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An Operator Control Theory Approach To The Shell Standard Control Problem

by

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Abstract

The Shell Standard Control Problem (SSCP), with its hard constraint specifications and the multiple performance objectives, is clearly the kind of problem to which the use of an on-line optimizing control algorithm like Quadratic Dynamic Matrix Control (QDMC) seems to be an appropriate approach. The presence of the hard constraints in the on-line optimization problem however, makes the overall system nonlinear even though the process dynamics are assumed linear. The Contraction Mapping Principle has been applied to the operator mapping the state of the system (plant+controller) at sampling point k to that at $k+1$ to obtain nominal and robust stability conditions for the nonlinear system. These conditions can be used to analyze the stability properties of the QDMC algorithm and to obtain design insights by examining their variation during simulations of the system.

1 Preliminaries

The hard constraint and performance specifications of the SSCP can be described as a QDMC problem in a relatively straightforward manner, as it is illustrated in [7]. This section will set some notation for later use.

The properties of the controller are independent of the type of model description used for the plant (see, e.g., [2]). The impulse response description is a convenient one:

$$y(k+1) = H_1 u(k) + H_2 u(k-1) + \dots + H_N u(k-N+1) \tag{1}$$

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where y is the output vector, u is the input vector and N is an integer sufficiently large for the effect of inputs more than N sample points in the past on y to be negligible.

The QDMC-type algorithms [1,3,4,2] use a quadratic objective function that includes the square of the weighted norm of the predicted error (setpoint - predicted output) over a finite horizon in the future as well as penalty terms on u or Δu:

$$\min_{u(\bar{k}),\ldots,u(\bar{k}+M-1)} \sum_{l=1}^{P} [\begin{array}{l} e(\bar{k}+l)^T\Gamma^2 e(\bar{k}+l) + u(\bar{k}+l-1)^T B^2 u(\bar{k}+l-1) \\ +\Delta u(\bar{k}+l-1)^T D^2 \Delta u(\bar{k}+l-1)] \end{array} \tag{2}$$

The minimization of the objective function is carried out over the values of $\Delta u(\bar{k})$, $\Delta u(\bar{k}+1)$,..., $\Delta u(\bar{k}+M-1)$, where $\bar{k}$ is the current sample point and M a specified parameter. The minimization is subject to possible hard constraints on the inputs u, their rate of change Δu, the outputs y and other process variables usually referred to as associated variables. The details on the formulation of the optimization problem can be found in the cited references. After the problem is solved on-line at $\bar{k}$, only the optimal value for the first input vector $\Delta u(\bar{k})$ is implemented and the problem is solved again at $\bar{k}+1$. The optimal $u(\bar{k})$ depends on the tuning parameters of the optimization problem, the current output measurement $y(\bar{k})$ and the past inputs $u(\bar{k}-1)$,..., $u(\bar{k}-N)$ that are involved in the model output prediction. Let f describe the result of the optimization:

$$u(k) = f(y(k), u(k-1), \ldots, u(k-N)) \tag{3}$$

The optimization problem of the QDMC algorithm can be written as a standard Quadratic Programming problem:

$$\min_{v} q(v) = \frac{1}{2} v^T G v + g^T v \tag{4}$$

subject to

$$A^T v \geq b \tag{5}$$

where

$$v = [\ \Delta u(\bar{k})\ \ \ldots\ \ \Delta u(\bar{k}+M-1)\ ]^T \tag{6}$$

and the matrices G, A, and vectors g, b are functions of the tuning parameters (weights, horizon, M, some of the hard constraints). The vectors g, b are also linear functions of $y(\bar{k})$, $u(\bar{k}-1)$,..., $u(\bar{k}-N)$. For the optimal solution v^* we have [5]:

$$\begin{bmatrix} G & -\hat{A} \\ -\hat{A}^T & 0 \end{bmatrix} \begin{bmatrix} v^* \\ \lambda^* \end{bmatrix} = - \begin{bmatrix} g \\ \hat{b} \end{bmatrix} \tag{7}$$

where $\hat{A}^T$, $\hat{b}$ consist of the rows of A^T, b that correspond to the constraints that are active at the optimum and λ^* is the vector of the Lagrange multipliers. The optimal $\Delta u(\bar{k})$, descibed by (3), corresponds to the first m elements of the v^* that satisfies (7), where m is the dimension of u.

The special form of the LHS matrix in (7) allows the numerically efficient computation of its inverse in a partitioned form [5]:

$$\begin{bmatrix} G & -\hat{A} \\ -\hat{A}^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} H & -T \\ -T^T & U \end{bmatrix} \tag{8}$$

Then

$$v^* = -Hg + T\hat{b} \tag{9}$$

$$\lambda^* = T^T g - U\hat{b} \tag{10}$$

and

$$u(\bar{k}) = u(\bar{k}-1) + \begin{bmatrix} I & 0 & \dots & 0 \end{bmatrix} v^* \stackrel{\text{def}}{=} f(y(k), u(k-1), \dots, u(k-N)) \tag{11}$$

2 Stability Conditions

Some recent work by the author [8] used the Operator Control Theory framework [6], to study the properties of the overall nonlinear system. In this approach, the stability and performance of the nonlinear system can be studied by applying the contraction mapping principle on the operator F that maps the "state" of the system (plant + controller) at sample point k to that at sample point $k+1$. The fact that the plant dynamics are assumed linear allows us to obtain results and carry out computations that are not yet feasible in the general case. We can define as the "state" of the system at sample point k the following vector

$$x(k) = \begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} \tag{12}$$

where

$$\begin{array}{lll} x_1(k+1) \stackrel{\text{def}}{=} u(k) & = & f(y(k), u(k-1), \dots, u(k-N)) \\ & = & f(H_1 u(k-1) + \dots + H_N u(k-N), \\ & & u(k-1), \dots, u(k-N)) \\ & \stackrel{\text{def}}{=} & \Psi(u(k-1), \dots, u(k-N)) \\ & = & \Psi(x(k)) \\ x_2(k+1) \stackrel{\text{def}}{=} u(k-1) & = & x_1(k) \\ \vdots & \vdots & \\ x_N(k+1) \stackrel{\text{def}}{=} u(k-N+1) & = & x_{N-1}(k) \end{array} \tag{13}$$

The "state" vector $x(k)$ is defined so that knowledge of it allows the computation of $x(k+1)$ by applying the plant and controller equations on it. Indeed the operator F that maps $x(k)$ to $x(k+1)$ is given by

$$x(k+1) = F(x(k)) = \begin{bmatrix} \Psi(x(k)) \\ x_1(k) \\ \vdots \\ x_{N-1}(k) \end{bmatrix} \tag{14}$$

Note, however, that although f is known, since it describes the on-line optimizing control algorithm and it involves only the process model, Ψ is not exactly known, because it involves the "true" plant impulse response coefficients H_1,..., H_N.

Convergence of the successive substitution $x(k+1) = F(x(k))$ to the unique fixed point of the contraction implies stability of the overall nonlinear system; fast convergence implies good performance. The use of the contraction mapping principle allows the development of conditions for robust stability and performance in terms of some induced matrix norm of the derivative F' of the above operator F.

Let J_i be a set of indices for the active constraints of (4) and J_1,..., J_n correspond to all possible active sets of constraints when all xs in the domain of F are considered. Every such J_i corresponds to an $\hat{A}_i$ and a $\hat{b}_i$. It was shown in [8] that for all xs that correspond to the same J_i and for which an infinitecimal change in their value does not change the set of active constraints, the derivative of Ψ and therefore of F exist and it has the same value that depends on the particular set J_i:

$$F'_{J_i} = \begin{bmatrix} (\nabla_{x_1}\Psi)_{J_i} & (\nabla_{x_2}\Psi)_{J_i} & \dots & (\nabla_{x_{N-1}}\Psi)_{J_i} & (\nabla_{x_N}\Psi)_{J_i} \\ I & 0 & \dots & 0 & 0 \\ 0 & I & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I & 0 \end{bmatrix} \tag{15}$$

where from (13) it follows that

$$(\nabla_{x_j}\Psi)_{J_i} = (\nabla_{x_j}f)_{J_i} + (\nabla_y f)_{J_i} H_j \tag{16}$$

The derivatives of f can be computed easily from (11):

$$(\nabla_{x_j}f)_{J_i} = \begin{bmatrix} I & 0 & \dots & 0 \end{bmatrix} (-H_{J_i}\nabla_{x_j}g + T_{J_i}\nabla_{x_j}\hat{b}_i) \tag{17}$$

where the derivatives of g, b_i are constant since g, b are linear functions of $y(\bar{k})$, $u(\bar{k}-1)$,..., $u(\bar{k}-N)$. The same expression as in (17) is also true for the derivative with respect to $y(\bar{k})$, the current measurement. Also note that in the case of x_1, the identity matrix I should be added to the RHS of (17).

It turns out that $F(x)$ is quasi-linear and that it is differentiable everywhere except the points where an infinitecimal change will change the set of active constraints at the optimum of (4). The following theorems were proven in [8]. The terms stability and instability of the control system are used in the global sense over the domain of F under consideration.

Theorem 1 *F is a contraction if and only if there exists a consistent matrix norm $||.||$, for which*

$$||F'_{J_i}|| < 1, \quad i = 1, \dots, n \tag{18}$$

The practical use of (18) is limited by the fact that finding an appropriate consistent norm is not a trivial task. The following three theorems provide conditions which are more readily computable.

Theorem 2 *The control system is asymptotically stable if*

$$||\ (\nabla_{x_1}\Psi)_{J_i} \quad (\nabla_{x_2}\Psi)_{J_i} \quad \dots \quad (\nabla_{x_N}\Psi)_{J_i}\ ||_\infty < 1, \quad i = 1, \dots, n \tag{19}$$

where

$$||B||_\infty = \max_i \sum_{j=1}^{N} |b_{ij}| \tag{20}$$

Note that for single-input single-output plants (19) becomes

$$\sum_{j=1}^{N} |\frac{\partial \Psi_{J_i}}{\partial x_j}| < 1, \quad i = 1, \ldots, n \tag{21}$$

which for the unconstrained case is simply a sufficient condition for the closed-loop poles to lie inside the Unit Circle.

Theorem 3 *F can be a contraction only if*

$$\rho(F'_{J_i}) < 1, \quad i = 1, \ldots, n \tag{22}$$

where $\rho(A)$ is the spectral radius of A. Note that if the optimization (4) is not subject to (5), then $n = 1$ and (22) becomes sufficient as well, because, given a matrix one can always find a consistent norm arbitrarily close to its spectral radius. The reason that (22) is not sufficient in general is that such a consistent norm is in general a different one for two different matrices (different J_is), while (18) requires the same norm for all i. In the case of $n = 1$, (22) translates to the requirement that the closed-loop poles of the system are located inside the Unit Circle.

If (22) is not true, then F is not a contraction. This however does not necessarily imply that the control system is unstable. The following theorem provides a condition that is sufficient for instability.

Theorem 4 *The control system is unstable if*

$$\rho(F'_{J_i}) > 1, \quad i = 1, \ldots, n \tag{23}$$

Theorem 4 can be used to predict instability of the overall nonlinear system. Theorem 3 on the other hand does not seem at a first glance to be of much use, since violation of (22) does not necessarily imply instability. From a practical point of view, however, violation of that condition for some i, should be taken as a very serious warning that the control system parameters should be modified. The reason is that when in the region of the domain of F that corresponds to that i, the system will behave as a virtually unstable system, the only hope for stability being to move to a region with $\rho(F'_{J_i}) < 1$. It might be the case that for a particular system in question this will always happen, making this system a stable one. But even in this case, a temporary unstable-like behavior might occur, thus making the control algorithm practically unacceptable.

From (16) we see that F'_{J_i} depends on the impulse response coefficient matrices $H_1, \ldots, H_N$ of the actual plant. These matrices are never known exactly and so in order to guarantee stability for the actual plant, one has to compute the conditions of Section 2 not just for the model, but for all possible plants. To do so, one needs to have some information on the possible modeling error associated with the H_is. Let $\mathcal{H}$ be the set of possible values for these coefficients. Then

Theorem 5 *The control system is asymptotically stable for all plants with coefficients in $\mathcal{H}$ if*

$$\sup_{\mathcal{H}} \| \ (\nabla_{x_1}\Psi)_{J_i} \quad (\nabla_{x_2}\Psi)_{J_i} \quad \ldots \quad (\nabla_{x_N}\Psi)_{J_i} \ \|_\infty < 1, \quad i = 1, \ldots, n \tag{24}$$

Theorem 6 *F can be a contraction for all plants with coefficients in $\mathcal{H}$ only if*

$$\sup_{\mathcal{H}} \rho(F'_{J_i}) < 1, \quad i = 1, \ldots, n \tag{25}$$

3 A Robust Linear Control Stabilization Interpretation of the Necessary Conditions

In order to carry out the maximizations over $\mathcal{H}$ described by (25), (24), one needs to parametrize the "uncertain" $H_1, \ldots, H_N$, in terms of a fewer "uncertain" parameters. For example, in the simple case where the linear plant dynamics are described by the transfer function $\frac{K}{\tau s+1}$, where K, τ, are within some ranges, we can write $H_1, \ldots, H_N$, as functions of K, τ, and compute $\sup_{\mathcal{H}}$ as $\sup_{K,\tau}$. However, the situation is usually more complex, a fact that makes the efficient parametrization of the modeling error in $H_1, \ldots, H_N$, a very important research topic.

The following re-formulation of the necessary conditions of the previous section, allows us to bypass the problem of dealing with uncertainty in the Hs directly, and use the tools that were developed for Robust Linear Control (e.g., the structured singular value) to treat any of the types of model error that can be handled by that theory. Consider a standard feedback controller $C(z)$. Then

$$u(z) = C(z)(r(z) - y(z)) \tag{26}$$

where r is the setpoint vector. Define

$$C_{J_i}(z) \stackrel{\text{def}}{=} - \left[I - (\nabla_{x_1}f)_{J_i} z^{-1} - \ldots - (\nabla_{x_N}f)_{J_i} z^{-N}\right]^{-1} (\nabla_y f)_{J_i} \tag{27}$$

Since the plant is assumed to be open-loop stable, for stability of this linear control system we need that the closed-loop transfer function between u and r or d (disturbance) be stable. From (26), (27) we get by using (1)

$$u(z) = - \left[I - (\nabla_{x_1}\Psi)_{J_i} z^{-1} - \ldots - (\nabla_{x_N}\Psi)_{J_i} z^{-N}\right]^{-1} (\nabla_y f)_{J_i} r(z) \tag{28}$$

where $(\nabla_{x_j}\Psi)_{J_i}$ is given by (16). Hence, stability of the linear unconstrained system under feedback control $C_{J_i}(z)$ is equivalent to stability of the transfer matrix in (28), which is equivalent to (22) since F'_{J_i} is the companion matrix of the denominator of (28). Hence we have

Theorem 7 *F can be a contraction only if all feedback controllers $C_{J_i}(z)$, $i = 1, \ldots, n$, produce a stable system when applied to the unconstrained process.*

Theorem 8 *F can be a contraction for all plants in a set Π, only if all feedback controllers $C_{J_i}(z)$, $i = 1, \ldots, n$, stabilize all plants in the set Π.*

The advantage of Thm. 8 over Thm. 6 lies in the fact that through Thm. 8 we can handle any set Π that Robust Linear Control theory can. This new interpretation of the conditions also indicates that robust performance conditions can be formulated for the same set of feedback controllers. For the sufficient conditions a similar formulation may be possible but it would probably involve some conservativeness.

4 Practical Interpretation of a Condition Violation

Conditions (22), (19) can be used to examine the nominal stability of the system for a particular selection of tuning parameters. An important question however is what are the implications if for a particular $\hat{A}_i$ the conditions are not satisfied. This would only be relevant if the particular combination of active constraints at the optimum can actually occur during the operation of the control system. The following is a procedure that can decide if a certain set of active constraints at the optimum is relevant.

Let $\check{A}^T$, $\check{b}$ consist of the rows of A^T, b that correspond to the inactive constraints at the optimum. Then by using (9), (10) we see that in order for such a combination to be possible at the optimum we need to have

$$\check{A}^T(-Hg + T\hat{b}) \geq \check{b} \tag{29}$$

$$T^T g - U\hat{b} \geq 0 \tag{30}$$

Since g, b are linear combinations of the past manipulated variables and the current measurement, (29), (30) can be combined with the hard constraints on the past us, the past Δus and the output $y(\bar{k})$ to constitute a system of linear inequalities that have to have a feasible solution over the values of the past inputs and the current measurement. Note that depending on the estimate of expected disturbances, one may wish to modify the bounds on $y(\bar{k})$ that are used in the above problem. If the problem has no feasible solution, then the fact that for that particular $\hat{A}$ the stability conditions are not satisfied, is of no practical importance.

Note that the above procedure can also serve to construct a sequence of possible past inputs that can lead to a situation during the operation of the control system where the stability conditions are not satisfied.

5 Analysis of Simulation Results

The computation of the stability conditions at all possible combinations of active constraints at the optimum of the on-line optimization problem can be extremely time-consuming and therefore a systematic method that does not have to check all possibilities is needed. Since no such method for checking the conditions is currently available, the following procedure for providing the designer with insights on tuning the controller parameters can be used.

For a given set of values for the tuning parameters, the designer can simulate the overall system for certain disturbances and/or setpoints that he considers of practical relevance. Such simulations can show instability or simply bad behavior at certain points during the simulation. This behavior which stops short of instability might be captured as a violation of condition (22) which is necessary for F to be a contraction. By computing these conditions at every sampling point during the simulation and by studying the robustness properties of the C_{J_i}s that correspond to the points where the conditions were violated, the designer may be able to improve the tuning parameters.

6 Illustrations

6.1 Robust Stability of a SISO process

Consider the process model

$$\tilde{p}(s) = \frac{1}{s+1} \tag{31}$$

A sampling time $T = 0.1$ will be used and the control algorithm will minimize on-line the objective function

$$\min_{u(\bar{k}),\ldots,u(\bar{k}+M-1)} \sum_{l=1}^{P} \left[e(\bar{k}+l)^T \Gamma^2 e(\bar{k}+l) + \Delta u(\bar{k}+l-1)^T D^2 \Delta u(\bar{k}+l-1) \right] \tag{32}$$

To allow the analytic study of the properties of the control system we shall choose the parameters to be $P = M = \Gamma = 1$. A choice of $D = 0$, when there are no hard constraints, will result in an IMC controller that inverts the model [3].

Let us now consider a model-plant mismatch caused by a delay term in the plant:

$$p(s) = \frac{e^{-0.15s}}{s+1} \tag{33}$$

For this plant, robust linear control theory can easily show that the control system will be unstable for $D = 0$. D has to ber increased over $D = 0.2$ to stabilize it. The choice $D = 0.4$ results in reasonable performance.

Our interest in this example has to do with the effect of hard constraints on its output. Let us specify a lower bound of -1 and an upper bound of $+1$ for y and include these constraints in the on-line optimization problem. Since the horizon $P = 1$, it is not possible for both to be active at the optimum. In this case $n = 3$, corresponding to (i) no active constraints, (ii) upper constraint active, (iii) lower constraint active. Analytic computaion of $c_{J_i}(z)$, $i = 1, 2, 3$, results in the expressions

$$\begin{aligned} c_{J_1}(z) = H_1/[\ & (D^2 + H_1^2) + (H_1 H_2 - H_1^2 - D^2) z^{-1} \\ & + H_1(H_3 - H_2) z^{-2} + \ldots + H_1(H_N - H_{N-1}) z^{-N+1} - H_1 H_N z^{-N}] \end{aligned} \tag{34}$$

$$\begin{aligned} c_{J_2}(z) = c_{J_3}(z) = 1/[\ & H_1 + (H_2 - H_1) z^{-1} \\ & + (H_3 - H_2) z^{-2} + \ldots + (H_N - H_{N-1}) z^{N+1} - H_N z^{-N}] \end{aligned} \tag{35}$$

One can easily see from these exressions that c_{J_2} and c_{J_3} correspond to an IMC controller that inverts the process model, the same as c_{J_1} for $D = 0$. The difference is that D does not appear in (35) and therefore this controller will be unstable when the model-plant mismatch is present. The question that arises now, is the one discussed in Section 4. For the case of the upper constraint and for a setpoint equal to zero, (30) predicts that if the system is at

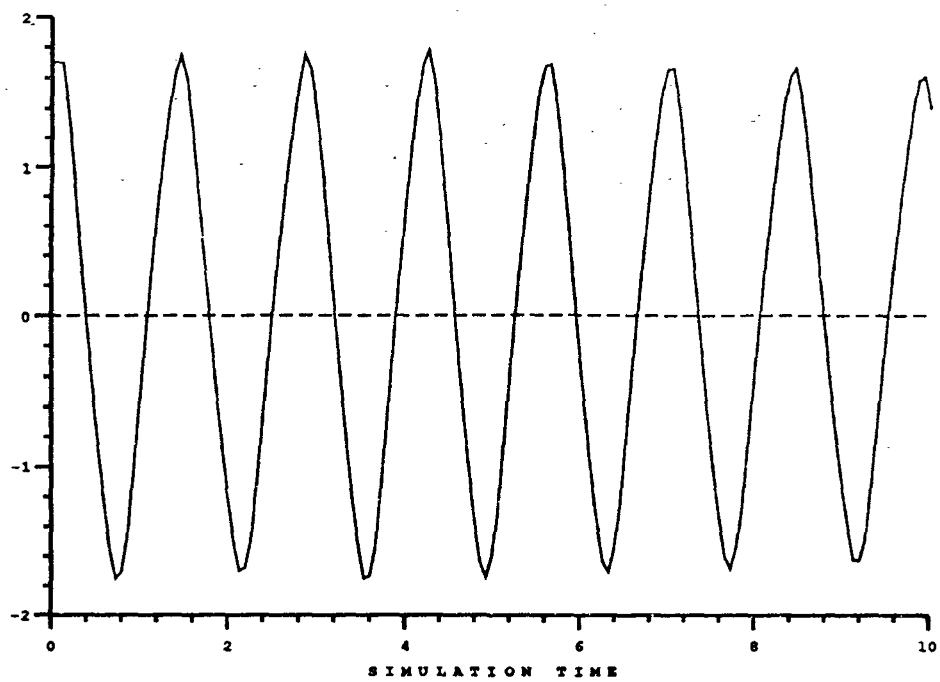

Figure 1: Constrained; D=0.4 and d=1.70

equilibrium, a disturbance of magnitude greater than 1.6 will result in an on-line optimization where the upper constraint is active. The system could however manage to return to the contraction region of no active constraints. Indeed for a disturbance of 1.7, as Fig. 1 shows, the system is still stable, although at the edge of instability. An increase of the disturbance to 1.75 however results in an unstable system as Fig. 2 shows. Note that $D = 0.4$ is being used; although D does not appear in (35), it does play a role on whether the constraints are active at the optimum. Both simulations use the plant of (33).

Let us now remove the constraints from the optimization problem and repeat the simulation for the same $d = 1.75$ and $D = 0.4$. The result is shown in Fig. 3. The response is reasonable and the constraints are virtually satisfied, although they were removed from the optimization problem. This example is not meant to suggest that output constraints should not be included in the optimization, but merely to point out that their effect should be studied carefully before their inclusion and to demonstrate that the stability conditions that were provided in this paper can predict this effect successfuly.

6.2 2×2 Subsystem of the SSCP

Let us consider the top 2×2 part of the Heavy Oil Fractionator of the SSCP [7]. This system has as outputs 1 and 2, the Top End Point and the Side End Point correspondingly. The

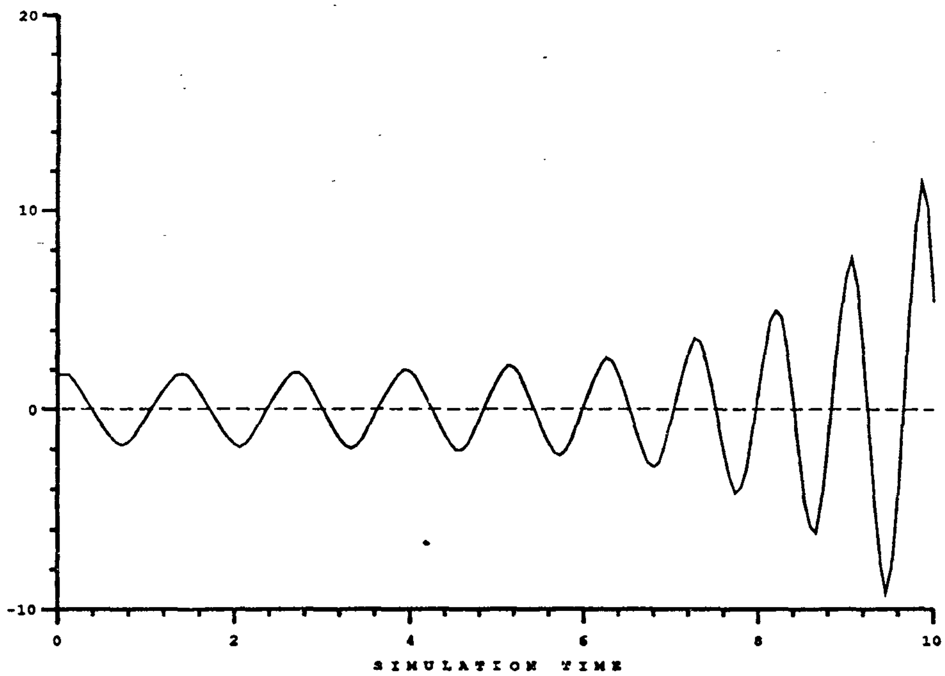

Figure 2: Constrained; D=0.4 and d=1.75

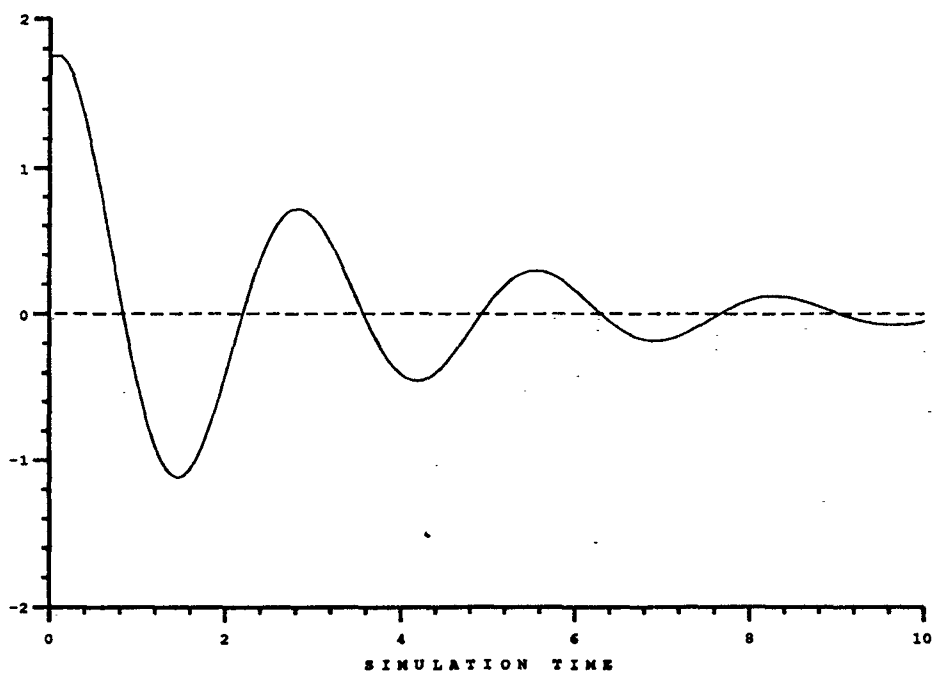

Figure 3: Unconstrained; D=0.4 and d=1.75

inputs are the Top Draw and the Side Draw. The transfer function of this subsystem is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{(4.05+2.11\epsilon_1)e^{-27s}}{50s+1} & \frac{(1.77+0.39\epsilon_2)e^{-28s}}{60s+1} \\ \frac{(5.39+3.29\epsilon_1)e^{-18s}}{50s+1} & \frac{(5.72+0.57\epsilon_2)e^{-14s}}{60s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{36}$$

where ϵ_1, ϵ_2 represent the model uncertainty and they can take any value between -1 and $+1$, 0 corresponding to the nominal model. A sampling time of $T = 6min$ is selected which results in lower and upper constraints of -0.3 and 0.3 for the changes in the inputs. Lower and upper constraints of -0.5 and 0.5 exist for all the inputs and outputs.

Our goal is to see how the stability conditions can be used to analyze simulation results. In the objective function of (2) we select $P = 6$, $M = 2$, $B = D = 0$. The minimization is carried out subject to the above described hard constraints. The Constraint Window for the outputs is 5-6 for the Top End Point and 3-4 for the Side End Point. Beginning the windows at earlier times may result in infeasibilities because of the longer time delays. It should be noted that this selection of parameters is meant as a simple one rather than an "optimal" one.

The simulation for no model-plant mismatch is shown in Fig. 4, where a disturbance in the form of simultaneous step changes of 0.5 in the Upper and the Intermediate Reflux Duties is used. The same disturbance is used in all simulations in this section. Use of the disturbance transfer function models yields the following output disturbance vector:

$$d(s) = \begin{bmatrix} \frac{1.20e^{-27s}}{45s+1} & \frac{1.44e^{-27s}}{40s+1} \\ \frac{1.52e^{-15s}}{25s+1} & \frac{1.83e^{-15s}}{20s+1} \end{bmatrix} \begin{bmatrix} 0.5/s \\ 0.5/s \end{bmatrix} \tag{37}$$

Note that the plot of ρ is the value of the necessary condition for the particular J_i occuring at the sample points during the simulation. When a model-plant mismatch is present, as in the following simulations, it is computed for the coefficients of the actual plant used in the simulation.

Next, a mismatch between the model and the plant is assumed, corresponding to $\epsilon_1 = -1$ and $\epsilon_2 = 1$. The simulation is shown in Fig. 5. By looking just at the outputs and inputs there is no indication of a potential problem. However by looking at the plot of ρ we see that the necessary condition is close to being violated during part of the simulation. It is simple to check that this part of the simulation corresponds to the case where at the optimum of the on-line optimization no constraint is active. The problem is not significant in this simulation because eventually, the lower constraint for the Top Draw becomes active at the optimum and we move to a well-behaved region. Let us now repeat the simulation of Fig. 5 but with the lower constraints for the inputs at -1 rather than -0.5. The simulation is shown in Fig.6 and this time the system sufers from persistent oscillations because the constraint does not become active early on. Figure 7 repeats the simulation of Fig. 6 but with a larger mismatch. We are using $\epsilon_1 = -1.2$ and $\epsilon_2 = 1.2$. This time we are in the instability region as the plots show. The question of interest at this point is how can one use the plot of ρ in Fig. 7 to make a parameter change so that the system is stabilized. From the previous simulations it is clear that one way would be to simply increase the value of the lower input constraint, i.e., use this constraint as a tuning parameter. What is important to note however is the following:

Tuning Observation

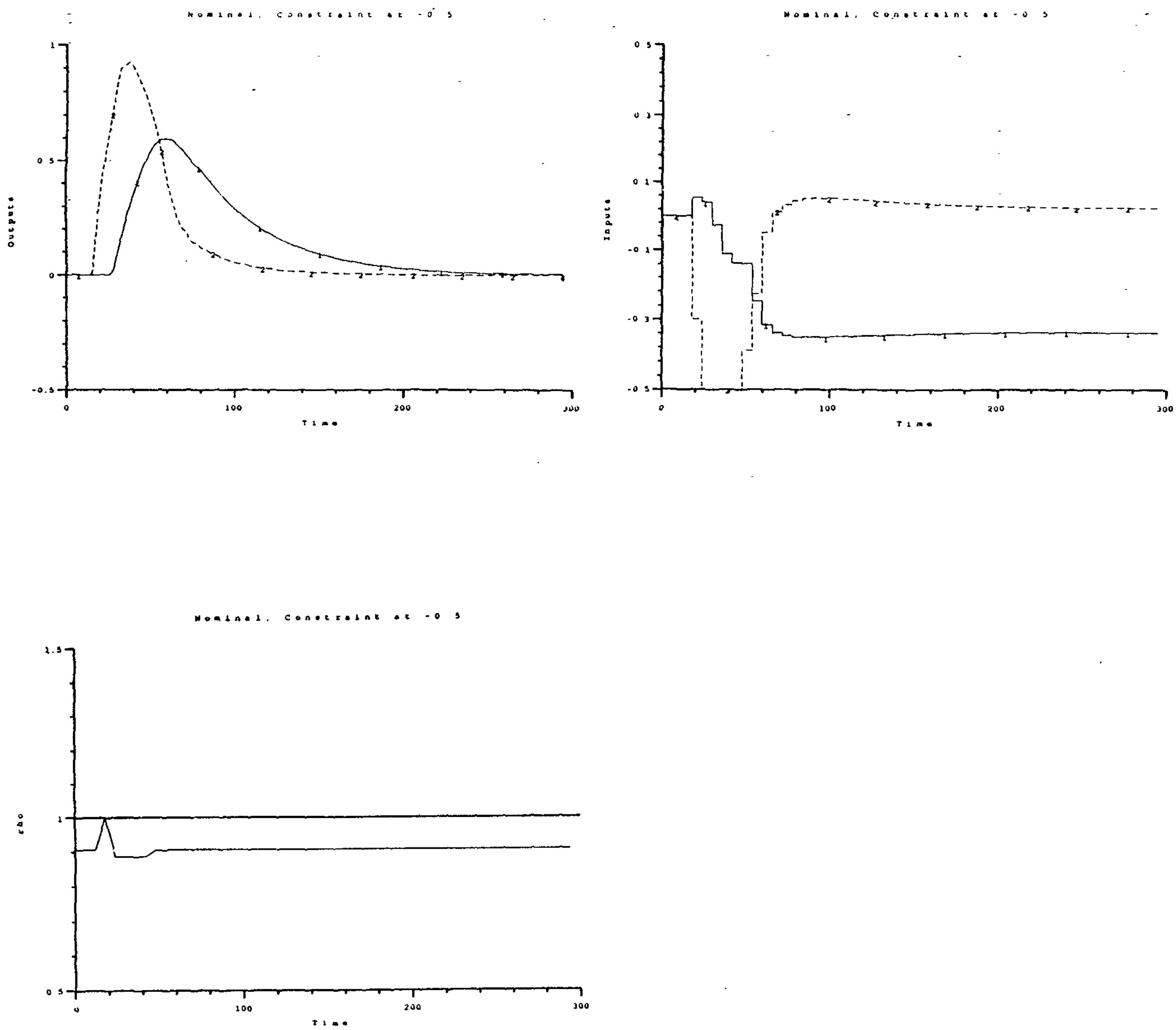

Figure 4: Nominal; lower input constraint at -0.5. (a) Outputs; (b) Inputs; (c) $\rho(F')$

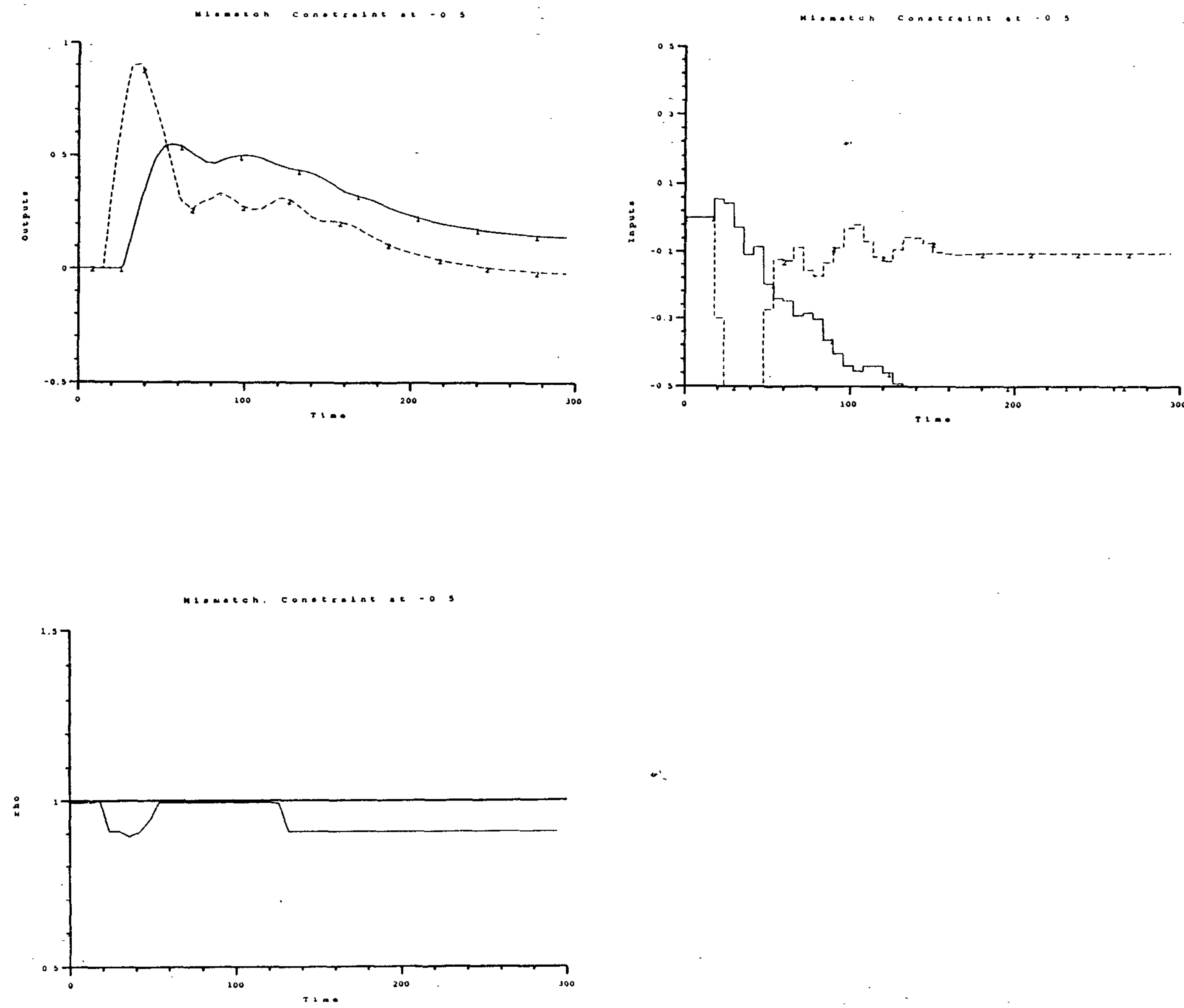

Figure 5: $\epsilon_1 = -1$, $\epsilon_2 = 1$; lower input constraint at -0.5. (a) Outputs; (b) Inputs; (c) $\rho(F')$

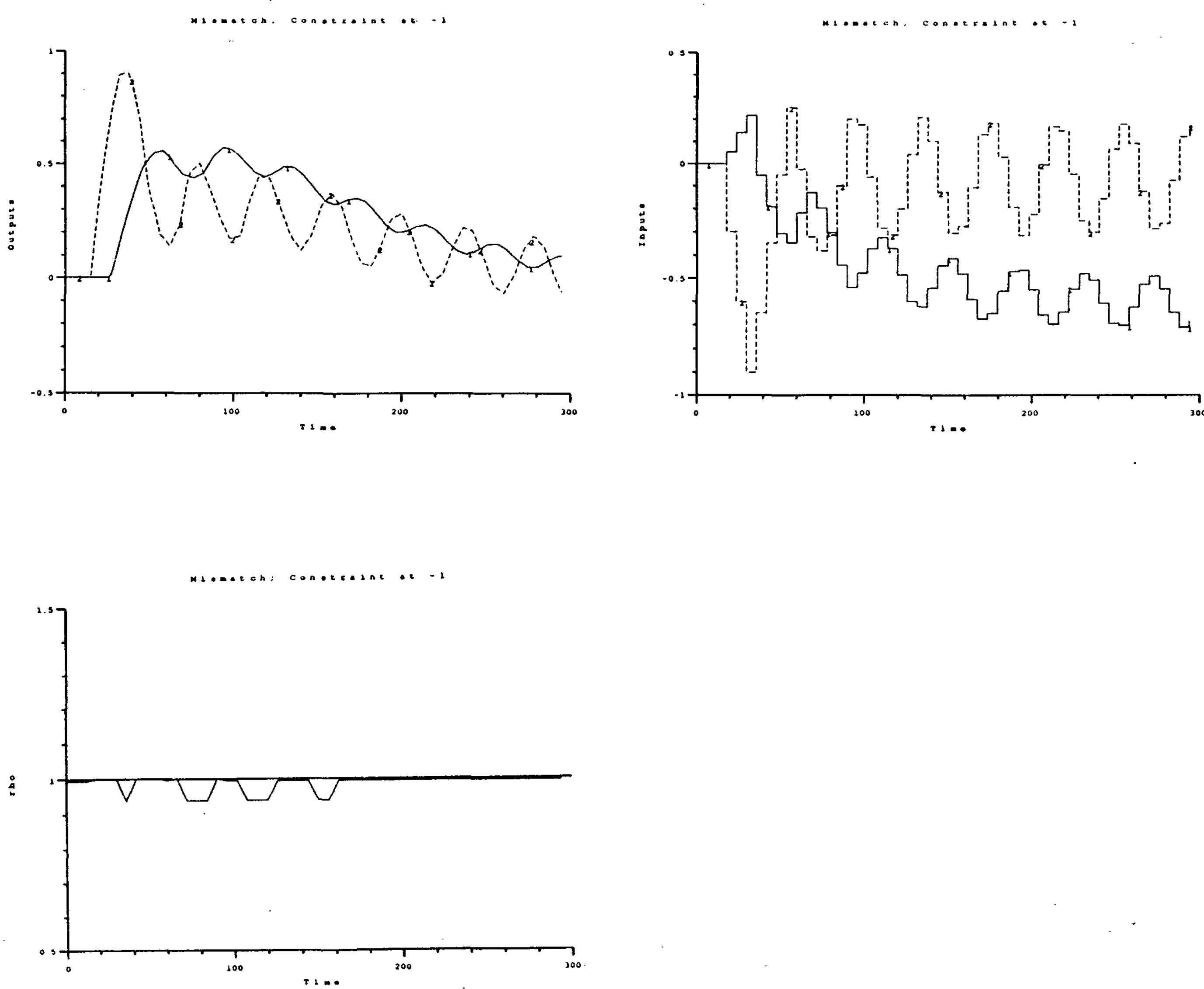

Figure 6: $\epsilon_1 = -1$, $\epsilon_2 = 1$; lower input constraint at -1. (a) Outputs; (b) Inputs; (c) $\rho(F')$

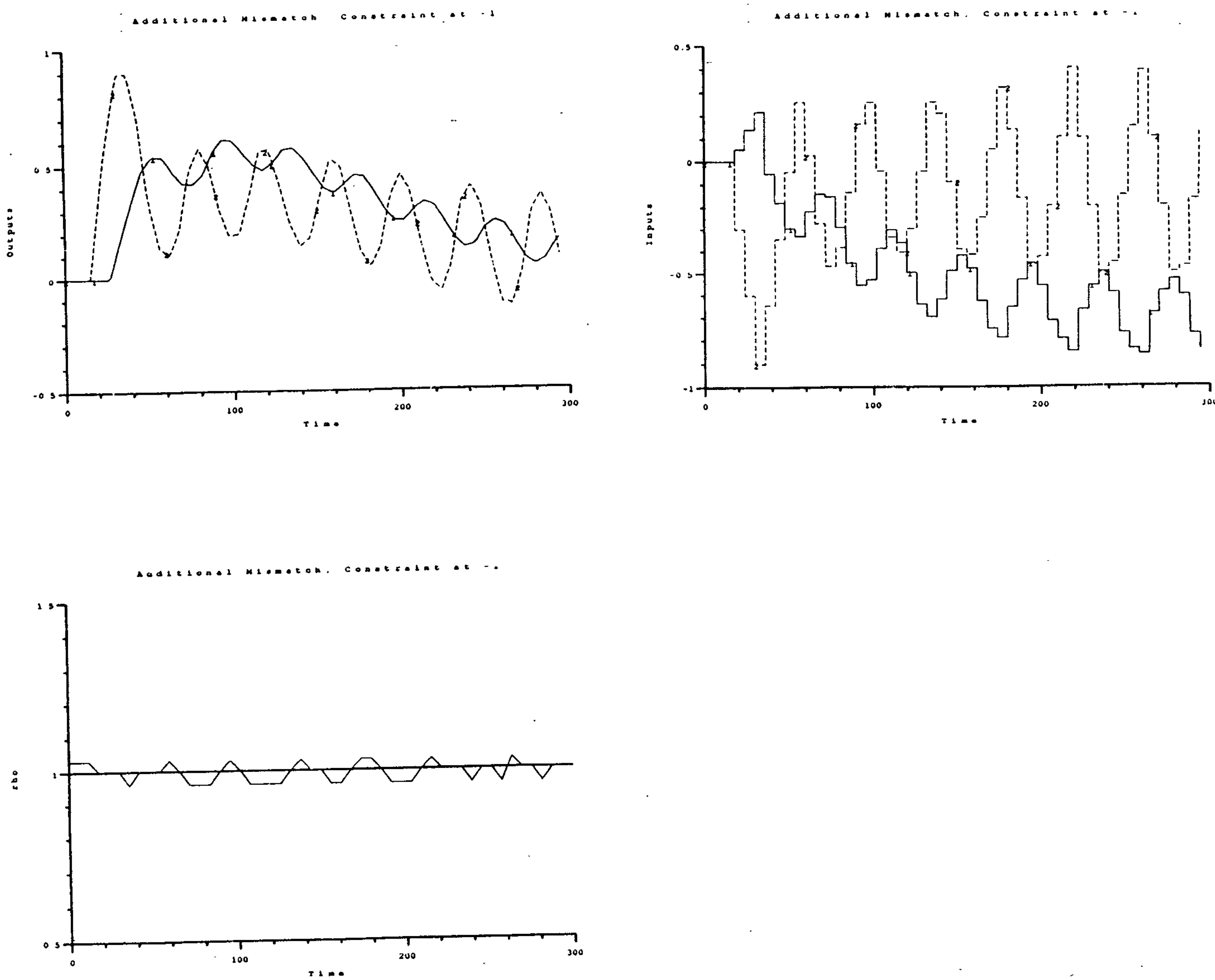

Figure 7: $\epsilon_1 = -1.2$, $\epsilon_2 = 1.2$; lower input constraint at -1. (a) Outputs; (b) Inputs; (c) $\rho(F')$

The values of the hard constraints do not appear in the expressions of the C_{J_i}s; hence they can influence stability only by keeping a destabilizing J_i from occuring. They cannot change a C_{J_i} into a stabilizing controller; this can be accomplished only by the parameters of the objective function.

Hence it seems that is safer to actually try to find values for the parameters of the objective function that make C_{J_1} stabilizing (where J_1 is defined to correspond to the case of no active constraints at the optimum), without changing the values of the hard constraints. But this is a problem that can be addressed through Robust Linear Control Theory. Use of the Structured Singular Value shows that a $B = 0.2$ stabilizes the system. The simulation is given in Fig. 8. Note that if the problematic C_{J_i} corresponded to some active constraints, the situation would stiil be treated through the same tools.

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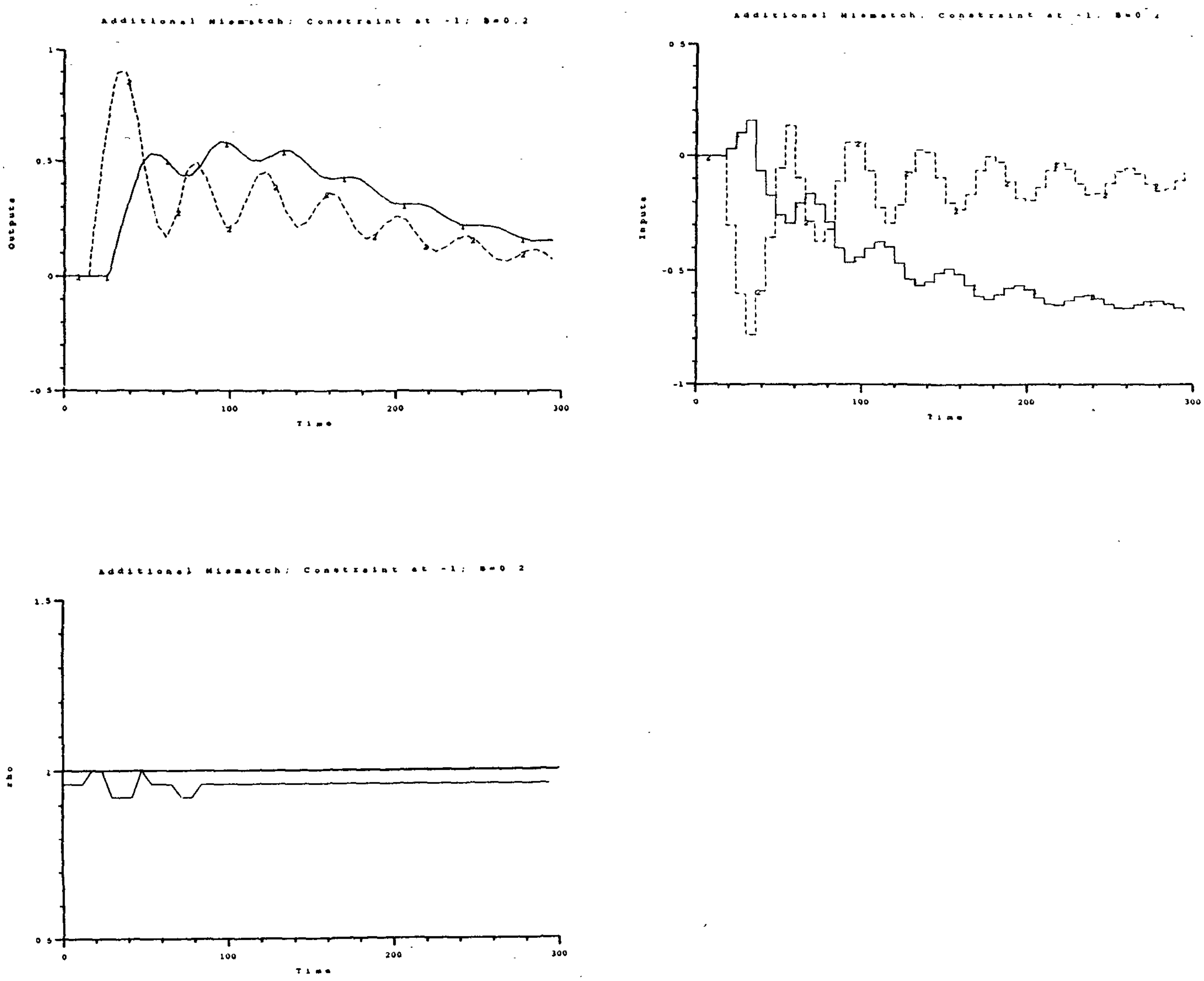

Figure 8: $\epsilon_1 = -1.2$, $\epsilon_2 = 1.2$; lower input constraint at -1; $B = 0.2I$. (a) Outputs; (b) Inputs; (c) $\rho(F')$