This dissertation explores the learning and risk mechanisms underlying the dynamics of route choice and activity scheduling decisions. With respect to route choice dynamics, the study models decision mechanisms related to travel time perception, learning, and risk attitudes, exploring their implications on system performance over time. This objective is accomplished by performing experiments using a network performance model, in this case an agent-based simulation model of individual experience given the collective effects arising from the interaction of the agents’ route choice decisions. In regards to activity scheduling decisions, the study examines the range of behavioral insights obtained from a modeling framework that views the individual scheduling process as a single-server queuing system, introducing the concept of activity stress. The study presents numerical experiments on this
framework using a discrete event simulation of an M/G/1 queuing system. Furthermore, an operational model of activity participation is estimated using observed activity schedules. The results indicate that travel time uncertainty and user perception of this uncertainty greatly affect the performance of the system over time, in particular the convergence of traffic flows. With respect to activity scheduling, the results overall indicate the significance of activity stress in motivating activity scheduling and participation decisions over time, with particular importance placed on the evolution of activity queue and activity schedule states over time. Results from studies investigating both route choice and activity scheduling behavior indicate the important role of decision dynamics for determining the behavior of users in complex information-rich environments.
LEARNING AND RISK PERCEPTION MECHANISMS IN ROUTE CHOICE AND ACTIVITY SCHEDULING DYNAMICS

By

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Chapter 1.0 Introduction and Motivation

The study of human decision making is central to the understanding of socio-economic systems, including transportation systems, as decisions determine a major part of human interactions. Therefore, understanding and modeling decisions are essential for the microscopic understanding of macroscopic social phenomena observed, such as social exchange, formation of groups, economic markets, and the dynamics of traffic flow and activity patterns. This study focuses on the last phenomena, namely the decisions driving the time-dependent flow of traffic and activities. An understanding of the human level dynamics or mechanisms governing traffic and activity patterns, by means of empirical and numerical studies, can lead to improved insights into the resulting macroscopic phenomena, and possibly the ability to derive them from these dynamics. This optimistic vision is motivated by the great success in the derivation of the structural and dynamic properties of matter from elementary physical interactions. Along a similar line of thought, this study seeks to apply the same optimistic vision and principles towards exploring and understanding traffic flows and human activity patterns over space and time, focusing on the underlying dynamic decisions.

Additionally, this study is conducted against a backdrop of public discussion and policies aimed at improving growing societal problems, such as traffic congestion and the resulting externalities that include public health, air pollution, and energy consumption. These policies focus on providing better travel demand management strategies, such as demand peak-spreading, telecommuting, versus capital improvements. Evaluating and determining the effectiveness of these policies requires
an understanding of travel and activity patterns over varying time frames. Along with this shift in policy perspective, the rapid spread of new information and communication technologies has profoundly affected the spatial and temporal boundaries of human activity, creating both new opportunities for improved travel experiences through telecommuting, e-shopping, and real-time information systems for trips and activities. However, assessing and forecasting the promise of these new technologies and services requires insight into the decision making of travelers under dynamic and complex information environments.

The motivation behind this study of traffic system user decisions, and in particular the interaction of these decisions with the system, is two-fold. First, from a scientific perspective, traffic systems are examples of socio-economic systems that exhibit macroscopic properties, patterns, or features as a result of microscopic decisions. The consequences of these decisions are determined collectively from the physical interactions of users with varying levels of information and technological capabilities. This study seeks to understand the microscopic user behaviors that lead to macroscopic behaviors of traffic. Second, from a practical standpoint, we seek the ability to devise strategies and policies for managing traffic systems for many compelling reasons, including concern for the quality of urban life. Severe traffic congestion and substandard air quality are symptoms of the same phenomena that result from the same underlying decision processes of individual system users.

The next section presents the main objectives of this study as they relate to the motivations discussed. The following section discusses the research approach to
address the study objectives. The third section discusses the main contributions of this research. The final section of this chapter outlines the remaining chapters of this study.

1.1 Research Objectives

The main objective of this study is to investigate the decision mechanisms or dynamics underlying travel and activity choices. In this study, travel is viewed as an integral component of a time-continuous activity pattern or schedule, viewed as a sequence of activities; it results from the interdependent choices of which activities to participate in, where, for how long, and in what sequence (which may include choices of start and finish times), along with travel choices such as mode and route choices. Due to the breadth of travel and activity decisions users make, this study focuses on understanding and modeling mechanisms related to two decisions: i) day-to-day route choice; and ii) activity scheduling and participation.

Both types of decisions share similarities as well as differences. Underlying both decisions are mechanisms that govern the integration of new information with past experiences (learning), leading to updated perceptions, and the choices that result from evaluating these updated perceptions of anticipated payoffs. However, route choice decisions are indicative of the spatial distribution of users across a network, whereas activity scheduling and participation decisions are more indicative of the temporal distribution of their activities. This study focuses on both decisions due to the similarities in their underlying mechanisms or dynamics, allowing a comparison
between the dynamics of two decisions that govern different dimensions (spatial and temporal) of travel-activity patterns.

In regards to day-to-day route choice decisions, two main objectives of this study are:

1) Develop a modeling framework for examining decision mechanisms that capture the following day-to-day route choice dynamics:

   i) travel time perception, in particular uncertainty;
   
   ii) travel time learning and updating, including the timing of updating;
   
   iii) risk perception; and

2) Investigate the interrelationship between the three mechanisms above on the day-to-day traffic flow evolution of networks.

The need for research on these two objectives stems from wide ranging applications in modeling commuter behavior, network state prediction, and traffic management and planning. A greater emphasis is placed on the first objective of representing and modeling route choice dynamics, in view of its significant role in determining several aspects of network performance over time.

The first objective related to the dynamics of route choice decisions focuses on the perception and updating of travel times, and the role of risk attitudes. With respect to travel time perception, this study investigates the implications of a travel time uncertainty updating mechanism based on concepts from Bayesian statistical
inference. Perceived travel time is viewed as consisting of mean and error components, both of which can be updated in light of new experiences (observations), using concepts from the Bayesian updating of probability distributions with new information. The error component is assumed to reflect the degree of uncertainty associated with a perceived travel time. In relation to updating the mean and error, or the second objective, (i) Bayesian updating, in addition to two other behavioral learning perspectives, (ii) reinforcement and (iii) belief (epistemic) learning, are also considered. To further address this objective, updating trigger mechanisms are modeled to account for the timing of learning and updating, possibly resulting from associated costs of updating or personal perceptions. Finally, with respect to the last objective concerning risk attitudes, this study proposes a mechanism for weighing the objective probabilities in relation to personal risk attitudes. In particular, risk seekers are assumed to overweigh objective probabilities of gains and under-weigh probabilities of losses, with the opposite for risk avoiders.

The second objective related to route choice is to examine the system performance implications of the mechanisms mentioned above, in particular the day-to-day evolution of traffic flows. Two principal types of descriptors are considered: i) day-to-day flow pattern of traffic, in particular convergence and ii) time until convergence, if any, is reached. The existence of an equilibrium state is commonly assumed in transportation planning practice. The particular focus on convergence addresses the validity of this assumption in relation to the decision mechanisms mentioned previously. More specifically, investigating the convergence of the system,
if any, under the behavioral mechanisms proposed, may offer insight into the validity of an equilibrium state commonly assumed in practice. Although no concrete conclusions about the existence of an equilibrium state in “real” networks can be made, the second objective is concerned primarily with how reasonable this assumption might be, given behaviorally plausible user decision mechanisms.

In regards to activity scheduling and participation decisions, three main objectives of this study are to:

1) Develop a framework for modeling activity scheduling dynamics, in particular considering the following aspects:

   (i) the interrelationship between the static (long-term) and dynamic (short-term) aspects of activity scheduling;

   ii) the role of perceived stress related to latent or queued activities, in particular activities scheduled but not completed or engaged;

   iii) the role of unplanned, possibly emergency activities that arise during schedule execution,

2) Explore the range of behavioral insights that can be obtained from viewing the activity scheduling process as a single-server queuing system; and
3) Provide empirical evidence to support the concept of “activity stress,” in particular, its role in activity participation decisions.

Interest in the activity scheduling process arises from the realization that an improved understanding of travel behavior and activity patterns requires more than a better account for observed outcomes; it requires better models of the mechanisms underlying these behaviors (Pas 1985; Kitamura 1988; Ettema and Timmermans 1997; McNally 2000). A better understanding of activity scheduling dynamics (such as rescheduling) may lead to improved insights into the scheduling of unplanned activities, an issue ignored in previous studies. Furthermore, empirical evidence suggests that activity scheduling is highly dynamic, occurring over varying time horizons, with significant amounts of revision and continuous preplanning, even during execution (Doherty 2000; Miller and Roorda 2003). Recognizing that observed travel patterns are the result of an (unobserved and latent) underlying activity scheduling decision process, a need exists for adequately accounting for these dynamics (Hirsh et al. 1986; Kitamura 1988; Hanson and Huff 1988).

The first and main objective related to activity scheduling dynamics is to provide a theoretical and conceptual framework for modeling the scheduling of activities, including activity participation, in relation to both planned and executed schedules, of particular interest is the role of activities scheduled but not necessarily completed or engaged. To account for these “latent” activities the main behavioral perspective adopted is that of a queuing system, with the individual as a “server” that undertakes
or participates in arriving activities. Additionally, within this framework, the effect of activities that arise during schedule execution can also be accounted for. The range of behavioral insights drawn from such a behavioral perspective (a queuing system) is further explored in this study, as stated in the second objective above. In particular, this study further investigates the interrelationship between activity scheduling/participation decisions and the evolution of the queue, such as the queue length and other properties. The time-dependent properties of the activity queue are examined since activity stress is related to state of the queue, under the assumptions of the activity scheduling modeling framework in this study. Under the third objective, empirical evidence of the concept of “activity stress” is provided to illustrate the operational potential of the proposed framework.

The next section presents an overview of the research methodology adopted for pursuing these objectives and tasks.

1.2 Overview of Research Methodology

To investigate route choice dynamics with respect to the objectives described above, a model was used consisting of two main components: a) the individual user decisions component, which includes route switching, the mechanisms for updating travel times, mechanisms that trigger updating and learning, and a mechanism for route selection based on the subjective weighing of objective probabilities of travel time improvements, and b) a network performance model, in this case an agent-based simulation model of individual experience given the collective effects arising from the interaction of the agents’ route choice decisions. The interactions are captured
using a simple cost function that yields the mean objective travel times on links given the corresponding flows. The resulting travel times are then used as input in the user decision component, generating a set of new route choice decisions for the next day, and so on. Experiments are conducted using this simulation model with a hypothetical network to analyze the day-to-day dynamics of the system under different behavioral mechanisms. Note that the performance modeling has been kept to a bare minimum of complexity in order to focus the study on the route choice mechanisms; it would have been possible to use a more elaborate traffic simulator, though that might reduce the clarity of the resulting insight.

In order to investigate the effect of different assumptions on activity scheduling and participation decisions on the time-dependent properties of the activity queue, a discrete-event simulation for an M/G/1 queuing systems was developed for evaluating the individual activity scheduling process under different activity service and selection rules. The simulation model consists of two basic events, an activity arrival and a completed activity departure, that alter the state of the system. A next-event *time advance* approach is used to advance the simulation clock. The process of advancing the clock from one event to the next is continued until a stopping condition is satisfied, in the case of this study a set number of completed activities. To provide empirical support for the concept of an “activity stress,” an activity participation threshold was estimated using data on observed activity participation decisions. The development of simulated maximum-likelihood estimation procedures for dynamic discrete choice models, such as kernel-logit (mixed-logit) and probit models has relaxed many limitations, such as time dependence and substitution patterns. These
procedures are adapted and applied to an activity participation model based on the concept of a stress-threshold over time.

1.3 Significance of Research Objectives and Contributions

As mentioned previously, this research seeks to understand the dynamics behind route choice and activity scheduling decisions. From a scientific standpoint, this research adds to our understanding of the interrelationships between the microscopic behaviors of users in a traffic system and the macroscopic behavior of the system in relation to travel and activity patterns over time. Transportation systems are complex nonlinear social decision systems, where agents (sometimes) make non-cooperative decisions, the consequences of which are determined collectively from the interactions between users with varying information availability, technological capabilities, and decision-making capabilities. Providing information (via ICT, ATIS, ITS, etc…) to these systems adds complexity, possibly increasing user interaction, increasing randomness and thus, unpredictability in system behavior. Additionally, information may also allow for the exertion of regulatory effects. Furthermore, these systems, termed “symplectic” systems, are more complex than physical systems (fixed rules), due to human behavior (Herman 1992).

From a more practical standpoint, our ability to understand qualitatively and describe mathematically these behavioral and physical processes and their interactions may permit us to devise strategies and policies to manage these systems and guide them along socially desirable paths. Practical applications include the evaluation of demand management strategies, such as demand peak spreading, congestion pricing, ICT-
based demand management (telecommuting), and feedback and education programs that may lead to long-term behavior adjustment.

The investigation of route choice dynamics examines the day-to-day behavior of traffic flows under different user decision mechanisms. Three aspects of route choice decisions are investigated: i) travel time perception; ii) travel time updating and learning; and iii) risk attitudes. Notwithstanding the work done on travel time perception in past research, the issue of perception updating has received less attention, due to its latent nature. Furthermore, risk attitudes, viewed as the weighing of objective outcome probabilities, have also been given little attention in the transportation field. Thus, this investigation seeks to contribute to our understanding of these decision dynamics, focusing on their implications on system performance, with particular emphasis on convergence. Specifically, understanding the convergence of the system, if any, under the behavioral mechanisms proposed, may offer insights into the validity of an assumed equilibrium state, commonly used in transportation planning practice. Although no concrete conclusions about the existence of an equilibrium state can be made, this study provides indications of the reasonableness of this assumption given behaviorally plausible user decision mechanisms.

With respect to activity scheduling, a framework is developed to address the dynamic aspects of individual activity scheduling, and address the shortcomings of past models, building on an analogy between the individual activity scheduling process and the operation of a single server queuing system. Past studies have addressed only
to a very limited extent, or ignored altogether, the effect of unplanned activities generated during execution, the role of latent activities, and adjustments made to the initial schedule of tentatively planned activities. This study contributes to our understanding of these factors by considering their implications on the empirical analysis of reported (observed) activity diaries. This study also extends past investigation on activity scheduling by utilizing results from queuing theory in studying the individual activity scheduling process. Finally, this study provides an operational model of activity participation to illustrate the amenability of the proposed framework towards being operational. This model further provides additional insight into the role of activity stress as it relates to activity participation decisions.

1.4 Outline of Remaining Chapters

The next chapter provides a review of relevant literature pertaining to the objectives discussed in this chapter. In particular studies related to route choice models, activity scheduling models, and behavioral dynamics are reviewed and discussed as they pertain to the research objectives. Chapter 3 presents models of route choice dynamics, including mechanisms for travel time perception and learning, and risk attitudes. Simulation experiment results investigating the effect of these mechanisms on system performance are presented and discussed in Chapter 4. Chapter 5 presents models of activity scheduling dynamics. Chapter 6 presents simulation results that illustrate the range of behavioral insights gained from the models presented in Chapter 5. Additionally, an estimated mode of activity participation is presented and discussed. Chapter 7 presents concluding remarks for this investigation.
Chapter 2.0 Background and Literature Review

This chapter discusses issues relevant to an investigation of route choice and activity scheduling dynamics, and presents a review of existing modeling approaches. There are three main goals of this review. First, current knowledge on user route choice behavior is synthesized with respect to the research objectives presented in Chapter 1. In particular, past studies on modeling the interdependence between user behaviors and system performance are visited, and studies that examine the effects of learning, in conjunction with risk and uncertainty perception, are discussed. Second, an attempt is made to synthesize current knowledge on activity scheduling and activity-based approaches to travel analysis. Along this line of thought, the effectiveness of current modeling approaches in capturing the dynamics of activity participation decisions over time will be discussed. Models of activity participation and time allocation taken from economics, regional science, and transportation are reviewed. The third objective is to recognize the essential characteristics of the dynamic processes under study, outline the approaches used by other researchers, and highlight their advantages and limitations with respect to the research issues of interest. This review is not intended to be comprehensive with respect to related streams of research. In view of the main objectives of this study, this chapter focuses on the following areas: i) approaches to modeling the interdependence between day-to-day commuter route choice decision and system performance, in relation to learning and uncertainty and risk perceptions; ii) approaches to modeling activity scheduling, activity participation, and time allocation; and iii) decision processes in complex dynamic environments.
2.1 Models of User Behavior and System Performance

The dominant approach for capturing the interdependence between user behavior and network (system) performance has been to solve for an assumed equilibrium under various assumptions on this behavior. For example, when users are assumed to select paths that minimize their perceived travel times, a stochastic user equilibrium flow pattern can result (Sheffi 1985). Although widely used in planning practice, equilibrium approaches have two main shortcomings. First, they rely heavily on the assumption that the equilibrium state exists, is unique, stable, and converges quickly, though no empirical evidence is available to support these assumptions. Second, the effect of factors such as heterogeneity in users’ behavior, learning and perception processes, and random variations in demand response and network characteristics are difficult to capture.

Extensions of the classical equilibrium framework that consider the day-to-day adjustment processes of traveler decisions were first explored in Beckmann, McGuire, and Winsten’s (1956) seminal contribution to network modeling. Day-to-day adjustment models of departure time and route decisions of commuters in response to experience and other information were proposed by Mahmassani and Chang (1986) and Mahmassani (1990). The consideration of day-to-day adjustment resulted in the development of disequilibrium approaches to investigate the transportation system’s dynamic evolution and properties. Cascetta (1989) proposed a Markov chain formulation for analyzing day-to-day route choice dynamics. Cascetta and Cantarella (1991) further extended this formulation to include within-day
dynamics, and more recently (1996) have derived conditions for the existence and uniqueness of an equilibrium state in various dynamic process models for probabilistic assignment. A “tatonnement” adjustment process model, based on optimal control theory, has also been proposed, but no user behavioral models were embedded in the equations describing the day-to-day dynamics (1994).

Another approach to investigating the relationship between travel choices and network performance, which is recently gaining attention, consists of studies that either simulate the network conditions in response to decisions from real "actual" commuters (Mahmassani et. al. 1986, Mahmassani 1990, Helbing et. al. 2002), or simulate both the network and individual user decisions (Mahmassani and Chang 1986, Peeta and Pasupathy 2001). These studies attempt to circumvent the difficulty faced by equilibrium and disequilibrium approaches in capturing user behavior at the desired level of richness, simultaneously with measurements of prevailing conditions. Additionally, these approaches provide the ability to investigate the dynamic system evolution, in particular convergence and stability, and the mechanisms underlying the day-to-day choice behavior of users. Although one shortcoming of the experimental approach is the difference between user behavior in a simulated environment and in a real network, for the daily commuting decision environment these experiments are quite amenable since all participants are working commuters themselves and the route choice decision is typically made daily (see also Mahmassani and Jou 2000). Aside from the relationship between travel choices and system properties, the perception
and integration or of travel information and experiences (learning behavior) has also been studied, though to a much lesser extent than traveler choice processes.

2.2 Models of Learning and Route Choice

Learning behavior involves the acquisition information or experiences, and relating them with current conditions and perceptions to make decisions. In the context of route choice, individuals continually learn about the travel times in a network as they make repeated choices and gain experiences day-to-day. Many dynamic system properties of traffic networks, such as the convergence, robustness, and existence of equilibrium states are affected by the learning behaviors of users. Thus, learning plays an important role from a network performance standpoint in driving the day-to-day evolution of flows. In the context of route choice, learning processes allow individuals to relate historical experiences with current travel time experiences, thus shaping their estimates or perceptions of travel times. Additionally, learning processes may lead to changes in the perceived uncertainty associated with the travel time estimates, consequently affecting risk attitudes and perceptions. Learning and risk attitudes are two interrelated parts of a decision making process. However the specific mechanisms operating behind their relationship in the context of individual route choice and network traffic flow evolution have not been fully investigated. Thus, since past experiences likely influence users’ perceptions of network performance, modeling the mechanisms by which users integrate or learn from past experiences and information from other sources is important.
Behavioral decision theorists (psychologists) have extensively addressed the integration of experience and information, and its role in decision-making (Einhorn and Hogarth 1981; Ariely and Carmon 2001; Wallsten et al. 2006). These studies have examined learning at the individual-person level, focusing on the effects of information acquisition and integration on decision making in both deterministic and uncertain environments. However, these studies have typically ignored the effect of other decision makers and different information environments. Information availability plays an important role in determining which theories are feasible in different environments. Economists have also investigated learning behavior experimentally and theoretically, but on a macroscopic scale. These studies examine the role of simple information adjustment rules in driving equilibrium processes in games under different information environments (Roth and Erev 1993; Crawford 1995; Camerer et al. 2002). Theoretical work in learning and games has generally relied on the mathematics of stochastic processes to prove theorems about the limiting properties of different rules (Weibull 1995, Fudenberg and Levine 1998). Learning strategies with realistic limiting properties are often regarded as useful models of “actual” learning, but if limiting behaviors take too long to unfold these limiting theorems are less useful than modeling the actual path of equilibration over time. Additionally, studies in the game theory literature are less concerned with the individual attributes of the players, paying less attention to the effect of learning on personal perceptions of payoffs and uncertainty. Learning in the context of machine learning has been aimed at determining classification based on new samples, and thus are more algorithmic than behavioral in nature (Mitchell 1996; Duda et al. 2001).
Thus, their applicability to actual human decision making is limited due to the intense information processing and calculation requirements of their rules.

Despite the importance of learning in the dynamics of route choice behavior, the subject has received limited attention from transportation researchers. Horowitz (1984) suggests a process where past experienced costs are integrated according to a weighted average, and finds that even under this reasonable rule, the system may not converge to an equilibrium state. Mahmassani and Chang (1986) examine a myopic adjustment and experience-based model of perceived travel time for departure time choice. Under the myopic adjustment rule, the perceived travel time is a function of the latest day’s outcome exclusively. The experience-based model is similar to the average rule suggested by Horowitz (1985). They find that convergence occurs only when all users are satisfied with their departure times within a tolerable limit, and interestingly that using the experience-based rule does not always lead to convergence as expected. Ben-Akiva et al. (1991) propose a model where the updated perceived travel time is a weighted average of the historically perceived travel time and the time provided by ATIS, where the weight indicates the relative importance of historical and information provided travel times. Although all the models previously described address travel time perception and updating mechanisms, these models do not account for the uncertainty or variance associated with travel times. The variance and uncertainty associated with travel time estimates are important, since they may significantly affect an individual's sense of a route's reliability. Additionally, the above studies assume that personal perceptions and attitudes do not vary with time.
To account for both the integration of travel times and the associated uncertainty, a Bayesian updating model has been proposed in the transportation literature (Kaysi 1991, Jha et al. 1998). A Bayesian statistical framework can account for updating both the estimate of the mean and variance in light of new information (DeGroot 1970). Recently, Jha et al. (1998) proposed a Bayesian framework for updating the perceived mean travel time and variance in light of experience and information. However, their study makes the key assumption that individuals update their perceived travel times whenever new information is obtained or new travel times are experienced. This assumption may be unreasonable since a cost may be associated with each update, making updating every time a new piece of information or experience is obtained infeasible. Additionally, individuals may only consider some experiences or information as salient or "new," precluding updating every experience. Thus, rather than updating any new experience, individuals may learn selectively, updating only under certain conditions or triggers. Jha et al. (1998) address the issue of updating travel times and the uncertainty associated in a day-to-day context, but do not address the mechanisms that trigger updating.

In addition to the perception of travel time uncertainty, risk perceptions also affect route choice dynamics. Risk perceptions affect the decision-making process in light of perceived uncertainty. Additionally, depending of the risk attitudes of individuals, the perceived gains and losses experienced from day-to-day may would differ across the population of users in the system.
2.3 Risk Perception in Route Choice Models

The effect of risk attitudes have been extensively examined in decision science, economic, and psychological studies concerning decision making under uncertainty. Decisions under uncertainty require assessment of two attributes: i) the desirability (or “value”) of possible outcomes and ii) their respective likelihood of occurrence. Under the classical theory of decision making under risk, the utility of each outcome is weighted by its probability of occurrence (Von Neumann and Morgenstern 1947; Bernoulli (1738) 1954). Expected utility theory (EUT) reflects attitudes toward risk through the shape of the decision maker’s utility function. Risk aversion is reflected in a concave utility function, while risk seeking is associated with a convex function.

The expected utility model lends itself to be operationalized and thus underlies much of the normative application of decision analysis in practice. However, experimental studies of actual decision under risk have shown that individuals often violate the expected utility model. An alternate perspective is provided by prospect theory and its extension to cumulative prospect theory (Kahneman and Tversky 1982). Under prospect theory the value function and the weighing function exhibit diminishing sensitivity: the marginal impact diminishes with distance from a reference point. This function overweighs small probabilities and under-weighs moderate and high probabilities, explaining risk attitudes encountered in experimental data (Kahneman and Tversky 1979; Payne et al. 1981; Wehrung 1989). Thus, risk is manifested through the weighing of objective probabilities. This weighing function has been estimated for gains and losses using median data (Kahneman and Tversky 1982).
Despite its conceptual attractiveness to behavioral decision theorists, prospect theory has not been operational using actual data sets.

The role of risk in travel behavior analysis has also received some, albeit limited, attention, particularly in conjunction with the uncertainty or reliability of travel times. Early work in the area of reliability examined the impact of congestion on the uncertainty of travel time in the context of departure time choice, using simulation experiments (Noland and Small 1995; Noland et al. 1998). However, the authors did not relate their measure of risk (probability of being late) to explanatory variables, and learning (or feedback) effects in the demand component of their simulation were also ignored, resulting in constant perception parameters (values of time) across iterations. Recently, learning and travel time uncertainty effects were considered in simulation experiments also concerning departure time choice (Ettema et al. 2005). Learning and adaptation effects were modeled using reinforcement type learning rules. Similar to other studies, the authors also show that considering travel time uncertainty or variance strengthens the predictive powers of models of user response to congestion. However, the study did not include the effect of different user types, such as risk takers and avoiders, or optimizers and satisficers.

In the context of route choice decisions, researchers have recently begun to focus on the effects of learning, travel time uncertainty, and risk perception. Both early and more recent laboratory experiments reveal that learning and uncertainty are important, showing that route switching behavior does depend on previously
experienced travel time differences and their perceived variances (Mahmassani and Liu, 1999; Nakayama et al. 1999; Srinivisan and Mahmassani, 2000; Mahmassani and Srinivasan, 2004; Avineri and Prashker 2003, 2005). Many studies have also examined risk and uncertainty in route choice at a more microscopic level, focusing on individual attitudes and perceptions, but not examining the system-wide network effects. Econometric methods for measuring users’ risk aversion and their application to survey data on route choice were recently examined (de Palma and Picard 2005). The authors highlight the significance of key socio-economic factors in explaining levels of risk aversion but not risk seeking. However, their methodology is consistent with situations where individuals tend to over or under evaluate the probability of risky events, hence confounding risk aversion and biased perceptions of probabilities. Route choice has also been modeled as a one-armed bandit problem (choice between a random and safe route), under different information regimes (Chancelier et al. 2007). Through numerical examples, the authors show that individuals reduce their uncertainty about travel times as a function of their risk aversion. More specifically, individuals who are risk neutral tend to select the random route and stay with it, while individuals who are more risk averse tend to pick the safe route more frequently with increasing risk averseness. Interestingly the authors show that users indifferent between the safe and random route after experiencing one or the other value learning more before settling on a final route choice (convergence). The authors’ approach allows study of the individual economic benefits of learning. However, it does not consider benefits from the choices of others (through the congestion resulting from the collective decisions of users). Although recent studies route choice have examined
and addressed travel time uncertainty and risk attitudes, they do not consider the joint effects of congestion and more importantly activity scheduling.

2.4 Models of Activity Analysis: Space-Time Geography

The origins of activity analysis trace back to Chapin’s theory of activities and urban land use (Chapin 1974) and Hägerstand’s space-time prism (Hägerstand 1970). Chapin argues that activity patterns arise from an individual’s endogenous propensity to participate in activities, further emphasizing the role of an individual’s perception of service and facility quality. Hägerstand also believes that activities arise from individual propensity to engage in activities, but in contrast emphasizes the importance of spatial-temporal constraints (space-time prism) in determining the feasibility of an activity pattern (Hägerstand 1970; Burns 1979). The space-time prism assumes that activity scheduling arises from exogenous spatial and temporal constraints imposed on the individual, while Chapin’s emphasizes endogenous factors, such as personal attributes and motivation. Thus, a key difference between Chapin’s theory and Hägerstand’s space-time prism lies in the relative focus on the type of constraints and factors (endogenous or exogenous) acting on the individual. Additionally, in the context of travel behavior analysis, although the space-time prism is elegant in its presentation, it may have inspired researchers to focus more on revealed travel behavior, while the underlying processes and mechanisms that lead to the observed behavior have received less attention.
More recently, researchers have revisited Hägerstrand’s (1970) conceptual framework of time-space geography (the space-time prism), which offers a means of integrating the spatial and temporal components of travel-related decisions underlying the concepts of travel demand and accessibility. Recker and his collaborators (Recker 1995; Recker et al. 2001) have taken a mathematical programming approach towards modeling the household travel-activity decision-making process in the household activity pattern problem (HAPP), similar to the Pick-up and Delivery Problem with Time-Windows PDPTW. They develop a new solution process based on dynamic programming methods to solve for the HAPP problem. Recker’s empirical application of the HAPP modeling approach suggests the potential of activity-based modeling approaches for accessing the limits and bounds of travel time and accessibility improvements from modifications in a household’s activity pattern. However, these models currently place assumptions and restrictions on the behavioral and uncertainty aspects of the modeling process, ignoring the stochastic nature of both activity participation and travel time.

2.5 Time Allocation and Time Use Models

Alternatively, activity patterns can also be viewed as an allocation problem where an individual allocates available time and money to engage in activities and travel, subject to income and time constraints. One of the earliest time allocation models assumes that individuals maximize utility as a function of time allocated to activities and consumption of goods during activities (Becker 1965), capturing the relationship between time allocated to work and the potential to consume goods. Although
Becker’s model is useful for understanding time and money allocation, spatial factors and consequently travel were ignored. A similar model that accounts for travel describes mode choice as the allocation of time and money (Truong and Hensher 1985), but ignoring the complexity of travel in the context of activities. To address these issues, Kraan (1996) proposes a more general model that describes the allocation of time and money to activities and trips with varying purposes, also accounting for associated travel distances and activity frequencies. However, Kraan’s model assumes that activity frequency and travel distances are independent, which is unrealistic since intuitively the frequency an activity and distance to the activity are related. Furthermore, the utility function did not include travel time, precluding examination of tradeoffs between travel time and time allocated to an activity.

The previously mentioned models capture the tradeoff between travel and activities, but provide no explanation as to why specific origin-destination trips are made. Jara-Diaz’ (1994) proposed a model that attempts to address this issue, by assuming that utility is maximized based on time allocated to activities, travel, and trips by specific modes, in relation to goods consumed at different locations. Time is allocated to trips made by a specific mode, capturing mode choice, and goods can be consumed at different destination zones and prices, capturing spatial effects. Also, travel times by specific modes are incorporated directly into the utility function. One limitation of this and other models is that the decision variables are time allocated to activities, mode choice per trip, goods consumed at each destination, work hours, and number of trips, precluding examination of decisions concerning the timing of activities and
decisions regarding which trips to make. Thus, although these models based on microeconomic theory describe and capture the allocation of time and money to activities and trips, and their associated tradeoffs, they do not address the temporal dimension of individual “scheduling” behavior. Activity patterns or schedules occur in both temporal and spatial contexts, and although time allocation models have partially addressed the spatial aspects, the order or timing of these activities have not been addressed.

More recently, greater attention has been given to time allocation among discretionary versus mandatory and in-home versus out-of-home activities (Kitamura et al. 1996; Bhat and Misra 1999; Yamamoto and Kitamura 1999). These studies analyze the tradeoffs in allocating time to different types of activities. However, these studies make no distinction between travel time and duration of out-of-home activities. To address this distinction, many researchers have begun investigating tradeoffs between time allocation to activities and associated travel times. Meloni et al. (2004; 2007) extended Kitamura’s formulation (1996) by defining an endogenous variable for trade-off between trip times and discretionary activities. Despite their account of travel times, these allocation models still do not adequately address the spatial dimensions of travel and activity patterns.

### 2.6 Econometric Models of Activity Patterns

Transportation researchers have for the most part used theoretical and conceptual frameworks from other areas and applied these to predicting activity patterns.
Transportation work dealing explicitly with travel and activity scheduling falls into one of two categories, econometric/utility-based or heuristic/rule-based.

Econometric or utility-based models rely on the assumption that individuals choose an activity pattern or schedule that maximizes their utility. The simplest model applies a Multinomial Logit (MNL) to the choice from a set of complete activity patterns (Adler and Ben-Akiva 1979). Aside from limitations stemming from the IIA property of the MNL, this early model makes other assumptions that are now considered behaviorally unrealistic. First, individuals are assumed to determine their activity patterns at one point in time, though scheduling is a continuous process, occurring over different time horizons. Second, individual’s planned and observed schedules are assumed equivalent. More realistically, original schedules are modified in light of unexpected activities that arise during execution. To address these assumptions Nested-Logit models of activity patterns have also been developed (Kawakami and Isobe 1989; Bowman and Ben-Akiva 1996; Wen and Koppelman 1999), differing from one another in the type of choices made in the hierarchical decision structures. Nested-Logit models break down the scheduling process into partial decisions embedded inside a hierarchical nested decision structure, thus assuming that choices made at high levels are influenced by the utility from lower level alternatives, operationalized as the “logsum” term. While these models can capture a more realistic decision process compared to MNL, a major limitation lies in how the temporal aspects, the timing and duration of activities, are represented. For example, in Bowman and Ben-Akiva’s model (1996), only four time periods are taken into account, limiting the temporal aspect of scheduling captured.
To better account for the time dimension and other shortcomings, Bhat et al. developed CEMDAP (2004) a comprehensive econometrically based model system of complete activity patterns that accounts for choice of type of activity, duration of activities, travel time to the activity, and their timings. Despite its completeness in modeling individual activity schedules, the model is primarily an econometric model system that accounts for observed outcomes, but is limited in explaining the process leading to these outcomes. A utility-based modeling framework that also models complete activity schedules is STARCHILD (Recker et al. 1986), which differs from other utility-based models by i) focusing explicitly on activities; ii) capturing the interrelationship between scheduling decisions and space-time characteristics of the transportation and activity systems; and iii) capturing the choice set formation process. STARCHILD assumes that individuals generate activity patterns that maximize utility, subject to constraints such as the travel availability and temporal feasibility. The utility of a pattern is assumed to be composed of utilities for its time-component parts: i) travel time to the activity; ii) waiting time for the activity to start; and iii) actual participation time. An important implication of these assumptions is that the disutility from the effort in scheduling activities may exceed the utility from combining multiple sojourns into a single trip, implying that the cost of scheduling influences the outcome of the scheduling process. This further implies that activity scheduling is not simply an optimization problem where travel time is minimized or utility is maximized, but a “satisficing” process that results in an acceptable activity schedule with acceptable scheduling effort.
2.7 Heuristic Approaches to Modeling Activity Scheduling

In contrast to econometric approaches, rule-based or heuristic approaches focus explicitly on the sequence of decisions that result during scheduling and are implemented as a set of condition-action (IF-THEN) rules. A key assumption of SCHEDULER (Garling et al. 1989) is that individuals carry out a heuristic search in scheduling activities. An individual first selects a set of activities to perform with the high priorities from “long-term memory” (LTM), including space-time information. These activities sequenced to satisfy time constraints and minimize distance traveled using a “nearest neighbor” heuristic. The schedule is “mentally” executed and conflicts are resolved, with higher priority replace low priority activities. Finally, very low priority activities fill in open time slots. A slight extension of the SCHEDULER model is GISICAS (Kwan 1997), which focuses more on scheduling in a spatial context under ATIS. In regards to the scheduling algorithm, GISICAS is similar to SCHEDULER, but the difference lies in the spatial search heuristics used, and its use of GIS to define feasible opportunity sets with respect to the current locations and the immediate spatial-temporal constraints. Another heuristic model is AMOS (Pendyala et al. 1998), which simulates the travel decisions of individuals and the schedule adaptation process. The adaptation process is viewed as a trial-and-error process in which the individual tries several different alternative activity-travel options until he/she reaches a satisfactory schedule. The model repeats this experimentation process to achieve stability. One obstacle faced by the model is the empirical validity of decision rules. One model system that addresses this issue is ALBATROSS (Arentze and Timmermans 2000), developed with the objective of elaborating on
previous work by deriving the choice heuristic rules from empirical activity diaries. Although conceptually appealing, a common concern faced by heuristic/rule-based models is the difficulty in calibrating them.

In recent years there has been a dramatic improvement in operational comprehensive activity-based models for travel demand analysis, in addition to analytical studies that examine the interrelationship between choices and explanatory variables. Despite an improved understanding of the interrelationship between activity-travel choices, the issue of dynamic activity generation has been given limited attention. Historically, trip and tour-based models predict trip and tour generation as a function of socio-demographic variables and land use-accessibility measures. Most activity-based models have adopted the same approach to modeling activity participation. For example, CEMDAP (Bhat et al. 2004) predicts activity generation as a function of age, gender, race, income, and other socio-demographic variables, in addition to the nature of work schedules and median income of residential zone. Activity generation is definitely much more complex. Under the activity generation model developed by Habib and Miller (2006), activity generation is synonymous with modeling activity-agenda formation within an econometric framework, where activity utility is composed of a “goal” and “process” components. However, in their model they only focus on “goal” utility, precluding analysis of the “process” utility of activities which reflects activity scheduling and re-scheduling. The Aurora model, which simulates adaptation behavior in scheduling, is one of the few models in which daily activity generation is a complex function of history, available time, and time pressure, focusing on the underlying behavioral mechanisms (Timmermans, et al., 2001; Joh et
al., 2002). However, the model does not capture the effects of socio-demographic variables or planning across varying time horizons. Furthermore, the validity of the model needs to be tested with real-world data.

Given the need for models that account for the dynamics of route choice and activity scheduling decisions, the next section reviews the literature on dynamic decision processes and cognitive mechanisms, focusing on learning behaviors and updating.

2.8 Behavior Dynamics: Decision Processes and Mechanisms

In the transportation literature, most models of travel and activity scheduling behavior, cross-sectional or longitudinal, are based on the utility maximization paradigm. The validity of this behavioral framework in modeling actual travel-activity behavior is questionable. First, this framework assumes that individuals evaluate all alternatives and select the one with the highest utility. However, the repetitive nature of choices in addition to attention conflicts during information acquisition, suggests the presence of heuristic search processes in user behavior (Chang and Mahmassani 1988; Garling 1998; Mahmassani and Srinivasan 2004). Furthermore, evidence exist which suggests inertial and habitual effects, in addition to the presence of “transaction” costs for implementing choices, unaccounted for under utility maximization (Liu and Mahmassani 1998; Timmermans et al 2001).

Given the limited number of modeling frameworks that recognize these effects and the unexplained variability in existing models, there is a need to better understand the cognitive and decision processes underlying observed travel-activity behavior.
Models of day-to-day route choice dynamics account for the adjustment in trip-maker choices from day-to-day in response to pre-trip information and past experience, leading to an inherently more dynamic representation that captures the daily adjustment process. Past studies have reported considerable variability in trip-making behavior from one day to the next (Hatcher and Mahmassani, 1992; Jou and Mahmassani 1998; Srinivasan and Mahmassani 2000). Possible sources of this variability include user characteristics, travel time uncertainty, and varying trip objectives. Dynamics and rhythms have also been reported in the activity scheduling literature. Huff and Hanson (1990) concluded that individual travel-activity patterns are characterized temporally by both repetition (routine) and variability (non-routine).

Mahmassani and Chang (1985, 1987) proposed a two-stage framework for analyzing day-to-day behavior. During the first stage, the commuter decides whether to switch routes and/or departure time on the next day, based on current experience and information. Conditional on this decision, in the second stage the user selects a new route or determines the magnitude of departure time adjustment. The authors also proposed that route choice and departure time decisions are based on bounded-rational behavioral rules. Under this framework, a user will switch routes only if the experienced travel time savings exceeds a pair of indifference bands, relative to travel time savings and minimum travel time savings. Similarly, users will adjust departure times only if the user arrives outside the corresponding schedule delay indifference band. These models were calibrated using data from interactive experiments. Insights from these models and experiments are summarized by Mahmassani (1990) as
follows: i) indifference bands vary with experienced congestion and information; ii) users are more likely to switch departure times over route; iii) users tolerate greater schedule delay when facing increasing travel time fluctuations; and iv) impacts of unsuccessful experiences are more drastic and longer than successful ones. Many of these findings have been independently validated based on travel diary surveys of commuters in actual systems (Mahmassani and Jou 1998). Although these studies considered the decision to stay or switch routes, they do not address the specific choice of which path to take.

From the perspective of route choice, route switching is a byproduct given one’s current route choice. Investigations on route choice under ATIS view route choice as the net outcome between (i) inertial effects that capture the seemingly inherent resistance towards switching and (ii) compliance or propensity towards the best path. Inertia reflects lower cognitive effort, information search and processing costs, switching costs, in addition to user’s familiarity and habits. Compliance reflects preference for more efficient routes, in addition to travel time savings and congestion avoidance. Srinivasan and Mahmassani (2000) modeled route selection under real-time information and investigated inertial and compliance mechanisms. Under their framework, a distinction between the decision situation where the current is the best path and other situations is made, with compliance meaning switching to the best path. The six possible combinations define hypothetical alternatives in a random utility model with a nested structure, in which the systematic utility component contains four terms corresponding to inertia, compliance, interaction between the two,
and a path-specific term. Analysis reveals that both inertia and compliance are significant mechanisms underlying route choice behavior. Inertia is negatively affected by congestion and travel time delay, while reinforced by information quality. Compliance is negatively affected by switching costs, while encouraged by travel time savings and information quality (Srinivasan and Mahmassani 2000).

While existing work on commuter dynamics provides some fundamental insights into the factors influencing day-to-day dynamics, it also suggests areas for further investigation. Additionally, previous work done presents and implements a general dynamic framework that can represent a variety of dynamic and stochastic processes. One limitation is that dynamic decision processes are not explicitly considered. Also additional research is required to extend the simple learning model proposed in this framework. The dynamic framework proposed by the authors may be generalized to explore cognitive decision processes and behavioral mechanism underlying commuter behavior. These mechanisms are naturally affected by perceptions of system performance from experience and information, in addition to the updating of these perceptions through passive and active learning processes.

### 2.9 Synthesis

This chapter briefly outlines and discusses current research on modeling travel-activity behavior dynamics, highlighting the deficiencies in existing approaches. Additionally opportunities for future investigations are identified.
Early studies on day-to-day trip-making dynamics have focused on modeling the
departure time and route switching decisions of commuters, under a “bounded-
rational” decision framework. Later studies extended these models to account for trip-
chaining and inertial effects and compliance in route selection. These studies have
identified, specified and estimated indifference thresholds relating to these commuter
decision dimensions, using empirical data from laboratory-like experiments, in
addition to field studies. Although valuable insights into complex human behavior
were gained, these studies did not address the underlying cognitive and decision
mechanism leading to these observations. More specifically, these models captured
many psychological aspects, such as response to positive and negative experiences,
correlation between choices across time, and the role of perceptions, but do not
explicitly address the mechanisms by which these aspects operate. Furthermore, these
studies investigated the timing of trips, but not the timing of the actual decisions,
which play an important role in determining the short-term and long-term
implications of decision dynamics.

An enormous literature exists on the study of human activity engagement that spans
several decades, ranging from studies in economics and geography, to sociology and
transportation. Significant progress towards characterizing the temporal and spatial
aspects of activities individually by their attributes has been made. These include
modeling frequency and time allocation associated with activities, in addition to
associated goods consumption and the interrelationships between travel-activity
choices. However, understanding the process behind activity scheduling requires a
better understanding of the decisions leading to the observed temporal and spatial aspects of activities, in addition to related processes, such as activity generation. Thus, a better understanding of the decisions behind activity scheduling and participation, including their timing in relation to each other, and their role in determining the spatial and temporal characteristics of activity patterns, rather than individual activity classes, is required.
Chapter 3.0 Models of Route Choice Dynamics

Investigating route choice decisions and the interaction of these decisions with the traffic system has both scientific and practical motivations. From a scientific standpoint, traffic systems are examples of complex nonlinear decisions systems, in which users make individual decisions, at times non-cooperatively, but the result or outcome of these decisions emerge collectively from the physical interactions of users with the system and each other. Understanding user decision processes and their interactions quantitatively and describing them mathematically are important from a practical standpoint. Traffic congestion and environmental problems affect the quality of human life, both of which result from the decisions of network users. Also, route choice decisions are essential to the planning and operation of transportation networks and systems, in terms of devising strategies and solutions to combat these system externalities. Thus, determining effective solutions to these problems relies on the ability to understand, predict and influence the space-time characteristics of users, which result from their decisions.

3.1 Objectives of Route Choice Models

This chapter presents models of route choice dynamics in traffic networks. The three dynamic mechanisms investigated and modeled in this chapter address the following behavioral aspects of day-to-day route choice decisions: i) travel time uncertainty perception; ii) travel time learning and iii) risk perception.

The route choice decision model presented in this study consists of two main components: i) a route selection component; and ii) a travel time perception and
learning component. Risk and uncertainty perception are captured within the second component. Three behavioral learning perspectives are examined: i) Bayesian inference; ii) reinforcement; and iii) belief (epistemic). Furthermore, the models presented also seek to capture the effect of risk attitudes on day-to-day traffic flows. Under the decision making framework in this study, users with different risk attitudes vary in their weighing of objective probabilities, in a manner similar to Prospect Theory (Tversky and Fox 1995). Results from simulation experiments conducted to investigate the system implications of the behavioral assumptions invoked by these models are presented in the next chapter. This chapter focuses primarily on presenting the models and discussing their behavioral implications.

The models presented aim to capture the interdependence between users’ travel time perception and learning/updating mechanisms (behavioral dynamics), and the day-to-day evolution of traffic flows. These individual (user) level models are embedded inside a microscopic (agent-based) simulation framework to investigate their collective effects on the day-to-day behavior of traffic flows. Experiments are conducted using this simulation model to examine the effect of (i) travel time perception updating/learning, (ii) updating trigger/terminate mechanisms, and (iii) risk attitudes on traffic flow evolution and other dynamic system properties, particularly convergence. This study extends past efforts by (i) introducing and comparing alternative formulations for the travel time perception and updating/learning process, (ii) investigating the mechanisms that trigger and terminate updating, (iii) investigating users’ perceived uncertainty in the network, (iv) capturing users’ risk attitudes in the learning process, and (v) capturing the effect of all the
above on the day-to-day network dynamics, in particular convergence. The simulation experiments provide an exploratory analysis on how different learning rules affect individual travel time perception over time, and the role of risk attitudes in perception, that may subsequently aid in designing experiments carried out in an interactive collaborative decision-making type laboratory, with actual users.

The remaining sections of this chapter specify the key elements and components of the modeling framework. A description of the simulation experiments performed, followed by presentation and discussion of key results, are found in the next chapter.

3.2 Route Choice Modeling Framework

Network traffic flow results from the interaction between users, their evaluation of past experiences, the resulting travel decisions, and the supply-side characteristics of the network. This section presents a route choice decision making framework that models and captures route selection, travel time learning/updating, and risk perception. Specifically, this framework consists of models of different mechanisms by which users integrate past with current experiences, and a mechanism that describes the weighing of objective probabilities. The detailed specifications of these models are presented in following sections.

3.2.1 Route Choice Decision Process

For a given day, an individual’s route choice yields an outcome or experience (travel time) that is a function of both the individual’s decision and those of other users in the system. The experience is integrated with past experiences through a travel time learning mechanism, consequently updating users’ perceptions of system
performance (network travel times). Based on the acceptability of the current travel time in light of past experiences, the individual will decide to switch routes or remain with the currently chosen one. Acceptability is based on the individual’s current perception of travel time, which depends on travel times experienced over a number of days, and the individual’s risk attitudes. Based on the perceived travel times and associated uncertainty, an individual weighs the chance of perceived success or failure resulting from switching routes. The success or failure is perceived, since individuals may be unsure of the accuracy of their own judgment. In the context of this study, the travel time uncertainty that users perceive arises from endogenous judgment errors, which affect the perceived error resulting from the stochastic behavior of the system. The day-to-day route choice decision process is illustrated in Figure 3.1.

**Figure 3.1: Route Choice Decision Process: Information Flows, Decision Flows, and Influence from Observed and Unobserved Components**
For a given day $d$, user $n$ experiences a travel time $T_{n,k}^{e,d}$ along chosen path $k$ in the network. Due to endogenous perception errors, this quantity may not be identical to the objective travel time. Based on an updating/learning mechanism, the user updates his perceived (updated) network travel times with the new experienced travel times. A route switching and consequently selection decision is made for the following day $(d+1)$ based on perceived travel times and individual risk attitudes. Users begin using a network with an initial perception of the travel times and with associated uncertainty. This initial perception could represent the user’s “best guess” of travel times, influenced by past experiences, information or other personal rules. The next section defines the different components of the perceived travel times, namely the updated and experienced travel times, both of which are perceived with judgment error.

### 3.2.2 Travel Time Perception

In this study, route switching decisions are made on the basis of perceived route travel times that vary across individuals and are updated in light of travel times experienced from day-to-day. Perception error is assumed to arise from endogenous factors that affect the user’s judgment of the accuracy of this travel time. Thus, the updated perceived travel time resembles a “learned” travel time or travel time in “memory” that is updated as new travel times are experienced. Perceived travel times are updated based on travel time experiences on a particular day $d$ and all days since the last update ($d = 0, 1, 2, \ldots, D-1$). The *updated* perceived travel time can be stated as follows:
\[ T_{n,k}^u = \tau_{n,k}^u + \varepsilon_{n,k}^u , \forall k \in K, n \in N , \] (3.1)

where

\( T_{n,k}^u \): *updated* perceived travel time for person *n* on route *k*

\( \tau_{n,k}^u \): mean *updated* perceived travel time

\( \varepsilon_{n,k}^u \): associated judgment error that is distributed Normal \( \sim N(0, \sigma_{n,k}^u) \)

Consequently, \( T_{n,k}^u \) is distributed Normal \( \sim N(\tau_{n,k}^u, \sigma_{n,k}^u) \), with the distribution varying across routes and individuals. As individuals experience new travel times, \( \tau_{n,k}^u \) and \( \sigma_{n,k}^u \) are updated accordingly through a learning mechanism, such as Bayesian updating or reinforcement learning. These learning mechanisms are described and discussed in section 3.4. Similar to \( T_{n,k}^u \), the perceived *experienced* travel time also consists of a mean and associated error, as follows:

\[ T_{n,k}^{e,d} = \tau_{n,k}^{e,d} + \varepsilon_{n,k}^{e,d} , \forall k \in K, n \in N , \] (3.2)

where

\( T_{n,k}^{e,d} \): perceived *experienced* travel time for person *n* on route *k*

\( \tau_{n,k}^{e,d} \): mean perceived *experienced* travel time

\( \varepsilon_{n,k}^{e,d} \): associated judgment error, distributed Normal \( \sim N(0, \sigma_{n,k}^{e,d}) \)

Consequently, \( T_{n,k}^{e,d} \) is distributed Normal \( \sim N(\tau_{n,k}^{e,d}, \sigma_{n,k}^{e,d}) \), with the distribution varying across each route for each individual, and also varying across individuals. In this study \( \tau_{n,k}^{e,d} \) is assumed to be the objective (actual) travel time on a particular route. Also the perceived *experienced* travel time is assumed to have the *same* error as the
updated perceived travel time \( (\sigma_{n,k}^{e,d} = \sigma_{n,k}^{u}) \). Behaviorally, this implies that individuals perceive their experienced travel times with the same error as the travel time they learn or update in memory, implying further that the uncertainty associated with the travel time judgments in memory carries over and influences the perception of experienced travel times. Thus, the experienced route travel time perceived by users reflects or is correlated with past experienced travel times for a particular route.

Experienced travel times are integrated with perceived travel times in memory through learning mechanisms. Additionally, individuals make route switching decisions (and consequently route choices) based on these perceived travel times, in conjunction with risk attitudes that affect the perception of gains and losses among routes in the choice set. Both learning mechanisms and risk attitudes play important roles in individuals’ route choices across time. The following sections present and describe the route switching and learning mechanisms, used in this study.

3.3 Route Switching Mechanism

In this study, users base their day-to-day route choice decisions on perceived travel times for the best and currently chosen routes. Consider a user \( n \) who selects route \( k \) on day \( d \), resulting in a perceived experienced travel time for the route \( T_{n,k}^{e,d} \). Given the perceived best travel time \( T_{n,\text{best}}^{d} \) on day \( d \), the user makes a route choice decision for the next day \((d+1)\) based on the difference between the perceived current and best travel times. If the difference is acceptable, the user will stay on the current path for the next day; otherwise, the user will switch to the route with the best perceived travel time. Thus, acceptability or tolerance is defined on the basis of the difference between
the current and best travel times. A mechanism for incorporating the concept of a "tolerance threshold" is stated as follows:

\[ T_{n,\text{savings}}^d = T_{n,\text{current}}^d - T_{n,\text{best}}^d \geq 0, \quad d = 1, 2, \ldots, D, \]  

(3.3)

Alternatively, the difference can be expressed in relative terms, as:

\[ \frac{T_{n,\text{savings}}^d}{T_{n,\text{best}}^d} \geq 0, \quad d = 1, 2, \ldots, D, \]  

(3.4)

\[ \delta_{nd} = \begin{cases} 1 & T_{n,\text{savings}}^d \geq \Delta_{nd} \geq 0 \\ 0 & \text{otherwise} \end{cases}, \]  

(3.5)

where

\( \delta_{nd} \): a binary indicator for route switching (0 = stay; 1 = switch)

\( \Delta_{nd} \): acceptability or tolerance threshold for travel time savings

Travel time savings as defined in Equation 3.3 or 3.4 are essentially the same behaviorally. However, Equation 3.4 is more plausible since it implies that users perceive travel time difference relative to a reference point (the best travel time \( T_{n,\text{best}}^d \)) rather than as an absolute difference, as implied by Equation 3.3. The threshold \( \Delta_{nd} \) defines the percent improvement over the current travel time to warrant switching routes. From the perspective of travel time sensitivity, users that are very sensitive to travel time differences have smaller \( \Delta_{nd} \) values compared to users insensitive to travel time differences, leading to a greater propensity towards switching. The travel time savings threshold \( \Delta_{nd} \) for user \( n \) on day \( d \) can be modeled as:
\[ \Delta_{nd} = \eta_{nd} \cdot \left( T_{n,\text{current}}^{d} \right) \]  

(3.6)

where

\( \eta_{nd} \): relative indifference threshold, as a fraction of current travel time \( T_{n,\text{current}}^{d} \)

Behaviorally, Equations 3.3 to 3.6 state that individuals with small values of \( \eta_{nd} \) (and consequently \( \Delta_{nd} \)) are less tolerant of small travel time differences compared with individuals with large values. If \( \eta_{nd} \) takes a value of zero, then individuals are intolerant of any difference in travel times and will switch for even the smallest travel time difference, which is behaviorally implausible. A person’s inherent travel time difference sensitivity (\( \eta_{nd} \) and \( \Delta_{nd} \)) may reflect judgments confidence and perceived feedback from experiences, in addition to inherent user preferences, destination activity conditions, and risk attitudes. The expressions above are similar to the earliness and tardiness thresholds for arrival and departure times used by Mahmassani and Chang (1986). The tolerance thresholds (\( \eta_{nd} \) and \( \Delta_{nd} \)) reflect a number of factors including individual attitudes and preferences, and thus should vary across the population over time. However, since the focus of this study is on the mechanisms for the perception and updating of the travel times, and not the switching mechanisms, \( \eta_{nd} \) is assumed to be equal for all users and fixed across time. Nonetheless, the actual threshold value \( \Delta_{nd} \) (Eq. 3.6) varies with the person since the experienced perceived travel times are different across the population. A similar switching model has been used extensively in various simulation studies (Mahmassani and Jayakrishnan 1991), and empirically verified in several laboratory experiments dealing with route
switching behavior of commuters under information received from ATIS (Mahmassani and Stephan 1988; Mahmassani and Liu 1999), though those other studies did not explicitly address the perception dimension.

### 3.4 Travel Time Learning Mechanisms

In the route choice context, learning is defined as the integration of new with past experiences and information. Information availability plays an important role in determining the feasibility of different learning mechanisms in different information environments and conditions (Camerer 2003). In addition to relating experiences with current choices, learning processes may also lead to changes in the uncertainty perceived by individuals, and consequently their risk perceptions over time.

In the context of day-to-day route choice, individuals update their perceived travel times in memory $T^u_{n,k}$ with new experienced travel times $T^{e,n}_{n,k}$ through different learning mechanisms. Recall from the previous section that the perceived travel time (experienced or updated) for a route $k$ consists of a mean $\tau_{n,k}$ and associated random error $\epsilon^u_{nk}$ distributed Normal $\mathcal{N}(0, \sigma_{nk})$. Thus, perceived travel times can be viewed as distributed Normal with a mean $\tau_{nk}$ and variance $\sigma_{nk}$. In this study, learning mechanisms seek to update both parameters (the mean and variance) associated with the updated travel time $T^u_{nk}$, given new travel times experiences. Several generic theories of learning or information updating have been proposed in the psychology, game theory, and machine learning literature, such as reinforcement, belief,
sophisticated (anticipatory), directional, Bayesian inference, and Boltzmann-type learning, each with different information requirements. In this study, three types of learning are considered: i) Bayesian inference; ii) reinforcement; and iii) belief (epistemic). Each of these learning types is presented next in the context of day-to-day route choice and discussed. (Note: hereafter the subscripts \( n \) for individual and \( k \) for route are dropped for convenience and clarity of exposition).

### 3.4.1 Bayesian Learning

Perceived updated travel times are updated in light of trip experiences for a particular day \( d \) and all days since the last update. The current discussion focuses on learning (updating) mechanisms for integrating travel time experiences with perceived travel times. The first model considers concepts from Bayesian statistical inference. Under Bayesian learning, the mean and variance (moments) of a distribution are updated given new samples. In the context of day-to-day travel time perception, Bayesian learning can be applied to the learning of perceived travel times in a network, where the mean perceived (updated) travel time \( \tau_{nk}^u \) and variance \( \sigma_{nk}^u \) are updated given new travel times experienced each day. The distributions of both the updated perceived travel time \( T_{n,k}^u \) and experienced travel times \( T_{n,k}^{e,d} \) are assumed to be normally distributed with a known variance. Under Bayesian learning, the posterior distribution of \( T_{n,k}^u \) (post-updating), in light of experienced travel times \( T_{n,k}^{e,d} \) (the sample) is assumed normally distributed with the following parameters (mean, variance, and weights) (DeGroot 1970):

\[
\tau_{nk}^u = \beta (\tau_{nk}^u) + (1 - \beta) \cdot (T_{nk}^{e,d})
\] (3.7)
\[
\sigma'_{nk} = \frac{\sigma''_{nk} \cdot \sigma_{nk}^{ed}}{\sigma_{nk}^{ed} + D_s \sigma''_{nk}} 
\]

(3.8)

\[
\beta = \frac{(\sigma''_{nk})^{-1}}{(\sigma''_{nk})^{-1} + D_s (\sigma_{nk}^{ed})} 
\]

(3.9)

where \(\tau''_{nk}\) and \(\sigma''_{nk}\) are the posterior mean and variance, respectively, of the updated perceived travel time; \(\tau''_{nk}\) and \(\sigma''_{nk}\) are the prior mean and variance of the perceived travel time; \(\bar{T}_{nk}^{ed}\) and \(\sigma_{nk}^{ed}\) are the sample mean and variance of the experienced travel times (sample) on day \(d\), where the sample consists of all travel times not integrated prior to day \(d\); and \(D_s\) is the number of experienced travel times in this sample. If the number of experiences is less than three \((D_s \leq 3)\), \(\sigma''_{nk}\) is assumed to be equal to \(\sigma''_{nk}\).

To appreciate the behavioral implication of Bayesian statistical updating, define a measure \(\alpha\) called “confidence” which is the inverse of the variance. The above expressions (Eq. 3.7 thru 3.9) can now be written as:

\[
\tau''_{nk} = \left[ \frac{\alpha_{nk}}{\alpha_{nk} + D_s \alpha_{nk}^{ed}} \right] \cdot \left( \tau''_{nk} + \left[ \frac{D_s \alpha_{nk}^{ed}}{\alpha_{nk} + D_s \alpha_{nk}^{ed}} \right] \cdot \bar{T}_{nk}^{ed} \right) 
\]

(3.10)

\[
\sigma'_{nk} = \frac{\sigma''_{nk} \cdot \sigma_{nk}^{ed}}{\sigma_{nk}^{ed} + D_s \sigma''_{nk}} = \frac{1}{1/\sigma_{nk}^{ed} + D_s/\sigma_{nk}^{ed}} = \frac{1}{\alpha + D_s \alpha^e} 
\]

(3.11)

\[
\alpha_{nk} = 1/\sigma_{nk}^{u} \quad \text{and} \quad \alpha_{nk}^e = 1/\sigma_{nk}^{ed} 
\]

(3.12 and 3.13)

The above expressions convey several key behavioral implications. First, as the variance of users’ perceived travel times increases \((\sigma'''_{nk} \text{ and } \sigma''_{nk} \text{ increase in value}),\)
“confidence” decreases (Eq. 3.12 and 3.13). Conversely, as the variance of the error terms decrease, the confidence in the mean travel times \( \tau_{nk}^u \) and \( \tau_{nk}^{ed} \) increases.

Second, according to Equation 3.10, the posterior updated perceived travel time is the weighted average of the prior updated perceived travel time and the sample mean of experienced perceived travel times, where the weights are proportional to the posterior confidence \( \alpha \) and the perceived experienced travel time sample confidence \( D_s \cdot \alpha_{nk}^e \). This leads to three important properties: (i) with every perceived experienced travel time, the variance associated with the updated perceived travel time will always decrease (since \( \sigma_{nk}^{ed} \) and \( D_s \) are always positive) and thus confidence always increases; (ii) the greater the number of travel times experienced, the greater the confidence associated with the distribution of the posterior updated perceived travel time \( \alpha'_{nk} \); and (iii) as the confidence associated with the posterior distribution of the updated perceived travel time \( \alpha_{nk} \) increases such that \( \alpha_{nk} >> \alpha_{nk}^e \), new experienced travel times no longer affect the users’ updated perceived travel times.

The second point further suggests that a trade-off exists between the frequency of updates and the number of experienced travel times before updating. Thus, an individual may either experience small samples of travel times and update frequently, or experience large samples and update less frequently, in order to reach a particular confidence level.

### 3.4.2 Reinforcement Learning

Under reinforcement learning, alternatives or routes are “reinforced” by their previous positive payoffs, possibly “spilling over” to similar alternatives (routes with
overlapping links) (Erev et al. 1999). In terms of perceived travel times defined previously, a reinforcement type learning rule for updating the mean and variance can be expressed as:

\[
\tau'_{nk} = (\beta) \cdot (\tau^u_{nk}) + (1 - \beta) \cdot \left( \frac{T_{ed}^{nk}}{C_{d}} \right)
\]  

\[
\sigma'_{nk} = \frac{\sigma^u_{nk} \cdot \sigma_{nk}^{ed+}}{\sigma^u_{nk} + C_{d} \sigma_{nk}^{ed+}} = \frac{1}{\sigma^u_{nk} + C_{d} \sigma_{nk}^{ed+}}
\]  

\[
\beta = \frac{\phi \cdot C}{\phi \cdot C + C_{d}'} \quad 0 \leq \phi \leq 1
\]  

where \( \tau'_{nk} \) and \( \sigma'_{nk} \) are the posterior mean and variance of the updated perceived travel time; \( \tau^u_{nk} \) and \( \sigma^u_{nk} \) are the prior mean and variance of the perceived travel time.

Unlike Bayesian learning where both positive payoffs and losses can potentially be integrated, reinforcement learning only considers positive payoffs when updating. Thus, in the case of reinforcement learning, \( T_{nk}^{ed+} \) and \( \sigma_{nk}^{ed+} \) are the sample mean and variance of the experienced travel times, where the sample consists of travel times not integrated that were below a reference travel time (payoff) on day \( d \); and \( C_{d}' \) is the number of experienced travel times in this sample. \( C \) is the sum of all previous \( C_{d}' \). If the number of experiences is less than three (\( C_{d}' \leq 3 \)), \( \sigma^u_{nk} \) is assumed to be equal to \( \sigma_{nk}^{ed+} \). Additionally, \( \phi \) is a parameter reflecting the weight placed on past payoffs.

Under reinforcement learning strategies, individuals update their perceptions based only on their own experiences, requiring information on received payoffs from their
own actual behavior only (Roth and Erev 1993). In the context of day-to-day route choice, travel times for a particular route are updated only when the route is selected and an improved travel time is obtained relative to a reference travel time, thus “reinforcing” the (positive) perception of the travel time for a particular route. This further suggests that the reference point an individual selects plays a crucial role in reinforcement learning, since it determines which experiences are perceived as positive payoffs (gains). According to the expressions above (Eqs. 3.14 to 3.16), reinforcement learning is also governed by the parameter $\phi$ ($0 \leq \phi \leq 1$) which determines the “strength of memory” or “rate of forgetting.” As $\phi$ increases in magnitude, the rate of forgetting decreases and past payoffs have greater influence on current travel time perceptions. The expression above also suggests that as the number of payoffs experienced (travel time gains) exceeds the sample size ($C >> C'_d$), the weight placed on previous payoffs increases, independent of the strength of memory (value of $\phi$). Finally, both $C$ and $C'_d$ are a function of the frequency of updating, suggesting that under reinforcement learning, a tradeoff exists between the rate of learning or degree of experimentation and the perception of travel times.

### 3.4.3 Belief Learning

Belief learning assumes that individuals form and update “beliefs” about the choices of other individuals and act or behave according to these beliefs (Crawford 1995). One example of belief learning in game theory is fictitious play, where individuals keep track of the relative frequency with which other individuals make choices. The relative frequencies are the “beliefs” individuals use to make their next choices. In
general, belief learning strategies assume that individuals formulate beliefs about other individuals’ choices and base their own choices on these beliefs, thus requiring information on these choices’ payoffs. In the context of day-to-day route choice, this learning rule can be expressed as:

\[
\tau_{nk}^u = (\beta) \cdot (\tau_{nk}^u) + (1 - \beta) \cdot (T_{Nk}^{ed}),
\]

(3.17)

\[
\sigma_{nk}^u = \frac{\sigma_{nk}^u \cdot \sigma_{Nk}^{ed}}{\sigma_{Nk}^{ed} + C'_{dk} \sigma_{nk}^u} = \frac{1}{1/\sigma_{nk}^u + C'_{dk} / \sigma_{Nk}^{ed}},
\]

(3.18)

\[
D_S = C + C'_{dk}
\]

(3.19)

\[
\beta = \frac{\phi \cdot C}{\phi \cdot C + C'_{dk}} \quad 0 \leq \phi \leq 1
\]

(3.20)

where \( \tau_{nk}^u \) and \( \sigma_{nk}^u \) are the posterior mean and variance of the updated perceived travel time; \( \tau_{nk}^u \) and \( \sigma_{nk}^u \) are the prior mean and variance of the perceived travel time.

Similar to Bayesian learning, both positive payoffs (gains) and losses can potentially be integrated under belief learning, as opposed to reinforcement learning which only considers positive payoffs. Thus, in the case of belief learning, \( T_{Nk}^{ed} \) and \( \sigma_{Nk}^{ed} \) are the sample mean and variance of the experienced travel times for a route \( k \), where the sample consists of travel times experienced by other users on that route; and \( C'_{dk} \) is the number of experienced travel times (across persons and times) in this sample; \( C \) is the sum of all previous \( C'_{dk} \). If the number of experiences is less than three (\( C'_{dk} \leq 3 \)), \( \sigma_{nk}^u \) is assumed to be equal to \( \sigma_{nk}^u \). Additionally, similar to reinforcement learning, \( \phi \) is a parameter reflecting the weight placed on past experiences (strength of memory).
Belief learning is similar to reinforcement learning in that a weighted average between past and current experiences is taken. The main departure lies in the source and type of information used to update past experiences. Belief learning uses experiences from all other users, whereas reinforcement focuses exclusively on the user’s own experiences. Thus, belief requires the perceived gains and losses by all other users, while reinforcement requires only the payoffs of the user of interest. Furthermore, in reinforcement learning, updating only occurs for chosen routes, since only personal experiences are used. Under belief learning, travel times for all routes can potentially be used for updating depending on the behavior of other users in the system of interest. This suggests that under belief learning, users may increase their confidence in network perceived travel times, since they are accounting for travel time experiences on routes not taken. Similar to reinforcement learning, $\phi$ suggests a trade-off between “strength of memory” and frequency of learning or sample size. In the game theory literature, many studies have shown that heterogeneity in beliefs across individuals lead to different equilibria in coordination games (Van Huyck et al. 1991). The adaptive dynamics in coordination games have been shown to produce results similar to experiments with belief learning models (Crawford 1995; Ho and Wiegelt 1996; Battalio et al 1999). Recently, Helbing et al. (2004) have shown that day-to-day route choice resembles coordination games, and that over time players learn to take turns on a two-link network. These studies suggest that belief learning can lead to coordinated system states. However, these studies used relatively small numbers of players, such that keeping track of the payoffs and actions of other players is practical.
3.4.4 Conceptual Comparison of Learning Models

The section provides a conceptual comparison of the three learning models previously discussed. The models are discussed in terms of the weights placed on past and current experiences or information, and sources of information.

The main departure for Bayesian learning from other learning rules is the weight placed on past experiences, particularly on recently sampled experienced travel times. Whereas reinforcement and belief learning assume that the weight placed on historic experiences is a characteristic of the individual (total number of experiences), Bayesian learning provides a statistical basis for determining these weights, as a function of the parameters (variance) of the sample. If “confidence” is assumed to be the inverse of variance, then as variance increases, confidence decreases. Conversely, as variance associated with the updated travel time decreases, confidence increases.

As noted previously, three important resulting properties are: i) with every experienced travel time, the variance associated with the updated travel time always decreases and confidence always increases (since $D_s$ and $\sigma_{nk}^{ed}$ are always positive); ii) as the number of experienced travel time increases, the confidence associated with the posterior travel time in memory increases; and iii) as the confidence associated with the posterior travel time in memory increases such that the confidence in memory is much greater than that of the sample, the effect of newly experienced travel times decreases.

Interestingly, Bayesian, belief, and reinforcement learning share two common properties: i) updated travel times are a weighted average of the prior travel time in
memory and the travel times recently experienced; and ii) these weights exhibit a trade-off between frequency of updates and size of each update sample. The point of departure between the different rules is the source of experiences used in learning. Reinforcement only updates with travel times from individual choices that can be viewed as gains (decrease in travel time). Belief learning allows travel times experienced by other individuals in the population. Bayesian learning does not specify the source of the sample (how the sample is constructed or taken). These similarities and differences suggest that, in the context of day-to-day route choice, these three rules may yield similar sensitivity to frequency of update, but may lead to different results when the sources of experience differ. Furthermore, all else being equal, Bayesian learning may lead to a different rate of convergence compared to belief and reinforcement learning since its weights are a function of the actual travel times experienced (through the use of sample variance) and not just the frequency of choice.

3.5 Travel Time Learning Trigger Mechanisms

The preceding discussion on learning models addressed the updating of perceived travel times in light of past experiences. However, the timing and frequency of updating was not addressed. Furthermore, previous studies have assumed that updating occurs with every new experience (Jha et al. 1998). This assumption may be unreasonable if costs are associated with both updating and experiencing a new route, in which case a trade-off may exist between the number of updates and experiences, and the corresponding gains in confidence. Additionally, users may have different
perception thresholds for “new” experiences, suggesting selectivity in their updating behavior.

Three trigger mechanisms are described hereafter, based respectively on the number of days elapsed since the last update (time-based), the relative difference in travel times and the achieved confidence level. Both of the latter two mechanisms are event-driven on the basis of exogenous or endogenous variables, respectively.

1) *Number of Days*. Under this mechanism, updating is based on the number of elapsed days, leading to a periodic updating process. This trigger can be expressed as:

\[
\pi_{nd} = \begin{cases} 
1 & \text{modulus}(d, M_n) = 0, \ d=1, 2, 3, \ldots, D \\
0 & \text{otherwise} 
\end{cases}
\]  

(3.21)

where \( \pi_{nd} \) is a binary variable that indicates updating and takes a value of 1 for every \( M_n \) th day, and 0 otherwise for individual \( n \); \( M_n \) is an integer constant; \( d \) is the day number. Thus, updates occur more frequently for low values of \( M_n \), and less frequently for high values. Consequently, the number \( M_n \) of travel times experienced between updates is small for low values of and large for high values. Although updating periodically or every *fixed* number of days seems behaviorally implausible, with the exception of updating every day, investigating the system behavior under this rule provides useful insights into the effect of varying the length of time between updates. Understanding this effect is important to understanding the effect of other mechanisms, since other more elaborate triggering mechanisms derive their effect
partly or wholly from varying the frequency of updates, and consequently the time period between updates.

A more realistic trigger mechanism may be based on experiences rather than a fixed time period, i.e. event-based rather than time-based. A mechanism based on experienced travel times is described next.

2) *Difference in Experienced Travel Time.* Updating here is based on the difference between the perceived experienced travel time and the mean updated travel time, relative to the updated travel time.

\[
\pi_{nd} = \begin{cases} 
1 & |T_{nk}^{c.d} - \tau_{nk}^u| \geq (\Delta_{nd} \cdot \tau_{nk}^u) \\
0 & \text{otherwise.}
\end{cases}, \quad 0 \leq \Delta_{nd} \leq 1 \tag{3.22}
\]

where \(\pi_{nd}\) is the indicator that equals 1 if updating is “triggered” and occurs on day \(d\), and 0 otherwise; \(\Delta_{nd}\) is a threshold that defines if an experienced travel time \(T_{nk}^{c.d}\) is “salient” relative to \(\tau_{nk}^u\) the mean perceived updated travel time; This mechanism is similar to the route switching mechanism (Eq. 3.4), the key difference being that the comparison for this mechanism is the mean perceived updated travel time and experienced travel time, making an inter-day versus intra-day comparison. Under this mechanism, users update selectively, only for salient travel times, measured as the relative difference between the experienced and perceived updated travel times. Behavioral decision theorists share this view of integrating experiences, which theorizes that individuals do not integrate all experiences, but focus only on a few
defining “gestalt” characteristics (Ariely and Zauberman 2000). Their view is that in combining experiences, individuals only extract salient features, such as the maximum and minimum values. The mechanism above is consistent with this view, since only when travel times are very different in magnitude from the learned travel time does updating occur. A high value of $\Delta_{\text{nd}}$ corresponds to a very “selective” individual who only integrates rarely, while a low value of $\Delta_{\text{nd}}$ corresponds to an individual who integrates frequently. Although $\Delta_{\text{nd}}$ is assumed to be the same across the population, each individual has a different cutoff or threshold since it is based on $(\Delta_{\text{nd}} \times \tau^u_{\text{nk}} )$ and $\tau^u_{\text{nk}}$ varies across the population. In this study, this issue was not considered because the focus is on the effect of updating and perception mechanisms on network performance and not necessarily the effect of threshold values.

The above two mechanisms are both trigger mechanisms for initiation of the updating process. Termination of the process also deserves attention. If a cost is associated with each update, individuals are unlikely to update endlessly. A terminating mechanism based on the confidence for the learned travel time is described next.

3) **Confidence of updated (learned) travel time.** Under this mechanism, updating occurs until the confidence in the travel times for all routes in the network has reached a desired level. Any updating that occurs after the desired level is reached will only improve the confidence, since confidence always increases with each update regardless of the sample size (by nature of rules for updating the variance). This trigger can be expressed as:
\[
\pi_{nd} = \begin{cases} 
1 & \alpha_{nk} \leq (\lambda_n \cdot \tau_{nk})^{-1}, \quad \lambda_n \geq 0 \\
0 & \text{otherwise.} 
\end{cases}
\] (3.23)

where \( \pi_{nd} \) is the indicator that equals 1 if updating occurs on day \( d \), if the confidence of the perceived travel time is below the desired level for user \( n \), and 0 otherwise; \( \lambda_n \) is a relative threshold interpreted as the variance of the perceived travel time over a segment of unit travel time. According to the above mechanism (Eq. 3.23), updating occurs as long as the confidence in travel time perceptions for a route is below a desired level \( (\lambda_n \cdot \tau_{nk})^{-1} \). The expression \( \lambda_n \cdot \tau_{nk} \) gives the variance that corresponds to the desired confidence level. The motivation behind a confidence-based learning mechanism is that confidence in the perceived (learned) travel time may be a good indicator of “familiarity” with a traffic network. Once an individual reaches a certain level of familiarity, learning ceases and he/she may become insensitive to new travel times. Thus, the mechanism above may serve as a model for describing the time required for an individual, from when he/she first enters a network, to become a regular commuter. Additionally, individuals may have different confidence requirements for different routes used for different purposes. To reflect this, the above mechanism could be modified such that learning ceases when the confidence on an individual route, rather than for all routes, reaches a desired level.

A behavioral issue closely related to learning and uncertainty perception is risk perception. Risk perception concerns the perception of uncertainty as it relates to the likelihood of an outcome. The issue of travel time uncertainty perception that arises from endogenous error in judgment has been examined and discussed in previous
sections. The next section discusses the mechanism by which this uncertainty relates to the perceived likelihood of gains and losses in route choice decisions over time.

3.6 Risk Perception Mechanism

Decision making in environments with uncertainty requires the evaluation of the desirability (gains and losses) of outcomes and their likelihood of occurrence. Day-to-day route choice decisions may be framed as a decision based on perceived differences between routes with respect to an experienced travel time that may be shorter or longer than the updated mean perceived travel time \( \tau_{nk}^u \), or other reference point. From the perspective of probability and statistics, for a particular route \( k \), an individual \( n \) perceives a travel time distributed with a mean \( \tau_{nk}^u \) and variance \( \sigma_{nk}^u \), which are updated as new experiences are acquired over time (learning). The mechanism by which users evaluate perceived travel times for routes in a network is presented and discussed in this section.

The classical framework for decision making under uncertainty is expected utility theory (EUT) which states that individuals weigh the utility of each outcome by its probability of occurrence (von Neumann and Morgenstern 1947). Under EUT, risk attitudes are explained through the shape (concavity or convexity) of an individual’s utility curve, where gains and losses are mapped through a utility function \( u(x) \), and \( x \) is the value (payoff or outcome) of pursuing an alternative (choice). Although the EUT framework has dominated decision making under risk in microeconomics and in normative decision analysis, experimental studies have consistently revealed
behaviors that are not compatible with EUT (Kahneman and Tversky 1979; Payne et al. 1981; Wehrung 1989). In particular, these experimental studies suggest that individuals tend to under-weigh outcomes that are merely probable in comparison with outcomes that are obtained with certainty, depending on whether the outcome is a gain or loss.

An alternative theory to account for these inconsistencies is Prospect Theory (PT). Under prospect theory, the prospect of a lottery is determined by taking the sum of the values of alternative outcomes weighted by their subjective probabilities of occurrence, and a choice is made based on these prospects; this is done in two phases. A lottery is defined as a probability distribution on a finite set of gains and losses. The first phase is an “editing” phase where outcomes of lotteries are coded as gains or losses relative to some reference point. The issues of reference point selection will be discussed later in this section. However, for the purpose of the current discussion, the reference point is taken as the perceived best travel time $T_{a,best}$ on day $d$, mentioned in Section 3.3. In the second phase, these gains and losses are evaluated using a value function $v(.)$ for the travel time differences and a weighing function $\Omega(.)$ for their objective probabilities (which returns the corresponding subjective probabilities), which jointly determine attitudes towards risk. Under Prospect Theory, individuals in general exhibit four different patterns of risk aversion and risk seeking behaviors (Kahneman and Tversky 1979, 1992; Tversky and Fox 1995): i) risk seeking for gains and ii) risk aversion for losses of low probabilities; and iii) risk aversion for gains and iv) risk seeking for losses of high probability.
The EUT and PT decision-making frameworks are similar in structure, since they both involve a weighted sum of the outcome values weighted by their likelihood of occurrence, but very different in content and interpretation. The main difference is that under PT, gains and losses are evaluated differently, both with regard to their value and respective probability of occurrence—subjective probabilities in PT do not necessarily obey the basic rules of probability. Different risk attitudes arise due to the asymmetric weighing of probabilities for gains and losses. Thus, in environments where individuals constantly update their distributions of travel time for a given route (learning), under the PT model of decision making, individual risk attitudes and consequently uncertainty play a more pronounced role in determining individual route choice, leading routes with greater travel times to be chosen due to less associated uncertainty or variance.

A mechanism by which individuals perceive the likelihood of outcomes is proposed in this study. Assuming that the perception of outcome likelihoods is correlated with risk attitudes, the mechanism presented addresses the role of risk attitudes in route choice decisions with perceived travel times. Under this mechanism, individuals are assumed to under-weigh or over-weigh the probability of gains and losses, independent of whether the probability is high or low. Individuals who under-weigh probabilities of gains and over-weigh probabilities of losses, independent of the magnitude of these gains and losses are viewed as risk-averse. Risk seeking individuals would exhibit the converse, under-weighing probabilities of losses and over-weighing probabilities of gains, independent of their magnitudes. In this study, a probability weighing function (Eq. 3.25) that weighs objective probabilities in this
matter is used, along with the following value function (Eq. 3.26). The risk mechanism investigated in this study is specified as follows:

\[ S(k) = \Omega^{\text{gain}}(\Pr(\Delta T_k \geq 0)) \cdot v\left(\operatorname{E}(\Delta T_k^+\right)) + \Omega^{\text{loss}}(\Pr(\Delta T_k < 0)) \cdot v\left(\operatorname{E}(\Delta T_k^-)\right) \]  

(3.24)

\[ \Omega(p) = \begin{cases} 
\left(1 - \frac{\pi}{\pi}\right) \cdot p & p \leq \pi, 0 \leq \pi \leq 1 \\
\left(\frac{\pi}{1 - \pi}\right) \cdot p - \left(1 - \frac{2\pi}{1 - \pi}\right) & p > \pi, 0 \leq \pi \leq 1 
\end{cases} \]  

(3.25)

\[ v(T^E) = \begin{cases} 
(T^E)^\alpha & \text{if } T^E = \operatorname{E}(\Delta T^+) \\
-A(T^E)^\alpha & \text{if } T^E = \operatorname{E}(\Delta T^-) 
\end{cases} \]  

(3.26)

where,

- \( S(k) \) is the “score” (analogous to the prospect) of choosing route \( k \);
- \( \Delta T_k \) is the difference between an anticipated travel time for route \( k \) and the reference point, taken as the best travel time (\( T_{k, \text{best}}^d - T_k^a \));
- \( \operatorname{E}(\Delta T_k^+) \) is the expected gain for \( \Delta T_k > 0 \), and consequently \( T_{k, \text{best}}^d > T_k^a \);
- \( \operatorname{E}(\Delta T_k^-) \) is the expected gain for \( \Delta T_k < 0 \), and consequently \( T_{k, \text{best}}^d < T_k^a \);
- \( \alpha \) and \( \lambda \) are parameters that determine the shape of the value function (Eq. 3.26);
- \( \pi \) is a parameter between \([0, 1]\) that determines the position of the inflection point of the probability weighing function (Eq. 3.25).

In this study, the score (similar to the prospect) of switching to a route \( k \) is determined according to Equation 3.24, as the weighted sum of the value of a gain and the value
of a loss. These values are weighted by their respective perceived probability of occurrence, which are the objective probabilities weighted subjectively according to Equation 3.25.

The probability weighing is governed by the parameter \( \pi \) which varies with risk attitude, and magnitudes of losses vs. gains. A risk averse individual would have a low \( \pi \) for losses (low \( \pi^{\text{loss}} \)), resulting in an overweighing of probabilities, and a high \( \pi \) for gains (\( \pi^{\text{gain}} \)), resulting in an under-weighing of probabilities, where \( \pi^{\text{gain}} \) and \( \pi^{\text{loss}} \) sum to one (\( \pi^{\text{gain}} + \pi^{\text{loss}} = 1 \)). Risk seekers would exhibit the converse. A plot of the weighing function (Eq. 3.25) for varying \( \pi^{\text{loss}} \) and consequently \( \pi^{\text{gain}} \) is shown below in Figure 3.2. In this study the value function (Eq. 3.26) is assumed to be concave for gains and convex for losses, determined by the shape parameter \( \alpha \). Note that the marginal impact from gains and losses diminishes with distance from a reference point. Given that the shape parameter \( \lambda \) is positive, the function is steeper for losses compared to gains.
3.7 Reference Travel Time Selection

Previously, learning mechanisms for integrating experiences with memory were presented. Additionally, risk and choice mechanisms for describing route choices based experienced and updated travel times under uncertainty were described. A key parameter in all the mechanisms presented is the reference travel time used. For example, in the route switching mechanism (Section 3.3) the best travel time (candidate route for switching) was compared to the experienced travel time; thus, the reference travel time in this case is the experienced travel time. If the route with the best travel time was better beyond a threshold, then the individual selects that route. Three alternative reference travel times are as follows:

![Weighted objective probability](image)

*Figure 3.2: Weighing functions for a risk averse individual (\(\pi^{\text{loss}} = 0.25; \pi^{\text{gain}} = 0.75\))*
where $\phi$ is a weighing parameter. The expressions above imply different types of route choice behavior. Under Equation 3.27, individuals base their day-to-day route choice ONLY on the updated perceived travel time, which is updated over time. Thus, although individuals may experience extremely long travel times for a route $k$ on a particular day $d$, the individual would not switch routes if this experience has little impact on the updated travel time, perhaps due to many experiences of short travel times. Equation 3.29 is the converse of this choice behavior and states that individuals will act (switch routes) based on experienced travel time for that day unless updating occurs on that day, in which case the updated travel time would be used. This is the switching mechanism implied in Section 3.3.

### 3.8 Concluding Remarks

In this chapter, mechanisms for day-to-day route switching, travel time learning, and risk perception were presented. These mechanisms allow an investigation of the dynamics of route choice decisions from day-to-day. Perceived travel times, either experienced or updated, are assumed to consist of a mean and variance. Learning
mechanisms were presented to examine the updating of these parameters in light of new travel experiences. Additionally, recognizing the cost incurred from each update, triggering mechanisms for updating were also presented to capture the timing of updating decisions. Finally, a mechanism for weighing objective probabilities of travel time gains and losses was presented. This study assumes that risk taking behavior is reflected through these weights. The next chapter presents simulation experiments that show the system implications of the different behavioral mechanisms presented in this chapter.
Chapter 4.0 Experiments of Day-to-Day Route Choice

This chapter describes the system features and related details of the simulation experiments, principal factors investigated, and specific properties and performance descriptors considered in this investigation. The individual level decision mechanisms presented in the previous chapter are embedded inside an agent-based simulation framework to allow examination of their relationship with system behaviors over time, such as traffic flow evolution. The motivation behind conducting these experiments is two-fold. First, these simulation experiments relax the existence of an equilibrium state, commonly assumed in planning practice, allowing the individual behavioral rules to drive the network flows from day-to-day. Although no concrete conclusions about the existence of an equilibrium state in “real” networks can be made, these experiments allow insights into how reasonable this assumption is given behaviorally plausible user behaviors. Second, these experiments also illustrate the importance of latent user attributes, such as perceived travel time uncertainty and risk attitudes, in influencing the behavior of traffic networks. Due to their latent nature, empirical investigations on the effects of these attributes are limited. Thus, these experiments seek to contribute to the body of knowledge about them.

The next section gives a description of the system used in the simulation experiments. The second section presents the experimental factors examined in these experiments. The final section presents the results and discusses their implication on day-to-day route choice behavior in traffic networks.
4.1 System Features

The network used for this study, shown in Figure 4.1, consists of 9 nodes and 12 links. Link cost-flow functions $c_{\text{link}} = c_{\text{link}}(f_{\text{link}})$ were used with a linearly varying cost beyond the value $e_{\text{link}} \cdot \text{cap}_{\text{link}}$, according to the following expressions for link $l$:

\[
\begin{align*}
    c_l = \begin{cases} 
        t_l^{\min} \cdot \left(1 + \frac{b_l f_l}{\text{cap}_l - f_l}\right) & f_l \leq e_l \cdot \text{cap}_l \\
        t_l^{\min} \cdot \left(1 + \frac{b_l e_l}{1 - e_l} + b_l \left(\frac{f_l/\text{cap}_l - e_l}{(1 - e_l)^2}\right)\right) & f_l > e_l \cdot \text{cap}_l 
    \end{cases}
\end{align*}
\]

(4.1)

where

- $t_l^{\min} \geq 0$ is the zero-flow travel time;
- $b_l \geq 0$ defines the slope of the curve;
- $\text{cap}_l \geq 0$ is the link capacity;
- $0 \leq e_l \leq 1$ defines the under saturation limit.

Links located near the center of the network have smaller capacities compared with links on the border, and thus their cost-flow functions are more sensitive to varying flows. Links along the border have larger free flow times compared with the links in the center. Nodes 1, 4, 5, 8, and 9 are origins and destinations and all possible OD pairs are connected. Parameter values, OD pairs and base demand values are given in Tables 4.1 and 4.2.
Figure 4.1: Network used in Simulation Experiment

Table 4.1: Link Characteristics and Parameters

<table>
<thead>
<tr>
<th>link</th>
<th>$t^{\text{min}}$</th>
<th>capacity</th>
<th>b</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>360</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>360</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>360</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>360</td>
<td>0.10</td>
<td>0.95</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>180</td>
<td>0.15</td>
<td>0.95</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>180</td>
<td>0.15</td>
<td>0.95</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>150</td>
<td>0.15</td>
<td>0.95</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>150</td>
<td>0.15</td>
<td>0.95</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>240</td>
<td>0.12</td>
<td>0.95</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>240</td>
<td>0.12</td>
<td>0.95</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
<td>240</td>
<td>0.12</td>
<td>0.95</td>
</tr>
<tr>
<td>12</td>
<td>15</td>
<td>240</td>
<td>0.12</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Table 4.2: OD Demand

<table>
<thead>
<tr>
<th>O-D</th>
<th>Routes</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-8</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>1-9</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>9-8</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>1-5</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>5-8</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>1-4</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4-8</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

In order to initiate the dynamics of the system, travel times for the initial iteration are specified according to the initial loading pattern, using the cost-flow functions. Consequently, the initial mean updated travel time is set to the initial travel time, and the variance set to $\beta \cdot \tau_0^\alpha$. $\beta$ is interpreted as the initial variance of the perceived travel time over a segment of unit travel time and is the same for all users. Thus, a large $\beta$ indicates that the initial overall level of uncertainty is high in the system, which is realistic for systems with many "new" users. $\tau_0^\alpha$ is the initial travel time in memory.

Note also that users' perceived travel times are generated by drawing (using Monte-Carlo simulation) from their respective normal distributions described in the last section. Users are loaded randomly across ODs and subsequently paths. Different probabilistic loading patterns could also be used. Other specifics that have been varied across simulations are discussed next.

### 4.2 Experimental Factors

The modeling elements investigated in the first set of simulation experiments for investigating travel time perception updating and updating triggering mechanisms can
be grouped into four categories: a) tolerance level for route switches; b) updating mechanisms; c) total usage level; and d) initial confidence.

The experimental factors investigated in the simulations relating to learning and risk attitudes can be grouped broadly into two categories: a) factors relating to learning and information integration mechanisms and b) factors relating to risk and route switching mechanisms. Furthermore, two scenarios were considered for experiments concerning risk attitudes. Under the first scenario every user in the population made route choice decisions using Equations 3.24 to 3.26, thus weighing the objective probabilities of outcomes subjectively. Under the second scenario, users only considered travel time differences between the best and current route within a tolerance threshold (similar to earlier experiments with trigger mechanisms) and switched routes independently of the perceived probability of success (travel time reduction). With respect to risk and route switching, two factors examined were: a) individual perception risk attitudes reflected through the degree of under or overweighing; and b) the relative percentage of risk seekers and avoiders. Three types of learning mechanisms used in this simulation are: a) Bayesian; b) reinforcement; and c) belief. A summary of experimental factors considered in these experiments are shown in Table 4.3.

### 4.2.1 Travel Time Perception Experimental Factors

**Tolerance Levels.** As mentioned in the last section, users switch routes only when the difference between the travel time on the best and current route exceed a tolerance level reflected in the parameter $\Delta_{nd}$ (Eq. 3.3 to 3.5). Simulations were run for $\Delta_{nd} =$
0.30 and 0.50. As $\Delta_{nd}$ value, users have greater tolerance for differences between the best and current travel times, and thus the system should in general converge with greater ease, all other factors being equal. All users are assumed to have the same $\Delta_{n}$ value. Although this may seem restrictive, recall that individuals use $(\Delta_{nd} \times \tau_{nk}^{u} )$ to determine their switching decisions, and since all individuals experience different travel times, the actual tolerance level varies across a population.

**Total Usage.** The total number of users for each OD pair is fixed for a given simulation. Past simulation studies have shown that networks under higher congestion or usage levels tend to experience greater difficulty in reaching convergence of flows in the network. The base usage level was set at $V = 180$, however simulation runs were made for $2V$, $3V$, and $4V$.

**Updating Mechanisms.** Simulations are performed for all three mechanisms described in the previous section. For the first mechanism, which is based on the number of days between updates ($M_n$), simulation runs were made for $M_n = 0$, 1, 3, 5, 7, 10, and 14. For the second mechanism, sensitivity to the threshold value was investigated. Simulation runs were made for $\Delta_{nd}^c = 0.07$ and 0.90. A low value indicates a situation where nearly every travel time is considered salient and thus updating takes place nearly every day, similar to a $M_n = 1$. However, for experiments that vary $M_n$, the travel times for all the paths were updated, while for experiments that vary $\Delta_{nd}^c$ only the chosen path was updated. Thus, a low $\Delta_{nd}^c$ corresponds to a more selective situation than $M_n = 1$. For the final mechanism, which is based on the confidence of the perceived updated travel time, $\lambda_n$ was set to $\lambda_n = 0.05$ and 0.90. The actual
confidence threshold used is \((\lambda_n \cdot \tau_{nk})^{-1}\). Thus, as \(\lambda_n\) increases, the required confidence level decreases, and updating stops sooner compared to a low value of \(\lambda_n\). Recall that in Bayesian updating confidence values always increase with every new sample.

**Initial Confidence.** The initial confidence reflects the overall uncertainty in travel time perception for the system. The initial confidence was set through the parameter \(\beta\), discussed at the end of Section 3.1, which is interpreted as the initial variance of the perceived travel time over a segment of unit travel time. Thus a high value for \(\beta\) indicates a high initial variance and lower confidence, and consequently users' perceived travel times will vary greatly initially. Under a Bayesian updating scheme, the variance will decrease with each update. Although less uncertainty in the perceived travel times is positive from the perspective of the individual user, this might not necessarily lead to faster convergence from the perspective of the system. For a low initial confidence, the system may converge more slowly or not at all compared to a high initial confidence level. The experiment results shed additional light on these complex interactions.

### 4.2.2 Risk and Learning Experimental Factors

According to the route switching mechanism presented in Eqs. 3.24 to 3.26, the route with the maximum “score” on day \(n\) is chosen. As previously discussed, the score is the weighted sum of expected gains and losses, weighted by their subjective probabilities (Eq. 3.25). Values are evaluated based on travel time differences from a reference point and determined according to Eq. 3.26. Additionally, the reference point for the decision to switch routes (Eq. 3.4) was set to either i) the updated travel
time \( (\tau_{nk}^u) \), ii) the experienced travel time \( (T_{nk}^{ed}) \), or iii) a weighted average of the two, evaluated according to Equations 3.27 to 3.29.

**Risk Attitudes.** The key parameters governing this risk mechanism are a) the concavity and convexity of the value function for gains and losses determined by \( \alpha_n \) in Eq. 3.25 and b) the degree to which individuals over and under weigh objective probabilities associated with gains and losses reflected through the parameters \( \pi_n^{\text{loss}} \) and \( \pi_n^{\text{gain}} \) that determine the inflection points in the probability weighing function for gains and losses, where \( 0 \leq \alpha_n, \pi_n^{\text{loss}}, \pi_n^{\text{gain}} \leq 1 \). Due to the symmetric nature of the weighing function assumed in this study (Eq. 3.24), only the \( \pi_n^{\text{loss}} \) for risk adverse individuals needs to be specified to determine \( \pi_n^{\text{gain}} \) for risk adverse and risk seeking individuals, as previously discussed. Thus, \( \pi_n^{\text{loss}} \) and \( \pi_n^{\text{gain}} \) determine the degree of risk attitude (level of aversion and seeking) for all individuals. According to prospect theory, risk adverse individuals over weigh probabilities of losses \( (\pi_n^{\text{loss}} < 0.5) \) and under weigh probabilities of gains \( (\pi_n^{\text{gain}} > 0.5) \), with the reverse for risk seekers. These parameters were normally distributed around the means \( \pi^{\text{loss}} \) and \( \alpha \). The following values for these two parameter were used: \( \pi^{\text{loss}} = \{0.10, 0.2, \ldots, 0.5\} \); \( \alpha = 0.3 \). Additionally the percentage of risk seekers in the population \( (\gamma_{\text{risk}} \cdot \text{total number of users}) \) were varied by setting \( \gamma_{\text{risk}} = \{0, 0.1, 0.2, \ldots, 1\} \).

**Learning Related Factors.** Simulations are performed on all three learning mechanisms previously described. The main parameter governing the reinforcement
and belief learning mechanisms is the weight $\phi \in [0,1]$ placed on historical experiences, as shown in Equations 3.14 to 3.20. The parameter $\phi$ reflects an individual’s memory strength or the degree he retains past experiences. As $\phi$ increases, the greater and individual’s memory, and the more weight placed on historical experiences. In this study $\phi$ was allowed to vary normally across the population for experiments related to risk attitudes, with a mean 0.5 and a variance of $\beta \phi$ where $\beta$ is the variance associated with a unit of $\phi$.

Population Factors. In addition to the factors described previously, two population related factors were also considered:

1. Population Level. Five different population levels were considered in this study for each OD (a set number of users was assigned to each OD). The base case was 100 users corresponding to a population factor of 1 ($V = 1$). Other population levels considered were: $V = \{1, 1.5, 2, \text{ and } 3\}$. Previous studies have shown that convergence is harder to obtain at higher levels of population.

2. Initial Uncertainty. Additionally, different levels of initial uncertainty were also considered. Uncertainty is measured by the initial beta used to determine the initial variance of travel time. Three different values were considered: $\beta = \{1, 2, \text{ and } 3\}$.
All simulation and corresponding parameter values are summarized in Table 4.3.

**Table 4.3: Experimental Factors Considered**

<table>
<thead>
<tr>
<th>Factors Relating to Experiments Considering Risk Attitudes</th>
<th>Factors Common to All Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Percentage of Risk Seekers and Avoiders</td>
<td>i) Demand Level (V)</td>
</tr>
<tr>
<td>ii) Degree of Risk Attitude ($\pi^{\text{gain}}$ and $\pi^{\text{loss}}$)</td>
<td>ii) Initial Uncertainty (variance - $\beta$)</td>
</tr>
<tr>
<td></td>
<td>iii) Perceived Travel Times</td>
</tr>
<tr>
<td></td>
<td>iv) Learning Mechanisms</td>
</tr>
</tbody>
</table>

### 4.3 Performance Measures and Properties

Three principle types of descriptors are considered:

1. *Day-to-day flow pattern of traffic, in particular convergence.* Convergence is reached when users have stopped switching routes for the remainder of the simulation. For cases where a strict convergence is unattainable, a plot of the day-to-day flow is shown to facilitate a qualitative analysis.

2. *Number of days until convergence.* The number of days till convergence is the number of days from the start of the simulation till convergence is reached. For cases where a strict convergence is unattainable, number of days till convergence is the number of days till the flows on all paths change within an acceptable tolerance level till the end of the simulation run.
3. *Day-to-day deviation of travel times from the user equilibrium travel times.*

Deviation from the user equilibrium travel times of determined and monitored from day-to-day as through the simulation run.

### 4.4 Simulation Results

First, simulation results from the experiments investigating travel time perception and updating are presented and discussed. Next, results from simulations relating to risk and learning are presented and discussed.

#### 4.4.1 Travel Time Perception and Updating

The results of different simulation runs, each corresponding to a different combination of assumptions regarding the factors discussed in the previous section, are presented and discussed in this section. For each case, the system was simulated for a period of 80 days. First the effect of varying inter-update periods on convergence is examined, as well as the effects of varying the initial confidence, usage level, and route switching threshold $\Delta_{nd}$. Second, the effect of selective updating on convergence are examined, in particular selectivity in the integrated travel times. Finally, the effect of terminating the updating process at different confidence levels is examined.

For each simulation, the state of each link after 80 days was recorded as one of three outcomes: convergence to a steady state $C$, regular oscillatory pattern, or no convergence $NC$. These outcomes are summarized in Table 4.4 for varying inter-update periods. The number of days after which steady state or oscillations were
obtained is indicated (number in parentheses), along with the number of updates (number in brackets). The values for other parameters are also presented.

*Varying Inter-Update Periods.* The results in Table 4.4 reveal several important trends. First, although lower perception uncertainty is desirable from an individual user perspective, it may delay or preclude system convergence. For initial $\beta = 0.5$ (low uncertainty, high confidence) and 5.0 (high uncertainty, low confidence) the system experienced greater difficulty converging compared to initial $\beta = 1.0$. Behaviorally, this implies that systems where the overall travel time perception error is low (mostly regular commuters) or high (mostly new commuters) have greater difficulty converging. A plausible explanation is that at very low initial perception uncertainty ($\beta = 0.5$) users may not be experiencing a wide range of travel times (because perception error is low in general). Under a Bayesian updating model for a stationary process, perception error becomes smaller with each update. Thus, as users continue to update, the experienced travel times are only marginally different across users. If all users perceive the same travel times, they will switch to the shortest path every day and the system will not converge, similar to an all-or-nothing assignment. At very high initial perception uncertainty ($\beta = 5.0$), although updating decreases perception error from day-to-day, the decrease may not be fast enough for travel times to become consistent, and thus flows do not converge. The mean updated travel time may have stopped varying from day-to-day, but the associated uncertainty is still relatively high such that users still perceive very different travel times and continue switching.
<table>
<thead>
<tr>
<th>Exp#</th>
<th>Usage</th>
<th>β</th>
<th>Days Between Updates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3V</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3V</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3V</td>
<td>0.5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3V</td>
<td>0.5</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>3V</td>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>3V</td>
<td>0.5</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>3V</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>3V</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>15</td>
<td>3V</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>3V</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>18</td>
<td>3V</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>19</td>
<td>3V</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>3V</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>3V</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>22</td>
<td>3V</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>23</td>
<td>3V</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>26</td>
<td>4V</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>27</td>
<td>4V</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>
Second, all other factors being the same, as the number of days between updates increases, the number of days till convergence decreases initially then increases, and the number of updates required for convergence decreases (Figs. 4.2 and 4.3; Table 4.4).

Figure 4.2: Days until Convergence vs. Inter-update Period Usage = 3V, $\beta = 1$, $\Delta_n = 0.30$

Figure 4.3: Updates until Convergence vs. Inter-update Period; Usage = 3V, $\beta = 1$, $\Delta^i = 0.30$
These trends are a consequence of Bayesian updating as expressed through Equations 3.7 to 3.9. As the period between updates increases, users obtain a larger sample of experienced travel times. This has two effects: (i) the mean value will stabilize more rapidly since the variability of the means across samples decreases with increasing sample size $N$; and (ii) the variance will decrease more rapidly for each update with a larger $N$. Thus, as the number of days between updates increases, the number of updates till convergence decreases (Fig. 4.3). However, the corresponding number of days till convergence does not decrease monotonically (Table 4.4, Experiments 7-14). Initially if the samples are small (short inter-update period) the number of updates required for convergence is high, since the information content associated with each update is limited to a single new observation. For large samples (long inter-update periods), the number of updates until convergence is small enough that the total number of days is actually less (Fig 4.3). The analysis suggests that an “optimum” level of new information content might contribute to faster system convergence following major system changes. The “optimum” level of information would be the amount of information that yields the minimum point on the curve in Figure 4.2, assuming that providing information affects the route switching propensity of individuals. Although Figure 4.3 only shows trends for two links, the same trend was found for all links, as Table 4.4 indicates.

Third, lower usage levels generally exhibit greater propensity towards convergence compared to higher usage levels, and systems are more likely to converge at high tolerable differences between the best and current travel times in route choice decisions compared to low tolerable differences (Table 4.4). Both these trends confirm prior
findings obtained under very different assumptions and learning mechanisms (Mahmassani and Chang 1986; Cascetta and Cantarella 1991). As expected, if users are willing to accept larger differences between the best and current travel times (not switch) then the chance of convergence would be greater. Convergence is less likely at high usage levels (4V) principally because the travel times are more sensitive to flow fluctuations the more congested the system is, as captured in the link flow-cost functions (and would be predicted by virtually all standard queuing or traffic flow models).

Selective Updating of Experienced Travel Times. Under the second updating rule (Eq. 3.22), users are only updating when experienced with travel times that differ from their expected values by a certain relative threshold \( \Delta_{\text{nd}} \), which is the percent difference between the mean updated travel time \( \tau_{nk}^{u} \) and \( T_{nk}^{\text{cd}} \) the perceived experienced travel time. Thus, a high \( \Delta_{\text{nd}}^{u} \) would reflect a very selective individual. Figures 4.4 through 4.11 show the evolution of path flows and travel times for two OD pairs over time for two different \( \Delta_{\text{nd}}^{u} \) values: 0.07 (unselective user) and 0.90 (selective).
Figure 4.4: Flow vs. Time for OD Pair 2; Usage = 3V, \( \beta = 1 \), \( \Delta_{nd} = 0.5 \), and for \( \Delta_{nd}' = 0.07 \)

Figure 4.5: Flow vs. Time for OD Pair 2; Usage = 3V, \( \beta = 1 \), \( \Delta_{nd} = 0.5 \), and for \( \Delta_{nd}' = 0.90 \)
Figure 4.6: Travel Time vs. Time for OD Pair 2; Usage = 3V, $\beta = 1$, $\Delta_{nd} = 0.5$, and $\Delta_{nd}' = 0.07$

Figure 4.7: Travel Time vs. Time for OD Pair 2; Usage = 3V, $\beta = 1$, $\Delta_{nd} = 0.5$, and $\Delta_{nd}' = 0.90$
Figure 4.8: Flow vs. Time for OD Pair 3; for Usage = 3V, $\beta = 1$, $\Delta nd = 0.5$, and $\Delta_{nd}^u = 0.07$

Figure 4.9: Flow vs. Time for OD Pair 3; for Usage = 3V, $\beta = 1$, $\Delta nd = 0.5$, and $\Delta_{nd}^u = 0.90$
Comparison of the figures reveals that a system with selective individuals is more likely to stabilize than a system with unselective individuals, all other parameters equal. Thus, a
system with individuals who update almost every travel time experienced is less likely to converge than a system with selective individuals. Another aspect of the update process that can help understand this phenomenon is that the perceived experienced travel times are drawn from a Gaussian distribution, and thus, while selective individuals update less frequently (because their travel times are located at the tails) than unselective individuals, their updates will generally be more efficient in terms of moving the updated mean in the right direction. The figures above depict the convergence for two OD pairs with two paths each. Since four of the seven OD pairs in the network have only one path, convergence for these pairs is not meaningful since there is no choice. For the one OD pair with four paths, convergence was obtained but with many more iterations than pairs with only two paths. Also from the plots of travel times over iterations, one can see that the final traffic flow state is not close to a user-equilibrium.

Terminating Based on Confidence. The last rule examined is a termination mechanism for updating. According to this rule, updating occurs every day until the confidence reaches a certain level for all paths, determined by $\lambda_n$, the variance of the perceived travel time over a segment of unit travel time (Eq. 3.23). Recall also that the actual confidence threshold used is $(\lambda_n \cdot \tau_{\text{diff}})^{-1}$. Thus, as $\lambda_n$ increases, the required confidence level decreases, and updating stops sooner compared to a low value of $\lambda_n$. Furthermore, in Bayesian updating, confidence values always increase with every new sample. Figures 4.12 through 4.19 show the evolution of path flows and travel times for OD pairs 2 and 3 for two different $\lambda_n$ values: 0.05 and 0.90. All other parameters being equal, the system is more likely to converge at a low $\lambda_n$ than at a high $\lambda_n$, which exhibits the “flip-flop” effect. As expected, a system with users that stop learning early is less likely to converge than a system with users that keep learning and
integrating new information. Premature termination of the learning process results in generally greater dispersion of the perceived travel times, independent of the user’s experience in the system, yet with no corresponding reduction in the user’s propensity to switch paths as actual travel time variability subsides. This behavior does not seem particularly plausible or consistent with actual observation, and argues in favor of learning models that recognize that perceptions evolve in conjunction with the user’s experience and behavior.

![Flow vs. Time for OD Pair 2 for Usage = 3V, β = 5, Δ_{nd} = 0.30, and λ_{nd} = 0.05](image)

**Figure 4.12:** Flow vs. Time for OD Pair 2 for Usage = 3V, β = 5, Δ_{nd} = 0.30, and λ_{nd} = 0.05
Figure 4.13: Flow vs. Time for OD Pair 2 for Usage = 3V, $\beta = 5$, $\Delta_{nd} = 0.30$, and $\lambda_{nd} = 0.90$

Figure 4.14: Travel Time vs. Time for OD Pair 2 for Usage = 3V, $\beta = 5$, $\Delta_{nd} = 0.30$, and $\lambda_{nd} = 0.05$
Figure 4.15: Travel Time vs. Time for OD Pair 2 for Usage = 3V, $\beta = 5$, $\Delta_{nd} = 0.30$, and $\lambda_{nd} = 0.90$

Figure 4.16: Flow vs. Time for OD Pair 3 for Usage = 3V, $\beta = 5$, $\Delta_{nd} = 0.30$, and $\lambda_{nd} = 0.05$
Figure 4.17: Flow vs. Time for OD Pair 3 for Usage=3V, \( \beta = 5 \), \( \Delta_{nd} = 0.30 \), and \( \lambda_{nd} = 0.90 \)

Figure 4.18: Travel Time vs. Time for OD Pair 3 for Usage = 3V, \( \beta = 5 \), \( \Delta_{nd} = 0.30 \), and \( \lambda_{nd} = 0.05 \)
**4.4.2 Risk and Learning**

The results from the simulation experiments relating to risk and learning are presented and discussed in this section, with respect to four factors. First the effects of varying demand levels and initial perception of uncertainty under Bayesian, reinforcement, and belief learning mechanisms are presented. The next set of experiments considers the effects of varying mean $\pi^{loss}$ values and different percentages of risk seekers and avoiders within the population. The third section considers the effects of varying initial travel time perception of uncertainty (variance) on the convergence of the system. Finally the effects of different reference travel times on convergence are examined.
Varying Demand Levels. In traffic systems, demand levels fluctuate over time, due to latent demand for travel and time-varying activity patterns. Past studies have shown that as demand levels increase, there is less propensity towards convergence (Mahmassani 1984; Chen and Mahmassani 2004). In the first set of experiments conducted in this study, demand levels were varied across different learning mechanisms. Demand levels are varied by increasing the base demand level (180 users according to Table 4.2) through a demand factor (V). Thus, \( V = 1 \) corresponds to the base demand level, while \( V = 2 \) corresponds to an increase in demand by a factor of two. The results from these experiments are shown in Table 4.5.

<table>
<thead>
<tr>
<th>Demand Level (V)</th>
<th>Bayesian</th>
<th>Reinforcement</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V = 1.00 )</td>
<td>11</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>( V = 1.50 )</td>
<td>10</td>
<td>61</td>
<td>7</td>
</tr>
<tr>
<td>( V = 2.00 )</td>
<td>15</td>
<td>NC</td>
<td>7</td>
</tr>
<tr>
<td>( V = 3.00 )</td>
<td>NC</td>
<td>NC</td>
<td>7</td>
</tr>
</tbody>
</table>

In these experiments, convergence was reached when flows change by two or less users from iteration to iteration. Under Bayesian and reinforcement learning, lower usage levels show a greater propensity towards convergence compared to high levels, confirming past results, but under different learning mechanisms. However, under a belief learning mechanism that updates using averages of experienced travel times across all users on a particular route, convergence appears less sensitive to demand levels. High demand levels show less
propensity towards convergence principally because the travel times are more sensitive to flow fluctuations the more congested the system is, as captured in the link flow-cost functions (and would be predicted by virtually all standard queuing or traffic flow models). Under belief learning, since users update using travel times averaged across all user experiences for a particular route, the effects of travel time fluctuation or variation across users may be reduced, leading to similar travel time perceptions across all users on a particular route, all else being equal. Finally, strict convergence under reinforcement learning was more difficult to obtain, relative to other learning mechanisms. One plausible explanation is that reinforcement is a selective updating mechanism that leads to updating only for experienced travel time gains (choices that lead to a reduction in travel times). Thus, under reinforcement learning, updating may occur less frequently and with smaller samples of experience in general compared to other learning mechanisms. One assumption of the learning rules used in this study is that with each update, the confidence increases (variance decreases), leading to perceived travel time distributions that become tighter around the mean with each update. Thus new experiences (travel times) have less an impact on users’ travel time perceptions. Under reinforcement learning since updating only occurs for travel time gains, the perceived travel time uncertainty (variance) may not decrease at the same rate as other mechanisms, thus leading to slower convergence compared to Bayesian and belief learning.

*Varying Initial Uncertainty.* Experiments were also conducted to examine the effects of the initial uncertainty, determined by the value of $\beta$, under each of the three learning rules. These results are shown for two demand levels ($V=1$ and $V=2$).
Table 4.6: Number of Iterations until Convergence for Different Initial Perceived Error ($\beta$); $\beta = \text{Variance Associated with a Unit of Travel Time for Two Demand Levels.}$

<table>
<thead>
<tr>
<th>Demand: $V=1$</th>
<th>Bayesian</th>
<th>Reinforcement</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta=1</td>
<td>11</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>Beta=2</td>
<td>13</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>Beta=3</td>
<td>15</td>
<td>38</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand: $V=2$</th>
<th>Bayesian</th>
<th>Reinforcement</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta=1</td>
<td>15</td>
<td>NC</td>
<td>7</td>
</tr>
<tr>
<td>Beta=2</td>
<td>16</td>
<td>NC</td>
<td>7</td>
</tr>
<tr>
<td>Beta=3</td>
<td>16</td>
<td>NC</td>
<td>7</td>
</tr>
</tbody>
</table>

The results above indicate that under Bayesian and Reinforcement learning, as the initial uncertainty increases, convergence in traffic flows is more difficult to obtain. One possible explanation for this is that given that users have a higher perceived uncertainty or judgment error, more new experiences are required to decrease this perception error. In general, reinforcement takes more time until convergence relative to Bayesian since, since the travel time experiences sampled under reinforcement learning only consists of travel time “gains” (reduction in travel time). Under belief learning, since users update using travel times averaged across all user experiences for a particular route, the effects of travel time fluctuation or variation across users may be reduced, leading to similar travel time perceptions across all users on a particular route, all else being equal. Finally, similar to the results in Table 4.5, higher demand levels lead to more difficulty with respect to convergence.

Risk Attitudes. Under a decision process that takes into account users’ perceptions of uncertainty, risk attitudes play important roles in the evaluation of travel time likelihoods for
route choices. The parameter $\pi^{\text{loss}}$ indicates the position of the inflection point in Equation 3.26, indicating the degree to which users’ subjectively overweigh or under-weigh probabilities. The set of experiments that examined the effects of risk attitude levels and the proportion of risk seekers and avoiders in the population, show that risk attitudes do affect the convergence of traffic systems. The results for Bayesian and belief learning mechanisms under a decision process that takes into account risk attitudes are presented in Figures 4.20 and 4.21.

Figure 4.20: Number of iterations until convergence as the mean $\pi^{\text{loss}}$ increases, for different percentages of risk seekers in the population: Belief Learning Experiments
Figure 4.21: Number of iterations until convergence as the mean $\pi^{\text{loss}}$ increases, for different percentages of risk seekers in the population: Belief Learning Experiments

Figure 4.20 shows that under Bayesian learning, as the mean degree of risk attitudes in the population become more extreme ($\pi^{\text{loss}}$ increases, leading to extremely risk averse and risk seeking individuals in the population), the propensity towards convergence is greater, relative to a lower $\pi^{\text{loss}}$. Furthermore, a high percentage of risk seeking individuals (90%) increases the propensity towards convergence under Bayesian learning, compared to a low percentage (10%). In the risk mechanism proposed in this study, risk seekers would under-weigh probabilities of losses and over weigh probabilities of gains. Thus, risk seekers may have a higher propensity towards switching to routes with larger perceived variances, unless the travel time gain between the current and alternative routes is huge. Risk avoiders on the other hand show greater propensity towards staying on routes with lower variances, despite the
possibility of a travel time gain for switching. One consequence of the Bayesian learning rule is that as users gain travel time experiences over time, their perceived variance decreases, thus users’ perceived travel times are insensitive to new experiences. One plausible explanation for the higher propensity towards convergence exhibited by systems with more risk seekers relative risk avoiders is that risk seekers switch at a greater frequency due to their propensity towards routes with huge variances, relative to risk avoiders, thus reducing their perceptions of travel time uncertainty at a greater rate compared to risk avoiders.

The results for belief learning (Fig. 4.21) show that although a system with a low percentage of risk seekers has a greater propensity towards convergence than one with a high percentage of risk seekers, the difference in propensities is less relative to the results from a Bayesian learning rule. Under belief learning perceived travel times are updated using averaged travel times across all users choosing the same route. Thus, the effects of travel time fluctuations or variation across users is reduced, leading to similar travel time perceptions across all users of a particular route, all else being equal, leading to a greater propensity towards convergence, compared to systems where individuals are perceiving different travel times.

*Initial Perceived Variance (β) and Risk.* In addition to examining risk attitudes, the effects of initial perceived travel time variance (β) or uncertainty were examined. The parameter β indicates the initial dispersion the perceived travel times. Thus, a higher β indicates greater initial perceived variance in the travel times (low confidence). The number of iterations until convergence for different percentages of risk seekers in the population and different values of initial perceived travel time variance are presented in Figures 4.14 and 4.15 for different initial values (β) for perceived travel time uncertainty.
The initial perceived dispersion (variance) of travel times seems to have no effect on convergence under Bayesian and belief learning. This departs from previous studies that
show that if the initial perceived variance is too low (low $\beta$), the system has a lower propensity towards converging since additional learning has marginal effects on the perceived variance (Chen and Mahmassani 2004). One plausible explanation for this difference is that risk attitudes are explicitly considered in this study. Some users may be very risk seeking, thus switching routes for any small probability of a travel time gain. Thus, a low perceived travel time variance may not have a pronounced effect since some risk seeking individuals would be switching in any case. Also, note that a low percentage of risk seeking users in the population does not necessarily indicate the absence of extremely risk seeking behaviors (high $\pi^{loss}$), since the values for $\pi^{loss}$ are drawn from a normal distribution. Thus, for any percentage of risk seeking users there would be users with a high degree of risk seeking behavior (high $\pi^{loss}$).

Finally, under Bayesian learning, as the percentage of users who are risk seeking increases, convergence appears easier to obtain. Also, convergence is easier to obtain under belief learning compared to Bayesian learning overall. These results are consistent with those observed in Figures 4.14 and 4.15. The effects of varying initial perceived travel time variance may be reduced due to the presence of risk seeking (and risk avoiding) users in the system that may switch routes or stay despite small probabilities of gains or losses.

*Reference Travel Time.* Finally, the effects of perceived travel times, more specifically the selection of a reference travel time affects convergence were examined. Perceived travel times refer the weight placed on travel times in memory and experienced travel times, when updating does not occur (Eqs. 3.27-3.29). These results show that as users place more weight on updated travel times, the propensity towards convergence increases, compared to users
who place more weight on recently experienced travel times. These results are shown in Figure 4.16.

Figure 4.24: Number of iterations until convergence as the percentage of risk seekers increases, for different types of perceived travel time, under Bayesian learning

The result show that under Bayesian learning, as the number of risk seekers in the population increases, convergence is more difficult to obtain in general, similar to other results obtained in this study. Risk seekers may exhibit greater switching, relative to risk avoiders, and leading to a greater spread from iteration to iteration, resulting in higher propensity towards convergence. Additionally, under Bayesian learning, as the experienced travel time is weighed more in an individual’s perceived travel time, convergence is more difficult to obtain.
Under the assumption that users that weigh their travel times from updated travel times more compared to experienced travel times have a higher propensity towards choosing the same route day-to-day. Also, as users choose (sample) the same route more frequently from day-to-day, their confidence in the perceived travel time for that route increases (variance decreases), and thus future experienced travel times will have less an impact. As users rely or place more weight on their updated travel times, their route choice behavior becomes more consistent from day-to-day, as shown in Figure 4.16.

4.5 Concluding Remarks

This study investigated the effect of different perception and learning mechanisms on the day-to-day behavior of traffic flows. Travel time perception and learning mechanisms were modeled using Bayesian statistical inference concepts, and were embedded in a microscopic (agent-based) simulation framework to investigate their collective effects on the day-to-day behavior of traffic flows. This study extended past work on travel time perception and learning by considering the travel time perception and learning process, the triggering and terminating mechanisms which govern it, and the effect of the above on the day-to-day dynamic behavior of a traffic network, in particular convergence. It represents a first step towards understanding the mechanisms behind the dynamics of route choice behavior, and thus is primarily exploratory in nature.

The results indicate that individuals’ perception of path travel times resulting from endogenous judgment error and the mechanisms for integrating them with past experiences both greatly affect the convergence of the system. Through the experiments conducted in this study, several important effects were observed. First, individuals’ overall travel time
perceptions strongly influence convergence of the traffic system. When the overall travel time perception error is low (mostly regular commuters) or high (mostly new commuters), system convergence is difficult to attain. Second, all other factors being the same, as inter-update period increases, the number of days till convergence decreases initially and then increases, and the number of updates required for convergence decreases. Additionally, the results suggest that an “optimum” level of new information content might contribute to faster system convergence. Third, a system with individuals that update almost every travel time experienced is less likely to converge than a system with selective individuals. Finally, premature termination of the learning process results in generally greater dispersion of the perceived travel times, independent of the user’s experiences, yet with no corresponding reduction in the user’s propensity to switch paths as actual travel time variability subsides. Overall these findings indicate that the perceived confidence associated with experienced travel times is an important factor in route choice decisions and should not be ignored. Additionally, these findings call into question the behavioral assumptions invoked in deterministic and stochastic equilibrium assignment models, in particular fixed and homogenous perception parameters, and have important implications for dynamic network performance models. Finally, note that convergence was a desired criterion in this paper, which assumes a fixed demand level. Under variable demand convergence is still sought. Although the system may not be at a strict user equilibrium (UE) state, there still exists a unique solution at which all users have minimized their “perceived” travel times. It can be shown that the equivalent mathematical program for variable demand is strictly convex and thus has one stationary point, which is a minimum (Sheffi 1985). Additionally, note that the
link-cost functions used in this paper were two-piece and thus discontinuous. This may be problematic since convergence is not guaranteed.

This study also examines the role of risk attitudes and individual perceptions of travel time on the day-to-day behavior of traffic flows. In this study a decision making mechanism in which risk attitudes are reflected through the subjective probability weights for gains and losses is used to examine the role of risk attitudes and travel time uncertainty on day-to-day route choice dynamics. Additionally, three learning types are considered: i) Bayesian; ii) reinforcement; and iii) belief. These learning and risk mechanisms are modeled and embedded inside a microscopic (agent-based) simulation framework to study their collective effects on the day-to-day behavior of traffic flows. Additionally, we also examined the role of risk seekers in driving system-wide properties of traffic networks over time.

The results show that explicitly considering risk attitudes and their effect on an individual’s perception of uncertainty does influence the convergence of traffic flows in a network. Risk attitudes affect route choice decisions by influencing how individuals perceive uncertainty and how uncertainty relates to route travel times experienced in the decision making process. The results show that the presence of risk seekers and avoiders may affect the route switching frequency of users, thus affecting the spread of users across route from day-to-day. More specifically, the results show that the percentage of risk seekers in the population affects the rate of convergence, possibly by affecting the rate of sampling taken by individuals and by adding variability in travel times for individuals who are not risk seeking. Additionally, for Bayesian learning, any mechanism that affects the rate of sampling will affect the rate of convergence. Convergence under Bayesian learning is a function of both the
perceived travel times and the perceived dispersion of these travel times. Risk attitudes affect the weight placed on the likelihood of gains and losses.

Reinforcement learning describes how travel times experienced are integrated, but does not explicitly say anything about how uncertainty changes over time. There is no assumption in reinforcement learning that individuals perceive less dispersion in travel times as more experiences are gained. Thus, unlike a system with Bayesian learners, convergence is in general more difficult to achieve. Additionally, since reinforcement learners only update travel time gains, the rate of sampling from day-to-day may not be high enough to lead to convergence. One assumption of the learning rules used in this study is that the propensity towards convergence increases as users’ perceived confidence in travel times increases (perceived variance decreases). Although belief learning faces the same issue, since it considers experiences of all users, this may serve to lead a system to faster convergence compared to reinforcement learning.

Finally, these results show that there are system-wide properties that are common to all cases, regardless of learning rule or the explicit consideration of risk attitudes. First as demand levels increase, convergence is more difficult to achieve. Second, as individuals rely more on their updated travel times when they are making route choice decisions, less switching among routes occurs and individuals choose a particular route more consistently. Since updated travel times only change with updating learning, they vary less over time with long travel times experienced.
Chapter 5.0 Models of Activity Scheduling Dynamics

Demand management strategies, such as telecommuting, demand peak spreading, congestion pricing, and advanced traveler information systems, have continued to evolve and gain momentum in the policy arena. However, assessing and evaluating the effectiveness of these strategies requires a better understanding and analysis of travel behavior over longer time frames that extend beyond the peak traffic periods.

The need to better understand travel behavior has led to the development of activity-based approaches for analyzing travel, characterized by an improved theoretical basis underlying demand forecasting methods and improved policy sensitivity from developed models (Kitamura 1988; Ettema and Timmermans 1997; McNally 2000). This class of approaches recognizes that travel results from the need to participate in activities over space and time. Under this behavioral paradigm, travel is an integral component of a time-continuous activity pattern or schedule, viewed as a sequence of activities; it results from the interdependent choices of which activities to participate in, where, for how long, and in what sequence (which may include choices of start and finish times), along with travel choices such as mode and route choices.

Although the forecasting capabilities of travel demand models have improved significantly, researchers increasingly realize that an improved understanding of travel behavior and activity patterns requires more than a better account for observed outcomes; it requires better models of the processes and mechanisms operating behind these behaviors (Pas 1985). Many researchers have recognized the need for in-depth research into the scheduling process underlying observed activity schedules, and the importance of the “rescheduling” process in
understanding the dynamics or behavioral changes over time that could result from the demand management measures mentioned previously (Axhausen and Garling 1992; Axhausen 1998; Doherty and Miller 2000). Additionally, a better understanding of individual activity scheduling dynamics (such as rescheduling) may lead to improved insight into the scheduling of unplanned activities, an issue ignored in previous studies. Furthermore, empirical evidence suggests that activity scheduling is highly dynamic, occurring over varying time horizons, with significant amounts of revision and continuous re-planning, even during execution (Doherty and Miller 2000; Miller and Roorda 2003). Recognizing that observed travel patterns are the result of an (unobserved and latent) underlying activity scheduling decision process, a need exists for models that adequately capture this dynamic process.

Past research suggests that experiences from past activity participation decisions may influence current decisions (Hirsh et al. 1986; Hanson and Huff 1988; Hamed and Mannering 1993; Doherty and Miller 2000). However, due to the limited availability of data beyond single day activity-trip diaries, the focus of past research on activity choice is on modeling observed complete schedules at the end of the day (Bowman and Ben-Akiva 1996; Wen and Koppelman 1999). Past research has also shown that there is significant day-to-day variability in travel-activity behavior based on the analysis of variability in multi-day travel-activity behavior (Kitamura 1988; Pas 1988). For example, Hanson and Huff (1988) show that a one-week record of travel does not capture behavior over the long-term, such as a five-week period. Activity-travel data collected over a period of time are still rare, but two recent datasets are Mobidrive (2002) collected over a six-week period, and the CHASE (2000) data set which looks more at adjustments made to schedules over time. Longitudinal observations
of individual activity behavior would allow estimation of dynamics models. Very few examples of dynamic disaggregate choice models exist in the activity-based literature. Hirsh et al. (1986) estimate a parametric model of dynamic scheduling for weekly shopping behaviors. Jou and Mahmassani (1998) looked at trip chaining in commuter trips, but not the actual scheduling of activities beyond the commute trip. A model of the dynamics of activity scheduling would allow researchers to address several questions regarding the state-dependency and heterogeneity of scheduling behavior, including the relationship between planned and executed schedules.

### 5.1 Objectives of Modeling Activity Scheduling Dynamics

The main objective of the present study on activity scheduling dynamics is to provide a theoretical and conceptual framework for modeling the temporal aspects of the activity scheduling process, including activity participation decisions. The main perspective from which the scheduling and participation process is viewed is that of a queuing system, with the individual as a “server” that conducts or engages in activities that arise over time (“arrivals”). The scope of the process considered in the modeling framework includes both static long-term (strategic) and dynamic short-term (tactical) aspects of activity scheduling and participation.

The modeling framework presented seeks to address and account for the effects of the following aspects and factors in the scheduling and participation decisions of individuals over time: (i) unplanned activities arising during schedule execution; (ii) planned activities arising prior to executing a schedule; and (iii) activities that have arrived but not scheduled or
serviced (queued). This modeling framework extends previous work on activity scheduling by (i) further considering the individual scheduling process, (ii) accounting for the interrelationship between the static (long-term) and dynamic (short-term) aspects of activity scheduling, and (iii) considering the role of “latent” activities waiting in queue on activity schedules and activity participation over time.

The next section presents concepts for activity classification and the characteristics of different activity classes used in this study. The following sections present and discuss the conceptual and modeling framework for activity scheduling. The following chapter presents results from simulation experiments as well as an operational model, estimated with actual data, which reflects the main concepts from the modeling framework presented in the current chapter.

5.2 Activity Classification and Characteristics

Different disciplinary perspectives have proposed different approaches to classifying personal activities. Economists typically focus on identifying and differentiating between market and non-market activities. Sociologists, in turn, divide personal activities into individual vs. social, or work vs. leisure. In the transportation area, a classification approach should differentiate between travel and non-travel related activities (Harvey, 2003). Yamamoto and Kitamura (1999) classified activities into two categories: mandatory, if an individual must execute that activity, vs. discretionary, if an individual has a choice to be engaged in it. In general, activities that are fixed in space and time have scheduling priority over those that are free in space/time. Mandatory activities have priority over discretionary
ones even if free in space/time (Kawakami and Isobe, 1990). Furthermore, Huff and Hanson (1990) concluded that individual travel-activity patterns are characterized temporally by both repetition (routine) and variability (non-routine). These concepts are applied in the framework presented in this study.

Activities are first differentiated into mandatory (or compulsory) and discretionary activities. In the short-term, individuals typically have no choice over participating in mandatory activities; their durations and frequencies are fixed. For example, work and medical appointments are mandatory activities since there is no choice in the short-term (such as within a day), over participating in them, only in the long-term (over a month or year). On the other hand, individuals typically have a choice over participating in discretionary activities in the short-term, such as leisure and maintenance activities. For example, a person may choose to shop for groceries or read a book with varying frequency over the course of a day or week (short-term). Maintenance activities are similar to mandatory activities, in the sense that they need to be completed, but with more flexibility in relation to frequency or timing. Resting and eating are both activities that are mandatory, but there is some flexibility over how often and when they can occur. Due to this relative flexibility, maintenance activities are considered to be discretionary activities in this study. A final type of activities that arises over time consists of emergency activities. Similar to mandatory activities, individuals have no choice over participation, but decisions are typically made in the short term soon after such emergencies arise. Their frequency is also lower compared to other activity types, but other attributes, such as their duration, have greater variability.
For the purpose of this study, all activities fall into one of two main categories: mandatory or discretionary, with maintenance and leisure activities falling under discretionary, and emergency under mandatory. From the perspective of the scheduling time frame used in this study, mandatory activities can be viewed as scheduled, and discretionary as unscheduled, reflecting differences in the timing of their participation decisions. This suggests that mandatory activities are scheduled prior to inserting unscheduled discretionary activities into the schedule. Thus, individuals initially begin each day with a “skeletal schedule” with mandatory activities scheduled and fixed, such as work and medical appointments. As the day progresses, discretionary activities are inserted depending on their feasibility both temporally and spatially. Since discretionary activities can potentially be impulse-driven or unplanned, they may exert greater influence on travel-activity patterns compared to other more routine activities, since the former exhibits greater uncertainty in time and space. The general classification of activities used is shown in Figure 5.1.

**Figure 5.1 Activity Classification**

An important characteristic of some maintenance and leisure activities is that they may be interrupted for another activity of significantly higher priority or preference, and resumed again when the higher priority activity is completed. For example, when reading for leisure or cleaning, individuals may stop momentarily to run a quick errand, and return to reading or
cleaning when finished. Despite their relevance for investigating activity scheduling, over time, “overlapping” activities are not pursued beyond the conceptual level in this study.

5.3 The Activity Scheduling Decision Process

This section presents an overview and conceptual description of the activity scheduling decision process modeled and investigated in this study. The components of the modeling framework for this process are given in the later sections of this chapter. Simulation and statistical estimation results relating to this model are presented in the following chapter.

Activity scheduling can be viewed as an activity queuing/service process, where the system consists of a single server (the individual) that services (conducts) activities that arrive according to a generation process. This queuing system can further be characterized as a priority queuing system with vacations and preemptions, reflecting the inherent preferences and scheduling perspectives of the individual.

At the start of a scheduling period, such as a day or week, an activity schedule is still in “skeletal” form, with intervals of time devoted to mandatory activities. From the perspective of a queuing system, these mandatory time intervals can be viewed as vacation intervals devoted to (planned) mandatory activities, during which the individual is unavailable to engage in unplanned non-mandatory activities. As the schedule is executed over time, new activities may arise. Depending on the availability and possibly preferences of the individual, activities may need to “wait in a queue” before being dealt with. An individual may be unavailable to deal with an activity due to current participation in a mandatory activity
(vacation interval) or other type of activity. Preferences influence the importance (priority) with which an individual views an activity. For example, an individual may view emergency activities as very important, thus never placing them in queue. Viewing activity scheduling as a queuing process allows for many intuitive behavioral scheduling decisions to be explained and possibly described quantitatively. For example, participating in high-stress activities (medical emergency) over currently engaged activity can be viewed as preemption. Furthermore, activities changing priority groups, such as from discretionary to mandatory, can be viewed as switching or “jockeying” between priority classes, possibly due to time-varying stress levels. A more specific and detailed conceptual framework of activity scheduling is presented next.

5.3.1 Activity Scheduling: Conceptual Framework

As previously explained, individual activity scheduling is conceptualized as a single server queuing system with the individual as a server that needs to service activities that arise according to some arrival process. Consider the “flow” or movement of activities and the schedule adjustments in response to their (activities) movements that occur over time for a person. At the beginning of a scheduling period, before executing the schedule, a person begins with a planned schedule with intervals of time devoted to mandatory activities. Assuming that the start and end times of mandatory intervals are inflexible, discretionary activities are only pursued outside these intervals. As the person executes the schedule over time, two events may occur: i) new activities may be generated (arrive), requiring either a) adjustments to the schedule (insertions, deletions, shifts) or b) changes in the activity queue; ii) old (existing) discretionary activities may require adjustment, resulting in adjustments to
the current schedule. This framework assumes that these adjustments occur only in the discretionary periods, since mandatory activities are likely fixed in time and space. This process, depicted in Figure 5.2, continues as the person executes the schedule and new information or activities are generated, until the end of the horizon of interest.

![Figure 5.2: Activity Scheduling Process](image)

The framework consists of two main components: (i) an activity generation process and (ii) an activity scheduling/participation process. The activity generation process determines the arrival pattern of activities, characterized by arrival times and frequencies. The order and duration of activity participation is governed by a decision process based on activity attributes and individual preferences. This decision process can vary in complexity. For example, an individual may use a simple rule, such as first-come-first-serve (FCFS) rule, or more elaborate (and realistic) rules based on individual stress and time use preferences.
Alternatively, this process may also be strategic/long-term (overall utility maximization) or tactical/heuristic (LIFO, SIRO, etc) in nature. An activity queue forms when the number of activities generated exceeds the capacity of the individual to accommodate them, with capacity being a function of personal attributes and abilities.

The conceptual framework presented above addresses several aspects of activity scheduling behavior that have not been adequately captured in previous studies, including i) the relationship between activity generation and participation behavior; ii) the role of latent activities; iii) the movement or flow of activities and the adjustment of schedules over time; and iv) the interrelationship between planned and executed schedules. For example, at times it may be necessary to generate and pursue a second activity as a complement or follow up to a prior activity, or specifically generate an activity to mitigate the effects of queued activities. Additionally, generating new activities during execution may also lead to adjustments to the planned schedule. Finally, within this conceptual framework, unplanned activities with high priority, such as medical emergencies or the onset of a new sale, may preempt any existing activity.

5.4 Activity Scheduling: Modeling Framework

Application of the above conceptual framework towards understanding activity scheduling behavior and evaluating transportation planning policies requires mathematical models that can capture these behaviors. Additionally, these models should be feasible and made operational with reasonable data requirements. The remaining sections of this chapter provide a modeling framework for describing these behaviors quantitatively. The following
chapter will address the issues related to making the framework operational within an econometric model estimation framework.

5.4.1 The Concept of Stress

To make the above framework operational, the concept of activity stress is introduced as a driving motivation for the scheduling behaviors described previously. The concepts of stress and time pressure are not new concepts in psychological studies of decision making over time. They are adapted in this study to the activity scheduling context.

Stress plays an important role in the relationship between environmental and psycho-social influences, and health (Dougall and Baum 2001). Due to its latency, multiple dimensions, and varying contexts of use, the construct of “stress” has been difficult to define. The classical definition of stress is the condition under which environmental demands exceed an individual’s adaptive capacity, resulting in physiological and psychological changes (Selye 1980). Since environmental demands can refer broadly to a number of activities and events occurring at different temporal scales, past and current methods for measuring stress exertion have varied, but typically involve qualitative methods coupled with basic statistical analysis, such as regression or factor analysis (Cohen, Kessler, and Gordon, 1995). Additionally, there have also been studies that examine stress from a biological perspective that link physiological data, commonly blood pressure, to stress and health (Matthews et al., 1986).

Past and recent methods for measuring stress and its relationship to changing environmental demands have also relied primarily on qualitative methods, such as checklists and interviews, due to the latent and multidimensional nature of stress. Additionally, statistical methods, such
as factor analysis have also been used to examine the relationship quantitatively. Although these methods have been effective in looking at stress in static contexts as a function of environmental demands, they are ill-suited for looking at dynamic situations where the interest is on how stress changes over time from day-to-day or within-day. Although many studies have examined daily and within-day stress, these studies have only applied methods used for episodic stress analysis (interviews and checklists) to a shorter time frame, inadequately capturing the volatile behavior of stress over time (Eckenrode and Bolger, 1995). Additionally, these methods have underutilized or ignored stress data from a biological perspective.

Furthermore, research in psychology has shown that the perceived consequences from engaging in an activity affect how activity choices are made (Garling et al 1996; Garling et al 1999). Time pressure is known to lead to adverse consequences for the quality of judgment, decision making, and problem solving (Edland and Svenson 1993). Additionally, time pressure has also been found to lead to psychological and physiological stress, with possible long-term health effects (Lundberg 1993, 1996).

**5.4.2 Activity Scheduling: Mechanisms**

Given an understanding of the concept of stress and pressure, in particular the factors that influence and characterize it, models of activity scheduling mechanisms can now be developed. This study focuses on the mechanisms for activity scheduling and participation that characterize scheduling dynamics. Although the mechanisms behind activity generation are also important, as shown in the conceptual framework previously presented, they are
addressed only to a limited degree in this study, to focus more on the dynamics of scheduling.

The first mechanism examined considers the decision to participate in an activity \( r \) at time \( t \), either existing and already in queue (an old activity) or recently generated (a new activity). In making this decision, the individual evaluates several factors, including i) attributes of the activity; ii) attributes of the schedule, including the activity queue; and iii) personal attributes and abilities. An activity participation mechanism based on the concept of activity stress is as follows:

\[
\delta_{nt}^r = \begin{cases} 
1 & \text{AST}_{nt}^r \geq \alpha_{nt} \geq 0 \quad \forall t = 1, \ldots, T, r = 1, \ldots, A, n = 1, \ldots, N \\
0 & \text{otherwise}
\end{cases}
\]  

(5.1)

where \( \delta_{nt}^r \) is an activity participation indicator (1 = participate; 0 = not participate) for person \( n \), activity \( r \), and at time \( t \); \( \text{AST}_{nt}^r \) is the stress associated with activity \( r \) at time \( t \); and \( \alpha_{nt} \) is the stress threshold for activity participation. According to the expression above (Eq. 5.1), if the stress of an activity \( r \) exceeds the stress threshold \( \alpha_{nt} \), person \( n \) will participate in activity \( r \), otherwise, the activity would remain in queue. Following the previous discussion on activity stress and time pressures, the perceived stress of an activity and activity stress threshold are defined as follows:

\[
\text{AST}_{nt}^r = f(X_n, S_{nt}, Z_r^r) + \epsilon_{nt}^r, \quad \forall t = 1, \ldots, T, r = 1, \ldots, A, n = 1, \ldots, N
\]  

(5.2)
\[ \alpha_{nt} = f(X_n, S_{nt}, Z'_{nt}) + \mu_{nt}, \forall t = 1, ..., T, r = 1, ..., A, n = 1, ..., N \]  
(5.3)

where \( X_n \) are person specific attributes; \( S_{nt} \) are schedule-related attributes; \( Z'_{nt} \) are activity specific attributes, and \( \varepsilon^r_{nt} \) and \( \mu_{nt} \) are associated errors that may result from observation or measurement, or unobserved variations in taste. This rule is similar to other boundedly rational rules, based on Simon’s (Simon 1955) notion of satisficing, developed for other travel decisions such as departure time or route switching (Mahmassani and Chang 1986).

The parameter \( \alpha_{nt} \) is the stress threshold and can be viewed as the amount of stress an individual can tolerate; it may also represent an exogenous level of aspiration. Depending on the value of \( \alpha_{nt} \), different scheduling behaviors are exhibited. A high value of \( \alpha_{nt} \) may indicate a person very tolerant of stress. Such a person would only participate in activities with high associated stresses, compared to a person with a low \( \alpha_{nt} \), who has less tolerance to stress and is willing to participate in any activity. Thus, this threshold reflects the inherent preferences and attitudes of the person \( n \), which may be a function of the schedule, activity or person specific attributes.

Similar to the decision to participate in an activity, a model of activity scheduling decisions may also be expressed as:

\[
\theta^r_{nt} = \begin{cases} 
1 & \text{AST}^r_{nt} \geq \omega^r_{nt} \geq 0 \quad \forall t = 1, ..., T, r = 1, ..., A, n = 1, ..., N \\
0 & \text{otherwise}
\end{cases} 
\]
(5.4)

\[ \omega^r_{nt} = f(X_n, S_{nt}, Z'_{nt}) + \nu^r_{nt}, \forall t = 1, ..., T, r = 1, ..., A, n = 1, ..., N \]
(5.5)
where $\theta^r_{nt}$ is an activity scheduling indicator ($1 = $ schedule; $0 = $ do not schedule); $\text{AST}^r_{nt}$ is the stress associated with activity $r$, as described previously; $\omega_{nt}$ is a stress threshold associated for scheduling; and $\upsilon_{nt}$ is the corresponding error. If the stress of an activity $r$ exceeds the threshold $\omega_{nt}$, person $n$ will schedule the activity $r$, otherwise, the activity would remain in queue or be ignored. Similar to $\alpha_{nt}$, the threshold $\omega_{nt}$ would also vary with person specific attributes $X_n$, schedule-related attributes $S_{nt}$, and activity specific attributes $Z^r_{nt}$.

These two mechanisms appear to be behaviorally very similar and nearly identical. First, both mechanisms are based on Simon’s concept of satisficing, such that person $n$ will pursue the action (participate or schedule) if the stress of activity $r$ exceeds the corresponding threshold. Second, in both cases the thresholds are a function of person, activity, and schedule specific attributes. The main difference between these two mechanisms (Eq. 5.1 and 5.4) lies in the behavioral implications and interpretation of their associated thresholds ($\omega_{nt}$ and $\alpha_{nt}$). The first mechanism (Eq. 5.1) addresses activity participation, which typically occurs over a shorter time horizon compared to scheduling decisions. Prior to actually participating in an activity, an individual may still “change his mind”. In contrast, scheduling decisions typically occur over longer time horizons on a more strategic level. Thus, the first mechanism would reflect more short-term perceptions and factors, while the second mechanism would reflect long-term aspirations. This difference also carries over to the interpretation of the thresholds. While the $\omega_{nt}$ threshold may reflect more strategic long-term objectives, such as the frequency of the activity or overall schedule flexibility, $\alpha_{nt}$ would
reflect more myopic factors. By measuring and estimating both of these thresholds jointly, one could examine the interrelationship between long and short-term scheduling decisions and associated dynamics. Analyses similar to those conducted in studies examining the relationship between pre-trip and en-route switching models (Mahmassani and Liu, 1999) could be carried out with the necessary activity scheduling data. The next section describes the different latent measures used in more detail.

5.4.3 Activity Scheduling: Latent Measures and Quantities

In the activity scheduling process, the decision to schedule and participate in activities that arise during schedule execution is driven by the stress of the activities in relation to a stress threshold that reflects the stress tolerance of a person. To provide further behavioral modeling insight into the measure of stress, this section takes a closer look at the composition of activity stress. Define the stress of an activity as a tradeoff between the utility from pursuing, either participating or scheduling, an activity and the stress from not pursuing it, expressed as follows (Note: From this point forward, pursuing an activity refers to either participating or scheduling the activity):

\[
\text{AST}_{rt}^t = U_{rt}^+ + U_{rt}^- , \quad \forall t = 1,\ldots,T, \ r = 1,\ldots,A
\]  (5.6)

where \(\text{AST}_{rt}^t\) is the stress from pursuing an activity \(r\) at time \(t\); \(U_{rt}^+\) is utility from pursuing the activity and \(U_{rt}^-\) is the stress from not pursuing. Depending on the behavior of interest,
AST\textsuperscript{r} may indicate a person’s net (or overall) inclination towards placing or leaving an activity in queue, or completely ignoring it if scheduled. Under this perspective $U_{rt}^{+}$ is interpreted as the potential utility derived from the activity itself, while $U_{rt}^{-}$ is the stress incurred from placing an activity in queue, and thus ignoring the activity. Details with respect to the composition and contributing factors of these two values are described and discussed next.

The utility $U_{rt}^{+}$ derived from a specific activity $r$ for person at time $t$ may be further decomposed into utilities from the time components of the activity, attributes of the person $n$, endogenous attributes of the activity, which includes the effect of activity $r$ on other activities $k \neq r$. Temporally, three interval durations are associated with an activity $r$: i) $D_{rt}$ the duration of the activity; ii) $W_{rt}$ the duration spent waiting to engage in the activity after it has been scheduled, and iii) $Q_{rt}$ the duration of time waiting in queue before the activity was scheduled. Each activity pattern or schedule can be viewed as a sum of these segments for each activity, with the total time $T_{rt}$ associated with activity $r$ at time $t$ given as:

$$T_{rt} = D_{rt} + W_{rt} + Q_{rt}, \forall t = 1, ..., T, r = 1, ..., A$$

(5.7)

where $D_{rt}$ is the time spent participating in activity $r$; $W_{rt}$ is the waiting time for activity $r$; and $Q_{rt}$ is the time activity $r$ spent in queue before being scheduled. Aside from the time components $T_{rt}$ of an activity, the utility of an activity may also be affected by intrinsic activity attributes, time-varying activity attributes, and attributes that relate the activity to
other activities, either in queue or scheduled. The utility for engaging in an activity can be expressed as follows:

\[ U_{rt}^+ = f(T_{rt}, Z_{rt}, X), \forall t = 1, \ldots, T, r = 1, \ldots, A \] (5.8)

\[ Z_{rt} = f(R_{rt}, H_{rt}), \forall t = 1, \ldots, T, r = 1, \ldots, A \] (5.9)

where, \( T_{rt} \) is the total time associated with an activity \( r \) at time \( t \); \( Z_{rt} \) are attributes, static and time-varying, of activity \( r \) and possibly the schedule if scheduling decisions are considered; \( R_{rt} \) are the time-varying attributes of activity at time \( t \); \( H_{rt} \) are the static attributes that relate activity \( r \) with other activities; and \( X \) are the attributes of the person. As stated previously, time components of an activity include the duration of the activity, the duration of the wait prior to the activity (after it is scheduled), and the amount of time spent in queue before the activity was scheduled. Intrinsic attributes of an activity \( r \) include attributes that do not vary with time, such as activity type, as well as time-varying attributes. For example, the level of priority of an activity may change with time. Examples of attributes that relate an activity to other activities include the relative degrees of complementary and of substitutability.

Although \( U_{rt}^+ \) represents the utility of derived from a specific activity \( r \), it can be broken down further into positive benefit and negative transaction cost components:

\[ U_{rt}^+ = U_{rt}^B + U_{rt}^C, \forall t = 1, \ldots, T, r = 1, \ldots, A \] (5.10)
where $U^B_n$ is the positive benefit component and $U^C_n$ is the negative transaction cost component. The focus of the following discussion is on the negative cost component, since the positive benefit results from the actual participation of an activity. The individual incurs transaction costs from fitting an activity into an existing schedule. For example, fitting an activity into an existing schedule may involve shifting already scheduled activities forward or backward in time, or deleting them completely. Even if there is sufficient space in the existing schedule, disutility is still incurred since the flexibility of the schedule might decrease. Thus, disutility results from any temporal change to the activities of an existing schedule, resulting from actions such as shifting, deleting, and placing an activity in queue. Shifting refers to changing the activity start and end times, possibly to accommodate new activities. Deletion refers to removing an activity from an existing schedule without placing it back into the queue. Finally, schedule transaction costs may also include changes in aggregate and latent schedule characteristics such as flexibility and efficiency. Given these components of scheduling, transaction costs, $U^C_n$ can be expressed as follows:

\[
U^C_n = f\left(C^{\text{TR}}_{rt}, C^{\text{AG}}_{rt}\right), \forall t = 1, \ldots, T, r = 1, \ldots, A
\]

(5.11)

where $C^{\text{TR}}_{rt}$ is the component that reflects costs from scheduling adjustments made to an existing schedule; and $C^{\text{AG}}_{rt}$ is the component that reflect the cost from changes in schedule attributes, such as flexibility and efficiency, induced by scheduling activity $r$.

The activity stress incurred $U^-_n$ from keeping activities in queue, or ignoring them completely, can be viewed as a cost the person incurs for not pursuing an activity. Activity
stress can be viewed as a kind of disutility incurred from an activity that is left unscheduled in queue. The person is assumed to be aware of all activities in queue. However, if the person makes no effort to schedule or participate in the activity, or these attempts have been unsuccessful, the activity induces stress (or pressure) on the person, the magnitude of which naturally depends on activity attributes. Also, this stress may vary with time as the deadline for completing the activity approaches. For example, consider a paper assignment for a student. Initially when the paper is assigned no effort may be made by the individual to schedule its completion, and the activity (write paper) may be placed in queue. However, as the deadline for submitting the paper approaches the activity may gain importance and cause stress on the individual. Thus, the stress an activity carries is a function of both intrinsic and temporal activity attributes, as well as attributes of the individual. The stress from an activity can be expressed in a similar fashion to $U^r_n$ as:

$$U^r_t = f(T^r_t, Z^r_t, X), \forall t = 1, ..., T, r = 1, ..., A$$  \hspace{1cm} (5.12)

$$Z^r_t = f(R^r_t, H^r_t), \forall t = 1, ..., T, r = 1, ..., A$$  \hspace{1cm} (5.13)

where, $T^r_t$ is the total time associated with an activity $r$ at time $t$; $Z^r_t$ are attributes, both static and time-varying, of activity $r$ and possibly the schedule if scheduling decisions are considered; $R^r_t$ are the time-varying attributes of activity at time $t$; $H^r_t$ are the static attributes that relate activity $r$ with other activities; and $X$ are the attributes of the person. Examples of activity attributes that affect the amount of stress production are the number of participants or its inherent priority. If an activity requires a large number of participants it is likely to cause
more stress in queue, since keeping it unscheduled also imposes stress on other participants. Likewise, an activity with a high priority imposes stress on the individual if it is left unscheduled in queue. Examples of temporal activity attributes that affect stress are the amount of time the activity has been in queue and the time until the activity is unavailable. Intuitively, the longer a high priority activity is left in queue, the more stress it imposes. Similarly, as the time until an activity is unavailable decreases, the stress the activity imposes on the individual is likely to increase. For example, as a paper submission deadline approaches (the available time for writing the paper decreases) it imposes more stress as a function of the available time.

5.5 Concluding Remarks

This chapter presented conceptual and modeling frameworks for investigating the dynamics of activity scheduling decisions over time. The perspective from which activity scheduling is viewed is that of a single-server queuing system in which the individual is a server that schedules and conducts/engages in arriving activities. Two decisions were examined, namely activity scheduling and participation, where the former represents a more long-term planning type decision, while the latter occurs over the short-term. This modeling perspective allows for many realistic scheduling behaviors not addressed in previous models, such as the interaction between long-term and short term activity scheduling decisions, the effects of latent activities generated but never participated or scheduled, and the effects of unplanned, possible emergency type activities.

To make this framework operational, the concept of activity stress and pressure was developed and characterized. Stress and time pressure have been investigated in the
psychology and health literature, and have been shown to influence the activity choices of individuals. Due to its latent nature, several challenges exist in measuring activity stress in either the dynamic or static sense. Consequently, several existing approaches to measuring stress over varying timeframes have relied on qualitative methods. Under the modeling framework presented in this chapter, stress plays an important driving motivation for activity scheduling and participation decisions. Thus, the degree to which the modeling framework presented can be made operational depends on the measurement of stress and time pressure, and their observed effects on activity schedules over time. These concepts are operationalized in the next chapter using actual activity/travel survey data.
Chapter 6.0 Activity Scheduling Dynamics: Simulation Experiments and Threshold Estimation

The purpose of this chapter is two-fold: i) first, simulation experiments are conducted to explore the range of behavioral insights that can be obtained from the modeling framework previously presented; and ii) second, the activity participation threshold presented in the previous chapter is estimated econometrically using empirical data from a travel-activity diary, illustrating the degree to which this modeling framework can be made operational. In particular, with respect to the first goal, simulation experiments are carried out for a single-server queuing system to explore the relationship between different scheduling rules and “service” (performance) measures, such as the length of the activity queue and the waiting time. These experiments also permit insight into the relationship between formal queuing theory and individual activity scheduling. Additionally, in order to assess the degree to which the modeling framework can be made operational, a simple specification of the activity participation stress threshold presented in the previous chapter is estimated using a dataset consisting of one day observations of individual activity schedules.

6.1 Simulation Modeling of Activity Scheduling

The development of and rationale for the theoretical and conceptual model itself, placed against the backdrop of previous contributions to activity scheduling and the growing body of contributions to activity-based travel demand modeling and forecasting approaches, was presented in the preceding chapter. In this chapter, the activity stress measures previously described are put into an operational format to explore the range of behavioral insights permitted and illustrate its amenability towards being operational.
The next section briefly revisits some of the relevant conceptual aspects of the queuing model of activity scheduling presented in the previous chapter. The next section also discusses details related to the simulation model used to conduct the numerical experiments, and is followed by discussion of the simulation results.

### 6.1.2 Basic Logic of the Simulation Model

A discrete-event simulation for an M/G/1 queuing systems was developed for evaluating the individual activity scheduling process under different activity service and selection rules. The simulation model consists of two basic events, an activity arrival and a completed activity departure, that alter the state of the system.

For the simulation developed in this study, a *next-event time advance* approach is used to advance the simulation clock. According to the *next-event time advance* approach the simulation clock is initialized to zero and the times of occurrence for future events are determined. The simulation clock is advanced to the time of the most imminent (first) of these future events. In light of the most imminent event occurring, the state of the system is updated, and the times of occurrence for future events are also updated. The process of advancing the clock from one event to the next is continued until a stopping condition is satisfied, in the context of this study a set number of completed activities. Note that successive jumps in the simulation clock are variable in size (duration).

In this study, activities are assumed to belong to a class $k$ that is assumed to experience inter-arrival times $A_{k1}$, $A_{k2}$, … that are independent and identically distributed (IID) exponential random variables. This study does not implement the distinction between strategic and
tactical decisions. Thus, all activities picked for service are assumed scheduled and engaged in the order of their service. After arriving, activities are serviced according to the scheduling rule defined in the next two subsections. Service times (activity durations) for these activities are generated for each activity class $k$, $S_{k1}$, $S_{k2}$, ..., independently of the inter-arrival times. An arriving activity that finds the individual busy participating in another activity joins the activity queue and waits its turn to be serviced. Upon completing an activity, the individual then selects the next activity to service from the queue (if any) according to the scheduling rules defined in later subsections. This process continues iteratively until a stopping condition is reached.

### 6.1.3 Stress Index for Simulation Experiments

In this study the amount of stress experienced from activities in queue are considered as a measure of an activity’s potential for leaving the queue, leading to person $n$ deciding to schedule/participate the activity. Stress is generally defined as follows:

\[
AST_{nt}^r = \beta_c + \beta_{nv}(R_{nt}) + \beta_{inv}(H_{nt}) + \beta_{v}(\epsilon_r), \forall t = 1, ..., T, r = 1, ..., A, n = 1, ..., N \tag{6.1}
\]

\[
1 = \beta_c + \beta_{nv} + \beta_{inv} + \beta_v \tag{6.2}
\]

$AST_{nt}^r$ is the stress from pursuing an activity $r$ at time $t$; $R_{nt}$ are the time-varying attributes of activity at time $t$; $H_{nt}$ are the time-invariant (static) attributes that relate activity $r$ with other activities; $\epsilon_r$ is a random term distributed Normal $\sim N(0, 1)$ that reflects the unobserved stress contributing attributes of activity $r$; $\beta_c$ is a constant ($0 \leq \beta_c \leq 1$) that reflects the inherent
stress of an activity; and $\beta_v$, $\beta_{inv}$, and $\beta_v$ are weights placed on the time-varying, time-invariant, and random term respectively, each weight between one and zero ($0 \leq \beta_v, \beta_{inv}, \beta_v \leq 1$).

In this study, the primary time varying activity attribute considered is the duration of time an activity spends in queue at time $T_{rt}^q$. To capture the perception of $T_{rt}^q$ with respect to other activities in queue the following two transforms of $T_{rt}^q$ are used:

\[
T_{n,1}^{q*} = \frac{T_{rt}^q}{\max_{r=1,...,A_Q}(T_{rt}^q)}
\]  
(6.3)

\[
T_{n,2}^{q*} = \left[\frac{T_{n,1}^{q*}}{\alpha}\right]^{\alpha_1-1} \times \left[\frac{T_{n,1}^{q*}}{\alpha}\right]^{\alpha_2-1}, \alpha = (\alpha_1 - 1)/(\alpha_1 + \alpha_2 - 2), \alpha_1 > 0, \alpha_2 > 0
\]  
(6.4)

where $\alpha_1$ and $\alpha_2$ are shape parameters for Equation 6.4, set to $\alpha_1=5$ and $\alpha_2=1.5$ in this study; $A_Q$ is the total number of activities in queue. According to the first expression, Equation 6.3, the contribution of the time an activity spends in queue to its stress increases with the length of its time in queue, relative to the activity with the longest wait time in queue. Thus, the activity with the longest wait time in queue, $\max(T_{rt}^q)$, contributes the most amount of stress. The first expression states that in general, the stress contribution from an activity in queue will increase monotonically with its time in queue. Unlike Equation 6.3, the second expression permits the stress contribution of an activity to decrease with a long enough wait.
time in queue; thus stress does not increase monotonically with time waiting in queue. In the context of activity scheduling behavior, Equation 6.4 implies that the longer an activity remains in queue, the greater the amount of stress the individual will experience from it, but after reaching a maximum stress level, stress would decrease either steeply or gradually over time. This stress behavior is plausible since the “salience” of an activity in queue may diminish over time, past a critical amount of time. For example, if an activity has an inflexible deadline, as time approaches the deadline and the activity is still not completed (still in queue), the stress will increase. However, after a deadline has passed, the stress from not completing the activity (leaving it in queue) may decrease, as the appeal of the activity or urgency for completing it decreases.

Aside from time-varying attributes, this study also considered time invariant activity attributes that contribute to activity stress. The main time invariant attribute considered in this study is the \( T_{\text{exp}} \). To scale \( T_{\text{exp}} \) to fall between one and zero, while preserving the order of magnitude, the following expression was used:

\[
T_{\text{exp}}^* = 1 - \left( 1 / \gamma \right) \cdot \exp \left( - T_{\text{exp}}^* / \gamma \right)
\]  

(6.5)

where \( \gamma \) is a scale parameter. According to Equation 6.5, as the expected activity duration \( T_{\text{exp}}^* \) increases, its stress contribution \( T_{\text{exp}}^* \) increases sharply initially and then gradually. Additionally, the stress contribution approaches a maximum value as \( T_{\text{exp}}^* \) approaches infinity. Behaviorally, this functional form suggests that at low expected activity durations, individuals are extremely sensitive to small changes in duration time and consequently stress
changes sharply for each additionally unit of time. The low sensitivity of the stress contribution \( T^*_n \) at high expected duration values is consistent with past studies on framing effects on individuals’ perceptions of values and costs (Kahneman and Tversky 1979). For very high expected durations, an additional unit time is insignificant relative to the duration, and thus the corresponding change in stress would also be small.

### 6.1.4 Scheduling/Participation Rule

The previous section discussed different measures of stress and attributes of the activities in queue that contribute to the stress an individual perceives from the activity. In this study, one scheduling rule was used based on the maximum stress. Under this mechanism, activities in queue are selected based on the activity that provides the maximum reduction of stress. Consequently, this results in the activity that exerts the largest amount of stress to be chosen. This mechanism can be expressed as follows:

\[
\eta_t = \begin{cases} 
1 & \text{if } \text{AST}_t^r \geq \text{AST}_w^r, \forall w \neq r \\
0 & \text{otherwise,} 
\end{cases}
\]  

(6.6)

where \( \eta_t \) is a binary variable that indicates scheduling/participation and takes a value of 1 if activity \( r \) is selected and removed from queue at time \( t \), and 0 otherwise. Behaviorally, the mechanism above states that an activity will be selected for scheduling/participation at time \( t \) if the stress relief it brings is the greatest relative to all activities in queue. One assumption made in the rule above is that at any time \( t \), all activities in queue are considered for participation; however, this may be an unrealistic assumption. If all activities in queue have very low stress levels (as indicated by their stress indices), then an individual will unlikely...
schedule/participate in an activity if the cost associated with removing the activity outweighs the amount of stress relief it brings. Furthermore, individuals likely consider multiple objectives when forming a schedule. For example, an individual might participate in activities to maximize the amount of stress relief, while minimizing the loss in time flexibility of the schedule. Such a rule would need to be multi-objective in its formulation, and is not within the immediate scope of the present investigation.

6.1.5 Experimental Factors

The experimental factors investigated in this simulation study can be grouped into two main categories: a) attributes of activity classes; and b) parameters and functional forms of stress and indices.

Attributes of Activity Classes: Class attributes considered in this study are the mean inter-arrival time, the mean service time, and inherent priority.

Parameters of the Stress Index and Functional Forms of Stress Measures: Different functional forms of stress (Eqs. 6.1 to 6.5) were considered, and their parameters or weights varied, as discussed in section 6.2.

6.1.6 Scheduling Process Performance Measures

The following performances measures were investigated using simulation experiments:

1. Expected Average Delay. The expected average delay in queue for a total of \( A_k \) activities of class \( k \) in the simulation \( d_k(A_k) \) is the average of all delays experienced by activities in the same class. Delay is defined as the time an activity must wait in queue before being
serviced. For a given simulation run resulting in activity service delays \( D_{k1}, D_{k2}, \ldots, D_{k, A_k} \) an estimator of \( d_k(A_k) \) is expressed simply as the statistical mean:

\[
d_k(A_k) = \frac{1}{A_k} \cdot \sum_{r=1}^{A_k} D_{kr}
\]  

(6.7)

which is an average of the \( A_k D_{kr} \)'s that are observed in the simulation. Note that delay can also take on a zero value and are counted in the average, since an individual with many zero delays may have a light activity load or may indicate an individual which can process many tasks quickly. Alternatively, the expected average delay for all activities in queue is also calculated.

2. Expected Number of Activities in Queue. The expected number of activities in queue, denoted by \( q(n) \), is taken over continuous time; however it is approximated as a weighted average, defined as follows:

\[
q(n) = \sum_{i=0}^{\infty} i \cdot p_i
\]  

(6.8)

\[
p_i = T_i / T(n)
\]  

(6.9)

\[
T(n) = T_0 + T_1 + T_2 + \ldots
\]  

(6.10)
where \( q(n) \) is the weighted average (over a total of \( n \) activities observes) over possible values of \( i \) (number of activities in queue); \( p_i \) is the observed portion of time during the simulation that there were \( i \) activities in queue; \( T_i \) is the total simulation time in which there were \( i \) activities in queue.

3. Expected Utilization of an Individual. The expected utilization of an individual indicates the level of “activity congestion” the individual experiences for activities. The expected utilization of an individual is the expected portion of time during the simulation (between time 0 and \( T(n) \)) that the individual is busy (not idle) denoted by \( u(n) \). From a single simulation run, \( u(n) \) can be computed similarly to the \( q(n) \) as a weighted average and expressed as:

\[
u(n) = \frac{T_B}{T(n)} \quad (6.11)\]

Where \( T_B \) is the total amount of time the individual is busy during the simulation, and \( T(n) \) is the total time of the simulation needed to observe a total of \( n \) activities.

6.2 Simulation Experiments of Activity Scheduling

In this section the results of different simulation runs, each corresponding to a different combination of assumptions regarding the factors discussed in the previous section, are presented and discussed. For each case, the system was simulated until 10,000 completed activities were reached. First the effect of varying the weight values (\( \beta \))s in the stress index previously defined in Equation 6.1 was examined. Second, the effect of different stress index
functional forms and parameters was examined. Finally, the effect of preemptive “emergency” activities was examined.

### 6.2.1 Varying Weights

The first set of simulation runs considered only one activity class. Equation 6.3 was used to determine the stress contribution from the waiting time in queue. The weights in Equation 6.1 were varied, such that in each case, only one weight was set to one and all other to zero. The following results were obtained.

| Table 6.1: Mean Inter-arrival Time = 5 mins; Mean Activity Duration = 4 mins |
|--------------------------------------------------|-----------------|-----------------|-----------------|-----------------|
| \( \beta_c = 1 \) (Const)                      | \( \beta_{tv} = 1 \) (Time-Varying) | \( \beta_{inv} = -1 \) (Time Invariant) | \( \beta_v = 1 \) (Random) |
| Average Delay in Queue                        | 14.6            | 14.6            | 7.26            | 14.5            |
| Average Number in Queue                       | 2.96            | 2.96            | 1.47            | 2.94            |
| Average Number in System                      | 3.77            | 3.77            | 2.28            | 3.75            |
| Fraction Spent > 4.5 mins                     | 0.82            | 0.82            | 0.66            | 0.73            |
| Fraction Time Queue > 1                       | 0.51            | 0.51            | 0.39            | 0.51            |
| Utilization                                    | 0.81            | 0.81            | 0.81            | 0.81            |
| Simulation Time (mins)                        | 49295.7         | 49295.7         | 49295.7         | 49295.7         |
| Activities Completed                          | 10000           | 10000           | 10000           | 10000           |

Note that when all activities have the same priority, the simulation model resorts to the FCFS rule to select activities. Thus in the case where \( \beta_c = 1 \) and all other weights equal zero, the simulation would follow a FCFS rule (since every activity has a stress of one). The results in Table 6.1 reveal that an individual who chooses activities based on the shortest activity duration (\( \beta_{inv} = -1 \)) in case of a single activity class, experiences shorter delays compared to an individual who chooses activities based on arrival priority (\( \beta_c = 1 \)) or time spent in queue in queue (\( \beta_{tv} = 1 \)). Thus, by always picking the shorter duration activity first, the individual can more quickly turn attention towards activities in queue, reducing the delays queued
activities experience. Additionally, both cases ($\beta_c = 1; \beta_v = 1$) yield results equivalent to steady-state results for an M/M/1 queue with a mean inter-arrival time of five minutes ($1/\lambda = 5$) and a mean service time of four minutes ($\mu = 4$). As mentioned earlier, in the case where $\beta_c = 1$ and all other weights equal zero, the simulation would follow a FCFS rule, since every activity has a stress of one. In the case where $\beta_v = 1$, by selecting the activity with the longest time in queue, the individual is implementing a rule similar to FCFS. In general, if there is already a queue existing, the activity that has the longest wait time in queue is the activity that arrives the earliest. Service according to a random term ($\beta_v = 1$) distributed Normal $\sim N(0,1)$ yields results very similar the first two cases ($\beta_{inv} = -1; \beta_v = 1$), but not strictly equivalent. These results are consistent with the result from queuing theory that states the mean wait time is independent of the service discipline, so long as the latter is not based on the service time, such as the SEPT (shortest expected processing time) rule (Larson and Odoni 1981).

Although a direct comparison of the actual values in the Table 6.1 and 6.3 is not possible, since different mean arrival rates and mean service rates were used for the two cases, the general trends in each table can still be examined. The parameters for each activity class are shown in Table 6.2. Table 6.3 shows that in the context of more than one activity class ($k = 5$) the results are similar to the case with a single class (Table 6.1). If an individual chooses to service activities based on the shortest expected duration first ($\beta_{inv} = -1$), the average delay experienced by the activities is less than if service occurred according to the other three cases ($\beta_c = 1; \beta_v = 1; \beta_v = 1$). Similarly, for the same case, the individual performs better with
respect to the other performance measures. Since no definite trends could be determined from varying the weights over different components, these results are not shown in the table.

Although the results are similar to those obtained from queuing theory, the behavioral implications in the context of activity scheduling are worth noting. First, only under the ideal condition that an individual completes activities in an order based on expected duration is the average delay time minimized. Realistically, individuals do not base their activity participation decisions only on the duration, but on other activity attributes such as time in queue, location, priority, and many others. Thus, a more realistic stress function would be one with varying weights across the different components (for example $\beta_c = 0.1$; $\beta_{tv} = 0.4$; $\beta_{tinv} = 0.3$; $\beta_v = 0.2$). Although this suggests that individuals select activities for participation in a suboptimal manner, there may be short periods where an individual tries to finish activities with a short duration first before pursuing ones with a longer duration.

### Table 6.2: Simulation Parameters of Different Activity Classes

<table>
<thead>
<tr>
<th>Activity Class</th>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Interarrival Time (mins)</td>
<td>μ_interarrival</td>
<td>40</td>
<td>25</td>
<td>30</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>Mean Duration Time (mins)</td>
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<td>1</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Activity Class</td>
<td>$\beta_c = 1$</td>
<td>$\beta_{iv} = 0$</td>
<td>$\beta_{inv} = 0$</td>
<td>$\beta_v = 0$</td>
<td>All Classes</td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>---------------</td>
<td>----------------</td>
<td>----------------</td>
<td>--------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average Delay in Queue</td>
<td>Average Number in Queue</td>
<td>Average Number in System</td>
<td>Fraction Spent &gt; 4.5 mins</td>
<td>Fraction Time Queue &gt; 1</td>
<td></td>
</tr>
<tr>
<td>$\beta_{c} = 0$</td>
<td>Average Delay in Queue</td>
<td>Average Number in Queue</td>
<td>Average Number in System</td>
<td>Fraction Spent &gt; 4.5 mins</td>
<td>Fraction Time Queue &gt; 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average Number in System</td>
<td>0.26</td>
<td>0.45</td>
<td>0.36</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Average Number in System</td>
<td>0.26</td>
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<td>0.36</td>
<td>0.06</td>
<td>0.03</td>
</tr>
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<td>0.49</td>
<td>0.55</td>
<td>0.07</td>
<td>0.08</td>
</tr>
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<td>0.43</td>
<td>0.55</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>Utilization</td>
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<td>0.07</td>
<td>0.04</td>
<td>0.02</td>
<td>0.14</td>
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<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
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<tr>
<td></td>
<td>Activities Completed</td>
<td>1783</td>
<td>2786</td>
<td>2221</td>
<td>2263</td>
<td>947</td>
</tr>
<tr>
<td>$\beta_c = 0$</td>
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<td>4.74</td>
<td>5.08</td>
</tr>
<tr>
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<td>0.21</td>
<td>0.16</td>
<td>0.16</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>Average Number in System</td>
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<td></td>
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<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Utilization</td>
<td>0.03</td>
<td>0.08</td>
<td>0.13</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>$\beta_v = 0$</td>
<td>Simulation Time (mins)</td>
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<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
</tr>
<tr>
<td></td>
<td>Activities Completed</td>
<td>1783</td>
<td>2786</td>
<td>2221</td>
<td>2263</td>
<td>947</td>
</tr>
<tr>
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<td>Average Delay in Queue</td>
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<td>3.58</td>
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<tr>
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<td>Average Number in Queue</td>
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</tr>
<tr>
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<td>Average Number in System</td>
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<td>0.41</td>
<td>0.41</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>Fraction Spent &gt; 4.5 mins</td>
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<td>0.36</td>
<td>0.52</td>
<td>0.54</td>
<td>0.77</td>
</tr>
<tr>
<td>$\beta_{inv} = -1$</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Utilization</td>
<td>0.03</td>
<td>0.08</td>
<td>0.13</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td>$\beta_v = 0$</td>
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<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
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<tr>
<td></td>
<td>Activities Completed</td>
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<td>2786</td>
<td>2221</td>
<td>2263</td>
<td>947</td>
</tr>
<tr>
<td>$\beta_c = 0$</td>
<td>Average Delay in Queue</td>
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<td>5.29</td>
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</tr>
<tr>
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<td>Average Number in Queue</td>
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<td>0.21</td>
<td>0.17</td>
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</tr>
<tr>
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<td>Average Number in System</td>
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<td>0.49</td>
<td>0.46</td>
<td>0.46</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
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<td>0.39</td>
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<td>0.56</td>
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<td>0.03</td>
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<tr>
<td></td>
<td>Utilization</td>
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<td>1783</td>
<td>2786</td>
<td>2221</td>
<td>2263</td>
<td>947</td>
</tr>
</tbody>
</table>
6.2.2 Alternative Functional Form for Time-Varying Attributes

In the set of experiments shown in Table 6.5, Equation 6.4 was used to compute the stress contribution from an activity in queue as function of its time in queue. Comparing the results in Table 6.4 with results in Table 6.5 shows that altering the functional form of the stress contribution from an activity’s queue time, as suggest by Equations 6.3 and 6.4, has no effect on the performance measures. One explanation for this result is that both Equations 6.3 and 6.4 increase monotonically as the time in queue increases, over a wide range of time values, for the time spent in queue. Since selection is based on choosing the activity with the larger relative stress compared to other activities, the actual scale or magnitude of an activity’s stress does not matter, so long as the relative order of stress is preserved. Although Equation 6.4 does not allow stress to increase monotonically over all values of time in queue, for parameters ($\alpha_1=5; \alpha_2=1.5$), the decrease in stress occurs for very large values of time in queue.
### Table 6.4: Experiments using Equation 6.3 for Capturing Stress Contribution from Time in Queue

<table>
<thead>
<tr>
<th>Activity Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>All Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_c = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Delay in Queue</td>
<td>5.14</td>
<td>5.08</td>
<td>5.07</td>
<td>4.74</td>
<td>5.08</td>
<td>5.01</td>
</tr>
<tr>
<td>Average Number in Queue</td>
<td>0.13</td>
<td>0.21</td>
<td>0.16</td>
<td>0.16</td>
<td>0.07</td>
<td>0.73</td>
</tr>
<tr>
<td>$\beta_{tv} = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Number in System</td>
<td>0.33</td>
<td>0.49</td>
<td>0.46</td>
<td>0.45</td>
<td>0.32</td>
<td>1.25</td>
</tr>
<tr>
<td>Fraction Spent &gt; 4.5 mins</td>
<td>0.34</td>
<td>0.43</td>
<td>0.57</td>
<td>0.58</td>
<td>0.79</td>
<td>0.52</td>
</tr>
<tr>
<td>$\beta_{tinv} = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction Time Queue &gt; 1</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
<td>0.18</td>
</tr>
<tr>
<td>Utilization</td>
<td>0.03</td>
<td>0.08</td>
<td>0.13</td>
<td>0.14</td>
<td>0.15</td>
<td>0.51</td>
</tr>
<tr>
<td>$\beta_v = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation Time (mins)</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
</tr>
<tr>
<td>Activities Completed</td>
<td>1783</td>
<td>2786</td>
<td>2221</td>
<td>2263</td>
<td>947</td>
<td>10000</td>
</tr>
</tbody>
</table>

### Table 6.5: Experiments using Eq.6.4 for Capturing Stress Contribution from Time in Queue ($\alpha_1=5;\alpha_2=1.5$)

<table>
<thead>
<tr>
<th>Activity Class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>All Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_c = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Delay in Queue</td>
<td>2.97</td>
<td>3.22</td>
<td>3.68</td>
<td>3.58</td>
<td>5.21</td>
<td>3.55</td>
</tr>
<tr>
<td>Average Number in Queue</td>
<td>0.08</td>
<td>0.13</td>
<td>0.12</td>
<td>0.12</td>
<td>0.07</td>
<td>0.52</td>
</tr>
<tr>
<td>$\beta_{tv} = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Number in System</td>
<td>0.27</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.32</td>
<td>1.03</td>
</tr>
<tr>
<td>Fraction Spent &gt; 4.5 mins</td>
<td>0.25</td>
<td>0.36</td>
<td>0.52</td>
<td>0.54</td>
<td>0.77</td>
<td>0.46</td>
</tr>
<tr>
<td>$\beta_{tinv} = -1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction Time Queue &gt; 1</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.13</td>
</tr>
<tr>
<td>Utilization</td>
<td>0.03</td>
<td>0.08</td>
<td>0.13</td>
<td>0.14</td>
<td>0.15</td>
<td>0.51</td>
</tr>
<tr>
<td>$\beta_v = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation Time (mins)</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
</tr>
<tr>
<td>Activities Completed</td>
<td>1783</td>
<td>2786</td>
<td>2221</td>
<td>2263</td>
<td>947</td>
<td>10000</td>
</tr>
</tbody>
</table>
6.2.3 Preemptive Activities

The final set of simulation experiments looked at the effect of emergency activities with preemption privileges. The class with preemption privileges is class $k=5$. Under preemption, an activity with a high priority can immediately enter service, thus preempting all other activities. The only exception occurs when the individual is already servicing an emergency activity, in which case the latter activity has to wait. An example of such occurrence is shown in Table 6.6. Note that the Average Delay in Queue, even for the emergency activity class, is never zero, since an emergency activity may need to wait for a previous emergency activity to finish before entering service. Overall, the results in Table 6.6 show that if an activity class has preemptive privileges (i.e. emergency activities) the performance measure for the preemptive class will improve, but other classes have reduced levels (they get worse). Thus, the greater the degree of preemption of an individual’s activity class, the more adverse is the effect for other activity classes without preemption privileges.
Table 6.6: Experiments with Preemptive Activities (in the red box) ($\alpha_1=5; \alpha_2=1.5$)

<table>
<thead>
<tr>
<th>$\beta_c = 0$</th>
<th>Average Delay in Queue</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>All Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average Number in Queue</td>
<td>0.13</td>
<td>0.21</td>
<td>0.16</td>
<td>0.16</td>
<td>0.07</td>
<td>0.73</td>
</tr>
<tr>
<td>$\beta_{iv} = 1$</td>
<td>Average Number in System</td>
<td>0.33</td>
<td>0.49</td>
<td>0.46</td>
<td>0.45</td>
<td>0.32</td>
<td>1.25</td>
</tr>
<tr>
<td>$\beta_{inv} = 0$</td>
<td>Fraction Spent &gt; 4.5 mins</td>
<td>0.34</td>
<td>0.43</td>
<td>0.57</td>
<td>0.58</td>
<td>0.79</td>
<td>0.52</td>
</tr>
<tr>
<td>$\beta_v = 0$</td>
<td>Average Number in System</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>Simulation Time (mins)</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
<td>68388.89</td>
</tr>
<tr>
<td></td>
<td>Activities Completed</td>
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<td>2276</td>
<td>2221</td>
<td>2263</td>
<td>947</td>
<td>10000</td>
</tr>
<tr>
<td>$\beta_c = 0$</td>
<td>Average Delay in Queue</td>
<td>13.72</td>
<td>13.57</td>
<td>12.64</td>
<td>13.24</td>
<td>3.93</td>
<td>12.29</td>
</tr>
<tr>
<td>$\beta_{iv} = 1$</td>
<td>Average Number in Queue</td>
<td>0.35</td>
<td>0.54</td>
<td>0.41</td>
<td>0.41</td>
<td>0.05</td>
<td>1.76</td>
</tr>
<tr>
<td>$\beta_{inv} = 0$</td>
<td>Average Number in System</td>
<td>0.59</td>
<td>0.86</td>
<td>0.74</td>
<td>0.74</td>
<td>0.55</td>
<td>2.33</td>
</tr>
<tr>
<td></td>
<td>Simulation Time (mins)</td>
<td>70055.51</td>
<td>70055.51</td>
<td>70055.51</td>
<td>70055.51</td>
<td>70055.51</td>
<td>70055.51</td>
</tr>
<tr>
<td></td>
<td>Activities Completed</td>
<td>1754</td>
<td>2276</td>
<td>2202</td>
<td>2254</td>
<td>1064</td>
<td>10000</td>
</tr>
</tbody>
</table>

6.3 Concluding Remarks for Simulation Experiments

Numerical experiments were carried out using a simulation model of an M/G/1 queuing system to explore the range of behavioral insight from an operational form of the modeling framework presented in the preceding chapter. Simulation experiments were conducted to explore the relationship between different scheduling rules and “service” measures, such as the length of the activity queue and the waiting time. These experiments also permit insight into the relationship between formal queuing theory and activity scheduling.

The results show that if selection is based on choosing the activity with the larger relative stress compared to other activities, the actual scale or magnitude of an activity’s stress does
not matter, so long as the relative order of stress is preserved. This result further suggests that under any rule where activities are selected based on the magnitude of stress relative to all other activities in queue, the performance of the individual (how queue length, average waiting time, etc...) will not matter so long as the order of preference is preserved.

6.4 Estimation of a Stress Threshold for Activity Participation

To illustrate the degree to which the modeling framework presented in the previous chapter can be made operational, a stress threshold for activity participation over the time period of a day was statistically estimated using actual data from a travel activity survey. In the next section a more precise definition of the activity participation problem previously presented is given. The following section presents an econometric model formulation of activity participation, based on the concept of "activity stress" discussed in the previous chapter. The remaining sections present and discuss the estimation results, including their behavioral interpretation and implications.

6.4.1 Definition of the Activity Participation Problem

In this section, attention is restricted to the activity participation problem. The model estimation methodology can be readily applied to scheduling decisions. However, due to data limitations, estimating a scheduling stress threshold for scheduling decisions is not possible, since only observation of the final executed schedule were available.

Assume that on a given day $d$, a person $n$ begins with a "skeletal" activity schedule composed of intervals devoted to mandatory activities, and all remaining intervals devoted to discretionary activities. These mandatory activities are assumed to be scheduled and fixed, in terms of start and stop times, following the description of mandatory activities given in other
studies. Although discretionary activities may also be scheduled at the beginning of the day, since their start and stop times are more flexible relative to mandatory activities, they are not included in the skeletal schedule. At the start of each discretionary interval $p$, the person continues to make activity participation decisions $\delta_n^{pt}$ until the next mandatory period is reached, where $t$ is a subscript that denotes the $t^{th}$ decision of interval $p$. At each decision, the person decides whether or not to pursue a queued activity ($\delta_n^{pt}=1$) or not ($\delta_n^{pt}=0$). This process continues until the end of the day is reached, or until all discretionary intervals are completed. A sample schedule evolution for a person with two discretionary time periods ($P=2$), two decisions in the first period ($T_1=2$) and four in the second period ($T_2=4$), is shown in Figure 6.1. Specifically, the schedule states at the beginning and end of the day, in addition to the end of each discretionary interval, are shown.

![Figure 6.1 Activity Participation Process](image)

### 6.5 Econometric Model Formulation of Stress Thresholds

The development of simulated maximum-likelihood estimation procedures for dynamic kernel logit (mixed-logit) and probit models (Train 2001; Srinivasan and Mahmassani 2006)
has relaxed many limitations, such as time dependence and substitution patterns. These procedures are amenable towards estimating an activity participation model based on the concept of a stress-threshold over time. This section develops an econometric model of activity participation over time based on the concept of a stress threshold.

6.5.1 Model Estimation Framework

Notation

Let $n$ be the subscript to denote the person, $n=1,\ldots,N$.

Let $t$ represent the decision stage in the discretionary interval $p$, $t=1,\ldots,T_p$.

Let $p$ be the discretionary interval index, $p=1,\ldots,P$.

Let $\delta_{nt}^{pt}$ represent the queued activity participation decision indicator for person $n$ at stage $t$, and in interval $p$.

Let $U_{nt}^{pt}$ represent the corresponding utility of participating in one more queued activity.

Let $V_{nt}^{pe}$ be the systematic component of queued activity participation utility $U_{nt}^{pt}$.

$\varepsilon_{nt}^{pt}$, $\tau_{nt}^{pe}$ represent the multivariate normal (MVN) and logistic error components of $U_{nt}^{pt}$.

$\varepsilon$ is the vector of $\varepsilon_{nt}^{pt}$ across decisions; $\tau$ is the vector of $\tau_{nt}^{pe}$ across decisions.

Behavioral Framework

At each decision stage $t$, person $n$ has two mutually exclusive actions available (binary choice): i) participate in one more queued activity; ii) not participate in a queued activity.

Decision stages are assumed to occur before each opportunity to participate in a queued activity, such as the end of a previous activity. We also assume that activities cannot overlap. Thus, starting another activity while participating in the first activity is not permitted. A
sequence of binary choices is easily represented as a set of $T_p$ dummy variables for each of the $p$ intervals ($p=1…,P$):

$$
\delta_{nt}^p = \begin{cases} 
1 & \text{participate in queued activity} \\
-1 & \text{otherwise}
\end{cases} \quad (6.12)
$$

Denote this sequence by $C_n = \{ \delta_{n1}^1, \ldots, \delta_{nT_p}^1, \ldots, \delta_{n1}^p, \ldots, \delta_{nT_p}^p \}$. \quad (6.13)

For a given discretionary interval, at each decision stage $t$, the person will participate in one more queued activity if $U_n^p t > \alpha_n^p$. Otherwise, if $U_n^p t \leq \alpha_n^p$, the person $n$ has chosen to participate in a non-queued activity instead of a queued activity. Furthermore, $U_n^p t$ will be a utility function representing the utility assigned to participating in one more queued activity at stage $t$ in period $p$; $\alpha_n^p$ is the **tolerable stress level** or the expected net value of continuing to participate in queued activities; $\alpha_n^p$ may also represent a totally exogenous level of aspiration. Defining the net utility of participating in one more activity as $U_n^p t$, this threshold for participating in queued activities is taken as 0 without loss of generality. Then the decision rule for participating in queued activities may be stated as:

$$
\delta_{nU}^p t \geq 0 \quad \forall t \quad (6.14)
$$

At any given stage $t$ the probability that the person will participate in a queued activity is:

$$
Pr(U_n^p t > \alpha_n^p) = Pr(\delta_{nU}^p U_n^p t > 0) \forall t. \quad (6.15)
$$

Although calibrating a sequence of binary decisions as mutually independent decisions is simpler, the actual presence of auto-correlation, state-dependence and heterogeneity effects
may lead to inconsistent estimates and erroneous inferences. Thus, the modeling framework must allow for specifying and testing these effects. To accomplish this, dynamic models are generally calibrated using a Mixed-Logit or (MNP) Probit estimation framework. However, these frameworks become computationally difficult with increasing number of alternatives and/or durations.

6.5.2 Dynamic Mixed (Kernel) Logit Formulation

To overcome computational difficulties typically associated with the MNP framework, a dynamic kernel logit (DKL) approach is used in this study. As with other random utility models, the utility for participating in a queued activity consists of systematic and random components. Assume that the systematic component is defined as a function of experiences captured through the attributes of the activity queue and schedule, including individual activities, short-term experiences reflected in the activity attributes at the current time period, decision maker attributes, and a set of unknown parameters to be estimated. The random component is composed of a normal error-term which is correlated across repeated decision instances of a given individual, and an independent and identically distributed logistic error-term as shown below:

\[ U_{pt}^n = V_{pt}^n + \varepsilon_{n}^pt + \tau_{n}^pt \]  

(6.16)

Let:

\[ \varepsilon = \left( \varepsilon_{n}^{11}, \varepsilon_{n}^{12}, ..., \varepsilon_{n}^{PT}, ..., \varepsilon_{n}^{PTp} \right) \]

\[ \varepsilon' \sim \text{MVN}(0, \Sigma_{e}) \]

\[ \tau = \left( \tau_{n}^{11}, \tau_{n}^{12}, ..., \tau_{n}^{PTp}, ..., \tau_{n}^{PTp} \right) \]
\[ \tau' \sim \text{i.i.d logistic} \ (0, \Sigma_n) = \left(0, \sigma_i^2 I\right) \]

\[ \sigma^2_i = \pi^2/(3\mu^2), \]

I is a \( PT_p \times PT_p \) unit matrix; and

\( \mu \) is the logit scale parameter (set to 1).

For decisions to participate in one more queued activity or not, for a given period \( p \) with \( T_p \) decision stages for person \( n \), the probability or likelihood of an observed sequence of decisions is:

\[
\begin{align*}
L(C_n) &= \text{Pr}\{ \delta_{n1}^{t1}, \ldots, \delta_{n Tp}^{t1}, \ldots, \delta_{n1}^{pt}, \ldots, \delta_{n Tp}^{pt}, \ldots, \delta_{n1}^{pT}, \ldots, \delta_{n Tp}^{pT} \} \\
&= \text{Pr}\{ \delta_{n}^{pT}, t = 1, \ldots, T_p, p = 1, \ldots, P \} \quad (6.17)
\end{align*}
\]

\[
\begin{align*}
L(C_n) &= \text{Pr}\{ \delta_{n1}^{11} U_{n1}^{11} > 0 \cap \delta_{n1}^{12} U_{n1}^{12} > 0 \cap \ldots \cap \delta_{n Tp}^{pT} U_{n Tp}^{pT} > 0 \} \\
&= \text{Pr}\{ \delta_{n}^{pT} \left( V_{n}^{pT} + \epsilon_{n}^{pT} + \tau_{n}^{pT} \right) > 0, p = 1, \ldots, P, t = 1, \ldots, T_p \} \quad (6.18)
\end{align*}
\]

Rewriting and substituting gives:

\[
\begin{align*}
L(C_n) &= \text{Pr}\{ \delta_{n}^{pt}, p = 1, \ldots, P, t = 1, \ldots, T_p \} \\
&= \text{Pr}\{ \delta_{n}^{pt} \left( V_{n}^{pt} + \epsilon_{n}^{pt} + \tau_{n}^{pt} \right) > 0, p = 1, \ldots, P, t = 1, \ldots, T_p \} \quad (6.19)
\end{align*}
\]

Conditioning on \( \epsilon \) gives:

\[
\begin{align*}
\text{Pr}\{ \delta_{n}^{pt}, p = 1, \ldots, P, t = 1, \ldots, T_p \} &= \int_{\epsilon} \text{Pr}\{ \delta_{n}^{pt}, p = 1, \ldots, P, t = 1, \ldots, T_p \mid \epsilon \} f(\epsilon) d\epsilon \quad (6.20)
\end{align*}
\]
\[ \Pr\left\{ \delta_{nt}^{pt}, p = 1, \ldots, P, t = 1, \ldots, T_p \right\} = \int_{\epsilon} \Pr\left\{ \delta_{nt}^{pt} \left( V_{nt}^{pt} + \epsilon_{nt}^{pt} + \tau_{nt}^{pt} \right) > 0, p = 1, \ldots, P, t = 1, \ldots, T_p \mid \epsilon \right\} f(\epsilon) d\epsilon \quad (6.21) \]

By conditioning on \( \epsilon \), \( \epsilon_{nt}^{pt} \) is known and can be treated as deterministic. For a given \( \epsilon \) the conditional deterministic utility \( W_{nt}^{pt} \) is given by:

\[ W_{nt}^{pt} = V_{nt}^{pt} + \epsilon_{nt}^{pt} \quad (6.22) \]

Simplifying gives:

\[ \mathcal{L}(C_n) = \int_{\epsilon_n} \Pr\left\{ \delta_{nt}^{pt} \left( W_{nt}^{pt} + \tau_{nt}^{pt} \right) > 0, p = 1, \ldots, P, t = 1, \ldots, T_p \mid \epsilon \right\} f(\epsilon) d\epsilon \quad (6.23) \]

The probability expression on the right-hand side is written as:

\[ \Pr\left\{ \delta_{nt}^{pt} \left( W_{nt}^{pt} + \tau_{nt}^{pt} \right) > 0, p = 1, \ldots, P, t = 1, \ldots, T_p \mid \epsilon \right\} = \prod_{p=1}^{P} \prod_{t=1}^{T_p} \left[ \Pr\left\{ \delta_{nt}^{pt} \left( W_{nt}^{pt} + \tau_{nt}^{pt} \right) > 0 \mid \epsilon \right\} \right] \quad (6.24) \]

\[ \Pr\left\{ \delta_{nt}^{pt} \left( W_{nt}^{pt} + \tau_{nt}^{pt} \right) > 0 \mid \epsilon \right\} = \frac{1}{1 + \exp\left( - \mu \cdot \delta_{nt}^{pt} \left( W_{nt}^{pt} \right) \right)} \quad (6.25) \]

The choice probability for a person \( n \) is:
\[
L(C_n) = \int_{\varepsilon_n} \prod_{p=1}^{P} \prod_{t=1}^{T_p} \left[ \frac{1}{1 + \exp(-\mu \cdot \delta_n^p(W_n^{pt}))} \right] f(\varepsilon) d\varepsilon
\]  

(6.26)

Assuming that there are \(N\) independent observations in the sample, the likelihood of observations for this sample can be expressed as:

\[
\prod_{n=1}^{N} L(C_n) = \prod_{n=1}^{N} \left[ \int_{\varepsilon_n} \prod_{p=1}^{P} \prod_{t=1}^{T_p} \left[ \frac{1}{1 + \exp(-\mu \cdot \delta_n^p(W_n^{pt}))} \right] f(\varepsilon) d\varepsilon \right]
\]  

(6.27)

The log-likelihood is expressed as:

\[
\sum_{n=1}^{N} \ln[L(C_n)] = \sum_{n=1}^{N} \ln \left[ \int_{\varepsilon_n} \prod_{p=1}^{P} \prod_{t=1}^{T_p} \left[ \frac{1}{1 + \exp(-\mu \cdot \delta_n^p(W_n^{pt}))} \right] f(\varepsilon) d\varepsilon \right]
\]  

(6.28)

6.5.3 Estimation Procedure

The likelihood in Equation 6.25 involves the computation of a \(PT_p\) dimensional MVN integral, and is computed using Monte Carlo simulation. The desired likelihood (Eq.6.25) is the expected value of the function \(h(\varepsilon)\). Thus, this likelihood is estimated as the average of the function \(h(\varepsilon)\) over several draws from the MVN distribution of \(\varepsilon\). The parameters that maximize the simulated log-likelihood are determined through non-linear optimization techniques as shown in Figure 6.2.

The maximum simulated log-likelihood estimator is consistent. However, the estimator's bias only vanishes with increasing number of Monte-Carlo draws (Revelt and Train, 1998). These
draws are generated using independent pseudo-random sequences. The covariance matrix of coefficient estimators was estimated using the negative expectation of the inverse of the Hessian matrix (matrix of second partial derivatives of the likelihood function with respect to the parameters, evaluated numerically).

Figure 6.2: Mixed-Logit Estimation Procedure

Initialize Estimated Parameters \( \theta(0) \);
Iteration Count = 0

Monte-Carlo Simulated Likelihood Function Computation
a. For each observation, draw \( R \) vectors of MVN error terms
b. For each draw compute the product of logit probabilities (kernel function)
c. Compute the multidimensional integral by averaging the product of logit probabilities over the \( R \) draws
d. Aggregate the likelihood in step c across all observations

Function Maximum?

Stop

Set \( I = I+1 \)

BFGS Quasi-Newton Procedure
1. Direction finding
2. Step-size computation
3. Hessian updating
4. Compute parameters \( \theta(I+1) \) for next iteration
6.5.4 Data Assembly

This section describes the data assembly procedure used in this study. The (BATS) 2000 Bay Area Travel Survey was used for estimating the activity participation stress tolerance threshold previously described. The key information needed to estimating this threshold from observed activity schedules are the i) activities completed over the schedule execution period (past decisions), in this case a day, and ii) state of the queue at the time of participation decisions (anticipated work load). Similar information would be needed to model activity scheduling decisions. However, due to the lack of observed data on these decisions, they could not be included. Other socio-demographics and travel-related data may also be used. The expected arrival rate was estimated based on socio-demographic attributes and used as an instrumental variable in the threshold model.

Data assembly begins with classifying the activities into mandatory and discretionary activities, and then further distinguishing the latter into activities that appear in queue and "impulse" activities. First, four types of activities were considered mandatory in this study: i) sleep; ii) work; iii) work-related; and iv) medical/health appointments. The last three were considered mandatory due to the fixed nature of their start and stop times in the time frame of a day. Although sleep may be regarded as more of a discretionary or maintenance activity, it was considered mandatory due to the repetitive nature of its position in a person's schedule (at the beginning and end of the day). All other activities were regarded as discretionary activities, and were further grouped into queued and non-queued activities. Queued activities included: i) meals; ii) personal service; iii) out-of-home shopping; and iv) household chores/personal care. Note that the general modeling framework presented previously could
still be applied in conjunction with more elaborate classifications, such as one that considers salient attributes of activities.

Participation decisions were taken to occur at the end of each discretionary activity, queued or non-queued, or at the end of a mandatory interval. Thus at each decision, the person makes the binary choice to participate in a queued activity or not. The number of queued activities completed was calculated at each decision to be a running total of the number of completed activities that were considered as "queued."

Finally, due to the large number of observations in the data set (116,773 decisions), observations were further segmented by the number of decisions a person makes in a day. For simulated-maximum likelihood estimation procedures, given such a huge number of decisions would have been infeasible in regards to computation time. Thus, a subset of observations consisting of persons who made five, six, seven, and eight decisions in a day were taken from the original set of observations, and used for estimation. The next section describes specification of the systematic component of the utility function. More specifically, a method for accounting for the generation of new activities, and thus, the changing state of the activity queue with each decision, is presented next.

### 6.5.5 Model Specification Issues

In this study, the net utility for participating in one more queued activity, during schedule execution, is given as:

\[ U_n^{pl} = V_n^{pl} + \varepsilon_n^{pl} + \tau_n^{pl} \]  

(6.29)
The main determinant of the stress an individual experiences is the activity queue as it evolves over time. This stress is assumed to increase and decrease as activities flow in and out of the activity queue, or the composition of activities in the queue changes. To reflect this, the systematic component of the net utility function is specified to reflect the flow of activities, and is expressed as:

\[
V_{n}^{pt} = f(Q_{npt}, X_{n}, \beta) \tag{6.30}
\]

\[
Q_{npt} = Q_{n,p,t-1}^C - Q_{npt}^C + Q_{npt}^G \tag{6.31}
\]

where

- \( Q_{npt} \) is the state of the activity queue at time \( t \).
- \( Q_{npt}^C \) is the total number of queued activities completed at time \( t \).
- \( Q_{npt}^G \) is the total number of queued activities generated at time \( t \).
- \( X_{n} \) are person-specific attributes.
- \( \beta \) is a vector of parameters to estimate.

At the initial time (\( t=0 \)), the state of the queue is assumed to be the total number of activities observed for that day, based on the observed completed activity schedules. Since activities generated during schedule execution are not observed, the following specification for the number of activities generated up to time \( t \) is used:

\[
Q_{npt}^G = \lambda_{n} \times T_{np,t-1}^A \tag{6.32}
\]
where $\lambda_n$ is the mean arrival rate of queued activities per unit time, and $T_{np,t-1}^{A}$ is the duration since the last activity arrival. The mean arrival rate $\lambda_n$ is determined using an instrumental variable approach. Implementing this approach, $\lambda_n$ is determined through a Poisson regression on a series of exogenous variables. The values of $\lambda_n$ predicted by the Poisson regression are then used in estimating the model in Eq. 6.28. The instrumental variable approach has been used successfully in the analysis of discrete/continuous data (Dubin, J., and D. McFadden 1984; Train 1986). Since the parameter $\lambda_n$ is nonnegative, a convenient parameterization is given by:

$$\lambda_n = \exp(X_n' \beta) \quad (6.33)$$

The motivation for the Poisson regression was to obtain a proxy for the number of queued activities generated over a time interval, using the instrumental variable approach. This approach addresses possible endogeneity issues that may arise from not accounting for generated activities over time. To further illustrate this issue, consider the case where only the total number of queued activities observed for a day is used as a proxy for the queue size throughout the day; thus the total number of queued activities over a day is assumed constant over time. This is illustrated in Table 6.7 for the example shown in Figure 6.1.
Table 6.7: Activities Completed and Left in Queue for the example in Figure 6.1.

<table>
<thead>
<tr>
<th>Time</th>
<th>Completed</th>
<th>Left in Queue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Under this proxy (total number of observed queued activities in the day), there is endogeneity between the number completed and the number left in queue. This occurs since more realistically, the total number of queued activities would vary over the day. One solution to this issue is to determine the number of queued activities generated at each decision. Thus to accomplish this, the mean arrival rate (activities per unit time) is estimated using a Poisson regression (Eqs. 6.32 and 6.33), and is multiplied by the time elapsed since the last queued activity was generated. Although this study assumes a constant rate of activity generation over time, a dynamic time varying rate could also be estimated given longitudinal activity data, and possibly individuals’ needs and goals over time. This expected rate was used as an instrument for determining the number of activities generated since the last generated activity.
Response and Preference Heterogeneity

Heterogeneity refers to the variability in the propensity of individuals to select an action, in this case to participate in a queued activity, and responsiveness to independent variables. Heterogeneity in this study is accommodated in two ways. Observed heterogeneity is accounted for by variations in preference sensitivity (intrinsic bias) and response sensitivity to exogenous factors among different user (market) segments. Unobserved preference heterogeneity is incorporated by a person-specific error term across choice instances, reflecting unobserved intrinsic bias towards participating in a queued activity. Unobserved response heterogeneity is represented through the use of random coefficients for a subset of important variables. This assumes that the response of a person to values taken by explanatory variables varies across the population. Accordingly, the parameters of the systematic specification (for a subset of variables) are assumed to be random variables across the population with a mean parameter $\beta_k$ and a standard deviation $\zeta_{\beta_k}$.

6.6 Estimation Results and Discussion

This section presents the results from estimated models of stress thresholds for activity participation described in the preceding section. The primary goal of these results is to i) illustrate the amenability of the activity scheduling framework previously presented, towards being operational; ii) show evidence in support of the concept of “activity stress” in empirical data; and iii) provide further insight into the activity scheduling process. Data for estimation was obtained from the Bay Area Travel Survey (BATS) 2000, which is an activity-travel survey for two days (per person), not necessarily from the same year. Due to the size of the
entire dataset (64,755 day-observations), and the implications for simulation-based econometric models, the models presented in this paper were estimated on a sub-sample of observations consisting of six, seven, and eight decisions per day. These levels were selected since they gave the highest number of observations, relative to other levels (one, two, three, etc… decision per day). To illustrate the similarities in the results across sub-samples, a pooled model was estimated distinguishing between the four levels, to allow direct comparison of the estimated coefficients of the model.

The first set of results show estimated parameters from Poisson regression models based on counts of the number of queued activities observed over the period of a day (1440 minutes), for the entire sample (64,755 day-observations). The second set of results compares the estimated parameters and implications for a pooled dataset consisting of six, seven, and eight decisions per day, under the assumption of independent decisions. The third set of results is from model estimated on the different sub-samples, for a mixed-logit model. In both sets, the estimated parameters as they relate to stress and attributes of the activity queue (activities generated and completed) are discussed.

6.6.1 Activity Arrival Rate Models

This section presents the estimated arrival rates (Eq. 6.33) based on the entire sample of day-observations, using a Poisson regression. Two models were estimated. The first model regresses the count of queued activities per day against a constant only; the second model regresses the count of queued activities per day against socio-economic attributes, such as gender, age, and if the day was a weekend.
Table 6.8: Poisson Regression: Model 1 is with a constant only; Model 2 is with socio-economic variables.

<table>
<thead>
<tr>
<th>Var #</th>
<th>Variable Description</th>
<th>Mean Value</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Coefficient</td>
<td>t-stat</td>
</tr>
<tr>
<td>1</td>
<td>Constant</td>
<td></td>
<td>1.20</td>
<td>556.59</td>
</tr>
<tr>
<td>2</td>
<td>Gender (0/1)</td>
<td>0.48</td>
<td>-0.14</td>
<td>-31.61</td>
</tr>
<tr>
<td>3</td>
<td>Age &gt; 30 (0/1)</td>
<td>0.68</td>
<td>0.24</td>
<td>50.06</td>
</tr>
<tr>
<td>4</td>
<td>Weekend (0/1)</td>
<td>0.13</td>
<td>0.07</td>
<td>11.85</td>
</tr>
</tbody>
</table>

Log-Likelihood: Model 1 = -133809.80, Model 2 = -131915.00
Sample Size: Model 1 = 64755, Model 2 = 64755

Based on the results above, the mean arrival rate is 3.32 activities per day. Activities included in the estimation of the arrival rate include all non-recreational discretionary activities. The results show that activities arrive at a higher rate for individuals over the age of thirty and on weekends. Also, males seem to experience lower arrival rates relative to females. These results suggest that persons over the age of thirty may experience higher stress relative to younger persons, due to a higher arrival rate of activities, given the same activity scheduling abilities. Similarly, according to the estimation results above, activities arrive at a much higher rate during the weekend, compared to the weekday, undoubtedly reflecting the fact that mandatory (work) activities occupy a much larger portion of the user’s service capacity on weekdays.

These results are based on observed counts of queued activities over the period of a day, and thus they only reflect these observations. Conceivably, activities can be generated at a higher rate than actually observed. As a result, the expected arrival rate (3.32 activities per day) may actually be higher. The purpose of the Poisson regression was to obtain a proxy for the number of queued activities generated over a time interval, using the instrumental variable approach. This is important to ensure that no endogeneity exists in the dataset, when
considering the state of the queue and the number of activities completed. The next two sections presents results from using this proxy as an instrument in estimating the pressure or stress towards participating in queued activities.

6.6.2 Evidence of Activity Stress and Pressure

To investigate the presence of stress and pressure in activity scheduling and participation, the stress threshold previously described was estimated only with indicator variables indicating the number of activities completed and the number of activities in queue. Socio-demographic variables are considered in later models. First a model using a pooled sample with all numbers of decisions per day (5-8) was estimated under the assumption of independent observations to examine differences in estimation results arising from differences in the number of decisions an individual makes in a day. Next, a mixed-logit model was estimated to relax some of the assumptions from the previously estimated models regarding homogeneity in response. The results are discussed in relation to their implication on activity stress and pressure over time.

Repeated Binary-Logit Model

This section presents estimation results under the assumption of independent error terms for the model previously presented. The motivation behind estimating a model under the independence assumption is that it allows estimation on a pooled sample, and permits a direct comparison between the segments based on the number of decisions made in a day. These estimation results are shown in Table 6.9. The results indicate that in general, as the number of activities completed increases, the stress or propensity towards participating in more queued activities decreases from a reference point of zero, independent of the number of
decisions observed on a particular day (Fig. 6.3). Furthermore, as the number of queued activities increases, taken from a reference point of zero, the stress decreases slightly then increases with each additional queued activity (Fig. 6.3). From the figures, it can be seen that the general trend of stress as a function of activities completed and activities left in queue is the same regardless of the number of decisions made per day.

Figure 6.3: Stress as a Function of Queued Activities Completed (top) and Left in Queue (bottom)
Table 6.9: Estimation Results for Repeated Binary Logit Decisions, Segmented by Number of Decisions per Day

<table>
<thead>
<tr>
<th>Var #</th>
<th>Variable Description</th>
<th>Coefficient</th>
<th>t-stat</th>
<th>Var #</th>
<th>Variable Description</th>
<th>Coefficient</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Alternative specific constant (for participation)</td>
<td>1.2597</td>
<td>4.1790</td>
<td>165</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Number of Queued Activities Completed</td>
<td>-1.3046</td>
<td>-24.8910</td>
<td>18</td>
<td>Completed 1 queued activity previously (0/1)</td>
<td>0.2090</td>
<td>2.3170</td>
</tr>
<tr>
<td>3</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>-0.0569</td>
<td>-1.4720</td>
<td>19</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>-0.0282</td>
<td>-0.4270</td>
</tr>
<tr>
<td>4</td>
<td>Completed 3 queued activity previously (0/1)</td>
<td>-0.0747</td>
<td>-1.9080</td>
<td>20</td>
<td>Completed 3 queued activity previously (0/1)</td>
<td>0.0257</td>
<td>0.3980</td>
</tr>
<tr>
<td>5</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.0083</td>
<td>-0.1670</td>
<td>21</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.1109</td>
<td>-1.6650</td>
</tr>
<tr>
<td>6</td>
<td>Number of Activities in Queue</td>
<td>-2.7398</td>
<td>-8.0260</td>
<td>22</td>
<td>1 Activity in Queue (0/1)</td>
<td>-0.8991</td>
<td>-1.4710</td>
</tr>
<tr>
<td>7</td>
<td>Completed 1 queued activity previously (0/1)</td>
<td>0.0805</td>
<td>1.3720</td>
<td>23</td>
<td>2 Activities in Queue (0/1)</td>
<td>0.5289</td>
<td>0.8640</td>
</tr>
<tr>
<td>8</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>-0.0445</td>
<td>-0.7600</td>
<td>24</td>
<td>3 Activities in Queue (0/1)</td>
<td>-0.5538</td>
<td>-6.6100</td>
</tr>
<tr>
<td>9</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.0746</td>
<td>-1.1430</td>
<td>25</td>
<td>&gt;3 Activities in Queue (0/1)</td>
<td>0.2668</td>
<td>5.3420</td>
</tr>
<tr>
<td>10</td>
<td>Number of Queued Activities Completed</td>
<td>-0.0413</td>
<td>-0.5010</td>
<td>26</td>
<td>Completed 1 queued activity previously (0/1)</td>
<td>0.1095</td>
<td>1.2570</td>
</tr>
<tr>
<td>11</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>0.0805</td>
<td>1.3720</td>
<td>27</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>0.0340</td>
<td>0.4500</td>
</tr>
<tr>
<td>12</td>
<td>Completed 3 queued activity previously (0/1)</td>
<td>-0.0445</td>
<td>-0.7600</td>
<td>28</td>
<td>Completed 3 queued activity previously (0/1)</td>
<td>0.0255</td>
<td>0.3440</td>
</tr>
<tr>
<td>13</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.0746</td>
<td>-1.1430</td>
<td>29</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.1736</td>
<td>-2.4330</td>
</tr>
<tr>
<td>14</td>
<td>Number of Activities in Queue</td>
<td>-0.9131</td>
<td>-2.0340</td>
<td>32</td>
<td>3 Activities in Queue (0/1)</td>
<td>-1.0653</td>
<td>-12.4840</td>
</tr>
<tr>
<td>15</td>
<td>Completed 1 queued activity previously (0/1)</td>
<td>0.7605</td>
<td>1.6940</td>
<td>33</td>
<td>&gt;3 Activities in Queue (0/1)</td>
<td>0.3352</td>
<td>5.4970</td>
</tr>
<tr>
<td>16</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>-0.2565</td>
<td>-4.0400</td>
<td>34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>0.0693</td>
<td>1.5160</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Log-Likelihood: -52069.58
Sample Size: 116773
Table 6.10: Estimation Results for Pooled Sample with no Distinction with respect to Number of Decisions Made

<table>
<thead>
<tr>
<th>Var #</th>
<th>Variable Description</th>
<th>Pooled Coefficient</th>
<th>Model t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Alternative specific constant (for participation)</td>
<td>1.2954</td>
<td>4.2860</td>
</tr>
<tr>
<td>2</td>
<td>Number of Queued Activities Completed (Indicator, baseline = not completed)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Completed 1 queued activity previously (0/1)</td>
<td>-1.2600</td>
<td>-37.5880</td>
</tr>
<tr>
<td>4</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>-0.0511</td>
<td>-2.1620</td>
</tr>
<tr>
<td>5</td>
<td>Completed 3 queued activity previously (0/1)</td>
<td>-0.1061</td>
<td>-4.5520</td>
</tr>
<tr>
<td>6</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.2346</td>
<td>-10.6040</td>
</tr>
<tr>
<td>6</td>
<td>Number of Activities in Queue (Indicator, baseline = not true)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1 Activity in Queue (0/1)</td>
<td>-2.9745</td>
<td>-8.9060</td>
</tr>
<tr>
<td>8</td>
<td>2 Activities in Queue (0/1)</td>
<td>2.7310</td>
<td>18.6500</td>
</tr>
<tr>
<td>9</td>
<td>3 Activities in Queue (0/1)</td>
<td>0.7586</td>
<td>28.0410</td>
</tr>
<tr>
<td>10</td>
<td>&gt;3 Activities in Queue (0/1)</td>
<td>1.2994</td>
<td>75.7720</td>
</tr>
</tbody>
</table>

Log-Likelihood Value: -52585.60
Sample Size: 116773

Figure 6.4 Stress as a Function of Activities in Queue and Activities Completed
Additionally, a pooled model that did not differentiate between numbers of decisions made per day was also estimated. These estimation results are presented in Table 6.10, with stress plotted in Figure 6.4. These results show a similar trend, the sample was segmented between the numbers of decisions made per day.

Overall, these results show that activity stress varies with the state of the queue and the activity schedule, with these states represented by the number of activities generated and completed respectively. The shape of the curves in the figures above indicate that in general, stress decreases with more completed activities, independent of the number of decisions made per day. This is behaviorally intuitive, since more activities completed indicates more activities leaving the queue, which translates into less pressure, assuming the arrival rate of activities does not exceed the participation rate of activities by a large margin. Similarly, the figures above also indicate that as the number of activities in queue increases, the stress experienced with each additional activity also increases. Furthermore, from the figures above, stress from activities in queue seem to be more sensitive to the actual number of activities (two activities in queue, three activities in queue, etc.), relative to stress relief resulting from completing activities. The stress from activities in queue seem to decrease initially then continually increase. This may reflect the fact that individuals prefer to have one activity waiting in queue over no activities at all, which indicates a completely idle person. These results are obtained under the assumption that each activity is homogenous and exerts the same amount of stress or provides the same amount of stress relief, independent of activity type, duration, or other criteria. However, more realistically activities may vary in the amount of stress they
provide, depending on their characteristics. For example, stress from an activity may actually oscillate over time. The next section provides results from a mixed logit model estimation. The motivation of the mixed-logit estimation was to relax the assumptions of the previous independent binary decision model, specifically with respect to the response from individuals across the population to the queue and activity schedule states.

Repeated Mixed-Logit Model

This section presents results from a mixed-logit estimation of activity participation. These results are shown in Table 6.11. Recall that the total sample size was too large (N=116,773) to estimate feasibly using simulated-maximum likelihood procedures. Thus, this sample was segmented by the number of decisions made per day. The results below are for two different segments: six and seven decisions per day. In these models, response and preference heterogeneity were also allowed. The coefficients on the indicators for completing one activity in queue and for having one activity in queue were assumed to be normally distributed across the population to capture variations in response (stress) to schedule and activity queue states. Furthermore, unobserved random preference heterogeneity was accounted for as well.

Similar to previous findings, the estimation results below indicate that stress or propensity towards participating in activities increases with more activities in queue. Also, stress decreases with completion of more queued activities, but with a less steep slope. One possible explanation is that individuals are more sensitive to stress associated with activities still in queue, relative to activities completed, suggesting that the former plays a stronger role in defining and motivating individual schedules over time.
Furthermore, these results suggest that stress is a latent variable that builds or accumulates over time. Thus, to better account for its effects, longitudinal data on individuals’ actual schedules as they evolve over time, including their needs and goals, may be required. The results also suggest the presence of “dynamics” underlying these scheduling decisions, leading to the accumulation and release of stress over time. These results are based on observed schedules at the end of the day. Given a richer data set that accounts for not only the final outcome of decisions, but also the scheduling decisions made during the day, the dynamics of this process may be captured more completely. Finally the estimated results also indicate that the only significant variation in response to the state of the queue occurs when the first (one) activity is completed, indicated by significance of the standard deviation $\zeta_2$. This further suggests that only in response to completed activities is there variation across the population.
Table 6.11: Mixed-Logit Estimation Results

<table>
<thead>
<tr>
<th>Var #</th>
<th>Variable Description</th>
<th># Decisions 6</th>
<th></th>
<th># Decisions 7</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Coefficient</td>
<td>t-stat</td>
<td>Coefficient</td>
<td>t-stat</td>
</tr>
<tr>
<td>1</td>
<td>Alternative specific constant (for participation)</td>
<td>-0.7652</td>
<td>-0.9728</td>
<td>2.8794</td>
<td>2.4583</td>
</tr>
<tr>
<td>2</td>
<td>Number of Queued Activities Completed (Indicator, baseline = not completed)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Completed 1 queued activity previously (0/1)</td>
<td>-1.3096</td>
<td>-19.8725</td>
<td>-1.0439</td>
<td>-12.0822</td>
</tr>
<tr>
<td>3</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>0.0192</td>
<td>0.4315</td>
<td>-0.0927</td>
<td>-1.6947</td>
</tr>
<tr>
<td>4</td>
<td>Completed 3 queued activity previously (0/1)</td>
<td>-0.1305</td>
<td>-2.9392</td>
<td>-0.0716</td>
<td>-1.3612</td>
</tr>
<tr>
<td>5</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.1011</td>
<td>-2.2417</td>
<td>-0.2023</td>
<td>-4.1711</td>
</tr>
<tr>
<td>6</td>
<td>Number of Activities in Queue</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1 Activity in Queue (0/1)</td>
<td>-1.6628</td>
<td>-1.8715</td>
<td>-5.3582</td>
<td>-4.0927</td>
</tr>
<tr>
<td>7</td>
<td>2 Activities in Queue (0/1)</td>
<td>3.3453</td>
<td>7.9955</td>
<td>3.1896</td>
<td>5.3851</td>
</tr>
<tr>
<td>8</td>
<td>3 Activities in Queue (0/1)</td>
<td>0.7936</td>
<td>15.1740</td>
<td>0.5306</td>
<td>6.8288</td>
</tr>
<tr>
<td>9</td>
<td>&gt;3 Activities in Queue (0/1)</td>
<td>1.4127</td>
<td>43.3344</td>
<td>1.6167</td>
<td>39.9185</td>
</tr>
<tr>
<td>10</td>
<td>Unobserved preference and response heterogeneity parameters</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Person-specific error standard deviation (ζ1)</td>
<td>0.0153</td>
<td>0.1453</td>
<td>-0.1545</td>
<td>-0.3599</td>
</tr>
<tr>
<td>11</td>
<td>Standard Deviation for var#2 (ζ2)</td>
<td>-0.1831</td>
<td>-2.4060</td>
<td>-0.3362</td>
<td>-3.7606</td>
</tr>
<tr>
<td>12</td>
<td>Standard Deviation for var#6 (ζ3)</td>
<td>0.0208</td>
<td>0.1921</td>
<td>-0.1252</td>
<td>-0.2606</td>
</tr>
<tr>
<td></td>
<td>Log-Likelihood Value</td>
<td>-14494.2742</td>
<td></td>
<td>-12223.8516</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sample Size</td>
<td>32394</td>
<td></td>
<td>26621</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.5: Stress as a Function of Activities Completed (top) and in Queue (bottom)
6.6.3 Estimation Results Related to Socio-Demographic Factors

To examine the effects of socio-demographic variables on the stress threshold, variables that indicate the number of queued activities completed were interacted with socio-demographic variables. In addition, an indicator variable was introduced to indicate whether the observation falls on a weekend or weekday. These results are shown in Table 6.12.

With respect to gender effects, males appear to perceive a greater disutility towards participating in more queued activities (Figure 6.6) after completing one or more activities, relative to females. Although the difference in disutility perceived by males relative to females is insignificant after completing three or more activities, males initially perceive greater disutility relative to females. Additionally, the results also suggest that the curve is steeper for males, thus for every additional activity in queue completed, males experience a disutility that rises more sharply compared to females. With respect to the stress threshold, it suggests that males are more content with completing fewer activities relative to females, who experience less disutility having completed the same number of activities from queue.

With respect to age effects, older individuals (> 30 years of age) perceived significantly less disutility after completing one or two queued activities, though his effect decreases in magnitude after completing three or more activities (Figure 6.7). In regards to the stress threshold, these results suggest that older individuals are more tolerant of activity stress than younger individuals.
Finally with respect to weekday versus weekend, the results indicate that individuals perceive more disutility in completing additional queued activities on weekends, suggesting less inclination towards completing more activities on weekends, relative to weekdays (Figure 6.8). One possible explanation for this is that weekends are typically perceived as "free" time. Since queued activities considered in this study were mostly on the "maintenance" side, individuals would in general be less favorable towards participating in many of these activities, such as going to the bank, on the weekends compared to weekdays. This suggests a further investigation that would look more at queued activities that are recreational and leisurely in nature.

![Figure 6.6: Gender Differences in Perceived Disutility](image-url)
Figure 6.7: Age Differences in Perceived Disutility

Figure 6.8: Weekend vs. Weekday Differences in Perceived Disutility
Table 6.12: Gender, Age, and Weekend Effects

<table>
<thead>
<tr>
<th>Var #</th>
<th>Variable Description</th>
<th>Pooled</th>
<th>Model</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Coefficient</td>
<td>t-stat</td>
</tr>
<tr>
<td>1</td>
<td>Alternative specific constant (for participation)</td>
<td>2.6079</td>
<td>94.7740</td>
</tr>
<tr>
<td></td>
<td><strong>Number of Queued Activities Completed</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>(Indicator, baseline = not completed)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Completed 1 queued activity previously (0/1)</td>
<td>-1.5130</td>
<td>-35.9410</td>
</tr>
<tr>
<td>3</td>
<td>Completed 2 queued activity previously (0/1)</td>
<td>-0.2075</td>
<td>-4.7270</td>
</tr>
<tr>
<td>4</td>
<td>Completed 3 queued activity previously (0/1)</td>
<td>-0.1427</td>
<td>-3.2460</td>
</tr>
<tr>
<td>5</td>
<td>Completed &gt;3 queued activity previously (0/1)</td>
<td>-0.0176</td>
<td>-0.3950</td>
</tr>
<tr>
<td></td>
<td><strong>Number of Queued Activities Completed Interacted with Sex (0/1)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>(Indicator, baseline = Female)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Completed 1 queued activity previously × Sex (0/1)</td>
<td>-0.1241</td>
<td>-3.8110</td>
</tr>
<tr>
<td>7</td>
<td>Completed 2 queued activity previously × Sex (0/1)</td>
<td>-0.0922</td>
<td>-2.0520</td>
</tr>
<tr>
<td>8</td>
<td>Completed 3 queued activity previously × Sex (0/1)</td>
<td>-0.0357</td>
<td>-0.8110</td>
</tr>
<tr>
<td>9</td>
<td>Completed &gt;3 queued activity previously × Sex (0/1)</td>
<td>-0.0090</td>
<td>-0.2180</td>
</tr>
<tr>
<td></td>
<td><strong>Number of Queued Activities Completed Interacted with Age (0/1)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>(Indicator, baseline = &lt; 30 years old)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Completed 1 queued activity previously × Age (0/1)</td>
<td>0.6368</td>
<td>19.0560</td>
</tr>
<tr>
<td>11</td>
<td>Completed 2 queued activity previously × Age (0/1)</td>
<td>0.0812</td>
<td>1.7590</td>
</tr>
<tr>
<td>12</td>
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<td>-0.0806</td>
<td>-1.7510</td>
</tr>
<tr>
<td>13</td>
<td>Completed &gt;3 queued activity previously × Age (0/1)</td>
<td>-0.2797</td>
<td>-6.0590</td>
</tr>
<tr>
<td></td>
<td><strong>Number of Queued Activities Completed Interacted with Weekend (0/1)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>(Indicator, baseline = Unemployed/Retired/Non-Student)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Completed 1 queued activity previously × Weekend (0/1)</td>
<td>-0.4060</td>
<td>-10.3120</td>
</tr>
<tr>
<td>15</td>
<td>Completed 2 queued activity previously × Weekend (0/1)</td>
<td>0.0588</td>
<td>1.0700</td>
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<td>Completed 3 queued activity previously × Weekend (0/1)</td>
<td>0.0816</td>
<td>1.4670</td>
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<td>17</td>
<td>Completed &gt;3 queued activity previously × Weekend (0/1)</td>
<td>0.1321</td>
<td>2.4160</td>
</tr>
</tbody>
</table>

Log-Likelihood Value: -58767.78

Sample Size: 116773
6.7 Concluding Remarks related to Threshold Estimation

This study presented a model of activity scheduling based on the concept of a single-server queue. Depending on person-specific attributes that affect the individual’s ability to complete activities, an activity queue may build and exert stress or pressure on the individual. As activities leave the queue, stress is released, though may accumulate again as more activities arise. To make this framework operational, the concept of stress was introduced as a motivator for scheduling decisions. The decision to participate in an activity is governed by a threshold that reflects the preferences and other endogenous characteristics of the individual, and possibly other external aspirations. In this study, the stress tolerance threshold was estimated to illustrate the degree to which this modeling framework can be made operational. Furthermore, an instrumental variable for accounting for activities generated was also used to capture the generation of activities over time, and to overcome endogeneity issues from assuming that the activity queue size is fixed over time.

The estimation results indicate that as the number of activities increases in the queue, more stress is perceived by the individual, and thus there is more pressure or propensity to participate in queued activities. Similarly, as activities are completed, and hence removed from the queue, the individual perceives his/her stress to decrease. These results hold regardless of the number of decisions made during a day. Accounting for response and preference heterogeneity also gives similar results. Overall, these results suggest the presence of underlying dynamics that govern the accumulation and release of stress over time. Future studies should further consider these dynamics, preferably with a richer longitudinal dataset. Future studies should also consider scheduling decisions, in addition
to participation decisions, and the relationship between activities scheduled and actually completed. Additionally, by formulating the activity scheduling process as a queuing system, it may allow future investigation into the “economics” of activity participation over time, with respect to interrelationships between the numbers of activities generated (demand), and the abilities of the individual to complete activities (supply), as reflected by the evolving state of the queue.
Chapter 7.0 Conclusions

The study of human decision making in traffic systems continues to be a challenging area of study, promising new opportunities for the efficient management of these systems and improvement to the quality of urban life. This study investigated the decision mechanisms underlying the dynamics of route choice and activity scheduling decisions. With respect to route choice dynamics, the main objective was to model and understand mechanisms related to travel time perception, learning, and risk attitudes, and to explore their implications on system performance over time. This objective was accomplished through performing experiments using a network performance model, in this case an agent-based simulation model of individual experience given the collective effects arising from the interaction of the agents’ route choice decisions. With respect to activity scheduling decisions, the main goal was to examine the range of behavioral insights obtained from a modeling framework that viewed the individual scheduling process as a single-server queuing system. The concept of "activity stress" was introduced to allow the framework to be operational. This study presented numerical experiments on this framework using a discrete event simulation of an M/G/1 queuing system. Furthermore, an operational model of activity participation was presented.

7.1 Main Contributions

This study led to several contributions in the area of travel behavior research. First, this research served to advance theories of individual learning in the dynamics of user behavior, in particular by introducing a new perspective on activity scheduling dynamics,
that of a queuing system. Additionally, this study augmented previous theories of route choice dynamics by explicitly considering learning processes with statistical and cognitive dimensions, and risk perceptions in a stochastic dynamic environment. This study also provided further consideration of trigger mechanisms, in both route choice learning dynamics and activity scheduling, with similar theoretical behavioral constructs, such as cumulative pressures and thresholds. Methodologically, this study went further than previous works in implementing micro-level rules for learning, and perception updating in context of traffic networks, to examine dynamic system properties, and build towards day-to-day analysis tools.

With respect to route choice dynamics, this study contributed extensively, in terms of breadth, to our understanding of both decision mechanisms and system dynamics, covering the following areas: i) the perception of uncertainty and risk; ii) the updating process for these perceptions; iii) the effect of both at a system performance level. Additionally, deeper insight was gained into i) the reasonableness of assumptions regarding an equilibrium state in networks, under plausible user behaviors; and ii) the day-to-day route choice process of users under different learning types. Finally, this study allowed for a better understanding of the timing of learning and updating, suggesting the need to examine the tradeoffs between the respective value of time savings, learning, and perceived uncertainty.

With regard to activity scheduling dynamics, this study provided a behavioral perspective, that of a single-server queuing system, that allowed an understanding of the
role of “latent” activities generated and the relationship between planned and executed activity schedules. This study also introduced the concept of “activity stress” and a means to capture its effects through an operational model. Finally, with respect to the conventional analysis of observed activity schedules, this study calls into question assumptions about differences between activities generated and those completed.

7.2 Route Choice Dynamics

In this study, mechanisms for travel time perception, travel time learning, and risk perception were presented. These mechanisms were used in an investigation of the dynamics of route choice decisions from day-to-day. Perceived travel times, either experienced or updated, are assumed to consist of a mean and variance. Learning or updating mechanisms were also presented to examine the updating of perceived travel times in light of new travel experiences. Also, recognizing that a cost may be incurred from each update, triggering mechanisms for updating were also presented to account for the timing of updating decisions. Finally, risk attitudes were accounted for in the route choice decision process through a mechanism for weighing objective probabilities of travel time improvements, assuming that risk taking behavior is reflected through these weights. Simulation experiments were conducted to study the system performance implications of these different behavioral mechanisms.

To investigate travel time perception and learning, mechanisms related to these behaviors were modeled using concepts from Bayesian statistics, and were embedded in a microscopic (agent-based) simulation framework to investigate their collective effects on
the day-to-day behavior of traffic flows. This study extended past work by further considering the perception and learning process, the triggering and terminating mechanisms which govern it, and the effect of the above on the day-to-day dynamic behavior of a traffic network, in particular convergence.

First, the results indicate that individuals’ perception of travel times and the mechanisms for integrating them with past experiences both greatly affect the convergence of the system. Several important effects were observed. When the overall travel time perception error is low (mostly regular commuters) or high (mostly new commuters), system convergence was more difficult to attain. Second with respect to the time until convergence, all other factors being the same, as inter-update period increases, the time until convergence decreases initially and then increases, and the number of updates required for convergence decreases. This result suggests that an “optimum” level of new information content might contribute to faster system convergence. Third, a system with users that update almost at every travel time experienced is less likely to converge than a system with selective users.

Overall these results indicate that the perceived confidence (or error) associated with experienced travel times is an important factor in route choice decisions and should not be ignored. Additionally, these findings call into question the behavioral assumptions invoked in deterministic and stochastic equilibrium assignment models, in particular fixed and homogenous perception parameters, and have important implications for dynamic network performance models.
Finally, note that convergence was a desired criterion in this study, which assumed a fixed demand level; however, under variable demand convergence is still sought. Although the system may not be at a strict user equilibrium (UE) state, there still exists a unique solution at which all users have minimized their “perceived” travel times. It can be shown that the equivalent mathematical program for variable demand is strictly convex and thus has one stationary point, which is a minimum (Sheffi 1985). Additionally, note that the link-cost functions used in this paper were two-piece and thus discontinuous, possibly being problematic since convergence is not guaranteed.

With respect to route choice, this study also examines the role of learning rules other than Bayesian learning, and risk attitudes in the day-to-day behavior of traffic flows. In this study a mechanism that assumes risk attitudes are reflected through the subjective probability weights for gains and losses is used to examine the role of risk attitudes on day-to-day route choice dynamics. Additionally, these three learning types are considered: i) Bayesian; ii) reinforcement; and iii) belief.

First, the results show that explicitly considering risk attitudes does influence the convergence of traffic flows in a network. Risk attitudes affect route choice decisions by influencing the perception of uncertainty and how this uncertainty relates to route travel times experienced in the decision making process. The presence of risk seekers and avoiders may affect the route switching frequency of users, thus affecting the spread of users across route from day-to-day. The results show that the percentage of risk seekers
in the population affects the rate of convergence, possibly by affecting the rate of sampling taken by individuals and by adding variability in travel times for individuals who are not risk seeking. The results also indicate that under Bayesian learning, any mechanism that affects the rate of sampling will affect the rate of convergence. Convergence under Bayesian learning is a function of both the perceived travel times and the perceived dispersion of these travel times.

Reinforcement learning describes how travel times experienced are integrated, but does not explicitly say anything about how uncertainty changes over time. Since reinforcement learners only update travel time gains, the rate of sampling from day-to-day may not be high enough to lead to convergence. One assumption of all the learning rules used is that the propensity towards convergence increases as users’ perceived confidence in travel times increases (perceived variance decreases). Under belief learning, since it considers experiences of all users, the system may go towards a faster convergence compared to reinforcement learning.

Finally, these results show that there are system-wide properties that are common to all cases, regardless of learning rule or the explicit consideration of risk attitudes. First as demand levels increase, convergence is more difficult to achieve. Second, as individuals rely more on their updated travel times when they are making route choice decisions, less switching among routes occurs and individuals choose a particular route more consistently. Since updated travel times only change with updating or learning, they vary less over time with long experienced travel times.
7.3 Activity Scheduling Dynamics

Numerical experiments were conducted using a simulation model of an M/G/1 queuing system to explore the range of behavioral insights that might be gained from a modeling framework that views the individual as a server in a queuing system, with activities arriving and forming a queue. Simulation experiments were carried out to explore the relationship between different scheduling rules and “service” (performance) measures, such as the length of the activity queue and the waiting time. These experiments also permit insight into the relationship between formal queuing theory and activity scheduling. The results show that if selection is based on choosing the activity with the larger relative stress compared to other activities, the actual scale or magnitude of an activity’s stress does not matter, so long as the relative order of stress is preserved.

A model of activity participation was also estimated using observed activity schedules. This model was estimated under a discrete choice framework, where individuals made repeated binary decisions about participating in one more queued activities. The estimation results indicate that individuals do experience stress when completing activities over time or when the activity queue grows. Specifically, as individuals complete more activities, and as the number in queue decreases, they have less inclination towards pursuing more queued activities. The opposite occurs when activity queues grow in size, with stress increasing with each additional activity. This further suggests that as individuals complete more activities in queue, their tolerance for stress increases if no new activities are added. However, since more activities are generated
over time, stress does not constantly decrease, but may vary with the evolving states of the queue and activity schedule. The results further indicate that socio-demographic variables may lead to variations in the perception of activity stress over time.

7.4 Application and Implication of Results

The applications and implications of this study in the area of travel demand management and travel behavior analysis are numerous, specifically for evaluating user behavior over the short term, in response to real-time information and new information communication technologies within transportation systems. Given the rapid spread and development of new personal real-time information communication devices, individuals continually expand their spatial and temporal boundaries for activities and consequently travel. Thus, understanding and improving activity patterns and related travel decisions within this growing complex dynamic information-rich environment requires models that can capture important aspects of these decisions, such as learning, information processing, and risk anticipation.

The models developed and presented in this study are amenable to capturing the dynamics that individual route choice and activity scheduling face within these dynamic environments. In this study the route choice models captured the dynamics of decisions with respect to learning, uncertainty perception, and risk perception, all of which play important roles in the integration of current with past experiences, in addition to the anticipation of future outcomes. Thus, the route choice model presented in this study permits an evaluation of the effects from real-time information on individual behaviors over time. Similarly, viewing activity scheduling as a queuing process permits evaluation
of real-time activity information on activity scheduling decisions occurring over time frames shorter than a day. In general the models developed in this study have wide applications for the understanding and evaluation of user behaviors in transportation systems, where users are faced with continuous real-time information, possibly through new technologies and shared experiences with other users.

7.5 Future Research Directions

Several directions for future research are suggested to extend and expand the findings of this study. With respect to route choice dynamics, due to the lack of empirical data on day-to-day route choices, the results in this study could only be exploratory in nature, relying on simulation. Future studies should further consider validation of the results shown in this study, using empirical data of day-to-day route choice decisions. Furthermore, closer examination of the validity of assumptions made with respect to the decision mechanisms, such as the weighing of objective probabilities could be accomplished using psychological experimentation.

With respect to activity scheduling, although both a conceptual framework and operational model were presented, significant additional work should be done on the rescheduling aspects of activity scheduling. Due to the lack of data on the schedule adjustment process, these dynamic aspects of activity scheduling could not be explored beyond a simulation approach. Future research should also consider the interrelationship between planned schedules and executed schedules.
Bibliography


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