TECHNICAL RESEARCH REPORT

On the Fairness of the Reverse-Link MAC Layer in cdma2000 1xEV-DO

by Peerapol Tinnakornsrisuphap, Christopher Lott

TR 2004-2

On the Fairness of the Reverse-Link MAC Layer in cdma2000 1xEV-DO

Peerapol Tinnakornsrisuphap †   and   Christopher Lott ‡

†Department of Electrical and Computer Engineering
University of Maryland, College Park

‡Qualcomm Inc.

Email: `peerapol@umd.edu, clott@qualcomm.com`

April 1, 2004

Abstract

We investigate the fairness of two reverse-link MAC algorithms in cdma2000 1xEV-DO High Rate Packet Data systems. Following the framework proposed by Kelly [1] for Internet congestion-control, we formulate a utility maximization problem and provide a simple sufficient condition for both algorithms to converge to the solution of this problem. Furthermore, we identify that the solution of this problem corresponds to the equal throughput fairness criteria, i.e., all the access terminals have equivalent throughput at the equilibrium.

1   Introduction

The third generation (3G) wireless standard cdma2000 1xEV-DO, developed by Third Generation Partnership Project 2 (3GPP2), is designed in response to an increasing demand for high-speed wireless data service. The technology presents a breakthrough in providing very high data rate downstream Internet access to users. Meanwhile, the upstream traffic channel also has become increasingly important due to development of new applications, such as camera phones, interactive games and videoconferencing.

The reverse traffic channel of cdma2000 1xEV-DO system utilizes Code Division Multiple Access (CDMA) physical layer architecture to share the available bandwidth. On top of the physical layer, a medium access control (MAC) layer is utilized to provide an adaptive scheme to adjust the transmission rate of the access terminals (ATs) to fairly share and efficiently utilize the available bandwidth. A simple rate control scheme for the reverse traffic channel MAC is introduced in [3] and is adopted as a part of the IS-856 standard. Subsequently, an enhanced scheme [4] has been adopted for IS-856 Rev A to further improve the system efficiency. Both of these MAC algorithms can be viewed as distributed, feedback-based resource allocation schemes where the interference level at the basestation transceiver (BTS) is limited and the transmission rate at the ATs is adjusted in response to the interference level.

In this paper, we investigate the rate control algorithm of both [3] and [4] in the reverse traffic channel (upstream) of cdma2000 1xEV-DO system. As is always the case in any distributed resource allocation mechanism (for example, the TCP congestion-control mechanism in the Internet), the key properties of such schemes are *fairness* and *stability*. Here we consider the reverse traffic channel in an isolated sector where the ATs have full data buffer and are not power-limited. Under the utility maximization criterion in the framework proposed by Kelly [1], we identify the implicit utility functions of the rate control algorithms. Further, we provide a simple sufficient condition for both algorithms to be asymptotically stable and to converge to the fair rate allocation.

This paper is organized as follows. Section 2 presents simple models of the reverse traffic channel MAC algorithms. A utility maximization problem is formulated in Section 3, and then the fairness and stability of the reverse traffic channel MAC algorithms are considered. Simulation results supporting the analysis are given in Section 4. Finally, the paper concludes with suggestion for future work in Section 5.

Some words on notation in use. The indicator function of an event A is given by $\mathbf{1}[A]$. For any $x, y, z \in \mathbb{R}$ and $y < z$, let $x \wedge y = \min(x, y)$, $x \vee y = \max(x, y)$ and $[x]_y^z = (x \wedge z) \vee y$, i.e., x is restricted to the range $[y, z]$.

2 The Reverse-Link MAC layer

In this section, we briefly describe the operation of the reverse traffic channel MAC layer in cdma2000 1xEV-DO system. The core of the reverse-link (RL) traffic channel is a distributed, feedback-based mechanism which can be separated into two components, i.e., the AT algorithm and the BTS algorithm. The AT algorithm autonomously adjusts each AT transmission rate/power according to the feedback signal from the BTS in order to maximize the throughput while keeping the interference level below a certain threshold.

We consider two RL MAC algorithms in this paper. First, we describe a simple model of the RL MAC algorithm in IS-856. Detailed description of the algorithm can be found in [3] and [5]. Later, a simplified model of the newly proposed *Enhanced* RL MAC algorithm [4] will be described.

Let N be the number of ATs that share the same BTS. Time is assumed to be slotted into contiguous timeslots. A timeslot is equal in duration to a subframe, which is the duration that each AT updates its transmission rate.

2.1 Access Terminal Overview

For AT $i = 1, \ldots, N$, denote $R_i(t)$ as its transmission rate in timeslot $[t, t+1)$. Let $P_i(t)$ denote the current transmit pilot power of AT i in the timeslot. The pilot power $P_i(t)$ is controlled by the power control algorithm which tries to equalize the received pilot power from each AT at the BTS. Let $T_i(t)$ denote the ratio of the total transmit power to the pilot power (T2P) in linear scale of AT i in the timeslot $[t, t + 1)$, i.e., the total transmit power of AT i in timeslot $[t, t + 1)$ will be $T_i(t)P_i(t)$.

2.1.1 IS-856 RL MAC

The set of permissible transmission rates in the IS-856 system is structured as $\mathcal{R} = \{0, R_{min}, 2R_{min}, \ldots, R_{max}/2, R_{max}\}$. In other words, for $n = 2, 3, \ldots$ the n-th member of the set has the value $2^{n-2}R_{min}$. In IS-856 system, $T_i(t)$ depends on the transmission rate, i.e.,

$$T_i(t) = F_R(R_i(t)), \tag{1}$$

for some $\mathcal{R} \to \mathcal{T}$ mapping F_R, where we denote the set of permissible T2P as $\mathcal{T} = \{1, T_{min}, \ldots, T_{max}\}$. Equivalently, we have the following relationship

$$R_i(t) = F_T(T_i(t)), \tag{2}$$

for some $\mathcal{T} \to \mathcal{R}$ mapping F_T. The set $\mathcal{T}, \mathcal{R}$ and the mappings F_R, F_T depend on the physical layer of the system.

The transmission rate in the next timeslot of AT i depends on the marking mechanism at the BTS and its current transmission rate. The marking mechanism signals the ATs that the load level at the BTS exceeds the set threshold by setting the *Reverse Activity Bit (RAB)*. Each AT then responds through the following probabilistic algorithm. If RAB is set, AT i reduces the transmission rate in the next timeslot by half with probability $p(R_i(t))$ for some $\mathbb{R}_+ \to [0, 1]$ mapping p. Otherwise, it retains its current transmission rate. On the other hand, if RAB is not set, AT i doubles its transmission rate in the next timeslot with probability $q(R_i(t))$ for some $\mathbb{R}_+ \to [0, 1]$ mapping q or keeps its transmission rate the same otherwise. If we represent the RAB bit AT i received in the beginning timeslot $[t + 1, t + 2)$ by $M(t + 1)$ (i.e., $M(t + 1) = 1$ implies that the RAB bit is set in the beginning of timeslot $[t + 1, t + 2)$ and $M(t + 1) = 0$ if not set), then the complete evolution of the transmission is

$$
\begin{aligned}
R_i(t + 1) \;=\; & M(t + 1)\mathbf{1}\left[U_i(t + 1) < p(R_i(t))\right] \left(\frac{R_i(t)}{2} \vee R_{min}\right) \\
& + M(t + 1)\mathbf{1}\left[U_i(t + 1) \geq p(R_i(t))\right] R_i(t) \\
& + (1 - M(t + 1))\mathbf{1}\left[U_i(t + 1) < q(R_i(t))\right] (2R_i(t) \wedge R_{max}) \\
& + (1 - M(t + 1))\mathbf{1}\left[U_i(t + 1) \geq q(R_i(t))\right] R_i(t), \qquad (3)
\end{aligned}
$$

where we let $\{U_i(t + 1), \; i = 1, 2, \ldots, t = 0, 1, \ldots\}$ be a collection of $[0, 1]$-uniform i.i.d. rvs.

2.1.2 Enhanced RL MAC

Enhanced RL MAC [4] adjusts and controls directly the AT transmit power instead of the transmission rate. This is done in part to improve the control of system loading and to adjust early termination goals in the hybrid ARQ.

Enhanced RL MAC also operates in a discrete-time fashion. For AT $i = 1, \ldots, N$, we still have a similar relationship between the transmission rate and power, i.e., $T_i(t) = F_R(R_i(t))$ for some $\mathcal{R} \to \mathcal{T}$ mapping F_R and $R_i(t) = F_T(T_i(t))$ for some $\mathcal{T} \to \mathcal{R}$ mapping F_T. The transmit power in the timeslot $[t + 1, t + 2)$ also responds to the RAB feedback information from the BTS algorithm similar to IS-856, however, there are now two rvs representing the transmit power: (i) the actual instantaneous transmission

T2P $T_i(t + 1)$ which is restricted to the discrete set $\mathcal{T}$ (ii) the allocated resource T2P $\hat{T}_i(t + 1)$ which is an $\mathbb{R}$-valued continuous rv. Let $\Delta T_i(t + 1)$ denote the difference in the allocated resource power between timeslot $[t, t+1)$ and $[t + 1, t + 2)$. Then

$$\Delta T_i(t + 1) = (1 - M(t + 1))g_u(\hat{T}_i(t)) - M(t + 1)g_d(\hat{T}_i(t)), \qquad (4)$$

where g_u, g_d are some $\mathcal{T} \to \mathbb{R}_+$ mappings. which control $\hat{T}$ ramping.

In order to determine the actual transmit power, a token bucket mechanism is utilized to map the continuous allocation $\hat{T}$ to the discrete allocation T. The token level $\beta_i(t)$ represents the available power budget that AT i can utilize at the end of the given timeslot $[t, t + 1)$. After determining the proposed transmit power, the token level is filled by $\hat{T}_i(t + 1)$. If we assume the ATs always transmit with the maximum allowable power, then

$$T_i(t + 1) = \max_{t \in \mathcal{T}} \left(t \leq (\beta_i(t) + \hat{T}_i(t + 1)) \wedge \beta_{max} \right), \qquad (5)$$

where β_{max} is the token bucket size. After the transmission, the token level is drained by $T_i(t + 1)$, the actual transmit power. So the token level at the end of the timeslot becomes

$$\beta_i(t + 1) = (\beta_i(t) + \hat{T}_i(t + 1)) \wedge \beta_{max} - T_i(t + 1). \qquad (6)$$

Note that $\beta_i(t) \geq 0$ for all $t = 1, 2, \ldots$.

In this manner, the variable T is chosen from a discrete set such that the average allocation matches that of the continuous variable $\hat{T}$, which is the resource allocated to the AT. The T value in effect dithers among discrete allocations to achieve the desired average power utilization.

2.2 Basestation Model

In each timeslot, the BTS receives the signal from all ATs. The accumulated power of the received signal is then used to calculate the *rise over thermal (RoT)* at the BTS, which represents the level of interference at the BTS. For proper system performance, the BTS needs to control the RoT to be below a certain threshold for the majority of timeslots, e.g., below 7 dB in 99% of the timeslots [7], to limit the level of interference while trying to maximize the throughput. In order to accomplish this, the BTS uses the aforementioned RAB bit to signal the ATs to reduce their transmission rate (or power) and

hence reduce the RoT level. The RoT in timeslot $[t, t + 1)$ is calculated as follows [6]:

$$Z(t) = 10 \log_{10} \left(1 + \sum_{i=1}^{N} \frac{T_i(t) P_R}{N_0 W} \right), \tag{7}$$

where $N_0 W$ represents the background noise power (including the intercell interference) in watts and we assume perfect power control, i.e., the pilot power from each AT at the BTS is exactly P_R watts – the power necessary for successful decoding.[1]

The mechanism in which the BTS tries to control the RoT value to be under a certain threshold is by setting the RAB to signal the ATs to reduce the rate/power whenever the RoT exceeds the threshold, i.e.,

$$M(t + 1) = \Gamma(Z(t)), \tag{8}$$

where $\Gamma : \mathbb{R}_+ \rightarrow \{0, 1\}$ is a step function with the threshold at Z_{thresh} dB, i.e.,

$$\Gamma(x) = \begin{cases} 0, & x < Z_{thresh} \\ 1, & x \geq Z_{thresh}. \end{cases} \tag{9}$$

Equivalently, the threshold and the measure can be done in term of load $X(t)$ as in [3]. The relationship between the RoT and load is given by [6]

$$Z(t) \approx \frac{1}{1 - X(t)}, \tag{10}$$

for $X(t) \in [0, 1]$. Throughout this paper, we will only focus on the threshold and measure of RoT as the equivalent mechanism in terms of load can be found using (10).

3   Fairness and the Utility Maximization Model

We now consider the problem of allocating the total power to pilot ratio (T2P) T_i for each AT i, $i = 1, \ldots, N$ as a *competitive market* problem in economics in order to formulate the fairness criteria of the algorithm. The

---

[1]In the actual system, the pilot power can fluctuate even with a perfect power control. However, such fluctuation is small when no single AT dominate the RoT at the BTS. Further discussion on this assumption is available in the appendix.

competitive market model comprises of scarce resource (margin of the RoT below the threshold in our problem) and two agents, i.e., the producers (BTS) and the consumers (ATs).

In our problem, each AT consumes a portion of the available interference power budget at the BTS. Since the acceptable interference level is limited, the BTS utilizes the RAB to signal the price of the resource to the ATs. In a competitive market, price is adjusted until the supply equals demand at which point the market is in equilibrium and the resulting allocation is *fair* and optimal. We use the utilitarian criterion (sometimes referred to as utility maximization), where the equilibrium is achieved for the allocation that results in the sum of the utilities to be the greatest. More details on microeconomic theory and competitive market model can be found in [9].

In this section, we first formulate a utility maximization problem following the framework proposed by Kelly [1] and later show that both of the RL MAC algorithms described in Section 2 approximate distributed algorithms which solve this utility maximization problem. Hence, both of the RL MAC algorithms are fair in the utilitarian criterion. Moreover, this fairness criteria is simple as each AT has equivalent throughput at the equilibrium.

3.1   The System Problem

For $i = 1, \ldots, N$, AT i has its own pilot channel, and its pilot power $P_i > 0$ is assumed to be perfectly controlled, i.e., the received pilot power at the BTS is equal to some constant P_R watts for each AT. The actual transmit power of AT i is determined by T2P factor $T_i > 0$ of the pilot power, i.e., the actual transmit power for AT i is $T_i P_i$. Here we assume that ATs are not power-limited.

The objective of the problem is to maximize the sum of the utility of the ATs which depends on T_i. Denote the utility function of AT i by $U_i :$ $\mathbb{R}_+ \rightarrow \mathbb{R}$. The utility function U_i represents the consumer (or AT) preference or satisfaction towards the commodity (T_i). The consumer's satisfaction typically increases with diminishing return as the amount of commodity the consumer receives increases.

Initially, we assume that U_i is increasing, differentiable and strictly concave. Further, we assume that the utility of AT i is equal to $U_i(T_i)$ with no regards to the actual transmit power. Under these assumptions, we can then derive the *fair* T2P allocations in Proposition 1 and Corollary 1 using a standard convex optimization technique. Then in Proposition 2, we derive the

7

implicit utility functions associated with the distributed RL MAC algorithms and identify a simple condition which guarantees that the assumptions on the utility function in Proposition 1 and Corollary 1 are satisfied.

The maximization problem is constrained by the requirement that the rise-over-thermal (RoT) threshold, i.e., the interference level, is effectively restricted to be below a threshold Z_{thresh} dB. Let $N_0 W$ be the noise power in the system, then this constraint is equivalent to [6]:

$$10 \log_{10} \left(1 + \sum_{i=1}^{N} \frac{T_i P_R}{N_0 W} \right) \leq Z_{thresh} \tag{11}$$

$$\Longleftrightarrow \quad \sum_{i=1}^{N} \frac{T_i P_R}{N_0 W} \leq 10^{Z_{thresh}/10} - 1$$

$$\Longleftrightarrow \quad \sum_{i=1}^{N} T_i \leq C, \tag{12}$$

where $C = \frac{N_0 W}{P}(10^{Z_{thresh}/10} - 1)$.

The objective of the problem is to maximize the sum of the utility. Therefore, we can pose the optimization problem as follows:

$$\max \sum_{i=1}^{N} U_i(T_i) \tag{13}$$

$$\text{subject to} \quad T_i \geq 0$$

$$\sum_{i=1}^{N} T_i \leq C.$$

The first result follows from a straightforward convex optimization.

Proposition 1 *Assuming $T_i \in \mathbb{R}_+$, $i = 1, \ldots, N$. Then the solution to the system problem (13) satisfies $U'_k(T_k) = U'_l(T_l)$, $k, l = 1, 2, \ldots, N$.*

Proof. The proposition follows directly by applying Lagrange multipliers. ∎

The following result is a simple corollary of Proposition 1. It states that when the utility functions are uniform, then the fair rate allocation is the equal rate (or T2P) allocation.

Corollary 1 *Assuming the condition in Proposition 1 with $U_1 = U_2 = \ldots = U_N$, then $T_k = T_l$ for $k, l = 1, \ldots, N$.*

The optimization problem (13) is similar to the classic utility maximization problem considered by Kelly [1]. In the subsequent work, it is shown that this utility maximization problem can be separated into the following users and network problem. For $i = 1, \ldots, N$, the user i problem is

$$USER_i(U_i; \lambda_i): \quad \max U_i(w_i/\lambda_i) - w_i \tag{14}$$
$$\text{subject to} \quad w_i \geq 0,$$

where w_i represents the budget of user i, and λ_i is the price/unit of the resource for user i. Therefore, $w_i/\lambda_i = T_i$ is the amount of resource available under this budget.

Let $\mathbf{w} = [w_1 \ w_2 \ \ldots \ w_N]$, the network problem is the following *proportional fair* problem.

$$NETWORK(C; \mathbf{w}): \quad \max \sum_{i=1}^{N} w_i \log(T_i) \tag{15}$$
$$\text{subject to} \quad \sum_{i=1}^{N} T_i \leq C$$
$$T_i \geq 0.$$

The solution to the network problem is to distribute the resource to each user proportional to the budget of the user.

It is shown in [1] that there always exists a shadow price vector $\lambda := [\lambda_1 \ \lambda_2 \ \ldots \ \lambda_N]$ such that λ_i, T_i solves $USER_i(U_i; \lambda_i)$ and $\mathbf{T} := [T_1 \ \ldots \ T_N]$ solves $NETWORK(C; \mathbf{w})$. Further, $T_1, \ldots, T_N$ are the unique solution to $SYSTEM(U, C)$.

3.2 The Distributed Algorithm

Since the total utility maximization problem can be separated into users and network problems, Kelly, Maulloo and Tan [2] propose a distributed algorithm based on a penalty function that solves a modified $NETWORK(C; \mathbf{w})$ problem. We modify this distributed algorithm in a form suitable to our problem. Consider the following dynamical system:

$$\frac{dT_i}{dt} = k(T_i(t)) \left(w_i - T_i(t) f \left(\sum_{j=1}^{N} T_j(t) \right) \right), \quad i = 1, \ldots, N, \tag{16}$$

where the penalty function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, increasing and not identically zero. For some fixed $\epsilon > 0$, the mapping $k : \mathbb{R}_+ \to (\epsilon, \infty]$ is a gain function. It is easy to see that the dynamical system (16) tries to equalize $T_i(t)f\left(\sum_{j=1}^{N} T_j(t)\right)$ to w_i.

Now considering the user problem, if $U_i, i = 1, \ldots, N$ is strictly concave and differentiable, then the solution to the user problem is $\lambda_i = w_i/T_i = U_i'(T_i)$. Suppose user i can vary its w_i smoothly depending on current value of T_i to keep track of its optimal solution, then $w_i(t) = T_i(t) \cdot U_i'(T_i(t))$. Substitute this into (16) and the complete dynamical system is

$$\frac{dT_i}{dt} = k\left(T_i(t)\right) T_i(t) \left(U_i'(T_i(t)) - f(\sum_{j=1}^{N} T_j(t)) \right), \quad i = 1, \ldots, N. \quad (17)$$

The following theorem is slightly modified from [2]. It shows that the dynamical system (17) converges to the approximated solution of the system problem.

Theorem 1 *The $\mathbb{R}_+^N \to \mathbb{R}$ mapping*

$$\mathcal{U}(\mathbf{T}) = \sum_{i=1}^{N} w_i \log T_i - \int_0^{\sum_{j=1}^{N} T_j} f(y)dy \quad (18)$$

is a Lyapunov function of system (17). The unique value of $\mathbf{T}$ maximizing $\mathcal{U}(\mathbf{T})$ is an equilibrium point where the system trajectories converge.

Proof. Note that $\mathcal{U}(\mathbf{T})$ is strictly concave with an interior maximum because of the assumptions on $w_i > 0$ and f. Therefore, the maximizing $\mathbf{T}$ is unique. Since $w_i = T_i(t) \cdot U_i'(T_i(t))$,

$$\frac{\partial \mathcal{U}(\mathbf{T})}{\partial T_i} = U_i'(T_i) - f(\sum_{j=1}^{N} T_j(t)), \quad (19)$$

then we have

$$\frac{d\mathcal{U}}{dt} = \sum_{i=1}^{N} \frac{\partial \mathcal{U}}{\partial T_i} \frac{dT_i(t)}{dt}$$

$$= \sum_{i=1}^{N} k(T_i(t))T_i(t) \left(U_i'(T_i(t)) - f(\sum_{j=1}^{N} T_j(t)) \right)^2. \quad (20)$$

10

Therefore, $\frac{d\mathcal{U}}{dt} > 0$ unless $\mathbf{T}$ maximizes $\mathcal{U}$. Thus, $\mathcal{U}$ is a Lyapunov function of (17) and the theorem follows directly. ∎

At the equilibrium point, $U_i'(T_i) = f(\sum_{j=1}^N T_j)$, $i = 1, \ldots, N$, and so $U_k'(T_k) = U_l'(T_l)$, $k, l = 1, \ldots, N$ similar to Proposition 1. Since $\mathcal{U}(\mathbf{T})$ can be approximated arbitrarily close to (15) depending on the choice of the penalty function f, hence $\mathbf{T}$ will converge to the optimal solution of the approximated network problem.

In the discrete-time system of cdma2000 1xEV-DO, we can interpret the dynamic (17) as follows. Let $T_i(t)$ be the value of the transmit-to-pilot gain in timeslot t of AT i, then

$$T_i(t+1) - T_i(t) \approx k(T_i(t))T_i(t)\left(U_i'(T_i(t)) - f(\sum_{j=1}^N T_j(t))\right), \qquad (21)$$

for $i = 1, \ldots, N$. The penalty function f feeds the information of the state of the network, i.e., the RoT level back to the ATs. For f being a step function such as in (9), the change in T_i can be written as follows:

$$
\begin{aligned}
&\Delta T_i(t+1) \\
=\ & T_i(t+1) - T_i(t) \qquad\qquad\qquad\qquad\qquad\qquad\qquad (22)\\
=\ & \begin{cases} \Delta T_{up,i}(T_i(t)) := k(T_i(t))T_i(t)U_i'(T_i(t)), & M(t+1) = 0 \\ \Delta T_{down,i}(T_i(t)) := k(T_i(t))T_i(t)\left(U_i'(T_i(t)) - 1\right), & M(t+1) = 1, \end{cases}
\end{aligned}
$$

where $M(t+1)$ is the *reverse activity bit* which will be set when the RoT rises above the set threshold of Z_{thresh} dB from (8) and (9).

Given $i = 1, \ldots, N$, the following derivation shows the relationship between $\Delta T_{up,i}(T)$ and $\Delta T_{down,i}(T)$ and the implicit utility function U_i.

Proposition 2 *Assume $\Delta T_{up}(T) > 0$ and $\Delta T_{down}(T) \leq 0$ for all $T > 0$. If the ratio $|\Delta T_{up}(T)/\Delta T_{down}(T)|$ is strictly decreasing as a function of T, then the RL MAC algorithm approximates a utility function which is increasing and strictly concave.*

Proof. From (22), we have

$$
\begin{aligned}
\Delta T_{up}(T) &\approx k(T)TU'(T) \\
\text{and}\quad \Delta T_{down}(T) &\approx k(T)T\left(U'(T) - 1\right).
\end{aligned}
$$

11

T	ΔT_{up}	ΔT_{down}	$\Delta T_{down}/\Delta T_{up}$	$k(T)$	$U'(T)$
4.37	0.442	0	0	0.101	1
6.73	0.294	-0.148	-0.501	0.066	0.666
11.44	0.365	-0.294	-0.806	0.058	0.553
23.13	1.551	-1.461	-0.942	0.130	0.515

Table 1: Approximated value of $U'(T)$ in IS-856 system with parameters from [3].

Therefore,

$$
\begin{aligned}
k(T) &\approx \frac{\Delta T_{up}(T) - \Delta T_{down}(T)}{T} \\
\text{and} \quad U'(T) &\approx \frac{\Delta T_{up}(T)}{\Delta T_{up}(T) - \Delta T_{down}(T)} \\
&= \frac{1}{1 + |\Delta T_{down}(T)/\Delta T_{up}(T)|},
\end{aligned}
\tag{23}
$$

where the last equality follows from $\Delta T_{up} > 0$ and $\Delta T_{down} \leq 0$ for all T. Since $U'(T)$ is positive and strictly decreasing as T increases under the assumption, we can conclude that U approximates an increasing and strictly concave function of T. ∎

The approximated utility function is implicitly defined in the dynamic operation of the T ramping under RAB control. In the actual system, all the ATs use the same RL MAC algorithm, i.e., $\Delta T_{up,k}(T) = \Delta T_{up,l}(T)$ and $\Delta T_{down,k}(T) = \Delta T_{down,l}(T)$ for $T \in \mathbb{R}_+$ and $k, l = 1, \ldots, N$. This implies that all ATs have the same utility function. The following corollary follows from this observation, Corollary 1 and Proposition 2.

Corollary 2 *Assuming conditions in Proposition 2 and $\Delta T_{up,k}(T) = \Delta T_{up,l}(T)$ and $\Delta T_{down,k}(T) = \Delta T_{down,l}(T)$ for $T \in \mathbb{R}_+$ and $k, l = 1, \ldots, N$, then the algorithm converges and at the equilibrium $T_k = T_l$, $k, l = 1, \ldots, N$.*

Proof. The result follows directly from Corollary 1 and Proposition 2. ∎

3.2.1 Examples

For IS-856, we use the values in Table 1 of [3] to approximate the value of $U'(T)$. Since the algorithm is probabilistic, we use the average increase/decrease of T instead, i.e., for any $R \in \mathcal{R}$

$$
\begin{aligned}
\Delta \bar{T}_{up}(F_R(R)) &= q(R)\left(F_R(2R \wedge R_{max}) - F_R(R)\right) \\
\Delta \bar{T}_{down}(F_R(R)) &= p(R)\left(F_R(\frac{R}{2} \vee R_{min}) - F_R(R)\right).
\end{aligned}
\tag{24}
$$

From (23), the approximated value of $U'(T)$ is given in Table 1. Since $|\Delta \bar{T}_{up}(T)/\Delta \bar{T}_{down}(T)|$ is monotonically decreasing in T, we can conclude from Proposition 2 that U in the IS-856 system approximates an increasing and strictly concave function and thus at the equilibrium $T_k = T_l$, $k, l = 1, \ldots, N$ from Corollary 1.

For Enhanced RL MAC, from (4) we can deduce that $\Delta T_{up}(T) = g_u(T)$ and $\Delta T_{down}(T) = -g_d(T)$ for some $\mathbb{R}_+ \to \mathbb{R}_+$ mappings g_u, g_d. Since the functions g_u, g_d are always chosen such that the ratio $g_u(T)/g_d(T)$ is strictly decreasing in T [4], the conditions in Proposition 2 are satisfied trivially and the result from Corollary 2 immediately follows.

4   Simulation Results

In this section, we present some simulation results supporting the theoretical findings in the previous section. According to Corollary 2, the distributed RL MAC algorithms are asymptotically stable and converge to the fair allocation under the condition that $|\Delta T_{up}(T)/\Delta T_{down}(T)|$ is strictly decreasing as a function of T. In this section, we simulate the RL MAC systems under this condition and demonstrate their fairness. Under the fairness criteria $T_k = T_l$, $k, l = 1, \ldots, N$, we can expect that the average throughput of the ATs are identical since the transmission rate of an AT is a function of T2P.

In order to quantify fairness under this criteria, we use the fairness index [11] which is given as

$$
\text{fairness index} = \frac{\left(\sum_{i=1}^{N} R_i\right)^2}{N \cdot \sum_{i=1}^{N} R_i^2},
\tag{25}
$$

where N is the number of ATs in the sector and R_i is the average throughput of AT i. The range of the fairness index is $[0, 1]$. The fairer the throughputs

are, the higher the index. The maximum value of the fairness index can be achieved if and only if $R_i = R_j$, $i, j = 1, \ldots, N$.

4.1 Simulator Description

In our simulator, a sector covers a hexagonal area. The access point contains two receive antennas which cover the sector. The simulation is performed on a single isolated sector. In the sector, ATs are randomly placed uniformly in the hexagonal area. The size of the sector is chosen according to a link budget that assumes the ATs are not power-limited, i.e., ATs always have enough power to transmit at their highest transmission rates R_{max}.

The simulator is discrete-time with the smallest time unit being a slot of 5/3 ms. A packet on the reverse-link takes 16 slots to transmit irrespective of the transmission rate. The power control algorithm is enabled with the command being set every slot for IS-856 and every subframe (four slots) for Enhanced RL MAC. The power control command is assumed to be perfectly transmitted from the BTS to the ATs but is delayed by one slot in either case. Each AT increases (resp. decreases) its pilot power by 1 dB for every up (resp. down) power control command. The target pilot SINR is adjusted through the outer loop power control [10] to achieve 1% packet error rate.

We assume that the signal transmitted by each AT receives an independent fading, simulated by a single path Rayleigh fading process. The process is assumed to be exponentially correlated in time with correlation given as a function of the terminal's speed. In these simulations, we assume the terminal is moving at the speed of 3 km/h.

The simulation is performed under a snapshot mode, i.e., ATs location along with path loss and shadowing are fixed throughout the duration of the simulation. Since we assume the ATs are not power-limited, AT location has very little effect on MAC behavior. ATs are assumed to have full buffer, i.e., they transmit as much data as the MAC algorithm allows.

The RAB is generated at the BTS by comparing the RoT to the threshold. If the RoT is greater than the threshold at the BTS, then RAB is set to one. Otherwise, it is set to zero. The threshold level is dynamically adjusted to maintain that RoT exceeds the 7dB threshold less than 1% of the timeslots. The number of ATs in the sector is 16. The duration of the simulation is 300,000 timeslots.
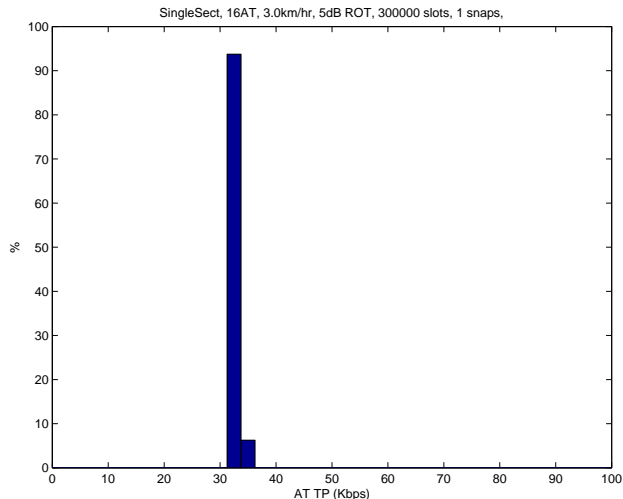
14

Figure 1: Example of rate distribution in IS-856 RL MAC when the transition probabilities satisfy the conditions in Proposition 2.

4.1.1 IS-856

The transition probabilities of the probabilistic MAC algorithm are set according to the values given in [3]. In Section 3.2.1, we have already demonstrated that these transition probabilities satisfy the conditions of Proposition 2.

The simulation result shows that the average AT throughput is 32.84 kbps while the standard deviation of the throughput is only 0.71 kbps. The fairness index of the throughput is 0.9996. The throughput distribution of this system is shown in Figure 1.

We then simulate another system with the transition probabilities in Table 2 which do not satisfy the condition in Proposition 2. For this system, the average AT throughput is reduced to 26.04 kbps while the standard deviation of the throughput is now 8.69 kbps. The fairness index of the throughput is now reduced to 0.9054. The throughput distribution of this system is shown in Figure 2.

	Data Rate (kbps)					
	0	9.6	19.2	38.4	76.8	153.6
q	1	1/16	1/16	1/2	1/2	0
p	0	1/2	1/2	1/16	1/16	1

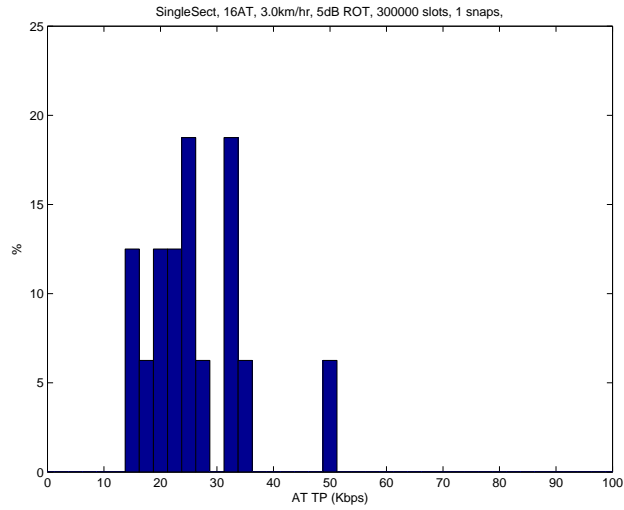Table 2: Unfair transition probabilities for IS-856 RL MAC.

Figure 2: Example of rate distribution in IS-856 RL MAC when the transition probabilities do not satisfy conditions in Proposition 2.
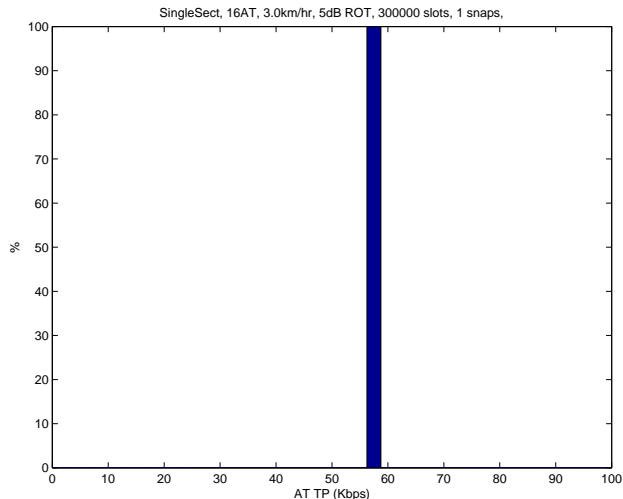
16

Figure 3: Example of rate distribution in Enhanced RL MAC when the conditions of Proposition 2 is satisfied.

4.1.2 Enhanced RL MAC

The simulation of Enhanced RL MAC is carried out over QUALCOMM's proprietary AT Class 7 specification. In this AT class, the function $g_u(.)$ and $g_d(.)$ in (4) are selected such that g_u/g_d is a strictly decreasing function. Therefore, the condition of Proposition 2 is trivially satisfied.

The result from the simulation shows that the average AT throughput is 57.03 kbps while the standard deviation of the throughput is 0.51 kbps. The fairness index of the throughput is 0.9999. The throughput distribution of this system is shown in Figure 3.

We also simulate Enhanced RL MAC with g_u, g_d selected such that the ratio $g_u/g_d = 1$ for all the values of T2P – this violates the sufficient conditions in Proposition 2. The average distribution of this simulation is shown in Figure 4. The average AT throughput is 51.21 kbps while the standard deviation of the throughput is 8.26 kbps. The fairness index of the throughput is 0.9762.
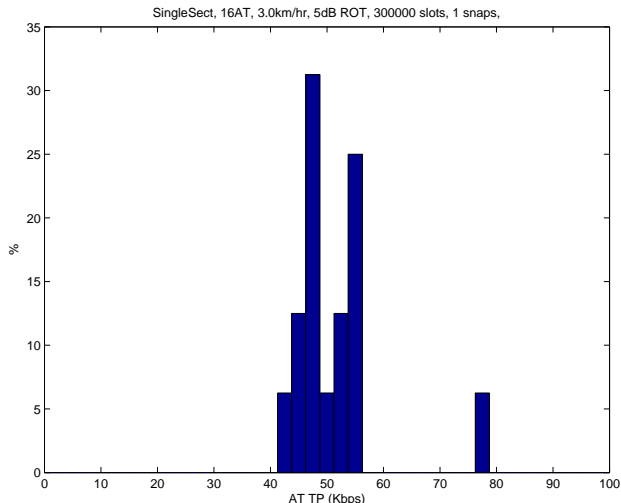
17

Figure 4: Example of rate distribution in Enhanced RL MAC when the ratio $g_u/g_d = 1$ for all values of T2P, i.e., the condition in Proposition 2 is not satisfied.

5 Conclusion

In this paper, we formulate a utility maximization problem and show a simple sufficient condition for the RL MAC algorithms in cdma2000 1xEV-DO to converge to the solution of this utility maximization problem. Moreover, we identify that the solution of this utility maximization problem is actually a simple equal throughput fairness criteria.

Using this framework, we can also extend the problem to the case when the ATs are power-limited. Preliminary study suggests that the solution to the power-limited scenario is similar to the *reverse water-filling* solution in rate distortion theory [12]. Further extension to the multi-sector scenario can also be done by formulating a problem similar to the optimization flow control problem with multiple bottlenecks [2].

References

[1] F. Kelly "Charging and Rate Control for Elastic Traffic", *European Transactions on Telecommunications*, vol. 8, pp. 33-37, 1997.

[2] F. Kelly, A. Maulloo, and D. Tan "Rate Control in Communication Networks: Shadow Prices, Proportional Fairness and Stability", *Journal of the Operations Research Society*, vol. 49, pp. 237-252, 1998.

[3] Sumantra Chakravarty, Rajesh Pankaj, and Eduardo Esteves "An Algorithm for Reverse Traffic Channel Rate Control for cdma2000 High Rate Packet Data Systems", *IEEE Globecom 2001*, November, 2001.

[4] Christopher Lott "Enhanced Rlmac for 1xEV-DO", *3GPP2 Standard Meeting*, April, 2003.

[5] Qiang Wu and Eduardo Esteves "The cdma2000 High Rate Packet Data System", in Chapter 4 of *Advances in 3G Enhanced Technologies for Wireless Communications*, March, 2002.

[6] Roberto Padovani "Reverse Link Performance of IS-95 Based Cellular Systems", *IEEE Personal Communications Magazine*, pp. 28-34, Third Quarter, 1994.

[7] R. Thomas Derryberry "1xEV-DV Evaluation Methodology" 3GPP2/TSG-C.R1002, *3GPP2 Technical Specification Group C*.

[8] Andrew J. Viterbi *CDMA : Principles of Spread Spectrum Communication*, Addison-Wesley Wireless Communications Series, 1995.

[9] Andreu Mas-Colell, Michael D. Whinston and Jerry R. Green *Microeconomic Theory*, Oxford University Press, 1995.

[10] Rashid Attar and Eduardo Esteves "A Reverse Link Outer-Loop Power Control Algorithm for cdma2000 1xEV Systems", International Conference on Communications (ICC), April, 2002.

[11] R. Jain, D. Chiu, and W. Hawe "A Quantitative Measure Of Fairness And Discrimination For Resource Allocation In Shared Computer Systems", DEC Research Report TR-301, September 1984.

[12] Thomas M. Cover and Joy A. Thomas *Elements of Information Theory*, Wiley Series in Telecommunications, NY, 1991.

A   On the Received Pilot Power at the BTS

Throughout this paper, we assume that the received pilot power at the BTS from AT i is exactly equal to P_R watts for all $i = 1, 2, \ldots, N$. As a result, the expression of the RoT is simplified as given in (7). Although this ignores the interplay between the power control algorithm and the rate control algorithm, it is a reasonable approximation under typical scenarios. This is the main topic of discussion of this appendix . Before we discuss further the relationship between these algorithms in Appendix A.2, a brief overview of the power control loop is given in Appendix A.1.

A.1   The Power Control Loop in cdma2000 1xEV-DO

In the cdma2000 1xEV-DO system, the pilot power from AT is controlled by two power control loops, i.e., the inner- and outer-loop power control. The inner-loop power control adjusts the pilot power according to a threshold, say γ_i, which is the level of SINR required for successful decoding at the desired error rate. We define the SINR seen at the BTS for AT i as $SINR_i$ which is given by

$$SINR_i = \frac{P_{i,R}}{N_0 W + \sum_{j=1, j \neq i}^{N} T_j P_{j,R}}, \tag{26}$$

where $P_{i,R}$ denote the received pilot power of AT $i = 1, \ldots, N$.

The inner-loop power control uses up/down power command to regulate the received pilot power $P_{i,R}$ such that the SINR level of AT i is close to the threshold γ_i, i.e., if $SINR_i > \gamma_i$, then the down power command is issued to the AT i and AT i subsequently reduces its transmitted pilot power by a predefined constant and vice versa for $SINR_i < \gamma_i$.

The threshold γ_i is adjusted through a much slower outer-loop power control to maintain a fixed decoding error percentage. More details on the outer-loop power control algorithm can be found in [10].

According to our setup, we can deduce the followings:

(a) By the virtue that ATs are uniform (as they are not power-limited) and the outer-loop power control algorithm operates on a much slower dynamics than MAC rate control algorithm, it is reasonable to assume that this threshold is identical for all mobiles in the long-run (especially if our goal is to equalize the throughput), i.e., $\gamma_1 = \ldots = \gamma_N = \gamma$ for some constant γ.

(b) The inner-loop power control algorithm operates on a much faster timescale than the MAC rate control algorithm, so the rate control algorithm experiences SINR around this threshold γ.

Throughout this paper, we assume that the power control is perfect and we have timescale decomposition, i.e., the setpoint SINR thresholds $\gamma_i, i = 1, \ldots, N$ are identical and fixed (relative to the dynamics of RL MAC rate control) at γ, and AT i instantaneously adapts its transmitted power such that its SINR level is at its threshold as soon as T_i is adjusted.

A.2 Interaction between Power Control and Rate Control Algorithms

Even with perfect power control and timescale decomposition, a complex interaction still arises from the following key observation. The received power $P_{i,R}$ required for a given AT i depends on the interference seen by that AT, which is a function of the T_j's. As other ATs adjust their T's, AT i must adjust its $P_{i,R}$ to keep a $SINR_i$ at the level γ. This process also applies to all other ATs until a new fixed point is found for all $P_{j,R}$, i.e., the received power from each AT has to be adjusted to be consistent with the current values of $T_i, i = 1, \ldots, N$. This, in turn, will also affect the dynamics of T's in the future as the RoT level depends on $P_{i,R}$'s.

A detailed analysis of the complete interaction is very complicated and is beyond the scope of this report. The analysis in this paper is conducted under the assumption that the received pilot power from each AT is fixed and identical to some constant P_R. Although this follows in part from the perfect power control/timescale decomposition assumptions, more justifications are needed. We argue that, under typical scenarios, the dynamic range of the received pilot power is small and its effect to the T2P adjustment is only marginal. Our arguments will be given after the following simple calculation.

Assuming perfect power control/timescale decomposition, we have $SINR_i$ equals to $\gamma_i = \gamma, i = 1, \ldots, N$. From (26), if the current RoT level is Z dB, we have

$$\gamma = \frac{P_{i,R}}{N_0 W + \sum_{j=1, j \neq i}^{N} T_j P_{j,R}} \tag{27}$$

$$= \frac{P_{i,R}}{N_0 W Z_{lin} - T_i P_{i,R}}, \tag{28}$$

21

where $Z_{lin} = 10^{Z/10}$. Therefore, the fixed point solution for $P_{i,R}$ under the given array of T2P's and RoT level is

$$P_i = \frac{\gamma}{1 + \gamma T_i} \cdot N_0 W \cdot Z_{lin} \ . \tag{29}$$

In this paper, the assumption that the received pilot power from all ATs equals to some constant P_R is reasonable under the standard assumption in microeconomics that no one consumer dominate the market, i.e., no significant contribution to the RoT level comes from a single AT. This standard market assumption in turn justifies the constant P_R assumption by the following reasons:

(R1) Typically, γ is a very small quantity. This is evident from (27) as it is a ratio between only the receive pilot power of AT i to total power – noise and total receive power from other ATs. Hence, the ratio $\frac{\gamma}{1+\gamma T_i}$ in (29) is close to γ for typical values of T_i's, assuming fixed γ which is adjusted through a much slower outer-loop power control algorithm.

Although there are certain cases where this argument does not hold, e.g., very small number of ATs with only one of them having very large T_i, it is not a scenario of a practical interest. Another way to say that AT i does not take up too much resource is to say that γT_i is small, which is a good approximation for a large number of AT's.

(R2) Assuming (R1), the fixed-point $P_{i,R}, i = 1, \ldots, N$ are all proportional to the RoT, Z_{lin}. This is acceptable in our framework as long as all $P_{i,R}$'s are identical, which follows from (R1). Specifically, we can renormalize P_i in term of Z_{thresh} in (11) and the remaining analysis follows readily. Hence, we can conclude that the dependence of $P_{i,R}$ on Z_{lin} does not contribute to the stability of the algorithm. Moreover, we note that the objective of the MAC rate control algorithm is to control the RoT Z around its threshold Z_{thresh}, and therefore the dynamic range of Z is small and so we simplify our formulation such that the received power is fixed at P_R for the clarity of the presentation and to focus on the behabior of MAC rate control algorithm.

(R3) The power control loop is a *positive* feedback loop with very small delay. As RoT increases, $P_{i,R}$ is increased to overcome higher level of interference. Effectively, this leads to further rise in the RoT and vice versa. The only possibility for this system can be stable is that the

positive feedback gain has to be very small comparing to the negative gain from the MAC rate control on T_i. This also help provide justification that the dynamics in the pilot power $P_{i,R}$ does not contribute much to the dynamic of the overall MAC layer.

A.3   Refinement of the analysis

Although (R1) is a reasonable assumption for the usual cases of interest, it puts an apriori assumption on the initial rate allocations from which the algorithm runs, and from which we demonstrate its convergence to equal allocation. Simulation results seem to suggest, however, that the algorithm will converge, under the conditions identified in this paper, regardless of the initial rate configuration.

The difficulties in relaxing this assumption arise from the fact that P_i is a function of all T_i's as well as the RoT Z_{lin} — the variable we wish to control. In our future work, we plan to reformulate the problem using (29) as the received power instead of the fixed value P_R. We expect that a similar analysis can be established by using the renormalization technique described in (R2).