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by

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Abstract

In this paper, a two-level decomposition method for optimal design is described. Using this method, an optimal design problem is decomposed into several subproblems in the first-level and a coordinating problem in the second-level. In the first-level, the subproblems are analyzed using the global monotonicity concepts, then in the second-level the analyses of the subproblems are coordinated to obtain the optimal solution. Two engineering design examples, namely a gear reducer (formulated and solved in the literature) and a flywheel (formulated and solved here), illustrate applications of the developed method.

1. Introduction

Since the publication of Dantzig and Wolfe (1960), there have been numerous pieces of literature describing algorithmic development and/or application of decomposition in mathematical programming. The reader may consult several of the major books on the subject including those by Lasdon (1970), Wismer (1971), and Himmelblau (1973). More recently, many engineering problems have been solved using decomposition methods including those in mechanical (Davis, 1978; Siddall and Michael, 1980; Johnson and Benson, 1984a and 1984b), structural (Kirsch, 1981; Haftka, 1984), and aerospace design (Sobieski, 1982; Sobieski et al., 1984; Barthelemy and Riley, 1986). Most of the existing decomposition algorithms have a two-level structure. They break down a problem into several smaller subproblems. In the first-level, these smaller subproblems are solved independently. Then in the second-level, the solutions of the subproblems are combined in a prespecified manner to obtain the solution to the original problem.

There are several reasons why a decomposition method should be used to obtain the optimal solution of an engineering problem. First, many engineering problems are by their nature decomposable to several subproblems. For example, a punch-press is a mechanical system composed of several subsystems (components) including a flywheel, crankshaft, connecting rod, etc. Using decomposition, each component is optimized at the first-level and then the components' solutions are coordinated at the second-level to obtain the optimal solution to the original problem which is the punch-press. Second, interdisciplinary nature of decomposition methods force even more interactions across disciplines (Sobieski and Haftka, 1986). For example, optimal design of an automobile may be obtained by analysis of its engine, aerodynamics, structure, materials, etc., independently and then they may be integrated using a decomposition-based optimization. Finally, decomposition methods fit well to distributed or even parallel processing capabilities which are typical of a modern computing environment.

In Section 2, we describe briefly a two-level decomposition method for which the motivation and detailed description together with some test results is given in Azarm and Li (1986). In Section 3, the decomposition method is applied to two mechanical design examples, namely a gear reducer, and a flywheel for which the derivation of the model is described. Finally, we present our concluding remarks in Section 4.

2. A Monotonicity-Based Decomposition Method (MBDM)

In this section, we present a proposed two-level decomposition method. It is an extension of the feasible model coordination method (Kirsch,1981) coupled with the global monotonicity analysis (Wilde, 1975; Papalambros and Wilde, 1979 and 1980). In this method ,the global monotonicity analysis has been utilized in the first-level subproblem(s) to identify the candidate active constraints. This information is then sent to the second-level problem which in turn finds a new point for an improved objective function using any conventional (single-level) nonlinear programming method. The new point is then sent back to the first-level subproblem(s) and the iteration process continues until an optimal solution is obtained.

We consider here the following problem which may be decomposed into two levels with several subproblems:

Minimize $f(y;x) = f_0(y) + \sum_{i=1}^{I} f_i(y;x_i)$

Subject to:

$$
\begin{aligned}
& g_l(y) \leq 0 && l = 1, \ldots, L \\
& g_{i,j}(y;x_i) \leq 0 && i = 1, \ldots, I \\
& && j = 1, \ldots, J
\end{aligned}
\tag{1}
$$

where f and g represent the objective and constraint functions, y represents the vector of *interaction* (global) variables - fixed between subproblems, and x_i represents the *noninteraction* (local) variable in subproblem i. Index j corresponds to the number of constraints in subproblem i. The formulation for subproblem i with x_i as the only local variable (y is fixed) is:

Minimize $f_i(y;x_i)$

Subject to:

$$
g_{i,j}(y;x_i) \leq 0 \qquad j = 1, \ldots, J. \tag{2}
$$

Suppose that in subproblem $i(1 \leq i \leq I)$ the objective function f_i is monotonic, e.g. increasing with respect to (w.r.t.) variable x_i. In addition, suppose that (without the loss of generality) in subproblem i the first J' constraints ($J' < J$) have opposite monotonicity w.r.t. x_i, e.g. decreasing, therefore based on the first rule of monotonicity analysis (Wilde,1975) they are the candidate active constraints. Also, suppose that the subproblem $i(1 \leq i \leq I)$ can be rewritten in the following form:

Minimize $f_i(y; x_i)$

Subject to:

$$
\begin{aligned}
& x_i \geq g'_{i,1}(y) \\
& \quad \cdot \\
& \quad \cdot \\
& \quad \cdot \\
& x_i \geq g'_{i,J}(y) \\
& x_i \leq g'_{i,(J'+1)}(y) \\
& \quad \cdot \\
& \quad \cdot \\
& \quad \cdot \\
& x_i \leq g'_{i,J}(y)
\end{aligned} \tag{3}
$$

then the second-level problem is written in the following form:

Minimize $f(y; x) = f_0(y) + \sum_{i=1}^{I} f_i(y; x_i^*)$

Subject to:

$$
g_l(y) \leq 0 \qquad\qquad l = 1, \cdots, L \tag{4}
$$

where

$$
x_i^* = max\{g'_{i,j} : 1 \leq j \leq J'\}. \tag{5}
$$

Those constraints, $g'_{i,j}(y; x_i)$ $(j > J')$, which are violated by the x_i^* should be transfered from the first-level subproblem(s) to the second-level problem. Also note that in subproblem $i(1 \leq i \leq I)$, for the case where the objective function is decreasing and the first J' constraints are increasing w.r.t. x_i, then:

$$
x_i^* = min\{g'_{i,j} : 1 \leq j \leq J'\}. \tag{6}
$$

The two-level iterative procedure for this method may be summarized as follows:

(1) Select the starting value for the global variables y,

(2) Use monotonicity analysis to find x_i^* (y is fixed), $1 \leq i \leq I$, for the subproblems,

(3) Find a new y such that $f(y; x^*)$ is decreased,

(4) Return to step (2) until the minimum for $f(y; x)$ is obtained.

An extension of the method for the more general case of several subproblems with several local variables is given by Azarm and Li (1986).

3. Examples

The methodology described in the previous section has been combined with the VMCON progrom (Crane et al., 1980) which is based on a sequential quadratic programming technique of the type suggested by Powell (1978). Two engineering design examples , namely a gear reducer and a flywheel, illustrate the applications of MBDM in this section. For each example, after decomposition the first-level subproblems are solved by the global monotonicity analysis and the second-level problem by the VMCON program.

Example 3.1: Gear Reducer

This example was modelled by Golinski (1970) and solved by several optimization schemes including those by Datseris (1982), Azarm (1984), and Li and Papalambros (1985). The nonlinear programming statement for this example is:

Minimize $f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$

Subject to:

(7)

g_1: $27x_1^{-1}x_2^{-2}x_3^{-3} \leq 1$

g_2: $397.5x_1^{-1}x_2^{-2}x_3^{-2} \leq 1$

g_3: $1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \leq 1$

g_4: $1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \leq 1$

g_5: $A_1/B_1 \leq 1100$

$$A_1 = [(\frac{745x_4}{x_2x_3})^2 + (16.9)10^6]^{0.5}$$

$$B_1 = 0.1x_6^3$$

g_6: $A_2/B_2 \leq 850$

$$A_2 = [(\frac{745x_5}{x_2x_3})^2 + (157.5)10^6]^{0.5}$$

$$B_2 = 0.1x_7^3$$

g_7:	$x_2x_3 \leq 40$	
g_8:	$5 \leq x_1/x_2 \leq 12$	$:g_9$
g_{10}:	$2.6 \leq x_1 \leq 3.6$	$:g_{11}$
g_{12}:	$0.7 \leq x_2 \leq 0.8$	$:g_{13}$
g_{14}:	$17 \leq x_3 \leq 28$	$:g_{15}$
g_{16}:	$7.3 \leq x_4 \leq 8.3$	$:g_{17}$
g_{18}:	$7.3 \leq x_5 \leq 8.3$	$:g_{19}$
g_{20}:	$2.9 \leq x_6 \leq 3.9$	$:g_{21}$
g_{22}:	$5.0 \leq x_7 \leq 5.5$	$:g_{23}$
g_{24}:	$(1.5x_6 + 1.9)x_4^{-1} \leq 1$	
g_{25}:	$(1.1x_7 + 1.9)x_5^{-1} \leq 1.$	

Here we select x_2, x_3, x_4 and x_5 as the global variables and decompose the problem into three subproblems. In the first-level, x_1, x_6, and x_7 are selected as the local variables for subproblem 1, 2, and 3, respectively:

Subproblem 1:
Minimize $f_1 = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934)$
Subject to:

(8)

$g_{1,1}$:	$x_1 \geq 27/(x_2^2x_3) = g'_{1,1}$
$g_{1,2}$:	$x_1 \geq 397.5/(x_2^2x_3^2) = g'_{1,2}$
$g_{1,3}$:	$x_1 \geq 5x_2 = g'_{1,3}$
$g_{1,4}$:	$x_1 \geq 26 = g'_{1,4}$
$g_{1,5}$:	$x_1 \leq 12x_2 = g'_{1,5}$
$g_{1,6}$:	$x_1 \leq 3.6 = g'_{1,6}.$

In this subproblem, constraints $g_{1,1}$, $g_{1,2}$, $g_{1,3}$, and $g_{1,4}$ are the candidate active constraints (f can be shown to be increasing w.r.t. x_1), Therefore:

$$x_1^* = max\{g'_{1,j} : j = 1,2,3,4\} \tag{9}$$

Subproblem 2:

Minimize $f_2 = -1.508x_1x_6^2 + 7.477x_6^3 + 0.7854x_4x_6^2$

Subject to:

(10)

$g_{2,1}$: $\quad x_6 \geq (A_1/100)^{1/3} = g'_{2,1}$

$\quad A_1 = [(745x_4x_2^{-1}x_3^{-1})^2 + 16.9 * 10^6]^{0.5}$

$g_{2,2}$: $\quad x_6 \geq (1.93x_2^{-1}x_3^{-1}x_4^{-3})^{0.25} = g'_{2,2}$

$g_{2,3}$: $\quad x_6 \geq 2.9 = g'_{2,3}$

$g_{2,4}$: $\quad x_6 \leq 3.9 = g'_{2,4}$

$g_{2,5}$: $\quad x_6 \leq (x_4 - 1.9)/1.5 = g'_{2,5}$.

Likewise:

$$x_6^* = max\{g'_{2,j} : j = 1,2,3\} \tag{11}$$

Subproblem 3:

Minimize $f_3 = -1.508x_1x_7^2 + 7.477x_7^3 + 0.7854x_5x_7^2$

Subject to:

(12)

$g_{3,1}$: $\quad x_7 \geq (A_2/85)^{1/3} = g'_{3,1}$

$\quad A_2 = [(745x_5x_2^{-1}x_3^{-1})^2 + 157.5 * 10^6]^{0.5}$

$g_{3,2}$: $\quad x_7 \geq (1.93x_2^{-1}x_3^{-1}x_5^{-3})^{0.25} = g'_{3,2}$

$g_{3,3}$: $\quad x_7 \geq 5.0 = g'_{3,3}$

$g_{3,4}$: $\quad x_7 \leq 5.5 = g'_{34}$

$g_{3,5}$: $\quad x_7 \leq (x_5 - 1.9)/1.1 = g'_{3,5}$.

Likewise:

$$x_7^* = max\{g_{3,j} : j = 1, 2, 3\}. \tag{13}$$

Therefore, the second-level problem may be written in the following form:

Minimize $f(x) = 0.7854x_1^*x_2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1^*(x_6^{*^2} + x_7^{*^2}) + 7.477(x_6^{*^3} + x_7^{*^3}) + 0.7854(x_4x_6^{*^2} + x_5x_7^{*^2})$

Subject to:

$$\begin{aligned} &x_2x_3 \leq 40 \\ &0.7 \leq x_2 \leq 0.8 \\ &17 \leq x_3 \leq 28 \\ &7.3 \leq x_4 \leq 8.3 \\ &7.3 \leq x_5 \leq 8.3. \end{aligned} \tag{14}$$

The two-level structure for this example is shown in Figure 1. The solution of this example together with the performance results of the VMCON program is given in Table 1, where:

NF/NG:	Number of objective or constraint functions evaluations
NDF/NDG:	Number of gradient of objective or constraint functions evaluations
CPU:	Central processing time in second
MBD–VM:	Combined MBDM and the VMCON program.

Example 3.2: Flywheel

Flywheel applications date back to several thousand years. A punch-press is one of those mechanical systems in which a flywheel is used, not only for power conservation but also as a mechanical filtering element in a circuit through which power is flowing. Indeed energy is stored in a flywheel by speeding it up and is delivered by it by slowing it down. It also acts as a smoothing or equalizing element.

Therefore, a flywheel plays an important role in the performance of the punch-press. The design of a flywheel and particularly its weight is often overestimated because of the lack of optimization in the design process. A step-by-step model for the design of a minimum weight cast flywheel, Figure 2(a), of a punch-press for punching holes in a plate is presented.

Objective Function:

The objective function is to minimize the weight of the flywheel (W_F), which is the sum of the weights of rim (W_R), hub (W_H), and spokes (W_S):

$$f \;=\; W_F \;=\; W_R \;+\; W_H \;+\; sW_S \tag{15}$$

Therefore, the objective function is:

$$f = 0.25\pi\gamma_F[8r_R h_R b_R + (D_{OH}^2 - D_{IH}^2)b_H + 0.5sk_S d_s^2(2r_R - h_R - D_{OH})] \tag{16}$$

Constraints:

In this section, constraints describing various physical as well as practical limitations in the design of flywheel are presented:

Shaft Stress:

This constraint imposes a lower bound on the inside diameter of the flywheel (D_{IH}). This is found by calculating the crankshaft stresses at the point where the flywheel is overhung, point C in Figure 2(b). At point C, only torsional stress due to punching is taken into account. Presumably, this stress is much larger than any other stress present at point C, with proper overload factor. The torque producing this stress is given by:

$$T \;=\; F_1 R \tag{17}$$

where

$$F_1 = F_{\max} sin(\theta)\{1 + cos(\theta)/[(l/R)^2 - sin^2(\theta)]^{1/2}\} \tag{18}$$

$$F_{\max} = \pi d t_P S_{up} n_{OL}. \tag{19}$$

Notice that point C is in fact under partial (half-cycle) torsional stress due to punching. Therefore, the mean (τ_m) and alternating (τ_a) shear stresses are considered as:

$$\tau_m \;=\; \tau_a \;=\; 1/2\tau_{\max} \tag{20}$$

where

$$\tau_{\max} = 16T_m/(\pi D_{IH}^3) \tag{21}$$

$$T_m = T/2 \tag{22}$$

$$\tau_m = \tau_a. \tag{23}$$

Using fatigue as the governing criterion (Juvinall, 1967), this constraint is:

$$(\tau_a/S_{ns} + \tau_m/S_{us}) \leq 1/S_f \tag{24}$$

which after simplification may be written in the following form:

$$D_{IH} \geq K_1 \tag{25}$$

where

$$K_1 = [4TS_f(1/S_{us} + 1/S_{ns})/\pi]^{1/3}. \tag{26}$$

Key Stress:

A key is used to secure the flywheel on the crankshaft and transfer torque between them. In determining the strength of the key the assumption is made that the forces are uniformly distributed throughout the key length. It is customary to base the strength of the key on failure by shearing (Shigley, 1972):

$$\tau_{kmax} = F/A \tag{27}$$

where

$$F = 2T/D_{IH} \quad , \quad A = b_H t_k \tag{28}$$

Again, as in the shaft stress constraint:

$$\tau_{ak} = \tau_{mk} = 1/2\tau_{kmax} \tag{29}$$

thus

$$(\tau_{ak}/S_{ns} + \tau_{mk}/S_{us}) \leq 1/S_{fkey} \tag{30}$$

which may be simplified into the following form:

$$b_H D_{IH} \geq K_2 \tag{30}$$

where

$$K_2 = TS_{fkey}(1/S_{us} + 1/S_{ns})/t_k. \tag{31}$$

Energy of Flywheel:

In order to find the energy of the flywheel, we should first find the total energy which is used per stroke. The force-displacement curve is illustrated by Figure 3(a), where the maximum force occurs around three-eighths of the depth of the plate (Hollowenko, 1955). It is customary to approximate the area under force-displacement curve by:

$$W = 1/2F_{\max}t_p. \tag{33}$$

The size of the motor required can be determined from the condition that the energy taken from the flywheel must be returned to it by the motor in a cycle. From Figure 3(b):

$$W = (\text{area})_{abcda} = (\text{area})_{efghde} \tag{34}$$

therefore, the lower bound on the flywheel energy (E_f) is (friction is neglected):

$$E_f = (\text{area})_{fghf} \simeq W - P_M t_1 \tag{35}$$

$$P_M = WN. \tag{36}$$

The kinetic energy (K.E.) of a body rotating about a fixed center is:

$$K.E. = 1/2I\omega^2 \tag{37}$$

since there is velocity change due to punching, the expression for change of energy is:

$$\Delta(K.E.) = 1/2[I(\omega_{\max}^2 - \omega_{\min}^2)] \tag{38}$$

or

$$\Delta(K.E.) = I(V_{\max}^2 - V_{\min}^2)/(2r_R^2) \tag{39}$$

which may be rewritten as:

$$\Delta(K.E.) = IV_{ave}^2 C_f/r_R^2 \tag{40}$$

where

$$C_f = (V_{\max} - V_{\min})/V_{ave} \quad , \quad V_{ave} = (V_{\max} + V_{\min})/2 = V \tag{41}$$

and

$$I = W_F r_R^2 / g_c \quad , \quad V = r_R \omega. \tag{42}$$

Thus the energy constraint is:

$$\Delta(K.E.) \geq E_f \tag{43}$$

which may be written in the following form:

$$k_3 r_R^{-3} h_R^{-1} b_R^{-1} \leq 1 \tag{44}$$

where

$$k_3 = E_f / \{[9\pi\gamma_F C_f/(4g_c)](2\pi N/60)^2\} \tag{45}$$

where the K.E. of the hub and spokes are assumed to be one-eighth of the one by rim concentraited at the mean radius of the rim (Bradford, 1940).

Rim Stresses:

The rim stresses have been developed due to combined effect of hoop tension and bending stresses. The factors taken into account include: (1) The number of spokes, which if increased, will decrease the rim span between the spokes and hence decrease the bending moment. (2) The relative thickness of the spokes, which if too thin, could offer little constraint to the expanding of the rim to its natural diameter under centrifugal force, and hence would cause little bending stress. Conversely, if they were too thick, they would restrain the rim, thereby setting up heavy bending stresses at the junctions of the rim and spokes. (3) The relative thickness of the rim to the diameter, which if large, would cause rim resistance to bending to be large and the bending stress small. Conversely, if small, the rim is subject to high bending stresses. The maximum combined stress at the rim of the flywheel having six spokes is given by (Machinery's Handbook, 1971):

$$S_R = V_1^2[1 + ((.56B - 1.81)/(3Q + 3.14))Q]/10 \tag{46}$$

where

$$Q = \pi k_s d_s^2/(4h_R b_R) \quad , \quad B = D_{OR}/h_R \tag{47}$$

$$D_{OR} = 2r_R + h_R \quad , \quad V_1 = D_{OR}(2\pi N/60)/24. \tag{48}$$

Therefore

$$S_R \leq S_{uc}/S_{fR}. \tag{49}$$

Spokes Stresses:

In the design of spokes it is assumed that cantilever action is predominant. The bending moment due to punching at the hub of the arms is taken to be:

$$M = T(D_{OR} - D_{OH})/(sD_{OR}) \tag{50}$$

for which the bending stress is:

$$S_1 = M/Z \tag{51}$$

where

$$Z = \pi k_s d_s^3/32. \tag{52}$$

Also it may be necessary to check the spokes for tension due to rim expansion. It may be safe to assume that each spokes is in tension due to one-half of the centrigufal force of that portion of the rim which it supports:

$$S_2 = (4W_R V^2)/(\pi s g_c k_s r_R d_s^2) \tag{53}$$

where

$$V = r_R \omega \tag{54}$$

therefore:

$$S_1 + S_2 \leq S_{uc}/S_{fS}. \tag{55}$$

Stability:

The hub length of the flywheel is usually taken greater than the shaft diameter for stability reasons(Shigley 1972).

$$D_{IH} \leq b_H. \tag{56}$$

Side constraints:

These are lower, $lb(.)$, and upper, $ub(.)$, bounds on the variables based on space and/or experience:

$$\begin{aligned}
lb(r_R) \leq r_R &\leq ub(r_R) \\
lb(h_R) \leq h_R &\leq ub(h_R) \\
lb(b_R) \leq b_R &\leq ub(b_R) \\
lb(D_{OH}) \leq D_{OH} &\leq ub(D_{OH}) \\
lb(D_{IH}) \leq D_{IH} &\leq ub(D_{IH}) \\
lb(b_H) \leq b_H &\leq ub(b_H) \\
lb(d_s) \leq d_s &\leq ub(d_s).
\end{aligned} \tag{57}$$

The complete model for this example is:

$$\text{Minimize } f = 0.25\pi\gamma_F[8r_R h_R b_R + (D_{OH}^2 - D_{IH}^2)b_H + 0.5sk_S d_S^2(2r_R - h_R - D_{OH})]$$

Subject to:

(57)

g_1:	$K_1/D_{IH} \leq 1$	
g_2:	$K_2/(D_{IH}b_H) \leq 1$	
g_3:	$K_3/(r_R^3 h_R b_R) \leq 1$	
g_4:	$S_R \leq S_{uc}/S_{fR}$	
where,	$S_R = V_1^2[1 + ((.56B - 1.81)/(3Q + 3.14))Q]/10$	
and,	$Q = \pi k_s d_s^2/(4h_R b_R)$	
	$B = D_{OR}/h_R$	
	$D_{OR} = 2r_R + h_R, \qquad V_1 = D_{OR}(2\pi N/60)/24$	
g_5:	$S_1 + S_2 \leq S_{uc}/S_{fS}$	
where,	$S_1 = 32(2r_R - h_R - D_{OH})/[s(2r_R + h_R)\pi k_s d_s^3]$	
	$S_2 = 8\pi\gamma_F h_R b_R(r_R\omega)^2/(\pi s g_c k_s d_s^2)$	
g_6:	$D_{IH}/b_H \leq 1$	
g_7:	$18 \leq r_R \leq 34$	$:g_8$
g_9:	$6 \leq h_R \leq 10$	$:g_{10}$
g_{11}:	$5 \leq b_R \leq 8$	$:g_{12}$
g_{13}:	$12 \leq D_{OH} \leq 16.5$	$:g_{14}$
g_{15}:	$6.5 \leq D_{IH} \leq 10$	$:g_{16}$
g_{17}:	$5.5 \leq b_H \leq 8.5$	$:g_{18}$
g_{19}:	$5 \leq d_s \leq 8$	$:g_{20}$

where r_R, h_R, b_R, D_{OH}, D_{IH}, b_H, and d_s are the dimensions of the flywheel, Figure 2(a), considered here as the design variables. All other parameters are given in the nomenclature.

Here, we choose h_R, b_R, D_{OH}, b_H, and d_s as the global variables and decompose the problem into two subproblems. D_{IH} and r_R are selected to be the local variables for the subproblems 1 and 2, respectively. Thus in the first-level we have:

Subproblem 1:

Minimize $f_1 = 0.25\pi\gamma_F(D_{OH}^2 - D_{IH}^2)b_H$

Subject to:

$$
\begin{aligned}
g_{1,1}&: \quad D_{IH} \le (b_H = g'_{1,1}) \\
g_{1,2}&: \quad D_{IH} \le (10 = g'_{1,2}) \\
g_{1,3}&: \quad D_{IH} \ge (6.5 = g'_{1,3}) \\
g_{1,4}&: \quad D_{IH} \ge (K_2 = g'_{1,4}) \\
g_{1,5}&: \quad D_{IH} \ge (K_3/b_H = g'_{1,5})
\end{aligned}
\tag{58}
$$

where $g_{1,1}, g_{1,2}$ are possible active constraints in this subproblem. Therefore:

$$D^*_{IH} = min\{g'_{1,1}, g'_{1,2}\}. \tag{59}$$

Subproblem 2:

Minimize $f_2 = 0.25\pi\gamma_F[8r_Rh_Rb_R + 0.5Sk_sd_s^2(2r_R - h_R - D_{OH})]$

subject to:

$$
\begin{aligned}
g_{2,1}&: \quad r_R \ge ([K_4/(h_Rb_R)]^{1/3} = g'_{2,1}) \\
g_{2,2}&: \quad r_R \ge (18 = g'_{2,2}) \\
g_{2,3}&: \quad r_R \le 34 \\
g_{2,4}&: \quad S_R \le S_{uc}/S_{fR} \\
g_{2,5}&: \quad S_1 + S_2 \le S_{uc}/S_{fS}
\end{aligned}
\tag{60}
$$

constraints $g_{2,1}$ and $g_{2,2}$ are the candidate active constraints. Thus:

$$r^*_R = max\{g'_{2,1}, g'_{2,2}\}. \tag{61}$$

The second-level problem is written in the following form:

$$\text{Minimize } f = 0.25\pi\gamma_F[8r_R^* h_R b_R + (D_{OH}^2 - {D_{IH}^*}^2)B_H + 0.5sk_S d_S^2(2r_R^* - h_R - D_{OH})]$$

Subject to:

$$\begin{aligned} &b \le h_R \le 10 \\ &5 \le b_R \le 8 \\ &12 \le D_{OH} \le 16.5 \\ &5.5 \le b_H \le 8.5 \\ &5 \le d_s \le 8. \end{aligned} \tag{62}$$

The two-level structure for this example is shown in Figure 4. The solution of this example together with the performance results of the VMCON program with and without combining with MBDM is given in Table 2.

4.Concluding Remarks

The merit of the decomposition method (MBDM) described here lies on the utilization of global monotonicity within a two-level framework. In order to apply the method the problem should be seperable, as is demonstrated in equation (1). In addition, the objective and constraint functions should be globally monotonic (within the feasible domain) w.r.t. the local design varibles. The proposed method is demonstrated with two engineering design examples, namely a gear reducer and a flywheel. In both examples, the global monotonicity analyses of the subproblems have been combined with a conventional (single-level) optimization algorithm. The results show the substantionally improved performance of the two-level approach over the single-level optimization algorithm. However this improved performance is at the expense of some initial analytical effort on the part of the design optimizer for decomposition and global monotonicity analysis of the subproblems. Finally, the work presented here set the groundwork for the extension of monotonicity analysis to optimal design of large-scale problems where local rather than global monotonicity analysis will be used.

5. Acknowledgement

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- 05113 and partially through NSF Grant CDR - 85 - 00108. This support is greatly appreciated.

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Nomenclature for The Flywheel Design:

Design Variables:

r_R	=mean radius of rim, in
h_R	=thickness of rim, in
b_R	=width of rim, in
D_{OH}	=outside diameter of hub, in
D_{IH}	=inside diameter of hub, in
b_H	=width of hub, in
d_s	=length of major axis of spoke's cross section

Parameters:

C_f	=coefficient of flywheel's speed fluctuation, .1
d	=diameter of the holes to be punched, .875 in
E_f	=lower bound on the energy of the flywheel, in-lb
$F_{\max}$	=maximum punching force, 1b
F_1	=tangential component of force on the crankthrow, 1b
g_c	=gravity acceleration, 386 in/sec^2
I	=mass moment of inertia, 1bm-in^2
k_s	=ratio of the minor to major axis of the spoke's cross section(ellipse),.5
l	=connecting rod length, 12 in
n_{OL}	=overload factor for punching, 2
N	=frequency of rotation of flywheel, 120 rpm
R	=crank radius, 4 in
s	=number of spokes, 6
S_f	=safety factor for the shaft, 2
S_{fkey}	=safety factor for the key, 2
S_{fR}	=safety factor for the rim, 11
S_{fS}	=safety factor for the spokes, 6
S_{uc}	=strength of the flywheel, 40,000 psi
S_{up}	=ultimate strength of the plate, 20,000 psi
S_{us}	=shear strength of the shaft, 160,000 psi
S_{ys}	=shear yield strength of the shaft, 100,000 psi
S_{ns}	=shear fatigue strength of the shaft, 30,000 psi
t_p	=plate thickness, .75 in

t_1	=punching time, .05 sec
t_k	=width of key, 1 in
T	=flywheel torque, in-1lb
V_1	=rim's outside speed, ft/sec
W	=total energy of punching, in-lb
P_M	=motor power, (in-lb)/sec
γ_F	=density of flywheel, .26 lb/in^3
θ	=punching angle, 50^o
ω	=rotational speed, rad/sec

List of Tables

List of Figures

	VMCON	MBD-VM
x_1^*	3.5	3.5
x_2^*	0.7	0.7
x_3^*	17	17
x_4^*	7.3	7.3
x_5^*	7.7153199	7.7153199
x_6^*	3.3502147	3.3502147
x_7^*	5.2866545	5.2866545
f^*	2994.4711	2994.4711
CPU TIME	4.24	2.18
NF/NG	65	16
NDF/NDG	8	3

Table 1

	VMCON	MBD–VM
r_R^*	23.108115	23.107961
h_R^*	6	6
b_R^*	5	5
D_{OH}^*	12	12
D_{IH}^*	8.5	8.5
b_H^*	8.5	8.5
d_s^*	5	5
f^*	1473.1101	1473.1101
CPU time	4.35 sec	2.44 sec
NF/NG	57	13
NDF/NDG	7	2

Table 2

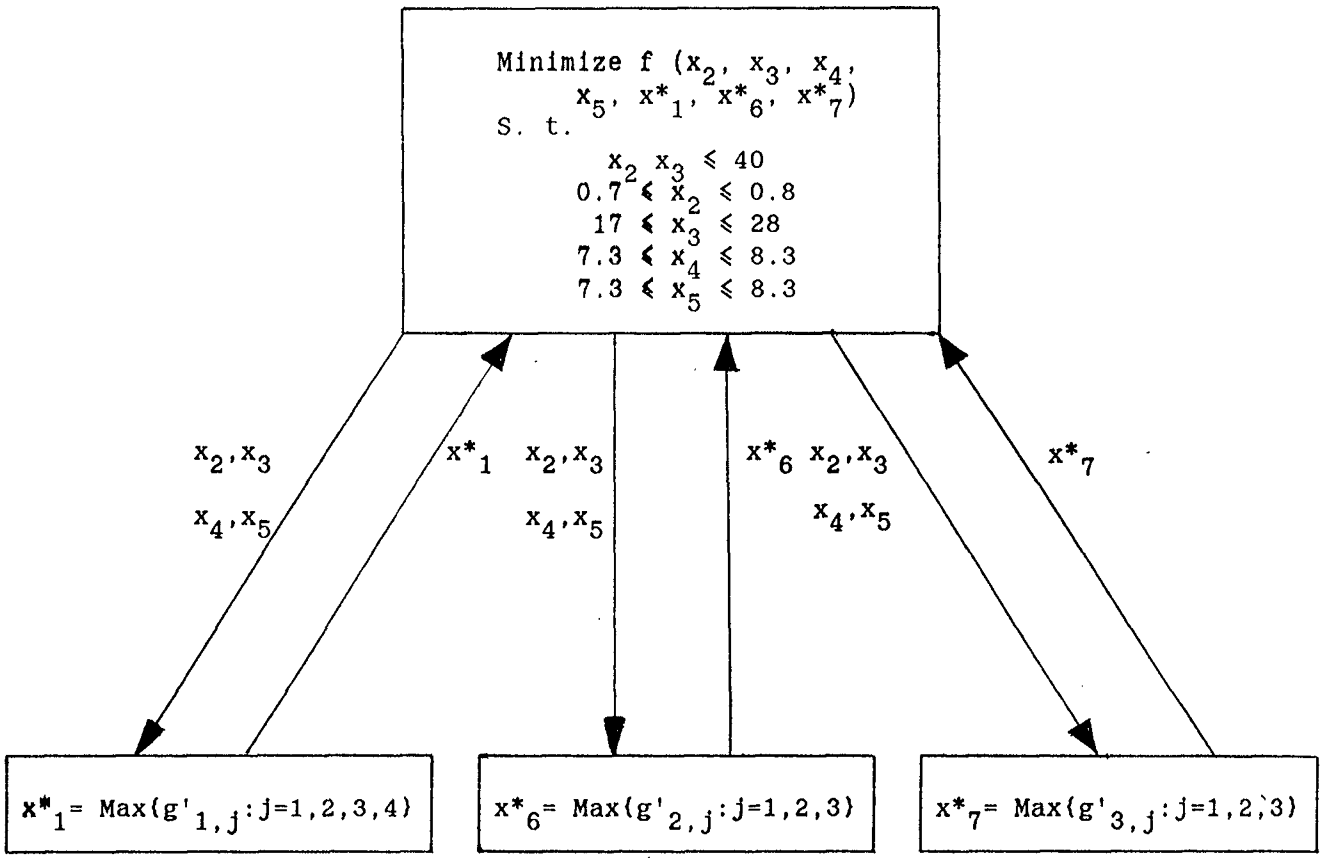
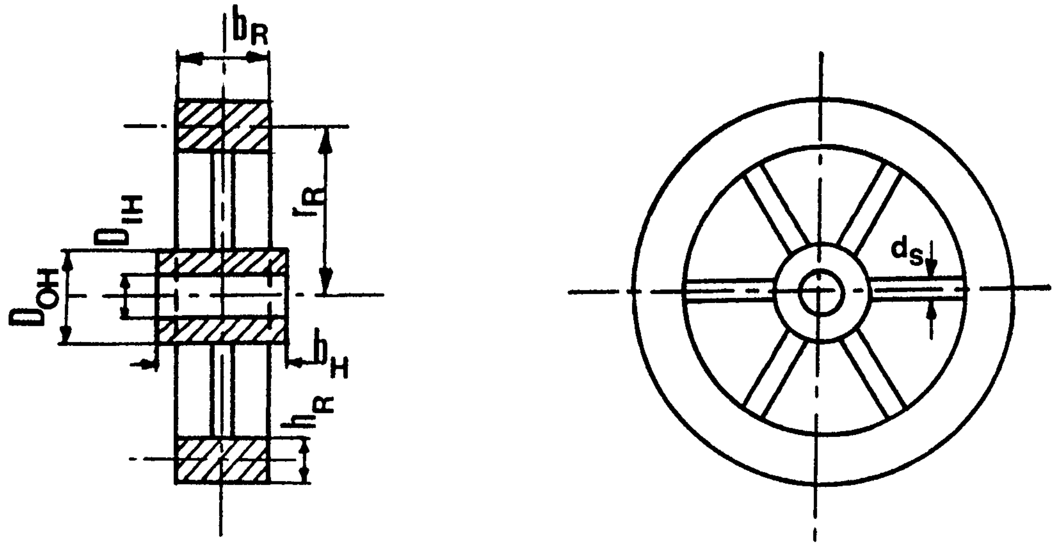
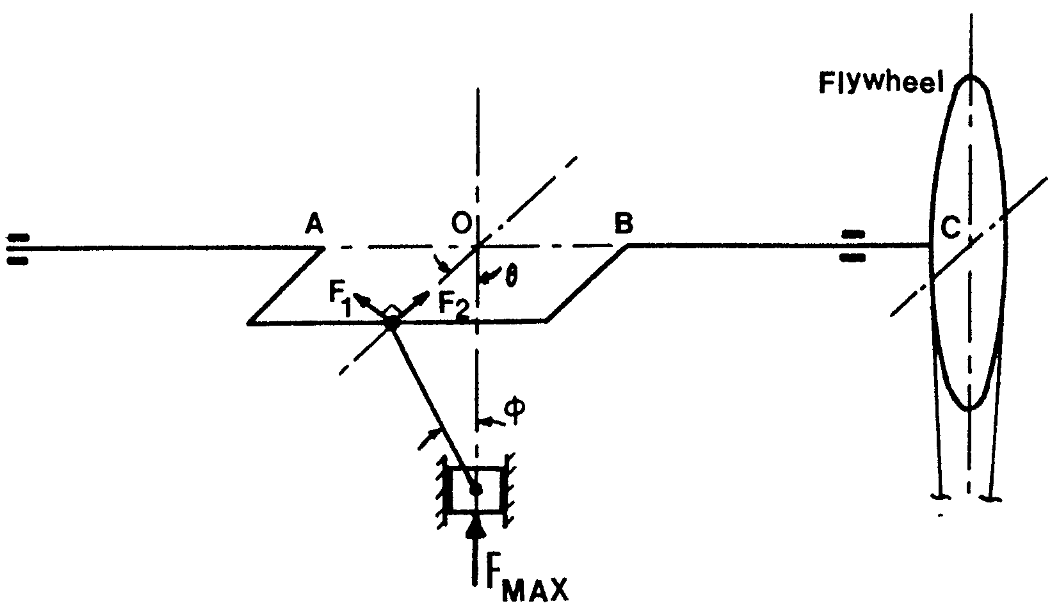

Minimize f (x2, x3, x4, x5, x*1, x*6, x*7)
S. t.
x2 x3 ⩽ 40
0.7 ⩽ x2 ⩽ 0.8
17 ⩽ x3 ⩽ 28
7.3 ⩽ x4 ⩽ 8.3
7.3 ⩽ x5 ⩽ 8.3
x2,x3
x4,x5
x*1
x2,x3
x4,x5
x*6
x2,x3
x4,x5
x*7
x*1= Max(g'1,j:j=1,2,3,4)
x*6= Max(g'2,j:j=1,2,3)
x*7= Max(g'3,j:j=1,2,3)

Figure 1

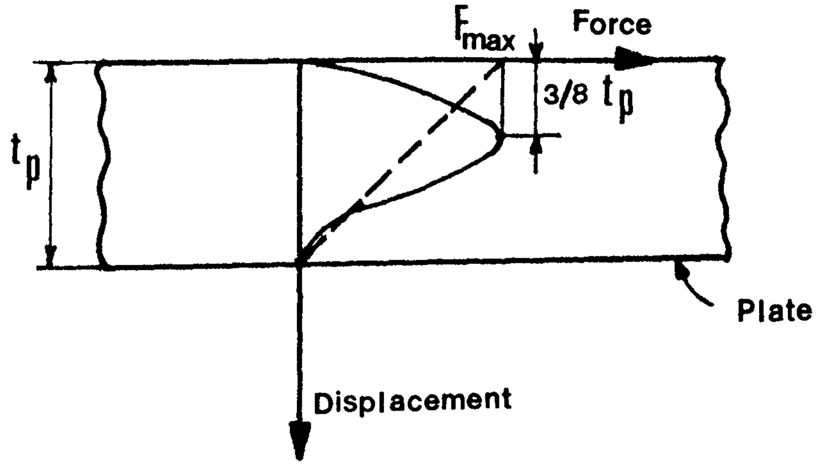


(a)



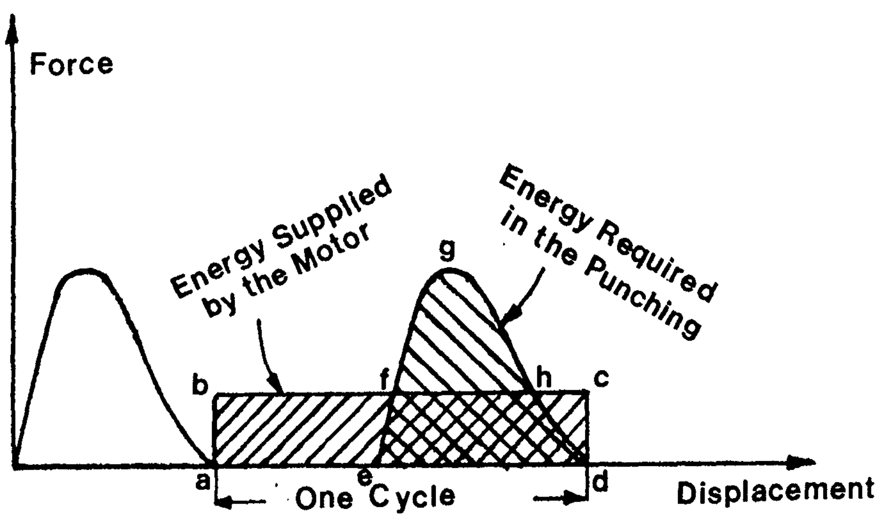
(b)

Figure 2



F_{max}
Force
3/8 t_p
t_p
Plate
Displacement

(a)



Force
Energy Supplied by the Motor
Energy Required in the Punching
g
b
f
h
c
a
e
d
One Cycle
Displacement

(b)

Figure 3

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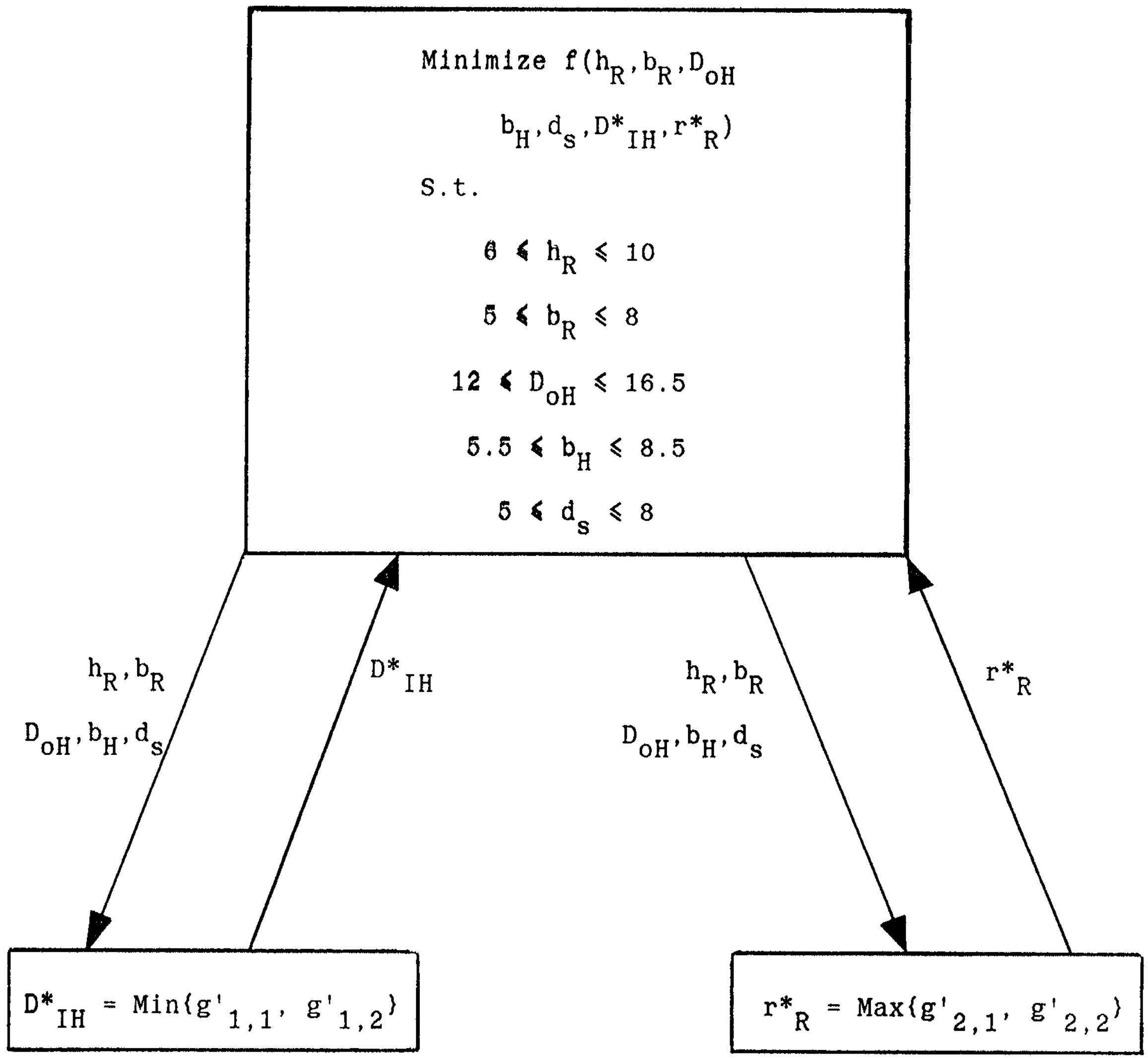
Minimize $f(h_R, b_R, D_{oH}, b_H, d_s, D^*_{IH}, r^*_R)$
s.t.
$6 \leq h_R \leq 10$
$5 \leq b_R \leq 8$
$12 \leq D_{oH} \leq 16.5$
$5.5 \leq b_H \leq 8.5$
$5 \leq d_s \leq 8$
h_R, b_R
D_{oH}, b_H, d_s
D^*_{IH}
h_R, b_R
D_{oH}, b_H, d_s
r^*_R
$D^*_{IH} = \mathrm{Min}\{g'_{1,1}, g'_{1,2}\}$
$r^*_R = \mathrm{Max}\{g'_{2,1}, g'_{2,2}\}$

Figure 4