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Digital Controller Design For The Pitch Axis Of The F-14 Using An H^{∞} Method

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ABSTRACT

An H^∞ based control system design procedure is developed. The basic idea is to produce a controller in which the closed-loop system is designed, via H^∞ methods similar to those of Kwakernaak, to be robust with respect to both disturbance rejection and insensitivity to parameter variations. This design algorithm is then applied to design a pitch axis controller for a digitized model of the $F-14$ aircraft. In this two-degree-of-freedom structure, the controller is designed via our H^∞ method. The pre-compensator is used to shape the output response so as to cause the entire system to achieve the desired performance. The robustness of the angle-of-attack step response under large parameter variations is also examined.

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1. Introduction

In this paper, a particular H^∞ design approach is coupled with multi-criterion optimization to design a pitch axis controller for the $F-14$. The design problem is an augmented version one of two CSS benchmark problems [15].

First, we design a discrete time controller even though the benchmark is in continuous time. We believe the digitized controller is more realistic given current technology as well as being a more interesting challenge because of the effects of sampling. Second, the benchmark, while specifying gain and phase margins, does not specify the expected ranges for the parameters of the plant. We have obtained estimates of the reasonable ranges of parameter variations and check our design against these values.

It must be said that, given the rapid advances in H^∞ techniques, there are now several other ways to go about solving our design problem. There are certainly better methods for computing H^∞ optimal controls. Some of them may well lead to better designs. There is still a significant advantage, at least in designing airplane controllers, to our approach. There is an explicit separation between the robustness and performance aspects of the design, since the designer is expected to adjust the design, in nearly real time, to the criticisms of a test pilot "flying" a simulator, it is important to not conflate the different aspects of the design.

It will be apparent, on reading the paper, that we have not yet arrived at a satisfactory design. However, we believe these intermediate results are of value in demonstrating the difficulty of the problem and the thought processes involved in trying to use H^∞ methods to solve a realistic design problem.

2. The $F-14$ Program

2.1 Statement of the original problem

A block diagram for the pitch axis control of the $F-14$ is given in Figure 1. The model is based on a linearization of the system about an operating point of Mach=0.71, altitude=35,000 ft and total air speed 690.4 ft/sec. All the parameter values are taken from [1].

In the original design, shown in Fig. 1, the controller structure was fixed in advance and the six parameters denoted by $\tau_c, \tau_w, k_\alpha, k_q, k_F, k_I$ were optimized by a method similar to that proposed by Levine and Athans [13]. The choice of controller structures was based on many years of experience in designing aircraft control systems. Most similar aircraft have similarly structured pitch axis controls.

The controller was designed to meet several specifications. The closed loop response to a commanded 1^o change in pitch should approximate the response of a second order system (i.e., the implicit model) with specified bandwidth and damping. More specifically, the angle-of-attack (α) response to such a step input is to match that of a given critically damped, second order system with damping 0.707 and bandwidth 2.49 rad/sec. The control system is also to minimize the response to wind gusts and to α-sensor (pitch sensor) noise. Models for both noise sources are shown in Fig. 1. Finally, the minimum allowable gain and phase margins are specified.

In order to understand some of the design choices described subsequently, it is important to know that candidate controllers are normally tested in a flight simulator. The designer is expected to be able to modify the design, in nearly real time, in response to comments from the pilot.

2.2 Reformulation of the $F-14$ Problem

The original controller is removed and will be replaced by a discrete time controller, as shown in Fig. 2. The controller design is divided into two separate parts. The feedback controller, $F(z)$, will be designed by an H^∞ technique so as to maximize the robustness of the closed loop system. Robustness, in this context, corresponds to achieving all of the control specifications other than matching the desired second order response. The pre-compensator, $C(z)$, will be chosen so that the whole system matches the desired second order response in one case. In a second case, the pre-compensator will be chosen to give a very good servo response.

In order to determine whether the resulting controller designs are robust enough for use in a real airplane, estimates of the likely variation in this parameters of the model were obtained. These are: $Z_\delta \pm 50\%$, $M_w \pm 25\%$, $M_q \pm 50\%$, $M_\delta \pm 25\%$, $\tau_\alpha \pm 10\%$ and

$Z_w \pm 20\%$. It is reasonable to require that the controlled system behave very well for all possible combinations of parameters in this range.

3. The Solution Method

The block diagram of the linear time invariant multivariable sampled-data system which we consider is given in Figure 2, $G(s) \in R(s)^{n\times m}$ is the $n \times m$ strictly proper continuous plant to be controlled, $F(z) \in R(z)^{m\times n}$ is the $m \times n$ digital feedback controller to be designed, ZOH is a zero order hold device, and $C(z)$ is a pre-compensator that is also to be designed. We assume that the system is well-posed and has no hidden oscillations. For realistic control design, minimizing only the sensitivity or only the complementary sensitivity suffers serious drawbacks. However, it is impossible to make both small at every frequency and a tradeoff is inevitable in the design process.

For the system given in Figure 2 and following Kwakernaak [2,3], we take a linear combination of quadratic terms of weighted $S(z)$ and $T(z)$, i.e.,

$$(\bar{W}_1 S)^*(\bar{W}_1 S) = S^*\bar{W}_1^*\bar{W}_1 S \equiv S^* W_1 S,$$
$$(\bar{W}_2 T)^*(\bar{W}_2 T) = T^*\bar{W}_2^*\bar{W}_2 T \equiv T^* W_2 T$$

where $S(z), T(z)$ are the system sensitivity matrix and the complementary sensitivity matrix respectively, and are defined as : $S(z) = [I_n + P(z)F(z)]^{-1}$, $T(z) = I_n - S(z)$. The matrices $W_1(z), W_2(z)$, defined by $W_1(z) = \bar{W}_1^*(z)\bar{W}_1(z)$; $W_2(z) = \bar{W}_2^*(z)\bar{W}_2(z)$, are frequency dependent weighting matrices. We want to design a digital controller $F(z) \in R(z)^{m\times n}$ to stabilize the given plant $G(s)$ and, at the same time, to minimize

$$(P) \qquad \min_{F(z)} ||(S^*W_1S + T^*W_2T)(e^{jwT_s})||_\infty$$

The following two assumptions are made throughout this paper.

(1) $G(s)$ is of least order r with poles $\{p_i, i = 1, 2, ..., r\}$ such that if $Re(p_i) = Re(p_j)$ then $Im(p_i - p_j) \neq 2k\pi/T_s$ $(k = 1, 2, ...)$.

(2) $\bar{W}_1(z), \bar{W}_2(z) \in R(z)^{n\times n}$ are causal, nonsingular, stable and have no poles on the unit circle.

The choice of weights is problem specific. In general, these user-defined weights are chosen to be stable, diagonal with the diagonal elements to be minimum phase, real and rational. With diagonal weights, each one of the input and the output signals can be weighted individually. Thus, the designer can easily trade-off the relative importance of the signals over the same or different frequency bands.

From the given wind gust model, we can estimate the possible frequency range of the wind gust disturbances (w_q, q_g) of the system. Since the effect of wind gusts and plant parameter variations are related to the system sensitivity matrix S, we choose the weight on S to be

$$\bar{W}_1(s) = 2.0(s + 2.0)^{-1} I_2.$$

Similarly, we estimate the main frequency range of the α-sensor noise. The effect of sensor noise is related to the system complementary sensitivity matrix T. Therefore, we choose the weight on T to be

$$\bar{W}_2(s) = (s + 2.0)(s + 10.0)^{-1} I_2.$$

By Tustin's transformation, the digitized forms for these two weights are

$$\bar{W}_1(z) = 0.1(z + 1.0)(2.1z - 1.9)^{-1} I_2 \text{ and } \bar{W}_2(z) = (1.05z - 0.05)(1.25z - 0.75)^{-1} I_2.$$

Let $\Omega(P)$ denote the set of all real rational controllers F that stabilize P (where P is a cascade connection of the plant $G(s)$ and the ZOH), then it is well-known that [4,5]

$$\begin{aligned}\Omega(P) &= \{(Y - R\tilde{N}_p)^{-1}(X + R\tilde{D}_p) \mid R \in M(RH^\infty),\ det(Y - R\tilde{N}_p) \neq 0\} \\ &= \{(\tilde{X} + D_p Q)(\tilde{Y} - N_p Q)^{-1} \mid Q \in M(RH^\infty),\ det(\tilde{Y} - N_p Q) \neq 0\}\end{aligned}$$

where (N_p, D_p), $(\tilde{D}_p,\ \tilde{N}_p)$ are any right and left coprime fractional factorization of P with the corresponding quadruple $(X, Y, \tilde{X}, \tilde{Y})$.

We use this kind of stabilizing controller as our controller structure so that system stability is guaranteed. To exploit H^∞, it is more convenient to use a ζ-transform where $\zeta = z^{-1}$. Therefore, all the matrices involved hereafter are transformed into real rational matrices in the variable ζ. Using the parametrization, the performance criterion can be expressed as

$$S^*W_1S + T^*W_2T$$
$$= [MN_pR\tilde{D}_p + MN_pX - M^{-*}W_1]^*[MN_pR\tilde{D}_p + MN_pX - M^{-*}W_1] +$$
$$W_1(W_1 + W_2)^{-1}W_2$$

where M is a spectral factor of $W_1 + W_2$. Note that the right hand side of this equation is a quadratic equation except that the "coefficients" are rational functions of ζ.

In [6], Helton mentioned the problem of $\min_{f \in D} \sup_\omega \Theta(\omega, f(j\omega))$ (where Θ is a given positive valued function, D is a feasible set) and he claimed that many problems taking this form have the property that an optimum f_{opt} will make the objective function $\Theta(\omega, f_{opt}(j\omega))$ constant in ω almost everywhere; that is, frequency independent. This is the so called "self flattening" property. Since our goal is design rather than optimality, we will confine the feasible set of $F(z)$ to be a subset of $\Omega(P)$, expressed as $\Omega'(P)$, which makes the objective function be self flattening. That is, we look for an equalizing solution. Therefore, our optimization problem is

$$\min_{F \in \Omega'(P)} ||S^*W_1S + T^*W_2T||_\infty \equiv \alpha_{opt}^2$$

The procedure of finding a non-equalizing solution is quite similar to that of the equalizing solution with some modifications [7].

In [8], Helton presented a method to solve broad-band circuit design problems, particularly gain equalization problems. We will use Helton's idea to solve for α_{opt}^2. We first state Helton's result.

Let $C(e^{j\theta}), P(e^{j\theta}), R(e^{j\theta}) \in (RL^\infty)^{n \times n}$ and P^2, R^2 be strictly positive definite. Define a disk $\Delta_C^{P,R}$ in the matrix function space $(RL^\infty)^{n \times n}$ to be the set of all $H(e^{j\theta}) \in (RL^\infty)^{n \times n}$ which satisfy $(H - C)P^2(H - C)^* \le R^2$, i.e.,

$$\Delta_C^{P,R} = \{H \in (RL^\infty)^{n \times n} | (H(e^{j\theta}) - C(e^{j\theta}))P^2(e^{j\theta})(H(e^{j\theta}) - C(e^{j\theta}))^* \le R^2(e^{j\theta});$$
$$\forall \theta \in [0, 2\pi]\}.$$

Then the question is : For such a given $\Delta_C^{P,R}$, does this set contain a function in $(RH^\infty)^{n\times n}$? The question is answered by the following theorem.

Theorem 1

For a given $\Delta_C^{P,R}$ defined as above, there is a function $H \in (RH^\infty)^{n\times n}$ in this set if and only if the maximum eigenvalue λ_{max}^2 of

$$(H_C[T_{P^{-2}}]^{-1}H_C^*)x = \lambda^2 T_{R^2}x$$

is less than or equal to one. (Here H_G and T_G are the Hankel and Toeplitz matrices generated by G with $G \in (RL^\infty)^{n\times n}$).

Suppose that $\tilde{F}$ is the optimal stabilizing controller with corresponding $(\tilde{S}, \tilde{T})$. The corresponding cost function has the form

$$\tilde{S}^*W_1\tilde{S} + \tilde{T}^*W_2\tilde{T} = \alpha_{opt}^2 I_n \quad \text{with} \quad \alpha_{opt}^2 \geq \alpha_o^2$$

where α_o^2 is an easily computed lower bound on α_{opt}^2 (see [9] for an algorithm for computing α_o^2). Consider the following problem

$$\tilde{S}^*W_1\tilde{S} + \tilde{T}^*W_2\tilde{T} = (\alpha_o^2 + \varepsilon)I_n \quad \text{where} \quad \varepsilon \geq 0. \tag{1}$$

Equation (1) can be written as

$$\begin{aligned}(MN_pR\tilde{D}_p + MN_pX - M^{-*}W_1)^*&(MN_pR\tilde{D}_p + MN_pX - M^{-*}W_1)\\ &= (\alpha_o^2 + \varepsilon)I_n - W_1(W_1 + W_2)^{-1}W_2.\end{aligned}$$

Define

$$\begin{aligned}C \equiv& M^{-*}W_1 \in (RL^\infty)^{n\times n},\\ Q_\epsilon \equiv& (\alpha_o^2 + \varepsilon)I_n - W_1(W_1 + W_2)^{-1}W_2 \in (RL^\infty)^{n\times n},\\ H \equiv& MN_pR\tilde{D}_p + MN_pX \in (RH^\infty)^{n\times n}.\end{aligned} \tag{2}$$

Then Equation (1) becomes

$$(H(e^{j\theta}) - C(e^{j\theta}))^*(H(e^{j\theta}) - C(e^{j\theta})) = Q_\epsilon(e^{j\theta}), \quad \theta \in [0, 2\pi]. \tag{3}$$

Note that here C is determined by weighting matrices W_1, W_2 only. Therefore, it is known in advance. The para-Hermitian matrix $Q_\epsilon(\zeta)$ is a function of ε and $Q_\epsilon(e^{j\theta}) > 0$ for $\varepsilon > 0, \theta \in [0, 2\pi]$.

Letting Equation (3) define a "disk" $\Delta_\epsilon = \Delta_C^{I,Q_\epsilon}$ (see above) then finding α_{opt}^2 is equivalent to solving the following problem $(P1)$:

$$\text{Find the smallest } \varepsilon \text{ such that } \varepsilon \geq 0, \Delta_\epsilon \cap (RH^\infty)^{n\times n} \neq \emptyset$$

Therefore, solving the controller design problem is equivalent to performing the following three steps :

(1) Solve $(P1)$.

(2) If ε_o solves $(P1)$ (i.e., $\alpha_{opt}^2 = \alpha_o^2 + \varepsilon_o$) then find the $\tilde{H} \in (RH^\infty)^{n\times n}$ which satisfies $(\tilde{H} - C)^*(\tilde{H} - C) = Q_{\epsilon_o}$.

(3) Based on this $\tilde{H}$, find the corresponding $\tilde{R}$.

To solve $(P1)$, we consider the following

Theorem 2

Consider problem $(P1)$ and its corresponding generalized eigenvalue problem

$$(H_C^* H_C)x = \lambda T_{Q_\epsilon} x \qquad (P2)$$

where C, Q_ϵ are defined in (2). Then, $\Delta_\epsilon \cap (RH^\infty)^{n\times n} \neq \emptyset$ if and only if $\lambda_{max}(\varepsilon) \leq 1$, where $\lambda_{max}(\varepsilon)$ is the largest eigenvalue of $(P2)$.

$\lambda_{max}(\varepsilon)$ is a smooth function of ε, making it fairly easy to computeε_o

We already know that the optimal stabilizing controller $\tilde{F}(z)$, parametrized in terms of the controller parameter matrix $\tilde{R}$, satisfies

$$[MN_p\tilde{R}\tilde{D}_p + MN_pX - M^{-*}W_1]^*[MN_p\tilde{R}\tilde{D}_p + MN_pX - M^{-*}W_1] = Q_{opt} \qquad (4)$$

with

$$Q_{opt}(\zeta) \equiv \alpha_{opt}^2 I_n - W_1(W_1 + W_2)^{-1}W_2 \in (RL^\infty)^{n\times n}.$$

From now on, we omit the independent variable ζ. The rational matrix $W_1(W_1+W_2)^{-1}W_2$ is para-Hermitian. By factorization of Q_{opt} into $\Sigma^*\Sigma$ and by separating the stable and unstable parts, Equation (4) can be written as (see [9] for details)

$$N_p\tilde{R} = [M^{-1}\Sigma\tilde{D}_p^{-1}]_+ + [(W_1+W_2)^{-1}W_1\tilde{D}_p^{-1}]_+ - [N_pX\tilde{D}_p^{-1}]_+,$$
$$[N_pX\tilde{D}_p^{-1}]_- = [(W_1+W_2)^{-1}W_1\tilde{D}_p^{-1}]_- + [M^{-1}\Sigma\tilde{D}_p^{-1}]_-. \tag{5}$$

Define $V \equiv [M^{-1}\Sigma\tilde{D}_p^{-1}]_+ + [(W_1+W_2)^{-1}W_1\tilde{D}_p^{-1}]_+ - [N_pX\tilde{D}_p^{-1}]_+$. Then, solving this parameter matrix $\tilde{R}(\zeta)$ reduces to solving a rational matrix equation problem, i.e.,

Given $N_p(\zeta) \in (RH^\infty)^{n\times m}$, $V(\zeta) \in (RH^\infty)^{n\times n}$ and both are strictly causal.

Solve $N_p(\zeta)\cdot\tilde{R}(\zeta) = V(\zeta)$ so that $\tilde{R}(\zeta) \in (RH^\infty)^{m\times n}$.

This rational matrix equation can be regarded as a stable exact model matching problem (SEMMP) which can be solved by existing methods such as [10,11].

Based on the results so far, the steps required to find the controller $\tilde{F}(z)$ to minimize the given H^∞-cost can be summarized as follows.

Controller Design:

Given $\bar{W}_1(z), \bar{W}_2(z) \Longrightarrow$ **Procedure (I)** $\Longrightarrow \alpha^2_{opt}$

Given nominal plant $P_o(z) \Longrightarrow$ **Procedure (II)** $\Longrightarrow \tilde{R}(z) \Longrightarrow \tilde{F}(z)$

Procedures (I) and (II) are given below.

Procedure (I)

Data : $\bar{W}_1, \bar{W}_2$ given, set $\varepsilon_o = 0$.

(1) Calculate $W_1 = \bar{W}_1^*\bar{W}_1, W_2 = \bar{W}_2^*\bar{W}_2$ and $(W_1+W_2)^{-1}W_2$.

(2) Find α_o^2 ([9]).

(3) Perform spectral factorization of W_1+W_2 so that $W_1+W_2 = M^*M$.

(4) Perform ζ-transform for all matrices involved.

(5) Set $C = (M^*)^{-1}W_1$.

(6) Set $k = 0$.

(7) Let $Q(\varepsilon_k) = (\alpha_o^2+\varepsilon_k)I_n - W_1(W_1+W_2)^{-1}W_2$.

(8) Perform spectral factorization of $Q(\varepsilon_k)$ so that $Q(\varepsilon_k) = L_k^*L_k$.

(9) Form $[CL_k^{-1}]_+$.

(10) Find the maximum Hankel singular value $\bar{\gamma}_k$ of $[CL_k^{-1}]_+$.

(11) If $\bar{\gamma}_k \leq 1$, set $\alpha_{opt}^2 = \alpha_o^2 + \varepsilon_k$, go to **Procedure (II)**.

(12) Calculate $\varepsilon_{k+1} = \varepsilon_k + (\bar{\gamma}_k - 1)/\bar{\gamma}_k$.

(13) Set $k = k + 1$, go to (7).

Procedure (II)

Data: ω_s, $P_o(z)$ given.

(1) Perform inverse ζ-transform for all matrices involved.

(2) Do right, left coprime factorization of $P_o(z)$.

(3) Find X, Y from known $\tilde{D}_p, \tilde{N}_p, D_p, N_p$.

(4) Let $Q = \alpha_{opt}^2 I_n - W_1(W_1 + W_2)^{-1}W_2$.

(5) Calculate $[\Sigma]_- = -[C]_-$.

(6) Find $[\Sigma]_+$ from steps (4) and (5).

(7) Set $\Sigma = [\Sigma]_+ + [\Sigma]_-$.

(8) Form the matrix $V = [M^{-1}\Sigma\tilde{D}_p^{-1}]_+ + [(W_1 + W_2)^{-1}W_1\tilde{D}_p^{-1}]_+ - [N_p X\tilde{D}_p^{-1}]_+$.

(9) Solve $N_p\tilde{R} = V$ so that $\tilde{R} \in R(z)^{m\times n}$ stable, causal.

(10) Form $\tilde{F} = (Y - \tilde{R}\tilde{N}_p)^{-1}(X + \tilde{R}\tilde{D}_p)$.

Stop.

The problem $(P2)$ can be changed to

$$(H_{CL^{-1}})^*(H_{CL^{-1}})x = \hat{\lambda}x$$

where $L(\varepsilon)$ is a spectral factor of Q_ε. The stable part of CL^{-1} determines the Hankel matrix $H_{CL^{-1}}$. The matrix $[CL^{-1}]_+$ is real rational, from Kronecker's Theorem, the induced Hankel matrix $H_{CL^{-1}}$ is of finite rank and its dimension equals the McMillan degree of $[CL^{-1}]_+$. That is,

$$\text{rank } (H_{CL^{-1}}) = \text{ McMillan degree } ([CL^{-1}]_+).$$

Therefore, finding the maximum eigenvalue of $(P2)$ is equivalent to finding the maximum singular value of $H_{CL^{-1}}$, i.e.,

$$\lambda_{max}(\varepsilon) = \hat{\lambda}_{max}(\varepsilon) = \bar{\sigma}^2(H_{CL^{-1}}).$$

From [12], this infinite dimensional Hankel matrix can be changed to a finite dimensional matrix with its dimension the McMillan degree of $[CL^{-1}]_+$. Therefore, by performing the Singular Value Decomposition (SVD) of this finite matrix, we can then get the "exact" $\hat{\lambda}_{max}(\varepsilon)$, thus, $\lambda_{max}(\varepsilon)$.

4. Design Results

4.1 Results of Procedure (I,II)

The maximum value of α^2 for which the constant term of the numerator polynomial of the determinant of the matrix $[\alpha^2 I_2 - W_1(W_1 + W_2)^{-1}W_2](\zeta)$ (denoted as $det\ [\alpha^2 I_2 - W_1(W_1 + W_2)^{-1}W_2](\zeta)$) is zero equals zero. For $\alpha^2 > 0.09$, there are four roots with two inside the unit circle, two outside the unit circle and at $\alpha^2 = 0.09$, there are four roots lying on the unit circle. Therefore, the maximum value of α^2 for which $det\ [\alpha^2 I_2 - W_1(W_1 + W_2)^{-1}W_2]|_{z=e^{j\theta}} = 0$ is 0.09. Thus, $\alpha_o^2 = 0.09$.

Following the Design Procedure (I), we find that $\epsilon_{opt} = 0.01561$. Therefore, $\alpha_{opt}^2 = 0.09 + 0.01561 = 0.10561$, i.e.,

$$\min_{F\in\Omega'(P)} ||S^*W_1S + T^*W_2T||_\infty = 0.10561$$

In an earlier design [14] where $\bar{W}_1, \bar{W}_2$ are both low-pass with $W_2(s) = 10.0(s + 10.0)^{-1}I_2$, $\omega_s = 62.8\ rad/sec$ and the truncation of Hankel and Toeplitz matrices (a less accurate computation) was used in Procedure (I), the optimal cost we found is 0.5 [9].

The calculation of procedure (II) is straightforward. In this $F-14$ design example, we cannot directly solve *step* (9) due to the unstability of $(N_p^T N_p)^{-1} N_p^T$, therefore, we used the algorithm given in [11] to find the controller parameter matrix $\tilde{R}(z)$. The stabilizing controller we found is a one by two real-rational, causal transfer matrix with order 14.

4.2 Analysis of Results

The prefilter $C(z)$ is a (2×1) transfer matrix and is used to shape the output response. For simplicity, we assume that it has the following form

$$C(z) = \begin{matrix} k_1\ (z+\alpha_1)(z+\beta_1)^{-1} \\ k_2\ (z+\alpha_2)(z+\beta_2)^{-1} \end{matrix}$$

with $|\beta_1| < 1$, $|\beta_2| < 1$. The prefilter structure used by Grumman is a first order s-domain transfer function. In this digitized design problem, we take the simplest possible structure for the corresponding z-domain transfer function. Based on this structure, we try to find the six design parameters so that $C(z)$ is stable and causes the overall system to achieve the desired performance. The parameters we found are: $k_1 = 0.16$, $\alpha_1 = 0.10$, $\beta_1 = -0.85$, $k_2 = -0.38$, $\alpha_2 = -0.73$ and $\beta_2 = -0.95$.

The angle-of-attack step response $\alpha(t)$ is shown in Figure 3 together with the response with $\xi = 0.707$, $\omega_n = 2.49$ rad/sec. From this figure, we see that the performance of $\alpha(t)$ of our designed system is much better than that of the original desired $\alpha(t)$. It has a faster response, no overshoot and reaches its steady state after $t = 1.15$ sec.

It should be apparent that we are just showing off. We could meet the design specification quite easily by appropriate choice of $C(z)$. The real problem is robustness and this is primarily governed by the closed loop portion of the system.

The robustness of the designed system is examined by changing the nominal parameter values by a prespecified amount. Figures 4, and 5 show the change in the α response from varying the parameters Z_δ, and M_ω by $\pm 50\%$,and $\pm 25\%$ respectively. From these figures, we see that the designed system is very robust against these parameter variations. The change in the α response due to variations in M_q of $\pm 50\%$ and Z_w by $\pm 20\%$ are similar.

Figure 6 shows the change in the α response from changing the actuator time constant τ_a from $+10\% \sim -6\%$. Again, it is very insensitive. However, we found that reducing $|\tau_a|$ by more than 6% will cause oscillations. To further reduce $|\tau_a|$ will even cause instability as shown in Figure 7 while increasing $|\tau_a|$ has no such phenomenon.

The α response under the variations as M_δ ranging from -25% to $+15\%$ is still quite robust. But increasing $|M_\delta|$ beyond $+15\%$ will deteriorate the α response and increasing further will cause instability, see Figure 8.

J.S. Yang's thesis [9] and [14] contain an earlier attempt at a controller design using a lower sampling frequency and with W_2 a low pass filter (see Section 4.1). This design was generally much more sensitive to parameter variations than the design shown here. However, the earlier design did not go unstable until τ_α and M_δ has been perturbed by more than -10% and + 25% respectively. Furthermore, the original design from Grumman is very robust and does not become unstable for the given range of parameter variations. Of course, it is a continuous time controller.

5. Conclusions

We believe there is good news and bad news in the results we have presented. The controller we have designed performs extremely well for the nominal parameters and the degradation in performance for substantial variations in the parameters is virtually too small to measure. This is certainly good news.

The bad news is that the controller is very high order and that the system is unstable for parameter values in the expected range of variation. The high order controller is a minor problem. A method is given in J.S. Yang's thesis [9] and in [14] for approximating the high order controller by a lower order controller. This technique, when applied to the earlier design, reduced a 12^{th} order controller to 6^{th} order with very little change in performance or robustness. We expect similar results for the present controller.

The instability raises important questions. How can we produce a design that does not exhibit this instability? One obvious possibility is to increase the nominal value of τ_α and decrease the nominal M_δ. Other possibilities involve changing the weights. We are trying both.

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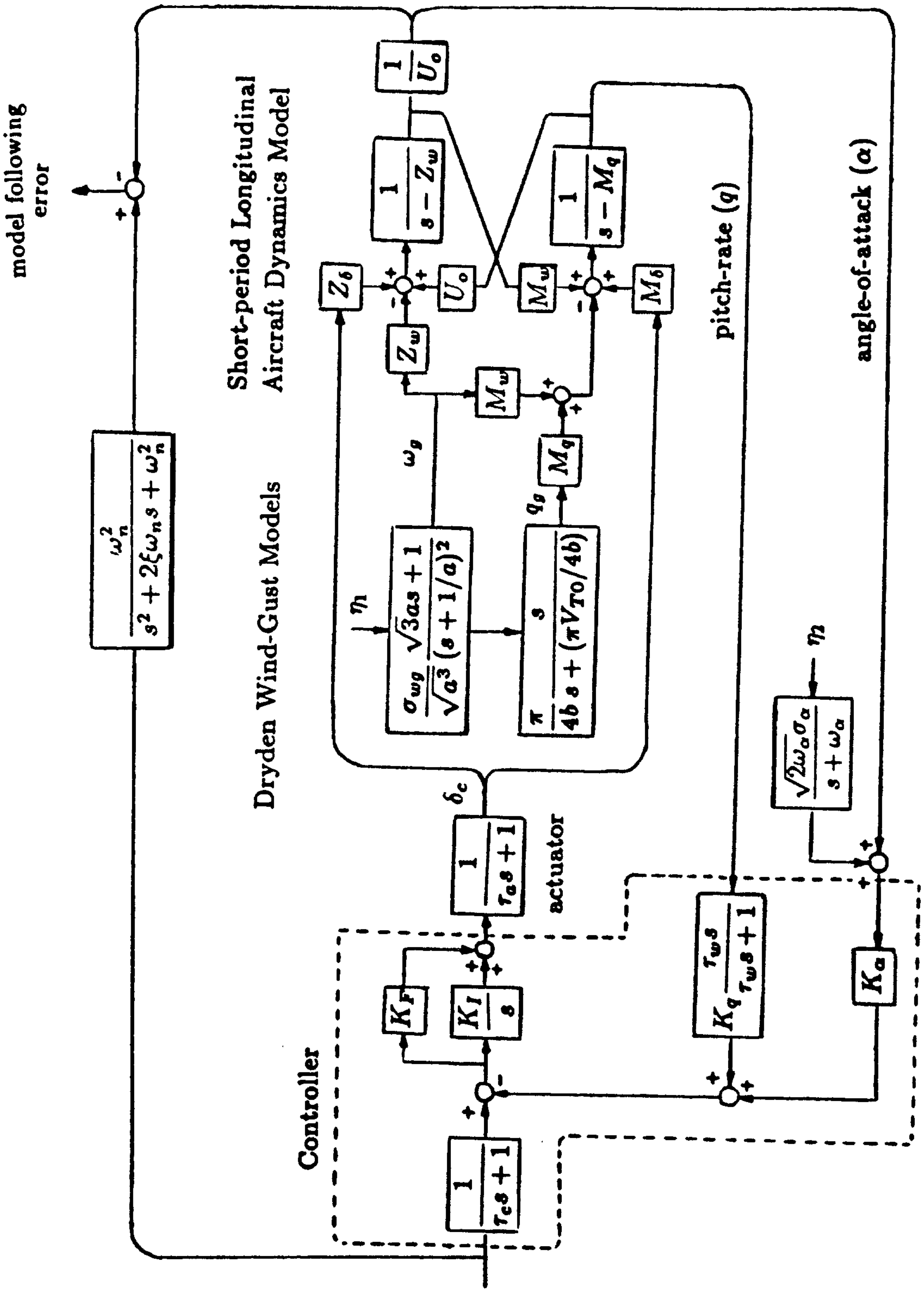

Figure 1.1 Block Diagram of the Grumman $F-14$ Control Problem.

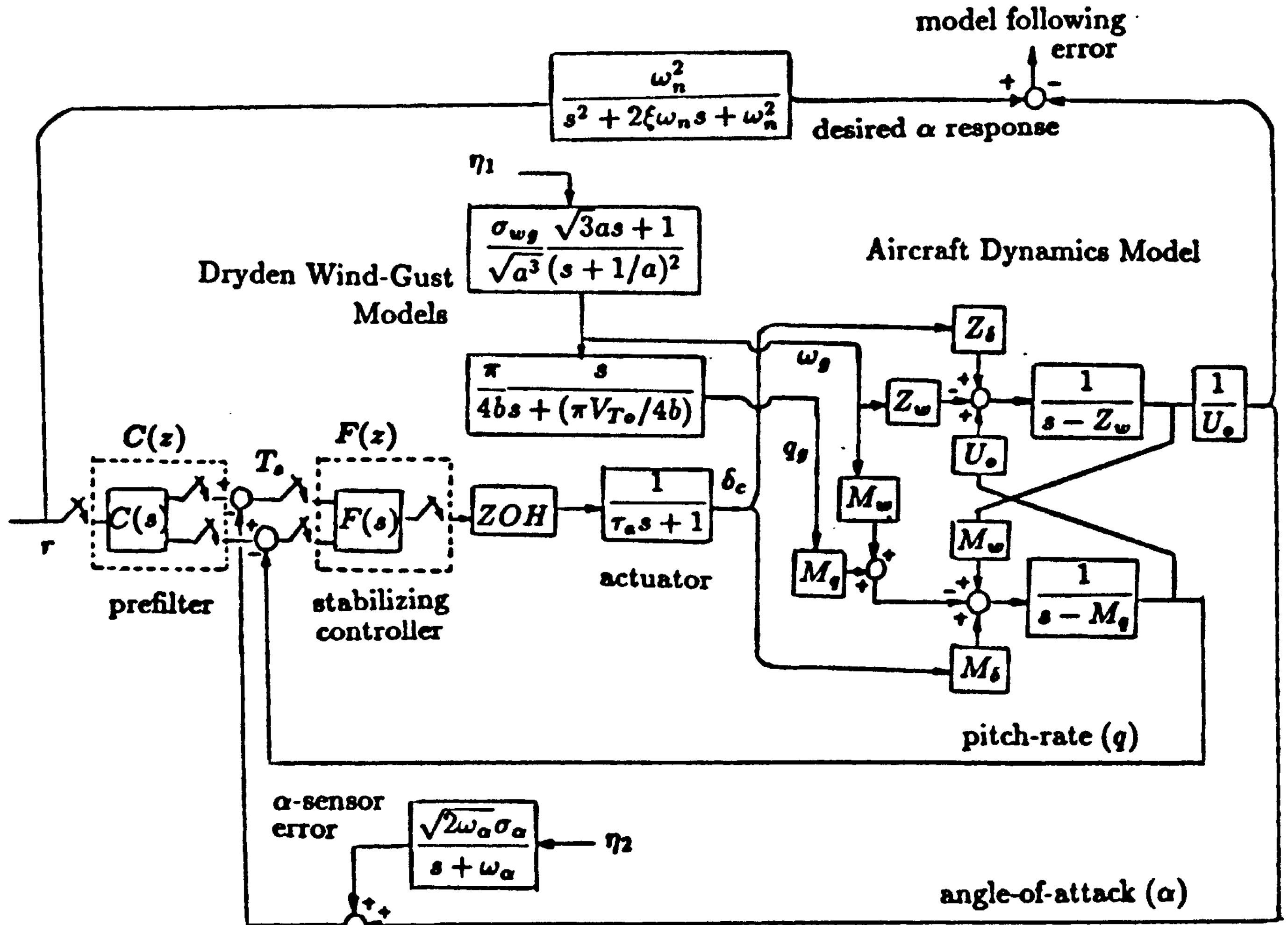

Figure 2 Block Diagram of the Modified [redacted] F-14 Problem.

Angle-of-Attack Step Response

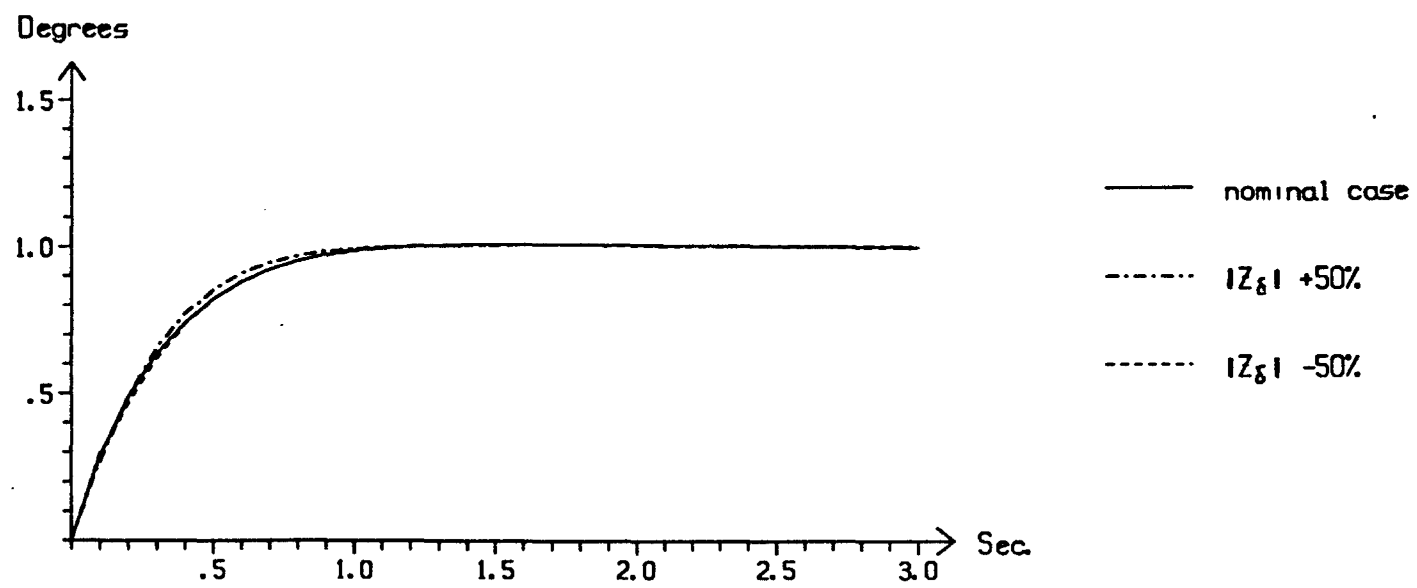

Fig. 4 α Step Response Under Parameter Z_δ ($\pm 50\%$) Variations

$k_1 = 0.16$, $\alpha_1 = 0.10$, $\beta_1 = -0.85$

$k_2 = -0.38$, $\alpha_2 = -0.73$, $\beta_2 = -0.95$

Step Response of Angle-of-Attack

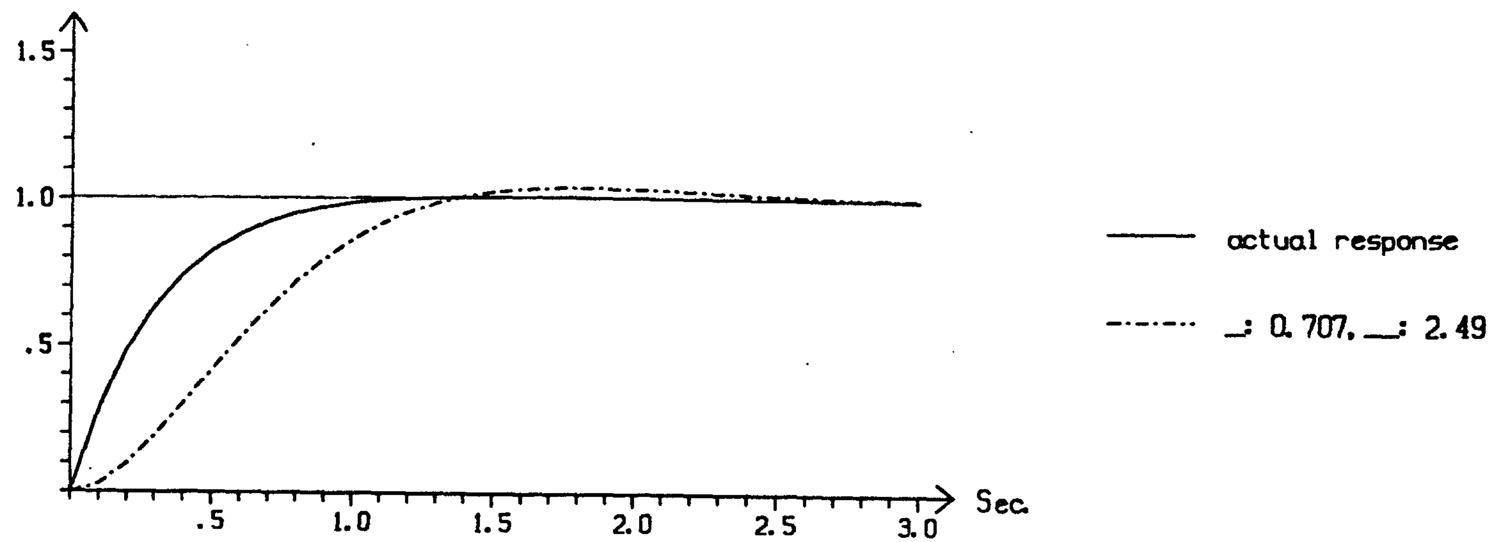

Fig. 3 Angle-of-Attack Step Response with Parameters Given Above

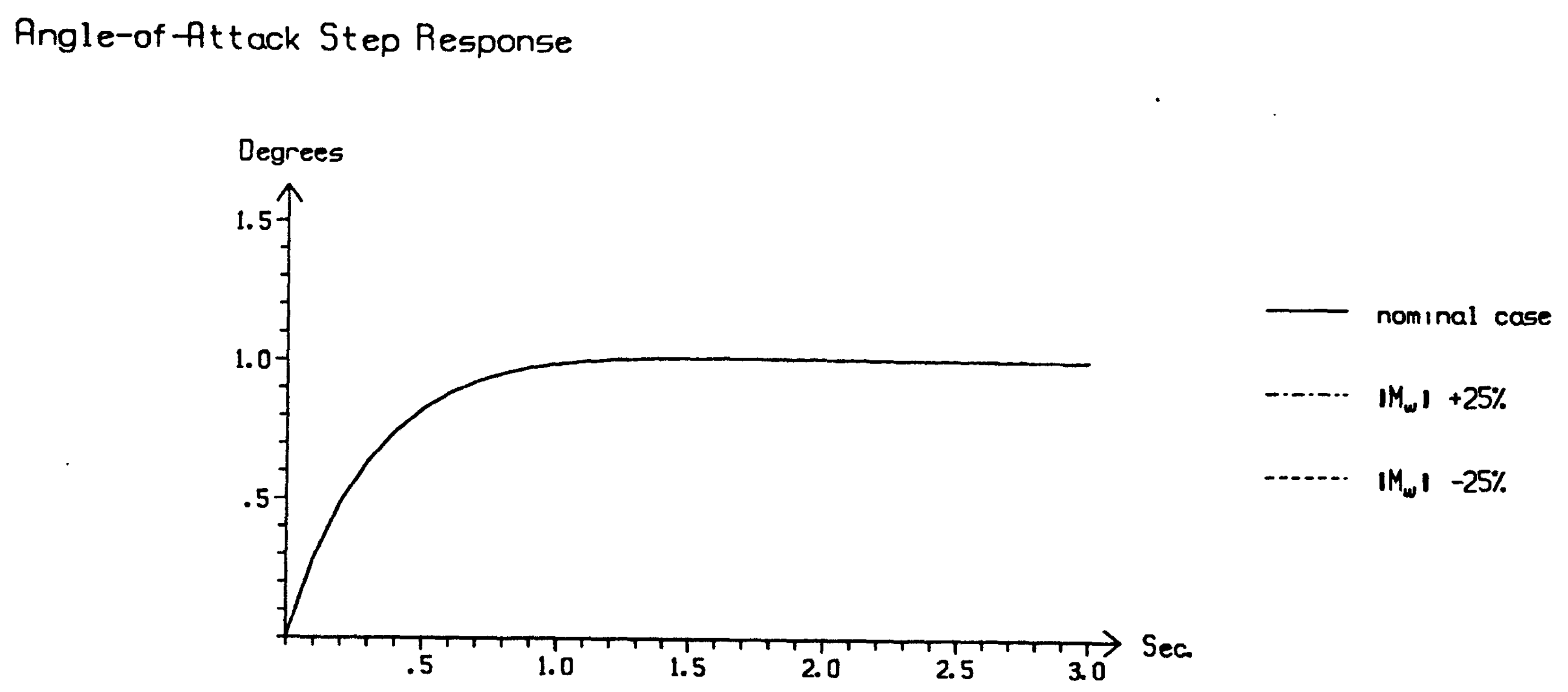

Fig. 5 α Step Response Under Parameter M_ω ($\pm$ 25%) Variations

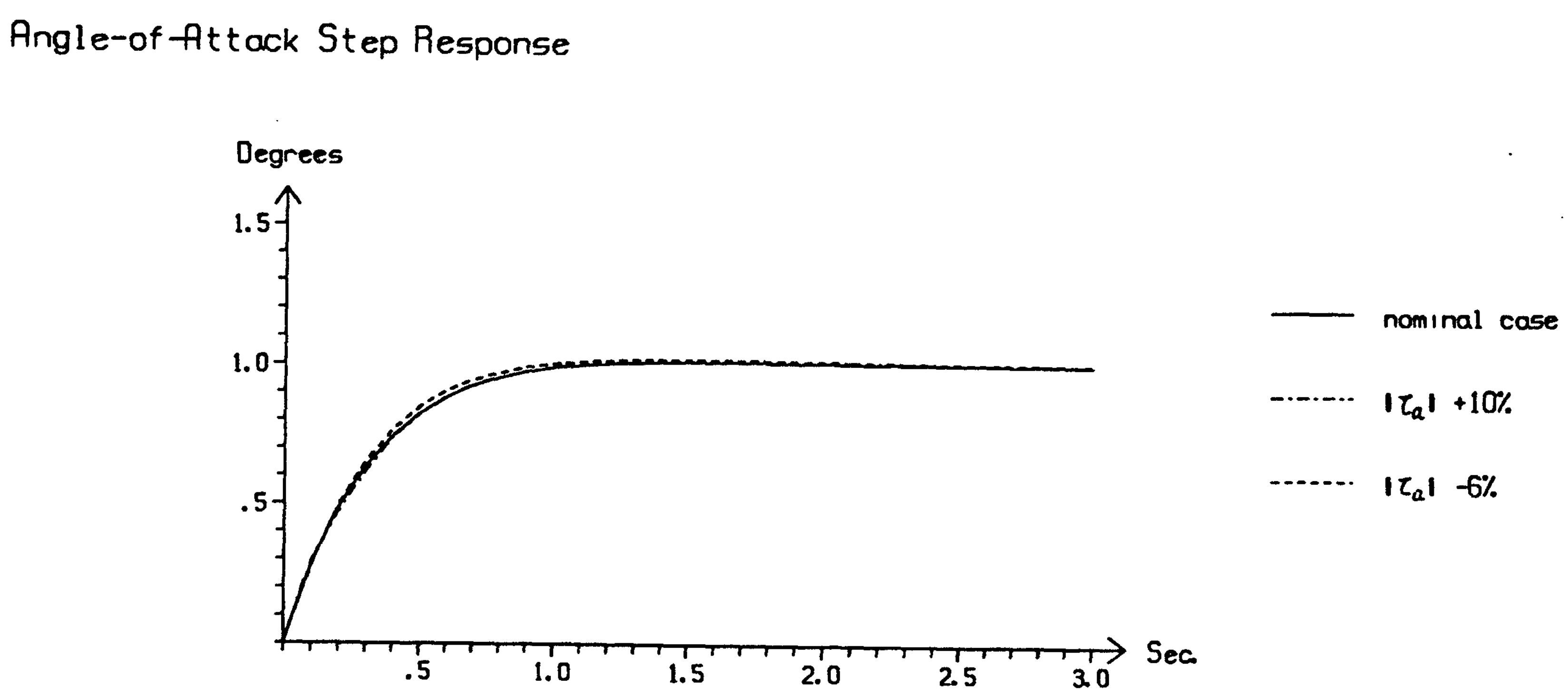

Fig. 13 (6) α Step Response Under Parameter τ_a (+10%, -6%) Variations

Angle-of-Attack Step Response

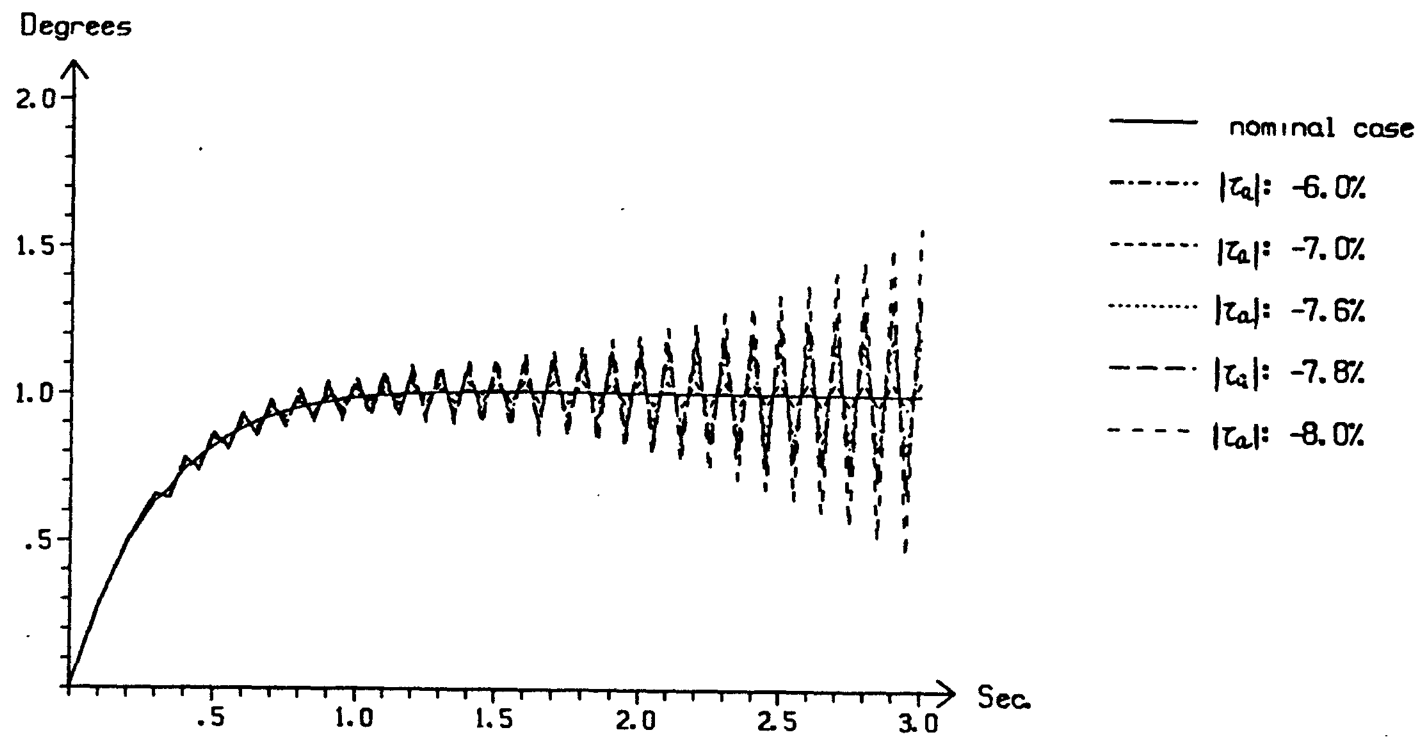

Fig. 7 α Step Response Under Parameter τ_a (-6% ~ -8%) Variations

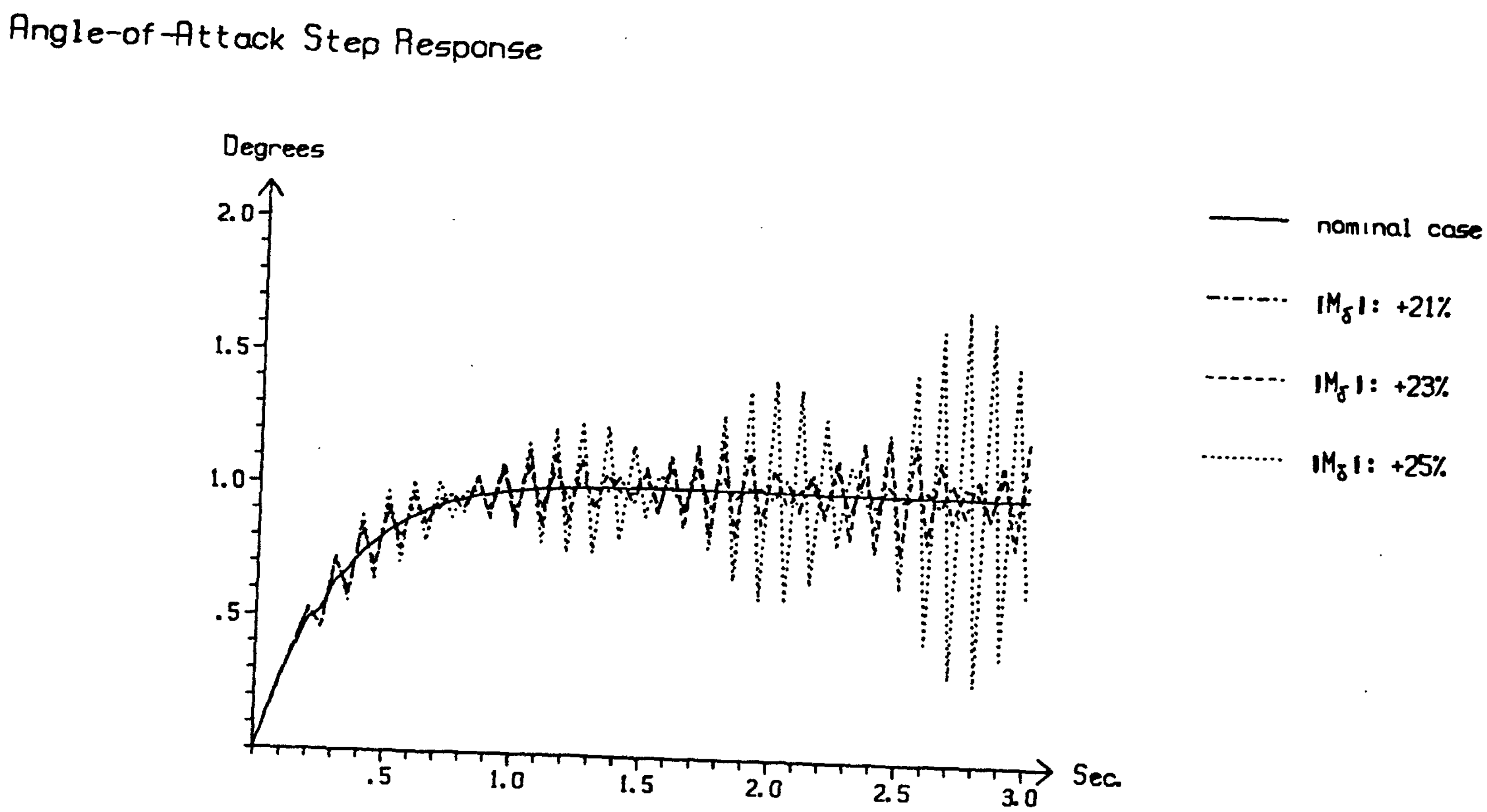

Fig. 8 α Step Response Under Parameter M_δ (+21% ~ +25%) Variations