TECHNICAL RESEARCH REPORT

Local Monitoring of the Internet Network

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Introduction

The internet, as well as other freely evolving networks has a topology that changes dynamically; therefore the topology is very complicated. Presently, there is an increase in dependency on the internet for several things such as communication, information traffic and many more, hence it is imperative that we prevent attackers from disrupting this network. To do this, it is essential to count on a mathematical model that can allow early detection (then ring the bell) of attacks to the network.. The mathematical tool that we are looking for to accomplish that early detection is based on the use of Tomography ideas. The last statement is based on the April 28-29, 2000 workshop on the interface between the mathematical sciences and three areas of computer science: network traffic modeling, computer vision, and data mining and search. This workshop was cosponsored by the Board on Mathematical Sciences (BMS) and the Computer Science and Telecommunications Board of the National Research Council.[O].

1.1 Tomographic approach (claim)

Let us state a general idea about how Tomography can be used to do monitoring and give some kind of diagnostic.

Consider a Riemannian manifold M and assume there is a perturbation on M represented by the function f. If we want to know f , one way to do it, it is to measure $\int_L f(x)dx$ over a family of geometric objects L, say geodesic lines for instance. The typical cases of this idea are the given by the Radon transform in the Euclidean space $\Re^2$ and the space $\Re^3$, where L denotes an arbitrary straight line, respectively an arbitrary affine space, a plane in $\Re^3$. Examples of this can be the CT (Computerized Tomography) and MRI scanners that model this way. [N3], [K1]. As we explain below, the corresponding natural tomographic tool for monitoring the internet, as well as other particular types of networks, should be based on Electrical Impedance Tomography (EIT). For it, let us give a brief explanation of EIT.

Consider Calderon's Inverse Conductivity Problem in the plane. (For details, see [U] and [S]) Let Ω be a bounded domain in $\Re^2$ with sufficiently smooth boundary. The electrical conductivity is a bounded and strictly positive nice function β on Ω and a current Ψ on the boundary is a function satisfying the condition

$$\int_{\partial\Omega} \Psi ds = 0$$

i.e. the the average of the values of Ψ on $\partial\Omega$ is zero. Let u be the function representing the induced potential which satisfies the next boundary problem with Neumann conditions

$$\begin{cases} div(\beta grad\ u) = 0, & in\ \ \Omega \\ \beta \frac{\partial u}{\partial n} = \Psi, & on\ \ \partial\Omega \end{cases} \tag{1}$$

where Ψ is given and n is the outer unit normal vector on $\partial\Omega$. This problem has a unique solution u where the uniqueness of u is up to an additive constant. The function u is the potential distribution on Ω so $grad\ u$ is the electrical field. If s represents the tangent vector to $\partial\Omega$, it holds that the tangential derivative of u, $\frac{\partial u}{\partial s}$, depends linearly on Ψ. So, for Ψ given and β, the unknown conductivity, there exists u which defines a map

$$\beta, \Psi \longrightarrow \frac{\partial u}{\partial s}$$

where β the only remaining function to be known.

If Λ_β stands for such a map

$$\Lambda_\beta : \Psi \longrightarrow \frac{\partial u}{\partial s} \tag{2}$$

then Λ_β is a bounded linear operator from the Sobolev space $H^\alpha(\partial\Omega)$ into itself, and β determines Λ_β.

Calderon's problem is to determine β from the knowledge of Λ_β, which we could think is determined experimentally by the choice of convenient currents Ψ. Given that β is to be found, then consider the map

$$\Lambda : \beta \longrightarrow \Lambda_\beta \tag{3}$$

It has been shown by Nachman ([N1] [N2]) that the map Λ is one-to-one, in other words, β is identifiable by the action of its corresponding map Λ_β. For our purposes, we will be more interested in the reconstruction (approximate reconstruction) of β from the knowledge of Λ_β , i.e., the problem is to find the inverse of the mapping (3). This problem is called inverse conductivity problem.

It turns out that this reconstruction corresponds to solving a tomographic problem. Let us briefly recall why. For the goals to achieve, let Ω the unit disk in $\Re^2$ where the unit disk is considered as a model of the hyperbolic plane H. More precisely the Poincare plane where the geodesic lines are the arcs of circles that are orthogonal to $\partial\Omega$. In this case, the Radon transform of a function f is given by

$$Rf(\zeta) = \int_\zeta f(s) ds$$

where ζ is a geodesic and ds is the infinitesimal element of hyperbolic length. For a function g of a single variable, we denote and define the hyperbolic convolution by

$$f * g(x) = \int_H f(y)g(d(x,y))dy \tag{4}$$

where d is the hyperbolic distance

$$ds = \frac{2\,|dz|}{(1 - |z|^2)}$$

and dz denotes the Euclidean distance in $\Re^2$.

One can then show that if β is approximately constant, that is

$$\beta(x) = \beta_o + \varepsilon b(x) + o(\varepsilon)$$

where εb small then one can recover εb testing with dipoles at different points of the boundary, namely, $\Psi_Z = \frac{\partial}{\partial Z}\delta_Z$ the tangential derivation of the delta function at the point $z \in \partial\Omega$. According to Berenstein and Casadio, [B], it can be shown that the perturbation εb satisfies a convolution equation of the form $k * \varepsilon b = \mu$ where $k(t)$ is given by

$$k(t) = \frac{\cosh^{-2}(t) - 3\cosh^{-4}(t)}{8\pi}$$

and μ is computed from the boundary data

Our purpose is try to connect to relate these ideas to the network monitoring problem. We first need to understand the network under the simplified model of being a tree. The key point is that it has been known for quite a long time that analysis on trees and analysis in the hyperbolic plane are closely related, and there are similar general principles for particular (reasonable) graphs.

At this point we take for granted that trees can be embedded in the hyperbolic plane as a union of geodesic lines in such a way that the embedding is isometric, i.e. the distance between adjacent embedded vertices is exactly 1, thus tomography in tree corresponds exactly to tomography in the hyperbolic plane, and that the $chip-firing-game$ corresponds exactly to EIT. Our claim is that this is the connection between the continuos version of tomography and the discrete one, and consequently the continuos approach to the internet topology and diagnosis and the discrete one.

Since we need to locally model the internet network, we need to establish an explicit correspondence between graphs and the hyperbolic space (embedding of graphs in H) in order to explain what tomography should be in terms of graphs, obtain inversion formulae, and other analogies such that to find out if there is an approximate inversion formula for the $chip - firing - game$ approach to detection of irregularities in the network.

It is important to keep in mind tomographic ideas to use the relation of EIT with the hyperbolic Radon transform, and the fact that we can use wavelets as a tool to invert the Radon transform as well as radial convolution operators to determine what multiresolution analysis (MRA) of the internet is.

This approach leads to develop some kind of *Radon* transform theory on graphs G, and because of the motivation given before, it will be necessary to study the discrete version of the Laplacian on graphs.

Why graphs?. Because computer networks are considered as nodes of graphs, and the discrete version of Laplacian is needed since nodes are separated.

The goal is clearly to develop some tools to detect attackers or intruders based on the discrete version of the Laplacian $\Im$ which expression is

$$\Im = \begin{cases} 1 \; if \; u = v \\ \frac{-1}{\sqrt{dudv}} \; if \;\; u \sim v \\ 0 \; otherwise \end{cases}$$

where u, v are vertices of G and du represents the degree of the vertex u.

To accomplish with it, the two following research topics are stated.

1 Eigenvalue bounds for the Laplacian.

2 Solve discrete version of the Dirichlet problem with Neumann boundary conditions (NBVP).

Regarding 1, eigenvalues of Laplacian give information about the graph such as vertices, number of nodes, edges, i.e. some approximation to the shape of the graph. References considered are [C1] and [C2] which have to do with the $chip - firing - game$.

In studying discrete version of L, the next result exists in the literature [C3].

$$\sum f(t) \leq \frac{1}{\sigma_1} N\sqrt{n}$$

where $\sum f(t)$ = total number of firings, N=number of chips, n=number of nodes, σ_1=first eigenvalue of $\Im$, $\sum f(t)$ can represent the number of times computers communicate , and a bound for σ_1 is so far $\sigma_1 \geq \frac{1}{Dn}$,with D the diameter of the graph G. (For detailed literature about graphs theory, see [C3]).

One of the goals is to obtain a better upper bound for $\sum f(t)$ as well as an lower bound which would make easier to detect intruders. To obtain a better bound, next improvement is obtained:

$$\sigma_1 \geq \frac{1}{2\sigma_2}$$

which doesn't depend on D anymore [C2].

Let $\widetilde{f}$ be the element representing the information density (distribution) in the network . Now, the primary goal is to detect some kind of perturbation of $\widetilde{f}$

We believe that by solving the discrete version of the *Dirichlet* problem with *Neumann* boundary conditions

$$\begin{cases} \Delta \widetilde{f} = 0 \;\; in \; S \\ \sum\limits_{x \sim y} \widetilde{f(x)} - \widetilde{f(y)} = \Psi(x), \; x \in \delta S \end{cases}$$

where Ψ represents some information at the boundary of the graph such as especial signals, delay, time to reach target in round trips, especial sent messages with codes , we would be able to obtain that perturbation of $\widetilde{f}$ from the given data $\Psi(x)$. S represents the subgraph and δS represents the vertex boundary of S.

To deal with this, some sort of *Radon* transform on graphs has also to be deøned.

Considering the NBVP, the next important result was reached at the seminar [S2]. Namely,

Theorem (NBVP for Δ) Let S be a subgraph of a host graph G with a nonempty (vertex) boundary, $\delta S \neq \emptyset$. Let

$$\begin{array}{l} f : \overline{S} = S \cup \delta S \to \Re \\ g : S \to \Re \\ \psi : \delta S \to \Re \end{array}$$

be functions satisfying that (the NBVP)

$$\begin{cases} \Delta f(x) = g(x), \ x \in S & (1) \\ \frac{\partial f(z)}{\partial n} = \psi(z), \ z \in \delta S & (2) \\ \int_{\delta S} \psi = \int_S g & (3) \end{cases}$$

then the solution f can be written as

$$f(x) = b_o + < G(x,\cdot), g >_S - < G(x,\cdot), \psi >_{\delta S}, \ x \in \overline{S}$$

where $< h, \ f >= \sum_{x \in X} h(x) f(x)$, b_o is an arbitrary constant, and G is the discrete Green function for a new graph $G_o = S \cup \delta S$ with no boundary.

Proof. By Green's Identity,

$$\int_{\delta S} \frac{\partial f}{\partial n} = \int_S \Delta f$$

the condition (3) is necessary for NBVP to have a solution. We rewrite (1) and (2) as

$$\begin{cases} \sum\limits_{y \in \overline{S}} [f(y) - f(x)] \frac{w(x,y)}{dx} = g(x), \ \ x \in S \\ \sum\limits_{y \in S} [f(y) - f(z)] \frac{w(y,z)}{dz'} = -\psi(z), \ \ z \in \delta S \end{cases} \qquad (5)$$

where $w(x,y)$ is the weight function on the edges which is either 1 or 0 depending on if x is adjacent or not to y

Considering $n = |S|$ and $m = |\delta S|$, (5) is a linear system consisting of $n+m$ equations and n unknowns for $x \in S$, but since we don't know the values of f at the boundary δS then (5) is actually a linear system with $n+m$ equations and $n+m$ unknowns. This system can not have a unique solution because the addition of a constant is again a solution for (5). In solving (5), a new graph

$G_o = S \cup \delta S$ with no boundary is considered, then for each $z \in \delta S$, dz' in (5) is the number of vertices in G_o which are adjacent to z, given that there are no adjacent vertices in δS. Hence, we may write $dz = dz'$, and then equation 2 in (5) is written as

$$\sum_{y \in V_o} [f(y) - f(z)] \frac{w(y,z)}{dz'} = -\psi(z), \;\; z \in \delta S \tag{6}$$

where V_o is the set of vertices in G_o. In addition, equation 1 in (5) can be written as

$$\sum_{y \in V_o} [f(y) - f(x)] \frac{w(x,y)}{dx} = g(x), \;\; x \in S \tag{7}$$

If $\Phi : V_o \to \Re$ is the function defined by

$$\Phi(x) = \begin{cases} g(x), \, x \in S \\ -\psi(x), \, x \in \delta S \end{cases}$$

then we obtain

$$\sum_{y \in V_o} [f(y) - f(x)] \frac{w(x,y)}{dx} = \Phi(x), \;\; x \in V_o \tag{8}$$

in other words

$$\Delta f(x) = \Phi(x), \, x \in V_o \tag{9}$$

where Δ is the discrete Laplacian for G_o. Considering T a diagonal matrix with entries $T(x,x) = dx, \; x \in V_o$, then for $h = T^{1/2} f$, (9) is equivalent to

$$\Im h(x) = T^{1/2} \Phi \tag{10}$$

where $\Im$ is the normalized Laplacian for G_o. From this, we can find eigenvalues $0 = \lambda_o < \lambda_1 \leq \lambda_2 \leq ..... \leq \lambda_{N-1}$, where $N = |V_o|$, and corresponding orthonormal eigenfunctions $\Phi_o, \Phi_1, \Phi_2 ...., \Phi_{N-1}$ for $\Im$.

It holds that $\Phi_o(x) = \sqrt{\frac{dx}{volG_o}}$, $x \in V_o$, so the solution h for (10) can be written as

$$h(x) = \sum_{i=0}^{N-1} a_i \Phi_i(x)$$

where a_i the Fourier coefficients. It follows that for $i \geq 1$,

$$(-\lambda_i) a_i = (-\lambda_i) < h, \Phi_i > = < T^{1/2} \Phi, \Phi_i >$$

hence

$$a_i = -\frac{1}{\lambda_i} < T^{1/2} \Phi, \Phi_i >, i = 1, 2, ..., N-1$$

Given that $0 = \lambda_o$, it follows that

$$0.a_o = \frac{1}{\sqrt{volG_o}} \left[\int_{\delta S} \psi - \int_S g \right]$$

which also tells us that the constant a_o can be chosen arbitrarily, then for $x \in V_o$

$$\begin{aligned} f(x) &= \frac{a_o}{\sqrt{volG_o}} + \sum_{i=1}^{N-1}(-\frac{1}{\lambda_i})(\sum_{y\in S} g(y)\Phi_i(y)\sqrt{dy})\frac{\Phi_i(y)}{\sqrt{dx}} \\ &\quad - \sum_{i=1}^{N-1}(-\frac{1}{\lambda_i})(\psi(y)\Phi_i(y)\sqrt{dy})\frac{\Phi_i(y)}{\sqrt{dx}} \\ &= b_o + < G(x,\cdot), g >_S - < G(x,\cdot), \psi >_{\delta S} \end{aligned}$$

where b_o is an arbitrary constant.

Corollary 1 The solution to NBVP is unique up to an additive constant.

Corollary 2

$$\begin{cases} \Delta f(x) = 0, & x \in S \\ \frac{\partial f}{\partial n}(z) = \psi(z), & z \in \delta S \\ \int_{\delta S} \psi = 0 \end{cases} \Longrightarrow f(x) = b_o - < G(x,\cdot), \psi >_{\delta S}, \;\; x \in \overline{S}$$

As it was said before, some tools will be used, and then here is a brief introduction to Multiresolution analysis and wavelets. For practical applications of the kind we have here, we would like to prove the same type of localization theorem in the hyperbolic geometry as that one existing for the Euclidean *Radon* transform. One of the first things to do is to define a suitable Multi Resolution Analysis (MRA) on the hyperbolic unit disk such that arbitrary functions can be approximated by using (hyperbolic) wavelets.

1.2 Multiresolution Analysis

In order to study the wavelet theory, a framework that characterizes the wavelets in a general way is multiresolution analysis (MRA). Given a function $f \in L_2(\Re)$, this can be represented in several ways. The differences among these ways of representing f depends on the details levels of f. The various levels of details are a consequence of the MRA.

Definition A family $\{V_j\}_{j\in Z} \subset L_2(\Re)$ of closed subspaces is called a MRA if it satisfies the following axioms:

1. $\{0\} \subset \cdots\cdot V_{-1} \subset V_0 \subset V_1 \cdots\cdot \subset L_2(\Re)$
2. $\overline{\bigcup_{j=-\infty}^{\infty} V_j} = L_2(\Re)$
3. $\exists \{\Phi(x-k)\}_{k\in Z}$ *an orthonormal basis for* V_0
4. $f \in V_j \Leftrightarrow f(2\cdot) \in V_{j+1}$
5. $\bigcap_{j=-\infty}^{\infty} V_j = \{0\}$

Axiom 5 can be dropped because can be obtained from the previous four axioms. Some authors ask for the existence of a Riesz basis for V_0 instead of an orthonormal basis, and this is based on the properties that they want the MRA to posses. The spaces V_j are called approximation spaces. Projections of a function $f \in L_2(\Re)$ onto V_j are approximations to f which converges to f as $j \to \infty$. If $f \in V_j$, then the requirement that the escaled function $f(2\cdot)$ is in V_{j+1} determines a relation between the nested spaces V_j. From axiom 3, it follows that

$$\left(\int_{-\infty}^{\infty} |\Phi(x)|^2 \, dx\right)^{1/2} = 1$$

It is also required that

$$\int_{-\infty}^{\infty} \Phi(x)dx = 1$$

Considering these nested spaces, define

$$W_j = \{f \in V_{j+1} / \ f \perp V_j\}$$

i.e. W_j is the orthogonal complement of V_j in V_{j+1} and since each V_j is a closed subspace of the Hilbert space $L_2(\Re)$, it follows that

$$\begin{aligned} V_{j+1} &= V_j \oplus V_j^{\perp} \\ &= V_j \oplus W_j \end{aligned} \tag{11}$$

If j and j_0 are two integers such that $j > j_0$, then applying (11) recursively it holds

$$V_j = V_{j_0} \oplus \left(\bigoplus_{j=j_0}^{j-1} W_j\right) \tag{12}$$

It follows that any function f in V_j can be expressed as a linear combination of functions in V_{j_0} and in W_j for $j = j_0 \cdots j-1$. If $s \in Z$ is such that $s > j$ then f will be expressed a a different linear combination; hence the function f can be analyzed separately at different scales. From (6), when $j_0 \to -\infty$ and $j \to \infty$, we obtain

$$\bigoplus_{j=j_0}^{j-1} W_j = L_2(\Re)$$

then the spaces W_j are mutually orthogonal.

Given that the family $\{\Phi(x-k)\}_{k\in Z}$ is an orthonormal basis for V_0 then from repeated application of axiom 4, it follows that $\left\{\Phi(2^j x - k)\right\}_{k\in Z}$ is an orthogonal basis for V_j. Since $\Phi(2^j x - k)$ is the function $\Phi(2^j x)$ translated by $k/2^j$, then Φ becomes narrower. It is possible to normalize the family $\left\{\Phi(2^j x - k)\right\}_{k\in Z}$. since

$$\left(\int_{-\infty}^{\infty} \left|\Phi(2^j x - k)\right|^2 dx\right)^{1/2} = 2^{-j} \left\|\Phi\right\|_2^2 = 2^{-j}$$

;therefore, $\left\{2^{j/2}\Phi(2^jx-k)\right\}_{k\in Z}$ is an orthonormal basis for V_j. In the same way, there is a function $\Psi(x)$ such that $\left\{2^{j/2}\Psi(2^jx-k)\right\}_{k\in Z}$ is an orthonormal basis for W_j. These functions are usually expressed as

$$\Phi_{j,k}(x) = 2^{j/2}\Phi(2^jx-k)$$

$$\Psi_{j,k}(x) = 2^{j/2}\Psi(2^jx-k)$$

and

$$\Phi_k(x) = \Phi_{0,k}(x)$$
$$\Psi_k(x) = \Psi_{0,k}(x)$$

Φ and Ψ are called the basic scaling function and the basic wavelet function respectively. Ψ is also called the mother wavelet, and by definition of the spaces W_j it follows that $\Psi_{j,k}$ and $\Phi_{j,k}$ are orthogonal. In addition, given that the spaces W_j are mutually orthogonal, it follows that the wavelets are orthogonal across scales. Hence,

$$\int_{-\infty}^{\infty}\Phi_{j,k}(x)\Phi_{j,l}(x)dx = \delta_{k,l}$$
$$\int_{-\infty}^{\infty}\Psi_{i,k}(x)\Psi_{j,l}(x)dx = \delta_{k,l}\delta_{i,j}$$
$$\int_{-\infty}^{\infty}\Phi_{i,k}(x)\Psi_{j,l}(x)dx = 0,\ j \geq i$$

It is not imperative that W_j be orthogonal to V_j. W_j can be chosen such that it is not orthogonal to V_j. If so, the wavelets that will be obtained are called bi-orthogonal wavelets.

1.3 Vanishing moments property

As we said earlier, the scaling functions can express the linear part of a function exactly. Further, they can represent polynomials of low degree, say P-1, exactly. Namely, assume it is required that

$$x^p = \sum_{k=-\infty}^{k=\infty} M_k^p\Phi(x-k), \quad x\in\Re, \quad p=0,1.....,P-1$$

with

$$M_k^p = \int_{-\infty}^{\infty} x^p\Phi(x-k)dx, \quad k\in Z, \quad p=0,1.....,P-1$$

which is called the p^{th} moment of $\Phi(x-k)$. By considering the inner product between x^p and $\Psi(x)$ it holds that

$$\int_{-\infty}^{\infty} x^p\Psi(x)dx = \sum_{k=-\infty}^{k=\infty} M_k^p\int_{-\infty}^{\infty}\Phi(x-k)\Psi(x)dx = 0$$

because Φ and Ψ are orthogonal. Hence, it follows

$$\int_{-\infty}^{\infty} x^p\Psi(x)dx = 0, \qquad x\in\Re, \quad p=0,1.....,P-1$$

which is called the vanishing moments property that we already mentioned.

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