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Robustness and Tuning of On-Line Optimizing Control Algorithms

by

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Abstract

A significant number of Model Based Process Control algorithms solve on-line an appropriate optimization problem and do so at every sampling point. The major attraction of such algorithms, like the Quadratic Dynamic Matrix Control (QDMC), lies in the fact that they can handle static nonlinearities in the form of hard constraints on the inputs (manipulated variables) of a process. The presence of such constraints as well as additional performance or safety induced hard constraints on certain outputs or states of the process, result in an on-line optimization problem that produces a nonlinear controller, even when the plant and model dynamics are assumed linear. This paper provides a theoretical framework within which the stability and performance properties of such algorithms can be studied.

1 Introduction

The problem of input saturation is of extreme importance for process control applications, because of its presence in almost every chemical system, even when the process dynamics can be assumed linear. In addition to the input constraints, safety and certain performance specifications also require the presence of hard constraints on some output and state variables. The urgency of rigorous theoretical work in this area has been repeatedly pointed

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out by the industry (e.g., [1]). An approach that has been tried in the chemical industry during the past few years is to on-line solve an appropriate optimization problem and to do so at every sampling point. The repeated application of such methods (e.g., Quadratic Dynamic Matrix Control (QDMC) [2] on industrial problems with considerable success indicate that sufficient degrees of freedom exist in these formulations. A drawback that has prohibited their widespread use is the fact that no exact tuning procedure for the optimization parameters exist and such tuning often has to be carried out on-line by experienced designers.

The presence of hard constraints in the on-line optimization problem produces a nonlinear controller even when the plant and model dynamics are assumed linear. The fact that the overall control system (plant + controller) is nonlinear makes the study of its properties quite involved, especially since no analytic expression is available for the controller. The problems are compounded when robustness with respect to model-plant mismatch is also considered, because no straighforward extension of the results of the Robust Linear Control Theory to this particular problem exists, even though the plant and model dynamics are assumed linear. Some efforts have been made recently [1,3] to achieve robustness by modifying the "min" optimization problem that is solved on-line to a "min max" problem that minimizes the objective function over all possible plants. One of the problems of this approach is that either the computations for solving the optimization problem are too time consuming to be carried out on-line at every sample point or to simplify the computations one has to use simplistic model uncertainty descriptions that are unrealistic. Another, potentially serious problem is the fact that these methods inherently assume that by solving the "min max" problem to obtain a sequence of future inputs (manipulated variables) and then implementing the first one and repeating the computation at the next sample point, one is guaranteed robust stability and performance, provided that a sufficiently long horizon is used in the objective function. However, feedback from an uncertain plant exists in reality and it is not taken into account in the formulation of the optimization problem, which is an open-loop minimization of the objective function over all possible plants. This fact can conceivably lead to performance deterioration and instability. Note that the situation is quite different from studying (and guaranteeing) a stabilizing control algorithm when no model error is present, in which case the assumption is reasonable, although not proven for the general case.

The problems discussed just above, cannot possibly be satisfactorily addressed without considering the problem in its proper nonlinear framework. It is the author's opinion that instead of augmenting the objective functions to add robustness, an action that dramatically increases the computational load and at the same time produces no rigorous robustness guarantees, one should study the problem in its nonlinear nature, obtain conditions that guarantee nominal and robust stability and performance and tune the parameters of the original optimization problems (e.g., QDMC) to satisfy them.

2 On-Line Optimization Problem

Although control algorithms of the type described in Section 1 have been applied to systems with nonlinear dynamic models (QDMC [4]), it is usually assumed that the dynamics are linear, the nonlinearity of the problem arising from the hard constraints. The properties of the controller are independent of the type of model description used for the plant (see, e.g., [5]). The impulse response description is a convenient one:

$$y(k+1) = H_1 u(k) + H_2 u(k-1) + \dots + H_N u(k-N+1) \tag{1}$$

where y is the output vector, u is the input vector and N is an integer sufficiently large for the effect of inputs more than N sample points in the past on y to be negligible.

The QDMC-type algorithms [2,6,7,5] use a quadratic objective function that includes the square of the weighted norm of the predicted error (setpoint - predicted output) over a finite horizon in the future as well as penalty terms on u or Δu. The minimization of the objective function is carried out over the values of $u(\bar{k})$, $u(\bar{k}+1)$, ..., $u(\bar{k}+M-1)$, where $\bar{k}$ is the current sample point and M a specified parameter. The minimization is subject to possible hard constraints on the inputs u, their rate of change Δu, the outputs y and other process variables usually referred to as associated variables. More details on the formulation of the optimization problem can be found in the cited references. After the problem is solved on-line at $\bar{k}$, only the optimal value for the first input vector $u(\bar{k})$ is implemented and the problem is solved again at $\bar{k}+1$. The optimal $u(\bar{k})$ depends on the tuning parameters of the optimization problem, the current output measurement $y(\bar{k})$ and the past inputs $u(\bar{k}-1)$,..., $u(\bar{k}-N)$ that are involved in the model output prediction. Let f describe the result of the optimization:

$$u(k) = f(y(k), u(k-1), \ldots, u(k-N)) \tag{2}$$

The optimization problem of the QDMC-type algorithms can be written as a standard Quadratic Programming problem [2]:

$$\min_{v} q(v) = \frac{1}{2} v^T G v + g^T v \tag{3}$$

subject to

$$A^T v = b \tag{4}$$

where

$$v = [\ u(\bar{k}) \ \ \ldots \ \ u(\bar{k}+M-1)\ ]^T \tag{5}$$

and the matrices G, A, and vectors g, b are functions of the tuning parameters (weights, horizon, M). The vectors g, b are also linear functions of $y(\bar{k})$, $u(\bar{k}-1)$,..., $u(\bar{k}-N)$. For the optimal solution v^* we have [8]:

$$\begin{bmatrix} G & -\hat{A} \\ -\hat{A}^T & 0 \end{bmatrix} \begin{bmatrix} v^* \\ \lambda^* \end{bmatrix} = - \begin{bmatrix} g \\ b \end{bmatrix} \tag{6}$$

where $\hat{A}$ consists of the rows of A that correspond to the constraints that are active at the optimum and λ^* is the vector of the Lagrange multipliers. The optimal $u(\bar{k})$, descibed by (2), corresponds to the first m elements of the v^* that satisfies (6), where m is the dimension of u.

3 Formulation of the Problem as a Contraction Mapping

The framework selected for the study of the properties of the overall nonlinear system is that of the Operator Control Theory [9]. In this approach, the stability and performance of the nonlinear system can be studied by applying the contraction mapping principle on the operator $\breve{F}$ that maps the "state" of the system (plant + controller) at sample point k to that at sample point $k+1$. The fact that the plant dynamics are assumed linear allows us to obtain results and carry out computations that are not yet feasible in the general case. We can define as the "state" of the system at sample point k the following vector

$$x(k) = \begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \end{bmatrix} \tag{7}$$

where

$$\begin{array}{llll} x_1(k+1) & \stackrel{\text{def}}{=} u(k) & = & f(y(k), u(k-1), \ldots, u(k-N)) \\ & & = & f(H_1 u(k-1) + \ldots + H_N u(k-N), \\ & & & u(k-1), \ldots, u(k-N)) \\ & & \stackrel{\text{def}}{=} & \Psi(u(k-1), \ldots, u(k-N)) \\ & & = & \Psi(x(k)) \\ x_2(k+1) & \stackrel{\text{def}}{=} u(k-1) & = & x_1(k) \\ & \vdots & \vdots & \\ x_N(k+1) & \stackrel{\text{def}}{=} u(k-N+1) & = & x_{N-1}(k) \end{array} \tag{8}$$

The "state" vector $x(k)$ is defined so that knowledge of it allows the computation of $x(k+1)$ by applying the plant and controller equations on it. Indeed the operator F that maps $x(k)$ to $x(k+1)$ is given by

$$x(k+1) = F(x(k)) = \begin{bmatrix} \Psi(x(k)) \\ x_1(k) \\ \vdots \\ x_{N-1}(k) \end{bmatrix} \tag{9}$$

Note, however, that although f is known, since it describes the on-line optimizing control algorithm and it involves only the process model, Ψ is not exactly known, because it involves the "true" plant impulse response coefficients H_1,..., H_N.

Convergence of the successive substitution $x(k+1) = F(x(k))$ to the unique fixed point of the contraction implies stability of the overall nonlinear system; fast convergence implies good performance. The use of the contraction mapping principle allows the development of conditions for robust stability and performance in terms of some induced matrix norm of the derivative F' of the above operator F.

4 Stability Conditions

We shall now proceed to obtain stability conditions for the overall nonlinear system by obtaining conditions under which the mapping described by F is a contraction. The terms stability and instability of the control system are used in the global sense over the domain of F under consideration.

Let us first examine the differentiability of F. From (9) it follows that this is equivalent to differentiability of $\Psi(x)$ and from (8) to differentiability of f. Let us assume that for some point x in the domain of F, an infinitesimal change in x (which results in a change of g, b in (3), (4)) does not change $\hat{A}$, i.e., the set of active constraints at the optimum does not change (note that A is independent of x). Then from (6) it follows that the derivative of Ψ exists and it has a constant value in a neighbourhood of that x.

Let J_i be a set of indices for the active constraints of (3) and $J_1,\ldots, J_n$ correspond to all possible active sets of constraints when all xs in the domain of F are considered. Every such J_i corresponds to an $\hat{A}_i$. Then from the above discussion, it is evident that for all xs that correspond to the same J_i and for which an infinitecimal change in their value does not change the set of active constraints, the derivative of Ψ and therefore of F exist and it has the same value that depends on the particular set J_i:

$$F'_{J_i} = \begin{bmatrix} (\nabla_{x_1}\Psi)_{J_i} & (\nabla_{x_2}\Psi)_{J_i} & \cdots & (\nabla_{x_{N-1}}\Psi)_{J_i} & (\nabla_{x_N}\Psi)_{J_i} \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \tag{10}$$

where from (8) it follows that

$$(\nabla_{x_j}\Psi)_{J_i} = (\nabla_{x_j} f)_{J_i} + (\nabla_y f)_{J_i} H_j \tag{11}$$

It is clear from the above discussion that $F(x)$ is quasi-linear and that it is differentiable everywhere except the points where an infinitecimal change will change the set of active constraints at the optimum of (3). It follows then that for F to be a contraction, it is necessary that

$$||F'_{J_i}|| \leq \theta < 1, \quad i = 1, \ldots, n \tag{12}$$

where $||.||$ is any consistent matrix norm[1], the same for all i. The above condition however, can be shown to be sufficient as well. Consider two points x^a, x^b and let the straight path connecting them in the domain of F be broken into the succesive segments $x^a \to x^1$, $x^1 \to x^2$,..., $x^l \to x^b$, the points of each of which correspond to the same J_i: J_{k_0}, J_{k_1},..., J_{k_l}, respectively. Then

$$\begin{aligned} ||F(x^a) &- F(x^b)|| \\ &= ||(F(x^a) - F(x^1)) + (F(x^1) - F(x^2)) + \ldots + (F(x^l) - F(x^b))|| \\ &= ||F'_{J_{k_0}}(x^a - x^1) + F'_{J_{k_1}}(x^1 - x^2) + \ldots + F'_{J_{k_l}}(x^l - x^b)|| \\ &= ||(a_0 F'_{J_{k_0}} + a_1 F'_{J_{k_1}} + \ldots + a_l F'_{J_{k_l}})(x^a - x^b)|| \\ &\leq (a_0 + a_1 + \ldots + a_l)\theta||x^a - x^b|| \\ &= \theta||x^a - x^b|| \end{aligned} \tag{13}$$

where a_j is the relative length of the respective segment as compared to $x^a \to x^b$. From (13) it follows that F is a contraction. The fact that there is only a finite number of J_is allows us to drop the θ from (12) to obtain:

Theorem 1 *F is a contraction if and only if there exists a consistent matrix norm $||.||$, for which*

$$||F'_{J_i}|| < 1, \quad i = 1, \ldots, n \tag{14}$$

The practical use of (14) is limited by the fact that finding an appropriate consistent norm is not a trivial task. The following two subsections provide conditions which are more readily computable. The third subsection formulates the respective robustness conditions.

4.1 Sufficient Condition

By selecting one particular consistent matrix norm and stating (14) for that norm, one can get a sufficient only condition.

Let us select the following norm, which can be shown to be a consistent one on $\mathbf{R}^{mN \times mN}$[10], where m is the plant dimension:

$$||A|| = ||DAD^{-1}||_\infty \tag{15}$$

[1]A consistent matrix norm has the property $||AB|| \leq ||A||\,||B||$.

where

$$||B||_\infty = \max_i \sum_{j=1}^{N} |b_{ij}| \tag{16}$$

$$D = \mathrm{diag}(I, \eta I, \eta^2 I, \ldots, \eta^{N-1} I) \tag{17}$$

$$0 < \eta < 1 \tag{18}$$

Then

$$DF'_{J_i}D^{-1} =$$
$$\begin{bmatrix} (\nabla_{x_1}\Psi)_{J_i} & (\nabla_{x_2}\Psi)_{J_i}\eta^{-1} & \ldots & (\nabla_{x_{N-1}}\Psi)_{J_i}\eta^{-(N-2)} & (\nabla_{x_N}\Psi)_{J_i}\eta^{-(N-1)} \\ \eta I & 0 & \ldots & 0 & 0 \\ 0 & \eta I & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & \eta I & 0 \end{bmatrix} \tag{19}$$

From (15)-(19) we get

$$\begin{aligned} & ||DF'_{J_i}D^{-1}||_\infty < 1 \\ \Leftrightarrow\ & ||\ (\nabla_{x_1}\Psi)_{J_i} \ \ (\nabla_{x_2}\Psi)_{J_i}\eta^{-1} \ \ \ldots \ \ (\nabla_{x_N}\Psi)_{J_i}\eta^{-(N-1)}\ ||_\infty < 1 \\ \Leftarrow\ & ||\ (\nabla_{x_1}\Psi)_{J_i} \ \ (\nabla_{x_2}\Psi)_{J_i} \ \ \ldots \ \ (\nabla_{x_N}\Psi)_{J_i}\ ||_\infty < \eta^{N-1} \end{aligned} \tag{20}$$

Since any η in $(0,1)$ will do and there is only a finite number of J_is, from (20) we can obtain:

Theorem 2 *The control system is asymptotically stable if*

$$||\ (\nabla_{x_1}\Psi)_{J_i} \ \ (\nabla_{x_2}\Psi)_{J_i} \ \ \ldots \ \ (\nabla_{x_N}\Psi)_{J_i}\ ||_\infty < 1, \quad i = 1, \ldots, n \tag{21}$$

Note that for single-input single-output plants (21) becomes

$$\sum_{j=1}^{N} |\frac{\partial \Psi_{J_i}}{\partial x_j}| < 1, \quad i = 1, \ldots, n \tag{22}$$

which for the unconstrained case is simply a sufficient condition for the closed-loop poles to lie inside the Unit Circle.

4.2 Instability Conditions

For every consistent matrix norm we have

$$\rho(A) \leq ||A|| \tag{23}$$

where $\rho(A)$ is the spectral radius of A, defined as $\rho(A) = \max_j |\lambda_j(A)|$, $\lambda_j(A)$, being the eigenvalues of A. Then from (14) and (23) we get

Theorem 3 *F is a contraction only if*

$$\rho(F'_{J_i}) < 1, \quad i = 1, \ldots, n \tag{24}$$

Note that if the optimization (3) is not subject to (4), then $n = 1$ and (24) becomes sufficient as well, because, given a matrix one can always find a consistent norm arbitrarily close to its spectral radius [10]. The reason that (24) is not sufficient in general is that such a consistent norm is in general a different one for two different matrices (different J_is), while (14) requires the same norm for all i. In the case of $n = 1$, (24) translates to the requirement that the closed-loop poles of the system are located inside the Unit Circle.

If (24) is not true, then F is not a contraction. This however does not necessarily imply that the control system is unstable. The following theorem provides a condition that is sufficient for instability.

Theorem 4 *The control system is unstable if*

$$\rho(F'_{J_i}) > 1, \quad i = 1, \ldots, n \tag{25}$$

The proof follows the argument that if a stable local equilibrium point existed, then for the J_i corresponding to that point we would have $\rho(F'_{J_i}) < 1$.

Theorem 4 can be used to predict instability of the overall nonlinear system. Theorem 3 on the other hand does not seem at a first glance to be of much use, since violation of (24) does not necessarily imply instability. From a practical point of view, however, violation of that condition for some i, should be taken as a very serious warning that the control system parameters should be modified. The reason is that when in the region of the domain of F that corresponds to that i, the system will behave as a virtually unstable system, the only hope for stability being to move to a region with $\rho(F'_{J_i}) < 1$. It might be the case that for a particular system in question this will always happen, making this system a stable one. But even in this case, a temporary unstable-like behavior might occur, thus making the control algorithm practically unacceptable. The example in Section 5 demonstrates a situation where violation of (24) is enough to produce an unstable system although (25) is not satisfied.

4.3 Robustness Conditions

From (11) we see that F'_{J_i} depends on the impulse response coefficient matrices H_1,..., H_N of the actual plant. These matrices are never known exactly and so in order to guarantee stability for the actual plant, one has to compute the conditions of Sections 4.2, 4.1, not just for the model, but for all possible plants. To do so, one needs to have some information on the possible modeling error associated with the H_is. Let $\mathcal{H}$ be the set of possible values for these coefficients. Then we can write the following conditions:

Theorem 5 *The control system is asymptotically stable for all plants with coefficients in* $\mathcal{H}$ *if*

$$\sup_{\mathcal{H}} \| \; (\nabla_{x_1}\Psi)_{J_i} \quad (\nabla_{x_2}\Psi)_{J_i} \quad \ldots \quad (\nabla_{x_N}\Psi)_{J_i} \; \|_\infty < 1, \quad i = 1, \ldots, n \tag{26}$$

Theorem 6 F *is a contraction for all plants with coefficients in* $\mathcal{H}$ *only if*

$$\sup_{\mathcal{H}} \rho(F'_{J_i}) < 1, \quad i = 1, \ldots, n \tag{27}$$

In order to carry out the maximizations over $\mathcal{H}$ described by (27), (26), one needs to parametrize the "uncertain" H_1,..., H_N, in terms of a fewer "uncertain" parameters. For example, in the simple case where the linear plant dynamics are described by the transfer function $\frac{Ke^{-sd}}{\tau s+1}$, where K, d, τ, are within some ranges, we can write H_1,..., H_N, as functions of K, d, τ, and compute $\sup_{\mathcal{H}}$ as $\sup_{K,d,\tau}$. However, the situation is usually more complex, a fact that makes the efficient parametrization of the modeling error in H_1,..., H_N, a very important research topic.

5 Illustration

Let us consider a system with the following transfer function:

$$P(s) = \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{-2e^{-5s}}{s+1} & \frac{-s+2}{(s+2)(s+1)} \end{bmatrix} \tag{28}$$

A sampling time $T = 0.5$ is used and the following objective function is minimized on-line:

$$\min_{u(\bar{k}),\ldots,u(\bar{k}+M-1)} \sum_{l=1}^{P} \left[e(\bar{k}+l)^T \Gamma^2 e(\bar{k}+l) + u(\bar{k}+l-1)^T B^2 u(\bar{k}+l-1) \right] \tag{29}$$

where $\bar{k}$ is the current sample point, e is the predicted difference between the setpoints and the plant outputs and Γ, B, are weights.

$$\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \tag{30}$$

is selected signifying that the first output is more important than the second.

Let us first consider the unconstrained problem. First we select $P = M = 2$, which is a selection that is expected [6,7] to produce an unstable control system if $B = 0$. The reason is the right-half plane (RHP) zero of $P(s)$. Indeed, one can easily check that for these values of the tuning parameters, we have $\rho(F'_{J_1}) > 1$, where J_1 corresponds to the case where no constraints are active at the optimum. Hence the necessary condition (24) predicts the instability. From theory [7] we know that by making B sufficiently large, we can stabilize the system. Indeed by making

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix} \tag{31}$$

the system is stabilized ($\rho(F'_{J_1}) < 1$, which is sufficient for $n = 1$). The fact that the RHP zero is pinned to the second plant output, made it unnecessary to increase the 11 element of B. The response to a unit step change in setpoint 1 is shown in Fig. 1. The steady-state offset in output 2 is expected from theory and can be avoided by modifying the control algorithm, but we will not do so to avoid the unnecessary complication of the example.

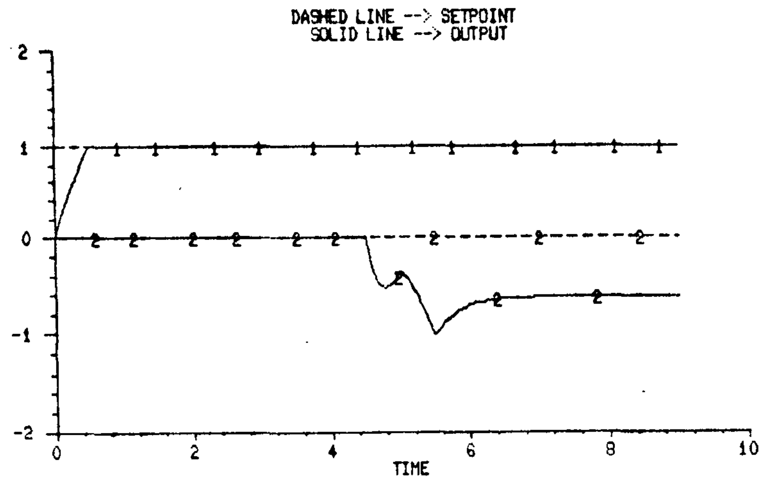

Figure 1: Unconstrained minimization.

Let us now assume that after looking at the response, the designer decides that a slight tightening of the specifications is in order, namely the addition in the optimization problem of a lower bound on output 2 at the value -0.9. Since output 2 only slightly violated this bound when the unconstrained algorithm was used, one might think that the response for the constrained algorithm should be almost the same as that in Fig. 1. This is not so, however. The response for the same setpoint change is shown in Fig. 2. The system is unstable. An instability warning was issued by the necessary condition for F to be a contraction (24), since $\rho(F'_{J_2}) > 1$, where J_2 corresponds to the case where the low constraint on output 2 is active at the optimum. Indeed by looking at a close-up of Fig. 2 in Fig. 3, we see that the system went unstable as soon as output 2 reached the low bound to which the on-line minimization was subject. The constraint remained active at the subsequent sample points and the system never stabilized.

A question that one may ask at this point is whether the use of a

$$B = \begin{bmatrix} 0 & 0 \\ 0 & \beta \end{bmatrix} \tag{32}$$

with a β larger than the previously used value of 0.1, will stabilize the system.

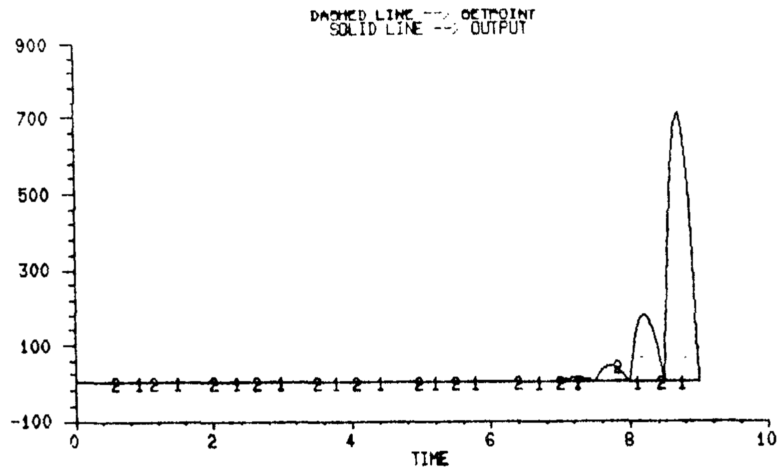

Figure 2: Minimization subject to lower bound constraint on output 2.

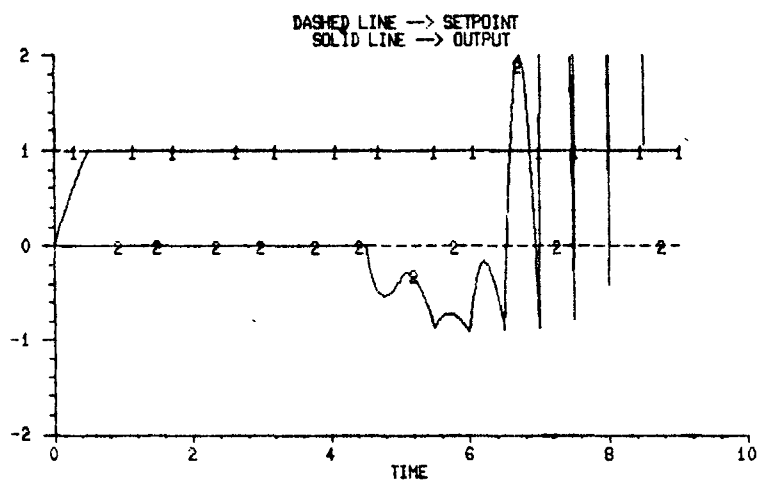

Figure 3: Close-up of Fig. 2.

We know that this would be the case for the unconstrained problem; however, for the constrained case that does not happen. By examining the analytic expression for F'_{J_2} one sees that β does not even appear in it and can therefore in no way influence the stability of the system when the constraint becomes active. When the constraint is reached, the algorithm puts as its higher priority keeping output 2 above the lower bound and to do so it inverts the 22 element of $P(s)$ and causes instability.

6 Conclusions

The main goal of this paper was to provide a theoretical framework for the study of the properties of control algorithms that are based on the on-line minimization of some objective function, subject to certain hard constraints. The selected framework seems to be quite promising since it allowed the derivation of necessary and/or sufficient conditions for nominal and robust stability of the overall nonlinear system. The simple example that was used demonstrated in a clear way that one cannot afford to neglect the nonlinear phenomena caused by the hard constraints to which the on-line optimization is subject. This example also indicates that inclusion of hard constraints on the plant outputs in the specifications can cause serious problems.

Future work in this framework should address the issue of performance, possibly by minimizing the contraction constant. Also the sufficient condition (21) should be used to obtain tuning guidelines for the optimization parameters P, M, Γ, B, so that stability is achieved for general or special cases.

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