ABSTRACT

Title of Dissertation: **ONLINE INVENTORY REPLENISHMENT AND FLEET ROUTING DECISIONS UNDER REAL-TIME INFORMATION**

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Logistics managers rely on increasingly sophisticated technologies to track demand and associated inventories, allowing rapid response to meet anticipated demand, avoid shortfalls while minimizing transportation and inventory carrying costs. This ability to respond gives rise to complex decision problems, characterized by combinatorial underlying problems under progressively unfolding demand. Real-time information also increases the ability to coordinate effectively inventory management and transportation service.

The advantages of coordinating inventory replenishment with vehicle routing decisions have long been recognized, giving rise to the inventory routing problem, which arises in the context of vendor-managed inventories. These typify an emerging class of collaborative logistics arrangements facilitated by information and communication technologies. The ability to coordinate inventory with routing decisions in real-time adds an important dimension to the problem. While fleet management decisions under real-time information have been studied extensively, coupling these with inventory replenishment decisions in real-time remains in the

early stages of conceptualization and development. The main objective of this dissertation is to examine effectiveness of policies for managing inventories taking into consideration the interaction between inventory replenishment, retailer sequencing and transportation cost.

A major motivation for the online inventory routing problem is the presence of uncertainty about future consumption rates at different facilities. The possibility of updating plans on a continuous basis, based on real-time information about demand realizations makes possible decisions to modify the set and/or the sequence of subsequent facilities to be visited, diverting a truck from its current destination to visit a different facility, and adjusting amounts to be delivered to subsequent customers in the route.

This dissertation proposes two decomposition approaches, in which a simplified version of either the inventory-control or the routing side is solved first, and then that solution is used as a soft constraint when solving the other side. For each approach, different operational polices are proposed, reflecting different degrees of sophistication in terms of technology and optimization capabilities. These operational policies are based on a rolling-horizon framework, wherein new plans are repeatedly generated, based on updated information. Finally, the performance of proposed strategies is simulated and the impacts of using sophisticated real-time strategies are discussed.

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By

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Dedication

To my parents,

and my family: Macarena, Tomás & Amelia

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Chapter 1: Introduction

1.1. ***Motivation***

This research studies real-time distribution strategies and their associated benefits for a two-level distribution system, from one depot to *N* retailers, wherein vehicle delivery routes can be updated using real-time information about current inventory levels and the status of the vehicle. This section discusses the need to conduct research in the field of logistics and distribution systems, especially in the area of real-time operations.

This research is motivated by three main considerations: (i) the importance of logistics and distribution systems in the national and local economy, (ii) the current trend to coordinate logistic operations, and (iii) the opportunities offered by current information and communication technologies (ICT) to operate and control a system in real-time. Those considerations are discussed in the following subsections.

1.1.1. The Importance of Logistics and Distribution Systems

Logistics and distribution systems are critical components of any modern economy wherein most products are consumed away from their production points. In the United States, during 2003, logistics activities accounted for $936 billion, equivalent to 8.5 percent of the nominal gross demographic product (GDP). The two main components of logistic costs are transportation and inventory costs, accounting for 63 percent ($593 billion) and 24 percent ($222 billion) respectively. Among transportation modes, the trucking industry represents 81 percent ($ 482 billion) of the total transportation expenses, which is more than half of the total logistic costs (Council_of_Logistic_Management, 2004). Furthermore, at the level of individual firms is estimated that distribution costs represent between 10 and 30 percent of a product's sales price (La Londe, 1994, Sharp and Goetschalckx, 1999, Ghiani et al., 2004). Therefore, given the large amount of resources involved, even slight improvements in logistic operations could have a significant impact on the overall economy. That explains, in part, why in the last two decades some of the largest and most successful companies, such as Dell and Wal-Mart, have transformed the role of logistic operations to that of a strategic weapon rather than just a support function to coordinate the movement and storage of products required to satisfy consumers' demands (Ball, 2002).

1.1.2. The Coordination of Logistic Operations

The second significant motivation for this research is the current trend to coordinate logistical functions, such as inventory control and transportation, in order to improve the efficiency of supply chains, taking advantage of different logistical operations synergies (Thomas and Griffin, 1996). The coordination of inventory and transportation operations is particularly relevant when customers are part of the same company or when vendor-managed inventory (VMI) strategies are employed (Campbell et al., 1998). Under VMI agreements the supplier, who could be a manufacturer or a distributor, takes control of buyers' inventory levels, ensuring that adequate service levels are maintained. Companies that have successfully implemented VMI agreements include Dell (Kapuscinski et al., 2004), Wall-Mart, Procter and Gamble, and Campbell Soup (Buzzell and Ortmeyer, 1995, Fisher, 1997, Lee, 2004, Mishra and Raghunathan, 2004). Some examples of distribution systems where inventory and routing decisions had been integrated are fuel oil delivery to gas stations, industrial gas distribution, beer and soft drink distribution, vending machine replenishment, cash replenishment to automatic teller machines (ATMs), and supermarket product replenishment.

1.1.3. Technologies that Enable Operations with Real-Time Information

The third significant motivation for this research are the opportunities offered by ongoing developments in information and communication technologies (ICT), which allow sharing information between different stages in a supply chain at progressively reduced costs (Rabah and Mahmassani, 2002). ICT developments can be divided into three groups:

a) communication and tracking of devices that allow automating the way information is input to computer systems and transmitted between them;
b) commercial vehicle operations (CVO) technologies that allow the control of a fleet of vehicles on a real-time basis; and
c) software and decision-support systems (DSS) that provide data processing capabilities at a particular facility.

These systems increase the speed and accuracy at which data is entered, gathered, and communicated, and they provide better real-time visibility about inventory levels throughout the distribution system and better control over a fleet of vehicles on a real-time basis.

a) Communication and Tracking Devices

Communication and tracking devices can be divided into two groups: data transmission between computer systems and physical transaction tracking.

The development of protocols and associated company standards to transmit business transaction data between computer systems—such as Electronic Data Interchange (EDI), and more recently eXchange Markup Language (XML) to communicate using the Internet (see www.rosettanet.org)—have facilitated information and data exchange between different computer systems, thereby avoiding paperwork.

In terms of physical transactions tracking, bar coding systems (see Masters and La Londe, 1994), and more recently, radio frequency identification (RFID) transponders and readers, which do not require the manual scanning of products, are instrumental in improving the speed and accuracy at which transactions and movements of products are recorded and updated in computer systems. The interest on RFID is rapidly increasing after the Department of Defense (DoD) and Wal-Mart mandated that all cases and pallets entering their systems must have RFID transponders after 2004 (Datta, 2003). However, because of privacy concerns, RFID is unlikely to soon replace bar coding at the final consumer level (Blanchard, 2003).

b) CVO Technologies

At the level of commercial vehicle operations (CVO), ICT developments that enable real-time operations include automatic vehicle identification (AVI), two-way communication systems, automatic vehicle location (AVL), and other related technologies, such as on board computers (OBC), and navigation devices (Regan et al., 1995).

i) AVI systems are basically applications of RFID technologies to CVO, consisting of adding a transponder to each tractor, which allow recognition it when passing through a reader. Some applications of AVI include electronic toll systems, weigh-in motion systems, and RFID at terminal gates for tracking tractor arrivals and departures.

ii) Two-way communications systems allow transferring voice and data between the dispatch center and drivers in real-time. Available two-way systems vary from VHF radios, cellular phones to satellite communications, and differ in kinds of communication permitted (voice and/or data), range of operation, and cost.

iii) AVL systems are used to map vehicle positions in real-time. The technology leader in this market is Global Positioning System (GPS) receivers, which can compute current position and speed within meters of accuracy by sending signals to four out of twenty-four GPS satellites and triangulating. Some leading GPS receivers suppliers include Novatel, Garmin, Navman, and Magellan.

iv) Among other related technologies, the current trend in the industry is to integrate on-board computers (OBCs), using generic PDAs, with AVL systems. Some integrated systems on the market include OmniTracs from Qualcomm, MobileMax from Aether Systems, VMX 8700 from Data Ltd Inc, Mobius TTS from Cadec Inc, g2x system from PeopleNet, i58sr and i88s from Motorola, and iPAQ PDA from Compaq.

c) Software and Decision-Support Systems

Among software and decision-support systems (DSS), the main development was the Enterprise Resource Planning (ERP) system. ERP systems use a centralized database to collect, manage, and share organizational information across business functions. As Rutner et al. (2003) state "ERP is becoming a widely accepted computerized process for handling data in American corporations with over 92% of companies using or in the process of implementing" it. ERP systems vary in terms of sophistication from a simple transactional database to multi-component decision-support systems (DSS), also known as ERP-II. Among the main ERP components are production scheduling, material requirements planning (MRP), financial management, inventory management, demand planning, transportation management, and human resource management. Market leading ERP vendors include SAP, Baan, PepleSoft, J.D. Edwards, and Oracle.

In addition to components included in ERP systems, there had been, during the last decade, an increasing interest in developing DSS for specific operational purposes, such as supply chain planning (SCP) systems, warehouse management systems (WMS), transportation management systems (TMS), and advance planning and scheduling (APS). However, notwithstanding their names, in most cases they lack true optimization capabilities and rely on simple heuristics to obtain feasible solutions (Fleischmann and Meyr, 2003, Simchi-Levi et al., 2003, p. 317). Some leading providers of such systems are Manugistic, i2, and Manhattan Associates.

Another important development is the spatial Geographic Information System (GIS) database, which allows presenting and manipulating geographically referenced data. GIS capabilities have been implemented in many graphical user interfaces

(GUI) used by DSSs. Some leading GIS providers include ESRI and Caliper Corporation.

In summary, all of the above-described ICT developments provide access to real-time information on the current state of the system— i.e. inventory levels at each facility and status of the fleet—which allows managers to make online decisions on a continuing basis to improve routing plans. Those developments give managers new opportunities to react faster to changes in predicted demand patterns or traffic conditions, and adjust plans accordingly. However, the operational decisions are complex, since the underlying problems are combinatorial and unfold in real-time, precluding the evaluation of all possible alternatives by the decision maker. Moreover, the stochastic nature of such systems implies that information about the state of the system is gradually revealed and cannot be accurately predicted in advance. Therefore, in order to take maximum advantage of the extensive quantities of real-time information made available by ICTs, supply-chain managers need to use information effectively. That requires the development of models and algorithms that can exploit the full potential of real-time information for distribution-logistic operations.

1.1.4. Other Motivations

Finally, as it will be discussed in Chapter 2, previous research on real-time fleet management has focus on how to serve load demands for transportation services that are exogenous to the system, in the context of dynamic vehicle-routing problems (Gendreau et al., 1999, Larsen et al., 2002) and pick up and delivery problems (Regan et al., 1995, Regan et al., 1996a, Yang et al., 2004, Kim et al., 2002a, Kim et al., 2004). In this research, routing decisions are coordinated with inventory control. In that fashion, it is expected that monitoring inventory levels would allow improving the forecast and coordination of transportation activities, giving the operator the option to visit a facility earlier than needed to take advantage of transportation savings. That could be particularly useful when demand is highly variable and/or unpredictable, which is normally the case when final consumers are separated by many echelons from the echelon considered, or as a consequence of the phenomenon known as the bullwhip effect (Lee et al., 1997, Fine, 1998, Chen et al., 2000).

Having established the main motivations for this research, then, the next section presents the specific problem studied.

1.2. Problem Statement

The focus of this research is on formulating inventory-routing problems (IRPs) in a stochastic dynamic environment with real-time information about current inventory levels, as well as delivery vehicle locations and status.

The specific distribution system considered is a two-level supply chain, in which a set of geographically dispersed facilities facing stochastic demands have to be repeatedly replenished from a central warehouse (or depot) over a long period of time. The facilities to be replenished could represent final customers, retailers who serve demand from final customers, or distribution centers from which a set of additional facilities are replenished. In this system, products are transported from the depot to the set of retailers by a vehicle with limited capacity, the plans for which can be updated with real-time information about the state of the system, thanks to modern information and communication capabilities. This problem is designated as the online inventory routing problem (OIRP) under real-time information. The OIRP is formulated and solved considering inventory allocation and transportation decisions together. As such, the OIRP considers the trade-off among transportation, inventory holding, and stock-out costs.

In this research—contrary to the common view in real-time fleet management problems where load demands are exogenous to the system—decisions to replenish inventory, by how much, in what sequence, and by which vehicle, are conducted in an integrated real-time decision framework. In addition a central-planer approach to the problem is assumed. That is, the system is assumed to be operated and controlled by a central decision maker who seeks to move inventories in the system in such a way as to maximize total expected profit in the long-run for the complete system. Moreover, the central decision maker operates with real-time information about the complete state of the system.

Key features of this OIRP are the presence of uncertainty about future consumption rates at different facilities and the possibility of updating plans based on accurate real-time information about the complete state of the system; i.e., accurate real-time knowledge of all local inventory levels, and location of and remaining load in each truck. That contrast with deterministic environments, in which decisions can be made with perfect hindsight, thus real-time operational capabilities would not modify the nature of the problem. The possibility of updating plans on a continuous basis, based on real-time information about demand realizations makes possible some additional decisions to update truck-route plans, such as modifying the set and/or the sequence of subsequent customers to be visited; diverting a truck from its current destination to visit a different facility; and adjusting amounts to be delivered to subsequent customers in the route.

Such a new operational environment could enable more efficient use of existing resources and increase system reliability. However, the design of efficient strategies to operate the system can be extremely difficult. On one hand, the dispatcher faces a fleet-routing and scheduling problem—which is combinatorial—to obtain new operational plans. Since, even simplified static and deterministic versions of the inventory-routing problem are computationally hard (Bertazzi et al., 2007), a trade off between quality of solutions and speed should be considered in the search of new plans. On the other hand, given that plans can be modified at any time, based on new information, the events and circumstances under which a plan update would be beneficial should be specified.

1.3. Research Context and Scope

The broad context for this research is product distribution and logistics operations in which a set of facilities need to be repeatedly refilled from a single facility with the same product over time. The problem studied entails the management of a fleet of trucks that moves the product from the depot to the set of retailers, combining deliveries to different facilities in the same route. This type of fleet operation is known as less than truck load (LTL), since a vehicle could transport loads for different facilities simultaneously. In this research while the vehicle is enroute reallocation of loads among retailers is considered. However, this dissertation does not address distribution operations with transshipments, that is, it is assumed that after products have been delivered to a particular facility, they cannot be reclaimed and reassigned to a different facility.

Distribution systems can be operated either centrally (i.e., vertically integrated) or decentrally, in which different agents control different parts of the chain. In this research the distribution system is assumed to be controlled and managed by a central agent who seeks the best performance for the complete system; therefore, this research does not consider coordination mechanisms to achieve system optimal decision in a decentralized supply chain. Accordingly, pricing and incentive mechanisms that could align the strategic decisions in a supply chain are out of the scope of this work. In addition, a single product is considered in the analysis. Hence, decisions related to the mix of products transported, where a supplier provides complement and substitute products, are not studied.

Generally, logistics decisions are classified according to the planning horizon involved, from longer- to shorter-term into strategic, tactical, and operational (see, for example Ghiani et al., 2004, Simchi-Levi et al., 2003). This research deals with real-time operational decisions. In particular, the possibilities opened by decisions during en-route distribution operations are studied. It is assumed that upper hierarchical (strategic and tactical) decisions about the system configuration are given, e.g., the set of facilities to be refilled from a particular distribution center, and the characteristics of the fleet of vehicles assigned to serve those facilities are not directly considered. Moreover, a single-vehicle approach to the problem is assumed. Hence, strategies that could split deliveries to a particular facility from more than one depot or truck are precluded and left for future research.

In this research, demand processes at different facilities are assumed to be the only source of uncertainty, even though, in real-world applications, particularly in urban areas, there are also uncertainties of traffic conditions that could lead to significantly varying travel times as a consequence of network congestion. In this research, travel time between facilities is assumed to be fixed and known. Moreover, time associated with loading and unloading operations is not considered; thus, the only source of delay in vehicle routes is travel time between facilities. In addition, is assumed that demands are known in probability distribution, and that these demand processes at retailers cannot be affected by the central decision maker.

This work investigates scenarios wherein plans could be continuously updated, based on accurate real-time information about fleet status and inventory levels at each facility. In those scenarios, in which the distribution system could be monitored and controlled on a real-time basis, the main issues studied are: when and how to update distribution plans, based on real-time information. The scenarios are compared with operations wherein some or all these information and communication technologies are not available.

Another important assumption in this research is that daily and weekly cycle operation characteristics are not taken into account, that is the system is assumed to be operating continuously, without interruption. Moreover, labor-related constraints are not considered. In short, it is assumed that the vehicle and all facilities are always in operation; i.e., deliveries can be scheduled at any time, with neither time windows for particular facilities nor restrictions on the number of hours that a driver can operate a vehicle. Therefore, delivery routes are constrained only by the vehicle's capacity to transport products.

The main focus of this research is on problem formulation and design of real-time strategies. The implementation and analysis of the proposed strategies in large size problems is out of the scope of this research and is left as a future endeavor.

1.4. <u>Research Objectives</u>

The main objectives of this research are to:

1. formulate and analyze the online inventory routing problem (OIRP), taking into account explicitly real-time information about fleet status and inventory levels at different facilities;
2. develop operational-control strategies to operate a distribution system in which transportation operations and inventory control are coordinated—the strategies should be tailored to different degrees of availability of real-time information associated to different scenarios in terms of ICT installed;
3. evaluate the benefits of the proposed real-time operational strategies and the value of using real-time information and sophisticated optimization techniques in a centrally operated distribution system, establishing the characteristics of distribution systems for which real-time operational capabilities would be more beneficial, in terms of demand variability, location and distance between facilities, and the relationship between inventory holdings, stock outs, and transportation costs.

In order to address those main objectives, the following specific tasks are considered:

1. Formulate the OIRP under real-time information about inventory levels at different facilities and fleet status (location and load remaining in the vehicle). This formulation should take into account the possibility of modifying delivery plans at any time, based on accurate real-time information about the state of the system.
2. Develop dynamic operational-control strategies or policies to operate a distribution system in which transportation and inventory control decisions are centralized. These strategies determine when and how to update operational plans for scenarios with different degree of real-time information.
3. Formulate local off-line problems and heuristics used to update distribution plans for different operational-control strategies.
4. Propose a methodology to evaluate the performance of the developed dynamic decision strategies.
5. Develop a simulation framework to analyze distribution operations from a central facility to a set of retailers facing stochastic demands under real-time information.
6. Identify evaluation benchmarks in a dynamic environment for one-to-many distribution systems.
7. Evaluate the competitive performance of different strategies under different degrees of sophistication in terms of the ICT used to operate the

system, particularly in the degree that current plans can be updated, quantifying the possible benefits of operations under real-time information.

8. Study the characteristics of distribution-system that could most benefit by implementing sophisticated strategies using real-time information—in terms of: i) the location and distance between facilities; ii) the variability in facilities' demands; iii) the relationship among lost sales, inventory holding, and transportation costs; iv) the presence of disruption in demand patterns; and v) other parameters, such as the ratio between the capacity of the truck and that of the facilities.

1.5. ***Main Contributions***

A primary contribution of this dissertation is to incorporate the processes that generate demands for transportation services in the study of real-time fleet operations. The specific contributions of this research are related with the main task presented, and include:

a) the formulation of the online inventory routing problem (OIRP), taking into account explicitly real-time information about fleet status and inventory levels at different facilities;
b) the development and design of operational-control strategies to operate a distribution system in which transportation operations and inventory control are coordinated tailored to different degrees of availability of real-time information associated to different scenarios in terms of ICT installed;

c) the formulation of local off-line problems and heuristics used to update distribution plans for different operational-control strategies;

d) the development of a methodology and a simulation framework to analyze and evaluate the performance of dynamic decision strategies in the context of one-to-many distribution systems;

e) the identification of evaluation benchmarks in a dynamic environment for distribution operations from a central facility to a set of retailers facing stochastic demands;

f) improve the understanding of the characteristics of distribution-system that could most benefit by implementing sophisticated control-strategies using real-time information, and portray the main expected benefits associated with those strategies.

1.6. ***Dissertation Structure***

This dissertation is organized as follows. After this introductory chapter, which presents the main motivations for research in this area introduces the specific problem studied, and states the research scope, main objectives and expected contributions, chapter 2 presents a review of related research in the literature. This background chapter is divided into three parts. First, a review of single item inventory models with particular attention to results used in this research is presented. Second, routing and scheduling problems are classified, and the problem studied in this research is place in the context of previous research on inventory routing problems (IRPs) and real-time fleet operations. Third, a summary of real-time combinatorial optimization approaches is presented.

Chapter 3 formulates the specific problem studied in this research. In this chapter the problem context and main assumptions are stated. In addition sources of complexity are explained and the two approaches being used to deal with this problem are introduced.

Chapter 4 presents the formulation and design of optimization based strategies, in which the inventory control side of the problem is solved *a priori* and its results are used as target levels when plans are updated. In these strategies an off-line optimization problem is formulated and employed to update routing and inventory allocation plans. It presents different control strategies based on different degrees of real-time information availability for controlling the system. This chapter formulates a local off-line problem, which is used to update distribution plans in all optimization based strategies. Also, it presents an optimization framework for adjusting policy parameters for each strategy.

Chapter 5 is dedicated to the formulation and design of *fixed-tour* based control strategies. In this case the routing side of the problem is solved *a priori*. These strategies are based on *a priori* set of routes to refill retailers with recourse actions depending on different degrees of real-time information capabilities for controlling the system. This chapter presents the rationale and characteristics of these strategies. In addition, it offers an analysis and optimization of policy parameters for each case.

Chapter 6 presents different experiments designed to evaluate and compare the set of proposed real-time policies. It describes the set of scenarios used, including scenarios with steady-state demand processes, and scenarios with sudden changes in demand patterns. Finally, it presents and discusses experimental results.

To conclude, the last chapter presents a summary of the main contributions, findings and results. In addition, it presents a list of possible extensions and directions for future research.

Chapter 2: Background Review and Previous Research

This chapter reviews previous research that relates to the problem studied in this research. This review is divided into three parts. Section 1 reviews previous work on single-item inventory control for a single facility. Section 2 presents a classification of vehicle-routing literature, and the main contributions in inventory-routing and real-time fleet operations are categorized and described. Section 3 presents a summary of real-time combinatorial optimization approaches.

2.1. Previous Work on Single-Item Inventory Models for a Single Facility

This section reviews the main results of research on single-item inventory models for a single facility used in our research. First, main sources of inventory costs are examined. Then the classic Economic Order Quantity (EOQ) model for deterministic demand is presented. Finally, periodic and continuous review models with stationary stochastic demands are reviewed.

2.1.1. Inventory Costs

Before reviewing material on minimizing inventory costs, the main costs associated with inventory are discussed. In general, inventory costs can be divided into three categories: ordering or procurement costs; inventory holding costs; and inventory shortage costs.

Ordering and procurement costs are associated with purchase, transport, and handling of products to a particular facility, and they include fixed costs for each order and variable costs per unit of product. Despite order size, there are fixed costs or setup costs per order. Fixed costs are explained by economies of scale in production and by consolidation of products for transportation and handling.

Inventory holding costs are related to products or material stored per unit of time. Those costs include opportunity costs of capital immobilized in inventory and in warehousing. Among warehousing costs are insurance of items, taxes, rent for warehouse space, maintenance, and handling costs. In addition, there are obsolescence costs in the case of perishable and seasonal goods. Obsolescence costs are not discussed in this review, which assumes a constant value of products distributed.

Shortage or stock-out costs are incurred when demands cannot be met. Shortage could also result in lost sales or backorders. When demands could be satisfied by a competitor, shortage could lead to lost-sales costs, which include profit lost from not selling the product, and could have a negative impact on future demands because of lost of consumer goodwill. On the other hand, when items are difficult to substitute, stock-outs may entail delayed demand satisfaction with associated backorder costs. In some instances, when products supplied are raw material for other production processes, stock-outs may lead to disruption of the entire production line.

Since shortage costs are hard to quantify, some inventory models service use levels of order fulfillment instead. Two common service-level performance measures used are percentage of demand fulfilled from on-hand inventory, also known as *fill rate*, and percentage of time with shortages. In those models orders are placed so that the expected service levels satisfy a specific target value. However, given that any service level used has an implicit shortage-cost value, and that our objective is to study the impact of different distribution strategies on the system performance without imposing restriction on cost trade-offs between different alternatives, service-levels approaches are not used in this research and, consequently, they are not reviewed in this section.

During the past century and particularly since the Second World War, inventory management and the trade-offs between different sources of inventory costs have been extensively studied. Good recent reviews of inventory-control literature can be found in Graves et al. (1993), Axsäter (2000), and Zipkin (2000). The next subsections review the main results for single-facility inventory systems used in our research.

2.1.2. EOQ Model for Constant Demand Rate

In the context of steady-state deterministic demand in a single facility, Harris (1913) introduced a simple model, known as the Economic Order Quantity (EOQ), to study the trade-off between inventory-holding and order costs. The EOQ model assumes that (i) demand is constant at rate $\bar{\mu}$ per unit of time, (ii) shortages are prohibited, (iii) orders are delivered complete and instantaneously with zero lead time, (iv) costs are constant and no discount rate of money is considered, (v) order costs are composed of a fixed part¸ K, per order and a variable part, c, per item ordered, and (vi) inventory-holding costs are accrued at a rate h per unit of time. Based on those assumptions is relatively easy to show that the optimal policy is to order a batch of size Q^*, also known EOQ, when the inventory level reaches zero.

$$Q^* = \sqrt{2 \cdot K \cdot \bar{\mu} / h} \tag{2.1}$$

Even though that is a simple model, its results are very robust with respect to demand rate and cost parameters (see for example Lee and Nahmias, 1993). Moreover, the third assumption could be relaxed to include deterministic lead times, in which case orders should be placed so that they arrive when the inventory level reaches zero.

The next subsection reviews the main results relevant to our research, among models with stationary stochastic demands.

2.1.3. Models with a Stationary Stochastic Demand Rate

One way to classify stochastic inventory models is in relation to their review process, i.e. when and how often are inventory levels reviewed and decisions made to place orders. Using that criterion, inventory models can be classified as either periodic-review or continuous-review models. The next subsections present the main results from the literature used in our research.

2.1.3.1. Periodic-Review Models

In periodic-review models, inventory level is known at the beginning of each period and orders can be made only at those epochs.

The most basic model in this group is the newsvendor or newsboy model in which the number of periods considered is only one. In that model, before a stochastic demand of size D is realized, a decision to stock y should be made. If h is the overage cost per unit of remaining inventory at end of the period, p is the penalty cost per unit of unsatisfied demand, and c is the cost of each unit ordered, then for a given demand δ the total cost at the end of the period is

$G(y) = cy + h(y-\delta)^+ + p(\delta - y)^+$, which is a convex function on *y*, where $(x)^+ = \max\{0, x\}$. It could be shown that the optimal stocking decision should satisfy the optimality conditions given by (see for example Lee and Nahmias, 1993):

$$\Pr(D \le S^*) = \frac{p}{h+p} \tag{2.2}$$

Where *p/*(*h*+*p*) is known as the critical ratio. Then the optimal policy for a newsvendor problem is to order up to S^* whenever the initial inventory level is below S^*, otherwise do not place an order. This is also known as "order up to" policy.

The model could be extended to include fixed-order costs, *K*, which are accrued only when an order is placed. In this case the optimal policy will place orders only when the initial inventory level is below a threshold $s < S^*$, given by the solution of $G(s) = G(S^*) + K$, i.e. orders are placed only when the expected benefits are higher than *K*. This policy is known as (*s*, *S*) policy, and can be stated: whenever the current inventory level is below the reorder point *s*, an order is placed to bring the inventory level to *S*; otherwise, do not place an order.

When multiple periods are considered, Scarf (1960) presented a finite-horizon model with fixed ordering costs and backlogging, and showed that an (*s*, *S*) policy, which might have different parameters at each period, is optimal when the value function is K-convex. For infinite-horizon problems, the optimality of the stationary (*s*, *S*) policy was shown by Iglehart (1963). In multi-period models, the concept of inventory position, defined as the sum of inventory on hand plus inventory in transit (already ordered), minus backorders, is normally used in the definition of inventory policies instead of initial inventory level. Figure 2.1 presents an (s, S) policy with a review period of size T.

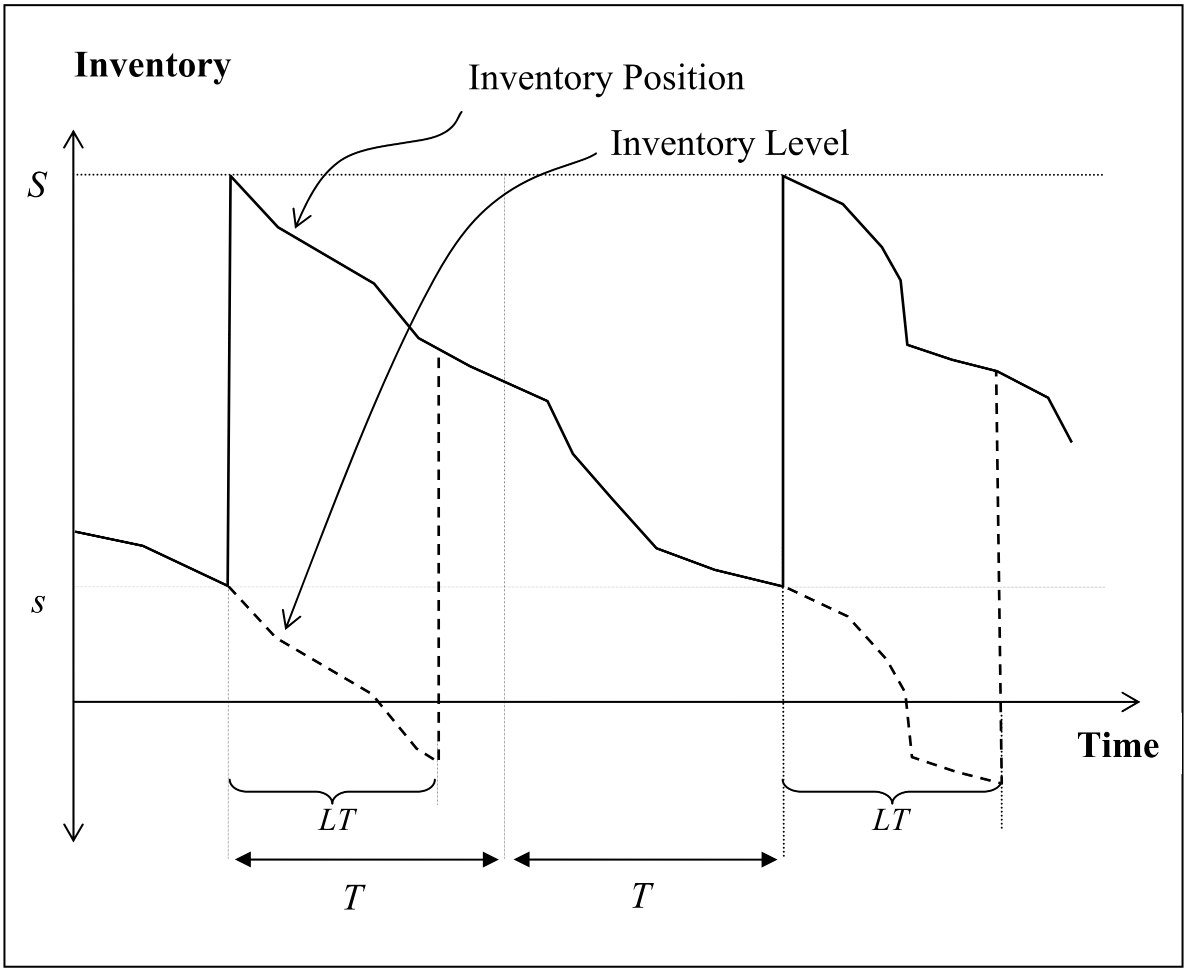


Figure 2- 1: (s, S) Policy under Periodic Review

2.1.3.2. Continuous-Review Models

In continuous-review models, inventory is always assessed and orders can be placed at any time. In particular, inventory replenishment decisions can be made as soon as new demands are served. The optimality results of (s, S) policies have been extended to the continuous review case (Beckmann, 1961, Zheng, 1991).

In order to analyze continuous-review inventory systems, a demand process should be specified. Following the notation in Lee and Nahmias (1993), a demand process can be described by the probability distributions of demand interarrival times and demand size at each demand arrival. In general is assumed that demand interarrival times are IID random variables with a finite mean $1/\lambda$. In addition, demand distribution at each demand epoch is assumed to have a probability mass function (pmf) $\psi(\cdot)$ and cumulative density function (cdf) $\Psi(\cdot)$ with finite mean θ. In the backlogging case, when demand follows a compound Poisson process, i.e. interarrival times are Poisson distributed and arrivals are a Poisson process, it can be demonstrated that the steady state distribution of the inventory position (*IP*) is:

$$\Pr(IP = k) = m_k \bigg/ \sum_{j=s+1}^{S} m_j \qquad \text{, for } j = s+1,\ s+2,\ \ldots,\ S-1 \qquad (2.3)$$

where $m_j = \sum_{k=j+1}^{S} m_k \cdot \psi(k-j)$ is the average number of visits to $IP = j$ during a replenishment cycle. Without loss of generality is assumed that $\psi(0) = 0$, otherwise the demand process can be replaced by an equivalent process with $\tilde{\lambda} = \lambda \cdot (1 - \psi(0))$ and $\tilde{\psi}(j) = \psi(j) \cdot (1 - \psi(0))$ for $j > 0$. In addition, it is required that not all demand sizes be a multiple of some integer larger than one. Based on expression (2.3), the steady state distribution of the inventory level (*IL*) could be computed as:

$$\Pr(IL = k) = \sum_{j=\max\{s+1,k\}}^{S} \Pr(IP = j) \cdot \Pr(D(LT) = j - k) \qquad \text{, for } k \le S \qquad (2.4)$$

where *D*(*LT*) is the total demand during a deterministic lead time of length *LT*. In case of unit demand sizes, i.e. demand according to a Poisson process, it can be shown that in a steady state the inventory position is uniformly distributed in (*s*+1, *s*+2, ..., *S*). That result can be extended to different IID interarrival time distributions, as long as the demands are unitary (Richards, 1975).

In scenarios with lost sales, results are harder to obtain, because when shortages occur, lost demands do not change the inventory position. However, results can be obtained using a standard simplifying assumption that the number of outstanding orders can be at most one, i.e. $s < Q = (S - s)$. Because of that simplification, inventory level and inventory position are always the same before an order is placed, which can be used as a renewal epoch. In that case, if the total demand during a time unit is approximated, using a normal distribution with mean μ and standard deviation σ, then demand during a deterministic lead time, $D(LT) \sim N(\tilde{\mu}, \tilde{\sigma}^2)$ with probability density function (pdf) $f_{D(LT)}(\cdot)$, where $\tilde{\mu} = LT \cdot \mu$ and $\tilde{\sigma} = \sqrt{LT} \cdot \sigma$. Based in these simplifications the expected cost per unit of time can be expressed as:

$$EC = \frac{E(\text{Cost per Cycle})}{E(\text{Cycle Length})} = \frac{h \cdot \left[\frac{Q}{2} + E\left(s - D(LT)\right)^{+}\right] + p \cdot E\left(s - D(LT)\right)^{-}}{\frac{1}{\mu}\left[Q + E\left(s - D(LT)\right)^{-}\right]} \quad (2.5)$$

where:

$$E\left(s - D(LT)\right)^{-} = \int_{s}^{\infty}\left(1 - \Phi\left(\tfrac{x - \tilde{\mu}}{\tilde{\sigma}}\right)\right)dx = \tilde{\sigma} \cdot G\left(\tfrac{s - \tilde{\mu}}{\tilde{\sigma}}\right) \quad (2.6)$$

$$E\left(s - D(LT)\right)^{+} = E\left(s - D(LT)\right) + E\left(s - D(LT)\right)^{-} = s - \tilde{\mu} + \tilde{\sigma} \cdot G\left(\tfrac{s - \tilde{\mu}}{\tilde{\sigma}}\right) \quad (2.7)$$

Therefore:

$$EC = \frac{h \cdot Q\left[\frac{Q}{2} + s - \tilde{\mu} + \tilde{\sigma} \cdot G\left(\frac{s - \tilde{\mu}}{\tilde{\sigma}}\right)\right] + \mu \cdot p \cdot \tilde{\sigma} \cdot G\left(\frac{s - \tilde{\mu}}{\tilde{\sigma}}\right)}{\left[Q + \tilde{\sigma} \cdot G\left(\frac{s - \tilde{\mu}}{\tilde{\sigma}}\right)\right]} \quad (2.8)$$

where $G(\cdot)$ is the loss function, which gives the expected number of units of demand lost as a function of the initial inventory level when demand is distributed normal standard, i.e. $G(y)=\int_y^\infty (x-y)\phi(x)dx$, where $\phi(\cdot)$ is the pdf of normal standard distribution.

As mentioned by Axsäter (2000, pp.73), expression (2.8) can be further simplified by neglecting the second term in the denominator and the same term in the first term in the numerator. In addition, the parameters should satisfy $p\mu > (h \cdot Q/2)$, otherwise, it would be not profitable to operate the system.

The optimization of the parameters of an (*s*, *S*) policy can be done using iterative procedures (see Axsäter, 2000), or using an efficient optimization procedure developed by Zheng and Federgruen (1991).

This section reviewed the most important results on inventory models for single item and single facility used in this research. The next section reviews the main literature in fleet routing and scheduling, particularly previous work on inventory-routing problems.

2.2. Background in Routing and Scheduling Problems

This section reviews previous work in routing and scheduling of vehicles relevant to this research. First, a classification of research on routing and scheduling problems is offered. The second subsection reviews and discusses previous research in inventory-routing problems (IRPs), i.e. vehicle routing in which inventory replenishment decisions are combined. The third subsection presents previous research in real-time fleet operations.

2.2.1. Classification of Fleet Routing and Scheduling Problems

Fleet routing and scheduling problems have received extensive and fruitful attention since the late nineteen fifties. Broad overview can be found in Bodin et al.(1983), Christofides (1985), Golden and Assad (1988), Fisher (1995), and more recently Toth and Vigo (2002). Problems found therein can be classified according to their main characteristics, as shown in Table 2.1 (Bodin et al., 1983, Assad, 1988, Psaraftis, 1988).

Table 2- 1: Classification of Fleet Routing and Scheduling Problems

Characteristics	Options
1. Fleet Size	Single vs. Multiple vehicles Fixed vs. Variable fleet size
2. Fleet Type	Homogeneous vs. Heterogeneous vehicle types Single vs. Multiple Compartments
3. Vehicle Terminals	Single vs. Multiple terminals (or depots)
4. Nature of Demands	Deterministic (known) vs. Stochastic demands Partial satisfaction of demand allowed vs. not allowed Customers with different priorities vs. same priorities
5. Location of Demands	At nodes vs. On arcs (or mixed)
6. Information on Parameters (demand and travel times)	Deterministic parameters vs. Uncertain (Stochastic) parameters All data known in advance (static problems) vs. Real-time inflow of data (dynamic problems)
7. Underlying network	Undirected vs. directed (or mixed) Euclidean vs. No-euclidean distances Deterministic vs. Stochastic travel times
8. Route Restrictions	Vehicle maximum capacity vs. vehicles with unlimited capacity Max route length (or time) vs. not imposed (unlimited) Max number of customer per route vs. not imposed (unlimited) Loading restrictions/equipments vs. unrestricted Vehicle type/site dependencies
9. Operations	Pure pickups or pure deliveries vs. mixed (pickups and deliveries) Split deliveries allowed vs. Split deliveries disallowed Truckloads (TL) vs. Less than truckloads (LTL) Single commodities vs. Multiple commodities
10. Costs	Variable or routing costs (per distance) Fixed operating costs (per vehicle in the fleet) Opportunity (penalty) costs associated to unserved demands
11. Objectives	Minimize total routing cost Minimize sum of fixed and variable costs Minimize number of vehicle required Maximize utility function based on service or convenience Maximize utility function based on customers priorities

Using the above classification scheme, the online inventory-routing problem (OIRP) studied in our research can be categorized in terms of the vehicle fleet used as a single vehicle with a single terminal (depot). The demand locations are known and occur at facilities that are represented by nodes of the underlying network. Travel times among those nodes are deterministic and known. Moreover, they are proportional to the distance traveled, according to a Euclidean metric. The demands at those nodes are stochastic and dynamically revealed in a real-time fashion. In addition, those facilities should be repeatedly refilled over time, in contrast with problems where immediate visits do not have a direct impact on future visits. Moreover, facilities might have different priorities, i.e., inventory holding and shortage costs can be different at different facilities. In terms of route restriction, truck capacity is the only route constraint considered. The type of operations considered is pure deliveries of a single commodity, in which visits to different facilities, can be combined in the same truck tour with the possibility of using truck loads (TL) and/or less than truck loads (LTL), and splitting deliveries to a particular customer into different tours. Finally, in term of objectives considered, an operational planning perspective is taken, in which long term decisions—such as fleet size, set of customers to be served from a particular depot, and assignment of vehicles to a set of customers—are considered as given. Therefore, at this operational level, the cost objectives to minimize are transportation costs proportional to travel distance, and inventory holding and shortage costs, which can be different among facilities.

In the next subsection the main references for inventory-routing problems (IRPs) studied in the literature are described and categorized.

2.2.2. Inventory Routing Problems (IRPs)

In general, inventory-routing problems (IRPs) are long-term dynamic control problems. Those types of problems are very difficult to formulate. In large practical cases, which are common in real-world applications, IRPs are almost impossible to solve to optimality, even with accurate data. Therefore, most approaches that deal with real-world problems address them in a short-term planning horizon, where the long-term effects are included, using some approximation, but some of the more complex features of IRPs are ignored.

Even though previous research on IRPs share some common elements, most of the problems presented in the literature address systems having different characteristics. A detailed review of the works is presented in Baita et al. (1998), Campbell et al. (1998), Campbell and Savelsbergh (2002), and Kleywegt et al. (2002). In Table 2.2, previous research is classified according to the following specific characteristics:

a) time horizon considered, which can be single period or multiple periods ─either finite or infinite number of periods─ with either discrete or continuous time;
b) demands, which can be deterministic or stochastic. In the deterministic case, demand can be constant or time-varying. In the stochastic case, demands can be either stationary or non-stationary;
c) objective can be profit maximization, or minimization of costs, which can include transportation costs (C_{Tr}), inventory holding costs (C_{IH}),inventory stock-out costs (C_{ISO}),and crew-associated costs (C_{Crew});

d) fleet size, which can be limited to a fixed number of vehicles (single or multiple vehicles), or variable numbers of vehicles;

e) route constraints, which can be related to vehicle capacity and/or maximum length of route, either an upper bound in route distance or time;

f) number of visits per vehicle tour, which can be to a single facility or to multiple facilities;

g) routes can be fixed-static, variable-static, or dynamic. In fixed-static, facilities are always visited in the same sequence. In variable-static, a new route is obtained according to the state of the system and then implemented without en-route modifications. In dynamic, en-route modifications are allowed, either changing the sequence of facilities to be visited, and/or the amount to be refilled at each facility in the tour;

h) information about inventory levels at each facility used to set up plans can be on-line accurate information about the state of the system, or forecasted information based on expected consumption since the previous visit to each facility. Some models assume systems with forecasted information about inventory levels that also receive on-line information about stock outs. Those models are denoted Forecasted & SO. That distinction is relevant in stochastic demand models, since in deterministic demand models, the state of the system is known at any time; and,

i) plan updates considered, which can be event driven, time driven, or mixed (event and time) driven. Among time driven updates the most common are rolling horizon (RH) approaches, in which plan updates can take place at each period or at regular intervals of time. In some cases a plan update is imputed, since the authors did not discuss how their formulations should be implemented.

After classification, some formulations and solution methods that are representative of the approaches proposed to deal with IRPs are highlighted.

Table 2- 2: Previous Research in Inventory-Routing Problems (IRPs)

Reference	Time Horizon	Demands	Objective to Min	Fleet size	Route Constraint	No. Visits	Routes	Information	Plan Update(*)
Bell et al.(1983)	Finite Discrete	Constant Deterministic	- Profit	Limited	Veh. Cap.	Multiple	Var.-Static	N/A	Periodically (RH)
Federgruen & Zipkin (1984)	Single Period	Stochastic	$C_{Tr} + C_{IH} + C_{ISO}$	Limited	Veh. Cap.	Multiple	Var.-Static	Initial State	N/A
Golden et al. (1984)	Finite Discrete	Non-Stationary Stochastic	$C_{Tr} + C_{ISO} + C_{Crew}$	Unlimited	Veh Cap-Time	Multiple	Var.-Static	Forecast & SO	Periodically (RH) &SO
Dror & Ball (1987)	Finite Discrete	Non-Stationary Stochastic	$C_{Tr} + C_{ISO}$	Limited	Veh. Cap.	Multiple	Var.-Static	Forecast	Periodically (RH) & SO
Chien et al. (1989)	Single Period	Deterministic	-Rev.+ $C_{Tr} + C_{ISO}$	Limited	Veh. Cap.	Multiple	Var.-Static	N/A	N/A
Anily and Federgruen (1990)	Infinite Discrete	Constant Deterministic	$C_{Tr} + C_{IH}$	Unlimited	Veh. Cap.	Multiple	Fixed-Static	N/A	N/A
Viswanathan & Mathur (1997)	Infinite Discrete	Constant Deterministic	$C_{Tr} + C_{IH}$	Unlimited	No & Veh Cap	Multiple	Fixed-Static	N/A	N/A
Bard et al. (1998a; 1998b)	Finite Discrete	Stationary Stochastic	$C_{Tr} + C_{ISO}$	Limited	Veh Cap-Time	Multiple	Var.-Static	Forecast & SO	Periodically (RH) & SO
Reiman et al. (1999)	Infinite Continuous	Stationary Stochastic	$C_{Tr} + C_{IH} + C_{ISO}$	Single	Veh. Cap.	Single and Multiple	Fixed-Static	On-Line	Continuous
Rabah & Mahmassani (2002)	Finite Discrete	Stationary Stochastic	$C_{Tr} + C_{IH} + C_{ISO}$	Single	Veh. Cap.	Multiple	Var.-Static	On-Line	Only at SO
Bertazzi et al 2002(2002)	Finite Discrete	Varying Deterministic	$C_{Tr} + C_{IH}$	Single	Veh Cap	Multiple	Var.-Static	N/A	N/A
Kleywegt et al. (2002)	Infinite Discrete	Stationary Stochastic	-Rev.+ $C_{Tr} + C_{IH} + C_{ISO}$	Limited	Veh. Cap.	Single	Var.-Static	On-Line	Each Period
Campbell & Savelsbergh (2004)	Finite Discrete	Constant Deterministic	C_{Tr}	Limited	Veh Cap-Time	Multiple	Var.-Static	N/A	Periodically (RH)
Kleywegt et al. (2004)	Infinite Discrete	Stationary Stochastic	-Rev.+ $C_{Tr} + C_{IH} + C_{ISO}$	Limited	Veh. Cap.	At most 2	Var.-Static	On-Line	Each Period
Adelman (2004)	Infinite Discrete	Stationary Stochastic	$C_{Tr} + C_{IH} + C_{ISO}$	Limited	Veh. Cap.	Multiple	Var - Static	On-Line	Each Period
Aghezzaf et al. (2006)	Infinite Continuous	Constant Deterministic	$C_{Tr} + C_{IH} + C_{Crew}$	Limited	Veh. Cap.	Multiple	Fixed-Static	N/A	N/A
Savelsbergh and Song (2007)	Finite Continuous	Constant Deterministic	C_{Tr}	Limited	Veh Cap	Multiple	Var.-Static	Initial State	Periodically (RH)

*: (RH= Rolling-horizon)

Bell et al. (1983) present a successful implementation of a decision-support system for the distribution of industrial gases, in which vehicle routes are designed solving a deterministic vehicle-routing problem (VRP) based on forecasted consumptions at each facility.

Federgruen and Zipkin (1984) formulate an IRP for a single day as a non-linear integer program. Their model assumes that the demand at the customers' site is stochastic, the depot has limited capacity, and cost functions are non-linear. That is a distinctive feature of the model, since depots with limited capacity are not normally included in other models. The non-linear cost function includes transportation costs, inventory holding costs, and shortage costs for each customer. The objective is to construct delivery routes that minimize the total cost incurred on the day under consideration. The solution approach starts with a feasible solution. Then, the procedure iteratively exchanges customers among routes. The main drawback of that formulation is: the consequences of today's decisions on following days are not considered.

Golden et al. (1984) also formulate a single day problem with the objective being to minimize total costs. The objective is sought while maintaining an adequate level of inventory, with a pre-specified target value, at each customer. To solve the problem, they use a heuristic that selects the set of customers to be visited based on an urgency measure for each customer. The urgency is determined by the ratio of the inventory level to inventory capacity. Customers who are below a set target level are scheduled to be visited. The solution is obtained by iteratively incorporating clients to a traveling salesman problem (TSP). Clients are incorporated until the vehicle transportation capacity is reached and no more clients need to be visited.

Even though the pre-specified target inventory levels for each customer is intended to take into account the long-term effect of short-term decisions, the main limitation of those two pioneer works is the limited planning horizon. In the nineteen nineties, as a result of increasing computer capabilities, new approaches to overcome that drawback were developed. Those approaches deal with long-term problems, and are described below.

Campbell and Savelsbergh (2004) propose a two-step integer programming solution approach based on a rolling-horizon framework. Even though they assume deterministic consumption rates at customers' sites, they consider safety stocks to handle the stochastic nature of demand. Additionally, they assume that an unlimited amount of product is available at the depot and they do not incorporate the inventory holding costs either at the depot or at retailers' sites.

Their problem is formulated as a two-phase Integer Program (IP). In Phase I, they determine when to deliver and how much to supply on each visit to each customer. In Phase II, they solve the following problems. First, they determine delivery routes for each day, which involves solving a vehicle-routing problem with time windows (VRPTW). Second, they construct vehicles routes and schedules for two consecutive days. The solution of Phase II is constrained: the quantities delivered to each customer should be equal to or greater than the solution provided by the first phase. Finally, they solve Phase I for one month and, using this solution, they solve Phase II for only the first two days.

This problem is combinatorial, because there are a large number of possible delivery routes, and the problem needs to be solved for a long planning horizon. In order to make this IP computationally tractable, they make additional assumptions. One of them is to consider only a small, but good, set of delivery routes. Such clusters are normally selected as a prepossessing step. Another assumption is that time periods towards the end of the planning horizon are aggregated. An additional assumption to reduce the number of integer variables is to relax integrality restrictions on the variables representing weekly decisions. Moreover, they also reduce the set of customers to those that require a delivery in the very short term, i.e. the next few days; customers with large impact on efficiency of the schedule, either with high demand or be being very distant from the depot; and customers that, though not require an immediate delivery, are near or in the same cluster as the first two types.

Bard et al. (1998a, 1998b) and Jaillet et al. (2002) study a similar two-phase approach. The main differences are: customer inventory levels are not continuously reviewed, and satellite facilities are considered. Satellite facilities are additional depots where vehicles can be reloaded, avoiding the necessity to return to the central depot. Another difference is that the objective function combines two criteria. First, the marginal costs associated with visiting customers on a different day than the optimal day between deliveries. And second, the minimization of daily transportation cost.

The first criterion, incremental cost, is calculated using the approximation presented in Jaillet et al. (2002). This is computed by first obtaining the optimal number of days between deliveries for each client. Then the cost of serving each client on a day different than the optimal day is computed as an incremental cost. That value is calculated, assuming that future deliveries are maintained according to the optimal interval.

As a second criterion in the objective function, they take into account routing customers within a given day. That is, this criterion attempts to minimize daily transportation cost.

The problem is solved in two steps, using a rolling-horizon framework. First, one-year optimal delivery days are computed for each customer. Second, an assignment problem is solved for the first two weeks. The second problem is solved considering transportation costs and incremental costs associated with visiting customers on days different from the one obtained in the first step. Finally, they implement the solution for the first week only.

Kleywegt et al. (2002, 2004) formulate a general IRP as a discrete time Markov decision process (MDP). They make six basic assumptions: (i) that inventories at customers' sites can be measured once a day at no additional cost; (ii) that unsatisfied demands are lost, i.e., not backlogged; (iii) that inventory holding costs are incurred at customers' sites for each unit of inventory per day; (iv) that the supplier obtains revenues every time he/she dispatches products to a retailer proportional to the quantity delivered; (v) that the depot has an unlimited supply of the product and its inventory holding costs are not considered; and (vi) that the supplier knows the cost associated with each decision before hand (i.e., there are no uncertainties with respect to transportation costs associated with each possible policy and to inventory holding costs at each customer site). Moreover, the supplier knows the values associated with shortage penalties.

The objective function is to maximize the discounted sum of net benefits over an infinite horizon. At each stage, net benefits take into account: the revenues obtained from the quantities delivered to each customer, the transportation costs of product from depot to customers, the inventory holding costs at each customer's site, and the expected costs associated with expected shortage penalties.

The Markov Decision process (MDP) associated with this problem is extremely hard to solve when there are more than four customers and a limited number of vehicles. To overcome the problem, the authors develop an approximation technique for the optimal value function. The approximation is based on decomposition per customer. The decomposition is easily computed given the smaller state space. Then an approximate value function is obtained by optimally assigning the fleet capacity by solving a non-linear knapsack problem.

The authors show near-to-optimality results for small problem-instances and better solutions than other approximate policies in larger problem-instances. The small problem-instances considered have up to five customers and have demands taking less than 10 discrete values per customer; the larger ones, up to 60 consumers and 30 vehicles but only up to two levels of demand per consumer.

Adelman (2004) also formulate a similar stochastic inventory-routing problem as a discrete time Markov decision process (MDP). The main difference between his work and Kleywegt et al. (2002, 2004) is that he presents a math-programming approach which uses dual prices of linear program relaxations to approximate the value function, instead of using a simulation-based approach. As in most approximate DP approaches, only instances with small state spaces can be solved. In addition, in his problem setting, lost sales are considered, and no constraints are explicitly imposed on the number of facilities visited per tour. Adelman presented results in which his approach outperforms Kleywegt et al.'s direct shipping policy.

These DP formulations are very interesting from a formulation standpoint. However, it is extremely difficult, normally impossible, to solve even for small problem-instances. The DP method "provides more benefit if the available information about the future is more accurate" (Kleywegt et al., 2002, p. 115). In reality, information about the future is not very accurate, since information about demand distributions is not exact. Therefore, in real-world applications the benefits of DP approaches are expected to be lower than those presented in simulations, where demand follows exact, known demand distributions.

The main difference between the problems found in the literature and the one addressed in this research is that all previous works have considered static vehicle routes, i.e. vehicle routes are not modified after they started until they are completed. However, with modern information and communication technologies, it would be possible to establish mid-route communication with the drivers to modify their plans (Regan et al., 1995, Regan et al., 1996a).

To summarize thus far, the main decisions addressed in IRPs have been established, the extension under real-time information has been introduced, and the main previous primary research in the area has been reviewed. The following section analyzes previous research on fleet operations under real-time information—research that is relevant to the IRP addressed in our research.

2.2.3. Real-Time Fleet Operations

Real-time techniques are important in a context where information about the state of the system is gradually revealed during the operation and cannot be accurately predicted in advance. That area of research for fleet management is relatively new, Psaraftis (1988) points out that by the end of the nineties not much had been published on real-time vehicle- routing problems. For recent surveys on dynamic vehicle-routing problems and related routing problems see Psaraftis (1995), Powell et al.(1995), Bertsimas and Simchi-Levi (1996), and Powell (2003).

The main two approaches followed to deal with operations under real-time information have been rolling-horizon methods and stochastic methods to address an infinite-horizon system under steady-state conditions.

The first approach uses a rolling-horizon framework (see for example Winston, 1994), in which a new problem-instance is solved as new information become available. But instead of implementing the solution for the complete planning horizon, the solution for only the first part of the planning period is implemented, and the process is repeated. During the time a new solution is computed the vehicles continue moving and new events could unfold; so, there is a trade-off between the time required to obtain a new solution and the quality of the solution (Ichoua et al., 2000). Moreover, given that the problems are NP-Hard, optimal solutions would lead to long computation times, which would make them impractical for real-size problems. Hence, normally fast heuristics have been implemented that take advantage of local operations, such as insertion.

One interesting implementation of the rolling-horizon approach to the general dynamic VRP with a time window is proposed by Gendreau et al. (1999) and extended by Ichoua et al. (2000) to include diversions. They propose a general heuristic strategy in which a tabu search procedure is continuously running, trying to improve the current solution, and new requests are handled with a faster local-search heuristic for inserting new demands. That strategy allows them to take acceptance or rejection decisions in a fixed amount of time. One of the interesting features of the implementation is the time projection used to update the state of the system. It is used to correctly reflect the initial conditions on the problem to be solved when a new demand is known. Instead of considering the actual state of the system at time t in the insertion procedure, they project it to a time $(t+\partial t)$ where ∂t is the time required in the optimization procedure.

In the context truckload (TL) pick up and delivery problems, Regan et al. (1995, 1996b) propose and investigate various local rules for the dynamic assignment of vehicles to loads under real-time information. The rules are easy to implement and fast to execute, but they could be improved, using formal optimization techniques. Yang et al. (1998, 2004) extend that work to consider re-optimization real-time policies; the main drawback of this approach is the computation time required, which limits the applicability of the approach to limited-size problems. To overcome those difficulties, Kim et al. (2002b) consider a two-phase optimization approach: in a first step, new demands are inserted if they are feasible to the truck with minimum insertion cost; then, in a second step, re-assignments of loads between different trucks are considered, but restricted to a subset of vehicles to maintain computation times stable.

Another application relevant to our research is the dynamic allocation of inventories for a fixed delivery route presented by Kumar et al. (1995). They compare static-allocation policies—where the replenishment quantities at each retailer are determined simultaneously for all retailers—and dynamic-allocations policies—where replenishment quantities are determined sequentially, upon arrival of the delivery vehicle at each retailer, on the basis of the inventory level at subsequent retailers in the fixed route. They show that even under the "dynamic-allocation assumption"—where the dynamic-allocation problem at each retailer is relaxed, allowing negative replenishment quantities—dynamic policies yield lower expected cost per replenishment and allocation cycle than static policies.

A second, more ambitious approach is the use of stochastic methods, in which, instead of reacting to new information, the future is forecast. Among stochastic methods there are two main categories: Stochastic Programming and Markov Decision Processes. The main literature in this area can be found in Powell et al. (1995), Powell (2003), and Gans and Van Ryzin (1999). Unfortunately, those approaches have computation time that grows exponentially with the size of the problem, making them more suitable for *a priori* plans than for real-time re-optimization.

2.3. Background in Real-Time Combinatorial Optimization

One of the characteristics of operations in real time is that information about problems needing to be solved by decision makers is dynamically revealed. That contrasts with traditional static optimization, in which it is normally assumed that all relevant data to solve a problem-instance is known in advance. In real-time operations decisions should be made without complete information about future outcomes, and since those outcomes are not known in advance, they could only be considered in a probabilistic sense at any decision epoch. In addition, plans or policies can be updated with online information about the state of the system. For that reason, the implementation of an operational-control strategy should establish (i) when, i.e. what events should trigger plan updates, and (ii) how to update plans.

In terms of plan-update epoch decisions, the most common operational strategies are (i) event-driven strategies, in which plan-updates are triggered whenever the state of the system satisfies certain criteria, (ii) time-driven strategies, in which plans are updated at regular time intervals, for example periodic review strategies on inventory control, and (iii) mixed strategies, in which event- and time-driven strategies are considered together.

With respect to how plans are updated, planning decisions can be classified as (i) reactive, by which the previous plan is locally modified to accommodate recent events, (ii) incremental, by which the previous plan is more than slightly modified, and (iii) deliberative (or re-plan), by which a completely new plan is built from scratch; this is normally performed when the state of the system significantly deviates from its forecast (Seguin et al., 1997, Grötschel et al., 2001b). In general, the recommended type of planning decision depends on a trade-off between the benefits of fast reaction to unpredicted events and the quality of the resulting solutions. Normally, the more time spent on evaluating alternatives, the better the plan selected.

In order to evaluate and compare different real-time operational strategies, there have been two main approaches proposed in the literature: competitive analysis and discrete-events simulation.

Competitive analysis is a form of worst-case analysis, in which the evaluation of each decision is based on the worst-possible sequence of events resulting from that decision. The main limitation of that approach is that, in many cases, results are unduly pessimistic. Even though some modifications have been presented to overcome that limitation, it is still complicated to obtain meaningful results for combinatorial problems. In addition, competitive analysis does not take into account real-time requirements of real-world systems in which the trade-off between solution quality and speed is a relevant issue. A detailed overview of competitive analysis and extensions is presented in Grötschel et al. (2001b).

The second approach to evaluating and comparing real-time strategies is discrete-events simulation, in which the operation of the system is mimicked under different operational strategies. Those experiments are conducted for different realizations of the same stream of events over long periods of time, and statistics about the performance of the system using different criteria are gathered (see for example Law and Kelton, 2000). The main advantages of simulation experiments are: they provide results for analytically intractable systems, and they provide a full range of statistics about system performance.

Chapter 3: Problem Definition and General Approach

This chapter provides a detailed formulation of the online inventory routing problem (OIRP) studied in this research and describes the general approach used to solve the problem. Section 1 states the problem context and specific assumptions. Section 2 introduces the main notation used and formulates the OIRP as a real-time combinatorial optimization problem. Section 3 presents the major sources of complexity, and the general approach used to deal with the OIRP.

3.1. Problem Context and Main Assumptions

The general characteristics of the problem studied were introduced in Chapter 1. This section presents a specific definition of the OIRP and the main assumptions related to thereto.

In the OIRP a two-echelon distribution system for a single product from one to many facilities is considered. The system is composed of a single-vehicle fleet with limited capacity, a single depot that keeps an infinite supply at no cost, where the vehicle is reloaded, and a set of N retailers which face independent and stochastic demand processes and which need to be repeatedly refilled over time. The vehicle moves products from the depot to the retailers, and can consolidate loads to different facilities on the same route. In addition, the vehicle can reallocate loads among retailers while en route, but transshipments are not allowed. Consequently, after products have been delivered to a particular facility, they cannot be reclaimed and reassigned to a different facility.

This research assumes that demands are the only source of uncertainty. Variability in travel times because of incidents, congestion in the network, or possible vehicle breakdowns are not considered. Moreover, it is assumed that loading and unloading time is negligible. Yet, real-time operational capabilities might be also provide benefits in those circumstances, allowing the operator of the system to respond faster to possible contingencies.

The system is assumed to be operating continuously, without interruption, that is, daily and weekly cycle operation characteristics are not taken into account. Moreover, labor-related constraints are not considered. In short, it is assumed that the vehicle and all facilities are always in operation; i.e., deliveries can be scheduled at any time, with neither time windows for particular facilities nor restrictions on the number of hours that a driver can operate a vehicle. Therefore, delivery routes are constrained only by the vehicle's capacity to transport products.

The system is operated by a central decision maker, whose objective is to move inventories in the system so as to maximize profit in the long-run. It is assumed that the demand processes at retailers cannot be affected by the decision maker decisions; that is, short-term pricing incentives are not considered. Therefore, the problem is equivalent to minimizing the expected total operating cost per unit of time.

The operating costs considered consist of transportation, inventory holding, and lost-sales penalty costs. Transportation costs are assumed to be only proportional to the total distance traveled by the vehicle. That is consistent with a hierarchical decision-making perspective, in which strategic and tactical decisions, such as the fleet size for a given day, are fixed and given. Accordingly, in the short-term operational problem studied here, fixed-fleet costs are considered to be sunk costs; hence, the relevant decision become how much to use those resources. In addition, transportation costs per unit of distance will not depend on the amount of load transported by the vehicle, which is a common assumption in the vehicle-routing literature (Christofides, 1985, Golden and Assad, 1988, Toth and Vigo, 2002). In relation to inventory costs, each retailer i accrues inventory-holding cost, h_i, per unit of inventory on hand per unit of time, and the demand during stock-out is lost with an associated penalty cost, p_i, for each unit of demand lost per retailer i. Those costs parameters are considered to be known and fixed for the planning horizon.

The central decision maker operates the system with real-time information about the complete state of the system. In other words, the central decision maker has accurate real-time knowledge of all local inventory levels, the location of the vehicle, and the load remaining in the vehicle. The decision maker also has real-time two-way communication with the truck driver and can update truck plans at any time. However, if the vehicle is traveling when an update decision is made, a time lag is imposed before the new plan can be implemented.

The main decisions available to the decision maker are related to truck plans and are defined by i) the sequence of facilities to be visited, ii) departure time from the depot, which is the only place where the truck can be idle, and iii) the amounts to be delivered (or picked up, in the case of the depot) to subsequent facilities on the route. Therefore, the main decision alternatives at a given plan update epoch are:

a) modify the set and/or the sequence of subsequent customers on the planned routes,
b) divert a truck from its current destination to a different facility, if the truck is traveling,
c) adjust amounts to be delivered to subsequent facilities on the route, or
d) change the amount of time spent at the depot.

Even though the idealized problem described here represents a simplified version of real-world logistic-distribution problems, in which some issues are deliberately ignored, its analysis can provides relevant insights about how to use real-time information and control capabilities in distribution operations, and the associated benefits therefrom.

3.2. __Problem Formulation__

This section formally presents the online inventory-routing problem (OIRP) being investigated. First, the main notation and parameters used to describe the problem are introduced. Second, the main variables and additional notation used to describe the OIRP are presented. Third, main constraints to be satisfied in the operation of the system are formally stated. Finally, the objective function of the OIRP is specified.

3.2.1. Preliminaries and Problem Parameters

In order to present the OIRP, the following general notation is used to describe the elements of the system. The set of retailers is designated as $\Im$, $\Im = \{1, 2, ..., i, ..., N\}$, and the set of all facilities (depot and retailers) as $\Im_0$,

$\Im_0 \equiv \Im \cup \{0\}$. Those *N*+1 facilities are denoted by sub-index *i*=0,1, 2, …, *N* (sub-index 0 is for the depot) and are located in a bounded subset in the Euclidean space. Those locations are denoted *l*(*i*) for $i \in \Im_0$. The function $d(\cdot,\cdot)$ gives the Euclidean distance between two facilities or between a facility and the vehicle location. Each retailer *i* has a maximum capacity to store inventory, κ_i, measured in the units of the single product considered. In addition, the vehicle has limited capacity, Υ, measured in the same units, and its assumed to travel at constant speed according to the Euclidean metrics. Without loss of generality, the vehicle speed is assumed to be one.

Each retailer *i* serves an independent demand process. In general, it is assumed that each facility serves a compound Poisson demand process, in which customer arrivals to retailers follow Poisson processes, and customers' demand sizes are independent discrete random variables. Demand processes have associated arrival rates $\lambda_i(t)$ for retailer *i* at time *t*, and associated probability mass function (pmf) $\psi_i^j(t)$, for the probability that a customer arriving at time *t* to retailer *i* has a demand size equal to *j*. In addition, unless otherwise noted, customer demand sizes are assumed to be Poisson distributed with mean $\theta_i(t)$. Thus the expected demand per unit of time at retailer *i* at time *t*, $\mu_i(t)$, can be calculated as $\mu_i(t) = \lambda_i(t) \cdot \theta_i(t)$. In these demand processes, arrivals times and demand sizes are denoted $\tau_{i,m}$ and $\delta_{i,m}$ respectively, for the mth customer arrival to facility *i*. Thus, the total number of customer arrivals to retailer *i* that have occurred by time *t* is

$A_i(t) = \max\{m \geq 0 : \tau_{i,m} \leq t\}$, and the total demand at customer *i* until time *t* is $D_i(t) = \sum_{m=1}^{A_i(t)} \delta_{i,m}$, including satisfied and lost demands.

The state of the system at time *t*, $X(t)$, can be described by the following parameters: (i) inventory levels at time *t*, $\iota(t) = (\iota_1(t),...,\iota_i(t),....,\iota_N(t))$, where $\iota_i(t)$ is the inventory level at facility *i* at time *t*, (ii) location of the truck at time *t*, $\ell(t)$, and (iii) load remaining in the truck at time *t*, $\nu(t)$. Hence, the state of the system at time *t* can be expressed as:

$$X(t) = [\iota(t) \quad \ell(t) \quad \nu(t)] \tag{3.1}$$

The decision maker can update plans at any epoch *t* based on $X(t)$ and past events, but without knowledge of future events. Plan updates are implemented immediately unless the vehicle is moving, in which case a time lag—between the epoch when a decision to update a plan is made and the plan is implemented—is considered. This is modeled using a time projection, which takes into account the time from the moment the decision to update the current plan is made until the new plan begins to be executed. Hence, instead of considering the actual state of the system at time *t* in the solution procedure, the state of the system is projected to a time $(t + \partial t)$, $\tilde{X}(t + \partial t)$, assuming expected consumption rates and truck current speed and destination, where ∂t is the projection time, which includes any solution procedure used to update plans and the time required for the driver to modify his current destination.

In the OIRP, there are three sets of cost parameters: (i) transportation cost per unit of distance traveled by the vehicle, *TC*, (ii) inventory holding costs at each retailer *i*, h_i for retailer *i*, and (iii) penalty associated with each unit of demand lost during stock-out, p_i is the at retailer i .

3.2.2. Decision Variables

In this OIRP system, the only decisions available to the decision maker are related to truck plans, and can be summarized as, when, and how truck plans are updated. As mentioned, a plan or policy, π , can be specified by the sequence of facilities to be visited, $\mathbf{s} = \begin{bmatrix} s_1 & s_2 & \dots & s_L \end{bmatrix}$, amounts to be delivered, $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \dots & q_L \end{bmatrix}$, and arrival times, $\mathbf{t} = \begin{bmatrix} t_1 & t_2 & \dots & t_L \end{bmatrix}$, to each one of those facilities, in which L is the length of the planning horizon in terms of number of visits programmed. Thus, a plan or policy can be written as:

$$\pi = \begin{bmatrix} \mathbf{s} & \mathbf{q} & \mathbf{t} \end{bmatrix} \tag{3.2}$$

In addition, since the state of the system is continuously monitored and plans can be updated at any time, plan update epochs are also decision variables. The sequence of update epochs are denoted $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \dots \end{bmatrix}$, in which u_n is the time of the n$^{\text{th}}$ plan update satisfying $u_{n+1} > u_n \geq 0$ for all *n*. Let $U(t) = \max\{n \geq 0 : u_n \leq t\}$ be the number of plan updates up to epoch *t*. Then $\pi(t) \equiv \pi_{U(t)}$ is the current plan at epoch *t*, and $u(t) \equiv u_{U(t)}$ is the time of the last plan update. Accordingly, the set of update epochs, $\{u_1, u_2, \dots, u(t)\}$, and associated policies $\{\pi_1, \pi_2, \dots, \pi(t)\}$, give the complete history of vehicle deliveries until epoch *t*.

The following additional notation is introduced, before introducing the main constraints to be satisfied in the OIRP. Let $H\left(\pi_n\right)$ be the number of facilities visited while following the n[th] policy, i.e., $H\left(\pi_n\right) = \max\left\{i : t_i(\pi_n) \le u_{n+1}\right\}$ which satisfies $H(\pi_n) \le L(\pi_n)$, and let $H\left(\pi(t)\right)$ be the number of facilities already visited under the current plan at epoch *t*. Let $B_i(t)$ be the total number of visits to facility *i* that have occurred by epoch *t*, which can be expressed as:

$$B_i(t) = \sum_{n=1}^{U(t)} \sum_{j=1}^{H(\pi_n)} \mathbf{1}\left(i = s_j(\pi_n)\right) \quad \text{, for } i \in \Im_0 \text{, and all } t \tag{3.3}$$

where $\mathbf{1}(\cdot)$ is an indicator function that takes the value one if the argument is true and zero otherwise. In addition, let $\rho_{i,k}$ and $q_{i,k}$ denote the k[th] arrival time and quantity refilled at facility *i*, respectively.

$$\rho_{i,k} = \left\{t_j(\pi_n) : B_i(u_n) < k \le B_i(u_{n+1}); s_j(\pi_n) = i; \rho_{i,k-1} < t_j(\pi_n) < \rho_{i,k+1}\right\} \tag{3.4}$$

$$q_{i,k} = \left\{q_j(\pi_n) : B_i(u_n) < k \le B_i(u_{n+1}); s_j(\pi_n) = i; \rho_{i,k-1} < t_j(\pi_n) < \rho_{i,k+1}\right\} \tag{3.5}$$

Thus, the total amount of products refilled to retailer *i* until time *t* is

$Q_i\left(t\right) = \sum_{k=1}^{B_i(t)} q_{i,k}$.

3.2.3. Main Constraints

The main constraints that must be satisfied in this OIRP are related to the dynamics of the system and could be stated as follows:

a) Inventory levels at each retailer are always non-negatives and less than their capacity:

$$0 \le \iota_i\left(t\right) \le \kappa_i \quad \text{, for } i \in \Im \text{, and all } t \tag{3.6}$$

b) Inventory levels at retailers decrease with consumptions and increase with deliveries:

$$\lim_{\varepsilon \to 0} \iota_i(\tau_{i,m} + \varepsilon) = \max\left\{0, \left(\iota_i(\tau_{i,m}) - \delta_{i,m}\right)\right\} \quad \text{, for } i \in \Im \text{, and all } m \tag{3.7}$$

$$\lim_{\varepsilon \to 0} \iota_i(\rho_{i,k} + \varepsilon) = \iota_i(\rho_{i,k}) + q_{i,k} \quad \text{, for } i \in \Im \text{, and all } k \tag{3.8}$$

c) The load remaining in the truck is always non negative:

$$\upsilon(t) \geq 0 \quad \text{, for all } t \tag{3.9}$$

d) The amount delivered to a retailer *i* is not greater than the load remaining in the vehicle at that delivery epoch, and the load remaining in the vehicle after the delivery is decreased by the quantity delivered.

$$q_{i,k} \leq \upsilon(\rho_{i,k}) \quad \text{, for } i \in \Im \text{, and all} \tag{3.10}$$

$$\lim_{\varepsilon \to 0} \upsilon(\rho_{i,k} + \varepsilon) = \upsilon(\rho_{i,k}) - q_{i,k} \quad \text{, for } i \in \Im \text{, and all} \tag{3.11}$$

e) The total amount delivered in a tour does not exceed its capacity.

$$\sum_{r=B_i(\rho_{0,k})}^{B_i(\rho_{0,k+1})} \sum_{i \in \Im} q_{i,r} \leq \Upsilon \quad \text{, for all } k \tag{3.12}$$

f) The location of the truck is modified whenever the truck is not idle, and the truck moves toward the next facility at unit speed, so that arrival times should satisfy:

$$\begin{aligned} & t_{h+1}(\pi_n) - t_h(\pi_n) \geq d\left(s_{h+1}(\pi_n), s_h(\pi_n)\right) \\ & \text{, for } h = 1, 2, \ldots, \left(H\left(\pi_n\right) - 1\right) \text{, and all } \pi_n \end{aligned} \tag{3.13}$$

If the decision space is restricted to send vehicle to the next facility and vehicle can only be idle at the depot, then this constraint should be satisfied with equality whenever $s_{h+1}(\pi_n) > 0$ and $s_h(\pi_n) > 0$. In addition, for this case, the vehicle location at the beginning of the n[th] plan can be written as:

$$\ell(u_n)=\begin{cases}\alpha\cdot l(s_H(\pi_{n-1}))+(1-\alpha)\cdot l(s_{H+1}(\pi_{n-1})) & \text{, if } \alpha\le 1\\ l(s_H(\pi_{n-1})) & \text{, otherwise}\end{cases} \tag{3.14}$$

, where $\alpha=\left(t_{H+1}(\pi_{n-1})-u_n\right)/d(s_H(\pi_{n-1}),s_{H+1}(\pi_{n-1}))$, and the location of the vehicle at time *t*:

$$\ell(t)=\begin{cases}\alpha\cdot l\left(s_H\left(\pi(t)\right)\right)+(1-\alpha)\cdot l\left(s_{H+1}\left(\pi(t)\right)\right) & \text{, if } \alpha\le 1\\ l\left(s_H\left(\pi(t)\right)\right) & \text{, otherwise}\end{cases} \tag{3.15}$$

, where $\alpha=\left(t_{H+1}\left(\pi(t)\right)-u(t)\right)/d\left(s_H\left(\pi(t)\right),s_{H+1}\left(\pi(t)\right)\right)$.

Finally, the set of policies until epoch *t*, $\left\{\pi_1,\pi_2,\ldots,\pi(t)\right\}$, that satisfy all these constraints for a given stream of demand realization,

$\boldsymbol{\delta}(t)=\left\{\delta_{1,1},\delta_{1,2},\ldots,\delta_{1,A_1(t)},\delta_{2,1},\delta_{2,2},\ldots,\delta_{2,A_2(t)},\ldots,\delta_{N,1},\delta_{N,2},\ldots,\delta_{N,A_N(t)}\right\}$, is denoted $\Omega(t)$.

3.2.4. Objective Function

The objective of the central decision maker is to move the inventories in the system so as to minimize the expected total operating cost, composed of transportation, inventory holding, and lost sales costs. Using the notation introduced in the previous subsection, those three components can be written as:

a) Total transportation costs until epoch *t*, *TTC*(*t*) =

$$TC\cdot\left\{\sum_{n=1}^{U(t)}\left[d\left(\ell(u_n),s_1(\pi_n)\right)+\sum_{h=2}^{H(\pi_n)}d\left(s_{h-1}(\pi_n),s_h(\pi_n)\right)\right]+d\left(s_H(\pi(t)),\ell(u(t))\right)\right\} \tag{3.16}$$

b) Total inventory holding costs until epoch *t*, *TIHC*(*t*) =

$$\sum_{i\in\Im}\left\{h_i\cdot\left(\sum_{k=0}^{B_i(t)}\left\{\begin{array}{l}\iota_i\left(\rho_{i,k}\right)\cdot\left[\rho_{i,k}-\tau_{i,A_i\left(\rho_{i,k}\right)}\right]+\\ \iota_i\left(\tau_{i,\left(A_i\left(\rho_{i,k}\right)+1\right)}\right)\cdot\left[\max\left\{t,\tau_{i,\left(A_i\left(\rho_{i,k}\right)+1\right)}\right\}-\rho_{i,k}\right]+\\ \sum_{m=A_i(\rho_{i,k})+2}^{\max\left\{A_i(\rho_{i,k+1}),A_i(t)\right\}}\iota_i\left(\tau_{i,m}\right)\cdot\left[\tau_{i,m}-\tau_{i,m-1}\right]\end{array}\right\}\right)\right\} \tag{3.17}$$

where:

$$\iota_i\left(\tau_{i,m+1}\right)=\sum_{k=B_i\left(\tau_{i,m}\right)}^{B_i\left(\tau_{i,m+1}\right)}q_{i,k}+\max\left\{0,\iota_i\left(\tau_{i,m}\right)-\delta_{i,m}\right\} \tag{3.18}$$

$$\iota_i\left(\rho_{i,k}\right)=\max\left\{0,\iota_i\left(\tau_{i,A_i\left(\rho_{i,k}\right)}\right)-\delta_{i,A_i\left(\rho_{i,k}\right)}\right\} \tag{3.19}$$

c) Total lost sales penalty costs until epoch t, $TLSC(t)$ =

$$\left\{p_i\cdot\sum_{k=1}^{B_i(t)}\max\left\{0,\left(\left(\sum_{m=A_i(\rho_{i,k})+1}^{\max\left\{A_i(\rho_{i,k+1}),A_i(t)\right\}}\delta_{i,m}\right)-q_{i,k-1}-\iota_i\left(\rho_{i,k-1}\right)\right)\right\}\right\} \tag{3.20}$$

Therefore the OIRP objective function can be written as:

$$\min_{\{u_n,\pi_n\}_{n=1,\ldots,U(t)}}\ \lim_{t\to\infty}\frac{1}{t}\mathrm{E}\left\{TTC\left(t\right)+TIHC\left(t\right)+TLSC\left(t\right)\right\} \tag{3.21}$$

where the set of update epochs and policies are restricted so as to use information about current state of the system and past events without knowledge of the future, and $\{u_n,\pi_n\}_{n=1,\ldots,U(t)}\in\Omega(t)$.

Thus far, the formal definition of the problem studied in this dissertation has been introduced. The next section discusses major sources of complexity, the general approach that is being used to deal with the OIRP, and the main limitations.

3.3. ***General Approach***

As previously stated, the OIRP does not seem to be tractable. Among the main difficulties in solving this control problem are:

a) Simplified static and deterministic versions of the problem are NP-Hard, i.e., given the complete stream of future demands at each retailer, the associated inventory-routing problem, to optimally schedule deliveries to retailers, is very difficult to solve. In addition to the sequencing complexity of the problem, in the scheduling of visits to facilities is difficult to correctly capture the effect of short-term decisions on long-term costs, since deliveries depend on the time and amount reloaded in the previous visit to that facility. For that reason, an optimal solution to the problem would require a long-term planning horizon; therefore, even for small problem-instances, it is unlikely that the problem could be solved to even near optimality in a reasonable time. That precludes the use of a complete static and deterministic IRP formulation for re-planning purposes in real-time operations.

b) In addition to the combinatorial challenge of the static version of the problem, demands are dynamic and stochastic, and decisions can be updated at any time. In fact—in contrast with other real-time fleet operation problems in which requests to the system are clearly decision epochs—in the OIRP, final customer-demand epochs occur so often that it would be infeasible to adjust plans at each one of them. Thus, update epochs are not clearly defined, and obtaining the best update epoch is a not trivial undertaking.

c) Because retailer deliveries can be combined on the same route, optimal policies to serve each retailer depend not only upon that retailer's inventory level, but also upon the state of the complete system. In fact, transportation costs to service a particular facility are not fixed, but depend upon the set of facilities served on the same route (Campbell et al., 1998). Moreover, since a single vehicle serves all retailers, the lead time to replenish a particular retailer might be affected by congestion, in terms of the number of additional deliveries that are scheduled before that visit.

d) The advances in real-time online combinatorial optimization neither provide tools to solve problems, such as the OIRP, to optimality nor give clear guidance on how to exploit online information in its operation (Grötschel et al., 2001b, Grötschel et al., 2001a).

e) Finally, as in most real-time combinatorial problems, there is a trade-off between the quality of a new plan and the response time at update epochs.

Those difficulties prevent solving the problem or finding an optimal policy directly from the formulation presented in the previous section. Instead, two approaches are proposed wherein either the inventory control side or the routing side of the problem are solved first. Those formulations take into account only a simplified version of the other side of the problem, but allow formulating optimization problems for that other side, in which those *a priori* solutions are used as soft-constraints. In the first approach, inventory reorder parameters are established for each facility and then used as target levels on a routing problem used to update plans. In the second case, the sequence in which facilities are visited is fixed, and then inventory allocation decisions are taken respecting that sequence.

On each approach, different operational policies are proposed, tailored for different degrees of sophistication in terms of ICT. Those operational policies are based on a rolling-horizon framework, wherein new operational plans are repeatedly generated, based on updated information about the state of the system, and they are implemented until the next update epoch is reached. In that scheme, operational strategies are defined by when and how plans are updated.

In terms of plan update epochs, three different operational-strategy cases are analyzed, based on different degrees of ICT capabilities considered. They can be ordered in terms of decreasing ICT requirements as: i) truck routes can be continuously updated, allowing for en-route diversions, ii) truck routes can be updated only at facilities (en-route diversion not being allowed), and iii) truck routes cannot be updated after the truck leaves the depot, i.e. truck plans can be updated only upon tour completions. In all cases, full information about the state of the system at plan update epochs is assumed.

In terms of optimization capabilities, two cases are considered for obtaining new plans: i) simple rules, and ii) specialized software that allows solving combinatorial problems on a real-time basis. In both cases, operational strategies should establish what rules and/or mathematical-program formulations will be used to obtain new plans. This research proposes mixed-integer programming (MIP) problem formulation for new plan generations.

Thus, different operational policies could be implemented, based on how often the off-line problem is solved and/or how many steps of the current solution are implemented before solving a new problem-instance with updated information about the state of the system.

In order to evaluate and compare real-time strategies, between the two approaches discussed in section 2.3 (competitive analysis and discrete event-simulation)—given the difficulties, which were previously discussed in point "a," related to obtaining analytical solutions for the deterministic IRP—competitive analysis approaches are discarded in our analysis. So, evaluation and comparison of proposed policies are conducted using discrete-event simulation experiments.

3.3.1. Simulation Framework

In discrete-event simulation, the state of the system is traced as the events that modify its status unfold. At each one of these events, the state of the system is updated and the simulation clock is advanced to the next event until the end of the simulation (Law and Kelton, 2000). Those events can be divided into two categories: stochastic and deterministic. In the OIRP, stochastic events include demand at retailers' sites; and deterministic events—which, in our case, are policy related, since only deterministic policies are studied—include truck arrival time at a facility with associated delivery; truck diversion; tour-completion time with associated replenishment; and tour beginning time.

Simulation experiments enable the analysis of the system in a replicable and controlled environment, in which different policies can be fairly compared under similar conditions. Different policies are compared, based on an identical stream of stochastic events, which are generated using the same set of random number seed generators (see, for example, Law and Kelton, 2000). Thus, results of simulation experiments reflect operation of the system under steady-state conditions for the particular set of parameters studied, in which statistics about the specific state of the system can be derived.

In this manner, simulation experiments allow comparison of the performance among proposed policies in a wide range of scenarios related to different combinations of problem parameters, such as demand rates, demand uncertainties, the existence of disruptive demand patterns, trade-off among cost parameters, vehicle and facilities capacities, geographic location of facilities, etc.

3.3.2. Benchmark Policies

An additional difficulty in evaluating real-time fleet operational strategies is that there are not settled benchmarks to compare proposed policies. As discussed by Kim (2003), "detailed specifications of the problem have a significant impact on the performance of a policy." Since the OIRP has not been studied before, two benchmark policies are introduced and developed.

The first benchmark policy, BENCH1, emulates what can be achieved operating the system in a decentralized manner with agents following optimal policies. In BENCH1, each retailer manages his own inventory, placing orders to a central supplier who, once a day, schedules deliveries for previous-day orders. In this case, on one hand, based on the orders received at the end of each day, the supplier creates routes solving a vehicle-routing problem (VRP). Then the VRP solution is implemented to make deliveries for her customers. On the other hand, each retailer will follow an optimal continuous-review policy to control his inventory, which in this case, as discussed in subsection 2.1.3, corresponds to an (*s, S*) policy. That is, each retailer will place an order of size *S-s* immediately if his inventory level is below *s*. The optimal parameters for an (*s*, *S*) policy can be obtained using Zheng and Federgruen algorithm (1991) or by exhaustive search over the feasible region.

The second benchmark policy, *most-urgent-next* (MUN) is based on a simple greedy decision rule. In MUN at each delivery epoch the vehicle is send next to refill the retailer closest to run out of inventory. To select the next retailer to be refilled, inventory levels are inspected and based on average consumption rates, the time at which each retailer would run-out of inventory if not visited is calculated. If the vehicle had enough load remaining to refill the selected retailer, it would be send directly to that location, otherwise it would go first to be refilled at the depot and then to that retailer. In that policy, each retailer *i* is refilled up to a pre-specified target level, S_i, or up to capacity κ_i. MUN implementation assumes that inventory levels are monitored and that routes are created so that the next delivery is decided upon refilling a retailer.

In these benchmark policies, operational control parameters, such as reorder levels, are adjusted based on the specific parameters of each scenario. The details will be presented in Chapter 6, as part of the simulation experiments.

3.3.3. Performance Measures

As stated in the objective function of the OIRP, the main performance index used to evaluate any proposed real-time strategy is the minimization of expected total costs per unit of time (week or day), which include expected inventory holding, stock-outs, and transportation costs. Those indicators contain common performance measures used in distribution operation, such as fill rates and fleet utilization. In the OIRP, fill rates, which measure the proportion of demand served from inventory on hand, are equivalent to stock-outs. Also, fleet utilization, which measures the percentage of time the vehicle is not idle, can be obtained directly from transportation costs, as the ratio between transportation costs per period and the cost of running the vehicle continuously during that period. In addition, other important performance measures, not included in the objective function, that are considered in our analysis include:

a) Variability of total costs per unit of time, which is measured as the standard deviation of weekly total costs, and can also be decomposed in its components. This is an indicator of reliability and consistency in the costs of operating the system using a particular strategy. In general, managers prefer strategies with high consistency, to avoid the additional burden of explaining bad outcomes and dealing with disruption operations.
b) Route lengths in terms of number of retailers visited and distance traveled. Even though the proposed models do not consider any restrictions on tour length, it is interesting to analyze whether the application of a particular strategy would lead to unreasonably long routes.

So far, the online inventory routing problem (OIRP) has been formulated, previous research that relates to it has been reviewed, and the general approach used to study this problem has been presented. In the next chapters, proposed real-time strategies are presented and then evaluated. The proposed real-time strategies are classified according to the technology used to update plans in (i) optimization based strategies, and (ii) simple-heuristic based strategies. In the first group, an off-line optimization problem is formulated and employed to update plans, and in the second group, simple-heuristic rules are used to update plans. In this manner, the first two tasks will be presented together in two chapters, one dedicated to optimization based strategies, and other devoted to simple-heuristic based strategies. Then, the last task, evaluation of proposed real-time strategies and analysis of benefits for different distribution-system scenarios, is presented in one chapter describing simulation experiments and results.

Chapter 4: Formulation and Design of Optimization Based Control Strategies for Fixed Inventory Target Levels

This chapter presents the formulation and design of optimization based strategies, in which the inventory control side of the problem is solved first without considering joint replenishment to different facilities. First, a local off-line problem is formulated which is then used in different real-time control strategies to update plans. Second, it presents different control strategies based on different degrees of real-time information availability for controlling the system. Also, it presents an optimization framework for adjusting policy parameters for each strategy.

4.1. Off-Line Optimization Problem Formulation

This section presents the local off-line problem used on optimization based control strategies to update plans. First, the general approach to locally update plan is presented. Second, optimization of refilling levels method is presented. Finally, a mathematical formulation of the routing problem used to update delivery plans is presented.

4.1.1. Preliminaries

One of the main difficulties in formulating this problem is to be able to capture the effect of short-term decisions on long-run costs. If the customers were visited in isolation of each other using direct deliveries from the depot, served by independent vehicles, the optimal policy for each customer could be computed. In this case, a well known result on inventory control for single items inventory systems with stochastic consumption rates, constant replenishment lead times, and standard cost assumptions is the optimality of (s, S) policies, see for example Axsäter (2000) and Zipkin (2000). In an (s, S) policy each time the inventory position (inventory on hand plus on order minus backorders) is below s a delivery is scheduled to send a quantity equal to S minus the inventory position, so the inventory position becomes equal to S.

However, since only one truck is serving all customers and customer deliveries can be combined on the same route, transportation (delivery) costs are not fixed. Indeed, they would depend on the set of customers that are served together on the same route. Then the optimal policy to serve each customer would depend not only on its inventory level, but also on the complete state of the system.

To deal with this problem, optimal refilling levels for each facility are first specified, assuming that there is no pattern of deliveries. These levels are then kept as targets to refill up to, on each delivery, when plans are generated. However, there are only penalties associated with violating them as they are not included as hard constraints in the off-line routing problem. The off-line routing problem generates a plan that stipulates for each customer the next delivery time and quantity to refill, based on reorder quantities and on the current state of the system, i.e. inventory levels at each facility, and location and load remaining on the truck. This plan is obtained by minimizing the sum of transportation costs, and expected lost sales penalty (*LSP*) costs, subject to visiting all customers once during the planning horizon.

In the next subsection, the method used to compute reorder quantities for each facility is presented, followed by the mathematical formulation of the off-line routing problem.

4.1.2. Optimization of Refilling Levels

To compute the target refilling levels for each facility, the sum of expected average cost for all facilities is minimized, assuming that there is no pattern of deliveries and the truck visits only one customer per route. That is the truck goes back to the depot after refilling a customer and from there it would go to the next customer if it were necessary. Since the truck visits only one customer per route, transportation costs associated with serving a particular customer are fixed. Even though customers are visited in isolation, given that there is a single truck to serve all of them, the possibility of waiting for service due to the truck serving other facilities is incorporated. Additionally, in contrast with traditional inventory systems where quantities are fixed after orders are made, in the system of interest quantities can be updated after arriving to a customer.

Then, using a policy that places orders when the inventory level is s and refills up to level S, the expected average cost (AC) at steady state at each facility could be calculated using the renewal reward theorem (see for example Wolff, 1989). In equation (4.2) the right hand side is only an approximation because the impact of expected stock-out time during the cycle is neglected from the cycle length and holding costs, (see Axsäter, 2000 pp. 65)

$$AC_i = \frac{\text{E[cost per cycle]}}{\text{E[cycle length]}} \tag{4.1}$$

$$AC_i \approx \frac{FTC_i + h_i\left(\frac{S_i - s_i}{\mu_i} + L_i\right)\left((s_i - L_i\mu_i) + \frac{1}{2}(S_i - s_i + L_i\mu_i)\right) + p_i \cdot \sigma_i\sqrt{L_i} \cdot G\left(\frac{s_i - L_i\mu_i}{\sigma_i\sqrt{L_i}}\right)}{\left(\frac{S_i - s_i}{\mu_i} + L_i\right)} \quad (4.2)$$

where FTC_i is the fixed transportation cost associated to serve retailer i (in isolation); $G(x)$ is the loss function that gives the expected number of lost sales at the end of a period with demand distributed N(0,1) given that the initial inventory level is x, see (Axsäter, 2000); and $L_i = T + d(0, i) + W_i$, is the sum of the review period, T, and the total lead time. The length of the review period, T, is the time between plan updates and depends on the policy implemented. The lead time is composed of travel time from the depot to retailer i, $d(0, i)$, and the expected waiting time for service for customer i, W_i. This expected waiting time could be expressed, as a function of reorder quantities, using the following recursive expression

$$W_i = \sum_{j \neq i}\left\{\underbrace{\Pr\{\text{cust. } j \text{ is in service}\}}_{\beta_j} \cdot \left(d(0,j)\right) + \underbrace{\Pr\{\text{cust. } j \text{ is waiting for service}\}}_{\gamma_j} \cdot \left(2d(0,j)\right)\right\} \quad (4.3)$$

$$\beta_j = \frac{2d(0,j)}{\left(\frac{S_j - s_j}{\mu_j}\right) + L_j} \quad (4.4)$$

, and

$$\gamma_j = \frac{W_j}{\left(\frac{S_j - s_j}{\mu_j}\right) + L_j} \quad (4.5)$$

where β and γ are the proportion of the cycle in which a retailer is being served and waits for service, respectively. To evaluate the waiting times for all facilities, given a vector of reorder quantities, a bisection procedure is used iteratively until the waiting times for all facilities are consistent.

Finally, to obtain the optimal reorder quantities, the sum of average cost for all facilities is minimized, subject to $L_i = T + d(0, i) + W_i$; equations (4.3), (4.4), and (4.5); and $\sum_i 2d(0,i) \cdot \left(\mu_i / (S_i - s_i) \right) < 1$. The first four are definitional constraints, and the fourth implies that truck utilization rate should be less than 100%. This problem is solved using a steepest decent numerical procedure, in which at each step the gradient is evaluated numerically. Then, the solutions found in this step are used as input parameters every time the off-line routing problem presented in the next subsection is called.

4.1.3. Mathematical Formulation of the Problem

In this off-line routing problem, the current inventory levels at all facilities are considered as given, and the load remaining and the distance to all facilities of the truck. When the truck is at the depot the load remaining is equal to the truck capacity. Additional input parameters in this formulation are transportation cost *TC* [$/hr]; inventory holding cost h_i [$/unit-day]; lost sales cost p_i [$/unit]; and order up to level S_i [units].

It is assumed that the central decision maker would try to follow the optimal reorder up to *S* policy for each customer. However, since patterns of deliveries are not considered, he/she would deviate from that policy to take advantage of transportation savings. In order to measure the impact on transportation and inventory cost of deviating from the reorder up to *S* policy, incremental inventory costs (*IIC*) for each facility are computed. These *IIC* are calculated as a one time deviation from the reorder up to *S* policy, assuming that after this deviation the optimal policy is resumed. These *IIC* can be expressed as the sum of expected incremental transportation costs (*ITC*), and expected lost sales penalty costs (*LSP*). Notice that the impacts on holding costs are only considered through the specification of reorder up to levels. To compute *ITC*, first notice that, if each retailer is considered in isolation, for a given consumption rate μ, the minimum transportation costs are achieved when deliveries arrive when the inventory level is zero and the quantity delivered is *S*. In this case transportation costs per unit of time are $FTC \cdot (\mu / S)$. Then *ITC*, associated with scheduling a delivery of size *q* units at time *t* after the current time, given that the current inventory level is ι, could be expressed as

$$ITC(q,t/\iota) = \left(\frac{FTC}{\left(\frac{q}{\mu}\right)} - \frac{FTC}{\left(\frac{S}{\mu}\right)} \right) \left(\frac{q}{\mu} \right) = FTC \cdot \left(\frac{\mu}{q} - \frac{\mu}{S} \right) \left(\frac{q}{\mu} \right) = FTC \cdot \left(1 - \frac{q}{S} \right) \quad (4.6)$$

where $0 \le q \le (S - \iota + \mu \cdot t)$, and *S* is the optimal reorder up to level. On the other hand, expected lost sale penalty (*LSP*) costs, associated with scheduling a delivery at time *t* after the current time, given that the current inventory level is ι, could be computed, approximating the distribution of total demand during *t* as $\mathrm{N}(t\mu, t\sigma^2)$, as:

$$LSP(t / \iota) = p \cdot \int_{\iota}^{\infty} (u - \iota) f_{D(t)}(u) du \quad (4.7)$$

$$LSP(t/\iota) = p \cdot \sigma\sqrt{t} \int_{\frac{\iota - t\mu}{\sigma\sqrt{t}}}^{\infty} \left(v - \left(\frac{\iota - t\mu}{\sigma\sqrt{t}} \right) \right) \phi(v) dv = p \cdot \sigma\sqrt{t} \cdot G\left(\frac{\iota - t\mu}{\sigma\sqrt{t}} \right) \quad (4.8)$$

Based on these *IIC* an off-line problem could be formulated similarly to a vehicle routing problem (VRP), where the next visit to each customer is scheduled based on his or her current inventory level, but adding in the objective function the *IIC*. This static off-line problem is formulated as minimizing the sum of *IIC* for all retailers and total transportation costs for the next delivery, subject to visiting all customers once during the planning period (next week) and inventory levels not exceeding order up to levels, *S*, for each retailer.

Thus, the variables of this problem are:

q_i^r : Quantity to be delivered to retailer *i* by the truck in its r^{th} tour, where tours are numbered from 0 (0 is the current tour).

$$x_{ij}^r = \begin{cases} 1, & \text{If facility } j \text{ is visited immediately after facility } i \text{ by the truck in its } r^{th} \text{ tour.} \\ 0, & \text{Otherwise.} \end{cases}$$

$$y_i^r = \begin{cases} 1, & \text{If facility } i \text{ is served by the truck in its } r^{th} \text{ tour.} \\ 0, & \text{Otherwise.} \end{cases}$$

t_i : Arrival time to retailer *i*. ($i \in \Im$).

t_0^r: Arrival time to the depot by the truck in its r^{th} tour. t_0^0 is the truck arrival time to the depot in its current tour.

In addition, the parameters of this model are:

ι_i : Retailer *i* current inventory level.

κ_i : Retailer *i* capacity to store inventory.

Υ : Truck capacity.

ν : Load remaining in the truck, which is equal to Υ when the truck is at the depot.

TC : Transportation cost per unit of distance traveled by the truck. This is measured in [$/hr], since the truck moves at constant speed.

h_i : Retailer *i* inventory holding cost [$/unit-day].

p_i : Retailer i lost sales cost per unit of demand not satisfied [$/unit].

S_i : Retailer i order up to level [units].

s_i : Retailer i reorder level [units].

ℓ : Facility where the truck is currently located. $\ell \in \{0,1,2,...,N,N+1\}$, it is equal to N+1 when truck is en-route. In this case, a dummy node, N+1, is created at the projected position.

d_{ij}: Distance from facility i to facility j. Notice that when the truck is en-route distance from the dummy node N+1 to all facilities should be included.

$\Re \equiv \{0,1,2,...,R\}$: Set of tours (routes) for the truck in the planning horizon, where R is the maximum number of tours not considering the current tour (r=0). Thus, $r \in \Re$.

H : Length of planning horizon (maximum number of hours of operation).

The mixed integer programming (MIP) formulation is presented below:

$$\text{Min.} \sum_{i\in\Im} LSP_i(t_i / \iota_i) + \sum_{i\in\Im}\sum_{r\in\Re} ITC_i(q_i^r, t_i/\iota_i)\cdot y_i^r + TC\cdot \sum_{i\in\Im_0}\sum_{j\in\Im_0:j\neq i} d_{ij}\cdot\left(\sum_{r\in\Re} x_{ij}^r\right) \quad (4.9)$$

$$= \sum_{i\in\Im}\left\{p_i\cdot\sigma_i\sqrt{t_i}\cdot G\left(\frac{\iota_i - t_i\mu_i}{\sigma_i\sqrt{t_i}}\right)\right\} + \sum_{i\in\Im}\sum_{r\in\Re} FTC_i\cdot y_i^r - \sum_{i\in\Im}\sum_{r\in\Re} FTC_i\cdot\left(\frac{q_i^r}{S_i}\right) + TC\cdot\sum_{i\in\Im_0}\sum_{j\in\Im_0:j\neq i} d_{ij}\cdot\left(\sum_{r\in\Re} x_{ij}^r\right) \quad (4.10)$$

Subject to:

$$\sum_{j\in\Im_0:j\neq i}\sum_{r\in\Re} x_{ij}^r = 1 \quad \text{, for } i\in\Im \quad (4.11)$$

$$\sum_{i\in\Im} x_{i0}^r = 1 \quad \text{, for } r = 1, 2, \ldots, R \quad (4.12)$$

$$\sum_{i \in \Im} x_{i0}^0 + x_{N+1,0}^0 = 1 \tag{4.13}$$

$$\sum_{j \in \Im_0 : j \neq \ell} x_{\ell j}^0 = 1 \tag{4.14}$$

$$\sum_{j \in \Im_0 : j \neq i} x_{ij}^0 = \sum_{j \in \Im_0 : j \neq i} x_{ji}^0 + 1\{i = \ell\} \quad \text{, if } \ell \neq N+1 \text{; for } i \in \Im \tag{4.15}$$

$$\sum_{j \in \Im_0 : j \neq i} x_{ij}^0 = \sum_{j \in \Im_0 : j \neq i} x_{ji}^0 + x_{N+1,i}^0 \quad \text{, if } \ell = N+1 \text{, for } i \in \Im \tag{4.16}$$

$$\sum_{j \in \Im_0 : j \neq i} x_{ij}^r = \sum_{j \in \Im_0 : j \neq i} x_{ji}^r \quad \text{, for } i \in \Im_0 \text{; } r = 1, 2, \ldots, R \tag{4.17}$$

$$\sum_{j \in \Im} x_{0j}^1 \leq \sum_{i=1}^{N+1} x_{i0}^0 \tag{4.18}$$

$$\sum_{j \in \Im} x_{0j}^r \leq \sum_{i \in \Im} x_{i0}^{(r-1)} \quad \text{, for } r = 2, 3, \ldots, R \tag{4.19}$$

$$\sum_{i \in \Im} x_{\ell i}^0 \geq \sum_{i \in \Im : i \neq j} x_{ij}^0 \quad \text{, for } j \in \Im \tag{4.20}$$

$$\sum_{i \in \Im} x_{0i}^r \geq \sum_{i \in \Im : i \neq j} x_{ij}^r \quad \text{, for } j \in \Im \text{; } r = 1, 2, \ldots, R \tag{4.21}$$

$$t_j \geq t_i + d_{ij} - M\left(1 - x_{ij}^r\right) \quad \text{, for } i \in \Im\text{; } j \in \Im\text{; } r \in \Re \tag{4.22}$$

$$t_j \leq t_i + d_{ij} + M\left(1 - x_{ij}^r\right) \quad \text{, for } i \in \Im\text{; } j \in \Im\text{; } r \in \Re \tag{4.23}$$

$$t_j \geq d_{\ell j} - M\left(1 - x_{\ell j}^0\right) \quad \text{, for } j \in \Im \tag{4.24}$$

$$t_j \leq d_{\ell j} + M\left(1 - x_{\ell j}^0\right) \quad \text{, if } \ell \neq 0 \text{, for } j \in \Im \tag{4.25}$$

$$t_j \geq t_0^r + d_{0j} - M\left(1 - x_{0j}^{(r+1)}\right) \quad \text{, for } j \in \Im \text{, } r \in \Re \tag{4.26}$$

$$t_\ell = 0 \quad \text{, if } \ell \neq N+1 \tag{4.27}$$

$$t_0^0 \geq d_{N+1,0} - M\left(1 - x_{N+1,0}^0\right) \quad \text{, if } \ell = N+1 \tag{4.28}$$

$$t_0^0 \leq d_{N+1,0} + M\left(1 - x_{N+1,0}^0\right) \quad \text{, if } \ell = N+1 \tag{4.29}$$

$$t_0^r \ge t_i + d_{i0} - M\left(1 - x_{i0}^r\right) \quad \text{, for } i \in \Im;\ r \in \Re \tag{4.30}$$

$$t_0^r \le t_i + d_{i0} + M\left(1 - x_{i0}^r\right) \quad \text{, for } i \in \Im;\ r \in \Re \tag{4.31}$$

$$t_0^r \ge t_0^{(r-1)} \quad \text{, for } r = 1, 2, \ldots, R \tag{4.32}$$

$$t_0^R \le \mathrm{H} \tag{4.33}$$

$$\sum_{j \in \Im_0 : j \ne i} x_{ij}{}^r = y_i{}^r \quad \text{, for } i \in \Im;\ r \in \Re \tag{4.34}$$

$$0 \le q_i^r \le \Upsilon \cdot y_i^r \quad \text{, for } i \in \Im;\ r \in \Re \tag{4.35}$$

$$\sum_{i \in \Im} q_i^r \le \Upsilon \quad \text{, for } r = 1, 2, \ldots, R \tag{4.36}$$

$$\sum_{i \in \Im} q_i^0 \le \nu \tag{4.37}$$

$$s_i - \iota_i + \mu_i t_i \le \sum_{r \in \Re} q_i^r \le S_i - \iota_i + \mu_i t_i \text{, for } i \in \Im : i \ne \ell \tag{4.38}$$

$$\min\{(s_\ell - \iota_\ell), \nu\} \le q_\ell^0 \le S_\ell - \iota_\ell \quad \text{, if } \ell \ne N+1 \tag{4.39}$$

$$x_{ij}^r \in \{0,1\} \text{ and } y_i^r \in \{0,1\} \quad \text{, for } i \in \Im_0;\ j \in \Im_0;\ r \in \Re \tag{4.40}$$

$$x_{N+1,j}^0 \in \{0,1\} \quad \text{, if } \ell = N+1 \text{, for } j \in \Im_0 \tag{4.41}$$

The objective function (4.9)-(4.10) minimizes the sum of the total expected lost sale penalties at each facility for the next scheduled visit, and the total transportation costs.

All tours start at the depot with a full truck, with the exception of the current tour, r=0, in which the truck starts at any facility or en-route (at node N+1), and the truck might not be full (its current load is ν).

Constraint (4.11) ensures that the next visit for each retailer is programmed. Equations (4.12) and (4.13) ensure that the truck returns to the depot in all its tours.

Constraint (4.14) dictates that the truck should leave from its current location. Constraints (4.15)-(4.16)-(4.17) give continuity of flow ensuring that the number of arrivals equals the number of departures at each node. Constraints (4.18)-(4.19) ensure that subsequent routes could be traveled only if the previous route is completed. Constraints (4.20)-(4.21) ensure that current route leave the initial node and subsequent routes leave the depot to visit retailers. Constraints (4.22) through (4.32) ensure that the arrival times at each facility are consistent with travel times between them and the initial conditions, where M is a big number. Constraint (4.33) dictates that the last route should be completed before the end of the planning horizon H . Constraint (4.34) relates facilities served on each route with its links. Constraints (4.35) guarantee that only customers visited from a particular route could receive deliveries from it. Constraints (4.36)-(4.37) ensure that the truck capacities are not exceeded and that the quantity delivered cannot exceed the load remaining in the vehicle. Constraints (4.38)-(4.39) guarantee that inventory levels should not exceed the order up to level, S_i and inventory level should be greater than s_i after refilling; however an exception is allowed at the current facility if the load remaining in the truck is insufficient (4.39).

As mentioned, the purpose of this formulation is to update truck plans making use of updated information about the state of the system. The next section describes three strategies that solve this formulation in a rolling horizon framework

4.2. Optimization Based Real-Time Strategies

The off-line problem described in the previous section will be used to determine how to update truck routes and inventory allocations in a rolling horizon framework. Different policies could be implemented based on how often the off-line problem is solved and/or how many steps of the current solution are implemented before solving a new instance with updated information about the state of the system. Three policies were implemented using the off-line IRP presented in the previous section. These are Replan at Tour Completions (RTC), Replan at Delivery Epochs (RDE), and Replan at Delivery Epochs with possible en-route diversions (RDE+div), which differ in how often the off-line problem is solved. These three policies are presented in what follows, ordered in terms of increasing ICT requirement.

4.2.1. Replan at Tour Completions (RTC) Strategy

In Replan at Tour Completions (RTC), the off-line IRP is solved each time the truck returns to the depot, i.e. completes a tour, and only the first route of the current solution is implemented. In this policy, the review period, T, used to compute the optimal refilling levels, is obtained as the expected distance on a tour over the set of retailers.

4.2.2. Replan at Deliver Epochs (RDE) Strategy

In Replan at Delivery Epochs (RDE), the off-line problem is called at delivery epochs. Each time a truck arrives to a facility, either a retailer or the depot (delivery epoch), an off-line IRP is solved and the solution implemented until the next delivery epoch. That is the amount specified by the solution is delivered at the current facility, and the truck is sent to the next facility specified by the solution. In this policy, the review period, T, used to compute the optimal refilling levels, is obtained as the expected distance between two retailers.

4.2.3. Replan at Deliver Epochs with possible en-route diversions (RDE+div) Strategy

In Replan at Delivery Epochs with possible en-route diversions (RDE+div), plans are updated at delivery epochs, as in RDE, but in addition plans are updated when demand disruptions occur. In this case, inventory levels are continuously monitored while the vehicle is traveling; whenever a facility's consumption since the last plan update is below or above 3 standard deviations from its expected demand, the current plan is updated. To update the plan, the state of the system, i.e. the location of the truck, and inventory levels assuming expected consumption rates, is first projected. Then based on the projected state of the system, an off-line routing problem is solved and the next step implemented. In this strategy, the truck could be diverted if in the new plan the next facility to be visited differs from the current destination.

In order to solve the off-line IRP formulation used in these strategies, the first term in equation is piecewise linearized, so that small instances can be efficiently solved using CPLEX 10.0 with default settings. This problem can be solved in a few seconds for most instances with less than six facilities and few minutes for instance with less than nine facilities. The design of heuristics to solve larger size instances is beyond the scope of this dissertation, and is left as a future extension.

Chapter 5: Formulation and Design of Fixed-Tour Based Control Strategies

This chapter states the rationale and characteristics of a set of control strategies wherein facilities are visited in a predetermined sequence, i.e. a set of *a priori* routes is used. As a part of fixed-tour strategies, recourse actions are introduced to illustrate what can be achieved with different degrees of real-time information available for controlling the system. In addition, an analysis and optimization of policy parameters is presented.

One of the main difficulties in implementing the set of policies presented in the previous chapter is that they require solving difficult combinatorial problems at decision epochs. In addition, even though metaheuristics can be developed to solve the proposed formulation, there is no proof that better strategies can obtained by allowing for complete flexibility at decision epochs or solving off-line problems with longer planning horizons. Moreover, in the proposed formulation, when inventory target levels are obtained, it is assumed that each facility is visited independent of the others. Possibly, if joint replenishment efforts were introduced, when inventory target levels were established, better performance of the system could be achieved.

This chapter deals with the formulation and design of fixed-tour based control strategies in order to better understand the impact of: i) restricting the set of feasible decisions at plan-update epochs, particularly restricting the sequence in which facilities are visited; and ii) coordinating visits to facilities that are close to each other.

Fixed-tour strategies discussed in this chapter are based on a common distribution strategy known as *milk-runs*, in which retailers are refilled, using fixed routes (see, for example, Chopra and Meindl, 2003, pp. 243-244).

Among the anticipated advantages of using fixed-tour strategies are the following:

i) Fixed-tour policies are simpler to implement in the field, since they permit drivers to know in advance what routes they will be driving. For that reason, they might be preferred, even when their expected performance might be inferior to a more sophisticate policy.

ii) A better formulation of the inventory-control side of the problem can be obtained when the sequence in which facilities are visited is fixed. In particular, when the time between replenishments to a particular retailer is constant, inventory replenishment levels can be reduced.

iii) If transportation costs were predominant in the total cost function, then using effective tours might be more effective than having the flexibility of repeatedly changing them during execution. Thus, tours with unnecessary zigzags are avoided.

iv) The scheduling of visits to near-by facilities is always coordinated to reduce transportation costs per visit.

However, restricting the flexibility to attend retailers who are close to stock-out will, presumably, increase stock-out penalties. That could be particularly costly when demand variability is high or demand forecasts are wrong.

The most simple of such policies is illustrated by the case in which no real-time communication capabilities are present. In that case, a *fixed-tour at regular-intervals* (FTRI) can be implemented. In FTRI a fixed delivery tour is implemented without updating plans, even when that might be profitable. However, when real-time communication capabilities are available, recourse action can be introduced to react to deviations from projected consumption patterns.

There are several ways to improve upon FTRI, when real-time information about inventory levels is available. In this research two of them that are studied:

- Update the intervals between delivery tours, based on updated information about inventory levels at retailers. Then, only the ability to monitor retailer inventory levels from the depot is required, since en-route vehicle plans would not be modified.
- Skip retailers whenever the expected total cost savings are greater than the expected increment of stock-out penalty costs. Skipping strategies can be implemented with or without vehicle-communication capabilities, as long as inventory levels can be centrally monitored. In the first case, skip decisions can be made en-route, based on updated information about inventory levels; and in the second case, only before leaving the depot. In this research, only en-route skipping decisions are studied.

In the following sections those strategies are discussed, explaining how decision parameters are obtained for each case.

5.1. Fixed Tour at Regular Intervals (FTRI) Strategy

In FTRI, facilities are visited following an *a priori* sequence, in which each retailer *i* is refilled to a pre-specified target level, S_i, or up to capacity κ_i. FTRI implementation assumes that inventory levels are not monitored and that routes are created so that deliveries do not exceed vehicle capacity, i.e. route failures are not permitted. One way to ensure that the vehicle does not run out product in route is to impose that the sum of target levels of retailers visited on the route is less than the vehicle capacity.

5.1.1. Optimization of Refilling Levels for FTRI Strategy

For an FTRI strategy, the expected cost per unit of time can be evaluated analytically, assuming that the total demand during a tour interval, at each retailer *i*, is normally distributed with mean $\mu_i' = \mu_i L_I$, and standard deviation $\sigma_i' = \sigma_i \sqrt{L_I}$, where L_I is the time interval between successive tours, which should be not smaller than the tour length, L_T, i.e. $L_I \geq L_T$. In that case, the expected cost per unit of time can be expressed as:

$$\begin{aligned} EC_{FTRI} = TC \cdot \left(\frac{L_T}{L_I} \right) &+ \sum_{i \in \Im} h_i \cdot \int_{-\infty}^{S_i} (S_i - x_i) \tfrac{1}{\sigma_i'} \phi\left(\tfrac{x_i - \mu_i'}{\sigma_i'} \right) dx_i \\ &+ \sum_{i \in \Im} h_i \cdot \int_{-\infty}^{S_i} (\tfrac{x_i}{2}) \tfrac{1}{\sigma_i'} \phi\left(\tfrac{x_i - \mu_i'}{\sigma_i'} \right) dx_i \\ &+ \sum_{i \in \Im} h_i \cdot \int_{S_i}^{\infty} (\tfrac{S_i^2}{2x_i}) \tfrac{1}{\sigma_i'} \phi\left(\tfrac{x_i - \mu_i'}{\sigma_i'} \right) dx_i \\ &+ \sum_{i \in \Im} \left(\tfrac{p_i}{L_I} \right) \cdot \int_{S_i}^{\infty} (x_i - S_i) \tfrac{1}{\sigma_i'} \phi\left(\tfrac{x_i - \mu_i'}{\sigma_i'} \right) dx_i \end{aligned} \tag{5.1}$$

In equation (5.1) the first term represents the total transportation costs per unit of time, which is the product of transportation costs per unit of time and the fraction of time that the vehicle is being used. The second term represents the inventory holding costs associated with the remaining inventory from the previous visit to each facility, also known as *safety stock*. The third and fourth terms account for inventory holding costs of products consumed during a cycle at each facility. Those two terms can be approximated, replacing S^2 by x^2 in the fourth term, obtaining $\sum_{i\in\Im} h_i \cdot \frac{\mu_i'}{2}$.

That gives an upper bound to the expected total costs—which, when the probability of stock-outs are very low, is a good approximation, since in that case the fourth term is insignificant, compared with the remaining terms. On the other hand, a lower bound can be obtained ignoring those two terms. That lower bound would be tight only for very long tour intervals, in which demand realizations would be orders of magnitude higher than S. In general, the expected costs should be closer to the upper bound, since in this research scenario with high lost-sale penalties are relevant. Finally, the last term represents the lost-sale penalties per unit of time at each facility. Thus, grouping terms, equation (5.1) can be approximated with these upper and lower bound expressions:

$$\begin{aligned} EC_{FTRI} \le\ & TC \cdot \left(\frac{L_T}{L_I} \right) + \sum_{i\in\Im} h_i \cdot \left\{ \left(S_i - \mu_i' \right) + \left(\tfrac{\mu_i'}{2} \right) \right\} \\ & + \sum_{i\in\Im} \left(\tfrac{p_i}{L_I} + h_i \right) \int_{S_i}^{\infty} (x_i - S_i) \tfrac{1}{\sigma_i'} \phi\left(\tfrac{x_i - \mu_i'}{\sigma_i'} \right) dx_i \end{aligned} \tag{5.2}$$

$$\begin{aligned} EC_{FTRI} \le\ & TC \cdot \left(\frac{L_T}{L_I} \right) + \sum_{i\in\Im} h_i \cdot \left(S_i - \left(\tfrac{\mu_i'}{2} \right) \right) \\ & + \sum_{i\in\Im} \left(\tfrac{p_i}{L_I} + h_i \right) \cdot \sigma_i' \cdot G\left(\tfrac{S_i - \mu_i'}{\sigma_i'} \right) \end{aligned} \tag{5.3}$$

$$EC_{FTRI} \geq TC \cdot \left(\frac{L_T}{L_I} \right) + \sum_{i \in \Im} h_i \cdot \left(S_i - \mu_i ' \right) + \sum_{i \in \Im} \left(\tfrac{p_i}{L_I} + h_i \right) \cdot \sigma_i ' \cdot G\left(\tfrac{S_i - \mu_i '}{\sigma_i '} \right) \tag{5.4}$$

, where $G(\cdot)$ is the loss function, which gives the expected number of units of demand lost as a function of the initial inventory level, when demand is normal standard distributed, i.e. $G(y) \equiv \int_y^\infty (x-y)\phi(x)dx = \phi(y) - y\left[1 - \Phi(y)\right]$, where $\phi(\cdot)$ and $\Phi(\cdot)$ are the pdf and cdf of the normal standard distribution respectively.

For a given tour interval, L_I, the refilling levels for each facility that minimize (5.1) can be obtained from the first order condition, since that function is convex. Lower and upper bounds are constructed using bounds on the second term in (5.5). In that manner, bounds are obtained for the optimal refilling levels, which are presented in (5.7).

$$\frac{\partial EC_{FTRI}}{\partial S_i} = h_i + h_i S_i \int_{S_i}^{\infty} \left(\tfrac{1}{x} \right) \tfrac{1}{\sigma_i '} \phi\left(\tfrac{x - \mu_i '}{\sigma_i '} \right) dx + \left(\tfrac{p_i}{L_I} + h_i \right) \left[\Phi\left(\tfrac{S_i - \mu_i '}{\sigma_i '} \right) - 1 \right] \tag{5.5}$$

$$h_i + \left(\tfrac{p_i}{L_I} + h_i \right) \left[\Phi\left(\tfrac{S_i - \mu_i '}{\sigma_i '} \right) - 1 \right] \leq \frac{\partial EC_{FTRI}}{\partial S_i} \leq h_i + \left(\tfrac{p_i}{L_I} \right) \left[\Phi\left(\tfrac{S_i - \mu_i '}{\sigma_i '} \right) - 1 \right] \tag{5.6}$$

$$\mu_i ' + \sigma_i ' \Phi^{-1}\left(\tfrac{p_i - h_i L_I}{p_i} \right) \leq S_i^* \leq \mu_i ' + \sigma_i ' \Phi^{-1}\left(\tfrac{p_i}{p_i + h_i L_I} \right) \tag{5.7}$$

Those target levels are the solution to the newsvendor problem presented in (2.2), which should not be a surprise, since in FTRI each retailer is visited in every tour and refilled up to *S*. In scenarios where inventory-holding costs are insignificant with respect to lost sale costs, i.e. *hL*<<*p*, target levels should be as high as possible. Then, depending on the problem parameters, either full truck loads or filling up to facilities' capacities would be the best strategy.

To obtain the best tour frequencies, the upper bound on optimal refilling levels presented in (5.7) is used to evaluate expected total costs (5.2). Thus, evaluating that expression for different values of L_I, the best tour frequency can be obtained. In Figures 5-1 to 5-3, the expected weekly costs as a function of tour intervals, for scenarios including seven retailers with the same demand parameters, are presented. As shown, scenarios with higher inventory holding costs and higher demand variability tend to be more sensitive to tour intervals. In particular, when retailer capacities to hold inventory are binding (see Figure 5-3) deviation from optimal tour frequency has a greater effect on expected costs.

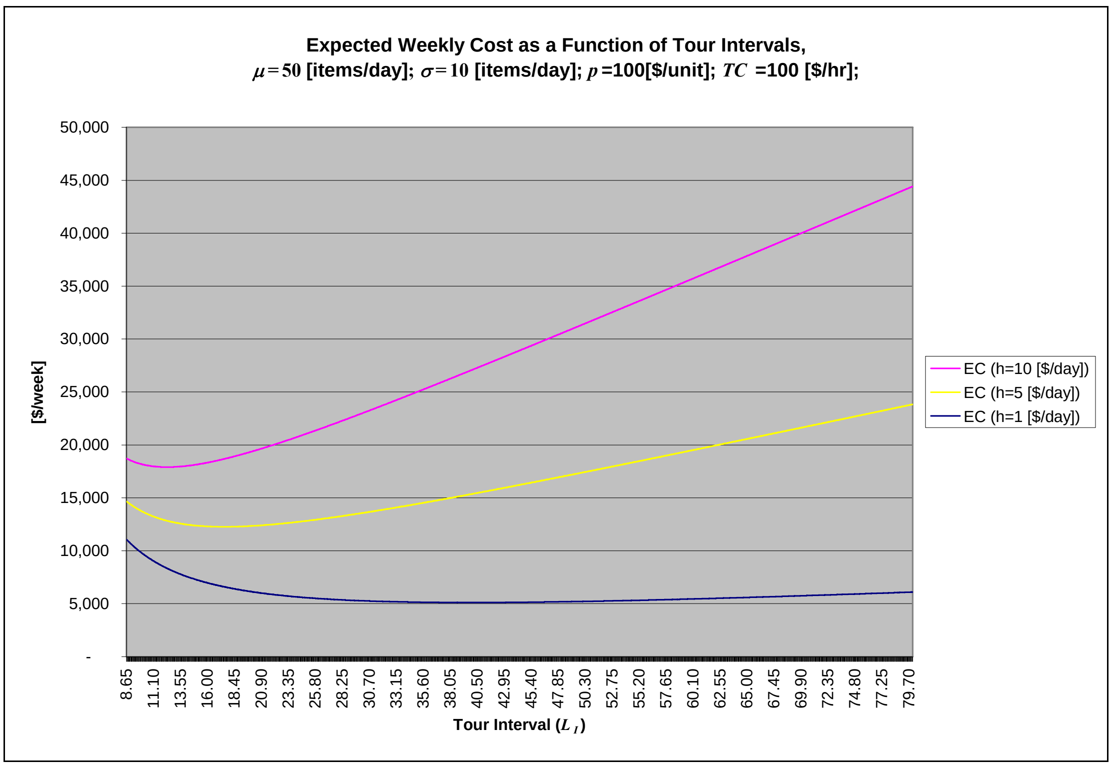


Figure 5-1: Expected Weekly Costs for FTRI as Function of L_I

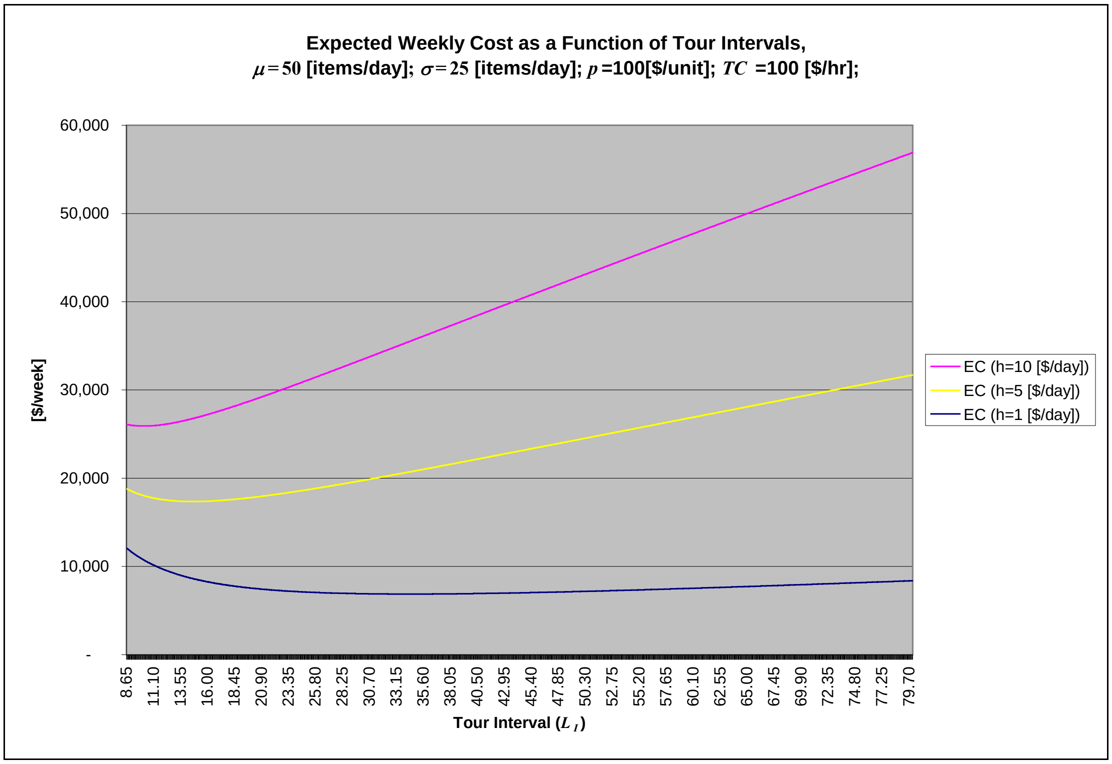


Figure 5-2: Expected Weekly Costs for FTRI as Function of L_I

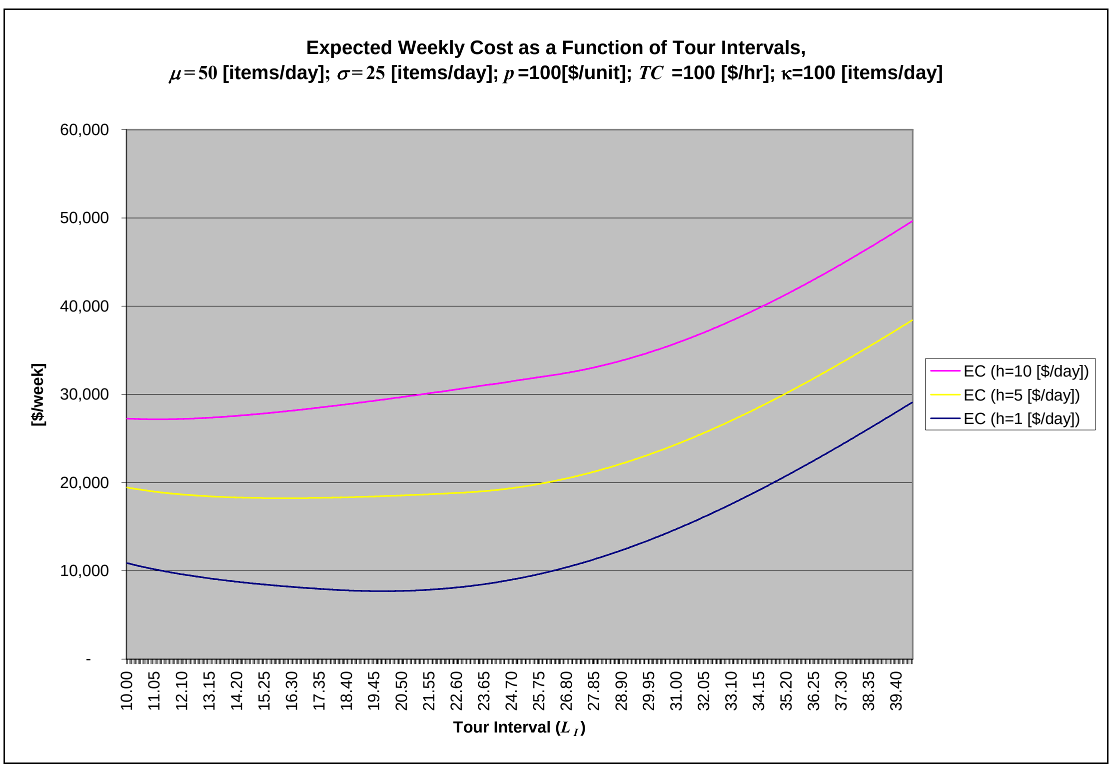


Figure 5-3: Expected Weekly Costs for FTRI as Function of L_I

5.1.2. Procedure for Obtaining FTRI-Strategy Tours

To construct the *a priori* tours used in FTRI, a simple route first-cluster second heuristic is implemented(Beasley, 1983). In this heuristic, *a priori* routes are constructed, following a four-step procedure:

i) obtain an optimal or close-to-optimal solution to the traveling-salesperson problem (TSP) over the set of facilities;

ii) assuming that the tour is traveled uninterruptedly (i.e., the time interval between visits to a particular retailer is equal to the tour travel time), obtain optimal refilling levels for the retailers in that tour, using the solution to the newsvendor problem presented in subsection 5.1.1;

iii) create different routes, dividing the tour into segments, using the Optimal Partitioning heuristic introduced by Beasley (1983); and

iv) optimize the time that the vehicle is idle at the depot between tours, using a bisection search method, in which the effect of different idle times is assessed numerically, evaluating the expected cost of the system per unit of time, as presented in subsection 5.1.1.

5.2. Fixed Tour Updating Intervals (FTUI) Strategy

When inventory levels at different facilities can be monitored in real-time, the tour frequency can be updated either to avoid stock-out penalties at some facilities or to save on transportation costs when consumption rates have been less than expected. This possibility of updating tour intervals is particularly attractive when demand variability is high, capacity constraints at retailer sites to store inventory are binding, or demand parameters are not well known.

In this strategy, whenever the vehicle is at the depot, a decision on waiting an additional time or departing immediately has to be continuously weighed until vehicle departure. How to evaluate the trade-off between expected benefits and costs and how to make that decision are covered in the next subsection.

5.2.1. Update of Truck Idle-Times on FTUI Strategy

To evaluate the effect of waiting-additional-time decisions, the total expected cost per unit of time until completion of the next tour is computed. In that computation, inventory costs associated with a particular retailer are accounted only until the next visit to that facility. That approach facilitates the analysis, since, for each facility, the time at which it is refilled is a renewal epoch for its demand process. That assumes that after a facility had been refilled, the probability of stock-out before tour completion is null, which is a reasonable assumption when lost-sale penalties are high, because in that case target levels are high enough to avoid the occurrence of stock-outs at the beginning of the cycle.

The expected costs per unit of time until completion of the next tour, *ECNT*, can be computed as a function of the additional time spend at the depot, t_0, given the current inventory level at each retailer, ι_i, as follows:

$$\begin{aligned}
ECNT(t_0/\iota) = & \left(\frac{TC\cdot L_T}{t_0+L_T}\right) \\
& +\sum_{i\in\Im} h_i \frac{1}{(t_0+\tilde{d}_{0i})}(t_0+\tilde{d}_{0i})\cdot\int_{-\infty}^{\iota_i}\left(\iota_i-\tfrac{1}{2}x_i\right)\frac{1}{\sigma_i\sqrt{(t_0+\tilde{d}_{0i})}}\,\phi\left(\frac{x_i-\mu_i(t_0+\tilde{d}_{0i})}{\sigma_i\sqrt{(t_0+\tilde{d}_{0i})}}\right)dx_i \\
& +\sum_{i\in\Im} h_i \frac{1}{(t_0+\tilde{d}_{0i})}(t_0+\tilde{d}_{0i})\cdot\int_{\iota_i}^{\infty}\left(\frac{\iota_i^2}{2x_i}\right)\frac{1}{\sigma_i\sqrt{(t_0+\tilde{d}_{0i})}}\,\phi\left(\frac{x_i-\mu_i(t_0+\tilde{d}_{0i})}{\sigma_i\sqrt{(t_0+\tilde{d}_{0i})}}\right)dx_i \\
& +\sum_{i\in\Im} p_i \frac{1}{(t_0+\tilde{d}_{0i})}\int_{\iota_i}^{\infty}\left(x_i-\iota_i\right)\frac{1}{\sigma_i\sqrt{(t_0+\tilde{d}_{0i})}}\,\phi\left(\frac{x_i-\mu_i(t_0+\tilde{d}_{0i})}{\sigma_i\sqrt{(t_0+\tilde{d}_{0i})}}\right)dx_i
\end{aligned} \tag{5.8}$$

, where $\tilde{d}_{0i} = \sum_{k=1}^{i} d_{k-1,k}$ is the time since the vehicle left the depot until retailer *i* is visited. Applying a procedure similar to the one used to obtain (5.2) upper and lower bounds for expression (5.8) can be constructed, as shown in equations (5.9) and (5.10) respectively.

$$\begin{aligned} ECNT(t_0 / \iota) \leq & \left(\frac{TC \cdot L_T}{t_0 + L_T} \right) + \sum_{i \in \Im} h_i \cdot \left(\iota_i - \tfrac{1}{2} \mu_i (t_0 + \tilde{d}_{0i}) \right) \\ & + \sum_{i \in \Im} \left(\tfrac{p_i}{(t_0 + \tilde{d}_{0i})} + h_i \right) \sigma_i \sqrt{(t_0 + \tilde{d}_{0i})} \cdot G\left(\tfrac{\iota_i - \mu_i (t_0 + \tilde{d}_{0i})}{\sigma_i \sqrt{(t_0 + \tilde{d}_{0i})}} \right) \end{aligned} \quad (5.9)$$

$$\begin{aligned} ECNT(t_0 / \iota) \geq & \left(\frac{TC \cdot L_T}{t_0 + L_T} \right) + \sum_{i \in \Im} h_i \cdot \left(\iota_i - \mu_i (t_0 + \tilde{d}_{0i}) \right) \\ & + \sum_{i \in \Im} \left(\tfrac{p_i}{(t_0 + \tilde{d}_{0i})} + h_i \right) \sigma_i \sqrt{(t_0 + \tilde{d}_{0i})} \cdot G\left(\tfrac{\iota_i - \mu_i (t_0 + \tilde{d}_{0i})}{\sigma_i \sqrt{(t_0 + \tilde{d}_{0i})}} \right) \end{aligned} \quad (5.10)$$

Figures 5-4 and 5-5 present plots of the expressions for scenarios with high and low inventory holding costs, where $ECNT_u(t_0)$ and $ECNT_l(t_0)$ are the upper and lower bounds presented in (5.9)-(5.10) respectively. In addition, each term is plotted: $TC(t_0)$ is the first term in both expressions, $EIHCNT_u(t_0)$ or $EIHCNT_l(t_0)$ is the second term in (5.9)-(5.10) respectively, and ELSPNT corresponds to the third term. As could be anticipated, both bounds are very tight when inventory holding costs are low.

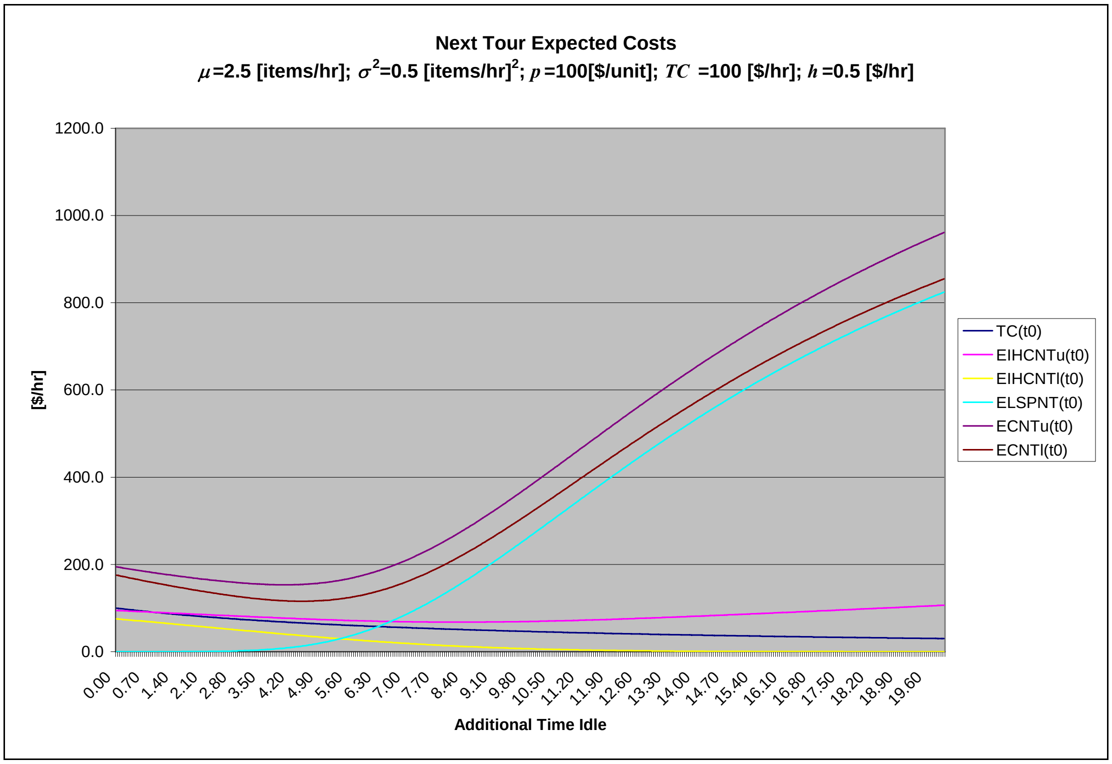


Figure 5-4: Next Tour Expected Costs per Unit of Time as a Function of t_0

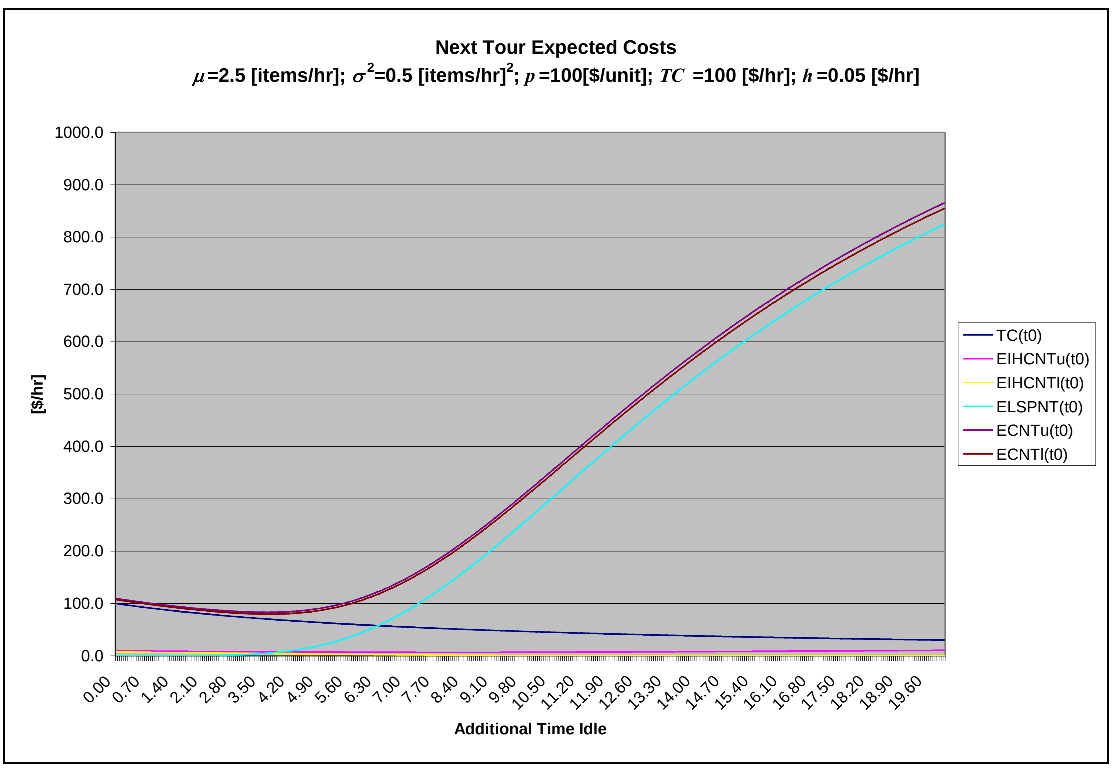


Figure 5-5: Next Tour Expected Costs per Unit of Time as a Function of t_0

Expressions (5.9) and (5.10) prevent obtaining optimal conditions analytically, and their minima do not coincide. In fact, it is easy to verify that the departure time that minimizes the upper bound function (5.9) is lower than the one that minimizes the lower bound function (5.10). In addition, the next-tour-expected cost function is asymmetric, and—for the set of parameters of interest—the increment in expected costs is lower when deviating the same amount from the optimal departure time to an earlier epoch than to a later one. For that reason, the upper bound function is used to decide whether to wait additional time at the depot.

5.2.2. Procedure Used to Implement Updating Interval Strategies

In order to implement updating-tour-interval decisions, the derivative of equation (5.9) is evaluated numerically, since the optimal departure time cannot be obtained analytically from that expression. Equation (5.11) presents the marginal increment in expected cost of the next tour.

$$\frac{\partial ECNT_u(t_0/\iota)}{\partial t_0} = \left(\frac{-TC \cdot L_T}{\left(t_0 + L_T\right)^2}\right) - \tfrac{1}{2}\sum_{i\in\Im} h_i \cdot \mu_i + \frac{\partial}{\partial t_0}\left(\sum_{i\in\Im}\left(\tfrac{p_i}{(t_0+\tilde{d}_{0i})} + h_i\right)\sigma_i\sqrt{(t_0+\tilde{d}_{0i})} \cdot G\left(\tfrac{\iota_i - \mu_i(t_0+\tilde{d}_{0i})}{\sigma_i\sqrt{(t_0+\tilde{d}_{0i})}}\right)\right) \tag{5.11}$$

The expression is evaluated repeatedly at small constant intervals of time, Δt, to decide when to leave the depot. Since expression (5.9) is quasi-convex, whenever (5.11) is positive during the interval, immediate departure is recommended. For that reason, to make a decision is necessary only to evaluate the equation at the end of the next time interval. Therefore, when updating-interval strategies are implemented, every time the vehicle is at the depot, expression (5.11) is repeatedly evaluated with updated inventory levels at $t_0 = \Delta t$, where Δt is the incremental time that the vehicle would wait at the depot at each decision epoch until departure.

5.3. ***Fixed Tour Skipping Retailer (FTSR) Strategy***

One of the main disadvantages of previously proposed fixed-tour strategies is that whenever replenishments are executed, all the facilities in the tour are visited, even though some facilities might have high inventory levels, which are probably adequate to last until the next delivery. Thus, savings could be achieved if those facilities were skipped in the current tour.

The decision to skip a particular retailer can be made before leaving the depot or enroute. If the decision about the set of retailers to skip is made at the beginning of the tour, the probability of skipping retailers near the end of the tour might be low compared to those near the beginning. In that case, tour orientation, i.e. the direction in which the tour is traveled, might become a relevant question. In order to simplify the analysis, skipping facilities in the tour is considered only in scenarios in which vehicles are equipped with two-ways communications capability. In that case, the decision to skip a particular retailer can be made up to the time of departure from the previous facility, when additional information about demand realization at facility will be available.

5.3.1. Skip Decision on FTSR Strategy

It is assumed that the decision to skip the next retailer is made without considering the inventory levels at the remaining facilities on the tour and, furthermore, assuming that they will be visited. That approach is taken to avoid addressing the combinatorial problem of choosing which set of the remaining retailers to visit, given the current conditions. That is, when making the decision to skip or visit the i^{th} facility, it is assumed that facilities $(i+1)^{\text{th}}$, $(i+2)^{\text{th}}$, etc. will be visited. In that context, the additional savings and costs of skipping retailer i when the vehicle is at retailer ℓ are computed as follows.

The expected incremental cost of skipping (EICS) retailer i, and the expected incremental cost savings of skipping (EISS) retailer i can be computed, given that the vehicle is at facility ℓ and the current inventory level at i. They are calculated in equations (5.12) and (5.13), respectively.

$$EICS_i(\iota_i / \ell) = LSP\left((L_I + d_{\ell,i}) / \iota_i\right) - LSP\left(d_{\ell,i} / \iota_i\right) - LSP\left(L_I / S_i\right) \qquad (5.12)$$

$$\begin{aligned}
EISS_i(\iota_i / \ell) &= TC \cdot \left(d_{\ell,i} + d_{i,i+1} - d_{\ell,i+1}\right) \\
&+ h_i \cdot \left\{
\begin{aligned}
&\int_{y=-\infty}^{\iota_i - x}\int_{x=-\infty}^{\iota_i} L_I\left(S_i - \iota_i + x\right)\frac{1}{\sigma_i^2\sqrt{L_I \cdot d_{\ell,i}}}\,\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)dxdy \\
&+\int_{y=\iota_i - x}^{S_i}\int_{x=-\infty}^{\iota_i}\left[\begin{aligned}&\left(L_I\left(S_i - \iota_i + x\right) - \tfrac{1}{2}\left(L_I - \tfrac{L_I(\iota_i - x)}{y}\right)(x + y - \iota_i)\right)\\ &\cdot \frac{1}{\sigma_i^2\sqrt{L_I \cdot d_{\ell,i}}}\,\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)\end{aligned}\right]dxdy \\
&+\int_{y=S_i}^{\infty}\int_{x=-\infty}^{\iota_i}\tfrac{1}{2}L_I\left(\frac{S_i^2}{y} - \frac{(\iota_i - x)^2}{y}\right)\frac{1}{\sigma_i^2\sqrt{L_I \cdot d_{\ell,i}}}\,\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)dxdy \\
&+\int_{y=-\infty}^{S_i}\int_{x=\iota_i}^{\infty} L_I\left(\tfrac{1}{2}y + (S_i - y)\right)\frac{1}{\sigma_i^2\sqrt{L_I \cdot d_{\ell,i}}}\,\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)dxdy \\
&+\int_{y=S_i}^{\infty}\int_{x=\iota_i}^{\infty} L_I\left(\frac{S_i^2}{2y}\right)\frac{1}{\sigma_i^2\sqrt{L_I \cdot d_{\ell,i}}}\,\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)dxdy
\end{aligned}
\right\}
\end{aligned} \qquad (5.13)$$

The EICS is related to the additional expected lost sales during the next tour cycle if facility i is skipped on the current tour. The EISS is the sum of the transportation costs saved by skipping i on the current tour, and inventory holding savings at i during the next tour. In equation (5.13), the last two terms are relevant only when the probability of a stock-out is not negligible if facility *i* is not visited on the current tour. If that were the case, for scenarios with relevant parameters, facility *i* should not be skipped, and EICS should be higher than EISS. A lower bound on the EISS can be constructed as follows:

$$
\begin{aligned}
EISS_i(\iota_i/\ell) \geq\ & TC\cdot\left(d_{\ell,i}+d_{i,i+1}-d_{\ell,i+1}\right) \\
& + h_i \cdot \left\{ \begin{array}{l}
L_I \left[\begin{array}{l} \left(S_i-\iota_i\right)\Phi\left(\frac{S_i-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)\Phi\left(\frac{\iota_i-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right) \\ + \underbrace{\int_{y=-\infty}^{S_i}\int_{x=-\infty}^{\iota_i} \frac{x}{\sigma_i^2\sqrt{L_I\cdot d_{\ell,i}}}\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)dxdy}_{>0} \end{array} \right] \\
-\frac{1}{2}\int_{y=\iota_i-x}^{S_i}\int_{x=-\infty}^{\iota_i}\left[\left(\underbrace{L_I-\frac{L_I(\iota_i-x)}{y}}_{<L_I}\right)\underbrace{(x+y-\iota_i)}_{<(S_i-\iota_i)}\cdot\frac{1}{\sigma_i^2\sqrt{L_I\cdot d_{\ell,i}}}\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)\right]dxdy \\
+\frac{1}{2}L_I\int_{y=S_i}^{\infty}\int_{x=-\infty}^{\iota_i}\left(\underbrace{\frac{S_i^2}{y}-\frac{(\iota_i-x)^2}{y}}_{>0}\right)\frac{1}{\sigma_i^2\sqrt{L_I\cdot d_{\ell,i}}}\phi\left(\frac{x-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\phi\left(\frac{y-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)dxdy
\end{array} \right\} \\
\geq\ & TC\cdot\left(d_{\ell,i}+d_{i,i+1}-d_{\ell,i+1}\right)+h_i\cdot\left\{\frac{1}{2}L_I\left[\left(S_i-\iota_i\right)\Phi\left(\frac{S_i-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)\Phi\left(\frac{\iota_i-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\right]\right\}
\end{aligned}
\tag{5.14}
$$

Thus, the decision to skip a customer is based on a threshold policy, i.e. whenever a retailer's current inventory level is above a certain level, it is skipped; otherwise it is visited. Those threshold-levels depend on the current facility ℓ at which the truck is located. They are computed as the critical levels that change the sign of the following expression.

$$
\begin{aligned}
Skip_i\left(\iota_i/\ell\right) =\ & TC\cdot\left(d_{\ell,i}+d_{i,i+1}-d_{\ell,i+1}\right)+h_i\cdot\left\{\frac{1}{2}L_I\left[\left(S_i-\iota_i\right)\Phi\left(\frac{S_i-\mu_i L_I}{\sigma_i\sqrt{L_I}}\right)\Phi\left(\frac{\iota_i-\mu_i d_{\ell,i}}{\sigma_i\sqrt{d_{\ell,i}}}\right)\right]\right\} \\
& - LSP\left((L_I+d_{\ell,i})/\iota_i\right)+LSP\left(d_{\ell,i}/\iota_i\right)+LSP\left(L_I/S_i\right)
\end{aligned}
\tag{5.15}
$$

If that expression was positive, facility *i* should be skipped, otherwise it should be visited.

5.3.2. Procedure Used to Implement FTSR Strategy

FTSR strategy is implemented using the same tours and reorder levels obtained for FTRI and FTUI strategies, and the procedure presented in 5.2.2 for updating tour intervals. In addition, the possibility of skipping a facility is always evaluated before departing to it based on expression (5.15). Thus, FTSR only differs from FTUI in that skipping decisions are considered before departing to a facility.

To compare all proposed operational strategies in this and the previous chapter, simulation experiments were designed and run, as described in the next chapter.

Chapter 6: Simulation Experiments

This chapter documents experiments designed to evaluate and compare proposed real-time policies. It describes the set of scenarios, including those with steady-state demand processes and those with sudden changes in demand patterns. Finally, it presents and discusses experiment results.

Simulation runs were performed with all eight real-time strategies presented in previous chapters; namely: i) the two benchmark policies, BENCH1 and MUN, ii) the three re-optimization strategies, RTC, RDE and RDE+div, presented in Chapter 4, and iii) the three fixed-tour strategies, FTRI, FTUI and FTSR, presented in Chapter 5. In addition, RDE+FT_S strategy was considered. RDE+FT_S is the same RDE strategy, but implemented using inventory target levels obtained for the FTRI strategy.

6.1. <u>Simulation Scenarios</u>

This section presents the main elements and defining parameters of the simulated scenarios. First, the set of fixed parameters used in all simulation is introduced. Second, the set of parameters considered in scenarios with steady-state demand processes is presented. Third, inventory reorder levels are obtained for each combination of strategy and scenario simulated. Finally, experiments with demand disruption at one facility are presented.

Because of limited computational resources, and since each simulation run takes hours of computer time—even days for some re-optimization strategies—the strategies were not tested under all possible parameters. For each strategy, simulations were performed for four cases of facility layouts, and 12 sets of parameters, representing typical cost settings, probabilistic scenarios, and constraints.

6.1.1. Set of Fixed Parameters

Distances between facilities are Euclidean and are measured in units of time [hours], because it is assumed, without loss of generality, that the vehicle moves at unit speed. All facilities are located in a square region, with side length of four hours, with the depot in the center of the square region, i.e. the depot is locate at (2, 2). In all cases simulated, seven retailers and one depot are considered. In Case 0, retailers are symmetrically distributed around the depot at 1.2 hours apart, and in Cases 1 to 3 they are randomly distributed in the region. Figures 6.1 through 6.4 show the locations of facilities for each case. In those figures the TSP tour visiting all facilities is drawn. The distances of those TSP tours are 8.65, 12.23, 8.38, and 8.69 hours, respectively. Facilities were renumbered so that their numbers coincide with their positions in the TSP tour.

In addition to the location of facilities for each case studied, the following set of parameters is considered as fixed: the vehicle capacity, Υ = 400 [units]; the length of the planning horizon used on the off-line problem for re-optimization strategies, H = 100 [hrs] which is also assumed to be the amount of working hours per week; lost sales penalty costs, p_i = 100 [$/unit] for all retailers; and the fixed transportation costs used to obtain refilling levels for re-optimization strategies, $FTC_i = 2 \cdot d_{0i} \cdot TC$, which is computed as twice the cost of a tour from the depot considering only that retailer.

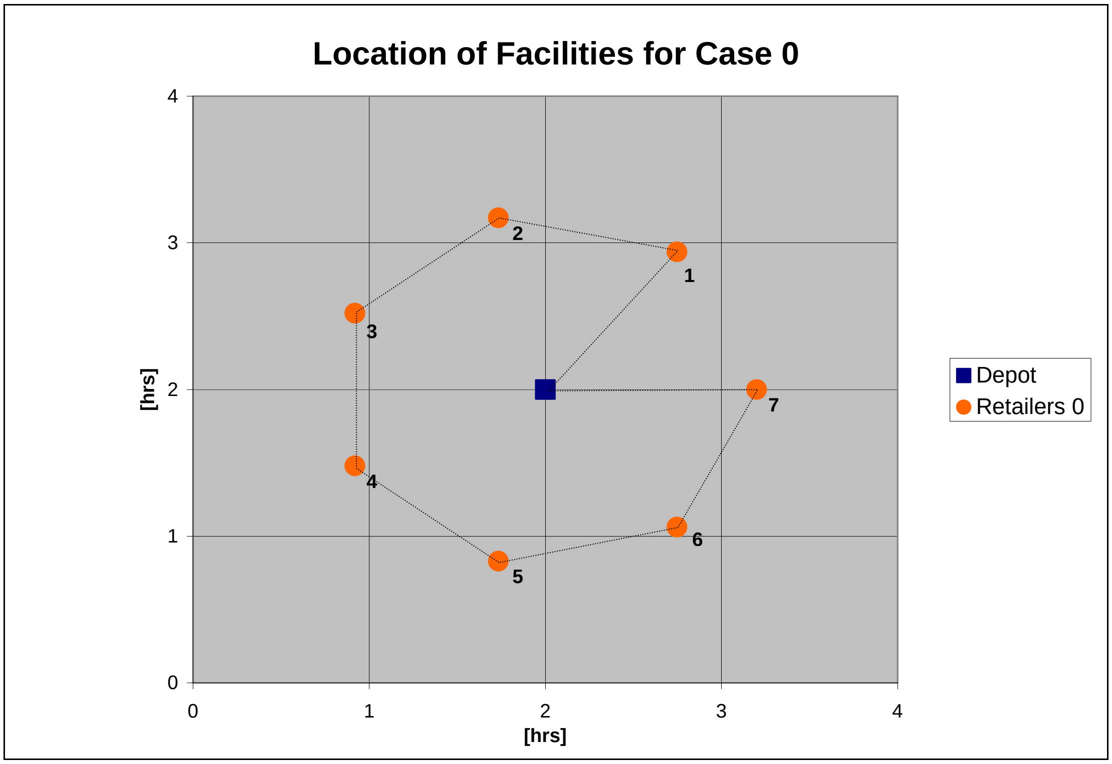


Figure 6- 1: Location of facilities for Case 0 (Symmetric case)

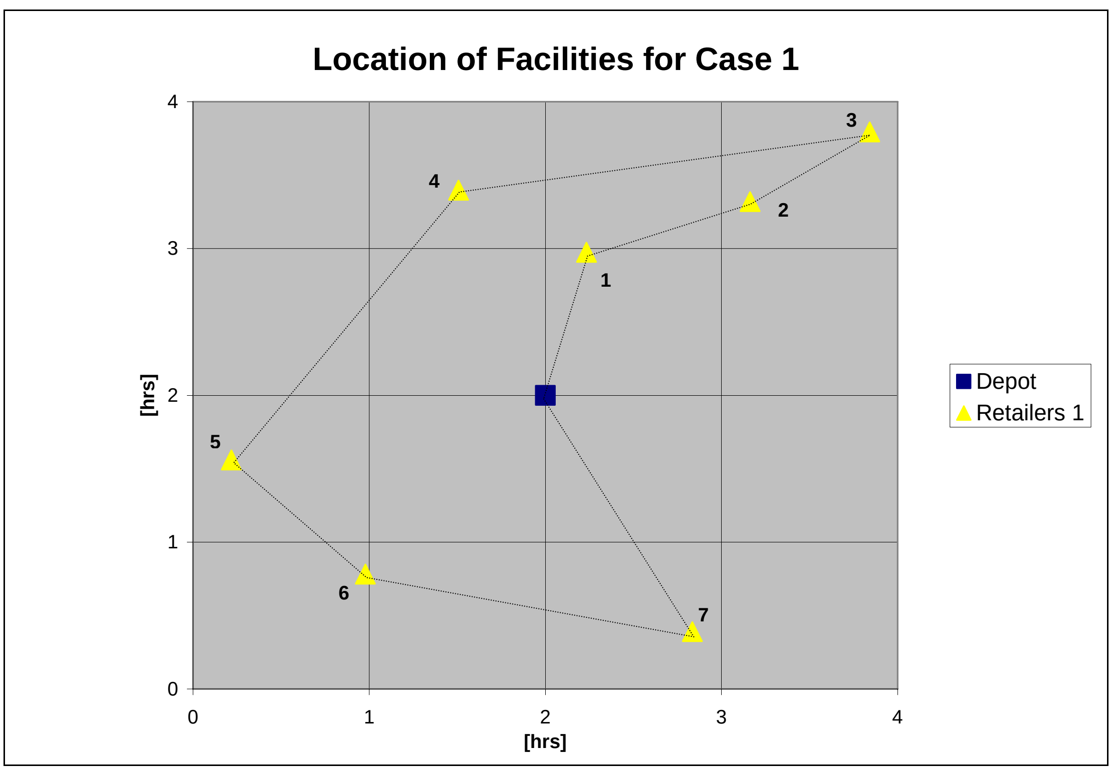


Figure 6- 2: Location of facilities for Case 1

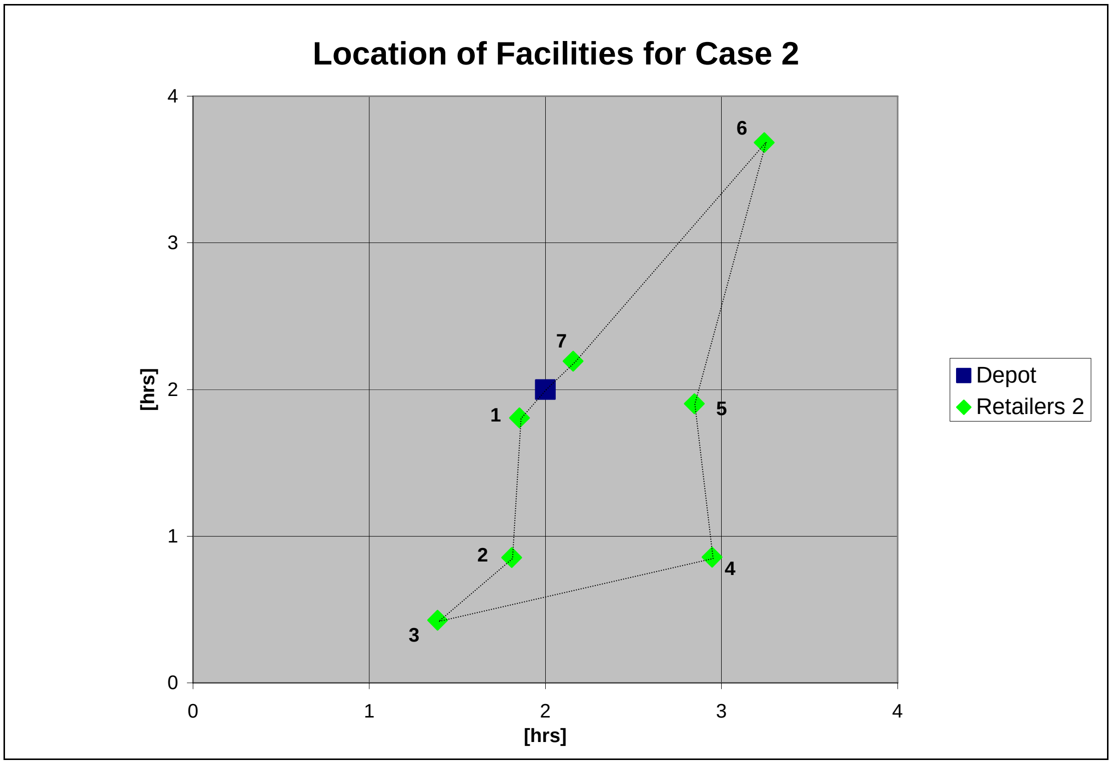


Figure 6- 3: Location of facilities for Case 2

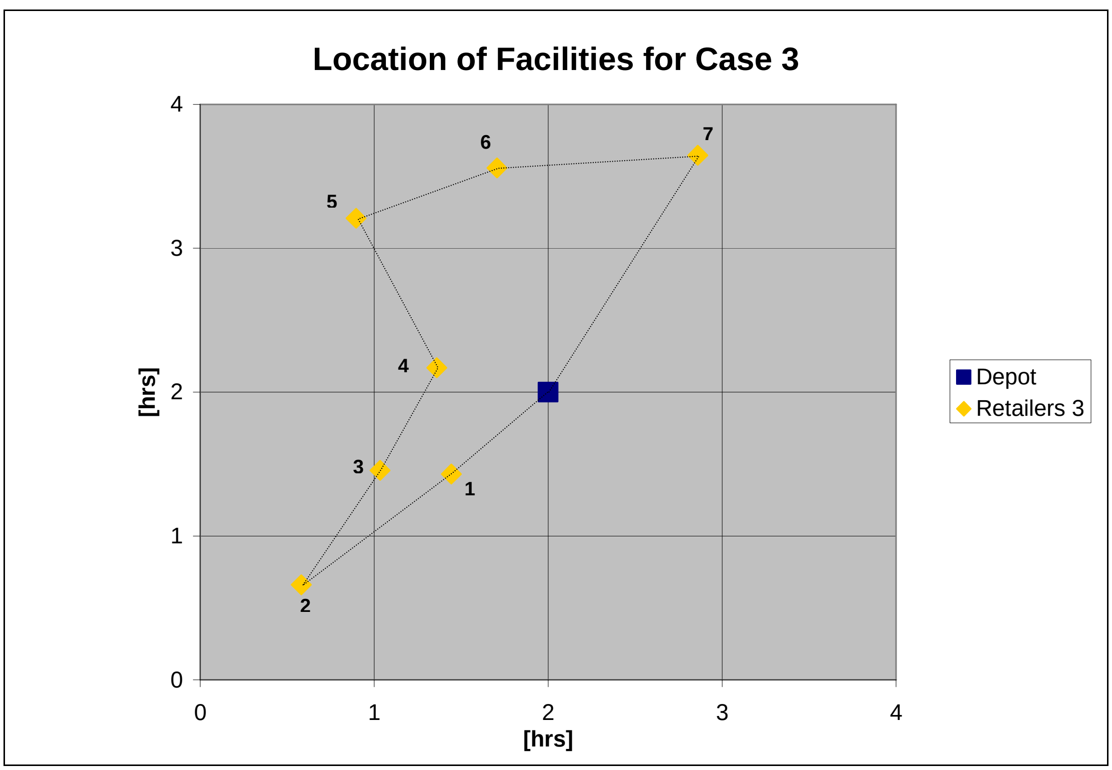


Figure 6- 4: Location of facilities for Case 3

6.1.2. Scenarios with Steady-State Demand Patterns

Two sets of scenarios were studied: i) products with high inventory-holding costs and no capacity constraints at retailers' sites, and ii) products with low inventory-holding costs and capacity constraints at retailers' sites. Scenarios with low inventory holding costs and no capacity constraints at retailers were not considered, since for those scenarios the best policy would be full-truck load deliveries.

For each set of scenarios, a base case was considered. The set of Parameters No. 1 is the base case for high inventory-holding costs scenarios, in which TC=100 [$/hr], h_i = 10 [$/unit-day] for all retailers (inventory holding cost), and demand parameters are the same for all retailers, and equal to λ_i=50 [arrivals/Day] and θ_i = 1 [units], for all i. This demand process can be approximated as $N(50,10^2)$ for daily periods. For low inventory-holding cost and limited capacity at retailers' sites scenarios, the set of Parameters No. 7 is the base case, in which h_i = 1 [$/unit-day], κ_i= 100[units] for all retailers, and the remaining parameters are the same as in the set of Parameters No. 1.

In order to analyze the impact of transportation costs and demand variability on the proposed policies, ten additional scenarios, in which those parameters vary, were studied. Table 6-1 shows the parameters for those remaining scenarios. In scenarios with high (low) inventory holding costs and without (with) capacity constraint at retailers' sites, set of Parameters No. 2 and No. 3 (No. 8 and No. 9) permits studying the effects of changes in transportation costs, and set of Parameters

No. 4 to No. 6 (No. 10 to No. 12) permits studying the effects of increments on demand variability.

Table 6- 1: Simulation Scenarios

Parameter Set	TC [$/hr]	h [$/day]	κ [units]	λ [arrivals/day]	θ [units]	Approx. $N(\mu,\sigma^2)$
1	100	10	∞	50	1	$N(50,10^2)$
2	33	10	∞	50	1	$N(50,10^2)$
3	300	10	∞	50	1	$N(50,10^2)$
4	100	10	∞	10.5	4.8	$N(50,17^2)$
5	100	10	∞	4.35	11.5	$N(50,25^2)$
6	100	10	∞	2.4	20.8	$N(50,33^2)$
7	100	1	100	50	1	$N(50,10^2)$
8	33	1	100	50	1	$N(50,10^2)$
9	300	1	100	50	1	$N(50,10^2)$
10	100	1	100	10.5	4.8	$N(50,17^2)$
11	100	1	100	4.35	11.5	$N(50,25^2)$
12	100	1	100	2.4	20.8	$N(50,33^2)$

In general, inventory-holding and lost-sale costs have important differences, depending on the nature of the product distributed. For that reason—and taking into account that this research deals with general distribution systems—the scenarios studied consider important variations of ratios between transportation and inventory costs. Thus, in the experimental design, the magnitude of each cost parameter is less relevant than the relationship among them.

In Appendix C, reorder parameters for each set of experiments, are presented. The reorder parameters of the (s, S) used in BENCH1 were obtained using Zheng and

Federgruen algorithm (1991). RTC and RDE reorder parameters were obtained using the procedure described in subsection 4.1.2. The only difference between RTC and RDE is the review period considered. In RTC the expected TSP length was used, and in RDE the expected distance between two facilities was used. For MUN strategy, RDE refilling up to levels, *S*, were used. Refilling levels for FTRI strategy were obtained using the procedure presented in subsection 5.1.1. The remaining fixed-tour strategies—FTUI and FTSR—used the same *S* parameters. The last columns of the tables in Appendix C, show the inventory target parameters used in the RDE+FT_S experiments. In those experiments the RDE strategy was executed with refilling up to levels obtained for FTRI.

6.1.3. Scenarios with Unpredicted Changes in Demand Patterns

In all previous scenarios was assumed that demand-process parameters could be precisely estimated; however, this is hardly ever true, particularly for products with a short life-cycle. Thus, the purpose of the experiments was to investigate the performance of different strategies under disruption demand patterns at a particular facility. In distribution systems when such disruptions are observed, it is difficult for the decision maker to update inventory target parameters, since in the short term those changes can be attributed to deviation in normal consumption patterns.

Two experiments were performed with unpredicted changes in demand patterns for the set of Parameters No. 1 and No. 7. These experiments were only performed for the Case 0, symmetric location of facilities, in order to isolate the effect of the demand disruption with respect to the location of facilities. For these experiments the arrival rate of customers to retailer 4 was doubled without updating demand parameters on each strategy.

6.2. <u>Simulation Results</u>

For every combination of strategy and set of parameters studied, simulations were carried for 30 replication runs of 100 hours (1 week) each, and all four Cases of facility layouts. For each set of parameters, different strategies were simulated with common random numbers, and the same initial conditions. The initial conditions for the first replication were the same for all strategies in the same scenario, starting with the vehicle at the depot, and initial inventory levels presented in Appendix C. The effect of initial conditions is only relevant up to the first visit to each facility, which is small compared to the length of each run to have significant effects. Moreover, those initial conditions were only used in the first replication, and results suggest that transient-state effects are negligible.

For each simulation run, the following measures of performance were examined: average transportation costs; average inventory holding costs; average lost sales penalty costs; and average tour length. For each one of those measures, interval estimates were obtained. Those results are presented in Appendix D in Tables D-1 through D-12. Based on those results, 95% confidence intervals for the average total cost per week were computed, and presented in Figures 6-5 through 6-16. In addition, the results for the two scenarios with unpredicted changes in demand patterns are presented in Tables D-13 and D-14, and Figures 6-23 and 6-24.

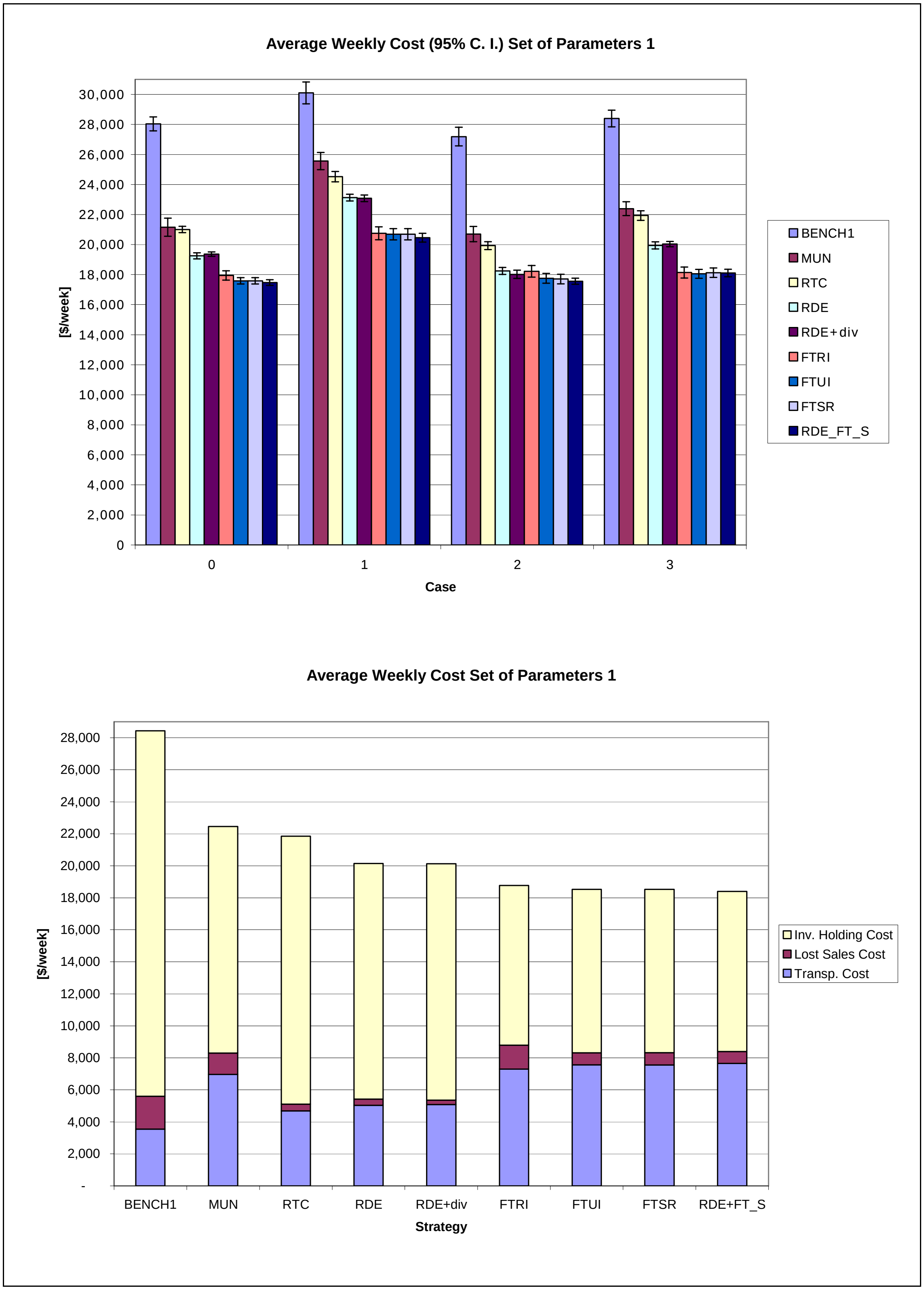


Figure 6- 5: Results for the Set of Parameters 1 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

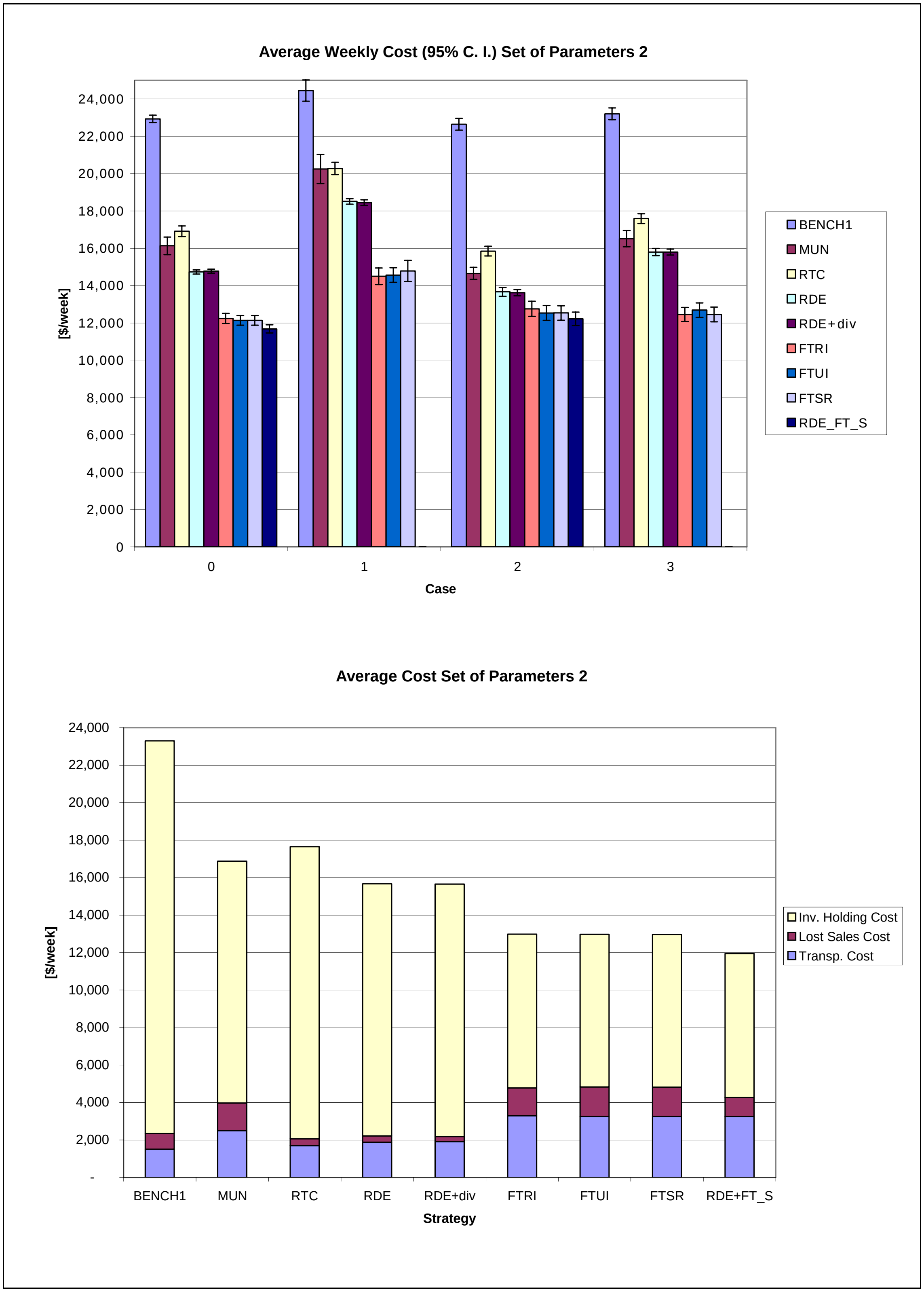


Figure 6- 6: Results for the Set of Parameters 2 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

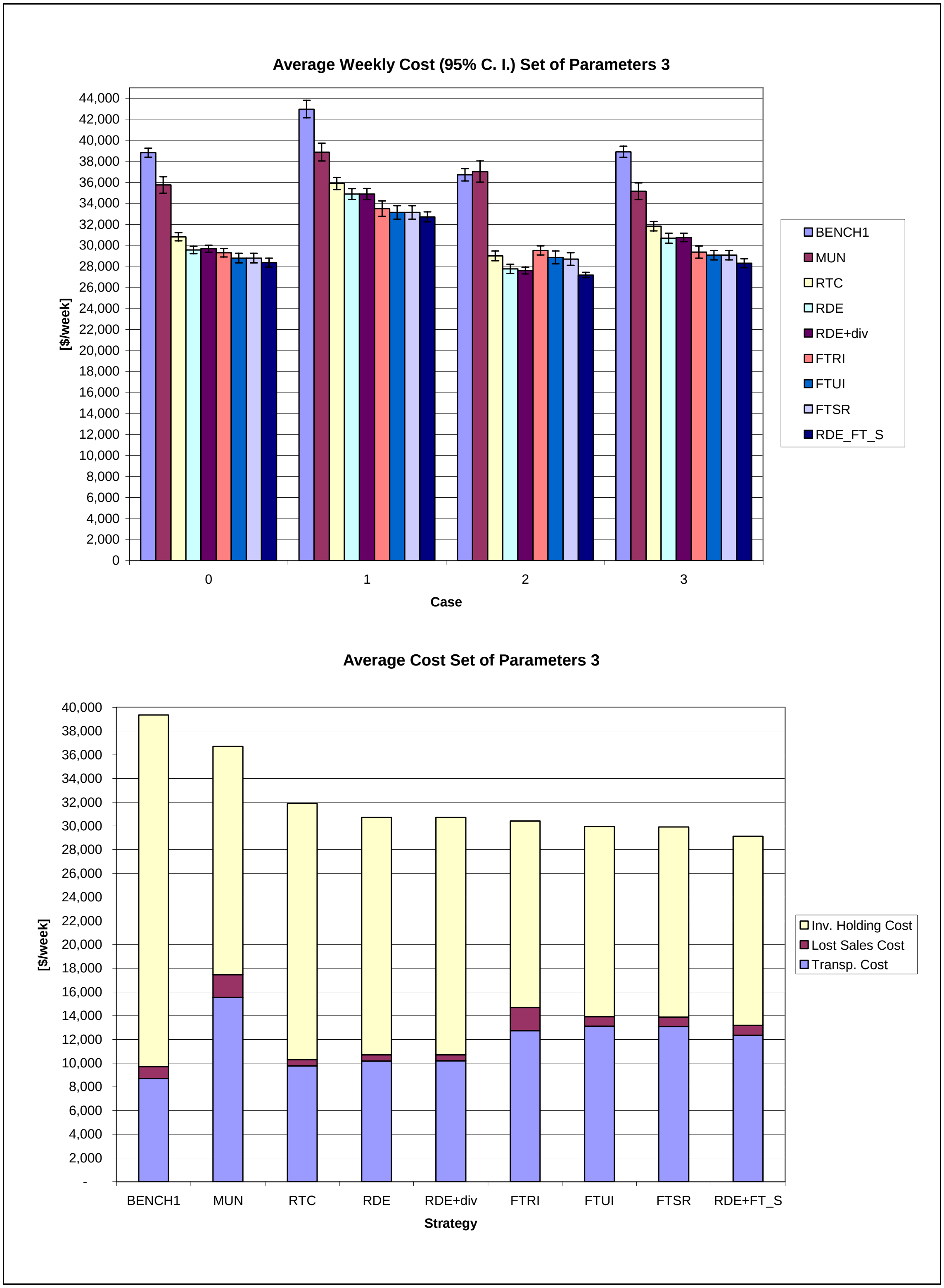


Figure 6- 7: Results for the Set of Parameters 3 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

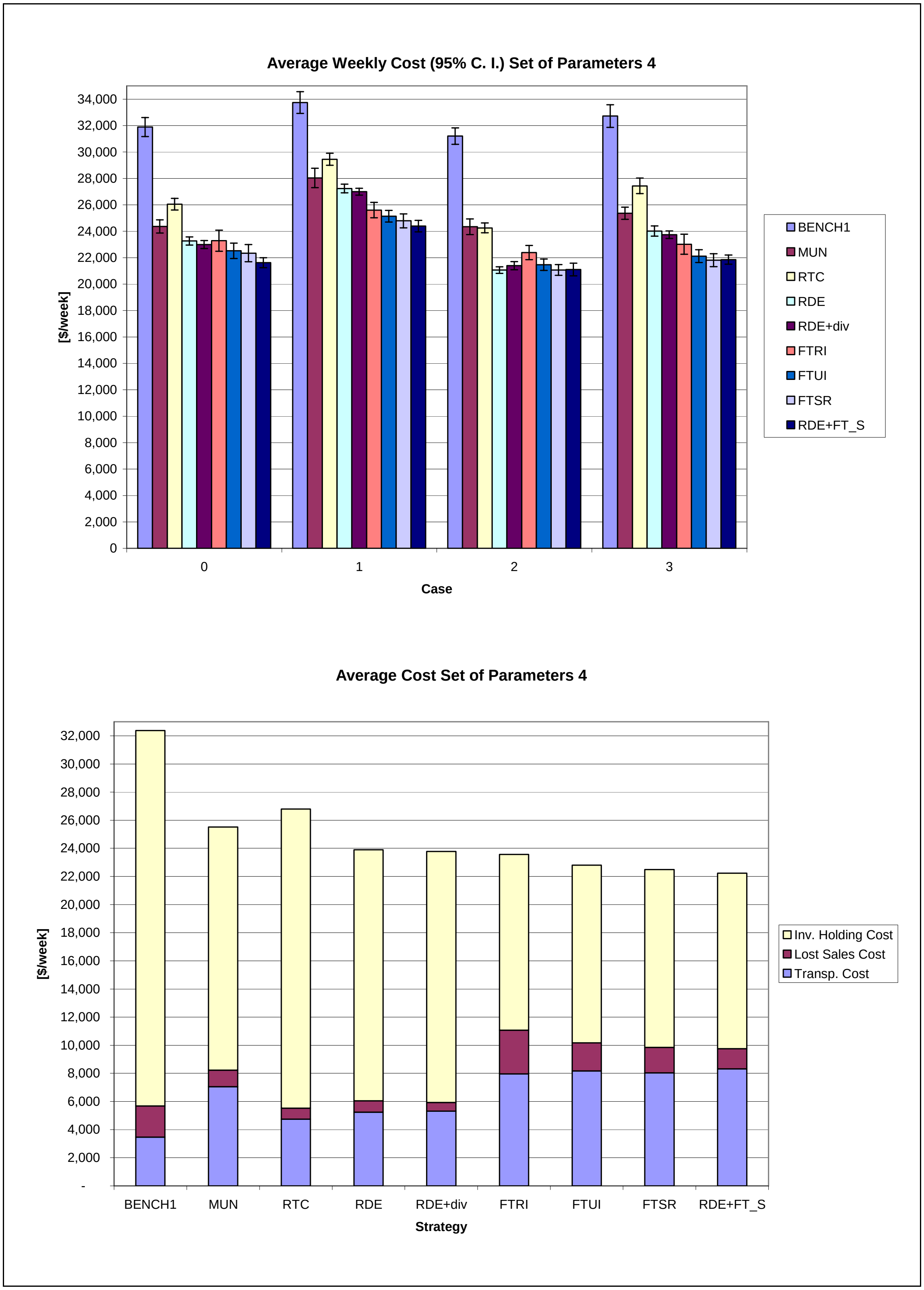
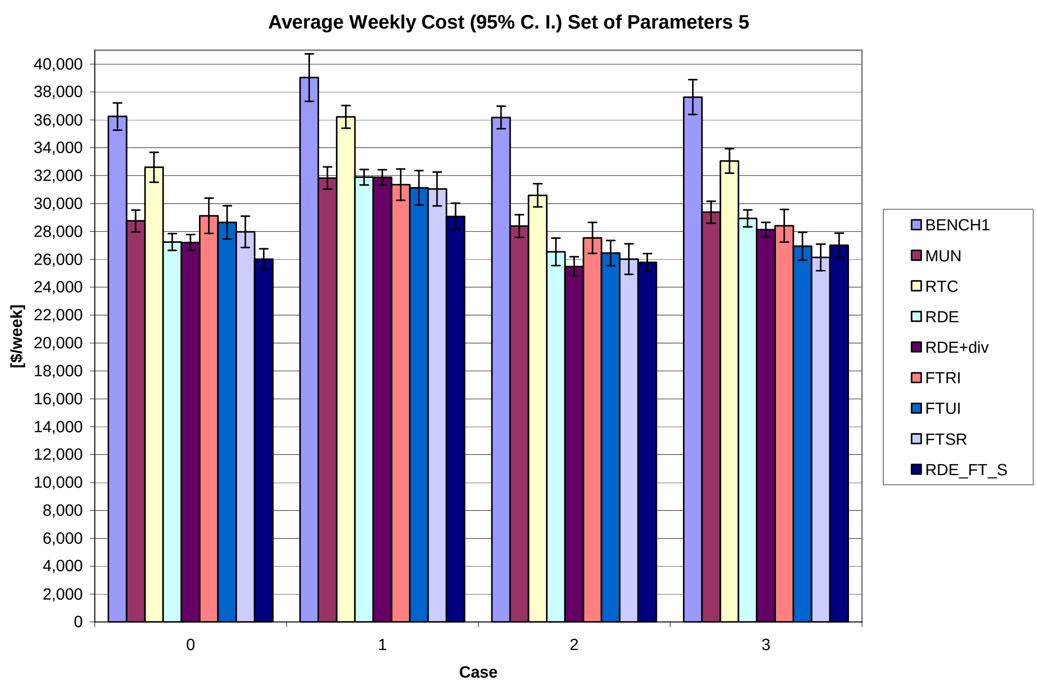


Figure 6- 8: Results for the Set of Parameters 4 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases



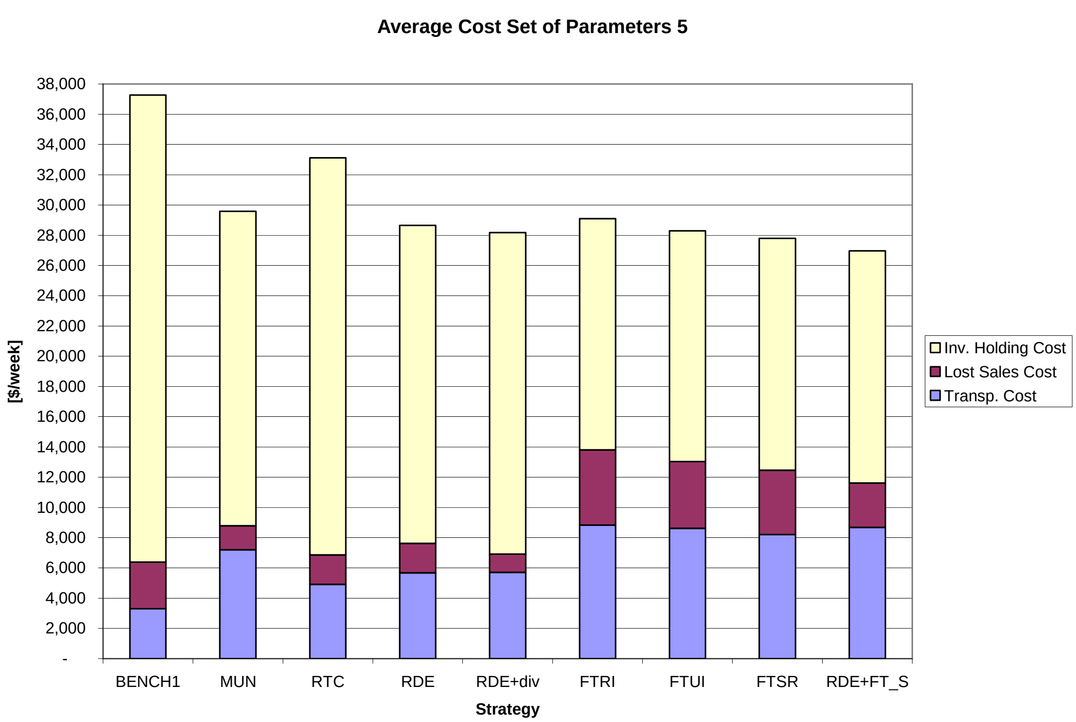


Figure 6- 9: Results for the Set of Parameters 5 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

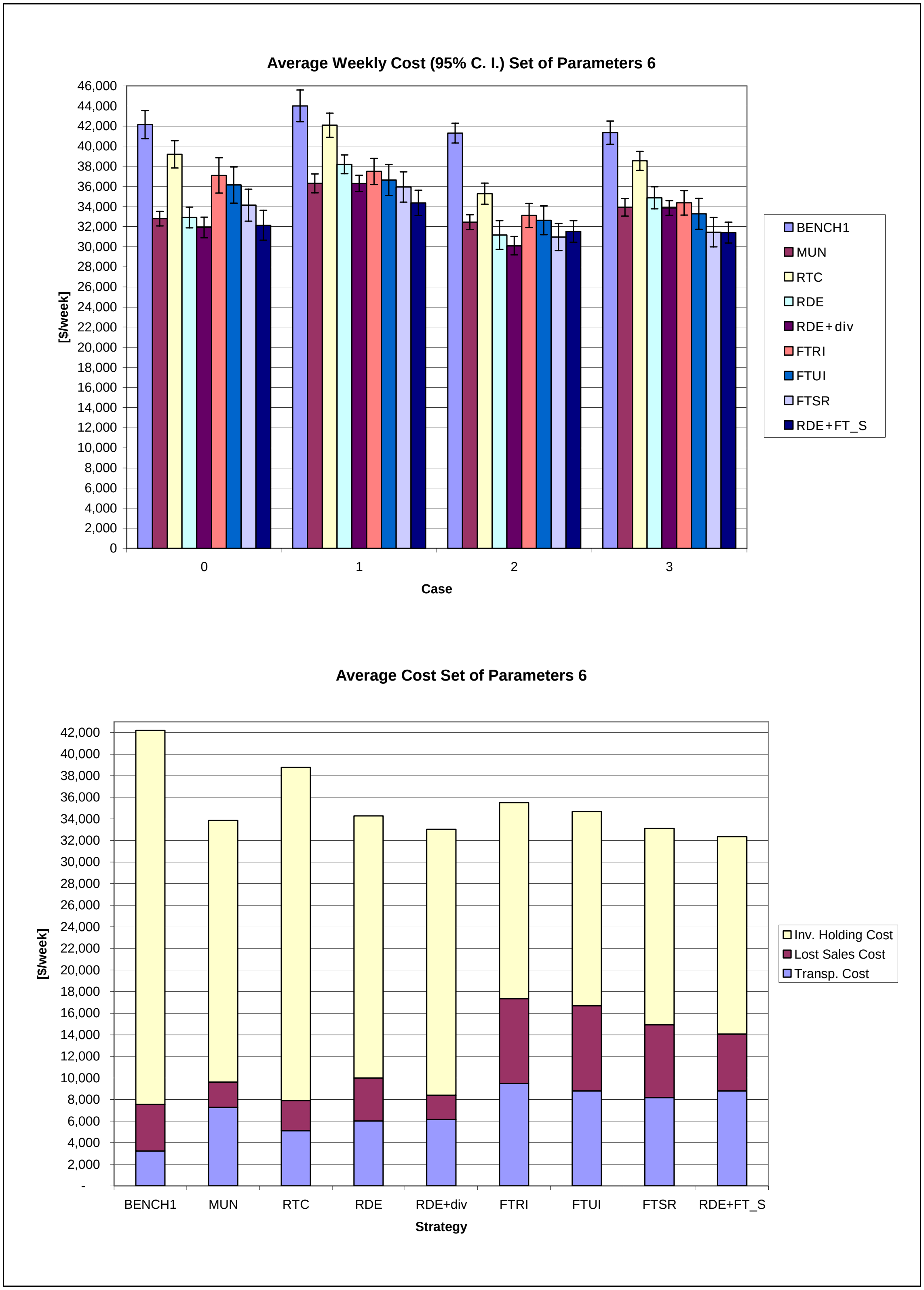


Figure 6- 10: Results for the Set of Parameters 6 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

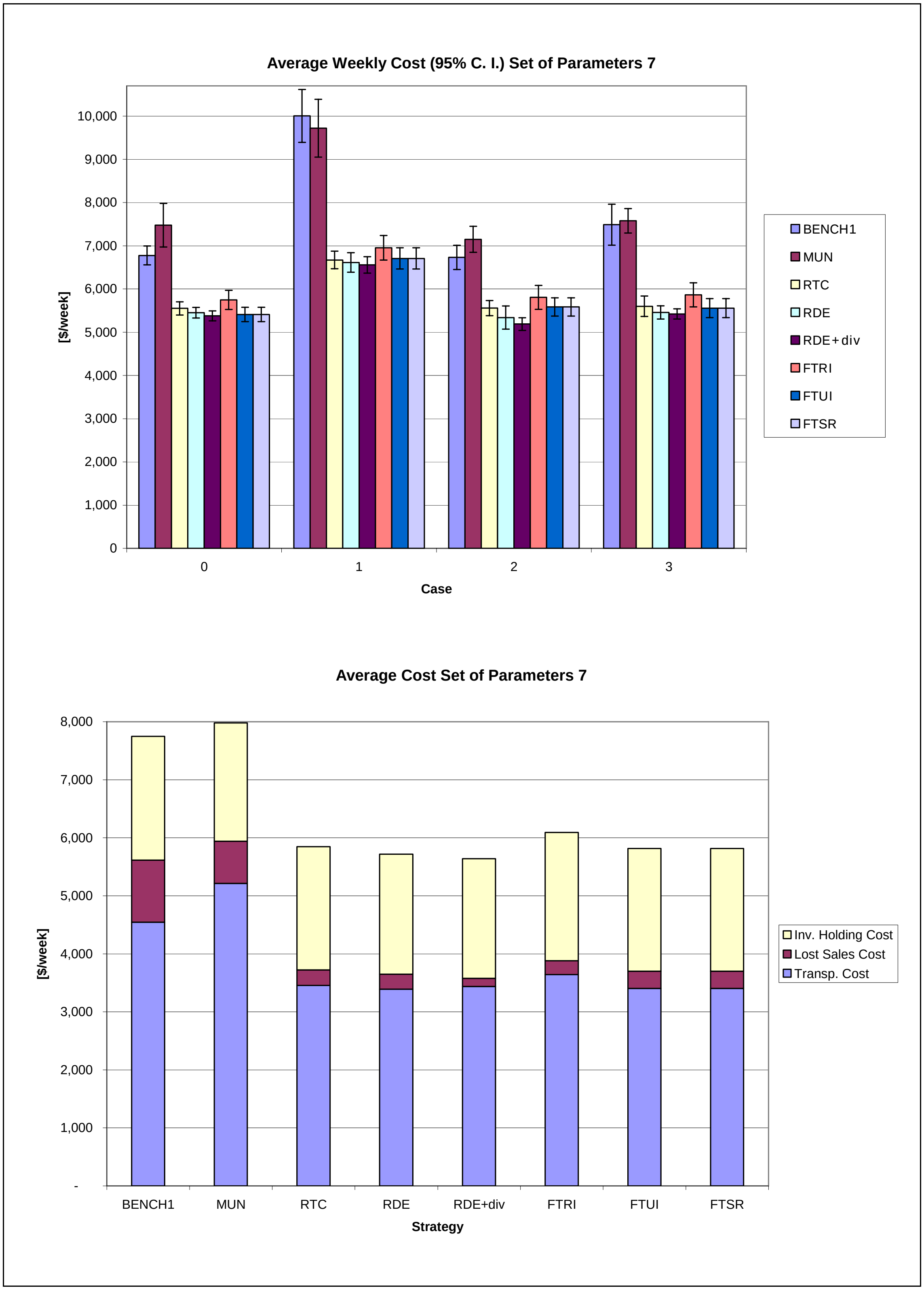


Figure 6- 11: Results for the Set of Parameters 7 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

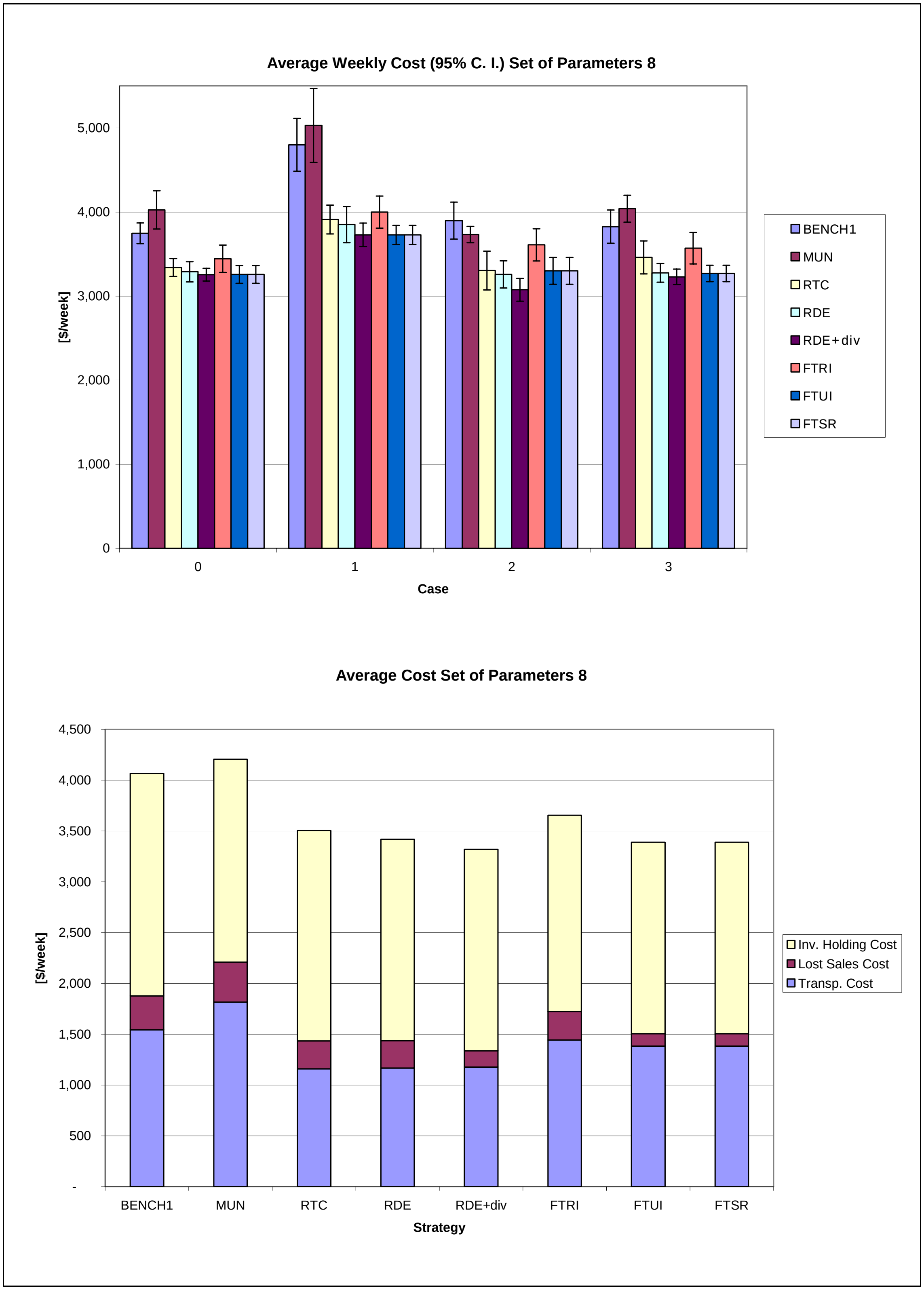


Figure 6- 12: Results for the Set of Parameters 8 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

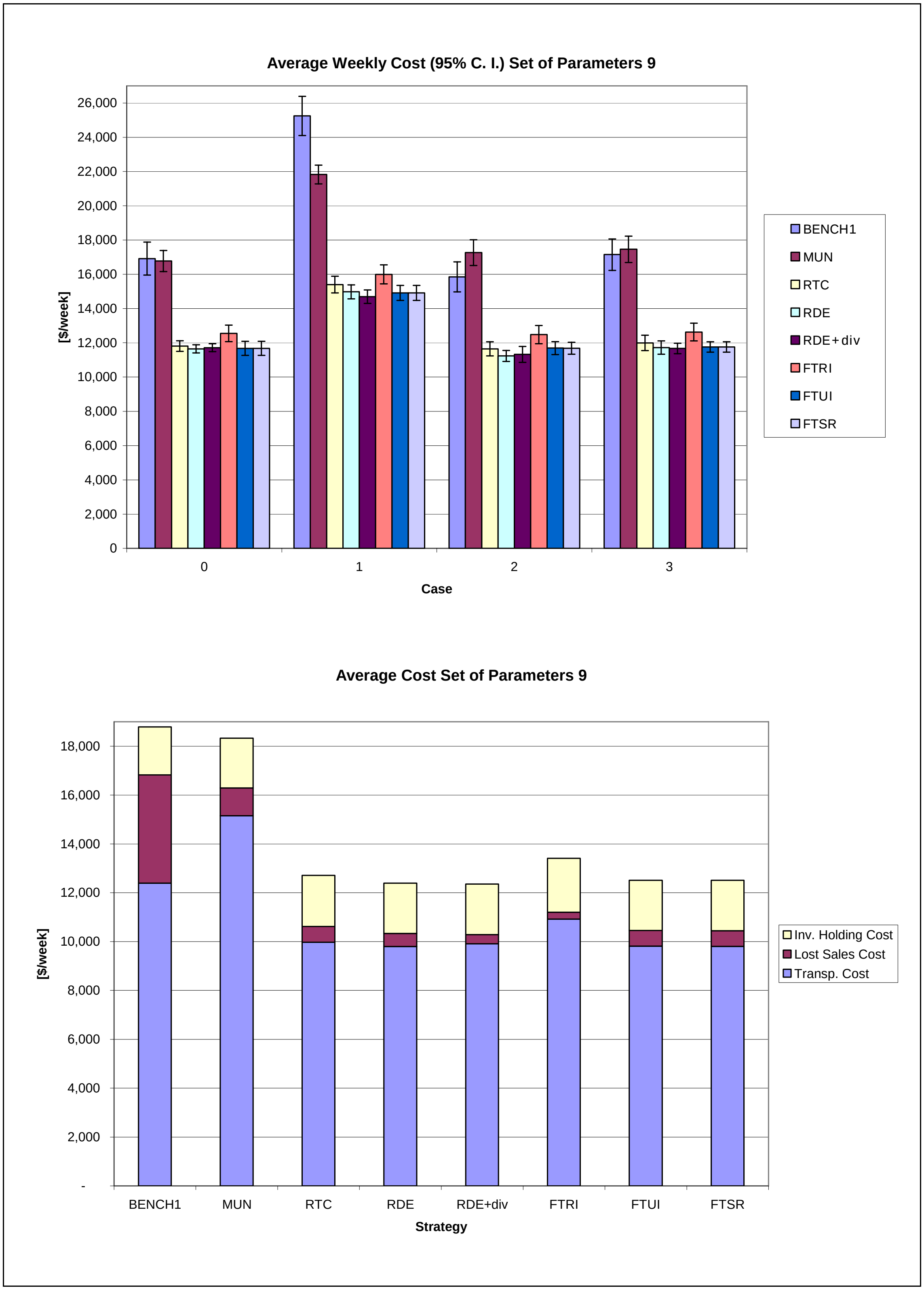


Figure 6- 13: Results for the Set of Parameters 9 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

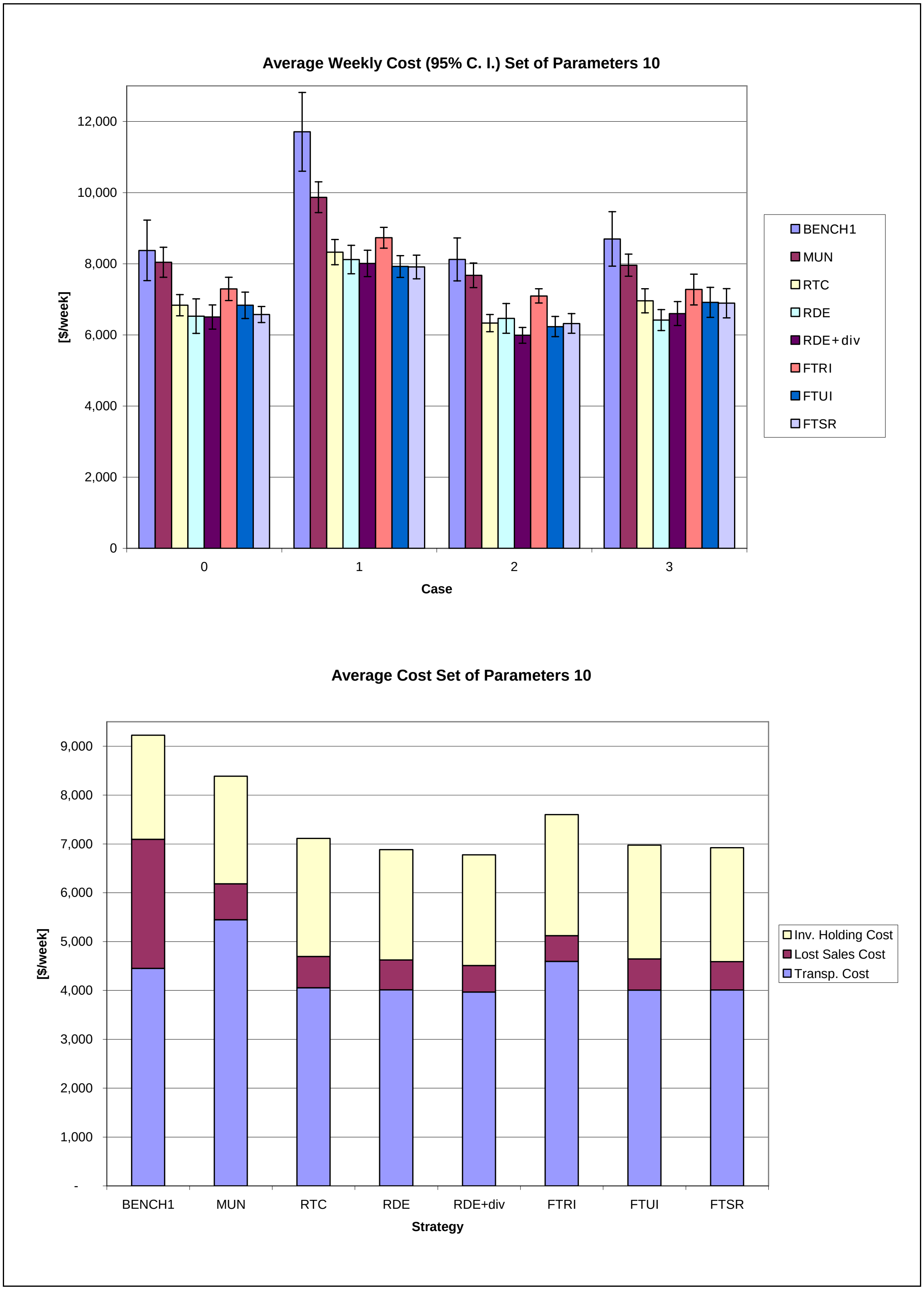


Figure 6- 14: Results for the Set of Parameters 10 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

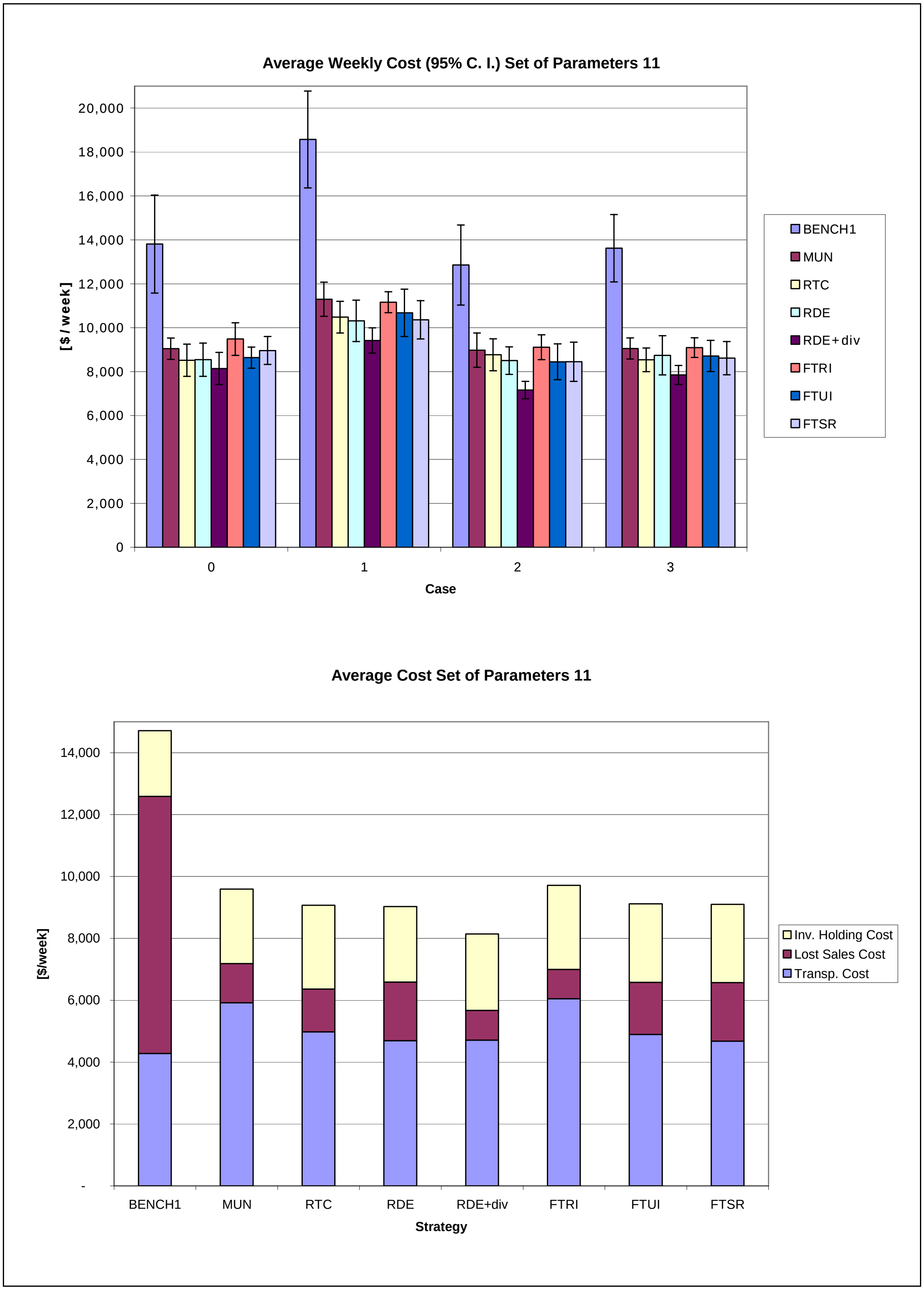


Figure 6- 15: Results for the Set of Parameters 11 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

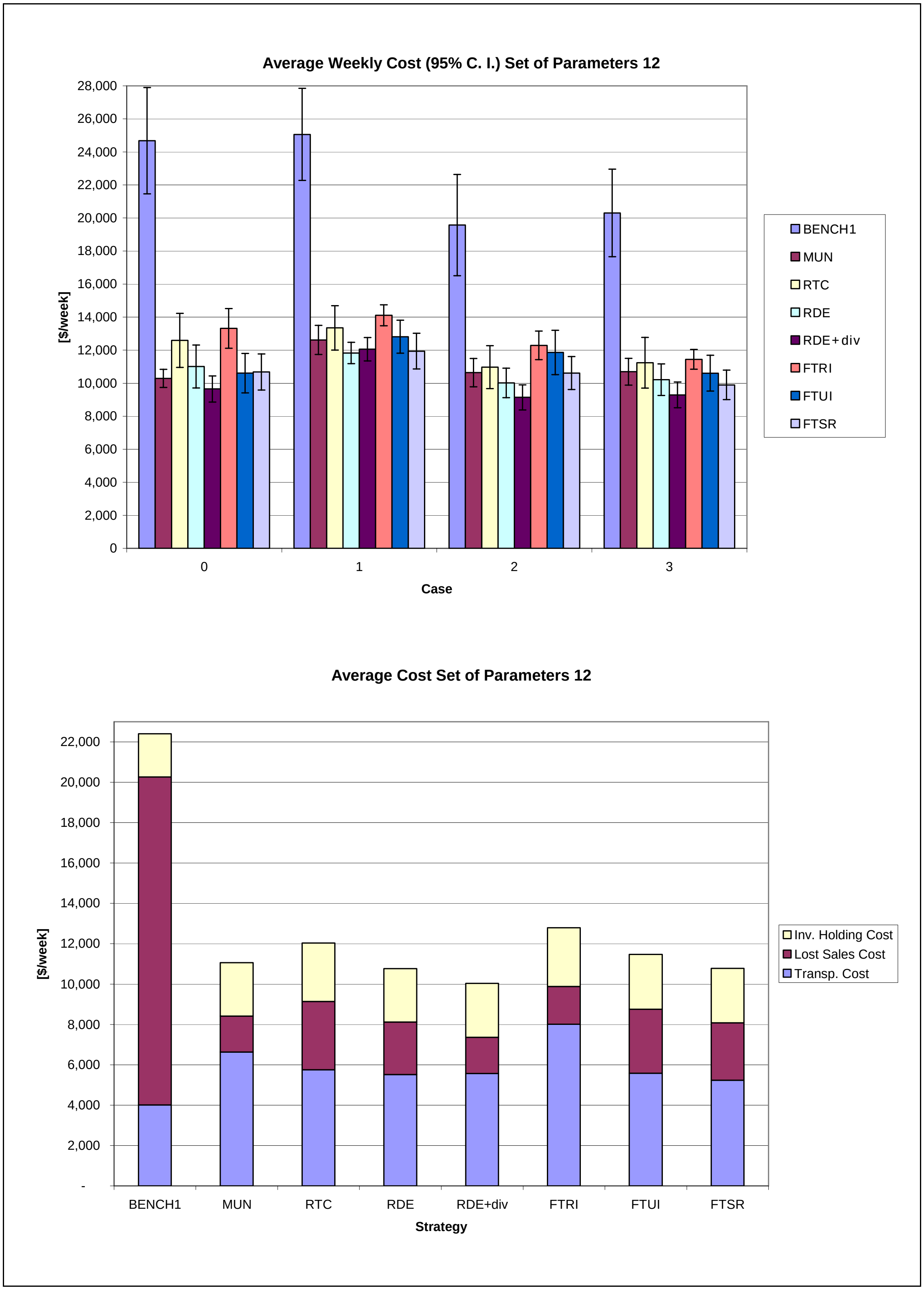


Figure 6- 16: Results for the Set of Parameters 12 with 95% C.I. for the mean, and Average Cost for each Strategy among all cases

6.3. Analysis of Results

Simulations were carried only under typical—and in some cases promising—system parameters, and run under idealized probabilistic distributions. Thus, the results presented are valid only for the range of values studied. Nevertheless, the simulated scenarios were intended to replicate real-world applications and the results are adequate to serve as general guidelines for them.

For all sets of parameters considered, the developed online strategies systematically outperformed benchmark strategies. The best proposed strategies achieved reductions in average total costs of approximately 30% and 15% compared against benchmark policies BENCH1 and MUN, respectively. The average cost improvements were computed as the average of:

$$\frac{\left(\text{Avg. Total Cost of Strategy} - \text{Avg. Total Cost of BENCH}\right)}{\text{Avg. Total Cost of BENCH}} \cdot 100\% \qquad (6.1)$$

for all sets of parameters and cases considered. Moreover, the optimal decentralized benchmark policy, BENCH1, was systematically outperformed by centralized strategies. That can be explained in part by the fact that BENCH1 tends to carry more inventories to be protected from longer lead times. In addition, all proposed strategies achieved less variability in average costs than the BENCH1 strategy.

In general, re-optimization strategies with appropriate inventory target levels had the lowest average costs. However, in many scenarios with moderate demand variability, there were no significant differences, in terms of average total cost, between the best re-optimization and the best fixed-tour strategies.

Among re-optimization strategies, those that update plans at delivery epochs, RDE and RDE+div, were the best strategies for the set of parameters considered. The possibility of diversions—either en-route or when the vehicle is idle at the depot—improves system performance in scenarios with low inventory-holding costs and high-demand variability. However, further research is needed to identify scenarios in which en-route diversion could be beneficial, since in RDE vehicle idle time at the depot is set upon arrival and not updated, even when that might be profitable. The benefits of re-planning at delivery epochs tend to be higher in cases where there are clusters of facilities close to each other and/or near to the depot, such as in Case 2.

Among fixed-tour strategies, the possibility of updating tour intervals offered benefits of up to 10% in scenarios with low inventory-holding costs and high demand variability. Moreover, for scenarios studied, the possibility of skipping retailers in the route produced small benefits in scenarios with high demand variability and small tour intervals. Otherwise, that possibility did not produce significant benefits, since in those scenarios the probabilities of skipping were too small. Among the cases of location of facilities, the possibility of skipping presented higher benefits in Case 2, wherein one facility had a high insertion cost in the tour.

6.3.1. Analysis of the Product Inventory Holding Costs Impact

The experiments were carried out for two sets of scenarios: i) products with high inventory-holding costs, and ii) products with low inventory-holding costs and capacity constraints at retailers' sites. As shown in Figure 6-17, a comparison of the two sets of scenarios illustrates that re-planning strategies tended to increase their benefits vs. benchmark policies when they were applied in scenarios with low inventory-holding costs, whereas fixed-tour strategies tended to have similar benefits.

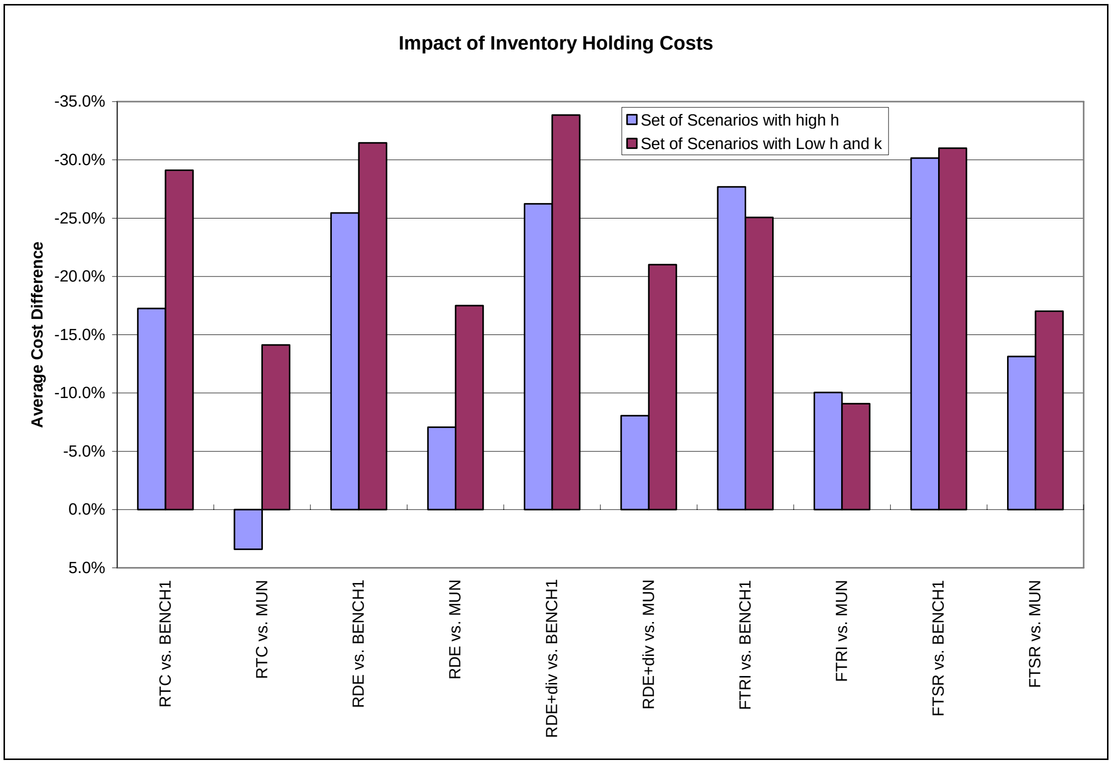


Figure 6- 17: Impact of Inventory Holding Cost

6.3.2. Analysis of Changes in Transportation Costs

In Figures 6-18 and 6-19, reductions in average total costs for different re-planning and fixed-tour strategies vs. the two benchmark policies are illustrated. In scenarios with high inventory-holding costs, the benefits of the proposed strategies tend to decrease (increase) as transportation costs increase, when compared with BENCH1 (MUN). That can be explained mainly by the fact that in BENCH1 there are longer lead-times, so facilities need to carry more inventory. Thus, the more relevant the inventory-holding costs, the worse the performance of BENCH1. In scenarios with low inventory-holding costs and retailer capacities, the benefit of the proposed strategies tends to increase, as transportation costs increase when compared to any of the benchmark policies. Thus, with the exception of BENCH1, for scenarios with high inventory-holding costs, the benefits of the proposed policies tend to increase as a function of the proportion of transportation costs in the total cost function.

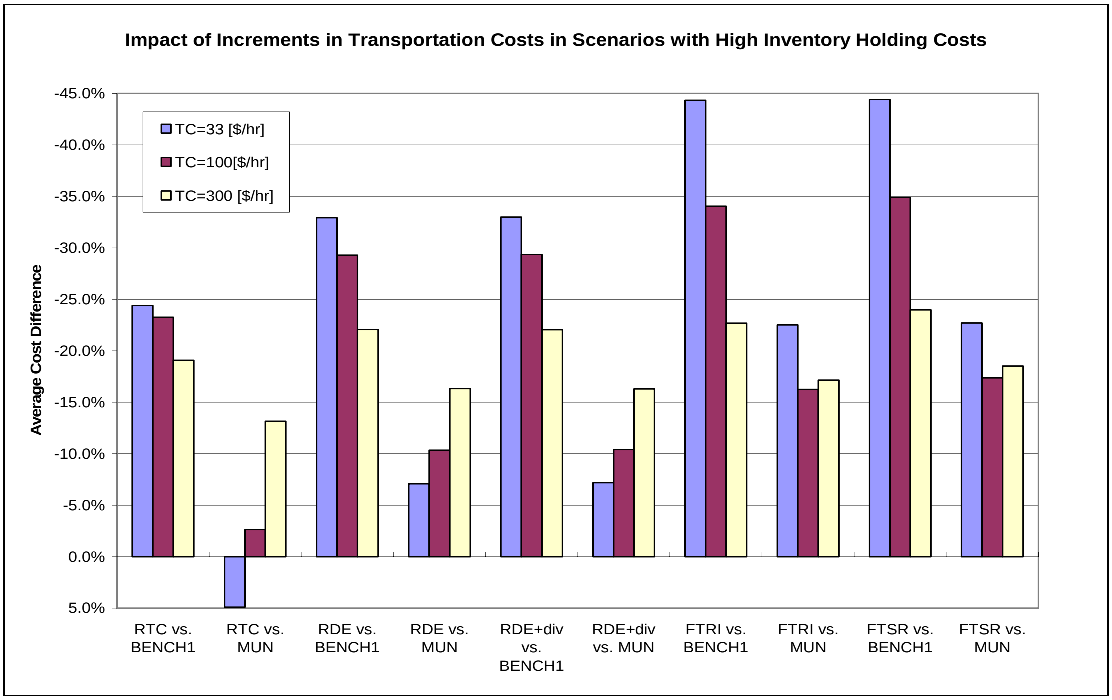


Figure 6- 18: Impact of Increments in Transportation Costs in Scenarios with High Inventory Holding Cost, Set of Parameters 1, 2 and 3

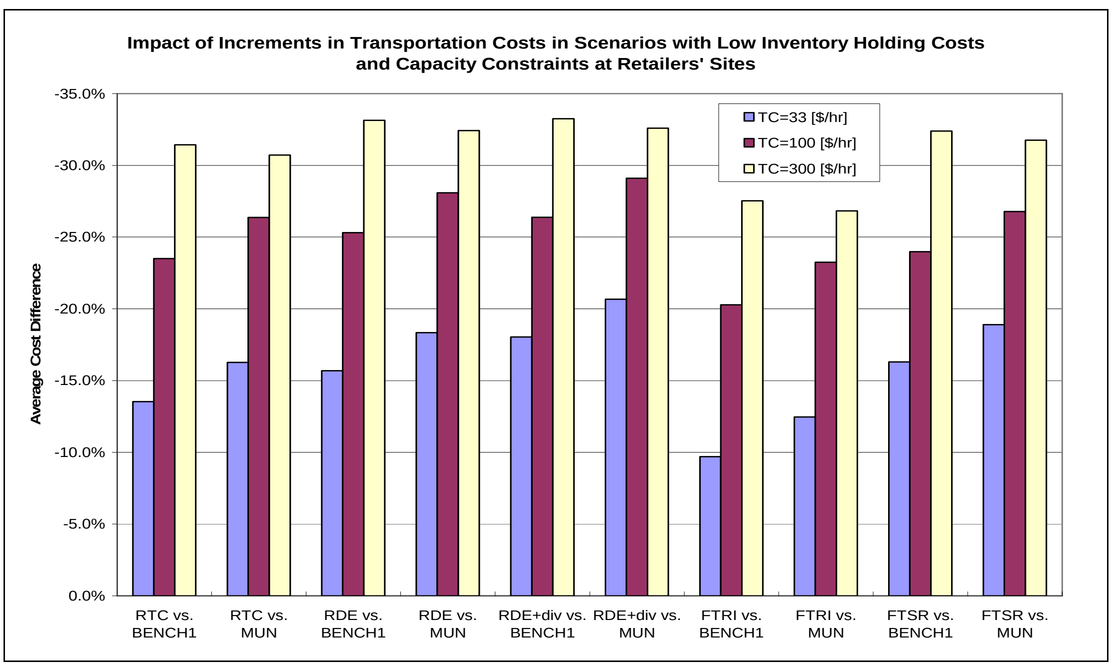


Figure 6- 19: Impact of Increments in Transportation Costs in Scenarios with Low Inventory Holding Cost, Set of Parameters 7, 8 and 9

Among re-planning strategies when transportation costs are more significant in the total costs, re-planning at delivery epochs is less beneficial, compared with re-planning only at tour completions. A comparison between RDE and RTC shows that RDE reduced by approximately 11%, 8%, and 4% the average total costs in scenarios with high inventory-holding costs, set of parameters 2 (TC=33), 1 (TC=100), and 3 (TC=300), respectively. In scenarios with low inventory-holding costs, the differences are less dramatic and remain relatively constant (around 2.5%) with respect to changes in transportation costs. Those values were computed in the same manner as Eq. (6.1), but replacing BENCH with RTC, for all sets of cases considered. Those differences could be explained mainly by the fact that in RTC higher inventory levels are maintained, therefore when inventory-holding costs are predominant in the total cost function, differences between RDE and RTC are higher.

6.3.3. Analysis of Tour Length

In Figure 6-20, the average number of visits per tour and the average tour length (in hours) are presented for each strategy. MUN was the only strategy with average tour lengths longer than 10 hours, because in that strategy the vehicle did not return to the depot until it was empty and might have delivered small quantities to facilities that were not near stock-out.

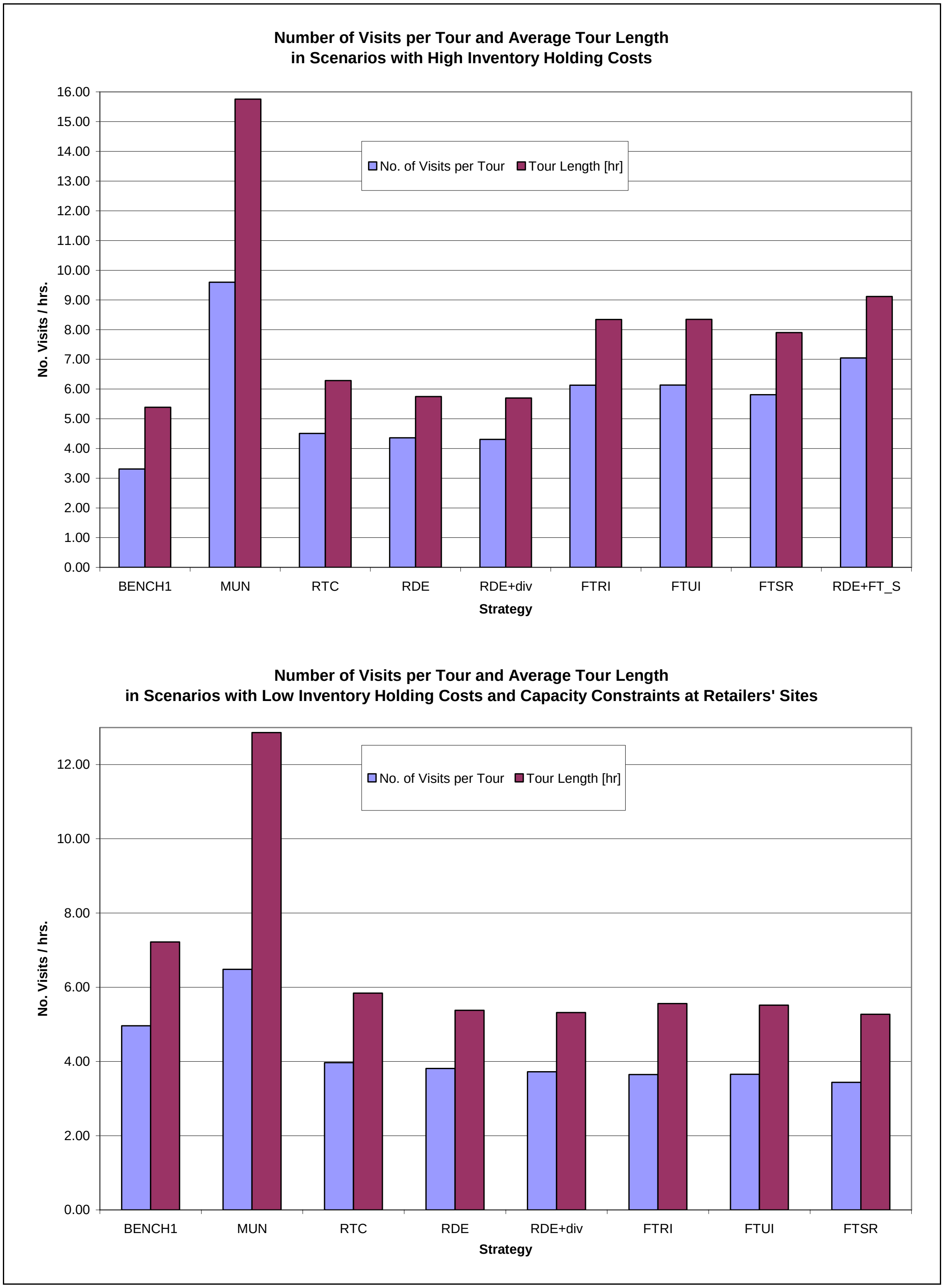


Figure 6- 20: Average Number of Visits per Tour and Average Tour Length

For scenarios with high inventory-holding costs, fixed-tour and RDE+FT_S (RDE with fixed-tour inventory-target levels) strategies tend to have longer routes and more visits than re-planning strategies with original inventory target levels, which is consistent with lower reorder quantities. It is interesting to observe that in those scenarios, fixed-tour and RDE+FT_S strategies produce higher vehicle-utilization rates. At the operational level studied in this research, higher utilization rates were good as long as they reduced total costs. However, when the system is designed, higher utilization rates might imply a larger fleet size. Hence, at a strategic-decision level, the trade-off between operational costs and fleet size fixed-costs should be taken into account.

In contrast, for scenarios with low inventory-holding costs, fixed-tour and re-planning strategies present similar average numbers of visits per tour and tour lengths, possibly because, in those scenarios, the quantities delivered in all proposed strategies are similar.

6.3.4. Analysis of Demand-Variability Impact

As shown in Figure 6-21, in scenarios with high inventory-holding costs, as demand variability increases, the benefits of the proposed strategies tend to decrease, compared to benchmark strategies. Conversely, in scenarios with low inventory-holding costs and capacity constraints, as demand variability increases, the benefits of the proposed strategies tend to increase vs. BENCH1 and decrease vs. MUN, as illustrated in Figure 6-22. As expected, MUN begins to be competitive in scenarios with very high demand variability.

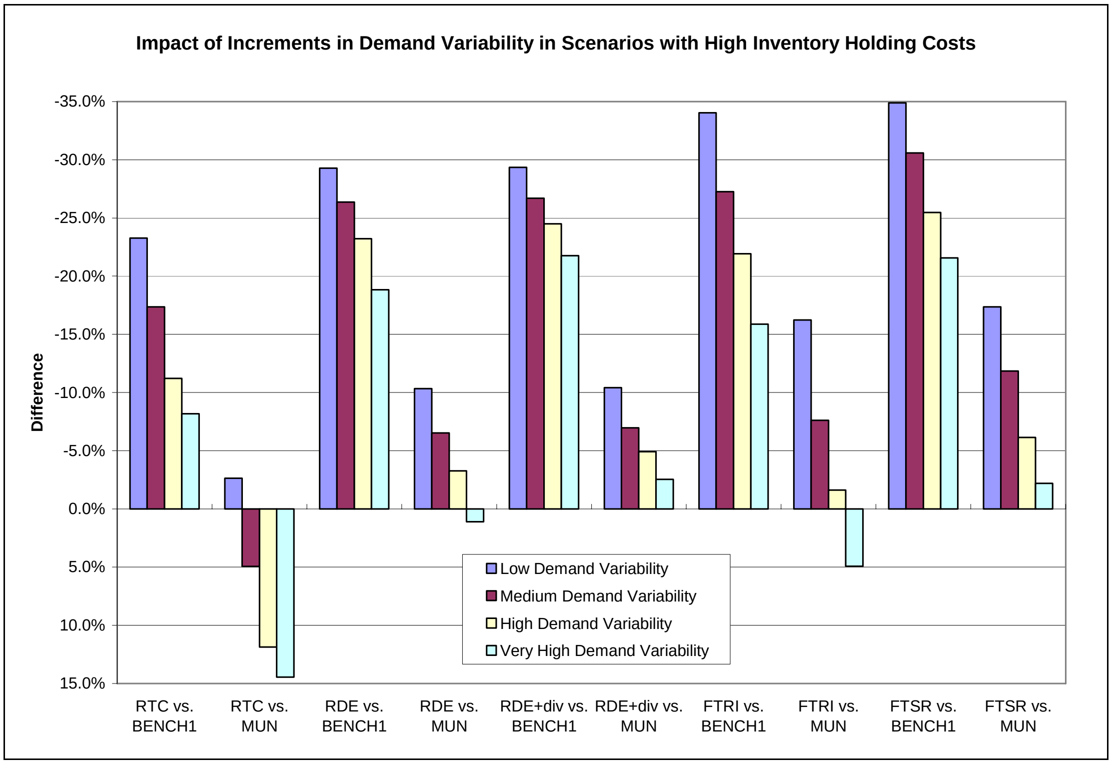


Figure 6- 21: Impact of Increments in Demand Variability in Scenarios with High Inventory-Holding Costs, Set of Parameters 1, 4, 5 and 6

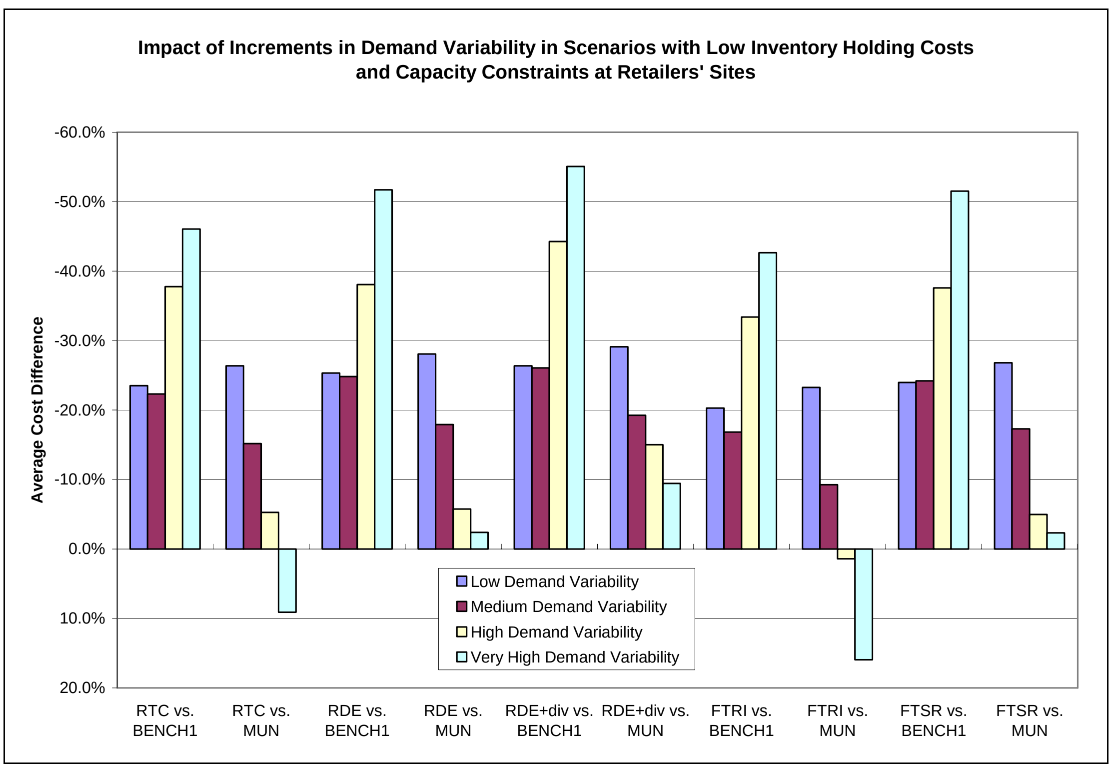


Figure 6- 22: Impact of Increments in Demand Variability in Scenarios with Low Inventory-Holding Costs, Set of Parameters 7, 10, 11 and 12

In scenarios with increased demand variability, i.e. those with set of Parameters No. 5, No. 6, No. 11, and No. 12, the advantage of re-planning strategies at delivery epochs, compared with re-planning only at tour completions, tends to be slightly higher than in scenarios with less demand variability.

Among fixed-tour strategies, in scenarios with low inventory-holding costs, the capability to update intervals offers increased benefits, compared to FTRI, as demand variability increases. In scenarios with high inventory-holding costs, the benefits of updating intervals, compared to regular intervals, remain relatively stable, since the reduction in lost sales tends to be compensated for by increments in holding costs.

The possibility of diversions—either en-route or when the vehicle is idle at the depot—improves system performance in scenarios with low inventory-holding costs and high demand variability. However, further research is needed to identify scenarios in which en-route diversion would be beneficial.

6.3.5. Analysis of Demand-Disruptions Scenarios

Finally, two scenarios for the set of Parameters No. 1 and No. 7, in which demand disruption occurs at a particular facility are studied. For those scenarios, the arrival rate of customers to Retailer 4 was doubled without updating the demand parameters on each strategy. The experiments were performed only for Case 0 (symmetric location of facilities) in order to isolate the impact of the demand disruption with its location.

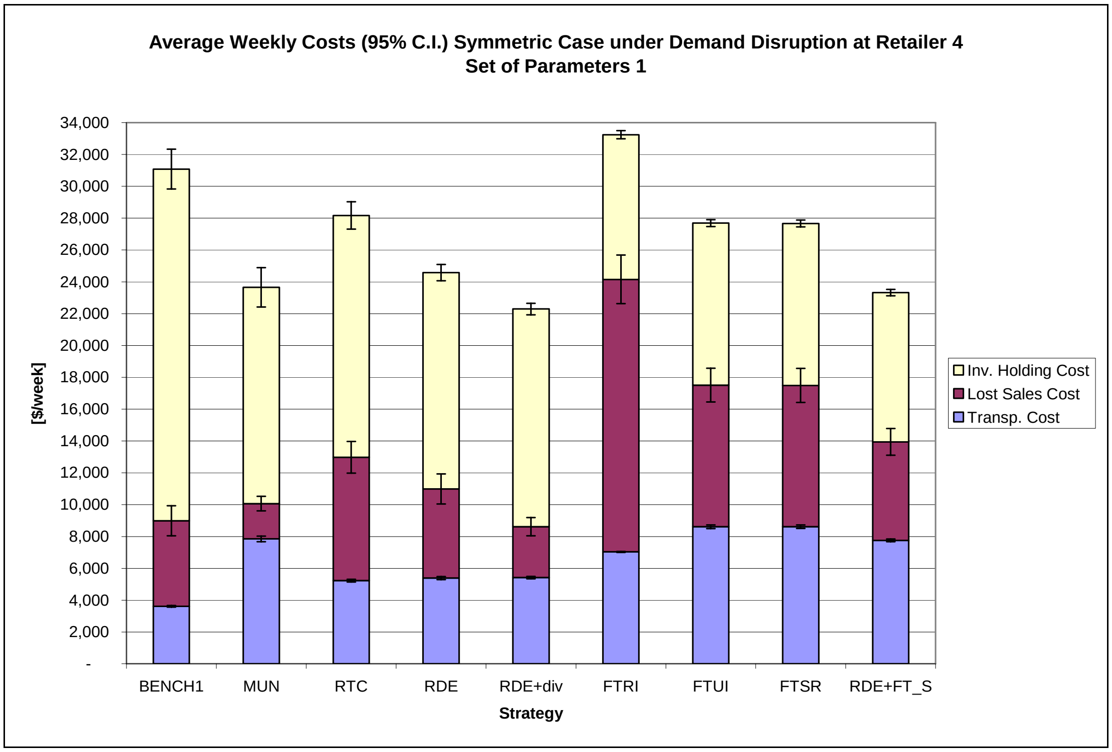


Figure 6- 23: Average Weekly Costs with 95% C.I. for the mean, under Twice Demand Arrivals at Retailer 4 and Without Inventory Target Update, for the Set of Parameters 1

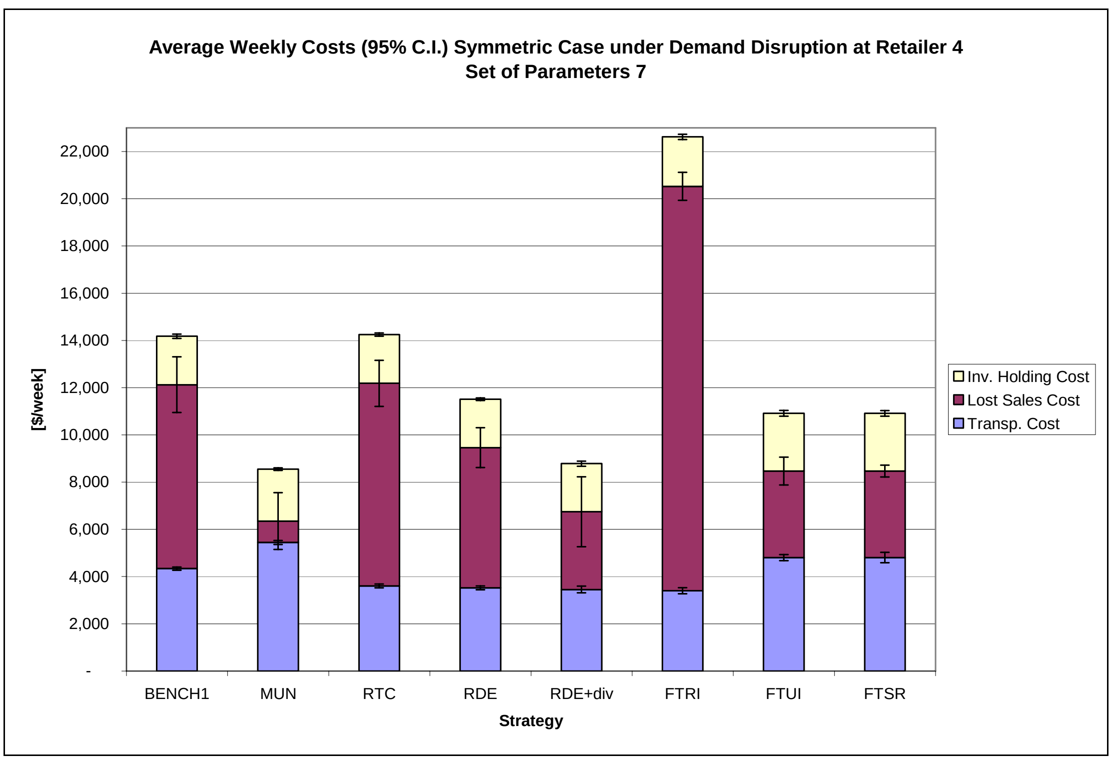


Figure 6- 24: Average Weekly Costs with 95% C.I. for the mean, under Twice Demand Arrivals at Retailer 4 and Without Inventory Target Update, for the Set of Parameters 7

As illustrated in Figure 6-23 and 6-24, the average total costs decomposed in transportation, lost sales, and inventory-holding costs for each policy. Also, 95% confidence intervals for the mean are presented under brackets for each component.

Re-planning strategies, particularly RDE+div and RDE+FT_S, systematically outperform fixed-tour strategies. That could be explained in part by the additional flexibility to react to demand disruptions in re-planning strategies; for example, it is possible to return to a facility already visited.

The MUN strategy had the best performance in scenarios with low inventory-holding costs and was among the best in scenarios with high inventory-holding costs. That is not surprising, since MUN is a greedy rule that cares well for the facility closest to running out of inventory. Nevertheless, when the parameters used in re-planning or fixed-tour strategies are accurate, MUN is systematically outperformed.

6.4. Summary of Main Results

The following is a summary of the main results reported in this chapter.

- The optimal decentralized benchmark policy, BENCH1, is systematically outperformed by centralized strategies, probably because BENCH1 tends to carry more inventory, thereby being protected from longer lead times.
- The developed online strategies systematically outperform benchmark strategies. The best proposed strategies achieved average-total-costs reductions of approximately 30% and 15%, respectively, compared with benchmark policies BENCH1 and MUN.

- Strategies that included complete re-planning at delivery epochs, with appropriate inventory-control parameters, had the best performance of the studied strategies for the set of parameters considered.
- Among re-planning strategies, those that updated plans at delivery epochs (RDE and RDE+div) were the best for the set of parameters considered. The possibility of diversions—either en-route or when the vehicle was idle at the depot—improve system performance in scenarios with low inventory-holding costs and high demand variability. However, further research is needed to identify scenarios in which en-route diversion would be beneficial.
- Among fixed-tour strategies, the capability to update tour intervals provided benefits of up to 10% in scenarios with low inventory-holding costs and high demand variability. Moreover, the possibility of skipping retailers on the route did not produce significant benefits in the scenarios studied.
- In scenarios with high inventory-holding costs and moderate demand variability, fixed-tour strategies performed among the best of those studied. In general, fixed-tour strategies are a very good benchmark when evaluating the implementation of more sophisticated real-time control strategies.

Chapter 7: Conclusions

This last chapter summarizes this dissertation's main contributions, findings, and conclusions. In addition, it lists possible extensions and directions for future research.

7.1. ***Summary of Contributions and Findings***

In this section the main objectives of this dissertation, which were outlined in Section 1.4, are reexamined, presenting related findings and conclusions.

The first objective of this research was to formulate and state the online inventory routing problem (OIRP) in a manner that explicitly took into account real-time information about fleet status and inventory levels at different facilities. That objective was achieved by providing a formal definition of the OIRP and discussing various problem features.

The second main objectives were the development and design of operational-control strategies, and the formulation of local off-line problems and heuristics used to update distribution plans within distribution systems, wherein transportation operations and inventory control operations were coordinated. Those objectives were accomplished by proposing two decomposition approaches in which a simplified version of either the inventory-control side or the routing side of the problem is solved first, and then that solution was used as a soft constraint when solving the other side. In the first approach, inventory-reorder parameters were first established for each facility, then used as target levels on a developed routing problem used to update plans. The main contributions in that approach were the formulation of a mathematical programming model for the short-term off-line IRP and the development of an inventory-control model that explicitly recognized queueing effects on multiple-orders. A key contribution in that MIP model was the development of an objective function that recognizes the operational trade-offs involved in online distribution decisions. In the second approach, the routing side of the problem was solved *a priori,* establishing *fixed-tours* to re-supply retailers; then, inventory allocation decisions were taken respecting that sequence. An important contribution of the second approach was the analytical derivation of expected costs for fixed tours at regular interval strategies. For both approaches, different rolling-horizon strategies were developed, tailored to different degrees of availability of real-time information associated with different scenarios in terms of the ICT installed.

The two final objectives in this research were: to improve the understanding of the relationship among problem parameters of a distribution system that could most benefit by implementing sophisticated control strategies; and to estimate the major benefits associated with implementing proposed real-time strategies. This last objective was accomplished primarily by simulation experiments for different scenarios. In order to fulfill that objective, a simulation framework was developed to analyze and evaluate the performance of the proposed dynamic-decision strategies for scenarios with: i) different facilities layout, ii) different relationship among cost parameters, and iii) different demand variabilities. In addition, two evaluation benchmarks for the OIRP were identified. The first was a decentralized system in which each agent followed optimal policies; namely, that each retailer applied an optimal single-echelon inventory-control policy, and that the vendor scheduled deliveries solving a VRP. The second benchmark took into consideration a centralized system with real-time communication capabilities in which a simple greedy rule was used to schedule the next delivery. Under that policy, at each delivery epoch, the vehicle is sent next to re-supply the retailer nearest to running out of inventory.

The main findings of those experiments can be summarized as follows:

- The developed online strategies systematically outperform benchmark strategies. Moreover, those strategies are able to reduce stock-out penalties better than the benchmarks considered. The best proposed strategies achieve reductions in average total costs of approximately 30% and 15% compared against benchmark policies BENCH1 and MUN respectively.
- Strategies that use complete re-planning at delivery epochs, with appropriate inventory-control parameters, have the best performance of the studied strategies for the set of parameters considered.
- Among re-planning strategies, those that update plans at delivery epochs, RDE and RDE+div, were the best strategies for the set of parameters considered. The possibility of diversions—either en-route or when the vehicle is idle at the depot—improve system performance in scenarios with low inventory-holding costs and high demand variability. However, further research is needed to identify scenarios in which en-route diversion would be beneficial.
- Among fixed-tour strategies, the possibility of updating tour intervals offers benefits of up to 10% in scenarios with low inventory-holding costs and high

demand variability. Moreover, the possibility of skipping retailers in the route did not produce significant benefits for scenarios studied.

- In scenarios with high inventory-holding costs and moderate demand variability, fixed-tour strategies perform among the best studied strategies. In general, fixed-tour strategies are a very good benchmark when evaluating the implementation of more sophisticated real-time control strategies.

7.2. ***Future Research and Extensions***

This dissertation presents the first study of the OIRP. Given the scope of this research, there are many promising extensions appropriate for attention in future research. The first set of extensions relate to topics that are either direct extensions of the problem studied or study aspects for which inconclusive answers were found.

- The results obtained in this dissertation provide a limited understanding of the benefits associated with en-route truck diversions. A more comprehensive understanding would require comparing strategies with diversions against similar strategies without diversions but including updates in idle time at the depot. Moreover, for scenarios with low stock-out probabilities, an analysis might require the design of different experiments in which those low probability events are sampled, using importance-sampling methods, for example.
- The improvement of the formulation of the inventory-control side of the problem in re-optimization strategies in order to incorporate synergies associated with serving clusters of retailers together.

- The development of hybrid strategies—those that integrate fixed-tour deliveries for normal conditions and the plan re-optimization approach when demand disruptions occur—would follow some promising avenues. In general, incorporating the possibility of updating idle time at the depot in re-optimization strategies is needed to better understand the advantages of re-routing vs. fixed-tour strategies.

The second set of extensions relate to topics that were beyond the scope of this dissertation. Such extensions can be divided into three groups: i) demand processes and types of operations considered, ii) types of uncertainties considered; and iii) design of distribution system for online operations. In terms of demand processes and types of operations allowed, the following can be considered:

a) The analysis of the performance of the proposed strategies could be extended to incorporate priorities for some facilities and scenarios with asymmetric demand patterns.
b) In this research a single product was considered in the analysis. However, most distribution systems deal with multiple products. An important extension might incorporate decisions related to the mix of products transported.
c) This research is assumed that demand processes at retailers could not be affected by the central decision maker. An extension to this research might deal with scenarios in which distribution decisions are combined with real-time pricing to, for example avoid stock-outs at particular facilities. Further refinements in that direction might consider scenarios in

which a supplier provides complement and substitute products and combine inventory and pricing decisions.

d) Incorporating daily- and weekly-cycle operational characteristics in the analysis, such as changes in demand patterns during the day and week, and end-of-cycle constraints, would be logical extensions. Moreover, including labor-related constraints in the vehicle routes, such as the number of hours that a driver can operate a vehicle, and time windows for particular facilities, would be worthy extensions.

e) Another possible extension would be to incorporate transshipment operations between retailers, since this dissertation assumed that products that had been delivered to one facility could not be reclaimed and reassigned to another facility.

In this research the only source of uncertainty were demand processes at different facilities. However, in real-world inventory-routing systems, additional activities are affected by uncertainties.

a) Traffic conditions are an important source of uncertainties, particularly in urban areas, as they can cause significantly varying travel times as a consequence of network congestion. Thus, extending this research to account for the uncertainty of travel times would seem to be a worthy extension.

b) In addition, time required for loading and unloading, which was not considered in this dissertation, is also affected by uncertainties. That is particularly relevant in maritime operations, in which ports are affected by weather and congestion which can significantly affect the amount of time spent on loading and unloading vehicles.

Finally, in terms of the design of distribution systems for online operations, there are topics that would extend this research, such as the following:

a) The implementation and analysis of the proposed strategies in large-scale inventory-routing systems. In particular, the development of solution approaches for re-planning strategies in scenarios with large number of retailers. Since for the proposed re-optimization formulation, with current technology, solution times grow exponentially with problem size, the implementation of re-planning strategies in large size instances probably would require the development of fast heuristics or metaheuristics.

b) The design of large-scale inventory-routing systems composed of multiple depots, retailers, and vehicles, to be operated with real-time ICT capabilities—in particular, the allocation of vehicles and retailers to depots. One example might be the analysis of strategies that could divide deliveries to a particular facility among more than one depot or truck.

c) The analysis of fleet-sizing decisions for real-time operation under different cost parameters.

d) The analysis of coordination mechanisms to achieve close-to-optimal system decisions in a decentralized inventory-routing system, such as pricing incentives that could align the strategic decisions of each player in a decentralized distribution system.

Appendix A: Notation

$\Im$: $\Im = \{1, 2, ..., i, ..., N\}$ set of retailers

$\Im_0$, $\Im_0 \equiv \Im \cup \{0\}$ set of all facilities (depot and retailers). Those N+1 facilities are denoted by sub-index i=0,1, 2, …, N (sub-index 0 is for the depot)

$l(i)$ locations for $i \in \Im_0$.

$d(\cdot,\cdot)$ function: gives the Euclidean distance between two facilities or between a facility and the vehicle location

κ_i :retailer i maximum capacity to store inventory

Υ : vehicle limited capacity.

$\lambda_i(t)$ Customers' arrival rates for retailer i at time t

$\psi_i^j(t)$: probability that a customer arriving at time t to retailer i has a demand size equal to j.

$\theta_i(t)$. mean of customer demand sizes when they are assumed to be Poisson distributed.

$\mu_i(t)$: the expected demand per unit of time at retailer i at time t, which can be calculated as $\mu_i(t) = \lambda_i(t) \cdot \theta_i(t)$.

$\tau_{i,m}$:Arrivals times for the m$^{\text{th}}$ customer arrival to facility i.

$\delta_{i,m}$ Demand sizes for the m$^{\text{th}}$ customer arrival to facility i.

$A_i(t) = \max\{m \geq 0 : \tau_{i,m} \leq t\}$: Total number of customer arrivals to retailer i that have occurred by time t

$D_i(t) = \sum_{m=1}^{A_i(t)} \delta_{i,m}$: Total demand at customer i until time t

$X(t)$: state of the system at time t. $X(t) = [\iota(t) \quad \ell(t) \quad \nu(t)]$.

$\iota_i(t)$: inventory level at facility i at time t.

$\iota(t) = (\iota_1(t), ..., \iota_i(t), ..., \iota_N(t))$: vector of inventory levels at time t

$\ell(t)$: Location of the truck at time t

$\nu(t)$: Load remaining in the truck at time t,.

$X(t) = [\iota(t) \quad \ell(t) \quad \nu(t)]$

TC: transportation cost per unit of distance traveled by the vehicle

h_i: Inventory holding costs at each retailer i

p_i: Penalty associated with each unit of demand lost during stock-out at retailer i

π : a plan or policy, which can be specified by $\pi = [\mathbf{s} \quad \mathbf{q} \quad \mathbf{t}]$

$\mathbf{s} = [s_1 \quad s_2 \quad \dots \quad s_L]$: Sequence of facilities to be visited.

$\mathbf{q} = [q_1 \quad q_2 \quad \dots \quad q_L]$: Amounts to be delivered.

$\mathbf{t} = \begin{bmatrix} t_1 & t_2 & \dots & t_L \end{bmatrix}$, arrival times to each one of those facilities, in which L is the length of the planning horizon in terms of number of visits programmed.

$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \dots \end{bmatrix}$: The sequence of update epochs in which u_n is the time of the nth plan update satisfying $u_{n+1} > u_n \geq 0$ for all n.

$U(t) = \max\{n \geq 0 : u_n \leq t\}$: number of plan updates up to epoch t.

$\pi(t) \equiv \pi_{U(t)}$ is the current plan at epoch t

$u(t) \equiv u_{U(t)}$ is the time of the last plan update

C_{Tr} : Transportation costs

C_{IH} : Inventory holding costs.

C_{ISO} : Inventory stock-out costs

C_{Crew} : Crew-associated costs

(s, S) policy, and can be stated: whenever the current inventory level is below the reorder point s, an order is placed to bring the inventory level to S; otherwise, do not place an order.

Appendix B: Glossary

APS advance planning and scheduling
ATMs: automatic teller machines
AVI automatic vehicle identification
AVL automatic vehicle location
CVO: commercial vehicle operations.
DP: Dynamic Programming
DSS: decision-support systems
EDI: Electronic Data Interchange
EOQ Economic Order Quantity
ERP: Enterprise Resource Planning
ERP-II: multi-component decision-support systems
GDP: gross demographic product
GIS Geographic Information System
GPS Global Positioning System
ICT: information and communication technologies
IID: independent and identically distributed
IP: Integer Program
IRPs inventory-routing problems
LTL: less than truck load
MDP: Markov decision process
MRP: material requirements planning
OBC on board computers
OIRP online inventory routing problem
PDA: personal digital assistant
RFID: radio frequency identification
RH: rolling horizon
SCP supply chain planning systems
SO: stock-outs
TL: truck loads
TMS transportation management systems
TSP: traveling salesman problem
VMI: vendor-managed inventory
VRP: vehicle-routing problem
VRPTW: vehicle-routing problem with time windows
WMS warehouse management systems
XML: eXchange Markup Language

Appendix C: Inventory Reorder Level Parameters

Table C- 1: Inventory Target Levels for Set of Parameters 1

Case	Retailer	$t(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	9.3	62.3	53.5	39.6	30.1	19.9	43.5	43.1	16.4	26.7
	2	35.7	62.3	53.5	39.6	30.1	19.9	43.5	43.1	16.4	26.7
	3	22.0	62.3	53.5	39.6	30.1	19.9	43.5	43.1	16.4	26.7
	4	4.4	62.3	53.5	39.6	30.1	19.9	43.5	43.1	16.4	26.7
	5	40.5	62.3	53.5	39.6	30.1	19.9	43.5	43.1	16.4	26.7
	6	5.3	62.3	53.5	39.6	30.1	19.9	43.5	43.1	16.4	26.7
	7	30.1	62.3	53.5	39.6	30.1	19.9	43.5	43.1	16.4	26.7
1	1	30.8	62.8	49.2	46.5	24.8	26.8	37.3	50.1	14.5	35.6
	2	47.6	61.2	63.8	46.1	39.9	27.2	54.0	50.1	21.4	28.7
	3	18.7	60.1	76.3	46.4	53.6	28.0	68.4	50.1	28.1	22.0
	4	9.5	61.7	58.9	46.1	34.8	27.0	48.4	50.1	19.0	31.1
	5	22.4	61.1	65.1	46.1	41.3	27.3	55.5	50.1	22.0	28.1
	6	12.1	61.5	60.9	46.1	36.8	27.1	50.6	50.1	19.9	30.2
	7	38.7	61.1	64.8	46.1	41.0	27.3	55.0	50.1	21.8	28.3
2	1	9.4	66.0	26.4	40.4	8.2	18.8	15.3	42.5	5.9	36.6
	2	10.9	62.4	52.7	38.5	29.7	18.8	42.6	42.5	16.0	26.5
	3	36.4	61.3	62.6	38.6	39.7	19.5	53.3	42.5	20.8	21.8
	4	48.2	61.7	59.0	38.5	36.0	19.2	49.4	42.5	19.0	23.5
	5	12.1	63.2	45.7	38.7	23.0	18.5	35.3	42.5	13.0	29.5
	6	58.5	60.7	69.2	38.9	46.6	20.0	60.5	42.5	24.2	18.3
	7	2.1	66.0	26.7	40.4	8.4	18.8	15.7	42.5	6.0	36.5
3	1	32.4	63.3	44.3	41.9	21.0	21.8	33.0	43.2	12.4	30.8
	2	18.9	60.9	67.0	41.6	43.7	22.8	57.8	43.2	23.0	20.2
	3	0.8	62.5	51.6	41.5	27.9	21.9	40.8	43.2	15.5	27.7
	4	18.9	63.8	40.8	42.2	17.9	21.8	29.3	43.2	11.0	32.2
	5	7.0	61.4	61.7	41.5	38.2	22.4	51.9	43.2	20.3	22.9
	6	30.3	61.5	60.8	41.5	37.2	22.4	51.0	43.2	19.9	23.3
	7	28.6	61.1	65.5	41.6	42.1	22.7	56.1	43.2	22.2	21.0

Table C- 2: Inventory Target Levels for Set of Parameters 2

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	6.5	65.0	32.5	45.1	18.6	26.0	30.5	33.0	16.4	16.6
	2	25.0	65.0	32.5	45.1	18.6	26.0	30.5	33.0	16.4	16.6
	3	15.4	65.0	32.5	45.1	18.6	26.0	30.5	33.0	16.4	16.6
	4	3.1	65.0	32.5	45.1	18.6	26.0	30.5	33.0	16.4	16.6
	5	28.4	65.0	32.5	45.1	18.6	26.0	30.5	33.0	16.4	16.6
	6	3.7	65.0	32.5	45.1	18.6	26.0	30.5	33.0	16.4	16.6
	7	21.2	65.0	32.5	45.1	18.6	26.0	30.5	33.0	16.4	16.6
1	1	21.9	65.4	30.1	53.3	17.5	35.0	26.6	42.9	14.5	28.4
	2	36.0	64.1	38.5	52.6	28.4	34.7	40.9	42.9	21.4	21.5
	3	14.7	63.2	45.6	52.5	40.0	35.2	53.7	42.9	28.1	14.8
	4	7.1	64.5	35.7	52.8	24.3	34.7	36.0	42.9	19.0	23.9
	5	17.0	64.0	39.2	52.5	29.6	34.8	42.2	42.9	22.0	20.9
	6	9.1	64.3	36.8	52.7	25.9	34.7	37.9	42.9	19.9	23.0
	7	29.4	64.0	39.0	52.6	29.3	34.8	41.8	42.9	21.8	21.0
2	1	5.3	68.1	17.0	44.7	6.1	24.9	8.6	32.3	5.9	26.4
	2	7.5	65.0	32.1	43.2	17.2	23.9	29.1	32.3	16.0	16.2
	3	25.8	64.2	37.8	43.1	24.3	24.3	37.7	32.3	20.8	11.5
	4	33.7	64.5	35.7	43.1	21.6	24.1	34.5	32.3	19.0	13.3
	5	8.0	65.7	28.1	43.5	13.1	23.8	23.3	32.3	13.0	19.3
	6	42.3	63.7	41.6	43.2	29.5	24.7	43.7	32.3	24.2	8.1
	7	1.2	68.0	17.2	44.6	6.2	24.9	8.9	32.3	6.0	26.3
3	1	22.0	65.9	27.3	48.0	13.2	28.6	22.4	33.1	12.4	20.7
	2	14.2	63.9	40.3	47.2	29.3	28.8	43.3	33.1	23.0	10.1
	3	0.6	65.2	31.4	47.6	17.3	28.5	28.9	33.1	15.5	17.6
	4	12.6	66.2	25.3	48.4	11.1	28.8	19.5	33.1	11.0	22.1
	5	5.1	64.3	37.3	47.2	25.0	28.6	38.3	33.1	20.3	12.8
	6	22.3	64.3	36.8	47.2	24.1	28.6	37.4	33.1	19.9	13.3
	7	21.3	64.0	39.4	47.2	27.9	28.8	41.8	33.1	22.2	10.9

Table C- 3: Inventory Target Levels for Set of Parameters 3

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	16.6	59.1	89.4	33.2	64.5	14.3	77.7	74.5	16.4	58.2
	2	63.8	59.1	89.4	33.2	64.5	14.3	77.7	74.5	16.4	58.2
	3	39.3	59.1	89.4	33.2	64.5	14.3	77.7	74.5	16.4	58.2
	4	7.8	59.1	89.4	33.2	64.5	14.3	77.7	74.5	16.4	58.2
	5	72.4	59.1	89.4	33.2	64.5	14.3	77.7	74.5	16.4	58.2
	6	9.5	59.1	89.4	33.2	64.5	14.3	77.7	74.5	16.4	58.2
	7	53.9	59.1	89.4	33.2	64.5	14.3	77.7	74.5	16.4	58.2
1	1	57.5	59.6	82.1	35.9	55.5	16.8	69.7	84.8	14.5	70.3
	2	83.4	57.8	107.3	36.1	79.9	17.7	94.6	84.8	21.4	63.4
	3	31.7	56.4	128.8	36.8	100.7	18.9	115.8	84.8	28.1	56.7
	4	16.9	58.4	98.9	35.9	71.8	17.3	86.3	84.8	19.0	65.8
	5	39.1	57.6	109.5	36.1	82.0	17.8	96.8	84.8	22.0	62.7
	6	21.5	58.1	102.2	36.0	75.0	17.5	89.6	84.8	19.9	64.9
	7	67.7	57.7	108.9	36.1	81.4	17.8	96.2	84.8	21.8	62.9
2	1	19.9	63.6	42.6	34.6	20.2	13.4	32.3	73.4	5.9	67.5
	2	19.8	59.2	88.0	33.0	63.4	14.0	77.1	73.4	16.0	57.4
	3	63.9	57.9	105.2	33.2	80.1	14.8	93.7	73.4	20.8	52.6
	4	85.4	58.4	98.9	33.1	74.0	14.5	87.4	73.4	19.0	54.4
	5	22.3	60.1	76.0	33.0	51.8	13.6	65.0	73.4	13.0	60.4
	6	102.0	57.1	116.6	33.6	91.2	15.5	105.5	73.4	24.2	49.2
	7	4.3	63.5	43.1	34.6	20.7	13.4	32.8	73.4	6.0	67.4
3	1	61.2	60.3	73.6	34.1	48.7	14.6	62.2	73.4	12.4	61.0
	2	32.9	57.4	112.7	34.5	86.4	16.2	100.6	73.4	23.0	50.4
	3	1.5	59.3	86.1	34.0	60.7	15.0	74.5	73.4	15.5	57.9
	4	36.4	60.9	67.6	34.3	42.9	14.5	56.3	73.4	11.0	62.4
	5	12.3	58.0	103.7	34.2	77.7	15.7	91.7	73.4	20.3	53.1
	6	53.7	58.1	102.1	34.2	76.2	15.7	90.2	73.4	19.9	53.5
	7	49.9	57.6	110.1	34.4	83.9	16.1	98.1	73.4	22.2	51.2

Table C- 4: Inventory Target Levels for Set of Parameters 4

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	10.4	70.1	56.9	45.4	35.8	22.9	48.5	48.7	23.7	25.1
	2	39.8	70.1	56.9	45.4	35.8	22.9	48.5	48.7	23.7	25.1
	3	24.5	70.1	56.9	45.4	35.8	22.9	48.5	48.7	23.7	25.1
	4	4.9	70.1	56.9	45.4	35.8	22.9	48.5	48.7	23.7	25.1
	5	45.2	70.1	56.9	45.4	35.8	22.9	48.5	48.7	23.7	25.1
	6	5.9	70.1	56.9	45.4	35.8	22.9	48.5	48.7	23.7	25.1
	7	33.6	70.1	56.9	45.4	35.8	22.9	48.5	48.7	23.7	25.1
1	1	36.0	70.8	52.7	51.8	31.2	29.0	43.6	56.0	21.2	34.8
	2	54.3	68.4	67.2	50.6	47.0	29.0	61.6	56.0	30.2	25.8
	3	21.2	66.5	79.6	50.4	61.6	29.5	77.6	56.0	38.7	17.2
	4	10.9	69.2	62.3	50.9	41.6	28.9	55.6	56.0	27.0	28.9
	5	25.6	68.2	68.5	50.6	48.5	29.0	63.3	56.0	31.0	24.9
	6	13.9	68.8	64.3	50.8	43.8	28.9	58.0	56.0	28.3	27.7
	7	44.2	68.2	68.1	50.6	48.1	29.0	62.8	56.0	30.8	25.2
2	1	11.2	76.0	30.0	48.2	12.2	22.9	18.2	48.1	9.2	39.0
	2	12.1	70.2	56.1	45.0	34.1	22.1	47.1	48.1	23.2	24.9
	3	40.1	68.6	66.0	44.6	44.5	22.6	58.7	48.1	29.4	18.7
	4	53.2	69.1	62.4	44.7	40.7	22.4	54.5	48.1	27.1	21.0
	5	13.5	71.5	49.2	45.5	27.3	22.0	39.3	48.1	19.1	29.0
	6	64.5	67.6	72.6	44.5	51.7	23.0	66.7	48.1	33.8	14.3
	7	2.4	75.9	30.3	48.1	12.4	22.9	18.6	48.1	9.3	38.8
3	1	37.1	71.8	47.8	48.0	26.7	24.8	37.7	48.8	18.4	30.4
	2	21.1	67.9	70.4	46.6	50.6	25.3	64.6	48.8	32.3	16.5
	3	0.9	70.4	55.0	47.2	33.9	24.8	46.1	48.8	22.5	26.3
	4	21.9	72.5	44.3	48.4	23.5	25.0	33.8	48.8	16.4	32.4
	5	7.8	68.7	65.1	46.7	44.8	25.1	58.2	48.8	28.8	20.0
	6	34.0	68.8	64.2	46.7	43.8	25.0	57.1	48.8	28.2	20.6
	7	31.9	68.1	68.9	46.6	48.9	25.3	62.8	48.8	31.3	17.5

Table C- 5: Inventory Target Levels for Set of Parameters 5

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	*S-s*	*s*	*S-s*	*s*	*S-s*	*S*	*s*	*S-s*
0	1	11.1	78.2	61.1	52.0	40.7	26.7	52.1	48.7	32.0	23.2
	2	42.7	78.2	61.1	52.0	40.7	26.7	52.1	48.7	32.0	23.2
	3	26.3	78.2	61.1	52.0	40.7	26.7	52.1	48.7	32.0	23.2
	4	5.2	78.2	61.1	52.0	40.7	26.7	52.1	48.7	32.0	23.2
	5	48.5	78.2	61.1	52.0	40.7	26.7	52.1	48.7	32.0	23.2
	6	6.4	78.2	61.1	52.0	40.7	26.7	52.1	48.7	32.0	23.2
	7	36.1	78.2	61.1	52.0	40.7	26.7	52.1	48.7	32.0	23.2
1	1	39.7	79.2	56.9	57.4	37.6	32.4	48.2	56.0	28.7	36.0
	2	59.3	75.8	71.3	55.5	54.3	32.0	67.2	56.0	40.2	24.5
	3	23.1	73.2	83.7	54.7	69.8	32.1	84.4	56.0	50.9	13.8
	4	11.9	76.9	66.5	56.0	48.6	32.0	60.8	56.0	36.3	28.4
	5	27.9	75.5	72.6	55.4	55.9	32.0	69.0	56.0	41.3	23.4
	6	15.2	76.4	68.4	55.8	50.9	32.0	63.3	56.0	37.8	26.9
	7	48.2	75.6	72.3	55.5	55.5	32.0	68.5	56.0	41.0	23.7
2	1	12.6	86.0	34.7	55.6	17.2	27.8	20.4	48.1	12.9	41.6
	2	12.9	78.4	60.3	51.2	40.5	26.1	50.3	48.1	31.4	23.1
	3	42.7	76.1	70.1	50.3	51.5	26.3	62.6	48.1	39.2	15.3
	4	56.7	76.9	66.5	50.6	47.5	26.2	58.1	48.1	36.3	18.2
	5	14.4	80.1	53.4	52.0	33.3	26.1	42.1	48.1	26.1	28.4
	6	68.8	74.6	76.7	50.0	59.3	26.6	71.1	48.1	44.8	9.7
	7	2.7	85.9	35.0	55.6	17.5	27.8	20.8	48.1	13.1	41.4
3	1	40.4	80.5	52.1	54.9	31.5	28.9	41.0	48.8	25.1	30.1
	2	22.8	75.1	74.5	52.3	56.3	28.7	69.5	48.8	42.9	12.4
	3	1.0	78.6	59.2	53.8	39.0	28.6	49.9	48.8	30.5	24.7
	4	23.9	81.5	48.6	55.5	28.2	29.1	37.0	48.8	22.6	32.6
	5	8.4	76.3	69.2	52.7	50.2	28.6	62.7	48.8	38.5	16.7
	6	36.6	76.5	68.4	52.8	49.2	28.6	61.6	48.8	37.8	17.4
	7	34.4	75.4	73.0	52.4	54.6	28.7	67.6	48.8	41.6	13.6

Table C- 6: Inventory Target Levels for Set of Parameters 6

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	12.0	85.4	65.6	58.3	44.8	30.0	56.4	48.7	40.3	21.1
	2	46.3	85.4	65.6	58.3	44.8	30.0	56.4	48.7	40.3	21.1
	3	28.5	85.4	65.6	58.3	44.8	30.0	56.4	48.7	40.3	21.1
	4	5.7	85.4	65.6	58.3	44.8	30.0	56.4	48.7	40.3	21.1
	5	52.6	85.4	65.6	58.3	44.8	30.0	56.4	48.7	40.3	21.1
	6	6.9	85.4	65.6	58.3	44.8	30.0	56.4	48.7	40.3	21.1
	7	39.1	85.4	65.6	58.3	44.8	30.0	56.4	48.7	40.3	21.1
1	1	43.7	86.7	61.4	63.5	42.3	35.3	53.0	56.0	36.3	38.2
	2	64.6	82.4	75.7	60.8	59.4	34.4	73.3	56.0	50.3	24.3
	3	25.1	79.2	87.9	59.3	75.5	34.2	91.9	56.0	63.1	11.5
	4	13.0	83.8	70.9	61.6	53.5	34.6	66.4	56.0	45.5	29.1
	5	30.4	82.1	76.9	60.7	61.0	34.3	75.2	56.0	51.6	23.0
	6	16.6	83.2	72.8	61.3	55.8	34.5	69.2	56.0	47.4	27.2
	7	52.6	82.2	76.6	60.7	60.6	34.4	74.7	56.0	51.2	23.4
2	1	14.1	94.7	39.9	63.1	20.9	32.1	22.9	48.1	16.6	44.2
	2	13.9	85.6	64.8	57.6	44.5	29.5	54.2	48.1	39.5	21.3
	3	46.0	82.8	74.5	56.2	55.8	29.5	67.4	48.1	49.1	11.8
	4	61.1	83.8	70.9	56.7	51.6	29.4	62.5	48.1	45.6	15.3
	5	15.6	87.8	58.0	58.8	37.3	29.8	45.5	48.1	33.1	27.7
	6	74.1	81.0	81.0	55.6	63.8	29.6	76.6	48.1	55.8	5.1
	7	3.1	94.5	40.2	63.0	21.2	32.0	23.4	48.1	16.9	44.0
3	1	44.2	88.2	56.7	61.1	36.3	32.3	45.0	48.8	31.9	29.5
	2	24.7	81.6	78.8	57.5	62.0	31.4	75.4	48.8	53.5	8.0
	3	1.1	86.0	63.7	59.7	44.0	31.8	54.3	48.8	38.5	22.9
	4	26.3	89.4	53.3	61.9	33.0	32.6	40.7	48.8	28.8	32.6
	5	9.1	83.0	73.6	58.1	55.6	31.5	68.0	48.8	48.2	13.2
	6	39.7	83.3	72.7	58.2	54.6	31.5	66.7	48.8	47.4	14.1
	7	37.3	82.0	77.3	57.7	60.2	31.5	73.3	48.8	52.0	9.5

Table C- 7: Inventory Target Levels for Set of Parameters 7

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	17.5	63.6	36.4	38.4	61.6	18.1	81.9	48.7	16.4	83.6
	2	67.2	63.6	36.4	38.4	61.6	18.1	81.9	48.7	16.4	83.6
	3	41.4	63.6	36.4	38.4	61.6	18.1	81.9	48.7	16.4	83.6
	4	8.2	63.6	36.4	38.4	61.6	18.1	81.9	48.7	16.4	83.6
	5	76.3	63.6	36.4	38.4	61.6	18.1	81.9	48.7	16.4	83.6
	6	10.0	63.6	36.4	38.4	61.6	18.1	81.9	48.7	16.4	83.6
	7	56.8	63.6	36.4	38.4	61.6	18.1	81.9	48.7	16.4	83.6
1	1	62.8	64.5	35.5	44.4	55.6	23.9	76.1	56.0	14.5	85.5
	2	66.5	61.6	38.4	44.4	55.6	24.6	75.4	56.0	21.4	78.6
	3	20.4	59.3	40.7	44.8	55.2	25.5	74.5	56.0	28.1	71.9
	4	14.8	62.6	37.4	44.4	55.6	24.3	75.7	56.0	19.0	81.0
	5	30.4	61.4	38.6	44.5	55.5	24.7	75.3	56.0	22.0	78.0
	6	18.1	62.2	37.8	44.4	55.6	24.4	75.6	56.0	19.9	80.1
	7	53.0	61.5	38.5	44.5	55.5	24.7	75.3	56.0	21.8	78.2
2	1	38.0	70.1	29.9	39.2	48.7	16.5	61.5	48.1	5.9	94.1
	2	21.0	63.8	36.2	38.5	61.5	18.2	81.8	48.1	16.0	84.0
	3	55.2	61.9	38.1	38.9	61.1	19.1	80.9	48.1	20.8	79.2
	4	79.3	62.6	37.4	38.7	61.3	18.8	81.2	48.1	19.0	81.0
	5	28.2	65.3	34.7	38.4	61.6	17.7	82.3	48.1	13.0	87.0
	6	77.6	60.6	39.4	39.2	60.8	19.7	80.3	48.1	24.2	75.8
	7	8.1	70.0	30.0	39.2	49.7	16.5	62.1	48.1	6.0	94.0
3	1	79.5	65.6	34.4	40.0	60.0	19.2	80.8	48.8	12.4	87.6
	2	25.9	61.0	39.0	40.5	59.5	20.9	79.1	48.8	23.0	77.0
	3	1.6	64.0	36.0	40.1	59.9	19.7	80.3	48.8	15.5	84.5
	4	52.4	66.3	33.7	40.0	60.0	19.0	81.0	48.8	11.0	89.0
	5	10.7	62.0	38.0	40.3	59.7	20.4	79.6	48.8	20.3	79.7
	6	47.4	62.2	37.8	40.3	59.7	20.4	79.6	48.8	19.9	80.1
	7	40.3	61.3	38.7	40.4	59.6	20.8	79.2	48.8	22.2	77.8

Table C- 8: Inventory Target Levels for Set of Parameters 8

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	17.2	68.4	31.6	40.7	59.3	19.4	80.6	94.9	14.5	80.4
	2	66.1	68.4	31.6	40.7	59.3	19.4	80.6	94.9	21.4	73.6
	3	40.7	68.4	31.6	40.7	59.3	19.4	80.6	94.9	28.1	66.9
	4	8.1	68.4	31.6	40.7	59.3	19.4	80.6	94.9	19.0	76.0
	5	75.1	68.4	31.6	40.7	59.3	19.4	80.6	94.9	22.0	72.9
	6	9.8	68.4	31.6	40.7	59.3	19.4	80.6	94.9	19.9	75.0
	7	55.9	68.4	31.6	40.7	59.3	19.4	80.6	94.9	21.8	73.1
1	1	51.9	69.0	31.0	47.4	52.6	26.0	62.9	82.3	5.9	76.5
	2	64.2	66.9	33.1	48.0	52.0	27.2	72.8	82.3	16.0	66.3
	3	19.6	65.3	34.7	48.7	51.3	28.4	71.6	82.3	20.8	61.6
	4	14.3	67.6	32.4	47.8	52.2	26.8	73.2	82.3	19.0	63.4
	5	29.4	66.7	33.3	48.1	51.9	27.3	72.7	82.3	13.0	69.3
	6	17.5	67.3	32.7	47.9	52.1	27.0	73.0	82.3	24.2	58.2
	7	51.2	66.8	33.2	48.1	51.9	27.3	72.7	82.3	6.0	76.4
2	1	20.9	73.3	26.7	41.7	20.2	17.7	33.9	82.4	12.4	69.9
	2	20.6	68.5	31.5	41.0	59.0	19.8	80.2	82.4	23.0	59.4
	3	54.0	67.1	32.9	41.7	58.3	21.0	79.0	82.4	15.5	66.8
	4	77.6	67.6	32.4	41.4	58.6	20.5	79.5	82.4	11.0	71.3
	5	23.2	69.6	30.4	41.1	53.5	18.8	67.6	82.4	20.3	62.1
	6	75.6	66.2	33.8	42.2	57.8	21.8	78.2	82.4	19.9	62.5
	7	4.5	73.2	26.8	41.7	20.8	17.7	34.4	82.4	22.2	60.2
3	1	63.0	69.8	30.2	42.9	50.0	20.5	64.1	83.6	16.4	67.2
	2	25.2	66.5	33.5	43.6	56.4	23.1	76.9	83.6	16.4	67.2
	3	1.5	68.7	31.3	42.6	57.4	21.2	75.4	83.6	16.4	67.2
	4	37.8	70.4	29.6	42.9	44.0	20.2	58.4	83.6	16.4	67.2
	5	10.4	67.2	32.8	43.3	56.7	22.5	77.5	83.6	16.4	67.2
	6	46.2	67.3	32.7	43.2	56.8	22.4	77.6	83.6	16.4	67.2
	7	39.2	66.7	33.3	43.5	56.5	22.9	77.1	83.6	16.4	67.2

Table C- 9: Inventory Target Levels for Set of Parameters 9

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	17.9	56.9	43.1	35.0	65.0	16.0	84.0	100.0	14.5	85.5
	2	68.9	56.9	43.1	35.0	65.0	16.0	84.0	100.0	21.4	78.6
	3	42.4	56.9	43.1	35.0	65.0	16.0	84.0	100.0	28.1	71.9
	4	8.4	56.9	43.1	35.0	65.0	16.0	84.0	100.0	19.0	81.0
	5	78.2	56.9	43.1	35.0	65.0	16.0	84.0	100.0	22.0	78.0
	6	10.2	56.9	43.1	35.0	65.0	16.0	84.0	100.0	19.9	80.1
	7	58.2	56.9	43.1	35.0	65.0	16.0	84.0	100.0	21.8	78.2
1	1	65.1	58.3	41.7	40.2	59.8	21.1	78.9	100.0	5.9	94.1
	2	69.4	53.4	46.6	39.6	60.4	21.3	78.7	100.0	16.0	84.0
	3	21.4	48.4	51.6	39.4	60.6	21.7	78.3	100.0	20.8	79.2
	4	15.4	55.1	44.9	39.8	60.2	21.2	78.8	100.0	19.0	81.0
	5	31.8	52.9	47.1	39.6	60.4	21.4	78.6	100.0	13.0	87.0
	6	18.9	54.4	45.6	39.7	60.3	21.3	78.7	100.0	24.2	75.8
	7	55.4	53.1	46.9	39.6	60.4	21.4	78.6	100.0	6.0	94.0
2	1	52.0	65.9	34.1	36.8	63.2	15.7	84.3	100.0	12.4	87.6
	2	21.5	57.2	42.8	35.2	64.8	16.1	83.9	100.0	23.0	77.0
	3	56.9	53.8	46.2	35.1	64.9	16.6	83.4	100.0	15.5	84.5
	4	81.6	55.1	44.9	35.1	64.9	16.4	83.6	100.0	11.0	89.0
	5	28.9	59.4	40.6	35.4	64.6	15.9	84.1	100.0	20.3	79.7
	6	80.2	51.4	48.6	35.1	64.9	17.1	82.9	100.0	19.9	80.1
	7	11.0	65.8	34.2	36.8	63.2	15.7	84.3	100.0	22.2	77.8
3	1	81.4	59.8	40.2	36.8	63.2	17.3	82.7	100.0	16.4	83.6
	2	26.8	52.3	47.7	36.3	63.7	18.1	81.9	100.0	16.4	83.6
	3	1.7	57.5	42.5	36.5	63.5	17.5	82.5	100.0	16.4	83.6
	4	53.6	60.9	39.1	37.1	62.9	17.2	82.8	100.0	16.4	83.6
	5	11.0	54.1	45.9	36.3	63.7	17.9	82.1	100.0	16.4	83.6
	6	48.9	54.5	45.5	36.3	63.7	17.8	82.2	100.0	16.4	83.6
	7	41.7	52.8	47.2	36.3	63.7	18.1	81.9	100.0	16.4	83.6

Table C- 10: Inventory Target Levels for Set of Parameters 10

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	16.3	69.4	30.6	47.3	52.7	23.7	76.3	100.0	21.2	78.8
	2	62.6	69.4	30.6	47.3	52.7	23.7	76.3	100.0	30.2	69.8
	3	38.6	69.4	30.6	47.3	52.7	23.7	76.3	100.0	38.7	61.3
	4	7.7	69.4	30.6	47.3	52.7	23.7	76.3	100.0	27.0	73.0
	5	71.1	69.4	30.6	47.3	52.7	23.7	76.3	100.0	31.0	69.0
	6	9.3	69.4	30.6	47.3	52.7	23.7	76.3	100.0	28.3	71.7
	7	52.9	69.4	30.6	47.3	52.7	23.7	76.3	100.0	30.8	69.2
1	1	65.1	70.8	29.2	56.0	44.0	21.1	78.9	100.0	9.2	90.8
	2	69.4	66.2	33.8	54.6	45.4	21.3	78.7	100.0	23.2	76.8
	3	21.4	62.4	37.6	53.8	46.2	21.7	78.3	100.0	29.4	70.6
	4	15.4	67.7	32.3	55.0	45.0	21.2	78.8	100.0	27.1	72.9
	5	31.8	65.8	34.2	54.5	45.5	21.4	78.6	100.0	19.1	80.9
	6	18.9	67.1	32.9	54.8	45.2	21.3	78.7	100.0	33.8	66.2
	7	55.4	65.9	34.1	54.5	45.5	21.4	78.6	100.0	9.3	90.7
2	1	40.5	79.6	20.4	49.6	50.4	22.9	65.6	100.0	18.4	81.6
	2	19.5	69.7	30.3	47.4	52.6	23.9	76.1	100.0	32.3	67.7
	3	51.6	66.5	33.5	47.0	53.0	24.5	75.5	100.0	22.5	77.5
	4	74.0	67.7	32.3	47.1	52.9	24.2	75.8	100.0	16.4	83.6
	5	26.2	72.0	28.0	47.8	52.2	23.5	76.5	100.0	28.8	71.2
	6	72.6	64.5	35.5	46.9	53.1	25.0	75.0	100.0	28.2	71.8
	7	8.7	79.5	20.5	49.6	50.4	22.9	66.5	100.0	31.3	68.7
3	1	73.1	72.5	27.5	50.1	49.9	25.7	74.3	100.0	23.7	76.3
	2	24.0	65.2	34.8	48.9	51.1	26.6	73.4	100.0	23.7	76.3
	3	1.5	70.1	29.9	49.6	50.4	25.9	74.1	100.0	23.7	76.3
	4	48.1	73.8	26.2	50.4	49.6	25.6	74.4	100.0	23.7	76.3
	5	9.9	66.8	33.2	49.0	51.0	26.3	73.7	100.0	23.7	76.3
	6	43.9	67.1	32.9	49.1	50.9	26.3	73.7	100.0	23.7	76.3
	7	37.4	65.7	34.3	48.9	51.1	26.5	73.5	100.0	23.7	76.3

Table C- 11: Inventory Target Levels for Set of Parameters 11

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	*S-s*	*s*	*S-s*	*s*	*S-s*	*S*	*s*	*S-s*
0	1	14.9	71.2	28.8	56.8	43.2	30.1	69.9	100.0	28.7	71.3
	2	57.3	71.2	28.8	56.8	43.2	30.1	69.9	100.0	40.2	59.8
	3	35.3	71.2	28.8	56.8	43.2	30.1	69.9	100.0	50.9	49.1
	4	7.0	71.2	28.8	56.8	43.2	30.1	69.9	100.0	36.3	63.7
	5	65.1	71.2	28.8	56.8	43.2	30.1	69.9	100.0	41.3	58.7
	6	8.5	71.2	28.8	56.8	43.2	30.1	69.9	100.0	37.8	62.2
	7	48.5	71.2	28.8	56.8	43.2	30.1	69.9	100.0	41.0	59.0
1	1	48.8	72.9	27.1	68.1	31.9	40.9	59.1	100.0	12.9	87.1
	2	53.4	67.0	33.0	64.4	35.6	39.4	60.6	100.0	31.4	68.6
	3	16.8	62.0	38.0	61.9	38.1	38.5	61.5	100.0	39.2	60.8
	4	11.8	69.0	31.0	65.5	34.5	39.8	60.2	100.0	36.3	63.7
	5	24.5	66.5	33.5	64.2	35.8	39.3	60.7	100.0	26.1	73.9
	6	14.5	68.2	31.8	65.1	34.9	39.7	60.3	100.0	44.8	55.2
	7	42.7	66.7	33.3	64.2	35.8	39.3	60.7	100.0	13.1	86.9
2	1	39.3	82.4	17.6	62.2	37.8	30.8	63.7	100.0	25.1	74.9
	2	17.9	71.5	28.5	56.9	43.1	30.3	69.7	100.0	42.9	57.1
	3	47.4	67.5	32.5	55.6	44.4	30.5	69.5	100.0	30.5	69.5
	4	67.9	69.0	31.0	56.0	44.0	30.4	69.6	100.0	22.6	77.4
	5	23.9	74.3	25.7	57.9	42.1	30.3	69.7	100.0	38.5	61.5
	6	67.0	64.9	35.1	55.0	45.0	30.8	69.2	100.0	37.8	62.2
	7	8.5	82.3	17.7	62.3	37.7	30.8	65.0	100.0	41.6	58.4
3	1	65.5	74.9	25.1	61.2	38.8	33.4	66.6	100.0	32.0	68.0
	2	21.9	65.8	34.2	57.5	42.5	33.0	67.0	100.0	32.0	68.0
	3	1.4	72.0	28.0	59.8	40.2	33.2	66.8	100.0	32.0	68.0
	4	43.0	76.4	23.6	62.0	38.0	33.6	66.4	100.0	32.0	68.0
	5	9.0	67.9	32.1	58.2	41.8	33.0	67.0	100.0	32.0	68.0
	6	39.9	68.2	31.8	58.3	41.7	33.0	67.0	100.0	32.0	68.0
	7	34.1	66.4	33.6	57.7	42.3	33.0	67.0	100.0	32.0	68.0

Table C- 12: Inventory Target Levels for Set of Parameters 12

Case	Retailer	$\iota(0)$	BENCH1		RTC		RDE		FTRI	RDE+FT_S	
			s	S-s	s	S-s	s	S-s	S	s	S-s
0	1	13.6	68.4	31.6	64.8	35.2	36.3	63.7	100.0	40.3	59.7
	2	52.3	68.4	31.6	64.8	35.2	36.3	63.7	100.0	40.3	59.7
	3	32.2	68.4	31.6	64.8	35.2	36.3	63.7	100.0	40.3	59.7
	4	6.4	68.4	31.6	64.8	35.2	36.3	63.7	100.0	40.3	59.7
	5	59.4	68.4	31.6	64.8	35.2	36.3	63.7	100.0	40.3	59.7
	6	7.8	68.4	31.6	64.8	35.2	36.3	63.7	100.0	40.3	59.7
	7	44.2	68.4	31.6	64.8	35.2	36.3	63.7	100.0	40.3	59.7
1	1	42.4	70.1	29.9	73.9	26.1	48.6	51.4	100.0	36.3	63.7
	2	48.3	63.9	36.1	68.0	32.0	45.3	54.7	100.0	50.3	49.7
	3	15.6	58.2	41.8	63.5	36.5	42.8	57.2	100.0	63.1	36.9
	4	10.5	66.0	34.0	69.8	30.2	46.3	53.7	100.0	45.5	54.5
	5	22.2	63.3	36.7	67.5	32.5	45.0	55.0	100.0	51.6	48.4
	6	13.0	65.2	34.8	69.1	30.9	45.9	54.1	100.0	47.4	52.6
	7	38.7	63.5	36.5	67.6	32.4	45.1	54.9	100.0	51.2	48.8
2	1	37.6	78.6	21.4	74.0	26.0	39.0	61.0	100.0	16.6	83.4
	2	16.2	68.7	31.3	65.1	34.9	36.7	63.3	100.0	39.5	60.5
	3	43.5	64.4	35.6	62.8	37.2	36.3	63.7	100.0	49.1	50.9
	4	62.1	66.0	34.0	63.6	36.4	36.4	63.6	100.0	45.6	54.4
	5	21.6	71.6	28.4	67.0	33.0	37.1	62.9	100.0	33.1	66.9
	6	61.8	61.5	38.5	61.5	38.5	36.1	63.9	100.0	55.8	44.2
	7	8.0	78.5	21.5	73.8	26.2	39.0	61.0	100.0	16.9	83.1
3	1	58.3	72.1	27.9	70.7	29.3	40.7	59.3	100.0	31.9	68.1
	2	20.1	62.5	37.5	63.9	36.1	38.6	61.4	100.0	53.5	46.5
	3	1.2	69.2	30.8	68.2	31.8	39.9	60.1	100.0	38.5	61.5
	4	38.1	73.5	26.5	71.9	28.1	41.2	58.8	100.0	28.8	71.2
	5	8.2	64.8	35.2	65.3	34.7	39.0	61.0	100.0	48.2	51.8
	6	36.3	65.2	34.8	65.5	34.5	39.1	60.9	100.0	47.4	52.6
	7	31.2	63.2	36.8	64.3	35.7	38.7	61.3	100.0	52.0	48.0

Appendix D: Detailed Results

Table D- 1: Simulation Results: Set of Parameters 1

TC= 100 [\$/hr], h_i= 50[\$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for all i

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	22,569	2,041	24,610	3,428	**28,038**
	St Dev	862	1,474	1,244	164	1,240
	CV	0.04	0.72	0.05	0.05	0.04
MUN	Mean	12,843	1,901	14,744	6,415	**21,159**
	St Dev	539	1,798	1,572	494	1,633
	CV	0.04	0.95	0.11	0.08	0.08
RTC	Mean	16,221	167	16,388	4,621	**21,008**
	St Dev	646	278	591	241	561
	CV	0.04	1.67	0.04	0.05	0.03
RDE	Mean	13,997	312	14,308	4,943	**19,252**
	St Dev	227	387	486	258	543
	CV	0.02	1.24	0.03	0.05	0.03
RDE+div	Mean	14,158	228	14,386	4,980	**19,366**
	St Dev	234	304	307	248	397
	CV	0.02	1.33	0.02	0.05	0.02
FTRI	Mean	9,703	1,211	10,914	7,032	**17,946**
	St Dev	203	895	833	72	825
	CV	0.02	0.74	0.08	0.01	0.05
FTUI	Mean	9,884	499	10,382	7,205	**17,587**
	St Dev	174	519	500	238	561
	CV	0.02	1.04	0.05	0.03	0.03
FTSR	Mean	9,884	499	10,382	7,205	**17,587**
	St Dev	174	519	500	238	561
	CV	0.02	1.04	0.05	0.03	0.03
RDE+FT_S	Mean	9,679	561	10,239	7,233	**17,472**
	St Dev	157	509	501	192	507
	CV	0.02	0.91	0.05	0.03	0.03
Strategy		Case 1				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	22,812	2,928	25,739	4,363	**30,103**
	St Dev	841	2,302	1,852	432	1,963
	CV	0.04	0.79	0.07	0.10	0.07
MUN	Mean	16,361	1,763	18,124	7,437	**25,561**
	St Dev	725	1,672	1,393	545	1,541
	CV	0.04	0.95	0.08	0.07	0.06
RTC	Mean	18,895	321	19,215	5,308	**24,523**
	St Dev	703	498	886	330	911
	CV	0.04	1.55	0.05	0.06	0.04
RDE	Mean	17,287	307	17,594	5,536	**23,130**
	St Dev	316	410	498	317	602
	CV	0.02	1.34	0.03	0.06	0.03
RDE+div	Mean	17,238	239	17,477	5,606	**23,083**
	St Dev	327	271	417	354	592
	CV	0.02	1.13	0.02	0.06	0.03
FTRI	Mean	10,953	1,606	12,558	8,197	**20,756**
	St Dev	249	1,191	1,136	117	1,139
	CV	0.02	0.74	0.09	0.01	0.05
FTUI	Mean	11,301	786	12,087	8,605	**20,693**
	St Dev	221	821	816	458	1,004
	CV	0.02	1.04	0.07	0.05	0.05
FTSR	Mean	11,301	786	12,087	8,605	**20,693**
	St Dev	221	821	816	458	1,004
	CV	0.02	1.04	0.07	0.05	0.05
RDE+FT_S	Mean	11,033	824	11,857	8,607	**20,464**
	St Dev	159	769	753	215	779
	CV	0.01	0.93	0.06	0.03	0.04

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	**Mean**	22,721	1,432	24,153	3,041	**27,194**
	St Dev	683	1,691	1,576	216	1,665
	CV	0.03	1.18	0.07	0.07	0.06
MUN	**Mean**	12,725	612	13,337	7,366	**20,703**
	St Dev	381	1,202	1,104	525	1,372
	CV	0.03	1.97	0.08	0.07	0.07
RTC	**Mean**	14,903	660	15,563	4,367	**19,930**
	St Dev	362	755	693	264	701
	CV	0.02	1.14	0.04	0.06	0.04
RDE	**Mean**	12,647	680	13,327	4,922	**18,249**
	St Dev	250	574	636	230	640
	CV	0.02	0.84	0.05	0.05	0.04
RDE+div	**Mean**	12,658	443	13,101	4,919	**18,019**
	St Dev	252	592	667	235	739
	CV	0.02	1.34	0.05	0.05	0.04
FTRI	**Mean**	9,549	1,746	11,295	6,928	**18,223**
	St Dev	188	1,117	1,039	121	1,031
	CV	0.02	0.64	0.09	0.02	0.06
FTUI	**Mean**	9,732	873	10,604	7,152	**17,756**
	St Dev	153	749	760	284	882
	CV	0.02	0.86	0.07	0.04	0.05
FTSR	**Mean**	9,721	877	10,597	7,111	**17,708**
	St Dev	148	743	746	270	843
	CV	0.02	0.85	0.07	0.04	0.05
RDE+FT_S	**Mean**	9,645	669	10,314	7,247	**17,561**
	St Dev	121	400	401	266	557
	CV	0.01	0.60	0.04	0.04	0.03
Strategy		**Case 3**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	**Mean**	23,221	1,821	25,041	3,355	**28,396**
	St Dev	901	1,620	1,410	279	1,488
	CV	0.04	0.89	0.06	0.08	0.05
MUN	**Mean**	14,696	1,046	15,742	6,650	**22,392**
	St Dev	616	1,234	981	555	1,244
	CV	0.04	1.18	0.06	0.08	0.06
RTC	**Mean**	16,938	534	17,473	4,466	**21,939**
	St Dev	528	667	855	282	851
	CV	0.03	1.25	0.05	0.06	0.04
RDE	**Mean**	14,960	257	15,217	4,727	**19,943**
	St Dev	337	576	535	222	627
	CV	0.02	2.24	0.04	0.05	0.03
RDE+div	**Mean**	15,020	212	15,232	4,805	**20,037**
	St Dev	308	326	410	198	478
	CV	0.02	1.54	0.03	0.04	0.02
FTRI	**Mean**	9,696	1,405	11,100	7,041	**18,142**
	St Dev	221	961	943	69	976
	CV	0.02	0.68	0.08	0.01	0.05
FTUI	**Mean**	9,920	839	10,759	7,295	**18,054**
	St Dev	160	747	768	328	803
	CV	0.02	0.89	0.07	0.04	0.04
FTSR	**Mean**	9,915	926	10,841	7,290	**18,131**
	St Dev	149	761	795	329	833
	CV	0.02	0.82	0.07	0.05	0.05
RDE+FT_S	**Mean**	9,679	932	10,611	7,498	**18,108**
	St Dev	175	765	729	259	673
	CV	0.02	0.82	0.07	0.03	0.04

30 replication with common random numbers
Results in [$/week]

Table D- 2: Simulation Results: Set of Parameters 2

TC*= 33 [$/hr], h_i= 50[$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for all *i

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	21,344	181	21,525	1,408	**22,932**
	St Dev	461	502	544	38	536
	CV	0.02	2.78	0.03	0.03	0.02
MUN	Mean	11,904	1,748	13,651	2,476	**16,128**
	St Dev	489	1,579	1,290	186	1,278
	CV	0.04	0.90	0.09	0.08	0.08
RTC	Mean	14,906	315	15,220	1,689	**16,910**
	St Dev	477	670	775	72	770
	CV	0.03	2.13	0.05	0.04	0.05
RDE	Mean	12,649	187	12,837	1,890	**14,727**
	St Dev	231	282	281	72	301
	CV	0.02	1.50	0.02	0.04	0.02
RDE+div	Mean	12,635	236	12,871	1,901	**14,772**
	St Dev	248	313	312	84	293
	CV	0.02	1.33	0.02	0.04	0.02
FTRI	Mean	7,802	1,141	8,943	3,299	**12,242**
	St Dev	160	726	714	18	715
	CV	0.02	0.64	0.08	0.01	0.06
FTUI	Mean	7,736	1,141	8,877	3,257	**12,134**
	St Dev	138	723	684	32	689
	CV	0.02	0.63	0.08	0.01	0.06
FTSR	Mean	7,736	1,141	8,877	3,257	**12,134**
	St Dev	138	723	684	32	689
	CV	0.02	0.63	0.08	0.01	0.06
RDE+FT_S	Mean	7,809	627	8,436	3,242	**11,678**
	St Dev	141	595	581	29	592
	CV	0.02	0.95	0.07	0.01	0.05
Strategy		**Case 1**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	20,641	1,886	22,527	1,920	**24,447**
	St Dev	699	1,950	1,512	72	1,535
	CV	0.03	1.03	0.07	0.04	0.06
MUN	Mean	15,338	2,274	17,612	2,632	**20,244**
	St Dev	802	2,434	2,076	189	2,081
	CV	0.05	1.07	0.12	0.07	0.10
RTC	Mean	17,971	432	18,402	1,873	**20,275**
	St Dev	703	801	869	119	891
	CV	0.04	1.86	0.05	0.06	0.04
RDE	Mean	16,246	282	16,527	1,980	**18,507**
	St Dev	313	330	412	124	399
	CV	0.02	1.17	0.02	0.06	0.02
RDE+div	Mean	16,212	243	16,455	1,987	**18,442**
	St Dev	297	367	415	91	403
	CV	0.02	1.51	0.03	0.05	0.02
FTRI	Mean	9,652	1,545	11,196	3,300	**14,496**
	St Dev	225	1,245	1,184	26	1,193
	CV	0.02	0.81	0.11	0.01	0.08
FTUI	Mean	9,580	1,717	11,297	3,268	**14,565**
	St Dev	211	1,067	1,061	32	1,059
	CV	0.02	0.62	0.09	0.01	0.07
FTSR	Mean	9,573	1,940	11,513	3,265	**14,778**
	St Dev	217	1,582	1,512	42	1,528
	CV	0.02	0.82	0.13	0.01	0.10

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	20,559	787	21,345	1,300	**22,646**
	St Dev	418	859	835	55	848
	CV	0.02	1.09	0.04	0.04	0.04
MUN	Mean	11,087	985	12,072	2,576	**14,648**
	St Dev	405	1,006	873	177	864
	CV	0.04	1.02	0.07	0.07	0.06
RTC	Mean	13,720	490	14,210	1,635	**15,845**
	St Dev	348	632	671	81	707
	CV	0.03	1.29	0.05	0.05	0.04
RDE	Mean	11,243	518	11,761	1,903	**13,664**
	St Dev	203	611	619	81	639
	CV	0.02	1.18	0.05	0.04	0.05
RDE+div	Mean	11,275	386	11,661	1,952	**13,613**
	St Dev	193	411	439	109	452
	CV	0.02	1.07	0.04	0.06	0.03
FTRI	Mean	7,610	1,847	9,457	3,298	**12,755**
	St Dev	124	1,140	1,106	25	1,098
	CV	0.02	0.62	0.12	0.01	0.09
FTUI	Mean	7,551	1,740	9,291	3,236	**12,526**
	St Dev	116	1,086	1,071	51	1,071
	CV	0.02	0.62	0.12	0.02	0.09
FTSR	Mean	7,573	1,713	9,286	3,244	**12,530**
	St Dev	110	1,042	1,048	43	1,054
	CV	0.01	0.61	0.11	0.01	0.08
RDE+FT_S	Mean	7,554	1,418	8,973	3,246	**12,219**
	St Dev	125	1,024	972	35	965
	CV	0.02	0.72	0.11	0.01	0.08

Strategy		Case 3				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	21,317	493	21,811	1,389	**23,199**
	St Dev	482	745	843	43	842
	CV	0.02	1.51	0.04	0.03	0.04
MUN	Mean	13,287	909	14,195	2,316	**16,511**
	St Dev	504	1,452	1,146	181	1,151
	CV	0.04	1.60	0.08	0.08	0.07
RTC	Mean	15,735	247	15,981	1,604	**17,585**
	St Dev	464	480	666	72	697
	CV	0.03	1.95	0.04	0.04	0.04
RDE	Mean	13,682	363	14,045	1,751	**15,795**
	St Dev	209	501	516	75	512
	CV	0.02	1.38	0.04	0.04	0.03
RDE+div	Mean	13,766	238	14,004	1,790	**15,794**
	St Dev	205	374	410	90	426
	CV	0.01	1.57	0.03	0.05	0.03
FTRI	Mean	7,768	1,382	9,150	3,300	**12,450**
	St Dev	155	1,026	1,012	18	1,007
	CV	0.02	0.74	0.11	0.01	0.08
FTUI	Mean	7,725	1,709	9,433	3,248	**12,682**
	St Dev	115	1,067	1,032	44	1,050
	CV	0.01	0.62	0.11	0.01	0.08
FTSR	Mean	7,733	1,462	9,195	3,256	**12,451**
	St Dev	115	1,071	1,037	46	1,053
	CV	0.01	0.73	0.11	0.01	0.08

30 replication with common random numbers
Results in [$/week]

Table D- 3: Simulation Results: Set of Parameters 3

TC= 300 [$/hr], h_i= 50[$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for all i

Strategy		Case 0 (Symmetric case) Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	29,895	488	30,383	8,441	**38,825**
	St Dev	642	731	761	688	1,156
	CV	0.02	1.50	0.03	0.08	0.03
MUN	Mean	19,248	829	20,077	15,671	**35,748**
	St Dev	707	1,386	1,114	1,459	2,103
	CV	0.04	1.67	0.06	0.09	0.06
RTC	Mean	20,780	368	21,147	9,666	**30,814**
	St Dev	628	577	743	749	1,050
	CV	0.03	1.57	0.04	0.08	0.03
RDE	Mean	19,086	448	19,534	10,027	**29,562**
	St Dev	470	547	517	682	931
	CV	0.02	1.22	0.03	0.07	0.03
RDE+div	Mean	19,167	452	19,619	10,063	**29,682**
	St Dev	437	483	538	709	914
	CV	0.02	1.07	0.03	0.07	0.03
FTRI	Mean	15,416	1,549	16,965	12,329	**29,293**
	St Dev	415	990	941	385	1,067
	CV	0.03	0.64	0.06	0.03	0.04
FTUI	Mean	15,565	709	16,274	12,506	**28,780**
	St Dev	365	518	639	831	1,226
	CV	0.02	0.73	0.04	0.07	0.04
FTSR	Mean	15,565	709	16,274	12,506	**28,780**
	St Dev	365	518	639	831	1,226
	CV	0.02	0.73	0.04	0.07	0.04
RDE+FT_S	Mean	15,828	702	16,530	11,829	**28,359**
	St Dev	391	766	771	732	1,155
	CV	0.02	1.09	0.05	0.06	0.04
Strategy		**Case 1** **Inv. Holding Cost**	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	30,680	1,703	32,383	10,581	**42,965**
	St Dev	980	1,742	1,813	845	2,226
	CV	0.03	1.02	0.06	0.08	0.05
MUN	Mean	21,104	1,529	22,633	16,244	**38,877**
	St Dev	1,063	1,710	1,426	1,885	2,254
	CV	0.05	1.12	0.06	0.12	0.06
RTC	Mean	24,095	609	24,705	11,183	**35,888**
	St Dev	874	934	1,132	897	1,555
	CV	0.04	1.53	0.05	0.08	0.04
RDE	Mean	22,833	529	23,362	11,523	**34,886**
	St Dev	536	624	800	1,062	1,364
	CV	0.02	1.18	0.03	0.09	0.04
RDE+div	Mean	22,800	505	23,305	11,578	**34,883**
	St Dev	738	682	966	1,012	1,448
	CV	0.03	1.35	0.04	0.09	0.04
FTRI	Mean	17,195	2,197	19,392	14,106	**33,499**
	St Dev	505	1,321	1,280	1,302	1,966
	CV	0.03	0.60	0.07	0.09	0.06
FTUI	Mean	17,715	725	18,440	14,691	**33,130**
	St Dev	686	606	934	1,198	1,705
	CV	0.04	0.84	0.05	0.08	0.05
FTSR	Mean	17,715	725	18,440	14,691	**33,130**
	St Dev	686	606	934	1,198	1,705
	CV	0.03	0.84	0.05	0.08	0.05
RDE+FT_S	Mean	17,336	1,014	18,350	14,356	**32,706**
	St Dev	405	1,077	1,026	839	1,269
	CV	0.02	1.06	0.06	0.06	0.04

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	**Mean**	28,522	727	29,250	7,470	**36,719**
	St Dev	661	1,167	1,231	716	1,562
	CV	0.02	1.60	0.04	0.10	0.04
MUN	**Mean**	16,594	4,798	21,393	15,627	**37,020**
	St Dev	781	3,346	2,759	2,391	2,722
	CV	0.05	0.70	0.13	0.15	0.07
RTC	**Mean**	19,246	771	20,017	8,978	**28,995**
	St Dev	655	1,172	1,023	796	1,247
	CV	0.03	1.52	0.05	0.09	0.04
RDE	**Mean**	17,627	553	18,180	9,578	**27,758**
	St Dev	428	765	888	819	1,218
	CV	0.02	1.38	0.05	0.09	0.04
RDE+div	**Mean**	17,561	469	18,031	9,566	**27,597**
	St Dev	460	417	537	663	869
	CV	0.03	0.89	0.03	0.07	0.03
FTRI	**Mean**	15,204	2,134	17,338	12,178	**29,516**
	St Dev	340	1,184	976	530	1,163
	CV	0.02	0.55	0.06	0.04	0.04
FTUI	**Mean**	15,473	809	16,282	12,565	**28,846**
	St Dev	351	911	1,054	787	1,620
	CV	0.02	1.13	0.06	0.06	0.06
FTSR	**Mean**	15,459	718	16,177	12,521	**28,698**
	St Dev	390	896	1,044	846	1,621
	CV	0.03	1.25	0.06	0.07	0.06
RDE+FT_S	**Mean**	15,373	604	15,977	11,189	**27,167**
	St Dev	270	554	549	627	691
	CV	0.02	0.92	0.03	0.06	0.03
Strategy		**Case 3**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	**Mean**	29,530	1,009	30,539	8,368	**38,907**
	St Dev	905	1,053	1,118	745	1,399
	CV	0.03	1.04	0.04	0.09	0.04
MUN	**Mean**	20,035	449	20,484	14,654	**35,138**
	St Dev	818	555	949	1,645	2,134
	CV	0.04	1.24	0.05	0.11	0.06
RTC	**Mean**	22,267	372	22,639	9,184	**31,823**
	St Dev	636	444	689	821	1,207
	CV	0.03	1.19	0.03	0.09	0.04
RDE	**Mean**	20,555	561	21,116	9,563	**30,678**
	St Dev	416	806	822	832	1,286
	CV	0.02	1.44	0.04	0.09	0.04
RDE+div	**Mean**	20,594	573	21,167	9,585	**30,751**
	St Dev	533	592	673	803	1,089
	CV	0.03	1.03	0.03	0.08	0.04
FTRI	**Mean**	15,115	1,938	17,053	12,309	**29,362**
	St Dev	356	1,619	1,498	482	1,548
	CV	0.02	0.84	0.09	0.04	0.05
FTUI	**Mean**	15,431	963	16,394	12,672	**29,067**
	St Dev	358	640	696	866	1,217
	CV	0.02	0.66	0.04	0.07	0.04
FTSR	**Mean**	15,431	963	16,394	12,672	**29,067**
	St Dev	358	640	696	866	1,217
	CV	0.02	0.66	0.04	0.07	0.04
RDE+FT_S	**Mean**	15,248	1,030	16,279	12,018	**28,296**
	St Dev	389	935	1,039	649	1,143
	CV	0.03	0.91	0.06	0.05	0.04

30 replication with common random numbers
Results in [$/week]

Table D- 4: Simulation Results: Set of Parameters 4

TC= 100 [$/hr], h_i= 50[$/week], λ_i= 10.5 [arrivals/day], θ_i= 4.8 [units] for all *i*

Strategy		Case 0 (Symmetric case) Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	26,668	1,924	28,592	3,293	**31,885**
	St Dev	833	1,779	1,870	238	1,922
	CV	0.03	0.92	0.07	0.07	0.06
MUN	Mean	16,535	923	17,458	6,903	**24,360**
	St Dev	531	996	1,118	514	1,339
	CV	0.03	1.08	0.06	0.07	0.05
RTC	Mean	20,424	863	21,287	4,757	**26,044**
	St Dev	636	989	1,109	331	1,169
	CV	0.03	1.15	0.05	0.07	0.04
RDE	Mean	17,050	1,018	18,068	5,197	**23,265**
	St Dev	355	855	798	303	820
	CV	0.02	0.84	0.04	0.06	0.04
RDE+div	Mean	16,934	727	17,660	5,326	**22,987**
	St Dev	436	804	769	333	831
	CV	0.03	1.11	0.04	0.06	0.04
FTRI	Mean	12,128	3,471	15,599	7,684	**23,283**
	St Dev	332	2,209	2,134	71	2,138
	CV	0.03	0.64	0.14	0.01	0.09
FTUI	Mean	12,348	2,139	14,488	8,030	**22,518**
	St Dev	197	1,472	1,484	478	1,569
	CV	0.02	0.69	0.10	0.06	0.07
FTSR	Mean	12,328	2,104	14,432	7,909	**22,341**
	St Dev	218	1,820	1,779	400	1,756
	CV	0.02	0.87	0.12	0.05	0.08
RDE+FT_S	Mean	12,169	1,533	13,702	7,919	**21,621**
	St Dev	161	1,112	1,124	380	1,013
	CV	0.01	0.73	0.08	0.05	0.05
Strategy		**Case 1 Inv. Holding Cost**	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	26,615	2,797	29,412	4,326	**33,738**
	St Dev	1,055	2,524	2,131	401	2,200
	CV	0.04	0.90	0.07	0.09	0.07
MUN	Mean	19,530	1,282	20,812	7,222	**28,034**
	St Dev	957	1,953	1,806	543	1,972
	CV	0.05	1.52	0.09	0.08	0.07
RTC	Mean	23,530	524	24,054	5,388	**29,442**
	St Dev	908	839	1,185	317	1,225
	CV	0.04	1.60	0.05	0.06	0.04
RDE	Mean	20,716	649	21,365	5,867	**27,231**
	St Dev	430	690	701	480	882
	CV	0.02	1.06	0.03	0.08	0.03
RDE+div	Mean	20,810	307	21,118	5,872	**26,990**
	St Dev	428	555	611	431	686
	CV	0.02	1.81	0.03	0.07	0.03
FTRI	Mean	13,594	3,147	16,741	8,857	**25,597**
	St Dev	317	1,654	1,596	85	1,562
	CV	0.02	0.53	0.10	0.01	0.06
FTUI	Mean	13,717	2,377	16,094	9,038	**25,132**
	St Dev	249	1,286	1,181	324	1,182
	CV	0.02	0.54	0.07	0.04	0.05
FTSR	Mean	13,730	2,077	15,807	8,976	**24,783**
	St Dev	239	1,440	1,402	353	1,418
	CV	0.02	0.69	0.09	0.04	0.06
RDE+FT_S	Mean	13,618	1,499	15,117	9,270	**24,387**
	St Dev	284	1,073	1,156	240	1,173
	CV	0.02	0.72	0.08	0.03	0.05

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	**Mean**	26,427	1,817	28,244	2,953	**31,196**
	St Dev	1,117	1,751	1,602	254	1,662
	CV	0.04	0.96	0.06	0.09	0.05
MUN	**Mean**	15,341	1,651	16,992	7,352	**24,343**
	St Dev	540	1,621	1,570	563	1,590
	CV	0.04	0.98	0.09	0.08	0.07
RTC	**Mean**	19,364	545	19,909	4,340	**24,249**
	St Dev	751	596	982	287	989
	CV	0.04	1.09	0.05	0.07	0.04
RDE	**Mean**	15,385	699	16,084	4,976	**21,060**
	St Dev	273	641	617	311	673
	CV	0.02	0.92	0.04	0.06	0.03
RDE+div	**Mean**	15,458	812	16,270	5,125	**21,395**
	St Dev	269	784	806	310	817
	CV	0.02	0.96	0.05	0.06	0.04
FTRI	**Mean**	12,047	2,750	14,797	7,586	**22,383**
	St Dev	263	1,477	1,427	65	1,437
	CV	0.02	0.54	0.10	0.01	0.06
FTUI	**Mean**	12,114	1,679	13,793	7,670	**21,462**
	St Dev	214	1,108	1,153	545	1,139
	CV	0.02	0.66	0.08	0.07	0.05
FTSR	**Mean**	12,205	1,386	13,591	7,474	**21,064**
	St Dev	234	1,054	1,089	497	1,089
	CV	0.02	0.76	0.08	0.07	0.05
RDE+FT_S	**Mean**	12,062	1,240	13,302	7,800	**21,102**
	St Dev	172	1,200	1,221	378	1,287
	CV	0.01	0.97	0.09	0.05	0.06
Strategy		**Case 3**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	**Mean**	27,080	2,313	29,393	3,318	**32,711**
	St Dev	849	2,482	2,267	255	2,309
	CV	0.03	1.07	0.08	0.08	0.07
MUN	**Mean**	17,761	860	18,621	6,730	**25,351**
	St Dev	682	965	1,089	550	1,238
	CV	0.04	1.12	0.06	0.08	0.05
RTC	**Mean**	21,753	1,165	22,918	4,513	**27,431**
	St Dev	943	1,582	1,557	294	1,565
	CV	0.04	1.36	0.07	0.07	0.06
RDE	**Mean**	18,207	911	19,118	4,893	**24,011**
	St Dev	439	1,205	1,020	305	1,031
	CV	0.02	1.32	0.05	0.06	0.04
RDE+div	**Mean**	18,213	578	18,791	4,949	**23,740**
	St Dev	510	672	627	322	764
	CV	0.03	1.16	0.03	0.07	0.03
FTRI	**Mean**	12,217	3,094	15,311	7,701	**23,012**
	St Dev	288	2,051	2,019	78	2,033
	CV	0.02	0.66	0.13	0.01	0.09
FTUI	**Mean**	12,343	1,825	14,168	7,948	**22,116**
	St Dev	230	1,194	1,265	397	1,304
	CV	0.02	0.65	0.09	0.05	0.06
FTSR	**Mean**	12,327	1,670	13,997	7,806	**21,803**
	St Dev	206	1,241	1,255	317	1,298
	CV	0.02	0.74	0.09	0.04	0.06
RDE+FT_S	**Mean**	12,117	1,467	13,584	8,262	**21,846**
	St Dev	213	997	890	349	941
	CV	0.02	0.68	0.07	0.04	0.04

30 replication with common random numbers
Results in [$/week]

Table D- 5: Simulation Results: Set of Parameters 5

TC= 100 [$/hr], h_i= 50[$/week], λ_i= 4.35 [arrivals/day], θ_i= 11.5 [units] for all *i*

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	30,862	2,273	33,135	3,105	**36,241**
	St Dev	1,292	2,761	2,590	243	2,615
	CV	0.04	1.21	0.08	0.08	0.07
MUN	Mean	19,999	1,509	21,508	7,251	**28,759**
	St Dev	1,043	1,693	1,803	598	2,105
	CV	0.05	1.12	0.08	0.08	0.07
RTC	Mean	25,127	2,488	27,615	4,981	**32,596**
	St Dev	989	2,867	2,857	437	2,860
	CV	0.04	1.15	0.10	0.09	0.09
RDE	Mean	19,988	1,895	21,883	5,358	**27,241**
	St Dev	419	1,540	1,473	436	1,615
	CV	0.02	0.81	0.07	0.08	0.06
RDE+div	Mean	20,272	1,383	21,654	5,558	**27,212**
	St Dev	485	1,386	1,370	381	1,499
	CV	0.02	1.00	0.06	0.07	0.06
FTRI	Mean	14,743	5,912	20,655	8,473	**29,128**
	St Dev	399	3,563	3,391	44	3,402
	CV	0.03	0.60	0.16	0.01	0.12
FTUI	Mean	14,877	5,311	20,188	8,462	**28,650**
	St Dev	335	3,080	3,135	504	3,199
	CV	0.02	0.58	0.16	0.06	0.11
FTSR	Mean	14,911	4,769	19,680	8,287	**27,968**
	St Dev	333	2,890	2,917	521	3,016
	CV	0.02	0.61	0.15	0.06	0.11
RDE+FT_S	Mean	14,923	2,847	17,770	8,240	**26,010**
	St Dev	264	2,001	2,041	419	2,038
	CV	0.02	0.70	0.11	0.05	0.08
Strategy		**Case 1**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	30,842	4,103	34,945	4,083	**39,028**
	St Dev	1,354	4,971	4,560	375	4,586
	CV	0.04	1.21	0.13	0.09	0.12
MUN	Mean	23,346	1,259	24,606	7,219	**31,825**
	St Dev	1,117	1,514	1,478	853	2,139
	CV	0.05	1.20	0.06	0.12	0.07
RTC	Mean	28,961	1,647	30,607	5,616	**36,223**
	St Dev	1,605	1,640	2,242	472	2,197
	CV	0.06	1.00	0.07	0.08	0.06
RDE	Mean	24,254	1,248	25,502	6,386	**31,888**
	St Dev	637	1,519	1,367	571	1,487
	CV	0.03	1.22	0.05	0.09	0.05
RDE+div	Mean	24,592	959	25,551	6,327	**31,878**
	St Dev	709	1,350	1,375	584	1,455
	CV	0.03	1.41	0.05	0.09	0.05
FTRI	Mean	16,810	4,568	21,378	9,969	**31,347**
	St Dev	548	3,081	3,014	74	3,015
	CV	0.03	0.67	0.14	0.01	0.10
FTUI	Mean	16,581	5,129	21,711	9,422	**31,132**
	St Dev	399	3,255	3,249	455	3,300
	CV	0.02	0.63	0.15	0.05	0.11
FTSR	Mean	16,700	5,142	21,842	9,205	**31,047**
	St Dev	412	3,053	3,027	512	3,263
	CV	0.02	0.59	0.14	0.06	0.11
RDE+FT_S	Mean	16,871	2,762	19,633	9,441	**29,074**
	St Dev	363	2,495	2,453	289	2,558
	CV	0.02	0.90	0.12	0.03	0.09

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	30,787	2,473	33,260	2,919	**36,179**
	St Dev	1,061	2,254	2,035	274	2,165
	CV	0.03	0.91	0.06	0.09	0.06
MUN	Mean	18,397	2,570	20,967	7,416	**28,384**
	St Dev	599	2,245	2,082	770	2,188
	CV	0.03	0.87	0.10	0.10	0.08
RTC	Mean	24,345	1,885	26,229	4,363	**30,593**
	St Dev	963	2,152	2,193	482	2,229
	CV	0.04	1.14	0.08	0.11	0.07
RDE	Mean	18,311	2,872	21,182	5,361	**26,543**
	St Dev	418	2,646	2,545	501	2,645
	CV	0.02	0.92	0.12	0.09	0.10
RDE+div	Mean	18,589	1,596	20,184	5,312	**25,496**
	St Dev	344	1,633	1,741	501	1,848
	CV	0.02	1.02	0.09	0.09	0.07
FTRI	Mean	14,752	4,408	19,160	8,381	**27,541**
	St Dev	350	3,131	2,988	0	2,988
	CV	0.02	0.71	0.16	0.00	0.11
FTUI	Mean	14,665	3,653	18,317	8,136	**26,453**
	St Dev	244	2,408	2,311	597	2,446
	CV	0.02	0.66	0.13	0.07	0.09
FTSR	Mean	14,742	3,923	18,665	7,352	**26,017**
	St Dev	247	2,908	2,907	578	2,944
	CV	0.02	0.74	0.16	0.08	0.11
RDE+FT_S	Mean	14,738	2,968	17,706	8,081	**25,787**
	St Dev	248	1,729	1,624	478	1,679
	CV	0.02	0.58	0.09	0.06	0.07
Strategy		Case 3				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	31,057	3,478	34,536	3,098	**37,634**
	St Dev	1,391	3,501	3,377	252	3,371
	CV	0.04	1.01	0.10	0.08	0.09
MUN	Mean	21,483	999	22,482	6,897	**29,378**
	St Dev	1,187	1,158	1,457	848	2,098
	CV	0.06	1.16	0.06	0.12	0.07
RTC	Mean	26,585	1,774	28,359	4,703	**33,062**
	St Dev	1,204	2,158	2,348	303	2,343
	CV	0.05	1.22	0.08	0.06	0.07
RDE	Mean	21,582	1,767	23,348	5,591	**28,940**
	St Dev	494	1,567	1,615	551	1,651
	CV	0.02	0.89	0.07	0.10	0.06
RDE+div	Mean	21,587	905	22,492	5,639	**28,131**
	St Dev	525	1,315	1,209	544	1,384
	CV	0.02	1.45	0.05	0.10	0.05
FTRI	Mean	14,928	4,988	19,916	8,495	**28,411**
	St Dev	375	3,258	3,147	64	3,152
	CV	0.03	0.65	0.16	0.01	0.11
FTUI	Mean	14,920	3,592	18,512	8,431	**26,943**
	St Dev	261	2,479	2,423	590	2,671
	CV	0.02	0.69	0.13	0.07	0.10
FTSR	Mean	14,987	3,168	18,155	7,986	**26,141**
	St Dev	263	2,473	2,491	558	2,569
	CV	0.02	0.78	0.14	0.07	0.10
RDE+FT_S	Mean	14,879	3,145	18,025	8,984	**27,009**
	St Dev	365	2,486	2,367	387	2,371
	CV	0.02	0.79	0.13	0.04	0.09

30 replication with common random numbers
Results in [$/week]

Table D- 6: Simulation Results: Set of Parameters 6

TC*= 100 [$/hr], h_i= 50[$/week], λ_i= 2.4 [arrivals/day], θ_i= 20.8 [units] for all *i

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	34,425	4,670	39,096	3,044	**42,140**
	St Dev	1,691	4,308	3,834	325	3,758
	CV	0.05	0.92	0.10	0.11	0.09
MUN	Mean	23,539	1,954	25,493	7,302	**32,795**
	St Dev	1,197	1,728	1,783	881	1,941
	CV	0.05	0.88	0.07	0.12	0.06
RTC	Mean	29,653	4,261	33,914	5,281	**39,195**
	St Dev	1,614	3,799	3,574	523	3,630
	CV	0.05	0.89	0.11	0.10	0.09
RDE	Mean	23,151	4,040	27,190	5,717	**32,907**
	St Dev	633	2,860	2,841	479	2,775
	CV	0.03	0.71	0.10	0.08	0.08
RDE+div	Mean	23,482	2,630	26,112	5,793	**31,906**
	St Dev	524	2,704	2,699	417	2,749
	CV	0.02	1.03	0.10	0.07	0.09
FTRI	Mean	17,395	10,358	27,753	9,330	**37,083**
	St Dev	547	4,903	4,723	49	4,719
	CV	0.03	0.47	0.17	0.01	0.13
FTUI	Mean	17,314	9,936	27,250	8,888	**36,137**
	St Dev	381	4,878	4,781	489	4,822
	CV	0.02	0.49	0.18	0.05	0.13
FTSR	Mean	17,550	8,015	25,565	8,568	**34,133**
	St Dev	480	4,378	4,287	565	4,244
	CV	0.03	0.55	0.17	0.07	0.12
RDE+FT_S	Mean	17,574	6,129	23,703	8,433	**32,136**
	St Dev	371	4,033	4,017	390	3,976
	CV	0.02	0.66	0.17	0.05	0.12
Strategy		Case 1				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	34,452	5,503	39,955	4,050	**44,005**
	St Dev	1,673	4,789	4,108	414	4,211
	CV	0.05	0.87	0.10	0.10	0.10
MUN	Mean	26,848	2,233	29,081	7,225	**36,306**
	St Dev	1,119	2,763	2,382	776	2,527
	CV	0.04	1.24	0.08	0.11	0.07
RTC	Mean	33,758	2,504	36,262	5,824	**42,086**
	St Dev	1,463	3,145	3,157	479	3,215
	CV	0.04	1.26	0.09	0.08	0.08
RDE	Mean	27,965	3,227	31,192	7,002	**38,194**
	St Dev	841	2,572	2,350	560	2,517
	CV	0.03	0.80	0.08	0.08	0.07
RDE+div	Mean	28,194	1,324	29,518	6,785	**36,303**
	St Dev	833	1,997	2,050	720	2,134
	CV	0.03	1.51	0.07	0.11	0.06
FTRI	Mean	20,375	7,116	27,491	9,998	**37,490**
	St Dev	669	3,654	3,461	127	3,494
	CV	0.03	0.51	0.13	0.01	0.09
FTUI	Mean	20,072	7,259	27,332	9,303	**36,635**
	St Dev	648	4,173	4,141	388	4,145
	CV	0.03	0.57	0.15	0.04	0.11
FTSR	Mean	20,161	6,841	27,002	8,939	**35,941**
	St Dev	725	4,156	4,052	560	4,053
	CV	0.04	0.61	0.15	0.06	0.11
RDE+FT_S	Mean	20,618	4,295	24,912	9,440	**34,352**
	St Dev	455	3,310	3,323	249	3,375
	CV	0.02	0.77	0.13	0.03	0.10

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	34,610	3,952	38,563	2,734	**41,297**
	St Dev	1,464	2,876	2,592	331	2,660
	CV	0.04	0.73	0.07	0.12	0.06
MUN	Mean	21,356	3,562	24,918	7,523	**32,441**
	St Dev	841	2,225	1,930	857	1,944
	CV	0.04	0.62	0.08	0.11	0.06
RTC	Mean	28,890	1,965	30,855	4,419	**35,274**
	St Dev	1,271	2,334	2,813	498	2,829
	CV	0.04	1.19	0.09	0.11	0.08
RDE	Mean	21,304	4,612	25,916	5,247	**31,163**
	St Dev	587	3,785	3,640	548	3,844
	CV	0.03	0.82	0.14	0.10	0.12
RDE+div	Mean	21,667	2,631	24,298	5,800	**30,098**
	St Dev	453	2,452	2,386	646	2,430
	CV	0.02	0.93	0.10	0.11	0.08
FTRI	Mean	17,423	6,490	23,913	9,197	**33,110**
	St Dev	540	3,432	3,233	56	3,225
	CV	0.03	0.53	0.14	0.01	0.10
FTUI	Mean	17,232	7,102	24,334	8,290	**32,624**
	St Dev	368	3,794	3,665	685	3,839
	CV	0.02	0.53	0.15	0.08	0.12
FTSR	Mean	17,452	6,159	23,612	7,353	**30,964**
	St Dev	388	3,829	3,662	573	3,612
	CV	0.02	0.62	0.16	0.08	0.12
RDE+FT_S	Mean	17,508	5,770	23,279	8,242	**31,521**
	St Dev	337	2,931	2,820	430	2,877
	CV	0.02	0.51	0.12	0.05	0.09
Strategy		**Case 3**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	35,055	3,169	38,224	3,119	**41,343**
	St Dev	1,409	2,945	3,135	290	3,112
	CV	0.04	0.93	0.08	0.09	0.08
MUN	Mean	25,234	1,656	26,891	7,029	**33,920**
	St Dev	1,309	1,979	1,978	789	2,347
	CV	0.05	1.19	0.07	0.11	0.07
RTC	Mean	31,198	2,420	33,618	4,926	**38,544**
	St Dev	1,359	2,973	2,462	423	2,527
	CV	0.04	1.23	0.07	0.09	0.07
RDE	Mean	24,729	4,071	28,800	6,065	**34,865**
	St Dev	762	3,254	2,833	634	2,964
	CV	0.03	0.80	0.10	0.10	0.09
RDE+div	Mean	25,215	2,420	27,635	6,213	**33,848**
	St Dev	609	2,029	1,893	642	1,937
	CV	0.02	0.84	0.07	0.10	0.06
FTRI	Mean	17,498	7,497	24,995	9,374	**34,369**
	St Dev	509	3,449	3,249	44	3,262
	CV	0.03	0.46	0.13	0.00	0.09
FTUI	Mean	17,321	7,267	24,588	8,688	**33,276**
	St Dev	309	3,887	3,871	717	4,118
	CV	0.02	0.53	0.16	0.08	0.12
FTSR	Mean	17,648	5,909	23,557	7,886	**31,443**
	St Dev	309	3,664	3,672	807	3,922
	CV	0.02	0.62	0.16	0.10	0.12
RDE+FT_S	Mean	17,465	4,871	22,336	9,068	**31,404**
	St Dev	388	2,871	2,712	408	2,787
	CV	0.02	0.59	0.12	0.05	0.09

30 replication with common random numbers
Results in [$/week]

Table D- 7: Simulation Results: Set of Parameters 7

TC= 100 [$/hr], h_i= 5[$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for all i

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,178	440	2,618	4,158	**6,776**
	St Dev	41	605	594	173	588
	CV	0.02	1.38	0.23	0.04	0.09
MUN	Mean	2,066	762	2,828	4,648	**7,476**
	St Dev	75	1,357	1,325	484	1,355
	CV	0.04	1.78	0.47	0.10	0.18
RTC	Mean	2,153	183	2,336	3,215	**5,551**
	St Dev	58	350	351	172	414
	CV	0.03	1.92	0.15	0.05	0.07
RDE	Mean	2,098	220	2,318	3,134	**5,452**
	St Dev	46	246	232	211	328
	CV	0.02	1.12	0.10	0.07	0.06
RDE+div	Mean	2,104	93	2,197	3,183	**5,380**
	St Dev	42	227	233	177	309
	CV	0.02	2.44	0.11	0.06	0.06
FTRI	Mean	2,226	119	2,345	3,402	**5,747**
	St Dev	54	350	349	383	592
	CV	0.02	2.94	0.15	0.11	0.10
FTUI	Mean	2,118	159	2,277	3,135	**5,412**
	St Dev	47	337	334	243	447
	CV	0.02	2.12	0.15	0.08	0.08
FTSR	Mean	2,118	159	2,277	3,135	**5,412**
	St Dev	47	337	334	243	447
	CV	0.02	2.12	0.15	0.08	0.08
Strategy		Case 1				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,004	2,128	4,132	5,872	**10,004**
	St Dev	67	1,633	1,590	196	1,643
	CV	0.03	0.77	0.38	0.03	0.16
MUN	Mean	2,024	1,299	3,323	6,397	**9,720**
	St Dev	87	1,892	1,845	680	1,800
	CV	0.04	1.46	0.56	0.11	0.19
RTC	Mean	2,135	177	2,311	4,360	**6,671**
	St Dev	74	341	341	306	548
	CV	0.03	1.93	0.15	0.07	0.08
RDE	Mean	2,098	307	2,405	4,208	**6,613**
	St Dev	44	479	467	334	606
	CV	0.02	1.56	0.19	0.08	0.09
RDE+div	Mean	2,087	184	2,271	4,286	**6,557**
	St Dev	45	292	286	339	508
	CV	0.02	1.59	0.13	0.08	0.08
FTRI	Mean	2,202	226	2,429	4,525	**6,953**
	St Dev	68	512	521	423	763
	CV	0.03	2.26	0.21	0.09	0.11
FTUI	Mean	2,132	273	2,405	4,303	**6,708**
	St Dev	66	398	413	390	654
	CV	0.03	1.46	0.17	0.09	0.10
FTSR	Mean	2,132	273	2,405	4,303	**6,708**
	St Dev	66	398	413	390	654
	CV	0.03	1.46	0.17	0.09	0.10

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	**Mean**	2,196	603	2,799	3,931	**6,731**
	St Dev	56	781	768	200	753
	CV	0.03	1.30	0.27	0.05	0.11
MUN	**Mean**	1,991	429	2,420	4,729	**7,149**
	St Dev	63	648	644	490	813
	CV	0.03	1.51	0.27	0.10	0.11
RTC	**Mean**	2,056	444	2,500	3,059	**5,559**
	St Dev	61	476	442	249	472
	CV	0.03	1.07	0.18	0.08	0.08
RDE	**Mean**	1,955	323	2,278	3,062	**5,339**
	St Dev	41	650	647	196	718
	CV	0.02	2.02	0.28	0.06	0.13
RDE+div	**Mean**	1,945	175	2,119	3,071	**5,191**
	St Dev	42	266	273	236	394
	CV	0.02	1.53	0.13	0.08	0.08
FTRI	**Mean**	2,201	320	2,521	3,287	**5,808**
	St Dev	45	571	567	365	746
	CV	0.02	1.79	0.23	0.11	0.13
FTUI	**Mean**	2,104	420	2,524	3,061	**5,584**
	St Dev	55	472	458	277	563
	CV	0.03	1.13	0.18	0.09	0.10
FTSR	**Mean**	2,104	420	2,524	3,061	**5,584**
	St Dev	55	472	458	277	563
	CV	0.03	1.13	0.18	0.09	0.10
Strategy		**Case 3**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	**Mean**	2,165	1,108	3,273	4,215	**7,489**
	St Dev	60	1,241	1,214	205	1,271
	CV	0.03	1.12	0.37	0.05	0.17
MUN	**Mean**	2,083	424	2,507	5,073	**7,579**
	St Dev	77	647	616	526	757
	CV	0.04	1.53	0.25	0.10	0.10
RTC	**Mean**	2,151	264	2,416	3,185	**5,600**
	St Dev	61	488	494	252	637
	CV	0.03	1.85	0.20	0.08	0.11
RDE	**Mean**	2,115	188	2,303	3,156	**5,458**
	St Dev	38	311	305	225	407
	CV	0.02	1.66	0.13	0.07	0.07
RDE+div	**Mean**	2,114	105	2,218	3,205	**5,424**
	St Dev	48	227	232	249	314
	CV	0.02	2.17	0.10	0.08	0.06
FTRI	**Mean**	2,212	302	2,514	3,348	**5,862**
	St Dev	67	512	528	360	748
	CV	0.03	1.69	0.21	0.11	0.13
FTUI	**Mean**	2,107	332	2,440	3,118	**5,558**
	St Dev	60	432	445	256	590
	CV	0.03	1.30	0.18	0.08	0.11
FTSR	**Mean**	2,107	332	2,440	3,118	**5,558**
	St Dev	60	432	445	256	590
	CV	0.03	1.30	0.18	0.08	0.11

30 replication with common random numbers
Results in [$/week]

Table D- 8: Simulation Results: Set of Parameters 8

TC*= 33 [$/hr], h_i= 5[$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for all *i

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,228	107	2,335	1,411	**3,746**
	St Dev	46	333	324	34	330
	CV	0.02	3.11	0.14	0.02	0.09
MUN	Mean	2,086	364	2,450	1,575	**4,025**
	St Dev	97	619	576	158	612
	CV	0.05	1.70	0.24	0.10	0.15
RTC	Mean	2,153	126	2,279	1,062	**3,341**
	St Dev	53	315	292	89	286
	CV	0.02	2.50	0.13	0.08	0.09
RDE	Mean	2,106	133	2,240	1,049	**3,288**
	St Dev	37	324	317	68	322
	CV	0.02	2.43	0.14	0.06	0.10
RDE+div	Mean	2,109	93	2,202	1,053	**3,255**
	St Dev	36	168	172	77	202
	CV	0.02	1.80	0.08	0.07	0.06
FTRI	Mean	1,897	146	2,043	1,401	**3,444**
	St Dev	47	430	415	81	439
	CV	0.02	2.94	0.20	0.06	0.13
FTUI	Mean	1,839	96	1,936	1,322	**3,258**
	St Dev	38	270	269	90	287
	CV	0.02	2.80	0.14	0.07	0.09
FTSR	Mean	1,839	96	1,936	1,322	**3,258**
	St Dev	38	270	269	90	287
	CV	0.02	2.80	0.14	0.07	0.09
Strategy		**Case 1**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	2,082	727	2,809	1,989	**4,799**
	St Dev	59	859	839	48	842
	CV	0.03	1.18	0.30	0.02	0.18
MUN	Mean	2,028	860	2,888	2,142	**5,030**
	St Dev	80	1,184	1,142	160	1,184
	CV	0.04	1.38	0.40	0.07	0.24
RTC	Mean	2,148	322	2,470	1,440	**3,910**
	St Dev	58	439	458	109	461
	CV	0.03	1.36	0.19	0.08	0.12
RDE	Mean	2,064	358	2,423	1,427	**3,850**
	St Dev	51	549	541	112	577
	CV	0.02	1.53	0.22	0.08	0.15
RDE+div	Mean	2,068	213	2,281	1,448	**3,729**
	St Dev	50	338	340	96	373
	CV	0.02	1.59	0.15	0.07	0.10
FTRI	Mean	2,103	300	2,403	1,596	**3,999**
	St Dev	64	484	474	117	511
	CV	0.03	1.61	0.20	0.07	0.13
FTUI	Mean	2,075	91	2,166	1,562	**3,728**
	St Dev	52	249	257	157	305
	CV	0.03	2.74	0.12	0.10	0.08
FTSR	Mean	2,075	91	2,166	1,562	**3,728**
	St Dev	52	249	257	157	305
	CV	0.03	2.74	0.12	0.10	0.08

Strategy		Case 2 Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,231	308	2,539	1,359	**3,897**
	St Dev	40	593	591	42	588
	CV	0.02	1.92	0.23	0.03	0.15
MUN	Mean	1,845	90	1,934	1,797	**3,731**
	St Dev	49	184	173	174	259
	CV	0.03	2.05	0.09	0.10	0.07
RTC	Mean	1,874	377	2,251	1,053	**3,304**
	St Dev	38	635	615	71	621
	CV	0.02	1.68	0.27	0.07	0.19
RDE	Mean	1,745	421	2,166	1,092	**3,258**
	St Dev	37	447	440	70	435
	CV	0.02	1.06	0.20	0.06	0.13
RDE+div	Mean	1,750	236	1,986	1,089	**3,075**
	St Dev	48	363	347	88	360
	CV	0.03	1.54	0.17	0.08	0.12
FTRI	Mean	1,860	367	2,227	1,382	**3,609**
	St Dev	40	527	510	90	514
	CV	0.02	1.44	0.23	0.07	0.14
FTUI	Mean	1,802	190	1,992	1,308	**3,300**
	St Dev	48	411	409	76	426
	CV	0.03	2.17	0.21	0.06	0.13
FTSR	Mean	1,802	190	1,992	1,308	**3,300**
	St Dev	48	411	409	76	426
	CV	0.03	2.17	0.21	0.06	0.13

Strategy		Case 3 Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,215	194	2,409	1,417	**3,826**
	St Dev	51	534	542	35	532
	CV	0.02	2.76	0.22	0.02	0.14
MUN	Mean	2,027	259	2,286	1,753	**4,039**
	St Dev	83	458	419	158	429
	CV	0.04	1.77	0.18	0.09	0.11
RTC	Mean	2,101	280	2,381	1,079	**3,460**
	St Dev	57	501	496	74	526
	CV	0.03	1.79	0.21	0.07	0.15
RDE	Mean	2,007	170	2,177	1,099	**3,276**
	St Dev	43	273	277	70	299
	CV	0.02	1.61	0.13	0.06	0.09
RDE+div	Mean	2,009	98	2,107	1,121	**3,228**
	St Dev	40	228	230	71	247
	CV	0.02	2.33	0.11	0.06	0.08
FTRI	Mean	1,861	310	2,172	1,397	**3,569**
	St Dev	43	507	495	78	503
	CV	0.02	1.63	0.23	0.06	0.14
FTUI	Mean	1,818	109	1,927	1,343	**3,270**
	St Dev	49	227	230	97	260
	CV	0.03	2.08	0.12	0.07	0.08
FTSR	Mean	1,818	109	1,927	1,343	**3,270**
	St Dev	49	227	230	97	260
	CV	0.03	2.08	0.12	0.07	0.08

30 replication with common random numbers
Results in [$/week]

Table D- 9: Simulation Results: Set of Parameters 9

TC*= 300 [$/hr], h_i= 5[$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for all *i

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,007	3,317	5,324	11,593	**16,917**
	St Dev	63	2,482	2,445	550	2,587
	CV	0.03	0.75	0.46	0.05	0.15
MUN	Mean	2,077	426	2,503	14,269	**16,772**
	St Dev	95	442	427	1,532	1,648
	CV	0.05	1.04	0.17	0.11	0.10
RTC	Mean	2,132	172	2,304	9,507	**11,811**
	St Dev	63	279	283	776	826
	CV	0.03	1.62	0.12	0.08	0.07
RDE	Mean	2,096	192	2,288	9,358	**11,646**
	St Dev	36	301	290	491	642
	CV	0.02	1.57	0.13	0.05	0.06
RDE+div	Mean	2,109	140	2,249	9,466	**11,715**
	St Dev	44	251	251	597	621
	CV	0.02	1.80	0.11	0.06	0.05
FTRI	Mean	2,226	118	2,344	10,205	**12,549**
	St Dev	54	344	343	1,150	1,296
	CV	0.02	2.91	0.15	0.11	0.10
FTUI	Mean	2,051	578	2,629	9,048	**11,677**
	St Dev	57	626	648	722	1,101
	CV	0.03	1.08	0.25	0.08	0.09
FTSR	Mean	2,051	578	2,629	9,048	**11,677**
	St Dev	57	626	648	722	1,101
	CV	0.03	1.08	0.25	0.08	0.09
Strategy		**Case 1**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	1,799	7,725	9,524	15,720	**25,244**
	St Dev	62	2,998	2,977	1,000	3,075
	CV	0.03	0.39	0.31	0.06	0.12
MUN	Mean	2,009	929	2,937	18,889	**21,826**
	St Dev	67	1,205	1,168	1,384	1,469
	CV	0.03	1.30	0.40	0.07	0.07
RTC	Mean	2,060	1,001	3,060	12,338	**15,399**
	St Dev	62	837	852	853	1,310
	CV	0.03	0.84	0.28	0.07	0.09
RDE	Mean	2,029	873	2,902	12,072	**14,974**
	St Dev	39	745	741	945	1,097
	CV	0.02	0.85	0.26	0.08	0.07
RDE+div	Mean	2,031	493	2,524	12,172	**14,696**
	St Dev	47	635	638	783	1,046
	CV	0.02	1.29	0.25	0.06	0.07
FTRI	Mean	2,200	220	2,420	13,574	**15,994**
	St Dev	67	475	486	1,270	1,483
	CV	0.03	2.16	0.20	0.09	0.09
FTUI	Mean	2,061	614	2,675	12,243	**14,918**
	St Dev	59	670	662	973	1,176
	CV	0.03	1.09	0.25	0.08	0.08
FTSR	Mean	2,061	614	2,675	12,243	**14,918**
	St Dev	59	670	662	973	1,176
	CV	0.03	1.09	0.25	0.08	0.08

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,056	3,174	5,229	10,616	**15,845**
	St Dev	69	2,274	2,233	748	2,359
	CV	0.03	0.72	0.43	0.07	0.15
MUN	Mean	2,022	2,323	4,345	12,921	**17,266**
	St Dev	101	2,007	1,966	1,470	2,024
	CV	0.05	0.86	0.45	0.11	0.12
RTC	Mean	2,089	685	2,774	8,874	**11,647**
	St Dev	61	703	678	753	1,110
	CV	0.03	1.03	0.24	0.08	0.10
RDE	Mean	2,058	430	2,488	8,747	**11,234**
	St Dev	46	563	549	619	871
	CV	0.02	1.31	0.22	0.07	0.08
RDE+div	Mean	2,065	380	2,445	8,881	**11,326**
	St Dev	55	1,100	1,085	662	1,236
	CV	0.03	2.89	0.44	0.07	0.11
FTRI	Mean	2,201	413	2,614	9,861	**12,475**
	St Dev	51	696	684	1,096	1,419
	CV	0.02	1.68	0.26	0.11	0.11
FTUI	Mean	2,060	720	2,780	8,910	**11,690**
	St Dev	63	779	768	666	1,010
	CV	0.03	1.08	0.28	0.07	0.09
FTSR	Mean	2,060	743	2,804	8,881	**11,684**
	St Dev	58	761	753	559	933
	CV	0.03	1.02	0.27	0.06	0.08
Strategy		**Case 3**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	1,991	3,514	5,505	11,643	**17,148**
	St Dev	66	2,453	2,412	667	2,464
	CV	0.03	0.70	0.44	0.06	0.14
MUN	Mean	2,050	880	2,930	14,527	**17,457**
	St Dev	89	1,169	1,116	1,470	2,068
	CV	0.04	1.33	0.38	0.10	0.12
RTC	Mean	2,096	721	2,816	9,178	**11,995**
	St Dev	62	807	831	689	1,194
	CV	0.03	1.12	0.30	0.08	0.10
RDE	Mean	2,072	617	2,688	9,036	**11,724**
	St Dev	48	637	632	733	1,041
	CV	0.02	1.03	0.23	0.08	0.09
RDE+div	Mean	2,056	493	2,550	9,121	**11,670**
	St Dev	46	569	570	570	821
	CV	0.02	1.15	0.22	0.06	0.07
FTRI	Mean	2,198	384	2,582	10,044	**12,626**
	St Dev	55	629	624	1,079	1,387
	CV	0.03	1.64	0.24	0.11	0.11
FTUI	Mean	2,064	631	2,695	9,059	**11,755**
	St Dev	61	644	640	571	810
	CV	0.03	1.02	0.24	0.06	0.07
FTSR	Mean	2,064	631	2,695	9,059	**11,755**
	St Dev	61	644	640	571	810
	CV	0.03	1.02	0.24	0.06	0.07

30 replication with common random numbers
Results in [$/week]

Table D- 10: Simulation Results: Set of Parameters 10

TC*= 100 [\$/hr], h_i= 5[\$/week], λ_i= 10.5 [arrivals/day], θ_i= 4.8 [units] for all *i

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,173	2,082	4,255	4,119	**8,374**
	St Dev	58	2,259	2,229	200	2,281
	CV	0.03	1.09	0.52	0.05	0.27
MUN	Mean	2,227	808	3,035	5,007	**8,042**
	St Dev	107	914	915	600	1,130
	CV	0.05	1.13	0.30	0.12	0.14
RTC	Mean	2,395	658	3,054	3,782	**6,836**
	St Dev	94	786	768	237	801
	CV	0.04	1.19	0.25	0.06	0.12
RDE	Mean	2,273	652	2,925	3,603	**6,528**
	St Dev	57	1,264	1,250	297	1,307
	CV	0.02	1.94	0.43	0.08	0.20
RDE+div	Mean	2,281	602	2,883	3,620	**6,503**
	St Dev	52	877	859	321	913
	CV	0.02	1.46	0.30	0.09	0.14
FTRI	Mean	2,464	526	2,990	4,302	**7,292**
	St Dev	77	843	823	266	878
	CV	0.03	1.60	0.28	0.06	0.12
FTUI	Mean	2,339	736	3,075	3,758	**6,833**
	St Dev	66	879	868	331	997
	CV	0.03	1.20	0.28	0.09	0.15
FTSR	Mean	2,335	489	2,824	3,751	**6,574**
	St Dev	69	448	452	372	607
	CV	0.03	0.92	0.16	0.10	0.09
Strategy		**Case 1**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	Mean	2,000	3,985	5,986	5,721	**11,707**
	St Dev	68	2,999	2,958	257	2,971
	CV	0.03	0.75	0.49	0.04	0.25
MUN	Mean	2,183	1,023	3,206	6,661	**9,867**
	St Dev	106	1,241	1,199	571	1,162
	CV	0.05	1.21	0.37	0.09	0.12
RTC	Mean	2,419	726	3,145	5,181	**8,326**
	St Dev	114	901	917	269	953
	CV	0.05	1.24	0.29	0.05	0.11
RDE	Mean	2,273	634	2,907	5,212	**8,119**
	St Dev	52	906	900	553	1,074
	CV	0.02	1.43	0.31	0.11	0.13
RDE+div	Mean	2,292	637	2,929	5,080	**8,009**
	St Dev	49	932	927	357	993
	CV	0.02	1.46	0.32	0.07	0.12
FTRI	Mean	2,484	525	3,009	5,722	**8,731**
	St Dev	50	744	726	265	784
	CV	0.02	1.42	0.24	0.05	0.09
FTUI	Mean	2,337	572	2,909	5,013	**7,922**
	St Dev	63	670	654	497	817
	CV	0.03	1.17	0.22	0.10	0.10
FTSR	Mean	2,341	529	2,870	5,040	**7,910**
	St Dev	60	670	660	532	889
	CV	0.03	1.27	0.23	0.11	0.11

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,199	2,081	4,280	3,842	**8,122**
	St Dev	75	1,643	1,630	196	1,613
	CV	0.03	0.79	0.38	0.05	0.20
MUN	Mean	2,161	541	2,702	4,971	**7,674**
	St Dev	110	707	663	487	926
	CV	0.05	1.31	0.25	0.10	0.12
RTC	Mean	2,408	466	2,874	3,459	**6,334**
	St Dev	110	578	591	271	645
	CV	0.05	1.24	0.21	0.08	0.10
RDE	Mean	2,190	690	2,881	3,586	**6,466**
	St Dev	43	946	949	484	1,120
	CV	0.02	1.37	0.33	0.13	0.17
RDE+div	Mean	2,195	364	2,559	3,431	**5,990**
	St Dev	54	520	504	284	596
	CV	0.02	1.43	0.20	0.08	0.10
FTRI	Mean	2,475	464	2,939	4,156	**7,095**
	St Dev	65	522	524	268	533
	CV	0.03	1.12	0.18	0.06	0.08
FTUI	Mean	2,305	341	2,646	3,587	**6,234**
	St Dev	75	598	598	406	759
	CV	0.03	1.75	0.23	0.11	0.12
FTSR	Mean	2,314	421	2,735	3,587	**6,322**
	St Dev	67	593	592	385	733
	CV	0.03	1.41	0.22	0.11	0.12

Strategy		Case 3				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,159	2,412	4,571	4,126	**8,697**
	St Dev	72	2,049	2,017	225	2,051
	CV	0.03	0.85	0.44	0.05	0.24
MUN	Mean	2,233	570	2,804	5,155	**7,959**
	St Dev	101	737	729	533	836
	CV	0.05	1.29	0.26	0.10	0.11
RTC	Mean	2,442	710	3,152	3,805	**6,957**
	St Dev	108	939	919	288	910
	CV	0.04	1.32	0.29	0.08	0.13
RDE	Mean	2,297	468	2,765	3,653	**6,418**
	St Dev	50	758	746	264	790
	CV	0.02	1.62	0.27	0.07	0.12
RDE+div	Mean	2,296	558	2,854	3,744	**6,598**
	St Dev	45	864	868	298	899
	CV	0.02	1.55	0.30	0.08	0.14
FTRI	Mean	2,483	592	3,075	4,201	**7,276**
	St Dev	81	1,196	1,159	237	1,167
	CV	0.03	2.02	0.38	0.06	0.16
FTUI	Mean	2,340	898	3,238	3,676	**6,914**
	St Dev	74	1,038	1,030	370	1,131
	CV	0.03	1.16	0.32	0.10	0.16
FTSR	Mean	2,342	882	3,223	3,666	**6,889**
	St Dev	69	1,027	1,010	357	1,107
	CV	0.03	1.16	0.31	0.10	0.16

30 replication with common random numbers
Results in [$/week]

Table D- 11: Simulation Results: Set of Parameters 11

TC*= 100 [$/hr], h_i= 5[$/week], λ_i= 4.35 [arrivals/day], θ_i= 11.5 [units] for all *i

Strategy		Case 0 (Symmetric case) Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,155	7,635	9,790	4,017	**13,807**
	St Dev	112	5,935	5,883	297	5,990
	CV	0.05	0.78	0.60	0.07	0.43
MUN	Mean	2,432	985	3,417	5,626	**9,042**
	St Dev	123	1,166	1,159	560	1,302
	CV	0.05	1.18	0.34	0.10	0.14
RTC	Mean	2,704	1,200	3,903	4,612	**8,515**
	St Dev	139	1,811	1,743	360	1,969
	CV	0.05	1.51	0.45	0.08	0.23
RDE	Mean	2,438	2,003	4,441	4,102	**8,543**
	St Dev	59	1,963	1,958	359	2,036
	CV	0.02	0.98	0.44	0.09	0.24
RDE+div	Mean	2,467	1,476	3,943	4,199	**8,142**
	St Dev	57	1,778	1,758	355	1,965
	CV	0.02	1.20	0.45	0.08	0.24
FTRI	Mean	2,705	1,108	3,814	5,670	**9,483**
	St Dev	84	1,945	1,899	236	1,999
	CV	0.03	1.76	0.50	0.04	0.21
FTUI	Mean	2,553	1,365	3,918	4,717	**8,635**
	St Dev	70	1,156	1,164	411	1,286
	CV	0.03	0.85	0.30	0.09	0.15
FTSR	Mean	2,548	1,812	4,360	4,600	**8,959**
	St Dev	82	1,789	1,778	339	1,711
	CV	0.03	0.99	0.41	0.07	0.19

Strategy		Case 1 Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	1,997	11,111	13,108	5,463	**18,571**
	St Dev	112	5,915	5,856	334	5,932
	CV	0.06	0.53	0.45	0.06	0.32
MUN	Mean	2,365	1,828	4,193	7,101	**11,294**
	St Dev	115	2,188	2,141	679	2,086
	CV	0.05	1.20	0.51	0.10	0.18
RTC	Mean	2,703	1,454	4,157	6,321	**10,479**
	St Dev	134	1,850	1,793	417	1,933
	CV	0.05	1.27	0.43	0.07	0.18
RDE	Mean	2,462	1,752	4,214	6,097	**10,311**
	St Dev	66	2,302	2,289	596	2,526
	CV	0.03	1.31	0.54	0.10	0.24
RDE+div	Mean	2,488	921	3,409	6,006	**9,414**
	St Dev	64	1,253	1,225	521	1,543
	CV	0.03	1.36	0.36	0.09	0.16
FTRI	Mean	2,719	897	3,616	7,541	**11,157**
	St Dev	64	1,247	1,228	173	1,277
	CV	0.02	1.39	0.34	0.02	0.11
FTUI	Mean	2,540	2,104	4,644	6,033	**10,677**
	St Dev	60	2,686	2,697	629	2,890
	CV	0.02	1.28	0.58	0.10	0.27
FTSR	Mean	2,514	2,067	4,582	5,778	**10,359**
	St Dev	77	2,309	2,318	617	2,340
	CV	0.03	1.12	0.51	0.11	0.23

Strategy		Case 2				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	**Mean**	2,199	6,968	9,167	3,689	**12,857**
	St Dev	84	4,807	4,759	282	4,894
	CV	0.04	0.69	0.52	0.08	0.38
MUN	**Mean**	2,406	1,310	3,716	5,258	**8,975**
	St Dev	133	2,227	2,155	756	2,089
	CV	0.06	1.70	0.58	0.14	0.23
RTC	**Mean**	2,728	1,732	4,460	4,306	**8,766**
	St Dev	131	1,873	1,841	374	1,947
	CV	0.05	1.08	0.41	0.09	0.22
RDE	**Mean**	2,398	1,974	4,372	4,129	**8,501**
	St Dev	58	1,550	1,561	620	1,679
	CV	0.02	0.79	0.36	0.15	0.20
RDE+div	**Mean**	2,436	653	3,088	4,073	**7,161**
	St Dev	61	940	944	577	1,060
	CV	0.03	1.44	0.31	0.14	0.15
FTRI	**Mean**	2,726	908	3,634	5,477	**9,111**
	St Dev	70	1,499	1,470	245	1,524
	CV	0.03	1.65	0.40	0.04	0.17
FTUI	**Mean**	2,533	1,565	4,098	4,350	**8,448**
	St Dev	72	2,134	2,135	526	2,182
	CV	0.03	1.36	0.52	0.12	0.26
FTSR	**Mean**	2,513	1,865	4,378	4,073	**8,451**
	St Dev	98	2,315	2,302	474	2,400
	CV	0.04	1.24	0.53	0.12	0.28
Strategy		**Case 3**				
		Inv. Holding Cost	**Lost Sales Cost**	**Total Inv. Cost**	**Transp. Cost**	**Total Cost**
BENCH1	**Mean**	2,144	7,516	9,659	3,960	**13,620**
	St Dev	88	4,136	4,083	197	4,123
	CV	0.04	0.55	0.42	0.05	0.30
MUN	**Mean**	2,415	946	3,361	5,691	**9,052**
	St Dev	85	1,372	1,337	654	1,289
	CV	0.04	1.45	0.40	0.11	0.14
RTC	**Mean**	2,715	1,144	3,859	4,678	**8,537**
	St Dev	140	1,398	1,353	398	1,444
	CV	0.05	1.22	0.35	0.09	0.17
RDE	**Mean**	2,459	1,823	4,282	4,458	**8,740**
	St Dev	71	2,178	2,142	530	2,402
	CV	0.03	1.19	0.50	0.12	0.27
RDE+div	**Mean**	2,502	784	3,287	4,562	**7,849**
	St Dev	74	1,138	1,130	376	1,162
	CV	0.03	1.45	0.34	0.08	0.15
FTRI	**Mean**	2,718	859	3,576	5,517	**9,093**
	St Dev	64	1,161	1,136	246	1,199
	CV	0.02	1.35	0.32	0.04	0.13
FTUI	**Mean**	2,532	1,715	4,247	4,464	**8,711**
	St Dev	69	1,943	1,930	524	1,901
	CV	0.03	1.13	0.45	0.12	0.22
FTSR	**Mean**	2,523	1,832	4,355	4,259	**8,613**
	St Dev	62	2,066	2,064	356	2,040
	CV	0.02	1.13	0.47	0.08	0.24

30 replication with common random numbers
Results in [$/week]

Table D- 12: Simulation Results: Set of Parameters 12

TC*= 100 [$/hr], h_i= 5[$/week], λ_i= 2.4 [arrivals/day], θ_i= 20.8 [units] for all *i

Strategy		Case 0 (Symmetric case) Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	**Mean**	2,137	18,784	20,921	3,758	**24,679**
	St Dev	145	8,613	8,512	363	8,639
	CV	0.07	0.46	0.41	0.10	0.35
MUN	**Mean**	2,673	1,174	3,847	6,441	**10,288**
	St Dev	118	1,487	1,460	671	1,474
	CV	0.04	1.27	0.38	0.10	0.14
RTC	**Mean**	2,906	4,250	7,156	5,436	**12,592**
	St Dev	149	4,277	4,239	471	4,401
	CV	0.05	1.01	0.59	0.09	0.35
RDE	**Mean**	2,626	3,741	6,367	4,641	**11,008**
	St Dev	82	3,575	3,563	385	3,475
	CV	0.03	0.96	0.56	0.08	0.32
RDE+div	**Mean**	2,660	2,214	4,874	4,779	**9,653**
	St Dev	66	2,021	2,034	517	2,128
	CV	0.02	0.91	0.42	0.11	0.22
FTRI	**Mean**	2,898	2,904	5,802	7,515	**13,317**
	St Dev	85	3,275	3,257	174	3,233
	CV	0.03	1.13	0.56	0.02	0.24
FTUI	**Mean**	2,728	3,364	6,092	4,518	**10,610**
	St Dev	80	3,075	3,090	544	3,212
	CV	0.03	0.91	0.51	0.12	0.30
FTSR	**Mean**	2,686	2,874	5,560	5,117	**10,677**
	St Dev	84	2,860	2,845	553	2,925
	CV	0.03	1.00	0.51	0.11	0.27

Strategy		Case 1 Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	**Mean**	2,017	17,912	19,928	5,131	**25,059**
	St Dev	127	7,455	7,373	349	7,482
	CV	0.06	0.42	0.37	0.07	0.30
MUN	**Mean**	2,571	2,220	4,791	7,824	**12,615**
	St Dev	127	2,385	2,345	733	2,368
	CV	0.05	1.07	0.49	0.09	0.19
RTC	**Mean**	2,866	3,121	5,986	7,362	**13,349**
	St Dev	129	3,558	3,512	560	3,604
	CV	0.05	1.14	0.59	0.08	0.27
RDE	**Mean**	2,652	1,920	4,572	7,253	**11,826**
	St Dev	68	1,667	1,654	605	1,739
	CV	0.03	0.87	0.36	0.08	0.15
RDE+div	**Mean**	2,660	1,946	4,606	7,451	**12,057**
	St Dev	71	1,832	1,814	543	1,889
	CV	0.03	0.94	0.39	0.07	0.16
FTRI	**Mean**	2,910	1,214	4,124	9,990	**14,115**
	St Dev	72	1,697	1,684	148	1,699
	CV	0.02	1.40	0.41	0.01	0.12
FTUI	**Mean**	2,706	2,934	5,640	7,170	**12,810**
	St Dev	73	2,656	2,641	815	2,677
	CV	0.03	0.91	0.47	0.11	0.21
FTSR	**Mean**	2,689	2,668	5,357	6,583	**11,940**
	St Dev	65	2,926	2,921	662	2,915
	CV	0.02	1.10	0.55	0.10	0.24

Strategy		Case 2 Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,232	13,941	16,173	3,398	**19,571**
	St Dev	122	8,261	8,195	303	8,219
	CV	0.05	0.59	0.51	0.09	0.42
MUN	Mean	2,617	2,381	4,997	5,639	**10,636**
	St Dev	132	2,314	2,324	711	2,297
	CV	0.05	0.97	0.47	0.13	0.22
RTC	Mean	2,926	3,228	6,154	4,816	**10,970**
	St Dev	130	3,391	3,354	486	3,479
	CV	0.04	1.05	0.54	0.10	0.32
RDE	Mean	2,616	2,712	5,328	4,683	**10,011**
	St Dev	74	2,434	2,426	759	2,389
	CV	0.03	0.90	0.46	0.16	0.24
RDE+div	Mean	2,669	1,805	4,474	4,667	**9,142**
	St Dev	78	1,884	1,861	604	2,051
	CV	0.03	1.04	0.42	0.13	0.22
FTRI	Mean	2,924	2,105	5,029	7,260	**12,289**
	St Dev	75	2,361	2,338	177	2,327
	CV	0.03	1.12	0.46	0.02	0.19
FTUI	Mean	2,718	3,925	6,642	5,219	**11,861**
	St Dev	78	3,435	3,410	658	3,598
	CV	0.03	0.88	0.51	0.13	0.30
FTSR	Mean	2,697	3,488	6,185	4,428	**10,613**
	St Dev	73	2,680	2,667	463	2,686
	CV	0.03	0.77	0.43	0.10	0.25

Strategy		Case 3 Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,168	14,369	16,537	3,766	**20,303**
	St Dev	97	7,092	7,049	289	7,109
	CV	0.04	0.49	0.43	0.08	0.35
MUN	Mean	2,711	1,360	4,071	6,624	**10,695**
	St Dev	136	1,965	1,967	792	2,176
	CV	0.05	1.44	0.48	0.12	0.20
RTC	Mean	2,917	2,907	5,824	5,410	**11,235**
	St Dev	121	4,036	4,015	386	4,119
	CV	0.04	1.39	0.69	0.07	0.37
RDE	Mean	2,668	2,066	4,734	5,480	**10,214**
	St Dev	78	2,432	2,423	650	2,565
	CV	0.03	1.18	0.51	0.12	0.25
RDE+div	Mean	2,710	1,175	3,886	5,405	**9,291**
	St Dev	69	1,753	1,732	744	2,099
	CV	0.03	1.49	0.45	0.14	0.23
FTRI	Mean	2,922	1,223	4,144	7,299	**11,443**
	St Dev	61	1,630	1,610	136	1,601
	CV	0.02	1.33	0.39	0.02	0.14
FTUI	Mean	2,721	2,469	5,190	5,416	**10,606**
	St Dev	92	2,897	2,890	571	2,905
	CV	0.03	1.17	0.56	0.11	0.27
FTSR	Mean	2,728	2,346	5,074	4,819	**9,892**
	St Dev	69	2,195	2,221	514	2,403
	CV	0.03	0.94	0.44	0.11	0.24

30 replication with common random numbers
Results in [$/week]

Table D- 13: Simulation Results: Set of Parameters 1, under Twice Demand Arrivals at Retailer 4 and Without Inventory Target Update

***TC*= 100 [\$/hr], h_i= 50[\$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for *i*={1,2,3,5,6,7} , λ_4= 100[arrivals/day], θ_4= 1 [units]**

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	22,099	5,370	27,470	3,615	**31,085**
	St Dev	3,358	2,535	3,445	139	3,438
	CV	0.15	0.47	0.13	0.04	0.11
MUN	Mean	13,594	2,216	15,810	7,845	**23,655**
	St Dev	3,336	1,225	3,197	463	3,332
	CV	0.25	0.55	0.20	0.06	0.14
RTC	Mean	15,196	7,751	22,946	5,226	**28,172**
	St Dev	2,308	2,670	3,133	247	3,097
	CV	0.15	0.34	0.14	0.05	0.11
RDE	Mean	13,592	5,602	19,194	5,384	**24,578**
	St Dev	1,380	2,540	2,960	268	2,947
	CV	0.10	0.45	0.15	0.05	0.12
RDE+div	Mean	13,673	3,203	16,876	5,416	**22,291**
	St Dev	980	1,549	1,600	209	1,627
	CV	0.07	0.48	0.09	0.04	0.07
FTRI	Mean	9,094	17,124	26,217	7,032	**33,249**
	St Dev	679	4,090	3,862	72	3,865
	CV	0.07	0.24	0.15	0.01	0.12
FTUI	Mean	10,180	8,900	19,079	8,613	**27,692**
	St Dev	567	2,839	3,033	298	3,168
	CV	0.06	0.32	0.16	0.03	0.11
FTSR	Mean	10,179	8,876	19,055	8,614	**27,669**
	St Dev	564	2,888	3,063	296	3,193
	CV	0.06	0.33	0.16	0.03	0.12

30 replication with common random numbers
Results in [\$/week]

Table D- 14: Simulation Results: Set of Parameters 7, under Twice Demand Arrivals at Retailer 4 and Without Inventory Target Update

***TC*= 100 [$/hr], h_i= 5[$/week], λ_i= 50[arrivals/day], θ_i= 1 [units] for *i*={1,2,3,5,6,7}, λ_4= 100[arrivals/day], θ_4= 1 [units]**

Strategy		Case 0 (Symmetric case)				
		Inv. Holding Cost	Lost Sales Cost	Total Inv. Cost	Transp. Cost	Total Cost
BENCH1	Mean	2,055	7,787	9,842	4,336	**14,177**
	St Dev	237	3,156	3,099	177	3,125
	CV	0.12	0.41	0.31	0.04	0.22
MUN	Mean	2,197	905	3,102	5,445	**8,547**
	St Dev	314	669	787	601	934
	CV	0.14	0.74	0.25	0.11	0.11
RTC	Mean	2,065	8,581	10,646	3,602	**14,247**
	St Dev	157	3,229	3,212	221	3,211
	CV	0.08	0.38	0.30	0.06	0.23
RDE	Mean	2,052	5,937	7,989	3,522	**11,511**
	St Dev	185	2,621	2,533	211	2,535
	CV	0.09	0.44	0.32	0.06	0.22
RDE+div	Mean	2,036	3,291	5,327	3,453	**8,780**
	St Dev	142	2,271	2,272	226	2,269
	CV	0.07	0.69	0.43	0.07	0.26
FTRI	Mean	2,097	17,119	19,216	3,402	**22,618**
	St Dev	295	3,987	4,002	383	4,115
	CV	0.14	0.23	0.21	0.11	0.18
FTUI	Mean	2,443	3,666	6,109	4,802	**10,912**
	St Dev	293	1,588	1,644	344	1,843
	CV	0.12	0.43	0.27	0.07	0.17
FTSR	Mean	2,444	3,666	6,110	4,802	**10,913**
	St Dev	327	1,588	1,654	344	1,855
	CV	0.13	0.43	0.27	0.07	0.17

30 replication with common random numbers
Results in [$/week]

Bibliography

ADELMAN, D. (2004) A price-directed approach to stochastic inventory/routing. *Operations Research,* 52**,** 499-514.

AGHEZZAF, E. H., RAA, B. & VAN LANDEGHEM, H. (2006) Modeling inventory routing problems in supply chains of high consumption products. *European Journal of Operational Research,* 169**,** 1048-1063.

ANILY, S. & FEDERGRUEN, A. (1990) One Warehouse Multiple Retailer Systems with Vehicle-Routing Costs. *Management Science,* 36**,** 92-114.

ASSAD, A. A. (1988) Modeling and Implementation Issues in Routing. IN GOLDEN, B. L. & ASSAD, A. A. (Eds.) *Vehicle Routing: Methods and Studies.* Amsterdam, Elsevier.

AXSÄTER, S. (2000) *Inventory control,* Boston, Mass.; London, Kluwer Academic.

BAITA, F., UKOVICH, W., PESENTI, R. & FAVARETTO, D. (1998) Dynamic routing-and-inventory problems: A review. *Transportation Research Part a-Policy and Practice,* 32**,** 585-598.

BALL, M. O. (2002) Research Directions and Educational Programs in Supply Chain Management. College Park, MD, Robert H. Smith School of Business, Univeristy of Maryland.

BARD, J. F., HUANG, L., DROR, M. & JAILLET, P. (1998a) A branch and cut algorithm for the VRP with satellite facilities. *Iie Transactions,* 30**,** 821-834.

BARD, J. F., HUANG, L. & JAILLET, P. (1998b) Decomposition approach to the inventory routing problem with satellite facilities. *Transportation Science,* 32**,** 189-203.

BEASLEY, J. (1983) Route First-Cluster Second Methods for the Vehicle Rotuing Problem. *Omega,* 11**,** 403-408.

BECKMANN, M. (1961) An Inventory Model for Arbitrary Interval and Quantity Distributions of Demand. *Management Science,* 8**,** 35-57.

BELL, W. J., DALBERTO, L. M., FISHER, M. L., GREENFIELD, A. J., JAIKUMAR, R., KEDIA, P., MACK, R. G. & PRUTZMAN, P. J. (1983) Improving the Distribution of Industrial Gases with an Online Computerized Routing and Scheduling Optimizer. *Interfaces,* 13**,** 4-23.

BERTAZZI, L., PALETTA, G. & SPERANZA, M. G. (2002) Deterministic order-up-to level policies in an inventory routing problem. *Transportation Science,* 36**,** 119-132.

BERTAZZI, L., SAVELSBERGH, M. & SPERANZA, M. G. (2007) Inventory Routing.

BERTSIMAS, D. J. & SIMCHILEVI, D. (1996) A new generation of vehicle routing research: Robust algorithms, addressing uncertainty. *Operations Research,* 44**,** 286-304.

BLANCHARD, D. (2003) Fears of "Big Brother" sidetrack Benetton's smart tag initiative. *Transportation and Distribution,* 44**,** 20.

BODIN, L., GOLDEN, B., ASSAD, A. & BALL, M. (1983) Special Issue - Routing and Scheduling of Vehicles and Crews - the State of the Art. *Computers & Operations Research,* 10**,** 1-&.

BUZZELL, R. D. & ORTMEYER, G. (1995) Channel Partnerships Streamline Distribution. *Sloan Management Review,* 36**,** 85-96.

CAMPBELL, A. M., CLARKE, L. W. & SAVELSBERGH, M. W. P. (1998) The inventory routing problem. IN CRAINIC, T. G. & LAPORTE, G. (Eds.) *Fleet Management and Logistics.* Dordrecht, The Netherlands, Kluwer Academic Publishers.

CAMPBELL, A. M. & SAVELSBERGH, M. (2002) Inventory Routing in Practice. IN TOTH, P. & VIGO, D. (Eds.) *The Vehicle Routing Problem.* SIAM Monographs on Discrete Mathematics and Applications.

CAMPBELL, A. M. & SAVELSBERGH, M. W. P. (2004) A decomposition approach for the inventory-routing problem. *Transportation Science,* 38**,** 488-502.

CHEN, F., DREZNER, Z., RYAN, J. K. & SIMCHI-LEVI, D. (2000) Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information. *Management Science,* 46**,** 436-443.

CHIEN, T. W., BALAKRISHNAN, A. & WONG, R. T. (1989) An Integrated Inventory Allocation and Vehicle-Routing Problem. *Transportation Science,* 23**,** 67-76.

CHOPRA, S. & MEINDL, P. (2003) *Supply Chain Management,* Upper Sadle River, NJ, Prentice Hall.

CHRISTOFIDES, N. (1985) Vehicle Routing. IN LAWLER, E. L., LENSTRA, J. K., RINNOOY KAN, A. H. G. & SHMOYDS, D. B. (Eds.) *The Traveling salesman problem: a guided tour of combinatorial optimization.* Chichester [West Sussex]; New York, John Wiley and Sons.

COUNCIL_OF_LOGISTIC_MANAGEMENT (2004) 15th Annual State of Logistic Report, accessed December 2004. IN MANAGEMENT, C. O. L. (Ed.).

DATTA, S. (2003) Radio frequency identification (RFID) made easy. MIT forum for supply chain innovation.

DROR, M. & BALL, M. (1987) Inventory Routing - Reduction from an Annual to a Short-Period Problem. *Naval Research Logistics,* 34**,** 891-905.

FEDERGRUEN, A. & ZIPKIN, P. (1984) A Combined Vehicle-Routing and Inventory Allocation Problem. *Operations Research,* 32**,** 1019-1037.

FINE, C. H. (1998) *Clockspeed: winning industry control in the age of temporary advantage,* Reading, Mass., Perseus Books.

FISHER, M. L. (1995) Vehicle Routing. IN BALL, M. O., MAGNANTI, T. L., MONMA, C. L. & NEMHAUSER, G. L. (Eds.) *Network routing.* Amsterdam; New York, Elsevier.

FISHER, M. L. (1997) What is the right supply chain for your product? *Harvard Business Review,* 75**,** 105-&.

FLEISCHMANN, B. & MEYR, H. (2003) Planning Hierarchy, Modeling and Advance Planning Systems. IN KOK, A. G. D. & GRAVES, S. C. (Eds.) *Supply chain management: design, coordination and operation.* Elsevier.

GANS, N. & VAN RYZIN, G. (1999) Dynamic vehicle dispatching: Optimal heavy traffic performance and practical insights. *Operations Research,* 47**,** 675-692.

GENDREAU, M., GUERTIN, F., POTVIN, J. Y. & TAILLARD, E. (1999) Parallel tabu search for real-time vehicle routing and dispatching. *Transportation Science,* 33**,** 381-390.

GHIANI, G., MUSMANNO, R. & LAPORTE, G. (2004) *Introduction to logistics systems planning and control,* Chichester, West Sussex; Hoboken, NJ, USA, J. Wiley.

GOLDEN, B., ASSAD, A. & DAHL, R. (1984) Analysis of a Large-Scale Vehicle-Routing Problem with an Inventory Component. *Large Scale Systems in Information and Decision Technologies,* 7**,** 181-190.

GOLDEN, B. L. & ASSAD, A. A. (Eds.) (1988) *Vehicle Routing: Methods and Studies,* Amsterdam, Elsevier.

GRAVES, S. C., RINNOOY KAN, A. H. G. & ZIPKIN, P. H. (1993) *Logistics of production and inventory,* Amsterdam; New York, North-Holland.

GRÖTSCHEL, M., KRUMKE, S. O. & RAMBAU, J. (2001a) Online Optimization of Complex Transportation Systemes. IN GRÖTSCHEL, M., KRUMKE, S. O. & RAMBAU, J. (Eds.) *Online optimization of large scale systems.* Berlin; New York, Springer.

GRÖTSCHEL, M., KRUMKE, S. O., RAMBAU, J., WINTER, T. & ZIMMERMANN, U. W. (2001b) Combinatorial Online Optimization in Real-Time. IN GRÖTSCHEL, M., KRUMKE, S. O. & RAMBAU, J. (Eds.) *Online Optimization of Large Scale Systems.* Berlin; New York, Springer.

HARRIS, F. W. (1913) How many parts to make at once. *Factory, The Magazine of Management,* 10**,** 135-136, 152.

ICHOUA, S., GENDREAU, N. & POTVIN, J. Y. (2000) Diversion issues in real-time vehicle dispatching. *Transportation Science,* 34**,** 426-438.

IGLEHART, D. L. (1963) Optimality of (S, S) Policies in the Infinite Horizon Dynamic Inventory Problem. *Management Science,* 9**,** 259-267.

JAILLET, P., BARD, J. F., HUANG, L. & DROR, M. (2002) Delivery cost approximations for inventory routing problems in a rolling horizon framework. *Transportation Science,* 36**,** 292-300.

KAPUSCINSKI, R., ZHANG, R. Q., CARBONNEAU, P., MOORE, R. & REEVES, B. (2004) Inventory decisions in Dell's supply chain. *Interfaces,* 34**,** 191-205.

KIM, Y. (2003) Hybrid Approaches To Solve Dynamic Fleet Management Problems. *Ph D dissertation, The University of Texas at Austin.*

KIM, Y., MAHMASSANI, H. S. & JAILLET, P. (2002a) Dynamic truckload truck routing and scheduling in oversaturated demand situations. *Transportation Network Modeling 2002***,** 66-71.

KIM, Y. J., MAHMASSANI, H. S. & JAILLET, P. (2002b) Two-phase optimization approaches to solve dynamic large fleet management problems. University of Texas at Austin.

KIM, Y. J., MAHMASSANI, H. S. & JAILLET, P. (2004) Dynamic truckload routing, scheduling, and load acceptance for large fleet operation with priority demands. *Transportation Network Modeling 2004***,** 120-128.

KLEYWEGT, A. J., NORI, V. S. & SAVELSBERGH, M. W. P. (2002) The Stochastic inventory routing problem with direct deliveries. *Transportation Science,* 36**,** 94-118.

KLEYWEGT, A. J., NORI, V. S. & SAVELSBERGH, M. W. P. (2004) Dynamic programming approximations for a stochastic inventory routing problem. *Transportation Science,* 38**,** 42-70.

KUMAR, A., SCHWARZ, L. B. & WARD, J. E. (1995) Risk-Pooling Along a Fixed Delivery Route Using a Dynamic Inventory-Allocation Policy. *Management Science,* 41**,** 344-362.

LA LONDE, B. J. (1994) Evolution of the integrated logistics concept. IN ROBESON, J. F., COPACINO, W. C. & HOWE, R. E. (Eds.) *The Logistics Handbook.* New York, The Free Press.

LARSEN, A., MADSEN, O. & SOLOMON, M. (2002) Partially dynamic vehicle routing - models and algorithms. *Journal of the Operational Research Society,* 53**,** 637-646.

LAW, A. M. & KELTON, W. D. (2000) *Simulation modeling and analysis,* Boston, McGraw-Hill.

LEE, H. L. (2004) The triple-A supply chain. *Harvard Business Review,* 82**,** 102-+.

LEE, H. L. & NAHMIAS, S. (1993) Single-Product, Single-Location Model. IN GRAVES, S. C., RINNOOY KAN, A. H. G. & ZIPKIN, P. H. (Eds.) *Logistics of production and inventory.* Amsterdam; New York, North-Holland.

LEE, H. L., PADMANABHAN, V. & WHANG, S. (1997) The bullwhip effect in supply chains. *Sloan Management Review,* 38**,** 93-102.

MASTERS, J. M. & LA LONDE, B. J. (1994) The role of new information technology in the practice of traffic management. IN ROBESON, J. F., COPACINO, W. C. & HOWE, R. E. (Eds.) *The Logistics Handbook.* New York, The Free Press.

MISHRA, B. K. & RAGHUNATHAN, S. (2004) Retailer- vs. vendor-managed inventory and brand competition. *Management Science,* 50**,** 445-457.

POWELL, W., JAILLET, P. & ODONI, A. (1995) Stochastic and Dynamic Networks and Routing. *in Ball, M.O., T.L. Magnanti, C.L. Monma. And G.L. Nemhauser (eds), Handbook in Operations Research and Management Science, Vol. 8, Network Routing, Elsevier, Amsterdam, pp. 141-296.*

POWELL, W. B. (2003) Dynamic Models of Transportation Operations. IN KOK, A. G. D. & GRAVES, S. C. (Eds.) *Supply chain management: design, coordination and operation.* Elsevier.

PSARAFTIS, H. N. (1988) Dynamic Vehicle Routing Problems. IN GOLDEN, B. L. & ASSAD, A. (Eds.) *Vehicle Routing: Methods and Studies.* Amsterdam, Elsevier.

PSARAFTIS, H. N. (1995) Dynamic vehicle routing: Status and prospects. *Annals of Operations Research,* 61**,** 143-164.

RABAH, M. & MAHMASSANI, H. S. (2002) Impact of information and communication technologies on logistics and freight transportation - Example of vendor-managed inventories. *Freight Transportation 2002***,** 10-19.

REGAN, A. C., MAHMASSANI, H. S. & JAILLET, P. (1995) Improving the efficiency of commercial vehicle operations using real-time information: potential uses and assignment strategies. *Transportation Research Record*, 188-198.

REGAN, A. C., MAHMASSANI, H. S. & JAILLET, P. (1996a) Dynamic decision making for commercial fleet operations using real-time information. *Transportation Research Record*, 91-97.

REGAN, A. C., MAHMASSANI, H. S. & JAILLET, P. (1996b) Dynamic dispatching strategies under real-time information for carrier fleet management. IN LESORT, J. B. (Ed.) *Transportation and Traffic Theory.*

REIMAN, M. I., RUBIO, R. & WEIN, L. W. (1999) Heavy traffic analysis of the dynamic stochastic inventory-routing problem. *Transportation Science,* 33**,** 361-380.

RICHARDS, F. R. (1975) Distribution of Inventory Position in a Continuous-Review (S,S) Inventory System. *Operations Research,* 23**,** 366-371.

RUTNER, S. M., GIBSON, B. J. & WILLIAMS, S. R. (2003) The impacts of the integrated logistics systems on electronic commerce and enterprise resource planning systems. *Transportation Research Part E-Logistics and Transportation Review,* 39**,** 83-93.

SAVELSBERGH, M. & SONG, J. H. (2007) Inventory routing with continuous moves. *Computers & Operations Research,* 34**,** 1744-1763.

SCARF, H. E. (1960) The optimality of (s, S) policies in the dynamic inventory problem. IN ARROW, K. A., KARLIN, S. & SUPPES, P. (Eds.)

Mathematical Methods in the Social Science. Standford, CA, Standford University Press.

SEGUIN, R., POTVIN, J. Y., GENDREAU, M., CRAINIC, T. G. & MARCOTTE, P. (1997) Real-time decision problems: An operational research perspective. *Journal of the Operational Research Society,* 48**,** 162-174.

SHARP, G. & GOETSCHALCKX, M. (1999) Online Logistics Tutorial. Georgia Institute of Technology, School of Industrial and System Engineering.

SIMCHI-LEVI, D., KAMINSKY, P. & SIMCHI-LEVI, E. (2003) *Designing and managing the supply chain: concepts, strategies, and case studies,* Boston, McGraw-Hill/Irwin.

THOMAS, D. J. & GRIFFIN, P. M. (1996) Coordinated supply chain management. *European Journal of Operational Research,* 94**,** 1-15.

TOTH, P. & VIGO, D. (2002) *The vehicle routing problem,* Philadelphia, PA, SIAM publishing.

VISWANATHAN, S. & MATHUR, K. (1997) Integrating routing and inventory decisions in one-warehouse multiretailer multiproduct distribution systems. *Management Science,* 43**,** 294-312.

WINSTON, W. L. (1994) *Operations research: applications and algorithms,* Belmont, Calif., Duxbury Press.

WOLFF, R. W. (1989) *Stochastic modeling and the theory of queues,* Englewood Cliffs, N.J., Prentice Hall.

YANG, J., JAILLET, P. & MAHMASSANI, H. (1998) On-Line algorithms for truck fleet assignment and scheduling under real-time information. *Transportation Research Record*, 107-113.

YANG, J., JAILLET, P. & MAHMASSANI, H. (2004) Real-time multivehicle truckload pickup and delivery problems. *Transportation Science,* 38, 135-148.

ZHENG, Y. S. (1991) A Simple Proof for Optimality of (S, S) Policies in Infinite-Horizon Inventory Systems. *Journal of Applied Probability,* 28, 802-810.

ZHENG, Y. S. & FEDERGRUEN, A. (1991) Finding Optimal (S, S) Policies Is About as Simple as Evaluating a Single Policy. *Operations Research,* 39, 654-665.

ZIPKIN, P. H. (2000) *Foundations of inventory management,* Boston, McGraw-Hill.