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Observers for Optimal Anticipatory Control of Ram Velocity in Injection Molding

by

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ABSTRACT

The application of optimal anticipatory control assumes that the entire state is available for feedback. However, in the case of injection molding this assumption is violated due to economic considerations. For that purpose, this paper discusses the effectiveness of optimal anticipatory control using estimates of the states obtained from a dynamic system, called observer. State estimation is essential to optimal control techniques since in general the states are used in some form of feedback and the number of measureable states is usually much less than the actual number of state variables, limited by the availability of cheap and rugged sensors and the ease of installation. Furthermore the measurements from the existing sensors may be corrupted by significant noise. In the present paper both the stochastic and the deterministic cases are investigated for application to optimal control of ram velocity in injection molding.

INTRODUCTION

Analog and digital PID controllers have long been used by the industry for univariate control of the various machine variables. This class of controllers are essentially of the input/output type and do not take into account the intermediate states. To achieve tighter control, state feedback control schemes such as optimal control, that take the form $\underline{u}(k) = f(\underline{x}(k),k)$, for a system governed by eq. 1 below, can be employed

$$\begin{aligned} \underline{x}(k+1) &= A\underline{x}(k) + B\underline{u}(k) + \underline{v}(k) \\ \underline{y}(k) &= C\underline{x}(k) + \underline{e}(k) \end{aligned} \qquad (1)$$

where $\underline{x}$, $\underline{y}$, and $\underline{u}$ are the state, output and the input vectors respectively. A, B and C are the system matrices, while $\underline{v}(k)$ and $\underline{e}(k)$ are the process disturbance and the measurement noise vectors.

Implementation of such a control law requires that the entire state be available at the discrete sampling instances. The states can be made available either through direct measurements or through estimation. Direct measurement of states would require additional instrumentation which due to the unavailability of cheap, rugged and accurate sensors is often unattractive as an option. Under such situations an approximation of the state vector is substituted for the unavailable states and the feedback control law takes the form $\underline{u}(k) = f(\underline{\hat{x}}(k),k)$, where $\underline{\hat{x}}$ is the estimate of the state vector. This approach results in the decomposition of the control design problem into two phases. The first phase is the design of the control law assuming that the state vector is available. This was the subject of our companion paper (1) wherein the form of the control law was determined through minimization of a quadratic function of the tracking error and the controller effort.

The second phase is concerned with the design of a system that produces an estimate of the state vector. This system in the deterministic setting is termed as an "Optimal Deterministic Observer", to distinguish it from the Kalman filter used in the stochastic setting. An observer is thus a dynamic system that estimates the states of a system in the presence of noise, and whose characteristics are somewhat free to be determined by the designer. Moreover, introduction of an observer does not change the closed loop eigenvalues of the original system but merely adjoins its own eigenvalues thus preserving the stability properties of the original system. Figure 1 shows the corresponding block diagram of a state controller with an observer. Inputs to the observer include the controller effort $\underline{u}(k)$ and the measurable output $\underline{y}(k)$. The observer then produces an estimate of the states which are fed back through the state controller that multiplies it by a time varying gain H to produce the actuator signal. The general form of the observer as proposed by figure 1 is

$$\underline{\hat{x}}(k+1) = A\underline{\hat{x}}(k) + B\underline{u}(k) + K[\underline{y}(k) - C\underline{\hat{x}}(k)]$$

$$\underline{y}(k) = C\underline{x}(k)$$

where K is the observer gain and all other terms have the same interpretation as eq. 1.

It is the manner in which the observer gain K is determined, that distinguishes one observer from the other. The objective of the Kalman filter used in this study it to minimize the expectation of the estimation error, while optimal deterministic observer minimizes a quadratic loss function that weighs the estimation error and the controller effort. Both the stochastic case and the deterministic cases will now be discussed in sequence.

STOCHASTIC CASE

The injection molding process can be considered as a stochastic process in view of the various disturbances present. Due to the rough industrial environment, ruggedness is sometimes more important than accuracy. The measurements from the available sensors would therefore often be corrupted by noise. Kalman and Bucy (2) treated the problem of state estimation for a linear finite dimensional dynamic plant when all the measurements are corrupted by white noise. They showed that for such a plant the "best" estimate of the states is the output from another linear finite-dimensional dynamic system called the state estimator, which is driven by the plant inputs and outputs. Many modifications have been made since the first paper was published by Kalman 25 years ago. In this study the injection molding process will be modelled as having both the measurement noise and the process disturbance. The form of Kalman filter used in this study was therefore the one that accounts for both noise sources.

Kalman filter theory

It is assumed that the injection molding state variables can be representated as a stationary stochastic vector, which can be described by the following Markov process

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) + \underline{v}(k)$$

with measurement equation

$$\underline{y}(k) = C\underline{x}(k) + \underline{e}(k)$$

where

$\underline{x}(k)$ = the state vector of dimension (mx1)

$\underline{y}(k)$ = the measurable output vector, the ram velocity, of dimension (rxl)

$\underline{u}(k)$ = the input vector, the servo-valve opening, of dimension (rxl)

$\underline{v}(k)$ = the process disturbance vector of dimension (mxl)

$\underline{e}(k)$ = the measurement noise vector of dimension (rxl)

A = the system matrix (mxm)

B = the input matrix (mxr)

C = the output matrix (rxm)

m = the number of states, in our case = 4

r = the number of outputs or inputs, in our case = 1.

The process disturbance $\underline{v}(k)$ represents random upsets not accouted for by the model. For ram velocity control system this would include errors in modelling, variations in the viscosity of the melt, the oil temperature and the melt temperature, each of which would result in a different ram velocity for a given valve opening. The process disturbance would also include deterioration in the performance of the servo-value due to wear with passage of time. Measurement noise is designated by $\underline{e}(k)$. In many practical applications velocity is determined by differentiation of the ram position signal provided by Linear Variable Differential Transformer (LVDT) or the potentiometer. Differentiation of the position signal would itself introduce considerable error. Measurement noise is therefore a major source of disturbance in our state estimation problem. It is assumed that $\underline{v}(k)$ and $\underline{e}(k)$ are discrete-time Gaussian white noise processes with zero mean and

$$E\{\underline{v}(k)\underline{v}^T(i)\} = \begin{cases} R_1 & i = k \\ 0 & i \neq k \end{cases} \qquad (2)$$

$$E\{\underline{e}(k)\underline{e}^T(i)\} = \begin{cases} R_2 & i = k \\ 0 & i \neq k \end{cases} \quad (3)$$

It is further assumed that

- $\underline{v}(k)$ and $\underline{e}(k)$ are statistically independent, which implies that

  $E\{\underline{v}(k)\underline{e}^T(i)\} = \underline{0}$ for all i and k

- the initial state $\underline{x}(0)$ has a Gaussian distribution with $E\{x(0)\} = \underline{m}_0$ and $\text{cov}\ \{\underline{x}(0)\} = R_0$
- the matrices R_0, R_1 and R_2 are positive semidefinite and are known apriori
- the model is reachable and observable

Our objective is to estimate the state vector $\underline{x}(k+1)$ based on the current measurement of ram velocity $\underline{y}(k)$ which is contaminated by white noise $\underline{e}(k)$ and the previous estimate of the states $\underline{x}(k)$ in presence of white noise $\underline{v}(k)$. The parameters of the observer are chosen so as to minimize the variance of the prediction error. Figure 2 shows the block diagram of the Kalman filter. The block diagram suggests the following form of the filter

$$\underline{\hat{x}}(k+1/k+1) = A\underline{\hat{x}}(k/k) + B\underline{u}(k) + K(k+1)\{\underline{y}(k+1) - C[A\underline{\hat{x}}(k/k) + B\underline{u}(k)]\}$$

where

$\underline{\hat{x}}(k+1/k+1)$ = the estimate of the state vector $\underline{x}$ at the $(k+1)^{th}$ sample given all the remaining data up to $(k+1)^{th}$ sample.

$K(k+1)$ = the time varying Kalman gain for the $(k+1)^{th}$ sample, and is determined recursively by the following algorithm (3).

$$K(k+1) = P(k/k-1)C^T[R_2 + CP(k/k-1)C^T]^{-1} \quad (4)$$

$$P(k/k-1) = AP(k-1/k-1)A^T + R_1 \quad (5)$$

$$P(k/k) = P(k/k-1) - K(k)CP(k/k-1) \quad (6)$$

$P(k/k-1)$ = the error covariance matrix that shows how the error propagates with time.

Further details are provided in the appendix.

Simulation Results

The effectiveness of a Kalman filter in estimating the states in the presence of additive white Gaussian noise will now be presented. Three possible noise combinations have been studied which include

a. process noise alone

b. measurement noise alone and

c. both the process noise and the measurement noise.

White noise with zero mean and a given variance has been simulated using a random number generator. The variance of the added noise is defined in terms of noise to signal percentage (NSP). Due to the unsteady state nature of the control task of following the ram velocity, it is difficult to define the variance of the additive white Gaussian noise in terms of the variance of the measured signal. It was therefore necessary to define the variance of the noise as a percentage of the signal at a given instance. For process disturbance the value of the final state was selected while for measurement noise the unit value of the reference signal was chosen. For example, process disturbance with a NSP of 10 means that the added noise has a variance approximately equal to 10% of the corresponding final state. However for measurement noise a NSP of 10 means that noise with variance equal to .1 is added to the actual measured ram velocity signal.

The effectiveness of the overall optimal control system using a Kalman filter to estimate the states in the presence of different noise levels has been evaluated by four performance measures. One was the sum of the squares of the errors in tracking a given profile (OBJE). The second one was concerned with the expenditure of total energy to the system. This was the sum of the squares of the controller effort at each sample (OBJU). The last two were concerned with the variability of the controller effort. The third one was the sum of the squares of the absolute values of the derivative of the controller effort (OBJD). The last one was the variance of the controller effort (VARU).

Table 1 provides the values of the four objectives for different disturbance/measurement noise levels and its combinations. Figure 3 compares the tracking characteristics of the optimal controller used in (1) along with a Kalman filter for state estimation, in the presence of measurement noise with a NSP of 10 and 20. The performance of the Kalman filter is satisfactory. It can be seen from figure 3 that the performance of the Kalman filter in the presence of significant measurement noise with a NSP of 20 is poor during the first few samples. This can be attributed to the fact that the estimation of the states is based on the available measurements of ram velocity and the higher the measurement noise the less is the information contained in the measured data and therefore greater is the deterioration in our ability to estimate the states. Poor estimates lead to poor control performance. However, after a few samples the performance is excellent and is comparable to the performance of the filter in the presence of lower level of measurement noise with a NSP of 10. This improvement in performance is however at the expense of increased variability in the controller effort during those samples.

Figure 4 shows the performance of the Kalman filter in the presence of process disturbance with a NSP of 10 and 30. Comparing the values of the performance measures for the process noise and the measurement noise in table 1, it is clear that the observer in our case deals better with the former than the latter for same noise levels. This can also be seen from figure 4. This suggests that modelling errors, material characteristic variations, deterioration in machine performance and other stochastic disturbances acting on the process can be suitably handled by a Kalman filter.

In an actual industrial environment both the measurement noise and the process disturbance are likely to be present simultaneously. The performance of the Kalman filter under such adverse situations is depicted in figure 5. Two noise combinations are shown. One is with a NSP of 10 for both the measurement noise and the process disturbance. The second combination consists of measurement noise with a NSP of 10 but a higher level of process disturbance with a NSP of 20. The Kalman filter is able to perform satisfactorily, though there is significant increase in the variability of the servovalve opening and the total energy input to the injection molding machine. This can be expected, as the Kalman filter has now to deal with two sources of noise.

DETERMINISTIC CASE

In situations where very high precision in molding is required it might be feasible to mount expensive accurate sensors. In such situations a reasonable assumption would be that "noise free" measurements are available. By "noise free" we mean that the measurement noise is significantly small. Under such situations it is in general possible to modify the Kalman-Bucy filter by the addition of differentiators and integrators to the original filter. Since in practice introduction of a differentiator is undesirable an alternate form of an observer called "optimal deterministic" observer has been used. As with the Kalman-Bucy filter, the optimal deterministic observer is provided with the plant inputs and the available outputs. An essential feature of the optimal deterministic observer used in this study is that the norm of the error in the estimate of the plant state vector is bounded by a decaying exponential function of time. Hence, the error approaches zero as time increases, and for this reason, the observer is often referred to as an asymptotic state estimator. Another advantage offered by this type of observer is that the same programming code as for the optimal regulator can be used. This fact will become clear from the following reasoning.

Optimal Deterministic Observer Theory

Injection molding plant under the assumption of noise free measurements can be modelled by the following state space representation

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k)$$

$$\underline{y}(k) = C\underline{x}(k) \qquad (9a)$$

where we assume that the servo-valve opening $\underline{u}(k)$ and the ram velocity $\underline{y}(k)$ are the only available error free measurements and that the state variables $\underline{x}(k)$ are observable. The block diagram of the optimal deterministic observer is shown in figure 6. Figure 6 shows that a model with the same structure as the process model is connected in parallel to it. State corrections $\Delta\underline{\hat{x}}(k)$ are generated by feeding back the weighted difference between the output signals from the model and the process. The resultant observer equation as given by Iserman in (4) is

$$\underline{\hat{x}}(k+1) = A\underline{\hat{x}}(k) + B\underline{u}(k) + K\Delta\underline{e}(k)$$

$$= A\underline{\hat{x}}(k) + B\underline{u}(k) + K[\underline{y}(k) - C\,\underline{\hat{x}}(k)] \quad (9b)$$

using eqs. 9a and 9b the estimation error $\underline{\tilde{x}} = \underline{x} - \underline{\hat{x}}$ can be given as

$$\underline{\tilde{x}}(k+1) = [A - KC]\,\underline{\tilde{x}}(k) \quad (10)$$

For asymptotic reduction of the estimation error given by eq. 10 we are required to find K such that $\lim_{k \to \infty} \underline{\tilde{x}}(k) = \underline{0}$. Inspection of eq. 10 suggests that the characteristic equation of the observer in discrete-time is

$$\det\,[zI - A + KC] \quad (11)$$

This has great resemblance to the characteristic equation of the closed loop state feedback optimal regulator with $\underline{u}(k) = -H\underline{\hat{x}}(k)$, which is of the form

$$\det\,[zI - A + BH] \quad (12)$$

Resemblance of the characteristic equations given by eqs. 11 and 12 suggests that the same code as for the optimal regulator used in (1), can be utilized for finding the optimal observer gain K, by proper transformation of the matrices. In order to find the required transformation, the characteristic equation of the observer has to be modified to a form that would allow comparison of the two characteristic equations. From matrix theory, $\det W = \det W^T$ where W is any matrix, therefore from eq. 11

$$\det\,[zI - A + KC] = \det\,[zI - A^T + C^T K^T] \quad (13)$$

and comparing eq. 13 with eq. 12 the following modifications result

$$A \to A^T,\ B \to C^T,\ H \to K^T \quad (14)$$

This shows that the same formulation as for the optimal regulator can be used, with the modifications suggested by eq. 14. Therefore the increase in the code and hence the storage requirement due to the implementation of this observer are not significant.

Simulation Results

An optimal deterministic observer is often termed as an asymptotic observer. The rate at which the estimation error is reduced is dependent upon the choice of the weighting matrix K. It is however difficult to select an optimal K as no general principles are available. It should be recalled that both the Kalman filter and the optimal deterministic observers have a similar basic structure and therefore performance equivalent to the Kalman filter should be possible by proper choice of K. This would correspond to the selection of K that would place the closed loop poles of the overall controlled system at the same locations as the Kalman filter. In our case through considerable simulations it was finally possible to fix K so as to achieve nearly the same performance as with the Kalman filter. However it was not possible to achieve equivalent performance for all the three cases simultaneously. The deterministic observer performed poorly in the presence of significant measurement noise. This can be seen from figure 7 which shows the performance of the optimal deterministic observer in the presence of measurement noise with a NSP of 10 and 20. More severe demands are imposed on the hydraulic valve than with the corresponding Kalman filter. The response of the deterministic observer to process disturbance alone and in the presence of both the noises are not shown as they are nearly the same as that with the Kalman filter given in figure 5 and 4 respectively. However, the values of the four objectives are provided in table 2.

CONCLUSIONS

The application of optimal anticipatory control to the injection molding process as proposed in our companion paper requires the use of observer theory to estimate the non-measurable states. In this present paper the effectiveness of two types of observers, the optimal deterministic one and the Kalman filter, to estimate the states in the presence of process disturbance and/or the measurement noise has been investigated. In most of the practical situations where optimal control is applied, observers are essential. The optimal deterministic observer,as the name suggests, can be used in deterministic situations or in situations where less noise is present. The Kalman filter is suggested for higher levels of noise.

Both the observers were found to work satisfactorily in conjunction with the optimal controller, in response to process disturbance. However the optimal deterministic observer performed poorly in the presence of significant measurement noise. It is therefore recommended that a Kalman filter be used in cases where measurement noise is dominant. The computer storage necessary for the state estimation is greatly reduced due to the recursive form of the filters. The storage requirements can be further reduced if only the steady-state gain is used.

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APPENDIX

Derivation of the Discrete Kalman Filter Algorithm (3).

The derivation of the Kalman filter algorithm can be based on several estimation methods such as maximum likelihood, Bayes method, recursive least squares and the minimum variance method. The following derivation uses minimum variance recursive estimation. For the purpose of clarity the observer that uses data up to time k to estimate the states at time k+1 of the form given below is derived

$$\underline{\hat{x}}(k+1/k) = A\underline{\hat{x}}(k/k-1) + B\underline{u}(k) + K(k)[\underline{y}(k) - C\underline{\hat{x}}(k/k-1)]$$

for a plant represented by the following state-space model

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) + \underline{v}(k)$$

$$\underline{y}(k) = C\underline{x}(k) + \underline{e}(k)$$

The reconstruction error $\underline{\tilde{x}} = \underline{x} - \underline{\hat{x}}$, at time k+1 is then given by

$$\begin{aligned}\underline{\tilde{x}}(k+1) &= \underline{x}(k+1) - \underline{\hat{x}}(k+1)\\ &= A\underline{\tilde{x}}(k) + \underline{v}(k) - K(k)[\underline{y}(k)-C\underline{\hat{x}}(k/k-1)]\\ &= [A - K(k)C]\underline{\tilde{x}}(k) + \underline{v}(k) - K(k)\underline{e}(k) \qquad \text{(A-1)}\end{aligned}$$

Our objective is to find the estimates taking into account the properties of the noise so as to minimize the variance of the estimation error denoted by

$$P(k) = E\{[\tilde{x}(k) - E\tilde{x}(k)][(\tilde{x}(k) - E\tilde{x}(k)]^T\} \qquad \text{(A-2)}$$

where

$$\begin{aligned}E\{\underline{\tilde{x}}(k+1)\} &= E\{[A-K(k)C]\underline{\tilde{x}}(k) + \underline{v}(k) - K(k)\underline{e}(k)\}\\ &= [A-K(k)C]E\{\underline{\tilde{x}}(k)\} + E\{\underline{v}(k)\} - K(k)E\{\underline{e}(k)\}\end{aligned}$$

By our assumption $E\{\underline{v}(k)\} = E\{\underline{e}(k)\} = \underline{0}$. Thus

$$E\{\underline{\tilde{x}}(k+1)\} = [A - K(k)C]E\{\underline{\tilde{x}}(k)\} \qquad \text{(A-3)}$$

Also given is $E\{\underline{x}(0)\} = \underline{m}_o$. Therefore the mean value of the reconstruction error is zero for all times $k > 0$ irrespective of K if $E\{\underline{\hat{x}}(0)\} = \underline{m}_o$. For example

$$E\{\underline{\tilde{x}}(k)\} = E\{\underline{x}(k) - \underline{\hat{x}}(k)\}$$

$$= E\{\underline{x}(k)\} - E\{\underline{\hat{x}}(k)\} = \underline{m}_o - \underline{m}_o = \underline{0}$$

Thus from eq. A-2

$$P(k+1) = E\{[\underline{\tilde{x}}(k+1)][\underline{\tilde{x}}(k+1)]^T\}$$

and from eq. A-1

$$P(k+1) = E\{[[A-K(k)C]\underline{\tilde{x}}(k)+\underline{v}(k)-K(k)\underline{e}(k)][[A-K(k)C]\underline{\tilde{x}}(k)+\underline{v}(k)-K(k)\underline{e}(k)]^T\}$$

$$= E\{[A-K(k)C]\underline{\tilde{x}}(k)\underline{\tilde{x}}(k)^T[A-K(k)C]^T+[A-K(k)C]\underline{\tilde{x}}(k)\underline{v}(k)^T$$

$$- [A-K(k)C]\underline{\tilde{x}}(k)\underline{e}(k)^TK^T(k)+\underline{v}(k)\underline{\tilde{x}}(k)^T[A-K(k)C]^T + \underline{v}(k)\underline{v}(k)^T$$

$$-\underline{v}(k)\underline{e}(k)^TK^T(k)-K(k)\underline{e}(k)\underline{\tilde{x}}(k)^T[A-K(k)C]^T - K(k)\underline{e}(k)\underline{v}(k)^T$$

$$+K(k)\underline{e}(k)\underline{e}(k)^TK^T(k)\}$$

$$= [A-K(k)C]E\{\underline{\tilde{x}}(k)\underline{\tilde{x}}(k)^T\}[A-K(k)C]^T+[A-K(k)C]E\{\underline{\tilde{x}}(k)\underline{v}(k)^T\}$$

$$- [A-K(k)C]E\{\underline{\tilde{x}}(k)\underline{e}(k)^T\}K^T(k)+E\{\underline{v}(k)\underline{\tilde{x}}(k)^T\}[A-K(k)C]^T$$

$$+ E\{\underline{v}(k)\underline{v}(k)^T\}-E\{\underline{v}(k)\underline{e}(k)^T\}K^T(k)-K(k)E\{\underline{e}(k)\underline{\tilde{x}}(k)^T\}[A-K(x)C]^T$$

$$- K(k)E\{\underline{e}(k)\underline{v}(k)^T\}+K(k)E\{\underline{e}(k)\underline{e}(k)^T\}K^T(k)$$

Now $\quad E\{\underline{v}(k)\underline{v}(k)^T\} = R_1$

$$E\{\underline{e}(k)\underline{e}(k)^T\} = R_2$$

$$E\{\underline{\tilde{x}}(k)\underline{\tilde{x}}(k)^T\} = P(k)$$

and all other expectations are zero as v(k), e(k) and $\tilde{x}(k)$ are uncorrelated statistically independent random variables. Thus

$$P(k+1) = [A-K(k)C]P(k)[A-K(k)C]^T + R_1 + K(k)R_2K^T(k) \qquad \text{(A-4)}$$

From eq.(A-4) it follows that if P(k) is positive semidefinite, then P(k+1) is also positive semidefinite. It is assumed that the criterion is to minimize the scalar $\underline{\alpha}^T P(k+1)\underline{\alpha}$, where $\underline{\alpha}$ is an arbitrary vector. It is also assumed that the optimal-gain vector, K, has been used up to time k-1.

From eq. (A-4)

$$\underline{\alpha}^T P(k+1)\underline{\alpha} = \underline{\alpha}^T\{AP(k)A^T + R_1 - K(k)CP(k)A^T - AP(k)C^TK^T(k)$$
$$+ K(k)[R_2 + CP(k)C^T]K^T(k)\}\underline{\alpha}$$

Completing the squares we will get

$$\underline{\alpha}^T P(k+1)\underline{\alpha} = \underline{\alpha}^T\{AP(k)A^T + R_1 - AP(k)C^T[R_2 + CP(k)C^T]^{-1}CP(k)A^T\}\underline{\alpha}$$
$$+ \underline{\alpha}^T\{[K(k) - AP(k)C^T[R_2 + CP(k)C^T]^{-1}][R_2 + CP(k)C^T]$$
$$[K(k) - AP(k)C^T[R_2 + CP(k)C^T]^{-1}]^T\}\underline{\alpha} \qquad \text{(A-5)}$$

The criterion in (A-5) has two terms. The first term is independent of K and the second term is non-negative as the matrix $[R_2 + CPC^T]$ is positive definite. The minimum is thus obtained if K is chosen such that the second term of (A-5) is zero.

This results in the following Kalman filter

$$K(k) = AP(k)C^T[R_2 + CP(k)C^T]^{-1}$$

$$P(k+1) = AP(k)A^T + R_1 - AP(k)C^T[R_2 + CP(k)C^T]^{-1}CP(k)A^T$$

Using similar reasoning it is possible to derive the filter which also uses $\underline{y}(k)$ to estimate $\underline{x}(k)$.

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Table 1. Comparison of the various objectives with a Kalman filter in presence of measurement and/or process disturbance

PROCESS DISTURBANCE NSP	MEASUREMENT NOISE NSP	OBJE	OBJU	OBJD	VARU
10	-	.082263	416.3475	12.1521	.68199
20	-	.201656	415.6577	16.3892	.68715
30	-	.413669	415.2759	20.7817	.70056
40	-	.717762	415.2146	25.3376	.71685
-	10	1.043234	417.9508	15.524	.69066
-	20	4.091946	422.1907	22.7318	.73486
-	30	9.198173	430.0666	30.3267	.81837
10	10	1.091284	418.7421	37.9946	.73669
20	10	1.002062	416.3911	33.8578	.72686
30	10	1.001828	414.3036	29.7637	.72506
10	20	4.570207	436.2188	76.7948	.90543
20	20	4.278620	432.4742	72.8024	.89088
30	20	4.079804	429.0166	68.9091	.87888

Table 2. Comparison of the various objectives with an optimal "deterministic" observer in presence of measurement and/or process disturbance.

PROCESS DISTURBANCE NSP	MEASUREMENT NOISE NSP	OBJE	OBJU	OBJD	VARU
10	-	.081894	416.3288	11.9437	.681483
20	-	.200027	415.5922	15.9857	.685917
30	-	.409939	415.1346	20.1876	.698325
40	-	.711071	414.9688	24.4745	.713340
-	10	1.26557	420.8568	39.7195	.736959
-	20	4.96106	438.1913	77.4551	.904162
-	30	11.07525	466.4404	111.6871	1.163479
10	10	1.09642	418.4238	35.8924	.73322
20	10	1.01070	416.2353	32.0016	.724567
30	10	1.01380	414.2813	28.2537	.723537
10	20	4.58561	434.5380	73.7246	.890126
20	20	4.30208	431.1447	70.0574	.878394
20	30	4.11046	428.0114	66.4393	.868966

Agrawal, Fig.1

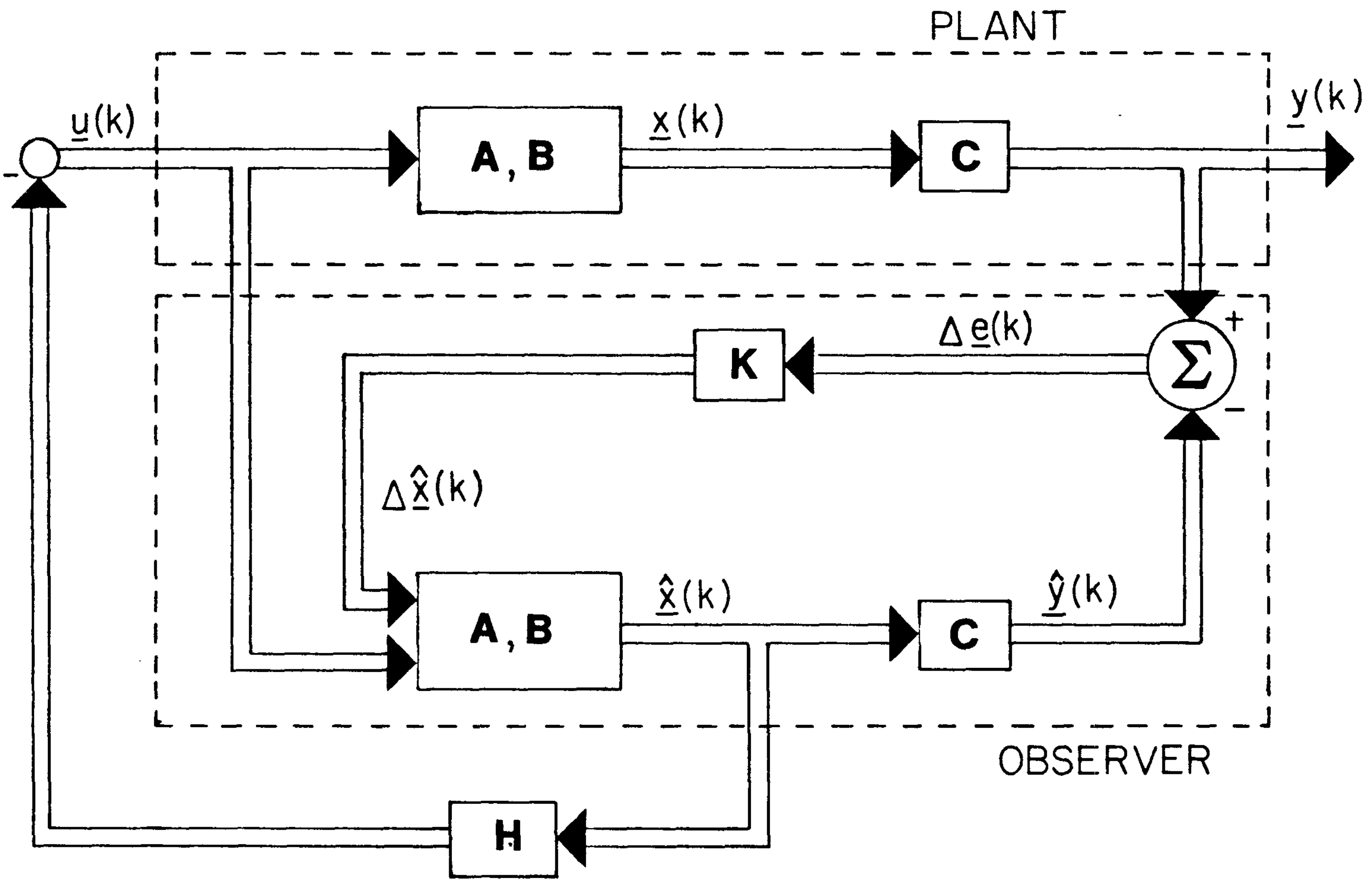

Fig. 1. Block diagram of a state controller with an observer.

Agrawal, Fig.2

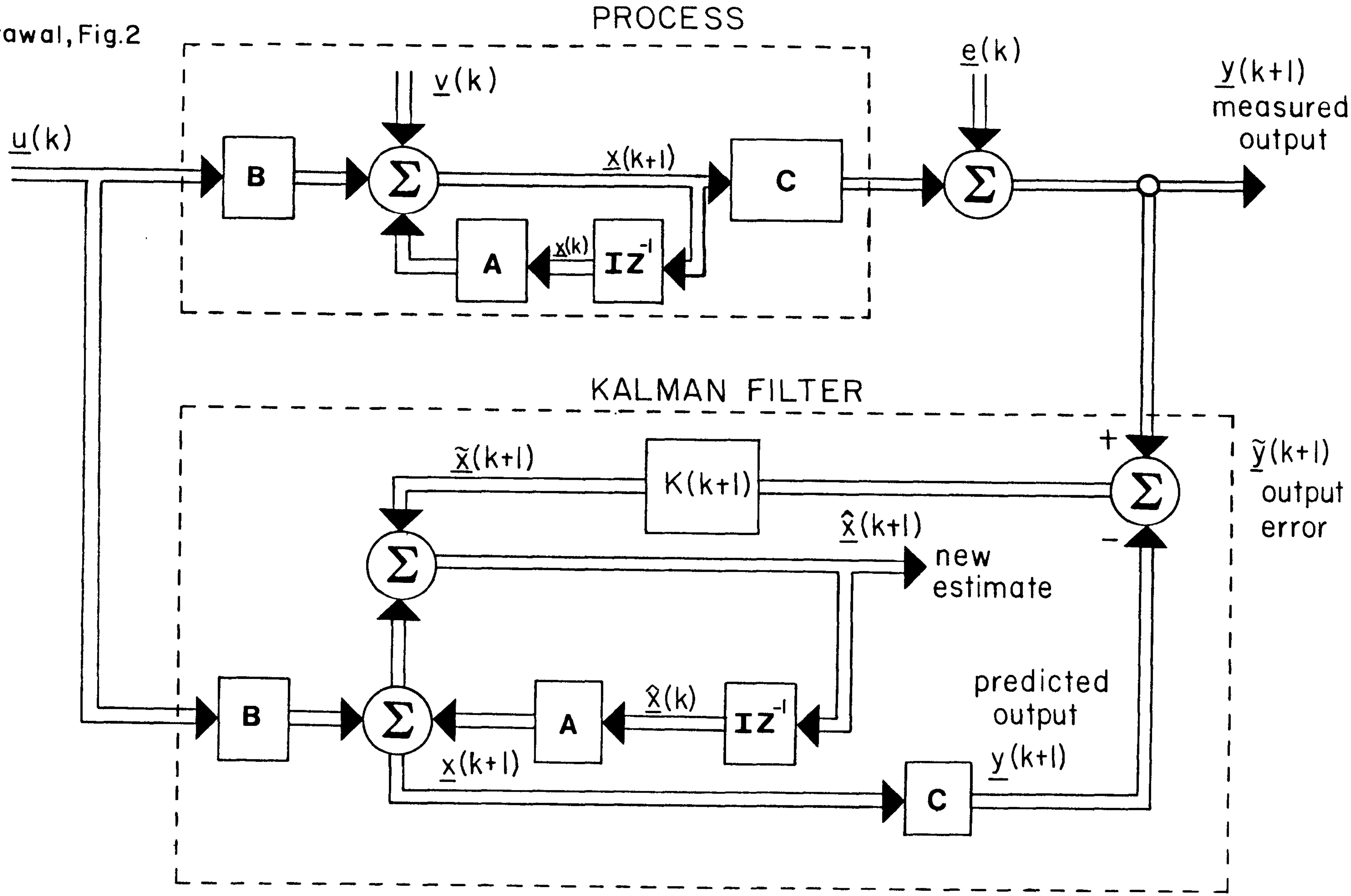

Fig. 2. Block diagram of a Kalman filter for state estimation.

Agrawal, Fig. 3

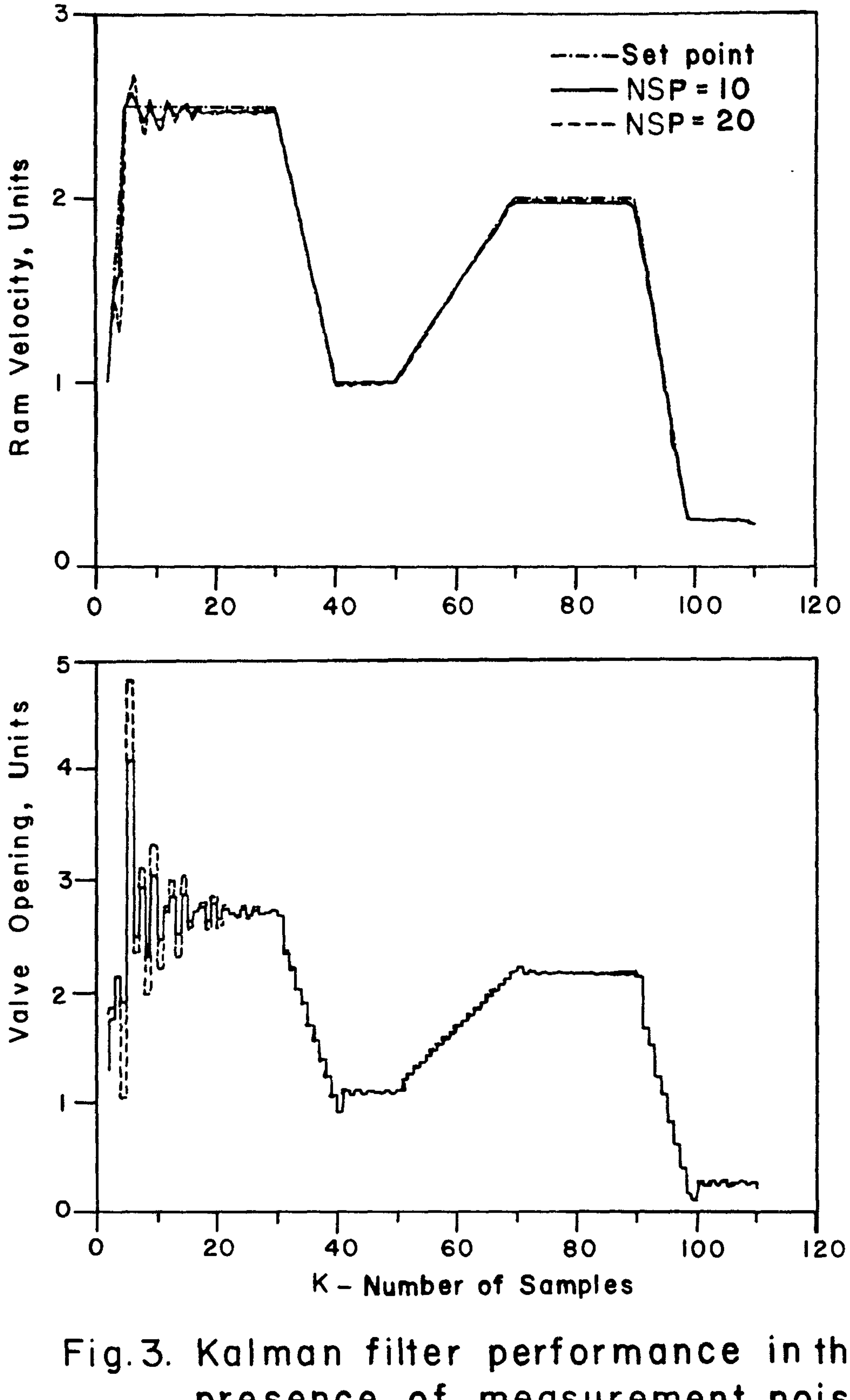

Fig. 3. Kalman filter performance in the presence of measurement noise.

Agrawal, Fig.4

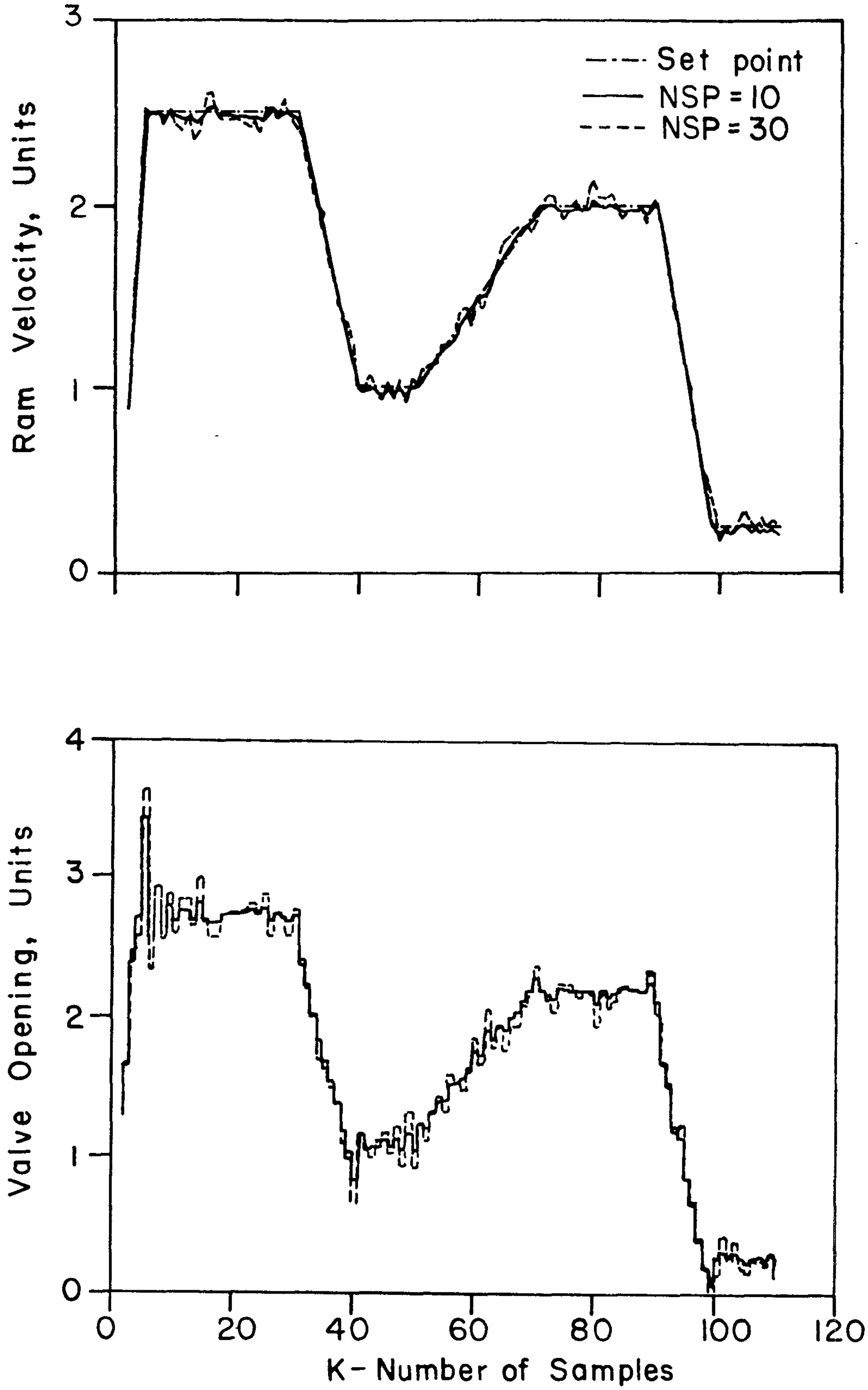

Fig.4. Kalman filter performance in the presence of process disturbance.

Agrawal, Fig. 5

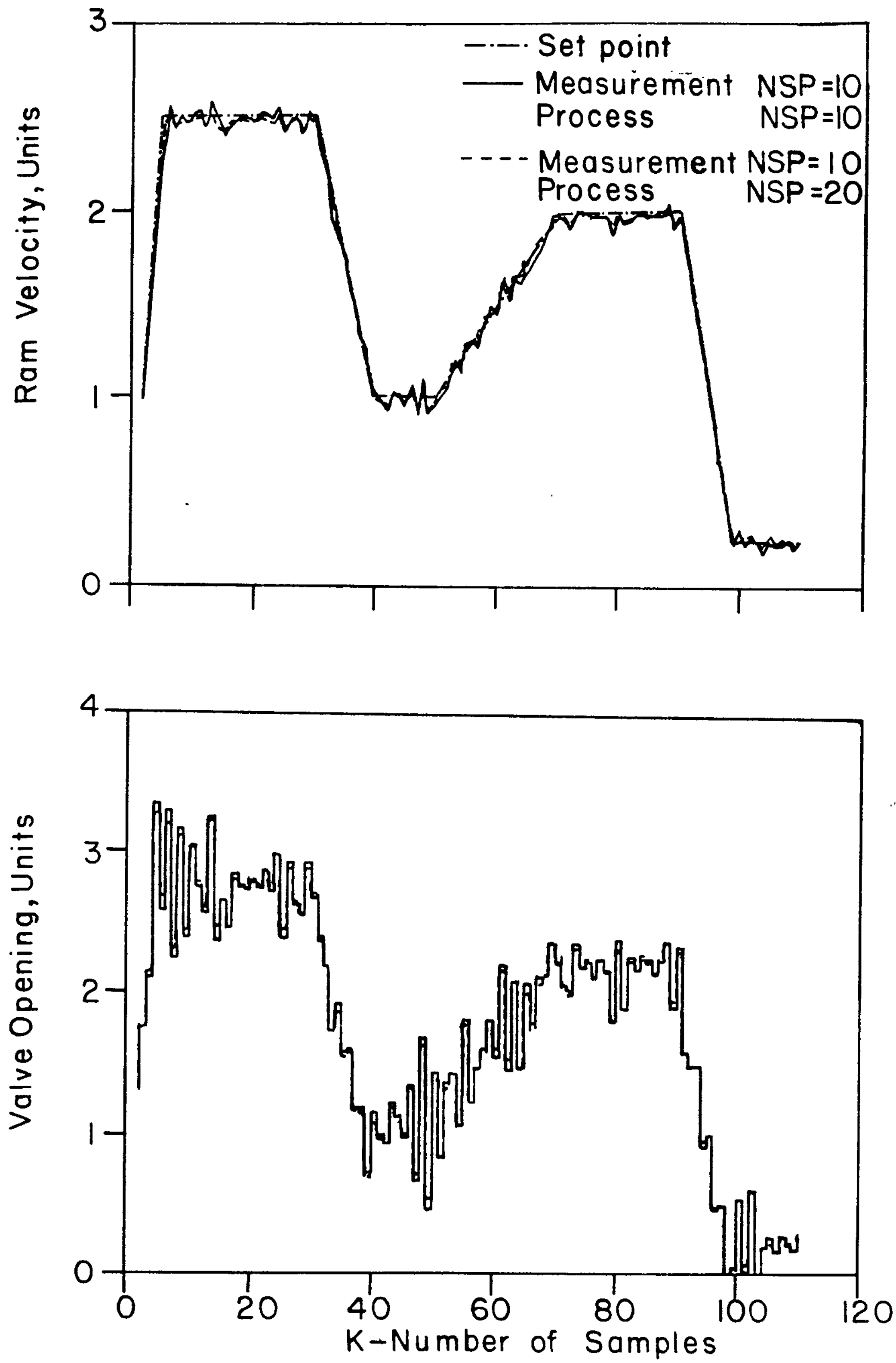

Fig. 5. Kalman filter performance in presence of both the measurement noise and the process disturbance.

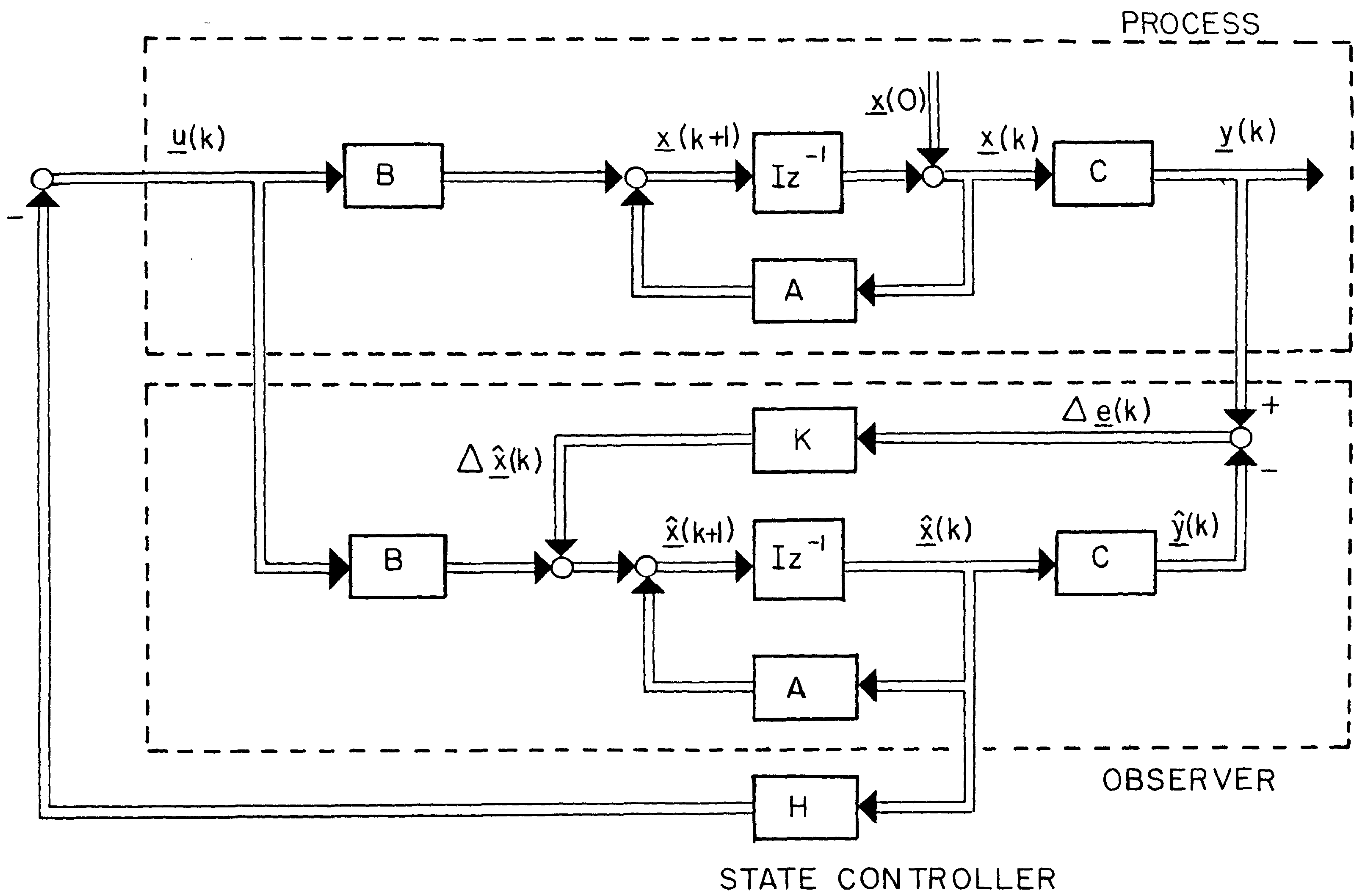

Fig.6. Block diagram of an optimal deterministic observer.

Agrawal, Fig. 7

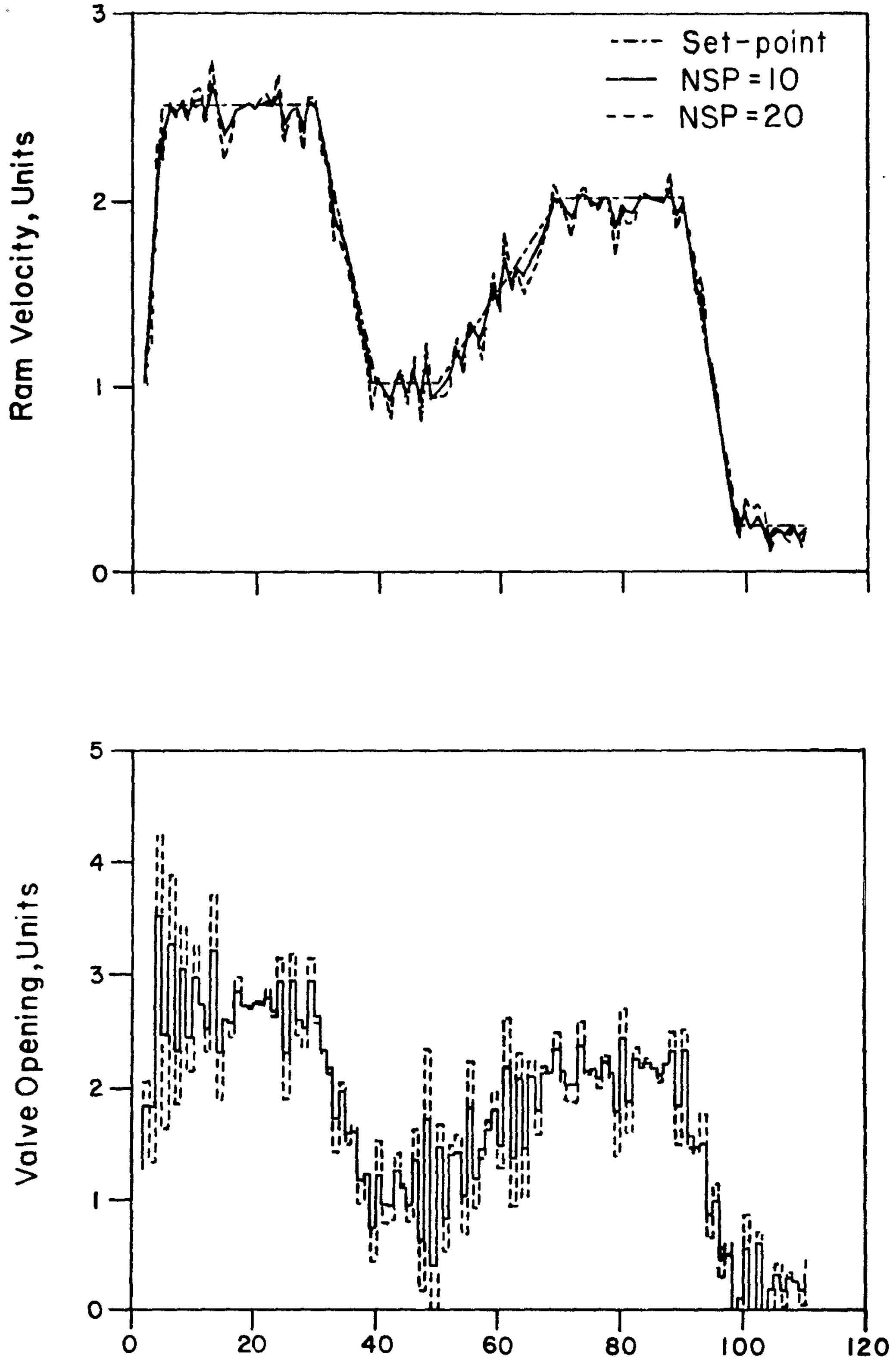

Fig. 7. Optimal deterministic observer performance in the presence of measurement noise.