}
\end{aligned}$$

where the second equality follows from the fact that $\{\mathcal{A}^{(n)}(1), (\mathcal{A}^{(n)}(1))^c\}$ is an event space, and the last equality is a consequence of the relation $(\mathcal{A}^{(n)}(1))^c \subset (\mathcal{B}^{(n)}(1))^c$.

Note that $\Pr [\mathcal{A}^{(n)}(2) \cap (\mathcal{A}^{(n)}(1))^c \mid \mathcal{B}^{(n)}(0)]$ is the probability that two nodes starting in the same subcell at timeslot $t = 0$, will (i) first move to different cells at timeslot $t = 1$ and (ii) then arrive at a common cell at timeslot $t = 2$. Clearly, this probability, denoted by ζ , is strictly positive and does not depend on n . Thus, from (5) and (6),

$$\Pr [I^{(n)}(2) = 1] \geq \frac{\underline{p} \cdot \zeta}{1 - \underline{p} \cdot (p_u^2 + p_d^2 + p_l^2 + p_r^2)} \geq \underline{p} \cdot \zeta > 0. \tag{7}$$

Equation (7) implies that if $h_1(n) \rightarrow \infty$ as $n \rightarrow \infty$, for all $\epsilon > 0$,

$$\begin{aligned}
\limsup_{n \rightarrow \infty} \Pr \left[\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}} \leq \epsilon \right] &= \limsup_{n \rightarrow \infty} \Pr \left[I^{(n)}(2) \leq \frac{\epsilon (h_1(n))^2}{p^{(n)}} \right] \\
&\geq \limsup_{n \rightarrow \infty} \Pr [I^{(n)}(2) \leq \epsilon (h_1(n))^2] \\
&\geq \limsup_{n \rightarrow \infty} \Pr [I^{(n)}(2) = 1] \\
&\geq \underline{p} \cdot \zeta, \tag{8}
\end{aligned}$$

where the second inequality is a consequence of the assumption that $\epsilon \cdot (h_1(n))^2 \rightarrow \infty$, and the last inequality follows from (7). Because (8) is true for all $\epsilon > 0$, we have

$$\lim_{\epsilon \downarrow 0} \left(\limsup_{n \rightarrow \infty} \Pr \left[\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}} \leq \epsilon \right] \right) \geq \underline{p} \cdot \zeta. \tag{9}$$

In order for the rvs $\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}}$ to converge in distribution to an exponential rv with parameter one as $n \rightarrow \infty$, the limit on the left-hand side of (9) must equal zero (because an exponential

rv is a non-negative continuous rv). Hence, (9) implies that the rvs $\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}}$ do not converge in distribution to an exponential rv with parameter one as $n \rightarrow \infty$.

B. Implication for the RW mobility model

Consider the case where the pmf $P^{(n)}$ is a discrete uniform distribution over the set $\mathcal{S}^{(n)}$ of subcells in a cell (as in the HRW mobility model described in subsection II-B). In this case, the finding above suggests that if the number of subcells in a cell does not grow unbounded, i.e., there exists some finite constant B such that $h_2(n) \leq B$ for all $n \geq 1$, then the rvs $\frac{I^{(n)}(2)}{(h_1(n))^2/p^{(n)}} = \frac{I^{(n)}(2)}{(h_1(n) \cdot h_2(n))^2}$ do *not* converge in distribution to an exponential rv as $n \rightarrow \infty$ because $p^{(n)} = (h_2(n))^{-2} \geq B^{-2}$ for all n .

As mentioned earlier, the RW mobility model is a special case of the HRW mobility model with $h_2(n) = 1$ for all $n \geq 1$. Therefore, (appropriately scaled) inter-meeting times under the RW mobility model do *not* converge in distribution to an exponential rv as $h_1(n) \rightarrow \infty$. However, when $\beta < 0.5$ in the HRW mobility model described in subsection II-B, the number of subcells in a cell, $n^{1-2\beta}$, grows unbounded and the (appropriately scaled) inter-meeting times converge to an exponential rv as $n \rightarrow \infty$.

VI. GENERALIZED HRW MOBILITY MODEL AND HETEROGENEOUS MOBILITY

In this section we extend our results on the HRW mobility model in Section IV in two directions: First, we generalize the HRW mobility model by allowing nodes to move to non-adjacent cells. Secondly, we remove the assumption that the mobility of the nodes is homogeneous. Our findings show that a similar approximation of inter-meeting times using exponential rvs is still valid in these cases as well. These results suggest that, under certain assumptions, the distribution of inter-meeting times is not sensitive to the details of nodes' mobility. We also briefly comment on the independence of nodes' mobility.

A. Generalized HRW mobility model

In the HRW mobility model studied in Section IV (and also in the RW mobility model examined in Section V), a move by a node from one timeslot to next is restricted to one of the four neighboring cells of the current location (see Fig. 2). We relax this constraint as follows: Suppose that the trajectory of a node is given by $\mathbf{L}^{(n)} = \{(C^{(n)}(t), S^{(n)}(t)); t = 0, 1, \dots\}$ as before. For each $n = 1, 2, \dots$, let $\{\Delta C^{(n)}(t); t = 0, 1, \dots\}$ be a sequence of i.i.d. rvs with some pmf $Q^{(n)}$ over the set $\{(i, j) \mid i, j \in \{-(h_1(n) - 1)/2, \dots, \lceil (h_2(n) - 1)/2 \rceil\}\} =: \Delta\mathcal{C}^{(n)}$. The transition of a node from the current cell $C^{(n)}(t)$ at timeslot t to another cell at timeslot $t + 1$ is now determined by $\Delta C^{(n)}(t)$. More precisely, a node in cell $C^{(n)}(t)$ at timeslot $t = 0, 1, \dots$, moves to the cell $C^{(n)}(t + 1) = C^{(n)}(t) + \Delta C^{(n)}(t)$ at timeslot $t + 1$.⁷

This model allows the node to remain in the same cell for more than one timeslot if $Q^{(n)}((0, 0)) = \Pr[\Delta C^{(n)}(t) = (0, 0)] > 0$. Moreover, when $Q^{(n)}(\Delta\mathbf{c}) > 0$ for all $\Delta\mathbf{c} \in \Delta\mathcal{C}^{(n)}$, a node located in some cell $C^{(n)}(t)$ at timeslot t can transition to any cell in $\mathcal{C}^{(n)}$ at timeslot $t + 1$. However, unlike in the i.i.d. mobility model [12], the probability with which a cell is selected for the following timeslot can depend on the current location of the node. The selection of a subcell $S^{(n)}(t + 1)$ within the selected cell $C^{(n)}(t + 1)$ for timeslot $t + 1$ is as described in subsection III-A for the HRW mobility model.

It is clear that $\mathbf{C}^{(n)} = \{C^{(n)}(t); t = 0, 1, \dots\}$ is a time homogeneous Markov chain with the state space $\mathcal{C}^{(n)}$, where the transition probabilities are determined by the pmf $Q^{(n)}$. The HRW mobility model described in subsection III-A is a special case of this generalized HRW mobility model with the probability $Q^{(n)}(\Delta\mathbf{c})$, $\Delta\mathbf{c} = (\Delta c_1, \Delta c_2) \in \Delta\mathcal{C}^{(n)}$, being strictly positive if and only if $\|\Delta\mathbf{c}\|_1 = |\Delta c_1| + |\Delta c_2| = 1$, where $\|\cdot\|$ denotes the L^1 -norm.

We impose the following assumptions on the Markov chain $\mathbf{C}^{(n)}$ and the conditional probability $p^{(n)}$.

⁷All additions are modulo $h_1(n)$.

Assumption 2: (i) For each $n = 1, 2, \dots$, the Markov chain $\mathbf{C}^{(n)}$ is irreducible and aperiodic.
(ii) $\lim_{n \rightarrow \infty} p^{(n)} = 0$, where $p^{(n)} = \sum_{\mathbf{s} \in \mathcal{S}^{(n)}} (P^{(n)}(\mathbf{s}))^2$ as defined in Section IV.

Since the state space $\mathcal{C}^{(n)}$ is finite for every $n = 1, 2, \dots$, Assumption 2(i) implies that the Markov chain $\mathbf{C}^{(n)}$ is also positive recurrent and, hence, ergodic [13, p.177]. Furthermore, it does not allow the case where the probability of staying in the same cell is one, i.e., $Q^{(n)}((0, 0)) < 1$. One can verify that the unique stationary distribution $\pi^{(n)}$ of the Markov chain $\mathbf{C}^{(n)}$ under the ergodicity assumption is the uniform distribution over the state space $\mathcal{C}^{(n)}$. This in turn tells us that, starting from *any* cell, the expected number of timeslots it takes to come back to the same starting cell, is equal to the number of cells $(h_1(n))^2$ in the network [7, Theorem (17), p.232].

Consider the same set-up we used in Section IV: For each $n = 1, 2, \dots$, there are two nodes $i = 0, 1$, that move according to the generalized HRW mobility model with some pmfs $P^{(n)}$ and $Q^{(n)}$. The trajectory of node $i = 0, 1$, is again denoted by

$$\mathbf{L}_i^{(n)} = \{L_i^{(n)}(t); t = 0, 1, \dots\} = \{(C_i^{(n)}(t), S_i^{(n)}(t)); t = 0, 1, \dots\} .$$

We assume that $\mathbf{L}_i^{(n)}$, $i = 0, 1$, are mutually independent, and let $\mathcal{I}^{(n)} = \{I^{(n)}(k); k = 1, 2, \dots\}$ be the sequence of inter-meeting times defined in subsection III-B. Assumption 2(i) guarantees that the two nodes will eventually meet with probability one, regardless of their initial locations.

We state the following theorem without a proof. The proof is a simple modification of that of Theorem 1.

Theorem 2: Under Assumption 2,

$$\lim_{n \rightarrow \infty} \Pr \left[\frac{I^{(n)}(2)}{(h_1(n))^2 / p^{(n)}} \leq x \right] = \begin{cases} 1 - e^{-x} , & x > 0 \\ 0 , & x \leq 0 . \end{cases} \quad (10)$$

Theorem 2 states that if the Markov chains $\mathbf{C}_i^{(n)}$, $i = 0, 1$, are ergodic and $p^{(n)}$ decreases to zero as $n \rightarrow \infty$, the inter-meeting times can be well approximated by exponential rvs for sufficiently large n without having to impose any further restrictions on the nodes' mobility

between cells. It is noteworthy that not only the limiting distribution is still exponential, but also the details of mobility between cells determined by the pmf $Q^{(n)}$ do not change the parameter of the limiting exponential distribution. Hence, one can view this as an *insensitivity result* because, under some assumptions, the details of transitions by the nodes between cells do not significantly affect the distribution of the inter-meeting times for sufficiently large n .

B. Heterogeneous mobility

Throughout our analysis in Section IV and in the previous subsection, we assumed that the mobility of the two nodes is homogeneous. In other words, we assumed that the pmfs $Q^{(n)}$ and $P^{(n)}$ for choosing the next cell and subcell, respectively, are identical for both nodes. Our results, however, continue to hold even when heterogeneous mobility is allowed among the nodes under some mild technical conditions: Suppose that $Q_i^{(n)}$ and $P_i^{(n)}$, $i = 0, 1$, denote node i 's pmfs for determining the next cell and subcell, respectively. We allow $P_0^{(n)}$ and $P_1^{(n)}$ and, also, $Q_0^{(n)}$ and $Q_1^{(n)}$ to be different.

The trajectory of node $i = 0, 1$, is given by

$$\mathbf{L}_i^{(n)} = \{L_i^{(n)}(t); t = 0, 1, \dots\} = \{(C_i^{(n)}(t), S_i^{(n)}(t)); t = 0, 1, \dots\}$$

as before. Again, $\mathbf{C}_i^{(n)} = \{C_i^{(n)}(t); t = 0, 1, \dots\}$, $i = 0, 1$, are time homogeneous Markov chains with the transition probabilities determined by $Q_i^{(n)}$. Define $\varsigma^{(n)} := \sum_{\mathbf{s} \in \mathcal{S}^{(n)}} (P_0^{(n)}(\mathbf{s}) \times P_1^{(n)}(\mathbf{s}))$, which is the probability that the two nodes are in contact given that they are in the same cell, i.e., $\varsigma^{(n)} = \Pr[L_0^{(n)}(t) = L_1^{(n)}(t) \mid C_0^{(n)}(t) = C_1^{(n)}(t)] = \Pr[S_0^{(n)}(t) = S_1^{(n)}(t)]$, $t = 0, 1, \dots$. We impose the following assumptions on the Markov chains $\mathbf{C}_i^{(n)}$, $i = 0, 1$, and the conditional probabilities $\varsigma^{(n)}$, $n = 1, 2, \dots$.

Assumption 3: (i) For each $n = 1, 2, \dots$, the Markov chains $\mathbf{C}_i^{(n)}$, $i = 0, 1$, are irreducible and aperiodic. (ii) $\varsigma^{(n)} > 0$ for all $n = 1, 2, \dots$, and $\lim_{n \rightarrow \infty} \varsigma^{(n)} = 0$.

If $\zeta^{(n)} = 0$, the two nodes never meet with probability one. Thus, in order to ensure that the nodes are in contact with a strictly positive probability when they are in the same cell and, hence, eventually meet with probability one, we need to assume $\zeta^{(n)} > 0$. In addition, one should note that, unlike in the homogeneous mobility case where the assumption $p^{(n)} \rightarrow 0$ necessarily means that the number of subcells increases unbounded (because $h_2(n) \rightarrow \infty$), Assumption 3(ii) does *not* require that the number of subcells in the network grow unbounded. In other words, even when there exists some *finite* bound on the total number of subcells in the network for all $n \geq 1$, Assumption 3(ii) can still be satisfied.

We state the following distributional convergence result without a proof, which is similar to that of Theorem 1.

Theorem 3: Under Assumption 3,

$$\lim_{n \rightarrow \infty} \Pr \left[\frac{I^{(n)}(2)}{(h_1(n))^2 / \zeta^{(n)}} \leq x \right] = \begin{cases} 1 - e^{-x}, & x > 0 \\ 0, & x \leq 0. \end{cases} \quad (11)$$

Theorem 3 tells us that even when the nodes' mobility is not homogeneous and the network size is not large, if the conditional probability $\zeta^{(n)}$ is small, inter-meeting times can still be well approximated by exponential rvs. This finding again suggests that the distribution of inter-meeting times is rather insensitive to the details of nodes' mobility and may resemble an exponential distribution under a broad set of assumptions, without having to assume a large network size.

C. Independence of nodes' mobility and the bounded domain of mobility

Although we assumed that the trajectories of the two nodes $\mathbf{L}_i^{(n)}, i = 0, 1$, are mutually independent throughout, we can relax this assumption as follows: Suppose that when two nodes meet, they coordinate their movements so that they can stay in contact while exchanging information. During this period they may not follow the (generalized) HRW mobility model. Once they complete the transfer of message(s), they resume following the (generalized) HRW

mobility model, independently of each other, until they meet again, at which point they repeat the process. It is clear that, under this assumption, the distribution of the inter-meeting times remains the same as before, whereas the number of consecutive timeslots they spend in contact after a meeting may change. Hence, all of our results still hold.

Cai and Eun [2] suggested that the bounded domain of mobility used in simulation may be one of main culprits for the emergence of an exponential tail in the distribution of inter-meeting times. Our results (Theorems 1 - 3), however, state that even as the domain becomes large (in fact, grows unbounded), the distribution of inter-meeting times exhibits an exponential tail. Thus, this suggests that the bounded domain alone does not account for the emergence of exponential tail in the distribution of inter-meeting times observed in simulation.

VII. SIMULATION

In this section we present simulation results to validate our analysis in the previous sections. First, we simulate the generalized HRW mobility model with two nodes and study the empirical distribution of the inter-meeting times. Then, we repeat the simulation under the RW mobility model.

A. The generalized HRW mobility model

In the first setting two nodes move according to the generalized HRW mobility model on a unit square area divided into $25 = (h_1)^2$ cells. Each cell is then further divided into $100 = (h_2)^2$ subcells. We assume that the pmf $Q(\Delta\mathbf{c})$, $\Delta\mathbf{c} \in \{(i, j) \mid i, j \in \{-2, -1, 0, 1, 2\}\}$ for selecting a next cell, is equal to $1/12$ if $1 \leq \|\Delta\mathbf{c}\|_1 \leq 2$ and 0 otherwise. In other words, a node in cell $C(t)$ at timeslot t moves to one of the 12 shaded cells in Fig. 4 with equal probability of $1/12$ at timeslot $t + 1$. We use the discrete uniform distribution for subcell selection with $P(\mathbf{s}) = 0.01$, $\mathbf{s} \in \{(a, b) \mid a, b \in \{0, 1, \dots, 9\}\}$.

A total of 80,008 inter-meeting times are collected in the simulation. Their histogram as well as the logarithm of the histogram are plotted in Fig. 5. The dotted line in Fig. 5(b) is the

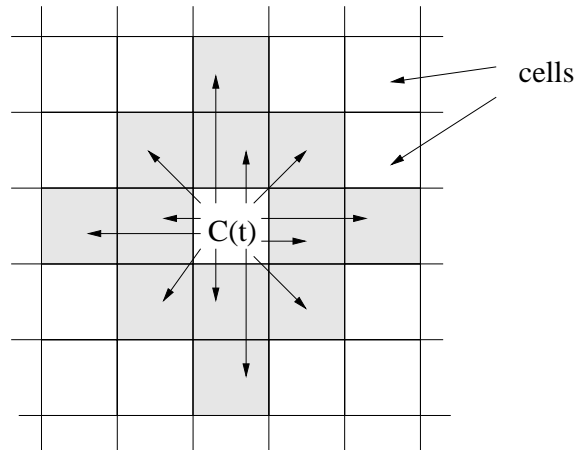


Fig. 4. Transition between cells by a node under the generalized HRW mobility model.

logarithm of the exponential fitting curve obtained from the collected inter-meeting times, using the *expfit* function in Matlab, which provided the fitting parameter of $\lambda = 4.003 \times 10^{-4}$. The plotted logarithm of the histogram and that of the exponential fitting curve suggest that indeed the distribution of the inter-meeting times closely resembles an exponential distribution with a mean $\lambda^{-1} \simeq (h_1 \times h_2)^2$, corroborating our finding in Theorem 2.

B. The RW mobility model

In the second setting we run the simulation with the RW mobility model described in subsection II-A by setting $h_1 = 19$ and $h_2 = 1$, i.e., one subcell in each cell. We collect a total of 103,791 inter-meeting times. Their histogram and the logarithm of the histogram are plotted in Fig. 6. It is clear from the plots that there is a high concentration of inter-meeting times close to the origin, which is consistent with our analysis in Section V. However, the plots also suggest that the tail of the distribution is still exponential. The logarithm of the exponential fitting curve for the tail with the fitting parameter $\lambda = 1.19 \times 10^{-3}$ is shown as the dotted line in Fig. 6(b). Clearly, in this case the parameter λ is quite different from $h_1^{-2} = 2.77 \times 10^{-3}$ due to the concentration of the distribution close to the origin.

VIII. CONCLUSION

We studied the distribution of inter-meeting times under the (generalized) HRW mobility model. We showed that when the conditional probability that two nodes are in contact given that they are in the same cell is small, the distribution of inter-meeting times can be well approximated by an exponential distribution. Moreover, this approximation holds even when the mobility of the two nodes is heterogeneous (under mild conditions). These findings indicate that the distribution of inter-meeting times is insensitive to the details of nodes' mobility, and an exponential distribution may provide a good approximation for it in a broad set of settings. Our findings are consistent with the recent observations that some distributions of inter-meeting times from simulation resemble exponential distributions.

APPENDIX I

PROOF OF LEMMA 1

The proof of the lemma will proceed as follows: First, we will show that the inter-contact times $X^{(n)}(k)$, $k \geq 2$, can be written as a random sum of i.i.d. rvs, where the number of rvs in the summation is geometrically distributed with parameter $p^{(n)}$. Second, using this observation, we will prove that the Laplace transforms of the scaled inter-contact times $\frac{X^{(n)}(2)}{(h_1^{(n)})^2/p^{(n)}}$ converge to that of an exponential rv with parameter one.

We first define the sequence of timeslots at which the two nodes visit the same *cell* together, i.e., $C_0^{(n)}(t) = C_1^{(n)}(t)$: Let $\mathcal{W}^{(n)} := \{W^{(n)}(k); k = 0, 1, \dots\}$ be a sequence of non-negative integers where (i) $W^{(n)}(0) = 0$ and, (ii) for $k \geq 1$,

$$W^{(n)}(k) = \inf \left\{ m \geq 0 \mid \sum_{t=0}^m \mathbf{1} \{C_0^{(n)}(t) = C_1^{(n)}(t)\} \geq k \right\} .$$

Since the two nodes must be in the same cell in order for them to be in contact, clearly the sequence $\mathcal{Z}^{(n)}$ of contact times defined in the proof of Theorem 1 is a subsequence of $\mathcal{W}^{(n)}$.

Also, for all $k = 1, 2, \dots$, define

$$Y^{(n)}(k) = W^{(n)}(k) - W^{(n)}(k-1) .$$

Then, the rvs $Y^{(n)}(k), k \geq 2$, denote the number of timeslots the two nodes need to arrive at a common cell since their last visit to the same cell together. For the same reason explained at the end of subsection III-B, the rvs $Y^{(n)}(k), k \geq 2$, are i.i.d.⁸

Let $A^{(n)}(0) = 0$ and, for $m \geq 1$,

$$A^{(n)}(m) = \sup \left\{ k \geq 1 \mid W^{(n)}(k) \leq Z^{(n)}(m) \right\} .$$

The rvs $A^{(n)}(m), m \geq 1$, denote the number of times the two nodes arrive at a common cell until the m -th *contact* occurs. For each $m = 1, 2, \dots$, define $B^{(n)}(m) := A^{(n)}(m) - A^{(n)}(m-1)$. The rvs $B^{(n)}(m), m \geq 2$, represent the number of visits to a common cell by the two nodes between the $(m-1)$ -th and m -th contacts. Recall that, given that the two nodes arrive at the same cell, they choose the same subcell within the cell (hence, are in contact) with probability $p^{(n)}$, independently of the past and the cell. As a result, the rvs $B^{(n)}(m), m \geq 2$, are i.i.d. geometric rvs with parameter $p^{(n)}$.

We can now rewrite the contact times $Z^{(n)}(k)$ and the inter-contact times $X^{(n)}(k)$ using the rvs $W^{(n)}(k)$ and $Y^{(n)}(k)$ as follows: For every $m \geq 1$,

$$Z^{(n)}(m) = \sum_{\ell=1}^{A^{(n)}(m)} Y^{(n)}(\ell) = W^{(n)}(A^{(n)}(m)) ,$$

and

$$X^{(n)}(m) = \sum_{\ell=A^{(n)}(m-1)+1}^{A^{(n)}(m)} Y^{(n)}(\ell) = \sum_{\ell=A^{(n)}(m-1)+1}^{A^{(n)}(m-1)+B^{(n)}(m)} Y^{(n)}(\ell) . \quad (12)$$

Equation (12) suggests that the inter-contact times $X^{(n)}(m), m \geq 2$, can be written as a sum of i.i.d. rvs $Y^{(n)}(\ell)$ and the number of rvs in the summation (i.e., $B^{(n)}(m)$) is geometrically distributed with parameter $p^{(n)}$ and independent of the rvs $Y^{(n)}(\ell), \ell \geq 2$. This completes the first step of the proof.

Recall that $p^{(n)} \rightarrow 0$ as $n \rightarrow \infty$ from Assumption 1(ii). Let $B^{(n)}$ be a geometric rv with parameter $p^{(n)}$, which is independent of rvs $Y^{(n)}(\ell), \ell \geq 1$. For every $s > 0$, the Laplace

⁸While $Y^{(n)}(k) \geq 1$ for all $k \geq 2$, $Y^{(n)}(1)$ can be zero when $W^{(n)}(1) = 0$. Thus, the distribution of $Y^{(n)}(1)$ is different from that of $Y^{(n)}(k), k \geq 2$.

transform (LT) of the rv $V^{(n)} := \sum_{\ell=1}^{B^{(n)}} Y^{(n)}(\ell+1)$ is given by [6, p.429]

$$\begin{aligned} \mathbf{E} \left[e^{-sV^{(n)}} \right] &= \mathbf{E} \left[e^{-s \sum_{\ell=1}^{B^{(n)}} Y^{(n)}(\ell+1)} \right] \\ &= \mathbf{E} \left[\mathbf{E} \left[e^{-s \sum_{\ell=1}^{B^{(n)}} Y^{(n)}(\ell+1)} \mid B^{(n)} \right] \right] \\ &= \mathbf{E} \left[\mathbf{E} \left[e^{-s Y^{(n)}(2)} \right]^{B^{(n)}} \right], \end{aligned} \quad (13)$$

where the last equality follows from the fact that $Y^{(n)}(\ell), \ell \geq 2$, are i.i.d.

Let $\phi^{(n)}(s) := \mathbf{E} \left[e^{-s Y^{(n)}(2)} \right]$ be the LT of the rvs $Y^{(n)}(\ell), \ell \geq 2$. Then, from (13) we have

$$\begin{aligned} \mathbf{E} \left[e^{-sV^{(n)}} \right] &= \sum_{b=1}^{\infty} \left(\phi^{(n)}(s)^b \cdot (1 - p^{(n)})^{b-1} \cdot p^{(n)} \right) \\ &= \frac{p^{(n)} \phi^{(n)}(s)}{1 - \phi^{(n)}(s) (1 - p^{(n)})} \\ &= \frac{p^{(n)} (1 - \mu^{(n)} s + s \xi^{(n)}(s))}{1 - (1 - \mu^{(n)} s + s \xi^{(n)}(s)) (1 - p^{(n)})}, \end{aligned} \quad (14)$$

where $\mu^{(n)} = \mathbf{E} \left[Y^{(n)}(2) \right]$ and $\xi^{(n)}(s) = \left(\phi^{(n)}(s) - (1 - \mu^{(n)} s) \right) / s$.

Using the fact that $\mathbf{C}_i^{(n)} = \{C_i^{(n)}(t); t = 0, 1, \dots\}$, $i = 0, 1$, are mutually independent ergodic Markov chains under Assumption 1(i), one can show that $\mu^{(n)} = (h_1(n))^2$ [7, p.232]. In addition, $|\xi^{(n)}(s)|$ is bounded as follows.

$$\begin{aligned} |\xi^{(n)}(s)| &= \left| \frac{\phi^{(n)}(s) - 1 + \mu^{(n)} s}{s} \right| \\ &= \left| \int_0^{\infty} \frac{e^{-sy} - 1 + s y}{s y} y dF^{(n)}(y) \right| \\ &\leq \int_0^{\infty} \left| \frac{e^{-sy} - 1 + s y}{s y} \right| y dF^{(n)}(y) \\ &\leq \Xi \cdot \int_0^{\infty} y dF^{(n)}(y) \\ &= \Xi \cdot \mu^{(n)}, \end{aligned} \quad (15)$$

where $F^{(n)}$ is the distribution of $Y^{(n)}(2)$, and the second inequality follows from the fact that

$\left| \frac{e^{-sy} - 1 + sy}{sy} \right|$ is upper bounded by a finite constant $\Xi > 0$.

Let $m^{(n)} = \mu^{(n)} \cdot \mathbf{E} [B^{(n)}] = \mu^{(n)}/p^{(n)}$. From (14), the LT of rv $V^{(n)}/m^{(n)}$ is

$$\begin{aligned} \zeta^{(n)}(s) &:= \mathbf{E} \left[e^{-s(V^{(n)}/m^{(n)})} \right] = \mathbf{E} \left[e^{-(s/m^{(n)})V^{(n)}} \right] \\ &= \frac{p^{(n)} \phi^{(n)}(s/m^{(n)})}{1 - \phi^{(n)}(s/m^{(n)})(1 - p^{(n)})} \\ &= \frac{p^{(n)} \left(1 - \mu^{(n)} s/m^{(n)} + s \xi^{(n)}(s/m^{(n)})/m^{(n)} \right)}{1 - (1 - \mu^{(n)} s/m^{(n)} + s \xi^{(n)}(s/m^{(n)})/m^{(n)}) (1 - p^{(n)})}. \end{aligned} \quad (16)$$

Substituting $m^{(n)} = \mu^{(n)}/p^{(n)}$ in (16) and after a little algebra, we can rewrite $\zeta^{(n)}(s)$ as follows.

$$\begin{aligned} \zeta^{(n)}(s) &= \frac{p^{(n)} \left(1 - \mu^{(n)} s p^{(n)}/\mu^{(n)} + s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)} \right)}{1 - (1 - \mu^{(n)} s p^{(n)}/\mu^{(n)} + s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)}) (1 - p^{(n)})} \\ &= \frac{p^{(n)} \left(1 - s p^{(n)} + s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)} \right)}{1 - ((1 - p^{(n)}) - (1 - p^{(n)}) s p^{(n)} + (1 - p^{(n)}) s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)})} \\ &= \frac{p^{(n)} \left(1 - s p^{(n)} + s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)} \right)}{p^{(n)} + (1 - p^{(n)}) s p^{(n)} - (1 - p^{(n)}) s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)}} \\ &= \frac{p^{(n)} \left(1 - s p^{(n)} + s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)} \right)}{p^{(n)} (1 + (1 - p^{(n)}) s - (1 - p^{(n)}) s \xi^{(n)}(s p^{(n)}/\mu^{(n)})/p^{(n)})} \\ &= \frac{1 - s p^{(n)} + s \xi^{(n)}(s p^{(n)}/\mu^{(n)}) p^{(n)}/\mu^{(n)}}{1 + (1 - p^{(n)}) s - (1 - p^{(n)}) s \xi^{(n)}(s p^{(n)}/\mu^{(n)})/p^{(n)}} \end{aligned} \quad (17)$$

In order to complete the second step of the proof, we first prove that, for each $n = 1, 2, \dots$, $\lim_{s \downarrow 0} \xi^{(n)}(s) = 0$. First, note that both the numerator, $(\phi^{(n)}(s) - 1 + \mu^{(n)} s)$, and the denominator, s , of $\xi^{(n)}(s)$ go to zero as $s \downarrow 0$ because $\lim_{s \downarrow 0} \phi^{(n)}(s) = 1$. Therefore, using L'Hôpital's rule, we obtain

$$\lim_{s \downarrow 0} \xi^{(n)}(s) = \lim_{s \downarrow 0} \frac{d\phi^{(n)}(s)/ds + \mu^{(n)}}{1}. \quad (18)$$

From the definition of $\phi^{(n)}(s) = \mathbf{E} [e^{-s Y^{(n)}(2)}]$, we have $\lim_{s \downarrow 0} (d\phi^{(n)}(s)/ds) = \mathbf{E} [-Y^{(n)}(2)] = -\mu^{(n)}$. Substituting this in (18) we get $\lim_{s \downarrow 0} \xi^{(n)}(s) = 0$.

Since $|\xi^{(n)}(s)|$ is upper bounded by $\Xi \cdot \mu^{(n)}$ for all $s > 0$ from (15) and $p^{(n)} \rightarrow 0$, hence $p^{(n)}/\mu^{(n)} \rightarrow 0$ and $\xi^{(n)}(s p^{(n)}/\mu^{(n)}) \rightarrow 0$, as $n \rightarrow \infty$ from Assumption 1(ii), we see that (17) satisfies the following convergence for all $s > 0$.

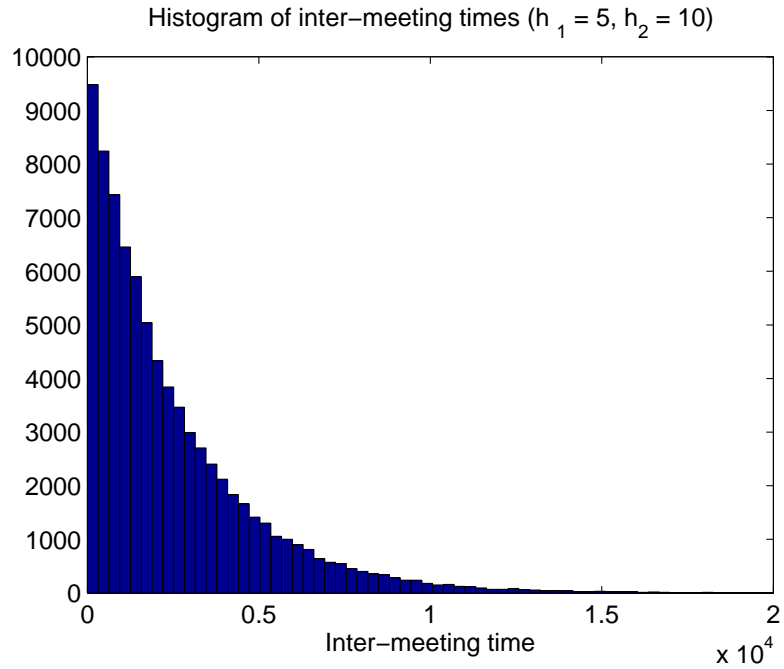
$$\zeta^{(n)}(s) \rightarrow \frac{1}{1+s} =: \zeta(s) \quad \text{as } n \rightarrow \infty$$

Note that $\zeta(s)$ is the LT of the exponential distribution with parameter one [6, p.430]. The facts that (i) $\zeta^{(n)}(s) \rightarrow \zeta(s)$ for all $s > 0$ and (ii) $\lim_{s \downarrow 0} \zeta(s) = 1$ together imply that the rvs $V^{(n)}/m^{(n)}, n = 1, 2, \dots$, converge in distribution to an exponential rv with parameter one (Theorem 1 [6, p.430] and Theorem 2 [6, p.431]). The lemma now follows from the observation that the rvs $V^{(n)}$ and $X^{(n)}(2)$ are identically distributed from (12), i.e., $V^{(n)} \stackrel{st}{=} X^{(n)}(2)$.

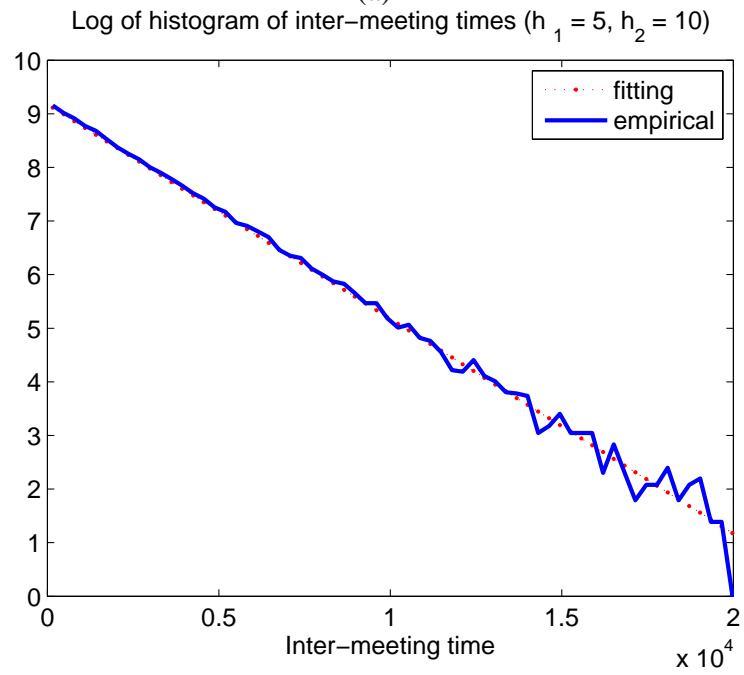
REFERENCES

- [1] A. Al-Hanbali, A. A. Kherani, R. Gorenvelt, P. Nain, and E. Altman, "Impact of mobility on the performance of relaying in ad hoc networks," *IEEE INFOCOM*, Barcelona (Spain), Apr. 2006.
- [2] H. Cai, and D. Y. Eun, "Crossing over the bounded domain: from exponential to power-law inter-meeting time in MANET" *ACM MobiCom*, Montreal (Canada), Sep. 2007.
- [3] A. Chaintreau, P. Hui, J. Crowcroft, C. Diot, R. Gass, and J. Scott, "Impact of human mobility on the design of opportunistic forwarding algorithms," *IEEE INFOCOM*, Barcelona (Spain), Apr. 2006.
- [4] A. Chaintreau, P. Hui, J. Crowcroft, C. Diot, R. Gass, and J. Scott, "Pocket switched networks: real-world mobility and its consequences for opportunistic forwarding," UCAM-CL-TR-617, Technical Report, Computer Laboratory, University of Cambridge, Feb. 2005.
- [5] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-delay trade-off in wireless networks," *IEEE INFOCOM*, Hong Kong (China), Apr. 2004.
- [6] W. Feller, *An Introduction to Probability Theory and Its Applications*, Vol. II, John Wiley & Sons, 1971.
- [7] G. Grimmett, and D. Stirzaker, *Probability and Random Processes*, third ed., Oxford University Press, 2001.
- [8] R. Groenevelt, P. Nain, and G. Koole, "The message delay in mobile ad hoc networks," *Performance Evaluation*, vol. 62(1-4), pp. 210-228, Oct. 2005.
- [9] P. Hui, A. Chaintreau, J. Scott, R. Gass, J. Crowcroft, and C. Diot, "Pocket switched networks and the consequences of human mobility in conference environments," *ACM SIGCOMM Workshop on Delay Tolerant-Networking*, Philadelphia (PA), Aug. 2005.
- [10] T. Karagiannis, J.-Y. Le Boudec, and M. Vojnovic, "Power law and exponential decay of inter contact times between mobile devices" *ACM MobiCom*, Montreal (Canada), Sep. 2007.
- [11] M. Kim, D. Kotz, and S. Kim, "Extracting a mobility model from real user traces," *IEEE INFOCOM*, Barcelona (Spain), Apr. 2006.
- [12] M. J. Neely, and E. Modiano. "Capacity and delay tradeoffs for ad-hoc mobile networks," *IEEE Trans. on Information Theory*, vol. 51(6), pp. 1917-1937, Jun. 2005.
- [13] S. M. Ross, *Stochastic Processes*, second ed., John Wiley & Sons, 1996.

- [14] G. Sharma, R. R. Mazumdar, and N. B. Shroff, "Delay and capacity trade-offs in mobile ad hoc networks: a global perspective," *IEEE INFOCOM*, Barcelona (Spain), Apr. 2006.
- [15] J. Su, A. Chin, A. Popivanova, A. Goel, and E. De Lara, "User mobility for opportunistic ad-hoc networking," *IEEE Workshop on Mobile Computing Systems and Applications (WMCSA)*, English Lake District (UK), Dec. 2004.
- [16] R. D. Yates, and D. J. Goodman, *Probability and Stochastic Processes*, second ed., John Wiley & Sons, 2005.

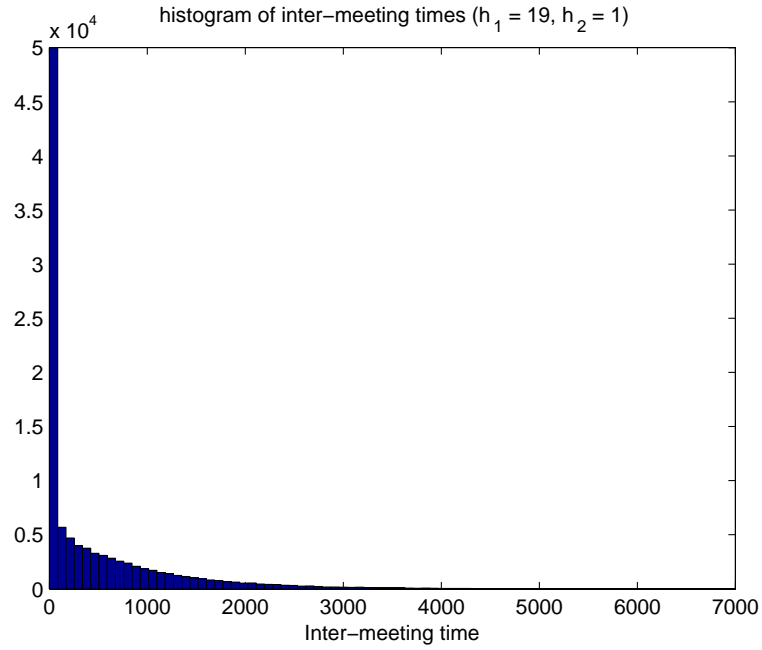


(a)

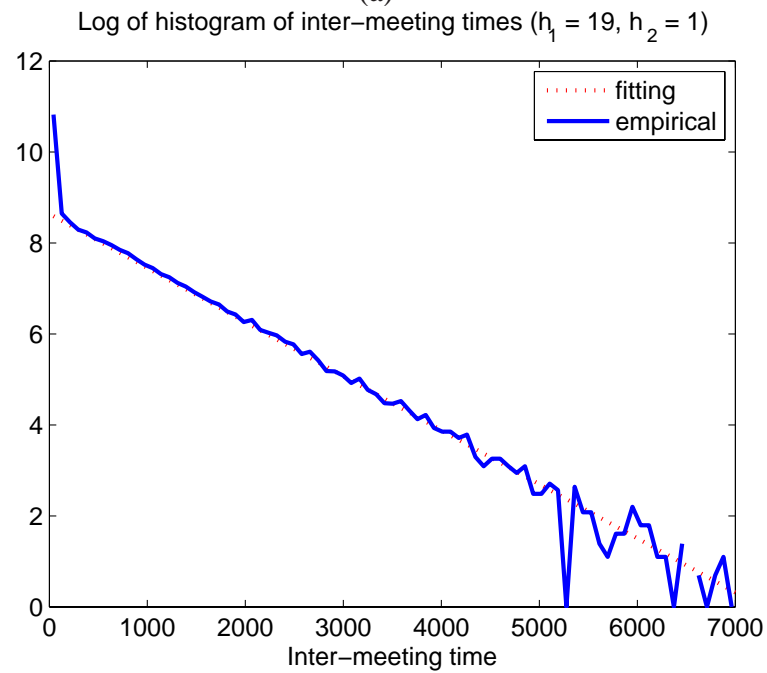


(b)

Fig. 5. Empirical distribution with $h_1 = 5$ and $h_2 = 10$. (a) Histogram of the inter-meeting times, (b) log plot of the histogram and exponential fitting.



(a)



(b)

Fig. 6. Empirical distribution under the RW mobility model ($h_1 = 19$ and $h_2 = 1$). (a) Histogram of the inter-meeting times, (b) log plot of the histogram and exponential fitting of the tail.