

# TECHNICAL RESEARCH REPORT

## Design of a Power Regulating Gearbox for Parallel Hybrid Electric Vehicles

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# DESIGN OF A POWER REGULATING GEARBOX FOR PARALLEL HYBRID ELECTRIC VEHICLES

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## **ABSTRACT**

A parallel-hybrid electric vehicle usually employs two power sources, an engine and an electric motor/generator, to drive a vehicle. Typically, the electric motor receives power stored in batteries to drive the vehicle at low vehicle speeds. Both the electric motor and the engine provide power to the vehicle for maximal acceleration and/or hill climbing. The engine alone drives the vehicle and, at the same time, charges the batteries via the electric motor/generator during highway cruising. And during braking periods, the kinetic energy of the vehicle is used to recharge the batteries. For such a parallel hybrid electric vehicle to function properly, an electro-mechanical system that manages the power flow among the two power sources and the vehicle is necessary. This paper describes the design of an innovative power regulating gearbox. The proposed mechanism can provide a vehicle with six different operating modes including a continuous variable transmission capability to optimize the performance, improve the fuel economy, and reduce the hazardous emissions. To illustrate the capability of the gearbox, kinematics, statics, and power flow analysis are performed for each of the six operation modes.

# 1 INTRODUCTION

Internal combustion (IC) engines are widely used as the power source for ground vehicles. Usually, the size of an IC engine used in a vehicle is many times greater than the average power requirement of the vehicle in order to meet various operating conditions. As a result, the IC engine cannot be operated at its optimal operating condition which leads to poor fuel economy and high emissions. These problems are particularly important in large cities where stop-and-go driving is common and where pollution is a major problem.

One way of reducing the pollutants is to employ electric vehicles powered by rechargeable batteries. However, all electric vehicles have rather limited applications. Generally, they suffer the problems of limited driving range, around 200 miles, insufficient power for acceleration and hill climbing, and prolonged recharging time.

An alternative approach is to employ a small engine to drive an electric generator for charging the batteries and an electric motor to drive the vehicle. Thus the problems of limited driving range and prolonged recharging time can be avoided. Batteries are used as the energy storage devices so that the engine can run at its most fuel-efficient condition most of the times. This type of vehicles is called the *serial hybrid electric vehicles*. However, serial hybrid electric vehicles are inefficient and, perhaps, uneconomical as compared to the *parallel hybrid electric vehicles* to be described later. Firstly, in a serial hybrid electric vehicle, power is transmitted from an engine via an electric generator, a battery charger, batteries, and an electric motor before it reaches the wheels. Such a long transmission line can consume enormous amount of power and make the vehicle inefficient. Secondly, it requires a huge electric motor to achieve adequate acceleration and hill climbing performance requirements. Thirdly, it also requires a separate electric generator. Thus, a serial hybrid electric vehicle can be rather expensive.

A more promising approach is the *parallel hybrid electric drive*. In a parallel hybrid electric vehicle, multiple power sources such as IC engine and electric motor/generator are used to simultaneously drive a vehicle. Hence, the size of the electric motor/generator can be much smaller than that of an all electric or serial hybrid electric vehicle. For example, an electric motor that can produce one-half of the maximal power required of a vehicle may

be sufficient. This way a small IC engine which runs near the optimal operating condition most of the times can be employed. The electric motor/generator may sometimes work as a motor and sometimes as a generator. Power generated by the IC engine can be used to drive the vehicle directly and to charge the batteries as needed. A critical component in the design of such a parallel hybrid electric vehicle is a gear box that can be used to regulate the power flow among the IC engine, the electric motor/generator, and the vehicle under various operating conditions. We call such a gear box a *power regulating gearbox* as opposed to the conventional automotive transmissions (Tsai, et al., 1988).

Numerous parallel hybrid electric vehicles have been proposed. The parallel hybrid drive system invented by Berman et al. (1971 and 1973) consists of an internal combustion engine, an electric generator, and an electric motor/generator coupled together by a complex power regulating gearbox in such a way that both IC engine and electric motor/generator can simultaneously provide torque to the wheels. The IC engine works as the first power source and a set of batteries as the second power source. The electric generator regulates the speed of the vehicle by varying its loading so that the IC engine can be operated at a constant speed. The electric motor/generator can function as a motor to add torque or as a generator to subtract torque from that produced by the engine. The power regulating gearbox can be operated in two different modes. In the first mode, the engine operates at a constant speed and relatively constant low power, and the amount of power delivered to the wheels is regulated by the loading of the electric generator. In the second mode, all engine power is directed to the output shaft and the engine operates in a variable throttle manner. The electric motor/generator acts as a motor to provide additional power, or as a generator to absorb excessive power depending on the driving condition of the vehicle.

The parallel drive vehicles disclosed by Hunt (1983 and 1984) require an IC engine as the primary power source, a conventional multiple speed transmission, a turbine driven electric generator and an alternator for additional battery charging capability. Kawakatsu (1982) also disclosed a parallel hybrid system which consists of an IC engine and two electric motors to allow more efficient use of the electric motors.

Many other parallel drive configurations utilizing an IC engine to drive one set of wheels and an electric motor to drive the other set have been proposed. See Shea (1979),

Kenyon (1984), Krohling (1986), Eller (1990) for examples. These configurations require a conventional transmission for the IC engine, a separate speed reducer for the electric motor, and an electric generator. Furthermore, a complex algorithm is needed for synchronizing the speeds and torques of the IC engine and electric motor.

The above disclosures all require multiple power sources, an electric generator, and a multiple-speed transmission to drive a vehicle. The cost and complexity involved in producing and controlling such complex hybrid electric vehicles may be prohibited. To overcome these deficiencies, Severinsky (1994) recently proposed an innovative approach using just one IC engine, one electric motor/generator, and a power regulating gearbox. The power regulating gearbox suggested by Severinsky is essentially the inverse of a limited slip differential gearbox. One of the side-gear of the differential is connected directly to the electric motor/generator, the other side-gear is connected to the engine via a double acting clutch, while the carrier is connected to the final reduction and differential unit. Although the gear box appears to be simple, slippage between the planet gears and the carrier will constantly consume energy and make the device inefficient. Moreover, the inclusion of a controllable clutch between the planet gears and their carrier can drastically increase the complex and cost of the device. This paper examines an alternative simple and efficient power regulating gearboxes for parallel hybrid electric vehicles.

## **2 PARALLEL HYBRID ELECTRIC VEHICLES**

Figure 1 shows the block diagram of a parallel hybrid electric drive system. An IC (internal combustion) engine 30 serves as one power source and a set of batteries 50 serves as the second power source. An electric motor/generator 40 can receive power from the batteries to drive the vehicle or take power from the engine or the vehicle to charge the batteries depending on the driving condition. A power regulating gearbox 20 is used to regulate the power flow among the IC engine 30, the electric motor/generator 40, and the vehicle. The electric motor/generator 40 can be connected to the power regulating gearbox 20 by a straight shaft or a gear reducer or any other mechanical coupling devices. A band or multi-disc clutch is installed between the shaft of the electric motor/generator and the casing of the gearbox for a reason to be explained later. The IC engine 30 is connected to the power

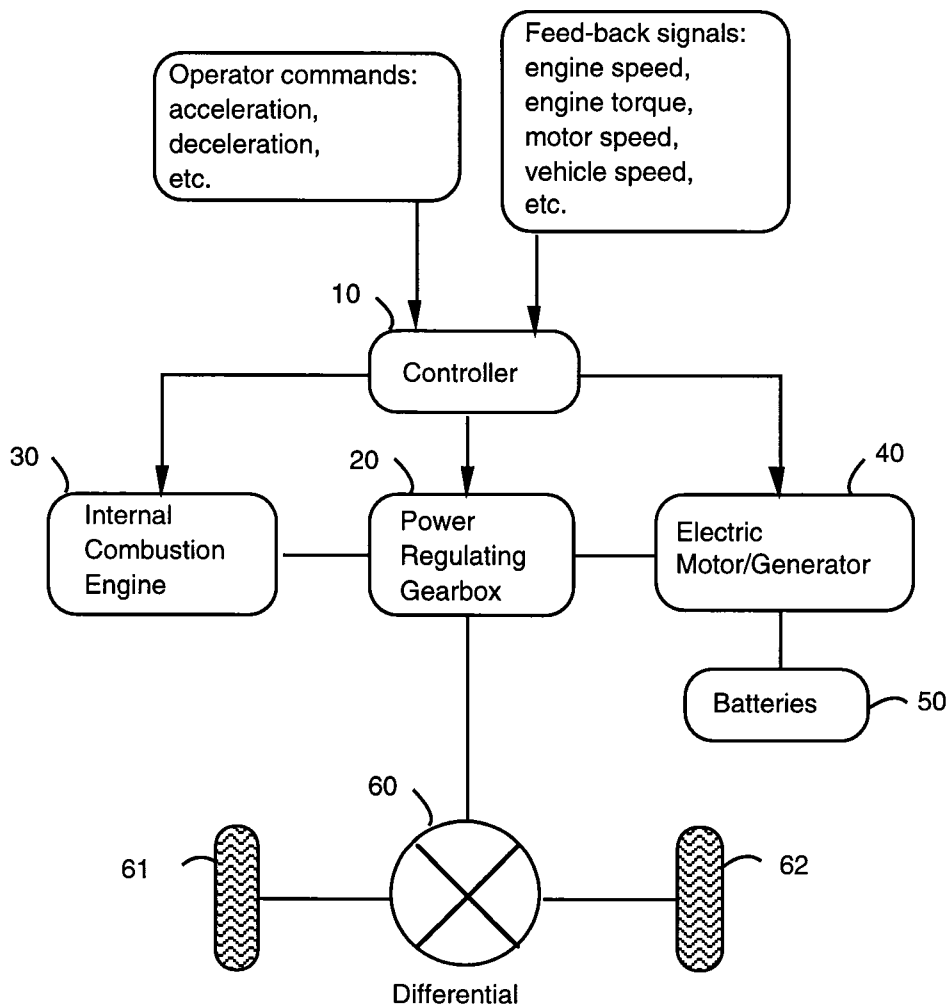


Figure 1: SCHEMATIC DIAGRAM OF A PARALLEL HYBRID ELECTRIC VEHICLE

regulating gearbox 20 by a rotating clutch not shown in the figure. The power regulating gearbox 20 receives power from the IC engine 30 and/or the electric motor/generator 40 and transmits it to the drive wheels 61 and 62 via a conventional differential gear box 60. A microprocessor controller 10 is used for the management of power flow among the IC engine 30, the electric motor/generator 40, and the vehicle.

The microprocessor controller 10 receives commands from the vehicle operator and feedback signals from the sensors, and controls the operations of the power regulating gearbox 20, the throttle opening and speed of the IC engine 30, and the speed and torque of the electric motor/generator 40. This way a parallel hybrid electric vehicle can be operated in several different modes to maximize its performance, to optimize its fuel consumption and to reduce its hazardous emissions.

Typically, the electric motor/generator 40 receives power stored in batteries 50 to drive the vehicle at low vehicle speeds, both the electric motor/generator 40 and IC engine 30 provide power to the vehicle during hill climbing or for maximum acceleration, the IC engine 30 alone drives the vehicle and charges the batteries 50 through the electric motor/generator 40 as needed during steady-state highway cruising, and the kinetic energy of the vehicle is used to charge the batteries 50 during the regenerative braking period.

### **3 POWER REGULATING GEARBOX**

Various planetary gear trains (Glover, 1964 and 1965, Buchsbaum and Freudenstein, 1970, Freudenstein, 1971, Tsai, 1987, Chatterjee and Tsai, 1994) can be use for regulating power flow among the IC engine 30, the electric motor/generator 40, and the vehicle. In general, the power regulating gear train should have two degrees of freedom, one for the IC engine 30 and the other for the electric motor/generator 40, with several control clutches. When one of the links is clutched to the casing of the gear box, the planetary gear train becomes a one-degree-of-freedom system so that the output torques of the IC engine 30 and the electric motor/generator 40 can be added or subtracted from one another without any restriction. When none of the links are clutched to the casing, the planetary gear train functions as a differential gearing so that with the engine 30 running at a constant speed, the vehicle speed can be varied by changing the speed of the electric motor/generator 40.

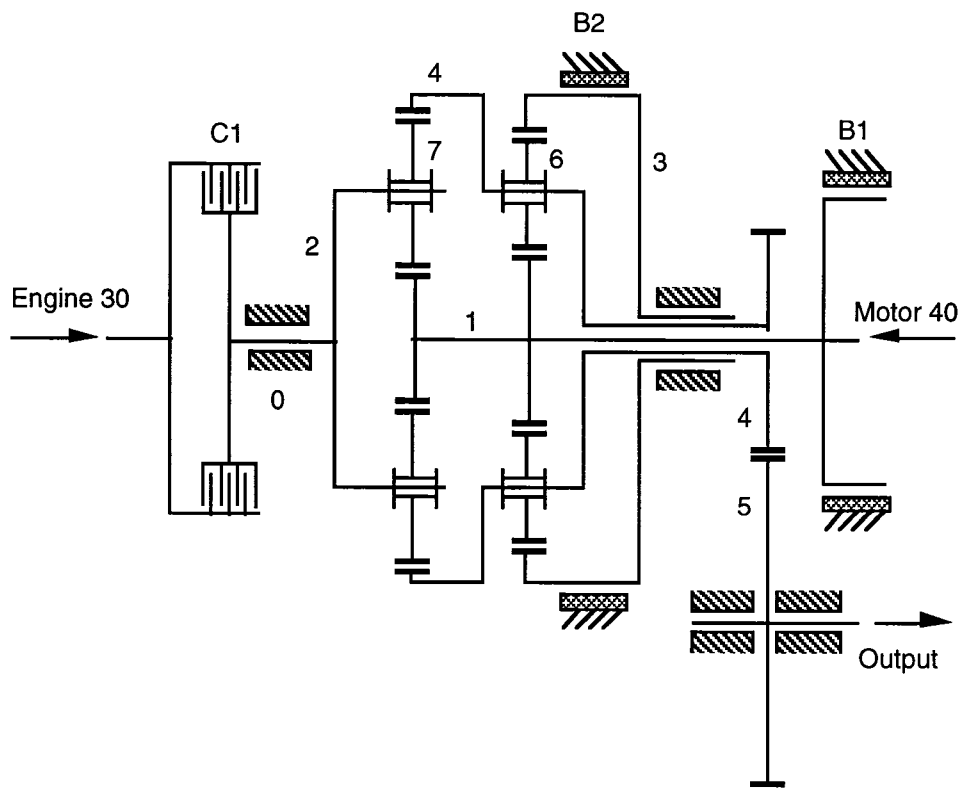


Figure 2: POWER REGULATING GEARBOX



Table 1: SIX MODES OF OPERATION

Mode	Clutch Engaged			Operating Conditions
	C1	B1	B2	
1			X	Electric motor alone
2	X		X	Power
3	X		X	Charging
4	X			CVT mode
5	X	X		Engine alone
6			X	Regenerative braking

Figure 2 shows the schematic diagram of a power regulating gearbox, wherein two short parallel heavy lines represent a gear mesh. The power regulating gearbox is made up of two simple planetary gear sets, a counter rotating gear pair, and three clutches. The planetary gear set shown to the left is called the *input planetary gear set* and the one shown to the right is called the *output planetary gear set*. Each planetary gear sets consists of a sun gear, an internal ring gear, and a carrier which supports several planet gears. A planetary gear set can be used to increase torque, increase speed, reverse the direction of rotation, or function as a differential drive. The two sun gears, both numbered 1, are rigidly connected together by one shaft which in turn is connected to the electric motor/generator 40 as shown in Fig. 2. The electric motor/generator can also be fixed to the casing of the gear box by a disc or band clutch B1. The ring gear 4 of the input planetary gear set is connected to the carrier of the output planetary gear set. The output carrier (input ring gear) is connected to the pinion of the final reduction gear pair, 4 and 5. Gear 5 of the final reduction gear pair is connected to the wheels via a conventional differential gearbox. The engine crankshaft 30 is coupled to the input carrier 2 by a multi-disc rotating clutch C1. The output ring gear 3 can spin freely or be grounded to the casing by a band clutch B2. Depending on which clutches are engaged, six modes of operation as listed in Table 1 are possible.

### 3.1 Electric Motor Alone

At low speeds or in city traffic, clutch B2 is engaged. With the output ring gear 3 serving as a reaction member, the electric motor/generator alone uses power stored in batteries to drive the vehicle.

## **3.2 Power Mode**

When maximum acceleration is desired or during hill climbing, clutches C1 and B2 are engaged simultaneously. The electric motor/generator starts the IC engine through the power regulating gearbox. Then both the IC engine and electric motor/generator simultaneously provide torque to the wheels to achieve maximum acceleration capability. Note that if a one-way clutch instead of a multi-disc rotating clutch C1 is used, then an additional starter is required to start the engine.

## **3.3 Charging Mode**

With clutches C1 and B2 engaged, the microprocessor controller may switch the electric motor/generator into a generator. Under this condition, the IC engine provides torque to drive the vehicle and, at the same time, charge the batteries through the electric motor/generator.

## **3.4 CVT Mode**

At moderate or high speeds, clutch B2 may be disengaged while keeping clutch C1 engaged. Under this clutching condition, part of the torque generated by the IC engine is directed to the wheels and the other part to the electric motor/generator for charging the batteries. It is possible to vary the speed of the vehicle by adjusting the loading of the electric motor/generator while running the engine at a constant speed. This way the power regulating gearbox functions as a continuous variable transmission (CVT).

## **3.5 Engine Alone**

During steady-state highway cruising, we may engage clutches B1 and C1. The IC engine alone drives the vehicle while the electric motor/generator is grounded to further improve the fuel economy.

## **3.6 Regenerative Braking**

During braking period, clutch B2 is engaged. The kinetic energy of the vehicle is used to charge the batteries through the electric motor/generator while the IC engine may be

disengaged from the gearbox if it is so desired.

## 4 KINEMATICS

The kinematics of a coupled planetary system can be easily analyzed by any of the well-known methods (Merritt, 1947, Freudenstein, 1971). In what follows, we shall employ the method of fundamental circuit equations introduced by Freudenstein (1971). Let  $i$  and  $j$  be a gear pair and  $k$  be the carrier. Then links  $(i, j, k)$  form a fundamental circuit and a fundamental circuit equation can be written as

$$\omega_i - \omega_k = \pm N_{ji}(\omega_j - \omega_k) \quad (1)$$

where  $\omega_i, \omega_j$  and  $\omega_k$  denote the angular velocities of link  $i, j$ , and  $k$ , respectively,  $N_{ji} = T_j/T_i$  denotes the gear ratio of the pair mounted on links  $j$  and  $i$ , where  $T_j$  and  $T_i$  denote the number of teeth on gears  $j$  and  $i$ . The sign in Eq. (1) is positive or negative according as the gear mesh is internal or external. We may consider  $\omega$  to be positive, if its rotation is in the counter clockwise direction as viewed from the right-hand-side of the schematic shown in Fig. 2.

Considering the multiple planet gears as one planet, the gear train shown in Fig. 2 contains four fundamental circuits:  $(7, 4, 2)$ ,  $(7, 1, 2)$ ,  $(6, 3, 4)$ , and  $(6, 1, 4)$ . The fundamental circuit equations can be written as

$$\omega_7 - \omega_2 = +N_{47}(\omega_4 - \omega_2) \quad (2)$$

$$\omega_7 - \omega_2 = -N_{17}(\omega_1 - \omega_2) \quad (3)$$

$$\omega_6 - \omega_4 = +N_{36}(\omega_3 - \omega_4) \quad (4)$$

$$\omega_6 - \omega_4 = -N_{16}(\omega_1 - \omega_4) \quad (5)$$

Eliminating  $\omega_6$  from Eqs. (4) and (5), yields

$$\omega_1 + N_{31}\omega_3 - (1 + N_{31})\omega_4 = 0 \quad (6)$$

Eliminating  $\omega_7$  from Eqs. (2) and (3), yields

$$\omega_1 + N_{41}\omega_4 - (1 + N_{41})\omega_2 = 0 \quad (7)$$

where  $N_{41} = N_{47}/N_{17}$  and  $N_{31} = N_{36}/N_{16}$ .

Equations (6) and (7) relate the angular velocities of links 1, 2, 3 and 4. Given angular velocities of any two links, we can solve Eqs. (6) and (7) for the other two.

## 5 STATICS AND POWER FLOW ANALYSIS

Numerous methods can be applied for the statics and power flow analyses of coupled planetary gear trains (Macmillan, 1964, Freudenstein and Yang, 1971, Sanger, 1972, Yang and Freudenstein, 1973, Yu and Beachley, 1985, Pennestri and Freudenstein, 1990a and 1990b). In what follows, we shall employ two basic equations for the statics and power flow analysis.

Let link 5 be excluded from the mechanism shown in Fig. 2. Under static equilibrium, summing the torques carried by the four coaxial links, 1, 2, 3 and 4, about their central axis of rotation, yields

$$\tau_1 + \tau_2 + \tau_3 + \tau_4 = 0 \quad (8)$$

Similarly, summing the power carried by the four coaxial links, yields

$$\tau_1\omega_1 + \tau_2\omega_2 + \tau_3\omega_3 + \tau_4\omega_4 = 0 \quad (9)$$

where  $\tau$  is positive, if it is applied in the counter clockwise direction as viewed from the right-hand-side of the schematic shown in Fig. 2.

Eliminating  $\tau_4$  from Eqs. (8) and (9), and making use of Eqs. (6) and (7), yields

$$N_{31}\tau_1 + \left(\frac{N_{31}}{1 + N_{41}}\right)\tau_2 - \tau_3 = 0 \quad (10)$$

Eliminating  $\tau_3$  from Eqs. (8) and (9), and making use of Eqs. (6) and (7), yields

$$(1 + N_{31})\tau_1 + \left(1 + \frac{N_{31}}{1 + N_{41}}\right)\tau_2 + \tau_4 = 0 \quad (11)$$

Equations (10) and (11) relate torques exerted on links 1, 2, 3 and 4 in terms of their gear ratios. Given torques exerted on links 1 and 2, we can find that exerted on links 3 and 4. We note that if both  $\tau_1$  and  $\tau_2$  are positive, then  $\tau_3$  is positive and  $\tau_4$  is negative. That is, with link 3 serving as a reaction member, torques applied by the electric motor/generator

and the IC engine are each amplified by a factor of  $(1 + N_{31})$  and  $(1 + \frac{N_{31}}{1+N_{41}})$ , respectively, before they are transmitted to the output link 4. Once torques exerted on the links are known, power carried by each link can be computed by the product of  $\tau$  and  $\omega$ .

## 6 NUMERICAL EXAMPLE

In this section, we shall demonstrate the feasibility of such a power regulating gearbox by a numerical example. Let the gear sizes of the input planetary gear set be  $T_1 = 42, T_7 = 26$ , and  $T_4 = 94$  and that of the output planetary gear set be  $T_1 = 42, T_6 = 26$ , and  $T_3 = 94$ . Then,  $N_{31} = N_{41} = 2.238$ . Assuming a highway cruising speed of 100 km/hr (62 miles/hr) is desired, then for a vehicle equipped with 78.74 cm (31 inches) diameter tires, the axle rotation speed is  $\omega_5 = 674$  rpm. If a final reduction ratio of 3.56 is used, then the rotation speed of link 4 is  $\omega_4 = 2,400$  rpm.

We now compute the angular velocities and torques of the links for each of the six operating modes.

### 6.1 Electric Motor Alone

In the first operation mode, clutch B2 is engaged while clutches B1 and C1 are disengaged. Under this clutching condition,  $\omega_3 = 0$  and  $\tau_2 = 0$ . Thus the input planetary gear set carries no load.

Substituting  $\omega_3 = 0$  into Eq. (6), yields

$$\omega_1 = (1 + N_{31})\omega_4 = 3.238\omega_4 \quad (12)$$

Substituting Eq. (12) into (7), yields

$$\omega_2 = (1 + \frac{N_{31}}{1 + N_{41}})\omega_4 = 1.691\omega_4 \quad (13)$$

Hence, the speed reduction ratio is 3.238 for the electric motor/generator and 1.691 for the IC engine.

Substituting  $\tau_2 = 0$  into Eqs. (10) and (11), yields

$$\tau_3 = N_{31}\tau_1 = 2.238\tau_1 \quad (14)$$

and

$$\tau_4 = -(1 + N_{31})\tau_1 = -3.238\tau_1 \quad (15)$$

Hence, using link 3 as a reaction member, torque generated by the electric motor/generator is amplified by 3.238 times at the output link 4. Including the final reduction, we obtain an overall reduction ratio of 11.5.

## 6.2 Power Mode

For maximal acceleration, both clutches C1 and B2 are engaged. Under this clutching condition  $\omega_3 = 0$ , but  $\tau_1$  and  $\tau_2$  are now both positive.

Since  $\omega_3 = 0$ , the speed relationships given by Eqs. (12) and (13) remain valid. However, since  $\tau_2$  is no longer zero, torque relationships are calculated from Eqs. (10) and (11) as

$$\tau_3 = N_{31}\tau_1 + \frac{N_{31}}{1 + N_{41}}\tau_2 = 2.238\tau_1 + 0.691\tau_2 \quad (16)$$

and

$$\tau_4 = -(1 + N_{31})\tau_1 - \left(1 + \frac{N_{31}}{1 + N_{41}}\right)\tau_2 = -(3.238\tau_1 + 1.691\tau_2) \quad (17)$$

Hence, the motor torque is amplified by 3.238 times and the engine torque is amplified by 1.691 times, and these two torques are added at the output link 4 to provide a maximum acceleration capability.

Finally, total power delivered to link 4 is given by

$$P_4 = \tau_4\omega_4 = -(3.238\tau_1 + 1.691\tau_2)\omega_4 = -(\tau_1\omega_1 + \tau_2\omega_2) \quad (18)$$

We conclude that power produced by the electric motor/generator and the IC engine are added at the output link 4. In this mode if we wish to accelerate the vehicle to a desired speed of 100 km/hr, i.e.,  $\omega_4 = 2,400$  rpm, then the electric motor/generator and the IC engine will be accelerated to the speeds of  $\omega_1 = 7,771$  rpm and  $\omega_2 = 4,058$  rpm, respectively, which are well within the operating range of an AC synchronous induction motor and a conventional IC engine.

### 6.3 Charging

With both clutches C1 and B2 engaged, the speed relationships are given by Eqs. (12) and (13), while the torque relations are given by Eqs. (16) and (17). However, the electric motor/generator is switched into a generator to charge the batteries. The electric motor/generator continues to rotate in the same direction with a reversal its applied torque, i.e.,  $\tau_1 < 0$ . Equation (17) can be rewritten as

$$\tau_4 = -[1.691\tau_2 - 3.238(-\tau_1)] \quad (19)$$

Hence, part of the torque (or power) generated by the IC engine is directed to the electric motor/generator to charge the batteries and the other part is directed to the output link 4 to drive the vehicle. The distribution of torque is dependent upon the loading of the generator.

### 6.4 CVT Mode

In the CVT mode, only clutch C1 is engaged. Hence, none of the links is held stationary and the gear train functions as a differential. Because of the gear train arrangement, torque exerted on the electric motor/generator will always have the same sense as that exerted on the output link 4. Hence, the electric motor/generator must be switched into a generator for regulating the speeds of the engine and the vehicle. The angular velocity of the electric motor is derived from Eq. (7) as

$$\omega_1 = -N_{41}\omega_4 + (1 + N_{41})\omega_2 = -2.238\omega_4 + 3.238\omega_2 \quad (20)$$

From Eq. (20), we conclude that the engine can be operated at any desired speed while the speed of the vehicle is regulated by the electric motor/generator. Since link 3 is free to rotate,  $\tau_3 = 0$  identically. The output planetary gear set carries no load.

Substituting  $\tau_3 = 0$  into Eq. (10), yields

$$\tau_1 = -\frac{1}{1 + N_{41}}\tau_2 = -0.3088\tau_2 \quad (21)$$

Substituting Eq. (21) into (11), yields

$$\tau_4 = (-1 + \frac{1}{1 + N_{41}})\tau_2 = -0.6912\tau_2 \quad (22)$$

Hence, 30.88 percent of the engine torque is directed to the electric motor/generator for charging the batteries while the remaining 69.12 percent is directed to the output link 4 for driving the vehicle.

Under the CVT mode, it is possible to run the IC engine at any desired speed while controlling the vehicle speed by changing the speed of the electric motor. For example, if we wish to drive the vehicle at 100 km/hr ( $\omega_4 = 3,000$  rpm) while running the IC engine at 2,400 rpm, then we can compute the speed of the electric motor by Eq. (20) as

$$\omega_1 = -2.238 \times 2,400 + 3.238 \times 3,600 = 6,286 \text{ rpm}$$

At this speed, assuming the engine torque is  $\tau_2 = 50$  Nm (36.9 ft-lb), then torque exerted by the electric motor/generator and the output link 4 are given by

$$\tau_1 = -0.3088\tau_2 = -15.44 \text{ Nm}$$

and

$$\tau_4 = -0.6912\tau_2 = -34.56 \text{ Nm}$$

The power generated by the IC engine, delivered to the electric motor/generator and the output link 4 are given by

$$P_2 = \tau_2\omega_2 = 18,850 \text{ Nm/s (or 25.26 hp)}$$

$$P_1 = \tau_1\omega_1 = -10,164 \text{ Nm/s (or -13.62 hp)}$$

and

$$P_4 = \tau_4\omega_4 = -8,686 \text{ Nm/s (or -11.64 hp)}$$

## 6.5 Engine Alone

With clutches B1 and C1 engaged, we have  $\omega_1 = 0$  and  $\tau_3 = 0$ . Substituting  $\omega_1 = 0$  into Eq. (7), yields

$$\omega_2 = \left(\frac{N_{41}}{1 + N_{41}}\right)\omega_4 = 0.691\omega_4 \quad (23)$$

This is known as the overdrive. Thus, for a highway cruising speed of 100 km/hr, the engine runs at  $\omega_2 = 0.691 \times 2,400 = 1,658$  rpm.



Since  $\tau_3 = 0$ , torque relations given by Eqs. (21) and (22) remain valid. Hence, to maintain the same output speed,  $\omega_4 = 2,400$  rpm, and the same output torque,  $\tau_4 = -34.56$  Nm, the engine needs to produce a torque of  $\tau_2 = \tau_4/0.691 = -50$  Nm. We note that shifting from the CVT mode to engine alone mode, the engine torque remains the same while its operating speed is drastically reduced. This is the most efficient mode for highway cruising.

## 6.6 Regenerative Braking

The kinematic and static torque relationships for the regenerative braking mode is identical to that of the electric motor alone mode, except for the fact that the torques have been reversed.

## 7 SUMMARY

Parallel hybrid electric vehicles offer a great potential for improved fuel economy, reduced emissions, and sufficient performance. It is shown that the proposed power regulating gearbox does have the potential for use as a power regulating device for parallel hybrid electric vehicles. Six modes of operations are shown to be feasible. Typically, the electric motor/generator alone drives the vehicle at low speeds or in traffic. To achieve a maximal acceleration or during hill climbing, both the IC engine and electric motor/generator provide torque to drive the vehicle. The electric motor/generator can also function as a speed controller to provide a continuous variable transmission capability. During steady-state highway cruising, the IC engine alone drives the vehicle. Further, the electric motor/generator can also function as a generator to charge the batteries at any speeds and during regenerative braking period. The capability for such a power regulating gearbox to provide six modes of operations is demonstrated with a numerical example.

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