Title of Dissertation: INTEGRATED ENVIRONMENTAL REGULATION WITH MULTIPLE POLLUTANTS AND JOINT ABATEMENT: THEORY AND AN APPLICATION TO AIR-QUALITY MANAGEMENT

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Despite calls for more integrated residual management, research on the performance of methods to regulate pollution has paid little attention to cases of multiple pollutant control. This study helps rectify this omission in multiple ways. In Chapter 1 I describe the many issues that arise in the control of multiple pollutants and identify where economics and policy have addressed them. In Chapter 2 I extend a well-known proof demonstrating that emissions taxes or cap-and-trade instruments may yield a Pareto optimal outcome in a general equilibrium setting to the case where there are multiple pollutants in the economy. Chapter 2 also includes an exploration of how changing the joint abatement relationship in a deterministic firm-level model affects emissions and allowance prices when taxes and/or cap-and-trade programs are used.
Chapter 3 extends a model commonly used in the instrument choice under uncertainty literature to the case of jointly abated pollutants. In the single pollutant case with uncertain abatement costs, Weitzman (1974) and others have demonstrated that the expected welfare from an emissions tax is likely not the same as for a tradable emissions cap and derive conditions under which each instrument is preferred to the other. I find that the criteria identifying the welfare-maximizing instrument in the single pollutant framework may be misleading in identifying the optimal set of instruments in a multiple pollutant framework. I also show that the optimal instrument for any one pollutant may depend on how the other pollutants are controlled.

A case study is then explored in Chapter 4. A market simulation model of the national electricity sector is employed to determine the optimal instrument combination to control sulfur dioxide and mercury emissions from coal-fired power plants. There are important, but uncertain, relationships in the abatement of these pollutants. The analysis shows that the optimal instrument combination consists of controlling mercury by a tax and sulfur dioxide by a tax. The cost of selecting a suboptimal instrument mix is between $90 and $190 million (2004 $) in 2020 depending on which suboptimal mix is adopted. Chapter 5 summarizes the findings and suggests issues for future research.
INTEGRATED ENVIRONMENTAL REGULATION WITH MULTIPLE POLLUTANTS AND JOINT ABATEMENT: THEORY AND AN APPLICATION TO AIR-QUALITY MANAGEMENT

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Dedication

To Mom and Dad, and Joan and David
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# Table of Contents

Dedication ..................................................................................................................... ii  
Acknowledgements ...................................................................................................... iii  
Table of Contents ......................................................................................................... iv  
List of Tables .............................................................................................................. vii  
List of Figures ............................................................................................................. viii  

**Chapter 1: The Basics of Multiple Pollutant Control** ...................................................1  
1.1 Introduction ..................................................................................................1  
1.2 Issues in Multiple Pollutant Control ............................................................2  
   1.2.1 Emissions versus Pollution: A Distinction ......................................2  
   1.2.2 Joint Pollution Damages ..................................................................2  
   1.2.3 Joint Emission Control Costs...........................................................4  
1.3 Multiple Pollutant Control in the Economics Literature..............................9  
1.4 Multiple Pollutant Control In Practice.......................................................14  
   1.4.1 Examples of Multiple Pollutant Control Problems.......................14  
   1.4.2 Regulatory Responses....................................................................16  
1.5 A Preview of the Dissertation....................................................................18  

**Chapter 2: Joint Abatement in a Deterministic Framework** ........................................22  
2.1 Introduction ................................................................................................22  
2.2 Achieving Pareto Optimality with Multiple Pollutants .............................23  
   2.2.1 The Model......................................................................................23  
   2.2.2 Characterizing a Market Equilibrium with Multiple Pollutants ....27  
   2.2.3 Comparing the Efficient and Market Outcomes ............................28  
2.3 Analytical Model of Joint Abatement........................................................32  
   2.3.1 Introduction....................................................................................32  
   2.3.2 Characterization of Firm Problem .................................................33  
   2.3.3 Both Pollutants Controlled by Tax Instruments.............................38  
   2.3.4 Summary of Effects of Changing Technological Relationship ....41  
Appendix 2.A Single Pollutant Example...............................................................43  
Appendix 2.B Alternative Instrument Combinations ............................................45  
   2.B.1 \( m \) controlled by a Quantity, \( s \) controlled by a Tax.......................45  
   2.B.2 \( m \) Controlled by a Tax, \( s \) Controlled by a Quantity .......................47  
   2.B.3 Both Pollutants Controlled by a Quantity.......................................50  

**Chapter 3: Analytical Model of Instrument Choice Under Uncertainty with Joint Abatement** ..........................................................52  
3.1 Introduction ................................................................................................52  
3.2 Review of the Single Pollutant Case..........................................................55  
   3.2.1 Graphical Analysis of Weitzman Result........................................55  
   3.2.2 Analytical Derivation of Weitzman Result....................................60  
3.3 The Multiple Pollutant Model....................................................................65  
   3.3.1 Quantity Instruments......................................................................68  
   3.3.2 Mix of Instruments.........................................................................69  
   3.3.3 Tax Instruments .............................................................................71  
   3.3.4 Expected Welfare for Each Instrument Mix .................................73
List of Tables

Table 2.1: Optimality Conditions with Multiple Externalities ........................................26
Table 2.2: Potential Instrument Mixes ........................................................................37
Table 2.3: Effect of an Increase in $\gamma$ on the Variables of Interest ...........................41
Table 3.1: Realized Levels of Abatement from Each Instrument Mix ............................73
Table 3.2: Expected Welfare from Each Instrument Mix .............................................74
Table 3.3: Summary of Special Cases .........................................................................103
Table 4.1: Characteristics that Define Boiler Categories for Hg Removal Efficiencies ..........................................................115
Table 4.2a: Model Results Summary given Expected Removal Efficiencies (2004$) ..................................................................................................131
Table 4.2b: Pollution Controls as a Share of Total Capacity given Expected Removal Efficiencies .............................................................................132
Table 4.3: Expected Welfare from Competing Instrument Mixes in 2020 (Million 2004$) ..........................................................................................137
Table 4.4: Emissions and Allowance Prices for Each Instrument Mix ...........................140
Table 4.5: Decomposing Welfare Changes Relative to Mean Removal Efficiency Case (Million 2004$) ..............................................................................143
Table A.1: Model Plants Representing Existing Capacity .............................................175
Table A.2: Model Plants Representing Potential Capacity ............................................176
Table A.3: Characteristics and Applicability of SO$_2$ Post-Combustion Controls in Haiku ........................................................................................................186
List of Figures

Figure 2.1: Graphical Representation of $g(.)$...............................................................36
Figure 3.1: Expected Welfare Maximizing Instruments...............................................57
Figure 3.2: Realized Welfare from the Tax and Quantity Instruments........................58
Figure A.1: Haiku Electricity Market Regions..............................................................172
CHAPTER 1: THE BASICS OF MULTIPLE POLLUTANT CONTROL

1.1 Introduction

The typical approach in U.S. environmental policy is to regulate pollutants individually. Yet there have been calls, both distant (Ayres and Kneese, 1969) and recent (National Research Council, 2004), for a more integrated approach to residual management. While the reasons for this recommendation are manifold, accounting for health and ecosystem interactions as well as control relationships among pollutants are the driving considerations. At the same time, many questions remain about how integrated pollution management should occur in practice and what regulatory methods are appropriate. To make this advice practicable, one needs to have an understanding of how the choice of regulatory techniques influences environmental and social welfare outcomes.

In this chapter, I describe what is meant by multiple pollutant control and offer some examples. I also briefly describe how economists have treated issues that arise in the control of pollutants that have joint effects and control costs and what is being done in the regulatory community to address these issues. The balance of this chapter outlines the questions considered in the remaining chapters of this dissertation, the general theme of which is an exploration of the desirability of competing market-based pollution control instruments, cap-and-trade and tax policies, when the regulator is uncertain of the abatement costs of jointly controlled pollutants.
1.2 Issues in Multiple Pollutant Control

1.2.1 Emissions versus Pollution: A Distinction

To decompose the various ways multiple pollutant issues present themselves we run into a nomenclature problem. The word “pollutant” is commonly used synonymously with “emissions”. In the following discussion a distinction is made between emissions, which is some byproduct that is disposed of in a common-property resource, and a pollutant, which is the bad that influences the quality of goods and the performance of production technologies that in turn affect welfare. While this definitional distinction is only important in this Chapter, it helps us decompose many of the issues that arise in multiple pollutant control.

1.2.2 Joint Pollution Damages

The easiest way to identify the issues that may arise when regulating multiple pollutants is to start with general forms of the joint damage and control costs functions. Let a damage function of \( Z \) pollutants be represented by:

\[
H(z_1,\ldots,z_Z)
\]

(1.1)

This damage function represents the willingness to pay or lost profits of those individuals or firms affected by the levels of \( z_j,\ldots,z_Z \) to avoid exposure to these pollutants.\(^1\) If there is any synergy in the damage caused by two pollutants then:

\[
\sum_{w=1}^{W} H_w(z_i)
\]

---

\(^1\) In a similar vein, one pollutant may cause \( W \) different harms that can be accounted for separately:
where $i \neq i'$. There is nothing restricting the sign of this cross-partial. There may be cases where increasing one pollutant increases the marginal damage of the other and cases where it reduces the marginal damage of the other.

Often, it is not simply the case that the level of a pollutant $z_i$ equals the sum of emissions over all sources of $z_i$. It may be the case that different types of emissions interact in the environment and form what are sometimes referred to as secondary pollutants. Furthermore, the damage caused by a particular quantity of emissions may be a function of the emitter’s location. Both of these extensions imply a more complex relationship between emissions and the level of $z_i$. The general relationship between the pollutant $z_i$ and a vector of $L$ types of emissions is represented by:

$$z_i \equiv Z_i(s_1, \ldots, s_L) \quad (1.3)$$

As discussed below, there are important examples where $Z_i(\cdot)$ is not only an interactive function of emissions (i.e. not additively separable) but is also non-convex. That said, often $Z_i(\cdot)$ takes an additively-separable form of the arguments $s_i$, where $s_i$ is the same substance as $z_i$ and $l$ indexes different emitters. When $Z_i(\cdot)$ is linear in the arguments $s_i$:

---

This form also implies separability in the willingness to pay for the qualities of the goods that are affected by $z_i$. While this is not a case of multiple pollutant control per se, it is the sort of problem that integrated pollution control regulations are intended to address.
the parameter $\phi_i$ (where $0 \leq \phi_i \leq 1$) is often called a dispersion or transfer coefficient.

The most well known relationship of this type is between emissions and the concentration of a pollutant at a monitoring site (Montgomery 1972).

A regulatory function similar to $Z_i(.)$ indicates the influence of different types of emissions on some indicator of environmental quality. If one were to be pedantic we would recognize that in most regulatory applications of $Z_i(.)$ the variable $z_i$ actually is an indicator of environmental quality in that it does not exactly correspond to the arguments influencing the damage function. Using the receptor site example, the damage to someone affected by emissions from various sources depends on the concentration of the pollutant where she is, not at the receptor site.

1.2.3 Joint Emission Control Costs

We now move to the joint control of multiple emissions. To understand the origins of joint control costs we begin with the profit maximization problem of a representative, yet unregulated, firm $k$ that generates a single product $y_k$.

\begin{equation}
\max_{y_k, x_{k1}, \ldots, x_{kN}, x_{k1}, \ldots, x_{kn}} p_k y_k - \sum_{n=1}^{N} c_{kn} x_{kn} \\
\text{s.t.} \\
y_k \leq f_k (x_{k1}, \ldots, x_{kN}, s_{k1}, \ldots, s_{km})
\end{equation}

\footnote{Note that we are ignoring the potential for emissions caused by households. Presumably accounting for this possibility is relatively unimportant to the following discussion.}
where $f_k(\cdot)$ is the production function of the firm, $x_{kn}$ are private inputs to production, and each $s_{km}$ represents an input, or emissions, of type $m$ that is disposed of in the environment.\(^3\),\(^4\) Emissions essentially represent the use of an environmental service and impose uncompensated costs on other agents in the economy. The parameters $p_k$ and $c_{k1}, \ldots, c_{kN}$ are respectively the prices of the product and inputs used by firm $k$. For the moment we assume that these prices are fixed.

Now let us suppose that a particular regulation $\Gamma_m$ on the emissions of type $m$ is under consideration. The regulation $\Gamma_m$ consists of individual restrictions

$\Gamma_{km}(s_{km})$ on the emissions of type $m$ from each firm $k$. The restrictions $\Gamma_{km}(s_{km})$ are also a function of regulatory parameters and may be a function of other choice variables in the firm’s objective function.\(^5\),\(^6\) For each firm $k$, the total cost of

\(^3\) We assume that the production function yields convex production sets and that the production of $y_j$ requires at least one privately priced good $x_j$ (as otherwise production of $y_j$ would be unconstrained given our assumption of constant prices). We write the production functions as if each $y_j$ may be a function of every type of emissions $m$ to keep the problem general, but it is of course possible that $\partial f_k / \partial s_{km} = 0$ for some $k$ and $m$.

\(^4\) Each $s_m$ in (1.3) corresponds to some $s_{km}$ if only one type of emissions from all firms is being controlled.

\(^5\) In their most general form these restrictions may even be a function of other emission types, but then the question of whether the problem is one of joint emission control is trivial.

\(^6\) Identifying forms where $\Gamma_{km}(s_{km})$ is some firm-level quantity restriction, like a performance standard, is straightforward. For a cap-and-trade program $\Gamma_m$ must also contain some balancing restriction across all regulated firms. Where $\Gamma_{km}(s_{km})$ represents a charge, say $t_{km}$ per unit of emissions, the restriction is of the form:

$$F_{km} - t_{km} s_{km} \geq 0$$

where the profit of source $k$ must also be adjusted by the fee paid, $F_{km}$. However, this fee must then be redistributed to agents in the economy and therefore is counted against profits for firm $k$ but not towards the cost of the regulation.
complying with $\Gamma_{km}(s_{km})$ is the profit from the solution to (1.5) minus the profit from the solution to:

$$\max_{y_k, x_{k1}, \ldots, x_{kN}, s_{k1}, \ldots, s_{kM}} \quad P_k y_k - \sum_{n=1}^{N} C_{kn} x_{kn}$$

s.t.

$$y_k \leq f_k(x_{k1}, \ldots, x_{kN}, s_{k1}, \ldots, s_{kM})$$

$$\Gamma_{km}(s_{km}) \geq 0$$

(1.6)

So, the cost of the regulation $\Gamma_m$ across all $K$ firms is:

$$C(\Gamma_m) \equiv \sum_{k=1}^{K} \pi_k(y_k, x_{k1}, \ldots, x_{kN}, s_{k1}, \ldots, s_{kM}) - \sum_{k=1}^{K} \pi_k(y_k, x_{k1}, \ldots, x_{kN}, s_{k1}, \ldots, s_{kM} | \Gamma_{km}(s_{km}))$$

(1.7)

where the first term on the right is the sum of all of the solutions to (1.5) while the second term is the sum of all of the solutions to (1.6).\textsuperscript{7,8} Now we can define the fundamental problem of joint emission control. Consider the case where a regulation $\Gamma_m$ on emissions of type $m'$ (e.g. a cap-and-trade program for sulfur dioxide) was imposed prior to $\Gamma_m$ (e.g. a performance standard on mercury). For our purposes, the definition of a joint emissions control problem is when the cost function for the regulation $\Gamma_m$ given $\Gamma_{m'}$:

\textsuperscript{7} Often, rather than a function of the form (1.7), emissions control costs are measured as changes in the cost of producing particular levels of $y_k$ from imposing $\Gamma_m$. Provided that output levels do not change considerably as a result of imposing $\Gamma_m$, such a measure is roughly equivalent to (1.7). Both measures ignore possible changes in consumer surplus from increasing the firms’ costs.

\textsuperscript{8} There is an analogue to the single pollutant-multiple harm scenario (i.e. footnote 1) in the emission control cost case. It is where there are multiple source types emitting one particular type of emissions. Often, separate regulations are established for each type of emitter without regard for how the other sources are regulated. This issue has also been grouped into the discussions of integrated pollutant control.
\[ C(\Gamma_m | \Gamma_{m'}) \]  

(1.8)

is different than (1.7). If \( C(\Gamma_m | \Gamma_{m'}) > C(\Gamma_m) \) then these pollutants are substitutes in their control. If \( C(\Gamma_m | \Gamma_{m'}) < C(\Gamma_m) \) they are complements in their control.9

An even more general form of the joint emission control cost function would allow goods and input prices to change.10 There, a joint emission control problem may arise when there are two competing technologies with different emission profiles generating the same product. If emissions from one of the technologies is subject to regulation, emissions from the competing technology will increase. This example demonstrates that it is not even necessary for a subset of the regulated sources to be emitting both types of emissions for a joint emission control problem to exist. While technological relationships between the use of inputs in the production of goods provides the clearest example of a joint emission control problem, such technical relationships are not a requisite.

There is one notable variety of joint emission control that is not captured by this definition. Emissions from one source may reduce the production possibilities,  

\[ 9 \text{ For a joint abatement problem to exist given our assumption that input and output prices do not change, both emission types must be used by at least one of the regulated sources. However, it is not necessary that } \frac{\partial^2 f_i(\cdot)}{\partial s_{km} \partial s_{km'}} \neq 0 \text{ for some } k \text{ for there to be a joint emission control problem. The production function may be separable in } s_{km} \text{ and } s_{km'}, \text{ but as the optimal levels of other inputs and output change, as a result of imposing } \Gamma_{m'}, \text{ the demand for } s_{km} \text{ may change. In this case imposing } \Gamma_{m'} \text{ will change the cost of } \Gamma_m. \]

\[ 10 \text{ We can now explain the purpose of holding output and input prices constant. This is so we can distinguish the situation where different types of emissions are being regulated from the situation where multiple firms with the same emission type are being regulated. In the later case, adding one element } \Gamma_{km'}(s_{km'}) \text{ to } \Gamma_m \text{ may result in a non-additive change in the cost of } \Gamma_m \text{ if the restriction on firm } k \text{ affects the prices faced by the other firms. However, this example is not one we would normally associate with a joint emission control problem.} \]
and thus affect the emissions, of another source. A setting often invoked in Baumol and Oates (1988) is that the source harmed by soot is a laundry, which in turn must increase its use of detergent to maintain production levels. What is not acknowledged is that the increased use of detergent will result in increased phosphorous emissions. If the phosphorus emissions affects the firm releasing the soot we have a case of “reciprocal externalities” (Cornes and Sandler, 1985). The optimal regulations on phosphorous and soot should therefore be jointly determined. Rarely is this sort of problem considered in a regulatory setting, but it has been acknowledged as applicable to transboundary pollution control.

So far, we have couched the joint emission control cost function in (perhaps excessively) general terms. We have been open to any type of regulation and any sort of interaction that causes the joint determination of emission levels. In the balance of this dissertation, we typically will be thinking of joint emission control problems that derive from relationships in production technologies. We will also have a narrow focus on the types of regulations under consideration. As economists have a well-reasoned preference for market-based regulations in a single pollutant framework, this research focuses solely on the performance of such policies.

In the key analytical contribution of this dissertation (Chapter 3), rather than apply an emissions control cost function of the form (1.7), I follow the convention in the literature on the choice of environmental regulations under uncertainty and use an analytical model that focuses on the abatement of emissions. Abatement is defined as the difference between emissions absent a regulation and emissions with a regulation. When a cap-and-trade or emission tax regulation is being
evaluated, it is fairly easy to understand and thus model these regulations in abatement space. In a deterministic model with a single emission type, a requirement of a minimum level of abatement is equivalent to a restriction on the allowable amount of emissions.\textsuperscript{11} However, when jointly controlled emissions are under consideration, this equivalence no longer holds. The uncontrolled level of one type of emissions depends on how intensely the other emission types are regulated.\textsuperscript{12} It also does not hold when there is uncertainty in the control costs of the regulated sources. These complications, primarily the latter one, will require us to operate in emissions space in the empirical portion of this dissertation.

1.3 Multiple Pollutant Control in the Economics Literature

In their simplest forms, the key issue with joint pollution damages or joint emission control costs is that optimal regulations must be simultaneously chosen. However, the importance of accounting for joint damages or control costs in the choice of regulations depends on the particular problem at hand. Often, the regulator may not jointly select regulations because the administrative cost of such an approach outweighs the benefits of integration.

Beyond the mere acknowledgement that efficiency requires the simultaneous choice of regulations in multiple pollutant control settings, more interesting and practical problems arise that deserve inquiry. However, there is

\textsuperscript{11} Of course in reality the world is uncertain. Therefore, regulations usually are not a function of abatement because it is impossible to observe what emissions would be absent a particular regulation.

\textsuperscript{12} It is even more complicated to view the effect on abatement of regulations other than emission cap-and-trade and tax programs. The form of the abatement cost function (i.e. costs measured in abatement space) depends on the forms of the regulations on all of the emission types.
relatively little normative and positive analysis of multiple pollutant control issues in the economics literature. The only theoretical studies that consider the implications of joint damages beyond a stylized general equilibrium setting, and the first notably in the context of the choice of environmental regulation under uncertainty, are Yohe (1977) and Kolstad (1987). I am aware of no empirical studies that truly consider joint damages of the form (1.1) where (1.2) holds. Of those that do consider multiple benefits, many are like the analysis in Banzhaf et al. (2004). In this paper the authors jointly estimate the optimal levels of sulfur dioxide (SO$_2$) and nitrogen oxides (NO$_X$) emissions from the electricity sector. However, the analysis is simplified in that the marginal damage of each pollutant is constant and therefore the optimal emission levels can be identified independently of each other (at least with respect to the damages they cause).

Most economics studies that consider integrated pollution management address complications that arise when different emission types determine the level of a secondary pollutant (i.e. examples of $Z_i(\cdot)$) or an environmental indicator. In these examples particular attention is often paid to the possibility of a non-convex relationship between emissions and the pollutant or indicator of interest. Beavis and

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13 Notably, however, Helfand, Berck and Maull (2003) saw fit to include discussions regarding multiple pollutant control issues in their chapter on pollution policy in the Handbook of Environmental Economics. They discuss the regulatory implications of multiple emissions that affect a set of environmental indicators, media-specific agencies, and using uniform regulations to control pollutants with heterogeneous damages and costs.

14 In the U.S. regulatory impact analyses are often required of new environmental regulations. These analyses typically estimate the damage reduction expected from the regulation. In my experience these analyses strictly consider damage functions of the form described in footnote 1.

15 For a brief discussion of the implications of non-convexities in the emissions/pollutant relationship see also Klassen (1996), p. 22.
Walker (1979) use an analytical model to assess the impact of multiple and interactive pollutants in general on the regulator’s ability to use emission taxes to achieve environmental targets at least cost. Endres (1986) also explores this setting and considers how the presence of such non-convexities affects the identification and relative performance of emission tax and cap-and-trade policies designed to attain a given environmental target at minimum cost.

Of the cases where $Z_i(\cdot)$ is not additively separable, perhaps the most well known example is the formation of ozone. Ozone is generated by a complex chemical reaction of volatile organic compounds (VOCs) and NOX in the atmosphere. Holding the emissions of one of these two substances fixed, as emissions of the other substance is increased the concentration of ozone may rise and then fall. This non-convexity implies that there may be multiple optima in the problem of determining the combination of VOCs and NOX that achieves a particular ozone concentration at least cost. Economic studies that evaluate the control of ozone include Repetto (1987), Braden and Proost (1996), Kim et al. (1998), Simpson and Eliassen (1999) and Schmieman et al. (2002).16 A common environmental indicator affected by a complex interaction of multiple pollutants is the concentration of dissolved oxygen in water. Studies that have evaluated policies for attaining a dissolved oxygen standard include Elofsson (2003), Lence (1991), and Carmichael and Strzepek (2000).17 There

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16 The innovation in Simpson and Eliassen (1999) and Schmieman et al. (2002) is that they consider additional environmental targets of the form (1.3) that are also affected by NOX emissions.

17 The studies of water quality management focus on different normative questions. Elofsson is particularly interested in how correlated stochastic emissions should be regulated, Lence is interested in setting optimal exchange rates for a single market covering multiple emissions while Carmichael
is also a significant literature on establishing optimal exchange rates for an emissions-trading program that is designed to maximize welfare or attain an environmental indicator when $Z_i(.)$ is non-linear (Bushnell and Friedman, 1994; Farrow et al., 2005; Førsund and Nævdal, 1998; Horan and Shortle, 2005; Hung and Shaw, 2005; Klaassen et al., 1994; and Schaltegger and Thomas, 1996).

On the cost side, some positive analyses have explored how a regulation to control one pollutant affects the emissions or cost of controlling other pollutants. The application that has generated the most interest from economists is the ancillary reduction in local pollutants that results from reducing carbon dioxide (CO$_2$) as part of a climate change policy (Boyd et al., 1995; Burtraw et al., 2003; Ekins, 1996; Feng et al., 2007; Michaelis, 1992). Related research explores political economy questions when the jointly controlled pollutants have different spatial effects. Caplan and Silva (2005) ask whether an efficient mechanism employing tradable emissions permits exists for the regulation of local and global pollutants that are complements in their control when participation in the global regulatory regime is voluntary. Caplan (2006) asks the same question but with the mechanism employing emission taxes. List and Mason (1999) consider a situation where the central government has the authority to regulate the global pollutant (i.e., participation is involuntary) but where the local authority is more sensitive to the effect of the level of one pollutant on the damages caused by the other. Thus, even when the local regulator does not account for the entire change in damages from indirectly changing the level of the global pollutant, it

and Strzepek focus on the effect of non-linear interactions between emissions in designing cost-minimizing regulations. Beavis and Walker’s (1979) analytical exploration was motivated by the example of dissolved oxygen and “isodeath” functions across multiple pollutants.
may be optimal for the local government to have the authority to regulate the global pollutant because of its information advantage.

Stepping away from political economy questions and evaluating the performance of market-based policies, Lence et al. (1988) use an engineering cost model to explore the performance of multiple and combined allowance markets for jointly controlled emissions. In their application they find that the cost of sequentially imposing caps for each emission type is low. While not a case of joint control costs as defined above, Montero (2001) identifies conditions where it is preferable to integrate pollution markets when the regulator is uncertain of the separate abatement costs of multiple emissions and this uncertainty in abatement costs may be correlated across the different emission types. Lutter and Shogren (2002) propose the use of tariffs on internationally traded CO₂ allowances where the tariff level is a function of the change in local emissions due to the seller’s abatement of CO₂. Finally, Eskeland (1997) conducts an empirical study evaluating the cost-effectiveness of regulatory proposals that yield reductions in jointly abated emissions.

Another multiple pollutant control setting of interest to economists, which has a similar flavor to the questions raised above with respect to jointly controlled

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18 Eskeland observes that in the management of jointly controlled emissions “not even cost-effectiveness analysis can be conducted without a value-based priority between emitted pollutants” (p. 1639). Davies and Mazurek (1998) make a similar claim stating that “degree of risk is the only such scale” to set rational priorities in the regulation of jointly controlled emissions (p. 17). However, any environmental indicator could satisfy the need for a common index for a cost-effectiveness analysis. That said, Eskeland’s or Davies and Mazurek’s welfare-based approaches are clearly appealing, but may be impractical.

A study similar to Eskeland’s is Lutter and Burtraw (2002) who consider the optimal mix of SO₂ and NOₓ emissions to achieve a given level of dollarized environmental damages. The objective function in this case does not require that these emissions are jointly controlled or yield joint benefits. However, these pollutants are indeed jointly controlled.
emission with different spatial effects, is when different types of emissions are substitutes that can be emitted into different media. Oates and Schwab (undated) take a normative approach to this problem and investigate how different agency structures (integrated or media-specific regulators) and regulatory approaches affect emissions and welfare. Greenstone (2003) offers an empirical exploration of this theme by estimating the effect of recently adopted air quality regulations on emissions to waterways. Aillery et al. (2005) is an addresses this issue empirical through the study of the costs and benefits of regulating emissions from manure.

Kolstad (1987) and Mendelsohn (1986) ask whether it is worthwhile to tailor regulations to emissions that have heterogeneous damages or control costs. They both explore this question in a theoretical framework and use similar models. Their analyses are general as, for example, the differences in damages may be because a particular type of emissions comes in a variety of forms (species) or is emitted in different locations. The papers differ in that Kolstad (1987) allows for joint benefits while Mendelsohn (1986) provides empirical evidence of the cost of homogeneous regulation.

1.4 Multiple Pollutant Control In Practice

1.4.1 Examples of Multiple Pollutant Control Problems

Examples abound where emissions jointly affect the level of a pollutant or some environmental indicator or that are jointly controlled. Examples of joint damage

19 These models are also quite similar to the one I used in Chapter 3
functions are more difficult to come by. Typically, where such effects have been identified the combination of the two pollutants has a linear influence on damages. That is, the combined effects are modeled using a linear function like (1.4). An example is the effect of different types of particulate emissions of a certain size on human health.

Examples of emissions that contribute to a secondary pollutant or an environmental indicator have already been mentioned (i.e. ozone and biological oxygen demand). Another example can be found in the control of SO$_2$, NO$_x$ and mercury (Hg). Hrabik and Watras (2002) show that increasing SO$_2$ and NO$_x$ deposition increases the damage from Hg by converting Hg to a form more readily taken up by aquatic species. Yet another example is that SO$_2$ emissions lead to regional cooling, offsetting the effect of pollutants that cause global warming. Even emissions that cause global warming have different impacts on climate change. For example, one ton of methane has 23 times the global warming potential as one ton of CO$_2$ (EIA, 2006).

On the cost side, there are numerous examples where emissions are jointly controlled. The example of interest in this dissertation is that technologies designed to

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20 It is easy to identify examples of emissions that cause multiple harms, however (i.e. footnote 1). For example, NO$_x$ and SO$_2$ both contribute to particulate pollution and acid deposition and NO$_x$ contributes to the formation of ozone. Often, emitters face multiple regulations on the same type of emissions with each regulation justified by a different effect. The regulation of VOCs is an example. States regulate sources of VOCs based on their contribution to ozone pollution while the federal government sets standards for VOCs that are toxic (the federal government even has multiple sources of legal authority for setting these standards) (Evans and Kruger, 2007). Note that in practice, however, the federal regulation is often the same as the state regulation as the states often simply take credit from federal regulations towards their ozone control obligations.

21 In this case the environmental indicator is temperature.
reduce SO₂, NOₓ and particulates from coal-fired power plants also lower Hg emissions. As alluded to above, reducing CO₂ emissions from electricity generators leads to reductions in NOₓ, SO₂ and other emissions. In these cases, the different types of emissions are complements in their control. An example where the jointly controlled emissions are substitutes is in the case of NOₓ and carbon monoxide emissions from gasoline engines.²² NOₓ control often makes combustion less efficient, which leads to an increase in carbon monoxide (Hass, 1975). While these examples address air emissions from fossil fuel combustion, many examples also exist for water pollution.

1.4.2 Regulatory Responses

Many European countries have already begun adopting more holistic approaches to regulating pollution (Hersch, 1996; Davies and Mazurek, 1998; Davies, 2001).²³ This approach is generally referred to as “integrated pollution control”, but as noted in Hersch (1996) the approach takes a variety of forms. At its most ambitious, it entails regulatory approaches that broadly consider the interconnectedness of ecological and economic systems. In reality, the integration is much less ambitious and typically focuses on the adoption of cross-media approaches to facility permitting. The European Union has required its members take such an approach to permitting in a 1996 Directive (European Union, 1996). The basic goals

²² For a whole slew of examples where pollutants are substitutes in their control, see Neligan (1975).
²³ That said, Simpson and Eliassen (1999) and Schieman et al. (2002) point out that when it comes to transboundary pollution control, the European approach has historically been of a single pollutant-single harm nature. This too is changing. A 1999 Protocol addresses the transboundary control of SO₂, NOₓ, VOCs and ammonia to reduce acid deposition, eutrophication, and ozone (United Nations, 1999).
of cross-media permitting include eliminating the incentive for regulators responsible for one media to adopt controls that shift emissions to another media, and improving the identification of emission sources at a facility (Davies and Mazurek, 1998). Notably, this endeavor is limited as it is focused solely on integration within facilities and not in setting environmental priorities and targets over multiple media. The hope, however, is that this more modest approach will ultimately lead to a regulatory system that does a better job of establishing priorities and encouraging pollution prevention (Davies, 2001).

In the U.S., the Environmental Protection Agency (EPA) “has conducted pilot efforts at integrated approaches for more than twenty years” (Davies and Mazurek, 1998, p. 18). There are a few current initiatives to evaluate the pros and cons of integrated pollution control and multiple-pollutant management.24 These include a study of the benefits and drawbacks of integrated pollution permitting in the United Kingdom (U.S.EPA, undated). The EPA also administers a grant program for developing countries that supports the adoption of regulations that integrate the control of emissions that cause global warming and those that cause local damages (U.S.EPA, 2007a). In response to the National Research Council (2004) report referenced at the beginning of this chapter, the EPA is encouraging states to adopt a “multipollutant control strategy” by “selecting a control strategy that optimizes the mix of control for multiple [air] pollutants” that should include at least ozone, particulates, and toxic air pollutants (Page, 2005).

24 Tellingly, these efforts themselves are not clearly integrated.
There are also efforts in the U.S. that, while not explicitly part of integrated or multiple pollutant initiatives, are still relevant to this discussion. For example, Title V of the 1990 Clean Air Act Amendments requires integrated air pollution permits for large stationary sources (U.S.EPA, 2007b). Some of the justifications for this initiative are reminiscent of those made in favor of integrated permitting in Europe. There are also examples of rules that acknowledge the benefit of integrating the control of multiple types of emissions and pollutants. For example, federal ozone regulations allow states to avoid tightening NOX or VOC emissions if such an approach may lead to an increase in ozone. While not yet proposed, the EPA is considering establishing a combined (secondary) ambient air quality standard for NOX and SO2.25

1.5 A Preview of the Dissertation

While there are many complications in multiple pollutant control that deserve more attention from the economics discipline, I select only one to address in this dissertation. An important research vein in environmental economics is the normative study of methods for regulating pollutants. This research is grouped under the rubric “instrument choice.” As noted above, typically economists advocate the use of price (tax) or quantity instruments (cap-and-trade programs) to achieve pollution control targets. The value of these instruments is in the incentives they provide to achieve collective emission reductions at least cost and their superiority in motivating technological development. In simple contexts, a regulator is indifferent between a

tax and a quantity approach (Baumol and Oates, 1988). However, complications may arise that differentially support these two instruments. One important complication that yields differences in the performance of these instruments is when the regulator is uncertain of the abatement costs of the polluting sources.\textsuperscript{26} Beyond the general theme of multiple pollutant control, the dissertation is specifically interested in how our understanding of instrument choice is affected when the regulator is uncertain of the abatement cost of jointly controlled pollutants.

The next chapter has two purposes. First, I demonstrate that the means for controlling pollution typically advocated by economists, cap-and-trade and pollution taxes, continue to yield Pareto optimal outcomes in an economy with multiple pollutants. This assures us that we have a basis to expect that these regulatory approaches are likely optimal when we move to an uncertain setting. While the proof is straightforward, laying out such a model helps one identify the various complications that might affect the regulation of multiple pollutants in a real-world setting. Chapter 2 also includes an exploration of how changing technological parameters affects emissions and allowance prices when two types of emissions are being controlled by taxes and/or cap-and-trade programs. Again, while relatively simple, this model will be used to interpret and help assure the reliability of some of the results from the simulation exercise described in Chapter 4.

\textsuperscript{26} Others include the presence of market distortions (Goulder, 2002). More complex instruments, like deposit-refund schemes, may be superior in the presence of uncertainty in the measurement of emissions.
I then explore the implication of abatement cost uncertainty on instrument choice in a two-pollutant framework in Chapter 3. The results from this model are compared to the findings from the single pollutant model most frequently attributed to Weitzman (1974). Weitzman and others have derived conditions under which a tax policy is preferable to a quantity policy and vice-versa. In this chapter I find that the well-known conditions that identify the optimal type of instrument to control one pollutant are no longer sufficient in the multiple pollutant case. However, a lesser-known way of identifying the optimal instrument in a single pollutant case is helpful for understanding the multiple pollutant case. I also evaluate whether our intuition regarding the optimal instrument to control one pollutant is still valid at the extremes of the shapes of the joint abatement cost function and the slopes of the abatement benefit functions. Along the way, the results are related to general equilibrium welfare analysis and the theory of second-best.

The motivation for this entire analysis derives from an important policy case. Regulations recently adopted by the EPA use cap-and-trade programs to control Hg, SO₂ and NOₓ emissions from the electricity sector. Technologies designed to abate SO₂, NOₓ and particulates from coal-fired power plants also reduce Hg emissions, but the extent to which they do so is uncertain. Chapter 4 explores the performance of different instrument combinations to control these pollutants using a detailed simulation model of the U.S. electricity sector (this model is described in the Appendix). An important ancillary contribution of this work is that it also provides the first estimate of the efficient level of Hg emissions from the electricity sector.
The final chapter concludes with a summary of the main findings of the dissertation and a brief discussion of directions for future research.
CHAPTER 2: JOINT ABATEMENT IN A DETERMINISTIC FRAMEWORK

2.1 Introduction

In the first part of this chapter I demonstrate that the regulatory approaches that may yield Pareto optimal outcomes in a single-pollutant economy, pollution tax and cap-and-trade programs, can also be used to generate Pareto optimal outcomes in an economy that generates multiple pollutants. This proof requires only a simply extension of a well-known general equilibrium model of an economy that generates a single pollutant. However with this result established, we can comfortably assume, when we introduce uncertainty in later chapters, that these regulatory approaches are those that would otherwise be recommended.

The second half of this chapter introduces a simple deterministic joint abatement model at the firm level. The model is designed to capture the essence of the technological relationship that motivates the analysis in the empirical portion of this dissertation. With this model we can explore how changing the abatement relationship affects emissions and allowance prices when either an emissions tax or cap-and-trade program is used to control each pollutant. The results from the analytical model will be compared to the results from the simulation exercise described in Chapter 4.
2.2 Achieving Pareto Optimality with Multiple Pollutants

2.2.1 The Model

In considering the challenge of regulating multiple pollutants, our first task is to establish that the regulatory instruments we traditionally believe can be used to generate a socially efficient outcome are applicable to the control of multiple pollutants. In a general equilibrium setting Baumol and Oates (1988) show that there exists a tax on the emissions of a single pollutant that assures a Pareto optimal outcome in a market setting. However, when there are multiple pollutants, is there now a set of emission taxes that can be used to assure a Pareto optimal outcome? The following analysis addresses this question.

The model used herein follows very closely the notation, order, assumptions and derivation in Baumol and Oates, Chapter 4 (1988). The primary difference is that now more than one pollutant is being emitted.\(^1\) Otherwise, we assume that the model and its characteristics, in particular those that assure a unique maximum in a single pollutant case, are the same. The following analysis also retains the “undepletable” characteristic of the pollutants.\(^2\) That is, the level of pollution affecting an individual or firm is the same level that affects all other individuals and firms. There is also no uncertainty and the model is static. Our final assumption is one that would not have meaning in the Baumol and Oates model: that the different

\(^1\) Tietenberg (1973a) and Ruff (1972) perform a very similar analysis to the one provided here. They both use general equilibrium models of a slightly different form to demonstrate the existence of a set of taxes on multiple pollutants that yield an efficient distribution of resources.

\(^2\) Freeman (1984) demonstrates that this assumption is not critical to the results.
pollutants do not interact in the environment. More will be said about this last assumption below.

The model is described by the following:

\( x_{ij} \): the amount of good (resource) \( i \) consumed by individual \( j \), \( (i=1,\ldots,n), (j=1,\ldots,m) \)

\( y_{ik} \): the amount of good (resource) \( i \) produced (used)\(^3\) by firm \( k \), \( (k=1,\ldots,h) \)

\( r_i \): the total quantity of resource \( i \) available to the community

\( s_{kl} \): the emission of pollutant \( l \) by firm \( k \) \( (l=1,\ldots,p) \)

\( z_l = \sum_{k=1}^{h} s_{kl} \): total emissions in the community of pollutant \( l \)

\( u'(x_{1j},\ldots,x_{nj},z_1,\ldots,z_p) \): individual \( j \)'s utility function

and

\( f^k(y_{1k},\ldots,y_{hk},s_{k1},\ldots,s_{kp},z_1,\ldots,z_p) \leq 0 \): firm \( k \)'s production set

We retain all of the assumptions in Baumol and Oates that assure a unique solution to this problem exists. Namely, that the preference functions are increasing in all \( x_{ij} \), quasi-concave, and twice differentiable. Furthermore, we assume each firm’s feasible production set is convex and twice differentiable.\(^4\)

A Pareto optimum can be found by maximizing the utility of a single individual, for convenience individual 1, subject to the restriction that there is no

\(^3\) Note there is no sign restriction on \( y_{ik} \) as it may be either an input to the firm, perhaps manufactured by another firm, or an output of the firm.

\(^4\) Note that Baumol and Oates say nothing about the change in the preference functions or the feasible production with respect to an increase in \( z_l \). We acknowledge here that the change may either be positive or negative. Therefore, the model may also represent the presence of both negative and positive externalities. Of course, for each \( z_l \) that on net has a positive external effect, the optimal tax on \( z_l \) will actually be a subsidy.
reduction in welfare of any other individual. The optimum is subject to the limitations on the production sets of the firms and the availability of resources. Our problem is then to maximize:

\[ u^I(x_{11}, \ldots, x_{n1}, z_1, \ldots, z_p) \]  

subject to:

\[ u^I(x_{1j}, \ldots, x_{nj}, z_1, \ldots, z_p) \geq u^{*j} \quad (j = 2, \ldots, m) \]

\[ f^k(y_{1k}, \ldots, y_{nk}, s_{k1}, \ldots, s_{kp}, z_1, \ldots, z_p) \leq 0 \quad (k = 1, \ldots, h) \]

\[ \sum_{i=1}^{m} x_{ij} - \sum_{h=1}^{h} y_{ik} \leq r_i \]

for all \( x_{ij} \geq 0, s_{kl} \geq 0, z_i \geq 0 \)

This problem can be solved using the Lagrangian:

\[ L = \sum_j \lambda_j [u^I(\cdot) - u^{*j}] - \sum_k \mu_k f^k(\cdot) + \sum_i \omega_i \left( r_i - \sum_j x_{ij} - \sum_k y_{ik} \right) \]  

where each \( \lambda_j, \mu_k, \) and \( \omega_i \) is a Lagrange multiplier. The optimality and complementary slackness conditions associated with the real choice variables for this problem may be found in the second column of Table 2.1.\(^5\) Again following Baumol and Oates, we simplify cross partial expressions so that \( u_{ij}^I = \frac{\partial u^I}{\partial x_{ij}}, f_{ik}^k = \frac{\partial f^k}{\partial y_{ik}}, \) etc.

\(^5\) We suppress the complementary slackness conditions associated with the utility, production set, and resource balancing restrictions and their respective Lagrange multipliers.
The specific distinction between this model and the one found in Baumol and Oates can be found in the last row of the second column of Table 2.1. Rather than there being a single pollutant that is controlled (i.e. \( p=1 \)), there is now a set of control conditions, one for each pollutant \( l \). Collectively the conditions found in the second column of Table 2.1 are necessary for a Pareto optimum, whose existence and uniqueness is guaranteed given our assumptions regarding the form of the utility functions and the production sets.

### Table 2.1: Optimality Conditions with Multiple Externalities

<table>
<thead>
<tr>
<th>Choice Variable</th>
<th>Pareto Optimality Conditions</th>
<th>Market Equilibrium</th>
<th>Optimal Prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{ij} )</td>
<td>( \lambda_j u_i^j - \omega_i \leq 0 )</td>
<td>( p_i - \alpha_j u_i^j \geq 0 )</td>
<td>( p_i = \omega_i )</td>
</tr>
<tr>
<td></td>
<td>( x_j (\lambda_j u_i^j - \omega_i) = 0 )</td>
<td>( x_j (p_i - \alpha_j u_i^j) = 0 )</td>
<td>( x_j )</td>
</tr>
<tr>
<td></td>
<td>( \forall i, j )</td>
<td>( \forall i, j )</td>
<td>( \forall i, j )</td>
</tr>
<tr>
<td>( y_{ik} )</td>
<td>(-\mu_k f_i^k + \omega_i = 0 )</td>
<td>( p_i - \beta_k f_i^k = 0 )</td>
<td>( p_i = \omega_i )</td>
</tr>
<tr>
<td></td>
<td>( \forall i, k )</td>
<td>( \forall i, k )</td>
<td>( \forall i, k )</td>
</tr>
<tr>
<td>( s_{kl} )</td>
<td>(-\mu_k f_i^k + \sum_j \lambda_j u_i^j - \sum_k \mu_{kl} f_i^k \leq 0 )</td>
<td>(-t_z - \beta_k f_i^k \leq 0 )</td>
<td>( t_z = \sum_j \lambda_j u_i^j - \sum_k \mu_{kl} f_i^k )</td>
</tr>
<tr>
<td></td>
<td>( s_{kl} \left(-\mu_k f_i^k + \sum_j \lambda_j u_i^j - \sum_k \mu_{kl} f_i^k \right) = 0 )</td>
<td>( s_{kl} \left(-t_z - \beta_k f_i^k \right) = 0 )</td>
<td>( \forall k, l )</td>
</tr>
<tr>
<td></td>
<td>( \forall k, l )</td>
<td>( \forall k, l )</td>
<td>( \forall k, l )</td>
</tr>
</tbody>
</table>
2.2.2 Characterizing a Market Equilibrium with Multiple Pollutants

Before we interpret the necessary conditions for a Pareto optimal outcome, and in particular what they say about the optimal pollution tax levels, we first explore the market conditions faced by consumers and producers. In this setting, we allow the regulator to impose a unique tax per unit of emissions on each pollutant \(l\). Consumers minimize their expenditures subject to their given utility identified above \((u'^j)\). Thus the Lagrangian for the representative individual \(j\) is:

\[
L_j = \sum_i p_ix_{ij} + \alpha_j \left[u'^j - u'(\cdot)\right]
\]

subject to: \(x_{ij} \geq 0\)

where \(\alpha_j\) is a Lagrange multiplier and the real choice variables are each \(x_{ij}\). The optimality and complementary slackness conditions associated with the real choice variables for this problem are reported in the first row of the third column of Table 2.1.

On the production side, the representative firm maximizes its profits subject to market prices and technological restrictions on its production possibilities. Furthermore, the regulator may impose a tax \(t_{z_i}\) on the representative firm’s emissions of each pollutant \(s_{z_i}\). The Lagrangian for the representative firm \(k\) is:

6 Unlike the treatment in Baumol and Oates, we avoid considering whether or not it is desirable to compensate or punish those exposed to emissions. We take it as a well-established result that they should not as those harmed by pollution fully internalize the effect of pollution on themselves.
\[ L_k \equiv \sum_i p_i y_{ik} - \sum_i t_{z_i} s_{\overline{i}} - \beta_k f^k(\cdot) \]  

subject to: \( s_{\overline{i}} \geq 0 \)

where \( \beta_k \) is a Lagrange multiplier and the real choice variables are \( y_{ik} \) and \( s_{\overline{i}} \). As in Baumol and Oates we use the notation \( \overline{k} \) to represent the firm that generates the particular share of pollutant \( l \) as opposed to the firms that are affected by it. The optimality conditions for the representative firm’s input and output choices are reported in the second row of the third column of Table 2.1. The optimality and associated complementary slackness conditions for this firm’s emission of pollutant \( z_i \) given the tax \( t_{z_i} \) are reported in the third row of the third column of Table 2.1.

2.2.3 Comparing the Efficient and Market Outcomes

As in the case where a single pollutant is being emitted, there exists a set of prices and optimally chosen taxes that replicate the social optimum. These are found in the last column of Table 2.1. We see in the cell in the lower right corner that, as in the single pollutant case, the regulator ought to choose a suite of taxes, where the tax on each pollutant \( l \) equals the sum of the marginal damages imposed on every agent in the economy from an additional increment of that pollutant at the efficient outcome. Described more succinctly, the optimal tax for each pollutant should equal the marginal social damage of that particular pollutant. Conditional on the regulator selecting these optimal tax levels, the economy will replicate the Pareto optimal distribution of resources and goods.
We can interpret the conditions for the optimal tax levels more cleanly if, as Baumol and Oates do, we view these conditions by considering their level relative to the value of one particular resource in this economy: time (i.e. labor and leisure). Let \( i^* \) represent this particular resource. Furthermore, we assume that each individual and each firm employs some of this resource so that \( x_{i,j} > 0 \) and \( y_{i,j} > 0 \). As such, the optimality conditions with respect to the use of labor/leisure hold with equality for each consumer and producer, implying:

\[
\lambda_j = \frac{\omega_j}{u'_j}, \quad \mu_k = \frac{\omega_k}{f'_k}
\]

Taking this information and incorporating it into the expression for the optimal tax for pollutant \( l \) we have:

\[
t_{z_l} = \omega_j \left[ -\sum_j \frac{u'_{z_l}}{u'_j} + \sum_k \frac{f'_{z_l}}{f'_k} \right] \quad (2.5)
\]

The expression \( u'_{z_l}/u'_j \) represents the compensation in the form of an amount of the good \( x_{i,j} \) required to maintain individual \( j \)'s utility given an increase in \( j \)'s exposure to \( z_l \) (holding all other goods consumed by \( j \) constant). This relationship can be easily shown:

\[
0 = du^j = u'_{z_l} dz_l + u'_{i,j} dx_{i,j} \Rightarrow \frac{u'_{z_l}}{u'_j} = \frac{dx_{i,j}}{dz_l}
\]

(2.6)
A similar expression can be derived for the compensation to firm $k$ in terms of the input $y_{ij}$ (which takes on a negative value) to keep the firm on its production frontier given an increase in $z_i$. Employing these expressions in (2.5) we have:

$$t_z = \omega_i \left[ \sum_j \frac{\partial x_{ij}}{\partial z_i} - \sum_k \frac{\partial y_{ik}}{\partial z_i} \right]$$

The expression on the right side of (2.7) equals the compensation, measured by the value of labor $\omega_i$, required by each agent affected by the pollutant $z_i$ to offset the damages they experience from the marginal emission of $z_i$. The marginal compensation required is actually the marginal damage to each agent.\(^7\) While we have not demonstrated so here, presumably it can also be shown that a set of tradable permits $\Xi_l$ $(l=1,\ldots,p)$ exist that yield prices equal to the optimal taxes.

While these results are not surprising, sometimes a simple model is powerful in allowing one to see what components of reality are missing and in turn to demonstrate areas for research. For example, as described in Chapter 1, partial equilibrium modes have considered complications where the emissions that the individuals and firms are exposed to is not simply equal to the sum of emissions released by the firms. This may be due to the distance between emitters and those affected by pollution, or because the emissions form a secondary pollutant that in turn affects the individuals and firms. Both of these extensions imply a more general form

\(^7\) Indeed, the term “compensation” here is a measure of the real welfare loss measured by the minimum willingness to accept of each affected individual and firm to be indifferent to its loss from the marginal contribution of $z_i$. It is not a claim that the individuals or firms should receive some payment for the damages they incur as a result of $z_i$. See footnote 6.
of the relationship between the exposure of individual $j$ to some pollutant $z_{jv}$ and a subset of the different discharges from the firms. In this model we could represent this general form $z_{jv} = Z_{jv}(s_{11}, \ldots, s_{1p}, \ldots, s_{hv}, \ldots, s_{hp})$, $(v=1, \ldots, r)$. Note that there may be $r$ of these pollutants and that $r$ is not necessarily equal (or less than) $p$, the number of emission types. Also, as mentioned in Chapter 1, the interactions may generate non-convexities such that they are multiple local maxima to the problem (2.2).\(^8,9\)

Another component missing from this model is any uncertainty on behalf of the regulator about the nature of damages and the demand for pollution by the regulated sources. Weitzman (1974) showed that in the presence of uncertainty regarding the abatement cost (benefit of pollution) of the emitting firms that the regulator may no longer be indifferent to using tax or quantity instruments to control pollution. As described in the previous chapter, the bulk of this dissertation considers extending the Weitzman model to multiple pollutants. But before we get to that point, we first consider an explicit and deterministic model of the abatement decisions of a

\[^{8}\text{Tietetenberg (1973b) uses a general-equilibrium model with a more general relationship between pollutants and the externalities of the form } z_{jv} = Z_{jv}(\cdot) = Z_{jv}(\cdot) \text{ for all } j \text{ (he refers to the pollutants } s_{jl} \text{ as } \text{“waste products” and the } z_{vl} \text{ as pollutants) to show that a set of taxes can be used to generate a Pareto optimal outcome. Despite his use of a general form of } Z_{jv}(\cdot), \text{ he does not acknowledge that the form of the } Z_{jv}(\cdot) \text{ functions themselves may create the possibility of multiple maxima or violate sufficiency conditions. Perhaps this is because he only interpreted } Z_{jv}(\cdot) \text{ capturing a spatial relationship between the } s_{jl} \text{'s and } z_{vl} \text{ that may change over time and so he does not consider the possibility that } Z_{jv}(\cdot) \text{ may not be additively separable. Ruff (1972) does consider the case where } Z_{jv}(\cdot) \text{ is not additively separable but also does not acknowledge that the relationship between emissions may yield non-convexities even though he is particular interested in the control of smog (ozone).}

\[^{9}\text{Another potential source of multiple local maxima in this problem is that the externalities themselves may introduce non-convexities in the social production possibilities set (see Chapter 8 of Baumol and Oates). While it is not technically accurate to suggest that increasing the number of pollutants increases the possibility that such non-convexities exist, the notion has some intuitive appeal.}

31
firm emitting multiple pollutants. This model will be used to inform the market simulation exercise that follows in Chapter 4.

2.3 Analytical Model of Joint Abatement

2.3.1 Introduction

Chapter 4 of this dissertation describes a market simulation exercise that explores a case where a regulator is choosing between tax and quantity instruments to control two pollutants but is uncertain of the joint abatement control costs of the regulated sources. That analysis is presented for a particularly policy-relevant case study: the control of mercury (Hg) and sulfur dioxide (SO₂) emissions from coal-fired power plants. There are pollution control technologies designed to control Hg and technologies designed to abate SO₂ from these plants. However, the pollution control technologies designed to abate SO₂ emissions also abate Hg emissions, although the extent to which they do is uncertain.10 We assume that this uncertainty only maintains for the regulator and that the regulated sources will know the effect of these controls on their Hg emissions by the time they make compliance decisions, if not earlier.

While the regulatory setting is uncertain from the perspective of the regulator, the market simulation model is deterministic. In order to capture the effect of uncertainty from the regulator’s perspective, the simulation model is solved for a

10 The actual policy setting is even more complicated than described here. Technologies used to control other common pollutants also have an uncertain effect on Hg emissions. To keep the discussion simple we only focus on the effect of SO₂ controls at this point. Chapter 4 describes the actual technology setting in detail.
number of possible outcomes of the effect of SO₂ controls on Hg emissions. The regulator assigns a probability weight to each possible outcome.

Before we get to the market simulation exercise, however, it is informative to understand in a simple model how changes in certain parameters affect emissions, prices and abatement intensity in this particular context. The following model is framed around the particular technology setting described above. There is a control specifically designed to abate SO₂ and another specifically designed to abate Hg. The control designed to abate SO₂ also abates Hg. In particular, we are interested in how changing the effectiveness of the SO₂ control in reducing Hg emissions affects the use of the abating inputs, the emissions of the pollutants if taxes are used, and the allowance prices if cap-and-trade programs are used. Restricting the model to represent a particular technology setting is both its strength and weakness. While it helps us judge and understand the reasonableness of the results from the simulation model, it does not provide a particularly broad insight into the regulation of multiple pollutants.

2.3.2 Characterization of Firm Problem

The following model provides some perspective into how we should think about the effect of changing the impact of SO₂ controls on Hg emissions. We start with a representative firm that is minimizing the cost of controlling two pollutants that arise from the given use of a dirty input, \( x \). The two pollutants are \( m \) and \( s \). Heuristically, we can think of \( x \) as coal, \( m \) as Hg and \( s \) as SO₂.
The total emission of $s$ is proportional to $x$ and can be reduced by a pollution control technology. The function $\sigma(l)$ represents the “emissions modification factor” of this abatement technology where $l$ is an input to the abatement process. The emissions modification factor is the percent of emissions that remain uncontrolled after the technology has been applied.\textsuperscript{11} The input $l$ is also chosen heuristically as lime is a key input to technologies called “scrubbers” that abate SO$_2$. Emissions of $s$ can be expressed as:\footnote{The emissions modification factor equals 1 minus the removal efficiency of the technology. The term removal efficiency, which equals the percentage of uncontrolled emissions that are abated, is more commonly used to describe the effectiveness of an abatement technique.}

$$s = x\sigma(l)$$

(2.8)

Similar to the emission of $s$, the emission of $m$ is also proportional to $x$. A technology designed to abate $m$ is characterized by the modification factor $\mu(a)$ . Here the variable name $a$ is chosen because activated carbon is the major variable input to reducing Hg emissions. At the same time, the input $l$ also reduces emissions of $m$ by the modification factor $\delta(yl)$ , where $y$ is some non-negative scalar. Changes to the scalar $y$ will be used to mimic the effect of changing the effectiveness of $l$ in abating $m$. The total emission of $m$ can thus be represented as:

$$m = x\mu(a)\delta(yl)$$

(2.9)

\footnote{A more accurate representation of emissions is:

$$s = \alpha_s x\sigma(l)$$

where $0<\alpha_s<1$ such that $s$ is a share of the weight of $x$. However, for expositional convenience we assume $\alpha_s=1.$}
Note the simple multiplicative form of these abatement technologies. This captures the idea that the abatement technologies are in sequence and that the use of one does not influence the performance of the other. This is a somewhat realistic assumption in the case of Hg and SO\textsubscript{2} controls. The use of scrubbers usually does not interfere significantly with the operation of an activated carbon injection (Hg removal) system and vice-versa. Where this assumption is a bit unrealistic is that the performance of a Hg control technology is somewhat dependent on the concentration, not just the total quantity, of the pollutant in the flue gas stream.

All of the emission modification factors are assumed to have the following properties:

\[ g'(\cdot) < 0 \quad (2.10) \]
\[ g''(\cdot) > 0 \quad (2.11) \]

\[ \frac{d^2}{d[i]^2} \ln(g(i)) = \frac{1}{[g(i)]^2} \left[ g''(i) g(i) - \left[ g'(i) \right]^2 \right] > 0 \quad (2.12) \]

\[ g(0) = 1 \quad (2.13) \]

\[ \lim_{i \to \infty} g(i) = 0 \quad (2.14) \]

where \( g(\cdot) \) is \( \{ \sigma(\cdot), \mu(\cdot), \delta(\cdot) \} \) and \( i \) is \( \{ l, a, \gamma \} \). The first two assumptions apply the intuition that the percentage reduction in emissions, \( 1 - g(\cdot) \), increases at a decreasing rate with an increase in the abating input \( i \). Abatement technologies generally have these characteristics (at least over a meaningful range) as physical and
chemical limits (such as reactive mixing) are approached. Figure 2.1 provides a graphical representation of \( g(.) \).

The third restriction requires a bit more explanation. What is being assumed is that the percentage change in the emissions modification factor decreases at an increasing rate as \( i \) increases. Thus, it gets more and more difficult, not just in absolute but in percentage terms, to lower the emissions modification factor with an additional unit of \( i \). This restriction has intuitive appeal as the emissions modification factor itself is a percentage representation (the percentage of uncontrolled emissions that remain).

Figure 2.1. Graphical Representation of \( g(.) \)

![Graph of g(i)](image)

The last two assumptions simplify the problem but are not necessary for the results that follow. As these functions should be thought of as emission modification factors, which are percentage representations, it is natural to treat them as bounded between 0 and 1. Even if (2.13) did not hold in practice, as long as the remaining assumptions hold, the function \( g(.) \) could be normalized as such. The
assumption (2.14) is additionally overly prescriptive as the maximum efficacy of the technology simply must be non-negative. Indeed, \( g(.) \) may even have a minimum at some finite \( i \), but considering such possibilities would unduly complicate the problem at this point.

Each pollutant may be controlled by either a single-valued tax or quantity restriction. There are thus four possible instrument mixes that the firm may face and they are summarized in Table 2.2 While this model has been described as the case of an individual firm, we can also think of it as representing the control of a group of regulated sources. Therefore we may then think of the shadow values on the quantity restrictions, either \( \bar{s} \) or \( \bar{m} \), as the respective allowance prices of these pollutants.

Table 2.2: Potential Instrument Mixes

<table>
<thead>
<tr>
<th></th>
<th>m instrument</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tax</td>
<td>quantity</td>
</tr>
<tr>
<td>s instrument</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tax</td>
<td>( t_m, t_s )</td>
<td>( \bar{m}, t_s )</td>
</tr>
<tr>
<td>quantity</td>
<td>( t_m, \bar{s} )</td>
<td>( \bar{m}, \bar{s} )</td>
</tr>
</tbody>
</table>

We are primarily interested in how increasing \( \gamma \) changes the use of the abating inputs \( l \) and \( a \) as well as the emissions and/or allowance price of the two pollutants. We begin by considering the case where both pollutants are controlled by a tax.
2.3.3 Both Pollutants Controlled by Tax Instruments

Let us start by analyzing the firm’s problem when each pollutant is controlled by a tax. Further, let us simplify the analytics and assume that the use of the polluting input, \(x\), equals 1.\(^{13}\) The input \(l\) is available at a constant cost, \(p_l\), while the per unit price of \(a\) is \(p_a\). The firm’s problem is thus:

\[
\min_{l,a} : p_l a + p_a a + t_x \sigma(l) + t_m \mu(a) \delta(\gamma l)
\]  
(2.15)

The necessary conditions for this problem are:

\[
p_l + t_x \sigma'(l) + t_m \gamma \mu'(a) \delta'(\gamma l) = 0
\]
\[
p_a + t_m \mu'(a) \delta(\gamma l) = 0
\]  
(2.16)

To be confident that these first-order conditions yield a global minimum, it is sufficient to know that the following Hessian matrix:

\[
\begin{bmatrix}
  t_x \sigma''(l) + t_m \gamma \mu''(a) \delta(\gamma l) & t_m \gamma \mu'(a) \delta'(\gamma l) \\
  t_m \gamma \mu'(a) \delta'(\gamma l) & t_m \mu''(a) \delta(\gamma l)
\end{bmatrix}
\]

is positive definite. That is, the following two restrictions must hold:

\[
t_m \gamma^2 \mu(a) \delta''(\gamma l) + t_x \sigma''(l) > 0
\]  
(2.17)

\[
t_m \left[ t_m \gamma^2 \left[ \mu(a) \delta(\gamma l) \mu''(a) \delta''(\gamma l) - \mu'(a)^2 \delta'(\gamma l)^2 \right] + t_x \sigma''(l) \mu''(a) \delta(\gamma l) \right] > 0
\]  
(2.18)

Our assumptions (2.10) through (2.12) assure that (2.17) and (2.18) are satisfied.

\(^{13}\) This assumption is maintained throughout the remainder of the analysis.
Again, we are interested in how changing $\gamma$ affects the use of the abating inputs. Applying standard comparative statics techniques we have:

$$
\frac{dl}{d\gamma} = \frac{-t_m l \delta'(\gamma l)}{t_m \gamma^2 \left[ \mu(a) \delta'(\gamma l) \mu''(a) \delta''(\gamma l) - \mu''(a)^2 \delta''(\gamma l)^2 \right] + t \sigma^*(l) \mu''(a) \delta(\gamma l)} \leq 0 
$$

$$
\frac{da}{d\gamma} = \frac{\mu'(a) \delta'(\gamma l)}{t_m \gamma^2 \left[ \mu(a) \delta'(\gamma l) \mu''(a) \delta''(\gamma l) - \mu''(a)^2 \delta''(\gamma l)^2 \right] + t \sigma^*(l) \mu''(a) \delta(\gamma l)} < 0
$$

The denominator of both of these expressions is positive as long as the second-order condition (2.18) holds. The numerator of (2.20) is negative implying that the use of the abating input $a$ falls as the effectiveness of $l$ in abating $m$ increases. Perhaps surprisingly, as can be seen in (2.19), an increase in the effectiveness of $l$ in abating $m$ does not necessarily imply an increase in the use of the abating input $l$. The numerator may be positive or negative. Note that a necessary condition for the numerator to be negative, and thus for $l$ to decrease as $\gamma$ increases is:

$$
\frac{\delta'(\gamma l)}{\delta''(\gamma l)} \left[ 1 + \frac{l \gamma \delta''(\gamma l)}{\delta''(\gamma l)} \right] > \frac{\mu''(a)^2}{\mu(a) \mu''(a)}
$$

While the right side of (2.21) is clearly positive and less than one given assumption (2.12), the left side can take any sign and may be greater than one.

This ambiguity actually has intuitive appeal. On one hand, there is an incentive to use more $l$ given that it is more effective in abating $m$; the tax burden of emitting $m$ can be lowered. On the other hand, less $l$ can be used to achieve the same
level of abatement, so reducing its use is desired. On net, these competing effects imply that the change in the use of $l$ is ambiguous.$^{14}$

We are also interested in how increasing $\gamma$ changes the emissions of the two pollutants. The change in the emission of $s$ (which is defined in (2.8)) is:

$$\frac{ds}{d\gamma} = \sigma'(l) \frac{\partial l}{\partial \gamma} \geq 0$$  \hspace{1cm} (2.22)$$

where $\partial l/\partial \gamma$ is defined by (2.19). We see that the change in $s$ is ambiguous as the change in $l$ is ambiguous. If the use of $l$ rises or falls, the emission of $s$ decreases or increases. The change in $m$, however, is unambiguous (taking the definition of $m$ from (2.9)):

$$dm = \mu'(a)\delta'(\gamma l) \frac{\partial a}{\partial \gamma} d\gamma + \gamma \mu(a)\delta'(\gamma l) \frac{\partial l}{\partial \gamma} d\gamma + l\mu(a)\delta'(\gamma l) d\gamma$$  \hspace{1cm} (2.23)$$

The crosspartials in this expression can be substituted with the comparative statics expressions (2.19) and (2.20) to yield:

$$\frac{dm}{d\gamma} = \frac{\delta(\gamma l)\delta'(\gamma l)[l\sigma'(l) - t_M\gamma \mu(a)\delta'(\gamma l)][\mu(a)\mu^*(a) - \mu'(a)^2]}{t_M\gamma^2[\mu(a)\delta(\gamma l)\mu^*(a)\delta^*(\gamma l) - \mu'(a)^2\delta'(\gamma l)^2] + t_1\sigma'(l)\mu^*(a)\delta(\gamma l)} < 0$$  \hspace{1cm} (2.24)$$

The denominator of this expression is positive given the second order conditions of this problem. In the numerator the first and second bracketed expressions are positive, while the term multiplying these bracketed expressions is negative, given our

---

$^{14}$ The possibility that the use of $l$ may fall with an increase in its effectiveness in abating $m$ is most easily seen in the case where $\mu(a) = 1$ and $t_M = 0$. Appendix 2.A reports this case.
assumptions (2.10)-(2.12). Therefore, the emission of $m$ falls as the effectiveness of $\gamma$ increases, as would be expected.

2.3.4 Summary of Effects of Changing Technological Relationship

Appendix 2.B records a similar analysis for the three other instrument mixes this firm may face. The collective results of this analysis are summarized in Table 2.3. The variables $\lambda_s$ and $\lambda_m$ are the shadow values on the emission constraints on $m$ and $s$. As noted above, we can interpret this firm-level problem as representing a composite abatement cost function for multiple regulated sources and that the shadow values represent the prices of tradable allowances.

Table 2.3: Effect of an Increase in $\gamma$ on the Variables of Interest

<table>
<thead>
<tr>
<th>Instrument Mix</th>
<th>$t_m, t_s$</th>
<th>$\bar{m}, t_s$</th>
<th>$t_m, \bar{s}$</th>
<th>$\bar{m}, \bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta l$</td>
<td>?</td>
<td>?</td>
<td>0</td>
<td>?$^1$</td>
</tr>
<tr>
<td>$\Delta a$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>?</td>
<td>?</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$\Delta m$</td>
<td>$\downarrow$</td>
<td>---</td>
<td>$\downarrow$</td>
<td>---</td>
</tr>
<tr>
<td>$\Delta \lambda_s$</td>
<td>---</td>
<td>---</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>$\Delta \lambda_m$</td>
<td>---</td>
<td>$\downarrow$</td>
<td>---</td>
<td>$\downarrow$</td>
</tr>
</tbody>
</table>

$^1$ The asymmetry in claims regarding $\Delta l$ when $s$ is controlled by a quantity is due to the number of constraints equaling the number of choice variables in the case where both pollutants are controlled by a quantity. See Appendix 2.B for details.

The message of this analysis is that changing the nature of the technological relationship between the level of $l$ and the emissions of $m$ does not have a clear effect on the emissions of $s$ if $s$ is controlled by a tax, or the shadow value of
if $s$ is controlled by a quantity. There are two competing influences on the use of $l$ as $\gamma$ increases. The first is to increase the use of $l$ as it has a greater impact on the level of $m$. The countervailing influence is that the level of $l$ already chosen (before $\gamma$ increased) has a greater effect on $m$ so now less $l$ is desired. As these competing influences affect the marginal benefit of using $l$ to control $m$, they also have an ambiguous effect on the shadow value of the constraint on $s$ when $s$ is controlled by a quantity.

Furthermore, we see in Appendix 2.B that, conditional on $m$ being controlled by a tax, the direction of change in $l$ (and thus $s$) if $s$ is controlled by a tax is dictated by the same expression that indicates whether $\lambda_s$ rises or falls. These expressions are (2.21) and (2.46) respectively. Perhaps more importantly, conditional on $s$ being controlled by a tax, the direction of the change in the level of $l$ as $\gamma$ changes may depend on whether $m$ is controlled by a tax or quantity. That the direction of $l$ may depend on whether $m$ is controlled by a tax or quantity as can be seen from a comparison of the expressions (2.21) and (2.38). This is true even if the quantity restriction on $m$ is chosen such that its shadow value equals the level of the tax.

An ancillary observation from this analysis is that increasing the abatement effectiveness of $l$ on $m$ unambiguously either lowers the emission of $m$ or the shadow value of a quantity restriction on $m$. Also, the use of $a$ falls if $\gamma$ increases regardless of how the pollutants are controlled. These results will be helpful in evaluating the results from the simulation modeling in Chapter 4.
Appendix 2. A Single Pollutant Example

How does changing the effectiveness of the abating input $l$ change the use of that input when only $m$ is being regulated? Here we ignore the level of emissions of $s$ (assume it is not regulated) and assume no unique technology for abating $m$ exists (so $\mu(a) \equiv 1$). Again, we let $x$ be the exogenously-determined polluting input, $p_l$ be the price of the abating input, and $\delta(\gamma l)$ be the emissions modification factor which has all of the restrictions (2.10)-(2.14). In this setting and with an emissions tax on $m$ the firm minimizes:

$$p_l l + t_m x \delta(\gamma l)$$  \hspace{1cm} (2.25)

We again simplify this problem and assume that the use of the polluting input, $x$, is 1. The first-order condition for this problem is:

$$p_l + t_m x \delta'(\gamma l) = 0$$  \hspace{1cm} (2.26)

The second order condition of this problem is satisfied:

$$t_m \gamma^2 \delta''(\gamma l) > 0$$  \hspace{1cm} (2.27)

Now we check to see what happens to the choice of the abating input $l$ if the abating effectiveness of $\gamma$ increases. Using (2.26), which implicitly defines the optimal level of $l$ as a function of $\gamma$, we have:

$$\frac{dl}{d\gamma} = \frac{-\gamma l \delta''(\gamma l) - \delta'(\gamma l)}{\gamma^2 \delta^*(\gamma l)} \leq 0$$  \hspace{1cm} (2.28)
In the numerator of (2.28) we see the competing influences of the effect of increasing $\gamma$ on the optimal level of $l$. The first part of the numerator is negative. This part captures how increasing the effectiveness of $l$ increases the inframarginal abatement of $m$, suggesting that the use of $l$ may be lowered. The bigger $\gamma l$ is already, the stronger this influence. The second part of the numerator is positive and captures the benefit of increasing the use of $l$ on the margin to reduce the total tax payment from emitting $m$. Rearranging (2.28) we have:

$$\frac{dl}{d\gamma} = -\frac{l}{\gamma} \left[1 + \frac{\delta'(\gamma l)}{\gamma l \delta^*(\gamma l)}\right] \leq 0$$

(2.29)

Where we can see that the effect of increasing $\gamma$ on $l$ depends on the elasticity of $\delta'(\gamma l)$ with respect to $\gamma l$. Note that the bracketed term in (2.29) is familiar as it is present in the expressions (2.19), (2.35) and (2.45) which indicate the sign of $dl/d\gamma$ or $d\lambda_c /d\gamma$ for the various instrument combinations.

---

15 Despite the ambiguity in the level of $l$ as $\gamma$ increases, the level of $m$ strictly falls as $l$ increases: $dm/d\gamma = -\delta'(\gamma l) \gamma \delta^*(\gamma l) < 0$. 

44
Appendix 2.B Alternative Instrument Combinations

2.B.1 \( m \) controlled by a Quantity, \( s \) controlled by a Tax

In this case the pollutant \( m \) is controlled by a quantity and the pollutant \( s \) is controlled by a tax. Here the firm faces the restriction:

\[
\mu(a)\delta(\gamma l) \leq \bar{m}
\]  
(2.30)

where \( \bar{m} \) is the quantity restriction on the pollutant \( m \). The firm’s problem can be expressed:

\[
\min_{l,a} : p_l l + p_a a + t_s \sigma(l) 
\]  
(2.31)

subject to the restriction (2.30). This problem can be represented by the Lagrangian:

\[
L(l,a,\lambda_m) \equiv p_l l + p_a a + t_s \sigma(l) + \lambda_m (\mu(a)\delta(\gamma l) - \bar{m})
\]  
(2.32)

The first-order conditions for this problem are\(^{16}\):

\[
\begin{align}
 p_l + t_s \sigma'(l) + \lambda_m \gamma \mu(a)\delta'(\gamma l) &= 0 \\
 p_a + \lambda_m \mu'(a)\delta(\gamma l) &= 0 \\
 \mu(a)\delta(\gamma l) - \bar{m} &= 0
\end{align}
\]  
(2.33)

These conditions describe a unique global minimum provided that the following bordered Hessian is positive-definite (has a negative determinant):

\[\text{\ldots}\]

\(^{16}\) We assume that the quantity restriction holds and ignore the complementary slackness condition for this problem.
\[
H_m = \begin{bmatrix}
0 & \gamma \mu(a) \delta'(\gamma l) & \mu'(a) \delta(\gamma l) \\
\gamma \mu(a) \delta'(\gamma l) & t, \sigma''(l) + \lambda_m \gamma^2 \mu(a) \delta''(\gamma l) & \lambda_m \gamma \mu'(a) \delta'(\gamma l) \\
\mu'(a) \delta(\gamma l) & \lambda_m \gamma \mu'(a) \delta'(\gamma l) & \lambda_m \gamma \mu''(a) \delta'(\gamma l)
\end{bmatrix}
\]

which indeed it is:

\[
|H_m| = -\delta(\gamma l) \left[ \gamma^2 \lambda_m \mu(a) \left[ \frac{\mu'(a)^2}{\delta'(\gamma l)} - \frac{\delta'(\gamma l)}{\delta''(\gamma l)} \right] + \sigma''(\gamma l) \left[ \frac{\mu(a) \mu''(a) - \mu'(a)^2}{\delta'(\gamma l)} \right] + t, \sigma'(l) \delta'(\gamma l) \right] < 0 \tag{2.34}
\]

The two central bracketed terms are positive given our assumption (2.12) and the last term in the bracketed expression is positive given our assumption (2.11). Notice that we introduced the label \(|H_m|\) to represent the second-order condition (2.34). This expression presents itself in the comparative statics expressions.

We now turn to the question of how the use of abating inputs \(l\) and \(a\), as well as the shadow value on the emission restriction \(m\), change as \(\gamma\) increases. These expressions can be summarized:

\[
\frac{dl}{d\gamma} = \frac{\lambda_m \gamma l \delta'(\gamma l)^2 \mu(a) - \mu(a) \mu''(a) - \mu'(a)^2}{|H_m|} \leq 0 \tag{2.35}
\]

\[
\frac{da}{d\gamma} = \frac{\mu(a) \delta'(\gamma l) \mu'(a) \left[ lt, \sigma''(l) - \gamma \lambda_m \mu(a) \delta'(\gamma l) \right]}{|H_m|} < 0 \tag{2.36}
\]

\[
\frac{d\lambda_m}{d\gamma} = \frac{-\lambda_m \delta'(\gamma l) \left[ \mu(a) \mu''(a) - \mu'(a)^2 \right] \left[ lt, \sigma''(l) - \gamma \lambda_m \mu(a) \delta'(\gamma l) \right]}{|H_m|} < 0 \tag{2.37}
\]
We see in (2.36) and (2.37) that the use of the abating input \( a \) falls as does the shadow value of the emission constraint \( \bar{m} \) when \( \gamma \) increases. The numerator in both of these expressions is positive given our assumptions (2.10)-(2.12) and the denominator as we have already established is negative. Neither finding is surprising as it is now easier to abate \( m \).

As in the case where both pollutants are controlled by a tax, we see in (2.35) that the effect of increasing \( \gamma \) on the use of the abating input \( l \) is indeterminate. If the following is true:

\[
2 - \frac{\delta (\gamma l)}{\gamma l \delta' (\gamma l)} \left[ 1 + \frac{\gamma l \delta^* (\gamma l)}{\delta' (\gamma l)} \right] \frac{\mu(a) \mu^*(a)}{\mu'(a)^2} > 0
\]  

(2.38)

then the use of \( l \) increases with an increase in \( \gamma \), otherwise the use of \( l \) falls or stays the same.\(^\text{17}\) The logic is as before. Now that each unit of \( l \) has a greater effect in reducing the emission of \( m \), less \( l \) is needed to maintain a given level of \( m \). However, the greater effectiveness of \( l \) allows the firm to substitute away from \( a \), suggesting that more \( l \) will be used. On net the effect on \( l \), and consequently on \( s \) (following (2.22)), is indeterminate.

2.B.2 \( m \) Controlled by a Tax, \( s \) Controlled by a Quantity

Next we consider the case where the firm faces a tax on the pollutant \( m \) and a restriction on the pollutant \( s \) that it be no greater than \( \bar{s} \). The restriction on \( s \) can be expressed:

\[ s \leq \bar{s} \]

\(^{17}\) Note that the right side of (2.38) must be greater than 1 given (2.12). We have not imposed any restriction on \( \delta (\gamma l) \) that would prevent the left side of (2.38) from being greater than 1.
\[ \sigma(l) \leq \bar{s} \quad (2.39) \]

The firm’s problem can be solved via the Lagrangian:

\[
L(l,a,\lambda) = p_l l + p_a a + \lambda [\sigma(l) - \bar{s}] + t_m \mu(a) \delta (\gamma l) \quad (2.40)
\]

This problem has the following first-order conditions\(^{18}\):

\[
\begin{align*}
    p_l + \lambda \sigma'(l) + t_m \gamma \mu(a) \delta'(\gamma l) &= 0 \\
    p_a + t_m \mu'(a) \delta(\gamma l) &= 0 \\
    \sigma(l) - \bar{s} &= 0
\end{align*}
\quad (2.41)
\]

If the determinant of the following bordered Hessian is negative, these first-order conditions describe a global minimum:

\[
H_{x} \equiv \begin{bmatrix}
0 & \sigma'(l) & 0 \\
\sigma'(l) & t_m \gamma \mu(a) \delta''(\gamma l) + \lambda \sigma''(l) & t_m \gamma \delta'(\gamma l) \mu'(a) \\
0 & t_m \gamma \delta'(\gamma l) \mu'(a) & t_m \delta(\gamma l) \mu''(a)
\end{bmatrix}
\]

The determinant is indeed negative:

\[
|H_{x}| = -t_m \delta(\gamma l) \sigma'(l)^2 \mu''(a) < 0 \quad (2.42)
\]

given our assumption (2.11). Note that we labeled the determinant \( |H_{x} | \) as \( s \) is being controlled by a quantity. We use this expression in summarizing the comparative statics analysis of the choice variables when \( \gamma \) is increased:

\[
\frac{dl}{d\gamma} = 0 \quad (2.43)
\]

\(^{18}\) Again we assume the constraint binds and ignore the complementary slackness condition.
\[
\frac{da}{d\gamma} = \frac{t_m l \delta'(\gamma l) \mu'(a) \sigma'(l)^2}{|H_\gamma|} < 0 \tag{2.44}
\]

\[
d\lambda_s = \frac{t_m^2 \gamma l \delta''(\gamma l)^2 \sigma'(l) \left[ \frac{\mu(a) \mu''(a)}{\gamma l \delta''(\gamma l)} \left[ 1 + \frac{\gamma l \delta''(\gamma l)}{\delta'(\gamma l)} \right] - \mu'(a)^2 \right]}{|H_\gamma|} \leq 0 \tag{2.45}
\]

Given that we are assuming that the constraint on the emission of \(s\) binds, it is expected that the level of \(l\) does not change in response to an increase in \(\gamma\). We see in (2.43) that this is the case. It is also unsurprising that the use of \(a\) falls with an increase in \(\gamma\) as we see in (2.44).

We see in (2.45) that the effect of increasing \(\gamma\) on the shadow value of the constraint (2.39) is ambiguous. The shadow value increases if the following expression holds:

\[
\frac{\delta(\gamma l)}{\gamma l \delta''(\gamma l)} \left[ 1 + \frac{\gamma l \delta''(\gamma l)}{\delta'(\gamma l)} \right] > \frac{\mu'(a)^2}{\mu(a) \mu''(a)} \,
\]

otherwise it decreases or stays the same. This ambiguity arises from the same competing effects that make the effect of changing \(\gamma\) on \(l\) ambiguous when \(s\) is controlled by a tax. In one sense, the value of using \(l\) to abate \(m\) has fallen because less \(l\) is needed to achieve a certain level of \(m\). This effect increases the shadow value of the constraint on \(s\) as there is a shift in the burden of using \(l\) to the control of \(s\). On the other hand, the marginal unit of \(l\) has an added benefit in reducing \(m\), which suggests that on the margin the benefit of using \(l\) shifts to controlling \(m\). This effect
lowers the shadow value of the constraint on $s$. On net these effects suggest that the change in $l$ is ambiguous.

Finally, we also want to know how the emission of the optimal level of $m$ changes as $\gamma$ adjusts. This change can be expressed (following the approach that generated (2.24)):

$$\frac{dm}{d\gamma} = \frac{l\delta' (\gamma l) }{\mu^*(a)} \left[ \mu(a)\mu''(a) - \mu'(a)^2 \right] < 0$$

(2.47)

Again this is an unsurprising, but comforting, outcome.

2.B.3 Both Pollutants Controlled by a Quantity

Finally we have the case where both pollutants are controlled by a quantity. The restrictions on emissions are expressed in (2.30) and (2.39). Note that except in a very unique case do these constraints simultaneously bind at the firm’s optimal choice of $l$ and $a$ as there are as many constraints as there are choice variables. As such the construction of a constrained optimization problem absent particular parameter values (such that we could identify which constraint binds) would not be meaningful.

An alternative method for dealing with this problem is to make the model even more realistic and impose yet another method for abating one or both of the pollutants. The simplest change would be to introduce a second abatement technology for the control of $s$, solving our degrees-of-freedom problem.\(^{19}\) Carrying out this

\(^{19}\) In the case of controlling SO$_2$ and Hg from coal-fired electricity generators the obvious change would be to have choice in the type of coal used. Different coal types have different Hg and sulfur
analysis would generate more dense expressions than already seen. At this point our
intuition is sufficient for arguing through this case.

For our purposes the most meaningful case is the one where the constraint
on \( m \) binds. This is the case where increasing \( \gamma \) would have an effect on the choice
variables \( a \) and \( l \).\(^{20}\) With an increase in \( \gamma \), both \( a \) and the shadow value of the
constraint on \( m \) fall. More importantly for this discussion, the change in \( l \) is
ambiguous. Now, if we assume that a constraint on the emissions of \( s \) indeed does
bind (because some technology exists to control \( s \) in addition to \( \sigma(l) \)), then as \( l \) rises
or falls the shadow value of the constraint on the emission of \( s \) will decrease or
increase.

\(^{20}\) This case is analogous to one where \( m \) is controlled by a quantity and \( s \) is controlled by a tax set
equal to zero.
CHAPTER 3: ANALYTICAL MODEL OF INSTRUMENT CHOICE UNDER UNCERTAINTY WITH JOINT ABATEMENT

3.1 Introduction

Weitzman (1974) was the first to demonstrate that when a planner is uncertain of the collective cost function of regulated sources, the expected welfare from an optimal price instrument is different from the expected welfare from an optimal quantity instrument. Furthermore, he shows that neither approach is necessarily superior, and identifies conditions sufficient for determining the preferred instrument given a particular regulatory setting. While Weitzman was speaking to a contemporary debate about whether price or quantity controls are preferable for managing a centrally planned economy or the production logistics of a firm, he recognized the applicability of his analysis to the choice between an emissions tax and a cap-and-trade program. Others who independently replicated his main results include Upton (1971), Adar and Griffin (1976) and Fishelson (1976).

Significant attention has been paid in the environmental economics literature to Weitzman’s and related analyses as of late, perhaps in response to the increasing adoption of incentive-based control polices. Stavins (1996) explores the likelihood and effect on instrument choice of a correlation between abatement cost
uncertainty and uncertainty in the benefits of abatement.¹ Recent extensions of the instrument choice under uncertainty literature include evaluations of tax and quantity instruments for the control of a stock pollutant (Hoel and Karp, 2001; Newell and Pizer, 2003). Other recent studies compare alternative regulatory instruments, such as index quantities (Newell and Pizer, 2006; Quirion, 2005) and non-linear taxes (Kaplow and Shavell, 2002; Shrestha, 2001) to the simple “single-valued” tax and quantity approaches.²

As Weitzman acknowledges, his analysis is germane to “one particular isolated economic variable that needs to be regulated” (P. 477).³ This chapter extends a standard instrument choice model to the control of multiple pollutants to see the extent to which insights from the single pollutant analysis continue to apply. While

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¹ Weitzman (1974) shows that uncertainty in the benefits of abatement do not influence the welfare ordering of the tax and quantity instrument except in the case where the uncertainty in benefits is correlated with the uncertainty in costs. One example of such a correlation occurs in the control of emissions from power plants. With warm weather, abatement costs are high when the damage caused by some of the pollutants from combustion is high.

² While there has been renewed interest in nonlinear taxes, there was significant interest in this approach after Weitzman’s study was published. For examples, see Ireland (1977), Karp and Yohe (1979) and Yohe (1981).

³ Other papers in the instrument choice literature that considers circumstances other than when an “isolated” economic variable is being regulated are Yohe (1976b, 1977a, 1981). None of these studies analyzes the choice of regulation when more than one variable is actually being regulated, however. While Koenig (1985) considers the simultaneous regulation of emissions and production (in the form of regulating exports), he assumes that the emissions are proportional to production. See also the studies described in footnote 16.
attention has been paid in the literature to the control of multiple pollutants, these studies employ a deterministic framework and rarely consider implications for instrument choice.

The following section reviews the Weitzman single pollutant analysis, both graphically and analytically, and the key relationship that determines whether price or quantity controls are preferable. It also reviews an interpretation of the Weitzman analysis that is attributable to Yohe (1978). Yohe’s interpretation facilitates understanding of the results that derive from the multiple pollutant case. The balance of the chapter lays out an extension of the single pollutant model to the case where two jointly-controlled pollutants are subject to regulation.

In the two-pollutant case, each pollutant may be controlled by a tax or quantity. The expected welfare expressions for each of the four possible instrument mixes are derived in turn, starting with the case where each pollutant is controlled by a quantity instrument. As there are four possible mixes, it is a challenge to compare expected welfare. Furthermore, as we will see, the expected welfare expression for the case where both pollutants are controlled by a tax is particularly complicated. Fortunately, the expected welfare expressions for the other three mixes are simple and familiar. Thus, the focus of the expected welfare comparisons is to simplify the expected welfare expression for the case where each pollutant is controlled by a tax. Finally, we look at special cases where the abatement benefit and cost functions take extreme forms to see which instrument mix is preferred in those settings. For example, we identify the optimal mix of instruments when the marginal benefit of abating each pollutant is constant.
The two-pollutant model demonstrates three related results. First, it shows that the relative efficiency of an instrument to control one pollutant depends on how jointly abated pollutants are being controlled. Second, the instrument suggested by the analysis for a single pollutant may be inappropriate if joint abatement is not explicitly considered. Third, it demonstrates that pair-wise comparisons of the expected welfare yielded by a subset of instrument combinations do not necessarily suggest the optimal combination of instruments. A consequence of these results is that all instrument combinations must be evaluated before the one that maximizes expected welfare is identified. However, the analysis confirms that when the shapes of the marginal abatement cost and benefit curves are at their extremes, the optimal instrument mix comports with intuition. For example, the optimal instrument mix when the marginal abatement cost of each pollutant is constant places a tax on each pollutant.

3.2 Review of the Single Pollutant Case

3.2.1 Graphical Analysis of Weitzman Result

To set the stage for the multiple pollutant case, we first graphically review the instrument choice analysis for a single pollutant. We assume a setting where the regulator, who wishes to maximize the expected net benefit of abatement, knows the benefit of abating pollution, but is uncertain of the costs of reducing pollution. While the regulator does not know the costs of abatement, she does have well-informed expectations regarding these costs. Furthermore, the regulated sources know their
control costs (the following section describes the information setting in greater
detail).

The regulator may choose to control pollution using an abatement target (a
quantity instrument) or may set a tax on pollution (a price instrument). The
regulated sources are assumed to minimize the cost of controlling pollution so that
they abate up to the point where the marginal cost of further control equals the tax. In
Figure 3.1 we see the marginal benefit of abatement curve (labeled $MB$) and the
expected marginal abatement cost curve (labeled $E[MAC]$) from the problem that the
regulator faces. The variable $m$ represents the level of abatement of the pollutant. The
tax that maximizes expected welfare is $t^*$ while the quantity that maximizes expected
welfare is $m^*$.  

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4 We are operating in abatement space, as opposed to emission space, as that is the tradition in this
literature.

5 It is a special case where it happens that the welfare maximizing tax level is set at the point where the
expected marginal abatement cost equals marginal benefits. For convenience, we assume this is the
case in the graphical analysis and, as we will see, in the analytical problem described in the following
section.
Figure 3.1: Expected Welfare Maximizing Instruments

Figure 3.2 shows the realized welfare for each instrument once the actual marginal abatement cost curve (labeled MAC) is revealed to the regulator. Figure 3.2 shows that abatement costs are lower than the regulator expected, but as we will see, whether costs are higher or lower than expected does not affect the relative performance of the two instruments.
The regulator now sees that the optimal level of abatement is $m^{**}$. If the regulator had chosen the quantity $m^*$, the welfare loss, relative to maximized net benefits, is equal to the area of the triangle labeled $\Lambda_m$. However, if the regulator had chosen to impose a tax $t^*$, abatement would be equal to $m(t^*)$, and the welfare loss relative to maximized net benefits would equal the area of the triangle labeled $\Lambda_t$.

In this case, we see that welfare would be higher if the tax instrument had been adopted. If the true marginal abatement cost curve were closer to the expected marginal abatement cost curve, the welfare losses relative to the optimum from both instruments would be smaller, but the tax instrument would still yield the higher welfare. Furthermore, if costs were actually higher than expected, such that the $MAC$
curve were above the $E[MAC]$ curve, the tax would still outperform the quantity instrument.

That the tax yields the highest welfare does not hold in general, however. If the marginal benefit of abatement curve pivoted around the expected optimum such that its slope were made steeper, we would see that the area of the triangle $\Lambda''$ would decrease while the area of the triangle $\Lambda'$ would increase. Eventually, the areas of the two triangles would be the same. This occurs when the slope of the marginal benefit curve equals the slope of the marginal abatement cost curve. If the marginal benefit curve were made steeper than the marginal abatement cost curve, then the welfare loss from the tax instrument would be greater than for the quantity instrument.

This is the basic insight of Weitzman (1974). Neither instrument necessarily outperforms the other in the face of abatement cost uncertainty. Whether a tax or a quantity instrument is preferred depends on the relative slopes of the marginal benefit and the marginal abatement cost curves. With either instrument the regulator is essentially forming a demand curve for abatement. In the case of a tax, this demand curve is perfectly flat, whereas it has infinite slope when a quantity policy is used. This explains why the tax policy performs best when the marginal benefit of abatement is relatively flat. When the marginal benefit curve is relatively flat, the tax policy generates a demand curve closer in shape to the marginal benefit curve than the quantity policy does. The opposite holds when the marginal benefit curve is relatively steep. We now show this result analytically.
3.2.2 Analytical Derivation of Weitzman Result

We begin with the collective abatement cost function of regulated sources by the function, which is represented by \( C(m;\theta) \) where \( \theta \), from the perspective of the regulator, is a vector of random variables. While unknown to the regulator, we assume that the information \( \theta \) is known to the regulated sources when compliance decisions are made. We can consider this a simple case of asymmetric uncertainty where the regulator does not expect that the benefit of collecting this information is worth the cost. Alternatively, \( \theta \) can be thought of as variables that fluctuate from period to period, and it is particularly difficult for the regulator to respond to these changes by continuously adjusting the instrument level.\(^6\)

The abatement cost function is assumed to have the following quadratic form:\(^7\)

\[
C(m;\theta) = [c_m + \theta_m]m + \frac{1}{2}c_{mm}m^2
\] (3.1)

The parameters \( c_m \) and \( c_{mm} \) are assumed to be non-negative, with the latter being the slope of the marginal abatement cost function. Rather than complicate the problem

\(^6\)The model shown here is actually static. So the idea here is that it captures a time frame in which the regulator cannot adjust the instrument. This is a fairly reasonable assumption over relatively short time horizons as regulations are rarely adjusted very quickly. Papers that have considered the choice of price and quantity instruments in the case where the regulator can learn about \( \theta \) over time include Moledina et al. (2003), Costello and Karp (2004), and Karp and Zhang (2005, 2006). These papers differ in their assumptions about the behavior of the regulatory and regulated entities (for example, whether the regulator or regulated entities are myopic).

\(^7\)Rather than assuming a functional form for (3.1) we could explicitly follow Weitzman (1974) and approximate \( C(m;\theta) \) with a second-order Taylor series expansion with unknown variables only in the first order terms. While the interpretation is slightly different, the parameters that determine the optimal mix of instruments are the same. Weitzman\’s approach is more general, but is only valid with \textquoteleft{}small\textquoteright{} errors and in the neighborhood of the optimal level of abatement. The approach here provides global results, but relies on a restrictive functional form. The approach taken here is closest to the demonstration in Baumol and Oates (1988) and Adar and Griffin (1976).
with a number of unknown variables, we assume there is a sole random variable, \( \theta_m \).

This variable shows up as an additive term in the marginal abatement cost curve for \( m \) and consequently different realizations of this variable shift the marginal abatement cost curve vertically.\(^8\) The properties \( E[\theta_m] = 0 \) and \( E[\theta_m^2] = \sigma_m^2 \) are imposed on the random variable. We will see that the instrument levels and the expected welfare they yield are not influenced by higher moments of \( \theta_m \).\(^9\)

The benefit of \( m \) is:

\[
B(m) \equiv b_m m - \frac{1}{2} b_{mm} m^2
\]  

(3.2)

Note that this function is deterministic and is solely dependent on the level of abatement. Consistent with the notion that the marginal benefit of abatement falls as abatement increases, the parameter \( b_{mm} \) is nonnegative. The parameter assumptions

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\(^8\) One common quibble with Weitzman’s analysis is that uncertainty is additive in the marginal cost functions. Others have argued that multiplicative uncertainty, where there is uncertainty in the slope of the marginal functions, may be a more realistic or consequential concern (Adar and Griffin, 1976; Watson and Ridker, 1984; Malcomson, 1978). As Adar and Griffin discuss, both cases may indeed be present and the extent to which either should be considered is an empirical question. (However, with multiplicative uncertainty the property of certainty equivalence, which is discussed in footnote 9, no longer holds.) Also, Weitzman (1978a) responds to Malcomson’s criticism that a simple approximation may be misleading by noting the same but further appeals to his original justification for using the Taylor series, Samuelson (1970), which shows that unless the errors are particularly large the first order approximation of the stochastic function is a sufficient representation. Yohe (1978) provides further evidence that the inclusion of higher-ordered terms is not likely to influence the choice of instrument. Regardless, the analysis herein is not intended to take sides in this particular debate. Even if multiplicative uncertainty were introduced into this problem, as long as additive uncertainty is retained, the relationships embedded in the comparisons below would still be present, albeit buried in more complicated analytics.

\(^9\) This functional form exhibits the property of certainty equivalence in that the optimal instrument levels in the presence of uncertainty are equal to those that would have been chosen by the regulator had he behaved as if he were certain of the abatement cost function with the unknown variable equal to its mean. A consequence of this form is that the optimal instruments are not functions of any moments of the random variable other than its mean, which allows for straightforward comparisons of expected welfare from the competing instruments.
for the benefit and cost functions assure a unique solution to the regulator’s problem. Further, we impose \( b_m > c_m \) so that regulatory intervention is desirable.

We can now explore the regulator’s problem, which is to maximize expected welfare:

\[
W(m) = E [B(m) - C(m; \theta)]
\] (3.3)

The quantity of abatement that maximizes this expression is \( m^* \). This can be achieved directly by a quantity instrument that imposes an abatement target equal to \( m^* \). In choosing the tax that maximizes expected welfare, the regulator anticipates that sources minimize their abatement costs and consequently reduce emissions until their marginal abatement cost equals the tax. Given some tax \( t_m \) and a realization of \( \theta_m \), the actual level of abatement of \( m \) is then determined by the implicit function:

\[
t_m = c_m + \theta_m + c_{mm} m
\] (3.4)

where the right side of (3.4) is the marginal abatement cost of \( m \). Using this expression the realized abatement of \( m \), which is unknown to the regulator until after the tax is levied, can be expressed as a function of the tax:

\[
m(t_m; \theta_m) = \frac{t_m - c_m - \theta_m}{c_{mm}} .
\]

Substituting \( m(t_m; \theta_m) \) into the expected welfare function (expression (3.3)) and maximizing with respect to \( t_m \) yields the optimal tax \( t_m^* \). Given the functional form of the problem, the expected quantity of abatement from using the optimal tax equals the level of abatement achieved by the optimal quantity instrument. That is:

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10 Implicit in the analytical model is a method for allocating abatement responsibility across regulated sources that minimizes their collective abatement cost. We assume that emission cap-and-trade programs are used where the emissions allowance market is frictionless and undistorted. Furthermore, we assume that the allowance allocation mechanism is non-distortionary.
Given the optimal instrument levels, a comparison of the expected welfare between the tax and the quantity instruments yields the following expression:\(^{11}\)

\[
W(t_m^*) - W(m^*) = \frac{\sigma_m^2}{2c_{nn}^2} \left[ c_{nn} - b_{nn} \right]
\]  
(3.6)

If this expression is positive, a tax approach is preferred, and, if it is negative, a quantity approach is preferred. As we saw in the graphical analysis, the critical parameters for instrument choice are the slope of the marginal abatement cost function, \(c_{nn}\), and the slope of the marginal benefit function, \(b_{nn}\). Again, if the slope of the marginal cost function is greater than the slope of the marginal benefit function, the control of emissions by a tax is the preferred policy. If the relative magnitude of these slopes is reversed, the quantity instrument is preferred. These are the basic results developed by Weitzman (1974).

A different interpretation of the expected welfare comparison between a tax and a quantity instrument that is masked by the comparison in (3.6) will be useful when we extend the problem to multiple pollutants. When a tax is used to control emissions, the level of abatement will decrease or increase depending on whether abatement costs are higher or lower than expected. This adjustment suggests that a tax instrument is preferable to a quantity instrument (where the level of abatement does not respond to the actual control costs). On the other hand, by using a tax to control emissions, the level of abatement is itself uncertain. In contrast, a quantity instrument has the advantage in that it avoids uncertainty in the level of abatement. Equation

\(^{11}\) For the sake of brevity \(W(t_m)\) is used in place of \(W(m(t_m))\).
(3.7) illustrates the intermediate expression leading to the formulation in equation (3.6) and makes this interpretation, which follows Yohe (1978), transparent:

\[
W(t^*_m) = W(m^*) + E \left[ -\theta_m \left( \frac{-\theta_m}{c_{mm}} \right) - \frac{1}{2} b_{mm} \left( \frac{-\theta_m}{c_{mm}} \right)^2 - \frac{1}{2} c_{mm} \left( \frac{-\theta_m}{c_{mm}} \right)^2 \right] \tag{3.7}
\]

In the bracketed expectation term in (3.7), we see one positive and two negative arguments. The first argument, labeled \( AA \), is positive and captures the direct relationship between the uncertain component of abatement costs and the realized level of abatement when a tax is used. In particular, if abatement costs are higher than expected, the level of abatement will fall and vice-versa. We label the first term the abatement adjustment effect.

The second two terms, labeled \( AU \), in the bracketed expression in (3.7) capture the penalty associated with the uncertainty in the realized level of abatement.

When a tax is used to control \( m \), so that \( m = m(t^*_m; \theta_m) \), the level of \( m \) is of course unknown. However, as reported above, its expected value is \( m^* \), which is the optimal quantity level. Due to the concavity of the benefit function, the expected benefit from controlling \( m \) by an optimal tax is then lower than if \( m \) were controlled by an optimal quantity. Furthermore, the expected benefit of controlling \( m \) by a tax decreases as the curvature of the benefit function, \( \frac{1}{2} b_{mm} \), increases. Similarly, the third term in the

\[12 \text{ The steps preceding the one shown in (3.7) begin with:} \]

\[
W(t^*_a) = E \left[ b_a m(t^*_a; \theta_a) - \frac{1}{2} b_{am} m(t^*_a; \theta_a)^2 - \frac{1}{2} c_{aa} m(t^*_a; \theta_a)^2 \right] - \left[ c_a + \theta_a \right] m(t^*_a; \theta_a) - \frac{1}{2} c_{aa} m(t^*_a; \theta_a)^2
\]

where \( m(t^*_a; \theta_a) = m^* - \theta_a / c_{aa} \) as reported in expression (3.5).
bracketed expression in (3.7) captures the cost of uncertainty as it relates to the convexity of the abatement cost function. The expected cost of controlling an uncertain abatement quantity is greater than the cost of controlling its expected value. We label the second and third terms the abatement uncertainty effect.

The net influence of the abatement adjustment and the abatement uncertainty effects dictate which instrument yields higher expected welfare. In the single pollutant case this comes down to a comparison of the relative slopes of the marginal abatement benefit and cost functions, which, as just noted, has significant intuitive appeal. In the literature, comparisons of marginal benefit and cost slopes have served as the basis for recommending the use of a particular instrument (Nordhaus, 2001; Banzhaf et al. 2004). However, this assumes that other regulations affecting emitting sources are held fixed or are not being simultaneously determined. As shown below, in a multiple pollutant framework with uncertain and joint abatement costs, a comparison of relative marginal slopes may be misleading in identifying the welfare maximizing set of instruments. However, differences in the magnitudes of the abatement adjustment and abatement uncertainty effects across instrument mixes continue to dictate the optimal mix of instruments.

### 3.3 The Multiple Pollutant Model

We now introduce the abatement of a second pollutant, $s$, into the collective abatement cost function.\(^\text{13}\)

\(^{13}\) The variables $m$ and $s$ are chosen heuristically, with $m$ representing the abatement of mercury and $s$ the abatement of sulfur dioxide. At certain points in this chapter it will be convenient to use the labels
\[ C(m,s;\theta) = [c_s + \theta_s]s + [c_m + \theta_m]m + \frac{1}{2} \left( c_{ss}s^2 + 2c_{sm}sm + c_{mm}m^2 \right) \]  
\tag{3.8}

The parameters \(c_s\), \(c_m\), \(c_{ss}\), and \(c_{mm}\) are all assumed to be non-negative. The parameter \(c_{ms}\) captures the joint abatement relationship. If \(c_{ms}\) is positive, abatement of one of the pollutants makes it more costly to abate the other. If on the other hand \(c_{ms}\) is negative, then abatement of one of the pollutants lowers the cost of controlling the other. An alternative view is that if \(c_{ms}\) is positive, the two pollutants are substitutes in abatement and if it is negative they are complements in abatement.

To keep the analysis tractable we assume only two, possibly related, random variables, \(\theta_m\) and \(\theta_s\). Analogous to the single-pollutant analysis, these variables are additive in their respective pollutant’s marginal abatement cost function such that a change in one of these variables results in a vertical shift in the marginal abatement cost function. Furthermore, we assume that \(E[\theta_m] = E[\theta_s] = 0\), \(E[\theta_m^2] = \sigma_m^2\), \(E[\theta_s^2] = \sigma_s^2\) and \(E[\theta_m \theta_s] = \sigma_{ms}\). The sign of the covariance of these two errors is unrestricted. We will see that the instrument levels and the expected welfare they yield are not influenced by higher moments of these variables.

Note that the parameter representing the cross partial of the cost function with respect to the abatement of the two pollutants \((c_{ms})\) is observed with certainty.

So, this is more a model with “uncertain costs and joint abatement”, as opposed to a model with “uncertain joint costs”. My decision to make \(c_{ms}\) known to the regulator is to refer to the particular pollutant that is being abated. When \(m\) and \(s\) are used it should be clear from the context whether we are referring to the level of abatement or the name of the pollutant being abated.
mainly for mathematical convenience. The expressions for expected welfare found below would be dense if \( c_{ms} \) was observed with error. Furthermore, despite the simplicity of the model, this formulation is sufficient to show that the relative-slopes condition is misleading in the case where there is uncertainty and jointness in the cost function. It also shows that any expected welfare ordering of the different instruments mixes is possible.

The benefit of reducing each of these pollutants is assumed to be unrelated to the level of abatement of the other. The abatement benefit function for \( m \) is as before in expression (3.2). The benefit of abating \( s \) is also assumed to have a quadratic form:

\[
B(s) = b_s s - \frac{1}{2} b_{ss} s^2
\]

where the parameters \( b_s \) and \( b_{ss} \) are nonnegative and \( b_s > c_s \).\(^{15}\)

The regulator decides on the appropriate combination of quantity and price instruments to control these two pollutants. We first derive the optimal levels of abatement where each pollutant is controlled by a quantity. We then consider a policy that controls one pollutant by a tax and the other by a quantity. Finally, we consider a

\[^{14}\] Like the single pollutant model, this two-pollutant model exhibits the property of certainty equivalence.

\[^{15}\] The relationship between \( b_s \) and \( c_s \) is made to assure that regulating \( s \) is desired if \( m \) is not regulated. However, this assumption is not necessary for the net-benefits of regulating these pollutants to be positive. As we see in expression (3.10) (in conjunction with the second-order condition with respect to the cost function) we need only that \( b_s > c_s \) or \( b_m > c_m \) to hold for regulatory intervention to be desirable.
policy where each pollutant is controlled by a tax. Welfare expressions are then derived for these instrument mixes and compared.\textsuperscript{16}

3.3.1 Quantity Instruments

In the case where quantities are used to control both pollutants, the regulator directly chooses the desired levels of abatement. Sources are also assumed to fully understand how abatement decisions will affect their emissions, so that the abatement constraint is adhered to. The regulator’s problem is to maximize expected welfare choosing $m$ and $s$:

\[
W(m,s) = \mathbb{E}\left[ b_m - c_m - \theta_m \right] m - \frac{1}{2} \left[ b_{mm} + c_{mm} \right] m^2 + \left[ b_s - c_s - \theta_s \right] s - \frac{1}{2} \left[ b_{ss} + c_{ss} \right] s^2 - c_{ms} ms \tag{3.10}
\]

A cost-minimizing equilibrium in the underlying emissions market is assured, along with the sign assumptions for $c_{ss}$ and $c_{mm}$, if $c_{ss} c_{mm} - c_{ms}^2 > 0$. This second order condition is assumed to hold. The first-order conditions of (3.10) can then be used to derive the optimal ex-ante abatement levels from the perspective of the regulator:

\[
m^* = \frac{\left[ b_m - c_m \right] \left[ b_{ss} + c_{ss} \right] - c_{ms} \left[ b_s - c_s \right]}{\left[ b_{ss} + c_{ss} \right] \left[ b_{mm} + c_{mm} \right] - c_{ms}^2} \tag{3.11}
\]

\[
s^* = \frac{\left[ b_s - c_s \right] \left[ b_{mm} + c_{mm} \right] - c_{ms} \left[ b_m - c_m \right]}{\left[ b_{ss} + c_{ss} \right] \left[ b_{mm} + c_{mm} \right] - c_{ms}^2} \tag{3.12}
\]

Before moving on to the cases where a tax is used to control one of the pollutants, we note two quick points. With uncertainty in the abatement cost function it is possible

\textsuperscript{16} Analytical models similar to the one presented here are found in Yohe (1976a) and (1977b). Yohe (1977b) discusses the performance of instruments to regulate the production of goods that have joint benefits and uncertain, but separate, costs. In an unpublished work, Yohe (1976a) looks deeper at the influence of joint benefits on instrument choice and presents an analytical model similar to the one presented here to begin to explore the case of uncertain joint production. However, Yohe does not look at the mixed instrument cases and does not identify the possibility that any expected-welfare ordering of the instrument mixes is possible.
that an abatement constraint may become slack if costs are substantially below what was expected. That is, actual abatement may be less than the optimal ex-ante level of abatement the regulator will impose (i.e. $m^*$ and/or $s^*$). In the analysis that follows, we assume that the actual abatement cost function is sufficiently close to the expected function so that there is no possibility that ex-post one of the abatement constraints would be slack.

The second point is that it is possible that the optimal quantity of abatement of one of the pollutants is negative. That is, it may be optimal for the emissions of one of the pollutants to increase. This may only occur when it is particularly difficult to simultaneously abate both pollutants (i.e., when $c_{ms} > 0$). If one of the pollutants causes significantly more damage, it may be acceptable for the emissions of the other to rise relative to the situation without regulatory intervention.

3.3.2 Mix of Instruments

Now let us consider the case where the regulator controls one pollutant with a tax while the other remains controlled by a quantity. Specifically, we consider the example where a tax is used to control the pollutant $s$ while a quantity instrument is used to control $m$. Affected sources are expected to minimize costs with respect to

---

17 Pizer (1997) refers to this possibility as “truncation.”

18 An example from the electricity sector is in the control of nitrogen oxide emissions. Ammonia emissions are a byproduct of a common nitrogen oxide abatement technology. In the absence of a nitrogen oxide control policy there would be no ammonia emissions. However, if nitrogen oxide were controlled with a cap, ammonia emissions would rise. In order to control this rise, the regulator may also control ammonia but still allow emissions to become positive. Hence, there is “negative abatement” of ammonia.

19 A symmetric case holds for when $m$ is controlled by a tax and $s$ is controlled by a quantity.
the tax on \( s \) which implies that abatement occurs until the marginal abatement cost of \( s \), given some quantity restriction \( m \), equals the tax on \( s \):

\[
t_s = c_s + \theta_s + c_{ss} s + c_{ms} m
\]  
(3.13)

The realized level of abatement of \( s \) can be expressed as a function of the random variable \( \theta_s \), the tax level \( t_s \), and the level of the quantity instrument \( m \):

\[
s(m, t_s; \theta) = \frac{c_s + \theta_s - t_s + c_{ms} m}{c_{ss}}
\]  
(3.14)

The regulator does not observe this level of abatement until after the tax is administered because the regulator cannot observe the actual value of \( \theta_s \) until then. So with uncertainty in the realized \( s \), the regulator chooses \( m \) and \( t_s \) that maximize:

\[
W(m, t_s) = E \left[ \frac{1}{2} \left[ b_m - c_m - \theta_m \right] m + \frac{1}{2} b_{mm} + c_{mm} \right] m^2
\]  
(3.15)

\[
- \frac{1}{2} b_{ss} + c_{ss} \right] s(m, t_s; \theta) - \frac{1}{2} b_{ss} + c_{ss} \right] s(m, t_s; \theta)^2
\]  
\left[ -c_{ms} m s(m, t_s; \theta) \right]

For the sake of brevity, we do not present the necessary conditions for this problem. However, we note that the choice variables that solve this problem are:

\[
t_s^* = \frac{b_m \left[ c_{ss} c_{mm} - c_{ms}^2 + c_{ss} b_{mm} \right] + b_s \left[ b_m - c_m \right] c_{ss} \left[ b_m + c_{mm} \right]}{b_{ss} + c_{ss} \left[ b_{ss} + c_{mm} \right] - c_{ms}^2}
\]  
(3.16)

for the optimal tax on \( s \) while the optimal abatement quantity of \( m \) is defined in equation (3.11). Given \( m^* \), the realized level of abatement of \( s \) given the optimal tax \( t_s^* \) is:

\[
s\left(m^*, t_s^*; \theta\right) = s^* - \frac{\theta_s}{c_{ss}}
\]  
(3.17)
Note that the expected level of $s$, given $m^*$ and $t^*_s$, is equal to the optimal choice of $s$ when both pollutants are controlled by a quantity. That is: \( E[s(m^*,t^*_s;\theta)] = s^* \) where $s^*$ is defined in equation (3.12).

Furthermore, we see in (3.17) that the deviation in the actual abatement of $s$ is strictly related to the error in the marginal cost of $s$ when $m$ is fixed. That is, the error $\theta_m$ does not affect the ex-post level of $s$ when $s$ is controlled by a tax. Looking again at the form of the total abatement cost function, focusing on the fact that the quantity of abatement of $m$ is fixed at any level $\bar{m}$, we have:

\[
C(\bar{m},s;\theta) = [c_s + \theta_s]s + [c_m + \theta_m]\bar{m} + \frac{1}{2}c_{ss}s^2 + 2c_ms\bar{m} + c_{mm}\bar{m}^2
\]  

(3.18)

As $m$ will not be adjusted by the regulated sources, the error $\theta_m$ does not work thorough the joint cost relationship $c_{ms}$ to affect in turn the realized abatement of $s$. With $m$ fixed, any realization of $\theta_m$ will only result in direct increases and decreases in total costs.\(^{20}\)

3.3.3 Tax Instruments

When both pollutants are regulated with an emissions tax, we anticipate that the regulated sources will abate emissions such that the realized marginal abatement cost of each pollutant is equal to its emissions tax:

\[
t_m = c_m + \theta_m + c_{mm}m + c_{ms}s \\
t_s = c_s + \theta_s + c_{ss}s + c_{ms}m
\]

(3.19)  
(3.20)

\(^{20}\) One way that the error $\theta_m$ may influence the abatement of $s$ is if it creates conditions where the abatement constraint on $m$ is not binding. In such a case, $m$ may be adjusted in order to influence the burden of complying with the tax on $s$. Again, we have assumed away such cases.
These optimality conditions for the regulated sources can be manipulated to show the realized abatement levels as functions of the taxes:

\[
m(t_m, t_s; \theta) = -\frac{c_{ms}}{c_{mm}c_{ss} - c_{ms}^2} \left[ t_s - c_s - \theta_s + c_{ss} \left( t_m - c_m - \theta_m \right) \right]
\]

(3.21)

\[
s(t_m, t_s; \theta) = -\frac{c_{ms}}{c_{mm}c_{ss} - c_{ms}^2} \left[ t_m - c_m - \theta_m + c_{ss} \left( t_s - c_s - \theta_s \right) \right]
\]

(3.22)

We see that given any taxes \( t_m \) and \( t_s \), the level of abatement of both pollutants will be affected by different realizations of \( \theta_m \) and \( \theta_s \). To find the optimal ex-ante tax levels, the regulator maximizes expected welfare with respect to the two taxes given (3.21) and (3.22):

\[
W(t_m, t_s) = E \left[ \left[ b_m - c_m - \theta_m \right] m(t_m, t_s; \theta) - \frac{1}{2} \left[ b_m + c_{mm} \right] m(t_m, t_s; \theta)^2 \right]
\]

\[
+ \left[ b_s - c_s - \theta_s \right] s(t_m, t_s; \theta) - \frac{1}{2} \left[ b_s + c_{ss} \right] s(t_m, t_s; \theta)^2
\]

\[
- c_{ms} \left[ t_m - c_m - \theta_m \right] s(t_m, t_s; \theta)
\]

(3.23)

When each pollutant is controlled by a tax, the optimal tax on \( s \) is the same as in (3.16). The optimal tax on \( m \) is an expression symmetric to (3.16). Conditional on the optimal tax levels derived from the solution to (3.23), the realized abatement levels are:

\[
m(t_m^*, t_s^*; \theta) = m^* + \frac{c_{ms} \theta_m - c_{ss} \theta_m}{c_{mm}c_{ss} - c_{ms}^2}
\]

(3.24)

\[
s(t_m^*, t_s^*; \theta) = s^* + \frac{c_{ms} \theta_m - c_{mm} \theta_m}{c_{mm}c_{ss} - c_{ms}^2}
\]

(3.25)

where \( m^* \) and \( s^* \) are defined by (3.11) and (3.12) respectively. Again, and attributable to the functional form of the problem, we see in (3.24) and (3.25) that the expected levels of abatement when tax instruments are used exclusively are equal to the ex-ante optimal abatement levels when quantity instruments are used exclusively. We also see how the random component associated with the cost of abating each pollutant
affects the realized abatement of the two pollutants. For example, and unsurprisingly, if \( \theta_s \) is particularly large and positive, which implies a direct shift upward in the marginal abatement cost of \( s \), then the optimal abatement of \( s \) falls. However, if \( \theta_m \) is large and positive, then the abatement of \( s \) by the regulated sources depends on the sign and the magnitude of \( c_{ms} \). The larger \( c_{ms} \) is in absolute value, the larger the effect of a particular realization of \( \theta_m \) is on the abatement of \( s \). If \( c_{ms} \) is sufficiently large such that the second order condition is just satisfied (\( |c_{ms}| \rightarrow \sqrt{c_{mm}c_{ss}} \)), the deviation in actual abatement from expected abatement becomes infinite.

### 3.3.4 Expected Welfare for Each Instrument Mix

Table 3.1 reports the realized abatement of the two pollutants under the four possible instrument mixes. We compare the expected welfare these policies yield by substituting the ex-post abatement levels in Table 3.1 into the expression for expected welfare.

\[ m_{\text{instrument}} \]

<table>
<thead>
<tr>
<th>Tax</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m + \frac{c_{ms}\theta_m - c_{ss}\theta_m}{c_{mm}c_{ss} - c_{ms}^2} )</td>
<td>( m^<em>, s^</em> - \frac{\theta_m}{c_{ss}} )</td>
</tr>
</tbody>
</table>

\[ s_{\text{instrument}} \]

<table>
<thead>
<tr>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m^* - \frac{\theta_m}{c_{mm}} )</td>
</tr>
</tbody>
</table>

\[ m^* \] and \( s^* \) are defined in expressions (3.11) and (3.12).
The expected welfare expressions for the different instrument mixes is provided in Table 3.2, where:

\[
W(m^*,s^*) = \left[ b_m - c_m \right] m^* - \frac{1}{2} \left[ b_{mm} + c_{mm} \right] m^{*2} + \left[ b_s - c_s \right] s^* - \frac{1}{2} \left[ b_{ss} + c_{ss} \right] s^{*2} - c_m m^* s^* \tag{3.26}
\]

Table 3.2: Expected Welfare from Each Instrument Mix

<table>
<thead>
<tr>
<th>m instrument</th>
<th>Tax</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W(m^<em>,s^</em>)$</td>
<td>$1 \left[ c_{mm} - b_{mm} \right] c_{ss} - \left[ b_m + c_m \right] c_{ss}^2$</td>
<td>$W(m^<em>,s^</em>) + \frac{\sigma^2}{2c_{mm}^2} \left[ c_{mm} - b_{mm} \right]$</td>
</tr>
<tr>
<td>$W(m^<em>,s^</em>)$</td>
<td>$\frac{1}{2} \left[ c_{mm} c_{ss} - c_{ss}^2 \right]$</td>
<td>$W(m^<em>,s^</em>) + \frac{\sigma^2}{2c_{ss}^2} \left[ b_{ss} + c_{ss} \right]$</td>
</tr>
</tbody>
</table>

The difference in expected welfare from any two sets of instruments can be determined from a comparison of the two appropriate cells in Table 3.2. We will often refer to these four policies by the set of instruments used: $\{t_m^*,t_s^*\}$, $\{m^*,t_s^*\}$, $\{t_m^*,s^*\}$ or $\{m^*,s^*\}$. As will be shown shortly, identification of the welfare maximizing combination of instruments requires an evaluation of all four cells as no one set of instruments strictly dominates another at the outset. We can already see in Table 3.2 that the relative efficiency of an instrument to control one pollutant depends on how the other pollutant is being controlled. For example, the difference in expected
welfare from using a tax to control \( m \) rather than a quantity depends on whether \( s \) is controlled by a tax or a quantity, even when these instruments to control \( s \) are chosen to optimize expected welfare.

It is fairly straightforward to show that any of the four possible instrument combinations may be preferred at the outset. This is easiest to show under the condition where \( c_{ms} \to 0 \). In this case, the expected welfare from the \( \{t_m^*, t_s^*\} \) policy converges to:

\[
W(t_m^*, t_s^*) \to W(m^*, s^*) + \left[ \frac{\sigma_m^2}{2c_{mm}} [c_{mm} - b_{mm}] + \frac{\sigma_s^2}{2c_{ss}} [c_{ss} - b_{ss}] \right]
\]

The bracketed term in (3.27) is the sum of the terms capturing the difference in expected welfare between the mixed instrument policies and the policy where a quantity is used to control each pollutant (see Table 3.2). We can see in (3.27) that this expression can be either positive or negative depending on the magnitudes of the slopes of the different marginal cost and marginal benefit functions. Recognizing this, we confirm, at least for arbitrarily small \( c_{ms} \), that no instrument mix is strictly dominated by one of the others at the outset.\(^{21}\)

\(^{21}\) Perhaps surprisingly, despite having information on the correlation between the errors of the marginal abatement cost functions, this information does not influence the preferred set of instruments. Quite simply, from the perspective of the regulator the two pollution control problems have separated. This result may change for more sophisticated regulatory instruments. For example, Montero (2001) considers the performance of an integrated multiple pollutant market when there is cost uncertainty in the control of each pollutant (i.e., there are two pollutants but they are not jointly abated). He shows that, it is possible that using a single cap with a trading ratio between the two pollutants yields higher expected welfare than if each pollutant were controlled by a separate quantity instrument. Assuming that the two pollutants have identical abatement cost and benefit functions in expectation, and that the random variables in the two cost functions are uncorrelated, then integrating the emission trading markets is preferred if the slope of the marginal benefit function is flatter than the slope of the marginal cost function. This is because the benefit of allowing the firms to adjust their pollution control in the face of unexpectedly high cost (what I call an abatement adjustment effect) outweighs the additional uncertainty in abatement that comes with integrating the markets (the abatement uncertainty effect).
We see in Table 3.2 that when deciding between a policy that uses a mix of instruments and one in which quantities are used exclusively, it is sufficient to compare the magnitude of the slopes of the marginal benefit and cost curves for the pollutant that may be controlled by a tax. That is, with the quadratic and additive uncertainty form of the abatement cost function, this comparison reduces to the one Weitzman and others identified (equation (3.6)). When the abatement of one of the pollutants is fixed, i.e. controlled by a quantity, the uncertainty in the marginal abatement cost for the other pollutant is limited to its “own” random variable. Therefore in this model, when one of the pollutants is controlled by a quantity, only the marginal benefit and cost slopes need to be compared for the pollutant potentially subject to the tax. The same cannot be said when comparing the expected welfare from the policy that uses a tax to control each pollutant to the expected welfare from the other instrument mixes. More will be said regarding these observations below.

3.4 Expected Welfare Comparisons

3.4.1 Comparisons Using a Single Random Variable

3.4.1.1 Comparing the Tax and Mixed Instrument Policies

Clearly the most complicated expected welfare expression is for the policy where a tax is used to control each pollutant. This complication arises because both

A similar possibility presumably extends to the model described here. This is because if Montero’s assumptions otherwise held, and the parameter values of the problem are such that that integrated pollutant trading dominates separate markets, then simply adding a small cross-partial term should not be able to change the result. What is interesting about having integrated pollutant markets is that, as with taxes but not with separate markets, there is both an abatement uncertainty and abatement adjustment effect, but unlike with taxes there is a fixed total quantity.
quantities are allowed to adjust. When there is some deviation in the random variable associated with one pollutant that motivates a change in the level of that pollutant, it also influences the cost of controlling the other pollutant. With that other pollutant controlled by a tax, its level may then also adjust. The simultaneous and reciprocal nature of these adjustments is captured in the expression for expected welfare when a tax is used to control each pollutant. With this model, it is the policy where taxes are used to control each pollutant that really shows how a simple relative-slopes comparison can be misleading when multiple pollutants are being controlled.\textsuperscript{22}

To begin to gain intuition regarding the form of $W(t_{m}^{*},t_{s}^{*})$, let us consider the case where the random variable $\theta_{i}$ is never particularly large and that the correlation in the errors is small. That is, we assume that $\sigma_{s}^{2} \approx 0$ and $\sigma_{ms} \approx 0$. Given these assumptions we first note from Table 3.1 that, conditional on $m$ being controlled by a quantity, the abatement of $s$ when controlled by a tax is approximately equal to the abatement of $s$ when controlled by a quantity. That is, one consequence of these assumptions is that $W(m^{*},t_{s}^{*}) \approx W(m^{*},s^{*})$.\textsuperscript{23}

With $\sigma_{s}^{2} \approx 0$ and $\sigma_{ms} \approx 0$, the difference in expected welfare between the $\{t_{m}^{*},t_{s}^{*}\}$ policy and the $\{m^{*},s^{*}\}$ policy can be expressed:

\textsuperscript{22} How it can be misleading would depend on the functional form of the problem. My primary concern is showing that even with this simple functional form it can be misleading.

\textsuperscript{23} In reality we might find this a peculiar outcome. That is, despite there being some random variable in the cost function, this variable does not affect the regulator’s choice of instrument to control $s$ when $m$ is controlled by a quantity. We would find it particularly odd if $c_{ms}$ were unknown to the regulator.
\[ W(t_m^*, t_s^*) - W(m^*, t_s^*) \approx \frac{c_{ss}^2 \sigma_m^2}{2(c_{mm} c_{ss} - c_{ms}^2)^2} \left[ c_{mm} - b_{mm} \right] - b_{ss} + c_{ss} \frac{c_{ms}^2}{c_{ss}^2} \]  

(3.28)

It can be seen in this expression that the \( \{ m^*, t_s^* \} \) policy is preferred to the \( \{ t_m^*, t_s^* \} \) policy if the slope of the marginal abatement benefit curve for \( m \), \( b_{mm} \), is steeper than the slope of the partial marginal abatement cost curve for \( m \), \( c_{mm} \). By partial we of course mean by ignoring, or not acknowledging, the possibility that \( s \) is adjusting at the same time. Applying the relative slope comparison would then suggest the correct instrument. However, if the marginal benefit curve of abating \( m \) were flatter than the own slope of the marginal cost of abating \( m \) (i.e. \( c_{mm} > b_{mm} \)), it may be possible that the relative slopes comparison suggests using a tax to control \( m \), when indeed it may be appropriate to control \( m \) by a quantity.

To gain some understanding as to why the hurdle for adopting a tax on \( m \) is now higher, we look at the difference in \( W(t_m^*, t_s^*) \) and \( W(m^*, t_s^*) \) before terms are collected to yield (3.28):

\[
\begin{align*}
W(t_m^*, t_s^*) - W(m^*, t_s^*) & \approx \\
E[-\theta_m] - \frac{c_{ss} \theta_m}{c_{mm} c_{ss} - c_{ms}^2} & - \frac{1}{2} b_{mm} E \left[ \frac{-c_{ss} \theta_m}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 & - \frac{1}{2} b_{ss} E \left[ \frac{-c_{ss} \theta_m}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 & - \frac{1}{2} c_{ss} E \left[ \frac{-c_{ss} \theta_m}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 \left( c_{ms} \theta_m \right) \left( c_{ms} \theta_m \right) \\
- \frac{1}{2} c_{ms} E \left[ \frac{-c_{ss} \theta_m}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 & - \frac{1}{2} c_{ms} E \left[ \frac{-c_{ss} \theta_m}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 & - \frac{1}{2} c_{ms} E \left[ \frac{-c_{ss} \theta_m}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 \left( c_{ms} \theta_m \right) \left( c_{ms} \theta_m \right) \\
\end{align*}
\]

(3.29)

The term \(-c_{ss} \theta_m \left[ c_{mm} c_{ss} - c_{ms}^2 \right]\) is the approximate adjustment from the expected level of \( m \) as a result of the realization of \( \theta_m \) when each of the pollutants is controlled by a tax. Likewise, \( c_{ms} \theta_m \left[ c_{mm} c_{ss} - c_{ms}^2 \right]\) is the approximate adjustment from the expected
level of $s$ as a result of the realization of $\theta_m$ when each of the pollutants is controlled by a tax. These adjustment terms can be seen in the expressions for the levels of $m$ and $s$ when each pollutant is controlled by a tax (i.e. the expressions (3.24) and (3.25)) subject to our assumptions that $\sigma_s^2 \approx 0$ and $\sigma_{ms} \approx 0$.

The first term in (3.29) is positive and captures the benefit of controlling $m$ by a tax in that the level of $m$ will, unlike when a quantity instrument is used, adjust to mitigate the impact of the difference between expected and actual abatement costs. As in the single pollutant case when costs are higher or lower than expected, $m$ will adjust accordingly.

The next two terms in (3.29) are negative and capture the influence of the concavity of the abatement benefit functions given that the abatement of both pollutants is uncertain. As in the single pollutant case, these two terms are negative. The last three terms of (3.29) capture the influence of the convexity of the abatement cost function coupled with the uncertainty in the abatement of both pollutants. Collectively these three terms are negative. Again, this argument appears in the single pollutant case, except that now abatement costs are a function of two pollutants.

Note that there is no term associated with the abatement of $s$ that is analogous to the first term in (3.29). That is, there is no direct benefit of adjusting $s$ in response to the variation in cost associated with controlling $m$. Therefore, when $s$ is controlled by a tax, it is no longer sufficient to compare the slope of the marginal benefit of abating $m$ with the slope of the partial marginal cost of abating $m$ when determining the optimal instrument to control $m$. Specifically, even if $c_{mm} > b_{mm}$, it
may be preferable to control $m$ by a quantity in order to avoid the cost of uncertainty (in terms of lower expected benefits and higher expected abatement costs) in the realized level of $s$.\(^{24}\)

3.4.1.2 Comparing the Quantity and Mixed Instrument Policies

In the previous section we learned that the added complication of $s$ changing creates conditions where even if $c_{mm} > b_{mm}$, it may be preferable to control $m$ by a quantity to avoid a variation in $s$. But does this mean that if the conditions are such that it is preferable to control $m$ by a tax even when $s$ is controlled by a tax, i.e.:

$$W(t_m^*, t_s^*) > W(m^*, t_s^*)$$

(3.30)

then the regulator can improve matters even further by controlling $s$ by a quantity? That is, if (3.30) is true with our single random variable case, then must the following also be true?

$$W(t_m^*, s^*) > W(t_m^*, t_s^*)$$

(3.31)

The answer is no. Note that the expression for the variation in $m$ when each pollutant is controlled by a tax explicitly acknowledges that the change in $m$ is influenced by the possibility that $s$ may change. The cost of the abatement uncertainty effect associated with $s$ when the $\{t_m^*, t_s^*\}$ policy is employed may be smaller than the cost associated with a lower abatement adjustment effect of $m$ when the $\{t_m^*, s^*\}$ policy is

\(^{24}\) One may wonder why the sign of (3.28) is independent of the sign of $c_m$. The intuition is that the unknown variable $\theta_m$ can be positive or negative. A positive sign of $\theta_m$ coupled with a negative sign of $c_m$ is equivalent to $\theta_m$ having a negative sign and $c_m$ having a positive sign. At this point in our discussion the direction of the joint abatement is irrelevant to the welfare ordering of the instruments. As can be seen in the expected welfare expression for the $\{t_m^*, t_s^*\}$ policy in Table 3.2, this symmetry no longer holds when the two random variables are correlated.
employed. To see this note that given $W(m^*, t_s^*) \approx W(m^*, s^*)$ the difference in expected welfare from using a tax to control $s$ when $m$ is controlled by a tax is:

$$W(t_m^*, t_s^*) - W(t_m^*, s^*) \approx \left[ W(t_m^*, t_s^*) - W(m^*, t_s^*) \right] - \left[ W(m^*, s^*) - W(t_m^*, s^*) \right]$$

$$\approx \left[ W(t_m^*, t_s^*) - W(m^*, t_s^*) \right] - \frac{\sigma_m^2}{2c_{mm}} \left[ c_{mm} - b_{mm} \right]$$

where $\left[ W(t_m^*, t_s^*) - W(m^*, t_s^*) \right]$ is described in (3.29). We can rewrite the expression (3.32) as:

$$W(t_m^*, t_s^*) - W(t_m^*, s^*) \approx \frac{1}{2} \left[ \frac{c_{ss}\sigma_m^2}{c_{mm}c_{ss} - c_{ms}^2} - \frac{\sigma_m^2}{c_{mm}} \right] \left[ c_{mm} - b_{mm} \right] - \left[ b_{ss} + c_{ss} \right] \frac{c_{ms}\sigma_m^2}{c_{mm}c_{ss} - c_{ms}^2}$$

(3.33)

As we saw in expression (3.29), a necessary condition for $W(t_m^*, t_s^*) > W(m^*, t_s^*)$, which we are assuming, is that $c_{mm} > b_{mm}$, or that the abatement adjustment effect of $m$ outweighs the abatement uncertainty effect from $m$ and $s$. We can now acknowledge the possibility that (3.33) may be positive, implying that just because a tax is the preferred instrument to control $m$ does not mean that it is preferable to control $s$ by a quantity.25

Under what conditions might the expression in (3.33) be positive? It must be the case that:

$$\frac{c_{ss}\sigma_m^2}{c_{mm}c_{ss} - c_{ms}^2} \gg \frac{\sigma_m^2}{c_{mm}}$$

(3.34)

---

25 The most straightforward way to see that (3.33) may be positive is to let $b_m = b_s = 0$. See section 3.4.4.1.
The left side of this expression is the variance in \(m\) when the \(\{t^*_m, t^*_s\}\) is employed and the right side is the variance in \(m\) when the \(\{t^*_m, s^*\}\) is employed. The left side is always larger than the right provided that the second order restriction holds. However, it is useful to note that it must be the case that controlling \(s\) by a tax must allow \(m\) to adjust much more when abatement costs are higher or lower than expected for the \(\{t^*_m, t^*_s\}\) policy to yield higher expected welfare than the \(\{t^*_m, s^*\}\) policy. It also helps if 

\[
c_{ms}^2 \sigma_m^2 / \left[ c_{mm} c_{ss} - c_{ms}^2 \right]^2
\]

is relatively small, which is the variance in \(s\) when both pollutants are controlled by a tax.

### 3.4.1.3 Relating Results to General Equilibrium Welfare Analysis

We return to the expected welfare comparison identifying the optimal instrument to control \(m\) when \(s\) is controlled by a tax (i.e. the difference between \(W(t^*_m, t^*_s)\) and \(W(m^*, t^*_s)\)) and note a mathematical relationship. Assuming that a tax is used to control \(s\), the rate of change in the additional cost of increasing \(m\) is:

\[
\frac{d^2C(m,s(m,t_s))}{dm^2} = \left[ c_{mm} c_{ss} - c_{ms}^2 \right] / c_{ss}
\]

(3.35)

Notionally we may think of this as the slope of the marginal cost of increasing \(m\) given some \(t_s\). \(^{26}\) If we adopted the expected welfare comparison for the single pollutant case (i.e. expression (3.6)) and replaced \(c_{mm}\) with this expression for the rate of change in costs, would we recover the expected welfare comparison between \(W(t^*_m, t^*_s)\) and \(W(m^*, t^*_s)\) (i.e. expression (3.28))? The answer is no. However, if we

\(^{26}\) The constancy of the slope with respect to \(m\) given any \(t_s\) is due to the functional form of the problem.
added to the slope of the marginal benefit of increasing $m$ an expression capturing the change in the marginal benefit of $s$ as a result of the change in $m$:

$$d^2 B(s(m,t_j))/dm^2 = -b_s [c_{ms}/c_{ss}].$$

(3.36)

then using the single pollutant relative-slopes template (expression (3.6)) would indeed yield expression (3.28).

The relationship in (3.35) has been acknowledged in the literature on the measurement of general equilibrium welfare changes from policies affecting private goods. A key finding of that literature is that it is possible to measure all of the welfare changes from a policy that directly affects a particular market solely in that market despite the fact that there are related markets that are indirectly affected (see Just et al., 2004 for a discussion). One can measure welfare changes using general equilibrium demand and supply functions in the directly affected markets. The slopes of the general equilibrium demand and supply curves differ from the slopes of the partial equilibrium demand and supply curves. We see that on the supply side of abatement, where the emitters internalize the effect of the change in $s$ on the cost of controlling $m$, that the slope of the general equilibrium supply of $m$ is that required to make a judgment as to the optimal instrument to control $m$. Unfortunately, however, this does not mean that we can ignore the “market” for $s$ as we could in an example where $m$ and $s$ are private goods. This is due to the joint public goods nature of the problem. The emitters of the two pollutants do not internalize the change in the benefits of abating $s$ and thus the effect of this change does not reveal itself in the “market” or space of $m$. Thus, we need to explicitly add to the partial abatement
benefit slope of $m$ the slope of the abatement benefit of $s$ as $m$ changes to recover the relative-slopes comparison in the single pollutant analysis.

3.4.2 Comparing Expected Welfare with Two Random Variables

We now reintroduce the second random variable $\theta_s$ back into the problem. Our purpose is not to carry out a full discussion of how competing abatement uncertainty and abatement adjustment effects, accounting for the covariance between $\theta_s$ and $\theta_m$, dictate which instrument is preferred. The discussion above demonstrates the utility of comparing the competing abatement uncertainty and abatement adjustment effects for the different mixes, but it is harder to demonstrate this welfare ordering result without reducing terms. Rather, our purpose is more limited in that we first show that perhaps unanticipated expected welfare orderings of the instrument mixes are possible. Indeed, parameters may be chosen such that any expected welfare ordering is possible. However, in so doing we can allude to how differences in the abatement adjustment and abatement uncertainty effects for the different instrument mixes are influencing the orderings.

We start by revisiting Table 3.2 and provide further insight into the expected welfare comparison between the policies that use mixed instruments and the policy that uses a quantity to control each pollutant. Next, we focus on comparisons between the policy that uses a tax to control each pollutant and the three other instrument combinations. We conclude this section by considering some special cases where the parameters take on particular values that are potentially of empirical
interest. For example, is it the case that the $\{t_m^*, t_s^*\}$ policy is preferred when the marginal benefit of abatement curves are flat?

3.4.2.1 Comparing the Quantity and the Mixed Instrument Policies

One may wonder why the cross-partial of the abatement cost function ($c_{ms}$) is not present in the expressions reporting the expected welfare difference between the $\{m^*,s^*\}$ mix and the $\{t_m^*,s^*\}$ and $\{m^*,t_s^*\}$ mixes. To understand this, first note that the expected value of the term $c_{ms}$ is captured in the expression $W(m^*,s^*)$, which is common to all three of these instrument mixes. For the $\{m^*,s^*\}$ and $\{m^*,t_s^*\}$ instrument mixes the variance in the term $c_{ms}$, which we can associate with the pollutant not potentially subject to a tax, is greater than for the policy where only quantity instruments are used. However, the expected value of this term is the same for the policies that use a quantity to control at least one of the pollutants. Any variation in this term will affect total costs, but by itself does not influence the choice of instruments for the pollutant that is potentially subject to a tax.

3.4.2.2 The Tax Policy with No Covariance in the Random Variables

It is fairly easy to see that one can pick parameters to create any expected welfare ordering of the $\{t_m^*,s^*\}$, $\{m^*,t_m^*\}$ and $\{m^*,s^*\}$ policies. It is a greater challenge to show that reasonable parameters can be chosen such that the expected welfare of the $\{t_m^*,t_s^*\}$ policy may be placed anywhere among any given expected welfare
ordering of the other three instrument mixes. This section and the following thus focus on the \( \{ t^*_m, t^*_s \} \) policy.

We now let the variance in \( \theta_s \) to be significantly large, but continue to assume that the covariance between the two random variables is small (i.e. \( \sigma_{sm} \approx 0 \)). In this case, expected welfare from using a tax to control each pollutant becomes:

\[
W(t^*_m, t^*_s) = W(m^*, s^*) + \frac{1}{2} \left( \frac{\sigma^2_m \left[ (c_{mm} - b_{mm}) c_{ss}^2 - (b_{ss} + c_{ss}) b_{ms}^2 \right]}{\sigma^2_s \left[ c_{ss}^2 - b_{ss}^2 \right]} \right)
\]

(3.37)

This is the only expected welfare expression that is affected by the assumption that \( \sigma_{sm} \approx 0 \). From an inspection of (3.37) a potentially surprising result appears possible. Let us say that the parameter values of this problem are such that the two policies that use a mix of instruments yield higher expected welfare than the policy that strictly uses quantity controls. For this to be true then the parameter values of the problem must be such that \( c_{mm} > b_{mm} \) and \( c_{ss} > b_{ss} \) as we see in Table 3.2. However, these two restrictions do not appear to assure that the \( \{ t^*_m, t^*_s \} \) policy is the preferred policy because of the strictly negative terms in the bracketed expression on the right side of (3.37). We will see in the following analysis that this is the case. Might it be possible that even when \( c_{mm} > b_{mm} \) and \( c_{ss} > b_{ss} \) that a policy that uses a tax to control each pollutant actually yields the lowest expected welfare? The answer is also yes. That said, it is also possible that the \( \{ t^*_m, t^*_s \} \) policy yields the highest expected welfare of the four instrument mixes.
To see these possibilities more easily, let us start by considering the conditions where the \( \{t_m^*, t_s^*\} \) policy yields the highest expected welfare. We define 
\[
\phi \equiv \frac{c_{ms}}{\sqrt{c_{mm} c_{ss}}} \quad \text{where } \phi \text{ has the same sign as } c_{ms} \text{ and } -1 < \phi < 1 \text{ to satisfy the second-order condition for the affected firms’ problem. The variable } \phi \text{ is a proxy for the influence of joint abatement. Furthermore, we define } \\
\omega_m \equiv \frac{b_{mm}}{c_{mm}} \text{ and } \\
\omega_s \equiv \frac{b_{ss}}{c_{ss}}. \text{ We assume that } 0 < \omega_m < 1 \text{ and } 0 < \omega_s < 1 \text{ so that the } \{m^*, t_s^*\} \text{ and } \{t_m^*, s^*\} \text{ policies are preferred to the } \{m^*, s^*\} \text{ policy. If the } \{t_m^*, t_s^*\} \text{ policy yields higher expected welfare than the two policies that use a mix of instruments, the following conditions must both hold:}
\]
\[
0 < \left[ \sigma_m^2 \phi^2 + \sigma_s^2 \frac{c_{mm}}{c_{ss}} \right] \left[ 1 - \omega_s - \omega_m \phi^2 - \phi^2 \right] - \sigma_m^2 \phi^2 2 \omega_m \left[ 1 - \phi^2 \right] \\
0 < \left[ \sigma_s^2 \phi^2 + \sigma_m^2 \frac{c_{ss}}{c_{mm}} \right] \left[ 1 - \omega_m - \omega_s \phi^2 - \phi^2 \right] - \sigma_s^2 \phi^2 2 \omega_s \left[ 1 - \phi^2 \right] \\
(3.38)
\]
The top condition assures that the \( \{t_m^*, t_s^*\} \) policy is preferred to the \( \{m^*, t_s^*\} \) policy, while the bottom condition assures that the \( \{t_m^*, t_s^*\} \) policy is preferred to the \( \{t_m^*, s^*\} \) policy. We see that if \( \omega_m \) and \( \omega_s \) are both sufficiently small, then the \( \{t_m^*, t_s^*\} \) policy is preferred to the other three. That \( \omega_m \) and \( \omega_s \) would be close to 0 suggests that it is relatively more important to allow the abatement levels of both pollutants to adjust in response to the realized values of \( \theta_m \) and \( \theta_s \). Furthermore, it suggests that the marginal benefit curves are fairly flat. The benefit of controlling each pollutant with a tax is thus particularly important.
We can also see that if the following conditions hold then the conditions in (3.38) are violated:

\[
0 > 1 - \omega_s - \omega_m \phi^2 - \phi^2 \\
0 > 1 - \omega_m - \omega_s \phi^2 - \phi^2
\]  

(3.39)

and thus the \( \{t_m^*, t_s^*\} \) policy yields a lower expected welfare than the policies that use a mix of instruments. We see from (3.39) that if the absolute value of \( \phi \), which captures the magnitude of joint abatement, is close enough to 1, then it is possible for the \( \{t_m^*, t_s^*\} \) policy to yield lower expected welfare than the policies that use a mix of instruments.\(^\text{27}\) Recall from our discussion of the expressions for the expected levels of \( m(t_m^*, t_s^*; \theta) \) and \( s(t_m^*, t_s^*; \theta) \) that the deviation in the realized abatement of the pollutants increases as the influence of joint abatement increases.\(^\text{28}\) If the variation in the realized levels of the abatement of both pollutants is large enough, then one of the policies that controls one pollutant by a tax and the other by a quantity is preferred to all.

So, we have established the possibility that the mixed instrument policies may be preferred to the policy that places a tax on each pollutant when all three of these mixes are superior to the policy that controls each pollutant by a quantity. It is a relatively straightforward step to show that it is also possible, in the circumstance

\(^{27}\) Also note that as \( \omega_m \) and \( \omega_s \) get closer to one the gain from the abatement adjustment effect is small relative to the cost associated with the abatement uncertainty effect.

\(^{28}\) Strictly speaking, this is only absolutely true for the abatement of both pollutants if there is no covariance between \( \theta_m \) and \( \theta_s \) (which we are assuming at this point). With a positive covariance between the random variables in the cost function the variance of the abatement of one of the pollutants may fall as the magnitude of joint abatement \( c_{ms} \) increases.
where the mixed instrument policies are preferred to the two policies that use the same instrument, for the policy that controls each pollutant by a quantity to be preferred to the policy that uses a tax to control each pollutant.\textsuperscript{29} Using our notation, the difference in the expected welfare from the $\{t_m^*, t_s^*\}$ policy minus the expected welfare from the $\{m^*, s^*\}$ policy is:

\[
W(t_m^*, t_s^*) - W(m^*, s^*) = \frac{1}{2\left(1 - \phi^2\right)^2} \left[ \frac{\sigma_{mm}^2}{c_{mm}} \left[ 1 - \omega_m - \omega_s \phi^2 - \phi^2 \right] + \frac{\sigma_{ss}^2}{c_{ss}} \left[ 1 - \omega_s - \omega_m \phi^2 - \phi^2 \right] \right] \quad (3.40)
\]

Clearly this expression may be negative if $\omega_m$, $\omega_s$, and $|\phi|$ are large enough. With $\omega_m$ and $\omega_s$ close to one, the benefit of the abatement of the two pollutants adjusting to the difference in expected cost is small, and with $|\phi|$ close to one the variance in the expected abatement of both pollutants is quite high.

For the record, we have shown in this section that the following welfare orderings are possible:

\[
W(t_m^*, t_s^*) > W(t_m^*, s^*) \geq W(m^*, t_s^*) > W(m^*, s^*) \quad (3.41)
\]
\[
W(t_m^*, s^*) \geq W(m^*, t_s^*) > W(t_m^*, t_s^*) > W(m^*, s^*) \quad (3.42)
\]

And, perhaps most surprisingly, so is the following welfare ordering:

\[
W(t_m^*, s^*) \geq W(m^*, t_s^*) > W(m^*, s^*) > W(t_m^*, t_s^*) \quad (3.43)
\]

We have not explicitly laid out all of the parameter realizations that lead to these ordering, but we have shown that increasing the influence of joint abatement relative to the differences between the slopes of the partial marginal abatement cost and

\textsuperscript{29} And thus is a more rapid way to demonstrate the immediately proceeding results.
benefit curves leads to the ordering in (3.43) when the random variables are not correlated.\textsuperscript{30}

3.4.2.3 The Tax Policy with Covariance in the Random Variables

The previous section might lead one to conclude, given some values for $b_{mm}$, $b_{ss}$, $c_{mm}$ and $c_{ss}$, that the presence of joint abatement would make it more likely that the policy that uses a tax to control each pollutant yields the lowest expected welfare. This would be the wrong conclusion, at least without first considering the case where the random variables are correlated (i.e. $\sigma_{sm} \neq 0$). This returns us to the full expression for the expected welfare from the $\{t^*_m, t^*_s\}$ instrument mix. We will see that in this case it is possible for the $\{t^*_m, t^*_s\}$ policy to yield the highest expected welfare even when the policy that controls each pollutant by a quantity, $\{m^*, s^*\}$, is preferred to both policies that use a mix of instruments.

To simplify our analysis we again turn to the variables $\omega_m$, $\omega_s$ and $\phi$ as defined above. However, we now assume $\omega_m \geq 1$ and $\omega_s \geq 1$ so that the $\{m^*, s^*\}$ policy is preferred to both policies that use a mix of instruments. The expected welfare difference between the $\{t^*_m, t^*_s\}$ policy and the $\{m^*, s^*\}$ policy can then be expressed:

\textsuperscript{30} The careful reader will note that the possibility of an ordering similar to (3.43), with $W(m^*, t^*_s)$ and $W(m^*, s^*)$ yielding approximately the same expected welfare, was identified in the section 3.4.1.2.
\[ W(t^*_m, t^*_s) - W(m^*, s^*) = \frac{1}{2c_{ms}c_{ss}[1-\phi^2]^2} \left[ \sigma^2_{mss} \left[ 1 - \omega_m - \omega_s \phi^2 - \phi^2 \right] \\
+ \sigma^2_{ss} \left[ 1 - \omega_s - \omega_m \phi^2 - \phi^2 \right] \\
- \sigma_{ms} 2\phi \sqrt{c_{mm}c_{ss}} \left[ 1 - \omega_s - \omega_m - \phi^2 \right] \right] \] (3.44)

This is still a difficult expression to handle. To see more clearly the possibility that the \( \{t^*_m, t^*_s\} \) policy may be preferred given our assumptions regarding the values of \( \omega_m \) and \( \omega_s \), let \( \omega_m = \omega_s = 1 \) so that the regulator is indifferent between the \( \{m^*, t^*_s\} \), \( \{t^*_m, s^*\} \) and \( \{m^*, s^*\} \) policies.\(^{31}\) Then, for expression (3.44) to be positive, the following condition must hold:

\[ \frac{\rho_{ms}}{\phi} > \frac{1}{1+\phi^2} \left[ \frac{\sigma^2_{mss}}{\sigma^2_{ss}} + \frac{\sigma^2_{ss} c_{mm}}{\sigma^2_{ms} c_{ss}} \right] \] (3.45)

where \( \rho_{ms} \) is the correlation coefficient between the two random variables. The right side of (3.45) is clearly positive, so we see that the condition is violated unless \( \sigma_{ms} \) and \( c_{ms} \) have the same sign (i.e. \( \rho_{ms}/\phi \) is positive). We will see why shortly. But first, to understand the conditions under which (3.45) holds, note that the bracketed term on the right is minimized when \( \frac{\sigma^2_{ss} c_{mm}}{\sigma^2_{ms} c_{ss}} = 1 \) in which case it equals 2. Therefore, we see that if the absolute value of \( \phi \) is sufficiently small and the absolute value of \( \rho_{ms} \) is sufficiently large (and again they both have the same sign), then it is possible for (3.45) to hold and thus for the following ordering:

\[ W(\{t^*_m, t^*_s\}) > W(\{m^*, s^*\}) = W(\{t^*_m, s^*\}) = W(\{m^*, t^*_s\}) \] (3.46)

to hold.

\(^{31}\)It is left to Appendix 3.A to demonstrate the possibility that the \( \{t^*_m, t^*_s\} \) policy may yield the highest expected welfare even when \( \omega_m > 1 \) and \( \omega_s > 1 \).
Now we return to the question as to why $\sigma_{ms}$ and $c_{ms}$ must have the same sign in order for the policy that uses taxes exclusively to yield the highest expected welfare when the other three instrument mixes yield the same expected welfare. The answer can be seen in the variance of the abatement levels when each pollutant is controlled by a tax. For a moment let us consider the abatement adjustment and the abatement uncertainty effects. With the $\{m^*, s^*\}$ policy yielding higher expected welfare than the $\{m^*, t^*_r\}$ and $\{t^*_m, s^*\}$ policies, it must be the case that for each policy where one of the pollutants is controlled by a tax that the cost of the abatement uncertainty effect is greater than the benefit of the abatement adjustment effect. For the abatement uncertainty effect to be “large”, the variance in level of the pollutant controlled by a tax must be large. Thus, if it is indeed the case that the $\{t^*_m, t^*_s\}$ yields higher expected welfare than the three other possible mixes, then it must be the case that the abatement uncertainty effect is relatively small than when only one of the pollutants is controlled by a tax. Appendix 3.B shows that a sufficient condition for $\{t^*_m, t^*_s\}$ to yield higher expected welfare is that the variance in abatement for both $m$ and $s$ is smaller under this policy than when each is the sole pollutant controlled by a tax. In order for the variance in the abatement quantities to be smaller when both pollutants are controlled by a tax, the influence of the correlation between the random variables must offset the effect of the joint abatement relationship. This occurs when

32 The small abatement uncertainty effect comes at a cost of a smaller abatement adjustment effect. What is happening when both pollutants are controlled by a tax is that there are two relatively small abatement adjustment effects offsetting two small abatement uncertainty effects.
the $\sigma_{ms}$ and $c_{ms}$ terms have the same sign. Furthermore, with both terms having the same sign, the covariance between $m(t_m^*, t_s^*; \theta)$ and $s(t_m^*, t_s^*; \theta)$ is smaller (than if they had the same magnitude but different signs), which reduces the abatement uncertainty effect associated with the term $-c_{ms}ms$.

To be clear, our focus on the possibility that the $\{t_m^*, t_s^*\}$ policy may yield the lowest expected welfare when the correlation of the random variables was small, and how it could be the best policy when the correlation was large, in both cases despite the expected welfare ordering of the other instrument mixes, was to isolate the importance of the correlation. While the presence of joint abatement itself would suggest that taxes are less preferred, the correlation in the random variables may offset or exacerbate the influence of joint abatement. It is probable that the random variables errors are correlated as they are generated by the same underlying primal cost-minimization problem.

3.4.3 Relating Results to Theory of Second-Best

Expected welfare orderings such as that found in expression (3.43), where the $\{t_m^*, t_s^*\}$ policy has the highest expected welfare followed by the $\{m^*, s^*\}$ policy, yields an observation related to other analyses of second-best regulatory conditions. Say the regulator was restricted to using a quantity instrument to control $s$ for some reason.\(^{33}\) With this welfare ordering, $m$ should then be controlled with a quantity

\[^{33}\text{Admittedly, the impetus for using a quantity to control } s \text{ may suggest yet another distortion that will differentially affect the performance of the tax and quantity instruments.}\]
instead of by a tax if expected welfare is to be maximized subject to this restriction. Generally speaking, the second-best literature focuses on how *levels* of regulatory instruments vary in the presence of preexisting distortions. The most common example in the economics of environmental policy is in the double-dividend literature which shows that the optimal pollution tax in the presence of preexisting labor tax distortions is less than the optimal suggested by the Samuelson conditions (i.e. marginal social benefit equaling marginal social cost of pollution) (Goulder, 2002). In the context of instrument choice under uncertainty, a study by Quirion (2004) shows that the comparative advantage of a tax over a quantity policy increases in the presence of a distortionary labor tax even when the quantity is auctioned off. In the analysis presented here, it is the instrument itself that may be suboptimal given a distortion in the choice of the instrument to control the other pollutant. An ordering of the form in (3.46) indicates that using a tax to control $m$ is suboptimal given some exogenous restriction that $s$ must be controlled by a quantity. Presumably the optimal instrument to control $m$ may also be affected if there is a restriction on the *level* of the instrument to control $s$. To my knowledge, this is the first demonstration where the regulatory instrument itself changes in a second-best setting where there is an exogenous restriction on the instrument used to control a related pollutant.

A study that finds a similar result is Oates and Schwab (undated). Recall from Chapter 1 that in their model there are two media-specific agencies, each regulating one pollutant. There is a dirty input that generates both pollutants, so the pollutants are substitutes, and a pollution abatement technology. They find that, conditional on one of the agencies imposing a technology standard, the other agency
should tax both the pollutant is responsible for and the dirty input. This result is similar to the findings of the optimal pollution tax literature. That literature finds that, if the optimal pollution tax can not be imposed (say, for political reasons), it may be optimal for the regulator to tax a polluting input. If the optimal tax could have been adopted, then it is not necessary to regulate the dirty input. The distinction in Oates and Schwab is that one regulator is imposing a suboptimal regulation on one pollutant, so the regulator of the other pollutant taxes both that pollutant and the dirty input that created it.

One distinction between my findings and those in Oates and Schwab is that in the model here there is the intent that the level of the instrument that is exogenously determined is intended to maximize (expected) welfare. In Oates and Schwab the regulator that moves first may not necessarily be interested in maximizing welfare and thus may impose a suboptimal method of control (like a technology standard) or may not pick a pollution tax or quantity level that accounts for how the level of the jointly abated pollutant changes.

However, the Oates and Schwab framework suggests another question that may be explored with the model herein. That is, what if the two pollutants were regulated by different agencies, and the instruments were adopted in sequence rather than simultaneously? While not explored here, given our second best findings, one can imagine that the instrument mix that results from the sequential adopting of controls may not be the optimal mix from simultaneously adopting the controls. This is particularly true if the regulator that moves first does not account for how the instrument it selects may influence the level of the other pollutant.
3.4.4 Special Cases

In this section we consider some special cases that may be of empirical interest. The key question is whether the intuition drawn from the single pollutant setting extends to the two-pollutant setting when the shapes of the marginal benefit and cost functions are at their extremes. Generally speaking, we will see that it does.

3.4.4.1 Constant Marginal Benefits

We first explore the case where the slopes of the two marginal benefit functions are zero. That is, where \( b_{mm} = b_{ss} = 0 \). It is immediately clear from Table 3.2 that in this case the mixed instrument policies yield higher expected welfare than the policy that strictly uses quantities. Presumably, however, in this case the policy that uses a tax to control each pollutant set equal to \( b_m \) or \( b_s \) yields the highest expected welfare. We will show that this is indeed true. We’ll compare \( W(t^*_m,t^*_s) \) to \( W(t^*_m,s^*) \) and it will be clear that the finding will hold (symmetrically) for a comparison between \( W(t^*_m,t^*_s) \) and \( W(m^*_m,t^*_s) \). Again, \( \rho_{ms} \) is the correlation coefficient between the two random variables and we use the definition \( \phi \equiv c_{ms}/\sqrt{c_{mm}c_{ss}} \) where \(-1 < \phi < 1\).

For \( W(t^*_m,t^*_s) \) to be greater than \( W(t^*_m,s^*) \) it must be true that the following expression is strictly positive:

\[
W(t^*_m,t^*_s) - W(t^*_m,s^*) = \frac{\sigma_m^2 c_{xx} + \sigma_s^2 c_{mm} - 2\rho_{ms} \phi \sigma_m \sigma_s \sqrt{c_{mm}c_{ss}}}{2c_{mm}c_{ss} [1 - \phi^2]} - \frac{1}{2} \frac{\sigma_m^2}{c_{mm}} \quad (3.47)
\]

Note that if it is at all possible for \( W(t^*_m,s^*) \) to be greater than \( W(t^*_m,t^*_s) \), then it must be possible when \( W(t^*_m,t^*_s) \) is at its lowest level. As such, we will compare expected
welfare, assuming the conditions $|\rho_{ms}|=1$ and $\rho_{ms}\phi>0$, which minimizes (3.47).

Expression (3.47) then becomes:

$$W\left(t_m^*,t_s^*\right)-W\left(t_m^*,s^*\right)=\frac{1}{2c_{mm}}\frac{1}{c_{ss}}\left[\sigma_s\sqrt{c_{mm}}-\phi\sigma_m\sqrt{c_{ss}}\right]^2$$

(3.48)

There is no reason to suspect that the squared bracketed term on the right side of this expression is an imaginary number, so we take it as positive. Therefore, the entire expression is positive and the expected welfare from the $\{t_m^*,t_s^*\}$ policy cannot be less than the expected welfare from the $\{t_m^*,s^*\}$ policy. A symmetric result derives from a comparison between the $\{t_m^*,t_s^*\}$ and the $\{m^*,t_s^*\}$ policy.

What if only one of the marginal benefits where constant, say $b_{ss} = 0$, which instrument mix would yield the highest expected welfare? In the single pollutant case this would argue for a tax to be used to control $s$, but with joint abatement this simple result is no longer clear. Right off we see that it is possible that $W\left(t_m^*,s^*\right)$ may be larger than $W\left(m^*,t_s^*\right)$. That is, despite $b_{ss} = 0$, it may be relatively more important to tax $m$ than $s$. However, this does beg the question as to whether, if indeed it is true that:

$$\frac{\sigma_m^2}{c_{mm}}[c_{mm}-b_{mm}]>\frac{\sigma_s^2}{c_{ss}}$$

(3.49)

which is required for $W\left(t_m^*,s^*\right)>W\left(m^*,t_s^*\right)$ when $b_{ss} = 0$; then must it also be true that $W\left(t_m^*,t_s^*\right)>W\left(t_m^*,s^*\right)$? The answer is yes, and the preceding proof demonstrates this.

Note that $b_{mm} = 0$ maximizes the left side of (3.49) (without violating the constraint
that $b_{nm}$ be non-negative). We already showed that $W(t_m^*, t_s^*) > W(m^*, t_s^*)$ when $b_{nm} = 0$ (and $b_{ss} = 0$) is positive in expression (3.48).

### 3.4.4.2 Infinite Marginal Benefit Slopes

The next special case to consider is when the slope of the marginal abatement benefit curve becomes infinitely steep. We will assume that $b_{nm} \to \infty$. In the single pollutant case this would suggest that $m$ should be controlled by a quantity. With our two-pollutant problem, this finding holds when $s$ is controlled by a quantity. However, it is not immediately clear from looking at Table 3.2 whether this result holds when $s$ is controlled by a tax. That is, must the following expression be strictly positive when $b_{nm} \to \infty$?

$$W(m^*, t_s^*) - W(t_m^*, t_s^*) = \frac{\sigma_s^2}{2c_{ss}} [c_{ss} - b_{ss}] + \frac{b_{mm} \left[ \sigma_m^2 c_{ss}^2 + \sigma_m^2 c_{ms}^2 - 2 \sigma_m c_{ms} c_{ss} \right]}{2 \left[ c_{mm} c_{ss} - c_{ms}^2 \right]^2}$$

$$- \frac{1}{2 \left[ c_{mm} c_{ss} - c_{ms}^2 \right]^2} \left[ \sigma_m^2 c_{mm} c_{ss}^2 - b_{ss} c_{ss} + c_{ss} c_{ss} \right]^2 + \sigma_s^2 \left[ c_{ss} - b_{ss} \right]^2$$

$$\left[ c_{mm} c_{ss} - c_{ms}^2 \right]^2 - \sigma_m^2 c_{ms}^2 - c_{mm} b_{ss} - c_{ms}^2 \right] \right] \right] \right]$$

(3.50)

The answer is yes. To see this, first note that the first and third terms on the right side of (3.50) do not contain $b_{nm}$ and thus are not relevant in the limit. The second term therefore determines the sign in the limit and is clearly conditional on the bracketed term in the numerator. The bracketed term in the numerator is strictly positive given:

$$\sigma_m^2 c_{ss}^2 + \sigma_s^2 c_{ms}^2 - 2 \sigma_m c_{ms} c_{ss} \leq \sigma_m^2 c_{ss}^2 + \sigma_s^2 c_{ms}^2 - 2 \sigma_m \sigma_s c_{ms} c_{ss} = \left[ \sigma_m c_{ss} - \sigma_s c_{ms} \right]^2$$

(3.51)

That is, as the covariance must be less than $\sigma_m \sigma_s$, the expression on the left must be less than the expression in the middle, which we see is a perfect square.
3.4.4.3 Constant Partial Marginal Costs

We now analyze the case where the slope of the partial marginal abatement cost function of $m$ approaches zero, i.e. as $c_{mm} \to 0$. In this case the only expected welfare relationship that is clear from an inspection of Table 3.2 is that

$$W(m^* , s^*) > W(t^*_m , s^*) .$$

In the single pollutant case this is the result we would expect, that $m$ be controlled by a quantity. We see that if $c_{ss} > b_{ss}$ then $W(m^* , t^*_s)$ is greater than $W(m^* , s^*)$. This does not change the instrument used to control $m$, of course.

What is not clear, however, is that, even when it is advantageous to control $s$ by a tax when $c_{mm} \to 0$, must it be true that it can never be the case that it is advantageous to control $m$ by a tax? That is, must it always be true that $W(m^* , t^*_s) > W(t^*_m , t^*_s)$ when $c_{mm} \to 0$ and $c_{ss} > b_{ss}$?

Before we explore this question, note that with the cost function this extreme case is a little more difficult to analyze because one must assure that the second order condition is satisfied. Thus, we must also say something about the magnitude of $c_{ms}$ in our comparisons. To get around this issue we again use our proxy variable

$$\phi \equiv c_{ms} / \sqrt{c_{mm}c_{ss}} .$$

To return to the question at hand, the issue is whether the following expression is strictly positive:

$$(3.52)$$

$$W(m^* , t^*_s) - W(t^*_m , t^*_s) = \left[ -\sigma_m^2 \left[ (c_{mm} - b_{mm})c_{ss}^2 - (b_{ss} + c_{ss}) \phi^2 c_{mm}c_{ss} \right] \right] \right] \left[ -\sigma_s^2 \phi \left[ (\phi^2 - 2)[c_{ss} - b_{ss}]c_{mm}^2 - (b_{mm} + c_{mm})c_{mm}c_{ss} \right] \right] + \sigma_s m 2 \phi \sqrt{c_{mm}c_{ss}} \left[ -b_{mm}c_{ss} - c_{mm}b_{ss} + [1 - \phi^2]c_{mm}c_{ss} \right]$$
when $c_{mm} \to 0$ and $c_{ss} > b_{ss}$. The large bracketed term on the right of this expression will dictate the sign of (3.52) as $c_{mm} \to 0$ (the expression multiplying the bracketed term is strictly positive). The bracketed term converges to the non-negative expression $\sigma_m^2[b_{mm}c_{ss}^2]$ as $c_{mm} \to 0$, implying that it is never advantageous to control $m$ by a tax when $c_{mm} \to 0$.

3.4.4.4 Infinite Partial Marginal Cost Slopes

Our next special case considers the situation where the slope of the partial marginal abatement cost function approaches infinity, so that $c_{mm} \to \infty$. In the single pollutant case this condition would argue for controlling $m$ by a tax. This clearly holds for the two-pollutant case when $s$ is controlled by a quantity. However, if we inspect the relationship between the two mixed instruments policies as $c_{mm} \to \infty$:

$$W(m^*, t^*_s) - W(t^*_m, t^*_s) = \frac{\sigma_m^2}{2c_{ss}}[c_{ss} - b_{ss}] - \frac{\sigma_m^2}{2c_{mm}^2}[c_{ss} - b_{ss}]$$

(3.53)

we see it is possible for the $\{m^*, t^*_s\}$ mix to yield higher expected welfare if $c_{ss} > b_{ss}$ because the far right term in (3.53) converges to 0 as $c_{mm} \to \infty$. Of course, this again begs the question, if $W(m^*, t^*_s)$ is greater than $W(t^*_m, t^*_s)$ when $c_{mm} \to \infty$, must it then be the case that $W(t^*_m, t^*_s)$ is strictly greater than $W(m^*, t^*_s)$ when $c_{mm} \to \infty$? This would return us to the single pollutant result that regardless of how $s$ is controlled, it is best to control $m$ by a tax when $c_{mm} \to \infty$. Given the form of the cost function it is

---

34 An interesting ancillary result is if the regulator is restricted to using a tax to control $m$ despite the fact that $c_{mm} \to 0$, it is always better to control $s$ by a tax despite the relative magnitude of $c_{ss}$ and $b_{ss}$.
indeed best to control \( m \) by a quantity in this case. This can be seen in the following expression:

\[
W(t^*_m,t^*_s) - W(m^*_m,t^*_s) = \frac{\sigma^2_m [c_{ss} - b_{ss}]}{2} \left[ \frac{c_{mm}^2}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 - \frac{1}{c_{ss}^2} \\
+ \frac{1}{2} \left[ \frac{\sigma^2_m [c_{mm} - b_{mm} c_{ss} + b_{mms} c_{ms}^2]}{c_{mm} c_{ss} - c_{ms}^2} \right]^2 + \frac{\sigma^2_y}{c_{ms}^2} \left[ -b_{mm} + c_{mm} c_{ms}^2 \right]^{-2} - \sigma_{ms}^2 \left[ \frac{c_{mm} c_{ss} - b_{mm} c_{ss} - b_{mms} c_{ms}^2}{c_{mm} c_{ss} - c_{ms}^2} \right]
\]

(3.54)

Note that the right-most term in (3.54) converges to zero as \( c_{mm} \to \infty \) (the denominator has a squared \( c_{mm} \) term while the largest exponent on \( c_{mm} \) in the numerator is 1). Thus the sign of this expression is dictated by the first term on the right side of (3.54). This term converges to zero as \( c_{mm} \to \infty \), but notably is non-negative for all values of \( c_{mm} \), so even as the limit is approached it is optimal to control each pollutant by a tax.

What is the cause of this indifference at the extreme value of \( c_{mm} \)?

Basically, as \( c_{mm} \to \infty \) changing the intercept of the partial marginal abatement cost of \( m \) has little effect on the location of the curve (this explains why \( \sigma^2_m \) and \( \sigma^2_{ms} \) are not in the first term on the right side of (3.54)). The random variable changes the intercept of this function, so as it varies the curve does not move considerably. In the extreme case of \( c_{mm} \to \infty \), the error essentially becomes irrelevant.\(^{35}\) Clearly we are bounded to the restrictions from the form of the abatement cost function and how it is influenced by the source of randomness in this case.

\(^{35}\) Note that this is true in the single pollutant case. As \( c_{mm} \to \infty \) indeed it is best to control \( m \) by a tax, but the difference in the expected welfare from the two competing instruments converges to 0.
3.4.4.5 Equal Partial Marginal Benefit and Cost Slopes

While an informative exercise, the four cases just discussed do not indicate any surprising results nor deviate from what would be expected in a single pollutant case. We now revisit one example where indeed the intuition from the single pollutant case no longer holds when there is joint abatement. This is the case where the slopes of the marginal benefit functions equal the respective slopes of the partial marginal cost functions. In the case where there is no joint abatement, when \( c_{mm} = b_{mm} \) and \( c_{ss} = b_{ss} \), the regulator is indifferent to whether a tax or a quantity is used to control each pollutant as all four instrument mixes yield the same expected welfare. In the joint abatement setting, expected welfare is still the same for the \( \{ t^*_m, s^* \} \), \( \{ m^*, t^*_s \} \) and \( \{ m^*, s^* \} \) policies. However, expected welfare from the \( \{ t^*_m, t^*_s \} \) policy is different from that from the other three mixes, and it is possible that it yields higher or lower expected welfare than the other three instrument combinations. We showed in section 3.4.2.3 that \( \sigma_{ms} \) and \( c_{ms} \) must have the same sign for the \( \{ t^*_m, t^*_s \} \) policy to be preferred.

A further inspection of expression (3.45) showed that the absolute value of the correlation coefficient of the random variables needs to be relatively large and the magnitude of the joint abatement relationship, \( |\phi| \), needs to be relatively small for this relationship to hold.

3.4.4.6 Summary of Special Cases

The results of this section are summarized in Table 3.3.
### Table 3.3: Summary of Special Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Conditions</th>
<th>Expected Welfare Maximizing Instrument Mix</th>
<th>with $c_{sm}=0$</th>
<th>with $c_{sm} \neq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfectly elastic marginal benefits</td>
<td>$b_{mm} = 0$</td>
<td>${t^<em>_m, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${t^<em>_m, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td>${t^<em>_m, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${t^<em>_m, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td></td>
</tr>
<tr>
<td>Perfectly inelastic marginal benefits</td>
<td>$\lim b_{mm} \to \infty$</td>
<td>${t^<em>_m, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${t^<em>_m, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td>${t^<em>_m, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${t^<em>_m, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td></td>
</tr>
<tr>
<td>Perfectly elastic partial marginal costs</td>
<td>$\lim c_{mm} \to 0$</td>
<td>${t^<em>_m, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${t^<em>_m, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td>${t^<em>_m, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${t^<em>_m, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td></td>
</tr>
<tr>
<td>Perfectly inelastic partial marginal costs</td>
<td>$\lim c_{mm} \to \infty$</td>
<td>${m^<em>, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${m^<em>, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td>${m^<em>, t^</em><em>s}$ if $c</em>{ss} &gt; b_{ss}$, ${m^<em>, s^</em><em>s}$ if $c</em>{ss} &lt; b_{ss}$</td>
<td></td>
</tr>
<tr>
<td>Marginal benefits and partial cost slopes are equal</td>
<td>$b_{mm} = c_{mm}$, $b_{ss} = c_{ss}$</td>
<td>All mixes yield same expected welfare</td>
<td>The ${t^<em>_m, s^</em>_s}$, ${m^<em>, t^</em>_s}$ and ${m^<em>, s^</em>_s}$ policies yield same expected welfare. Generally speaking, the ${t^<em>_m, t^</em>_s}$ policy yields the highest expected welfare if the correlation coefficient of the random variables is larger, but has the same sign, as the joint abatement relationship.</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.5 Summary of Analytics

There are two key implications to draw from this analysis, and they arise *despite* the fact that we are using a particular functional form. The first is that, in the presence of joint abatement, a simple comparison of relative slopes without accounting for the way in which other pollutants are controlled may be misleading.
This was most easily seen in the example where the regulator is deciding on how to control $m$ where there is only one error, $\theta_m$, and the pollutant $s$ is controlled by a tax. The appropriate comparison is not the relative slopes of the marginal benefit of $m$ and the marginal cost of $m$. The effect on $s$, which varies, must also be accounted for. However, a weighting of the abatement adjustment and abatement uncertainty effects from using tax instruments continues to identify the optimal instrument mix.

The second is that pairwise comparisons of instruments, even when they consider the way in which the other pollutant is controlled, do not necessarily suggest the combination of instruments that yields the highest expected welfare. For example, if both policies with mixed instruments yield higher expected welfare than the policy that strictly uses quantity instruments, it is not necessarily true that the policy that strictly uses taxes is preferred. Indeed, it may be the worst option of the four. This result implies that the optimal method of controlling one pollutant may be conditional on the way the other pollutant is controlled.
Appendix 3.A Further Exploration of Welfare Orderings

In section 3.4.2.3 we showed that the following welfare ordering was possible:

\[ W\{t_m^*, t_s^*\} > W\{m^*, s^*\} = W\{t_m^*, s^*\} = W\{m^*, t_s^*\} \]  

(3.55)

In this appendix we want to demonstrate the possibility that the policy that uses a tax to control each pollutant may be preferred when the policy that uses a quantity to control each pollutant strictly yields the second highest expected welfare. For convenience let \( c_{mm} - b_{mm} = c_{ss} - b_{ss} = \omega \) where \( \omega \) is negative. This implies that \( W\{m^*, s^*\} > W\{m^*, t_s^*\} \) and \( W\{m^*, t_s^*\} > W\{t_m^*, s^*\} \). We simplify further by letting \( c = c_{mm} = c_{ss} \) and \( \sigma^2 = \sigma_m^2 = \sigma_s^2 \). These conditions are not necessary for the result to hold, but they make the possibility easier to see. Again we use the definition \( \phi = c/c_{ms} \) where \(-1 < \phi < 1\). For the expected welfare from the \( \{t_m^*, t_s^*\} \) policy to be higher than the expected welfare from the \( \{m^*, s^*\} \) policy, the following constraint must hold (where \( \rho_{ms} \) is the correlation coefficient between the two errors):

\[
\frac{\sigma^2}{c^2\left[1-\phi^2\right]^2} \left[ \omega \left[ 1 - 2\rho_{ms}\phi + \phi^2 \right] + \phi c \left[ \rho_{ms} - 2\phi + \rho_{ms}\phi^2 \right] \right] > 0
\]  

(3.56)

It is not transparent that this expression can be positive for negative \( \omega \). However, if let \( \rho_{ms} = 1 \) then (3.56) becomes:

\[
\frac{\sigma^2 \left[ \omega + \phi c \right]}{c^2 \left[1-\phi^2\right]} > 0
\]  

(3.57)

If \( \omega \) is only slightly negative, it is possible for (3.57) to hold if \( \phi \) is positive and \( c \) is large.
Appendix 3.B Reduced Variance in Quantities Sufficiency

Condition

In this Appendix we show that a sufficient condition for the following welfare ordering to hold:

\[ W\{t^*_m, t^*_s\} > W\{m^*, s^*\} = W\{t^*_m, s^*\} = W\{m^*, t^*_s\} \]  \hspace{1cm} (3.58)

is that the variances in the abatement quantities for the policy that uses a tax to control each pollutant are lower than when each pollutant is the one solely controlled by a tax. Recall that we showed that expression (3.58) (i.e., expression (3.45)) holds if the following is true (where \( \phi \equiv c_{ms}/\sqrt{c_{mm}c_{ss}} \) and \(-1 < \phi < 1\), \( b_{mm}/c_{mm} = b_{ss}/c_{ss} = 1 \) and \( \rho_{ms} \) is the correlation coefficient between the random variables):

\[ \frac{\rho_{ms}}{\phi} > \frac{1}{1 + \phi^2}\left[\sqrt{\sigma_{ms}^2c_{ss}/\sigma_{mm}^2c_{ss}} + \sqrt{\sigma_{ms}^2c_{mm}/\sigma_{ss}^2c_{ss}}\right] \]  \hspace{1cm} (3.59)

Where this expression is the same as expression (3.46).

Now we compare the variances in the quantities when each pollutant is controlled by a tax and when only one pollutant is controlled by a tax. In order for the covariance in \( m \) to be smaller when each pollutant is controlled by a tax compared to when only \( m \) is controlled by a tax, the following relationship must hold:

\[ \text{var}(m(t^*_m, t^*_s; \theta)) < \text{var}(m(t^*_m, s^*; \theta)) \Rightarrow \frac{\sigma_{ms}^2c_{ms}^2 + \sigma_{ss}^2c_{ss}^2 - 2\sigma_{ms}c_{ms}c_{ss}}{c_{mm}c_{ss} - c_{ms}^2} < \frac{\sigma_{ms}^2}{c_{mm}^2} \]  \hspace{1cm} (3.60)

Using our simplifications/restrictions this condition reduces to:

\[ \frac{\rho_{ms}}{\phi} > \frac{1}{2}\left[2 - \phi^2\right]\sqrt{\sigma_{ms}^2c_{ss}/\sigma_{mm}^2c_{ss}} + \sqrt{\sigma_{ms}^2c_{mm}/\sigma_{ss}^2c_{ss}} \]  \hspace{1cm} (3.61)

The condition symmetric to (3.61) for \( s \) is:
If (3.61) and (3.62) both hold then the following condition also holds:

\[
\frac{\rho_{\text{mm}}}{\phi} > \frac{1}{2} \left[ \sqrt{\sigma^2_{\text{ms}}/\sigma^2_{\text{mm}}} + \left[ 2 - \phi^2 \right] \sqrt{\sigma^2_{\text{sm}}/\sigma^2_{\text{ss}}} \right]
\]  

(3.62)

Note that the right side of (3.63) is greater than the right side of (3.59) as

\[
3 - \phi^2 < 1\left[ 1 + \phi^2 \right].
\]
CHAPTER 4: INTEGRATED POLLUTION CONTROL WITH
JOINT ABATEMENT: AN APPLICATION TO AIR-
QUALITY REGULATION

4.1 An Application to the Electricity Sector

The control of mercury (Hg), sulfur dioxide (SO₂) and nitrogen oxides
(NOₓ) pollution from the electricity sector presents a relevant and ripe regulatory
context for analyzing the effect of uncertain joint abatement costs on instrument
choice. Federal regulations affecting these pollutants have recently been promulgated
and legislative proposals are currently being considered that would lead to reductions
in these pollutants. At the same time, there is considerable uncertainty as to the effect
of post-combustion technologies designed to abate SO₂, NOₓ and particulate matter
emissions on Hg emissions. The preceding analysis indicates the need for careful
analysis of integrated instrument choice when pollutants are jointly abated and
control costs are uncertain.

In this chapter we carry out such an analysis using a detailed linked partial-
equilibrium simulation model of the U.S. electricity sector. We use this model to
estimate the effect of using competing instrument mixes to control Hg and SO₂
emissions from this sector on expected welfare. By linked we mean that the model
captures the relationship between multiple markets for electricity and the fuels used to
generate electricity over time and space. This model was developed at Resources for
the Future to analyze the effect of changes in environmental and pricing rules on this
sector. The following analysis represents the first time it has been used explicitly to model the effect of uncertainty on the optimal level and performance of a regulation.\(^1\)

The modeling approach herein is unlike most others concerned with the effect of uncertainty on instrument choice in a particular regulatory setting. Previous studies have captured uncertainty through stochastic components of estimated cost functions (Watson and Ridker, 1984) and distributions of parameters drawn from deterministic computable general equilibrium models applied to reduced form analytical models (Newell and Pizer, 2003; Hoel and Karp, 2001). Pizer (2002) incorporates stochastic variables in a reduced form general equilibrium model to estimate the welfare gains of controlling carbon by a tax or quantity instrument. In the model used here, uncertainty is represented through correlated variations in control technology parameters across emitting sources in a detailed market simulation model.\(^2\)

The study described herein is most like Kolstad (1987) and Dowlatabadi and Harrington (1990). Kolstad (1987) uses a market simulation model of SO\(_2\) emitting industries in the four-corners region of the U.S. to compare the properties of a tax, quantity and command-and-control policy. In the Kolstad (1987) model the source of uncertainty in the cost of controlling emissions comes from uncertainty in

\(^1\) The model has been used to explore how uncertainty in, for example, natural gas prices affects the performance of a planned pollution control policy. That is, in the form of sensitivity analyses (see for example Burtraw et al., 2003a). The study described in Burtraw et al. (2006b), which was initiated after this study, also uses this model to evaluate the performance of a pollution control policy when there is uncertainty in a particular component of the model.

\(^2\) There is at least one empirical study of the performance of landing fees and harvest quotas for the management of fish stocks that incorporates parameter uncertainty into a reduced form harvest model (Hannesson and Kennedy, 2005). This study considers the management of a single species.
the final (inelastic) demand for goods. Similarly, Dowlatabadi and Harrington (1990) use a regional model of electricity generators meeting a demand target at least cost. They consider the effect of uncertainty in various parameters (fuel input, demand, plant performance, etc.) on the production cost of given competing regulatory approaches (permits, tax, and plant-level performance standards) to control SO₂. Their model does not consider the benefits of abatement (which varies with the tax and standards policy) when evaluating the competing instruments. Neither of these studies (nor the ones in the previous paragraphs) considers the joint selection of instruments when multiple pollutants are being controlled.

For reasons described below, the following analysis is somewhat limited given the challenge of accounting for parameter uncertainty in an otherwise large and deterministic market model. However, the key components required for a more thorough analysis are identified. Furthermore, the following presentation does demonstrate how the relative efficiency of an instrument to control one pollutant depends on how other jointly abated pollutants are being controlled and provides a sense of the extent of welfare loss that might be expected from choosing the wrong mix of instruments. An important ancillary contribution of this work is that it also provides the first estimate of the efficient level of Hg emissions from the electricity sector.

In the next section we briefly describe the current regulatory environment for the control of these pollutants. This is followed by a discussion of the uncertainty surrounding Hg emissions and its control. We then describe how the benefits and costs of emissions are measured. With this information, we (roughly) identify the
optimal levels of the instruments and estimate and compare the expected welfare for each instrument mix. We then discuss the effect of the different instrument mixes on allowance prices, emissions, the adoption of pollution control technologies, and the distribution of welfare.

4.2 Policy Background

In May 2005 the U.S. Environmental Protection Agency (EPA) adopted the Clean Air Interstate Rule (CAIR), which uses cap-and-trade programs to control SO$_2$ and NO$_X$ emissions (Federal Register, 2005a). The rule comes into effect in 2009 and is intended to facilitate regional compliance with tighter particulate and ozone ambient air quality standards by reducing inter-state transport of these pollutants. The affected sources are large electricity generating and industrial sources in the eastern and central U.S. Coal-burning power plants bear the largest burden of this regulation as they account for nearly all of the SO$_2$ and 75% of the NO$_X$ emissions from the electricity sector.

The Clean Air Mercury Rule (CAMR) was also finalized in May 2005 and applies a cap-and-trade program for Hg emissions to coal-burning power plants (Federal Register, 2005b). Hg is a neurotoxin where fetal exposure has been shown

---

3 That Hg is controlled with a cap-and-trade program is controversial for at least two reasons. The first is simply that Hg is a toxic pollutant with potentially local effects and therefore may be a poor candidate for emissions trading on a national-scale (Bellas and Lange, 2005). The second is based on a legal interpretation of the 1990 Clean Air Act Amendments. Some have argued that the Amendments require a technology-based performance standard to control Hg once it was designated as “hazardous” by the EPA in 2000. The EPA addressed this possibility and actually proposed a rule that would have required compliance with a performance standard at the same time it proposed the rule embodying a cap-and-trade approach. However, given the EPA’s interpretation of how the standard should be set, projected emissions from the proposed standards rule is greater than from the proposed cap-and-trade
to cause IQ loss. The EPA was under court order to develop this rule after determining in December, 2000 that Hg emitted from these sources qualifies as a hazardous pollutant subject to regulation under the 1990 Clean Air Act Amendments. That said, one should not interpret the close timing of the adoption of CAIR and CAMR rules as mere serendipity. It was indeed acknowledged by the EPA that the affected sources could better plan their compliance strategies if adoption of these rules was coordinated.4

Most of the Hg emitted by coal burning facilities is in one of two species or forms. “Elemental” Hg is relatively inert and has a wide deposition pattern, while “oxidized”, or “speciated”, Hg deposits close to its source and is easily converted to methylmercury in aquatic systems (Landis et al. 2004). Methylmercury accumulates in the tissue of predators, including humans, and is known to disrupt fetal development and neurological function. Despite the differing effect of the two forms, the CAMR rule employs an emissions trading program that does not differentiate between them.

A highly debated element of CAMR was the total allowance allocation. The EPA believes that technologies specifically designed to control Hg emissions will not begin to be commercially viable until after 2010. As such, the EPA was

4 Many of the details of these programs are nontrivial, but also are not particularly relevant to the analysis herein. Key design elements include; decentralized allowance allocation, restrictions on banking, integration with Title IV of the 1990 Clean Air Act Amendments, the geographic extent of each market, and separate trading programs for annual and seasonal NOX emissions. For a summary of the two rules see Burtraw et al. (2006a).
reluctant to impose a cap from 2010 to 2018 on Hg tighter than the level of emissions already anticipated.\textsuperscript{5} One particularly significant source of uncertainty in estimating future Hg emissions is the extent to which post-combustion controls for conventional pollutants abate Hg.\textsuperscript{6}

There are two ways this source of uncertainty manifests itself in projections of Hg emissions. The first, which received the most attention during the development of CAMR, is through the effect of NO\textsubscript{X} and SO\textsubscript{2} control retrofits expected for compliance with CAIR. The ancillary Hg reduction from these controls is often referred to in the regulatory process as their Hg “co-benefit”. When a wet SO\textsubscript{2} scrubber is used in combination with a selective catalytic reduction (SCR) system, the most common NO\textsubscript{X} post-combustion control, they can reduce Hg emissions up to 98\%.\textsuperscript{7} At this point the combined effect of these two controls on Hg emissions is relatively well known (EPRI, 2004).\textsuperscript{8}

\textsuperscript{5} EPA decided on an annual allocation of 38 tons of Hg emissions from 2010 to 2017. This is the expected amount of Hg emissions in 2010 given the CAIR rule. Starting in 2018 the allocation will be 15 tons. While the allocation from 2010 to 2017 is based on expected Hg emissions absent CAMR, the Hg allowance price is not expected to be zero as the CAMR allowances may be banked for future use. Thus, Hg emissions are actually expected to be below the cap from 2010 to 2017 as affected sources rely more on post-combustion controls to abate NO\textsubscript{X} and SO\textsubscript{2} than they would absent the CAMR rule (2005a).

\textsuperscript{6} It has been known for a long time that post-combustion controls have an important effect on Hg emissions. Watson (1978) predicted Hg emissions to 2025 assuming that SO\textsubscript{2} scrubbers reduce Hg by about a third. Watson also assumed that by 2000 all boilers firing high sulfur coal would be retrofit with scrubbers while all boilers that fire low-sulfur coal would be retrofit by 2015.

\textsuperscript{7} Wet scrubbers more readily collect Hg when it is in its oxidized form. An SCR system does not reduce Hg independently but converts a significant share of elemental Hg to oxidized Hg. This conversion explains why the collection of Hg by SO\textsubscript{2} scrubbers improves when an SCR system is present.

\textsuperscript{8} When the EPA began considering whether to regulate Hg emissions from power plants, few coal boilers in the U.S. had SCR systems. As such, relatively little was known about the effect of SCRs on Hg emissions. Since then approximately a third of the U.S. coal-fired capacity has installed this
The amount of Hg currently emitted by coal-fired boilers also is uncertain. This is the second, and somewhat neglected, way Hg emissions projections are affected by uncertainty regarding the influence of controls designed to abate other pollutants. Even with the tighter restrictions on NOX and SO2 emissions required by CAIR, a considerable share of the coal-fired generating capacity, about 25% in 2020 by EPA’s central estimate, will not have NOX or SO2 post-combustion controls (U.S.EPA 2005b). The extent to which additional SO2 and NOX controls will be adopted depends in part on how much Hg would be emitted and thus how much must be abated. One way to view uncertainty in the marginal abatement cost function in the instrument choice literature is that the regulator is unsure of the baseline level of emissions. For a discussion of this interpretation see Newell and Pizer (2000).

4.3 Characterizing Uncertainty in Mercury Emissions

In 1999 the EPA requested coal chemistry analyses from 80 randomly selected coal-fired boilers to better understand the effect of conventional pollutant controls on Hg emissions. These data show that the percentage of Hg reduced is a function of the type of post-combustion technologies used to control SO2, NOX and particulate emissions as well as the boiler’s design and type of coal that is fired (U.S.EPA 2002a).

The EPA uses these data to estimate representative Hg removal efficiencies for different boiler categories for the purpose of regulatory modeling (U.S.EPA 2002a; 2005d). The removal efficiency equals one minus the ratio of Hg emitted to technology. These retrofits are in response to the recently enacted summertime NOX cap-and-trade program that affects electricity generators in 19 midwestern and eastern states and Washington, D.C.
the Hg embodied in the coal. The boiler categories are defined by each unique combination of boiler design, coal rank, and SO₂, NOₓ and particulate pollution control listed in Table 4.1. The representative removal efficiency for each category is equal to the unweighted average Hg removal efficiency from all of the flue gas measurements taken from the boilers in that category. For those categories where no boiler was surveyed, EPA assumes that the Hg removal efficiency from a similar category is applicable. In the following analysis we adopt EPA’s approach for grouping boilers and estimating their representative removal efficiencies, with the exception of those categories that include both a wet SO₂ scrubber and SCR control where our estimates of the representative removal efficiencies are drawn from EPRI (2004).

Table 4.1: Characteristics that Define Boiler Categories for Hg Removal Efficiencies

<table>
<thead>
<tr>
<th>Boiler Design (Firing Type)</th>
<th>Particulate Control</th>
<th>NOₓ Control</th>
<th>SO₂ Control</th>
<th>Coal Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulverized Coal</td>
<td>Hot-Side (HS) Electrostatic Precipitator (ESP)</td>
<td>SCR</td>
<td>Wet</td>
<td>Bituminous</td>
</tr>
<tr>
<td>Stoker</td>
<td>Cold-Side (CS) ESP</td>
<td>None/Other</td>
<td>Dry</td>
<td>Subbituminous</td>
</tr>
<tr>
<td>Cyclone/Wet</td>
<td>Fabric Filter (FF)</td>
<td>SCR</td>
<td>None</td>
<td>Lignite</td>
</tr>
<tr>
<td>Fluidized Bed</td>
<td>CSESP+FF</td>
<td>None/Other</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>HSESP+FF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Particulate Scrubber</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>None/Other</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The data show that even within a particular boiler category the Hg reduction varies considerably. For example, the average Hg removal efficiency of pulverized coal boilers that fire bituminous coal and have a cold-side electrostatic precipitator but no SO₂ or NOₓ controls is 46% with a standard deviation of 23%.
This is a non-trivial uncertainty given that currently about 75GW of coal-fired capacity (about a quarter of the total U.S. coal-fired capacity) fits this description.

The estimated variance in the removal efficiencies for each of the boiler categories may simply suggest that the boilers in each category have different, but similar, removal efficiencies. If the removal efficiencies vary across boilers, but on average agree with the regulator’s expectations, then the different instrument mixes will, generally speaking, yield the same welfare level. This is because the shape of the social abatement cost function is known (provided there is a sufficiently large number of sources) even when the ordering of polluters along the function may be unknown.\(^9\)\(^10\) When the removal efficiencies are correlated among boilers in each category, and across categories, then this is no longer the case. Rather, the shape of the expected social abatement cost function is itself uncertain and thus the different instrument mixes would no longer yield the same expected welfare. This is the way we will interpret the sample variances of the removal efficiencies for each boiler category. Furthermore, we will assume that these removal efficiencies are perfectly

---

\(^9\) Admittedly, uncertainty in the variance itself could matter to the performance of a particular instrument mix (and thus across mixes) given that different boilers in the same category can have different utilization rates (i.e., those blessed with a higher removal efficiency and thus lower Hg emissions may operate more). In reality we would likely see a greater variance in the utilization rates the greater the variance in the removal efficiency. However, the model only uses a single removal efficiency for each category.

\(^10\) The presence of such randomness and the inability of the regulator to identify each source type being the primary justification for employing incentive based regulatory policies.
correlated across the categories: when the removal efficiency for one category is high so are the removal efficiencies for all other categories.\textsuperscript{11}

These are not completely baseless assumptions for how one can interpret the variances in the removal efficiencies. For example, one would expect that if, for example, fabric filters contribute to greater Hg removal than expected within one category then the actual removal efficiency would also be higher than expected for other categories that have boilers controlled by fabric filters. Admittedly, this is not the way the data were analyzed, with representative removal efficiencies estimated as functions of individual boiler attributes. But it seems, even if the variance in the observed data may overestimate any common source of error across all of the boilers, that common sources of error are likely to exist.

4.4 Framework for Analysis

In the following analysis net benefits are analyzed in emissions space. This is different from the theoretical analysis in Chapter 3, where the analysis focused on levels of abatement. However, the analysis is fundamentally the same. Emission benefits are equivalent to a reduction in abatement costs while the social damage from emissions is equivalent to the forgone benefits of abatement as emissions increase. Changing to emissions space simplifies the description of the problem and

\textsuperscript{11} This approach bounds the effect of uncertainty on welfare provided the model is monotonic in the same direction for all of the removal efficiencies, which we would expect it is. For a proof of this conjecture, see Kurowicka and Cooke (2006).
allows us to present welfare estimates without having to impose a particular counterfactual emissions baseline from which to measure abatement.¹²

4.4.1 Emissions Benefits

4.4.1.1 The Electricity Sector Model (Haiku)

A simulation model of the electricity market in the contiguous U.S. is used to estimate the benefits of SO₂ and Hg emissions. The model, developed by Resources for the Future and known as “Haiku”, yields partial-equilibrium welfare changes expected to result from imposing environmental and electricity pricing policies on the sector. Haiku is an iterative tâtonnement model in the prices for both outputs and major inputs to this sector.¹³ Welfare is measured as the sum of consumer and producer surplus in the electricity market adjusted by any changes in government revenues.¹⁴ The model also captures the effect of these policies on the production and prices of key inputs to the sector such as coal and natural gas. The model allows for dynamic investment and compliance behavior and is usually solved for two to four simulation years over a 25-year time horizon.

¹² Recall that Newell and Pizer (2000) note that one interpretation of uncertainty in an abatement cost function is that there is uncertainty in the baseline level of emissions. By operating in emissions space we avoid this problem. We also avoid the problem, also noted by Newell and Pizer, that uncertainty in the baseline level of emissions also means that there is uncertainty in the benefit of abatement that is therefore correlated with uncertainty in the abatement cost function. Admittedly, as we will see, the slopes of our damage functions make this last concern irrelevant.

¹³ For additional description of Haiku see the Appendix of this dissertation. For references to its use in peer-reviewed literature see Banzhaf et al. (2004). For each unique combination of model parameters, the version of the model used herein takes approximately 50 hours to solve (converge) on a computer with a 2GHz processor with 1GB of RAM.

¹⁴ As suggested by Harberger (1964, 1971) the use of a partial equilibrium model will fully measure the welfare change from imposing a policy affecting the electricity sector, provided income effects are small and distortions in related markets are insignificant. Parry, 2005, suggests that distortions in the labor market are important for measuring the welfare consequences of policies affecting this sector.
The model divides the contiguous states of the U.S. into 20 regional electricity markets roughly defined by historic regional electricity reliability council boundaries with some further spatial disaggregation. Electricity demand is price responsive and distinguished by consumer class (residential, industrial, commercial), season (summer, winter, spring/fall), and time of day (baseload, shoulder, peak, and superpeak). As such there are 720 distinct retail electricity markets in each simulation year. Interregional trade is modeled endogenously subject to transmission capacity constraints between regions. The wholesale electricity market is assumed to be competitive so that prices are based on the relationship between the marginal cost of generation in different regions. Retail markets can be characterized by either average (cost-of-service) or marginal cost pricing. Deviating from reality, the following analysis assumes that the retail electricity pricing structure in each region is competitive with time-of-day pricing. This is an attempt to avoid unanticipated results driven by suboptimal pricing policies that are endemic in the electricity sector and generate second-best welfare ordering effects when new policies are imposed (Parry, 2005).

Electricity generators are represented by “model plants” defined by nine criteria: location, vintage (existing or new), prime mover, fuel, relative operating cost (for those generators using natural gas or nuclear fuel), and, for coal-fired boilers, coal demand region, capacity, and the expected presence of SO₂ and NOₓ post-combustion abatement controls in 2010. Up to 85 economic and design variables are used to characterize a model plant. These variables are usually weighted averages of
observations drawn from the individual generators the model plant represents. The total capacity of a model plant is the sum of the capacity of its constituent generators.

Electricity generator dispatch at any instance is determined by a model plant’s short-run operating cost. Capital stock investment and retirement, as well as investment in pollution control technologies, are determined by the expected profitability of generation assets over time. Assumptions regarding the performance of new generation capacity and pollution control technologies are drawn from a variety of sources.

4.4.1.2 Modeling Pollution Abatement Options

Haiku tracks emissions of SO$_2$, NO$_X$, Hg and carbon dioxide from each model plant. Considerable emphasis is placed on the ability of the model to represent the common compliance options available to electricity generators to reduce these pollutants. To comply with regulations controlling SO$_2$, NO$_X$, and Hg, coal-fired model plants may install post-combustion controls, reduce their generation, or change the coal they fire. The different coal types are characterized by their rank (heat content), location where they are mined, and sulfur and Hg contents. The post-combustion technologies for SO$_2$ control include wet and dry scrubbers. The coal model plants may adopt SCR or Selective Non-Catalytic Reduction (SNCR) technologies to abate NO$_X$ emissions. Typically, the coal model plants may install activated carbon injection (ACI), which is specifically designed to abate Hg. However, in the following analysis ACI is not available as a control technology under the stark assumption that it will not be commercially available. Finally, we note that plant managers are presumed to make investment decisions knowing the operating
and pollution control costs, including the Hg removal efficiencies, of their plant and all other generators.

4.4.2 Emission Damages

The damage functions for SO$_2$ and NO$_X$ are drawn from Banzhaf et al. (2004). Banzhaf et al. link the Haiku model to an atmospheric transport and health effects model to estimate efficient emission taxes for the control of SO$_2$ and NO$_X$ in 2010.$^{15}$ The damage functions they trace out are non-convex and thus yield non-linear marginal damage functions. The non-convexity arises because of the incongruity of the instrument, a nationally uniform fee, and the varying local effect of the pollutants.$^{16}$ However, the authors find that the marginal damage functions for the two pollutants are generally flat. Using their central estimates, the marginal damage from a ton of SO$_2$ in 2010 is about $3,911 while the marginal damage from a ton of NO$_X$ is about $1,229 (all prices and welfare measures reported herein are in 2004 $).$^{17}$

The marginal damage function for Hg is drawn from Rice and Hammitt (2005). They estimate the annual benefit of Hg reductions expected from the adoption of a particular legislative proposal to control SO$_2$, NO$_X$ and Hg emissions from the

$^{15}$ While a variety of health afflictions are accounted for in the calculation of damages, their analysis does not capture the benefits of improved ecosystem health.

$^{16}$ As the emissions fee increases, the damage caused by the marginally controlled source, depending on its location, may be higher or lower than the previously controlled source.

$^{17}$ Despite the expectation that damages will increase with population growth between 2010 and 2020, the respective years of interest in the Banzhaf et al. and the present study, no attempt is made here to adjust the marginal damage estimates accordingly. A similar criticism can be made for the Hg damage estimate used below, which is an average of the average damage estimates for 2010 and 2020.
electricity sector. Their analysis is limited to human health effects and assumes that the sole pathways for Hg intake are commercial and freshwater fish consumption. The authors identify a variety of human health afflictions that may be caused by Hg, from IQ losses due to fetal exposure to cardiovascular damages that lead to premature mortality. They provide an ordinal ranking of the likelihood that Hg causes these different health effects. This ranking is based on their interpretation of the strength of scientific consensus regarding these potential effects. The following analysis assumes that all of the potential consequences of Hg exposure that the authors identify actually occur. The more speculative benefits, which are those associated with premature mortality, are included in the emission damage estimates to help ensure that the influence of different instrument mixes on expected welfare can be identified. They were also included to calculate a likely lower bound on the efficient level of Hg emissions.

Rice and Hammitt assume that “equilibria currently exist between deposited Hg and fish methylmercury concentrations and between fish methylmercury concentrations and methylmercury exposures to individuals who consume these fish” (xvi). Further, they assume that any changes in deposition will “lead to linear and proportional changes in fish Hg concentrations.” These assumptions abstract from the notions that Hg is a stock pollutant, that there is a lag between Hg deposition and fish uptake, and that fish consumers can influence their
own exposure. The study also abstracts from the complication that Hg is emitted in multiple forms, and the shares of these different forms may change as total emissions change. However, it is the only study that provides detailed estimates of the possible impact of changes in Hg deposition on both IQ and cardiovascular health. While Rice and Hammitt account for the effect of emission location on damages, our analysis ignores spatial heterogeneity in damages. The consequence of all of these assumptions is that the implied marginal damage of Hg is constant.

While we are assuming that all of the potential damages Rice and Hammitt (2005) identify actually occur, for consistency a lower value of a statistical life is used. In particular, the value of a statistical life used herein is drawn from Mrozek and Taylor (2002) ($2.46 million). This is the same value of a statistical life used in Banzhaf et al. (2004) and is 40% of the one used by Rice and Hammitt (2005) and by EPA (U.S.EPA 2005a). In the following analysis the marginal damage per pound of Hg is $40,940 or $2,560 per ounce.

18 Hoel and Karp (2001) and Newell and Pizer (2003) explore instrument choice in the presence of abatement cost uncertainty for a single stock pollutant. Both find that as the persistence of the pollutant increases, ceteris paribus, quantities are favored. Intuitively, increased persistence increases the slope of the marginal damage function, as current emissions would have an even greater effect on future damages.

19 For a more detailed description of the adjustment to the Rice and Hammitt (2005) estimates using the value of statistical life used in Banzhaf et al. (2004), see Palmer et al. (2005).

20 This level of marginal damages is much higher than estimated elsewhere. For example, the EPA estimated that the average damage of the Hg remaining after CAIR comes into effect is about $3,400 per pound in 2020 (U.S.EPA 2005c; U.S.EPA 2006; Griffiths et al., 2006). The central estimate from Gayer and Hahn (2006) is clearly even lower (although difficult to determine exactly as they report damages over multiple periods). Both of these newer studies only consider IQ loses due to prenatal exposure when estimating damages. Of course, the Rice and Hammitt estimate for damages associated with lost IQ is also lower ($5,066 per pound).
Constant marginal damages clearly take some of the excitement out of the instrument choice analysis. Emissions taxes set equal to constant marginal damages cannot be improved upon on efficiency grounds. In part, constant marginal damages are an artifact of the underlying epidemiological and valuation analyses, which often only estimate or have sufficient power to report average effects. Therefore, the form of the damage functions assumed presently is not necessarily a true representation of the relationship between emissions and health and ecosystem damages.

4.4.3 Expected Welfare

4.4.3.1 Determining Optimal Instrument Levels

Now we bring together the benefits and the costs of emissions to determine the optimal levels for the different instrument mixes and compare their performance. We simplify the analysis to the choice of instruments to control Hg and SO2 emissions from coal-fired boilers. The pair \( \{I_m, I_s\} \) identifies the instruments controlling Hg and SO2 where the subscripts indicate the particular pollutant and each instrument \( I \) can either be a cap \( Q \) or a tax \( T \) on emissions.

Where quantity policies are used to control SO2 or Hg, all allowances are allocated via an efficient auction, where the auction price is equal to the marginal value product of emissions across affected generators, and allowances may not be

21 With taxes equal to marginal damages, all of the prices in the model would be considered equal to their respective input’s opportunity cost regardless of the realization of the Hg removal efficiencies.

22 Even assuming the form is correct, of course there is significant uncertainty in the level of marginal damages of emissions for all three pollutants. However, we have ignored uncertainty in the level of damages as this uncertainty, unless it is correlated with the uncertainty in the benefits of emissions, is irrelevant to determining the optimal mix of instruments (although not necessarily the levels of these instruments). More will be said about uncertainty in the level of damages in Chapter 5.
banked. Allocation via an auction is assumed to keep symmetry between the
treatment of tax revenues and the rents accruable to the emission allowances.\textsuperscript{23}

For each model plant there is a performance parameter indicating the
expected Hg removal efficiency given every possible pollution control and coal type
combination that the plant may use. The model selects the cost minimizing
combination of controls and fuel type to comply with a particular regulation given
this information. It is the regulator’s uncertainty in the values of these removal
efficiencies that we are interested in capturing. The regulator will be able to observe
whether the removal efficiencies are higher or lower than expected by observing the
prices of the tradable allowances prices if the pollutants are controlled by quantities
or the emission levels of the pollutants if they are controlled by taxes.\textsuperscript{24}

The vector of Hg removal efficiencies is represented by \( \theta \). The expression:

\[
 w(I_m, I_s; \tilde{\theta})
\]

\[ (4.1) \]

\textsuperscript{23} An alternative justification typically invoked for assuming that allowances are allocated when
comparing tax and quantity policies for electricity generators is that given average cost pricing
grandfathering allowances (giving them away for free) leads to additional distortions in the market (see
Parry, 2005). However, we are presently assuming marginal cost pricing of electricity.

\textsuperscript{24} So, implicit in our modeling is that the regulator may observe total emissions with near certainty
after the regulations are adopted. That is, some mechanism for monitoring emissions at each facility
has been put in place that is more sophisticated than the approach that the EPA used to estimate the Hg
removal efficiencies (see Section 4.3). This is a reasonable assumption. The CAMR program includes
a set of detailed requirements describing how Hg emissions should be monitored. The measurement of
SO\textsubscript{2} and NO\textsubscript{X} emissions from coal-fired boilers is already quite sophisticated given requirements of the
1990 Clean Air Act Amendments.
represents total welfare from a particular instrument mix given some removal efficiency realization $\tilde{\theta}$.\textsuperscript{25} Welfare is defined as the sum of consumer surplus, producer surplus and government revenues from the pollution instruments minus the total pollution damages.\textsuperscript{26}

Optimally, we would like to solve for the level of instruments that maximize expected welfare for each instrument pair $\{T_m, T_s\}$, $\{Q_m, Q_s\}$, $\{T_m, Q_s\}$ and $\{Q_m, T_s\}$:

$$W(I_m, I_s) \equiv E\left[w(I_m, I_s; \theta)\right]$$

(4.2)

and compare the results to determine which mix yields the highest expected welfare. Given the complexity of the model, such an approach is essentially impossible. An alternative method is to solve the model for multiple draws from the removal efficiency distribution using a number of different possible caps (or taxes) on each pollutant type and then use the resulting observations to estimate a stochastic and

\textsuperscript{25} A more accurate, but cumbersome, representation of welfare is as a function of the level of emissions, $q$, where emissions are influenced by a cap or a tax: $w(q_m(I_m, I_s; \tilde{\theta}), q_s(I_m, I_s; \tilde{\theta}), \tilde{\theta})$.

\textsuperscript{26} In the analytical model in the previous chapter the abatement cost function could be taken as an approximation of the cost to producers given that any change in production and output price, and thus to consumer surplus in the final product markets, was negligible. In this analysis the effect on consumer welfare is non-negligible and thus must be accounted for. There are two observations worth mentioning. First, the fact that Weitzman did not consider more general changes when measuring welfare effects is relatively unimportant. The fundamental point of Weitzman’s analysis is that under uncertainty the instruments yield different expected welfare. That we also have to account for changes in consumer surplus does not change this. Second, we will also make an assumption of fixed demand below for modeling convenience, but this also allows us to think of the objective function in the electricity sector model as one of minimizing the cost of production. Thus the combined change in producer and consumer surplus given a change in $\theta$ equals the change in cost of producing a certain amount of electricity.
continuous joint emissions benefit function.\textsuperscript{27} With this function in hand, one could then consider the expected welfare from a variety of emission damage functions.

Given the solution times of the model, even this method would be resource intensive (but admittedly not impossible). Instead, our approach is to consider the performance of the different instrument mixes in the neighborhood of the social optimum given our deterministic pollution damage functions. Furthermore, rather than solving the model for multiple draws from the removal efficiency distribution, we solve the model using a few choice sets of removal efficiencies and assign probability weights to them.

Before moving on to the calculation of expected welfare, we note some important assumptions in the following analysis. First, we limit the analysis to estimating welfare in the year 2020 to avoid complications that arise from analyzing emission profiles over time. For this reason we also prevent allowances from being banked. We further abstract from differences in the temporal emission profiles by ignoring emission reductions leading up to full compliance with the regulations. Also note that while our focus is on instrument choice for Hg and SO\textsubscript{2} control, the results are conditional on assumptions regarding the presence of other environmental regulations. We impose a tax on NO\textsubscript{X} emissions from coal-fired boilers equal to the central estimate of marginal damage ($1,229) reported in Banzhaf et al. (2004). As such, we should think of NO\textsubscript{X} emissions as any other input with a completely elastic supply (while we calculate the revenues collected from the NO\textsubscript{X} tax, this amount is

\textsuperscript{27} This is the approach taken in Pizer (2002), Dowlatabadi and Harrington (1990) and Hannesson and Kennedy (2005).
exactly equal to the opportunity cost of emitting NOX). At the same time, we remove the influence of existing regulations that directly influence emissions of the pollutants of interest, such as the Title IV SO2 trading program and the recently adopted CAIR and CAMR rules. However, pollution control technologies already adopted in response to these regulations are taken as given. They may be supplemented but cannot be removed.

Given constant marginal damages, presumably the policy where each pollutant is subject to a tax yields the highest expected welfare. That is, despite the modeling limitation preventing us from determining the optimal instrument levels for all four instrument mixes, we know the optimal tax levels for the \(\{T_m,T_s\}\) mix. Therefore we first estimate welfare from this policy, assuming that the removal efficiencies equal their expected value, \(\theta\) :

\[ w(T_m, T_s; \bar{\theta}) \]

(4.3)

Tables 4.2a and 4.2b summarize the key results from estimating (4.3) using Haiku. For comparison, the tables include the results for 2020 of EPA’s electricity sector analysis of the combined CAMR and CAIR rule. Although there are significant differences between the regulatory programs being modeled, the general concordance in the results between the models provides confidence in the reasonableness of their projections.\(^{28,29}\) The significant difference in the average electricity price is in part

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\(^{28}\) The reader keenly interested in the policy implications of this comparison might immediately note the similarity in Hg emissions and prices across the two policies. However, too direct a comparison is inadvisable for a few reasons. First, SO2 emissions under the CAIR policy are much higher. If SO2 emissions under CAIR were lower, then Hg price would also be lower. Furthermore, Hg emissions are
attributable to our assumption that electricity is priced at marginal cost throughout the electricity system.

We see that at these tax levels the model predicts that 98% of the coal capacity has SO2 scrubbers. The share that does not have scrubbers is prevented from installing them.\textsuperscript{30} Further, we see that a large share of the coal capacity is controlled by the combination of wet scrubbing and SCR. The expectation was that changing the removal efficiencies would result in more or less adoption of these controls. However, their ubiquity (particularly of scrubbers) at the efficient tax levels significantly tempers their importance as marginal control options when the Hg removal efficiencies are adjusted. That said, the utilization of coal plants with falling for CAMR after 2020 as a bank of Hg allowances is being drawn down. In part, the future scarcity of Hg emissions is reflected in the Hg allowance price, explaining why Hg emissions between the two models are close despite lower SO2 reductions in the CAIR/CAMR EPA analysis.

Another relevant comparison is the central case in Banzhaf et al. (2004), which used an earlier version of the Haiku model. In that analysis, SO2 emissions are about 1.05 million tons at a tax of $3,911 while NOX emissions were 1.4 million tons at a tax of $1,229 (see Table 1 in Banzhaf et al.). The Banzhaf et al. emission estimates are in the ballpark of the estimates here. Important differences between the analyses are that Banzhaf et al. did not impose a Hg policy and had lower electricity demand (given its focus on 2010) and natural gas prices.

\textsuperscript{29} One may wonder what levels of abatement are implied by the taxes imposed. Learning this information would require an additional run of the model (it is not enough to look at the amount of these pollutants in the coal consumed as the types of coal consumed change with the regulation) and assumptions about the regulations that would be in place absent these taxes. However, for SO2 and NOX emissions, we may look to Banzhaf et al. (2004) for a rough approximation as to baseline levels of these pollutants. The baseline level of SO2 is approximately 8.5 million tons while the baseline level of NOX is approximately 3.7 million tons. These estimates are rough because they simulate an earlier regulatory year and assume different relative fuel prices than the version of the model used herein. Furthermore, the uncontrolled level of SO2 assumes that a NOX tax ($1,229) is in place and the uncontrolled level of NOX assumes that an SO2 tax ($3,911) is in place. The abatement levels are those reductions that would occur beyond the Title IV SO2 and NOX acid rain programs and the NOX SIP Call (so, the baseline emissions levels are not actually uncontrolled).

Banzhaf et al. (2004) do not report Hg emissions. However, note that even if the taxes on all three pollutants were zero, the level of uncontrolled Hg emissions would still vary with the assumed Hg removal efficiency levels. This is because existing controls, like those reducing particulates, would still have an uncertain effect on Hg emissions.

\textsuperscript{30} Boilers associated with capacity less than 100 megawatts may not install post-combustion controls.
different combinations of controls does change with changes to the removal efficiencies.

While we have described the optimal tax levels, we have not determined the optimal instrument levels for the other three instrument mixes. As discussed above, determining the optimal instrument levels for these three mixes would be extremely resource intensive. That said, theoretically the optimal taxes when both pollutants are controlled by a tax are those that should be used when a variety of instruments are being used (i.e \( \{ T_m, Q_s \} \) or \( \{ Q_m, T_s \} \)).\(^{31}\) This leaves us with the task of finding “optimal” quantities for the other three instrument mixes. We take the resulting emissions of SO\(_2\) and Hg in

Table 4.2a as the optimal levels of the quantity instruments. That is, we take as optimal the levels of instruments that maximize:

\[
\max_{I_m, I_s} \theta(I_m, I_s; \theta)
\]

instead of those that maximize (4.2). By not using the optimal quantities, we are imposing a bias against the instrument mixes that use quantity restrictions. If chosen optimally, they would yield welfare at least as great as reported below.

\(^{31}\) As we saw in Chapter 3, with constant marginal damages, a tax equal to the marginal damage for a pollutant cannot be improved upon when the other jointly controlled pollutant is controlled by a quantity. This is because varying the tax level for the pollutant controlled by a tax cannot influence the level of emissions of the pollutant controlled by a quantity (presuming the quantity constraint is binding). (Note that if both pollutants are controlled by a tax, and one of the taxes was at a suboptimal level, then it may be the case that the tax for the other pollutant should not be set equal to marginal damages in order to influence the emissions of the first pollutant.)
Table 4.2a: Model Results Summary given Expected Removal Efficiencies (2004 $)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Electricity Price ($/MWh)</td>
<td>$83</td>
<td>$73</td>
</tr>
<tr>
<td>Generation (1000 GWh)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coal</td>
<td>2,428</td>
<td>2,365</td>
</tr>
<tr>
<td>Natural Gas+Oil</td>
<td>1,018</td>
<td>1,265</td>
</tr>
<tr>
<td>Nuclear</td>
<td>817</td>
<td>809</td>
</tr>
<tr>
<td>Renewables (Including Hydro)</td>
<td>552</td>
<td>408</td>
</tr>
<tr>
<td>TOTAL</td>
<td>4,816</td>
<td>4,847</td>
</tr>
<tr>
<td>Emissions(^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SO(_2) (million tons)</td>
<td>National 1.085</td>
<td>4.433</td>
</tr>
<tr>
<td></td>
<td>Policy Affected 1.057</td>
<td>4.214</td>
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<tr>
<td>NO(_X) (million tons)</td>
<td>National 1.533</td>
<td>2.212</td>
</tr>
<tr>
<td></td>
<td>Policy Affected 1.170</td>
<td>1.254</td>
</tr>
<tr>
<td>Hg (tons)</td>
<td>National 34.27</td>
<td>28.28</td>
</tr>
<tr>
<td></td>
<td>Policy Affected 21.15</td>
<td>24.20</td>
</tr>
<tr>
<td>Emission Tax/Allowance Price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SO(_2) (per ton)</td>
<td>$3,911</td>
<td>$1,306</td>
</tr>
<tr>
<td>NO(_X) (per ton)</td>
<td>$1,229</td>
<td>$1,486</td>
</tr>
<tr>
<td>Hg (per pound)</td>
<td>$40,940</td>
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<tr>
<td>Abatement Expenditure (billion $)</td>
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<tr>
<td>Benefits (billion $)(^3)</td>
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<tr>
<td>Consumer Surplus(^4)</td>
<td>---</td>
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<tr>
<td>Producer Surplus</td>
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<tr>
<td>SO(_2) Revenue</td>
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<tr>
<td>NO(_X) Revenue</td>
<td>$1.44</td>
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<tr>
<td>Hg Revenue</td>
<td>$1.73</td>
<td>$0</td>
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<td>TOTAL</td>
<td>---</td>
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</tr>
<tr>
<td>Damages (billion $)(^5)</td>
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<td></td>
</tr>
<tr>
<td>SO(_2)</td>
<td>$4.13</td>
<td>$16.48</td>
</tr>
<tr>
<td>NO(_X)</td>
<td>$1.44</td>
<td>$1.26</td>
</tr>
<tr>
<td>Hg</td>
<td>$1.73</td>
<td>$1.98</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$7.30</td>
<td>$19.72</td>
</tr>
</tbody>
</table>

1. Source: U.S.EPA 2005a and 2005b. Allowance prices are for CAIR cap-and-trade programs for SO\(_2\) and NO\(_X\). These programs are not national in scope.
2. Policy Affected emissions are those emitted by the sources subject to regulation. For example, in our analysis only coal plants are subject to the tax on NO\(_X\) while in the EPA analysis large coal, natural gas and oil-fired units in the eastern and central U.S. are subject to the NO\(_X\) cap-and-trade program.
3. The EPA’s electricity sector model does not report welfare estimates as an economist conceives of them. The EPA reports the cost of a policy as the increase in generation costs to meet a particular demand, not as changes in producer and consumer surplus.
4. Haiku uses constant inelastic electricity demand curves. Therefore consumer surplus is unbounded. However, differences in consumer surplus can be measured.
5. These are damages attributable to emissions from the policy affected sources. Furthermore, the EPA does not report damages from emissions. The total damage functions used to calculate damages from the Haiku analysis is used to calculate damages from emissions predicted by the EPA analysis.
Table 4.2b: Pollution Controls as a Share of Total Capacity given Expected Removal Efficiencies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wet Scrubber Only/SNCR</td>
<td>8%</td>
<td>18%</td>
</tr>
<tr>
<td>Wet w/ SCR</td>
<td>85%</td>
<td>49%</td>
</tr>
<tr>
<td>Wet w/ ACI</td>
<td>---</td>
<td>2%</td>
</tr>
<tr>
<td>Dry Scrubber Only/SNCR</td>
<td>2%</td>
<td>---</td>
</tr>
<tr>
<td>Dry w/ SCR</td>
<td>3%</td>
<td>---</td>
</tr>
<tr>
<td>Dry w/ ACI</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>ACI Only</td>
<td>---</td>
<td>2%</td>
</tr>
<tr>
<td>SCR Only</td>
<td>0%</td>
<td>6%</td>
</tr>
<tr>
<td>None/SNCR</td>
<td>2%</td>
<td>22%</td>
</tr>
</tbody>
</table>

1. The EPA does not distinguish between scrubber type when reporting model results.

4.4.3.2 Varying the Mercury Removal Efficiencies

As discussed above, we solve the model using a few choice sets of Hg removal efficiencies and assign probability weights to them. Specifically, we estimate welfare for each instrument mix given three different sets of removal efficiencies. For each boiler category \( i \), the removal efficiency \( \theta_i \) can either be at its mean \( \bar{\theta}_i \), mean plus standard deviation \( \bar{\theta}_i + \sigma_i \), or mean minus standard deviation \( \bar{\theta}_i - \sigma_i \). As noted above we thus assume that all boilers within a particular category have the same Hg removal efficiency and that removal efficiencies are perfectly correlated across boiler categories. Furthermore, we assume a symmetric distribution of these errors for each boiler category. An exception is in cases where the standard deviation encompasses either a negative removal efficiency or an efficiency greater than one. In these cases the bounds of 0 and 1 are used instead.

The next question is how we should weight the different welfare outcomes given each Hg removal efficiency realization. We propose two methods, both of which are necessarily approximations. One method is to find probability weights for
the discrete three-outcome distribution of $\theta$ that assure that its standard deviation and mean preserve the mean and standard deviation of the continuous removal efficiency distribution $\theta$. These probability weights are then applied to the three possible welfare outcomes to calculate expected welfare from each instrument mix. To preserve the mean of the continuous distribution, which we label $f(\theta)$, for the discrete distribution, labeled $g(\theta)$, the following must hold:

$$\int \theta f(\theta) = \bar{\theta} = \gamma [\bar{\theta} + \sigma] + [1 - 2\gamma] \bar{\theta} + \gamma [\bar{\theta} - \sigma]$$

(4.5)

To make things easier, we have assumed that the discrete distribution is symmetric so that $\gamma$ is the probability weight on the two tails of $g(\theta)$. Clearly the restriction (4.5) is not sufficient to find a unique $\gamma$. However, when we further assume that the standard deviation of the two distributions is the same, we obtain:

$$\sqrt{\int \theta^2 f(\theta)} = \sigma = \sqrt{\gamma [\bar{\theta} + \sigma - \bar{\theta}]^2 + [1 - 2\gamma] [\bar{\theta} - \sigma]^2 + \gamma [\bar{\theta} - \sigma - \bar{\theta}]^2}$$

(4.6)

Solving (4.5) and (4.6) simultaneously we find that $\gamma = .5$. Thus, with this approach, the two extreme removal efficiencies cases are weighted by .5, while a zero weight is applied to welfare from the mean removal efficiency to find the expected welfare for each instrument mix. We refer to this as the “discrete distribution” approach below.

The problem with the discrete distribution approach is that it is concerned with preserving the moments of $\theta$ and not in preserving the moments of the distribution of expected welfare, which is really the function we are interested in. The above approach would work fairly well as an approximation if there were little curvature in the objective function with respect to $\theta$. A priori, there is no reason to
believe that this is a quality of the optimization problem represented by the simulation model and damage estimates with respect to $\theta$ (i.e., $w(I_m, I_s; \theta)$). Our second approach attempts to capture curvature in this problem using a second-order Taylor series approximation.

We begin deriving the Taylor series approach with an expansion of the welfare function around the mean removal efficiencies for each of the $N$ boiler categories:

$$w(I_m, I_s; \theta) \approx w(I_m, I_s; \bar{\theta}) + \sum_{i}^{N} \sum_{j}^{N} \frac{\partial^2 w(I_m, I_s; \theta)}{\partial \theta_i \partial \theta_j} \sigma_{ij} \theta_i \theta_j$$

(4.7)

Thus expected welfare can be expressed:

$$W(I_m, I_s; \theta) \equiv E[w(I_m, I_s; \theta)] \approx w(I_m, I_s; \bar{\theta}) + .5 \sum_{i}^{N} \sum_{j}^{N} \frac{\partial^2 w(I_m, I_s; \theta)}{\partial \theta_i \partial \theta_j} \sigma_{ij} \theta_i \theta_j$$

(4.8)

where $\sigma_{ij}$ is the covariance between the removal efficiency for the boiler categories $i$ and $j$ (or, when $i = j$, the variance of $\theta$). As we do not explore the last term in (4.8) explicitly, we are not ready to make use of this form. First, we do not know the value of $\frac{\partial^2 w(I_m, I_s; \theta)}{\partial \theta_i \partial \theta_j} \sigma_{ij}$ for any boiler categories $i$ and $j$, because we do not have an explicit form of the function $w(I_m, I_s; \theta)$. We could linearly approximate

$$\frac{\partial^2 w(I_m, I_s; \theta)}{\partial \theta_i \partial \theta_j} \sigma_{ij}$$

if we solved the model for a set of each $\theta_i$ and $\theta_j$, but clearly such an approach would be very model intensive given that there are multiple boiler categories. Given our assumption that the removal efficiencies are perfectly positively
correlated across the boiler categories \((\rho_{ij} = 1)\), we do know \(\sigma_i^2\) for all \(i\) from the sample data and could find \(\sigma_{ij}\). We thus make a further approximation of (4.8):

\[
E\left[W(I_m, I_s; \theta)\right] \approx w(I_m, I_s; \bar{\theta}) + 0.5 \hat{w}'' N \hat{\sigma}_h^2
\]

(4.9)

where the parameters \(N, \hat{w}''\), and \(\hat{\sigma}_h^2\) require description. The parameter \(N\) is the number of distinct boiler categories. In the raw data, there are truly only 31 boiler categories for which the EPA collected data. As discussed above, for the categories where no data were collected, the EPA assigned the removal efficiencies for similar categories. Furthermore, we remove 15 categories for which there is little representation in the underlying data, given that we do not think changing these removal efficiencies would affect welfare significantly. “Little representation” is taken to mean less than 1% of capacity in the input data or potentially as a result of the policies. This leaves us with 16 distinct boiler categories. The parameter \(\hat{w}''\) is a linear approximation of the change in the increase in welfare as all of the removal efficiencies increase. Thus, \(\hat{w}''/N\) is being treated as an approximate average of each term \(\frac{\hat{\sigma}_h^2}{\hat{w}''(\theta)} w(I_m, I_s; \theta)\bigg|_{\theta=\bar{\theta}}\).

The parameter \(\hat{\sigma}_h^2\) is the variance of the realized average Hg removal efficiency for each instrument mix (these values are reported below) and is assumed to be the average variance and covariance across the removal efficiencies for the different boiler categories. Given that the observed removal efficiencies are endogenous to the model and affected by the particular instrument mix being applied, the variation captured by \(\hat{\sigma}_h^2\) may be lower or higher than what would be suggested.
from equation (4.8). Admittedly, even with the appeal of this approach in capturing
the non-linearity of the objective function, it is quite rough given the multitude of
approximations required by (4.9).

Before moving on to the results, we first note a few additional assumptions
and modeling decisions for the model runs where the removal efficiencies deviate
from their mean. In part, to further simplify the analysis and reduce the noise in the
solutions and run times, the electricity generation, fuel prices, and inter-regional
transmission are held constant at the levels found in the solution to (4.3).\textsuperscript{32} As
discussed in footnote 26, by keeping electricity generation constant, the electricity
sector model can be thought of as strictly solving a cost-minimization problem.

4.5 Comparing Instrument Mixes

4.5.1 Expected Welfare Comparisons

We are now ready to look at the effect of the different instrument mixes on
expected welfare. The first three rows of Table 4.3 report, for each instrument mix,
welfare given each Hg removal efficiency realization as measured by the change in
welfare from the case where removal efficiencies are at their means (expression (4.3)
). Let us first compare welfare across instrument mixes for each removal efficiency
realization. When the removal efficiencies are both greater or lower than the mean,
the instrument mix that yields the highest welfare is the one where each pollutant is
controlled by a tax. This is to be expected given our assumption of constant marginal

\textsuperscript{32} A further justification for keeping fuel prices is constant is that one does not have to account for
welfare changes in the fuel markets as a result of price effects. Essentially the supply of fuels is
assumed to be perfectly elastic.
damages for these pollutants and that all other inputs are treated as if they are priced at their opportunity cost. Furthermore, the welfare ordering of the remaining instrument mixes is the same for the two extreme removal efficiency realizations. The \( \{T_m, Q_s\} \) instrument mix yields the second-highest welfare followed by the \( \{Q_m, T_s\} \) mix and finally the \( \{Q_m, Q_s\} \) mix.

Table 4.3: Expected Welfare from Competing Instrument Mixes in 2020
(Million 2004$)

<table>
<thead>
<tr>
<th>Hg Removal Efficiencies</th>
<th>Instrument Mix</th>
<th>Mean</th>
<th>Mean + Std. Dev.</th>
<th>Mean - Std. Dev.</th>
<th>Expected Welfare</th>
<th>Expected Welfare Loss Relative to Optimal Mix $^2$</th>
<th>Taylor Series Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( T_m, T_s )</td>
<td>( T_m, Q_s )</td>
<td>( Q_m, T_s )</td>
<td>( Q_m, Q_s )</td>
<td>Mean</td>
<td>Mean + Std. Dev.</td>
</tr>
<tr>
<td>Welfare $^1$</td>
<td></td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$.745</td>
<td>$.844</td>
</tr>
<tr>
<td>Discrete Distribution Approach</td>
<td>Expected Welfare</td>
<td>$180$</td>
<td>$90$</td>
<td>$30$</td>
<td>$0$</td>
<td>$.745</td>
<td>$.844</td>
</tr>
<tr>
<td>Removal Efficiency</td>
<td>Expected Welfare Loss Relative to Optimal Mix $^2$</td>
<td>0</td>
<td>-$90</td>
<td>-$150</td>
<td>-$180</td>
<td>.745</td>
<td>.844</td>
</tr>
</tbody>
</table>

1. Welfare measured as change in welfare from average removal efficiency case due to unboundedness of consumer surplus. See Table 4.2a and 4.2b.
2. Values may not sum due to rounding.

Comparing across the removal efficiencies, we see that for three of the instrument mixes, raising the removal efficiencies by one standard deviation has a larger effect on welfare than lowering the removal efficiencies by one standard.
This is somewhat surprising because one might imagine that there are diminishing marginal welfare benefits to increasing the removal efficiencies. However, the Hg removal efficiencies are bounded between 0 and 1 and many of the boiler categories (on a generation capacity basis prior to adoption of the policies) have removal efficiency ranges that are constrained by the lower bound.

As the $\{T_m, T_s\}$ instrument mix is the preferred policy regardless of the removal efficiency level, it is also the mix that maximizes expected welfare. Owing to the consistent ordering of welfare given the different Hg removal efficiencies, we also see that the $\{T_m, Q_s\}$ instrument mix yields the second highest expected welfare, followed by the $\{Q_m, T_s\}$ and then the $\{Q_m, Q_s\}$ mixes. The analytical model in the previous chapter suggests that this ordering of expected welfare is possible and indeed in the case of constant marginal damages is one of only two feasible orderings. It also appears that at these instrument levels it is more important to have the correct Hg control rather than the correct SO2 control, as expected welfare from the $\{T_m, Q_s\}$ mix is higher than that for the $\{Q_m, T_s\}$ mix. This is likely attributable to the significant amount of SO2 controls that are installed at this level of control. Changes in generation patterns, coal consumption and NOX control are relatively more important for the control of Hg than changes in the number of SO2 controls given that so many SO2 controls are installed.

---

33 Although this expectation is not as clean cut as the declining marginal welfare improvement from increasing the removal efficiencies must come from reduced benefits of emitting given that marginal damages are assumed to be constant.
The \( \{T_m,T_s\} \) instrument mix leads to a $90 million increase in welfare over the next-best instrument mix in 2020 using the discrete distribution estimate and $160 million using the Taylor series estimate. If both pollutants are regulated by a cap and trade program, as traditionally has been the case in the U.S., the loss from not adopting the \( \{T_m,T_s\} \) mix is $180 million using the discrete distribution estimate or $190 million using the Taylor series estimate.

The expected cost of selecting the incorrect instrument may even be higher given that rarely are such regulations immediately changed. For example, it took 15 years before the national cap on SO2 emissions was lowered. If we expect the expected cost of selecting the incorrect set of instruments to accrue annually for a 15-year period starting in 2020, the expected current (2007) cost of selecting the \( \{Q_m,Q_s\} \) mix over the \( \{T_m,T_s\} \) mix at a 5% social discount rate is $1,040 million or $1,100 million depending on the estimate used.

4.5.2 Emissions, Allowance Prices and Emission Controls

Table 4.4 reports the allowance prices and emissions of SO2, NOx and Hg for each instrument mix and each removal efficiency realization. The change in emissions and allowance price for each pollutant as a result of changing the Hg removal efficiencies varies based on the pollutant in question and the instrument mix. Clearly as the Hg efficiencies go up, we expect Hg emissions to fall if Hg is controlled by a tax or the allowance price to fall if Hg is controlled by a quantity. We see this outcome in Table 4.4. However, it is not clear what should happen to the SO2
emissions or allowance price or to NOX emissions when the removal efficiencies change.

To begin to understand these effects, let us first consider what may happen to SO2 controls and emissions if SO2 controls become more effective at abating Hg. Furthermore, let us consider the specific instrument mix where Hg is controlled by a quantity and SO2 is controlled by a tax. On one hand, the benefit of installing SO2 emission controls goes up so there is a greater incentive to use these controls to comply with the Hg constraint. As a result SO2 emissions fall. On the other hand, when the removal efficiencies increase, fewer SO2 controls are needed achieve the Hg cap, so the incentive to install SO2 controls falls. In this case SO2 emissions may go up. On net, the effect of raising the Hg removal efficiency of SO2 controls on SO2 emissions and the amount of SO2 controls installed is ambiguous. A similar ambiguity arises with respect to the allowance price when SO2 is controlled by a
quantity. This possible ambiguity in the effect of increasing Hg removal efficiencies on SO2 emissions and allowance prices is demonstrated using the simple analytic model in Chapter 2.

Referring now to Table 4.4, let us first consider the instrument mixes where Hg is controlled by a tax. Note that the higher the Hg removal efficiencies are, the lower SO2 emissions are when SO2 is controlled by a tax. The relationship is not so straightforward when SO2 is controlled by a quantity. With Hg removal efficiencies lower than their mean, it is the allowance price of SO2 that increases (as opposed to emissions). Yet, the allowance price also increases when the removal efficiencies are higher than their mean. However, like the SO2 emissions decrease with the \( \{T_{m},T_{s}\} \) mix, this change is small.

Things get a little more complicated when Hg is controlled by a quantity. With SO2 controlled by a tax, SO2 emissions are higher when the removal efficiencies are higher than their mean. Less SO2 scrubbing is needed to reduce Hg so SO2 emissions increase. Yet, with this instrument mix, SO2 emissions are also higher when the Hg removal efficiencies are lower than their mean. What might explain this result is that the reduction in the effectiveness of SO2 controls in reducing Hg dominates so that less capacity is scrubbed. Hg is then abated in another manner (in particular we see more NOX controls in this case). We see a similar result when each pollutant is controlled by a quantity instrument. The allowance price for SO2 is slightly higher when the Hg removal efficiencies are higher than their mean. Yet, when the Hg removal efficiencies are lower than their mean, the SO2 allowance price again is
higher. In this case it is the reduced joint benefit of reducing SO\textsubscript{2} emissions that dominates, shifting the marginal benefit of scrubbing towards the abatement of SO\textsubscript{2}.

So, while it is possible for a change in the amount of capacity that installs scrubbers to explain the changes in SO\textsubscript{2} emissions and prices, are we seeing such a change in the capacity of installed scrubbers? In fact, the reasons for the changes in SO\textsubscript{2} emissions are more nuanced than comparing the shares of coal capacity that are scrubbed. There is a decrease in the capacity and generation of scrubbed coal-fired units as the removal efficiencies increase regardless of the instrument mix. It is changes in the coals that are fired, which have different Hg and sulfur contents, and the amount of NO\textsubscript{X} controls that explain the changes in SO\textsubscript{2} prices and emissions in Table 4.4.

For completeness, Table 4.5 is provided to show the different components of expected welfare for when the removal efficiencies are above and below the mean. The bolded lines in Table 4.5 correspond to the second through fourth row of Table 4.3. There are a few items to observe in Table 4.5. First, regardless of the direction of change in the removal efficiencies, producer surplus falls in all but one case and consumer surplus always rises.\textsuperscript{34} This is a bit surprising; one would expect that lowering abatement costs given fixed demand would make producers better off, but it may be because the mean removal efficiency case was not fully converged and thus the relative distribution of consumer and producer surplus (but not the total) was not fully sorted out.

\textsuperscript{34} This is consistent with the fact that electricity price also falls relative to the mean removal efficiency case for all of the runs represented in Table 4.5.
An interesting result that can be seen in Table 4.5 is that consumers have the highest expected welfare from the $\{T_m, Q_s\}$ mix (ignoring for a moment the distribution of government revenues). Thus consumers prefer an instrument mix that is suboptimal from the standpoint of society. In fact, the optimal mix is the least preferred by consumers. A key reason $\{T_m, T_s\}$ mix is optimal is that it places the lowest burden on producers. We make this observation without trying to explain its cause. However, it does suggest that perhaps greater attention should be paid in the
literature to the distributional effects of instrument choice under uncertainty (Kelley (2005) being a notable exception).

4.6 Conclusions

The preceding analysis demonstrates the importance of the joint selection of pollution control instruments for an important policy case. We estimate differences in expected welfare from different instrument mixes for controlling SO₂ and Hg emissions from coal-fired power plants. Expected welfare is defined as the benefit of emissions as measured by the sum of welfare in the electricity market plus net government revenues minus the damages from pollution. As in the analytical model from the first half of the paper, the instrument levels are (roughly) chosen to maximize expected welfare. The specific source of uncertainty is in the effect of conventional pollution control technologies on Hg emissions.

The optimal instrument combination places a tax on SO₂ and a tax on Hg emissions. The expected welfare cost of selecting a suboptimal instrument mix in 2020 is between $90 and $190 million (2004 $) depending on which suboptimal instrument mix is adopted. While these estimates are subject to important caveats described above, this analysis provides empirical support for the proposition that pollutants should not be regulated independently, as is the tradition in U.S. pollution control policy.

An ancillary contribution of this analysis is that we see that the optimal level of Hg emissions, assuming it causes extensive mortality effects, is about 21 tons. This suggests that the annual Hg allowance allocation of 15 tons (starting in
2018) from the recently adopted CAMR is too low. However, one should also account for the differences in the level of jointly controlled emissions before making this case. Furthermore, one should also explicitly take into account the fact that Hg is a stock pollutant with potentially very localized effects.
CHAPTER 5: CONCLUSIONS

5.1 Introduction

The purpose of this chapter is to summarize the key findings of the previous chapters and to offer some suggestions for future research. The last section provides some concluding thoughts.

5.2 Summary of Results

Chapter 2 opens with an exploration of whether emission taxes can yield a Pareto optimal outcome when an economy creates multiple pollutants. As shown, carefully chosen taxes may indeed yield an efficient outcome in this setting. The regulator must choose a suite of taxes where the tax on each pollutant equals the sum of the marginal damages imposed on every agent in the economy from an additional increment of that pollutant. This optimality condition for the level of each tax thus dictates the optimum in both the single and multiple-pollutant case. The simple yet encompassing form of the general equilibrium model also allows one to identify real-world complications that may affect the ability of the regulator to set these taxes. One of the complications discussed in Chapter 2 (as well as in Chapter 1) is that the relationship between emissions and the pollutants that affect the welfare of agents in the economy may be complex.

The second half of Chapter 2 presents a simple deterministic model of joint abatement. In this model, the joint abatement relationship is due to the presence of a technology designed to control one pollutant that happens to also abate another pollutant. I perform comparative statics on a parameter that captures the intensity of
this abatement relationship. Each pollutant may be regulated by an emission tax or a cap-and-trade program so there are four possible instrument combinations. While increasing this parameter has expected effects on the emissions or allowance price of the pollutant for which the technology was not designed (they fall) and in the use of the technology specifically designed for that pollutant (it too falls), the rest of the story is more nuanced. The change in the use of the technology that abates both pollutants is ambiguous and may even depend on how the pollutants are being controlled. Furthermore, the effect on the allowance price or emissions of the pollutant for which the jointly abating technology is designed is also ambiguous. This exercise assisted in our understanding of the results of the simulation study in Chapter 4.

The core analytical findings of this dissertation are found in Chapter 3. In the abstract model used therein, a regulator wishes to maximize welfare from the abatement of two jointly controlled pollutants where she is certain of the benefits of abating these pollutants but is uncertain of their joint abatement cost. She may use either a tax or a quantity to control each pollutant. A particular functional form is used that exhibits properties similar to the models used by Weitzman (1974) and others in the single pollutant case. The model shows that the instrument suggested by the analysis for a single pollutant may be inappropriate if joint abatement is not explicitly considered. However, an interpretation of the single pollutant model attributable to Yohe (1978) continues to suggest the optimal mix of instruments in the two-pollutant case. This said, our intuition regarding which instrument is optimal to
control one of the pollutants is still valid at the extremes of the shape of the joint
abatement cost function and values of the slopes of the marginal benefit functions.

Unsurprisingly, the two-pollutant model indicates that the relative
efficiency of an instrument to control one pollutant depends on how other jointly
abated pollutants are being regulated. Perhaps more surprisingly, I find that the
optimal instrument to control one pollutant may be conditional on the way the other
pollutant is controlled. In the most extreme case, pair-wise comparisons of instrument
mixes may not suggest the optimal combination of instruments. For example, pair-
wise comparisons might suggest that it is better to use a cap-and-trade program to
control both pollutants because this combination leads to higher welfare than the two
combinations that tax one of the pollutants and control the other with a cap-and-trade
program. However, it may be the case that in this setting taxing both pollutants yields
the highest expected welfare. This very general finding is present despite the fact that
a particular functional form is used in the analysis.

This last finding also has implications for the theory of second best. If the
regulator is restricted to controlling a pollutant by one type of instrument, then the
optimal control on the other pollutant may change. This finding is novel. As is
typically the case in the theory of the second best, it is levels of otherwise suboptimal
instruments that change, not the actual type of instrument used.

The analytical model demonstrates that theoretically the simultaneous
selection of the instruments to control jointly abated pollutants is desirable when the
regulator is uncertain of their control costs. However, a question remains as to
whether this issue is empirically meaningful. In Chapter 4 I use a market simulation
model to explore a contemporary policy question as an example. Specifically, the EPA has recently adopted regulations on mercury (Hg), sulfur dioxide (SO₂) and nitrogen oxide (NOₓ) emissions from coal-fired power plants. One of the key issues in the development of these regulations was that EPA was uncertain of the extent to which technologies designed to abate SO₂, NOₓ and particulate matter reduce Hg emissions. Using a market simulation model of the U.S. electricity sector, I estimate differences in expected welfare from different instrument mixes for controlling SO₂ and Hg emissions from coal-fired power plants. Expected welfare is defined as the benefit of emissions as measured by the sum of welfare in the electricity market plus net government revenues minus the damages from pollution. As in the analytical model from Chapter 3, the instrument levels are (roughly) chosen to maximize expected welfare.

The optimal instrument combination places a tax on SO₂ and a tax on Hg emissions. The expected welfare cost of selecting a suboptimal instrument mix in 2020 is between $90 and $190 million (2004$) depending on which suboptimal instrument mix is adopted. An ancillary contribution of this analysis is that we see that the optimal level of Hg emissions, assuming it causes extensive mortality effects, is about 21 tons. While these estimates are subject to important limitations that deserve further attention, this analysis provides an empirical perspective on the importance of simultaneously selecting pollution control instruments for jointly abated pollutants in an important policy case.
5.3 Directions for Further Research

Clearly there are some issues raised in the previous chapters that deserve a bit more attention. For example, more could be said about how the analytical results in Chapter 3 relate to our understanding of general equilibrium supply and demand functions. In this context, it is likely informative to contrast the results in Chapter 3 to the case where one of the two arguments in the abatement function is actually a private good, that the regulated firms are supplying, with market forces determining its level. With respect to the simulation modeling, it would useful to explore how the magnitudes of the welfare comparisons change given different beliefs about the level of the marginal damages of all three of the pollutants of interest.

While there are clearly parts of the analysis in the proceeding chapters that could be extended a little further, the purpose of this section is to think even bigger.1 As I worked on this dissertation a number of questions have struck me that deserve further inquiry. While other researchers had already asked some of these questions (with different levels of completeness and success), others have not yet been explored. To start with, there are some game-theoretic and political economy issues worth exploring in the context of abatement cost uncertainty. For example, as discussed in Chapter 3, using the model therein how might instrument choice be affected if the regulation for each pollutant were adopted in sequence by two different agencies? Presumably this depends on whether an agency responsible for one of the

1 Thinking the most broadly takes us outside the question of instrument choice under uncertainty for jointly abated pollutants. A similar multi-good extension might be useful to the tariffs versus quotas debate in the trade literature and the landing/hunting fee versus quota in the species management literature.
pollutants accounts for any consequent changes in the levels or cost of controlling jointly controlled pollutants. Oates and Schwab (undated) raise this point with respect to their model of two media-specific agencies where each regulate one of two pollutants that are substitutes in their control. In their model, one of the agencies is a follower in that it takes the other agency’s behavior as given. The agency that is the first-mover may choose suboptimal regulations for some exogenous reason. The choice of regulation of the follower depends on whether or not it takes into account how its regulation will affect the level of the pollutant it is not ultimately responsible for.\(^2\) Extensions might explore why the first-mover chose a seemingly suboptimal regulation in the first place. A further extension would be to consider the case where one of the regulators is simply uncertain about how the other regulator will control the pollutant it is responsible for.\(^3\)

It is even likely worthwhile to extend this question somewhat to include issues outside the setting of pollution control. For example, the environmental regulation may depend on how or whether the antitrust agency regulates a polluting monopolist. If the monopolist were unregulated, perhaps the environmental regulator would prefer allocating allowances based on the firm’s output. In a dynamic setting, this is equivalent to subsidizing and thus increasing production, which in a more pure setting is suboptimal. However, for a monopolist production is presumably

\(^2\) Studies in the double-dividend literature face a similar issue. Goods taxes causing preexisting distortions are often assumed out of the environmental regulator’s control, but that the environmental regulator still accounts for any change in the distortions they cause (for example, see Quirion 2004).

\(^3\) A similar problem, but in the case of a single regulator, is where the regulator that does not yet have the authority (say because there is abatement benefit uncertainty) to control one of the pollutants but does have an expectation that he will be able to in the future.
inefficiently low in the first place, so the implicit subsidy may be welfare improving. 4

As described in Chapter 1 there are papers on the topic of multiple agencies or jurisdictions regulating jointly abated pollutants, but again few of them incorporate instrument choice in a full game-theoretic framework and none with cost or benefit uncertainty.

Extensions of the single pollutant model under abatement cost uncertainty that have proven worthwhile may also be worth exploring in a multiple pollutant setting. For example, Pizer and Newell (2003) point out in the context of regulating a stock pollutant under abatement cost uncertainty that it may be optimal to switch the instrument used to control the pollutant from a tax to a quantity at some point in time. 5 In the case where one pollutant is a stock pollutant and the other is a flow pollutant, might it also be true that it is optimal to switch the instrument used to control the flow pollutant at some point? I think the second-best results of Chapter 3 suggest that it might. As noted in Chapter 4, the case of one of the pollutants being a flow and the other being a stock is actually more relevant to the example of the joint control of Hg, which is a stock pollutant, and SO2 which essentially can be considered a flow pollutant.

4 Another example where the choice of instrument may not be tax or quantity set at the Pigovian level is when there is no regulation targeting externalities derived from research and development on abatement techniques. See for example Fischer et al. (2003).

5 One question I have been asked regularly about the preceding analysis is how the introduction of a safety valve on one of the pollutants might fit into the welfare ordering of the instrument mixes. The answer is likely simple: it would yield higher expected welfare than any of the four instrument mixes as demonstrated by Roberts and Spence (1976) in the single pollutant case. This is because at least one of the damage functions would be more closely mimicked by the combination of instruments. Of course, if the safety valve were triggered the effect on the other pollutant would depend on the parameters of the model.
Even in the single pollutant case where there is abatement cost uncertainty, there are additional questions to be asked. As highlighted in Chapter 4 the ordering of expected consumer surplus and producer surplus does not follow the ordering of expected total surplus across the instrument mixes. It would be useful to explore systematically the conditions where consumers and producers prefer one instrument to another. The general equilibrium extension described in the first paragraph of this section could be applied to this question.

There are also questions outside the instrument choice under uncertainty literature to be asked regarding the management of jointly controlled pollutants. For example, how do competing regulations perform if one jointly controlled pollutant is relatively more difficult to monitor than another? Might there be some mechanism that takes advantage of knowing that the emissions of one of the pollutants is more readily observable to achieve an efficient outcome in the regulation of both? Of course, if we just think of emissions as inputs, findings in the existing literature that note that a single pollutant can be indirectly monitored from observing the use of a particular input may be directly applicable. Another example is to evaluate critically a claim frequently made in favor of integrating pollution permits over a more piecemeal approach. Namely, that integrating the design and implementation of regulations across pollutants that are related in the generation is more likely to lead to the adoption of production methods that are less pollution-intensive (Davies, 2001).

To me, one of the bigger issues in the preceding analysis is that the model used in Chapter 3 is somewhat unsatisfying in that the dual abatement cost function hides the underlying technological relationships and sources of uncertainty. By
sources of uncertainty I mean unknown goods or input prices, technological parameters, etc. So, despite the discovery that any welfare ordering of instruments is possible, suggesting a new result germane to the theory of second best, the model gives little tangible sense of when this might occur (particularly as these results depend on errors and the correlation of these errors whose origins are not transparent). A model that is less abstract and provides a specific technological form and source of uncertainty would be helpful in this regard. While such an analytical model could be designed to yield closed form solutions, the welfare comparisons of the different instrument mixes might not be transparent. For this I would use Monte Carlo techniques to estimate differences in expected welfare.\(^6\) Such a study would be a useful intermediate step between the model in Chapter 3 and the detailed simulation analysis in Chapter 4. Indeed, this was my original intent when I started constructing the model at the end of Chapter 2. I have even identified a useful functional form of the emissions modification factors (here as an example of \(\delta(\lambda l)\)):

\[
\frac{g}{(\lambda l)^{\delta} - g} \tag{5.1}
\]

that satisfies the assumptions (2.10)-(2.14) on page 35 when \(g > 0\) and \(0 < G \leq 1\).\(^7\) Of course, I would also need to propose a form of the emissions damage function. Again,

\(^6\) Hannesson and Kennedy (2005) perform an analysis similar to the one proposed here in the case of managing a single-species fishery using quotas or landing fees. They are interested in what types of uncertainties, for example in the population dynamics of the species and the cost of harvesting, favor one of these instruments over the other.

\(^7\) I briefly considered using estimated quadratic abatement cost functions to perform such an analysis. However, I could find only two studies that estimated abatement cost functions using a stochastic framework (I primarily reviewed the studies referenced in Pizer and Kopp, 2005). Hartman et al.
the purpose of such a study is to provide a more concrete understanding of when different instrument mixes might be preferred in cases of multiple pollutant control.

An important policy issue is to understand the consequence of the EPA’s decision to regulate Hg using a single cap-and-trade program when its two forms or species behave quite differently. As discussed in Chapter 4, elemental Hg is relatively inert and has a wide deposition pattern while oxidized (speciated) Hg is readily taken up by aquatic animals and precipitates rapidly out of the atmosphere. This question is germane to the theoretical literature on uniformly regulating pollutants that have heterogeneous damages (Mendelsohn, 1986; Kolstad, 1987). The simulation model used in this dissertation is well suited for such an analysis. It is, relatively speaking, straightforward to account for both elemental and oxidized Hg emissions in the model. It is also possible to adjust the level of allowable NOₓ and SO₂ emissions to see how such adjustments affect the relative emissions of the two different forms of Hg. The EPA’s recently adopted CAIR rule is expected to yield more post-combustion control retrofits for NOₓ than SO₂. This suggests that a small increase in the allowable NOₓ emissions may lead to a reduction in oxidized Hg, which is the more harmful form. To make this analysis more complete, rather than simply evaluating emission patterns, the market model could be linked to an air dispersion and health-effects model. It must be said, however, that this is an ambitious step, and

(1997) appear to have estimated such function using the data from the Pollution Abatement Costs and Expenditures Survey (PACE) survey conducted by the U.S. Census for the EPA. However, the lead author is unsure that this is the case because the functions were not made publicly available and have been lost (personal communication with David Wheeler, Lead Economist, Development Research Group, World Bank, May 5, 2006). Newell, Pizer and Shih (2004) also use the PACE data to estimate a joint quadratic abatement cost function for the cement industry, but find that the unconstrained function is non-convex. This may be unsurprising as regulations in the U.S. are not designed to yield cost-effective emission reductions.
not simply because our understanding of the behavior of Hg in the environment and its health effects is relatively incomplete.

A similar question that can be addressed with the electricity sector simulation model is what are the effects of recently adopted state controls on Hg emissions. Many states are adopting much tighter regulations on Hg than imposed by the recently adopted federal regulation. However, because the federal regulation is a cap-and-trade program, the Hg emissions that will not be emitted in one state will be emitted elsewhere. The simulation model can be used to estimate the resulting spatial and temporal pattern of Hg emissions, as well as for the jointly abated SO₂ and NOₓ emissions, as a result of the adoption of the state regulations.

5.4 Conclusions

The motivation for this dissertation was to understand the consequences of uncertainty in control of Hg on the performance of competing regulations for the control of Hg and SO₂. One question not yet addressed is whether the analytical model in Chapter 3 should have suggested to us something about the magnitude of the welfare comparisons in Chapter 4. As we will see, it would have been difficult to anticipate how varying the Hg removal efficiencies would affect the performance of the competing instrument mixes. Perhaps a simpler question is whether the model in Chapter 3 sufficiently represents the uncertain control cost relationship that is modeled in Chapter 4.

To understand whether the model in Chapter 3 should have suggested something about the estimates in Chapter 4, we start by looking back to the Weitzman
single pollutant analysis. There, we derived a measure of the welfare cost of using a suboptimal instrument:

\[ W(t^*_m) - W(m^*) = \frac{\sigma^2}{2\sigma^2_m} \left[ \epsilon_{mm}^m - b_{mm} \right] \]  

First, we see in this expression that the larger the variance, the larger the welfare difference. But note, this is not the variance of some primary parameter (like of the price of a fuel input). Rather, it is the variance of the intercept term of the marginal abatement cost function. How much the marginal abatement cost might deviate from its expected location depends on a number of factors, including all of the moments of the unknown parameters and their importance to the cost of controlling pollution.

Of course, a similar argument can be made in the case of the two-pollutant model, where there are two uncertain parameters and a covariance between them. Even if we suspected that the functional form of the cost function in Chapter 3 would sufficiently represent the relationship between controlling SO\(_2\) and Hg, there still would have been no way for us to know ex ante how the uncertainty in the Hg removal efficiencies would manifest themselves in the uncertain parameters in the cost function. Therefore, we could not have known, using the model in Chapter 3, how important uncertainty in the removal efficiencies would be to instrument choice.

However, now that we have our empirical results from Table 4.4 of Chapter 4, we can get a sense as to how much the demand for the pollutants move around as the removal efficiencies are changed. Recall that the Hg demand function is equivalent to the Hg marginal abatement cost function, where the dependent variable is “emissions” instead of “emissions reduced” (i.e., abatement). We see that the demand
for Hg does move around quite a bit, as the marginal value product of Hg emissions (given a quantity target for SO₂) is more than four times higher when the removal efficiencies are low compare to when the removal efficiencies are high. The variance is 26,600 (in dollars). At the same time, the demand for SO₂ does not move around much.⁸

Returning to expression (5.2), we see that if the marginal benefits of abatement are constant, then the welfare difference reduces to a ratio of the variance to the slope of the marginal abatement cost function, divided by two (i.e., \( \sigma_m^2 / 2c_{mm} \)). In this case, the steeper the marginal abatement cost function, the lower the welfare difference between the two instruments. With a very steep slope, the difference in welfare from using the two instruments would be minimal. A similar result holds for the two-pollutant model. The difference in expected welfare from using a tax to control each instrument and a quantity to control each instrument, when the marginal benefit of abatement of the two pollutants is constant, is:

\[
W\left( t^*_m, t^*_s \right) - W\left( m^*, s^* \right) = \frac{\sigma_m^2 c_{ss} + \sigma_s^2 c_{ss} - 2\rho_{ms} \sigma_m \sigma_s c_{ms}}{2 \left[ c_{mm} c_{ss} - c_{ms}^2 \right]}
\]

(5.3)

As the partial slope of the marginal abatement cost of \( m \) goes to infinity (i.e., \( c_{mm} \to \infty \)), this expression converges to:

---

⁸To take this analysis further, one could perform an estimation exercise using the results in Table 4.4 of Chapter 4 to find parameter values for an emissions benefit function equivalent in form to the emission cost function in Chapter 3. There are 5 parameters to estimate in the demand function (not counting the uncertain parameters), and 8 sets of prices and quantities in Table 4.4. If the emissions demand functions fit the data well, and the parameter values of the second order terms do not vary much, then one could argue that the emissions benefit function is a reasonable approximation for the empirical question in Chapter 4. (We must be careful as to how we interpret these demand functions as they would assume that the output and price of electricity may change).
Furthermore, if the slope of the marginal abatement cost of \( s \) also goes to infinity, then (5.4) converges to zero. So, with flat marginal benefit curves and steep marginal abatement cost curves, the expected welfare difference from the two instrument mixes is small.

If we thought that the cost function in Chapter 3 was a sufficient representation of the cost relationship for the abatement of SO\(_2\) and Hg in Chapter 4, then from the observations from the proceeding paragraph we might expect that the welfare cost from choosing the wrong instrument would be fairly small. First, note that assuming a constant marginal benefit of abatement is equivalent to assuming a constant marginal damage function, as we do in Chapter 4. Furthermore, with policies as stringent as those modeled in Chapter 4, it is likely that we are in the steep parts of the partial marginal abatement cost curves for SO\(_2\) and Hg. Banzhaf et al. (2004) explore the entire shape of the marginal abatement cost function for SO\(_2\) and find that is very steep in the neighborhood where the tax on SO\(_2\) is $3,911/ton.\(^9\) For Hg, the slope of the demand curve can be estimated from Table 4.4 in Chapter 4, assuming that SO\(_2\) is controlled by a quantity. (Again, the slope of the emissions demand function is the negative of the slope of the marginal abatement cost function) The slope of the demand function for Hg, with either high or low removal is about -$0.1

\[^9\] There are numerous caveats that come with this observation given the different economic and regulatory conditions considered in Banzhaf et al. (2004). For example, they assume that no Hg regulation is in place. However, generally speaking it is safe to say that the SO\(_2\) marginal cost function is steep in this neighborhood.
per pound. In absolute terms, this is a small number, but to say whether it means that costs are relatively flat or steep requires additional context. A more informative statistic is the arc elasticity of the Hg demand function, which is between -.47 and -.64 (for the high and low removal efficiencies, respectively). This range of the arc elasticity suggests that the marginal abatement cost function of mercury is fairly steep. In sum, perhaps one should not be surprised then that the welfare differences measured Chapter 4 were not all that large. This all said, one should not make too much of these observations without knowing whether the functional form of the problem in Chapter 3 satisfactorily describes the joint control costs for SO₂ and Hg.

A related and important question is whether one would argue, given the welfare cost estimates in Chapter 4, that it should be a high priority for the EPA to spend more resources from a limited research budget to understand better the effect of controls designed to abate other pollutants on Hg emissions. That is, how might we think about the results in Chapter 4 if we viewed this problem as a value of information question? Frankly, I think it is probably not that important for the EPA to gather more data on the effect of the conventional pollution control technologies on Hg emissions. While I think that the estimate of the welfare loss from using the wrong instrument mix, $90 to $190 million a year, is nothing to scoff at, there are more important priorities. Furthermore, in some ways the deck was stacked in favor of finding a large welfare loss from using suboptimal instrument mixes in the simulation analysis of Chapter 4, because the variance in the removal efficiencies were assumed to be perfectly correlated across sources, which likely will not be the
case. At least with this particular source of uncertainty, in reality the real welfare consequence of choosing a suboptimal mix might be smaller.

In my mind, the biggest priority for further research was just hinted at: the bounds of uncertainty on the damages from NO\textsubscript{X}, SO\textsubscript{2} and Hg pollutants are large despite the numerous studies that have attempted to narrow these estimates. For example, Banzhaf et al. (2004) report standard deviation bounds of about $1,500 to $5,500 per ton of SO\textsubscript{2} reduced and $800 to $1,700 per ton of NO\textsubscript{X} reduced (1999 $).

While it may be true that measuring the damages from pollution deserves still greater attention from economists, there are many other potentially fruitful avenues for research. The purpose of this dissertation was to make a tightly focused, but thorough, contribution to our understanding of the optimal regulation of jointly controlled pollutants. As the research ideas proposed above and the research review from Chapter 1 both suggest, there is still much to understand. This is true even though issues that arise in the control of multiple pollutants have been acknowledged for a long time.
APPENDIX A: THE HAIKU ELECTRICITY SECTOR MODEL

A.1 Introduction

The purpose of this appendix is to describe the market simulation model used in Chapter 4 of this dissertation. This model, known as Haiku, is a simulation model of the electricity market in the contiguous states of the U.S. Its primary purpose is to facilitate the study of national and regional environmental and market structure policies that affect this sector. To this end, Haiku yields partial-equilibrium measures of welfare changes expected to result from policy and market changes in the electricity sector. The model also provides insights into the effects of these changes on production and prices in important related markets such as those for coal and natural gas. In this way Haiku can be viewed as a linked partial-equilibrium model with the links being the relationships between the markets.

Haiku was developed and is maintained by research staff at Resources for the Future. The initial development of Haiku derived from a need to provide reliable emission estimates to study the effects of policies to reduce acidification. An expected attendant use was to analyze the environmental consequences of opening regional retail electricity markets to competition (Palmer et al. 2002). The model has been expanded and refined over time. Additional applications of the Haiku model include estimating reductions in conventional pollutants ancillary to reductions in carbon dioxide (CO₂) emissions (Burtraw et al. 2003b), analyzing the efficiency and distributional consequences of alternative allocation schemes for carbon emission
allowances (Burtraw et al. 2002) and estimating the efficient national level of sulfur
dioxide (SO₂) and nitrogen oxide (NOₓ) emissions from the electricity sector
(Banzhaf et al. 2004).¹

Stewardship of the model is a collaborative effort led by Dallas Burtraw and Karen Palmer. Anthony Paul and I have major long-term responsibilities in the management, construction and design of the model. The model has also received major contributions from Ranjit Bharvirkar, Danny Kahn, David Lankton and Erin Mansur and a number of interns.

My primary responsibilities include collecting, condensing, and assembling the raw supply and demand data. This includes the characterization of the operating costs and performance of all of the existing generators in the model. I also am responsible for characterizing the performance of many of the potential (new) generator types. For renewable technologies this includes determining their total regional availability. For nuclear technologies this includes tracking changes to the industry including the widespread adoption of capacity increases in recent years. I also have a lead role in developing the pollution control algorithms in the model, particularly in the characterization of the abatement technologies. It is also my responsibility to stay current with state-level environmental regulations affecting this

¹ The emission estimates generated by Haiku were then used in the Tracking Analysis Framework model. The Tracking Analysis Framework model traces the effect of regional changes in emissions of SO₂ and NOₓ to changes in environmental quality and health outcomes. The model also provides estimates of the welfare consequences of these changes. It was developed as part of the National Acid Precipitation Assessment Program to study the cost and benefits of the SO₂ trading program created by the 1990 Clean Air Act Amendments. The Tracking Analysis Framework model is still frequently used in studies that employ Haiku.
sector and where necessary assure that the effect of these rules are captured in the model.

The following section briefly describes the philosophy of constructing a model like Haiku and using it for economic analysis. This is followed by a description of the major components of the model and an overview of how the model works. This section also provides insight into the sophistication of the model in the types of policies it can represent. The closing section of this Appendix briefly describes the software used to code, manage, and run the model.

A.2 Modeling Philosophy

A simulation model is a laboratory designed for the exploration of a particular set of research questions in an empirically representative way. In economics, simulation models are typically used because they allow a high level of complexity in the representation of affected markets. This allows the researcher to explore, that is simulate, states with conditions dramatically different than what has already been observed and to trace the effects of a policy through numerous linkages. The comprehensiveness of a simulation model is subject to constraints on funding, data, and computational power. The model must also remain manageable and interpretable to be of use.

With all simulation models, decisions must be made about the information to include and the appropriate amount and method of data aggregation. The scope of information to include is, of course, greatly influenced by the intended uses of the model. For Haiku, a characterization of electricity supply and demand consistent with
economic theory is needed at the least. Yet questions remain about how completely one wants to represent these two sides of the market. For example, to what extent should associated fuel markets be accounted for in the determinants of electricity supply? This is a question of the appropriate breadth of the model. Another aspect of model scope to determine is the amount of information (unique variables) required to sufficiently represent each dimension of the market.

The availability of data also influences the scope of the model. Data used to parameterize Haiku are drawn from a variety of sources. These sources generally fall into two classes. In some cases, parameters are estimated using data from individual agents. Estimates calculated for other simulation models, as well as those found in engineering and market studies, are also used. These alternative sources of parametric data have their pros and cons. Parameters calculated from raw, agent-level, data can be tailored to specific needs of the model. While the relevance of these estimates is transparent to the researcher, challenges include building familiarity with the method of data collection and understanding what the data represent. Additional drawbacks include the need to maintain and check the accuracy of large data sets. With information gathered from engineering and market studies, much of the work collecting and aggregating raw data is already complete. The same can be said for parametric estimates culled from similar models. However, the researcher must still become familiar with how these estimates were generated in order to determine their
appropriateness for the model.\(^2\) The standard of appropriateness is not always strict: often the model structure is dictated by the estimates that are available.

The researcher must also determine how to make use of large primary datasets to represent important market dynamics. A common technique in simulation modeling is using a representative agent to capture the behavior of a group of similar individual agents.\(^3\) The use of representative agents reduces solution times and the need for computational power. At the same time, the construction of representative agents presents two important data aggregation questions. The first is determining how many categories of individuals, and thus number of representative agents, the model should have. Often there are obvious criteria for establishing different categories, such as the existence of relevant discrete differences among individuals. However, discrete differences may be more important in one situation than another and not all meaningful variations are particularly discrete.

There is also the matter of aggregating variables of interest into representative parameters when constructing representative agents. The appropriate method of calculating a representative value, usually some type of mean, is not always obvious and there can be a number of candidate weighting schemes. Additionally, while the criteria used to establish the categories of representative

\(^2\) For Haiku, existing parameter estimates are frequently drawn from other models and studies in cases where professional expertise or proprietary data are required to make reasonable performance projections. The performance functions for retrofit pollution controls in Haiku, described in later sections of this chapter, are drawn from the documentation of a similar simulation model. For each technology, the original source engineering studies were reviewed to determine how these functions should be applied in Haiku.

\(^3\) In Haiku, ‘model plants’ are the representative agents that depict a category of similar individual generators. The following section includes a description of the model plants in Haiku.
agents are selected so that variations across the entire population are captured, it may still be worthwhile to capture important variations within the groups depicted by a single representative agent. Again, it is a challenge to know what types of variations are important and how best to represent them in the model.

The previous paragraphs lay out some of the important general questions that a researcher faces when constructing a simulation model. The narrative describes each modeling decision as if it were a sequential step in the construction of the model. Yet these decisions cannot be so easily decomposed; they often occur simultaneously and iteratively. The discussion also avoided providing any particular decision rules that could be applied to answer these questions. There are rarely explicit rules that dictate the correct scope and structure of a model other than trying to remain consistent with economic theory and the incentives that the firm managers face. But note that, in economics, the challenge of knowing how completely to represent a problem is not unique to simulation modeling. The theoretician must decide which dimensions of a problem to include in the analysis, and the applied welfare economist must determine the appropriate set of markets to analyze.

As a solution to the question of the appropriate method of aggregation and model design, one might conceivably test the multitude of competing model specifications and test the importance of every variable of possible influence. In addition, one could check the sensitivity of results to every reasonable method of representing and aggregating these data. Clearly such a process is not prudent. Practicality suggests that modelers must often use their own experience and intuition when determining appropriate model structure and methods of data aggregation. This
is not to say that different model specifications are not tested. But even then it is a matter of the researcher’s perspective and judgment in determining which competing specifications are worth testing.

This discussion of how to test the specification of the model leads us to the question of the appropriate benchmark of performance to which the model should be compared. In the case of Haiku one can test the agreement between the predictions of different model specifications and actual market outcomes. Specifications that yield prices closer to what was actually observed are generally favored. Another test of model reliability is whether changes in model specifications yield results consistent with the researcher’s expectations.4 For example, in Haiku one could impose a decline in the supply of natural gas and see if the model predicts higher electricity prices and an increase in the consumption of substitute fuels as a result. These approaches are limiting in that they do not necessarily indicate which specifications maximize the predictive power of the model.

One way to increase confidence in the predictions of the model is to compare them to forecasts from similar models. To this end, Haiku was among a stable of models evaluated through studies hosted by the Stanford Energy Modeling

4 I avoid using the phrase ‘consistent with economic theory’ here because it is too limiting. It is more like testing to see if the results of the model are consistent with economic doctrine, economic intuition, or with otherwise informed priors. Simply because some predictions of the model are unanticipated does not mean they are inconsistent with economic theory. Indeed the ability to discover unanticipated results that turn out to be theoretically consistent, simply in a less transparent way, is one of the virtues of simulation models.
The Forum brings together similar models to analyze a particular energy or environmental policy and compare their forecasts. The purpose of this effort is to bring attention to those salient insights common to the models. It also provides a structure to explore the advantages and drawbacks of competing modeling approaches. Haiku also is among the models being used in the Renewable Energy Modeling Analysis Partnership. The Partnership, which began in 2006 and is modeled after the Energy Modeling Forum process, brings together modelers and analysts from government and private institutions to compare a set of models and the results they generate with respect to policies promoting renewable energy. The purpose is to understand whether the results the models generate are generally due to structural differences or changes in assumptions (i.e., relative prices). I am a lead investigator in the use of Haiku for the Partnership.

Since its initial development, Haiku has expanded along with the need to capture more subtle market dynamics. The development of a large simulation model is a continual process. As such, questions of scope, design, and aggregation are continually faced as the model is extended, modified, and updated. The following section sheds some light on a few of the most important solutions to these challenges in Haiku. It includes an outline of the key elements of the model’s structure and a description of the aggregation scheme of constituent generators to represent supply. Following sections explain how abatement controls for SO₂, NOₓ and mercury (Hg)

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5 Specifically, Haiku was among the models that participated in the fora on *Prices and Emissions in a Restructured Electricity Market* and *A Competitive Electricity Industry* (Energy Modeling Forum, 1998; 2001).
are represented in the model. In addition, these sections serve to facilitate a more detailed understanding of the modeling results in Chapter 4.

A.3 A Description of Haiku

As mentioned in the introduction, Haiku is a simulation model designed to explore the affects of environmental and market structure policies on the U.S. electricity sector. The model is typically used to estimate changes in the sector over the next 20 to 25 years. The major elements of the Haiku electricity sector model are briefly described in this section. We begin with a description of how the national electricity market is separated into regional ones. This is followed by a brief description of the demand side of the electricity market. A more detailed treatment of market supply is then provided.

A.3.1 Regional Markets

Give that Haiku is national in scope, we need to aggregate electricity markets spatially to reduce the computational demands of the model. Figure A.1 shows the boundaries of the 20 Haiku market regions. In most cases the regional electricity markets in Haiku are roughly defined by the historic boundaries of the regional (and sometimes sub-regional) councils of the North American Electric Reliability Council. These regional markets are used because there is readily available information on the transmission constraints between these regions and because they typically reflect a control area for management of the transmission grid. In some areas of the country, particularly in the Northeast, these regions have been split up further. For example, the Mid-Atlantic Area Council region has been further sub-divided into
areas based on whether or not the areas belong to states participating in the Regional Greenhouse Gas Initiative.

Intraregional electricity transmission is unconstrained. However, the model does roughly account for line-losses, the difference between electricity generation and consumption explained by the loss as heat in the transmission lines, within the region. Interregional trade is subject to constraints on the transmission capacity between regions. Subject to these constraints, the amount of trade between regions is determined endogenously.
Figure A.1: Haiku Electricity Market Regions
A.3.2 Demand

Electricity demand is distinguished by consumer type, season, and time blocks that are constructed to capture important fluctuations in electricity demand. The customer types are residential, commercial, and industrial, and the seasonal periods are summer, winter, and spring/fall. The different time blocks represent base, shoulder, peak and superpeak loads (i.e., demand) for electricity. Respectively these represent 70%, 25%, 4% and 1% of the hours of each season.

For each consumer class, time block, season and region demand is characterized using a constant elasticity functional form:

\[ Q = AP^e \] (A.1)

The variables in \( Q \) and \( P \) have their standard interpretation. The elasticities, \( e \), are drawn from parametric data provided by the Energy Information Administration (EIA). While the elasticities may be different for each separate consumer class and market, due to data limitations they often are not. The parameter \( A \) is calibrated using historic information on demand and prices along with the parameter \( e \). The parameter \( A \) is then adjusted to capture anticipated increases in demand over time.

In addition to the markets for electricity generation, Haiku also captures the presence of capacity reserve markets in each market region. These markets exist to assure that there is reserve power on hand in case demand is higher than expected over a short time-scale (hourly or daily). While Haiku is deterministic, such that these
reserves are never tapped into, it is still necessary to account from revenues from these markets to fully capture the revenue sources of new and existing generators.

A.3.3 Supply

A.3.3.1 The Model Plant Approach

To tractably depict the composition of electricity supply, generation sources are represented by ‘model plants’. The model plants are designed to be representative of a certain type of existing or potential generator capacity within a region.6 By capacity we mean the maximum potential generation, measured in the maximum number of megawatt hours it can produce in an hour, of the model plant. For existing capacity, model plants are constructed using information from a set of technologically similar constituent generators within each Haiku market region. Potential capacity is the electricity generation capability that the model can suggest would be built in response to demand growth or other changes in the market. Each potential type of generation technology is represented by a different model plant.

Table A.1 lists the model plants used to represent existing capacity. Again, the characteristics and economic performance of each model plant vary by Haiku market region and depend on the characteristics of the constituent generators they represent. The existing generators are grouped by the following seven indices (other than market region and vintage), with all but the last four represented in Table A.1: prime mover, fuel used, relative efficiency, and for coal-fired boilers, coal demand

6 The term ‘model plant’ is a bit of misnomer. They are actually ‘model generators’ in that they are to represent a set of similar electricity generators, not plants.
region, size (capacity), and the presence of existing SO₂ and NOₓ post-combustion control technologies as of 2010. Each of these categories will be discussed in turn.

Table A.2 lists those model plants that represent potential capacity in Haiku. Potential generators are distinguished by prime mover, fuel used, relative efficiency and coal demand region.

Table A.1: Model Plants Representing Existing Capacity

<table>
<thead>
<tr>
<th>Prime Mover</th>
<th>Fuel</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined Cycle</td>
<td>Natural Gas</td>
<td></td>
</tr>
<tr>
<td>Combined Cycle</td>
<td>Oil</td>
<td></td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>Natural Gas</td>
<td>Efficient</td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>Natural Gas</td>
<td>Inefficient</td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>Oil</td>
<td></td>
</tr>
<tr>
<td>Conventional Hydro</td>
<td>Water</td>
<td></td>
</tr>
<tr>
<td>Pumped Storage</td>
<td>Water</td>
<td></td>
</tr>
<tr>
<td>Renewable</td>
<td>Wind</td>
<td></td>
</tr>
<tr>
<td>Steam</td>
<td>Biomass (Dedicated)</td>
<td></td>
</tr>
<tr>
<td>Steam‡</td>
<td>Coal</td>
<td></td>
</tr>
<tr>
<td>Steam</td>
<td>Geothermal</td>
<td></td>
</tr>
<tr>
<td>Steam</td>
<td>Natural Gas</td>
<td>Efficient</td>
</tr>
<tr>
<td>Steam</td>
<td>Natural Gas</td>
<td>Inefficient</td>
</tr>
<tr>
<td>Steam</td>
<td>Nuclear</td>
<td>Efficient</td>
</tr>
<tr>
<td>Steam</td>
<td>Nuclear</td>
<td>Inefficient</td>
</tr>
<tr>
<td>Steam</td>
<td>Oil</td>
<td></td>
</tr>
<tr>
<td>Steam/CT</td>
<td>MSW/LFG</td>
<td></td>
</tr>
</tbody>
</table>

‡ There are up to 15 different coal model plants in each Haiku market region. Coal units are grouped by the coal demand region they are located in, whether they have post-combustion SO₂ and NOₓ controls (and if so, what type), and their capacity. See the text for details.
Table A.2: Model Plants Representing Potential Capacity

<table>
<thead>
<tr>
<th>Prime Mover</th>
<th>Fuel</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined Cycle</td>
<td>Natural Gas</td>
<td>Conventional</td>
</tr>
<tr>
<td>Combined Cycle (Duct Burner)</td>
<td>Natural Gas</td>
<td>Conventional</td>
</tr>
<tr>
<td>Combined Cycle</td>
<td>Natural Gas</td>
<td>Advanced</td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>Natural Gas</td>
<td>Conventional</td>
</tr>
<tr>
<td>Combustion Turbine</td>
<td>Natural Gas</td>
<td>Advanced</td>
</tr>
<tr>
<td>Integrated Gasification Turbine</td>
<td>Biomass (Dedicated)</td>
<td></td>
</tr>
<tr>
<td>Combined Cycle (IGCC)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal Combustion</td>
<td>Landfill Gas</td>
<td></td>
</tr>
<tr>
<td>Steam</td>
<td>Coal</td>
<td></td>
</tr>
<tr>
<td>Steam</td>
<td>Geothermal</td>
<td></td>
</tr>
<tr>
<td>Renewable</td>
<td>Wind</td>
<td></td>
</tr>
</tbody>
</table>

The primemover is the description of the technology or method used to generate electricity. The most common prime mover types are gas (combustion) and steam-driven turbines. As their name suggests, steam turbines are operated by steam generated from some heating processes. In the case of fossil fuels, combustion occurs in a boiler and steam is generated using the resultant heat. With combustion turbines, the exhaust gas from the combustion process is used to rotate the turbine directly. Combustion turbines typically use clean fuels because of the direct interface between the exhaust gas and the turbine.\textsuperscript{7} Combined cycle systems are essentially a combination of these two technologies. In these systems air heated to a high temperature is first used to turn a combustion turbine. The exhaust that exits the combustion turbine (often referred to as waste heat) is then used to create steam,\textsuperscript{7} Internal combustion engines are grouped with combustion turbines in the creation of model plants.
which in turn is used to run a steam turbine. Other prime movers uniquely correspond to the fuel they use, such as wind turbines.

The relevance of the fuel index is assumed to be self-explanatory. However, one may realize that the categories are somewhat limiting. For example, combustion turbines often can fire both natural gas and light oil. Such dual-fuel capability is not modeled in Haiku. Constituent generators are categorized by the primary fuel they combust, and the model limits them to consuming only that fuel. Similarly, the costly process of converting a coal-fired boiler to one that fires a liquid fossil fuel (called repowering) is not represented.

In certain cases constituent generators are also distinguished by their relative efficiency. Existing natural gas-fired combustion and steam turbines are divided into efficient and inefficient model plants based on their heat rate.\(^8\) The purpose of this division is to capture the heterogeneity in the efficiency of these generator types. Units with a heat rate higher than the capacity-weighted mean heat rate of like units in the region are considered inefficient, while those with lower heat rates are considered efficient. The determination of the relative efficiency of nuclear units is based on a multi-factor analysis. The primary distinguishing characteristics between efficient and inefficient nuclear units are their relative fixed and variables operating and maintenance costs and their availability for generation through the 1990’s. Finally, the relative efficiency of potential (new) generators is based on

\(^8\) The heat rate is the amount of energy, usually measured as million British thermal units (Btus), required as an input in order to obtain a certain number of kilowatt-hours of electricity. The lower the heat rate of a generator, the more efficient it is.
expectations of performance improvements over time. As such, the model cannot forecast the construction of “advanced” units until later years.

For the purpose of constructing model plants, coal generators are divided into 15 different categories in each Haiku market region. The generators are grouped based on their combination of existing SO₂ and NOₓ post-combustion controls, their capacity, and the “coal demand region” that they are located in. The coal demand regions are a construction that Haiku adopted from the National Energy Modeling System (NEMS) model.¹ In the NEMS model the U.S. is divided into 13 different coal demand regions. These regions determine the types of coal available to the constituent generators. A Haiku market region may overlap portions of different coal demand regions. In each Haiku market region, potential coal units are usually allowed to locate in any of the coal demand regions. More will be said about the meaning and purpose of the coal demand regions in the section on fuel markets.

A.3.3.2 Characteristics of Model Plants

A.3.3.2.1 Data Sources

Each model plant is represented by up to 85 unique parameters. These include the seasonal capacities, heat rate, operating costs, direct costs of pollution control retrofits, outage (availability) rates, and emission rates for each model plant. The performance characteristics and availability of generators that represent potential capacity are primarily adopted from the NEMS model. The performance of existing

¹ NEMS is an integrated multi-sector simulation model of the key energy-intensive markets in the U.S. It is used to generate the projections found in the popular Annual Energy Outlook (EIA 2007) as well as to perform policy analyses requested by Congress and various executive agencies.
generators is constructed from a mix of survey and compliance data from the population of grid-connected generators, and, when such data is unavailable, parametric data drawn from other sources.

The construction of existing model plants begins with a database of the entire population of grid-connected electricity generators in the U.S. The EIA collects prime mover, fuel, capacity, and location information from these sources via the Annual Electric Generator Reports (EIA 2000; 2002; 2005). When raw data is used to characterize the performance of model plants, the data are typically drawn from information collected by a variety of government agencies. Furthermore, when additional plant or generator-level data is available, it is typically from plants with large steam boilers due to their large share of total generation in the U.S. For example, fuel use and generation from all large steam boilers is collected by the EIA using the Steam-Electric Plant Operation and Design Report (EIA, 2001b).

Information on the population and basic characteristics of generators, including their historic emission rates and fuel use, is supplemented by data collected by the EPA to monitor compliance with the acid rain program created by the 1990 Clean Air Act Amendments (U.S.EPA, 2001). Information on operating and maintenance costs (O&M) for large steam plants is taken from the Federal Energy Regulatory Commission’s Form 1 (Platts, 2002).10

10 These are the ‘big picture’ data sets in that they broadly cover a large share of generating capacity. However, many other data sets are used to construct components of the model. For example, survey data from the Nuclear Regulatory Commission is used to identify planned capacity increases to existing nuclear generators (Nuclear Regulatory Commission, 2005); data on historic availability of generators comes from North American Electric Reliability Council (North American Electric Reliability Council, 2003), and information on historic generation from hydropower comes from data
Due to restrictions in survey coverage, certain variables of interest are not available for all generators. Missing data are a particular problem for small generators or generators not historically subject to cost-of-service (i.e. average cost) pricing. Often, parametric estimates are drawn from the literature or other models to compensate for this missing information. The two models that are the most common sources of parametric data used in Haiku are NEMS and, to a lesser extent, the EPA’s Integrated Planning Model (IPM) model (U.S.EPA, 2002b, 2005d). When parametric data from other sources is unavailable, alternative sources include using industry or regional averages or other statistical techniques to impute the missing values from the available generator-level data.

A.3.3.2.2 Data Aggregation

The task of generating representative statistics for the model plants follows the preparation of the generator-level data. The method of calculating these statistics may have an important influence on model performance, as there is often considerable heterogeneity in the variables of interest among the generators that comprise a model plant. One must also be sensitive to how the constituent generator data were collected. For example, constituent generator observations are typically annual means. There are few data that capture the fluctuation in these variables over

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11 The raw data are usually cleaned and compiled using the statistical program STATA. STATA is adopted for this task given its transparency, ability to handle large data sets, and ease in embedding documentation into the programs that construct the parametric data.
the course of the year. Also, typically the observations are based on one year of data; rarely are time-series data used to characterize an individual generator.\footnote{12 The costs of operating nuclear units are an important exception.}

While the functional relationships underlying the constituent generator data are sometimes complex, typically simple aggregation schemes are used. The majority of the variables of interest are represented solely by capacity-weighted arithmetic means. This simple approach is sensible for variables that are themselves ratios to total capacity (such as fixed O&M costs). This is also a reasonable approach when the utilization of the constituent generators is expected to be proximate or when the variable does not fluctuate considerably with the utilization of a generator (at least within expected operating ranges).

The heat rate and fixed and variable O&M costs of a model plant receive a more careful treatment. In addition to a mean, a standard deviation of these variables is also generated. This information is used to capture the expectation that as the utilization of a model plant increases, more costly and less efficient generators are expected to provide a larger share of total generation. The variances of these variables are used to create an increasing cost function for the model plant.

Parametric estimates of operating costs and variances drawn from engineering studies are used to represent most generator types. Typically these costs are invariant across the country, although they do vary based on the generation technology being used and generator capacity. As mentioned above, given their large contribution to total electricity generation and, consequently, due to greater data
availability from these sources, the costs and efficiencies of fossil and nuclear steam model plants is based on data from their constituent units. The average and variance fixed O&M cost and heat rate for these model plants are capacity-weighted estimates from the average O&M costs of the constituent generators of the model plant. The average and variance of the variable O&M cost are capacity-weighted estimates from the constituent generators.\(^\text{13}\)

The aggregation scheme described in the previous paragraphs is considered acceptable as the model predicts production, prices and emissions sufficiently close to those observed historically. Clearly, in this case constituent generators are assumed to be operating close to their historic utilization, so perhaps this outcome is not surprising if the remainder of the model is constructed appropriately. Whenever forecasted utilization of the constituent generators deviates far from historic utilization the reliability of model estimates becomes more questionable. As discussed above, there are additional techniques to increase confidence in model performance, such as comparing results to the prediction of other models and testing whether model results are consistent with expectations.

\(^{13}\) Other candidate approaches are to calculate generation or capacity-weighted mean and variances for variable O&M cost. A generation-weighted mean would seem the most sensible as the variable O&M cost is reported per unit of generation (i.e, $/megawatt hour). This approach would put more weight on units with lower variable O&M costs because those are the units that run more. However, at levels of utilization exceeding historic utilization, these estimates would likely underestimate variable costs. A capacity-weighted variance would get around this problem by implicitly assuming that an increase in generation is shared evenly across all units (assuming variable O&M cost for each generator is constant in the relevant range, which is not a poor assumption). However, we might expect that units with historically lower utilization will be run more as demand increases over time, so we adopt an unweighted mean and variance as they are typically higher than the capacity-weighted mean and variance of the variable O&M cost.
A.3.3.3 Fuel Markets

In addition to electricity, Haiku is capable of simultaneously solving for equilibrium prices in important fuel markets. The distribution and cost of coal, natural gas, and biomass fuels have historically been important factors in debates on how emissions from the electricity sector should be regulated.\[^{14}\] This is because fuel choice is an important factor in determining a generator’s compliance strategy with pollution control requirements. In the U.S., coal-fired generators are a significant source of CO\(_2\), particulate matter, NO\(_X\), SO\(_2\) and Hg emissions and are the primary source of all these pollutants from the electricity sector. Natural gas does not contain any Hg or sulfur and emits less CO\(_2\) and NO\(_X\) than coal. Therefore, switching from coal to natural gas generation, through both changes in the relative utilization of existing generators and the turnover of the stock of generating capital, is an important strategy for complying with regulations that affect these pollutants.

The supply functions for coal, natural gas and biomass fuels in Haiku are adopted from the NEMS model. For coal, there are separate supply functions for each major production regions in the U.S. Coal is treated as a heterogeneous commodity. The average heat content and levels of impurities distinguish the coal from each region. Heat content is a measure of the intrinsic energy stored in Btus, in this case, in a ton of coal. The impurities include the percentage of sulfur and Hg in a ton of coal. This information is used to calculate the uncontrolled emissions of these pollutants.\[^{15}\]

\[^{14}\] While oil is used to generate electricity, it accounts for a relatively small share of total generation.
\[^{15}\] The NO\(_X\) emissions rate is not particularly sensitive to the coal used. Rather, uncontrolled emissions of this pollutant are particularly sensitive to the combustion temperature and characteristics within a
To simplify the model, the coal from each production (supply) region is not available in every coal demand region. Each coal demand region is limited to using those coals where transportation infrastructure exists to provide delivery or where transportation capacity is likely to develop in the face of tightening emission standards.

A.3.3.4 Pollution Abatement Technologies in Haiku

In addition to being able to adjust their fuel use, the model allows coal-fired model plants to install post-combustion control technologies to reduce emissions of SO$_2$, NO$_X$ and Hg.$^{16}$ Of the 85 or so variables used to characterize each model plant, about 50 are used to capture the emissions and cost of controlling these pollutants, with over half of these associated with the emission and the cost of controlling Hg.

The cost and performance of abatement technologies for these pollutants are taken from the assumptions in the EPA’s IPM model (U.S.EPA, 2002b; 2005d). This model is used to support EPA’s modeling of air pollution policies that affect the electricity sector. As such, many of the assumptions therein are publicly available. Furthermore, the studies that underlie these functions are publicly available, which allows us to gain a more complete understanding of their suitability for use in Haiku.

Each model plant that has yet to retrofit a SO$_2$ or NO$_X$ post-control may choose the types of controls it installs. For SO$_2$ controls, which are often called

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$^{16}$ The model also accounts for the presence of existing and planned post-combustion controls. This information is collected and compiled from a variety of sources, including trade press, EIA (2000; 2001b; 2002) and U.S.EPA (2001).
“scrubbers”, the model plants have a choice between two different types. These types vary in their cost of installation, operation, percentage of uncontrolled SO2 emissions reduced (the removal efficiency), and coal type that they may be used with. See Table A.3 for a description of these technologies (in Haiku, the cost and performance functions describing the two wet scrubber types are combined).\textsuperscript{17} For NOX controls, the model plant may install a Selective Catalytic Reduction system. This technology is assumed to remove NOX emissions by 90%. The model also tracks the presence of Selective Non-Catalytic Controls, which cost less than Selective Catalytic Reduction systems but are assumed to reduce NOX emissions by only 35%. The installation and operating costs of the SO2 and NOX controls are also a function of the capacity of the generator. When a model plant representing coal capacity adopts a particular control, it is assumed that all of the constituent generators adopt this control and that the controls cannot be removed later.

\textsuperscript{17} Note that the coal use restrictions are not applied to units constructed prior to 2010.
Table A.3: Characteristics and Applicability of SO$_2$ Post-Combustion Controls in Haiku†

<table>
<thead>
<tr>
<th>Technology</th>
<th>Type‡</th>
<th>Coal Sulfur Content Restriction</th>
<th>Removal Efficiency</th>
<th>Size Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lime Spray Drying</td>
<td>Dry</td>
<td>Greater than 0.4%, less than &lt;2% sulfur</td>
<td>90%</td>
<td>Greater than 550 megawatts</td>
</tr>
<tr>
<td>Limestone Forced Oxidation</td>
<td>Wet</td>
<td>Greater than 2% sulfur§</td>
<td>95%</td>
<td>Greater than 100 megawatts</td>
</tr>
<tr>
<td>Magnesium Enhanced Lime</td>
<td>Wet</td>
<td>Less than 2.5% sulfur</td>
<td>96%</td>
<td>Greater than 100 megawatts, less than 550 megawatts</td>
</tr>
</tbody>
</table>

† Adopted from U.S.EPA (2002b), Table 5.2. In Haiku, the cost and performance functions describing the two wet scrubber types are combined.

‡ The key factors distinguishing wet from dry processes is that in wet processes the scrubber waste is wet and the flue gas leaving the absorber, the compartment where SO$_2$ emissions are taken up, is saturated with water. This distinction is important because the extent to which a scrubber reduces Hg emissions depends on the type of process.

§ U.S.EPA (2002b) also indicates that this control is further limited to bituminous coal, but that is likely because only bituminous coals have a sulfur content this high.

As alluded to above, estimating Hg emissions from coal-fired plants is more challenging than estimating SO$_2$ or NO$_X$ emissions. Hg emissions are a function of the coal used, because of the coal’s Hg content and because of its heat content, and the other pollutant controls in use. As described in Chapter 4, each combination coal type and pollution control combination has its own removal efficiency, which is the percentage reduction in uncontrolled Hg emissions. The primary source of the expected (average) removal efficiencies used in the IPM model is U.S.EPA (2002a). U.S.EPA (2002a) also reports the standard deviations of these estimates, which are used in the analysis described in Chapter 4.
There is also an Hg-specific abatement technology, called Activated Carbon Injection, which Haiku typically allows coal-fired boilers to install. The cost function from this control is also taken from IPM. However, for reasons described in Chapter 4 this option was not allowed for the analysis in this dissertation.

A.3.4 Regulatory Regimes

Haiku is flexible in that it can incorporate and model a variety of environmental regulations and electricity pricing regimes. We discuss the electricity pricing polices first.

A.3.4.1 Electricity Pricing

The wholesale electricity market in the U.S. was opened to competition by legislation passed in 1992. The legislation requires owners of transmission assets to make their capability accessible, at cost-based rates, to other generators. Haiku assumes that this market structure yields competitive wholesale prices, where the prices are set at the marginal cost of generation.

At the retail level, markets can be characterized by average or marginal cost pricing. Traditionally, electricity prices have been set at the average cost of service, and this is known simply as average cost or cost-of-service pricing. In average cost pricing regimes prices often vary based on season and by consumer class. Haiku is designed to capture these subtleties.

Retail electricity pricing in some states is based on the marginal cost of generation. Here we are referring solely to the generation aspect of electricity supply. Electricity transmission and distribution services continue to be priced at the average
cost of providing them. In reality, and in Haiku, marginal cost pricing typically does not imply that consumers face the instantaneous marginal cost of generation. Rather, consumers are assumed to face one of two pricing mechanisms: one that captures the variation in the marginal cost of generation over the course of a day and another that does not. Under time-of-day pricing, consumers are assumed to face the marginal cost that occurs during that time block. Consumers are assumed to know these prices ahead of time. Where prices are assumed not to vary over the course of the day, the different consumer classes are expected to face an average of the marginal cost of generation over the time blocks that is weighted by their share of consumption in those time blocks.

This simple description, while sufficiently informative for the purposes of becoming familiar with Haiku, belie the complexity of the different pricing regimes, both in the sense of how they are represented in Haiku and how they are implemented in practice. The importance of these different regimes is clear, however. As is well known, the method of electricity pricing has important static and dynamic efficiency implications on the performance of the electricity market. Recent research employing Haiku has shown that the method of electricity pricing also has important implications to the performance of pollution control policies (Burtraw et al. 2002).

A.3.4.2 Environmental Regulation

Haiku is particularly powerful in its ability to represent the variety of pollution abatement requirements that are proposed and adopted. The model estimates

\[ \text{________________________} \]

\[ ^{18} \text{Price differences across consumer classes can still arise from differences in the transmission and distribution costs of serving these customers.} \]
CO₂, SO₂, Hg and NOₓ emissions from the sector.¹⁹ To comply with regulations controlling these pollutants, model plants may install post-combustion controls, reduce their generation, or sometimes change the fuel they use. The presence of cap-and-trade and emission tax regulation can be represented for any of these pollutants. Prescriptive, or command-and-control, regulations may also be imposed on emitters of these pollutants. For example, all potential units are assumed to be subject to new source performance standards for these pollutants. Existing units may be subject to restrictions on their emissions per unit of fuel use or generation and/or a policy akin to the performance requirements for sources that make major modifications (i.e., New Source Review). This later approach takes the form of a limitation on how much a model plant can increase its generation above historic utilization without installing post-combustion controls. The model also allows these regulations to be limited to particular seasons and regions of the country or a particular set of model plants.

Emission trading programs can be further distinguished by whether or not banking is allowed or by the method of allowance allocation. Allocation schemes using auctions, updating, gratis or a mix of these methods can be imposed. The auction approach assumes that the auction is efficiently designed in that the clearing price equals the marginal value product of emissions across affected generators. However, a variety of methods for the distribution of the auction revenues can be modeled, some of which may distort the market. Under an updating allocation scheme the share of total allowances received by a source is updated annually and based on

¹⁹ In addition to regulations on emissions, Haiku also has the capability of modeling national and regional subsidies (tax credits) of a variety of forms to encourage the use of particular technologies as well as restrictions on the composition of the generation mix, such as renewable portfolio standards.
its share of total generation or fuel use in some recently preceding period. With gratis approaches allocations for all future periods are fixed\textsuperscript{20} and known at the beginning of the program. Emission taxes can also be represented as having rebates schemes with similar incentive effects as these allowance allocation methods.

Existing air pollution control requirements typically imposed in Haiku include regulations derived from Title I and Title IV of the 1990 Clean Air Act Amendments.\textsuperscript{21} Title IV of the Amendments created the well-known SO\textsubscript{2} emissions trading program. It also induced regulations limiting emission rates for NO\textsubscript{X}. Further controls on NO\textsubscript{X} derive from Title I of the Amendments, which is the section that addresses compliance with ambient air quality standards. Primary among these is the NO\textsubscript{X} SIP Call, a 22 state summer-time NO\textsubscript{X} cap-and-trade program designed to reduce interstate transport of this pollutant, a problem which confounds local efforts to comply with ozone concentration standards. As mentioned above, performance standards for new sources, as well as those sources that presumably undergo major modifications, are also imposed. Finally, a variety of other local and state-level requirements are also represented in the model.

A.3.5 Equilibrium and Convergence

The primary purpose of Haiku is to estimate partial-equilibrium welfare changes expected to result from imposing environmental and electricity pricing policies affecting the electricity sector. However, the goal in solving and coding the

\textsuperscript{20} That is, in terms of the incentives they generate, they are lump-sum.

\textsuperscript{21} For reasons described in Chapter 4, these regulations are not imposed in the analysis in this dissertation.
model is to find simultaneous market equilibria by modeling incentive-compatible behavior on the part of electricity generators. The choice variables in the model include, for each model plant, capacity, generation, fuel (and fuel type for coal-fired boilers) and pollution control adoption. Generator dispatch is based on minimization of short-run variable costs of generation. That is, a plant’s decision to produce electricity at any instance is determined by its short-run operating cost. Capital stock investment and retirement, as well as investment in pollution control technologies, are determined by the expected profitability of generation assets over time.

Haiku is an iterative tâtonnement model in the prices for both outputs and major inputs to this sector. At the beginning of each iteration, a set of prices is announced, with one price for each market. The model then determines the optimal quantities of generation, fuel use, etc. for each model plant at those prices. For each market, the model then compares evaluates the price electricity demanders are willing to pay (or, in the case of input markets, the price producers are willing to accept) at that quantity. A combination of this new set of prices and the original set of prices is used for the next iteration. The weight placed on the old and new set of prices in the determination of the set of prices used for the next iteration is a user input. The weights (or steps) are typically set to emphasize the new set of prices as the number of iterations increases. The model is determined to be in equilibrium when all of the prices are stable or fluctuate systematically around a fixed point for a large number of iterations.
A.4 Modeling Platform

The Haiku model is coded using the software *Analytica*. Each variable, index, or result is illustrated by an object in *Analytica* that appears as a node within an influence diagram. Hence, the screen views of the model itself provide additional documentation, and each object in the model has a description field that provides detail about how each object is used. The model algorithm is managed from iteration to iteration using *Visual Basic* and it is solved using the *Analytica Decision Engine* software.
BIBLIOGRAPHY


J. B. Bushnell and E. J. Friedman, Linearly Exchangable Permits for the Efficient Control of Multiple Pollutants, Program on Workable Energy Regulation, University of California Energy Institute, Berkeley (1994).


Federal Register, *Rule to Reduce Interstate Transport of Fine Particulate Matter and Ozone (Clean Air Interstate Rule); Revisions to Acid Rain Program; Revisions to the NOX SIP Call; Final Rule*, Washington, DC (2005a).


R. Hannesson and J. Kennedy, Landing fees versus fish quotas, Land Economics (2005), 81, 518-529.


I. Parry, Fiscal Interactions and the Costs of Controlling Pollution from Electricity, RAND Journal of Economics (2005), 36, 849-869.


U.S.EPA, *Basic Facts [Title V Permits]*,  
(2007b).


