Technical Research Report

SYSTEMS RESEARCH CENTER

Supported by the National Science Foundation Engineering Research Center Program (NSFD CD 8803012), Industry and the University

Bifurcations, Chaos and Crises in Power System Voltage Collapse

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TR 92-72

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Manuscript: June 16, 1992

Abstract

Bifurcations occurring in power system models exhibiting voltage collapse have been the subject of several recent studies. Although such models have been shown to admit a variety of bifurcation phenomena, the view that voltage collapse is triggered by possibly the simplest of these, namely by the (static) saddle node bifurcation of the nominal equilibrium, has been the dominant one. The authors have recently shown that voltage collapse can occur "prior" to the saddle node bifurcation. In the present paper, a new dynamical mechanism for voltage collapse is determined: *the boundary crisis of a strange attractor* or synonymously *a chaotic blue sky bifurcation.* This determination is reached for an example power system model akin to one studied in several recent papers. The identified mechanism for voltage collapse amounts to the disappearance of a strange attractor through collision with a coexisting saddle equilibrium point. This mechanism results in solution trajectories containing both an oscillatory component (as predicted by recent analytical work), and a sharp, steady drop in voltage (as observed in the field). More generally, blue sky bifurcations (not necessarily chaotic) are identified as important mechanisms deserving further consideration in the study of voltage collapse.

Keywords: Voltage collapse; boundary crisis; chaos; power systems; bifurcations; nonlinear systems; stability.

1 Introduction

Voltage collapse in electric power systems has recently received significant attention in the literature. (see, e.g., [1] for a synopsis). This has been attributed to increases in power demand which result in operation of an electric power system near its stability limits. A commonly held view is that voltage collapse arises at a saddle node (static) bifurcation of equilibrium points (see, e.g., [2], [3]). Dobson and Chiang [3] postulated a dynamic mechanism for voltage collapse tied to the saddle node bifurcation, which stresses the role of the center manifold of the system model at the bifurcation. In the same paper, they introduced a simple example power system containing a generator, an infinite bus and a nonlinear load. The saddle node bifurcation mechanism for voltage collapse postulated in [3] was investigated for this example in subsequent papers, including [3] and [8].

The presence of a saddle node bifurcation in a dynamical system does not preclude the presence of other, possibly more complex, bifurcations. Thus, the recent papers [4], [5], [7], [8] have shown that indeed other bifurcations occur in the example power system model studied in [3]. Other bifurcations which were found in this model include Hopf bifurcations from the nominal equilibrium, a cyclic fold bifurcation, period doubling bifurcations, as well as a period doubling cascade leading to chaotic behavior. (See, e.g., [14] and references therein for a general discussion of these phenomena.) Other papers have also studied bifurcations in voltage dynamics in other power system models [22], [23], [24].

The fact has therefore now been established that a variety of bifurcations, static and dynamic, occur in power system models exhibiting voltage collapse. The main theme of this paper, which continues the work reported in [5], [6], is to determine *the implications of these bifurcations for the voltage collapse phenomenon.* In our paper [5], a link was suggested between the voltage collapse phenomenon and the occurrence of dynamic bifurcations. Specifically, in that paper we showed the possible role of an oscillatory transient in voltage collapse. This was achieved using the model of [3]. In the present paper, we continue the study of dynamic bifurcations and voltage collapse, showing the dominant role of a *boundary crisis of a strange attractor* [16], [17] or synonymously *a chaotic blue sky bifurcation* [14] in voltage collapse. The present paper is based on our paper [6] where the role of a boundary crisis of a strange attractor in voltage collapse was first reported. The power system example used in the present paper and in [6] is similar to that of [3], but differs from it in several ways detailed in Section 3.

The remainder of the paper proceeds as follows. In the next section, we give a brief summary of some local bifurcations and more importantly, the blue sky bifurcations. In Section 3, we present the power system model which is used in the ensuing analysis. This model and the model of [3] differ only in the choice of parameter values. Bifurcations occurring in this model, including the emergence of a strange attractor and its disappearance in a boundary crisis (or a chaotic blue sky bifurcation), are studied. In Section 5, the implications of the bifurcations studied in Section 4 for the voltage collapse phenomenon are discussed. Conclusions are collected in Section 6.

2 Local and Blue Sky Bifurcations

2.1 Local bifurcations

Bifurcations, especially catastrophic bifurcations, play a decisive role in the voltage collapse mechanisms to be studied in this paper. Therefore, it is felt that a brief summary of some bifurcations which arise is in order. Generally, we are interested in nonlinear autonomous systems

$$\dot{x} = f(x, \mu) \tag{1}$$

Suppose that (1) possesses an equilibrium point $x_0(\mu)$ for a range of values of the parameter μ of interest. We assume that this is an asymptotically stable equilibrium for a large portion of this range. Thus, the equilibrium can qualify as a possible operating condition for the physical system (say, a power system) modeled by (1). When the power system operates in a highly stressed environment, it is possible for the equilibrium $x_0(\mu)$ to lose stability for some parameter value μ_c. At such a loss of stability, the nonlinear system (1) typically undergoes a local bifurcation. Such a bifurcation can give rise to new equilibria or periodic

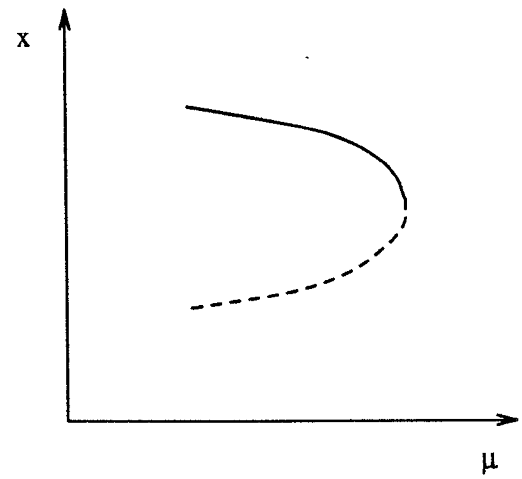

Figure 1: Saddle node bifurcation

orbits of (1).

In the case that the bifurcation of $x_0(\mu)$ is the merging of this nominal, stable equilibrium with another, unstable equilibrium, the bifurcation is said to be a saddle node bifurcation. Figure 1 depicts a saddle node bifurcation for the case of a scalar state vector. Of course, the terminology saddle node bifurcation is tailored for systems of dimension two or higher, but the figure addresses the scalar case for simplicity.

To state a theorem on saddle node bifurcation, we consider the system (1) where f is sufficiently smooth and $f(0,0) = 0$. Express the expansion of $f(x, \mu)$ in a Taylor series about $x = 0, \mu = 0$ in the form

$$f(x, \mu) = Ax + b\mu + Q(x, x) + \cdots \tag{2}$$

Note that $A = D_x f(0,0)$ is simply the Jacobian matrix of f at the origin for $\mu = 0$. The next hypothesis is basic to the saddle node bifurcation, as well as other stationary bifurcations of equilibria.

(SN1) The Jacobian A possesses a simple zero eigenvalue.

If (SN1) holds, denote by r (resp. l) the right column (resp. left row) eigenvector of the critical Jacobian A corresponding to the zero eigenvalue. Normalize r and l by setting the first component of r to 1 and then chosing l so that $lr = 1$. (This may require one to interchange the position of the first state variable with that of another state variable.) The next hypothesis, along with (SN1), ensures that Eq. (1) undergoes a saddle node bifurcation from the origin at $\mu = 0$.

(SN2) $lb \neq 0$ and $lQ(r,r) \neq 0$.

The precise statement is given in the following theorem. Note that usually this result is stated for a one-dimensional reduced system model, whereas we give a statement which applies directly to a general n-dimensional system [15].

Theorem 1 *(Saddle Node Bifurcation Theorem) With the notation and assumptions above, if (SN1) and (SN2) hold, then there is an $\epsilon_0 > 0$ and a function*

$$\mu(\epsilon) = \mu_2 \epsilon^2 + O(\epsilon^3) \tag{3}$$

such that $\mu_2 \neq 0$ and for each $\epsilon \in (0, \epsilon_0]$, Eq. (1) has a nontrivial equilibrium $x(\epsilon)$ near 0 for $\mu = \mu(\epsilon)$. The bifurcation point $x = 0$ for $\mu = 0$ is unstable.

This is the type of bifurcation which has been linked to voltage collapse in [3], [8]. From the point of view of this work, the most important feature of the saddle node bifurcation is the *disappearance, locally, of any stable bounded solution of the system* (1). In the following subsections, we give other, more complicated (nonlocal) examples of bifurcations displaying this same feature.

Suppose that the instability of the nominal equilibrium $x_0(\mu)$ is the result of a pair of eigenvalues of the system linearization crossing the imaginary axis in the complex plane. Then, as is well known [13], [12], generically it will be the case that a small amplitude periodic orbit of (1) emerges from the equilibrium $x_0(\mu)$. The following hypotheses are invoked in the theorem below.

(H1) The Jacobian $D_x f(0, \mu)$ possesses a pair of complex-conjugate simple eigenvalues $\lambda(\mu) = \alpha(\mu) + i\omega(\mu)$, $\overline{\lambda(\mu)}$, such that $\alpha(0) = 0$, $\alpha'(0) \neq 0$ and $\omega_c := \omega(0) > 0$.

(H2) $\pm i\omega_c$ are the only pure imaginary eigenvalues of the critical Jacobian $D_x f(0, 0)$.

Theorem 2 *(Hopf Bifurcation Theorem) Suppose the vector field f of system (1) is sufficiently smooth and $f(0, \mu) \equiv 0$. Given (H1) and (H2) above, then the following hold:*

(a) *(Existence) There is a $\epsilon_0 > 0$ and a smooth function $\mu(\epsilon) = \mu_2\epsilon^2 + O(\epsilon^3)$, such that for each $\epsilon \in (0, \epsilon_0]$ there is a nonconstant periodic solution $p_\epsilon(t)$ of system (1) near $x_0(\mu)$ for $\mu = \mu(\epsilon)$. The period of $p_\epsilon(t)$ is a smooth function $T(\epsilon) = 2\pi\omega_c^{-1}[1 + T_2\epsilon^2] + O(\epsilon^3)$, and its amplitude grows as $O(\epsilon)$.*

(b) *(Uniqueness) If $\mu_2 \neq 0$, there is a $\epsilon_1 \in (0, \epsilon_0]$ such that for each $\epsilon \in (0, \epsilon_1]$, the periodic orbit p_ϵ is the only periodic solution of system (1) for $\mu = \mu(\epsilon)$ lying in a neighborhood of $x_0(\mu(\epsilon))$.*

(c) *(Stability) Exactly one of the characteristic exponents of $p_\epsilon(t)$ approaches 0 as $\epsilon \downarrow 0$, and it is given by a real smooth function $\beta(\epsilon) = \beta_2\epsilon^2 + O(\epsilon^3)$. The relationship*

$$\beta_2 = -2\alpha'(0)\mu_2 \tag{4}$$

holds. Moreover, if all eigenvalues of $D_x f(0, 0)$ besides $\pm i\omega_c$ have negative real parts, then $p_\epsilon(t)$ is orbitally asymptotically stable with an asymptotic phase if $\beta(\epsilon) < 0$ but is unstable if $\beta(\epsilon) > 0$.

The two theorems above are of course not sufficient to address all types of jump phenomena in nonlinear systems, of which voltage collapse is a special case. Besides these local bifurcations, other bifurcations involving considerations which are not localized in state space near an equilibrium point, can and do arise. However, since an electric power system normally functions at a stable operating *point*, it is this condition (an equilibrium) which must yield, after one or many bifurcations, the decisive bifurcation after which collapse ensues. Generally speaking voltage collapse may be linked with the sudden loss of stable bounded solutions of a power system modeled by (1) in the vicinity of a pre-collapse operating condition. Next we discuss mechanisms of global bifurcation, namely blue sky bifurcations, by which a limit cycle or a strange attractor may disappear through interaction with other invariant sets.

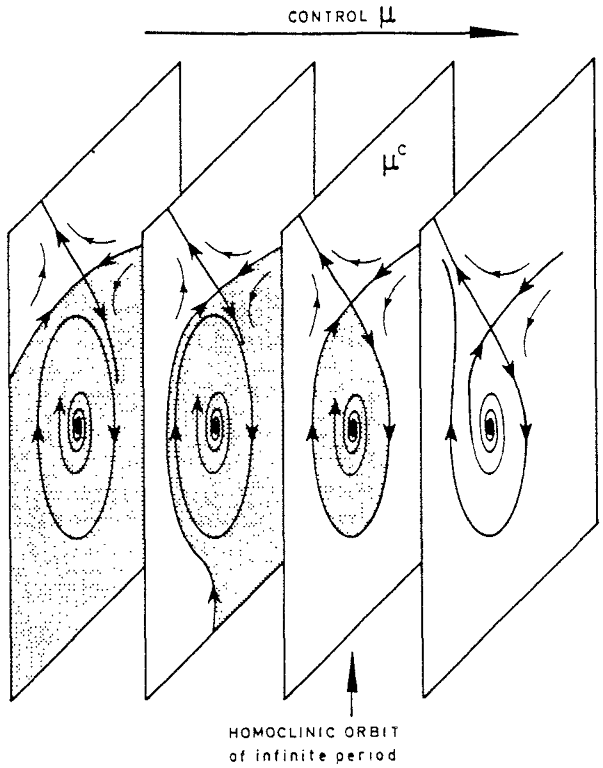

Figure 2: The blue sky catastrophe in which a homoclinic connection results in a limit cycle disappearing into the blue. [Caption and figure from Thompson and Stewart [14] (Fig. 13.5, p.271)]

2.2 Blue sky bifurcations

Thompson and Stewart [14] (p.268) refer to a type of global bifurcation in state space involving the discontinuous disappearance of a limit cycle as a *blue sky catastrophe* (see Figure 2). This is not a local bifurcation. However it does possesses the feature noted above for the saddle node bifurcation, namely the disappearance of the attractor by collision with a saddle, in this case a saddle equilibrium point. Here, we find it useful to refer to this and other bifurcations also as blue sky bifurcations since they possess the same feature, namely the disappearance of a (stable) solution of the system (1) by a collision with a saddle point or orbit. Thus in this sense saddle node bifurcation may be viewed as a blue sky bifurcation of a stable equilibrium.

The blue sky bifurcation for a periodic orbit is the sudden disappearance of a limit cycle trough a collision with a saddle equilibrium point. Prior to the critical parameter value μ_c at which the collision occurs, a saddle equilibrium coexists with the limit cycle. At μ_c the limit cycle and a branch of both the stable and unstable manifolds of the saddle point coincide, forming a *homoclinic connection.* Past μ_c the limit cycle no longer exists. Collision with a saddle fixed point is the typical mechanism by which a limit cycle can abruptly vanish from state space. This blue sky bifurcation can take two forms: the disappearance of a stable limit cycle and the disappearance of an unstable limit cycle. Moreover, it is a global, discontinuous or catastrophic bifurcation. It also serves as a prototype of a blue sky bifurcation for a strange attractor or a boundary crisis of a strange attractor, as we shall see next.

One type of global bifurcation involving a strange attractor is the sudden death of the attractor. Such a blue sky bifurcation occurs commonly for strange attractors of differential equations. Like the blue sky bifurcation for a limit cycle, a chaotic blue sky bifurcation involves a collision with an object of saddle type and is analogous to the blue sky disappearance of a limit cycle discussed above. The chaotic blue sky bifurcation is also known by another term, namely the *boundary crisis of a strange attractor.*

The term *crisis* was introduced in [16], [17], and applies to sudden qualitative changes in strange attractors with quasistatic changes in parameters. A crisis involving the sudden destruction of a strange attractor through collision with a saddle point, an unstable periodic orbit, or the stable manifold of such, is known as a *boundary crisis.* In a boundary crisis, a strange attractor exists for parameter values up to the critical value, at which the collision takes place. Subsequent to this value, the strange attractor no longer exists, but it leaves a signature, namely a transient chaotic motion. The transient chaotic motion appears chaotic for a relatively long time (depending on the initial condition), and then suddenly experiences a sharp excursion either to another, probably distant attractor, or to infinity. This excursion occurs through a tunnel in state space which necessarily follows the unstable manifold of the saddle point or orbit with which the collision takes place. The result is a discontinuous, catastrophic disappearance of the strange attractor. Such a bifurcation is always a global

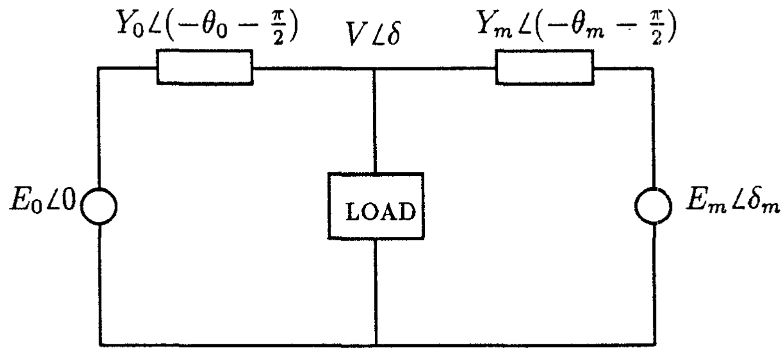

Figure 3: Power system model

bifurcation involving a homoclinic or heteroclinic event. In this paper, we shall use the terms chaotic blue sky bifurcation and boundary crisis of a strange attractor synonymously, although boundary crisis was originally introduced for one-dimensional maps.

3 A Power System Model

The power system under consideration in this paper is related to one previously considered by Dobson and Chiang [3]. Dobson and Chiang [3] employed a simple power system example to illustrate ideas, also presented in [3], which relate to the dynamics of voltage collapse. In [3], the example was chosen such that a nominal equilibrium undergoes a saddle node bifurcation as a reactive power loading parameter Q_1 is varied. Their example system includes a capacitor in parallel with a nonlinear load. (See Figure 3 of [3].) The capacitor is included to raise the voltage magnitude to nearly 1 per unit. The parameter Q_1 is taken as the bifurcation parameter of the system. It is found that the value of this parameter at the saddle node bifurcation is approximately 11.41 per unit. This is a rather high value, and is a consequence of inclusion of the capacitor in the example system. It seems that it is rather difficult to reach this level of reactive load at normally encountered power factors.

For this reason, in the present study we modify the power system example of [3], mainly through deletion of the capacitor from the system of [3]. This reduces the reactive power load parameter prior to collapse to approximately 2.56 per unit, while reducing the voltage magnitude to approximately 0.65 per unit. The resulting power system model is depicted in Figure 3. Possible compromises in which a capacitor is included to raise the voltage magnitude without a large increase in load, are not given priority in this work. It follows from [3] that the system dynamics (with no capacitor) is governed by the following four differential equations ($P(\delta_m, \delta, V)$, $Q(\delta_m, \delta, V)$ are specified below):

$$\dot{\delta}_m = \omega \tag{5}$$

$$M\dot{\omega} = -d_m\omega + P_m - E_m V Y_m \sin(\delta_m - \delta) \tag{6}$$

$$K_{qw}\dot{\delta} = -K_{qv2}V^2 - K_{qv}V + Q(\delta_m, \delta, V) - Q_0 - Q_1 \tag{7}$$

$$\begin{aligned} TK_{qw}K_{pv}\dot{V} = {} & K_{pw}K_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V \\ & +K_{qw}(P(\delta_m, \delta, V) - P_0 - P_1) \\ & -K_{pw}(Q(\delta_m, \delta, V) - Q_0 - Q_1) \end{aligned} \tag{8}$$

The notation is basically identical to that of [3], with the caveat that in the present paper there is no need for primed quantities. Primes are used in [3] to indicate Thévenin equivalent circuit values, a step which is made unnecessary since the capacitor is no longer included in the system. (See [3] for details.)

The load includes a constant PQ load in parallel with an induction motor. The real and reactive powers supplied to the load by the network are

$$P(\delta_m, \delta, V) = -E_0 V Y_0 \sin(\delta) + E_m V Y_m \sin(\delta_m - \delta) \tag{9}$$

$$Q(\delta_m, \delta, V) = E_0 V Y_0 \cos(\delta) + E_m V Y_m \cos(\delta_m - \delta) - (Y_0 + Y_m)V^2 \tag{10}$$

Most of the parameter values used in the present study agree with those of [3]. The parameters given in [3] correspond to a large generator. Our choice of parameter values corresponds to a medium sized generator (500MW). Of the parameter values used here, those which differ from values given in [3] are as follows:

$$M = 0.01464,\ Q_0 = 0.3,\ E_m = 1.05,\ Y_0 = 3.33,\ \theta_0 = 0 \text{ and } \theta_m = 0.$$

Those which coincide with values given in [3] are as follows:

$K_{p\omega} = 0.4,\ K_{pv} = 0.3,\ K_{q\omega} = -0.03,\ K_{qv} = -2.8,\ K_{qv2} = 2.1,$

$T = 8.5,\ P_0 = 0.6,\ P_1 = 0.0,$

$E_0 = 1.0,\ C = 12.0,\ Y_m = 5.0,\ P_m = 1.0,\ d_m = 0.05.$

All values are in per unit except for angles, which are in degrees.

4 Bifurcation Analysis

In this section, the results of a bifurcation analysis of the model (5)-(10) are given. The software package AUTO [21] is employed to assist this analysis. Figure 4 shows the dependence of the voltage magnitude V at system equilibrium points, as well as the stability of these equilibria, as a function of the bifurcation parameter Q_1. A solid line corresponds to stability of an equilibrium, while a dashed line corresponds to instability. Figure 5 depicts a blown-up bifurcation diagram, detailing some of the bifurcations which occur in the boxed region of Figure 4.

To simplify the discussion, note first that Fig. 4 depicts two bifurcations, and Fig. 5 depicts a total of five *additional* bifurcations. These seven bifurcations are labeled HB①, SNB②, CFB③, PDB④, PDB⑤, BSKY⑥ and BSKY⑦. Each of the seven bifurcations shown in Figures 4 and 5 is of one of the following types, with the corresponding acronyms:

- HB: Hopf bifurcation
- SNB: Saddle node bifurcation
- CFB: Cyclic fold bifurcation
- PDB: Period doubling bifurcation
- BSKY: Blue sky bifurcation

The technical connotations of these terms in the context of this paper will be clarified in the sequel. For ease of reference, we denote the values of the parameter Q_1 at which the bifurcations ①-⑦ occur by $Q_1^{①}$-$Q_1^{⑦}$, respectively. For $Q_1 < Q_1^{①}$, a stable equilibrium point exists with voltage magnitude in the neighborhood of 0.7. (Upper left in Fig. 4.) As Q_1 is increased, an unstable ("subcritical") Hopf bifurcation is encountered at the point

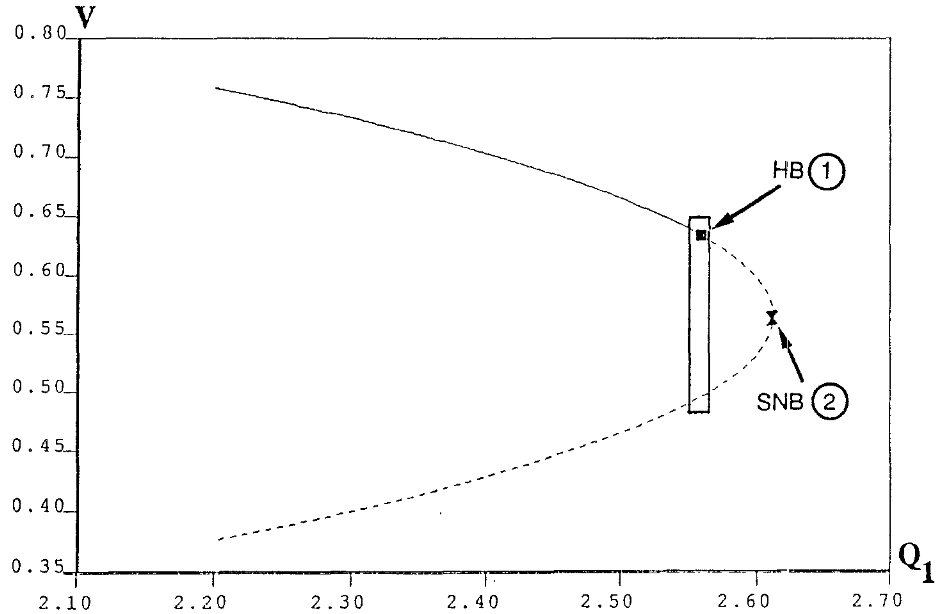

Figure 4: V vs. Q_1 at system equilibria

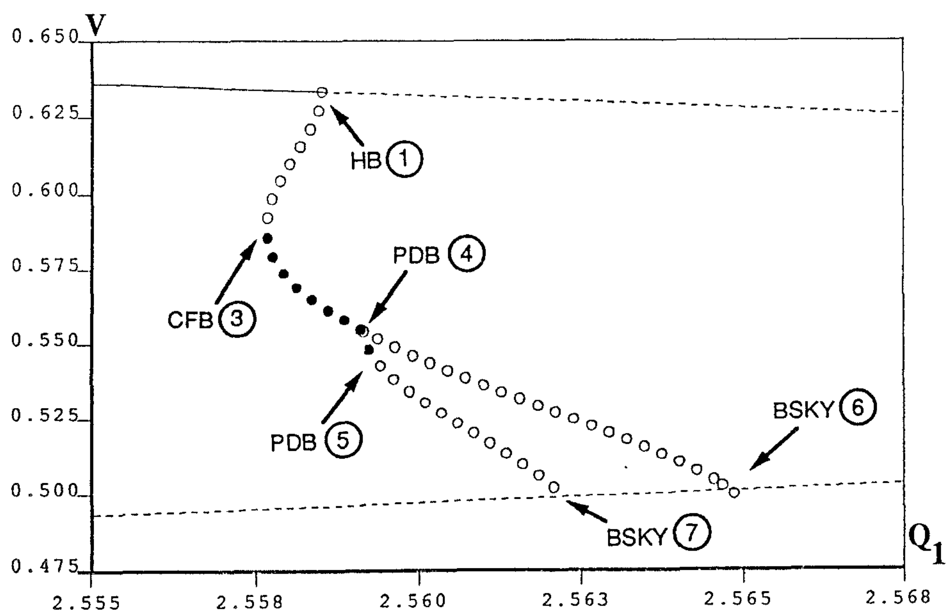

Figure 5: Magnified bifurcation diagram for boxed region in Fig. 4

labeled HB① in Fig. 4. This coincides with the point labeled HB① in Fig. 5. As Q_1 is increased further, the nominal stationary point (now unstable) disappears in the saddle node bifurcation (SNB② in Fig. 4) at $Q_1 = Q_1^{②}$.

Besides the bifurcations of the nominal equilibrium described in the foregoing, the periodic solution emerging from the Hopf bifurcation at HB① itself undergoes bifurcations. The bifurcation HB① is a subcritical Hopf bifurcation, resulting in a family of unstable periodic solutions occurring for Q_1 slightly less than $Q_1^{①}$. In Fig. 5, the minimum of the variable V for members of this family of periodic solutions is indicated by the circles appearing from ① and extending to the left. Open circles indicate instability of the periodic orbits. At $Q_1 = Q_1^{③}$, the unstable periodic solution undergoes a *cyclic fold bifurcation.* Thus, in Fig. 5, the continuation of the sequence of circles of periodic solutions for Q_1 near $Q_1^{③}$ exists for Q_1 slightly greater than $Q_1^{③}$. A cyclic fold bifurcation is simply a saddle node bifurcation of periodic solutions. Thus, the unstable periodic solution gains stability at $Q_1 = Q_1^{③}$. The solid circles emanating from CFB③ depict the continuation of the periodic solutions; they are solid to indicate stability.

This stable periodic orbit born at CFB③ loses stability at the *period doubling bifurcation* PDB④. At this bifurcation, a new periodic orbit appears which initially coincides with the original orbit, except that it is of exactly twice the period. The original orbit necessarily loses stability at such a bifurcation. The branch of period-doubled orbits is also shown in Fig. 5. This branch, depicted by the solid circles emanating to the right from PDB④, undergoes a further period doubling bifurcation in short order. This occurs at PDB⑤ in Fig. 5. These two period doubling bifurcations are followed by a cascade of period doubling bifurcations, resulting in a strange attractor for some values of Q_1. These further period doublings are not depicted in Fig. 5. However, Fig. 5 shows the continuation of the periodic orbits appearing at the cyclic fold bifurcation CFB③ and the period doubling bifurcation PDB④. Note that each of these periodic orbits disappears in a collision with the unstable (saddle) low voltage equilibrium point. These collisions are known by various names, including the blue sky bifurcation [14]. Thus, the disappearance of these orbits is indicated by BSKY⑥ and

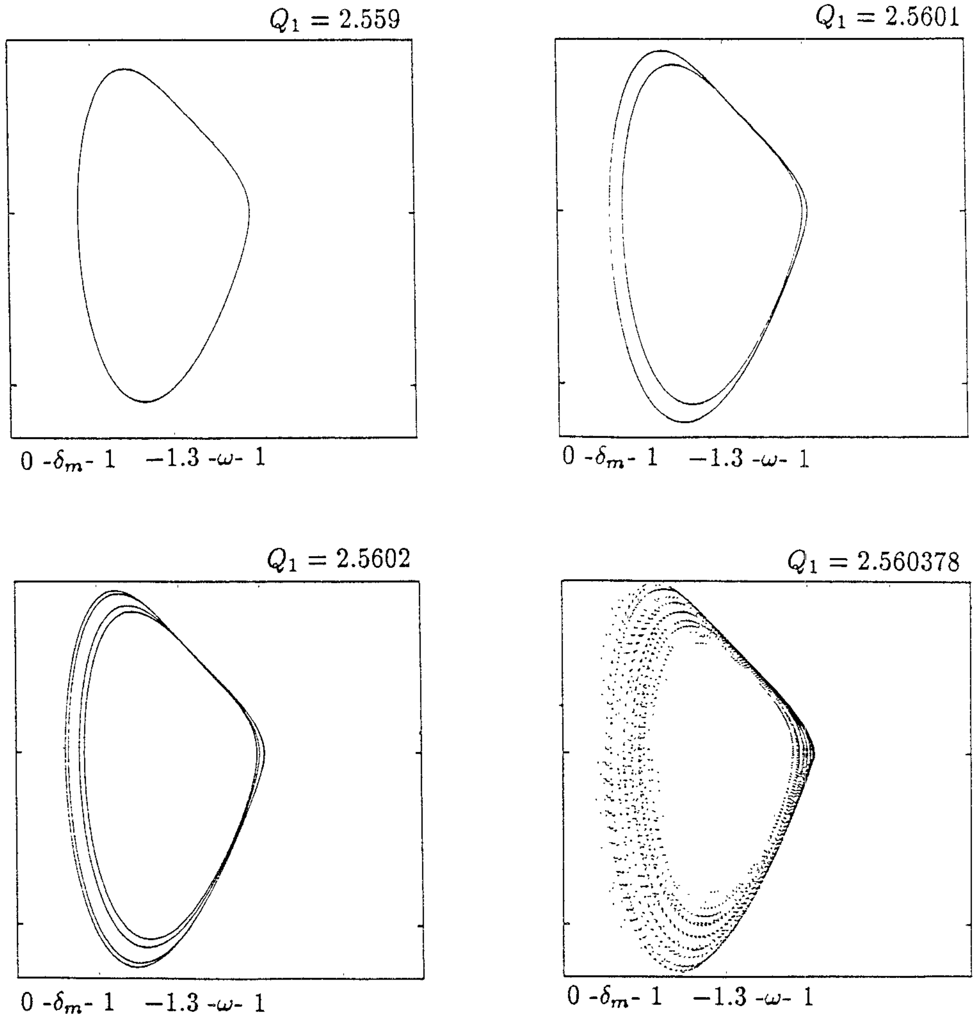

Figure 6: Period doubling cascade to chaos

BSKY⑦ in Fig. 5. Since the system undergoes a cascade of period doubling bifurcations, one expects that each period doubled orbit undergoes such a blue sky bifurcation.

Figure 6 depicts several stable periodic orbits in the sequence of period doublings discussed above, along with the strange attractor resulting from the period doubling cascade.

Let us pause to consider the implications of the bifurcations studied above for the system dynamics, assuming the parameter Q_1 is quasistatically increased. For the 'usual' values of the parameter Q_1, the system operates at the stable equilibrium. As the parameter is increased, the equilibrium loses stability at the Hopf bifurcation point, giving rise to an unstable periodic orbit. Since this orbit gains stability at the cyclic fold bifurcation, a stable periodic orbit surrounds the equilibrium at and slightly beyond the Hopf bifurcation point. The system then operates at this stable periodic orbit. For greater values of the parameter Q_1, this periodic solution also loses stability, but in doing so gives birth to a new stable (period doubled) periodic orbit. This scenario repeats itself in a cascading fashion, each time making available a stable periodic orbit, until a strange attractor emerges. The system operates on the strange attractor until the strange attractor disappears ("crisis"). After this crisis, there is no stable invariant set in the vicinity of the nominal equilibrium at which to operate. Thus, the system must now undergo a large transient excursion. In the next section, this excursion ("voltage collapse") is tied to the disappearance of the strange attractor.

5 Boundary Crises, Chaotic Blue Sky Bifurcations and Voltage Collapse

The bifurcations uncovered in the foregoing analysis, and especially the sudden disappearance of the strange attractor, are crucial to the understanding of voltage collapse for the model power system under consideration. We claim that, for the model under study, voltage collapse is triggered by the *boundary crisis of the strange attractor* [16], [17] or *chaotic blue sky bifurcation* [14], i.e., its sudden destruction through collision with the low voltage saddle point.

Recall that a boundary crisis or a chaotic blue sky bifurcation involves the sudden destruction of a strange attractor through collision of the strange attractor with a saddle point, an unstable periodic orbit, or the stable manifold of such. In a boundary crisis or a chaotic blue sky bifurcation, a strange attractor exists for parameter values up to the critical value, at which the collision takes place. Subsequent to this value, the strange attractor no longer exists, but it leaves a signature, namely a transient chaotic motion. The transient chaotic motion appears chaotic for a relatively long time (depending on the initial condition), and then suddenly experiences a sharp excursion either to another, probably distant attractor, or to infinity. This excursion occurs through a tunnel in state space which necessarily follows the *unstable manifold* of the saddle point or orbit with which the collision takes place. Note the distinction with the center manifold based view adopted in [3], [8].

With this knowledge of boundary crises, we can return to the example at hand. An attracting invariant set exists for parameter values Q_1 up to the critical value, Q_1^*, at which the boundary crisis takes place. *Voltage collapse occurs precisely at the parameter value* $Q_1 = Q_1^*$. This gives a clear alternative to the previous view [3] that the critical value of Q_1 at which voltage collapse occurs is that associated with the saddle node bifurcation, $Q_1 = Q_1^{\textcircled{2}}$. Note that one may view the Hopf bifurcation as an signal of impending voltage collapse, especially considering the very short interval in parameter space between the Hopf bifurcation (at $Q_1 = Q_1^{\textcircled{1}} = 2.55919...$) and the boundary crisis of the strange attractor (at $Q_1 = Q_1^* = 2.560378...$). Note also that the view adopted here is in agreement with that offered in our recent paper [5], which also shows an example in which voltage collapse occurs prior to the saddle node bifurcation.

Significantly, *the current proposal serves to reconcile two seemingly contradictory pieces of evidence*:

- First, the steady, sharp decrease in voltage observed in practice in voltage collapse; and
- Second, the presence of nonlinear phenomena such as oscillations and chaotic motion in dynamic models used in the study of voltage collapse, which is determined from analysis.

Figure 7 shows a projection of the dynamics onto the ω, V plane for a value of Q_1 slightly below Q_1^*. Figure 8 shows the same projection, for a value of Q_1 slightly greater than Q_1^*. The low voltage saddle point has one real positive eigenvalue, one real negative eigenvalue and a pair of complex conjugate eigenvalues with negative real part. Thus in both figures, the unstable manifold and only a section of the stable manifold of the low saddle point are shown. These manifolds are important to understand the underlying dynamics near crisis.

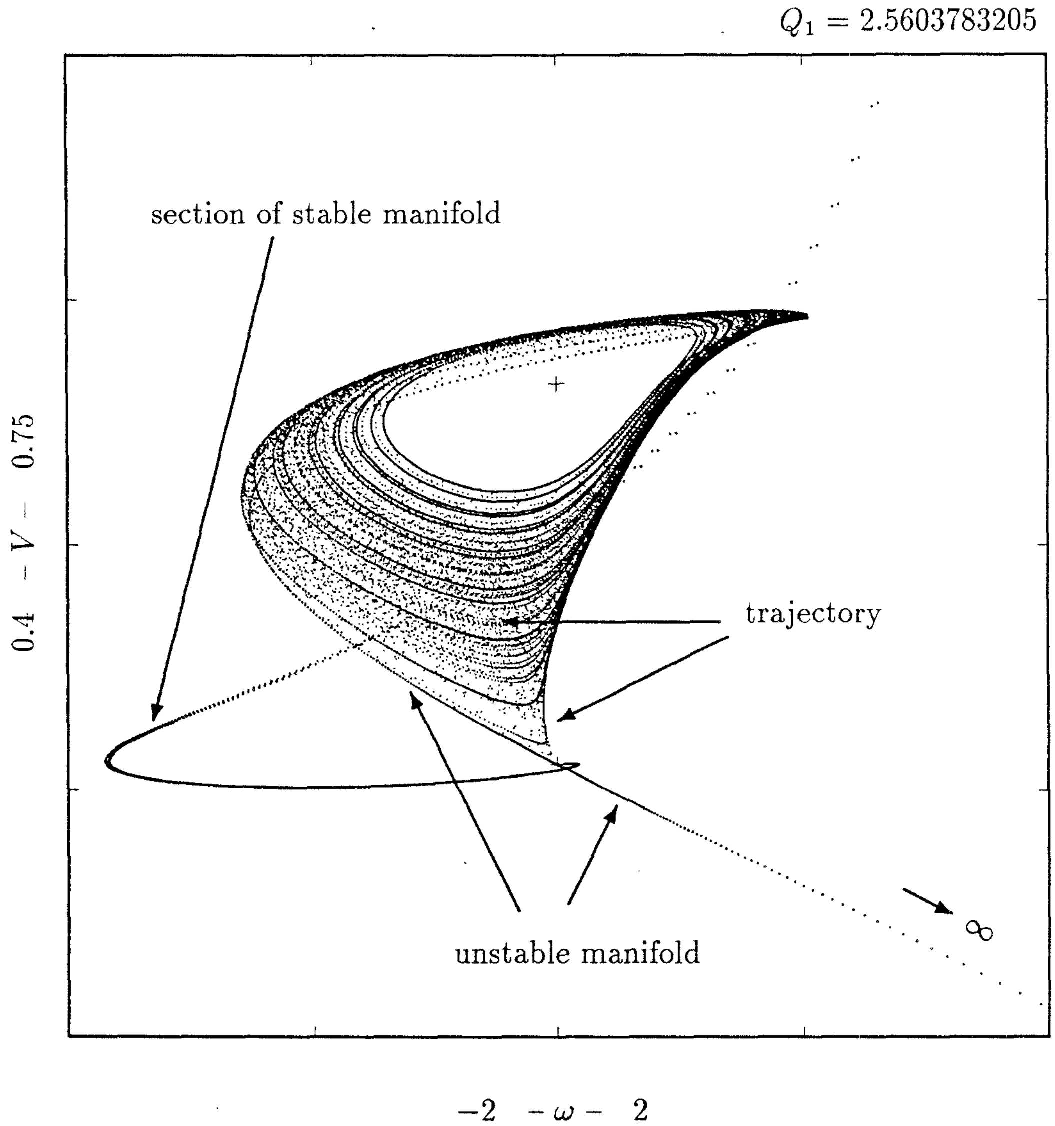

Figure 7: Just before boundary crisis: Strange attractor, unstable and stable manifolds of lower saddle point

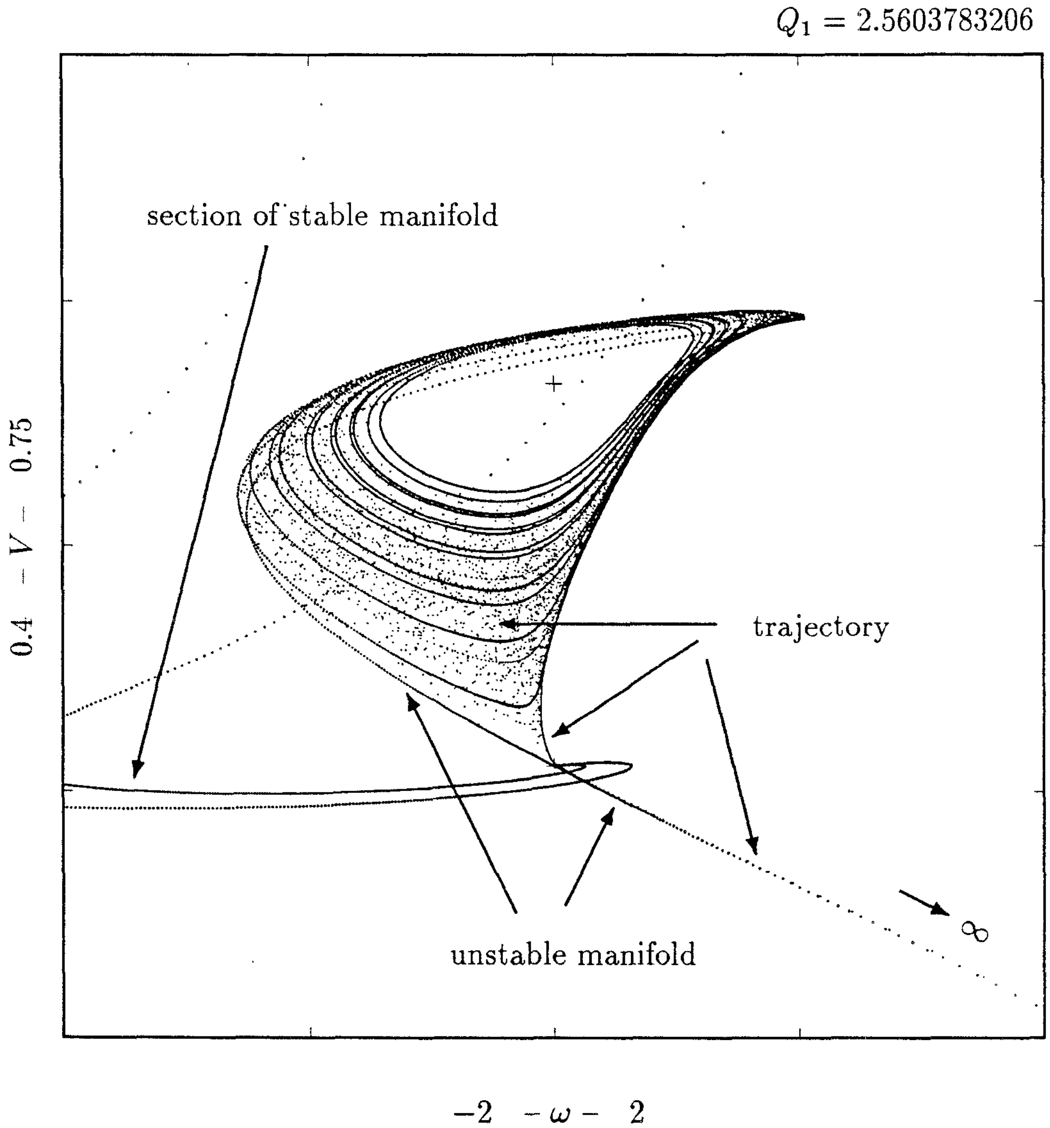

Figure 8: Just after boundary crisis: Transient chaos, unstable and stable manifolds of lower saddle point

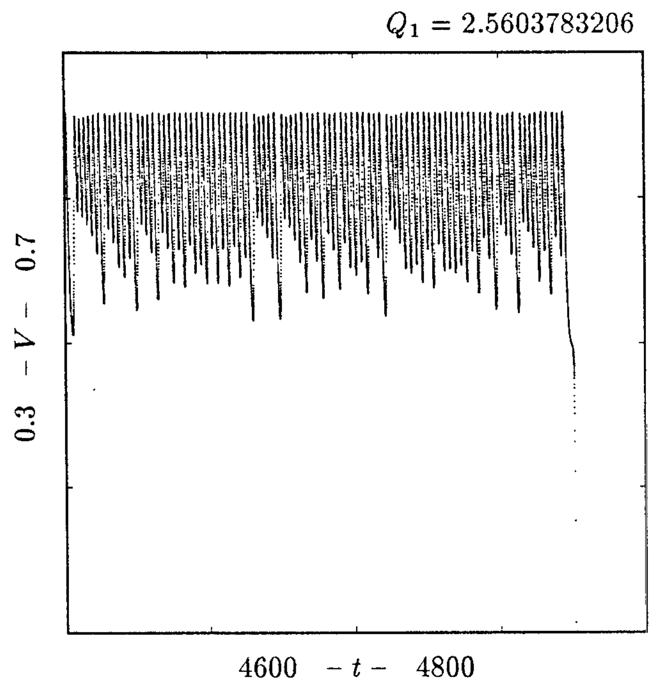

Figure 9: A time simulaton of transient chaos and voltage collapse just after boundary crisis

At $Q_1 = Q_1^*$ the closure of one of the branches of the unstable manifold of the low voltage saddle point is the strange attractor, while the stable manifold of the low voltage saddle point forms the boundary of the basin of attraction for the strange attractor for $Q_1 < Q_1^*$ [17]. In Figure 7, the chaotic invariant set is a strange attractor, i.e., is stable. Since the strange attractor is bounded, system trajectories are confined to a bounded subset of the state space. One of such trajectories is shown as thousands of points that flesh out the strange attractor. In Figure 8, the strange attractor no longer exists, and is replaced by a transient chaotic motion followed by passage of system trajectories near the low voltage saddle point, after which the trajectories follow the other branch of the unstable manifold of the saddle point. This results in a sharp decrease ("collapse") in the system voltage. Figure 9 represents a time simulation of V vs. time for the same value of Q_1 as that of Figure 8. The transient chaotic behavior and the pronounced collapse are illustrated in Figure 9. (Note the small magnitude of the transient oscillatory behavior.)

6 Related Voltage Collapse Phenomena

In this section, we discuss the mechanism of voltage collapse in the example of Dobson and Chiang [3]. That example also admits strange attractors (as reported in [8], [7] and [5]). In that example, the strange attractor nearest the first Hopf instability disappears as Q_1 is quasistatically increased. Unlike the analogous observation in this paper, however, the disappearance occurs prior to the first Hopf bifurcation. Thus in the model of [3], voltage collapse can follow two different routes. In the interval of parameter values $10.85... < Q_1 < 10.89434...$, hysteresis is present, i.e., in addition to the stable equilibrium, there is another coexisting attractor (see [5]). The coexisting attractor, depending on the parameter value, is either a stable limit cycle or a strange attractor. In this interval, if the system is perturbed away from the stable equilibrium, the system may still operate on the coexisting attractor. Then as Q_1 is quasistatically increased, the coexisting attractor undergoes further bifurcations, including a crisis at $Q_1^* = 10.89434...$ leading to the disappearance of the attractor, at which point voltage collapse occurs. On the other hand, even after the disappearance of the strange attractor, the nominal equilibrium of the system is still stable until the Hopf bifurcation. Hence another possible mechanism of voltage collapse, in this example, is linked to the subcritical Hopf bifurcation as suggested in [5]. As Q_1 passes the Hopf bifurcation point, the excursion of voltage exhibits increasing oscillations and then a sharp decrease. Note that subcritical Hopf bifurcation is a form of catastrophic bifurcation. Hence in this example, voltage collapse is triggered either by a boundary crisis or by a catastrophic Hopf bifurcation.

Finally, we remark that the crisis leading to the disappearance of the strange attractor is different from the one discussed in previous section. In the model of this paper, the crisis occurs as the strange attractor collides with the low voltage saddle point. In the example of [3], however, analysis and computations show that the crisis does not involve the low voltage saddle point. We surmise that it is rather the unstable limit cycle born through the subcritical Hopf bifurcation that collides with the strange attractor. Nevertheless, the mechanism is still a boundary crisis.

7 Conclusions

Voltage collapse is triggered by catastrophic bifurcations, in particular the blue sky bifurcations, for sample power system models. These blue sky bifurcations include the blue sky bifurcation of a limit cycle and the boundary crisis of a strange attractor. Boundary crises of stranger attractors near the collapse have indeed been identified, a numerical example in which such a bifurcation triggers the collapse has been given. The importance of these bifurcations and of the boundary crisis (or chaotic blue sky bifurcation) to the voltage collapse phenomenon have been argued. Moreover, the results serve to reconcile two seemingly contradictory pieces of evidence on the nature of voltage collapse, one based on analysis and the other on empirical evidence.

Acknowledgment

The authors are grateful to Drs. R. A. Adomaitis, J. C. Alexander and H.-C. Lee for several valuable discussions. This research has been supported in part by the NSF's Engineering Research Centers Program: NSFD CDR-88-03012, by NSF Grant ECS-86-57561, by the TRW Foundation, and by the Power Engineering Program at the Dept. of Electrical Engineering, University of Maryland.

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