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Adaptive Blind Multi-Channel Equalization for Multiple Signals Separation

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ABSTRACT

This paper investigates adaptive blind equalization for multiple-input and multiple-output (MIMO) channels and its application to blind separation of multiple signals received by antenna arrays in communication systems. The performance analysis is presented for the CMA equalizer used in MIMO channels. Our analysis results indicate that double infinite-length MIMO-CMA equalizer can recover one of input signals, remove the intersymbol interference (ISI), and suppress the rest signals. In particular, for the MIMO FIR channels satisfying certain conditions, the MIMO-CMA FIR equalizer is able to remove the ISI and co-channel interference regardless of the initial setting of the blind equalizer. To recover all input signals simultaneously, a novel MIMO channel blind equalization algorithm is developed in this paper. The global convergence of the new algorithm for MIMO channels is proved. Hence, the new blind equalization algorithm for MIMO channels can be applied to separate and equalize the signals received by antenna arrays in communication systems. Finally, Computer simulations are presented to confirm our analysis and illustrate the performance of the new algorithm.

SP EDICS

SP 3.4 Statistical multichannel filtering,
SP 3.6.2 Parameter estimation: multichannel time series.

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I. Introduction

The use of array signal processing in wireless communications under the framework of spatial division multiple access (SDMA) has been of great interest recently. In such situation, the sensors or antennas may receive a superposition of several signals via many channels from many moving sources. The system can be modeled as a *multiple-input multiple-output* (MIMO) system. One of the most crucial problems is not only to separate these signals, but also simultaneously equalize the MIMO channel such that high quality communications can be achieved. The signals separation in other MIMO systems, such as in speech processing, seismic exploration, and the analysis of biological systems, is also an important issue. To separate the signals and at the same time, remove the channel distortion, blind channel equalization techniques have been very effective.

For single-input single-output (SISO) systems, lots of blind identification algorithms [5], [7], [17], [26] and blind equalization algorithms [1], [3], [9], [10], [19], [20], [21], [24], [25], [30] have been proposed. Most of these algorithms exploit higher-order statistics of channel output. Among various algorithms, Godard algorithm (GA)[9], also known as the constant modular algorithm (CMA) [24], [25], is one of the best and simplest adaptive blind equalization algorithms. It has been shown [6], [21] that, for double infinite-length equalizers, CMA will always converge to a global minimum regardless of initial values. The local convergence properties of the CMA, when implemented with FIR equalizers, are observed and analyzed in [4], [14], [23] and the references therein.

Single-input multiple-output (SIMO) systems can be viewed as fractionally-spaced sampled communication systems or antenna arrays received only one input signal. The fractionally-spaced equalizer has been originally proposed to suppress timing sensitivity [8], [28]. The convergence performance of decision-feedback fractionally-spaced equalizer is investigated in [16]. The Godard algorithm, or CMA, can also be used in SIMO communication systems. The convergence of fractionally-spaced CMA adaptive blind equalizer is studied in [12], [15]. Recently, fractionally-spaced CMA adaptive blind equalizer under symbol timing offsets is considered in [27].

The equalization of MIMO transmission systems is studied in [18], [32] when the MIMO channel impulse response is known. Several iteration algorithms for blind estimation and separation of MIMO FIR channels have been developed in [29]. However, there is no proof on the global convergence of those iteration algorithms. The blind identification and multiple signal separation algorithms based on the higher-order statistics have been presented in [22], [31], [33]. However, there is no report on the adaptive blind equalization for MIMO channels. As indicated above, the CMA is one of the most popular algorithms used in SISO and SIMO systems. An interesting question is how effective the CMA is when it is used in MIMO systems, such as in mobile communications, to remove intersymbol interference (ISI), co-channel interference (CCI) or adjacent-channel interference (ACI). This paper first investigates the possibility of the use of CMA blind equalizer in MIMO channels. Our analysis demonstrates that the CMA blind equalizer is able to recover only one of the input signals, suppress the rest of signals. Furthermore, under certain condition, the CMA FIR equalizer, regardless of equalizer's initial setting, can perfectly recover one of the input signals from the outputs of the MIMO FIR channels. Therefore, the CMA can be used in mobile communication systems to remove ISI, CCI, and ACI. Then, we develop a new adaptive blind equalization algorithm for MIMO channels, to simultaneously recover all the input signals and at the same time to remove the ISI.

The remaining part of this paper is organized as follows. In Section II, we formulate the blind MIMO equalization problem and introduce a necessary and sufficient condition for an MIMO channel to have a bounded-input and bounded-output (BIBO) stable equalizer that can achieve distortionless reception. Then in Section III, we present the convergence analysis of the CMA blind equalizer used in MIMO channels. Our study indicates that most of the good convergence properties of CMA for SISO channels or SIMO channels still preserve for MIMO channels. Next, in Section IV, we develop a novel blind equalization algorithm to recover all input signals simultaneously. We prove the global convergence of the proposed algorithm. Finally, we present computer simulations to confirm our analysis results and illustrate the performance of the new algorithm in Section V.

II. Blind Equalization for MIMO Channels

Antenna arrays can be used in mobile communication systems to improve the communication quality and increase communication capacity. The antenna array received the superposition of several wide-band signals can be modelled as an MIMO system shown in Figure 1. The d complex sequences $a_1[n], \cdots, a_d[n]$ are sent through different channels with impulse responses $h_{ij}[n]$ for $i = 1, \cdots, M$ and $j = 1, \cdots, d$ ($d \leq M$). We will assume in this paper that the input sequences satisfy

$$E\{a_i[n]\} = E\{a_i^2[n]\} = 0, \tag{1}$$

and

$$2m_2^2 - m_4 > 0, \tag{2}$$

where

$$E\{|a_i[n]|^2\} = m_2, \;\; E\{|a_i[n]|^4\} = m_4. \tag{3}$$

If we define the *output vector* $\mathbf{x}[n]$, the *channel impulse response matrix* $H[n]$, and the *input vector* $\mathbf{a}[n]$ respectively as

$$\mathbf{x}[n] \triangleq \begin{pmatrix} x_1[n] \\ \vdots \\ x_M[n] \end{pmatrix}, \; H[n] \triangleq \begin{pmatrix} h_{11}[n] & \dots & h_{1d}[n] \\ \vdots & \vdots & \vdots \\ h_{M1}[n] & \dots & h_{Md}[n] \end{pmatrix}, \text{ and } \mathbf{a}[n] \triangleq \begin{pmatrix} a_1[n] \\ \vdots \\ a_d[n] \end{pmatrix}, \tag{4}$$

then the channel output vector $\mathbf{x}[n]$ can be expressed as

$$\mathbf{x}[n] = H[n] * \mathbf{a}[n], \tag{5}$$

where $*$ denotes the convolution of the matrix (or vector) sequences. For general matrix sequences $(a_{ij}[n])$ and $(b_{ij}[n])$, their convolution is defined as

$$(a_{ij}[n]) * (b_{ij}[n]) \triangleq (\sum_k a_{ik}[n] * b_{kj}[n]). \tag{6}$$

Equation (5) can also be written in Z-transform as

$$\mathbf{x}(z) = H(z)\mathbf{a}(z), \tag{7}$$

where $\mathbf{x}(z)$, $\mathbf{a}(z)$ and $H(z)$ are the $Z-$transform of $\mathbf{x}[n]$, $\mathbf{a}[n]$ and $H[n]$, respectively. For MIMO FIR channels, $H(z)$ is a polynomial matrix.

To recover the input signal $\mathbf{a}[n]$, a linear channel equalizer is applied to the channel output $\mathbf{x}[n]$ as in Figure 2, whose objective is to achieve distortionless reception. That is, to find $G[n]$ such that

$$G[n] * H[n] = \delta[n] I_d, \tag{8}$$

or simply

$$G(z)H(z) = I_d, \tag{9}$$

where I_d is a $d \times d$ identity matrix and $G[n]$ is the *equalizer matrix* defined as

$$G[n] \triangleq \begin{pmatrix} g_{11}[n] & \cdots & g_{1M}[n] \\ \vdots & \cdots & \vdots \\ g_{d1}[n] & \cdots & g_{dM}[n] \end{pmatrix}, \tag{10}$$

and $G(z)$ is the Z-transform of $G[n]$. Initially, we may take the filters in Figure 2 as being bounded-input and bounded-output (BIBO) stable and potentially non-causal (double-infinite) so as to deal with MIMO channels of non-causal inverse. In blind equalization, the original sequences $a_i[n] \in \mathcal{A}_i$ for $i = 1, \cdots, d$ are unknown to the receivers except for their statistical properties over the known alphabet sets $\mathcal{A}_i$. Usually, the signal constellations $\mathcal{A}_1 = \mathcal{A}_2 = \cdots = \mathcal{A}_d$ are symmetric such that the statistics of the input signals a_i for $i = 1, \cdots, d$ reflects the same symmetry. Thus, the recoverable signals from blind equalization will similarly subject to a phase ambiguity and a permutation ambiguity. Therefore, the best possible result of blind MIMO equalizers would be

$$G(z)H(z) = PD(z), \tag{11}$$

where P is a $d \times d$ permutation matrix and $D(z)$ is a diagonal matrix defined as

$$D(z) = \mathrm{diag}\{e^{\jmath\theta_1} z^{-n_1}, \cdots, e^{\jmath\theta_d} z^{-n_d}\}, \tag{12}$$

where $\theta_i \in [-\pi,\ \pi]$ and n_i is an integer for $i = 1, \cdots, d$. The equalizers with $G(z)$ satisfying (11) are called the *distortionless reception equalizer* for channel $H(z)$. It is obvious that the distortionless reception equalizer for a given MIMO channel is not necessarily unique.

Not all channels have a BIBO stable distortionless reception equalizer. A channel is said to satisfy the *distortionless reception condition* if there exists a BIBO stable distortionless reception equalizer for such channel. A single-input signal-output (SISO) channel satisfies distortionless reception condition if and only if the Z-transform of the channel impulse response has no zero on the unit circle. For MIMO channels, the following theorem gives a necessary and sufficient condition for the existence of BIBO stable distortionless reception equalizers.

Theorem 1: There exists a BIBO stable, linear, and distortionless reception equalizer for an MIMO channel if and only if

$$det(H^H(e^{\jmath\omega})H(e^{\jmath\omega})) \neq 0, \text{ for all } \omega \in [-\pi, \pi]. \tag{13}$$

Proof: If $\det(H^H(e^{\jmath\omega})H(e^{\jmath\omega})) \neq 0$ for all $\omega \in [-\pi, \pi]$, then $(H^H(e^{\jmath\omega})H(e^{\jmath\omega}))^{-1}$ exists and

$$G(e^{\jmath\omega}) = (H^H(e^{\jmath\omega})H(e^{\jmath\omega}))^{-1}H^H(e^{\jmath\omega}) \tag{14}$$

is the Fourier transform of a BIBO stable equalizer satisfying (11).

Conversely, if there is a $\omega_0 \in [-\pi, \pi]$ such that

$$\det(H^H(e^{\jmath\omega_0})H(e^{\jmath\omega_0})) = 0, \tag{15}$$

then $H(e^{\jmath\omega_0})$ will not be of full-rank. If there is a $G(e^{\jmath\omega})$ satisfying (11), then

$$PD(e^{\jmath\omega_0}) = G(e^{\jmath\omega_0})H(e^{\jmath\omega_0}) \tag{16}$$

would be singular. This is a contradiction since $PD(e^{\jmath\omega_0})$ is nonsingular from its definition. Therefore, the BIBO stable, linear, and distortionless equalizer $D(e^{\jmath\omega})$ does not exist in this case.

■

From Theorem 1, a necessary condition for an MIMO channel to have distortionless reception equalizer is $M \geq d$, that is, the number of channel outputs is no less than the number of channel inputs. In what follows, we will always assume that the discussed MIMO channels satisfy the distortionless reception condition.

III. Convergence of the CMA used in MIMO Channels

In this section, we will investigate the performance of the CMA equalizer used in MIMO channels. The MIMO-CMA blind equalizer discussed in this section is illustrated in Figure 3. After each channel output, a linear BIBO stable filter is used. The filter coefficients are adjusted to minimize the Godard cost function [9], [24], [25]

$$C(y[n]) = \frac{1}{4}E\{(|y[n]|^2 - r)^2\}, \tag{17}$$

where r is the dispersion constant defined as

$$r = \frac{m_4}{m_2}. \tag{18}$$

A. CMA Equalizer for MIMO Channels

From Figure 3, the channel output can be expressed as

$$y[n] = \sum_{i=1}^{d} \sum_{k=-\infty}^{\infty} a_i[n-k]s_i[k], \tag{19}$$

where $s_i[n]$ is the impulse response of the equalized system corresponding to the i-th input signal related to $h_{mi}[n]$ and $g_m[n]$ by

$$s_i[n] = \sum_{m=1}^{M} \sum_{k=-\infty}^{\infty} h_{mi}[k]g_m[n-k]. \tag{20}$$

Using (19), the Godard cost function defined in (17) can be expressed as

$$C(y[n]) = \frac{1}{4}[-(2m_2^2 - m_4)\sum_{i,n}|s_i[n]|^4 + 2m_2^2(\sum_{i,n}|s_i[n]|^2)^2 - 2m_4\sum_{i,n}|s_i[n]|^2 + m_4^2/m_2^2]. \tag{21}$$

If we denote

$$\mathbf{s} \triangleq (\cdots, s_d[-1], s_1[0], \cdots, s_d[0], s_1[1], \cdots), \tag{22}$$

then the Godard cost function (21) is a functional of $\mathbf{s}$, that has a similar form to that of SISO channel [6], [14]. Hence, if the MIMO channel satisfies the distortionless reception condition, and if the length of the equalizer in Figure 3 is double-infinite, following the Foschini's arguments [6], it can be easily shown that the only minimum points of the MIMO-CMA equalizer in Figure 3 are

$$|s_i[n]|^2 = \delta[n - n_d, i - i_0], \text{ for some integers } n_d \text{ and } i_0, \tag{23}$$

where $\delta[n, i]$ is defined as

$$\delta[n, i] \triangleq \begin{cases} 1 & \text{if } n = 0 \text{ and } i = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

This implies that the MIMO-CMA equalizer will converge to one of the input signal with only a time-delay and phase ambiguity and suppress the rest of input signals.

Furthermore, the other convergence properties are also preserved. Before stating these properties, we first give some relevant definitions.

The *attainable set* S_a for a given (finite or infinite) equalizer is defined as

$$S_a \triangleq \{\mathbf{s} : s_i[n] = \sum_{m=1}^{M} \sum_{k} h_{mi}[n-k] g_m[k], \sum_{k} |g_m[k]| < \infty\}. \tag{25}$$

In the above definition, the range of k relys on the length of the equalizer. It is obvious that S_a depends on the parameters of the channels.

The *unique global minimum set cone* $S_{i,n}$ is defined as

$$S_{i,n} \triangleq \{\mathbf{s} : |s_i[n]| > |s_j[k]| \text{ for all } i \neq j \text{ or } n \neq k \text{ and } \sum_{j,k} |s_j[k]| < \infty\}. \tag{26}$$

With the above definitions we can state the convergence properties of finite-length MIMO-CMA equalizers as follows.

Theorem 2: Let S_a be the attainable set of a given finite-length MIMO-CMA equalizer.

1. *If the initial equalizer parameters setting are such that the initial equalized system impulse response vector $\mathbf{s}^{in} \in S_a \bigcap S_{i,n}$ and its output satisfies the kurtosis condition*

$$\frac{kurt(y_n)}{kurt(a_n)} > 0.5, \tag{27}$$

then under a very small minimization step-size, the equalizer will cause $\mathbf{s}$ *to converge to a minimum point inside $S_a \bigcap S_n$. In the above expression,*

$$kurt(x) \triangleq \frac{K(x)}{\sigma_x^4}, \tag{28}$$

where σ_x^2 is the variance of x and $K(x) = E\{|x|^4\} - 2\sigma_x^4$ is the kurtosis of complex random variable x satisfying $E\{x^2\} = 0$.

2. *Denoting $E_{i,n} = \{\mathbf{s} : s_i[n] = e^{\jmath\phi},\ \phi \in [-\pi,\ \pi] \text{ and } s_j[k] = 0 \text{ if } j \neq i \text{ or } k \neq n\}$*
   (a) *If $E_{i,n} \subset S_a \bigcap S_{i,n}$, then there is only one minimum set $E_{i,n} \subset S_a \bigcap S_{i,n}$ while there is no minimum point on the boundary $S_{i,n}$.*
   (b) *If $E_{i,n}$ is near $S_a \bigcap S_{i,n}$, then there must exist only one minimum set in $S_a \bigcap S_{i,n}$ near $E_{i,n}$ while all other possible minima are near the boundary of $S_{i,n}$.*

Theorem 2 is basically the generalization of Theorem 6.2 and 6.2 in [14] for SISO channels, or Theorem 3.2 in [15] for SIMO channels. Its proof is similar to that of Theorem 6.2 and 6.3 in [14], therefore, it is omitted here.

According to Theorem 2.1, if we want to use the MIMO-CMA equalizer to recover the i-th input signal, and remove intersymbol interference (ISI) and co-channel interference, we need to select the initial setting of the equalizer such that $\mathbf{s}^{in} \in S_{i,n}$ for some n and the initial channel output satisfies the kurtosis condition.

Theorem 2.2 indicates the locations of the minima of the MIMO-CMA equalizers. Based on this part of the theorem, the initialization strategy discussed in [6], [14] can also be used for the MIMO-CMA blind equalizers.

B. CMA FIR equalizer for MIMO FIR Channels

In practice, most of the MIMO channels can be approximated as FIR MIMO channels. Without lose of generality, we can assume

$$h_{mi}[n] = 0 \text{ for } n < 0 \text{ or } n > L, \tag{29}$$

for $i = 1, \cdots, d$ and $m = 1, \cdots, M$, where L is the length of the MIMO FIR channel. The length of the impulse response $s_i[n]$ of the equalized system is $L + K - 1$ if an MIMO-CMA FIR equalizer with length K is used for the MIMO FIR channel. Let the parameters of the FIR equalizer be

$$g_m[n] = 0 \text{ for } n < 0 \text{ or } n > K, \tag{30}$$

for $m = 1, \cdots, M$. The relationship between the equalizer parameters $g_m[n]$ and the impulse response of the equalized system $s_i[n]$ can be expressed as

$$\mathbf{s}_{L+K-1} = \mathbf{g}_K \mathcal{H}_K, \tag{31}$$

where

$$\mathbf{s}_{L+K-1} \triangleq (s_1[0], \cdots, s_d[0], \cdots, s_1[L+K-2], \cdots,\ s_d[L+K-2]), \tag{32}$$

$$\mathbf{g}_K \triangleq (g_1[0], \cdots, g_M[0], \cdots, g_1[K-1], \cdots,\ g_M[K-1]), \tag{33}$$

and

$$\mathcal{H}_K \triangleq \begin{pmatrix} H[L-1] & H[L-2] & \dots & H[0] & & & \mathbf{0} \\ & H[L-1] & \ddots & \ddots & H[0] & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ \mathbf{0} & & & H[L-1] & \ddots & \ddots & H[0] \end{pmatrix}. \tag{34}$$

The singularity of $\mathcal{H}_K$ plays a crucial role in the convergence of the MIMO-CMA FIR equalizer. The relationship between the rank of the generalized Sylvester matrix $\mathcal{H}_K$ and the reducibility of $H(z)$ has been studied in multivariable control literature [2], [11]. Before stating the relationship, we first give the definition of the irreducibility of a matrix polynomial.

A $M \times d$ $(M > d)$ polynomial matrix $H(z)$ is said to be *irreducible* [2], [11] if there is no $d \times d$ polynomial matrix $R(z)$ with non-constant $\det(R(z))$, such that $H(z) = \tilde{H}(z)R(z)$, where $\tilde{H}(z)$ is an $M \times d$ polynomial matrix.

Using the results in [2], [11], we can prove the following lemma (see Appendix A for details).

Lemma 1: Let $H[L-1]$ *be of full column-rank, then* $\mathcal{H}_K$ *is of full column-rank for all* $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$, *if and only if* $H(z)$ *is irreducible.*

With the above lemma, we are able to prove the following convergence theorem for the CMA FIR equalizers used in MIMO FIR channels.

Theorem 3: For an MIMO FIR channel of length L, if $H(z)$ *is irreducible with* $H[L-1]$ *being of full rank, then any MIMO-CMA FIR blind equalizer with length* $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ *can achieve global convergence regardless of the its initial setting.*

Proof: Since $\mathcal{H}_K$ is of full column-rank for all $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ from Lemma 1, therefore, $\mathcal{H}_K^H \mathcal{H}_K$ is invertible, and for any $\mathbf{s}_{L+K-1} \in \mathcal{C}^{d(L+K-1)}$, there exists

$$\mathbf{g}_K = \mathbf{s}_{L+K-1} (\mathcal{H}_K^H \mathcal{H}_K)^{-1} \mathcal{H}_K^H, \tag{35}$$

such that $\mathbf{s}_{L+K-1} = \mathbf{g}_K \mathcal{H}_K$. Hence, $E_{i,n}$'s for $i = 1, \cdots, d$ and $n = 0, \cdots, \ L + K - 2$ are in attainable set S_a. From Theorem 2.2.a, the only minimum set of the MIMO-CMA FIR equalizer in $S_{i,n}$ is $E_{i,n}$. Since $S_a \bigcap S_{i,n}$ is empty for $i = 1, \cdots, d$ and $n < 0$ or $n > L + K - 2$, the CMA FIR equalizer has no other (local) minimum. Therefore, regardless of the initial setting of the equalizer, the equalizer will converge to one of the global minima of the equalizer. ■

The above theorem illustrates a very nice convergence property of the MIMO-CMA FIR equalizer used in MIMO channels. It indicates that the MIMO-CMA FIR equalizer can recover one of the input signals, remove ISI, and suppress CCI and ACI if the channel satisfies the condition stated in Lemma 1.

IV. A New Blind Equalization Algorithm for MIMO channels

We have studied the convergence of the CMA blind equalizer used in MIMO channels in the previous section and have shown that the output of the MIMO-CMA blind equalizer can recover one of the input signals and suppress the interference from the rest of the input signals. In this section, we will propose a new blind equalization algorithm, which can recover all input signals simultaneously. Without lost of generality, we will assume $d = 2$ in this section. The algorithm developed in this section can be easily extended to $d > 2$ case.

A. Algorithm Development

Consider the blind equalizer shown in Figure 2. If we adjust the equalizer parameters for each individual channel to minimize the Godard cost function in (17), then according to the analysis of Section III, the equalizer outputs $y_1[n]$ and $y_2[n]$ will be one of the input signals but we do not know which of the input signals. Note that $y_1[n]$ and $y_2[n]$ are either the same as or different from each other depending on the initial setting of the equalizer. Hence, to develop an algorithm that can simultaneously recover all input signals, we may have to modify the Godard cost function.

The new cost function for the adaptive blind equalization of MIMO channels is given as follows:

$$C_{MIMO} \triangleq C(y_1[n]) + C(y_2[n]) - c_o K(y_1, y_2), \tag{36}$$

where $c_o \geq m_4/(2m_2^2 - m_4)$, and $K(y_1, y_2)$ is a functional of $y_1(k)$ and $y_2(k)$ for all $k \leq n$ defined as

$$\begin{aligned} K(y_1, y_2) \quad \triangleq \quad & \frac{1}{2} \sum_{k=-\infty}^{-1} Cum(y_1[n], y_1^*[n], y_2[n+k], y_2^*[n+k]) \\ & + \frac{1}{2} \sum_{k=0}^{\infty} Cum(y_1[n-k], y_1^*[n-k], y_2[n], y_2^*[n]), \end{aligned} \tag{37}$$

with $Cum(y_1, y_1^*, y_2, y_2^*)$ being the cumulant of random complex variables y_1, y_1^*, y_2 and y_2^* defined as

$$Cum(y_1, y_1^*, y_2, y_2^*) \triangleq E\{|y_1|^2|y_2|^2\} - E\{|y_1|^2\}E\{|y_2|^2\} - |E\{y_1 y_2^*\}|^2, \tag{38}$$

for random variables y_i satisfying

$$E\{y_i\} = E\{y_i^2\} = 0 \text{ for } i = 1,\ 2. \tag{39}$$

It is the last term in (36) that makes it possible for the equalizer to converge to distinct input signals at each output of the equalizer.

Using the stochastic gradient method to search for minimum points of the new cost function, we can implement the new algorithm as

$$g_{jm}^{(n)}[k] = g_{jm}^{(n-1)}[k] - \mu((|y_j[n]|^2 - r)y_j[n] - c_o z_j[n])x_m^*[n-k], \tag{40}$$

for $j = 1,\ 2$ and $m = 1, \cdots, M$, where μ is a small step-size, $g_{jm}^{(n)}[k]$ is the k-th parameter of the jm-th filter after the n-th iteration, and $z_i[n]$'s are given by

$$z_1[n] = \sum_{l=0}^{\infty}(|y_2[n-l]|^2 y_1[n] - E\{|y_2[n-l]|^2\}y_1[n] - E\{y_1[n]y_2^*[n-l]\}y_2[n-l]), \tag{41}$$

$$z_2[n] = \sum_{l=0}^{\infty}(|y_1[n-l]|^2 y_2[n] - E\{|y_1[n-l]|^2\}y_2[n] - E\{y_2[n]y_1^*[n-l]\}y_1[n-l]). \tag{42}$$

If the ensemble average in the above expressions is substituted by empirical average as in [21], the resulting algorithm can be expressed as

$$g_{jm}^{(n)}[k] = g_{jm}^{(n-1)}[k] - \mu((|y_j[n]|^2 - r)y_j[n] - c_o z_j[n])x_m^*[n-k], \tag{43}$$

$$z_1[n] = \sum_{l=0}^{\infty}(|y_2[n-l]|^2 y_1[n] - < |y_2[n-l]|^2 >_n y_1[n] - < y_1[n]y_2^*[n-l] >_n y_2[n-l]), \tag{44}$$

$$z_2[n] = \sum_{l=0}^{\infty}(|y_1[n-l]|^2 y_2[n] - < |y_1[n-l]|^2 >_n y_2[n] - < y_2[n]y_1^*[n-l] >_n y_1[n-l]), \tag{45}$$

where

$$< |y_2[n-l]|^2 >_n = (1-\epsilon) < |y_2[n-1-l]|^2 >_{n-1} + \epsilon |y_2[n-1]|^2, \tag{46}$$

$$< |y_1[n-l]|^2 >_n = (1-\epsilon) < |y_1[n-1-l]|^2 >_{n-1} + \epsilon |y_1[n-1]|^2, \tag{47}$$

$$< y_1[n]y_2^*[n-l] >_n = (1-\epsilon) < y_1[n-1]y_2^*[n-1-l] >_{n-1} + \epsilon y_1[n]y_2^*[n-l], \tag{48}$$

and

$$< y_2[n]y_1^*[n-l] >_n = (1-\epsilon) < y_2[n-1]y_1^*[n-1-l] >_{n-1} + \epsilon y_2[n]y_1^*[n-l]. \tag{49}$$

with ϵ being the forgetting factor.

Remark: We have developed MIMO adaptive blind equalization algorithm for $d = 2$. For $d > 2$ case, the cost function C_{MIMO} can be extended to

$$C_{MIMO} = \sum_{i=1}^{d} C(y_i[n]) - c_o \sum_{i,j=1\ i \neq j}^{d} K(y_i, y_j). \tag{50}$$

The global convergence and local convergence properties discussed below can be similarly generalized to this case.

B. Global Convergence

We will prove the global convergence of the new algorithm here. Let

$$s_{ij}[n] \triangleq \sum_{m=1}^{M} g_{jm}[n] * h_{mi}[n] \tag{51}$$

then the channel output can be written as

$$y_j[n] = \sum_{i=1,2} \sum_{l=-\infty}^{\infty} a_i[l] s_{ij}[n-l], \tag{52}$$

for $j = 1, 2$. According to the definition in (37), we have

$$\begin{aligned} & Cum(y_1[n], y_1^*[n], y_2[n+k], y_2^*[n+k]) \qquad (53)\\ = \; & Cum(\sum_{i=1}^{2}\sum_{l} a_i[l]s_{i1}[n-l], \sum_{i=1}^{2}\sum_{l} a_i^*[l]s_{i1}^*[n-l], \\ & \qquad \sum_{i=1}^{2}\sum_{l} a_i[l]s_{i2}[n+k-l], \sum_{i=1}^{2}\sum_{l} a_i^*[l]s_{i2}^*[n+k-l]) \\ = \; & \sum_{i_1,i_2,i_3,i_4=1}^{2} \sum_{l_1,l_2,l_3,l_4} s_{i_1 1}[n-l_1]s_{i_2 1}^*[n-l_2]s_{i_3 2}[n+k-l_3]s_{i_4 2}^*[n+k-l_4] \\ & \qquad Cum(a_{i_1}[l_1], a_{i_2}^*[l_2], a_{i_3}[l_3], a_{i_4}^*[l_4]). \end{aligned}$$

Since we have assumed $a_i[n]$'s are independent for different i or n, then

$$\begin{aligned} & Cum(a_{i_1}[l_1], a_{i_2}^*[l_2], a_{i_3}[l_3], a_{i_4}^*[l_4]) \qquad (54)\\ = \; & \begin{cases} m_4 - 2m_2^2 & \text{for } i_1 = i_2 = i_3 = i_4 \text{ and } l_1 = l_2 = l_3 = l_4, \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

Therefore,

$$Cum(y_1[n], y_1^*[n], y_2[n+k], y_2^*[n+k]) = (m_4 - 2m_2^2) \sum_{i=1,2} \sum_{n=-\infty}^{\infty} |s_{i1}[n]|^2 |s_{i2}[n+k]|^2. \tag{55}$$

From (55), (37) can be expressed as

$$K(y_1, y_2) = \frac{1}{2}(m_4 - 2m_2^2) \sum_{i=1,2} \prod_{j=1,2} (\sum_{k} |s_{ij}[k]|^2). \tag{56}$$

Substituting (56) and (21) into (36), we obtain

$$
\begin{aligned}
C_{MIMO} &= \frac{1}{4}\sum_{j=1,\,2}[-(2m_2^2-m_4)\sum_{i,n}|s_{ij}[n]|^4+2m_2^2(\sum_{i,n}|s_{ij}[n]|^2)^2-2m_4\sum_{i,n}|s_{ij}[n]|^2] \\
&+\frac{1}{2}c_o(2m_2^2-m_4)\sum_{i=1,2}\prod_{j=1,2}(\sum_n|s_{ij}[n]|^2)+\frac{1}{2}m_4^2/m_2^2. \qquad (57)
\end{aligned}
$$

Using (57), we are able to find the minima of C_{MIMO} according to the following lemma which is proved in Appendix B.

Lemma 2: Let

$$
\begin{aligned}
f(t_{11},t_{12},t_{21},t_{22}) &= \frac{1}{4}[-(2m_2^2-m_4)(t_{11}^2+t_{12}^2+t_{21}^2+t_{22}^2) \\
&+2m_2^2[(t_{11}+t_{21})^2+(t_{12}+t_{22})^2] \\
&2c_o(2m_2^2-m_4)(t_{11}t_{12}+t_{21}t_{22}) \\
&-2m_4(t_{11}+t_{12}+t_{21}+t_{22})+2m_4/m_2^2]. \qquad (58)
\end{aligned}
$$

For any $c_0 \geq m_4/(2m_2^2-m_4)$, *the only minima of* $f(t_{11},t_{12},t_{21},t_{22})$ *on* $[0,+\infty)^4$ *are*

$$(t_{11},t_{12},t_{21},t_{22})=(1,0,0,1) \text{ and } (t_{11},t_{12},t_{21},t_{22})=(0,1,1,0). \qquad (59)$$

The global convergence of the new blind equalization algorithm is indicated by the following theorem.

Theorem 4: The MIMO blind equalizer using the cost function defined in (36) will converge to one of its global minimum regarless of its initial setting, if the equalizer length and the channel parameters satisfy one of the following two conditions:

1. *The MIMO channel satisfies distortionless reception condition and an infinite-length MIMO equalizer is used, or*
2. $H(z)$, *the Z-transform of the impulse response of the MIMO channel of length* L, *is an irreducible polynomial matrix with* $H[L-1]$ *being nonsingular, and the length of the equalizer* $K \geq \lceil\frac{(L-1)d}{M-d}\rceil$.

Proof: 1. Assuming that the MIMO channel satisfies distortionless reception condition and a double infinite-length MIMO equalizer is used.

Define a 2×2 impulse response matrix sequence $S[n]$ of the equalized system as

$$S[n] = \begin{pmatrix} s_{11}[n] & s_{21}[n] \\ s_{12}[n] & s_{22}[n] \end{pmatrix}. \tag{60}$$

According to the assumption, for any 2×2 matrix sequence $\{S[n]\} \in \ell^1(\mathcal{C}^{2\times 2})$, there is an MIMO equalizer sequence $\{G[n]\} \in \ell^1(\mathcal{C}^{2\times M})$, such that

$$S[n] = G[n] * H[n]. \tag{61}$$

Hence, the blind equalizer using the new algorithm will converge to some minimum of the functional C_{MIMO} on $\ell^1(\mathcal{C}^{2\times 2})$.

If $s_{12}[n]$, $s_{21}[n]$, and $s_{22}[n]$ for all n are fixed, then the necessary condition for C_{MIMO} to attain its minima is

$$\frac{\partial C_{MIMO}}{\partial \bar{s}_{11}[n]} = 0, \tag{62}$$

for $i,\ j = 1,\ 2$, where $\bar{s}$ denotes the complex-conjugate of s. Since

$$\frac{\partial C_{MIMO}}{\partial \bar{s}_{11}[n]} = s_{11}[n][-(2m_2^2 - m_4)|s_{11}[n]|^2 + 2m_2^2 t_{11} + c_o(2m_2^2 - m_4)t_{12} - m_4], \tag{63}$$

the possible stationary points of C_{MIMO} are

$$|s_{11}[n]|^2 = \begin{cases} \frac{c_o(2m_2^2 - m_4)t_{12} - m_4}{2m_2^2 M - (2m_2^2 - m_4)} & \text{if } n \in I, \\ 0 & \text{otherwise,} \end{cases} \tag{64}$$

where I is some set containing M integers and

$$t_{11} = \sum_k |s_{11}[k]|^2, \text{ and } t_{12} = \sum_k |s_{12}[k]|^2. \tag{65}$$

Indeed, following Foschini's [6] arguments, $s_{11}[n]$ in (64) is not the minimum if $M \geq 2$. Hence, the possible minima of the new cost function C_{MIMO} satisfy

$$|s_{11}[n]|^2 = t_{11}\delta[n - n_{11}] \tag{66}$$

where t_{11} is some non-negative real number, and n_{11} is an integer. Using the similar arguments as given in the above, we can obtain that the necessary condition for C_{MIMO} to attain its minima is

$$|s_{ij}[n]|^2 = t_{ij}\delta[n - n_{ij}] \tag{67}$$

for $i = 1,\ 2$, where t_{ij} for $i,\ j = 1,\ 2$ are some non-negative real number, and n_{ij} for $i,\ j = 1,\ 2$ are some integers.

Substituting (67) into (57) and applying Lemma 2, we obtain the only possible minima of the new cost function C_{MIMO}. They are

$$|s_{11}[n-n_{11}]|^2 = |s_{22}[n-n_{22}]|^2 = \delta[n],\ |s_{12}[n]|^2 = |s_{21}[n]|^2 = 0$$
$$|s_{11}[n]|^2 = |s_{22}[n]|^2 = 0,\ |s_{12}[n-n_{12}]|^2 = |s_{21}[n-n_{21}]|^2 = \delta[n] \quad (68)$$

for some integers n_{11}, n_{12}, n_{21}, and n_{22}.

When $s_{ij}[n]$ satisfies one of the conditions in (68), $C(y_1[n])$ and $C(y_2[n])$ attain their global minima simultaneously. At the same time, $-c_o K(y_1, y_2) = 0$, this implies that it attains minimum. Therefore, $s_{ij}[n]$'s in (68) are the only global minima of the new cost function C_{MIMO}. Hence, the MIMO equalizer will converge to one of the global minima regardless of its initial setting.

2. Assuming that $H(z)$, the Z-transform of the impulse response of the MIMO channel with length L, is an irreducible polynomial matrix with $H[L-1]$ being nonsingular, and the length of the equalizer $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$.

Define the MIMO equalizer matrix as

$$\mathcal{G}_K \triangleq (G[0],\ G[1], \cdots,\ G[K-1]) \quad (69)$$

and the MIMO equalized system matrix as

$$\mathcal{S}_{L+K-1} \triangleq (S[0],\ S[1], \cdots,\ S[L+K-2]). \quad (70)$$

Then

$$\mathcal{S}_{L+K-1} = \mathcal{G}_K \mathcal{H}_K. \quad (71)$$

According to Lemma 1, $\mathcal{H}_K$ for $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ is of full column rank. Hence, for any $\mathcal{S}_{L+K-1} \in \mathcal{C}^{2\times 2(L+K-1)}$, the MIMO equalizer matrix $\mathcal{G}_K = \mathcal{S}_{L+K-1}(\mathcal{H}_K^H \mathcal{H})^{-1}\mathcal{H}_K^H$ satisfies (71). Using similar arguments to the proof of the first part, we obaitn that the minima of the cost function C_{MIMO} on $\mathcal{C}^{2\times 2(L+K-1)}$ are

$$|s_{11}[n-n_{11}]|^2 = |s_{22}[n-n_{22}]|^2 = \delta[n],\ |s_{12}[n]|^2 = |s_{21}[n]|^2 = 0 \quad (72)$$
$$|s_{11}[n]|^2 = |s_{22}[n]|^2 = 0,\ |s_{12}[n-n_{12}]|^2 = |s_{21}[n-n_{21}]|^2 = \delta[n]$$

for $0 \leq n_{ij} \leq L+K-1$ and they are all global minima. Hence, the MIMO equalizer will converge to one of the global minima regardless of its initial condition.

■

From Theorem 4, the new algorithm is able to recover all input signals simultaneously. Furthermore, the MIMO FIR channel satisfying certain condition can be perfectly equalized by MIMO FIR equalizers employing the new algorithm.

C. Local Convergence

Similar to the blind equalizers for SISO channels, the new MIMO blind equalizer algorithm also suffers from the local convergence if the MIMO equalizers or channels under consideration does not satisfy the conditions in Theorem 4. As an example to illustrate such a problem, let us consider an AR MIMO channel with $d = M = 2$ and the transfer functions

$$h_{11}(z) = h_{22}(z) = \frac{1}{1 - z^{-1}\alpha}, \quad 0 < |\alpha| < \frac{1}{\sqrt{3}}, \text{ and } h_{12}(z) = h_{21}(z) = 0. \tag{73}$$

It is easy to check that $H(z)$ satisfies the distortionless reception condition using Theorem 1. If an MIMO FIR equalizer with length $K = 2$ is used to equalize this MIMO channel, then

$$G(z) = \begin{pmatrix} 1 - \alpha z^{-1} & 0 \\ 0 & 1 - \alpha z^{-1} \end{pmatrix} \tag{74}$$

can perfectly equalize this AR MIMO channel. However, as discussed in [14], there is a $\gamma \neq 0$ such that

$$g_{11}[n] = g_{22}[n] = \gamma\delta[n-1], \; g_{12}[n] = g_{21}[n] = 0 \tag{75}$$

is a local minimum of both $C(y_1[n])$ and $C(y_2[n])$. Since $-c_o K(y_1, y_2) = 0$ in this case, it is a global minimum of $-c_o K(y_1, y_2)$. Therefore, the $g_{ij}[n]$ in (75) is a local minimum of C_{MIMO}.

V. Computer Simulations

In order to confirm the analysis results and illustrate the effectiveness of the proposed algorithm, we present two computer simulation examples.

In our simulations, the input signals $a_i[n]$ are independent of each other for any different i nad n, and they are uniformly distributed over $\{\pm 1, \pm \jmath\}$. The channel noise is complex white Gaussian with zero mean and variance determined by the signal-to-noise ratio (SNR).

A. Convergence of the MIMO-CMA blind equalizer

In this simulation example, we have $d = 2$ and $M = 3$. The channel impulse response is given by

$$H[0] = \begin{pmatrix} -1.9522 & -0.5706 \\ -0.5666 & 0.4246 \\ -1.1293 & 0.7666 \end{pmatrix} \text{ and } H[1] = \begin{pmatrix} 1.0691 & -1.8841 \\ -0.7926 & 0.0598 \\ 0.3569 & -0.2744 \end{pmatrix} \tag{76}$$

and $SNR = 30dB$.

An MIMO-CMA FIR equalizer is used for the MIMO FIR channel. The length of the equalizer is $K = 3$ with initial setting $g_m[n] = \delta[m-1, n-1]$ and the step-size $\mu = 0.0005$. Figure 4 is the impulse responses of the equalized system after 10,000 iterations. From this figure, the MIMO-CMA FIR equalizer is able to recover the second input signal, remove the ISI and suppress the first signal. Figure 5 illustrates the changing of IT during iterations. The IT here is an index of measurement for intersymbol interference and co-channel interference defined as

$$IT = \frac{\sum_{i,n} |s_i[n]|^2 - \max_{i,n} |s_i[n]|^2}{\max_{i,n} |s_i[n]|^2}. \tag{77}$$

The simulation results in these two figures confirm Theorem 3.

B. Convergence of the new MIMO blind equalizer

In this simulation example, we choose $d = 2$ and $M = 4$. The channel impulse responses are shown in Figure 6 and $SNR = 30dB$.

The length of the equalizer used in our simulation is 20 with initial setting $g_{11}[n] = g_{22}[n] = \delta[n-10]$. The step-size is $\mu = 0.0001$ and the forgetting factor is given by $\epsilon = 0.01$.

Figure 7 is the impulse response $s_{ij}[n]$ of the equalized system after 20,000 iterations. In Figure 8, 1,000 channel outputs and 1,000 equalizer outputs are shown. Figure 9 illustrates the intersymbol and co-channel interference of the equalizer outputs, which is defined as

$$IT_j = \frac{\sum_{i,n} |s_{ij}[n]|^2 - \max_{i,n} |s_{ij}[n]|^2}{\max_{i,n} |s_{ij}[n]|^2}. \tag{78}$$

According to Figure 7, the two input signals are separated. The first equalizer recovers the first signal and the second recovers the second signal. From the simulation results, our new blind equalization algorithm can simultaneously reconstruct the input signals and remove intersymbol and co-channel interference effectively.

VI. Conclusions

This paper investigated the blind equalization of MIMO channels for multiple signals separation. We studied the convergence of the CMA blind equalizer used in MIMO channels. We demonstrated that the CMA blind equalizer is able to recover one of input signals and suppress the rest of input signals. Hence, by proper initialization, the CMA blind equalizer can be used in mobile communication systems to recover the desire signal, remove intersymbol interference, and suppress co-channel interference and adjacent channel interference. To recover all input signals simultaneously, we proposed a new blind equalization algorithm to separate all the input signals and at the same time equalize the MIMO channel. The global convergence of the new algorithm is illustrated theoretically and by computer simulation. The proposed algorithm not only can be applied in multiple signals separation in array processing, but also can be used in diverse fields of engineering including speech processing, data communication, sonar array processing, and in the analysis of biological systems.

Appendix A: Proof of Lemma 1

Since $H[L-1]$ is of full-rank, it has a full-rank $d \times d$ minor. Without lose of generality, we assume that $C[L-1] = (h_{ij}[L-1])_{i,j=1}^{d}$ is a full-rank mimor. Then there is a matrix E, of which all elements are complex-value constants, with $\det(E) = 1$, such that $C[L-1]E$ is upper triangular with non-zero diagonal elements. Let $C[n] = (h_{ij}[n])_{i,j=1}^{d}$ for $n = 1, \cdots, L-2$, and

$$C(z) = \sum_{n=0}^{L-1} C[n]z^{-n}, \qquad (A.1)$$

then

$$\begin{aligned} \det(C(z)) &= \det(C(z)E)\det(E^{-1}) \\ &= \det(C(z)E). \end{aligned} \qquad (A.2)$$

Let

$$C(z)E = \begin{pmatrix} p_{11}(z) & \cdots & p_{1d}(z) \\ \cdots & \cdots & \cdots \\ p_{d1}(z) & \cdots & p_{dd}(z) \end{pmatrix}, \qquad (A.3)$$

then

$$\partial(p_{ij}(z)) \begin{cases} \leq L-1 & \text{if } i > j, \\ = L-1 & \text{if } i = j, \\ \leq L-2 & \text{if } i < j, \end{cases} \tag{A.4}$$

where $\partial(p_{ij}(z))$ denotes the degree of polynomial $p_{ij}(z)$. Hence, the degree of $\det(C(z))$ is

$$\partial(\det(C(z))) = (L-1)d. \tag{A.5}$$

Then from Corollary 1 of [2], $H(z)$ is irreducible if and only if for $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$,

$$\begin{aligned} \text{rank}(\mathcal{H}_K) &= Kd + \partial(det(C(z))) \\ &= (K+L-1)d, \end{aligned} \tag{A.6}$$

which means that $\mathcal{H}_K$ for $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ is of full-rank.

Appendix B: Proof of Lemma 2

Direct calculation yields

$$\nabla^2 f = \frac{1}{2} \begin{pmatrix} m_4 & 2m_2^2 & c_o(2m_2^2 - m_4) & 0 \\ 2m_2^2 & m_4 & 0 & c_o(2m_2^2 - m_4) \\ c_o(2m_2^2 - m_4) & 0 & m_4 & 2m_2^2 \\ 0 & c_o(2m_2^2 - m_4) & 2m_2^2 & m_4 \end{pmatrix}. \tag{B.1}$$

Since $\nabla^2 f(t_{11}, t_{12}, t_{21}, t_{22})$ is not positive-definite, $f(t_{11}, t_{12}, t_{21}, t_{22})$ has no minimum inside $[0, +\infty)^4$. Therefore, its possible minima must be on the boundary of $[0, +\infty)^4$.

Without lose of generality, assume $t_{22} = 0$ and let $f_1(t_{11}, t_{12}, t_{21}) = f(t_{11}, t_{12}, t_{21}, 0)$. The minimum of $f_1(t_{11}, t_{12}, t_{21})$ is $(t_{11}, t_{12}, t_{21}) = (0, 1, 1)$ when $c_o > m_4/(2m_2^2 - m_4)$. Therefore, $(t_{11}, t_{12}, t_{21}, t_{22}) = (0, 1, 1, 0)$ may be a minimum of $f(t_{11}, t_{12}, t_{21}, t_{22})$ on $[0, +\infty)^4$. Indeed it is a minimum of $f(t_{11}, t_{12}, t_{21}, t_{22})$ since for any $\epsilon_{11}, \epsilon_{12}, \epsilon_{21}, \epsilon_{22}$ with $\epsilon_{11}, \epsilon_{12} \geq 0$ and $\epsilon_{11}^2 + \epsilon_{12}^2 + \epsilon_{21}^2 + \epsilon_{22}^2$ being small enough,

$$f(\epsilon_{11}, 1 + \epsilon_{12}, 1 + \epsilon_{21}, \epsilon_{22}) > f(0, 1, 1, 0). \tag{B.2}$$

Because of the symmetricity of $f(t_{11}, t_{12}, t_{21}, t_{22})$, $(t_{11}, t_{12}, t_{21}, t_{22}) = (1, 0, 0, 1)$ is another minimum of $f(t_{11}, t_{12}, t_{21}, t_{22})$.

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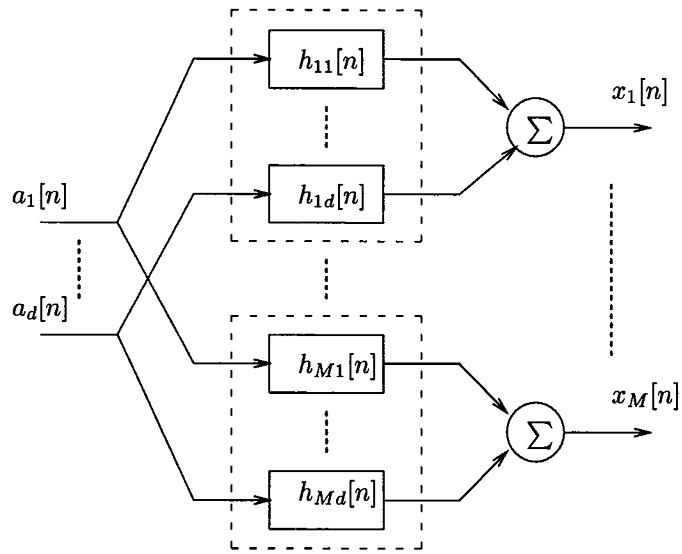

Fig. 1. Multiple-input and multiple-output channel

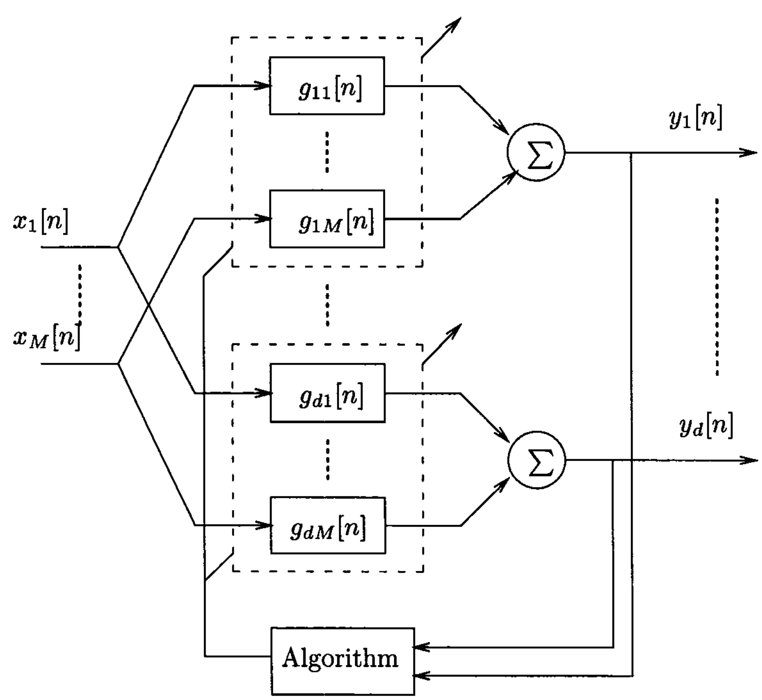

Fig. 2. Adaptive blind multiple-channel equalizer.

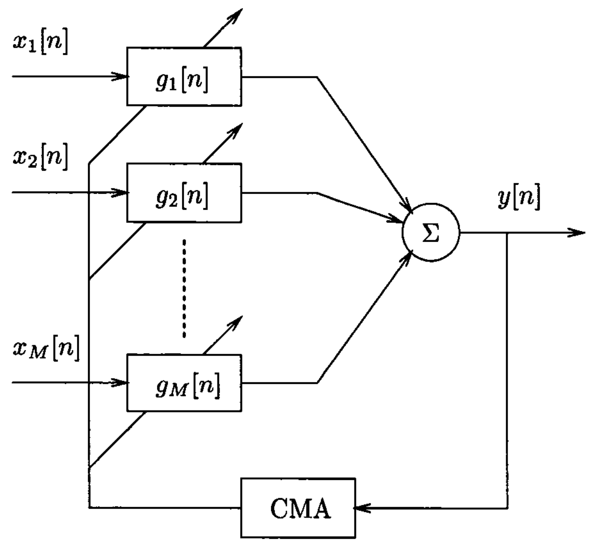

Fig. 3. The MIMO-CMA blind equalizer.

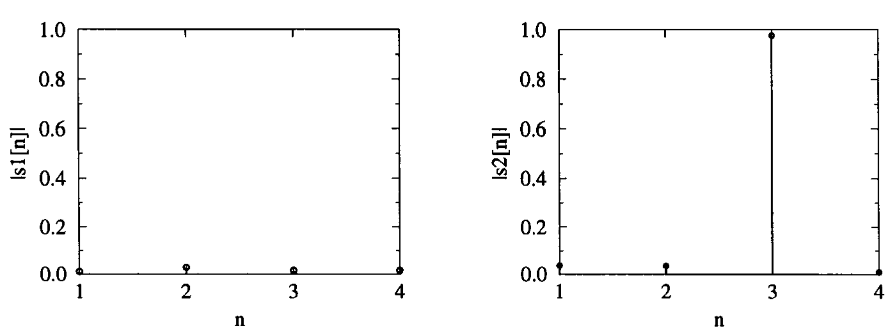

Fig. 4. The impulse response of the equalized system after 10,000 iterations.

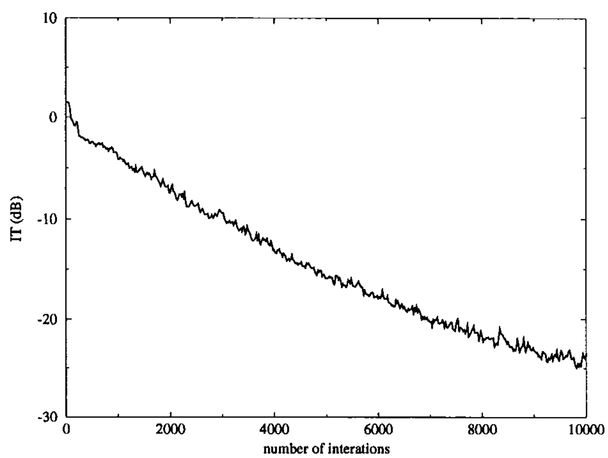

Fig. 5. Convergence of the MIMO-CMA blind equalizer

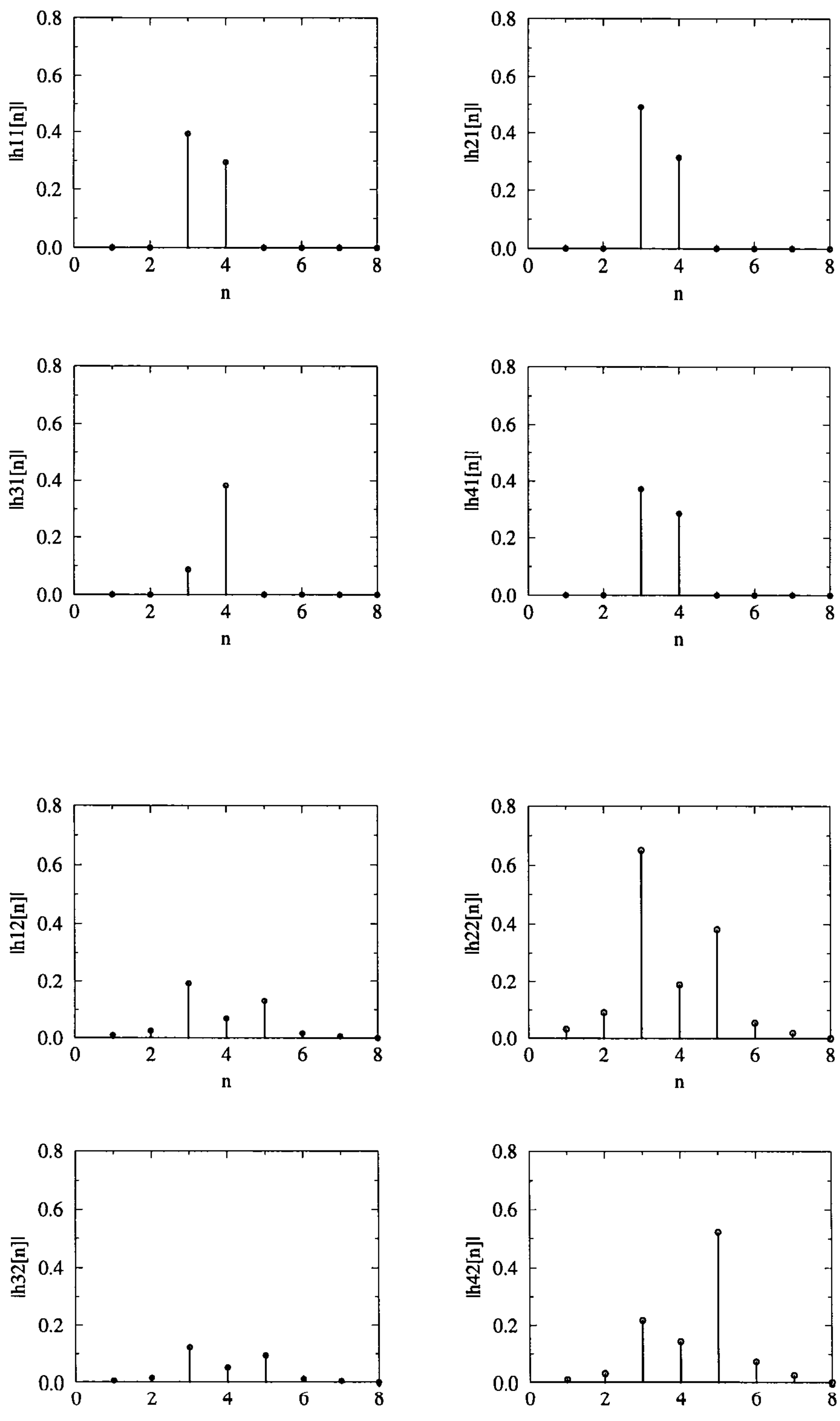

Fig. 6. The impulse response of the MIMO FIR channel

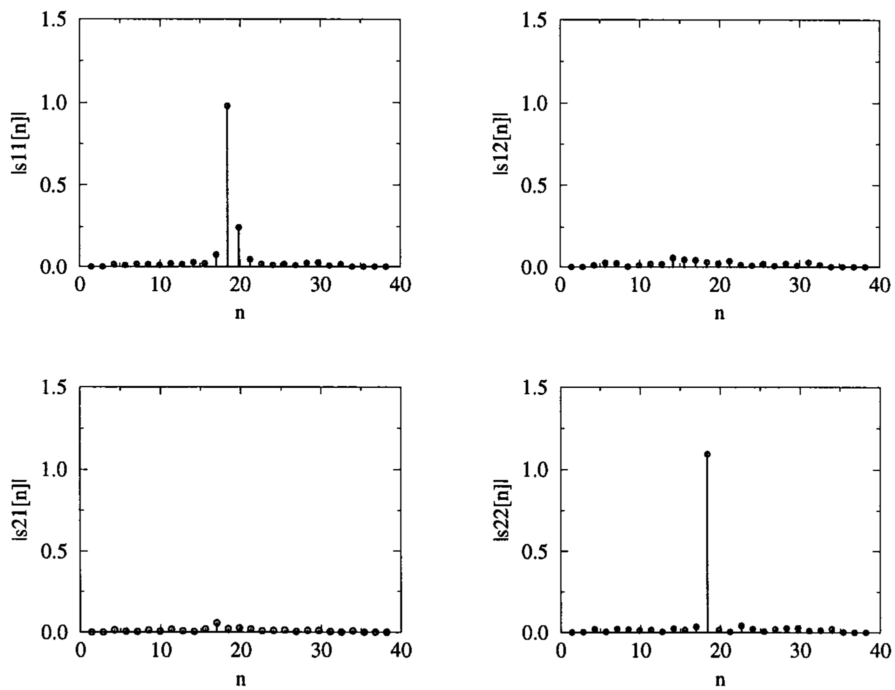

Fig. 7. The impulse response of the equalized system after 20,000 iterations

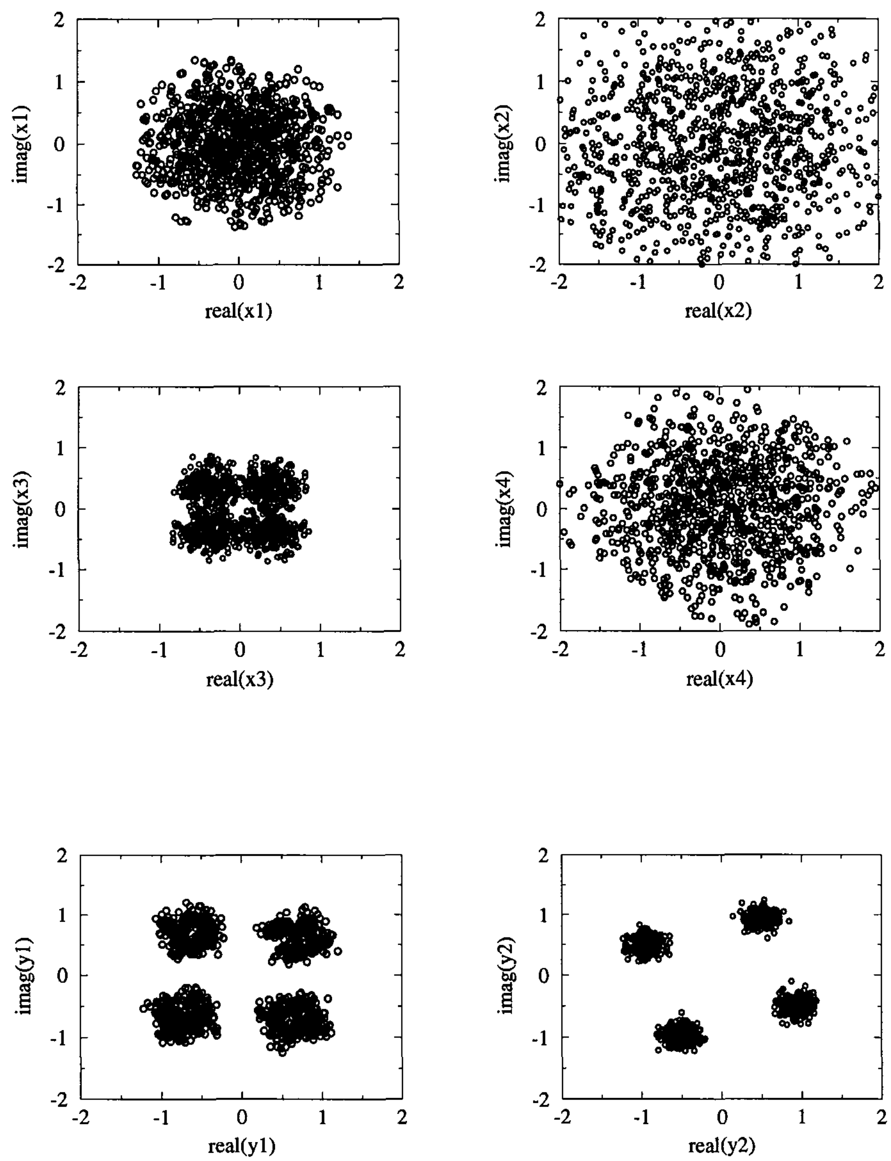

Fig. 8. 1000 inputs $x_1[n]$, $x_2[n]$, $x_3[n]$ and $x_4[n]$ and outputs $y_1[n]$, $y_2[n]$ of the new MIMO blind equalizer after 20,000 iterations.

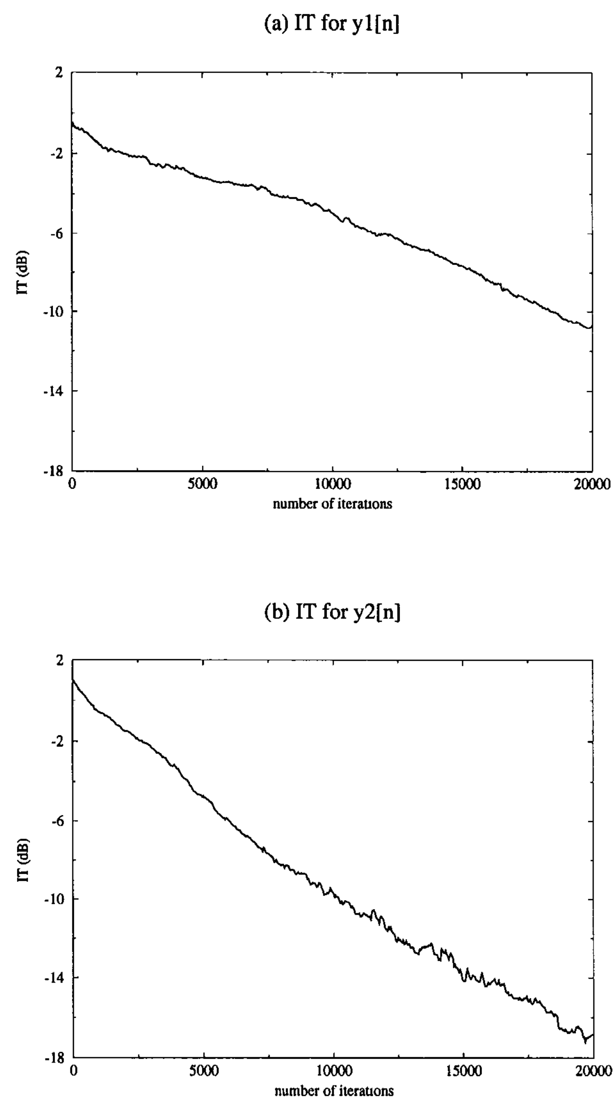

Fig. 9. Convergence of the new algorithm for the MIMO channel