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**CHEMICAL PROCESS SYSTEMS  
LABORATORY**

Short-Cut Analysis of Pressure Control in  
Steam Headers

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RESEARCH REPORT

# **CHEMICAL PROCESS SYSTEMS ENGINEERING LABORATORY**

SHORT-CUT ANALYSIS OF PRESSURE CONTROL IN STEAM HEADERS

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# “Short-Cut Analysis of Pressure Control In Steam Headers”

by

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## Abstract

Almost all chemical/petrochemical plants have steam networks to supply power and heating requirements. Good control of steam systems can have important economic implications. In this paper control of pressure in the high pressure header is analyzed. A simple short-cut model is used. The steam system is represented by a third order transfer function with widely separated time constants. This model compares very well with a detailed non-linear simulation and requires only readily available process information. The question of tuning PI controllers for the integrating pressure loop is treated in detail. A root locus analysis is used to develop tuning rules. Typical upsets such as boiler and turbine trips are evaluated for various steam systems. In each case the tuning rules provide very satisfactory control performance.

## 1. Introduction

The steam system of a plant is its prime energy source. In recent years, as energy and equipment costs increased, efficient steam systems became more important in the overall economics of process plants. A key design objective is to develop plants which can be maintained near their desired operating conditions. A detailed simulation can predict accurately the dynamic behavior of a steam system to various upsets. Many models have been proposed for various types of boilers <sup>(1)</sup> <sup>(2)</sup> <sup>(3)</sup>. However a dynamic simulation is costly to develop and to use particularly when it applies to complex steam networks. A short-cut analysis can then be a good alternative to study steam systems. Such an analysis provides information on the dynamics of the system and gives insight into key parameters for which the control performance is most sensitive <sup>(4)</sup> <sup>(5)</sup>. Furthermore,

simple tuning rules can be developed for the controllers. A typical steam system is shown in figure 1. The boilers are drum-type, oil or gas fired boilers. The objective is to maintain the HP header pressure within specific limits using the fuel load of the control boiler as a manipulative variable.

## 2. Short - cut model

The purpose of a short-cut analysis is to give a simple representation of the process to be studied, so that key parameters can be identified and tuning rules determined. A detailed simulation was first developed <sup>(4)</sup>. The system consisting of boilers, headers and turbines was described by a set of 35 non-linear equations, algebraic and differential. Accurate responses can be expected from this detailed approach. It is shown <sup>(4)</sup> that the pressure loop of a steam system can be accurately modeled as linear. In this paper, we use the short-cut model to develop tuning rules for the pressure controllers. The responses are given for this short-cut model and compared with the detailed non-linear simulation. Figure 2 shows a simplified representation of the boiler unit. The short-cut analysis leads

Table 1 : Time constants

# boilers	$\tau_1$ (sec)	$\tau_2$ (sec)	$\tau_w$ (sec)	$\tau_F$ (sec)
1	119.	1.009	5.08	10.
4	115.3	0.78	5.08	10.

to the following relationship relating the header pressure  $P_s$  to the heat input  $Q_r$ :

$$\frac{\overline{P}_s}{\overline{Q}_r} = \frac{K_q}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (1)$$

The time constants  $\tau_1$  and  $\tau_2$  are functions of the design parameters and operating conditions.  $\tau_1$  is a strong function of the header size. Further  $\overline{Q}_r$  and  $\overline{W}_f$  are related by:

$$\frac{\overline{Q}_r}{\overline{W}_f} = \frac{K_w}{(\tau_w s + 1)(\tau_F s + 1)} \quad (2)$$

where  $\tau_w$  and  $\tau_F$  are the tube wall and fire box time constants respectively.  $K_w$  is the gain given by:

$$K_w = \frac{\Delta Q_r}{\Delta W_f} \approx \frac{Q_r}{W_f} \quad (3)$$

$\tau_1$ ,  $\tau_2$ ,  $\tau_w$  and  $\tau_F$  are given in table 1 for a single-boiler system, and a multi-boiler system using 4 identical boilers. Typically the time constants satisfy the following relationship:

$$\tau_1 \gg \tau_w, \tau_F \gg \tau_2 \quad (4)$$

Since  $\tau_2$  has a negligible effect on the dynamics of the system, its effect will be ignored. This assumption gives:

$$\frac{\overline{P}_s}{\overline{W}_f} = \frac{K}{(\tau_1 s + 1)(\tau_w s + 1)(\tau_F s + 1)} \quad (5)$$

where :

$$K = K_w K_q \quad (6)$$

The gain  $K$  is a strong function of the number of boilers. If all boilers are identical  $K(n)$  is related to  $K(1)$  by:

$$K(n) = \frac{K(1)}{n} \quad (7)$$

This short cut model has been compared to a detailed dynamic simulation for various types of upsets (4). In each case, a very good prediction of the dynamic behavior was obtained from the short cut analysis. An example is given in figure 3. The header pressure responses are given for a step disturbance of -30% in demand for a single-boiler system, which may correspond to a turbine trip. The dotted and solid lines represent the short-cut and detailed non-linear simulations respectively.

### 3. Application to controller tuning

The open loop transfer function relating the control variable  $P_s$  and the manipulative variable  $W_f$  is :

$$G = \frac{K}{(\tau_1 s + 1)(\tau_w s + 1)(\tau_F s + 1)} \quad (8)$$

where :

$$\tau_1 \gg \tau_w, \tau_F$$

A limiting case occurs as the header volume gets very large.  $\tau_1$  is then considered as infinite. The transfer function becomes:

$$G' = \frac{K'}{s(\tau_w s + 1)(\tau_F s + 1)} \quad (9)$$

The ultimate parameters for these 2 systems are:

For G:

$$\omega_u = \sqrt{\frac{\tau_1 + \tau_w + \tau_F}{\tau_1 \tau_w \tau_F}} \quad (10)$$

$$K_{c_u} = \frac{(\tau_1 \tau_w + \tau_1 \tau_F + \tau_w \tau_F) \omega_u^2 - 1}{K} \quad (11)$$

For G':

$$\omega'_u = \frac{1}{\sqrt{\tau_F \tau_w}} \quad (12)$$

$$K'_{c_u} = \frac{\omega_u^2}{K'} (\tau_w + \tau_F) \quad (13)$$

As shown later, the application of Ziegler-Nichols tuning rules to these systems results in a very oscillatory behavior. Therefore Ziegler-Nichols rules are not applicable. A root locus analysis is used to determine the controller settings. New tuning rules are developed in this section. They apply to third order systems with widely separated time constants and to third order systems with a pure integrator. The block diagram is given in figure 4. With a PI controller the open loop transfer function is:

$$T_{open\ loop} = \frac{K K_c (s + \frac{1}{\tau_I})}{s(\tau_1 s + 1)(\tau_w s + 1)(\tau_F s + 1)} \quad (14)$$

$T_{open\ loop}$  has 4 poles  $-\frac{1}{\tau_1}$ ,  $-\frac{1}{\tau_w}$ ,  $-\frac{1}{\tau_F}$  and 0, and one zero:  $-\frac{1}{\tau_I}$ . The PI controller contributes a pole at the origin and a zero at  $-\frac{1}{\tau_I}$ . The closed loop stability is determined by the equation:

$$1 + T_{open\ loop} = 0 \quad (15)$$

which leads to:

$$(\tau_a s + 1)(\tau_b s + 1)(\tau_c^2 s^2 + 2\xi \tau_c s + 1) = 0 \quad (16)$$

where the time constant  $\tau_a$  and  $\tau_b$  are functions of  $K$ ,  $\tau_1$ ,  $\tau_w$ ,  $\tau_F$  and of the controller settings  $K_c$  and  $\tau_I$ .  $\tau_c$  is given by:

$$\tau_c = \sqrt{\frac{\tau_1 \tau_w \tau_F \tau_I}{K K_c \tau_a \tau_b}} \quad (17)$$

and  $\xi$  by:

$$\xi = \frac{1}{2} \tau_c \left( \frac{\tau_1 \tau_w + \tau_1 \tau_F + \tau_w \tau_F}{\tau_1 \tau_w \tau_F} - \frac{1}{\tau_a} - \frac{1}{\tau_b} \right) \quad (18)$$

2 roots are real:  $-\frac{1}{\tau_a}$  and  $-\frac{1}{\tau_b}$  and two roots are complex conjugate, given by:

$$s = \frac{-\xi \pm i \sqrt{1 - \xi^2}}{\tau_c} \quad (19)$$

The root locus diagram has 4 branches (loci):

- ◊ One from  $-\frac{1}{\tau_w}$  to  $-\infty$
- ◊ One from  $-\frac{1}{\tau_F}$  to  $-\frac{1}{\tau_I}$
- ◊ Two symmetric loci breaking from the real axis between 0 and  $-\frac{1}{\tau_1}$  to reach asymptotes whose angle is  $\pm \frac{\pi}{3}$ . The center of gravity is at

$$\frac{1}{3} \left( \frac{1}{\tau_1} + \frac{1}{\tau_w} + \frac{1}{\tau_F} - \frac{1}{\tau_I} \right).$$

For a system with a pure integrator instead of the lag ( $\tau_1 s + 1$ ), the diagram has the same shape. The origin is then a double pole from which two branches leave. The two constants  $\tau_a$  and  $\tau_b$  introduce two exponential terms in the pressure function. These two terms decrease very rapidly. The decay ratio will then depend essentially on the remaining second order term. As an approximation, we write a quarter decay ratio criterion equal to that for a second order system, i.e.:

$$\exp \left( -2 \pi \frac{\xi}{\sqrt{1 - \xi^2}} \right) = 0.25 \quad (20)$$

where:

$$\frac{\xi}{\sqrt{1 - \xi^2}} = \left| \frac{Re(s)}{Im(s)} \right| = \mu \quad (21)$$

On each diagram the straight line  $y = -\frac{1}{\mu} x$  is drawn. The gain for which the quarter decay ratio criterion is satisfied is given at the intersection of the straight line and the complex root locus. Figures 5, 6 and 7 show the root locus diagram for 3 different reset times. The quarter decay ratio criterion gives a good estimate of the controller gain  $K_c$  that should be used. The reset time  $\tau_I$  still has to be found. As  $\tau_I$  increases, the complex loci shift away from the imaginary axis. Figure 8 gives the real part of the complex root for which the quarter decay ratio is obtained as a function of the reset time. For small values of  $\tau_I$ , the distance to the right-half plane (RHP) increases very quickly as  $\tau_I$  increases and then levels off as  $\tau_I$  exceeds 70 seconds. This behavior can be interpreted as follows. In the first part of the curve, the system gains in stability

as the complex loci move away from the RHP. In the last part, there is no significant improvement in stability when  $\tau_I$  increases. The system just becomes more sluggish. A good choice for  $\tau_I$  appears to be in the transition part of the curve around 55 seconds for this particular example. The following settings, determined from this root locus analysis give a very satisfactory response for the single-boiler system considered here:

$$K_c = 0.055$$

$$\tau_I = 57 \text{ sec}$$

From these results as well as from various other tests, the tuning rules are derived as follows:

$$\tau_I = 1.35 P_u \quad (22)$$

$$K_c = 0.3 K_{c_u} \quad (23)$$

where

$$P_u = \frac{2 \pi}{\omega_u}$$

In these expressions  $P_u$  and  $K_{c_u}$  are functions of  $\tau_1$  and  $K_p$  which are representative of a particular steam system. They will vary as the design parameters and operating conditions vary. These tuning rules are compared with Ziegler-Nichols rules for the 3 following cases:

- ◊ Case 1:  $\tau_1 = 119 \text{ sec}$ ,  $\tau_w = 5.08 \text{ sec}$ ,  $\tau_F = 5 \text{ sec}$
- ◊ Case 2:  $\tau_1 = 119 \text{ sec}$ ,  $\tau_w = 5.08 \text{ sec}$ ,  $\tau_F = 10 \text{ sec}$
- ◊ Case 3:  $\tau_w = 5.08 \text{ sec}$ ,  $\tau_F = 10 \text{ sec}$  and pure integrator

The corresponding controller settings are summarized in table 2. The responses are shown in figures 9 to 11. A very oscillatory behavior is displayed when the controller

Table 2 : Controller tunings

Case	$P_u$	$K_{c_u}$	$\tau_I$	$K_c$	$\tau_I(\text{Z-N})$	$K_c(\text{Z-N})$
1	30.42	0.2378	41.06	0.07134	25.35	0.107
2	42.2	0.185	57.	0.055	35.17	0.08325
3	44.8	0.164	60.48	0.0491	37.3	0.738

settings recommended by Ziegler-Nichols are used. With the tuning rules developed above, the responses are very satisfactory for many different cases.

4. Comparison of closed loop responses with a detailed simulation

The closed loop responses obtained from the short cut model (dotted line) are compared with the detailed non-linear simulations (solid line) for various cases. In each case, the controller settings are given by the tuning rules. As shown in the 3 following cases, these settings give very satisfactory results when applied to the non-linear simulation. For each disturbance considered, the header pressure can be calculated from the short-cut model as an explicit function of time <sup>(4)</sup>.

4.1 **First case: Step in fuel flow**

The steam system consists of 4 boilers. A step of 0.3  $\frac{lbm}{sec}$  is applied to the fuel flow. For a step in  $W_f$  we have:

$$\bar{d} = \overline{W_f} \quad (24)$$

and:

$$G_d = G \quad (25)$$

$$T_{closed\ loop} = \frac{G}{1 + G G_c} \quad (26)$$

or

$$\frac{\overline{P_s}}{\overline{W_f}} = \frac{\left(\frac{\tau_I}{K_c}\right) s}{(\tau_a s + 1)(\tau_b s + 1)(\tau_c^2 s^2 + 2 \xi \tau_c s + 1)} \quad (27)$$

Since

$$\overline{W_f} = \frac{\alpha}{s} \quad (28)$$

where  $\alpha$  is the size of the step, one gets:

$$\overline{P_s} = \frac{\alpha \tau_I}{K_c} \frac{1}{\underbrace{(\tau_a s + 1)(\tau_b s + 1)(\tau_c^2 s^2 + 2 \xi \tau_c s + 1)}_{\Gamma(s)}} \quad (29)$$

$\Gamma(s)$  can be expanded as:

$$\Gamma(s) = \frac{K_1}{\tau_a + 1} + \frac{K_2}{\tau_b + 1} + \frac{K_3 s + K_4}{\tau_c^2 s^2 + 2 \xi \tau_c s + 1} \quad (30)$$

where:

$$\begin{aligned} K_1 &= [\Gamma(s) (\tau_a s + 1)]_{at\ s=-\frac{1}{\tau_a}} \\ K_2 &= [\Gamma(s) (\tau_b s + 1)]_{at\ s=-\frac{1}{\tau_b}} \\ K_3 &= -\frac{\tau_c^2}{\tau_a \tau_b} (K_1 \tau_b + K_2 \tau_a) \\ K_4 &= 1 - K_1 - K_2 \end{aligned}$$

$P_s$  is then given in the time domain by:

$$P_s(t) = P_s^o + \alpha \left(\frac{\tau_I}{K_c}\right) p(t) \quad (31)$$

with:

$$\begin{aligned} p(t) &= \frac{K_1}{\tau_a} e^{-\frac{t}{\tau_a}} + \frac{K_2}{\tau_b} e^{-\frac{t}{\tau_b}} + \frac{K_3}{\tau_c^2} \cos\left(\sqrt{1 - \xi^2} \frac{t}{\tau_c}\right) \\ &e^{-(\xi \frac{t}{\tau_c})} + \frac{K_4 - K_3 \frac{\xi}{\tau_c}}{\tau_c \sqrt{1 - \xi^2}} \sin\left(\sqrt{1 - \xi^2} \frac{t}{\tau_c}\right) e^{-(\xi \frac{t}{\tau_c})} \end{aligned} \quad (32)$$

Equations (22) and (23) provide the controller settings as:

$$\tau_I = 56.9\ sec : \text{Reset time}$$

$$K_c = 0.216 : \text{Controller gain}$$

The corresponding responses are shown in figure 12. A good prediction of the peak and of the pseudo-period of oscillation is obtained from the short-cut simulation.

4.2 **Second case: Boiler trip**

One of the 4 boilers of the steam system fails. The controller settings are the same as in Case 1. Figure 13 compares the corresponding response with the one obtained from the detailed simulation. The short cut method gives a good prediction of the peak at about 616 psi. A discrepancy exists for the pseudo-period.

4.3 **Third case: Turbine trip**

A single-boiler system is simulated. A step of -30% in steam demand is applied. Here again the pressure can be calculated as a direct function of time <sup>(4)</sup>. The controller settings are:

$$K_c = 0.055$$

$$\tau_I = 57\ sec$$

The responses are shown in figure 14 and compared with the detailed non-linear simulation. The control performance and prediction are again very satisfactory.

5. Summary and conclusions

The control of the HP header of a steam system is discussed. A short-cut model is used to design the controllers. Tuning rules are developed for pressure loops using a root locus analysis. These rules give very good control performance for a wide range of processes. The root locus analysis presented in this paper can be used in the same way for other specific systems. Short-cut methods appear as a good alternative to dynamic non-linear simulations. They

provide useful insight into the dynamics of the system and an accurate prediction of the dynamic behavior, as various upsets occur. The short-cut approach is a simple way to determine the control performance of many steam systems and the influence of key parameters on this control performance. The short-cut approach can be a helpful tool for designing and studying control systems of steam networks.

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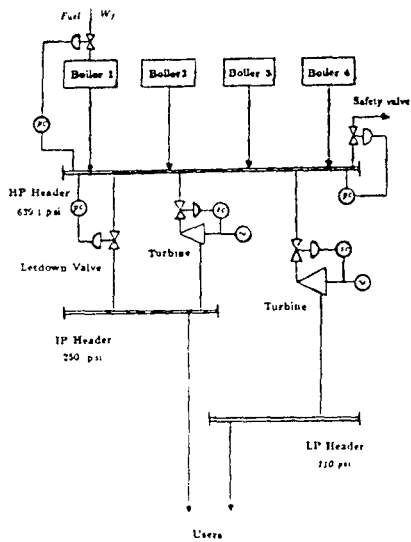


fig. 1

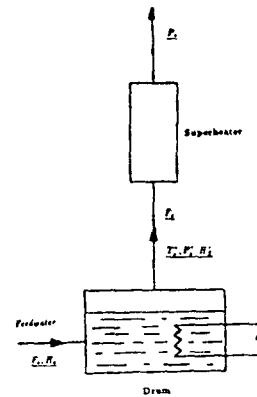


fig. 2

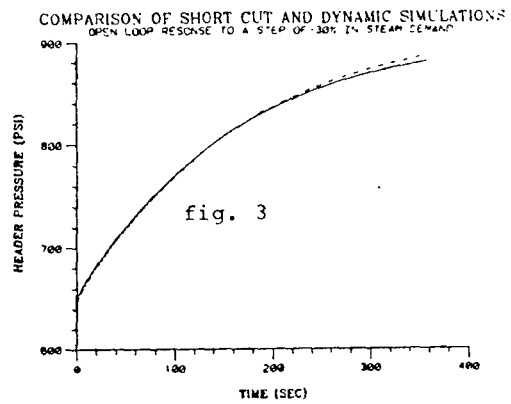


fig. 3

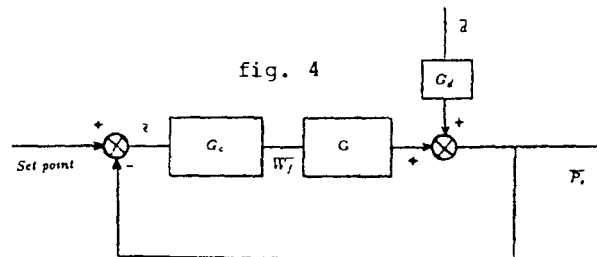


fig. 4

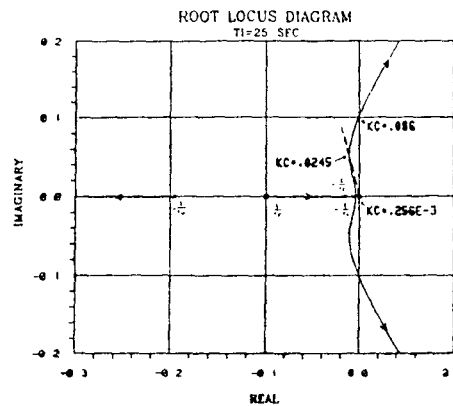
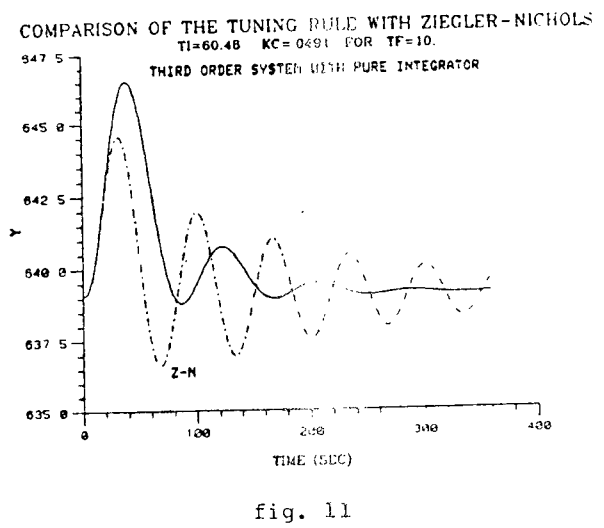
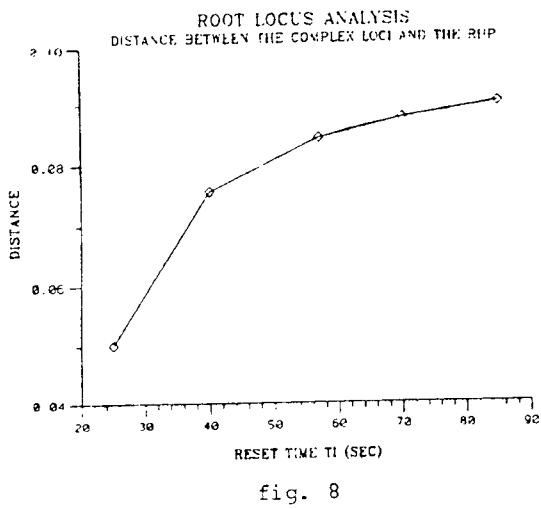
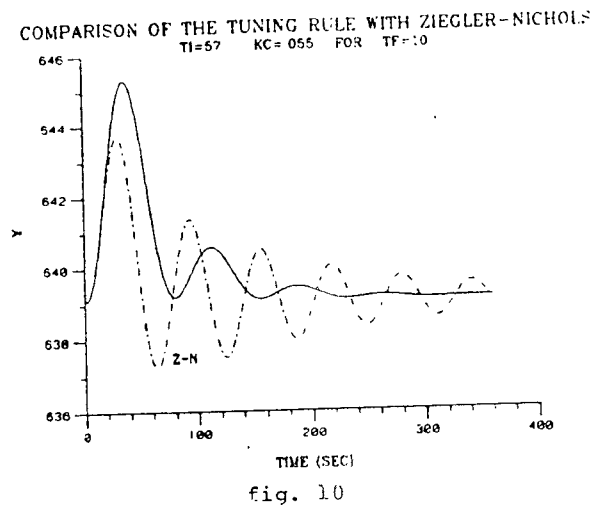
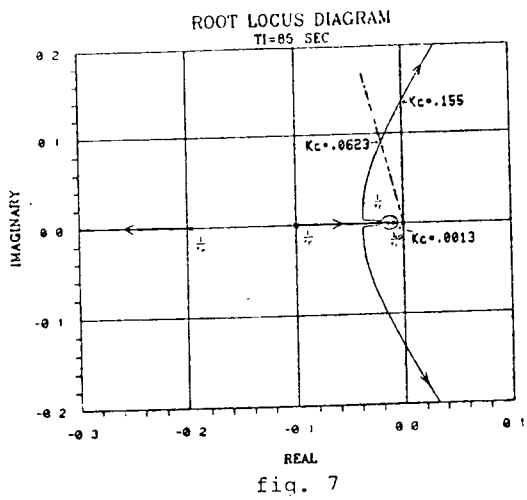
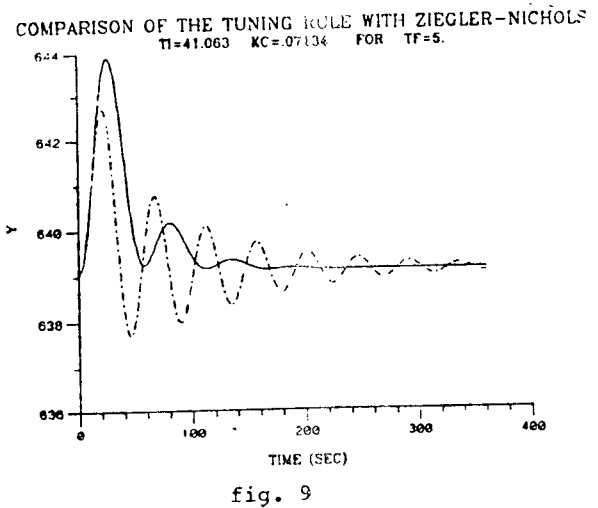
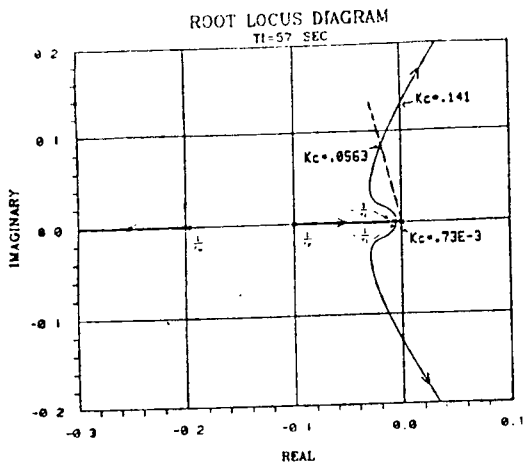


fig. 5





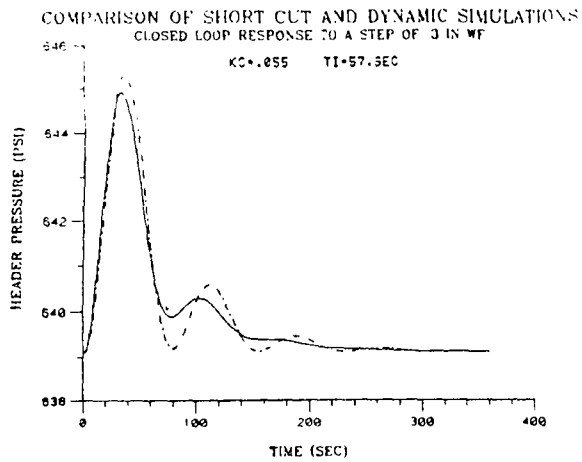


fig. 12

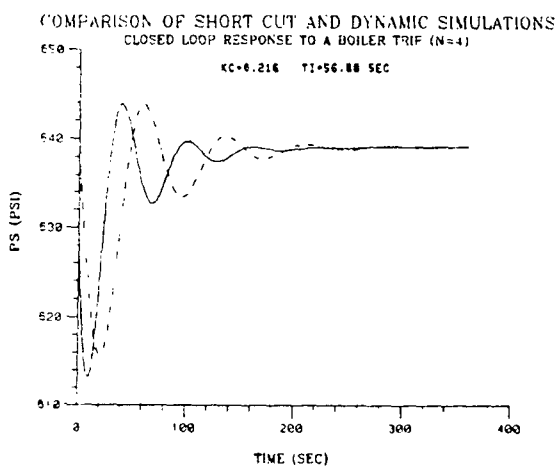


fig. 13

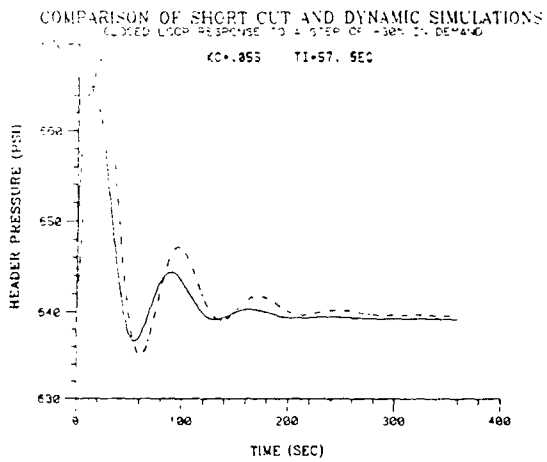


fig. 14