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Control of Nonlinear Phenomena at the Inception of Voltage Collapse

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Abstract

Nonlinear phenomena, including bifurcations and chaos, occurring in power system models exhibiting voltage collapse have been the subject of several recent studies. These nonlinear phenomena have been determined to be crucial factors in the inception of voltage collapse in these models. In this paper, the problem of controlling voltage collapse in the presence of these nonlinear phenomena is addressed. The work focuses on an example power system model that has been studied in several recent papers. The bifurcation control approach is employed to modify the bifurcations and to suppress chaos. The control law is shown to result in improved performance of the system for a greater range of parameter values.

Keywords: Voltage stability; power systems; bifurcation; chaos; control systems.

I. Introduction

Voltage collapse in electric power systems has recently received significant attention in the literature (see, e.g., [1] for a synopsis). This research has been motivated by increases in power demand which result in operation of electric power systems near their stability limits. A number of physical mechanisms have been identified as possibly leading to voltage collapse. From a mathematical perspective, voltage collapse has been viewed as arising from a *bifurcation* of the power system governing equations as a parameter is varied through some critical value. In a number of papers (e.g., [2], [3], [4], [15], [16]), voltage collapse is viewed as an instability which coincides with the disappearance of the steady state operating point as a system parameter, such as a reactive power demand, is quasistatically varied. In the language of bifurcation theory, these papers link voltage collapse to a *fold* or *saddle node bifurcation* of the nominal equilibrium point.

Dobson and Chiang [3] postulated a mechanism for voltage collapse tied to the saddle node bifurcation, stressing the role of the center manifold of the system model at the bifurcation. In the same paper, they introduced a simple example power system containing a generator, an infinite bus, and a nonlinear load. The saddle node bifurcation mechanism for voltage collapse [3] was investigated for this example in subsequent papers, including [4] and [9]. On the other hand, instead of attributing voltage collapse to a single bifurcation mechanism, [10] refined the term of voltage collapse to distinguish transitions resulting from finite-sized disturbances in state space from bifurcations in parameter space. Thus in the terminology of [10], the saddle node bifurcation mechanism of voltage collapse is classified as a *parametric voltage collapse.*

An essential distinction exists between the mathematical formulation of voltage collapse problems and transient stability problems. In studying transient stability, one often is interested in whether or not a given power system can maintain synchronism (stability) after being subjected to a physical disturbance of moderate or large proportions. The faulted power system in such a case has been perturbed in a severe way from steady-state, and one studies whether the post-fault system returns to the initial steady-state (regains synchronism). In the parametric voltage collapse scenario, however, the disturbance may be viewed as a slow change in a system parameter, such as a power demand. Thus, the disturbance does not necessarily perturb the system away from steady-state. The steady-state varies continuously with the changing system parameter, until it disappears at a saddle node bifurcation point. It is therefore not surprising that the saddle node bifurcation has been studied as a possible route to voltage collapse.

However, the presence of a saddle node bifurcation in a dynamical system does not preclude the presence of other, possibly more complex, bifurcations. In fact, the recent papers [5], [6], [8], [9] have shown that other bifurcations do occur in the example power system model studied in [3]. The bifurcations found in this model include Hopf bifurcations from the nominal equilibrium, a cyclic fold bifurcation, period doubling bifurcations, as well as a period doubling cascade leading to chaotic behavior. (See, e.g., [19] and references therein for a general discussion of these phenomena.) Other papers have also studied bifurcations in voltage dynamics in other power system models [25], [26], [10].

The fact has therefore now been established that a variety of bifurcations, static and dynamic, occur in power system models exhibiting voltage collapse. The purpose of this paper, which continues the work reported in [6], [7], is to determine the implications of these bifurcations for the voltage collapse phenomenon and to address the issue of voltage collapse control. In our paper [6], a link was suggested between the voltage collapse phenomenon and the occurrence of dynamic bifurcations: specifically, we showed the possible role of an oscillatory transient in voltage collapse. In the present paper, we continue the study of dynamic bifurcations and voltage collapse, showing the possible role of catastrophic bifurcations, including crises of strange attractors [20], [21] in voltage collapse.

Upon revealing the various bifurcations (static and dynamical, local and global) and the associated rich dynamics, one would naturally ask the question: what can we do about voltage collapse? More specifically, what is the possible role of feedback control in such situation? In the present paper, we report some positive results in this direction. In previous work [11], [12], [14] problems of *bifurcation control* have been addressed. This involves design

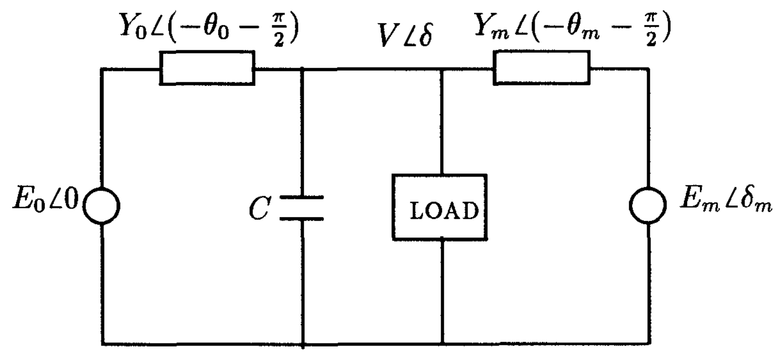

Figure 1: Power system model

of feedback controls to modify the stability and amplitude of bifurcated solutions in general nonlinear control systems. The latter work has been applied to control problems in high angle of attack flight, stall of jet engines, and oscillatory behavior of tethered satellites. The latter part of the present paper shows the utility of the bifurcation control approach in the control of voltage collapse. The resulting controllers can be linear, nonlinear, or composite linear-plus-nonlinear depending on the desired objective. It is demonstrated that voltage collapse can be postponed through modifying the bifurcations and suppressing the chaos and crises.

The remainder of the paper proceeds as follows. In the next section, we present the power system model which is used in the ensuing analysis. In Section 3, bifurcations occurring in this model, including the emergence of two strange attractors and their disappearance in boundary crises (or chaotic blue sky catastrophes), are reviewed and studied. The implications of these bifurcations for the voltage collapse phenomenon are discussed. In Section 4, the issue of voltage collapse control is investigated using the bifurcation control approach. Conclusions are collected in Section 5.

II. The Model

The power system under consideration in this paper, previously considered by Dobson and Chiang [3], is depicted in Figure 1. The three nodes of the equivalent circuit in Fig. 1 are an infinite busbar, a generator node represented by a constant voltage behind a reactance, and a load busbar. It follows from [3] that the system dynamics is governed by the following four differential equations ($P(\delta_m, \delta, V)$, $Q(\delta_m, \delta, V)$ are specified below):

$$\dot{\delta}_m = \omega \tag{1}$$

$$M\dot{\omega} = -d_m\omega + P_m + E_m V Y_m \sin(\delta - \delta_m - \theta_m) + E_m^2 Y_m \sin\theta_m \tag{2}$$

$$K_{qw}\dot{\delta} = -K_{qv2}V^2 - K_{qv}V + Q(\delta_m, \delta, V) - Q_0 - Q_1 \tag{3}$$

$$TK_{qw}K_{pv}\dot{V} = K_{pw}K_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V + K_{qw}(P(\delta_m, \delta, V) - P_0 - P_1) - K_{pw}(Q(\delta_m, \delta, V) - Q_0 - Q_1) \tag{4}$$

All symbols and parameters are the same as in [3]. The load includes a constant PQ load in parallel with an induction motor. The real and reactive powers supplied to the load by the network are

$$P(\delta_m, \delta, V) = -E_0{}'VY_0{}'\sin(\delta + \theta_0') - E_m V Y_m \sin(\delta - \delta_m + \theta_m) + (Y_0' \sin\theta_0' + Y_m \sin\theta_m)V^2 \tag{5}$$

$$Q(\delta_m, \delta, V) = E_0{}'VY_0{}'\cos(\delta + \theta_0{}') + E_m V Y_m \cos(\delta - \delta_m + \theta_m) - (Y_0' \cos\theta_0' + Y_m \cos\theta_m)V^2 \tag{6}$$

III. System Dynamics and Voltage Collapse

III.1 Bifurcations Analysis

In this subsection, the result of a bifurcation analysis of the model (1)-(6) is presented (See also [6], [7], [9]). The continuation/bifurcation software package AUTO [24] is employed to assist this analysis. A representative bifurcation diagram for the system (1)-(6) appears in Fig. 2. This diagram relates the voltage magnitude V to the bifurcation parameter Q_1 (the reactive power demand).

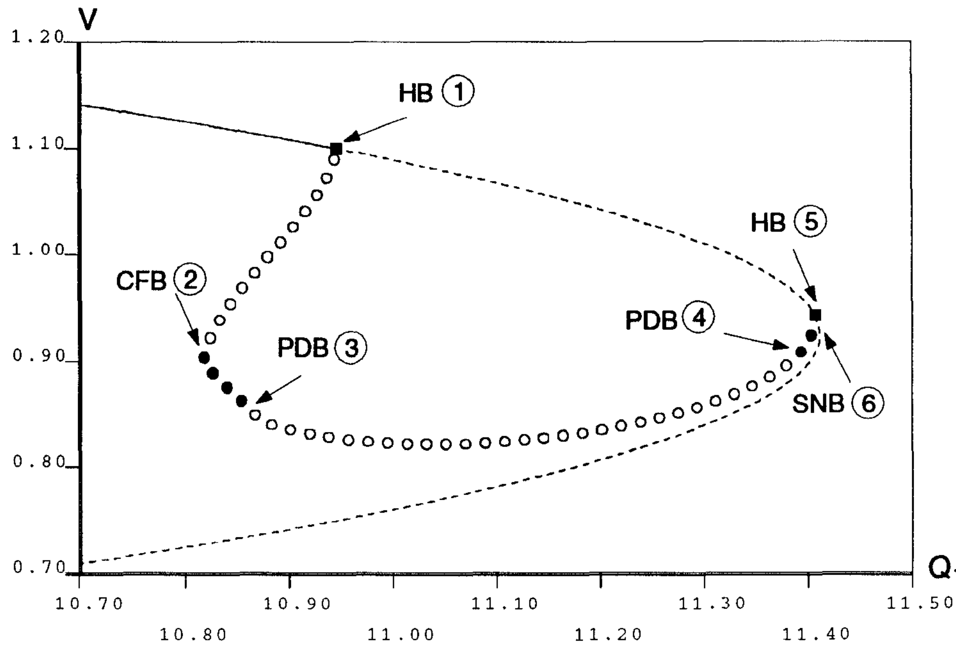

Figure 2: Bifurcation diagram of open loop system

To simplify the discussion, note first that in Fig. 2 six bifurcations are depicted. These are labeled HB①, CFB②, PDB③, PDB④, HB⑤, and SNB⑥. For simplicity, we may also refer to these bifurcations by their numbers ①-⑥, respectively. The acronyms have the following meanings:

- HB: Hopf bifurcation
- SNB: Saddle node bifurcation
- CFB: Cyclic fold bifurcation
- PDB: Period doubling bifurcation

For ease of reference, we denote the values of the parameter Q_1 at which the bifurcations ①-⑥ occur by $Q_1^{①}$-$Q_1^{⑥}$, respectively. In Fig. 2, a solid curve represents a locus of locally asymptotically stable equilibrium points, while a dashed curve corresponds to an unstable branch. For $Q_1 < Q_1^{①} = 10.9461...$, a stable equilibrium point exists with voltage magnitude in the neighborhood of 1.2 (upper left in Fig. 2.). As Q_1 is increased, an unstable ("subcritical" in the sense of increasing Q_1) Hopf bifurcation is encountered at the point labeled HB① in Fig. 2. As Q_1 is increased further, the stationary point regains stability at

$Q_1 = Q_1^{⑤}$ during a stable ("supercritical" in the sense of decreasing Q_1) Hopf bifurcation. This stable equilibrium merges with another, unstable stationary branch and disappears in the saddle node bifurcation labeled SNB⑥ in Fig. 2. The numerical computations show that the family of periodic solutions emerging from the Hopf bifurcation at ① and the family of periodic solutions emerging from the Hopf bifurcation at⑤ are one and the same.

Besides the bifurcations of the nominal equilibrium described in the foregoing, the periodic solutions born during the Hopf bifurcations at ① and ⑤ themselves undergo (secondary) bifurcations. In Fig. 2, the circles denote the minimum value of the variable V of periodic solutions. Open circles indicate unstable periodic solutions; filled circles denote stable orbits. Because HB① is a subcritical bifurcation, the periodic solution branch emanating from this point consists of unstable limit cycles. At $Q_1 = Q_1^{②}$, the unstable periodic solution branch undergoes a *cyclic fold bifurcation* CFB②. Since this bifurcation is a saddle-node bifurcation of limit cycles, we should expect the branch to turn around and give rise to a segment of locally asymptotically stable periodic solutions for some range of $Q_1 > Q_1^{②}$.

The stable periodic orbit born at CFB② loses stability at the *period doubling bifurcation* point PDB③. During this bifurcation, a new periodic orbit appears which initially coincides with the original orbit, except that it is of exactly twice the period. The original orbit necessarily loses stability during this bifurcation. The branch of period-doubled orbits is not shown in Fig. 2, nor are any further bifurcating solutions from that branch. However, note that an analogous period doubling bifurcation sequence (but in reverse order, w.r.t. Q_1) is found to occur from the periodic obits emanating from HB⑤; this bifurcation is labeled PDB④ in Fig. 2.

Simulations of the system in the parameter range corresponding to the "Hopf window" indicate the presence of additional bifurcations of periodic orbits, and of aperiodic (and apparently chaotic) orbits. This is expected. There are further period doublings (not shown) just beyond the period doublings PDB③ and PDB④ indicated in Fig 2. This indicates there are two period doubling cascades, which ultimately result in chaotic orbits. Indeed, one surmises there are an infinite number of periodic branches, of higher and higher period, paralleling the single branch shown, extending from PDB③ to PDB④. These consist entirely of unstable orbits, save for the regions near PDB③ and ④ where the period-doubling sequences take place. The post-period doubling chaos is stable—there are two strange attractors for a certain range of the parameter values in the Hopf window. One is near HB① in the approximate range $Q_1 = 10.89 - 10.894$, the other is near HB⑤ in the approximate range $Q_1 = 11.377 - 11.3825$. Note that, in [8], Liapunov exponents and power spectra are calculated as evidence for the presence of strange attractors. More interestingly, the strange attractor near the HB① disappears suddenly at $Q_1^* = 10.89434...$ and the strange attractor near the HB⑤ disappears at approximately $Q_1^\dagger = 11.376$. In chaos literature, the sudden death of these strange attractors is known as a *boundary crisis.*

The term *crisis* was introduced in [20], [21] to describe sudden qualitative changes in strange attractors with quasistatic changes in parameters. A crisis involving the destruction of a strange attractor through collision with a saddle point or a saddle periodic orbit, (or, equivalently, the stable manifold of either) is known as a *boundary crisis.* In a boundary crisis, a strange attractor exists for parameter values up to the critical value, at which the collision takes place. Subsequent to this value, the strange attractor no longer exists, but it leaves its signature: *transient chaos.* The transient motion appears chaotic for an arbitrarily long time (depending on the initial condition), and then suddenly experiences a sharp excursion to another, possibly distant attractor (which may be at infinity). Here we surmise for the strange attractor nearer to HB① it is the unstable limit cycle born through the subcritical Hopf bifurcation that collides with the strange attractor. For the strange attractor near HB⑤, it is the low voltage saddle point that collides with the strange attractor.

III.2 Voltage Collapse

Analysis of the bifurcation scenario discussed in the foregoing section is important for organizing our understanding of the dynamics of voltage collapse for the model power system under consideration. To consider the implications of the bifurcations studied above in terms of the system dynamics, assume that the parameter Q_1 is quasistatically increased. For the 'usual' values of the parameter Q_1, the system operates at the stable equilibrium, At $Q_1^{②}$, we cross the cyclic fold bifurcation point. At this point, a stable/unstable limit cycle pair is born and coexists with the stable equilibrium point. The sets of initial conditions which asymptotically approach each of the two attractors is separated by the stable manifold of the saddle limit cycle born during CFB②. As Q_1 varies from $Q_1^{②}$ to $Q_1^{③}$, this periodic solution also loses stability, but in doing so gives birth to a new stable (period doubled) periodic orbit. This scenario repeats itself in a cascading fashion, each time making available a stable periodic orbit, until a strange attractor emerges. This strange attractor then disappears at Q_1^*. Notice that all these bifurcations take place prior to the HB①. Thus, in the interval of $Q_1^{②} - Q_1^*$, there exists a *partial* hysteresis loop, i.e., in addition to the stable equilibrium, there is another coexisting attractor. This means voltage collapse can occur through two different routes in this system.

In the interval of $Q_1^{②} - Q_1^*$, if the system is perturbed away from the stable equilibrium, the system may settle down to the coexisting attractor. The coexisting attractor, depending on the parameter value, is either a stable limit cycle or a strange attractor. Then as Q_1 is quasistatically increased, a boundary crisis of the strange attractor is encountered at Q_1^*, at which point voltage collapse occurs. On the other hand, even after the disappearance of the strange attractor, the nominal equilibrium of the system is still stable until the Hopf bifurcation HB①. Hence another possible mechanism of voltage collapse is linked to the subcritical Hopf bifurcation as suggested in [6]. As Q_1 passes the Hopf bifurcation point, the excursion of voltage exhibits increasing oscillations and then a sharp decrease. Note that subcritical Hopf bifurcation is a form of catastrophic bifurcation. Hence in this example, voltage collapse is triggered either by a boundary crisis or by a catastrophic Hopf bifurcation.

We remark that for parameter values very near the saddle node bifurcation there exists a range of stable operating conditions for the system. Depending on the parameter value, this stable operating condition can be a stable equilibrium, a stable periodic orbit, or a strange attractor. However, it is also true that the system would in all likelihood not be operating at these conditions, since voltage collapse would probably have already occurred as a result of the mechanisms discussed above.

III.3 Related Voltage Collapse Phenomena

In [7], we studied system (1)-(6) with some modifications, mainly through the deletion of the capacitor from the system. It is found that in the modified system, the voltage collapse is triggered by the boundary crisis of a strange attractor. In the modified system, there is only one Hopf bifurcation, a subcritical bifurcation analogous to HB①. The unstable limit cycle born during this bifurcation also undergoes a cyclic fold bifurcation, giving a stable limit cycle which persists beyond the Hopf bifurcation point, giving a *true* hysteresis loop. Eventually, the limit cycle proceeds through a period doubling cascade, giving rise to a strange attractor, which ultimately is destroyed in a boundary crisis.

In [10], it was shown that in a rudimentary but representative power system model, a (parametric) voltage collapse occurs following either a subcritical Hopf bifurcation or a saddle node bifurcation as control gain and load are varied as parameters. The model is essentially two dimensional, therefore there is no strange attractor. Also the model in [10] is subject to algebraic system constraints in the form of load flow equations and thus is a differential-algebraic system. Despite the existence of singularities in the state space due to the constraints, the occurrence of the parametric voltage collapse is still triggered by catastrophic bifurcations, namely subcritical Hopf and saddle node bifurcations. In the classification of voltage collapse phenomena in [10], another type of voltage collapse is the so-called dynamic (state space) voltage collapse. As discussed in the introduction of this paper, dynamic voltage collapse occurs when the post fault state lies outside the transient stability region and so the quantification of this nonlinear stability characteristic requires computing the basins of attraction of the stable equilibria. In this work, we focus on the parametric aspect of the voltage collapse phenomena. Of course, the understanding of voltage dynamics will not be complete without performing analysis in the entire state space and the entire parameter space (or a relevant region), which is beyond the scope of this paper.

Generally speaking voltage collapse may be linked with the sudden loss of stable bounded solutions of a power system model in the vicinity of a pre-collapse operating condition. In the next section, we consider using feedback control to postpone the occurrence of voltage collapse or to improve the operability in the presence of a rich structure of bifurcations.

IV. Bifurcation Control of Voltage Collapse

In this section, we consider local control of voltage collapse at its inception. That is, we design controllers which can delay the occurrence of voltage collapse, as opposed to controllers for recovery from voltage collapse. The controllers we seek do not involve forced system operation in parameter ranges where voltage collapse does not occur, but are designed to work in the parameter ranges of difficulty. In order to control voltage collapse in power system models such as the one studied in the preceding section, one has to design control laws to deal with bifurcations, chaos and crises. In our work [13], [14], we have shown that control laws which significantly reduce the amplitude of a bifurcated solution, or significantly enhance its stability over a nontrivial parameter range, are viable tools in the taming of chaos. Here similar techniques will be employed to control the bifurcations, chaos, and crises. In doing so we expect to increase the stability margin of the system in parameter space. In other words, voltage collapse will be 'postponed' so that stable operation of the system will be allowed beyond the point of impending collapse in the open loop system. In particular, the control laws are designed to increase two types of stability margin: a *static stability margin* and a *dynamic stability margin*. The static stability margin is measured in parameter space from the point where the nominal equilibrium loses its stability. The dynamic stability margin is measured in parameter space from the point where voltage collapse takes place. As discussed in the previous section, these two stability margins are not necessarily the same. In the system model under study, the static stability margin is measured from the Hopf bifurcation point $Q_1^{①}$; the dynamic stability, however, is measured either from $Q_1^{①}$ or the crisis point Q_1^* depending on the route to collapse.

In this particular model, the stable equilibrium point loses its stability through the subcritical Hopf bifurcation HB①. The subcriticality of the Hopf bifurcation has several negative effects on the system: the system may exhibit a jump from the stable equilibrium to the coexisting attractor under perturbation, and the boundary crisis is also a direct consequence of the subcriticality of the Hopf bifurcation. Moreover, the region of attraction of the stable equilibrium is bounded by the stable manifold of the unstable limit cycle, and so this region shrinks as criticality is approached. These factors motivate the design of feedback control laws directed at the Hopf bifurcation which reduce the negative effects and increase the stability margin of the system in parameter space. As shown in [13], [14], such control action can also suppress the chaos and crises by 'squeezing' the period doubling cascades. Next we present a brief summary of the bifurcation control approach in the context of Hopf [11] and period doubling [14] bifurcation control and then proceed to use these techniques in the voltage collapse control problem.

IV.1 Nonlinear Bifurcation Control

Consider a one-parameter family of nonlinear autonomous control systems

$$\dot{x} = f_\mu(x, u). \tag{7}$$

where $x \in I\!R^n$ is the state vector, $\mu \in I\!R$ is the system parameter, f_μ is a smooth map from $I\!R^n \times I\!R$ to $I\!R^n$, and u is a scalar input. Local bifurcation control deals with the design of smooth control laws $u = u(x)$ which stabilize a bifurcation occurring in the one-parameter family of systems (7). These control laws exist generically, even if the critical eigenvalues of the linearized system at the equilibrium of interest are uncontrollable. The feedback control designs of [11] transform a subcritical (unstable) Hopf bifurcation to a supercritical and hence stable, bifurcation.

The design procedure aims to ensure the asymptotic stability of the Hopf bifurcation point as well as orbital asymptotic stability of the periodic solutions emerging from the bifurcation point for a range of parameter values. Suppose for $u = 0$, x_{e,μ_c} is the nominal equilibrium of (7) with $\mu = \mu_c$ and the following hypothesis holds: The Jacobian matrix

$D_x f_{\mu_c}(x_{e,\mu_c}, 0)$ has a simple pair of nonzero pure imaginary eigenvalues $\lambda_1(\mu_c) = j\omega_c$ and $\lambda_2(\mu_c) = -j\omega_c$ with $\omega_c \neq 0$ and the transversality condition

$$\frac{\partial Re[\lambda(\mu_c)]}{\partial \mu} \neq 0 \tag{8}$$

is satisfied, and all the remaining eigenvalues are in the open left half complex plane. The Hopf Bifurcation Theorem [17], [18] asserts the existence of a one-parameter family $p_\epsilon, 0 < \epsilon \leq \epsilon_0$ of nonconstant periodic solutions of system (7) emerging from $x = x_{e,\mu_c}$ at the parameter value μ_c for ϵ_0 sufficiently small. The periodic solution $p_\epsilon(t)$ occurring at parameter values $\mu(\epsilon)$ have period near $2\pi{\omega_c}^{-1}$. Exactly one of the characteristic exponents of p_ϵ governs the asymptotic stability and is given by a real, smooth and even function

$$\beta(\epsilon) = \beta_2\epsilon^2 + \beta_4\epsilon^4 + \cdots. \tag{9}$$

That is, p_ϵ is orbitally asymptotically stable if $\beta(\epsilon) < 0$ but is unstable if $\beta(\epsilon) > 0$. Generically the local stability of the bifurcated periodic solutions p_ϵ is typically decided by the sign of the coefficient β_2. Note the sign of β_2 also determines the stability of the critical equilibrium point x_{e,μ_c}. Therefore, a feedback control law $u = u(x)$ which renders $\beta_2 < 0$ will stabilize both the critical equilibrium point x_{e,μ_c} and the bifurcated periodic solutions. An algorithm for computing the "stability coefficient" β_2 can be found in, e.g., [11].

It is well known that only the quadratic and cubic terms occurring in a nonlinear system undergoing a Hopf bifurcation influence the value of β_2. Thus only the linear, quadratic, and cubic terms in an applied control u have potential for influencing β_2. If the critical mode is controllable, a linear stabilizing feedback exists. Interestingly it is shown in [11] that a cubic stabilization feedback also exists in such case. On the other hand, if the critical mode is uncontrollable, the system may still be stabilizable by a quadratic feedback control law. See [11] for details.

Now suppose the periodic solution emerging from the Hopf bifurcation point undergoes a cascade of period doubling bifurcations to chaos. As shown in [14], nonlinear feedback control laws can be designed which influence the degree of stability and amplitude of a given period-doubled orbit. If the amplitude of the periodic orbit can be constrained sufficiently, then the cascade of period doublings to chaos can be eliminated.

IV.2 Voltage Collapse Control

In this subsection, we carry out the design of controllers for voltage collapse control in the model (1)-(4). Consider system (1)-(4) subject to control u by adding a control function u to the end of the right hand side of Eq. (4) to give

$$\dot{\delta}_m = \omega \tag{10}$$

$$M\dot{\omega} = -d_m\omega + P_m + E_m V Y_m \sin(\delta - \delta_m - \theta_m) + E_m^2 Y_m \sin\theta_m \tag{11}$$

$$K_{qw}\dot{\delta} = -K_{qv2}V^2 - K_{qv}V + Q(\delta_m, \delta, V) - Q_0 - Q_1 \tag{12}$$

$$T K_{qw} K_{pv} \dot{V} = K_{pw}K_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V + K_{qw}(P(\delta_m, \delta, V) - P_0 - P_1) - K_{pw}(Q(\delta_m, \delta, V) - Q_0 - Q_1) + u(\omega) \tag{13}$$

where $P(\delta_m, \delta, V)$, $Q(\delta_m, \delta, V)$ are given by (5) and (6).

Note that the control is implemented by injecting a speed signal into the load node. Feedback signals which are some dynamic function of the speed ω are injected into the excitation system, a scheme that is widely used and is known as a power system stabilizer (PSS). The speed signal needs no washout since it does not affect the system equilibrium structure at steady state. Note also that such a controller does not affect the position of the saddle node bifurcation SNB⑥.

One control law design transforms a subcritical Hopf bifurcation into a supercritical bifurcation and ensures a sufficient degree of stability of the bifurcated periodic solutions so that chaos and crises are eliminated. These control laws allow stable operation very close to the point of impending collapse (saddle node bifurcation). Because the critical mode in this case is controllable, a purely cubic control is designed to handle all these tasks. Another control design involves changing the critical parameter value at which the Hopf bifurcations occur by a linear feedback control. Because of the special structure of the system under study, this linear feedback law eliminates the Hopf bifurcations and the resulting chaos and crises. Thus, the linearly controlled system can operate at a stable equilibrium up to the saddle node bifurcation.

IV.2.1 Nonlinear Bifurcation Control

To render the Hopf bifurcation HB① supercritical, we employ a cubic feedback with measurement of ω. The closed loop is Eq. (10)-(13) and u is of the form

$$u = k_n \omega^3 \tag{14}$$

where $k_n > 0$ is the (scalar) cubic feedback gain.

Values of k_n which give a supercritical HB are determined by computing the stability coefficient β_2 of the closed loop system. Since transforming HB① to a supercritical Hopf bifurcation is strictly a local result, computational analysis techniques must be used assess the effects of the nonlinear control on the global dynamical behavior of the closed-loop system. As shown in [13], [14], larger values of the gain k_n not only enhance the stability of the bifurcation but also result in *a reduced amplitude* of the stable limit cycle over a range of parameter values. Recalling the discussion in subsection IV.1, if the amplitude of the periodic orbit can be constrained sufficiently, then the cascade of period doublings to chaos can be eliminated. Fig. 3 shows a bifurcation diagram for the closed loop system with control gain $k_n = 0.5$. In the closed-loop system, HB① is rendered supercritical. Moreover, the period doubling bifurcations, including the two period doubling cascades and the resulting two strange attractors and their crises are all eliminated. The benefits of changing HB① to a supercritical bifurcation can be seen in Fig. 5 where the dynamic response of the system to increasing Q_1 to a value beyond HB① is shown. Transient trajectory (a) shows the increasing oscillations and ultimate voltage collapse without control. With nonlinear control and same initial conditions, however, we see that voltage settles to a small amplitude oscillation in trajectory (b) rather than collapsing.

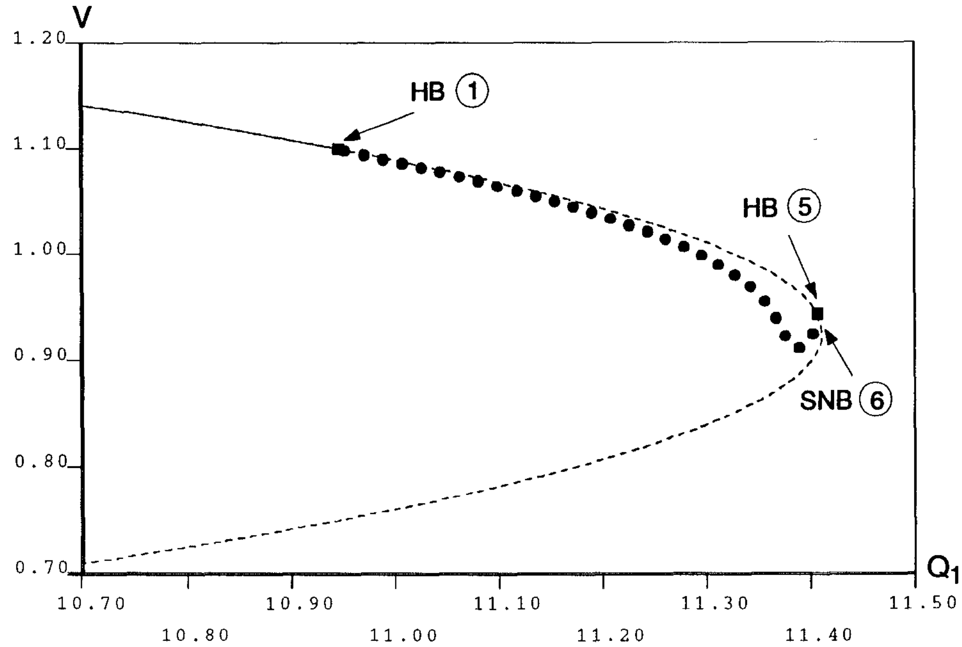

Figure 3: Bifurcation diagram of closed loop system with cubic control ($k_n = 0.5$)

Evidently such a control has a very favorable effect on the voltage collapse dynamics. By transforming the Hopf bifurcation HB① to a supercritical bifurcation, the multistability near the HB① is eliminated and hence the occurrence of jump behavior of the system operating point under perturbation is prevented. More significantly, the system can operate at a small amplitude limit cycle as Q_1 crosses the previous collapse point $Q_1^{①}$ and this transition can be done in a continuous fashion. Also because of the supercriticality of the Hopf bifurcation in the closed loop system, the region of attraction of the nominal stable equilibrium is increased. As Q_1 increases further, the nominal equilibrium regains stability at $Q_1 = Q_1^{⑤}$ through the supercritical Hopf bifurcation HB⑤. The operation of the system takes yet another continuous transition from the small periodic orbit to the stable equilibrium as Q_1 crosses $Q_1^{⑤}$. The system can operate at the stable equilibrium until the saddle node bifurcation point SNB⑥ is encountered. Then a sharp voltage collapse will take place.

Note that by introducing a new type operating condition (a stable small amplitude limit cycle), though the static stability margin of the system remains the same as in the open loop case, the dynamic stability margin is increased up to the saddle node bifurcation point.

Thus, in the closed loop system, the fatal voltage collapse then occurs at the saddle node bifurcation point as postulated in [3].

IV.2.2 Linear Bifurcation Control

Since the critical mode is controllable, a linear stabilizing control exists for the stabilization of the Hopf bifurcation point. However, in the context of voltage collapse control, one has to consider the effect of such control over a range of parameters. The effect may be difficult to determine since linear feedback will affect all the eigenvalues and eigenvectors. In particular, a high gain linear feedback may well destabilize modes that are open-loop stable. Also it should not be surprising that in some situations a linear feedback which locally stabilizes an equilibrium may in result in globally unbounded behavior [12]. For small feedback gains, however, one can expect that the bifurcation will reappear at a different parameter value. Fortunately, in this particular example, linear feedback with measurement of ω in the form

$$u = k_l \omega \tag{15}$$

can impart desirable effects on the system. In (15), $k_l > 0$ is the (scalar) linear feedback gain.

Since system (10)-(13) is a parametrized system, it is very difficult to study the effect of the linear control (15) over a range of parameter values by standard pole placement techniques directed at a particular equilibrium for a particular parameter value. However, if we consider the control gain k_l as a second parameter to the system in addition to the parameter Q_1, the effect of linear control can be tracked with two-parameter continuations of the Hopf bifurcation points (HB① and HB⑤). Recall that the control design does not affect the position of SNB⑥.

Fig. 4a shows a two-parameter (k_l and Q_1) curve of the Hopf bifurcation points ① and ⑤. It can be seen that as k_l increases from 0, HB① and HB⑤ move closer to each other with HB① having a much faster pace. As k_l increases further, HB① merges with HB⑤ leading to their disappearance. Figs. 4b, 4c and 4d show the Hopf window shrinks and ceases to exist. The benefits of eliminating the Hopf window are seen in trajectory (c) of Fig. 5, where

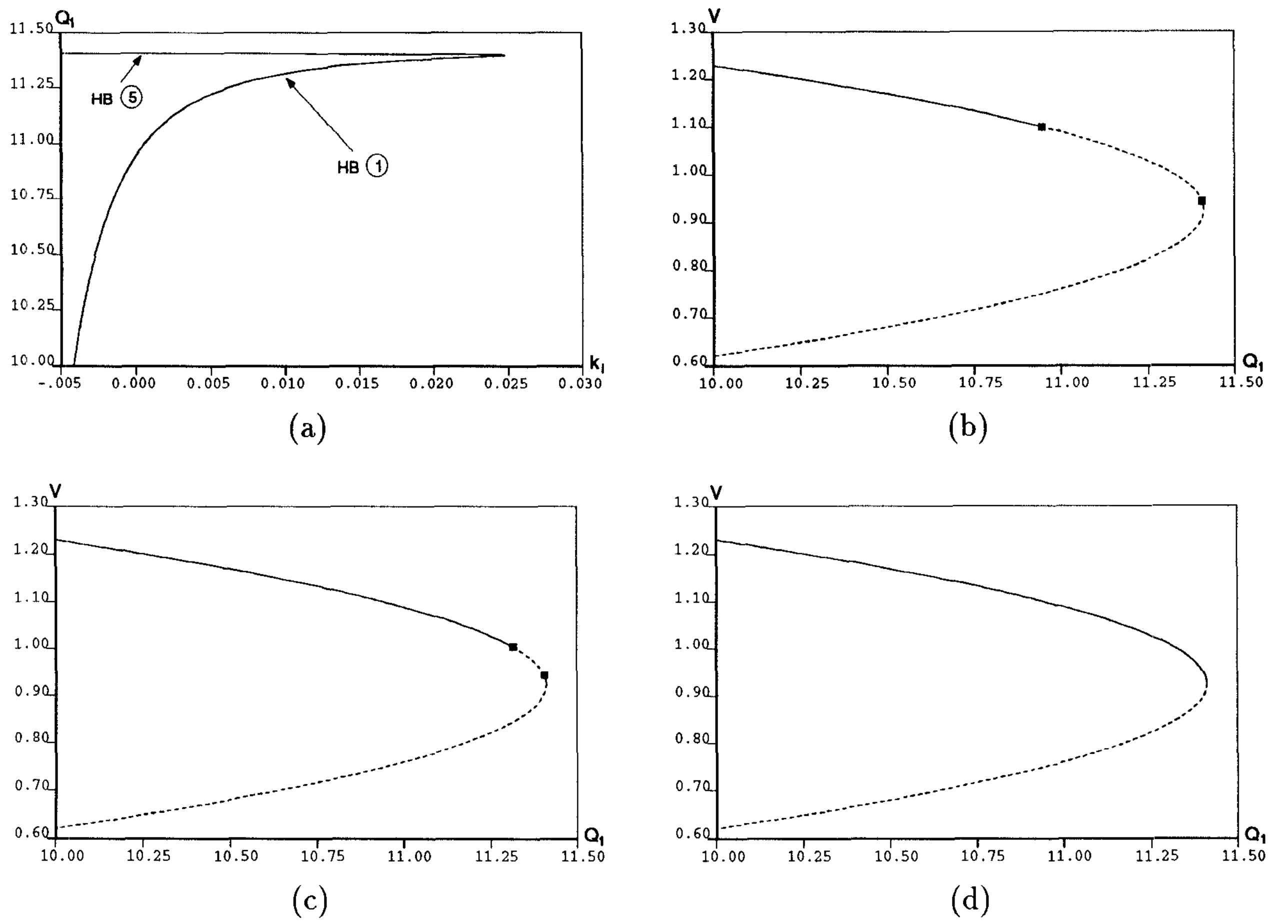

Figure 4: With linear bifurcation control: (a) two-parameter curves of the Hopf bifurcation points; bifurcation diagrams with (b) $k_l = 0$ (no control), (c) $k_l = 0.01$, (d) $k_l = 0.025$.

increasing Q_1 to a point which is beyond the location of the original HB① results in the system settling down to the original, high voltage equilibrium branch. The initial conditions of trajectory (c) coincide with those of (a) and (b).

With linear bifurcation control (15), both the static and dynamic stability margins can be increased. When the Hopf bifurcations cease to exist (approximately for $k_l > 0.0245$), the static and dynamic stability margins are maximized in that the system can operate at a stable nominal equilibrium up to the saddle node bifurcation. Also, all the dynamic bifurcations, including Hopf bifurcations, period doubling bifurcations, period doubling cascade to chaos and crises, are extinguished.

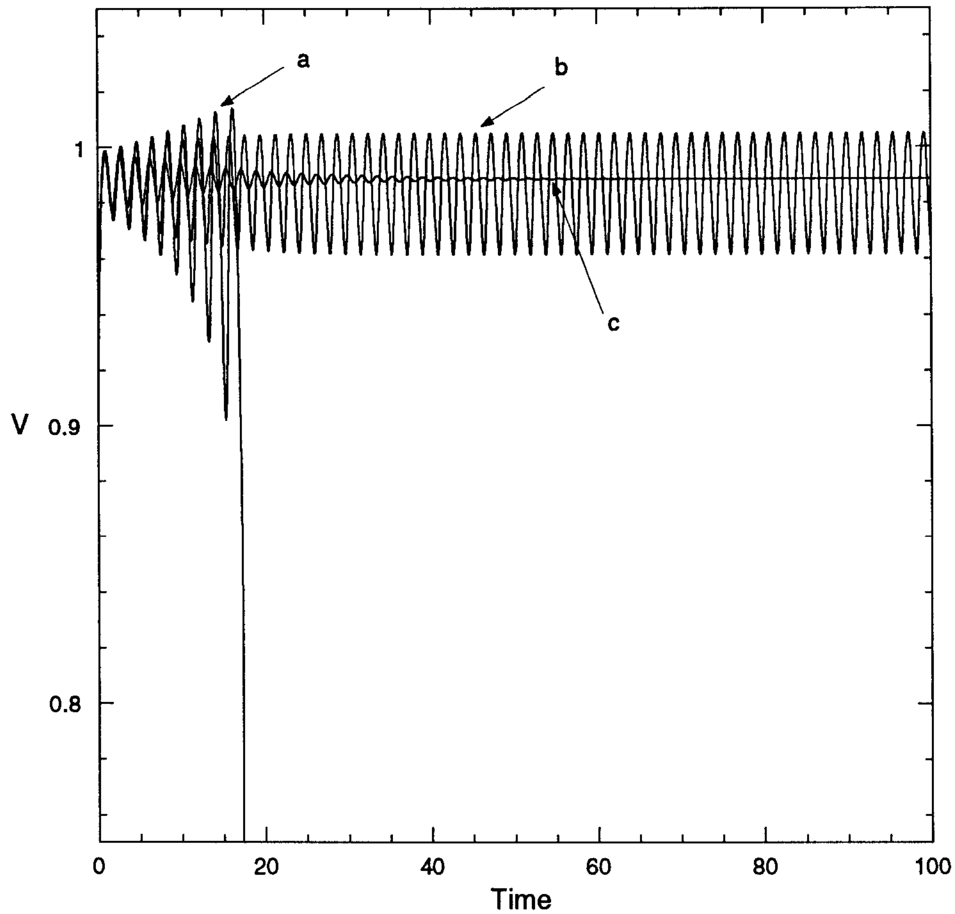

Figure 5: Dynamic responses of the system at $Q_1 = 11.35$: **a.** without control; **b.** with nonlinear control $k_n = 0.5$; **c.** with linear control $k_l = 0.025$.

IV.2.3 Composite Bifurcation Control

The two types of control law given above, namely the cubic control (14) and the linear control (15), can be combined to result in a composite control law. The closed loop is Eq. (10)-(13) and u is of the form

$$u = k_l \omega + k_n \omega^3 \tag{16}$$

where $k_l > 0$ and $k_n > 0$ are the (scalar) linear and nonlinear feedback gains, respectively.

With this composite control, the designer has the freedom to choose proper static and dynamic stability margins. Besides the flexibility in terms of achievable behavior of the system under such control (over a range of parameter values), the nonlinear term in the control may be used to improve the transient response.

V. Conclusions

Nonlinear phenomena have been studied for a power system dynamic model which has previously been used to illustrate voltage collapse. It was found that for this model, the nominal operating point undergoes dynamic bifurcations prior to the static bifurcation to which voltage collapse has been attributed. These dynamic features, including chaos and crises, result in a reduced stability margin in parameter space. Moreover, the issue of voltage collapse control is addressed. It is demonstrated that, via the bifurcation approach, both the static and dynamic stability margins can be increased and the voltage collapse can be postponed by feedback control of the nonlinear phenomena. The resulting controllers are not explicit functions of the bifurcation parameter, and are effective over a range of parameter values. Although the relative importance of the effects of these nonlinear phenomena in general power systems under stressed conditions is still a topic of continuing research, the bifurcation control approach appears to be a viable technique for control of these systems.

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Figure Captions

Fig. 1. Power system model

Fig. 2. Bifurcation diagram of open loop system

Fig. 3. Bifurcation diagram of closed loop system with cubic control ($k_n = 0.5$)

Fig. 4. With linear bifurcation control: (a) two-parameter curves of the Hopf bifurcation points; bifurcation diagrams with (b) $k_l = 0$ (no control), (c) $k_l = 0.01$, (d) $k_l = 0.025$

Fig. 5. Dynamic responses of the system at $Q_1 = 11.35$: **a.** without control; b. with nonlinear control $k_n = 0.5$; **c.** with linear control $k_l = 0.025$