ABSTRACT

Title of dissertation: PHYSICS BEYOND THE STANDARD MODEL: SUPERSYMMETRY AND EXTRA DIMENSIONS

Ken Hsieh
Doctor of Philosophy, 2007

Dissertation directed by: Professor Markus A. Luty
Department of Physics

After briefly describing the Standard Model and its hierarchy problem, I focus on supersymmetry as an elegant solution to the hierarchy problem and describe the Minimal Supersymmetric Standard Model. I then present two well-motivated models of supersymmetry-breaking based on the ideas of gauge- and anomaly-mediated supersymmetry-breaking. These two models are constructed to alleviate the little hierarchy problem of the Minimal Supersymmetric Standard Model, and can naturally offer distinct collider signatures. I describe typical spectra of these two models and discuss their collider signatures at the Large Hadron Collider.

I also present a model based on universal extra dimensions that solves both the problems of proton decay and neutrino masses in the minimal framework. The model offers an interesting dark matter candidate as an admixture of Kaluza-Klein modes, and I analysis the range in parameter space that may give the observed relic density of dark matter and the prospects of direct detection within the viable parameter space.

PHYSICS BEYOND THE STANDARD MODEL: SYPERSYMMETRY AND EXTRA DIMENSIONS

by

Ken Hsieh

Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2007

Advisory Committee:
Professor Markus A. Luty, Chair/Advisor
Professor Rabindra N. Mohapatra, Co-Advisor
Professor Sarah Eno
Professor Xiangdong Ji
Professor Jian-Guo Liu

Acknowledgement

When I came to the University of Maryland in August 2002, graduation five or six years away seemed an eternity away. Alas, five years seem to have suddenly passed. These five years have been a journey of tremendous growth for me in all aspects, and there are countless people that deserve my most sincere gratitude.

I would like to thank my advisor Dr. Markus Luty for guiding me through this process. His suggestions of hard problems and his constant encouragement are integral parts of my academic growth. From Markus I see the enthusiasm and persistence of doing physics, and at the same time having fun doing it. I am deeply impressed by his deep insights and his ability to explain abstract concepts in a physical manner. I would have a successful career if I only inherit a fraction of his abilities.

I would also like to thank my co-advisor Dr. Rabindra Mohapatra. Like Markus, Rabi also suggests hard problems and offer constant encouragement in face of hardship. Rabi's quick, sharp insights and strong intuitive that guide to the answers never cease to impress me, and I hope I have inherited even a fraction of his wisdom.

I also thank all other faculty members of the group: Dr. Kevork Abazajian, Dr. S. James Gates, Dr. Wally Greenberg, Dr. Young Suh Kim, and Dr. Jogesh Pati, for providing such a warm and friendly atmosphere. I especially would like to thank Jogesh for his wonderful lectures on particle physics.

I am fortunate to have collaborated with Salah Nasri and Junhai Kang, whom guided and helped my growth through this Ph. D. process with extremely helpful suggestions and warm encouragements.

I am fortunate to share office with Haibo Yu, Willie Merrell, and Parul Rastogi. Not only are they great officemates, they are always available to hear and answer my quick questions, inside and outside of physics. I like to thank fellow students of the group Yingchuan Li, Nickolas Setzer, Sogee Spinner, and Yi Cai for the constant discussion. I would especially like to thank Haibo and Yingchuan for the heated debates that I hope will persist after we have parted ways.

I thank my family for all the love and support on this long journey. Finally, I am at a lost of words to thank my best friend, Xiaoli Huang, whose love brightens my everyday, past and future.

Table of Contents

List of Tables

List of Figures

List of Abbreviations

AMSB	anomaly-mediated supersymmetry-breaking
CDM	cold dark matter
CDMS	Cryogenic Dark Matter Search
DGMSB	D-type (Dirac) gauge-mediated supersymmetry-breaking
GMSB	gauge-mediated supersymmetry-breaking
GAMSB	gauge- and anomaly-mediated supersymmetry-breaking
KK	Kaluza-Klein
LHC	Large Hadron Collider
MSSM	minimal supersymmetric Standard Model
QCD	quantum chromodynamics
QED	quantum electrodynamics
RGE	renormalization group equation
SM	Standard Model
SSB	spontaneous symmetry breaking
SUGRA	supergravity
SUSY	supersymmetry
UED	universal extra dimensions

Chapter 1

INTRODUCTION

The world as we know it so far contains four fundamental interactions: gravitational, electromagnetic, weak, and strong forces. While the gravitational and electromagnetic forces are familiar to us in everyday life (with distance scales larger than 10^{-4} cm) because of their long-range nature, the weak and strong forces only play important roles on the distance scale roughly the size of the nuclei, about 10^{-13} cm. Along the same vein, although all the physical objects we encounter in everyday life can be constructed from protons, neutrons, and the electrons, we know there are other elementary objects, such as muon (a heavier version of the electron) and neutrino. Furthermore, we know that protons and neutrons are not elementary: they are in turn made of more fundamental objects called quarks that manifest themselves as the constituents of protons and neutrons at an energy scale of roughly 10 GeV (a length scale of 10^{-16} cm). The field of theoretical high energy physics, or particle physics, seeks to explain the phenomena we observe regarding these building blocks of nature that, for now, we believe to be fundamental.

The Standard Model (SM) of particle physics is a framework that describes the interactions between these fundamental particles from 10 GeV up to the energy scale of 100 GeV (a length scale of 10^{-17} cm). While one of its main ingredients, the Higgs boson, still evades detection, many of the predictions of the SM have been confirmed with flying colors over the past twenty years. Despite its success, the Standard Model leaves many open questions and motivations, both theoretically and experimentally, for beyond-the-Standard-Model (BSM) physics.

- Theoretically, we expect such BSM physics to exist at an energy scale immediately above 100 GeV if the SM is to be free from fine-tuning to one part in 10^{32}. Such extreme fine-tuning of fundamental parameters is analogous to finding a top perfectly balanced at its tip: though technically possible, one suspects there is an underlying balancing mechanism. Furthermore, a crucial ingredient of the SM, spontaneous symmetry breaking (SSB) of electroweak symmetry, is triggered by hand. We expect a more fundamental theory to give an explanation of this mechanism.

- Experimentally, we observe the mixing between different types of neutrinos that is forbidden within the SM. We also observe that galaxies seem to be much more massive (as inferred from the observed velocities of stars in the galaxies) than what we infer from the number of visible stars (by more means than light) stars that make up the galaxy. Apparently, most of the mass of a galaxy comes dark matter, a term that has been coined to describe the mysterious, invisible (thus dark) component of the Universe. Although the SM, combined with the standard model of cosmology, gives an excellent account of the observed densities of the various elements, it does not give an explanation of the observed amount of dark matter.

The expectation that dark matter is an (new) elementary particle that weakly (and thus rarely) interacts gives the mass of the dark matter particle to be of the order 100 GeV. New physics at energy scales immediately above 100 GeV may also alleviate the extreme fine-tuning that exists in the SM. Given these motivations, and the Large Hadron Collider (LHC) is scheduled to probe this unexplored energy scale, there are many exciting ideas and predictions of such BSM physics.

Supersymmetry (SUSY) is a promising example of such BSM physics that we may discover experimentally at the LHC. It extends the Poincar*é* group (the group that describes the translation and rotation of space-time) to the super-Poincar*é* group and relates fermionic states (states with half-integer spins) to bosonic states (states with integer spin). By incorporating the SM as part of a supersymmetric theory, many attractive features emerge. First and foremost, the large radiative corrections that give rise to the fine-tuning problem are elegantly cancelled by the large radiative corrections from the corresponding superpartners. In additional to this elegant cancellation, the Minimal Supersymmetric Standard Model (MSSM) naturally contains a dark matter candidate that may be observed directly at the LHC. The MSSM can also give a natural mechanism for electroweak symmetry breaking (EWSB) that occurs in the SM. Supersymmetric theories may also be embedded into grand unified theories (GUT) that may unite the strong, weak, and electromagnetic forces in a single framework. With these features of SUSY, and the prospect of discovering it at the LHC, we now live in an exciting time.

This dissertation is a collection of models of BSM physics based on supersymmetry (SUSY) and extra dimensions, and the possible experimental signatures of these models. In Chapter 2, I present a brief overview of the SM with a discussion of its success and failures. I also give a description of SUSY and the MSSM in Chapter 2 as background to the research presented in the subsequent Chapters. Chapters 3 through 5 are the essence of my thesis as they represent the three main projects I have undertaken during my graduate studies. In Chapter 3, I describe a well-motivated, simple extension to the MSSM constructed with Markus A. Luty. In Chapter 4, I discuss another extension to the MSSM that, in addition to having many strong theoretical motivations, can potentially drastically alter conventional collider signatures. In Chapter 5, I switch gears and discuss the work I have done, with Rabindra N. Mohapatra and Salah Nasri, on the dark matter of a six-dimensional (6D) universal extra-dimensional (UED) model. Chapter 6 concludes my thesis and offers my outlook for these models.

Chapter 2

BACKGROUND - THE SM AND THE MSSM

2.1 A Simpler Model - Quantum Electrodynamics

Consider the Lagrangian of a free, massive electron field denoted by Ψ_e with mass m_e,

$$\mathcal{L} = \overline{\Psi}_e \gamma^\mu \partial_\mu \Psi_e, -m_e \overline{\Psi}_e \Psi_e, \tag{2.1}$$

which is invariant under the transformation

$$\Psi \to e^{iaeQ_e} \Psi, \tag{2.2}$$

where Q_e is a constant that is characteristic to the electron. Since the parameter a is independent of the space-time coordinates, the transformation of Eq. (2.2) is a *global* transformation. Suppose now that we demand the Lagrangian be invariant under a *local* transformation, with $a = a(x)$,

$$\Psi \to e^{ia(x)eQ_e} \Psi, \tag{2.3}$$

then the original Lagrangian in Eq. (2.1) is no longer invariant,

$$\mathcal{L} \to \overline{\Psi}_e \gamma^\mu \partial_\mu \Psi_e, -m_e \overline{\Psi}_e \Psi_e - ieQ_e \partial_\mu a \overline{\Psi}_e \gamma^\mu \Psi_e. \tag{2.4}$$

The additional piece, as expected, arises from the derivative acting on $a(x)$. The invariance of the Lagrangian under local transformation can be restored by introducing another field A_μ (the photon) that interacts with the electron,

$$\mathcal{L}_{\text{int}} = eQ_e\overline{\Psi}\gamma^\mu\Psi A_\mu, \tag{2.5}$$

and transforms via

$$A_\mu \to A_\mu - i\partial_\mu a(x). \tag{2.6}$$

Invariance under Eq. (2.6) allows a kinetic energy term for the photon

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2.7}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, but forbids a mass term for the field A_μ with the form of $m_A^2 A_\mu A^\mu$.

In summary, the insistence of invariance under *local* transformations in Eq. (2.3) leads to the introduction of a massless gauge field A_μ that transforms as Eq. (2.6) and with an interaction of the form Eq. (2.5). The resulting Lagrangian can be written as

$$\mathcal{L}_{\text{QED}} = i\overline{\Psi}\gamma^\mu D_\mu\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m_e\overline{\Psi}\Psi, \tag{2.8}$$

where

$$D_\mu = \partial_\mu - iQeA_\mu, \tag{2.9}$$

and the invariance under the transformations Eqs. (2.3) and (2.6) is manifest in this notation since D_μ transforms as

$$D_\mu \to e^{ia(x)Q_e}D_\mu. \tag{2.10}$$

In a particle physicist's jargon, the transformations Eqs. (2.3) and (2.6) are called *gauge transformations* and the principle of gauge invariance results in the Lagrangian Eq. (2.8). Since Eq. (2.3) is a phase transformation that is an element of the $U(1)$ group, quantum electrodynamics (QED) is a $U(1)$ gauge theory.

2.2 The Standard Model of Particle Physics

The Standard Model [1] of particle physics is a quantum field theory based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ (denoted by $G(321)$ for short) with the matter content

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3, 2, +\tfrac{1}{6}), \quad d_R^c \sim (\overline{3}, 1, -\tfrac{1}{3}), \quad u_R^c \sim (\overline{3}, 1, +\tfrac{2}{3}),$$
$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2, -\frac{1}{2}), \quad e_R^c \sim (1, 1, 1), \tag{2.11}$$

where the quantum numbers in parenthesis correspond respectively to the three subgroups of $G(321)$. This matter content in the SM come in three copies, or families. The gauge bosons of the SM are denoted as

$$B_\mu \sim (1, 1, 0), \quad W_\mu^A \sim (1, 3, 0), \quad G_\mu^A \sim (8, 1, 0), \tag{2.12}$$

for the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ subgroups of $G(321)$, respectively. The group $G(321)$ is spontaneously broken by the vacuum expectation value (vev) of the Higgs boson

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, +\tfrac{1}{2}), \tag{2.13}$$

and masses for the fermions and the gauge bosons are generated through the Higgs mechanism [2].

The SM Lagrangian can be broken down into three parts

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{Gauge}} + \mathcal{L}_{\mathrm{Yukawa}} - V_{\mathrm{Higgs}}, \tag{2.14}$$

where $\mathcal{L}_{\mathrm{Gauge}}$ contains the kinetic energy and the gauge interactions of the matter content, $\mathcal{L}_{\mathrm{Yukawa}}$ contains the Yukawa interactions between the Higgs and the fermions (quarks and leptons), and V_{Higgs} is the Higgs potential that spontaneously breaks the electroweak gauge symmetry.

The gauge interactions in the SM are constructed in a similar manner as in QED, by promoting the derivative of the kinetic energy term to a covariant derivative (see Eq. (2.9)),

$$\partial_\mu \to D_\mu = \partial_\mu + i g_3 \frac{\lambda^A}{2} G^A_\mu + i g_2 \frac{\sigma^A}{2} W^A_\mu + i Y g_{\mathrm{Y}} B_\mu, \tag{2.15}$$

where λ^A for $A = 1, 2 \ldots 8$ are the Gell-Mann matrices so that $\lambda/2$ are the generator of the $SU(3)$ group and σ^A with $A = 1, 2, 3$ are the Pauli-matrices. We then have the gauge interactions

$$\begin{aligned}
\mathcal{L}^{\rm SM}_{\rm Gauge} = & \; i\overline{Q}_L\gamma^\mu\left[\partial_\mu - ig_3\frac{\lambda^A}{2}G^A_\mu - ig_2\frac{\sigma^A}{2}W^A_\mu - i\frac{1}{6}g_{\rm Y}B_\mu\right]Q_L \\
& + i\overline{u^c_R}\gamma^\mu\left[\partial_\mu - ig_3\frac{\lambda^A}{2}G^A_\mu + i\frac{2}{3}g_{\rm Y}B_\mu\right]u^c_R \\
& + i\overline{d^c_R}\gamma^\mu\left[\partial_\mu - ig_3\frac{\lambda^A}{2}G^A_\mu - i\frac{1}{3}g_{\rm Y}B_\mu\right]d^c_R \\
& + i\overline{L}_L\gamma^\mu\left[\partial_\mu - ig_2\frac{\sigma^A}{2}W^A_\mu + i\frac{1}{2}g_{\rm Y}B_\mu\right]L_L \\
& + i\overline{e^c_R}\gamma^\mu\left[\partial^\mu - ig_{\rm Y}B_\mu\right]e^c_R \\
& + \left[\partial^\mu + ig_2\frac{\sigma^A}{2}W^{A\mu} - i\frac{1}{2}g_{\rm Y}B^\mu\right]\phi^\dagger\left[\partial_\mu - ig_2\frac{\sigma^A}{2}W^{A\mu} + i\frac{1}{2}g_{\rm Y}B^\mu\right]\phi.
\end{aligned} \tag{2.16}$$

The SM Yukawa couplings are (with family indices suppressed)

$$\mathcal{L}_{\rm Yukawa} = \overline{Q}y_u\phi u_R + \overline{Q}y_d\tilde{\phi}d_R + \overline{L}y_e\phi e_R + {\rm h.c.}, \tag{2.17}$$

where y_u, y_d, and y_e are 3×3 matrices in family space, and $\tilde{\phi} = i\sigma_2\phi^*$.

The Higgs potential has the form

$$V_{\rm Higgs} = m_H^2(\phi^\dagger\phi) - \lambda(\phi^\dagger\phi)^2, \tag{2.18}$$

with $m_H^2 < 0$ and $\lambda > 0$. Since $m_H^2 < 0$, $\phi = 0$ is not a stable extremum of the potential, and the vacuum expectation value (vev) of the potential is

$$\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \tag{2.19}$$

where $v = \sqrt{-m_H^2/\lambda}$, and the canonical Higgs fields are

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(\phi^0 + v + ia_0) \end{pmatrix}. \tag{2.20}$$

Once the $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken, there are mass terms for the gauge bosons that arise from their interactions with the Higgs bosons (the last line of Eq. (2.16)),

$$\mathcal{L}_{\text{gauge-Higgs}} = \begin{pmatrix} B_\mu & W_{3\mu} \end{pmatrix} \begin{pmatrix} \frac{1}{4}g_1^2 v^2 & -\frac{1}{4}g_1 g_2 v^2 \\ -\frac{1}{4}g_1 g_2 v^2 & \frac{1}{4}g_2^2 v^2 \end{pmatrix} \begin{pmatrix} B^\mu \\ W_3^\mu \end{pmatrix}. \tag{2.21}$$

This matrix has determinant zero, therefore one of its eigenvalues is zero. Upon diagonalizing the mass matrix by the transformation

$$U = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix}, \tag{2.22}$$

where

$$\tan\theta_w = \frac{g_1}{g_2}, \tag{2.23}$$

we have the mass eigenstates

$$\begin{aligned} A_\mu &= \cos\theta_w B_\mu + \sin\theta_w W_{3\mu}, \\ Z_\mu &= -\sin\theta_w B_\mu + \cos\theta_w W_{3\mu}, \end{aligned} \tag{2.24}$$

with masses

$$\begin{aligned} m_{A_\mu}^2 &= 0, \\ m_{Z_\mu}^2 &= \frac{1}{4}(g_1^2 + g_2^2)v^2. \end{aligned} \tag{2.25}$$

Experimentally, v=246 GeV [3]. The charged $SU(2)_L$ gauge bosons also receive mass through spontaneous symmetry-breaking,

$$m_{W_\mu^\pm}^2 = \frac{1}{4}g_2^2 v^2, \tag{2.26}$$

where

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right). \tag{2.27}$$

It is important to note that these masses are generated through spontaneous symmetry breaking, and not inserted by hand. The correlation of the masses and couplings between the Higgs and gauge bosons ensures that the theory is unitary at high scales, whereas mass terms for the gauge bosons inserted by hand would violate unitarity at a high energy scale. Three degrees of freedom from the Higgs doublet serve as the longitudinal modes of the three massive gauge bosons, and only one real Higgs boson, h^0, remains with the mass

$$m_{h^0}^2 = 2\lambda v^2. \tag{2.28}$$

Expressing the gauge interactions of Eq. (2.16) in terms of the mass eigenstates $W^\pm$, Z, and A, we have the neutral-current interactions

$$\begin{aligned}
\mathcal{L}_{NC} &= \sqrt{g_1^2+g_2^2}\left\{T_3^f - \left(T_3^f + Y_f\right)\frac{g_1^2}{g_1^2+g_2^2}\right\}\overline{f}\gamma^\mu Z_\mu f + \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}(T_3^f + Y^f)\overline{f}\gamma^\mu A_\mu f \\
&= \frac{e}{\sin\theta_w\cos\theta_w}\left\{T_3^f - Q_f\sin^2\theta_w\right\}\overline{f}\gamma^\mu Z_\mu f + eQ_f\overline{f}\gamma^\mu A_\mu f, \qquad (2.29)
\end{aligned}$$

where $f = (u_L,\ u_R,\ d_L,\ d_R,\ e_L,\ e_R,\ \nu_L)$, $T_3^f = \pm 1/2$ is the third component of weak-isospin of the left-handed fermion f_L ($T_3^f = 0$ for f_R), and Y^f the hypercharge of the fermion. The charge assignments are given as in the description of the matter content in Eq. (2.11), and listed in Table 2.1. We have also defined e, the strength of electromagnetic interaction

$$e \equiv g_2\sin\theta_w = g_1\cos\theta_w = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}. \tag{2.30}$$

Table 2.1: The isospin and hyper-charge assignments of the fermionic matter content of the SM.

f	T_3^f	Y_f
u_L	$+\frac{1}{2}$	$\frac{1}{6}$
u_R^c	0	$-\frac{2}{3}$
d_L	$\frac{1}{2}$	$\frac{1}{6}$
d_R^c	$\frac{1}{2}$	$\frac{1}{3}$
e_L	$-\frac{1}{2}$	$\frac{1}{2}$
e_R^c	0	1
ν_L	$+\frac{1}{2}$	$\frac{1}{2}$

The charged current interactions are given by

$$\mathcal{L}_{CC} = \frac{g_2}{\sqrt{2}} \left(\overline{u}_L \gamma^\mu W_\mu^- d_L + \overline{e}_L \gamma^\mu W_\mu^- \nu_L \right) + \text{h.c.}. \tag{2.31}$$

After EWSB, with $\langle \phi^0 \rangle = v/\sqrt{2}$, the Yukawa interactions generate the masses of the quarks and charged leptons,

$$m_u = y_u \frac{v}{\sqrt{2}}, \quad m_d = y_d \frac{v}{\sqrt{2}}, \quad m_e = y_e \frac{v}{\sqrt{2}}. \tag{2.32}$$

2.3 Open Questions to the Standard Model

Even though many of the predictions of the SM have been confirmed with spectacular precision [4], there are both experimental evidences and theoretical open questions that the SM does not answer. Much, if not all, of BSM physics is aimed at accommodating these experimental results and/or answering the theoretical open questions.

Hierarchy Problem

While the masses of the fermions and gauge bosons are *protected* by chiral and gauge symmetries, respectively, there is no symmetry that protects the Higgs boson from receiving large radiative corrections. In particular, the Higgs boson receives quadratic corrections from its Yukawa interactions

$$\delta m_H^2 = \frac{y_t^2}{8\pi^2}\Lambda^2. \tag{2.33}$$

Such quadratic corrections also arise from gauge and quartic interactions. Since the only known cutoff scales of the SM, such the GUT scale or the Planck scale, are all many orders of magnitude higher than the weak scale, there must then be a spectacular cancellation between the radiative corrections and the corresponding counter-terms to keep $\sqrt{-m_H^2}$ near the weak scale. Avoiding such unnatural cancellation has been perhaps the strongest motivation to believe that the SM is only an effective theory, and new physics will surface at a scale below 10 TeV.

Quantization of Electric Charge

The exact cancellation of the electric charges between the proton and the electron is crucial in our universe: even a slight deviation from exact cancellation would cause ordinary matter to be unstable. The hypercharge assignment of the matter content of the SM is given by hand in such a way that the SM produces the observed electric charge. As the hypercharge is a gauged $U(1)_Y$ group, the charge assignment is continuous. The quantization of electric charge suggests that the fundamental gauge group of Nature is a non-Abelian, allowing only discrete charge, and the SM gauge group only emerges after breaking of this more fundamental group. In other words, the SM is extended to a grand unified theory (GUT) [5]. We do not concern ourselves with GUTs in this thesis, but only note some references to this rich subject.

Families/Flavor Problem

The SM involves three copies of matter content with hierarchical masses spanning seven orders of magnitude between the mass of the top quark to the neutrino masses. Current experiments strongly constrain the existence of a fourth family up to masses of 100 GeV for charged leptons, 45 GeV for neutral leptons, and 200 GeV for quarks [6]. The SM can not answer why there are three generations of matter or whether there exits a fourth generation. As with the quantization of the electric charge, the family/flavor problem may require extending the group structure of the SM to impose group structure among the flavors.

Neutrino Masses and Oscillations

In the SM, unless there are unrenormalizable interactions, neutrinos are strictly massless because there are no right-handed neutrinos. However, there is strong experimental evidence of small, but non-zero, neutrino masses. The smallness of neutrino masses can be explained elegantly through the seesaw mechanism [7] and much model-building has center around the explanation of the mixing patterns of the lepton sector. We do not discuss the model-building and phenomenology of neutrino masses in this thesis, but only note here that the seesaw mechanism can be implemented straightforwardly in our models of SUSY-breaking.

Dark Matter

The rotational curves of galaxies [8], combined with other cosmological observations [9], suggest that baryonic matter only accounts for roughly 4% of the energy budget of the universe, and that nearly one quarter of the mass of the observable universe comes in the form of dark matter. If dark matter is an elementary particle, the SM would have to be extended as there is no viable dark matter candidate within the SM. Although neutrinos are massive and interact only weakly, the density of neutrinos as calculated under the standard model of cosmology can not account for the observed dark matter [10]. This suggests that extensions to the SM should contain a heavy, stable particle that has so far eluded detection in laboratories.

2.4 Supersymmetry

Within the SM, the hierarchy problem arises because the mass of the Higgs boson receives quadratic corrections that must be cancelled through fine-tuning to one part in $M_{\mathrm{GUT}}^2/M_{\mathrm{weak}}^2 \sim 10^{28}$. In fact, such quadratic corrections to the masses of elementary bosons are generic in quantum field theory. To solve the hierarchy problem, one then needs a symmetry that would forbid either a mass term of elementary scalars or such quadratic corrections. There are many ways to solve the hierarchy problem, and some examples are considering Higgs as a pseudo-Goldstone boson (little Higgs and/or twin Higgs) of broken symmetries; considering Higgs as composites of fermions instead of an elementary scalar (technicolor); and forbidding quadratic corrections through a symmetry that links scalars with fermions, thus scalar masses with fermion masses. In the last method, the symmetry introduced that relates a fermion and a boson is supersymmetry, and it will be the main focus of this Thesis. In particular, we will focus on the Minimal Supersymmetric Standard Model (MSSM) and its extensions. There are many excellent reviews and texts about supersymmetry and the phenomenology of the MSSM [11], and this section of the Thesis follow closely with these works.

As supersymmetry is a symmetry between fermions and bosons, the generators of supersymmetry must carry half-integer spin. These generators can be written in terms of Weyl spinors Q and their conjugates $\overline{Q}$, that satisfy

$$\begin{aligned}\{Q_\alpha, Q_\beta\} = \left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} &= 0;\\ \left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} &= 2\sigma^\mu_{\alpha\dot\beta}\frac{\partial}{\partial x_\mu}.\end{aligned} \tag{2.34}$$

Since no two fermions (scalars) can be related to the same scalar (fermion) partner under SUSY transformation, a supersymmetric model must contain an equal number of fermionic and scalar degrees of freedom. The Lagrangian of a supersymmetric theory can be constructed from superfields, objects that contain a fermion, its scalar superpartner, and an auxiliary field. In constructing the MSSM and the extensions to the MSSM discussed in this dissertation, we would need two types of superfield: a (left) chiral superfield and a vector superfield. We describes these two types of superfield in turn.

A left chiral superfield Φ can be written as

$$\Phi = \phi + \sqrt{2}\theta^\alpha \psi_\alpha + \frac{1}{2}\theta^\alpha \theta^\beta \epsilon_{\alpha\beta} F \tag{2.35}$$

where ϕ is the scalar component of Φ, ψ the left-handed fermionic, and F the auxiliary field. $\theta^{\alpha,\beta}$ are Grassmann variables with $\alpha, \beta = 1, 2$ with $\theta^1\theta^1 = \theta^2\theta^2 = 0$, and ϵ is the antisymmetric tensor, with $\epsilon_{12} = 1$. We will use the notation $\theta^2 \equiv \frac{1}{2}\theta^\alpha\theta^\beta\epsilon_{\alpha\beta} = \theta^1\theta^2$. We denote the conjugate of θ^α is denoted by $\overline{\theta}^{\dot{\alpha}}$. Using dimensional analysis, with the mass dimension of $[\phi == 1$ and $[\psi] = \frac{3}{2}$, we have $[\theta] = -\frac{1}{2}$ and $[F] = 2$.

A vector superfield is one that is self-conjugate $V \equiv V^\dagger$ and can be written as

$$\begin{aligned} V = &\left(1 + \theta^2\overline{\theta}^2 \partial_\mu \partial^\mu\right) C \\ &+ \left(i\theta + \theta^2 \sigma^\mu \overline{\theta} \partial_\mu\right) \chi + \left(-i\overline{\theta} + \overline{\theta}^2 \sigma^\mu \overline{\theta} \partial_\mu\right) \chi^\dagger \\ &+ i\theta^2(M + iN) - i\overline{\theta}^2(M - iN) - \theta\sigma_\mu\overline{\theta}A^\mu \\ &+ 2i\theta^2\overline{\theta}\lambda^\dagger - 2i\overline{\theta}^2\theta\lambda + \theta^2\overline{\theta}^2 D, \end{aligned} \tag{2.36}$$

where C, M, N, are real scalars, χ and λ are Weyl spinors, A^μ is a vector field, and D will turn out to be an auxiliary field. We choose a gauge called the "Wess-Zumino" gauge that removes many unphysical degrees of freedom

$$\chi = C = M = N = 0, \tag{2.37}$$

and we are left with

$$V_{WZ} = -\theta\sigma_\mu\overline{\theta}A^\mu + 2i\theta^2\overline{\theta\lambda} - 2i\overline{\theta}^2\theta\lambda + \theta^2\overline{\theta}^2 D. \tag{2.38}$$

Using the superfields, the Lagrangian is constructed by taking the component of products of superfields having the highest dimension. Equivalently, these terms involve the most powers of Grassmann variables because they are the only object with negative dimension). This is because the variation under SUSY transformation of such component contains a total space-time derivative which vanishes as a surface term upon integration over space-time, leaving the action invariant.

We can construct the free theory with the highest component of $\Phi^\dagger\Phi$. Now, we have chosen a specific representation of Φ in Eq. (2.35) so that we have no $\overline{\theta}$ dependence. Writing $\Phi^\dagger$ in this specific representation would induce space-time derivatives so that $\Phi^\dagger$ will have a $\overline{\theta}^2\theta$ component as well as a $\overline{\theta}^2$ component that is naively expected by simply taking the conjugate of Eq. (2.35). Taking the highest $(\theta^2\overline{\theta}^2)$ component of $\Phi^\dagger\Phi$ gives

$$\mathcal{L}_{\text{free}} = \left[\Phi^\dagger\Phi\right]_{\theta^2\overline{\theta}^2} = -\phi\partial_\mu\partial^\mu\phi^* - i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi + FF^*, \tag{2.39}$$

which contains kinetic energy terms for ϕ and ψ, while leaving F as an auxiliary field.

Supersymmetric masses and Yukawa interactions are constructed from the highest component of the superpotential (W), sum of products of superfields. As the superpotential can involve only chiral superfields, the highest component involves only θ^2. For examples, if we have the superpotential

$$W = M\Phi_1\Phi_2 + \Phi_1\Phi_2\Phi_3, \tag{2.40}$$

the resulting contribution to the Lagrangian is

$$\mathcal{L}_{\text{Mass}} = [M\Phi_1\Phi_2]_{\theta^2} + \text{h.c.} = M(\phi_1 F_2 + \phi_2 F_1 - \psi_1\psi_2) + \text{h.c.}, \tag{2.41}$$

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} = [\Phi_1\Phi_2\Phi_3]_{\theta^2} + \text{h.c.} &= \phi_1\phi_2 F_3 + \phi_1 F_2\phi_3 + F_1\phi_2\phi_3 \\ &\quad - \psi_1\phi_2\psi_3 - \phi_1\psi_2\psi_3 - \psi_1\psi_2\phi_3 + \text{h.c.}.\end{aligned} \tag{2.42}$$

Adding Eq. (2.39) with Eqs. (2.41) and (2.42), we can express the auxiliary fields in terms of the other scalar fields in the model through the equation of motion of F

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial F_1} &= 0 = F_1^* + M\phi_2 + \phi_2\phi_3 \;\rightarrow\; F_1^* = -\frac{\partial W}{\partial \phi_1}, \\ \frac{\partial \mathcal{L}}{\partial F_2} &= 0 = F_2^* + M\phi_1 + \phi_1\phi_3. \;\rightarrow\; F_2^* = -\frac{\partial W}{\partial \phi_2},\end{aligned} \tag{2.43}$$

where the derivative is taken by considering the superpotential in Eq. (2.40) as a holomorphic function of the scalar fields, rather than chiral superfields. Solving Eq. (2.43) for $F_{1,2}$ and substituting back the results into Eqs. (2.39), (2.41) and (2.42), we have both scalar and fermion mass terms of the form

$$\mathcal{L} \supset -M^2(\phi_1^*\phi_1 + \phi_2^*\phi_2) - M(\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger), \tag{2.44}$$

and there are also quartic interactions in addition to the usual Yukawa (scalar-fermion-fermion) interactions. The contributions of the SUSY-invariant masses and

Yukawa couplings to the potential can be written in a compact manner

$$
\begin{aligned}
V_F &= -F_i F_i^* + F_i^* \frac{\partial W}{\partial \phi_i} + F_i \frac{\partial W^*}{\partial \phi_i^*} \\
&= F_i F_i^*. \qquad (2.45)
\end{aligned}
$$

To have gauge interactions, we need two ingredients: the kinetic energy term for the gauge bosons (and their superpartners), and the gauge interactions. The kinetic energy terms are constructed from the θ^2 component of $\mathcal{W}\mathcal{W}$, where $\mathcal{W}$ is a chiral superfield constructed from the vector superfield

$$
\mathcal{W}_\alpha = -i\lambda + \left(D - \frac{i}{2}\sigma_{\mu\nu}F_{\mu\nu} \right)\theta + \theta^2 \sigma^\mu \partial_\mu \lambda^\dagger. \qquad (2.46)
$$

The kinetic term then contains

$$
\mathcal{L} = \frac{1}{4}\left[\mathcal{W}\mathcal{W}\right]_{\theta^2} = -i\lambda^\dagger \sigma_\mu \partial_\mu \lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D^2. \qquad (2.47)
$$

Notice that D is an auxiliary field, as alluded earlier.

The gauge interactions in the SM are constructed by promoting the derivative of the kinetic term to a covariant derivative. In SUSY, the gauge interactions are constructed by inserting the real superfield in the kinetic term

$$
\begin{aligned}
&[\Phi^*\Phi]_{\theta^2\bar{\theta}^2} \to \left[\Phi^* e^{2gV}\Phi\right]_{\theta^2\bar{\theta}^2} \\
&\qquad = |D_\mu\phi|^2 - i\psi^\dagger\sigma_\mu D^\mu \psi + g\phi^* D\phi + ig\sqrt{2}(\phi^*\lambda\psi - \lambda^\dagger\psi^\dagger\phi) + |F|^2, \qquad (2.48)
\end{aligned}
$$

where D_μ is the usual covariant derivative as in the case of the SM.

Combining Eqs. (2.47) and (2.48), we can solve for the auxiliary field D in terms of the scalar fields

$$
\frac{\partial \mathcal{L}}{\partial D} = 0 = D + g\phi^*\phi \to D = -g\phi^*\phi. \qquad (2.49)
$$

The auxiliary field D then contributes to the potential

$$V_D = -\frac{1}{2}D^2 - g\phi^*\phi D = \frac{g^2}{2}\left(\phi^*\phi\right)^2. \tag{2.50}$$

2.5 The Minimal Supersymmetric Standard Model

The MSSM promotes the Weyl fermions of the SM to a chiral superfield that contains the associated scalars, called sfermions or sfermions, labelled with a tilde ($\tilde{}$) above the corresponding fermion. We take all the fermionic components of the superfields to be left-handed, and denote the superfields with capital letters

$$\begin{aligned} Q_L &= \begin{pmatrix} U_L \\ D_L \end{pmatrix} \sim (3, 2, +\tfrac{1}{6}), \quad D_R^c \sim (\overline{3}, 1, -\tfrac{1}{3}), \quad U_R^c \sim (\overline{3}, 1, +\tfrac{2}{3}), \\ L_L &= \begin{pmatrix} \nu_L \\ E_L \end{pmatrix} \sim (1, 2, -\frac{1}{2}), \quad E_R^c \sim (1, 1, 1). \end{aligned} \tag{2.51}$$

It is important to note here that one should take the names U_R^c, D_R^c, and E_R^c as whole, and not the charge conjugation of a superfield. The super- and sub-scripts on their labels indicates that while the fermionic components are left-handed, they will turn out to be the charge-conjugate of the right-handed components of the SM fermions.

The gauge bosons of the SM are contained in vector superfields, containing the vector gauge bosons and their fermionic superpartners, the gauginos, labelled by λ_i with $i = 1, 2, 3$ for the three gauge groups of the SM. The Higgs gauge bosons are the scalar components of the Higgs chiral superfields, and two copies of the Higgs superfields are needed with opposite hypercharges to ensure that the theory is free of gauge-anomalies. For the Higgs bosons, it is the fermionic components, Higgsinos, that are labelled with a tilde

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \sim (1, 2, +\tfrac{1}{2}), \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \sim (1, 2, -\tfrac{1}{2}). \tag{2.52}$$

The Yukawa interactions of the MSSM are given through the superpotential

$$W = y_t Q_L H_u U_R^c + y_d Q_L H_u D_R^c + y_\tau L_L H_d E_R^c + \mu H_u H_d. \tag{2.53}$$

As described in the previous section, these couplings in the superpotential not only give the usual fermion-fermion-scalar Yukawa interactions, there are also quartic scalar interactions. The μ-term in the superpotential gives a SUSY-invariant mass to the Higgs bosons and the Higgsino. Prior the EWSB, the SM matter content is massless. As we will see, the MSSM provides a mechanism of EWSB dynamically, and both the neutral scalar components of H_u and H_d will receive vev's, denoted by (v_u and v_d, respectively). However, even after EWSB, we would expect superpotential of the form

$$W = m_e L_L E_R^c \tag{2.54}$$

that gives the *same* mass to the electron and its superpartner, the s-electron. Since we do not observe a fundamental scalar with electric charge of unity having the same mass as the electron, SUSY must be broken with the SM as an effective theory below the scale of SUSY-breaking. The SUSY-breaking effects are *soft*, meaning that the cancellation of quadratic correction of scalar masses, ensured by exact SUSY, is still present even in the presence of SUSY-breaking. The soft SUSY-breaking terms contain masses for the scalars and gauginos, and trilinear interactions. Since SUSY-breaking terms are all dimensionful, they do not affect the quadratic corrections to scalar masses. In the MSSM, the soft-breaking terms are

$$\begin{aligned}\mathcal{L}_{\text{soft}} = & m_{\tilde{Q}}^2 |\tilde{Q}|^2 + m_{\tilde{u}}^2 |\tilde{u}_R^c|^2 + m_{\tilde{d}}^2 |\tilde{d}_R^c|^2 \\ & + m_{\tilde{L}}^2 |\tilde{L}_L|^2 m_{\tilde{e}}^2 |\tilde{e}_R^c|^2 \\ & + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + B_\mu \mu (H_u H_d + \text{h.c.}) \\ & + \frac{M_1}{2} \lambda_1 \lambda_1 + \frac{M_2}{2} \lambda_2 \lambda_2 + \frac{M_3}{2} \lambda_3 \lambda_3 + \text{h.c.} \\ & + a_t \tilde{Q}_L H_u \tilde{u}_R^c + a_d \tilde{Q}_L H_u \tilde{d}^c + a_\tau \tilde{L}_L H_d \tilde{e}^c + \text{h.c.}. \end{aligned} \tag{2.55}$$

2.5.1 Spectrum of the MSSM

Higgs Bosons

Including the soft-breaking terms, the electrically neutral Higgs bosons have the potential

$$\begin{aligned}V_{\text{Higgs}} = & (|\mu|^2 + m_{H_u}^2) |H_u^0|^2 + (|\mu|^2 + m_{H_d}^2) |H_d^0|^2 - B_\mu \mu H_u^0 H_d^0 + \text{h.c.} \\ & \frac{1}{8} \left(g_2^2 + g_1^2 \right) \left(|H_u^0|^2 - |H_d^0|^2 \right)^2 . \end{aligned} \tag{2.56}$$

The conditions for spontaneous breaking of symmetry while ensuring that the potential is bounded from below are

$$\begin{aligned} & B_\mu^2 |\mu|^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2), \quad \text{(spontaneous EWSB)} \\ & 2B_\mu |\mu| < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \quad \text{(bounded from below)} \end{aligned} \tag{2.57}$$

With these conditions satisfied, both H_u^0 and H_d^0 will develop vev's. Let us denote

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}}, \langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}}, \tan\beta = \frac{v_u}{v_d}, \tag{2.58}$$

then from the couplings of H_u^0 and H_d^0 to the Z_μ gauge boson, we have

$$M_Z^2 = \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 + v_d^2). \tag{2.59}$$

So we see that v_u and v_d are related to the SM vev via the relations

$$v^2 = v_u^2 + v_d^2, \tag{2.60}$$

$$v_u = v \sin\beta, \quad v_d = \cos\beta. \tag{2.61}$$

At tree level, the minimization conditions are

$$\begin{aligned} |\mu|^2 + m_{H_d}^2 &= B_\mu \mu \tan\beta - (M_Z^2/2)\cos 2\beta, \\ |\mu|^2 + m_{H_u}^2 &= B_\mu \mu \cot\beta + (M_Z^2/2)\cos 2\beta. \end{aligned} \tag{2.62}$$

Once the potential is minimized and H_u and H_d develop vev's, three of the Higgs degrees of freedom, G^0 and $G^\pm$ will be eaten by Z and $W^\pm$, respectively, to become the longitudinal modes. The five physical degrees of freedom that remain, h^0, H^0, A^0, and $H^\pm$, have masses at tree-level

$$m_{h^0}^2 = m_Z^2 \cos^2 2\beta, \tag{2.63}$$

$$m_{H^0}^2 = \mu^2 + \mathcal{O}(m_Z^2), \tag{2.64}$$

$$m_{A^0}^2 = B_\mu \mu + \mathcal{O}(m_Z^2), \tag{2.65}$$

$$m_{H^\pm}^2 = \mu^2 + \mathcal{O}(m_Z^2), \tag{2.66}$$

where we have assumed that $\mu^2 \ll m_Z^2$, which is a good approximation given the lower experimental bound on μ [12]. In terms of the real and imaginary parts of the complex Higgs fields,

$$\begin{aligned} H_u^0 &= \frac{1}{\sqrt{2}}\left(h_u^0 + i a_u^0 + v_u\right), \\ H_d^0 &= \frac{1}{\sqrt{2}}\left(h_d^0 + i a_d^0 + v_d\right), \end{aligned} \tag{2.67}$$

the mass eigenstates have the mixture

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix}, \tag{2.68}$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} a_u^0 \\ a_d^0 \end{pmatrix}, \tag{2.69}$$

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h_u^0 \\ a_d^0 \end{pmatrix}, \tag{2.70}$$

where the mixing angle α is given by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_{A^0}^2 + M_Z^2}{m_{H^0}^2 - m_{h^0}^2}, \quad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_{A^0}^2 - M_Z^2}{m_{H^0}^2 - m_{h^0}^2}. \tag{2.71}$$

Whereas the masses of H^0, A^0, and $H^\pm$ have no lower bound, the mass of h^0 is bounded from above at tree level by M_Z, and violates current LEP2 experimental bound of $m_{\text{Higgs}}^{\text{SM}} > 114.4$ GeV [13]. However, there are large radiative corrections coming from the heavy s-top masses

$$\delta m_h^2 = \frac{3 y_t^4 v^2}{8\pi^2}\left(\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12 m_{\tilde{t}}^4}\right), \tag{2.72}$$

where $m_{\tilde{t}}^2 = \sqrt{m_{\tilde{t}_L}^2 m_{\tilde{t}_R}^2}$ and $X_t = A_t + \mu\cot\beta$. The dependence of m_{h^0} on $m_{\tilde{t}}$ is only logarithmic so, even with moderate mixing of $A_t = m_{\tilde{t}}$, s-top masses of the order

1 TeV are needed to evade the Higgs bound. This introduces the so-called little hierarchy problem that will be discussed heavily in the next section.

Neutralino and Chargino Spectrum

The MSSM also contains fermionic superpartners to the gauge and Higgs bosons

$$\begin{aligned} \text{Higgsino:} \quad & \tilde{h}_u^0,\ \tilde{h}_u^+,\ \tilde{h}_d^0,\ \tilde{h}_d^-, \\ \text{Gaugino:} \quad & \lambda_1,\ \lambda_2^0,\ \lambda_2^\pm,\ \lambda_3. \end{aligned} \tag{2.73}$$

We first discuss the spectrum of electrically charged fermions first, and then the electrically neutral ones.

Before EWSB, $\tilde{h}_u^+$ and $\tilde{h}_d^-$ would form a Dirac fermion with a Dirac mass of μ because of the supersymmetric mass term $\mu H_u^+ H_d^-$ in the superpotential. The charged winos, $\lambda_2^\pm$, have a SUSY-breaking gaugino mass of M_2. However, after EWSB, the Higgs vev introduces mixing among the charged states, and we have the mass matrix

$$M_{\text{chargino}} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta m_W \\ \sqrt{w} c_\beta m_W & -\mu \end{pmatrix} \tag{2.74}$$

Diagonalizing Eq. (2.74) gives the mass eigenstates called charginos, denoted by $\chi_i^\pm$ with $i = 1, 2$ and $m_{\chi_1^\pm} < m_{\chi_2^\pm}$.

In the electrically neutral sector, before EWSB, $\tilde{h}_u^0$ and $\tilde{h}_d^0$ would form a Dirac fermion with a Dirac mass of μ, and λ_1 and λ_2^0 have Majorana SUSY-breaking masses of M_1 and M_2, respectively. After EWSB, in the basis $(\lambda_1, \lambda_2^0, \tilde{h}_u^0, \tilde{h}_d^0)$, we have the mass matrix

$$M_{\text{neutralino}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix}. \tag{2.75}$$

Diagonalizing Eq. (2.75) gives the mass eigenstates χ_i^0 for $i = 1, 2, 3, 4$, collectively called neutralinos, with χ_1^0 being the lightest of the four.

Sfermions

The masses of the scalar superpartners are mainly determined by the SUSY-breaking masses if the trilinear couplings are small. Assuming that the trilinear interactions are proportional to the Yukawa matrices, there may be significant mixing between the left and right s-tops. The s-top mass matrix, in the basis $\{\tilde{t}_L, \tilde{t}_R\}$, is

$$\begin{pmatrix} m_{\tilde{t}_L}^2 + m_t^2 & y_t v(A_t - \mu \cot\beta) \\ y_t v(A_t - \mu \cot\beta) & m_{\tilde{t}_R}^2 + m_t^2 \end{pmatrix}. \tag{2.76}$$

Diagonalizing this matrices gives the two s-top states $\tilde{t}_1$ and $\tilde{t}_2$.

2.5.2 Features of the MSSM

Gauge Unification

With the added particle content, the RGE evolution of the gauge couplings in the MSSM differ from that of the SM. When the three gauge couplings are evolved to higher scales, they magically all intersect at one point at an energy scale of 10^{16} GeV, strongly hinting yet another scale of new physics and encourages us to believe in SUSY and GUT.

Dark Matter

R-parity, or matter parity, is a Z_2 symmetry that is conserved in the MSSM. The R-parity assignment is such that all the particles of the SM and the extra Higgs boson states are even under R-parity, while the s-fermions, the gauginos, and the Higgsinos are odd under R-parity. With R-parity conserved, the lightest R-odd state is absolutely stable and can serve as a possible dark matter candidate. The R-parity is assigned such that the SM fermions and the Higgs and gauge bosons have even R-parity, so the lightest R-odd state must be the lightest superpartner (LSP). Such state is usually the lightest neutralino. Since the neutralino is a mixture of bino, wino, and the Higgsino, the precise properties of the lightest neutralino as dark matter such as its annihilation and elastic nuclear cross-sections depend crucially on the particular mixtures. If the LSP is indeed dark matter, it would leave large missing energy at colliders such as LHC, and can be detected at direct search experiments such as the Cryogenic Dark Matter Search [14].

The Little-Hierarchy Problem

For large $\tan\beta \geq 7$, H_u^0 essentially acts as the Higgs field of the SM and has the potential (suppressing the superscript 0)

$$V = \frac{1}{2}(|\mu|^2 + m_{H_u}^2)(h_u^2 + a_u^2) + \frac{1}{32}(g_2^2 + g_1^2)(h_u^4 + 2h_u^2 a_u^2 + a_u^4). \tag{2.77}$$

With $(|\mu|^2 + m_{H_u}^2) < 0$, the electroweak vacuum expectation value (vev) is given by

$$v^2 = \langle h_u^0 \rangle^2 = -\frac{\mu^2 + m_{H_u}^2}{\frac{1}{8}(g_2^2 + g_1^2)}. \tag{2.78}$$

With a_u eaten by Z-boson, the Higgs boson has a mass of

$$m_{h_u^0}^2 = \frac{1}{4}(g_2^2 + g_1^2)v^2 = M_Z^2, \tag{2.79}$$

which saturates bound in Eq. (2.66) as we are working the large $\tan\beta$ limit.

As noted earlier, this lightest Higgs boson in the MSSM acquires large radiative corrections from large s-top masses, so that the MSSM is not currently ruled out by the LEP2 bound of $m_h > 114.4$ GeV. However, large s-top masses also induce large radiative corrections to the soft-breaking $m_{H_u}^2$,

$$\delta m_{H_u}^2 = -\frac{3y_t^2}{8\pi^2}\left[m_{\tilde{t}}^2\left(\ln\frac{\Lambda^2}{m_{\tilde{t}}^2} + 1\right) + \frac{1}{2}A_t^2 \ln\frac{\Lambda^2}{m_{\tilde{t}}^2}\right], \tag{2.80}$$

where Λ is the scale of SUSY-breaking. Since the combination of $m_{H_u}^2 + |\mu|^2$ essentially serves as m_H^2 in the SM potential, this large radiative correction to $m_{H_u}^2$ also induces fine-tuning,

$$\frac{\delta \ln v^2}{\delta \ln m_{H_u}^2} \sim \frac{6y_t^2}{8\pi^2}\frac{m_{\tilde{t}}^2}{M_Z^2}\ln\frac{\Lambda^2}{m_{\tilde{t}}^2}, \tag{2.81}$$

for $\Lambda \sim 100$ TeV and $m_{\tilde{t}} \sim 1.5$ TeV (as required to evade the LEP2 bound), the MSSM is fine-tuned to about one part in 100.

The μ-Problem

In the MSSM, the size of soft SUSY-breaking terms must also be near the weak scale to solve the hierarchy problem in the SM. Because of EWSB in the MSSM, the μ-term also has to be of the same magnitude as the soft terms (see Eq. (2.62)). However, μ is not a SUSY-breaking parameter. Rather, it is a SUSY-invariant parameter appearing in the superpotential, whose natural scale is GUT scale or the Planck scale. The unnatural smallest of μ is the μ-problem. Note that the μ-problem is not as disastrous as the hierarchy problem of the SM because μ does not receive large radiative corrections. As a SUSY-invariant parameter, the radiative corrections to μ are proportional to μ itself. Thus, once we have a reasonable explanation why μ is of the order 1 TeV at tree level, we do not have to worry about μ being dragged to higher scales by large radiative corrections.

2.6 Models of SUSY-Breaking

The MSSM contains roughly 120 free parameters compared to the 19 free parameters needed to define the SM, and most of these new free parameters reside in the SUSY-breaking sector. The general study of this 120-dimensional parameter spacing is forbiddingly difficult. Consequently, there are many models of SUSY-breaking developed over the years. In addition to facilitate studies of the (constrained) parameter space, many of the SUSY-breaking theories have predictions outside of the MSSM. The easiest way to derive SUSY-breaking effects of many theories is through spurion analysis [23][15], but here we only give the derived results.

2.6.1 Constrained MSSM

The constrained MSSM (CMSSM) reduces the free parameters of the MSSM to five parameters

$$(m_0^2, m_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu)), \tag{2.82}$$

where m_0^2, $m_{1/2}$, and A_0 serve as universal boundary conditions for the soft scalar masses, gaugino masses, and scaled trilinear interactions, respectively, as inputs at a high scale, usually M_{GUT} or M_{Planck}. The scalar mass matrices are assumed to be diagonal in the same basis that diagonalizes the fermion masses. Once the soft terms are evolved to the weak scale through the MSSM RGEs (listed in Appendix A.4), B_μ and $|\mu|$ are fixed by requiring successful EWSB with the corresponding $\tan\beta$.

Because of its simplicity, CMSSM is the most widely studied and analyzed model of SUSY-breaking. However, since much correlation is built into the resulting spectrum, many interesting features of the vast parameter space are left unexplored.

2.6.2 Gauge-Mediated SUSY-Breaking

Gauge-mediated SUSY-breaking (GMSB) is achieved through a heavy (masses of 100 TeV) vector-like supermultiplet whose interactions with a SUSY-breaking sector is communicated to the MSSM particles through gauge interactions. The SUSY-breaking can be parameterized through the Lagrangian

$$\Delta\mathcal{L} = \int d^2\theta (M + \theta^2 F)\tilde{P}P + \text{h.c.}, \tag{2.83}$$

where $\tilde{P}$ and P are vector-like superfields ($\overline{\mathbf{5}}$ and $\mathbf{5}$ of $SU(5)$, for example). We demand that $F < M^2$ to ensure that the scalars of $\tilde{P}$ and P have positive squared-masses. SUSY-breaking is parameterized by F that splits fermion and scalar masses of $\tilde{P}$ and P. The SUSY-breaking gaugino and scalar masses are generated at the scale M through one- and two-loop diagrams, respectively, involving $\tilde{P}$ and P. The boundary conditions of the gaugino masses are

$$M_{\lambda_i} = k_i(N_5 + 3N_{10})\frac{g_i^2(M)}{16\pi^2}\frac{F}{M} \quad (i = 1, 2, 3), \tag{2.84}$$

with $k_1 = 5/3$, and $k_2 = k_3 = 1$. N_5 and N_{10} are the number of $\mathbf{5}$ and $\mathbf{10}$ of $SU(5)$ multiplets comprising of the messenger $\tilde{P}$ and P. The general boundary conditions for the SUSY-breaking sfermion masses are

$$m_{\tilde{f}}^2 = 2\sum_{i=1}^{3} C_i^{\tilde{f}} k_i \frac{g_i^4}{(16\pi^2)^2}(N_5 + 3N_{10})\left(\frac{F}{M}\right)^2, \tag{2.85}$$

where C_i^f is the quadratic Casimir of $\tilde{f}$ under the gauge group i. For the fundamental representation of $SU(N)$, $C = (N^2 - 1)/(2N)$, and $C = Y^2$ for the $U(1)_Y$ group. The explicit forms of the soft scalar masses are given in Appendix B.2.

Notice that the boundary conditions are flavor-independent. Therefore, one can align the basis of the sfermions to be the same as the fermions without inducing off-diagonal elements in the soft-masses at the scale M. The trilinear couplings are not generated at the scale M. The initial conditions of the gaugino and soft-scalar masses are then evolved through the MSSM RGEs (listed in the Appendix A.4) to the weak scale. The RGEs will generate non-zero trilinear couplings at the weak scale. At the weak scale, μ and B_μ are determined through the constraints of successful EWSB and $\tan\beta$. Even though the RGE evolution will induce off-diagonal elements in the soft-mass matrices through the Yukawa couplings, their effects to the flavor-changing processes in the SM are suppressed.

2.6.3 Anomaly-Mediated SUSY-Breaking

To discuss anomaly-mediated SUSY-breaking (AMSB) [16, 17] requires background on supergravity (SUGRA), the supersymmetrized version of Einstein gravity. The discussion of the formal subject of SUGRA is outside the scope of this thesis, and we only point the readers to the references [18] and state the results. In anomaly mediation, SUSY breaking comes from non-vanishing value of the auxiliary component of the conformal compensator ϕ

$$\phi = 1 + \theta^2 F_\phi, \tag{2.86}$$

that is introduced to have local conformal invariance in super-gravity. [1] The nonzero value of F_ϕ breaks superconformal to the super-Poincaré group. The formalism of SUGRA dictates that couplings of ϕ (and thus F_ϕ) are fixed by conformal invariance. Specifically, the effects of ϕ can be described by replacing the running scale

$$\ln\mu \to \ln\frac{\mu}{|\phi|} = \ln\mu - \frac{1}{2}(\theta^2 F_\phi + \text{h.c.}). \tag{2.87}$$

Therefore, when one includes radiative corrections to the Lagrangian of the MSSM, the effects of F_ϕ is manifest. For any running coupling $g(\mu)$, the running scale is

[1] This is analogous to the introduction of the photon in order to have invariance under local gauge transformations.

introduced through its β-function, β_g the

$$g(\mu) = g(\mu_0) + \beta_g(\ln \mu - \ln \mu_0). \tag{2.88}$$

Since ϕ is tied to the running scale through the replacement in Eq. 2.87, the SUSY-breaking effects of F_ϕ will be proportional to a β-function.

Calculating the SUSY-breaking effects through the conformal compensator in a general SUSY theory, we obtain

$$m^{2(\text{AMSB})}_{\tilde{Q}_i} = -\frac{|F_\phi|^2}{4} \sum_j \left(\frac{\partial \gamma_i}{\partial g_j} \beta_{g_j} \frac{\partial \gamma_i}{\partial y_j} \beta_{y_j} \right), \tag{2.89}$$

$$A^{(\text{AMSB})}_i = +F_\phi \beta_{y_i}, \tag{2.90}$$

$$m^{(\text{AMSB})}_\lambda = -\frac{F_\phi}{g} \beta_g, \tag{2.91}$$

where β_g and β_y are the β-functions of the gauge and Yukawa couplings, respectively, and γ is the anomalous dimension

$$\beta_g \equiv \frac{\partial g}{\partial \ln \mu}, \tag{2.92}$$

$$\beta_y g \equiv \frac{\partial y}{\partial \ln \mu}, \tag{2.93}$$

$$\gamma \equiv \frac{\partial \ln Z}{\partial \ln \mu}, \tag{2.94}$$

where Z is the wavefunction renormalization. In the particular case of the MSSM, the explicit forms of the soft terms are given in Appendix B.2. As with GMSB, the soft scalar masses for the first two-generations are nearly degenerate, and flavor-changing neutral currents of the SM processes are suppressed.

Unlike GMSB, which gives boundary conditions that need to be evolved through the MSSM RGEs to obtain a spectrum at the weak scale, AMSB gives the RG trajectories of the SUSY-breaking terms. Thus the predictions of AMSB are independent of any high energy physics except for an overall scale factor. Unfortunately, the soft masses of the sleptons turn out to be negative, and the framework of AMSB must be extended. There are many models and extensions to AMSB [19, 20, 21], and in Chapter 3 we will offer an alternative to these models.

Chapter 3

MIXED GAUGE- AND ANOMALY-MEDIATED SUSY-BREAKING

3.1 Model

As we discuss in the previous Chapter, AMSB and GMSB are attractive mechanisms for breaking SUSY without flavor problems. In AMSB, SUSY is broken by the vev of a supergravity auxiliary field $\langle F_\phi \rangle$, whose couplings to matter are governed by scale covariance, and are hence naturally flavor-blind. It defines a preferred renormalization group (RG) trajectory for all SUSY breaking couplings in terms of a single SUSY breaking scale $\langle F_\phi \rangle \sim 10$ TeV. Unfortunately, the slepton mass parameters are negative in the MSSM. In this Chapter, we propose a solution to this problem based on an idea due to Nelson and Weiner [19], which built on early work by Pomarol and Rattazzi [20]. Nelson and Weiner considered a theory with extra vectorlike fields P and $\tilde{P}$ and added a coupling of the form[1]

$$\Delta\mathcal{L} = \int d^4\theta \, \frac{\phi}{\phi^\dagger} c P \tilde{P} + \text{h.c.} \tag{3.1}$$

This gives rise to a Dirac fermion mass $c\langle F_\phi \rangle$ and scalar mass terms

$$V = |c\langle F_\phi \rangle|^2 (|P|^2 + |\tilde{P}|^2) + \Big(c|\langle F_\phi \rangle|^2 P\tilde{P} + \text{h.c.} \Big) \,. \tag{3.2}$$

[1]Couplings of this form with P and $\tilde{P}$ replaced by the MSSM Higgs fields contribute to the Giudice-Masiero mechanism for generating the MSSM μ term [22].

The scalar mass-squared terms are positive for $|c| > 1$. Assuming $|c| \sim 1$, this is a supersymmetry breaking threshold at the scale $\langle F_\phi \rangle$, which gives SUSY breaking threshold corrections of order $g^2 \langle F_\phi \rangle / 16\pi^2$ to SUSY breaking masses, taking them off the AMSB RG trajectory. As shown in Ref. [19], the leading threshold corrections to the scalar masses vanish, and the slepton mass-squared terms are therefore still negative at the scale $\langle F_\phi \rangle$. One can get positive slepton masses at the weak scale only by having a large number of messengers (5 or more $\mathbf{5} \oplus \bar{\mathbf{5}}$'s), which generates large gaugino masses at the messenger scale ~ 10 TeV, which in turn generates positive slepton masses from running between the messenger scale and the weak scale. However, the resulting theories generally have charged slepton LSP, and the large number of messengers destroys perturbative unification.

In this Chapter we consider a very simple extension of this model that has a more attractive phenomenology. The model consists of the MSSM plus a singlet S in addition to the vectorlike fields P, $\tilde{P}$. We include the most general interactions with dimensionless coefficients. The additional terms in the Lagrangian are therefore

$$\Delta \mathcal{L} = \int d^4\theta \frac{\phi^\dagger}{\phi} \left(\frac{1}{2} c_S S^2 + c_P P \tilde{P} \right) + \text{h.c.} \tag{3.3}$$

$$+ \int d^2\theta \left[\frac{\lambda_S}{3!} S^3 + \lambda_P S P \tilde{P} \right] + \text{h.c.} \tag{3.4}$$

A superpotential coupling of the form $S H_u H_d$ is assumed to be absent.[2] For $|c_S| < 1$ the potential for S has a local maximum at $S = 0$, so $\langle S \rangle \neq 0$. This gives rise to a

[2]For example, it may be forbidden by a discrete R symmetry $S(\theta) \mapsto -S(i\theta)$, $P(\theta) \mapsto +P(i\theta)$, $\tilde{P}(\theta) \mapsto +\tilde{P}(i\theta)$, $H_u(i\theta) \mapsto +H_u(i\theta)$, $H_d(i\theta) \mapsto -H_d(i\theta)$, $u^c(\theta) \mapsto -u^c(i\theta)$, with all other fields even.

more general threshold with none of the problems of the minimal model.

The scalar potential that arises from Eq. (3.3) is

$$V = \left| c_S \langle F_\phi^\dagger \rangle S + \tfrac{1}{2}\lambda_S S^2 + \lambda_P P \tilde{P} \right|^2 + |c_P \langle F_\phi^\dagger \rangle + \lambda_P S|^2 (|P|^2 + |\tilde{P}|^2) \tag{3.5}$$

$$+ |\langle F_\phi \rangle|^2 \left(\tfrac{1}{2} c_S S^2 + c_P P \tilde{P} \right) + \text{h.c.} \tag{3.6}$$

The potential is quadratic in P, $\tilde{P}$, so we look for a minimum with $\langle P \rangle = \langle \tilde{P} \rangle = 0$. In Appendix B.1, we minimize the potential for real couplings and VEVs. We show that the global minimum preserves CP for

$$c_S < 0 \tag{3.7}$$

and then we obtain

$$\langle S \rangle = -\frac{\langle F_\phi \rangle}{2\lambda_S} \left(3c_S + \sqrt{c_S(c_S - 8)} \right), \tag{3.8}$$

$$\left\langle \frac{F_S}{S} \right\rangle = \frac{\langle F_\phi \rangle}{4} \left(-c_S + \sqrt{c_S(c_S - 8)} \right). \tag{3.9}$$

This gives rise to a mass term for P, $\tilde{P}$ that can be conveniently written as

$$\Delta \mathcal{L} = \int d^2\theta \, \phi \mathcal{M} P \tilde{P} + \text{h.c.}, \tag{3.10}$$

where

$$\mathcal{M} = M[1 + \theta^2 r \langle F_\phi \rangle], \tag{3.11}$$

In this parameterization $r \neq 0$ parameterizes the deviation from a supersymmetric threshold, *i.e.* $r = 0$ gives a pure anomaly-mediated spectrum below the messenger scale. The model of Nelson and Weiner has $r = -2$. We then have

$$M = c_P(1 + X)\langle F_\phi \rangle, \tag{3.12}$$

$$r = -\frac{2 + \frac{1}{4}X\left(c_S + 4 - \sqrt{c_S(c_S - 8)} \right)}{1 + X}, \tag{3.13}$$

where

$$X = \frac{\lambda_P \langle S \rangle}{c_S \langle F_\phi \rangle} = -\frac{\lambda_P}{2 c_P \lambda_S} \left(3 c_S + \sqrt{c_S (c_S - 8)} \right). \tag{3.14}$$

This shows that all values of M and r are allowed, since $1+X$ can be small and have either sign. (Note that this does not require any Yukawa couplings to be large.) In order to avoid a negative mass eigenvalue for the scalars P, $\tilde{P}$ at the minimum, we require

$$|(r+1)\langle F_\phi \rangle| < |M|. \tag{3.15}$$

We now evaluate the threshold contributions to the standard model fields due to the P fields. The general formulas can be obtained from the methods of Refs. [23, 24]. The soft SUSY breaking terms can be parameterized by higher superspace components of dimensionless couplings via

$$m_0^2 = -\frac{\partial}{\partial \theta^2} \frac{\partial}{\partial \bar{\theta}^2} \ln Z, \tag{3.16}$$

$$m_{1/2} = \frac{1}{g} \frac{\partial}{\partial \theta^2} g, \tag{3.17}$$

$$\lambda A = -2 \frac{\partial}{\partial \theta^2} \lambda, \tag{3.18}$$

where all couplings are taken to be real superfields. In the present model, all SUSY breaking is contained in the conformal compensator and the P, $\tilde{P}$ mass term, so we have

$$\frac{\partial}{\partial \theta^2} = \frac{1}{2} \langle F_\phi \rangle \left(r \frac{\partial}{\partial \ln M} - \frac{\partial}{\partial \ln \mu} \right). \tag{3.19}$$

Note that this implies the presence of mixed anomaly- and gauge-mediated terms for scalar masses, as first pointed out in Ref. [20]. In this way, we can obtain expressions for the soft masses at the scale M in the effective theory where P and $\tilde{P}$ have been integrated out:

$$m_0^2(M) = \frac{1}{4}\langle F_\phi\rangle^2 \left\{ -r^2 \frac{\partial \gamma'}{\partial g_i'}\beta_i' + 2r(r+1)\frac{\partial \gamma}{\partial g_i}\beta_i' - (r+1)^2 \frac{\partial \gamma}{\partial g_i}\beta_i \right\}, \tag{3.20}$$

$$m_{1/2}(M) = \frac{1}{g}\langle F_\phi\rangle \left[r\beta_g' - (r+1)\beta_g \right], \tag{3.21}$$

$$A(M) = -\frac{1}{\lambda}\langle F_\phi\rangle \left[r\beta_\lambda' - (r+1)\beta_\lambda \right]. \tag{3.22}$$

Here primed (unprimed) quantities refer to the theory above (below) the scale M. The anomalous dimensions are defined by

$$\beta_i = \frac{\partial g_i}{\partial \ln \mu}, \qquad \gamma = \frac{\partial \ln Z}{\partial \ln \mu}. \tag{3.23}$$

The expression for the scalar masses can be simplified in the case of fields with no Yukawa couplings to messengers, for which $\gamma' = \gamma$. We then have

$$m_0^2(M) = m_{0\,\mathrm{AMSB}}^2 + \frac{1}{4}r(r+2)\langle F_\phi\rangle^2 \frac{\partial \gamma}{\partial g_i}\Delta\beta_i, \tag{3.24}$$

where

$$m_{0\,\mathrm{AMSB}}^2 = -\frac{1}{4}\langle F_\phi\rangle^2 \frac{\partial \gamma}{\partial g_i}\beta_i. \tag{3.25}$$

and $\Delta\beta = \beta' - \beta$. Similarly, we can write

$$m_{1/2}(M) = m_{1/2\,\mathrm{AMSB}} + \frac{r}{g}\langle F_\phi\rangle \Delta\beta_g \tag{3.26}$$

$$A(M) = A_{\mathrm{AMSB}} - \frac{r}{\lambda}\langle F_\phi\rangle \Delta\beta_\lambda. \tag{3.27}$$

These expressions explicitly display the fact that the soft masses reduce to the AMSB values in the limit $r \to 0$. The scalar masses (but not gaugino masses and

A terms) also reduce to their AMSB values for $r \to -2$, as in the model of Nelson and Weiner. In the generalized model, all soft masses reduce to the gauge-mediated values in the limit $r \to \infty$ with $r\langle F_\phi \rangle$ held fixed. For general r, the SUSY breaking spectrum in this model interpolates continuously between anomaly mediation and gauge mediation with a messenger scale of order 10 TeV (assuming all dimensionless couplings are order unity).

As with the case of pure gauge- and anomaly-mediated SUSY breaking, Eqs. (3.20)–(3.22) are leading order results in a power series with subleading corrections suppressed by $\mathcal{O}((\langle F_\phi \rangle / M^2)^2)$ and $\mathcal{O}((r\langle F_\phi \rangle / M^2)^2)$. In the present class of models, it is natural to have $M \sim \langle F_\phi \rangle, r\langle F_\phi \rangle$, where these effects may be important. They have been calculated for the case of pure gauge mediation, where they are known to be numerically small unless the SUSY breaking is tuned to be close to the instability limit $F/M^2 \to 1$ [25]. Because these corrections are UV finite, they do not depend on the regulator, and therefore depend on the conformal compensator only through the superfield mass of the messengers (see Eq. (3.10)). We can therefore use the results for gauge mediation with the replacement $F/M^2 \to (r+1)\langle F_\phi \rangle / M^2$. Since the stability limit is $|(r+1)\langle F_\phi \rangle / M^2| < 1$ here as well, the corrections are small in the absence of fine tuning.

3.2 Spectra and Numerical Results

We now discuss the SUSY breaking spectrum that results from this model. We assume that the messengers come in complete $SU(5)$ multiplets, so that the gauge coupling unification in the MSSM is not an accident. The simplest possibility is then that the messengers consist of N copies of $\mathbf{5} \oplus \bar{\mathbf{5}}$. For perturbative unification, we require $N \leq 4$. Under the standard model gauge group, these decompose into a doublet and a triplet, each of which can have different couplings c_P and λ_P (see Eq. (3.3)). These give rise to different values for r for the doublet and triplet messengers, and hence different SUSY breaking masses for colored and uncolored superpartners. We assume for simplicity that the N messengers have the same coupling (*e.g.* there can be an unbroken $SU(N)$ symmetry in the messenger sector). This can be relaxed to obtain even more general spectra.

For large r, the spectrum is close to that of gauge mediation. However, because SUSY breaking is driven by anomaly mediation, the gravitino mass is naturally of order $\langle F_\phi \rangle$, alleviating the gravitino problem of typical GMSB theories. This may not be large enough for large r, but it is possible (and natural) to have masses for the gravitino and other gravitational moduli that are parametrically larger than $\langle F_\phi \rangle$ with SUSY breaking dominated by anomaly mediation [26].

The simplest model is completely specified at high energies by M, F_ϕ, r_2, r_3, N, and μ. One parameter is eliminated by requiring that the Higgs VEV takes its experimentally determined value, so this model has four continuous and one discrete parameter.[3] Of these, the dependence on the messenger scale is only logarithmic, since it just sets the scale for the RG running down to the weak scale. Explicit formulas for soft masses are presented in Appendix B.

[3] The top quark Yukawa coupling is fixed by demanding that the top quark mass has its measured value.

For illustration, the spectrum of superpartner masses at the messenger scale is shown in Fig. 3.1 as a function of $r = r_2 = r_3$ for $M = 50$ TeV, for $N = 1$ and $N = 4$ respectively. For $r < 0$ we can obtain positive slepton mass-squared parameters, but the right-handed sleptons are lighter than the bino, giving rise to charged slepton LSP. We therefore focus our attention on $r > 0$. The spectra are still qualitatively similar to gauge- and anomaly mediation in the sense that colored superpartners are heavier than uncolored ones. For example, obtaining positive slepton mass-squared parameters requires $r \gtrsim 1$, which then implies $m_{\tilde{q}} \gtrsim 5m_{\tilde{\ell}}$.

Quite different possibilities exist if $r_2 \neq r_3$. In Fig. 3.2 we show an example spectrum with $N = 1$ and $r_3 = -1$. We again require $r > 0$ to avoid a slepton LSP. We see that the spectrum is more degenerate, and the $SU(2)_W$ contribution to superpartner masses is comparable to $SU(3)_C$. For $r_2 \gtrsim 2$, the superpartners charged under $SU(2)_W$ are the heaviest, followed by the gluino, then right-handed scalars and the Bino. Such spectra open up new regions of SUSY parameter space that may be interesting to explore. These spectra have a light stop, and therefore requires an additional contribution to the Higgs quartic. Possibilities include a "fat" Higgs [27] or large D terms from exotic gauge interactions [28].

We give some representative points in parameter space in Table 3.1, assuming $r_2 = r_3$ for simplicity. At the scale M we evaluate the soft-breaking parameters using Eqs. (3.20)–(3.22), and evolve them down using the MSSM RG equations to the stop mass scale $m_{\tilde{t}}$. (Since we have small mixing in the stop sector, we simply use the common stop mass.) At the scale $m_{\tilde{t}}$, we determine the μ parameter by minimizing the one-loop effective potential. This includes the largest 2-loop corrections to the

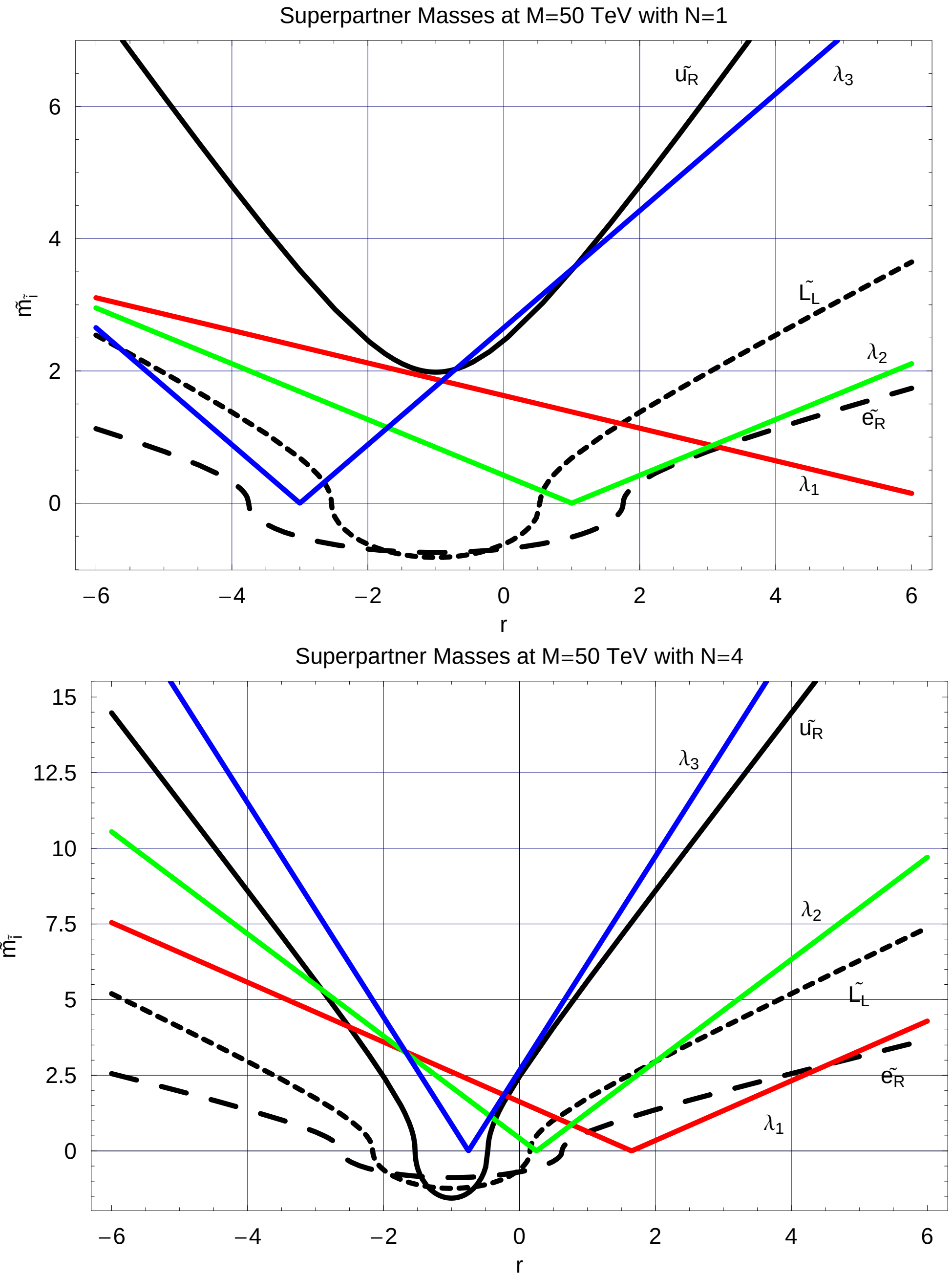

Figure 3.1: Spectrum of superpartner masses as a function of $r = r_2 = r_3$ for $M = 50$ TeV, and $N = 1$ (top) and $N = 4$ (bottom). For gaugino masses we plot $|M|$ and for scalar masses, we plot $|m^2|^{1/2} \times \mathrm{sgn}(m^2)$. All masses are in units of $F_\phi/(16\pi^2)$.

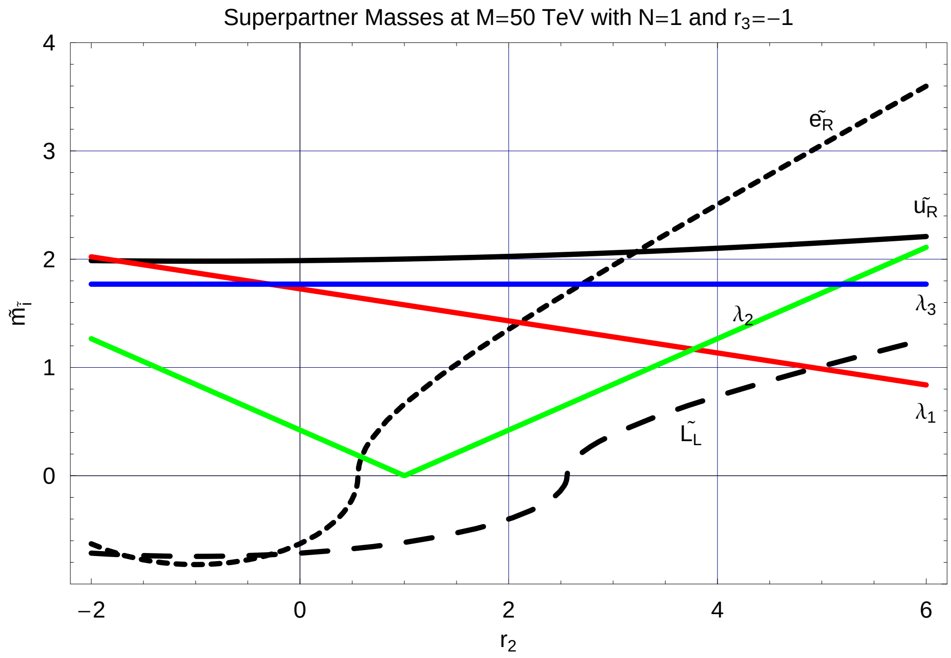

Figure 3.2: Spectrum of superpartner masses as a function of r_2 for $N = 1$, $M = 50$ TeV, and $r_3 = -1$. For gaugino masses we plot $|M|$ and for scalar masses, we plot $|m^2|^{1/2} \times \mathrm{sgn}(m^2)$. All masses are in units of $F_\phi/(16\pi^2)$.

effective potential because we use a value of y_t that includes 1-loop QCD corrections [29]. We then add by hand the 2-loop QCD threshold corrections to the higgs mass $m_{h^0}^2$, although this is a small correction (< 2 GeV) for small stop mixing.

The spectra given in Table 3.1 satisfy all experimental constraints. The most severe constraint is the LEP Higgs mass bound $m_{h^0} > 114.4$ GeV. Because we do not have large mixing in the stop-sector, we require $m_{\tilde{t}} \sim 1$ TeV to satisfy the Higgs mass bound, and the experimental constraints on the sleptons and LSP are easily satisfied. As we have large stop masses, these models are fine-tuned.

We quantify the fine tuning by the sensitivity of the Higgs to varying parameters at the GUT scale. The Higgs mass is quadratically sensitive to the stop mass, but this is not a fundamental parameter in this model. The most sensitive fundamental parameter is $g_3(M_{\rm GUT})$, so we define

$$\text{Fine tuning} \equiv \frac{g_3(M_{\rm GUT})}{v} \frac{\partial v}{\partial g_3(M_{\rm GUT})} = \frac{\partial \ln v}{\partial \ln g_3(M_{\rm GUT})}. \tag{3.28}$$

Because the sensitivity is through the stop mass, the tuning increases quadratically with the stop mass, while the lightest Higgs mass increases only logarithmically. This means that the fine tuning increases exponentially as a function of the lightest Higgs mass. This phenomenon is intrinsic to the MSSM, not just the present model, and is illustrated in Fig. 3.3. Note that the fine-tuning is somewhat less for a large number of messengers, since QCD is non-asymptotically free in this case with $N = 4$, and therefore the sensitivity to $g_3(M_{\rm GUT})$ is reduced.

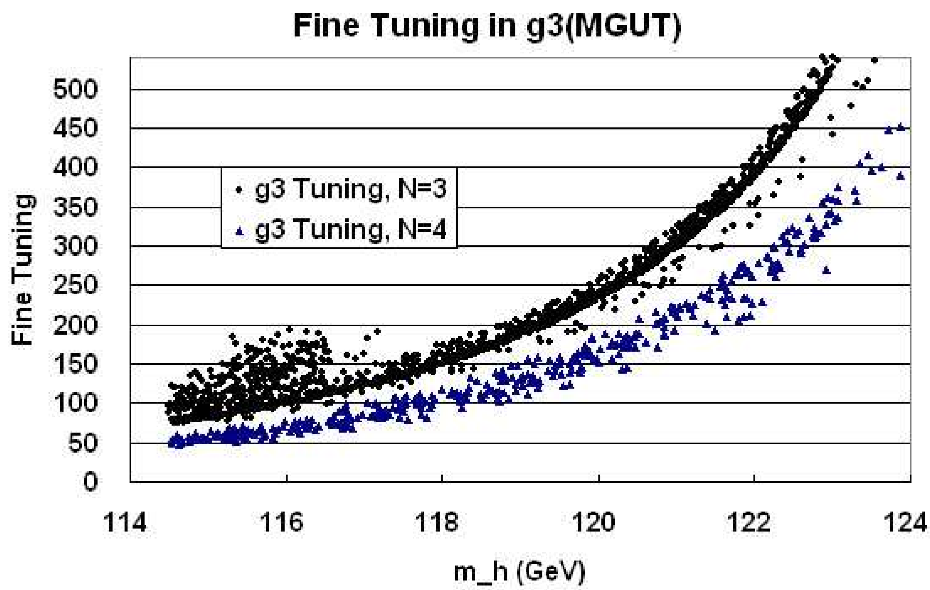

Figure 3.3: Fine-tuning in $g_3(M_{GUT})$ as a function of lightest Higgs mass m_{h^0} for models with $r > 0$ for $N = 3$ and 4.

3.3 Conclusion

We have constructed a well-motivated minimal model that naturally breaks SUSY in a flavor-blind way with a messenger scale near 10 TeV. The minimal model with one messenger has four continuous parameters and one discrete parameter, and can give rise to spectra that are very different from scenarios considered in the literature. These include "compact" spectra with colored superpartners close in mass to uncolored superpartners, a feature of the spectrum that may help with SUSY naturalness.

Table 3.1: Sample MSSM spectra. All masses are in GeV. The main text gives the definition of fine-tuning.

	Point 1	Point 2
N	1	4
r	14.6	6.45
F_ϕ	7.19 TeV	6.34 TeV
M	201 TeV	81 TeV
μ	485	425
$\tan\beta$	17.1	17.7
m_{h^0}	115	115
$m_{\tilde{l}L}$	380	330
$m_{\tilde{l}R}$	190	150
$m_{\tilde{q}L}$	1220	1170
$m_{\tilde{u}R}$	1170	1130
$m_{\tilde{d}R}$	1065	1120
$m_{\tilde{t}1}$	1070	1050
$m_{\tilde{t}2}$	1180	1150
$m_{\tilde{g}}$	880	1280
$m_{\tilde{\chi}_1^0}$	80	165
Tuning	170	55

Chapter 4

MINIMAL D-TYPE GAUGE-MEDIATED SUSY-BREAKING

4.1 Motivation

As noted in Chapter 2, the MSSM suffers from the little hierarchy problem because the s-top masses are required to be at least one order of magnitude heavier than the weak scale, causing a fine-tuning of (Eq. (2.81))

$$\frac{\delta \ln v^2}{\delta \ln m_{H_u}^2} \sim \frac{6y_t^2}{8\pi^2}\frac{m_{\tilde{t}}^2}{M_Z^2}\ln\frac{\Lambda^2}{m_{\tilde{t}}^2}. \tag{4.1}$$

Large stop masses are generic in theories of supersymmetric breaking such as CMSSM, GMSB, and AMSB, where the soft masses are generated at some high scale, and the predictions near the electroweak scale are made through the evolution of the MSSM RGEs. This evolution usually leads to a hierarchy between the masses of s-leptons and s-quarks

$$\frac{m_{\tilde{q}}^2}{m_{\tilde{l}}^2} \sim \frac{g_3^4}{g_1^4} \sim 100. \tag{4.2}$$

This hierarchy, along with the experimental lower bound on the s-lepton masses of $m_{\tilde{l}} > 100$ GeV, implies s-quarks with masses of the order 1 TeV, and gives a fine-tuning by Eq. (4.1) of about one part in 60 for $\Lambda \sim 40$ TeV.

As we note in Chapter 2, phenomenologically, heavy s-tops are needed to satisfy the LEP2 bound of $m_h > 114.4$ GeV [13]. In the MSSM, the mass of the lightest CP-even Higgs boson is given by the formula

$$m_h^2 = M_Z^2 \cos^2 2\beta + \frac{3y_t^4 v^2}{8\pi^2}\left(\ln\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2} - \frac{X_t^4}{12m_{\tilde{t}}^4}\right). \tag{4.3}$$

In GMSB and AMSB, where it is typical to have large $\tan\beta$ and small trilinear A_t, one would indeed need stop masses of roughly 1.5 TeV to satisfy the LEP2 bound, with a fine-tuning of one part in 70 for $\Lambda = 10$ TeV. However, even when one treats the soft terms as free parameters without appealing to any model of SUSY-breaking, it is possible to satisfy the LEP2 bound without heavy stops by having large trilinear couplings $A_t \sim 2m_{\tilde{t}}$. In this scenario, the fine-tuning in Eq. (4.1) is tamed to one part in 20 and even the fine-tuning with respect to A_t term

$$\frac{\delta \ln v^2}{\delta \ln A_{\tilde{t}}^2} \sim \frac{3y_t^2}{8\pi^2}\frac{A_{\tilde{t}}^2}{M_Z^2}\ln\frac{\Lambda^2}{m_{\tilde{t}}^2}. \tag{4.4}$$

is under control to one part in 20. The fact that large A_t can satisfy LEP2 bounds with the least fine-tuning in the MSSM has led to models of SUSY-breaking that may give to rise large A_t terms. Unfortunately, in these models, it is difficult to achieve both large A_t terms *and* light stops required to minimize fine-tuning, as having light stops typically leads to sleptons that violate experimental bounds via the hierarchy Eq. (4.2).

Therefore, although the LEP2 bounds on the Higgs mass do not necessarily imply heavy stops and the MSSM in its most generality is compatible with fine-tuning as little as one part in 20, simple models of SUSY-breaking typically have problems reaching this corner of parameter space. Instead, one is forced to live with heavy stops, and thus fine-tuning, via the hierarchy of Eq. (4.2) from the MSSM RGEs and the experimental bounds on the slepton masses.

In this Chapter, we present a solution to the little hierarchy problem of the MSSM by making gauginos Dirac particles instead of Majorana ones, and dub our model D-Type (or Dirac) gauge-mediated SUSY-breaking (DGMSB). Dirac gaugino masses have been considered in the literature before [30] [31]. However, our model differs from these model in that we keep all the Dirac gauginos light, and D-type SUSY-breaking is the dominant contribution to the soft scalar masses. With Dirac gauginos, the fine-tuning of the Higgs sector is reduced because the RGEs of the soft mass terms are modified, and one no longer has the relation Eq. (4.2). In fact, the RGEs of squark masses vanish at one-loop (for the first two families) and squark masses receive a large *negative* contribution through the RGE evolution at two-loop order. This causes the sleptons to be heavier than the squarks at low scales, and one can easily satisfy the slepton mass bound of $m_{\tilde{l}} > 100$ GeV while having squark masses of the order 500 GeV. We will proceed to construct the D-type SUSY-breaking sector in Section 4.2. In Section 4.3, we discuss how the SUSY-breaking can lead to Dirac gaugino masses and the MSSM soft masses. In Section 4.4, we analyze the Higgs and dark matter sector of the model, and present several viable spectra in Section 4.5. Finally, in Section 4.6, we discuss some collider phenomenology of Dirac gauginos and point out the striking differences between this model and the existing, typical SUSY-breaking models.

4.2 D-Type SUSY Breaking

We first construct the hidden sector of the model. Consider the O'Raighfeartaigh model with superpotential

$$W = \frac{\lambda_+}{2} N_- \Sigma_+^2 + \frac{\lambda_-}{2} N_+ \Sigma_-^2 + \kappa S(\Sigma_+\Sigma_- - u^2). \tag{4.5}$$

This has a $U(1)_X$ symmetry

$$X(\Sigma_\pm) = \pm 1,\ X(N_\pm) = \pm 2,\ X(S) = 0, \tag{4.6}$$

which we will gauge. This model breaks SUSY:

$$F_{N_\pm}^\dagger = \frac{\partial W}{\partial N_\pm} = \frac{\lambda_-}{2}\Sigma_\mp^2, \tag{4.7}$$

$$F_S^\dagger = \frac{\partial W}{\partial S} = \kappa(\Sigma_+\Sigma_- - u^2), \tag{4.8}$$

so the F-flat conditions are not compatible. We look for a vacuum where $\langle D_X \rangle \neq 0$, where

$$D_X = |\Sigma_+|^2 - |\Sigma_-|^2 + 2(|N_+|^2 - |N_-|^2). \tag{4.9}$$

Note that in order to get $\langle D_X \rangle \neq 0$, it is crucial to break charge conjugation symmetry under which

$$\Sigma_+ \leftrightarrow \Sigma_+, \quad N_+ \leftrightarrow N_-, \quad S \leftrightarrow S. \tag{4.10}$$

The only gauge-invariant renormalizable couplings that are not included in the superpotential are S^2, S^3, N_+N_-, and SN_+N_-. They can be forbidden by a $U(1)_R$ symmetry. Because of the linear term in S we have

$$R(S) = 2, \tag{4.11}$$

and hence $R(\Sigma_+\Sigma_-) = 0$. If we write

$$R(\Sigma_\pm) = \pm r, \tag{4.12}$$

then we have

$$R(N_\pm) = 2 \pm r. \tag{4.13}$$

Now the R charges of the terms we want to forbid are

$$R(S^2) = 4,\ R(S^3) = 6,\ R(N_+N_-) = 4,\ R(SN_+N_-) = 6. \tag{4.14}$$

The superpotential above is therefore the most general renormalizable one compatible with $U(1)_X \times U(1)_R$. A discrete subgroup of $U(1)_R$ is also sufficient to forbid the unwanted terms.

We look for a minimum with

$$\langle S\rangle = \langle N_\pm\rangle = 0, \qquad \langle \Sigma_+\rangle \neq \langle \Sigma_+\rangle \neq 0. \tag{4.15}$$

Note that since

$$F^\dagger_{\Sigma_\pm} = \frac{\partial W}{\partial \Sigma_\pm} = \lambda_+ N_\mp \Sigma_\pm + \kappa S \Sigma_\mp, \tag{4.16}$$

this implies $\langle F_{\Sigma_\pm}\rangle = 0$. This means that if the messengers are coupled to Σ_+ and Σ_-, they will not give any F-type breaking of SUSY in the visible sector.

The classical potential that follows from the superpotential is

$$\begin{aligned} V = {} & \tfrac{1}{4}\lambda_+^2|\Sigma_+|^4 + \tfrac{1}{4}\lambda_-^2|\Sigma_-|^4 + \kappa^2|\Sigma_+\Sigma_- - u^2|^2 \\ & + |\lambda_+ N_- \Sigma_+ + \kappa S \Sigma_-|^2 + |\lambda_- N_+ \Sigma_- + \kappa S \Sigma_+|^2 \\ & + \frac{g_X^2}{2}\left(|\Sigma_+|^2 - |\Sigma_-|^2 + 2|N_+|^2 - 2|N_-|^2\right)^2 \end{aligned} \tag{4.17}$$

Assuming a vacuum of the form Eq. (4.15), the only nontrivial minimization conditions are then

$$0 = \frac{\partial V}{\partial \Sigma_\pm^\dagger} = \tfrac{1}{2}\lambda_\pm^2 \Sigma_\pm^2 \Sigma_\pm^\dagger + \kappa^2 \Sigma_\mp^\dagger (\Sigma_+ \Sigma_- - u^2) \pm g_X^2 (|\Sigma_+|^2 - |\Sigma_+|^2) \Sigma_\mp. \tag{4.18}$$

We write

$$\Sigma_\pm = \sigma_\pm e^{i\alpha_\pm}, \tag{4.19}$$

with $\sigma_\pm$ real and $0 \le \alpha_\pm < \pi$ (so that $\sigma_\pm$ can be either positive or negative). Then the minimization conditions can be written

$$\tfrac{1}{2}\lambda_\pm^2 \sigma_\pm^3 = -\kappa^2 (\sigma_+ \sigma_- - u^2 e^{-i(\alpha_+ + \alpha_-)}) \sigma_\mp \pm g_X^2 (\sigma_+^2 - \sigma_-^2) \sigma_\pm. \tag{4.20}$$

This depends only on the phase $\theta = \alpha_+ + \alpha_-$. Taking the imaginary part of either equation immediately gives $\theta = 0$. (We assume that u^2 is real and positive, which can be achieved by rephasing S.) The fact that the difference $\alpha_+ - \alpha_-$ is undetermined is because there is a Nambu-Goldstone boson in the system. (This is eaten by the massive $U(1)_X$ gauge boson.) We can rewrite the minimization conditions as

$$\left(\tfrac{1}{2}\lambda_\pm^2 + g_X^2\right) \sigma_\pm^3 = \left[\kappa^2 u^2 - (\kappa^2 - g_X^2) \sigma_+ \sigma_-\right] \sigma_\mp. \tag{4.21}$$

Taking the ratio of these equations gives two solutions

$$\frac{\sigma_+}{\sigma_-} = \pm x, \qquad x = \left(\frac{\lambda_-^2 + 2g_X^2}{\lambda_+^2 + 2g_X^2} \right)^{1/4}. \tag{4.22}$$

We see that $\langle \Sigma_+ \rangle \ne \langle \Sigma_+ \rangle$ as long as $\lambda_- \ne \lambda_+$. From this we obtain

$$\sigma_+^2 = \frac{\kappa^2 u^2}{\tfrac{1}{2} x (\lambda_+^2 + 2g_X^2) \pm x^{-1} (\kappa^2 - g_X^2)}. \tag{4.23}$$

The condition that $\sigma_+^2 > 0$ implies

$$\left[(\lambda_+^2 + 2g_X^2)(\lambda_-^2 + 2g_X^2)\right]^{1/2} > \pm 2(g_X^2 - \kappa^2). \tag{4.24}$$

To get $\langle D_X \rangle \neq 0$, we need $r \neq 0$, which requires $\lambda_+ \neq \lambda_-$ (to break C). Note that it is natural to have $\langle D_X \rangle \ll \langle \Sigma_\pm \rangle^2$ by choosing $\lambda_+ \simeq \lambda_-$ because charge conjugation symmetry becomes exact when $\lambda_+ = \lambda_-$. This vacuum has all the features we want.

A crucial question is whether the stationary point is at least a local minimum. It is easy to see that in the limit $g_X \to 0$, the potential cannot decrease away from the stationary point. This is because the first three terms in Eq. (4.17) that depend only on $\Sigma_\pm$, are locally minimized, and the remaining terms vanish at the stationary point and are positive definite. Therefore, for sufficiently small g_X we only need to compute the contribution of the D terms along directions in field space that are massless in the limit $g_X \to 0$. (We know that there is at least one such direction from the general properties of O'Raifeartaigh models.) This simplifies the calculation considerably and we find that there are regions of parameter space where the scalar mass-squared terms are all positive.

Note that there is a folk theorem that it is not possible to have SUSY breaking with $D \gg F$ without Fayet-Iliopoulos terms, which are not allowed by supergravity. This model does not contradict the folk theorem, since $\langle F_N \rangle \gtrsim \langle D_X \rangle$. However, the fact that $\langle F_{\Sigma_\pm} \rangle = 0$ means that D-type breaking dominates in the visible sector if the messengers couple only to $\Sigma_\pm$. It is completely natural for the messengers not to couple to N and N_-, since these have different $U(1)_X$ charges.

4.3 SUSY-Breaking Masses in DGMSB

4.3.1 Dirac Gaugino Mass

The messengers must be in vector-like representations of the standard model gauge group so that their fermion components can be heavy without breaking electroweak symmetry. We take the messengers to be in complete $SU(3)^3 \times Z_3 \equiv SU(3)_c \times SU(3)_L \times SU(3)_R \times Z_3$ representations so that gauge coupling unification in the MSSM is not an accident. They must also be charged under $U(1)_X$ in order to feel SUSY breaking from the $U(1)_X$ D term. The simplest possibility is for the messengers to have quantum numbers under $SU(3)^3 \times U(1)_X$

$$P_\pm \sim \mathbf{3}^3_{\pm x}, \tag{4.25}$$

$$\tilde{P}_\pm \sim \bar{\mathbf{3}}^3_{\pm 1/2}, \tag{4.26}$$

and

$$\mathbf{3}^3_\pm \equiv (\mathbf{3}, \mathbf{1}, \mathbf{1})_{\pm 1/2} \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1})_{\pm 1/2} \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3})_{\pm 1/2}. \tag{4.27}$$

The gaugino partner is the fermionic part of an adjoint chiral superfield, Ξ, of $SU(3)^3$ and we assume it is uncharged under $U(1)_X$:

$$\Xi \sim (\mathbf{8}, \mathbf{1}, \mathbf{1})_0 \oplus (\mathbf{1}, \mathbf{8}, \mathbf{1})_0 \oplus (\mathbf{1}, \mathbf{1}, \mathbf{8})_0. \tag{4.28}$$

This is the minimal case, where we do not have to add a partner to Ξ to cancel $U(1)_X$ gauge anomalies.

In order to write the gaugino mass diagram, the gaugino partner must have superpotential coupling to the messengers:

$$\Delta W = \sum_{\pm} h_{\pm} \tilde{P}_{\mp} \Xi P_{\pm}. \tag{4.29}$$

The most general supersymmetric mass term we can write for the messengers is

$$\Delta W = \sum_{\pm} \left[M_{\pm} \tilde{P}_{\mp} P_{\pm} + y_{\pm} \Sigma_{\mp} \tilde{P}_{\pm} P_{\pm} \right], \tag{4.30}$$

consistent with our $U(1)_X$ assignments to $P_{\pm}$ and $\tilde{P}_{\pm}$.

In order to generate a nonzero Dirac gaugino mass, we need to break SUSY and charge conjugation. Let us discuss them in turn.

- SUSY-breaking is clearly felt by the messengers through SUSY-violating scalar masses in the $U(1)_X$ D term potential:

$$V_D = \frac{g_X^2}{2} \left\{ |\Sigma_+|^2 - |\Sigma_-|^2 + x \left(|P_+|^2 - |P_-|^2 + |\tilde{P}_+|^2 - |\tilde{P}_-|^2 \right) \right\}^2. \tag{4.31}$$

- Charge conjugation must be defined to change the sign of the standard model gaugino field in order to forbid a Dirac gaugino mass.

$$\Sigma_+ \leftrightarrow \Sigma_-, \quad P_+ \leftrightarrow \tilde{P}_-, \quad \tilde{P}_+ \leftrightarrow P_-, \quad \Xi \mapsto \Xi. \tag{4.32}$$

  This is broken by $\langle \Sigma_+ \rangle \neq \langle \Sigma_- \rangle$. Note that because charge conjugation interchanges fields with conjugate standard model representations, it is broken by standard model interactions involving ordinary quarks and leptons. This is a very small breaking of charge conjugation invariance that is not relevant for the low-order diagrams we are considering.

We conclude that the interactions above are sufficient to break all the symmetries required to generate a nonzero gaugino mass.

In components, the diagram that generates a Dirac gaugino mass is shown in Figure 4.1 [1], where λ is the fermion component of the standard model gauge multiplet, and ξ is the fermion component of the adjoint chiral superfield Ξ. The fields in the loops are messengers. In superfields, the Dirac gaugino mass term

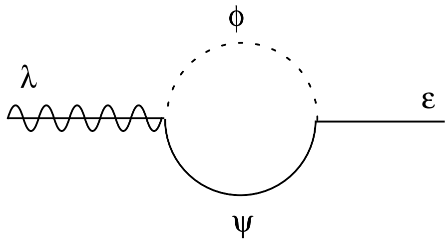

Figure 4.1: Component diagram that gives rise to a Dirac mass for the gaugino.

corresponds to the effective operator

$$\Delta\mathcal{L}_{\text{eff}} \sim \int d^2\theta \, \frac{1}{8\pi^2} \, \frac{1}{M} X^\alpha \, \text{tr}(W_\alpha \Xi) + \text{h.c.}, \tag{4.33}$$

where W_α is the standard model field strength, X_α is the $U(1)_X$ field strength. SUSY is broken by

$$\langle X_\alpha \rangle \sim \theta_\alpha \langle D_X \rangle \neq 0, \tag{4.34}$$

and it is easy to see that this gives a Dirac gaugino mass of order

$$m_{1/2} \sim \frac{1}{8\pi^2} \, \frac{\langle D_X \rangle}{M}. \tag{4.35}$$

To avoid having possible tadpole term

$$W = M^2 \Xi_{U(1)}, \tag{4.36}$$

[1]All the Feynman diagrams of this Thesis are generated by JaxoDraw [32].

in the superpotential, we choose the mass scale of messengers $M_\pm$ to be at or above the GUT scale, where such tadpole terms are forbidden by $SU(3)^3$ gauge symmetry. For simplicity, we pick $M_+ > M_-$ and $M_- = M_{\text{GUT}}$. Evaluating the diagram Fig. 4.1 directly, we obtain a Dirac gaugino mass

$$m_{\text{Dirac}} = -\frac{3g_{\text{GUT}}}{16\pi^2}\left(\frac{h_+}{M_+} + \frac{h_-}{M_-}\right)\langle D_X\rangle, \tag{4.37}$$

where g_{GUT} is the gauge coupling of the $SU(3)^3$ group.

Once $SU(3)^3$ breaks to the SM gauge group, we have the decompositions

$$\begin{aligned}
(\mathbf{8},\mathbf{1},\mathbf{1}) &= (\mathbf{8},\mathbf{1},\mathbf{1}) \\
(\mathbf{1},\mathbf{8},\mathbf{1}) &= (\mathbf{1},\mathbf{3},\mathbf{0}) \oplus (\mathbf{1},\mathbf{1},\mathbf{0}) \oplus (\mathbf{1},\mathbf{2},\tfrac{\mathbf{1}}{\mathbf{2}}) \oplus (\mathbf{1},\mathbf{2},-\tfrac{\mathbf{1}}{\mathbf{2}}) \\
(\mathbf{1},\mathbf{1},\mathbf{8}) &= 4(\mathbf{1},\mathbf{1},\mathbf{0}) \oplus \mathbf{2}(\mathbf{1},\mathbf{1},\mathbf{1}) \oplus \mathbf{1}(\mathbf{1},\mathbf{1},-\mathbf{1}).
\end{aligned} \tag{4.38}$$

We see that the chiral adjoint Ξ of $SU(3)^3$ decompose into chiral adjoint of the SM, a set of vector-like (under the SM gauge group) bachelors (as their fermionic components do not marry to gauginos of the MSSM), and SM singlets. We assume that below the messenger thresholds (near or above the GUT scale), the bachelors obtain supersymmetric mass terms below 10 TeV while the gauginos that are singlets under the SM gauge group obtain Majorana masses of the order M_{GUT}. That is, below the messenger threshold, only those particles that carry the SM quantum numbers survive, and only the fermionic components of Ξ that transform as adjoint of the SM marry with the corresponding gauginos to form Dirac particles. The Dirac gaugino masses and the MSSM scalar masses evolve through the corresponding RGEs from M_{GUT} to M_{weak}.

To compute the RGE of the Dirac gaugino masses, we note that the Dirac gaugino mass computed in Eq. (4.37) is the physical mass, and it is related to the gaugino mass $m_{1/2}$ of Eq. (4.35) by

$$m_{\text{Dirac}} \sim \frac{g}{\sqrt{Z_\Xi}} m_{1/2}, \tag{4.39}$$

where Z_Ξ is the wavefunction renormalization of the chiral adjoint Ξ. Thus, m_{Dirac} evolves with scale due to running of Z_Ξ and g, with the RGE

$$\begin{aligned} \frac{d}{dt} m_{\text{Dirac}} &= m_{\text{Dirac}} \left(\frac{1}{g}\frac{dg}{dt} - \frac{1}{2}\frac{d \ln Z_\Xi}{dt} \right) \\ &= m_{\text{Dirac}} \frac{1}{(16\pi^2)} \left(b g^2 - 2 C_\Xi g^2 \right), \end{aligned} \tag{4.40}$$

where b is the coefficient of the gauge coupling beta function

$$\beta_g = \frac{d}{dt} g = \frac{b}{(16\pi^2)} g^3, \tag{4.41}$$

and C_Ξ is the Casimir index of Ξ. The three Dirac gauginos in the model then have the RGEs

$$\frac{d}{dt} m_1 = m_1 \frac{16}{(16\pi^2)} g_1^2, \tag{4.42}$$

$$\frac{d}{dt} m_2 = m_2 \frac{1}{(16\pi^2)} \frac{5}{2} g_2^2, \tag{4.43}$$

$$\frac{d}{dt} m_3 = m_3 \frac{1}{(16\pi^2)} \left(-\frac{8}{3} \right) g_3^2. \tag{4.44}$$

4.3.2 Stability of the S-gaugino

Note that once we have the interactions Eqs. (4.29) and (4.30), we necessarily also allow a one-loop mass to the scalar components of Ξ, which we call s-gauginos and denote by η, through the diagrams shown in Figure 4.2. In terms of SUSY

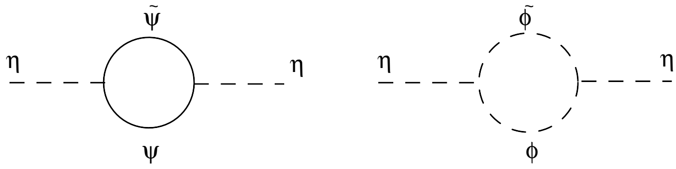

Figure 4.2: Component diagrams that give sgaugino masses.

spurions, the η mass corresponds to the operator

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^2\theta \, \frac{1}{8\pi^2} \frac{1}{M^2} X^\alpha X_\alpha \, \mathrm{tr}(\Xi^2) + \text{h.c.} \tag{4.45}$$

It is not hard to see that this operator is allowed whenever the Dirac gaugino mass operator is allowed, and gives a mass of order

$$m_\eta^2 \sim \frac{1}{8\pi^2} \frac{\langle D_X \rangle}{M^2}. \tag{4.46}$$

The operator in Eq. (4.45) gives masses of the form $\eta^2 + \text{h.c.}$, and such masses tend to destabilize the η potential. In the Appendix C we minimize the η effective potential and extract η mass terms of the form

$$V = m_\eta^2 |\eta|^2 + (B_\eta \eta^2 + \text{h.c.}) + \mathcal{O}(\eta^3), \tag{4.47}$$

where

$$m_\eta^2 = \frac{1}{8\pi^2} \frac{D(m_+^2 - m_-^2)|h_- M_+ - h_+ M_-|^2}{(|M_+|^2 - |M_-|^2)^2} \left(-2 + \frac{|M_+|^2 - |M_-|^2}{|M_+|^2 - |M_-|^2} \ln \frac{|M_+|^2}{|M_-|^2} \right), \tag{4.48}$$

$$B_\eta = -\frac{D^2}{32\pi^2} \left(\frac{h_+^2}{|M_+|^2} + \frac{h_-^2}{|M_-|^2} \right) + \frac{D}{16\pi^2} \frac{(m_+^2 - m_-^2)}{(|M_+|^2 - |M_-|^2)} \left(c_1 + c_2 \ln \frac{|M_+|^2}{|M_-|^2} \right), \tag{4.49}$$

and

$$c_1 = h_+^2 \frac{M_+^\dagger}{M_+} - h_-^2 \frac{M_-^\dagger}{M_-} + 2\frac{(h_+ M_+^\dagger - h_- M_-^\dagger)^2}{|M_+|^2 - |M_-|^2},$$
$$c_2 = -2\frac{M_+^\dagger M_-^\dagger (h_- M_+ - h_+ M_-)(h_+ M_+^\dagger - h_- M_-^\dagger)}{(|M_+|^2 - |M_-|^2)^2}. \tag{4.50}$$

Treating $M_\pm$ as real variables, the two η (mass)^2 eigenvalues have the form $m_\eta^2 \pm 2B_\eta$. Since the Yukawa couplings $h_\pm$ can take either sign, we find that it is possible to have both η mass eigenvalues positive. As in the case of their fermion partners, we assume that those components of η that carry non-zero SM quantum numbers survive to a scale below 10 TeV to ensure the unification of gauge couplings.

Let us now consider the RGEs of m_η^2. In addition to gauge interactions, the η-fields have Yukawa interactions to the messengers $\tilde{P}$ and P. However, below the messenger threshold (which also breaks $SU(3)^3$ to $G(321)$), η has only gauge interactions to the $G(321)$ group. However, Dirac gaugino masses can not enter the RGEs of m_η^2, and there are only 2-loop contributions of the form

$$\frac{dm_\eta^2}{d\ln\mu} \sim \frac{g^4}{(16\pi^2)^2} m_\eta^2, \tag{4.51}$$

Since these are only 2-loop effects, we will assume that the SUSY-breaking masses to η stay constant with scale. The scalar partners to the bachelors will, however, receive additional supersymmetric mass terms that do evolve with scale.

4.3.3 Soft Masses of the S-Fermions

The MSSM particle content are embedded into a **27** of $SU(3)^3$,

$$\mathbf{27}_{SU(3)^3} = (\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1}) \oplus (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}). \tag{4.52}$$

Once $SU(3)^3$ is broken into $G(321)$, $\mathbf{27}$ decomposes as

$$
\begin{aligned}
(\mathbf{1},\mathbf{3},\overline{\mathbf{3}})_{SU(3)^3} &= (\mathbf{1},\mathbf{2},\tfrac{\mathbf{1}}{\mathbf{2}}) \oplus \mathbf{2}(\mathbf{1},\mathbf{2},-\tfrac{\mathbf{1}}{\mathbf{2}}) \oplus (\mathbf{1},\mathbf{1},\mathbf{1}) \oplus \mathbf{2}(\mathbf{1},\mathbf{1},\mathbf{0}),\\
(\overline{\mathbf{3}},\mathbf{3},\mathbf{1})_{SU(3)^3} &= (\overline{\mathbf{3}},\mathbf{1},-\tfrac{\mathbf{2}}{\mathbf{3}}) \oplus \mathbf{2}(\overline{\mathbf{3}},\mathbf{1},\tfrac{\mathbf{1}}{\mathbf{3}}),\\
(\mathbf{3},\overline{\mathbf{3}},\mathbf{1})_{SU(3)^3} &= (\mathbf{3},\mathbf{2},\tfrac{\mathbf{1}}{\mathbf{6}}) \oplus (\mathbf{3},\mathbf{1},-\tfrac{\mathbf{1}}{\mathbf{3}}).
\end{aligned}
\tag{4.53}
$$

We will assume that below the messenger scale only the MSSM matter content (quarks, leptons, and two Higgs doublet) and a singlet, denoted by S, survive. In addition to the Yukawa couplings in the superpotential in Eq. (2.53), we replace the μ-term in the superpotential by the couplings of the next-to-minimal supersymmetric Standard Model (NMSSM)

$$
\Delta W = \lambda S H_u H_d + \frac{\kappa}{3} S^3. \tag{4.54}
$$

.

The MSSM soft scalar masses below one messenger threshold is

$$
m_{\tilde{Q}}^2 = -\left[\ln M^\dagger M\right]_{\theta^2\bar{\theta}^2} \frac{1}{2}(\gamma' - \gamma), \tag{4.55}
$$

which involves one-loop matching contribution to the gauge couplings as well as the differences in anomalous dimension at two-loop order (γ is the same at one loop order between the fundamental and effective theory). The highest components of the SUSY-breaking spurions are

$$
\left[\ln M_{1,2}^\dagger M_{1,2}\right]_{\theta^2\bar{\theta}^2} = \pm 2\langle X\rangle \frac{|m_+|^2 - |m_-|^2}{|M_+|^2 - |M_-|^2}, \tag{4.56}
$$

and the change in the anomalous dimension of a chiral superfield Φ_i is generally,

$$
(16\pi^2)^2(\gamma' - \gamma) = 4 \sum_k C_k(i)\big(a_k - \Delta S_k(R)\big) g_k^4, \tag{4.57}
$$

where a_k is the matching contribution of the gauge coupling at the threshold

$$g_{i,\text{fund}}^2 - g_{i,\text{eff}}^2 = \frac{a_i}{(16\pi^2)} g_i^4. \tag{4.58}$$

For our case of SUSY-breaking mediated above M_{GUT}, with $SU(3)^3 \times Z_3$ as the unification group, the change in the anomalous dimension is

$$(16\pi^2)^2 \Delta\gamma = 3\tfrac{16}{3}(a_{\text{GUT}} - 1) g_{\text{GUT}}^4. \tag{4.59}$$

The multiplying factor of 3 arises because under any $SU(3)$ of $SU(3)^3$, we have three pairs of fundamental+antifundamental. So we have

$$m_{\mathbf{27}}^2 = \frac{4}{(16\pi^2)^2} \frac{(m_+^2 - m_-^2)}{(M_+^2 - M_-^2)} \left[g_{\text{GUT}}^4(M_+) - g_{\text{GUT}}^4(M_-) \right] \langle D_X \rangle. \tag{4.60}$$

It may seem that $m_{\mathbf{27}}^2$ is a three-loop quantity since, in addition to the explicit two-loop factors in Eq. (4.60), it is proportional to the difference in gauge couplings at two different scales, which is proportional to the β-function. However, g_{GUT} is semi-perturbative above M_{GUT} and the difference $(g_{\text{GUT}}^4(M_+) - g_{\text{GUT}}^4(M_-))$ is numerically large, of the same order as $g_{\text{GUT}}^4(\mu = M_{\text{GUT}})$.

In the MSSM, the RGEs of the soft-masses of the first two generations involve only the Majorana gaugino masses at 1-loop order. In DGMSB, the Dirac masses can not enter through the RGEs of the soft-masses, and there are only 2-loop contributions from the *eta* masses. These 2-loop contributions to the RGEs have two important consequences:

- First, although these contributions are of 2-loop order, they are of the form

$$\frac{dm_{\tilde{f}}^2}{d\ln\mu} \sim \frac{g^4}{(16\pi^2)^2} m_\eta^2. \tag{4.61}$$

Since m_η^2 is parametrically larger than $m_{\tilde{f}}^2$ by $16\pi^2$, such effects are as important as 1-loop effects.

- Second, contrary to the contributions from Majorana gaugino masses, these terms are positive, causing the soft masses to decrease when evolved to the weak scale. Since these terms are proportional to the gauge couplings, the squarks will be lighter than sleptons at the weak scale, which have important collider consequences.

Since $m_\eta^2/m_{\tilde{f}}^2 \sim 16\pi^2$, we only keep m_η^2 in the RGEs of the soft scalar masses. The relevant terms for the RGEs are

$$\frac{d}{dt}m_{\tilde{f}}^2 = \frac{8}{(16\pi^2)^2}\sum_a g_a^4 C_a^{\tilde{f}} S_a(\Xi) m_{\eta,a}^2, \tag{4.62}$$

where $C_a^{\tilde{f}}$ is the quadratic Casimir of the s-fermion $\tilde{f}$ under the gauge group a, $S_a(\Xi)$ is the Dynkin index of the fields of Ξ that survive down to near the weak scale. Although the components of η have different physical masses, it is only the soft masses of η that enter the RGEs of the MSSM soft masses, and soft-masses of η do not evolve with scale, as discussed in the previous section. Therefore, the RGEs to the MSSM soft masses are all scaled by the same factor m_η^2. For the MSSM soft masses, we have the beta functions

$$\frac{dm_{H_u}^2}{d\ln\mu} = \left(10g_1^4 + 18g_2^4\right)\frac{m_\eta^2}{(16\pi^2)^2} + \frac{1}{16\pi^2}(3X_t) + \frac{1}{16\pi^2}X_\lambda, \tag{4.63}$$

$$\frac{dm_{H_d}^2}{d\ln\mu} = \left(10g_1^4 + 18g_2^4\right)\frac{m_\eta^2}{(16\pi^2)^2} + \frac{1}{16\pi^2}X_\lambda, \tag{4.64}$$

$$\frac{dm_{\tilde{Q}_L^i}^2}{d\ln\mu} = \left(10g_1^4 + 18g_2^4 + 8g_3^4\right)\frac{m_\eta^2}{(16\pi^2)^2} + \frac{\delta_{i3}}{16\pi^2}X_t \tag{4.65}$$

$$\frac{dm_{\tilde{u}_R^i}^2}{d\ln\mu} = \left(\frac{40}{9}g_1^4 + 8g_3^4\right)\frac{m_\eta^2}{(16\pi^2)^2} + \frac{delta_{i3}}{16\pi^2}(2X_t), \tag{4.66}$$

$$\frac{dm_{\tilde{D}_R}^2}{d\ln\mu} = \left(\frac{10}{9}g_1^4 + 8g_3^4\right)\frac{m_\eta^2}{(16\pi^2)^2}, \tag{4.67}$$

$$\frac{dm_{\tilde{L}_L}^2}{d\ln\mu} = \left(10g_1^4 + 18g_2^4\right)\frac{m_\eta^2}{(16\pi^2)^2}, \tag{4.68}$$

$$\frac{dm_{\tilde{L}_L}^2}{d\ln\mu} = \left(40g_1^4\right)\frac{m_\eta^2}{(16\pi^2)^2}, \tag{4.69}$$

$$\frac{dm_S^2}{d\ln\mu} = \frac{1}{16\pi^2}(2X_\lambda + X_\kappa), \tag{4.70}$$

where i on $\tilde{Q}_L^i$ and $\tilde{u}_L^i$ denotes generations, and we approximate only the top Yukawa coupling, λ, and κ to be non-zero. We also define

$$\begin{aligned} X_t &= 2y_t^2(m_{H_u}^2 + m_{\tilde{Q}_L^3}^2 + m_{\tilde{u}_R^3}^2), \\ X_\lambda &= 2\lambda^2(m_{H_u}^2 + m_{H_d}^2 + m_S^2), \\ X_\kappa &= 2\kappa^2(3m_S^2), \end{aligned} \tag{4.71}$$

for the ease of notation in the RGEs above.

4.3.4 Trilinear Couplings and the B_μ-term

Trilinear soft terms are forbidden in DGMSB. It it easiest to see this through the spurion analysis. The D-type SUSY-breaking comes from two spurions

$$\langle V_X \rangle \sim \theta^2 \overline{\theta}^2 \langle D_X \rangle, \tag{4.72}$$

$$\langle X_\alpha \rangle \sim \theta_\alpha \langle D_X \rangle. \tag{4.73}$$

To have the effective operators for trilinear couplings, we have to insert the chiral superfield X_α to the superpotential. We have to insert X_α twice to contract the spinor indices and also to absorb the two powers of θ from the integral. Dimensional analysis then gives the appropriate power suppression

$$\mathcal{L}_{\text{eff}} \sim \int d^2\theta \frac{1}{8\pi^2} \frac{X_\alpha X^\alpha}{M^3} Q_i Q_j Q_k + \text{h.c.} \sim \frac{1}{8\pi^2} \frac{\langle D_X \rangle^2}{M^3} \tilde{Q}_i \tilde{Q}_j \tilde{Q}_k + \text{h.c.}. \tag{4.74}$$

Relative to the Dirac gaugino masses that are of the weak scale, the trilinear interactions have an additional suppression of $\langle D_X \rangle / M^2 \sim 10^{-16}$. Therefore, there are no trilinear interactions for all practical purposes.

However, we can still have an effective $B_\mu \mu$-term

$$\mathcal{L}_{\text{eff}} \sim \int d^2\theta \frac{1}{16\pi^2} \frac{X_\alpha X^\alpha}{M^2} H_u H_d + \text{h.c.} \sim \frac{1}{16\pi^2} \frac{\langle D_X \rangle^2}{M^2} H_u H_d + \text{h.c.}. \tag{4.75}$$

The resulting $B_\mu \mu$-term is too large by one loop factor of $16\pi^2$. However, if the couplings above the messenger scale that generate the $B_\mu \mu$ term are not $SU(3)^3$ invariant, we would expect them to be suppressed at the messenger scale. Since B-terms in general ($B_\mu \mu$ is a particular case) do not enter the RGEs of the soft-masses nor the gaugino masses, we will simply treat them as free parameters at the weak scale.

4.4 Higgs Bosons and Dark Matter in DGMSB

4.4.1 Higgs Mass

Although we have successfully constructed a SUSY-breaking scenario that gives small s-top masses while having viable s-lepton masses, we do not have the trilinear coupling A_t that is crucial to satisfy the LEP2 bounds on the Higgs boson. While there is some mixing between the s-top masses from the μ-term, these terms are suppressed by $\tan^{-1}\beta$. Therefore, the DGMSB model must be extended to either gives a large Higgs mass or arrange for dominant cascade decays of the Higgs boson that may have escaped detection at LEP [33]. This is the main reason why we have added a singlet in addition to the MSSM spectrum. The superpotential of Eq. (4.54) contributes to the potential for the neutral bosons

$$V_{F,\mathrm{NMSSM}} = |\lambda H_u H_d + \kappa S^2|^2 + |\lambda|^2 |S|^2 (|H_u|^2 + |H_d|^2), \tag{4.76}$$

and we have the SUSY-breaking contributions

$$V_{\mathrm{Breaking}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + B_H(H_u H_d + \mathrm{h.c.}) + B_S(S^2 + \mathrm{h.c.}). \tag{4.77}$$

Since S is a part of **27** of $SU(3)^3$, m_S^2 has the same boundary condition as the other soft-breaking scalar masses, and m_S^2 can be driven to negative values at the weak scale for large enough λ and κ. As scalar components of S develops a vev, we dynamically generate the μ-term of the MSSM and give mass to the fermionic component of S.

In the NMSSM potential, the tree-level bound on the mass of the lightest CP-even boson no longer applies. Though there are no compact formulas for the masses, we note that we have an additional quartic interaction of the Higgs

$$\Delta V = |\lambda|^2 |H_u|^2 |H_d|^2, \tag{4.78}$$

in addition to the usual quartic gauge interactions, and we can have tree-level masses that are larger than M_Z with small $\tan\beta$ and large λ. With the radiative corrections from s-tops taken into account, the resulting spectra can be consistent with LEP2 bounds on the Higgs mass. We show example spectra in the next section.

4.4.2 Dark Matter

The lightest superpartner of a typical spectrum in DGMSB is the Dirac bino. Naïvely, the Dirac bino as dark matter is ruled out by direct detection experiments. However, the complete story is here more subtle. With the mixing through EWSB processes, the two states of the Dirac bino are necessarily split slightly into two Majorana mass eigenstates. The true lightest state, being a Majorana fermion, has a p-wave suppressed elastic cross-section with the nucleon. On the other hand, for the relic density of the LSP one can essentially treat the LSP as a Dirac bino, without taking into account the mass-splitting induced by EWSB effects.

In the MSSM, the annihilation cross-section of the Majorana bino is p-wave suppressed, leading to large relic density that is inconsistent with observed measurements. To have viable bino-dominated dark matter in the MSSM requires fine-tuning in the MSSM spectrum [34] such that either (a) there is delicate mixing between the bino with the Higgsino and/or wino that have large annihilation cross-sections

[35], (b) there is a nearby state (usually the s-tau), such that there is significant co-annihilation cross section, and/or (c) there is a state (usually the CP-odd Higgs boson A^0) whose mass is almost exactly twice the mass of the bino so that the bino annihilation cross-section has an s-wave resonance.

Compared with the Majorana bino, pseudo-Dirac bino as LSP is a much more natural candidate as dark matter because the annihilation cross-section of the pseudo-Dirac bino is not p-wave suppressed. This can be seen in Fig. 4.3, where we plot the dark matter relic density for both the Majorana and Dirac bino as a function of a common sfermion mass. The relic density of the Majorana bino is calculated using MicrOMEGAs 2.0 [36]. Even though neither case comes close to the observed relic density, the Dirac bino has a much lower relic density because it annihilates efficiently without p-wave suppression. The lower relic density of the pure Dirac bino (compared to that of the Majorana bino) means that, for the LSP to have the observed relic density, there are no longer needs for dedicate mixing with the wino and Higgsino and special channels for co-annihilation and/or s-wave resonance.

4.5 Example Spectra

To compute the spectra, we have to specify the following free parameters

$$h_{\pm},\ m_{\pm},\ m_D,\ M_{\pm},\ \tan\beta,\ \lambda,\ \kappa. \tag{4.79}$$

The Yukawa couplings of the MSSM are then fixed by the SM spectrum and $\tan\beta$. For all our numerical analysis, we take M_- to be M_{GUT}, which is the scale that g_1

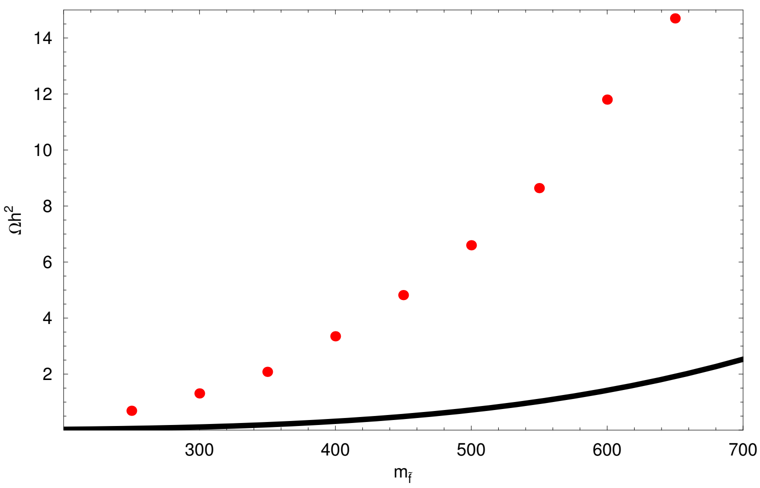

Figure 4.3: The dark matter relic density Ωh^2 for Majorana (red dots) and Dirac (black line) bino as a function of a common sfermion mass. The observed dark matter relic abundance is $\Omega h^2 = 0.127\,{}^{+0.007}_{-0.013}$. Both of the theoretical curves have a $m_{\tilde{f}}^4$ dependence, but the Dirac bino gives a much lower relic abundance because it annihilates efficiently without p-wave suppression.

and g_2 unify, and fix M_+ to be 5×10^{17} GeV. We specify $\tan\beta$, λ, and κ at weak scale with the constraint that they do not encounter Landau poles before M_-. At the scale M_-, we specify the values $h_\pm$, $m_\pm$ and m_D so that the s-gauginos have positive squared-masses, and compute the initial conditions of the soft masses and Dirac gaugino masses. These boundary conditions are then evolved to the weak scale, where the three minimization conditions are used to determine B_H, B_S, and $\langle S \rangle$. Once the potential is minimized, we check that the current experimental lower bounds of Higgs and superpartners are satisfied.

We include some sample spectra in Tables 4.1 and 4.2, and note some main features.

- At the input level, M_{GUT}, the gaugino masses are independent of the scalar masses, as in the case of CMSSM. However, gaugino masses in DGMSB do not feed into the RGEs of the scalar masses, as noted. The unified values of gaugino masses at M_{GUT} and their RGEs gives us the relation at the weak scale

$$M_1 : M_2 : M_3 \sim 1 : 1.9 : 4.5. \tag{4.80}$$

- The sleptons are heavier than the squarks, as discussed earlier. However, the sfermions (squarks and sleptons) have a rather compact spectra in DGMSB, a feature not present in generic models of SUSY-breaking.

4.6 Collider Signatures

The combination of Dirac gauginos and inverted s-fermion spectra (s-quarks lighter than s-leptons) have interesting consequences at hadron colliders such as the LHC. As the detailed, quantitative study of the collider signatures of Dirac gauginos are still under study, we here offer only some general remarks. Although we will focus on the features of the spectra listed in the previous section, some of our remarks apply to general spectra of DGMSB.

4.6.1 Suppression of Same-Sign Di-muon

The same-sign di-lepton signal is a useful signal for the MSSM (in the search of light s-tops, for example). However, in DGMSB theories, the (pseudo-)Dirac nature

	S1	S2	S3
h_+	-0.1	-0.05	-0.1
h_-	-0.8	-0.4	-0.8
m_+	9×10^{11}	-11×10^{11}	-10×10^{11}
m_-	-8×10^{11}	10×10^{11}	9×10^{11}
m_D	8×10^{10}	3.5×10^{10}	3×10^{10}
η_1	2070	1050	1740
η_2	4770	2300	3740
λ_1	150	56	80
λ_2	250	94	140
λ_3	640	245	360
$\tilde{l}_L$	1270	1280	1010
$\tilde{l}_R$	1310	1290	1040
$\tilde{Q}_{L1}$	930	1210	740
$\tilde{Q}_{L3}(\tilde{t}_1)$	680	990	540
$\tilde{u}_{R1}$	990	1220	790
$\tilde{u}_{R3}(\tilde{t}_2)$	430	710	340
$\tilde{d}_{R1}$	1000	1220	800
$\tilde{d}_{R3}$	1000	1220	800

Table 4.1: First portion of typical spectra of DGMSB, see the caption of Table 4.2 for more details. All masses are in units of GeV.

	S1	S2	S3
h_1	118	122	115
h_2	1330	1505	1048
h_3	1420	1700	1136
a_1	1180	1460	930
a_2	1380	1590	1110
$\mu(\lambda\langle S\rangle)$	820	990	660
$\tilde{S}$	1400	1700	1120

Table 4.2: Generic spectra of DGMSB models, continued from Table 4.1. We fix the parameters $\tan\beta = 2.2$,$\lambda = 0.7$, and $\kappa = 0.6$ at the weak scale. We denote the neutral CP-even (odd) bosons by h (a), the charged Higgs bosons and Higgsinos have masses given by μ. All masses are in GeV.

of the gauginos necessarily suppresses same-sign dilepton signals. While a Majorana gluino can decay to both squark-antifermion and antisquark-fermion pairs, a Dirac gluino can only decay through one such channel because it is distinct from its anti-particle. Therefore, when gluinos-anti-gluino pairs are produced at the LHC, the gluino (anti-gluino) would decay to squark-antiquark (anti-squark quark). As all the gauginos are Dirac, the sign of the lepton in the gluino decay chain is necessary opposite to that of the anti-gluino decay chain [2]

4.6.2 Three-Body Decays of the Gluino and the Wino

In a typical MSSM spectrum, where we have

$$m_{\tilde{g}} > m_{\tilde{q}} > m_{\chi_2^0} > m_{\tilde{l}} > m_{\chi_1^0}, \tag{4.81}$$

one can measure various parameters of the MSSM via the cascade decay shown in Figure 4.4. We do not distinguish between s-fermion (fermion) and anti-sfermion (anti-fermion) because various combinations are possible due to the Majorana nature of the gluino and neutralinos.

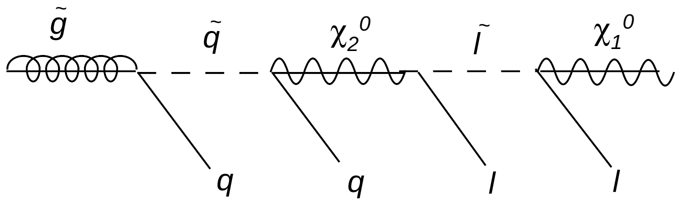

Figure 4.4: Typical cascade of MSSM gluino.

[2]Like-sign quarks or leptons may still occur through the interactions with the Higgsinos, but these are suppressed by Yukawa couplings.

In DGMSB, the sleptons are heavier than the squarks, and the decay chain in Figure 4.4 is no longer valid. In fact, in DGMSB, it is natural to have the spectra

$$m_{\tilde{l}} > m_{\tilde{q}} > m_{\tilde{g}} > m_{\chi_2^0} > m_{\chi_1^0}, \tag{4.82}$$

where the gaugino masses are Dirac masses. Such spectra have striking signatures. First, as all the sfermions are heavier than all the gauginos, the gluino and χ_2^0 (dominantly the Dirac wino) must now undergo three-body decay through virtual sfermions. The phase-space suppression of three-body decays translates into long lifetimes for both the gluino and χ_2^0. Furthermore, the gluino will preferentially decay into the wino through

$$\tilde{g} \rightarrow \overline{q}q\tilde{w}, \tag{4.83}$$

with the wino also undergoing a three-body decay

$$\tilde{w} \rightarrow \overline{q}q\tilde{b}, \overline{l}l\tilde{b}. \tag{4.84}$$

Compared to the case of the MSSM, this is a relatively short decay chain.

4.6.3 Singly-Produced $SU(3)$-Adjoint

In the MSSM, the superpartners must be pair produced because of R-parity[3]. However, in DGMSB, because the fermionic component of Ξ marries with the gaugino and must necessarily be R-parity odd, η must be R-parity even and can be singly

[3] Although there are additional (compared to the SM) states that are R-parity even (such as the extra Higgs boson), the additional colored states are all R-parity odd, and must be pair-produced.

produced. Although the production channel of η involves one loop, the production cross-section can still be significant[4], and the s-wave resonance may be observable.

4.7 Conclusions

In this Chapter we have constructed a model D-type gauge-mediated SUSY-breaking. This model is inspired by the little hierarchy problem, and solves it by having Dirac gauginos. Dirac gauginos in a supersymmetric theory lead to many interesting consequences at the LHC. In particular, the sleptons are heavier than squarks and like-sign leptons signals would be much suppressed. We believe the collider phenomenology of this model offers an interesting alternative to currently studied scenarios, and are working to extend the current tools to adapt to this model.

During a completion of this work, a similar D-type gauge mediation model was independently put forth by Antoniadis *et. al.* [37].

[4]This is similar to Higgs production from two gluons at one-loop.

Chapter 5

EXTRA DIMENSIONS

One of the most interesting idea in string theories is that our physical world does not contain only three spatial dimensions. It is possible for our physical world to have additional spatial dimensions as long as they are *compactified*, and their existence can be tested through the behavior of elementary particles. Such extra dimensions, if they exist, may help explain why the weak scale is much lower than the GUT scale or the Planck scale, and solve the hierarchy problem [44][45][46][47]. The ideas and ways to visualize these extra dimensions can be found in many popular texts, as well as technical works. Here I only offer one analogy of how there can be small, extra dimensions, that is hidden from us in everyday life. Imagine that we are looking at an electrical cable from afar (much further than the radius of the wire) so that the cable seems to be a one-dimensional string. Upon closer inspection, when our distance to the wire is comparable to the radius of the wire, we realize that the wire has a finite, non-zero thickness, and every single point of the string (when we look from afar) is in fact a ring of a tiny radius (when we take a closer look) that connects to form the wire. Similarly, if we can *look* closer at our physical world, we may find that each point of our three-dimensional space may contain one or more such rings.

The situation regarding extra dimensions is all the more exciting with the prospect that we may discover their existence at the LHC. It turns out that a quantum field that lives in the extra dimension will appear as a collection of quantum fields, called Kaluza-Klein (KK) modes, in the effective four-dimensional world, with equally-spaced masses. One particular class of extra dimensional models, known as universal extra dimension (UED) [49], proposes that all the fields in the SM are allowed to propagate in flat extra dimensions. The minimal UED (mUED) model has been studied extensively. It offers an interesting dark matter candidate in the KK mode of the photon [38][39], and its collider signatures may fake those of the MSSM [48]. However, since mUED is expected only be an effective theory up to a scale of about 10 or 100 TeV, it may have the problems of proton decay or neutrino masses when the SM gauge group is enlarged. Both of these problems can be solved in a model proposed by Mohapatra and Perez-Lorenzana [51] that implements left-right symmetry with two extra, flat dimensions. The orbifolding of the model, $T_2/Z_2 \times Z_2'$, suppresses both proton decay and neutrino masses in a simple manner. Furthermore, the model proposes an interesting dark matter candidate: an admixture of the KK modes of the photon and sterile neutrinos. In this Chapter, we present a detailed study of the dark matter properties of the model. We map out its viable parameter space, and calculate the direct detection rates of dark matter in this region of parameter space.

5.1 6D UED

We choose the gauge group of the model to be $SU(3)_c \times SU(2)_L \times SU(2)_{\rm R} \times U(1)_{\rm B\text{-}L}$ with matter content per generation as follows:

$$\begin{aligned} \mathcal{Q}_{1,-}, \mathcal{Q}'_{1,-} &= (3,2,1,\tfrac{1}{3}); \quad \mathcal{Q}_{2,+}, \mathcal{Q}'_{2,+} = (3,1,2,\tfrac{1}{3}); \\ \psi_{1,-}, \psi'_{1,-} &= (1,2,1,-1); \quad \psi_{2,+}, \psi'_{2,+} = (1,1,2,-1); \end{aligned} \tag{5.1}$$

where, within parenthesis, we have written the quantum numbers that correspond to each group factor, respectively and the subscript gives the six dimensional chirality to cancel gravitational anomaly in six dimensions. We denote the gauge bosons as G_M, $W^{\pm}_{1,M}$, $W^{\pm}_{2,M}$, and B_M, for $SU(3)_c$, $SU(2)_L$, $SU(2)_{\rm R}$ and $U(1)_{\rm B\text{-}L}$ respectively, where $M = 0,1,2,3,4,5$ denotes the six space-time indices. We will also use the following short hand notations: Greek letters $\mu, \nu, \cdots = 0,1,2,3$ to denote usual four dimensions indices, as usual, and lower case Latin letters $a, b, \cdots = 4,5$ for those of the extra space dimensions. We will also use $\vec{y}$ to denote the (x_4, x_5) coordinates of a point in the extra space.

First, we compactify the extra x_4, x_5 dimensions into a torus, T^2, with equal radii, R, by imposing periodicity conditions, $\varphi(x_4, x_5) = \varphi(x_4 + 2\pi R, x_5) = \varphi(x_4, x_5 + 2\pi R)$ for any field φ. This has the effect of breaking the original $SO(1,5)$ Lorentz symmetry group of the six dimensional space into the subgroup $SO(1,3) \times Z_4$, where the last factor corresponds to the group of discrete rotations in the x_4-x_5 plane, by angles of $k\pi/2$ for $k = 0,1,2,3$. This is a subgroup of the continuous $U(1)_{45}$ rotational symmetry contained in $SO(1,5)$. The remaining $SO(1,3)$ symmetry gives the usual 4D Lorentz invariance. The presence of the surviving Z_4 symmetry leads to suppression of proton decay [50] as well as neutrino mass [51].

Employing the further orbifolding conditions :

$$
\begin{aligned}
Z_2 \;&:\; \mathbf{y} \to -\mathbf{y} \\
Z_2' \;&:\; \begin{cases} (x_4, x_5)\,' \to -(x_4, x_5)\,' \\ \mathbf{y}\,' = \mathbf{y} - (\pi R/2, \pi R/2) \end{cases}
\end{aligned}
\tag{5.2}
$$

We can project out the zero modes and obtain the KK modes by assigning appropriate $Z_2 \times Z_2'$ quantum numbers to the fields.

In the effective 4D theory the mass of each mode has the form: $m_N^2 = m_0^2 + \frac{N}{R^2}$; with $N = \vec{n}^2 = n_1^2 + n_2^2$ and m_0 is the Higgs vacuum expectation value (vev) contribution to mass, and the physical mass of the zero mode.

We assign the following $Z_2 \times Z_2'$ charges to the various fields:

$$
\begin{gathered}
G_\mu(+,+);\; B_\mu(+,+);\; W^{3,\pm}_{1,\mu}(+,+);\; W^3_{2,\mu}(+,+);\; W^\pm_{2,\mu}(+,-); \\
G_a(-,-);\; B_a(-,-);\; W^{3,\pm}_{1,a}(-,-);\; W^3_{2,a}(-,-);\; W^\pm_{2,a}(-,+).
\end{gathered}
\tag{5.3}
$$

For quarks we choose,

$$
\begin{aligned}
Q_{1\mathrm{L}} &\equiv \begin{pmatrix} u_{1\mathrm{L}}(+,+) \\ d_{1\mathrm{L}}(+,+) \end{pmatrix};\; Q'_{1\mathrm{L}} \equiv \begin{pmatrix} u'_{1\mathrm{L}}(+,-) \\ d'_{1\mathrm{L}}(+,-) \end{pmatrix}; \\
Q_{1\mathrm{R}} &\equiv \begin{pmatrix} u_{1\mathrm{R}}(-,-) \\ d_{1\mathrm{R}}(-,-) \end{pmatrix};\; Q'_{1\mathrm{R}} \equiv \begin{pmatrix} u'_{1\mathrm{R}}(-,+) \\ d'_{1\mathrm{R}}(-,+) \end{pmatrix}; \\
Q_{2\mathrm{L}} &\equiv \begin{pmatrix} u_{2\mathrm{L}}(-,-) \\ d_{2\mathrm{L}}(-,+) \end{pmatrix};\; Q'_{2\mathrm{L}} \equiv \begin{pmatrix} u'_{2\mathrm{L}}(-,+) \\ d'_{2\mathrm{L}}(-,-) \end{pmatrix}; \\
Q_{2\mathrm{R}} &\equiv \begin{pmatrix} u_{2\mathrm{R}}(+,+) \\ d_{2\mathrm{R}}(+,-) \end{pmatrix};\; Q'_{2\mathrm{R}} \equiv \begin{pmatrix} u'_{2\mathrm{R}}(+,-) \\ d'_{2\mathrm{R}}(+,+) \end{pmatrix};
\end{aligned}
\tag{5.4}
$$

and for leptons:

$$\psi_{1\mathrm{L}} \equiv \begin{pmatrix} \nu_{1\mathrm{L}}(+,+) \\ e_{1\mathrm{L}}(+,+) \end{pmatrix}; \quad \psi'_{1\mathrm{L}} \equiv \begin{pmatrix} \nu'_{1\mathrm{L}}(-,+) \\ e'_{1\mathrm{L}}(-,+) \end{pmatrix};$$

$$\psi_{1\mathrm{R}} \equiv \begin{pmatrix} \nu_{1\mathrm{R}}(-,-) \\ e_{1\mathrm{R}}(-,-) \end{pmatrix}; \quad \psi'_{1\mathrm{R}} \equiv \begin{pmatrix} \nu'_{1\mathrm{R}}(+,-) \\ e'_{1\mathrm{R}}(+,-) \end{pmatrix};$$

$$\psi_{2\mathrm{L}} \equiv \begin{pmatrix} \nu_{2\mathrm{L}}(-,+) \\ e_{2\mathrm{L}}(-,-) \end{pmatrix}; \quad \psi'_{2\mathrm{L}} \equiv \begin{pmatrix} \nu'_{2\mathrm{L}}(+,+) \\ e'_{2\mathrm{L}}(+,-) \end{pmatrix};$$

$$\psi_{2\mathrm{R}} \equiv \begin{pmatrix} \nu_{2\mathrm{R}}(+,-) \\ e_{2\mathrm{R}}(+,+) \end{pmatrix}; \quad \psi'_{2\mathrm{R}} \equiv \begin{pmatrix} \nu'_{2\mathrm{R}}(-,-) \\ e'_{2\mathrm{R}}(-,+) \end{pmatrix}. \tag{5.5}$$

The zero modes i.e. (+,+) fields corresponds to the standard model fields along with an extra singlet neutrino which is left-handed. They will have zero mass prior to gauge symmetry breaking. The singlet neutrino state being a left-handed (instead of right-handed as in the usual case) has important implications for neutrino mass. For example, the conventional Dirac mass term $\bar{L}H\nu_{\mathrm{R}}$ is not present due to the selection rules of the model and Lorentz invariance. Similarly, $L\tilde{H}\nu_{2L}$ is forbidden by gauge invariance as is the operator $(LH)^2$. Thus neutrino mass comes only from much higher dimensional terms.

For the Higgs bosons, we choose a bi-doublet, which will be needed to give masses to fermions and break the standard model symmetry and a pair of doublets

(Z_2, Z_2')	Particle Content
$(++)$	$Q_{1\mathrm{L}};\ u_{2\mathrm{R}};\ d'_{2\mathrm{R}};\ \psi_{1\mathrm{L}};\ e_{2\mathrm{R}};\ \nu'_{2\mathrm{L}};$ $G_\mu;\ B_\mu;\ W^{3,\pm}_{1,\mu};\ W^3_{2,\mu};$ $\phi^0_u;\ \phi^-_u;\ \chi^0_{\mathrm{R}}$
$(+-)$	$Q'_{1\mathrm{L}};\ u'_{2\mathrm{R}};\ d_{2\mathrm{R}};\ \psi'_{1\mathrm{R}};\ \nu_{2\mathrm{R}};\ e'_{2\mathrm{R}};$ $W^\pm_{2,\mu};$ $\phi^+_d;\ \phi^0_d;\ \chi^-_{\mathrm{R}}$
$(-+)$	$Q'_{1\mathrm{R}};\ u'_{2\mathrm{L}};\ d_{2\mathrm{L}};\ \psi'_{1\mathrm{L}};\ \nu_{2\mathrm{L}};\ e'_{2\mathrm{R}};$ $W^\pm_{2,a};$ $\chi^0_{\mathrm{L}};\ \chi^-_{\mathrm{L}}$
$(--)$	$Q_{1\mathrm{R}};\ u_{2\mathrm{L}};\ d'_{2\mathrm{L}};\ \psi_{1\mathrm{R}};\ \nu_{1\mathrm{R}};\ e_{2\mathrm{L}};$ $G_a;\ B_a;\ W^{3,\pm}_{1,a};\ W^3_{2,a}$

Table 5.1: Particle content of 6D model separated by $Z_2 \times Z_2'$ parities.

$\chi_{L,R}$ with the following $Z_2 \times Z_2'$ quantum numbers:

$$\phi \equiv \begin{pmatrix} \phi_u^0(+,+) & \phi_d^+(+,-) \\ \\ \phi_u^-(+,+) & \phi_d^0(+,-) \end{pmatrix}; \quad \chi_L \equiv \begin{pmatrix} \chi_L^0(-,+) \\ \\ \chi_L^-(-,+) \end{pmatrix}; \quad \chi_{\mathrm{R}} \equiv \begin{pmatrix} \chi_{\mathrm{R}}^0(+,+) \\ \\ \chi_{\mathrm{R}}^-(+,-) \end{pmatrix}, \tag{5.6}$$

and the following charge assignment under the gauge group,

$$\begin{aligned} \phi &= (1,2,2,0), \\ \chi_L &= (1,2,1,-1), \quad \chi_{\mathrm{R}} = (1,1,2,-1). \end{aligned} \tag{5.7}$$

At the zero mode level, only the SM doublet (ϕ_u^0, ϕ_u^-) and a singlet χ_{R}^0 appear. The vacuum expectation values (vev) of these fields, namely $\langle \phi_u^0 \rangle = v_w$ and $\langle \chi_{\mathrm{R}}^0 \rangle = v_{\mathrm{R}}$, break the SM symmetry and the extra $U(1)_Y'$ gauge group, respectively. A diagram that illustrates the lowest KK modes of all the particles and their masses is shown in Fig. 5.1 with the following identification of modes in Table 5.1.

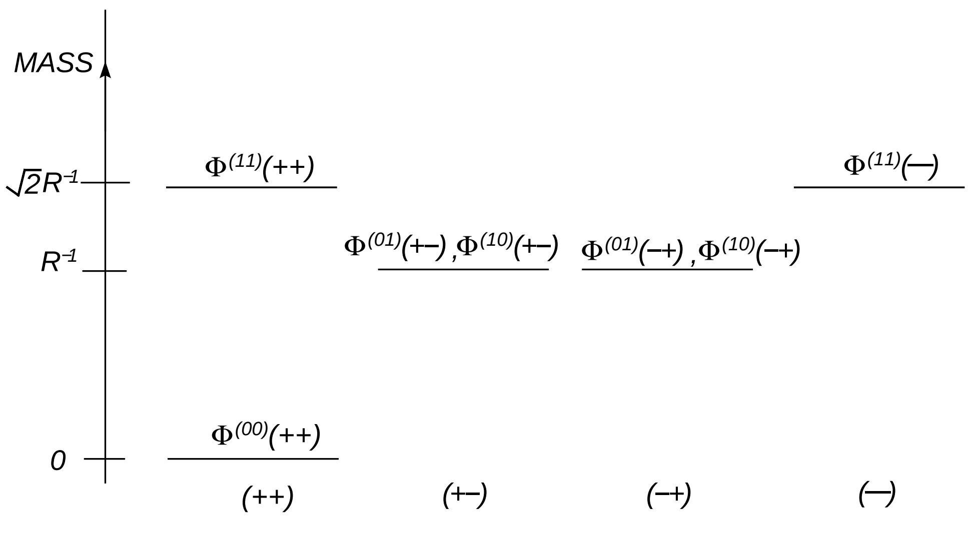

Figure 5.1: The masses of lowest KK-modes of 6D model.

The most general Yukawa couplings in the model are

$$h_u\bar{Q}_1\phi Q_2 + h_d\bar{Q}_1\tilde{\phi}Q_2' + h_e\bar{\psi}_1\tilde{\phi}\psi_2 + h_u'\bar{Q}_1'\phi Q_2' + h_d'\bar{Q}_1'\tilde{\phi}Q_2 + h_e'\bar{\psi}_1'\tilde{\phi}\psi_2' + h.c.; \quad (5.8)$$

where $\tilde{\phi} \equiv \tau_2\phi^*\tau_2$ is the charge conjugate field of ϕ. A six dimensional realization of the left-right symmetry, which interchanges the subscripts: $1 \leftrightarrow 2$, is obtained provided the 3×3 Yukawa coupling matrices satisfy the constraints: $h_u = h_u^\dagger$; $h_u' = h_u'^\dagger$; $h_e = h_e^\dagger$; $h_e' = h_e'^\dagger$; $h_d = h_d'^\dagger$. At the zero mode level one obtains the SM Yukawa couplings

$$\mathcal{L} = h_u\bar{Q}\phi_u u_R + h_d\bar{Q}\tilde{\phi}_u d_R + h_e\bar{L}\tilde{\phi}_u e_R + h.c. \quad (5.9)$$

It is important to notice that in the above equation $h_{u,e}$ are hermitian matrices, while h_d is not. The vev of ϕ_u gives mass to the charged fermions of the model. As far as the neutrino mass is concerned, the lowest dimensional gauge invariant operator in six-D that gives rise to neutrino mass after compactification has the form $\psi_{1,L}^T\phi\psi_{2,L}\chi_R^2$ and leads to neutrino mass $m_\nu \simeq \lambda\frac{v_{wk}v_R^2}{M_*^2(M_*^2R)^3}$. For $M_*R \sim 100$ and $M_* \sim 10$ TeV, $v_R \sim 2$ TeV and $\lambda \sim 10^{-3}$, we get neutrino masses of order $\sim$ eV without fine tuning. Furthermore, it predicts that the neutrino mass is Dirac (predominantly) rather than Majorana type.

As there are a large number of KK modes, one may worry whether or not electroweak precision constraints in terms of S and T parameters are satisfied. It has been shown that in the minimal universal extra dimension (mUED) the KK contributions to the T parameter almost cancel for heavier standard model Higgs [49, 54]. However, it was found that in the MUED for Higgs mass heavier than 300 GeV the lightest Kaluza-Klein particle is the charged KK Higgs [55]. The abundance of such charged massive particles are inconsistent with big bang nucleosynthesis as well as other cosmological observations for masses less than a TeV [56]. This lead to the conclusion that the compactification scale $1/R > 400$ GeV for $m_H > 300$ GeV. To our knowledge, there has been no such analysis for the $6-$D models similar to ours, and it is outside the scope of the current paper to perform a complete analysis regarding the electroweak constraints. Therefore, we leave the investigation of this open issue for future work.

5.1.1 Spectrum of Particles

Once the extra dimensions are compactified, the KK modes are labelled by the quanta of momenta in the extra dimensions. As we have two such extra spatial dimensions, the KK modes are labelled by two integers, and we will denote a KK mode as $\phi^{(mn)}$, where m (n) is the momentum in the quantized unit of R^{-1} along the fifth (sixth) dimension. A detailed expansion of a field in the 6D theory into KK mode is presented in the Appendix. Generally, $\phi^{(mn)}$ would receive a (mass)^2 of the order $(m^2 + n^2)R^{-2}$.

Gauge and Higgs Particles at the Zeroth KK Level

In the gauge basis, we have the zero-mode gauge bosons: $B^{(00)}_{\text{(B-L)}\mu}, W^{\pm,3(00)}_{L,\mu}$, and $W^{3(00)}_{R,\mu}$. After symmetry-breaking, we will have the usual gauge bosons of the SM: one exactly massless gauge boson, $A^{(00)}_{\mu}$, one pair of massive, charged vector boson $W^{\pm,(00)}_{L,\mu}$, and one massive neutral gauge boson $Z^{(00)}_{\mu}$. In addition, we will have another neutral gauge boson $Z'^{(00)}_\mu$, as well as mixing between $Z^{(00)}_\mu$ and $Z'^{(00)}_\mu$.

In this subsection we calculate the zeroth-mode gauge boson masses and mixings from Higgs mechanism (and drop the (00) superscript throughout this subsection). The relevant terms are

$$\mathcal{L}_h = \mathrm{Tr}[(D_\mu\phi)^\dagger D_\mu\phi] + (D^\mu\chi_{\mathrm{R}})^* D_\mu\chi_{\mathrm{R}} + (D^\mu\chi_L)^* D_\mu\chi_L \tag{5.10}$$

where

$$\begin{aligned}
D_\mu\phi &= \partial_\mu\phi - ig_{\mathrm{L}}(\overrightarrow{\tau}\cdot\overrightarrow{W_{L\mu}})\phi + ig_{\mathrm{R}}\phi(\overrightarrow{\tau}\cdot\overrightarrow{W_{\mathrm{R}\mu}}),\\
\phi &= \begin{pmatrix} \phi_u^0 & \phi_d^+ \\ \phi_u^- & \phi_d^0 \end{pmatrix},\\
D_\mu\chi_L &= \left(\partial_\mu - ig_{\mathrm{L}}(\overrightarrow{\tau}\cdot\overrightarrow{W_{L,\mu}}) + i(\frac{1}{2})g_{\mathrm{BL}}B_{\text{(B-L)},\mu}\right)\begin{pmatrix} \chi_L^0 \\ \chi_L^- \end{pmatrix},\\
D_\mu\chi_{\mathrm{R}} &= \left(\partial_\mu - ig_{\mathrm{R}}(\overrightarrow{\tau}\cdot\overrightarrow{W_{\mathrm{R},\mu}}) + i(\frac{1}{2})g_{\mathrm{BL}}B_{\text{(B-L)},\mu}\right)\begin{pmatrix} \chi_{\mathrm{R}}^0 \\ \chi_{\mathrm{R}}^- \end{pmatrix},\\
\overrightarrow{\tau}\cdot\overrightarrow{W_\mu} &= \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix}.
\end{aligned} \tag{5.11}$$

With vev of the fields $\langle\phi_u^0\rangle = v_w$ and $\langle\chi_{\mathrm{R}}^0\rangle = v_{\mathrm{R}}$, we obtain the following mass terms for the gauge bosons:

$$
\mathcal{L} = \frac{1}{2}v_w^2(W^+_{L,\mu}W^-_{L,\mu}) + \frac{1}{2}(v_w^2+v_{\rm R}^2)(W^+_{R,\mu}W^-_{R,\mu})
$$

$$
+\frac{1}{2}\begin{pmatrix} W^3_{L,\mu} & W^3_{R,\mu} & B_{(\text{B-L})\mu}\end{pmatrix}\begin{pmatrix} \frac{1}{2}g_{\rm L}^2v_w^2 & -\frac{1}{2}g_{\rm L}g_{\rm R}v_w^2 & 0 \\ -\frac{1}{2}g_{\rm L}g_{\rm R}v_w^2 & \frac{1}{2}g_{\rm R}^2(v_w^2+v_{\rm R}^2) & -\frac{1}{2}(g_{\rm R}g_{\rm BL})v_{\rm R}^2 \\ 0 & -\frac{1}{2}(g_{\rm R}g_{\rm BL})v_{\rm R}^2 & \frac{1}{2}g_{\rm BL}^2v_{\rm R}^2 \end{pmatrix}\begin{pmatrix} W_L^{3,\mu} \\ W_R^{3,\mu} \\ B^\mu_{(\text{B-L})}\end{pmatrix}. \tag{5.12}
$$

The exact expressions of the mass eigenvalues and the compositions of the eigenstates (A_μ, Z_μ, Z'_μ) in terms of $(B_{(\text{B-L})\mu}, W^3_{L,\mu}, W^3_{R,\mu})$ are rather complicated, and we make the approximation of $v_{\rm R} \gg v_w$. In this approximation, we find the relations,

$$
\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix} = U_G^\dagger \begin{pmatrix} W^0_{1,\mu} \\ W^0_{2,\mu} \\ B_\mu \end{pmatrix}, \tag{5.13}
$$

where

$$
U_G^\dagger = \begin{pmatrix} \sin\theta_w & \cos\theta_w & 0 \\ \cos\theta_w & -\sin\theta_w & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\theta_{\rm R} & \cos\theta_{\rm R} \\ 0 & \cos\theta_{\rm R} & -\sin\theta_{\rm R} \end{pmatrix}, \tag{5.14}
$$

and

$$
\tan\theta_{\rm R} \equiv \frac{g_{\rm BL}}{g_{\rm R}}, \quad g_{\rm Y}^2 \equiv \frac{g_{\rm BL}^2 g_{\rm R}^2}{g_{\rm BL}^2+g_{\rm R}^2}, \quad \text{and} \quad \tan\theta_w \equiv \frac{g_{\rm Y}}{g_{\rm L}}. \tag{5.15}
$$

It is easy to understand U_G intuitively. In the limit $v_w \ll v_{\rm R}$, the symmetry-breaking occurs in two stages, corresponding to the two matrices in U_G. First, we have $SU(2)_L \times SU(2)_{\rm R} \times U(1)_{\text{B-L}} \to SU(2)_L \times U(1)_Y$, where a linear combination of

$B_{(\text{B-L}),\mu}$ and $W^3_{R,\mu}$ acquire a mass to become Z'_μ, while the orthogonal combination, $B_{Y,\mu}$, remains massless and serves as the gauge boson of the residual group $U(1)_Y$. Then we have the standard electroweak breaking of $SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}$, giving us massive Z_μ and the massless photon A_μ. Using U_G, we can simplify the mass matrix enormously

$$U_G^\dagger \mathcal{M}^2 U_G = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_Z^2 & -\frac{g_{\text{R}}^2}{\sqrt{(g_{\text{L}}^2+g_{\text{Y}}^2)(g_{\text{R}}^2+g_{\text{BL}}^2)}} M_Z^2 \\ 0 & -\frac{g_{\text{R}}^2}{\sqrt{(g_{\text{L}}^2+g_{\text{Y}}^2)(g_{\text{R}}^2+g_{\text{BL}}^2)}} M_Z^2 & M_{Z'}^2 \end{pmatrix}, \tag{5.16}$$

where we have defined the mass eigenvalues (up to $\mathcal{O}(v_w/v_{\text{R}})^2$)

$$\begin{aligned} M_Z^2 &= \left(g_{\text{L}}^2 + g_{\text{Y}}^2\right) \frac{v_w^2}{2} \\ M_{Z'}^2 &= (g_{\text{BL}}^2 + g_{\text{R}}^2) \frac{v_{\text{R}}^2}{2} + \frac{g_{\text{R}}^4}{(g_{\text{BL}}^2 + g_{\text{R}}^2)} \frac{v_w^2}{2}. \end{aligned} \tag{5.17}$$

Here we see that we have explicitly decoupled A_μ, and it remains massless exactly. Although we have defined M_Z to be same as the tree-level mass of Z-boson of the Standard Model, here Z_μ is strictly speaking not an eigenstate because of the $Z - Z'$ mixing. Such mixing would be important, as we will see, for the calculation of relic density and the direct detection rates of the dark matter of the model. However, in the limit $v_{\text{R}}^2 \gg v_w^2$ that we will be working with, we can treat the defined masses and states in Eq. (5.17) as eigenvalues and eigenstates, and treat the mixing terms perturbatively in powers of (v_w^2/v_{R}^2).

The only zero-mode Higgs bosons in the model are $\phi_u^{0(00)}, \phi_u^{-(00)}$, and $\chi_{\text{R}}^{0,(00)}$. Four of the six degrees of freedoms are eaten and the remaining physical Higgs particles are the real parts of $\phi_u^{0(00)}$ and $\chi_{\text{R}}^{0,(00)}$. The masses of these particles are determined from the potential and are free parameters, whose values, however, do not affect the calculations of the relic density and direct detection rates of the dark matter.

Gauge and Higgs Particles at the First KK Level

We first consider the question of whether KK modes of Higgs bosons acquire vevs. The zero modes Higgs bosons acquire vevs due to negative mass-squared terms in the potential. The higher KK modes of the Higgs bosons $\phi^{(\mathrm{mn})}$, however, have an additional mass-squared contribution of the form $(m^2 + n^2)R^{-2}$. Therefore, if the negative mass-squared term in the potential is smaller in magnitude than R^{-2}, then none of the higher Higgs KK modes would acquire vevs. We will assume this is the case in our calculations, and the only fields that acquire vevs are $\phi_u^{0(00)}$ and $\chi_{\mathrm{R}}^{0,(00)}$, the zero-modes of neutral Higgs fields.

Here we will only consider the details of those gauge bosons in the (11) KK modes, and in this subsection it is understood that we have the superscript (11). That is, we do not consider the (01) and (10) modes of $W^{\pm}_{R,\mu,5,6}$. For a compact notation that will be convenient later on, for the scalar partners (G_5 and G_6) of a generic vector gauge boson (G_μ), we form the combinations

$$G_{(\pm)} \equiv \frac{1}{\sqrt{2}}(G_5 \pm G_6). \tag{5.18}$$

In the absence of Higgs mechanism, $G_{(+)}$ will be eaten by G_μ at the corresponding KK-level, while $G_{(-)}$ will be left as a physical degree of freedom. Qualitatively, $W^{\pm}_{R,\mu}$ eats a linear combination of $W^{\pm}_{R,(+)}$, $W^{\pm}_{R,(-)}$, and χ_{R}^{-} (all fields with the superscript

(01) and (10)), while the two remaining orthogonal directions are left as physical degrees of freedom.

At the (11)-level, before symmetry breaking, we have the modes

$$\begin{aligned}
\text{Neutral Gauge Bosons:} &\quad W^3_{L,\mu}, W^3_{R,\mu}, B_\mu \\
\text{Neutral Scalars:} &\quad W^3_{L,(+)}, W^3_{L,(-)}, W^3_{R,(+)}, W^3_{R,(-)}, B_{(+)}, B_{(-)}, \phi^0_u, \chi^0_{\rm R} \\
\text{Charged Gauge Bosons:} &\quad W^\pm_{L\mu} \\
\text{Charged Scalars:} &\quad W^\pm_{L,(+)}, W^\pm_{L,(-)}, \phi^-_u. \qquad (5.19)
\end{aligned}$$

Three (two) linear combinations of the neutral (charged) scalars would be eaten, leaving seven (four) degrees of freedom (note that the Higgs fields are complex). Since only the zero-mode Higgs acquire vevs, the Higgs mechanism contribution to the mass matrix of the neutral gauge bosons is same as that in Eq. (5.12), and we have an additional contribution of $2R^{-2}\mathbf{1}_{3\times3}$. We can diagonalize the (mass)^2 matrix up to $\mathcal{O}(v_w^2/v_{\rm R}^2)$ using the same unitary matrix U_G and obtain the eigenvalues to be those in Eq. (5.17) with the additional $2R^{-2}$.

Of the neutral scalars, we have several sets of particles that do not mix with members of other sets at tree level:

$$\begin{aligned}
\text{Set 1:} &\quad \tfrac{1}{\sqrt{2}}\mathrm{Re}[\phi^0_u], \tfrac{1}{\sqrt{2}}\mathrm{Re}[\chi^0_{\rm R}] \\
\text{Set 2:} &\quad W^3_{L,(-)}, W^3_{R,(-)}, B_{(-)} \\
\text{Set 3:} &\quad W^3_{L,(+)}, W^3_{R,(+)}, B_{(+)}, \tfrac{1}{\sqrt{2}}\mathrm{Im}[\phi^0_u], \tfrac{1}{\sqrt{2}}\mathrm{Im}[\chi^0_{\rm R}]. \qquad (5.20)
\end{aligned}$$

The squared-mass of particles in Set 1 are simply $2R^{-2}$ in addition to the squared-masses of corresponding particles at (00)-modes. The mass matrix of particles in

Set 2 are exactly that of the neutral gauge bosons, with a lightest mode of $A_{(-)}$ with mass $m_{A_{(-)}} = m_{A_\mu} = \sqrt{2}R^{-1}$. Three linear combinations of particles in Set 3 are eaten, and the two remaining particles have masses that will depend on the Higgs potential. As is the case with the zeroth-modes, as long as these Higgs are heavier than the lightest gauge bosons, the values of their masses will not affect our results about the dark matter of the model.

Spectrum of Matter Fields

Because there is no yukawa coupling between the Higgs doublet χ_{R} and matter, at tree level all mass terms arise from the momentum in the extra dimensions and v_w. The structure of the yukawa couplings, with the $Z_2 \times Z_2'$ orbifolding ensures that the zero-mode matter fields have the SM spectrum. As for the higher modes, the mass terms arising from the extra dimension connect the left- and right-handed components of a 6D chiral fermion $\Psi_\pm$, where $\pm$ denotes 6D chirality. The mass terms arising from electroweak symmetry-breaking, however, connects left- and right-handed components of two different 6D chiral fields. Taking the electron as an example, the mass matrix of the electron KK modes in the basis $\{e_{1L}\ e_{1R}\ e_{2L}\ e_{2R}\}$

(with e_{1L} and e_{2R} having zero modes) is

$$\mathcal{M}_{e^{(1)}} = \begin{pmatrix} \bar{e}_{1L} & \bar{e}_{1R} & \bar{e}_{2L} & \bar{e}_{2R} \end{pmatrix} \begin{pmatrix} 0 & R^{-1} & 0 & y_e \frac{v_2}{\sqrt{2}} \\ R^{-1} & 0 & y_e \frac{v_2}{\sqrt{2}} & 0 \\ 0 & y_e \frac{v_2}{\sqrt{2}} & 0 & -R^{-1} \\ y_e \frac{v_2}{\sqrt{2}} & 0 & -R^{-1} & 0 \end{pmatrix} \begin{pmatrix} e_{1L} \\ e_{1R} \\ e_{2L} \\ e_{2R} \end{pmatrix}$$
$$= \begin{pmatrix} \bar{e}_1 & \bar{e}_2 \end{pmatrix} \begin{pmatrix} R^{-1} & y_e \frac{v_2}{\sqrt{2}} \\ y_e \frac{v_2}{\sqrt{2}} & -R^{-1} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}. \tag{5.21}$$

Generalizing this, we see that the (mn) modes have masses

$$m_{f(mn)} = \left(\frac{N^2}{R^2} \pm m^2_{f(00)} \right)^{1/2}, \tag{5.22}$$

where $N^2 = m^2 + n^2$ and $m^2_{f(00)} = y_f^2 \frac{v_w^2}{2}$ is the zero-mode mass of the fermion.

Possible Dark Matter Candidates

In order to see the dark matter candidates in our model, we look at the spectrum of the KK modes (see figure 5.1). There are two classes of KK modes of interest whose stability is guaranteed by KK parity: the ones with $(-,-)$ and $(\pm,\mp)$ $Z_2 \times Z_2'$ quantum numbers. The former have mass $\sqrt{2}R^{-1}$ and the latter R^{-1}. We see from figure 5.1 that the first class of particles are the first KK mode of the hypercharge gauge boson B_Y and the second are the right handed neutrinos $\nu_{\mathrm{2L,\,2R}}$. The presence of the RH neutrino dark matter makes the model predictive and testable as we will see quantitatively in what follows. The basic idea is that ν_{R} annihilation proceeds primarily via the exchange of the Z' boson. So as the Z' boson mass gets larger, the annihilation rate goes down very fast (like $M_{Z'}^{-4}$) and the ν_2's overclose the Universe.

Also since there are lower limits on the Z' mass from collider searches [62], the ν_2's contribute a minimum amount to the $\Omega_{\rm DM}$. This leads to a two-component picture of dark matter and also adds to direct scattering cross section making the dark matter detectable. Below we make these comments more quantitative and present our detailed results.

5.1.2 Dark Matter Candidate I: KK Modes of the Sterile Neutrino

Annihilation channels of $\nu_{\rm 2L,2R}$

Since the yukawa couplings are small, except for the top-quark coupling, we only consider annihilations through gauge-mediated processes. For completeness, we first list the couplings between matter fields and the neutral vector gauge bosons. For matter fields charged under $SU(2)_1$, we have

$$\begin{aligned}\mathcal{L}^{SU(2)_1}_{\overline{f}fB} = (\overline{q}\gamma^\mu P_L q)\Bigg[&\left(T^3_L + \frac{Y_{BL}}{2}\right)\left(\frac{g_{\rm L}g_{\rm R}g_{\rm BL}}{\sqrt{g^2_{\rm L}g^2_{\rm R}+g^2_{\rm BL}g^2_{\rm L}+g^2_{\rm BL}g^2_{\rm R}}}\right)A_\mu \\ &+\left(\frac{T^3_L g^2_{\rm L}(g^2_{\rm R}+g^2_{\rm BL}) - \frac{Y_{BL}}{2}g^2_{\rm R}g^2_{\rm BL}}{\sqrt{g^2_{\rm L}g^2_{\rm R}+g^2_{\rm BL}g^2_{\rm L}+g^2_{\rm BL}g^2_{\rm R}}\sqrt{g^2_{\rm BL}+g^2_{\rm R}}}\right)Z_\mu \\ &+\left(\frac{Y_{BL}}{2}\frac{g^2_{\rm BL}}{\sqrt{g^2_{\rm BL}+g^2_{\rm R}}}\right)Z'_\mu\Bigg]\,. \end{aligned} \tag{5.23}$$

And for matter fields charged under $SU(2)_2$,

$$\begin{aligned}\mathcal{L}^{SU(2)_2}_{\overline{f}fB} = (\overline{q}\gamma^\mu P_{\rm R} q)\Bigg[&\left(T^3_R + \frac{Y_{BL}}{2}\right)\left(\frac{g_{\rm L}g_{\rm R}g_{\rm BL}}{\sqrt{g^2_{\rm L}g^2_{\rm R}+g^2_{\rm BL}g^2_{\rm L}+g^2_{\rm BL}g^2_{\rm R}}}\right)A_\mu \\ &+\left(-T^3_R - \frac{Y_{BL}}{2}\right)\left(\frac{g^2_{\rm R}g^2_{\rm BL}}{\sqrt{g^2_{\rm L}g^2_{\rm R}+g^2_{\rm BL}g^2_{\rm L}+g^2_{\rm BL}g^2_{\rm R}}\sqrt{g^2_{\rm BL}+g^2_{\rm R}}}\right)Z_\mu \\ &+\left(\frac{-T^3_R g^2_{\rm R} + \frac{Y_{BL}}{2}g^2_{\rm BL}}{\sqrt{g^2_{\rm BL}+g^2_{\rm R}}}\right)Z'_\mu\Bigg]\,, \end{aligned} \tag{5.24}$$

where $T^3_{\rm L} = \pm\frac{1}{2}$ and $T^3_{\rm R} = \pm\frac{1}{2}$ are the quantum number for the $SU(2)_1$ and $SU(2)_2$ groups respectively. We choose this notation because $SU(2)_1$ is to be identified with $SU(2)_L$ of the Standard Model, even though there are right-handed particles that are charged under the $SU(2)_1$ group. Also, $Y_{\rm BL} = +1/3$ for quarks and $Y_{\rm BL} = -1$ for leptons.

Using these formulas, the gauge interaction of the dark matter candidates $\nu_{2L,2R}$ is given by the six-dimensional Lagrangian

$$\mathcal{L}_\nu = -\frac{1}{2}(\overline{\nu}\gamma^\mu\nu)\frac{g^2_{\rm R} + g^2_{\rm BL}}{\sqrt{g^2_{\rm BL} + g^2_{\rm R}}}Z'_\mu. \tag{5.25}$$

We first notice that $\nu_{\rm 2L,2R}$ couple as a Lorentz vector. Second, we see that $\nu_{\rm 2L,2R}$ do not couple to A_μ nor Z_μ as expected because $\nu_{\rm 2L,2R}$ are singlets under the SM gauge group. There is a small coupling between $\nu_{\rm 2L,2R}$ and Z_μ due to $Z'_\mu - Z_\mu$ mixing. For the purpose of evaluating annihilation cross sections, we can safely ignore this mixing, as we will show. However, this mixing will be important when we consider the direct detection of $\nu_{\rm 2L,2R}$. In addition, we have the charged-current interaction, similar to the SM case

$$\mathcal{L}_{\rm CC} = \frac{g_{\rm R}}{\sqrt{2}}\left(\overline{\nu}_2\gamma^\mu W^+_{2,\mu}P_{\rm R}e_2 + \overline{e}_2\gamma^\mu W^-_{2,\mu}P_{\rm R}\nu_2\right). \tag{5.26}$$

Even though e_2 is a Dirac spinor, its left-handed component has $Z_2 \times Z'_2$ charge of $e_{2L}(--)$, and the annihilation is kinematically forbidden.

Although we have two independent Dirac fermions for dark matter, $\nu_2^{(10)}$ and $\nu_2^{(01)}$, they couple the same way to Z'_μ and have the same annihilation channels. The only difference is that, for charged current processes, $\nu^{(01)}$ ($\nu^{(10)}$) couples to $W^{\pm,(01)}_{2,\mu}$ ($W^{\pm,(10)}_{2,\mu}$). The dominant contribution to the total annihilation cross section of $\nu_{2L,R}$

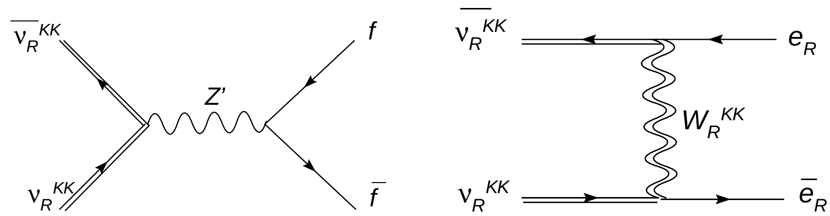

Figure 5.2: Diagrams of annihilation channels of the lightest KK sterile neutrino to the SM fermion-antifermion pairs.

is s-channel process mediated by Z'_μ, as shown in figure 5.2. The thermal-averaged cross section for $\langle\sigma(\overline{\nu}_2\nu_2 \to \overline{f}f)v_{\rm rel}\rangle$, where f is any chiral SM fermion except the right-handed electron $e_{\rm R}$, is

$$\sigma(\overline{\nu}_2\nu_2 \to \overline{f}f)v_{\rm rel} = \frac{g^2_{(\overline{\nu}\nu Z'_\mu)}g^2_{(\overline{f}fZ'_\mu)}}{12\pi}\frac{s+2M_\nu^2}{(s-M_{Z'}^2)^2}, \tag{5.27}$$

and with $s = 4M_\nu^2 + M_\nu^2 v_{\rm rel}^2$, we expand in $v_{\rm rel}^2$,

$$\sigma(\overline{\nu}_2\nu_2 \to \overline{f}f)v_{\rm rel} = \frac{g^2_{(\overline{\nu}\nu Z'_\mu)}g^2_{(\overline{f}fZ'_\mu)}}{2\pi}\frac{M_\nu^2}{(4M_\nu^2-M_{Z'}^2)^2}\left[1+v_{\rm rel}^2\left(\frac{1}{6}-\frac{2M_\nu^2}{4M_\nu^2-M_{Z'}^2}\right)\right]. \tag{5.28}$$

For the final state $\overline{e}_{\rm R}e_{\rm R}$, we have a t-channel process through charged-current in addition to the s-channel neutral-current process (see figure 5.2). The cross-section therefore involves three pieces: two due to the s and t channels and another from the interference, denoted by σ_{ss}, σ_{tt} and σ_{st} respectively. Of these, σ_{ss} has the same

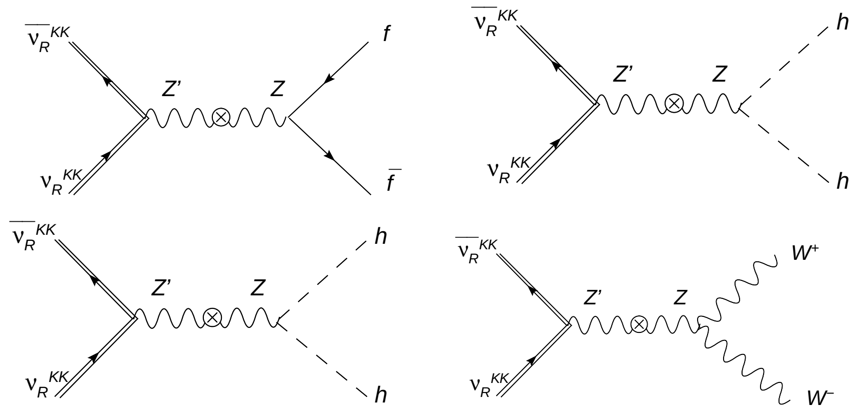

Figure 5.3: Diagrams of annihilation channels of the lightest KK sterile neutrino to the SM particles through $Z - Z'$ mixing.

form as Eq. (5.28), and we have

$$\sigma(\overline{\nu}_2\nu_2 \to \overline{e}_{\rm R}e_r)_{tt}v_{\rm rel} = \frac{g_{\rm R}^4}{32\pi}\frac{M^2}{(M^2+M_{W_{\rm R}}^2)^2}\left[1+v_{\rm rel}^2\left(\frac{3M^4+M^2M_{W_{\rm R}}^2+M_{W_{\rm R}}^4}{3(M^2+M_{W_{\rm R}}^2)^2}\right)\right],$$
$$\sigma_{st}v_{\rm rel} = \frac{g_{(\overline{\nu}\nu Z'_\mu)}g_{(\overline{e}_{\rm R}e_{\rm R}Z'_\mu)}g_{\rm R}^2}{4\pi(4M^2-M_{Z'}^2)(M^2+M_{W_{\rm R}}^2)}\Big[M^2 - v_{\rm rel}^2\times$$
$$\frac{M^2(40M^4+M_{W_{\rm R}}^2M_{Z'}^2+8M^2M_{W_{\rm R}}^2-7M^2M_{Z'}^2)}{12(4M^2-M_{Z'}^2)(M^2+M_{W_{\rm R}}^2)}\Big] \tag{5.29}$$

Due to $Z - Z'$, there can also be annihilation of KK neutrino into the SM Higgs, charged bosons, as well as fermion-antifermion pairs. The diagrams for these processes are shown in figure 5.3. In the limit that $v_w \ll v_{\rm R}$, we can work to the leading-order in the expansion of $\mathcal{O}(v_w^2/v_{\rm R}^2)$, where we can estimate these processes by treating the $Z - Z'$ mixing as a mass-insertion. In terms of Feynman diagrams, these annihilation channels are s-channel processes, where a pair KK neutrino annihilates into a Z'-boson, which propagates to the mixing vertex, converting Z' to

Z, which then decays into h^*h (both neutral and charged), massless W^+W^- or $\overline{f}f$. Compared to the amplitude of annihilation of KK neutrino into the SM fermions without $Z-Z'$ mixing, the annihilation through mixing have effectively a replaced propagator

$$\frac{1}{(s-M_{Z'}^2)} \to \frac{1}{(s-M_{Z'}^2)}\delta M^2 \frac{1}{(s-M_Z^2)} \tag{5.30}$$

where

$$\delta M^2 \equiv \frac{g_{\rm R}^2}{\sqrt{(g_{\rm L}^2+g_{\rm Y}^2)(g_{\rm R}^2+g_{\rm BL}^2)}} M_Z^2, \tag{5.31}$$

is the off-diagonal element in the $Z-Z'$ (mass)^2 matrix. Since $s \sim 4M_\nu^2 = 4R^{-2}$, the annihilation cross section into transverse gauge bosons and the Higgs bosons are suppressed by a factor of $M_Z^4/s^2 \sim (100GeV)^4/16(500GeV)^4 \sim 10^{-4}$, and can therefore be neglected. The same is true for the annihilation to fermion-antifermion pairs of the SM; we can ignore the effects of $Z-Z'$ mixing in these channels. As for the longitudinal modes, the ratio of annihilation cross-sections of the longitudinal modes of the gauge bosons to the one single mode of the SM fermion-antifermion pair is roughly

$$\frac{\sigma(\nu^{\rm KK}\nu^{\rm KK} \to W^+W^-)}{\sigma(\nu^{\rm KK}\nu^{\rm KK} \to \overline{f}f)} \sim \left(\frac{\delta M^2}{m_W^2}\right)^2. \tag{5.32}$$

This ratio is about $\frac{1}{2}$ for $g_{\rm R} = 0.7 g_{\rm L}$. As there is only one annihilation mode into the longitudinal modes of the charged gauge bosons, whereas there are many annihilation channels to the SM fermion-antifermion pairs, the total annihilation cross section is dominated by the SM fermion-antifermion contributions.

Co-annihilation Contributions to the Relic Density of $\nu_{2L,2R}$

In the MUED model, the KK mode of the left-handed electron, e_L^{KK}, is expected to be nearly degenerate with the KK mode of the left-handed neutrino. The self- and co-annihilation contribution of e_L^{KK} has been studied in the literature [38] [43], where it is shown that including such effects do not significantly alter the qualitative results, and that ν_L^{KK} with a slightly different mass can still account for the observed relic density. (However, ν_L^{KK} is ruled out by the direct detection experiments. This will be discussed in detail in Section 5.1.5.)

For our current model, the story is different. As can be seen in Eq. (5.5), $e_{2L,2R}$, the partners of $\nu_{2L,2R}$ under $SU(2)_2$, carry different quantum numbers under the $Z_2 \times Z_2'$ orbifold, and thus do not have (10) nor (01) modes. There are states that are nearly degenerate with $\nu_{2L,2R}$, such as the e' states. However, these states interact with $\nu_{2L,2R}$ only through Yukawa interactions, which can be ignored. Therefore, we expect effects of self- and co-annihilation with $\nu_{2L,2R}$ nearby states to be even smaller than the MUED case, and ignore all such effects in our analysis.

Important Differences in Comparison to Standard Analysis

We note here that our analysis of the annihilation channels for $\nu_{2L,2R}^{KK}$ differ from those of [38] and [39] for ν_L^{KK} in two important ways. First, in their analysis, the s-channel process is mediated by Z-boson of the SM, whose mass can be ignored, whereas we have s-channel processes mediated by Z', whose mass is significantly larger than the mass of our dark matter candidate in the region of interest. Second, to a good approximation we can discard t,u-channel processes mediated by charged gauge bosons $W_2^{\pm}$, because $m^2_{W_2^{\pm}}$ has contributions both from R^{-1} and v_{R}. To see this, let us make the approximation $m^2_{W^{\pm}} = m^2_{Z'} + R^{-2}$, then we compare the cross section involving the product of a t or u diagram with a s-channel diagram σ_{st} with that coming from the square of an s-channel diagram σ_{ss},

$$\frac{\sigma_{ss}}{\sigma_{st}} \approx \frac{m_\nu^2 + m^2_{W^{\pm}}}{4m_\nu^2 - m^2_{Z'}} = \frac{2(R^{-1})^2 + m^2_{Z'}}{4(R^{-1})^2 - m^2_{Z'}}. \tag{5.33}$$

Then $\sigma_{ss} \gg \sigma_{st}$ would require that

$$\frac{2(R^{-1})^2 + m^2_{Z'}}{4(R^{-1})^2 - m^2_{Z'}} \gg 1 \quad \rightarrow \quad m^2_{Z'} \gg 2(R^{-1})^2, \tag{5.34}$$

which is satisfied in the region of interest in the parameter space. Similarly, the cross section involving two $t-$ or u-channel diagrams, σ_{tt}, σ_{uu} or σ_{tu} is small compared to σ_{ss}.

5.1.3 Dark Matter Candidate II: KK Modes of the Scalar and Vector Gauge Boson

The lightest (11) mode is either $B^{(11)}_{(\text{-})}$ or $B^{(11)}_\mu$, depending on radiative corrections. Although in Reference [61] found that $B^{(11)}_\mu$ is heavier than $B^{(11)}_{(\text{-})}$, this result is specific the choice of orbifold in that particular case, and may not apply to $Z_2 \times Z_2'$ orbifold that we have here. Instead of performing the radiative corrections to determine which of the two particles is lighter, we will do a phenomenological study exploring both of these cases. To simplify the notation, we will often discard the (11) superscript in the fields.

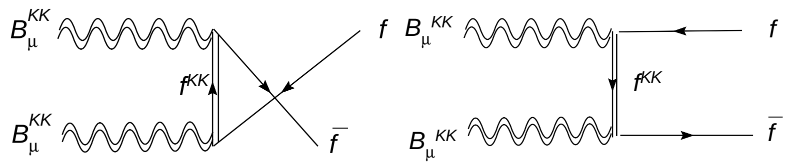

Figure 5.4: Annihilation channels of a pair of KK vector photon into SM fermion-antifermion pair.

(Co)-Annihilation Channels of $B_\mu^{(11)}$

When $v_w \ll R^{-1}$, $B_\mu^{(11)}$ is same as $A_{Y,\mu}^{(11)}$, the KK mode of the photon up to small mixing effects. The annihilation channels and cross sections of $B_\mu^{(11)}$ have been studied in detail in [38] and [43], and in this subsection we summarize their results.

$B_\mu^{(11)}$ can annihilate itself into a fermion-antifermion pair through t- and u-channel processes mediated by the (11) mode of the fermion (figure 5.4). It is important to note that the left- and right-handed fermions of the SM have separate massive KK modes with vector-like couplings to the zero-mode fermion and $B_\mu^{(11)}$. The annihilation cross section can be written as

$$\sigma(B_\mu^{(11)} B_\mu^{(11)} \to \overline{f} f) = g_1^4 (Y_L^2 + Y_{\mathrm{R}}^4) N_c \frac{10(2M_f^2 + s)\mathrm{ArcTanh}(\beta) - 7s\beta}{72\pi s^2 \beta^2}, \tag{5.35}$$

where $M_f = \sqrt{2}R^{-1}$ is the mass of the KK-fermion exchanged, N_c is the color factor in the final state (3 for quarks and 1 for leptons), and $Y_{L,R}$ is the hypercharge of the left- and right-handed fermion. Summing over all the SM fermions gives

$$\sum_{f\in\mathrm{SM}} N_c (Y_L^4 + Y_{\mathrm{R}}^4) = 3(Y_{e_L}^4 + Y_{e_{\mathrm{R}}}^4 + Y_{\nu_L}^4 + 3(Y_{u_L}^4 + Y_{u_{\mathrm{R}}}^4 + Y_{d_L}^4 + Y_{d_{\mathrm{R}}}^4)) = \frac{95}{18}. \tag{5.36}$$

There are also annihilation channels to Higgs through t- and u-channel processes mediated by a (11) mode of the Higgs boson as well as a quartic interaction. The annihilation cross section is given by

$$\sigma(B_\mu^{(11)} B_\mu^{(11)} \to h^* h) = \frac{g_1^4 Y_\phi^4}{6\pi\beta s}, \tag{5.37}$$

where $Y_\phi = 1/2$ is the hypercharge of the Higgs doublet. By summing over two complex Higgs doublets, we have taken into account the annihilation into the longitudinal zero modes of the W and Z gauge bosons.

In the MUED, the KK mode with mass closest to $B_\mu^{(1)}$ is the KK mode of the right-handed electron $e_{\mathrm{R}}^{(1)}$ when radiative corrections are included [59]. However, compared to the case without co-annihilation, the qualitative results of the relic density due to $B_\mu^{(1)}$ remains the same when one includes the co-annihilation $e_{\mathrm{R}}^{(1)} B_\mu^{(1)} \to e_R A_\mu$ [38]. As pointed out by [38], this is because there are only two channel of such co-annihilation, leading to a small co-annihilation cross section, and thus small change in the relic density for a fixed R^{-1}.

In our case, we expect $B_{(\text{-})}^{(11)}$ (which has no MUED analog) to be close in mass to $B_\mu^{(11)}$ in addition to $e_{1R}^{(11)}$ (the analog of $e_{\mathrm{R}}^{(1)}$ in MUED). Furthermore, the co-annihilation $B_\mu^{(11)} B_{(\text{-})}^{(11)} \to XX$ is significant as $B_\mu^{(11)} B_{(\text{-})}^{(11)}$ can annihilate to all the SM fermions through t- and u-channel processes mediated by a KK fermion. The co-annihilation cross section to fermion-antifermion pair is

$$\sum_{f \in \mathrm{SM}} \sigma(B_\mu^{(11)} B_{(\text{-})}^{(11)} \to \overline{f} f) = g_1^4 \frac{95}{18} \frac{\mathrm{ArcTan}(\beta)}{12\pi s \beta^2}. \tag{5.38}$$

Although the co-annihilation effect was overlooked in [42], the most important conclusions of our previous work remain the same, as we will show later.

(Co)-Annihilation Channels of $B^{(11)}_{(\text{-})}$

The coupling of $B^{(11)}_{(\text{-})}$ to matter fields in the full 6D-Lagrangian is given by (in four-component notation)

$$\begin{aligned}\mathcal{L}^{\text{6D}} &= \frac{g_1 Y}{\sqrt{2}} B_{\text{Y,-}} \left[\overline{\Psi}_-(i\gamma_5 - \mathbf{1})\Psi_- + \overline{\Psi}_+(i\gamma_5 + \mathbf{1})\Psi_+\right] \\ &= \frac{g_1 Y}{\sqrt{2}} B_{Y,-} \left[(-i-1)\overline{\Psi}_{\text{-L}}\Psi_{\text{-R}} + (i-1)\overline{\Psi}_{\text{-R}}\Psi_{\text{-L}} + (-i+1)\overline{\Psi}_{\text{+L}}\Psi_{\text{+R}} + (i+1)\overline{\Psi}_{\text{+R}}\Psi_{\text{+L}}\right].\end{aligned} \tag{5.39}$$

In terms of KK-modes, $B^{(11)}_{(\text{-})}$ will couple to fermion fields in (00-fermion)(11-fermion) pairs, and its annihilation channels to fermions will proceed through $t-$ and $u-$processes mediated by a KK-fermion. The annihilation cross section is

$$\sum_{f\in\text{SM}} \sigma(B^{(11)}_{(\text{-})} B^{(11)}_{(\text{-})} \to \overline{f} f) = g_1^4 \frac{95}{18} \frac{2(2M_f^2 + s)\text{ArcTan}(\beta) - 3s\beta}{2\pi s \beta^2}. \tag{5.40}$$

In the non-relativistic limit, this cross-section is p-wave suppressed. There is also annihilation to a pair of Higgs bosons through the quartic coupling

$$\mathcal{L}^{\text{4D}} = g_1^2 Y_H^2 B^{(11)}_{(\text{-})} B^{(11)}_{(\text{-})} H^{\dagger(00)} H^{(00)}, \tag{5.41}$$

and this gives a cross section of

$$\langle \sigma v_{\text{rel}} \rangle = \frac{g_1^4 Y_H^4}{2\pi s}. \tag{5.42}$$

Because the annihilation of $B^{(11)}_{(\text{-})}$ to fermion modes is p-wave suppressed, the relic density resulting $B^{(11)}_{(\text{-})}$ self-annihilation channels would in general be too high. Therefore, we must rely on co-annihilation channels such as $B^{(11)}_{\mu} B^{(11)}_{(\text{-})} \to XX$ to obtain observed relic density, as we will see in the next section.

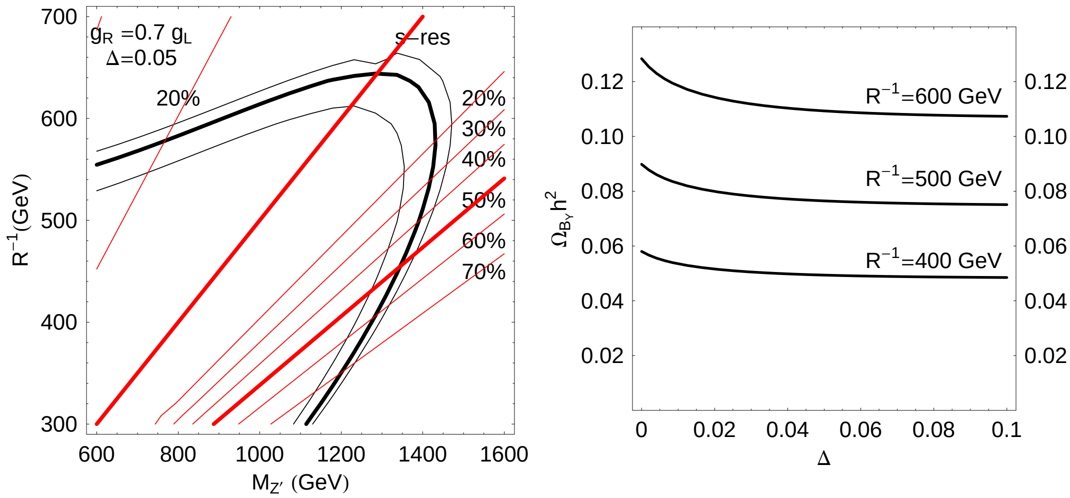

Figure 5.5: The plot on the left shows the contour in the $R^{-1} - M_{Z'}$ plane that corresponds to $\Omega_{\nu_{L,R}} h^2 + \Omega_{B_Y} h^2$ being the observed dark matter. The intersection of the red lines with the contour indicate the fraction of KK neutrinos in the dark matter. The plot on the right shows $\Omega_{B_Y} h^2$ as a function of Δ for various values of R^{-1}.

5.1.4 Numerical Results of Relic Density

The main free parameters of our theory are R^{-1} and $M_{Z'}$, and the mass-splitting $\Delta \equiv (M_{B_\mu^{(11)}} - M_{B_{(-)}^{(11)}})/M_{B_{(-)}^{(11)}}$. In addition, we have $g_{\rm R}$ or $g_{\rm BL}$ as a free parameter as long as we can satisfy the constraint

$$g_1^2 = \frac{g_{\rm BL}^2 g_{\rm R}^2}{g_{\rm BL}^2 + g_{\rm R}^2}.$$

$B_\mu^{(11)}$-$\nu_{\rm 2L,2R}^{(01)}$ Dark Matter

For $\Delta = 0.05$, we present the allowed region in $R^{-1} - M_{Z'}$ space that gives the observed dark matter relic density in the first plot of figure 5.5. Since both $B_\mu^{(11)}$ and

$\nu^{(01)}_{2\mathrm{L},2\mathrm{R}}$ can independently give the correct relic density without co-annihilation from other modes with almost degenerate mass, varying Δ does not affect the qualitative results of what we present below. The independence of $\Omega_{B^{(11)}_\mu} h^2$ on Δ as can be seen in the second plot of Figure 5.5. For small values of $M_{Z'}$, the annihilation of $\nu^{(01)}_{2\mathrm{L},2\mathrm{R}}$ is efficient and most of the dark matter is $B^{(11)}_\mu$ having a mass of roughly $\sqrt{2}R^{-1} \sim 700$ GeV. In fact, along the line $2M_{\nu^{(01)}} = 2R^{-1} = M_{Z'}$, the annihilation of $\nu^{(01)}_{2\mathrm{L},2\mathrm{R}}$ has an s-channel resonance, and its contribution to dark matter relic density is minimal. Away from the line of s-channel resonance, the contribution of $\nu^{(01)}_{2\mathrm{L},2\mathrm{R}}$ to the relic density increases, and R^{-1} decreases so as to decrease the relic density due to $B^{(11)}_\mu$, keeping the total relic density within the allowed range.

The current experimental bound on the massive, neutral, vector boson is $M_{Z'} > 800$ GeV. If we further impose the bound that $R^{-1} > 400$ GeV, the allowed region in the parameter space is very limited.

$B^{(11)}_{(-)}$-$\nu^{(01)}_{2\mathrm{L},2\mathrm{R}}$ Dark Matter

As stated earlier, $B^{(11)}_{(-)}$ by itself can not annihilate efficiently enough to account for the observed relic abundance. However, there is significant co-annihilation process $B^{(11)}_{(-)} B^{(11)}_\mu \to \overline{f} f$. In Figure 5.6, we show contours that give the observed relic density for various values of Δ. We see that when $B^{(11)}_{(-)}$ and $B^{(11)}_\mu$ are nearly degenerate to less than 5%, then the distribution of dark matter among $\nu^{(01)}_{2\mathrm{L},2\mathrm{R}}$ and $B^{(11)}_{(-)}$ is similar to the previous case. When the mass splitting between $B^{(11)}_{(-)}$ and $B^{(11)}_\mu$ is larger than 5%, however, the model is ruled out as we can not obtain the

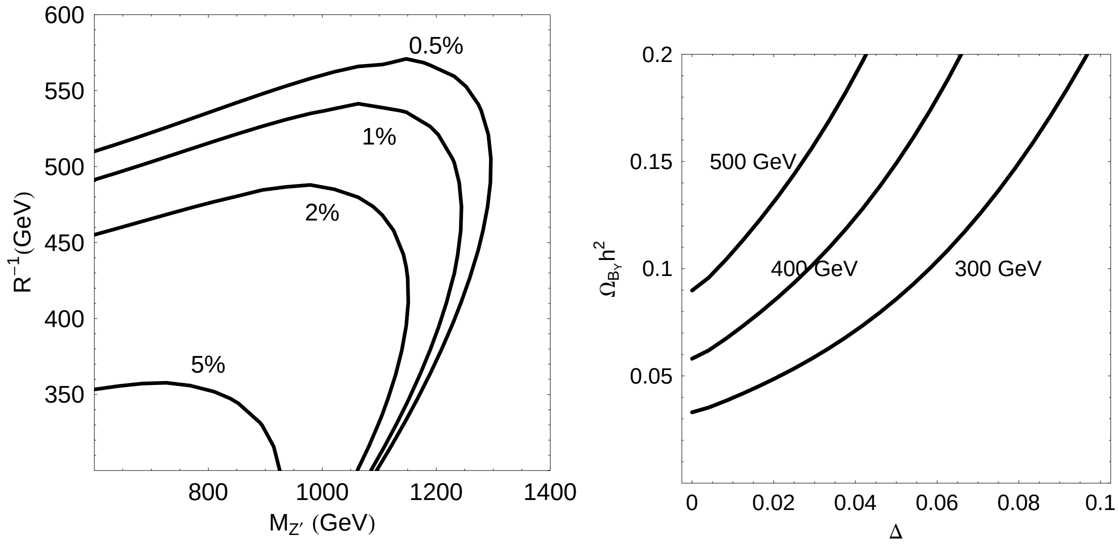

Figure 5.6: The plot on the left shows the allowed region in the parameter space that gives rise to the observed dark matter relic density for $g_R = 0.7g_L$ and different values of Δ. On the right, we plot the relic density due to KK scalar photon as a function of the mass-splitting Δ for various values of R^{-1}.

observed relic density without violating $R^{-1} > 400$ GeV bound. When $B^{(11)}_{(\text{-})}$ and $B^{(11)}_{\mu}$ are nearly degenerate, $\nu^{(\text{KK})}_{\text{2L,2R}}$ can still contribute significantly to the observed relic density when $M_{Z'}$ is about 1.2 TeV and $R^{-1} \sim 400$ GeV.

5.1.5 Direct Detection of Two-Component Dark Matter

As we have a two-component dark matter, the total dark matter-nucleon cross section is given by

$$\sigma_n = \kappa_{\nu_\text{R}} \sigma_{\nu_\text{R}} + \kappa_B \sigma_B, \tag{5.43}$$

where $\sigma_{\nu_\text{R}(B)}$ is the spin-independent KK neutrino (hypercharge vector or pseudoscalar)-nucleon scattering cross section, and

$$\kappa_{\nu_\text{R}} \equiv \frac{\Omega_{\nu_\text{R}} h^2}{\Omega_{\nu_\text{R}} h^2 + \Omega_B h^2}, \tag{5.44}$$

is the fractional contribution of the KK neutrino relic density to the total relic density of the dark matter. κ_B is similarly defined. As pointed out in Ref. [38], σ_B is of the order $\sigma_B \sim 10^{-10}$ pb, and we will find that $\sigma_{\nu_\text{R}} \gg \sigma_B$. Therefore, it is a good approximation to take σ_n as

$$\sigma_n \approx \kappa_{\nu_\text{R}} \sigma_{\nu_\text{R}}. \tag{5.45}$$

The elastic cross section between $\nu_{\text{2L,2R}}$ and a nucleon inside a nucleus $N(A, Z)$ is given by

$$\sigma_n = \frac{b_N^2 m_n^2}{\pi A^2}, \tag{5.46}$$

where $b_N = Zb_p + (A - Z)b_n$ and $b_{p,n}$ is the effective four-fermion coupling between $\nu_{\text{2L,2R}}$ and a nucleon. They are given by $b_p = 2b_u + b_d$ and $b_n = b_u + 2b_d$. In our case, although $\nu_{2\mathrm{L},2\mathrm{R}}$ only couples to Z'_μ at leading order, we have to taken into account the $Z-Z'$ mixing. We can including the effects of mixing up to order of $\mathcal{O}(M_Z^2/M_{Z'}^2)$ by treating the mixing as perturbations and include one vertex mixing. In this case, we have

$$b_q = \frac{1}{2M_{Z'}^2} g_{(\overline{\nu_2}\nu_2 Z')} \left[(g_{(\overline{q}_L q_L Z')} + g_{(\overline{q}_\mathrm{R} q_\mathrm{R} Z')}) - (g_{(\overline{q}_L q_L Z)} + g_{(\overline{q}_\mathrm{R} q_\mathrm{R} Z)}) \frac{\delta M^2}{M_Z^2} + \mathcal{O}\left(\frac{M_Z^2}{M_{Z'}^2}\right) \right], \tag{5.47}$$

where

$$\delta M^2 \equiv \frac{g_\mathrm{R}^2}{\sqrt{(g_\mathrm{L}^2 + g_\mathrm{Y}^2)(g_\mathrm{R}^2 + g_\mathrm{BL}^2)}} M_Z^2 \tag{5.48}$$

is the mixing between Z and Z' (see Eq. (5.16)).

The prospects of direct detection of $B_\mu^{(11)}$ has been studied extensively, and the calculated detection rates are beyond the reach of current experiments. As for $B_{(\text{-})}^{(11)}$, because there is no s-wave for elastic scattering $B_{(\text{-})}^{(11)} N \to B_{(\text{-})}^{(11)} N$, the cross-section is suppressed by a factor of $v_\mathrm{rel}^2 \sim 10^{-5}$. Therefore, we expect that the direct detection rates of $\nu_{2\mathrm{L},2\mathrm{R}}^{(10)}$ will dominate that of both $B_\mu^{(11)}$ and $B_{(\text{-})}^{(11)}$. This is one of the main points of our work: the lightest KK-mode of sterile neutrino as dark matter candidate could be detected directly in the current and the next rounds of direct-detection experiments if its relic density is significant compared to the observed total relic density, in contrast to other dark matter candidates in the literature, such as the neutralino of MSSM or the lightest KK-mode of the photon.

In Figure 5.7, we show the direct detection cross section as a function of $M_{Z'}$ for both cases where $B_\mu^{(11)}$ and $B_{(\text{-})}^{(11)}$ is the lighter of the two. The horizontal lines correspond to the upper bounds on σ_n from CDMS II for dark matter candidates

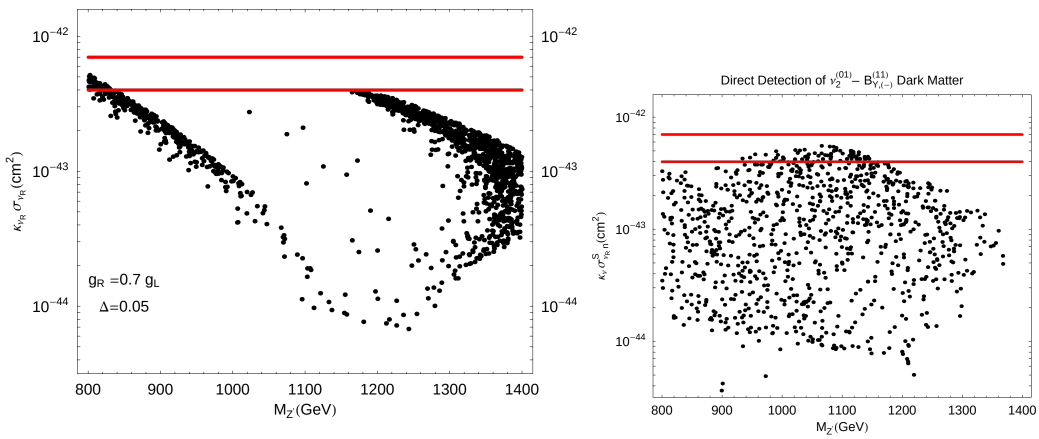

Figure 5.7: The plot on the left (right) shows the dark matter-nucleon cross-section as a function of $M_{Z'}$ for the case where KK vector (scalar) photon is lightest (11) mode. The plots scan over different values of R^{-1} and $-0.05 < \Delta < 0$ that gives the observed relic density. The horizontal lines correspond to the upper bounds on σ_n from CDMS II for dark matter candidates with masses 300 and 500 GeV

with masses 300 and 500 GeV, which are about $4\times10^{-43}\text{cm}^2$ and $7\times10^{-43}\text{cm}^2$, respectively.

A particularly interesting region in the parameter space is $R^{-1} \sim 400$ GeV and $M_{Z'} \sim 1200$ GeV. Here, the KK-sterile neutrino contributes to roughly half of the relic density. This admixture of dark matter is just below the current experimental bound from direct detection, as shown in Figure 5.5, when we use the CDMS II bound that dark matter-nucleon spin-independent cross-section must not exceed $4 \times 10^{-43}\text{cm}^2$ for a 400 GeV dark matter.

5.1.6 Some phenomenological implications

In this section, we give a qualitative comparison of the phenomenological implications of this model with those of the conventional left-right symmetric models [63]. It has long been recognized that two important characteristic predictions of the left-right models are the presence of TeV scale W_R and Z' gauge bosons which can be detectable in high energy colliders [64]. In addition to the collider signatures of generic UED models [65], two predictions characteristic of the model discussed here which differ from those of the earlier models are : (i) The mass of Z' has an upper bound of about 1.5 TeV and a more spectacular one where (ii) the W_R in this model, being a KK excitation, does not couple to a pair of the known standard model fermions which are zero modes. This property of the W_R has a major phenomenological impact and will require a completely new analysis of constraints on this e.g. the well known $K_L - K_S$ mass difference constraint on M_{W_R} [66] does not apply here since the mixed $W_L - W_R$ exchange box graph responsible for the new contribution to $K_L - K_S$ mass difference does not exist. The box graph where both exchange particles are W_R's exists but its contribution to the $\Delta S = 2$ Hamiltonian is suppressed compared to the left-handed one by a factor $\left(\frac{M_{W_L}}{M_{W_R}}\right)^4$ and gives only a very weak bound on M_{W_R}.

Also, the bounds from muon and beta decay [67] are nonexistent for the same reason because there is no tree level W_R contribution to these processes. Furthermore, in this model, there is no $W_L - W_R$ mixing unlike the conventional left-right models.

Because of this property, the decay modes and production mechanism of the W_R are also very different from the case of the conventional left-right model, while the decay modes and production mechanism of the Z' remain the same. We do not discuss the Z' case which has been very widely discussed in literature.

The W_R will have a mass given by the formula $M_{W_R}^2 \sim cos^2\theta_R M_{Z'}^2 + R^{-2}$. Furthermore, it can only be pair produced and will decay to $u'_{\rm 2L,R}\bar{d'}_{\rm 2L,R}$, $u_{\rm 2L,R}\bar{d}_{\rm 2L,R}$, $\bar{e}_{\rm 2L,R}\nu_{\rm 2L,R}$, and $\bar{e'}_{\rm 2L,R}\nu'_{\rm 2L,R}$. For sub-TeV W_R, only the decay modes $\bar{e'}_{\rm 2L}\nu'_{\rm 2L}$ and $u_{\rm 2R}\bar{d}_{\rm 2R}$ will dominate depending on the precise value of W_R mass and the R^{-1}. The leptonic decay mode will look very similar to the supersymmetric case where pair-produced sleptons will decay to a lepton and the neutralino. The hadronic channel will however look different from the squark case.

5.2 Conclusions

In summary, we have studied the profile of cold dark matter candidates in a Universal Extra Dimension model with a low-scale extra W_{R} and Z'. There are two possible candidates: $\nu_{\mathrm{R}}^{\mathrm{KK}}$ and either the $B_{(\text{-})}^{(11)}$ or $B_{\mu}^{(11)}$ depending on which one receives less radiative corrections[1]. We have done detailed calculation of the relic density of these particles as a function of the parameters of the model which are g_{R}, R^{-1} and $M_{Z'}$. We find upper limits on these parameters where the above KK modes can be cold dark matter of the universe. In discussing the relic abundance, we have considered the co-annihilation effect of nearby states. We also calculate the direct detection cross-section in current underground detectors for the entire allowed parameter range in the model and we find that, for the case where KK neutrino contributes significantly to the total relic density, the lowest possible value of the cross-section predicted by our model is accessible to the current and/or planned direct search experiments. Therefore, the most interesting region of our model can not only be tested in the colliders but also these dark matter experiments. Combined with LHC search for the Z' of left-right model, dark matter experiments could rule out this model.

[1] During the completion of this Thesis, the radiative corrections to the spectrum of our model has been analyzed in [68]

Chapter 6

CONCLUSIONS AND OUTLOOK

In this Thesis, we have built two models based on SUSY aimed to alleviate the little hierarchy problem. The first model is a framework that generalizes both GMSB and AMSB. While its resulting spectra are qualitatively similar to the GMSB spectra, it can potentially lead to a compact spectrum. The second model views the little hierarchy problem in a different light, and makes the gauginos Dirac to change the RG evolution of the sfermion masses. The resulting spectra are not only inverted, they also contain Dirac gluino and a singlet-bino mixture dark matter. Because of the inverted spectra and Dirac gluino, we expect to see many less lepton signals from the LHC.

We have also studied the dark matter properties of a UED model that solves both the problems of proton decay and neutrino mass – both present in generic UED models. We find that the parameter space is strongly constrained by the observed dark matter relic density and bounds from direct detection. We also find a theoretical lower bound of the direct detection rate as well as a mass range for an extra Z' boson. Both of these tests can be used to confirm or rule out the UED model.

However, much work remains to be done and many issues need to be investigated. While the GAMSB model in Chapter 3 gives a similar spectrum to several other GMSB-motivated models, our DGMSB model in Chapter 4 offers very different collider signatures from the convention ones. A concrete study of the collider signatures of DGMSB will require mapping out the viable parameter space, particularly in the Higgs and light neutralino sectors. Because of the Dirac nature of gauginos, many of the existing tools of the high-energy physics community will have to be substantially modified. The UED model of Chapter 6 requires a study of the radiative corrections to find out which of $B^{(11)}_{(\text{-})}$ and $B^{(11)}_{\mu}$ is lighter, and thus be a component of the dark matter. Although we cover both cases in this Thesis, the identification of the lightest state would be useful for further studies.

We live in an exciting time. All the models in the Thesis, and many more specific realizations of BSM physics, will confront experimental results from the LHC in a few years. We also live in a time that requires a joint effort from both theorists and experimentalists to interpret and analyze the results from the LHC. Whatever the LHC results are, it will bring new life to the field and we hope to soon uncover the Nature's secrets beyond the SM.

Chapter A

RENORMALIZATION GROUP EQUATIONS

A.1 Overview

Renormalization is the re-parameterization of the parameters of the quantum field theory such that the calculated physical quantities are finite order-by-order in perturbation theory.

In this appendix, we present the RGEs in a bottom-up order, starting with the effective theory of QED and QCD, proceed onto the SM, and then MSSM.

A.2 Effective Theory below Mass of Z-boson

Below the mass scale of Z-boson, an effective theory can be obtained from the SM by integrating out the massive gauge bosons $W^{\pm}, Z$, and the top quark. This effective theory is described by the gauge symmetry $\mathrm{SU}(3)_C \times \mathrm{U}(1)_{\mathrm{em}}$ with the gauge couplings g_3 and e, respectively, and contains the weak interactions through the effective four-fermion interactions.

$$
\begin{aligned}
\frac{d}{d\ln\mu}g_3 &= \left(\frac{2}{3}(n_u+n_d)-11\right)\frac{g_3^3}{16\pi^2}+\left(\frac{38}{3}(n_u+n_d)-102\right)\frac{g_3^5}{(16\pi^2)^2} \\
&+\left(\frac{8}{9}n_u+\frac{2}{9}n_d\right)\frac{g_3^3e^2}{(16\pi^2)^2} \\
&+\left(\frac{5033}{18}(n_u+n_d)-\frac{325}{54}(n_u+n_d)^2-\frac{2857}{2}\right)\frac{g_3^7}{(16\pi^2)^3}, \qquad \text{(A.1)}\\
\frac{d}{d\ln\mu}e &= \left(\frac{16}{9}n_u+\frac{4}{9}n_d+\frac{4}{3}n_l\right)\frac{e^3}{16\pi^2}+\left(\frac{64}{27}n_u+\frac{4}{27}n_d+4n_l\right)\frac{e^5}{(16\pi^2)^2} \\
&+\left(\frac{64}{9}n_u+\frac{16}{9}n_d\right)\frac{e^3g_3^2}{(16\pi^2)^2}, \qquad \text{(A.2)}
\end{aligned}
$$

where n_u, n_d, and n_l are the number of up-type quarks, down-type quarks, and leptons, respectively.

A.3 The Standard Model

The RGEs of the Standard Model can be found in several places ([69]), and here we merely summarize the main results. The Yukawa couplings of the Standard Model can be defined using the convention

$$
\mathcal{L}_{\rm SM} = \overline{Q}_L\tilde{\Phi}\mathbf{Y}_u^{\rm SM,\dagger}u_R+\overline{Q}_L\Phi\mathbf{Y}_d^{\rm SM,\dagger}d_R+\overline{L}_L\Phi\mathbf{Y}_e^{\rm SM,\dagger}e_R+\text{h.c.}-m_\Phi^2\Phi^\dagger\Phi-\frac{1}{2}\lambda(\Phi^\dagger\Phi)^2, \tag{A.3}
$$

where the family (flavor) indices have been suppressed, and where Q and L are the SU(2) quark and lepton doublet, respectively,

$$
Q_L=\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad L_L=\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}. \tag{A.4}
$$

The Higgs scalar doublet and its SU(2) conjugate are denoted by Φ and $\tilde{\Phi}$, respectively

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \tilde{\Phi} = i\sigma_2 \Phi. \tag{A.5}$$

u_R, d_R, and e_R are the SU(2) singlets. The Yukawa couplings are 3×3 matrices whose structure have important consequences for flavor- and CP-violation. However, for the purpose of our study, we will make the approximation that only the Yukawa couplings of the third generation is significant,

$$\mathbf{Y}_u^{\rm SM} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \widehat{y}_t \end{pmatrix}, \mathbf{Y}_d^{\rm SM} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \widehat{y}_d \end{pmatrix}, \mathbf{Y}_e^{\rm SM} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \widehat{y}_\tau \end{pmatrix}. \tag{A.6}$$

The hat on the Yukawa couplings is used to distinguish between the Yukawa couplings of the SM and the MSSM. With this approximation, the β-function of the Yukawa couplings are

$$(16\pi^2)\frac{d}{d\ln\mu}\widehat{y}_t = \widehat{y}_t\left(\frac{9}{2}\widehat{y}_t^2 + \frac{3}{2}\widehat{y}_d^2 + \widehat{y}_\tau^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right), \tag{A.7}$$

$$(16\pi^2)\frac{d}{d\ln\mu}\widehat{y}_d = \widehat{y}_d\left(\frac{9}{2}\widehat{y}_d^2 + \frac{3}{2}\widehat{y}_t^2 + \widehat{y}_\tau^2 - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right), \tag{A.8}$$

$$(16\pi^2)\frac{d}{d\ln\mu}\widehat{y}_\tau = \widehat{y}_\tau\left(\frac{5}{2}\widehat{y}_\tau^2 + 3\widehat{y}_t^2 + 3\widehat{y}_d^2 - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right). \tag{A.9}$$

The two-loop β-function of the gauge couplings are

$$(16\pi^2)\frac{d}{d\ln\mu}g_1 = \frac{41}{6}g_1^3 + \frac{g_1^3}{16\pi^2}\left(\frac{199}{18}g_1^2 + \frac{27}{6}g_2^2 + \frac{44}{3}g_3^2 - \frac{17}{6}\widehat{y}_t^2 - \frac{5}{6}\widehat{y}_d^2 - \frac{15}{6}\widehat{y}_\tau^2\right), \tag{A.10}$$

$$(16\pi^2)\frac{d}{d\ln\mu}g_2 = \frac{19}{6}g_2^3 + \frac{g_2^3}{16\pi^2}\left(\frac{3}{2}g_1^2 + \frac{35}{6}g_2^2 + 12g_3^2 - \frac{3}{2}\widehat{y}_t^2 - \frac{3}{2}\widehat{y}_d^2 - \frac{1}{2}\widehat{y}_\tau^2\right), \tag{A.11}$$

$$(16\pi^2)\frac{d}{d\ln\mu}g_3 = -7g_3^3 + \frac{g_3^3}{16\pi^2}\left(\frac{11}{6}g_1^2 + \frac{9}{2}g_2^2 - 26g_3^2 - 2\widehat{y}_t^2 - 2\widehat{y}_d^2\right). \tag{A.12}$$

The β-function of the Higgs quartic coupling is

$$\begin{aligned}(16\pi^2)\frac{d}{d\ln\mu}\lambda &= 12\lambda^2 + \lambda\left(12\widehat{y}_t^2 + 12\widehat{y}_d^2 + 4\widehat{y}_\tau^2 - 3g_1^2 - 9g_2^2\right)\\ &\quad + \frac{9}{4}\left(\frac{1}{3}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^4\right) - 12\widehat{y}_t^4 - 12\widehat{y}_d^4 - 4\widehat{y}_\tau^4\end{aligned} \tag{A.13}$$

A.4 The MSSM

The β-functions for a general SUSY model can be found in several places [70], and here we only list the beta functions for those couplings and SUSY-breaking parameters that appear in the MSSM. Again, we take the approximation that only the 33 entry of the Yukawa matrices are nonzero.

The 1-loop β-function of the gauge couplings are,

$$\frac{d\ g_1}{d\ln\mu} = \frac{11}{16\pi^2}g_1^3, \tag{A.14}$$

$$\frac{d\ g_2}{d\ln\mu} = \frac{1}{16\pi^2}g_2^3, \tag{A.15}$$

$$\frac{d\ g_3}{d\ln\mu} = -\frac{3}{16\pi^2}g_3^3. \tag{A.16}$$

Correspondingly, the gaugino masses have the β-functions

$$\frac{d\ M_1}{d\ln\mu} = \frac{11}{8\pi^2}M_1, \tag{A.17}$$

$$\frac{d\ M_2}{d\ln\mu} = \frac{1}{8\pi^2}M_2, \tag{A.18}$$

$$\frac{d\ M_3}{d\ln\mu} = -\frac{3}{8\pi^2}M_3. \tag{A.19}$$

The 1-loop β-functions for the couplings that appear in the superpotential are

$$16\pi^2\frac{d\ y_t}{d\ln\mu} = y_t\left(6y_t^2 + y_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2\right), \tag{A.20}$$

$$16\pi^2\frac{d\ y_b}{d\ln\mu} = y_b\left(6y_b^2 + y_t^2 + y_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{9}g_1^2\right), \tag{A.21}$$

$$16\pi^2\frac{d\ y_\tau}{d\ln\mu} = y_\tau\left(4y_\tau^2 + 3y_b^2 - 3g_2^2 - 3g_1^2\right), \tag{A.22}$$

$$16\pi^2\frac{d\ \mu}{d\ln\mu} = \mu\left(3y_t^2 + 3y_b^2 + y_\tau^2 - 3g_2^2 - g_1^2\right). \tag{A.23}$$

The corresponding SUSY-breaking trilinear and B_μ terms have the β-functions

$$16\pi^2\frac{d\ a_t}{d\ln\mu} = (16\pi^2)\frac{a_t}{y_t}\beta_{y_t} + 12y_t^2a_t + 2y_ty_ba_b + y_t\left(\frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{26}{9}g_1^2M_1\right), \tag{A.24}$$

$$16\pi^2\frac{d\ a_b}{d\ln\mu} = (16\pi^2)\frac{a_b}{y_b}\beta_{y_b} + 12y_b^2a_b + 2y_ty_ba_t + 2y_by_\tau a_\tau + y_b\left(\frac{32}{3}g_3^2M_3 + 6g_2^2M_2 + \frac{14}{9}g_1^2M_1\right), \tag{A.25}$$

$$16\pi^2\frac{d\ a_\tau}{d\ln\mu} = (16\pi^2)\frac{a_\tau}{y_\tau}\beta_{y_\tau} + 8y_\tau^2a_\tau + 6y_\tau y_ba_b + y_\tau\left(6g_2^2M_2 + 6g_1^2M_1\right), \tag{A.26}$$

$$16\pi^2\frac{d\ B_\mu}{d\ln\mu} = 6y_ta_t + 6y_ba_b + 2y_\tau a_\tau + 6g_2^2M_2 + \frac{26}{9}g_1^2M_1. \tag{A.27}$$

The soft, SUSY-breaking scalar masses have the β-functions,

$$16\pi^2 \frac{d\ m_{H_u}^2}{d\ln\mu} = -6g_2^2 M_2^2 - 2g_1^2 M_1^2 + 3X_t, \tag{A.28}$$

$$16\pi^2 \frac{d\ m_{H_d}^2}{d\ln\mu} = -6g_2^2 M_2^2 - 2g_1^2 M_1^2 + 3X_b + X_\tau, \tag{A.29}$$

$$16\pi^2 \frac{d\ m_{\tilde{Q}^i}^2}{d\ln\mu} = -\frac{32}{3}g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{9}g_1^2 M_1^2 + \delta_{i3}(X_t + X_b), \tag{A.30}$$

$$16\pi^2 \frac{d\ m_{\tilde{u}^i}^2}{d\ln\mu} = -\frac{32}{3}g_3^2 M_3^2 - \frac{32}{9}g_1^2 M_1^2 + \delta_{i3}(2X_t), \tag{A.31}$$

$$16\pi^2 \frac{d\ m_{\tilde{d}^i}^2}{d\ln\mu} = -\frac{32}{3}g_3^2 M_3^2 - \frac{8}{9}g_1^2 M_1^2 + \delta_{i3}(2X_b), \tag{A.32}$$

$$16\pi^2 \frac{d\ m_{\tilde{L}^i}^2}{d\ln\mu} = -6g_2^2 M_2^2 - g_1^2 M_1^2 + \delta_{i3}(X_\tau), \tag{A.33}$$

$$16\pi^2 \frac{d\ m_{\tilde{e}^i}^2}{d\ln\mu} = -6g_2^2 M_2^2 - 8g_1^2 M_1^2 + \delta_{i3}(2X_\tau), \tag{A.34}$$

where

$$X_t = 2y_t^2(m_{H_u}^2 + m_{\tilde{Q}^3}^2 + m_{\tilde{u}^3}^2) + 2a_t^2, \tag{A.35}$$

$$X_b = 2y_b^2(m_{H_d}^2 + m_{\tilde{Q}^3}^2 + m_{\tilde{d}^3}^2) + 2a_b^2, \tag{A.36}$$

$$X_\tau = 2y_\tau^2(m_{H_d}^2 + m_{\tilde{L}^3}^2 + m_{\tilde{e}^3}^2) + 2a_\tau^2. \tag{A.37}$$

Chapter B

DETAILS OF THE MIXED GAUGE- AND ANOMALY-MEDIATED MODEL

B.1 Minimization of the Potential in Eq. (3.6)

We minimize Eq. (3.5) assuming that all couplings are real. It is useful to write the potential as

$$V = \lambda_S^2 \left\{ \left| \tfrac{1}{2} S + \frac{c\langle F_\phi \rangle}{\lambda_S} \right|^2 |S|^2 + \left(\frac{c_S \langle F_\phi \rangle}{\lambda_S} \right)^2 \left(\frac{1}{2c_S} S^2 + \text{h.c.} \right) \right\} \tag{B.1}$$

and use units where $c_S \langle F_\phi \rangle / \lambda_S = 1$. We see that the phase structure is completely determined by the dimensionless parameter

$$\xi = \frac{1}{c_S}. \tag{B.2}$$

Writing

$$\langle S \rangle = s e^{i\theta}, \tag{B.3}$$

where s and θ are real, we have

$$\frac{V}{\lambda_S^2} = (1 + \xi \cos 2\theta) s^2 + s^3 \cos\theta + \tfrac{1}{4} s^4. \tag{B.4}$$

This is stationary in θ for

$$s = 0 \quad \text{or} \quad \sin\theta = 0 \quad \text{or} \quad \cos\theta = -\frac{s}{4\xi}. \tag{B.5}$$

We consider these cases one at a time.

The case $\sin\theta = 0$ is equivalent to $\langle S \rangle = s =$ real. In that case, we find stationary points

$$s = s_\pm = \tfrac{1}{2}\left[-3 \pm \sqrt{1-8\xi}\right]. \tag{B.6}$$

Consistency therefore requires $\xi < \frac{1}{8}$. It is easy to check that

$$V(s_-) < V(s_+), V(0) \quad \text{for} \quad \xi < 0, \tag{B.7}$$

$$V(0) < V(s_-), V(s_+) \quad \text{for} \quad 0 < \xi < \tfrac{1}{8}. \tag{B.8}$$

It remains only to consider the third condition in Eq. (B.5). In this case, the stationary points are

$$s = \tilde{s}_\pm = \pm 2\sqrt{\frac{\xi(\xi-1)}{2\xi-1}}. \tag{B.9}$$

Reality of s and $|\cos\theta| \leq 1$ are satisfied only if

$$\xi \geq 0. \tag{B.10}$$

We have $V(\tilde{s}_+) = V(\tilde{s}_-)$, as we expect since CP is spontaneously broken. We can check that

$$V(0) < V(\tilde{s}_\pm) \quad \text{for} \quad \xi < 1, \tag{B.11}$$

$$V(\tilde{s}_\pm) < V(0) \quad \text{for} \quad \xi > 1. \tag{B.12}$$

We conclude that

$$\langle S \rangle = \begin{cases} s_- & \text{for} \quad \xi < 0 \\ 0 & \text{for} \quad 0 < \xi < 1 \\ \tilde{s}_\pm e^{i\theta_\pm} & \text{for} \quad \xi > 1, \end{cases} \tag{B.13}$$

where

$$\cos\theta_{\pm} = \mp\sqrt{\frac{\xi - 1}{4\xi(2\xi - 1)}}. \tag{B.14}$$

Restoring the units, we obtain the formulas used in the main text.

B.2 Formulas for Soft Masses

In this appendix, we give some explicit one-loop formulas for SUSY breaking masses in the model of Chapter 3, as well as the cases GMSB and AMSB, which arise as special cases. The beta functions for the MSSM gauge couplings with N_2 doublets and N_3 triplets are

$$\beta_i = \frac{b_i}{16\pi^2} g_i^3, \tag{B.15}$$

where

$$b_3 = -3 + N_3, \tag{B.16}$$

$$b_2 = 1 + N_2, \tag{B.17}$$

$$b_1 = 11 + N_2 + \tfrac{2}{3} N_3. \tag{B.18}$$

The one-loop anomalous dimensions are

$$\gamma_{Q3} = \frac{1}{16\pi^2}\left[\tfrac{16}{3}g_3^2 + 3g_2^2 + \tfrac{1}{9}g_1^2 - 2y_t^2\right], \tag{B.19}$$

$$\gamma_{u3} = \frac{1}{16\pi^2}\left[\tfrac{16}{3}g_3^2 + \tfrac{16}{9}g_1^2 - 4y_t^2\right], \tag{B.20}$$

$$\gamma_{d3} = \frac{1}{16\pi^2}\left[\tfrac{16}{3}g_3^2 + \tfrac{4}{9}g_1^2\right], \tag{B.21}$$

$$\gamma_{L} = \frac{1}{16\pi^2}\left[3g_2^2 + g_1^2\right], \tag{B.22}$$

$$\gamma_{e} = \frac{1}{16\pi^2}\left[4g_1^2\right], \tag{B.23}$$

$$\gamma_{Hu} = \frac{1}{16\pi^2}\left[3g_2^2 + g_1^2 - 6y_t^2\right], \tag{B.24}$$

$$\gamma_{Hd} = \frac{1}{16\pi^2}\left[3g_2^2 + g_1^2\right], \tag{B.25}$$

For the quark fields of the first and second generation, the top Yukawa coupling contribution should be dropped. We do not include the other Yukawa couplings, since they are negligible for small $\tan\beta$. The beta function for the top Yukawa coupling is

$$\beta_{yt} = \frac{y_t}{16\pi^2}\left[6y_t^2 - \tfrac{16}{3}g_3^2 - 3g_2^2 - \tfrac{13}{9}g_1^2\right]. \tag{B.26}$$

These formulas can be used to compute the MSSM soft masses using Eqs. (3.24)–(3.27) in the main text. In the one-loop approximation, the contributions from the doublet and triplet messengers just add, and we obtain *e.g.*

$$m^2_{\tilde{Q},\mathrm{AMSB}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[16g_3^4 - 3g_2^4 - \tfrac{11}{9}g_1^4 + 2y_t(16\pi^2\beta_{y_t})\right], \tag{B.27}$$

$$\Delta m^2_{\tilde{Q}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[r_3(r_3+2)N_3\left(\tfrac{16}{3}g_3^4 + \tfrac{2}{27}g_1^4\right)\right.$$
$$\left. + r_2(r_2+2)N_2\left(3g_2^4 + \tfrac{1}{9}g_1^4\right)\right], \tag{B.28}$$

$$m^2_{\tilde{u}_R,\mathrm{AMSB}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[16g_3^4 - \tfrac{176}{9}g_1^4 + 4y_t(16\pi^2\beta_{y_t})\right], \tag{B.29}$$

$$\Delta m^2_{\tilde{u}_R} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[\tfrac{16}{3}g_3^4 r_3(r_3+2)N_3\right.$$
$$\left. + \tfrac{16}{9}g_1^4\left(\tfrac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.30}$$

$$m^2_{\tilde{d}_R,\mathrm{AMSB}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[16g_3^4 - \tfrac{44}{9}g_1^4\right], \tag{B.31}$$

$$\Delta m^2_{\tilde{d}_R} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[\tfrac{16}{3}g_3^4 r_3(r_3+2)N_3\right.$$
$$\left. + \tfrac{4}{9}g_1^4\left(\tfrac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.32}$$

$$m^2_{\tilde{L},\mathrm{AMSB}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-3g_2^4 - 11g_1^4\right], \tag{B.33}$$

$$\Delta m^2_{\tilde{L}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[3g_2^4 r_2(r_2+2)N_2\right.$$
$$\left. + g_1^4\left(\tfrac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.34}$$

$$m^2_{\tilde{e}_R,\mathrm{AMSB}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-44g_1^4\right], \tag{B.35}$$

$$\Delta m^2_{\tilde{e}_R} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[4g_1^4\left(\tfrac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.36}$$

$$m^2_{H_u,\mathrm{AMSB}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-3g_2^4 - 11g_1^4 + 6y_t(16\pi^2\beta_{y_t})\right], \tag{B.37}$$

$$\Delta m^2_{H_u} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[3g_2^4 r_2(r_2+2)N_2\right.$$
$$\left. + g_1^4\left(\tfrac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right], \tag{B.38}$$

$$m^2_{H_d,\mathrm{AMSB}} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[-3g_2^4 - 11g_1^4\right], \tag{B.39}$$

$$\Delta m^2_{H_d} = \frac{1}{2}\frac{\langle F_\phi\rangle^2}{(16\pi^2)^2}\left[3g_2^4 r_2(r_2+2)N_2\right.$$
$$\left. + g_1^4\left(\tfrac{2}{3}r_3(r_3+2)N_3 + r_2(r_2+2)N_2\right)\right]. \tag{B.40}$$

For the squarks of the first and second generation, we drop the top Yukawa coupling contribution.

From the results above, we can derive the GMSB results of the soft scalar masses by taking the limit $r \to \infty$ and rF_ϕ staying fixed, as discussed in Chapter 3. We also replace N_3 and N_2 by N, the number of messenger $\overline{\mathbf{5}} \oplus \mathbf{5}$ vector-like multiplets.

$$m^2_{\tilde{Q},\mathrm{GMSB}} = \frac{N}{2}\frac{(F/M)^2}{(16\pi^2)^2}\left[\tfrac{16}{3}g_3^4 + 3g_2^4 + \tfrac{5}{27}g_1^4\right], \tag{B.41}$$

$$m^2_{\tilde{u}_R,\mathrm{GMSB}} = \frac{N}{2}\frac{(F/M)^2}{(16\pi^2)^2}\left[\tfrac{16}{3}g_3^4 + \tfrac{80}{27}g_1^4\right], \tag{B.42}$$

$$m^2_{\tilde{d}_R,\mathrm{GMSB}} = \frac{N}{2}\frac{(F/M)^2}{(16\pi^2)^2}\left[\tfrac{16}{3}g_3^4 + \tfrac{20}{27}g_1^4\right], \tag{B.43}$$

$$m^2_{\tilde{L},\mathrm{GMSB}} = \frac{N}{2}\frac{(F/M)^2}{(16\pi^2)^2}\left[3g_2^4 + \tfrac{5}{3}g_1^4\right], \tag{B.44}$$

$$m^2_{\tilde{e}_R,\mathrm{GMSB}} = \frac{N}{2}\frac{(F/M)^2}{(16\pi^2)^2}\left[\tfrac{20}{3}g_1^4\right], \tag{B.45}$$

$$m^2_{H_u,\mathrm{GMSB}} = \frac{N}{2}\frac{(F/M)^2}{(16\pi^2)^2}\left[3g_2^4 + \tfrac{5}{3}g_1^4\right], \tag{B.46}$$

$$m^2_{H_d,\mathrm{GMSB}} = \frac{N}{2}\frac{(F/M)^2}{(16\pi^2)^2}\left[3g_2^4 + \tfrac{5}{3}g_1^4\right]. \tag{B.47}$$

The gaugino masses are given by

$$m_{\lambda_1} = \frac{\langle F_\phi \rangle}{16\pi^2}\left(-11 + \tfrac{2}{3}r_3N_3 + r_2N_2\right), \tag{B.48}$$

$$m_{\lambda_2} = \frac{\langle F_\phi \rangle}{16\pi^2}\left(-1 + r_2N_2\right), \tag{B.49}$$

$$m_{\lambda_3} = \frac{\langle F_\phi \rangle}{16\pi^2}\left(3 + r_3N_3\right), \tag{B.50}$$

where the first term in the parenthesis is the AMSB contribution while the remaining terms are contributions from the doublet and triplet messengers.

Chapter C

STABILITY OF THE S-GAUGINOS

In this Appendix we calculate the mass of η, the s-gaugino, through its Coleman-Weinberg potential, and compare the results with those obtained from the methods of the analytic continuation.

C.1 Coleman-Weinberg Potential for η, the Chiral Adjoint

The Coleman-Weinberg potential of η has the form

$$V_{CW} = \frac{1}{64\pi^2}\sum_i (-1)^{2s_i} M_i^4 \left[\ln\frac{M_i^2}{\mu^2} - \frac{3}{2}\right], \tag{C.1}$$

where M_i is an η-dependent mass eigenvalue, and s_i is the spin of the field i. For our case at hand, the Coleman-Weinberg potential will sum through the contribution of the messengers, because the mass of the messengers depends on the field η. The Coleman-Weinberg potential of η can be constructed by making the substitutions (see Eqs. (4.29) and (4.30))

$$M_\pm \to M_\pm + h_\pm \eta, \tag{C.2}$$

which we will do at the very end of the calculation.

For $\langle \eta \rangle = 0$, the SUSY-preserving $(\text{mass})^2$ values are

$$M_1^2 = \frac{1}{2}\Big[|M_+|^2 + |M_-|^2 + |m_+|^2 + |m_-|^2 + \sqrt{(|M_+|^2 + |m_+|^2 - |M_-|^2 - |m_-|^2)^2 + 4|M_+ m_-^\dagger + \cdots} \tag{C.3}$$

$$M_2^2 = \frac{1}{2}\Big[|M_+|^2 + |M_-|^2 + |m_+|^2 + |m_-|^2 - \sqrt{(|M_+|^2 + |m_+|^2 - |M_-|^2 - |m_-|^2)^2 + 4|M_+ m_-^\dagger + \cdots} \tag{C.4}$$

We note that in the limiting case of $m_- = m_+ = 0$, we obtain $M_{1,2}^2 = M_\pm^2$.

Expanding the SUSY-preserving $(\text{mass})^2$ eigenvalues to $\mathcal{O}(m_\pm^2)$, we have

$$\begin{aligned}
|M_1|^2 &= |M_+|^2 + \frac{M_+ M_- m_+^\dagger m_-^\dagger + M_+^\dagger M_-^\dagger m_- m_+}{|M_+|^2 - |M_-|^2} + \frac{|M_+|^2}{|M_+|^2 - |M_-|^2}(|m_+|^2 + |m_-|^2) + \mathcal{O}\left(\frac{m_\pm^4}{M_\pm^2}\right) \\
|M_2|^2 &= |M_-|^2 - \frac{M_+ M_- m_+^\dagger m_-^\dagger + M_+^\dagger M_-^\dagger m_- m_+}{|M_+|^2 - |M_-|^2} - \frac{|M_-|^2}{|M_+|^2 - |M_-|^2}(|m_+|^2 + |m_-|^2) + \mathcal{O}\left(\frac{m_\pm^4}{M_\pm^2}\right)
\end{aligned} \tag{C.5}$$

Since we will extract the η dependence, it is important that we keep the $M_\pm$ as complex variables. However, we can treat $m_\pm$ as real variables. On the other hand, it is crucial to treat $m_\pm$ as complex variables when we extend to superspace.

Including the effects of $D \equiv m_D^2$, we have the $(\text{mass})^2$ matrices of the scalars

$$\mathcal{M}_\phi^2 = \begin{pmatrix} |M_+|^2 + |m_+|^2 + D & m_- m_+^\dagger + m_+^\dagger M_- \\ m_-^\dagger m_+ + m_+ M_-^\dagger & |M_-|^2 + |m_-|^2 - D \end{pmatrix}, \tag{C.6}$$

$$\mathcal{M}_{\tilde\phi}^2 = \begin{pmatrix} |M_+|^2 + |m_-|^2 - D & m_+^\dagger M_+ + m_- M_-^\dagger \\ m_+ M_+^\dagger + m_-^\dagger M_- & |M_-|^2 + |m_+|^2 + D \end{pmatrix}. \tag{C.7}$$

Expanding the scalar mass eigenvalues, we obtain

$$
\begin{aligned}
m_{\phi_1}^2 &= |M_1|^2 + D - 2D\frac{(m_- M_+^\dagger + m_+^\dagger M_-)(m_-^\dagger M_+ + m_+ M_-^\dagger)}{|M_+|^2 - |M_-|^2} + \mathcal{O}\left(\frac{m_\pm^4}{M_\pm^2}, \frac{m_\pm^2 D^2}{M_\pm^4}\right), \\
m_{\phi_2}^2 &= |M_2|^2 - D + 2D\frac{(m_- M_+^\dagger + m_+^\dagger M_-)(m_-^\dagger M_+ + m_+ M_-^\dagger)}{|M_+|^2 - |M_-|^2} + \mathcal{O}\left(\frac{m_\pm^4}{M_\pm^2}, \frac{m_\pm^2 D^2}{M_\pm^4}\right), \\
m_{\tilde{\phi}_1}^2 &= |M_1|^2 - D + 2D\frac{(m_+ M_+^\dagger + m_-^\dagger M_-)(m_+^\dagger M_+ + m_- M_-^\dagger)}{|M_+|^2 - |M_-|^2} + \mathcal{O}\left(\frac{m_\pm^4}{M_\pm^2}, \frac{m_\pm^2 D^2}{M_\pm^4}\right), \\
m_{\tilde{\phi}_2}^2 &= |M_2|^2 + D - 2D\frac{(m_+ M_+^\dagger + m_-^\dagger M_-)(m_+^\dagger M_+ + m_- M_-^\dagger)}{|M_+|^2 - |M_-|^2} + \mathcal{O}\left(\frac{m_\pm^4}{M_\pm^2}, \frac{m_\pm^2 D^2}{M_\pm^4}\right).
\end{aligned}
\tag{C.8}
$$

With the fermion mass eigenvalues of Eq. (C.5) and scalar mass eigenvalues in Eq. (C.8), we can extra the $Dm_\pm^2$ and D^2 coefficients of the Coleman-Weinberg potential (setting $m_\pm$ to be real for simplicity)

$$
\begin{aligned}
V_{CW} = &\frac{1}{8\pi^2}\frac{D(m_+^2 - m_-^2)}{|M_+|^2 - |M_-|^2}\left(|M_+|^2 \ln\frac{|M_+|^2}{\mu^2} - |M_-|^2 \ln\frac{|M_-|^2}{\mu^2}\right) \\
&+ \frac{D^2}{16\pi^2}\left(\ln\frac{|M_+|^2}{\mu^2} + \ln\frac{|M_-|^2}{\mu^2}\right).
\end{aligned}
\tag{C.9}
$$

Making the substitutions

$$M_\pm \to M_\pm + h_\pm \eta, \tag{C.10}$$

we can extract η mass of the form

$$V = m_\eta^2 |\eta|^2 + (B_\eta \eta^2 + \text{h.c.}) + \mathcal{O}(\eta^3), \tag{C.11}$$

where

$$m_\eta^2 = \frac{1}{8\pi^2}\frac{D(m_+^2 - m_-^2)|h_- M_+ - h_+ M_-|^2}{(|M_+|^2 - |M_-|^2)^2}\left(-2 + \frac{|M_+|^2 - |M_-|^2}{|M_+|^2 - |M_-|^2}\ln\frac{|M_+|^2}{|M_-|^2}\right), \tag{C.12}$$

$$B_\eta = -\frac{D^2}{32\pi^2}\left(\frac{h_+^2}{|M_+|^2} + \frac{h_-^2}{|M_-|^2}\right) + \frac{D}{16\pi^2}\frac{(m_+^2 - m_-^2)}{(|M_+|^2 - |M_-|^2)}\left(c_1 + c_2 \ln\frac{|M_+|^2}{|M_-|^2}\right), \tag{C.13}$$

and

$$c_1 = h_+^2 \frac{M_+^\dagger}{M_+} - h_-^2 \frac{M_-^\dagger}{M_-} - 2\frac{(h_+ M_+^\dagger - h_- M_-^\dagger)^2}{|M_+|^2 - |M_-|^2},$$
$$c_2 = -2\frac{M_+^\dagger M_-^\dagger (h_- M_+ - h_+ M_-)(h_+ M_+^\dagger - h_- M_-^\dagger)}{(|M_+|^2 - |M_-|^2)^2}. \tag{C.14}$$

Treating $M_\pm$ as real (thus $B_\eta = B_\eta^\dagger$) and decompose η as

$$\eta = \frac{1}{\sqrt{2}}(\mathrm{Re}[\sqrt{2}\,\eta] + i\mathrm{Im}[\sqrt{2}\,\eta]), \tag{C.15}$$

the real and imaginary parts of η have masses

$$m^2(\mathrm{Re}[\sqrt{2}\,\eta]) = m_\eta^2 + 2B_\eta,$$
$$m^2(\mathrm{Im}[\sqrt{2}\,\eta]) = m_\eta^2 - 2B_\eta. \tag{C.16}$$

It is possible to have both of these eigenvalues positive.

Treating $M_\pm$ as real and taking the limit $M_+ \gg M_-$, we have the masses

$$m_\eta^2 \sim \frac{D}{8\pi^2}\frac{h_-^2(m_+^2 - m_-^2)}{M_+^2}\left(-2 + \ln\frac{M_+^2}{M_-^2}\right), \tag{C.17}$$

$$B_\eta \sim -\frac{D^2}{32\pi^2}\frac{h_-^2}{M_-^2} + \frac{D}{16\pi^2}\frac{(m_+^2 - m_-^2)}{M_+^2}(3h_+^2 - h_-^2). \tag{C.18}$$

C.2 Analytic Continuation to Superspace

Treating the wavefunction renormalization Z as a spurion, the soft-masses of the scalar component of a chiral superfield is related to the highest component of $\ln Z$

$$m_\eta^2 = -\left[\ln Z_\eta\right]_{\theta^2\bar{\theta}^2}. \tag{C.19}$$

We consider the case of $M_1 \gg M_2$. For the ease of notation, we will in general use double-prime ($''$) to denote quantities of the fundamental theory above M_1; primed quantities ($'$) refer to those in the effective theory between M_1 and M_2, and un-primed for quantities in the effective theory below M_2. In this case, the wavefunction renormalization of η at a scale $\mu < M_2$ is given by

$$\ln Z(\mu) = \ln Z''(\Lambda) + \gamma'' \ln \frac{|M_1|}{\Lambda} + \gamma' \ln \frac{|M_2|}{|M_1|} + \gamma \ln \frac{\mu}{|M_2|}, \tag{C.20}$$

where γ'' $(\gamma')[\gamma]$ is the anomalous dimension of the fundamental theory above M_1 (effective theory between M_1 and M_2)[below M_2].

The anomalous dimensions are given by

$$\gamma'' = -\frac{1}{8\pi^2}\left(\frac{|h_+|^2}{Z_\eta Z''_{\tilde{P}_-} Z''_{P_+}} + \frac{|h_-|^2}{Z''_\eta Z''_{\tilde{P}_+} Z''_{P_-}}\right), \tag{C.21}$$

$$\gamma' = -\frac{1}{8\pi^2}\left(\frac{|h_-|^2}{Z'_\eta Z'_{\tilde{P}_+} Z'_{P_-}}\right), \tag{C.22}$$

$$\gamma = 0, \tag{C.23}$$

where Z are the wavefunction renormalization for the various fields. Note that $\gamma = 0$, so the η masses do not run (at one-loop) once all the messengers have been integrated out. To the order we are interested in, we have

$$Z''_P = 1,\ Z''_\eta = Z'_\eta = 1, \tag{C.24}$$

where P is any messenger. However, in the effective theory below M_1, we have

$$Z'_{\tilde{P}_+} = 1 + \frac{m_+^\dagger e^{-2X} m_+}{M_+^\dagger M_+}, \tag{C.25}$$

$$Z'_{P_-} = 1 + \frac{m_-^\dagger e^{+2X} m_-}{M_+^\dagger M_+}. \tag{C.26}$$

Putting everything together, Eq. (C.20) becomes

$$\ln Z(\mu) = \ln Z''(\Lambda) - \frac{1}{8\pi^2}\left(|h_+|^2 + |h_-|^2\right)\ln\frac{|M_1|}{\Lambda} - \frac{|h_-|^2}{8\pi^2}\left(1 - \frac{m_+^\dagger e^{-2X} m_+}{M_+^\dagger M_+} - \frac{m_-^\dagger e^{+2X} m_-}{M_+^\dagger M_+}\right)\ln\frac{|M_2|}{|M_1|}, \tag{C.27}$$

and the $\theta^2\overline{\theta}^2$ components are

$$[\ln|M_1|]_{\theta^2\overline{\theta}^2} = D\frac{|m_+|^2 - |m_-|^2}{|M_+|^2 - |M_-|^2}, \tag{C.28}$$

$$[\ln|M_2|]_{\theta^2\overline{\theta}^2} = -D\frac{|m_+|^2 - |m_-|^2}{|M_+|^2 - |M_-|^2}, \tag{C.29}$$

$$\left[e^{2X}\right]_{\theta^2\overline{\theta}^2} = 2D. \tag{C.30}$$

The $|\eta|^2$ mass then have the form

$$m_\eta^2 = \frac{D}{8\pi^2}\frac{(|h_+|^2 - |h_-|^2)(|m_+|^2 - |m_-|^2)}{|M_+|^2 - |M_-|^2} + \frac{D}{8\pi^2}|h_-|^2\frac{(|m_+|^2 - |m_-|^2)}{|M_+|^2}\ln\frac{|M_-|^2}{|M_+|^2}. \tag{C.31}$$

Chapter D

KK MODE EXPANSION UNDER $T^2/Z_2 \times Z_2'$

In this Appendix we present many details of the calculations in Chapter 5 of this Thesis, including the expansion of the KK modes, as well as the normalization of the various fields and couplings under the orbifold $T^2/Z_2 \times Z_2'$.

D.1 Fields on $T^2/Z_2 \times Z_2'$

For convenience of type-setting, we define the functions

$$
\begin{aligned}
c(i,j) &\equiv \cos\frac{ix^5 + jx^6}{R}, \qquad & s(i,j) &\equiv \sin\frac{ix^5 + jx^6}{R}, \\
c'(i,j) &\equiv \cos\frac{ix'^5 + jx'^6}{R}, \qquad & s'(i,j) &\equiv \sin\frac{ix'^5 + jx'^6}{R},
\end{aligned} \tag{D.1}
$$

And for reference we will make use of this integral

$$
\int_0^{2\pi R} dx^5 \int_0^{2\pi R} dx^6 c(i,j)c(m,n) = \int_0^{2\pi R} dx^5 \int_0^{2\pi R} dx^6 s(i,j)s(m,n) = 2\pi^2 R^2 \delta_{im}\delta jn \tag{D.2}
$$

for positive integers i, j, m and n, extensively. We have the compactified space $2\pi R \times 2\pi R$ by imposing the periodic boundary conditions on the fields

$$
\phi(x^\mu, x^4, x^5) = \phi(x^\mu, x^4 + 2\pi R, x^5) = \phi(x^\mu, x^4, x^5 + 2\pi R). \tag{D.3}
$$

The periodic boundary conditions mean that we can write the fields in the form of

$$
\phi(x^\mu, x^4, x^5) = \sum_{n,m} \left(c(n,m)\varphi^{(nm)}(x^\mu) + s(n,m)\tilde{\varphi}^{(nm)}(x^\mu) \right). \tag{D.4}
$$

On top of the periodic boundary conditions, we impose two orbifolding symmetries on our theory

$$Z_2 : \mathbf{y} \to -\mathbf{y}, \quad Z_2' : \mathbf{y}' \to -\mathbf{y}'. \tag{D.5}$$

with $\mathbf{y}' = \mathbf{y} - (\pi R/2, \pi R/2)$. Demanding that the Lagrangian be invariant under the orbifolding symmetries, we can assign parities to the fields under the discrete transformations and remove roughly half of the KK modes in Eq. (D.4). The choices of signs are motivated by the desired phenomenology. In our case, we have two orbifolding symmetries, so we can assign two signs to a given field. There are four possibilities: $(+, \pm)$ and $(-, \pm)$, and we examine each case separately.

For $(+, \pm)$ case, we have the general expansion

$$\begin{aligned}\phi^{(+,\pm)}(x^\mu, x^4, x^5) &= \frac{1}{2\pi R} \sum_{n,m=0}^{\infty} \left(c(n,m) \varphi^{(nm)} \right) \\ &= \frac{1}{2\pi R} \sum_{n,m=0}^{\infty} \left[\left(c'(n,m) \cos \frac{(m+n)\pi}{2} \right. \right. \\ &\qquad \left. \left. - s'(n,m) \sin \frac{(m+n)\pi}{2} \right) \varphi^{(nm)} \right]. \end{aligned} \tag{D.6}$$

So we see that for $(+,+)$ fields, we need $n+m$ and $n-m$ to be even.

For $(+,-)$ fields, we need $n+m$ and $n-m$ to be odd. For $(-, \pm)$ case, we have the general expansion

$$\begin{aligned}\phi^{(-,\pm)}(x^\mu, x^4, x^5) &= \frac{1}{2\pi R} \sum_{n+m\geq 1}^{\infty} \left(s(n,m) \varphi^{(nm)} \right) \\ &= \frac{1}{2\pi R} \sum_{n+m\geq 1}^{\infty} \left[\left(s'(n,m) \cos \frac{(m+n)\pi}{2} \right. \right. \\ &\qquad \left. \left. + c'(n,m) \sin \frac{(m+n)\pi}{2} \right) \varphi^{(nm)} \right]. \end{aligned} \tag{D.7}$$

So we see that for $(-,+)$ fields, we need $n+m$ and $n-m$ to be odd. For $(-,-)$ fields, we need $n+m$ and $n-m$ to be even. Of course, for the $(-,\pm)$ cases, we can not have $(m,n)=(0,0)$ mode.

D.2 Normalization of fields and couplings

Matter fields

The dark matter candidates of the theory are the first KK modes of the neutrinos charged under $SU(2)_2$. They have the $Z_2 \times Z_2'$ charges: $\nu_{\text{2L}}(-,+), \nu_{\text{2R}}(+,-)$. If we let $\overrightarrow{n}=(n,m)$, we see each of $\nu_{\text{2L,2R}}$ has two independent modes: $\overrightarrow{n}=(1,0)$ and $\overrightarrow{n}=(0,1)$. These are two independent Dirac particles in the sense that there is no mixing at tree level in the effective 4D theory.

We expand the kinetic energy term

$$\begin{aligned}\mathcal{L}_{\text{6D-KE}} &= i\overline{\Psi}\Gamma^M\partial_M\Psi \\ &= i\begin{pmatrix}\overline{\Psi}_- & \overline{\Psi}_+\end{pmatrix}\begin{pmatrix}0 & \gamma^\mu\partial_\mu + i\gamma_5\partial_5 + \partial_6 \\ \gamma^\mu\partial_\mu + i\gamma_5\partial_5 - \partial_6 & 0\end{pmatrix}\begin{pmatrix}\Psi_+ \\ \Psi_-\end{pmatrix} \\ &= i\overline{\Psi}_-(\gamma^\mu\partial_\mu + i\gamma_5\partial_5 + \partial_6)\Psi_- + i\overline{\Psi}_+(\gamma^\mu\partial_\mu + i\gamma_5\partial_5 - \partial_6)\Psi_+. \qquad \text{(D.8)}\end{aligned}$$

Note that Ψ is an eight-component object, while $\Psi_\pm$ are four-component, six-dimensional chiral spinors. We denote six-dimensional chirality by $\pm$ and four-dimensional chirality by L, R. Each six-dimensional chiral spinor is vector-like in the four-dimensional sense, and each is a Dirac spinor. Since our dark matter candidate is of (-1) 6D-chirality, we only deal with the first part of the kinetic energy term, and drop the subscript.

Since we are after the coefficients, we expand in detail the first KK mode of the dark matter candidate.

$$
\begin{aligned}
\mathcal{L}_{\text{6D-KE}} &\supset i\overline{\Psi}_{-}(\gamma^\mu\partial_\mu + i\gamma_5\partial_5 + \partial_6)\Psi_{-} \\
&= i\begin{pmatrix}\Psi_L^\dagger & \Psi_{\mathrm{R}}^\dagger\end{pmatrix}\begin{pmatrix} i\partial_5+\partial_6 & \sigma^\mu\partial_\mu \\ \overline{\sigma}^\mu\partial_\mu & -i\partial_5+\partial_6\end{pmatrix}\begin{pmatrix}\Psi_{\mathrm{R}} \\ \Psi_L\end{pmatrix} \\
&= i(\Psi_L^\dagger\sigma^\mu\partial_\mu\Psi_L + \Psi_{\mathrm{R}}^\dagger\overline{\sigma}^\mu\partial_\mu\Psi_{\mathrm{R}} + \Psi_L^\dagger(i\partial_5+\partial_6)\Psi_{\mathrm{R}} + \Psi_{\mathrm{R}}(-i\partial_5+\partial_6)\Psi_L) \qquad \text{(D.9)}
\end{aligned}
$$

At this point, we use the KK-expansions. Noting the charge assignments $\nu_{2L}(-,+), \nu_{2R}(+,-)$, we expand the fields as

$$
\begin{aligned}
\nu_{\mathrm{2R}} &= \frac{1}{\sqrt{2\pi R}}\left(c(1,0)\nu_{\mathrm{2R}}^{(10)}(x^\mu) + c(0,1)\nu_{\mathrm{2R}}^{(01)}(x^\mu)\right), \\
\nu_{\mathrm{2L}} &= \frac{1}{\sqrt{2\pi R}}\left(s(1,0)\nu_{\mathrm{2L}}^{(10)}(x^\mu) + is(0,1)\nu_{\mathrm{2L}}^{(01)}(x^\mu)\right). \qquad \text{(D.10)}
\end{aligned}
$$

The four-dimensional effective Lagrangian is obtained by inserting the expansion of Eq. (D.10) into Eq. (D.9), and integrate x^5 and x^6 from 0 to $2\pi R$. Following this procedure, we obtain

$$
\begin{aligned}
\mathcal{L}_{\text{4D-eff}} = \int_0^{2\pi R} dx^5 \int_0^{2\pi R} dx^6 \mathcal{L}_{\text{6D-KE}} = i\,\big(&\nu_{\mathrm{2L}}^{\dagger(10)}\sigma^\mu\partial_\mu\nu_{\mathrm{2L}}^{(10)} + \nu_{\mathrm{2L}}^{\dagger(01)}\sigma^\mu\partial_\mu\nu_{\mathrm{2L}}^{(01)} \\
&+\nu_{\mathrm{2R}}^{\dagger(10)}\overline{\sigma}^\mu\partial_\mu\nu_{\mathrm{2R}}^{(10)} + \nu_{\mathrm{2R}}^{\dagger(01)}\overline{\sigma}^\mu\partial_\mu\nu_{\mathrm{2R}}^{(01)}\big) \\
&-\frac{1}{R}\left(\left(\nu_{\mathrm{2L}}^{\dagger(10)}\nu_{\mathrm{2R}}^{(10)} - \nu_{\mathrm{2R}}^{\dagger(10)}\nu_{\mathrm{2L}}^{(10)}\right) - \left(\nu_{\mathrm{2L}}^{\dagger(01)}\nu_{\mathrm{2R}}^{(01)} - \nu_{\mathrm{2R}}^{\dagger(01)}\nu_{\mathrm{2L}}^{(01)}\right)\right).
\end{aligned}
\qquad \text{(D.11)}
$$

From this calculation, we see that we have two independent Dirac neutrinos that do not mix with each other: $\nu^{(01)}$ and $\nu^{(10)}$.

Following the same procedure, we can find the normalization of scalars.

$$\begin{aligned}
\Phi(+,+) &= \frac{1}{2\pi R}\phi^{(00)} + \frac{1}{\sqrt{2}\pi R}\sum_{m,n}\cos\frac{mx^5+nx^6}{R}\phi^{(mn)},\\
\Phi(+,-) &= \frac{1}{\sqrt{2}\pi R}\sum_{m,n}\cos\frac{mx^5+nx^6}{R}\phi^{(mn)},\\
\Phi(-,+) &= \frac{1}{\sqrt{2}\pi R}\sum_{m,n}\sin\frac{mx^5+nx^6}{R}\phi^{(mn)},\\
\Phi(-,-) &= \frac{1}{\sqrt{2}\pi R}\sum_{m,n}\cos\frac{mx^5+nx^6}{R}\phi^{(mn)}.
\end{aligned} \tag{D.12}$$

Again, the rules of m, n in the previous section for fermions apply to the scalars.

Gauge bosons

As we are only interested in the normalization of the gauge fields, we consider a generic gauge boson, $\mathcal{A}_M$, associated with an $U(1)$ symmetry. We then have the expansion

$$\begin{aligned}
\mathcal{L}_{gauge} &= -\frac{1}{4}F^{MN}F_{MN}\\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{5\mu}F^{5\mu} - \frac{1}{2}F_{6\mu}F^{6\mu} - \frac{1}{2}F_{56}F^{56}\\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\
&\quad + \frac{1}{2}(\partial_\mu\mathcal{A}_5\partial^\mu\mathcal{A}_5 + \partial_5\mathcal{A}_\mu\partial_5\mathcal{A}^\mu - \partial_5\mathcal{A}_\mu\partial^\mu\mathcal{A}_5 - \partial_\mu\mathcal{A}_5\partial_5\mathcal{A}^\mu)\\
&\quad + \frac{1}{2}(\partial_\mu\mathcal{A}_6\partial^\mu\mathcal{A}_6 + \partial_6\mathcal{A}_\mu\partial_6\mathcal{A}^\mu - \partial_6\mathcal{A}_\mu\partial^\mu\mathcal{A}_6 - \partial_\mu\mathcal{A}_6\partial_6\mathcal{A}^\mu)\\
&\quad - \frac{1}{2}(\partial_5\mathcal{A}_6\partial_5\mathcal{A}_6 + \partial_6\mathcal{A}_5\partial_6\mathcal{A}_5 - 2\partial_5\mathcal{A}_6\partial_6\mathcal{A}_5)
\end{aligned} \tag{D.13}$$

Notice that we have made the changes $\mathcal{A}^{5,6} = -\mathcal{A}_{5,6}, \partial^{5,6} = -\partial_{5,6}$, so that $\mathcal{A}_{5,6}$ should be treated as real, scalar fields. We will work with this equation for the various gauge particles.

As with the case of the neutral gauge bosons in our theory, we assign the $Z_2 \times Z_2'$ parities to be $\mathcal{A}_\mu(++)$ and $\mathcal{A}_{5,6}(--)$, and obtain the following lowest KK modes:

$$\begin{aligned} &A_\mu^{(00)},\ A_\mu^{(11)},\ A_\mu^{(20)},\ A_\mu^{(02)}, \\ &A_5^{(11)},\ A_5^{(20)},\ A_5^{(02)}, \\ &A_6^{(11)},\ A_6^{(20)},\ A_6^{(02)}, \end{aligned} \tag{D.14}$$

and the KK-mode expansions for these states

$$\begin{aligned} \mathcal{A}_\mu &= \frac{1}{2\pi R} A_\mu^{(00)} + \frac{1}{\sqrt{2}\pi R}\left(\cos\frac{2x^5}{R}A_\mu^{(20)} + \cos\frac{2x^6}{R}A_\mu^{(02)} + \cos\frac{x^5+x^6}{R}A_\mu^{(11)}\right), \\ \mathcal{A}_{5,6} &= \frac{1}{\sqrt{2}\pi R}\left(\sin\frac{2x^5}{R}A_{5,6}^{(20)} + \sin\frac{2x^6}{R}A_{5,6}^{(02)} + \sin\frac{x^5+x^6}{R}A_{5,6}^{(11)}\right). \end{aligned} \tag{D.15}$$

One may check that the normalization gives canonical fields for the scalars $A_{5,6}^{\mathrm{KK}}$.

For the four-dimensional Lagrangian we insert the expansion of Eq. (D.15) into Eq. (D.13) and integrate over x^5 and x^6. In additional to canonical kinetic energy terms, we obtain the masses of these modes.

$$\begin{aligned} \mathcal{L} = &\frac{1}{2}\left[\frac{4}{R^2}A_\mu^{(20)}A^{\mu,(20)} + \frac{4}{R^2}A_\mu^{(02)}A^{\mu,(02)} + \frac{2}{R^2}A_\mu^{(11)}A^{\mu,(11)}\right] \\ &-\frac{1}{2}\left[\frac{4}{R^2}(A_5^{(20)})^2 + \frac{4}{R^2}(A_6^{(02)})^2 + \frac{1}{R^2}(A_5^{(11)})^2 + \frac{1}{R^2}(A_6^{(11)})^2 - 2\frac{1}{R^2}A_5^{(11)}A_6^{(11)}\right] \end{aligned} \tag{D.16}$$

We note first that we have some massless modes in $A_6^{(02)}$ and $A_5^{(20)}$. This can be traced to the fact that we do not have terms $\partial_5 A_5$ and $\partial_6 A_6$ in $F_{\mathrm{MN}}F^{\mathrm{MN}}$. As in the case of 5D UED models, these modes are eaten by the corresponding KK modes of $A_\mu^{(\mathrm{mn})}$ so they can become massive. Here we note that the linear combination of $\frac{1}{\sqrt{2}}(A_5^{(11)} + A_6^{(11)})$ is also massless, and is eaten by $B_{\mu(11)}$. Generally, at each KK level, one linear combination of $A_5^{\rm KK}$ and $A_6^{\rm KK}$ (and any corresponding KK modes of Higgs particle, if there is Higgs mechanism) is eaten by $A_\mu^{\rm KK}$, while the orthogonal KK combination remains a physical mode, and is a potential DM candidate if it is indeed the lightest KK mode.

Normalization of couplings

In six-dimensional Lagrangian, both the yukawa and gauge couplings are dimensionful. We find the correct normalization by equating the 4D couplings to the effective 4D coupling resulting from integrating over x^5 and x^6. For example, consider a generic yukawa interaction in the 6D theory

$$\mathcal{L}_{\text{6D-Yukawa}} = y^{\rm 6D}\overline{\Psi}_1\Phi\Psi_2$$

where $y^{\rm 6D}$ has dimension $[M]^{-1}$. The coupling involving the zero-modes in the effective theory is then

$$\mathcal{L}_{\text{4D-Yukawa}} = \int_0^{2\pi R} dx^5 \int_0^{2\pi R} dx^6 \frac{y^{\rm 6D}}{(2\pi R)^3}\overline{\psi}_1^{(00)}\phi^{(00)}\psi_2^{(00)} = \frac{y^{\rm 6D}}{2\pi R}\overline{\psi}_1^{(00)}\phi^{(00)}\psi_2^{(00)}. \tag{D.17}$$

So effectively we have $y^{\rm 4D} = y^{\rm 6D}(2\pi R)^{-1}$. Note that this is general: for the SM couplings in the 4D effective theory, all fields are (00) and have a normalization $(2\pi R)^{-1}$, so the effective 4D couplings obtained after integrating over x^5 and x^6 are simply the 6D couplings multiplied by $(2\pi R)$. By the same reasoning, we also have $\lambda^{\rm 6D} = (2\pi R)^2\lambda^{\rm 4D}$ for the quartic coupling in the potential.

In general, the coupling between higher modes will come with extra factors resulting from integrating over x^5 and x^6. However, the most important case for our purpose of calculating annihilation diagrams involve couplings between two fermions and a boson where exactly one of field is a (00) mode, and the two other fields are both (mn) mode with m, n nonzero. Suppose we have a coupling in the 6D Lagrangian of the form $\mathcal{L}^{\text{6D}} = g^{\text{6D}}\overline{\Psi}\Phi\Psi$, where g^{6D} has dimension of $[M]^{-1}$, and we impose that $g^{\text{6D}} = g^{\text{4D}}(2\pi R)$. In the 4D effective theory we have $\mathcal{L}^{\text{4D}} = g^{\text{4D}}\overline{\psi}^{(00)}\phi^{(mn)}\psi^{(mn)}$, where the lower-case fields are the KK-modes of the corresponding 6D fields in capital letters. The effective coupling between the KK modes $g^{\text{4D}}\overline{\psi}^{(00)}\phi^{(mn)}\psi^{(mn)}$ in the effective 4D theory is then

$$\mathcal{L}_{\text{4D-eff}} = \int_0^{2\pi R} dx^5 \int_0^{2\pi R} dx^6 \frac{g^{\text{4D}}(2\pi R)}{(2\pi R)(\sqrt{2}\pi R)^2} c^2(m,n)\overline{\psi}_1^{(00)}\phi^{(mn)}\psi^{(mn)} = g^{\text{4D}}\overline{\psi}_1^{(00)}\phi^{(mn)}\psi^{(mn)}. \tag{D.18}$$

So we see that there is no additional factors compared to the case with all (00)-modes.

Bibliography

[1] S. L. Glashow, Nucl. Phys. **22**, 579 (1961).
A. Salam and J. C. Ward, Phys. Lett. **13**, 168 (1964).
S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[2] P. W. Higgs, Phys. Rev. Lett. **13**, 508 (1964).
P. W. Higgs, Phys. Lett. **12**, 132 (1964).
P. W. Higgs, Phys. Rev. **145**, 1156 (1966).

[3] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **96**, 032002 (2006) `[hep-ph/0510099]`.
T. van Ritbergen and R. G. Stuart, Phys. Rev. Lett. **82**, 488 (1999) `[hep-ph/9808283]`.
ALEPH Collaboration, Phys. Rept. **427**, 257 (2006) `[hep-ex/0509008]`.

[4] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

[5] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974) [Erratum-ibid. D **11**, 703 (1975)].
R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 2558 (1975).
H. Georgi and S. L. Glashow, Phys. Today **33N9**, 30 (1980).

[6] Lepton and quark summary tables of [4].

[7] P. Minkowski, Phys. Lett. B **67**, 421 (1977).
M. Gell-Mann, P. Ramond and R. Slansky, *Supergravity* (P. van Nieuwenhuizen & D. Z. Freedman et al. eds), North Holland Publ. Co., Amsterdam, 1980, p. 315
R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981).

[8] V. C. Rubin and W. K. Ford Jr., Astrophys. J. **159**, 379 (1970).
V. C. Rubin, N. Thonnard and W. K. Ford Jr., Astrophys. J. **238**, 471 (1980).

[9] D. N. Spergel et al., `[astro-ph/0603449]`.

[10] E. W. Kolb and M. S. Turner, Front. Phys. **69**, 1 (1990).

[11] S. P. Martin, `[hep-ph/9709356]`.
M. Drees, `[hep-ph/9611409]`.
M. Dine, `[hep-ph/9612389]`.

[12] Supersymmetric particle searches of [4].

[13] A. Heister *et al.* [ALEPH Collaboration], Phys. Lett. B **526**, 191 (2002) [arXiv:hep-ex/0201014].
J. Abdallah *et al.* [DELPHI Collaboration], Eur. Phys. J. C **32**, 145 (2004) [hep-ex/0303013].
M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. B **519**, 33 (2001) [hep-ex/0102025].
G. Abbiendi *et al.* [OPAL Collaboration], Eur. Phys. J. C **26**, 479 (2003) [hep-ex/0209078].
R. Barate *et al.* [LEP Working Group for Higgs boson searches], Phys. Lett. B **565**, 61 (2003) [arXiv:hep-ex/0306033].

[14] CDMS Collaboration, D. S. Akerib, *Phys. Rev. Lett.***96** (2006) 011302 [astro-ph/0509259].

[15] M. A. Luty, [hep-th/0509029].

[16] L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999) [hep-th/9810155].

[17] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998) [hep-ph/9810442].

[18] J. Wess and J. Bagger, "Supersymmetry and supergravity," *Princeton, USA: Univ. Pr. (1992) 259 p*

[19] A. E. Nelson and N. T. Weiner, [hep-ph/0210288].

[20] A. Pomarol and R. Rattazzi, JHEP **9905**, 013 (1999) [hep-ph/9903448].

[21] Z. Chacko, M. A. Luty, I. Maksymyk and E. Ponton, JHEP **0004**, 001 (2000) [hep-ph/9905390].

[22] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).

[23] G. F. Giudice and R. Rattazzi, Nucl. Phys. B **511**, 25 (1998) [hep-ph/9706540].

[24] N. Arkani-Hamed, G. F. Giudice, M. A. Luty and R. Rattazzi, Phys. Rev. D **58**, 115005 (1998) [hep-ph/9803290].

[25] S. Dimopoulos, G. F. Giudice and A. Pomarol, Phys. Lett. B **389**, 37 (1996) [hep-ph/9607225];
S. P. Martin, Phys. Rev. D **55**, 3177 (1997) [hep-ph/9608224].

[26] M. A. Luty, Phys. Rev. Lett. **89**, 141801 (2002) [hep-th/0205077].

[27] R. Harnik, G. D. Kribs, D. T. Larson and H. Murayama, Phys. Rev. D **70**, 015002 (2004) [hep-ph/0311349];
S. Chang, C. Kilic and R. Mahbubani, Phys. Rev. D **71**, 015003 (2005) [hep-ph/0405267].

[28] P. Batra, A. Delgado, D. E. Kaplan and T. M. P. Tait, JHEP **0402**, 043 (2004) [hep-ph/0309149].

[29] J. R. Espinosa and R. J. Zhang, Nucl. Phys. B **586**, 3 (2000) [hep-ph/0003246].

[30] P. J. Fox, A. E. Nelson and N. Weiner, JHEP **0208**, 035 (2002) [hep-ph/0206096].

[31] L. M. Carpenter, P. J. Fox and D. E. Kaplan, [hep-ph/0503093].

[32] D. Binosi and L. Theussl, *Comput. Phys. Commun.***161** (2004) 76 [hep-ph/0309015].

[33] S. Chang, P. J. Fox and N. Weiner, JHEP **0608**, 068 (2006) [hep-ph/0511250].

[34] S. F. King and J. P. Roberts, JHEP **0609**, 036 (2006) [hep-ph/0603095].

[35] N. Arkani-Hamed, A. Delgado and G. F. Giudice, Nucl. Phys. B **741**, 108 (2006) [hep-ph/0601041].

[36] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, Comput. Phys. Commun. **176**, 367 (2007) [hep-ph/0607059].

[37] I. Antoniadis, K. Benakli, A. Delgado and M. Quiros, [hep-ph/0610265].

[38] G. Servant and T. M. P. Tait, *Nucl. Phys. B***650**(2003) 391 [hep-ph/0206071];

[39] H. C. Cheng, J. L. Feng and K. T. Matchev, *Phys. Rev. Lett.* **89** (2002) 211301 [hep-ph/0207125].

[40] K. Agashe and G. Servant, *Phys. Rev. Lett.***93** (2004) 231805 [hep-ph/0403143].

[41] R. N. Mohapatra and V. L. Teplitz, *Phys. Rev. D***62** (2000) 063506 (2000) [astro-ph/0001362];
R. Foot, *Int. J. Mod. Phys. D***13** (2004) 2161 [astro-ph/0407623];
Z. Berezhiani, P. Ciarcelluti, D. Comelli and F. L. Villante, *Int. J. Mod. Phys. D***14** (2005) 107 [astro-ph/0312605];
A. Y. Ignatiev and R. R. Volkas, *Phys. Rev. D***68** (2003) 023518 [hep-ph/0304260].

[42] K. Hsieh, R. N. Mohapatra and S. Nasri, *Phys.Rev.***D 74** (2006) 066004. [hep-ph/0604154]

[43] K. Kong and K. T. Matchev, *JHEP***01** (2006) 038 [hep-ph/0509119].

[44] I. Antoniadis, *Phys. Lett. B***246** (1990) 377.

[45] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phys. Lett. B***436** (1998) 257 [hep-ph/9804398].

[46] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *Phys. Lett. B***429** (1998) 263 [hep-ph/9803315].

[47] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].

[48] H. C. Cheng, K. T. Matchev and M. Schmaltz, Phys. Rev. D **66**, 056006 (2002) [hep-ph/0205314].

[49] T. Appelquist, H. C. Cheng and B. A. Dobrescu, *Phys. Rev.***D 64** (2001) 035002 [hep-ph/0012100].

[50] T. Appelquist, B. A. Dobrescu, E. Ponton and H. U. Yee, *Phys. Rev. Lett.***87** (2001) 181802 [hep-ph/0107056].

[51] R. N. Mohapatra and A. Perez-Lorenzana, *Phys. Rev.***D 67** (2003) 075015 [hep-ph/0212254].

[52] T. Appelquist, B. A. Dobrescu, E. Ponton and H. U. Yee, *Phys. Rev.***D 65** (2002) 105019 [hep-ph/0201131].

[53] B. A. Dobrescu and E. Poppitz, *Phys. Rev. Lett.***87** (2001) 031801 [hep-ph/0102010].

[54] I. Gogoladze and C. Macesanu, *Phys. Rev.***D 74** (2006) 093012 [hep-ph/0605207]

[55] M. Kakizaki, S. Matsumoto and M. Senami, *Phys. Rev.***D 74** (2006) 023504 [hep-ph/0605280].

[56] A. Gould, B. T. Draine, R. W. Romani and S. Nussinov, *Phys. Lett.***B 238** (1990) 337;
S. Dimopoulos, D. Eichler, R. Esmailzadeh and G. D. Starkman, *Phys. Rev.***D 41** (1990) 2388;
A. Kudo and M. Yamaguchi, *Phys. Lett.* **B 516** (2001) 151 [hep-ph/0103272].

[57] T. Appelquist and H. U. Yee, *Phys. Rev.***D 67** (2003)055002 (2003) [hep-ph/0211023].

[58] F. Burnell and G. D. Kribs, *Phys. Rev.***D 73** (2006) 015001 [hep-ph/0509118].

[59] H. C. Cheng, K. T. Matchev and M. Schmaltz, *Phys. Rev.***D 66** (2002) 036005. [hep-ph/0204342].

[60] Particle Data Group collaboration, S. Eidelman et al., *Phys. Lett.***B 592** (2004) 1.

[61] E. Ponton and L. Wang, *JHEP***11** (2006) 018 [hep-ph/0512304].

[62] For a review, see Particle Data Group, J. Erler and P. Langacker, *Electroweak model and constraints on new physics Zeit. fur Phys.***C 15** (2000) 1.

[63] J. C. Pati and A. Salam, *Phys. Rev.* **D 10** (1974) 275;
R. N. Mohapatra and J. C. Pati, *Phys. Rev.***D 11**, (1975) 566;
G. Senjanović and R. N. Mohapatra, *Phys. Rev.***D 12** (1975) 1502.

[64] For discussion of W_R and Z' properties of the left-right models, see B. Kayser and J. Gunion, *Proceedings of the Snowmass workshop, 1984*, ed. R. Donaldson et al.;
M. Cvetic and P. Langacker, *Phys. Rev.***D 46** (1992) 4943; *Erratum-ibid.* **D 48** (1993) 4484 [hep-ph/9207216]; T. G. Rizzo, [hep-ph/0610104].

[65] C. Macesanu, C. D. McMullen and S. Nandi, *Phys. Lett.***B 546** (2002) 253 [hep-ph/0207269].

[66] G. Beall, M. Bander and A. Soni, *Phys. Rev. Lett.* **48** (1982) 848.

[67] M. A. B. Beg, R. V. Budny, R. N. Mohapatra and A. Sirlin, *Phys. Rev. Lett.***38** (1977) 1252, *Erratum-ibid.***39** (1977) 54.

[68] A. T. Azatov, [hep-ph/0703157].

[69] H. Arason, D. J. Castano, B. Keszthelyi, S. Mikaelian, E. J. Piard, P. Ramond and B. D. Wright, Phys. Rev. D **46**, 3945 (1992).

[70] D. J. Castano, E. J. Piard and P. Ramond, Phys. Rev. D **49**, 4882 (1994) [hep-ph/9308335].
S. P. Martin and M. T. Vaughn, Phys. Rev. D **50**, 2282 (1994) [hep-ph/9311340].