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Locators and Sensors for Automated Coordinate Checking Fixtures

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Locator and Sensor Placement for Automated Coordinate Checking Fixtures

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Abstract

This article proposes a systematic method for the optimal design of sensor locations for an automated Coordinate Checking Fixture (CCF). The fixture can be employed for making at-machine assessments of the dimensional accuracy of manufactured components. Coordinate measurements obtained by the sensors built into the fixture can be utilized in estimating geometric parameters of a manufactured part. Two important issues that arise in the design of a CCF are the optimal number of sensors to be used and the best locations for each sensor. The proposed method uses statistical analyses of the Fisher information matrix and the prediction matrix to obtain an optimal set of sensors from an initial candidate set. Sensors are placed at locations that maximize the determinant of the Fisher information matrix for best parameter estimation, while the sensor of the least contribution to the measurement objective is iteratively eliminated. With the benefit of physical insight, the design procedure results in a balanced decision for the ultimate placement of sensors. The developed method also addresses the problem of selection of part locators for part localization in the CCF. Examples are provided for illustration of the developed procedure for automotive space frame extrusion parts.

1 Introduction

For a process of manufacturing quality components, especially in metal forming, it is often necessary to keep a close vigilance on the dimensions and tolerance compliance of output parts. A good example is the aluminum extrusions used in the recent space-frame automotive body structures (Ashley 1994). The aluminum-alloy extrusions are generally formed into complex three dimensional configurations on bending machines to meet the required streamlined shapes of modern cars. Their functional and assembly requirements necessitate tight dimensional and shape tolerances. Deviations in the settings of manufacturing process parameters frequently cause the resulting components to be dimensionally unacceptable for assembly operations.

Thus, there is a need for an efficient inspection system which could constantly monitor the manufacturing process and provide corrective feedback useful in maintaining the process parameters within acceptable limits. An important element of such a system is its ability to characterize the geometric variations in the parts, from their detected dimensional errors using a coordinate measuring device. A Coordinate Measuring Machine (CMM) is often used to play a significant role in obtaining the required coordinate data.

Since CMMs are rather fragile and expensive to be placed directly on the shop floor, the manufactured parts to be inspected have to be transported to a CMM inspection laboratory. The CMM has then to be programmed suitably to measure locations of surface points of interest to the inspection. Consequently, the process of part inspection becomes tedious and time consuming. It may often lead to a significant amount of idle or lag time for the manufacturing task, particularly during its set-up and process calibration stages. For periodic monitoring, the delay would hamper the production rate of the process. This loop of bringing parts to the CMM and then bringing the analysis results back to the machine tool may prove to be a major bottleneck, especially when frequent inspection is required in a high volume production.

This article suggests an alternative technique of using automated Coordinate Checking Fixtures (CCFs) for at-machine measurement and analysis of manufactured parts. The goal is to eliminate part transporting and machine down-time for the process re-calibration with valuable dimensional measurement feedback. The article describes the general concept of Coordinate Checking Fixtures and outlines major issues in their design and implementation. The majority of the article focuses on the most fundamental and critical design problem, namely, placement of a set of position sensors for automated coordinate measurements and selection of appropriate part locators and a clamping scheme for part restraint. These discussions are put in the context of the manufacture of the automotive space-frame extrusions. Applications of the CCFs are, of course, not limited to the specific example.

2 Automated Coordinate Checking Fixtures

Similar in many respects to the conventional check fixtures, automated Coordinate Checking Fixtures (CCFs) also have a set of locating and clamping elements to secure the part in the fixture, by spatially constraining the six rigid body degrees of freedom of the fixtured part. Traditionally, 6 locator points are chosen as a set of 3, 2 and 1 points, respectively, in three orthogonal planes. Therefore, they are traditionally referred to as the "3-2-1" points. Additionally, the automated CCF has a set of position sensors embedded in the fixture for measuring the coordinates of certain points targeted on the part surface. Figure 1 shows the schematic of an automated CCF meant for dimension detection of a simple bent extrusion component. Such fixtures can be conveniently placed at-machine, and configured to provide the required data input for a suitable process of manufactured part analysis like manufactured part modeling (Wang 1995). Prompt feedback can be obtained, so that the necessary adjustments to the process can be made.

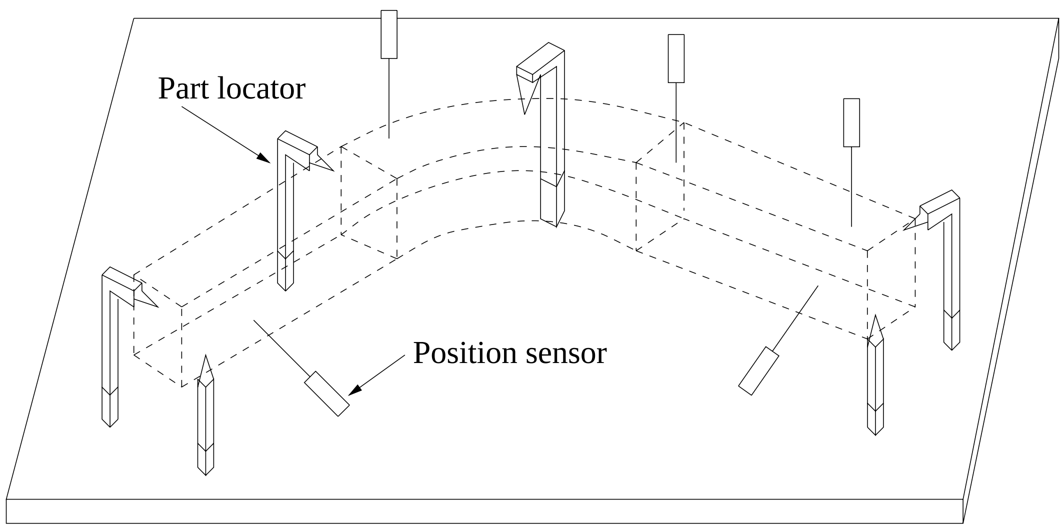

Figure 1: Schematic of an automated CCF for a V-shaped extrusion.

Among major design issues, a suitable sensing technique has to be chosen for an automated CCF. For automotive structure applications, dial-indicators with electronic interface, miniature position probes and displacement transducer such as LVDTs are cost-effective and have sufficient accuracy. Vision systems and laser probes among other types of non-contact measuring devices may also be suitable for the purpose. In any case, the term "sensor" in this article refers to the suitable sensing device without specification of a particular type.

Perhaps the most important issue related to the design of automated CCFs is the placement of the sensors and the locators. Variational characteristics of the manufactured part being tested are described by a mathematical model. The characteristic parameters or properties are estimated by an analysis process in which measured dimensional variations are compared with predictions made by the model. The premise is that if agreement can be attained, the parameters are uniquely and accurately estimated. This premise depends significantly on the number of sensors employed and on the spatial placement of the sensors as well as of the locators used for constraining the part. This is the fundamental problem investigated in the research work reported in this article. Other issues in a detailed CCF design, such as mechanical structures, locator geometry, clamping mechanisms, and data acquisition, are more related to the practical use of a specific CCF in question. These issues are not the focus of the article and are not addressed here.

In the following, an overview of related research in the fields of part localization, model fitting, and optimal measurement location studies is given first. A mathematical formulation of the proposed methodology of optimal sensor placement is then described. The design of the part locating and clamping elements is also discussed. Then, a step-by-step procedure to implement the design approach is detailed. Finally, examples are provided to illustrate the approach in an application to the automotive space-frame extrusions.

3 Previous Work

The problem of sensor and locator placement for Coordinate Checking Fixtures (CCFs) is closely related to conventional fixture design which focuses on part localization. There is substantial literature on general fixture design. The basic concept of fixturing is form closure (Reuleaux 1963), which has been extensively studied in recent years in fixture design (Asada 1985, Mani 1986, Chou 1989, DeMeter 1994) and in robotics (Lakshminarayana 1978, Mishra 1986, Nguyen 1986, Li 1986, Trinkle 1986). The problem has also been investigated for modular fixture design (Brost 1994, Thompson 1986, Hoffman 1987). This problem is equivalent to that of part localization studied in (Sahoo 1991).

The use of an automated CCF is tightly related to the concept of modeling and analysis of manufactured parts. Etesami (1988) presented a modeling scheme for manufactured parts using perfect form representation of surface features, curves and datum coordinate systems. Stelson (1995) presented a statistical method for finding the maximum allowable bend angle error for the required tolerance specifications for multi-bent tubes. Wang (1995) proposed the use of a variational model for representing a manufactured part, comprising critical shape and rigid body location parameters. The model is constructed by fitting CMM data through a least squares optimization. This manufactured part modeling analysis could be integrated with the automated Coordinate Checking Fixture (CCF) proposed in this article to develop an at-machine inspection and analysis system.

Techniques for choosing suitable measurement/sensor locations have been applied in various areas, including machine vision (Cowan 1988, Yi 1995) and manufacturing systems for fault diagnosis in automobile assembly (Khan 1995) and for CMM measurements (Choi 1996, Gupta 1997).

Kammer (1991) presented a methodology for selection of an optimum set of sensor locations for on-orbit modal identification for large space structures. Initially a large candidate sensor set is chosen. The sensor locations are then ranked depending upon their contribution to the linear independence of the modal partitions, and sensors with smaller contributions are eliminated. Effects of sensor noise and model uncertainty can be also considered (Kammer 1992). Fadale (1995) proposed two approaches to optimal sensor locations in heat transfer problems, a procedure involving the maximization of the determinant of the Fisher information matrix and a sensitivity analysis that takes into account of interaction between parameters. These two procedures were shown to lead to identical designs in some special cases.

The approach presented in this article for the optimal placement of locators and sensors in automated Coordinate Checking Fixtures (CCFs) is based on and extends the methods of (Fadale 1995) and (Kammer 1991). Properties that are unique to the automated CCF design are analyzed. The aim of the proposed design procedure is to achieve a *balanced design* in making the best parameter estimation with a minimal number of sensors.

4 Mathematical Formulation

A Coordinate Checking Fixture (CCF) is similar to a conventional fixture whose primary function is to locate and hold a manufactured part for manufacturing, assembly, and inspection operations. A main difference of the CCF is that a set of position sensors are incorporated in the CFF for coordinate measurement of the part being fixtured. There are essentially two major purposes for the measurements. The first purpose is for the geometric quality inspection of the part. In this case, tolerance requirements may be specified at a set of pre-defined locations on the part. Thus, the sensors on the CCF are positioned at these pre-specified locations and their readings would indicate if the part being inspected meets the point-wise tolerance specifications. This approach to tolerance conformance inspection has been practiced in the automotive industry quite often.

Another application of an automated CCF is to use the coordinate measurements to estimate a set of geometric parameters that characterize geometric deviations of the part from its nominal shape. The geometric deviations may be resulted during the manufacturing processes, and their estimation may prove to be valuable for process monitoring and correction. For this purpose, a mathematical model characterizing geometric deviations is necessary and the locations of sensors and locators of the CCF play a significant role.

The mathematical model describing geometric variations of a manufactured part is referred to as a manufactured part model (Wang 1995). In the model the extent of shape deviations from the ideal shape are specified by the domain of a set of shape descriptors $\{\phi\}_{n\times 1}$, that parameterize a collection of lines, surfaces, and other geometric elements that describe the part. The interest in using an automated CCF is to estimate these n shape parameters from the coordinate data collected by the sensors placed in the checking fixture. The CCF must also have a set of locators and a clamping device to firmly securing the part in the fixture. Six point locators (or equivalent) are sufficient in theoretical consideration.

This section describes a theoretical formulation and provides a basis for the method of locator and sensor location optimization presented in the following sections. The method for locator placement uses rigid body constraining properties of the fixture, while the sensor placement is based on description relating the deviations of the spatial (rigid body) and geometric (shape) parameters of the part to the changes in readings of sensors located at various positions on the part surface. In the formulation, part locators can be considered equivalent to position sensors, with the special condition that they are always in contact with the inspected part. Therefore they register a zero reading at all times.

With the above equivalence in mind, consider the CCF to be equipped with m sensors and 6 point locators. Let the manufactured part of interest have n geometric (shape) parameters $\{\phi\}_{n\times 1}$ apart from the 6 rigid body parameters $\{t\}_{6\times 1}$, 3 for rotation and 3 for translation. Due to the special property of the part locators, if a nominal (without any geometric error) part is placed in the fixture, it will fit perfectly with none of the sensors registering any displacement or deviation, provided that the sensors are suitably calibrated before use. On the contrary, an imperfect part would have to be adjusted a bit before it can be securely constrained. As a result, its position and orientation in the fixture would differ from the nominal case. In effect, changes (imperfections) $\{\delta\phi\}_{n\times 1}$ in the shape parameters would cause deviations $\{\delta t\}_{6\times 1}$ in the position and orientation of the part. Accompanying these would be corresponding changes $\{\delta d\}_{(m+6)\times 1}$ in the sensor readings.

Figure 2 illustrates the change in a sensor reading. If it is assumed that the sensor measures displacement in a direction perpendicular to the nominal part surface, then the change in the reading of the sensor is that component of the distance between the nominal and new sensor positions, which is along the normal to the nominal surface. The sensor estimates the position change of the surface point on the part.

Thus, the measured coordinate changes can be related to the shape and location deviations by the following general linear approximation:

$$\left\{ \begin{array}{c} \delta d_{6\times 1} \\ \delta d_{m\times 1} \end{array} \right\} = \left[\begin{array}{cc} \{J_{11}\}_{6\times 6} & \{J_{12}\}_{6\times n} \\ \{J_{21}\}_{m\times 6} & \{J_{22}\}_{m\times n} \end{array} \right] \left\{ \begin{array}{c} \delta t_{6\times 1} \\ \delta\phi_{n\times 1} \end{array} \right\} \tag{1}$$

where the entire $(6+m)\times(6+n)$ Jacobian matrix $[J]$ represents the extent to which the rigid body and the parameter changes contribute to changes in the sensor readings. Specific expressions for the Jacobian matrix depend on the details of the manufactured part model used. Wang (1995) described such a model for automotive space-frame extrusions. In general, the model is case dependent.

In this linear approximation, $\{\delta d\}_{6\times 1}$ correspond to perturbations in position registered at the locator points, while $\{\delta d\}_{m\times 1}$ correspond to the position changes measured by the actual m position sensors. It should be noted that the locators are treated as sensors in the equation. Since the part locators must completely constrain the part while it is placed in the fixture, each of these six locator points will be in contact with the part surface irrespective of the nature of geometric variations in the

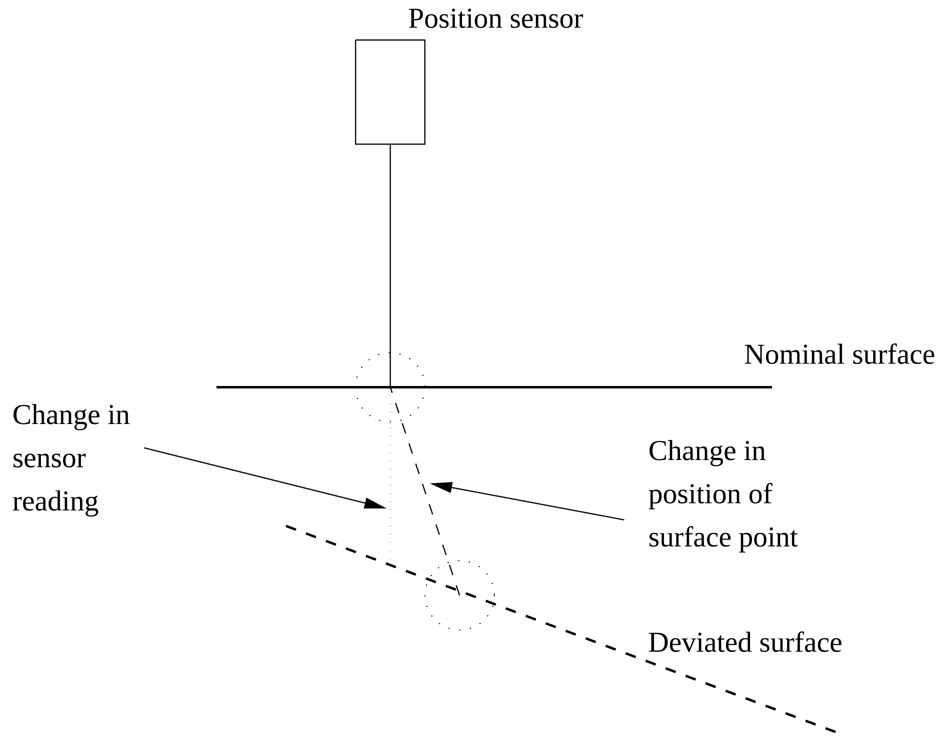

Figure 2: Illustration of the change in sensor reading.

particular component being tested. Thus $\{\delta d\}_{6\times 1}$ must always be the zero vector, i.e., $\{\delta d\}_{6\times 1} = \{0\}_{6\times 1}$. Therefore, the first six equations in Eq. 1 can be rewritten as

$$\{0\}_{6\times 1} = [J_{11}]_{6\times 6}\ \{\delta t\}_{6\times 1} + [J_{12}]_{6\times n}\ \{\delta \phi\}_{n\times 1} \tag{2}$$

One important requirement of the Coordinate Checking Fixture (CCF) is that the part being inspected is forced to comply with the (spatial) localization constraints imposed by the fixture's part locators. Thus, if the part is free of geometric errors ($\{\delta\phi\} = \{0\}$), then it should position itself at its nominal location ($\{\delta t\} = \{0\}$) in the fixture. Applying this condition in Eq. 2 gives the following:

$$[J_{11}]_{6\times 6}\ \{\delta t\}_{6\times 1} = \{0\}_{6\times 1} \tag{3}$$

For this equation to hold for $\{\delta t\} = \{0\}$, it becomes obvious that matrix $[J_{11}]_{6\times 6}$ must be of *full rank*, a *non-singularity* condition to be always ensured in CCF design. This full rank property of $[J_{11}]$ is later used in the design procedure to identify suitable part locators for the CCF. Furthermore, for non-singular $[J_{11}]$, Eq. 2 yields the following relation:

$$\{\delta t\}_{6\times 1} = -[J_{11}]^{-1}[J_{12}]\ \{\delta \phi\}_{n\times 1} \tag{4}$$

Eq. 4 defines an important relation between the variations in the geometric (shape) of the part and the variations in its spatial (rigid-body) location within the fixture. Whenever a geometric error exists in the part, there will be a perturbation in the location of the part from its nominal location once the part is firmly constrained in the fixture. As a result, any point on the surface of the part may deviate from its nominal position. If a surface point is targeted by a position sensor, the sensor may register a measurement of the resulting deviation. Based on the mathematical model of the manufactured part, the shape variations may be estimated from the positional measurements of a sufficient set of sensors.

This cause and effect relation is mathematically described as follows, by substituting Eq. 4 into the lower m equations in Eq. 1,

$$\{\delta d\}_{m\times 1} = \mathcal{J}\{\delta\phi\}_{n\times 1} \tag{5}$$

where

$$\mathcal{J} = ([J_{22}] - [J_{21}][J_{11}]^{-1}[J_{12}])_{\ m\times n} \tag{6}$$

This matrix $\mathcal{J}$ is referred to as the sensitivity matrix as it relates the deviations in the n geometric (shape) parameters of the part to the changes in the m sensor readings. Eq. 5 represents the final form of the parameter estimation model proposed in the paper.

5 Shape Parameter Estimation

One of the most common procedures for parameter estimation is the least-squares technique in which the sum of the squares of the differences between measurements and the predictions made by the system model, using assumed parameters, is minimized. For linear model described by Eq. 5, the least squares estimator of the parameters $\{\delta\hat{\phi}\}$ is obtained by minimizing $\|\{\delta d\} - \mathcal{J}\{\delta\hat{\phi}\}\|^2$, which yields the normal equations

$$(\mathcal{J}^T\mathcal{J})\,\{\delta\hat{\phi}\} = \mathcal{J}^T\{\delta d\} \tag{7}$$

For non-singular $(\mathcal{J}^T\mathcal{J})$ these equations can be solved to obtain estimate parameters

$$\{\delta\hat{\phi}\} = (\mathcal{J}^T\mathcal{J})^{-1}\mathcal{J}^T\{\delta d\} \tag{8}$$

6 Sensor Placement

The above parameter estimation yields approximate values of the parameters. The degree of approximation is related to a number of factors, including the accuracy of the measurements and the uncertainty in model parameters. It must be recognized that all measurements are subject to noise. In addition, the measurements upon which the estimation is based may not be sensitive to the unknown parameters. Generally, different locations on the surface of the part have different sensitivity levels. Therefore, in consideration of both factors, it is important to locate sensors at the points of minimum noise effect and of maximum sensitivity. This is the first objective of the sensor placement design for automated CCFs.

The second objective is to make as efficient use as possible of a given number of sensors. In theory, if agreement can be attained for a large number of measurements, the parameters are uniquely and accurately determined. In practice, however, space and cost limitations may severely limit the number of sensors that can be placed on the CCF. The second goal is to achieve a balanced design by ensuring that each sensor has a comparable contribution (relative to the other sensors) towards the total information content of the design. A measure of the information content of the design is to be defined below. Sensors that do not contribute significantly (in a relative sense) to the design's information content need either to be re-positioned in the fixture, or eliminated from the sensor set.

In order to satisfy these objectives, suitable metrics that measure progress towards these goals have to be identified. These metrics would facilitate to develop an optimal design procedure to address the concerns of *how many* sensors to use and *where* to place them. This is the essential aim of the paper.

6.1 Fisher Information Matrix

The sensor placement requires the solution of an optimal experimental design problem. The first step is to define an optimal criterion that minimizes the estimation errors due to sensor noise $\{\varepsilon\}$. Let's assume that all sensors are exposed to the same noise level and the sensor noise is uncorrelated, i.e., $\boldsymbol{V}(\{\varepsilon\}) = \boldsymbol{I}\sigma^2$, where $\boldsymbol{V}$ is the covariance matrix and σ^2 is the variance of the noise respectively. Then the covariance of the least squares estimator $\{\delta\hat{\phi}\}$ is

$$\boldsymbol{V}\{\delta\hat{\phi}\} = \sigma^2(\mathcal{J}^T\mathcal{J})^{-1} = \boldsymbol{M}^{-1} \tag{9}$$

where $\boldsymbol{M}$ is the $n \times n$ Fisher information matrix (Fedorov 1972).

The information matrix $\boldsymbol{M}$ characterizes the total error in the estimated parameters. Therefore, the optimal sensor placement problem entails finding sensor locations for which the estimation error is minimized in some sense. Various criteria exist for the optimization of the Fisher information matrix to ensure the minimum estimation error. Major criteria include Cond($\boldsymbol{M}$), Trace($\boldsymbol{M}$) (A-Optimality), the maximum eigenvalue of $\boldsymbol{M}^{-1}$ (E-Optimality), and Det($\boldsymbol{M}$) (D-Optimality) (Box 1987, p. 490). Ensuring that the important and necessary information content of the model (Eq. 5) is embodied within the sensor set is the primary concern of the optimal sensor placement design.

The least squares approach is based on a deterministic view point of the system. The problem can be considered from a probabilistic point of view with an alternative approach of the Maximum Likelihood estimation. It is known that, under the restrictions of the noise of the measurements, the maximum likelihood approach reduces to the least-squares approach (Fedorov 1972). In the perspective of the maximum likelihood approach, however, the information matrix $\boldsymbol{M}$ is a measure of the total sensitivity of the sensor measurements to perturbations in all the unknown geometric (shape) parameters. Thus, an optimal $\boldsymbol{M}$ should be sought to maximize the total sensitivity. This leads to a conclusion that maximizing sensitivity to parameter variations and minimizing effects due to noise could result in the same sensor locations. Fedorov (1972) provided a detailed presentation of the basic properties of the Fisher information matrix.

6.2 The Prediction Matrix

The second goal of the sensor placement design is concerned with efficient and easy extraction and presentation of the important and necessary information of the model, i.e., the parameters to be estimated. Solutions to this problem are subjected to a practical limit on the number of sensors to be used. It is clear that in the multi-parameter system, the response of one sensor could be affected by more than one parameter. Also, the measurement of one sensor may have a dominant influence on the inferences drawn from all sensor measurements. The significance and influence of sensors can be examined by an analysis of the predicted responses and residuals of the least-squares estimation.

The predicted (or fitted) values of the changes in the sensor readings are defined by the model Eq. 5 with the estimated parameters $\{\delta\hat{\phi}\}$. Substituting Eq. 8 into Eq. 5, one obtains

$$\{\delta\hat{d}\} = \mathcal{J}(\mathcal{J}^T\mathcal{J})^{-1}\mathcal{J}^T\,\{\delta d\} = \boldsymbol{H}\,\{\delta d\} \tag{10}$$

where $\boldsymbol{H} = \mathcal{J}(\mathcal{J}^T\mathcal{J})^{-1}\mathcal{J}^T$. The $m \times m$ matrix is known as the *prediction matrix* or the *hat matrix* simply because it maps $\{\delta d\}$ into $\{\delta\hat{d}\}$.

The prediction matrix has properties of symmetry and idempotence ($\boldsymbol{H} = \boldsymbol{H}^2$) (Hoaglin 1978). The trace of the prediction matrix is equal to its rank, which is the number (n) of parameters in the model of Eq. 5,

$$\text{Rank }(\boldsymbol{H}) = \text{Trace }(\boldsymbol{H}) = \sum_{i=1}^{m} h_{ii} = n \tag{11}$$

It is immediately clear that the diagonal elements of $\boldsymbol{H}$ satisfy that $0 \leq h_{ii} \leq 1$ and the off-diagonal elements have limits that $-0.5 \leq h_{ij} \leq 0.5$ for $i,\ j = 1,\ 2,\ \cdots,\ m$ and $i \neq j$.

One feature of great importance is that $\boldsymbol{H}$ provides a measure of influence of a sensor data on the sensor data values predicted by the model Eq. 5, with the diagonal elements being the most direct indicators. Each of the diagonal elements, h_{ii}, indicate the relative contribution of each of the m sensors towards the detection of dimensional information from the part. Sensors contributing more information to the model have higher values of their corresponding diagonal element in $\boldsymbol{H}$. This knowledge about the roles of individual sensors in a given sensor set can help identify members that do not have an appreciable contribution to the information content of the design.

If $h_{ii} = 0$, then the corresponding sensor (i) does not contribute and the shape variations of the part is not even observable from the sensor location. On the other hand, if $h_{ii} = 1$, $\delta\hat{d}_i$ is determined by one sensor reading (δd_i) alone; hence the model dedicates one parameter to this particular sensor. For any h_{ii} between 0 and 1, for example, if $h_{ii} = 0.5$, prediction $\delta \hat{d}_i$ is determined by an equivalent of two sensors. Therefore, h_{ii} can be generally interpreted as to relate the influence of an equivalent number of sensors on the prediction $\delta \hat{d}_i$. Similarly, h_{ij} can be regarded as the amount of influence each sensor j has on the model prediction at location i (Chatterjee 1988).

6.3 Balanced Design for Sensor Placement

The task now is to choose a suitable optimization criterion which would use the insights provided by the the Fisher information matrix $\boldsymbol{M}$ and the prediction matrix $\boldsymbol{H}$, and achieve the two objectives that were laid down above. The criterion is to facilitate an optimal design procedure to address the concerns of *how many* sensors to use and *where* to place them.

The design criterion of our choice is the D-optimality, or the *determinant criterion* (Fedorov 1972, Box 1987), for which the determinant of the Fisher information matrix $|\boldsymbol{M}|$ is to be maximized. A design based on this criterion is also known as a *balanced design* (Bates 1988, p. 126), because it has two important and suitable features.

First, the D-optimal design minimize the variance of the least squares estimate of the parameters, and thus reduces the effects on the parameter estimates of the sensor noise. This can be best explained through the use of the confidence region of the n estimated parameters $\{\delta\hat{\phi}\}$. From Eq. 9, the uncertainty of the estimated parameters can be described by a joint confidence region expressed as

$$(\{\delta\phi\} - \{\delta\hat{\phi}\})^T \, (\mathcal{J}^T \mathcal{J}) \, (\{\delta\phi\} - \{\delta\hat{\phi}\}) = \text{Constant} \tag{12}$$

The volume of this ellipsoidal region is proportional to $|\boldsymbol{M}|^{-1/2}$ (Bates 1988, p. 124), since the inverse of the matrix $\boldsymbol{M}$ is the covariance matrix of the estimated parameters (Eq. 9). Thus, by maximizing $|\boldsymbol{M}|$, one in effect minimizes the confidence region in the parameter space, and consequently maximizes the signal-to-noise ratio. This achieves our first design objective.

Second, the D-optimal design reduces the interactions between parameters, so the effects of the parameter changes are better separated among the given sensors (Bates 1988, p. 124). Again from a geometric point of view, the determinant criterion implies that the columns of $\mathcal{J}$ are made to be as orthogonal as possible. This makes particular sensors more sensitive to changes in particular parameters. Recall that the elements in the prediction matrix of Eq. 10 are coefficients weighting the relative contributions of the sensors employed in the sensor design model with each diagonal term being the most direct indicator for the particular corresponding sensor. Thus, maximizing the determinant criterion causes the off-diagonal elements of the prediction matrix $\boldsymbol{H}$ to diminish, making the diagonal terms dominant.

In addition, the D-optimality attempts to make all h_{ii} near the average value (n/m) of the diagonals. Consequently, a balanced design is achieved with each sensor contributing comparably to the total information content of the design. In fact, it is known that the D-optimal criterion of maximizing $|\boldsymbol{M}|$ is equivalent to minimizing the maximum value of the prediction diagonals (h_{ii}) (Box 1987, p. 492). The latter is often referred to as G-optimality (Box 1987). Each h_{ii} is proportional to the variance of the corresponding predicted response $\delta\hat{d}_i$ of Eq. 10. Thus, the D-optimality makes sensors that have a significant role become conspicuous, and makes it easier to identify sensors which are not contributing appreciably to the sensor design, so that they may be eliminated from the sensor set. Therefore, the D-optimality criterion helps achieve the second design objective as well. A computational procedure is now made possible for both selecting a suitable size of sensors and finding their optimal locations.

7 The Design Procedure

The above analysis allows us to develop a computational procedure for determining a set of optimum sensor locations for identification of shape parameters of a part with the automated CCF. The above sensor location analysis requires that suitable part locators are already chosen and are represented in the sensor model of Eq. 5. The locator placement problem is to be discussed in the next section.

The computational procedure for the design of a set of optimum sensor locations is as follows:

Step 1: Initially, a candidate set of sensor positions is selected at various locations on the part surfaces. The candidate set should be large enough to include all of the accessible part surfaces associated with the geometric parameters to be identified. However, it is assumed that the placement of all the initial sensors is impossible; therefore, the initial candidate set must be reduced to a minimum limited by a required confidence level in the estimation results or to an allotted number of sensors.

Step 2: The D-optimality criterion is applied to maximize $|\boldsymbol{M}|$ with respect to the locations of the sensors. The Fisher information matrix is defined according to the sensor system of Eq. 5.

Step 3: For the the optimal sensor locations obtained, their prediction matrix $\boldsymbol{H}$ is then calculated. The diagonal elements of the prediction matrix represent the fractional contribution of each sensor to the parameter estimation. The sensor corresponding to the smallest h_{ii} diagonal value is then eliminated from the sensor set.

Step 4: For the remaining sensors, the procedure is continued following steps 2 through 3, iteratively deleting the sensor that makes the least contribution to the current sensor set in each cycle of the optimization. This leads finally to a minimal number of sensors required.

It is now time to consider the effect of the deletion of one sensor location during the iterative process on the determinant of the Fisher information matrix $|\boldsymbol{M}|$ which is an indication of the information retained in the model (Eq. 5). If one sensor, the ith, is deleted, then the information matrix with the remaining sensors is denoted to be $\boldsymbol{M}_i$. It is known that the determinant of the information matrix $|\boldsymbol{M}_i|$ decreases such that

$$|\boldsymbol{M}_i| = (1 - h_{ii})\ |\boldsymbol{M}| \tag{13}$$

where h_{ii} is the ith diagonal element of the prediction matrix $\boldsymbol{H}$ prior to the deletion (Chatterjee 1988).

Clearly, this relation indicates that h_{ii} represents the exact fractional change in the determinant of the information matrix if the ith sensor is deleted. Hence, the deletion of the sensor associated with the smallest h_{ii} value in the current sensor set would result in the smallest possible reduction of the determinant of the Fisher information matrix, which in turn leads to the smallest impact on the error of the parameter estimation. Once this sensor is deleted (in Step 3), locations of the remaining sensors are further optimized (in Step 2), allowing all sensors to reach their maximum potential to achieve a balanced design of sensor placement at each iteration.

It should be noted that this is a greedy algorithm for a mixed discrete/continuous optimization problem. It does not guarantee that the global optimum will be reached. However, by deleting only the sensor of the least impact , the loss of information is always kept to be minimum at each iteration of deletion. If more than one sensor is deleted in each iteration, the number of cases to be considered at a time will be combinatorially exploded. Further, more rapid reduction of $|\boldsymbol{M}|$ tends to result in a lesser optimal solution. In the examples presented below, the single deletion design procedure is shown to be effective.

The decision of when to stop the iterative deletion process depends on the designer's constraints with respect to sensor cost and information potential of the obtained sensor set. Usually, it is good to stop when there is a drastic decrease in the objective function ($|\boldsymbol{M}|$) upon successive deletion(s). The minimum required number of sensors in the design (to prevent the sensor model from becoming under-determined) is equal to the number of geometric (shape) parameters considered in the model, i.e., $n = m$.

8 Part Locator Placement

The primary role of the part locators is to completely constrain the rigid-body degrees of freedom of the part. This is equivalent to the full-rank condition expressed in Eq. 3. Typically, part locators are pre-defined by a tolerance specification scheme (e.g., the datum requirements) or by the use of heuristic rules such as the traditional "3-2-1" part localization technique.

Alternatively, the sensor design procedure described above may also be utilized for the locator selection. In this case, the matrix $[J_{11}]$ in Eq. 1 concerning with only the perturbations in the six rigid body parameters ($\{t\}_{\ 6\times 1}$) is considered to construct the relevant Fisher information matrix and the prediction matrix. In a similar manner, starting from a candidate set of locators, the D-optimal design procedure would result in a final set of 6 locator points on the part surface, where the effect of changes in the spatial position and orientation of the part are prominent. Each locator also has a balanced contribution towards the firm localization of the part within the fixture. This procedure for locator placement in a CCF is illustrated in Section 10.

9 Numerical Implementation

Major aspects of the computational implementation of the optimal sensor placement procedure are related to the D-optimality based optimization in Step 2. The spatial locations of the sensors on the part surfaces form the design variables. Each candidate sensor is restricted to vary its location within a specified surface. Thus, the sensor position is represented by two variables (u, v) of a surface parameterization. Further, the domain of these parametric variables can be represented generally as

$$(u, v) \in [0, 1] \tag{14}$$

The spatial (x, y, z) location of the point can be mapped from the (u, v) parameters with appropriate coordinate transformations.

Thus, the sensor location optimization of Step 2 of the design procedure is states as

$$\begin{aligned} \max_{u_i, v_i} \quad & |\boldsymbol{M}|, \quad \text{subject to} \\ (u_i, v_i) \in & [0, 1], \ \forall \text{ sensors } i = 1, \ldots, m \end{aligned} \tag{15}$$

A routine for Constrained Optimization By Linear Approximations (COBYLA) developed by Powell (1992) is used in the current implementation. Since the objective function ($|\boldsymbol{M}|$) has a non-smooth, non-linear nature, the existence of derivatives at all points in the design space is not guaranteed. This makes the optimization difficult for available optimization routines to handle. COBYLA does not require evaluation of derivatives of objective and constraint functions, and iteratively optimizes the objective by forming a simplex at each step. A merit function that penalizes the greatest constraint violation helps in improving the design variables obtained. A trust region bound that limits the change in the design vector has to be prescribed before the optimization starts. In the course of the optimization, this bound is gradually decreased, resulting in a shrinking simplex region. The size of this simplex region is eventually reduced to the accuracy desired, to obtain optimal estimates of the design variables. Performance of the method has been evaluated in two test examples illustrated below.

10 Examples

The proposed design procedure has been applied to the design of a Coordinate Checking Fixture (CCF) for two automotive space frame components. The first extrusion considered has a rectangular cross-section with a single in-plane bend. The second example is a more complex extrusion also with a rectangular cross-section, but with two bends in different planes.

10.1 Example 1

The first hollow extrusion considered here is a V-shaped bent extrusion with the nominal angle of bend being 90 degrees and a rectangular cross-section as shown in Figure 3. The part has three segments and 12 surfaces over which the candidates can be placed for obtaining suitable part locators and position sensors with the design procedure proposed here.

It is obvious that 6 point locators can be easily determined with the conventional "3-2-1" part localization approach. As an alternative, the optimal locator design procedure described in Section 8 is used and illustrated here. First, 18 initial candidate locators are placed uniformly to cover most of the part surface of the extrusion, as depicted in Figure 3. Figure 4 shows the optimized locations obtained before any locators are deleted (Step 2), and Figure 5 shows the final 6 optimum locations obtained after 12 locators are deleted iteratively. At each iteration, the D-optimal locations are first found, and then the prediction matrix diagonal values (h_{ii}s) are analyzed. The locator with the least corresponding h_{ii} value is then deleted.

As shown in Figure 5, the final 6 optimum locators selected appear to be different from a conventional "3-2-1" arrangement. This locator placement results in better balance among the locators with the contributions of each locator being comparable. During the process, the objective function shows a downward trend (Figure 6), as a single locator gets deleted and consequently the information content of the design decreases. Note that, except for the first iteration cycle (from P to Q on the graph), the optimization improves little in increasing the values of the objective function. In Figure 6, symbols $\times$ and $+$ indicate initial and optimum objective function value, respectively. The appearance of symbol $*$ indicates a near overlap between the initial and optimum objective function values. This is a general trend of the procedure and is to be discussed in the end of the section.

With these 6 part locators, it is ready for selecting optimal positions of sensors for estimation of the bend angle. For the single parameter in the model, at least one sensor is needed. The positions of the 6 part locators obtained (Figure 5) were used to construct the Jacobian matrix (Eq. 6). An initial set of 12 sensor locations are selected as shown in Figure 7. Most of these positions were chosen on surfaces which could be intuitively perceived to be affected by changes in the angle of bend. For the sake of illustration, 3 sensors were placed on the top and bottom surfaces of the extrusion, where they would not be of much use in detecting deviations in the bend angle. Figures 8 - 10 show the optimal sensor locations obtained before any deletions, after 4, and the final set after 11 position

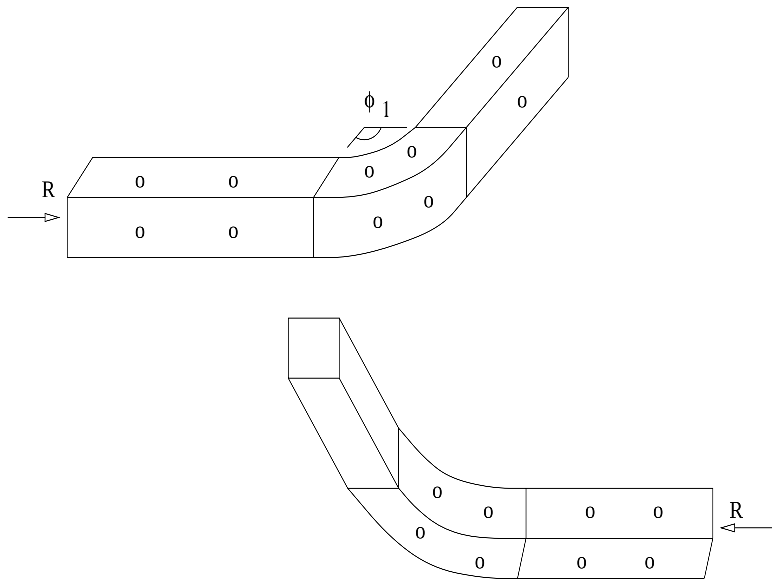

Figure 3: Example 1: 18 initial part locator locations.

sensors are deleted, respectively. The graph of the objective function across these design iterations is shown in Figure 11, again noting the appearances of symbol $*$ indicating near overlap of initial and optimum objective function values. Figure 12 depicts the variations in the prediction matrix values during the 11 iterations.

During the course of the design, sensors that have relatively lower contribution to the design are eliminated one-by-one. The first sensor to be deleted is the one in the left lower corner of the part. This location is coincident with a part locator and thus would register no positional changes. Its deletion does not result in any decrease in $|\mathbf{M}|$ as indicated by points B and C in Figure 11. Next three sensors to be deleted are the ones that lie on the top and bottom surfaces of the extrusion, as expected, because they do not contribute to the detection of the bend angle. This is evident in Figure 11 marked between D and E. Sensors which are part of groups crowding near the same location are also successively removed. The procedure reaches at a single sensor as the minimum number required to detect changes in the single shape parameter, the bend angle.

The graph of the objective (Figure 11) shows the high initial rise in the objective function from point A to B as the sensor locations are improved before any deletions are made. Upon deleting one sensor, the design is further improved appreciably from point C to D in the optimization for that iteration. From point F (4 deletions) onwards, the downward slope of the graph becomes detectable, and after point G (8 deletions), the drop in the objective upon further deletions drastically increases. Points F and G are examples of the sensor placement, where the designer may decide to stop the procedure, and use the sensor set obtained at that point, 7 and 4 sensors respectively. This decision depends on the compromise that the designer wishes to achieve between the information potential of the design, and the issues of cost and increased fixture complexity from retaining additional sensors.

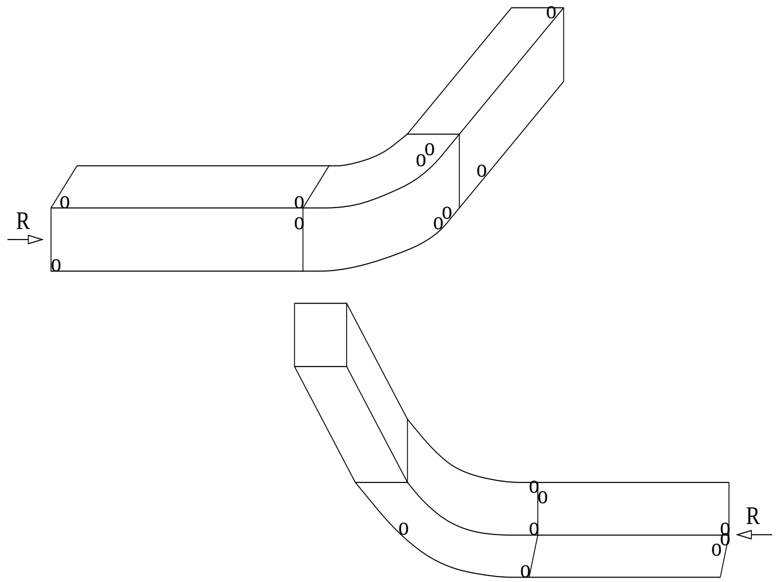

Figure 4: Example 1: Optimum locator locations before any deletions.

Figure 12 provides another perspective of the sensor selection process. In each iteration, the prominent sensor candidates are with relatively higher prediction diagonal values shown in Figure 12. Sensors like those that lie on the top/bottom surfaces tend to have very low contributions and are ended up being removed in course of the design iterations to trim down the sensor set.

To obtain a better perspective on the choice of a suitable size for the position sensor set, consider Figure 13 which plots the variation of the average and scaled (to the interval [0,1]) deviation of sensor prediction values (h_{ii} s) after each deletion. As more sensors are deleted, the average prediction value (n/m) of the sensor set rises, while the deviation in the prediction values among individual sensors gradually drops. The cross-over point, where these two curves intersect is a reasonable choice for stopping the sensor deletion process and accepting the sensor set obtained at that point. From Figure 13, this point is observed to correspond to the case of 7 deletions made, or of 5 sensors remained. Another heuristic for set size selection is obtained from a thumb rule proposed by Hoaglin (1978). Their prescription of a desirable prediction value level, calculated as ($2n/m$), can be applied to a sensor model with n parameters and m initial candidate sensors. An average prediction value close to this ($2n/m$) value can be considered to be desirable in the obtained sensor set. In the model for the present example, we have 1 parameter and 12 initial candidates, giving the value of this heuristic as 0.1667. Thus, from Figure 13, this heuristic suggests that the sensor set obtained after 6 deletions should be used in the final fixture design. The validity of these two stopping-points is evident from an inspection of the objective function history graph (Figure 11), which shows that the objective function value drops appreciably upon further deletions.

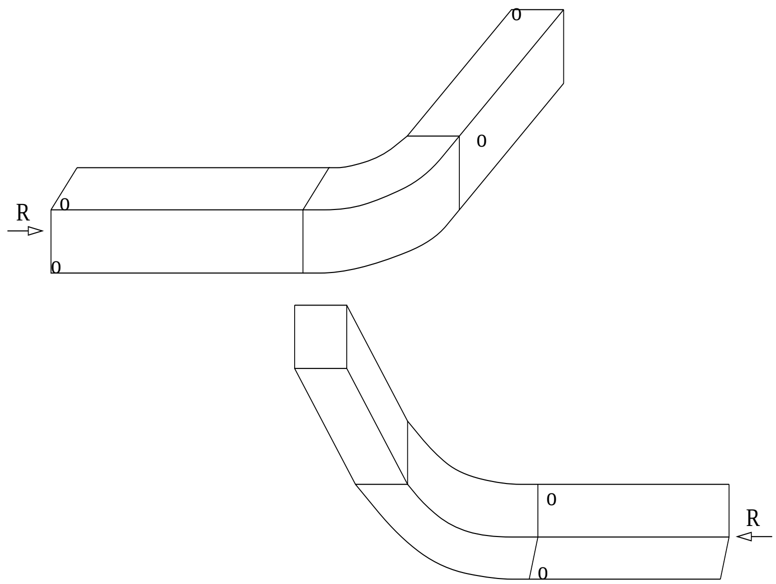

Figure 5: Example 1: 6 final optimum locators after 12 iterative deletions.

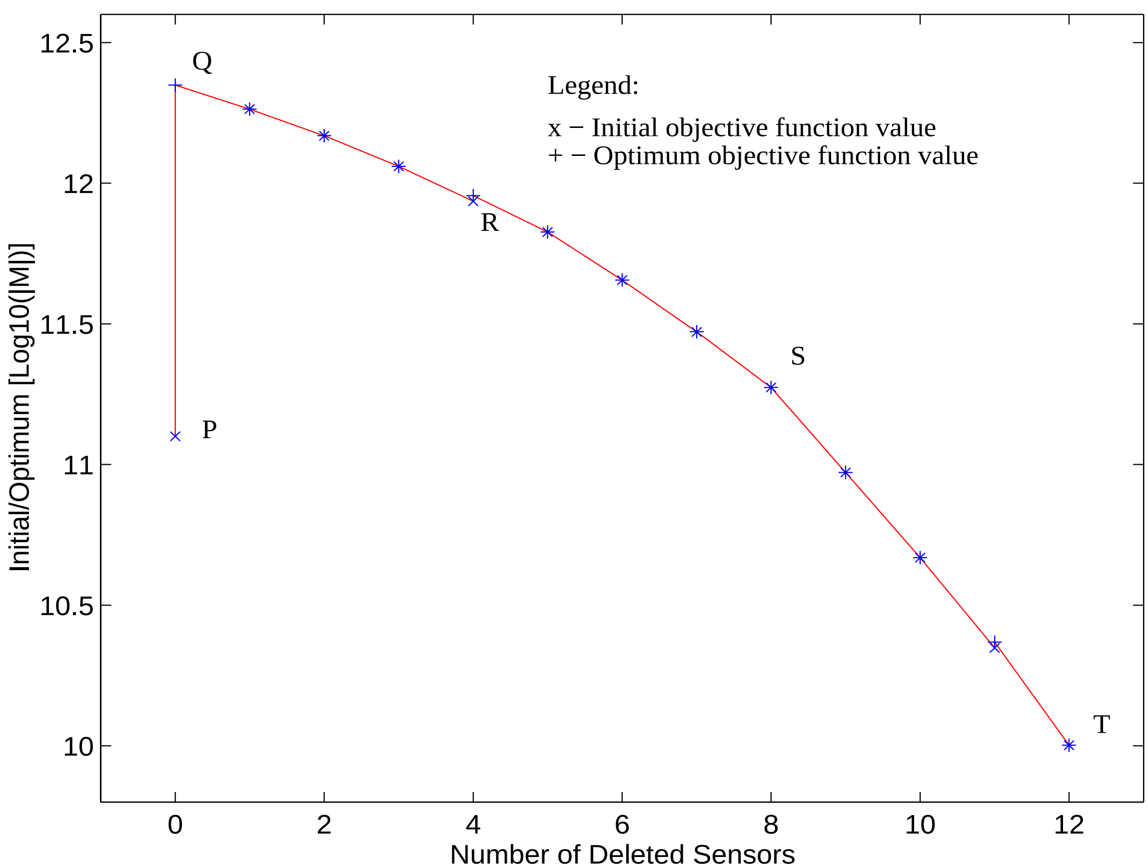

Figure 6: Example 1: Objective function history for part locator selection.

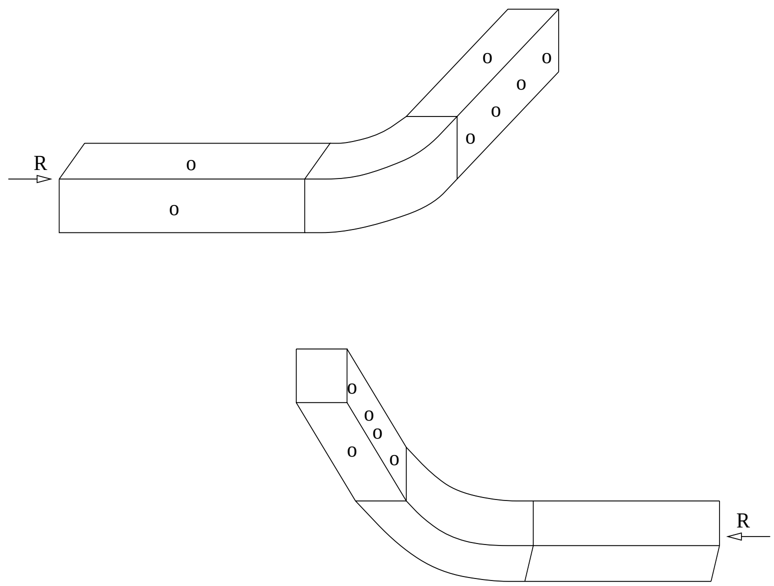

Figure 7: Example 1: 12 initial sensor locations.

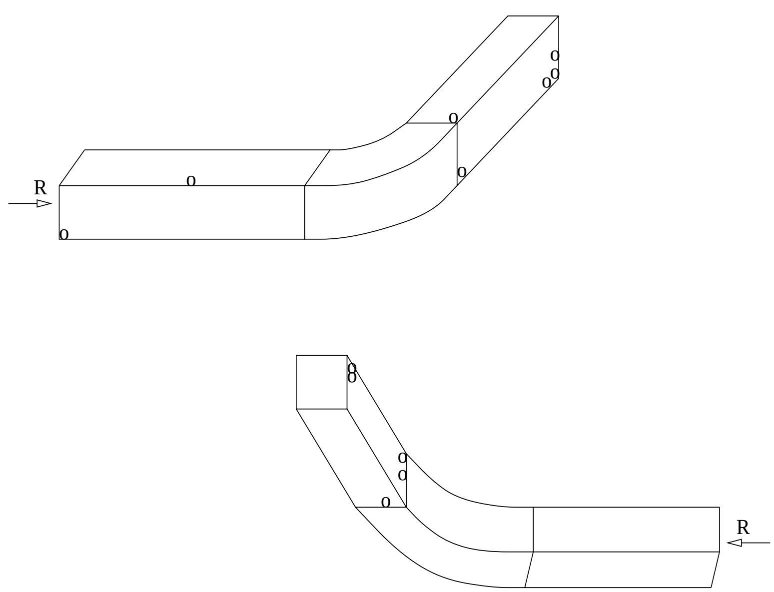

Figure 8: Example 1: Optimum sensor locations before any deletions.

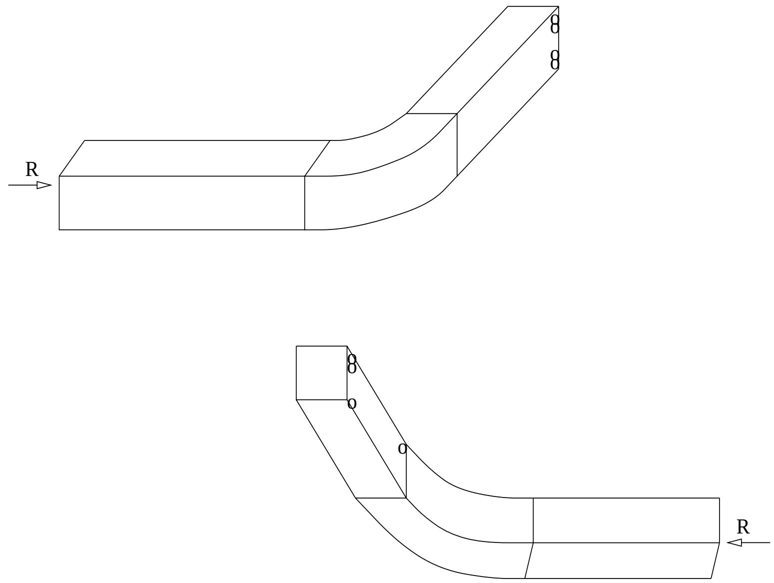

Figure 9: Example 1: Optimum 8 sensor locations after 4 deletions.

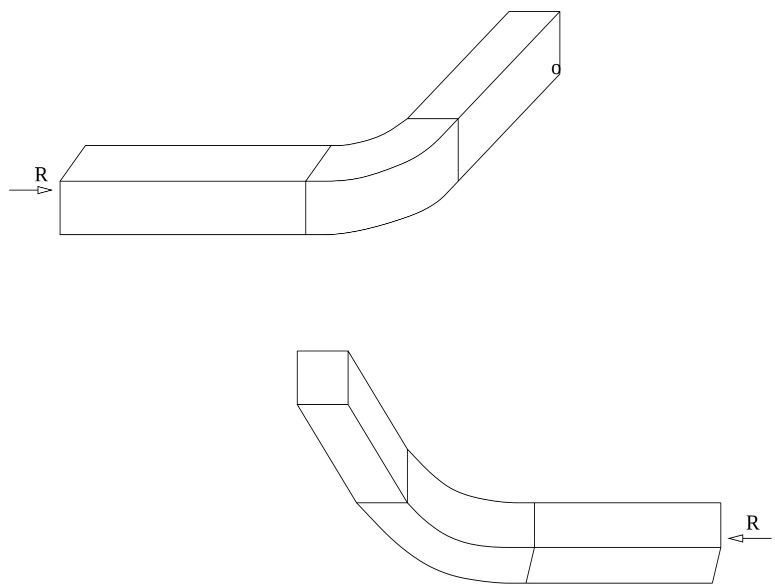

Figure 10: Example 1: Final optimum sensor location (1) after 11 deletions.

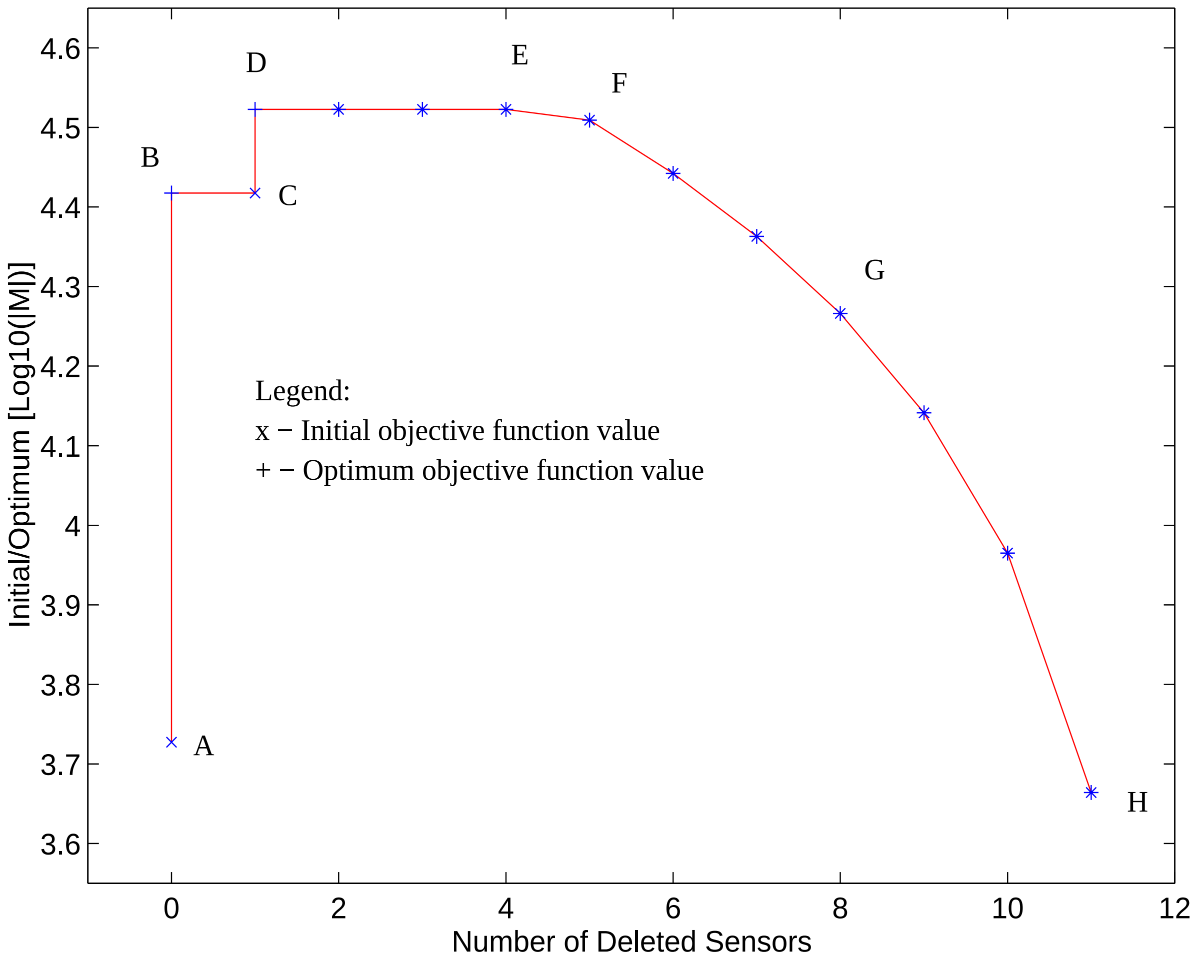

Figure 11: Example 1: Objective function history for sensor selection.

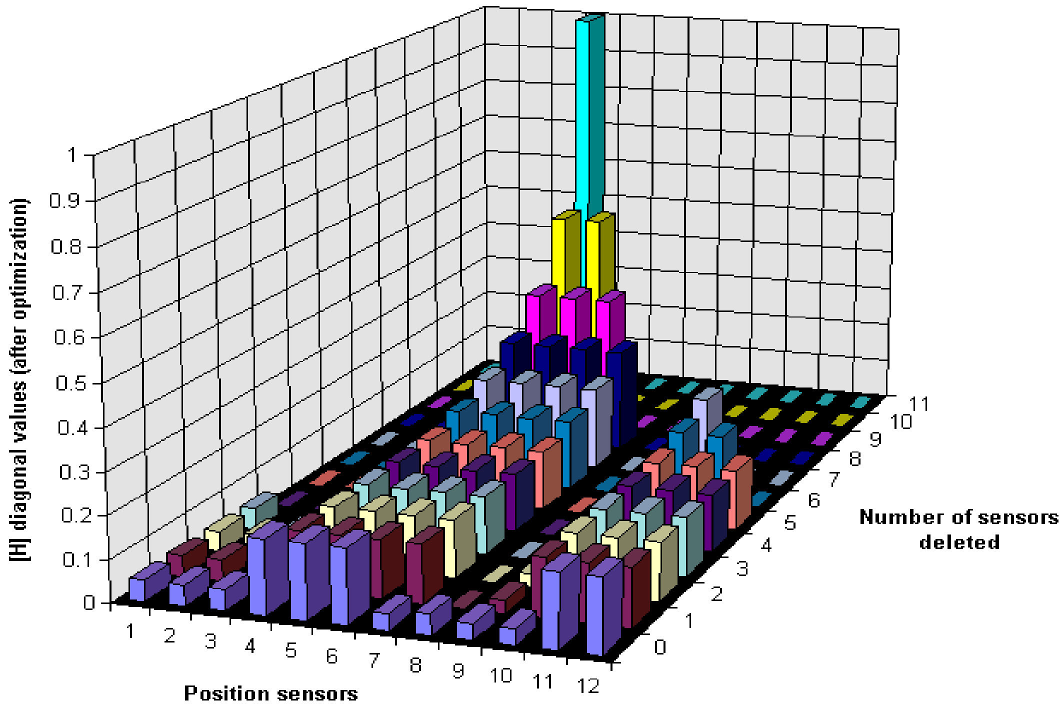

Figure 12: Example 1: Prediction diagonal values during sensor selection.

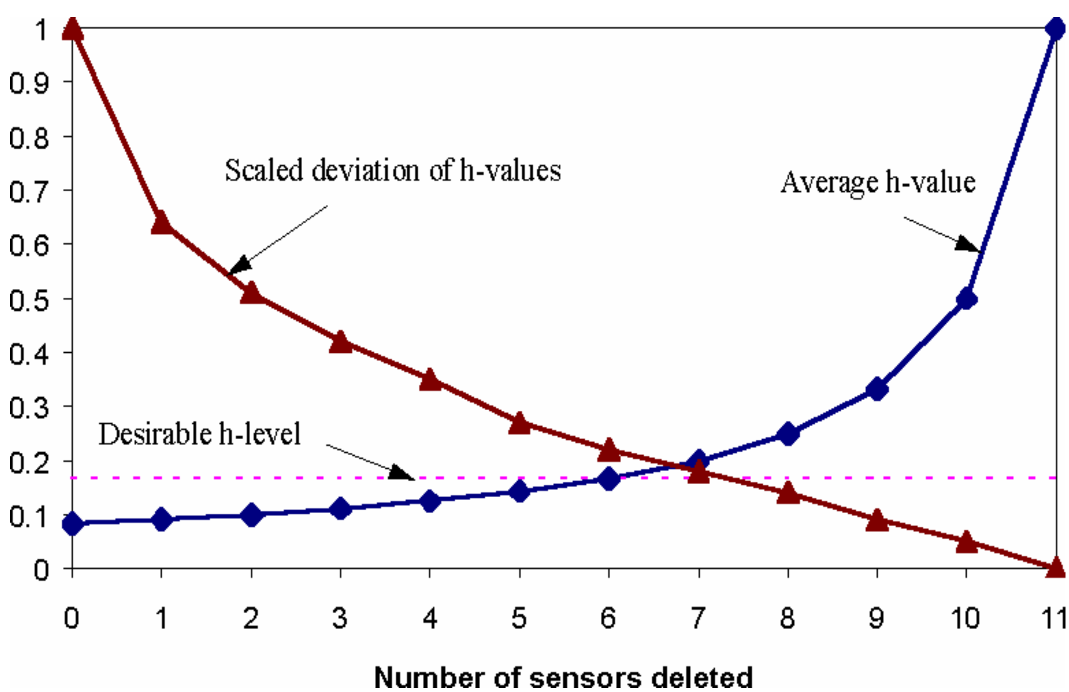

Figure 13: Example 1: Changes in the prediction values.

10.2 Example 2

Figure 14 shows the part for another example. The part is composed of two straight segments, joined together by a combination of two circular (bend) segments. The circular segments are placed consecutively, with the second bend starting immediately following the first bend, but in different planes. The nominal values of the bend angles are 12.82 degrees and 48.09 degrees respectively.

For part locator selection, an initial set of 22 locators were placed (Figure 15) in an effort to cover the surface of the part as extensively as possible. Figures 16 and 17 represent the optimized locator locations obtained before any deletions, and after 16 locator deletions which yields the optimum six-locator set, respectively.

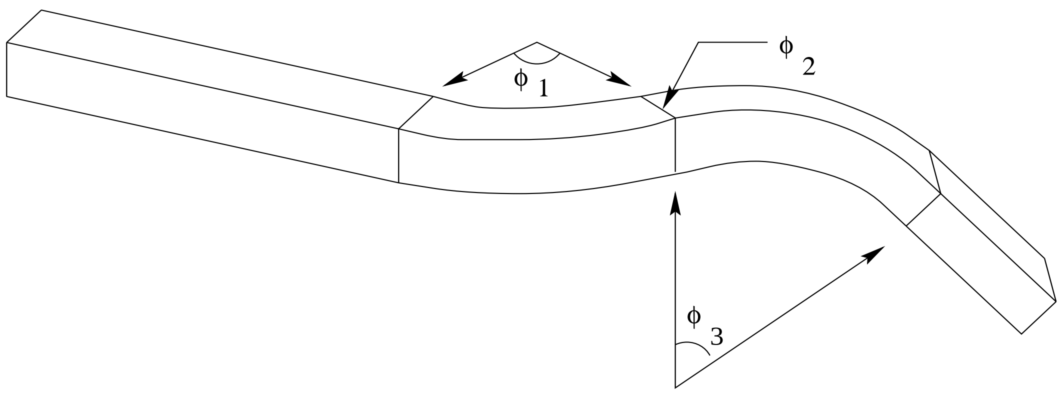

Figure 14: Multi-bent extrusion of example 2.

The shape parameters of interest are the two bend angles, and the length between the two bends (Figure 14). In the nominal design, this length is a “zero length”, because there is no nominal segment between the two (circular) segments. Thus, any deviation in this geometry is of interest while setting or resetting the extrusion’s manufacturing operation.

Sixteen initial sensor locations were made as shown in Figure 18. Figures 19 and 20 show the optimized locations of the sensors obtained before any deletions, and the final set obtained after 13 deletions, respectively. The behavior of the objective function during the course of the sensor selection iterations is plotted in Figure 21, and the prediction values are captured in Figure 22.

Figure 23 plots the variation of the average prediction value and scaled (to the interval [0,1]) prediction value deviation after successive sensor deletions. As explained in Example 1, the cross-over point of these two curves gives a reasonable estimate of the sensor set size to use in the final design. For Example 2, this cross-over point is found to lie in the vicinity of 7 sensor deletions. Another estimate is obtained from the heuristic ($2n/m$) for the desirable prediction value level (Hoaglin 1978). The value of this heuristic for the example is found to be 0.375, which suggests halting the sensor trimming process after 8 deletions. Indeed, observing the corresponding points (respectively points E and F) in Figure 21 provides evidence of their suitability as stopping-points in the sensor selection procedure, as further deletions cause notable deterioration of the ob-

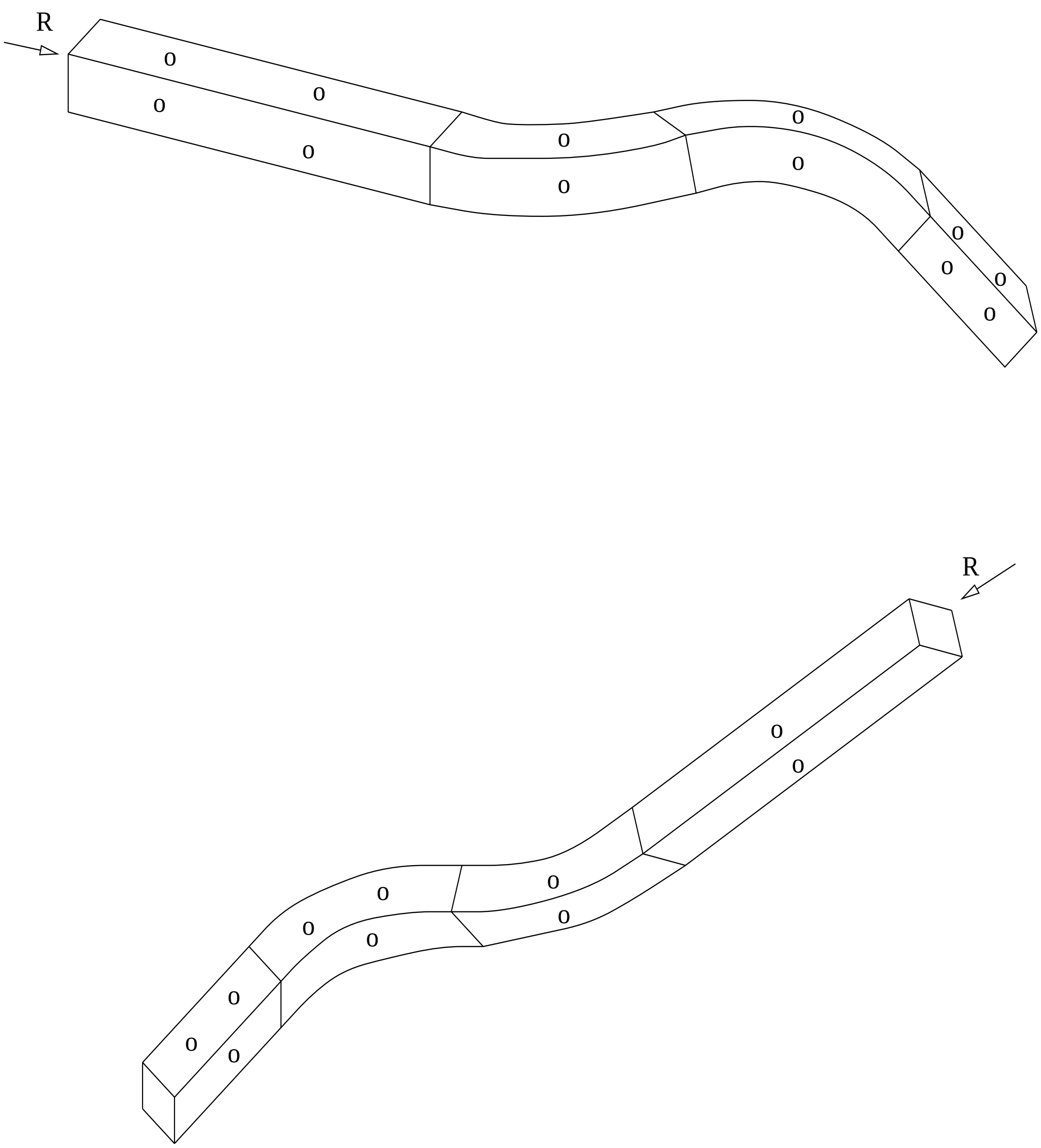

Figure 15: Example 2: Initial 22 locator locations.

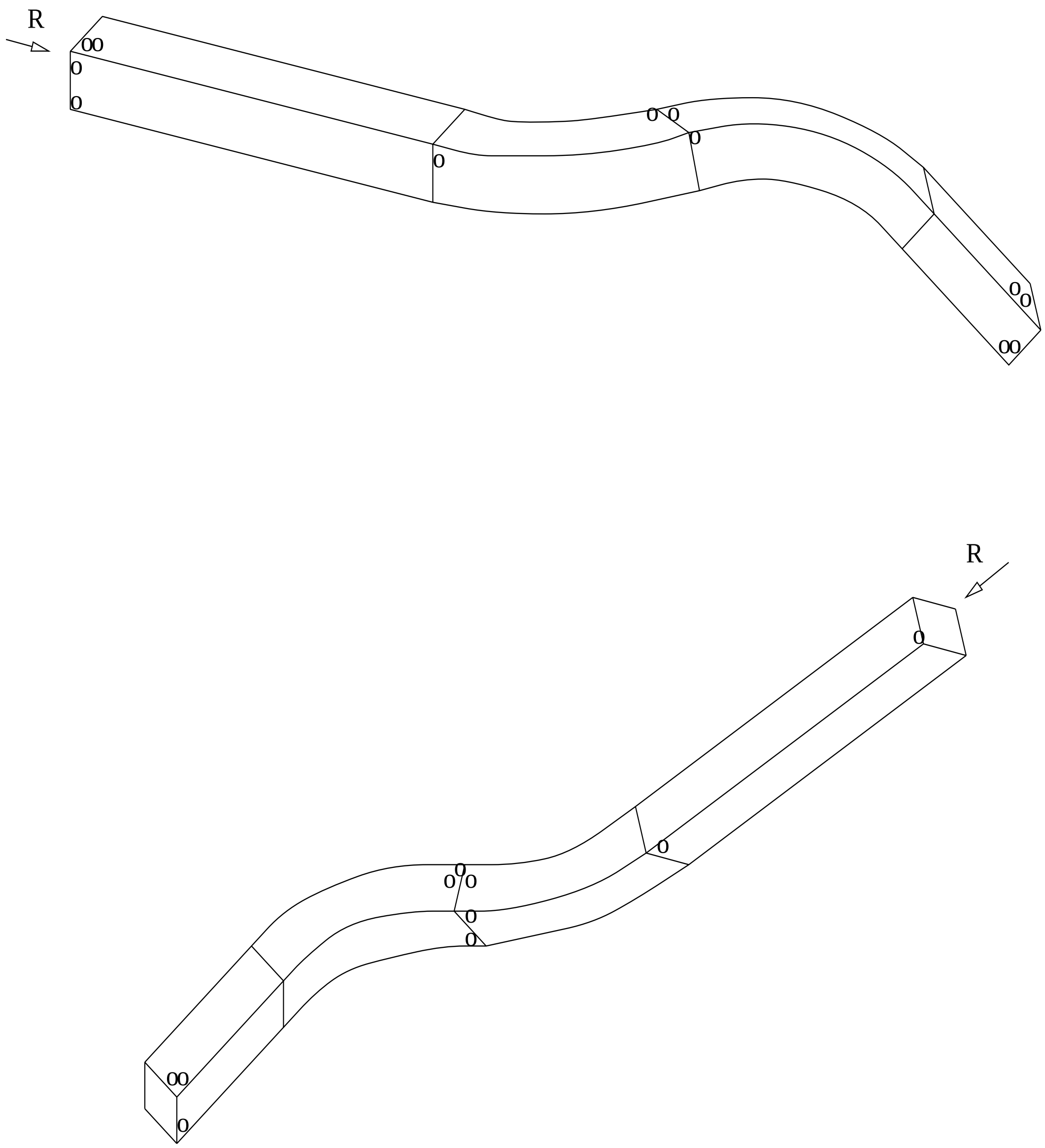

Figure 16: Example 2: Optimum locator locations before any deletions.

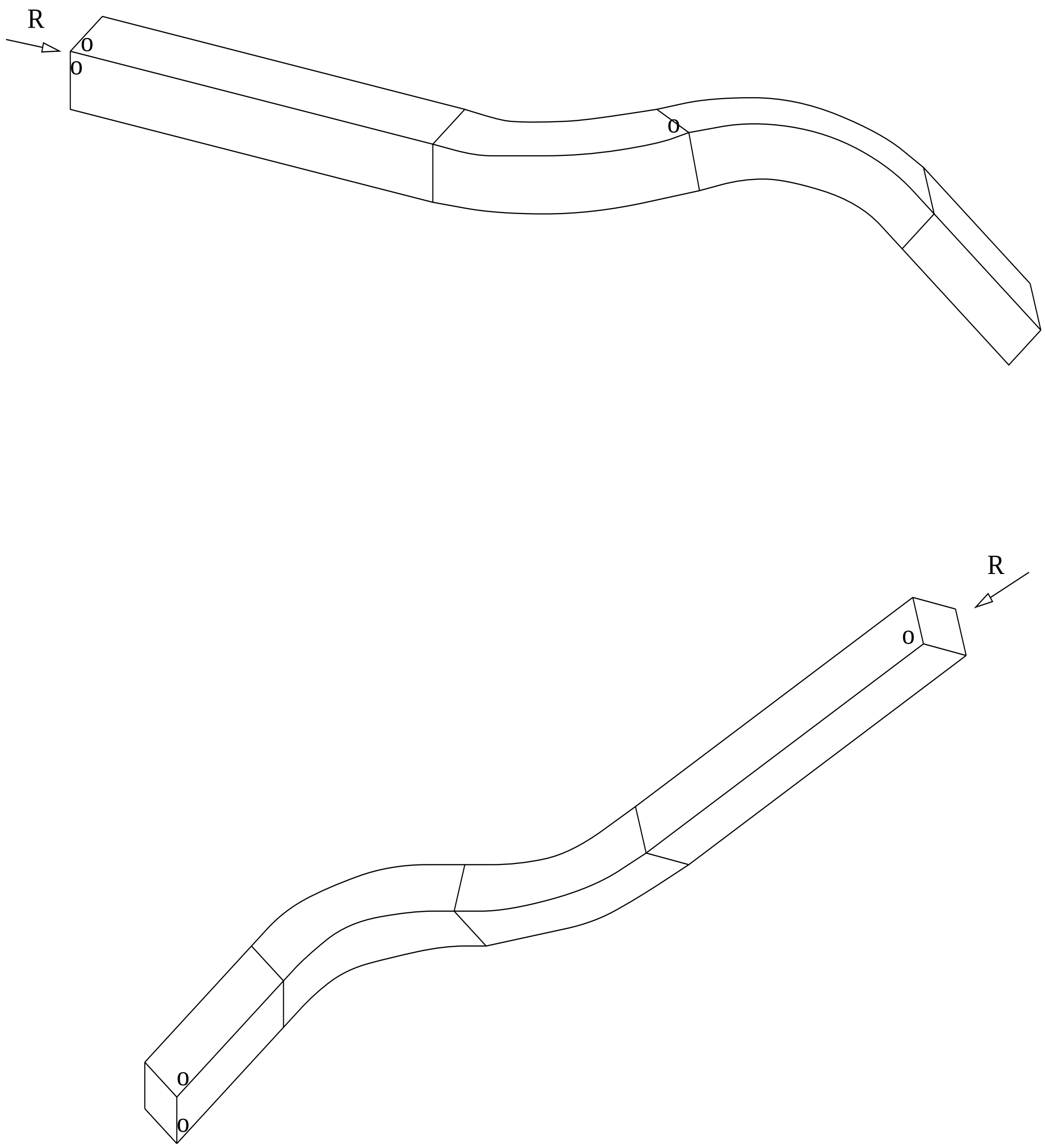

Figure 17: Example 2: Final 6 optimum locators.

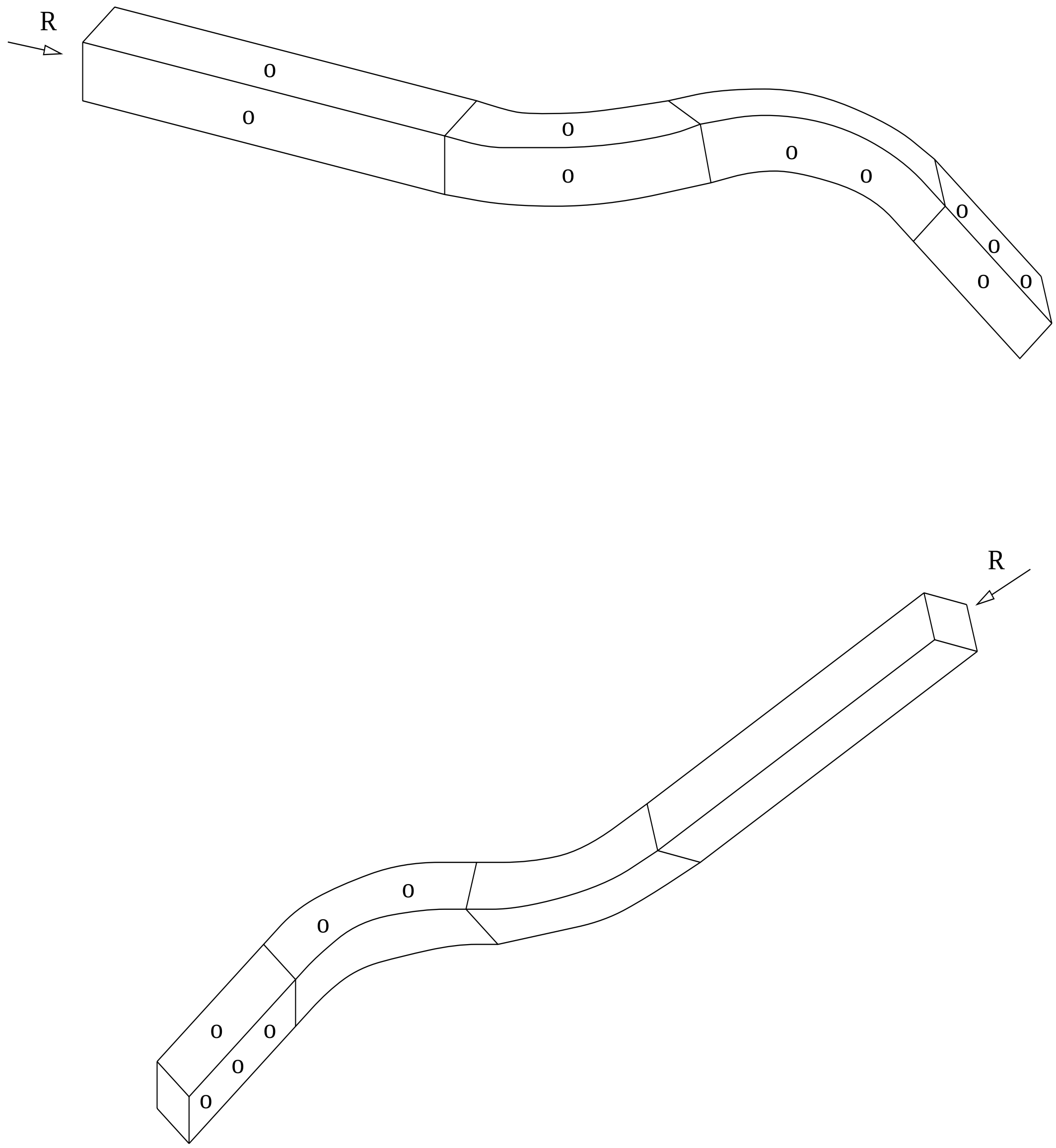

Figure 18: Example 2: Initial sensor locations.

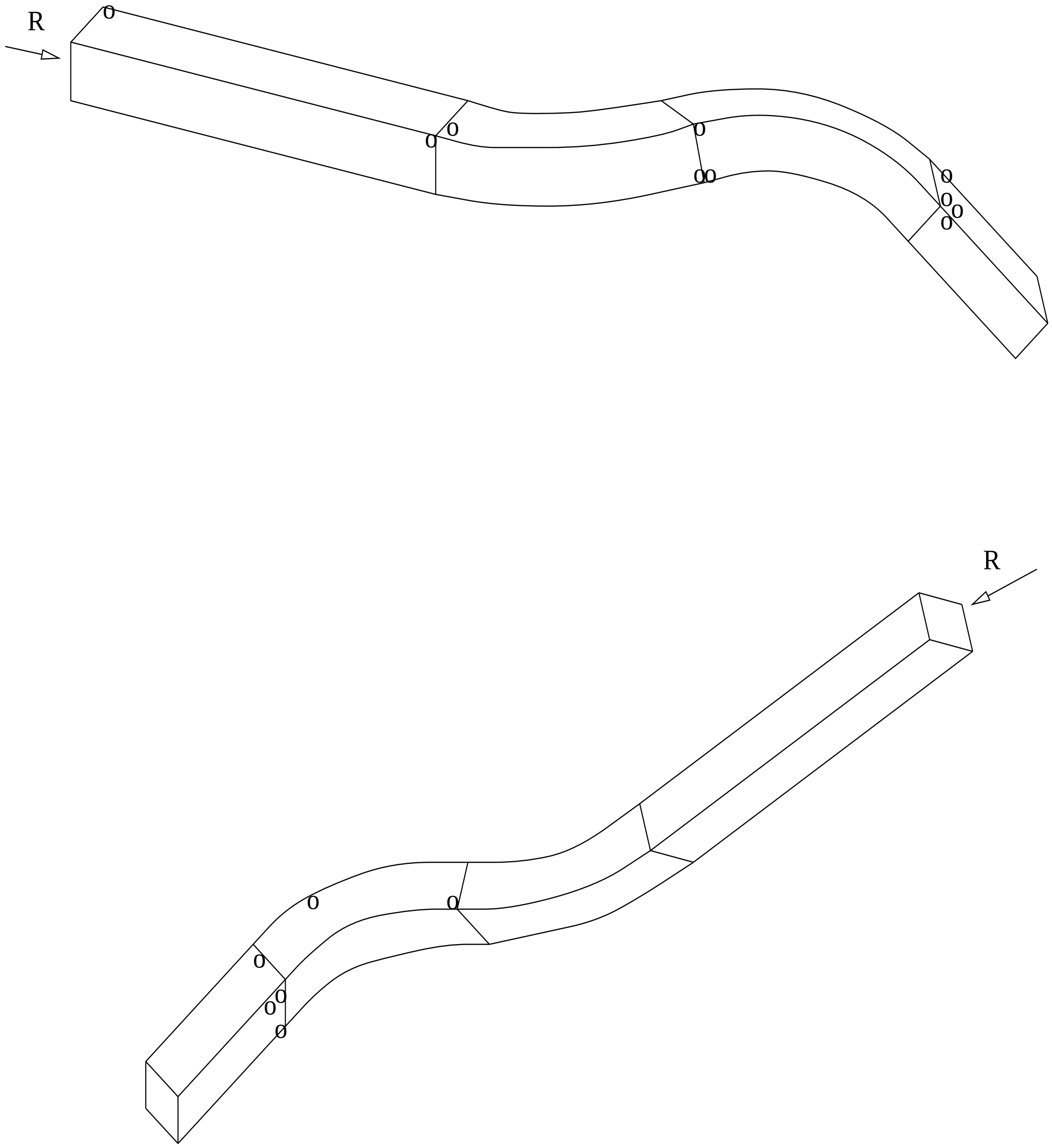

Figure 19: Example 2: Optimum sensor locations before any deletions.

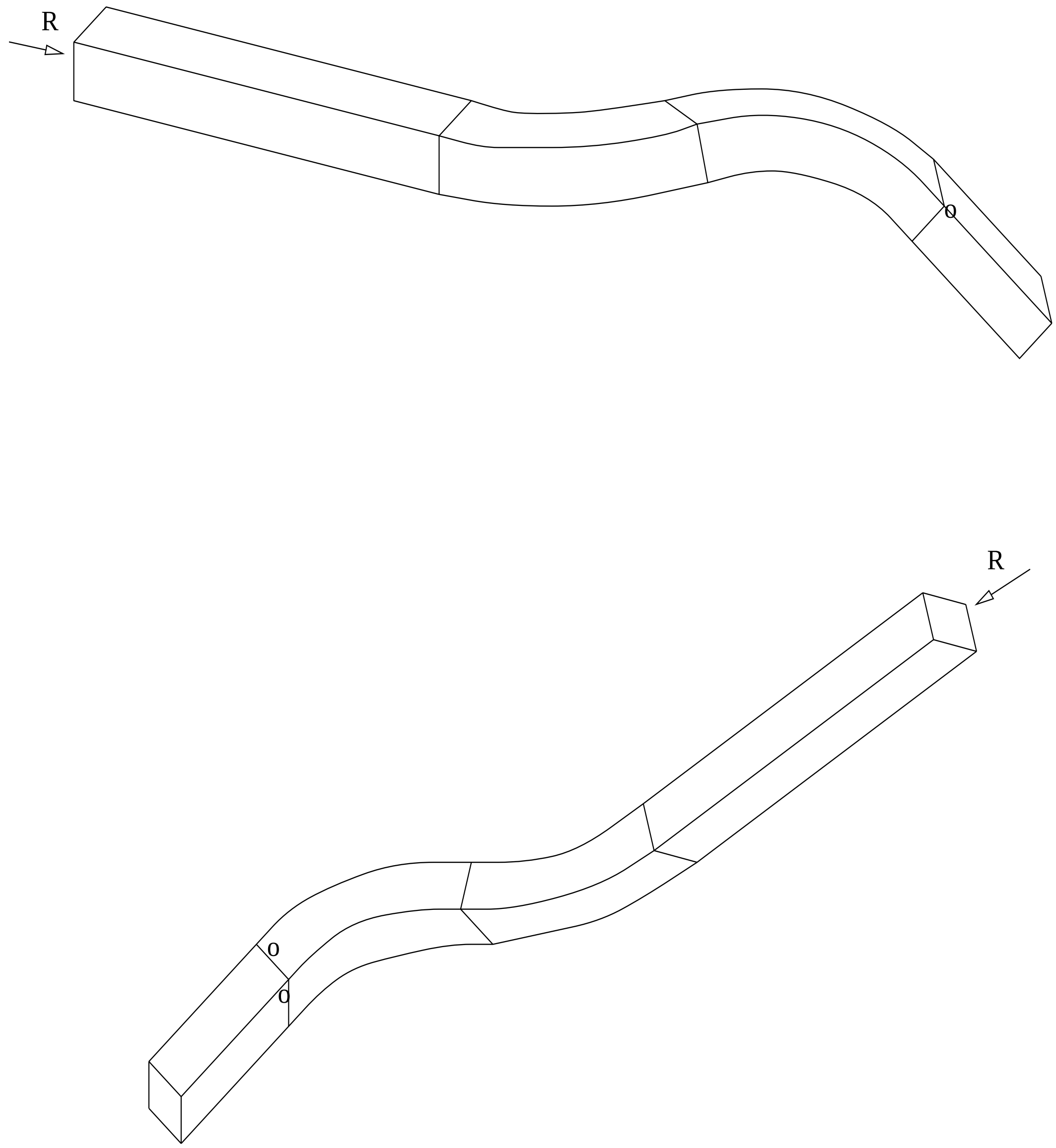

Figure 20: Example 2: Final 3 optimum sensor locations.

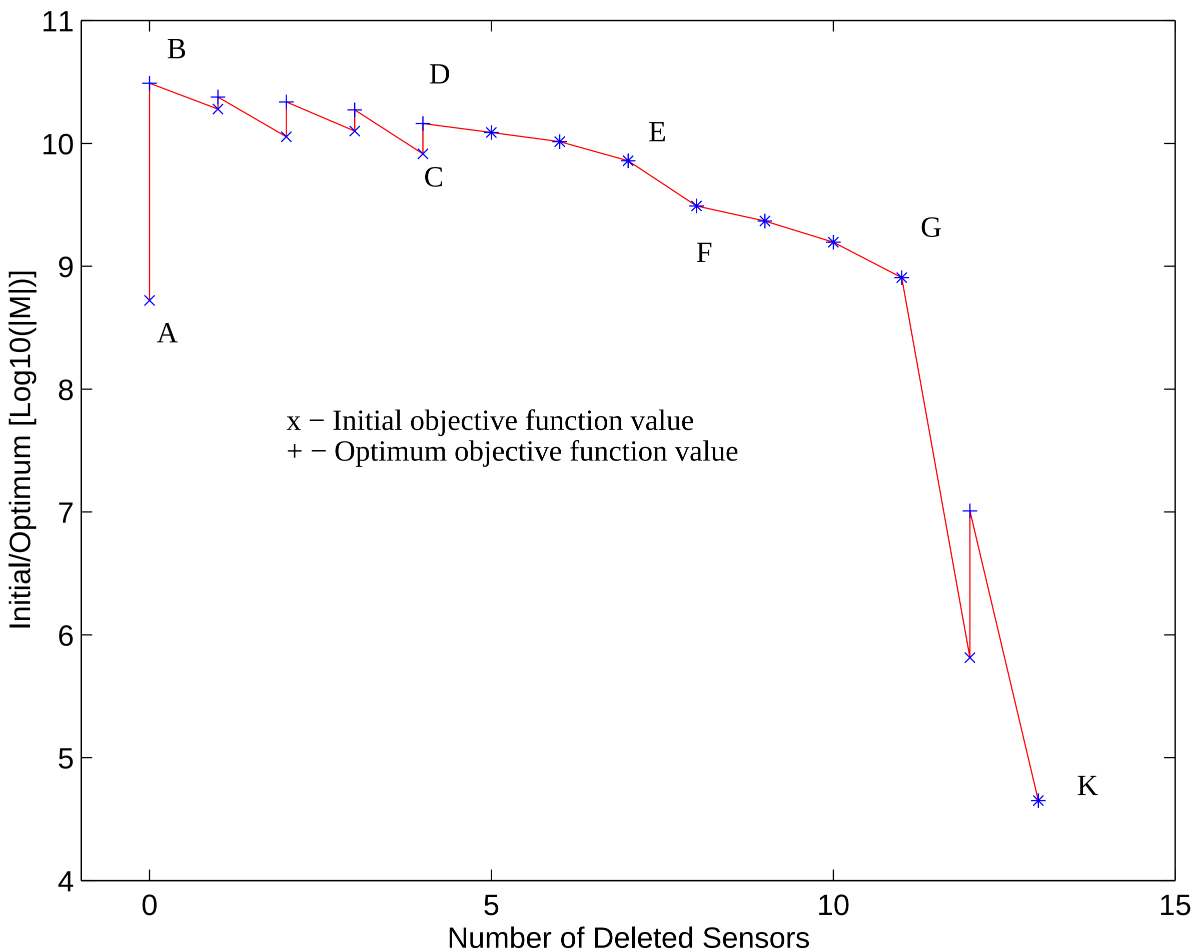

Figure 21: Example 2: Objective function history for sensor selection.

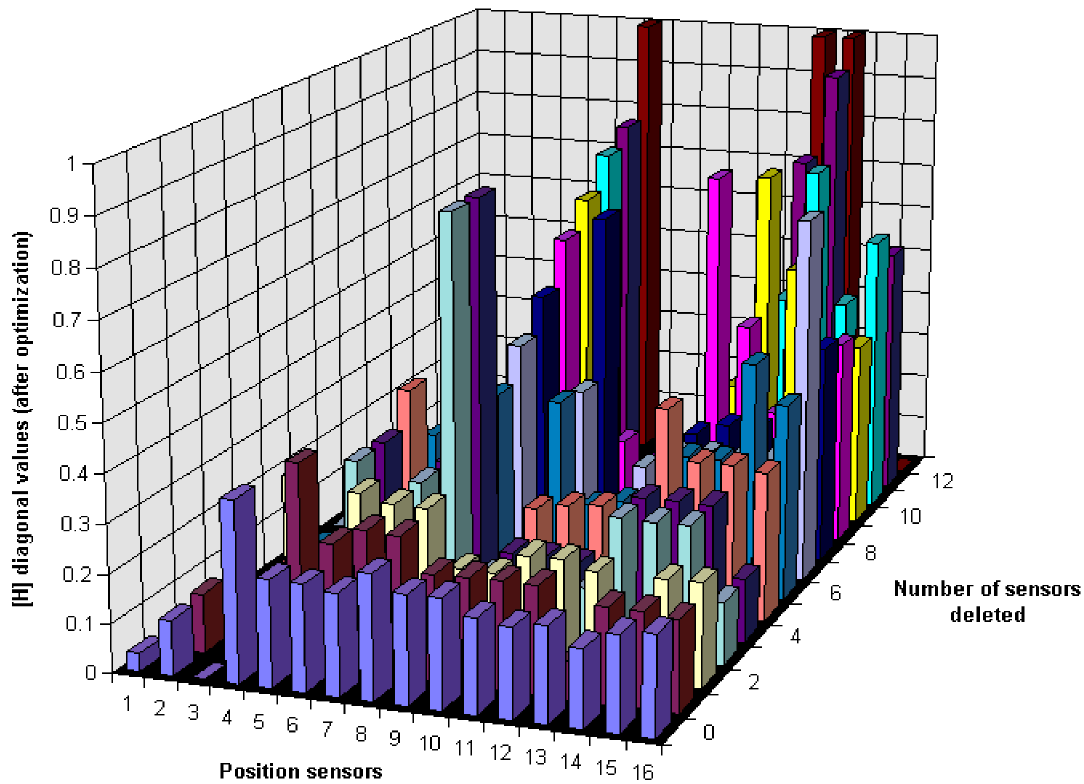

Figure 22: Example 2: Prediction value history for sensor selection.

jective function value of the design. However, these considerations should be weighted along with the important practical issues of design cost and complexity before a compromise stopping-stage in the design can be decided upon.

10.3 Discussions

For either the locator or the sensor design, optimized locations of the locators (or sensors) are often found near (or approaching) the edges of the respective surfaces. It is also observed that the locators (or sensors) tend to get clumped together near each other on a particular surface, especially in the initial stages of the design procedure, when the number of positions is large. These observations are associated with a unique feature of the D-optimal design that the solution is replicated. If the model has n parameters to be determined, then there exist n number of distinct solutions or “hot spots”. When the m candidate points (locators or sensors) are more than the number n of parameters, then the D-optimal design usually results in replications of the n distinct optimal locations (Bates 1988, p. 125). The locator (or sensor) clusters are formed due to the existence of local optimum corresponding to these “hot spots”. During the design iterations, the locations chosen for deletion are

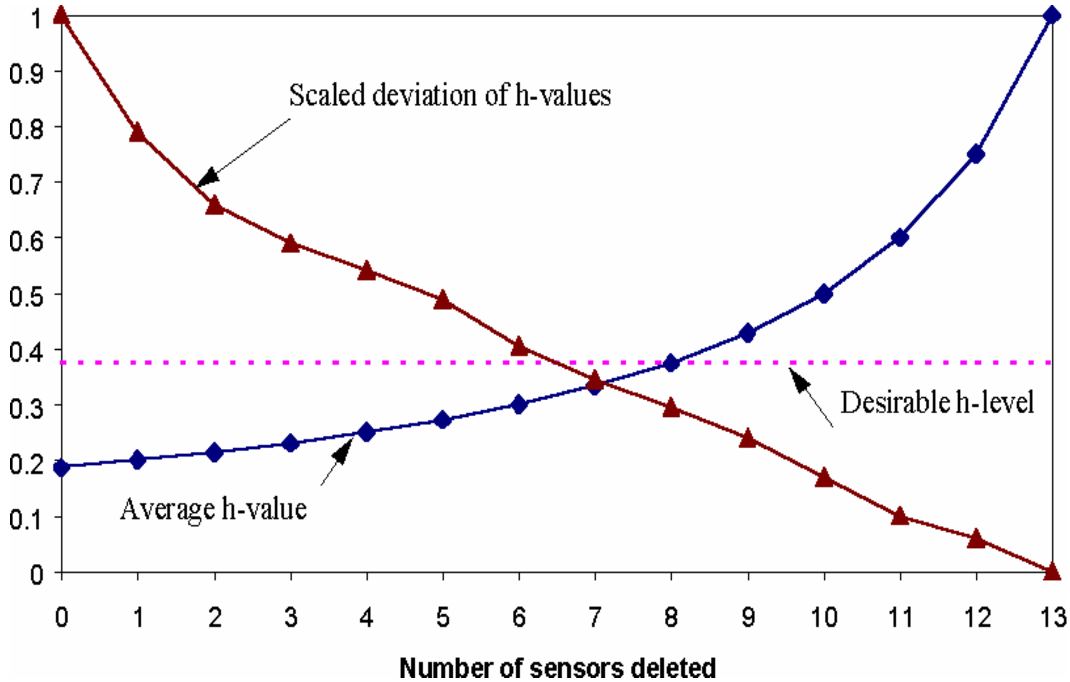

Figure 23: Example 2: Changes in prediction values.

mostly from those multiple candidates.

For this reason it is not necessary to use a large number of initial candidates. As the number of design variables increases, the iterative design procedure becomes inefficient. The design process would successively eliminate redundant candidates whose positions are essentially determined during the first few design iterations. As such, for the fixture design examples, up-to around 25 sensors or locators are used in the current implementation. This is also partially due to the limitation of COBYLA in handling large number of variables.

The computations for the both the examples were performed on a Sun workstation. The computations for the first example with initial 18 candidates took approximately 9 minutes of execution time. For the second example, the procedure took approximately 15 minutes. The increased time consumption for the second example can be attributed partly to the greater complexity of the part shape, and also to the larger size (22) of the initial candidate set.

11 Conclusions

In this article, we have proposed a systematic design procedure for optimal placement of sensors and locators for an automated Coordinate Checking Fixture (CCF). The primary issues of the design are deciding on *how many* sensors to use and *where* to place each of them. For measuring the goodness of a design, two metrics from the statistical analyses, the Fisher information matrix ($\boldsymbol{M}$) and the prediction matrix ($\boldsymbol{H}$) have been used in the proposed procedure. As a measure of the information content of the sensors, the determinant ($|\boldsymbol{M}|$) should be maximized for a given sensor set. Accordingly, the D-optimality criterion yields a balanced design with minimum effects of sensor noise and maximum sensitivity to variations in the part parameters. The diagonal values of the prediction matrix ($\boldsymbol{H}$) are directly representative of the relative contribution of corresponding sensors to the information provided by the sensor set. These prediction values were used in the design procedure to eliminate the sensor of the least significance iteratively for obtaining an optimal size of sensors from a candidate set. The procedure can also be used for the determination of locations of part locators of the CCF.

The contributions of this article are towards development of the concept of using automated Coordinate Checking Fixtures (CCFs). With built-in sensors, a CCF may be used at the site of the manufacturing operation to inspect manufactured components for specification compliance. An analysis of the measured coordinate data allows the process engineer to efficiently calibrate the manufacturing operation during set-up and to make any necessary re-adjustments during periodic monitoring, using real-time feedback that provides indications of any process-drifts. The CCF may be considered as an alternative to a Coordinate Measuring Machine (CMM). The flexibility of at-machine measurement and analysis provided by the CCF eliminates the need to transport batches of manufactured parts to a CMM laboratory and reduces the machine down-time.

Acknowledgments

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