SRC TR 88-10

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by

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January 1988

Abstract

This paper describes applications of global and local monotonicity analysis within a decomposition framework. We present a general formulation and solution procedure, based on a bottom-level global monotonicity analysis, for a design optimization problem which is decomposed into three levels of subproblems. We then perform an optimality test to prove that the optimality conditions for the decomposed subproblems will form the optimality conditions of the overall problem. Furthermore, applications of a two-level decomposition method is presented in which an overall global monotonicity analysis or first-level local monotonicity analysis is performed. Well-known examples illustrate applications of the methods.

1. Introduction

Design optimization is at a rather critical stage in that many large and/or complex real-world design problems occur which can not be effectively handled using conventional methods. Special technique and/or modification of the conventional methods is necessary to obtain optimal solutions of such design problems. One possible means of solution involves a hierarchical decomposition of the problem into a number of smaller subproblems each with its own objective and constraints. In this type of decomposition, interconnection between subproblems is usually multi-level, see Figure 1, where a given-level subproblem coordinates the lower-level subproblem(s) and in turn is coordinated by a higher-level subproblem. One of the main advantages of the multi-level decomposition methods is that they fit well into the multidisciplinary framework of a design process where a number of engineering disciplines interact (Mesarovic et al. 1970; Sobieski and Haftka, 1987). Furthermore, they allow parallel processing, and use of different specialized optimization techniques on various portions of the problem.

A decomposition-based design optimization method usually consists of two steps. First, the integrated or nondecomposed problem (objective and constraint functions) is partitioned into a hierarchical two- or multi-level subproblems. To be successful in the first step, the integrated problem should be formulated in such a way that it will be fully or at least partially decomposable. Once the problem is formulated, there are usually several alternatives to decompose the problem. Perhaps, one of the most obvious alternatives is the physical decomposition of the problem. For example, optimal design model of an aircraft should be decomposable into its main components, namely, wing, fuselage, landing gear, engine, etc.

Second, starting with the lowest-level, the subproblems are solved independently. The solutions to the subproblems at a given level are then coordinated by the upper-level problems, i.e., subproblems are forced to select solutions corresponding to an overall optimum. In general, the lower- and the upper-level subproblems are solved iteratively. Each of the lower-level subproblems is itself a constrained design optimization problem and should be solved many times before the solutions to the upper-level problems are obtained. The success and effectiveness of the second step depends on how simple and independent are the solutions to the lower-level subproblems.

There exists a variety of decomposition-based approaches for solving a given design optimization problem. In general, these techniques fall into two different methods, namely, the goal coordination and the model coordination (Wismer, 1971; Kirsch, 1981). The model coordination method, in particular, is more attractive for engineering design, since the iteration process may be terminated whenever it is desired with a feasible even though nonoptimal solution. Several engineering problems have been solved using decomposition methods including those in chemical (Wilde, 1965), mechanical (Siddall and Michael, 1980), structural (Kirsch, 1981; Haftka, 1984), and aerospace design (Sobieski et al., 1984; Barthelemy and Riley, 1986; Wrenn et al. 1987).

In a recent paper, Azarm and Li (1987a) proposed a two-level decomposition approach, an extension of the model coordination method, based on the global monotonicity concepts. The proposed approach applied to a number of problems including those in mechanical design (Azarm and Li, 1987b). The present paper is an extension of that effort. We present here a three-level formulation of a separable design optimization problem. A three-level solution procedure is then suggested based on the global monotonicity analysis. An optimality test is performed to see whether the optimality conditions of the decomposed three-level subproblems is the same as the optimality conditions of the overall problem. The three-level decomposition is illustrated by a gear-reducer example. Finally we demonstrate, with examples, two methods for a two-level decomposition. In one of the methods, global monotonicity is applied to the first- and second-level subproblems, while in the other, local monotonicty is applied to the first-level subproblems.

2. Formulation and Procedure

The formulation and procedure which we present in this section is for a three-level problem, but in fact it can be generalized to any number of levels. The approach is essentially an extension of the two-level formulation and procedure given by Azarm and Li (1987a and 1987b). We consider the following nonlinearly constrained design optimization problem:

$$\text{Minimize } \{f(z): g(z) \le 0\} \quad (1)$$

where z is an n-vector of design variables, f and g are the objective and the vector of inequality constraints, respectively. We assume that the equality constraints have been eliminated already, for example, through direct elimination or through an appropriate projection of the problem to the subspace of equality constraints.

Furthermore, we assume that the problem is decomposable into three levels, namely, the top-level, the middle-level, and the bottom-level. The top-level is composed of one subproblem (or problem). However, each of the middle and the bottom levels may be composed of several subproblems. In each subproblem, the variables are partitioned into two groups, namely, the local and the global variables. We define the global variables to be those which are taken to be fixed in a subproblem, and the local variables to be those which are taken to be changed in the subproblem. The definition for local and global variables given here is slightly different from the ones given previously by Azarm and Li (1987a and 1987b). For example, in Figure 2 which shows a three-level decomposition, we have set $z = (y,x)^t$, where y represents the vector of top-level local design variables - fixed in the lower levels, and x represents the vector of top-level global design variables - fixed in the top-level. Likewise, we have set $x = (u,v)^t$, where u represents the vector of middle-level local design variables, and v together with y represent the vector of middle-level global design variables. Finally, in the bottom-level, v represents the vector of bottom-level local design variables, and u together with y represent the vector of bottom-level global design variables.

We assume that the objective function f is in the following additively separable form:

$$f(y,u,v) = f_0(y) + \sum_{i=1}^{I} [f_i(y,u_i) + \sum_{j=1}^{J} f_{i,j}(y,u_i,v_{i,j})] \quad (2)$$

where i, j are indices corresponding to the number of middle-level subproblems, number of bottom-level subproblems with respect to (w.r.t.) the middle-level subproblem i, respectively (see, Figure 2).

In addition, we assume that the inequality constraints are in the following form:

$$
\begin{aligned}
& g_h(y) \le 0 && h = 1,\ldots, H \\
& g_{i,k}(y,u_i) \le 0 && i = 1,\ldots, I \\
& g_{i,j,l}(y,u_i,v_{i,j}) \le 0 && j = 1,\ldots, J \\
& && k = 1,\ldots, K \\
& && l = 1,\ldots, L
\end{aligned}
\tag{3}
$$

where h, k, and l are indices corresponding to the number of inequality constraints in the top-level problem, number of inequality constraints in middle-level subproblem (i), and number of inequality constraints in bottom-level subproblem (i,j), respectively (Figure 2).

The formulation of the bottom-level subproblem (i,j) with y and u_i as the vectors of the bottom-level global design variables, and $v_{i,j}$ as the only bottom-level local design variable is:

$$
\begin{aligned}
& \text{Minimize } f_{B_{i,j}}(y,u_i,v_{i,j}) = f_{i,j}(y,u_i,v_{i,j}) \\
& \text{Subject to:} \\
& \qquad g_{i,j,l}(y,u_i,v_{i,j}) \le 0, \quad l = 1,\ldots, L
\end{aligned}
\tag{4}
$$

The formulation of the middle-level subproblem (i) with y and $v^*_{i,j}$ (found from the bottom-level subproblem (i,j)) as the vector of middle-level global design variables and u_i as the vector of middle-level local design variables is:

$$
\begin{aligned}
& \text{Minimize } f_{M_i}(y,u_i,v^*_{i,j}) = f_i(y,u_i) + \sum_{j=1}^{J} f_{i,j}(y,u_i,v^*_{i,j}) \\
& \text{Subject to:} \\
& \qquad g_{i,k}(y,u_i) \le 0, \quad k = 1,\ldots, K
\end{aligned}
\tag{5}
$$

Finally, the formulation of the top-level problem with u_i^* and $v_{i,j}^*$ as the vector of top-level global design variables (found from the lower levels), and y as the vector of top-level local design variables is:

$$\text{Minimize } f_T(y,u_i^*,v_{i,j}^*) = f_0(y) + \sum_{i=1}^{I} [f_i(y,u_i^*) + \sum_{j=1}^{J} f_{i,j}(y,u_i^*,v_{i,j}^*)]$$

Subject to: (6)

$$g_h(y) \le 0, \quad h = 1,\ldots, H$$

The iterative procedure used for the above decomposed problem is summarized below:

Given an initial point as the current point, $z^o = (y^o,u^o,v^o)^t$,

Begin step A, for i = 1,..., I,

begin step B, for j = 1,..., J,

(B.1) for a given $(y^o,u_i^o)^t$, use global monotonicity analysis to find $v_{i,j}^*$ for the bottom-level subproblem (i,j),

(B.2) find a new u_i^o, for a given y^o, from the middle-level subproblem (i) such that f_{M_i} is decreased,

(B.3) return to step (B.1) until the minimum for f_{M_i} is obtained, for a given y^o,

end step B,

(A.1) find a new y^o from the top-level subproblem such that f_T is decreased,

(A.2) go to step A until the minimum for f is obtained,

End step A.

Note that in the procedure suggested here, the middle- and the top-level subproblems are solved by a conventional optimization method (Powell, 1978).

3. Optimality Test

In this section we will check whether the first-order Karush-Kuhn-Tucker (KKT) optimality conditions for the decomposed problems, eqs. (4)-(6), will form the KKT conditions of the nondecomposed (overall) problem, eqs. (2)-(3). The regularity assumption of the points under consideration is that the gradient vectors of active constraints are linearly independent at the regular points.

Now, let $\bar{z}$ be a solution (local minimum) to eq. (1). We partition $\bar{z}$ in the following way:

$$\bar{z} = (\bar{y},\bar{u},\bar{v})^t \tag{7}$$

where

$$\bar{y} = (\bar{y}_1,\ldots,\bar{y}_t,\ldots,\bar{y}_{T'})^t \tag{8}$$

$$\bar{u} = (\bar{u}_1,\ldots,\bar{u}_i,\ldots,\bar{u}_I)^t \tag{9}$$

$$\bar{u}_i = (\bar{u}_{i,1},\ldots,\bar{u}_{i,m},\ldots\bar{u}_{i,M})^t \tag{10}$$

and

$$\bar{v} = (\bar{v}_1,\ldots,\bar{v}_i,\ldots,\bar{v}_I)^t \tag{11}$$

$$\bar{v}_i = (\bar{v}_{i,1},\ldots,\bar{v}_{i,j},\ldots\bar{v}_{i,J})^t \tag{12}$$

or

$$\bar{v} = (\bar{v}_{1,1},\ldots,\bar{v}_{1,J},\ldots,\bar{v}_{i,1},\ldots,\bar{v}_{i,j},\ldots,\bar{v}_{i,J},\ldots\bar{v}_{I,1},\ldots\bar{v}_{I,J})^t \tag{13}$$

We start with the KKT conditions for the bottom-level subproblems to form partially the KKT conditions of the nondecomposed problem. We then extend our argument to show that the KKT conditions of the upper-level subproblems will form the remaining KKT conditions of the nondecomposed problem.

The KKT conditions for the bottom-level subproblem (i,j), eq. (4), may be expressed as follows (all evaluations performed at ($\bar{z}$):

$$(\partial f_{i,j}/\partial v_{i,j}) + \sum_{l=1}^{L} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial v_{i,j}) = 0 \tag{14}$$

$$\lambda_{i,j,l}g_{i,j,l} = 0, \quad \lambda_{i,j,l} \geq 0 \tag{15}$$

$$g_{i,j,l} \leq 0, \qquad l=1,\ldots,L \tag{16}$$

Summing up eq. (14) for all the bottom-level subproblems, where i=1,..., I and j=1,...,J, will result in:

$$\sum_{j=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial v_{i,j} + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial v_{i,j}) = 0 \tag{17}$$

Likewise, the KKT conditions for the middle-level subproblem (i), eq. (5), may be expressed as follows:

$$\partial f_{M_i}/\partial u_{i,m} + \sum_{k=1}^{K} \lambda_{i,k}(\partial g_{i,k}/\partial u_{i,m}) = 0 \tag{18}$$

$$\lambda_{i,k}g_{i,k} = 0, \qquad \lambda_{i,k} \geq 0 \tag{19}$$

$$g_{i,k} \leq 0 , \qquad k=1,\ldots,K \tag{20}$$

where m=1,...,M and

$$\partial f_{M_i}/\partial u_{i,m} = \partial f_i/\partial u_{i,m} + \sum_{j=1}^{J} \partial f_{i,j}/\partial u_{i,m}$$

$$+ \sum_{j=1}^{J} (\partial f_{i,j}/\partial v^*_{i,j})(\partial v^*_{i,j}/\partial u_{i,m}) \tag{21}$$

Now, suppose in subproblem (i,j), constraint $g_{i,j,l}$ is regionally (or globally) active (Azarm and Papalambros, 1984a):

$$g_{i,j,l}(y, u_i, v^*_{i,j}) = 0 \tag{22}$$

since $v_{i,j}$ is the only local variable in subproblem (i,j), eq. (14) may be written as (considering eqs. (15) and (22)):

$$\partial f_{i,j}/\partial v^*_{i,j} + \lambda_{i,j,l}(\partial g_{i,j,l}/\partial v^*_{i,j}) = 0 \quad (23)$$

so that for i=1,...,I and j=1,...,J, we have

$$\partial f_{i,j}/\partial v^*_{i,j} = -\lambda_{i,j,l}(\partial g_{i,j,l}/\partial v^*_{i,j}) \quad (24)$$

Furthermore, since y is constant in the middle-level subproblems, we may also consider y to be constant in eq. (22), so that for every feasible neighborhood of a feasible point and for m=1,...,M, we have:

$$\partial g_{i,j,l}/\partial u_{i,m} + (\partial g_{i,j,l}/\partial v^*_{i,j})(\partial v^*_{i,j}/\partial u_{i,m}) = 0 \quad (25)$$

which results in

$$\partial v^*_{i,j}/\partial u_{i,m} = -(\partial g_{i,j,l}/\partial v^*_{i,j})^{-1}(\partial g_{i,j,l}/\partial u_{i,m}) \quad (26)$$

considering eqs. (26), (24), and (21), we may rewrite eq. (18) as follows:

$$\partial f_i/\partial u_{i,m} + \sum_{j=1}^{J} \partial f_{i,j}/\partial u_{i,m} + \sum_{j=1}^{J} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial u_{i,m})$$

$$+ \sum_{k=1}^{K} \lambda_{i,k}(\partial g_{i,k}/\partial u_{i,m}) = 0, \quad m=1,\ldots,M \quad (27)$$

summing up eq. (27) for all the middle-level subproblems

$$\sum_{i=1}^{I} \partial f_i/\partial u_{i,m} + \sum_{i=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial u_{i,m} + \sum_{i=1}^{I} \sum_{j=1}^{J} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial u_{i,m})$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_{i,k}(\partial g_{i,k}/\partial u_{i,m}) = 0, \quad m=1,\ldots,M \quad (28)$$

The argument remains valid if $g_{i,j,l}$ is not active, in which case based on eq. (15), the corresponding Lagrange multiplier is zero ($\lambda_{i,j,l}=0$). Therefore, eq. (28) may be rewritten in the following form:

$$\sum_{i=1}^{I} \partial f_i/\partial u_{i,m} + \sum_{i=1}^{I}\sum_{j=1}^{J} \partial f_{i,j}/\partial u_{i,m} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial u_{i,m})$$

$$+ \sum_{i=1}^{I}\sum_{k=1}^{K} \lambda_{i,k}(\partial g_{i,k}/\partial u_{i,m}) = 0, \ m=1,\ldots,M \qquad (29)$$

Finally, the KKT conditions for the top-level problem, eq. (6), may be expressed as follows:

$$\partial f_T/\partial y_t + \sum_{h=1}^{H} \lambda_h \partial g_h/\partial y_t = 0 \qquad (30)$$

$$\lambda_h g_h = 0, \qquad \lambda_h \geq 0 \qquad (31)$$

$$g_h \leq 0, \qquad h=1,\ldots,H \qquad (32)$$

where $t=1,\ldots,T'$, and

$$\partial f_T/\partial y_t = \partial f_0/\partial y_t + \sum_{i=1}^{I} \partial f_i/\partial y_t + \sum_{i=1}^{I}\sum_{m=1}^{M} (\partial f_i/\partial u^*_{i,m})(\partial u^*_{i,m}/\partial y_t)$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{J} \partial f_{i,j}/\partial y_t + \sum_{i=1}^{I}\sum_{j=1}^{J}\Big[\sum_{m=1}^{M} (\partial f_{i,j}/\partial u^*_{i,m})(\partial u^*_{i,m}/\partial y_t)$$

$$+ (\partial f_{i,j}/\partial v^*_{i,j})(\partial v^*_{i,j}/\partial y_t)\Big] \qquad (33)$$

Now, suppose a subset of the constraints in the lower-level subproblems is active, namely, in the bottom-level subproblem (i,j), $i=1,\ldots,I$ and $j=1,\ldots,J$, constraint $g_{i,j,l}$ is regionally (or globally) active:

$$g_{i,j,l}(y,u^*_i,v^*_{i,j}) = 0 \qquad i=1,\ldots,I \quad j=1,\ldots,J \qquad (34)$$

and in the middle-level subproblem (i), $i=1,\ldots,I$, there are M regionally (or globally) active constraints:

$$g_{i,k}(y,u^*_i) = 0, \qquad i=1,\ldots,I \qquad k=1,\ldots,M \tag{35}$$

then, from eq. (34)

$$\partial g_{i,j,1}/\partial y_t + \sum_{m=1}^{M} (\partial g_{i,j,1}/\partial u^*_{i,m})(\partial u^*_{i,m}/\partial y_t)$$

$$+ (\partial g_{i,j,1}/\partial v^*_{i,j})(\partial v^*_{i,j}/\partial y_t) = 0, \qquad i=1,\ldots,I \qquad j=1,\ldots,J \tag{36}$$

and from eq. (35)

$$\partial g_{i,k}/\partial y_t + \sum_{m=1}^{M} (\partial g_{i,k}/\partial u^*_{i,m})(\partial u^*_{i,m}/\partial y_t) = 0 \qquad i=1,\ldots,I \qquad k=1,\ldots,M \tag{37}$$

where $t=1,\ldots,T'$. We now use the vector notation, defined by eqs. (7)-(13), to rewrite eqs. (33), (36), and (37) as follows:

$$\partial f_T/\partial y_t = \partial f_0/\partial y_t + \sum_{i=1}^{I} \partial f_i/\partial y_t + \sum_{i=1}^{I} (\partial f_i/\partial u^*_i)^t(\partial u^*_i/\partial y_t) + \sum_{i=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial y_t$$

$$+ \sum_{i=1}^{I} \sum_{j=1}^{J} (\partial f_{i,j}/\partial u^*_i)^t(\partial u^*_i/\partial y_t) + \sum_{i=1}^{I} \sum_{j=1}^{J} (\partial f_{i,j}/\partial v^*_{i,j})(\partial v^*_{i,j}/\partial y_t) \tag{38}$$

and

$$\partial g_{i,j,1}/\partial y_t + (\partial g_{i,j,1}/\partial u^*_i)^t(\partial u^*_i/\partial y_t)$$

$$+ (\partial g_{i,j,1}/\partial v^*_{i,j})(\partial v^*_{i,j}/\partial y_t) = 0, \qquad i=1,\ldots,I \qquad j=1,\ldots,J \tag{39}$$

and

$$\partial g_i/\partial y_t + (\partial g_i/\partial u^*_i)\ (\partial u^*_i/\partial y_t) = 0 \qquad i=1,\ldots,I \tag{40}$$

where

$$(\partial f_i/\partial u^*_i) = (\partial f_i/\partial u^*_{i,1},\ldots,\ \partial f_i/\partial u^*_{i,M})^t \tag{41}$$

$$(\partial u^*_i/\partial y_t) = (\partial u^*_{i,1}/\partial y_t,\ldots,\ \partial u^*_{i,M}/\partial y_t)^t \tag{42}$$

$$(\partial f_{i,j}/\partial u^*_i) = (\partial f_{i,j}/\partial u^*_{i,1},\ldots,\partial f_{i,j}/\partial u^*_{i,M})^t \tag{43}$$

$$(\partial g_i/\partial y_t) = (\partial g_{i,1}/\partial y_t,\ldots,\partial g_{i,M}/\partial y_t)^t \tag{44}$$

$$(\partial g_i/\partial u^*_i) = \begin{matrix} \partial g_{i,1}/\partial u^*_{i,1} & \cdots & \partial g_{i,1}/\partial u^*_{i,M} \\ & \vdots & \\ \partial g_{i,M}/\partial u^*_{i,1} & \cdots & \partial g_{i,M}/\partial u^*_{i,M} \end{matrix} \tag{45}$$

$$(\partial g_{i,j,1}/\partial u^*_i) = (\partial g_{i,j,1}/\partial u^*_{i,1},\ldots,\partial g_{i,j,1}/\partial u^*_{i,M})^t \tag{46}$$

from eq. (39), we have

$$\partial v^*_{i,j}/\partial y_t = -(\partial g_{i,j,1}/\partial v^*_{i,j})^{-1}[(\partial g_{i,j,1}/\partial y_t)+(\partial g_{i,j,1}/\partial u^*_i)^t$$

$$(\partial u^*_i/\partial y_t)], \qquad i=1,\ldots,I$$

$$j=1,\ldots,J \tag{47}$$

and from eq. (40)

$$\partial u^*_i/\partial y_t = -(\partial g_i/\partial u^*_i)^{-1}(\partial g_i/\partial y_t), \qquad i=1,\ldots,I \tag{48}$$

substituting eq. (48) into (47)

$$\partial v^*_{i,j}/\partial y_t = -(\partial g_{i,j,1}/\partial v^*_{i,j})^{-1}[(\partial g_{i,j,1}/\partial y_t) - (\partial g_{i,j,1}/\partial u^*_i)^t (\partial g_i/\partial u^*_i)^{-1}(\partial g_i/\partial y_t)] \qquad i=1,\ldots,I \quad j=1,\ldots,J \tag{49}$$

then, substituting eqs. (48) and (49) into eq. (38) to have

$$\partial f_T/\partial y_t = \partial f_o/\partial y_t + \sum_{i=1}^{I} \partial f_i/\partial y_t - \sum_{i=1}^{I} (\partial f_i/\partial u^*_i)^t (\partial g_i/\partial u^*_i)^{-1}(\partial g_i/\partial y_t)$$

$$+ \sum_{i=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial y_t - \sum_{i=1}^{I} \sum_{j=1}^{J} (\partial f_{i,j}/\partial u^*_i)^t (\partial g_i/\partial u^*_i)^{-1}(\partial g_i/\partial y_t)$$

$$- \sum_{i=1}^{I} \sum_{j=1}^{J} \{(\partial f_{i,j}/\partial v^*_{i,j})(\partial g_{i,j,1}/\partial v^*_{i,j})^{-1}[(\partial g_{i,j,1}/\partial y_t)$$

$$-(\partial g_{i,j,1}/\partial u^*_i)^t (\partial g_i/\partial u^*_i)^{-1}(\partial g_i/\partial y_t)]\} \tag{50}$$

or

$$\partial f_T/\partial y_t = \partial f_o/\partial y_t + \sum_{i=1}^{I} \partial f_i/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} [-(\partial f_{i,j}/\partial v^*_{i,j})$$

$$(\partial g_{i,j,1}/\partial v^*_{i,j})^{-1}(\partial g_{i,j,1}/\partial y_t)] + \sum_{i=1}^{I} \{[-(\partial f_i/\partial u^*_i)^t - \sum_{j=1}^{J} (\partial f_{i,j}/\partial u^*_i)^t$$

$$- \sum_{j=1}^{J} (-\partial f_{i,j}/\partial v^*_{i,j})(\partial g_{i,j,1}/\partial v^*_{i,j})^{-1}(\partial g_{i,j,1}/\partial u^*_i)^t]$$

$$(\partial g_i/\partial u^*_i)^{-1}\}(\partial g_i/\partial y_t) \tag{51}$$

eq. (51) may be written in the following form (considering eq. (24)):

$$\partial f_T/\partial y_t = \partial f_o/\partial y_t + \sum_{i=1}^{I} \partial f_i/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial y_t)$$

$$+ \sum_{i=1}^{I} \{[-(\partial f_i/\partial u^*_i)^t - \sum_{j=1}^{J} (\partial f_{i,j}/\partial u^*_i)^t - \sum_{j=1}^{J} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial u^*_i)^t]$$

$$(\partial g_i/\partial u^*_i)^{-1}\}(\partial g_i/\partial y_t) \tag{52}$$

We may simplify eq. (52), by showing that its last term is $\sum_{i=1}^{I} \sum_{k=1}^{M} \lambda_{i,k}$ $(\partial g_{i,k}/\partial y_t)$. To do that, we rewrite eq. (27) for all the middle-level subproblems using our vector notation:

$$\partial f_i/\partial u^*_i + \sum_{j=1}^{J} \partial f_{i,j}/\partial u^*_i + \sum_{j=1}^{J} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial u^*_i)$$

$$+ (\partial g_i/\partial u^*_i)^t \ \boldsymbol{\lambda}_i = 0 \tag{53}$$

where

$$\boldsymbol{\lambda}_i = (\lambda_{i,1},\ldots,\lambda_{i,k},\ldots,\lambda_{i,M})^t \tag{54}$$

hence, from eq. (53)

$$\boldsymbol{\lambda}_i = [(\partial g_i/\partial u^*_i)^t]^{-1}[-(\partial f_i/\partial u^*_i) - \sum_{j=1}^{J} (\partial f_{i,j}/\partial u^*_1)$$

$$- \sum_{j=1}^{J} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial u^*_i)] \tag{55}$$

or

$$\boldsymbol{\lambda}_i^t = [-(\partial f_i/\partial u^*_i)^t - \sum_{j=1}^{J} (\partial f_{i,j}/\partial u^*_i)^t - \sum_{j=1}^{J} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial u^*_i)^t]$$

$$(\partial g_i/\partial u^*_i)^{-1} \tag{56}$$

so, based on eqs. (56), (54), and (44), eq. (52) is written as follows:

$$\partial f_T/\partial y_t = \partial f_o/\partial y_t + \sum_{i=1}^{I} \partial f_i/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} \lambda_{i,j,l}$$

$$(\partial g_{i,j,l}/\partial y_t) + \sum_{i=1}^{I} \sum_{k=1}^{M} \lambda_{i,k}(\partial g_{i,k}/\partial y_t) \tag{57}$$

Again, the argument remains valid if some of the lower-level constraints are not active, i.e., $g_{i,j,l}<0$ and/or $g_{i,k}<0$, in which case the corresponding Lagrange multiphers are zero, i.e., $\lambda_{i,j,l} = 0$ and/or $\lambda_{i,k} = 0$. Thus, eq. (30) may be written in the following form (considering eq. (57)):

$$\partial f_o/\partial y_t + \sum_{i=1}^{I} \partial f_i/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} \partial f_{i,j}/\partial y_t + \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L} \lambda_{i,j,l}(\partial g_{i,j,l}/\partial y_t)$$

$$+ \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_{i,k}(\partial g_{i,k}/\partial y_t) + \sum_{h=1}^{H} \lambda_h \partial g_h/\partial y_t = 0 \tag{58}$$

Therefore, eqs. (17), (29), (58) together with eqs. (15), (16), (19), (20), (31), and (32) derived for all the subproblems will form the KKT conditions for the overall (nondecomposed) problem, eqs. (2)-(3).

4. Example: Global Monotonicity-Based Decomposition (GMBD)

In this section we present a well-known example from the literature (Golinski, 1970), namely a gear reducer, which has been solved by several optimization methods including those by Datseris (1982), Azarm (1984), Li and Papalambros (1985). More recently, Azarm and Li (1987b) solved this example using a two-level global monotonicity-based decomposition. Here, we will show that by using different decomposed subproblems than the ones adopted by Azarm and Li (1987b), we can solve the problem using the two- as well as the three-level decomposition method. In addition, we will show that with the insight obtained from the decomposition, we can solve the problem analytically as well.

Here, for convenience, the nonlinear programming statement for this example is given:

Minimize $f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2)$

$+ 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$

Subject to: (59)

g_1: $27x_1^{-1}x_2^{-2}x_3^{-1} \le 1$

g_2: $397.5x_1^{-1}x_2^{-2}x_3^{-2} \le 1$

g_3: $1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \le 1$

g_4: $1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \le 1$

g_5: $A_1/B_1 \le 1100$

$$A_1 = [(\frac{745x_4}{x_2x_3})^2 + (16.9)10^6]^{0.5}$$

$$B_1 = 0.1x_6^3$$

g_6: $A_2/B_2 \le 850$

$$A_2 = [(\frac{745x_5}{x_2x_3})^2 + (157.5)10^6]^{0.5}$$

$$B_2 = 0.1x_7^3$$

g_7: $x_2x_3 \le 40$

g_8: $5 \le x_1/x_2 \le 12$ $:g_9$

g_{10}: $2.6 \le x_1 \le 3.6$ $:g_{11}$

g_{12}: $0.7 \le x_2 \le 0.8$ $:g_{13}$

g_{14}: $17 \le x_3 \le 28$ $:g_{15}$

g_{16}: $7.3 \le x_4 \le 8.3$ $:g_{17}$

g_{18}: $7.3 \le x_5 \le 8.3$ $:g_{19}$

g_{20}: $2.9 \le x_6 \le 3.9$ $:g_{21}$

g_{22}: $5.0 \le x_7 \le 5.5$ $:g_{23}$

g_{24}: $(1.5x_6 + 1.9)x_4^{-1} \le 1$

g_{25}: $(1.1x_7 + 1.9)x_5^{-1} \le 1.$

4.1. Two-Level Decomposition

The gear reducer considered here, see Figure 3, consists of two subsystems, namely, shaft and bearings 1 and shaft and bearings 2. These two subsytems are selected to correspond to subproblems 1 and 2, respectively. The two-level physically decomposed problem has x_4 and x_6 as the local variables for subproblem 1, x_5 and x_7 as the local variables for subproblem 2, and x_1, x_2, x_3 as the local variables for the top-level problem:

Subproblem 1:

Minimize $f_1(x_1,x_2,x_3,x_4,x_6) = -1.508x_1x_6^2 + 7.477x_6^3 + 0.7854x_4x_6^2$

Subject to: (60)

g_3: $1.93x_2^{-1}x_3^{-1}x_4^{3}x_6^{-4} \le 1$

g_5: $A_1/B_1 \le 1100$

g_{16}: $7.3 \le x_4 \le 8.3$ $:g_{17}$

g_{20}: $2.9 \le x_6 \le 3.9$ $:g_{21}$

g_{24}: $(1.5x_6 + 1.9)x_4^{-1} \le 1$

Subproblem 2:

Minimize $f_2(x_1,x_2,x_3,x_5,x_7) = -1.508x_1x_7^2 + 7.477x_7^3 + 0.7854x_5x_7^2$

Subject to: (61)

g_4: $1.93\, x_2^{-1}\, x_3^{-1}x_5^{3}x_7^{-4} \le 1$

g_6: $A_2/B_2 \le 850$

g_{18}: $7.3 \le x_5 \le 8.5$ $:g_{19}$

g_{22}: $5 \le x_7 \le 5.5$ $:g_{23}$

g_{25}: $(1.1\, x_7 + 1.9)\, x_5^{-1} \le 1$

In subproblem 1, it can be easily verified that the objective function is increasing w.r.t. x_4 and x_6 within the feasible range of $x_4 \ge 7.3$, $x_6 \ge 2.9$, and $x_1 \le 3.6$. Hence according to the first rule of monotonicity analysis (see, for example, Papalambros and Wilde, 1988), w.r.t. x_6 constraints g_3, g_5, and g_{20} are the candidate active constraints. In order to find the dominant active constraint, we rearrange g_3, g_5, and g_{20} as follows:

$$g_3: \quad x_6 \ge (1.93\, x_2^{-1}x_3^{-1}x_4^{3})^{1/4} \tag{62}$$

$$g_5: \quad x_6 \ge (A_1/110)^{1/3} \tag{63}$$

$$g_{20}: \quad x_6 \ge 2.9 \tag{64}$$

Then, we find the lower and upper bounds of the right-hand sides of eqs. (62) and (63) using the available bounds on the variables x_2, x_3, and x_4:

$$2.406 \le (1.93x_2^{-1}x_3^{-1}x_4^{3})^{1/4} \le 3.103 \tag{65}$$

$$3.346 \le \quad (A_1/110)^{1/3} \qquad \le 3.352 \tag{66}$$

From equations (62)-(66), we conclude that g_5 is the active constraint and g_3, and g_{20} are the redundant constraints. Likewise, it can be verified that in subproblem 1, w.r.t. x_4 constraints g_{16} and g_{24} are the candidate active constraints. If we rearrange g_{24} as follows:

$$g_{24}: \qquad x_4 \ge 1.5x_6 + 1.9 \tag{67}$$

Since g_5 is active, by substituting the lower and upper bounds of variable x_6 from eq. (66), we can find the lower and upper bounds of the right-hand side of equation (67):

$$6.918 \le 1.5x_6 + 1.9 \le 6.928 \tag{68}$$

which if compared with $x_4 \ge 7.3$, will result in g_{16} as the active constraint and g_{24} as the redundant constraint.

Therefore, in subproblem 1 constraints g_5 and g_{16} are found to be active:

$$g_5: \qquad x_6^* = \{[(745\ x_4^*/(x_2x_3))^2 + 16.9 \times 10^6]^{0.5} \ /\ 110\}^{1/3} \tag{69}$$

$$g_{16}: \qquad x_4^* = 7.3 \tag{70}$$

In subproblem 2, w.r.t. x_7 constraints g_4, g_6, g_{22} are the candidate active constraints. By a similar analysis, as in subproblem 1, we have:

$$g_4: \qquad x_7 \ge (1.93x_2^{-1}x_3^{-1}x_5^{3})^{1/4} \tag{71}$$

$$g_6: \qquad x_7 \ge (A_2/85)^{1/3} \tag{72}$$

$$g_{22}: \qquad x_7 \ge 5 \tag{73}$$

where

$$2.406 \ge (1.93x_2^{-1}x_3^{-1}x_5^{3})^{1/4} \quad \le 3.103 \tag{74}$$

$$5.28568 \le (A_2/85)^{1/3} \quad \le 5.28686 \tag{75}$$

From equations (70)-(75) we conclude that g_6 is active. Likewise, w.r.t. x_5 constraints g_{18} and g_{25} are the candidate active constraints. However, since g_6 is active, then we can use the lower bound of x_7 from equation (75) into the right-hand side of g_{25}:

$$g_{25}: \qquad x_5 \geq 1.1x_7 + 1.9 \geq 1.1\ (5.28568) + 1.9 \approx 7.714 \tag{76}$$

which if compared with g_{18}, results in g_{25} as the active constraint.

Therefore, in subproblem 2, constraints g_6 and g_{25} are found to be active:

$$g_6: \qquad [(745x_5^*/(x_2x_3))^2 + 157.5x10^6]^{0.5}\ /0.1x_7^* = 850 \tag{77}$$

$$g_{25}: \qquad (1.1x_7^* + 1.9)x_5^{*-1} = 1 \tag{78}$$

The second-level problem is written in the following form:

Minimize $f(x) = 0.7854x_1x_2^2\ (3.3333x_3^2 + 14.9334x_3 - 43.0934)$

$$-1.508x_1(x_6^{*2} + x_7^{*2}) + 7.477(x_6^{*3} + x_7^{*3}) + 0.7854(x_4^*x_6^{*2} + x_5^*x_7^{*2})$$

Subject to: (79)

g_1: $27x_1^{-1}x_2^{-2}x_3^{-1} \leq 1$

g_2: $397.5\ x_1^{-1}x_2^{-2}x_3^{-2} \leq 1$

g_7: $x_2x_3 \leq 40$

g_8: $5 \leq x_1/x_2 \leq 12$:g_9

g_{10}: $2.6 \leq x_1 \leq 3.6$:g_{11}

g_{12}: $0.7 \leq x_2 \leq 0.8$:g_{13}

g_{14}: $17 \leq x_3 \leq 28$:g_{15}

where x_4^*, x_5^*, x_6^*, and x_7^* are found from subproblems 1 and 2 as shown in

Figure 4. The second-level problem is solved by a single-level optimization method (Powell, 1978).

4.2. Three-Level Decomposition

In the gear reducer example, if we continue to decompose each subproblem into two levels, we will then end up with a three-level decomposed problem. To do that, subproblem 1 is decomposed into two levels with x_4 (corresponds to bearings 1) and x_6 (corresponds to shaft 1) to be the middle- and the bottom-level local variables, respectively. Likewise, subproblem 2 is decomposed into two levels with x_5 (corresponds to bearings 2) and x_7 (corresponds to shaft 2) to be the middle- and bottom-level local variables. Hence, the original gear reducer problem is decomposed into three levels with the bottom-level subproblems (1,1) and (2,1) as follows:

Subproblem (1,1):

Minimize $f_{B_{1,1}} = 1.508x_1x_6^2 + 7.477x_6^3 + 0.7854x_4x_6^2$

Subject to: (80)

g_3: $x_6 \geq (1.93x_2^{-1}x_3^{-1}x_4^3)^{1/4} = g'_{1,3}$

g_5: $x_6 \geq (A_1/110)^{1/3} = g'_{1,5}$

g_{20}: $x_6 \geq 2.9 = g'_{1,20}$

g_{21}: $x_6 \leq 3.9 = g'_{1,21}$

g_{24}: $x_6 \leq (x_4 - 1.9)/1.5 = g'_{1,24}$

Subproblem (2,1):

Minimize $f_{B_{2,1}} = -1.508x_1x_7^2 + 7.477x_7^3 + 0.7854x_5x_7^2$

Subject to: (81)

g_4: $x_7 \geq (1.93x_2^{-1}x_3^{-1}x_5^3)^{1/4} = g'_{2,4}$

g_6: $x_7 \geq (A_2/85)^{1/3} = g'_{2,6}$

g_{22}: $x_7 \geq 5 = g'_{2,22}$

g_{23}: $x_7 \leq 5.5 = g'_{2,23}$

g_{25}: $x \leq (x_5 - 1.9)/1.1 = g'_{2,25}$

Application of the global monotonicity analysis to the bottom-level subproblems (1,1) and (2,2) results in:

$$x_6^* = \max \{g'_{1,3}, g'_{1,5}, g'_{1,20}\} \quad (82)$$

and

$$x_7^* = \max \{g'_{2,4}, g'_{2,6}, g'_{2,22}\} \quad (83)$$

The middle-level subproblems 1 and 2 are:

Middle-Level Subproblem 1:

$$\begin{aligned} &\text{Minimize } f_{M_1} = -1.508x_1x_6^{*2} + 7.477x_6^{*3} + 0.7854x_4x_6^{*2} \\ &\text{Subject to:} \\ &\qquad 7.3 \leq x_4 \leq 8.3 \end{aligned} \quad (84)$$

Middle-Level Subproblem 2:

$$\begin{aligned} &\text{Minimize } f_{M_2} = 1.508x_1x_7^{*2} + 7.477x_7^{*3} + 0.7854x_5x_7^{*2} \\ &\text{Subject to:} \\ &\qquad 7.3 \leq x_5 \leq 8.3 \end{aligned} \quad (85)$$

The top-level problem is the same as the one given by equation (79). Figure 5 shows the three-level structure for this example. The middle- and top-level subproblems are solved by a single-level optimization method (Powell, 1978).

4.3. Analytical Approach

In the previous section, we only applied the global monotonicity analysis to the lowest-level subproblems. However, if the mathematical formulation of the problem allows, it is desirable to apply the global monotonicity to the upper-level subproblems as well. Indeed the simpler the upper-level subproblems are made, the less number of times (iterations) will be needed to solve the lower-level subproblems. Furthermore, as we will show in this section, in some cases the application of global monotonicity analysis and subsequent simplication of the lower- and the upper-level subproblems may lead us directly to the optimum.

Let us first consider the gear reducer example without the two practical constraints g_{24} and g_{25}. They are design conditions on the shafts based on experience, as was referred to by Li and Papalambros (1985) in a similar but a single-level analysis of the example. We use the results of our analysis obtained from subproblems 1 and 2 of section 4.1 to find x_4^*, x_6^*, x_7^* using eqs. (70), (69), (77), respectively. In case of x_5^*, since constraint g_{25} has not been considered, we can concluded that from subproblem 2 constraint g_{18} is active, i.e., $x_5^* = 7.3$. We now apply the global monotonicity analysis to the second-level problem which is given by eq. (79). The second-level problem has the local variables, x_1, x_2, and x_3 and the global variables x_4, x_5, x_6, x_7. It can be easily shown that $\partial f/\partial x_i$, i = 1,2,3, is positive within the feasible domain (note that x_4^* and x_5^* are fixed, and from eqs. (69) and (77), x_6^* and x_7^* are explicit functions of x_1, x_2, x_3). Hence, based on the monotonicity rules, constraints g_1, g_2, g_8, and g_{10} are the candidate active constraints w.r.t. x_1. We rearrange these constraints to obtain:

$$g_1: \qquad x_1 \geq (27/(x_2^2 x_3)) = g'_1 \tag{86}$$

$$g_2: \qquad x_1 \geq (397.5/(x_2^2 x_3^2)) = g'_2 \tag{87}$$

$$g_8: \qquad x_1 \geq 5x_2 = g'_8 \tag{88}$$

g_{10}: $x_1 \geq 2.9$ (89)

where $1.86 \leq g'_1 \leq 3.24$, $0.79 \leq g'_2 \leq 2.8$, and $g'_8 \geq 3.5$. Hence, we can conclude that constraint g_8 is active. Likewise, it can be verified that w.r.t. x_2 and x_3 constraints g_{12} and g_{14} are active. Thus the constraint-bound solution of this example is $x^* = (3.5, 0.7, 17, 7.3, 7.3, 3.35, 5.29)^t$ which corresponds to the active set (g_5, g_6, g_8, g_{12}, g_{14}, g_{16}, g_{18}), and is similar to the one reported by Li and Papalambros (1985).

Now consider the gear reducer example with the practical constraints g_{24} and g_{25}. In this case, from eqs. (77)-(78), x_5^* and x_7^* are implicit (rather than explicit) functions of x_2 and x_3. Hence, we can not directly apply the monotonicity rules to the second-level problem, as was the case without considering g_{24} and g_{25}. However, we may use a well-known operation, namely, constrained derivatives (Beightler et at., 1979), to find monotonicity properties of the second-level problem w.r.t. x_2 and x_3. To use this operation here, we set $z = (y,x)^t$ where x and y represent the global variables and the local variables of the second-level problem, respectively. If g is the m-vector of the lower-level active constraints, then we can find the monotonicity of the upper-level problem w.r.t. its local variables using the constrained derivative (or reduced gradient) operator:

$$(\partial/\partial y) = (\partial/\partial y) - (\partial/\partial x)(\partial g/\partial x)^{-1}(\partial g/\partial y) \tag{90}$$

The quantities $(\partial g/\partial x)$ and $(\partial g/\partial y)$ are the Jacobian matrices of the vector function $g(y,x)$.

Using the constrained-derivatives operation on variables x_1, x_2, and x_3, we can verify that the objective function of the second-level problem, eq. (26), is increasing w.r.t. x_1, x_2, and x_3 within the feasible range of x_4, x_5, x_6, and x_7. Hence, as it was found previously, constraints g_8, g_{12}, g_{14} are active. The constraint-bound solution for the problem is $x^* = (3.5, 0.7, 17, 7.3, 7.72, 3.35, 5.29)^t$ which corresponds to the active set (g_5, g_6, g_8, g_{12}, g_{14}, g_{16}, g_{25}). Again, this solution is similar to the one reported by Li and Papalambros (1985).

5. Examples: Local Monotonicity-Based Decomposition (LMBD)

In this section we present two examples which have been solved by a two-level monotonicity-based decomposition method. Here we have applied local monotonicity (Azarm and Pabalambros, 1984b), rather than global monotonicity (Papalambros and Wilde, 1979 and 1988), to the first-level subproblems. LMBD is particularly attractive for those problems in which the global monotonicity analysis on the lower-level subproblems requires extensive algebraic manipulations.

The first example is the gear reducer presented in section 4. The two-level decomposition of the example considered is similar to the one presented in section 4.1. However, here the first-level subproblems are solved using the ACCME program (Azarm and Papalambros, 1984b) while the second-level problem is solved by a sequential quadratic programming method (Powell, 1978).

The second example is a 22-bar truss subjected to 10 kN load on its edge (Kirsch, 1981), see Figure 6. The design objective is to minimize the volume (or the weight) of the truss. The constraints require that the stresses in the truss elements will not exceed 100,000 kN/m^2. The design variables are y, shown on the Figure, and the members'cross-sectional areas (x_n). Also, $l_n(y)$ is the length of the nth member, which is a function of y, and $|\sigma_n(x_n,y)|$ is the absolute value of the stress in the nth member which is a function of y and x_n. The problem may be formulated as:

$$\text{Minimize } f(y,x) = \sum_{n=1}^{22} x_n l_n(y)$$

$$\text{Subject to:} \qquad (91)$$

$$|\sigma_n(x_n,y)| \le 100{,}000$$

$$x_n > 0$$

$$2 \le y \le 6$$

where n=1,..., 22. The 22 first-level subproblems, to be solved for a given y, are as follows (n=1,...,22):

$$\text{Minimize } f_{B_i}(y, x_n) = x_n l_n$$

$$\text{Subject to:} \qquad (92)$$

$$|\sigma_n(x_n, y)| \leq 100{,}000$$

$$x_n > 0$$

The analysis of truss is done by a finite element program. The subproblems are then solved independently using the ACCME program (Azarm and Papalambros, 1984b). The solution of the first-level subproblems are coordinated by a second-level problem formulated as follows:

$$\text{Minimize } f(y, x) = \sum_{n=1}^{22} x_n^* \, l_n(y)$$

$$\text{Subject to:} \qquad (93)$$

$$2 \leq y \leq 6$$

The second-level problem has only one variable, hence it can be solved by a one-dimensional optimization method. The iteration history for this example using the one-dimensional (Reklaitis et al., 1983) golden-section or the interval-halving method on the second-level problem is presented in Figure 7.

6. Concluding Remarks

Several monotonicity-based decomposition methods for solving a design optimization problem which is decomposable into several subproblems has been presented. These methods should make possible solutions of problems previously too difficult to handle by monotonicity analysis within a single-level framework.

One disadvantage of these methods, when coupled with a conventional single-level optimization method, is the possibility of discontinuous behavior of bottom-level derivatives (see, eqs. (82) and (83)) if they are needed by the upper-level subproblems. This can be resolved by using an optimization method on the upper-level subproblems which does not require derivatives from the lower-level subproblems. Another solution is to use a penalty function approach of the type suggested by Haftka (1984). The other disadvantage is that for a given value of the upper-level local variables, one or more of the lower-level subproblems may have no solution. One possible remedy is to transfer violated constraints from the lower- to upper-level subproblems.

In general, global monotonicity analysis of subproblems (done manually here) is desirable, because of the generality of knowledge contained in such an analysis. However, for large problems, the manual operation is likely to be tedious and cause mistakes. To overcome this problem, a symbolic manipulatin program (see, for example, MACSYMA, 1983) may be used. If global monotonicity analysis is not possible, as is the case in the 22-bar truss example where analytical expressions for the subproblems are unavailable, then local monotonicity analysis can be used.

Finally, the question of how to model a decomposable (or how to decompose a) design optimization problem remains open. It seems that this question should be addressed during the design modelling stage. Development of a systematic methodology (or a set of guidelines) which will lead the designer to a decomposable model should be investigated. Furthermore, once the model is formulated, several alternatives may exist for decomposition. The criterion and/or strategy for selecting the best alternative remains to be investigated.

7. Acknowledgement

The work of the first author was supported through NSF Grant DMC-85-05113. The work of the second author was supported partially through NSF Grant CDR-85-05113 and partially through NSF Grant CDR-85-00108.

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List of Figures

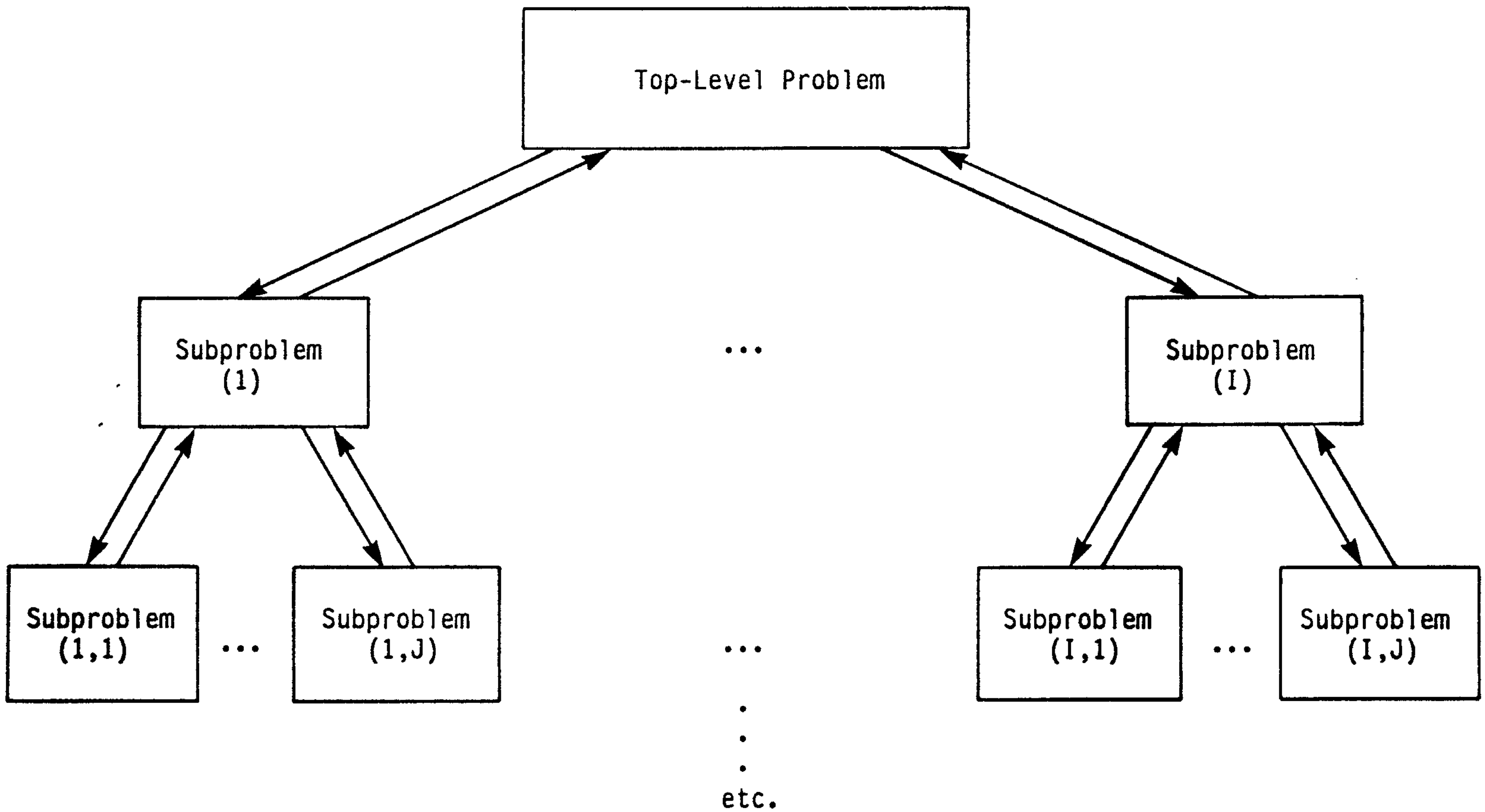

Top-Level Problem
Subproblem (1)
...
Subproblem (I)
Subproblem (1,1)
...
Subproblem (1,J)
...
Subproblem (I,1)
...
Subproblem (I,J)
etc.

Figure 1

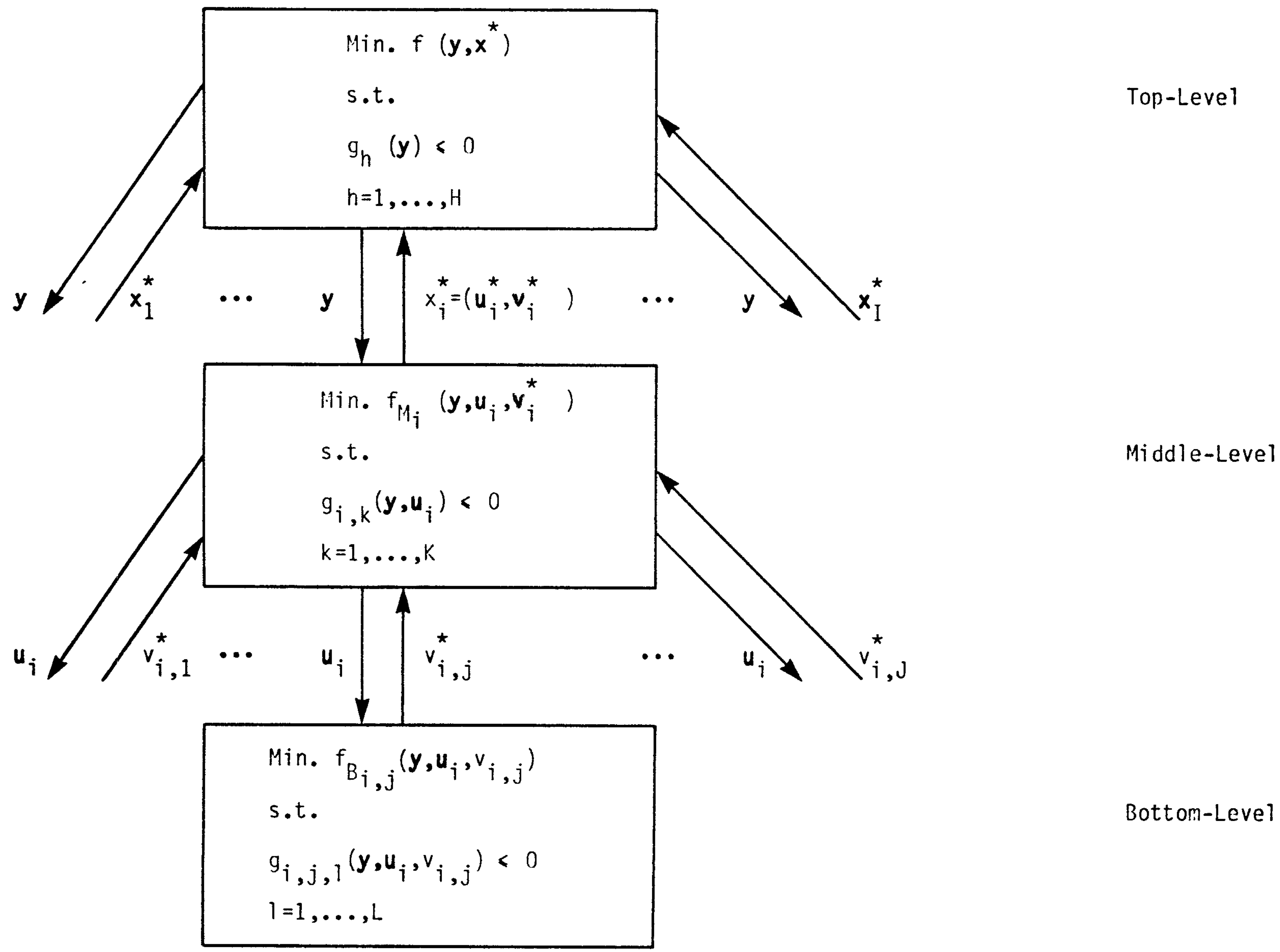

Min. f (y,x*)
s.t.
g_h (y) < 0
h=1,...,H
Top-Level
y
x*_1
...
y
x*_i=(u*_i,v*_i)
...
y
x*_I
Min. f_{M_i} (y,u_i,v*_i)
s.t.
g_{i,k}(y,u_i) < 0
k=1,...,K
Middle-Level
u_i
v*_{i,1}
...
u_i
v*_{i,j}
...
u_i
v*_{i,J}
Min. f_{B_{i,j}}(y,u_i,v_{i,j})
s.t.
g_{i,j,l}(y,u_i,v_{i,j}) < 0
l=1,...,L
Bottom-Level

Figure 2

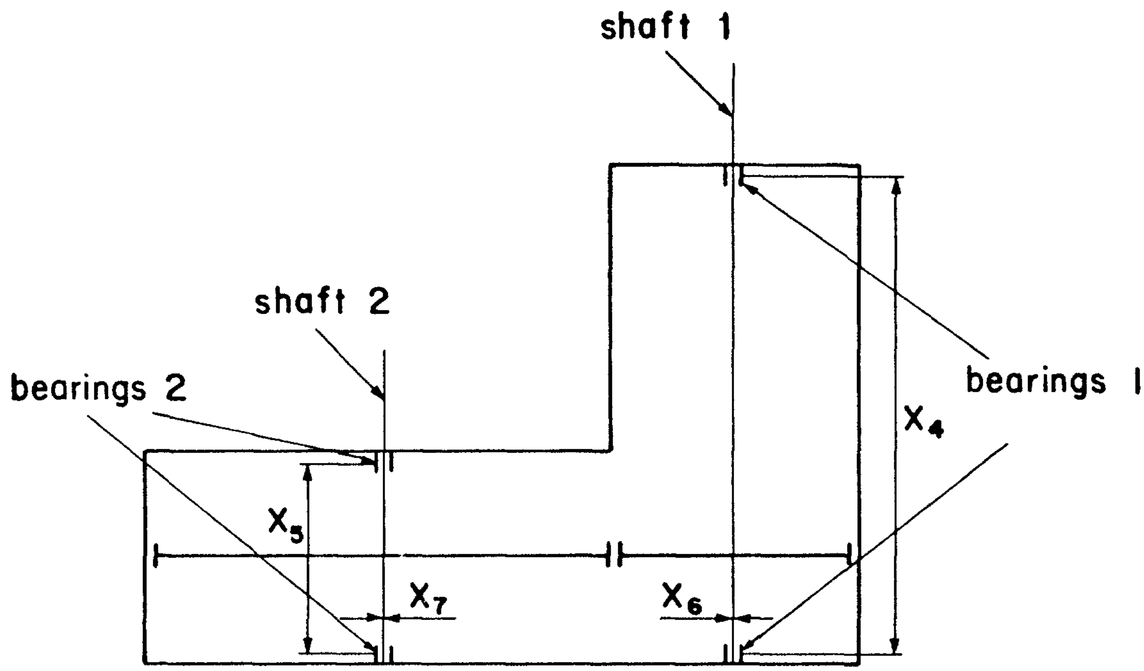


Figure 3

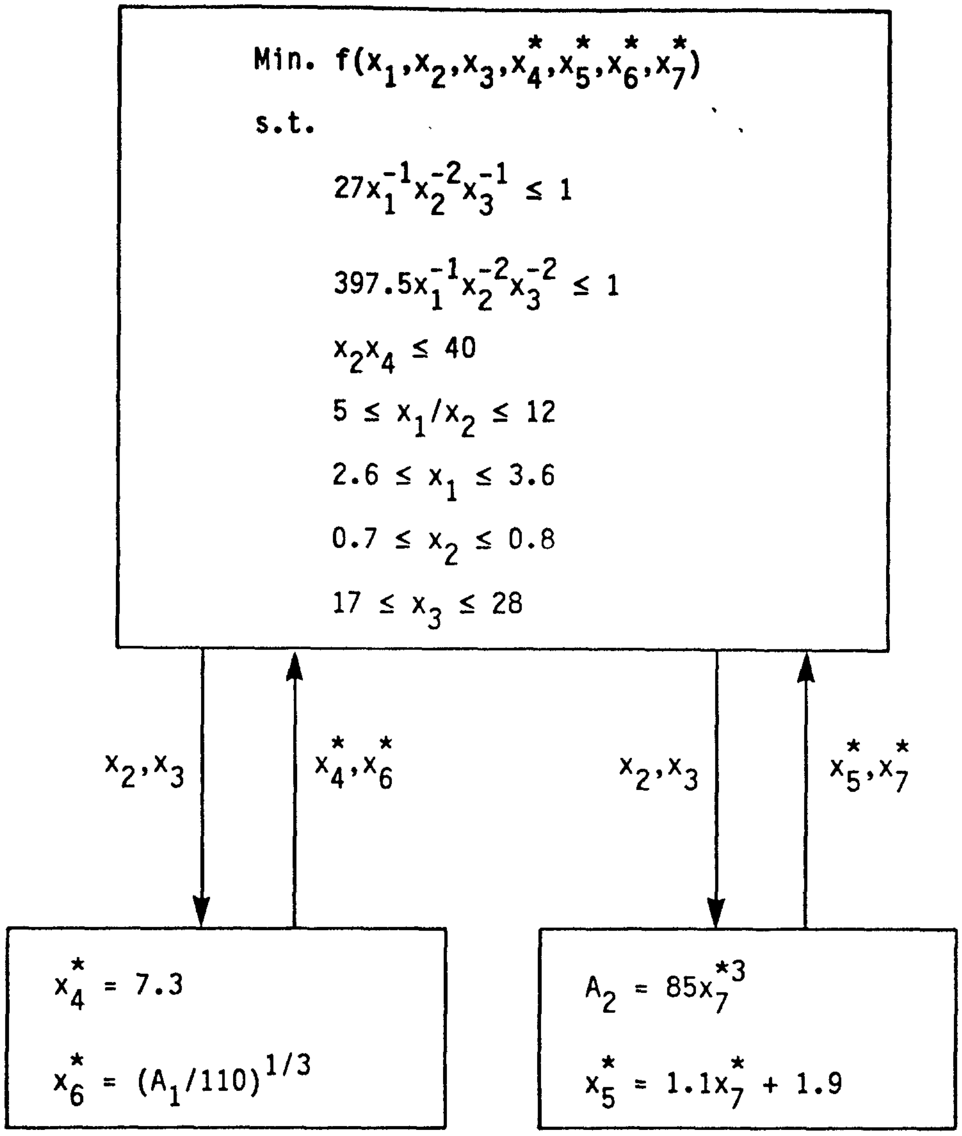
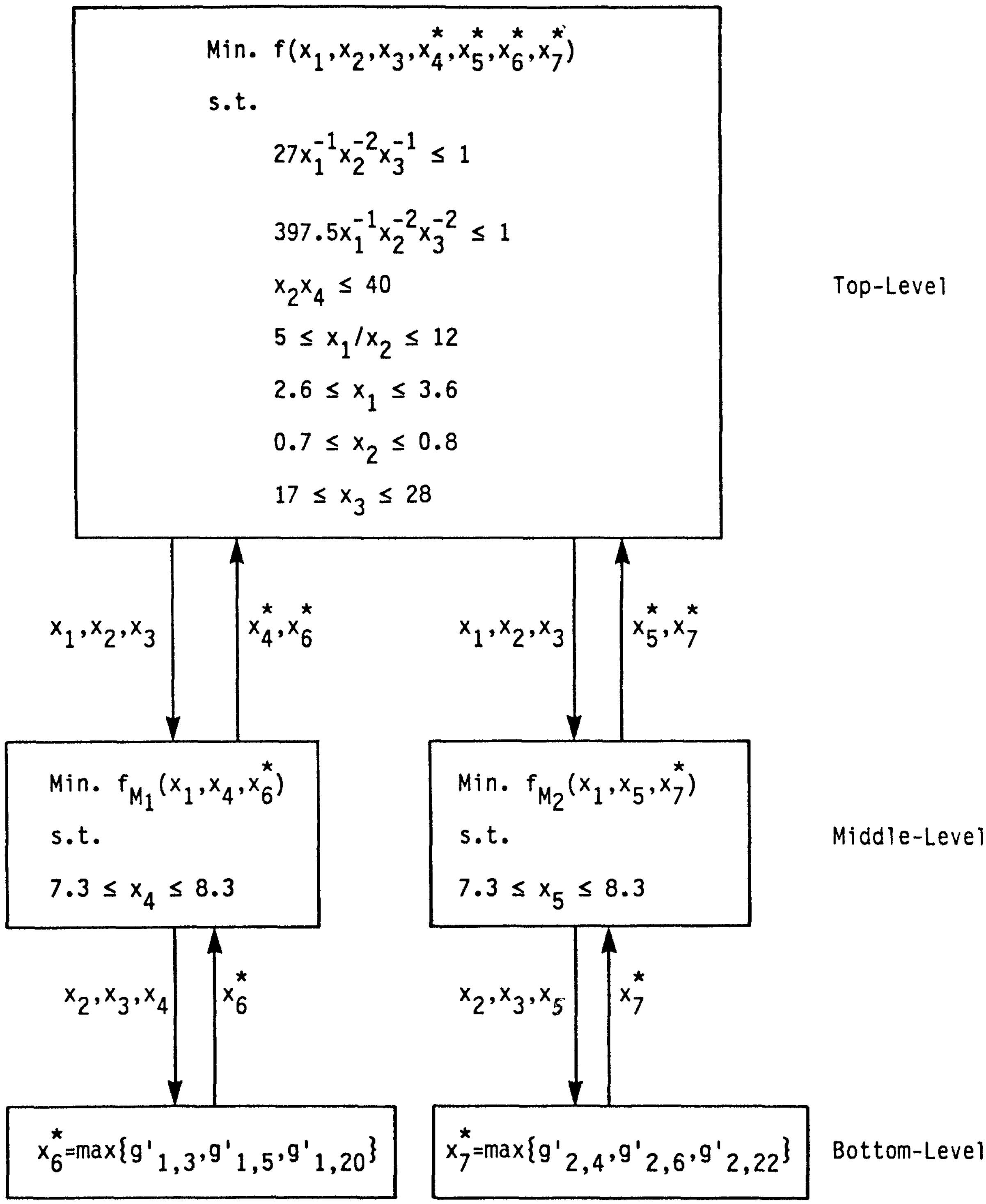

Min. $f(x_1,x_2,x_3,x_4^*,x_5^*,x_6^*,x_7^*)$
s.t.
$27x_1^{-1}x_2^{-2}x_3^{-1} \le 1$
$397.5x_1^{-1}x_2^{-2}x_3^{-2} \le 1$
$x_2x_4 \le 40$
$5 \le x_1/x_2 \le 12$
$2.6 \le x_1 \le 3.6$
$0.7 \le x_2 \le 0.8$
$17 \le x_3 \le 28$
x_2,x_3
x_4^*,x_6^*
x_2,x_3
x_5^*,x_7^*
$x_4^* = 7.3$
$x_6^* = (A_1/110)^{1/3}$
$A_2 = 85x_7^{*3}$
$x_5^* = 1.1x_7^* + 1.9$

Figure 4



Min. f(x1,x2,x3,x4*,x5*,x6*,x7*)
s.t.
27x1^-1x2^-2x3^-1 ≤ 1
397.5x1^-1x2^-2x3^-2 ≤ 1
x2x4 ≤ 40
5 ≤ x1/x2 ≤ 12
2.6 ≤ x1 ≤ 3.6
0.7 ≤ x2 ≤ 0.8
17 ≤ x3 ≤ 28
Top-Level
x1,x2,x3
x4*,x6*
x1,x2,x3
x5*,x7*
Min. fM1(x1,x4,x6*)
s.t.
7.3 ≤ x4 ≤ 8.3
Min. fM2(x1,x5,x7*)
s.t.
7.3 ≤ x5 ≤ 8.3
Middle-Level
x2,x3,x4
x6*
x2,x3,x5
x7*
x6*=max{g'1,3,g'1,5,g'1,20}
x7*=max{g'2,4,g'2,6,g'2,22}
Bottom-Level

Figure 5

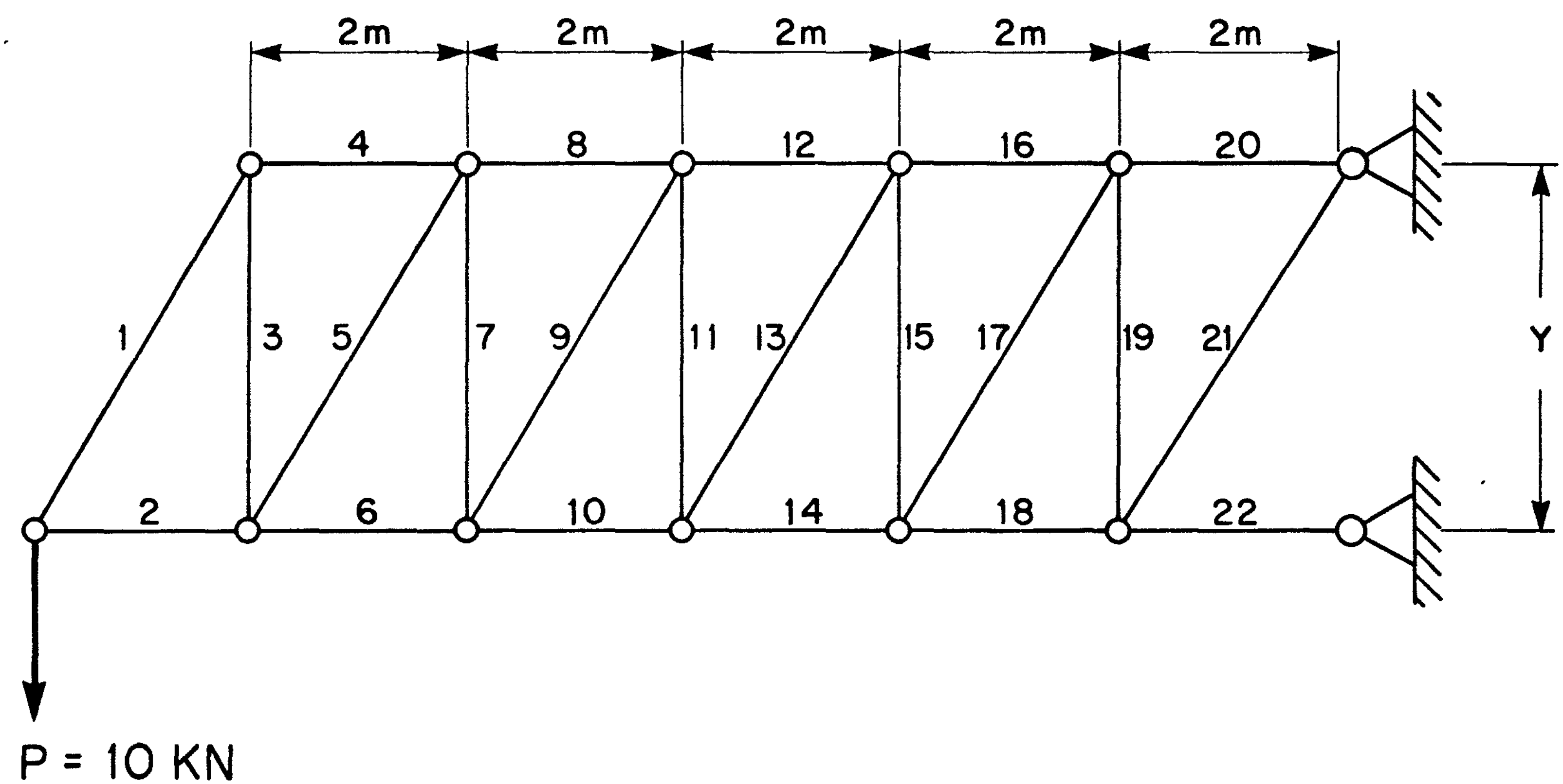

2m
2m
2m
2m
2m
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
Y
P = 10 KN

Figure 6

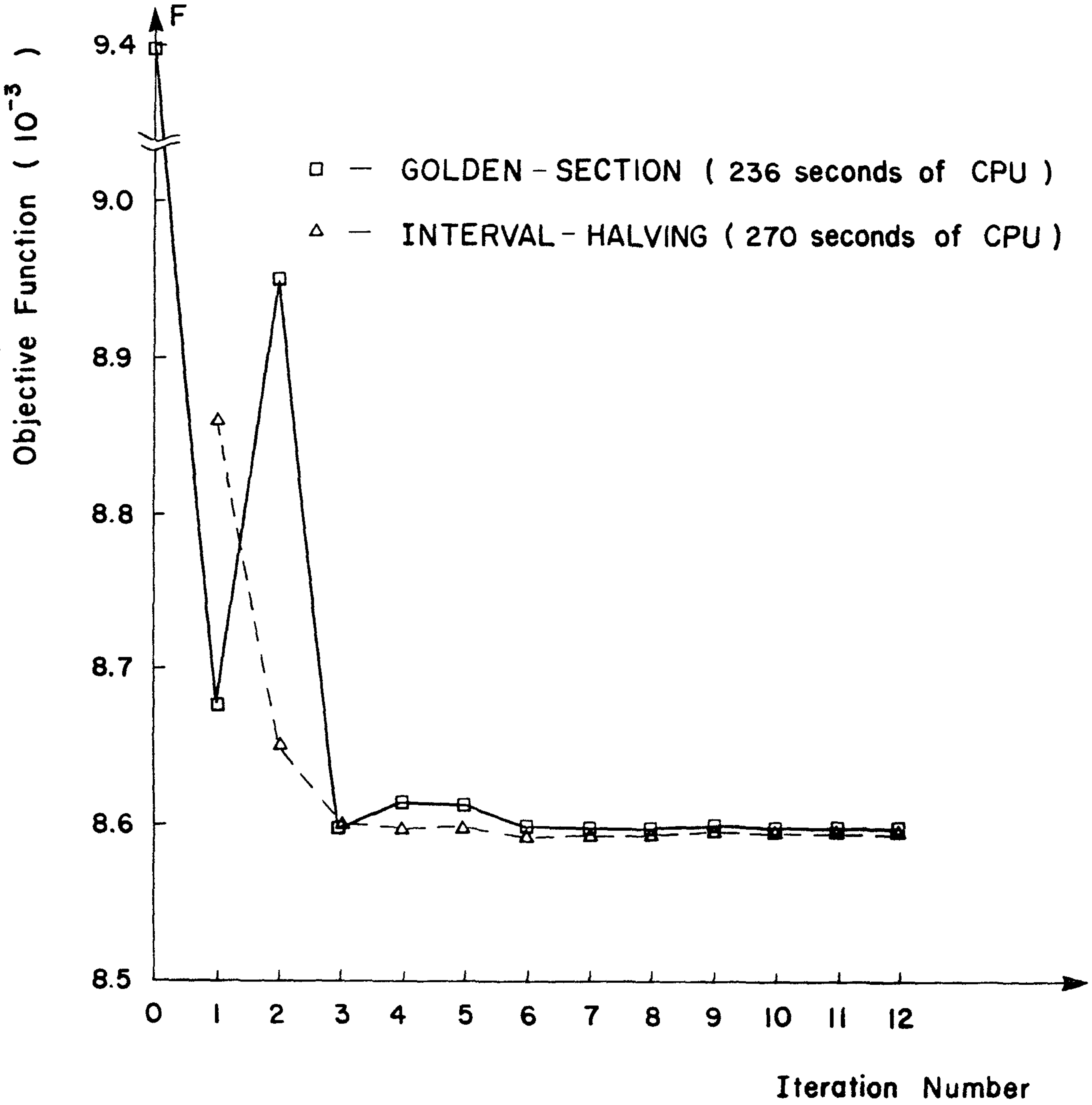

F
Objective Function (10⁻³)
9.4
9.0
8.9
8.8
8.7
8.6
8.5
□ — GOLDEN - SECTION (236 seconds of CPU)
△ — INTERVAL - HALVING (270 seconds of CPU)
0 1 2 3 4 5 6 7 8 9 10 11 12
Iteration Number

Figure 7