SRC TR 86-35

Optimal Anticipatory Control of Ram Velocity in Injection Molding

by

I.O. Pandelidis and A.R. Agrawal

Optimal Anticipatory Control of Ram Velocity in Injection Molding

I.O. PANDELIDIS and A.R. AGRAWAL

Department of Mechanical Engineering

University of Maryland

College Park, Maryland - 20742

Optimal Anticipatory Control of Ram Velocity in Injection Molding

I.O. PANDELIDIS and A.R. AGRAWAL

Department of Mechanical Engineering

University of Maryland

College Park, Maryland - 20742

ABSTRACT

This paper discusses a computer control system for ram velocity of an injection molding machine using optimal state feedback based on the linear quadratic control theory. A new approach for the selection of appropriate weighting matrices is presented in this context. The simulation results reveal that the optimal controller has improved performance over the conventional PID controller presently used, having faster speed of response, significantly better tracking performance and better noise filtering properties. The execution speed and the core storage requirements would allow implemetation even on a small online computer.

I. INTRODUCTION

Injection molding is one of the major processes used by the plastics industry. Applications of plastics are increasing at a very fast rate and with it are increasing the demands imposed on the material manufacturers, machine producers and the process control engineer. Improvements are being made in all the above three areas. However, with the introduction of low-cost high performance VLSI chips the third option is very attractive. Control algorithms, which only a few years ago would have required a main-frame computer can now be implemented online through a minicomputer at marginal cost.

Applications of advanced control schemes to injection molding are presently very limited. It is the aim of this research to show the effectiveness of an optimal control strategy to track down a specified ram velocity setpoint profile.

Use is made of the separation theorem to justify the decomposition of the overall control design procedure into two phases. The present paper provides the results of controller design assuming that the full state vector is available for feedback. The companion paper (1) then presents the results of the controller design based on the estimated states.

The organization of this paper has been done as follows. Section II provides the motivation for selecting ram velocity as a controlled variable and for employing optimal anticipatory control. Section III, provides a brief introduction to the theory of optimal control. Section IV, details the dynamic model employed for ram velocity. Theoretical and analytical aspects of selecting the weighting matrices are then given in section V. Section VI presents the simulation results. The issue of disturbance rejection is discussed in section VII. Practical aspects of implementing this algorithm are provided in section VIII. Conclusions are finally stated in section IX.

II. MOTIVATION

This section first provides the motivation for selection of ram velocity as the controlled variable and then proceeds with the motivation for employing optimal anticipatory control.

The ultimate motivation of applying controls is to achieve a high product quality. It is therefore necessary to quantify "quality" into measurable quantities in terms of process variables that closely correlate to the final product specifications. It is well known that ram velocity is such a variable, as it affects the shear rates and the shear stresses produced in the product. These are critical in determining quality characteristics such as shrinkage, warpage, and impact strength. Ram velocity also plays an important role in improving problems such as welds, burns, short shorts, splay and flash. Injection time, a measure of productivity, can be shortened by proper velocity profiling (2). Surface finish is also improved with decreased injection time as it prevents material from setting before it fully packs. Proper control of ram velocity further aids in improving part resolution, the ability of the material to conform to the shape of the cavity through rapid and uniform filling of the cavity. Accurate control of ram velocity is thus essential during the filling phase and was therefore selected as the controlled variable.

Moreover, it is now possible through microprocessor control to regulate the entire filling phase by controlling the injection speed of the ram so as to follow a fixed trajectory aimed at achieving the following (3).

- high ram velocity during filling of the less critical areas such as runners, resulting in minimum heat loss and shortened injection time

- slower ram speed when the melt reaches the gates to eliminate jetting
- velocity adjusted so as to maintain constant surface flow front speed, to prevent inconsistences in flow pattern that can cause surface tension resulting in shrinkage on the adjacent surfaces of the molded part
- significant reduction of the ram velocity just prior to the moment the cavity is filled to prevent overpacking and/or flashing.

Figure 1 shows a typical ram velocity profile to fill a mold of increasing cross-section initially and having a constant cross-section thereafter. The profile for this part can be divided into several sections. During section I which corresponds to runner fill, velocity is very rapidly increased and then is held constant. This results in minimum injection time and heat loss in the material. During section II the velocity is rapidly reduced to eliminate jetting at the gate. Once the melt enters the cavity the aim is to maintain a constant flow front speed. Section III corresponds to filling of the increasing cross-section portion of the cavity while section IV is concerned with filling of the constant cross-section portion of the cavity. The ram velocity profile should increase to maintain constant velocity of the melt across the mold surface during section III, while the ram velocity should be maintained at a constant during section IV. The velocity is then reduced during section V to eliminate flashing and/or overpacking.

All of the above objectives have to be achieved in the short duration of time available for mold filling. Moreover molds vary in degrees of complexity and therefore require different injection velocity profiles to attain the desired part quality. The injection velocity profile is also a function of the material being processed. To prevent burning in heat sensitive materials the velocity should not be increased once the material is flowing through the smallest restriction such as the gate (2).

The above considerations impose stringent requirements on the control scheme to follow the desired reference trajectory. The conventional controllers such as a P.I.D. employed presently by the industry attempt to achieve the above by responding in a causal way. This approach however does not utilize the known information about the future reference inputs, and therefore has a suboptimal performance. As a consequence the output response is often characterized by unnecessarily large deviations from the reference profile. A new approach is presented here called optimal anticipatory control. The proposed form of the optimal controller acts in two stages in order to utilize the knowledge of the anticipated future reference values. In the first stage a modified reference signal is generated while in the second stage a time varying feedback gain matrix is calculated. The generation of the signal and the gain matrix are both fairly computationally intensive. However the calculations can be done off-line, and the results stored in the memory of the computer controller. The on-line computer processing requirements are therefore not significantly greater than those of the conventional controllers.

III. OPTIMAL CONTROL THEORY

In recent years there has been a considerable interest in the optimal control methods aimed at minimizing or maximizing a criterion which is a quadratic function of the states and the controller effort while simultaneously satisfying the physical constraints of the system. Use is made of the full state information rather than only the output which in most cases is a subset of the state vector. As compared to the other design methods such as pole-placement where the closed-loop poles are specified, optimal control provides better understanding in that, it minimizes a loss function which is our final aim.

The objective of the optimal anticipatory control employed in this study is to minimize the deviation of the actual output from the desired, prespecified reference. In mathematical terms this objective gets translated into the following form

$$J = \frac{1}{2}\,||\underline{e}_{kf}||^2_S + \frac{1}{2}\sum_{k=0}^{k_f-1}\{||\underline{e}_k||^2_Q + ||\underline{u}_k||^2_R\}$$

which is equivalent to

$$J = \frac{1}{2}\,\{\underline{e}^T(k_f)S\underline{e}(k_f) + \sum_{k=0}^{k_f-1}\{\underline{e}^T(k)\ Q(k)\underline{e}(k) + \underline{u}^T(k)R(k)\underline{u}(k)\}$$

where

$\underline{e}_k = \underline{z}_k - \underline{y}_k$; the tracking error at the k^{th} sample

$\underline{Z}_k$ = the desired output at the k^{th} sample

$\underline{y}_k$ = the actual output at the k^{th} sample

K_f = finite final time

S = positive semidefinite weighting matrix for final time assumed zero in our case

Q = positive semidefinite weighting matrix of the states.

R = the weighting matrix of the controlling variable and should be positive definite.

It is through the introduction of the weighting matrices Q and R that the designer has the freedom to weigh controller effort against the tracking error. Selection of these matrices is of considerable importance and will be detailed in section V.

Figure 2 shows the block diagram of the proposed control scheme. $\underline{Z}_k$ is the reference value of the ram velocity at the k^{th} sampling instance. P_{kf} and $\underline{g}_{kf}$ are the gain matrix and the correction vector at the final time determined through the solution of the boundary conditions. Details about the boundary conditions and the derivation are provided in the appendix. Once the controller (shown by dashed lines) is supplied with the values of $\underline{Z}_k$, $\underline{g}_{kf}$ and P_{kf},it calculates by backward recursion all the values of the correction vector $\underline{g}$ and the gain matrix P from k = Kf to k = 0. Knowing this and the value of the state vector at any instance the controller effort at that instance can then be calculated.

As can be seen from figure 2, the application of optimal control requires a model of the form

$$\begin{aligned} \underline{x}(k+1) &= A\underline{x}(k) + B\underline{u}(k) \\ \underline{y}(k) &= C\underline{x}(k) \end{aligned} \tag{1}$$

where

$\underline{x}(k+1)$ = the state vector at the $(k+1)^{th}$ sample

$\underline{x}(k)$ = the state vector at the k^{th} sample

$\underline{y}(k)$ = the output at the k^{th} sample

A = system matrix

B = control matrix

C = output matrix

The following section discusses the model used for this study.

IV. MODEL DESCRIPTION

Several models have been cited in the literature. The simplest one is by Fara (4) who reports a model with only a gain term. The model is given by

$$C_t = G \bullet M_t \qquad (2)$$

where

C_t = controlled variable, the ram velocity at sampling instance t

M_t = manipulated variable, the percentage change in the valve opening at sampling instance t

G = the process gain corresponding to different step changes in the valve opening.

Fara has also given a time series model of the form

$$C(k) = W_0\, M(k-1) + (1-\theta B)a(k) \qquad (3)$$

where

M(k-1) = the valve opening at the $(k-1)^{th}$ sample

B = the backward shift operator i.e. $B \bullet M(k) = M(k-1)$

a(k) = the random noise value at the k^{th} sample

W_0 and θ = the model and noise parameters respectively.

As opposed to a model with only a gain term Wang, et al.(5) have reported a fourth order model of the form

$$G(s) = \frac{C(s}{M(s)} = \frac{2.144 \times 10^{11}}{(s+125)(s+1138)[(s+383)^2+(1135)^2]} \qquad (4)$$

where

M, the manipulated variable, is a measurable voltage signal proportional to the current input to the servovalve and

C, the controlled variable, is a measurable voltage signal proportional to the ram velocity during injection.

The model given by eq. 4 was initially determined analytically and was then further verified experimentally by Wang et al. As such the model forms a basis for all our simulations.

As the proposed control scheme is for computer application, digital control algorithms have been employed. It was therefore necessary to find the corresponding discrete time state space model of the form given by eq. 1, corresponding to the transfer function given by eq. 4. The discrete state space model corresponding to a sampling time of 5 millisecond is

$$\underline{x}(kh+h) = \begin{bmatrix} 7.3408 \bullet 10^{-1} & 1.8323 \bullet 10^{-3} & 1.2657 \bullet 10^{-6} & 5.2109 \bullet 10^{-10} \\ -1.0636 \bullet 10^{+2} & -2.6707 \bullet 10^{-1} & 5.0630 \bullet 10^{-4} & 2.0843 \bullet 10^{-7} \\ -4.2545 \bullet 10^{4} & -5.0683 \bullet 10^{2} & -7.9748 \bullet 10^{-1} & 8.3375 \bullet 10^{-5} \\ -1.7018 \bullet 10^{7} & -2.0273 \bullet 10^{5} & -7.1899 \bullet 10^{2} & -9.666 \bullet 10^{-1} \end{bmatrix} \bullet \underline{x}(kh)$$

$$+ \begin{bmatrix} 1.7605 \bullet 10^{-12} \\ 1.83134 \bullet 10^{-10} \\ -1.3518 \bullet 10^{-7} \\ -1.3745 \bullet 10^{-4} \end{bmatrix} \bullet u(kh)$$

$$\underline{y}(kh) = [2.144 \bullet 10^{11} \quad 0 \quad 0 \quad 0\,]\ \underline{x}(kh) + 1.396 \bullet 10^{-1}\ \underline{u}(kh) \qquad (5)$$

Once the model is selected, the next important issue to be resolved is that of deciding upon the values of the weighting matrices. The following section provides the method of selecting the weights for the controller effort and the states.

V. SELECTION OF WEIGHTING MATRICES

In many practical situations it is difficult to find a natural choice of the relative weights of the various states and the manipulated variables. In such cases the closed loop system obtained by the solution of optimal control law, is analysed with respect to transient response, frequency response, robustness and so on. The elements of the loss functions are modified until the desired performance is obtained. This procedure seems like a strange use of optimization theory and one might be tempted to ask why other methods such as direct search over the feedback gain or pole-placement are not used instead. The reason is that the application of LQ theory will automatically guarantee a stable closed loop systems with reasonable margins. In addition to that, it is fairly easy to establish the influence that the weighting matrices have on the properties of the closed loop system. In our study the weighting matrices were determined first by theoretical considerations and further refined by optimization. Theoretical and analytical methods for determining the weighting matrices will now be discussed.

a. <u>Theoretical considerations:</u>

For a process governed by eq. 1

$$\underline{x}_k^T Q \underline{x}_k = y_k^T L y_k \tag{6}$$

where $y_k = \underline{C}\,\underline{x}_k$.

Therefore

$$y_k^T L y_k = \underline{x}_k^T \underline{C}^T L \underline{C}\,\underline{x}_k \tag{7}$$

Substituting eq.7 into eq.6 gives

$$\underline{x}_k^T Q \underline{x}_k = \underline{x}_k^T \underline{C}^T L \underline{C}\,\underline{x}_k$$

Thus $Q = \underline{C}^T L \underline{C}$ and $L = 1$ for SISO case

Thus $Q = \underline{C}^T \underline{C}$ (8)

For the particular model given by eq. 5, eq. 8 suggests that Q(1,1) be $4.596*10^{22}$.

b. Analytical considerations:

The value of the matrix Q suggested by eq. 8 is excessively large and would require a correspondingly large r. It was therefore necessary to carry out actual simulations to decide the optimal scaled values for the weighting matrices. However eq. 8 suggests that Q has only one nonzero element Q(1,1). Thus there are only two parameters, Q(1,1) and r that need to be selected. For optimization purposes only the relative weights or the ratio Q(1,1)/r is important. Thus the number of design parameters have been reduced to only one. Four criteria were used to determine the optimal values of the weighting matrices. These are listed below:

- the sum of the squares of the errors over the whole injection cycle i.e.

$$OBJE = \sum_{k=0}^{N} \underline{e}(k)^2 \qquad (9.1)$$

  where N is the last time step of the filling phase.

- the second criterion is the sum of the squares of the controller efforts over one cycle i.e.

$$OBJU = \sum_{k=0}^{N} \underline{u}(k)^2 \qquad (9.2)$$

- the third one is the sum of the absolute values of the derivatives of the controller effort i.e.

$$OBJD = \sum_{k=0}^{N-1} \{ABS[\underline{u}(k+1)-\underline{u}(k)]\} \qquad (9.3)$$

- the last one is the variance of the controller effort i.e.

$$VARU = E\{(\underline{u}-\overline{\underline{u}})^2\} \quad (9.4)$$

where E is the expectation operator.

In our case the manipulated variable is the opening of the valve and the life of the valve depends upon the rate at which it is operated. The third and the fourth objectives have been selected in view of this. Naturally, the smaller the value of each of the four criterion, the better the control strategy is judged to be. Figure 3 shows the variations of the four objectives w.r.t. the ratio Q(1,1)/r.

The final values were found by actually carrying out constrained optimization using the augmented lagrangian method (6). The profile of figure 1 was chosen arbitrarily for optimization. Only the first objective given by 9.1 was used as the objective function and the remaining criteria given by equations 9.2, 9.3, and 9.4 were designated as constraints. The sum of the squares of the controller effort was not a part of the objective as our aim is not to minimize the controller effort, but rather to use the controller valve to its maximum. The optimum values selected were Q(1,1) = 500 for r=5.

VI. SIMULATION RESULTS

In order to demostrate the effectiveness of the proposed anticipatory control algorithm a PID controller was also applied to the same model with the same sampling period. Reference profiles have been simplified for simulation purposes. This was necessary to explicitly demonstrate the effectiveness of the controllers to respond to the profiles with numerous set-points. The PID controller parameters were initially determined by optimization and further refined through simulation. The PID controller equation used was

$$u(k)=.45\bullet e(k)+.2331\bullet e(k-1)-.3057\bullet e(k-2)+u(k-1) \quad (10)$$

Figure 4 shows the results of the PID and the optimal controllers to track a profile which consists of a simple ramping of the ram velocity followed by a sudden increase in velocity at the 50^{th} sample in the form of a step with subsequent decrease to the initial velocity. The optimal controller is able to track the ramp portion of the profile virtually without any error. It is clear from figure 4 that the optimal controller also responds well to the step change. Improved performance is at the cost of the increased variability of the controller effort. However, table 1 shows that the value of the sum of the squares of the controller effort, which is representative of the total energy input to the system, is nearly the same.

Figure 5 shows the results obtained from the application of the PID and the optimal controllers to the profile of figure 1. The optimal controller outperforms the PID controller, as expected. The OBJE value is reduced from 10.86 for PID control to .052 for optimal control. The optimal controller tracks the step with some steady-state error and with increased variability of the valve opening. It is possible to reduce such errors by proper modification of the weighting matrices. Figure 3 suggests that if slight deterioration in performance can be tolerated, considerable reduction in the controller effort and its variability can be obtained. It is therefore at the discretion of the designer to select the objective and decide the weighting matrices accordingly. In the present context the same weighting matrices were used for all the profiles in order to provide a common ground for comparison with the PID controller.

In practice the velocity profile is specified as setpoints w.r.t. the percentage of stroke or time. Figure 6 shows the tracking of a profile obtained by joining such discrete points by a series of steps. The optimal controller again outperforms the PID controller, especially during the transition from one set-point to another. The sluggish response of the PID controller is clearly visible. However, the improved performance of the optimal controller is at the cost of higher variablity of the valve opening. A remedy to this would be to smoothen out the transition from one set-point to another. This can be achieved by joining the set-points by a series of ramps. Figure 7 compares the tracking characteristics of the PID and the optimal controllers to track a profile obtained by joining the same discrete points by a series of ramps. As seen from figure 7 and table 1, the optimal controller effort and its variablity is significantly reduced. The intermediate reference points could easily be generated by the microcomputer.

From the above discussion it follows that with the increasing complexity of the velocity profile a PID controller progressively becomes inefficient and the implementation of an optimal controller becomes more imperative.

VII. DISTURBANCE REJECTION

For the purpose of comparing the disturbance rejection capabilities of both the PID and the optimal controllers, the model given by eq. 1, can be modified to include process disturbance as follows:

$$\underline{x}(k+1)=A\underline{x}(k)+B\underline{u}(k)+\underline{v}(k)$$
$$\underline{y}(k) = C\underline{x}(k) \qquad (11)$$

where

$v(k)$ is a discrete-time Gaussian white noise with zero mean and variance σ^2, all other terms having the same interpretation as given by eq.1.

Figure 8 shows the tracking characteristics of the PID and the optimal controllers to track the profile of figure 1 in the presence of process disturbance $\underline{v}(k)$ with variance equal to 10% of the final state value. Considerable deterioration is seen in the PID controller performance relative to the optimal controller performance. Table 2 compares the values of the four objectives for three different noise levels. It can be seen that there is a considerable increase in the variability of the controller effort while the controller effort itself decreases steadily for both the PID and the optimal controllers. Figure 9 shows the performance of the PID and the optimal controllers in the presence of process disturbance with variance equal to 20% of the final state value. The optimal controller still performs satisfactorily.

VIII. PRACTICAL ASPECTS OF IMPLEMENTATION

Once the model is specified the gain matrix P can be precomputed and stored in memory. The correction vector $\underline{g}$ can be precomputed once the reference profile is specified. The optimal controller requires that a time varying feedback gain K be used. However, this would require a higher computational flexibility of the controller than those presently available in the market. In order to eliminate this complexity, only the steady state value of the gain matrix K was used.

Implementation of this scheme can then be accomplished rather simply with a control device capable of multiplying the actual measured values of the ram velocity by a constant value of K and sum the result with the vector $\underline{g}$, to produce the control $\underline{u}$. Implementation of this algorithm requires that all states be known at the sampling instances. However this requirement is not a very restrictive one. Initially while designing the optimal controller it can be assumed that all the states are known, and then an observer can be designed for estimation of all the non-measurable states. Thus it does not really matter for the design of an optimal controller, whether the states come out of the same system or from another dynamic system called an observer. The companion paper provides further details about observers.

IX. CONCLUSIONS

The effectiveness of optimal anticipatory control for controlling the ram velocity has been shown. In all the simulations the optimal controller was able to track the reference trajectory much closer than the PID controller. In terms of disturbance rejection also the optimal anticipatory controller performed in a superior manner. As the computationally intensive work can be done off-line, the on-line computational capacity requirements are not significantly higher than those of the PID controllers and can be done by the online microcomputer controller. The memory storage requirements are also very low as the steady-state gain is adequate. Moreover, due to its anticipatory nature, the proposed controller will enable many more set points to be programmed than presently possible by the PID controllers.

AKNOWLEDGEMENT

The financial support for this study was made available by a grant from NSF, Grant No. CDR 8500108, through the Systems Research Center of the University of Maryland. The computer time for this study was made available by the computer science center of the University of Maryland.

APPENDIX

Derivation of the Discrete Optimal Anticipatory Control Algorithm

Let the discrete state-space system be described by the following difference equation

$$\underline{x}_{K+1} = \underline{A}_K \underline{x}_K + \underline{B}_K \underline{u}_K$$

$$\underline{y}_K = C_K \underline{x}_K \qquad \text{(A-1)}$$

Our objective is to find the control that minimizes the cost function (for fixed Kf)

$$J = \frac{1}{2} || \underline{e}_{Kf} ||_S^2 + \frac{1}{2} \sum_{K=0}^{Kf-1} \{ || \underline{e}_K ||_{Q_K}^2 + || \underline{u}_K ||_{R_K}^2 \}$$

where $\underline{e}_K = \underline{z}_K - \underline{y}_K$

$$= \underline{z}_K - C_K \underline{x}_K$$

Thus $J = \frac{1}{2} || \underline{z}_K - C_K \underline{x}_K ||_S^2 + \frac{1}{2} \sum_{K=0}^{Kf-1} \{ || \underline{z}_K - C_K \underline{x}_K ||_{Q_K}^2 + || \underline{u}_K ||_{R_K}^2 \}$

In order to fulfill our objective, use is made of the discrete maximum principle wherein a Hamiltonian of the form given below is defined.

$$H_K = \frac{1}{2} [\underline{z}_K - C_K \underline{x}_K]^T Q_K [\underline{z}_K - C_K \underline{x}_K] + \frac{1}{2} \underline{u}_K^T R_K \underline{u}_K + \underline{\lambda}_{K+1}^T [A_K \underline{x}_K + B_K \underline{u}_K]$$

The optimal control input is determined from the solution of certain necessary conditions given below

$$\frac{\partial H_K}{\partial \underline{u}_K} = 0 + R_K \underline{u}_K + B_K^T \underline{\lambda}_{K+1} = 0$$

Thus $\underline{u}_K = - R_K^{-1} B_K^T \underline{\lambda}_{K+1}$ (A-2)

and $\frac{\partial H_K}{\partial \underline{x}_K} = \underline{\lambda}_K$

thus

$$C_K^T Q_K C_K \underline{x}_K - C_K^T Q_K \underline{z}_K + A_K^T \underline{\lambda}_{K+1} = \underline{\lambda}_K \tag{A-3}$$

with the terminal conditions

$$\underline{\lambda}_{Kf} = C_{Kf}^T S C_{Kf} \underline{x}_{Kf} - C_{Kf}^T S \underline{z}_{Kf} \tag{A-4}$$

Substituting eq. A-2 into the state equation given by eq. A-1 gives

$$\underline{x}_{K+1} = A_K \underline{x}_K - B_K R_K^{-1} B_K^T \underline{\lambda}_{K+1} \tag{A-5}$$

Let us assume a solution of the form

$$\underline{\lambda}_K = P_K \underline{x}_K - \underline{g}_K \text{ or}$$

$$\underline{\lambda}_{K+1} = P_{K+1} \underline{x}_{K+1} - \underline{g}_{K+1} \tag{A-6}$$

where $\underline{g}_K$ is the correction vector

Substituting eq. A-6 in eq. A-5 gives

$$\underline{x}_{K+1} = A_K \underline{x}_K - B_K R_K^{-1} B_K^T [P_{K+1} \underline{x}_{K+1} - \underline{g}_{K+1}]$$

$$= [I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} [A_K \underline{x}_K + B_K R_K^{-1} B_K^T \underline{g}_{K+1}] \tag{A-7}$$

Substituting back eq. A-7 into eq. A-6 gives

$$\underline{\lambda}_{K+1} = P_{K+1} [I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} [A_K \underline{x}_K + B_K R_K^{-1} B_K^T \underline{g}_{K+1}] - \underline{g}_{K+1}$$

$$= \{P_{K+1} [I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} A_K\} \underline{x}_K$$

$$+ P_{K+1} [I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} B_K R_K^{-1} B_K^T \underline{g}_{K+1} - \underline{g}_{K+1} \tag{A-8}$$

Rewriting eq. A-3

$$\underline{\lambda}_K = C_K^T Q_K C_K \underline{x}_K - C_K^T Q_K \underline{z}_K + A_K^T \underline{\lambda}_{K+1}$$

and substituting this into eq. A-6 gives

$$P_K \underline{x}_K - \underline{g}_K = C_K^T Q_K C_K \underline{x}_K - C_K^T Q_K \underline{z}_K + A_K^T \underline{\lambda}_{K+1}$$

Thus

$$\underline{\lambda}_{K+1} = A_K^{-T}\{[P_K - C_K^T Q_K C_K]\underline{x}_K - \underline{g}_K + C_K^T Q_K \underline{z}_K\} \qquad \text{(A-9)}$$

Comparing eq. A-8 and eq. A-9 gives the following identities

$$P_{K+1}[I + BR^{-1}B_K^T P_{K+1}]^{-1} A_K = A_K^{-T}[P_K - C_K^T Q_K C_K]$$

$$\rightarrow P_K = A_K^T P_{K+1}[I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} A_K + C_K^T Q_K C_K \qquad \text{(A-10)}$$

and

$$P_{K+1}[I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} B_K R_K^{-1} B_K^T \underline{g}_{K+1} - \underline{g}_{K+1} = A_K^{-T}\{-\underline{g}_K + C_K^T Q_K \underline{z}_K\}$$

$$\rightarrow \underline{g}_K = A_K^T\{I - P_{K+1}[I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} B_K R_K^{-1} B_K^T\}\underline{g}_{K+1} + C_K^T Q_K \underline{z}_K \qquad \text{(A-11)}$$

From eq. A-6

$$\underline{\lambda}_{Kf} = P_{Kf}\underline{x}_{Kf} - \underline{g}_{Kf} \qquad \text{(A-12)}$$

Comparing eq. A-12 with eq. A-4 gives the following boundary conditions

$$P_{Kf} = C_{Kf}^T S C_{Kf} \qquad \text{(A-13)}$$

$$\underline{g}_{Kf} = C_{Kf}^T S \underline{z}_{Kf} \qquad \text{(A-14)}$$

Calculation of the Controller Effort

From eq. A-10

$$P_K - C_K^T Q_K C_K = A_K^T P_{K+1}[I+B_K R_K^{-1} B_K^T P_{K+1}]^{-1} A_K \qquad \text{(A-15)}$$

and from eq. A-11

$$\underline{g}_K - C_K^T Q_K \underline{z}_K = A_K^T\{I - P_{K+1}[I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} B_K R_K^{-1} B_K^T\}\underline{g}_{K+1} \qquad \text{(A-16)}$$

Substituting eq. A-15 and eq. A-16 into eq. A-9 and then eq. A-9 into eq. A-2 gives

$$\underline{u}_K = -R_k^{-1} B_K^T\{P_{K+1}[I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} A_K \underline{x}_K$$

$$+ P_{K+1}[I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1} B_K R_K^{-1} B_K^T \underline{g}_{K+1} - \underline{g}_{K+1}\}$$

$$= -R_K^{-1} B_K^T\{P_{K+1}[I + B_K R_K^{-1} B_K^T P_{K+1}]^{-1}[A_K x_K + B_K R_K^{-1} B_K^T \underline{g}_{K+1}] - \underline{g}_{K+1}\}$$

REFERENCES

1. A.R. Agrawal and I.O. Pandelidis,"Observers for Optimal Anticipatory Control of Ram Velocity in Injection Molding",this issue.
2. D.L. Smith,SPE ANTECH Tech. Papers,21,225(1975).
3. D.B. Hunkar,SPE ANTECH Tech. Papers,21,161(1975).
4. D.A. Fara,M.Eng.Thesis,McGill Univ.,Montreal(1983).
5. K.K. Wang,et al.,Cornell Univ.,Progress Report #10(1984).
6. G.V. Reklaitis,A. Ravindran and K.M. Ragsdell,"Engineering Optimization",John Wiley and Sons,New York(1983).

Table 1. Comparison of the various objectives for the PID and the optimal controllers.

CONTROLLER TYPE	OBJECTIVE TYPE	SET POINT PROFILE - GIVEN BY FIGURE NUMBERS			
		4	5	6	7
PID Optimal	OBJE OBJE	1.54221 .01422	10.86320 .05204	10.08032 .12977	2.01980 .08601
PID Optimal	OBJU OBJU	90.03169 89.99966	419.67495 417.34689	808.72374 808.89719	707.42070 693.14822
PID Optimal	OBJD OBJD	3.36152 8.52512	7.32564 9.20823	7.00191 37.68219	6.20290 6.70195
PID Optimal	VARU VARU	.26072 .27348	.63330 .67306	.48147 .53850	.66301 .63594

Table 2. Comparison of the disturbance rejection property of the PID and the optimal controllers.

CONTROLLER TYPE	PROCESS DISTURBANCE AS % OF FINAL STATE	OBJE	OBJU	OBJD	VARU
PID Optimal	10 10	11.043553 .081793	416.4632 416.3289	7.9373 12.0277	.636329 .681594
PID Optimal	20 20	11.385976 .198923	413.4191 415.6056	8.8004 16.1829	.634904 .686480
PID Optimal	30 30	11.890491 .403427	410.5427 415.1771	9.8118 20.5428	.640624 .700191

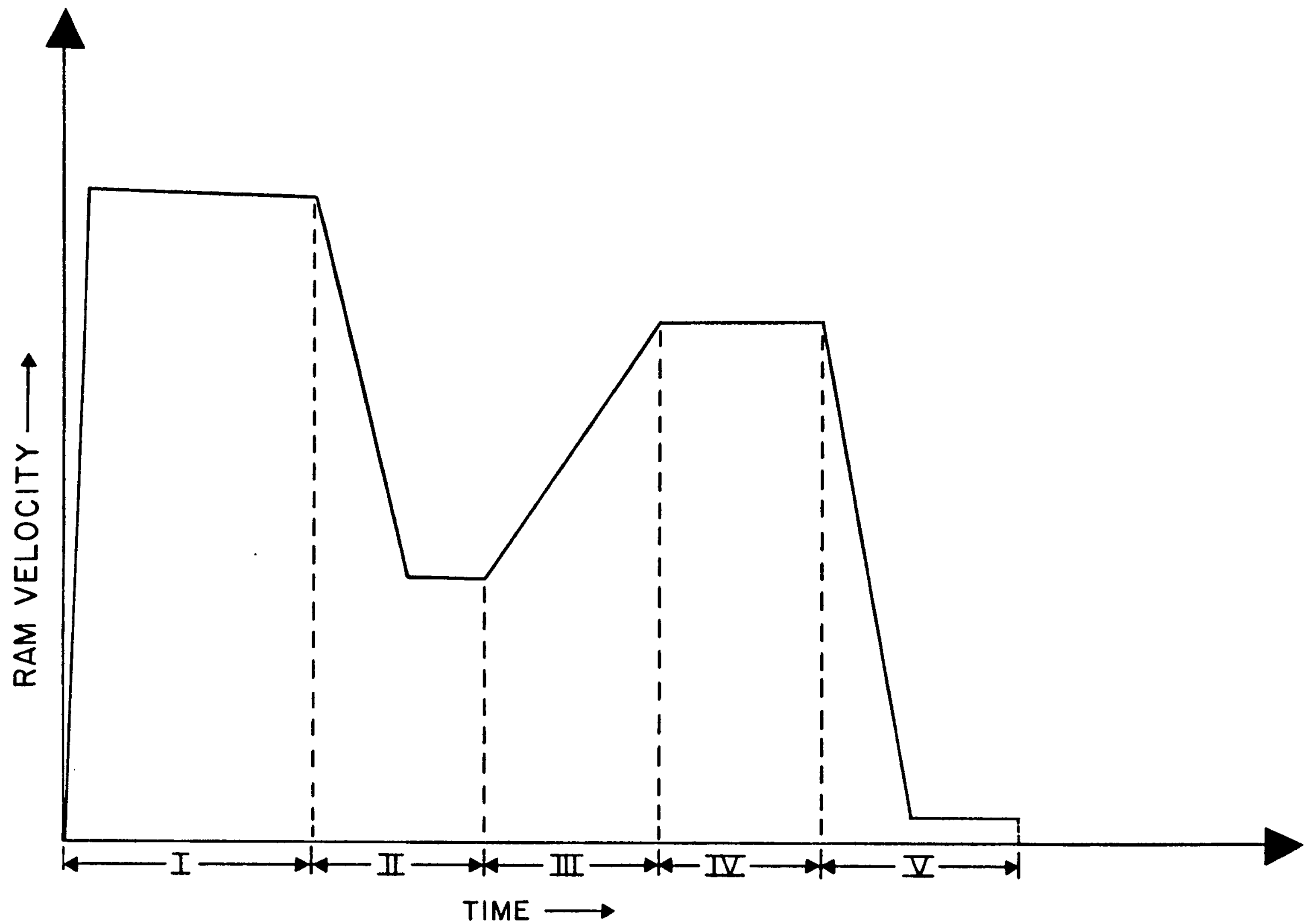

Fig. I. A typical ram velocity profile.

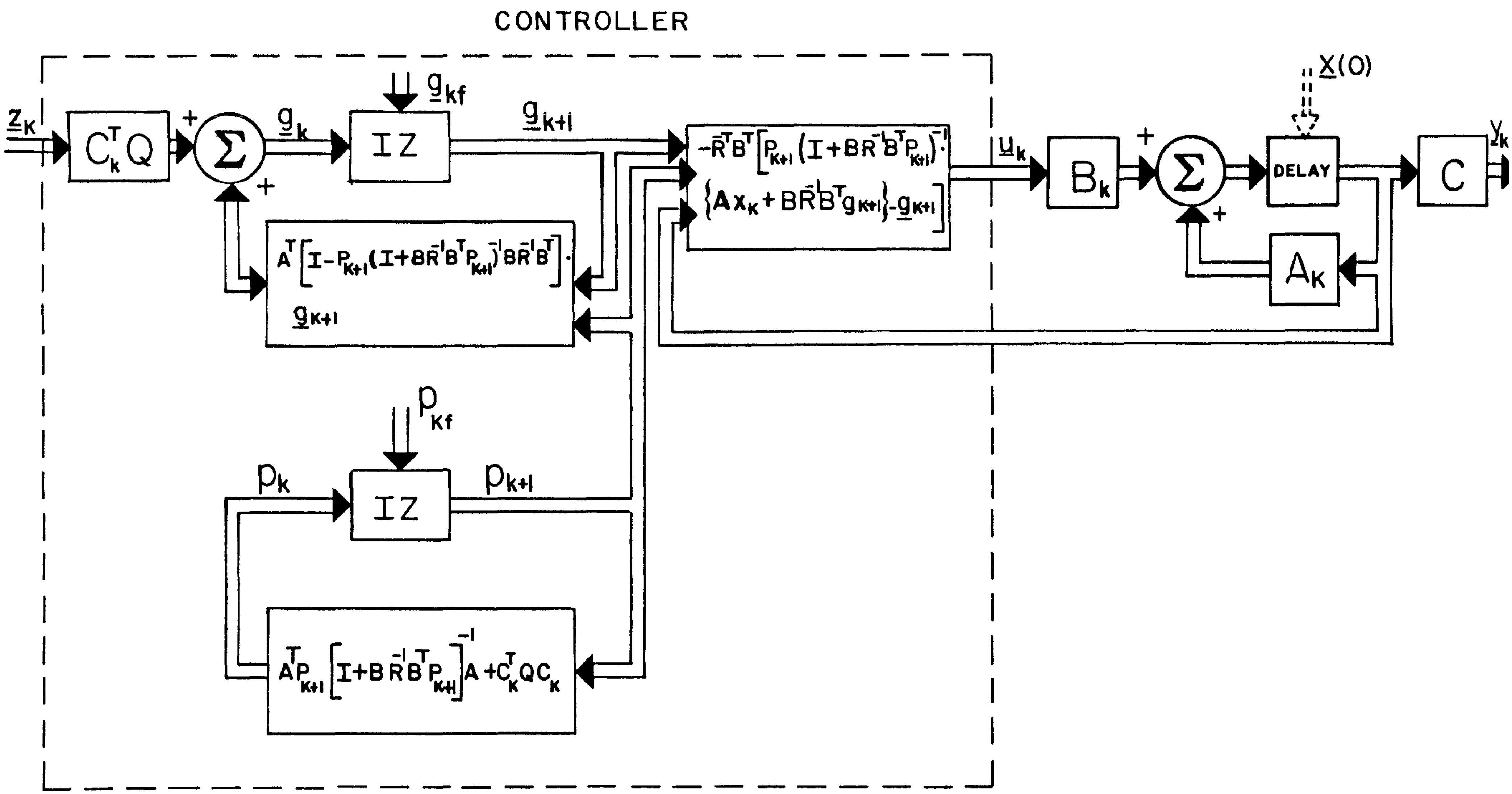

Fig.2. Block diagram of an optimal tracking control algorithm.

Pandelidis, Fig. 3

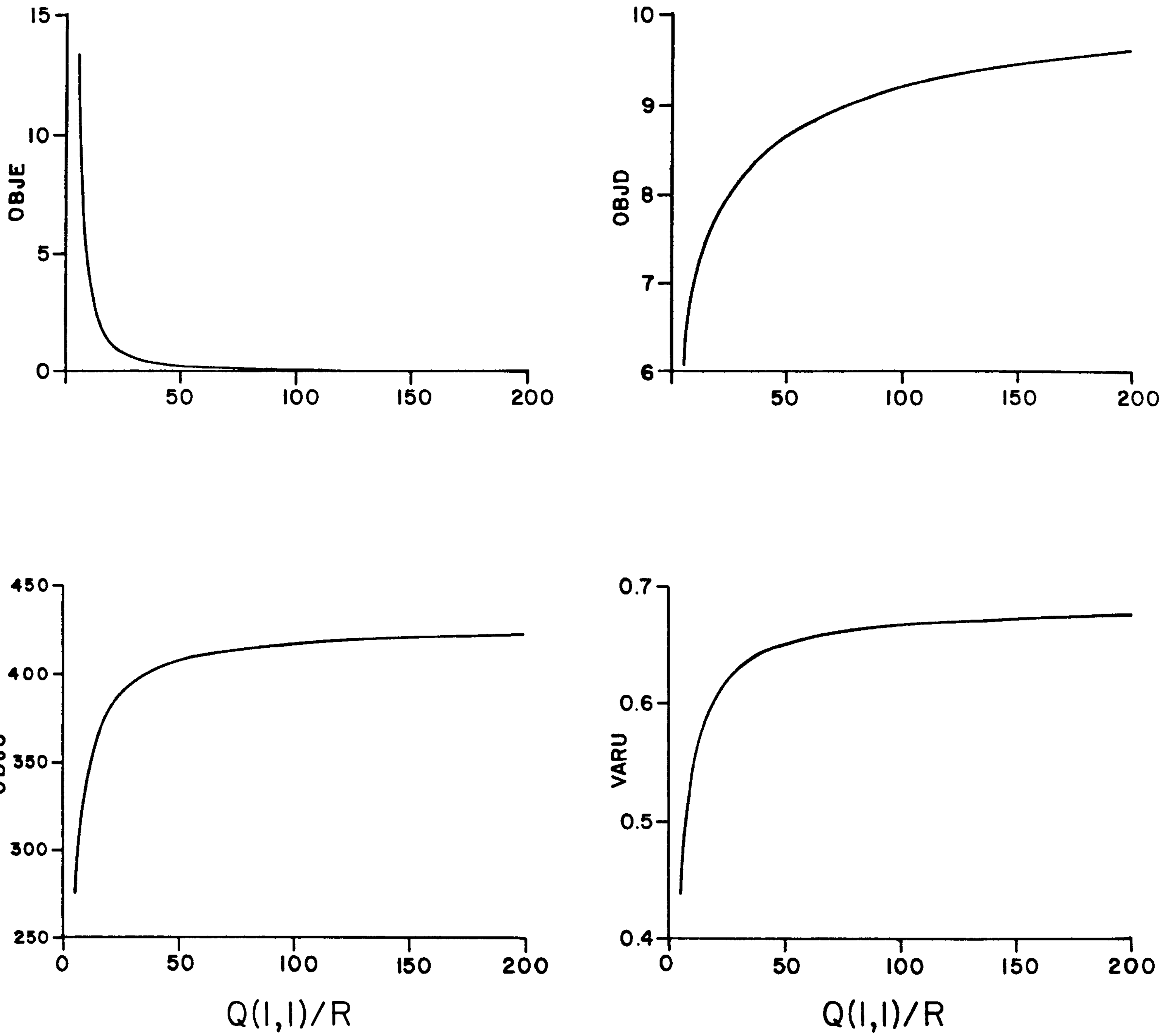

Fig. 3. Effect of the variation in the ratio of the weighting matrices on the values of the various performance measures.

Pandelidis, Fig.4

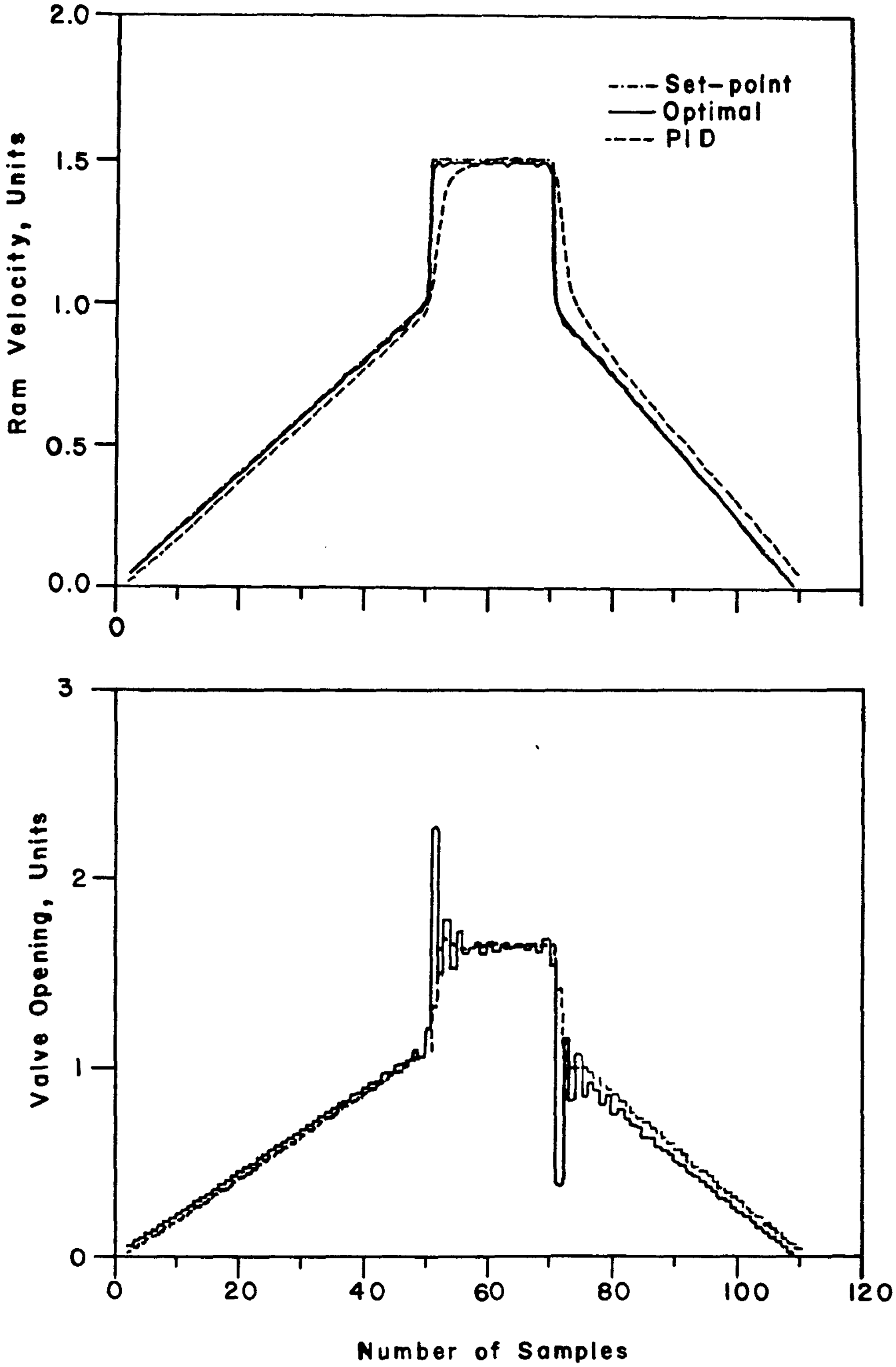

Fig.4 . Ram velocity response with the PID and optimal controllers to a set point velocity profile.

Pandelidis, Fig.5

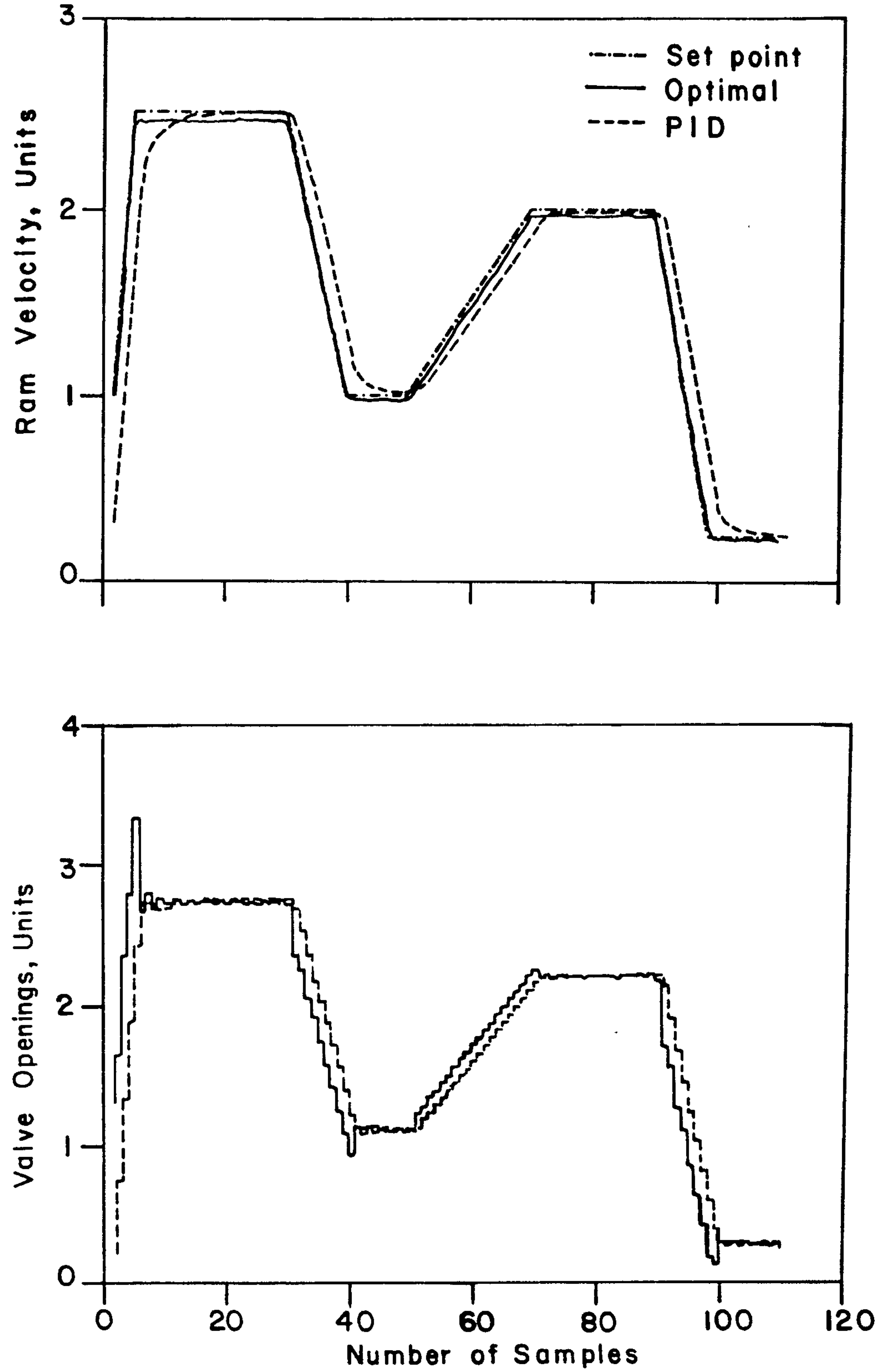

Fig.5. Ram velocity response with the PID and optimal controllers to a set-point velocity profile of figure 1.

Pandelidis, Fig.6

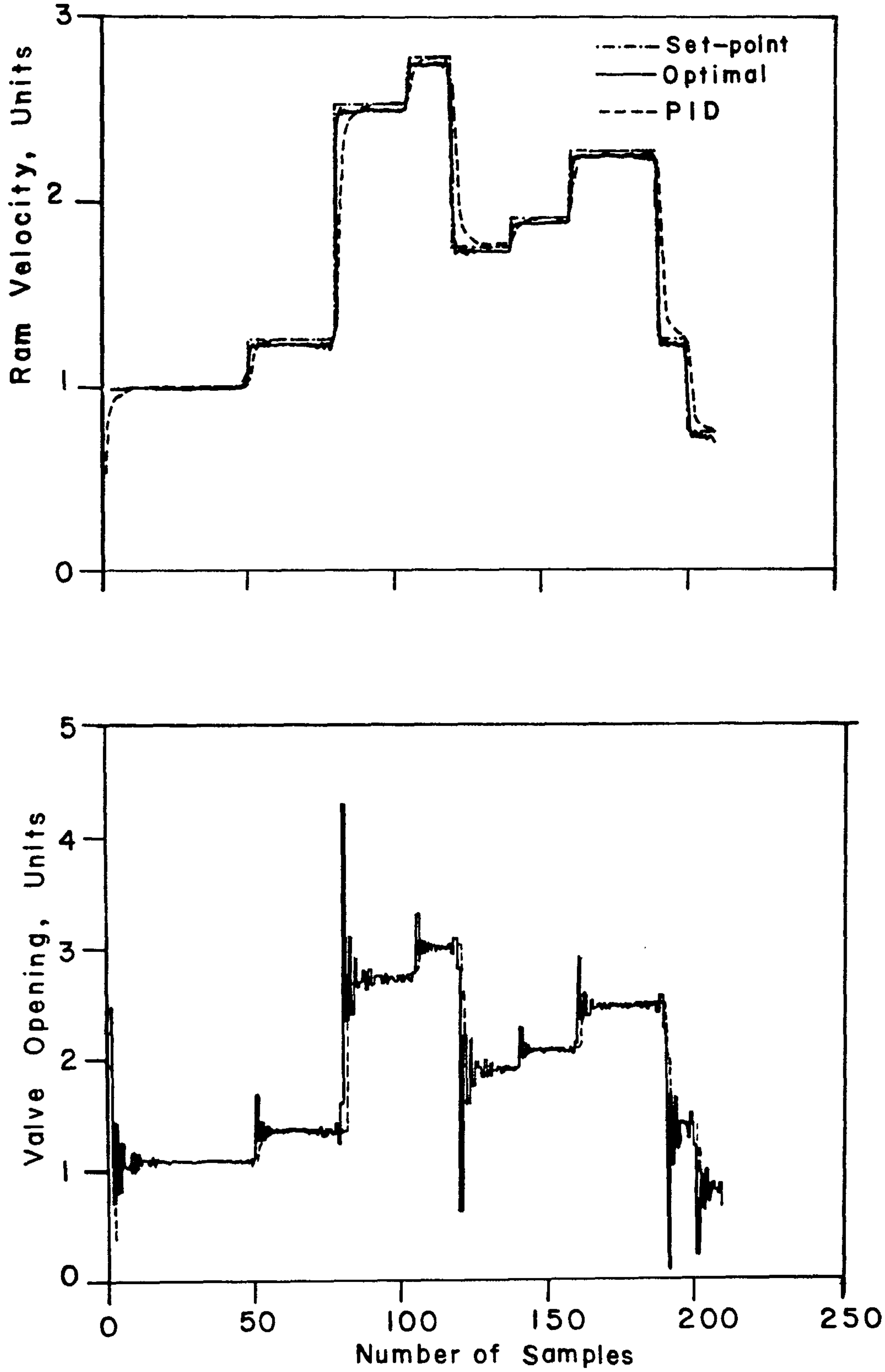

Fig.6. Ram velocity response with the PID and optimal controllers to a profile obtained by joining the velocity set-points by a series of steps.

Pandelidis, Fig. 7

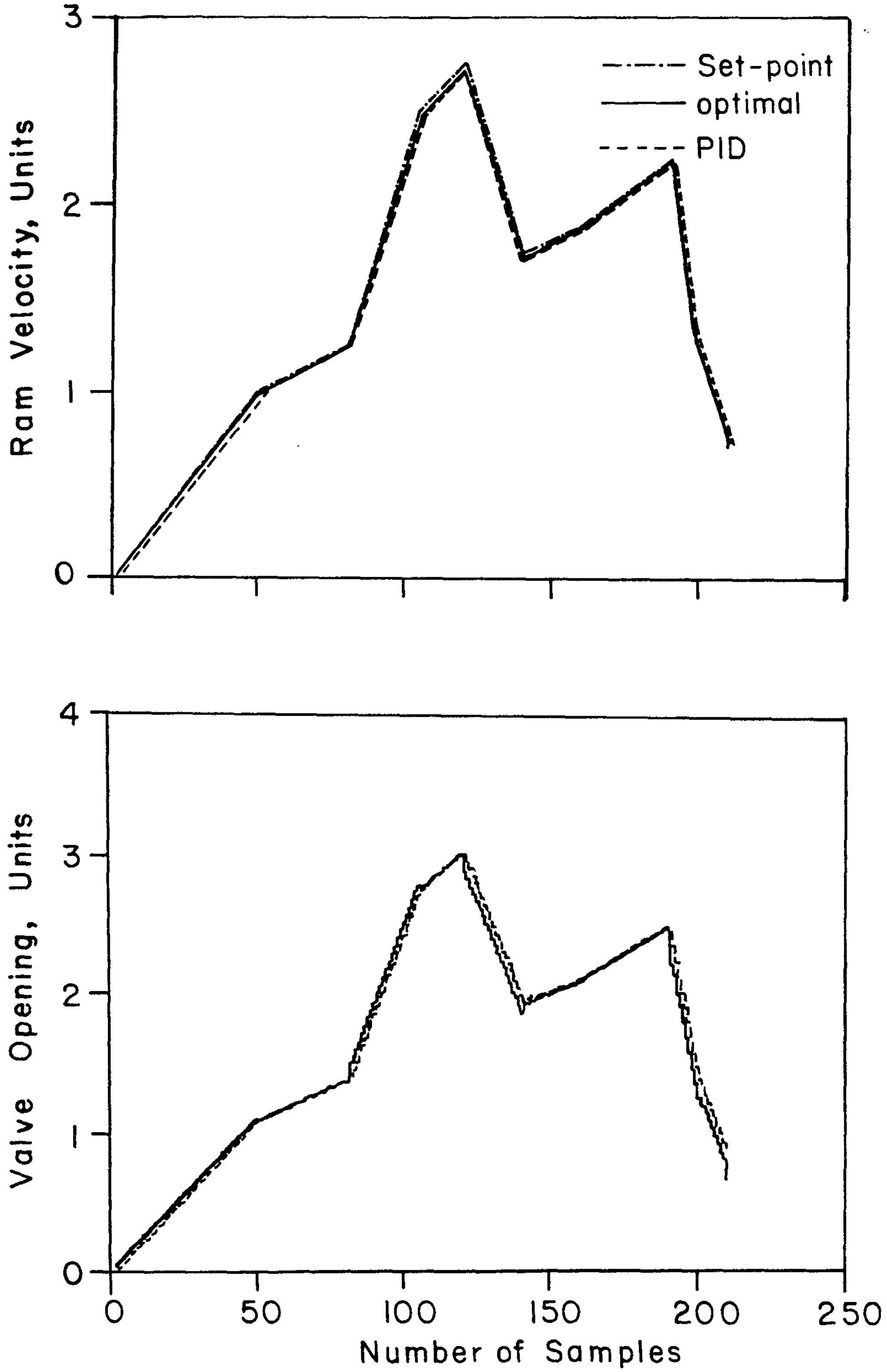

Fig. 7. Ram velocity response with the PID and optimal controllers to a profile obtained by joining the velocity set points by a series of ramps.

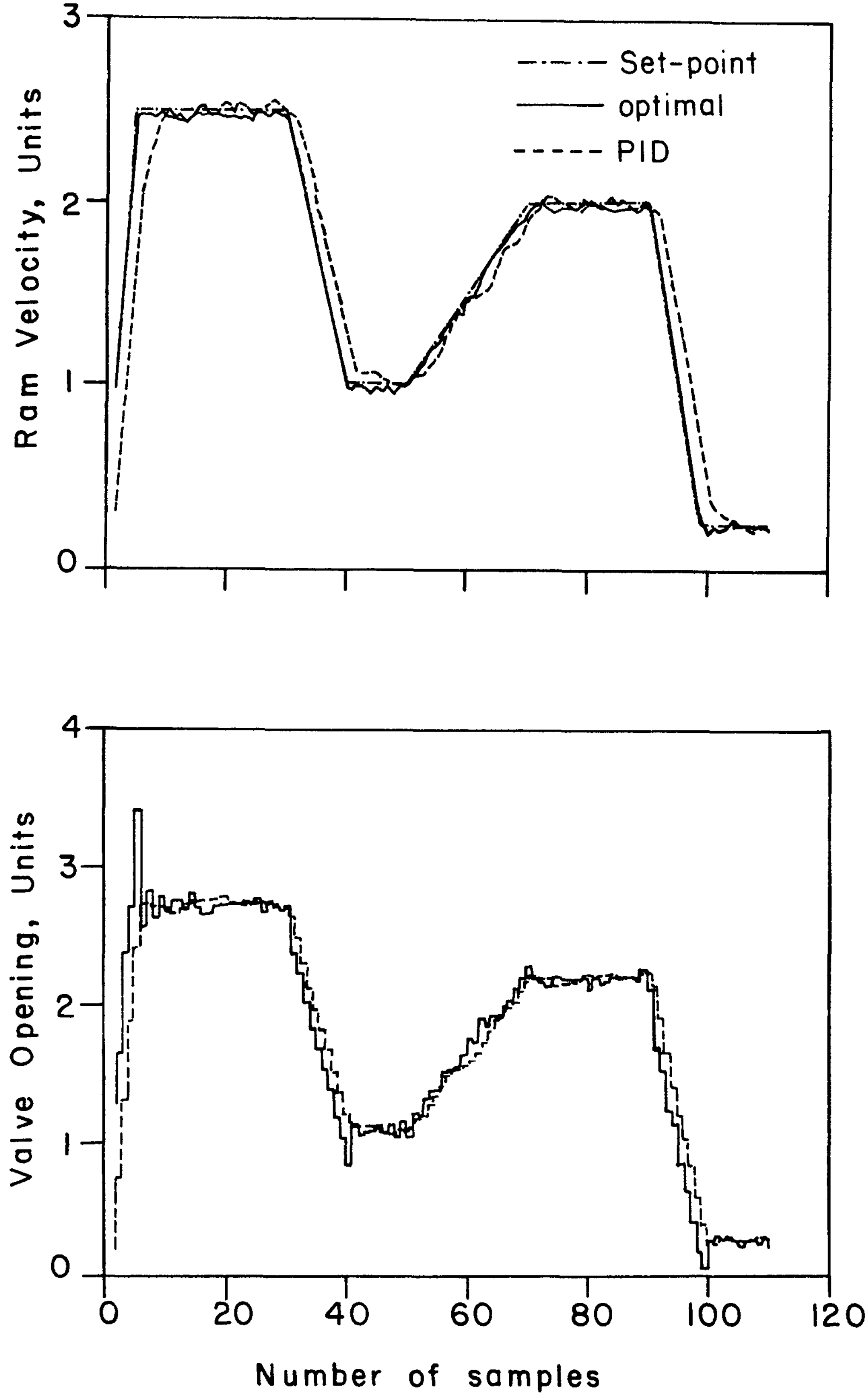

Fig.8. Ram velocity response with the PID and optimal controllers to the profile of fig.1, in presence of load disturbance with variance equal to 10% of the final state.

Pandelidis, Fig.9

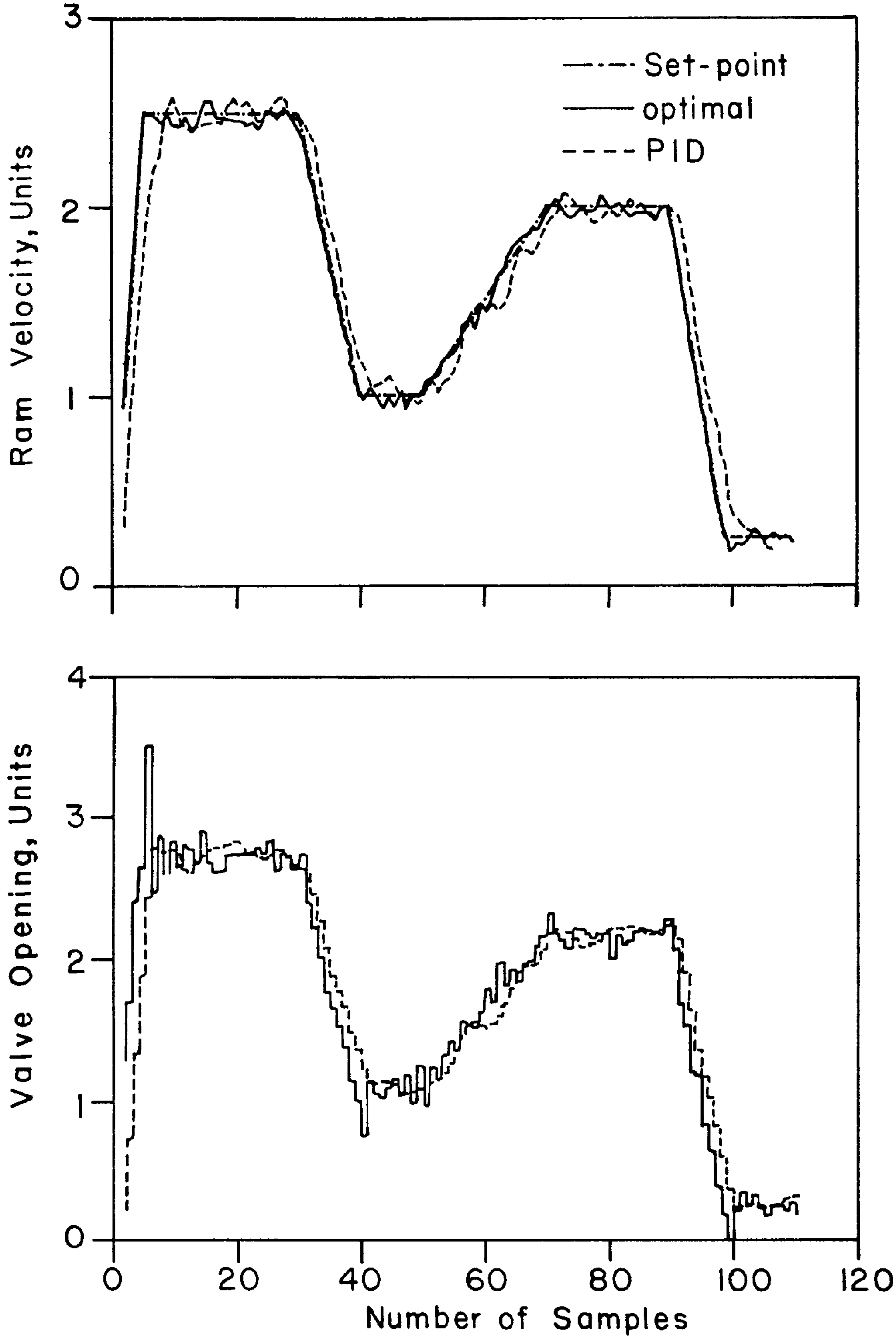

Fig. 9. Ram velocity response with the PID and optimal controllers to the profile of fig. 1, in presence of load disturbance with variance equal to 20% of the final state.