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Bounding On-Off Sources -- Variability Ordering and Majorization to the Rescue

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Bounding on-off sources – Variability ordering and majorization to the rescue

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Abstract—We consider the problem of bounding the loss rates of the aggregation of on-off sources in a bufferless model by the loss rates associated with the aggregation of i.i.d. on-off sources. We use well known results from the theory of variability orderings to establish a conjecture of Rasmussen et al., a recent upper bound of Mao and Habibi, and to discuss a new conjectured upper bound by these authors.

If the traffic is offered for transmission over a link operating at C bps, only $\min(R(t), C)$ bps can be accommodated, and in the absence of any buffer, the remaining $(R(t) - C)^+$ bps represents the instantaneous loss rate over that link. Under (1) the (average) loss rate of the source $\{R(t), t \geq 0\}$ over the C bps link is well defined and given by

I. INTRODUCTION

Traffic burstiness has long been considered a key factor for provisioning link and buffer resources at ATM multiplexers. In a first step, these issues can be addressed with the help of a simple *bufferless* model fed by fluid-like input traffic. An information source is then characterized by its \mathbb{R}_+ -valued rate process $\{R(t), t \geq 0\}$, so that the source bursts at time $t \geq 0$ with an instantaneous rate of $R(t)$ bps, say for sake of definiteness. For obvious practical reasons, it is customary to require the constraint $0 \leq R(t) \leq P$ ($t \geq 0$) where P is the peak rate of the source.

A. Loss rates

In most situations of interest, the rate process $\{R(t), t \geq 0\}$ can be assumed ergodic (as we do from now on) in the sense that for all $x \geq 0$,

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mathbf{1}[R(t) \leq x] dt = \mathbf{P}[R \leq x] \quad a.s. \quad (1)$$

for some \mathbb{R}_+ -valued rv R . If the rate process is stationary and ergodic, then (1) holds with the steady-state rate variable R determined through the weak convergence $R(t) \Rightarrow_t R$. Under (1) the source admits an average rate given by

$$m(R) := \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T R(t) dt = \mathbf{E}[R] \quad a.s. \quad (2)$$

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$$\begin{aligned} L(R; C) &:= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (R(t) - C)^+ dt \\ &= \mathbf{E}[(R - C)^+] \quad a.s. \end{aligned} \quad (3)$$

B. Multiplexing many sources

While the definition (3) for $L(R; C)$ might appear too poor a marker of source behavior to be of any use, its evaluation is nevertheless helpful for dimensioning link capacity or as the basis for a Call Admission Control (CAC) procedure [1], [2], [5]. In the latter instance, traffic carried on the link is typically obtained by multiplexing several independent information sources. If N sources $\{R_n(t), t \geq 0\}$ ($n = 1, \dots, N$) are multiplexed on a link operating at C bps, the total instantaneous rate is given by

$$R(t) = R_1(t) + \dots + R_N(t), \quad t \geq 0.$$

Under appropriate ergodic assumptions, it follows that

$$\begin{aligned} L(R; C) &= L(R_1 + \dots + R_N; C) \\ &= \mathbf{E}[(R_1 + \dots + R_N - C)^+] \end{aligned} \quad (4)$$

where R_1, \dots, R_N are the steady-state rate variables for the component sources.

As indicated already in [2], [3], evaluating (4) can be computationally prohibitive even in the simplest of cases

¹We write $x^+ = \max(x, 0)$ for any scalar x .

due to the large number of sources that need to be multiplexed at any given time. This difficulty is further exacerbated when the component sources are statistically dissimilar (as is the case in practice) [2]. This state of affairs has prompted a search for *upper bounds* on loss rates which are *computationally efficient*, and yet sufficiently *tight* to provide good approximations.

C. On-off sources

Most of the efforts have been carried out for the class of on-off sources (e.g., [1], [2], [3], [5]). A source with rate process $\{R(t), t \geq 0\}$ is said to be a (generalized) on-off source if $R(t)$ alternates between two states, namely $R(t) = 0$ (resp. $R(t) = P$) when the source is silent (resp. active) at time $t \geq 0$. Under the ergodic assumption (1), such an on-off source admits a steady-state rate R with finite range $\{0, P\}$. In fact, it is easy to see that

$$\mathbf{P}[R = P] = 1 - \mathbf{P}[R = 0] = f(R)$$

where $f(R)$ is the activity factor of the source defined by

$$f(R) := \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mathbf{1}[R(t) > 0] dt \quad a.s. \quad (5)$$

For on-off sources, we have $m(R) = f(R)P$, so that such sources are fully characterized by the pairs $(P, f(R))$ or $(P, m(R))$. We find it useful to represent the steady state rate R of the on-off source $\{R(t), t \geq 0\}$ with peak rate P and activity factor $f(R)$ as

$$R =_{st} PB(f(R))^2$$

where for p in $[0, 1]$, $B(p)$ denotes a $\{0, 1\}$ -valued (Bernoulli) random variable (rv) with $\mathbf{P}[B(p) = 1] = p$. We then refer to such an on-off source as the on-off source $(P, B(f(R)))$.

D. Earlier bounds and new results

Consider N independent on-off sources $(P_n, B(f_n))$ with peak rate P_n and activity factor f_n ($n = 1, \dots, N$); the resulting steady state rate for the aggregate traffic is

$$\sum_{n=1}^N P_n B_n(f_n) \quad (6)$$

where $B_1(f_1), \dots, B_N(f_N)$ are independent Bernoulli rvs. The upper bounds derived in the literature on loss rates (4) for the aggregate traffic (6) can be interpreted as

²For two \mathbb{R} -valued rvs X and Y with the same distribution, we write $X =_{st} Y$.

loss rates for an aggregation of *fewer*, say $L \leq N$, i.i.d. on-off sources $(P_{\text{new}}, B_\ell(f_{\text{new}}))$ with common peak rate P_{new} and activity factor f_{new} ($\ell = 1, \dots, L$). The resulting steady state rate for the aggregate traffic is now

$$\sum_{\ell=1}^L P_{\text{new}} B_\ell(f_{\text{new}}). \quad (7)$$

where $B_1(f_{\text{new}}), \dots, B_L(f_{\text{new}})$ are i.i.d. Bernoulli rvs.

Rasmussen et al. [5, p. 353] conjectured that when the sources in (6) have identical peak rates, say P , but possibly different activity parameters, the aggregation (7) of N *homogeneous* on-off sources with identical peak rate $P_{\text{new}} = P$ and activity parameter $f_{\text{new}} = N^{-1}(f_1 + \dots + f_N)$, provides an upper bound. This conjecture was recently established by Mao and Habibi [3, Thm. 1] from basic principles. These authors also establish another upper bound [3, Thm. 3], this time for N *homogeneous* on-off sources, by replacing them with a *reduced* number of homogeneous on-off sources. Finally, they conjecture the validity of an upper bound [3, Conjecture 1] which generalizes both the upper bound of Rasmussen et al. and their upper bound.

Here, we establish a general comparison result for weighted sums of independent Bernoulli rvs [Section IV]. In Section V we show how this general result can be used to readily derive the conjecture of Rasmussen et al., and the upper bound of Mao and Habibi, and to discuss their conjectured upper bound. The proper framework for addressing these issues (and comparisons in general) is one that combines stochastic orderings [6] with the notion of majorization [4]: The variability orderings we use are tailored made for comparing loss rates [Prop. 1], while majorization is useful for formally comparing degrees of heterogeneity. The relevant definitions and facts are given in Section II. This is followed in Section III by a discussion of three simple operations that reduce variability. An elementary application of this material readily yields the general comparison result in Section IV.

II. STOCHASTIC ORDERINGS AND MAJORIZATION

The basic tools are introduced in this section.

A. Variability orderings

For \mathbb{R} -valued rvs X and Y , we say that X is smaller than Y in the convex (resp. increasing convex) ordering if

$$\mathbf{E}[\varphi(X)] \leq \mathbf{E}[\varphi(Y)] \quad (8)$$

for all mappings $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ which are convex (resp. increasing and convex) provided the expectations in (8) exist; we write $X \leq_{cx} Y$ (resp. $X \leq_{icx} Y$). We refer to these

orderings as the *variability* orderings. Additional material on these orderings can be found in the monographs [6].

B. Key facts

We now present well-known facts that help shape the approach taken here. First, an equivalent definition of the convex increasing ordering [6, Thm. 1.3.1, p. 9].

Proposition 1: For \mathbb{R} -valued rvs X and Y with finite expectations, we have $X \leq_{icx} Y$ if and only if

$$\mathbf{E}[(X - a)^+] \leq \mathbf{E}[(Y - a)^+], \quad a \in \mathbb{R}.$$

Proposition 1 makes it clear why the variability orderings are likely vehicles for carrying out the comparisons discussed earlier. Put simply, establishing the comparison $L(R_1; C) \leq L(R_2; C)$ for *all* values of C between the loss rates of two information sources with steady-state rates R_1 and R_2 is *equivalent* to the comparison $R_1 \leq_{icx} R_2$.

Next, we explore the impact of the constraint $\mathbf{E}[X] = \mathbf{E}[Y]$ [6, Thm. 1.3.1, p. 9].

Proposition 2: For \mathbb{R} -valued rvs X and Y with finite expectations, we have $X \leq_{cx} Y$ if and only if $X \leq_{icx} Y$ and $\mathbf{E}[X] = \mathbf{E}[Y]$.

Finally, the convex ordering is closed under independent addition [6, p. 9].

Proposition 3: Consider two sets of mutually independent \mathbb{R} -valued rvs X_1, \dots, X_N and Y_1, \dots, Y_N . If $X_n \leq_{cx} Y_n$ for each $n = 1, \dots, N$, then

$$X_1 + \dots + X_N \leq_{cx} Y_1 + \dots + Y_N.$$

C. Majorization

Let K denote some given positive integer. For any vector $\mathbf{x} = (x_1, \dots, x_K)$ in \mathbb{R}^K , let $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(K)}$ denote the components of \mathbf{x} arranged in increasing order. For vectors \mathbf{x} and \mathbf{y} in \mathbb{R}^K , we say that \mathbf{x} is *majorized* by \mathbf{y} , and write $\mathbf{x} \prec \mathbf{y}$, whenever the conditions

$$\sum_{i=1}^k x_{(i)} \geq \sum_{i=1}^k y_{(i)}, \quad k = 1, 2, \dots, K \quad (9)$$

hold with

$$\sum_{i=1}^K x_i = \sum_{i=1}^K y_i. \quad (10)$$

Additional information regarding majorization can be found in the monograph [4]. Note that for any \mathbf{x} in \mathbb{R}^K , we have $x_{\text{av}} \mathbf{e} \prec \mathbf{x}$ with $\mathbf{e} = (1, \dots, 1)$ in \mathbb{R}^K , and

$$x_{\text{av}} = \frac{1}{K}(x_1 + \dots + x_K).$$

III. REDUCING VARIABILITY

Below we identify three operations that reduce variability, thus leading to comparisons in the ordering \leq_{cx} .

A. Normalized Bernoulli rvs

We begin with a comparison result for renormalized Bernoulli rvs. Recall that for p in $[0, 1]$, $B(p)$ denotes an $\{0, 1\}$ -valued rv with $\mathbf{P}[B(p) = 1] = p$.

Lemma 1: The collection of rvs $\{p^{-1}B(p), p \in (0, 1]\}$ is monotone decreasing in the convex ordering, i.e.,

$$q^{-1}B(q) \leq_{cx} p^{-1}B(p), \quad p < q.$$

In other words, increasing p makes $p^{-1}B(p)$ more variable.

Proof. We need to show that

$$\mathbf{E}[\varphi(p^{-1}B(p))] \leq \mathbf{E}[\varphi(q^{-1}B(q))], \quad p < q \quad (11)$$

for any convex mapping $\varphi : \mathbb{R} \rightarrow \mathbb{R}$, where

$$\mathbf{E}[\varphi(p^{-1}B(p))] = p(\varphi(p^{-1}) - \varphi(0)) + \varphi(0), \quad p \in (0, 1].$$

Hence, it suffices to establish (11) for convex mappings $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ such that $\varphi(0) = 0$. However, under this constraint, it is well known that $x \rightarrow x^{-1}\varphi(x)$ is non-decreasing on $(0, \infty)$ and the conclusion follows. ■

B. Heterogeneity decreases variability

For \mathbf{p} in $[0, 1]^K$, we define the rv $S_K(\mathbf{p})$ as the sum

$$S_K(\mathbf{p}) \equiv \sum_{k=1}^K B_k(p_k)$$

where the Bernoulli rvs $B_1(p_1), \dots, B_K(p_K)$ are assumed mutually independent.

Lemma 2: For vectors \mathbf{p} and \mathbf{q} in $[0, 1]^K$, it holds that $S_K(\mathbf{q}) \leq_{cx} S_K(\mathbf{p})$ whenever $\mathbf{p} \prec \mathbf{q}$.

Proof. For any integer-convex mapping $\varphi : \mathbb{N} \rightarrow \mathbb{R}$, we define the mapping $\Phi_K : [0, 1]^K \rightarrow \mathbb{R}$ by

$$\Phi_K(\mathbf{p}) \equiv \mathbf{E}[\varphi(S_K(\mathbf{p}))], \quad \mathbf{p} \in [0, 1]^K. \quad (12)$$

It is well known [4, F.1, p. 360] that the mapping Φ_K is Schur-concave in that the condition $\mathbf{p} \prec \mathbf{q}$ implies $\Phi_K(\mathbf{q}) \leq \Phi_K(\mathbf{p})$, and the conclusion $S_K(\mathbf{q}) \leq_{cx} S_K(\mathbf{p})$

follows from the definition of the convex ordering \leq_{cx} . ■ For any positive integer $L \leq N$, it holds that

$$\sum_{n=1}^N P_n B_n(f_n) \leq_{cx} \frac{NP^*}{L} \sum_{\ell=1}^L B_\ell(f^*) \quad (18)$$

where the rvs $B_1(f^*), \dots, B_L(f^*)$ are i.i.d. Bernoulli rvs.

The next result, originally due to Hoeffding [4, p. 359], is an immediate consequence of Lemma 2.

Lemma 3: For any vector \mathbf{p} in $[0, 1]^K$, it holds that $S_K(\mathbf{p}) \leq_{cx} S_K(p_{av} \mathbf{e})$ where $p_{av} = \frac{1}{K}(p_1 + \dots + p_K)$.

C. Linear combinations

Let $\{X_n, n = 1, 2, \dots\}$ denote a sequence of i.i.d. \mathbb{R} -valued rvs. The following result is an easy consequence of Proposition B.2 in [4, p. 287]; see also B.2.b in [4, p. 288].

Lemma 4: For each positive integer K , it holds that

$$\sum_{k=1}^K a_k X_k \leq_{cx} \sum_{k=1}^K b_k X_k \quad (13)$$

whenever $\mathbf{a} \prec \mathbf{b}$ in \mathbb{R}^K .

An immediate corollary to Lemma 4 is obtained by taking positive integers $L < K$, and $\mathbf{a} = K^{-1}(1, \dots, 1)$ and $\mathbf{b} = L^{-1}(1, \dots, 1, 0, \dots, 0)$ in $[0, 1]^K$.

Lemma 5: For positive integers $L < K$, it holds that

$$\frac{1}{K} \sum_{k=1}^K X_k \leq_{cx} \frac{1}{L} \sum_{\ell=1}^L X_\ell. \quad (14)$$

This last result was first derived by Marshall and Proschan [4, B.2.c, p. 288], and formalizes the notion that averaging decreases variability.

IV. THE MAIN RESULT

Consider N independent on-off sources as described in Section I-C, where for each $n = 1, \dots, N$, the n^{th} source $(P_n, B_n(f_n))$ has peak rate P_n and activity factor f_n so that its average rate m_n is given by

$$m_n = P_n f_n.$$

As these N sources are multiplexed, the resulting total average rate is simply

$$m_{\text{total}} = m_1 + \dots + m_N. \quad (15)$$

Proposition 4: With P^* selected so that

$$\max_{n=1, \dots, N} P_n := P_{\max} \leq P^*, \quad (16)$$

set

$$f^* := \frac{m_{\text{total}}}{NP^*}. \quad (17)$$

Thus, the aggregation of heterogeneous independent on-off sources can be upper bounded in the convex ordering by an aggregation of fewer related i.i.d on-off sources.

Proof. For each $n = 1, \dots, N$, define

$$f_n^* := \frac{P_n}{P^*} f_n = \frac{m_n}{P^*}$$

and note that

$$P_n f_n = P^* f_n^* = m_n,$$

so that f_n^* lies in $(0, 1]$ since $f_n^* \leq f_n$. From this last equality we conclude by Lemma 1 that

$$f_n^{-1} B_n(f_n) \leq_{cx} f_n^{*-1} B_n(f_n^*). \quad (19)$$

With this in mind, we now get

$$\begin{aligned} \sum_{n=1}^N P_n B_n(f_n) &= \sum_{n=1}^N P_n f_n \left(f_n^{-1} B_n(f_n) \right) \\ &= \sum_{n=1}^N P^* f_n^* \left(f_n^{-1} B_n(f_n) \right) \\ &\leq_{cx} \sum_{n=1}^N P^* f_n^* \left(f_n^{*-1} B_n(f_n^*) \right) \\ &= P^* \sum_{n=1}^N B_n(f_n^*) \end{aligned} \quad (20)$$

where the inequality follows from (19) via Lemma 3.

Next, we observe that

$$\frac{1}{N} \sum_{n=1}^N f_n^* = \frac{1}{N} \sum_{n=1}^N \frac{m_n}{P^*} = f^*.$$

Invoking Lemma 3 we then find that

$$\begin{aligned} P^* \sum_{n=1}^N B_n(f_n^*) &\leq_{cx} P^* \sum_{n=1}^N B_n(f^*) \\ &= NP^* \frac{1}{N} \sum_{n=1}^N B_n(f^*) \\ &\leq_{cx} NP^* \frac{1}{L} \sum_{\ell=1}^L B_\ell(f^*) \end{aligned} \quad (21)$$

where the second comparison follows from Lemma 5. Combining (20) and (21) readily leads to (18). ■

V. APPLICATIONS

Proposition 4 will now be used to discuss the conjecture of Rasmussen et al., the upper bound of Mao and Habibi and their conjectured upper bound. Given N independent on-off sources $(P_n, B_n(f_n))$ ($n = 1, \dots, N$), all these results express bounds of the form

$$\sum_{n=1}^N P_n B_n(f_n) \leq c_x P_{\text{new}} \sum_{\ell=1}^L B_\ell(f_{\text{new}}) \quad (22)$$

with i.i.d. on-off sources $(P_{\text{new}}, B_\ell(f_{\text{new}}))$ ($\ell = 1, \dots, L$) for appropriate constants $P_{\text{new}} \geq P_{\text{max}}$ and f_{new} in $(0, 1]$, and some positive integer $L \leq N$.

A. The bound by Rasmussen et al. [5]

Assume $P_1 = \dots = P_N =: P_c$, and apply Proposition 4 with $P^* = P_c = P_{\text{max}}$ and $L = N$. Direct inspection yields

$$f^* = \frac{m_{\text{total}}}{NP^*} = \frac{1}{N} \sum_{n=1}^N f_n$$

and the bound of Rasmussen and al. is obtained in the form (22) with $L = N$, $f_{\text{new}} = f^*$ and $P_{\text{new}} = P_c$. As should be clear from Lemma 3, this bound is simply a well-known stochastic comparison result for sums of Bernoulli rvs due to Hoeffding [4, p. 359].

B. The bound by Mao and Habibi [3, Thm. 3]

Assume $P_1 = \dots = P_N =: P_c$ and $m_1 = \dots = m_N =: m_c$, whence $P_{\text{max}} = P_c$, $m_{\text{total}} = Nm_c$ and

$$f_n = \frac{m_n}{P_n} = \frac{m_c}{P_c}, \quad n = 1, \dots, N.$$

With $P^* \geq P_c$ and $L = \lceil \frac{N}{U} \rceil$ for some positive integer U , it is plain that $L \leq N$ while (17) yields

$$f^* = \frac{m_{\text{total}}}{NP^*} = \frac{m_c}{P^*}.$$

Now select $P^* \geq P_c$ so that $\frac{NP^*}{L} = UP_c$; this is always possible by taking $P^* = \frac{U}{N} \lceil \frac{N}{U} \rceil P_c$. Applying Proposition 4 under these conditions, we get Theorem 3 in [3] in the form (22) with $L = \lceil \frac{N}{U} \rceil$, $f_{\text{new}} = f^*$ and $P_{\text{new}} = UP_c$.

C. The conjecture by Mao and Habibi [3, Conjecture 1]

Consider the inequality (18) of Proposition 4 with P^* and f^* selected as in (16)-(17), and $L \leq N$. A given target value $P_{\text{new}} \geq P_{\text{max}}$ for the peak rate is achieved in the upper bound of (18) when selecting the positive integer $L \leq N$ so that

$$\frac{NP^*}{L} = P_{\text{new}}, \quad (23)$$

in which case (18) becomes (22) with $f_{\text{new}} = f^*$ determined once P^* has been selected.

Reductions in computations are achieved by selecting the *smallest* admissible value of L , say L_{min} , so that P^* given via (23) satisfies (16). These constraints yield

$$L_{\text{min}} := \min\{L = 1, \dots, N : \frac{L}{N} P_{\text{new}} \geq P_{\text{max}}\} \quad (24)$$

whence L_{min} and the corresponding P^* are now given by

$$L_{\text{min}} = \lceil N \frac{P_{\text{max}}}{P_{\text{new}}} \rceil \quad \text{and} \quad P^* = \frac{L_{\text{min}}}{N} P_{\text{new}}, \quad (25)$$

so that

$$f_{\text{new}} = f^* = \frac{m_{\text{total}}}{NP^*} = \frac{m_{\text{total}}}{L_{\text{min}} P_{\text{new}}}. \quad (26)$$

The bound conjectured in [3] is also of the form (22) but with L_{MH} terms instead where

$$L_{\text{MH}} = \lceil \frac{P_{\text{total}}}{P_{\text{new}}} \rceil \quad \text{with} \quad P_{\text{total}} = P_1 + \dots + P_N. \quad (27)$$

We note that $L_{\text{MH}} \leq L_{\text{min}}$. Were we to apply Proposition 4 with L_{MH} terms, we would get (22) with

$$f_{\text{new}} = \frac{m_{\text{total}}}{NP^*} \quad \text{and} \quad P_{\text{new}} = \frac{NP^*}{L_{\text{MH}}} \quad (28)$$

whenever $P^* \geq P_{\text{max}}$. Unfortunately, if the strict inequality $L_{\text{MH}} < L_{\text{min}}$ holds, the definition of L_{min} precludes the existence of $P^* \geq P_{\text{max}}$ such that the second equality in (28) holds for any given value of P_{new} . Thus, in general, the conjectured bound of Mao and Habibi, if correct, is not a byproduct of Proposition 4. In fact we suspect that the conjectured bound is in error, being ‘‘too tight,’’ and should be replaced by the *provably correct* upper bound (22) characterized by (25) and (26).

REFERENCES

- [1] R.J. Gibbens, F.P. Kelly and P.B. Key, ‘‘A decision-theoretic approach to call admission control in ATM networks,’’ *IEEE Journal on Selected Areas in Communications* **JSAC-13** (1995), pp. 1101–1114.
- [2] T.-H. Lee, K.-C. Lai and S.-T. Duann, ‘‘Design of a real-time call admission controller for ATM networks,’’ *IEEE Transactions on Networking* **TON-4** (1996), pp. 758–765.
- [3] G. Mao and D. Habibi, ‘‘A tight upper bound for heterogeneous on-off sources,’’ Globecom 2000, San Francisco (CA), November 2000.
- [4] A.W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York (NY), 1979.
- [5] C. Rasmussen, J.H. Sorensen, K.S. Kvols and S.B. Jacobsen, ‘‘Source-independent acceptance procedures in ATM networks,’’ *IEEE Journal on Selected Areas in Communications* **JSAC-9** (1991), pp. 351–358.
- [6] D. Stoyan, *Comparison Methods for Queues and Other Stochastic Models*, English Translation (D.J. Daley, Editor), J. Wiley & Sons, New York (NY), 1984.