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Adaptive Output Tracking of Invertible MIMO Nonlinear Systems

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Abstract

In this paper we discuss some initial results on the adaptive tracking of MIMO nonlinear systems which do not have a well defined vector relative degree. First, we consider systems that are right-invertible with linear parametric uncertainty in their dynamics. Second, we consider the case where the system is not necessarily invertible. Simulation results for both schemes are presented.

I. Introduction

There has been considerable research on the application of nonlinear adaptive control theory for improving the feedback linearization in the input-output response of nonlinear systems under parameter uncertainties. Most of the current research is based on linearization by means of coordinate changes and assumes the existence of a (vector) relative degree at a point of interest. The nonlinear systems to which these systems can be applied are characterized by very restrictive coordinate-free geometric conditions [KKM91a, KKM91b, TKKS91, CB90, Akh89, KKM89, MKK90, TKMK89, SI89, NA88, SC86], with primary focus on SISO systems.

It is known that the possibility of using state feedback for input-output linearization is not restricted to systems with a certain relative degree, but holds for a broader class of nonlinear systems [Isi89]. In particular, one can utilize the well-known structure algorithm developed by Hirschorn and Singh for the inversion of multi-input multi-output nonlinear systems and construct a right-inverse system. The inverse system then can be used as a decoupling prefilter that produces the input to the original system such that the outputs track a desired path. Moreover, using output feedback and precompensation, asymptotic functional reproducibility can be achieved. In this paper, we present a new adaptive control scheme for MIMO nonlinear systems which is based on the inversion algorithm developed by Hirschorn [Hir79] and Singh [Sin81].

II. Adaptive Control of Invertible MIMO Nonlinear Systems

Consider the following nonlinear system with m inputs and l outputs with $l \leq m$:

$$\begin{aligned} \dot{x}(t) &= A(x) + \sum_{i=1}^{m} u_i \cdot B_i(x) \qquad x \in M \\ y(t) &= C(x(t)) \end{aligned} \tag{1}$$

where the state space is a connected, n-dimensional, real analytic manifold; $A, B_i \in V(M)$ is the real vector space of the real analytic vector fields on M, $u_i \in U$ is the class of real analytic functions from $[0\ \infty)$ into $\Re$, and $C(\cdot) : M \rightarrow \Re^l$ is a real analytic mapping. The output of this system can be made to track various signals depending on controls u_i and the choice of initial states. We are interested in deriving a control law $u = (u_1, u_2, \ldots, u_m)$ for asymptotic tracking of a given signal $y_m = f(\cdot)$ under parametric uncertainty in (1) so that the output $y(t, u, x_0)$ of (1) converges to y_m as $t \rightarrow \infty$.

The inversion algorithm, developed by Hirschorn [Hir79] and Singh [Sin81], gives a systematic scheme to obtain a sequence of systems associated with (1). These systems are derived by performing a series of simple operations such as differentiation, row ordering, and row reduction on the output $y(\cdot)$ of system (1). In what follows, we skip the details of the algorithm and summarize the end results. Associated with (1), we construct a sequence of systems in the form of the following system k:

$$\begin{aligned} \dot{x}(t) &= A(x) + B(x).u \qquad x \in M_k \\ z_k(t) &= C_k(x) + D_k(x).u \end{aligned} \tag{2}$$

where $B(x) = (B_1, B_2, \ldots, B_m)$, M_k is an open dense submanifold of M, $D_k(x)$ has all but the first r_k rows zero and has rank r_k for all $x \in M_k$. The tracking order β of the system (1) is defined as the least positive integer k such that $r_k = l$ or $\beta = \infty$ if $r_k < l$ for all $k > 0$. Hence, $D_\beta(x)$ is an $l \times m$ matrix with rank l ($l \leq m$) for all $x \in M_\beta$. Therefore, if $\beta < \infty$, any given analytic function $f(\cdot)$ is functionally reproducible in the sense of [Sin84] by system β. $z_k(t)$ is partitioned in the form:

$$z_k(t) = \begin{bmatrix} \bar{z}_k(t) \\ \hat{z}_k(t) \end{bmatrix} = \begin{bmatrix} \bar{C}_k(x) \\ \hat{C}_k(x) \end{bmatrix} + \begin{bmatrix} D_{k1}(x) \\ 0 \end{bmatrix} .u(t) \tag{3}$$

where rank $D_{k1}(x) = r_k$, for all $x \in M_k$, and $\bar{z}_k(t)$ and $\bar{C}_k(x)$ consist of the first r_k elements of $z_k(t)$ and C_k. $z_k(t)$ can also be written in terms of the derivatives of the output $y(t, u, x_0)$ up to kth order:

$$z_k(t) = \begin{bmatrix} \bar{z}_k(t) \\ \hat{z}_k(t) \end{bmatrix} = \begin{bmatrix} H_k(x) \\ J_k(x) \end{bmatrix} Y_k(t) \tag{4}$$

where:

$$Y_k(t) = [(y^{(1)})^T, (y^{(2)})^T, \ldots, y^{(k)})^T]^T$$

The system β is:

$$\begin{aligned} \dot{x}(t) &= A(x) + B(x).u \qquad x \in M_\beta \\ z_\beta(t) &= C_\beta(x) + D_\beta(x).u \end{aligned} \tag{5}$$

with

$$z_\beta(t) = H_\beta(x) \cdot Y_\beta(t) \tag{6}$$

and $J_\beta(x) = 0$. We can rewrite this as:

$$z_\beta(t) = N(x) \begin{bmatrix} y_1^{(n_1)} \\ \vdots \\ y_l^{(n_l)} \end{bmatrix} + M(x)\tilde{y} \tag{7}$$

where n_i and N_i are the lowest and highest order derivatives of $y^{(i)}$ appearing in (6),

$$\tilde{y} = \left[y_1^{(n_1+1)}, \cdots, y_1^{(N_1)}, y_2^{(n_2+1)}, \cdots, y_l^{(N_l)}\right]^T \tag{8}$$

and $N(x)$ is an $l \times l$ nonsingular matrix with determinant of $N(x) = \pm 1$ for all $x \in M_\beta$.

With perfect knowledge of A, B and C in (1), (5) and (7) can be utilized to achieve asymptotic tracking by applying the following control law introduced in [Sin84]:

$$u(t) = D_\beta^\dagger(x) \cdot \{-C_\beta(x) + M(x)\tilde{y} + N(x) \cdot K\} \tag{9}$$

where:

$$K \triangleq \begin{bmatrix} {y_m}^{(n_1)} + \sum_{j=0}^{n_1-1} p_{1j}({y_m}_1^{(j)} - y_1^{(j)}) + v_1(t) \\ \vdots \\ {y_m}^{(n_l)} + \sum_{j=0}^{n_l-1} p_{lj}({y_m}_l^{(j)} - y_l^{(j)}) + v_l(t) \end{bmatrix}$$

and $D_\beta^\dagger(x)$ is the pseudoinverse of $D_\beta(x)$, p_{ij} are some constant coefficients, and $v_i(t)$ is a servocompensator of the form:

$$\dot{v}_i = \gamma_{i0}({y_m}_i(t) - y_i(t))$$

for robustness against disturbances in the system. Coefficients p_{ij} and γ_{i0} are chosen such that all the roots of the corresponding characteristic polynomial have negative real parts:

$$\sum_{j=0}^{n_i+1} \gamma_{ij} \cdot s^j = 0 \qquad i = 1, \ldots, l$$

with $\gamma_{ij} = p_{i,j-1}, j = 1, \ldots, n_i + 1$.

Control law (9) does not guarantee the internal stability of system (1) under this feedback. For the states remain bounded, all the unobservable modes of the system under such feedback must remain stable, and of course, all the states must remain in M_β. These conditions must also hold for adaptive version of the above control law discussed bellow.

We are interested in solving the decoupling/tracking problem under parametric uncertainty in the original system, *i.e.* in $A(x), B(x)$, or $C(x)$. Consider system (1) under parametric uncertainty:

$$\begin{aligned} \dot{x}(t) &= A(x,\theta) + \sum_{i=1}^{m} u_i \cdot B_i(x,\theta) \qquad x \in M \\ y(t) &= C(x(t),\theta) \end{aligned} \tag{10}$$

where θ represents the vector containing unknown parameters. Recall that following the above inversion algorithm, system (k) in (2) was obtained by a sequence of linear operations (row ordering and reduction) on the original system. Hence, the β system under parametric uncertainties will be:

$$\begin{aligned} \dot{x}(t) &= A(x,\theta) + B(x,\theta) \cdot u \qquad x \in M_\beta \\ z_\beta(t) &= C_\beta(x,\bar{\theta}) + D_\beta(x,\bar{\theta})u \end{aligned} \tag{11}$$

where $\bar{\theta}$ is now possibly a new vector of unknown constants that is related to the original vector θ, but it may be of higher dimension, and

$$z_\beta(t) = H_\beta(x(t),\bar{\theta}) \cdot Y_\beta(t) \tag{12}$$

which may be rewritten in the form:

$$z_\beta(t) = N(x(t),\bar{\theta}) \cdot \hat{y} + M(x(t),\bar{\theta}) \cdot \tilde{y} \tag{13}$$

where $\tilde{y}$ was defined in (8) and:

$$\hat{y} = \left[y_1^{(n_1)}, \cdots, y_l^{(n_l)}\right]^T \tag{14}$$

and, as before, n_i and N_i are the lowest and highest order derivatives of $y^{(i)}$ appearing in (12). Recall also that $N(x(t),\bar{\theta})$ is an $l \times l$ nonsingular matrix with determinant of $N(x(t),\bar{\theta}) = \pm 1$ for all $x \in M_\beta$. We now make the following assumption, which together with the algorithmic assumptions made above ($l \leq m$ and $\beta < \infty$) represents the class of smooth nonlinear MIMO systems that our adaptive tracking scheme is applicable to.

Assumption 1 *The vector fields $C_\beta(x(t),\bar{\theta})$, $D_\beta(x(t),\bar{\theta})$ and $H_\beta(x(t),\bar{\theta})$ in (11) and (12) depend linearly on the unknown parameters $\bar{\theta}$.*

Next, we introduce the following control law which is the same as the control law introduced in [Sin84] for asymptotic reproducibility of nonlinear systems, except that the uncertain parameters θ in system (10) have been replaced by the adjustable parameters $\hat{\theta}$ that are on line estimates of the true parameters $\bar{\theta}$ with some updating rules yet to be determined:

$$u(t) = D_\beta^\dagger(x,\hat{\theta}) \cdot \left[-C_\beta(x,\hat{\theta}) + M(x,\hat{\theta})\tilde{y} + N(x,\hat{\theta}) \cdot K\right] \tag{15}$$

where:

$$K = \begin{bmatrix} y_{m1}^{(n_1)} + \sum_{j=0}^{n_1-1} p_{1j}(y_{m1}^{(j)} - y_1^{(j)}) \\ \vdots \\ y_{ml}^{(n_l)} + \sum_{j=0}^{n_l-1} p_{lj}(y_{ml}^{(j)} - y_l^{(j)}) \end{bmatrix} \tag{16}$$

where $D_\beta^\dagger(x)$ is the pseudoinverse of $D_\beta(x)$, and p_{ij} are some constant coefficients such that the roots of the corresponding characteristic polynomials have negative real parts. Using Lyapunov stability theory, we will now derive a suitable updating rule for the adjustable parameter vector $\bar{\theta}$ such that output tracking is achieved.

From assumption (1), we have:

$$\begin{aligned} C_\beta(x(t),\bar{\theta}) &= \sum_{i=1}^{p} C_{\beta i}(x)\cdot\bar{\theta}_i + C_{\beta 0}(x) = \tilde{C}_\beta(x)\cdot\bar{\theta} + C_{\beta 0}(x) \\ D_\beta(x(t),\bar{\theta})\cdot u &= \sum_{i=1}^{p} D_{\beta i}(x,u)\cdot\bar{\theta}_i + D_{\beta 0}(x,u) = \tilde{D}_\beta(x,u)\cdot\bar{\theta} + D_{\beta 0}(x,u) \\ M(x(t),\bar{\theta})\cdot\tilde{y} &= \sum_{i=1}^{p} M_i(x,\tilde{y})\cdot\bar{\theta}_i + M_0(x,\tilde{y}) = \tilde{M}(x,\tilde{y})\cdot\bar{\theta} + M_0(x,\tilde{y}) \\ N(x(t),\bar{\theta})\cdot\hat{y} &= \sum_{i=1}^{p} N_i(x,\hat{y})\cdot\bar{\theta}_i + N_0(x,\hat{y}) = \tilde{N}(x,\hat{y})\cdot\bar{\theta} + N_0(x,\hat{y}) \end{aligned} \tag{17}$$

Substituting (15) into (11) and using (13) for $z_\beta(t)$ gives:

$$N(x(t),\bar{\theta})\hat{y} + M(x(t),\bar{\theta})\tilde{y} = C_\beta(x(t),\bar{\theta}) + D_\beta(x(t),\bar{\theta})u$$

$$N(x,\hat{\theta})[K-\hat{y}] = \tilde{C}_\beta(x)\cdot\phi + \tilde{D}_\beta(x,u)\cdot\phi - \tilde{M}(x,\tilde{y})\cdot\phi - \tilde{N}(x,\hat{y})\cdot\phi$$

$$N(x,\hat{\theta})[K-\hat{y}] = W^1(x,u,\tilde{y},\hat{y})\cdot\phi \tag{18}$$

where $\phi = \hat{\theta} - \bar{\theta}$. Since $N(x,\hat{\theta})$ is nonsingular by construction with determinant ± 1, we have:

$$[K-\hat{y}] = N^{-1}(x,\hat{\theta})\cdot W^1(x,u,\tilde{y},\hat{y})\cdot\phi = W^2(x,u,y^{(i)},\hat{\theta})\cdot\phi \tag{19}$$

Now let $\epsilon = [e_1, \dot{e}_1, \ldots, e_1^{(n_1-1)}, e_2, \ldots, e_l^{(n_l-1)}]^T$ and choose p_{ij} such that the corresponding characteristic polynomials $\sum_{j=0}^{n_i-1} p_{ij} \cdot s^j = 0$ are asymptotically stable. Consider the following Lyapunov candidate function:

$$V = \epsilon^T R \epsilon + \phi^T \Omega \phi$$

where $\Omega^T = \Omega = diag(1/g_i) > 0$ and $R = R^T > 0$ is the solution of the Lyapunov equation:

$$R \cdot P + P^T \cdot R = -Q$$

for some $Q = Q^T > 0$ and:

$$P = diag(P_i)$$

$$P_i = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -p_{i0} & -p_{i1} & & \ldots & -p_{i,n_i-1} \end{pmatrix}$$

Note that:

$$\dot{\epsilon} = P \cdot \begin{bmatrix} e_1 \\ \vdots \\ e_1^{(n_1-1)} \\ e_2 \\ \vdots \\ e_l^{(n_l-1)} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ W_1^2 \\ 0 \\ \vdots \\ W_l^2 \end{bmatrix} \cdot \phi = P\epsilon + W\phi \tag{20}$$

Then, the derivative of V evaluated along the solution trajectories of the error equation (20) is:

$$\dot{V} = -\epsilon^T Q \epsilon + 2\epsilon^T R W(x, u, y^{(i)}, \hat{\theta})\phi + 2\phi\Omega\dot{\phi}$$

Taking parameter update laws as:

$$\dot{\phi} = -\Omega^{-1} \cdot W^T \cdot R \cdot \epsilon \tag{21}$$

gives:

$$\dot{V} = -\epsilon^T Q \epsilon \leq -\gamma \|\epsilon\|^2 \leq 0$$

This proves that V is bounded. Hence e_i and $\hat{\theta}_i$ are bounded, and $\dot{V}$ is bounded and integrable. If, moreover, the system is internally stable, then $\|e_i\| \rightarrow 0$ as $t \rightarrow \infty$. Of course, for the system be internally stable under such feedback, all the unobservable modes must remain stable. In fact, if the zero dynamics of the system is not asymptotically stable then it is possible that for some reference signals, the tracking control law producing a linear input-output response may result in unbounded unobservable states. To achieve asymptotic tracking for *all* reference signals, sufficient conditions are typically too restrict and often hard to meet. Given a specific class of reference signals one might search for *bounded-input bounded-state* (BIBS) property of the unobservable subsystem under the above decoupling feedback control, treating the output y as input. This is a generalization of BIBS assumption in [KKM91b]. This subsystem is obtained using the generalized normal form transformation of [Isi89]. The following theorem summarizes the main result of this section:

Theorem 1 *Suppose that the system described by (10) has a finite tracking order ($\beta < \infty$), has at least as many inputs as there are outputs ($l \leq m$), and that assumption* 1 *holds (linear parameter dependence). Then given any smooth bounded signal $y_m = [y_{m1}, \ldots, y_{ml}]$ with bounded derivatives up to order $n_i - 1$, with the control law (15) and (21), and p_{ij} chosen such that the corresponding characteristic polynomials are asymptotically stable, if* *the resulting unobservable subsystem is BIBS with respect to output as its input, the output $y(\cdot)$ of (10) approaches y_m.*

Theorem (1) is the adaptive version of tracking theorems of [Sin84, Hir81, Sin80] which are based on constructing a right inverse system and hold for finite time. In order to guarantee the output tracking with internal stability, it is possible to modify control law (9) and its adaptive counterpart, (15) and (21), using generalized normal transformation of [Isi89]. This transformation is readily given by the structure algorithm discussed above. Results based on this approach are given in another paper [GB92].

To illustrate the proposed design technique, in the next section we will consider its potential application to the control of nonlinear system arising in the outer-loop design of an aircraft.

III. Applications to Aircraft Control Problem

Consider the nonlinear system arising in the outer-loop design of an aircraft [Ass73, Sin80]:

$$\dot{x} = \begin{bmatrix} x_2 \\ 0 \\ 0 \\ (\frac{-g}{v_0}) \cdot Sin^2(x_1) + x_3 Cos(x_1) \\ (\frac{g}{2v_0}) \cdot Sin(2x_1) + x_3 Sin(x_1) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$y = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix} \tag{22}$$

with:

$$x = (\phi, p, q_w, \gamma, \psi)^T$$

where ϕ is roll angle, p is the roll rate, q_w is the wind referenced pitch rate, γ is the vertical path flight angle, and ψ is the horizontal path flight angle of the airplane. g is the gravitational constant, and v_0 is the air speed. The objective of the outer-loop design is to decouple γ and ψ. It is desired to design a robust control law $u(t) = [u_1, u_2]^T$ such that under parametric uncertainty and slow variations in v_0, γ and ψ will remain decoupled and follow pilot command inputs.

Let's define $\theta_1 = (g/2v_0)$ and using the structure algorithm, one gets:

$$\begin{aligned} z_3(t) &= \begin{bmatrix} -x_2 \cdot (2\theta_1 Sin(2x_1) + x_3 Sin(x_1)) \\ x_3 {x_2}^2 \, tan(x_1) sec(x_1) \end{bmatrix} \\ &\quad + \begin{bmatrix} cos(x_1) & 0 \\ x_2 sec(x_1) & 2\theta_1 + x_3 sec(x_1) \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ z_3(t) &\triangleq C_3(x, \bar{\theta}) + D_3(x, \bar{\theta}) u \end{aligned} \tag{23}$$

Also:

$$\begin{aligned} z_3(t) &= \begin{bmatrix} 1 & 0 \\ -x_2 sec^2(x_1) & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1^{(2)} \\ y_2^{(3)} \end{bmatrix} + \begin{bmatrix} 0 \\ -tan(x_1) \end{bmatrix} y_1^{(3)} \\ z_3(t) &\triangleq N(x)\hat{y} + M(x)\tilde{y} \end{aligned} \tag{24}$$

In view of (15), let's choose the following control law:

$$u(t) = D_3^{-1}(x,\hat{\theta}) \cdot \left[-C_3(x,\hat{\theta}) + M(x)\cdot\tilde{y} + N(x)\cdot K\right] \tag{25}$$

where:

$$D_3(x,\hat{\theta}) \triangleq \begin{bmatrix} cos(x_1) & 0 \\ x_2 sec(x_1) & 2\hat{\theta}_1 + x_3 sec(x_1) \end{bmatrix}$$

$$C_3(x,\hat{\theta}) \triangleq \begin{bmatrix} -2\hat{\theta}_1 x_2 Sin(2x_1) + x_2 x_3 Sin(x_1) \\ x_3 {x_2}^2\, tan(x_1) sec(x_1) \end{bmatrix} \tag{26}$$

and:

$$K = \begin{bmatrix} {y_m}_1^{(2)} + \alpha_{11}\dot{e}_1 + \alpha_{10}e_1 \\ {y_m}_2^{(3)} + \alpha_{22}\ddot{e}_2 + \alpha_{21}\dot{e}_2 + \alpha_{20}e_2 \end{bmatrix} \tag{27}$$

where $e \triangleq y_m - y$. Applying (25) to (23), using (24) for $z_3(t)$ and regrouping terms gives:

$$\begin{bmatrix} \ddot{e}_1 + \alpha_{11}\dot{e}_1 + \alpha_{10}e_1 \\ e_2^{(3)} + \alpha_{22}\ddot{e}_2 + \alpha_{21}\dot{e}_2 + \alpha_{20}e_2 \end{bmatrix} = \begin{bmatrix} -2x_2 Sin(2x_1) \\ -2{x_2}^2 sin(2x_1) sec^2(x_1) + 2u_2 \end{bmatrix} \cdot \phi \tag{28}$$

where $\phi \triangleq \hat{\theta}_1 - \theta_1$. Now let $\epsilon = [e_1, \dot{e}_1, e_2, \dot{e}_2, \ddot{e}_2]^T$ and choose α_{ij} such that the corresponding characteristic polynomials are asymptotically stable, for example:

$$\alpha_{11} = 20, \alpha_{10} = 100, \alpha_{22} = 30, \alpha_{21} = 300, \alpha_{20} = 1000$$

which results in five poles on $s = -10$. Consider the following Lyapunov candidate function:

$$V = \epsilon^T R \epsilon + 1/g \cdot \phi^T \cdot \phi$$

where $g > 0$ is the adaptation gain for parameter θ, and $R = R^T > 0$ is the solution of the Lyapunov equation:

$$R \cdot P + P^T \cdot R = -Q$$

with $Q = Q^T > 0$ chosen for our simulation to be $Q \triangleq 1000 \cdot I_5$ where I_5 is the 5×5 identity matrix, and:

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -100 & -20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1000 & -300 & -30 \end{pmatrix} \tag{29}$$

Using (28), we have the following relationship:

$$\dot{\epsilon} = P \cdot \epsilon + \begin{pmatrix} 0 \\ -2x_2 sin(2x_1) \\ 0 \\ 0 \\ -2{x_2}^2 sin(2x_1) sec^2(x_1) + 2u_2 \end{pmatrix} \cdot \phi \triangleq P \cdot \epsilon + W(x, u) \cdot \phi \tag{30}$$

The derivative of V along the solution trajectories of (22) is:

$$\dot{V} = -\epsilon^T Q \epsilon + 2\epsilon^T \cdot R \cdot W(x, u) \cdot \phi + 2g\phi \cdot \dot{\phi}$$

The design procedure of last section applied to this system results in the updating law:

$$\dot{\phi} = -1/g \cdot W^T \cdot R \cdot \epsilon \tag{31}$$

The generalized normal form for this system is obtained with new coordinates as ($\xi = (\xi_1, \cdots, \xi_5)^T$):

$$\xi = (h_4, f_4, h_5, f_5, \psi)^T$$

with $\psi \triangleq -tan(X_1) \cdot \mathcal{L}f_4 + \mathcal{L}f_5$ where the transformation Φ given by $x \rightarrow \xi$ is a local diffeomorphism. This system does not have any zero dynamics and the BIBS condition of theorem (1) is automatically satisfied. Hence, since $\dot{e}_i$ and e_i are bounded and $\|e_i\| \in \mathcal{L}_\infty$, we conclude that $\|e_i\| \to 0$ as $t \to \infty$, and consequently: $y_i \to y_{m_i}$ as $t \to \infty$.

Figures (1) and (2) show the vertical and horizontal flight path angles tracking the command inputs. Figure (3) indicates that the errors (ϵ) converge to zero. Figure (4) shows the response (all the states) of the closed-loop aircraft to pilot command inputs. It is clear that the responses are stable and decoupled, and adaptive output tracking is achieved.

IV. Noninvertible Systems

We now extend our results to MIMO nonlinear systems that do not necessarily have a finite tracking order β, hence not invertible in the sense defined in [Hir79, Sin81]. Consider the nonlinear system given in (10) with m inputs and l outputs and assume $l \leq m$. We obtain the following system by differentiating y:

$$\begin{aligned} \frac{dy}{dt} &= \dot{y}(t) = dc_{x(t)}(\dot{x}(t)) \\ &= dc_{x(t)}\left(A(x,\theta) + \sum_{i=1}^{m} u_i \cdot B_i(x,\theta)\right) \\ &= (AC)(x,\bar{\theta}) + \sum_{i=1}^{m} u_i (B_i C)(x,\bar{\theta}) \end{aligned} \tag{32}$$

Define $D(x,\bar{\theta}) \triangleq [B_1C(\cdot), B_2C(\cdot), \ldots, B_mC(\cdot)]$, an $l \times m$ matrix for each $x \in M$, and with this notation we write:

$$\dot{y} = AC(x,\bar{\theta}) + D(x,\bar{\theta})u \qquad x \in M \tag{33}$$

where θ represents the vector containing unknown parameters in the system, $\bar{\theta}$ is a new vector of unknown constants that is related to the original vector θ in (10), possibly of higher dimension, and it is assumed that $\bar{\theta}_i^{\prime}s$ appear linearly in (33). Note that this was the first step in the structure algorithm used in section (II) with $r_1 \triangleq max_{x \in M}\{rankD(x, \bar{\theta})\}$. If $r_1 = l$, then we have the case where $\beta = 1$, and one can apply the design scheme developed in section (II) to this system since the $l \times m$ matrix $D(x, \bar{\theta})$ is of full rank on $M_1 \triangleq \{x : rankD(x, \bar{\theta}) = r_1\}$ and $D \cdot D^{\dagger} = I$ on M_1, where $D^{\dagger}$ is the pseudoinverse of D. In the case where $r_1 < l$, the structure algorithm would continue to the next step as explained before since $D^{\dagger}$ no longer exist. However, consider a matrix $\hat{D}(x, \bar{\theta}, \hat{\vartheta})$ where ϑ is a vector of some "fictitious" parameters appearing linearly in $\hat{D}(x, \bar{\theta}, \hat{\vartheta})$ and are injected in $D(x, \bar{\theta})$ such that $rank\hat{D}(x, \bar{\theta}, \hat{\vartheta}) = l$ for all $x \in M$ with $\hat{\vartheta}$ an estimate of ϑ. So: $\hat{D}(x, \bar{\theta}, 0) = \hat{D}(x, \bar{\theta})$. Define $\hat{\theta}$ to be the estimates of the vector $\eta = [\bar{\theta}^T, \vartheta^T]^T$ and $\phi \triangleq \hat{\theta} - \eta$. The idea now is to find updating laws for $\hat{\theta}$ such that $y_i \to y_{m_i}$ while $rank\ \hat{D}(x, \hat{\theta})$ remains constant. To do this we will assign small values to the gains corresponding to $\hat{\vartheta}$, the estimates of the fictitious parameters, so that they change very slowly. Moreover, one can use a suitable projection algorithm in order to prevent the convergence of these parameters to their true values (usually zero) and $rank\ \hat{D}(x, \hat{\theta})$ remains constant. The control will then be based on estimates of the fictitious parameters with updating rules determined such that the stability is preserved and the tracking is achieved. This procedure can also be viewed as a dynamic state feedback control.

With this in mind consider (33) and in view of (15)

$$u(t) = \hat{D}^{\dagger}(x, \hat{\theta}) \cdot \left[-C(x, \hat{\theta}) + \ddot{y}_m + \alpha_1 \dot{e} + \alpha_0 e\right] \tag{34}$$

where $\hat{\theta}$ are estimated parameters with updating laws to be determined, and $e \triangleq y_m - y$.

Applying (34) to (33) gives:

$$\begin{aligned} \ddot{e} + \alpha_1 \dot{e} + \alpha_0 e &= \left[\hat{D}(x,\hat{\theta}) - D(x,\theta)\right] \cdot u + C(x,\hat{\theta}) - C(x,\theta) \\ \ddot{e} &= -\alpha_1 \dot{e} - \alpha_0 e + W^1(x,u,\hat{\theta}) \cdot \phi \end{aligned} \tag{35}$$

We have:

$$\dot{\epsilon} = P \cdot \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \vdots \\ \dot{e}_l \end{bmatrix} + \begin{bmatrix} 0 \\ W_1^1 \\ 0 \\ \vdots \\ W_l^1 \end{bmatrix} \cdot \phi = P\epsilon + W\phi \tag{36}$$

where $\epsilon \triangleq [e_1, \dot{e}_1, e_2, \ldots, \dot{e}_l]^T$ and:

$$P = diag(P_i)$$

$$P_i = \begin{pmatrix} 0 & 1 \\ -\alpha_0 & -\alpha_1 \end{pmatrix}$$

Consider the following Lyapunov candidate function:

$$V = \epsilon^T R \epsilon + \phi^T \Omega \phi$$

where $\Omega^T = \Omega = diag(1/g_i) > 0$ and $R = R^T > 0$ is the solution of the Lyapunov equation:

$$R \cdot P + P^T \cdot R = -Q$$

for some $Q = Q^T > 0$. Taking the derivative of V evaluated along the solution trajectories of (36) and the updating laws:

$$\dot{\phi} = -\Omega^{-1} \cdot W^T \cdot R \cdot \epsilon \tag{37}$$

gives:

$$\dot{V} = -\epsilon^T Q \epsilon \leq -\gamma \|\epsilon\|^2 \leq 0$$

Hence, since $\dot{e}_i$ and e_i are bounded and $\|e_i\| \in \mathcal{L}_\infty$, we conclude that $\|e_i\| \to 0$ as $t \to \infty$. Consequently: $y_i \to y_{m_i}$ as $t \to \infty$.

Of course, as in any adaptive control strategy, the matrix $\hat{D}(x, \hat{\theta})$ has to be monitored, on line, to remain nonsingular as long as there are nonzero errors in the system. Assuming these adjustable parameters for the fictitious parameters do not converge to their true values (zero), we can state the following theorem:

Theorem 2 *Suppose that (33) is linear in $\bar{\theta}_i$. Then given any smooth bounded signal $y_m = [y_{m1}, \ldots, y_{ml}]$ with bounded derivatives such that the estimates of ϑ_i do not converge to zero (nonpersistently exciting signal), with the control law (37) and (34), $n_i \geq 2$, and p_{ij} chosen such that the corresponding characteristic polynomials are asymptotically stable, the output $y(\cdot)$ of (10) tracks y_m, for all $x_0 \in M_\beta$.*

Note that if one chooses $n_i = 2$ in (34), then the control law (34) with (37) is of a PID type controller coupled with state feedback. In order to have asymptotic tracking with internal stability, x_0 and y_m need to be such that all the states remain in a compact subset of M_β in the operating region of interest. Sufficient conditions to globally achieve this are typically very restrictive in general. The application of this theorem is, however, more useful for a given nonlinear system where the boundedness of states can be shown explicitly under this feedback, or where the operating region of interest is known to be contained in the domain of internal stability of our system. An example, where this is easily the case is treated in the next section. It is also possible to use other Lyapunov type functions to guarantee that the estimates ϑ_i do not converge to zero. We refer to [GB92] for further detail.

V. Example 2

Although the proposed schemes here are intended more for MIMO nonlinear systems, for the sake of comparison and to illustrate the design procedure, we apply the scheme developed in the last section to the problem considered in [KKM91b] in which the output y of the system:

$$\begin{aligned} \dot{z}_1 &= z_2 + \theta z_1^2 \\ \dot{z}_2 &= z_3 + u \\ \dot{z}_3 &= -z_3 + y \\ y &= z_1 \end{aligned} \tag{38}$$

is required to track the reference signal $y_r = 0.1sin(t)$. Differentiating y gives (from (32)):

$$\dot{y} = z_2 + \theta z_1^2 \tag{39}$$

where $D \equiv 0$, and we introduce a fictitious parameter in (39) with true value zero and estimate $\hat{\theta}_2$, and apply in view of (34):

$$u(t) = \frac{1}{\hat{\theta}_2}\left\{-z_2 - \hat{\theta}_1 z_1^2 + \dot{y}_m + \alpha(y_m - y)\right\} \tag{40}$$

to (38) with $\alpha > 0$. We have from (35) and (39):

$$\begin{aligned} \dot{y} &= z_2 + \theta z_1^2 - \hat{\theta}_2 u + \left\{-z_2 - \hat{\theta}_1 z_1^2 + \dot{y}_m + \alpha(y_m - y)\right\} \\ \dot{e} &= -\alpha e + z_1^2 \phi_1 + u\phi_2 \\ \dot{e} &= -\alpha e + \left[z_1^2, u\right] \cdot \phi \triangleq -\alpha \cdot e + W(z_1, u) \cdot \phi \end{aligned} \tag{41}$$

where $\phi_1 \triangleq (\hat{\theta}_1 - \theta), \phi_2 \triangleq \hat{\theta}_2$, and $\phi \triangleq [\phi_1, \phi_2]$. From (37), we have:

$$\begin{aligned} \dot{\phi} &= -\Omega^{-1} \cdot W^T \cdot e \\ \dot{\phi}_1 &= -g_1 z_1^2 e = -g_1 y^2 (y_m - y) \\ \dot{\phi}_2 &= -g_2 u e \end{aligned} \tag{42}$$

Clearly boundedness of $y = z_1$ is guaranteed. In fact, in systems of this type, where all the observable states are chained as in [KKM91b], it suffices to go up to the first derivative of the output in the control law. In practice, this can be easily and effectively implemented using PID controllers. Note that since the unobservable subsystem z_3 is BIBS with y regarded as input, boundedness of z_3 follows. Moreover, the boundedness of z_2 is clear from (38) with the control law (40) in place. Hence, $\dot{e}$ is bounded and $y \to y_m$ as $t \to \infty$ for any initial condition $y(0)$.

For simulation, we chose $\alpha = 100$, $g_1 = 1, g_2 = 10^{-4}$, $\theta = 1, \hat{\theta}_1(0) = 0.6$, and $\hat{\theta}_2(0) = 0.03$. The results of the simulation, shown in Fig(5) and Fig(6), indicate that the tracking error converges to zero as predicted. Compare to the results claimed in [KKM91b], the rate of convergence is fast, and the results hold for any initial conditions as shown in [KKM91b]. In [KKM91b], this problem was also solved, for comparison, using another adaptive scheme developed in [SI89], and it was shown that the later scheme works only locally when $e(0) < 0.45$.

VI. Conclusion

In this paper, we have described some initial results of a research on adaptive control in

MIMO nonlinear systems where the vector relative degree is not defined. For right-invertible systems, we utilized the structure algorithm of Hirschorn and Singh for the inversion of the nonlinear input-output map under parametric uncertainty such that adaptive tracking was achieved. For non-invertible systems, we presented an algorithm based on introducing some fictitious parameters and update laws such that tracking was achieved. The resulting control law could also be considered as a dynamic feedback control for the original system.

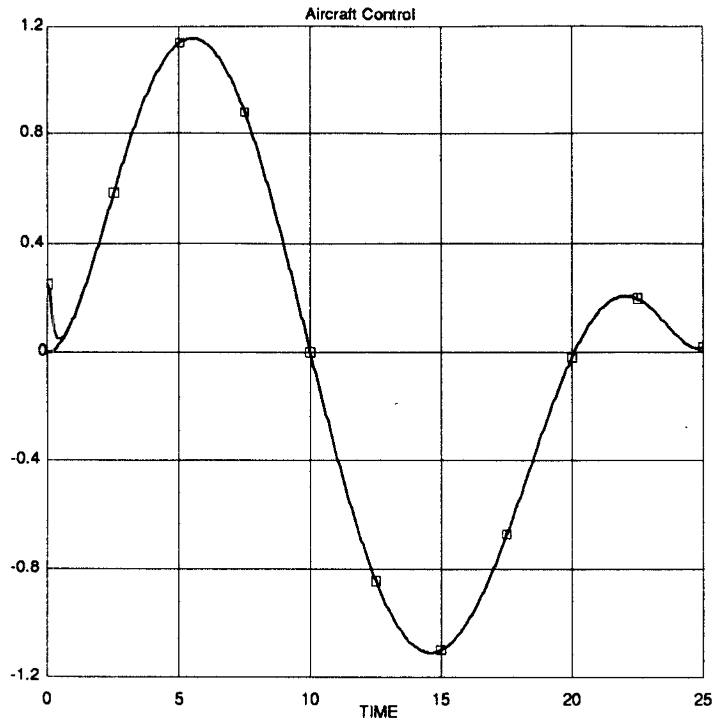

Figure 1: Desired Model and Plant Output Trajectories: y_{m1} and y_1, $e_1(0) = \%20$

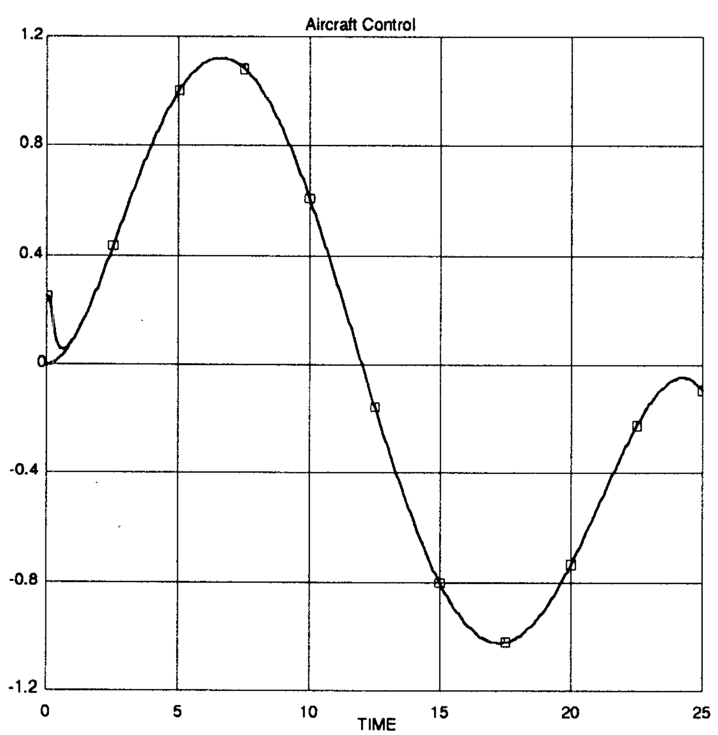

Figure 2: Desired Model and Plant Output Trajectories: y_{m2} and y_2, $e_2(0) = \%20$

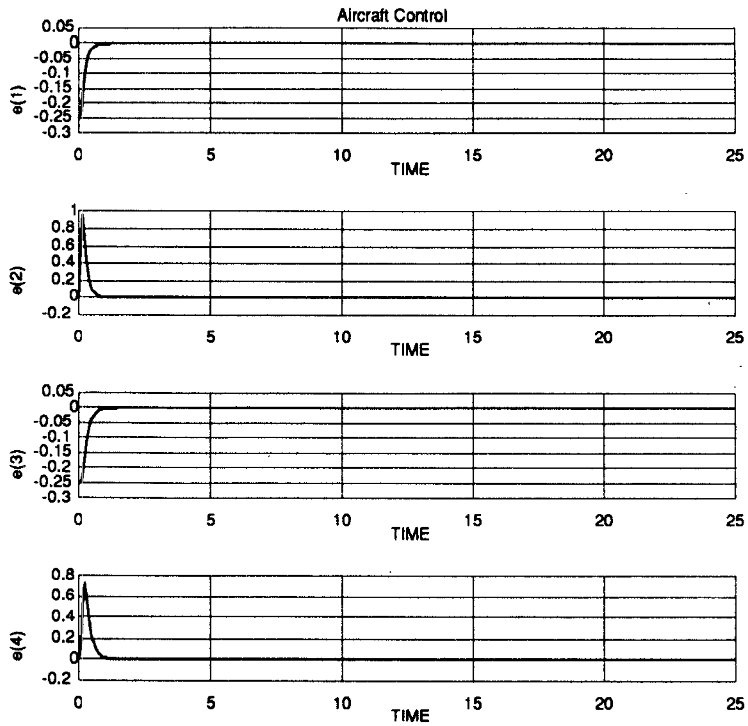

Figure 3: Error Trajectories, $e_i(0) = \%20$

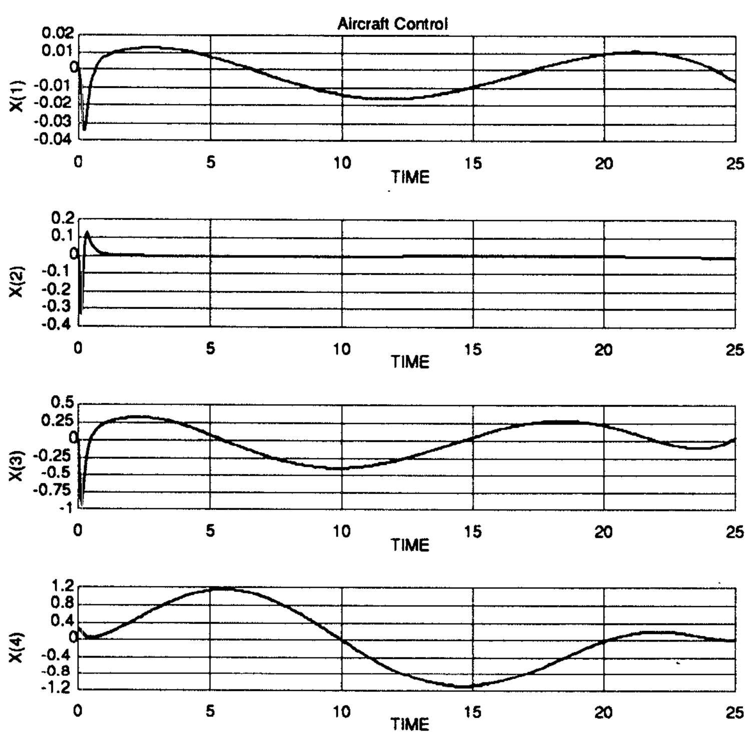

Figure 4: State Trajectories, $e_i(0) = \%20$

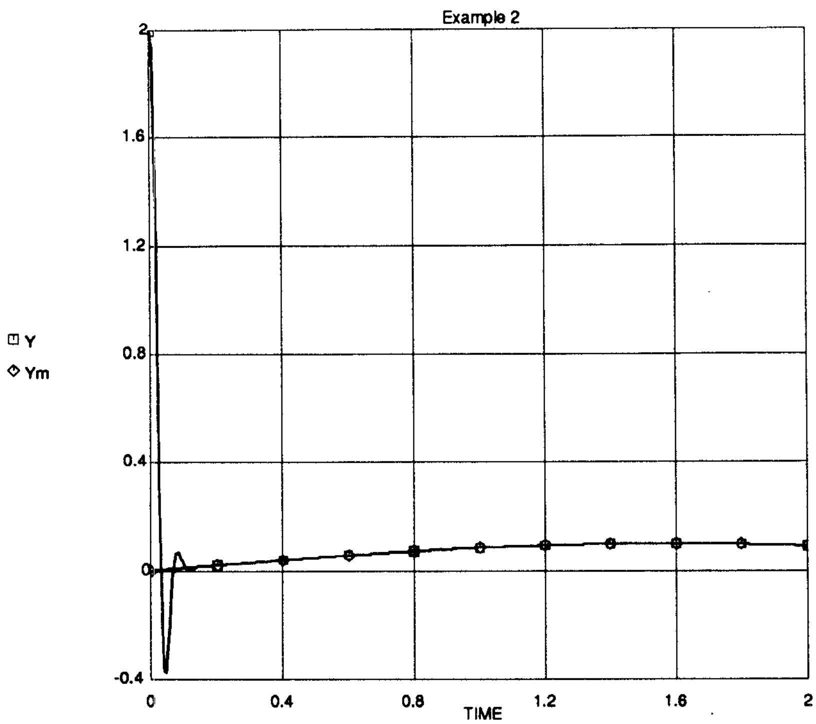

Figure 5: Desired Model and Plant Output Trajectories: y_m and y in example 2 $e(0) = 2$

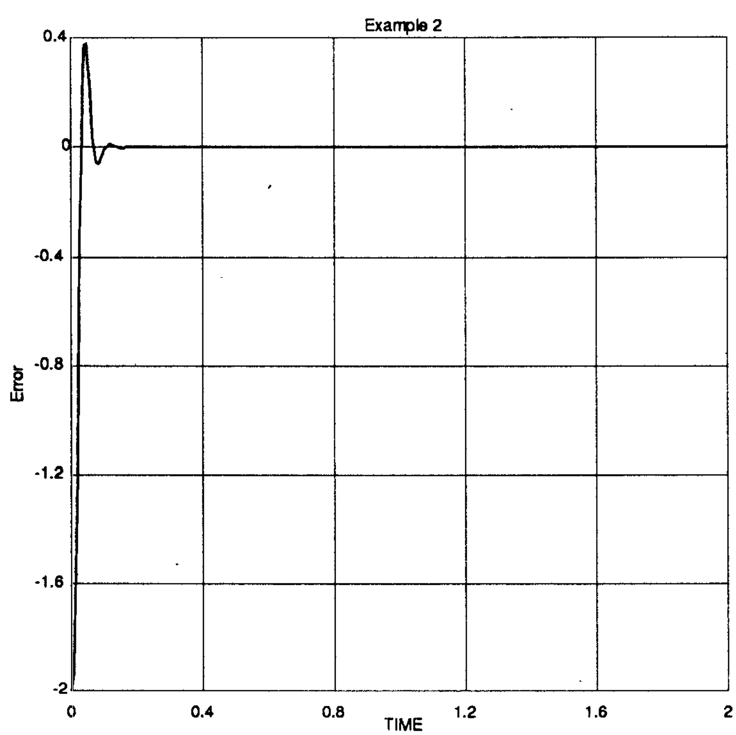

Figure 6: Error Trajectories, in example 2 $e(0) = 2$

References

[Akh89] O. Akhrif. Nonlinear adaptive control with applications to flexible structures. *Ph. D. Dissertation, University of Maryland at College Park*, 1989.

[Ass73] S. J. Asseo. Decoupling of a class of nonlinear systems and its applications to an aircraft control problem. *J. of Aircraft*, 10:739–747, 1973.

[CB90] G. Campion and G. Bastin. Indirect adaptive state feedback control of linearly parametrized nonlinear systems. *Int. J. Adapt. Contr. Signal Proc.*, 4:345–358, September 1990.

[GB92] R. Ghanadan and G. L. Blankenship. Adaptive control of mimo nonlinear systems. *In preparation*, 1992.

[Hir79] R. M. Hirschorn. Invertibility of multivariable nonlinear systems. *IEEE Trans. Automatic Control*, AC-24:855–865, 1979.

[Hir81] R. M. Hirschorn. Output tracking in multivariable nonlinear systems. *IEEE Trans. Automatic Control*, AC-26:593–595, 1981.

[Isi89] A. Isidori. *Nonlinear Control Systems.* Springer Verlag, Berlin, 1989.

[KKM89] I. Kanellakopoulos, P. V. Kokotovic, and R.. Marino. Robustness of adaptive nonlinear control under an extended matching condition. *in Proc. IFAC Symp. Nonlinear Control System Design, Capri, Italy*, pages 192–197, June 1989.

[KKM91a] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse. An extended direct scheme for robust adaptive nonlinear control. *Automatica*, 27:247–255, March 1991.

[KKM91b] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse. Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Trans. Automatic Control*, AC-36, No. 11:1241–1253, 1991.

[MKK90] R. Marino, I. Kanellakopoulos, and P. V. Kokotovic. Adaptive tracking for feedback linearizable siso systems. *Proc. 6th Yale Workshop on Adaptive and Learning Systems, New Haven, CT*, pages 1–6, 1990.

[NA88] K. Nam and A. Arapostathis. A model-reference adaptive control scheme for pure-feedback nonlinear systems. *IEEE Trans. Automatic Control*, 33:803–811, September 1988.

[SC86] J. E. Slotine and J. A. Coetsee. Adaptive sliding controller synthesis for nonlinear systems. *Int. J. Control*, 43:1631–1651, June 1986.

[SI89] S. S. Sastry and A. Isidori. Adaptive control of linearizable systems. *IEEE Trans. Automatic Control*, AC-34:1123–1131, 1989.

[Sin80] S. N. Singh. Decoupling of invertible nonlinear systems with state feedback and precompensation. *IEEE Trans. Automatic Control*, AC-25, No. 6:1237–1239, 1980.

[Sin81] S. N. Singh. A modified algorithm for invertibility in nonlinear systems. *IEEE Trans. Automatic Control*, AC-26:895–598, 1981.

[Sin84] S. N. Singh. Asymptotic reproducibility in nonlinear systems and attitude control of gyrostat. *IEEE Trans. Aerospace and Electronic Systems*, AES-20:94, 1984. No. 2.

[TKKS91] A. Teel, R. Kadiyala, P. V. Kokotovic, and S. S. Sastry. Indirect techniques for adaptive input output linearization of nonlinear systems. *Int. J. Control*, 53:193–222, January 1991.

[TKMK89] D. G. Taylor, P. V. Kokotovic, R. Marino, and I. Kanellakopoulos. Adaptive regulation of nonlinear systems with unmodeled dynamics. *IEEE Trans. Automatic Control*, AC-34:405–412, April 1989.