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The Development of an Atlas of Bevel-Gear-Type Spherical Wrist Mechanisms

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ABSTRACT

In this paper, fundamental functional requirements for the design of bevel-gear-type, spherical wrist mechanisms have been established. Using these functional requirements, all the admissible kinematic structures of spherical wrist mechanisms with eight or less links have been identified and catagorized, and a representative wrist mechanism corresponding to each kinematic structure has been sketched for the purpose of demonstration. Some of these mechanisms have already been used in industrial robots, while others appear to be new and deserve further study.

1. INTRODUCTION

In order to position and orient an object in a three-dimensional space, a manipulator must have at least six degrees of freedom. Usually, the first three degrees of freedom are used for controlling of the position (gross motion), and the last three for controlling of the orientation (local motion). For this reason, the linkage associated with the first three moving links is called the major linkage, or the arm, and the portion associated with the last three moving links is called the minor linkage, or the wrist. Pieper (10) showed that a manipulator with three adjacent joint axes intersecting at a point leads to a closed form solution for the inverse kinematics. Hence, a spherical wrist makes the decoupling of position and orientation possible.

In addition to the need for a robot wrist to perform a spherical motion, recent trend in wrist technology has required the wrist to be dexterous in its workspace. Since 1940s many researchers have devoted themselves in creating simple and cost effective wrist mechanisms which can fulfill the above requirements. Rosheim (12) provides a thorough survey in the current developments of wrist actuactor technology. We may find that it is difficult to design a wrist mechanism which satisfies all the important demands such as dexterity, simplicity, and cost effectiveness. Some wrist mechanisms are claimed to be singularity-free. However, they often suffer from the problems of complicated (heavy) mechanical construction, less stiffness and less precision (8,14). On the other hand, bevel-gear-type wrist mechanisms, such as those used in PUMA, Bendix, Cincinnati-Milacron T3 robots, are considered to be simple, reliable, and cost effective, but they generally contain singularities in their workspace. Furthermore, because the actuactors can be mounted remotely from the wrist center, they suffer less vibration and dynamics problems. We can ask ourselves the following question. For a given number of links, how many spherical bevel-gear-type wrist mechanisms exist? To answer this question, one can resort to the aide of graph theory (5). In what follows, functional requirements for the design of bevel-gear-type, spherical wrists will be established. Then all the admissible kinematic structures of bevel-gear-type, spherical wrist mechanisms will be identified.

2. ROBOT WRIST CHARACTERISTICS

It is helpful to summarize some important characteristics associated with the design of wrist mechanisms. As it is known, dexterity is a very important factor in designing robot wrist mechamism. In addition, compactness and simplicity are also essential requirements. In general, an ideal wrist configuration should possess the following characteristics:

- A minimum of three degrees of freedom
- Small size and light weight
- Remote drive system
- High accuracy and repeatability
- High mechanical stiffness
- Large workspace
- End effector possess a spherical motion
- Low manufacturing cost

In response to the above characteristics, some of them call upon proper dimension of mechanical parts which is known as dimensional synthesis. Many researchers have been working on this area (1,3,6,7). Others call upon proper configuration of the mechanism which is known as structural synthesis.

In this study, we shall focus ourselves on the structural synthesis. Performance evaluation of the mechanisms shall not be discussed here.

We shall concentrate on the creation of bevel-gear-type wrist mechanisms. First, functional requirements for the design of spherical wrist mechanisms will be established. Then these requirements will be used for the selection of gear trains, from an atlas of gear trains developed earlier (16), that are suitable for the design of wrists.

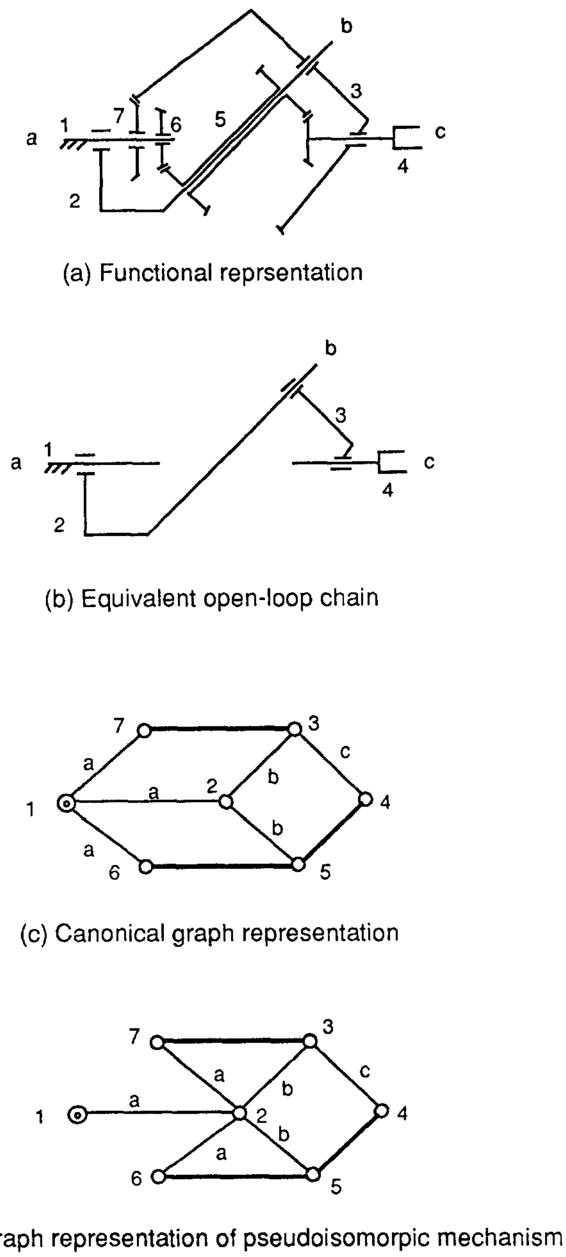
3. FUNDAMENTAL REQUIREMENTS

Before establishing functional requirements for the bevel-gear-type spherical wrist mechanisms, let's examine the topological characteristics associated with an existing one. A typical bevel-gear-type spherical wrist mechanism is the Cincinnati-Milacron 3-Roll wrist designed by Stakehouse (13). Aside from its special arrangement of axis orientation which permits full range of rotation, we shall look into its kinematic model. Figure 1(a) shows the schematic representation; Figure 1(b) shows the equivalent open-loop chain (15), which is obtained by deleting the drive trains from the mechanism; And Figure 1(c) shows the canonical graph representation of the mechanism. In the graph representation, a link is denoted by a vertex and a joint is denoted by an edge. A turning pair is represented by a thin edge and a gear pair by a heavy edge. See Tsai and Lin (16) for the definitions of the graph elements in an epicyclic gear train. A kinematic analysis reveals that the mechanism has three degrees of freedom. The three input links are links 2, 6 and 7 and the output link is link 4.

In the equivalent open-loop chain, four links (including ground link) are serially connected, and there are three axes of rotation intersecting at one common point which is known as the wrist center. Since the equivalent opened-loop chain is imbedded in the mechanism, its graph representation must be a sub-graph of the graph representing the whole mechanism. Examining Figure 1(c), we note that the sub-graph for the equivalent open-loop chain is the thin-edged path, 1-2-3-4 connecting from the ground link to the end-effector. The three labels a, b and c in the thin-edged path represent the locations of the joint axes of rotation in space.

One advantage of the bevel-gear-type spherical wrist is that the actuactors can be located remotely from the wrist center, which makes the wrist mechanism light weight and compact. This is accomplished by the use of gear-trains which permit the transfer of power from the ground level to mid-level and then the end-effector level. As shown in Figure 1(c), there are two gear-edged paths starting from the ground level, one ended at the end-effector level, and another ended at the mid-level. Relative motions of link 4 with respect to link 3, and link 3 with respect to link 2 are accomplished by the two transmission lines defined by the gear-edged paths 7-3 and 6-5-4 (4). Relative motion of link 2 with respect to link 1 is accomplished by driving link 2 directly.

Furthermore, since links 1, 2, 6 and 7 are coaxial, the mechanism shown in Figure 1 can be rearranged into one with an articulation point (16) as shown in Figure 1(d). We concluded that, in general, a three degrees of freedom wrist can be constructed by putting a non-fractionated two-degree-of-freedom gear train in series with a fixed link. The third degree-of-freedom is simply obtained by rotating the two-degree-of-freedom gear train about the fixed link. Thus we can simplify the synthesis problem by evaluating the atlas of two degree-of-freedom gear-trains developed earlier.



(a) Functional reprsentation

(b) Equivalent open-loop chain

(c) Canonical graph representation

(d) Graph representation of pseudoisomorpic mechanism

Figure 1 Kinematic model of the Cincinnati-Milacron 3-roll wrist

From the above discussions, the fundamental requirements for a graph of two-degree-of-freedom gear train to be admissible for the construction of a spherical wrist can be summarized as follows:

R1. There must be three or more vertices connected together with thin edges of the same label. These vertices are potential candidates for input links.

R2. In the graph, there must be at least two heavy-edged paths, called the transmission lines (4), originated from the input vertices. Each input vertex can be the starting point for only one of the transmission lines. On the other hand, two transmission lines can meet at a common vertex other than the input vertex.

R3. There must exist at least three edge levels in one of the thin-edged paths originated from the input vertices.

R4. A link is considered as a redundant link, if removal of the link and its associated joints from the kinematic chain does not change the degrees of freedom of the mechanism. Note that the input links and the end-effector link are not to be removed. We shall consider only those mechanisms with no redundant links.

The first requirement has been derived from the fact that the size and inertia of the wrist must be as small as possible. The input links must be coaxial to permit all the actuactors be connected to the base link. The second requirement states that power is to be transmitted from the input links to other links via gear meshes. The third requirement states that there should be at least three different levels from the base link to the end-effector link. The three articulation points enable the wrist to perform a three degree-of-freedom motion. However, if there are more than three levels, then the articulation points are coupled, and only three of them are independent. The fourth requirement is set primarily for the reason of efficiency. A redundant link does nothing but waste energy. Elimination of redundant links results in a mechanism having the same degrees of freedom and with fewer links. Hence such mechanisms should be discarded.

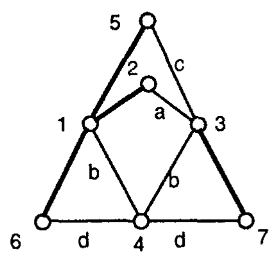


Figure 2 Two D.O.F epicyclic gear train

To illustrate the above design principles, we take one graph from the atlas of two-degree-of-freedom gear trains developed by Tsai and Lin (16) as an example. Figure 2 shows that links 6, 4 and 7 are coaxial at level d, and links 1, 3, and 4 are coaxial at level b. Both sets of links are potential candidates for the input links at this stage. Using 6, 4 and 7 as input links, we may then choose link 2 or 5 as the end-effector link, since there are three edge levels in the thin-edged paths, (6-4-3-2), (7-4-3-2), (6-4-3-5), and (7-4-3-5). In addition, there are two transmission lines originated from vertices 6 and 7. Hence, links 6, 4 and 7 are considered as a set of acceptable input links. Links 1, 3, and 4 are excluded from further consideration as input links since fundamental requirement R3 can not be satisfied. Although the kinematic chain using links 6, 4 and 7 as input links satisfies the first three fundamental requirements, it still can't be considered as a qualified wrist mechanism structure. Because once we choose link 2 as the end-effector link, link 5 becomes a redundant link and vice versa.

4. DEVELOPMENT OF THE ATLAS

All the graphs listed in the atlas of two-degree-of-freedom gear chains (16) have been analyzed. A computer program was written to test the first three requirements. Those graphs which satisfy R1 to R3 are then further examined to identify the existence of redundant links. If a graph satisfies all the fundamental requirements, then it is said to be an admissible graph.

For each admissible graph, a base link is added in series and coaxial with one of the input links to form a three-degree-of-freedom mechanism. A mechanism containing three or more coaxial links can have multiple graph representations known as pseudoisomorphic graphs. It is helpful to redraw the graphs in canonical form defined in Tsai (15), In the canonical representation, all the thin-edged paths originated from the base link have distinct edge labels, and the distance along the thin-edged path, from the base link to any other link, is shortest. In this way, we note that the equivalent open-loop chain is defined by a thin-edged path which starts from the base link and ends at end-effector link. In this paper, starting from the base link to the end-effector link, we shall label the thin edges in the equivalent open-loop chain in an alphabetic order. Labels which are not in an alphabetic order represent axes which are not coaxial with any of the axes in equivalent open-loop chain. It has been shown (4) that for each graph, there is a unique structure matrix describing the relation between angular velocities of the input links and the joint angular velocities of the equivalent open-loop chain. That is,

$$\dot{\phi} = A\,\dot{\theta}, \qquad (1)$$

where $\dot{\phi}$ is angular velocity vector of the input links, and

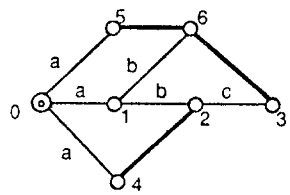
$\dot{\theta}$ is joint velocity vector of the equivalent open-loop chain.

The matrix A is an nxn square matrix if the number of articulation points in the mechanism is equal to n, where n is the number of degrees of freedom. For example, the input-output relation for the Cincinnati-Milacron 3-Roll wrist, Figure 1(a), is given by:

$$\begin{bmatrix} \dot{\theta}_{61} \\ \dot{\theta}_{71} \\ \dot{\theta}_{21} \end{bmatrix} = \begin{bmatrix} 1 & N_{56} & N_{45}\,N_{56} \\ 1 & N_{37} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_{21} \\ \dot{\theta}_{32} \\ \dot{\theta}_{43} \end{bmatrix} \qquad (2)$$

where $\theta_{ij} = \theta_i - \theta_j$ and $N_{ij} = N_i/N_j$ is the gear tooth ratio.

Figures 3 through 6 show the results obtained from the analysis. Figure 3 shows the graph with seven vertices and its corresponding structure matrix. Figures 4(a) through 4(f) show the graphs with eight vertices and their corresponding structure matrices. We note that Figures 4(a) through 4(c) have the same structure matrix as that of Figure 3. In fact, mechanisms constructed from Figures 4(a) through 4(c) are the so-called "derived mechanisms" of Figure 3. See Chang and Tsai (4) for the definition of derived mechanism. Figure 5 shows a representative wrist mechanism constructed from the graph of Figure 3. Figures 6(a) through 6(f) show the representative wrist mechanisms constructed from the graphs of Figures 4(a) through 4(f). All the wrist mechanisms are sketched to have spherical motion.



$$\begin{bmatrix} \# & \# & \# \\ \# & \# & 0 \\ \# & 0 & 0 \end{bmatrix}$$

Figure 3 Graph with seven vertices

Figure 5 is the canonical representation of the Cincinnati-Milacron 3-Roll wrist, which has the least number of links. Figure 6(a) is the canonical representation of PUMA wrist. We note that the PUMA wrist mechanism is constructed by adding an idler gear between links 6 and 3 of the mechanism shown in Figure 5.

Figure 6(d) is identified as the structure of the Bendix wrist mechanism. The remaining gear trains are believed to be new kinematic structures for wrist mechanisms.

Note that in Figure 6(e), there are four links that are coaxial with base link. However, only three input links are required. Among the four coaxial links, (1, 4 and 5) is not a valid set of input links. If links 1, 5 and 6 are chosen as input links, then link 4 becomes a redundant link, and if links 1, 4 and 6 are chosen as input links, then link 5 becomes a redundant link. In either cases, the function of the resulting mechanism is the same as that of Figure 5. However, if links 4, 5 and 6 are chosen as input links, then the resulting mechanism will have different functional characteristics.

The mechanism shown in Figure 6(f) has four articulation points, that is, the number of articulation points is greater than the number of degree-of-freedom by one. Further analysis shows that, although it has four articulation points, the relative motions of the links associated with the joints are not all independent. For this reason, the structural matrix shown in Figure 4(f) has been augumented by one. The fourth row of the matrix equated to zero shows the internal dependence (coupling) of the joint angles or joint velocities. This constraint guarantees that the number of inputs is equal to the number of effective joints, and the solution of the system is determinate.

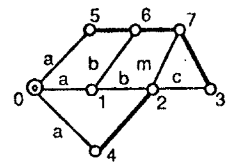
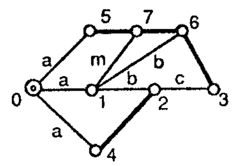
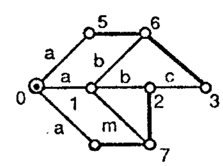
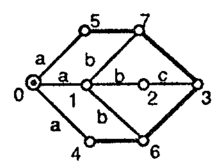
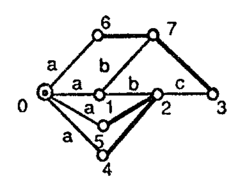
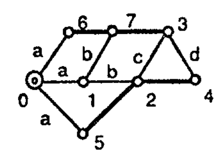
Graphs		Structure Matrices
	(a)	$\begin{bmatrix} \# & \# & \# \\ \# & \# & 0 \\ \# & 0 & 0 \end{bmatrix}$
	(b)	$\begin{bmatrix} \# & \# & \# \\ \# & \# & 0 \\ \# & 0 & 0 \end{bmatrix}$
	(c)	$\begin{bmatrix} \# & \# & \# \\ \# & \# & 0 \\ \# & 0 & 0 \end{bmatrix}$
	(d)	$\begin{bmatrix} \# & \# & \# \\ \# & \# & \# \\ \# & 0 & 0 \end{bmatrix}$
	(e)	$\begin{bmatrix} \# & \# & \# \\ \# & \# & 0 \\ \# & \# & 0 \end{bmatrix}$
	(f)	$\begin{bmatrix} \# & \# & \# & 0 \\ \# & \# & 0 & 0 \\ \# & 0 & 0 & 0 \\ 0 & 0 & \# & \# \end{bmatrix}$

Figure 4 Graphs with eight vertices

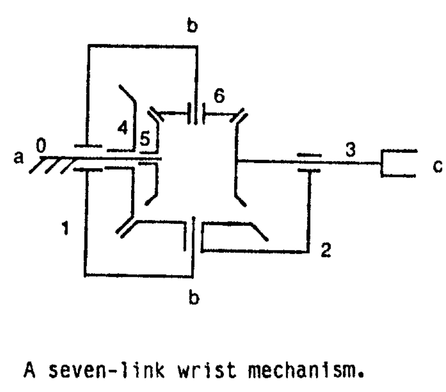


Figure 5 A seven-link wrist mechanism.

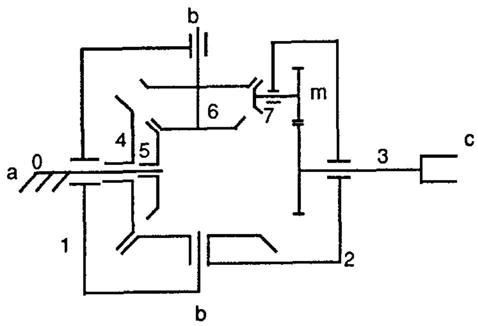


Fig. 6(a)

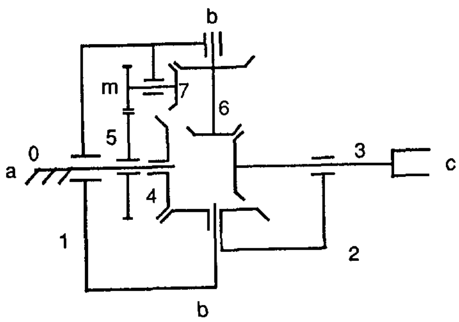


Fig. 6(b)

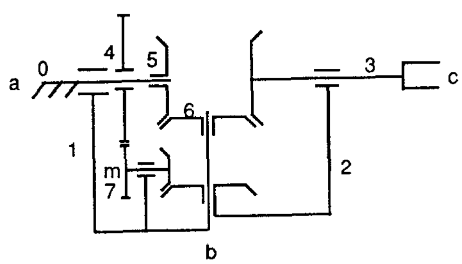


Fig. 6(c)

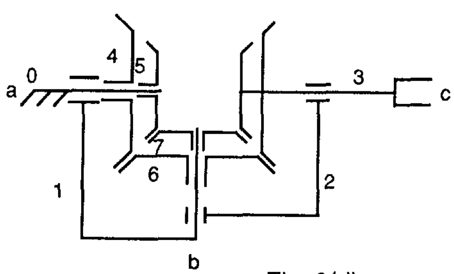


Fig. 6(d)

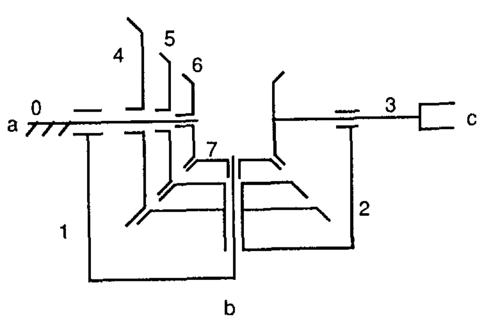


Fig. 6(e)

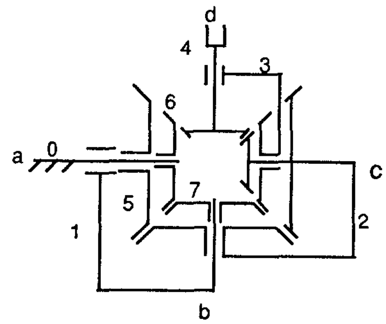


Fig. 6(f)

Figure 6 Eight-link wrist mechanisms constructed from Figs. 4(a) - 4(f)

5. SUMMARY

The kinematic structure of bevel-gear-type robotic wrist mechanism has been analyzed. Based on the analysis, a set of fundamental requirements for the sturctural synthesis of bevel-gear-type wrist mechanisms has been established. All the admissible kinematic structures of spherical wrist mechanisms with seven and eight links have been identified and a representative mechanism corresponding to each kinematic structure has been sketched. It is interesting to note that a three degrees of freedom wrist can have four articulation points.

Although wrist mechanisms presented in this paper are all constructed by having a true two-degree-freedom gear train in series with a base link, they can also be designed by using true three-degree-of-freedom gear trains. However, this will results in a more complicated design.

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