

TECHNICAL RESEARCH REPORT

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TR 2006-12



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Lifetime Maximizing Adaptive Power Control in Wireless Sensor Networks

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Abstract—Network lifetime is one of the most critical performance measures for wireless sensor networks. Various schemes have been proposed to maximize the network lifetime. In this paper we consider the lifetime maximization problem via a new approach: adaptive power control. We focus on the sensor networks that consists of a sink and a set of homogeneous wireless sensor nodes, which are randomly deployed according to a uniform distribution. Each node has the same initial energy and the same data generation rate. We formally analyze the lifetime maximizing adaptive power control problem by dividing the network into different layers and then modelling it as a linear programming problem, where the goal is to find an optimal way to adjust the transmission power and split the traffic such that the maximum energy consumption speed among all layers is minimized, and therefore the network lifetime is maximized. One surprising observation from the numerical results is that when every node can reach the sink directly, the optimal solution for each node is to send traffic either to its next inner layer or to the sink directly. This observation has also been justified by the theoretical analysis. The numerical results also show that the lifetime elongation can still be significant even when only those nodes in the innermost few layers are allowed to adaptively adjust their transmission power. We then propose a fully distributed algorithm, the Energy-Aware Push Algorithm (EAPA), and show through simulation that it can dramatically extend the network lifetime.

I. INTRODUCTION

Wireless sensor networks have drawn extensive attention in recent years due to potential applications that include environmental monitoring, industrial sensing and diagnostics, battlefield surveillance, target tracking, search and rescue, and disaster relief. However, owing to their compact form and extremely low cost, wireless sensor nodes are usually severely energy constrained. Furthermore, in a large-scale wireless sensor network

with thousands of nodes, replacing batteries will be either extremely difficult or even impossible, especially in harsh areas such as battlefields. This leads to one of the most critical performance measures for wireless sensor networks: *network lifetime*.

Aiming at maximizing the network lifetime of wireless sensor networks, various types of techniques have been proposed in the past few years. One way is to design energy-aware routing protocols, such as the schemes proposed in [1]–[3]. Another way is to apply energy-aware sleep scheduling, such as [4]–[6], which have addressed how and when to put the sensor nodes into sleep mode to reduce energy consumption. By performing in-network data aggregation which exploits the redundancy among the collected data, the network lifetime can also be extended, as demonstrated in [7]–[9]. By applying energy-efficient clustering and hierarchical routing, that is, dividing the network into multiple clusters and routing in a hierarchical structure, the network lifetime can also be extended, as shown in [10]–[12]. Moving the sink around to collect data from sensors to balance the energy consumption at sensor nodes is also a possible option [13]. Another straightforward way to extend the network lifetime is to allocate more resources into specific areas to relieve the bottleneck effect, as illustrated in [14]–[16].

In this paper, the lifetime maximization issue in wireless sensor networks is attacked by a totally different approach: adaptive power control. That is, we adaptively adjust the sensor nodes' transmission power based on the traffic pattern. Besides being able to dramatically extend the network lifetime when working alone, the approach proposed in this paper can also work together with the existing network lifetime maximization techniques to further extend the network lifetime. Although there exists considerable literature on applying power control in wireless ad hoc networks and sensor networks [17]–[20], few of them touch the lifetime maximization problem.

In this work we focus on the following wireless

sensor network model: the network consists of a set of homogeneous wireless sensor nodes and a sink, where each node needs to periodically send sensed data to the sink. Network lifetime is the time elapsing between network deployment and the moment when the first node dies [1]. If all sensor nodes have the same initial energy, which is usually the case, then the nodes around the sink will usually run out of energy very quickly due to the fact that they also need to relay packets for the other nodes. When this happens, nodes away from the sink may be left with considerable unused energy. Thus, the energy in the network is not efficiently utilized.

To utilize the network energy in a more efficient way, in this paper we suggest a new approach: Provided the network connectivity can be maintained, we will adaptively adjust the nodes' transmission power such that the fewer the number of packets a node needs to transmit, the higher the transmission power it uses. By increasing its transmission power, such a node reduces the relaying burden it imposes on other nodes. If the adjustment can be done in a proper way, the network lifetime can be significantly extended. One possible drawback of such adaptive power control is that it may increase interference. However, since nodes in a sensor network usually have a very low duty-cycle, with appropriate channel assignment and scheduling, such effects can usually be avoided or at least minimized.

To formally analyze the lifetime maximizing adaptive power control problem, we first divide the network into different layers according to the distance from the sink. Then the problem can be modeled as a linear program, where the goal is to find an optimal way for each layer to adjust its transmission power in order to maximize the network lifetime. After examining the numerical results, it is surprising to see that when there is no maximum transmission range constraint, the traffic of each node (including both the traffic it generates and the traffic it relays) only needs to be split into two parts, one is transmitted to the next layer, and the other to be directly sent to the sink. This observation has also been justified by our theoretical analysis. Another surprising observation is that the lifetime extension can still be significant even when we only allow nodes in the innermost layers to adaptively adjust their transmission power. For example, if we only allow the nodes in the innermost two layers to adjust their transmission power, the network lifetime can be extended by at least 50%.

Besides deriving the numerical results for the linear program model, which yields a centralized solution, we have also designed a fully distributed algorithm,

the Energy-Aware Push Algorithm (EAPA), for each node to dynamically split the traffic and adjust the transmission power. The simulation results show that the network lifetime can be dramatically extended by applying this algorithm. In fact, the results obtained using the distributed algorithm closely approach the optimal centralized results.

The rest of this paper is organized as follows. Section II describes the network model and the problem formulation. Section III presents the numerical results. The theoretical analysis is presented in Section IV. The distributed algorithm description and the simulation results are given in Section V. Finally Section VI concludes this paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper we focus on extending the network lifetime of wireless sensor networks. We consider the scenario where a set of homogeneous wireless sensor nodes are randomly deployed inside a (circular) area with a sink located in the center of the area. Each sensor node periodically submits the collected/sensed information to the sink. Based on the transmission power constraint as well as other factors, such as residual energy, each sensor node can either directly transmit the data to the sink, or request some other sensor nodes to relay the data.

Since in many situations the deployed sensor nodes are homogenous, in this paper we assume that all sensor nodes have the same initial energy, denoted by E . We assume that the data generation rate is the same for all sensor nodes. Since the sensors usually have very low-duty cycles, we will not consider interference, which can usually be handled through proper scheduling. In this paper, as suggested in [1], we refer to *network lifetime* as the time elapsing between network deployment and the moment when the first node dies.

Due to the rapid signal strength attenuation in wireless communication, shorter transmission range is usually preferred when only transmitting power consumption is considered. However, to maintain a certain connectivity, the transmission range should also exceed certain threshold [21]. Furthermore, since receiving power may also play an important role in energy consumption in short range wireless communication systems, smaller transmission range may not always be better than longer transmission range. By including the effect of receiving power consumption, there usually exists an optimal transmission range such that the total energy consumption can be minimized, as demonstrated in [22]. In this work we assume that a minimum transmission

range r_{min} is assigned to each sensor node. We also assume that each sensor node can adaptively adjust the transmission range, but only to integer multiples of r_{min} . Meanwhile, each sensor node will also have a maximum transmission range constraint, denoted by r_{max} .

The energy consumption model under consideration is as follows: Given the path loss exponent α , the power consumption with the transmission range r_{min} is normalized as 1; then for transmission range r , which is a multiple of r_{min} , the power consumption is given by

$$P(r) = \left(\frac{r}{r_{min}} \right)^\alpha. \quad (1)$$

In this paper we investigate how to extend the lifetime by adaptively adjusting the sensors' transmission power. Given that all nodes have the same initial energy, this is equivalent to minimizing the maximum energy consumption rate among all sensors by adjusting the sensors' transmission power. Then we can model this problem as a MIN-MAX problem. Given the minimum transmission range r_{min} , we can divide the sensor nodes in the network into different layers: for each sensor node, it belongs to the i^{th} layer if its distance to the sink lies in the range $((i-1) \times r_{min}, i \times r_{min}]$. To simplify the problem formulation, we further abstract the network model by assuming that the sensors in each layer are (deterministically) uniformly deployed. The results presented in Section V justify this assumption by showing that the lifetime extension obtained based on this model can be regarded as a (tight) upper-bound for the general case where nodes are randomly deployed.

We first consider a special one-dimensional case, where the sensors are equally spaced deployed along a line with the sink located at the center of line. We assume that each sensor will generate/collect g bits of data per unit time. Let $x_{i,j}$ ($i > j$) denote the amount of data (in bits) that each sensor in layer i will send to the nodes in layer j per unit time with transmission range $(i-j)r_{min}$, that is, the transmission power is $x_{i,j}(i-j)^\alpha$. Let N denote the total number of layers in the network. Then the lifetime maximizing adaptive power control problem can be modeled as a linear programming problem:

$$\min P \quad s.t. \quad (2)$$

$$\sum_{k=i+1}^N x_{k,i} + g = \sum_{j=0}^{i-1} x_{i,j}, \quad 1 \leq i \leq N \quad (3)$$

$$\sum_{j=0}^{i-1} x_{i,j} \times (i-j)^\alpha \leq P, \quad 1 \leq i \leq N \quad (4)$$

$$x_{i,j} \geq 0, \quad 1 \leq i \leq N, \quad 0 \leq j < i \quad (5)$$

Here condition (3) is for traffic conservation, that is, the amount of transmitted traffic should be equal to the amount of received plus generated traffic. Condition (4) indicates that the energy consumption rate of all nodes should be no more than P , where the node with the maximum energy consumption rate will determine the network lifetime. Condition (5) is introduced to guarantee that the solutions are feasible.

Now we study the more general two-dimensional situation. In this case, for all the sensors in the same layer (e.g., i^{th} layer), we assume they are deterministically uniformly distributed in the region between the circles of radii $(i-1)r_{min}$ and ir_{min} . The sink is located at the center of the disk. In the two-dimensional network, the number of sensors in different layers is different, and it is readily verified that if nodes are uniformly deployed, then the ratio between the number of sensors in layer i and layer j is $\frac{2i-1}{2j-1}$. Due to the symmetry of the network topology and traffic pattern, we can generally assume that when each sensor in layer i sends $x_{i,j}$ traffic to layer j , each sensor in layer j will receive $\frac{2i-1}{2j-1}x_{i,j}$ traffic from layer i in average. Similar to the one-dimensional case, the lifetime maximizing adaptive power control problem can be modeled as a linear programming problem:

$$\min P \quad s.t. \quad (6)$$

$$\sum_{k=i+1}^N \frac{2k-1}{2i-1} x_{k,i} + g = \sum_{j=0}^{i-1} x_{i,j}, \quad 1 \leq i \leq N \quad (7)$$

$$\sum_{j=0}^{i-1} x_{i,j} \times (i-j)^\alpha \leq P, \quad 1 \leq i \leq N \quad (8)$$

$$x_{i,j} \geq 0, \quad 1 \leq i \leq N, \quad 0 \leq j < i \quad (9)$$

III. NUMERICAL RESULTS

In this section, the numerical results will be presented for the lifetime maximizing adaptive power control problem described in section II. The MIN-MAX consuming power will be computed through linear programming; then the network lifetime can be computed. The baseline network lifetime is the case without power control, that is, every node always uses r_{min} as transmission range. We normalize $r_{min} = 1$; then the network with radius N has N layers.

Fig. 1 shows the results when there is no maximum transmission range constraint for all nodes. In the figure, *network radius* identifies the network size; *extended lifetime percentage* is the lifetime extended ratio by using adaptive power control, that is, if the extended lifetime is x%, then the whole lifetime is (1+x%) times the lifetime without power control. Fig. 1 shows that the

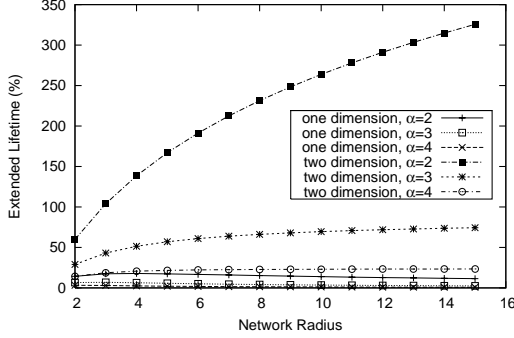


Fig. 1. Extended lifetime when there is no maximum transmission range constraint for all nodes

adaptive power control scheme is more effective for the two-dimensional network than for the one-dimensional network. For the two-dimensional network, the nodes around the sink need to relay more traffic, so they are more critical than their counterpart in the one-dimensional network in terms of energy consumption. Thus, when we smooth the energy consumption rate by adjusting transmission power, we can get more gain in the two-dimensional network. The results also show that the lower the path loss exponent, the greater the extended lifetime. When the path loss exponent is higher, the benefit of increased transmission range is reduced since more power is required to maintain a larger transmission range. So the proposed power control technique is less effective when the path loss exponent is higher.

Fig. 1 also shows that in the two-dimensional network, the network lifetime can be extended about 50% when the path loss exponent is 3, and around 25% when the path loss exponent is 4. When the path loss exponent is 2, the larger the network, the more the network lifetime can be extended, and when the network radius is 15, the network lifetime can be extended 340%. So far we do not know if there is an upper bound for the lifetime extension when path loss exponent is 2 in the two-dimensional network.

Fig. 2 shows the results for the MIN-MAX problem with one more constraint: the maximum transmission range for each node is 2. This can be modelled as a linear programming problem by adding constraints $x_{i,j} = 0, i - j > 2$ to equations (2-5) and (6-9). Similar to Fig. 1, Fig. 2 shows that when dimension is higher or the path loss exponent is lower, the network lifetime can be extended more. It also shows that when $r_{max} = 2$ for all nodes, the extended network lifetime percentage is almost the same for different network sizes in the two-dimensional case: 75%, 33% and 14% for path loss

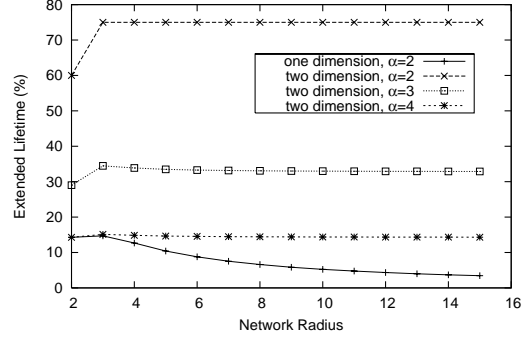


Fig. 2. Maximum transmission range for each node is 2

exponent being 2, 3, and 4 respectively.

We studied this phenomena and found that for the $\alpha = 2$ case, when the transmission range for the nodes beyond layer 3 is constrained to be r_{min} , the lifetime extension bound of 75% is still achievable. We assume there are k nodes in the first layer and then $(2i-1)k$ nodes in the i^{th} layer, each node generates g traffic and there are N layers in the network. To achieve the bound for the $\alpha = 2$ case, the nodes in the 1^{st} layer need to transmit $\frac{4}{7}N^2kg$ traffic to the sink; the nodes in the 2^{nd} layer need to transmit $\frac{3}{7}N^2kg$ traffic to the sink and transmit none to the 1^{st} layer; the nodes in the 3^{rd} layer need to transmit $\frac{4}{7}(N^2-1)kg$ traffic to the 1^{st} layer and $\frac{3}{7}(N^2-3)kg$ traffic to the 2^{nd} layer. Similarly, when $\alpha = 3$, the bound 32.8% is achievable when only the inner 4 layers are allowed to increase their transmission range beyond r_{min} ; and when $\alpha = 4$, the bound 12.8% is achievable when only the inner 3 layers are considered. This means that when the maximum transmission range is constrained to 2, the network lifetime is mainly determined by how the inner 4 layers transmit traffic.

The other interesting observation is that the extended lifetime decreases when the network size increases for the one-dimensional case. Through the analysis in the two-dimensional case, we know that when the maximum transmission range is 2, only the inner 3 layers count. On the other hand, in the one-dimensional case, the larger the network size, the smaller the relative traffic difference between the inner 4 layers when all nodes use same transmission range. So the gain by adjusting the transmission range is smaller when the network size is larger.

Fig. 3 shows the results for the two-dimensional network with path loss exponent equal to 2. We set maximum transmission range to be 2, 3, 4 respectively, as an additional constraint. The results show that the higher the maximum transmission range, the higher the

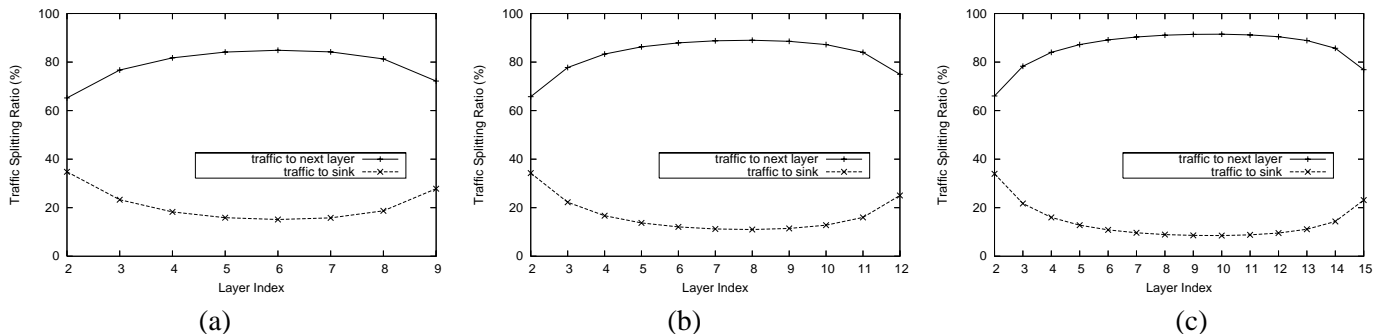


Fig. 4. Traffic splitting ratio for each layer. Traffic is split between the next inner layer and the sink. Two-dimensional network with path loss exponent $\alpha = 2$. (a) network radius = 9 (b) network radius = 12 (c) network radius = 15

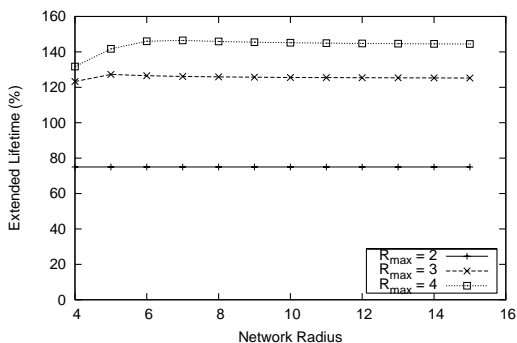


Fig. 3. Different maximum transmission range for two-dimensional network with path loss exponent $\alpha = 2$

extended network lifetime. This is easy to understand since the higher the maximum transmission range, the more flexibility in power control. The other observation is that the extended lifetime is almost the same for different network size, 75% for $r_{max} = 2$, 125% for $r_{max} = 3$ and 160% for $r_{max} = 4$. Similar to the explanation for Fig. 2, this is because when there is a maximum transmission range constraint for all nodes, the network lifetime is mainly determined by how the several innermost layers work.

Fig. 4 illustrates one interesting observation for the numerical result: when there is no constraint on the maximum transmission range, all nodes in the network will send traffic either to the sink or to the nodes in the next inner layer. Fig. 4 shows the results for the two-dimensional network with path loss exponent being 2. The traffic splitting ratio for each layer is shown for three different size networks in subfigures (a), (b), (c) respectively. Though the splitting ratio is different, all three subfigures have similar shape: the nodes in the middle layers send traffic to next inner layer with higher ratio, the nodes in the layers either near the sink or near the boundary send traffic to next inner layer with lower

ratio. It can be explained as follows: nodes in layers near the boundary have less traffic, so they can afford to send a higher percentage of traffic directly to the sink; nodes in layers closer to the sink can afford to send a higher percentage of traffic directly to the sink since their distance to the sink is small.

Fig. 5 shows the results for the comparison between the case where power control is only allowed in the inner i layers—i.e., the transmission range for layers beyond i is 1—and the case where it is allowed in all layers but the maximum transmission range is i . The results show that the normalized lifetime for the case where only the inner i layers is considered is less than the case when r_{max} is i since the case $r_{max} = i$ is more flexible than the case where power control is restricted to the inner i layers. Although the performance when power control is limited to the inner layers is not as good as that obtained when setting the maximum transmission range constraint, it is still attractive considering that the scheme is much simpler to implement. The results show that the network lifetime can be extended 50%, 91% and 126% when power control is limited to the inner 2, 3, and 4 layers respectively. The results also show that the difference between the two cases is smaller when the path loss exponent is higher. It is worth pointing out that when only the inner i layers are considered to adjust transmission power dynamically, the same traffic splitting property is observed as in Fig. 4 for the inner i layers, that is, each layer sends traffic either to the sink or the next inner layer.

IV. THEORETICAL ANALYSIS

The results in Section III suggest the following conjecture: when a sensor node can send traffic to the sink directly, then it should either send the traffic to the sink directly, or send to its next inner layer. In this section, we will formally prove this conjecture.

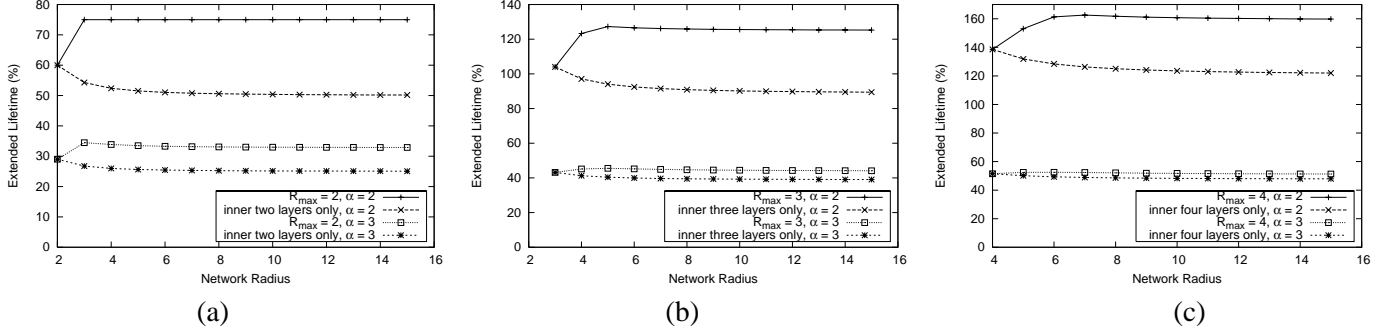


Fig. 5. Comparison between constraining transmission power adjustment to innermost several layers versus imposing a maximum transmission range constraint. Two-dimensional network. (a) only consider inner 2 layers vs $R_{max} = 2$ (b) only consider inner 3 layers vs $R_{max} = 3$ (c) only consider inner 4 layers vs $R_{max} = 4$

Let us first consider the one dimensional situation. We consider the following linear optimization problem.

$$\min P \quad s.t. \quad (10)$$

$$\sum_{k=i+1}^N x_{k,i} + g_i = \sum_{j=0}^{i-1} x_{i,j}, \quad 1 \leq i \leq N \quad (11)$$

$$\sum_{j=0}^{i-1} x_{i,j} \times (i-j)^\alpha \leq P, \quad 1 \leq i \leq N \quad (12)$$

$$x_{i,j} \geq 0, \quad 1 \leq i \leq N, \quad 0 \leq j < i \quad (13)$$

The optimization problem described in equations 10-13 is slightly more general than the problem described in equation 2-5 by using g_i in equation 11 instead of g . g_i represents the generation rate for the i^{th} layer.

Theorem 1: When $\alpha \geq 1$, there always exists an optimal solution to the optimization problem 10-13 with the following form: $\{x_{i,i-1} \geq 0, x_{i,0} \geq 0, x_{i,j} = 0, 1 \leq i \leq N, 1 \leq j \leq i-2\}$.

Proof: We refer to $\{x_{i,i-1} \geq 0, x_{i,0} \geq 0, x_{i,j} = 0, 1 \leq i \leq N, 1 \leq j \leq i-2\}$ as standard form. We will show that any optimal solution can be transformed into a solution in standard form without losing optimality.

Let $\{x_{i,j}\}$ be an optimal solution. If this optimal solution is not in the standard form, then we can transform $\{x_{i,j}\}$ to $\{\tilde{x}_{i,j}\}$ such that $\{\tilde{x}_{i,j}\}$ is in the standard form. The whole procedure is as follows:

We iteratively apply the following procedure: find the first link $x_{n,n-r}$ with the following properties:

- $\{x_{i,i-1} \geq 0, x_{i,0} \geq 0, x_{i,j} = 0, 0 < i < n, 1 \leq j \leq i-2\}$, that is, for all $i < n$, except $x_{i,0}$ and $x_{i,i-1}$, no other links can have non-zero value.
- For all $0 < j < n-r, x_{n,j} = 0$.

Next we show how to redistribute $x_{n,n-r}$ to the other links without increasing the MIN-MAX power. Specifically, we will redistribute the traffic on links $(n, n-r), (n, 0), (n, n-r+1), (n-r+1, 0), (n-r+1, n-r)$ in such a way that no traffic will go through the link $(n, n-r)$ and the MIN-MAX power among all layers does not increase. For layer $n-r$, its initial power is:

$$\begin{aligned} P_{n-r} &= (n-r)^\alpha x_{n-r,0} + x_{n-r,n-r-1} \\ &\geq x_{n-r+1,n-r} + x_{n,n-r} + \sum_{i=n+1}^N x_{i,n-r} + g_{n-r} \quad (14) \end{aligned}$$

where $x_{n-r+1,n-r} + x_{n,n-r} + \sum_{i=n+1}^N x_{i,n-r} + g_{n-r} = x_{n-r,0} + x_{n-r,n-r-1}$ is the traffic that layer $n-r$ needs to transmit.

We split traffic $x_{n,n-r}$ into two parts $\Delta x_{n,n-r+1}$ and $\Delta x_{n,0}$ which will be sent to the layer $n-r+1$ and the sink respectively. To conserve traffic and to keep the layer n power consumption unchanged, we need to have

$$\begin{cases} \Delta x_{n,0} + \Delta x_{n,n-r+1} = x_{n,n-r} \\ n^\alpha \Delta x_{n,0} + (n-r)^\alpha \Delta x_{n,n-r+1} = n^\alpha x_{n,n-r} \end{cases} \Rightarrow \begin{cases} \Delta x_{n,n-r+1} = \frac{n^\alpha - r^\alpha}{n^\alpha - (n-r)^\alpha} x_{n,n-r} \\ \Delta x_{n,0} = \frac{r^\alpha - (n-r)^\alpha}{n^\alpha - (n-r)^\alpha} x_{n,n-r} \end{cases} \quad (15)$$

After this traffic rerouting, the power consumed by layer n does not change. However, the incoming traffic of layer $n-r+1$ has been increased. Therefore we need to adjust layer $n-r+1$'s traffic too. We intend to keep the power consumption of layer $n-r+1$ the same, so we try to increase $x_{n-r+1,n-r}$ by $\Delta x_{n-r+1,n-r}$ and decrease $x_{n-r+1,0}$ by $\Delta x_{n-r+1,0}$. Traffic conservation and power consumption invariance imply that

$$\begin{cases} \Delta x_{n-r+1,n-r} - \Delta x_{n-r+1,0} = \Delta x_{n,n-r+1} \\ \Delta x_{n-r+1,n-r} = (n-r+1)^\alpha \Delta x_{n-r+1,0} \end{cases}$$

$$\Rightarrow \begin{cases} \Delta x_{n-r+1,n-r} = \frac{(n-r+1)^\alpha}{(n-r+1)^{\alpha-1}} \Delta x_{n,n-r+1} \\ \Delta x_{n-r+1,0} = \frac{1}{(n-r+1)^{\alpha-1}} \Delta x_{n,n-r+1} \end{cases} \quad (16)$$

Then there are two possible scenarios:

- Scenario 1: $\Delta x_{n-r+1,0} \leq x_{n-r+1,0}$
- Scenario 2: $\Delta x_{n-r+1,0} > x_{n-r+1,0}$

For scenario 1, $\{x_{i,j}\}$ is updated as follows:

$$\begin{aligned} x_{n,n-r}^1 &= 0 \\ x_{n,n-r+1}^1 &= x_{n,n-r+1} + \Delta x_{n,n-r+1} \\ x_{n,0}^1 &= x_{n,0} + \Delta x_{n,0} \\ x_{n-r+1,n-r}^1 &= x_{n-r+1,n-r} + \Delta x_{n-r+1,n-r} \\ x_{n-r+1,0}^1 &= x_{n-r+1,0} - \Delta x_{n-r+1,0} \\ x_{i,j}^1 &= x_{i,j}, \text{ for other } i, j \text{ and } i > n-r \\ x_{i,j}^1 &= \frac{\sum_{k=i+1}^N x_{k,i}^1 + g_i}{\sum_{k=i+1}^N x_{k,i} + g_i} x_{i,j}, i \leq n-r \end{aligned}$$

After updating, the traffic for layers over $n-r+1$ keeps same except layer n , so their power consumption do not change. The power consumption of layers n and $n-r+1$ do not change, and the incoming traffic of layer $n-r$ is changed by $\Delta x_{n-r+1,n-r} - x_{n,n-r} = \left(\frac{(n-r+1)^\alpha}{(n-r+1)^{\alpha-1}} \cdot \frac{n^\alpha - r^\alpha}{n^\alpha - (r-1)^\alpha} - 1 \right) x_{n,n-r}$. If $\frac{(n-r+1)^\alpha}{(n-r+1)^{\alpha-1}} \cdot \frac{n^\alpha - r^\alpha}{n^\alpha - (r-1)^\alpha} \leq 1$, the incoming traffic of layer $n-r$ will not increase. Thus, the power consumption of layer $n-r$ will not increase. Now, we show $\frac{(n-r+1)^\alpha}{(n-r+1)^{\alpha-1}} \cdot \frac{n^\alpha - r^\alpha}{n^\alpha - (r-1)^\alpha} \leq 1$.

$$\begin{aligned} &\frac{(n-r+1)^\alpha}{(n-r+1)^{\alpha-1}} \cdot \frac{n^\alpha - r^\alpha}{n^\alpha - (r-1)^\alpha} \leq 1 \\ \Leftrightarrow &[(n-r+1)r]^\alpha + (r-1)^\alpha \geq [(n-r+1)(r-1)]^\alpha + n^\alpha \end{aligned} \quad (17)$$

Noting that $(n-r+1)r+r-1 = (n-r+1)(r-1)+n = C$ where C is constant, equation (17) is equivalent to

$$(C-r+1)^\alpha + (r-1)^\alpha \geq (C-n)^\alpha + n^\alpha \quad (18)$$

Consider the function $f(x) = (C-x)^\alpha + x^\alpha$. This function is convex since $f''(x) \geq 0$ when $\alpha \geq 1$ and it is symmetric about $x = \frac{C}{2}$. It is easy to verify that the larger the value of $|C-2x|$, the larger the value of $f(x)$. So the inequality (17) is equivalent to

$$|(n-r+1)r - (r-1)| \geq |(n-r+1)(r-1) - n| \quad (19)$$

It is easy to check that inequality (19) holds for all $0 < n \leq N$, $0 < r < n-1$, so $\frac{(n-r+1)^\alpha}{(n-r+1)^{\alpha-1}} \cdot \frac{n^\alpha - r^\alpha}{n^\alpha - (r-1)^\alpha} \leq 1$, and $\Delta x_{n-r+1,n-r} \leq x_{n,n-r}$, that is, the incoming traffic of layer $n-r$ does not increase. We then recursively update the traffic from layer $n-r$ to layer 1. Since the

incoming traffic for layer $n-r$ does not increase, the traffic for all layers 1 to $n-r$ do not increase either, then their power consumption do not increase. Thus the MIN-MAX power does not increase in scenario 1.

Now let us consider scenario 2: $\Delta x_{n-r+1,0} > x_{n-r+1,0}$. In this scenario, we cannot decrease $x_{n-r+1,0}$ by the whole amount $\Delta x_{n-r+1,0}$. Consequently, $\{x_{i,j}\}$ is updated as follows:

$$\begin{aligned} x_{n,n-r}^1 &= 0 \\ x_{n,n-r+1}^1 &= x_{n,n-r+1} + \Delta x_{n,n-r+1} \\ x_{n,0}^1 &= x_{n,0} + \Delta x_{n,0} \\ x_{n-r+1,n-r}^1 &= x_{n-r+1,n-r} + x_{n-r+1,0} + \Delta x_{n,n-r+1} \\ x_{n-r+1,0}^1 &= 0 \\ x_{i,j}^1 &= x_{i,j}, \text{ for other } i, j \text{ and } i > n-r \\ x_{i,j}^1 &= \frac{\sum_{k=i+1}^N x_{k,i}^1 + g_i}{\sum_{k=i+1}^N x_{k,i} + g_i} x_{i,j}, i \leq n-r \end{aligned}$$

After updating, the power consumption of layer n keep same. The power consumption of layer $n-r+1$ is

$$P_{n-r+1}^1 = x_{n-r+1,n-r} + x_{n-r+1,0} + \Delta x_{n,n-r+1}.$$

Next we will show $P_{n-r+1}^1 \leq P_{n-r}$. Since $\Delta x_{n-r+1,0} > x_{n-r+1,0}$, we have

$$\begin{aligned} x_{n-r+1,0} + \Delta x_{n,n-r+1} &< \Delta x_{n-r+1,0} + \Delta x_{n,n-r+1} \\ &= \Delta x_{n-r+1,n-r} \leq x_{n,n-r} \end{aligned} \quad (20)$$

We then have

$$P_{n-r+1}^1 < x_{n-r+1,n-r} + x_{n,n-r} \leq P_{n-r} \quad (21)$$

So the power consumption of layer $n-r+1$, P_{n-r+1}^1 , is smaller than the original MIN-MAX power.

The incoming traffic of layer $n-r$ is changed by $x_{n-r+1,0} + \Delta x_{n,n-r+1} - x_{n,n-r} < 0$. Thus, the incoming traffic of layer $n-r$ is decreased, so the power consumption of layer $n-r$ will not increase. The power consumption of all other layers do not increase either. Therefore, in scenario 2, after updating, the MIN-MAX power does not increase either.

Thus, after this procedure, $\{x_{i,j}\}$ is updated to $\{x_{i,j}^1\}$ by redistributing traffic on links $(n, n-r)$, $(n, 0)$, $(n, n-r+1)$, $(n-r+1, 0)$, $(n-r+1, n-r)$ to delete the traffic on $(n, n-r)$, and the MIN-MAX power does not increase. We keep executing this procedure until the solution is in the standard form. Since each application of this procedure does not increase the MIN-MAX power, the theorem is proved. ■

Similarly, we consider two-dimensional case:

$$\min P \quad s.t. \quad (22)$$

$$\sum_{k=i+1}^N \frac{2k-1}{2i-1} x_{k,i} + g_i = \sum_{j=0}^{i-1} x_{i,j}, \quad 1 \leq i \leq N \quad (23)$$

$$\sum_{j=0}^{i-1} x_{i,j} \times (i-j)^\alpha \leq P, \quad 1 \leq i \leq N \quad (24)$$

$$x_{i,j} \geq 0, \quad 1 \leq i \leq N, \quad 0 \leq j < i \quad (25)$$

Theorem 2: When $\alpha \geq 1$, there always exists an optimal solution to the optimization problem 22-25 with the following form: $\{x_{i,i-1} \geq 0, x_{i,0} \geq 0, x_{i,j} = 0, 1 \leq i \leq n, 1 \leq j \leq i-2\}$

Proof: Using a similar procedure as in the proof of Theorem 1, it is easy to show this theorem holds. ■

Remarks: based on above analysis we can see that no matter what kind of traffic generation pattern, Theorem 1 and 2 always holds.

V. DISTRIBUTED ALGORITHM AND PERFORMANCE

In this section, we propose a fully distributed algorithm, *Energy Aware Push Algorithm* (EAPA), to adaptively adjust the transmission power for each sensor. We also present simulation results for EAPA.

A. Energy Aware Push Algorithm

From Theorem 2, we know that the optimal solution for the lifetime maximizing power control problem has the property that a layer either sends its traffic directly to the sink or to the next inner layer, provided the sink is reachable for this layer. Furthermore, the object is to minimize the maximum energy consumption rate so that the lifetime is maximized. This suggests that if the residual energy of a node is more than the residual energy of the corresponding next hop node(s) in the next layer toward the sink, it should send its packets directly to the sink; otherwise, it should send the packets to the node(s) in the next layer toward the sink.

The proposed algorithm EAPA works as follows: Suppose the routing table is given according to some routing scheme that specifies for each node in layer i , one or more next hop nodes that are within a distance r_{min} from the given node. We refer to the next hop nodes as the *parents* of the given node. Each node keeps record of the residual energy of its parent(s). This can be done by letting the nodes broadcast their residual energy periodically. When a node wants to send some packets, it first checks whether its residual energy is higher than all of its parents; if it is, then it sends the packets to the

sink directly. Otherwise it sends the packets to the parent with highest residual energy. The formal description is in Algorithm 1.

Algorithm 1 Energy Aware Push Algorithm

Input: Initial energy E , transmission power P_0 for sending traffic using transmission range r_{min} , transmission power $i^\alpha P_0$ for sending traffic to the sink and the routing table, i.e., a list \mathcal{P} maintaining its parents

```

1:  $E_{residual} = E$ 
2: while ( $E_{residual} > 0$ ) do
3:   Let the total traffic need to be sent is  $T$ 
4:   if ( $T \cdot P_0 > E_{residual}$ ) then
5:     break
6:   end if
7:   Find the parent  $p$  with maximum residual energy
   from  $\mathcal{P}$  with residual energy  $E_{residual}^p$ 
8:   if ( $E_{residual}^p \geq E_{residual}$ ) then
9:     send traffic  $T$  to  $p$ 
10:     $E_{residual} = E_{residual} - T \cdot P_0$ 
11:   else if ( $E_{residual} \geq T \cdot i^\alpha P_0$ ) then
12:     send traffic  $T$  to the sink
13:     $E_{residual} = E_{residual} - T \cdot i^\alpha P_0$ 
14:   else
15:     send traffic  $T$  to  $p$ 
16:     $E_{residual} = E_{residual} - T \cdot P_0$ 
17:   end if
18: end while

```

In Algorithm 1, we assume the routing table is fixed, the residual energy for each parent is updated periodically and the transmission range can only be adjusted to integer multiples of r_{min} , that is, when the distance to the destination is in $((i-1)r_{min}, ir_{min}]$, transmission range ir_{min} is used.

B. Simulation Results

Geographic routing [23] is simply forwarding data packets to the neighbor geographically closest to the destination. We modify geographic routing slightly and refer to it as *enhanced geographic routing*. In enhanced geographic routing, each node will select two neighbors (within r_{min}) that are geographically closest to the sink as its parents and forward data packets to the parent with higher residual energy. In the simulation, we use enhanced geographic routing to compute the routing table for each sensor node.

The simulation is set up as follows: Sensor nodes are randomly deployed in a disk according to 2-D uniform

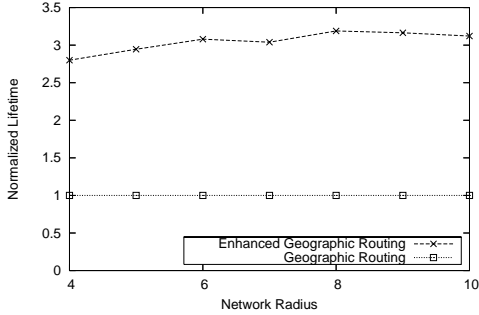


Fig. 6. Comparison between geographic routing and enhanced geographic routing

distribution; the size of the disk is variable; the sink is located at the center of the disk; the node density is 25; path loss exponent α is 2; minimum transmission range r_{min} is 1; each sensor has 1,000,000 units initial energy; each sensor generates one packet per unit time; transmitting one data packet using minimum transmission range requires one unit of energy. Each point on the figure is the average performance of 100 different randomly generated networks.

We first compare the network lifetime difference between traditional geographic routing and the enhanced geographic routing. Fig. 6 shows the comparison between geographic routing and enhanced geographic routing. We normalize the lifetime for geographic routing as 1. It shows that the lifetime for enhanced geographic routing is almost 3 times of that geographic routing. This is because the nodes' positions are asymmetric due to the randomness, even though the nodes are deployed according to uniform distribution. Due to the asymmetry of the nodes' positions, there may exist some bottleneck nodes that are required to relay packets for many nodes when traditional geographic routing is used. When enhanced geographic routing is used, this kind of bottleneck effect is diminished so the network lifetime is significantly increased.

Fig. 7 shows the extended network lifetime by using EAPA in various scenarios. The baseline is the network lifetime using enhanced geographic routing only. It shows that when there is no maximum transmission range constraint, the normalized lifetime will increase when the network size increases, which is consistent with the numerical results, and the lifetime can be extended almost 350% when network radius is 10. It also shows that when only the nodes in the several inner layers execute EAPA, the network lifetime still can be extended significantly: 75% for inner 2 layers case, 130% for inner

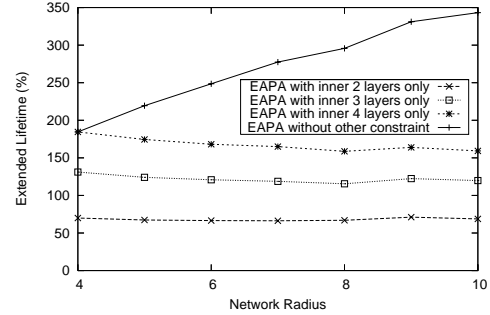


Fig. 7. Normalized lifetime for EAPA in various scenarios

3 layers case and 175% for inner 4 layers. And similar to numerical result, when the power control is restricted to several inner layers, the normalized network lifetime is independent of network size.

By observing Fig. 7 closely, we found that the extended lifetime by applying EAPA is even higher than the numerical result obtained by solving the linear program. On the surface, this seems surprising, since the linear program yields a centralized algorithm. We use Fig. 8 to explain this phenomena. In each subfigure, there are 3 lines. We normalize the ratio between network lifetime with power control and without power control computed from the numerical result as 1. We use two different baselines to show the EAPA result: the simulation baseline which is obtained from simulations on random networks by using enhanced geographic routing; the theoretical baseline which is computed theoretically based on the ideal network which is described in section II. It turns out the simulation baseline is less than the theoretical baseline which is easy to understand: due to asymmetry in the random network, there exist some bottlenecks that significantly reduce the network lifetime. Due to the low simulation baseline, when we compute the extended lifetime based on the simulation baseline, we get a higher ratio. Had we used traditional geographic routing (instead of enhanced geographic routing) to obtain the baseline, the bottlenecks would have been more severe and the ratios for EAPA correspondingly higher.

VI. CONCLUSION

Battery powered wireless sensor networks are extremely energy constrained. To conquer this problem, various schemes have been proposed. In this paper we study the lifetime maximization problem through a new approach: adaptive power control. We formulate this lifetime maximizing problem as a linear program. The

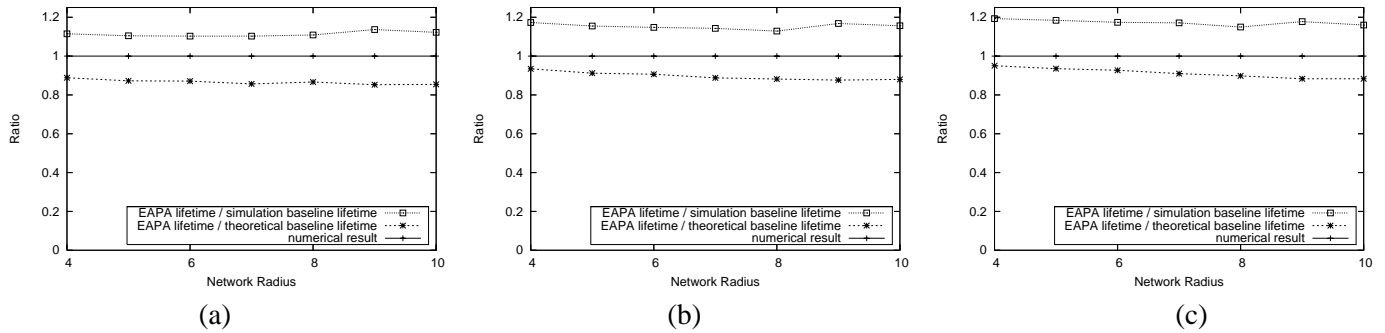


Fig. 8. Using numerical results as baseline vs using simulation results as baseline for EAPA (a) EAPA restricted to inner 2 layers (b) EAPA restricted to inner 3 layers (c) EAPA restricted to inner 4 layers

numerical results obtained suggest a surprising conjecture, namely that if a node can reach the sink directly, the optimal way for it to split the traffic is to either send to the next layer toward the sink (i.e., using the minimum transmission range) or send directly to the sink. We then theoretically analyze this optimization problem and prove the conjecture. Besides the centralized linear programming model, we also proposed a fully distributed algorithm: Energy-Aware Push Algorithm. The simulations show that EAPA can extend the network lifetime dramatically; even when we impose the restriction that only nodes around the sink can adaptively adjust their transmission power, the resulting lifetime extension is still significant.

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