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Systematic Enumeration of Parallel Manipulators

by Lung-Wen Tsai

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Abstract

In this paper, a systematic methodology for enumeration of the kinematic structures of parallel manipulators is presented. Parallel manipulators are classified into planar, spherical, and spatial mechanisms. The classification is followed by an enumeration of the kinematic structures according to the degrees of freedom and connectivity listing. In particular, a class of parallel manipulators with pure translational motion capability is enumerated.

Nomenclature

C: cylindric joint
C_k: connectivity of limb k which is defined as the degrees of freedom associated with all the joints of limb k
F: degrees of freedom of a mechanism
P: prismatic joint
R: revolute joint
S: spherical joint
U: universal joint
m: number of limbs in a parallel manipulator
n: number of links in a mechanism
j: number of joints in a mechanism, assuming that all the joints are binary
f_i: degrees of freedom associated with joint i
L: number of independent loops in a mechanism
λ: freedom of the space in which a mechanism is intended to function

1 Introduction

The development of parallel manipulators can be dated back to the early 1960's when Gough and Whitehall (1962) first devised a six-linear jack system for use as a universal tire testing machine. Later, Stewart (1965) developed a platform manipulator for use as an aircraft simulator. Recently, there has been an ever increasing interest in the study of parallel manipulators (Clearly and Arai, 1991; Fichter, 1986; Grffis and Duffy, 1989; Husain and Waldron, 1994; Innocenti and Parenti-Castelli, 1990; Lin et al., 1994; Mohamed and Duffy, 1985; Nanua et al., 1990; Raghavan, 1993; Zhang and Song, 1994; etc.).

A parallel manipulator typically consists of a moving platform that is connected to a fixed base by several limbs. The number of limbs is usually equal to the number of degrees of freedom (DOF) of the moving platform such that each limb requires only one actuator and all actuators can be mounted on or near a fixed base. As a result of this special kinematic structure, parallel manipulators can be designed with relatively low inertia, high stiffness, large payload, and high speed capability. These advantages continue to motivate research and development as evidenced by machining centers recently developed by Giddings & Lewis and Ingersoll Milling Machine Co. (Aronson, 1996).

Although parallel manipulators have been studied extensively, most of the studies have concentrated on the Stewart-Gough type manipulator. The Stewart-Gough type manipulator, however, has a relatively small workspace and complex mechanical design. Furthermore, its direct kinematics is extremely difficult to solve. Therefore, it may be advantageous to explore other types of parallel manipulators with reduced complexity (Lee and Shah, 1987; Pierrot et al., 1990, Tsai, 1996; Tsai and Tahmasebi, 1993; Tsai and Stamper, 1996). However, most of these manipulators were developed on an ad hoc basis. Hunt (1983) first studied the structural kinematics of parallel manipulators. And this appears to be the only literature on the classification of parallel manipulators.

In this paper, a systematic methodology for enumeration of the kinematic structures of parallel manipulators is introduced. Parallel manipulators are classified into planar, spherical, and spatial mechanisms. Then, the kinematic structures of parallel manipulators are enumerated according to their degrees of freedom and connectivity listings.

2 Design Methodologies

Mechanical design is the process of creating synthesized solutions in the form of products or systems that satisfy customer's needs (Ullman, 1992). It can be thought of as a mapping from a functional space to a physical space. The design process can be divided into three interrelated phases: (1) product specification and planning phase, (2) conceptual design phase, and (3) product design phase. During the product specification and planning phase, we identify the customer's needs and translate them into engineering specifications in terms of the functional requirements, time and money available for the development, and plan the project accordingly. In the conceptual design phase, we generate as many design alternatives as possible, and seek for a most promising concept for product development. A rough idea of how the product will function and what it looks like is developed. In the product design phase, dimensional synthesis, design analysis, design optimization, prototype demonstration, and engineering documentation are produced.

The design process is iterative in nature and the solutions are usually not unique. It should be noted that although the third phase is usually the most time consuming phase, most of the manufacturing cost of a product is committed by the end of conceptual design phase. According to a survey, 75 percent of the manufacturing cost of a typical product is committed during the first two phases. Hence, great care must be taken during the product specification and conceptual design phases.

The conceptual design of a mechanism is traditionally accomplished by the designer's ingenuity and experience. Perhaps, a more efficient approach is to make use of atlases of mechanisms. Mechanisms are classified according to their functional characteristics and used as a primary source of ideas for the designers. This approach, however, cannot ensure the identification of all possible design alternatives, nor does it necessarily lead to an optimum design. Recently, a new approach based on the concept of separation of kinematic structure from function has been evolved (Freudenstein and Maki, 1979). The kinematic structure contains the essential information about which link is connected to which other link by what type of joint. It can be employed for systematic enumeration of mechanisms, if the mechanism structure characteristics are properly understood. The systematic methodology can be summarized as follows:

S1. Identify the functional requirements from the customer's needs.

S2. Determine the nature of desired motion (planar, spherical, or spatial mechanism), degrees of freedom, type and complexity of the mechanisms of interest.

S3. Identify the structural characteristics associated with some of the desired functional requirements.

S4. Enumerate all the possible kinematic structures which satisfy the above structural characteristics.

S5. Sketch the corresponding mechanisms and screen out isomorphic mechanisms, if any. Two mechanisms are said to be isomorphic if there is a one-to-one correspondence between their links and joint which preserve the connection between links.

S6. Use the remaining functional requirements to eliminate all the infeasible solutions. This results in a set of candidate mechanisms.

S7. Move onto the product design phase.

In general, the systematic design methodology consists of two engines: a *generator* and an *evaluator* as shown in Fig. 1. Specifically, some of the functional requirements are transformed into structural characteristics. These structural characteristics are incorporated as rules in the generator. The generator enumerates all possible solutions via a combinatorial analysis. The remaining functional requirements are incorporated as evaluation criteria in the evaluator to screen out infeasible solutions (Chatterjee and Tsai, 1994). This results in a set of candidate mechanisms. Finally, a most promising candidate is chosen for product design. The process may be iterated several times until a final product is achieved. This method of synthesis has been successfully applied for the structural synthesis of automotive transmission mechanisms, variable-stroke engine mechanisms, robotic wrist mechanisms, etc. (Chatterjee and Tsai, 1994; Freudenstein and Maki, 1983; Lin and Tsai, 1989).

How many of the desired functional requirements should be incorporated in the generator is a matter of engineering compromise. The more functional requirements are translated into structural characteristics

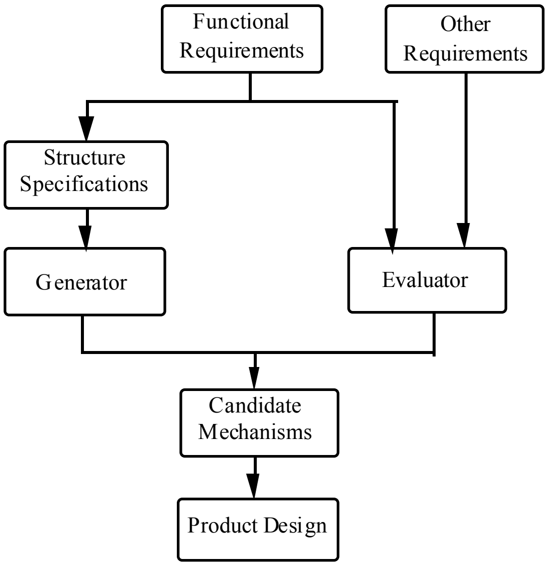

Figure 1: Flow chart of a systematic design methodology.

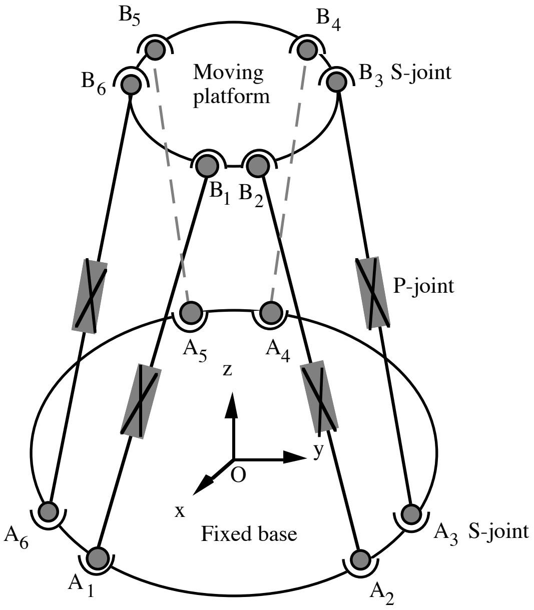

Figure 2: A typical parallel manipulator.

and incorporated in the generator, the less work is required of the evaluator. However, this may make the generator too complex to develop. Thomas Edison said “Genius is one percent inspiration and ninety-nine percent perspiration.” Inspiration can occur more readily only if perspiration is properly directed and focused. The systematic methodology presented in this paper is intended to help better organize the perspiration so that the inspiration can take place early in the design process.

3 Functional Requirements

In this study, we are interested in the enumeration of the kinematic structures of parallel manipulators. The functional requirements of such mechanisms can be hypothetically stated as follows:

F1. The mechanisms of interest are closed-loop mechanisms. Specifically, they consists of a moving platform that is connected to a fixed base by several limbs as shown in Fig. 2. The moving platform is to be used as the end-effector

F2. They possess multiple degrees of freedom. The number of DOF depends on the intended application.

F3. The number of limbs is preferable equal to the number of DOF such that only one actuated joint is required of each limb and the load on the moving platform can be shared by all actuators.

F4. All actuators are to be mounted on or near the fixed base. This condition implies that there is a base-connected revolute or prismatic joint in each limb, or a prismatic joint which is adjacent to a base-connected joint.

F5. Other specific design requirements.

4 Structural Characteristics

In this section, we translate as much functional requirements into structural structural characteristics as possible. It turns out that many of the desired structural characteristics can be derived from the basic kinematic equations.

Except for some overconstrained mechanisms, the degrees of freedom of a mechanism is governed by:

$$F = \lambda(n - j - 1) + \sum_i f_i. \tag{1}$$

The relationship between the number of independent loops, the number of links, and the number of joints in a mechanisms is given by Euler's equation:

$$L = j - n + 1. \tag{2}$$

Eliminating n and j from Eqs. (1) and (2), yields the loop-mobility criterion as:

$$\sum f_i = F + \lambda L. \tag{3}$$

We assume that each limb in a parallel manipulator is made up of a simple open-loop chain, and the number of limbs is equal to the number of DOF of the moving platform. Hence, it can be shown that

$$m = F = L + 1. \tag{4}$$

Let the connectivity, C_k, of the kth limb be defined as the DOF associated with all the joints on that limb. Then, it becomes obvious that

$$\sum_{k=1}^{m} C_k = \sum_{i=1}^{j} f_i. \tag{5}$$

Substituting Eq. (3) into Eq. (5), and then eliminating L by making use of Eq. (4), we obtain

$$\sum_{k=1}^{m} C_k = (\lambda + 1)F - \lambda. \tag{6}$$

To ensure proper mobility and controllability of the moving platform, the connectivity of each limb should not be greater than the motion parameter nor less than the DOF of the moving platform. That is

$$\lambda \geq C_k \geq F. \tag{7}$$

Equations (1), (4), (6) and (7) completely characterize the structural topology of a parallel manipulator.

5 Enumeration Of Parallel Manipulators

As mentioned earlier, the systematic design methodology consists of two engines: a generator and an evaluator. By incorporating Eqs. (4), (6) and (7) in the generator, functional requirements F1, F2, and F3 are automatically satisfied. The F4 and other requirements, if any, are more suitable for use as evaluation criteria. In what follows, we enumerate the kinematic structures of parallel manipulators according to their nature of motion and degrees of freedom.

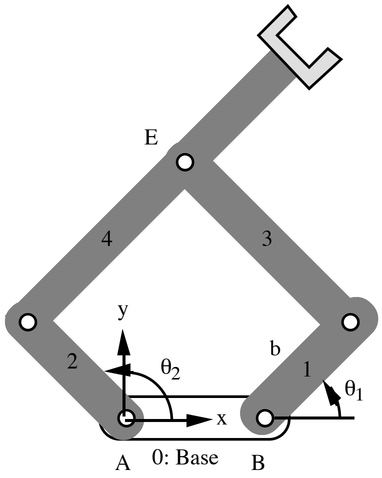

Figure 3: A planar 2-DOF, $RR - RRR$, parallel manipulator.

5.1 Planar Parallel Manipulators

For planar manipulators, we have $\lambda = 3$. Let revolute and prismatic joints be the available joint types. Then, all the revolute joint axes must be perpendicular to the plane of motion and the prismatic joint axes must lie on the plane of motion.

5.1.1 Planar Two-DOF Manipulators

For two-DOF manipulators, Eq. (4) yields $m = F = 2$ and $L = 1$. Equation (6) yields $\sum_{k=1}^{2} C_k = 5$. Hence, planar two-DOF manipulators are single-loop mechanisms, and the degrees of freedom associated with all the joints must be equal to five. Furthermore, Eq. (7) states that the connectivity in each limb is limited to no more than three and no less than two. Hence, one of the limbs is a single link and the other limb is a two-link chain. These two limbs together with the end-effector and the base link form a five-bar linkage.

A simple combinatorial analysis yields the following possible closed-loop five-bar linkages: $RRRRR$, $RRRRP$, $RRRPP$, and $RRPRP$. Any link of the five-bar linkages can be chosen as the base link. Once the base link is chosen, any of the two links that is not adjacent to the base link can be assigned as the end-effector link. For example, Fig. 3 shows a planar $RR - RRR$ manipulator.

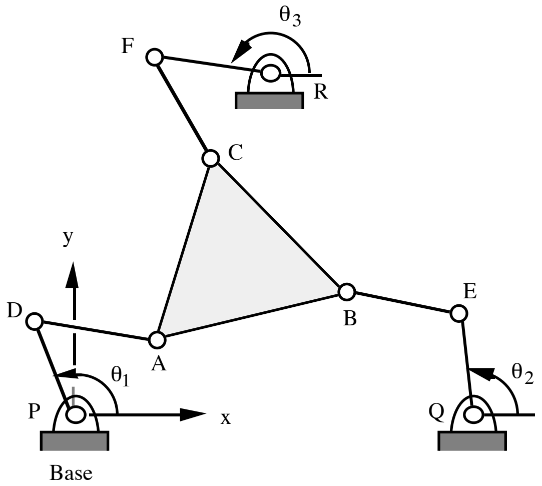

Figure 4: A planar 3-DOF, 3-*RRR*, parallel manipulator.

5.1.2 Planar Three-DOF Manipulators

For planar three-DOF parallel manipulators, Eq. (4) yields $m = F = 3$ and $L = 2$. Substituting $\lambda = 3$ and $F = 3$ into Eq. (6), we obtain:

$$C_1 + C_2 + C_3 = 4F - 3 = 9. \tag{8}$$

Furthermore, Eq. (7) reduces to

$$3 \geq C_k \geq 3. \tag{9}$$

Hence, the connectivity of each limb should be equal to three. That is, each limb has three degrees of freedom in its joints. Using revolute and prismatic joints as the available kinematic pairs, we obtain seven possible limb configurations: RRR, RRP, RPR, PRR, RPP, PRP, and PPR. The PPP combination is rejected due to the fact that it does not permit rotation of the end-effector. Theoretically, any of the above configurations can be used as a limb. Hence, there are potentially $7^3 = 343$ possible planar 3-DOF parallel manipulators. However, if we limit ourselves to those manipulators with identical limb structures, then the number of feasible solutions reduces to seven.

For examples, Fig. 4 shows a planar 3-DOF parallel manipulator using the *RRR* limb configuration and Fig. 5 shows another manipulator using the *PRP* limb configuration (Mohammadi et al., 1993).

5.2 Spherical Parallel Manipulators

The motion parameter for spherical mechanisms is also equal to three. Hence, the connectivity requirement for spherical parallel manipulators is identical to that of planar parallel manipulators. However, revolute joint is the only permissible joint type for the construction of spherical linkages. In addition, all the joint axes must intersect at a common point, called the *spherical center*. Therefore, the only possible two-DOF spherical manipulator is the five-bar $RR - RRR$ manipulator. Similarly, the only possible three-DOF spherical manipulator is the 3-*RRR* manipulator as shown in Fig. 6. Several articles related to the design and analysis of spherical parallel manipulators can be found in (Gosselin and Angeles, 1989 and 1990; Gosselin and Hamel, 1994; Innocenti and Parenti-Castelli, 1993; Wohlhart, 1994).

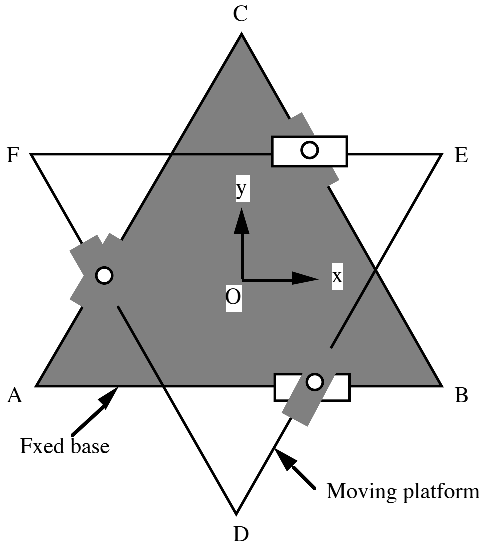

Figure 5: A planar 3-DOF, 3-*PRP*, parallel manipulator.

We note that spherical geared robotic mechanisms form an entirely different class of manipulators (Lin and Tsai, 1989; Chang and Tsai 1989) which are not included in this study.

5.3 Spatial Parallel Manipulators

For spatial manipulators, we have $\lambda = 6$. Equations (6) and (7) reduce to

$$\sum_{k}^{m} C_k = 7F - 6 \tag{10}$$

and

$$6 \geq C_k \geq F. \tag{11}$$

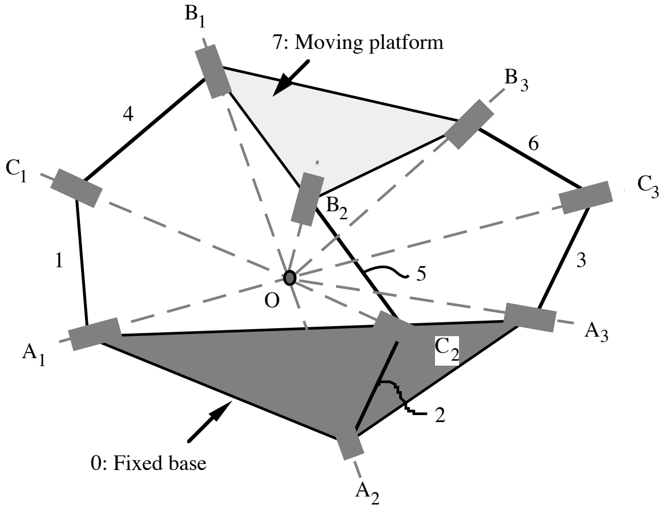

Figure 6: A spherical three-DOF, 3-RRR, manipulator.

Table 1: Classification of spatial parallel manipulators

Degrees of freedom F	Sum of all joint freedoms $\sum_i f_i$	Connectivity listing $C_k, k = 1, 2, 3 \cdots$
2	8	4,4 5,3 6,2
3	15	5,5,5 6,5,4 6,6,3
4	22	6,6,5,5 6,6,6,4
5	29	6,6,6,6,5
6	36	6,6,6,6,6,6

Table 2: Feasible limb configurations for spatial three-DOF manipulators

Type	Kind
201	RRS, RSR, RPS, RSP, PSR, PRS SPR, PPS, PSP, SPP
120	UPU, RUU, PUU

Solving Eqs. (10) and (11) simultaneously for positive integers of C_k, we can classify spatial parallel manipulators according to their degrees of freedom and connectivity listings as shown in Table 1.

The number of links incorporated in each limb can be any as long as the sum of all joint freedoms is equal to the required connectivity. The maximum number of links occurs when all the joints are one-DOF joints. Obviously, this will result in a large number of possible manipulators. In what follows, three- and six-DOF manipulators will be enumerated to illustrate the methodology.

5.3.1 Three-DOF Translational Platforms

We first enumerate three-DOF manipulators with pure translational motion characteristics. To narrow down the search domain, we shall limit ourselves to manipulators with three identical limb structures. Furthermore, we assume that each limb consists of two links and three joints. Referring to Table 1, the (5, 5, 5) connectivity listing becomes the only feasible solution. Hence, the degrees of freedom associated with all the joints of a limb should be equal to five. Let revolute, prismatic, universal, and spherical joints be the applicable joint types. A combinatorial analysis yields thirteen feasible kinds of limbs which can be categorized into two types as listed in Table 2.

For each type of limbs listed in Table 2, the first digit denotes the number of one-DOF joints, the second digit represents the number of two-DOF joints, and the third digit stands for the number of three-DOF joints. Thus, type 201 implies that there are two one-DOF, zero two-DOF, and one three-DOF joints in each limb, and type 120 indicates that there are one one-DOF, two two-DOF, and zero three-DOF joints in each limb. The joints listed for each kind of limbs, starting from the left to the right, correspond to a base-connected joint, an intermediate joint, and a moving-platform connected joint, respectively. Since it is preferable to have a ground-connected revolute or prismatic joint, or an intermediate prismatic joint for actuation purpose, SRR, SRP, URU, UUR, and UUP configurations are excluded from consideration.

Next, we consider the condition for the moving platform to possess pure translational motion characteristics. Intuitively, each limb should provide one constrain to the rotational degrees of freedom of the moving platform. Furthermore, the overall constraints provided by the three limbs should completely immobilize the rotation of the moving platform. Since a spherical joint cannot provide any constraint on the rotation of the moving platform, the entire type 201 limb configurations are excluded from further

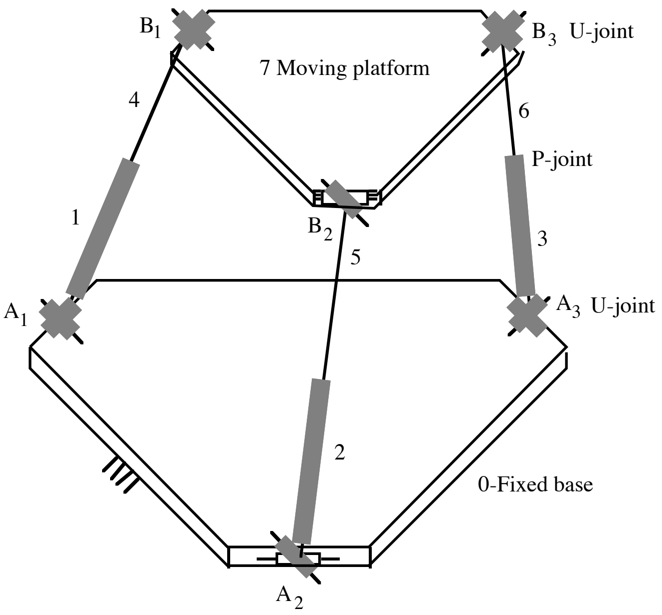

Figure 7: A spatial three-DOF, 3-*UPU*, translational platform.

consideration. Hence, we are left with three feasible limb configurations: *UPU*, *RUU*, and *PUU*.

To achieve pure translation, the axes of the two universal joints must be arranged in a special configuration. Specifically, the two inner revolute joint axes of the *U*-*U* pair must be parallel to each other, and the two outer revolute joint axes must also be parallel to each other. The prismatic joint of the *UPU* limb can be directed along a line connecting the two universal centers. The prismatic joint of the *PUU* limb can be directed along any direction. The base-connected revolute joint of the *RUU* limb must be parallel to the two outer joint axes of the *U*-*U* pairs.

Figures 7, 8, and 9 show the schematic diagrams of three parallel manipulators with the 3-*UPU*, 3-*PUU* and 3-*RUU* limb configurations, respectively. The kinematics of the 3-*UPU* manipulator was studied in detail by Tsai (1996). The 3-*RUU* manipulator was evolved into a mechanism with two short links and a planar parallelogram in each limb as shown in Fig. 10 (Stamper et al., 1997).

5.3.2 Six-DOF Parallel Manipulators

In this section, we briefly discuss on the enumeration of six-DOF parallel manipulators. Again, we limit ourselves to manipulators with six identical limb

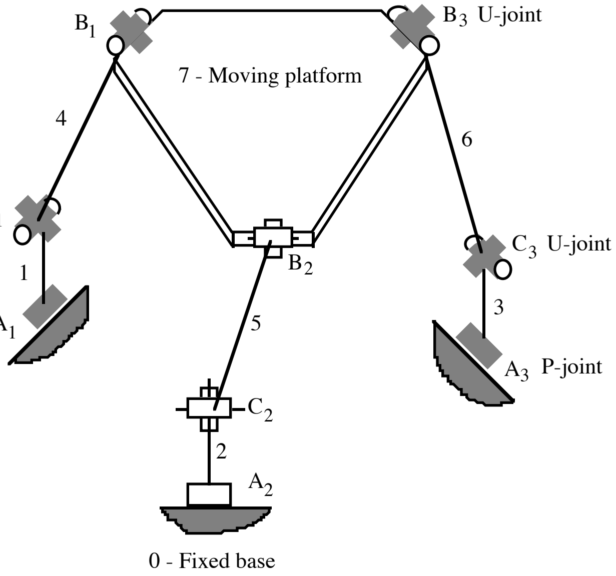

Figure 8: A spatial three-DOF, 3-*PUU*, translational platform.

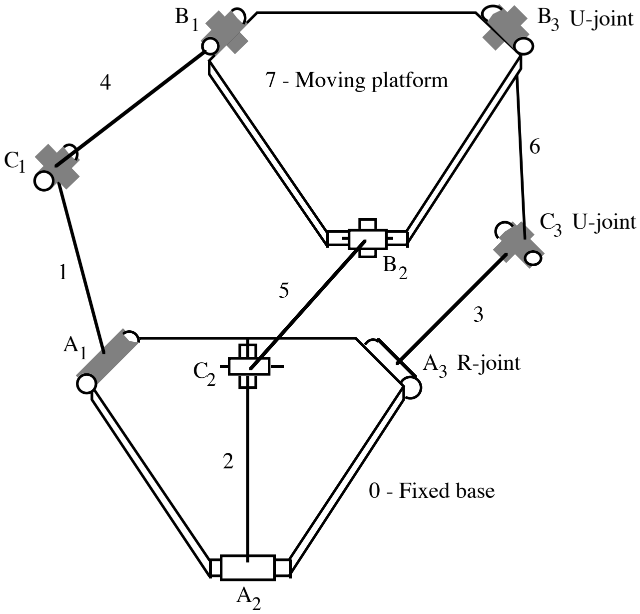

Figure 9: A spatial three-DOF, 3-*RUU*, translational platform.

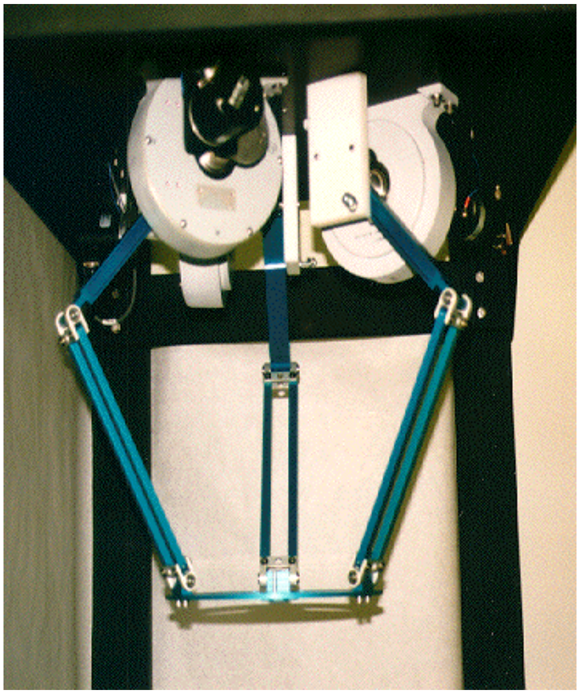

Figure 10: Prototype of a three-DOF translational platform.

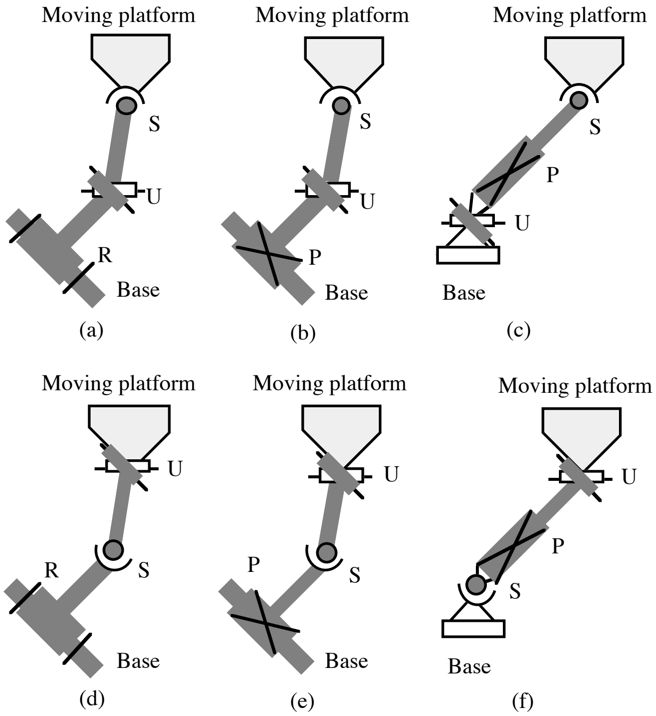

Figure 11: Six six-DOF limb configurations.

structures. We also assume that each limb consists of two links and three joints.

Referring to Table 1, we see that the (6,6,6,6,6,6) connectivity listing is the only solution. That is, the degrees of freedom associated with all the joints of a limb should be equal to six. Since there are three joints, the only possible solution is the type 111 limb which means that each limb consists of one of each of one-, two-, and three-DOF joints. Let revolute, prismatic, universal, and spherical joints be the applicable joint types. Then, there exist six feasible limb configurations: RUS, RSU, PUS, PSU, SPU, and UPS. Configurations SRU, SUR, URS, USR, SUP, and USP are excluded because they do not contain a base-connected revolute or prismatic joint, or an intermediate prismatic joint. Figure 11 shows six such limb configurations.

Note that the universal joints shown in Fig. 11 can be replaced by a spherical joint. This results in a passive degree of freedom, allowing the link to spin freely about a line passing through the centers of the two spheres. Thus, RSS, PSS and SPS are also feasible limb configurations. Furthermore, if a cylindric joint is used, then UCU and SCS limbs with the cylindric joint axis passing through the centers of the universal and spherical joints, respectively, are also feasible configurations.

6 Summary

A methodology for systematic enumeration of mechanisms is presented. Parallel manipulators are classified into planar, spherical, and spatial mechanisms. The structural characteristics associated with such parallel manipulators are identified. Then, these structural characteristics are employed for the enumeration of the kinematic structures using combinatorial analysis. To further demonstrate the methodology, a class of three-DOF parallel manipulators with pure translational motion characteristics is developed.

In the above derivation, we have excluded the cylindric and helical joints as two possible joint types. We have also limited ourselves to two major links in each limb and identical limb kinematic structures. Obviously, if these limitations are removed, the number of feasible solutions will grow exponentially.

We note that it is entirely conceivable to design a parallel manipulator with fewer number of limbs than the number of degrees of freedom. For such a

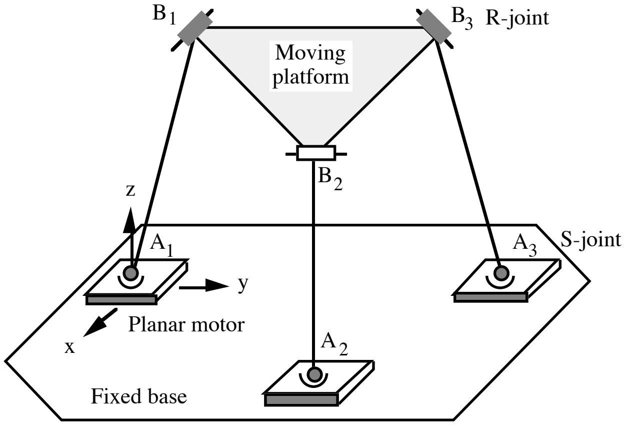

Figure 12: A six-DOF parallel manipulator with three supporting limbs.

design, more than one actuators would be needed for each limb. For example, Tahmasebi and Tsai (1995) developed a six-DOF parallel mini-manipulator with three supporting limbs. In this manipulator, each limb is driven by a bi-directional planar motor as shown in Fig. 12.

It is also conceivable to construct a parallel manipulator with more number of limbs than than the number of degrees of freedom. In this case, the connectivity of some of the limbs should be equal to the motion parameter, λ, so that they do not add any constraint to the moving platform. Figure 13 shows a three-DOF manipulator with six limbs. The three UPU Limbs, A_2B_2, A_4B_4 and A_6B_6, provide three constraints to the moving platform, while the prismatc joints in the three SPS limbs, A_1B_1, A_3B_3 and A_5B_5, are driven by three linear actuators. This arrangement has the advantage of separating the function of constraint from that of actuation.

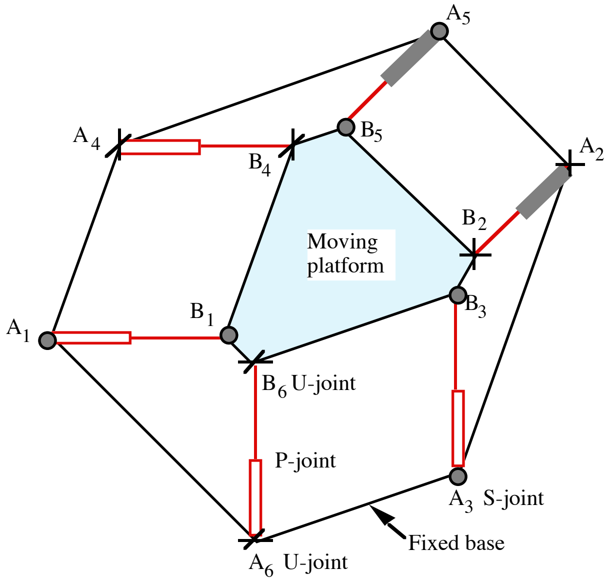

Figure 13: A three-DOF translational platform with six supporting limbs.

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