



SRC TR 88-90

**TECHNICAL
RESEARCH
REPORT**

**Multi-Level Design Optimization
Using Global Monotonicity**

by

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MULTI-LEVEL DESIGN OPTIMIZATION USING GLOBAL MONOTONICITY ANALYSIS

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ABSTRACT

This paper describes application of global monotonicity analysis within a multi-level design optimization framework. We present a general formulation and solution procedure, based on a bottom-level global monotonicity analysis, for a design optimization problem which is decomposed into three levels of subproblems. A well-known gear reducer example illustrates application of the method.

1. Introduction

One possible means of solution to a large and/or complex design optimization problem is a hierarchical decomposition of the problem into a number of smaller subproblems each with its own objective and constraints. In this type of decomposition, interconnection between subproblems is usually multi-level, Figure 1, where a given-level subproblem coordinates a lower-level subproblem(s) and in turn is coordinated by a higher-level subproblem. One of the main advantages of the multi-level design optimization methods is that they fit well

into the multi-disciplinary framework of design process where a number of engineering disciplines interact in order to obtain an integrated optimum design (Mesarovic et al. 1970; Sobieski and Haftka, 1987). Furthermore, these methods allow parallel processing, which is typical of a modern computing environment, and use of different specialized optimization techniques on various portions of the problem.

A multi-level design optimization procedure usually consists of two steps. First, the integrated or undecomposed problem (objective and constraint functions, also called the design optimization model) is partitioned into a hierarchy of two- or multi-level subproblems. To be successful in the first step, the integrated problem should be formulated in such a way that it will be fully or at least partially decomposable. While there is no systematic procedure available for the first step, one possible alternative is the one according to the physical make-up of the problem. For example, design optimization model of an aircraft might be decomposable into its main components, namely, wing, fuselage, landing gear, engine, etc. Second, starting with the lowest-level, the subproblems are solved independently. The solutions to the subproblems at a given level are then coordinated by the upper-level problems, i.e., subproblems are forced to select solutions corresponding to an overall optimum. In general, the lower- and the upper-level subproblems are solved iteratively. Each of the lower-level subproblems might be a constrained design optimization problem and should be solved several times before the solutions to the upper-level problems are obtained. The success and effectiveness of the second step often depends on how simple and independent are the solutions to the lower-level subproblems.

There exists a variety of decomposition-based approaches for solving a given design optimization problem. In general, these techniques fall into two different methods, namely, the goal coordination and the model coordination methods (Wismer, 1971; Kirsch, 1981). The model coordination method, in particular, is more attractive for engineering design optimization, since the iteration process may be terminated whenever it is desired with a feasible, even though nonoptimal, solution. Several engineering optimization problems have been solved using decomposition-based optimization methods including those in chemical (Wilde, 1965), mechanical (Siddall and Michael, 1980), structural (Kirsch, 1981; Haftka, 1984), and aerospace design (Sobieski et al., 1984; Barthelemy and Riley, 1986; Wrenn et al. 1987).

In a recent paper, Azarm and Li (1988) proposed a two-level design optimization approach, an extension of the model coordination method (Kirsch, 1981). The proposed approach applied to a number of problems including those in mechanical design optimization (Azarm and Li, 1987). The present paper is an extension of that effort. We present here a three-level formulation of a separable design optimization problem. A three-level solution procedure is then suggested based on the global monotonicity analysis (Papalambros and Wilde, 1988). The formulation and solution procedure which we present in this paper can be generalized to any number of levels. A well-known gear reducer example is used as a demonstration example.

2. Formulation

We consider the following nonlinearly constrained design optimization problem:

$$\text{Minimize } \{f(z): g(z) \leq 0\}$$

(1)

where z is an n -vector of design variables, f and g are the objective and the vector of inequality constraints, respectively. To simplify the problem, the equality constraints have been eliminated. However, in case of their presence, they may be handled either through direct elimination or constrained derivatives (Wilde and Beightler, 1967).

We assume that the problem is decomposable into three levels, namely, the top-level, the middle-level, and the bottom-level. The top-level is composed of one subproblem (or problem). However, the middle- or the bottom-level may be composed of several subproblems. In each subproblem, the variables are partitioned into two groups, namely, the local and the global variables. We define the global variables to be those quantities which are taken to be fixed in a subproblem, and the local variables to be those quantities which are taken to be changed in the subproblem. Quantities which are variables or fixed in each level are shown in Table 1. We have set $z = (y, x)^t$, where y represents the vector of top-level local design variables - fixed in the lower levels, and x represents the vector of top-level global design variables - fixed in the top-level. Likewise, we have set $x = (u, v)^t$, where u represents the vector of middle-level local design variables, and v together with y represent the vector of middle-level global design variables. Finally, in the bottom-level, v represents the vector of bottom-level local design variables, and u together with y represent the vector of bottom-level global design variables.

We assume that the objective function f is in the following additively separable form:

$$f(y,u,v) = f_0(y) + \sum_{i=1}^I [f_i(y,u_i) + \sum_{j=1}^J f_{i,j}(y,u_i,v_{i,j})] \quad (2)$$

where i, j are indices corresponding to the number of middle-level subproblems, number of bottom-level subproblems with respect to (w.r.t.) the middle-level subproblem i , respectively (Figure 2).

In addition, we assume that the inequality constraints are in the following form:

$$\begin{aligned} g_h(y) &\leq 0 & h &= 1, \dots, H \\ g_{i,k}(y, u_i) &\leq 0 & i &= 1, \dots, I \\ g_{i,j,l}(y, u_i, v_{i,j}) &\leq 0 & j &= 1, \dots, J \\ & & k &= 1, \dots, K \\ & & l &= 1, \dots, L \end{aligned} \quad (3)$$

where h, k , and l are indices corresponding to the number of inequality constraints in the top-level problem, number of inequality constraints in the middle-level subproblem (i), and number of inequality constraints in the bottom-level subproblem (i, j), respectively (Figure 2).

The formulation of the bottom-level subproblem (i, j) with y and u_i as the vectors of the bottom-level global design variables (fixed in the bottom-level), and $v_{i,j}$ as the bottom-level local design variable is:

$$\begin{aligned} &\text{Minimize}_{v_{i,j}} f_{B_{i,j}}(y, u_i, v_{i,j}) = f_{i,j}(y, u_i, v_{i,j}) \\ &\text{Subject to: } g_{i,j,l}(y, u_i, v_{i,j}) \leq 0, \quad l = 1, \dots, L \end{aligned} \quad (4)$$

The formulation of the middle-level subproblem (i) with y and $v_{i,j}^*$ (found from the bottom-level subproblem (i, j)) as the vector of middle-level global design variables (fixed in the middle-level) and u_i as the vector of middle-level local

design variables is:

$$\begin{aligned} \text{Minimize}_{u_i} f_{M_i}(y, u_i, v_{i,j}^*) &= f_i(y, u_i) + \sum_{j=1}^J f_{i,j}(y, u_i, v_{i,j}^*) \\ \text{Subject to: } g_{i,k}(y, u_i) &\leq 0, \quad k = 1, \dots, K \end{aligned} \quad (5)$$

Finally, the formulation of the top-level problem with u_i^* and $v_{i,j}^*$ as the vector of top-level global design variables (found from the lower levels, fixed in the top-level), and y as the vector of top-level local design variables is:

$$\begin{aligned} \text{Minimize}_y f(y, u_i^*, v_{i,j}^*) &= f_0(y) + \sum_{i=1}^I [f_i(y, u_i^*) + \sum_{j=1}^J f_{i,j}(y, u_i^*, v_{i,j}^*)] \\ \text{Subject to: } g_h(y) &\leq 0, \quad h = 1, \dots, H \end{aligned} \quad (6)$$

Figure 2 shows in three levels, the bottom-level subproblem (i,j), the middle-level subproblem (i), and the top-level problem.

3. Solution Procedure

The iterative solution procedure used for the decomposed problem is summarized below:

Given an initial point as the current point,

$$z^0 = (y^0, u^0, v^0)^t,$$

Begin step A, for $i = 1, \dots, I$,

begin step B, for $j = 1, \dots, J$,

(B.1) for a given $(y^0, u_i^0)^t$, use global monotonicity analysis to find $v_{i,j}^*$ from the bottom-level subproblem (i,j),

(B.2) find a new u_i^0 , for a given y^0 , from the middle-level subproblem (i) such that f_{M_i} is decreased,

(B.3) return to step (B.1) until the minimum for f_{M_i} is obtained, for a

given y^0 ,

end step B,

(A.1) find a new y^0 from the top-level problem such that f is decreased,

(A.2) go to step A until the minimum for f is obtained,

End step A.

The middle- and top-level subproblems are solved by a conventional (single-level) optimization method while the bottom-level subproblems are solved by the global monotonicity analysis. To do that, we assume in subproblem (i,j) the objective function $f_{B_{i,j}}$ is increasing w.r.t. the variable $v_{i,j}$ and the first L' constraints ($L' < L$) are decreasing w.r.t. $v_{i,j}$, then we can show that (Azarm and Li, 1988):

$$v_{i,j}^* = \max \{g'_{i,j,l} : 1 \leq l \leq L'\} \quad (7)$$

where $g'_{i,j,l}$ is obtained by rewriting constraint $g_{i,j,l}(y,u,v_{i,j}) \leq 0$ in the form of $v_{i,j} \geq g'_{i,j,l}(y,u)$. Likewise, when the objective function of subproblem (i,j) is decreasing w.r.t. $v_{i,j}$ and the first L' constraints are increasing w.r.t. $v_{i,j}$, then:

$$v_{i,j}^* = \min \{g'_{i,j,l} : 1 \leq l \leq L'\} \quad (8)$$

The assumption that the bottom-level objective function is increasing/decreasing, and the bottom-level constraints are decreasing/increasing, w.r.t $v_{i,j}$ Within a given range is not unrealistic. In fact, many engineering design optimization problems have one or more such design variables (Papalambros and Wilde, 1988).

Note that if the middle-level subproblems (or the top-level problem) are solved by a derivative-based optimization method, then we may need to obtain:

$$df_{M_i} / du_i = \partial f_{M_i} / \partial u_i + (\partial f_{M_i} / \partial v_{i,j}^*) (\partial v_{i,j}^* / \partial u_i) \quad (9)$$

This derivative may display a discontinuous behavior as a result of eq. (7) or (8), unless the active constraints found from the bottom-level subproblems are unchanged (Lootsma and Ragsdell, 1988).

4. A Gear Reducer Example

In this section, we present a well-known gear reducer example, Figure 3, which was first formulated by Golinski (1970) and solved by several optimization methods including those by Datsoris (1982), Azarm (1984), Li and Papalambros (1985). Here we present the final design optimization model. The reader may consult the cited reference for further information.

In the gear reducer example, the design objective is to minimize the overall volume (or weight). The design variables for the example are as follows:

- x₁ = gear face width (cm)
- x₂ = teeth module (cm)
- x₃ = number of teeth of pinion
- x₄ = distance between bearings 1 (cm)
- x₅ = distance between bearings 2 (cm)
- x₆ = diameter of shaft 1 (cm)
- x₇ = diameter of shaft 2 (cm)

And, the constraints are as follows:

- g₁ : Upper bound on the bending stress of the gear tooth.

- 92 : Upper bound on the contact stress of the gear tooth.
- 93-94 : Upper bounds on the transverse deflection of the shaft.
- 95-96 : Upper bounds on the stresses of the shaft.
- 97-923 : Dimensional restrictions based on space and/or experience.
- 924-925 : Design condition for the shaft based on experience.

Finally, the nonlinear programming statement for this example is presented:

$$\begin{aligned} \text{Minimize } f(x) = & 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) \\ & + 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \end{aligned} \quad (10)$$

subject to:

$$g_1: 27x_1^{-1}x_2^{-2}x_3^{-1} \leq 1$$

$$g_2: 397.5x_1^{-1}x_2^{-2}x_3^{-2} \leq 1$$

$$g_3: 1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \leq 1$$

$$g_4: 1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \leq 1$$

$$g_5: A_1/B_1 \leq 1100$$

$$A_1 = \left[\left(\frac{745x_4}{x_2x_3} \right)^2 + (16.9)10^6 \right]^{0.5}$$

$$B_1 = 0.1x_6^3$$

$$g_6: A_2/B_2 \leq 850$$

$$A_2 = \left[\left(\frac{745x_5}{x_2x_3} \right)^2 + (157.5)10^6 \right]^{0.5}$$

$$B_2 = 0.1x_7^3$$

$$g_7: x_2x_3 \leq 40$$

$$g_8: 5 \leq x_1/x_2 \leq 12 \quad : \quad g_9$$

$$g_{10}: 2.6 \leq x_1 \leq 3.6 \quad : \quad g_{11}$$

$$g_{12}: 0.7 \leq x_2 \leq 0.8 \quad : \quad g_{13}$$

$$g_{14}: 17 \leq x_3 \leq 28 \quad : \quad g_{15}$$

$$g_{16}: 7.3 \leq x_4 \leq 8.3 \quad : \quad g_{17}$$

$$g_{18}: 7.3 \leq x_5 \leq 8.3 \quad : \quad g_{19}$$

$$g_{20}: 2.9 \leq x_6 \leq 3.9 \quad : \quad g_{21}$$

$$g_{22}: 5.0 \leq x_7 \leq 5.5 \quad : \quad g_{23}$$

$$g_{24}: (1.5x_6 + 1.9)x_4^{-1} \leq 1$$

$$g_{25}: (1.1x_7 + 1.9)x_5^{-1} \leq 1.$$

4.1. Three-Level Design Optimization

The gear reducer example is decomposed into two subsystems, namely, shaft and bearings 1, and shaft and bearings 2. Each of these two subsystems may be

further decomposed, hence resulting in a three-level decomposed problem which is based on the physical make-up of the gear reducer. Here, this decomposition preserves: (1) the additively separable form of the objective function from the top- to the bottom-level subproblems (equation (2)), (2) the separable form of the constraints from the top- to the bottom-level subproblems (equation (3)), and (3) the simplicity of the bottom-level subproblems such that the global monotonicity analysis can be easily performed.

4.1.1. Formulation and Solution

The gear reducer is decomposed into three levels with the bottom-level subproblems (1,1) and (2,1) as follows:

Subproblem (1,1): Find the diameter (x_6) of shaft 1

$$\text{Minimize } f_{B_{1,1}} = -1.508x_1x_6^2 + 7.477x_6^3 + 0.7854x_4x_6^2 \quad (11)$$

Subject to:

$$g_3: \quad x_6 \geq (1.93x_2^{-1}x_3^{-1}x_4^3)^{1/4} = g'_{1,3}$$

$$g_5: \quad x_6 \geq (A_1/110)^{1/3} = g'_{1,5}$$

$$g_{20}: \quad x_6 \geq 2.9 = g'_{1,20}$$

$$g_{21}: \quad x_6 \leq 3.9 = g'_{1,21}$$

$$g_{24}: \quad x_6 \leq (x_4 - 1.9)/1.5 = g'_{1,24}$$

in which x_6 is a variable; x_1 , x_2 , x_3 , and x_4 are fixed.

Subproblem (2,1): Find the diameter (x_7) of shaft 2

$$\text{Minimize } f_{B_{2,1}} = -1.508x_1x_7^2 + 7.477x_7^3 + 0.7854x_5x_7^2 \quad (12)$$

Subject to:

$$g_4: \quad x_7 \geq (1.93x_2^{-1}x_3^{-1}x_5^3)^{1/4} = g'_{2,4}$$

$$\begin{aligned}
g_6: \quad x_7 &\geq (A_2/85)^{1/3} = g'_{2,6} \\
g_{22}: \quad x_7 &\geq 5 = g'_{2,22} \\
g_{23}: \quad x_7 &\leq 5.5 = g'_{2,23} \\
g_{25}: \quad x_7 &\leq (x_5 - 1.9)/1.1 = g'_{2,25}
\end{aligned}$$

in which x_7 is a variable; x_1 , x_2 , x_3 , and x_5 are fixed.

It can be easily demonstrated that application of the global monotonicity analysis to the bottom-level subproblems (1,1) and (2,1) results in:

$$x_6^* = \max \{g'_{1,3}, g'_{1,5}, g'_{1,20}\} \quad (13)$$

and

$$x_7^* = \max \{g'_{2,4}, g'_{2,6}, g'_{2,22}\} \quad (14)$$

The middle-level subproblems 1 and 2 are:

Middle-Level Subproblem 1: Find the distance (x_4) between bearings 1

$$\text{Minimize}_{x_4} f_{M1} = -1.508x_1x_6^{*2} + 7.477x_6^{*3} + 0.7854x_4x_6^{*2}$$

$$\text{Subject to: } 7.3 \leq x_4 \leq 8.3 \quad (15)$$

where x_4 is a variable; x_1 , and x_6^* are fixed.

Middle-Level Subproblem 2: Find the distance (x_5) between bearings 2

$$\text{Minimize}_{x_5} f_{M2} = -1.508x_1x_7^{*2} + 7.477x_7^{*3} + 0.7854x_5x_7^{*2}$$

$$\text{Subject to: } 7.3 \leq x_5 \leq 8.3 \quad (16)$$

where x_5 is a variable; x_1 , and x_7^* are fixed.

Finally, the top-level problem is:

$$\text{Minimize}_{x_1, x_2, x_3} f(x) = 0.7854x_1x_2^2 (3.3333x_3^2 + 14.9334x_3 - 43.0934)$$

$$\begin{aligned}
& -1.508x_1(x_6^{*2} + x_7^{*2}) + 7.477(x_6^{*3} + x_7^{*3}) \\
& + 0.7854(x_4^*x_6^{*2} + x_5^*x_7^{*2})
\end{aligned}$$

Subject to:

(17)

$$\begin{aligned}
g_1: & 27x_1^{-1}x_2^{-2}x_3^{-1} \leq 1 \\
g_2: & 397.5 x_1^{-1}x_2^{-2}x_3^{-2} \leq 1 \\
g_7: & x_2x_3 \leq 40 \\
g_8: & 5 \leq x_1/x_2 \leq 12 & :g_9 \\
g_{10}: & 2.6 \leq x_1 \leq 3.6 & :g_{11} \\
g_{12}: & 0.7 \leq x_2 \leq 0.8 & :g_{13} \\
g_{14}: & 17 \leq x_3 \leq 28 & :g_{15}
\end{aligned}$$

where x_1 , x_2 , and x_3 are variables; x_4^* , x_5^* , x_6^* , and x_7^* are fixed.

The initial point selected for this example is $x^0 = (2.6, 0.7, 17, 7, 7, 2.9, 5)^t$ which is infeasible, and gives $f(x^0) = 2335 \text{ (cm}^3\text{)}$. The final solution from the iterative procedure is $x^* = (3.5, 0.7, 17, 7.3, 7.71, 3.35, 5.29)^t$ which gives $f(x^*) = 2994 \text{ (cm}^3\text{)}$. This solution is identical to the one reported by Li and Papalambros (1985) who used a single-level optimization approach based on the global monotonicity analysis. However, the approach presented here is a multi-level one and applicable to a complex problem which is decomposable into several simple subproblems.

5. Concluding Remarks

The multi-level optimization method presented here should make possible solutions of problems previously too difficult to handle by the global monotonicity analysis within a single-level framework. However, for large problems, the

global monotonicity analysis (done manually here) is likely to be tedious and cause mistakes. To overcome this problem, a symbolic manipulation program (see, for example, MACSYMA, 1983) may be used. If global monotonicity analysis is not possible then local monotonicity analysis should be used (Azarm, 1984).

One disadvantage of the method presented here, when coupled with a conventional optimization method, is the possibility of discontinuous behavior of derivatives at the bottom-level subproblems (see, equations (7) or (8)). This can be resolved by using an optimization method on the upper-level subproblems which does not require derivatives from the lower-level subproblems. Another solution is to use a penalty function approach of the type suggested by Haftka (1984).

6. Acknowledgement

The work of the first author was supported through NSF Grant DMC-85-05113. The work of the second author was supported partially through NSF Grant DMC-85-05113 and partially through Engineering Research Centers Program NSF-CDR-88003012. The authors wish to thank the anonymous reviewers of this paper for their suggestions for improvements.

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Figure 1 Structure of a Multi-Level Decomposition

Figure 2 Decomposition Structure of a Three-Level Problem

Figure 3 A Gear Reducer

Figure 4 Three-Level Decomposition of a Gear Reducer

Levels	Quantities			
	y	x	u	v
Top-Level	var.	fixed		
Middle-Level	fixed	$x=(u,v)^t$	var.	fixed
Bottom-Level	fixed		fixed	var.

Table 1 Variable/Fixed Quantities in Each Level

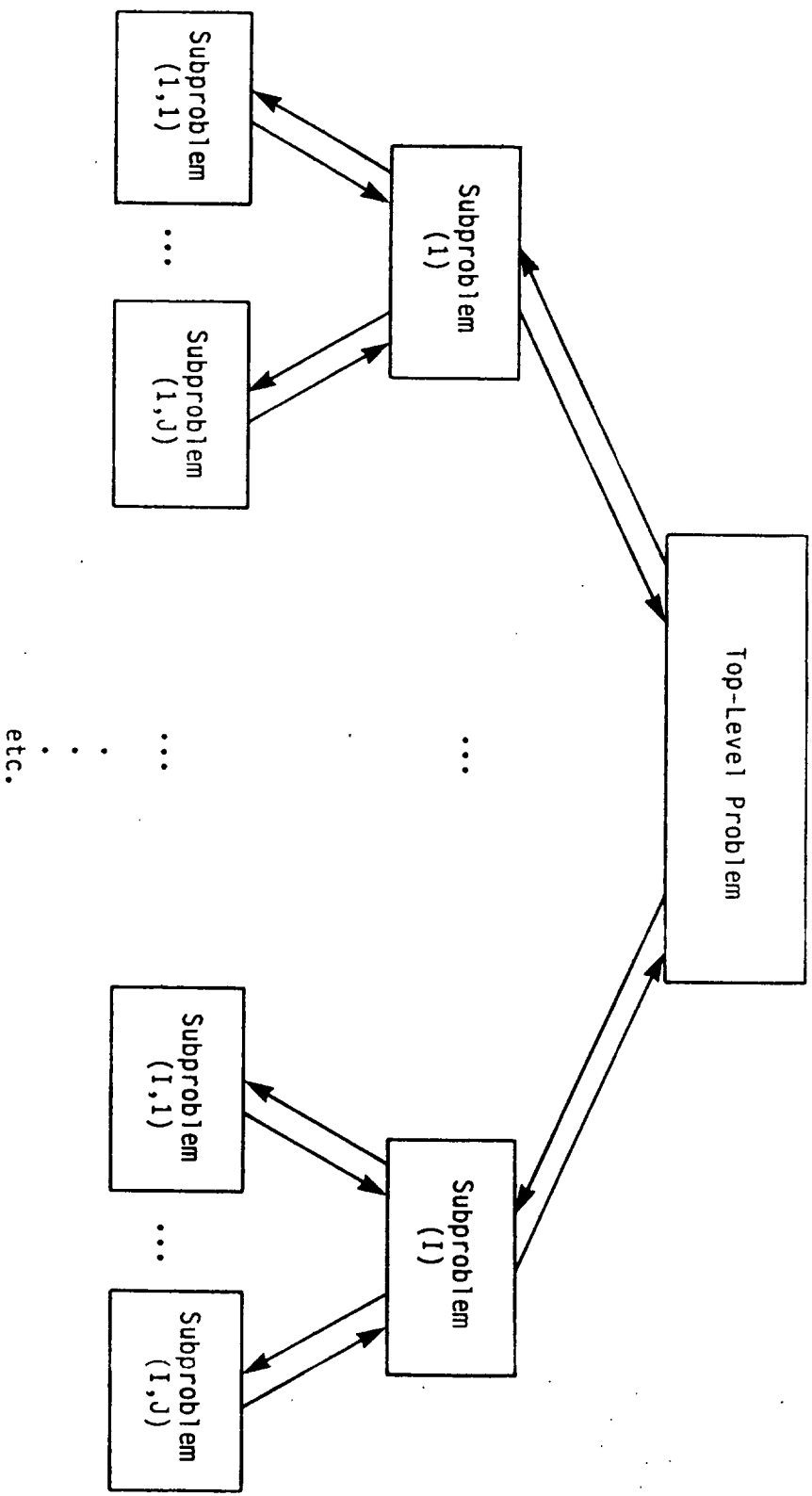


Figure 1 Structure of a Multi-Level Decomposition

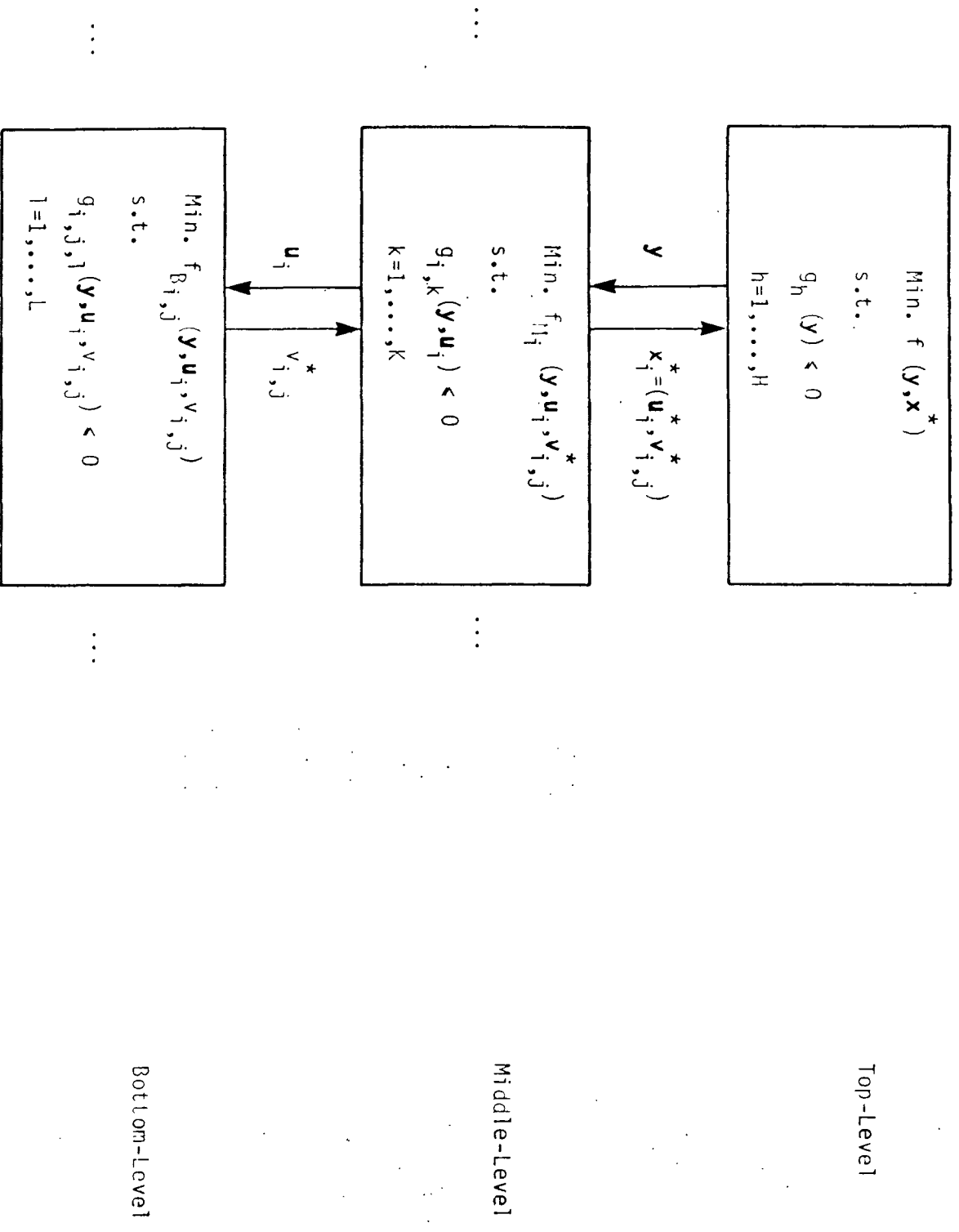


Figure 2. Decomposition Structure of a Three-Level Problem

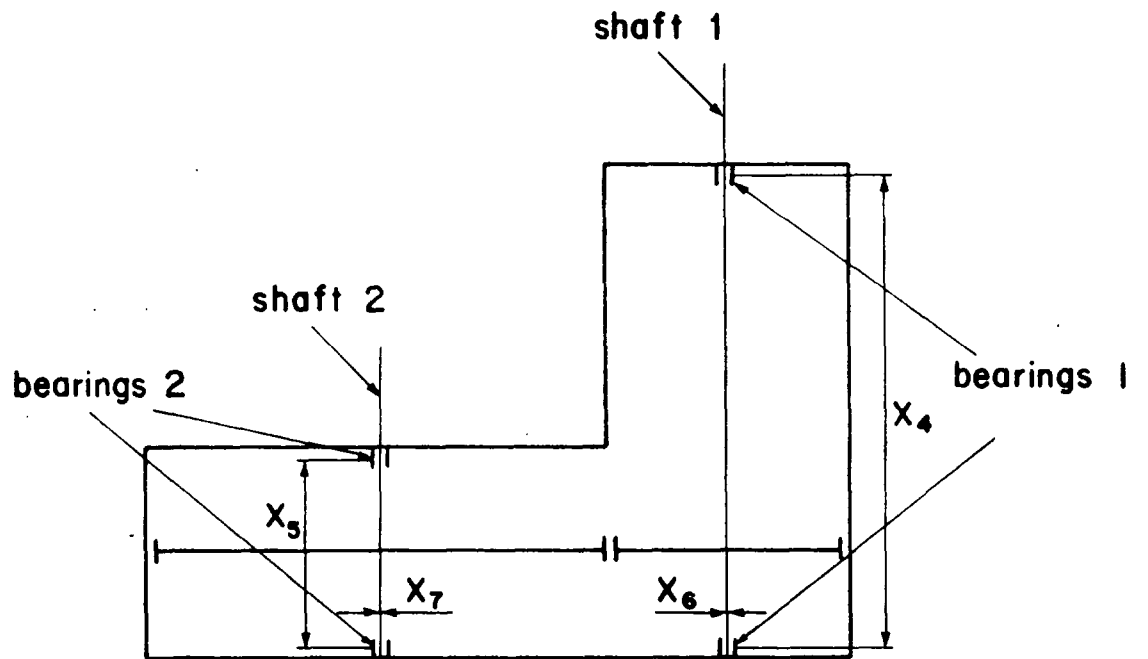


Figure 3 A Gear Reducer

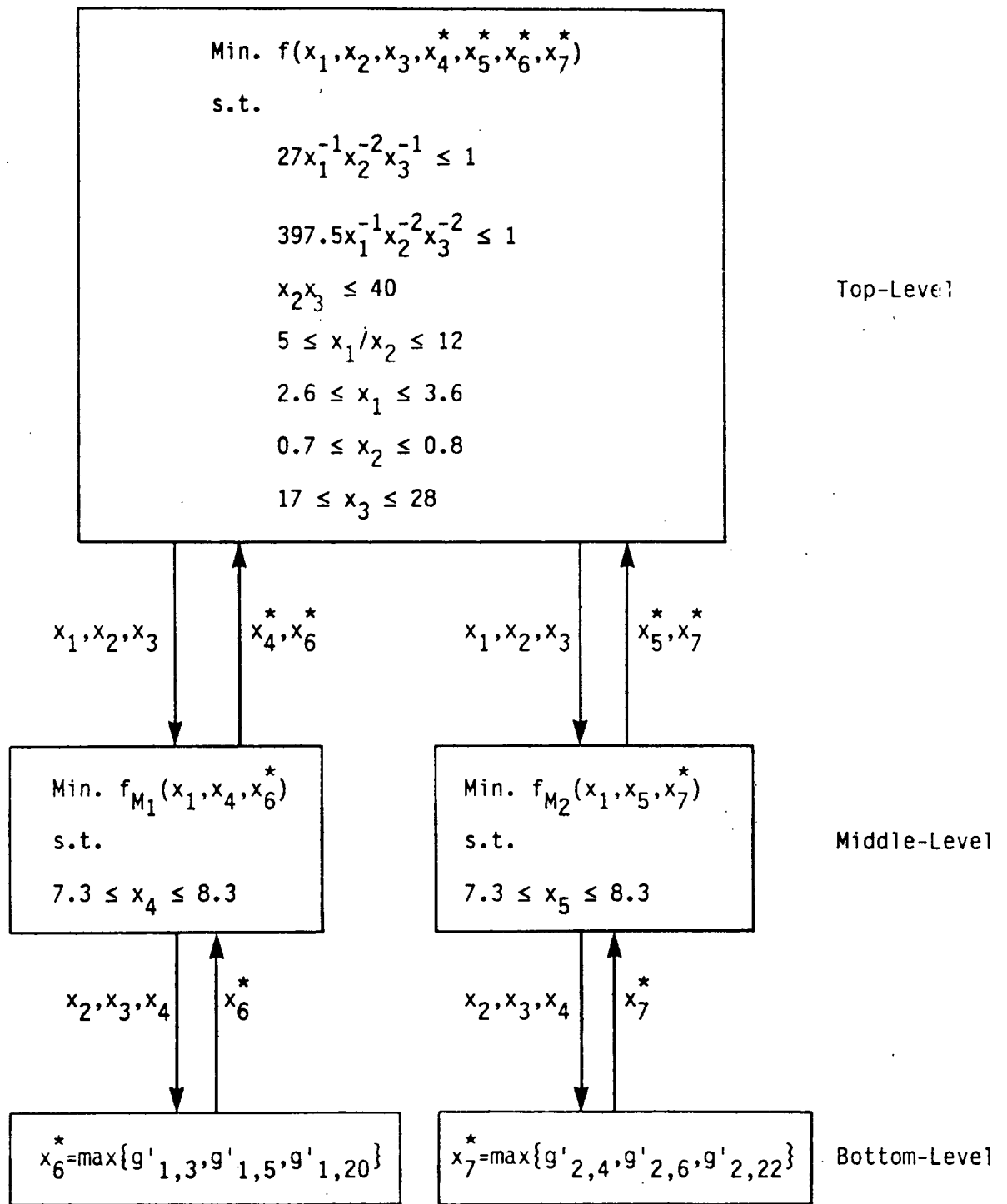


Figure 4 Three-Level Decomposition of a Gear Reducer

